Epipolar Geometry

This homework is intended provide students with the opportunities to obtain an epipolar line using two rectified image.

The zip file containing sample images and code is **hw6.zip** to PLATO.

Hand in all parts of this assignment in PLATO (both the code and report PDF file as specified). To get full marks, your functions (i.e., *.py files) must not only work correctly, but also must be clearly documented with sufficient comments for others to easily use and understand the code. You will lose marks for insufficient or unclear comments. In this assignment, you also need to hand in scripts showing tests of your functions on all the cases specified as well as the images and other answers requested. The scripts and results (as screenshots or otherwise) should be pasted into a single PDF file and clearly labeled. Note that lack of submission of either the code or the PDF will also result in loss of points.

The assignment

1. **Fundamental Matrix Estimation**

   We provide two pairs of rectified images for use in this task within the data folder. And we also provide a *.npy* file of each image name with corresponding feature points of the image.

   Each point is given in the form of a homogenous coordinate system of 3xN

   (35 points) The fundamental matrix F is defined by $x'^T F x = 0$ for any pair of matches x and x' in two images. Let $x = (u, v, 1)^T$ and $x' = (u', v', 1)^T$, $F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$ each match gives a linear equation

   $$uu'f_{11} + vu'f_{12} + u'f_{13} + uv'f_{21} + vv'f_{22} + v'f_{23} + uf_{31} + vf_{32} + f_{33} = 0$$

   Therefore, for all corresponding points, it can be generalized as follows.

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

And you can find F matrix instead of solving $Af = 0$, we seek f to minimize $|Af|$, least eigenvector for A^TA.

In *'compute_fundamental'*, use 8-point algorithm to compute the fundamental matrix. However, the provided points are larger than 8, so it is an overdetermined linear system.

So, get a least square solution for the fundamental matrix F, and ensure that $rank(F) = 2$ by SVD where the last singular value is set to be zero. Let $\mathrm{F} = \mathrm{U}\Sigma\mathrm{V}^T$, where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \rightarrow \Sigma' = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then $\mathrm{F}' = \mathrm{U}\Sigma'\mathrm{V}^T$ is the solution

HINT : You can easily get it by using the functions `np.linalg.svd` and `np.diag` by Numpy

2. Compute epipoles

(30 points) In *'compute_epipoles'*, compute the epipoles of two images based on the fundamental matrix

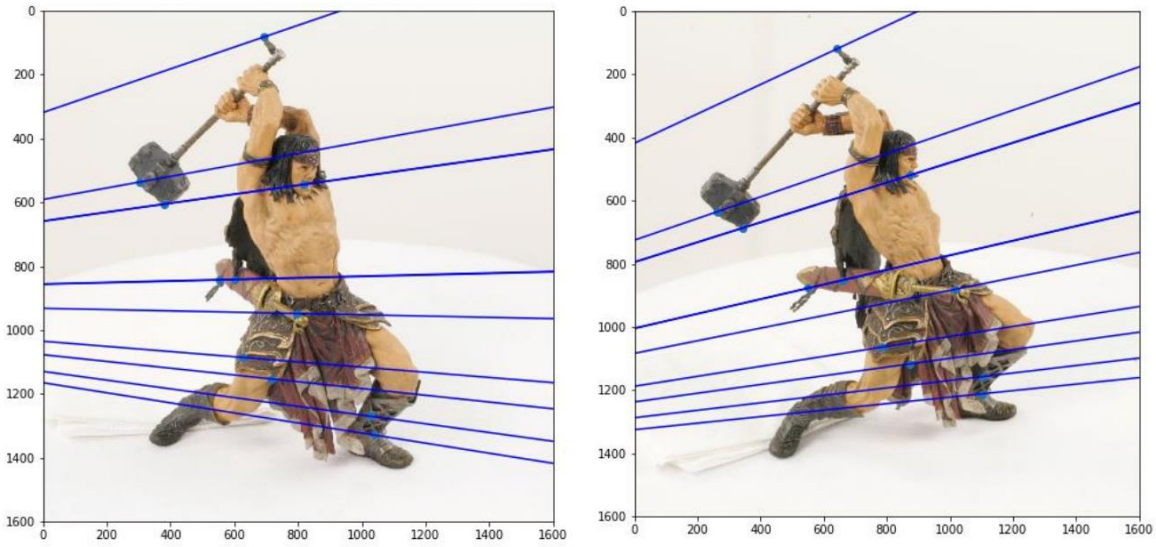
You can get the epipole through the following expressions with the fundamental matrix previously obtained.

$$Fe_1 = 0, \qquad F^Te_2 = 0$$

Note : Epipoles must be normalized using homogenous coordinate.

3. Epipolar lines

(35 point) In 'draw_epipolar_lines', draw the epipolar lines on the images using the outputs from the above methods. Compare your result image to the picture below.



Note : Be sure to use a variety of colors to represent the epipolar lines in your resulting image.

You should attach both pairs('warrior' and 'graffit') of resulting images to the report.

Deliverables

You will hand in your assignment in PLATO. You should hand in one zip file including two files, a file containing your code (i.e., *.py file). This file must have sufficient comments for others to easily use and understand the code. In addition, hand in a PDF document showing scripts (i.e., records of your interactions with the Python shell) showing the specified tests of your functions as well as the images and other answers requested. The PDF file has to be organized and easily readable / accessible. Assignments are to be handed in before 11:59pm on their due date