NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

Session 10-1

Ampere's Circuital Law

Learning Objectives

- State Ampere's circuital law;
- Apply Ampere's circuital law to solve problems; and
- State the steps for applying Ampere's law.

Magnetic Field Due to a Line Current

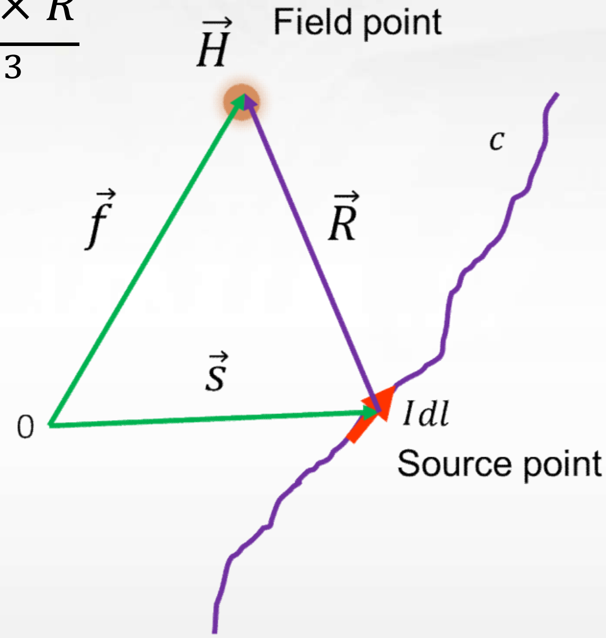
- From Biot-Savart law, we know

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

where

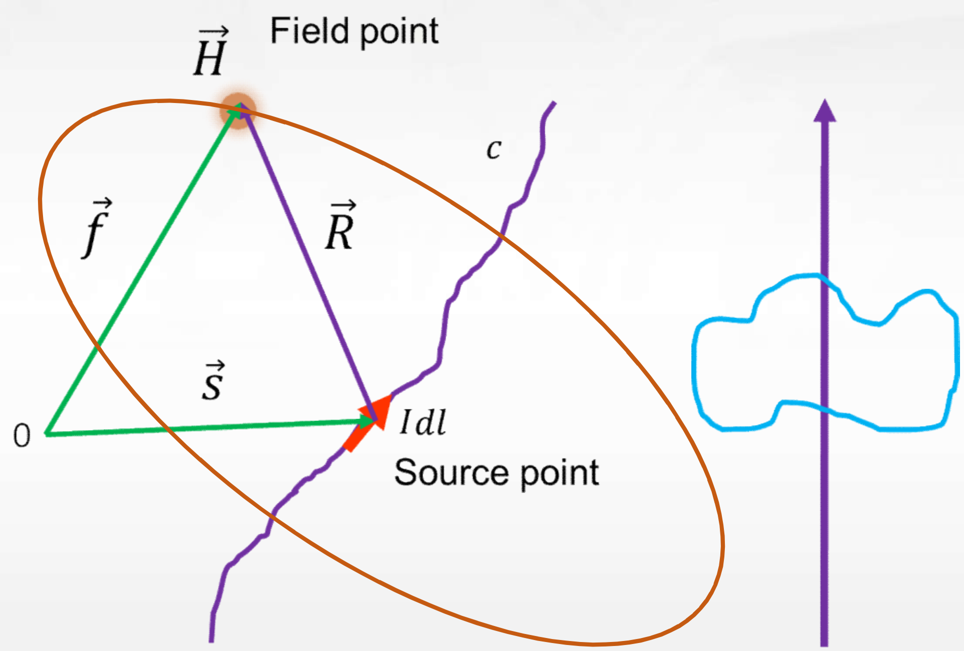
$$\vec{R} = \vec{f} - \vec{s}$$

$$R = |\vec{f} - \vec{s}|$$



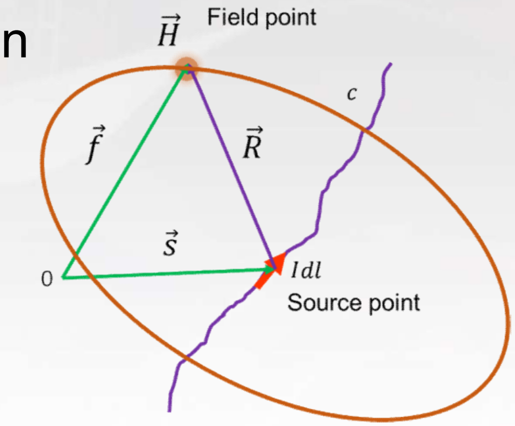
Question

What is the line integral of the magnetic field over a closed path around the line current?



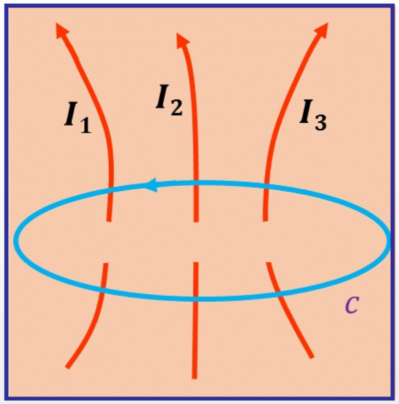
Ampere's Circuital Law

From Biot-Savart law, one may derive the following relation



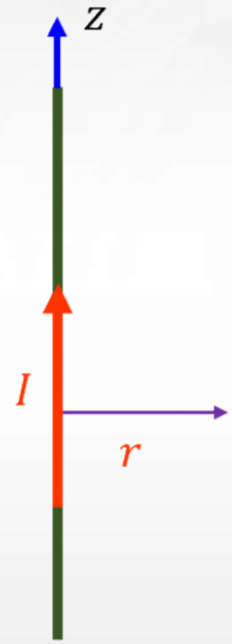
$$\boxed{\oint_C \vec{H} \cdot \vec{d}l = I = \iint_S \vec{J} \cdot \vec{d}s}$$

- This is known as *Ampere's circuital law*.
- It states: *the line integral of the magnetic field intensity around any **closed** path is equal to the **net** current enclosed by that path.*



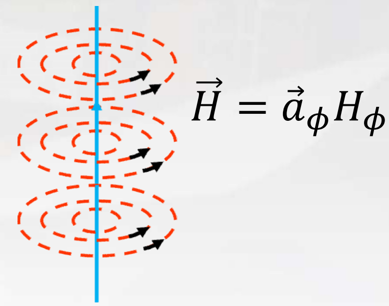
Example 1

A direct current I flows in an infinitely long straight thin wire. Find the magnetic field intensity at a point at a radial distance r away from the wire.



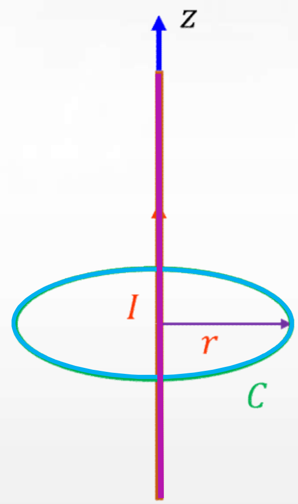
Example 1 Solution

- ❑ It is known that the magnetic field lines generated by a line current encircle the line current (right hand rule)



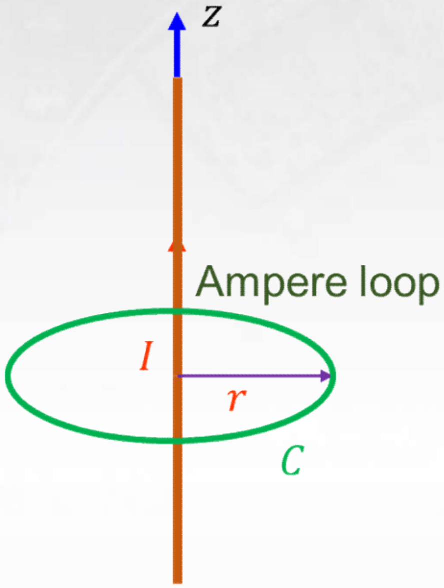
- ❑ Thus, in cylindrical coordinates,

$$\vec{H} = \vec{a}_\phi H_\phi \qquad \boxed{\oint_c \vec{H} \cdot \vec{d}l = I}$$



Step 1: Construct a closed path C (called an Ampere loop) as a circle passing through the field point as shown in the figure.

Example 1 Solution (cont'd)



$$\vec{H} = \vec{a}_\phi H_\phi$$

$$\vec{d}l = \vec{a}_\phi r d\phi$$

Step 2: Use Ampere's law:

$$\oint_C \vec{H} \cdot \vec{d}l = I$$

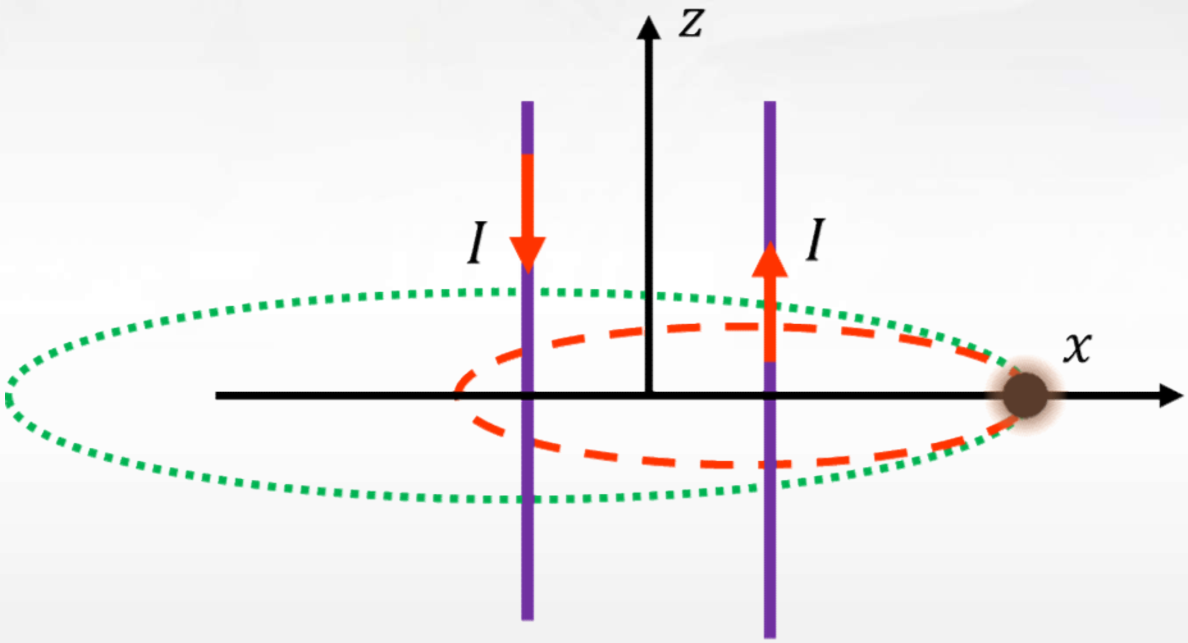
$$\int_0^{2\pi} \vec{a}_\phi H_\phi \cdot \vec{a}_\phi r d\phi = H_\phi 2\pi r = I$$

which gives

$$H_\phi = \frac{I}{2\pi r}, \vec{H} = \frac{I}{2\pi r}\vec{a}_\phi \ \text{A/m}$$

Question

How can Ampere's law be used to determine the magnetic field due to a two-wire transmission line?



Three Steps for Applying Ampere's Law

The application of Ampere's law involves the following three steps:

$$\boxed{\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}}$$

(1) Construct an Ampere loop, most often a circular loop, with symmetry about the current distribution

(2) Work out the line integral $\oint_C \vec{H} \cdot \vec{dl}$

(3) Find the total current enclosed by the Ampere loop and equate this to the line integral value found in step (2).

Summary

- Ampere's circuital law is stated as $\oint_C \vec{H} \cdot \vec{d}l = I = \iint_S \vec{J} \cdot \vec{d}s$;
- Ampere's law can be used to find the magnetic field due to a straight line current in a way which is simpler than the application of Biot-Savart law;
- There are three simple steps for applying Ampere's law.

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Session 10-2

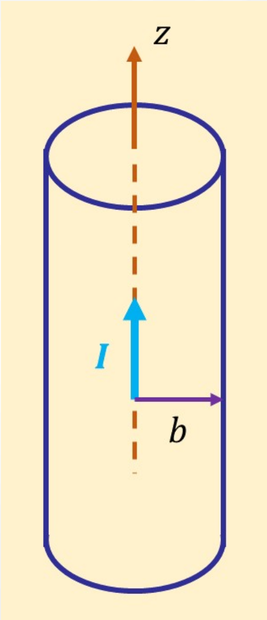
Ampere’s Circuital Law (2)

Learning Objectives

- ❑ Apply Ampere's circuital law to solve problems;
- ❑ Decide situations in which Ampere's law can be applied; and
- ❑ Obtain the differential form of Ampere's law.

Example 2

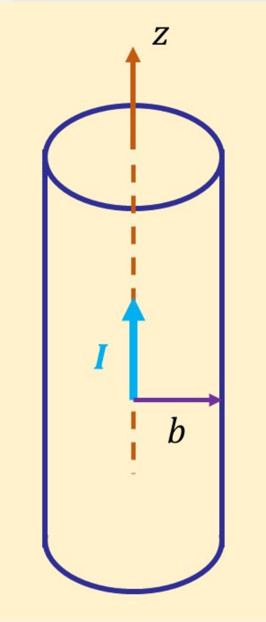
An infinitely long, straight conductor with a circular cross-section of radius b carries a current I with uniform current density.
Determine the magnetic field intensity everywhere.



Example 2 Solution

Since the current is uniformly distributed over the circular cross-section of the conductor, we obtain the current density

$$\vec{J} = \frac{I}{\pi b^2}\vec{a}_z$$

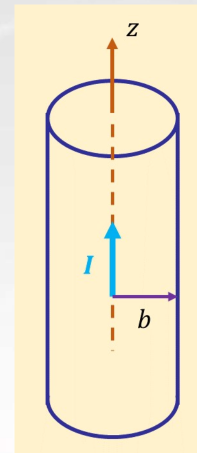


Similar to Example 1, we know from the right-hand rule and circular symmetry that

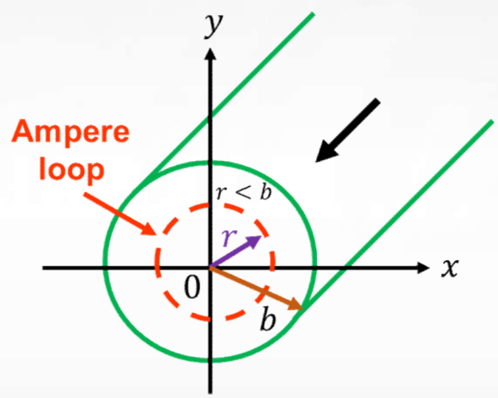
$$\vec{H} = \vec{a}_\phi H_\phi$$

Example 2 Solution (cont'd)

We then employ Ampere's law to determine H_ϕ for both $r < b$ and $r > b$ regions.



Case 1: $r < b$:



We choose a circle of radius r as the Ampere loop, as shown in the figure ($r < b$).
We apply Ampere's law, around this loop.

Example 2 Solution (cont'd)

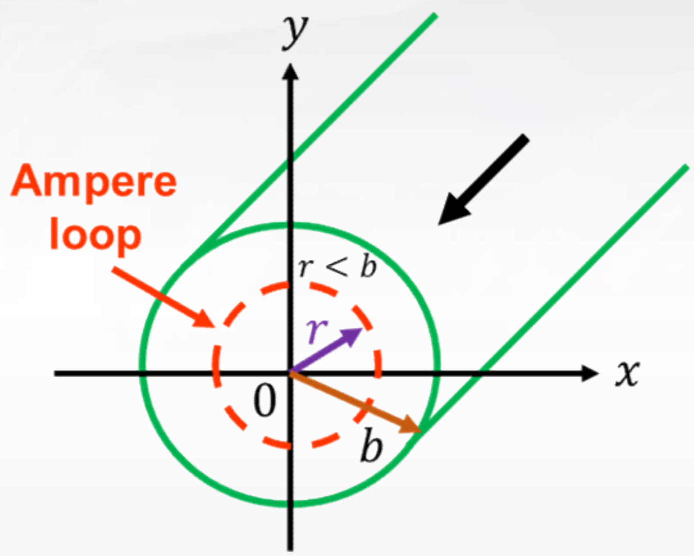
$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$\oint_C \vec{H} \cdot \vec{dl} = \int_0^{2\pi} \vec{a}_\phi H_\phi \cdot \vec{a}_\phi r d\phi = H_\phi 2\pi r$$

$$\iint_S \vec{J} \cdot \vec{ds} = \int_0^{r}\int_0^{2\pi} \frac{I}{\pi b^2} \vec{a}_z \cdot \vec{a}_z r d\phi dr$$

$$= \frac{I}{\pi b^2} \pi r^2 = \frac{I r^2}{b^2}$$

$$\vec{H} = \vec{a}_\phi \frac{rI}{2\pi b^2} \text{ A / m}$$

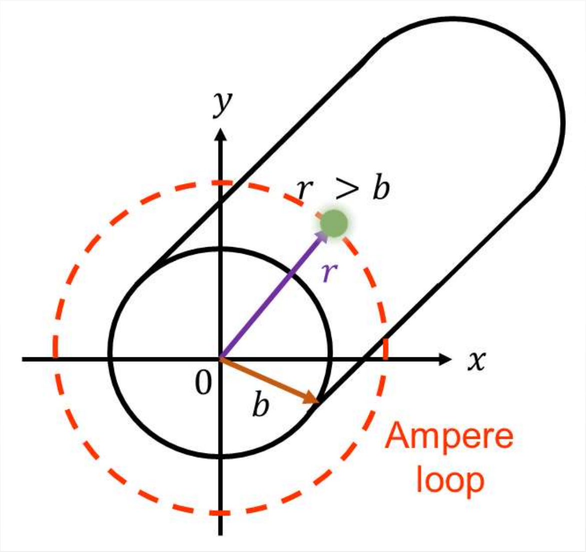


Example 2 Solution (cont'd)

Case 2: $r > b$,

$$\oint_C \vec{H} \cdot \vec{d}l = I = \iint_S \vec{J} \cdot \vec{d}s$$

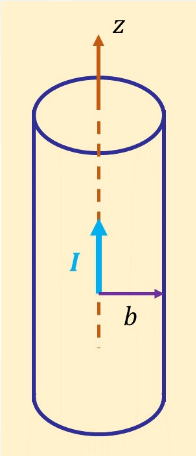
We choose a circle of radius r ($> b$) as the Ampere loop.



$$\oint_C \vec{H} \cdot \vec{d}l = H_\phi 2\pi r$$

$$\iint_S \vec{J} \cdot \vec{d}s = \frac{I}{\pi b^2} \pi b^2 = I$$

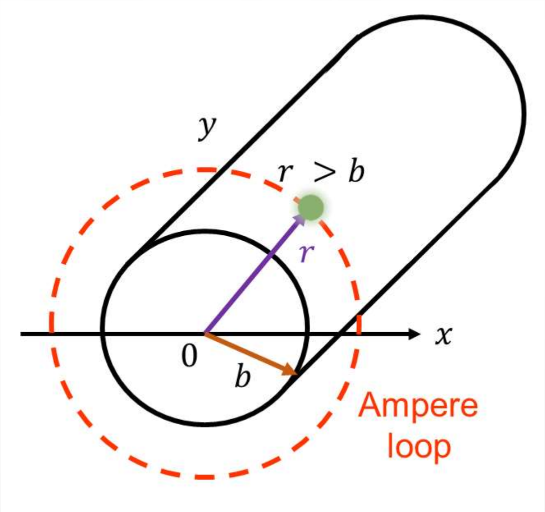
$$\vec{H} = \vec{a}_\varphi \frac{I}{2\pi r} \text{ A / m}$$



Example 2 Solution (cont'd)

Note:

- For $r > b$, the total current enclosed by the Ampere loop is the current flowing through the entire cross-section of the cylinder, which is I; and
- Because there is no current outside the cylindrical conductor.



Quiz

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Hayt and Buck, Ch 8, Q2:

The magnetic field intensity inside a cylindrical rod with uniformly distributed current density depends on the distance r from the axis of the rod as

A: $H \propto r$

B: $H \propto 1 / r$

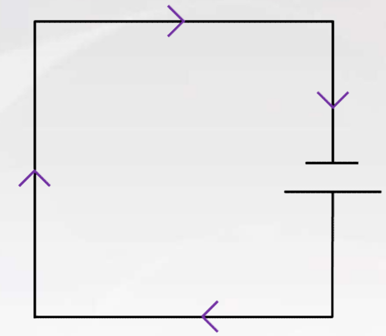
C: $H \propto 1 / r^2$

D: $H \propto const$

$$\vec{H} = \vec{a}_\phi \frac{rI}{2\pi b^2}$$

Usefulness of Ampere's Law

- Ampere's law is *valid* for **any practical** circuit since it is a direct consequence of Biot-Savart law.



- Ampere's law is *very useful* in determining magnetic fields due to uniform currents of **infinite** extent, for example
  - infinitely long line currents, or,
  - long, cylindrically symmetric currents

- Why Ampere's law is *not useful* to determine the magnetic field due to a **finite** line current?

  In this case, for a practical circuit, magnetic field is not symmetric.

Differential Form of Ampere's Law

Using Stokes' theorem $\oint_C \vec{H} \cdot \vec{d}l = \iint_S (\nabla \times \vec{H}) \cdot \vec{d}s$

one can readily obtain $\iint_S \nabla \times \vec{H} \cdot \vec{d}s = \iint_S \vec{J} \cdot \vec{d}s$

which leads to the differential form of Ampere's circuital law:

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

It is seen that *unlike* the static electric field intensity $\vec{E}$, the static magnetic field intensity $\vec{H}$ **is** *rotational*.

Quiz

Hayt and Buck, Ch 8, Q3:

At a point P, the static magnetic field is given by $\vec{H}(P) = 4z\vec{a}_y - 4y\vec{a}_z$ (A / m). Find the electric current density $\vec{J}(P)$ (in $\mathrm{A} / \mathrm{m}^2$) at P.

A: $\vec{J} = 0\ \mathrm{A/m^2}$

B: $\vec{J} = -4\vec{a}_y\ \mathrm{A/m^2}$

C: $\vec{J} = 8\vec{a}_x\ \mathrm{A/m^2}$

D: $\vec{J} = -8\vec{a}_x\ \mathrm{A/m^2}$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

Summary

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- Application of Ampere's circuital law to solve another problem;
- Ampere's law is very useful in determining magnetic fields due to cylindrically symmetric currents of infinite extent;
- The differential form of Ampere's law is $\nabla \times \vec{H} = \vec{J}$.

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Session 10-3

Magnetic Flux Density

Learning Objectives

- Define magnetic flux density in free space and in magnetic materials;
- Obtain magnetic flux passing through a surface;
- State and explain the solenoidal property of magnetic flux; and
- Compare static electric and magnetic field quantities and the laws that govern these quantities.

Magnetic Flux Density

In free space, the **magnetic flux density** is defined as

$$\vec{B} = \mu_0 \vec{H}$$

The constant $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$ is the permeability of free space.

where $\vec{B}$ is measured in Webers per square meter $(\mathrm{Wb/m^2})$, or a new unit called Tesla $(\mathrm{T} = \mathrm{Wb/m^2})$.

Magnetic Flux Density (continued)

For a magnetic material, the magnetic flux density $\vec{B}$ is related to the magnetic field intensity $\vec{H}$ as follows:

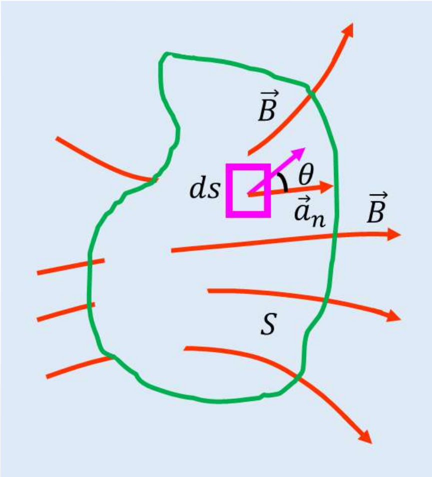
$$\boxed{\vec{B} = \mu\vec{H} = \mu_r\mu_0\vec{H}}$$

where μ is the permeability of the material, and μ_r is the relative permeability.

Magnetic Flux

The magnetic flux density vector $\vec{B}$, as the name implies, is a member of the flux-density family of vector fields.

Similar to the electric flux density $\vec{D}$, let us consider the magnetic flux passing through a surface S:

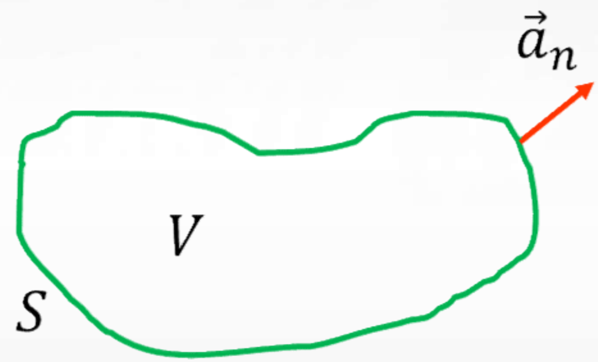


$$\Phi_m = \iint_S \vec{B} \cdot \vec{ds}$$

Magnetic Flux (continued)

Question:

$$\Phi_m = \oiint_S \vec{B} \cdot \vec{ds} = ? \quad (S \text{ is a closed surface})$$



Magnetic Flux (continued)

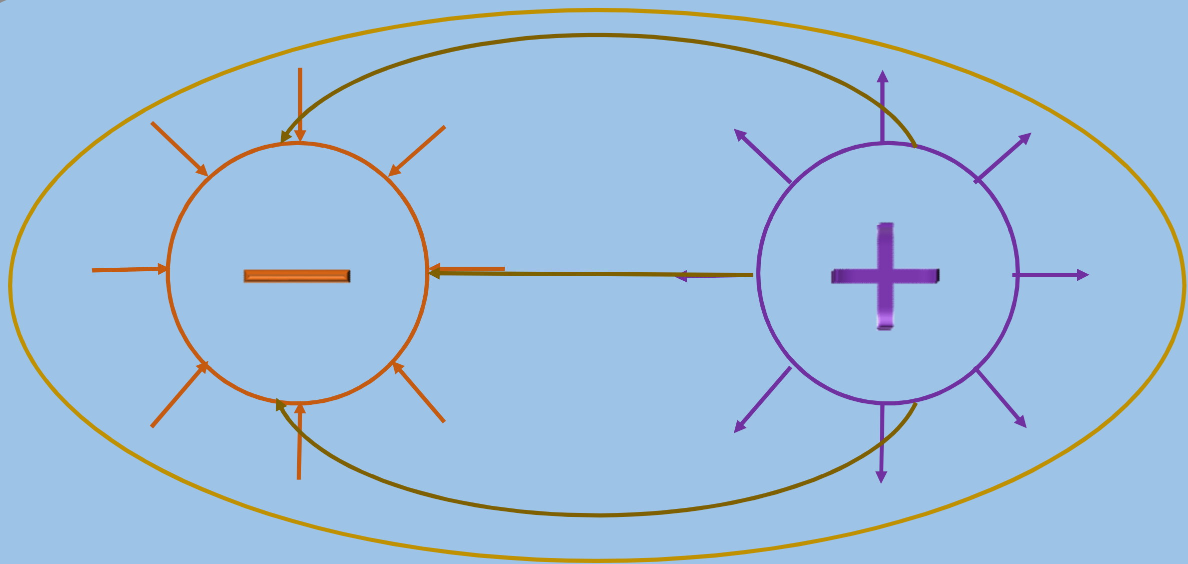
- The analogy between electric flux and magnetic flux reminds us of Gauss's law; and
- Which states that the total electric flux out of any closed surface is equal to the total charge enclosed.

$$\oiint_S \vec{D} \cdot \vec{ds} = Q = \iiint_V \rho_v dv$$

$\vec{B}$

- Electric flux lines *begin* and *terminate* on positive and negative charges, respectively.
- **Unlike electric charge, no isolated magnetic charge has been discovered!**

Magnetic Flux (continued)



$$\oiint_S \vec{B} \cdot \vec{d}s = Q_m \to 0$$

…minds us of

…d surface is

Solenoidal Property of Magnetic Flux

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Since there is no isolated magnetic charge:

$$\Phi_m = \oiint_S \vec{B} \cdot \vec{ds} = 0 \qquad (S \text{ is a closed surface})$$

Gauss' Law for magnetic flux

By the application of the Divergence theorem

$$\oiint_S \vec{B} \cdot \vec{ds} = \iiint_v (\nabla \cdot \vec{B}) dv \Rightarrow \nabla \cdot \vec{B} = 0$$

we see that $\boxed{\nabla \cdot \vec{B} = 0}$ One of the Maxwell's equations

Therefore, the magnetic flux lines are always closed. Alternatively stated, the magnetic field or flux density is always *solenoidal*.

Static Electric and Magnetic Fields

	Electric Field	Magnetic Field
Intensity	$\vec{E}(\mathrm{V/m})$	$\vec{H}(\mathrm{A/m})$
Flux Density	$\vec{D}(\mathrm{C/m^2})$	$\vec{B}(\mathrm{T = Wb/m^2})$
First Law	Coulomb's Law $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\vec{a}_R$	Biot-Savart Law $\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl}\times\vec{a}_R}{R^2}$
Second Law	Gauss's Law $\oiint_S \vec{D}\cdot\vec{ds} = Q = \iiint_V \rho_v dv$	Ampere's Law $\oint_C \vec{H}\cdot\vec{dl} = I = \iint_S \vec{J}\cdot\vec{ds}$
Curl	$\nabla\times\vec{E} = 0$	$\nabla\times\vec{H} = \vec{J}$
Divergence	$\nabla\cdot\vec{D} = \rho_v$	$\nabla\cdot\vec{B} = 0$
Force	$\vec{F}_e = q\vec{E}$	$\vec{F}_m = q\vec{u}\times\vec{B}$

Illustrative display

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- Hayt and Buck, Ch. 8, Illustration 1 (access in NTU Learn, self and peer assessments)
- Magnetic field lines for a number of configurations.

Summary

- The magnetic flux density in free space is defined as $\vec{B} = \mu_0 \vec{H}$.
- For a magnetic material, the magnetic flux density is related to the magnetic field intensity as follows:

$$\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$$

- The magnetic flux passing through a surface is defined as $\Phi_m = \iint_S \vec{B} \cdot \vec{ds}$.
- The solenoidal property of magnetic flux: magnetic flux lines are always closed and are always solenoidal.

Summary

- Below is a table of the comparison of static electric and magnetic field:

	Electric Field	Magnetic Field
Intensity	$\vec{E}(\mathrm{V/m})$	$\vec{H}(\mathrm{A/m})$
Flux Density	$\vec{D}(\mathrm{C/m^2})$	$\vec{B}(\mathrm{T = Wb/m^2})$
First Law	Coulomb's Law $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\vec{a}_R$	Biot-Savart Law $\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2}$
Second Law	Gauss's Law $\oiint_S \vec{D}\cdot\vec{ds} = Q = \iiint_V \rho_v dv$	Ampere's Law $\oint_C \vec{H}\cdot\vec{dl} = I = \iint_S \vec{J}\cdot\vec{ds}$
Curl	$\nabla \times \vec{E} = 0$	$\nabla \times \vec{H} = \vec{J}$
Divergence	$\nabla \cdot \vec{D} = \rho_v$	$\nabla \cdot \vec{B} = 0$
Force	$\vec{F}_e = q\vec{E}$	$\vec{F}_m = q\vec{u} \times \vec{B}$

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EE3001 Engineering Electromagnetics

Session 11-1

Faraday's Law

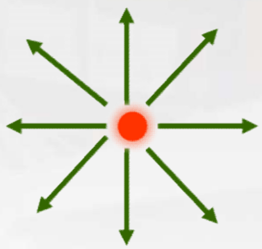
Learning Objectives

- State Faraday's law and its differential form.

Review of Static Electric and Magnetic Fields

- ❑ Static electric field $\dfrac{d\vec{E}}{dt} = 0$

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \times \vec{E} = 0 \qquad \vec{D} = \varepsilon \vec{E}$$

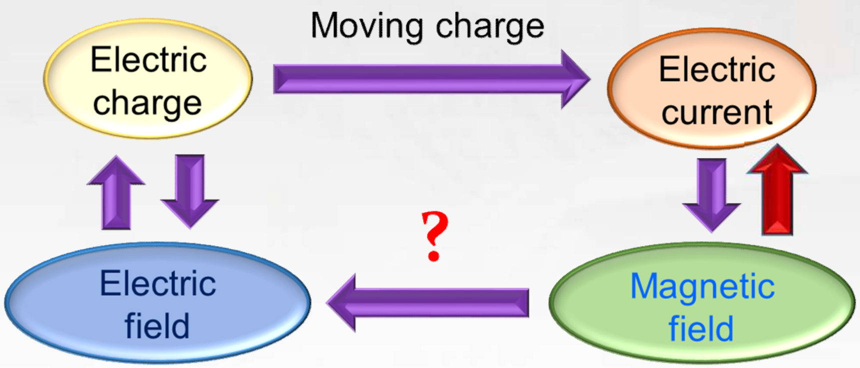


Note: Static means *constant* with time. For example, electric field due to stationary charges, and magnetic field due to steady direct currents.

- ❑ Static magnetic field $\dfrac{d\vec{B}}{dt} = 0$

$$\nabla \times \vec{H} = \vec{J} \qquad \nabla \cdot \vec{B} = 0 \qquad \vec{B} = \mu \vec{H}$$

Electric and Magnetic Field Linkages



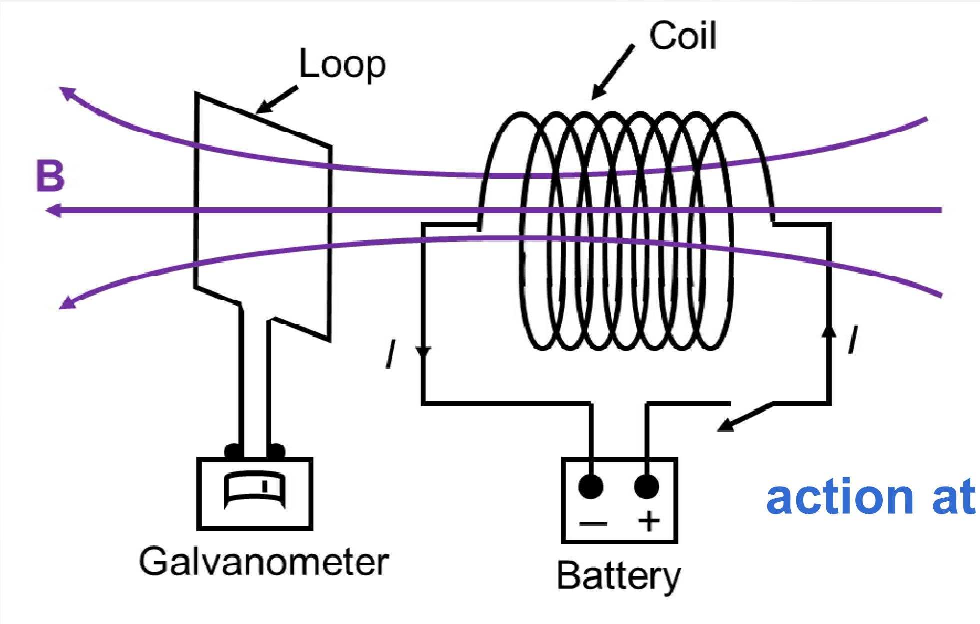
We already know that

- Electric charge produces electric field; and
- Moving charge, i.e., an electric current, produces a magnetic field.

Question: Can a magnetic field generate an electric current or electric field?

Phenomenon of Electromagnetic Induction

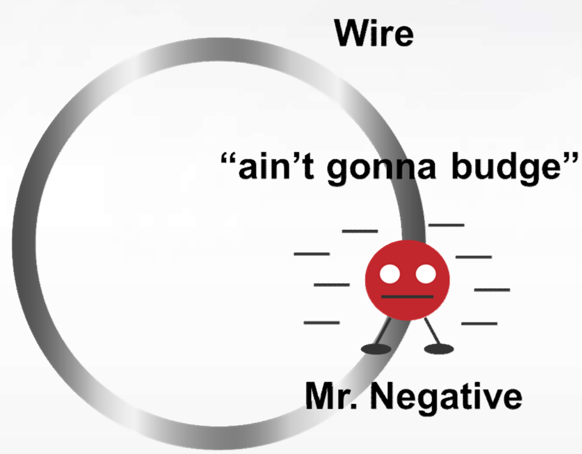
In 1831, Michael Faraday discovered experimentally that a current was induced in a closed conducting loop when the magnetic flux linking the loop changed with time.



action at a distance!

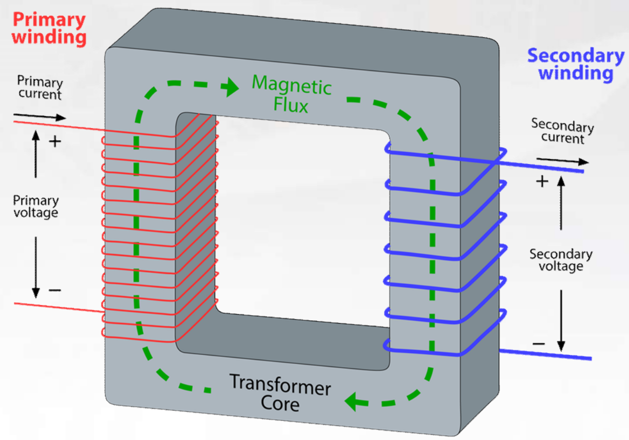
Movement of a Magnet

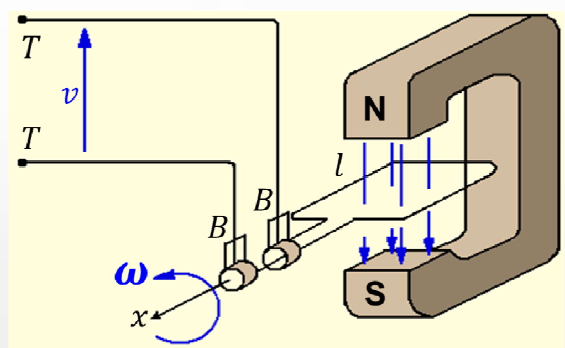
Movement of a magnet induces an electric current in the nearby conducting loop because the magnetic flux linking the loop changes when the magnet moves.



Do you know?

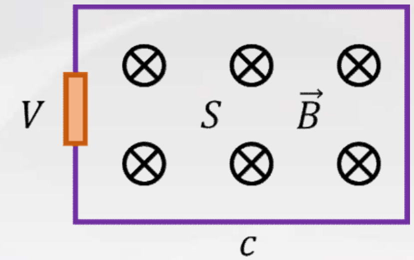
- The principle of operation of:
  - Transformers
  - Audio/video tape-replay
  - Generation of electricity



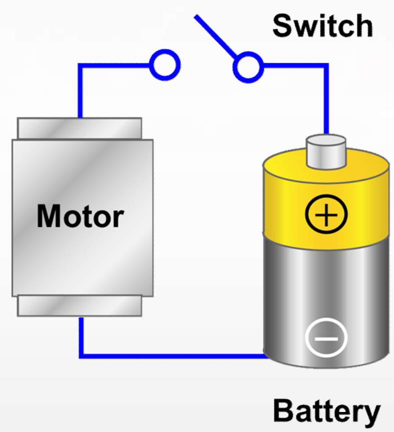


Electromotive Force (emf)

- **Observation:** Consider a rectangular conducting loop placed in a time-varying magnetic field. This results in an **induced current**.

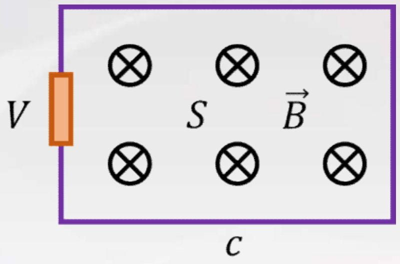


- **Electric field** is induced to drive the electrons in the wire.
- The induced electric field is called electromotive force (emf).
- This emf drives the current in the same way as a battery does.



Faraday's Law

The line integral of the induced electric field around a closed path equals the negative time rate of change of the magnetic flux linked to that path:



$$emf = \oint_C \vec{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_s \vec{B} \cdot \vec{ds}$$

One of the Maxwell's equations

Faraday's Law: Differential Form

Faraday's law:

$$\boxed{\oint_C \vec{E}\cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{s}}$$

By using Stokes' theorem

$$\oint_C \vec{E}\cdot d\vec{l} = \iint_S (\nabla\times\vec{E})\cdot d\vec{s} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{s}$$

one gets the differential form of Faraday's law

$$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

! **One of the Maxwell's equations**

Notes on Faraday's Law

1. The electric field induced by a time-varying magnetic flux density is no longer irrotational (or conservative), since

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$$

This is in contrast to the electrostatic case, where the static electric field is always irrotational, i.e.

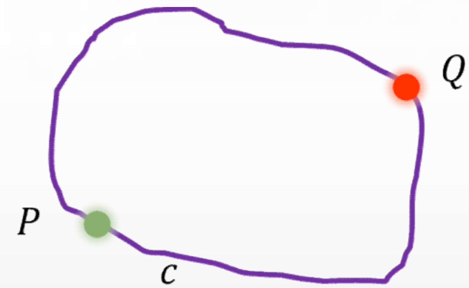
$$\nabla \times \vec{E}_{static} = 0$$

Notes on Faraday's Law

2. The value of the integral $\int_P^Q \vec{E} \cdot \vec{d}l$ depends on the choice of the path between points P and Q. This is in contrast to the electrostatic case, where the line integral of static electric field is independent of the choice of the path.

   Further,

$$\oint_C \vec{E} \cdot \vec{d}l = -\frac{d}{dt} \iint_S \vec{B} \cdot \vec{d}s \neq 0$$



Summary

- Faraday's law is stated as $\oint_C \vec{E} \cdot \vec{d}l = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \vec{B} \cdot \vec{d}s$ and its differential form is $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

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Session 11-2

Application of Faraday's Law: An Example

Learning Objectives

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- Describe the three cases for electromagnetic induction;
- Describe the steps for application of Faraday's Law; and
- Calculate induced emf in an example.

Demonstration of Electromagnetic Induction

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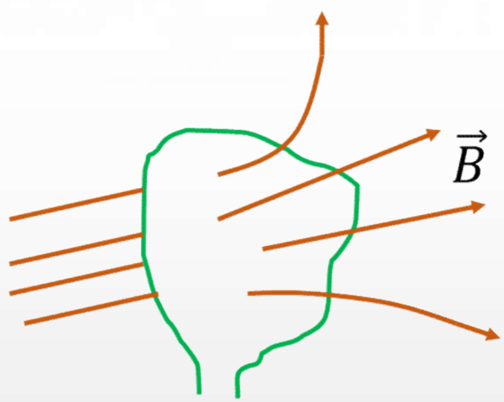
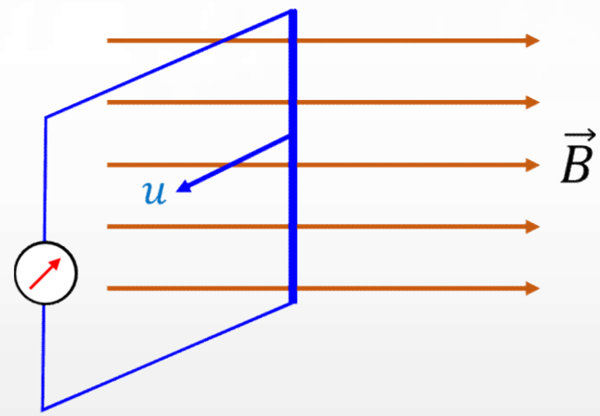
http://www.dnatube.com/video/3061/Faradays-Law-of-Induction (2:11 minutes)

Click the above link to view the video.

Three Cases for Electromagnetic Induction

- A stationary circuit in a time-varying magnetic field (transformer emf).
- A moving circuit (or conductor) in a static magnetic field, such that the magnetic flux linked to the circuit changes with time (motional emf).
- A combination of 1 and 2: a moving circuit in a time-varying magnetic field.

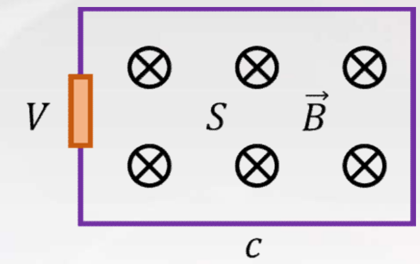




Calculation of Electromotive Force (emf)

- **Step 1:** Find the magnetic flux passing through the surface S.

$$\Phi_m = \iint_S \vec{B} \cdot \vec{d}s$$



- **Step 2:** Take the negative derivative of the magnetic flux with respect to time t to obtain the induced emf

$$\boxed{emf = -\frac{d\Phi_m}{dt}}$$

- For the case when there are N turns of the loop C, we have

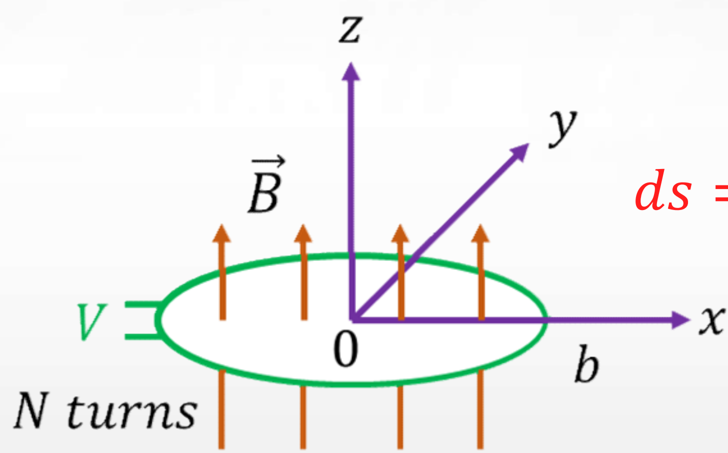
$$emf_{total} = N(emf_{one\ turn})$$

Example 1

A circular loop of N turns and radius b sits in the x-y plane under the illumination of

$$\vec{B} = \vec{a}_z B_0 \left(1 - \frac{r}{b}\right) \sin(\omega t) \ \mathrm{Wb/m^2}$$

Determine the induced emf.



Example 1 Solution

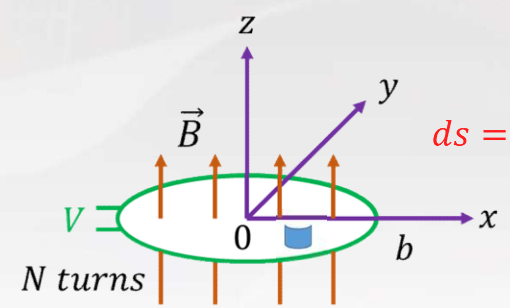
Solution:

The magnetic flux passing through the circular loop is

$$\Phi_m = \iint_S \vec{B} \cdot \vec{ds} = \int_0^{2\pi} \int_0^{b} \vec{a}_z B_0 \left(1 - \frac{r}{b}\right) \sin(\omega t) \cdot \vec{a}_z r dr d\phi$$

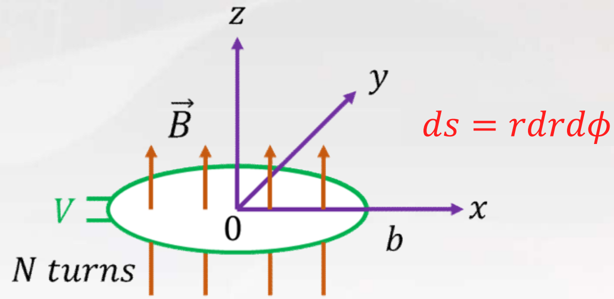
$$= \int_0^{2\pi} \int_0^{b} B_0 \left(1 - \frac{r}{b}\right) \sin(\omega t)\, r dr d\phi$$

$$= 2\pi \frac{b^2}{6} B_0 \sin(\omega t) = \frac{\pi b^2}{3} B_0 \sin(\omega t) \quad \text{Wb}$$



Example 1 Solution (contd.)

The induced voltage (emf) in the N-$turn$ loop is



$$emf = -N\frac{d\Phi_m}{dt} = -N\frac{d}{dt}\left\{\pi\frac{b^2}{3}B_0\sin(\omega t)\right\}$$

$$= -N\pi\frac{b^2}{3}B_0\omega\cos(\omega t) \ \ \mathrm{V}$$

Summary

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- The three cases for electromagnetic induction are:
  - A stationary circuit in a time-varying magnetic field (transformer emf).
  - A moving circuit (or conductor) in a static magnetic field, such that the magnetic flux linked to the circuit changes with time (motional emf).
  - A combination of 1 and 2: a moving circuit in a time-varying magnetic field.
- The steps for application of Faraday's Law; and
- Calculation of induced emf in an example illustrating case 1 of electromagnetic induction.

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Session 11-3

Lenz's Law

Learning Objectives

- State and apply Lenz's law.

Lenz's Law

Lenz's Law: The direction of any electromagnetic induction effect is such as to oppose the cause of the effect.

Cause → Effect

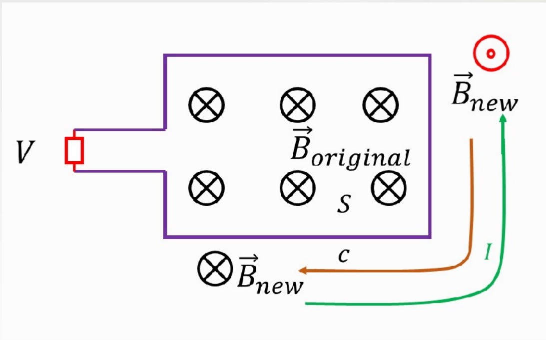
Time-varying $\vec{B}_{original}$ → Induced electric current

Induced electric current → (Right-hand rule) → New magnetic field $\vec{B}_{new}$

New magnetic field $\vec{B}_{new}$ → Time-varying $\vec{B}_{original}$

$\vec{B}_{new}$ opposes the change of $\vec{B}_{original}$

Lenz's Law: Two Cases



- If $\vec{B}_{original}$ (going into the paper) is increasing, the new $\vec{B}$ should be in a direction *to oppose the change* in the original $\vec{B}$; therefore the induced current must be in the **counter-clockwise** direction; and
- If $\vec{B}_{original}$ is decreasing, the new $\vec{B}$ should be in the same direction as the original $\vec{B}$, and therefore the induced current must be in the **clockwise** direction.

Summary

- State and apply Lenz's law.

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Session 11-4

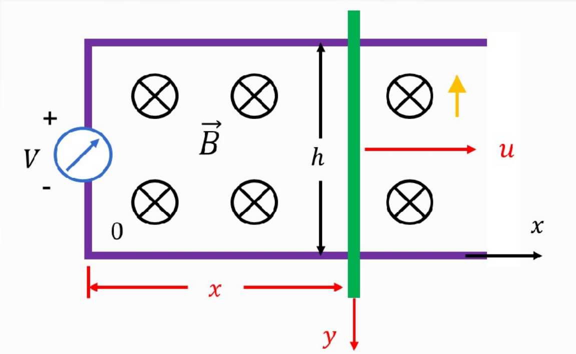
More Examples on Faraday's Law

Learning Objectives

- Apply Faraday's Law to calculate emf in more examples; and
- Decide the direction of flow of induced current using Lenz's Law.

A Moving Metallic Bar in a Static $\overrightarrow{B}$: Example 2

A metallic bar is sliding over a pair of conducting rails in a uniform magnetic field $\vec{B}$ at a constant velocity u along the $+x$ direction. The bar and the parallel conducting rails form a closed loop, through which a magnetic field is passing.



The magnetic flux linking the loop is

$$\Phi_m = \iint_S \vec{B} \cdot \vec{d}s = BS = Bhx \quad \text{Wb}$$

A Moving Metallic Bar in a Static $\vec{B}$: Example 2

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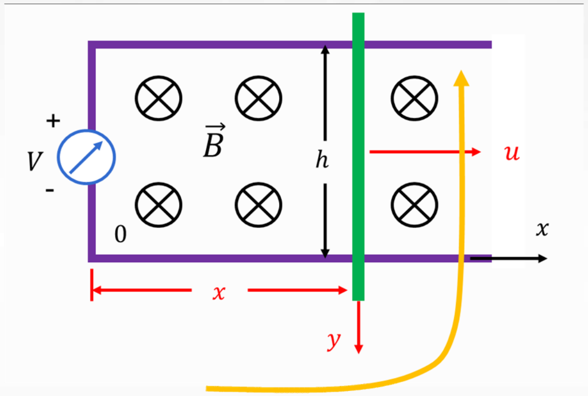
The induced emf is then equal to the negative time-derivative of the magnetic flux:

$$emf = -\frac{d\Phi_m}{dt} = -\frac{d(Bhx)}{dt} = -Bh\frac{d(x)}{dt} = -Bhu \quad \text{V}$$

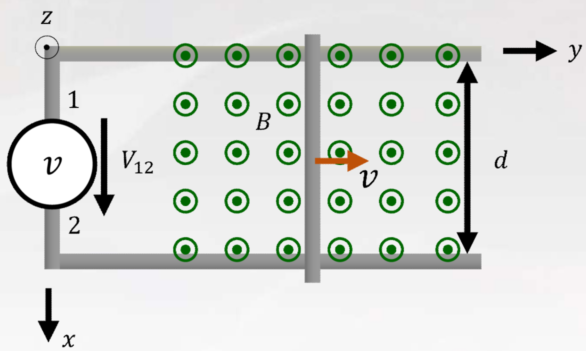


The direction of the induced current can be determined by Lenz's Law, **counter-clockwise**, as shown in the diagram.

Quiz

Hayt and Buck, Ch 10, Q2:

An external force is applied to a conducting bar supported by conducting rails, with which the bar is in perfect electrical contact (see figure). The bar moves with a constant velocity $v = 40a_y$ m/s. A steady magnetic field is present: $B = 0.5a_z$ T. The width between the supporting rails is $d = 0.5$ m. Find the voltage $V_{12} = V_1 - V_2$ measured by an ideal voltmeter ($R_V \to \infty$).



$$emf = -Bhu \to -Bdv$$

A: $V_{12} = 20$ V

B: $V_{12} = -20$ V

C: $V_{12} = 10$ V

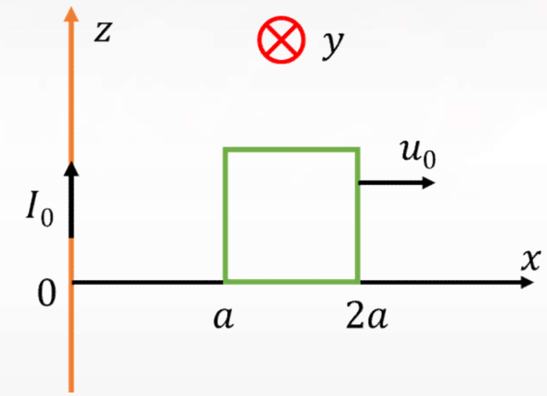
D: $V_{12} = -10$ V

Calculation of emf: Example 3

An infinitely long wire in air carrying a direct current I_0 is shown in the figure below. A square loop of side length a is moving along the x-axis at a constant velocity u_0 starting at $t = 0$ from the position shown in the figure.

Determine:

- the induced emf in the loop at time t;
- the magnetic flux density due to the line current at a point $(x, 0,0)$;
- the magnetic flux passing through the square loop at time t.

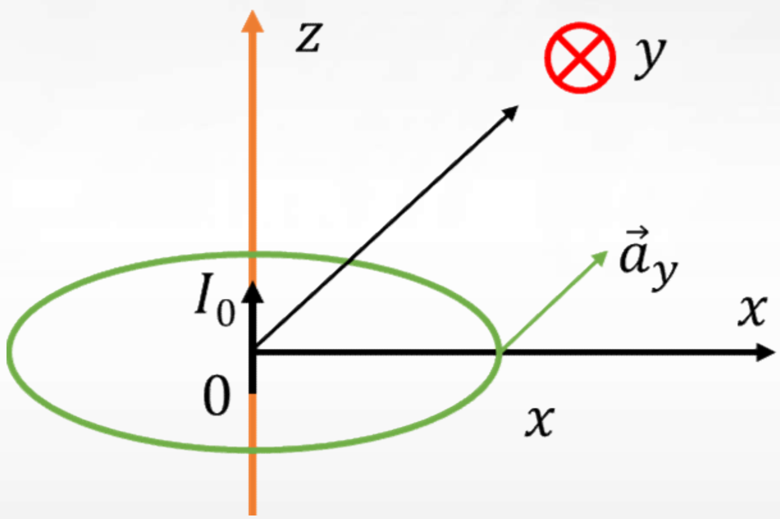


Example 3: Solution

1. The magnetic field due to the line current at a point $(x, 0,0)$ can be found by using Ampere's law:

$$\vec{H} = \frac{I_0}{2\pi r}\vec{a}_\varphi$$

$$\vec{H} = \frac{I_0}{2\pi x}\vec{a}_y$$



$$\vec{B} = \frac{\mu_0 I_0}{2\pi x}\vec{a}_y$$

Example 3: Solution

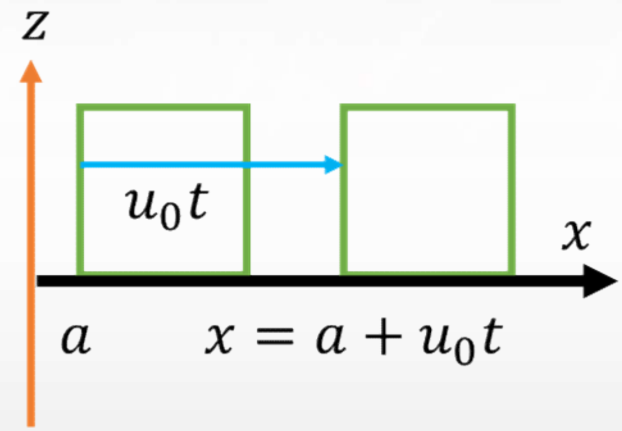
2. At time $t > 0$, the left edge of the square loop is at $x = a + u_0 t$ (original position 'a' plus the shift $u_0 t$ over the time period t).

The magnetic flux passing through the square loop is then

$$\Phi_m = \iint_s \vec{B} \cdot \vec{d}s = \int_x^{x+a} \int_0^a \frac{\mu_0 I_0}{2\pi x} \vec{a}_y \cdot \vec{a}_y dz dx$$

$$= \frac{\mu_0 I_0 a}{2\pi} \int_x^{x+a} \frac{1}{x} dx = \frac{\mu_0 I_0 a}{2\pi} \ln \frac{x+a}{x}$$

$$\Phi_m = \frac{\mu_0 I_0 a}{2\pi} \ln \frac{2a + u_0 t}{a + u_0 t} \quad \text{Wb}$$



Example 3: Solution

3. The induced emf in the loop at time t is then

$$\Phi_m = \frac{\mu_0 I_0 a}{2\pi} \ln \frac{2a + u_0 t}{a + u_0 t} \quad \text{Wb}$$

$$emf = -\frac{\partial \Phi_m}{\partial t} = -\frac{\mu_0 I_0 a}{2\pi}\left(\frac{u_0}{2a + u_0 t} - \frac{u_0}{a + u_0 t}\right) \text{V}$$

Summary

- Apply Faraday's Law to calculate emf in more examples; and
- Decide the direction of flow of induced current using Lenz's Law.

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Session 12-1

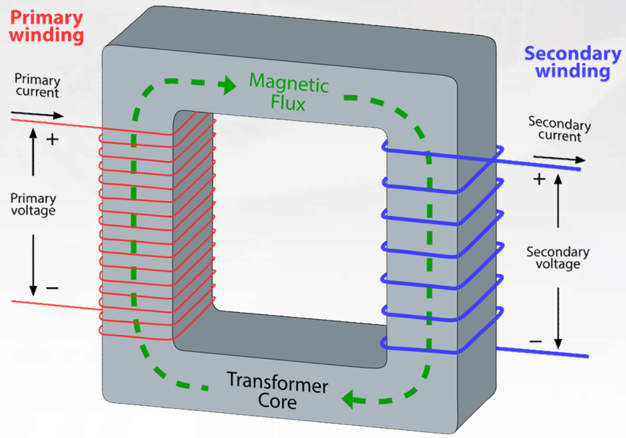
More Examples on Faraday's Law

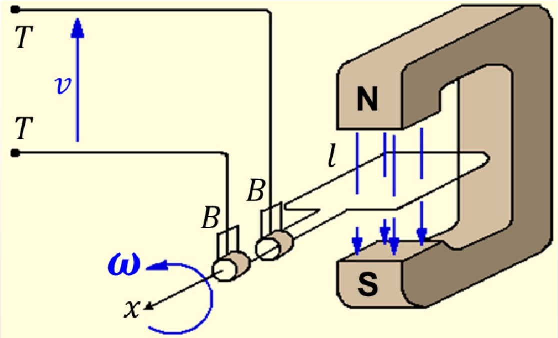
Learning Objectives

- Apply Faraday's law to calculate emf in more examples.

Do You Know?

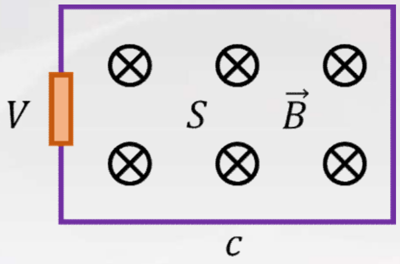
- The principle of operation of:
  - Transformers
  - Audio/video tape-replay
  - Generation of electricity





Faraday's Law

The line integral of the induced electric field around a closed path equals the negative time rate of change of the magnetic flux linked to that path:



$$emf = \oint_C \vec{E} \cdot \vec{d}l = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \iint_S \vec{B} \cdot \vec{d}s$$

One of the Maxwell's equations

Three Cases for Electromagnetic Induction

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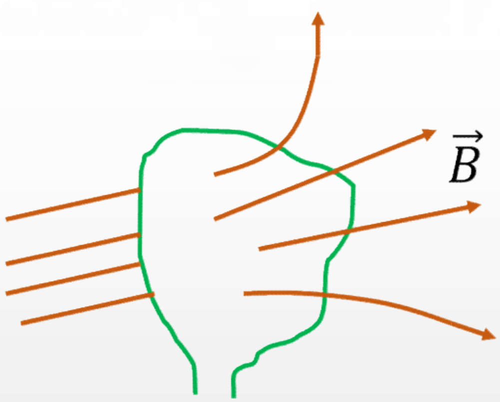
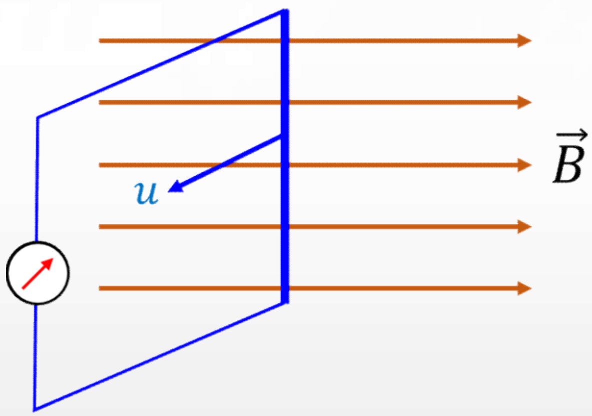
1) A stationary circuit in a time-varying magnetic field (transformer emf).

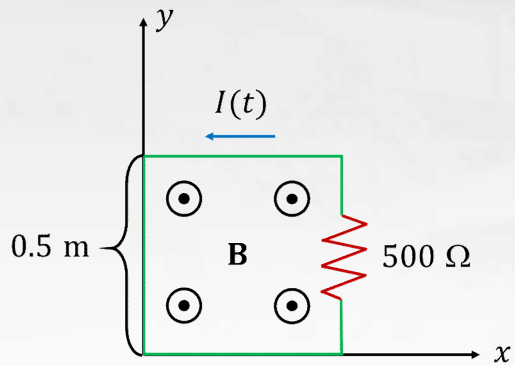
2) A moving circuit (or conductor) in a static magnetic field, such that the magnetic flux linked to the circuit changes with time (motional emf).

3) A combination of 1 and 2: a moving circuit in a time-varying magnetic field.





Example (Hayt & Buck, 10.6)



A perfectly conducting filament containing a small $500-\Omega$ resistor is formed into a square, as illustrated in the Figure.

Find $I(t)$ if

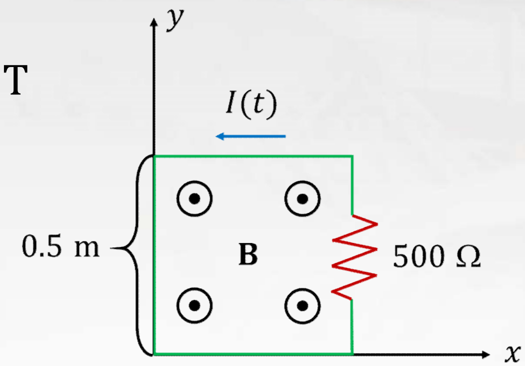
a) $\vec{B} = 0.3\cos\left(120\pi t - 30^\circ\right)\vec{a}_z$ T

b) $\vec{B} = 0.4\cos[\pi(ct - y)]\vec{a}_z$ μT

where $c = 3 \times 10^8$ m/s

Example Part (a) Solution

(a) $\vec{B} = 0.3\cos(120\pi t - 30^\circ)\vec{a}_z \mathrm{T}$



The magnetic flux passing through the square loop is

$$\Phi_m = \iint_S \vec{B} \cdot \vec{ds} = (0.3)(0.5)^2 \cos(120\pi t - 30^\circ)\,\mathrm{Wb}$$

Then the current will be

$$I(t) = \frac{emf}{R} = -\frac{1}{R}\frac{d\Phi_m}{dt} = \frac{(120\pi)(0.3)(0.25)}{500}\sin(120\pi t - 30^\circ)$$

$$= 57\sin(120\pi t - 30^\circ)\,\mathrm{mA}$$

Question: What if $\vec{B}$ points in the direction $\vec{a}_y$?

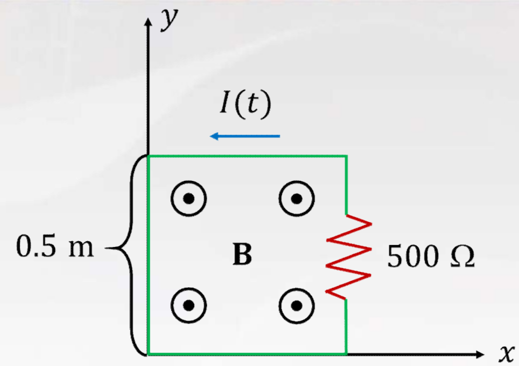
Example Part (b) Solution

(b) $\vec{B} = 0.4\cos[\pi(ct - y)]\vec{a}_z\ \mu\text{T}$

Since the magnetic flux density varies with y, the magnetic flux passing through the square loop is

$$\Phi_m = \iint_S \vec{B} \cdot \vec{ds} = (0.5)(0.4)\int_0^{0.5} \cos(\pi ct - \pi y)\, dy$$

$$= -\frac{0.2}{\pi}\left[\sin\left(\pi ct - \frac{\pi}{2}\right) - \sin(\pi ct)\right]\ \mu\text{Wb}$$



Then the current will be

$$I(t) = \frac{emf}{R} = -\frac{1}{R}\frac{d\Phi_m}{dt} = \frac{(0.2)c}{500}\left[\cos\left(\pi ct - \frac{\pi}{2}\right) - \cos(\pi ct)\right]\mu\text{A}$$

$$= \frac{0.2(3 \times 10^8)}{500}[\sin(\pi ct) - \cos(\pi ct)]\ \mu\text{A}$$

$$= 120[\sin(\pi ct) - \cos(\pi ct)]\text{mA}$$

Summary

- Apply Faraday's law to calculate emf in more examples.

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Session 12-2

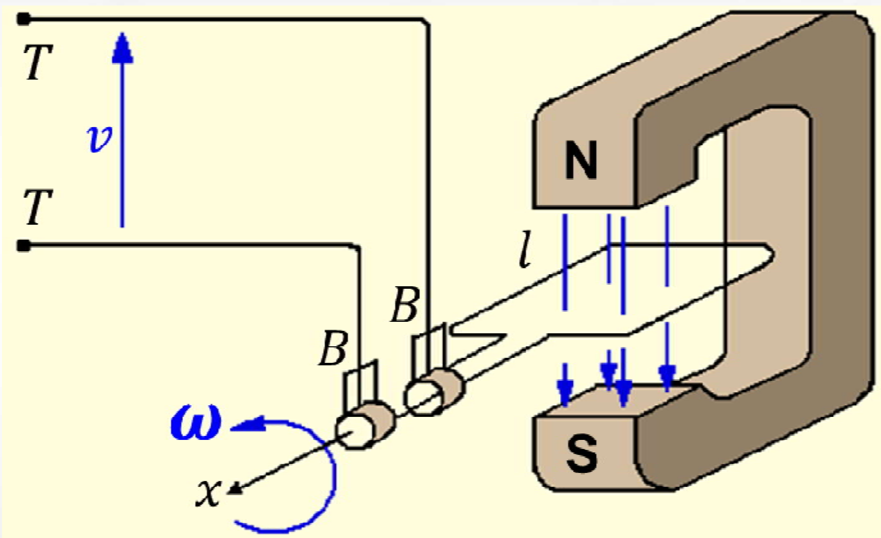
Applications of Electromagnetic Induction - 1

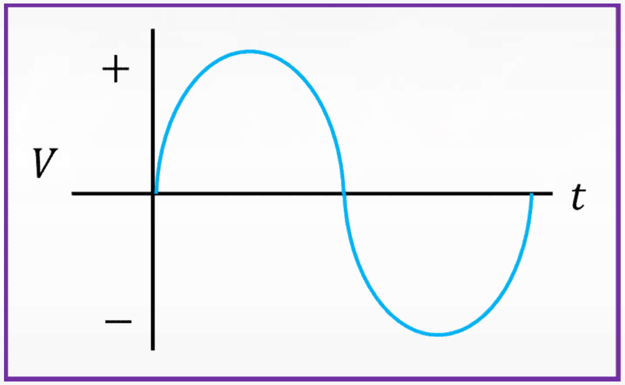
Learning Objectives

- Explain the application of electromagnetic induction in electricity generator; and
- Calculate the voltage generated in an electricity generator.

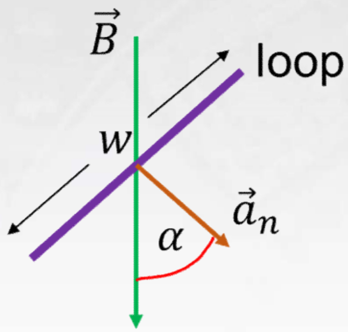
Electricity (AC Voltage) Generator

This diagram shows a simple schematic of an electricity generator. The rectangular loop (length l and width w) is rotating about the x-axis at an angular velocity of $\boldsymbol{\omega}$.





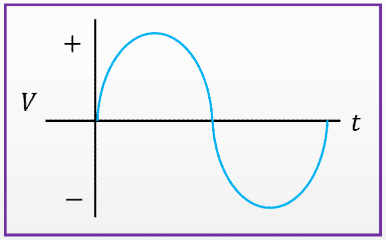
Electricity (AC Voltage) Generator (cont'd)



The magnetic flux $\Phi_m = \vec{B} \cdot \vec{S} = BS\cos\alpha$
where $S = lw$ and $\alpha = \boldsymbol{\omega} t$ is the angle between $\vec{B}$ and the normal to the loop surface, assuming that $\alpha = 0$ at time $t = 0$.

$$emf = -\frac{d}{dt}\left(\vec{B} \cdot \vec{S}\right) = -\frac{d}{dt}[BS\cos(\omega t)] = \omega BS\sin(\omega t)$$

It is seen that the induced voltage across the two ports of the rotor is naturally an AC voltage.



Animation Display

- Hayt and Buck, Ch. 10, Animation 1 (access in NTU Learn, self and peer assessment)

- AC voltage generator.

Quiz

Hayt and Buck, Ch 10, Q1:

A square loop of area $A = 100\ \mathrm{cm}^2$ and $N = 200$ turns rotates in an external steady magnetic field $B = 1.2\vec{a}_x\ \mathrm{T}$. The axis of rotation is $\vec{a}_z$. The loop rotates at a rate of $6000\ \mathrm{RPM}$ (revolutions per minute). What is the amplitude of the voltage e_m induced at the terminals of the loop (at open circuit)?

$$emf = -N\frac{d}{dt}(\vec{B}\cdot\vec{S}) = -N\frac{d}{dt}[BS\cos(\omega t)] = N\omega BS\sin(\omega t)$$

A: $e_m = 1508\ \mathrm{V}$

B: $e_m = 1120\ \mathrm{V}$

C: $e_m = 965\ \mathrm{V}$

D: $e_m = 557\ \mathrm{V}$

Summary

- ❑ Explain the application of electromagnetic induction in electricity generator; and
- ❑ Calculate the voltage generated in an electricity generator.

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Session 12-3

Applications of Electromagnetic Induction - 2

Learning Objectives

- ❑ Explain the application of electromagnetic induction in magnetic recording and replay; and
- ❑ Solve a design problem related to audio tape-recorder.

Magnetic Recording and Replay - 1

Audio/ Video Tape

- ❑ Valdemar Poulsen, a Danish engineer, invented magnetic recording in **1900**.
- ❑ He recorded speech on a thin steel wire with a simple electromagnet.
- ❑ Magnetic tapes were developed as an alternative to wires in the **1940s**.
- ❑ Videotapes were introduced in the late **1950s**.
- ❑ Tape speeds for video recording (past the magnetic head) are ~$5\ \mathrm{m/s}$, compared with only $0.3\ \mathrm{m/s}$ for audio.

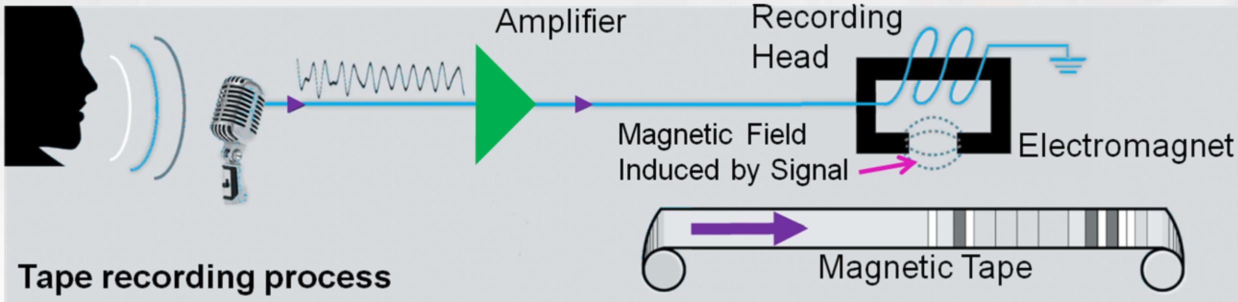
Magnetic Recording and Replay - 2

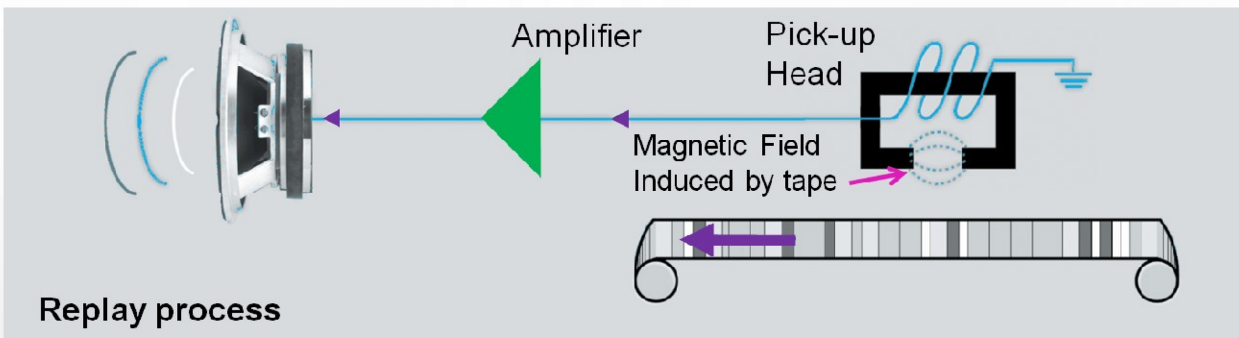
Floppy disk

Hard disk

- Other types of magnetic recording media: the flexible plastic disks called "floppies," the hard disks made of glass or aluminum, etc.

Magnetic Recording and Replay - 3





Example (Sem 2, AY 2012-13)

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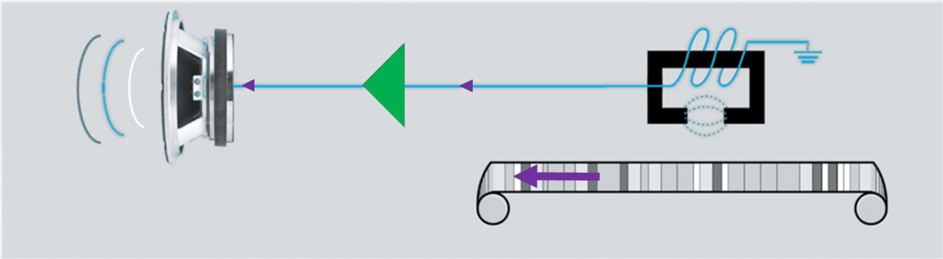
- A circular pick-up coil in an audio tape-recorder has a radius of $2\ \mathrm{mm}$ and its axis coincides with the z-axis. The coil is exposed to a magnetic flux density



$$\vec{B} = B_o \cos(6\pi \times 10^3 t)\vec{a}_z\ \mathrm{Wb/m^2}$$

If $B_o = 0.01\mathrm{Wb/m^2}$ and the pick-up coil must produce a peak voltage of $5\ \mathrm{mV}$, calculate the minimum number of turns required in the coil.

Example (Sem 2, AY 2012-13)

$$\vec{S} = \pi r^2 \vec{a}_z = \pi(2 \times 10^{-3})^2 \vec{a}_z$$

$$\vec{B} = B_o \cos(6\pi \times 10^3 t)\, \vec{a}_z$$

$$\Phi_m = \iint_S \vec{B} \cdot d\vec{s} = \vec{B} \cdot \vec{S} = BS = 4\pi \times 10^{-6} B_o \cos(6\pi \times 10^3 t)\ \mathrm{Wb}$$

$$\mathrm{V}_{ind} = -\frac{d\Phi_m}{dt}$$

$$= 4\pi \times 10^{-6} \times 0.01 \times 6\pi \times 10^3 \times \sin(6\pi \times 10^3 t)$$

$$\mathrm{V}_{ind,peak} = 24\pi^2 \times 10^{-5}\mathrm{V} = 2.37\ \mathrm{mV}$$

∴ To produce a peak voltage of $5\ \mathrm{mV}$, minimum number of turns $N = 3$.

Summary

- Explain the application of electromagnetic induction in magnetic recording and replay; and
- Solve a design problem related to audio tape-recorder.