

**NANYANG TECHNOLOGICAL UNIVERSITY****SEMESTER 1 EXAMINATION 2017-2018****EE2002 - ANALOG ELECTRONICS**

November / December 2017

Time Allowed: 2½ hours

**INSTRUCTIONS**

1. This paper contains 5 questions and comprises 10 pages.
  2. Answer ALL questions.
  3. All questions carry equal marks.
  4. This is a closed book examination.
  5. Unless specifically stated, all symbols have their usual meanings.
  6. A List of Formulae is provided in Appendix A on pages 8 to 10.
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1. (a) For the ideal Op-Amp in negative feedback shown in Figure 1(a) on page 2, derive the expression for the closed-loop gain  $A_{VCL} = (V_{OUT}/V_{IN})$ .

*Note: Parallel resistance of  $R_a$  and  $R_b$  can be written as  $R_a // R_b$  without expanding it.*

(10 Marks)

Note: Question No. 1 continues on page 2.

2

- (a) Determine the Q-point for the transistor.

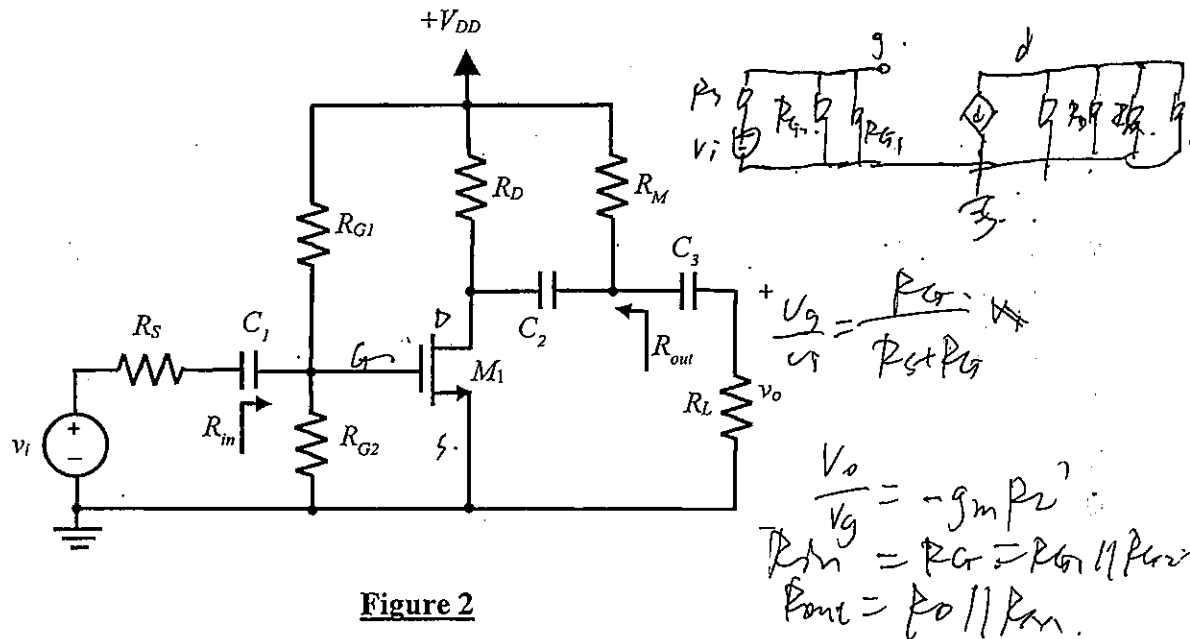
(5 Marks)

- (b) Determine the voltage gain  $A_v = \frac{v_o}{v_i}$ , the input resistance  $R_{in}$  and the output resistance  $R_{out}$  of the amplifier.

(12 Marks)

- (c) Determine the input small signal range for this amplifier.

(3 Marks)



**Figure 2**

3. For the differential amplifier circuit shown in Figure 3 on page 4. The voltage sources,  $v_1$  and  $v_2$ , represent the ground referenced ac input signals such that the differential input signal  $v_{id} = v_1 - v_2$  and the common-mode input signal  $v_{ic} = \frac{v_1 + v_2}{2}$ . The differential amplifier is biased by a constant current source with a DC current  $I_{SS}$  of  $300 \mu\text{A}$  and an ac resistance  $R_{SS}$  of  $175 \text{ k}\Omega$ . The matched pMOS transistors,  $M_1$  and  $M_2$ , have  $K_p = 250 \mu\text{A}/\text{V}^2$ ,  $V_{TP} = -1.5 \text{ V}$  and  $\lambda = 0.01 \text{ V}^{-1}$ . The output  $v_{out}$  is taken from the drain terminal of  $M_1$  to drive the load resistance  $R_L$ .

Note: Question No. 3 continues on page 4.

$$\frac{V_p}{V_g} = -\frac{i\omega R}{V_g} \quad (a)$$

If the DC voltages at the gates of  $M_1$  and  $M_2$  are both zero, determine the Q-point of the transistors. Verify that the pMOS transistors are operating in the saturation mode. (6 Marks)

(6 Marks)

$$= \frac{-g_m R}{1 + g_m R_{ss}}$$

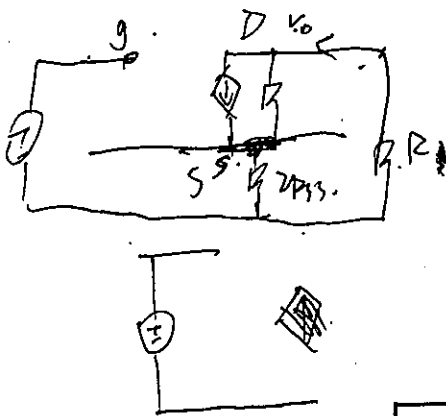
(b) For single-ended output and  $R_L = \infty$ , determine the differential-mode gain, common-mode gain and common-mode rejection ratio (CMRR) of the amplifier. (10 Marks)

(10 Marks)

(c) Determine the differential-mode input resistance and single-ended output resistance of the amplifier.

$$V_{gs} = V_g - V_s.$$

(4 Marks)



$$M_b = i \times R$$

$$V_3 = i_x Z_{R_{SS}}$$

$$+ (1 - g_m v_{gs}) r_o$$

$$= V_{TC} - i \times R_{SS}$$

$$= \frac{V_{TC} - i \times 2R_{SS}}{R_D + 12} \cdot 120$$

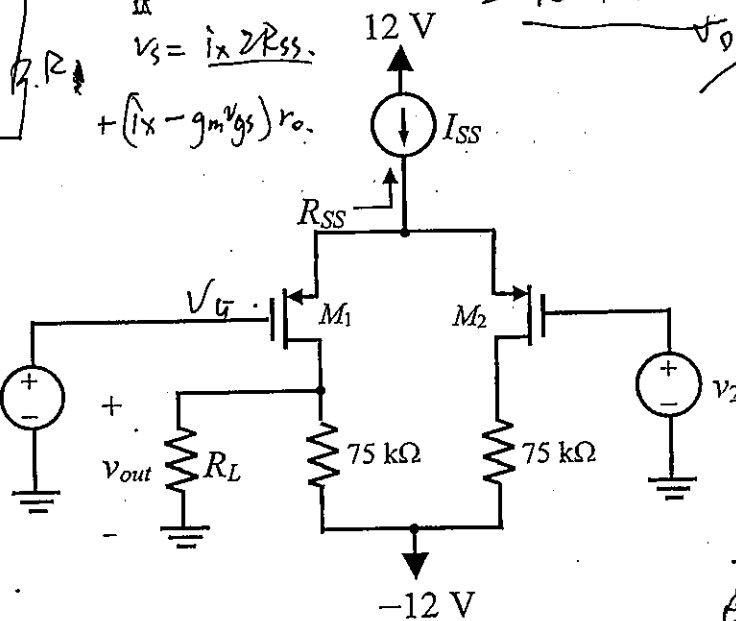
$$V_b = \bigcirc \quad \underline{V_{ic}}$$

$$= \approx g_{\mu\nu} g_{\alpha\beta} \underline{R}.$$

$$= g_m (v_{ic} - v_s) R.$$

$$= g_m v_{ic} R - g_m i_{x2} R$$

4.  $= g_m^{NiCl_2} - g_m^{Ni}$   
(a) Consider the



**Figure 3**

4. (a) Consider the current mirror circuit shown in Figure 4(a) on page 5. Assume all transistors are in saturation and ignore Early effect. It is given that  $I_{REF} = 100 \mu A$ ,  $V_{DD} = 10 V$ ,  $K'_n = K'_p = K' = 10 \mu A/V^2$ ,  $|V_{TP}| = |V_{TN}| = |V_T| = 0.6 V$  and (W/L) sizes of the transistors are written next to each of them in the figure. Also, the drain current equation for a MOSFET is given by:

$$I_D = \frac{K'}{2} \left( \frac{W}{L} \right) (V_{GS} - V_T)^2.$$

$$\Rightarrow |V_{GS}| = |V_T| + \sqrt{\frac{I_D}{K_n \left(\frac{W}{L}\right)}} =$$

Note: Question No. 4 continues on page 5.

- (i) Find the values of  $I_{out1}$  and  $I_{out2}$ . (2 Marks)

- (ii) What are the values of  $V_A$  and  $V_B$ ? (5 Marks)

- (b) For the same circuit in Figure 4(a), include Early effect now and do a small signal analysis to find an algebraic expression of the impedance  $R_{OUT}$  on the drain node of  $M_4$  (with voltage  $V_B$ ). Your answer should be in terms of  $g_m$  and  $r_o$  of the MOSFET. Indicate clearly the  $g_m$  and  $r_o$  of the  $i$ -th transistor as  $g_{m,i}$  and  $r_{o,i}$ , respectively. For example, the  $g_m$  and  $r_o$  for transistor  $M_2$  should be written as  $g_{m,2}$  and  $r_{o,2}$ , respectively. (7 Marks)

- (c) Consider the circuit in Figure 4(b) on page 6. If  $V_{BE} = 0.7$  V,  $V_A = 50$  V,  $\beta = 60$  and  $A_{E2} = 5A_{E1}$  where  $A_E$  denotes emitter area, find the value of  $I_{out}$  in mA correct up to two decimal places. (6 Marks)

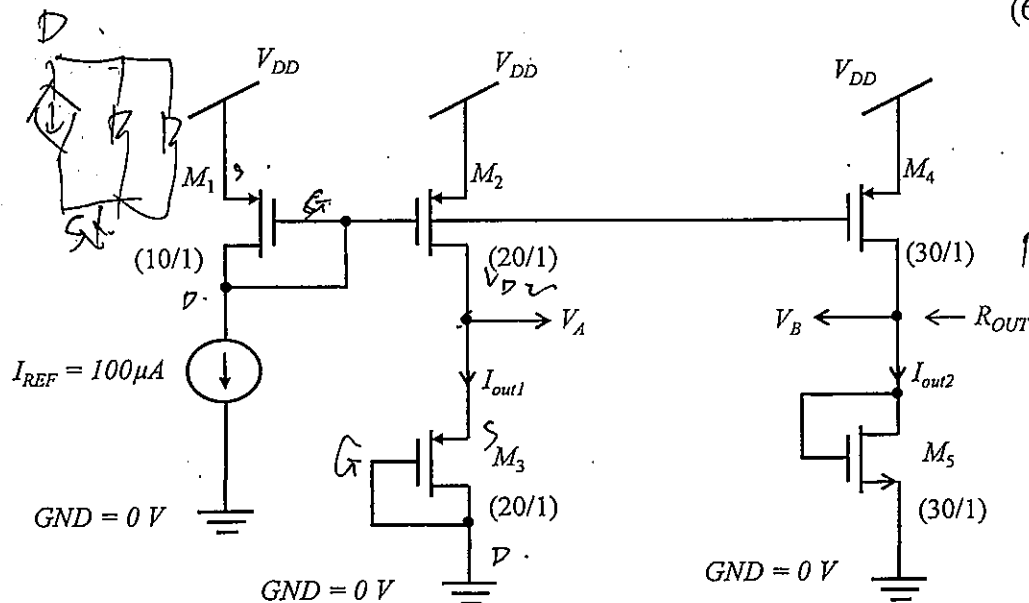
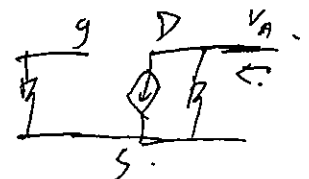


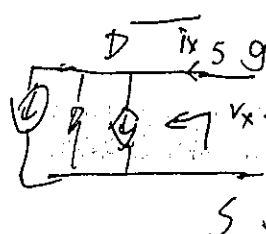
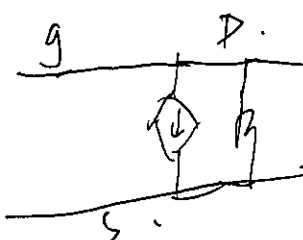
Figure 4(a)

$$I_{out1} = I_{D2} = \frac{k}{2} \left( \frac{W}{L} \right)_2 (V_{GS} - V_T)^2$$

$$\frac{V_{GS}}{V_{GS}}$$



Note: Question No. 4 continues on page 6.



$$i_x = g_m v_{gs}$$

$$= g_m v_x$$

$$R_{in} = \frac{V_A}{I_X} = \frac{1}{g_m}$$

$$I_{REF} = \frac{V_{DD} - V_{CE1}}{R}$$

$$I_{S0} \left( \frac{A_{E1}}{A} \right) e^{\frac{V_{BE}}{V_T}} \left( 1 + \frac{V_{CE}}{V_A} \right) + \frac{I_{S0}}{\beta} \left( \frac{A_{E2}}{A} \right) e^{\frac{V_{BE}}{V_T}} \left( 1 + \frac{V_{CE}}{V_A} \right)$$

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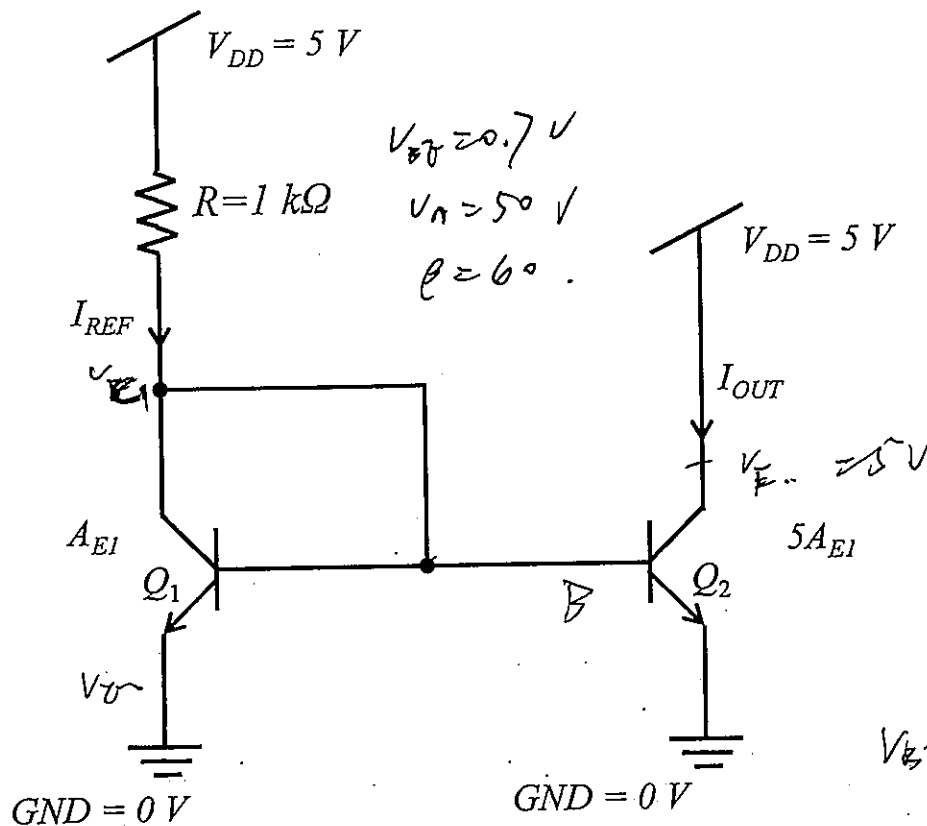


Figure 4(b)

5. (a) Consider the filter circuit shown in Figure 5(a) on page 7 and assume that the Operational Amplifier is ideal. Find an algebraic expression of the transfer function  $H(s) = V_{out}/V_{in}$  of this circuit in terms of the resistors and capacitors  $R_1, R_2, R_3, C_1$  and  $C_2$ . (6 Marks)
- (b) How many poles and zeros are there in the transfer function  $H(s)$  in part (a)? What are their (values) in terms of the resistors and capacitors  $R_1, R_2, R_3, C_1$  and  $C_2$ ? (6 Marks)
- (c) If  $R_1 = 100 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega, R_3 = 1 \text{ k}\Omega, C_1 = 10 \text{ nF}$  and  $C_2 = 1 \text{ nF}$ , find the magnitudes of poles and zeros found in part (b) of this question. (3 Marks)

Note: Question No. 5 continues on page 7.

$$as [b + c(1+ds)].$$

$$s[(R_2 + R_3)C_1 R_1 + C_1 R_1 R_2]$$

$$ab_3 + a_3 s(1+ds)$$

$$abc_3 + a_3 c s + a_3 d s^2$$

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- (d) For the circuit in Figure 5(b) with input at  $v_s$  and output at  $v_L$ , determine which capacitor contributes to the upper or higher cut-off frequency and explain the reason for your choice.

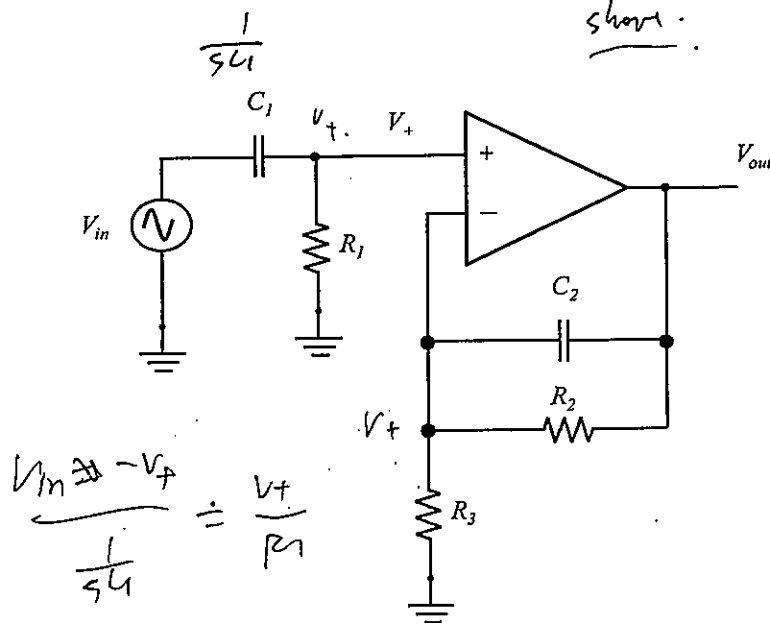
$$sC_1 R_1 R_2 + sC_1 R_1 R_3 + s^2 C_1 R_1 R_3 R_2 C_2$$

$$(R_2 + R_3) s C_1 R_1 + s^2 C_1 R_1$$

(3 Marks)

- (e) Use the OCTC method to find the resistance seen by the contributing capacitor found in part (d) of this question.

show



$$\frac{C_1 R_1 (R_2 + R_3)}{C_1 R_1 R_2 R_3 C_2}$$

$$\frac{V_{in} - V_+}{\frac{1}{sC_1}} = \frac{V_+}{R_1}$$

$$\frac{V_{out} - V_+}{(\frac{1}{sC_2} || R_2)} = \frac{V_+}{R_3}$$

$$\frac{V_{in}}{V_+} = \frac{\frac{1}{sC_1}}{R_1} + 1$$

Figure 5(a)

$$V_+ = \frac{R_1}{R_1 + \frac{1}{sC_1}}$$

$$\frac{V_{out}}{V_+} = \frac{(\frac{1}{sC_2} || R_2) + R_3}{R_3}$$

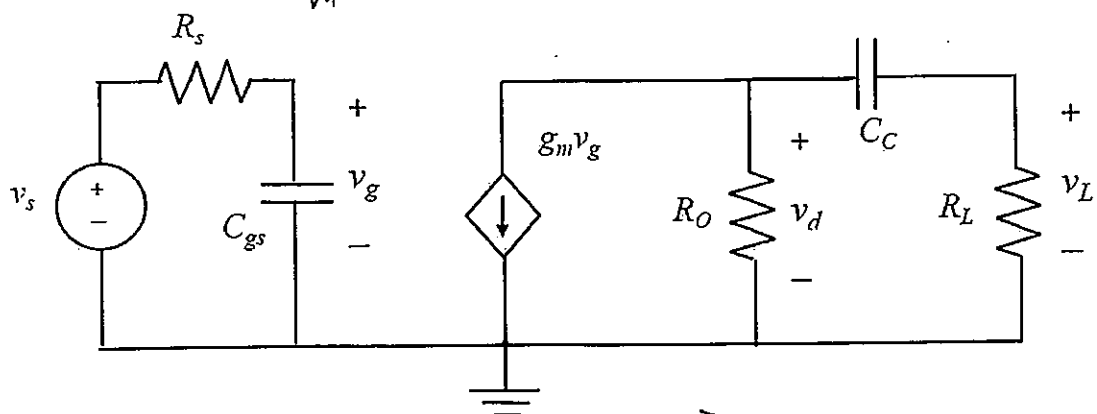


Figure 5(b)

$$\frac{\frac{R_2}{sC_2} + R_3}{\frac{1}{sC_2} + R_2} \times \frac{R_1}{R_1 + \frac{1}{sC_1}}$$

$$R_1 \left[ \frac{R_2}{sC_2} + R_3 \left( \frac{1}{sC_2} + R_2 \right) \right]$$

$$R_3 \left( \frac{1}{sC_2} + R_2 \right) \left( R_1 + \frac{1}{sC_1} \right)$$

$$sC_1 R_1 [R_2 + R_3(1 + R_2 sC_2)]$$

$$\frac{R_1 R_2 C_1 + R_3 (C_1 + sC_1 C_2 R_2)}{R_3 (C_1 + sC_1 C_2 R_2) (R_1 sC_2 C_1 + C_2 + R_1 R_2 + R_1 R_3 (1 + sC_2 R_2))}$$

$$sC_1 R_1 R_2 + sC_1 R_1 R_3 + s^2 C_1 R_2 R_1 R_3$$

## Appendix A

### List of Selected Formulae

#### Op-Amps:

Closed-Loop Negative Feedback Inverting Gain,  $A_{VCL} = \frac{v_o}{v_i} = -\frac{R_f}{R_i}$

Figure (a)

Closed-Loop Negative Feedback Non-Inverting Gain,  $A_{VCL} = \frac{v_o}{v_i} = \left(1 + \frac{R_f}{R_i}\right)$

Figure (b)

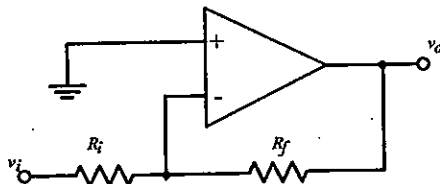


Figure (a)

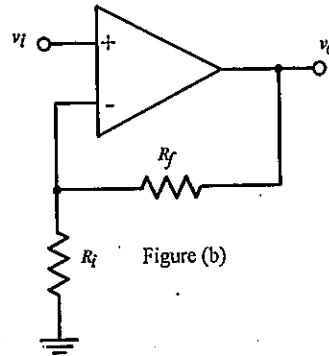


Figure (b)

Op-Amp's Slew Rate,  $SR \geq \left| \frac{dv_o}{dt} \right|_{\max} = A_{VCL} \omega a_m = A_{VCL} a_m 2\pi f$ ,

where  $v_i = a_m \sin(\omega t)$ ,  $v_o = A_{VCL} a_m \sin(\omega t)$  and  $\left| \frac{dv_o}{dt} \right| = A_{VCL} \omega a_m \cos(\omega t)$

Op-Amp's frequency response:  $A_{VOL}(jf) = \frac{A_o}{\left(1 + \frac{jf}{f_p}\right)}$

Gain-Bandwidth Product:  $A_o f_p = f_o = \frac{1}{\beta} (BW)_{CL}$

where  $\frac{1}{\beta} = \frac{R_f + R_i}{R_i}$

$$t_r = \frac{0.35}{(BW)_{CL}}$$

#### Diodes:

$$v_D \approx nV_T \ln\left(\frac{i_D}{I_S}\right) \text{ or } i_D \approx I_S e^{\left(\frac{v_D}{nV_T}\right)}$$

where  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$$\text{Diode conductance: } g_D = \frac{1}{r_D} = \frac{I_D}{nV_T}$$

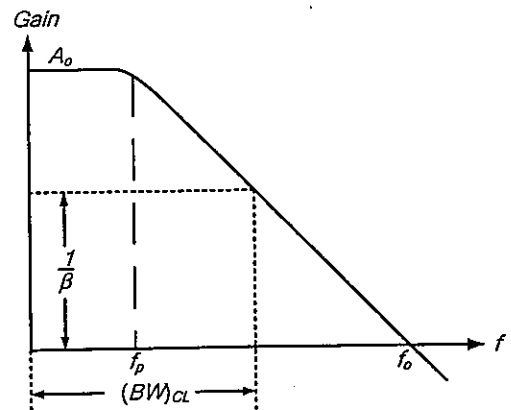


Figure (c)



**BJT in Forward Active Region:**

Ignore early effect:  $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right)$

With early effect:  $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right) \left(1 + \frac{|v_{CE}|}{V_A}\right)$

where  $I_S$ : Saturation current,

$V_T$ : Thermal voltage, assume 25 mV at room temperature,

$V_A$ : Early voltage.

For npn transistor,  $|v_{BE}| = v_{BE}$  and  $|v_{CE}| = v_{CE}$ ;

For pnp transistor,  $|v_{BE}| = v_{EB}$  and  $|v_{CE}| = v_{EC}$ .

**Small-signal model parameters of BJT:**

$$g_m = \frac{I_C}{V_T}, r_\pi = \frac{\beta}{g_m} \text{ and } r_o = \frac{V_A + |V_{CE}|}{I_C} \approx \frac{V_A}{I_C}$$

where  $I_C$ : DC collector current at Q-point

$V_{CE}$ : DC collector-emitter voltage at Q-point

Criterion for small-signal operation of BJT:  $|v_{be}| \leq 0.2 V_T$

**MOSFET in Saturation Region:**

Criterion:  $V_{DS} \geq V_{GS} - V_{TN}$  for NMOS;

$|V_{DS}| \geq |V_{GS}| - |V_{TP}|$  for PMOS

where  $V_{TN}, V_{TP}$ : Threshold voltage;

$V_{DS}$ : DC drain-source voltage,

$V_{GS}$ : DC gate-source voltage.

Ignore channel-length modulation effect:  $i_D = \frac{K_n}{2} (v_{GS} - V_{TN})^2$  for NMOS,

$$i_D = \frac{K_p}{2} (|v_{GS}| - |V_{TP}|)^2 \text{ for PMOS.}$$

With channel-length modulation effect:  $i_D = \frac{K_n}{2} (v_{GS} - V_{TN})^2 (1 + \lambda v_{DS})$  for NMOS,

$$i_D = \frac{K_p}{2} (|v_{GS}| - |V_{TP}|)^2 (1 + \lambda |v_{DS}|) \text{ for PMOS.}$$

where  $\lambda$ : channel length modulation parameter,

For NMOS  $K_n = K'_n \left(\frac{W}{L}\right)$  and  $K'_n = \mu_n C_{ox}$ ; For PMOS  $K_p = K'_p \left(\frac{W}{L}\right)$  and  $K'_p = \mu_p C_{ox}$ .

**MOSFET in Triode Region:**

Criterion:  $V_{DS} < V_{GS} - V_{TN}$  for NMOS;

$|V_{DS}| < |V_{GS}| - |V_{TP}|$  for PMOS

Ignore channel-length modulation effect:  $i_D = K_n \left( v_{GS} - V_{TN} - \frac{v_{DS}}{2} \right) v_{DS}$  for NMOS,

$$i_D = K_p \left( |v_{GS}| - |V_{TP}| - \frac{|v_{DS}|}{2} \right) |v_{DS}| \text{ for PMOS.}$$

With channel-length modulation effect:

$$i_D = K_n \left( v_{GS} - V_{TN} - \frac{v_{DS}}{2} \right) v_{DS} (1 + \lambda v_{DS}) \text{ for NMOS,}$$

$$i_D = K_p \left( |v_{GS}| - |V_{TP}| - \frac{|v_{DS}|}{2} \right) |v_{DS}| (1 + \lambda |v_{DS}|) \text{ for PMOS.}$$

### Small-signal model parameters of MOSFET

For NMOS:  $g_m = \sqrt{2K_n I_D (1 + \lambda V_{DS})} \approx \sqrt{2K_n I_D}$  and  $r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \approx \frac{1}{\lambda I_D}$

For PMOS:  $g_m = \sqrt{2K_p I_D (1 + \lambda |V_{DS}|)} \approx \sqrt{2K_p I_D}$  and  $r_o = \frac{\frac{1}{\lambda} + |V_{DS}|}{I_D} \approx \frac{1}{\lambda I_D}$

where  $I_D$ : DC drain current at Q-point  
 $V_{DS}$ : DC drain-source voltage at Q-point

Criterion for small-signal operation:

For NMOS:  $|v_{gs}| \leq 0.2(V_{GS} - V_{TN})$

For PMOS:  $|v_{gs}| \leq 0.2(|V_{GS}| - |V_{TP}|)$

where  $V_{GS}$ : DC gate-source voltage at Q-point.

### Miller Effect

The equivalent shunt input capacitance:  $C_x = C \times (1 + A_v)$

The equivalent shunt output capacitance:  $C_y = C \times (1 + \frac{1}{A_v})$

where  $-A_v$ : the gain of the voltage amplifier

$C$ : the original capacitance between the input and output terminals of the voltage amplifier

### Frequency Response

By using OCTC and SCTC methods, the upper cut-off frequency is estimated by  $\omega_{H-3dB} \approx \frac{1}{\sum_i C_i R_i}$

and the lower cut-off frequency is estimated by  $\omega_{L-3dB} \approx \sum_i \frac{1}{C_i R_i}$

where  $C_i$ : the contributing capacitor for the cut-off frequency

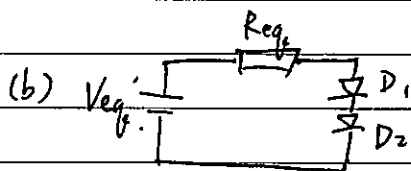
$R_i$ : the equivalent resistance seen by the capacitor  $C_i$

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$$1. (a) \frac{V_{in} - V_+}{V_+} = \frac{R_1}{R_2}, \quad \frac{V_x - V_-}{R_6} = \frac{V_x}{R_5 + R_6}, \quad \frac{V_{out} - V_x}{R_8} = \frac{V_x}{(R_5 + R_6) \parallel R_7}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{(R_1 + R_2) R_5 [R_8 + (R_5 + R_6) \parallel R_7]}{R_2 (R_5 + R_6) [(R_5 + R_6) \parallel R_7]}$$



According to Thevenin Theorem.

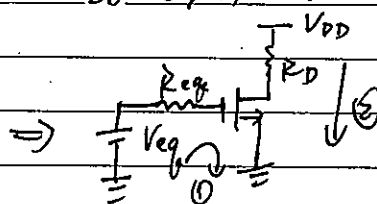
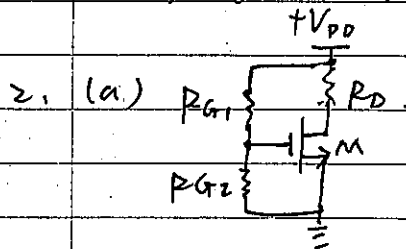
$$V_{eq} = \frac{R_2 \parallel (R_3 + R_4)}{R_1 + R_2 \parallel (R_3 + R_4)} V_s \cdot \frac{R_4}{R_3 + R_4} = 3V.$$

$$R_{eq} = ((R_1 \parallel R_2) + R_3) \parallel R_4 = 3k\Omega.$$

$$\text{Then, } I_D = \frac{3V - 2V_D}{3k\Omega} \quad \text{and} \quad V_D = nV_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln \frac{I_D}{15 \times 10^{-9} A}$$

$$V_D = 0.7V \rightarrow I_D = 53.3 \text{ mA} \rightarrow V_D = 0.632V \rightarrow I_D = 57.9 \text{ mA}$$

$$\rightarrow V_D = 0.634V \rightarrow I_D = 57.7 \text{ mA} \rightarrow V_D = 0.634V \rightarrow I_D = 57.7 \text{ mA}.$$

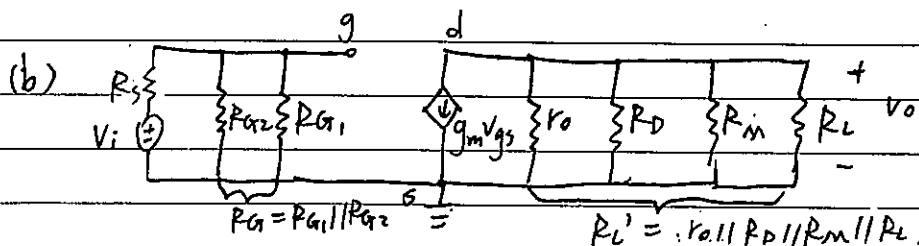


$$V_{eq} = \frac{R_{G2}}{R_{G1} + R_{G2}} V_{DD} = 3.5V.$$

$$R_{eq} = R_{G1} \parallel R_{G2} = 2.125 \text{ M}\Omega.$$

①  $V_{eq} = V_{gs}$

②  $V_{ds} = V_{DD} - I_D R_D \Rightarrow V_{ds} = 3.75V, I_D = 6.25 \text{ mA}$

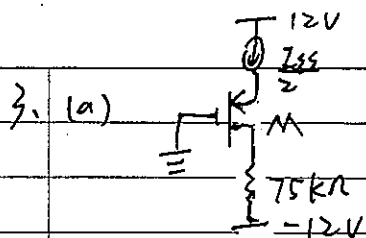


$$\frac{V_o}{V_g} = \frac{V_d}{V_g} = \frac{-g_m V_{gs} R_L'}{V_{gs}} = -g_m R_L', \quad A_{vt} = \frac{V_o}{V_g} \cdot \frac{V_g}{V_i} = -g_m R_L' \cdot \frac{R_g}{R_{in} + R_g}$$

$$R_{in} = R_g = R_{G1} \parallel R_{G2}, \quad R_{out} = R_D \parallel R_M \parallel r_o \approx R_D \parallel R_M$$

(c)  $|V_{in}| \leq 0.2 (V_{gs} - V_{TN}) (1 + g_m R_{S1}) \left( \frac{R_{in} R_g}{R_{in}} \right)$

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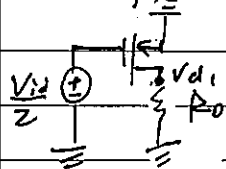


$$I_D = I_S = \frac{I_{SS}}{2} = 150 \mu A = \frac{K_p}{2} (|V_{GS}| - |V_{TP}|)^2$$

$$\Rightarrow |V_{GS}| = 2.60 V$$

$$\Rightarrow |V_{DS}| = V_{DD} - I_D R_D - (-|V_{GS}|) = 3.35 V > |V_{GS}| - |V_{TP}|$$

(b) Differential-mode.



$$V_{d1} = -g_m \left( \frac{V_{id}}{2} \right) R_D$$

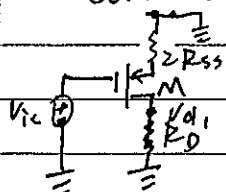
$$A_{dm-se} = \frac{V_{d1}}{V_{id}} = -\frac{g_m R_D}{2}$$

(c)

$$R_{in} = R_{id} = \infty$$

$$R_{out} = r_o || R_D \approx R_D$$

Common-mode



$$V_{d1} = -g_m V_{GS} R_D$$

$$V_{ic} = V_{GS} + g_m V_{GS} 2R_{SS}$$

$$\Rightarrow A_{cm-se} = \frac{V_{d1}}{V_{ic}} = \frac{-g_m R_D}{1 + 2g_m R_{SS}} \approx -\frac{R_D}{2R_{SS}}$$

$$CMRR = \left| \frac{A_{dm-se}}{A_{cm-se}} \right| = g_m R_{SS}$$

4. (a)  $I_{out1} = \frac{K'_1}{2} \left( \frac{W}{L} \right)_1 (|V_{GS}| - |V_T|)^2$

$$I_{out2} = \frac{K'_2}{2} \left( \frac{W}{L} \right)_2 (|V_{GS}| - |V_T|)^2$$

$$\text{and } I_{REF} = \frac{K'_1}{2} \left( \frac{W}{L} \right)_1 (|V_{GS}| - |V_T|)^2 = 100 \mu A$$

$$\text{thus, } I_{out1} = 200 \mu A, I_{out2} = 300 \mu A$$

$$|V_{GS}| = |V_T| + \sqrt{\frac{2I_{REF}}{K'_1 \left( \frac{W}{L} \right)_1}} = 2.014 V$$

$$V_A = V_{S3} = |V_{GS}| + V_{S3} = 2.014 V + 0 V = 2.014 V$$

$$V_B = V_{GS} = |V_{GS}| + V_{S5} = 2.014 V + 0 V = 2.014 V$$

(b) Didn't answer.

(c)  $I_{C1} = I_S \left( 1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}, I_{C2} = I_S \left( 1 + \frac{V_{CE2}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}$

$$I_{B1} = \frac{I_{C1}}{\beta_1} = \frac{I_S \left( 1 + \frac{V_{CE1}}{V_A} \right) e^{\frac{V_{BE}}{V_T}}}{\beta \left( 1 + \frac{V_{CE1}}{V_A} \right)} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}, I_{B2} = \frac{I_{C2}}{\beta_2} = \frac{I_S}{\beta} e^{\frac{V_{BE}}{V_T}}$$

$$\text{Since } I_S = I_{S0} \left( \frac{A_{FE}}{A} \right) \text{ and } A_{FE2} = 5 A_{FE1}$$

$$\Rightarrow MR = \frac{I_{out}}{I_{REF}} = \frac{5}{1 + \frac{5+1}{\beta}} = 4.545$$

$$I_{REF} = \frac{V_{DD} - V_{CE1}}{R} = \frac{V_{DD} - V_B}{R} = \frac{V_{DD} - V_{BE}}{R} = \frac{5V - 0.7V}{1k\Omega} = 4.3 mA$$

$$I_{out} = MR \cdot I_{REF} = 19.55 mA$$

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$$5. (a) \frac{V_{in} - V_+}{\frac{1}{sC_1}} = \frac{V_+}{R_1}, \quad \frac{V_{out} - V_+}{\frac{1}{sC_2} \parallel R_2} = \frac{V_+}{R_3}$$

$$\Rightarrow \frac{V_+}{V_{in}} = \frac{sC_1 R_1}{1 + sC_1 R_1}, \quad \frac{V_{out}}{V_+} = \frac{R_2 + R_3(1 + sC_2 R_2)}{R_3(1 + sC_2 R_2)}$$

$$\Rightarrow H(s) = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_+} \cdot \frac{V_+}{V_{in}} = \frac{sC_1 R_1}{1 + sC_1 R_1} \cdot \frac{R_2 + R_3(1 + sC_2 R_2)}{R_3(1 + sC_2 R_2)}$$

$$= \frac{sC_1 R_1 (R_2 + R_3 + sC_2 R_2 R_3)}{R_3 (1 + sC_1 R_1)(1 + sC_2 R_2)}$$

$$= \frac{C_1 R_1 (R_2 + R_3)}{R_3} \cdot \frac{s \left( 1 + \frac{R_2 + R_3}{C_2 R_2 R_3} \right)}{\left( 1 + \frac{s}{1/C_1 R_1} \right) \left( 1 + \frac{s}{1/C_2 R_2} \right)}$$

cb) zeros:  $0, -\frac{R_2 + R_3}{C_2 R_2 R_3}$

poles:  $-\frac{1}{C_1 R_1}, -\frac{1}{C_2 R_2}$

(d) Short  $C_{gs}$ ,  $V_g = 0 \Rightarrow$  reduce voltage gain  $\Rightarrow C_{gs}$  contributor.

Short  $C_c$ ,  $V_L = V_d$ , increases  $\Rightarrow$  increase voltage gain  $\Rightarrow C_c$  helper.

(e)  $R_{C_{gs}} = R_s$

