NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

November / December 2019 Time Allowed: $2\frac{1}{2}$ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 4.

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1. Consider the block matrix

$$P = \begin{bmatrix} B & 0 \\ C & A \end{bmatrix},$$

where A and B are square matrices, C and 0 are arbitrary and zero matrices of appropriate dimensions.

(a) Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 8 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A. Show your working clearly.

(10 Marks)

(b) Show that the eigenvalues of matrix A are also the eigenvalues of P. Hence, find the eigenvectors of P corresponding to the eigenvalues of A. You may assume that B is an arbitrary n-by-n matrix. Justify your answers.

(5 Marks)

Note: Question 1 continues on page 2.

(c) Let $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$. Find the remaining eigenvalues of matrix P.

(5 Marks)

(d) Let $C = \begin{bmatrix} 0 & -2 \\ -2 & -1 \end{bmatrix}$. Find the eigenvector of matrix P corresponding to one of the eigenvalues of matrix B found in part (c).

(5 Marks)

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 7 \\ 4 & 11 & 21 & 36 \\ 3 & 21 & 43 & 70 \\ 2 & 16 & 46 & 74 \end{bmatrix}.$$

(a) Use elementary row operations to reduce A to the row echelon form. Hence, find the determinant of A.

(10 Marks)

(b) Based on your working in part (a), or otherwise, find a matrix E that will transform A to the row echelon matrix you obtained in part (a). In other words, find E such that the product EA is the row echelon matrix you obtained in part (a).

(5 Marks)

(c) Consider the system

$$B\mathbf{x} = \mathbf{b},$$

where B is the first three columns of A, i.e.,

$$B = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 11 & 21 \\ 3 & 21 & 43 \\ 2 & 16 & 46 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}.$$

Determine:

(i) the condition(s) for b_1, b_2, b_3 and b_4 such that the system $B\mathbf{x} = \mathbf{b}$ is consistent.

(ii) the null-space and row space of B.

(10 Marks)

3. (a) Given $f(z) = |z^2| + \left|\frac{1}{z}\right|$, where $z = x + iy$, determine:

   (i) the limit of $f(z)$ as $z \to i$,

   (ii) if $f(z)$ is continuous at $z = i$.

   Using the Cauchy-Riemann equations, comment on the differentiability and analyticity of $f(z)$.

   (13 Marks)

   (b) Evaluate

$$\oint_C \left[5e^{2z} + z - 1 + \frac{z}{(z-1)^2(z^2 - 5z + 6)}\right] dz$$

   along the following paths C (counter-clockwise), where

   (i) C is the circle $|z| = \frac{1}{2}$.

   (ii) C is the circle $|z - 1| = \frac{1}{2}$.

   (iii) C is the circle $|z - 2i| = 3$.

   (12 Marks)

4. (a) For any $f(x, y, z)$,

$$\iint_S \text{curl (grad } f) \cdot d\mathbf{A} = 0.$$

   Justify the truth of this equation with proof(s).

   (6 Marks)

   (b) A vector field $\mathbf{F}(x, y, z) = z\,\mathbf{i} + xz\,\mathbf{j} + x\,\mathbf{k}$ cuts a planer surface $S : 3x + 2y + 6z = 6$, $x \geq 0$, $y \geq 0$, $z \geq 0$. Evaluate $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{A}$.

   (11 Marks)

   (c) Hence, using the result from part 4(b), or otherwise, find the work done in moving a particle along the straight line from $(0, 0, 1)$ to $(2, 0, 0)$.

   (8 Marks)

Appendix A

1. Complex Analysis

   (a) Complex Power: $z^c = e^{c \ln z}$

   (b) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$

   (c) Cauchy-Riemann equations:

   $$u_x = v_y, v_x = -u_y, \quad \text{or} \quad u_r = \frac{1}{r}v_\theta, v_r = \frac{-1}{r}u_\theta$$

   (d) Cauchy Integral Formula:

   $$\int_C \frac{f(z)}{(z-z_o)^m}\,dz = \frac{2\pi i}{(m-1)!}\frac{d^{(m-1)}}{dz^{(m-1)}}f(z)|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

   (a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

   (b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

   (c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

   (d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

   (e) Gauss Theorem: $\iiint_T \nabla\cdot\mathbf{F}\,dv = \iint_S \mathbf{F}\cdot\mathbf{n}\,dA$

   (f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dA = \int_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.