Lecture 2: Electromagnetic Principles and Actuators

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Explain the characteristics of magnetisation curve (B-H curve) in three important stages, including linear region, knee region and saturation region.
- ❖ Apply the principles of magnetic circuit and Ampere's Law to derive equations to calculate the magnetomotive force, reluctance, magnetic flux and flux density.
- ❖ Identify the procedures for developing magnetic equivalent circuits.
- ❖ Apply electric circuit theory to analyse and solve magnetic circuit problems.

Ferro-magnetism: Magnetisation Characteristics

❖ Magnetisation characteristics (B-H curve):

- Fig. 13 shows the variation of flux density B in ferro-magnetic material for increasing values of H.

- Initially, B increases almost linearly with H as more domains are aligned, but the increase in B slows down at higher values of H when left with lesser domains for alignment. Finally, B saturates to a certain value B_m when all the domains have aligned.

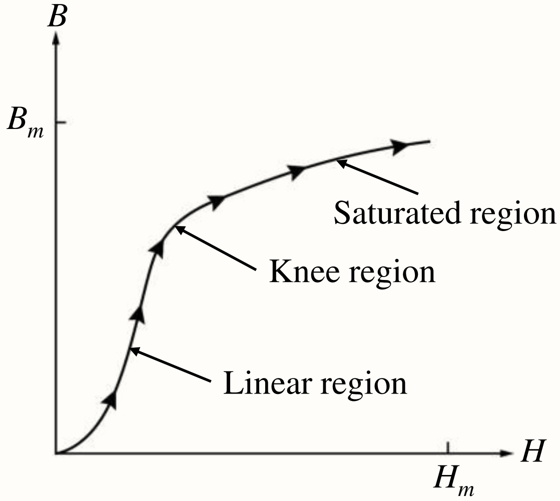

Fig. 13. Typical B-H curve.

Ferro-magnetism

- This curve is called the **Magnetisation Curve** or **B-H Curve**, and the important stages of the characteristics are indicated in Fig. 13:

  - Linear region, where B increases almost linearly with H.
  - Knee region, where the increase in B slows down significantly.
  - Saturation region, when B stops increasing and practically flattens.

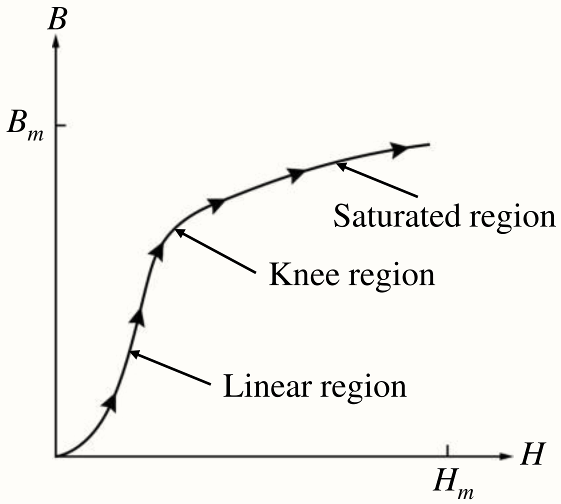

Fig. 13. Typical B-H curve.

Typical Magnetisation (B-H) Curves

- The B-H curves for three common magnetic materials are shown in Fig. 14.
- Note that different materials saturate at different levels of flux density.
- The initial portions of the curves (for lower values of B) are almost linear, where $B=\mu H$ with constant μ which is valid for the linear regions of the curves only.
- The permeability μ changes rapidly after the knee point (in the saturation region).

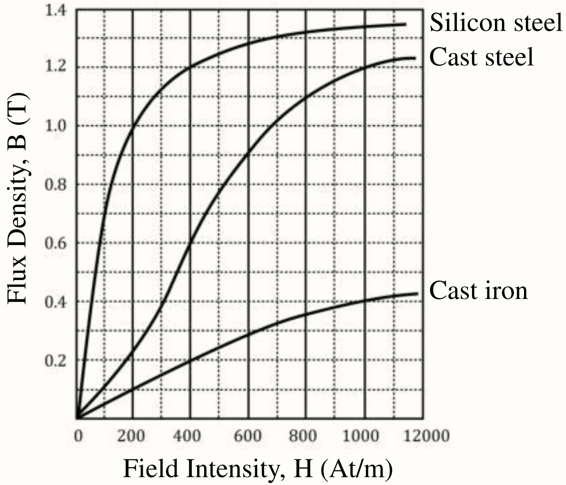

Fig. 14. Typical B-H curves.

Typical Magnetisation (B-H) Curves

- To keep the analysis simple, this course will consider the magnetic system to operate in the linear region, unless otherwise stated.

Permeability

- Since $B=\mu H$, the permeability μ of the material can be obtained as the ratio of B/H at each point of the magnetisation curve.

  - For example, at a point where $B=B_1$, and $H=H_1$: $\mu_1=B_1/H_1$ (**H/m**) as shown in Fig. 15.

- The value of μ increases with H to a max value, and then decreases steadily after saturation sets in.

- The value of μ remains approximately constant (within narrow limits) in the operating range of the flux density B.

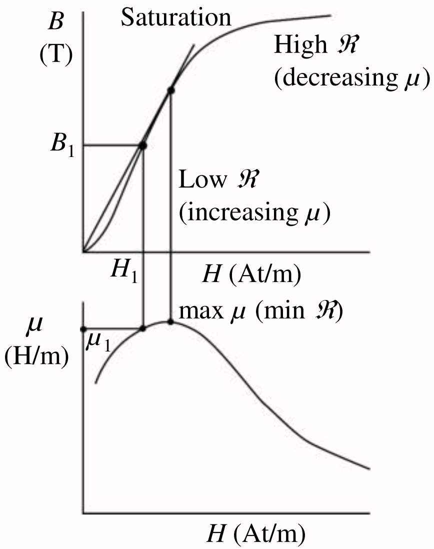

Fig. 15. Variation of permeability.

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Magnetic Circuits

- The flow of magnetic flux (φ) in a magnetic circuit created by the current flowing in a coil may be analysed as the flow of current in an electric circuit. Consider the magnetic circuit shown in Fig. 16.

- Common assumptions:
  - The flux is restricted to the magnetic core (i.e., no leakage of flux).
  - The magnetic flux density (B) is uniform within the magnetic core, which is taken as the flux density along the mean path. ($B=\varphi/A$)

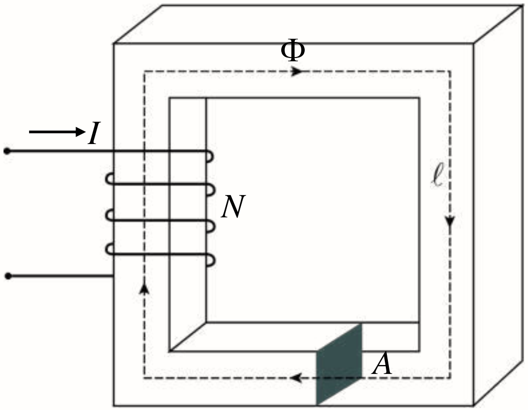

Fig. 16. Magnetic circuit.

Magnetic Circuits

- Applying Ampere's Law along the mean path,

$$\oint_c \vec{H}\,\overrightarrow{d\ell} = I_{enc}$$

Since $\oint_c \vec{H}\,\overrightarrow{d\ell} = H\oint_c \overrightarrow{d\ell} = H\ell$

and $I_{enc} = NI$

We get $H\ell = NI$

so that $B = \mu H = \dfrac{\mu NI}{\ell}$

and flux $\varphi = BA = \dfrac{\mu ANI}{\ell} = \dfrac{NI}{(\ell/\mu A)} = \dfrac{F}{\Re} \Rightarrow F = NI = \varphi\Re$

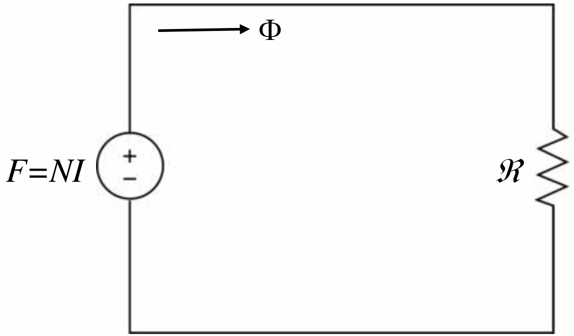

Fig. 17. Magnetic equivalent circuit.

Magnetic Circuits

- $F=NI$ is called the ***Magnetomotive Force*** (***mmf***), analogous to electromagnetic force (*emf*). Note that this depends purely on the electrical properties of the coil or the winding.

- $\mathfrak{R}=\ell/\mu A$ is called the ***Reluctance***, analogous to resistance. Note that this is purely a property of the magnetic core material and the structure.

- $\varphi=F/\mathfrak{R}$ or $F=\varphi\mathfrak{R}$ is called the ***Ohm's Law*** for the magnetic equivalent circuit, which may be drawn as shown in Fig. 17.

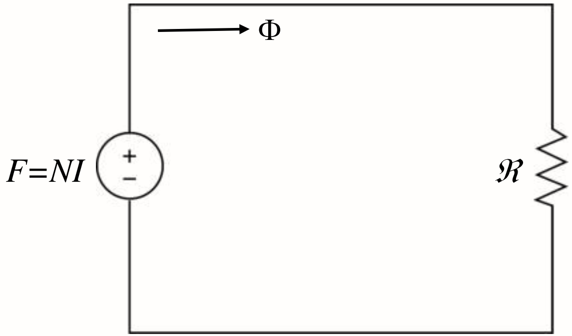

Fig. 17. Magnetic equivalent circuit.

Magnetic Equivalent Circuits

- Procedures for developing magnetic equivalent circuits:
  - Coils represent sources. Use Right Hand Rule to specify the direction of flux.
  - Trace the mean path followed by the flux.
  - Reluctance of various sections with different flux in them must be evaluated separately.
  - For a section with the same flux, reluctances may have to be found separately for different sub-sections, if they have different medium (core material, air gap, etc.) and different cross-sectional area.

Magnetic Equivalent Circuits

- When the magnetic circuit consists of two or more closed loops, Kirchhoff's Voltage Law (KVL) can be applied:

  $NI = \sum H_i \ell_i = \sum \varphi_i \mathfrak{R}_i$, for each closed loop.

- Magnetic circuits can be analysed using magnetic equivalent circuits as long as the circuit remains linear, i.e., the permeability μ remains constant.

Example 2

a) Put in the source and its direction (see Figs. 18 and 19).

b) Identify and trace the paths of φ, φ_1 and φ_2.

c) Find the reluctances of different sections using $\mathfrak{R} = \ell/\mu A$.
Reluctances of sections, $\boldsymbol{be}$, $\boldsymbol{bcde}$ and $\boldsymbol{bafe}$ must be found separately.

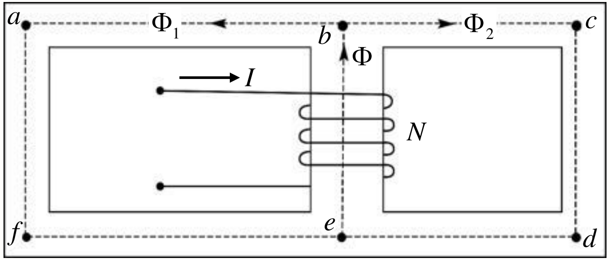

Fig. 18. Magnetic circuit.

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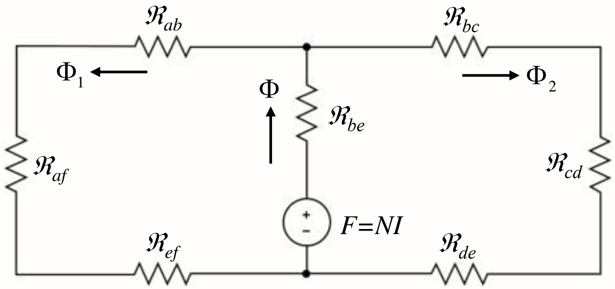

Fig. 19. Magnetic equivalent circuit.

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Example 2

d) Although the same flux flows through sub-sections, bc, cd, de, their reluctances may be found separately if (i) different materials are involved, and (ii) the cross sectional areas are different. Similarly, for sub-sections ba, af and fe.

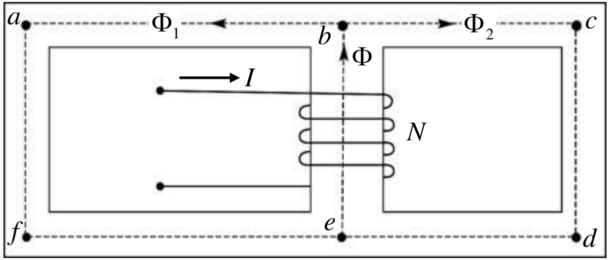

Fig. 18. Magnetic circuit.

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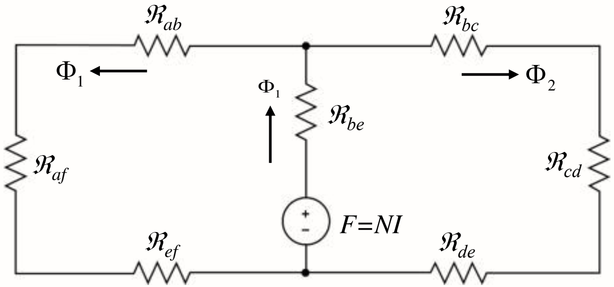

Fig. 19. Magnetic equivalent circuit.

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Magnetic Circuit Problems

- The basic tasks in the analysis of various magnetic circuits are (see Fig. 20):
  - To determine the mmf (or the current in the coil) necessary to establish a given flux or flux density.
  - To find the flux density in different sections for a given mmf (or a given current).

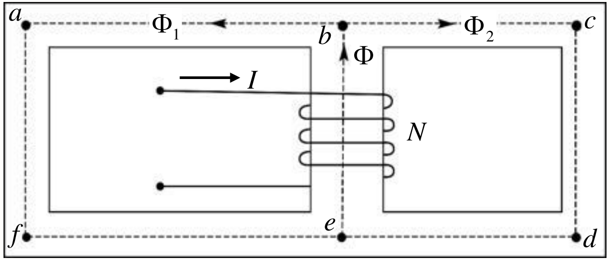

Fig. 20. Magnetic equivalent circuit.

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Magnetic Circuit Problems

❖ Solution techniques:

- For linear magnetic circuits (i.e., under constant μ), both these problems can be solved by using the magnetic equivalent circuit and applying the electric circuit analysis methods.

- For a nonlinear circuit, it is still straightforward to calculate the mmf required to establish the necessary flux. But, calculation of flux density for a given mmf requires an iterative solution.

Example 3 (Part 1)

Draw the magnetic equivalent circuit for the magnetic circuit with the dimensions as shown in Fig. 21. The depth of the core is 10 cm and μ_r of the material is known to be 1400. If the coil has 310 turns, estimate

a) the current required in the coil to obtain a flux density of 0.8 T in the coil, and

b) the flux density at different sections for a current of 2.8 A in the coil.

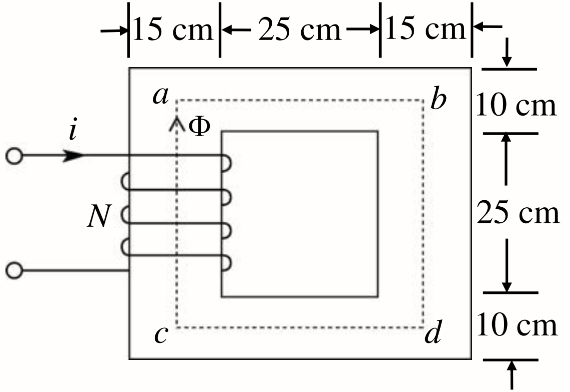

Fig. 21. Magnetic circuit.

(Solutions →)

Example 3 (Part 1) – Solutions

The cross-sectional areas of different sections are

$$A_{ab} = A_{cd} = 10 \times 10 = 100 \text{ cm}^2$$

$$A_{ac} = A_{bd} = 15 \times 10 = 150 \text{ cm}^2$$

The lengths of various sections are

$$\ell_{ab} = \ell_{cd} = 25 + 2 \times 15 / 2 = 40 \text{ cm}$$

$$\ell_{ac} = \ell_{bd} = 25 + 2 \times 10 / 2 = 35 \text{ cm}$$

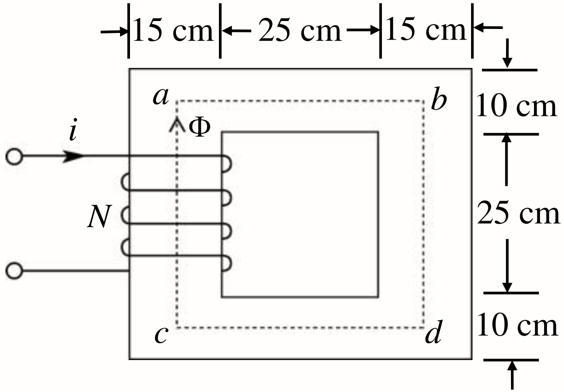

Fig. 21. Magnetic circuit.

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Example 3 (Part 1) – Solutions

The magnetic equivalent circuit is drawn as shown in Fig. 22, where

$$\mathfrak{R}_1 = \mathfrak{R}_3 = \frac{35\times10^{-2}}{1400\times4\pi\times10^{-7}\times150\times10^{-4}} = 13263\ \mathrm{H}^{-1}$$

$$\mathfrak{R}_2 = \mathfrak{R}_4 = \frac{40\times10^{-2}}{1400\times4\pi\times10^{-7}\times100\times10^{-4}} = 22736\ \mathrm{H}^{-1}$$

Total equivalent reluctance

$$\mathfrak{R}_{eq} = 2(\mathfrak{R}_1 + \mathfrak{R}_2) = 72000\ \mathrm{H}^{-1}$$

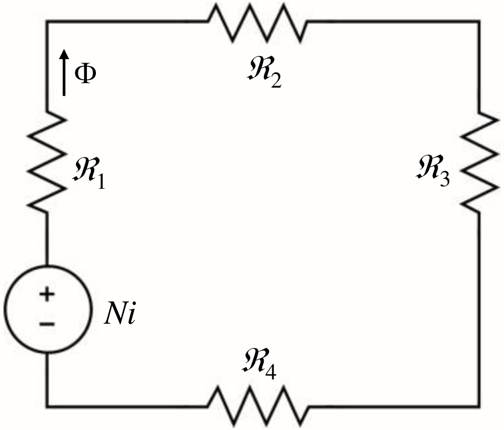

Fig. 22. Magnetic equivalent circuit.

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Example 3 (Part 1) – Solutions

a) For a flux density of 0.8 T inside the coil (see Fig. 23),

$$\varphi_{ac} = B_{ac} \times A_{ac} = 0.8 \times 150 \times 10^{-4} = 0.012 \text{ Wb}$$

$$Ni = \varphi_{ac} \times \mathfrak{R}_{\text{eq}} = 0.012 \times 72000 = 864$$

Therefore, $i = 864 / 310 = 2.787 \text{ A}$

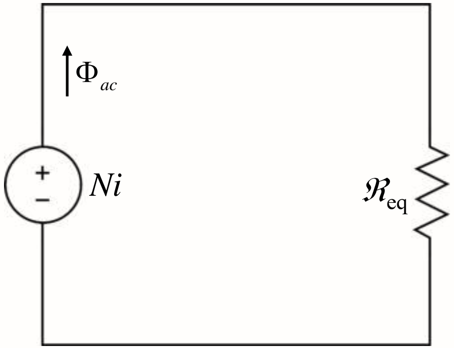

Fig. 23. Magnetic equivalent circuit.

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Example 3 (Part 1) – Solutions

b) For a coil current of 2.8 A (see Fig. 24),

Flux $\varphi = Ni / \mathfrak{R}_{eq} = 310 \times 2.8 / 72000 = 0.012 \text{ Wb}$

$$B_{\text{ab}} = B_{cd} = 0.012 / 100 \times 10^{-4} = 1.20 \text{ T}$$

$$B_{\text{ac}} = B_{bd} = 0.012 / 150 \times 10^{-4} = 0.80 \text{ T}$$

Thus, for linear circuits, tasks (a) and (b) are equivalent.

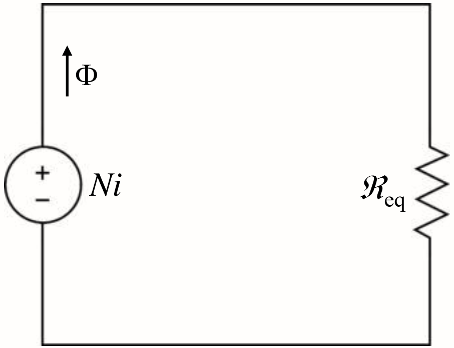

Fig. 24. Magnetic equivalent circuit.

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Example 3 (Part 2)

If the core material in this example is cast steel with the B-H curve as shown in Fig. 25, find

a) the current required in the coil to obtain a flux density of 0.8 T in the coil.

b) The flux density at different sections for a current of 2.8 A in the coil.

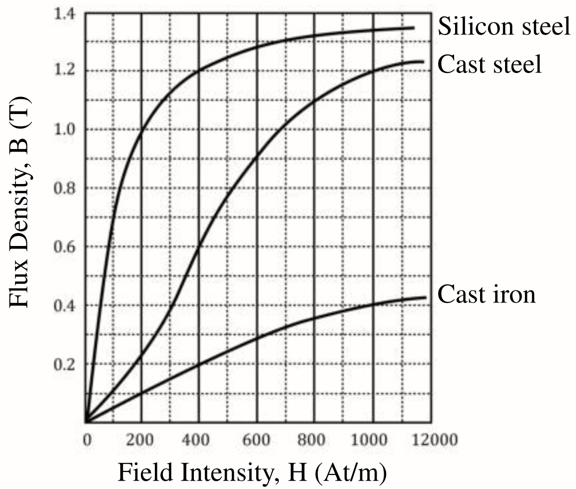

Fig. 25. Magnetisation (B-H) curves.

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(Solutions →)

Example 3 (Part 2) – Solutions

From Example 3 (Part 1), as shown in Fig. 26.

$$A_{ab} = A_{cd} = 10 \times 10 = 100 \text{ cm}^2$$

$$A_{ac} = A_{bd} = 15 \times 10 = 150 \text{ cm}^2$$

$$\ell_{ab} = \ell_{cd} = 25 + 2 \times 15 / 2 = 40 \text{ cm}$$

$$\ell_{ac} = \ell_{bd} = 25 + 2 \times 10 / 2 = 35 \text{ cm}$$

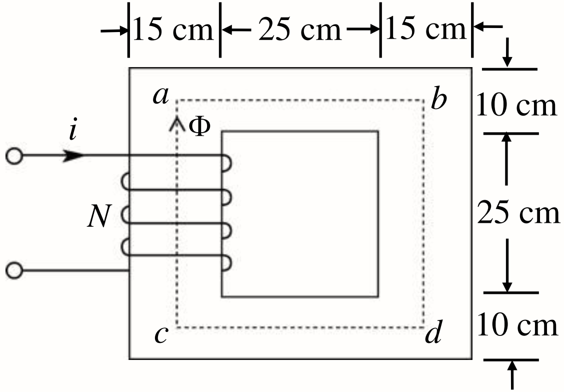

Fig. 26. Magnetic circuit.

Example 3 (Part 2) – Solutions

a) For a flux density of 0.8 T inside the coil,

$$B_{ac} = B_{bd} = 0.8 \text{ T}$$

From the B-H curve, $H_{ac} = H_{bd} = 450 \text{ A/m}$

Also, $\varphi_{ca} = B_{ca} \times A_{ca} = 0.8 \times 150 \times 10^{-4} = 0.012 \text{ Wb}$

$$= \varphi_{ab} = \varphi_{bd} = \varphi_{dc}$$

$$B_{ab} = B_{cd} = 0.012 / 100 \times 10^{-4} = 1.2 \text{ T}$$

From the B-H curve, $H_{ab} = H_{cd} = 1000 \text{ A/m}$

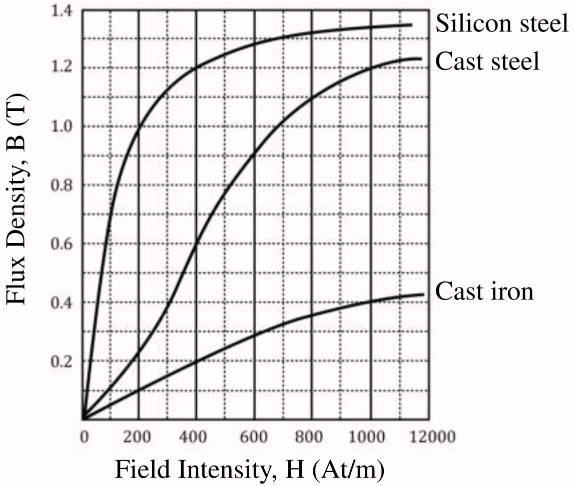

Fig. 25. Magnetisation (B-H) curves.

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Example 3 (Part 2) – Solutions

Therefore,

$$Ni = \sum H_i \ell_i = 2(450 \times 0.35) + 2(1000 \times 0.4) = 1115 \text{ At}$$

$$\therefore i = 1115/310 \text{ A} = 3.6 \text{ A}$$

It should be noted that this current is quite different from the one obtained with the linear model.

This is because $B = 1.2$ T goes well into saturation, and the linear model assuming constant μ is no longer valid.

Example 3 (Part 2) – Solutions

b) Finding the flux density for a given coil current needs iterative solution and will not be discussed. But, it should be clear that at 2.8 A, the flux density in the coil will be significantly lower than 0.8 T.

- However, this course will consider only the linear operation of magnetic circuits where the magnetic equivalent circuits are valid. In such cases, all the electric circuit analysis techniques can be used in the magnetic equivalent circuits.

Summary

In this lecture, you have learnt:

- The characteristics of magnetisation curve (B-H curve) comprising three important stages including linear region, knee region and saturation region.

- Ampere's Law applied to derive the equations to calculate the magnetomotive force, reluctance, magnetic flux and flux density.

- The procedures for developing magnetic equivalent circuits and the electric circuit theories applied to analyse and solve magnetic circuit problems.

References

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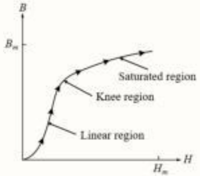
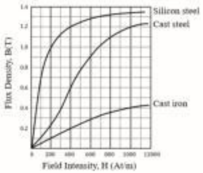
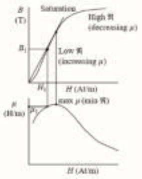
1	4 and 5		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 80), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
2	6, 23 and 25		Reprinted from *AC Circuits and Machines*, (p. 93), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
3	8		Reprinted from *AC Circuits and Machines*, (p. 94), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.

References

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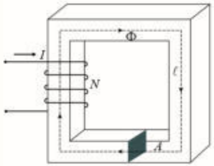
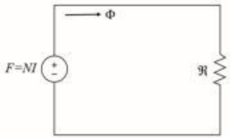
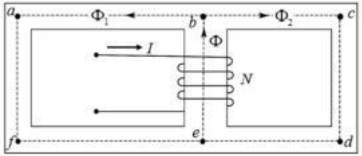
4	9		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 87), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
5	10 and 11		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p.87), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
6	14, 15 and 16		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 89), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.

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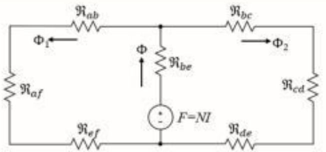
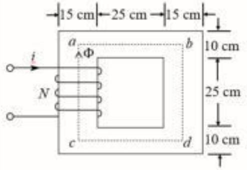
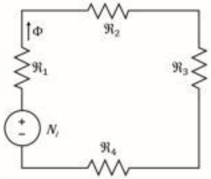
7	14 and 15		Reprinted from *AC Circuits and Machines*, (p. 97), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
8	18, 19 and 24		Reprinted from *AC Circuits and Machines*, (p. 134), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
9	20		Reprinted from *AC Circuits and Machines*, (p. 99), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.

References

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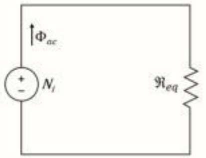
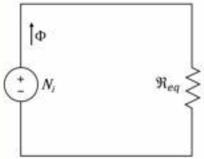
10	21		Reprinted from *AC Circuits and Machines*, (p. 99), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
11	22		Reprinted from *AC Circuits and Machines*, (p. 99), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.