NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) An equilateral triangle loop of side length a is centered at the origin in the xy plane. The triangle loop carries a uniform charge distribution with line charge density ρ_l in free space.

(i) Express the distance h between the triangle center and its base in terms of a.

(ii) Using Coulomb's law, determine the electric field intensity $\vec{E}(z)$ along the z axis due to the triangle loop.

Note: $$\int \frac{1}{\left(x^2+u^2\right)^{3/2}}dx = \frac{x}{u^2\sqrt{x^2+u^2}}$$

(14 Marks)

Note: Question No. 1 continues on page 2.

(b) Let the charges on the triangle loop of part (a) be moving to form a steady current I in the counter-clockwise direction (as viewed from $z > 0$). Determine the magnetic field intensity $\vec{H}(z)$ along the z axis due to the triangle loop current.

(11 Marks)

2. (a) A circular loop of radius a containing two series resistors with resistance values of R_1 and R_2 is centered at the origin in the xy plane in free space. The loop is subjected to a time-varying magnetic field intensity of the form (for time $t \geq 0$)

$$\vec{H} = \exp(-t)\vec{a}_x + \cos(t)\vec{a}_y + \sin(t)\vec{a}_z \quad \text{A/m}.$$

(i) Derive the magnetic flux Φ_m passing through the loop and the induced voltage V_{emf} at time $t \geq 0$.

(ii) Derive the induced current I as well as the voltages V_1 and V_2 across the resistors at time $t \geq 0$. Sketch a diagram to indicate the current direction and label the voltage polarities assumed in your answers.

(12 Marks)

(b) A 100 MHz plane wave is propagating in a lossy medium with relative permittivity $\varepsilon_r = 10$, conductivity $\sigma = 2$ S/m and relative permeability $\mu_r = 1$.

(i) Using the good conductor approximation, determine the skin depth δ and loss tangent $\tan\delta$. Discuss the physical significance of both δ's.

(ii) Without using the good conductor approximation, calculate the propagation constant γ and the values of both δ's.

(13 Marks)

3. (a) A 60 MHz uniform plane wave (UPW) in free space occupying the region $z \leq 0$ is given by:

$$\tilde{E}_i(z,t) = \vec{a}_y \; 0.3 \cos\left(\omega t - k_i z + \frac{\pi}{3}\right) \text{ V/m}.$$

The UPW is incident normally on a planar interface with a lossy medium having $\mu = \mu_o$, $\varepsilon = 4.5\varepsilon_o$ and $\sigma = 0.9$ S/m occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) ω and k_i.

(ii) The attenuation constant α for the lossy medium.

(iii) The position z at which the average power density of the transmitted wave drops to 5% of its value at $z = 0$.

(13 Marks)

(b) The magnetic field intensity of a uniform plane wave (UPW) propagating in free space $(z \leq 0)$ is given by:

$$\vec{H}_i(x,z) = \left(-5\vec{a}_x + 12.5\vec{a}_z\right)e^{-j(10x+4z)} \text{ mA/m}.$$

The UPW is obliquely incident on a second medium made of lossless dielectric having $\mu = \mu_o$ and $\varepsilon = 1.4\varepsilon_o$ at $z = 0$, and occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) The electric field intensity of the incident UPW, i.e., $\vec{E}_i(x,z)$.

(ii) The magnitude of transmitted magnetic field intensity in the second medium, i.e., H_{ot}.

(12 Marks)

4. (a) A 12.5-cm long lossless transmission line operating at a frequency of 1.2 GHz has a characteristic impedance $Z_0 = 100\ \Omega$ and a phase velocity $u_p = 2.25 \times 10^8$ m/s. The line is terminated in a load $Z_L = 70 + j60\ \Omega$.

Assume that the load end is located at $z = 0$ and the source end at $z = -\ell$, where ℓ is the length of the transmission line.

Find the following and state any assumption(s) made:

(i) The wavelength λ on the transmission line.

(ii) The reflection coefficient $\Gamma(z)$ in polar form at $z = 0$ and $z = -\ell$.

(iii) The position z at which the magnitude of current on the line is maximum.

(iv) The average power delivered to the load if the magnitude of maximum current on the line is $|I|_{\max} = 5$ A.

(20 Marks)

(b) An unknown load Z_L' is connected to the transmission line in part (a) and it causes a standing wave ratio (SWR) of 3.3 and a 180° phase shift on the reflected voltage wave. Determine the unknown load Z_L'.

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \dfrac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r \partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (r A_r)}{r \partial r} + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\parallel = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\parallel = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \, e^{-j\beta z} + V_o^- \, e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \, e^{-j\beta z} - V_o^- \, e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.