

## Lecture 2

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### EE3010: Electrical Devices and Machines

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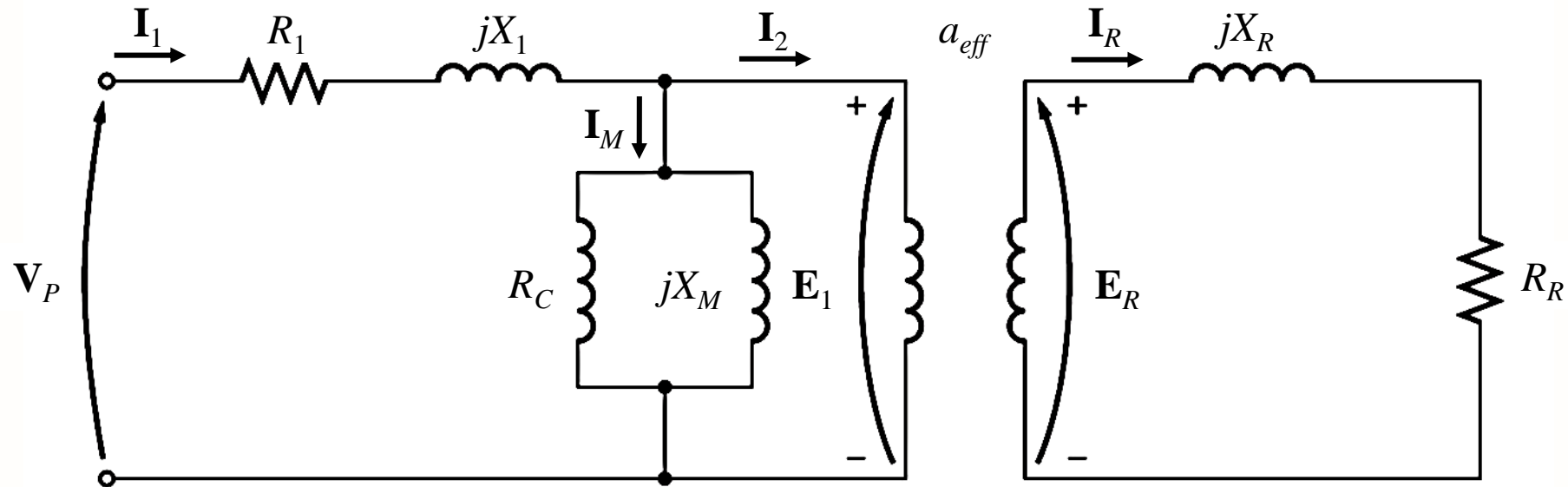
# Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Describe the equivalent circuit of an induction motor.
- ❖ Describe the power flows and power flow diagram of an induction motor.
- ❖ Calculate the various powers and motor efficiency of an induction motor from given power losses and operating conditions.

# The Equivalent Circuit of an Induction Motor

- ❖ Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer action, the equivalent circuit of an induction motor is very similar to that of a transformer.



Per phase equivalent circuit of an induction motor, with rotor and stator connected by an ideal transformer of turns ratio  $a_{eff}$ .

- ❖ The circuit parameters per phase:
- $R_1$  : Stator winding resistance
  - $X_1$  : Stator leakage reactance
  - $R_R$  : Rotor winding resistance
  - $X_R$  : Rotor leakage reactance
  - $a_{eff}$  : Effective turns ratio coupling  $E_1$  and  $E_R$
  - $R_C$  : Resistance representing core loss
  - $X_M$  : Magnetising reactance

# The Rotor Circuit Model

- ❖ In an induction motor, when a voltage is applied to the stator windings, a voltage is induced in the rotor windings.
- ❖ The induced voltage is 0 V when the rotor moves at synchronous speed.
- ❖ When the rotor is stationary (i.e. locked-rotor or blocked-rotor conditions), the largest voltage and rotor frequency are induced in rotor.

# The Rotor Circuit Model

❖ At any other speed,

- The rms value of the induced voltage generated in the rotor as it is swept by the rotating flux is

$$E_R = 4.44(f_r)N_r\phi_{\max} = 4.44(sf_s)N_r\phi_{\max} = sE_{RO}$$

where  $E_{RO} = 4.44(f_s)N_r\phi_{\max}$  is the rotor induced voltage at locked-rotor conditions ( $f_r = f_s$ ,  $s = 1$ ).

- The reactance depends on the inductance and the frequency of the voltage and current in the rotor.

$$X_R = \omega_r L_R = 2\pi(f_r)L_R = 2\pi(sf_s)L_R = sX_{RO}$$

where  $X_{RO}$  is the blocked-rotor reactance.

# Example 1

The frequency and induced voltage in the rotor of a certain six-pole wound-rotor induction motor, whose shaft is blocked, are 60 Hz and 100 V, respectively. Determine the corresponding values when the rotor is running at 1100 r/min.

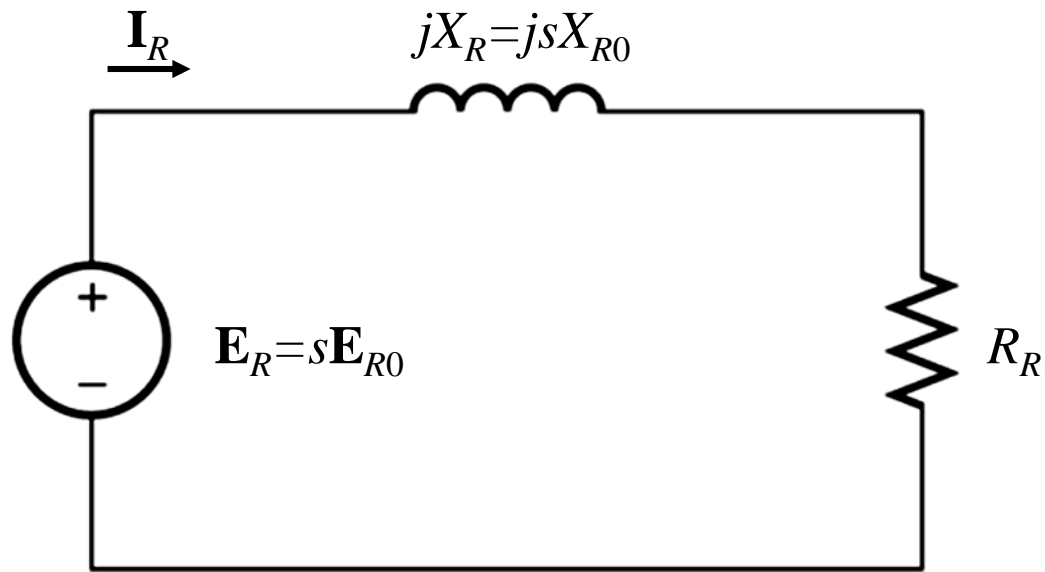
$$n_{sync} = \frac{120 f_s}{p} = \frac{120(60)}{6} = 1200 \text{ r/min}$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} = 0.0833$$

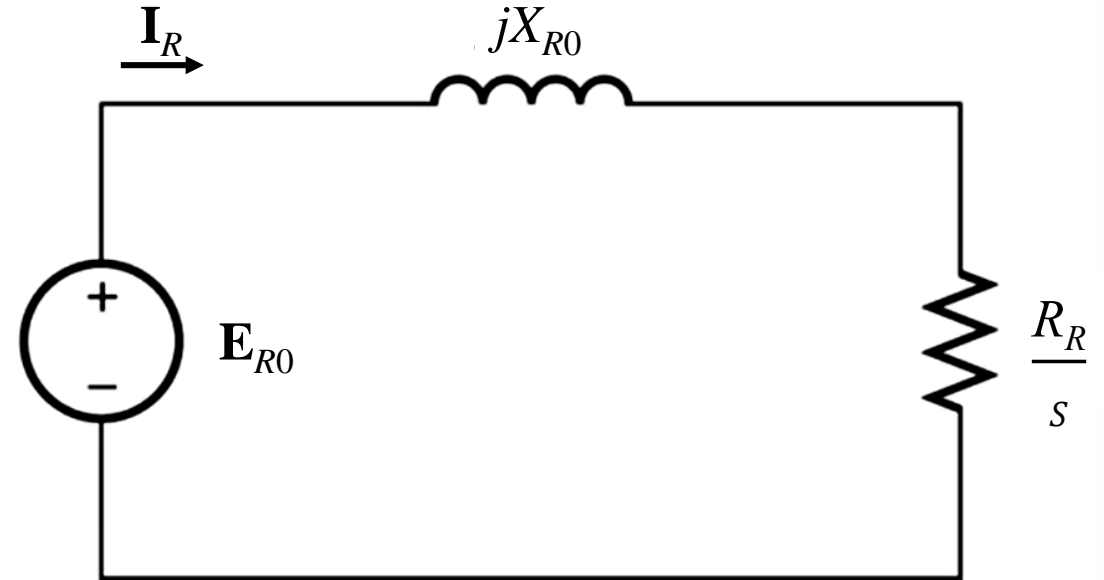
$$f_r = s f_s = 5 \text{ Hz}$$

$$E_R = s E_{RO} = 8.33 \text{ V}$$

# The Rotor Circuit Model



The rotor circuit model of an induction motor.



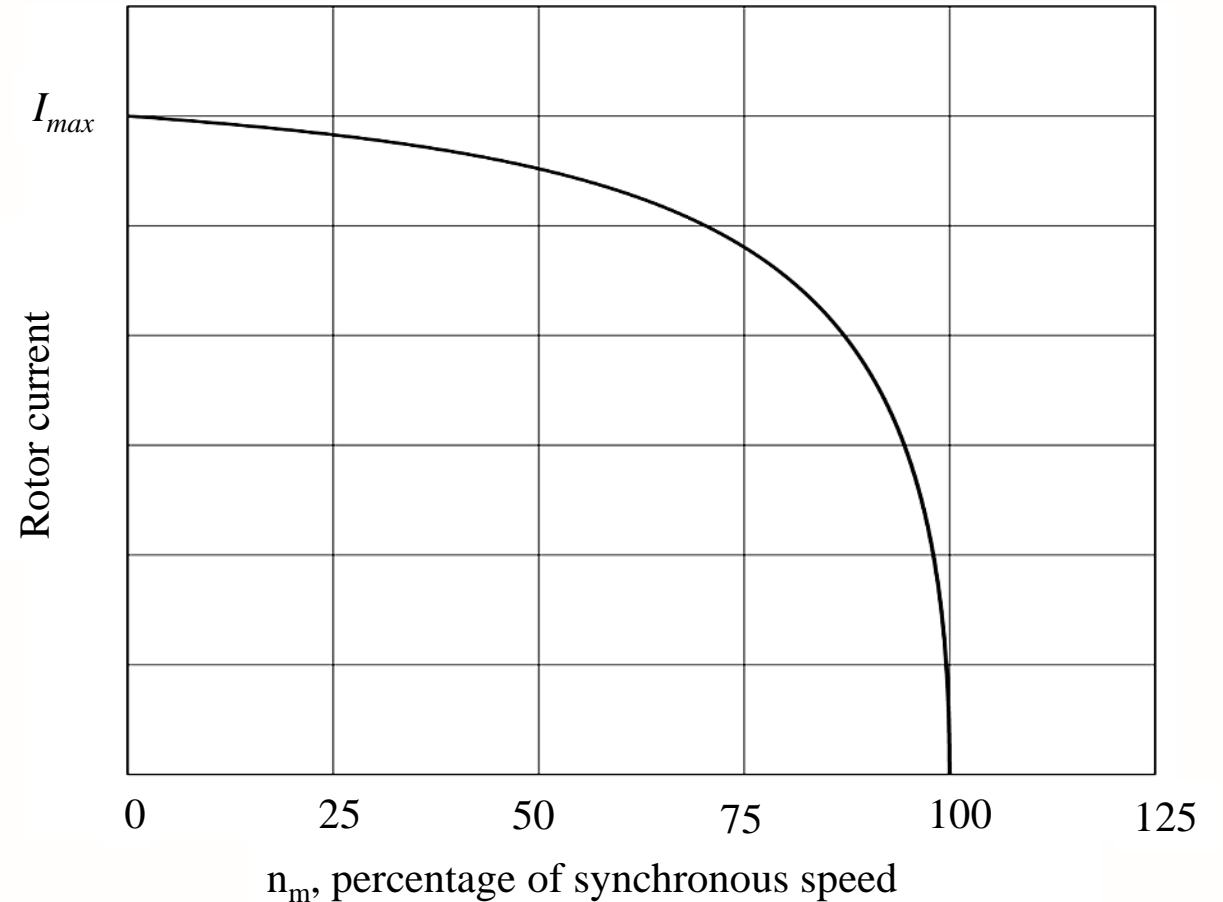
The rotor circuit model with all the frequency (slip) effects concentrated in resistor  $\frac{R_R}{s}$ .



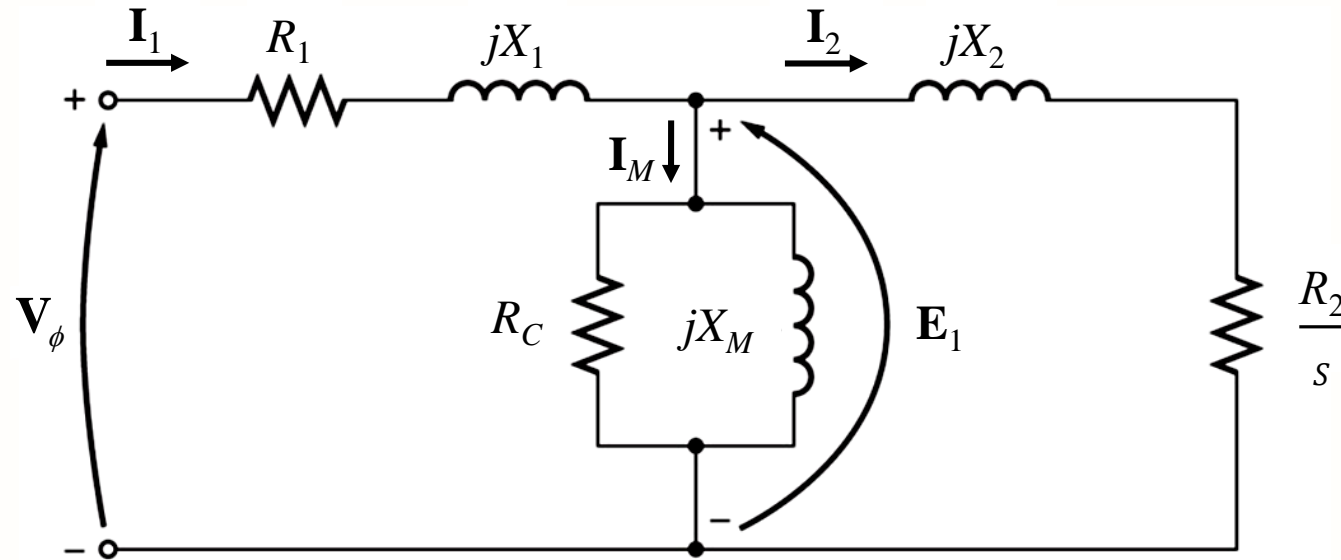
# The Rotor Circuit Model

❖ Rotor current as a function of rotor speed

$$\mathbf{I}_R = \frac{\mathbf{E}_{RO}}{R_R / s + jX_{RO}}$$



# The Final Equivalent Circuit of an Induction Motor



The per phase equivalent circuit of an induction motor referred to the stator.

- ❖ The core losses (W) are usually lumped together with friction, windage and stray losses and called the rotational losses.
- ❖ Hence, unless specified otherwise,  $R_C$  will not appear in the equivalent circuit subsequently.

- ❖ The referred rotor parameters to the stator side are:

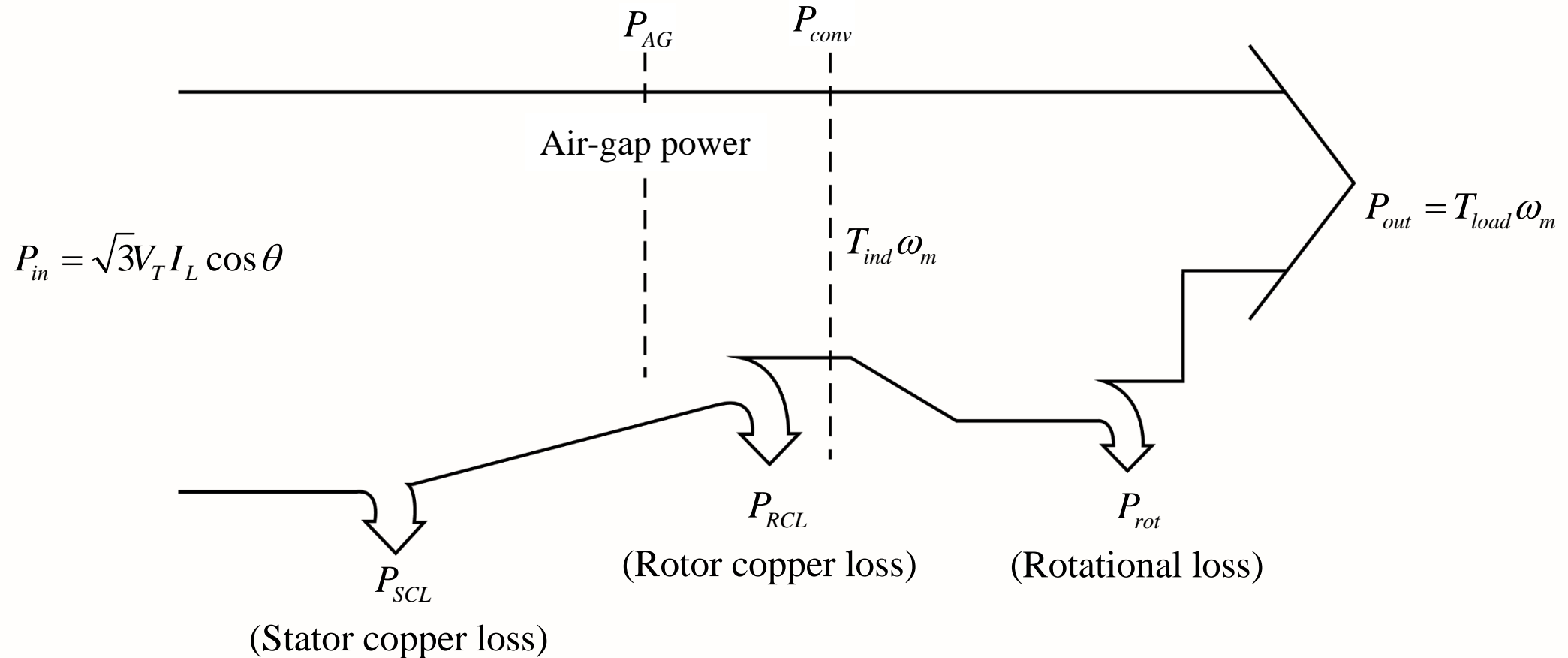
$$\mathbf{E}_1 = a_{eff} \mathbf{E}_{RO} ,$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{eff}} \text{ where } \mathbf{I}_R = \frac{\mathbf{E}_{RO}}{R_R / s + jX_{RO}}$$

$$R_2 = a_{eff}^2 R_R , \quad X_2 = a_{eff}^2 X_{RO}$$

$$a_{eff} = \text{effective turns ratio}$$

# Losses and Power-Flow Diagram in Induction Motors



Power flow diagram of an induction motor.

- ❖ Induction motor basically is a rotating transformer.
- ❖ Input is 3-phase supply.
- ❖ Output is mechanical power.
- ❖ Air gap power is the power transferred to the rotor of the machine across the air gap between the stator and the rotor.
- ❖ The converted power is the power converted from electrical to mechanical form.
- ❖ The core losses are usually lumped together with the friction, windage and stray load losses and called the rotational losses.

## Example 2

A 480 V, 60 Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 pf lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The total rotational losses which includes the friction, windage and core losses are 2400 W, and the stray losses are negligible. Find:

- a) The air-gap power  $P_{AG}$
- b) The converted power  $P_{conv}$
- c) The output power  $P_{out}$
- d) The efficiency of the motor  $\eta$

(Solutions →)

$$\begin{aligned} \text{a)} \quad P_{\text{in}} &= \sqrt{3} V_{\text{line}} I_{\text{line}} \cos \theta \\ &= \sqrt{3} (480) (60) 0.85 = 42.4 \text{ kW} \end{aligned}$$

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 40.4 \text{ kW}$$

$$\text{b)} \quad P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}} = 39700 \text{ W}$$

$$\text{c)} \quad P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 37.3 \text{ kW}$$

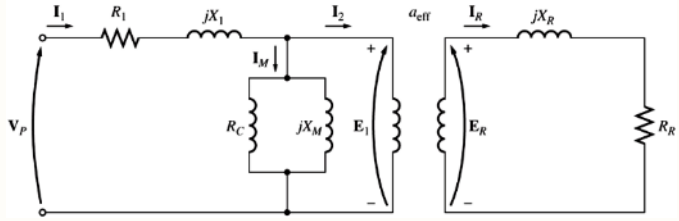
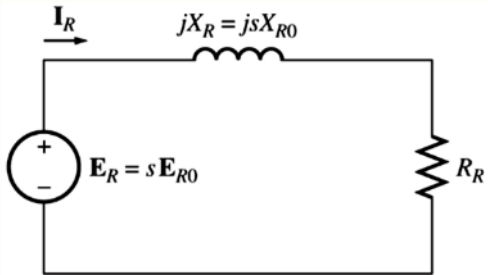
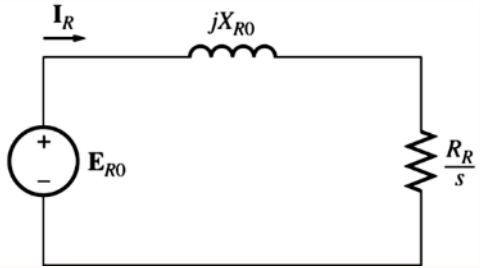
$$\text{d)} \quad \text{The efficiency of the motor } \eta = \frac{P_{\text{out}}}{P_{\text{in}}} (100\%) = 88\%$$

In this lecture, you have learnt:

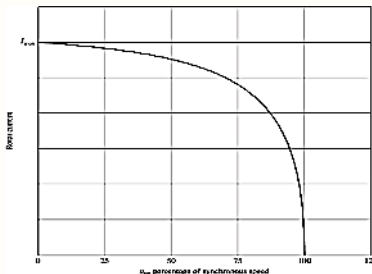
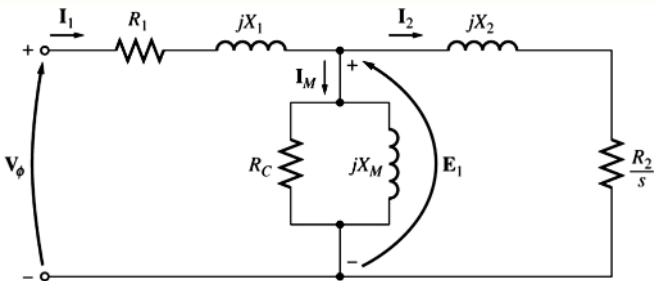
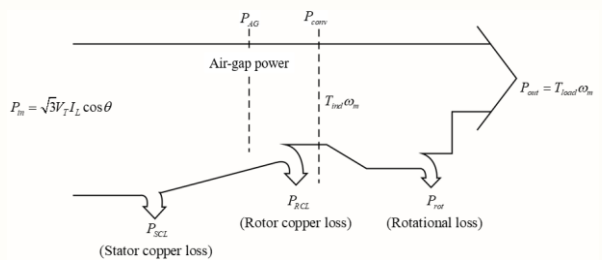
- ❖ The equivalent circuit of an induction motor turns out to be very similar to the equivalent circuit of a transformer.
- ❖ The power flow diagram of an induction motor.
- ❖ The computations of the various powers and motor efficiency of an induction motor from given power losses and operating conditions.



# References

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