EE3001 Engineering Electromagnetics

This session is about

35. Analysis and Design of Transmission Line

Learning Objectives

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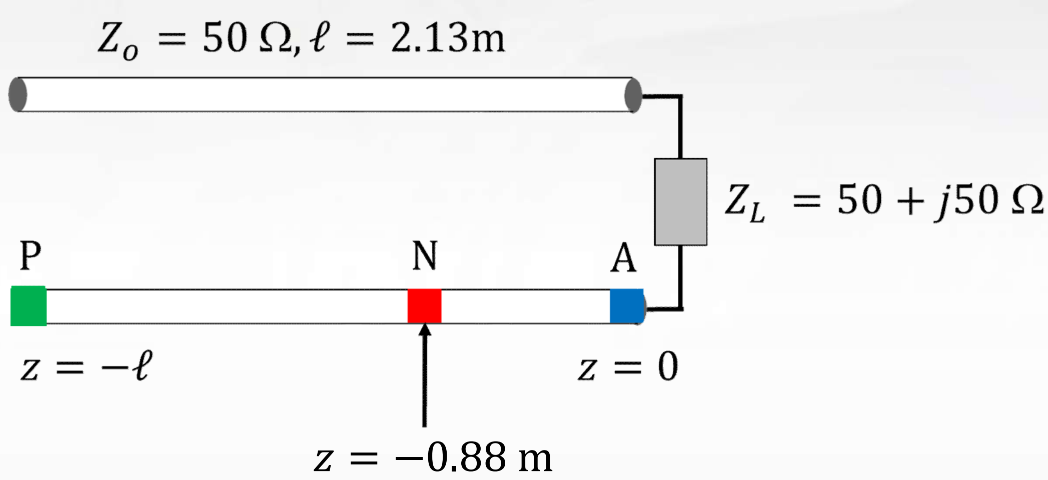
- ❑ Plot $\hat{Z}_{in}$ - circle using $\hat{Z}_{in}(z)$ or $\Gamma(z)$ at any position z;
- ❑ Plot $\hat{Z}_{in}$ - circle using SWR on the transmission line; and
- ❑ Given $\hat{Z}_L$ and ℓ/λ, determine $Z_{in}(z)$; and
- ❑ Given $\hat{Z}_{in}(z)$ and ℓ/λ, determine $\hat{Z}_L$ (unknown load impedance); and
- ❑ Given $\hat{Z}_{in}$ and $\hat{Z}_L$, determine ℓ/λ of a Transmission line.

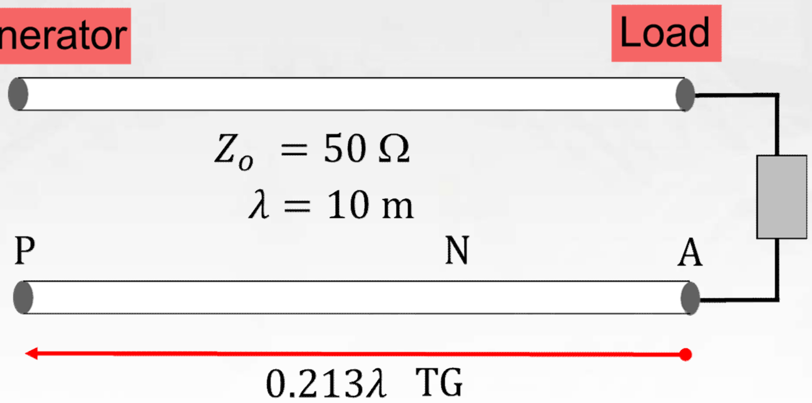
Example 1

- $f = 30$ MHz, $u_p = 3 \times 10^8$ m/s $[f\lambda = u_p]$
- Determine Z_{in} & Γ at A, P and N



- $Z_o = 50\ \Omega$, $\beta = \frac{\omega}{u_p} = \frac{2\pi \times 30 \times 10^6}{3 \times 10^8} = 0.2\pi$ rad/m $[\lambda = 10\text{ m}]$

Example 1



$$\hat{Z}_L = \frac{Z_L}{Z_o} = \frac{50 + j50}{50} = 1 + j$$

$$[r = 1, x = +1]$$

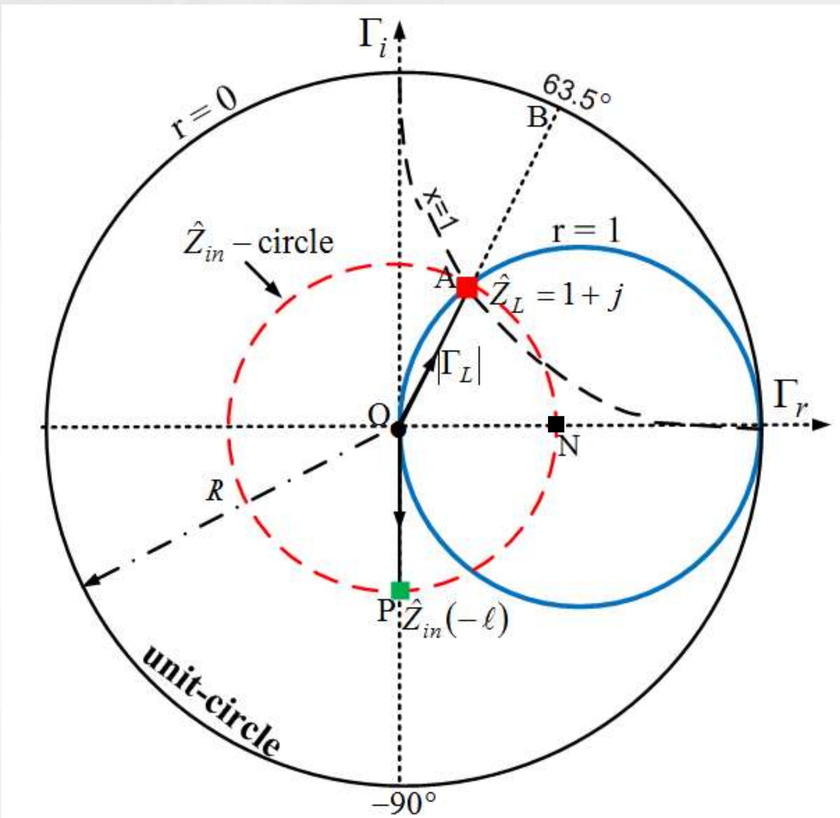
Note: All impedance divide by Z_o and distance divide by λ

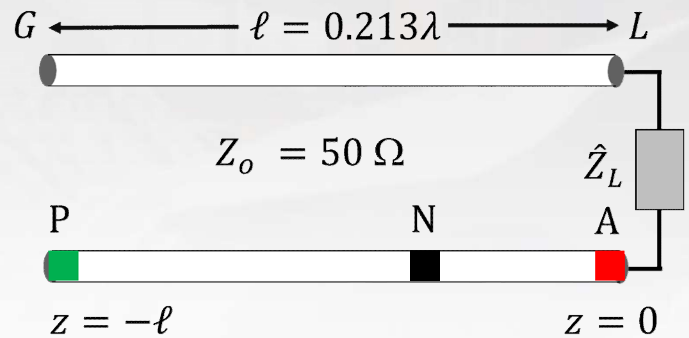
$$\frac{AP}{\lambda} = \frac{2.13}{10\text{m}} = 0.213 \rightarrow \text{A to P} = 0.213\lambda \text{ Towards Generator}$$

$$= 0.213 \text{ WTG}$$

$$\frac{AN}{\lambda} = \frac{0.88\text{m}}{10} = 0.088 \rightarrow \text{A to N} = 0.088 \text{ WTG}$$

Example 1



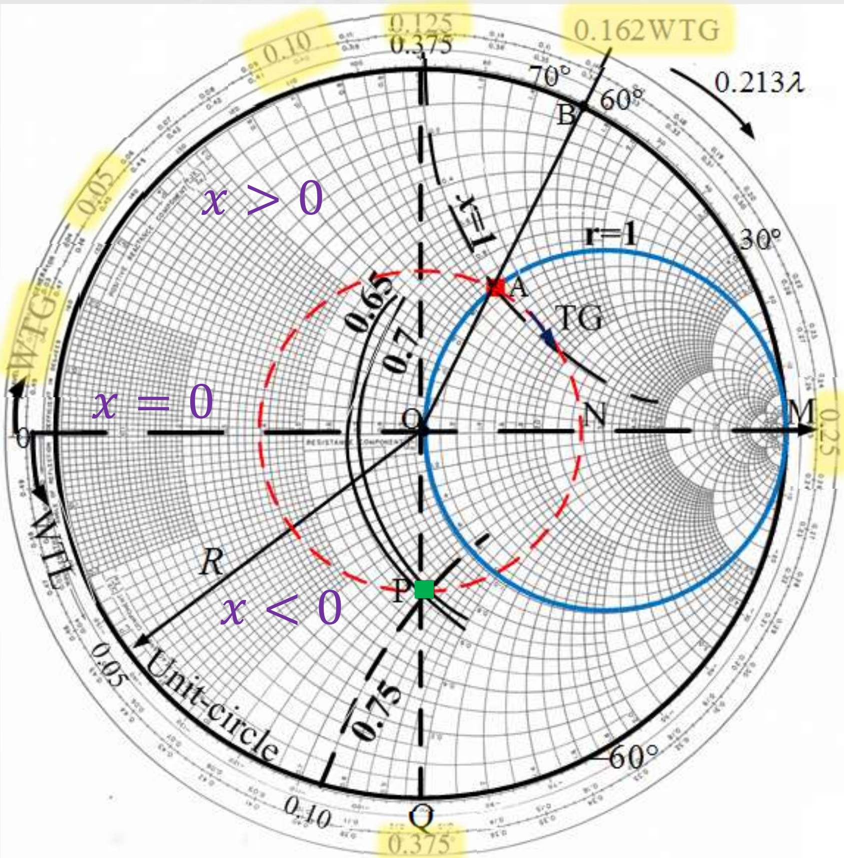


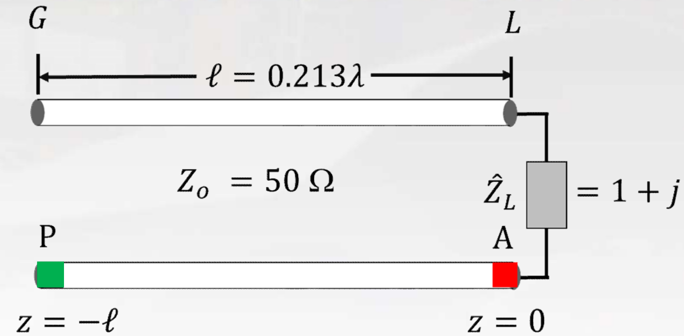
$\hat{Z}_L = 1 + j \Rightarrow r = 1, x = +1$

Plot point A on Smith Chart

- ❑ @A: $|\Gamma_L| = \frac{\text{OA}}{\text{OB}} = 0.45;\ \theta_\Gamma = 63.5^\circ$
- ❑ Plot $\hat{Z}_{in}$−circle: radius = OA center at O
- ❑ P $\xleftarrow[0.213\lambda]{\text{TG}}$ A

Example 1





@A: $\hat{Z}_L = 1 + j$

$r = 1, x = +1$

A@ 0.162 WTG

P@ 0.162 WTG

$+$ 0.213

0.375 WTG

@P: $|\Gamma| = 0.45, \theta_\Gamma = -90^\circ$

$\Gamma = 0.45\angle - 90^\circ$

$r = 0.675, x = -0.75$

$\hat{Z}_{in} = 0.675 - j0.75$

$Z_{in} = \hat{Z}_{in} Z_o = 33.8 - j37.5\ \Omega$

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35. Analysis and Design of Transmission Line (2)

Example 1

- Use of Smith Chart to determine $\hat{Z}_{in}(z)$

- All $\hat{Z}_{in}(z)$ along the transmission line $-\ell \leq z \leq 0$ lie on the $\hat{Z}_{in}$-circle, to find $\hat{Z}_{in}$ at a specific point such as point **P,** we need a **reference point on the line** e.g. $\hat{Z}_L \equiv$ point A can be used as the reference.

- To determine $\hat{Z}_{in}$ @ point P, we move from the reference point, A, to P, a distance of 0.213 **Wavelength Towards Generator(WTG).**
  NOTE: **WTG scale** starts at extreme left and increases from 0 to 0.5 in a **clockwise direction**.

- The **initial position** of $\hat{Z}_L$ [Point A] can be obtained from the **WTG scale** i.e. A @ 0.162 WTG

Example 1

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- P on WTG scale: $(0.162 + 0.213) = 0.375$ [along straight line OQ].
- The intersection between the straight line OQ and the $\hat{Z}_{in}$-circle gives Point P.
- @ **Point P**

$$r = 0.675, \quad x = -0.75 \quad ; \quad |\Gamma| = \frac{\text{OP}}{\text{OQ}} = 0.45$$

$$\hat{Z}_{in} = r + jx = 0.675 - j0.75 \quad ; \quad \theta_\Gamma = -90°$$

$$Z_{in} = \hat{Z}_{in}\, Z_o = 33.8 - j37.5\ \Omega \quad ; \quad \Gamma = 0.45\angle -90°$$

Example 1

- To obtain $\hat{Z}_{in}$ & Γ at point N [AN = 0.088 WTG], we may start from A @ 0.162 WTG, moves 0.088 WTG i.e. **N is located at $(\mathbf{0.162} + \mathbf{0.088}) = \mathbf{0.25}$ WTG** [along straight line OM].

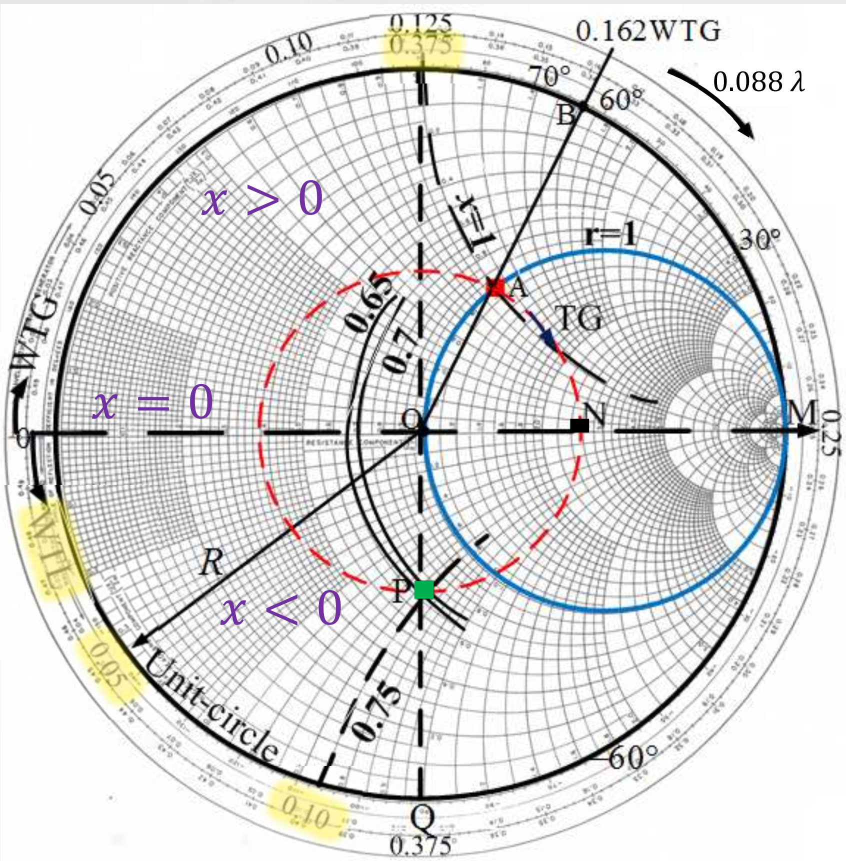
- Intersection between $\hat{Z}_{in}$-circle and OM gives Point N.

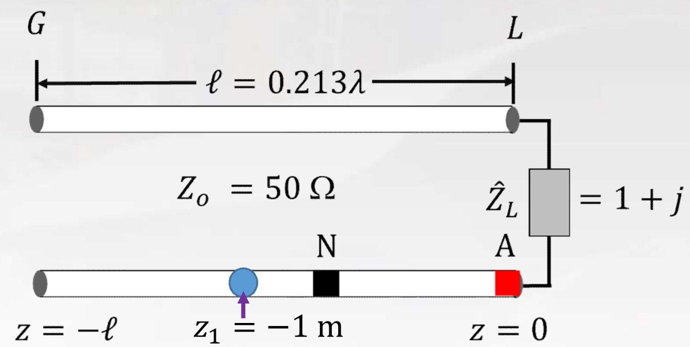
- **@ Point N**
  $r = 2.62,\ x = 0;$ $\Gamma = 0.45\angle 0°$;
  $\hat{Z}_{in} = 2.62 + \mathrm{j}0 \rightarrow Z_{in} = \hat{Z}_{in} Z_o = 131\ \Omega = R_{max} + j0$ @ $\theta_\Gamma = 0^\circ$

- **Exercise:** Find $\hat{Z}_{in}$ and Γ @ $z_1 = -1$ m $Az_1 = 0.1$ WTG [A @ 0.162 WTG]

- **Ans:** $\hat{Z}_{in} = 2.54 - j0.42$, $Z_{in} = 127 - j21\ \Omega$ $\Gamma = 0.45\angle - 9°$ [z_1 @ 0.262 WTG]

Example 1





@A: $\hat{Z}_L = 1 + j$

$r = 1, x = +1$

A@ 0.162 WTG

N@ 0.162 WTG

+ 0.088

0.25WTG

@N: $|\Gamma| = 0.45, \theta_\Gamma = 0^\circ$

$\Gamma = 0.45\angle 0^\circ$

$r = 2.62, x = 0$

$\hat{Z}_{in} = 2.62 - j0$

$Z_{in} = \hat{Z}_{in} Z_o = 131\ \Omega$

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35. Analysis and Design of Transmission Line (3)

Example 1

- **Use of Smith Chart to determine positions of Voltage maximum and minimum along Transmission Line** $-\ell \le z \le 0$ $[\ell = 0.213\lambda]$

- Find the distance from the load [Point A] to the first voltage maximum.

  $@\, z_{max}, \theta_\Gamma = 0° \; [\text{i.e. } z_{max} \equiv \text{Point N}]$

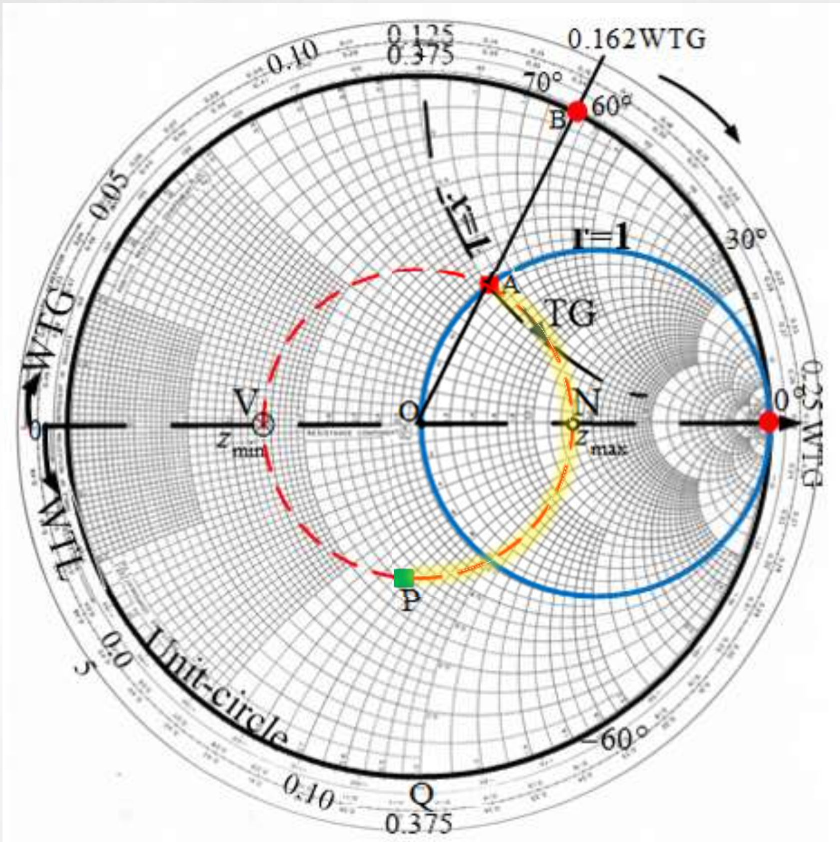
  $\therefore d_{max} = |\text{A} \to \text{N}| = (0.25 - 0.162)\lambda = 0.088\lambda.$ $[z_{max} = -0.088\lambda]$

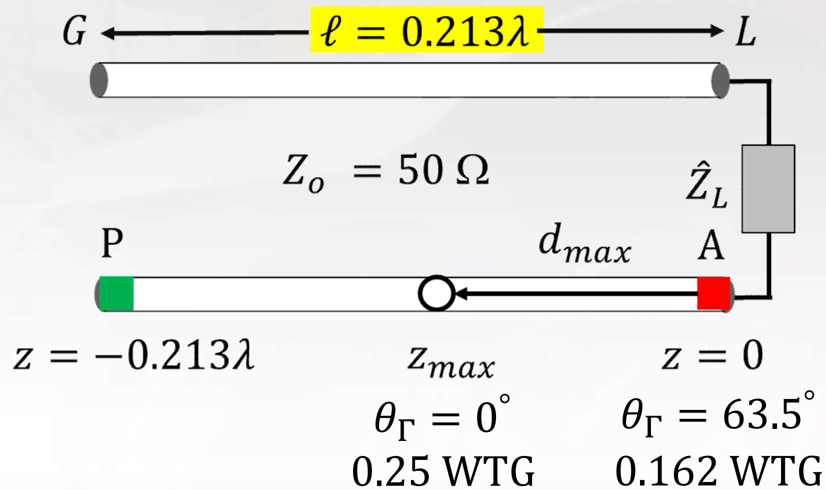
- Find the position of voltage minimum on the transmission line.

  $\therefore d_{min} = (d_{max} + 0.25)\lambda = 0.338\lambda > \ell = 0.213\lambda$ $[z_{min} \cancel{=} -0.338\lambda]$

  The position of voltage minimum is the point nearest to Point V $(\theta_\Gamma = \pm 180^\circ)$.
  Since Point V is nearest to Point P, $z_{min} \equiv \text{Point P}$.

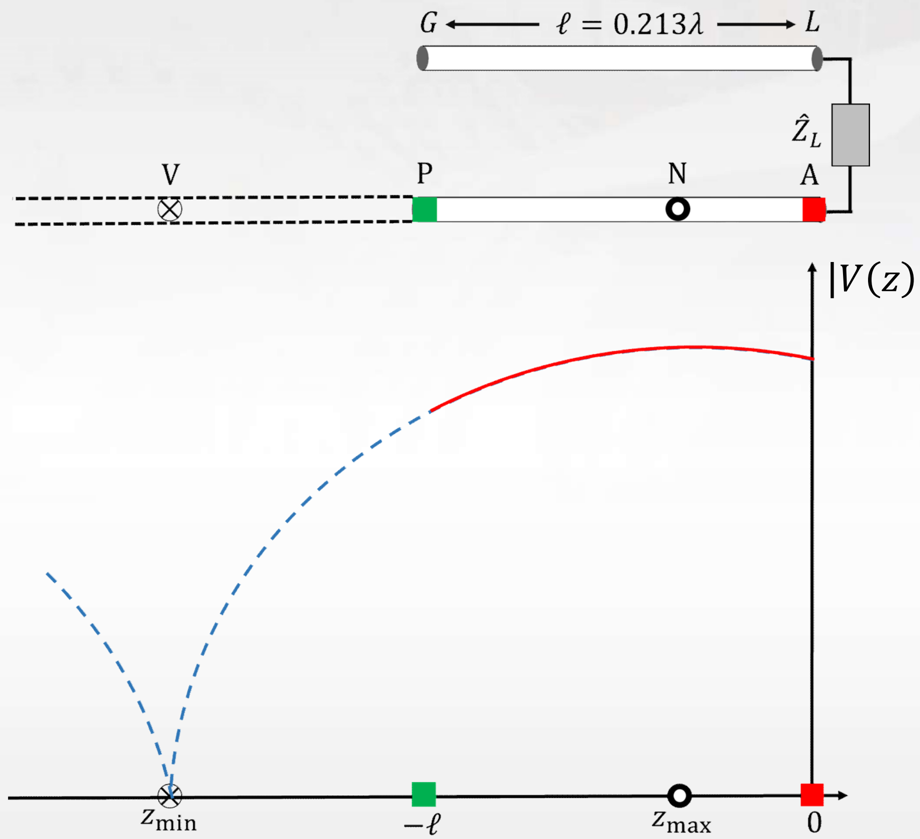
Example 1





$$d_{max} = (0.25 - 0.162)\lambda = 0.088\lambda$$
$$z_{max} = -d_{max} = -0.088\lambda$$
$$z_{min} = -(d_{max} + 0.25) = -0.338\lambda$$

Example 1



Example 1

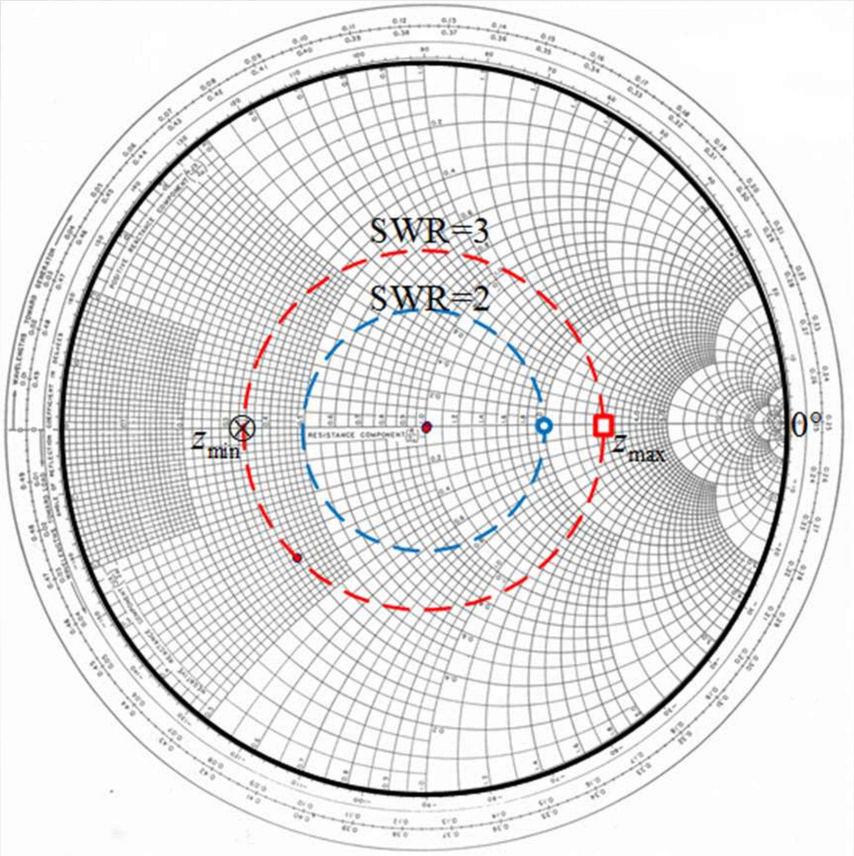
- SWR $= \frac{|V|_{max}}{|V|_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.45}{1-0.45} = 2.62$
- SWR can also be read directly from the $\hat{Z}_{in}$-circle:

$$\boxed{\hat{Z}_{in} = \frac{1+|\Gamma_L|\angle\theta_\Gamma}{1-|\Gamma_L|\angle\theta_\Gamma} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \text{SWR} \ @ \ \theta_\Gamma = 0}$$

$@ \ \theta_\Gamma = 0°$: $r = 2.62, x = 0 \rightarrow \hat{Z}_{in} = 2.62 + j0 = \text{SWR} + j0$

- REMARKS: Knowing SWR of a transmission line e.g. $|V|_{max} = 6\text{V}$ and $|V|_{min} = 2\text{V}$, we have SWR = 3 and $\hat{Z}_{in} = 3 + j0 \ @ \ \theta_\Gamma = 0°$ $[r = 3, x = 0]$ can be plotted on the Smith Chart and $\hat{Z}_{in}$-circle can be drawn.

Example 1

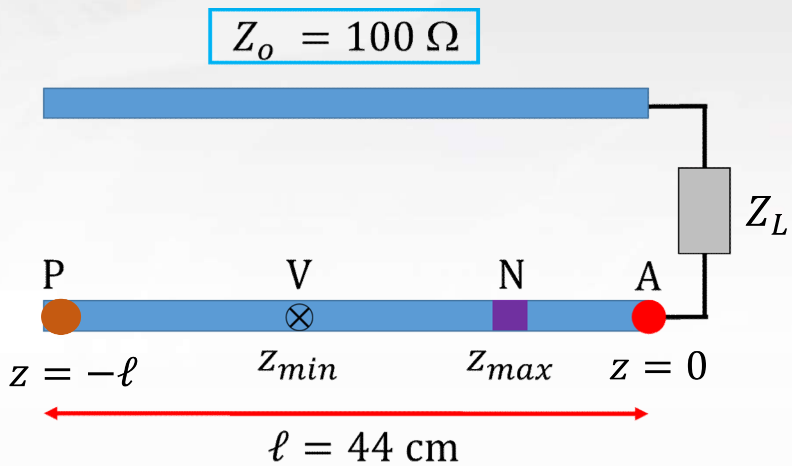


$SWR = 2$: $\hat{Z}_{in} = 2 + j0$ @ $\theta_\Gamma = 0$

$SWR = 3$: $\hat{Z}_{in} = 3 + j0$ @ $\theta_\Gamma = 0$

Example 2

- $|V|_{\max} = 57\text{ V}\ @z_{\max} = -10\text{ cm}, |V|_{\min} = 15\text{ V}\ @z_{mi} = -21\text{ cm},\ \ Z_o = 100\ \Omega$
  $\ell = 44$ cm find λ and $Z_{in}(-\ell)$.



(i) $\boxed{\frac{\lambda}{4} = |z_{\max} - z_{\min}| \rightarrow \lambda = 44\text{ cm}}$

$$\text{SWR} = \frac{|V|_{\max}}{|V|_{\min}} = \frac{57}{15} = 3.8$$

(ii) V @ 0 WTG

$\text{VP} = 44 - 21 = 23\text{ cm} = 0.523\lambda$

$\text{P @ } (0 + 0.523) = 0.523 \equiv 0.023\text{ WTG}$

$\hat{Z}_{in} = 0.28 + j0.13$

$Z_{in}(-\ell) = \hat{Z}_{in}\ Z_o = 28 + j13\ \Omega$

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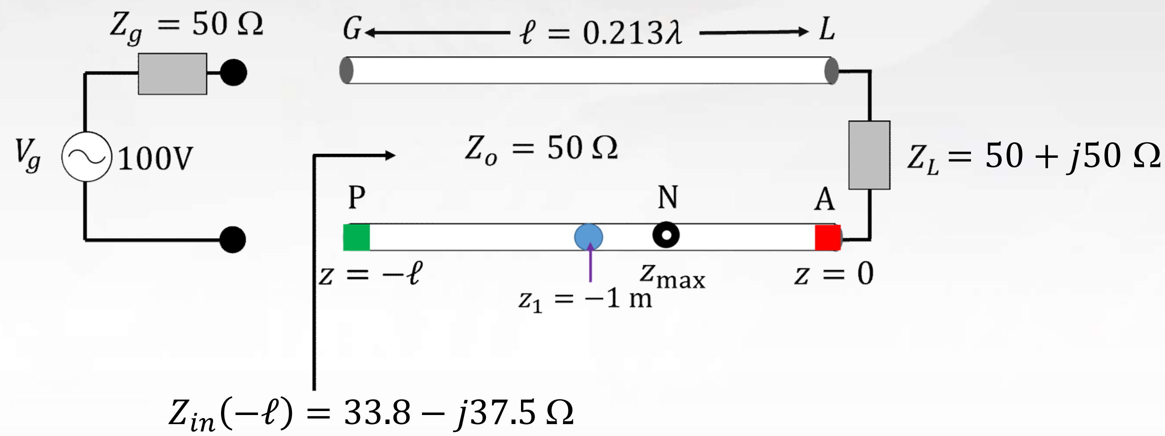
35. Analysis and Design of Transmission Line (4)

Example 3

- Consider the transmission line as shown below, find $P_{in}(z)$ and $|V(z)|$ at points P, z_1, N and A.



Given: $V_g = 100$ V, $Z_g = 50\ \Omega$, $Z_o = 50\ \Omega$, $\ell = 0.213\lambda$, $Z_L = 50 + j50\ \Omega$

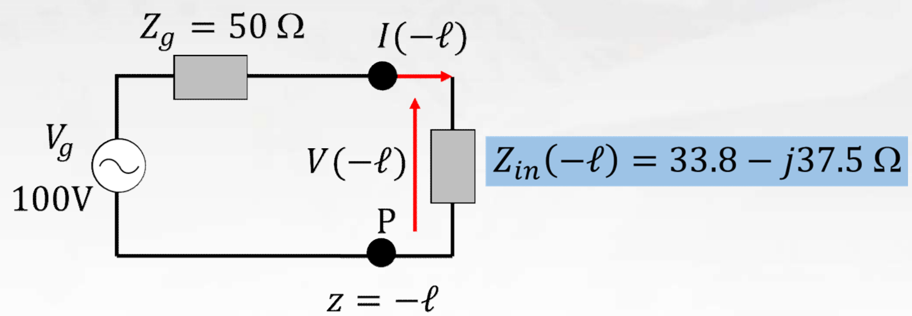
The input impedance @P : $Z_{in}(-\ell) = 33.8 - j37.5\ \Omega$ [Page 35-8]

@N : $Z_{in}(z_{\max}) = 131\ \Omega$ [Page 35-10]

@z_1: $Z_{in}(z_1) = 127 - j21\ \Omega$

Example 3

$|V(z)|$ and $P_{in}(z)$ @ Point P can be found using equivalent circuit at $z = -\ell$



$$V(-\ell) = \frac{Z_{in}}{Z_g + Z_{in}} \times V_g = 50 - j22.4 = 54.8\angle - 24^\circ \qquad \text{[Point P]}$$

$$P_{in}(-\ell) = \frac{1}{2}|I(-\ell)|^2\,\mathrm{Re}(Z_{in}) = \frac{1}{2}\left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}(Z_{in})$$

$$= \frac{1}{2}\left|\frac{100}{50 + (33.8 - j37.5)}\right|^2 \times (33.8) = 20\text{ W} = P_{in}(z) = P_L$$

Example 3

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$$\boxed{P_{in}(z) = \frac{1}{2}\left|\frac{V(z)}{Z_{in}(z)}\right|^2 \mathrm{Re}(Z_{in}(z))} \rightarrow |V(z)| = \sqrt{\frac{2P_{in}(z) \times |Z_{in}(z)|^2}{\mathrm{Re}(Z_{in})}} \qquad (5.8)$$

- At z_1, $Z_{in}(z_1) = 127 - j21\ \Omega$; $P_{in}(z_1) = 20\ \mathrm{W}$

$$|V(z_1)| = \sqrt{\frac{2(20) \times (127^2 + 21^2)}{127}} = 72.2\ \mathrm{V}$$

- At N: $z = z_{\max}$, $Z_{in}(z_{\max}) = 131\ \Omega$; $P_{in}(z_{\max}) = 20\ \mathrm{W}$

$$P_{in}(z_{\max}) = \frac{|V|^2_{\max}}{2R_{\max}} = 20\ \mathrm{W} \rightarrow |V|_{\max} = 72.4\ \mathrm{V}$$

- At A: $z = 0$, $Z_L = 50 + j50\ \Omega$, $P_{in}(0) = P_L = 20\ \mathrm{W}$, $|V_L| = 63.2\ \mathrm{V}$

Example 3

REMARKS: Using (3) to obtain $V(z)$ and $V(z,t) = \mathrm{Re}[V(z)e^{j\omega t}]$

$V(z) = V_o^+ e^{-j\beta}\ [1 + \Gamma_L\ e^{+j2\beta z}]$ (3) [Page 28-8]

$\Gamma_L = 0.45\angle 63.5^\circ$ can be found using Smith Chart or equation (5)

V_o^+ can be determined by substituting $V(-\ell) = 54.8\angle - 24^\circ$ V,

$\Gamma_L = 0.45\angle 63.5^\circ$ and $\beta\ell = 2\pi\frac{\ell}{\lambda} = 2\pi(0.213)$ into (3):

$V(-\ell) = V_o^+ e^{+j\beta\ell}[1 + \Gamma_L e^{-j2\ \ \ell}]$ (7) [Page 28-13]

$V_o^+ = 50\angle - 76.7^\circ$

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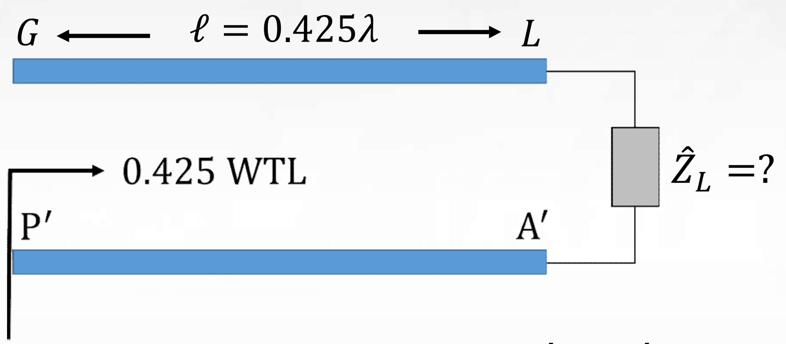
35. Analysis and Design of Transmission Line (5)

Example 4

- Use of Smith Chart to determine unknown load Z_L

- Given $Z_{in} = 67.5 - j75\ \Omega$, $Z_o = 100\ \Omega$ and $\ell = 0.425\lambda$, find Z_L and Γ_L.



$\hat{Z}_{in} = 0.675 - j0.75$

$[r = 0.675, x = -0.75]$

$$@\ \mathrm{P}':\ |\Gamma| = \frac{\mathrm{OP}'}{\mathrm{OQ}'} = 0.45$$

$$\theta_\Gamma = -90^\circ$$

P′ to A′ $= 0.425$ WTL

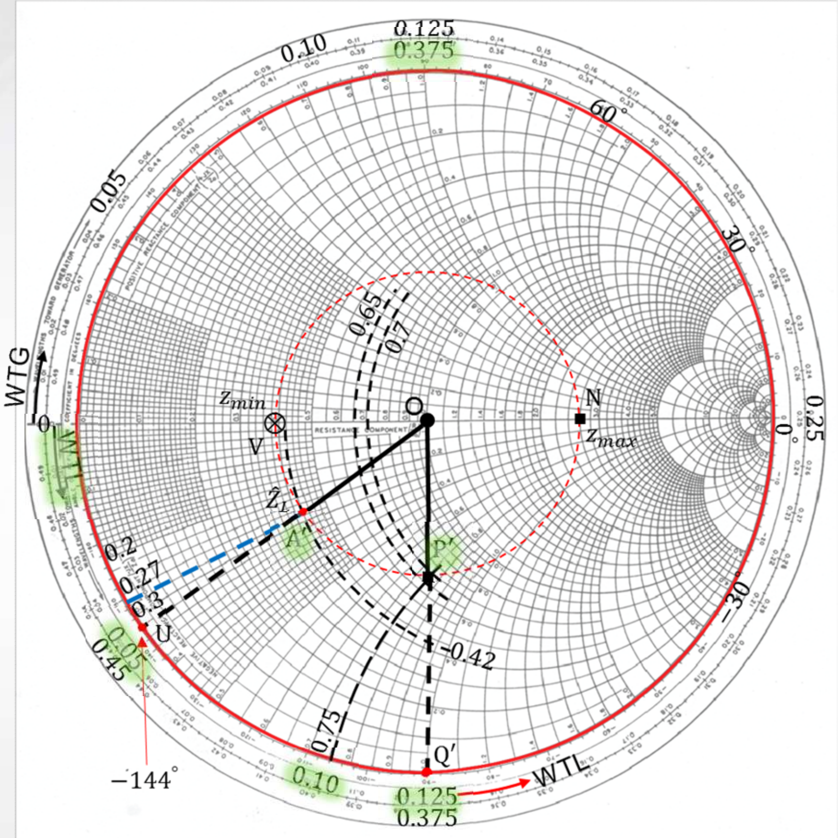
P′ @ 0.125 WTL

A′ @ 0.125 WTL

$+\ 0.425$

0.550 WTL ≡ 0.05 WTL

Example 4



$r = 0.675$ and $x = -0.75$ @ P′

P′ @ 0.125 WTL, $\ell = 0.425\lambda$

A′ @ 0.550 WTL ≡ 0.05 WTL

@ A′: $\Gamma_L = 0.45\angle -144^{\circ}$

$r = 0.42, \quad x = -0.27$

$\hat{Z}_L = 0.42 - j0.27$

$Z_L = \hat{Z}_L Z_o = 42 - j27\ \Omega$

Example 4

- $\hat{Z}_{in} = Z_{in}/Z_o = 0.675 - j0.75$ (Plot Point P′)
- Once P’ is plotted on the chart, $\hat{Z}_{in}$-circle can be drawn.
- To find $\hat{Z}_L$, we need to move $\mathrm{P}' \rightarrow \mathrm{A}'$, 0.425 **Wavelength Towards Load (WTL).**
- **NOTE:** Wavelength Towards Load **(WTL)** scale starts from 0 to 0.5 in **anti-clockwise direction.**
- The **initial position** of $\hat{Z}_{in}$ [Point P′] on the **WTL scale** can be obtained i.e. P′@ 0.125 WTL
- A′ on WTL: (0.125 + 0.425) = 0.55 ≡ 0.05 WTL [along OU].
- The intersection between OU and $\hat{Z}_{in}$-circle gives Point A’.

Example 4

- Q: Find the position of voltage minimum, z_{min}, on the transmission line.

- From Point A’ ($z = 0$) move TG: $\mathrm{A}' \xrightarrow{\mathrm{TG}} \mathrm{V} \rightarrow \mathrm{N} \rightarrow \mathrm{P}'$

  A' @ 0.45 WTG, V @ 0 WTG $\equiv 0.5$ WTG $\rightarrow \mathrm{A}'\mathrm{V} = (0.5 - 0.45)\lambda = 0.05\lambda$

  Point V is located 0.05λ from Point $\mathrm{A}'(z = 0)$:

  $z_{min} = -0.05\lambda$

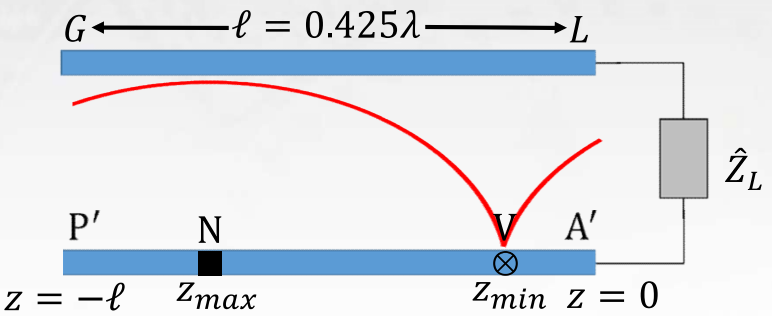
- From Point P’ ($z = -0.425\lambda$) move TL: $\mathrm{P}' \xrightarrow{\mathrm{TL}} \mathrm{N} \rightarrow \mathrm{V} \rightarrow \mathrm{A}'$

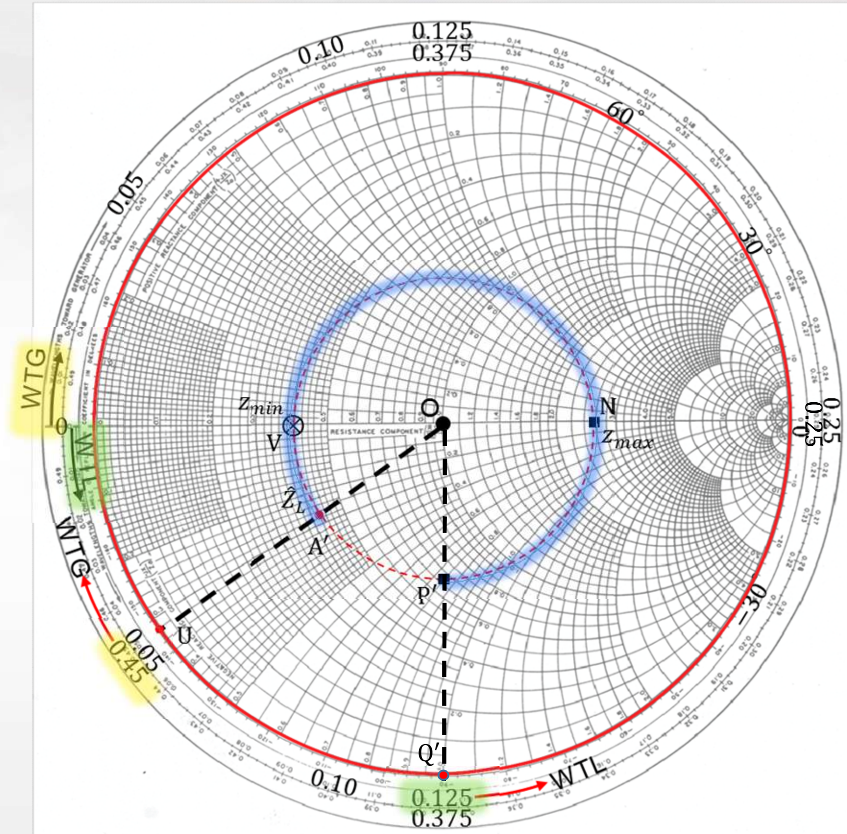
  P'@ 0.125 WTL, V @ 0.5 WTL $\rightarrow \mathrm{P}'\mathrm{V} = (0.5 - 0.125)\lambda = 0.375\lambda$

  Point V is located 0.375λ from Point $\mathrm{P}'(z = -0.425\lambda)$:

  $z_{min} = -(0.425 - 0.375)\lambda = -0.05\lambda$

Example 4





A′ @ 0.45 WTG

V @ 0.5 WTG

$\mathrm{A'V} = 0.05\lambda$

$z = -0.05\lambda$

P′ @ 0.125 WTL

V @ 0.5 WTL

P′V @ 0.375λ

$$z = -(0.425 - 0.375)\lambda = -0.05\lambda$$

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35. Analysis and Design of Transmission Line (6)

Example 5

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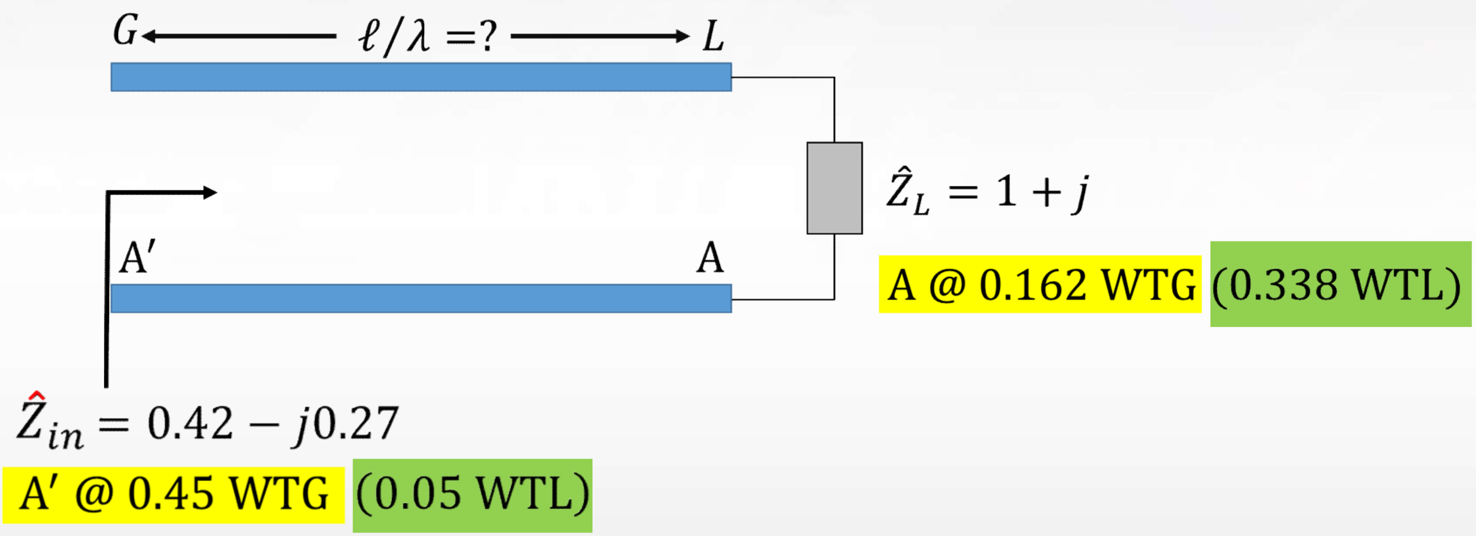
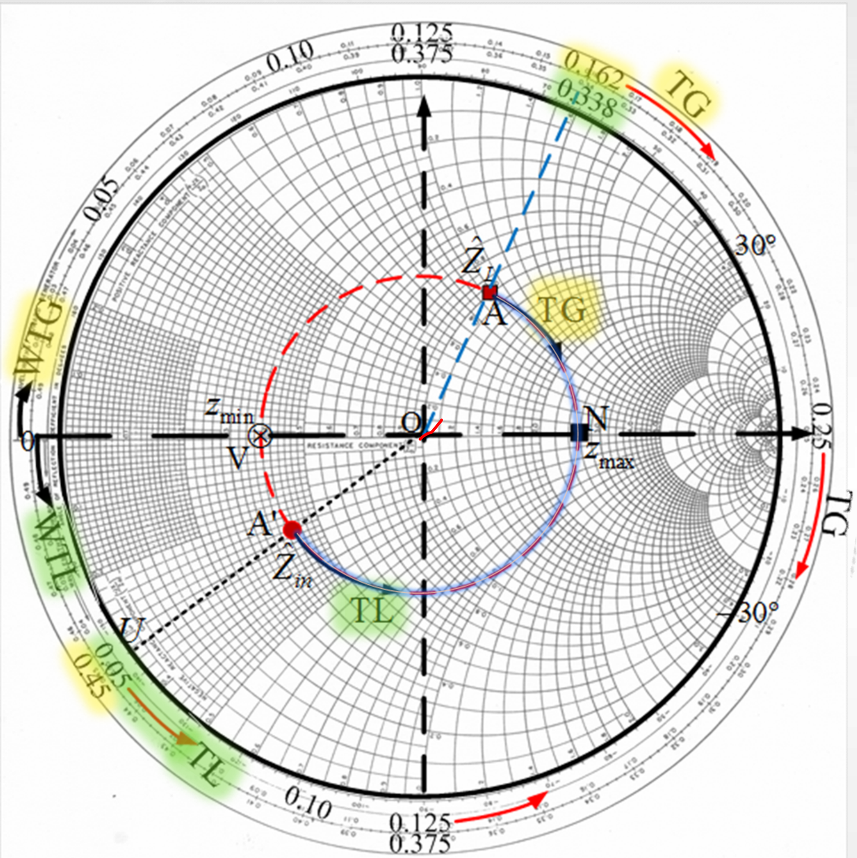
- **Use of Smith Chart to determine ℓ/λ**
- Given $\hat{Z}_{in} = 0.42 - j0.27$ and $\hat{Z}_L = 1 + j1$, determine the electrical length ℓ/λ of the transmission line.



Example 5



WTG: A → A′

0.162 WTG → 0.45 WTG

$$\frac{\ell}{\lambda} = (0.45 - 0.162)$$
$$= 0.288$$

WTL: A′ → A

0.05 WTL → 0.338 WTL

$$\frac{\ell}{\lambda} = (0.338 - 0.05)$$
$$= 0.288$$

Summary

- Plot $\hat{Z}_{in}$ - circle using

i. $\hat{Z}_{in}(z) = r + jx$ e.g. $\hat{Z}_{in}(0) = \hat{Z}_L = 1 + j1$ [P_1 ■]

P_1 @ 0.162 WTG

ii. $\Gamma(z) = |\Gamma|\angle\theta_\Gamma$ e.g. $\Gamma(-\ell) = 0.45\angle - 90^\circ$ [P_2 ●]

P_2 @ 0.125 WTL

iii. $\text{SWR} = \frac{|V|_{\text{max}}}{|V|_{\text{mi}}} = 2.62 \rightarrow \hat{Z}_{in} = 2.62 \;@\; \theta_\Gamma = 0$ [P_3 ■]

$$r = 2.62, x = 0$$

P_3 @ 0.25 WTL, 0.25 WTG

Summary

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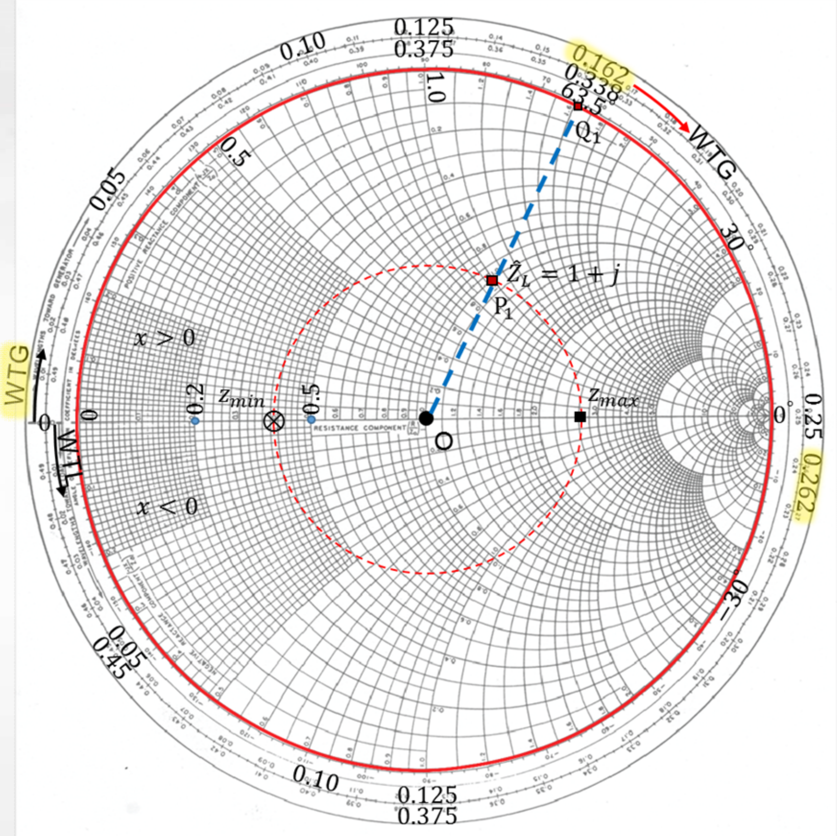
i. $\hat{Z}_{in}(z) = r + jx$ e.g. $\hat{Z}_{in}(0) = \hat{Z}_L = 1 + j1$ [P_1 ■]

P_1 @ 0.162 WTG



Summary

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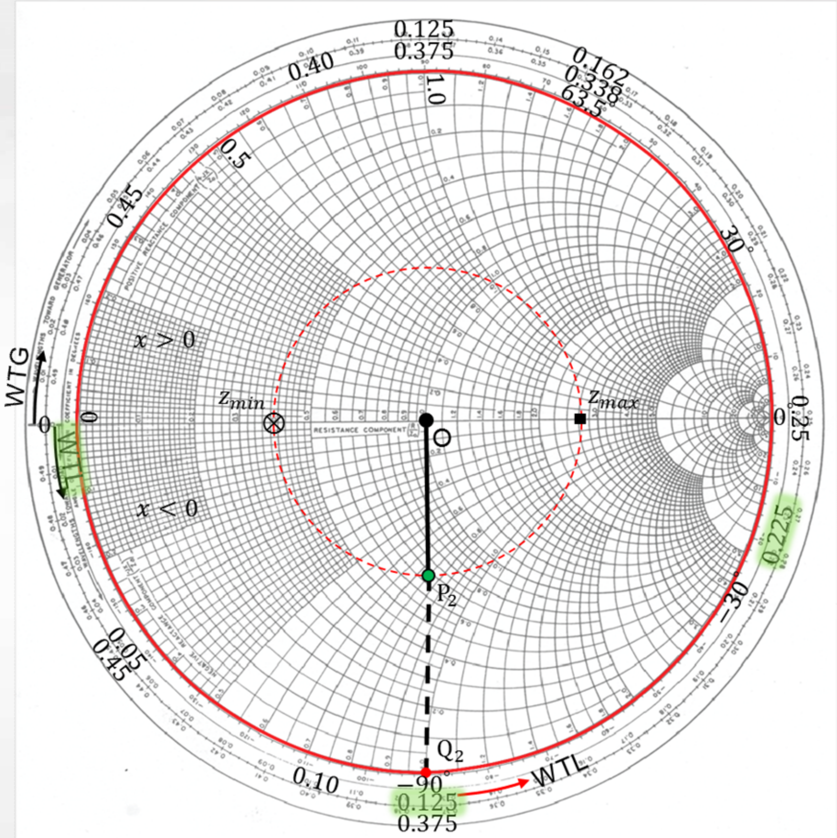
ii. $\Gamma(z) = |\Gamma|\angle\theta_\Gamma$ e.g. $\Gamma(-\ell) = 0.45\angle - 90^\circ$ [P_2]

P_2 @ 0.125 WTL

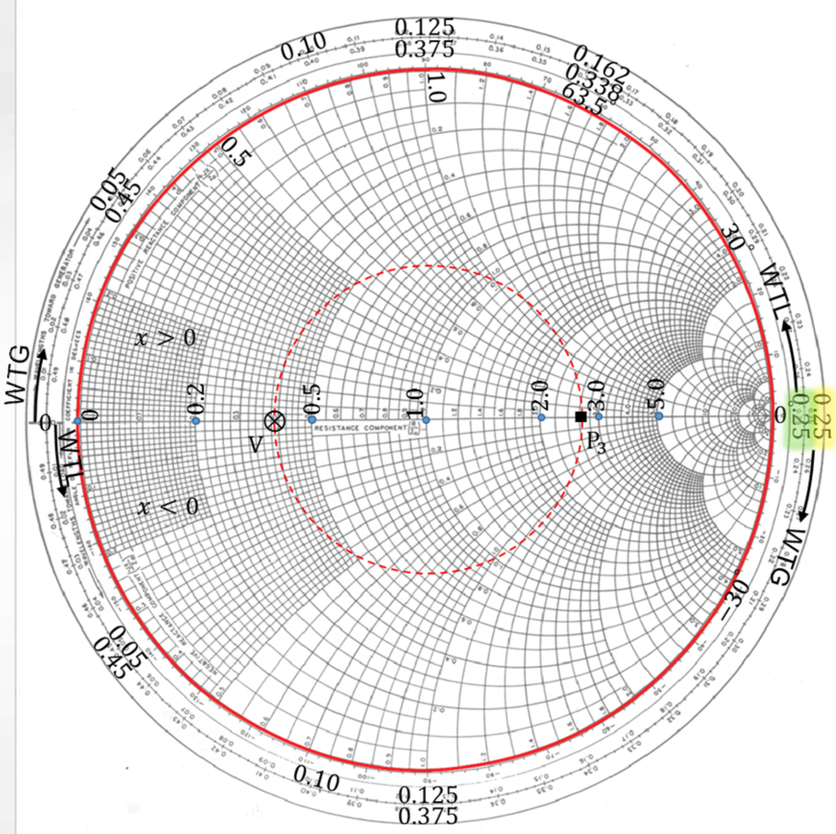


Summary

iii. SWR $= \frac{|V|_{\max}}{|V|_{min}} = 2.62 \rightarrow \hat{Z}_{in} = 2.62 \text{ @ } \theta_\Gamma = 0$ [P_3 ■]

$$r = 2.62, x = 0$$

P_3 @ 0.25 WTL, 0.25 WTG

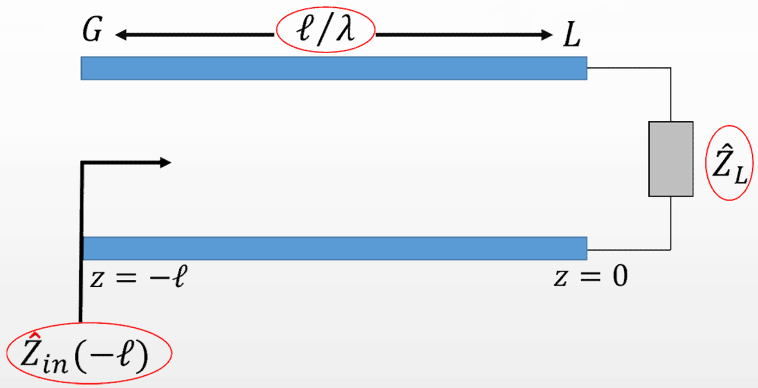


Summary

- Smith chart relates the following 3 variables: $\hat{Z}_{in}(z)$, $\hat{Z}_L$ and ℓ/λ.

  Given any two, the 3rd variable can be determined quickly.

1. Given $\hat{Z}_L$ and ℓ/λ, determine $\hat{Z}_{in}(-\ell)$ at the source end.
2. Given $\hat{Z}_{in}(z)$ and ℓ/λ, determine $\hat{Z}_L$ (unknown load impedance)
3. Given $\hat{Z}_{in}(-\ell)$ and $\hat{Z}_L$, determine ℓ/λ of a Transmission line.



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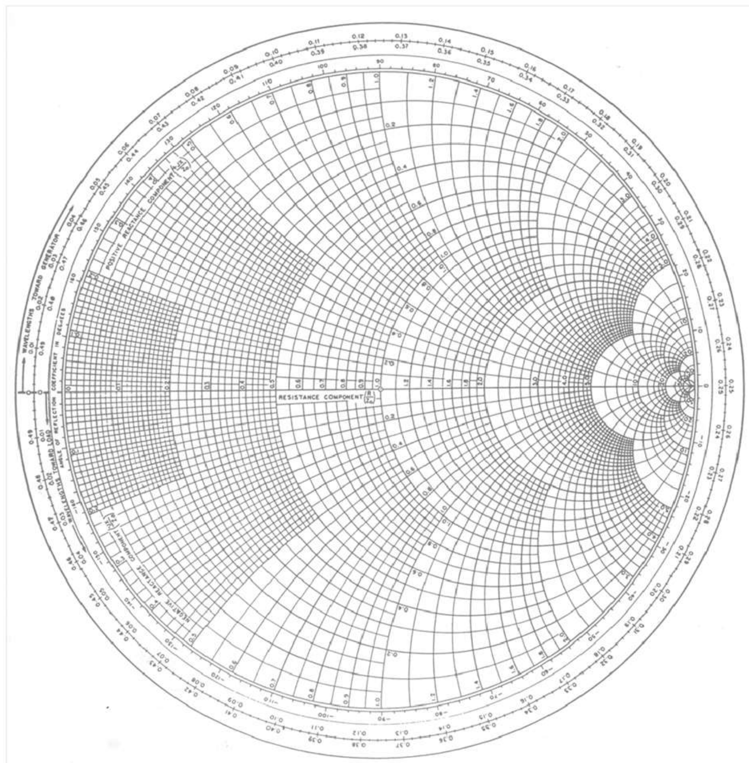
36. Summary of Smith Chart

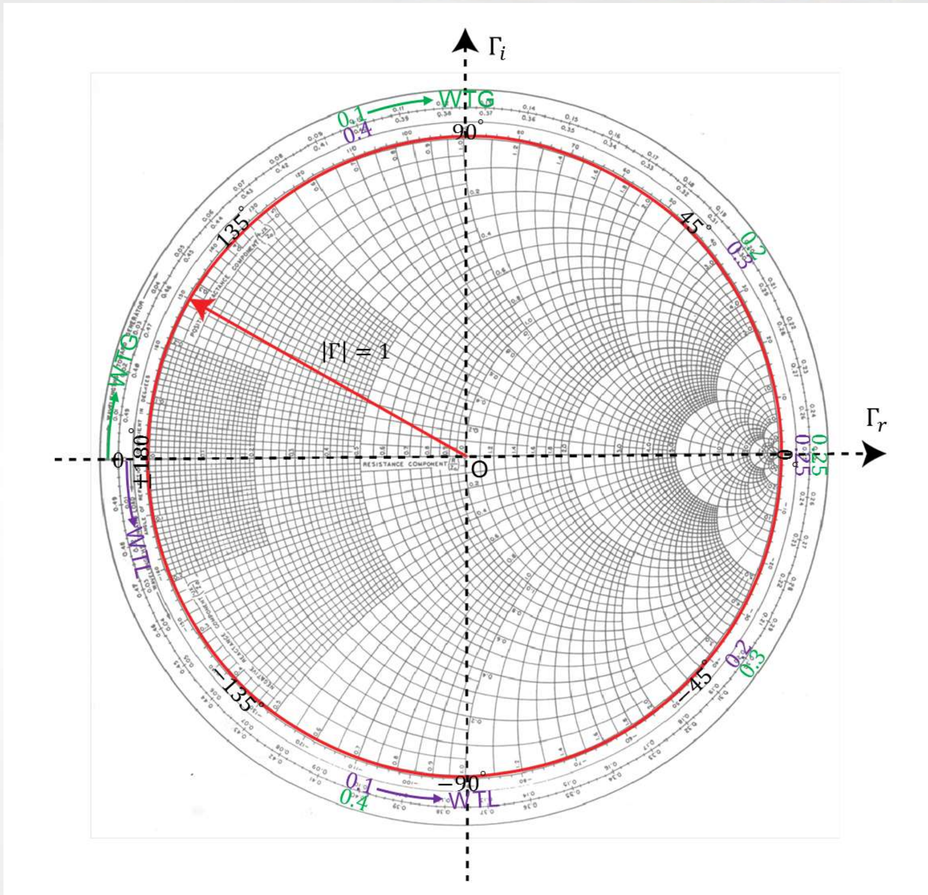
Smith Chart: Γ −Plane and $\hat{Z}_{in}(z)$ −Plane



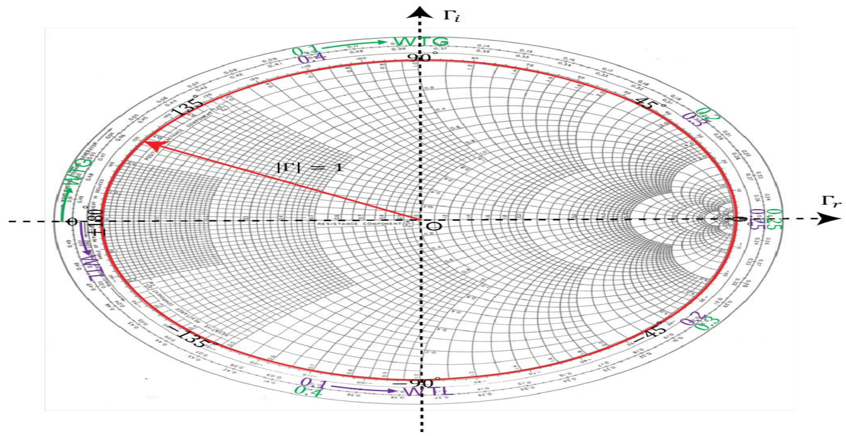
Smith Chart: Γ −Plane and $\hat{Z}_{in}(z)$ −Plane

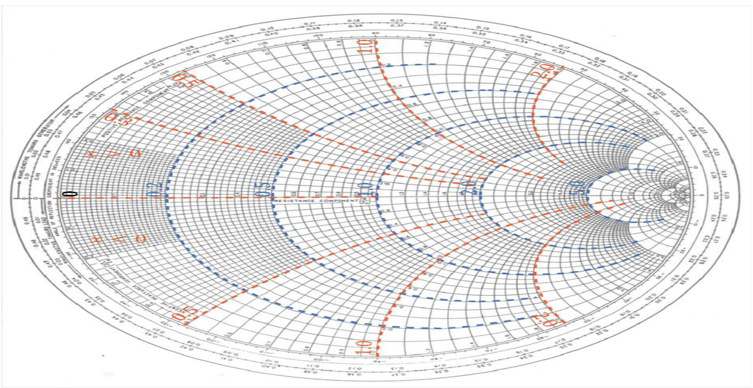
Γ −Plane

$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$



Smith Chart: Γ –Plane and $\hat{Z}_{in}(z)$ –Plane





$\hat{Z}_{in}$ –Plane

$\hat{Z}_{in}(z) = r + jx$

Smith Chart: Γ –Plane and $\hat{Z}_{in}(z)$ –Plane

$$@P: |\Gamma| = \frac{OP}{OQ} = 0.73$$

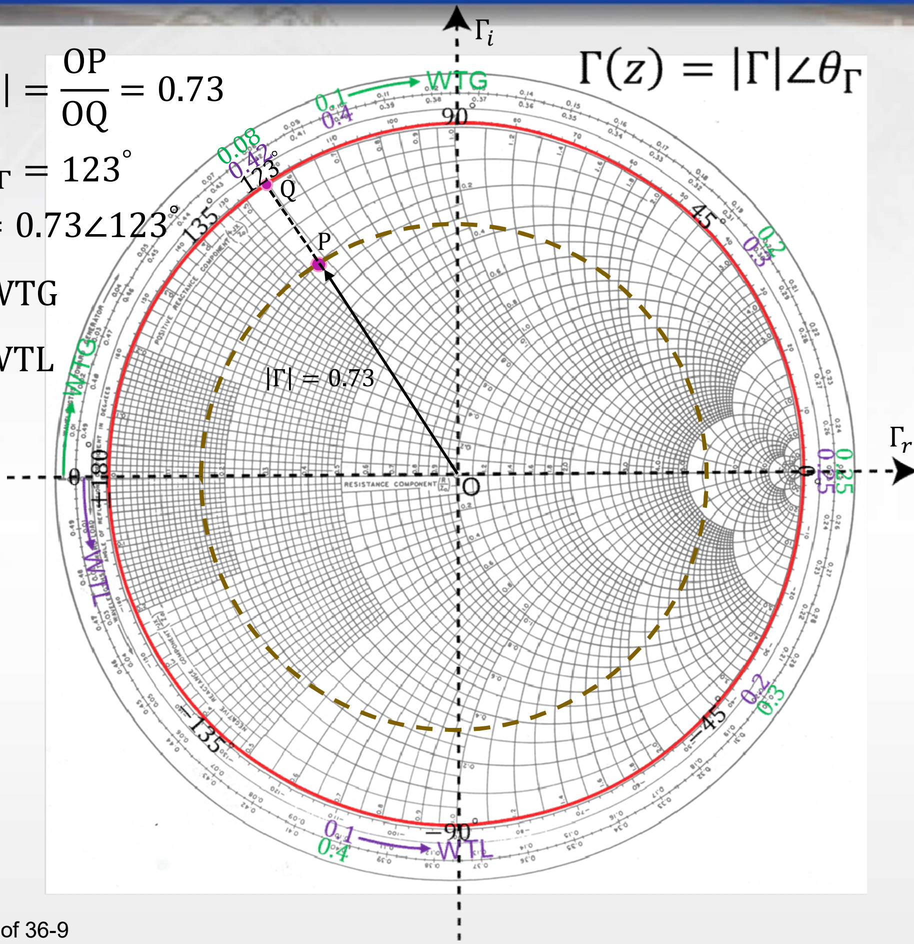
$$\theta_\Gamma = 123^\circ$$

$$\Gamma(z) = 0.73\angle 123^\circ$$

0.08 WTG

0.42 WTL

$$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$$



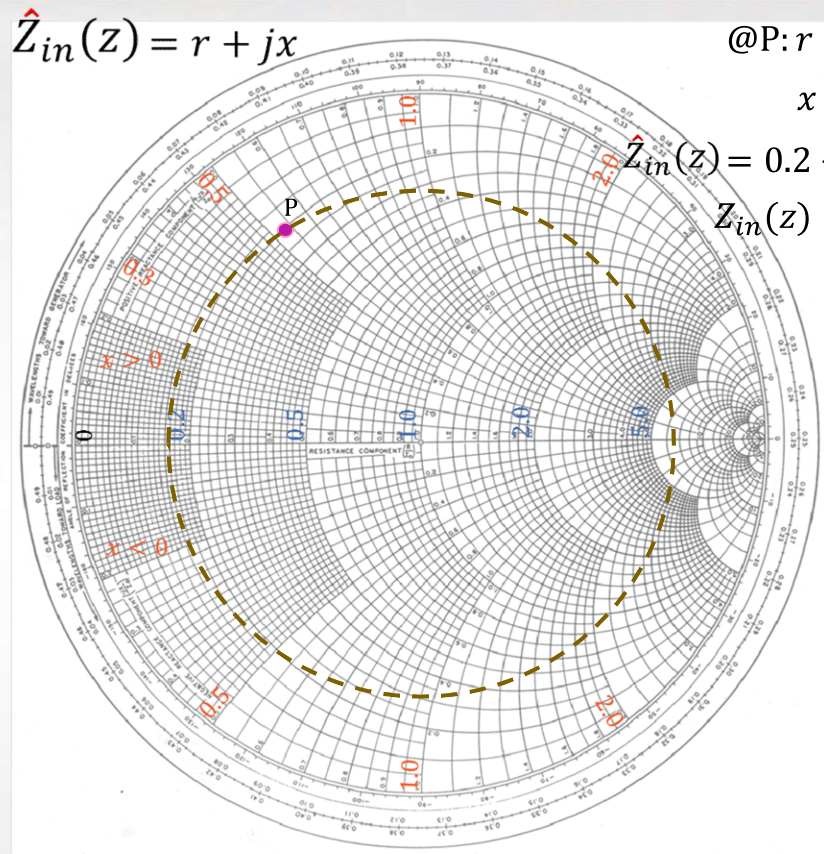
$$\hat{Z}_{in}(z) = r + jx$$

$$@P: r = 0.2$$

$$x = 0.525$$

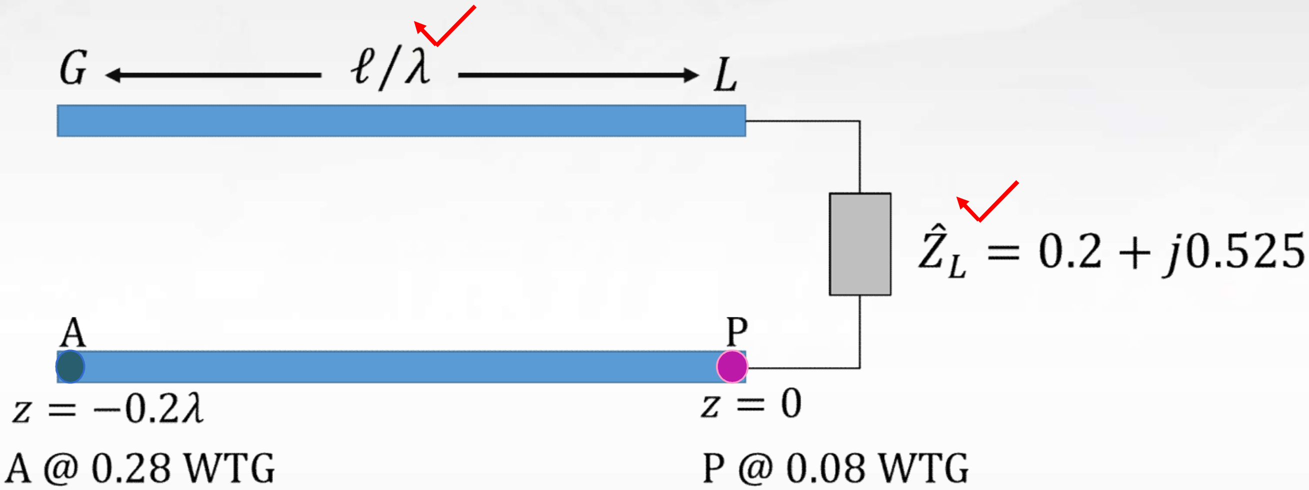
$$\hat{Z}_{in}(z) = 0.2 + j0.525$$

$$Z_{in}(z) = \hat{Z}_{in}\, Z_o$$



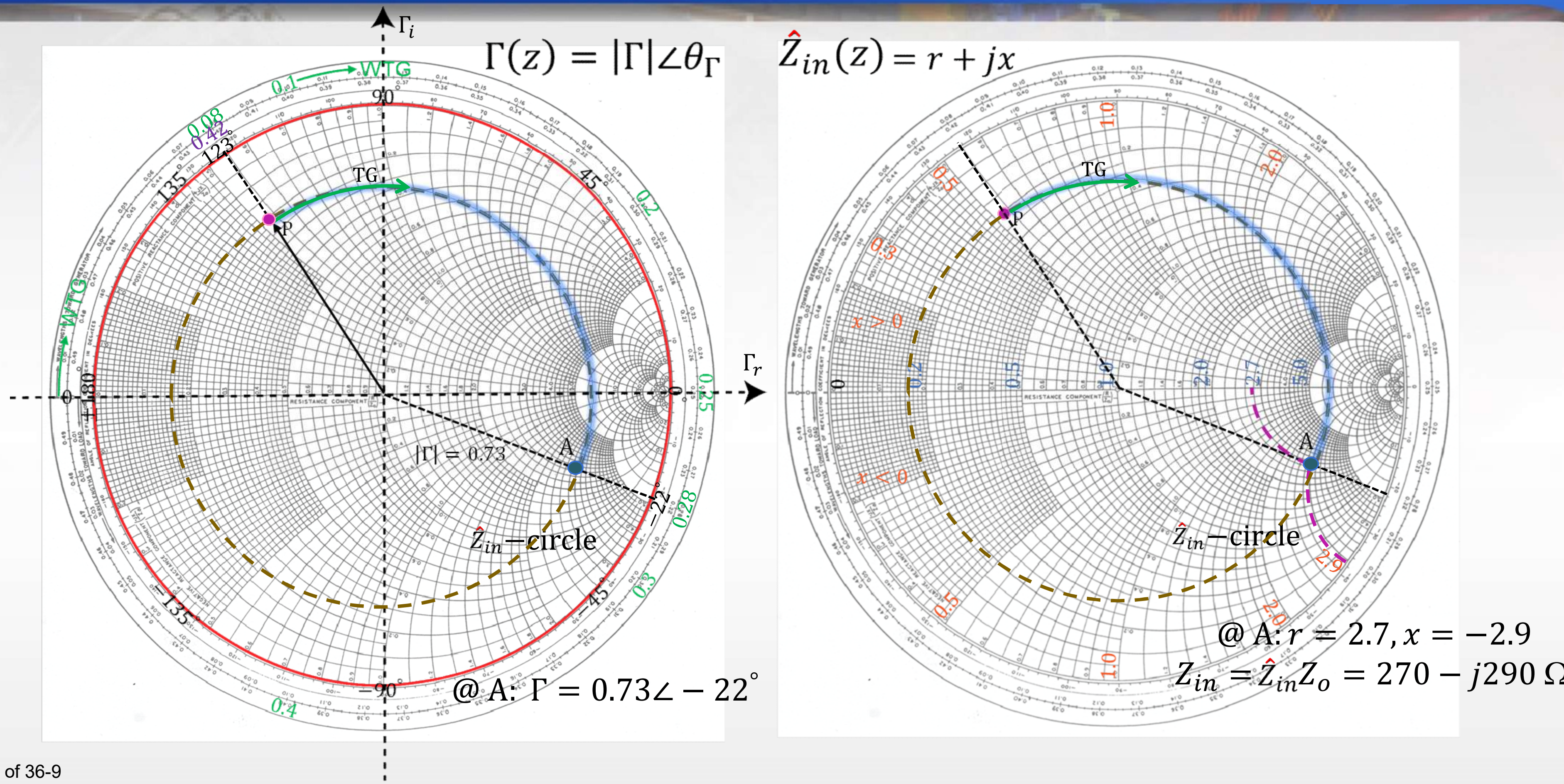
Example 1

- Given $Z_L = 20 + j52.5\ \Omega$, $Z_o = 100\ \Omega$, $\ell = 0.2\lambda$, find $\Gamma(-\ell)$ and $Z_{in}(-\ell)$.



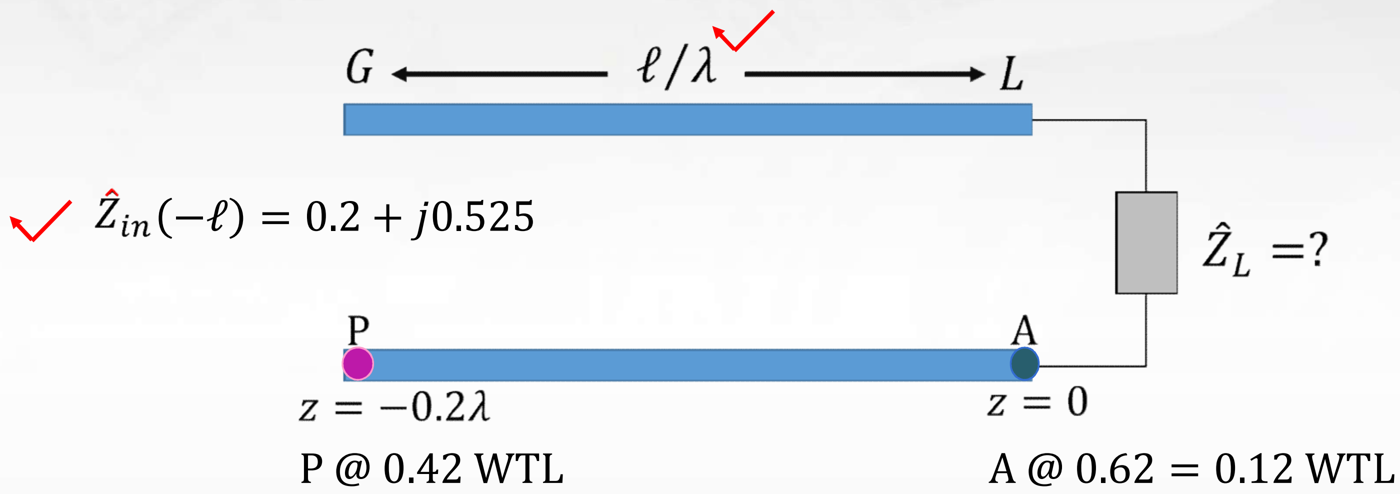
- Ans: $\Gamma(-\ell) = 0.73\angle -20.9^{\circ}$, $Z_{in}(-\ell) = 276 - j309\ \Omega$

Example 1


$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$
$\hat{Z}_{in}(z) = r + jx$
WTG
TG
P
A
$|\Gamma| = 0.73$
$\hat{Z}_{in}$−circle
Γ_i
Γ_r
@ A: $\Gamma = 0.73\angle - 22^\circ$
@ A: $r = 2.7, x = -2.9$
$Z_{in} = \hat{Z}_{in} Z_o = 270 - j290\ \Omega$

Example 2

- Given $Z_{in}(-\ell) = 20 + j52.5\ \Omega$, $Z_o = 100\ \Omega$, $\ell = 0.2\lambda$, find Γ_L and Z_L.



- Ans: $\Gamma_L = 0.73\angle -93^{\circ}$, $Z_L = 29 - j90.6\ \Omega$

Example 2

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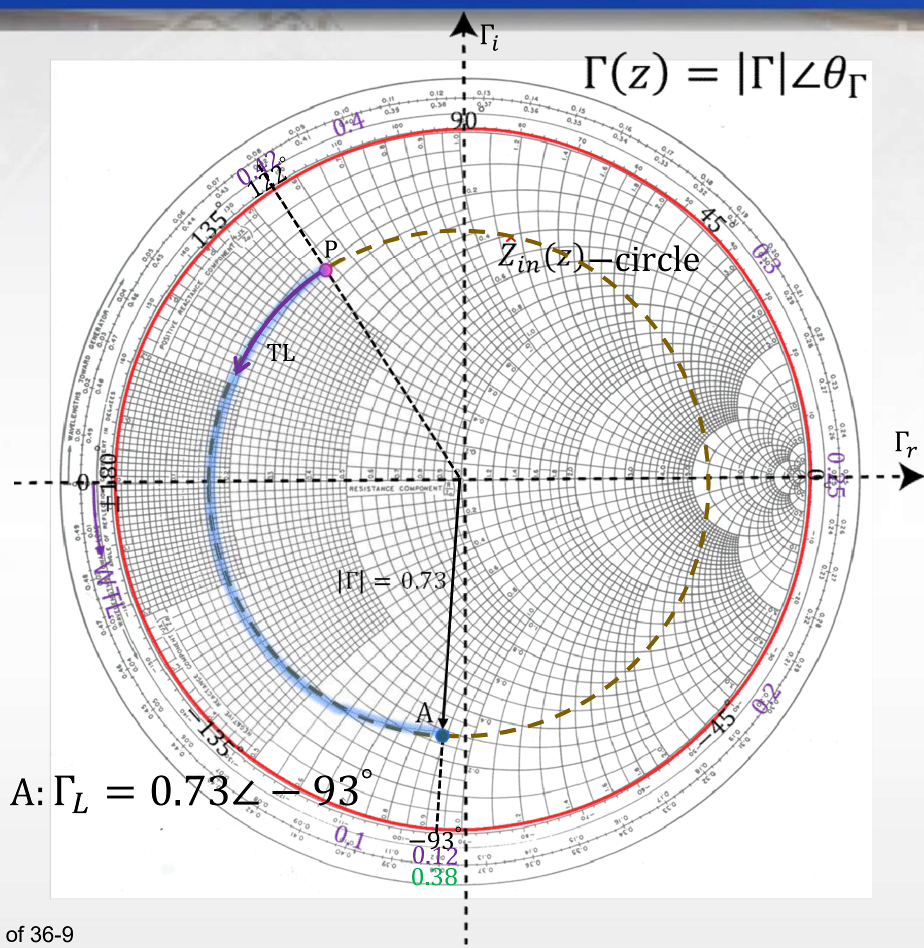
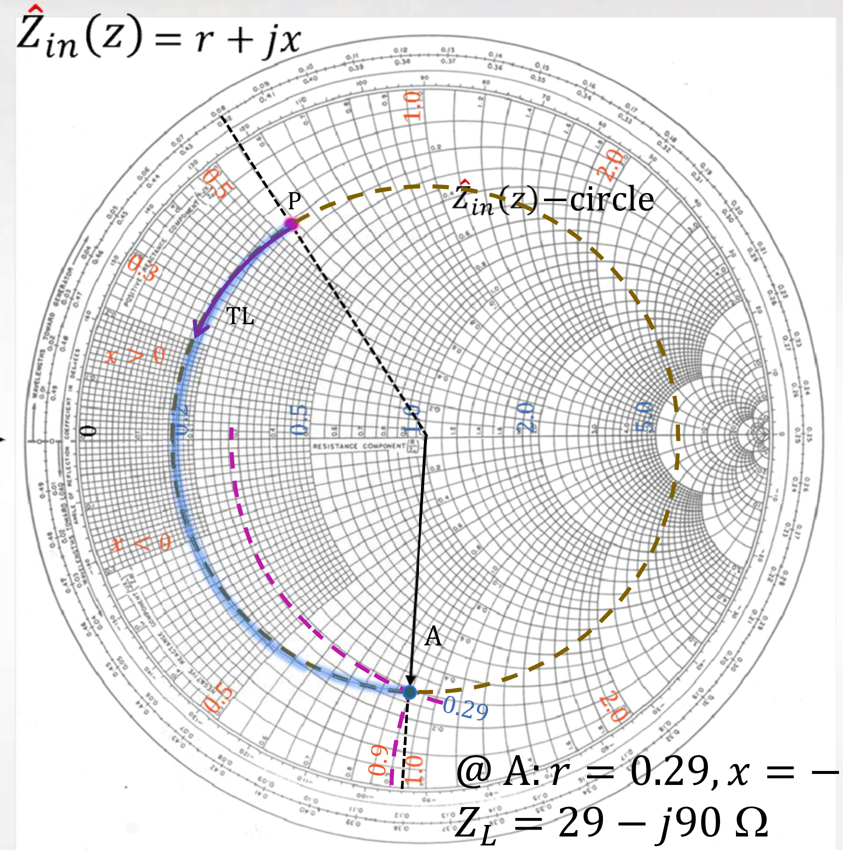
$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$



@ A: $\Gamma_L = 0.73\angle-93^{\circ}$

$\hat{Z}_{in}(z) = r + jx$

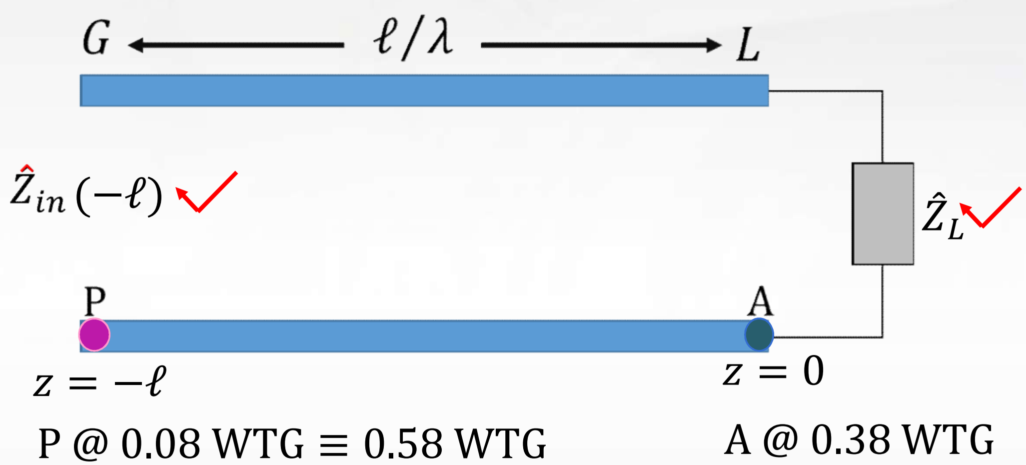


@ A: $r = 0.29, x = -0.9$

$Z_L = 29 - j90\ \Omega$

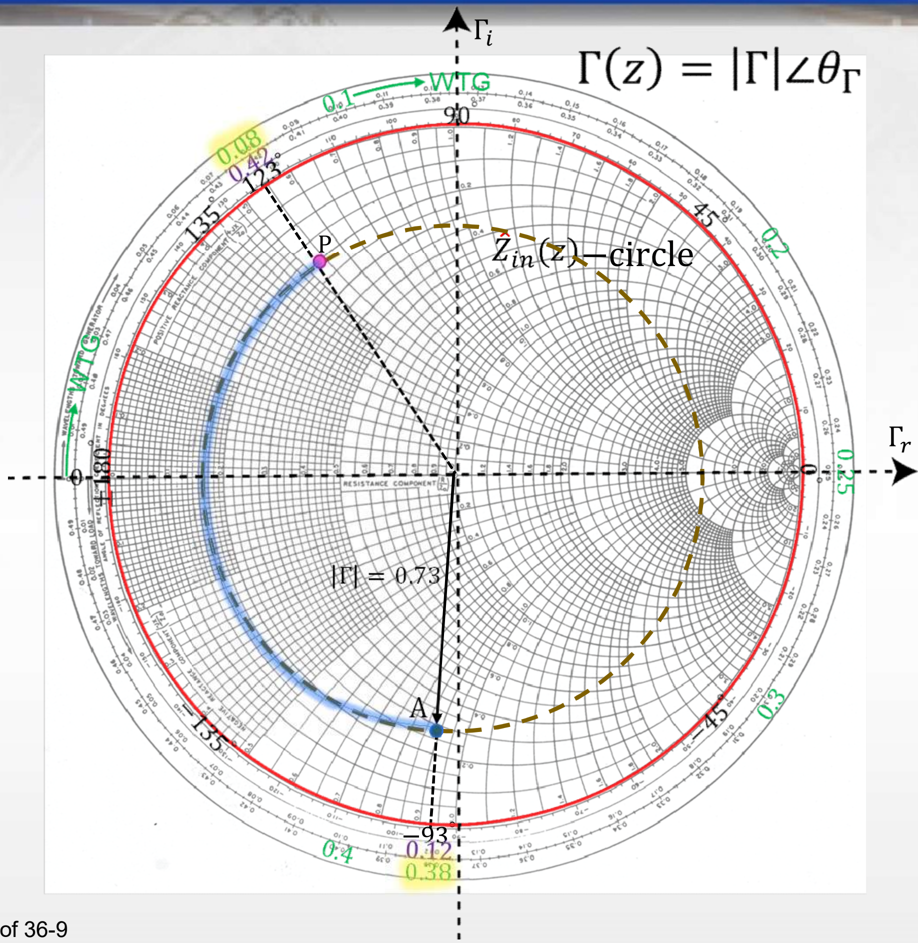
Example 3

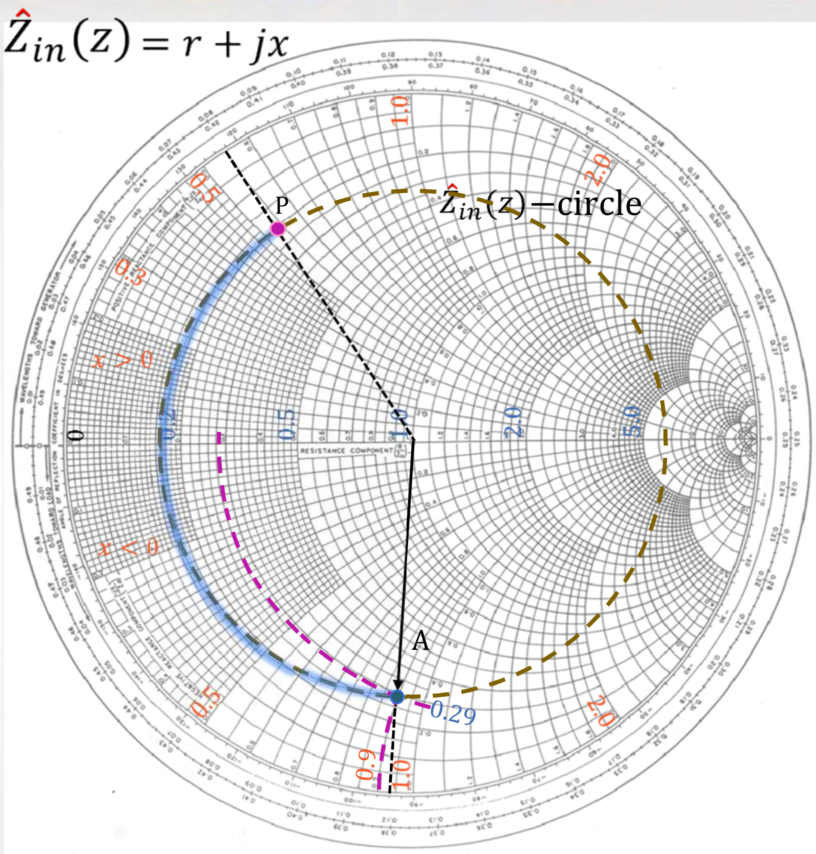
- Given $\hat{Z}_{in}(-\ell) = 0.2 + j0.525$, $\hat{Z}_L = 0.29 - j0.91$, find ℓ/λ.



- Ans: $\ell/\lambda = 0.2$

Example 3


$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$
Γ_i
Γ_r
WTG
P
A
$\hat{Z}_{in}(z)$−circle
$|\Gamma| = 0.73$
123°
−93
0.08
0.42
0.12
0.38
90
45
135
−45
−135
±180
0
0.1
0.2
0.25
0.3
0.4


$\hat{Z}_{in}(z) = r + jx$
$\hat{Z}_{in}(z)$−circle
P
A
0.29
$x > 0$
$x < 0$
0.2
0.5
1.0
2.0
5.0
0.9

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37. Appendix for Part II

Appendix for Part II

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❑ Reflection and Transmission of Electromagnetic Wave:

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}; \qquad \tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}; \qquad \tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}; \quad \tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}; \quad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Appendix for Part II

- **Transmission Line Equations:**

$$V(z) = V_o^+ e^{-j\beta} + V_o^- e^{+j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{+j\beta z}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \leq z \leq 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} Z_o$$