

NANYANG TECHNOLOGICAL UNIVERSITYSEMESTER 2 EXAMINATION 2018-2019EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2019

Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 4.

1. (a) Consider the following system of equations:

$$(1 + \lambda)x_1 + x_2 + x_3 = 0$$

$$x_1 + (1 + \lambda)x_2 + x_3 = 3$$

$$x_1 + x_2 + (1 + \lambda)x_3 = \lambda.$$

Determine the values of  $\lambda$  such that the system has

- (i) a unique solution,
- (ii) no solution,
- (iii) many solutions. Also, find these solutions in this case.

(12 Marks)

- (b) Suppose that matrices  $A, B$  and  $C$  satisfy  $[I_3 - C^{-1}B]^T C^T A = I_3$ , where

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$I_3 - C^{-1}B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Note: Question 1 continues on page 2.

and  $I_3$  stands for a  $3 \times 3$  identity matrix. Without calculating the inverse of  $C$ , show how  $A$  can be determined. Hence determine  $A$ .

(8 Marks)

(c) Given that

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \quad \text{and} \quad A = \mathbf{a}\mathbf{b}^T,$$

determine  $A^{2000}$ .

(5 Marks)

2. A linear system is given as

$$A\mathbf{x} = \mathbf{b}$$

where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ a^2 & 4 & 3a \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) Use elementary row operations to determine the rank of the matrix  $A$  if

(i)  $a = 2\sqrt{2}$

(ii)  $a = 4$

Hence, determine the condition imposed on  $a$  so that a unique solution can be obtained for any vector  $\mathbf{b}$ .

(10 Marks)

(b) (i) Determine the eigenvalues of matrix  $A$  in terms of  $a$ .(ii) Determine the eigenvectors for the case of  $a = \sqrt{3}$ .

(11 Marks)

(c) Consider the linear system described by  $B\mathbf{x} = \mathbf{b}$ , where

$$B = \begin{bmatrix} -5 & 1 & 0 \\ a^2 & -3 & 3a \\ 0 & 0 & -5 \end{bmatrix}.$$

Note: Question 2 continues on page 3.

By using the results in part (b)(i), determine the values of  $a$  so that a unique solution can be obtained.

Hint: First determine the relationship between  $A$  and  $B$  by subtracting the matrices.

(4 Marks)

3. (a) (i) Using the Cauchy-Riemann equations, comment on the differentiability and analyticity of  $f(z) = \cos^2 z$ .

- (ii) Hence, or otherwise, evaluate  $\int_1^{2\pi} \cos^2 z \, dz$  along the straight-line path from  $z = i$  to  $z = 2\pi$ .

(12 Marks)

- (b) Evaluate  $\int_0^\pi \frac{\sin 2\theta}{2 + \cos 2\theta} d\theta$ .

(13 Marks)

4. (a) Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  for  $\mathbf{F}(x, y, z) = xy \mathbf{i} + \cos y \mathbf{j} + e^z \mathbf{k}$ , along the straight-line path  $C$  from  $(3, 0, -2)$  to  $(4, 2, -1)$ .

(8 Marks)

- (b) Using a suitable spherical surface parameterization, evaluate  $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{A}$  for surface  $S: x^2 + y^2 + z^2 = a^2$ ,  $x \geq 0$ , and  $\mathbf{F}(x, y, z) = -z \mathbf{j} + y \mathbf{k}$ .

(10 Marks)

- (c) Using Stokes' Theorem, evaluate  $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{A}$  for surface  $S: x^2 + y^2 + z^2 = a^2$ ,  $x \geq 0$ , and  $\mathbf{F}(x, y, z) = -z \mathbf{j} + y \mathbf{k}$ .

b)  $x = a \cos u \sin v$  ;  $-\frac{1}{2}\pi \leq u \leq \frac{1}{2}\pi$   
 $y = a \sin u \sin v$  ;  $0 \leq v \leq \pi$   
 $z = a \cos v$

(7 Marks)

$\mathbf{r}_u = -a \sin u \sin v \mathbf{i} + a \cos u \sin v \mathbf{j}$

$\mathbf{r}_v = a \sin u \cos v \mathbf{i} + a \cos u \cos v \mathbf{j} - a \sin v \mathbf{k}$

$\mathbf{N} = -a^2 \cos u \sin v \mathbf{i} - a^2 \sin u \cos v \mathbf{j} - a^2 \sin v \cos u \mathbf{k}$

$\nabla \times \mathbf{F} = \left( \frac{\partial (-z)}{\partial y} - \frac{\partial (\cos y)}{\partial z} \right) \mathbf{i} - \left( \frac{\partial (-z)}{\partial x} - \frac{\partial (\cos y)}{\partial z} \right) \mathbf{j} + \left( \frac{\partial (\cos y)}{\partial x} - \frac{\partial (-z)}{\partial y} \right) \mathbf{k}$

Appendix A

## 1. Complex Analysis

(a) Complex Power:  $z^c = e^{c \ln z}$

(b) De Moivre's Formula:  $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

(c) Cauchy-Riemann equations:

$$u_x = v_y, \quad v_x = -u_y, \quad \text{or} \quad u_r = \frac{1}{r} v_\theta, \quad v_r = -\frac{1}{r} u_\theta$$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_0)^m} dz = \frac{2\pi i}{(m-1)!} \left. \frac{d^{(m-1)}}{dz^{(m-1)}} f(z) \right|_{z=z_0}$$

2. Vector Analysis. Let  $\mathbf{F} = F_1 \mathbf{i} + F_2 \mathbf{j} + F_3 \mathbf{k}$ .

(a) Scalar Triple Product:  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$

(b) Gradient:  $\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$

(c) Divergence:  $\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl:  $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Divergence Theorem:  $\iiint_V \nabla \cdot \mathbf{F} dv = \oint_S \mathbf{F} \cdot \mathbf{n} dA$

(f) Stokes Theorem:  $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dA = \oint_C \mathbf{F} \cdot d\mathbf{r}$

END OF PAPER

$$1) \begin{bmatrix} (1+\lambda) & 1 & 1 \\ 1 & (1+\lambda) & 1 \\ 1 & 1 & (1+\lambda) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ \lambda \end{bmatrix}$$

$$\begin{bmatrix} (1+\lambda) & 1 & 1 & 0 \\ 1 & (1+\lambda) & 1 & 3 \\ 1 & 1 & (1+\lambda) & \lambda \end{bmatrix}$$

$$R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 1 & (1+\lambda) & \lambda \\ 1 & (1+\lambda) & 1 & 3 \\ (1+\lambda) & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1(\lambda+1) \end{array} \begin{bmatrix} 1 & 1 & (1+\lambda) & \lambda \\ 0 & \lambda & -\lambda & 3-\lambda \\ 0 & -\lambda & -2\lambda-\lambda^2 & -\lambda(\lambda+1) \end{bmatrix} \Rightarrow \begin{array}{l} R_3 \leftarrow R_3 + R_2 \\ \Rightarrow \end{array} \begin{bmatrix} 1 & 1 & (1+\lambda) & \lambda \\ 0 & \lambda & -\lambda & 3-\lambda \\ 0 & 0 & -3\lambda-\lambda^2 & 3-2\lambda-\lambda^2 \end{bmatrix}$$

(i) Unique solution condition

$$① -3\lambda - \lambda^2 \neq 0$$

$$(-3-\lambda)\lambda \neq 0$$

$$\lambda_1 \neq 0 \text{ and } \lambda_2 \neq -3$$

(ii) no solution

$$① -3\lambda - \lambda^2 = 0 \quad \cap \quad ② 3-2\lambda-\lambda^2 \neq 0$$

$$\lambda = 0 \quad \lambda = -3$$

$$(3+\lambda)(1-\lambda) \neq 0$$

$$\lambda \neq -3 \quad \lambda \neq 1$$

the solution is  $\lambda_1 = 0$

(ii) many solution

$$① -3\lambda - \lambda^2 = 0 \quad \cap \quad ② 3-2\lambda-\lambda^2 = 0$$

$$\lambda = 0 \quad \lambda = -3$$

$$\lambda = -3 \quad \lambda = 1$$

the solution is  $\lambda = -3$

$$b) \cancel{[C^{-1}I_3]} - [C^{-1}[C \cdot I_3 - B]]^T C^T A = I_3$$

$$[C - B]^T C^{-T} C^T A = I_3$$

$$[C - B]^T I_3 A = I_3$$

$$I_3 \cdot A = [C - B]^T \cdot I_3$$

$$A = [C - B]^T = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$1c) A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [2 \ 1 \ -3]$$

$$A^2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [2 \ 1 \ -3] \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [2 \ 1 \ -3] = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [1] [2 \ 1 \ -3]$$

$$A^{2000} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [1] \dots [1] [2 \ 1 \ -3] = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 2 & -6 \\ 2 & 1 & -3 \end{bmatrix}$$

\* noted that  $[1]$  is scalar

$$2a) i) a = 2\sqrt{2}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} E_1: R_2 \leftarrow R_2 - 2R_1 \quad \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} E_2: R_3 \leftarrow R_3 - \frac{2}{2}R_2 \quad \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Rank of matrix  $A = 1$

$$ii) a = 4$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} E_1: E_2 \leftarrow E_2 - 2E_1 \quad \begin{bmatrix} 2 & 1 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Rank of matrix  $A = 3$

$$\begin{bmatrix} 2 & 1 & 0 & | & b_1 \\ a^2 & 4 & 3a & | & b_2 \\ 0 & 0 & 2 & | & b_3 \end{bmatrix} R_1 \leftarrow 0.5R_1 \quad \begin{bmatrix} 1 & 0.5 & 0 & | & 0.5b_1 \\ a^2 & 4 & 3a & | & b_2 \\ 0 & 0 & 2 & | & b_3 \end{bmatrix} R_2 \leftarrow R_2 - a^2 R_1 \quad \begin{bmatrix} 1 & 0.5 & 0 & | & 0.5b_1 \\ 0 & 4 - 0.5a^2 & 3a & | & b_2 - 0.5b_1 \\ 0 & 0 & 2 & | & b_3 \end{bmatrix}$$

$$4 - 0.5a^2 \neq 0$$

$$(2\sqrt{2} + a)(2\sqrt{2} - a) = 0$$

$$a_1 \neq 2\sqrt{2} \text{ and } a_2 \neq -2\sqrt{2}$$

$$b) i) \begin{vmatrix} \lambda - 2 & -1 & 0 \\ -a^2 & \lambda - 4 & -3a \\ 0 & 0 & \lambda - 2 \end{vmatrix} = 0$$

$$(\lambda - 2)^2 (\lambda - 4) - (\lambda - 2)a^2 = 0$$

$$(\lambda - 2)^2 (\lambda^2 - 6\lambda + 8 - a^2) = 0$$

$$\lambda = 2 \text{ and } \lambda = \frac{6 + \sqrt{36 - 4(8 - a^2)}}{2} \text{ and } \lambda = \frac{6 - \sqrt{36 - 4(8 - a^2)}}{2}$$

$$ii) \lambda = 2$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -3 & -2 & -3\sqrt{3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_2 = 0, x_1 + \sqrt{3}x_3 = 0$$

$$V_1 = \begin{bmatrix} -\sqrt{3} \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = \frac{6 + \sqrt{36 - 4(8 - a^2)}}{2} = 5$$

$$\begin{bmatrix} 3 & -1 & 0 \\ -3 & 1 & -3\sqrt{3} \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & 0 & -3\sqrt{3} \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_3 \neq 0, 3x_1 - x_2 = 0$$

$$V_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

$$\lambda = \frac{6 - \sqrt{36 - 4(8 - a^2)}}{2} = 1$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -3 & -3 & -3\sqrt{3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_2 = 0, x_3 \neq 0$$

$$V_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

2 c)

$$3 a) i) \cos^2 z = \frac{(e^{iz} + e^{-iz})^2}{4} = \frac{(e^{i(x+iy)} + e^{-i(x+iy)})^2}{4} = \frac{(e^{ix} \cdot e^{-y} + e^{-ix} \cdot e^y)^2}{4} =$$

$$= \frac{1}{4} (e^{-2y} \cdot e^{2ix} + 2 + e^{2ix} \cdot e^{2y})$$

$$= \frac{1}{4} (e^{-2y} \cos 2x + e^{-2y} \sin 2x i + 2 + e^{2y} \cos 2x - e^{2y} i \sin 2x)$$

~~let~~ let:

$$U = \frac{1}{4} (e^{-2y} \cos 2x + 2 + e^{2y} \cos 2x), \quad V = \frac{1}{4} (e^{-2y} \sin 2x + e^{2y} (-\sin 2x))$$

$$U_x = \frac{1}{4} (-2e^{-2y} \sin 2x - 2e^{2y} \sin 2x), \quad V_x = \frac{1}{4} (2e^{-2y} \cos 2x - 2e^{2y} \cos 2x)$$

$$U_y = \frac{1}{4} (-2e^{-2y} \cos 2x + 2e^{2y} \cos 2x), \quad V_y = \frac{1}{4} (-2e^{-2y} \sin 2x - 2e^{2y} \sin 2x)$$

Condition

$$U_x = V_y$$

$$\frac{1}{4} (-2e^{-2y} \sin 2x - 2e^{2y} \sin 2x) = \frac{1}{4} (-2e^{-2y} \sin 2x - 2e^{2y} \sin 2x)$$

satisfied

$$U_y = -V_x$$

$$\frac{1}{4} (-2e^{-2y} \cos 2x + 2e^{2y} \cos 2x) = -\frac{1}{4} (2e^{-2y} \cos 2x - 2e^{2y} \cos 2x)$$

satisfied

$f(z)$  is analytic and differentiable at every point

$$ii) \begin{cases} -1 + 2 \cos^2 z = \cos 2z \\ \cos^2 z = \frac{\cos 2z + 1}{2} \end{cases} = \int_{2\pi}^{2\pi} \frac{\cos 2z + 1}{2} dz$$

$$= \frac{1}{2} \left( \frac{1}{2} \sin 2z + z \right) \Big|_{2\pi}^{2\pi}$$

$$= \frac{1}{2} \left( \frac{1}{2} \sin 4\pi + 2\pi \right) - \frac{1}{2} \left( \frac{1}{2} \sin 2i - i \right)$$

$$= \frac{1}{2} \left( 2\pi - \frac{1}{2} \sin 2i - i \right)$$

$$b) z = e^{i2\theta}$$

$$\frac{dz}{z} = d\theta$$

$$\frac{1}{2i} \int_0^{2\pi} \frac{(z - \frac{1}{z})}{2 + \frac{1}{2}(z + \frac{1}{z})} \cdot \frac{dz}{z} = 0 \int_0^{2\pi} \frac{z^2 - 1}{z^2 + 4z + 1} \cdot \frac{dz}{(2i\theta)^2 \cdot \frac{2z}{z^2}}$$

$$\int_0^{2\pi} \frac{z^2 - 1}{z^2 + 4z + 1} \cdot \frac{dz}{2z}$$

$$\int \frac{z^2 - 1}{(z + 2 + \sqrt{3})2z} + \frac{1}{2} \int \frac{z^2 - 1}{z^2 + 4z + 1}$$

$$\pi i \left( \frac{-z^2 - 1}{(z + 2 + \sqrt{3})2z} \right) \Big|_{z=-2+\sqrt{3}} + \pi i \left( \frac{-z^2 - 1}{2(z^2 + 4z + 1)} \right) \Big|_{z=1} = 0$$

$$c) a) \nabla \times F = i \left( \frac{d(e^z)}{dy} - \frac{d(\cos y)}{dz} \right) + j \left( -\frac{d(x^2)}{dz} + \frac{d(e^z)}{dx} \right) + k \left( \frac{d(\cos y)}{dx} - \frac{d(xy)}{dy} \right)$$

$$= -xk$$

$$x = 3+t$$

$$y = 2t$$

$$z = -2+t \quad 0 \leq t \leq 1$$

$$r = (3+t)i + 2tj + (-2+t)k$$

$$dr = (i + 2j + k) dt$$

$$\int_0^1 [(3+t)2ti + \cos 2tj + e^{-2+t}k] \cdot (i + 2j + k) dt$$

$$\int_0^1 [(3+t)2t + 2\cos 2t + e^{-2+t}] dt$$

$$3t^2 + \frac{2}{3}t^3 + \sin 2t + e^{-2+t} \Big|_0^1$$

$$3 + \frac{2}{3} + \sin 2 + e^{-1} - e^{-2}$$

$$\frac{11}{3} + \sin 2 + \frac{e^{-1}}{e^2}$$

$$b) x = a \cos u \cdot \sin v; \quad -\frac{1}{2}\pi \leq u \leq \frac{1}{2}\pi$$

$$y = a \sin u \cdot \sin v; \quad 0 \leq v \leq \pi$$

$$z = a \cos v$$

$$r_v = a \cos u \cdot \sin v \cdot i + a \cos u \sin v \cdot j$$

$$r_v = a \cos u \cos v i + a \sin u \cos v j - a \sin v k$$

$$N = +a^2 \sin v (\cos u \sin v i + \sin u \sin v j + \cos v k)$$

$$\nabla \times f = i \left( \frac{d(y)}{dy} - \frac{d(-z)}{dz} \right) - j \left( \frac{d(y)}{dx} - \frac{d(x)}{dz} \right) + k \left( \frac{d(-z)}{dx} - \frac{d(x)}{dy} \right)$$

$$= i(1 - (-1)) - j(0 - 0) + k(0 - 0)$$

$$= 2i$$

$$\iint \nabla \times F \cdot dA = \iint \nabla \times F \cdot N(u, v) du dv$$

$$\int_0^{\frac{1}{2}\pi} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \begin{pmatrix} 2i \\ 0 \\ 0 \end{pmatrix} \cdot a^2 \sin v \cdot \begin{pmatrix} \cos u \sin v \\ \sin u \sin v \\ \cos v \end{pmatrix} du dv = 2a^2 \int_0^{\frac{1}{2}\pi} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sin^2 v \cos u du dv$$

$$= 2a^2 (+1+1) \int_0^{\frac{1}{2}\pi} \sin^2 v dv = 2a^2 (2) \cdot \frac{1}{2}\pi = 2\pi a^2$$

$$c) x=0; y = a \cos \theta; z = a \sin \theta; \quad 0 \leq \theta \leq 2\pi$$

$$r = a \cos \theta j + a \sin \theta k; \quad dr = -a \sin \theta j + a \cos \theta k$$

$$= \int_0^{2\pi} \begin{pmatrix} 0 \\ -a \sin \theta \\ a \cos \theta \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -a \sin \theta \\ a \cos \theta \end{pmatrix} d\theta = \int_0^{2\pi} a^2 (\sin^2 \theta + \cos^2 \theta) d\theta$$

$$= a^2 \theta \Big|_0^{2\pi} = 2\pi a^2$$



	Date	No.
2. (c)(i)	AREA anotherProgram, CODE, READONLY	(ii) Counter for outer loop:
	ENTRY	r0 (i)
	MOV r0, #10 $\Rightarrow$ MOV r0, #3	Counter for inner loop:
	MOV r2, #0	r1 (j)
	ADD r4, #0 X MOV r4, #0 (?)	i = 10
	loop1	j = 1
	MOV r3, #0	for (i=10; i>3; i--)
	ADD r2, r2, #1	for (j=1; j>5; j++)
	MOV r1, #1	Outer loop = 7x
	loop2	Inner loop = 5x
	CMP r1, #5 X CMP r1, #5	Total loop = 7x5
	BGT somewhere	= 35 times
	MUL r3, r2, r1	(Not sure)
	ADD r4, r4, r3	(iii) look at the lines with
	ADD r1, r1, #1	" $\Rightarrow$ "
	B loop2	(Not sure)
	somewhere	
	SNB r0, r0, #1 $\Rightarrow$ ADD r0, r0, #1	
	CMP r0, #3 X CMP r0, #3 $\Rightarrow$ CMP r0, #10	
	BGT loop1 $\Rightarrow$ BLT loop1	
	stop B stop	
	END	

3. (a) (i)	56 bytes = 28 half words
	12 x 4 bytes + 4 x 2 bytes = 56 bytes
(ii)	if R2 > R3 $\rightarrow$ update R2 if R4 < R3 $\rightarrow$ update R4
	R2 $\rightarrow$ store the smallest number R4 $\rightarrow$ store the biggest number
	R2 = 0x88991122 R4 = 0x76543210
(iii)	R0 $\rightarrow$ loop counter R1 $\rightarrow$ pointer which points to memory address R3 $\rightarrow$ temporary variable to store data from memory
(iv)	DCD 0x2001, 0x0010, 0x3210 (v) Total memory = 108 bytes
	DCD 0x0019, 0x1122, 0x5678
	DCD 0x1122, 0x100, 0x1100
	DCD 0x6633, 0x763, 0x646
	DCW 32000, 123, -654, 888

3. (b)

Starting address of FIQ : 0X0000001C

Starting address of IRQ : &lt;32MB (Unsure)

Modification of the link register :

For FIQ :  $lr = lr - 4$  (it also holds for IRQ).  $lr = PC - 4$ 

3. (c)

R3  $\rightarrow$  0X7FFFFFFFR2  $\rightarrow$  0X88779955R1  $\rightarrow$  0X40000FF8R0  $\rightarrow$  0X2323888sp  $\rightarrow$  0X40000FF0

4. (a)

Assembly code

AREA arithmetic, CODE, READONLY

ENTRY

ADR R4, table ; point to the address of the first element of 'table'

LDR R0, [R4] ; load value of a

LDR R1, [R4, #4] ; load value of b

LDR R2, [R4, #8] ; load value of c

ADD R5, R0, R1

SUB R5, R5, R2

STR R5, [R4, #12] ; to store the value of x in memory

stop B stop

SPACE

table DCD 10, 20, 6

END

Inline Assembly Function

\_\_inline int arithmetic (int a, int b, int c) {

int x, temp;

\_\_asm {

ADD temp, a, b

SUB x, temp, c };

return x;

}

4. (b) MOV r7, r1

LSR r7, #4

ADD r3, r4, r7

PUSH {r0-r4, r8, PC} (ii)

MOV r7, r4

LSL r7, #2

LDR r0, [r3, r7]

STR r0, [r2], #2 (iv)

loop SUB r3, r5

CMP r3, #0

BGE Greater

BLT Stop

Greater ADD r2, #1

B Loop

stop B stop