EE2003 Semiconductor Fundamentals

Ideal Diode Equation

Review of Basic Concepts

- By now, you should be able to do the following:
  - Explain why a p-n junction conducts only under forward bias and not under reverse bias.
  - Draw the energy band diagrams of a pn junction under forward and reverse biases.
  - How should the various formulae derived in section 2 be modified under an applied bias.
  - Relate the ‘excess’ minority carrier concentrations at the respective depletion edges to the applied voltage.
  - Explain the meaning of low-level injection.

Further Objective

- To analyze the p-n junction under an externally applied voltage bias **quantitatively**
  - To obtain a mathematical expression that relates the current flow to the applied bias.
- The problem at hand is one that concerns the transport (diffusion) of ‘excess’ carriers and could be analyzed by solving the continuity equations for electrons and holes.

Assumptions

- The n and p regions are uniformly doped all the way to the metallurgical junction, i.e. the p-n junction is abrupt.
- The lengths of the n an p regions are infinitely long compared to the minority carrier diffusion lengths.
- The applied voltage is sufficiently low such that low-level injection is valid.
- There is negligible or no voltage drop across the quasi-neutral n and p regions (i.e. no electric field in these regions).
- Thermal equilibrium prevails at the end contacts.

Assumptions

- There are no other external sources of excitation except the applied voltage.
- The excess carriers do not recombine in the space charge region (forward bias).
- No generation of carriers takes place in the space charge region (reverse bias).
- The p-n junction has reached steady state.

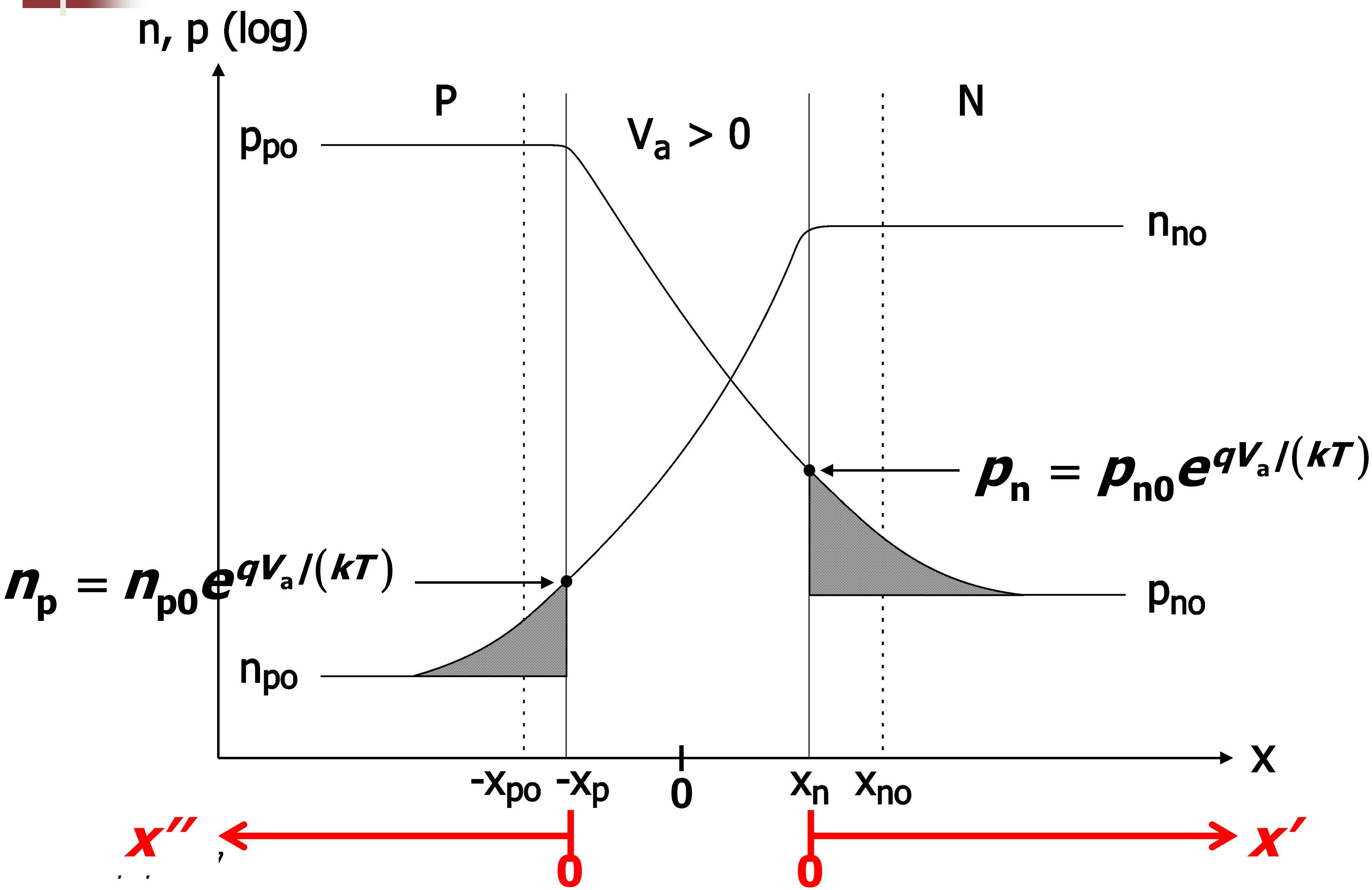
'Excess' Hole Distribution

- The steady-state 'excess' hole distribution in the quasi-neutral n region can be obtained by solving the continuity equation:

$$\frac{\partial \boldsymbol{p}_{\mathbf{n}}}{\partial \boldsymbol{t}} = -\mu_{\mathbf{p}}\left(\xi \frac{\partial \boldsymbol{p}_{\mathbf{n}}}{\partial \boldsymbol{x}'} + \boldsymbol{p}_{\mathbf{n}} \frac{\partial \xi}{\partial \boldsymbol{x}'}\right) + \boldsymbol{D}_{\mathbf{p}} \frac{\partial^2 \boldsymbol{p}_{\mathbf{n}}}{\partial \boldsymbol{x}'^2} + \boldsymbol{G}_{\mathbf{p}} - \frac{\Delta \boldsymbol{p}_{\mathbf{n}}}{\tau_{\mathbf{p}}}$$

- In order to solve this equation analytically, we need to reduce it to a simpler form using some of the assumptions made earlier.

Carrier Distributions – Forward Bias



'Excess' Hole Distribution

- Steady-state operation ⇒ $\partial p_n / \partial t = 0$
- No other external sources of excitation ⇒ $G_p = 0$
- No electric field in quasi-neutral region ⇒ $\xi = 0$
- Uniform doping ⇒ $\partial^2 p_n / \partial x' = \partial^2 \Delta p_n / \partial x'$
- Simplified continuity equation:

$$D_p \frac{\partial^2 \Delta p_n}{\partial x'^2} - \frac{\Delta p_n}{\tau_p} = 0$$

$$\frac{\partial^2 \Delta p_n}{\partial x'^2} = \frac{\Delta p_n}{D_p \tau_p} = \frac{\Delta p_n}{L_p^2}$$

‘Excess’ Hole Distribution

- The general solution is of the form:

$$\Delta p_n(x') = C_1 e^{-x'/L_p} + C_2 e^{x'/L_p}$$

- C_1 and C_2 are integration constants that can be easily determined by applying the appropriate boundary conditions at $x' = 0$ (depletion edge) and $x' \rightarrow \infty$ (end contact).

'Excess' Hole Distribution

- **Boundary condition 1:** Thermal equilibrium prevails at the end contact.

$$\Delta p_n(x' \to \infty) = 0$$

- Note that $\exp(-x'/L_p) \to 0$ as $x' \to \infty$.
- However, $\exp(x'/L_p) \to \infty$ as $x' \to \infty$.
- Since thermal equilibrium prevails at the end contact, **constant C_2 must be zero** in order for Δp_n to be zero at the end contact.

‘Excess’ Hole Distribution

- **Boundary condition 2:**

$$\Delta p_n(x'=0) = p_n - p_{no}$$
$$= p_{no}\left(e^{qV_a/(kT)} - 1\right)$$

- Therefore,

$$C_1 = p_{no}\left(e^{qV_a/(kT)} - 1\right)$$

‘Excess’ Hole Distribution

$$\Delta p_n(x') = p_{no}\left(e^{qV_a/(kT)} - 1\right) \cdot e^{-x'/L_p}$$

- In a long-base diode, the ‘excess’ holes in the quasi-neutral n region decays exponentially with distance.
- The constant $L_p = (D_p \tau_p)^{1/2}$ has a unit of distance and is commonly referred to as the **minority carrier (hole) diffusion length**.

‘Excess’ Electron Distribution

- Through a similar approach, the ‘excess’ electron distribution in the quasi-neutral p region can be obtained:

$$\Delta n_p\left(x''\right) = n_{po}\left(e^{qV_a/(kT)} - 1\right) \cdot e^{-x''/L_n}$$

Current Components – Forward Bias

- When a p-n junction is forward biased, electrons (holes) are injected from the n (p) to the p (n) region.

- Owing to a difference in the minority carrier concentration at the depletion edge and the end contact, the excess holes and electrons diffuse towards the end contacts after they have been injected into the respective n and p regions.

- The diffusion process gives rise to the minority carrier current components, as shown by the solid lines.

Current Components – Forward Bias

- Minority-carrier current components:

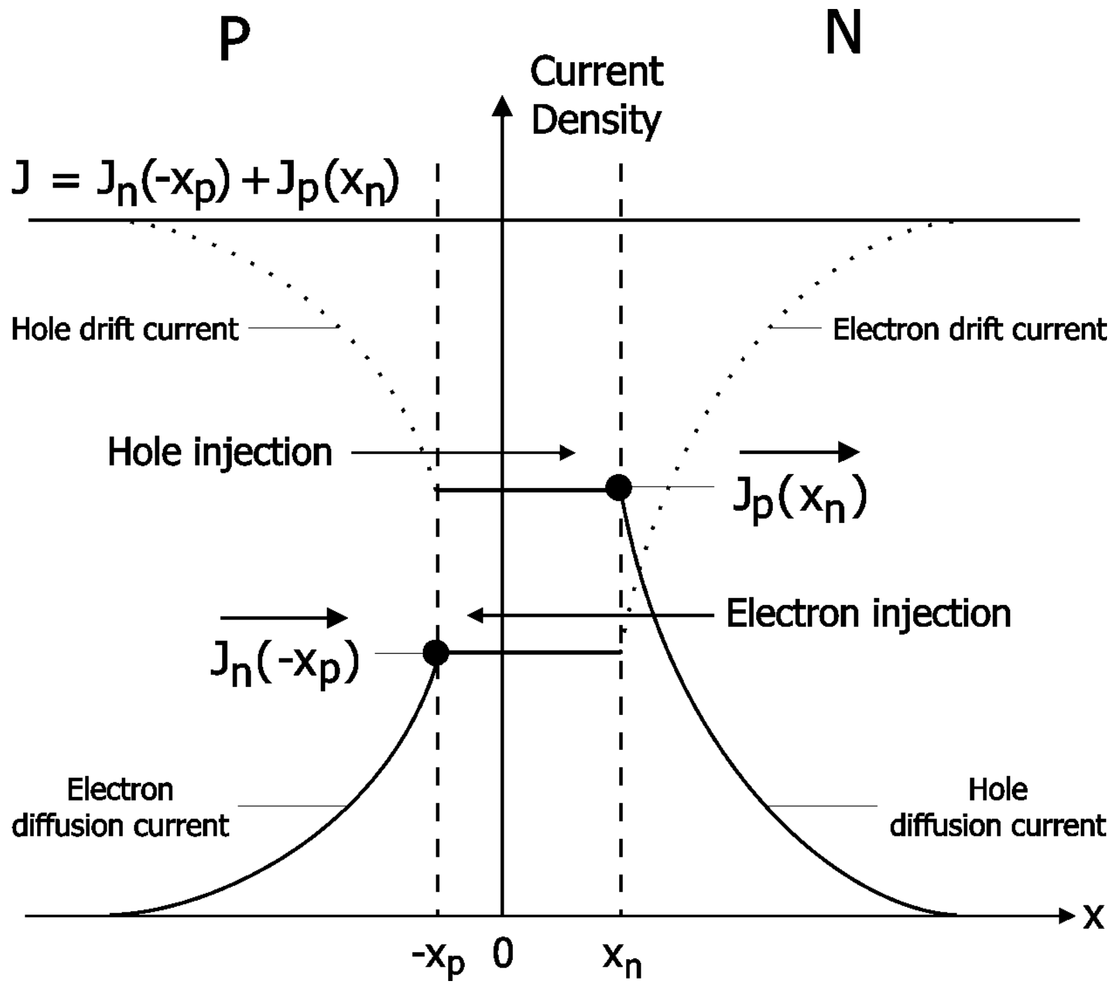
$$J_p(x') = qD_p \left| \frac{\partial p_n}{\partial x'} \right| = q \cdot \frac{D_p}{L_p} \cdot p_{no}\left(e^{qV_a/kT} - 1\right) \cdot e^{-x'/L_p}$$

$$J_n(x'') = qD_n \left| \frac{\partial n_p}{\partial x''} \right| = q \cdot \frac{D_n}{L_n} \cdot n_{po}\left(e^{qV_a/kT} - 1\right) \cdot e^{-x''/L_n}$$

  - The electron or hole diffusion current is not a constant, but varies as a function of distance from the edge of the depletion region.
  - This is because as the minority carriers diffuse along the length of the quasi-neutral region, they recombine with the majority carriers.

$$\partial^2 p_n/\partial x'^2 = \partial^2 \Delta p_n/\partial x'^2; \partial^2 n_p/\partial x''^2 = \partial^2 \Delta n_p/\partial x''^2$$

Current Components – Forward Bias



Current Components – Forward Bias

- Since there is no recombination in the space charge region, the minority-carrier currents flowing through it are constant.
- The individual minority-carrier current flow is completed by the majority carrier component, which supplies the charge injection into the respective n and p regions (dotted lines).
- The sum of the majority- and minority-carrier current components at any point gives the total current, which is independent of position and time under steady state condition.

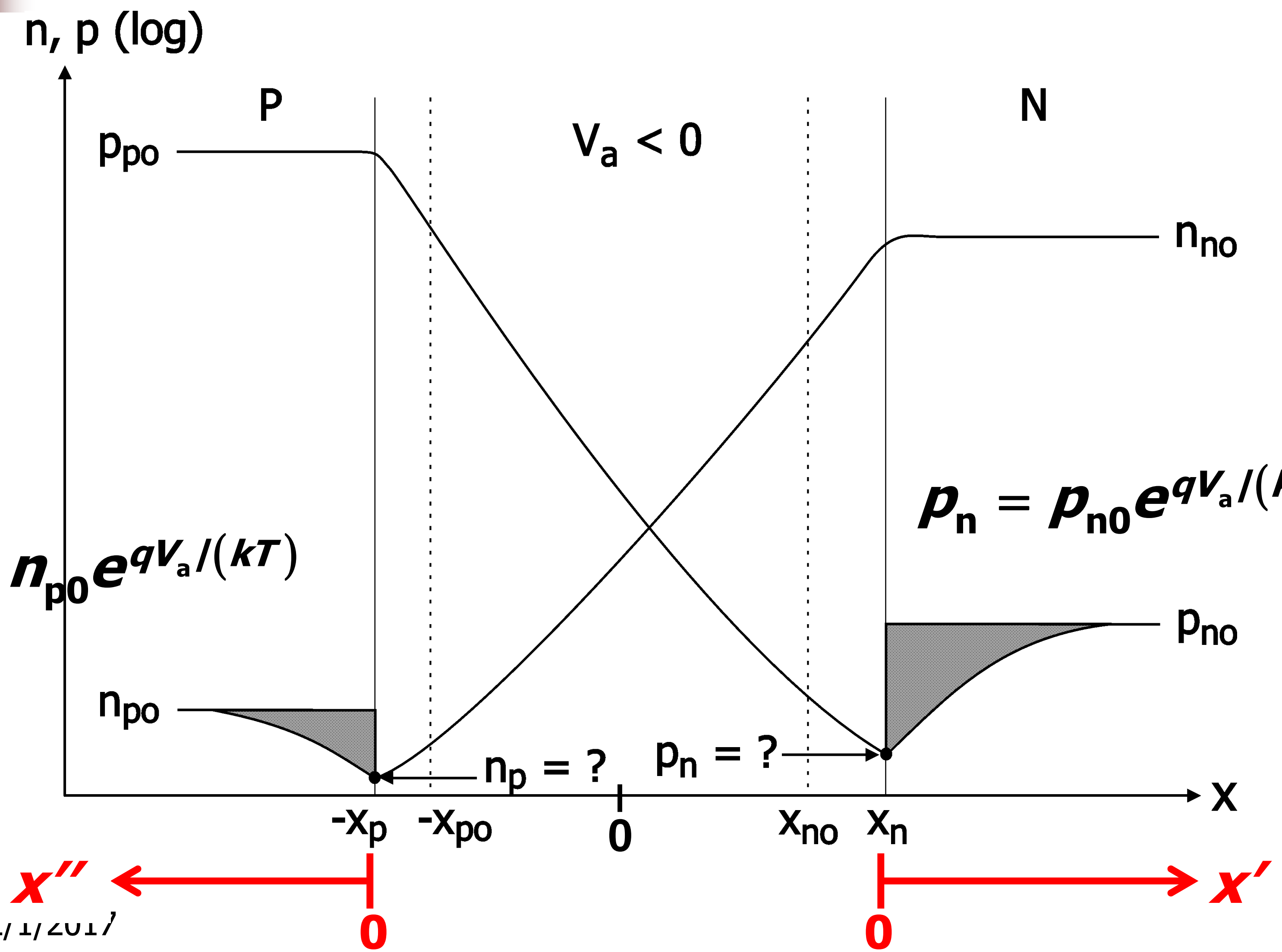
Current Components – Forward Bias

- Total diode current:

$$
\begin{aligned}
J &= J_p\left(x' = 0\right) + J_n\left(x'' = 0\right) \\
&= q\left(\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right)\left(e^{qV_a/(kT)} - 1\right) \\
&= J_0\left(e^{qV_a/(kT)} - 1\right)
\end{aligned}
$$

- This is the **ideal diode equation**.

Carrier Distributions – Reverse Bias



$$n_p = n_{p0} e^{qV_a/(kT)}$$

$$p_n = p_{n0} e^{qV_a/(kT)}$$

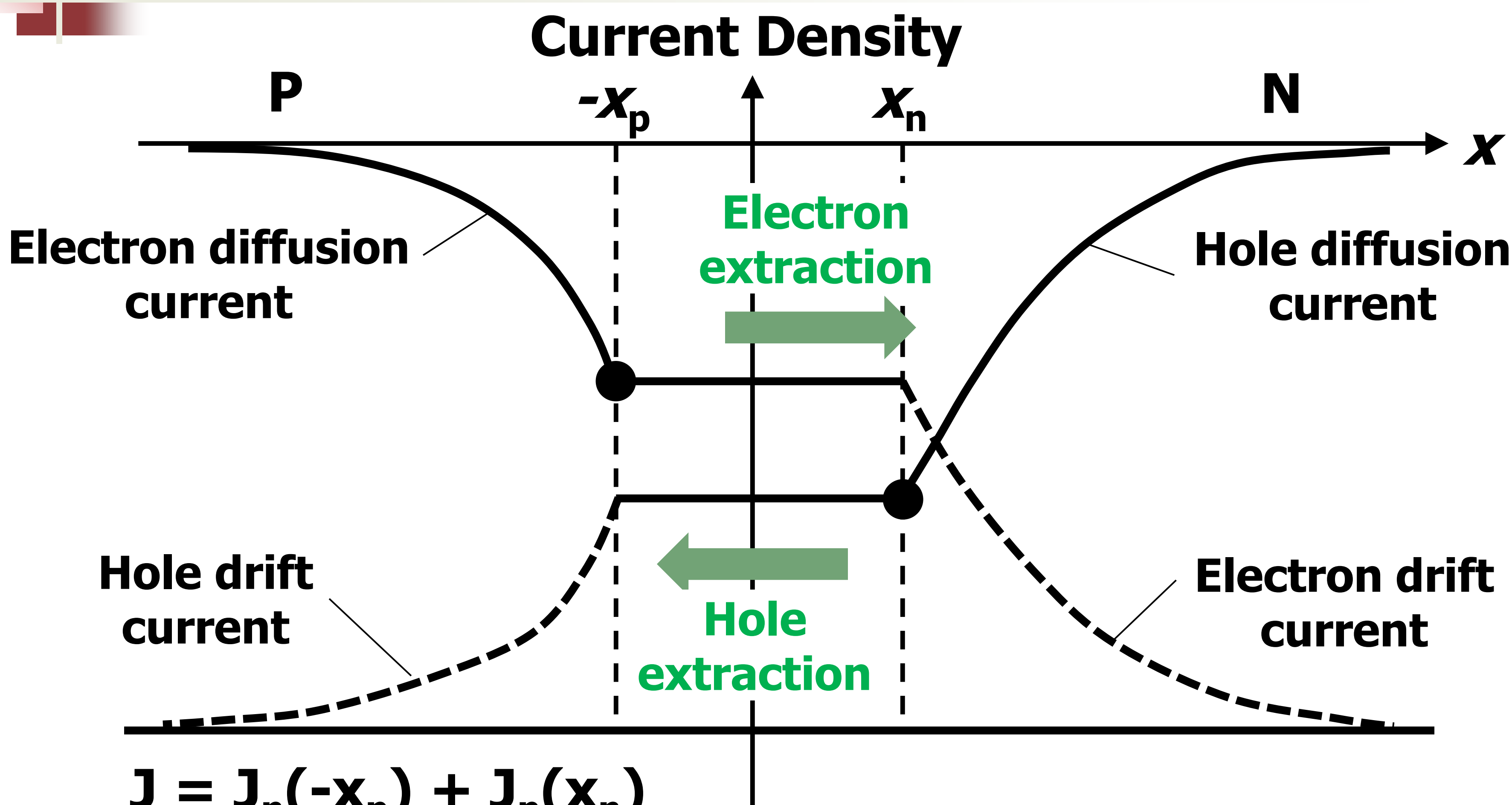
Current Components – Reverse Bias

- Under reverse bias, minority carriers at the edge of the depletion region are extracted across the depletion region.
- This causes a difference in the minority carrier concentration at the depletion edge and the rest of the quasi-neutral region.
- Hence, minority carriers, which are within one diffusion length from the depletion edge, diffuse to the depletion region.
- This diffusion process gives rise to a **small** minority-carrier current component.

Current Components – Reverse Bias

- Upon reaching the depletion region, the minority carriers are extracted across to the opposite region, where they become the majority carriers.
- Since there is no generation within the space charge region, the minority carrier current would be a constant throughout the region.
- The current flow is completed by the drift of the majority carriers towards the end contact of the opposite region.
- Except for a reversal in current direction, the same current distribution plot on page 16 describes the current flow of a reverse-biased pn junction.

Current Components – Reverse Bias



$J = J_n(-x_p) + J_p(x_n)$

Ideal Diode Equation

$$J = J_0\left(e^{qV_a/(kT)} - 1\right)$$

$$J_o = q\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right) = qn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

- The constant J_o is commonly known as the **reverse saturation current density**.
- J_o depends on the dopant concentrations, carrier diffusivities and lifetimes.
- It also depends on the temperature and bandgap energy of the semiconductor material through the intrinsic carrier concentration, n_i. Recall: $n_i = \sqrt{N_c N_v} \cdot e^{-E_g/2kT}$

Ideal Diode Equation

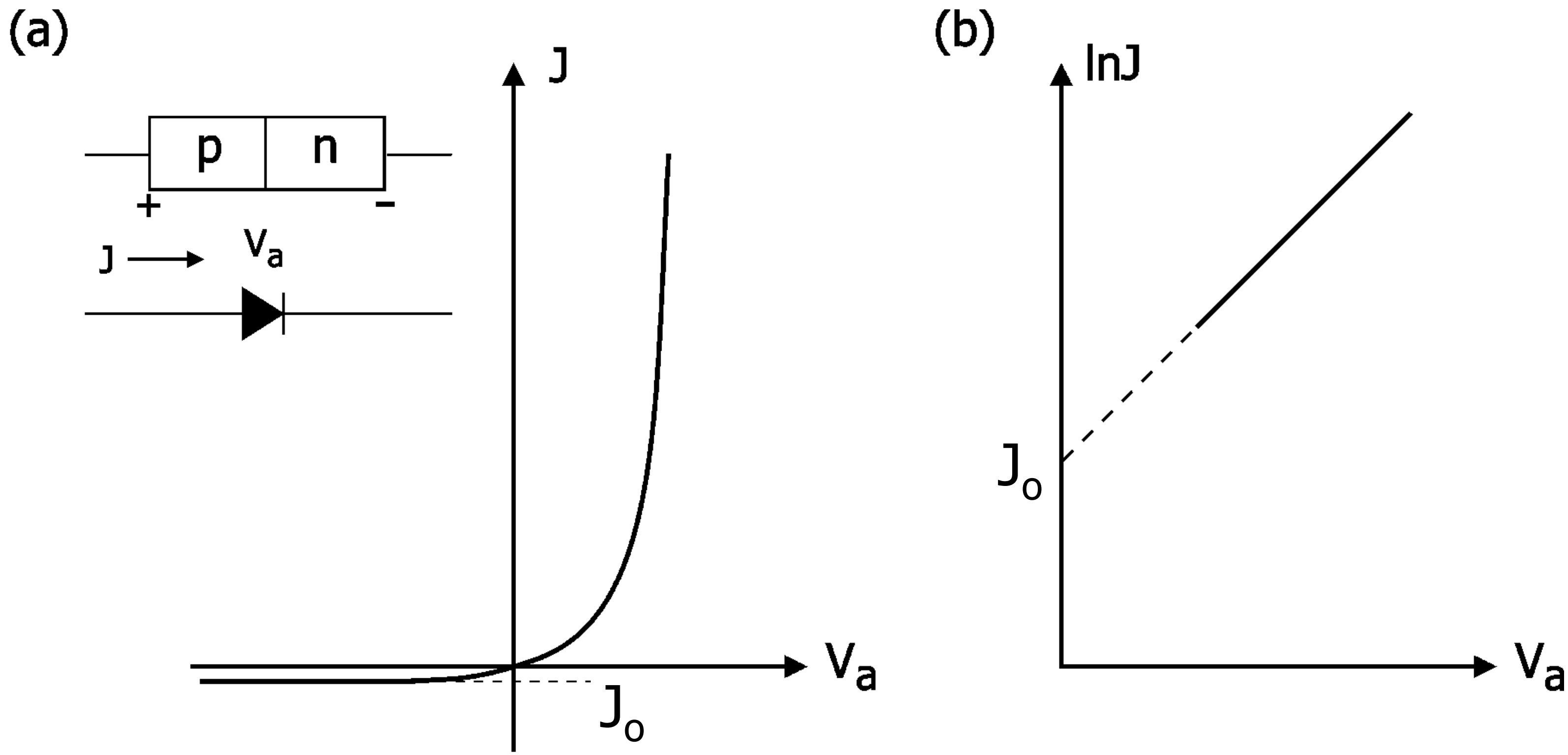
- **Forward bias:**
  - For an applied forward voltage of a few kT/q or larger, the exponential term in the bracket is much greater than 1. The ideal diode equation becomes:

$$J \approx J_0 e^{qV_a/(kT)} \Rightarrow \ln J \approx \ln J_0 + \frac{qV_a}{kT}$$

  - Plotting $\ln J$ versus V_a should therefore yield a straight line with intercept $\ln J_0$ and slope $q/(kT)$.
- **Reverse bias:**
  - For an applied reverse voltage of a few kT/q or larger (note that V_a is negative in this case), the exponential term is much smaller than 1. Hence,

$$J \approx -J_0$$

Ideal Diode Characteristics



(a) Current-voltage (I-V) characteristics of an ideal pn junction diode. (b) Forward I-V characteristics plotted on a log-linear scale.

Example 1

- To determine the reverse saturation current density at $T = 300$ K, of an ideal silicon p-n junction with the following parameters:
  - $N_A = N_D = 10^{16}$ cm^{-3}
  - $D_p = 10$ cm^2/s
  - $D_n = 25$ cm^2/s
  - $\tau_p = \tau_n = 0.5$ μs
- Assume $n_i = 1.5\text{x}10^{10}$ cm^{-3} at $T = 300$ K.

Example 1

- The reverse saturation current density is given by

$$J_o = qn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

which could be re-written as

$$D\tau = L^2$$

$$\frac{D}{L} = \frac{L}{\tau} = \frac{\sqrt{D\tau}}{\tau} = \sqrt{\frac{D}{\tau}}$$

$$J_o = qn_i^2\left(\frac{1}{N_D}\cdot\sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_A}\cdot\sqrt{\frac{D_n}{\tau_n}}\right)$$

- Substituting the parameters gives J_0 = 4.15x10^{-11} A/cm^2.
- Comment:
  - The reverse saturation current density is very small. If the cross-sectional area is 10^{-4} cm^2, for example, the current would be 4.15x10^{-15} A or 4.15 fA.

Example 2

- Consider a silicon p-n junction diode at $T = 300$ K. Design the diode such that $J_n = 20$ A/cm^2 and $J_p = 0.2$ A/cm^2 at $V_a = 0.65$. The following parameters apply:
  - $D_p = 10$ cm^2/s
  - $D_n = 25$ cm^2/s
  - $\tau_p = \tau_n = 0.5$ μs
- Assume $n_i = 1.5\text{x}10^{10}$ cm^{-3} at $T = 300$ K.

Example 2

- The electron diffusion current density:

$$J_n = q \cdot \frac{D_n}{L_n} \cdot \frac{n_i^2}{N_A}\left(e^{qV_a/(kT)} - 1\right) = q \cdot \sqrt{\frac{D_n}{\tau_n}} \cdot \frac{n_i^2}{N_A}\left(e^{qV_a/(kT)} - 1\right)$$

- Substituting in the numbers

$$20 = \left(1.6 \times 10^{-19}\right) \cdot \sqrt{\frac{25}{5 \times 10^{-7}}} \cdot \frac{\left(1.5 \times 10^{10}\right)^2}{N_A}\left(e^{0.65/0.0259} - 1\right)$$

yields $N_A = 1.01\text{x}10^{15}$ cm^{-3}.

- To obtain N_D, the same method can be used. However, it may be useful to note that

$$J_n \propto \frac{1}{N_A} \cdot \sqrt{\frac{D_n}{\tau_n}}, \quad J_p \propto \frac{1}{N_D} \cdot \sqrt{\frac{D_p}{\tau_p}}$$

Example 2

- Taking the ratio of the two current densities

$$\frac{J_n}{J_p} = \frac{N_D}{N_A} \cdot \sqrt{\frac{D_n}{\tau_n}} \cdot \sqrt{\frac{\tau_p}{D_p}}$$

yields

$$N_D = N_A \cdot \frac{J_n}{J_p} \cdot \sqrt{\frac{D_p}{D_n}}$$

$$= \left(1.01 \times 10^{15}\right)\left(\frac{20}{0.2}\right)\sqrt{\frac{10}{25}}$$

$$= 6.39 \times 10^{16}\ \text{cm}^{-3}$$

Example 2

- Comments:
  - The magnitude of the electron and hole current densities through a diode can be varied by changing the doping concentrations in the device.
  - In general, for a diode with one region more heavily doped than the other, the current would be dominated by the injection of carriers from the heavily doped region to the lowly doped region. In the above example, $N_D > N_A$, and hence the current is mainly made up of electron diffusion current in the lowly doped p region.