NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

This session is about

14. Reflection and Transmission of UPW at Oblique Incident

Learning Objectives

- Determine the direction of propagation, direction of E and H fields and the amplitude of E and H fields for UPW that travel along an arbitrary direction;
- Given E field of a UPW propagating in an arbitrary direction, determine the corresponding H field and vice versa; and
- Determine the instantaneous and time-average Poynting vector from the E and H field.

Normal Incidence vs Oblique Incidence

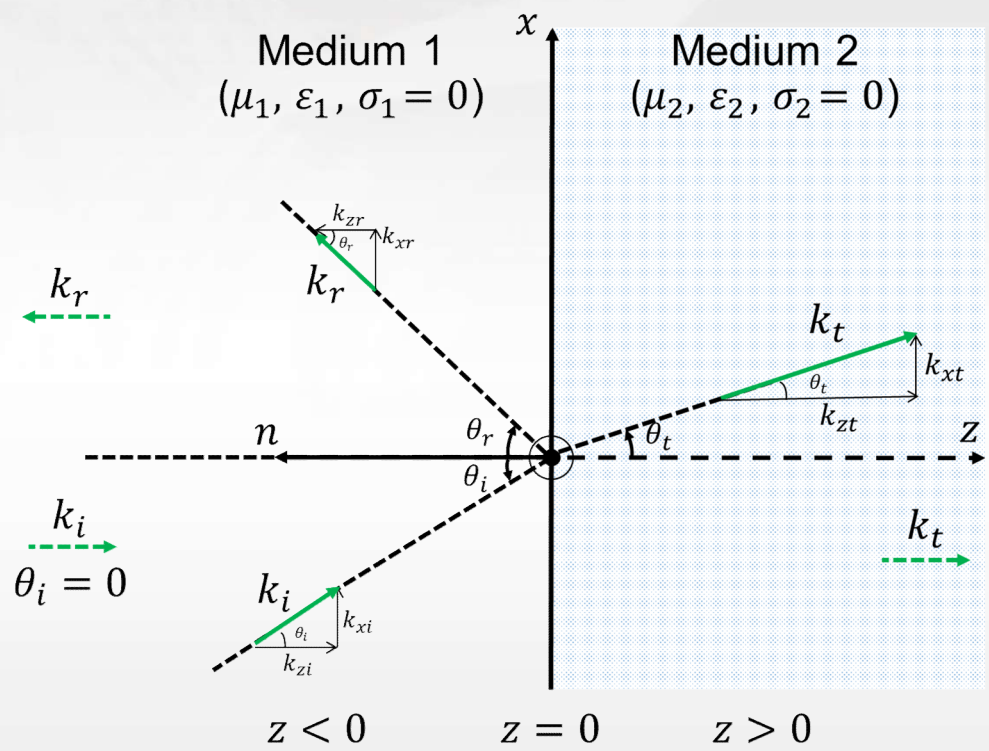
Oblique Incidence $(\theta_i \neq 0)$ at a Plane Boundary

- Given $\vec{E}_i \rightarrow \vec{k}_i,\ \vec{a}_{E_i},\ E_{oi}$

  $\vec{k}_i \rightarrow \vec{k}_r$ & $\vec{k}_t$

  $\vec{a}_{E_i} \rightarrow \vec{a}_{E_r}$ & $\vec{a}_{E_t}$

  $E_{oi} \rightarrow E_{or}$ & E_{ot}



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14. Reflection and Transmission of UPW at Oblique Incident (2)

Normal Incidence vs Oblique Incidence

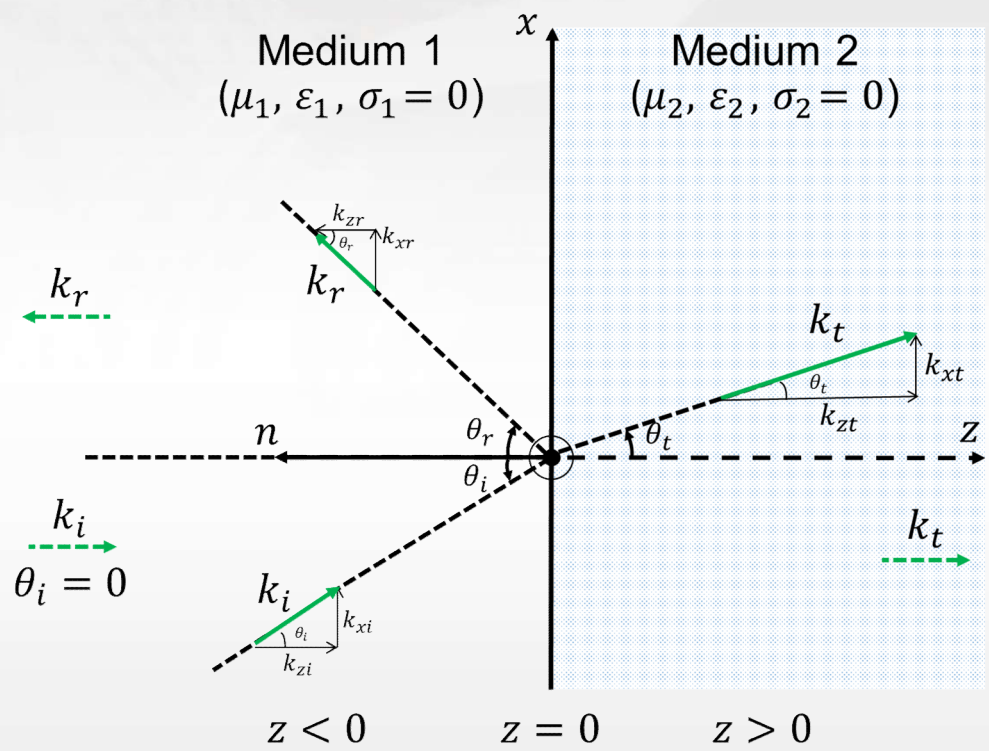
Oblique Incidence $(\theta_i \neq 0)$ at a Plane Boundary

- Given $\vec{E}_i \rightarrow \vec{k}_i,\ \vec{a}_{E_i},\ E_{oi}$

  $\vec{k}_i \rightarrow \vec{k}_r$ & $\vec{k}_t$

  $\vec{a}_{E_i} \rightarrow \vec{a}_{E_r}$ & $\vec{a}_{E_t}$

  $E_{oi} \rightarrow E_{or}$ & E_{ot}



UPW Travels Along an Arbitrary Direction

- **UPW travels along an arbitrary direction:**

UPW travels along $+z$ direction: $\vec{a}_k = +\vec{a}_z$

$$\vec{E}(z) = \vec{a}_E\, E_o\, e^{-jkz}$$

UPW travels along $+k$ direction:

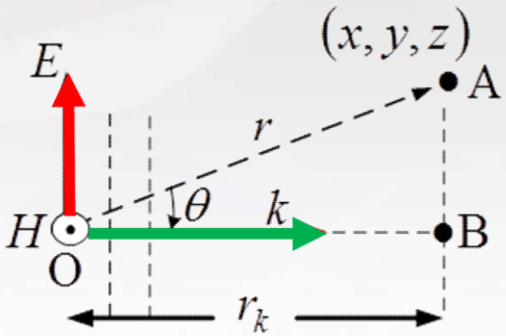
$$\vec{E}(r_k) = \vec{a}_E\, E_o\, e^{-jkr_k}$$

$$r_k = \vec{a}_k \cdot \vec{r} \longrightarrow \boxed{kr_k = k\vec{a}_k \cdot \vec{r} = \vec{k} \cdot \vec{r}}$$

$$\vec{k} = \vec{a}_x\, k_x + \vec{a}_y\, k_y + \vec{a}_z\, k_z \equiv k\vec{a}_k$$

$$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$$

$$kr_k = \vec{k} \cdot \vec{r} = k_x\, x + k_y\, y + k_z\, z$$



$$r_k = r\cos\theta = \vec{a}_k \cdot \vec{r}$$

$$k = \omega\sqrt{\mu\,\varepsilon}$$

UPW travels along an arbitrary $\vec{k}$ direction: $e^{-jkr_k} = e^{-j\vec{k}\cdot\vec{r}} = e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$

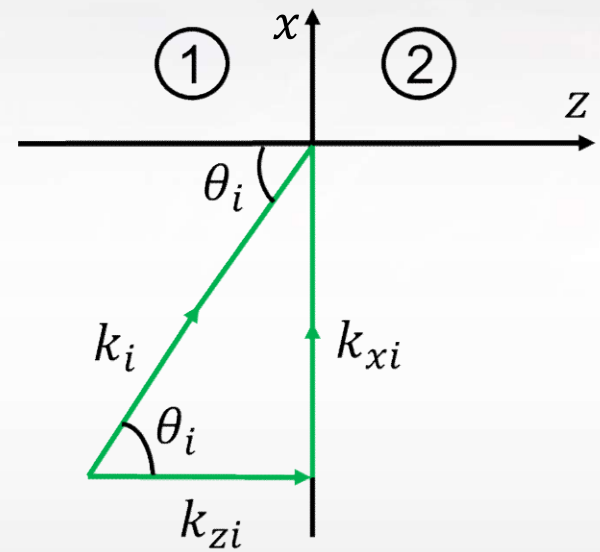
Example

Determine the direction of propagation $\vec{a}_k$ of the following UPW:

$$\vec{E}_i = \vec{a}_{E_i}\, E_{oi}\, e^{-j80}\ \ e^{-j60z} \equiv \vec{a}_{E_i}\, E_{oi}\, e^{-jk_{xi}x}\, e^{-jk_{yi}y}\, e^{-jk_{zi}z}$$

ANS:

- $k_{xi} = 80;\ k_{yi} = 0;\ k_{zi} = 60 \rightarrow \vec{k}_i = \vec{a}_x\, 80 + \vec{a}_z\, 60$
- $k_i = \sqrt{80^2 + 60^2} = 100 \qquad \rightarrow\ \vec{a}_{k_i} = \vec{a}_x\, 0.8 + \vec{a}_z\, 0.6$
- $k_i = \omega\sqrt{\mu_1\, \varepsilon_1} \rightarrow f \qquad \tan\theta_i = \frac{k_{xi}}{k_{zi}}$



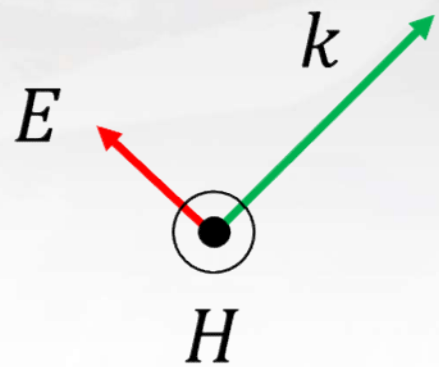
General Expressions of E and H fields

$$\vec{E} = E_o \; e^{-jk_x x} \; e^{-jk_y y} \; e^{-jk_z z} \; \vec{a}_E$$

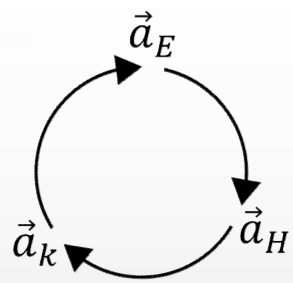
$$\vec{H} = \frac{E_o}{\eta} \; e^{-jk_x x} \; e^{-jk_y y} \; e^{-jk_z z} \; \vec{a}_H$$

$$\vec{k} = \vec{a}_x \, k_x + \vec{a}_y \, k_y + \vec{a}_z \, k_z = \vec{a}_k \, k$$

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \omega \sqrt{\mu \, \varepsilon} = \frac{2\pi}{\lambda}$$



$$\vec{a}_E \perp \vec{a}_H \perp \vec{a}_k$$



$$\vec{a}_E = \vec{a}_H \times \vec{a}_k$$

$$\vec{a}_H = \vec{a}_k \times \vec{a}_E$$

General Expressions of E and H fields

General Expressions of E and H fields

$$\vec{E} = E_o \, e^{-jk_x x} \, e^{-jk_y y} \, e^{-jk_z z} \, \vec{a}_E$$

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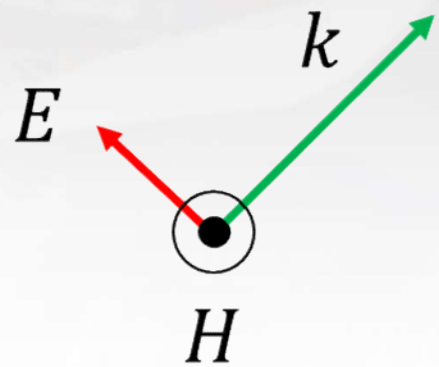

- $\vec{E}$ and $\vec{H}$ contain the following information

1) **Direction of Propagation** $(\vec{a}_k)$

2) **Direction of E and H** $(\vec{a}_E \text{ and } \vec{a}_H)$

3) **Amplitude of E and H** $(E_o = |E_o| \angle\phi_e;\ H_o = |H_o| \angle\phi_h)$

Summary

- For a UPW travels along an arbitrary $\vec{k}$ direction, the general expressions of E and H fields are:
  - $\vec{E} = E_o\, e^{-jk_x x}\, e^{-jk_y y}\, e^{-jk_z z}\, \vec{a}_E$;
  - $\vec{H} = \frac{E_o}{\eta}\, e^{-jk_x x}\, e^{-jk_y y}\, e^{-jk_z z}\, \vec{a}_H$;
  - Direction of Propagation ($\vec{a}_k$);
  - Direction of E and H ($\vec{a}_E$ and $\vec{a}_H$); and
  - Amplitude of E and H ($E_o = |E_o|\,\angle\phi_e$; $H_o = |H_o|\,\angle\phi_h$).

- Given E field of a UPW propagating in an arbitrary direction, determine the corresponding H field and vice versa.

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EE3001 Engineering Electromagnetics

This session is about

15. Examples of UPW at Oblique Incidence

Example 1

The electric field of a UPW in **air** is

$$\vec{E} = (3\vec{a}_x - 4\vec{a}_z)\, e^{-j(80x+60z-\pi/3)} \quad \text{V/m} \qquad (21)$$

Determine the **direction of propagation**, **direction of $\boldsymbol{E}$**, and **amplitude of $\boldsymbol{E}$**.

Solution:

Air: $u_p = 1/\sqrt{\mu_o\, \varepsilon_o} = 3 \times 10^8 \ \text{m/s}; \ \eta = \sqrt{\mu_o/\varepsilon_o} = 120\pi\ \Omega$

$$\vec{E} = (3\vec{a}_x - 4\vec{a}_z)\, e^{+j\pi/3}\, e^{-j80}\ \ e^{-j60z} = \vec{a}_E\, E_o\ \ e^{-jk_x x}\, e^{-jk_y y}\, e^{-jk_z z}$$

$k_x = 80;\ k_y = 0;\ k_z = 60 \ \rightarrow \ \vec{k} = \vec{a}_x\, 80 + \vec{a}_z\, 60$

$$k = \sqrt{{k_x}^2 + {k_z}^2} = \sqrt{80^2 + 60^2} = 100$$

Example 1 Solution

- $\vec{k} = \vec{a}_x\, 80 + \vec{a}_z\, 60, \quad k = 100$
- $\vec{a}_k = \dfrac{\vec{k}}{k} = \vec{a}_x\, 0.8 + \vec{a}_z\, 0.6$ **[Direction of Propagation]**
- $(3\,\vec{a}_x - 4\,\vec{a}_z)\; e^{+j\pi/3} = \dfrac{(3\,\vec{a}_x - 4\,\vec{a}_z)}{5}\; 5\, e^{+j\pi/3} = \vec{a}_E\, E_o$
- $\vec{a}_E = \dfrac{(3\,\vec{a}_x - 4\,\vec{a}_z)}{5} = 0.6\,\vec{a}_x - 0.8\,\vec{a}_z$ **[Direction of $\boldsymbol{E}$]**
- $E_o = 5\, e^{+j\pi/3} \equiv 5\angle 60^\circ$ V/m **[Amplitude of $\boldsymbol{E}$]**

Example 1 Part II

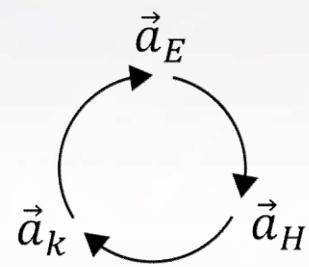
Q: What is the frequency of the wave?

$$k = 100 = \omega\sqrt{\mu\,\varepsilon} = \frac{2\pi f}{u_p} \Rightarrow f = 4.77\ \text{GHz}$$

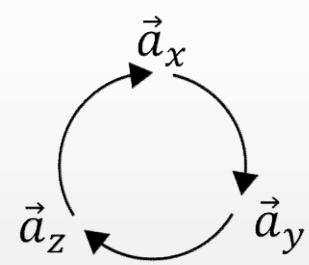
Q: Find the corresponding magnetic field.

$$\vec{E} = E_o\, e^{-j8} \quad e^{-j60}\ \vec{a}_E = 5\angle 60^\circ\, e^{-j8} \quad e^{-j60z}\ \vec{a}_E$$

$$\vec{H} = \frac{E_o}{\eta}\, e^{-j80x}\, e^{-j60z}\ \vec{a}_H = \frac{1\angle 60^\circ}{24\pi}\, e^{-j80x}\, e^{-j60z}\ \vec{a}_H$$



$$\vec{a}_H = \vec{a}_k \times \vec{a}_E = (0.8\,\vec{a}_x + 0.6\,\vec{a}_z) \times (0.6\,\vec{a}_x - 0.8\,\vec{a}_z) = \vec{a}_y$$

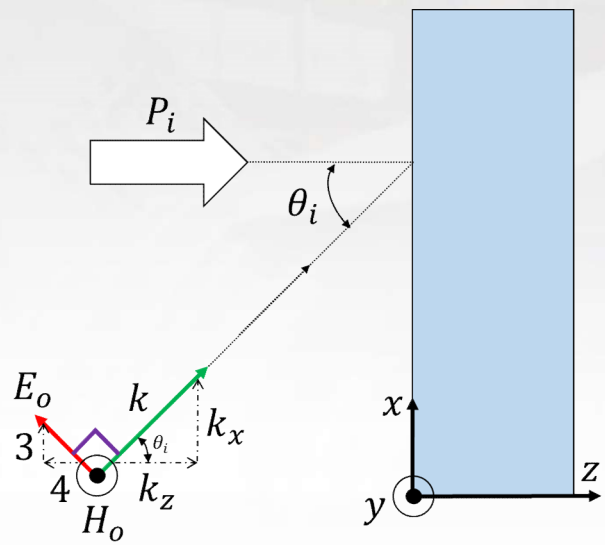


Example 1 Part III

$$\vec{k} = \vec{a}_x\, 80 + \vec{a}_z\, 60$$

$$\tan\theta_i = \frac{k_x}{k_z} = \frac{80}{60}$$

$$\theta_i = 53.1^\circ$$



Q: Find the time-average Poynting vector.

$$\vec{S}_i = \vec{a}_k \frac{|E_0|^2}{2\eta} = (\vec{a}_x\, 0.8 + \vec{a}_z\, 0.6) \times 0.033\ \mathrm{W/m^2}$$

$$= \vec{a}_x\, 0.0265 + \vec{a}_z\, 0.020\ \mathrm{W/m^2}$$

$$[S_i = 0.033]$$

$$P_i = S_i \cos\theta_i = 0.02$$

Example 2

Repeat Example 1

$$\vec{E} = j\,5\,e^{-j(80x+6\quad)}\,\vec{a}_y \quad \text{V/m} \qquad (22)$$

ANS

$\vec{a}_k = \vec{a}_x\,0.8 + \vec{a}_z\,0.6$

$E_o = 5\,\angle 90^\circ \ \text{V/m}$

$\vec{a}_E = \vec{a}_y$

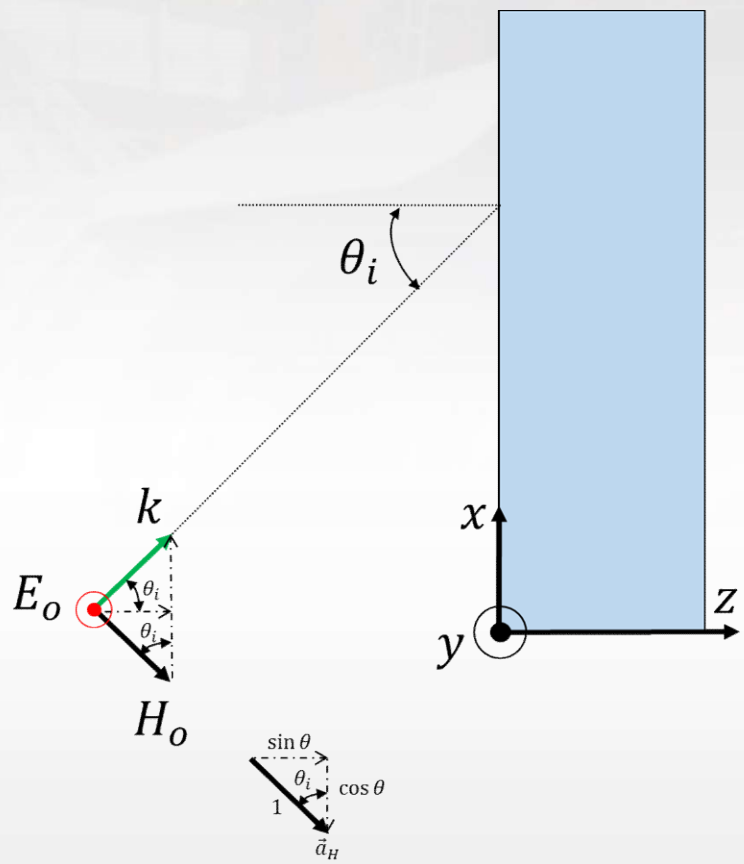
$f = 4.77 \ \text{GHz}$

$H_o = \dfrac{1}{24\pi}\,\angle 90^\circ$

$\theta_i = 53.1^\circ$

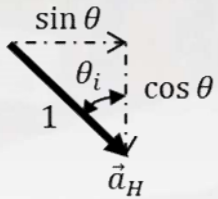
$\vec{a}_H = -\,\vec{a}_x\,0.6 + \vec{a}_z\,0.8$

$\vec{S} = \vec{a}_x\,0.0265 + \vec{a}_z\,0.020 \ \text{W/m}^2$

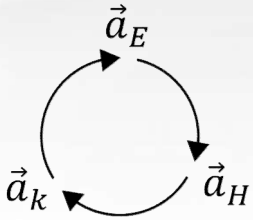


Example 2 Solution

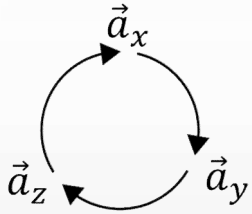
- $\vec{a}_H = \vec{a}_z \sin\theta_i - \vec{a}_x \cos\theta_i$



- $\vec{a}_H = \vec{a}_k \times \vec{a}_E = (\vec{a}_x\, 0.8 + \vec{a}_z\, 0.6) \times \vec{a}_y$
  $= \vec{a}_z\, 0.8 - \vec{a}_x\, 0.6$



- $\vec{a}_H = \vec{a}_k \times \vec{a}_E = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0.8 & 0 & 0.6 \\ 0 & 1 & 0 \end{vmatrix}$
  $= \vec{a}_z\, 0.8 - \vec{a}_x\, 0.6$



Remarks

- In the case of **oblique incidence** $(\theta_i \neq 0)$ of UPW at a plane boundary, we will restrict ourselves to **lossless Media** $(\sigma_1 = \sigma_2 = 0)$:

$$\vec{S} = \vec{a}_k \frac{|E_o|^2}{2\,\eta} \ \mathrm{W/m^2}$$

- **For normal incident** $(\theta_i = 0)$ at a plane boundary, we considered both **lossless and lossy Media** $(\mathrm{e.g.}\ \sigma_1 = 0, \sigma_2 \neq 0)$:

$$\vec{S}_2 = \frac{1}{2}\,\mathrm{Re}\left(\vec{E}_t \times \vec{H}_t^{\,*}\right) \ \mathrm{W/m^2} = \vec{S}_2(0)\, e^{-2\alpha z}$$

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EE3001 Engineering Electromagnetics

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16. UPW Oblique Incidence at a Plane Boundary

Learning Objectives

- Define plane of incidence and identify the tangential components of E and H fields at the interface;
- State the general expressions of the incident, reflected and transmitted E and H fields; and
- Apply law of reflection and Snell's law to determine the angle of reflection, angle of transmission and the direction of propagation of reflected and transmitted waves.

Oblique Incidence at a Plane Boundary

- Given $\vec{E}_i = \vec{a}_{E_i} E_{oi} e^{-jk_{xi}x} e^{-jk_{zi}z}$
- $\vec{a}_{ki}$ $\vec{a}_{E_i}, E_{oi}$ and $\theta_i = \tan^{-1}\frac{k_{xi}}{k_{zi}}$
- $\vec{k}_i \rightarrow \vec{k}_r$ & $\vec{k}_t$
- $\theta_i \longrightarrow \theta_r$ & θ_t
- $k_i = k_r = k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$
- $k_t = k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$

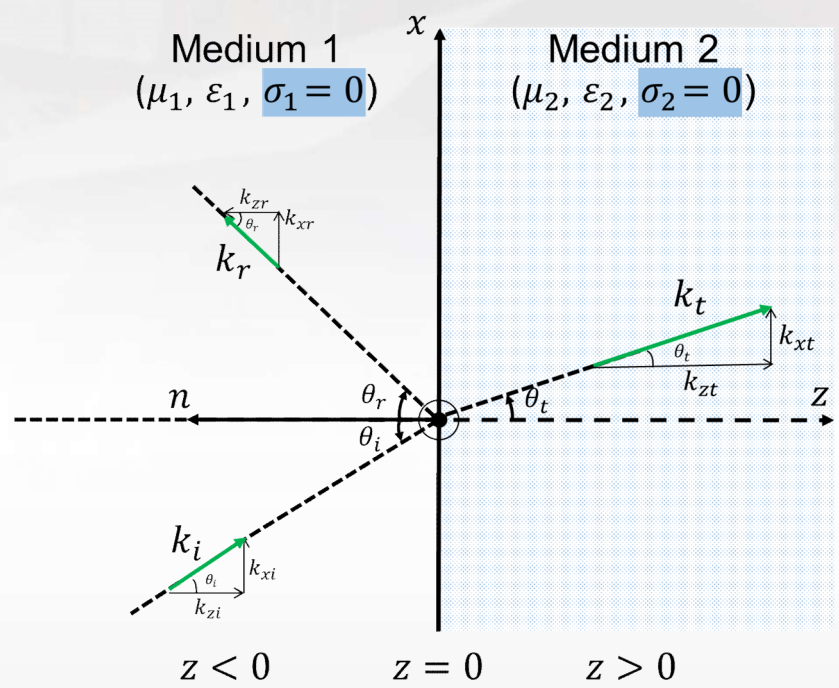


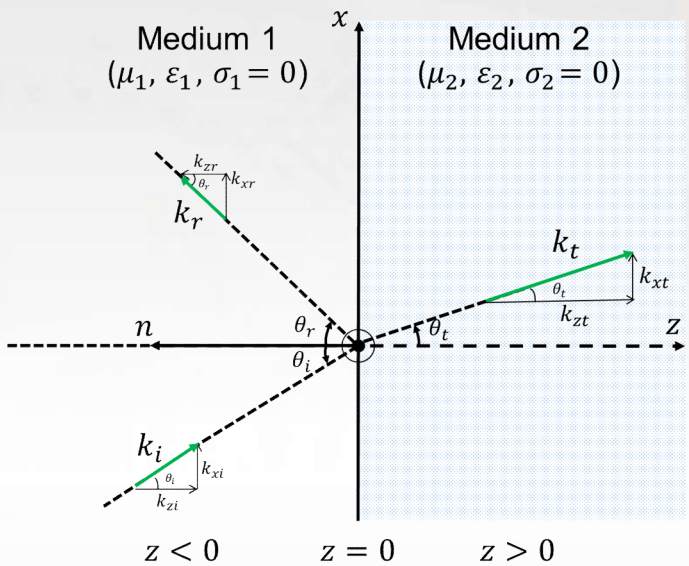
Figure 3.4 $[k_{yi} = 0]$

Oblique Incidence at a Plane Boundary

Refer to Figure 3.4:

- $\vec{k}_i, \vec{k}_r$ and $\vec{k}_t$ (θ_i, θ_r and θ_t) all lie in xz –plane – referred to as "**Plane of Incidence**".
- Plane of Incidence $\equiv xz -$ plane
- Boundary @ $z = 0$: $xy -$ plane
- Tangential components $\equiv \vec{a}_x$ and $\vec{a}_y$ components
- Normal Component $\equiv \vec{a}_z$

Incident, Reflected and Transmitted Waves

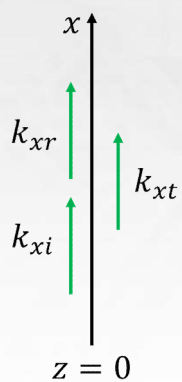


- $\vec{E}_i = \vec{a}_{E_i} E_{oi} \, e^{-jk_{xi}x} \, e^{-j \ \ zi z}$, $\quad \vec{E}_t = \vec{a}_{E_t} E_{ot} \, e^{-jk_{xt}x} \, e^{-j \ \ zt z}$

- $\vec{E}_r = \vec{a}_{E_r} E_{or} \, e^{-jk_{xr}x} \, e^{+jk_{zr}z}$

Boundary Conditions

- **Boundary conditions at** $z = 0$:
- $\vec{E}_i = \vec{a}_{E_i} E_{oi} \, e^{-jk_{xi}x}$
- $\vec{E}_r = \vec{a}_{E_r} E_{or} \, e^{-j \ \ xr x}$
- $\vec{E}_t = \vec{a}_{E_t} E_{ot} \, e^{-j \ \ xt x}$



$$u_{pi} = \frac{\omega}{k_{xi}}$$

$$u_{pr} = \frac{\omega}{k_{xr}}$$

$$u_{pt} = \frac{\omega}{k_{xt}}$$

- At $z = 0$, incident, reflected and transmitted waves all travel along the $+x$ direction.
- **Boundary conditions @** $\boldsymbol{z = 0}$: $\vec{E}_{\tan 1} = \vec{E}_{\tan 2} \qquad \forall -\infty < x < \infty$

  The only way the boundary condition will be <u>satisfied for all x</u> is that all the waves must travel at exactly the same velocity i.e.

$$k_{xi} = k_{xr} = k_{xt} \qquad [e^{-j \ \ xi x} = e^{-jk_{xr}x} = e^{-jk_{xt}x}]$$

Law of Reflection and Snell's Law

$$\boxed{k_{xi} = k_{xr} = k_{xt}} \qquad (24)$$

- $k_{xi} = k_{xr} \longrightarrow \; k_i \sin(\theta_i) = k_r \sin(\theta_r) \qquad [k_i = k_r = \omega\sqrt{\mu_1 \varepsilon_1}]$

$$\therefore \boxed{\theta_r = \theta_i} \qquad \textbf{[Law of Reflection]} \qquad (25)$$

- $k_{xi} = k_{xt} \longrightarrow \; k_i \sin(\theta_i) = k_t \sin(\theta_t) \qquad [k_t = \omega\sqrt{\mu_2 \varepsilon_2}]$

$$\boxed{\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{k_i}{k_t} = \sqrt{\frac{\mu_1\ \varepsilon_1}{\mu_2\ \varepsilon_2}}} \qquad \textbf{[Snell's Law]} \qquad (26)$$

Law of Reflection and Snell's Law

- We have derived **Law of Reflection** and **Snell's Law** from a consideration of **direction of propagation** of incident, reflected and transmitted waves. **No mentioned** has been made on the **polarisation** of the waves. Thus,

Law of Reflection and Snell's Law are independent of wave polarisation.

- It is important to note that:

Law of Reflection and **Snell's Law** are **general**. They apply to **any two media**, and for **any angle of incidence**.

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EE3001 Engineering Electromagnetics

This session is about

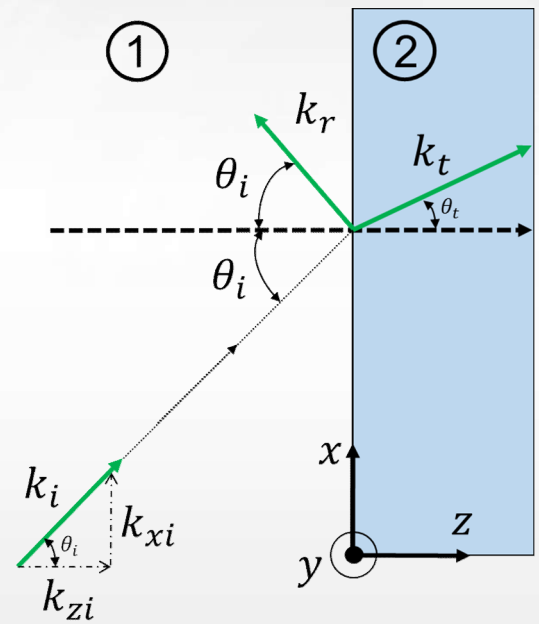
16. UPW at Oblique Incidence at a Plane Boundary (2)

Example

A uniform plane wave travelling in free space ($\mu_1 = \mu_o, \varepsilon_1 = \varepsilon_o$) occupying the region $z \leq 0$ is incident at a plane interface with a lossless dielectric medium ($\mu_2 = \mu_o, \varepsilon_2 = 4\varepsilon_o$) occupying the region $z \geq 0$. The electric field of the UPW is given by

$$\vec{E}_i = j\,5\,e^{-j80}\;\; e^{-j60z}\,\vec{a}_y \;\; \text{V/m}$$

Determine $\theta_i,\ \theta_t,\ \vec{k}_i,\ \vec{k}_r\ \&\ \vec{k}_t$.



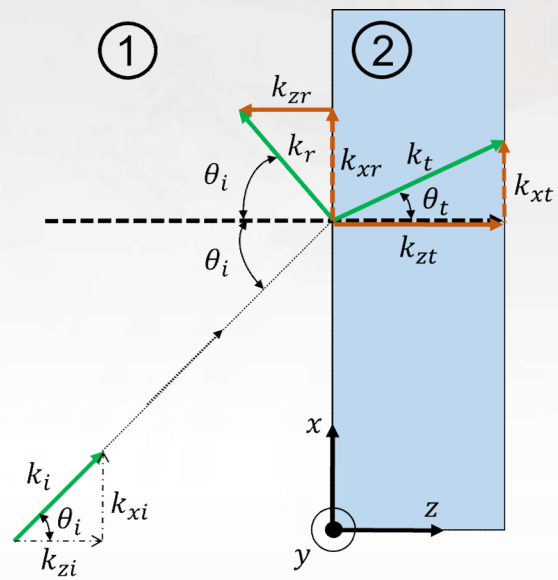
Example Solution

- **Ans:** $[(\mu_1 = \mu_o,\ \varepsilon_1 = \varepsilon_o);\ (\mu_2 = \mu_o,\ \varepsilon_2 = 4\varepsilon_o)]$
- $\vec{k}_i = \vec{a}_x\, k_{xi} + \vec{a}_z\, k_{zi} = \vec{a}_x\, 80 + \vec{a}_z\, 60$
- $\tan\theta_i = \frac{k_{xi}}{k_{zi}} = \frac{80}{60} \rightarrow \theta_i = 53.1°$
- Snell's Law: $\frac{\sin(\theta_t)}{\sin(\theta_i)} = \sqrt{\frac{\mu_1\,\varepsilon_1}{\mu_2\,\varepsilon_2}} = \frac{1}{2} \rightarrow \theta_t = 23.6°$ $[\theta_i = 53.1°]$
- $k_1 = k_r = k_i = \sqrt{k_{xi}^2 + k_{zi}^2} = 100 = \omega\sqrt{\mu_1\,\varepsilon_1} \rightarrow \omega = 3 \times 10^{10}\ \text{rad/s}$
- $k_2 = k_t = \omega\sqrt{\mu_2\,\varepsilon_2} = 3 \times 10^{10}\sqrt{\mu_o\, 4\,\varepsilon_o} = 200$

Example Solution

$k_1 \Rightarrow k_r$

$k_2 \Rightarrow k_t$



- $\vec{k}_i = \vec{a}_x\, 80 + \vec{a}_z\, 60$
- $\vec{k}_r = \vec{a}_x\, k_1 \sin\theta_i - \vec{a}_z\, k_1 \cos\theta_i = \vec{a}_x\, 80 - \vec{a}_z\, 60 \quad [k_1 = 100]$
- $\vec{k}_t = \vec{a}_x\, k_2 \sin\theta_t + \vec{a}_z\, k_2 \cos\theta_t = \vec{a}_x\, 80 + \vec{a}_z\, 183 \quad [k_2 = 200]$

Supplementary: NIS

Medium 1 $(\mu_1,\ \varepsilon_1,\ \sigma_1 = 0)$ – **Medium 2** $(\mu_2,\ \varepsilon_2,\ \sigma_2 \neq 0)$

$$\varepsilon_2 \rightarrow \varepsilon_{2c} = \varepsilon_2\left(1 - j\frac{\sigma_2}{\omega\ \varepsilon_2}\right);\ \ k_2 = \omega\sqrt{\mu_2\ \varepsilon_{2c}}$$

Law of reflection $(\theta_r = \theta_i)$ and Snell's Law would be true in general but the transmission angle θ_t becomes a **complex number**, thereby losing its simple geometrical interpretation.

- $\sin\theta_t = \sqrt{\frac{\mu_1\ \varepsilon_1}{\mu_2\ \varepsilon_{2c}}}\sin\theta_i;\ \cos\theta_t = \sqrt{1 - \sin^2\theta_t}$
- $\vec{E}_t = \vec{a}_{E_t}\ E_{ot}\ e^{-jk_{xt}x}\ e^{-jk_{zt}z} = \vec{a}_{E_t}\ E_{ot}\ e^{-jk_t\sin\theta_t x}\ e^{-jk_t\cos\theta_t z}$
- $\vec{E}_t = \vec{a}_{E_t}\ E_{ot}\ e^{-jk_{xt}x}\ e^{-j(k'_{zt} - jk''_{zt})z} = \vec{a}_{E_t}\ E_{ot}\ e^{-k''_{zt}z}\ e^{-jk_{xt}x}\ e^{-jk'_{zt}z}$

$$\boxed{\tan\theta_t = \frac{k_{xt}}{k'_{zt}}}$$

$[k_{xt} = k_{xi}]$

Summary

- Define plane of incidence and identify the tangential components of E and H fields at the interface.
- State the general expressions of the incident, reflected and transmitted E and H fields.
- Apply law of reflection and Snell's law to determine the angle of reflection, angle of transmission and the direction of propagation of reflected and transmitted waves.
- Law of Reflection and Snell's Law are general. They apply to any two media, and for any angle of incidence.

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EE3001 Engineering Electromagnetics

This session is about

17. Polarisation of UPW with respect to the Plane of Incidence

Learning Objectives

- Define parallel and perpendicular polarisation of UPW with respect to the plane incidence;
- Determine the parallel and perpendicular polarised components of a UPW at oblique incidence; and
- Identify polarisation (linear, circular or elliptical) of UPW from the parallel and perpendicular polarised components.

Oblique Incidence at a Plane Boundary

- **Plane of Incidence** $\equiv xz -$ plane

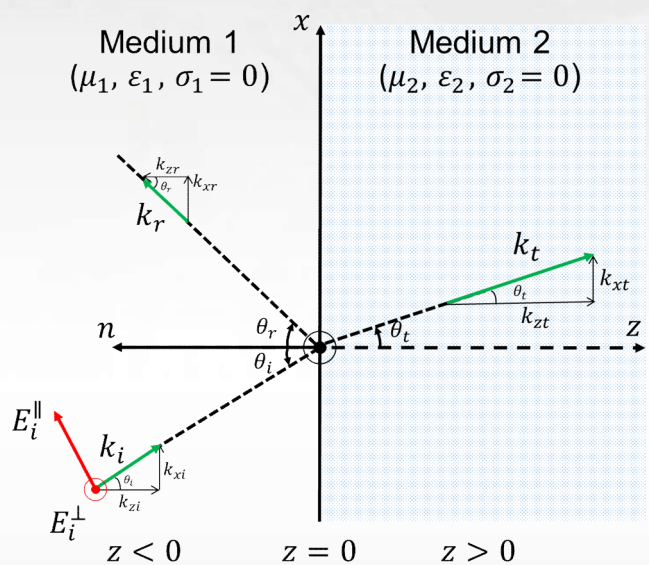
- We have shown that the **direction of propagation** of reflected and transmitted waves ($\vec{k}_r$ and $\vec{k}_t$) **do not depend on polarisation of the incident wave**:

$$\theta_i \rightarrow \theta_r \,\&\, \theta_t; \quad \vec{k}_i \rightarrow \vec{k}_r \,\&\, \vec{k}_t$$

- We are going to show that the **Amplitude** of the reflected and transmitted waves (E_{or} and E_{ot}) **depend on the polarisation of the incident wave with respect to the Plane of Incidence** i.e. whether direction of electric field $\vec{a}_{E_i}$ is $\parallel$ or $\perp$ with respect to the Plane of Incidence.

Plane of Incidence

- **Plane of Incidence** is the plane formed by $\vec{k}_i$, $\vec{k}_r$ and $\vec{k}_t$.



Plane of Incidence $\equiv xz -$ plane $\quad [k_y = 0]$

Parallel and Perpendicular Polarisation

- $\parallel$ – **Polarisation:** $\vec{a}_{E_i}$ is in the xz – plane

$$\vec{E}_i^{\parallel} = (3\,\vec{a}_x - 4\,\vec{a}_z)\, e^{-j(80x+60z-\pi/3)} \qquad (21) \qquad \text{[Page 15-2]}$$

- $\perp$ – **Polarisation:** $\vec{a}_{E_i} = \vec{a}_y$

$$\vec{E}_i^{\perp} = j\,5\,\vec{a}_y\, e^{-j(80x+6\quad)} \qquad (22) \qquad \text{[Page 15-6]}$$

REMARKS

- Incidence, reflected and transmitted waves all travel parallel to the xz – plane i.e. Plane of Incidence $\equiv$ xz – plane .
- $\parallel$ – or $\perp$ – polarisation refers to the direction of E with respect to the plane of incidence.

Polarisation of UPW (Arbitrary Direction)

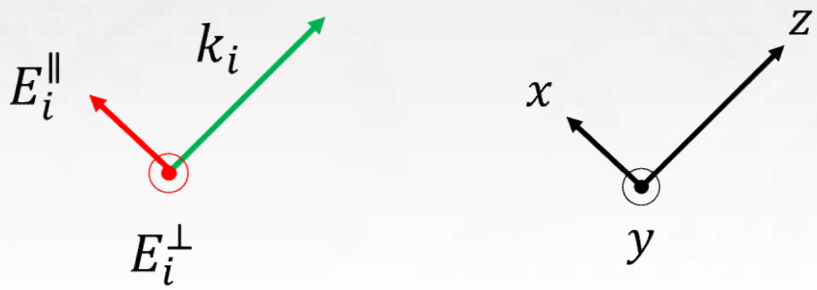
Polarisation (Linear, Circular or Elliptical) of a UPW propagates in an arbitrary direction

Given any incident wave, we can easily resolve it into ∥ – and ⊥ – polarised components:

- $\vec{E}_i = \left(3\, e^{j\pi/3}\, \vec{a}_x + j\, 5\, \vec{a}_y - 4\, e^{j\pi/3} \vec{a}_z\right) e^{-j(80x+60z)} = \vec{E}_i^{\parallel} + \vec{E}_i^{\perp}$ $\quad [k_y = 0]$
- $\vec{E}_i = \left(3\, e^{j\pi/3}\, \vec{a}_x - 4\, e^{j\pi/3} \vec{a}_z\right) e^{-j(80x+60z)} + \left(j\, 5\, \vec{a}_y\right) e^{-j(80x+60z)}$
- $\vec{E}_i^{\parallel} = \left(3\, e^{j\pi/3}\, \vec{a}_x - 4\, e^{j\pi/3} \vec{a}_z\right) e^{-j(80x+60z)}$ [Page 15-2: $E_{oi}^{\parallel} = 5\,\angle 60^{\circ}$]
- $\vec{E}_i^{\perp} = j\, 5\, \vec{a}_y\, e^{-j(80x+60z)}$ [Page 15-6: $E_{oi}^{\perp} = 5\,\angle 90^{\circ}$]

Polarisation of UPW

Polarisation (Linear, Circular or Elliptical) of a UPW



$\lvert E_o^{\parallel}\rvert = 0$ OR $\lvert E_o^{\perp}\rvert = 0$	→ Linear.
$\lvert \phi_{\perp} - \phi_{\parallel}\rvert = 0, 180^{\circ}$	→ Linear.
$\lvert \phi_{\perp} - \phi_{\parallel}\rvert = 90^{\circ}$ <u>AND</u> $\lvert E_o^{\parallel}\rvert = \lvert E_o^{\perp}\rvert$	→ Circular.
Other	→ Elliptical.

[Page 4-13]

Example

Determine the polarisation (linear, circular or elliptical) of the following UPW:

(i) $\vec{E}_i = \left(3\, e^{j\pi/3}\, \vec{a}_x + j\, 5\, \vec{a}_y - 4\, e^{j\pi/3}\, \vec{a}_z\right) e^{-j(80x+60z)}$

(ii) $\vec{E}_i = \left(3\, \vec{a}_x + j\, 5\, \vec{a}_y - 4\, \vec{a}_z\right) e^{-j(80x+60z)}$

(iii) $\vec{E}_i = \left(3\, \vec{a}_x + 5\, \vec{a}_y - 4\, \vec{a}_z\right) e^{-j(80x+6\quad)}$

ANS:

(i) $E_{oi}^{\parallel} = 5\,\angle 60°$ & $E_{oi}^{\perp} = 5\,\angle 90°$ (Elliptical)

(ii) $E_{oi}^{\parallel} = 5\,\angle 0°$ & $E_{oi}^{\perp} = 5\,\angle 90°$ (Circular)

(iii) $E_{oi}^{\parallel} = 5\,\angle 0°$ & $E_{oi}^{\perp} = 5\,\angle 0°$ (Linear)

Summary

- The amplitude of the reflected and transmitted wave (E_{or} and E_{ot}) depend on the polarisation of the incident wave with respect to the Plane of Incidence i.e. whether direction of electric field $\vec{a}_{E_i}$ is ∥ or ⊥ with respect to the Plane of Incidence.

- Determine the parallel and perpendicular polarised components of the incident wave at oblique incidence.

- Identify polarisation (linear, circular or elliptical) from the parallel and perpendicular polarised components.

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This session is about

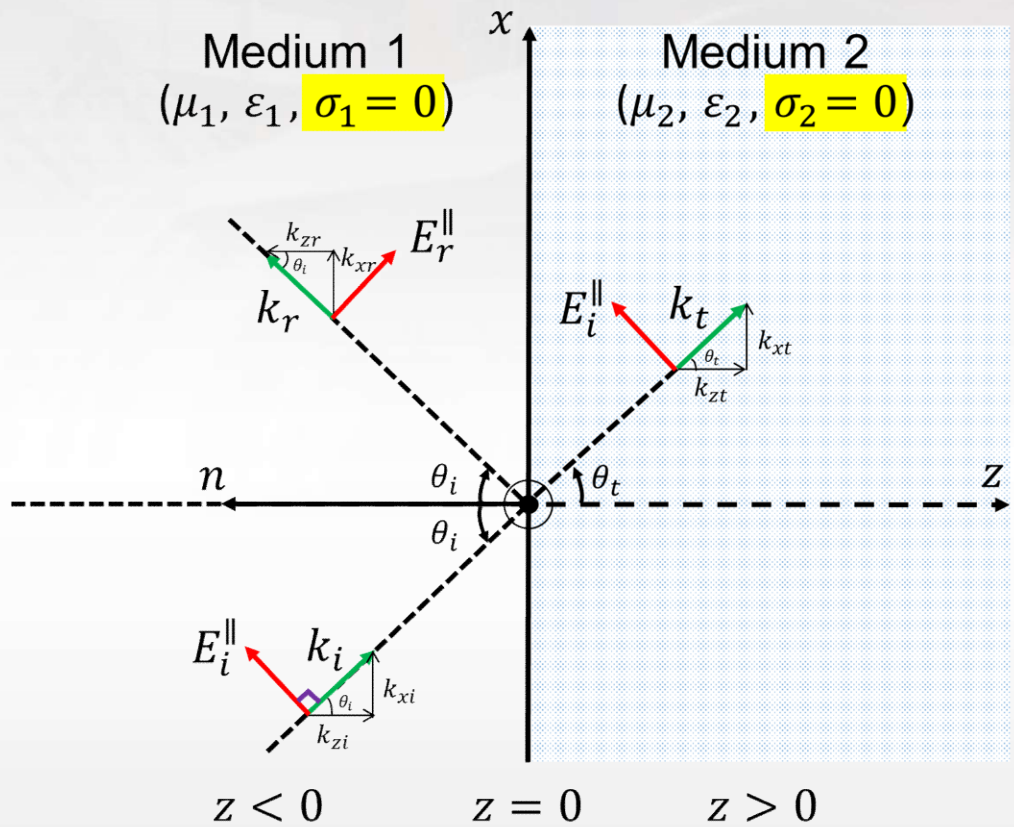
18. Oblique Incidence: Parallel Polarisation

Learning Objectives

- Determine the directions of reflected and transmitted E field, i.e. $\vec{a}_{Er}^{\parallel}$ and $\vec{a}_{Et}^{\parallel}$, for an incident UPW with parallel polarisation;
- Apply the reflection and transmission coefficients, i.e. $\Gamma_{\parallel}$ and $\tau_{\parallel}$, to obtain the amplitudes of the reflected and transmitted E field at the interface, i.e. $E_{or}^{\parallel}$ and $E_{ot}^{\parallel}$;
- Determine the reflected and transmitted E field, i.e. $\vec{E}_r^{\parallel}$ and $\vec{E}_t^{\parallel}$; and
- Obtain $\vec{H}_r^{\parallel}$ and $\vec{H}_t^{\parallel}$ from $\vec{E}_r^{\parallel}$ and $\vec{E}_t^{\parallel}$ respectively.

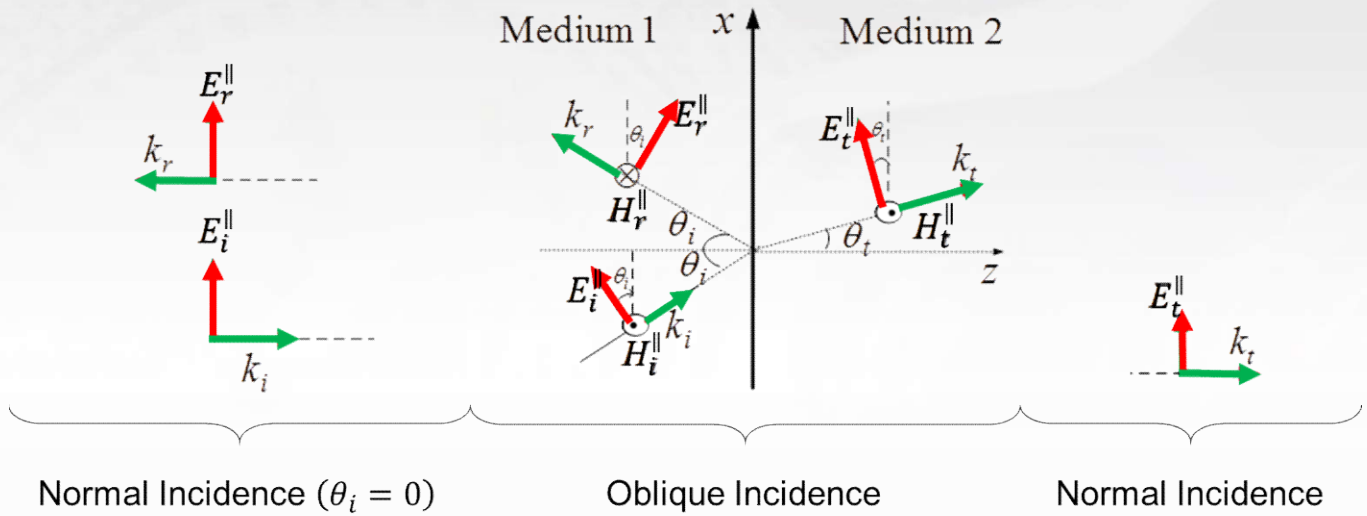
Parallel Polarisation

- $\vec{E}_i^{\parallel} = \vec{a}_{Ei}^{\parallel}\ E_{oi}^{\parallel}\ e^{-jk_{xi}x}\ e^{-jk_{zi}z}$
- $\vec{k}_i,\ \vec{a}_{Ei}^{\parallel},\ E_{oi}^{\parallel}$
- $\vec{k}_i \rightarrow \vec{k}_r\ \&\ \vec{k}_t$
- $\vec{a}_{Ei}^{\parallel} \rightarrow \vec{a}_{Er}^{\parallel}\ \&\ \vec{a}_{Et}^{\parallel}$
- $E_{oi}^{\parallel} \rightarrow E_{or}^{\parallel} = \Gamma_{\parallel}\ E_{oi}^{\parallel}\ \&\ E_{ot}^{\parallel} = \tau_{\parallel}\ E_{oi}^{\parallel}$



Parallel Polarisation

Parallel Polarisation: e.g. $\vec{E}_i^{\parallel} = \left(3\, e^{j\pi/3}\, \vec{a}_x - 4\, e^{j\pi/3}\, \vec{a}_z\right) e^{-j(80x+60z)}$



Problem: Given $\vec{E}_i^{\parallel} = \vec{a}_{E_i}^{\parallel}\, E_{oi}^{\parallel}\, e^{-jk_{xi}x}\, e^{-jk_{zi}z}$, find

$\vec{E}_r^{\parallel} = \vec{a}_{E_r}^{\parallel}\, E_{or}^{\parallel}\, e^{-jk_{xr}x}\, e^{+jk_{zr}z}$ and $\vec{E}_t^{\parallel} = \vec{a}_{E_t}^{\parallel}\, E_{ot}^{\parallel}\, e^{-jk_{xt}x}\, e^{-j\ \ _{zt}z}$

Parallel Polarisation

- Step 1: Given $\vec{E}_i^{\parallel} = \vec{a}_{E_i}^{\parallel} E_{oi}^{\parallel} e^{-jk_{xi}x} e^{-jk_{zi}z}$ (21) [Page 15-2]

  Determine $\vec{k}_i$, $\vec{a}_{E_i}^{\parallel}$, $E_{oi}^{\parallel}$

- Step 2: $\vec{k}_i = \vec{a}_x k_{xi} + \vec{a}_z k_{zi} \rightarrow \tan\theta_i = \frac{k_{xi}}{k_{zi}}$ θ_i [Page 16-9]

  Snell's Law: $\rightarrow \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}}$ θ_t

  $\vec{k}_r = \vec{a}_x k_r \sin\theta_i - \vec{a}_z k_r \cos\theta_i$ $[k_1 = \omega\sqrt{\mu_1 \varepsilon_1} = k_i = k_r]$

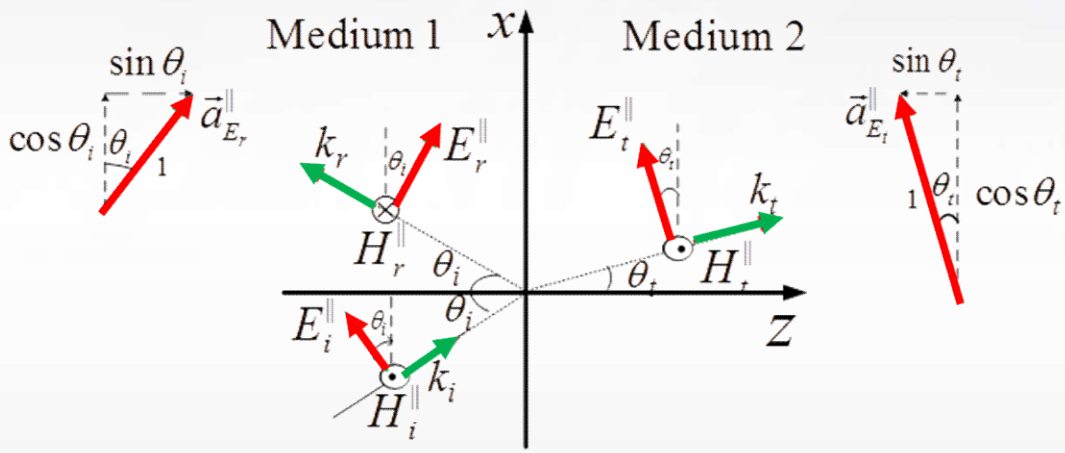
  $\vec{k}_t = \vec{a}_x k_t \sin\theta_t + \vec{a}_z k_t \cos\theta_t$ $[k_2 = \omega\sqrt{\mu_2 \varepsilon_2} = k_t]$

  $k_{xr}, k_{zr}, k_{xt}, k_{zt}$

Parallel Polarisation

- Step 3: $\vec{a}_{E_i}^{\parallel} \rightarrow \vec{a}_{E_r}^{\parallel}$ & $\vec{a}_{E_t}^{\parallel}$

$$\vec{E}_r^{\parallel} = \vec{a}_{E_r}^{\parallel} E_{or}^{\parallel} e^{-jk_{xr}x} e^{+jk_{zr}z} \text{ and } \vec{E}_t^{\parallel} = \vec{a}_{E_t}^{\parallel} E_{ot}^{\parallel} e^{-jk_{xt}x} e^{-j\ zt z}$$



$$\vec{a}_{E_r}^{\parallel} = \vec{a}_x \cos\theta_i + \vec{a}_z \sin\theta_i, \qquad \vec{a}_{E_t}^{\parallel} = \vec{a}_x \cos\theta_t - \vec{a}_z \sin\theta_t$$

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This session is about

18. Oblique Incidence: Parallel Polarisation (2)

Parallel Polarisation

- Step 4: $E_{oi}^{\parallel} \rightarrow E_{or}^{\parallel} = \Gamma_{\parallel}\, E_{oi}^{\parallel},\ E_{ot}^{\parallel} = \tau_{\parallel}\, E_{oi}^{\parallel}$

$$\vec{E}_r^{\parallel} = \vec{a}_{E_r}^{\parallel}\, E_{or}^{\parallel}\, e^{-jk_{xr}x}\, e^{+jk_{zr}z} \text{ and } \vec{E}_t^{\parallel} = \vec{a}_{E_t}^{\parallel}\, E_{ot}^{\parallel}\, e^{-jk_{xt}x}\, e^{-jk_{zt}z}$$

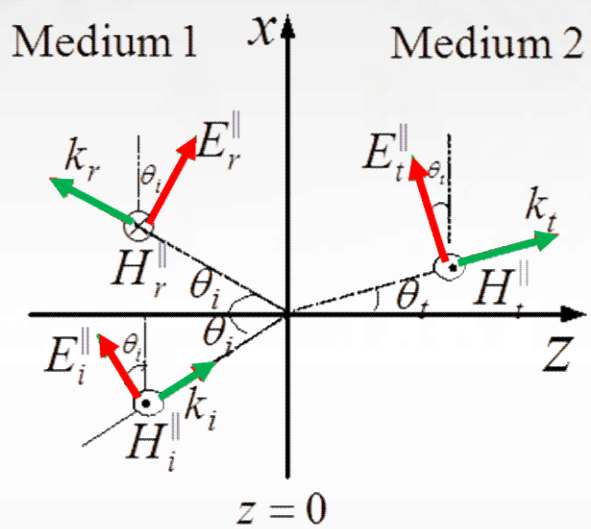
- $\Gamma_{\parallel}$ and $\tau_{\parallel}$ can be determined using **Boundary Conditions** i.e. **tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are continuous at the boundary** $(z = 0)$.

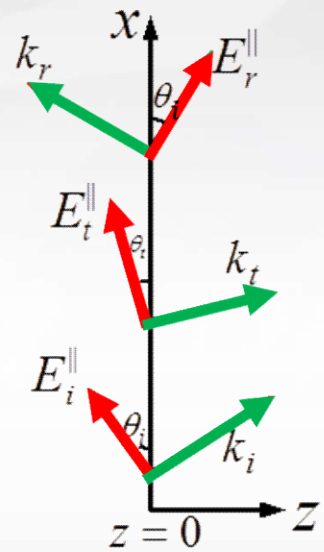
- Tangential components $\equiv \vec{a}_x$ and $\vec{a}_y$ components

- $\boxed{k_{xi} = k_{xr} = k_{xt}} \rightarrow e^{-jk_{xi}x} = e^{-jk_{xr}x} = e^{-j\ \ _{xt}x}$ [Page 16-7]

Parallel Polarisation: Tangential Components

- **Parallel Polarisation**:





- Tangential components $\equiv \vec{a}_x$ and $\vec{a}_y$ components

Example: $\vec{E}^{\parallel}_{\tan 2} = \vec{a}_x \, E^{\parallel}_{ot} \cos\theta_t \; e^{-j \;\; xt x} \; e^{-jk_{zt}z}$

Parallel Polarisation: Tangential Components

Medium 1:

- $\vec{E}_{\tan 1} = \vec{a}_x E_{oi}^{\parallel} \cos\theta_i \; e^{-jk_{xi}x} \, e^{-jk_{zi}z} + \vec{a}_x E_{or}^{\parallel} \cos\theta_i \; e^{-jk_{xr}x} \, e^{+jk_{zr}z}$

$$\vec{H}_{\tan 1} = \vec{a}_y \frac{E_{oi}^{\parallel}}{\eta_1} e^{-jk_{xi}x} \, e^{-j \;\; {}_{zi}z} - \vec{a}_y \frac{E_{or}^{\parallel}}{\eta_1} e^{-jk_{xr}x} \, e^{+jk_{zr}z}$$

Medium 2:

- $\vec{E}_{\tan 2}^{\parallel} = \vec{a}_x E_{ot}^{\parallel} \cos\theta_t \; e^{-jk_{xt}x} \, e^{-jk_{zt}z}$

$$\vec{H}_{\tan 2}^{\parallel} = \vec{a}_y \frac{E_{ot}^{\parallel}}{\eta_2} e^{-jk_{xt}x} \, e^{-jk_{zt}z}$$

- $\boxed{k_{xi} = k_{xr} = k_{xt}} \rightarrow e^{-jk_{xi}x} = e^{-jk_{xr}x} = e^{-jk_{xt}x}$ [Page 16-7]

Parallel Polarisation: Boundary Conditions

Boundary Conditions $(z = 0)$**:**

- $\vec{E}_{\tan 1} = \vec{E}_{\tan 2}$

$$E_{oi}^{\parallel} \cos\theta_i + E_{or}^{\parallel} \cos\theta_i = E_{ot}^{\parallel} \cos\theta_t \qquad (27)$$

- $\vec{H}_{\tan} = \vec{H}_{\tan 2}$

$$\frac{E_{oi}^{\parallel}}{\eta_1} - \frac{E_{or}^{\parallel}}{\eta_1} = \frac{E_{ot}^{\parallel}}{\eta_2} \qquad (28)$$

- If η_1, η_2, $E_{oi}^{\parallel}$, $\theta_i \rightarrow \theta_t$, are known, $E_{or}^{\parallel}$ and $E_{ot}^{\parallel}$ can be solved using (27) and (28) to give:

Reflection and Transmission Coefficients

$$E_{or}^{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} E_{oi}^{\parallel} = \Gamma_{\parallel} E_{oi}^{\parallel}$$

$$E_{ot}^{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} E_{oi}^{\parallel} = \tau_{\parallel} E_{oi}^{\parallel}$$

$$\boxed{\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}} = \frac{E_{or}^{\parallel}}{E_{oi}^{\parallel}} \qquad (29)$$

$$\boxed{\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}} = \frac{E_{ot}^{\parallel}}{E_{oi}^{\parallel}} \qquad (30)$$

- Equations (29) and (30) are called **Fresnel's equations**.

$$\Gamma_{\parallel}, \tau_{\parallel} = f\ (\eta_1, \eta_2, \theta_i, \theta_t)$$

Reflection and Transmission Coefficients

- (29) and (30) are given in the Appendix.

- <u>At normal incidence</u>: $\theta_i = \theta_t = 0$, $\Gamma_\parallel$ and $\tau_\parallel$ reduce to Γ and τ given in equations (3) and (4), respectively.

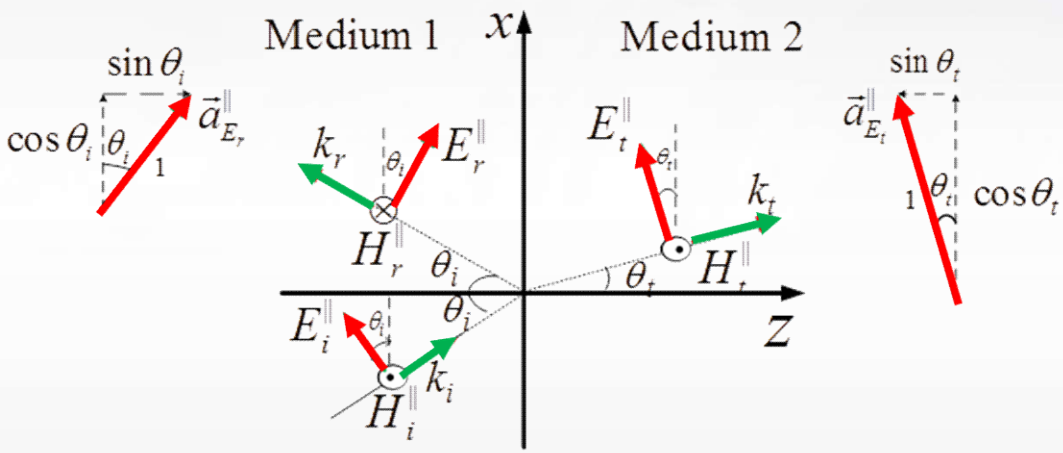
$$\Gamma_\parallel = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{3}$$

$$\tau_\parallel = \tau = \frac{2\eta_2}{\eta_2 + \eta_1} \tag{4}$$

Parallel Polarisation: Numerical Example

Given $\vec{E}_i^{\parallel} = \left(3\, e^{j\pi/3}\, \vec{a}_x - 4\, e^{j\pi/3}\, \vec{a}_z\right) e^{-j(80x+60z)}$, find $\vec{E}_t^{\parallel}$

- $\vec{E}_t^{\parallel} = \vec{a}_{Et}^{\parallel}\, E_{ot}^{\parallel}\, e^{-j\ \ _{xt}x}\, e^{-jk_{zt}z}$



Parallel Polarisation: Numerical Example

- $\vec{E}_i^{\parallel} = \left(3e^{j\pi/3}\,\vec{a}_x - 4\,e^{j\pi/3}\,\vec{a}_z\right) e^{-j(80x+60z)} \rightarrow \vec{E}_t^{\parallel} = \vec{a}_{E_t}^{\parallel}\,E_{ot}^{\parallel}\,e^{-j(k_{xt}x+k_{zt}z)}$

Step 1: $\vec{k}_i = \vec{a}_x\,80 + \vec{a}_z\,60,\;\; \vec{a}_{E_i}^{\parallel} = \vec{a}_x\,0.6 - \vec{a}_z\,0.8,\;\; E_{oi}^{\parallel} = 5\angle\pi/3$

Step 2: $\vec{k}_i = \vec{a}_x\,k_{xi} + \vec{a}_z\,k_{zi} \rightarrow \vec{k}_t = \vec{a}_x\,k_{xt} + \vec{a}_z\,k_{zt}$ [Page 16-9]

[$\theta_i = \tan^{-1}(\frac{k_{xi}}{k_{zi}}) = 53.1° \rightarrow \theta_t$ using Snell's Law]

[$k_{xt} = k_2 \sin\theta_t,\;\; k_{zt} = k_2\,\cos\theta_t$]

Step 3: $\vec{a}_{E_i}^{\parallel} \rightarrow \vec{a}_{E_t}^{\parallel} = \vec{a}_x\,\cos\theta_t - \vec{a}_z\,\sin\theta_t$ [Page 18-6]

Step 4: $\vec{E}_{oi}^{\parallel} \rightarrow \vec{E}_{ot}^{\parallel} = \tau_{\parallel}\,\vec{E}_{oi}^{\parallel}$ [$\tau_{\parallel} = f\,(\eta_1, \eta_2, \theta_i, \theta_t)$] [Page 18-11]

Summary

- Determine the directions of reflected and transmitted E field, i.e. $\vec{a}_{Er}^{\parallel}$ and $\vec{a}_{Et}^{\parallel}$, for an incident UPW with parallel polarisation;
- Apply the reflection and transmission coefficients, i.e. $\Gamma_{\parallel}$ and $\tau_{\parallel}$, to obtain the amplitudes of the reflected and transmitted E field at the interface, i.e. $E_{or}^{\parallel}$ and $E_{ot}^{\parallel}$;
- Determine the reflected and transmitted E field, i.e. $\vec{E}_r^{\parallel}$ and $\vec{E}_t^{\parallel}$; and
- Obtain $\vec{H}_r^{\parallel}$ and $\vec{H}_t^{\parallel}$ from $\vec{E}_r^{\parallel}$ and $\vec{E}_t^{\parallel}$ respectively.

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This session is about

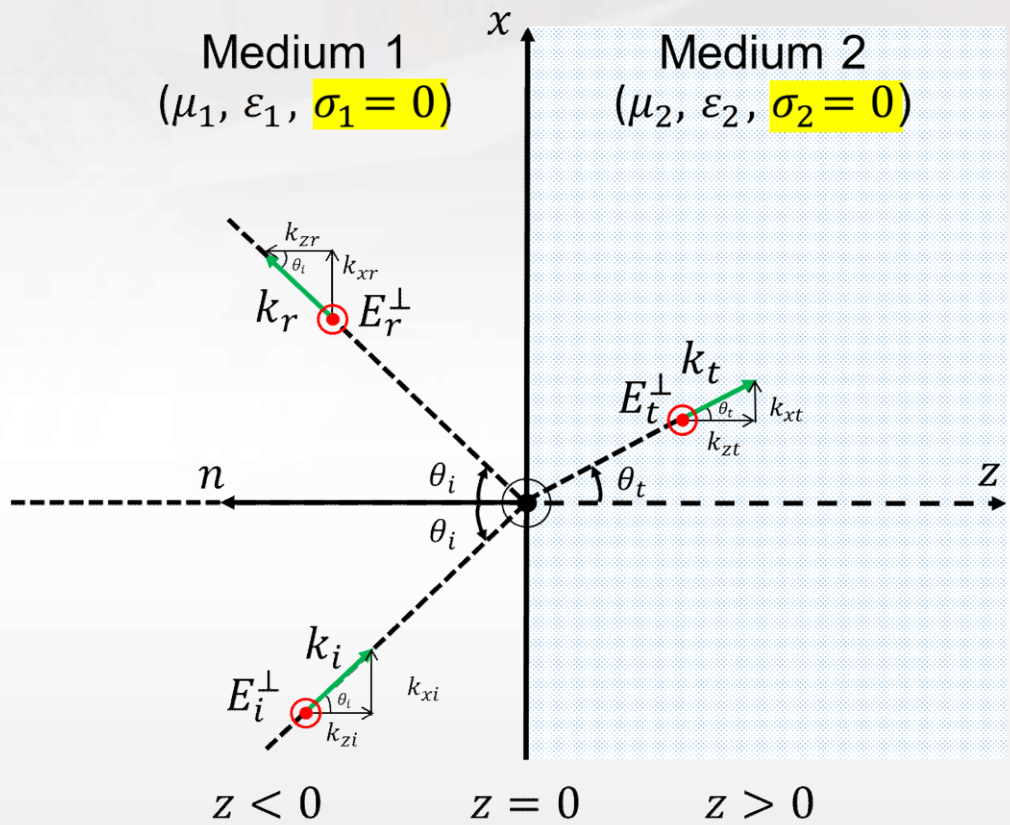
19. Oblique Incidence: Perpendicular Polarisation

Learning Objectives

- Determine the directions of reflected and transmitted E field, i.e. $\vec{a}_{Er}^{\perp}$ and $\vec{a}_{Et}^{\perp}$, for an incident UPW with perpendicular polarisation;
- Apply the reflection and transmission coefficients, i.e. $\Gamma_{\perp}$ and $\tau_{\perp}$, to obtain the amplitudes of the reflected and transmitted E field at the interface, i.e. $E_{or}^{\perp}$ and $E_{ot}^{\perp}$;
- Determine the reflected and transmitted E field, i.e. $\vec{E}_r^{\perp}$ and $\vec{E}_t^{\perp}$; and
- Obtain $\vec{H}_r^{\perp}$ and $\vec{H}_t^{\perp}$ from $\vec{E}_r^{\perp}$ and $\vec{E}_t^{\perp}$ respectively.

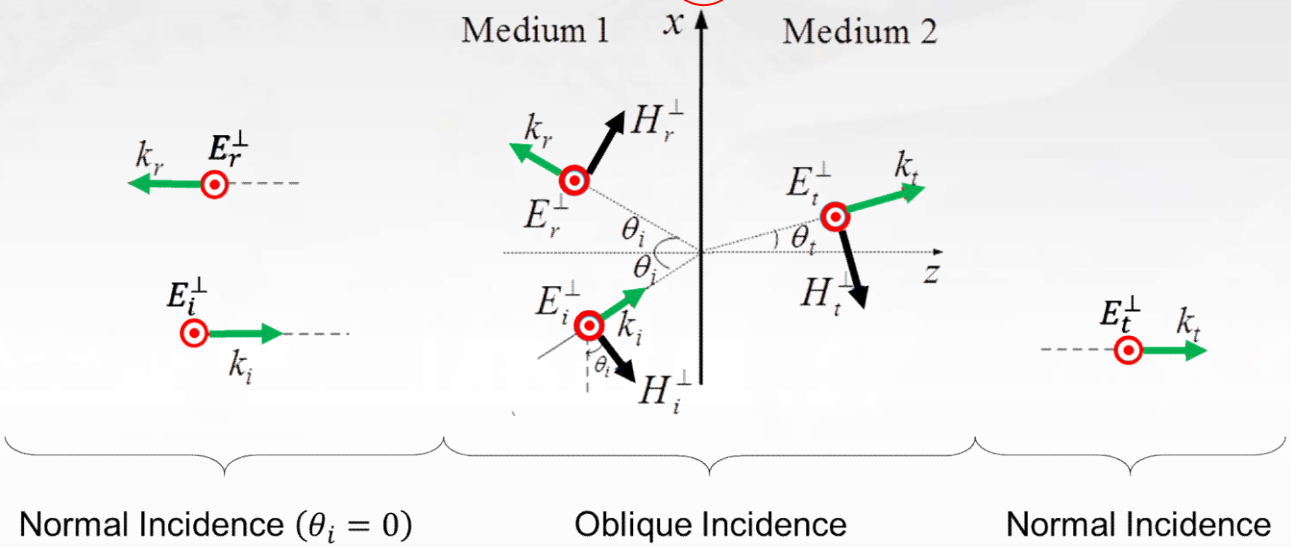
Perpendicular Polarisation

- $\vec{E}_i^{\perp} = \vec{a}_{Ei}^{\perp}\ E_{oi}^{\perp}\ e^{-jk_{xi}x}\ e^{-jk_{zi}z}$
- $\vec{k}_i,\ \vec{a}_{Ei}^{\perp},\ E_{oi}^{\perp}$
- $\vec{k}_i \rightarrow \vec{k}_r\ \&\ \vec{k}_t$
- $\vec{a}_{Ei}^{\perp} \rightarrow \vec{a}_{Er}^{\perp}\ \&\ \vec{a}_{Et}^{\perp}$
- $E_{oi}^{\perp} \rightarrow E_{or}^{\perp} = \Gamma_{\perp}\, E_{oi}^{\perp}\ \&\ E_{ot}^{\perp} = \tau_{\perp}\, E_{oi}^{\perp}$



Perpendicular Polarisation

- **Perpendicular Polarisation:** e.g. $\vec{E}_i^{\perp} = \vec{a}_y 10\angle 30° e^{-j(80x+6\quad)}$



- **Problem**: Given $\vec{E}_i^{\perp} = \vec{a}_{E_i}^{\perp} E_{oi}^{\perp} e^{-j\ \ _{xi}x} e^{-jk_{zi}z}$, find

- $\vec{E}_r^{\perp} = \vec{a}_y E_{or}^{\perp} e^{-jk_{xr}x} e^{+j\ \ _{zr}z}$ and $\vec{E}_t^{\perp} = \vec{a}_y E_{ot}^{\perp} e^{-j\ \ _{xt}x} e^{-j\ \ _{zt}z}$

Perpendicular Polarisation

- Step 1: Given $\vec{E}_i^{\perp} = \vec{a}_{E_i}^{\perp} E_{oi}^{\perp} e^{-jk_{xi}x} e^{-j\ \ _{zi}z}$ [Page 15-6]

  Determine $\vec{k}_i$, $\vec{a}_{E_i}^{\perp}$, $E_{oi}^{\perp}$

- Step 2: $\vec{k}_i = \vec{a}_x k_{xi} + \vec{a}_z k_{zi} \rightarrow \tan\theta_i = \frac{k_{xi}}{k_{zi}}$ θ_i [Page 16-9]

  Snell's Law: $\rightarrow \frac{\sin\theta_t}{\sin\ \ _i} = \sqrt{\frac{\mu_1\,\varepsilon_1}{\mu_2\,\varepsilon_2}}$ θ_t

  $\vec{k}_r = \vec{a}_x k_r \sin\theta_i - \vec{a}_z k_r \cos\theta_i$ $[k_1 = \omega\sqrt{\mu_1\varepsilon_1} = k_i = k_r]$

  $\vec{k}_t = \vec{a}_x k_t \sin\theta_t + \vec{a}_z k_t \cos\theta_t$ $[k_2 = \omega\sqrt{\mu_2\varepsilon_2} = k_t]$

  $k_{xr}, k_{zr}, k_{xt}, k_{zt}$

Perpendicular Polarisation

- Step 3: $\vec{a}_{E_i}^{\perp} = \vec{a}_y \rightarrow \vec{a}_{E_r}^{\perp} = \vec{a}_y \;\&\; \vec{a}_{E_t}^{\perp} = \vec{a}_y$

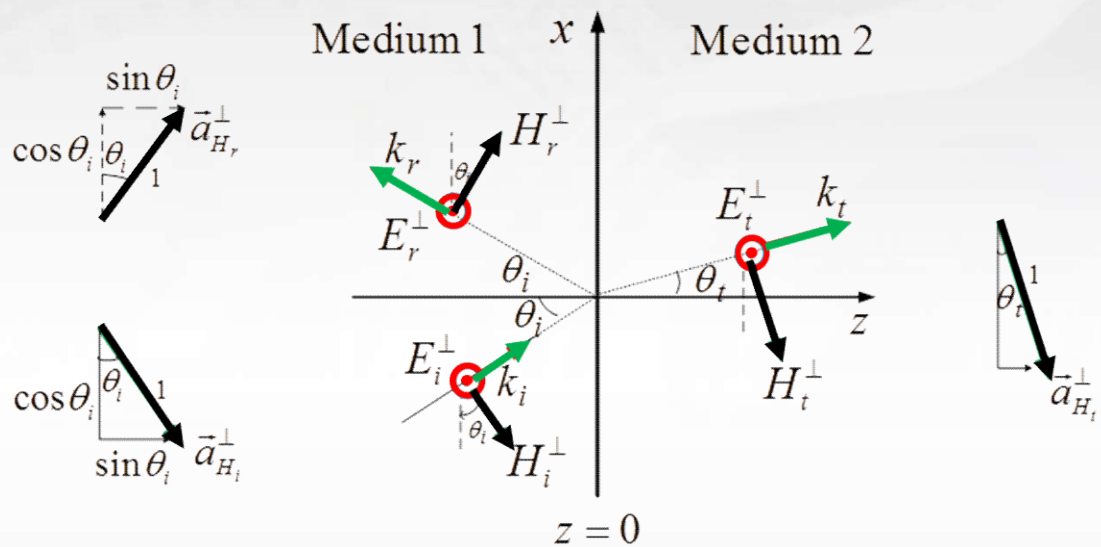
$$\vec{E}_r^{\perp} = \vec{a}_y\, E_{or}^{\perp}\, e^{-jk_{xr}x}\, e^{+jk_{zr}z} \text{ and } \vec{E}_t^{\perp} = \vec{a}_y\, E_{ot}^{\perp}\, e^{-jk_{xt}x}\, e^{-jk_{zt}z}$$

- Step 4: $E_{oi}^{\perp} \rightarrow E_{or}^{\perp} = \Gamma_{\perp}\, E_{oi}^{\perp},\;\; E_{ot}^{\perp} = \tau_{\perp}\, E_{oi}^{\perp}$

- $\Gamma_{\perp}$ and $\tau_{\perp}$ can be determined using **Boundary Conditions at** $z = 0$ i.e. **tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are continuous across the boundary** at $z = 0$.

Perpendicular Polarisation

- **Perpendicular Polarisation**:



- Tangential components $\equiv \vec{a}_x$ and $\vec{a}_y$ components

- $k_{xi} = k_{xr} = k_{xt} \rightarrow e^{-jk_{xi}x} = e^{-jk_{xr}x} = e^{-jk_{xt}x}$ [Page 16-7]

Perpendicular Polarisation

Medium 1:

$$\vec{E}_{\tan} = \vec{a}_y E_{oi}^{\perp} e^{-j\ \ xi x} e^{-jk_{zi}z} + \vec{a}_y E_{or}^{\perp} e^{-jk_{xr}x} e^{+jk_{zr}z}$$

$$\vec{H}_{\tan 1} = -\vec{a}_x \frac{E_{oi}^{\perp}}{\eta_1} \cos\theta_i\ e^{-jk_{xi}x} e^{-jk_{zi}z} + \vec{a}_x \frac{E_{or}^{\perp}}{\eta_1} \cos\theta_i\, e^{-jk_{xr}x} e^{+jk_{zr}z}$$

Medium 2:

$$\vec{E}_{\tan 2} = \vec{a}_y E_{ot}^{\perp} e^{-jk_{xt}x} e^{-jk_{zt}z}$$

$$\vec{H}_{\tan} = -\vec{a}_x \frac{E_{ot}^{\perp}}{\eta_2} \cos\theta_t\ e^{-jk_{xt}x} e^{-jk_{zt}z}$$

Perpendicular Polarisation

Boundary Conditions: @ $z = 0$

- $\vec{E}_{\tan 1} = \vec{E}_{\tan 2}$

$$E_{oi}^{\perp} + E_{or}^{\perp} = E_{ot}^{\perp} \qquad (32)$$

- $\vec{H}_{\tan 1} = \vec{H}_{\tan 2}$

$$\frac{E_{oi}^{\perp}}{\eta_1} \cos\theta_i - \frac{E_{or}^{\perp}}{\eta_1} \cos\theta_i = \frac{E_{ot}^{\perp}}{\eta_2} \cos\theta_t \qquad (33)$$

If η_1, η_2, $E_{oi}^{\perp}$, $\theta_i \rightarrow \theta_t$, are known, $E_{or}^{\perp}$ and $E_{ot}^{\perp}$ can be solved using (32) and (33) to give:

Perpendicular Polarisation

$$\Gamma_\perp = \frac{E_{or}^\perp}{E_{oi}^\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad (34)$$

$$\tau_\perp = \frac{E_{ot}^\perp}{E_{oi}^\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad (35)$$

- **Note:** $\theta_i = 0 \rightarrow \theta_t = 0$,

$$\Gamma_\perp = \Gamma_\parallel = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau_\perp = \tau_\parallel = \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Fresnel's Equations

FRESNEL'S EQUATIONS: $\Gamma_{\parallel},\ \Gamma_{\perp},\ \tau_{\parallel},\ \tau_{\perp} = f(\eta_1, \eta_2, \theta_i, \theta_t)$

- $\Gamma_{\parallel} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \dfrac{E_{or}^{\parallel}}{E_{oi}^{\parallel}}$ (29)

- $\tau_{\parallel} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \dfrac{E_{ot}^{\parallel}}{E_{oi}^{\parallel}}$ (30)

- $\Gamma_{\perp} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\ _i + \eta_1 \cos\theta_t} = \dfrac{E_{or}^{\perp}}{E_{oi}^{\perp}}$ (34)

- $\tau_{\perp} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \dfrac{E_{ot}^{\perp}}{E_{oi}^{\perp}}$ (35)

Summary

- Determine the directions of reflected and transmitted E field, i.e. $\vec{a}_{Er}^{\perp}$ and $\vec{a}_{Et}^{\perp}$, for an incident UPW with perpendicular polarisation;
- Apply the reflection and transmission coefficients, i.e. $\Gamma_{\perp}$ and $\tau_{\perp}$, to obtain the amplitudes of the reflected and transmitted E field at the interface, i.e. $E_{or}^{\perp}$ and $E_{ot}^{\perp}$;
- Determine the reflected and transmitted E field, i.e. $\vec{E}_r^{\perp}$ and $\vec{E}_t^{\perp}$; and
- Obtain $\vec{H}_r^{\perp}$ and $\vec{H}_t^{\perp}$ from $\vec{E}_r^{\perp}$ and $\vec{E}_t^{\perp}$ respectively.

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

This session is about

20. Example of UPW with Perpendicular Polarisation

Example

The magnetic field intensity of a uniform plane wave (UPW) propagating in free space $(z \leq 0)$ is given by

$$\vec{H}_i(z) = (-0.1\,\vec{a}_x + 0.25\,\vec{a}_z)\,e^{-j(5x+2z)} \ \text{A/m}$$

The wave is obliquely incident on a second medium $(z \geq 0)$ made of a lossless dielectric $(\mu = \mu_o,\ \varepsilon = 1.5\,\varepsilon_o)$.

Determine the following:

(i) The electric field of the incident UPW i.e. $\vec{E}_i(z)$.

(ii) The transmitted electric field intensity i.e. $\vec{E}_t(z)$.

Solution to Example

- $\vec{H}_i(z) = (-0.1\,\vec{a}_x + 0.25\,\vec{a}_z)\,e^{-j(5x+2z)}$ A/m $\quad \left[H_{oi} = \sqrt{0.1^2 + 0.25^2} = 0.269\right]$

$[E_{oi} = \eta_1 H_{oi}, \eta_1 = 120\pi\ \Omega]$

(i) $\vec{E}_i^{\perp}(z) = \vec{a}_y\,\eta_1\,H_{oi}\,e^{-j(5x+2z)} = \vec{a}_y\,101.5\,e^{-j(5x+\quad)}$ V/m $\quad [k_1 = 5.39 \rightarrow f]$

(ii) $\vec{E}_t^{\perp}(z) = \vec{a}_y\,E_{ot}\,e^{-j(k_{xt}x+k_{zt}z)}$ $\quad [k_2 = \omega\sqrt{\mu_2\varepsilon_2} = 6.6]$

$\tan\theta_i = \frac{k_{xi}}{k_{zi}} \rightarrow \theta_i = 68.2°$; Snell's Law: $\theta_t = 49.3°$

$k_{xt} = k_2 \sin\theta_t = 5, \quad k_{zt} = k_2 \cos\theta_t = 4.3$

$\theta_i = 68.2°, \quad \theta_t = 49.3°, \quad \eta_1 = 120\pi\ \Omega, \quad \eta_2 = 308\ \Omega \rightarrow \tau_\perp = 0.635$

$E_{ot} = \tau_\perp\,E_{oi} = 0.635 \times 101.5 = 64.5$

- $\vec{E}_t^{\perp}(z) = \vec{a}_y\,64.5\,e^{-j(5x+4.3z)}$ V/m