NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

This session is about

28. Analysis of Transmission Line

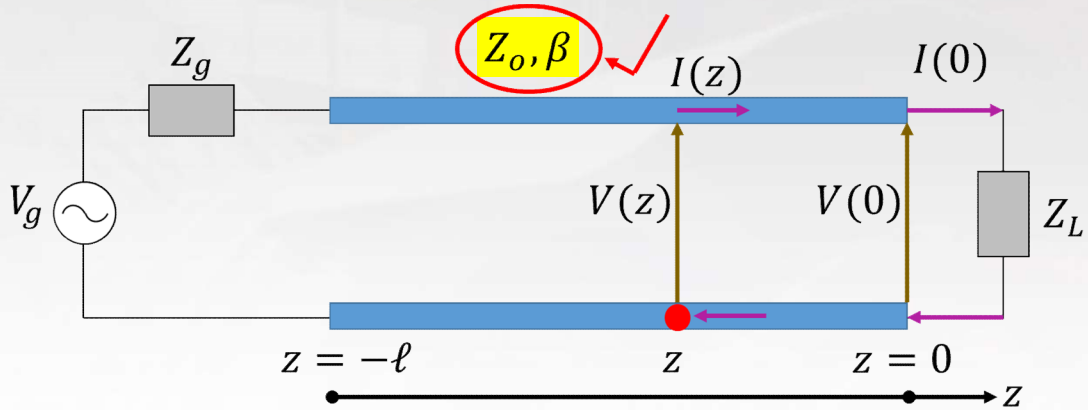
Learning Objectives

- Given a transmission line, determine reflection coefficient and input impedance at various positions, i.e. $\Gamma(z)$, $Z_{in}(z)$; and

- Determine voltage and current at various positions, i.e. $V(z)$, $I(z)$, and power deliver to the load.

Transmission Line Equations

- $V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$
- $I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{+j\beta z}$
- $\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$
- $\Gamma(z) = \Gamma_L e^{+j2\beta z}$
- $Z_{in}(-\ell) = \frac{Z_L + jZ_o tan\beta\ell}{Z_o + jZ_L tan\beta\ell} Z_0,$



$-\ell \leq z \leq 0$

- Example: A TL with $Z_o = 50\ \Omega$ and $u_p = 1.5 \times 10^8$ m/s operating at

$$f = 10MHz: \ \beta = \frac{\omega}{u_p} = \frac{2\pi f}{u_p} \rightarrow \beta = 0.42 \text{ rad/m or } \lambda = 15 \text{ m}$$

Analysis of Transmission Line

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Problem Statement

- Given $V_g = 100,\ Z_g = 100\Omega,\ Z_L = 50 + j50\Omega,\ Z_o = 50\Omega,\ \beta = 0.42\ \mathrm{rad/m}$ and $\ell = 2\mathrm{m}$, find the following:

  (i) $V(z)$ and $I(z)$ at $z = 0, -0.5\ell, -\ell$.

  (ii) $Z_{in}(z)$ at $z = 0, -0.5\ell, -\ell$.

  (iii) Power flow at $z = 0, -0.5\ell, -\ell$.

  (iv) z_{max}: $|V|_{max}$ and $|I|_{min}$; z_{min} : $|V|_{min}$ and $|I|_{max}$

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28. Analysis of Transmission Line (2)

Analysis of Transmission Line

- **Analysis of Transmission Line**

- Under **Time-Harmonic conditions** (Single frequency Steady State), the total voltage and current at any position z on a T-Line i.e. $V(z), I(z)$ is given by the sum of the incident and reflected components:

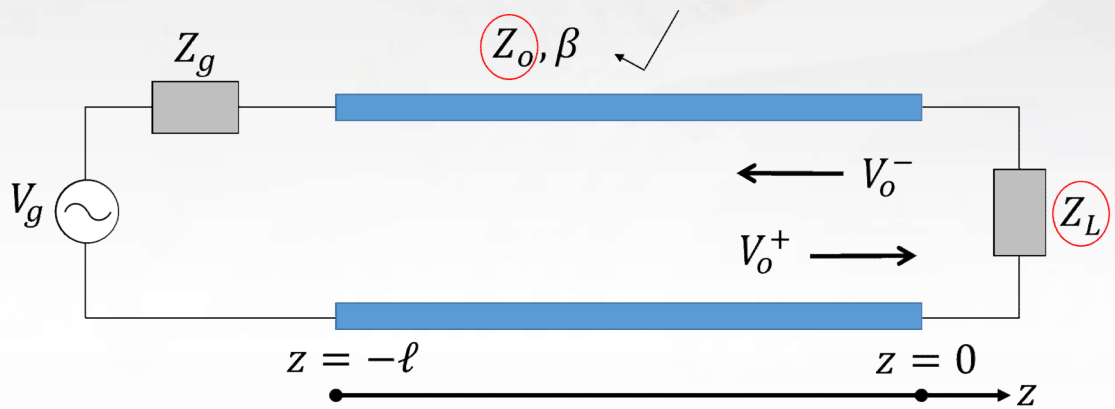
- $$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z} \quad (1)$$

  $$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{+j\beta z} \quad (2)$$

- V_o^+ and V_o^- are the amplitude of the incident and reflected voltage wave at $z = 0$ respectively.

Analysis of Transmission Line

- V_o^+ and V_o^- in (1) and (2) depend on the **boundary conditions** at the **source and the load ends** of the line.



- UPW: $\Gamma = \frac{E_{or}}{E_{oi}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ solely depend on boundary condition at $z = 0$.
- Similarly, we define reflection coefficient $\Gamma_L = \frac{V_o^-}{V_o^+}$ and we expect Γ_L can be solved by using the boundary condition at the load end $(z = 0)$.

Analysis of Transmission Line

- Substitute $V_o^- = \Gamma_L V_o^+$ into (1) and (2):

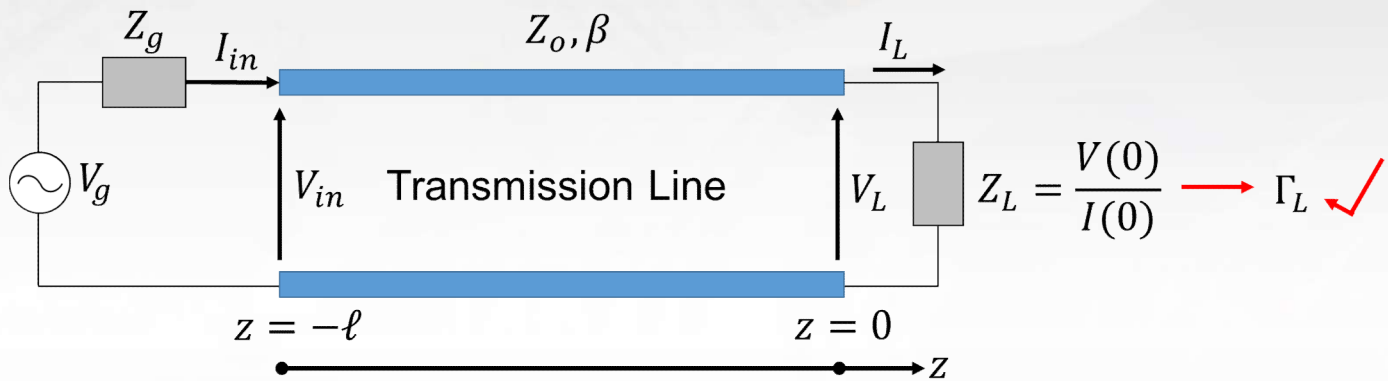
$$V(z) = V_o^+ e^{-j\beta z} + \Gamma_L V_o^+ e^{+j\beta z} = V_o^+ e^{-j\beta z}[1 + \Gamma_L e^{+j2\beta z}] \qquad (3)$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{\Gamma_L V_o^+}{Z_o} e^{+j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z}[1 - \Gamma_L e^{+j2\beta z}] \qquad (4)$$

$$-\ell \leq z \leq 0$$

Analysis of Transmission Line

- Using (3) and (4), Γ_L can be solved using boundary condition at the load end; V_o^+ can be obtained from the condition at the source-end.



$$V(z) = V_o^+ e^{-j\beta z}\,[1 + \Gamma_L e^{+j2}\quad] \qquad (3)$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z}\,[1 - \Gamma_L e^{+j2\beta}\ ] \qquad (4)$$

Analysis of Transmission Line

- **At the Load end:** $z = 0$
- Transmission Line equations, (3) and (4) give:
- $V_L = V(z = 0) = V_o^+(1 + \Gamma_L)$ (3a)
- $I_L = I(z = 0) = \frac{V_o^+}{Z_o}(1 - \Gamma_L)$ (4a)
- V_L and I_L must satisfied Ohm's Law at the load end $z = 0$.
- $\frac{V_L}{I_L} = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} = Z_L$
- $\boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}}$, (5)

$$\boxed{\frac{P_r}{P_i} = |\Gamma_L|^2}$$

Analysis of Transmission Line

REMARKS: $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$, $\frac{P_r}{P_i} = |\Gamma_L|^2$

- Γ_L is a function of mismatch between Z_L and Z_o. The larger the difference between Z_L and Z_o, the larger the $|\Gamma_L|$.
- Γ_L does not depend on the source $[V_g, Z_g]$ and the length ℓ of the line.
- $Z_L = Z_o$ (Matched Load) $\longrightarrow \Gamma_L = 0$ & $P_r = 0$ and $P_t \equiv 100\%$
- $Z_L = 0$ (Short) $\longrightarrow \Gamma_L = -1$ & $\frac{P_r}{P_i} = |\Gamma_L|^2 = 100\%$
- $Z_L = \infty$ (Open) $\longrightarrow \Gamma_L = 1$ & $\frac{P_r}{P_i} = |\Gamma_L|^2 = 100\%$

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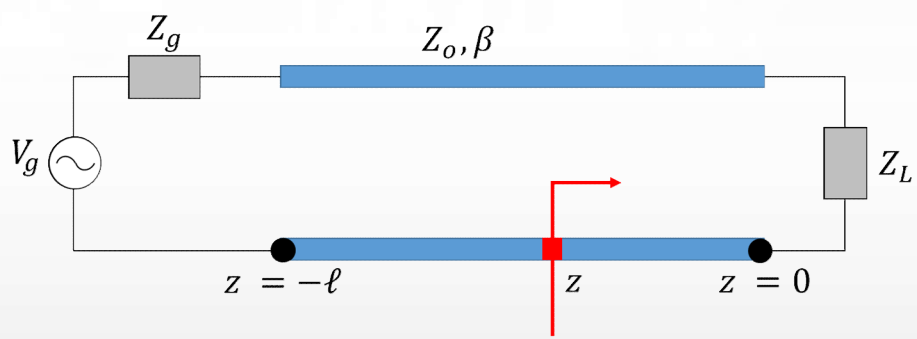
28. Analysis of Transmission Line (3)

Analysis of Transmission Line

- Once Γ_L is known, **<u>Input impedance</u>** $Z_{in}(z)$ at any position z along the transmission line can be determined:

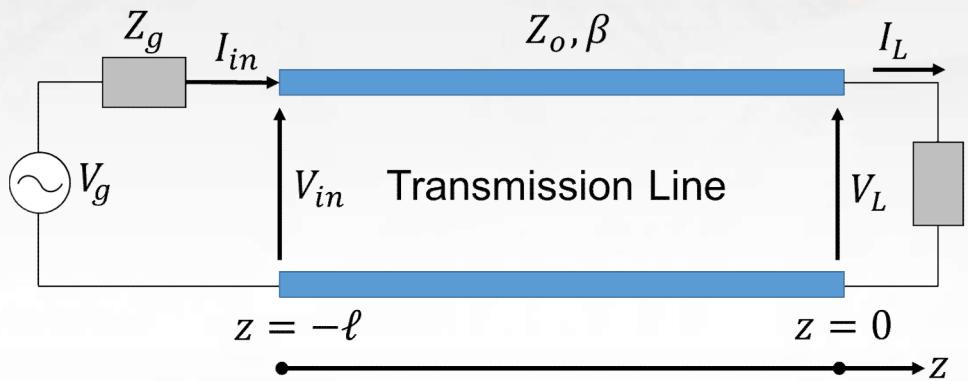
$$\frac{(3)}{(4)} \equiv \boxed{Z_{in}(z) = \frac{V_{(z)}}{I(z)} = Z_o \frac{1+\Gamma_L e^{+j2\beta z}}{1-\Gamma_L e^{+j2\beta z}} = R + jX} \quad -\ell \le z \le 0 \qquad (6)$$



$$Z_{in}(z) = Z_o \frac{1+\Gamma_L\, e^{+j2\beta z}}{1-\Gamma_L\, e^{+j2\beta z}}$$

Analysis of Transmission Line

- Using (3) and (4), Γ_L can be solved using boundary condition at the load end; V_o^+ can be obtained from the condition at the source-end.



$$Z_L = \frac{V(0)}{I(0)} \longrightarrow \Gamma_L$$

$$V(z) = V_o^+ e^{-j\beta z}\,[1 + \Gamma_L e^{+j2\beta}\,] \qquad (3)$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z}\,[1 - \Gamma_L e^{+j2\beta z}] \qquad (4)$$

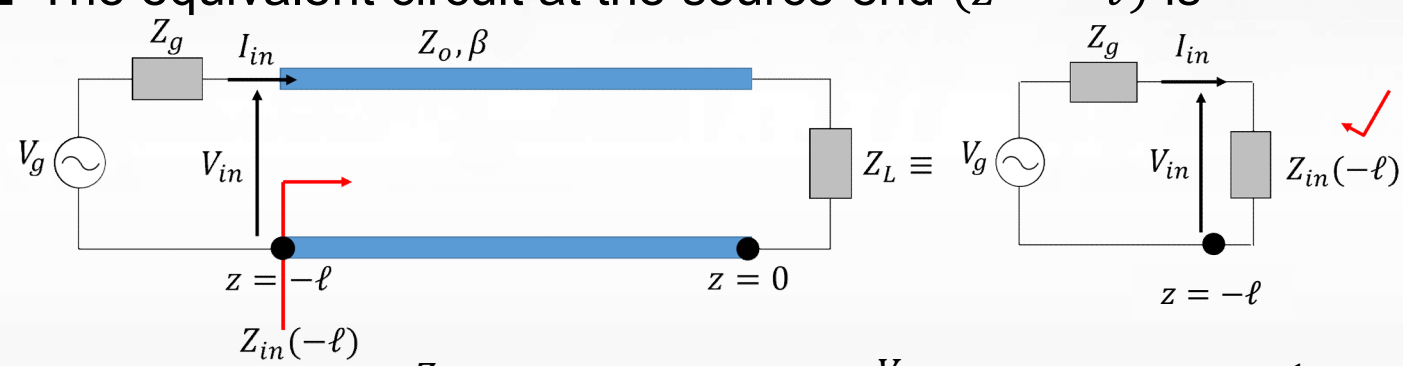
Analysis of Transmission Line

- **At the Source end:** $z = -\ell$ $[\ell > 0]$

$$Z_{in}(-\ell) = Z_o \frac{1 + \Gamma_L e^{-j2\beta\ell}}{1 - \Gamma_L e^{-j2\beta\ell}} = R_{in} + jX_{in}$$

$$\left[\beta\ell = 2\pi\frac{\ell}{\lambda}\right]$$

- The equivalent circuit at the source end $(z = -\ell)$ is



- $V(-\ell) = V_{in} = \frac{Z_{in}}{Z_{in}+Z_g} V_g,$ $\quad I_{in} = \frac{V_g}{Z_{in}+Z_g},$ $\quad P_{in} = \frac{1}{2}|I_{in}|^2 \mathrm{Re}(Z_{in})$

Analysis of Transmission Line

- The value of V_o^+ can be determined by using $V(-\ell)$ **at the source end**. Substitute $z = -\ell$ into (3):

$$V(-\ell) = \frac{Z_{in}}{Z_{in}+Z_g} V_g = V_o^+ e^{+j\beta\ell}\left[1 + \Gamma_L e^{-j2\beta\ell}\right] \quad (3)$$

- $\therefore \boxed{V_o^+ = \frac{V(-\ell)}{e^{j\beta\ell}\left[1+\Gamma_L e^{-j2\beta\ell}\right]}}$ (7) $\boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}}$ (5)

- NOTE: Γ_L and V_o^+ allow us to obtain $V(z)$ and $I(z)$ at any position z along the transmission line.

$$V(z) = V_o^+ e^{-j\beta z}\left[1 + \Gamma_L e^{+j2\beta z}\right] \quad (3)$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z}\left[1 - \Gamma_L e^{+j2\beta z}\right] \quad (4)$$

Summary

- Given a transmission line, determine voltage, current and input impedance at various positions, i.e. $V(z)$, $I(z)$ and $Z_{in}(z)$; and

- Determine power deliver to the load.

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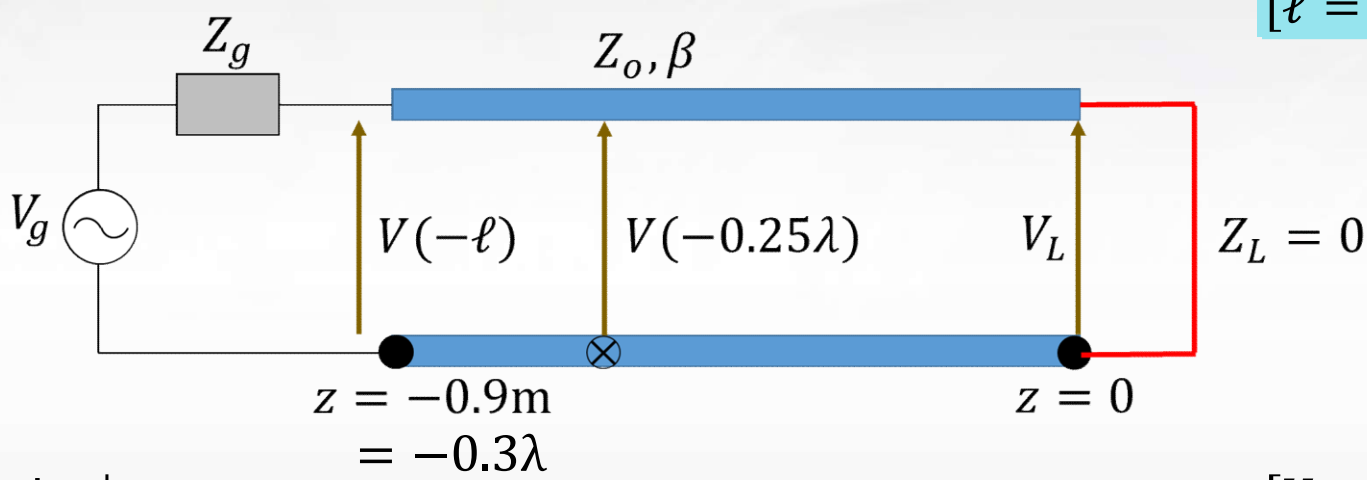
29. Examples of Analysis of Transmission Line

Example 1

- **Example:** $V_g = 100,\ Z_g = 50\,\Omega,\ Z_o = 50\,\Omega,\ \ell = 0.9\text{ m},\ Z_L = 0$

$$f = 100\text{MHz},\ u_p = 3\times 10^8\ \text{m/s} \rightarrow \beta = 2\pi/3$$

$[\lambda = 3\text{ m}]$
$[\ell = 0.3\lambda]$



Find
(a) Γ_L and V_o^+

$[V_L = 0]$

(b) $V(z)$ @$z = 0,\quad z = -0.25\lambda$ and $z = -\ell = -0.3\lambda$

Solution to Example 1

- $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = -1$ (5)

- $Z_{in}(-\ell) = Z_o \frac{1+\Gamma_L\, e^{-j2\beta\ell}}{1-\Gamma_L\, e^{-j2\beta\ell}} = -j154\,\Omega$ (6)

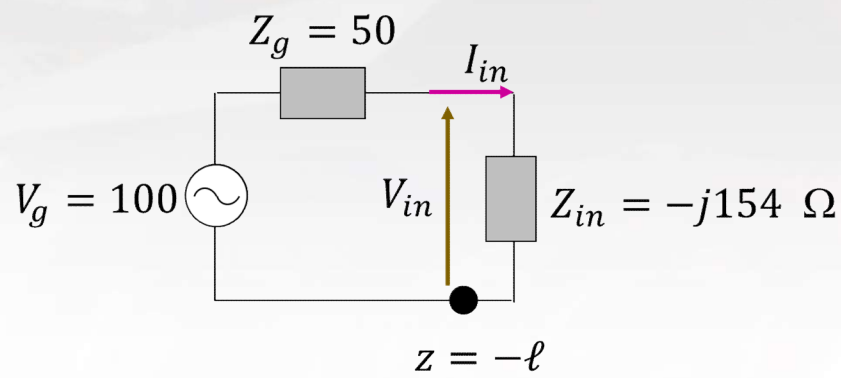
- $V(-\ell) = V_{in} = \frac{Z_{in}}{Z_{in}+Z_g} V_g = 95.1\angle -18^\circ$

- $V(z) = V_o^+\, e^{-j\beta z}\left[1 + \Gamma_L\, e^{+j2\beta z}\right]$ (3)

$$\left[z = -\ell = -0.9,\quad V(-\ell) = 95.1\angle -18^\circ,\ \Gamma_L = -1,\ \beta\ell = 0.6\pi \equiv 108^\circ\right]$$

- $V_o^+ = \frac{V(-\ell)}{e^{j\beta\ell}\left[1+\Gamma_L\, e^{-j2\beta}\right]} = \frac{95.1\angle -1^\circ}{1\angle 108^\circ\left[1.9\angle -18^\circ\right]} = 50\angle -108^\circ$ (7)

- $V(z = -0.25\lambda) = V_o^+ e^{-j\beta z}\left[1 + \Gamma_L\, e^{+j2\beta z}\right] = 100\angle -18^\circ$



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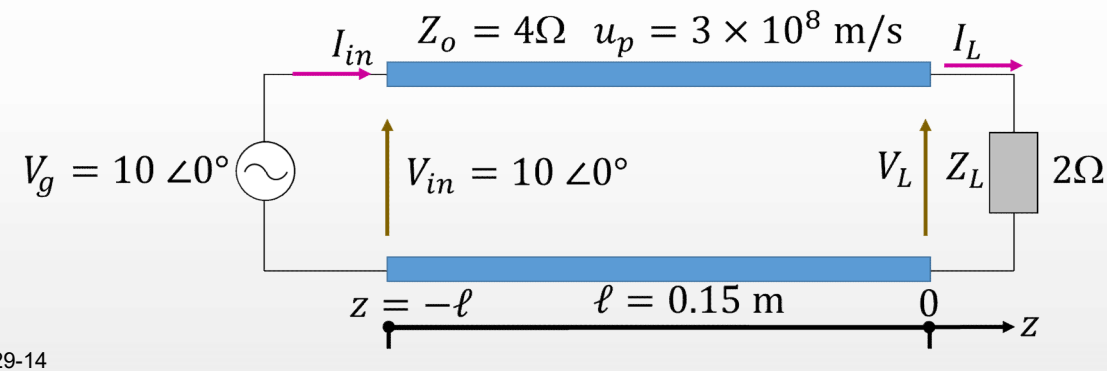
29. Examples of Analysis of Transmission Line (2)

Example 2

- A sinusoidal voltage source $V_g = 10\cos(\omega t)$ V, with zero internal impedance $[Z_g = 0]$ is connected to a $Z_o = 4\ \Omega$ **air-filled** transmission line of length $\ell = 15$ cm. The line is terminated with a load $Z_L = 2\ \Omega$ as shown in the figure.
  - (a) Find $V_{in},\ I_{in},\ V_L,\ I_L,\ P_{in}$ and P_L at $f = 50$ Hz $\quad \left[\beta = \frac{\pi}{3} \times 10^{-6}\right]$
  - (b) Find $V_{in},\ I_{in},\ V_L,\ I_L,\ P_{in}$ and P_L at $f = 500$ MHz $\quad \left[\beta = \frac{10\pi}{3}\right]$



Circuit Theory:

$V_{in} = V_L = 10\ \angle 0°$

$I_L = \frac{V_L}{Z_L} = 5\ \angle 0° = I_{in}$

$P_{in} = P_L = 25$ W

Solution to Example 2

- **(a) Solution:** $f = 50\ \text{Hz}$; $\ell = 0.15\ \text{m}$; **electrical length** $\ell/\lambda = ?$

- For low frequency applications e.g. $f = 50\ \text{Hz}$. $\left[\beta = \frac{\pi}{3} \times 10^{-6}\right]$
- $f\lambda = u_p = 3 \times 10^8\ \text{m/s} \rightarrow \lambda = 6000\ \text{km}$

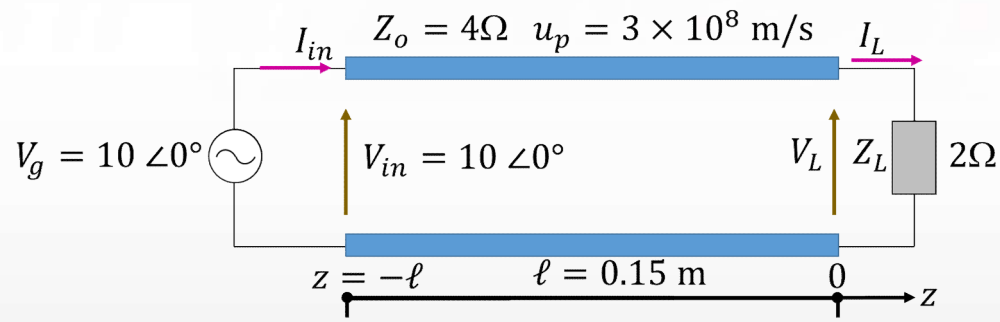
$$V(z) = V_o^+ \cancel{e^{-j\beta z}}\left[1 + \Gamma_L \cancel{e^{+j2}}\right] \cong V_o^+[1 + \Gamma_L] = V_{in} = V_L \qquad \text{(3b)}$$

$$I(z) = \frac{V_o^+}{Z_o} \cancel{e^{-j\beta z}}\left[1 - \Gamma_L \cancel{e^{+j2}}\right] \cong \frac{V_o^+}{Z_o}[1 - \Gamma_L] = I_{in} = I_L \qquad \text{(4b)}$$

- (3b) and (4b) show that if $e^{-j\beta z} \simeq 1$ and $e^{+j2\beta z} \simeq 1\ \forall\ -\ell \leq z \leq 0$, i.e. incident and reflected waves along the transmission line can be neglected. V & I are independent of position z.

Solution to Example 2

- $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = -\frac{1}{3}$ (5) $[Z_L = 2\ \Omega; Z_o = 4\ \Omega]$
- $V(-\ell) = 10\ \angle 0 \quad @z = -\ell$
- $V(-\ell) = V_o^+[1 + \Gamma_L] = 10\ \angle 0 \rightarrow V_o^+ = 15\ \angle 0\ \text{V}$
- $V(z) = 15\ \angle 0 \left[1 - \frac{1}{3}\right] = 10\ \angle 0\ \text{V}$
- $I(z) \cong \frac{15}{4}\ \angle 0 \left[1 + \frac{1}{3}\right] = 5\ \angle 0\ \text{A}$
- $P_{in} = P_L = \frac{1}{2}\text{Re}(V_L\ I_L{}^*) = 25\ \text{W}$

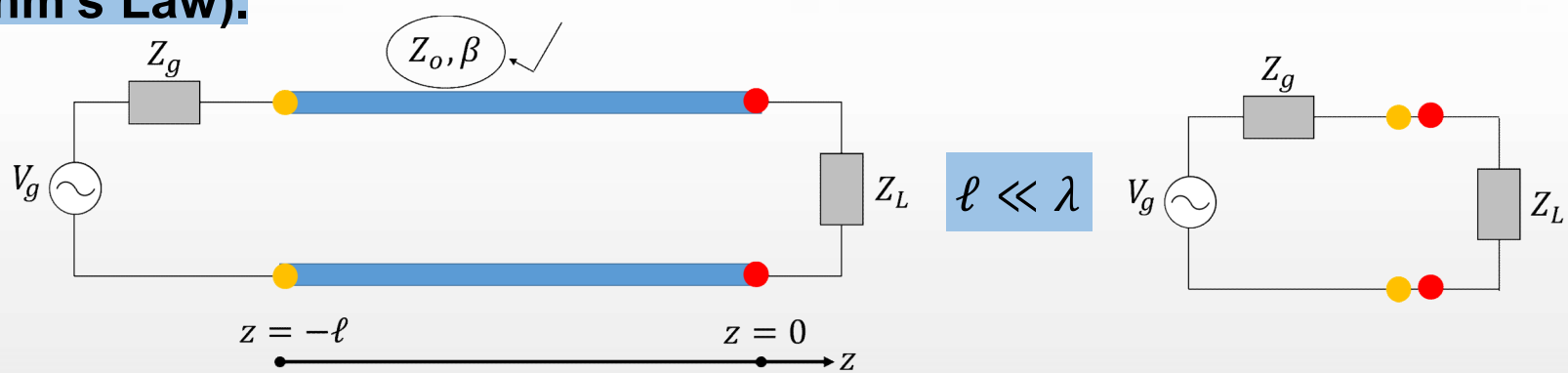


Solution to Example 2

Conclusion:

- If **electrical length** of the transmission line is very small i.e. $\ell/\lambda \ll 1$ (say $\ell < 0.01\lambda$), we may approximate $e^{-j\beta z} \simeq 1$, $e^{+j2\beta z} \simeq 1$ i.e. the incident and reflected waves along the transmission line can be neglected.

- **Transmission line equations** (3) and (4) → **Lumped Circuit Theory (KVL, KCL, Ohm's Law).**



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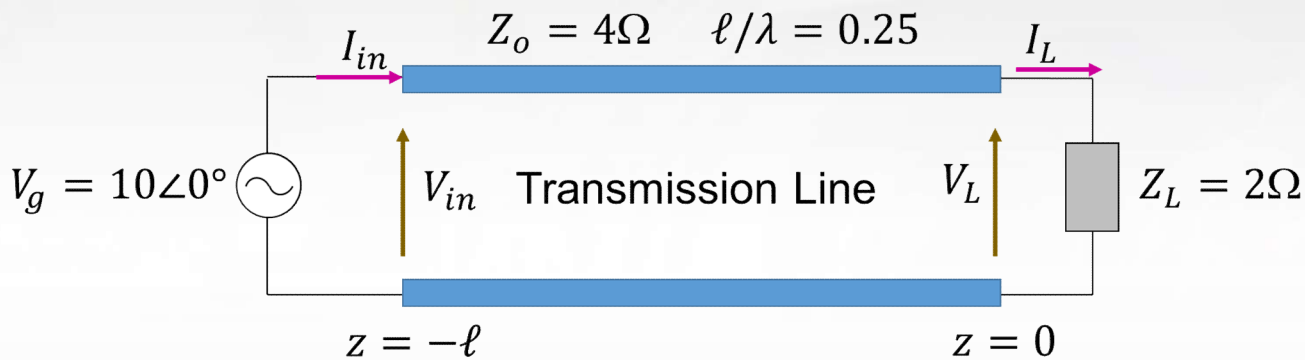
29. Examples of Analysis of Transmission Line (3)

Solution to Example 2

- **(b) Solution:** $f = 500\ \text{MHz};\ \ell = 0.15\ \text{m};\ \ell/\lambda = ?$
- $f\lambda = u_p = 3 \times 10^8\ \text{m/s} \rightarrow \lambda = 0.6\ \text{m}\ \therefore \ell/\lambda = 0.25$

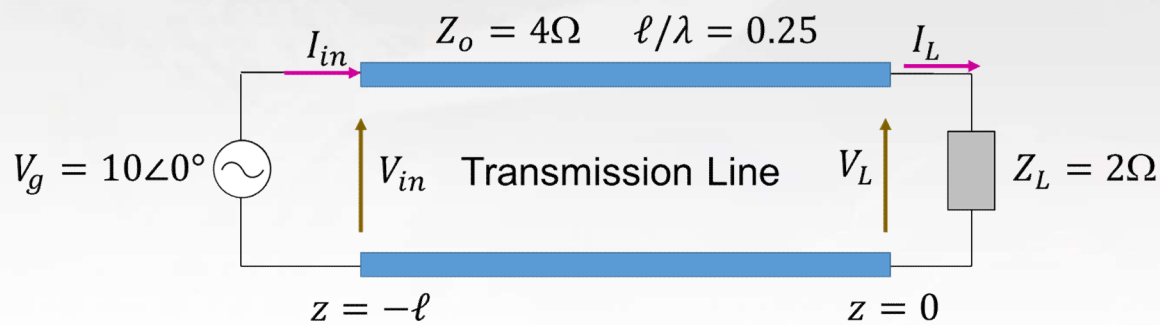


- $V(z) = V_o^+ e^{-j\beta}\left[1 + \Gamma_L e^{+j2\beta z}\right]$ (3)
- $I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z}\left[1 - \Gamma_L e^{+j2\beta z}\right]$ (4)

Solution to Example 2

- Given $V_g = 10\text{ V};\ Z_L = 2\ \Omega;\ Z_o = 4\ \Omega;\ \ell/\lambda = 0.25$
- At the load end $[z = 0]$,

$$\boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = -0.33}$$



- At the source end $[z = -\ell]$,

$$V(-\ell) = V_o^+ e^{+j\beta\ell}\left(1 + \Gamma_L e^{-j2\beta\ell}\right)$$

$$\left[\beta\ell = 2\pi\frac{\ell}{\lambda} = \frac{\pi}{2}\right]$$

$$10\angle 0 = V_o^+ e^{+j\pi/2}\left(1 - 0.33 e^{-j\pi}\right) = j1.33 V_o^+$$

$$\boxed{V_o^+ = -j7.5\text{ V}}$$

$$\left[e^{j\theta} = \cos\theta + j\sin\theta\right]$$

Solution to Example 2

- **Load end:** $(z = 0)$ $[\Gamma_L = -0.33, V_o^+ = -j7.5]$

$$V_L = V_o^+(1 + \Gamma_L) = -j7.5(1 - 0.33) = -j5 \text{ V}$$

$$I_L = \frac{V_o^+}{Z_o}(1 - \Gamma_L) = \frac{-j7.5}{4} \times \frac{4}{3} = -j2.5 \text{ A}$$

$$P_L = 1/2 \operatorname{Re}(V_L I_L{}^*) = 1/2 \operatorname{Re}(-j5 \times j2.5) = 6.25 \text{ W}$$

$$V_L(t) = \operatorname{Re}\left(V_L e^{j\omega t}\right) = 5 \sin(\omega t), \qquad [V_{in}(t) = 10 \cos(\omega t)]$$

$$I_L(t) = 2.5 \sin(\omega t)$$

- NOTE: $Z_L = \frac{V_L}{I_L} = 2\ \Omega$

Solution to Example 2

- **Source end:** $(z = -\ell = -0.25\lambda)$ $\quad [\Gamma_L = -0.33,\ V_o^+ = -j7.5,\ \beta\ell = \pi/2]$

$$V_{in} = V_o^+ e^{+j\beta\ell}\left(1 + \Gamma_L e^{-j2\beta\ell}\right) = 10\ \text{V} \qquad [V_{in}(t) = 10\cos(\omega t)]$$

$$I_{in} = \frac{V_o^+}{Z_o} e^{+j\beta\ell}\left[1 - \Gamma_L e^{-j2\beta\ell}\right]$$

$$= \frac{-j7.5}{4} j[1 - 0.33] = 1.25\text{A} \qquad [I_{in}(t) = 1.25\cos(\omega t)]$$

- $P_{in} = 1/2\,\text{Re}(V_{in} I_{in}^*) = 6.25\ \text{W} = P_L$

Lossless Transmission Line: $P_L = P_{in} = P(z)$ $\qquad -\ell \le z \le 0$

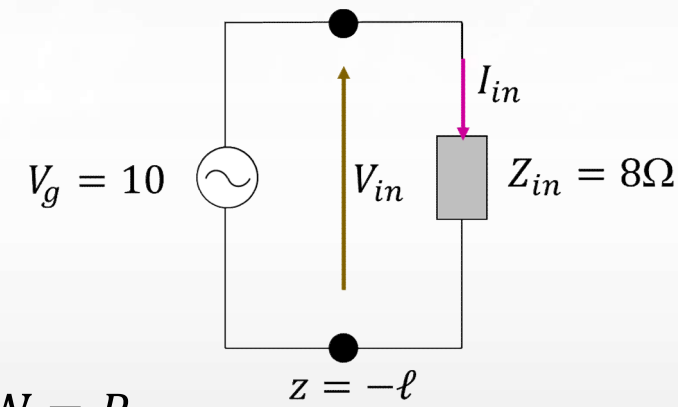
- NOTE: $Z_{in} = \frac{V_{in}}{I_{in}} = 8\ \Omega$ $\qquad \left[Z_{in}(-\ell) = Z_o \frac{1+\Gamma_L e^{-j2\beta\ell}}{1-\Gamma_L e^{-j2\beta\ell}} \quad (6)\right]$ Page 28-11

Solution to Example 2

- At the source end of the line, $z = -\ell$, the generator V_g looking into the line sees an **input impedance** Z_{in}

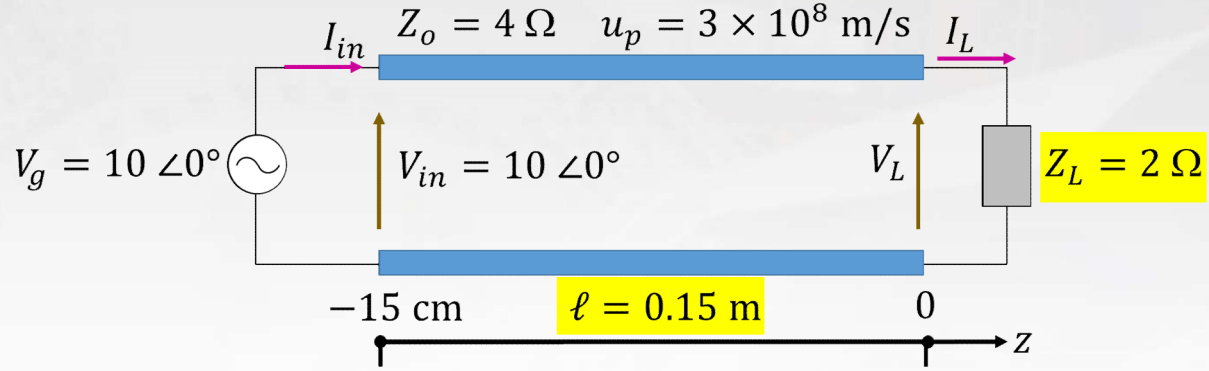
$$Z_{in} = \frac{V_{in}}{I_{in}} = 8\,\Omega \leftarrow [Z_L = 2\,\Omega]$$

- As far as the generator is concerned, the impedance at the source end is $\boxed{Z_{in} = 8\,\Omega \neq 2\,\Omega}$.



- $P_{in} = \frac{1}{2}|I_{in}|^2 \mathrm{Re}(Z_{in}) = \frac{1}{2}\left|\frac{V_{in}}{Z_{in}}\right|^2 \mathrm{Re}(Z_{in}) = 6.25\mathrm{W} = P_L$

Summary



- $f = 50$ Hz $\rightarrow \ell \ll \lambda$, $P_L = 25$ W $[Z_{in} \simeq 2\,\Omega = Z_L]$
- $f = 20$ MHz $\rightarrow \ell = 0.01\lambda$, $P_L = 24.7$ W $[Z_{in} = 2.0 + j0.2]$
- $f = 200$ MHz $\rightarrow \ell = 0.1\lambda$, $P_L = 12.2$ W $[Z_{in} = 2.7 + j1.9]$
- $f = 500$ MHz $\rightarrow \ell = 0.25\lambda$, $P_L = 6.25$ W $[Z_{in} = 8\,\Omega]$

Summary

- $f = 1000\ \text{MHz} \quad \rightarrow \ \ell = 0.5\lambda, \quad P_L = 25\ \text{W}$ $\quad [Z_{in} = 2\ \Omega]$
- $f = 1500\ \text{MHz} \quad \rightarrow \ \ell = 0.75\lambda, \quad P_L = 6.25\ \text{W}$ $\quad [Z_{in} = 8\ \Omega]$
- It can be shown that Z_{in} repeat every $\frac{\lambda}{2}$
- $P_{in} = \frac{1}{2}|I_{in}|^2 \text{Re}(Z_{in}) = P_L$ also repeat every $\frac{\lambda}{2}$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

This session is about

30. Reflection Coefficient $\Gamma(z)$ and Standing Waves on Transmission Lines

Learning Objectives

- Describe the general properties of reflection coefficient on transmission line;
- Apply standing waves equations to obtain $z_{\max}$, z_{min}, $|V|_{\max}$, $|V|_{min}$, $|I|_{\max}$, $|I|_{mi}$; and
- Transform complex load impedance into a purely resistive input impedance.

General Properties of Reflection Coefficient

- **General Properties of Reflection Coefficient** $\Gamma(z)$ [Page 12-5 to 12-8]

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z} \qquad -\ell \le z \le 0 \qquad (1)$$

- $\Gamma(z)$ at any position z is defined as

$$\Gamma(z) = \frac{V_o^- e^{+j\beta z}}{V_o^+ e^{-j\beta}} = \Gamma_L e^{+j2\beta z}, \qquad (11)$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma_L| e^{j\theta_o} \qquad @\ z = 0 \qquad (5)$$

$$\boxed{\Gamma(z) = |\Gamma_L| e^{j(\theta_o + 2\beta z)} = |\Gamma_L| \angle \theta_\Gamma} \qquad [\beta = \frac{2\pi}{\lambda}]$$

- $|\Gamma(z)| = |\Gamma_L| \equiv$ constant & $\boxed{\theta_\Gamma = \theta_o + 2\beta z} = \theta_o + \frac{4\pi}{\lambda} z$ (12)
- $\Gamma(z)$ **repeats every** $\frac{\lambda}{2}$

Summary of Standing Waves Equations

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❑ **SUMMARY:**

$[-\ell \le z \le 0]$

1. $z_{\max}$ & z_{min} depend on θ_o

$$\theta_\Gamma = \theta_o + 2\beta z_{\max} = 0, -2\pi$$
$$\theta_\Gamma = \theta_o + 2\beta z_{min} = \pi, -\pi$$

2. $z_{\max} \xleftrightarrow{\lambda/4} z_{mi}$

3. $|V|_{\max} \overset{@z_{max}}{\Leftrightarrow} |I|_{min}$ and $|V|_{min} \overset{@z_{min}}{\Leftrightarrow} |I|_{ma}$

4. $|V|_{\max}, |V|_{min}$ depend on $|\Gamma_L|$ & $|V_o^+|$

$$|V|_{\max} = |V_o^+|(1 + |\Gamma_L|)$$
$$|V|_{mi} = |V_o^+|(1 - |\Gamma_L|)$$

5. $\mathrm{SWR} = \frac{|V|_{\max}}{|V|_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$

$$\frac{P_r}{P_i} = |\Gamma_L|^2$$

REMARKS: $Z_{in} = R_{max} + j0 \ @z_{max}$; $Z_{in} = R_{min} + j0 \ @z_{min}$

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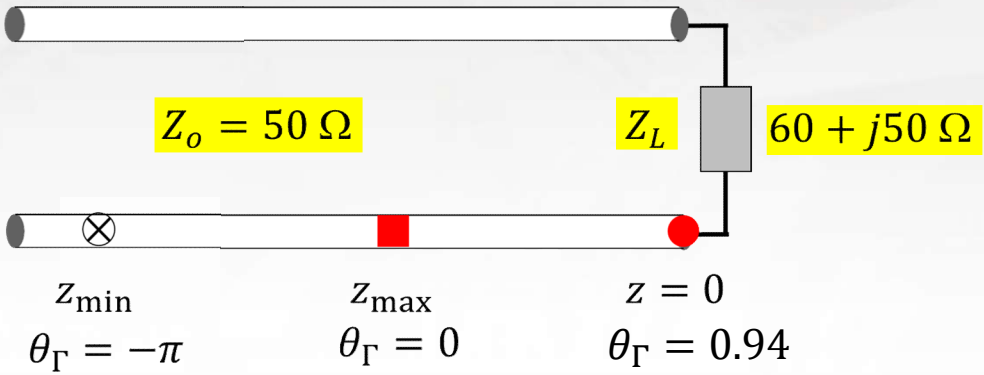
EE3001 Engineering Electromagnetics

This session is about

30. Reflection Coefficient $\Gamma(z)$ and Standing Waves on Transmission Lines (2)

Example 1

- **Example:** Find $z_{\max}$ and z_{min} for the TL given below:



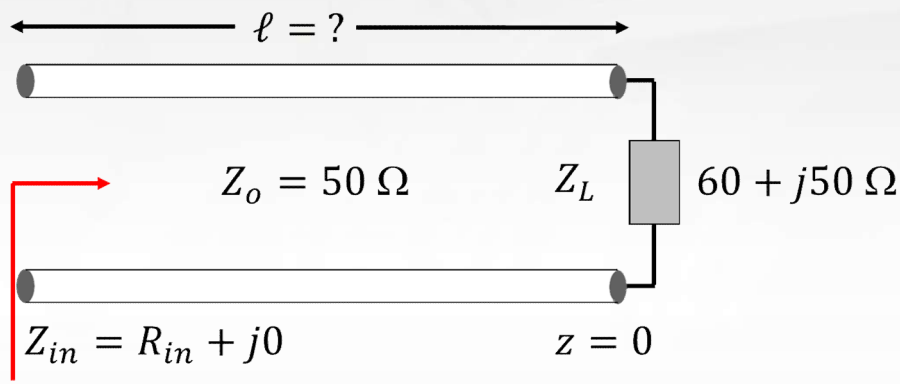
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.42\angle 54^\circ = 0.42\angle 0.94\text{rad} = |\Gamma_L|\angle\theta_o$$

$$\theta_\Gamma = \theta_o + 2\beta z_{\max} = 0 \longrightarrow z_{\max} = -\frac{\theta_o}{2\beta} = -\frac{0.94}{4\pi}\lambda$$

$$\therefore z_{\max} = -0.075\lambda,\ z_{\min} = -(0.075 + 0.25)\,\lambda = -0.325\lambda$$

Example 2

- **Example:** Find ℓ to transform a complex load Z_L into a purely resistive input impedance i.e. $Z_{in} = R_{in} + j0$.



$$Z_{in}(z) = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = Z_o \frac{1 + |\Gamma_L|\angle\theta_\Gamma}{1 - |\Gamma_L|\angle\theta_\Gamma} = R_{in} + j0 \quad (6)$$

z_{max} z_{min} z_{max}

NOTE: $Z_{in}(z)$ is real when $\Gamma(z)$ is real i.e. when $\theta_\Gamma = 0, -\pi, -2\pi, \cdots$

$$\Gamma(z) = |\Gamma_L| \cos\theta_\Gamma + j|\Gamma_L| \sin\theta_\Gamma$$

Example 2

$\theta_\Gamma = 0$: $\Gamma(z) = 0.42\angle 0 = 0.42 = |\Gamma_L|$ $[\ell = |z_{\max}| = 0.075\lambda]$

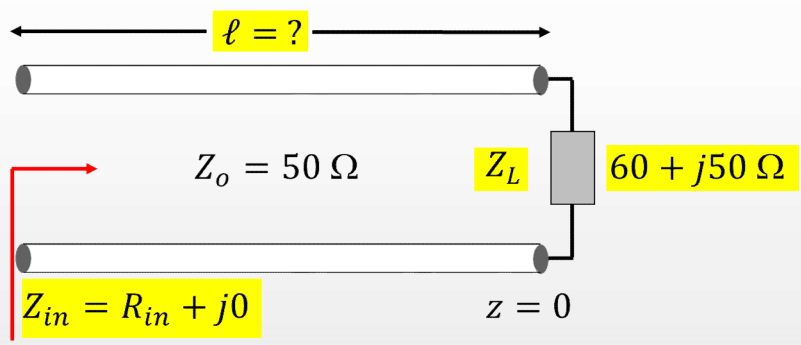
$@z_{\max}$

$$Z_{in}(z_{\max}) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)} = Z_o \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 123\ \Omega = R_{\max} + j0$$

$\theta_\Gamma = -180^\circ$: $\Gamma(z) = 0.42\angle -180^\circ = -0.42 = -|\Gamma_L|$ $[\ell = |z_{min}| = 0.325\lambda]$

$@z_{min}$

$$Z_{in}(z_{\min}) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)} = Z_o \frac{1-|\Gamma_L|}{1+|\Gamma_L|} = 20.3\ \Omega = R_{min} + j0$$



Summary

- Describe the general properties of reflection coefficient on transmission line;
- Apply standing waves equations to obtain $z_{\max}$, z_{min}, $|V|_{\max}$, $|V|_{min}$, $|I|_{\max}$, $|I|_{min}$; and
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