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A Hierarchical Communication Network Within A Financial Agent-Based Model

Design, Implementation and Analysis

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Abstract

Parting from the simple dynamics arising from traders congregating to buy and sell different assets under a commonly agreed-upon 'fair price,' financial markets have grown to become some of the complex systems ever developed by humanity. Consequently, developing models that can accurately explain or replicate their behavior has become a vital subject of academic focus. In this dissertation, we contribute to this growing body of literature by presenting an innovative model of social interaction dynamics and their effect on financial markets.

In particular, we propose that a tree-like hierarchical structure represents a simple and effective way to model the emergent behavior of financial markets, especially markets where there exists a pronounced intersection between social media influences and investor behavior. To show this, we present a 'Hierarchical Model,' under which traders communicate amongst one another through a hierarchical network. This model undergoes detailed analyses, evidencing that its behavior conforms to realistic standards. Subsequently, we show that slight modifications to the model result in the successful replication of multiple phenomena; providing evidence that this novel design can concisely, effectively, and simultaneously model a variety of events observable in real financial markets.

The bulk of the work behind this dissertation can be summarized as follows:

- Dedicated over 50 hours to researching the field of financial market simulations and Agent-Based Modeling.
- Wrote over 800 lines of code implementing the Agent-Based Models detailed in Lux (1995) and Meine and Vvedensky (2023).
- Spent no less than 200 hours exploring the design and implementation of a new model to simulate trader interaction within financial markets, as presented in Chapter 5. The end result consisted of 1500 lines of carefully constructed Python code.
- Validated the model by simulating over 10,000 independent markets, producing over 24,000,000,000 market observations. This required approximately 20 days of uninterrupted computation time.

Dedication and Acknowledgements

Dedicated to my grandpa for teaching me the value of hard work, and to my grandma for teaching me the virtue of patience. To my mom and dad: thank you for always being there for me, I know I don't always make it easy.

Special thanks to my supervisor, Dr. John Cartlidge, for his continued assistance and support throughout the writing of this dissertation.

Declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Taught Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, this work is my own work. Work done in collaboration with, or with the assistance of others, is indicated as such. I have identified all material in this dissertation which is not my own work through appropriate referencing and acknowledgement. Where I have quoted or otherwise incorporated material which is the work of others, I have included the source in the references. Any views expressed in the dissertation, other than referenced material, are those of the author.

Gonzalo Bohorquez, Wednesday 8th May, 2024

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Ethics Statement

This project did not require ethical review, as determined by my supervisor, Dr. John Cartlidge.

Chapter 1

Introduction

The main observation motivating this dissertation is that a hierarchical structure serves as an intuitive, yet highly explanatory, way to model social network dynamics. This idea leads to the creation of a model of financial markets wherein traders exchange information through a hierarchical network. We then explore the structure's ramifications regarding the market's behavior, as well as demonstrate this simple structure's versatility through its ability to function as a robust base for a wide variety of models, each simulating different phenomena.

First and foremost, to visualize a hierarchical social network structure, consider how a student might receive and process information:

1. A student's ideas are heavily affected by their teachers.
2. Meanwhile, these teachers are influenced by broader educational frameworks, developed by experts in the field.
3. In turn, these experts draw upon research, theories, and societal needs when designing their educational programs.
4. While these theories may be affected by broader societal factors such as cultural norms, economic demands, and political ideologies.

Notice that, as we go up the tree, factors become progressively broader and affect an exponentially greater number of people. For example, while a teacher might strongly influence 100 students, broader cultural norms could affect every student in the country to one degree or another. Although simple, this example illustrates the intuitive motivation behind modeling the dispersion of ideas in a hierarchical, cascade-like manner, such that top-level ideas 'trickle down' until they eventually reach any given individual.

This idea is by no means new; models containing similar structures are widely used across a variety of academic fields. However, despite several studies showcasing the usefulness of incorporating hierarchical network structures into opinion-diffusion models, there is a distinct lack of enthusiasm regarding their use as models for the communication between agents *within financial markets.* In this paper, we reframe the use of hierarchical structures to show that they are especially versatile and adept at simulating markets that are heavily influenced by social media (such as cryptocurrencies). In particular, we were able to achieve this by:

- Detailing the definition and implementation of an (Agent-Based) 'Hierarchical Model,' such that the agents within it interact via a hierarchical communication structure.
- Using empirical data (stylized facts) to demonstrate the model's ability to replicate financial markets with a high degree of realism.
- Empirically exploring the best values for the model's parameters, thus paving the way for future studies.
- Using the model as a base for additional simulations of a multitude of different phenomena, evidencing its versatility and the simplicity with which it can be extended.

Chapter 2

Contextual Background

2.1 Financial Markets

Financial markets are systems where participants (i.e. investors/traders) can buy, sell, and exchange various financial instruments[1]. The primary function of financial markets is to facilitate the efficient allocation of resources, providing a mechanism for pricing assets, raising capital, and transferring risk.

Most modern markets operate under a Continuous Double Auction (CDA), wherein traders can post 'bids' (to buy an asset at a certain price), or 'offers' (to sell the asset at a certain price); when someone posts an offer which satisfies[2] an existing bid (or vice-versa), a transaction is completed and the market price is updated to reflect this change (Smith et al., 2003).

2.1.1 Fundamental Value

In vastly simplified terms, an asset's 'fundamental value' refers to its inherent or "true" value, as determined by a variety of economic factors (e.x. the financial health of a company, economic conditions, prospects of future earnings, etc...). Investors interested in fundamental value aim to purchase assets when they believe the market price is below their true value and sell them when they exceed this value.

2.1.2 Speculation

When someone buys an asset because they think its price will go up, rather than because they think the asset is under-valued with respect to its 'fundamental value,' they are said to be engaging in speculation.

In other words, speculative investors are market participants who engage in trading with the primary objective of making profits from fluctuations in asset prices, rather than the underlying economic value of the assets themselves.

2.1.3 Volatility

Volatility refers to the degree of variation in the price of an asset over time. High volatility means the price of the asset can change dramatically over a short period in either direction, which can be a sign of uncertainty or rapid changes in market perceptions. Refer to Section 4.1 for technical details on how volatility is measured throughout this dissertation.

2.2 Agent-Based Modelling

Agent-Based Modeling (ABM) is a paradigm that focuses on simulating the actions and interactions of autonomous agents to assess their effects on a system. This approach enables researchers to analyze complex systems in a variety of fields, including economics, biology, social sciences, and computer science (Niazi and Hussain, 2011). Models created under this paradigm are referred to as Agent-Based Models (ABMs).

[1] Such as stocks, bonds, or currencies.

[2] A bid "satisfies" an offer if its price is below the offer price. Similarly, an offer "satisfies" a bid if its price is above the bid price.

At the core of ABM is the concept of agents, which are defined by their attributes and behaviors. Agents operate in an environment, interact with other agents, and adapt their behaviors based on their experiences and interactions. These agents can range from simple entities with rudimentary rules to sophisticated models that mimic human decision-making processes; although ABMs usually follow the "KISS" (*Keep It Simple, Stupid*) approach, as simpler agents make it easier for researchers to understand and analyze the mechanisms behind a simulation's emergent behavior.

One of the key strengths of the ABM paradigm lies in its ability to model systems from the bottom up[3]. This means starting from the individual level and observing how complex patterns and behaviors emerge from simple rules and interactions. This emergent behavior is a hallmark of complex systems and is often difficult to predict with top-down approaches (Bonabeau, 2002). From there, inferences and hypotheses regarding an overarching system's behavior can be made by adjusting the simulation's parameters and observing the outcomes.

Financial markets, which are essentially a system comprised of a variety of individuals (i.e. investors) making autonomous decisions (i.e. buying or selling an asset), can be succinctly modeled via ABMs. In this dissertation, we primarily consider the application of ABMs for the purpose of studying market dynamics and investor behavior.

2.3 Herding Models

Herding models, often studied within the context of financial markets, delve into the phenomenon where individuals in a group tend to follow the actions or behaviors of others, sometimes disregarding their own private information or signals (Lux, 1995; Bikhchandani et al., 1992).

The foundation of herding models is the idea that individuals, when faced with decisions, often look to the behavior of their peers as a source of information. This can be rational, especially in situations where direct information is costly or difficult to obtain, or where the actions of others are seen as an aggregation of private information; however, herding can also lead to irrational outcomes, where individuals ignore their private signals to their detriment and the detriment of the group as a whole.

In the context of herding models as a subset of ABMs, agents are modeled as deciding their actions based on both their private signals (as in usual ABMs) and the observed actions of others (the herding input) (Lux and Marchesi, 1999). A crucial point to keep in mind throughout the rest of this dissertation is that the interplay between these elements can lead to an equilibrium where herding occurs, even if it leads to suboptimal outcomes (for markets, these suboptimal outcomes may include features such as market instability, or an overall reduction in returns).

One classic example of herding in financial markets is the bubble and crash phenomenon, where investors rush to buy assets due to observed increases in their prices (fueled by the actions of others) rather than the intrinsic value of the assets. This can lead to inflated prices and eventually, a market crash when the bubble bursts.

A very illustrative example of this phenomenon's potentially catastrophic effects is the period now referred to as "Tulip Mania," which occurred in Holland during the 17th century. In the early 1630s, the price of tulip bulbs began to rise rapidly due to their increasing cultural significance. By 1637, luxurious houses in Amsterdam (the most expensive city in the world during that time) were worth less than a single tulip bulb. By 1638, tulip bulbs were worth less than a tenth of what they were worth the year prior (Goldgar, 2019). This example highlights the seemingly nonsensical degree to which the herding effect can affect the decisions of what can otherwise be assumed to be rational agents, emphasizing the significance of ABMs that take these influences into account.

2.4 Opinion Dynamics

Opinion dynamics are mathematical models that describe the processes through which individuals in a group influence one another and reach consensus[4] or disagreement[5] over time. In the context of herding ABMs, these dynamics simulate how the beliefs of speculative traders evolve based on the opinions and

[3] Note that the goal of ABMs is to attempt to explain complex dynamics that arise from simplistic agents interacting within a system, rather than the creation of agents with the intent of having them solve a complex problem (Niazi and Hussain, 2011).

[4] Refer to Deffuant et al. (2002), which defined the Relative Agreement Model (RAM), simulating how agents influence each other to achieve a common consensus opinion over time.

[5] Refer to Meadows and Cliff (2014) for the Relative Disagreement Model, which simulates how social networks may become increasingly polarized over time, rather than approaching a common consensus.

behaviors of others within their network. Typically, these models incorporate factors such as the strength of influence between agents, the resistance of agents to change their views, and the impact of external information (Acemoglu and Ozdaglar, 2011).

2.4.1 Social Media

In the context of opinion dynamics, the study of opinions within social media is particularly notable for a variety of reasons:

- Social media provides researchers with troves of information, much more than could be acquired through the study of any alternative means of communication. This makes results acquired from social media more numerous, and thereby more significant, than any available alternative (Willaert et al., 2020).
- Social media is becoming increasingly influential in modern society (Das et al., 2014). This means social media (and how opinions within it spread) is rapidly approaching a state in which developing a thorough understanding of it is vitally important, both from an academic point of view as well as in an economic sense.
- The inherent nature of social media networks results in interesting phenomena worthy of study. For example, these networks expose individuals to an extremely broad and diverse set of influences, including not only peers but also celebrities, influencers, and algorithmically curated content, which can lead to rapid shifts in public opinion and the emergence of echo chambers (see Section 2.5.4). Additionally, the anonymity and lack of accountability on social media can encourage more extreme expressions of opinion, which can escalate polarization as well as foment the spread of misinformation.

2.5 Social Media within Financial Markets

2.5.1 General Outline

Social media has revolutionized the speed and reach of information dissemination within financial markets. With traditional barriers to information lowered, investors can access and share market-related data instantaneously. The importance of this effect is showcased in studies such as Bollen et al. (2011), which suggests that Twitter mood can predict stock market movements, highlighting how collective emotions expressed online may correlate with the market's behavior.

2.5.2 Volatility

Sentiment expressed through social media has been shown to significantly impact market volatility. Positive news on social platforms can lead to rapid price increases, while negative news can cause sudden declines. For instance, Gilbert and Karahalios (2010) demonstrated a direct link between the sentiment on social media platforms and subsequent movements in the stock market, suggesting that increases in subjective expressions of anxiety could predict an increase in volatility for the S&P 500 index.

2.5.3 Cryptocurrency Markets

Cryptocurrencies or "cryptos" are digital currencies that act as mediums of exchange, and do not rely on any centralized authority (such as a government) in order to fulfill this function. These assets notable since most scholars suggest that they have no fundamental value[6] (Caginalp and Caginalp, 2018; Kukacka and Kristoufek, 2023; Haykir and Yagli, 2022), which essentially means that speculative pressures are behind any and all fluctuations in the market price of a cryptocurrency[7]. By most accounts, this is not how standard markets behave, and thus the study of cryptocurrency market dynamics tends to differ significantly from the study of traditional assets such as stocks or bonds.

[6] Or have a fundamental value with a negligible effect on their market price.

[7] Note, however, that 'fundamental value' is a very difficult (if not impossible) thing to quantify. As with anything that cannot be determined with unquestionable accuracy, the idea that assets such as Bitcoin have no intrinsic value is somewhat disputed, with studies such as Li and Wang (2017) taking an opposing view to the mainstream notion that their price is merely based on speculation.

In this dissertation, cryptocurrencies are of special interest since there is abundant research suggesting that these assets are subject to the effects of social media much more strongly than their traditional counterparts (Mai et al., 2018; Steinert and Herff, 2018; Phillips and Gorse, 2018). Since the main aim of the model(s) introduced in this paper is to reproduce such an effect, we will be comparing the behavior produced by our model to the behavior of some relevant cryptocurrencies as a way to gauge the model's accuracy and/or validity.

2.5.4 Echo Chamber Effect

The Echo Chamber Effect refers to the idea that social media platforms can create environments where investors are exposed predominantly to opinions and information that reinforce their existing beliefs (Cinelli et al., 2021). Such environments can lead to overconfidence and exacerbate market anomalies, as investors might ignore contrary evidence or broader market signals. Barber and Odean (2008) found that individual investors tend to trade more aggressively under the influence of overconfidence, which could be heightened by echo chambers in social media settings, leading to suboptimal trading decisions.

Furthermore, studies such as Jiao et al. (2020) and Cookson et al. (2022) suggest that the Echo Chamber Effect is more prevalent for positive opinions than for negative ones; which leads to social media communities displaying disproportionately bullish tendencies regarding the specific assets or asset classes most frequently discussed within them.

2.5.5 Pump & Dump Schemes

Pump & Dump schemes revolve around artificially inflating the price of an asset through exaggerated or entirely fabricated statements. Once the asset has been bought by unsuspecting buyers (drawn in by the hype), the schemers sell their holdings at the elevated prices, leading to a sudden collapse in the asset's market price and significant losses for the investors who bought the asset without prior knowledge of the underlying scheme (Li et al., 2021).

Lund (2022) suggests that these schemes are often facilitated by the reach and anonymity inherent to social media, where it is easier to spread misleading information quickly and to a large audience.

Chapter 3

Technical Background

3.1 Stylized Facts of Financial Markets

In empirical research, stylized facts refer to broad patterns observed across a variety of real datasets. These patterns persist with such consistency that they are widely acknowledged and utilized as foundational benchmarks for data produced by theoretical models. Unlike idiosyncratic data points or anomalies, these facts are general observations that regularly and consistently emerge from the analysis of empirical data.

Stylized facts are useful since they serve as basic tools for the development and analysis of theoretical models. Basically, these facts are a benchmark that models must account for or explain. This serves a vital purpose in fields such as model development, which have the potential to become very abstract and lose their connection to real-world phenomena. Particularly, financial markets tend to exhibit unpredictable behavior[1], making stylized facts an extremely useful way to feasibly validate whether the behavior of a financial market model conforms to reality.

The subsections below illustrate some stylized facts and their technical details. Note, however, that this list is by no means meant to be exhaustive or fully representative of the discourse surrounding each stylized fact. Instead, we simply provide the minimum necessary context needed to understand the usage of stylized facts within this dissertation.

3.1.1 Asset Returns

Most stylized facts make reference to the returns of an asset. Returns refer to how much the asset's price has changed in the past T time-steps. In other words, returns are measured over a time-lag T; such that, at time-step i:

$$R(i, T) = p(i) - p(i - T) \tag{3.1}$$

where $p(i)$ is the price at time-step i and $R(i, T)$ represent the asset's linear returns at that time-step for time-lag T.

Note that returns can also be expressed in: log[2]; Eq. 3.2, absolute; Eq. 3.3, or squared terms; Eq. 3.4.

$$\ln(R(i, T)) = \ln(p(i)) - \ln(p(i - T)) \tag{3.2}$$

$$|R(i, T)| = |\ln(p(i)) - \ln(p(i - T))| \tag{3.3}$$

$$R(i, T)^2 = (\ln(p(i)) - \ln(p(i - T)))^2 \tag{3.4}$$

3.1.2 Fat Tails in Log Returns

The "fat tails" phenomenon is a stylized fact observed for an asset's returns, indicating that large deviations from the mean are more common than would be expected under a normal distribution.

In a normal distribution, described by the probability density function (pdf):

[1] As opposed to physical systems, whose exact behavior can be predicted given their initial conditions.
[2] Whenever returns are mentioned without specification, assume we are referring to the log returns with $T = 1$.

$$f(x) = \frac{1}{2\pi\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3.5}$$

where μ is the mean and σ is the standard deviation, the tails (extreme values far from the mean) decline exponentially, which indicates an exceedingly low probability of extreme events; however, empirical observations of financial returns show that these extreme events occur more frequently than the normal distribution would predict. This discrepancy is captured by "fat tails," which are mathematically characterized by leptokurtic distributions; i.e. distributions with higher kurtosis than the normal distribution, which has a kurtosis value of 3.

Instead of raw kurtosis values, a common way to model the fat-tails of financial returns is through a Pareto distribution, such that the return's tails are modelled as decaying according to the equation:

$$f(x) = 1 - ax^{-\alpha} \tag{3.6}$$

3.1.3 Return to Gaussianity in Log Returns

The "Return to Gaussianity" or "Aggregational Gaussianity" phenomenon refers to the empirical observation that, while returns with $T = 1$ display leptokurtosis (see Sections 3.1.1 and 3.1.2), as T increases the distribution of the returns approaches a normal distribution.

In more mathematical terms, the excess kurtosis[3] of the returns is expected to approach 0 as T becomes larger.

3.1.4 Volatility Clustering

Autocorrelation refers to the correlation of a time series with its own past and future values. It is usually measured via an AutoCorrelation Function (ACF). The ACF has a lag value (τ), which determines how far into the past to look when calculating the correlation between two data-points (i.e. such that the two data-points are τ units apart).

For financial returns, the ACF typically shows insignificant autocorrelation, suggesting that returns are largely unpredictable based on past returns; however, when applied to the absolute or squared returns, the ACF often reveals significant positive autocorrelation, suggesting that large changes tend to be followed by large changes (in absolute value).

This phenomenon relates to volatility clustering, which is the idea that asset returns have a tendency to cluster based on their magnitude/significance. That is, large changes tend to be followed by large changes; while small changes are usually followed by small changes.

3.1.5 Slow Decay of Autocorrelation in Absolute Returns

This stylized fact is related to the observation that the aforementioned autocorrelation in the absolute returns of an asset tends to decay slowly, following a power law such that:

$$f(x) = at^{\beta} \tag{3.7}$$

where t is the lag in the ACF and $\beta \in [0.2, 0.4]$ (Cont, 2001).

3.2 Lux-Marchesi Agent-Based Model

3.2.1 Overview

The Lux-Marchesi model was introduced in Lux and Marchesi (1999), a seminal paper that paved the way for the use of ABMs in the study of financial markets. Due to its remarkable simplicity and versatility, the Lux-Marchesi Model is still frequently used as a basic building block in modern-day research into financial ABMs.

The Lux-Marchesi Model is populated by 2 types of trader: *chartists* and *fundamentalists*. Additionally, chartists are further split into 2 roles: *optimists* and *pessimists*. Fundamentalists trade based on the current price of the asset in relation to its fundamental price (i.e. buying if the asset is under-priced and

[3]Excess kurtosis refers to kurtosis values above those of a normal distribution; e.x. data with a kurtosis of 4 has an excess kurtosis of $4 - 3 = 1$.

selling if it is over-priced). Chartists trade based on speculation: pessimists are bearish and always sell, while optimists are bullish and always buy.

The above is significant since it showcases how the Lux-Marchesi model diverges from traditional financial theories: rejecting the notion of a single, rational representative agent, and instead attempting to model the complexity of real-world markets, where individuals with varying strategies and expectations interact.

Moreover, a key feature of the Lux-Marchesi model is the dynamic nature of its agents' roles. Traders are not statically bound to their initial roles of fundamentalists, optimists, or pessimists. They can switch their trading strategies based on the success of their current approach, reflecting the adaptive and evolving nature of real market participants.

The model also incorporates a mechanism for the diffusion of opinions among chartists, which can lead to herding behavior. Although simple and somewhat unrefined[4], this feature enhances the model's realism, allowing it to capture the psychological aspects of trading and the impact of collective sentiment on market movements.

3.2.2 Parameters

Table 3.1 contains an exhaustive list of all the (constant) parameters defined upon initializing the Lux-Marchesi model[5], alongside brief descriptions for each of them.

Name	Description
N	Number of traders in the market
α_1	How much chartists are influenced by the opinion of other chartists
α_2	How much chartists are influenced by changes in the asset price
α_3	How much traders are influenced by a role's profit
v_1	How often pessimists try to become optimists and viceversa
v_2	How often fundamentalists try to become chartists and viceversa
β	How often the market price changes
r	Dividends paid by the asset
R	Returns from alternative investments
s	Factor by which a fundamentalist's profit is reduced (Discount Factor)
p_f	Fundamental value
σ	Magnitude of fundamental value fluctuations
μ	Noise when price changes due to excess demand/supply
γ	How strongly fundamentalists react to deviations from the fundamental price
t_c	How much of the asset is bought or sold (by optimists or pessimists respectively)
δt	Time interval. One δt represents a time-step, the simulation runs for $\frac{1}{\delta t}$ time-steps per time-unit
δt'	Compare the current price and the price from δt' time-steps ago to determine how fast the price is changing

Table 3.1 Parameters for the Lux-Marchesi Model

[4]In this dissertation we explore how to incorporate a more complex system of opinion dynamics into the Lux-Marchesi model.

[5]Throughout this dissertation, the capitalized terms "Sets," "Parameter Sets," and "Standard Parameter Sets" all refer to the parameter sets illustrated in Table 3.1.

3.2.3 Price Dynamics

The probability that the price changes at any given time-step is determined by the excess demand from the traders in the market.
The probability that the price rises is determined by the equation:

$$\pi_{\uparrow p} = max[0, \beta(ED + \mu)] \tag{3.8}$$

Conversely, the probability that the price falls is determined by the equation:

$$\pi_{\downarrow p} = min[0, -\beta(ED + \mu)] \tag{3.9}$$

where ED represents the excess demand in the market:

$$ED = ED_c + ED_f \tag{3.10}$$

ED_c represents the total demand from chartists and is determined by the total number of optimists (n_+) and the total number of pessimists (n_-); ED_f represents the total demand exerted by fundamentalists and is determined by the number of fundamentalists (n_f), as well as whether fundamentalists are buying or selling the asset.

$$ED_c = (n_+ - n_-) * t_c \tag{3.11}$$

$$ED_f = n_f * \gamma * (p_f - p) \tag{3.12}$$

When the price does change, the magnitude by which it changes is always $0.2\delta_t$[6].

3.2.4 Fundamental Price

Note that most analyses/implementations of the Lux-Marchesi model set $\sigma = 0$, presumably because a constant fundamental price greatly facilitates the analysis and visualization of acquired results; however, if $\sigma > 0$, at every time-unit the fundamental price randomly fluctuates according to the equation:

$$[p_f(t) - p_f(t-1)] \sim \mathcal{N}(0, \sigma^2) \tag{3.13}$$

3.2.5 Excess Profit

Agents in the Lux-Marchesi model are simple since they only have two options: either buying or selling the asset. Excess profit is a measure of the opportunity cost inherent to these two options; i.e. if an agent bought the asset, its excess profit is how much it made from the purchase minus how much it would have made from not buying the asset, and vice-versa for the excess profit if it decided to sell. The excess profit for fundamentalists (EP_f), optimists (EP_+), and pessimists (EP_-) is determined by the following set of equations:

$$EP_f = s\left|\frac{p_f - p}{p}\right| \tag{3.14}$$

$$EP_+ = \left(r + \frac{\dot{p}}{v_2}\right)/p - R \tag{3.15}$$

$$EP_- = R - \left(r + \frac{\dot{p}}{v_2}\right)/p \tag{3.16}$$

where $\dot{p}$ is the price trend, determined by comparing the current price with the price from δ_t' time-steps ago:

$$\dot{p} = \left(p_t - p_{t-\delta_t'}\right)/\delta_t' \tag{3.17}$$

[6] There is no clear consensus on what this value should be, in this case, $0.2\delta_t$ has been chosen semi-arbitrarily and does not significantly deviate from most alternatives presented in the literature.

3.2.6 Transition Pressure

There are a total of 6 possible ways for traders to transition from one role to another: pessimist↔optimist, optimist↔fundamentalist, and pessimist↔fundamentalist. Each of these transitions is affected by different **transition pressures**.

U_1 represents the pressure exerted by the pessimist↔optimist transition, with positive values of U_1 meaning optimists feel (more) pressure to become pessimists and negative values meaning pessimists feel (more) pressure to become optimists.

$$U_1 = \alpha_1 x + \alpha_2 \frac{\dot{p}}{v_1} \tag{3.18}$$

where x represents the overall market sentiment (i.e. the difference between the number of optimists and the number of pessimists):

$$x = \frac{n_+ - n_-}{n_+ + n_-} \tag{3.19}$$

U_{21} represents the pressure exerted by the optimist↔fundamentalist transition, with positive values of U_{21} meaning optimists feel (more) pressure to become fundamentalists and negative values meaning fundamentalists feel (more) pressure to become optimists.

$$U_{21} = \alpha_3 \left(EP_f - EP_+\right) \tag{3.20}$$

U_{22} is similar to U_{21}, it represents the pressure exerted by the pessimist↔fundamentalist transition and is defined by the equation:

$$U_{22} = \alpha_3 \left(EP_f - EP_-\right) \tag{3.21}$$

3.2.7 Transition Probability

With the transition pressures defined, we can finally define equations for the transition probabilities. These represent the probability that, at any given time step, a trader switches from one state to another.

Note that transition probabilities are not (always) symmetrical (i.e. the probability that an optimist becomes a fundamentalist cannot be represented in terms of the probability that a fundamentalist becomes an optimist). So we define 6 transition equations:

For the optimist↔pessimist transitions:

$$\pi_{+\to-} = v_1 \cdot \frac{n_c}{N} \cdot e^{-U_1} \cdot \delta_t \tag{3.22}$$

$$\pi_{-\to+} = v_1 \cdot \frac{n_c}{N} \cdot e^{U_1} \cdot \delta_t \tag{3.23}$$

For the optimist↔fundamentalist transitions:

$$\pi_{+\to f} = v_2 \cdot \frac{n_+}{N} \cdot e^{-U_{21}} \cdot \delta_t \tag{3.24}$$

$$\pi_{f\to+} = v_2 \cdot \frac{n_f}{N} \cdot e^{U_{21}} \cdot \delta_t \tag{3.25}$$

And for the pessimist↔fundamentalist transitions:

$$\pi_{-\to f} = v_2 \cdot \frac{n_-}{N} \cdot e^{-U_{22}} \cdot \delta_t \tag{3.26}$$

$$\pi_{f\to-} = v_2 \cdot \frac{n_f}{N} \cdot e^{U_{22}} \cdot \delta_t \tag{3.27}$$

3.3 Meine et al. Hierarchical Model

Meine and Vvedensky (2023) present a model (subsequently referred to as the Meine et al. model) based on the Lux-Marchesi model as well as a hierarchical model of crashes proposed in Sornette and Johansen (1998).

Essentially, the Meine et al. model incorporates a hierarchical communication structure to the Lux-Marchesi model. This structure arranges the market participants in a tree-like structure, such that each trader above level 0 (i.e. all non-leaf nodes) has c children. The hierarchy is updated in two distinct steps:

- In the **backward pass**, the state of all non-leaf traders is dictated by which state the majority of its children are a part of.
- In the **forward pass**, the state of the top trader is propagated downwards until reaching the leaf nodes.

The leaf nodes themselves set their state according to the basic Lux-Marchesi equations; however, their behavior is also affected by the hierarchy, with strength proportional to a constant b. This effect is constrained to being positive, such that the number of traders of a given type in the hierarchy increases a leaf node's probability of switching to that state, but doesn't decrease the rate at which they choose other states. The corresponding equations for the transition probabilities are:

$$\pi'_{ab} = \pi_{ab} \cdot 2^{bn_a} \tag{3.28}$$

$$\pi'_{ba} = \pi_{ab} \cdot 2^{bn_b} \tag{3.29}$$

where a, b are any given trader type and n_a, n_b are the number of traders of each of those types (respectively) in the hierarchy above the leaf node in question.

3.4 Speculative Market Bubbles

3.4.1 Definition

Speculative bubbles are a surprisingly lively subject of academic debate, with differing views on their precise definition and the mechanisms by which they occur. Economically, a speculative bubble has generally been described as a situation where asset prices exceed their fundamental value by a large margin, driven primarily by investor behavior rather than intrinsic worth (Kindleberger et al., 2005); notwithstanding, this definition is contested, as there is a significant debate surrounding the idea that bubbles are a result of irrational market behaviors, as opposed to rational responses to changes in expectations about future market conditions (Shiller, 2015).

For the purposes of this dissertation, a speculative bubble will be defined as a period during which asset prices display explosive behavior, primarily driven by investor sentiment and market speculation, rather than fundamental factors. This simplified definition will help with the analysis of how speculative bubbles develop and burst within our models.

3.4.2 Detection

3.4.2.1 ADF Test

The Augmented Dickey-Fuller (ADF) test, originally proposed in Dickey and Fuller (1979), is a widely used statistical test for determining whether a given time series is stationary. A time series is considered stationary if its statistical properties, such as mean, variance, and autocorrelation, are constant over time.

The ADF test is an enhancement of the Dickey-Fuller test, which was also proposed in Dickey and Fuller (1979). The Dickey-Fuller test focuses on a simple model that does not account for complex dynamics in the time series data, such as serial correlation in the residuals from one period to the next. On the other hand, the ADF test includes lagged differences of the time series in its regression model. This addition allows the ADF test to control for serial correlation, making it a more robust and reliable method for testing the presence of a unit root in a wider array of time series data.

The null hypothesis H_0 of the ADF test states that the time series has a unit root, meaning it is non-stationary. Consequently, rejecting the null hypothesis would mean that the time series is determined to be stationary. Crucially, however, the ADF statistic can be either negative or positive, with negative values indicating stationarity; while positive values indicate non-stationarity or "explosive" behavior.

3.4.2.2 SADF Test

The supremum/sup-ADF (SADF) test (Phillips et al., 2011) involves conducting a series of ADF tests over a (forward-expanding) window of the time series data. This approach allows for the detection of unit roots within sub-samples of the data, which accommodates the idea that market conditions may change over time. The 'supremum' aspect of the SADF test comes from taking the maximum test statistic from this series of tests, thus capturing the strongest evidence of a unit root or explosive behavior within segments of the time series. From there, explosive behavior is deemed to have been found if the SADF statistic exceeds a certain critical value[7].

In mathematical terms, the SADF test can be expressed as:

$$SADF(r_0) = \sup_{r_w \in [r_0,1]} ADF_0^{r_w} \tag{3.30}$$

where r_0 is the minimum window size, and ADF_a^b represents an ADF test starting at point a and ending at point b.

The Backwards SADF (BSADF) test is an additional version of the SADF with a window that starts from the end of the time series and expands toward the front. Such that:

$$BSADF(r_0) = \sup_{r_w \in [r_0,1]} ADF_{1-r_w}^{1} \tag{3.31}$$

3.4.2.3 GSADF Test

The Generalized SADF (GSADF) test (Phillips et al., 2015) represents a further improvement over the SADF and ADF tests. This test allows for a variable starting point to the rolling window from the BSADF test, and thus can capture explosive behavior for any sub-sample of the time series data. The GSADF is mainly used for, and has proven successful at, identifying and accounting for multiple bubbles within the same dataset (Phillips et al., 2015).

In mathematical terms, the GSADF test can be represented as:

$$GSADF(r_0) = \sup_{r_1 \in [r_0,1]} \left\{ \sup_{r_w \in [r_0,1]} ADF_{r_w - r_1}^{r_w} \right\} \tag{3.32}$$

Similarly to the SADF test, the GSADF test must exceed a critical value for the dataset to be determined to contain explosive behavior (see Section 4.2.).

3.4.2.4 PWY Method

The PWY method (sometimes referred to as the PWY procedure) is used to, not only detect, but also timestamp bubbles in time series data.

To determine the starting point of a bubble, the BSADF test is conducted with increasingly larger starting points until explosive behavior is detected. This point, $\hat{r}^e$, is deemed the origination point of the bubble.

$$\hat{r}^e = \inf_{r_2 \in [r_0,1]} \{s : ADF_{r_2} > cv\} \tag{3.33}$$

Next, the test is conducted further until explosive behavior is no longer detected. This point, $\hat{r}^f$, is deemed the termination point of the bubble.

$$\hat{r}^f = \inf_{r_2 \in [\hat{r}^e + \log(T)/T, 1]} \{s : ADF_{r_2} < cv\} \tag{3.34}$$

where T is the length of the data, and $\log(T)/T$ is a (semi-arbitrarily) chosen minimum bubble size.

[7]The specific values used in this dissertation are illustrated in Section 4.2

Chapter 4

Materials and Methods

4.1 Volatility

4.1.1 Raw Volatility

To measure the volatility of an asset, we simply measure its standard deviation:

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}} \tag{4.1}$$

where μ is the mean market price and x_i is the market price at time-step i.

4.1.2 Fundamental Deviation

An arguably more informative metric, rather than how much the asset deviates from its own mean, is how much the asset deviates from its associated fundamental value. We measure this via a variable F_σ, such that:

$$F_\sigma = \sqrt{\frac{\sum(x_i - f_i)^2}{N}} - \sqrt{\frac{\sum(x_i - \mu)^2}{N}} \tag{4.2}$$

where f_i is the asset's fundamental value at time-step i.

Note that we subtract the overall deviation from the fundamental value by the asset's standard deviation so the volatility's magnitude doesn't overshadow the fundamental deviation metric.

4.2 GSADF Test and PWY Method

Although there are some more specialized/complex tests for detecting bubbles, the GSADF test is at the forefront of modern literature. Due to its remarkable simplicity (especially considering the quality of results it provides), as well as it being one of the most documented and well-researched bubble detection frameworks available, this test will be used throughout this thesis whenever tests for explosive behavior are conducted.

For the GSADF method with finite samples, we derive our significance level and r_0 values from Phillips et al. (2011). These values are illustrated in Table 4.1.

Conversely, for the PWY procedure, we follow the methodology set out in Phillips et al. (2012). That is, we identify significant bubbles through the asymptotic SADF critical values, illustrated in Table 4.2.

4.3 Stylized Facts

4.3.1 Fat Tails

Fat tails are usually represented by excess kurtosis (i.e. leptokurtosis, see Section 3.1); however, as explained in Lux and Marchesi (2000), kurtosis is a somewhat ambiguous concept, and it is not entirely clear how to compare the kurtosis statistics obtained for various time series.

	SADF			GSADF		
	90%	95%	99%	90%	95%	99%
$T = 100$, $r_0 = 0.190$	0.98	1.30	1.99	1.65	2.00	2.57
$T = 200$, $r_0 = 0.137$	1.12	1.40	1.90	1.84	2.08	2.70
$T = 400$, $r_0 = 0.100$	1.19	1.49	2.05	1.92	2.20	2.80
$T = 800$, $r_0 = 0.074$	1.25	1.53	2.03	2.10	2.34	2.79
$T = 1600$, $r_0 = 0.055$	1.28	1.57	2.22	2.19	2.41	2.87

Table 4.1 Critical values for the finite-sample SADF and GSADF tests

	SADF			GSADF		
	90%	95%	99%	90%	95%	99%
$r_0 = 0.190$	1.10	1.37	1.88	1.67	1.89	2.37
$r_0 = 0.137$	1.12	1.41	2.03	1.78	2.01	2.48
$r_0 = 0.100$	1.20	1.49	2.07	1.97	2.19	2.69
$r_0 = 0.074$	1.21	1.51	2.06	1.99	2.20	2.62
$r_0 = 0.055$	1.23	1.51	2.06	2.08	2.30	2.74

Table 4.2 Critical values for the asymptotic SADF and GSADF tests

For this reason, rather than measuring kurtosis, it is standard practice to measure fat-tailedness by assuming that the tails[1] of the returns decay according to a Pareto distribution: $1 - ax^{-\alpha}$. The absolute returns of most real assets present a tail decay such that $2 \leq \alpha \leq 6$, with lower values of α representing fatter tails (Cont, 2001).

To estimate α, we use methodology originally proposed in Hill (1975), where, for a tail of size m out of a total of n observations:

$$\alpha_H = \frac{1}{\frac{1}{m}\sum_{i=1}^{m}\left[\ln(R_{n-i-1}) - \ln(R_{n-m})\right]} \tag{4.3}$$

As is standard practice, whenever results regarding tails are discussed, we will present values for the 2.5%, 5%, and 10% right tails of the absolute returns.

4.3.2 Volatility Clustering

To showcase volatility clustering, we present the ACF for the absolute and squared returns for $\tau = 10$ and $T = 70$. This result is expected to be larger than 0, which means that, at any given time-step, there is a correlation between the current returns of the asset and its returns from 10 time-steps ago. To compute the ACF, we import the `acf` function from the `statsmodels.api.tsf` library.

4.3.3 Return to Gaussianity

To showcase the return to Gaussianity feature, we present the kurtosis of returns with $T = 1$, $T = 10$ and $T = 50$. We expect the excess kurtosis to be large when $T = 0$, and approach 0 as T gets larger (i.e. we expect the distribution of the returns to approach a normal distribution for large values of T).

To compute the excess kurtosis of the returns, we use the `norm.pdf` function from `scipy.stats` to fit the returns to a normal distribution, after which we use the `pandas.kurt` function to compute the distribution's kurtosis[2].

[1] "Tails" refers to the most extreme values in a dataset. The x% right tail refers to the x% largest values in the dataset.
[2] There are more rigorous ways to perform this analysis, such as via GARCH models; however, for the purposes of our

4.3.4 Slow Decay of Autocorrelation

To showcase the slow decay of the ACF, we present the decay of the absolute returns with $T = 70$ as τ gets larger.

This value is calculated by using the `scipy.optimize.curve_fit` function to fit an exponential decay function $f(x) = bx^{-a}$ to the ACF of the absolute returns.

4.4 Lux-Marchesi Model

	Set I	**Set II**	**Set III**	**Set IV**
$\boldsymbol{N}$	500	500	500	500
$\boldsymbol{\alpha_1}$	0.6	0.9	0.75	0.8
$\boldsymbol{\alpha_2}$	0.2	0.25	0.25	0.2
$\boldsymbol{\alpha_3}$	0.5	1	0.75	1
$\boldsymbol{v_1}$	3	4	0.5	2
$\boldsymbol{v_2}$	2	1	0.5	0.6
$\boldsymbol{\beta}$	6	4	2	4
$\boldsymbol{r}$	0.004	0.004	0.004	0.004
$\boldsymbol{R}$	0.0004	0.0004	0.0004	0.0004
$\boldsymbol{s}$	0.75	0.75	0.75	0.75
$\boldsymbol{p_f}$	10	10	10	10
$\boldsymbol{\sigma}$	0	0	0	0
$\boldsymbol{\mu}$	0.05	0.1	0.1	0.05
$\boldsymbol{\gamma}$	0.01	0.01	0.02	0.01
$\boldsymbol{t_c}$	0.02	0.015	0.02	0.01
$\boldsymbol{\delta t}$	0.01	0.01	0.01	0.01
$\boldsymbol{\delta t}$'	0.002	0.002	0.002	0.002

Table 4.3 Standard Parameter Sets for the Lux-Marchesi Model

4.4.1 Implementation

The Lux-Marchesi model was implemented in `Python 3.1`, using guidance from the original Lux-Marchesi paper (Lux and Marchesi, 1999), as well as following some implementation details found in Liu (2021).

With regard to specific details, there is not much debate surrounding how to implement the Lux-Marchesi model; our implementation only contains some minor 'contentious features' which are listed below:

- When an agent tries to transition into two different roles simultaneously, one is chosen at random.
- Although some recent implementations of the Lux-Marchesi model contain a maximum possible value for the number of chartists, our implementation has no such cap[3].
- At every time-step, agents switch their state before the price is updated.

analyses, a simple reference value to represent the kurtosis is sufficient.

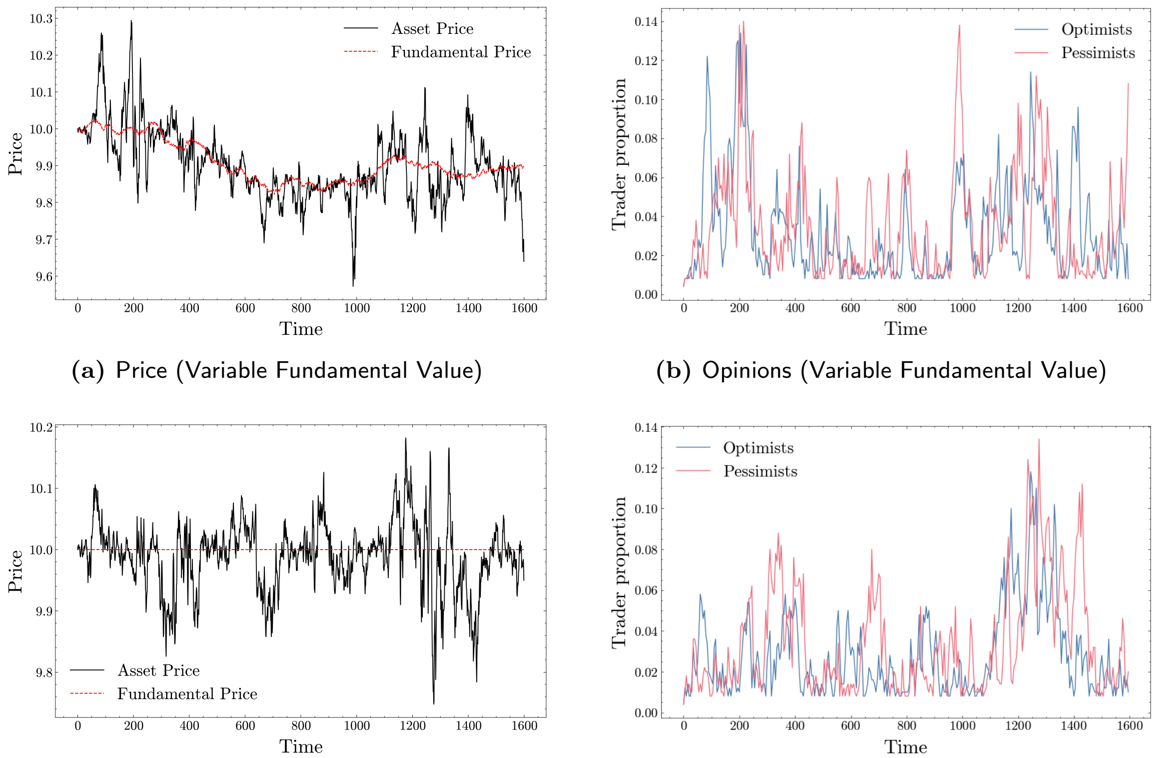
[3]For context, the chartist cap is usually placed to prevent unstable behavior; however, it remains unimplemented in this dissertation since we are generally interested in discovering instances where our model leads to such behavior.

- When the simulation starts, the market's population consists of 10% chartists and 90% fundamentalists.

4.4.2 Parameter Sets

Lux and Marchesi (1999) propose 4 Standard Parameter Sets to be used as constants in the Lux-Marchesi model. Since most subsequent analyses and implementations of the Lux-Marchesi model utilize these exact parameters, we will also make use of them in order to maintain as much consistency with pre-existing studies as possible. The values used for each of the aforementioned parameter sets are illustrated in Table 4.3.

4.4.3 Brief Analysis



(a) Price (Variable Fundamental Value)

(b) Opinions (Variable Fundamental Value)

(c) Price (Constant Fundamental Value)

(d) Opinions (Constant Fundamental Value)

Figure 4.1 Sample Runs of the Lux-Marchesi Model. Showing the market price (left) and associated opinion distributions (right). Notice that the price deviates from the fundamental value, and opinions showcase strong herding tendencies.

Figs. 4.1a and 4.1c show how the asset price evolves in the Lux-Marchesi model. As we can see, this price presents consistent deviations away from the fundamental value, which suggests that the market and the agents within it are not behaving optimally.

Furthermore, Figs. 4.1b and 4.1d show how the proportion of each agent type evolves in the previously mentioned markets. From these graphs, we can verify that herding behavior is indeed occurring, since large spikes in optimism and pessimism can clearly be observed at multiple points during the simulations.

Lastly, we verify that the model acts in accordance with the stylized facts discussed in Section 3.1. This is important since it allows us to verify the accuracy of our implementation of the Lux-Marchesi model[4], as well as providing baseline values under which to compare subsequent analyses using our stylized fact-related methodology. Table 4.4 showcases these results, from which we can observe:

[4]Even though Parameter Set I deviates from expected behavior quite significantly at some points, this is in line with other analyses of the model, which show that this parameter set displays unusually high levels of kurtosis alongside a rapidly decaying ACF (Lux and Marchesi, 2000).

Table 4.4 Lux-Marchesi Model Stylized Facts

	Set I	**Set II**	**Set III**	**Set IV**
Tail Decay (2.5%)	11.68	5.08	7.81	4.68
Tail Decay (5%)	5.58	3.39	4.28	3.48
Tail Decay (10%)	3.09	2.41	3.08	2.46
Kurtosis ($T = 1$)	2.18	10.22	2.98	2.15
Kurtosis ($T = 10$)	0.96	2.02	2.1	1.89
Kurtosis ($T = 50$)	0.59	0.96	0.73	0.54
Log AC ($\tau = 10$)	0.082	0.13	0.24	0.15
Squared AC ($\tau = 10$)	0.062	0.18	0.19	0.11
Abs AC Decay	0.98	0.57	0.61	0.74

*Each cell corresponds to the average of 50 simulations with $4 * 10^4$ time-steps.
*AC stands for Autocorrelation
*Kurtosis values represent **excess** kurtosis

- **Fat tails:** From the tail[5] decay generally ranging from 2 to 6.
- **Return to Gaussianity:** From the excess kurtosis at $T = 50$ being close to 0, while the excess kurtosis at $T = 1$ is larger than 2.
- **Slow decay of autocorrelations:** From the decay in the ACF of the absolute returns being smaller than 1[6].
- **Volatility clustering:** From the absolute and squared returns displaying an autocorrelation significantly larger than 0 when $\tau = 10$.

4.5 Meine et al. Model

Given that analyses of the Meine et al. model have never been published before (in papers other than Meine and Vvedensky (2023) itself), a great deal of care was taken to detail its implementation and analysis thoroughly. As a consequence, the full analysis is quite long, so, for brevity, this section will simply consist of a summary of the full version (which can be found in Appendix A).

In essence, implementing the Meine et al. model is quite challenging since its details are defined somewhat ambiguously. Furthermore, the parameter sets used in the original paper are not publicly available[7], so it is not feasible to accurately replicate the results contained within it.

Although analyses of (our best approximation of) the model detailed in the paper provide some interesting results, there is scant evidence suggesting that the model is a sufficiently accurate or robust representation of the phenomenon being explored in this dissertation. Hence, we conclude that amendments must be made to the model before it can be presented as a representation of social media dynamics within financial markets.

[5] The Lux-Marchesi model presents no differentiation between positive and negative price changes, so the tails are the largest absolute values of the returns (Lux and Marchesi, 2000).
[6] Note that these values are slightly larger than usual, which is most likely due to the nonstandard methodology used to calculate them.
[7] The original authors were contacted regarding the parameter sets but, as of writing this, they are yet to respond.

Chapter 5

Model Proposal

5.1 Motivation

When attempting to model the interplay between social networks and financial markets, hierarchies continually emerge as a simple yet effective way to represent several interesting phenomena (refer to Chapter 7 for examples). Modeling social network dynamics via hierarchies is a well-established practice that has proven to be quite powerful through several studies (Watts et al., 2002; Clauset et al., 2008). Furthermore, papers such as Zhang et al. (2017) and Dang et al. (2019) showcase the substantial improvements that hierarchical designs provide when it comes to modeling social media networks specifically.

However, despite the apparent usefulness of hierarchical network structures, there are almost no papers exploring their use as a way to model communication between agents in a financial ABM[1]. Out of the papers that do discuss hierarchically structured financial ABMs, none of them discuss them in the context of the influence of social media on financial markets and the phenomena that arise as a result. This chapter represents the first step towards filling this gap in the literature; through the careful definition and discussion of the 'Hierarchical[2] Model,' which, in turn, is based primarily on the Meine et al. model.

5.2 Changes to the Meine et al. Model

Parting from the Meine et al. model as implemented in Section 4.5, we proceed to modify several of its aspects with the intent of arriving at a model that can more accurately represent social media networks. The sections below illustrate these changes, as well as carefully motivating the reasoning behind each of them.

5.2.1 No Hierarchy = Lux-Marchesi

A significant hindrance when it comes to analyzing the Meine et al. model arises as a result of the fact that no arrangement of its parameters aligns its behavior perfectly with that of the basic Lux-Marchesi model. This means that there is no 'base' set of results against which to compare data gathered from this model and no 'base' set of parameters that can progressively be tweaked to explore how changes to the model affect its behavior from the ground up.

[1]The Google Scholar search for *(abm OR "agent based model") AND "financial market" AND (intitle:hierarchy OR intitle:hierarchical)* yields ten unique results:

- One of these is the Meine et al. model.
- Two others are Alfarano et al. (2009), and Alfarano et al. (2011); a working paper and its published counterpart showing that an ABM with a hierarchical-like structure leads to increased volatility in simulated markets, and underscoring the relation between these hierarchical structures and social network dynamics; other than the Meine et al. model, these models are the closest approximation to the model presented in this dissertation.
- Similarly, Theodosopoulos (2016) presents a hierarchically structured ABM similar to our model, with a heavy emphasis on rigorous mathematical analysis of its emergent features.
- Out of the remaining six papers, one mentions ABMs solely in their cited papers, three mention ABMs in their main text but are not centered around the design of an ABM, and two design an ABM wherein the agents are not, or only partially, organized in a hierarchical fashion.

[2]Assume that when the term "Hierarchical Model" is capitalized we are referring to the model defined in this chapter.

This can be fixed by simply limiting the influence of communities such that their state only affects traders, and making it so any and all other aspects of the market are affected solely by the traders themselves. Under this simple change, when $b = 0$, communities have no effect on traders OR on the market, and thus the market behaves exactly as it would under the standard Lux-Marchesi prescription.

5.2.2 Fundamentalists are Not Affected by Herding

The Meine et al. model is based on the Sornette–Johansen model, which, in turn, aims to model the organization of all traders operating within a market. Herein lies probably the most important distinction between our model and the Meine et al. model, as we are attempting to capture the organization and dynamics of traders *who are subject to speculative influences*, whilst the former model assumes all traders exchange information through a hierarchical structure.

Hence, we stick to the paradigm set out by the Lux-Marchesi model and only allow chartist traders to be affected by speculative influences. In effect, we amend the herding equation showcased in Eq. 3.18, redefining x as the hierarchical influence (instead of assuming all chartist traders in the market exert equal force upon one another). Furthermore, we replace α_1 with a new variable b, which represents the hierarchy's strength.

5.2.2.1 Bounded Hierarchical Influence

To further resemble the Lux-Marchesi model, we constrain the factor by which b is multiplied to be between -1 and 1, reflecting the fact that $x = \frac{n_+ - n_-}{n_+ + n_-}$ is also bounded by these two constants.

5.2.3 Negative Influences Affect the Simulation

In the Meine et al. model, community interactions are restricted to a positive influence. To quote the original paper, this means that "when traders choose a role, they will increase other traders' rates to choose the same role, but they will not lower their rate to choose different roles"[3].

This is an approximation with respect to the more intuitive alternative, which would be to follow the paradigm set out by the original Lux-Marchesi herding equations (Eqs. 3.18, 3.22 and 3.23) such that a role's popularity negatively influences the rate at which traders transition away from it:

$$\pi'_{ab} = \pi_{ab} 2^{b(n_b - n_a)} \tag{5.1}$$

Meine et al. justify this approximation via the following reasoning:

1. Considering both influences results in "individual driving forces for each trader," the calculation of which is computationally expensive.
2. Only considering positive influences prevents "dead traders" (i.e. traders that remain locked into one group/role). Since transition rates are not diminished, it is always possible for a trader to change groups.

We argue that these benefits do not outweigh their associated drawbacks, especially when the changes from Section 5.2.2 are taken into account (i.e. only chartists are affected by the hierarchy).

To understand our reasoning, it is crucial to consider that a community's influence on its children accumulates over time: a community that switches to a role but then immediately switches back will not have as much of a disruptive effect as a community that remains in that role for an extended period of time. Hence, the 'overall impact' of a role is a product of the transition rate to the role and the average time that communities stay in it, which is inversely proportional to the transition rate away from the role. This idea allows us to define a new variable that represents the 'overall impact' that any given community of role a has on the traders below it, such that:

$$I_a = \frac{\pi_{ba} + \pi_{ca}}{\pi_{ab} + \pi_{ac}} \tag{5.2}$$

[3]At the risk of sounding pedantic, please note that this is not strictly true since when a trader chooses a role they, by definition, withdraw from their previous role, and thus lower the probability of other traders choosing it.

Since the new hierarchy only affects chartists (see Section 5.2.2), the transition into a fundamentalist can be disregarded without loss of generality:

$$I_a = \frac{\pi_{ba}}{\pi_{ab}} \tag{5.3}$$

We now consider how the total influence of a role differs between the alternative ways of calculating hierarchical influence when a trader decides to switch to that role.

When influences are only positive:

$$I_a = \frac{\pi_{ba}}{\pi_{ab}} \propto \frac{2^{bn_a}}{2^{bn_b}} = 2^{b(n_a-n_b)}$$

...and a community switches from role b to role a:

$$\begin{aligned} I'_a = \frac{\pi_{ba}}{\pi_{ab}} &\propto \frac{2^{b(n_a+1)}}{2^{b(n_b-1)}} \\ &= 2^{2b+b(n_a-n_b)} \\ &= 4^b 2^{b(n_a-n_b)} = 4^b I_a \end{aligned}$$

When influences are both positive and negative:

$$I_a = \frac{\pi_{ba}}{\pi_{ab}} \propto \frac{2^{b(n_a-n_b)}}{2^{b(n_b-n_a)}} = 2^{2b(n_a-n_b)}$$

...and a community switches from role b to role a:

$$\begin{aligned} I'_a = \frac{\pi_{ba}}{\pi_{ab}} &\propto \frac{2^{b((n_a+1)-(n_b-1))}}{2^{b((n_b-1)-(n_a+1))}} \\ &= 2^{4b+2b(n_a-n_b)} \\ &= 16^b 2^{2b(n_a-n_b)} = 16^b I_a \end{aligned}$$

The equations above show that, for any given role a, a community's influence is 2^b times greater when both positive and negative influences are taken into account; however, when a community changes from role b to role a, the change to role a's influence increases by a factor of 4^b if both these influences are considered[4]. Therefore, increasing b has a much stronger impact on a trader's sensitivity to changes within the hierarchy than it has on how much the trader is affected by the hierarchy as a whole.

This is a very notable effect, as it makes hierarchical influences more significant whilst keeping their effect on the model's transition probabilities bounded. Given that influence from the hierarchy is the main effect being explored in this dissertation, we must conclude that solely considering positive influences is not a satisfactory approximation for our model. Consequently, we redefine the transition pressure for chartists as:

$$U'_1 = U_1 + b\ln(2)(n_+ - n_-) \tag{5.4}$$

Which results in transition probabilities:

$$\pi'_{+-} = \pi_{+-} 2^{-b(n_+-n_-)} \tag{5.5}$$

$$\pi'_{-+} = \pi_{-+} 2^{b(n_+-n_-)} \tag{5.6}$$

[4] Influence increases by 4^b in the former case, and 16^b in the latter case, so the overall increase in rate of change is $16^b/4^b = 4^b$.

5.2.4 Communities are not Homogeneous

As opposed to the Meine et al. model, where communities have a discrete state dictated by the majority of their children, our model distributes the influence of each child equally, such that even traders in the minority state have an effect on the overall state of the community. This can be represented by a community's state being a vector $[o,\ p,\ f]$ where o, p, and f represent the proportion of each individual component of the community's overall state[5].

The idea outlined above might represent the most controversial change in this chapter. In practical terms, non-homogenous communities are required in order to speed up computation; otherwise, it is necessary to simulate a massive number of communities in order to achieve proper results. Notwithstanding, both the Meine et al. approach and the approach outlined in this section could be argued to be 'reasonable[6]'.

5.3 Summary

Our model diverges significantly from the model it was initially based on, and in the end, approximates the Lux-Marchesi model much more closely than it does the Meine et al. model. As such, a definition of the model proposed in this dissertation without mention of the Meine et al. model provides a concise and clear way to summarise its key features. This section illustrates such a summary, containing all the key information required for a full implementation of the model proposed in this chapter.

5.3.1 Structure

We propose a financial ABM wherein agents interact via a hierarchical communication structure; this structure is populated by two types of agents: *Communities* and *Traders*. Traders exist within the bottom level of the hierarchy whilst communities populate all levels above the traders.

Eqn. (5.7) shows the number of traders (N_t), and communities (N_c) in the model, where the hierarchy has L levels, and every community has c children.

$$N_t = c^{L-1}, \quad N_c = \frac{c^k - 1}{L - 1} - N. \tag{5.7}$$

5.3.2 Traders

Traders are very similar to agents from the Lux-Marchesi model. The only difference is that, instead of having perfect knowledge about the overall distribution of opinions in the market, chartists only have access to information provided to them through the hierarchical network.

As such, the pressure exerted by the optimist↔pessimist transition becomes:

$$U_1 = b\left(\frac{C_o - C_p}{C_o + C_p}\right) + \alpha_2 \frac{\dot{p}}{v_1} \tag{5.8}$$

where b represents 'Hierarchy Strength' and C_o, C_p represent the (parent's) community proportion of optimists and pessimists respectively.

5.3.3 Communities

Communities do not store a discrete state but rather a vector: $[o,\ p,\ f]$, where o, p, and f are positive, continuous variables representing the optimist, pessimist, and fundamentalist states respectively.

For simplicity, we also let traders behave like a homogeneous community such that:

$$\begin{aligned} \mathit{Optimist} &= [1, 0, 0] \\ \mathit{Pessimist} &= [0, 1, 0] \\ \mathit{Fundamentalist} &= [0, 0, 1] \end{aligned}$$

Communities update their state in two distinct phases:

[5] For simplicity, and unless stated otherwise, when communities and traders are both discussed as interacting with or upon the same element of the model, traders can be assumed to behave like a homogeneous community; i.e. *Trader*(Optimist) = *Community*{o: 1, p: 0, f: 0}.

[6] Although in Chapter 7 it becomes apparent that this change results in a clear improvement regarding the explanatory value of the model.

1. In the **backward pass**, the state of a community is determined by the average of its children's states. So, for each child d:
$$C_{[o,\ p,\ f]} = \frac{1}{c}\sum d_{[o,\ p,\ f]} \tag{5.9}$$

2. Then, in the **forward pass**, the state of each community gets updated to account for the state of their respective parent p, such that:
$$C'_{[o,\ p,\ f]} = \frac{1}{2}C_{[o,\ p,\ f]} + \frac{1}{2}p_{[o,\ p,\ f]} \tag{5.10}$$

5.3.4 Overall Market Dynamics

At every time-step, the backward pass is followed by the forward pass, which is then followed by traders switching their state according to transition probabilities defined in Eqs. 5.5 and 5.6. Finally, the market price is updated following the paradigm set out in Section 3.2.3, with the caveat that, while communities exist to provide a medium for agents to communicate amongst one another, traders are responsible for buying or selling the asset and only traders affect the asset's market price.

Chapter 6

Empirical Analysis

6.1 Introduction

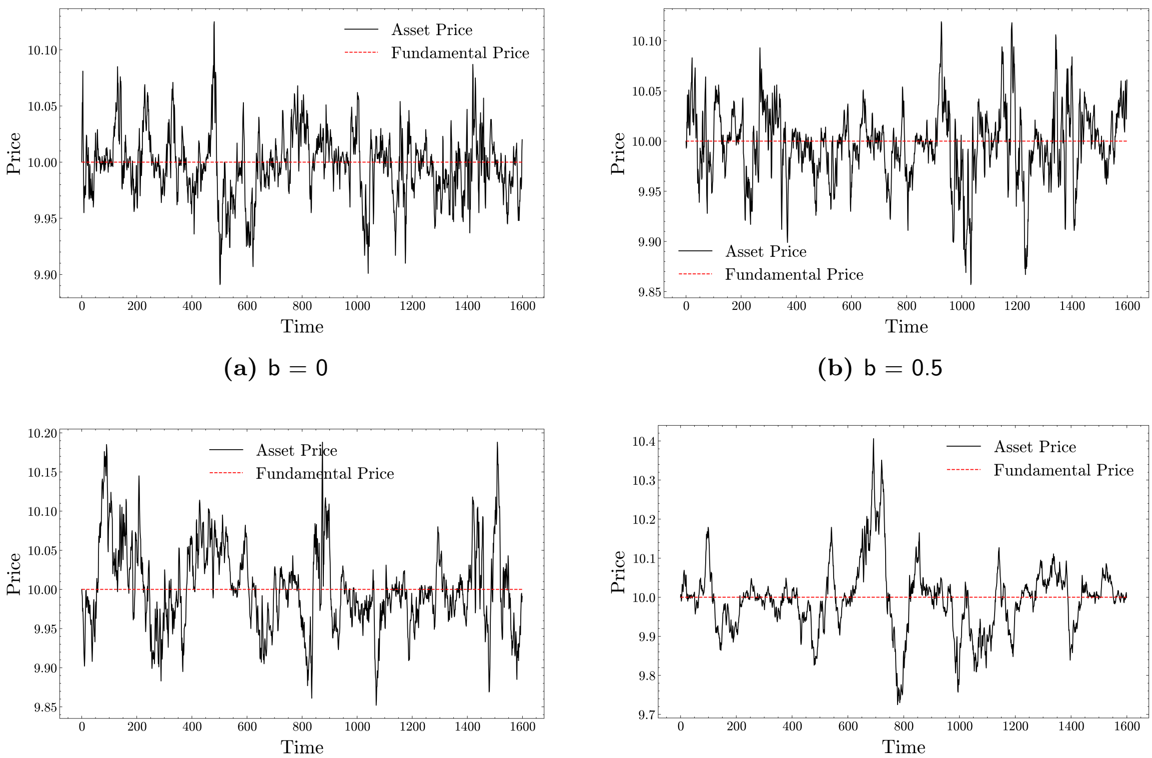
Having defined the Hierarchical Model, we proceed to empirically analyze its behavior. Broadly, the following chapter can be divided into three key objectives:

1. To explore the best parameters for the Hierarchical Model, thus facilitating subsequent analyses and/or implementations.

2. To provide comprehensive evidence that this model behaves in a reasonable and realistic manner.

3. To explore the model's overall behavior, and illustrate its most notable features.

6.2 Hierarchy Strength



(a) b = 0

(b) b = 0.5

(c) b = 1

(d) b = 2

Figure 6.1 Sample Runs of the Hierarchical Model for Varying Hierarchy Strength. Showcasing that hierarchy strength is associated with higher volatility and pronounced deviations from the fundamental value.

Fig. 6.1 shows some sample runs[1] under increasing values of b. As we can see, price movements increase in magnitude as the hierarchy's effect becomes more pronounced, with the market price staying close to the fundamental value when b is small but significantly deviating from it when b is large.

More rigorous analysis reveals that: for all parameter sets, an increase in b is associated with the asset's market price displaying a significant increase in volatility (see Table 6.1). Furthermore, all parameter sets display high levels of explosive behavior when $b = 2$, with parameter sets III and IV also displaying explosive behavior when $b = 1$ (see Table 6.2).

These findings are consistent with the results showcased in Alfarano et al. (2011), which finds that a hierarchical structure results in increased volatility. Furthermore, note that our overarching goal is to model social media opinion dynamics within financial markets, and most research into this topic suggests that a strong social-media influence results in speculative markets with high volatilities (see Section 2.5.2). We contend that the fact that our model is able to replicate these real-world observations successfully is a positive sign of its validity.

Table 6.1 Average Volatility for Different Hierarchy Strengths. Showing higher volatility as b increases.

b	Set I	Set II	Set III	Set IV
0.0	5.7	4.7	1.6	3.9
0.1	6.5	5.9	1.7	3.6
0.25	7.0	5.5	2.2	4.0
0.5	8.4	5.6	2.1	4.5
1.0	9.4	6.7	2.8	5.7
2.0	15.4	9.1	4.4	8.9

*Each cell corresponds to 50 simulations with $8 * 10^4$ time-steps.
*Volatility values are scaled by a factor of 10^{-2}

Table 6.2 Percentage of Explosive Runs for Different Hierarchy Strengths. Showing a higher frequency of explosive instances as b increases.

b	Set I	Set II	Set III	Set IV
0.0	0%	0%	4%	0%
0.1	4%	2%	4%	0%
0.25	4%	6%	8%	6%
0.5	4%	4%	12%	6%
1.0	8%	2%	14%	15%
2.0	50%	44%	34%	22%

*Each cell corresponds to 50 simulations with $8 * 10^4$ time-steps.

6.3 Hierarchy Shape

Running a brief analysis of the model under different values of L and c results in massive variations in the market's behavior; however, once the number of traders in the market (N_t) is accounted for and kept as close to 500 as possible, the effect of changing these values is greatly diminished.

There are only a few combinations of L and c that lead to N_t being reasonably close to 500 (these values are illustrated in Table B.1 in the appendix). Out of these, barring some minor exceptions, all combinations display similar behavior[2].

[1] These figures show markets under a constant fundamental value to ease with the visualization of results, samples with a non-constant fundamental value display the same behavior (see Fig. B.1 in the appendix for examples).
[2] The average volatility for different hierarchy shapes is showcased in Table B.2.

Table 6.3 Hierarchical Model stylized Facts (b = $3\alpha_1$)

	Set I	Set II	Set III	Set IV
Tail Decay (2.5%)	9.84	5.16	5.22	5.16
Tail Decay (5%)	5.6	3.81	4.21	3.82
Tail Decay (10%)	4.41	2.68	3.24	2.83
Kurtosis ($T = 1$)	0.56	1.64	1.15	1.35
Kurtosis ($T = 10$)	1.46	1.22	1.12	1.22
Kurtosis ($T = 50$)	0.73	0.96	0.017	0.16
Log AC ($\tau = 10$)	0.089	0.13	0.52	0.45
Squared AC ($\tau = 10$)	0.077	0.18	0.49	0.40
Abs AC Decay	0.99	0.57	0.68	0.67

*Each cell corresponds to 50 simulations with $4 * 10^4$ time-steps.
*AC stands for Autocorrelation
*Kurtosis values represent **excess** kurtosis

6.4 Standard Parameter Values

For further analyses, it is crucial to define the standard values to be used when modeling the hierarchy, since running each analysis for every possible combination of parameters would take an exorbitantly long time. The sections below illustrate what values these constants will take throughout the rest of this dissertation, as well as provide some discussion regarding why each specific value was chosen.

6.4.1 Hierarchy Shape

As discussed in Section 6.3, as long as the number of traders remains close to 500, the specific topology that the hierarchy takes does not make much difference regarding the market's behavior. Consequently, (unless stated otherwise) the standard will be for each community to have 5 children, and each hierarchy to have 5 levels. This strikes a good balance between the depth and breadth of the hierarchy, which should allow for more generalized analyses going forward.

6.4.2 Hierarchy Strength

Since b acts as a 'replacement' for α_1, each parameter set's b is defined in terms of its respective value for α_1 (e.x. we define $b = 3\alpha_1$ rather than $b = 1.8$).

The implied goal is for each parameter set's hierarchy to be as strong as possible (so the behavior resulting from the hierarchy's influence is as salient as possible). On the other hand, we are also interested in maintaining realism, so we constrain b to values that result in markets that behave in a way consistent with the stylized facts discussed in Section 3.1.

Results arising from a strong hierarchy deviate from the Lux-Marchesi model quite significantly, all the while still looking 'realistic'; notwithstanding, as we can observe in Table 6.3, the stylized facts for parameter sets I and II deviate from standard behavior at some key junctures, most notably in their failure to return to Gaussianity when $T = 50$[3].

Setting $b = 2\alpha_1$ mitigates the deviations discussed above, and produces stylized facts consistent with our expected values (see Table B.3 in the appendix). Hence, for parameter sets III and IV we shall use $b = 3\alpha_1$ as the default value for hierarchy strength; while parameter sets I and II will take $b = 2\alpha_1$ as their standard value.

Defining the hierarchy strengths in this way also carries the advantage that, while parameter sets III and IV will frequently display explosive behavior, parameter sets I and II will rarely do so (see Table 6.2). This adds significance to each individual parameter set, and makes it more likely to discover interesting behavior in experiments going forward.

[3]Parameter Set I's behavior deviates significantly from other facts too, but this is also present in the standard Lux-Marchesi analysis (see Table 4.4)

6.5 Stylized Facts

Table 6.4 showcases the stylized facts resulting from runs under the standard b values proposed in the section above. As we can see, except for a few minor exceptions, the stylized facts fall within their expected ranges.

One thing of note, however, is the elevated autocorrelation that this model displays when compared to the original Lux-Marchesi model. This indicates a higher degree of volatility clustering, which in turn indicates a lower degree of market efficiency (Hameed et al., 2009). This is a positive sign, since strong social media effects, such as those seen in cryptocurrency markets, are commonly believed to reduce the efficiency of the market upon which they act (Al-Yahyaee et al., 2018; Bundi and Wildi, 2019).

Table 6.4 Hierarchical Model stylized Facts (Standard b Values)

	Set I	**Set II**	**Set III**	**Set IV**
Tail Decay (2.5%)	7.57	4.72	5.22	5.16
Tail Decay (5%)	5.56	3.48	4.21	3.82
Tail Decay (10%)	3.41	2.72	3.24	2.83
Kurtosis ($T = 1$)	0.54	1.79	1.15	1.35
Kurtosis ($T = 10$)	0.99	1.72	1.12	1.22
Kurtosis ($T = 50$)	0.47	0.37	0.017	0.16
Log AC ($\tau = 10$)	0.055	0.29	0.52	0.45
Squared AC ($\tau = 10$)	0.047	0.4	0.49	0.40
Abs AC Decay	1.21	0.77	0.68	0.67

*Each cell corresponds to 50 simulations with $4 * 10^4$ time-steps.
*AC stands for Autocorrelation
*Kurtosis values represent **excess** kurtosis

6.6 General Analysis and Discussion

6.6.1 Parameter Set I

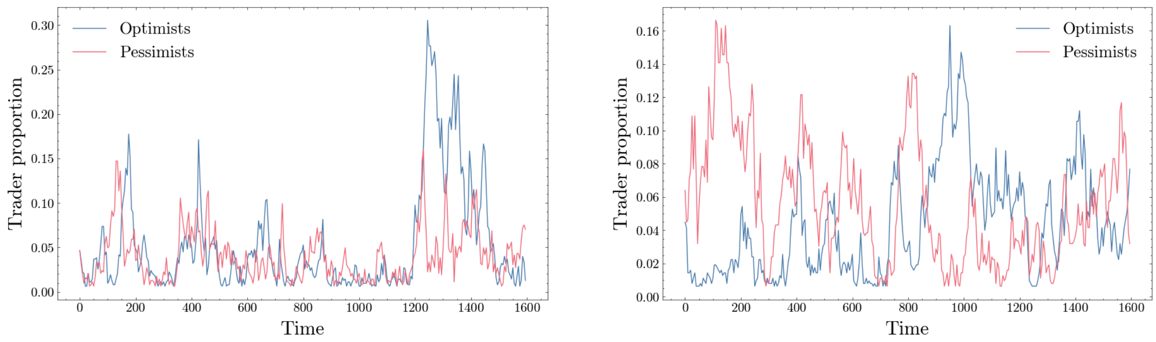
Preliminary analysis quickly reveals that (virtually) none of the discussions or conclusions presented below apply to Parameter Set I. In fact, Parameter Set I presents almost the same traits as the original Lux-Marchesi model. This, coupled with the fact that this parameter set does not follow the conventional stylized facts that all other standard sets do (neither in our model nor the Lux-Marchesi model), makes the analysis of Parameter Set I redundant and simply distracts from the main point(s) being made. Consequently, no further analyses or conclusions will be drawn from this parameter set, and we can proceed under the assumption that sets II, III, and IV are the only parameter sets under consideration.

6.6.2 Heterogenous Opinions

For all parameter sets, our model displays heavily heterogeneous opinions within the chartist population. That is to say, the number of optimists is significantly larger than the number of pessimists (and vice versa) at various points throughout the simulations.

These herding mechanics are significantly different from those of the basic Lux-Marchesi model. Rather than chartists displaying herding tendencies based mostly on the number of chartists in the market, the herding influence of optimists and pessimists individually is much stronger than the influence of chartists as a whole; while chartists still display herding behavior[4], they tend to agglomerate based on their role rather than their type.

[4]Herding behavior can be shown to exist since the proportion of optimists/pessimists at any given time-step is an excellent predictor of that role's proportion in the subsequent time-step.



(a) Opinion Distribution (Parameter Set II) (b) Opinion Distribution (Parameter Set III)

Figure 6.2 Opinion Distributions within the Chartist Population in the Hierarchical Model. Showcasing instances with heavily pronounced differences (left at T=1200-1400, right at T=0-200; 750-850; 950-1025) between the number of optimists and the number of pessimists (i.e. heterogeneity within the chartist population), a feature not present in the Lux-Marchesi model.

6.6.3 Bubbles and Explosive Behavior

Figure 6.3 PWY Procedure Applied Over the Hierarchical Model. Showing examples of explosive periods (shaded yellow) being prolonged in time (left), and occurring close to one another (right).

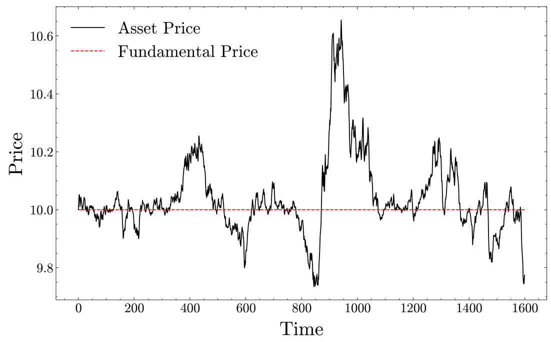
Conducting the PWY procedure[5] (see Section 4.2) for markets simulated under our standard values of b reveals frequent instances of explosive behavior. These instances are more common in Parameter Sets III and IV than in Parameter Set II, although they still occur in the latter Parameter Set with more frequency than in the basic Lux-Marchesi model.

Furthermore, the observed instances of explosive behavior are usually either: sustained in time, or occur in bursts that are themselves close together in time. This suggests that these bubbles are not occurring due to random chance, but rather due to a strong endogenous influence acting on the market price. Presumably, this influence is the chartist's behavior, which would indicate that the herding effect explored in Section 6.6.2 is having a significant effect on the asset price, an effect which can be detected

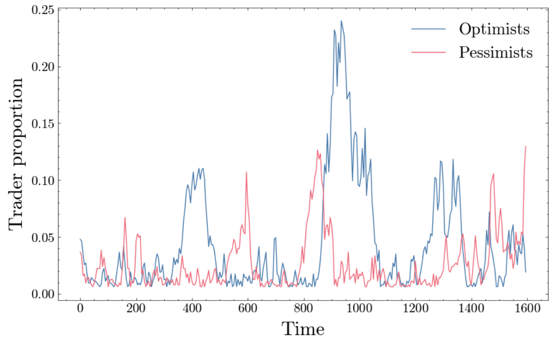
[5]Unless stated otherwise, assume the PWY method is carried out at the 90% significance level.

via statistical tests without utilizing specific knowledge regarding the distribution of each trader type within the market.

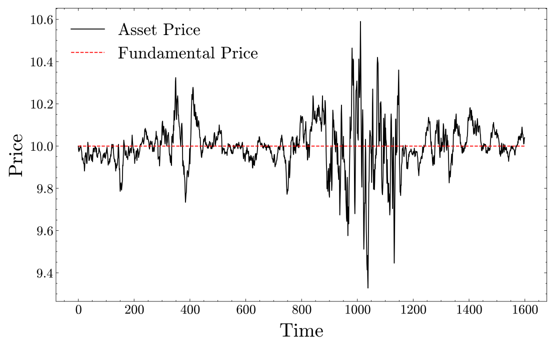
6.6.4 Comparison to the Lux-Marchesi Model



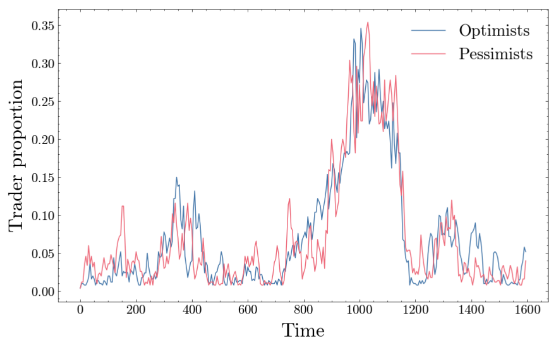
(a) Hierarchical Model Market Price



(b) Hierarchical Model Opinion Distributions



(c) Lux-Marchesi Model Market Price



(d) Lux-Marchesi Model Opinion Distributions

Figure 6.4 Lux-Marchesi Model vs. Hierarchical Model. Notice that in the Lux-Marchesi model, an increase in the number of chartists (bottom-right, T=800-1200) is associated with the asset displaying higher volatility (bottom-left, T=800-1200); whereas, in the Hierarchical Model, this increase (top-right, T=850-1000) is associated with the asset deviating from the fundamental value (top-left, T=850-1000). Also, note the heterogeneity of opinion present within the Hierarchical Model's chartist population.

At an objective level[6], there are two main differences between the behavior of the Hierarchical Model and the standard Lux-Marchesi model:

1. In the original Lux-Marchesi model, increases in the proportion of chartists are associated with increased volatility; however, in the Hierarchical Model increases in the proportion of chartists are related to large and prolonged deviations from the fundamental price.

2. Our model displays more heterogeneity of opinion between chartists; while, in the Lux-Marchesi model, the number of optimists is very strongly correlated with the number of pessimists at any given time step.

[6] As opposed to a purely subjective opinion, where I would argue that the behavior of the Hierarchical Model approaches real-world patterns much more closely than the Lux-Marchesi model does. This rings especially true for cryptocurrencies; for which, at times, the Hierarchical model produces behavior with an extremely strong resemblance to the trends that these assets could be reasonably expected to display in real life.

Chapter 7

Additional Models

7.1 Introduction

This chapter represents arguably the most important defense of the utility and significance of the model proposed in this dissertation. The aim is to evidence the observation that led to the creation of the model in the first place: that a hierarchical network represents a natural and intuitive way to model several phenomena observed in financial markets, especially those where social media has a strong influence on the actions of the traders within them (such as cryptocurrencies).

In order to do this, we present simple modifications to the Hierarchical Model that can be used to replicate a variety of the aforementioned phenomena. The goal of the sections below is not to provide an in-depth analysis of each phenomenon's characteristics or prove that the Hierarchical model is at the forefront of modern literature when it comes to modeling them. Rather, the aim is to showcase the versatility of the model, since under very minor and rudimentary changes to its overall structure and/or dynamics, it can satisfactorily represent a variety of interesting effects simultaneously.

7.2 Network Efficiency

7.2.1 Definition

We can model efficient networks by modifying the community forward pass (see Eq. 5.10); where, instead of a perfectly symmetrical division between the community's opinion and their parent's opinion, the parent's opinion gets amplified by a constant ϕ, such that:

$$C'_{[o,\ p,\ f]} = \frac{1}{2} C_{[o,\ p,\ f]} + \phi \cdot p_{[o,\ p,\ f]} \tag{7.1}$$

where higher values of ϕ represent less network decay, which is equivalent to higher network efficiency.

Under this change, the Hierarchical Model can be replicated by setting $\phi = \frac{1}{2}$. Furthermore, the model approaches the basic Lux-Marchesi model as ϕ tends to infinity.

7.2.2 Analysis

Fig. 7.1 shows the volatility and percentage of runs displaying explosive behavior for different values of ϕ[1]. As we can see, volatility and explosive behavior are heavily correlated with ϕ. In other words, as the network becomes more efficient (i.e. the network decay becomes smaller), the market becomes more volatile and prone to bubble formation.

7.2.3 Discussion

As illustrated in Section 7.2.2, volatility and explosive behavior decrease as decay increases (i.e. ϕ becomes smaller). These results are not what I initially expected. My initial hypothesis was that less effective

[1]The displayed values are those of Parameter Set IV, similar figures for Parameter Sets II and III can be found in the appendix (Figs. B.2 and B.3).

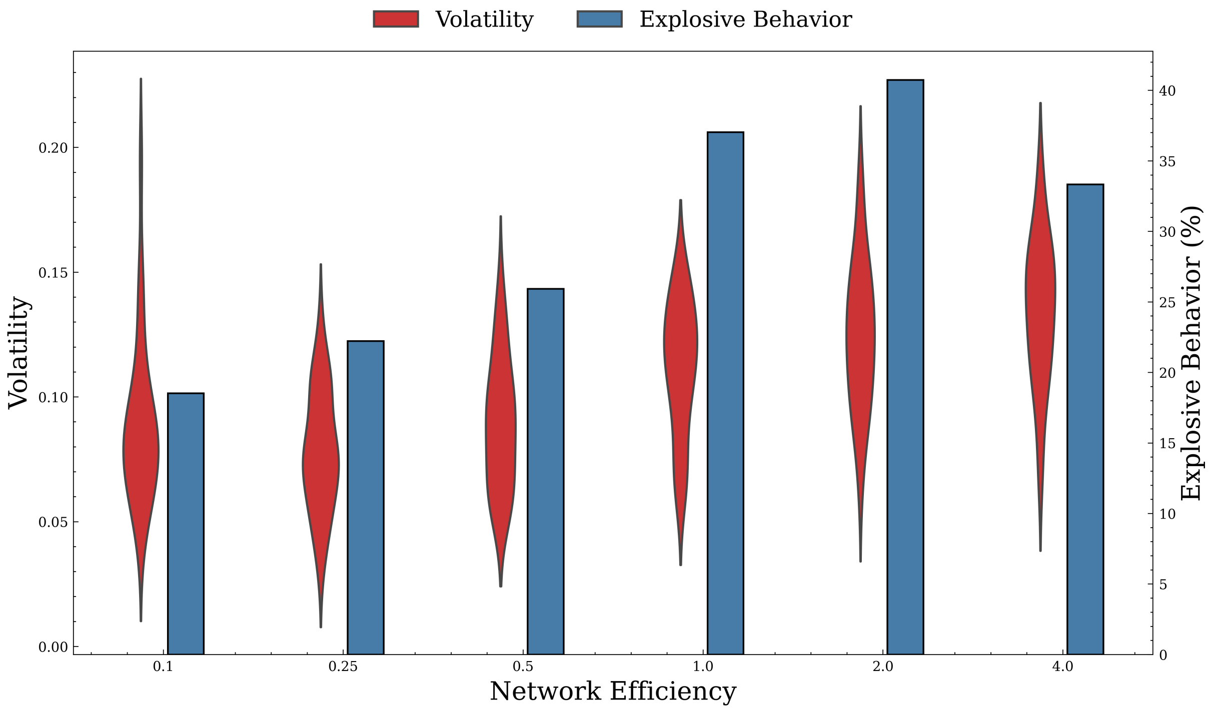


Figure 7.1 Volatility and Explosive Behavior for Different Network Efficiencies (Parameter Set IV). Showing that volatility and explosive behavior increase as ϕ increases.

inter-community communication would lead to more heterogenous beliefs, which would, in turn, lead to higher variance in the pricing of the asset.

In hindsight, it is clear why this hypothesis was incorrect. Explosive behavior and volatility come as a result of herding behavior, which requires a majority of traders to have the same (or at the very least similar) beliefs. Hence, homogenous beliefs within the market should be a predictor of explosive behavior, and consequently, communities not being able to communicate with each other makes it harder for traders to reach a consensus regarding overall opinion, which leads to less herding behavior.

These results support the idea that social media increases the volatility of an asset. Given that social media networks are an extremely efficient way to communicate ideas between agents, their 'decay factor' is presumably much lower than for any other alternative form of communication. Thus, we would expect that traders communicating via these networks would display herding behavior since they would have an easier time reaching an overall 'opinion consensus' due to the efficiency with which any given trader can share their opinion with every other trader in the network.

This also explains why speculative bubbles have become more common in the modern age. In the past, sharing opinions within large communities was an extremely inefficient process, with ideas being spread almost exclusively through the remarkably fallible process of word of mouth. With the invention of the printing press and eventually the internet, it became exponentially easier for opinions and ideas to spread, thereby facilitating the spontaneous emergence of homogeneous beliefs within large populations.

7.3 Pump & Dump Schemes

7.3.1 Definition

We model pump & dump schemes through the process of a community becoming 'corrupted': such that between time-steps T_0 and T_1, a corrupted community's (forward pass) signal conveys a deceptively high proportion of optimists to its children.

Hence, if community p is (subject to becoming) corrupted and the current time step t is between T_0 and T_1, the forward pass equation (see Eq. 5.10) becomes:

$$C'_{[o,\ p,\ f]} = \frac{1}{2}C_{[o,\ p,\ f]} + \frac{1}{2}p_{[S*(o+p+f),\ p,\ f]} \tag{7.2}$$

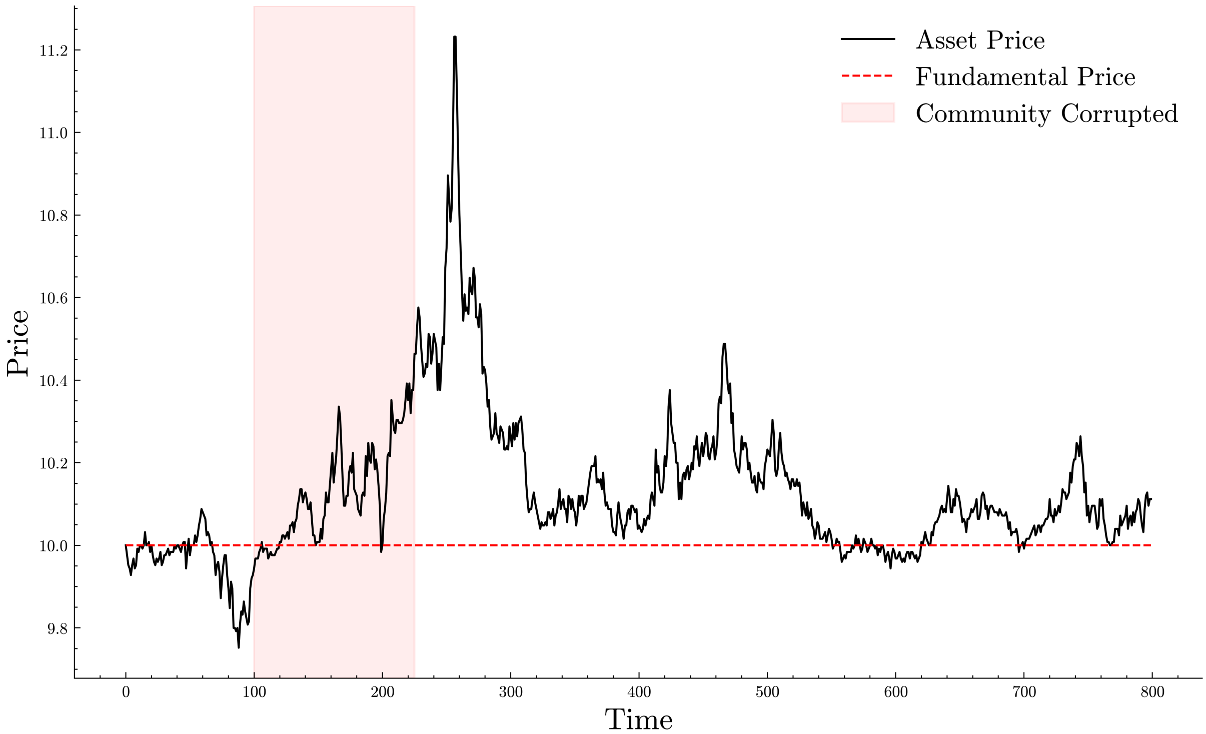


Figure 7.2 Pump & Dump Model. Where a community becoming corrupted (shaded red) results in a very pronounced peak in the asset price. Notice that the peak (T=275) occurs after the community ceases to be corrupted (T=225).

where S represents the **Signal Strength** and $S * (o + p + f)$ is the magnitude of the signal emitted by the corrupted community.

7.3.2 Analysis

We define a pump & dump scheme as being 'successful' if the maximum price reached during or after the community is corrupted is larger than the maximum price reached by 95% of uncorrupted simulations with equal parameters[2].

For our Standard Parameter Sets, the success rate of pump & dump schemes (with reasonable signal strengths[3]) is very low, insignificantly larger than it would be due to random chance.

Notwithstanding, some arrangements of parameters lead to more frequently successful pump & dump schemes. Broadly, markets under which pump & dump schemes have a significant success rate can be divided into 4 classes:

- **Speculative Markets:** Markets where the importance that traders place on the actual price trend of the asset (α_2) or the profit made by other traders (α_3) is much lower than the importance they place on one another's type (b).
- **Volatile Markets:** Markets where the price changes quickly. This change's effect is further exacerbated if the price is heavily influenced by the actions of speculative traders (i.e. t_c is large), or only mildly influenced by fundamentalist traders (i.e. γ is small).
- **Chartist-Heavy Markets:** Markets where chartist traders are exceedingly unlikely to become fundamentalists (i.e. s is small).
- **Nascent Markets:** Pump & dump schemes are generally more successful when T_0 is small, since the market is young and presumably still unstable.

[2]Each pump & dump run was compared to 50 uncorrupted simulations.
[3]Reasonable meaning $S \leq 50$.

For successful schemes, there is not necessarily a 1-to-1 correlation between T_0 and the price pumping; the largest peak in price frequently occurs a significant number of time-steps after T_1 and/or T_0.

Furthermore, the bubble crashes regardless of whether the corrupted signal is still ongoing. This essentially means that no corrupted signal is required for the bubble to pop; i.e. the 'dump phase' of the pump & dump scheme will occur even if $T_1 = \infty$.

7.3.3 Discussion

The fact that pump & dump schemes are not successful under the Standard Parameter Sets should be expected of any reasonable model, since these schemes are relatively infrequent, especially when it comes to established markets with a carefully monitored fundamental value.

Furthermore, the markets for which these schemes are successful (speculative, volatile, nascent, and chartist-heavy) reflect the real-world characteristics of markets that are (generally understood to be) susceptible to pump & dump schemes.

The two ideas discussed above, coupled with the fact that most of the samples analyzed are quite similar to real-world instances of pump & dump schemes, provide strong tentative evidence that this simple model is relatively successful at modeling pump & dump schemes.

7.4 Echo Chambers

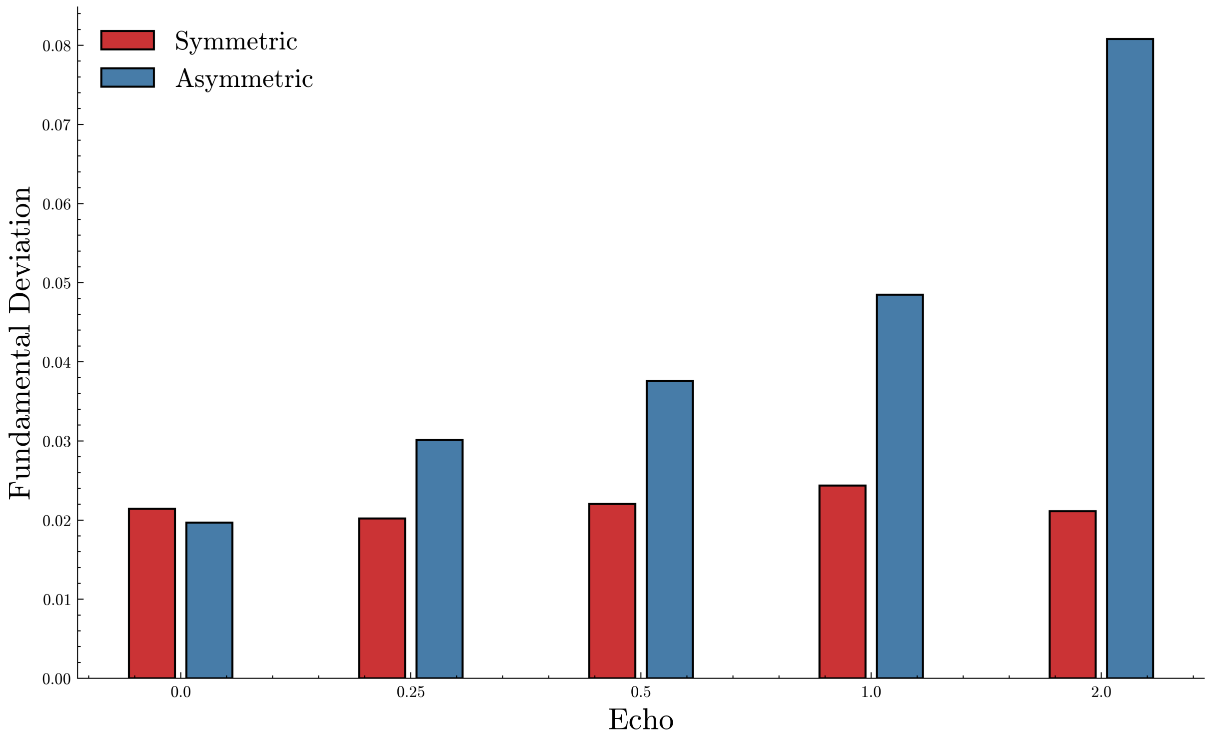


Figure 7.3 Symmetric vs Asymmetric Echo Chamber Model. Note that the asymmetric model exhibits a strong correlation between hierarchy strength and fundamental deviation, a relation not present for its symmetric counterpart.

7.4.1 Definition

As seen in Section 2.5.4, participants in online communities tend to replicate popular opinions through a phenomenon commonly dubbed the "Echo Chamber Effect". We can simulate this effect via a variable $\mathcal{E}$, such that the influence of a trader's opinion gets multiplied by $\mathcal{E}$ if the trader's opinion conforms to the majority opinion within his parent community.

Additionally, we define two distinct models for the echo chamber effect:

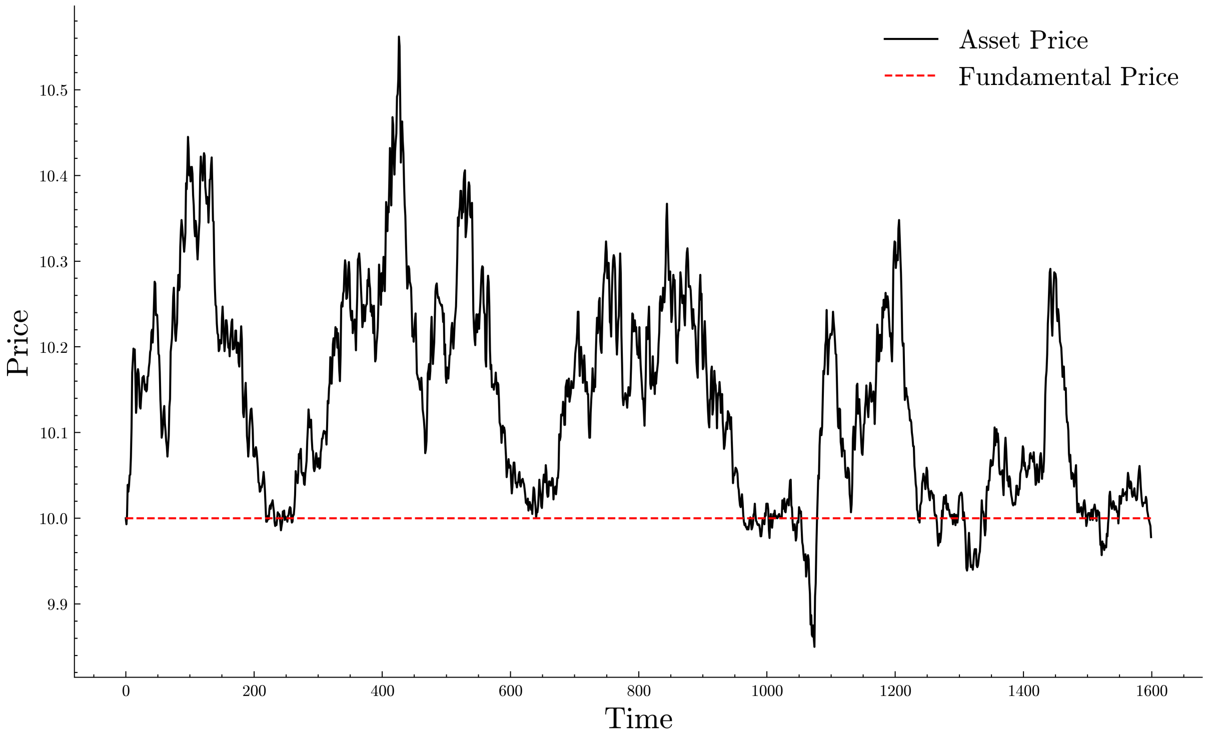


Figure 7.4 Sample Run of the Asymmetric Echo Chamber Model. Showcasing an asset price with very high volatility that consistently remains above the fundamental value.

- **Asymmetric Model:** Where only optimists are affected by $\mathcal{E}$. So, in the first step of the backward pass (see Eq. 5.9), optimists behave like a homogeneous community such that:

$$Optimist = \begin{cases} [\mathcal{E}, 0, 0] & \text{if optimists} > \text{pessimists} \\ [1, 0, 0] & \text{otherwise} \end{cases}$$

- **Symmetric Model:** Where all chartists are affected by $\mathcal{E}$. In the first step of the backward pass, optimists behave in the manner outlined above, and pessimists behave like a homogeneous community such that:

$$Pessimist = \begin{cases} [0, \mathcal{E}, 0] & \text{if pessimists} > \text{optimists} \\ [0, 1, 0] & \text{otherwise} \end{cases}$$

7.4.2 Analysis

For small (but non-zero) values of b, increasing $\mathcal{E}$ in both the symmetric and asymmetric echo chamber models has a similar effect to that of simply increasing the hierarchy strength.

However, for large values of b, these two models begin to differ significantly. Namely, whilst the symmetric model's effect on the market's behavior becomes negligible, the asymmetric model is associated with large and persistent deviations between the asset's market price and its fundamental value. This observation is showcased in Fig. 7.3.

7.4.3 Discussion

Looking at some sample runs for the asymmetric model, we observe behavior eerily similar to that of cryptocurrency markets: extremely high volatility and large speculative periods that are wholly unexplained by any fluctuations in the asset's corresponding fundamental value. Fig. 7.4 shows one of these sample runs for Parameter Set IV with $\mathcal{E} = 3$.

Suppose one imagines that the fundamental value lies at 0 rather than 10; this model seems to very succinctly represent the idea that cryptocurrencies with no fundamental value can still be massively over-inflated with respect to their market valuation, as well as simultaneously modeling these assets' propensity towards explosive and unstable behavior.

In summary, this is a straightforward model, and almost certainly misses many complexities regarding the dynamics of echo chambers. Notwithstanding, it approximates their apparent behavior decently well[4], and could easily be extended to model some of the more intricate features associated with this phenomenon.

[4]Of particular note is this model's resemblance to the patterns displayed by cryptocurrency markets.

Chapter 8

Conclusion

This paper presents the 'Hierarchical Model,' which relies on modifications to two established financial ABMs proposed in Lux and Marchesi (1999) and Meine and Vvedensky (2023). We prove that this model conforms to realistic standards by comparing it against several stylized facts of financial markets. After this, we explore the most suitable parameters under which to run the model, providing a comprehensive overview for its subsequent implementation and analysis. Finally, we present simple modifications to the model, which can be used to simulate several emergent events/phenomena in financial markets, especially those wherein social media influence plays a significant role.

It is important to keep in mind that the additional models presented in (the second-to-last chapter of) this dissertation are very straightforward, and, for any individual phenomenon, can be outperformed by models specifically tailored to replicate that specific phenomenon's empirical behavior. Notwithstanding, the goal of this dissertation is not to present a flawless model of any individual feature of social media dynamics within financial markets, but rather to showcase how a variety of phenomena can be simultaneously and succinctly modeled via a hierarchical communication structure. With that in mind, we believe the analyses and results presented in this dissertation achieved this goal satisfactorily.

In the form of the simplified models mentioned above, this paper provides a clear baseline for future work, wherein more extensive evidence could be gathered regarding each model's ability to explain their respective phenomenon's behavior, as well as exploring the possibility of adding complexity to them where appropriate. Furthermore, we only present 3 such phenomena, begging the question as to what other market features could be simulated through simple modifications to the basic hierarchical structure presented previously.

As a final note on future work, we would like to refer the reader to Zhang et al. (2017), a study that heavily inspired the hierarchical design present in this dissertation. Notice that their model incorporates the notion of "topics," such that each community varies with respect to the strength with which different topics affect them. Under relatively simple modifications, the Hierarchical Model could also include "topics," which could then be used to model different assets or asset classes[1]. This would enhance the realism of the model, since, in actuality, different (online) communities discuss a wide variety of assets, and the 'attention' paid to any individual asset tends to vary over time.

[1]Whereas, as it stands, the Hierarchical Model is only able to simulate the behavior of a singular asset.

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Appendix A

Meine et al. Model Implementation and Analysis

A.1 Critical Analysis

There are 2 main factors that hinder the implementation and subsequent analysis of the Meine et al. model:

- **Lack of parameter sets:** Meine and Vvedensky (2023) does not reference the parameter sets used to acquire their results. This makes it exceedingly difficult to verify if our implementation is acting in accordance with the model's expected behavior (since it is almost impossible to tell what exactly the model's expected behavior is without a specific set of parameters).
- **No mention of experiment number:** in a similar vein, the paper produces some interesting results, but makes no mention of how many experiments were run before acquiring them. This makes it hard to replicate the aforementioned results, since one cannot tell how often the phenomena showcased in the paper should be produced by the replicating model.

A.2 Implementation

The implementation of the Meine et al. model is quite straightforward. Parting from the base Lux-Marchesi model, the new model can be quickly simulated by adding a new `Community` class which has c children and sets its state according to the state that the majority of its children follow. After that, we implement the forward and backward passes. We finalize the implementation by modifying the chartist transition equations to reflect Eqs. 3.28 and 3.29, such that the state of each community affects the trader's transition probabilities.

There is only one contentious issue, relevant to calculating the number of parent communities for each (bottom-level) trader, which is discussed in the subsection below.

A.2.1 Above?

Meine et al. refer to traders at the bottom of the hierarchy as being affected by "all traders above them"; there are two distinct interpretations as to what this could mean:

1. Trader a is above Trader b if b's level is larger than that of a.
2. Trader a is above Trader b if a is a direct descendant of b (i.e. a can be reached from b via repeated transitions from parent to child).

Although common wisdom would seem to dictate that the latter interpretation is correct, the language in Meine et al.'s paper is quite unclear.

Leaving as little room for discussion regarding the model's implementation will greatly enhance the credibility of our subsequent empirical analysis, especially since its results differ somewhat significantly from those in the original paper (see Appendix A.3). For this purpose, we use a simple process of elimination to discard the former option; and thus show that the latter interpretation must be the correct one.

Consider the following theorem on the distribution of trader types within the hierarchy:

Lemma A.2.1. *The proportion of a trader type (optimist, pessimist, or fundamentalist) at level $l+1$ is approximately equal to the proportion of that trader type at level l (see Appendix B for proof).*

Theorem A.2.2. *The total proportion of a trader type in the hierarchy can be approximated by the proportion of that trader type at level* 0.

From Theorem A.2.2, we can deduce that if all the traders with a level larger than a bottom-level trader affect that trader's state, this would be tantamount to setting each trader's state based on the overall proportion of states at level 0, which would be equivalent to the basic Lux-Marchesi dynamic. Hence, we conclude that this assumption must be incorrect, given that the (implicit) purpose of this model is to present some sort of difference when compared to the Lux-Marchesi model. As such we implement the model based on the second interpretation outlined at the beginning of this (sub)section.

A.3 Empirical Analysis

It is important to keep in mind that exactly replicating Meine et al.'s experiments is not possible due to their parameter set not being publicly available; notwithstanding, replication with any of the Standard Parameter Sets (see Table 4.3) shows some enlightening trends, which are discussed below.

Note that, unless stated otherwise, all figures shown in the subsections below come from models with Standard Parameter Set IV, 6 levels, and 3 children per node. Any observation or result discussed in writing can be assumed to generalize to all Standard Parameter Sets.

A.3.1 (Meta) Stable Opinion Distributions

There are 4 states where the model achieves an equilibrium in which the distribution of opinions remains relatively stable for a large or indefinite number of time steps:

1. **Fundamentalist Homogeneity:** Fundamentalists make up the vast majority of traders. Characterized by the asset price having a low volatility and remaining close to the fundamental value.

2. **Chartist Homogeneity:** Chartists make up the vast majority of traders, and there is a (roughly) even split between the proportion of optimists and pessimists. Characterized by the asset price having a high volatility but remaining close to the fundamental value.

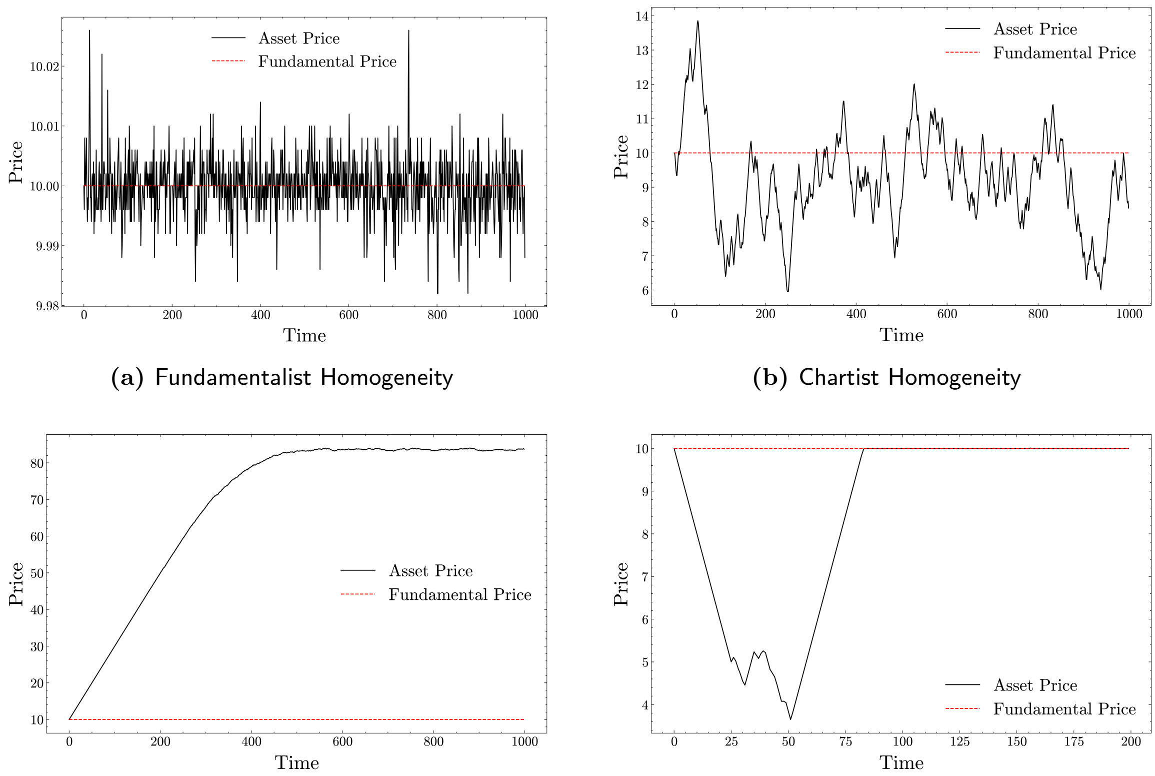
3. **Optimist Dominance:** Similarly to the *Chartist Homogeneity* state, chartists make up the vast majority of traders; however, optimists outnumber pessimists within the chartist population. Characterized by a steeply rising asset price that eventually settles at a price point significantly above the fundamental value.

4. **Pessimist Dominance:** As in the *Optimist Dominance* state, chartists make up the vast majority of traders; however, pessimists outnumber optimists within the chartist population. Characterized by a steeply descending asset price that eventually gets close to 0, at which point the market switches to a more stable state (usually *Fundamentalist Homogeneity*).

As discussed in the following sections, which of these states is reachable is determined by the magnitude of b. Within the reachable states, the specific one that any given run reaches is based on the initial distribution of each trader type.

A.3.2 Small b

When $b < 0.05$, a clear trend emerges; the model quickly converges into either one of the *Chartist Homogeneity* or the *Fundamentalist Homogeneity* states. Further analysis reveals that markets that start with a majority of fundamentalists stabilize at fundamentalist homogeneity, whereas markets that start with a significant majority of chartists stabilize at Chartist Homogeneity. Markets where the initial distribution of traders is similarly split between chartists and fundamentalists are mostly random; they tend to stabilize at *Fundamentalist Homogeneity*; although they often stabilize at *Chartist Homogeneity* too.



(a) Fundamentalist Homogeneity

(b) Chartist Homogeneity

(c) Optimist Dominance

(d) Pessimist Dominance

Figure A.1 Sample Asset Price Evolution for Different Market States

These states are strongly stable, with spontaneous changes from one state to another being extremely infrequent. Additionally, convergence into these states is almost immediate upon starting the simulation, with no intermediate metastable states.

In order to understand this result, it is important to define some further theorems on the distribution of trader types with this hierarchical structure:

Lemma A.3.1. *If fundamentalists constitute the majority opinion at level l, the proportion of fundamentalists at level $l+1$ will be bigger than or equal to the proportion of fundamentalists at level l.*

Theorem A.3.2. *If fundamentalists constitute the majority opinion at level 0, the total proportion of fundamentalists in the hierarchy will be bigger than or equal to the proportion of fundamentalists at level 0.*

Lemma A.3.3. *If the proportion of fundamentalists at level l is smaller than $\frac{1}{c}$, the proportion of fundamentalists at level $l+1$ will be smaller than the proportion of fundamentalists at level l.*

Corollary A.3.3.1. *If the proportion of chartists at level l is larger than $\frac{c-1}{c}$, the proportion of chartists at level $l+1$ will be bigger than or equal to the proportion of chartists at level l.*

Theorem A.3.4. *If the proportion of chartists at level 0 is larger than $\frac{c-1}{c}$, the total proportion of chartists in the hierarchy will be bigger than or equal to the proportion of chartists at level 0.*

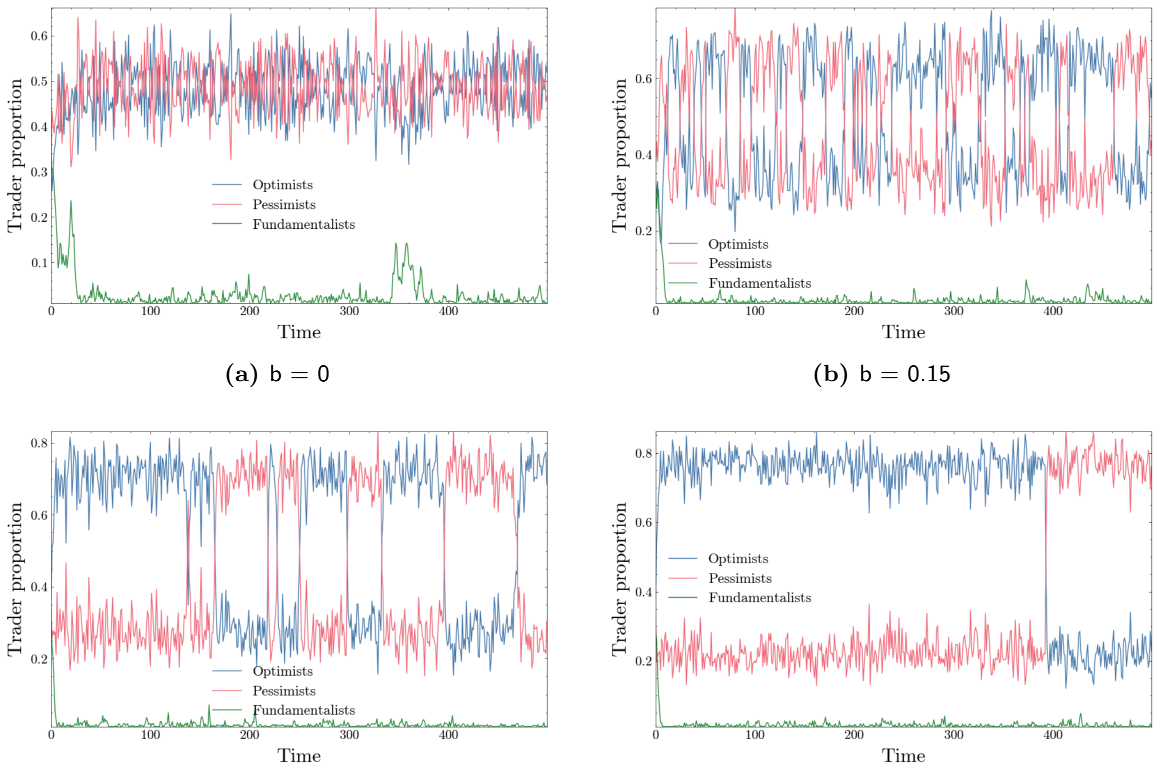
Theorem A.3.2 sheds some light on the previously observed convergence into a *Fundamentalist Homogeneity* state: if the majority of traders at level 0 are fundamentalists, the total proportion of fundamentalists in the hierarchy will be larger than at level 0. Since the transition probability from optimist/pessimist to fundamentalist is a multiple of the proportion of fundamentalists in the hierarchy (see Section 3.2.7), traders at level 0 have a higher chance of becoming fundamentalists than they would in the original Lux-Marchesi model.

Convergence into a *Chartist Homogeneity* state is similarly explained by Theorem A.3.4, with distributions where a majority of the traders at level 0 are chartists tending towards becoming exclusively populated by chartists.

Essentially, this hierarchical model has introduced additional pressure for traders to switch to the majority opinion. This creates a sort of 'feedback loop' where traders in the majority group at level 0 influence traders above to switch to that group, who then influence traders at level 0, who then influence the traders above, etc... For all practical purposes, when $b \approx 0$ this can be thought of as the only additional force acting on the model.

Experimentally, we can see that this new force overpowers the stochastic dynamics inherent to the standard Lux-Marchesi model, which explains why the new simulations quickly stabilize at states where (almost) all traders form part of the same group.

A.3.3 Medium-Sized *b*



(a) b = 0 **(b)** b = 0.15

(c) b = 0.2 **(d)** b = 0.25

Figure A.2 Sample Markets in the Chartist Homogeneity State.

For moderately sized values of b, the *Fundamentalist Homogeneity* state stays relatively consistent regarding its behavior. The only difference is that this state seems to become much more stable than when $b \approx 0$, with the price displaying even lower volatility and the proportion of chartist agents remaining consistently close to the minimum value allowed in the simulation.

The *Chartist Homogeneity* state does undergo significant changes however; in fact, the line between this state and the *Optimist/Pessimist Dominance* states becomes increasingly blurred as b grows larger. In Fig. A.2a we can observe that when $b = 0$, the proportion of optimists and pessimists remains relatively consistent, ranging from $0.4 - 0.6$; however, as b increases this range progressively gets larger, with Fig. A.2b showing a range of $0.3 - 0.7$, and Figs. A.2c and A.2d showing a range of $0.2 - 0.8$. Furthermore, the frequency with which optimists and pessimists switch to a state where they are the majority role gets progressively smaller as b increases, with the majority role switching rapidly and sporadically when b is close to 0, whilst only rarely switching as b grows larger.

In order to understand these observations, consider the original Lux-Marchesi model; where the chance to switch from an optimist to a pessimist, and vice-versa, are multiplied by $\frac{n_c}{N}$. This arrangement results in optimists and pessimists agglomerating based on the number of chartists in the market, rather than the number of optimists or pessimists individually.

In Meine et al.'s model, however, the new hierarchical influence distinguishes between these two trader types, so there is an additional force hindering the stability of the *Chartist Homogeneity* state, with the market becoming more inclined towards becoming either entirely populated by optimists, or entirely populated by pessimists.

A.3.4 Large *b*

For larger values of b, the distinction between the *Chartist Homogeneity* state and the *Optimist/Pessimist Dominance* states is clear; with the former no longer being identifiable when the model is run. Simulations with these high values of b are characterized by a lack of switching from the *Optimist Dominance* state to the *Pessimist Dominance* state and vice-versa. Instead, the *Optimist Dominance* state becomes strongly stable, whilst the *Pessimist Dominance* state shows stability until the asset price gets close to 0, at which point it switches to a *Fundamentalist Homogeneity* state.

Appendix B

Additional Data

Proof of Lemma A.2.1

Proof. Let $f(l) = x$ be the proportion of traders of type k at level l, where $0 \leq x \leq 1$
Let $g(l)$ be the expected proportion of traders of type k at level $l+1$
Let $h(l)$ be the difference between $f(l)$ and $g(l)$

For 3 traders per group:
$g(l) = \mathbb{P}(\text{2k in group}) + \mathbb{P}(\text{3k in group}) = \binom{3}{2}x^2(1-x) + \binom{3}{3}x^3 = 3x^2 - 2x^3$

$h(l) = 3x^2 - 2x^3 - x$
$h(l)$ has maximum point at $x = \frac{1+\sqrt{15}}{6}$

$f(0) = g(0) = 0$
$f(1) = g(1) = 1$
$f(\frac{1+\sqrt{15}}{6}) = 0.81 \approx g(\frac{1+\sqrt{15}}{6}) = 0.91$

Similarly, for 5 traders per group:
$g(l) = \mathbb{P}(\text{3k in group}) + \mathbb{P}(\text{4k in group}) + \mathbb{P}(\text{5k in group}) = \binom{5}{3}x^3(1-x)^2 + \binom{5}{4}x^4(1-x) + \binom{5}{5}x^5 = 6x^5 - 15x^4 + 10x^3$

$h(l) = 6x^5 - 15x^4 + 10x^3 - x$
$h(l)$ has maximum point at $x = 0.76$

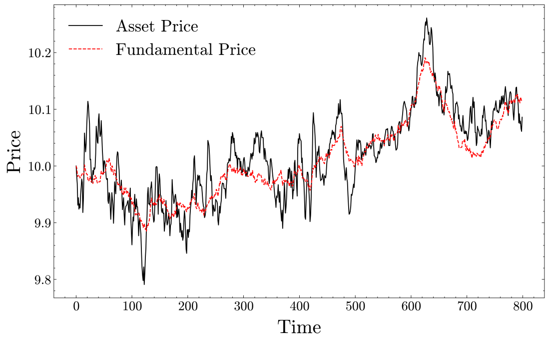
$f(0) = g(0) = 0$
$f(1) = g(1) = 1$
$f(0.76) = 0.76 \approx g(0.76) = 0.91$

For the extreme values of x, $f(l) = g(l)$
For any critical (maximum) points of $f(l) - g(l)$, $f(l) \approx g(l)$
Hence, $f(l) \approx g(l)$

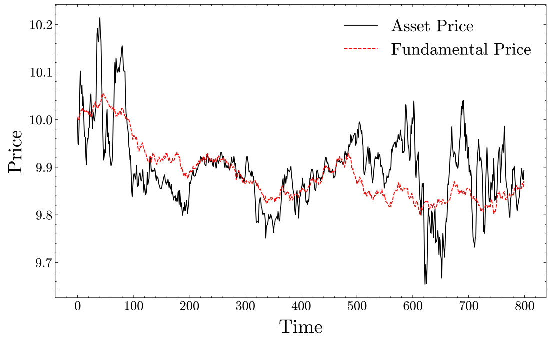
So the expected proportion of traders at level $l+1$ is approximately equal to the proportion of traders at level l. Since this is true for the *traders per group* values suggested by Meine et al., this conclusion can be generalized to their entire model. □

Note: the proof above only holds under the assumption that traders of type k are randomly distributed across level l. This assumption can be made without loss of generality since subsequent calculations refer to the ***expected*** *value of the proportion of traders at level $l+1$, and not their proportion for any individual run (where their distribution might be skewed).*

Figures referenced in Section 6.2



(a) b = 0



(b) b = 1

Figure B.1 Sample Runs of the Hierarchical Model for Varying b (Non-Constant Fundamental Value).

Tables referenced in Section 6.3

L	c	N_t	N_c
2	**500**	500	1
3	**23**	529	24
4	**8**	512	73
5	**5**	625	156
7	**3**	729	364
10	**2**	512	511

Table B.1 Hierarchy Shapes and their Respective Number of Agents and Communities

L	c	Set II	Set III	Set IV
2	**500**	5.6	1.7	4.5
3	**23**	9.5	2.5	9.1
4	**8**	8.4	2.6	8.7
5	**5**	10.4	3.1	9.7
7	**3**	8.1	2.6	7.7
10	**2**	8.8	3.1	8.4

*Each cell corresponds to 25 simulations with $8 * 10^4$ time-steps.
*Volatility values are scaled by a factor of 10^{-2}

Table B.2 Average Volatility for Different Hierarchy Shapes. With results showing that most shapes lead to markets with similar volatilities.

Table referenced in Section 6.4.2

	Set I	Set II	Set III	Set IV
Tail Decay (2.5%)	7.57	4.72	5.31	4.55
Tail Decay (5%)	5.56	3.48	4.41	3.65
Tail Decay (10%)	3.41	2.72	3.26	2.77
Kurtosis ($T = 1$)	0.54	1.79	1.03	1.48
Kurtosis ($T = 10$)	0.99	1.72	1.34	1.28
Kurtosis ($T = 50$)	0.47	0.37	0.18	0.33
Log AC ($\tau = 10$)	0.055	0.29	0.46	0.33
Squared AC ($\tau = 10$)	0.047	0.4	0.49	0.27
Abs AC Decay	1.21	0.77	0.67	0.75

*Each cell corresponds to 50 simulations with $4 * 10^4$ time-steps.
*AC stands for Autocorrelation
*Kurtosis values represent **excess** kurtosis

Table B.3 Hierarchical Model stylized Facts (b $= 2\alpha_1$)

Figures referenced in Section 7.2.2

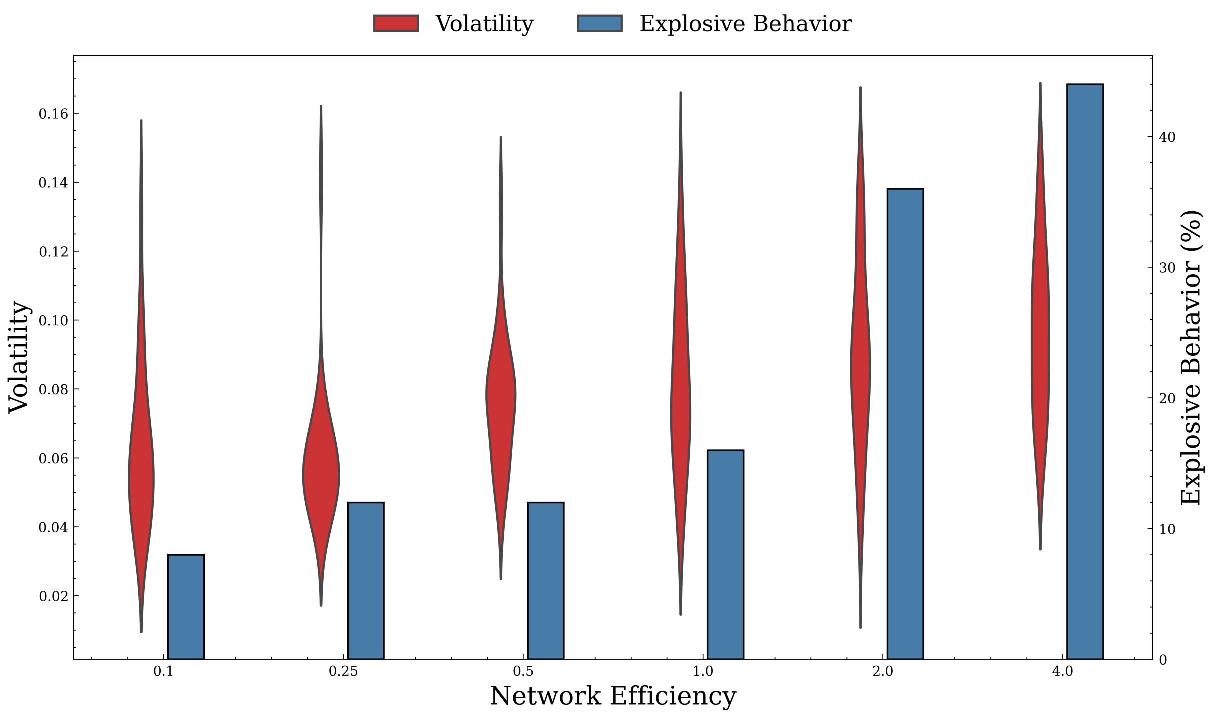


Figure B.2 Volatility and Explosive Behavior for Different Network Efficiencies (Parameter Set II).

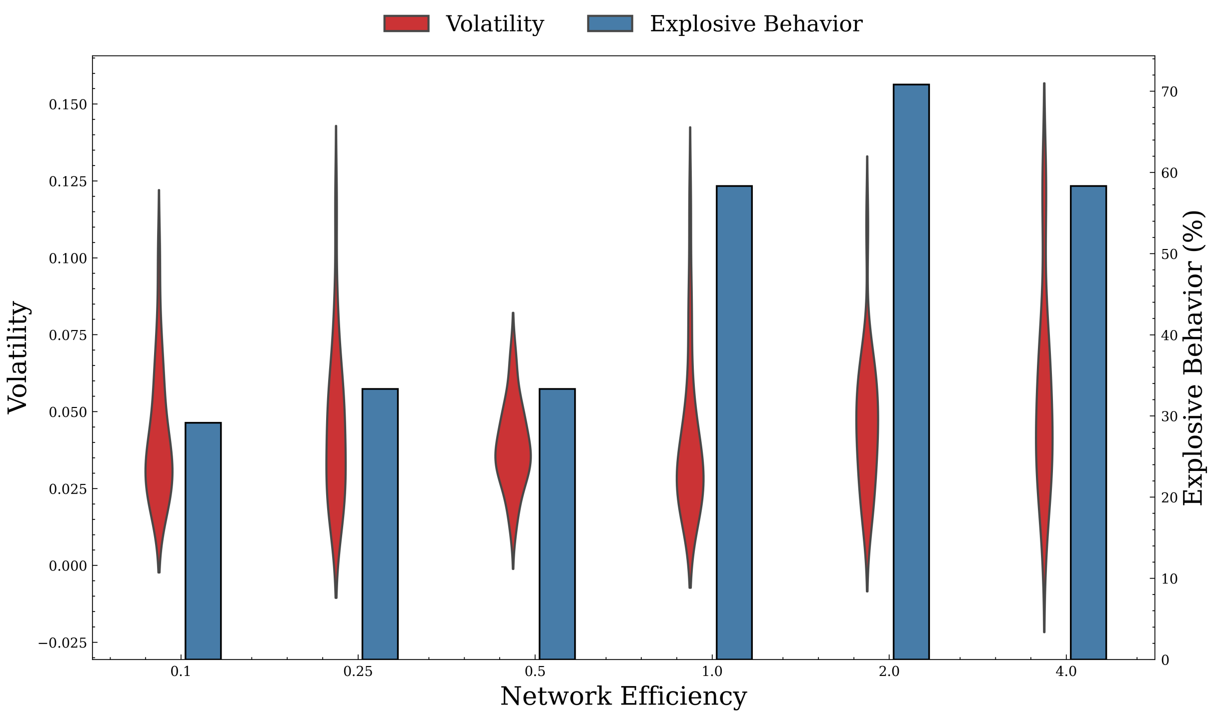


Figure B.3 Volatility and Explosive Behavior for Different Network Efficiencies (Parameter Set III).