EE3001
Engineering Electromagnetics

Information

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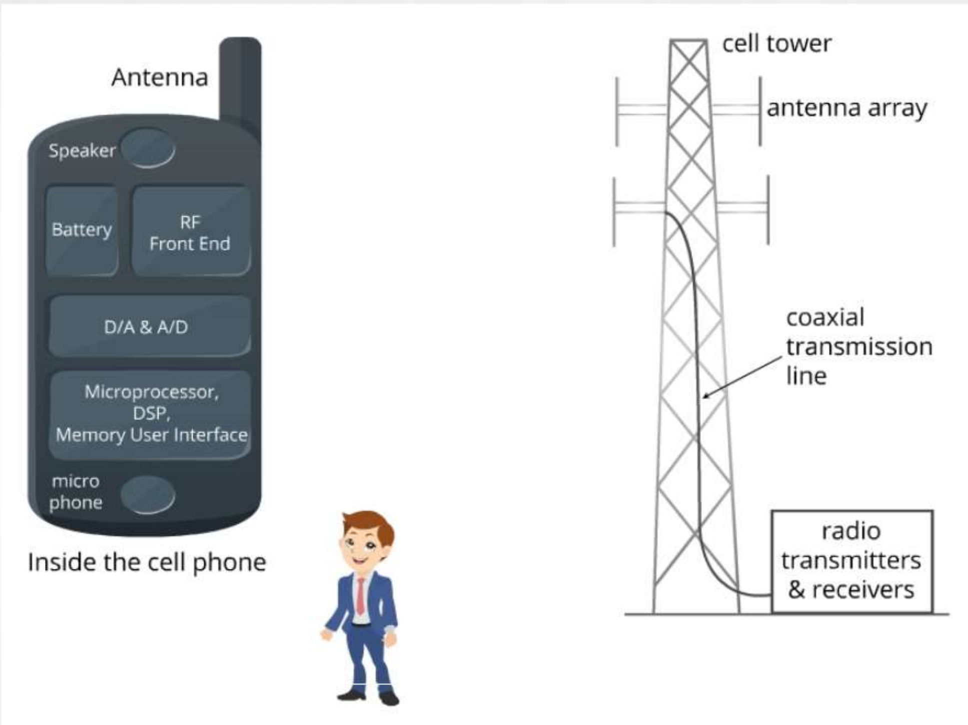
Activity

How many functions can your smart-phone perform?

- Telephone; messaging; contacts;
- Internet;
- Camera;
- Music; radio;
- Personal assistant (e.g., SIRI);
- Games; calculator; weather; torch;
- Clock; alarms; calendar; notes;
- Route-guidance; nearest place of interest;
- Shopping; banking…and much more

Cellular Communication System

Basic operation mechanism of the hand phone:



Course Outline

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Course Objective:

Describe the properties of electromagnetic waves

Part I (18 Hours)

- ❑ Electric and Magnetic Fields
- ❑ Maxwell's Equations
- ❑ Wave Equation and Uniform Plane Waves

Part II (20 Hours)

- ❑ Electromagnetic Power Flow
- ❑ Reflection and Transmission of Electromagnetic Waves
- ❑ Transmission Lines

Information

Related Courses

- ❑ EE2007 (Engineering Mathematics II):
  - Co-requisite; covers Complex Variables and Vector Calculus
- ❑ FE1002 (Physics Foundation For Electrical and Electronic Engineering):
  - Covers Electricity and Magnetism

Appendix

- ❑ There is important information in the slides in the appendix (if any).
- ❑ Please go through these slides on your own.

Text Books and Reference

Text Books

- Hayt, Jr. W. H., & Buck, J. A. (2012). *Engineering electromagnetics* (8th ed.). New York, NY: McGraw Hill. (QC670.H426 2012)
- Sadiku, M. N. O. (2015). *Elements of electromagnetics* (6th ed.). Oxford, OX: Oxford University Press. (QC760.S125 2007)

Reference

- Ulaby, F. T. (2007). *Fundamentals of applied electromagnetics* (5th ed.). Upper Saddle River, NJ: Pearson Education. (QC760.U36 2007)

Note: *Preparatory notes, lecture notes, tutorial questions, quiz questions/solutions, and solutions to drill questions are available on NTULearn.*

EE3001 Engineering Electromagnetics

Session 1-2

Introduction to Engineering Electromagnetics

Learning Objectives

- State the significance of electricity and magnetism;
- Define Engineering Electromagnetics; and
- Describe the concept of field.

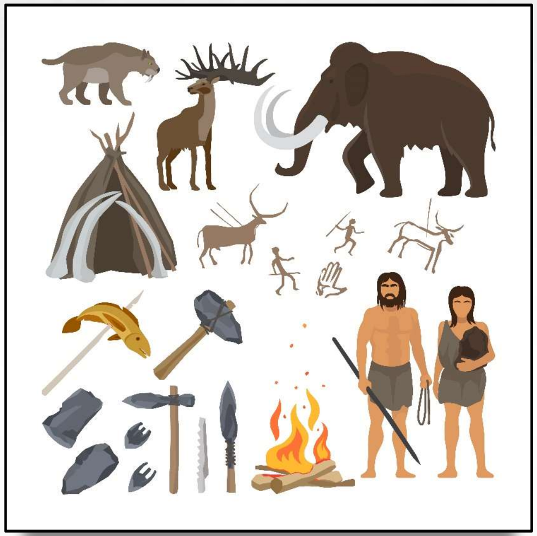
Activity

List 5 most important <u>technology-based</u> items you use **every day**:

- ❑ Cell-phone
- ❑ Electricity
- ❑ Computer
- ❑ Transport
- ❑ Television

Significance of Electricity and Magnetism

No Electricity & Magnetism

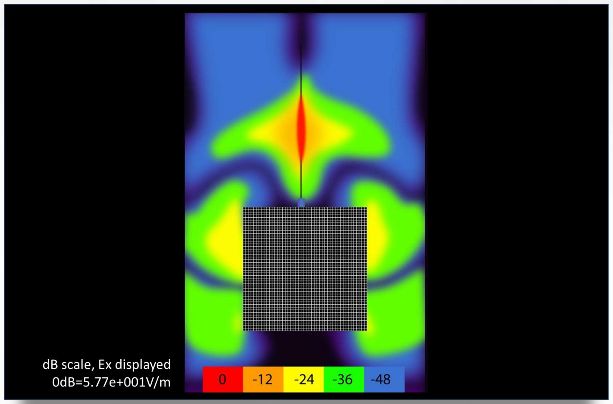


Modern Civilisation

Modern civilisation relies heavily on electricity and magnetism.

Engineering Electromagnetics

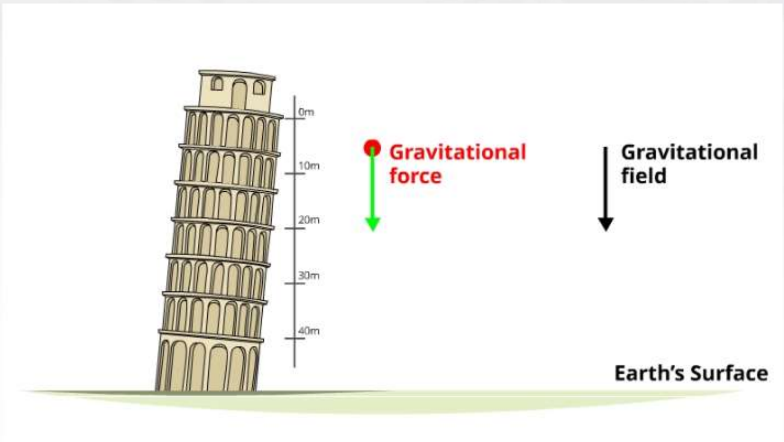
What is electromagnetics?
It is the study of electric and magnetic phenomena that are caused by electric charges at rest or in motion.



What is engineering electromagnetics?
It is the engineering applications of electric and magnetic phenomena.

Concept of Field

Experiment at the top of the Leaning Tower of Pisa:



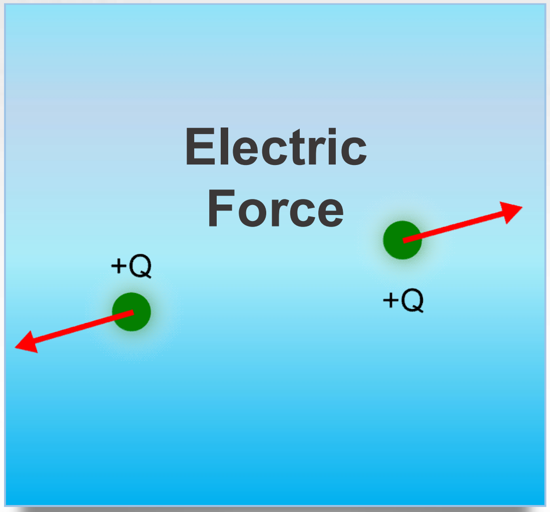
What is a field?

A field is usually associated with its corresponding force.

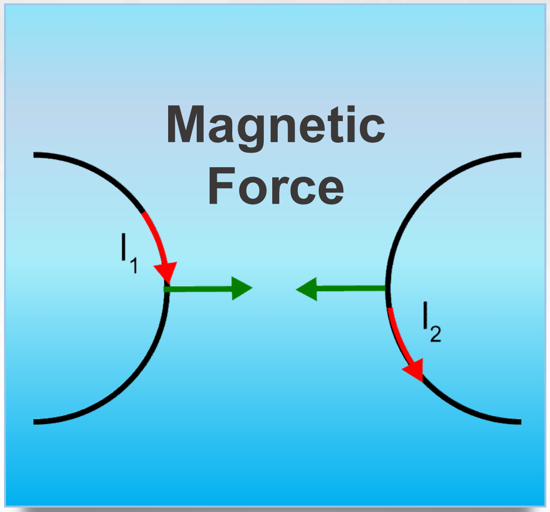
Field is a set of values assumed by a physical quantity:

- at various points in a region of space
- at various instants of time

Concept of Field



Electric Field



Magnetic Field

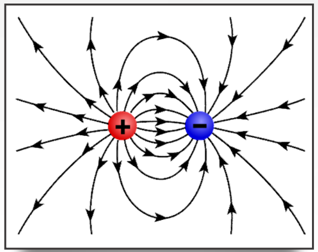
Electric force and magnetic force can be understood in terms of electric field and magnetic field.

Concept of Field

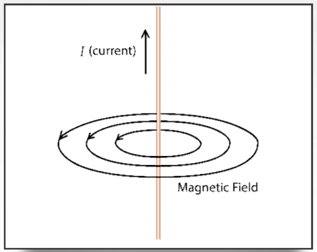
When do we need to describe phenomena in terms of fields?

1. There is action at a distance (or, action without a direct physical contact).

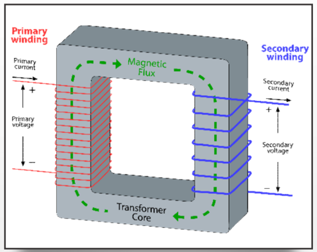
Electric field due to charge



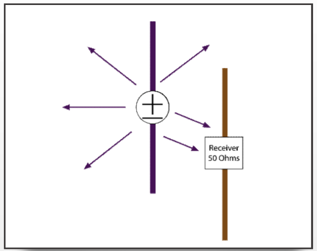
Magnetic field due to current



Electromagnetic induction (Transformers)



Radiation and coupling (Antennas)

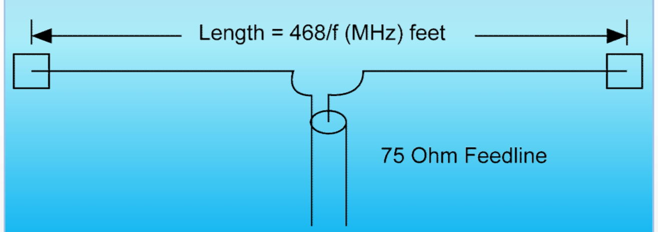


Concept of Field

Any other situation when we need to describe electrical phenomena in terms of fields?

Examples of other situations when we describe electrical phenomena in terms of fields:

An open-circuit!



Dipole antenna

$$L = \frac{468}{f} = \frac{468}{234} = 2 \text{ feet}$$

$$\lambda = \frac{3 \times 10^8}{234 \times 10^6} = 4 \text{ feet}$$

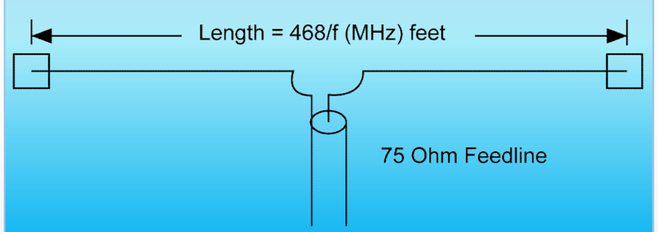
Length is comparable to the wavelength.

Concept of Field

Any other situation when we need to describe electrical phenomena in terms of fields?

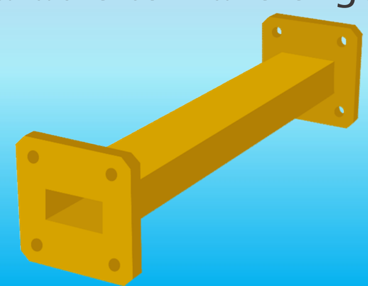
Examples of other situations when we describe electrical phenomena in terms of fields:

An open-circuit!
Length is comparable to wavelength.



Dipole antenna

A short-circuit!
Cross-section dimensions comparable to wavelength.

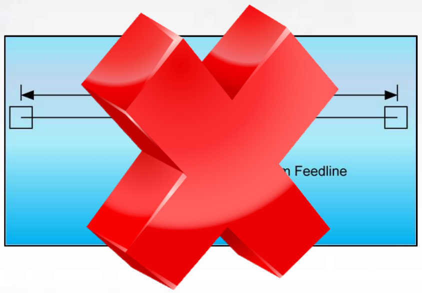


Waveguide

Concept of Field

When do we need to describe phenomena in terms of fields?

2. It is difficult or impossible to analyse the phenomena in terms of voltage and current.



In other words, Kirchhoff's voltage and current laws are no longer accurate!

When the circuit dimensions are of the same order of magnitude as the wavelength.

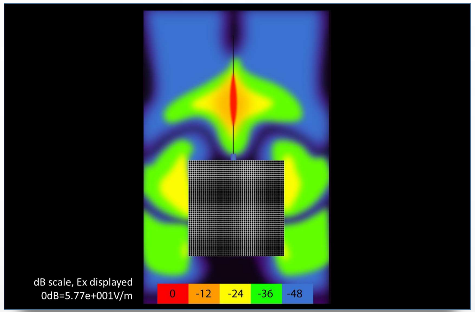
High frequencies: $wavelength = \dfrac{velocity\ of\ wave\ propagation}{frequency}$

Typically, the frequency will be in the range of GHz.

Concept of Field

What are the mathematical tools to describe fields?

- Fields are directed quantities
  - Vectors
    - Vector algebra
    - Coordinate system (Cartesian, Cylindrical, Spherical, etc.)
- Fields are functions of multiple variables



  - 3 spatial coordinates (e.g., x, y, and z)
  - Time (t)
- Calculus of multiple variables

Summary

- ❑ Without electricity and magnetism, we would not have modern civilisation.
- ❑ Engineering Electromagnetics is the engineering applications of electric and magnetic phenomena.
- ❑ The concept of field:
  - Required when there is ‘action at a distance’; and
  - Also required when circuit dimensions become comparable to wavelength.

EE3001 Engineering Electromagnetics

Session 1-3

Applications of Engineering Electromagnetics

Learning Objectives

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- List the applications of engineering electromagnetics;
- Describe the situations whereby simple circuit theory is not applicable;
- Explain how to reduce electromagnetic interference; and
- Explain what will happen when one of the components in an electromagnetic system is not functioning properly.

Applications of Engineering Electromagnetics

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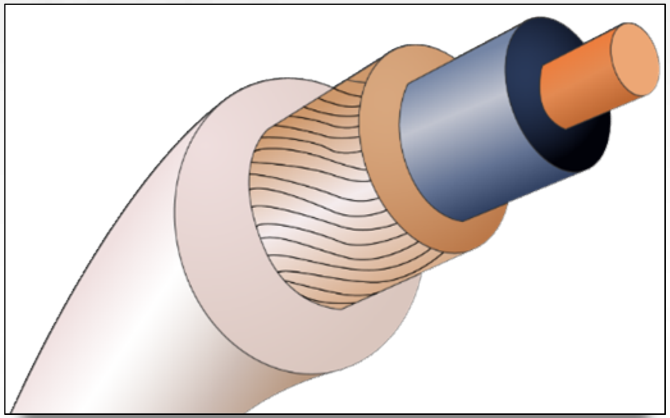
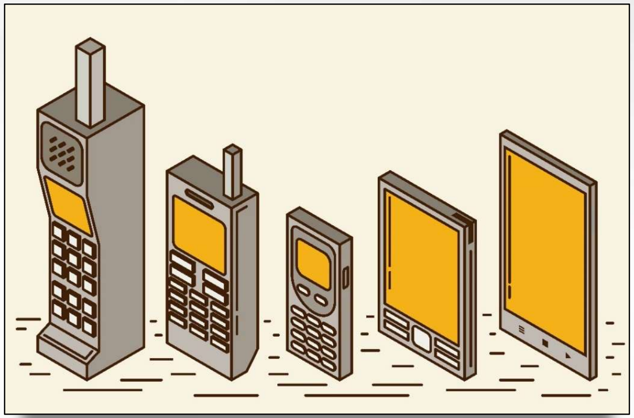
Transmission Lines

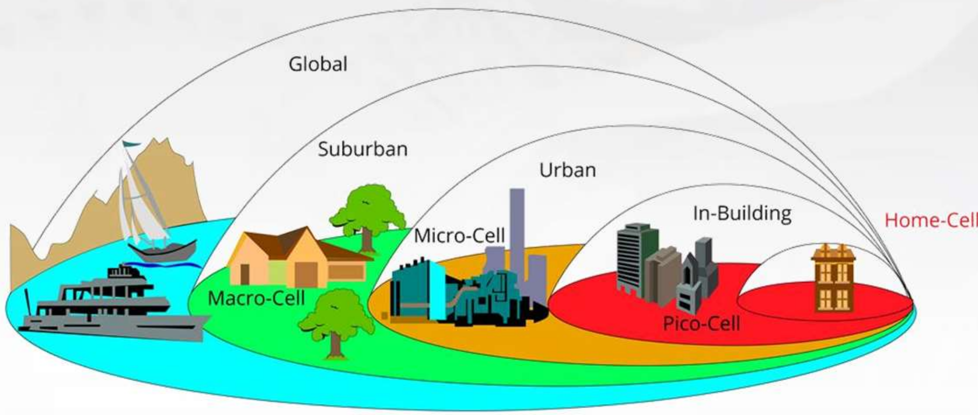


Antennas & Propagation (Wireless Communication)



Applications of Engineering Electromagnetics

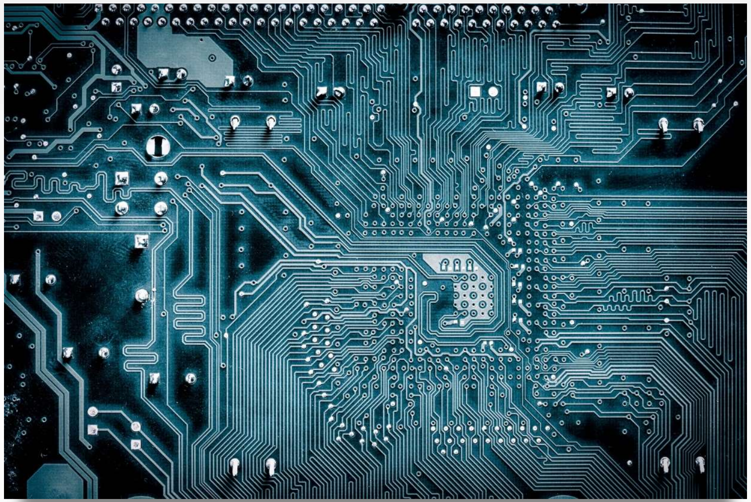
Wireless + Optical Communications



- ❑ Explosive expansion of optical fibre communication systems.
- ❑ Concepts of electromagnetics are crucial in understanding optical fibres and optoelectronic components.

Applications of Engineering Electromagnetics

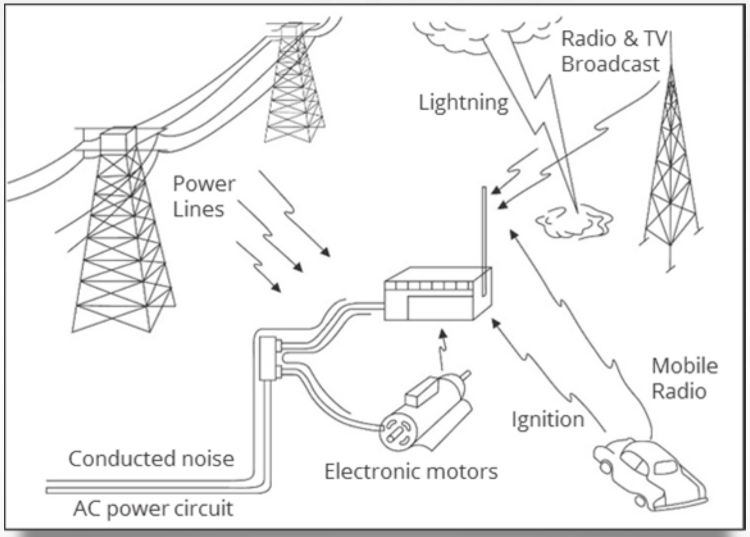
High-speed electronic circuits



As the clock rates get faster, circuit theory is insufficient to describe system performance.

Applications of Engineering Electromagnetics

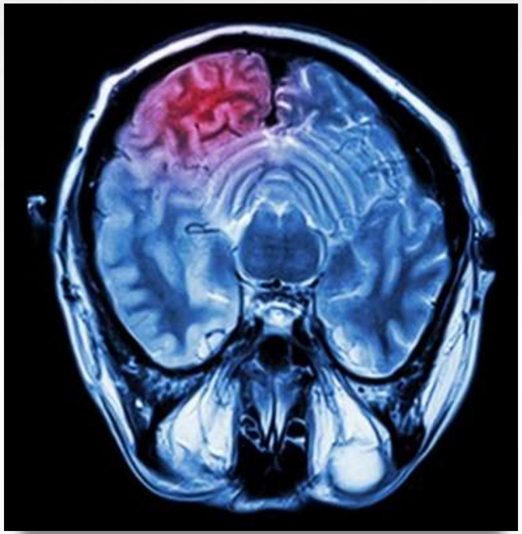
Issues of electromagnetic interference (EMI) and electromagnetic compatibility (EMC)



These are beginning to limit the performance of electronic systems.

Applications of Engineering Electromagnetics

Biology and medicine (life sciences)



Are cell-phone signals harmful?

For more information, see http://www.ece.northwestern.edu/ecefaculty/Allen1.html and then click "Why Study E&M"

- ❑ Detection and treatment of tumors
- ❑ Treatment of depression

Applications of Engineering Electromagnetics

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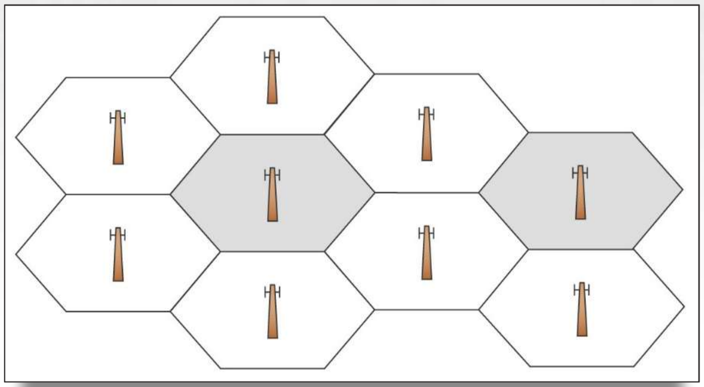
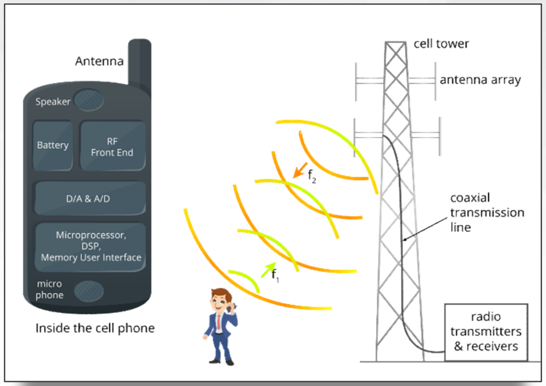
EM concepts in cellular communication system





- ❑ Waves propagate in space and material media.
- ❑ Waves are radiated and received by antennas.
- ❑ Waves propagate in transmission lines such as coaxial cables.
- ❑ Efficient circuits require impedance matching.

Applications of Engineering Electromagnetics

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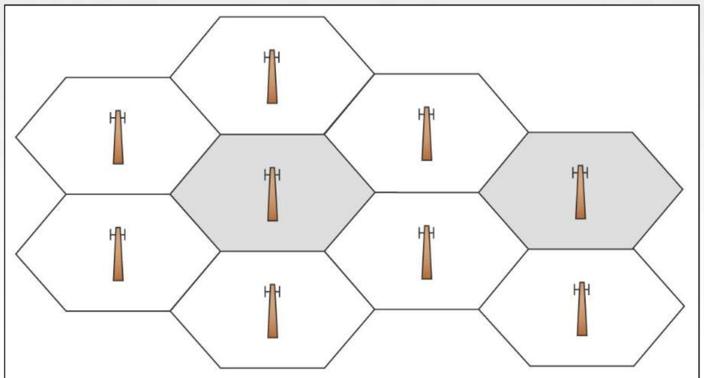
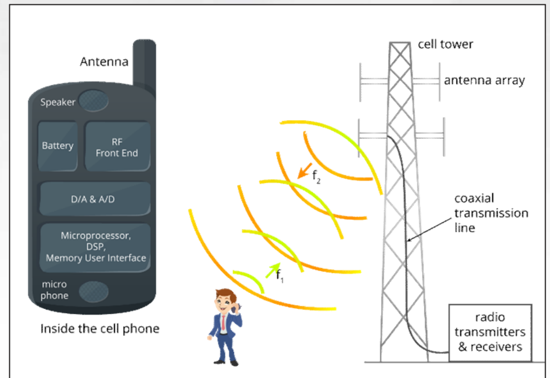
EM concepts in cellular communication system





- ❑ RF circuits are analysed and designed using EM concepts.
- ❑ Communication between towers may use wireless or optical communication.
- ❑ Noise and interference between electronic components.

iPhone 4 Antenna Problems

- ❑ Problems occurred when a hand or finger covered the antenna area on the left side of the iPhone.
- ❑ Interference between our bodies and the circuit of the antenna caused the iPhone 4 to lose signal strength.

Summary

- The application of engineering electromagnetics can be found in:
  - Transmission lines;
  - Wireless communication;
  - Optical communication;
  - High-speed electronic circuits;
  - Solution of electromagnetic interference; and
  - Life sciences.

- At higher frequencies, the circuit dimensions are comparable with wavelength. Therefore, simple circuit theory does not remain accurate.

Summary

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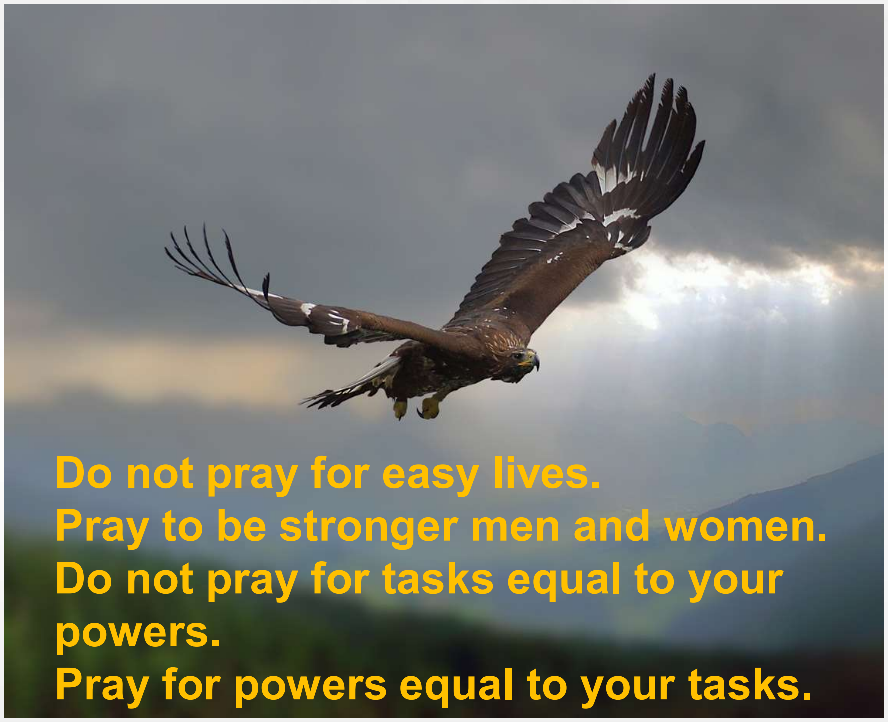
- By understanding the source of interference, i.e., the interfering electric and magnetic fields, we can reduce the electromagnetic interference.
- When one of the electromagnetic components is not functioning very well, it will affect the entire system.

Quote

All these applications require a better understanding of electromagnetic fields.



EE3001 Engineering Electromagnetics

Session 2-1

Interesting EM Applications

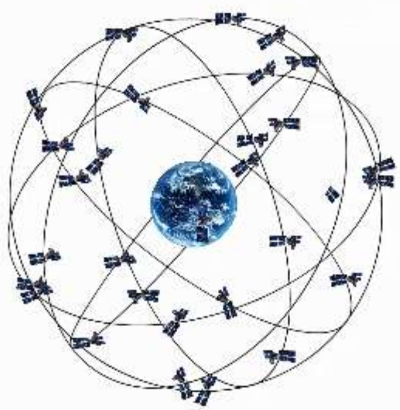
Learning Objectives

- ❑ Explain the principle of operation of the Global Positioning System;
- ❑ State the electromagnetic concepts that are involved in the Global Positioning System;
- ❑ Explain the idea of wireless power transmission; and
- ❑ State the key part played by electromagnetic concepts in wireless power charging.

Global Positioning System (GPS)

Space segment:

- 31 satellites, in 6 planes;
- Orbital altitude of ~20,000 km; and
- Each circles Earth every 12 hours;
- Each continuously transmits coded time signals.



Global Positioning System (GPS)

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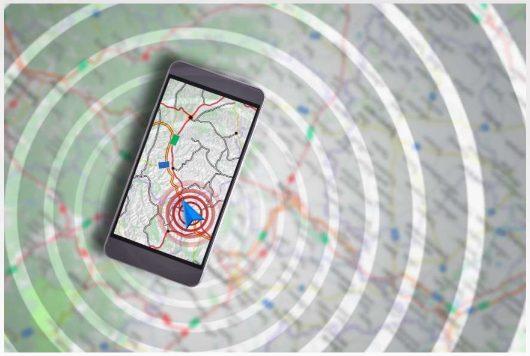
User segment:

- ❑ Hand-held or vehicle-mounted receivers receive and process multiple satellite signals; this allows them to determine their own location.

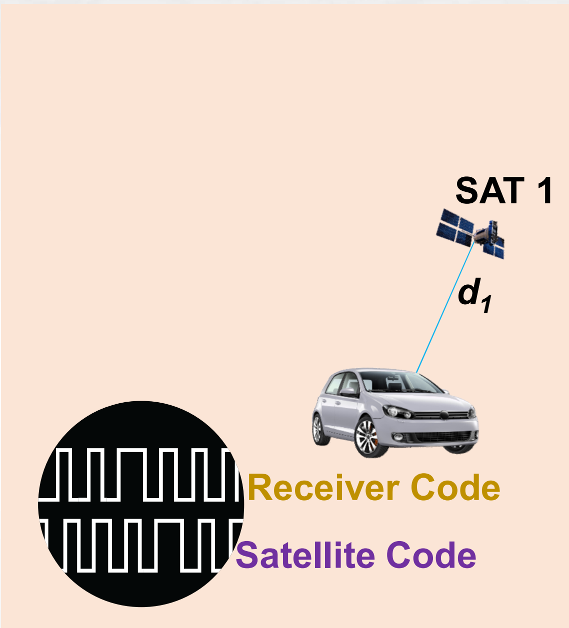


Ground network:

- ❑ Five ground stations, distributed around the world, that monitor the satellites and provide them with updates on their precise orbital location.

Global Positioning System (GPS)

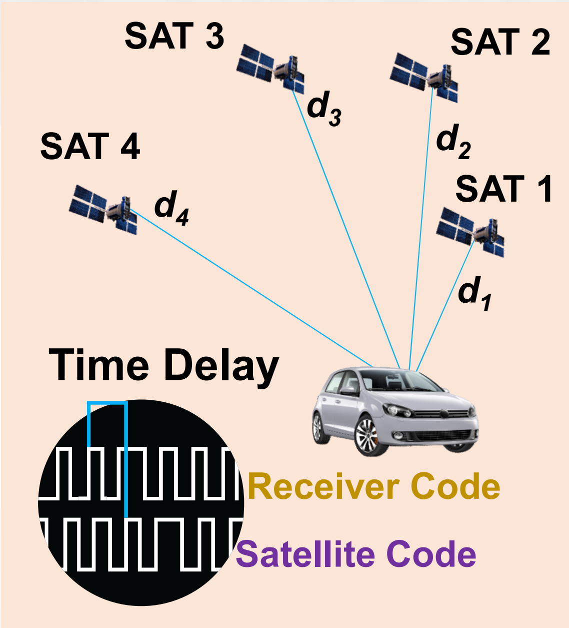
Principle of operation:



- Electromagnetic wave propagation through atmosphere (including ionosphere), buildings, cars, etc.;
- Antennas to transmit/receive signals from multiple satellites;
- Microwave cables, circuits and components to generate and decode signals.

Global Positioning System (GPS)

Principle of operation:



- The distances are established by measuring the time it takes the signals to travel from the satellites to the GPS receivers, and then multiplying them by the speed of light $c = 3 \times 10^8\ \mathrm{m/s}$.
- By using 'triangulation', the receiver can figure out its own location.

Electromagnetic concepts used in GPS:

- Properties of propagating electromagnetic waves; and
- Design of antennas and circuits.

Appendix

Wireless Power Transmission

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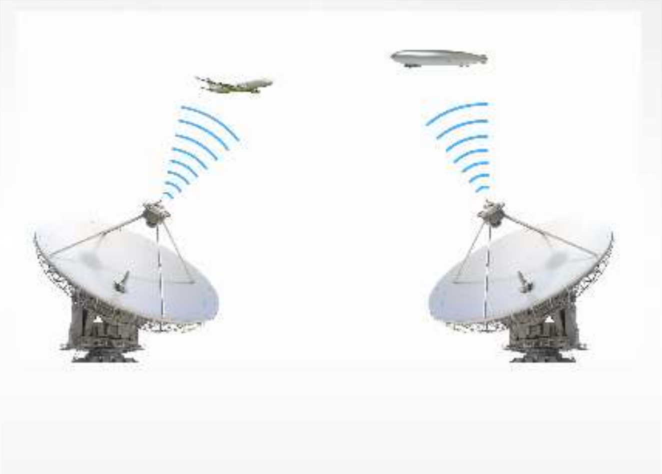
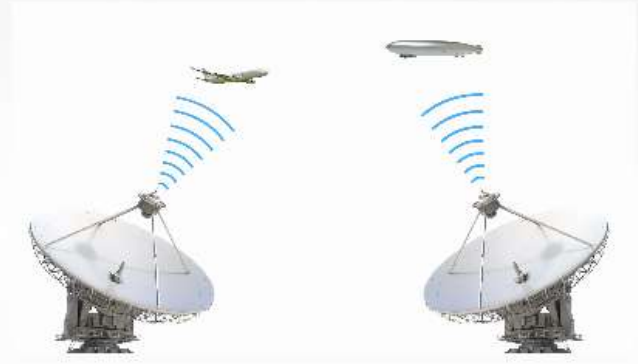
- Unmanned aircraft, airships and drones are used for:
  - Surveillance;
  - Broadcast; and
  - Relay.
- Microwave beamers on the ground;



- Rectennas on UAVs for microwave-to-dc conversion;

Wireless Power Transmission

- ❑ Wireless power transmission; and
- ❑ System considerations:
  - Frequency;
  - Antenna size;
  - Efficiency of conversion; and
  - Risk to life.



Wireless Power Charging

Devices that need charging:

computer

phone

camera

electric vehicles

Implantable devices:

health monitoring

delivery of medicine

pace-makers

Wireless Power Charging

Sensors that are used for:

structures

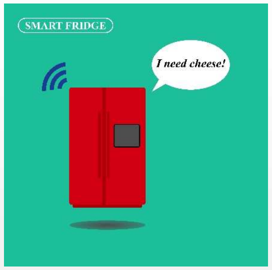
toxic environment

assisting elderly

smart homes

smart cities

internet-of-things



Wireless Power Charging

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Electric Vehicles:

- ❑ All electric vehicles:
  - Eliminate noxious fumes ☺
  - Must carry large batteries ☹
  - Limited range (100-200 km) ☹
  - Need periodical charging (full charge takes 6-8 hours) ☹

Wireless Power Charging

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Electric Vehicles:

- ❑ On-the-move charging!
  - On-line Electric Vehicle (OLEV) developed in Korea
  - Power transmission through electromagnetic induction

Appendix

Summary

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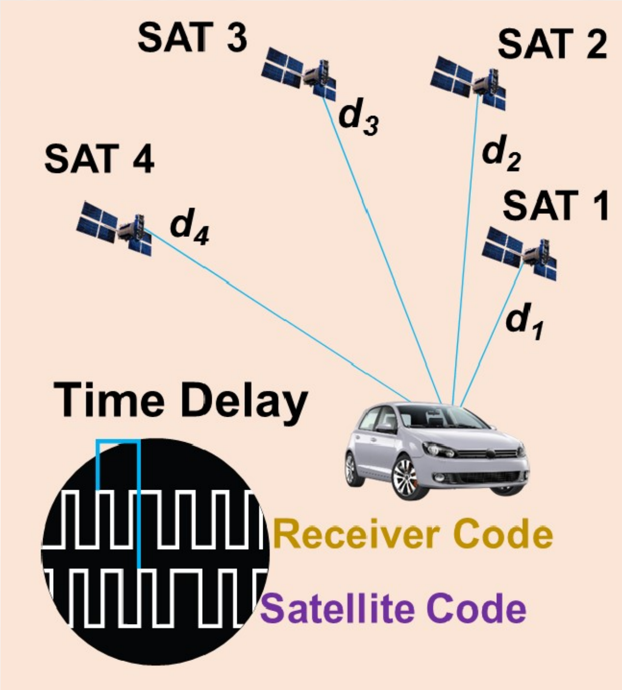
- The electromagnetic concept in GPS:
  - Properties of the propagating electromagnetic waves; and
  - Knowledge about antennas and RF circuits.

- Wireless power transmission:
  - Able to charge the batteries of the unmanned airborne vehicles by wireless power transmission;
  - Use of antennas to transmit power; and
  - Allow the unmanned airborne vehicles to stay afloat and work for a longer duration.

- Phenomenon of electromagnetic induction is involved in wireless power charging.

Appendix

Principle of operation:

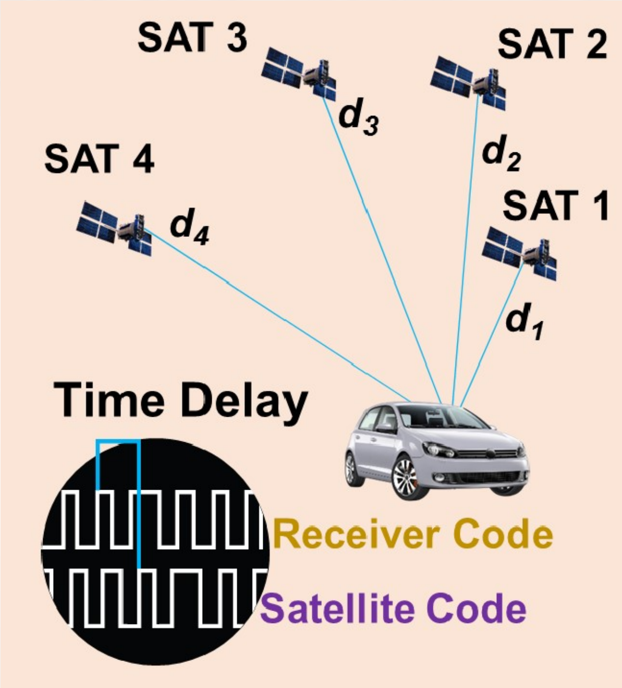


- Triangulation: Determination of the location (x_0, y_0, z_0) of any object in 3-D space, knowing the distances d_1, d_2 and d_3 between that object and three other independent points in space of known locations (x_1, y_1, z_1) to (x_3, y_3, z_3).

Next

Appendix

Principle of operation:

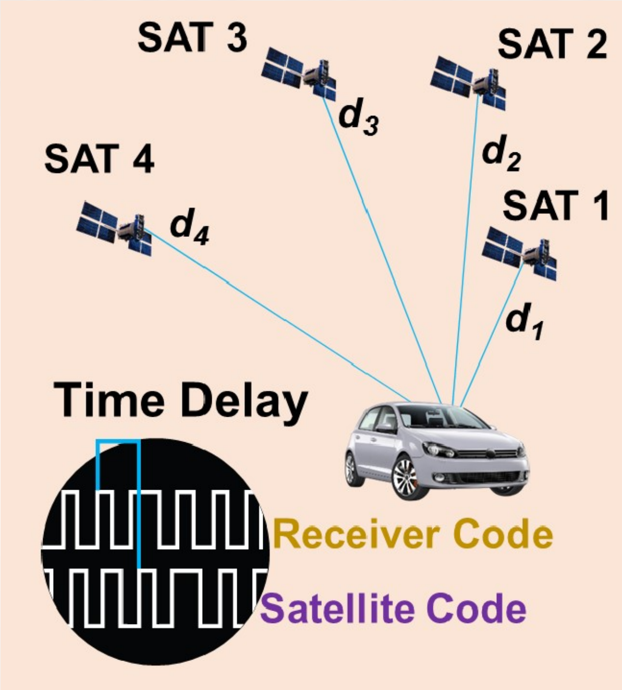


- The distances are established by measuring the time it takes the signals to travel from the satellites to the GPS receivers, and then multiplying them by the speed of light $c = 3 \times 10^8$ m/s.
- To correct for the time error of a GPS receiver, signal from a fourth satellite is needed.

Next

Appendix

Principle of operation:



- ❑ GPS provides a location inaccuracy of about 30 m, both horizontally and vertically.
- ❑ Accuracy can be improved to a few meters by using precision code, and to within 1 m by differential GPS.

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Appendix

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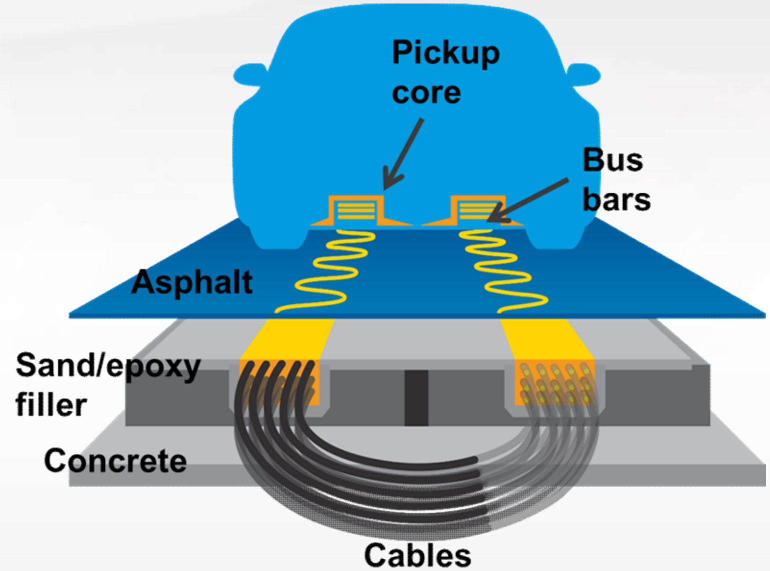
Wireless Power Charging of Electric Vehicles:

OLEV - Tram in a 2.2 km loop in Seoul Zoo:

- Transmitting coils embedded in part of road
- Receiving coils in cars
- Power transmission through electromagnetic induction between coils



Next

Appendix

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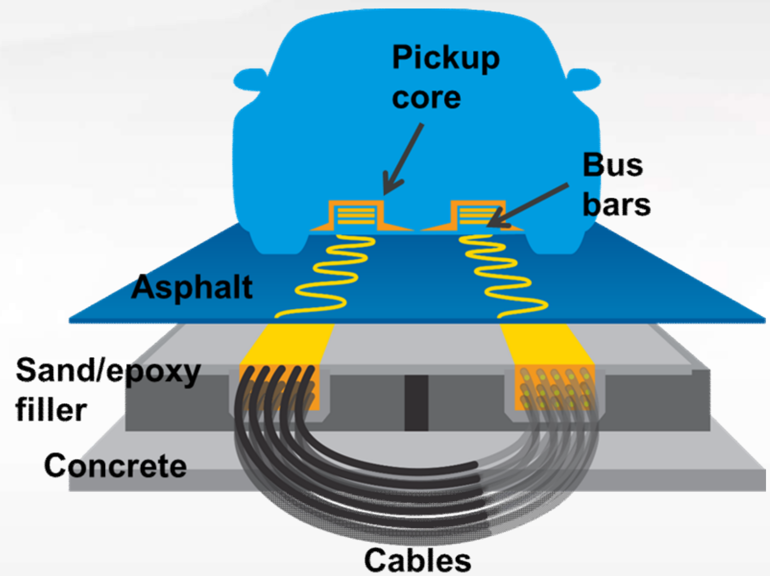
Wireless Power Charging of Electric Vehicles:

- ❑ Efficiency enhanced (~75%) by magnetic resonance coupling
- ❑ Transmitter's frequency tuned to the pickup circuit



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EE3001 Engineering Electromagnetics

Session 2-2

Basics of Vector Analysis - 1

Learning Objectives

- Define scalar and vector quantities;
- Carry out vector addition and vector subtraction;
- Define dot and cross product; and
- Explain unit vector.

Review of Vector Analysis

- This session (and lecture 6) review the basics of vector analysis.

- You are strongly encouraged to refer to:
  - The **Preparatory Notes** in NTULearn for a quick introduction;

  and
  - Chapter 1 of the first text book (Hayt & Buck)

  or
  - Part 1 of the second text book (Sadiku)

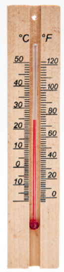
  or
  - Notes for EE2007 – Engineering Mathematics II – for detailed explanation and derivation.

Scalar Quantities

A scalar quantity may be represented by a single real number.

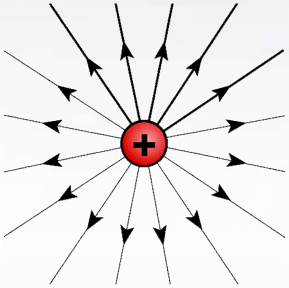
Examples of scalars:

temperature

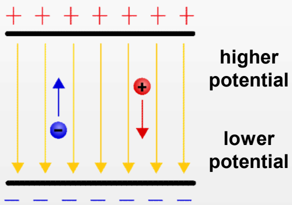


height

electric charge



electric potential

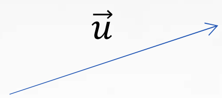


electromagnetic energy

Vector Quantities

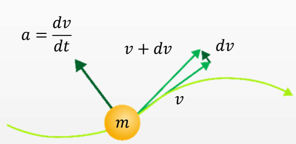
A vector quantity has both a magnitude and a direction in space.

A vector can be represented by a directed arrow segment. The length of the arrow describes the magnitude of the vector, and its direction represents the direction of the vector.

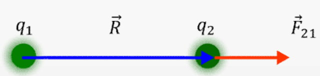


Examples of vectors:

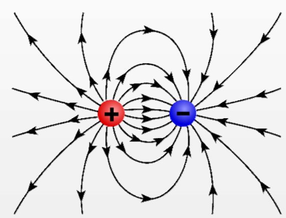
velocity



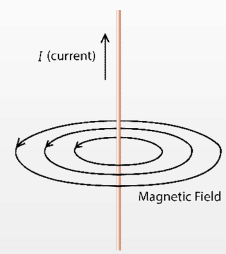
force



electric field

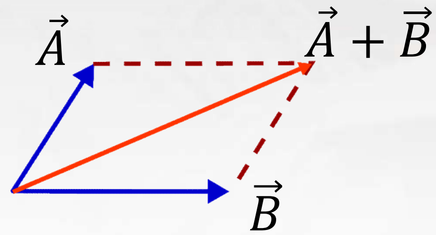


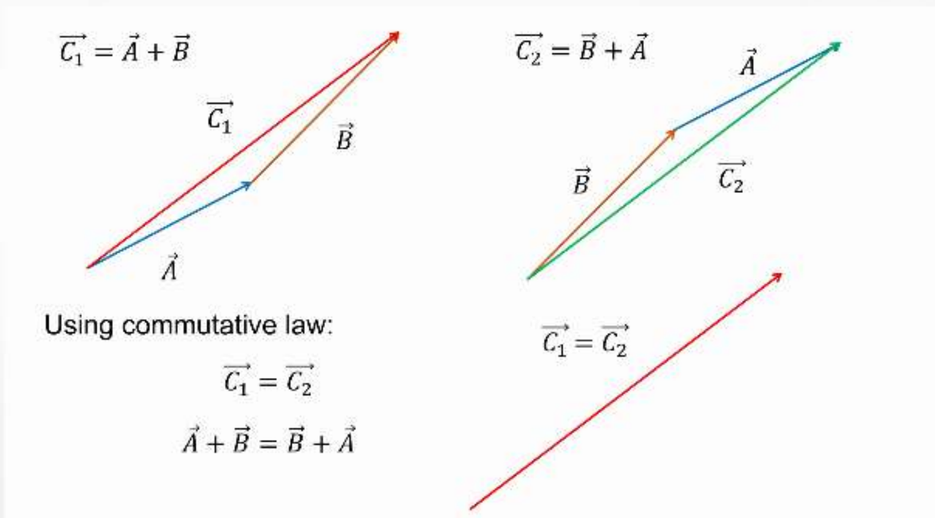
magnetic field



Vector Addition

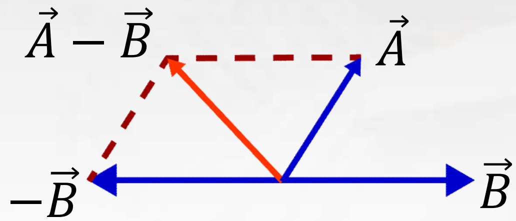
Vector addition follows the parallelogram rule.

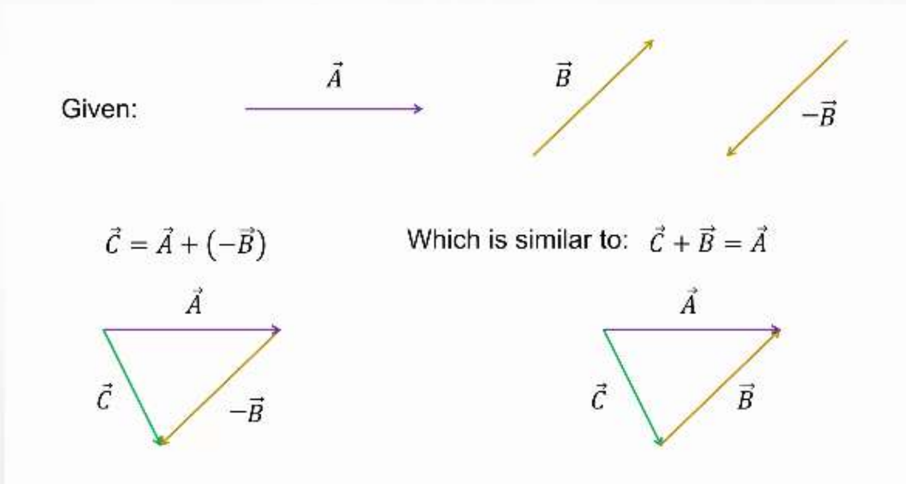




Vector Subtraction

Vector subtraction can be converted to addition.





Multiplication of Vectors

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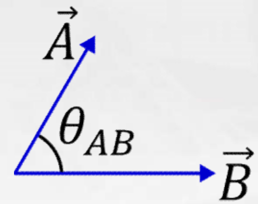
Definition of scalar or dot product:

$$\vec{A} \cdot \vec{B} = AB \cos \theta_{AB}$$

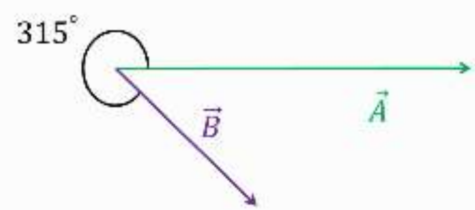


Note: If two vectors are perpendicular to each other, then their dot product must be zero.

Given: $|\vec{A}| = 2$ and $|\vec{B}| = 1$

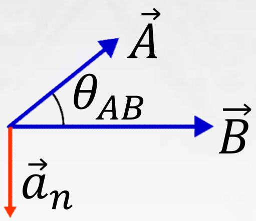
The dot product of the two vectors is:

$$\vec{A} \cdot \vec{B} = \mathrm{A}\,\mathrm{B} \cos \theta$$
$$= 2 \times 1 \times \cos 315^{\circ}$$
$$= 1.414$$



Cross Product

Definition of vector or cross product:

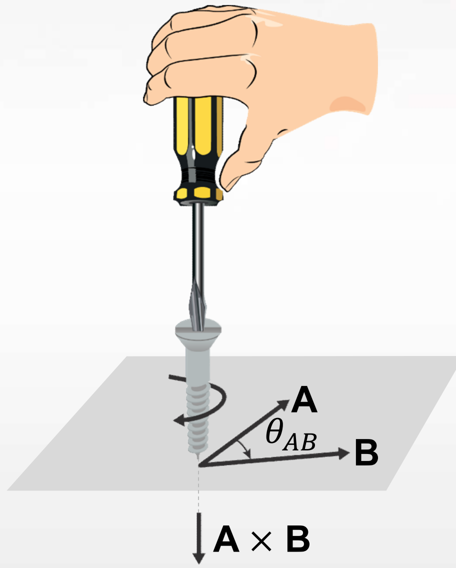


$$\vec{A} \times \vec{B} = \vec{a}_n AB \sin \theta_{AB}$$

$\vec{a}_n$

Right-hand rule:

The direction of $\vec{a}_n$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, and follows the thumb of the right hand when the fingers rotate from $\vec{A}$ to $\vec{B}$.



Cross Product

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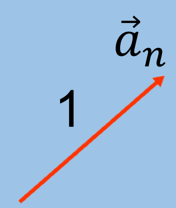
Unit vector, $\vec{a}_n$, has a magnitude of 1.

$$|\vec{a}_n| = 1$$



Unit vectors have only the direction information.

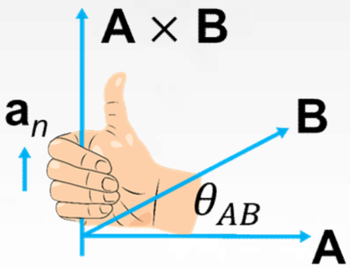
An **arbitrary vector** can be expressed as:

$$\vec{A} = \vec{a}_A A$$

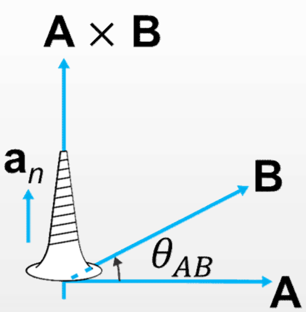
Cross Product

Direction of $\vec{A} \times \vec{B}$ and $\vec{a}_n$ using:

Right-hand rule



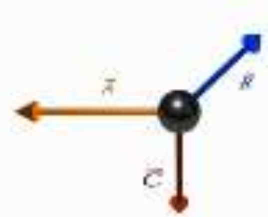
Right-handed-screw rule



Given: $|\vec{A}| = 1$ and $|\vec{B}| = 1$

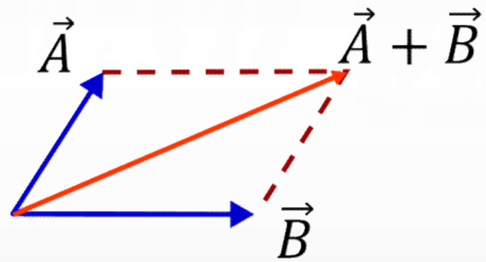
The cross product of the two vectors is:

$$\begin{aligned}\vec{C} &= \vec{A} \times \vec{B} \\ &= \vec{a}_n AB \sin\theta_{AB} \\ &= \vec{a}_n \times 1 \times 1 \times \sin 234^\circ \\ &= -0.809\vec{a}_n\end{aligned}$$



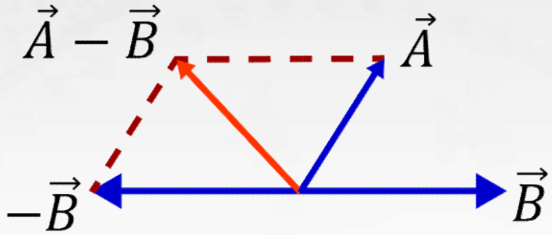
Summary

- A scalar quantity may be represented by a single real number.
- A vector quantity has both a magnitude and a direction in space.
- Vector addition follows the parallelogram rule and can be illustrated as:



Summary

- Vector subtraction can be converted to addition and can be illustrated as:



- Scalar or dot product is defined as:

$$\vec{A} \cdot \vec{B} = AB \cos\theta_{AB}$$

- Cross product is defined as:

$$\vec{A} \times \vec{B} = \vec{a}_n AB \sin\theta_{AB}$$

- A unit vector has a magnitude of one and the only information that it has is direction.

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EE3001 Engineering Electromagnetics

Session 2-3

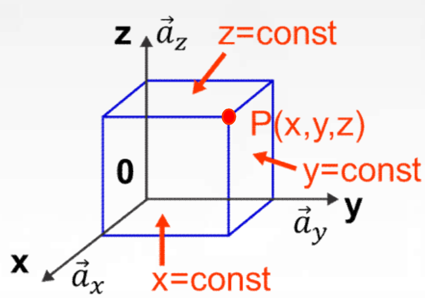
Basics of Vector Analysis - 2

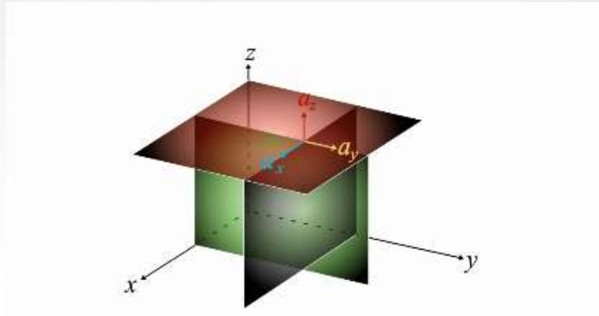
Learning Objectives

- State the coordinates and unit vectors used in Cartesian Coordinate System;
- Describe the components of an arbitrary vector in the Cartesian Coordinate System; and
- State the three differential elements of length and their combinations to get differential elements of area and the differential element of volume.

Cartesian Coordinate System

The purpose of a coordinate system is to locate a point or an object in the three-dimensional space uniquely.

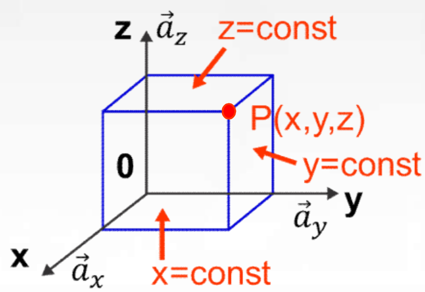




- Three coordinates: (*x, y, z*)
- Three unit vectors: $\vec{a}_x$ $\vec{a}_y$ $\vec{a}_z$
- Similar to: $\hat{i}$ $\hat{j}$ $\hat{k}$

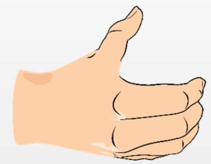
Cartesian Coordinate System

The use of a coordinate system is to locate a point or an object in the three-dimensional space uniquely.



Important: The arrangement of the coordinate axes *x*, *y* and *z* must satisfy the right-hand rule.

$$\vec{a}_x \times \vec{a}_y = \vec{a}_z$$



Components of a Vector

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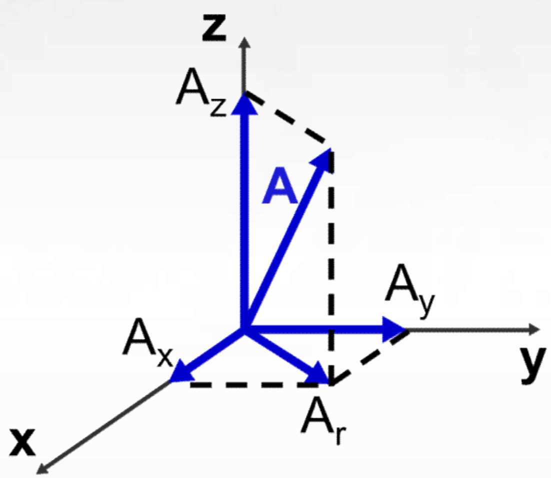
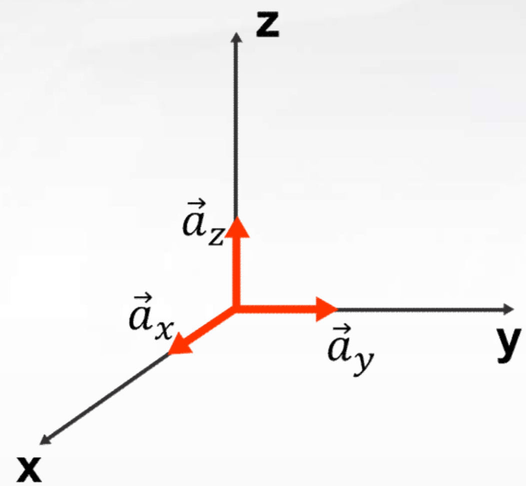
An arbitrary vector can be decomposed into three components in Cartesian coordinates.



$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$



Base vectors

Example 1

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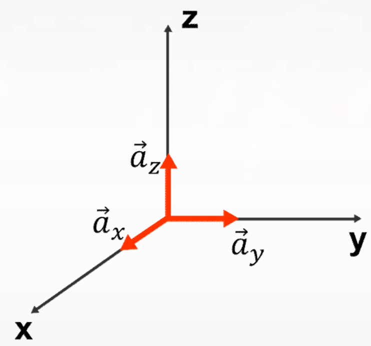
Assume two vectors:

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \qquad \vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$$

Dot Product

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}} \qquad \boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$



Example: $\vec{a}_x \cdot \vec{a}_y = 0$

Question: $\vec{a}_x \cdot \vec{a}_x = ?$

Example 2

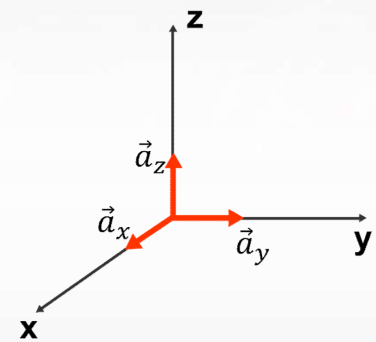
Assume two vectors:

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \qquad \vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$$

Cross Product

$$\boxed{\vec{A} \times \vec{B} = \vec{a}_n AB \sin\theta_{AB}}$$

$$\boxed{\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$



Example: $\vec{a}_x \times \vec{a}_y = \vec{a}_z$

Question: $\vec{a}_x \times \vec{a}_x = ?$

Quiz

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Hayt and Buck, Ch 1, Q1:

The cross product of $8\hat{a}_x + 6\hat{a}_y$ (first vector) and $9\hat{a}_x - 4\hat{a}_z$ (second vector) is

A: $72\hat{a}_x - 24\hat{a}_z$

B: $-24\hat{a}_x + 32\hat{a}_y - 54\hat{a}_z$

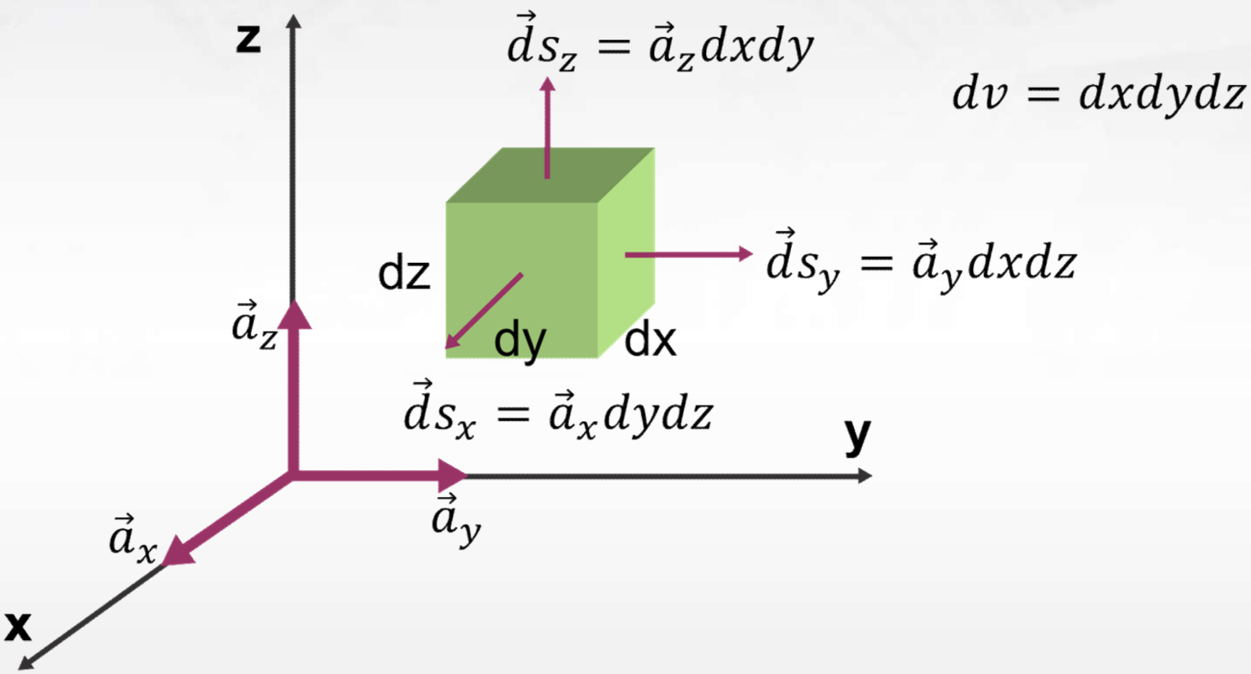
C: $72\hat{a}_x + 24\hat{a}_y + 16\hat{a}_z$

D: $24\hat{a}_x - 32\hat{a}_y + 54\hat{a}_z$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Coordinates: Differential Elements

Differential elements of length : dx, dy, dz



Summary

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- In the Cartesian coordinate system, the three coordinates are x, y and z. The respective unit vectors are $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$.

- An arbitrary vector in the Cartesian coordinate system has components A_x, A_y, and A_z. These components are along the x, y, and z directions respectively.

- The three differential elements of area are:
  - $ds_x = dydz$
  - $ds_y = dxdz$
  - $ds_z = dxdy$

EE3001 Engineering Electromagnetics

Session 3-1

Electric Charge Distributions - 1

Learning Objectives

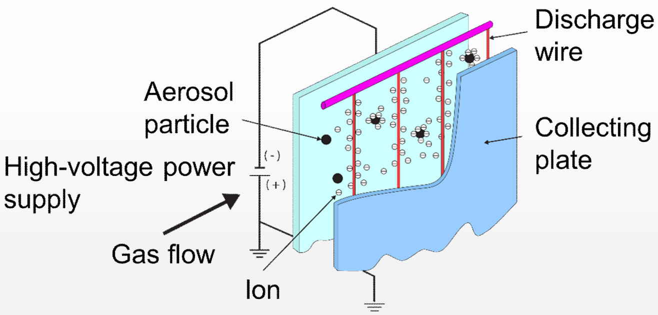
- State some applications that have their operation based on force between electric charges;
- Explain electric charge using the structure of atom;
- State the four types of basic electric charge distributions;
- Define line charge density and the total charge contained in a line charge distribution; and
- Define surface charge density and the total charge contained in a surface charge distribution.

Applications of Electric Force

Electrostatic precipitator:

- ❑ Found in thermal power plants where electricity is generated by burning coal;
  - In these plants huge stacks of smoke come out of the chimneys.

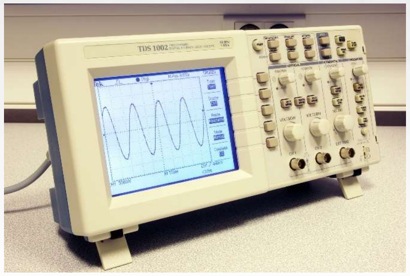
- ❑ Electrostatic precipitator is a filtration device that removes fine particles from the smoke by using the force of an induced electrostatic charge.



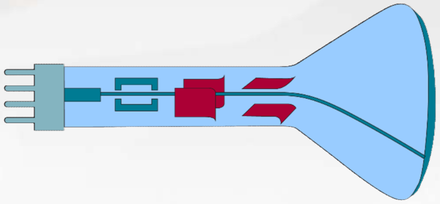
Applications of Electric Force

Photocopier

Oscilloscope



Cathode Ray Tube



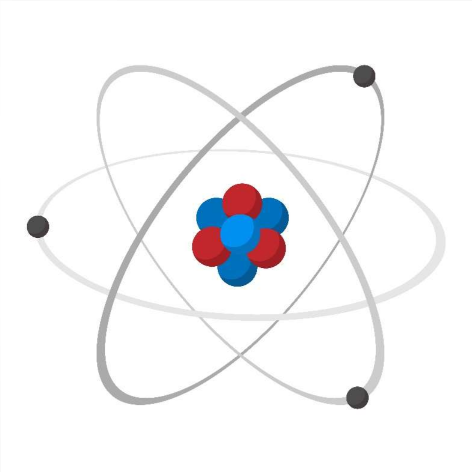
The operation of these devices is based on the force between electric charges.

How much is the charge on objects of different shapes?

Electric Charge

Electric charge, like mass, is one of the fundamental attributes of the particles of which matter is made.

- The structure of atoms that comprise matter can be described in terms of three types of particles:
  - positively charged proton;
  - uncharged neutron; and
  - negatively charged electron.

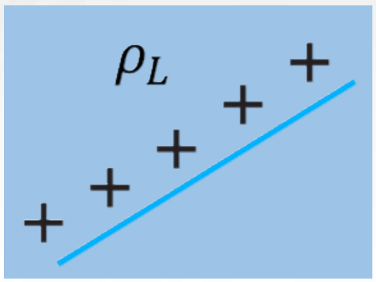


- There are two kinds of charges:
  - positive (+); and
  - negative (-).

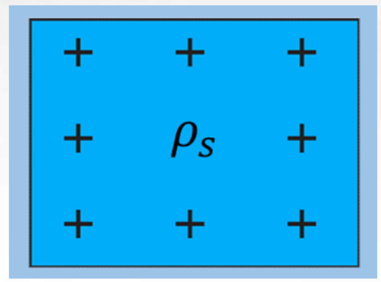
Different Types of Charge Distributions

Basic shapes of objects that can be charged:

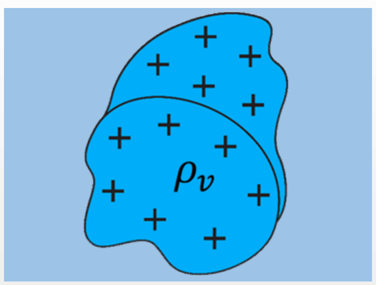
Line charge



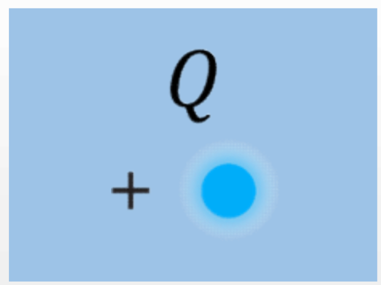
Surface charge



Volume charge



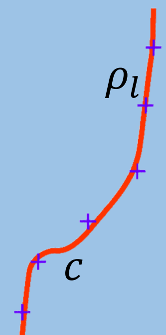
Point charge



Different Types of Charge Distributions

Basic sha

Line charge



- Line charge density ρ_l (C/m).
- The line charge density is defined as:

$$\rho_l = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L}$$

- The total charge contained in the entire contour C is:

$$\boxed{Q = \int_C \rho_l \, dl}$$

Different Types of Charge Distributions

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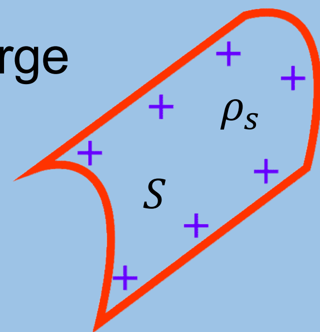
Basic sha

X

Surface charge



- Surface charge density ρ_s (C/m^2).
- The surface charge density is defined as:

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S}$$

- The total charge contained on the entire surface S is:

$$\boxed{Q = \iint_S \rho_s \, ds}$$

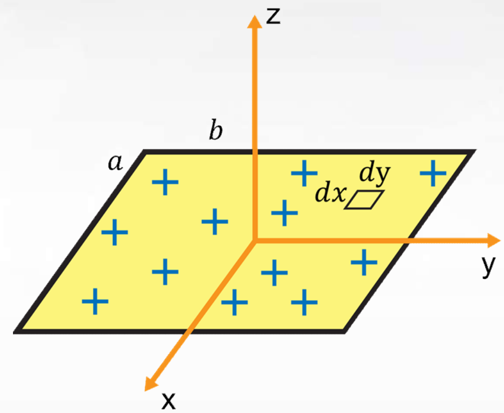
Example

Different Types of Charge Distributions

Surface Charge Distribution Example:

Given a surface charge distribution over a rectangular plate ($-\frac{a}{2} \le x \le \frac{a}{2}$, $-\frac{b}{2} \le y \le \frac{b}{2}$, and $z = 0$) as follows:

$$\rho_s = \rho_0 \left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)$$



Determine the total charge on the entire rectangular plate.

Solution

Different Types of Charge Distributions

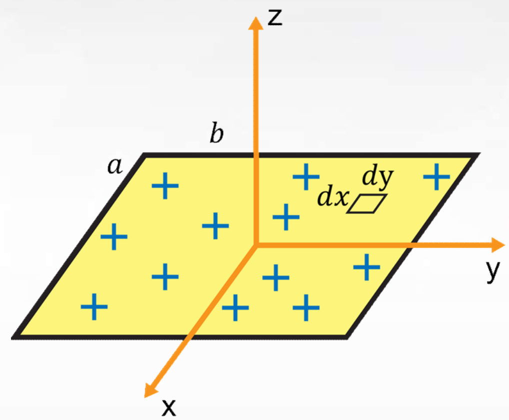
Surface Charge Distribution Solution:

$$Q_{total} = \iint_S \rho_S \, ds = \int_{x=-\frac{a}{2}}^{\frac{a}{2}} \int_{y=-\frac{b}{2}}^{\frac{b}{2}} \rho_s \, dydx$$

$$= \rho_0 \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(1 - \frac{x}{a}\right) dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \left(1 - \frac{y}{b}\right) dy$$

$$= \rho_0 \left(x - \frac{x^2}{2a}\right)\Bigg|_{-\frac{a}{2}}^{\frac{a}{2}} \left(y - \frac{y^2}{2b}\right)\Bigg|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \rho_0 ab \text{ (C)}$$



$$\rho_s = \rho_0 \left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)$$

Back

Summary

- ❑ Electrostatic precipitator, photocopier, oscilloscope and cathode ray tube etc. have operation that is based on force between electric charges.

- ❑ To describe electric charge using the structure of atoms:
  - If there is an excess of electrons on a body, it becomes negatively charged; and
  - If there is a deficiency of electrons, it becomes positively charged.

Summary

- The basic types of charge distributions are:
  - Line charge;
  - Surface charge;
  - Volume charge; and
  - Point charge.
- Line charge density is defined as $\rho_l = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L}$. The total charge contained in the entire contour is defined as $Q = \int_C \rho_l \, dl$.
- Surface charge density is defined as $\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S}$. The total charge contained on the entire surface is defined as $Q = \iint_S \rho_s \, ds$.

EE3001 Engineering Electromagnetics

Session 3-2

Electric Charge Distributions - 2

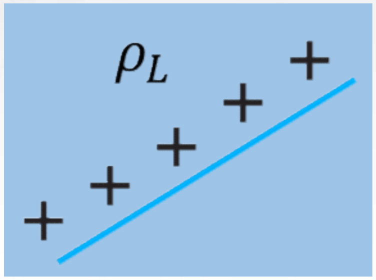
Learning Objectives

- ❑ Define volume charge density and the total charge contained on the entire volume;
- ❑ Explain the concept of point charge; and
- ❑ Solve problems involving volume charge distributions.

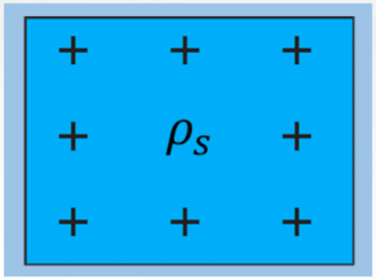
Different Types of Charge Distributions

Basic shapes of objects that can be charged:

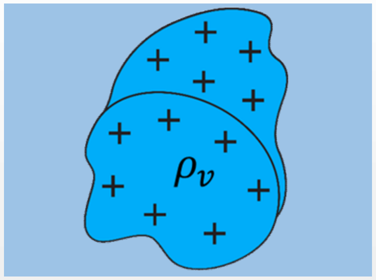
Line charge



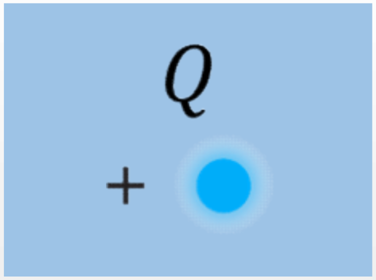
Surface charge



Volume charge



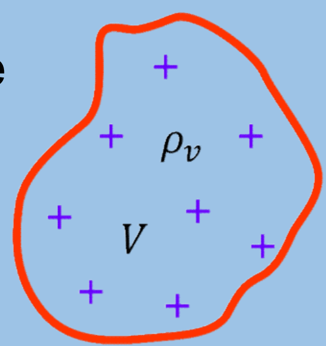
Point charge



Different Types of Charge Distributions

Basic sha

Volume charge



X

- Volume charge density ρ_v (C/m^3).
- The volume charge density is defined as:

$$\rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}$$

- The total charge contained in the entire volume V is:

$$\boxed{Q = \iiint_V \rho_v \, dv}$$

Quiz

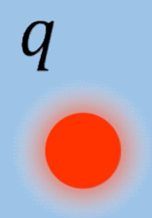
Example

Different Types of Charge Distributions

Basic sha

X

A point charge is an idealised concept:



- ❑ It has no volume;
- ❑ It is located at a specific point; and
- ❑ It maintains a certain amount of charge.

Quiz

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Hayt and Buck, Ch 2, Q4:

Charge Q is uniformly distributed in a sphere of radius a_1. How is the volume charge density going to change if this same charge occupies a sphere of radius $a_2 = a_1/4$?

$$\rho_v = \frac{Q}{V}$$

A: It will increase 4 times.

B: It will increase 64 times.

C: It will increase 16 times.

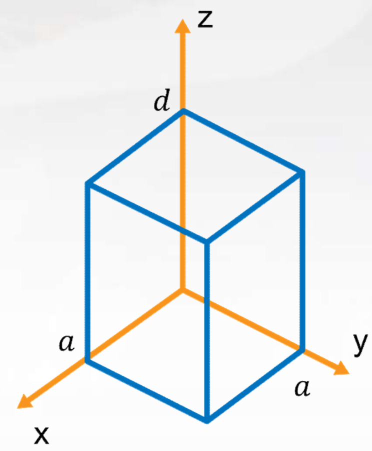
D: It will decrease 2 times.

Back

Volume Charge Distribution Example

Given a volume charge distribution over a square cylinder ($0 \leq x \leq a$, $0 \leq y \leq a$, and $0 \leq z \leq d$) as follows:

$$\rho_v = \left[1 - \left(\frac{x}{a}\right)^2\right]\left[1 - \left(\frac{y}{a}\right)^2\right]\left(1 - \frac{z}{d}\right)$$



Determine the total charge contained in the entire square cylinder.

Solution

Volume Charge Distribution: Solution

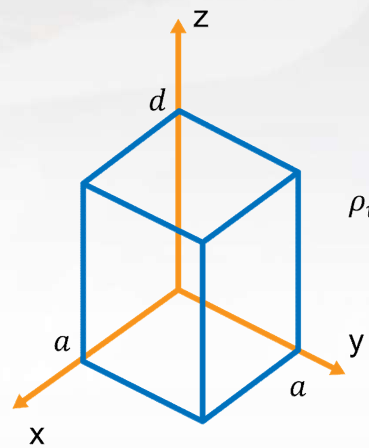
Volume Charge Distribution Solution:

$$Q_{total} = \iiint_V \rho_v \, dv = \int_0^d \int_0^a \int_0^a \rho_v \, dxdydz$$

$$= \int_0^d \int_0^a \int_0^a \left[1 - \left(\frac{x}{a}\right)^2\right]\left[1 - \left(\frac{y}{a}\right)^2\right]\left(1 - \frac{z}{d}\right) dxdydz$$

$$= \int_0^a \left[1 - \left(\frac{x}{a}\right)^2\right] dx \int_0^a \left[1 - \left(\frac{y}{a}\right)^2\right] dy \int_0^d \left(1 - \frac{z}{d}\right) dz$$

$$= \left(a - \frac{a^3}{3a^2}\right)\left(a - \frac{a^3}{3a^2}\right)\left(d - \frac{d^2}{2d}\right) = \frac{2a^2 d}{9} \text{ (C)}$$



$$\rho_v = \left[1 - \left(\frac{x}{a}\right)^2\right]\left[1 - \left(\frac{y}{a}\right)^2\right]\left(1 - \frac{z}{d}\right)$$

Back

Summary

- Volume charge density is defined as $\rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}$. The total charge contained in the entire volume is defined as $Q = \iiint_V \rho_v \, dv$.

- A point charge has no volume. It is located at a specific point and carries a certain amount of charge.

- The expression used to calculate total charge for a volume charge distribution is:
  - $Q_{total} = \iiint_V \rho_v \, dv$

EE3001 Engineering Electromagnetics

Session 3-3

Coulomb's Law

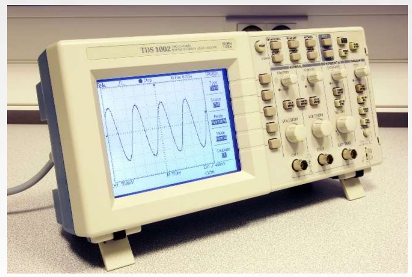
Learning Objectives

- Determine the force between two stationary point charges using Coulomb's Law;
- Express the force between two stationary point charges as a vector;
- Explain the relationship between vectors $\vec{R}$ and $\vec{a}_R$; and
- Relate the force acting on a charge q_2 due to charge q_1 and the force acting on q_1 due to q_2.

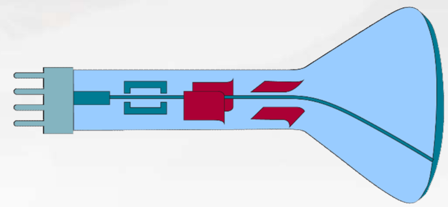
Applications of Electric Force

Photocopier

Oscilloscope



Cathode Ray Tube



The photocopier, oscilloscope and cathode ray tube operate based on the forces between electric charges.

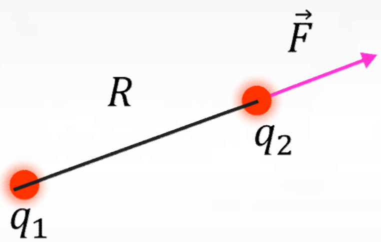
How much is the charge on objects of different shapes?

How much is the force between charged objects of different shapes?

Coulomb's Law

Coulomb's law was experimentally deduced by a French engineer, Charles Coulomb, in 1785.
http://en.wikipedia.org/wiki/Charles-Augustin_de_Coulomb

- ❑ The force between two stationary point charges, q_1 and q_2:



$$F = k\frac{q_1 q_2}{R^2}$$

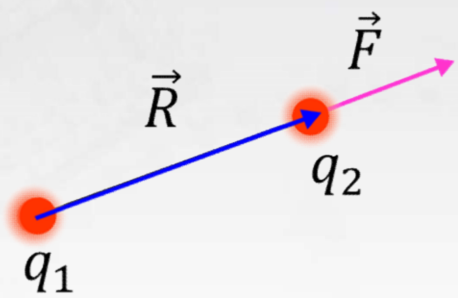
where,

- The constant of proportionality, k, is:

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ (m/F)$$

- Permittivity of free space: $\varepsilon_0 = \dfrac{1}{36 \ \times 10^9} \ (F/m)$

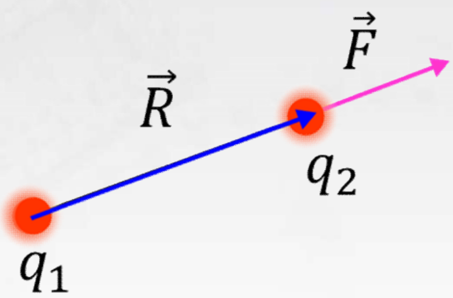
Coulomb's Law: Vector Expression



$$F = k\frac{q_1 q_2}{R^2}$$

- $\vec{F}$ is along the line joining q_1 and q_2:
  - It is **repulsive** if q_1 and q_2 are of **same sign**; and
  - It is **attractive** if q_1 and q_2 are of **opposite sign**.

Coulomb's Law: Vector Expression



$$F = k\frac{q_1 q_2}{R^2}$$

- Coulomb's law can be put in the following vector form:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\vec{a}_R = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{R}}{R^3}$$

$\vec{R}$ and $\vec{a}_R$

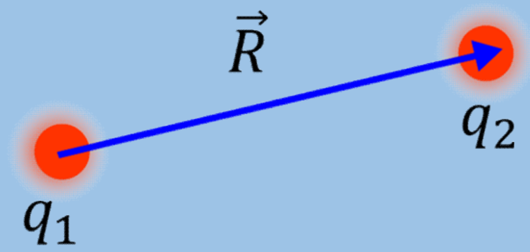
where,

- $\vec{F}$ is the force acting on charge q_2 due to charge q_1.
- $\vec{a}_R$ is the unit vector from point charge q_1 to point charge q_2.

$\vec{F}_{12}$

Coulomb's Law: Vector Expression

Vector $\vec{R}$ extends from q_1 to q_2.



The relationship between $\vec{R}$ and $\vec{a}_R$:

- ❑ $\vec{R} = \vec{a}_R R$
- ❑ $\vec{a}_R = \dfrac{\vec{R}}{R}$

- ❑ $\vec{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{R^2}\vec{a}_R = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2 \vec{R}}{R^3}$

❑ Coulo

$\vec{F} =$

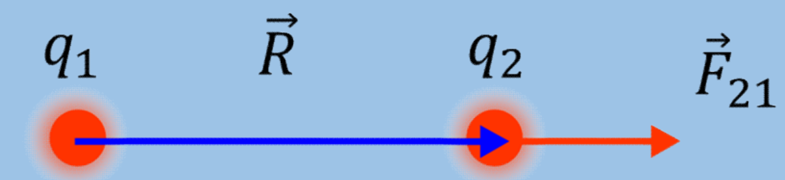
where,

- $\vec{F}$ is
  to c
- $\vec{R}$ is
  poi

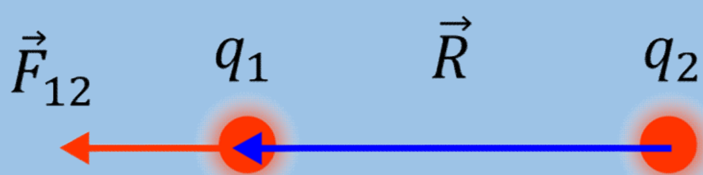
Coulomb’s Law: Vector Expression

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

❑ Force acting on q_2 due to q_1:



❑ Force acting on q_1 due to q_2:



❑ Coulo

$\vec{F} =$

where,

- $\vec{F}$ is
  to c
- $\vec{R}$ is
  poi

Quiz

Hayt and Buck, Ch 2, Q1:

Q_1 and Q_2 are two point charges, which are a distance 8 cm apart. The force acting on Q_2 is given by $F_{21} = \hat{a}_y 9 \times 10^{-12}$ N.

Now we replace Q_2 with a charge of the same magnitude but opposite polarity $Q_3 = -Q_2$, and we move Q_3 a distance 24 cm away from Q_1. What is the force vector F_{31} acting on Q_3?

A: $F_{31} = \hat{a}_y 3 \times 10^{-12}$ N

C: $F_{31} = -\hat{a}_y 3 \times 10^{-1}$ N

B: $F_{31} = -\hat{a}_y 1 \times 10^{-12}$ N

D: $F_{31} = \hat{a}_y 1 \times 10^{-12}$ N

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\vec{a}_R = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{R}}{R^3}$$

Example of Coulomb's Law

Interactive Display

- Hayt and Buck, Ch.2, Interactive 1 (access in NTULearn, 'Self and Peer Assessment')
- The forces of attraction/repulsion between two point charges Q_1 and Q_2.

Demonstration of Coulomb's Law

https://www.youtube.com/watch?v=B5LVoU_a08c

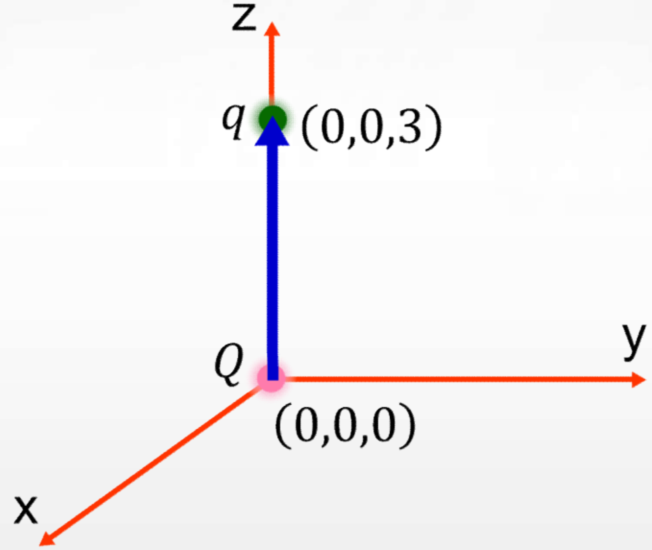
Uploaded by uclaphysicsvideo

Example of Coulomb's Law

Example 1

A charge q is located at $(0,0,3)m$ and has a charge of $10\ \mu\text{C}$. Another charge Q is located at $(0,0,0)$ and has a charge of $-300\ \mu\text{C}$. Determine the force exerted on q due to Q.

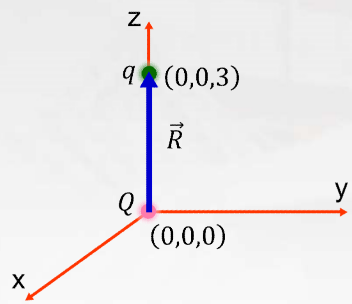
Force:
Attractive or repulsive?



Example of Coulomb's Law

Solution to Example 1

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\vec{a}_R = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{R}}{R^3}$$



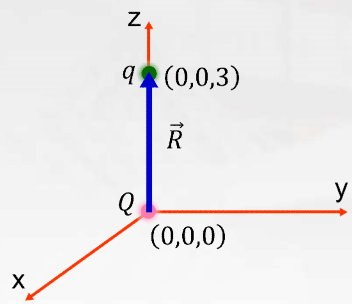
The distance vector $\vec{R}$ from point charge Q to point charge q is:

$$\vec{R} = (0-0)\vec{a}_x + (0-0)\vec{a}_y + (3-0)\vec{a}_z = 3\vec{a}_z$$

Example of Coulomb's Law

Solution to Example 1

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\vec{a}_R = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{R}}{R^3}$$



Using Coulomb's law, we obtain:

$$\vec{F} = 9 \times 10^9 \times \frac{-300 \times 10^{-6} \times 10 \times 10^{-6} \times (3\vec{a}_z)}{3^3}$$

$$= -3\vec{a}_z \text{ (N)}$$

It is seen that the force is attractive due to the opposite signs of the charges Q and q.

Appendix

Summary

- The force between two stationary point charges, q_1 and q_2, can be defined as $F = k\frac{q_1 q_2}{R^2}$.
  - The constant of proportionality, k, is $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ (m/F)$; and
  - Permittivity of free space is defined as $\varepsilon_0 = \frac{1}{36\pi\times10^9}\ (F/m)$.

- Coulomb's law in the vector form is stated as

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\vec{a}_R = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \vec{R}}{R^3}.$$

Summary

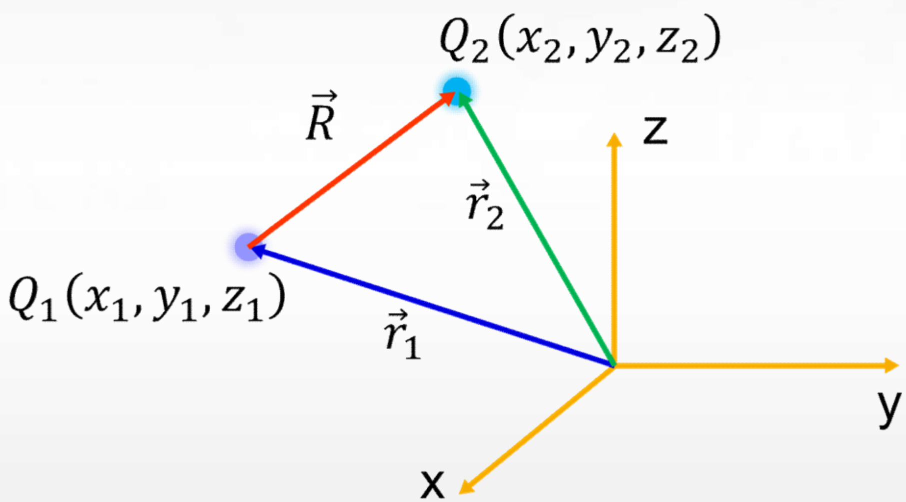
- The relationship between $\vec{R}$ and $\vec{a}_R$:
  - $\vec{R} = \vec{a}_R R$
  - $\vec{a}_R = \frac{\vec{R}}{R}$

- If the force acting on q_2 due to q_1 is $\vec{F}_{21}$, then the force acting on q_1 due to q_2 is equal and opposite to $\vec{F}_{21}$.

Appendix

Coulomb's Law: General Case

Q_1 is located at (x_1, y_1, z_1) and Q_2 is located at (x_2, y_2, z_2).
Determine the force on Q_2 due to Q_1.



Next

Appendix

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Solution to Coulomb's Law: General Case

Using Coulomb's law, we obtain:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2 \vec{R}}{R^3}$$

$$\vec{r}_1 = x_1\vec{a}_x + y_1\vec{a}_y + z_1\vec{a}_z$$

$$\vec{r}_2 = x_2\vec{a}_x + y_2\vec{a}_y + z_2\vec{a}_z$$

$$\vec{R} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

$$R = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Back