Daneshvarian Maryam Sadat 2016-12631

(I was typing of all the hand written of math parts but, unfortunately my word did stopped responding then all what I typed deleted so I had not enough time so did rewrite the removed parts by drawing with pen on word.)

Theory Questions

#1

The light reflected from a Lambertian surface depends on the spectral properties of the surface reflectance and of the illumination incident on the surface. In this surface, the light is simply the product of the spectral power distribution of the light source with the percent spectral reflectance of the surface.

Assuming one single point source light, illumination, surface reflection and sensor function, combine together in forming a sensor response, the light reflected from source at location x is proportional to illumination striking surface reflectance at location x and it is projected on the sensor array. The precise power of the reflected light is governed by the dot-product of unit vector corresponding to the surface normal at x and direction of the light source.

In consequence, if the light indent at x is a combination of 2 point source lights with lighting directions s1 and s2 then all lighting vectors can be combines into single effective direction vector. This means that the camera response to two lights equals the sum of the responses to each individual light.

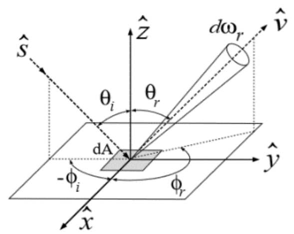
Reference : Computer Vision - ECCV'98: 5th European Conference on Computer Vision, Freiburg, Germany, June 2-6, 1998, Proceedings

#2

The icosahedro has 20 triangles as its faces , when projected onto the sphere, covers the entire sphere so one face covers 1/20 of the sphere. The sphere has solid angule 4pi, therefore one face of icosahedro covers 4pi/20 equals to pi/5

#3

In lambertian surface radiance is independent of the viewing direction of the sensor; it appears equally bright from all directions so Its BRDF is fr where p is the albedo of the surface, it is actually the fraction of incident energy that is reflected by the surface.



By conservation of energy

Conservation of energy:

$$\int f(\theta_i, \phi_i, \theta_r, \phi_r) \cos\theta_r \, d\omega_i \leq 1$$

Here as

$$\boxed{\text{BRDF} : f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L^{surf}(\theta_r, \phi_r)}{E^{surf}(\theta_i, \phi_i)}}$$

Then we have $f_r = \dfrac{\rho}{\Phi}$ $\quad L = \dfrac{d^2 \Phi_r}{dA \cos\theta_r \, dw}$

$$f_r = (\theta_r, \theta_i, \phi_r - \phi_i) = \frac{dL_r}{dE}$$

$$L_r = (\theta_r, \phi_r; \theta_i, \phi_i) = \frac{d^2 \Phi_r}{dA \cos\theta_r \, dw_r}$$

So as phi is equal to pi for surface then we can say that albedo should between zero and 1

#4

$x_0 = a$
$y_0 = b$
$z_0 = 0$

$$\underbrace{(x-a)^2 + (y-b)^2 + z^2 - R^2}_{F(x,y,z)} = 0 \qquad F(x,y,z) = 0$$

$$\frac{\partial F}{\partial x} = 2(x-a) \qquad \frac{\partial F}{\partial y} = 2(y-b) \qquad \frac{\partial F}{\partial z} = 2z$$

$$\nabla F = \langle 2(x-a), 2(y-b), 2z \rangle$$

$$\nabla F(u,v) = \langle 2(u-a), 2(v-b), 2z \rangle$$

As point $P(u,v)$ is on surface and origin $o(a,b)$ is on (x,y,z) then z will be equal to R

$$\vec{N} = \frac{\nabla F(u,v,R)}{\|\nabla F(u,v,R)\|}$$

Normal unit vector of sphere surface on Image coordinate

$$\|\nabla F(u,v,R)\| = \sqrt{4(u-a)^2 + 4(v-b)^2 + 4R^2}$$

#5

Lambertian Case :

K source brightness

ρ surface albedo (reflectance)

c normalization constant

$$I = \frac{\rho}{\pi} kC \cos\theta_i = \rho n s$$

Image intensity:

Let $\frac{\rho}{\pi} kC = 1$ then $I = \cos\theta_i = n \cdot s$

So if we take 3 images then we will have

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$
$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$
$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

If write in a matrix then we have :

$$\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

If we consider s as 3*3 matrix and n and I as 3*1 vectors then $\dot{n} = s^{-1} I$

Then $\rho = |\dot{n}|$ and finally:

$$n = \frac{\dot{n}}{|\dot{n}|} = \frac{\dot{n}}{\rho}$$

For more than three light sources will have better results :

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_N^T \end{bmatrix} \rho \mathbf{n}$$

$$I = S\dot{n}$$

$$s^T I = s^T s \dot{n}$$

$$\dot{n} = (s^T s)^{-1} s^T I$$