NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) Three parallel circular loops of radius a are located at $z = 0, +d, -d$ respectively in free space with their centers aligned along the z axis. Each circular loop has uniform charge distribution with line charge density ρ_l.

   (i) Find the electric potential $V_0(z), V_{+d}(z), V_{-d}(z)$ along the z axis due to each circular loop located at $z = 0, +d, -d$ respectively.

   (ii) Determine the total electric potential $V(z)$ and the electric field intensity $\bar{E}(z)$ along the z axis due to all three circular loops.

(13 Marks)

Note: Question No. 1 continues on page 2.

(b) Assume that a steady current I flows in each circular loop of part (a) in the counter clockwise direction (as viewed from $z > d$).

(i) Determine the magnetic field intensity $\vec{H}_0(z), \vec{H}_{+d}(z), \vec{H}_{-d}(z)$ along the z axis due to each current loop located at $z = 0, +d, -d$ respectively.

(ii) Find the total magnetic field intensity at the origin.

(12 Marks)

2. (a) Figure 1 shows a square loop of side length L which has its left side lying at rest along the y axis in free space at time $t = 0$. The whole loop is accelerated with constant acceleration a_0 toward $+x$ direction while being subjected to magnetic flux density of the form $\vec{B} = \dfrac{-\mu_0}{2\pi x}\vec{a}_z$ T.

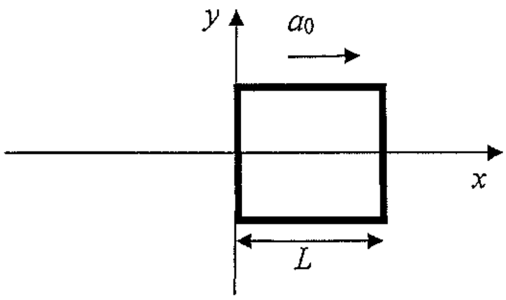


Figure 1

(i) Derive the expression of magnetic flux Φ_m passing through the loop at time t.

(ii) Determine the induced voltage V_{emf} in the loop at time t.

(10 Marks)

(b) A 1-MHz plane wave is propagating along the +z direction in a lossy nonmagnetic medium with relative permittivity ε_r and conductivity σ.

(i) Let us assume that the medium is a very good conductor. What can you say about the relationship between the real and imaginary parts of propagation constant γ? What is the phase angle of γ? Also, what can you say about the relationship between the real and imaginary parts of intrinsic impedance η_c? What is the phase angle of η_c?

Note: Question No. 2 continues on page 3.

(ii) Given that $\varepsilon_r = 1$ and $\sigma = 0.01$ S/m, calculate the real and imaginary parts, as well as the magnitude and phase angle of γ and η_c. Are the results consistent with your answers in part (i)? How can you tell whether the assumption of very good conductor is valid or not?

(15 Marks)

3. (a) The electric field of a uniform plane wave (UPW) in air occupying the region $z \leq 0$ is given by:

$$\vec{E}_i(z) = \left(50\vec{a}_x - j140\vec{a}_y\right) e^{-j6\pi z} \ V/\text{m}$$

The UPW is incident normally on a planar interface with a lossy medium having intrinsic impedance $\eta_c = 150\angle 0.2\ \Omega$ occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) The frequency of the UPW.

(ii) The polarization (Linear, Circular or Elliptical) of the UPW. Briefly explain your answer.

(iii) The reflection coefficient Γ (in polar form) at the planar interface.

(iv) The position z at which the total electric field in the air medium is minimum i.e., z_{min}.

(12 Marks)

(b) The magnetic field of a uniform plane wave (UPW) propagating in free space $(z \leq 0)$ is given by:

$$\vec{H}_i = \vec{a}_y 75 e^{-j(0.35x + 0.25z)} \text{ mA/m}$$

The UPW is obliquely incident on a lossless dielectric having $\mu = \mu_o$, $\varepsilon = 1.6\varepsilon_o$ at $z = 0$, and occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) The angle of incidence θ_i (in degrees) and angle of transmission θ_t (in degrees) of the incident UPW.

(ii) The time-average Poynting vector of the incident and reflected waves, i.e., $\vec{S}_i$ and $\vec{S}_r$.

(13 Marks)

4. (a) A 50 Ω air-filled transmission line is terminated in a load $Z_L = 68 - j27\ \Omega$. The line is operating at 900 MHz.

Find the following:

(i) The reflection coefficient Γ_L (in polar form) and the standing wave ratio (SWR) due to this load.

(ii) The shortest length of the line to make the impedance at the input end purely resistive and as large as possible.

(iii) A 75 Ω generator with maximum power available $P_{av} = 5$ W is connected to the transmission line in part (ii). Find the average power delivered to the load.

(13 Marks)

(b) A 50-cm long lossless transmission line having characteristic impedance $Z_0 = 75\ \Omega$ and a phase velocity $u_p = 3\times10^8$ m/s is open-circuited at the load end. A generator having an open-circuit voltage $V_g(t) = 100\cos(4\pi\times10^8 t - \pi/6)$ V and an internal impedance $Z_g = 75\ \Omega$ is connected to the transmission line.

Assume that the load is located at $z = 0$ and the generator is at $z = -\ell$ where ℓ is the length of the transmission line.

Find the following and state any assumption(s) made:

(i) The input impedance at the source end of the transmission line, i.e., $Z_{in}(-\ell)$.

(ii) The time-domain expression for the voltge at $z = -\ell$, i.e., $V(-\ell, t)$.

(12 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\parallel = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\parallel = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \; e^{-j\beta z} + V_o^- \; e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \; e^{-j\beta z} - V_o^- \; e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

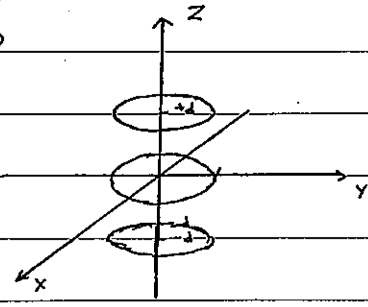
$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

36

EE3001 - ENGINEERING ELECTROMAGNETICS 2019/2020 Sem 1

1 a) i)



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_\ell \, d\ell}{R}$$

$$V_0(z) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_\ell \, d\ell}{\sqrt{z^2+a^2}} \qquad d\ell = a\,d\theta$$

$$V_0(z) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\rho_\ell \, a\, d\theta}{\sqrt{z^2+a^2}} = \frac{\rho_\ell (2\pi) a}{4\pi\varepsilon_0\sqrt{z^2+a^2}} = \frac{\rho_\ell a}{2\varepsilon_0\sqrt{z^2+a^2}}$$

$$V_{+d}(z) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_\ell \, d\ell}{\sqrt{(z')^2+a^2}}$$

$(z') = z-d \qquad \rho_\ell = a\,d\theta$

$$V_{+d}(z) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\rho_\ell \, a\, d\theta}{\sqrt{a^2+(z-d)^2}} = \frac{\rho_\ell (2\pi)(a)}{4\pi\varepsilon_0\sqrt{a^2+(z-d)^2}} = \frac{\rho_\ell a}{2\varepsilon_0\sqrt{a^2+(z-d)^2}}$$

$$V_{-d}(z) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_\ell \, d\ell}{\sqrt{(z^*)^2+a^2}}$$

$z^* = z+d \qquad \rho_\ell = a\,d\theta$

$$V_{-d}(z) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\rho_\ell \, a\, d\theta}{\sqrt{a^2+(z+d)^2}} = \frac{\rho_\ell (2\pi)(a)}{4\pi\varepsilon_0\sqrt{a^2+(z+d)^2}} = \frac{\rho_\ell a}{2\varepsilon_0\sqrt{a^2+(z+d)^2}}$$

1 a) ii) $\vec{E} = -\nabla V$

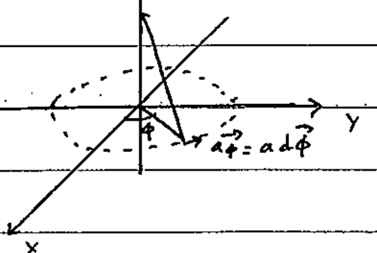
$$= -\vec{a_x}\frac{\partial V}{\partial x} - \vec{a_y}\frac{\partial V}{\partial y} - \vec{a_z}\frac{\partial V}{\partial z}$$

$$V = \frac{\rho_\ell a}{2\varepsilon_0}\left(\frac{1}{\sqrt{a^2+z^2}} + \frac{1}{\sqrt{a^2+(z+d)^2}} + \frac{1}{\sqrt{a^2+(z-d)^2}}\right)$$

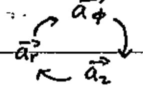
$$\frac{\partial V}{\partial x} = 0 \qquad \frac{\partial V}{\partial y} = 0 \qquad \frac{\partial V}{\partial z} = \frac{\rho_\ell a(2z)(-1/2)}{2\varepsilon_0(z^2+a^2)^{3/2}} + \frac{\rho_\ell a(2)(z+d)(-1/2)}{2\varepsilon_0((z+d)^2+a^2)^{3/2}} + \frac{\rho_\ell(a)(2)(z-d)(-1/2)}{2\varepsilon_0((z-d)^2+a^2)^{3/2}}$$

$$\frac{\partial V}{\partial z} = \frac{-\rho_\ell a}{2\varepsilon_0}\left(\frac{z}{(z^2+a^2)^{3/2}} + \frac{z+d}{(z^2+a^2)^{3/2}} + \frac{(z-d)}{(z^2+a^2)^{3/2}}\right)$$

1 b) i)



$\vec{r} = z\vec{a_z} \qquad \vec{s} = a\vec{a_r}$



$\vec{R} = \vec{r} - \vec{s} = z\vec{a_z} - a\vec{a_r}$

$d\vec{\ell} = a\,d\phi\,\vec{a_\phi} \qquad R = \sqrt{a^2+z^2}$

$\vec{a_\phi}\times\vec{a_z} = \vec{a_r} \qquad \vec{a_\phi}\times(-\vec{a_r}) = \vec{a_z} \Rightarrow$ by symmetry $\vec{a_\phi}$ and $\vec{a_r} = 0$

$$\vec{H_0}(z) = \frac{I}{4\pi}\int_0^{2\pi}\frac{a\,d\phi\,\vec{a_\phi}(z\vec{a_z}-a\vec{a_r})}{(a^2+z^2)^{3/2}} = \frac{I}{4\pi}\left(\frac{2\pi a^2}{(a^2+z^2)^{3/2}}\vec{a_z}\right) = \frac{Ia^2\vec{a_z}}{2(a^2+z^2)^{3/2}} \qquad z_1' = z^* - d$$

$$\vec{H_{+d}}(z) = \frac{I}{4\pi}\int_0^{2\pi}\frac{a\,d\phi\,\vec{a_\phi}(z\vec{a_z}-a\vec{a_r})}{(a^2+z_1'^2)^{3/2}} = \frac{I}{4\pi}\left(\frac{2\pi a^2\vec{a_z}}{(a^2+(z-d)^2)^{3/2}}\right) = \frac{Ia^2\vec{a_z}}{2(a^2+(z-d)^2)^{3/2}} \qquad z_2' = z+d$$

$$\vec{H_{-d}}(z) = \frac{I}{4\pi}\int_0^{2\pi}\frac{a\,d\phi\,\vec{a_\phi}(z\vec{a_z}-a\vec{a_r})}{(a^2+z_2'^2)^{3/2}} = \frac{I}{4\pi}\left(\frac{2\pi a^2\,\vec{a_z}}{(a^2+(z+d)^2)^{3/2}}\right) = \frac{Ia^2\vec{a_z}}{2(a^2+(z+d)^2)^{3/2}}$$

1(b)ii) at $z=0$

$$H_{total} = H_0 + H_{+d} + H_{-d}$$
$$= \frac{Ia^2}{2}\left(\frac{1}{a^3} + \frac{1}{(a^2+d^2)^{3/2}} + \frac{1}{(a^2+d^2)^{3/2}}\right)$$
$$= \frac{Ia^2}{2}\left(\frac{1}{a^3} + \frac{2}{(a^2+d^2)^{3/2}}\right)$$

2(a)i) $\vec{B} = -\frac{\mu_0}{2\pi x}\vec{a_z}$ $\quad d\vec{A} = \vec{a_z}\,dA \quad \vec{a_z}\cdot\vec{a_z} = 1 \quad dA = dx\,dy$

$$\phi = \iint \vec{B}\cdot d\vec{A} = -\iint \frac{\mu_0}{2\pi x}(\vec{a_z}\cdot\vec{a_z})\,dA = -\frac{\mu_0}{2\pi}\iint \frac{1}{x}\,dx\,dy$$

=> define the boundaries

$$\frac{1}{2}a_0t^2 \le x \le L + \frac{1}{2}a_0t^2 \qquad -\frac{L}{2} \le y \le \frac{L}{2}$$

$$\phi_m = -\int_{-L/2}^{L/2}\int_{\frac{1}{2}a_0t^2}^{L+\frac{1}{2}a_0t^2}\left(\frac{\mu_0}{2\pi x}\right)dx\,dy \qquad \int \frac{1}{x} = \ln(x) + k$$

$$\phi_m = -\frac{\mu_0}{2\pi}\int_{-L/2}^{L/2}\ln\left(\frac{L+\frac{1}{2}a_0t^2}{\frac{1}{2}a_0t^2}\right)dy$$

$$\phi_m = \frac{-\mu_0 L}{2\pi}\ln\left(\frac{L+\frac{1}{2}a_0t^2}{\frac{1}{2}a_0t^2}\right)$$ => the flux direction is out of the exam paper ⊙ due to negative sign

2(a)ii) $V = -N\frac{\partial\phi}{\partial t}$ $\quad N=1$

$$\frac{\partial\phi}{\partial t} = \frac{\mu_0 L}{2\pi}\left(\frac{\frac{1}{2}(a_0t)(2)}{\frac{1}{2}a_0t^2} - \frac{\frac{1}{2}(a_0t)(2)}{L+\frac{1}{2}a_0t^2}\right) = \frac{\mu_0 L}{2\pi}\left(\frac{a_0t}{\frac{1}{2}a_0t^2} + -\frac{a_0t}{L+\frac{1}{2}a_0t^2}\right)$$ => sign doesn't matter only sign of direction

$$e.m.f = \frac{\mu_0 L}{2\pi}\left(\frac{a_0t}{L+\frac{1}{2}a_0t^2} - \frac{2}{t}\right)\ V$$

=> direction of current is counter-clockwise of loop this is due to lenz law as flux ~~into~~ out of paper will reduce, this induces voltage that produces more flux out of the paper

2b) (i) +z direction $\Rightarrow \vec{E} = \vec{a}_x E_0^+ e^{-jkz}$ → Example

$\hookrightarrow -k$

Very good conductor $\Rightarrow \sigma >> \omega\varepsilon$ $f = 10^6$ Hz

Complex propagation constant $\Rightarrow \gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ ① $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$

$\sqrt{j} = \frac{1+j}{\sqrt{2}}$ $\gamma = \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma + j\omega\varepsilon} = \alpha + j\beta = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = \frac{1+j}{\delta}$

Phase angle $\Rightarrow \angle 45^\circ$

$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} = e^{j\pi/4}\sqrt{\frac{\mu\omega}{\sigma}} \Rightarrow \eta_c = \frac{1+j}{\sigma\delta}$ → Same values

$\angle(\eta_c) = 45^\circ$ ②

(ii) $\varepsilon_r = 1$ and $\sigma = 0.01$ S/m $\sigma/\omega\varepsilon > 20 \Rightarrow$ good conductor. $\omega = 2\pi \times 10^6$ Hz

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j7.895(0.01 + j5.56\times10^{-5})}$

$= (0.07895 \angle \pi/2)^{1/2} = 0.198 + 0.199j$ rad/m

$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = (4.3894 \ldots + 789.543939j)^{1/2}$

$= \sqrt{789.556 \angle 89.68^\circ} = 28.1 \angle 44.34^\circ\ \Omega$

$\angle(\eta_c) \approx 45^\circ$

Because the indicators for good conductors ~~(is)~~ are $\alpha \approx \beta$ and $\angle(\eta_c) = \pi/4$ or 45° hence the assumption is true

3) a) $E(z) = (50\vec{a}_x - j140\vec{a}_y)\, e^{-j6\pi z}$ V/m $\qquad \eta_2 = 150\angle 0.2\ \Omega$

(i) $k = 6\pi \quad 6\pi = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{1}{3}$ m $\qquad k = \frac{2\pi}{\lambda} \Rightarrow$ Wave number

$\lambda \cdot f = 3\times10^8$ m/s

$\frac{1}{3} f = 3\times10^8$ Hz

$f = 9\times10^8$ Hz $= 900$ MHz $\Rightarrow$ frequency

(ii) The polarization of UPW is elliptically polarized because the $|\phi_y - \phi_x| = 90°$ but $|E_{0y}| \neq |E_{0x}|$

Note: $\phi_y = 90° \quad \phi_x = 0°$ $\qquad |E_{0y}| = 140$ and $|E_{0x}| = 50$

(iii) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{150\angle 0.2 - 120\pi}{150\angle 0.2 + 120\pi} = 0.44185 \angle 2.95$ rad.

(iv) $z_{min} \Rightarrow \theta_0 + 2k z_{min} = \pi - ...$

$2\times 6\pi z_{min} = \pi - 2.95$

$z_{min} = 0.0163$ m $\left[-0.0163 - \frac{n\lambda}{6} \Rightarrow n = 0, 1, 2, ... \right]$ (general form)

3b) $\vec{H} = \vec{a_y}\, 75\, e^{-j(0.35x+0.25z)}$ mA/m $\mu = \mu_0$, $\varepsilon = 1.6\varepsilon_0$

(i) Because the magnetic field has only y-component, the magnetic field is perpendicularly polarized $\#$

Use the angle of incidence θ_i (in degrees) and the angle of transmission θ_t (in degrees) formula from the appendix.

$k_i = 0.35a_x + 0.25a_z \Rightarrow \vec{a_{ki}} = 0.8137a_x + 0.5812\vec{a_z}$ $\tan\theta_i = \frac{k_{xi}}{k_{zi}} = \frac{0.35}{0.25} \Rightarrow \theta_i = 0.9505$ rad or $54.462°$ $\#$ $\#$

$k_i = 0.4301$

Electric field is parallel polarized

$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \Rightarrow \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_0\varepsilon_0}{\mu_0 \times 1.6\varepsilon_0}} = 0.79056$ $\Gamma_{//} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = 0.0200$

$\sin\theta_t = 0.64331$

$\theta_t = 40.039°$ $\#$ $E_{or} = \Gamma E_{oi} = 7.0608$ $\#$

$E_{oi} = 75 \times 120\pi \times 10^{-3} = 28.261$ $\#$

(ii) $\vec{S_i} = \vec{a_{ki}} \frac{|E_{oi}|^2}{2\eta}$ W/m² $= \vec{a_{ki}} \frac{128.261^2}{2 \times 120\pi}$ W/m² $= 1.0593\, \vec{a_{ki}}$ W/m² $\Rightarrow a_{ki} = 0.8137a_x + 0.5812\vec{a_z}$ $\#$

$\vec{S_r} = \vec{a_{kr}} \frac{|E_{or}|^2}{2\eta}$ W/m² $= a_{kr} \frac{0.31947}{2 \times 120\pi}$ W/m² $= 0.0004\, a_{kr}$ W/m² $a_{kr} = 0.8137a_x - 0.5812\vec{a_z}$ $\#$ $\#$

$= 4.237 \times 10^{-4} a_{kr}$ W/m²

$\vec{S_i} = 0.862\vec{a_x} + 0.6156\vec{a_z}$ W/m² $\#$

$\vec{S_r} = 0.0003\vec{a_x} - 0.0002$ W/m² $\Rightarrow 3.447 \times 10^{-4}\vec{a_x} - 2.4626 \times 10^{-4}\vec{a_z}$ W/m² $\#$ $\#$

4 a) $Z_0 = 50\,\Omega$ $\quad Z_L = 68-27j\,\Omega$ $\quad f = 900\,MHz$ $\quad \lambda . f = 3\times10^8\,m/s \Rightarrow \lambda = 1/3\,m$ $\quad \beta = 2\pi/\lambda = 6\pi\ rad/m$ #

(i) Reflection coefficient $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{18-27j}{118-27j} = 0.2680\ \angle -0.75785$ #

$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 1.7322$ #

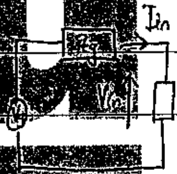
(ii) Using Smith Chart $\Rightarrow$ $z_L = \frac{68-27j}{50} = 1.36 - 0.54j = 0.316\lambda$ TG # . Generator is at 0.75λ TG

Shortest length of the line for it to be purely resistive $\Rightarrow$ $(0.75-0.316)\ \times 1/3\,m = 14.47\ cm$ # from the generator or $-14.47\,cm$ #

1st method.

(iii) $Z_g^* = 75\,\Omega$ $\quad P_{av} = 5\,W$ $\quad \therefore 5W = \frac{V_g^2}{8Z_g}$ $\quad \Rightarrow V_g = 54.772\ V$ # $\quad I_{in} = \frac{V_g}{Z_{in}+Z_g} = 0.339A$ #

$\therefore |Z_{in}(-l)| = |Z_0 \frac{1+\Gamma_L e^{-2\beta l}}{1-\Gamma_L e^{-2\beta l}}| = 86.612\,\Omega$



$V_L = Z_{in}(l)\ I_{in} = 29.36\ V$

$\therefore P_{avg}$ for the load $\Rightarrow \frac{1}{2} Re(V_L I_L^*) = \frac{1}{2} Re(29.36 \times 0.339) = 4.97$ Watt #

2nd method

Calculate equivalent $\Gamma_G = \frac{Z_{in}(-l) - Z_g}{Z_{in}(-l) - Z_g} = 0.0719$

$P_{avL} = (1 - |\Gamma_G|^2) \times P_{av} = 4.97$ watt

4(b) $Z_0 = 75\Omega$ $\omega = 4\pi \times 10^8 \Rightarrow f = 2\times10^8$ $\lambda . f = 10^8 \times 3$ $\beta = 2\pi/\lambda = 4\pi/3$

$l = 0.5m$ $V_{g(t)} = 100\cos(4\pi \times 10^8 t - \pi/6) V$ $\lambda = 3/2 m$

$u_p = 3\times10^8 m/s$ $Z_g = 75\Omega$

(i) $$Z_{in(-l)} = \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)} Z_0$$ Open Circuit $\Rightarrow \Gamma_L = 1 \Rightarrow Z_L = \infty$

$$Z_{in(-l)} = Z_0 * \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}} = 25\sqrt{3}i \Rightarrow 25\sqrt{3} \angle \pi/2 \ \Omega$$

(ii) $$V_{(-l)} = \frac{Z_{in}}{Z_{in}+Z_g} V_g \Rightarrow \frac{1}{2}\angle\frac{1}{3}\pi \times 100\cos(4\pi\times10^8 t - \pi/6)$$

$$= 50\cos(4\pi\times10^8 t + \pi/6) \text{ Volt}$$

Tips and Tricks for EE3001:

- Understand the use of appendix
- For uniform plane wave, make sure to understand the formula of for example, wave number k.
- Make sure to read the questions very carefully especially question 3 and 4 regarding uniform plane wave and transmission line (especially parallel and perpendicular polarized)
- Understand the use of smith chart for question 4