Lecture 8: Transformers

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Explain the transformer ratings with respect to rated load, rated voltages and rated currents.
- Analyse the performance of a practical transformer by examining the non-ideal features of a practical transformer.
- Solve simple problems with respect to the ideal transformers and practical transformers.

Transformer Ratings

- The typical nameplate rating of a transformer may look like the form:
  5 kVA, 500/250 V, 50 Hz.

- The nominal **apparent power** rating of 5 kVA indicates that the transformer can deliver up to 5 kVA in a continuous basis. This is also referred to as the ‘**full load**’ or ‘**rated load**’.

- The primary and the secondary nominal **rated voltages** are 500 V and 250 V respectively. Ratings do not imply the actual operating voltages.

Transformer Ratings

- A transformer is designed to operate at 50 Hz. Operating at different frequency may affect the flux density in the core and the core losses. $V \propto B_m \times f$

- The magnitudes of nominal **rated currents** (**full-load currents**) are:

$$I_1 = \frac{5000 \text{ VA}}{500 \text{ V}} = 10 \text{ A}, \text{and } I_2 = \frac{5000 \text{ VA}}{250 \text{ V}} = 20 \text{ A}$$

- **Turns ratio *a*** is not generally specified on the nameplate, but calculated from the ratio of the primary rated voltage to the secondary rated voltage of the transformer. i.e., $a = \frac{V_1}{V_2} = \frac{500}{250} = 2$

Example 1

An ideal transformer has 150 turns on the primary and 750 turns on the secondary. The primary is connected to a 240-V, 50-Hz source. The load connected at the secondary draws a current of 4 A at 0.8 pf (lag). Determine

a) The turns ratio,

b) The current in the primary,

c) The power supplied to the load, and

d) The flux in the core.

(Solutions →)

Example 1 – Solutions

a) The turns ratio $a = \frac{150}{750} = 0.2$

b) Since $I_2 = 4 \text{ A}$, then $I_1 = \frac{I_2}{a} = \frac{4}{0.2} = 20 \text{ A}$

c) Since $V_1 = 240 \text{ V}$, then $V_2 = \frac{V_1}{a} = \frac{240}{0.2} = 1200 \text{ V}$

Therefore, power supplied to the load:

$$P_2 = V_2 I_2 \cos\varphi = 1200 \times 4 \times 0.8 = 3840 \text{ W}$$

d) Since $E = 4.44 f \varphi_m N$, then

$$\varphi_m = \frac{E_1}{4.44 fN_1} = \frac{V_1}{4.44 fN_1} = \frac{240}{4.44 \times 50 \times 150} = 7.21 \text{ mWb}$$

Practical (Non-ideal) Transformer

- Practical transformers deviate from the ideal transformer in many respects. The non-ideal features of a practical transformer have to be appropriately modelled in order to analyse the performance of a practical transformer.
  - Both the windings, primary and secondary, have resistances R_1 and R_2 associated with them (see Fig. 9), which incur power losses of $I_1^2 R_1$ and $I_2^2 R_2$, and voltage drops of $I_1 R_1$ and $I_2 R_2$.

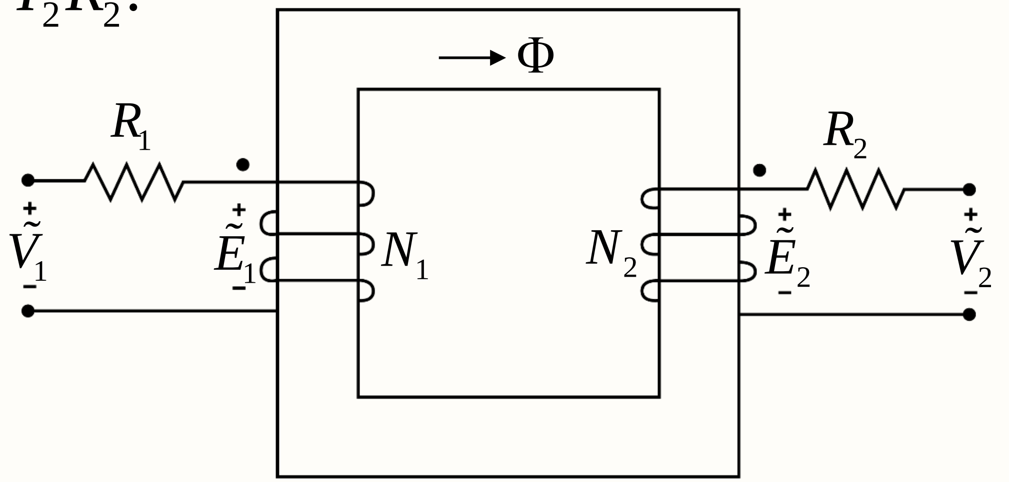


Fig. 9. A practical transformer.

Practical (Non-ideal) Transformer

- **Solution:** These effects in practical transformers may be modelled by representing the winding resistances by lumped resistances R_1 and R_2 in series with the respective winding outside the transformer.
  Note that $V_1 \neq E_1$ and $V_2 \neq E_2$.

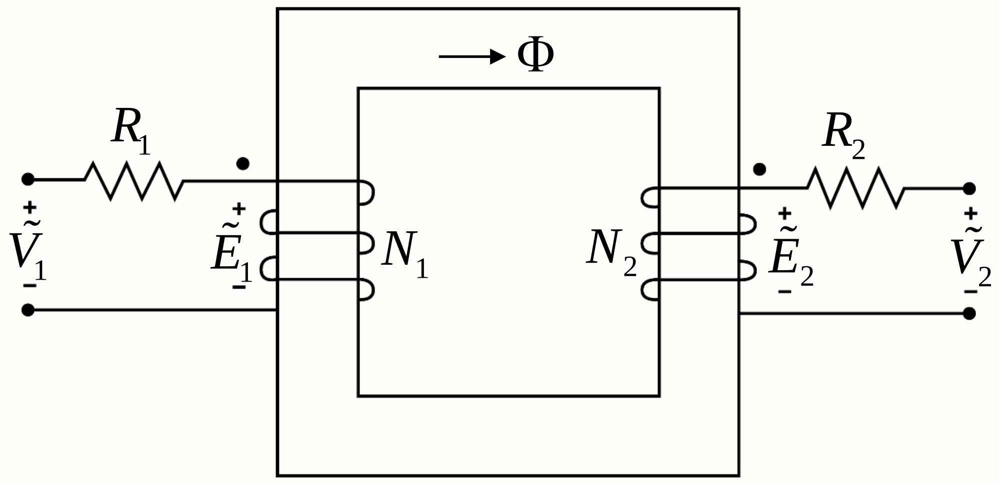


Fig. 9. A practical transformer.

Practical (Non-ideal) Transformer

❖ In addition to the mutual flux Φ_m, there are leakage fluxes $\Phi_{\ell 1}$ and $\Phi_{\ell 2}$, which link only the primary and the secondary windings respectively as shown in Fig. 10.

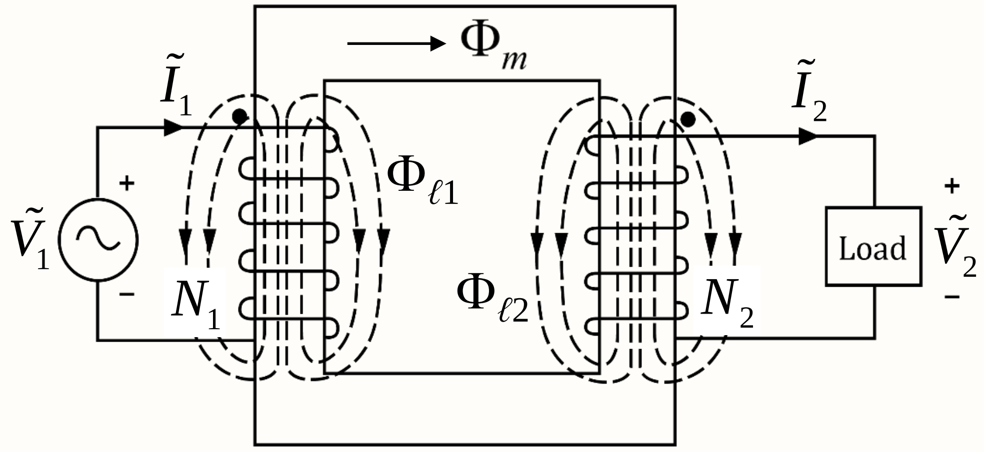


Fig. 10. Transformer with mutual and leakage fluxes.

Practical (Non-ideal) Transformer

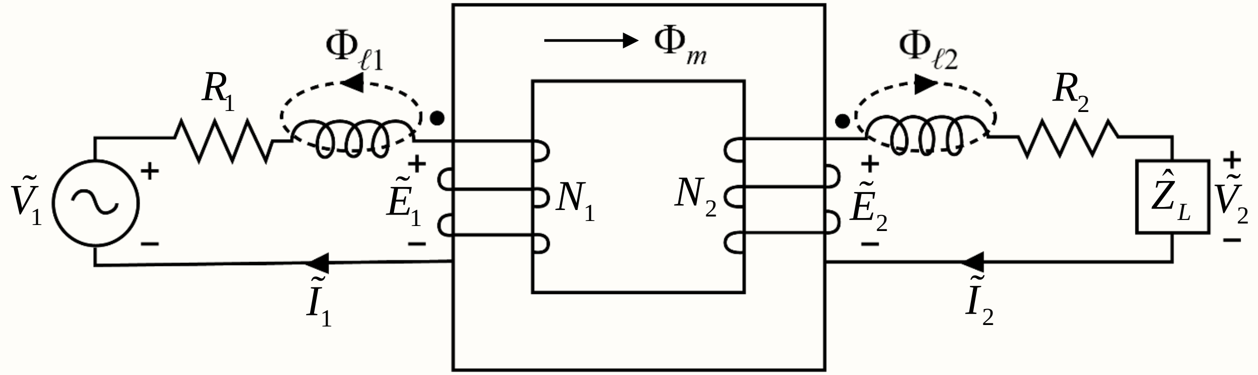


Fig. 11. Hypothetical windings showing mutual and leakage flux linkages separately.

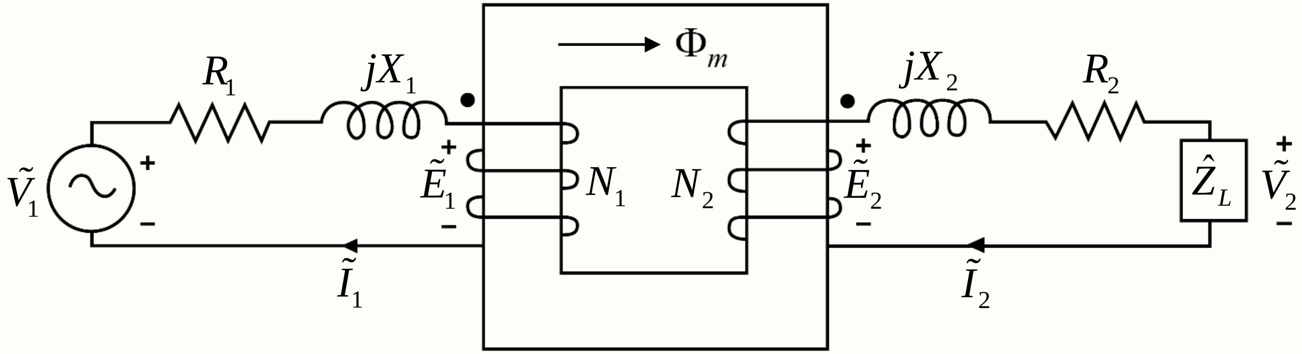


Fig. 12. A non-ideal transformer with winding resistances and leakage reactances.

Practical (Non-ideal) Transformer

- **Solution:** Each winding can be artificially represented by two coils, one producing the mutual flux and the other producing the leakage flux as shown in Fig. 11. The latter is represented outside the transformer proper. The coils producing the leakage flux can then be represented by approximate leakage inductances or leakage reactances as shown in Fig. 12.

Practical (Non-ideal) Transformer

- For the non-ideal transformer incorporating the coil resistances and the leakage reactances as shown in Fig. 13, it should be noted that $V_1 \neq E_1$ and $V_2 \neq E_2$.

  But, $\dfrac{E_1}{E_2} = \dfrac{I_2}{I_1} = \dfrac{N_1}{N_2} = a$

- Further, it should be noted that $V_1 = E_1 + I_1 \times (R_1 + jX_1)$, and $E_2 = V_2 + I_2 \times (R_2 + jX_2)$.

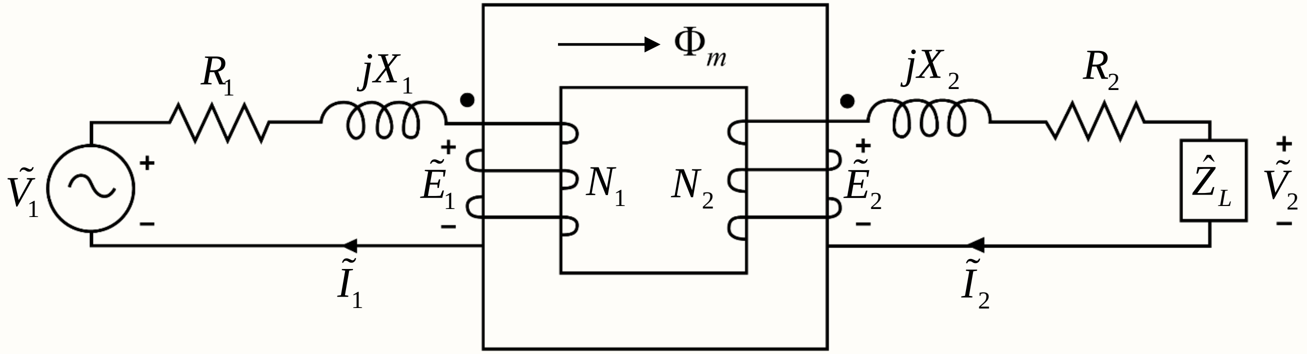


Fig. 13. A non-ideal transformer with winding resistances and leakage reactances.

Practical (Non-ideal) Transformer

❖ Non-ideal core material

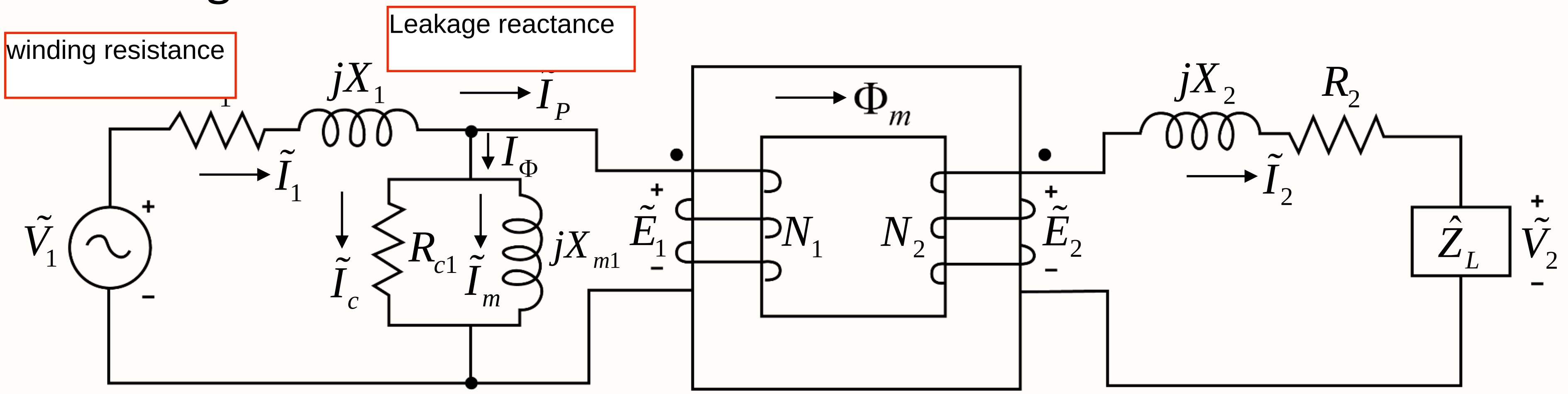
- requires some current to establish the mutual flux, and
- incurs core losses consisting of hysteresis and eddy current losses.

❖ Therefore, the primary winding will draw some current $\boldsymbol{I}_{\Phi}$, even when there is no load connected to the secondary winding in order to establish the mutual flux and to supply the core losses.

Practical (Non-ideal) Transformer

Due to finite permeability

- **Magnetising current** I_m should lag the induced voltage by 90º. So, it can be modelled as the current passing through a reactance jX_{m1} connected across E_1 as shown in Fig. 14.



where I_c – core loss current, R_{c1} – core loss resistance, and jX_{m1} – magnetising reactance

Fig. 14. Equivalent circuit of a practical transformer.

Practical (Non-ideal) Transformer

- The **magnetic** or **core losses** P_c consist of
  - Hysteresis loss: $P_h = K_h B_m{}^n f$, $n \approx (1.5 \sim 2.5)$
  - Eddy current loss: $P_e = K_e B_m{}^2 f^2$

  where B_m is the maximum flux density in the core and f is the supply frequency.

- In case of transformers, the frequency f remains constant so that

  $P_h = K_h B_m{}^n f \propto B_m{}^2$ (take $n \approx 2$) and,

  $P_e = K_e B_m{}^2 f^2 \propto B_m{}^2$

  Thus, $P_c = (P_e + P_h) \propto B_m{}^2$

Practical (Non-ideal) Transformer

- Since $E_1 = 4.44 N_1 A B_m f$ and for constant frequency f, $E_1 \propto B_m \Rightarrow E_1^{\,2} \propto B_m^2$

  Therefore, $P_c \propto E_1^{\,2}$

- Therefore, the **total core loss** can be modelled as the loss incurred by a resistance $\boldsymbol{R_{c1}}$ connected across E_1, so that the core loss

$$P_c = \frac{E_1^2}{R_{c1}} = I_c^{\,2} R_{c1}$$

- And, the no-load current

$$I_\Phi = I_c + I_m = \frac{E_1}{R_{c1}} + \frac{E_1}{jX_{m1}}$$

Practical (Non-ideal) Transformer

- The magnetic core is not commonly shown in the equivalent circuit, but indicated symbolically as shown Fig. 15.

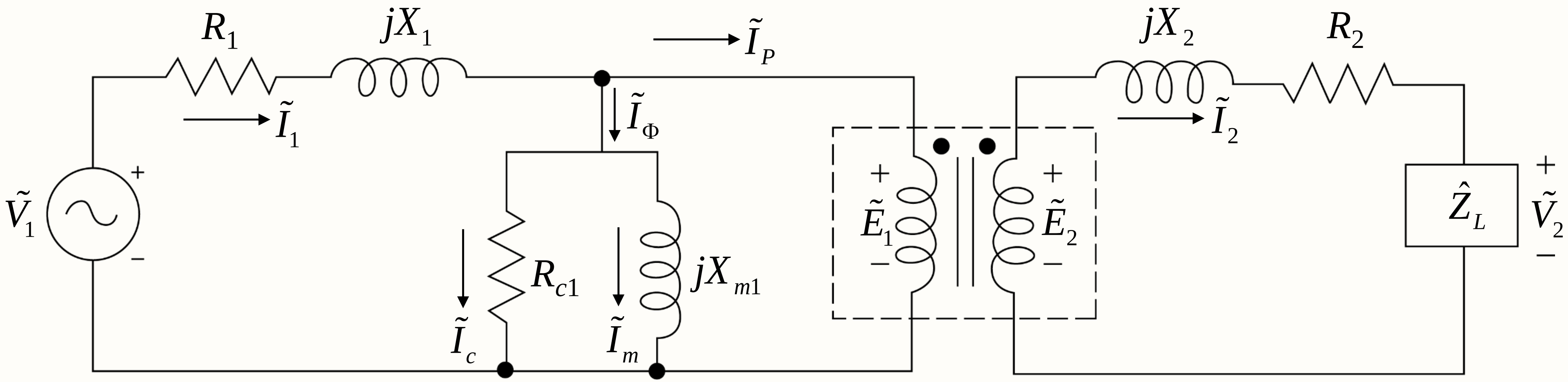


Fig. 15. Equivalent circuit of a practical transformer.

Example 2

A 23-kVA, 2300/230-V, 60-Hz transformer has the following parameters:

$R_1 = 4\ \Omega,\ R_2 = 0.04\ \Omega,\ X_1 = 12\ \Omega,\ X_2 = 0.12\ \Omega,$

$R_{c1} = 20\ \text{k}\Omega$, and $X_{m1} = 15\ \text{k}\Omega$

Draw the actual equivalent circuit of the transformer. If the transformer delivers 75% of its rated load at 0.866 pf (lag) at rated voltage, determine

a) the input current,

b) the input voltage,

c) the total copper loss,

d) the core loss, and

e) the input power.

(Solutions →)

Example 2 – Solutions

The equivalent circuit is shown in Fig. 16 with the parameters listed below.

$R_1 = 4\ \Omega,\ R_2 = 0.04\ \Omega$

$X_1 = 12\ \Omega,\ X_2 = 0.12\ \Omega$

$R_{c1} = 20\ \text{k}\Omega,$ and $X_{m1} = 15\ \text{k}\Omega$

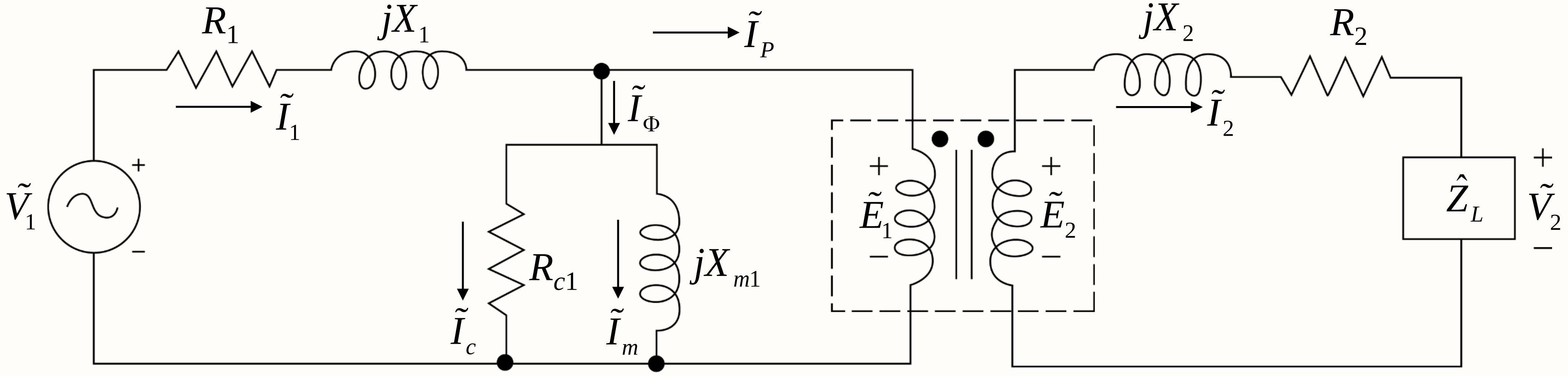


Fig. 16. Equivalent circuit of a transformer.

Example 2 – Solutions

$V_2 = 230\ \text{V}$ (**rated value**)

Let $V_2 = 230\angle 0°\ \text{V}$ (**reference**)

Why conjugate the current?

Load $S_2 = 0.75 \times 23\ \text{kVA} \Rightarrow S_2 = 0.75 \times 23\angle 30°\ \text{kVA}$, since $\cos^{-1} 0.866 = 30°$

Load current $I_2 = (S_2 / V_2)^* = 0.75 \times 23 \times 1000\angle -30° / 230\angle 0°$

$= 75\angle -30°$ A

$E_2 = V_2 + I_2(R_2 + jX_2) = 230\angle 0° + 75\angle -30° \times (0.04 + j0.12) = 237.18\angle 1.52°$

Turns ratio $a = 2300 / 230 = 10$

Therefore, $E_1 = aE_2 = 2371.8\angle 1.52°\ \text{V}$, and $I_p = I_2 / a = 7.5\angle -30°\ \text{A}$

Example 2 – Solutions

Then,

$$I_c = \frac{E_1}{R_{c1}} = \frac{2371.8\angle 1.52^\circ}{20000} = 0.1186\angle 1.52^\circ \text{ A}$$

$$I_m = \frac{E_1}{jX_{m1}} = \frac{2371.8\angle 1.52^\circ}{j15000} = 0.1581\angle -88.48^\circ \text{ A}$$

$$\begin{aligned} I_1 &= I_p + I_c + I_m \\ &= 7.5\angle -30^\circ + 0.1186\angle 1.52^\circ + 0.1581\angle -88.48^\circ = 7.683\angle -30.55^\circ \text{ A} \end{aligned}$$

and, $V_1 = E_1 + I_1(R_1 + jX_1) = 2371.8\angle 1.52^\circ + 7.683\angle -30.55^\circ \times (4 + j12)$

$$= 2447.6\angle 2.97^\circ \text{ V}$$

Example 2 – Solutions

Now the required quantities can be listed or calculated as

a) the input current $I_1 = 7.683\ \mathrm{A}$

b) the input voltage $V_1 = 2447.6\ \mathrm{V}$

c) the total copper loss: $I_1{}^2 R_1 + I_2{}^2 R_2 = 7.683^2 \times 4 + 75^2 \times 0.04 = 461.1\ \mathrm{W}$

d) the core loss:

$$\frac{E_1{}^2}{R_{c1}} = \frac{2371.8^2}{20000} = 281.3\ \mathrm{W}$$

or alternatively, the core loss:

$$I_c{}^2 R_{c1} = 0.1186^2 \times 20000 = 281.3\ \mathrm{W}$$

Example 2 – Solutions

e) The input power:

$$V_1 I_1 \cos\varphi = 2447.6 \times 7.683 \times \cos(2.97° + 30.55°) = 15677.4 \text{ W}$$

$$\text{Also, input power} = \text{output power } (S_2 \cos\varphi = V_2 I_2 \cos\varphi) + \text{copper loss} + \text{core loss}$$

$$= 0.75 \times 23 \times 1000 \times 0.866 + 461.1 + 281.3 = 15680.9 \text{ W}$$

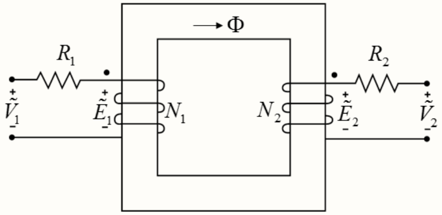
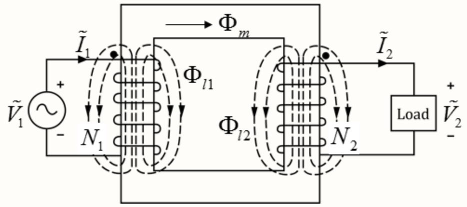
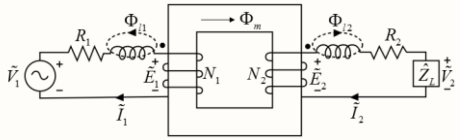
- ❖ Check the example in the book, the same problem has been solved for a leading power factor. Compare the effects of power factor.

Summary

In this lecture, you have learnt:

- The transformer ratings with respect to rated load, rated voltages and rated currents.
- The performance of a practical transformer by examining the non-ideal features of a practical transformer.
- Solving simple problems with respect to the ideal transformers and practical transformers.

References

No.	Slide No.	Image	Reference
1	8 and 9		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 214), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
2	10		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 214), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
3	11		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 215), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.

References

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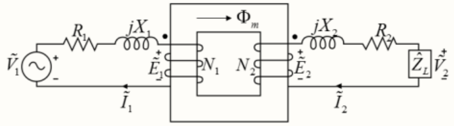
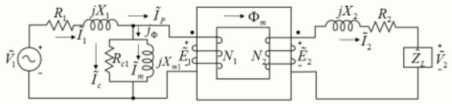
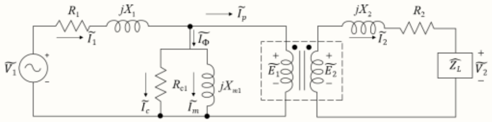
4	11 and 13		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 215), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
5	15		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 217), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
6	18 and 20		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 218), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.