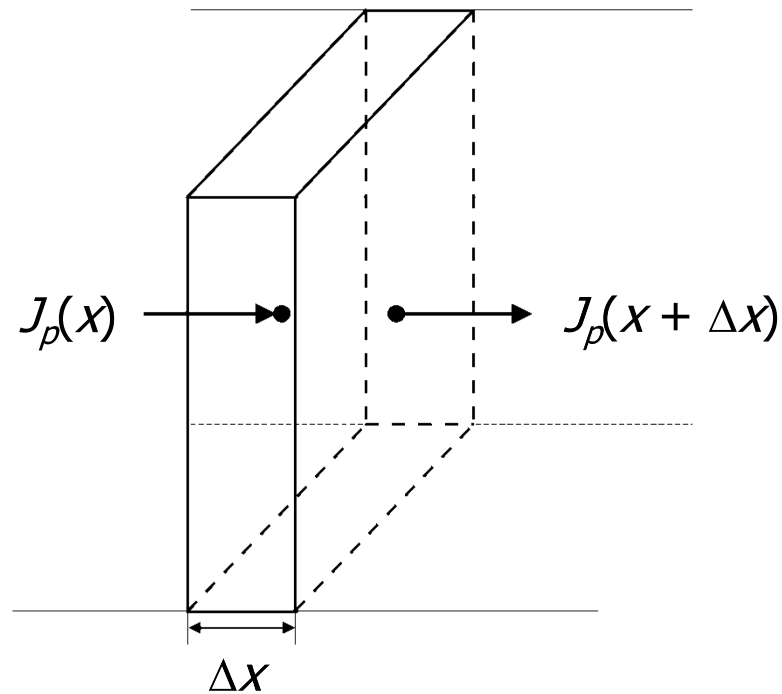
EE2003 Semiconductor Fundamentals

Continuity Equations

Continuity Equation

- The **continuity equation** is a mathematical expression for the **conservation of charges** at any point in a semiconductor.
- One dimensional analysis:
  - Consider an elemental volume of unit cross-sectional area and length Δx.
  - $J_p(x)$ and $J_p(x + \Delta x)$ are the hole current densities at the cross-sectional plane x and $x+\Delta x$ respectively.

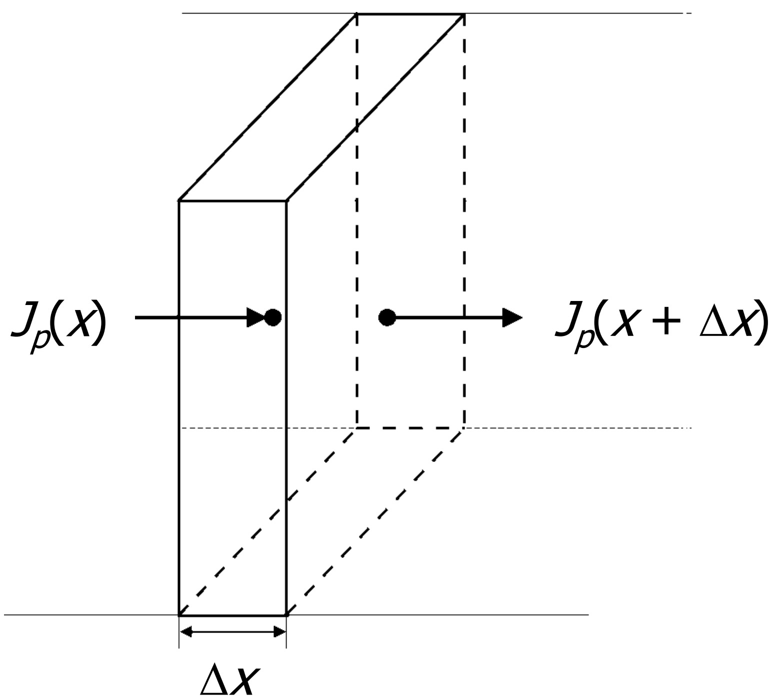


An elemental volume of a piece of semiconductor, showing the flow of holes across the two cross-sectional planes.

Continuity Equation

- The hole concentration within the elemental volume could be changed if:
  - $J_p(x)$ is not equal to $J_p(x+\Delta x)$, i.e. the number of holes flowing into the elemental volume is not equal to that flowing out of it.
  - There is **net** recombination inside the elemental volume.
  - Additional holes are produced by an external light source (i.e. generation due to an external excitation).

Continuity Equation



No. of holes entering on the left

$$= \frac{J_p(x) \cdot A}{q}$$

No. of holes leaving on the right

$$= \frac{J_p(x+\Delta x) \cdot A}{q}$$

Change in number of holes

$$= \frac{J_p(x) \cdot A - J_p(x+\Delta x) A}{q}$$

Continuity Equation

Recall the definition of first-order differential:

$$\frac{df(x)}{dx} = \lim_{\Delta x \to \infty} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- **Continuity equation for holes:**

$$\frac{\partial p}{\partial t} = \frac{1}{q}\left[\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x}\right] + G_L + G_{th} - R$$

$$= -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} + G_L - \frac{\Delta p}{\tau_p}$$

Recall: Net recombination rate, $R - G_{th} = \Delta p / \tau_p$

- The difference in the hole flux entering and leaving the volume.
- The rate at which holes are generated by an external source.
- The net recombination rate within the volume.

Continuity Equation

- **Continuity equation for electrons:**

$$\frac{\partial n}{\partial t} = \frac{1}{-q}\left[\frac{J_n(x) - J_n(x + \Delta x)}{\Delta x}\right] + G_L + G_{th} - R$$

$$= \frac{1}{q}\frac{\partial J_n(x)}{\partial x} + G_L - \frac{\Delta n}{\tau_n}$$

- Recall:

$$J_p(x) = qp\mu_p\xi - qD_p\frac{\partial p}{\partial x}$$

$$J_n(x) = qn\mu_n\xi + qD_n\frac{\partial n}{\partial x}$$

Continuity Equation

- **Continuity equations:**

$$\frac{\partial p}{\partial t} = -\mu_p \left(p \frac{\partial \xi}{\partial x} + \xi \frac{\partial p}{\partial x} \right) + D_p \frac{\partial^2 p}{\partial x^2} + G_L - \frac{\Delta p}{\tau_p}$$

$$\frac{\partial n}{\partial t} = \mu_n \left(n \frac{\partial \xi}{\partial x} + \xi \frac{\partial n}{\partial x} \right) + D_n \frac{\partial^2 n}{\partial x^2} + G_L - \frac{\Delta n}{\tau_n}$$

Continuity Equation

- In general, the continuity equations are complex functions of space and time. Numerical methods are needed to solve them.
- However, certain simplifying assumptions, depending on the conditions of the experiment, may be applied to obtain analytical solutions.
  - No electric field ($\xi = 0$).
  - No external generation source, i.e. $G_L = 0$.
  - Steady state, i.e. $\partial p/\partial t$, $\partial n/\partial t = 0$.

Continuity Equation

- The above assumptions reduce the continuity equations to second-order differential equations:
  - N-type semiconductor: $$\frac{\partial^2 p_n}{\partial x^2} = \frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{D_p \tau_p}$$
  - P-type semiconductor: $$\frac{\partial^2 n_p}{\partial x^2} = \frac{\partial^2 \Delta n_p}{\partial x^2} = \frac{\Delta n_p}{D_n \tau_n}$$
- In uniformly doped semiconductors,

$$\frac{\partial p_n}{\partial x} = \frac{\partial}{\partial x}(p_{n0} + \Delta p_n) = \frac{\partial \Delta p_n}{\partial x}, \frac{\partial n_p}{\partial x} = \frac{\partial}{\partial x}(n_{p0} + \Delta n_p) = \frac{\partial \Delta n_p}{\partial x}$$

(Note: Subscript 'n' or 'p' denotes the doping type)

Example 1

- **Steady-state** excess carrier concentration in a **uniformly doped** n-type semiconductor subjected to **uniform photo-generation**.
  - Incident light induces uniform electron-hole pair generation at a rate of G_L ($cm^{-3}s^{-1}$) $\Rightarrow$ zero carrier-concentration gradient.
  - No electric field present.
  - Steady-state $\Rightarrow$ $\partial p/\partial t$, $\partial n/\partial t = 0$.
  - Continuity equation for holes (assuming low-level injection):

$$G_L - \frac{\Delta p_n}{\tau_p} = 0$$

  - Steady-state excess hole concentration:

$$\Delta p_{n,ss} = G_L \tau_p$$

$$\cancelto{0}{\frac{\partial p_n}{\partial t}} = -\mu_p\left(p_n\frac{\partial \xi}{\partial x} + \xi\frac{\partial p_n}{\partial x}\right) + D_p\frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

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Example 1

$$\frac{\partial p_{n0}}{\partial x} = 0; \frac{\partial \Delta p_n}{\partial x} = 0 \Rightarrow \frac{\partial p_n}{\partial x} = 0$$

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  - Incident light induces uniform electron-hole pair generation at a rate of G_L (cm^{-3}s^{-1}) $\Rightarrow$ zero carrier-concentration gradient.
  - **No electric field present.**
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Example 2

- Continue from Example 1: Decay of photo-generated carriers in an n-type semiconductor with time.
  - Assume steady-state has been reached before the **light source is removed at t = 0 s**.
  - Note that $G_L = 0$ for $t \geq 0$s. The continuity equation for holes (assuming low-level injection) becomes

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(t) = C e^{-t/\tau_p}$$

  - The initial condition: $\Delta p_n(t\!=\!0) = G_L \tau_p$ (from Example 1) enables the integration constant C to be determined.

$$\Delta p_n(t) = G_L \tau_p e^{-t/\tau_p}$$

$$\frac{\partial p_n}{\partial t} = -\mu_p\left(p_n \cancelto{0}{\frac{\partial \xi}{\partial x}} + \xi \cancelto{0}{\frac{\partial p_n}{\partial x}}\right) + D_p \cancelto{0}{\frac{\partial^2 p_n}{\partial x^2}} + \cancelto{0}{G_L} - \frac{\Delta p_n}{\tau_p}$$

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$$\Delta p_n(t) = G_L\tau_p e^{-t/\tau_p}$$

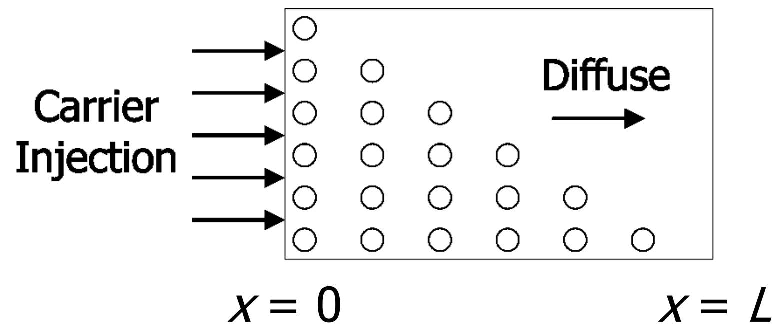
Example 2

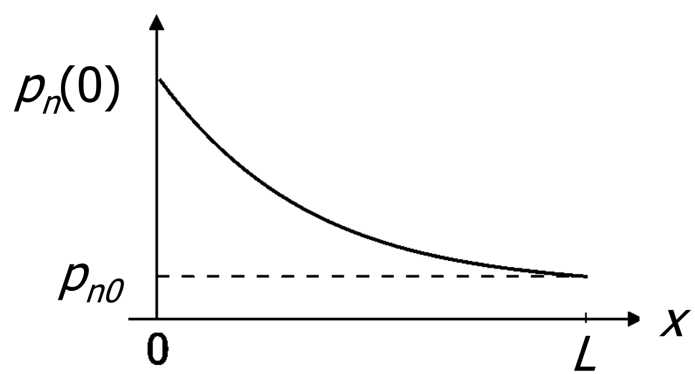
- Recall that the conductivity or conductance of the sample is proportional to the carrier concentrations. If the decay in conductance can be monitored as a function of time (in the form of a current decay), the minority carrier lifetime is easily determined.

Example 3

- **Steady-state** carrier concentration distribution with carrier injection happening at one end of a **uniformly doped** n-type semiconductor sample.

- Assumptions:
  - Hole generation occurs only at the plane $x = 0$.
  - Thermal equilibrium prevails at the other end of the sample.
  - The electric field is very small and may be neglected, $\xi = 0$.





$$\cancelto{0}{\frac{\partial p_n}{\partial t}} = -\mu_p \left(p_n \cancelto{0}{\frac{\partial \xi}{\partial x}} + \xi \cancelto{0}{\frac{\partial p_n}{\partial x}} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + \cancelto{0}{G_L} - \frac{\Delta p_n}{\tau_p}$$

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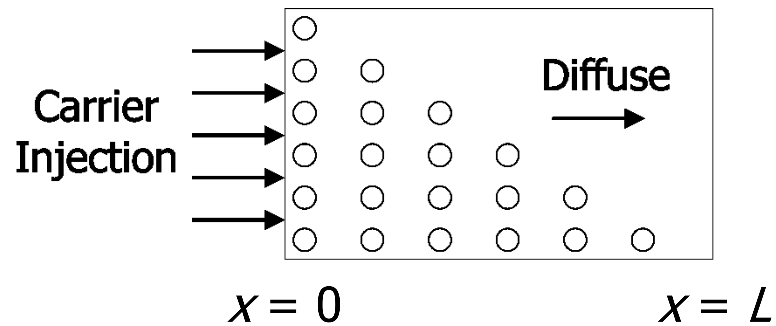
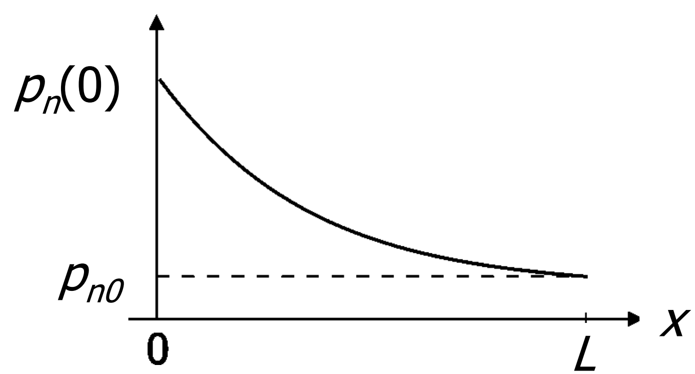




Recall:

$$D_p \rightarrow \mathrm{cm^2 s^{-1}}$$

$$\tau_p \rightarrow \mathrm{s}$$

$$L_p = \left(D_p \tau_p\right)^{1/2} \rightarrow \mathrm{cm}$$

Example 3

- Under steady state:

$$D_p \frac{\partial^2 p_n}{\partial x^2} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{\tau_p}$$

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{L_p^2}, L_p = \left(D_p \tau_p\right)^{1/2}$$

- This is a second-order differential equation, whose solution is of the form:

$$\Delta p_n(x) = C_1 e^{-x/L_p} + C_2 e^{x/L_p}$$

Example 3

- Two **boundary conditions** are needed to determine integration constants C_1 and C_2.
  - At $x = 0$, $\Delta p_n(x = 0) = \Delta p_n(0)$.
  - At $x = L$, $\Delta p_n(x = L) = 0$.
- Assuming a long-base sample, i.e. $L \rightarrow \infty$,

$$C_1 = \Delta p_n(0), C_2 = 0$$

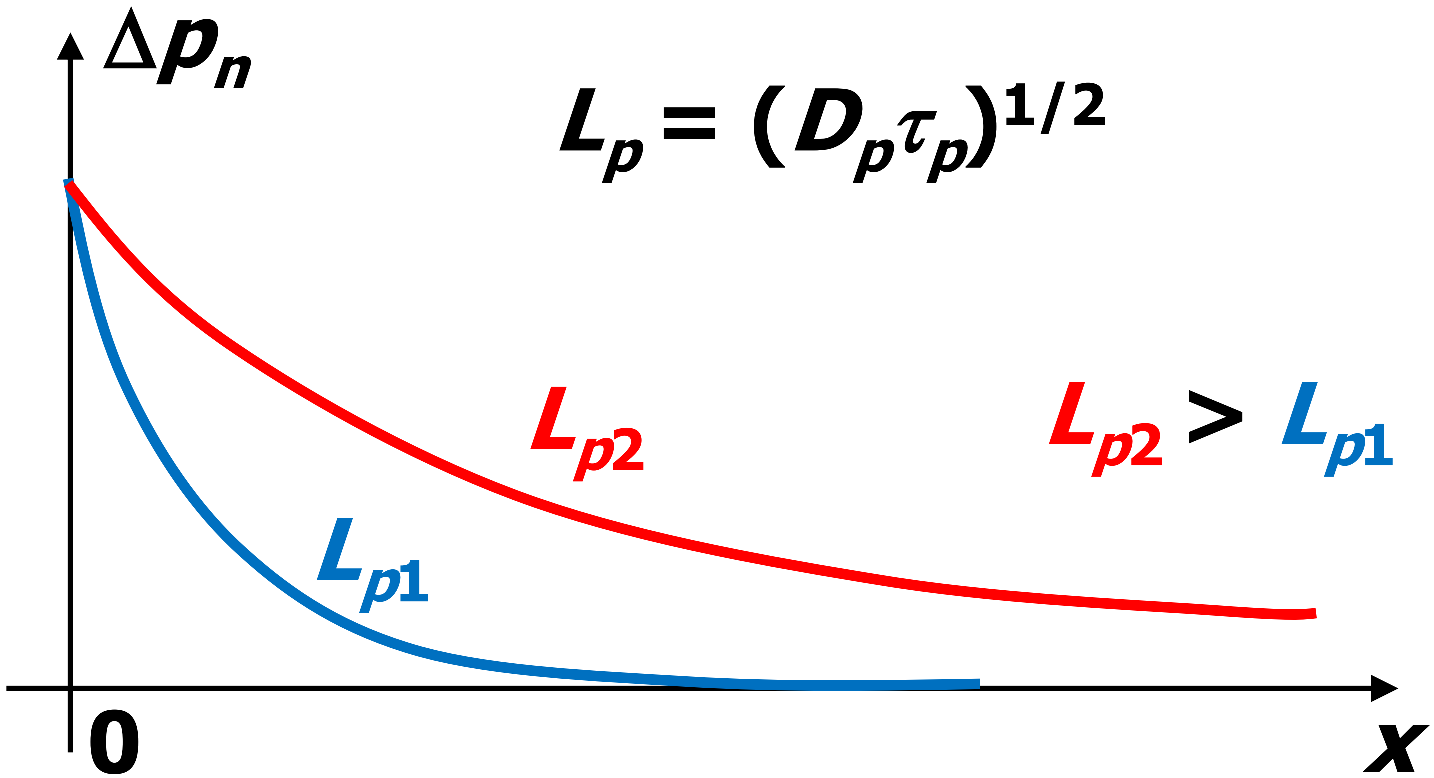
- The lateral hole distribution is an exponentially decaying function:

$$\Delta p_n(x) = \Delta p_n(0) e^{-x/L_p}$$

- L_p is the **minority carrier diffusion length**. At $x = L_p$, 1/e of the initial amount remains. Physically, L_p refers to the average distance a hole (minority carrier) diffuses before recombining.

Example 3

$$L_p = (D_p \tau_p)^{1/2}$$



Summary

- Continuity Equation
  - Meaning
  - Purpose
  - Application
  - Analytical solution (possible only by simplification based on reasonable assumption(s))