NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2017 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 8 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae and Physical Constants are provided in Appendix A on pages 6 to 8.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) A wire in free space, carrying uniform line charge density $\rho_l = 2$ nC/m, lies on the z-axis between $z = 1$ m and $z = 3$ m as shown in Figure 1 on page 2. Find the electric field intensity $\vec{E}$ at P(4 m, 0, 0).

It is given that $\int \frac{dz}{(a+z^2)^{1.5}} = \frac{z}{a\sqrt{a+z^2}}$ and $\int \frac{zdz}{(b+z^2)^{1.5}} = \frac{-1}{\sqrt{b+z^2}}$

(13 Marks)

Note: Question No. 1 continues on page 2.

14.

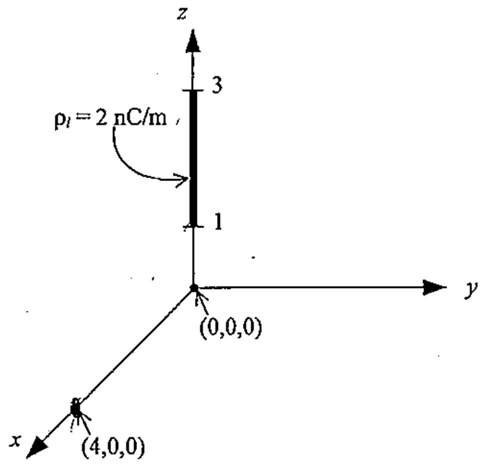

Figure 1

(b) A thin wire carries a steady current of 10 A which is directed from $+\infty$ to the origin along the x-axis and then back to $+\infty$ along the y-axis as shown in Figure 2. Find $\vec{H}$ at P(0, 0, 1 m). (Hint: Use Biot-Savart Law.)

It is given that $\int \frac{dz}{(a+z^2)^{1.5}} = \frac{z}{a\sqrt{a+z^2}}$

(12 Marks)

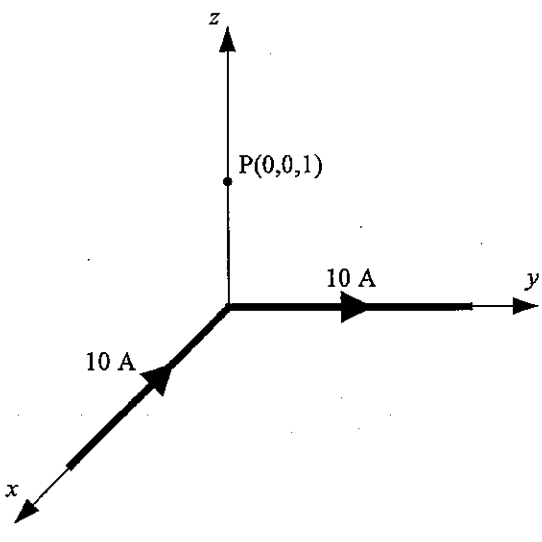

Figure 2

2. (a) For a certain medium with $\mu = 10^{-5}$ H/m, $\varepsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$, find m such that each of the following pairs of fields satisfies Maxwell's equations:

(i) $\vec{D} = 6\vec{a}_x - 2y\vec{a}_y + 2z\vec{a}_z$ nC/m^2, $\vec{H} = mx\vec{a}_x + 10y\vec{a}_y - 25z\vec{a}_z$ A/m.

(ii) $\vec{E} = (20y - mt)\vec{a}_x$ V/m, $\vec{H} = (y + 2\times10^6 t)\vec{a}_z$ A/m.

(12 Marks)

(b) The magnetic field intensity of a uniform plane wave propagating in a non-magnetic medium is

$$\tilde{H}(z,t) = \vec{a}_y 10\, e^{-10\pi z}\cos\left(5\pi\times10^7 t - 10\pi z\right) \text{ mA/m}$$

(i) Determine the wavelength, phase velocity, conductivity and complex intrinsic impedance.

(ii) This material is used for providing electromagnetic shielding to an electronic device operating at this frequency. What should be the thickness of this material to ensure that the field amplitude outside the shield drops to 1% of the value inside the shield? Ignore the effect of reflections at the interface.

(13 Marks)

3. (a) Determine the polarisation (linear, circular or elliptical) and the time average Poynting vector of a uniform plane wave (UPW) given by

$$\tilde{E}(y,t) = \vec{a}_x 80\sin\left(2\pi\times10^8 t + ky + 35°\right) - \vec{a}_y 80\cos\left(2\pi\times10^8 t + ky - 55°\right) \text{V/m}$$

Assume the UPW is travelling in a lossless medium having $\mu = \mu_o$ and $\varepsilon = 2.1\varepsilon_o$.

(8 Marks)

Note: Question No. 3 continues on page 4.

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(b) The time domain expression for the magnetic field of a UPW travelling in free space $(z \le 0)$ is given by

$$\tilde{H}_i(z,t) = \vec{a}_x 20\cos\left(\pi \times 10^7 t - k_i z\right) \text{ mA/m}$$

The UPW is incident normally on a plane interface at $z = 0$ with a lossy medium having intrinsic wave impedance $\eta_c = 200\angle 10°\ \Omega$ occupying the region $z \ge 0$.

Find the following:

(i) The wavenumber k_i in free space.

(ii) The reflection coefficient at the planar interface, i.e., Γ.

(iii) The time-domain expression for the reflected magnetic field, i.e., $\tilde{H}_r(z,t)$.

(9 Marks)

(c) The phasor expression for the electric field of a UPW in free space occupying the region $z \le 0$ is given by

$$\vec{E}_i(x,z) = \vec{a}_y 500 e^{-j2\pi(6x+2z)} \text{ V/m}$$

The UPW is incident at a plane interface with a lossless dielectric medium having $\mu = \mu_o$ and $\varepsilon = 2.8\varepsilon_o$ occupying the region $z \ge 0$.

Find the following:

(i) The polarisation of the incident UPW with respect to the plane of incidence. Briefly explain your answer.

(ii) The percentage of average incident power transmitted into the lossless dielectric medium.

(8 Marks)

4. (a) A lossless transmission line has a characteristic impedance $Z_o = 75\ \Omega$. When the transmission line is terminated with a short-circuit, the input impedance is measured to be $-j25\ \Omega$. When the transmission line is terminated with an unknown impedance Z_L, the input impedance is measured to be $135 - j30\ \Omega$.

Find the following and state any assumption(s) made.

(i) The electrical length ℓ/λ of the transmission line.

(ii) The unknown load impedance Z_L.

(iii) The standing wave ratio (SWR) when the line is terminated with Z_L.

(10 Marks)

(b) A generator having an open-circuit voltage $V_g(t) = 45\cos(2\pi \times 10^9 t)$ V and an internal impedance $Z_g = 50\ \Omega$ is connected to a 20-cm long lossless transmission line having characteristic impedance $Z_o = 50\ \Omega$ and a phase velocity $u_p = 3\times10^8$ m/s. The transmission line is terminated with a load impedance $Z_L = 40 + j60\ \Omega$.

Assuming that the load is located at $z = 0$ and the generator is located at $z = -\ell$ where ℓ is the length of the transmission line, find the following and state any assumption(s) made:

(i) The input impedance $Z_{in}(z)$ at $z = -\ell$.

(ii) The average power delivered to the load.

(iii) The position(s) of maximum magnitude of current on the line.

(iv) The magnitude of maximum current on the line.

(15 Marks)

The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

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Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

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Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_{/\!/} = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_{/\!/} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \, e^{-j\beta z} + V_o^- \, e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \, e^{-j\beta z} - V_o^- \, e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

EE3001 - ENGINEERING ELECTROMAGNETICS
SEM 1 - 2017 - 2018

1a/ $\vec{E} = \int \frac{dQ}{4\pi\varepsilon_0 R^3}\vec{R} = \int \frac{\rho_l\, dz}{4\pi\varepsilon_0 (4^2+z^2)^{3/2}}(4\vec{a_x} - z\vec{a_z})$

$= \frac{\rho_l}{4\pi\varepsilon_0}\left[4\vec{a_x}\int_1^3 \frac{dz}{(16+z^2)^{1.5}} - \vec{a_z}\int_1^3 \frac{z\,dz}{(16+z^2)^{1.5}}\right]$

$= \frac{2\times10^{-9}}{4\pi\varepsilon_0}\left[4\vec{a_x}\left(\frac{z}{16\sqrt{16+z^2}}\right)_1^3 - \vec{a_z}\left(\frac{-1}{\sqrt{16+z^2}}\right)_1^3\right]$

$= 1.58\,\vec{a_x} - 0.76\,\vec{a_z} \quad (V/m)$

1b/ $\vec{H_x} = \frac{1}{4\pi}\int \frac{I\vec{dl}\times\vec{R}}{R^3} = \frac{1}{4\pi}\int \frac{10\,(dy\,\vec{a_y})\times(\vec{a_z} - y\vec{a_y})}{(1+y^2)^{3/2}}$

$= \frac{10\vec{a_x}}{4\pi}\int_0^\infty \frac{dy}{(1+y^2)^{1.5}} = \frac{10\vec{a_x}}{4\pi}\left[\frac{y}{1\sqrt{1+y^2}}\right]_0^\infty = \frac{10}{4\pi}\vec{a_x}$

$\vec{H_x}$ created by wire on y-axis, $\vec{H_y}$ created by wire x-axis.
Similarly $\vec{H_y} = \frac{10}{4\pi}\vec{a_y}$

$\vec{H} = \vec{H_x} + \vec{H_y} = \frac{10}{4\pi}(\vec{a_x} + \vec{a_y}) \quad (A/m)$

2a/ (i) $\nabla\cdot\vec{B} = 0$

$\nabla\cdot\vec{H} = 0$

$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$

$m + 10 - 25 = 0$

$m = 15$

(ii) $\rho_v = 0 \Rightarrow \vec{J} = 0$

$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J} \Leftrightarrow \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & y+2\times10^6 t \end{vmatrix} = \varepsilon\frac{\partial\vec{E}}{\partial t}$

$\Leftrightarrow \vec{a_x} = -\varepsilon m\,\vec{a_x} \Leftrightarrow m = \frac{1}{-\varepsilon} = -250\times10^6$

2b/ (i) $jk = 10\pi + j\,10\pi = \alpha + j\beta$

$\lambda = \frac{2\pi}{\alpha} = \frac{2\pi}{10\pi} = 0.5\ (m)$

$u_p = \frac{\omega}{\alpha} = \frac{5\pi \times 10^7}{10\pi} = 5 \times 10^6\ (m/s)$

$\alpha = \beta \Leftrightarrow$ good conductor

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \Leftrightarrow \sigma = \frac{2\alpha^2}{\omega\mu} = \frac{2\times(10\pi)^2}{5\pi\times10^7\times\mu_0} = 10\ (\Omega/m)$

$\eta_c = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\frac{\pi}{\alpha}} = \sqrt{\frac{5\pi\times10^7\times\mu_0}{10}}\, e^{j\frac{\pi}{10\pi}} = 4.44\, e^{0.1j}\ (\Omega)$

(ii) $\vec{S}(z) = \vec{S}(0)\, e^{-2\alpha z} \Leftrightarrow e^{-2\alpha z} = 1\%$

$z = \frac{-\ln(0.01)}{2\times10\pi} = 0.073\ (m)$

3a/ $\vec{E} = \vec{a_x}\, 80\cos(2\pi\times10^8 t + ky - 55) - \vec{a_y}\, 80\cos(2\pi\times10^8 t + ky - 55)$

$\Delta\varphi = 0,\ |E_y| = |E_x| \Rightarrow$ linear polarization

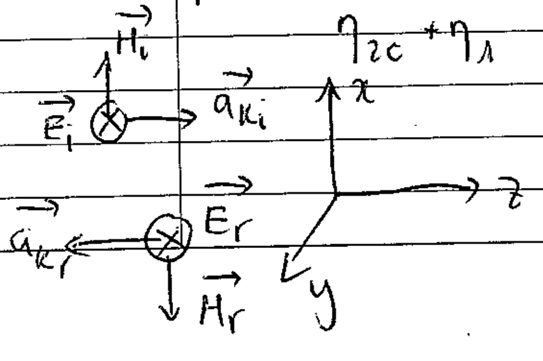
travelling direction: $\vec{a_k} = -\vec{a_y}$

$\vec{S} = \vec{a_k}\frac{|E_0|^2}{2\eta} = -\vec{a_y}\frac{(80\sqrt{2})^2}{2\sqrt{\frac{\mu_0}{2.1\varepsilon_0}}} = -24.62\,\vec{a_y}\ (W/m^2)$

b/ $k_i = \omega\sqrt{\mu_0\varepsilon_0} = \pi\times10^7\sqrt{\mu_0\varepsilon_0} = 0.105$

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73$

$\Gamma = \frac{\eta_{2c} - \eta_1}{\eta_{2c} + \eta_1} = \frac{200\angle10^\circ - 376.73}{200\angle10^\circ + 376.73} = 0.319\angle165.6^\circ$



$H_r = H_i \times \Gamma = 20e^{-jkz}\times 0.319\angle165^\circ$

$H_r = 6.37\angle165\, e^{-jkz}$

$\vec{H_r}(z,t) = -\vec{a_x}\, 6.37\cos(\pi 10^7 t - 0.105z + 165^\circ)$

3(c) (i) linear polarization because only got $\vec{a_y}$ component

(ii) $\theta_i = \tan^{-1}\frac{6}{2} = 71.57°$

$\sqrt{\varepsilon_0\mu_0}\sin\theta_i = \sqrt{\varepsilon_r\varepsilon_0\mu_r\mu_0}\sin\theta_t$

$\theta_t = \sin^{-1}\left(\sin 71.57 \times \frac{1}{\sqrt{2.8}}\right) = 34.54°$

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73\,(\Omega)$

$\eta_2 = \sqrt{\frac{\mu_0}{2.8\varepsilon_0}} = 225.14\,(\Omega)$

$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.627$

$\frac{P_t}{P_i} = 1-|\Gamma|^2 = 0.6069 = 60.69\%$

4a/(i) Short-circuit: $Z_{in}(-l) = \frac{Z_0}{j\tan\beta l}$

$\tan\beta l = \frac{Z_0}{jZ_{in}(-l)} = \frac{75}{j(-j25)} = 3$

$2\pi\frac{l}{\lambda} = 1.249$

$\frac{l}{\lambda} = 0.199$

(ii) unknow Z_L:

$Z_{in}(-l) = \frac{Z_L + j3Z_0}{Z_0 + 3jZ_L} Z_0$

$135 - j30 = \frac{Z_L + j\times3\times75}{75 + 3j\times Z_L}\times 75$

$Z_L = 46.23 + 26.71j$

(iii) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1}{\sqrt{10}}\angle 2.17\,\text{rad}$

$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 1.925$

$$\beta = \frac{\omega}{u_p} = \frac{20\pi}{3}$$

4b / $Z_{in}(-l) = \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} Z_0 = \frac{(40+60j) + j50\tan\left(0.2 \times \frac{20\pi}{3}\right)}{50 + j(40+60j)\tan\left(0.2\times\frac{20\pi}{3}\right)} \times 50$

$= 51.90 - 69j \ (\Omega)$

$P_L = \frac{1}{2} \times |I|^2 \times Re(Z_{in})$ (assume lossless medium)

$= \frac{1}{2} \times \left| \frac{V_g}{Z_g + Z_{in}} \right|^2 Re(Z_{in})$

$= \frac{1}{2} \left| \frac{45}{50 + 51.9 - 69j} \right|^2 \times 51.9 = 3.457 (W)$

(iii) Max current at min voltage z_{min}

$\theta_0 + 2\beta z_{min} = \pi, -\pi.$

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.56\angle 1.14$

$1.14 + 2 \times \frac{20\pi}{3} z_{m} = -\pi$

$z_{min} = -0.1022 \ (m)$

Max current at 10.22 (cm)

(iv) $I(z) = \frac{1}{Z_0}\left(V_0^+ e^{-\beta t} - V_0^- e^{+j\beta z}\right)$

$= \frac{V_0^+}{Z_0} e^{-j\beta t}\left(1 - \Gamma_L e^{+j\beta z 2}\right)$

- Find V_0^+:

$V(-l) = \frac{Z_{in}}{Z_{in} + Z_g} V_g = V_0^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right)$

$\frac{51.9 - 69j}{51.9 - 69j + 50} \times 45 = V_0^+ e^{j\frac{20 \times 0.2\pi}{3}}\left(1 + 0.56\angle 1.14 \times e^{-j2\times\frac{20\pi}{3}\times 0.2}\right)$

$V_0^+ = 22.54\angle 2.10 \ (V)$

- Calculate I_{max} $(I(z_{min}))$:

$I_{max} = \frac{22.54\angle 2.1}{50} e^{j\frac{20\pi}{3}\times 0.1022}\left(1 - 0.56\angle 1.14 \times e^{-j2\times\frac{20\pi}{3}\times 0.1022}\right)$

$= 0.7\angle -1.99$

$|I_{max}| = 0.7 \ A$