

Lecture 12

6 CARRIER TRANSPORT

Net flow of electrons and holes in a semiconductor generate currents, and the process by which these charged particles move is called transport.

The carrier transport phenomena is the foundation for determining the current-voltage characteristic of semiconductor devices.

- There are two basic transport mechanisms in a semiconductor crystal depending on the driving force:
 - Drift : movement of charge due to electric fields
 - Diffusion : flow of charge due to concentration gradients
- In addition, the electrons and holes are in constant motion due to thermal energy. However, the motion due to the thermal energy is random and does not lead to current flow.

- Strictly speaking, when there is a net current flow due to drift and/or diffusion phenomena, the semiconductor is no longer in equilibrium.

In this chapter, however, we will assume that the thermal equilibrium condition is not substantially disturbed. The presence of an electric field does not generate extra or excess carriers (electrons and/or holes).

6.1 Thermal motion

Above 0 K, there is always random motion of particles due to thermal energy.

- The higher the temperature, the greater the thermal energy causing the thermal motion
- At temperature T , the average thermal energy of particles in 3 dimensions (3-D) is

$$E_{thermal\ (3-D)} = \frac{3}{2} k_B T \quad (6.1)$$

($\frac{1}{2} k_B T$ per degree of freedom)

- The average thermal velocity v_{th} of the particle can be obtained by equating eqn. (6.1) to the kinetic energy of the motion

$$\frac{3}{2}k_B T = \frac{1}{2}mv_{th}^2 \quad \therefore v_{th} = \sqrt{\frac{3k_B T}{m}} \quad (6.2)$$

- From eqn. (6.2), the average thermal velocity of electrons at room temperature ($T = 300$ K) is about 10^5 m/s (360,000 km/h). Here, we have used the electron rest mass.
 - this electron kinetic energy corresponds only to several hundredths of an electron volt, and it is not sufficient for electron even to leave semiconductor crystal
- In the absence of any external forces/influences, the electron (or hole) takes part only in the random thermal motion
- there is no directed motion whatsoever, there being no fixed direction which the electron (or hole) would prefer.

- although electrons (or holes) move randomly with a very great velocity, the velocity of the directed motion equals zero.

Fig. 6.1 illustrates the motion of a hole in a semiconductor with and without an electric field ξ .

- without the electric field, there is not net movement of the hole
- with the electric field, there is net movement of the hole

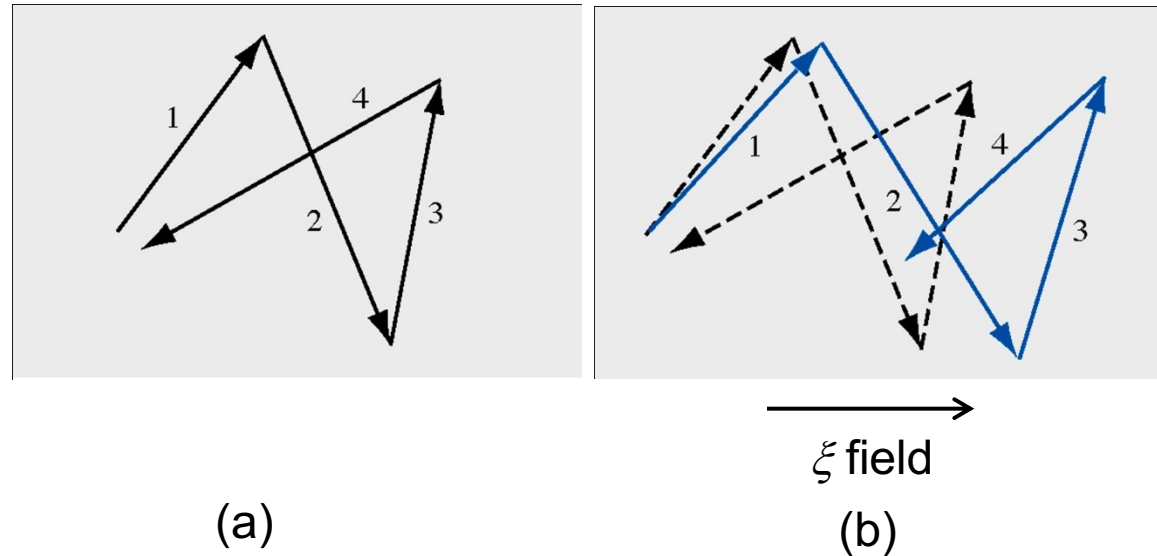


Fig. 6.1 Typical motion behavior of a hole in a semiconductor a) without an electric field and b) with an electric field

<http://pveducation.org/pvcdrom/pn-junction/movement-of-carriers-in-semiconductors>

6.2 Drift current

An electric field applied to a semiconductor produces a force on electrons and holes so that they will experience net acceleration and net movement.

- Directed motion is added on top of the thermal random motion of the carriers. This is illustrated in Fig. 6.1.b.
- Net movement of charge due to an electric field is called drift
- Net drift of charges give rise to a drift current

6.2.1 Mobility

Consider the motion of a hole in the presence of an electric field ξ . The equation of motion is

$$F = q\xi = m_p^* a \quad (6.3)$$

Here, F is the force experienced by the hole, q is the magnitude of the electric charge, a is the acceleration, ξ is the electric field and m_p^* is the effective mass of the hole

- If the electric field ξ is constant with time (i.e. does not change with time), velocity will increase linearly with time. Remember, the velocity at time t is

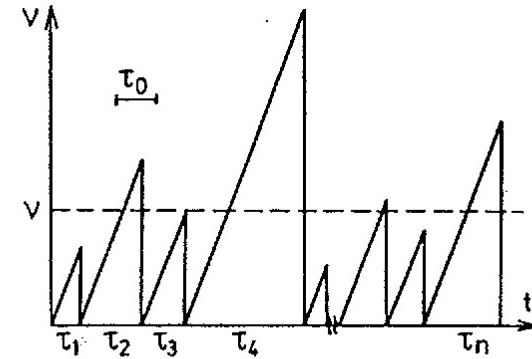
$$v(t) = v_0 + at \quad (6.4)$$

where v_0 is the initial velocity (velocity at $t = 0$)

- however, charged carriers in a semiconductor will experience collisions with
 - thermally vibrating lattice atoms
 - ionized impurity atoms and
 - defects
- after a collision the carrier can move in any direction, and this means that the velocity of directed motion after the collision is equal to zero.
- It follows, from eqn. (6.4), the velocity in the directed motion at time t is

$$v(t) = at \quad (6.5)$$

- Let us denote the time between collisions be τ . Since the collisions are quite accidental, the times between collisions τ can also be quite different.



The velocity of the directed motion of the electron (or the hole) in the electric field versus time at random scatterings.

The velocity of the directed motion of a charge carrier $v(t)$ in the presence of an electric field ξ versus time τ is shown in Fig. 6.2.

Fig. 6.2

- Although the times between collisions are not the same (as shown in Fig. 6.2), there is an average time between collisions which may be denoted by τ_c . Consequently, there is corresponding average velocity in the directed motion, called drift velocity, v_d . From eqn. (6.5), we then have

$$v_d = a \tau_c \quad (6.6)$$

- For a hole, we may then use the expression in eqn. (6.3) to substitute the acceleration a in eqn. (6.6). We obtain the drift velocity for hole

$$v_{dp} = \frac{q \tau_{cp}}{m_p^*} \xi \quad (6.7)$$

where τ_{cp} is the mean time between collisions for holes

- Note that in deriving eqn. (6.7) we have assumed low electric fields where τ_{cp} does not depend on ξ .

In eqn. (6.7), the hole drift velocity v_{dp} is directly proportional to the electric field ξ . We can define the hole mobility μ_p by writing

$$v_{dp} = \mu_p \xi \quad (6.8)$$

where

$$\mu_p = \frac{q \tau_{cp}}{m_p^*} \quad (6.9)$$

- mobility describes how well a particle will move due to an electric field
- unit of mobility is usually expressed in $\text{cm}^2/\text{V-s}$

We can apply the same analysis for electrons. Since electrons are negatively charged, the equation of motion of an electrons in the presence of an electric field ξ is

$$F = -q\xi = m_n^* a \quad (6.10)$$

- The electron drift velocity is

$$v_{dn} = a \tau_{cn} \quad (6.11)$$

where τ_{cn} is the mean time between collisions for electrons.

From eqn. (6.10), we obtain

$$v_{dn} = -\frac{q \tau_{cn}}{m_n^*} \xi \quad (6.12)$$

- We then write
$$v_{dn} = -\mu_n \xi \quad (6.13)$$

where
$$\mu_n = \frac{q \tau_{cn}}{m_n^*} \quad (6.14)$$

- From eqn. (6.13), the electron drift velocity is in opposite direction to the applied electric field.
- From eqn. (6.14), the electron mobility μ_n is a positive quantity

Table 6.1 list typical mobility values at $T = 300$ K of some semiconductors with low doping concentration

We see that $\mu_p < \mu_n$, this means for the same electric field, electrons move faster than holes.

Table 6.1

	μ_n (cm ² /V-s)	μ_p (cm ² /V-s)
Si	1350	480
GaAs	8500	400
Ge	3900	1900

Let us now look at the behavior of the mobility.

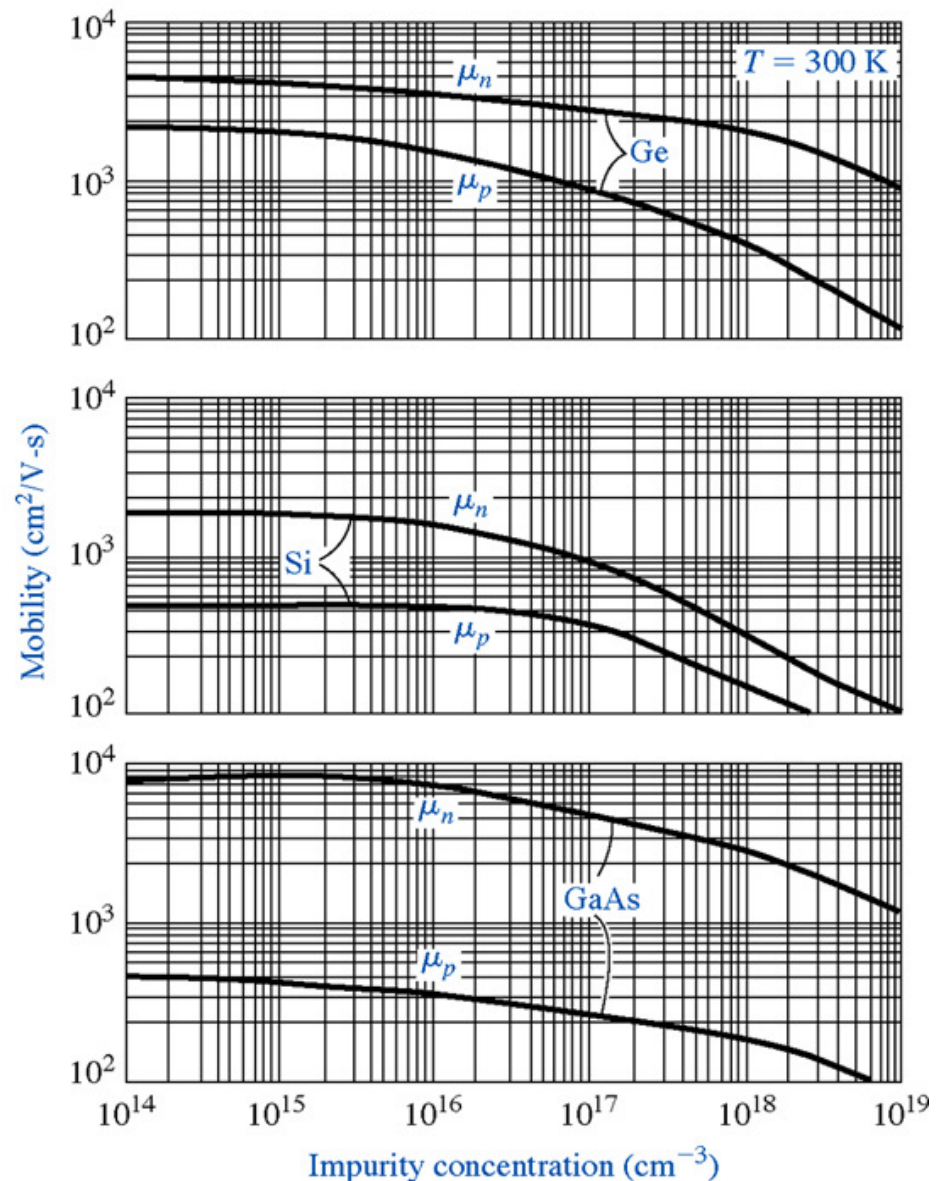
- We have earlier seen that charged particles in a semiconductor will be involved in collisions or scatterings with
 - thermally vibrating lattice atoms
 - ionized impurity atoms, and
 - defects

- It is interesting and important to note here, that the thermal vibration of lattice atoms, impurity atoms and defects disrupt the perfect periodic potential in the crystal.
 - The scattering is caused by the imperfection of the periodic potential.
 - A perfect periodic potential will allow the charge carriers (electrons and holes) to move unimpeded, or with no scattering, in the crystal. This can be understood from the wavefunctions discussed in Appendix D section D.2..
- Let us now discuss the scattering due to lattice vibrations in more detail.
 - Above 0 K, the atoms in a semiconductor vibrate about their lattice position in the crystal.
 - The thermal vibrations cause disruption of the periodic potential, resulting in an interaction between electrons or holes and the vibrating atoms and hence scattering.

- This type of scattering is referred to lattice scattering or phonon scattering.
- The lattice vibrations are expected to increase with increasing temperature, which implies that the number of scatterings per unit time increases. Consequently, the average time between collision decreases and hence, from eqns. (6.9) or (6.14), the mobility decreases.
- Next, let us discuss the scattering due to impurity atoms.
 - As we have seen earlier, the impurity atoms are all ionized at room temperature
 - Again, the ionized impurities disrupt the periodic potential causing the scattering. Here, the interaction between the carriers and the ionized impurities is the coulomb interaction.
 - This scattering is referred to ionized impurity scattering.

- We will expect that the number of scatterings per unit time to increase with increasing impurity concentration, which implies that the average time between scattering decreases. Again, from eqns. (6.9) or (6.14), the mobility will decrease.

Fig. 6.3 shows the electron and hole mobilities versus impurity concentrations of some semiconductor materials at 300 K.



Electron and hole mobilities versus impurity concentration for Ge, Si and GaAs at 300 K

Fig. 6.3

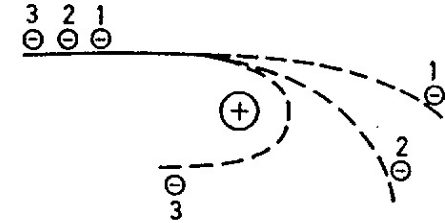
- We now ask a question, what is the effect of temperature on the impurity scattering.

- As the temperature increases, the random thermal velocity of carriers increases.

- This would reduce the time the carriers spend in the vicinity of the ionized impurity center.

- The less time spent in the vicinity of a coulomb force, the smaller the scattering effect and thus increasing the mobility due to the impurity scattering. Fig. 6.4 shows the schematic of the scattering effect with the carrier velocity.

- So, in contrast to the lattice scattering, the mobility due to the impurity scattering increases with increasing temperature.



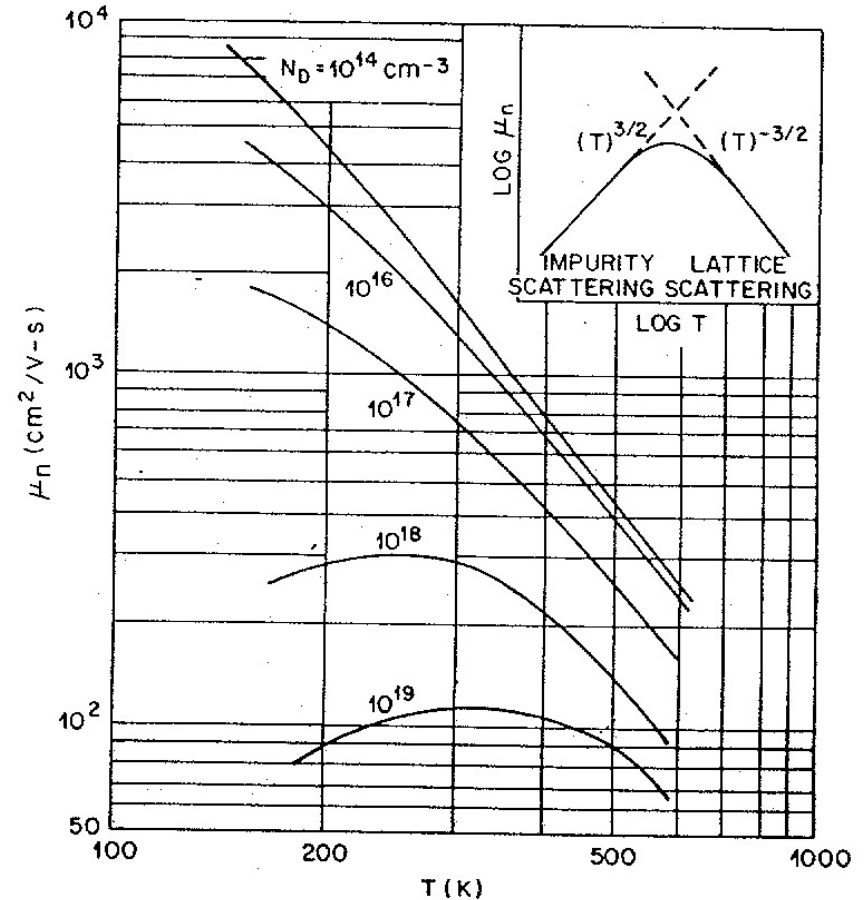
The carrier scattering by a charged impurity center. Electrons (1, 2 and 3) fly up to the impurity center along the same trajectory but with different velocities. Electron 1 has the greatest velocity; the attraction of the impurity atom in practice does not affect its initial trajectory. Electron 3 has the lowest velocity; the attraction of the charged impurity makes it change its route backwards.

Fig. 6.4

- We have discussed the effect of temperature on the mobility due to the lattice and ionized impurity scatterings.

Fig. 6.5 shows the electron mobility versus temperature of an n-type semiconductor.

- We firstly see that the mobility increases as the donor concentration decreases. This behavior has been discussed earlier (see Fig. 6.3).



Electron mobility in silicon versus temperature for various donor concentrations. Insert shows the theoretical temperature dependence of electron mobility.

Fig. 6.5

- For each mobility curve, we see that the mobility first increases and then on reaching a maximum begins to fall. So, we can divide into two regions, low and high temperature regions
- At high temperature region, the mobility decreases with increasing temperature. This indicates that at this region, the lattice scattering dominates.
- At low temperature region, the mobility increases with increasing temperature, this shows that at this region the impurity scattering dominates.

Key takeaways (Lecture #12)

- There are two basic transport mechanisms in a semiconductor crystal depending on the driving force:
 - Drift : movement of charge due to electric fields
 - Diffusion : flow of charge due to concentration gradients

- Drift velocity:

$$v_{dp} = \frac{q \tau_{cp}}{m_p} \xi \quad v_{dp} = \mu_p \xi$$

$$v_{dn} = -\frac{q \tau_{cn}}{m_n} \xi \quad v_{dn} = -\mu_n \xi$$

- At high temperature region, the mobility decreases with increasing temperature → lattice scattering dominates
- At low temperature region, the mobility increases with increasing temperature → impurity scattering dominates

Lecture 13

6.2.2 Drift current density

Consider a semiconductor sample with **hole density p** moving with drift velocity v_{dp} in the presence of an electric field ξ

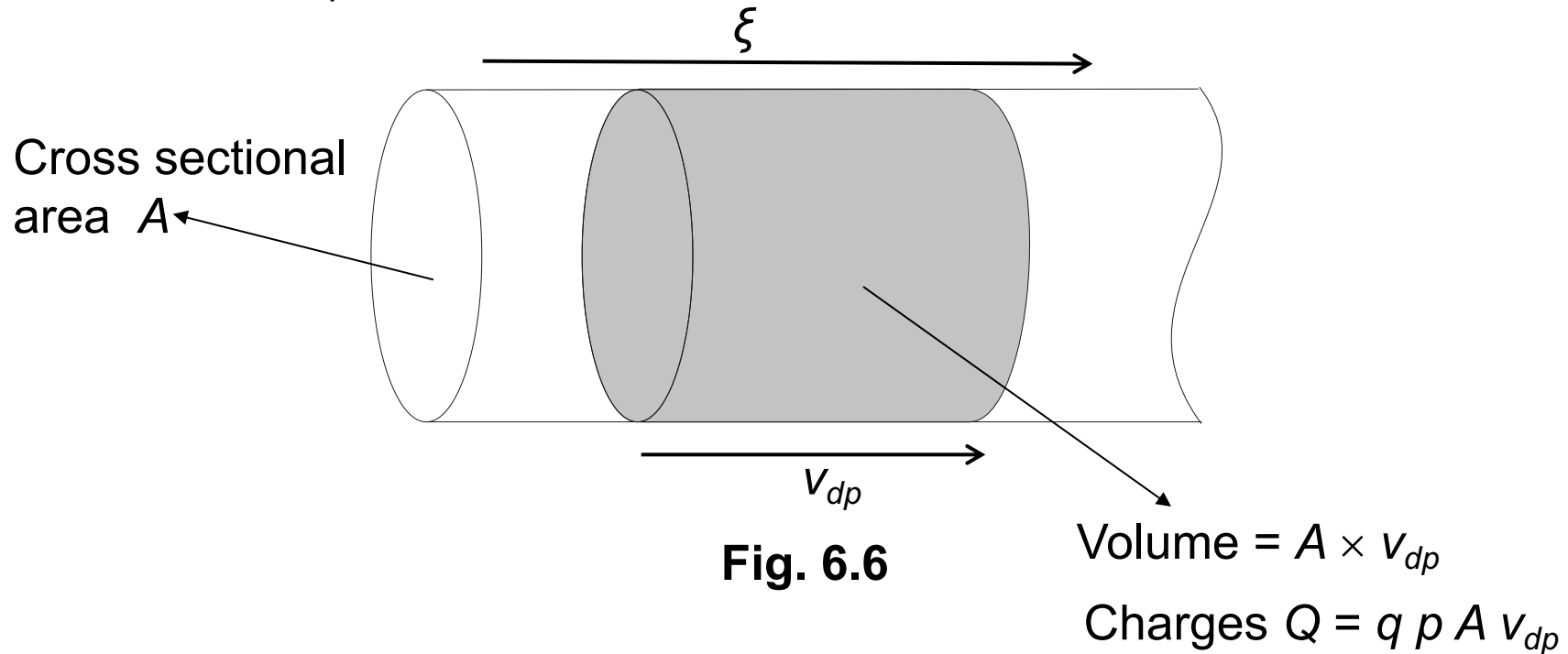


Fig. 6.6

- From Fig. 6.6, the number of charges in the volume $A \times v_{dp}$ is

$$Q = q p A v_{dp} \quad (6.15)$$

- Since the drift velocity is v_{dp} , in one second, the charges Q in the volume will cross the cross sectional A .
 - By the definition of a current = the number of charge crossing an area per second
 - It follows that the hole drift current

$$I_{p\ drift} = q\ p\ A\ v_{dp} \quad (6.16)$$

- Hence, the hole drift current density is

$$J_{p\ drift} = q\ p\ v_{dp} \quad (6.17)$$

Remember, the current density is the number of charge crossing a unit area per second.

- Using eqn. (6.8), we can substitute v_{dp} in eqn. (6.17). We then obtain

$$J_{p\ drift} = q\ p\ \mu_p\ \xi \quad (6.18)$$

We can apply the same analysis for electrons.

- If we do that we obtain the electron drift current density

$$J_{n\ drift} = -\ q\ n\ v_{dn} \quad (6.19.a)$$

Here, the negative (– ve) sign is because the electron is negatively charged. The electron drift current density $J_{n\ drift}$ is in the opposite direction to the electron drift velocity v_{dn} .

- From the expression for v_{dn} in eqn. (6.13), we get

$$J_{n\ drift} = -q\ n\ (-\mu_n\ \xi) = q\ n\ \mu_n\ \xi \quad (6.19.b)$$

- In semiconductors the electrical conduction is due to both electron and hole currents.
 - Therefore, the total drift current density J_{drift} is the sum of the individual electron and hole drift current densities

$$J_{drift} = J_{n\ drift} + J_{p\ drift}$$

$$J_{drift} = q\ \mu_n\ n\ \xi + q\ \mu_p\ p\ \xi = (q\ \mu_n\ n + q\ \mu_p\ p)\ \xi \quad (6.20)$$

- We may write the total drift current density J_{drift} in eqn. (6.20) as

$$\boxed{J_{drift} = \sigma \xi} \quad (6.21)$$

where $\boxed{\sigma = q\mu_n n + q\mu_p p} \quad (6.22)$

- σ is the electrical conductivity of the semiconductor material.
 - the unit is $(\text{ohm-cm})^{-1}$ or $(\Omega\text{-cm})^{-1}$
- Resistivity ρ is the reciprocal of conductivity

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n + q\mu_p p} \quad (6.23)$$

- the unit is (ohm-cm) or $(\Omega\text{-cm})$

- For the case of an extrinsic semiconductor, where the majority carrier concentration greatly exceeds the minority carrier concentration, we can write as follow

- For p-type where $p \gg n$,
$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_p p} \quad (6.24.a)$$

- For n-type where $n \gg p$,
$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n} \quad (6.24.b)$$

Example 6.1

Consider a bar of semiconductor as shown in Fig. 6.7 with an applied voltage V that produces a current I . Using eqn. (6.21) derive the expression $V = IR_R$ where R_R is the resistance of the semiconductor.

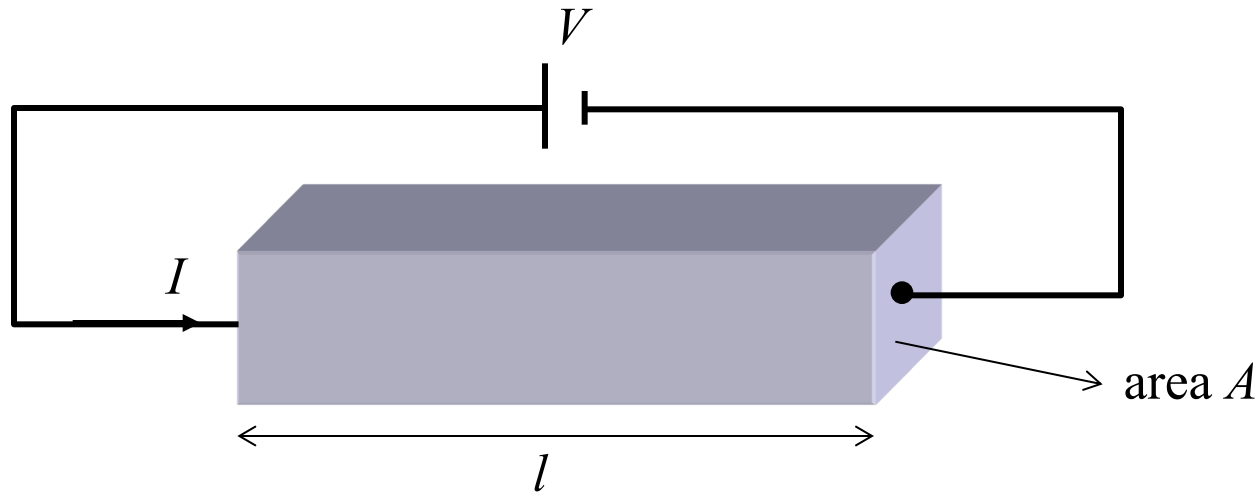


Fig. 6.7

The current density is $J = \frac{I}{A}$ and the electric field is $\xi = \frac{V}{l}$

From eqn. (6.21) $J = \sigma \xi$

$$\frac{I}{A} = \sigma \frac{V}{l}$$

$$V = \left(\frac{1}{\sigma} \frac{l}{A} \right) I$$

$$V = \left(\rho \frac{l}{A} \right) I = IR_R \quad \text{which is Ohm's law}$$

where $R_R = \rho \frac{l}{A}$

<http://pveducation.org/pvcdrom/pn-junction/drift>

6.3 Diffusion current

The word *diffusion* comes from Latin word “diffusio” (overflow, penetration). This notion can be applied to gases, liquids and solids.

- Diffusion is a flow of particles (e.g. electrons and holes) from a high concentrations region towards a low concentration region. This process is a consequence of random thermal motions of particles (atoms, molecules, electrons, holes etc.)
- To analyse the diffusion process, consider one-dimensional case where an electron density of a semiconductor varies in the x -direction as shown in Fig. 6.8.
- Here, the semiconductor temperature is assumed to be uniform, so that the average thermal energy, and hence the average thermal velocity v_{th} , of electrons is independent of x .

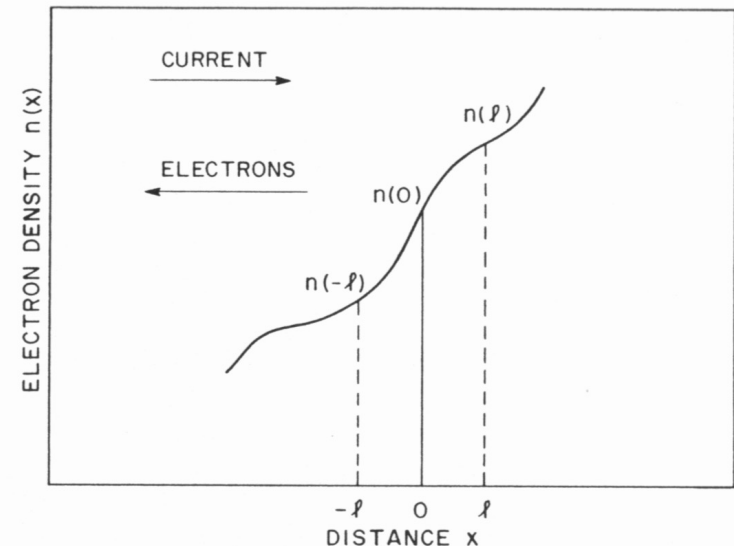
- Having a thermal velocity v_{th} and a mean time between collisions t_{cn} , the electron will consequently have a mean free path l , that is the average distance an electron travels between collisions. This is given by

$$l = v_{th} \tau_{cn} \quad (6.25)$$

- We can express the electron concentration at $x = l$ and $x = -l$ about the electron concentration at $x = 0$ (linear approximate and consider the slope in Fig. 6.8)

$$n(+l) = n(0) + l \frac{dn}{dx} \quad (6.26.a)$$

$$n(-l) = n(0) - l \frac{dn}{dx} \quad (6.26.b)$$



Electron concentration versus distance; l is the mean free path. The directions of electron and current flows are indicated by arrows.

Fig. 6.8

- On the average, at any instant of time, half of the electron concentration at $x = l$ will be traveling to the left and half of the electron concentration at $x = -l$ will be traveling to the right crossing the $x = 0$ plane.

Therefore, the average number of electrons crossing the plane at $x = 0$ per unit area per unit time in the $+x$ direction (or the electron flux) is

$$\phi_n = \frac{1}{2} n(-l) v_{th} - \frac{1}{2} n(l) v_{th} \quad (6.27)$$

Using eqn. (6.26)
$$\phi_n = \frac{1}{2} v_{th} \left\{ \left[n(0) - l \frac{dn}{dx} \right] - \left[n(0) + l \frac{dn}{dx} \right] \right\}$$

$$\phi_n = -v_{th} l \frac{dn}{dx} = -D_n \frac{dn}{dx} \quad (6.28)$$

where $v_{th} l = D_n \quad (6.29)$

is the electron diffusivity or the electron diffusion coefficient. The unit is cm^2/s .

- Since each electron has a charge of $-q$, the electron diffusion current density is

$$J_{n \text{ diff}} = -q\phi_n = q D_n \frac{dn}{dx} \quad (6.30)$$

- We can apply the same analysis to holes. We obtain the average number of holes crossing the plane at $x = 0$ per unit area per unit time in the $+x$ direction (or the hole flux) is

$$\phi_p = -D_p \frac{dp}{dx} \quad (6.31)$$

Since each hole has a charge of $+q$, the hole diffusion current density is

$$J_{p \text{ diff}} = q\phi_p = -q D_p \frac{dp}{dx} \quad (6.32)$$

here D_p is the hole diffusivity or the hole diffusion coefficient and has units of cm^2/s .

- Fig. 6.9 shows the diffusion current density and the concentration of charge carriers.

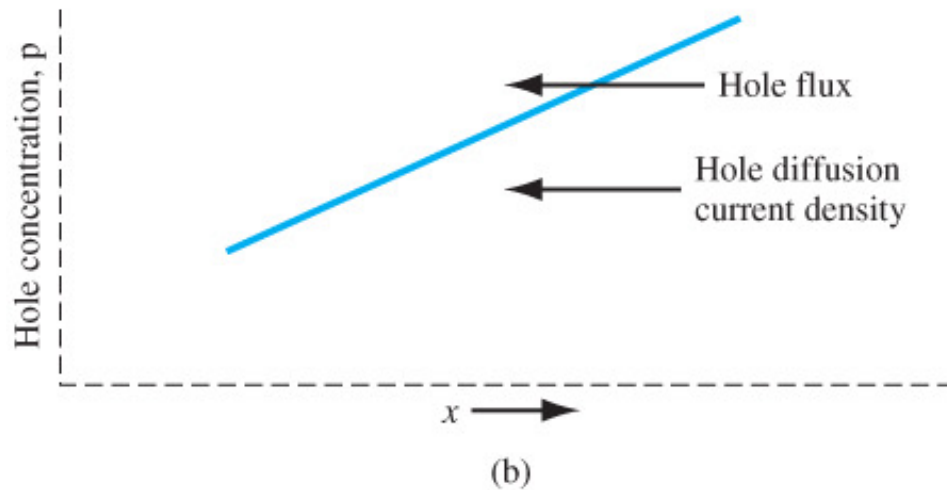
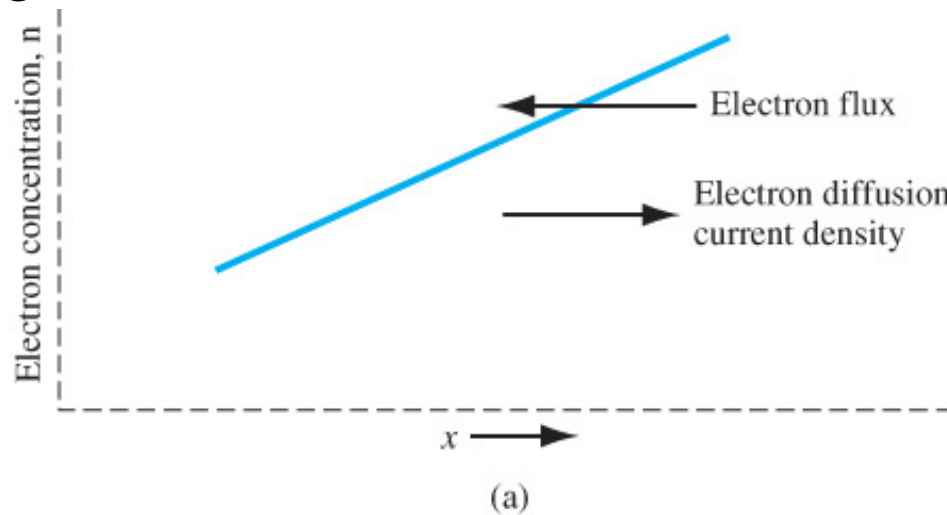


Fig. 6.9

Figure 5.11 | (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

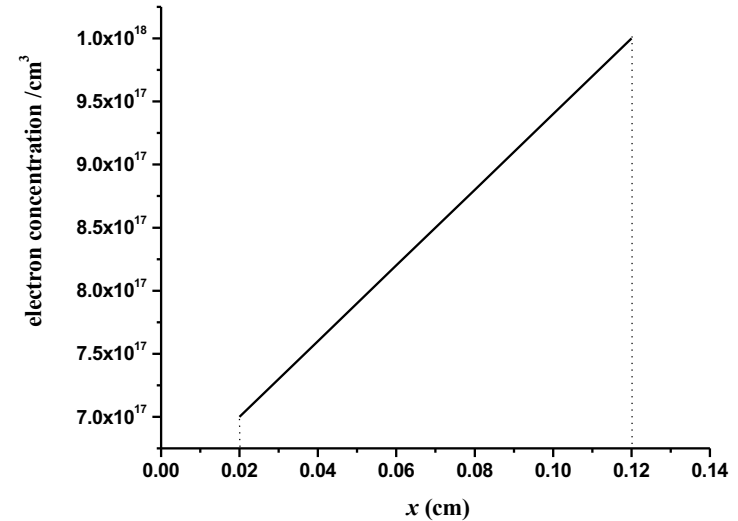
- For electrons (see Fig. 6.9.a) the concentration gradient (dn/dx) is positive. The electrons diffuse from high to low concentration, i.e. in the negative x direction. Since electrons are negatively charged, the current is in the positive x direction.
- In Fig. 6.9.b, the hole concentration gradient (dp/dx) is again positive. The holes diffuse from high to low concentration, i.e. in the negative x direction. Since holes are positively charged, the current is in the negative x direction.

<http://pveducation.org/pvcdrom/pn-junction/diffusion>

Example 6.2

In an n-type GaAs semiconductor at $T = 300$ K, the electron concentration varies linearly from $7 \times 10^{17} / \text{cm}^3$ to $1 \times 10^{18} / \text{cm}^3$ over a distance of 0.10 cm. Calculate the electron diffusion current density given that the electron diffusion coefficient D_n is $225 \text{ cm}^2/\text{s}$.

From eqn. (6.30)



Example 6.2

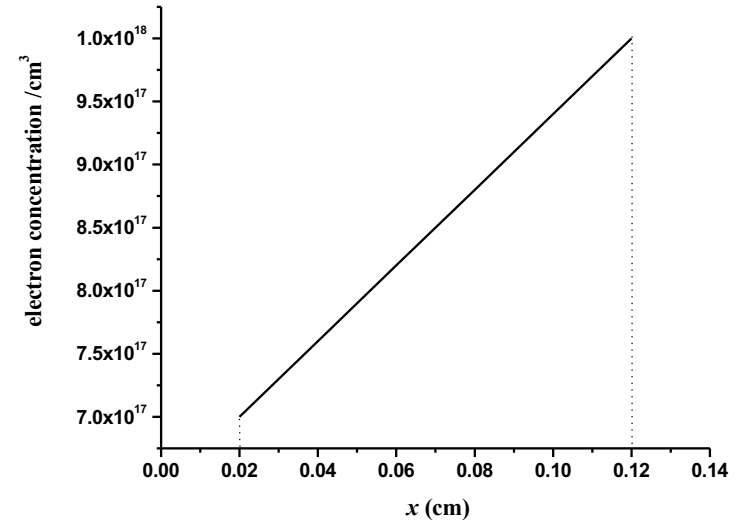
In an n-type GaAs semiconductor at $T = 300$ K, the electron concentration varies linearly from $7 \times 10^{17} / \text{cm}^3$ to $1 \times 10^{18} / \text{cm}^3$ over a distance of 0.10 cm. Calculate the electron diffusion current density given that the electron diffusion coefficient D_n is $225 \text{ cm}^2/\text{s}$.

From eqn. (6.30)

$$J_{n \text{ diff}} = q D_n \frac{dn}{dx}$$

$$J_{n \text{ diff}} = (1.6 \times 10^{-19}) (225) \frac{(1 \times 10^{18} - 7 \times 10^{17})}{0.10}$$

$$J_{n \text{ diff}} = 108 \text{ A/cm}^2$$



Having obtained the drift and diffusion current density for electrons and holes, we can have the total current density

- The total electron current density is

$$J_n = J_{n \text{ drift}} + J_{n \text{ diff}} = q \mu_n n \xi + q D_n \frac{dn}{dx} \quad (6.33)$$

- The total hole current density is

$$J_p = J_{p \text{ drift}} + J_{p \text{ diff}} = q \mu_p p \xi - q D_p \frac{dp}{dx} \quad (6.34)$$

In semiconductors the charge carriers are electrons and holes, therefore the total current density is

$$J_{total} = J_n + J_p \quad (6.35)$$

6.4 The Einstein Relation

There is a simple relation between diffusion coefficient of any particle and its mobility.

- We recall the electron diffusion coefficient in eqn. (6.29)

$$v_{th} / = D_n$$

- Using eqn. (6.25) we can write the electron diffusion coefficient as

$$D_n = v_{th}^2 \tau_{cn} \quad (6.36)$$

- For one dimensional case, the average thermal energy at temperature T is

$$E_{thermal (1-D)} = \frac{1}{2} k_B T \quad (6.37)$$

- Hence, the electron kinetic energy is

$$\frac{1}{2} m_n^* v_{th}^2 = \frac{1}{2} k_B T \quad (6.38)$$

$$\text{and } \therefore v_{th}^2 = \frac{k_B T}{m_n^*} \quad (6.39)$$

- Substituting v_{th} in eqn. (6.39) into eqn. (6.36), we obtain

$$D_n = \frac{k_B T}{m_n^*} \tau_{cn} \quad (6.40)$$

- Recall eqn. (6.14), the electron mobility is $\mu_n = \frac{q \tau_{cn}}{m_n^*}$

combining eqns. (6.14) and (6.40) we obtain $D_n = \frac{k_B T}{q} \mu_n$

(6.41.a)

- Similarly, the hole diffusion coefficient and mobility is related by

$$D_p = \frac{k_B T}{q} \mu_p \quad (6.41.b)$$

- The relation between the diffusion coefficient and mobility in eqn. (6.41) is known as the Einstein relation. The diffusion coefficient and mobility are not independent parameters.
 - The motion of charge carriers under an electric field and the diffusion process are based on the same mechanism: by the collisions of the charge carriers which occur within the average time intervals τ_{cn} (or τ_{cp}) with the average thermal velocity v_{th}

Table 6.2 lists typical mobility and diffusion coefficient values of electrons and holes at $T = 300$ K (mobility in $\text{cm}^2/\text{V}\cdot\text{sec}$ and diffusion coefficient in cm^2/sec)

Table 6.2

	μ_n	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium Arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

Problems

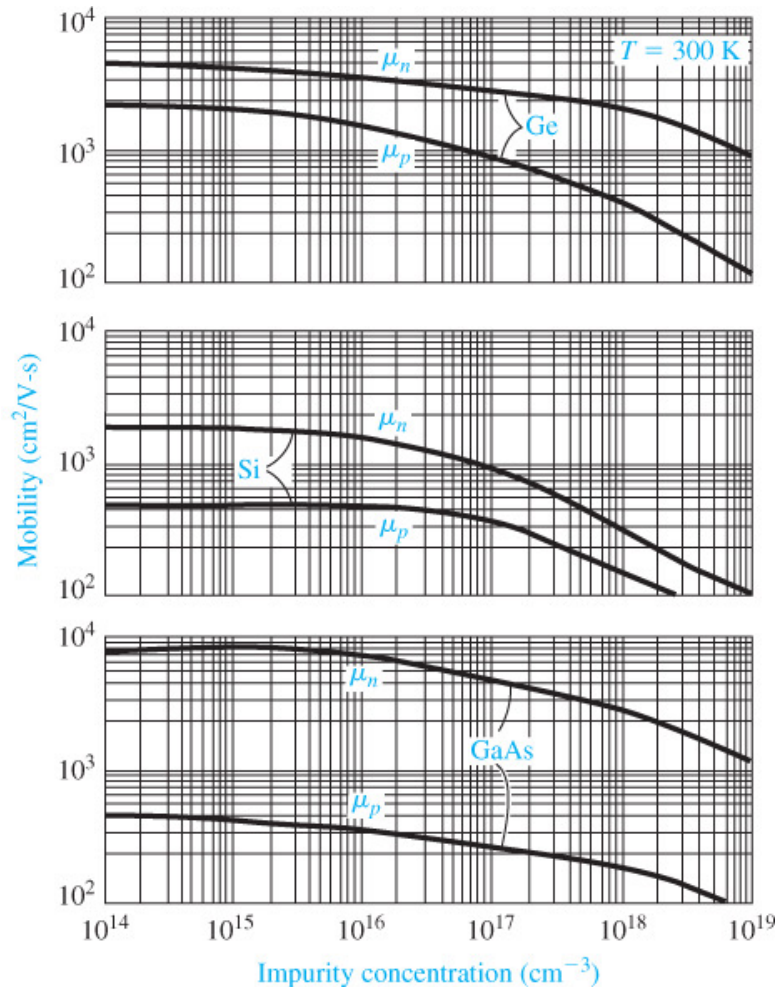
Q1: A drift current density of 75 A/cm² is required in a device using p-type silicon when an electric field of 120 V/cm is applied at 300K. Determine the required impurity doping concentration to achieve this specification. (Ex 5.1)

$$J_{drf} \cong e\mu_p p_o E = e\mu_p N_a E$$

$$75 = (1.6 \times 10^{-19})(480)N_a(120)$$

$$\text{which yields } N_a = 8.14 \times 10^{15} \text{ cm}^{-3}$$

Q2: A compensated p-type silicon material at $T = 300\text{K}$ has impurity doping concentration of $N_a = 2.8 \times 10^{17} \text{ cm}^{-3}$ and $N_d = 8 \times 10^{16} \text{ cm}^{-3}$. Determine the (a) hole mobility, (b) conductivity, and (c) resistivity. (Ex 5.3)



(a) For

$$N_I = N_a + N_d = 2.8 \times 10^{17} + 8 \times 10^{16} \\ = 3.6 \times 10^{17} \text{ cm}^{-3}, \\ \Rightarrow \mu_p = 200 \text{ cm}^2/\text{V-s}$$

$$(b) \sigma = e\mu_p(N_a - N_d) \\ = (1.6 \times 10^{-19})(200)(2 \times 10^{17}) \\ \sigma = 6.4 (\Omega \text{-cm})^{-1}$$

$$(a) \rho = \frac{1}{\sigma} = \frac{1}{6.4} = 0.156 \Omega \text{-cm}$$

Figure 5.3 | Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300\text{K}$.

(From Sze [14].)

Q3: The hole density in silicon is given by $p(x) = 10^{16} \exp(-x/L_p)$ for $x \geq 0$ where $L_p = 2 \times 10^{-4}$ cm. Assume that the hole diffusion coefficient is $D_p = 8$ cm²/s. Determine the hole diffusion current density at (a) $x=0$, (b) $x=2 \times 10^{-4}$ cm, and (c) $x=10^{-3}$ cm. (Ex 5.5)

$$\begin{aligned}
 J_p &= -eD_p \frac{dp}{dx} \\
 &= -eD_p \frac{d}{dx} \left[10^{16} e^{-x/L_p} \right] \\
 &= -eD_p (10^{16}) \left(\frac{-1}{L_p} \right) e^{-x/L_p} \\
 &= \frac{+eD_p (10^{16})}{L_p} e^{-x/L_p} \\
 &= \frac{(1.6 \times 10^{-19})(8)(10^{16})}{2 \times 10^{-4}} e^{-x/L_p} \\
 J_p &= 64 \exp\left(\frac{-x}{L_p}\right)
 \end{aligned}$$

(a) For $x = 0$,

$$J_p = 64 \text{ A/cm}^2$$

(b) For $x = 2 \times 10^{-4}$ cm,

$$J_p = 64 \exp\left(\frac{-2 \times 10^{-4}}{2 \times 10^{-4}}\right) = 23.54$$

A/cm²

(c) For $x = 10^{-3}$ cm,

$$J_p = 64 \exp\left(\frac{-10^{-3}}{2 \times 10^{-4}}\right) = 0.431 \text{ A/cm}$$

2

Q4: In GaAs, the donor impurity concentration varies as $N_{do} \exp(-x/L)$ for $0 \leq x \leq L$, where $L=0.1 \mu\text{m}$ and $N_{do} = 5 \times 10^{16} \text{ cm}^{-3}$. Assume $\mu_n = 6000 \text{ cm}^2/\text{Vs}$ and $T = 300\text{K}$. (a) Derive the expression for the electron diffusion current density vs. distance over the given range of x . (b) Determine the induced electric field that generates a drift current density that compensates the diffusion current density. (5.43)

(a) We have

$$J_{diff} = eD_n \frac{dn}{dx} = eD_n \frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)} \cdot N_{do} \exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n \left(\frac{kT}{e} \right) = (6000)(0.0259)$$

or

$$D_n = 155.4 \text{ cm}^2/\text{s}$$

Then

$$J_{diff} = \frac{-(1.6 \times 10^{-19})(155.4)(5 \times 10^{16})}{(0.1 \times 10^{-4})} \exp\left(\frac{-x}{L}\right)$$

or

$$J_{diff} = -1.243 \times 10^5 \exp\left(\frac{-x}{L}\right) \text{ A/cm}^2$$

(b)

$$0 = J_{drf} + J_{diff}$$

Now

$$J_{drf} = e\mu_n nE$$

$$= (1.6 \times 10^{-19})(6000)(5 \times 10^{16}) \left[\exp\left(\frac{-x}{L}\right) \right] E$$

or

$$J_{drf} = (48) \left[\exp\left(\frac{-x}{L}\right) \right] E$$

We have

$$J_{drf} = -J_{diff}$$

so

$$(48) \left[\exp\left(\frac{-x}{L}\right) \right] E = 1.243 \times 10^5 \exp\left(\frac{-x}{L}\right)$$

which yields

$$E = 2.59 \times 10^3 \text{ V/cm}$$

Key takeaways (Lecture #13)

- Drift current:

$$J_{drift} = J_{n\ drift} + J_{p\ drift}$$

$$J_{drift} = q \mu_n n \xi + q \mu_p p \xi = (q \mu_n n + q \mu_p p) \xi$$

- Diffusion current:

$$J_{n\ diff} = -q \varphi_n = q D_n \frac{dn}{dx} \quad J_{p\ diff} = q \varphi_p = -q D_p \frac{dp}{dx}$$

- Einstein Relation:

$$D_n = \frac{k_B T}{q} \mu_n \quad D_p = \frac{k_B T}{q} \mu_p$$