EE2010
Signals and Systems
Part III

1.1 Types of Modulation

with Instructor:

A/P Teh Kah Chan

Recommended Readings

Textbook

1. S. Haykin and B. V. Veen, *Signals and Systems*, Wiley, 2nd Edition, 2003. (TK5102.5.H419)

References

1. A. V. Oppenheim and A. S. Willsky, *Signals and Systems,* Prentice-Hall, 2nd Edition, 1997. (QA402.P62)
2. B. P. Lathi, *Linear Systems and Signals,* Oxford University Press, 1st Edition, 2002. (TK5102.5.L352)

Outline of Signals & Systems- Part 3

Modulation

1.1 **Types of Modulations** ☜

1) Basic Communication System☜
2) Three Types of Modulations☜
3) Main Purposes of Modulations☜

1.2 Amplitude Modulation

1) Time Domain Description
2) Frequency Domain Description
3) Generation of AM Signal
4) Demodulation of AM Signal

1.3 Angle Modulation

1) Frequency Modulation
2) Phase Modulation

1) Basic Communication System

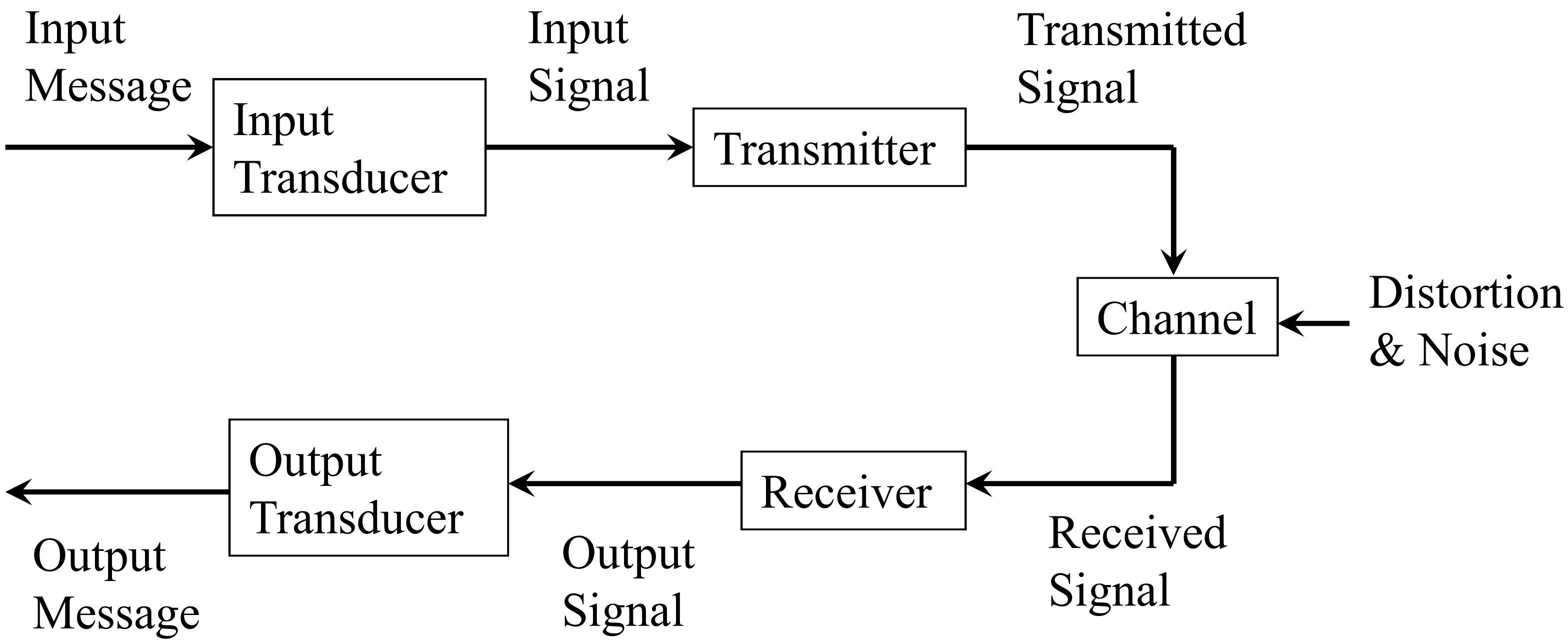

Figure 1: A basic communication system block diagram

1) Basic Communication System

Input/Output Transducer

- To convert the message into an electrical signal and vice versa

Transmitter

- To convert electrical signal into a form that is suitable for transmission through a physical channel
- This process is called modulation

Channel

- The physical medium that is used to send the signal from transmitter to receiver

Receiver

- To recover the message signal from the received signal
- This process is called demodulation

1) Basic Communication System

Definitions

- Modulation: A process in which the modulator systematically alters a carrier signal in accordance with a modulating signal which represents the message
- Modulating signal $m(t)$: The message or information signal to be sent, i.e. voice signal
- Carrier signal $c(t)$: A sinusoidal wave given by

$$c(t) \quad = \quad A_c \cos(2\pi f_c t + \phi_c)$$

where A_c is the carrier amplitude f_c is the carrier frequency, and ϕ_c is the carrier phase angle

- Modulated signal $x(t)$: The resultant signal of the modulating process

2) Types of Modulations

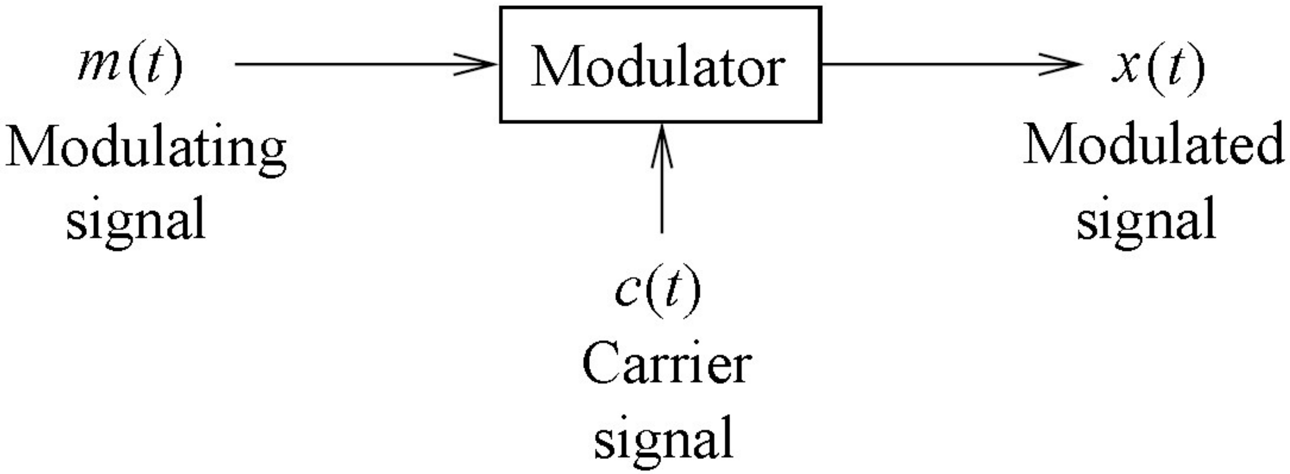

Figure 2: An illustration of modulation process

2) Types of Modulations

Amplitude Modulation (AM): The carrier amplitude is varied with the message signal

$$x_{AM}(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

Frequency Modulation (FM): The carrier frequency is varied with the message signal

$$x_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right]$$

Phase Modulation (PM): The carrier phase angle is varied with the message signal

$$x_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

2) Types of Modulations

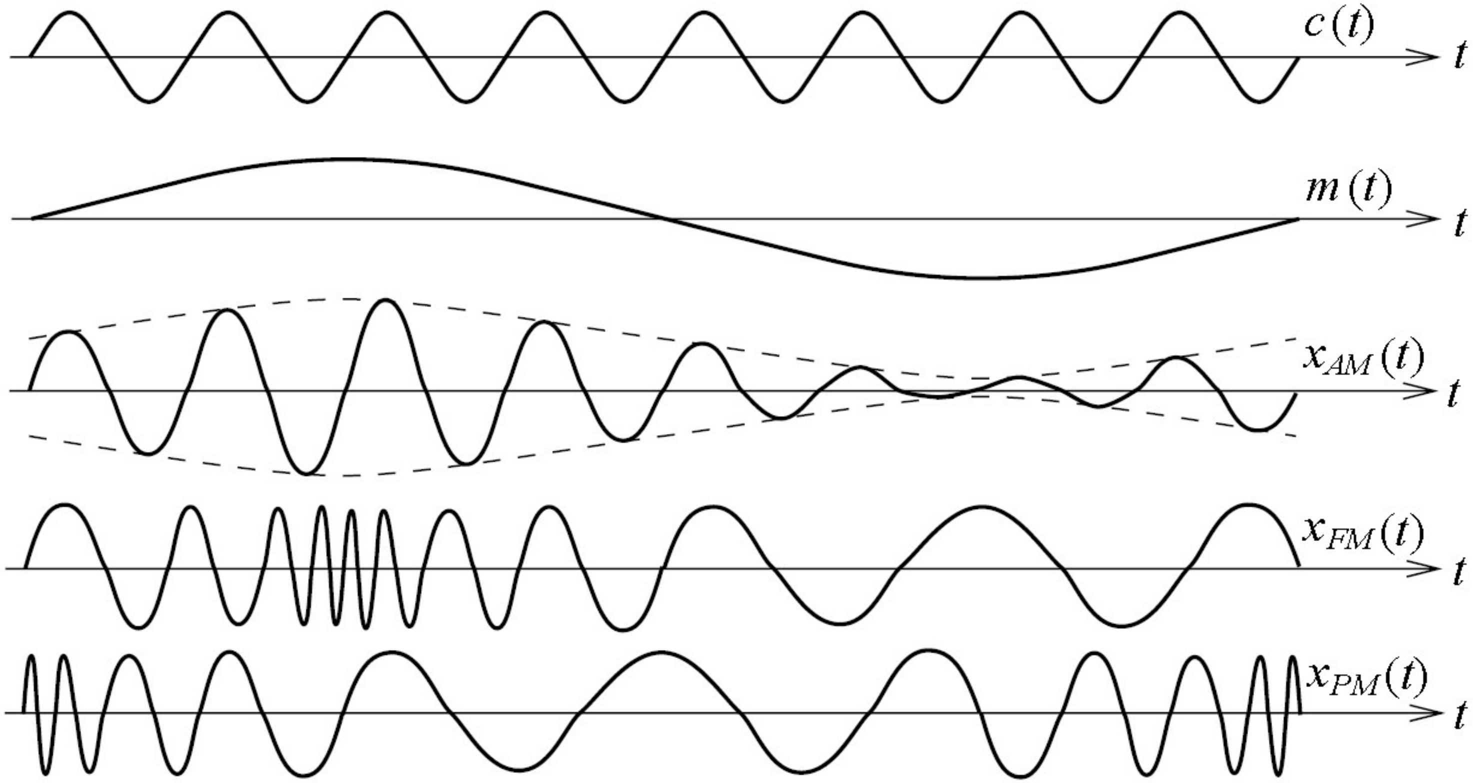

Figure 3a: Waveforms of various modulated signals

2) Types of Modulations

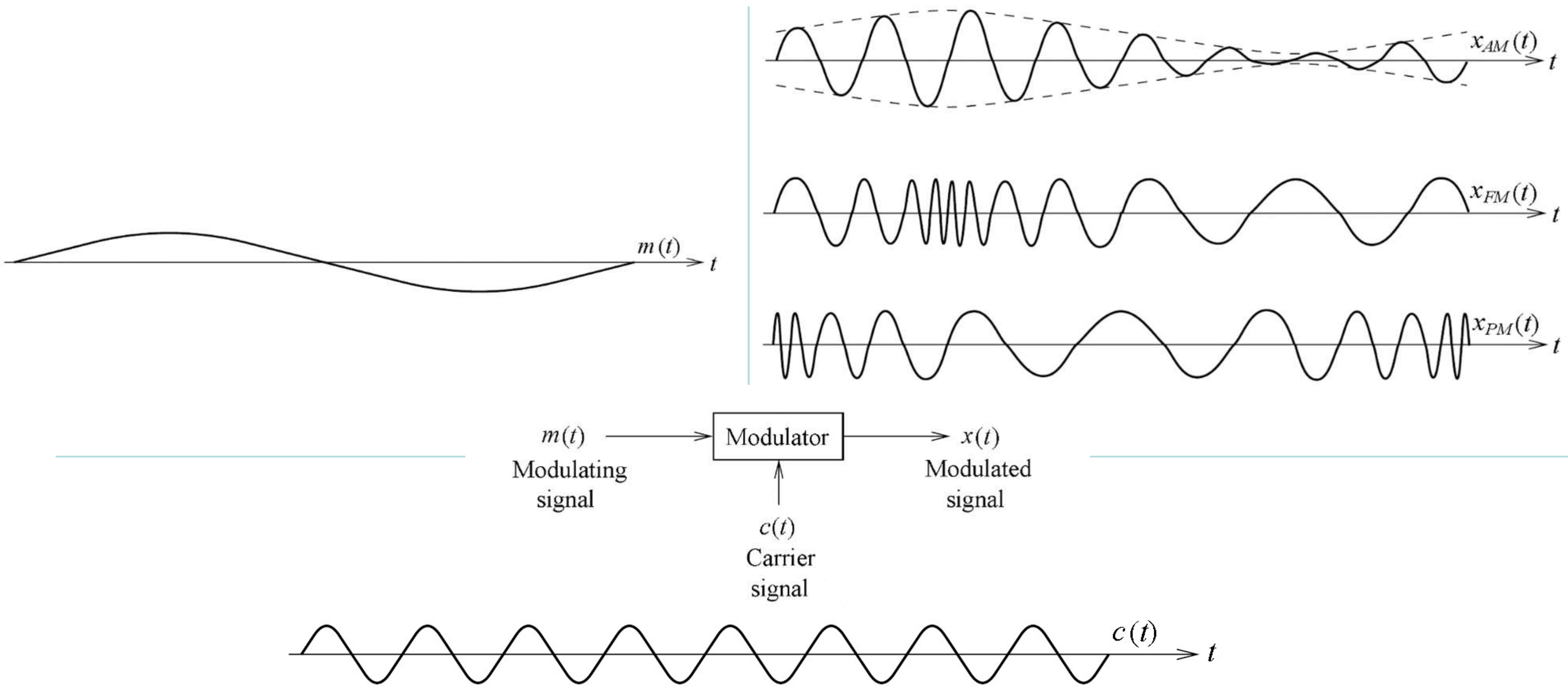

Figure 3b: Waveforms of various modulated signals

3) Main Purpose of Modulation

1) To shift the spectral content of message signal to operating frequency band

2) To allow efficient transmission

3) To permit the use of multiplexing

4) To provide better utilization of radio frequency spectrum

1.1 Types of Modulations Summary 1

- 1) Communication System Block Diagram
- 2) Types of Modulations
  - Amplitude Modulation (AM)

$$x_{AM}(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

  - Frequency Modulation (FM)

$$x_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right]$$

  - Phase Modulations (PM)

$$x_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

- 3) Main Purposes of Modulations

Please proceed with the next activity.

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1.2 Amplitude Modulation I

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Outline of Signals & Systems- Part 3

Modulation

1.1 Types of Modulations✓✓

1) Basic Communication System✓
2) Three Types of Modulations✓
3) Main Purposes of Modulations✓

1.2 Amplitude Modulation ☜

1) Time-Domain Description☜
2) Frequency-Domain Description ☜
3) Generation of AM Signal
4) Demodulation of AM Signal

1.3 Angle Modulation

1) Frequency Modulation
2) Phase Modulation

1.2 Amplitude Modulation

- Amplitude modulation (AM) is the process in which the carrier amplitude is varied in proportion to message signal $m(t)$
- The expression of the conventional or full AM is given by

$$x_{AM}(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

- A_c is the carrier amplitude
- k_a is a constant called the amplitude sensitivity factor of the modulator
- $m(t)$ is the modulating signal
- f_c is the carrier frequency
- ϕ_c is the carrier phase angle which is assumed to be zero

1.2 Amplitude Modulation

- The maximum absolute value of $k_a m(t)$ is defined as the modulation index μ
- Undermodulation, which is governed by the condition

$$|k_a m(t)| \quad \leq \quad 1, \quad \text{for all } t$$

- Overmodulation, which is governed by the condition

$$|k_a m(t)| \quad > \quad 1, \quad \text{for some } t$$

1) Time-Domain Description

Example 1: Consider a sinusoidal modulating signal $m(t) = A_m \cos(2\pi f_m t)$ where A_m and f_m are, respectively, the amplitude and frequency of the modulating signal. Determine the modulation index and sketch the corresponding time-domain AM waveform.

The amplitude modulated signal is given by

$$\begin{aligned} x_{AM}(t) &= A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \\ &= A_c\left[1 + k_a A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t) \end{aligned}$$

Thus the modulation index is given by $\mu = k_a A_m$

The waveforms of x_{AM}(t) for both undermodulation and overmodulation cases are shown in Figure 4.

1) Time-Domain Description

Example 1: $m(t) = A_m \cos(2\pi f_m t)$

For the undermodulation case, let A_{max} and A_{min} denote the maximum and minimum values of the positive envelope of the modulated wave, we have

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

Solving for μ yields

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

1) Time-Domain Description

Example 1: $m(t) = A_m \cos(2\pi f_m t)$

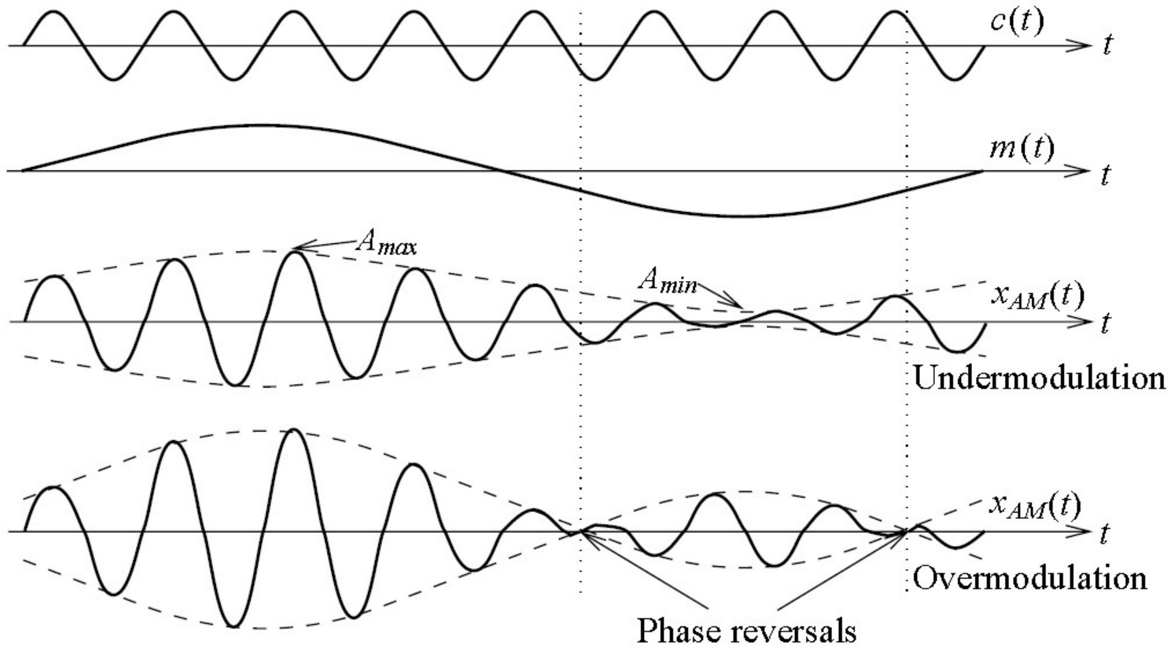

Figure 4: Waveforms of various AM signals

Spectrum of AM Signal

- The time-domain expression of the full AM is given by

$$\begin{aligned} x_{AM}(t) &= A_c\,[1 + k_a m(t)]\cos(2\pi f_c t) \\ &= A_c\cos(2\pi f_c t) + k_a A_c m(t)\cos(2\pi f_c t) \end{aligned}$$

- Performing the CT Fourier Transform on $x_{AM}(t)$ we obtain the spectrum of the full AM signal given by

$$X_{AM}(f) = \frac{A_c}{2}\,[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}\,[M(f - f_c) + M(f + f_c)]$$

where $M(f)$ is the FT of $m(t)$

Spectrum of AM Signal

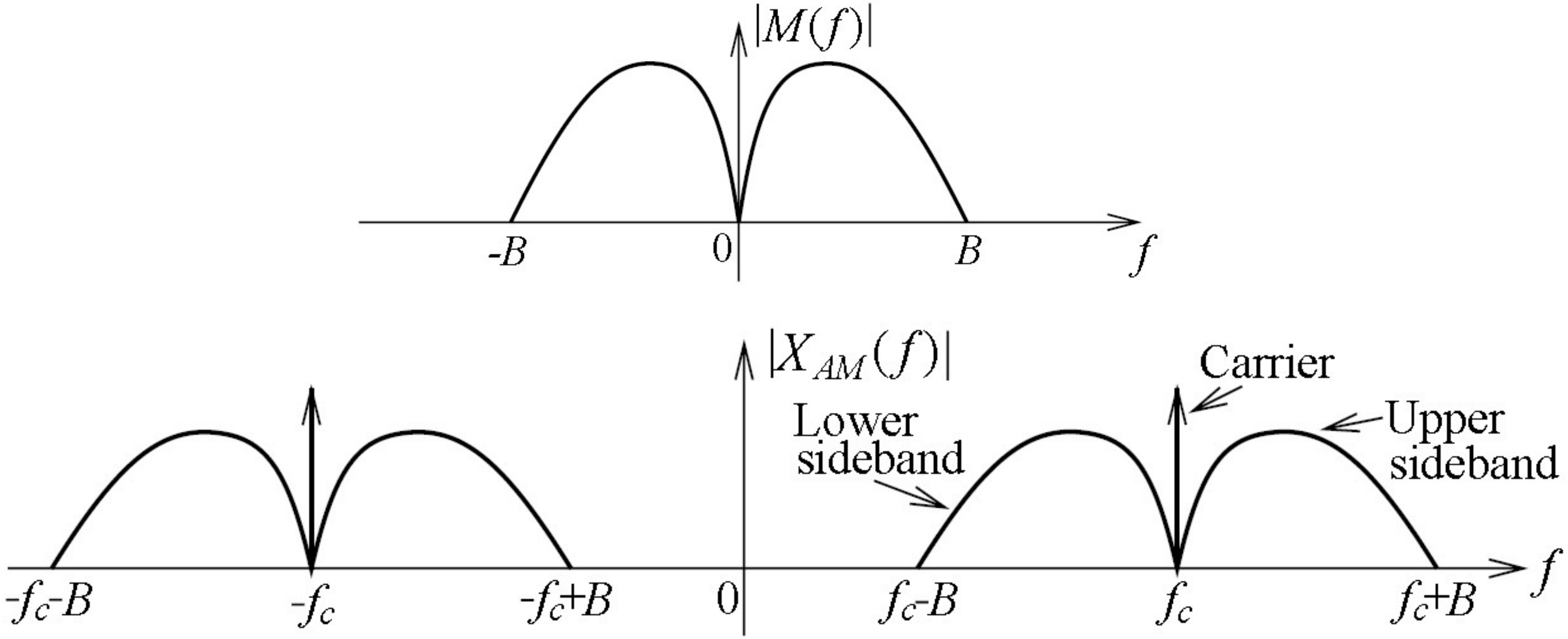

Figure 5: Spectrum of AM signal

Spectrum of AM Signal

From the AM spectrum shown in Figure 5, we observe:

- The AM spectrum consists of the carrier component at frequency f_c, plus two symmetrical sidebands centered at f_c
- These two symmetrical sidebands are called the upper sideband and lower sideband
- If the bandwidth of the modulating signal is B, the bandwidth of the AM signal is $2B$
- For $x_{\mathrm{AM}}(t)$ to contain no distortion, it is required that $f_c > B$, else distortion occurs
- In practice, $f_c >> B$

2) Frequency-Domain Description

Example 2: For the same AM signal in Example 1, sketch the spectrum of the AM wave and find the carrier and sideband power.

In this case, $x_{AM}(t) \quad = \quad A_c\,[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$

Thus, the spectrum is given by

$$\begin{aligned} X_{AM}(f) \quad = \quad & \frac{A_c}{2}\,[\delta(f - f_c) + \delta(f + f_c)] \\ & + \frac{\mu A_c}{4}\,[\delta(f - f_c - f_m) + \delta(f - f_c + f_m)] \\ & + \frac{\mu A_c}{4}\,[\delta(f + f_c + f_m) + \delta(f + f_c - f_m)] \end{aligned}$$

The spectrum is shown in Figure 6.

Power Efficiency of AM Signal

Example 2:

The carrier power is

$$P_c = \frac{A_c^2}{2}$$

The sideband power is

$$P_s = \frac{\mu^2 A_c^2}{4}$$

Thus, the power efficiency is given by

$$\frac{P_s}{P_s + P_c} = \frac{\mu^2}{2 + \mu^2}$$

Since the maximum value of μ for undermodulation condition is $\mu = 1$, thus the maximum power efficiency for the full AM is equal to 33.33%

Spectrum of AM Wave

Example 2:

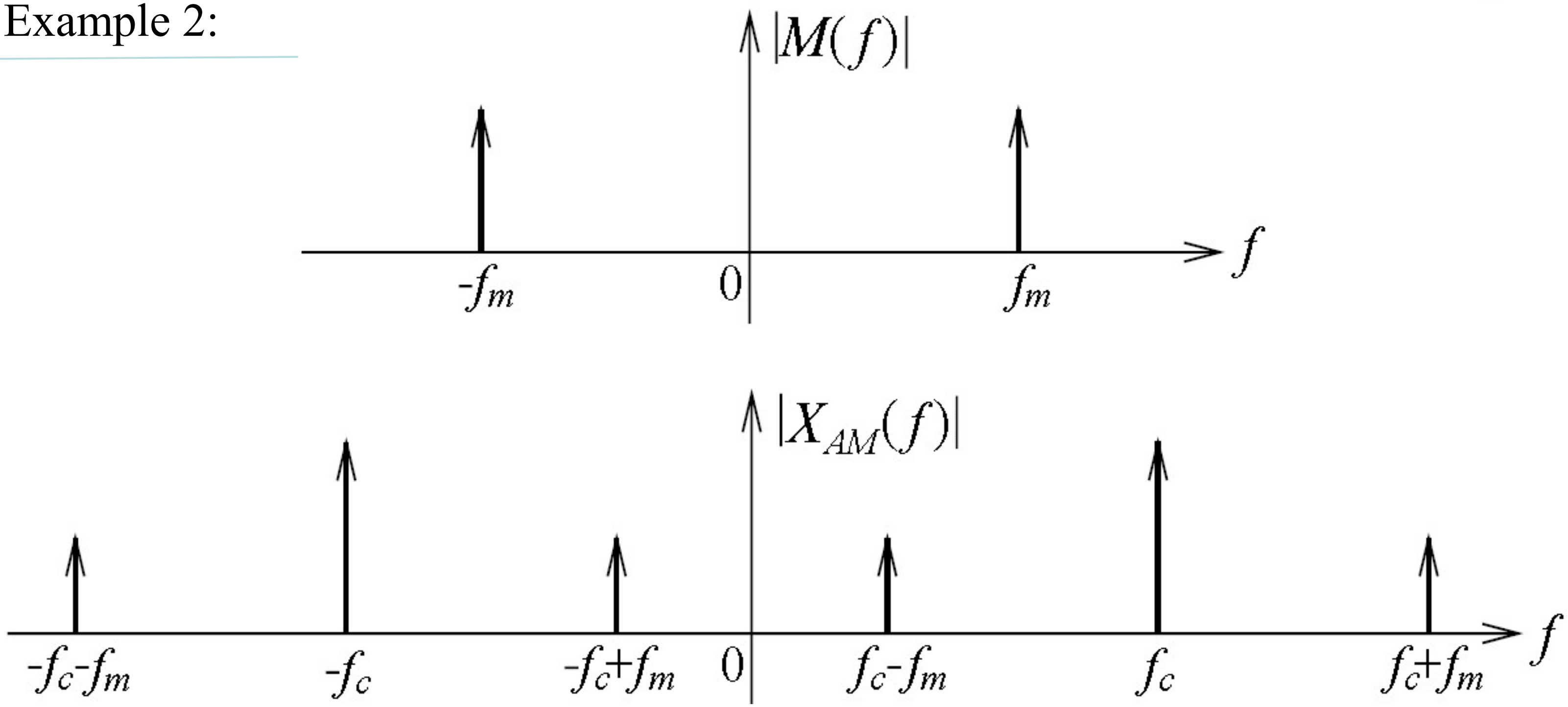

Figure 6: Spectrum of sinusoidally modulated AM signal

1.2 Amplitude Modulation I Summary 2

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1) Time-Domain Waveform of AM Signal

- Modulation Index of AM Signal

$$\mu = |k_a m(t)|_{\max} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

2) Frequency-Domain Spectrum of AM Signal

$$X_{AM}(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

- Power Efficiency of AM Signal

$$\eta = \frac{P_s}{P_s + P_c} = \frac{\mu^2}{2 + \mu^2}$$

Please proceed!

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1.2 Amplitude Modulation II

1.3 Angle Modulation

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1.1 Types of Modulations✓✓

1) Basic Communication System✓
2) Three Types of Modulations✓
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1.2 Amplitude Modulation

1) Time-Domain Description ✓
2) Frequency-Domain Description ✓
3) **Generation of AM Signal**☜
4) **Demodulation of AM Signal**☜

1.3 Angle Modulation

1) **Frequency Modulation**☜
2) **Phase Modulation**☜

3) Generation of AM Signal

- The time-domain expression of the full AM is given by

$$
\begin{aligned}
x_{AM}(t) &= A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \\
&= k_a\left[m(t) + B\right] A_c \cos(2\pi f_c t)
\end{aligned}
$$

where the constant bias is given by $B = {}^{1}/_{k_a}$

- Thus the AM signal can be generated using the system shown in Figure 7, which consists of a simple adder and a multiplier
- The modulation index can be controlled by adjusting the bias B

3) Generation of AM Signal

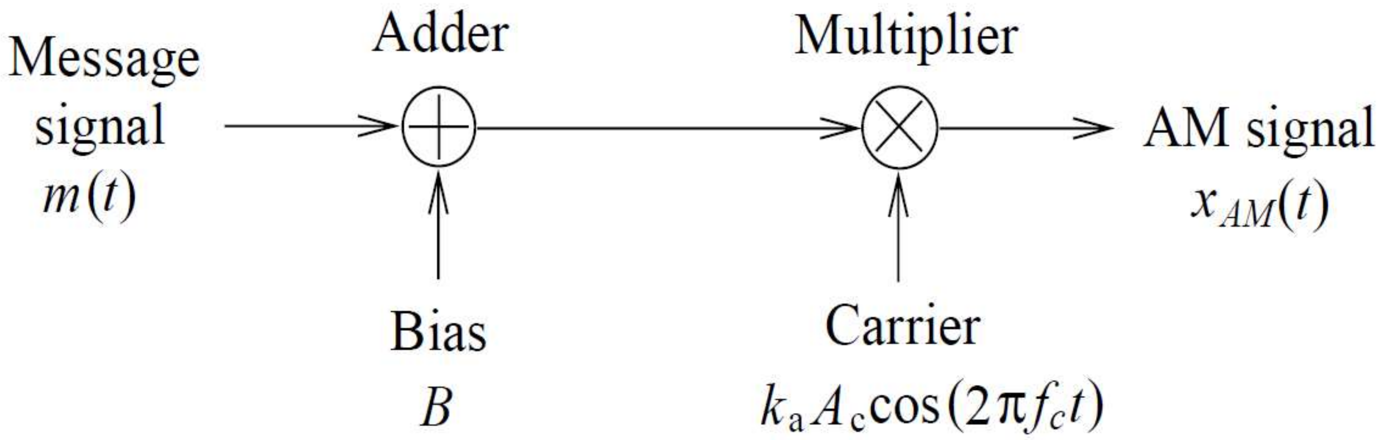

Figure 7: Generation of AM signal

4) Demodulation of AM Signal

- Demodulation is a process used to recover the original
- modulating signal from the modulated signal
- The envelope detector is commonly used to demodulate the full AM signal
- An envelope detector produces an output signal that follows the envelope of the input signal waveform
- A simple envelope detector consists of a diode and a resistor-capacitor filter as shown in Figure 8

4) Demodulation of AM Signal

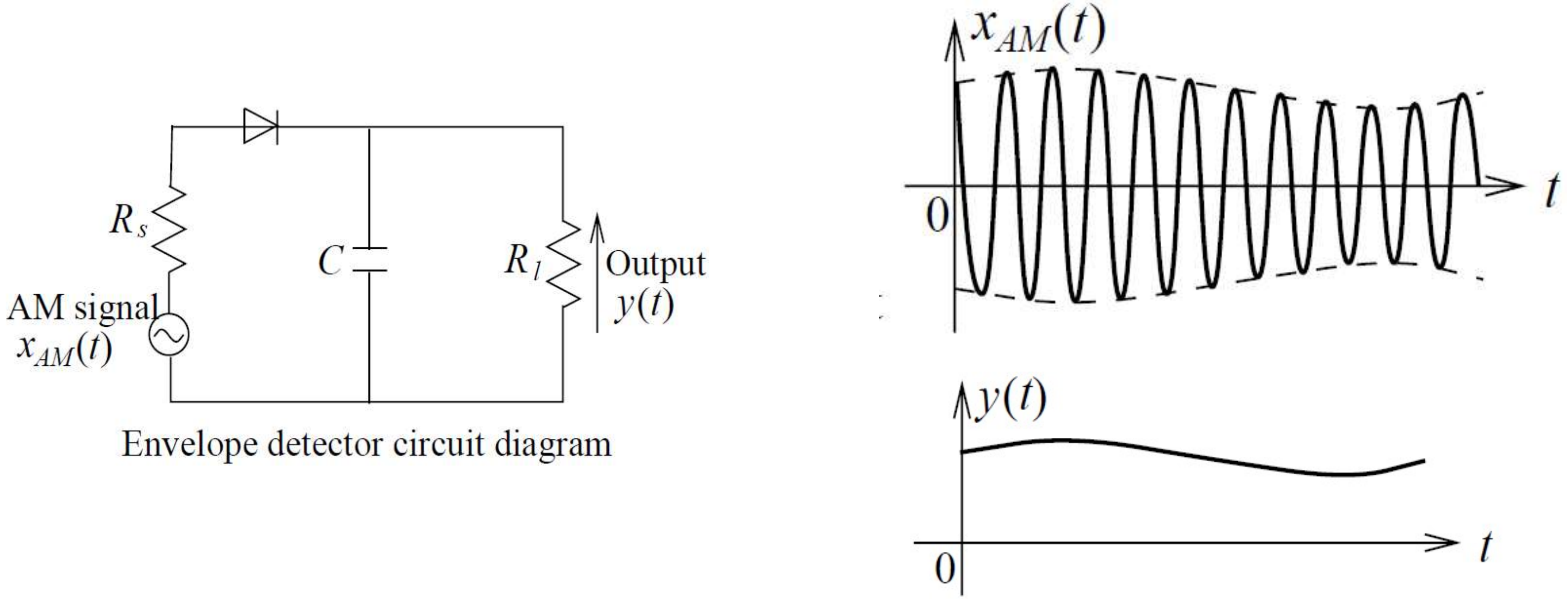

Figure 8: Envelope detector circuit diagram and signal waveforms

Other Types of Amplitude Modulation

- Double sideband suppressed carrier AM
- Quadrature AM
- Single sideband AM
- Vestigial sideband AM

1.3 Angle Modulation

- Two types of modulations
  1) Frequency Modulation (FM): The carrier frequency is varied with the message signal

$$x_{FM}(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right]$$

where k_f is the frequency sensitivity factor of the modulator

  2) Phase Modulation (PM): The carrier phase angle is varied with the message signal

$$x_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

where k_p is the phase sensitivity factor of the modulator

1.3 Angle Modulation

Example 3: Assuming the modulating signal is a sinusoidal signal,

i.e. $m(t) = A_m\cos(2\pi f_m t)$,

determine the corresponding expressions for the resultant FM and PM signals.

The FM signal expression is given by

$$\begin{aligned} x_{FM}(t) &= A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right] \\ &= A_c \cos\left[2\pi f_c t + k_f \int_0^t A_m \cos(2\pi f_m \tau)d\tau\right] \\ &= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{2\pi f_m}\sin(2\pi f_m t)\right] \\ &= A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \end{aligned}$$

where $\beta = \frac{k_f A_m}{2\pi f_m}$ is called the modulation index of the FM modulator.

1.3 Angle Modulation

Example 3:

Similarly, the corresponding PM signal expression is given by

$$\begin{aligned} x_{PM}(t) &= A_c \cos\left[2\pi f_c t + k_p m(t)\right] \\ &= A_c \cos\left[2\pi f_c t + k_p A_m \cos(2\pi f_m t)\right] \\ &= A_c \cos\left[2\pi f_c t + \beta_p \cos(2\pi f_m t)\right] \end{aligned}$$

where $\beta_p = k_p A_m$ is called the modulation index of the PM modulator

1.3 Angle Modulation

Example 4: A periodic rectangular wave m(t) as shown in Figure 9 frequency modulates a carrier of frequency f_c = 1 kHz and amplitude A_c = 2 Volts. The frequency sensitivity factor of the FM modulator is $k_f = 2\pi \times 100$ rad/Volt.
Sketch the resultant FM signal waveform.

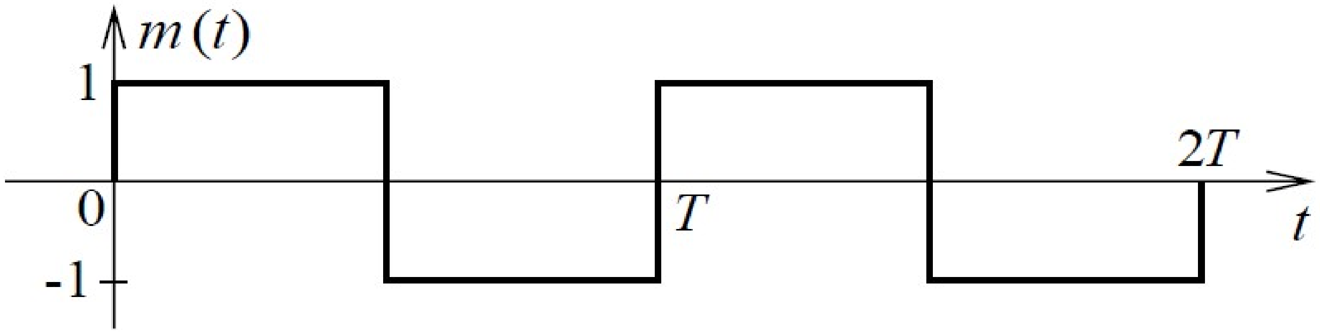

Figure 9: Example on frequency modulation

1.3 Angle Modulation

Example 4: A periodic rectangular wave m(t) as shown in Figure 9 frequency modulates a carrier of frequency f_c = 1 kHz and amplitude A_c = 2 Volts. The frequency sensitivity factor of the FM modulator is $k_f = 2\pi \times 100$ rad/Volt. Sketch the resultant FM signal waveform.

The FM signal expression is given by

$$
\begin{aligned}
x_{FM}(t) &= A_c \cos\left[2\pi f_c t + k_f \int_0^t m(\tau)d\tau\right] \\
&= 2\cos\left[2\pi 10^3 t + 2\pi \times 100 \int_0^t m(\tau)d\tau\right]
\end{aligned}
$$

The instantaneous frequency of the FM signal is given by

$$
\begin{aligned}
f_i(t) &= \frac{1}{2\pi} \times \frac{d}{dt}\left[2\pi 10^3 t + 2\pi \times 100 \int_0^t m(\tau)d\tau\right] \\
&= 10^3 + 100m(t)
\end{aligned}
$$

The waveform of $x_{FM}(t)$ is shown in Figure 10.

1.3 Angle Modulation

Example 4: Sketch the resultant FM signal waveform.

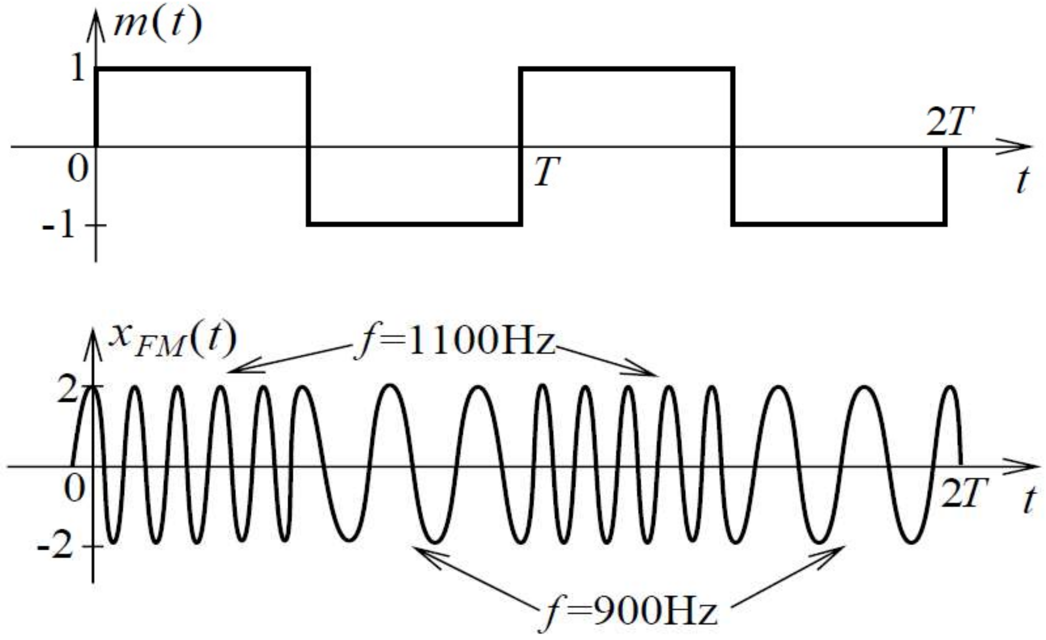

Figure 10: Waveform of FM signal

Modulation Summary 3

- ❑ Generation of AM Signal
- ❑ Demodulation of AM Signal
- ❑ Other Types of Amplitude Modulations
- ❑ Angle Modulations
  - o Frequency Modulation (FM)

$$x_{FM}(t) = A_c \cos[2\pi f_c t + k_f \int_0^t m(\tau)d\tau]$$

  - o Phase Modulation (PM)

$$x_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

Please proceed!