We denote by $\mathbf{e}_k$, $k = 1, \ldots, n$, the unit vector whose kth element is 1 and whose other elements are 0. We use the following special matrices:

$$Z \equiv \sum_{i=1}^{n-1} e_{i+1} e_i^T = \begin{pmatrix} 0 & & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & & \vdots & \vdots \\ \vdots & & \ddots & 0 & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}, \qquad J \equiv \sum_{i=1}^{n} e_{n-i+1} e_i^T = \begin{pmatrix} 0 & \cdots & \cdots & 0 & 1 \\ \vdots & & \cdot & 1 & 0 \\ \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & 1 & \cdot & & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{pmatrix}.$$

$$H(\theta) = \frac{1}{\cos\theta} \begin{bmatrix} 1 & -\sin\theta \\ -\sin\theta & 1 \end{bmatrix}$$

$$\mathbf{u}^T = (t_0\,,\, t_1\,,\, \ldots\,,\, t_{n-1}) / \sqrt{t_0}\,,$$
$$\mathbf{v}^T = (0\,,\, t_1\,,\, \ldots\,,\, t_{n-1}) / \sqrt{t_0}\,.$$

Algorithm FACTOR(T):

Set $\mathbf{u}_1 = \mathbf{u}$, $\mathbf{v}_1 = \mathbf{v}$.

For $k = 1, \ldots, n-1$ calculate $\mathbf{u}_{k+1}$, $\mathbf{v}_{k+1}$ such that

$$\mathbf{u}_{k+1}\mathbf{u}_{k+1}^T - \mathbf{v}_{k+1}\mathbf{v}_{k+1}^T = Z\mathbf{u}_k\mathbf{u}_k^T Z^T - \mathbf{v}_k\mathbf{v}_k^T\,,$$
$$\mathbf{e}_{k+1}^T \mathbf{v}_{k+1} = 0\,.$$

Then $T = U^T U$, where $U = \sum_{k=1}^{n} \mathbf{e}_k \mathbf{u}_k^T$. Hay 2 fprmas de obtener u

In fact we have not one algorithm but a class of factorization algorithms, where each algorithm corresponds to a particular way of realizing the elementary downdating

(3.3a) $$\sin\theta_k = \mathbf{e}_{k+1}^T \mathbf{v}_k / \mathbf{e}_k^T \mathbf{u}_k\,,$$

(3.3b) $$\cos\theta_k = \sqrt{1 - \sin^2\theta_k}\,,$$

and

(3.4) $$\begin{bmatrix} \mathbf{u}_{k+1}^T \\ \mathbf{v}_{k+1}^T \end{bmatrix} = H(\theta_k) \begin{bmatrix} \mathbf{u}_k^T Z^T \\ \mathbf{v}_k^T \end{bmatrix}.$$

Método 1

Thus, we may rewrite (3.6) as

(3.7a) $$\mathbf{v}_{k+1} = (\mathbf{v}_k - \sin\theta_k Z\mathbf{u}_k) / \cos\theta_k\,,$$

(3.7b) $$\mathbf{u}_{k+1} = -\sin\theta_k \mathbf{v}_{k+1} + \cos\theta_k Z\mathbf{u}_k\,.$$

Método 2

Note that equation (3.7a) is the same as the second component of (3.4). However, (3.7b) differs from the first component of (3.4) as it uses $\mathbf{v}_{k+1}$ in place of $\mathbf{v}_k$ to define $\mathbf{u}_{k+1}$. It is possible to construct an alternative algorithm by using the first component of (3.5) to define $\mathbf{u}_{k+1}$. This leads to the following formulas:

(3.8a) $$\mathbf{u}_{k+1} = (Z\mathbf{u}_k - \sin\theta_k \mathbf{v}_k) / \cos\theta_k\,,$$

(3.8b) $$\mathbf{v}_{k+1} = -\sin\theta_k \mathbf{u}_{k+1} + \cos\theta_k \mathbf{v}_k\,.$$

Similarly, from (3.7a) and (3.7b) we can obtain a *scaled mixed* elementary downdating algorithm via

$$\sin\theta_k = \beta_k \mathbf{e}_{k+1}^T \mathbf{x}_k / \alpha_k \mathbf{e}_k^T \mathbf{w}_k \ ,$$
$$\alpha_{k+1} = \alpha_k \cos\theta_k \ ,$$
$$\beta_{k+1} = \beta_k / \cos\theta_k \ ,$$

and

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \frac{\sin\theta_k \alpha_k}{\beta_k} Z\mathbf{w}_k \ ,$$
$$\mathbf{w}_{k+1} = -\frac{\sin\theta_k \beta_{k+1}}{\alpha_{k+1}} \mathbf{x}_{k+1} + Z\mathbf{w}_k \ .$$

The stability properties of scaled mixed algorithms are similar to those of the corresponding unscaled algorithms [12].

$$\mathbf{u}_k = \alpha_k \mathbf{w}_k \quad \text{and} \quad \mathbf{v}_k = \beta_k \mathbf{x}_k \ , \tag{4.3}$$

we obtain

$$T = W^T D^2 W \ ,$$

where

$$W = \sum_{k=1}^{n} \mathbf{e}_k \mathbf{w}_k^T \ ,$$
$$D = \sum_{k=1}^{n} \alpha_k \mathbf{e}_k \mathbf{e}_k^T \ .$$

Con este tercer método U = W raiz(D)

FUENTE:

ON THE STABILITY OF THE BAREISS AND RELATED TOEPLITZ FACTORIZATION ALGORITHMS*

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