NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) Three semi-infinitely long straight wires are shown in Figure 1 on page 2. The wires have an infinitesimally small gap between them at the origin. The wires are uniformly charged with line charge density of $+\rho_l$, $-\rho_l$, and $+\rho_l$, respectively.

$\vec{E}$

(i) What is the direction of the combined electric field intensity due to wires 1 and 2? Explain your answer briefly.

(ii) Determine the combined electric field intensity at point $(0, 0, z_0)$ $(z_0 > 0)$ due to the three wires.

It is given that $\int \frac{zdz}{(b+z^2)^{1.5}} = \frac{-1}{\sqrt{b+z^2}}$

(13 Marks)

Note: Question No. 1 continues on page 2.

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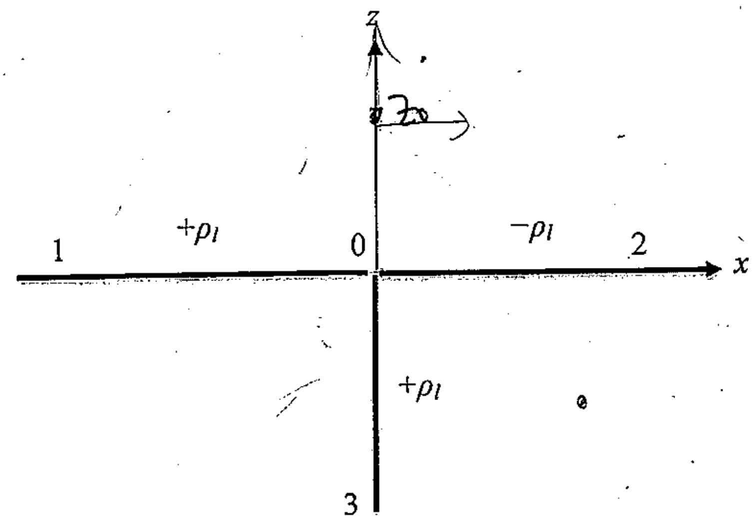

Figure 1

(b) A thin conducting wire of length $4w$ forms a square loop in the xy-plane. A direct current I flows in the wire along the counter-clockwise direction looking from the $+z$ direction.

(i) What is the direction of the overall magnetic field due to the loop? Explain your answer briefly.

(ii) Find the overall magnetic field intensity at point $(0, 0, z_0)$ $(z_0 > 0)$.

It is given that $\int \frac{dz}{(a+z^2)^{1.5}} = \frac{z}{a\sqrt{a+z^2}}$

(12 Marks)

2. (a) A circular conducting loop of radius $a = 0.2$ m lies in the $y = 0$ plane with its centre at the origin.

(i) Find the voltage induced in the loop if it lies in a magnetic flux density

$$\vec{B} = 0.05\cos\left(10^3 t\right)\left(\vec{a}_y + 2\vec{a}_z\right) \text{ T.}$$

(ii) Using a diagram, show the direction of the induced current in the loop at $t = 0$. Give reasons for your answer.

(12 Marks)

Note: Question No. 2 continues on page 3.

(b) A 4-GHz uniform plane wave is propagating in a lossless medium. The medium has a relative permittivity $\varepsilon_r = 8$ and a relative permeability $\mu_r = 2$. It is observed that the wave has the following electric field components:

$$E_y = 2\cos(\omega t - kx) \quad \text{V/m} \quad \text{and} \quad E_z = 3\cos(\omega t - kx) \quad \text{V/m.}$$

(i) Determine the magnetic field components of the wave.

(ii) Determine the angles made by the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ with respect to the z-axis.

(iii) Determine if Gauss' Law is satisfied by $\vec{E}$ and $\vec{H}$. Give brief reasons for your answer.

(13 Marks)

3. (a) The electric field of a uniform plane wave (UPW) in free space can be expressed as follows:

$$\vec{E} = \left(E_x\vec{a}_x + E_y\vec{a}_y + E_z\vec{a}_z\right)e^{-j(8x+6z)} \text{ V/m.}$$

(i) Determine the direction of time-average Poynting vector.

(ii) Write down the corresponding magnetic field expression.

(iii) For the electric field to represent a linearly polarized UPW, state all the pertaining conditions in terms of its components (i.e., $E_x = |E_x|\angle\phi_x, E_y = |E_y|\angle\phi_y, E_z = |E_z|\angle\phi_z$). You should consider all possibilities of having none, one or two of the components being zero.

(14 Marks)

(b) A uniform plane wave in air (occupying the region $z \le 0$) is obliquely incident at an angle $\theta_i = 30°$ onto a lossless medium (occupying the region $z \ge 0$) with $\varepsilon = 2.1\varepsilon_0$ and $\mu = \mu_0$.

(i) Determine the angle of transmission θ_t.

(ii) Calculate the reflection and transmission coefficients for parallel polarization, i.e., $\Gamma_{\parallel}$ and $\tau_{\parallel}$.

(iii) An analytical relation between the coefficients above is given by

$$1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Check whether this relation is satisfied using your answers above.

(iv) Discuss briefly how the relation above may be derived analytically and whether it is applicable for a lossy medium.

(11 Marks)

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4. (a) A transmission line in air has characteristic impedance $Z_0 = 50\,\Omega$ and length $l = 0.4\lambda$ (with λ being the wavelength). It is connected to a generator at $z = -l$ and terminated with an unknown load at $z = 0$. The generator has an open-circuit voltage $V_g = 10\,\text{V}$ and an internal impedance $Z_g = 50\,\Omega$. The ratio between the power delivered to the load P_L and the maximum available power from the generator P_{av} is given as

$$\frac{P_L}{P_{av}} = \frac{8}{9}$$

(i) What are the maximum available power from the generator P_{av} and the power delivered to the load P_L?

(ii) Determine the magnitude of load reflection coefficient $|\Gamma_L|$ and the standing wave ratio (SWR).

(iii) It is found that the first voltage maximum occurs at $z_{\max} = -0.35\lambda$. Determine the phase angle of load reflection coefficient $\theta_0 = \angle\Gamma_L$ (in degrees) and the load impedance Z_L.

(iv) Determine the input reflection coefficient Γ_{in} at $z = -l$.

(16 Marks)

(b) The load in part (a) is subsequently removed and the transmission line is left open-circuited.

(i) What are the input reflection coefficient Γ_{in} and the input impedance Z_{in} at $z = -l$?

(ii) Determine all the positions of voltage minima $z_{\min}$ on the transmission line.

(9 Marks)

Note: State any assumption made in the above. The Smith chart may be used in the solutions for one or both parts of this question. Please put the Smith chart inside (not outside) the answer script and tie it with a thread.

$$\frac{\cancel{8}\ 50 + R(z_{in})}{\cancel{Rein}\ R(z_{in})} = \frac{9}{8} \qquad \cancel{5}\ V_g^2$$

$$9R = \cancel{8}\ 400 + 8R$$

$$R = 400$$

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

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Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_{\perp} = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_{\perp} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_{\|} = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_{\|} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \ e^{-j\beta z} + V_o^- \ e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \ e^{-j\beta z} - V_o^- \ e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

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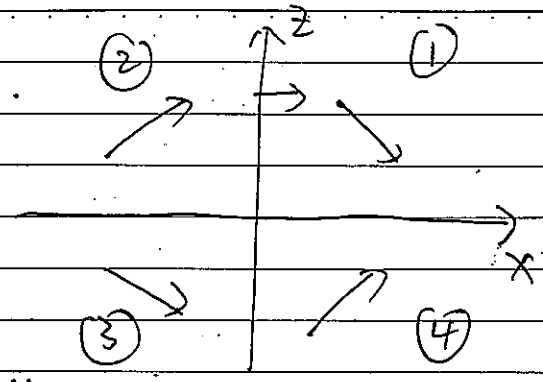
PYP Solution

1 (a) (i)



on Z-axis: points to the right

② & ④ quadrant: points to the upper right

① & ③ quadrant: points to the lower right

(ii) line 1:

$\vec{r} = (0, 0, z_0)$ $\quad \vec{s} = (x, 0, 0)$ $\quad x < 0$ $\quad \vec{R} = \vec{r} - \vec{s} = -x\vec{a_x} + z_0\vec{a_z}$

$dl = dx$ $\quad |\vec{R}| = \sqrt{x^2 + z_0^2}$

$$\vec{E_1} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{0} \frac{\rho_L \vec{R}}{R^3}\, dl = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\infty}^{0} \frac{(-x\vec{a_x} + z_0\vec{a_z})}{(x^2 + z_0^2)^{\frac{3}{2}}}\, dx$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left(\int_{-\infty}^{0} \frac{-x\vec{a_x}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx + \int_{-\infty}^{0} \frac{z_0\vec{a_z}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx\right)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left(\int_{0}^{\infty} \frac{x\vec{a_x}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx + \int_{-\infty}^{0} \frac{z_0\vec{a_z}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx\right)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left(\left[\frac{-\vec{a_x}}{\sqrt{z_0^2+x^2}}\right]_0^{\infty} + \left[\frac{z_0\vec{a_z}\, x}{z_0^2\sqrt{z_0^2+x^2}}\right]_{-\infty}^{0}\right)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left(\frac{\vec{a_x}}{\sqrt{z_0^2}} + 1\right)$$

line 2:

$\vec{r} = (0, 0, z_0)$ $\quad \vec{s} = (x, 0, 0)$ $\quad \vec{R} = \vec{r} - \vec{s} = -x\vec{a_x} + z_0\vec{a_z}$ $\quad x > 0$

$dl = dx$ $\quad |\vec{R}| = \sqrt{x^2 + z_0^2}$

$$\vec{E_2} = \frac{1}{4\pi\varepsilon_0}\int_{0}^{\infty} \frac{-\rho_L \vec{R}}{R^3}\, dl = \frac{-\rho_L}{4\pi\varepsilon_0}\int_{0}^{\infty} \frac{(-x\vec{a_x} + z_0\vec{a_z})}{(x^2 + z_0^2)^{\frac{3}{2}}}\, dx$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0}\left(\int_{0}^{\infty} \frac{-x\vec{a_x}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx + \int_{0}^{\infty} \frac{z_0\vec{a_z}}{(x^2+z_0^2)^{\frac{3}{2}}}\, dx\right)$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0}\left(\left[\frac{\vec{a_x}}{\sqrt{x^2+z_0^2}}\right]_0^{\infty} + \left[\frac{\vec{a_z}\, x}{z_0\sqrt{x^2+z_0^2}}\right]_0^{\infty}\right) = \frac{-\rho_L}{4\pi\varepsilon_0}\left(\frac{-\vec{a_x}}{\sqrt{z_0^2}} + 1\right)$$

line 3:

$\vec{r} = (0, 0, z_0)$ $\quad \vec{s} = (0, 0, z)$ $\quad \vec{R} = \vec{r} - \vec{s} = (z_0 - z)\vec{a_z}$ $\quad z < 0$

$dl = dz$ $\quad |\vec{R}| = \sqrt{(z_0 - z)^2}$

$$\vec{E_3} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{0} \frac{\rho_l \vec{R}}{R^3}\, dl = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-\infty}^{0} \frac{(z_0 - z)\,\vec{a_z}}{(z_0 - z)^3}\, dz$$

$$= \frac{\rho_l \vec{a_z}}{4\pi\varepsilon_0}\int_{-\infty}^{0} \frac{1}{(z_0 - z)^2}\, dz = \frac{\rho_l \vec{a_z}}{4\pi\varepsilon_0}\left[\frac{-1}{z_0 - z}\right]_{-\infty}^{0}$$

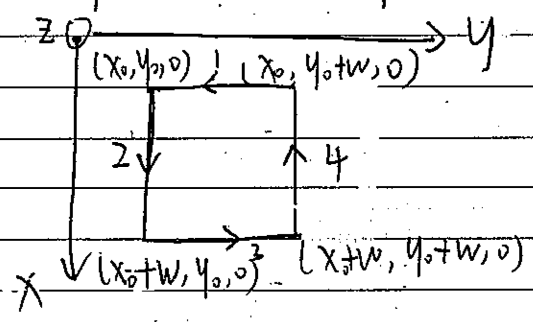
$$= \frac{\rho_l \vec{a_z}}{4\pi\varepsilon_0}\left(\frac{-1}{z_0}\right) = \frac{-\rho_l \vec{a_z}}{4\pi\varepsilon_0 z_0}$$

$$\vec{E}_{tot} = \vec{E_1} + \vec{E_2} + \vec{E_3} = \frac{\rho_l}{4\pi\varepsilon_0}\left(\frac{\vec{a_x}}{z_0} + 1 + \frac{\vec{a_x}}{z_0} - 1 - \frac{\vec{a_z}}{z_0}\right)$$

$$= \frac{\rho_l}{4\pi\varepsilon_0}\left(\frac{2\vec{a_x}}{z_0} - \frac{\vec{a_z}}{z_0}\right) \ V/m$$

1 (b) (i) According to right-hand rule, the overall magnetic field is to the +z direction.

(ii) The loop is at the position shown below



line 1 $\vec{r} = (0, 0, z_0)$ $\vec{s} = (x_0, y, 0)$ $\vec{R} = \vec{r} - \vec{s} = (-x_0, -y, z_0)$

$|\vec{R}| = \sqrt{x_0^2 + y^2 + z_0^2}$ $\quad d\vec{l} = -dy\,\vec{a_y}$

$$d\vec{l} \times \vec{R} = -dy\,\vec{a_y} \times (-x_0\vec{a_x} - y\vec{a_y} + z_0\vec{a_z}) = -(x_0\vec{a_z}\,dy + z_0\vec{a_x}\,dy)$$

$$\vec{H_1} = \frac{1}{4\pi}\int_c \frac{I\,d\vec{l}\times\vec{R}}{R^3} = \frac{-I}{4\pi}\int_{y_0}^{y_0+w} \frac{(x_0\vec{a_z} + z_0\vec{a_x})}{(x_0^2 + z_0^2 + y^2)^{\frac{3}{2}}}\, dy$$

$$= \frac{-I(x_0\vec{a_z} + z_0\vec{a_x})}{4\pi}\int_{y_0}^{y_0+w} \frac{dy}{(x_0^2 + z_0^2 + y^2)^{\frac{3}{2}}}$$

$$= \frac{-I(x_0\vec{a_z} + z_0\vec{a_x})}{4\pi}\left[\frac{y}{(x_0^2 + z_0^2)\sqrt{x_0^2 + z_0^2 + y^2}}\right]_{y_0}^{y_0+w}$$

$$= \frac{-I(x_0\vec{a_z} + z_0\vec{a_x})}{4\pi}\left[\frac{y_0 + w}{(x_0^2 + z_0^2)\sqrt{x_0^2 + z_0^2 + (y_0+w)^2}} - \frac{y_0}{(x_0^2 + z_0^2)\sqrt{x_0^2 + z_0^2 + y_0^2}}\right]$$

line 2

$\vec{r} = (0,0,z_0) \quad \vec{s} = (x, y_0, 0) \quad \vec{R} = \vec{r} - \vec{s} = (-x, -y_0, z_0)$

$\vec{R} = \sqrt{x^2+y_0^2+z_0^2} \qquad d\vec{l} = dx\,\vec{a_x}$

$d\vec{l} \times \vec{R} = dx\,\vec{a_x} \times (-x\,\vec{a_x} - y_0\,\vec{a_y} + z_0\,\vec{a_z}) = -(y_0\vec{a_z} + z_0\vec{a_y})dx$

$$\vec{H_2} = \frac{I}{4\pi}\int_{x_0}^{x_0+w} \frac{d\vec{l}\times\vec{R}}{R^3} = \frac{-I}{4\pi}\int_{x_0}^{x_0+w} \frac{(y_0\vec{a_z} + z_0\vec{a_y})}{(x^2+y_0^2+z_0^2)^{\frac{3}{2}}}\,dx$$

$$= \frac{-I(y_0\vec{a_z} + z_0\vec{a_y})}{4\pi}\left[\frac{x}{(y_0^2+z_0^2)\sqrt{y_0^2+z_0^2+x^2}}\right]_{x_0}^{x_0+w}$$

$$= \frac{-I(y_0\vec{a_z} + z_0\vec{a_y})}{4\pi}\left(\frac{x_0+w}{(y_0^2+z_0^2)\sqrt{y_0^2+z_0^2+(x_0+w)^2}} - \frac{x_0}{(y_0^2+z_0^2)\sqrt{y_0^2+z_0^2+x_0^2}}\right)$$

line 3

$\vec{r} = (0,0,z_0) \quad \vec{s} = (x_0+w, y, 0) \quad \vec{R} = \vec{r} - \vec{s} = (-(x_0+w), -y, z_0)$

$|\vec{R}| = \sqrt{(x_0+w)^2+y^2+z_0^2} \qquad d\vec{l} = dy\,\vec{a_y}$

$d\vec{l}\times\vec{R} = (x_0+w)\vec{a_z} + z_0\vec{a_x}$

$$\vec{H_3} = \frac{I}{4\pi}\int_{y_0}^{y_0+w} \frac{(x_0+w)\vec{a_z} + z_0\vec{a_x}}{[(x_0+w)^2+y^2+z_0^2]^{\frac{3}{2}}}\,dy$$

$$= \frac{I[(x_0+w)\vec{a_z} + z_0\vec{a_x}]}{4\pi}\left[\frac{y}{((x_0+w)^2+z_0^2)\sqrt{(x_0+w)^2+z_0^2+y^2}}\right]_{y_0}^{y_0+w}$$

$$= \frac{I[(x_0+w)\vec{a_z} + z_0\vec{a_x}]}{4\pi}\left[\frac{y_0+w}{[(x_0+w)^2+z_0^2]\sqrt{(x_0+w)^2+z_0^2+(y_0+w)^2}} - \frac{y_0}{[(x_0+w)^2+z_0^2]\sqrt{(x_0+w)^2+z_0^2+y_0^2}}\right]$$

line 4

$\vec{r} = (0,0,z_0) \quad \vec{s} = (x, y_0+w, 0) \quad \vec{R} = \vec{r}-\vec{s} = (-x, -(y_0+w), z_0)$

$|\vec{R}| = \sqrt{x^2+(y_0+w)^2+z_0^2} \qquad d\vec{l} = -dx\,\vec{a_x} \qquad d\vec{l}\times\vec{R} = [(y_0+w)\vec{a_z} + z_0\vec{a_y}]dx$

$$\vec{H_4} = \frac{I}{4\pi}\int_{x_0}^{x_0+w}\frac{d\vec{l}\times\vec{R}}{R^3} = \frac{I[(y_0+w)\vec{a_z} + z_0\vec{a_y}]}{4\pi}\int_{x_0}^{x_0+w}\frac{dx}{(x^2+(y_0+w)^2+z_0^2)^{\frac{3}{2}}}$$

$$= \frac{I[(y_0+w)\vec{a_z} + z_0\vec{a_y}]}{4\pi}\left[\frac{x_0+w}{[(y_0+w)^2+z_0^2]\sqrt{(x_0+w)^2+(y_0+w)^2+z_0^2}} - \frac{x_0}{[(y_0+w)^2+z_0^2]\sqrt{x_0^2+(y_0+w)^2+z_0^2}}\right]$$

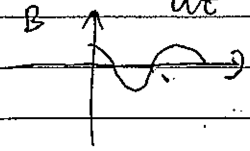
$\vec{H}_{tot} = \vec{H_1} + \vec{H_2} + \vec{H_3} + \vec{H_4}$

2(a)(i) $\phi_m = \iint_S \vec{B}\cdot d\vec{S} = 0.05(\vec{a_y}+2\vec{a_z})\cdot\vec{a_y}\cos(10^3 t)(0.04\pi)$ Wb

$= 2\times10^{-3}\cos(10^3 t)$ Wb

$emf = -\frac{d\phi_m}{dt} = 2\times10^{-3}\times10^{3}\sin(10^3 t) = 2\sin(10^3 t)$

(ii) (not sure)



$t=0^-$: no magnatic flux

$t=0^+$: magnatic flux increase from 0 in $\vec{a_y}$ direction, according to Lenz's Law, the direction of the induced current is in clockwise direction looking from y axis

2(b)(i) $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\times120\pi = 60\pi\ \Omega$ $\quad \vec{a_k}=\vec{a_x}$

$\vec{E} = (2\vec{a_y}+3\vec{a_z})\cos(wt-kx)$ V/m

$\vec{H} = \vec{a_x}\times\frac{\vec{E}}{\eta} = \left(\frac{1}{30\pi}\vec{a_z} - \frac{1}{20\pi}\vec{a_y}\right)\cos(wt-kx)$ A/m

(ii) angle of $\vec{E}$: $\tan^{-1}\left(\frac{2}{3}\right) = 33.69°$

angle of $\vec{H}$: $\tan^{-1}\left(\frac{1/20\pi}{1/30\pi}\right) = 56.31°$

(iii) (not sure) Gauss's Law is not satisfied by $\vec{H}$ because $\nabla\cdot\vec{H} \neq 0 \Rightarrow \nabla\cdot\vec{B} \neq 0$

not for $\vec{E}$ because $\nabla\cdot\vec{D} = \nabla(\varepsilon\vec{E}) \neq \rho$

3(a)(i) $0.8\vec{a_x}+0.6\vec{a_z} = \vec{a_k}$

(ii) $\eta = 120\pi\ \Omega$ $\quad \vec{H} = \frac{1}{\eta}\vec{a_k}\times\vec{E}$

$= \frac{e^{-j(8x+6z)}}{120\pi}\cdot(0.8E_y\vec{a_z} - 0.8E_z\vec{a_y} + 0.6E_x\vec{a_y} - 0.6E_y\vec{a_x})$

(iii) plane of Incidence $\equiv$ xz-plane.

$\parallel$-Polarisation : $\vec{a_E}$ in xz-plane

$\vec{E_\parallel} = (|E_x|e^{j\phi_x}\vec{a_x} + |E_z|e^{j\phi_z}\vec{a_z})e^{-j(8x+6z)}$

$E_\parallel = \sqrt{|E_x|^2 e^{j2\phi_x} + |E_z|^2 e^{j2\phi_z}}$ (not sure)

$\perp$-Polarization

$\vec{E_\perp} = |E_y|e^{j\phi_z}\cdot e^{-j(8x+6z)}$

$|E_\perp| = |E_y|\angle\phi_z$

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① One component is 0

$E_x = 0$: $E^{\parallel} = |E_z| \angle \phi_z \quad E^{\perp} = |E_y| \angle \phi_y$

$\Rightarrow |E_z| = 0$ OR $|E_y| = 0$ OR $|\phi_z - \phi_y| = 0$ OR $|\phi_z - \phi_y| = 180°$

$\Rightarrow$ linear

$E_y = 0$: linear

$E_z = 0$: $E^{\parallel} = |E_x| \angle \phi_x \quad E^{\perp} = |E_y| \angle \phi_y$

$\Rightarrow |E_x| = 0$ OR $|E_y| = 0$ OR $|\phi_x - \phi_y| = 0$ OR $|\phi_x - \phi_y| = 180°$

$\Rightarrow$ linear

② two components are 0

$E_x = E_y = 0$: $|E^{\perp}| = 0 \Rightarrow$ linear

$E_x = E_z = 0$: $|E^{\parallel}| = 0 \Rightarrow$ linear

$E_y = E_z = 0$: $|E^{\perp}| = 0 \Rightarrow$ linear

③ none components is zero

If $|E_x| = |E_z| = 0 \Rightarrow |E_{\parallel}| = 0 \Rightarrow$ linear

If $|E_y| = 0 \Rightarrow |E^{\perp}| = 0 \Rightarrow$ linear

If $|\phi_{\perp} - \phi_{\parallel}| = 0$ OR $180° \Rightarrow$ linear

3(b)(i) $\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \sqrt{\frac{1}{2.1}} \Rightarrow \theta_t = \sin^{-1}\left(\sqrt{\frac{1}{2.1}} \times \sin 30°\right) = 20.18°$

(ii) $\Gamma_{\parallel} = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} = -0.144$

$\eta_2 = 120\pi\sqrt{\frac{1}{2.1}} = 260.15\,\Omega \quad \cos(\theta_t) = \cos(20.18°) = 0.939$

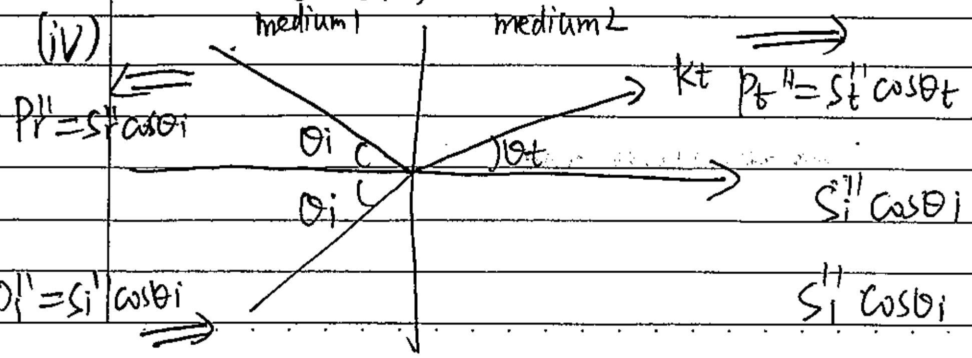
$\cos(\theta_i) = \cos(30°) = \sqrt{3}/2$

$\tau_{\parallel} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} = 0.7895$

(iii) $1 + \Gamma_{\parallel} = 0.856$

$\tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right) = 0.856 \Leftarrow$ Power Conservation $P_i^{\parallel} = P_t^{\parallel} + P_r$

(iv)



$S_i^{\parallel}\cos\theta_i = S_r^{\parallel}\cos\theta_i + S_t^{\parallel}\cos\theta_t$

$S_i^{\parallel}\cos\theta_i + S_i^{\parallel}\Gamma_{\parallel}\cos\theta_i = S_i^{\parallel}\tau_{\parallel}\cos\theta_t$

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It's applicable for lossy medium because Power conservation is still hold at the interface.

4(a) I am not sure for Q4

(i) $\frac{P_L}{P_{av}} = \frac{Z_{in}}{Z_g + Z_{in}} = \frac{8}{9} \Rightarrow Z_{in} = 400\Omega$

$P_L = \frac{1}{2}\left|\frac{V_g}{Z_g+Z_{in}}\right|^2 Re(Z_{in}) = \frac{1}{2}\left|\frac{10}{450}\right|^2 \times 400 = 0.0988\,W$

$P_{av} = 0.111\,W$

(ii) $Z_{in}(-l) = 400 = Z_0 \cdot \frac{1+\Gamma_L e^{+2j\beta z}}{1-\Gamma_L e^{+2j\beta z}}$

$\Rightarrow \Gamma_L = \frac{7}{9} e^{j1.6\pi}$

$|\Gamma_L| = 7/9 \Rightarrow SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 8$

(iii) $\theta_0 + 2\beta z_{max} = \theta_0 + 2 \cdot 2\pi/\lambda \cdot (-0.35\lambda) = 0 \Rightarrow \theta_0 = 1.4\pi$

$\theta_0 - 1.4\pi = -2\pi \Rightarrow \theta_0 = -0.6\pi = -108^\circ$

$\frac{Z_L}{Z_0} = \frac{1+\Gamma_L}{1-\Gamma_L} = 0.387 - j1.4123$

$Z_L = 50(0.387 - j1.4123) = 19.35 - j70.615$

(iv) $\Gamma(z) = \Gamma_L e^{+j2\beta z}$

$= |\Gamma_L| \cdot e^{j(\theta_0 + 2\beta z)} \doteq \frac{7}{9} e^{-j2.2\pi}$

(b)

(i) $Z_{oc} = \frac{Z_0}{j\tan\beta l} = \frac{-50j}{\tan(0.8\pi)} = -68.82j$

$\Gamma_L = \frac{Z_{oc}-Z_0}{Z_{oc}+Z_0} = e^{-j1.257}$

$\Gamma(-l) = \Gamma_L e^{+j2\beta z} = e^{-j2.857\pi}$

(ii)

$\theta_r = \theta_0 + 2\beta z_{min} = -\pi \Rightarrow z_{min} = -0.15\lambda$

Good luck for your exams! ^^