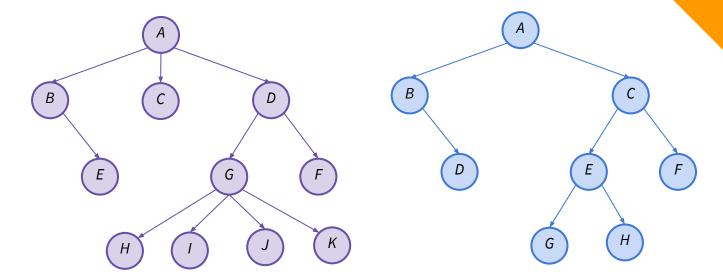
# Unidad 5: Árboles

Algoritmos y Estructuras de Datos

Concepto y propiedades Árboles Binarios Árboles Binarios de búsqueda

# Árboles - Concepto y propiedades

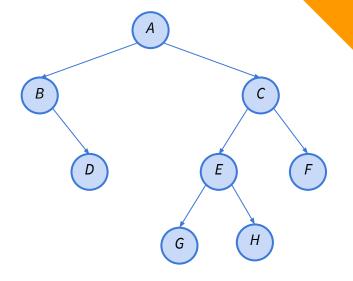
- Raíz
- Padre
- Hijo
- Rama
- Hoja
- Profundidad
- Nivel
- Hermano
- Descendiente
- Grado



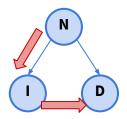
# Árboles binarios - Implementación

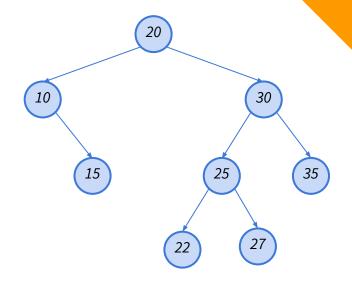
```
btn {
    datatype value
    btn left
    btn right
}
```

```
struct btn { // btn = binary tree node
   int value;
   struct btn *left;
   struct btn *right;
};
```

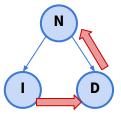


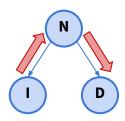
Pre-Order





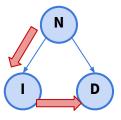
Post-Order



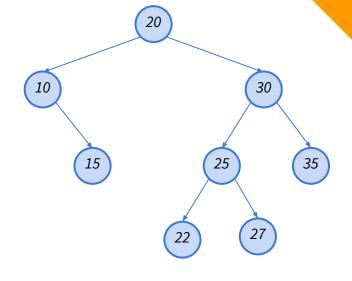


### Pre-Order

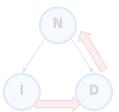
void preOrder(btn node) {

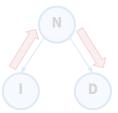


}



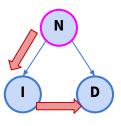
Post-Order

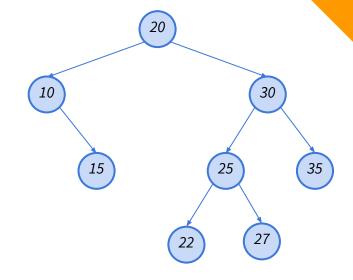




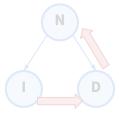
### Pre-Order

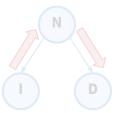
```
void preOrder(btn node) {
    printNode(node)
```





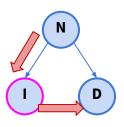
Post-Order

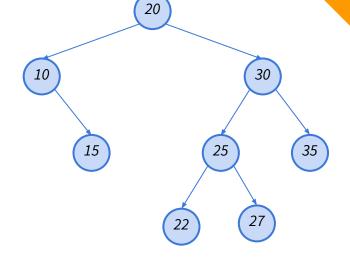




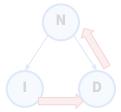
### Pre-Order

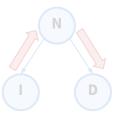
```
void preOrder(btn node) {
          printNode(node)
          preOrder(node.left)
```



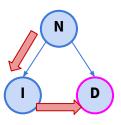


Post-Order

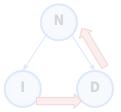


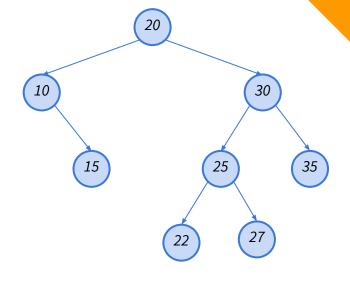


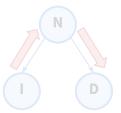
### Pre-Order



Post-Order

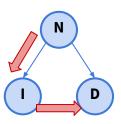




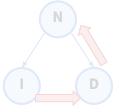


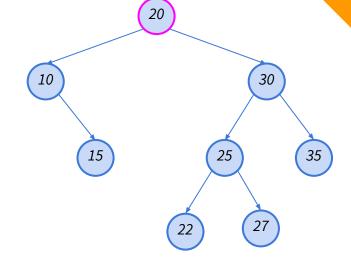
### Pre-Order

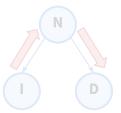
```
void preOrder(btn node) {
          printNode(node)
          preOrder(node.left)
          preOrder(node.right)
```



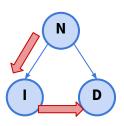
Post-Order



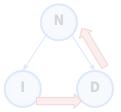


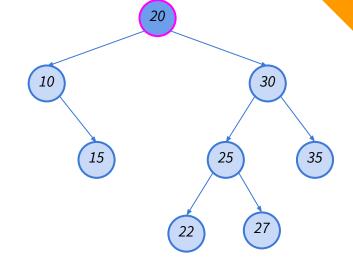


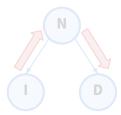
### Pre-Order



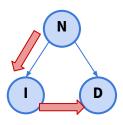
Post-Order



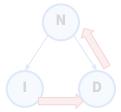


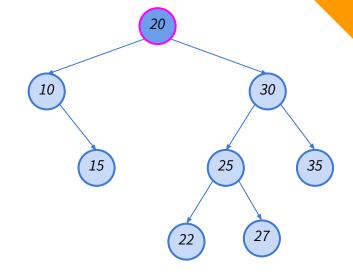


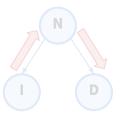
### Pre-Order



Post-Order

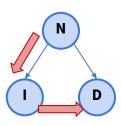


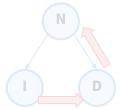




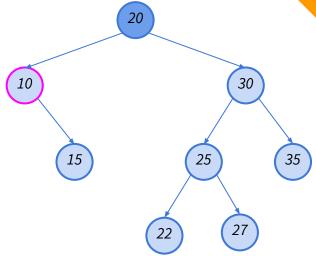
### Pre-Order

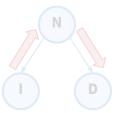
```
void preOrder(btn node) {
           printNode(node)
           preOrder(node.left)
           preOrder(node.right)
→ 20
```



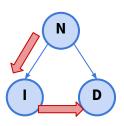




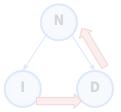


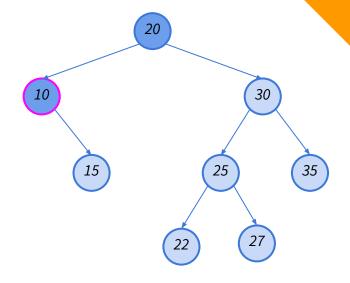


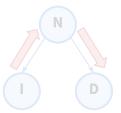
### Pre-Order



Post-Order

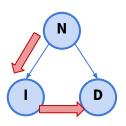




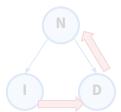


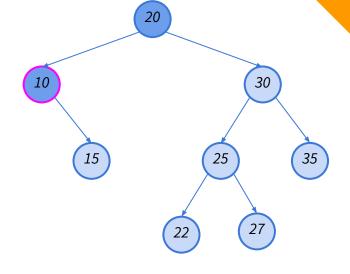
### Pre-Order

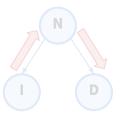
→ 20 10



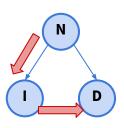
Post-Order





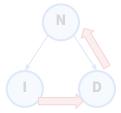


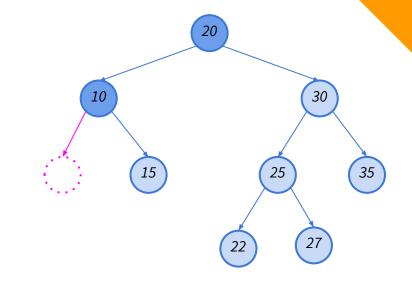
### Pre-Order

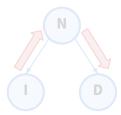


Post-Order

→ 20 10

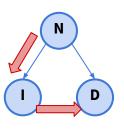


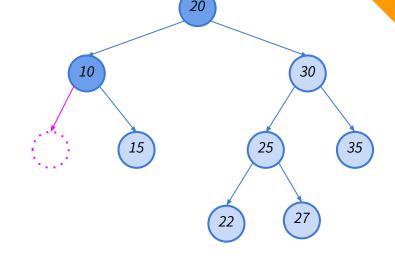




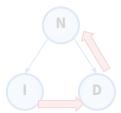
### Pre-Order

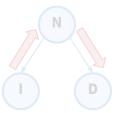
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```





Post-Order



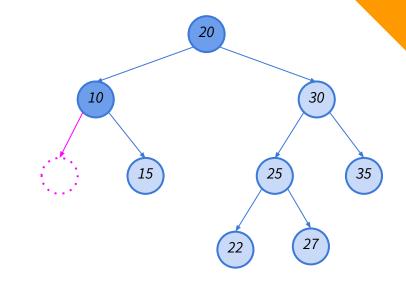


```
Pre-Order

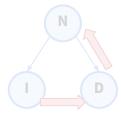
node != NULL

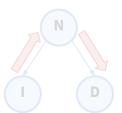
void preOrder(btn r e) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}

→ 20 10
```



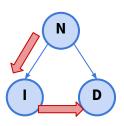
Post-Order



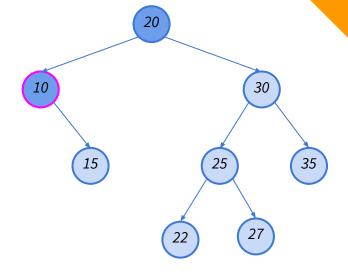


### Pre-Order

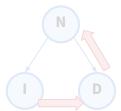
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

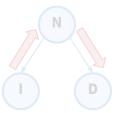


→ 20 10



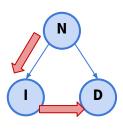
Post-Order



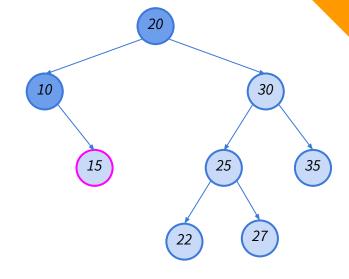


### Pre-Order

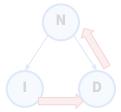
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

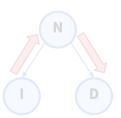


→ 20 10



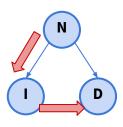
Post-Order



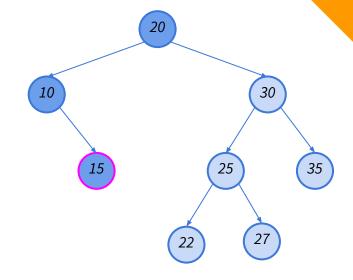


### Pre-Order

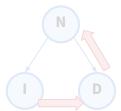
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

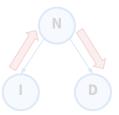


→ 20 10 15



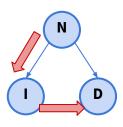
Post-Order



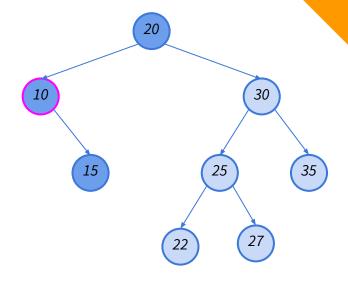


### Pre-Order

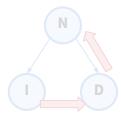
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

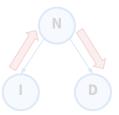


→ 20 10 15



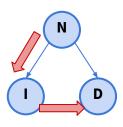
Post-Order



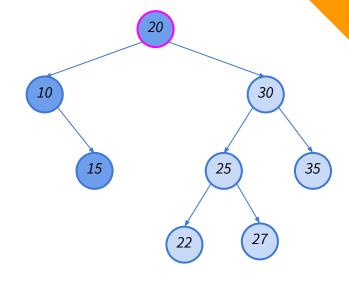


### Pre-Order

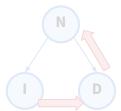
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

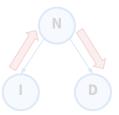


→ 20 10 15



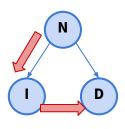
Post-Order



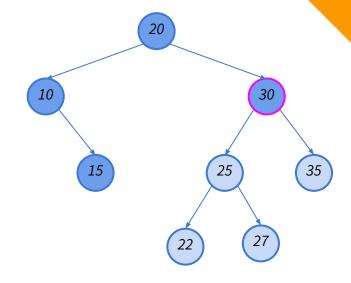


### Pre-Order

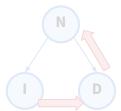
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

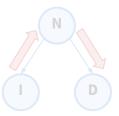


 $\rightarrow$  20 10 15 30



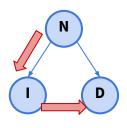
Post-Order



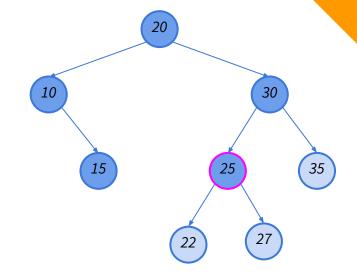


### Pre-Order

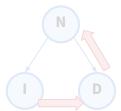
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

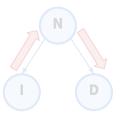


→ 20 10 15 30 25



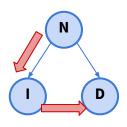
Post-Order



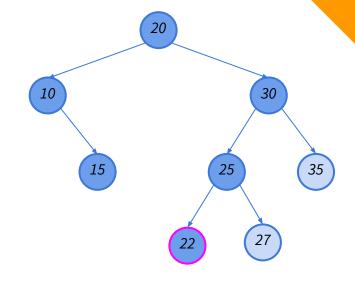


#### Pre-Order

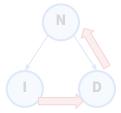
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

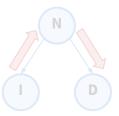


→ 20 10 15 30 25 22



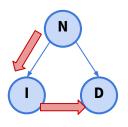
Post-Order



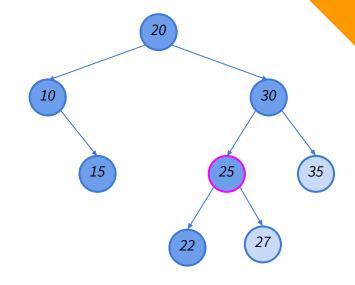


### Pre-Order

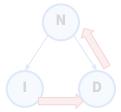
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

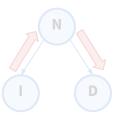


→ 20 10 15 30 25 22



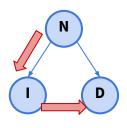
Post-Order



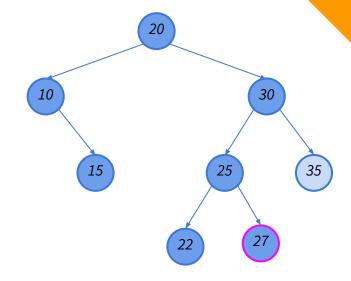


### Pre-Order

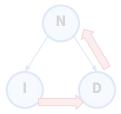
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

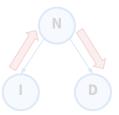


→ 20 10 15 30 25 22 27



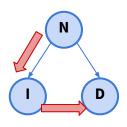
Post-Order



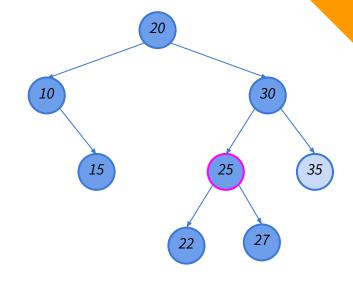


### Pre-Order

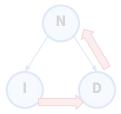
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

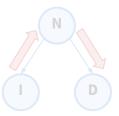


→ 20 10 15 30 25 22 27



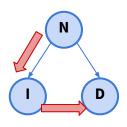
Post-Order



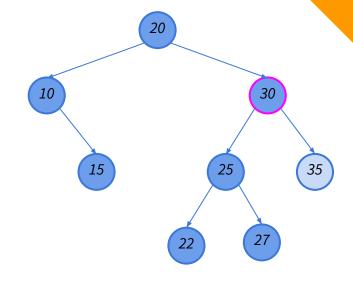


### Pre-Order

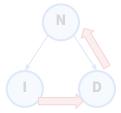
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

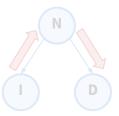


→ 20 10 15 30 25 22 27



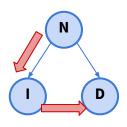
Post-Order



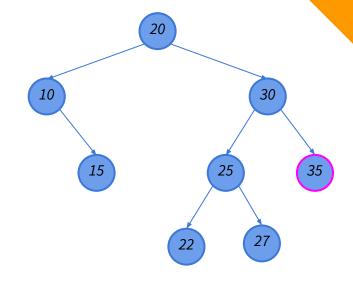


### Pre-Order

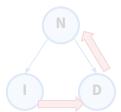
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

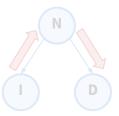


→ 20 10 15 30 25 22 27 **35** 



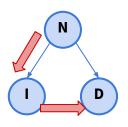
Post-Order



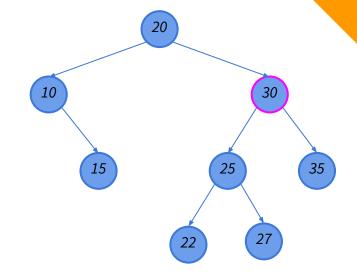


### Pre-Order

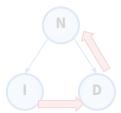
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

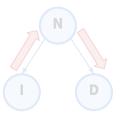


→ 20 10 15 30 25 22 27 35



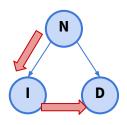
Post-Order



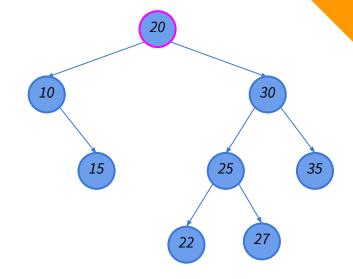


### Pre-Order

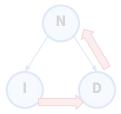
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

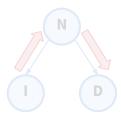


→ 20 10 15 30 25 22 27 35



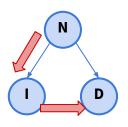
Post-Order



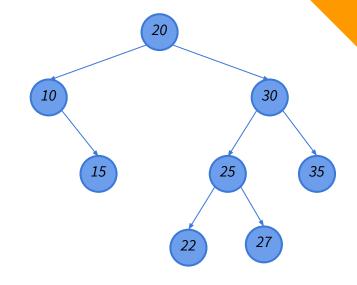


### Pre-Order

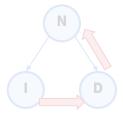
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

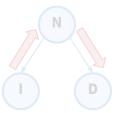


→ 20 10 15 30 25 22 27 35



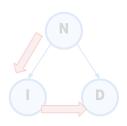
Post-Order



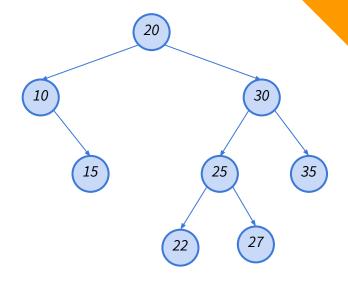


#### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

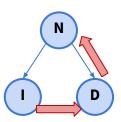


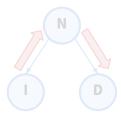
→ 20 10 15 30 25 22 27 35



#### Post-Order

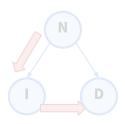
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



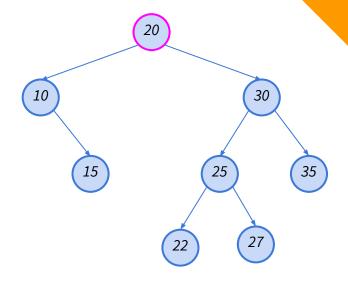


#### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

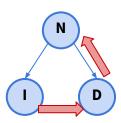


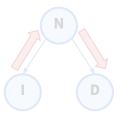
→ 20 10 15 30 25 22 27 35



#### Post-Order

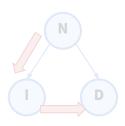
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



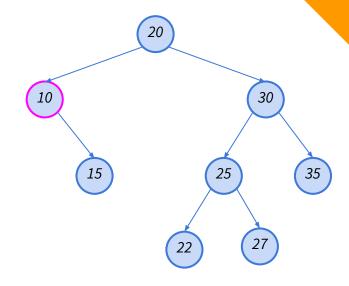


#### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

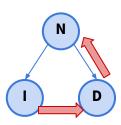


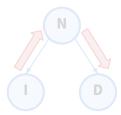
→ 20 10 15 30 25 22 27 35



#### Post-Order

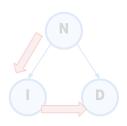
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



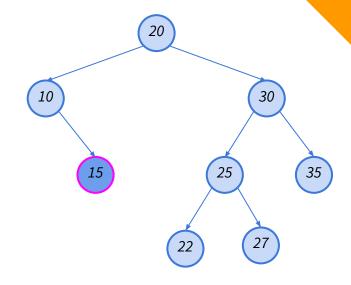


### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

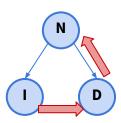


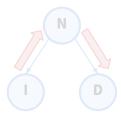
→ 20 10 15 30 25 22 27 35



### Post-Order

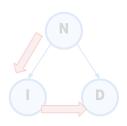
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



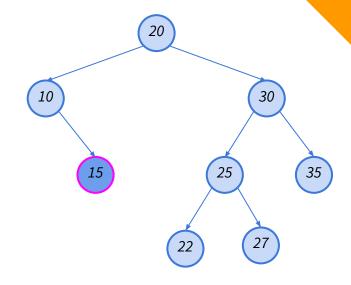


### Pre-Order

```
void preOrder(btn node) {
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        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

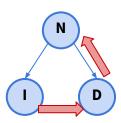


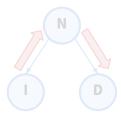
→ 20 10 15 30 25 22 27 35



### Post-Order

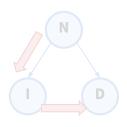
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```





### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



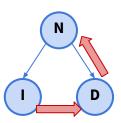
→ 20 10 15 30 25 22 27 35

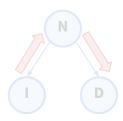
# 10 30 15 25 35

### Post-Order

15 **10** 

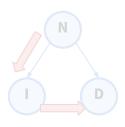
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```





### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



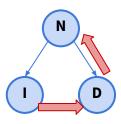
→ 20 10 15 30 25 22 27 35

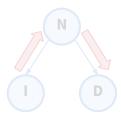
# 20 30 15 25 35

### Post-Order

15 10

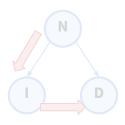
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



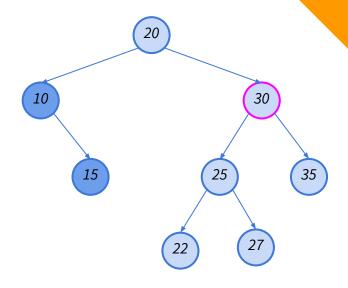


### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



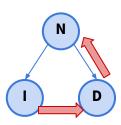
→ 20 10 15 30 25 22 27 35

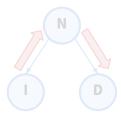


### Post-Order

15 10

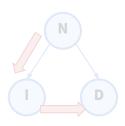
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



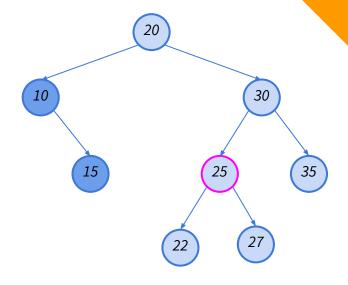


### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



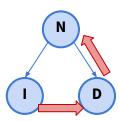
→ 20 10 15 30 25 22 27 35

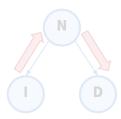


### Post-Order

15 10

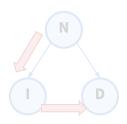
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



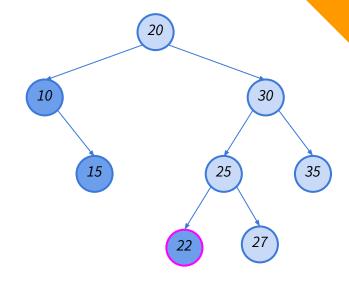


### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



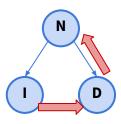
→ 20 10 15 30 25 22 27 35

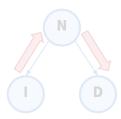


### Post-Order

15 10 <mark>22</mark>

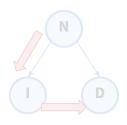
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```





### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



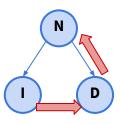
→ 20 10 15 30 25 22 27 35

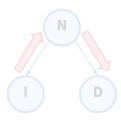
# 20 30 15 25 35

### Post-Order

15 10 22

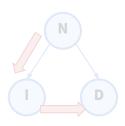
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



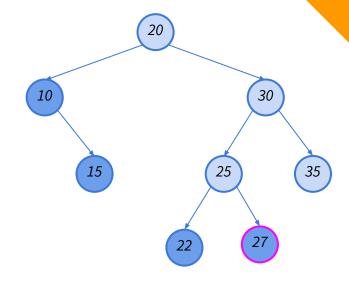


### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



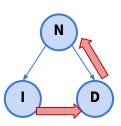
→ 20 10 15 30 25 22 27 35

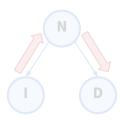


### Post-Order

15 10 22 27

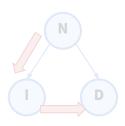
```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```





### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

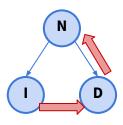


→ 20 10 15 30 25 22 27 35

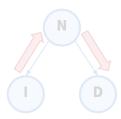
# 20 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



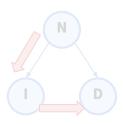
In-Order



→ 15 10 22 27 25

### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

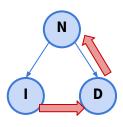


→ 20 10 15 30 25 22 27 35

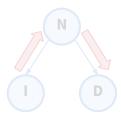
# 20 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



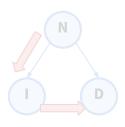
In-Order



→ 15 10 22 27 25

### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

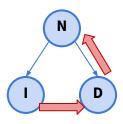


→ 20 10 15 30 25 22 27 35

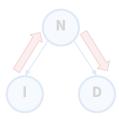
# 10 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



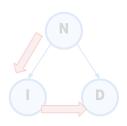
In-Order



→ 15 10 22 27 25 35

### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

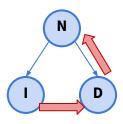


→ 20 10 15 30 25 22 27 35

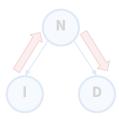
# 10 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



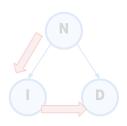
In-Order



**≥** 15 10 22 27 25 35 **30** 

### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

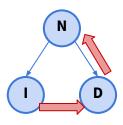


→ 20 10 15 30 25 22 27 35

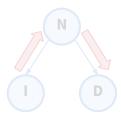
# 20 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



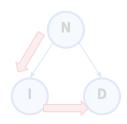
In-Order



≥ 15 10 22 27 25 35 30 20

### Pre-Order

```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```

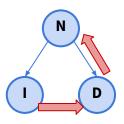


→ 20 10 15 30 25 22 27 35

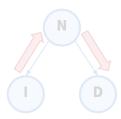
# 20 30 15 25 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```



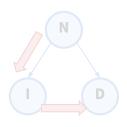
In-Order



≥ 15 10 22 27 25 35 30 20

### Pre-Order

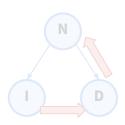
```
void preOrder(btn node) {
    if (notEmpty(node)){
        printNode(node)
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



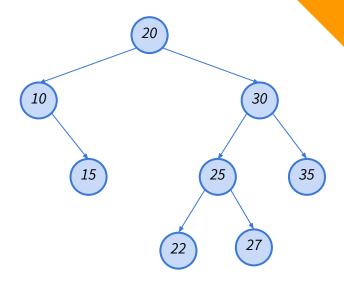
→ 20 10 15 30 25 22 27 35

### Post-Order

```
void postOrder(btn node) {
    if (notEmpty(node)){
        postOrder(node.left);
        postOrder(node.right);
        printNode(node);
    }
}
```

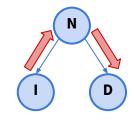


2 27 25 35 30 20



### In-Order

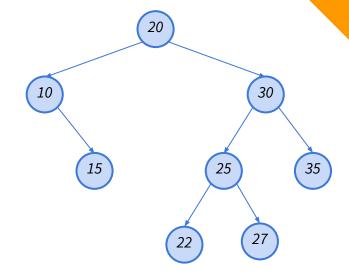
```
void inOrder(btn node) {
    if (notEmpty(node)){
        inOrder(node.left);
        printNode(node);
        inOrder(node.right);
    }
}
```



→ 10 15 20 22 25 27 30 35

Binary Search Tree (BST)

- Para todo nodo
  - Todos los descendientes de la rama izquierda tienen un valor menor, y
  - Todos los descendientes de la rama derecha tienen un valor mayor



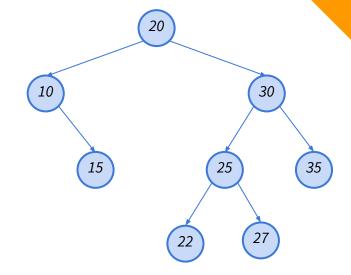
### In-Order

```
void inOrder(btn node) {
    if (notEmpty(node)){
        inOrder(node.left);
        printNode(node);
        inOrder(node.right);
    }
}
```

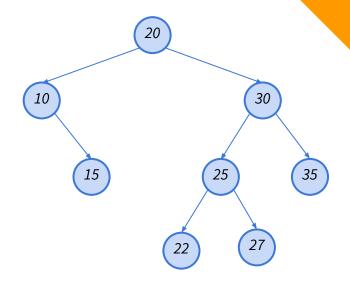
→ 10 15 20 22 25 27 30 35

Buscar un nodo

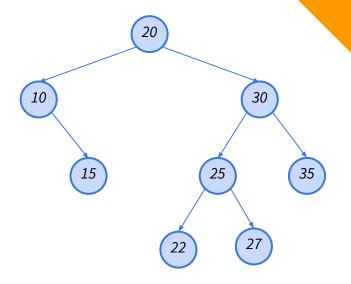
- Para todo nodo
  - Todos los descendientes de la rama izquierda tienen un valor menor, y
  - Todos los descendientes de la rama derecha tienen un valor mayor
- Buscar
  - Iterativo
  - Recursivo



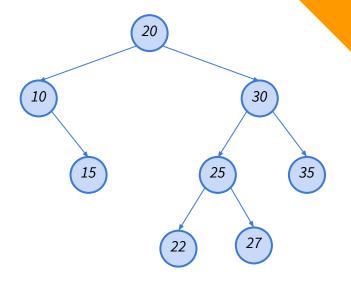
```
btn {
    datatype value
    btn left
    btn right
}
btn find(btn node, int value){
```



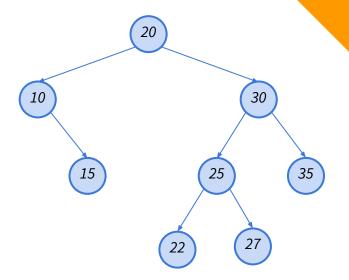
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
          if (node.value > value) {
               node = node.left
```



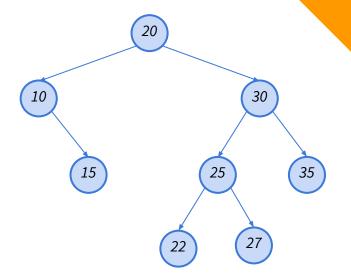
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while
                              (node.value != value) {
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
```



```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while
                              (node.value != value) {
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```

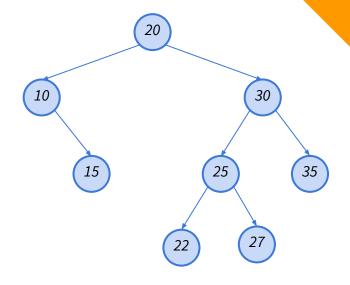


```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
    while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
              node = node.right
     return node
```



Buscar un nodo - Iterativo

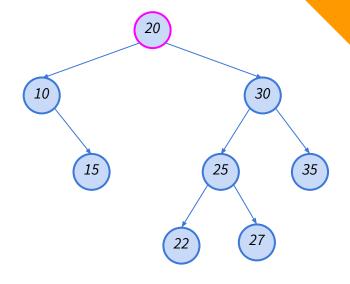
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

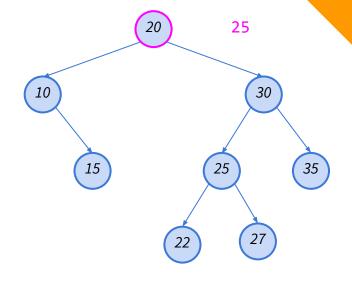
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

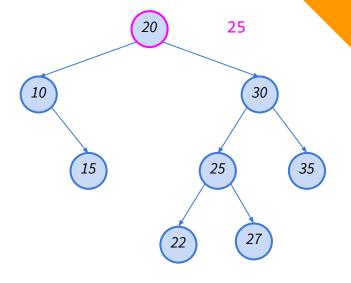
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

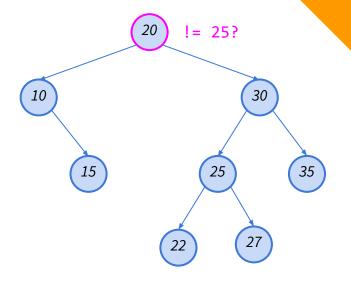
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

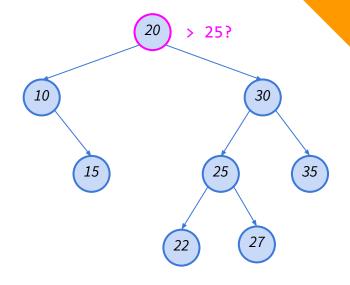
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

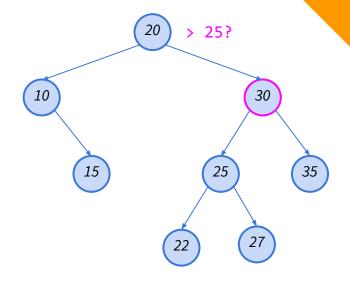
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

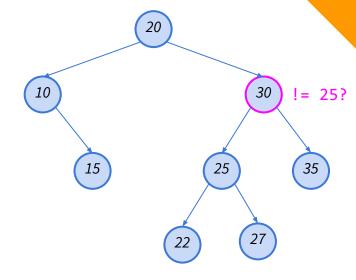
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

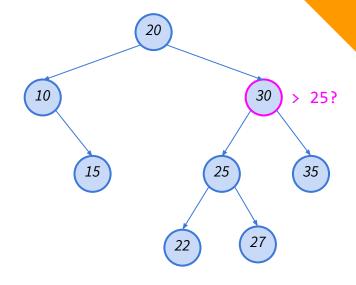
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

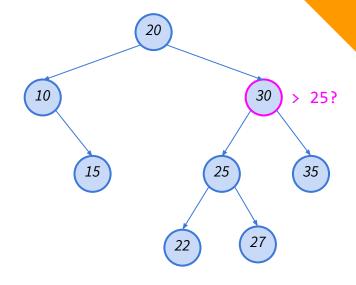
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

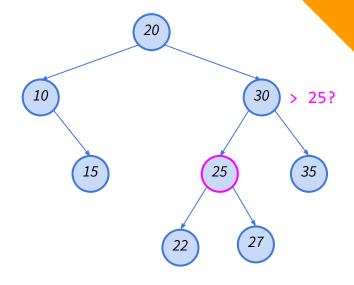
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

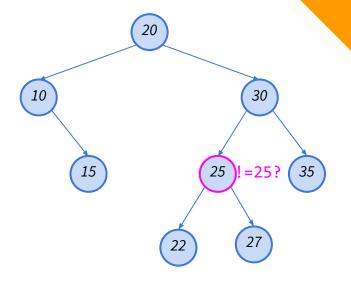
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

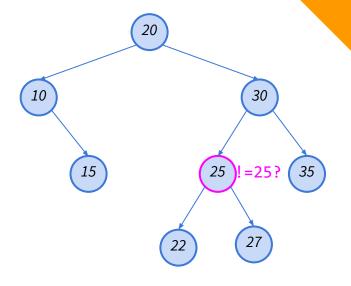
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

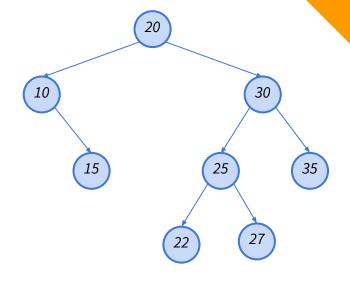
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

Buscar un nodo - Iterativo

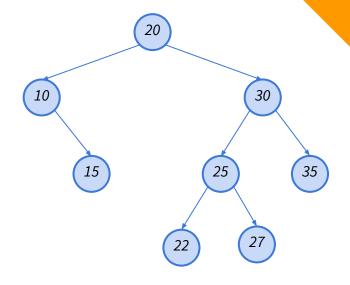
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

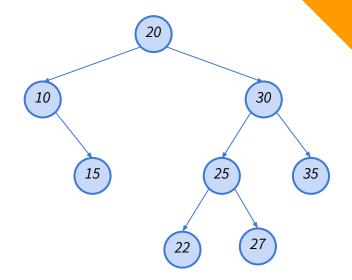
Buscar un nodo - Iterativo

```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     while (notEmpty(node) && (node.value != value)){
          if (node.value > value) {
               node = node.left
          } else {
               node = node.right
     return node
```



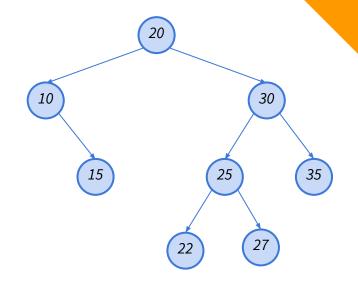
```
btn b = find(root, 25)
if (notEmpty(b)) {
...
```

```
btn {
    datatype value
    btn left
    btn right
}
btn find(btn node, int value){
```



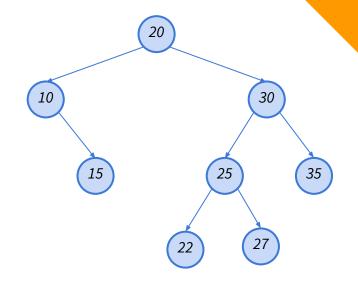
```
btn {
    datatype value
    btn left
    btn right
}

btn find(btn node, int value){
    if (node.value = value) {
        return node
    }
}
```

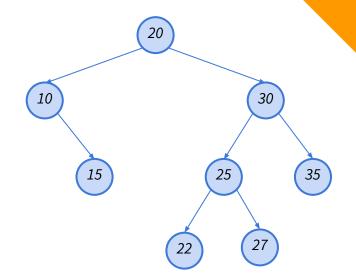


```
btn {
    datatype value
    btn left
    btn right
}

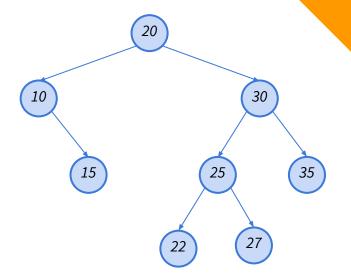
btn find(btn node, int value){
    if (node.value = value) {
        return node
    } else if (node.value > value) {
            return find (node.left, value)
    }
}
```



```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
          if (node.value = value) {
               return node
          } else if (node.value > value) {
               return find (node.left, value)
          } else {
               return find (node.right, value)
```

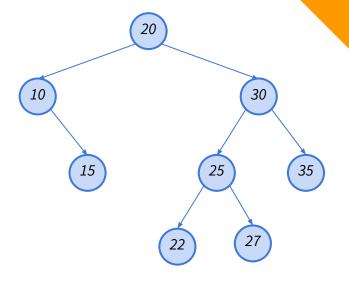


```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
     if (notEmpty(node)){
          if (node.value = value) {
               return node
          } else if (node.value > value) {
               return find (node.left, value)
          } else {
               return find (node.right, value)
     } else {
          return NULL
```



Buscar un nodo - Recursivo

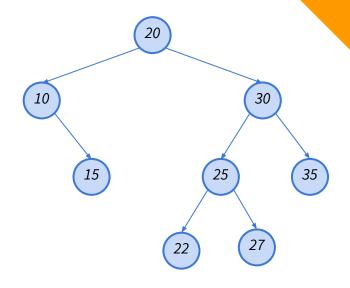
```
btn {
     datatype value
     btn left
     btn right
btn find(btn node, int value){
          if (isEmpty(node) || (node.value = value)) {
               return node
          } else if (node.value > value) {
               return find (node.left, value)
          } else {
               return find (node.right, value)
```



```
btn b = find(root, 15)
if (notEmpty(b)) {
...
```

Buscar un nodo

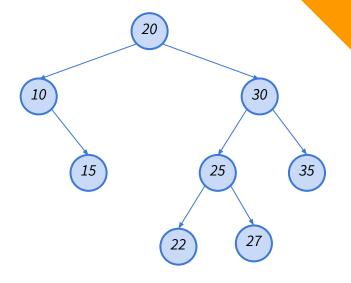
```
btn {
     datatype value
         left
     btn
     btn right
btn find(btn node, int value){
          if (isEmpty(node) | (node.value = value)) {
               return node
          } else if (node.value > value) {
               return find (node.left, value)
          } else {
               return find (node.right, value)
```



```
struct btn *find(struct btn *node, int value) {
   if ((node == NULL) || (node->value == value)) {
      return node;
   } else if (node->value > value) {
      return find(node->left, value);
   } else {
      return find(node->right, value);
   }
}
```

Agregar un Nodo

```
btn {
    datatype value
    btn left
    btn right
}
btn insert(btn node, int value){
```



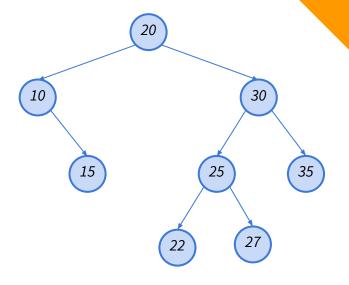
#### <u>Invocación</u>

```
root = insert(root, 12)
```

Agregar un Nodo

```
btn {
    datatype value
    btn left
    btn right
}

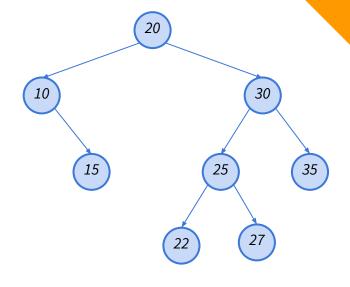
btn insert(btn node, int value){
    if (isEmpty(node)){
        btn aux = NEW()
        aux.value = value
        aux.left = NULL
        aux.right = NULL
        return aux
    }
}
```



```
root = insert(root, 12)
...
```

Agregar un Nodo

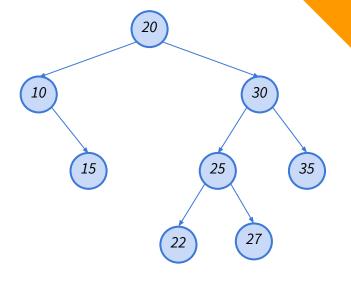
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
```



```
root = insert(root, 12)
...
```

Agregar un Nodo

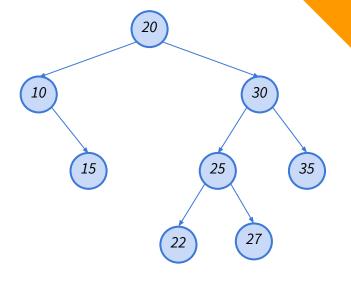
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
```



```
root = insert(root, 12)
...
```

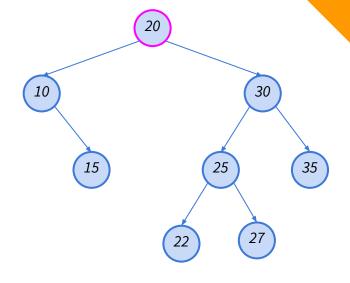
Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
root = insert(root, 12)
...
```

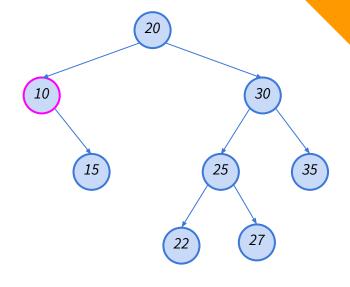
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

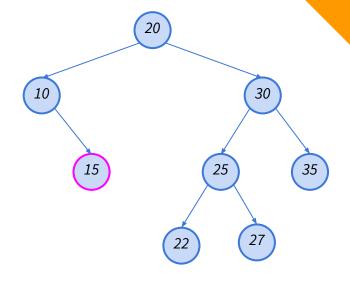
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```

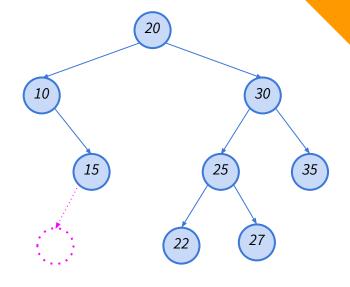


```
Invocación
```

```
root = insert(root, 12)
...
```

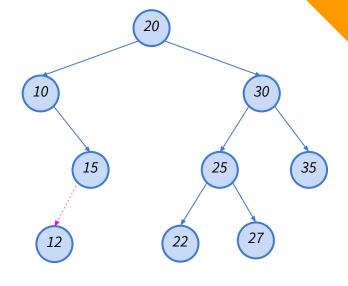
Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
root = insert(root, 12)
...
```

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```

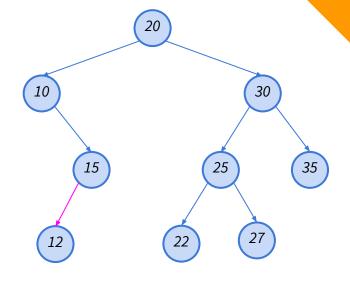


```
Invocación
```

```
root = insert(root, 12)
...
```

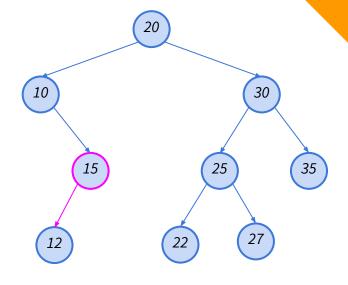
Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
root = insert(root, 12)
...
```

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```

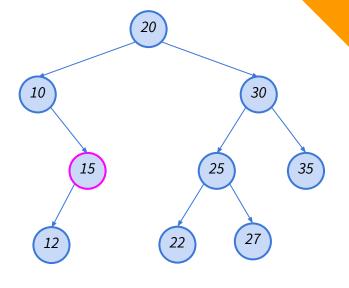


```
Invocación
```

```
root = insert(root, 12)
...
```

Agregar un Nodo

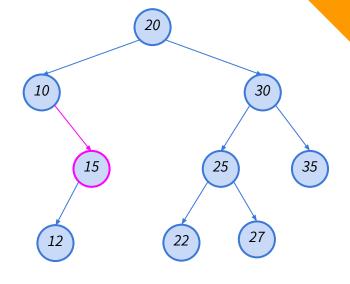
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
root = insert(root, 12)
...
```

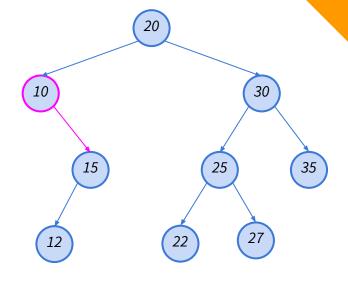
Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
root = insert(root, 12)
...
```

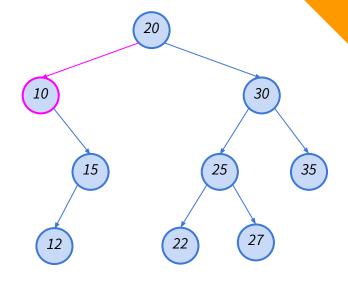
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

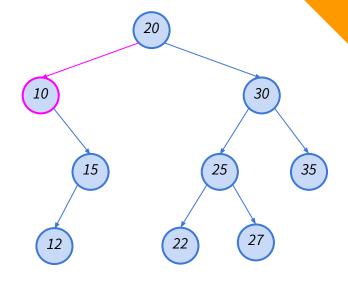
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

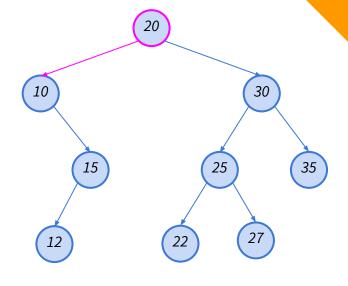
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

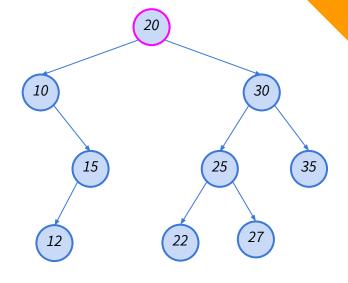
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

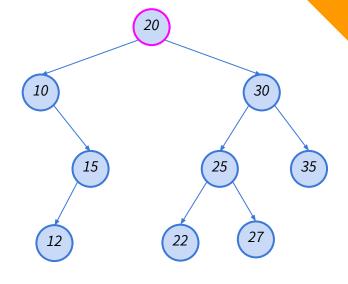
```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```



```
Invocación
```

```
root = insert(root, 12)
...
```

```
btn {
     datatype value
     btn left
     btn right
btn insert(btn node, int value){
     if (isEmpty(node)){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.right = insert(node.right, value)
          return node
```

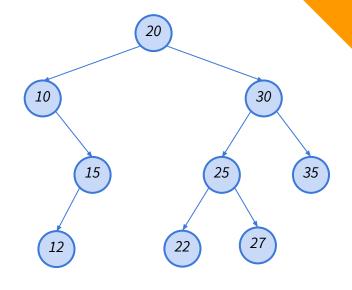


```
Invocación
```

```
root = insert(root, 12)
...
```

Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn create(btn node, int value){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
btn insert(btn node, int value){
     if (isEmpty(node)){
          return create(node, value)
     } else {
          if (node.value > value) {
               node.left = insert(node.left, value)
          } else if (node.value < value) {</pre>
               node.left = insert(node.right, value)
          return node
```

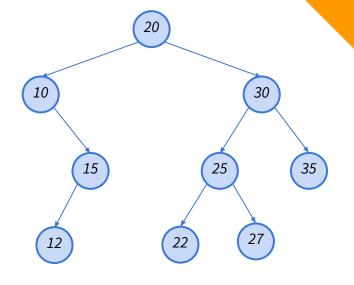


#### <u>Invocación</u>

```
root = insert(root, 12)
...
```

Agregar un Nodo

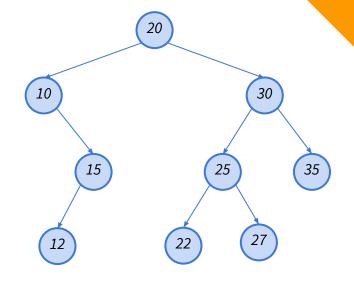
```
btn {
     datatype value
     btn left
     btn right
btn create(btn node, int value){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
btn insert(btn node, btn new){
     if (isEmpty(node)){
          return new
     } else {
          if (node.value > new.value) {
               node.left = insert(node.left, new)
          } else if (node.value < value) {</pre>
               node.left = insert(node.right, new)
          return node
```



```
root = insert(root, create(12))
...
```

Agregar un Nodo

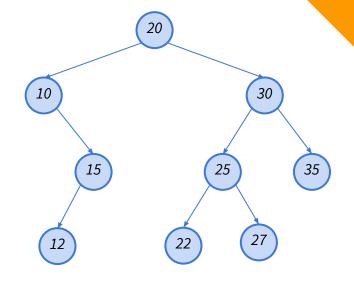
```
btn {
     datatype value
     btn left
     btn right
btn create(btn node, int value){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
bool insert(btn node, btn new){
     if (isEmpty(new)) return TRUE;
     if (isEmpty(node)){
          node = new
          return TRUE
     } else {
          if (node.value > new.value) {
               return insert(node.left, new)
          } else if (node.value < value) {</pre>
               return = insert(node.right, new)
          } else {
               return FALSE
```



```
btn new = create(12)
if (!insert(root, new)){
    free(new)
}
...
```

Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn create(btn node, int value){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
bool insert(btn fnode, btn new){
     if (isEmpty(new)) return TRUE;
     if (isEmpty(node)){
          node = new
          return TRUE
     } else {
          if (node.value > new.value) {
               return insert(node.left, new)
          } else if (node.value < value) {</pre>
               return = insert(node.right, new)
          } else {
               return FALSE
```



```
btn new = create(12)
if (!insert(root, new)){
    free(new)
}
...
```

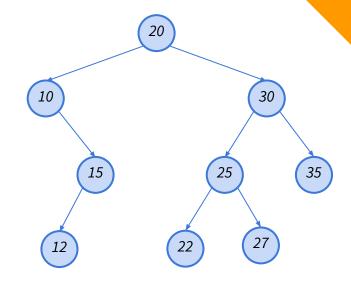
Agregar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn create(btn node, int value){
          btn aux = NEW()
          aux.value = value
          aux.left = NULL
          aux.right = NULL
          return aux
bool insert(btn inode, btn new){
     if (isEmpty(new)) return TRUE;
     if (isEmpty(node)){
          node = new
          return TRUE
     } else {
          if (node.value > value) {
               return insert(node.left, new)
          } else if (node.value < value) {</pre>
               return = insert(node.right, new)
          } else {
               return FALSE
```

C

```
struct btn *create(int value) {
    struct btn *n =
             (struct btn *)malloc(sizeof(struct btn));
    n->value = value;
    n->left = NULL;
   n->right = NULL;
int insertOver(struct btn **node, struct btn *new) {
   if (node == NULL) return 0;
   if (new == NULL) return 1;
   if ((*node) == NULL) {
        *node = new;
        return 1;
       if ( (*node)->value > new->value) {
            return insertOver(&((*node)->left), new);
       } else if ((*node)->value < new->value) {
           return insertOver(&((*node)->right), new);
        new = create(12);
        if (!insertOver(&root, new)){
            printf("\n Duplicado: %d", new->value);
            free(new);
       };
```

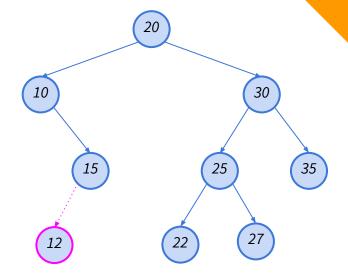
Eliminar un Nodo



Eliminar un Nodo

### Casos:

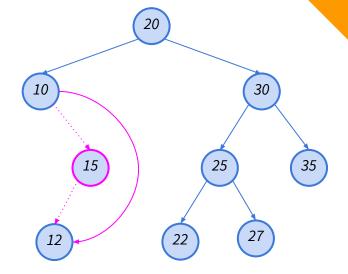
1. Cuando el nodo es una hoja



Eliminar un Nodo

#### Casos:

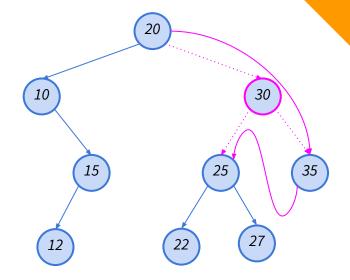
- 1. Cuando el nodo es una hoja
- 2. Cuando el nodo tiene un solo hijo



Eliminar un Nodo

#### Casos:

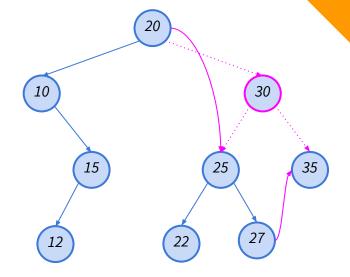
- 1. Cuando el nodo es una hoja
- 2. Cuando el nodo tiene un solo hijo
- 3. Cuando el nodo tiene ambos hijos



Eliminar un Nodo

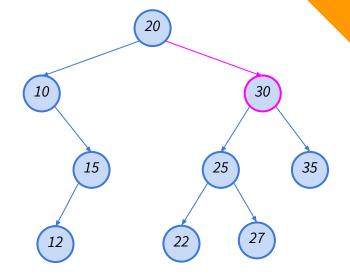
#### Casos:

- 1. Cuando el nodo es una hoja
- 2. Cuando el nodo tiene un solo hijo
- 3. Cuando el nodo tiene ambos hijos



Eliminar un Nodo

```
btn {
    datatype value
    btn left
    btn right
}
btn delete(btn \underline){
```

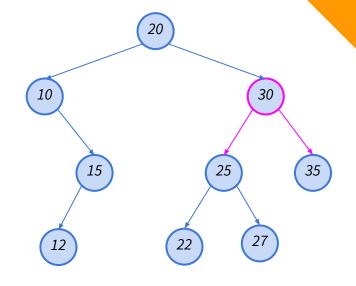


#### <u>Invocación</u>

```
btn deleted
deleted = delete(root.right)
...
```

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn fnode){
          btn aux = node
          insert (aux.right, aux.left)
```

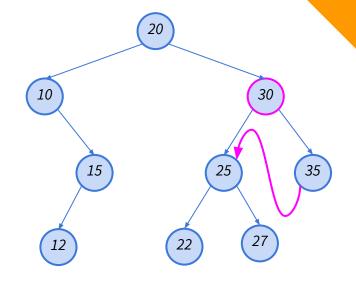


#### <u>Invocación</u>

```
btn deleted
deleted = delete(root.right)
```

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn fnode){
          btn aux = node
          insert (aux.right, aux.left)
```

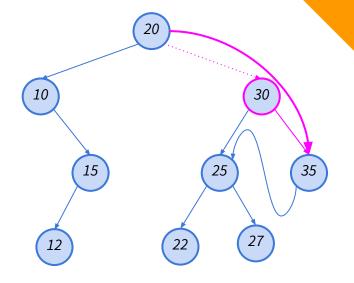


#### *Invocación*

```
btn deleted
deleted = delete(root.right)
...
```

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn fnode){
          btn aux = node
          insert (aux.right, aux.left)
          node = aux.right
```

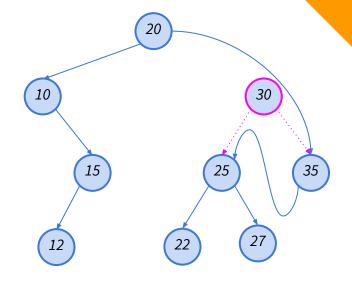


#### *Invocación*

```
btn deleted
deleted = delete(root.right)
...
```

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn ↑node){
          btn aux = node
          insert (aux.right, aux.left)
          node = aux.right
          aux.right = NULL
          aux.left = NULL
```

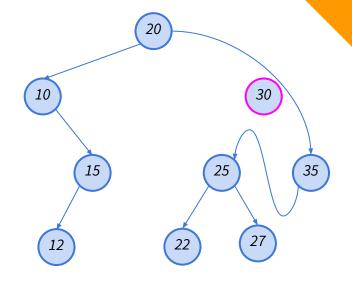


#### *Invocación*

btn deleted
deleted = delete(root.right)

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn ↑node){
          btn aux = node
          insert (aux.right, aux.left)
          node = aux.right
          aux.right = NULL
          aux.left = NULL
          return aux
```

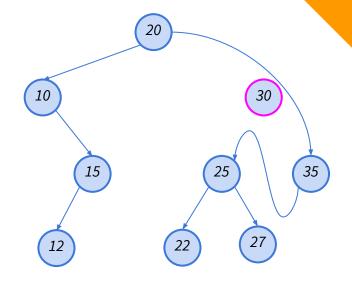


#### *Invocación*

```
btn deleted
deleted = delete(root.right)
...
```

Eliminar un Nodo

```
btn {
     datatype value
     btn left
     btn right
btn delete(btn ↑node){
     if (isEmpty(node)){
          return NULL
     } else {
          btn aux = node
          insert (aux.right, aux.left)
          node = aux.right
          aux.right = NULL
          aux.left = NULL
          return aux
```

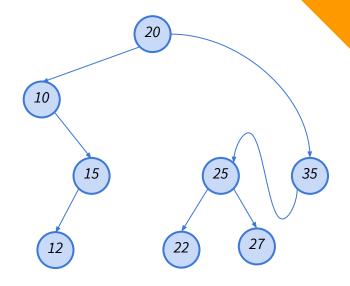


#### *Invocación*

btn deleted
deleted = delete(root.right)

Eliminar un Nodo

¿Y si eliminamos la raíz?

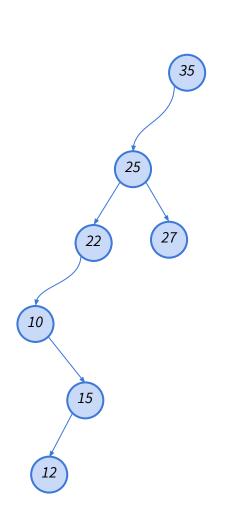


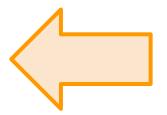
#### <u>Invocación</u>

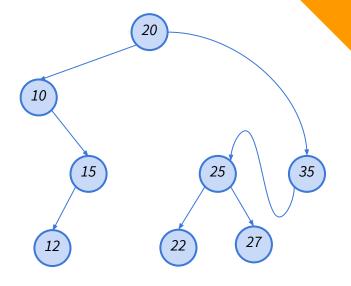
```
btn deleted
deleted = delete(root)
```

Eliminar un Nodo

¿Y si eliminamos la raíz?





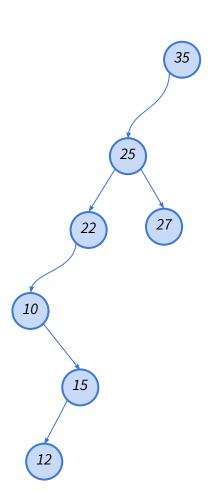


#### <u>Invocación</u>

btn deleted
deleted = delete(root)

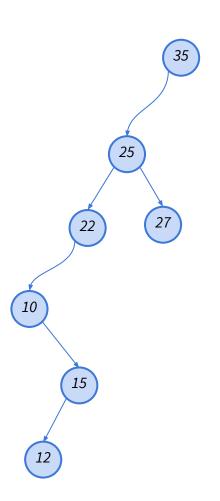
Concepto de equilibrio

¿Sigue siendo un ABB?

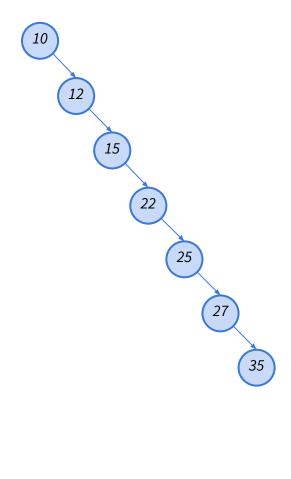


Concepto de equilibrio

¿Sigue siendo un ABB?



¿Y si el el orden de inserción fuera 10, 12, 15, 22, 25, 27, 35?



# Unidad 5: Árboles

Algoritmos y Estructuras de Datos

Concepto y propiedades Implementaciones Árboles Binarios Árboles Binarios de búsqueda