NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2016 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae and Table of Physical Constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) Three line charges, each of length L, are arranged in the form of an equilateral triangle in the xy-plane with the centre of the triangle located at the origin, as shown in Figure 1 (page 2). The line charges carry uniform charge densities ρ_{l1}, ρ_{l2}, and ρ_{l3}, respectively.

   (i) If $\rho_{l1} = \rho_0$, determine the electric field intensity at the centre of the triangle due to line charge 1 alone.

   (ii) If $\rho_{l2} = \rho_0$ and $\rho_{l3} = -\rho_0$, determine the total electric field intensity at the centre of the triangle due to line charges 2 and 3. (Hint: See if you can make use of the result of part (i).)

   It is given that $\int \frac{dx}{\left(a^2+x^2\right)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{a^2+x^2}}$.

(13 Marks)

Note: Question No. 1 continues on page 2.

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(b) Let the equilateral triangle shown in Figure 1 now represent a wire loop carrying a dc current I in the clockwise direction (when viewed from $+z$ direction).

(i) Determine the magnetic field intensity at point (0, 0, z) due to the entire loop.

(ii) What major changes, if any, are expected in the value of the magnetic field intensity if it is evaluated at points other than the z-axis? Justify your answer.

(12 Marks)

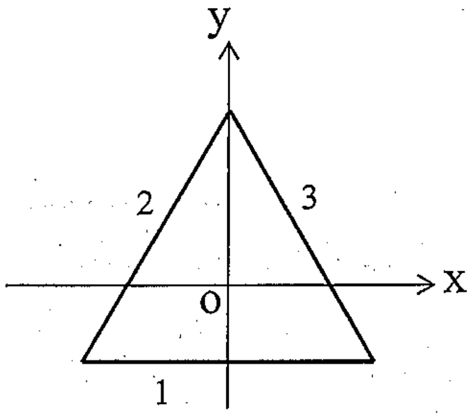

Figure 1

2. (a) A 30 cm by 40 cm rectangular loop rotates at 130 rad/s in a magnetic flux density 0.06 Wb/m^2 normal to the axis of rotation. The loop has 50 turns.

(i) Derive an expression for the time-varying voltage induced between the terminals of the loop. State any assumptions made.

(ii) Calculate the amplitude of the induced voltage.

(9 Marks)

(b) The electric and magnetic field intensities of a uniform plane wave travelling in a lossy medium are given by

$$\tilde{\mathbf{E}}(z,t) = 20\pi\, \vec{\mathbf{a}}_x\, e^{-90\pi z} \cos(9\pi \times 10^9 t - 156\pi z - \frac{\pi}{6}) \text{ V/m}$$

$$\tilde{\mathbf{H}}(z,t) = \vec{\mathbf{a}}_y\, e^{-90\pi z} \cos(9\pi \times 10^9 t - 156\pi z - \frac{\pi}{3}) \text{ A/m}$$

Note: Question No. 2 continues on page 3.

Determine:

(i) The attenuation constant α, phase constant β, and the complex intrinsic impedance η_c.
(ii) The relative permittivity ε_r, relative permeability μ_r, and the conductivity σ of the lossy medium.
(iii) Whether the Gauss' Law for electric flux density is satisfied in this case. Briefly justify your answer.

(16 Marks)

3. (a) The electric field of a uniform plane wave (UPW) travelling in free space ($z \le 0$) has the form of

$$\vec{E}_i(z) = \left(j100\vec{a}_x + 100\vec{a}_y\right)e^{-j\frac{\pi}{75}z} \text{ V/m}$$

The UPW is incident normally on a plane interface at $z = 0$ with a lossy medium ($\mu = \mu_o$, $\varepsilon = 2\varepsilon_o$ and $\sigma = 0.03$ S/m) occupying the region $z \ge 0$.

Determine the following and state any assumption(s) made:

(i) The polarization (linear, circular or elliptical) and the direction of propagation $\vec{a}_k$ of the UPW.
(ii) The time-average Poynting vector of the incident UPW, i.e., $\vec{S}_i$.
(iii) The attenuation constant α in the lossy medium.
(iv) The distance travelled by the transmitted wave in the lossy medium when the average power density drops to 1% of its value at $z = 0$.

(13 Marks)

(b) A uniform plane wave (UPW) in free space occupying the region $z \le 0$ is incident at a plane interface with a lossless dielectric medium having $\mu = \mu_o$ and $\varepsilon = 1.8\varepsilon_o$, occupying the region $z \ge 0$. The phasor magnetic field of the UPW is given by

$$\vec{H}_i(x,z) = \left(80\vec{a}_x + N\vec{a}_z\right)e^{-j(3\pi x + 2\pi z)} \text{ mA/m}$$

Note: Question No. 3 continues on page 4.

Find the following:

(i) The frequency of the UPW and the value of N.
(ii) The corresponding phasor electric field $\vec{E}_i(x,z)$ of the incident UPW.
(iii) The time-average power reflected from a 3 m^2 area at $z = 0$.

(12 Marks)

4. (a) A 50 cm long lossless transmission line operating at a frequency of 300 MHz has a characteristic impedance $Z_o = 50\ \Omega$ and a phase velocity $u_p = 2.25 \times 10^8$ m/s. The line is terminated in a load $Z_L = 35 + j30\ \Omega$. Assuming that the load end is located at $z = 0$ and the source end at $z = -\ell$ where ℓ is the length of the line, find the following and state any assumption(s) made:

(i) The wavelength λ on the transmission line.
(ii) The reflection coefficient $\Gamma(z)$ at $z = 0$
(iii) All the positions z at which the input impedance $Z_{in}(z)$ is real.
(iv) The input impedance $Z_{in}(z)$ at all the positions z in part (iii).
(v) The average power delivered to the load if the magnitude of minimum voltage on the line $|V|_{min} = 20$ V.

(20 Marks)

(b) An unknown resistive load R_L connected to a 100 Ω lossless transmission line causes a Standing Wave Ratio (SWR) of 1.5 with no phase shift on the reflected voltage wave. Determine the unknown resistive load.

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

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1/a i) $\vec{f}=0, \quad \vec{s}=x\vec{a_x}-\frac{\sqrt{3}}{6}L\vec{a_y}$

$\vec{R}=\vec{f}-\vec{s}=-x\vec{a_x}+\frac{\sqrt{3}}{6}L\vec{a_y}$

$R=\sqrt{x^2+\frac{1}{12}L^2}$

$\vec{E}=\frac{1}{4\pi\varepsilon_0}\int_l \frac{\rho_l\vec{R}\,dl}{R^3}$

$=\frac{\rho_0}{4\pi\varepsilon_0}\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{-x\vec{a_x}+\frac{\sqrt{3}}{6}L\vec{a_y}}{(x^2+\frac{1}{12}L^2)^{\frac{3}{2}}}dx$

Due to symmetry, only in $\vec{a_y}$ direction

$\vec{E}=\frac{\rho_0}{4\pi\varepsilon_0}\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{\frac{\sqrt{3}}{6}L\vec{a_y}}{(x^2+\frac{1}{12}L^2)^{\frac{3}{2}}}dx$

$=\frac{\rho_0}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}}{6}L\vec{a_y}\left[\frac{x}{\frac{1}{12}L^2\sqrt{\frac{1}{12}L^2+x^2}}\right]_{-\frac{1}{2}L}^{\frac{1}{2}L}$

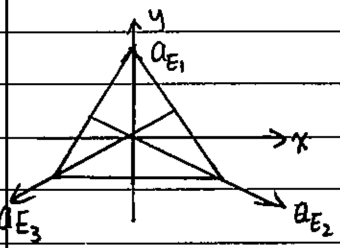
$=\frac{\rho_0}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}}{6}L\vec{a_y}\ \frac{L}{\frac{1}{12}L^2\cdot\sqrt{\frac{1}{12}L^2+\frac{1}{4}L^2}}$

$=\frac{\rho_0}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}}{6}L\vec{a_y}\ \frac{L}{\frac{1}{12}L^2\cdot L\sqrt{\frac{1}{3}}}$

$=\frac{\rho_0}{4\pi\varepsilon_0}\cdot\frac{\sqrt{3}}{6}L\vec{a_y}\ \frac{12\sqrt{3}L}{L^3}$

$=\frac{3\rho_0}{2\pi\varepsilon_0 L}\vec{a_y}$

ii)



$\vec{a_{E_1}}=\vec{a_y}$

$\vec{a_{E_2}}=\vec{a_x}\cos30^\circ-\vec{a_y}\sin30^\circ=0.866\vec{a_x}-\frac{1}{2}\vec{a_y}$

$\vec{a_{E_3}}=-\vec{a_x}\cos30^\circ-\vec{a_y}\sin30^\circ=-0.866\vec{a_x}-\frac{1}{2}\vec{a_y}$

$\vec{E_2}=\frac{3\rho_0}{2\pi\varepsilon_0 L}(0.866\vec{a_x}-\frac{1}{2}\vec{a_y})$

$\vec{E_3}=\frac{3(-\rho_0)}{2\pi\varepsilon_0 L}(-0.866\vec{a_x}-\frac{1}{2}\vec{a_y})$

$\vec{E_2}+\vec{E_3}=\frac{3\rho_0}{2\pi\varepsilon_0 L}(0.866\vec{a_x}-0.5\vec{a_y}+0.866\vec{a_y}+0.5\vec{a_y})$

$=\frac{3\rho_0}{2\pi\varepsilon_0 L}\times1.732\vec{a_x}$

$=\frac{2.598\rho_0}{\pi\varepsilon_0 L}\vec{a_x}$

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bi) Line 1

$\vec{r} = z\vec{a_z}$

$\vec{s} = x\vec{a_x} - \frac{\sqrt{3}}{6}L\vec{a_y}$

$\vec{R} = \vec{r} - \vec{s} = z\vec{a_z} - x\vec{a_x} + \frac{\sqrt{3}}{6}L\vec{a_y}$

$R = \sqrt{z^2 + x^2 + \frac{1}{12}L^2}$

$\vec{dl} = -\vec{a_x}\,dx$

$$\vec{H_1} = \frac{1}{4\pi}\int_c \frac{I\vec{dl}\times\vec{R}}{R^3}$$

$$= \frac{I}{4\pi}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} \frac{(-\vec{a_x})\times(z\vec{a_z} - x\vec{a_x} + \frac{\sqrt{3}}{6}L\vec{a_y})\,dx}{(z^2+x^2+\frac{1}{12}L^2)^{\frac{3}{2}}}$$

$$= \frac{I}{4\pi}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} \frac{-z\vec{a_y} - \frac{\sqrt{3}}{6}L\vec{a_z}\,dx}{(z^2+x^2+\frac{1}{12}L^2)^{\frac{3}{2}}}$$

Due to symmetry there is only in $\vec{a_z}$ direction

[diagram (aii)]

$$\vec{H_1'} = \frac{I}{4\pi}\cdot\left(-\frac{\sqrt{3}}{6}L\right)\vec{a_z}\int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{dx}{(z^2+x^2+\frac{1}{12}L^2)^{\frac{3}{2}}}$$

$$= \frac{I}{4\pi}\left(-\frac{\sqrt{3}}{6}L\right)\vec{a_z}\left[\frac{x}{(z^2+\frac{1}{12}L^2)\sqrt{z^2+\frac{1}{12}L^2+x^2}}\right]_{-\frac{1}{2}L}^{\frac{1}{2}L}$$

$$= \frac{I}{4\pi}\left(-\frac{\sqrt{3}}{6}L\right)\vec{a_z}\cdot\frac{L}{(z^2+\frac{1}{12}L^2)\sqrt{z^2+\frac{1}{12}L^2+\frac{1}{4}L^2}}$$

$$= -\frac{\sqrt{3}}{24\pi}\vec{a_z}\frac{L^2}{(z^2+\frac{1}{12}L^2)\sqrt{z^2+\frac{1}{3}L^2}}$$

$$\vec{H_t} = 3\vec{H_1'} = -\frac{\sqrt{3}}{8\pi}\vec{a_z}\frac{L^2}{(z^2+\frac{1}{12}L^2)\sqrt{z^2+\frac{1}{3}L^2}}$$

ii) At points other than z-axis $\vec{H}$ will have components $\vec{a_x}$ and/or $\vec{a_y}$ other than $\vec{a_z}$ as the 3 vectors in section (aii) can not cancel each other any more.

2/ai) $\Phi = \vec{B} \cdot \vec{S}$

$= 0.06 \cos wt \times 0.3 \times 0.4$

$= 7.2\times10^{-3} \cos 130t \ \text{Wb}$

$V = -N \frac{d\Phi}{dt}$

$= -50 \times 7.2\times10^{-3} \times 130 (-\sin 130t)$

$= 46.8 \sin 130t \ V$

ii) rms value $= \frac{46.8}{\sqrt{2}} = 33.093V$

bi) $\alpha = 90\pi \ Np/m$

$\beta = 156\pi \ rad/m$

$E_0 = 20\pi \angle -\frac{\pi}{6} = 20\pi \angle -30^\circ$

$H_0 = 1 \angle -\frac{\pi}{3} = 1 \angle -60^\circ$

$\eta_c = \frac{E_0}{H_0} = 20\pi \angle 30^\circ \ \Omega$.

ii) $\gamma = jw\sqrt{\mu\varepsilon_r}$ $\eta_c = \sqrt{\frac{\mu}{\varepsilon_r}}$

$\gamma \cdot \eta_c = jw\mu = jw\mu_0\mu_r$

$(90\pi + j156\pi)(20\pi\angle 30^\circ) = j9\pi\times10^9 \times 4\pi\times10^{-7}\mu_r$

$j35550.3 = j35531\mu_r$

$\mu_r \approx 1$

$\eta_c = \sqrt{\frac{\mu}{\varepsilon_r}}$

$\sqrt{\varepsilon_r} = \frac{\sqrt{\mu}}{\eta_c}$

$\varepsilon_r = \frac{\mu}{(\eta_c)^2} = \frac{4\pi\times10^{-7}}{(20\pi\angle 30^\circ)^2} = 1.5915\times10^{-10} - 2.7566\times10^{-10}j = \varepsilon - j\frac{\sigma}{w}$

$\varepsilon = \varepsilon_0\varepsilon_r = 1.5915\times10^{-10}$ $\varepsilon_r = 18$

$\frac{\sigma}{w} = 2.7566\times10^{-10}$ $\sigma = 7.794 \ (\Omega\text{-m})^{-1}$

iii) Gauss' Law only for static E-field. Here $\sigma \neq 0$, so it is not valid.

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3/a i) $E_{0x} = 100 \angle 90^\circ$

$E_{0y} = 100 \angle 0^\circ$

$|E_{0x}| = |E_{0y}|$

$|\phi_x - \phi_y| = 90^\circ$

$\therefore$ it is circular polarized.

$\vec{a_k} = \vec{a_z}$

ii) $\vec{S_x} = \frac{|E_{0x}|^2}{2\eta_1} \vec{a_k} = \frac{100^2}{2\times 120\pi} \vec{a_k} = 13.26 \vec{a_z}\ W/m^2$ | $\eta_1 = 120\pi$ free space

$\vec{S_y} = \vec{a_z}\, 13.26\ W/m^2$

$\vec{S_i} = \vec{S_x} + \vec{S_y} = \vec{a_z}\, 26.52\ W/m^2$

iii) $\frac{\sigma}{\varepsilon w} = \frac{0.03}{2\times\frac{1}{36\pi}\times10^{-9}\times W}$

$k_1 = \frac{\pi}{75} = W\sqrt{\varepsilon_0 \mu_0} = \frac{W}{c}$

$W = \frac{\pi c}{75}$

$= \frac{0.03}{2\times\frac{1}{36}\pi\times10^{-9}\times\frac{\pi}{75}\times3\times10^8}$

$= 135 > 20$ (good conductor)

$\alpha = \sqrt{\sigma\mu\pi f}$

$= \sqrt{0.03\times4\pi\times10^{-7}\times\pi\times\frac{\pi c}{75}\div2\pi}$

$= 0.4867\ Np/m.$

iv) $e^{-2\alpha z} = 0.01$

$e^{-2\times0.4867 z} = 0.01$

$z = 4.731\ m$

b i) $\vec{k_i} = 3\pi\vec{a_x} + 2\pi\vec{a_z}$

$k_i = \sqrt{(3\pi)^2 + (2\pi)^2}$

$= 11.327$

$= W\sqrt{\varepsilon_0\mu_0}$

$f = \frac{11.327c}{2\pi} = 5.408\times10^8\ Hz$

$\vec{H}\cdot\vec{k_i} = 0$

$(80\vec{a_x} + N\vec{a_z})\cdot(3\pi\vec{a_x} + 2\pi\vec{a_z}) = 0$

$N = -120$

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ii) $\vec{E_i} = \eta_1 \cdot \vec{H_i} \times \vec{a_k}$ free space $\eta_1 = 120\pi\,\Omega$.

$= 120\pi\,(80\vec{a_x} + 120\vec{a_z}) \times \left(\frac{3\pi\vec{a_x} + 2\pi\vec{a_z}}{11.327}\right) e^{-j(3\pi x + 2\pi z)}$

$= 120\pi \left(\frac{-160\pi\vec{a_y} - 360\pi\vec{a_y}}{11.327}\right) e^{-j(3\pi x + 2\pi z)}$

$= -54371\,\vec{a_y}\, e^{-j(3\pi x + 2\pi z)}$

iii) $\theta_i = \tan^{-1}\frac{3\pi}{2\pi} = 56.31^\circ$

$\theta_t = \sin^{-1}\left(\sqrt{\frac{1}{1.8}}\sin 56.31^\circ\right) = 38.33^\circ$

$\vec{E_i}$ in $\vec{a_y}$ direction, $\perp$ polarization.

$\Gamma_\perp = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$

$= \dfrac{\frac{120\pi}{\sqrt{1.8}}\cos 56.31^\circ - 120\pi\cos 38.33^\circ}{\frac{120\pi}{\sqrt{1.8}}\cos 56.31^\circ + 120\pi\cos 38.33^\circ}$

$= -0.3097$

$P_r = |\Gamma_\perp|^2 \cdot P_i$

$= |\Gamma_\perp|^2 \cdot \dfrac{|E_i|^2}{2\eta_1}\cos\theta_i$

$= (0.3097)^2 \cdot \dfrac{(54371)^2}{2\times 120\pi}\cos 56.31^\circ$

$= 208600\ W/m^2$

$P_r = 208600 \times 3$

$= 625800\ W$

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4(a)i) $\lambda = \frac{u_p}{f} = \frac{2.25\times10^8}{300\times10^6} = 0.75m.$

ii) $\hat{Z}_L = \frac{35+j30}{50} = 0.7+j0.6$

from smith chart, $\Gamma_L = \frac{3.2}{8.7}\angle 97°$

$= 0.3678\angle 97°$

iii) Z_L @ 0.115λ WTG.

$Z_1 = -(0.25\lambda - 0.115\lambda) = -0.135\lambda$

$= -0.10125\,m$

$Z_2 = -0.10125 - \frac{\lambda}{4} = -0.28875\,m$

$Z_3 = -0.10125 - \frac{\lambda}{2} = -0.47625\,m.$

iV) @ Z_1, Z_3 : $Z_{max} = 2.2\times50 = 110\Omega$

@ Z_2 : $Z_{min} = 0.46\times50 = 23\Omega.$

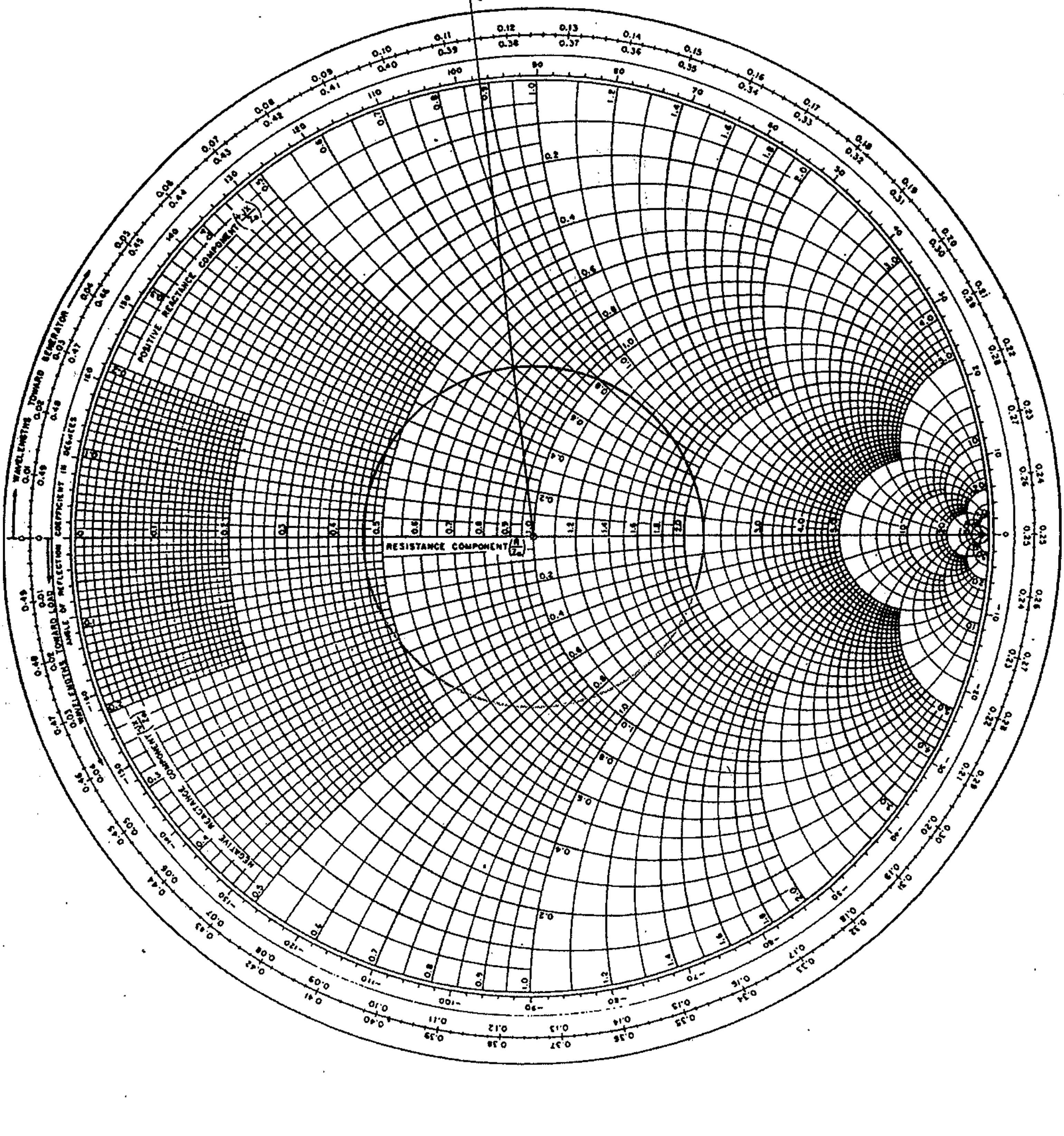
V) $P_{in} = \frac{|V_{min}|^2}{Z_{min}} = \frac{20^2}{23} = 17.39W$

b) $R_L = 1.5\times100$

$= 150\Omega.$

as V_0^+ and V_0^- no phase difference.

$\Gamma_L = \eta\angle 0°.$



WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
POSITIVE REACTANCE COMPONENT
NEGATIVE REACTANCE COMPONENT
RESISTANCE COMPONENT