

# DECOMPOSITION TECHNIQUES IN POPULATION HEALTH RESEARCH

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## Recap: Kitagawa

$$\Delta CDR = \underbrace{\sum_x \left( \frac{M_x(t_2) + M_x(t_1)}{2} \right) \left( \frac{N_x(t_2)}{N(t_2)} - \frac{N_x(t_1)}{N(t_1)} \right)}_{\text{Changes in x-composition}} + \underbrace{\sum_x \left( \frac{\frac{N_x(t_2)}{N(t_2)} + \frac{N_x(t_1)}{N(t_1)}}{2} \right) (M_x(t_2) - M_x(t_1))}_{\text{Changes in rates}} \quad (1)$$

## Recap: Vaupel & Canudas-Romo (2002)

$$\dot{\hat{v}} = \frac{\partial}{\partial y} \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx}$$

## Recap: Vaupel & Canudas-Romo (2002)

$$\begin{aligned}\dot{\bar{v}} &= \frac{\partial}{\partial y} \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \\ &= \bar{\dot{v}} + \frac{\int_0^\infty v(x)\dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx} \\ &\quad - \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \frac{\int_0^\infty \dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx}\end{aligned}$$

## Recap: Vaupel & Canudas-Romo (2002)

$$\begin{aligned}\dot{\bar{v}} &= \frac{\partial}{\partial y} \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \\ &= \bar{\dot{v}} + \frac{\int_0^\infty v(x)\dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx} \\ &\quad - \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \frac{\int_0^\infty \dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx} \\ &= \bar{\dot{v}} + E(v\dot{w}) - E(v)E(\dot{w})\end{aligned}$$

## Recap: Vaupel & Canudas-Romo (2002)

$$\begin{aligned}\dot{\bar{v}} &= \frac{\partial}{\partial y} \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \\&= \bar{\dot{v}} + \frac{\int_0^\infty v(x)\dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx} \\&\quad - \frac{\int_0^\infty v(x)w(x)dx}{\int_0^\infty w(x)dx} \frac{\int_0^\infty \dot{w}(x)w(x)dx}{\int_0^\infty w(x)dx} \\&= \bar{\dot{v}} + E(v\dot{w}) - E(v)E(\dot{w}) \\&= \bar{\dot{v}} + Cov(v, \dot{w})\end{aligned}$$

# Recap: Vaupel & Canudas-Romo (2002)

$$\dot{\bar{v}} = \underbrace{\bar{\dot{v}}}_{\text{Direct component}} + \underbrace{\text{Cov}(v, \dot{w})}_{\text{Structural or compositional component}}$$

## Recap: Vaupel & Canudas-Romo (2003)

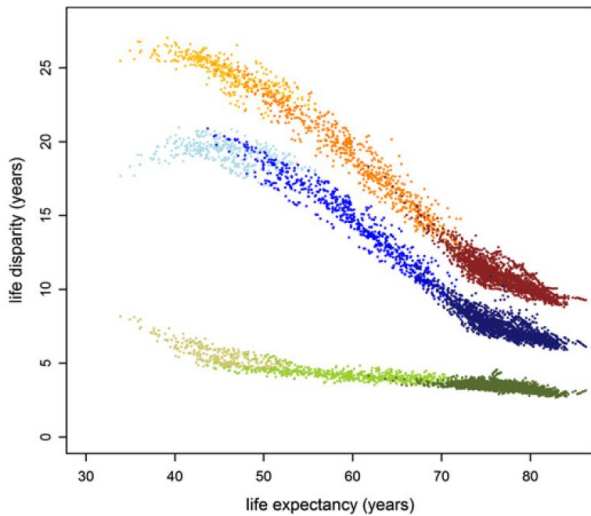
$$\dot{e}_o(t) = \int_0^{\infty} \rho(x) e(x) f(x) dx \quad (2)$$

can be written as:

$$\dot{e}_o(t) = \bar{\rho}(t) e^{\dagger}(t) + Cov(\rho, e_x) \quad (3)$$

where  $e^{\dagger} = \int_0^{\infty} e(x) f(x) da$  is the average life lost at time of death.





Source: Vaupel et al 2011

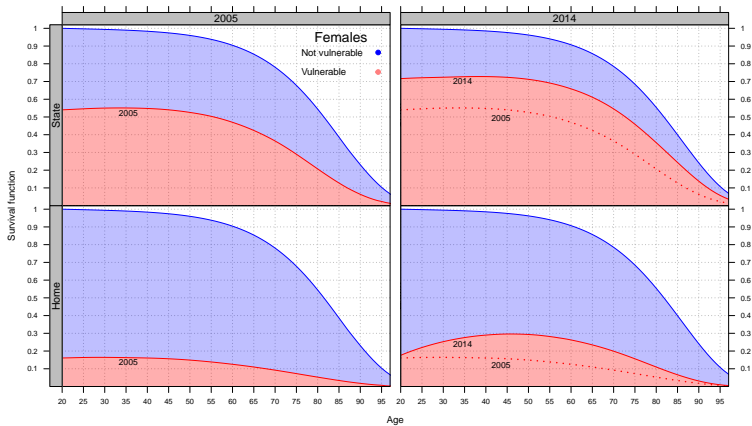
## Recap: Sullivan method

Life expectancy at age  $x$  is defined as

$$e(x) = \frac{\int_x^{\infty} \ell(a) da}{\ell(x)}$$

Then we can define disability-free life expectancy as

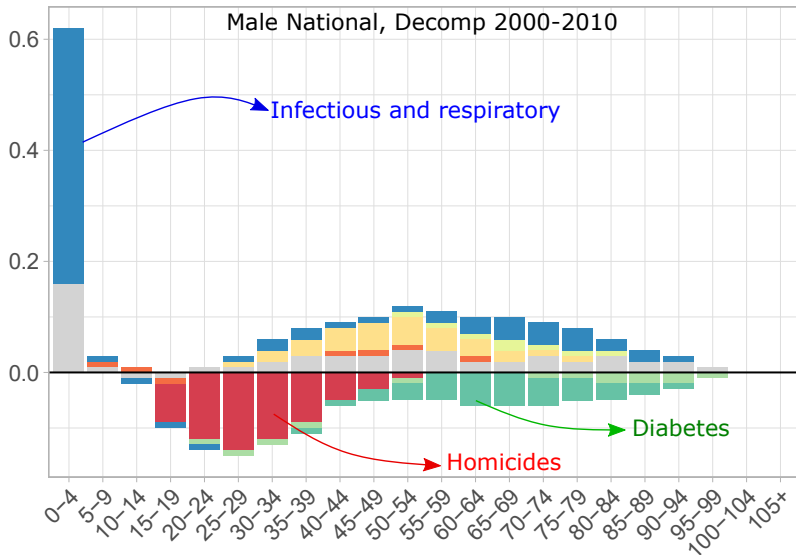
$$e^{DF}(x) = \frac{\int_x^{\infty} [1 - \pi(a)] \ell(a) da}{\ell(x)}$$



## Recap: Arriaga 1984

Effects of mortality change by age groups on life expectancies  
( $\sum_n \Delta_x = \text{Total change}$ ):

$${}_n\Delta_x = \underbrace{\frac{\ell_x^1}{\ell_0^1} \left( \frac{{}_nL_x^2}{\ell_x^2} - \frac{{}_nL_x^1}{\ell_x^1} \right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{\ell_0^1} \left( \frac{\ell_x^1}{\ell_x^2} - \frac{\ell_{x+n}^1}{\ell_{x+n}^2} \right)}_{\text{Indirect and interaction effects}}$$



Recap: Cause-deleted life tables (based on Chapter 4, Preston et al 2001)

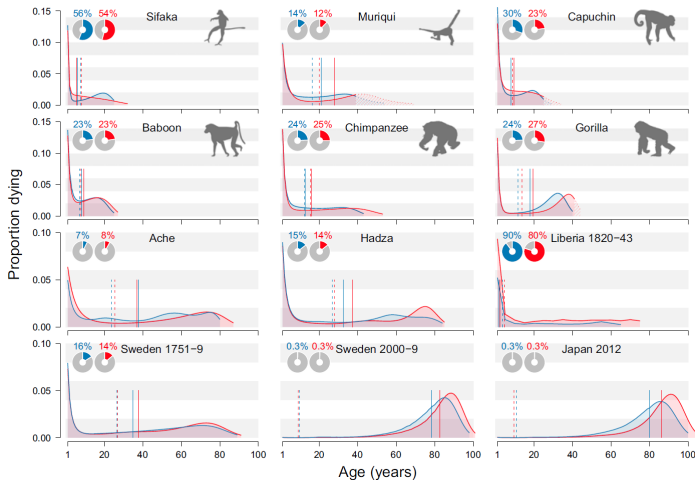
$${}_n^*p_x^i = [{}_np_x]^{R_i}$$

$$= [{}_np_x]^{\frac{{}_nD_x^i}{{}_nD_x}}$$

Now with this simple relation we can create  
hypothetical scenarios.

[https://population-health.shinyapps.io/  
saudi-arabia-health-profile/](https://population-health.shinyapps.io/saudi-arabia-health-profile/)

# Recap: Lifespan variation indicators



## Recap: Horiuchi et al 2008

$$y_2 - y_1 = \sum_{i=1}^n c_i$$

$c_i$  is the total change in  $y$  produced by changes in the  $i$ -th covariate,  $x_i$ .

Important: Theoretical foundation for decomp analysis: implies that even if a dependent variable is not an additive function of its covariates, a change in the dependent variable can be expressed as a sum of effects of the covariates.



# Recap: Horiuchi et al 2008

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## The impact of violence on Venezuelan life expectancy and lifespan inequality

Jenny García, José Manuel Aburto 

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**Abstract**  
**Background**  
Venezuela is one of the most violent countries in the world. According to the United Nations, homicide rates in the country increased from 32.9 to 61.9 per 100 000 people between 2000 and 2014. This upsurge coincided with a slowdown in life expectancy improvements. We estimate mortality trends and quantify the impact of violence-related deaths and other causes of death on life expectancy and lifespan inequality in Venezuela.