NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2021 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains 4 question and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae is provided in Appendix A on page 4.

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1. (a) A linear system of equations with unknowns x_i; $i = 1,2,3$, is given by

$$
\begin{aligned}
x_1 + 2x_2 + ax_3 &= 2 \\
3x_1 + bx_2 + 3x_3 &= b \\
-2x_1 - 4x_2 - 2x_3 &= c
\end{aligned}
$$

(i) Determine values of a, b and c for which the linear system is inconsistent.

(ii) Determine values of a, b and c for which the linear system has a unique solution.

(iii) Determine values of a, b and c for which the linear system has a one-parameter family of solutions.

(iv) Determine values of a, b and c for which the linear system has a two-parameter family of solutions.

(10 Marks)

(b) Find the inverse of the following matrix

$$
\begin{bmatrix} cos2\theta & sin2\theta \\ -sin2\theta & cos2\theta \end{bmatrix}
$$

(10 Marks)

Note: Question No. 1 continues on page 2.

(c) Let A, B be $n \times n$ matrices. If $AB = 0$, but $A \neq 0$ as well as $B \neq 0$, prove that $rank(A) < n$, and $rank(B) < n$.

(5 Marks)

2. (a) Describe the column space of the matrix A below:

$$A = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$$

(5 Marks)

(b) Consider a stochastic process $X_{n+1}=AX_n$, n = 1, 2, 3, ……, where A is the state transition matrix shown below and X_n is the state vector.

$$A = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$$

(i) Prove that there is a steady state for X_n when $n \to \infty$.

(12 Marks)

(ii) Find the steady state probability vector.

(8 Marks)

3. (a) Suppose that $f(z)=5my^3 + 5nx^2y + i5(x^3 + 2lxy^2)$. Find the constants m, n and l such that $f(z)$ is differentiable for all z. Hence find the derivative of $f(z)$, expressing your answer in terms of z.

(10 Marks)

(b) Evaluate the following integrals:

(i) $\oint_C \left[e^{z\sin|z|} + \frac{z^2+\cos z}{|z|^{10}} + \frac{\sin 2z}{z^2}\right]dz$, $C: |z| = \frac{\pi}{2}$ counterclockwise.

(ii) $\int_0^\infty \frac{dx}{1+4x^2}$

(10 Marks)

(c) Solve the equation $\bar{z} = z^{n-1}$ where $n \geq 3$ is an integer.

(5 Marks)

4. (a) The point (2, 3, a) is on the surface $2z-xy=4$. Determine a and find all the unit normal vectors of the surface at this point.

(6 Marks)

Note: Question No. 4 continues on page 3.

(b) Consider the force field $\mathbf{F}(x, y, z) = y\cos(xy)\,\mathbf{i} + x\cos(xy)\mathbf{j} + \sin z\mathbf{k}$. Show that the work done in moving an object from point $(1, \frac{\pi}{2}, \frac{\pi}{2})$ to $(5, \frac{\pi}{5}, \pi)$ in this field is independent of path and hence determine the work done. What can you say about these two points?

(13 Marks)

(c) The vector function $\mathbf{v}(x,y,z)$ has continuous second-order partial derivatives. Show that $\nabla \cdot \nabla \times \mathbf{v} = 0$.

(6 Marks)

Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c \ln z}$

(b) Euler's Formula: $e^{ix} = cos\, x + i\, sin\, x$

(c) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(d) Cauchy-Riemann equations:

$$u_x = v_y,\ v_x = -u_y,\ \text{or } u_r = \frac{1}{r}v_\theta,\ v_r = \frac{-1}{r}u_\theta$$

(e) Derivative, if exists: $f'(z) = u_x + iv_x = e^{-i\theta}\,(u_r + iv_r)$

(f) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\,\frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\Big|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Divergence Theorem: $\iiint_T \nabla.\mathbf{F}dv = \oiint_S \mathbf{F}\cdot\mathbf{n}dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

EE2007 ENGINEERING MATHEMATICS II
IM2007 ENGINEERING MATHEMATICS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

EE2007 PYP AY20/21 SEM 2

Q1. a) [10]

$$\left[\begin{array}{ccc|c} 1 & 2 & a & 2 \\ 3 & b & 3 & b \\ -2 & -4 & -2 & c \end{array}\right] \xrightarrow[R_3 = R_2 + R_3 - R_1]{R_2 = R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 2 & a & 2 \\ 0 & b-6 & 3-3a & b-6 \\ 0 & b-6 & 1-a & b+c-2 \end{array}\right]$$

$$\xrightarrow[R_3 = R_3 - R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & a & 2 \\ 0 & 6-b & 3a-3 & 6-b \\ 0 & 0 & 2a-2 & c+4 \end{array}\right]$$

(i) For inconsistent linear system, no solution

$(2a-2)\,x_3 = c+4$ (LHS 0 , RHS $\neq 0$)

$\therefore\ a=1,\ c\neq -4,\ b\in\mathbb{R}$ #

(ii) For unique solution,

$2a-2\neq 0 \quad 6-b\neq 0$

$\therefore\ a\neq 1,\ b\neq 6,\ c\in\mathbb{R}$ #

(iii) For one-parameter family of solutions,

$2a-2 = c+4 = 0$

$\therefore\ a=1,\ c=-4,\ b\neq 6$ #

(iv) For two-parameter family of solutions,

$2a-2 = c+4 = 0$ AND $6-b=0$

$\therefore\ a=1,\ b=6,\ c=-4$ #

Q1.b) [10]

$$\left[\begin{array}{cc|cc} \cos 2\theta & \sin 2\theta & 1 & 0 \\ -\sin 2\theta & \cos 2\theta & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2 : \frac{R_2}{\sin 2\theta}]{R_1 : \frac{R_1}{\cos 2\theta}} \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ -1 & \cot 2\theta & 0 & \operatorname{cosec} 2\theta \end{array}\right]$$

$$\xrightarrow[R_2 : R_2 + R_1]{} \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ 0 & \tan 2\theta + \cot 2\theta & \sec 2\theta & \operatorname{cosec} 2\theta \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ 0 & \frac{\tan^2 2\theta + 1}{\tan 2\theta} & \sec 2\theta & \operatorname{cosec} 2\theta \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ 0 & \frac{\sec^2 2\theta}{\tan 2\theta} & \sec 2\theta & \operatorname{cosec} 2\theta \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ 0 & \frac{1}{\sin 2\theta \cos 2\theta} & \sec 2\theta & \operatorname{cosec} 2\theta \end{array}\right]$$

$$\xrightarrow[R_2 = R_2(\sin 2\theta \cos 2\theta)]{} \left[\begin{array}{cc|cc} 1 & \tan 2\theta & \sec 2\theta & 0 \\ 0 & 1 & \sin 2\theta & \cos 2\theta \end{array}\right]$$

$$\xrightarrow[R_1 : R_1 - R_2 \tan 2\theta]{} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1 - \sin^2 2\theta}{\cos 2\theta} & -\sin 2\theta \\ 0 & 1 & \sin 2\theta & \cos 2\theta \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & \cos 2\theta & -\sin 2\theta \\ 0 & 1 & \sin 2\theta & \cos 2\theta \end{array}\right]$$

#

Alternative Solution :

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}^{-1} = \frac{\cancel{1}^{\,1}}{\cancel{\cos^2 2\theta + \sin^2 2\theta}} \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

#

Q1. c)

[5] Let A, B be $n \times n$ matrices, $A \neq 0$, $B \neq 0$, $AB = 0$

$\det(AB) = \det(A)\det(B) = 0$

Assume $\det(B) \neq 0$, $\det(A) = 0$,

$\Rightarrow B^{-1}$ exists $\Rightarrow \text{rank}(B) = n$

$\therefore A = AI = A(BB^{-1}) = (AB)B^{-1} = 0B^{-1} = 0$

But, $A \neq 0 \Rightarrow$ contradiction [applies to $\det(A) \neq 0$ and $\det(B) = 0$]

$\therefore \det(B) = \det(A) = 0$,

A, B does not have unique solution for every $Ax = b$, $Bx = b$

$\therefore \text{rank}(A) < n$ and $\text{rank}(B) < n$ [shown]

#

Q2. a)

[5] $$A = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0.4 & 0.3 & 0.3 & 0.1a \\ 0.3 & 0.6 & 0.1 & 0.1b \\ 0.3 & 0.1 & 0.6 & 0.1c \end{array}\right] \xrightarrow[R_3 = 10R_3]{R_1 = 10R_1,\ R_2 = 10R_2} \left[\begin{array}{ccc|c} 4 & 3 & 3 & a \\ 3 & 6 & 1 & b \\ 3 & 1 & 6 & c \end{array}\right]$$

$$\xrightarrow[R_3 = R_3 - R_2]{R_2 = R_2 - \frac{3}{4}R_1} \left[\begin{array}{ccc|c} 4 & 3 & 3 & a \\ 0 & 3.75 & -1.25 & b - \frac{3}{4}a \\ 0 & -5 & 5 & c - b \end{array}\right]$$

$$\xrightarrow{R_3 = R_3 + \frac{5}{3.75}R_2} \left[\begin{array}{ccc|c} 4 & 3 & 3 & a \\ 0 & 3.75 & -1.25 & b - \frac{3}{4}a \\ 0 & 0 & 10/3 & c - b + \frac{4}{3}(b - \frac{3}{4}a) \end{array}\right]$$

$\therefore$ columnspace of $A = \mathbb{R}^3$ #

* Note: Q2b)(i) requires you to prove 1 is an eigenvalue, and for (ii) find the eigenvector for 1, which gives 20 marks. The mark allocation for this question is particularly weird.

Q2. b) (i)

[12] Sub $\lambda = 1$: $\begin{vmatrix} 0.4-1 & 0.3 & 0.3 \\ 0.3 & 0.6-1 & 0.1 \\ 0.3 & 0.1 & 0.6-1 \end{vmatrix} = 0$

$\therefore$ There exist steady state for X_n when $n \to \infty$. [shown] #

[Alternative solution for Q2b)(i)]

Q2.b)(i) [12]

$$(A-\lambda I)\underset{\sim}{x} = 0$$

$$\det(A-\lambda I) = 0$$

$$\begin{vmatrix} 0.4-\lambda & 0.3 & 0.3 \\ 0.3 & 0.6-\lambda & 0.1 \\ 0.3 & 0.1 & 0.6-\lambda \end{vmatrix} = 0$$

$$(0.4-\lambda)\left[(0.6-\lambda)^2 - 0.1^2\right] - 0.3\left[0.3(0.6-\lambda) - 0.3(0.1)\right] + 0.3\left[0.1(0.3) - 0.3(0.6-\lambda)\right] = 0$$

$$(0.4-\lambda)\left[0.36 - 1.2\lambda + \lambda^2 - 0.01\right] - 0.3^2\left[0.6-\lambda-0.1\right] + 0.3^2\left[0.1 - 0.6 + \lambda\right] = 0$$

$$(0.4-\lambda)\left[\lambda^2 - 1.2\lambda + 0.35\right] + 0.09(-1+2\lambda) = 0$$

$$\cancel{0.4\lambda^2} - 0.\cancel{48}\lambda + 0.14 - \cancel{\lambda^3} + \cancel{1.2\lambda^2} - \cancel{0.35}\lambda - 0.09 + 0.\cancel{18}\lambda = 0$$

$$\lambda^3 - 1.6\lambda^2 + 0.65\lambda - 0.05 = 0$$

$$(\lambda-1)(\lambda-0.5)(\lambda-0.1) = 0$$

Eigenvalues : $\lambda_1 = 1$, $\lambda_2 = 0.5$, $\lambda_3 = 0.1$

$\because$ $\lambda_1 = 1$ is an eigenvalue, $X_{n+1} = AX_n$
there exist a steady state for X_n when $n \to \infty$. #

(ii) [8]

$\lambda_1 = 1$:

$$\left[\begin{array}{ccc|c} -0.6 & 0.3 & 0.3 & 0 \\ 0.3 & -0.4 & 0.1 & 0 \\ 0.3 & 0.1 & -0.4 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - R_2]{R_2 = 2R_2 + R_1} \left[\begin{array}{ccc|c} -0.6 & 0.3 & 0.3 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0.5 & -0.5 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 = R_3 + R_2]{} \left[\begin{array}{ccc|c} -0.6 & 0.3 & 0.3 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $-0.5x_2 + 0.5x_3 = 0$, $-0.6x_1 + 0.3t + 0.3t = 0$

$x_2 = x_3 = t$, $x_1 = t$ $\Rightarrow \underset{\sim}{v_1} = \begin{pmatrix}1\\1\\1\end{pmatrix}t$

As $n \to \infty$, $X_\infty = AX_\infty$

$\because$ $X_\infty = \begin{pmatrix}1\\1\\1\end{pmatrix}$,

$\therefore$ $\underset{\sim}{v_p} = \frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$ #

Q3. a) $f(z) = 5my^3 + 5nx^2y + i5(x^3 + 2lxy^2)$

[10]

$U_x = 10nxy \qquad V_x = 15x^2 + 10ly^2$

$U_y = 15my^2 + 5nx^2 \qquad V_y = 20lxy$

For $f(z)$ to be differentiable $\forall z$,

$U_x = V_y \qquad U_y = -V_x$

$10nxy = 20lxy \qquad 15my^2 + 5nx^2 = -15x^2 - 10ly^2$

By comparison,

$\cancel{10}n = 2\cancel{0}l \qquad 5n = -15, \qquad 15m = -10l$

$\therefore l = -\frac{3}{2} \qquad n = -3 \qquad m = 1$ #

$\therefore f'(z) = U_x + iV_x$

$= -30xy + i(15x^2 - 15y^2)$

$= 15i(x^2 + 2xyi - y^2)$

$= 15i(x+iy)^2$

$= 15z^2 i$ #

b) (i)

[10]

$\oint_C e^{z\sin|z|}\, dz = 0$ since $e^{z\sin|z|}$ is analytic everywhere.

$\oint_C \frac{z^2 + \cos z}{|z|^{10}}\, dz = 0$ since $z^2 + \cos z$ is analytic everywhere.

$\oint_C \frac{\sin 2z}{z^2}\, dz = \frac{2\pi i}{1!} \frac{d}{dz}(\sin 2z)\Big|_{z=0}$, $C: |z| = \frac{\pi}{2}$ ccw

$= 2\pi i(2\cos \cancel{2z}^{1})\Big|_{z=0}$

$= 4\pi i$

$\therefore \oint_C \left[e^{z\sin|z|} \cancel{+ \frac{z^2 + \cos z}{z^{10}}}_{0} + \frac{\sin 2z}{z^2}\right] dz = 4\pi i$ #

(ii)

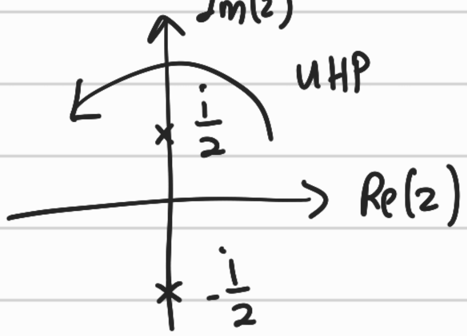
① $f(x) = \frac{1}{1+4x^2} = \frac{p(x)}{q(x)}$, $q(x) \neq 0$

② degree of $q(x) = 2 \geq$ degree of $p(x) + 2 = 0 + 2$

Poles of $f(z) = \frac{1}{1+4z^2}$: $1 + 4z^2 = 0$

$(2z - i)(2z + i) = 0$

$\int_0^\infty \frac{dx}{1+4x^2} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{1+4x^2} dx$ since $\frac{1}{1+4x^2}$ is an even function



$= \frac{1}{2} \oint_{UHP} f(z)\, dz$

$= \frac{1}{2} \oint_{UHP} \frac{1}{1+4z^2} dz$

$= \frac{1}{2} \oint_{UHP} \frac{1}{(2z+i)(2z-i)} dz$

$= \frac{1}{4} \oint_C \frac{1/2z+i}{z - \frac{i}{2}} dz$

$= \frac{1}{4}(2\pi i) \left[\frac{1}{2z+i} \right]_{z = \frac{i}{2}}$

$= \frac{\pi i}{2} \left(\frac{1}{2i} \right) \quad = \frac{1}{4}\pi$

#

c) $\bar{z} = z^{n-1}$, Let $z = re^{i\theta}$

[5] $re^{-i\theta} = r^{n-1} e^{i\theta(n-1)} = r^{n-1} e^{in\theta - i\theta}$, $n \geq 3$

$r^{2-n} = e^{in\theta - \cancel{i\theta + i\theta}} = e^{i\theta n}$

$r^{2\cancel{-n+n}} = (re^{i\theta})^n$

$r^2 = r^n e^{i\theta n} \quad \forall n$, $r^2 = r = r^3 = r^4 \ldots \Rightarrow r = 1$

$z^n = 1$

$\therefore z = e^{\frac{i2\pi k}{n}}$ $k = 0, 1, 2, \ldots n-1$

Q4. a) For a :

[6]

$$2a - 3(2) = 4$$

$$\therefore a = 5 \quad \#$$

Let $f(x,y,z) = 2z - xy$.

$$\nabla f = \frac{\partial}{\partial x}\underset{\sim}{i} + \frac{\partial}{\partial y}\underset{\sim}{j} + \frac{\partial}{\partial z}\underset{\sim}{k}$$

$$= (-y)\underset{\sim}{i} - x\underset{\sim}{j} + 2\underset{\sim}{k}$$

At $(2,3,5)$, $\nabla f = -3\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$

$$\|\nabla f\| = \sqrt{3^2 + 2^2 + 2^2} = \sqrt{17}$$

$\therefore$ unit normal vector $= \dfrac{\nabla f}{\|\nabla f\|} = \pm\dfrac{1}{\sqrt{17}}\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ #

b) $\vec{F}(x,y,z) = y\cos(xy)\,\underset{\sim}{i} + x\cos(xy)\,\underset{\sim}{j} + \sin z\,\underset{\sim}{k}$

[13]

$$\therefore \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}\overset{0}{\cancel{(\sin z)}} - \frac{\partial}{\partial z}\overset{0}{\cancel{(x\cos xy)}}\right] - j\left[\frac{\partial}{\partial x}\overset{0}{\cancel{(\sin z)}} - \frac{\partial}{\partial z}\overset{0}{\cancel{(y\cos xy)}}\right]$$

$$+ \underset{\sim}{k}\left[\frac{\partial}{\partial x}(x\cos xy) - \frac{\partial}{\partial y}(y\cos xy)\right]$$

$$= \underset{\sim}{k}(\cos xy - xy\sin xy - \cos xy + xy\sin xy) = \underset{\sim}{0} \quad \#$$

$\because \nabla \times \vec{F} = 0$, $\vec{F}$ is conservative, ie.

Work done is independent of path taken . [shown] #

$$V = \int \vec{F} \cdot (\partial x\,\underset{\sim}{i} + \partial y\,\underset{\sim}{j} + \partial z\,\underset{\sim}{k})$$

$$= \int y\cos xy\,\partial x + \int x\cos xy\,\partial y + \int \sin z\,\partial z$$

$$= \frac{\cancel{y}}{\cancel{y}}\sin xy + \frac{\cancel{x}}{\cancel{x}}\sin xy - \cos z$$

$$= \sin xy - \cos z + C$$

$$\therefore WD = V(5,\tfrac{\pi}{5},\pi) - V(1,\tfrac{\pi}{2},\tfrac{\pi}{2})$$

$$= (\overset{0}{\cancel{\sin \pi}} - \cos\pi) - (\sin\tfrac{\pi}{2} - \overset{0}{\cancel{\cos\tfrac{\pi}{2}}}) = 0 \quad \#$$

$\therefore$ The two points are equipotential . #

Q4. c) let $\underset{\sim}{x} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$,

[6]

$$\nabla \times \underset{\sim}{x} = \underset{\sim}{i}\left[\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(y)\right] - \underset{\sim}{j}\left[\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(x)\right] + \underset{\sim}{k}\left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right]$$

$$= \underset{\sim}{i}\left[\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right] - \underset{\sim}{j}\left[\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right] + \underset{\sim}{k}\left[\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right]$$

$$\therefore \text{LHS} = \nabla \cdot (\nabla \times \underset{\sim}{x})$$

$$= \frac{\partial}{\partial x}\left[\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right] - \frac{\partial}{\partial y}\left[\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right]$$

$$= \cancel{\frac{\partial^2 z}{\partial x \partial y}} - \cancel{\frac{\partial^2 y}{\partial x \partial z}} - \cancel{\frac{\partial^2 z}{\partial x \partial y}} + \cancel{\frac{\partial^2 x}{\partial y \partial z}} + \cancel{\frac{\partial^2 y}{\partial x \partial z}} - \cancel{\frac{\partial^2 x}{\partial y \partial z}}$$

$= 0 = \text{RHS}$ [shown] #