NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2022 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae is provided in Appendix A on page 4.

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1. (a) Solve the linear system of equations below:

$$\begin{aligned} 10x + 4y - 2z &= 14 \\ -3w - 15x + y + 2z &= 0 \\ w + x + y &= 6 \\ 8w - 5x + 5y - 10z &= 26 \end{aligned}$$

(7 Marks)

(b) Consider a matrix

$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

(i) Find the rank and nullity of A.

(6 Marks)

(ii) Describe the column space of A.

(5 Marks)

(iii) Determine the null space of A.

(7 Marks)

2. As shown in Figure 1, in the x,y-Cartesian coordinates P $\in \mathbf{R}^2$ is represented with a vector pointing from the origin to (x_1, y_1).

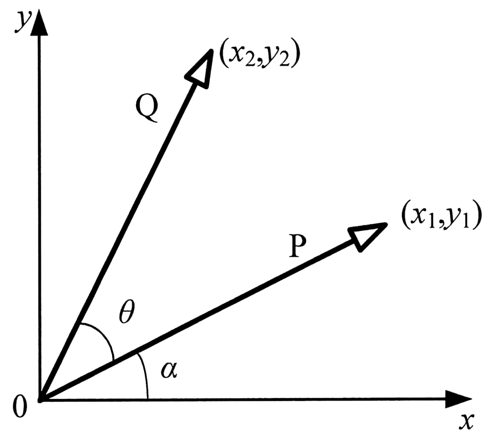

Figure 1

(a) Show that linear transformation AP = Q with

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},\ P = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix},\ Q = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

is a counterclockwise rotation of P about the origin, where θ is the angle of rotation.

(Note: $x_1 = r\cos\alpha,\ y_1 = r\sin\alpha,\ and\ r = \sqrt{x_1^2 + y_1^2}$)

(4 Marks)

(b) Determine whether column vectors of A are linearly independent.

(4 Marks)

(c) Diagonalize matrix A.

(11 Marks)

(d) Determine all entries of A^n.

(6 Marks)

3. (a) For the following function, determine whether it has limit when $z \to 1$ and is continuous at the point $z = 1$

$$f(z) = \begin{cases} 1, & z = 1 \\ \mathrm{Im}\left[\frac{z}{1+|z-1|}\right] & z \neq 1 \end{cases}$$

(5 Marks)

Note: Question 3 continues on page 3.

(b) Use the Cauchy-Riemann equations to determine the analyticity of the following functions. Find their derivatives at the points where they are differentiable.

(i) $f_1(z) = x^2 - iy^2$

(ii) $f_2(z) = z^{100} + i(x^2 - y^2 + 2ixy)^5$

(10 Marks)

(c) Evaluate the integral

$$\frac{1}{2\pi i}\oint_C [2 + (z + \frac{1}{z})]f(z)\frac{1}{z}dz,$$

where $C: |z| = 1$ counterclockwise and $f(z) = \frac{4}{4+z^{1000}}$. Hence use the result to find the following integral

$$\int_0^{2\pi} f(e^{i\theta}) \cos^2(\frac{\theta}{2}) d\theta$$

(10 Marks)

4. (a) Given that

$$\boldsymbol{F} = 2x^2yz^2\boldsymbol{i} + (x^2z^2 + \sin y)\boldsymbol{j} + 2x^2y\boldsymbol{k},$$

determine $\nabla(\nabla \cdot \boldsymbol{F}) \times (\nabla \times \boldsymbol{F})$.

(10 Marks)

(b) Find the work done in moving a particle in a force field given by $\boldsymbol{F} = (2z - 3y)\boldsymbol{i} + (3x - z)\boldsymbol{j} + (y - 2x)\boldsymbol{k}$ from point (0, 0, 0) to point (1, 2, 4) along the following paths:

(i) the straight lines from (0, 0, 0) to (1, 0 0) to (1, 2, 0) and then to (1, 2, 4);

(ii) the straight line joining (0, 0, 0) and (1, 2, 4).

(11 Marks)

(c) Find the outward flux of the vector field

$$\boldsymbol{A} = (\boldsymbol{x} - y^2 + \boldsymbol{z})\boldsymbol{i} + (\boldsymbol{y} + \sin\boldsymbol{z} + x^2)\boldsymbol{j} + (\boldsymbol{z} - \boldsymbol{x} + \cos\boldsymbol{y})\boldsymbol{k}$$

across the surface of the cube bounded by planes at $x = 0$, $x = 3$, $y = 0$, $y=3$, $z = 0$ and $z = 3$.

(4 Marks)

Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c \ln z}$

(b) Euler's Formula: $e^{ix} = cos\, x + i\, sin\, x$

(c) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(d) Cauchy-Riemann equations:

$u_x = v_y,\ v_x = -u_y$, or $u_r = \frac{1}{r}v_\theta,\ v_r = \frac{-1}{r}u_\theta$

(e) Derivative, if exists: $f'(z) = u_x + iv_x = e^{-i\theta}(u_r + iv_r)$

(f) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\Big|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Divergence Theorem: $\iiint_V \nabla.\mathbf{F}dV = \oiint_S \mathbf{F}\cdot\mathbf{n}dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dA = \oint_C \mathbf{F}\cdot\mathbf{dr}$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.