

# Lecture 12: Transformers

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**EE3010: Electrical Devices and Machines**

School of Electrical and Electronic Engineering

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By the end of this lecture, you should be able to:

- ❖ Describe the design and operating principles of autotransformers.
- ❖ Analyse an autotransformer to determine the power transferred by induction and the power delivered by conduction.
- ❖ Explain how the autotransformers can transfer more power capacity than the conventional two-winding transformers.
- ❖ Apply the operating principles of three-phase transformer banks and three-phase transformers in power supply systems.

# Autotransformer

- ❖ In normal transformers, the windings are electrically isolated and the energy transfer from one winding to another occurs completely through magnetic induction as shown in Fig. 44.

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} = a = \frac{N_1}{N_2}, \text{ and } E_1 I_1 = E_2 I_2 = S$$

is the total power delivered through induction.

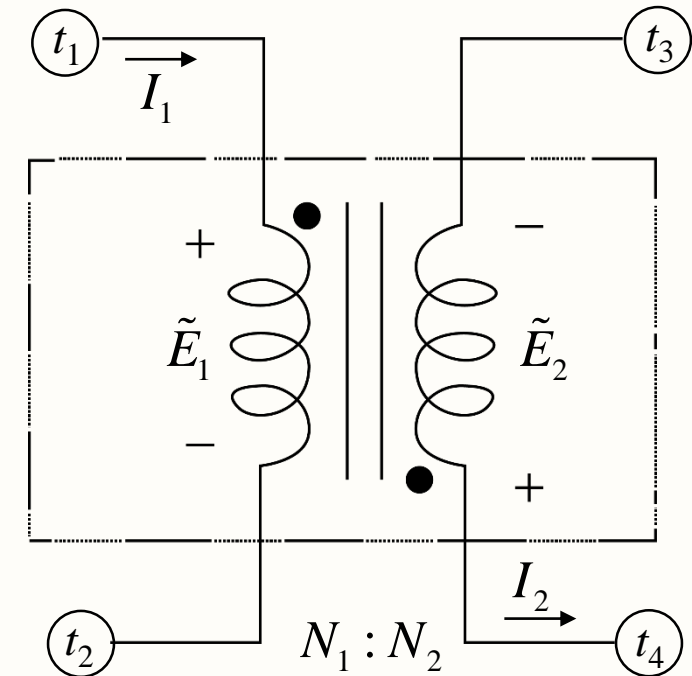


Fig. 44. Two-winding transformer.

- ❖ The windings of a transformer can, however, be electrically connected, so that the energy transfer may occur both through **conduction**, as well as magnetic **induction**. Then, the total power delivered can be enhanced (increased) by making proper electrical connections between the two windings. Transformers with such connections are called **autotransformers**.
- ❖ Commonly, the two-windings are connected in series. Then
  - Any two terminals may be the input, and
  - Any two terminals may be the output.
- ❖ With proper connections, it is possible to achieve several combinations of input-output voltages from a single two-winding transformer.

# Autotransformer

- ❖ Four common autotransformer configurations:

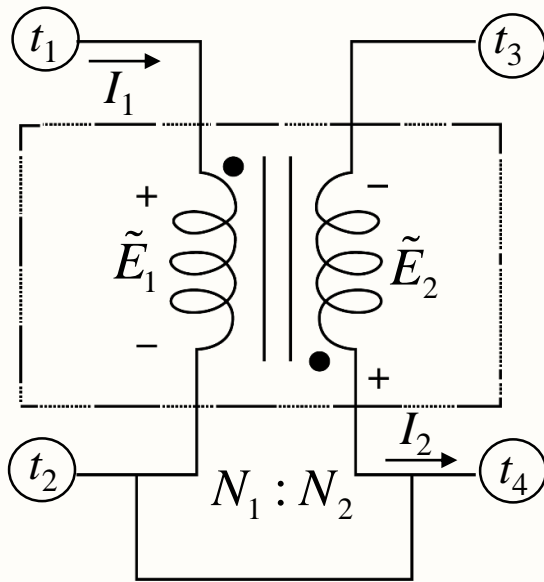


Fig. 45. Two-winding transformer.

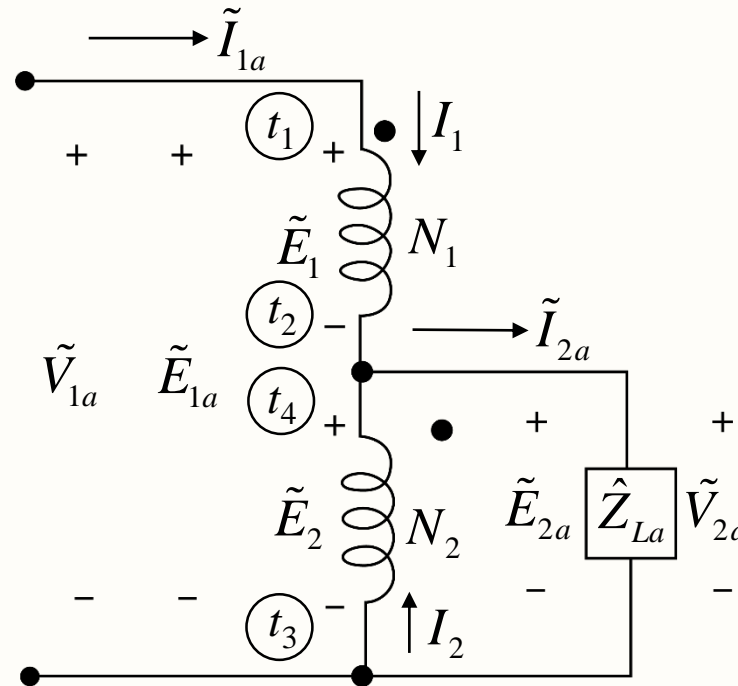


Fig. 46. Configuration 1:  
 $(V_1 + V_2)/V_2$  step-down connection.

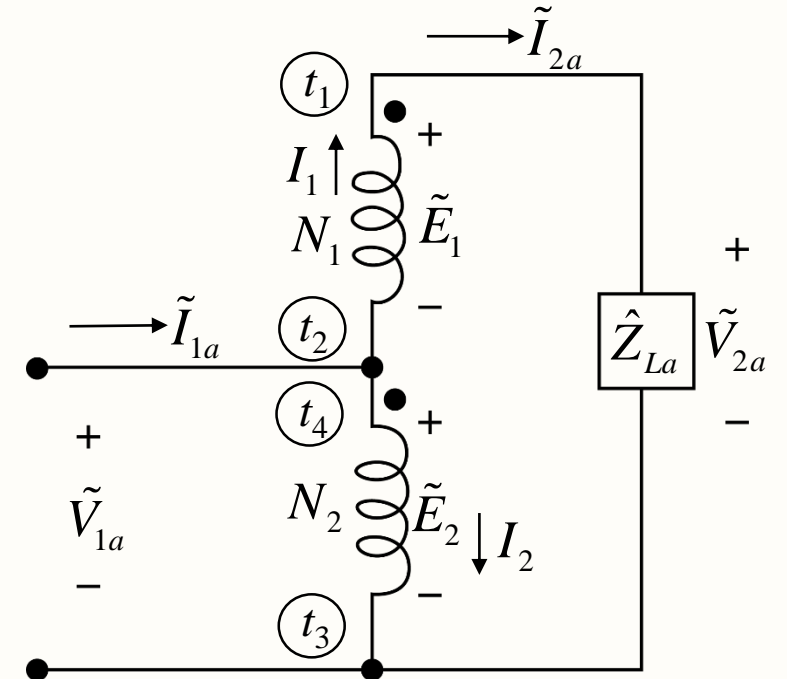


Fig. 47. Configuration 2:  
 $V_2/(V_1 + V_2)$  step-up connection.

# Autotransformer

❖ Four common autotransformer configurations:

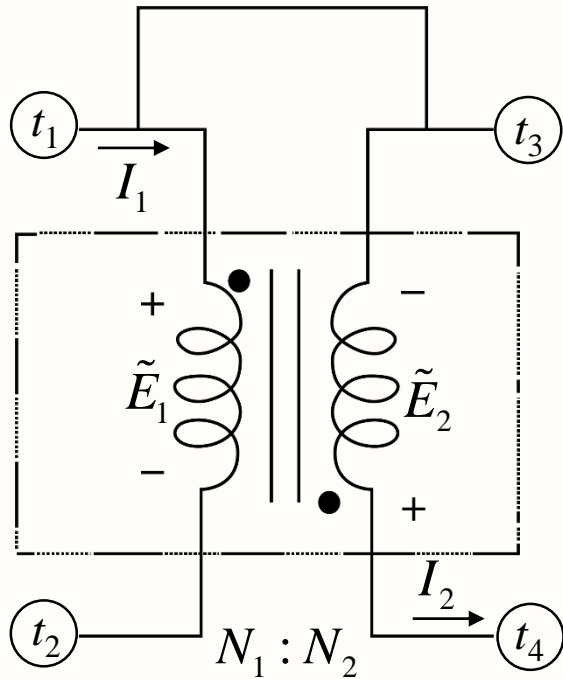


Fig. 48. Two-winding transformer.

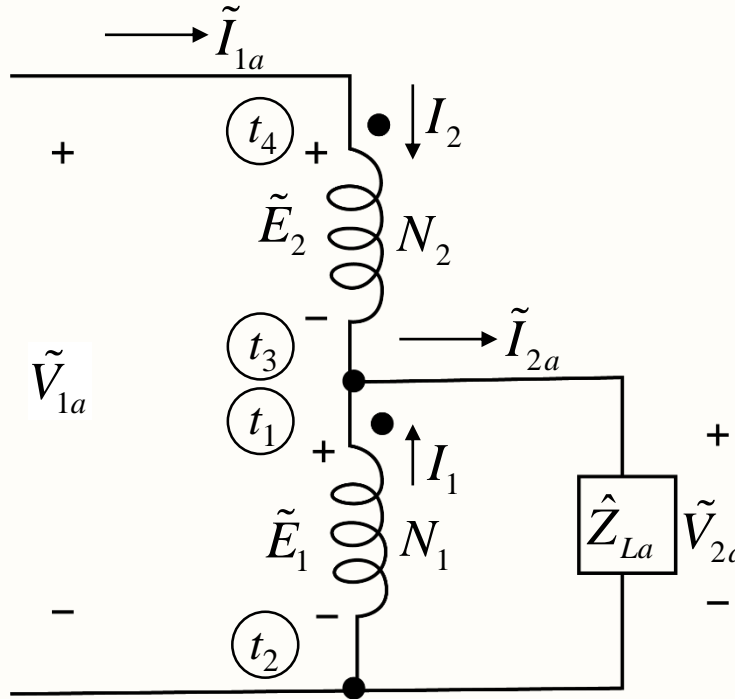


Fig. 49. Configuration 3:  
 $(V_1 + V_2)/V_1$  step-down connection.

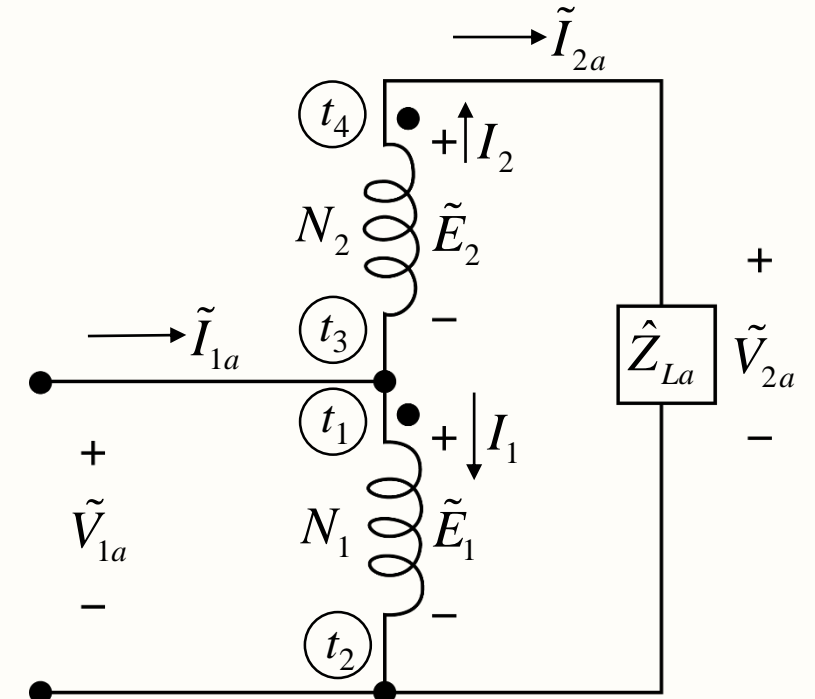


Fig. 50. Configuration 4:  
 $V_1/(V_1 + V_2)$  step-up connection.

# Autotransformer

- ❖ One common form of autotransformer is a single winding transformer with a sliding point, with variable output voltage, as shown in Fig. 51.

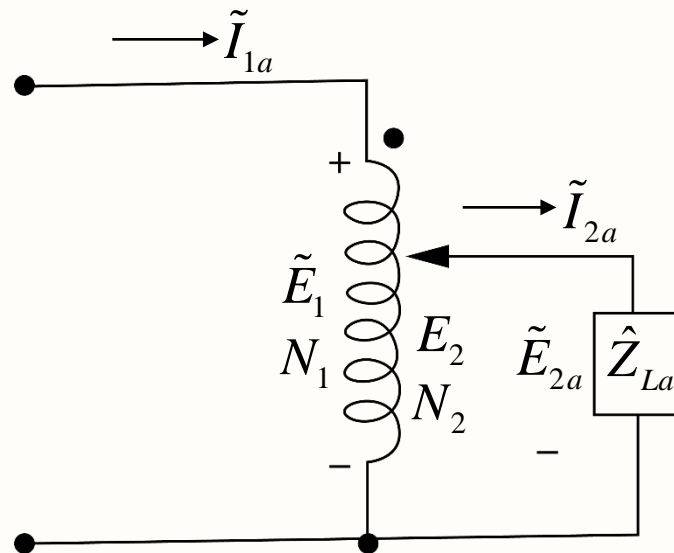


Fig. 51. A single winding transformer with variable output voltage.



- ❖ Autotransformers offer several advantages:
  - They can deliver more power for the same size.
  - They provide several voltage transformation ratios - voltage flexibility.
  - They are more efficient.
  - They are cheaper.
  - They take lesser excitation current.
- ❖ Their main disadvantage is the loss of electrical isolation between the primary and the secondary windings.

# Autotransformer Analysis

- ❖ Consider a particular autotransformer configuration, as shown in Fig. 53.

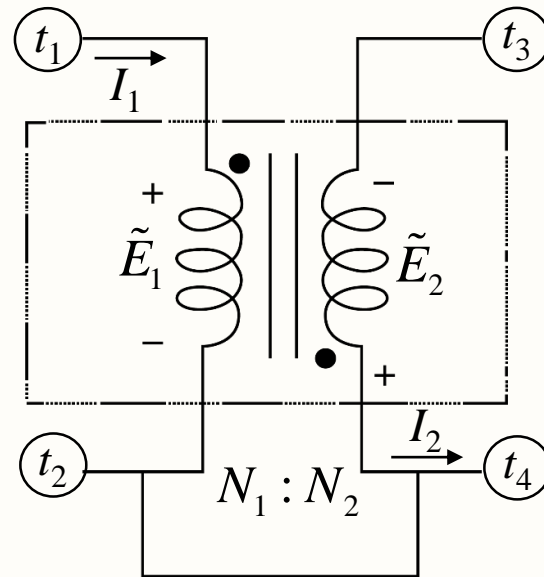


Fig. 52. Two-winding transformer.

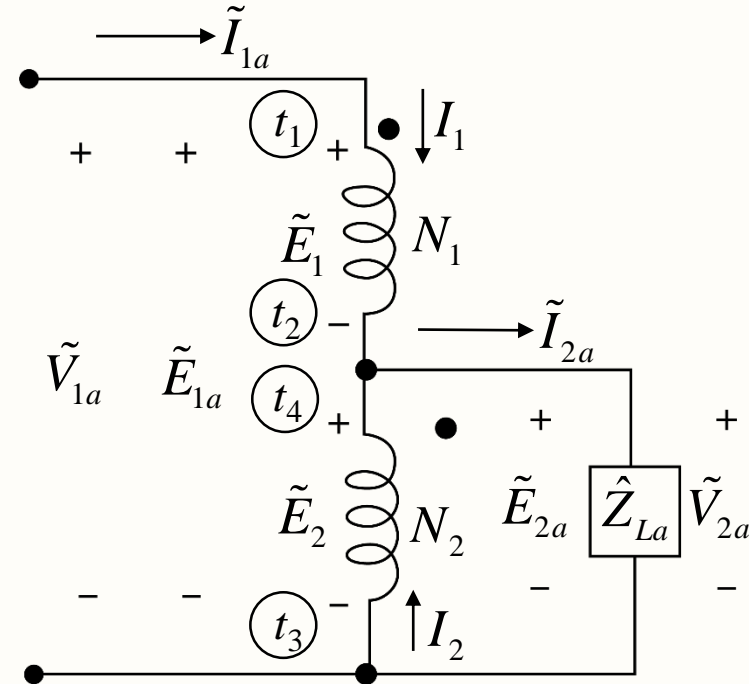


Fig. 53. Configuration 1:  
 $(V_1 + V_2)/V_2$  step-down connection.

- ❖ The following notations are used in the analysis:
  - $E_1$  and  $I_1$  are the winding 1 voltage and current.
  - $E_2$  and  $I_2$  are the winding 2 voltage and current.
  - $E_{1a}$  and  $I_{1a}$  are the autotransformer primary voltage and current.
  - $E_{2a}$  and  $I_{2a}$  are the autotransformer secondary voltage and current.

❖ For the two-winding transformer:

- $\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$
- $E_1 I_1 = E_2 I_2 = S$
- If current enters the dotted terminal in one coil, the currents must come out of the dotted terminal in the other coil.

# Autotransformer Analysis

❖ For the autotransformer,

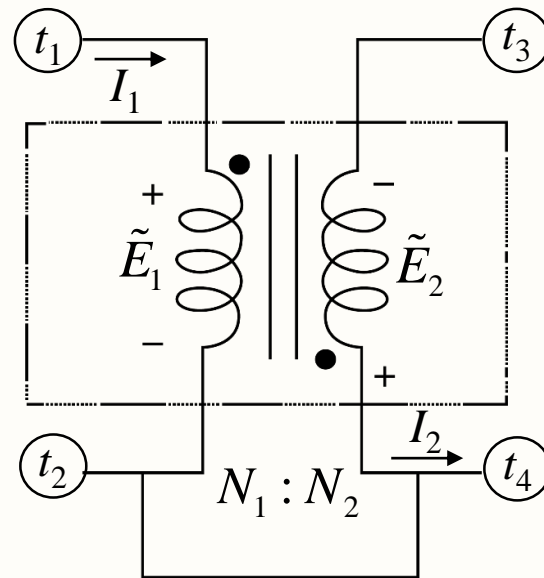


Fig. 54. Two-winding transformer.

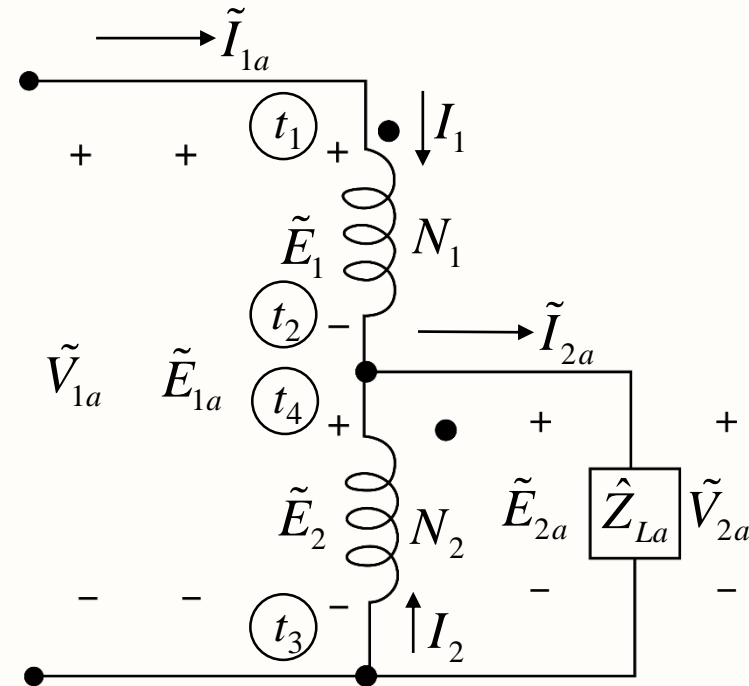


Fig. 55. Configuration 1:  
 $(V_1 + V_2)/V_2$  step-down connection.

# Autotransformer Analysis

- The autotransformer voltage ratio is

$$\frac{V_{1a}}{V_{2a}} = \frac{E_1 + E_2}{E_2} = \frac{E_1}{E_2} + 1 = a + 1 \quad \left( \frac{E_{1a}}{E_{2a}} = \frac{E_1 + E_2}{E_2} \right)$$

- The autotransformer current ratio is

$$\frac{I_{2a}}{I_{1a}} = \frac{I_1 + I_2}{I_1} = 1 + \frac{I_2}{I_1} = 1 + a$$

Therefore,

$$\frac{V_{1a}}{V_{2a}} = \frac{I_{2a}}{I_{1a}} \Rightarrow V_{1a} I_{1a} = V_{2a} I_{2a}, \text{ i.e., } S_{1a} = S_{2a}$$

- ❖ The input of the autotransformer is

$$S_{1a} = E_{1a} I_{1a} = (E_1 + E_2) I_1 = E_1 I_1 + E_2 I_1$$

- ❖ Similarly, the output of the autotransformer is

$$S_{2a} = E_{2a} I_{2a} = E_2 (I_1 + I_2) = E_2 I_2 + E_2 I_1$$

- ❖ The first terms of the input and the output of the autotransformer:

$$E_1 I_1 = E_2 I_2 = S$$

is the power transferred through regular transformer action, and is called the power transferred through **induction**.

- ❖ The second term:

$$E_2 I_1 = \left(\frac{E_1}{a}\right) I_1 = \left(\frac{1}{a}\right) E_1 I_1 = \frac{S}{a}$$

is the additional power delivered because of the connection as autotransformer. This is the power delivered by **conduction**.

- ❖ The total power delivered by the autotransformer is thus:

$$S_{1a} = S_{2a} = S + \frac{S}{a}$$

- ❖ It should be noted that the rating, i.e., the capacity of the transformer is apparently enhanced by an amount  $S/a$  when connected as an autotransformer as shown in Fig. 55.



- ❖ It can be seen through similar analyses of other autotransformer configurations that the rated power transfer by induction remains the same  $S$  for all configurations, while the rated power transfer by conduction is different for different configurations.
- ❖ Exercise: Analyse configurations of Figs. 47, 49 and 50, and determine the general expressions for power delivered through conduction for these configurations.

## Example 7

A 24-kVA, 2400/240-V transformer is to be connected as an autotransformer. Sketch the different possible connections. For each connection, determine

- a) the input voltage,
- b) the output voltage,
- c) the nominal rating of the autotransformer,
- d) the power delivered conductively, and
- e) The power transformed magnetically.

(Solutions →)

# Example 7 – Solutions

Consider a possible connection shown in Fig. 57.

$$E_1 = 2400 \text{ V}, \quad E_2 = 240 \text{ V}$$

$$\text{Turns ratio } a = \frac{2400}{240} = 10$$

Power rating  $S = 24 \text{ kVA}$

Therefore, the current ratings of the two-windings are

$$I_1 = \frac{S}{E_1} = 10 \text{ A, and } I_2 = \frac{S}{E_2} = 100 \text{ A}$$

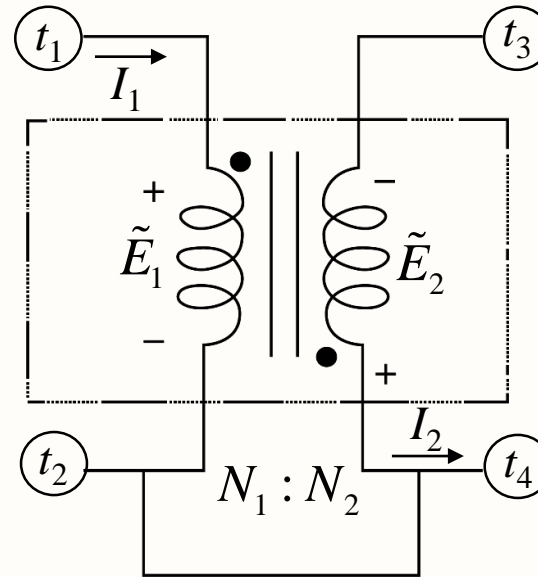


Fig. 56. Two-winding transformer.

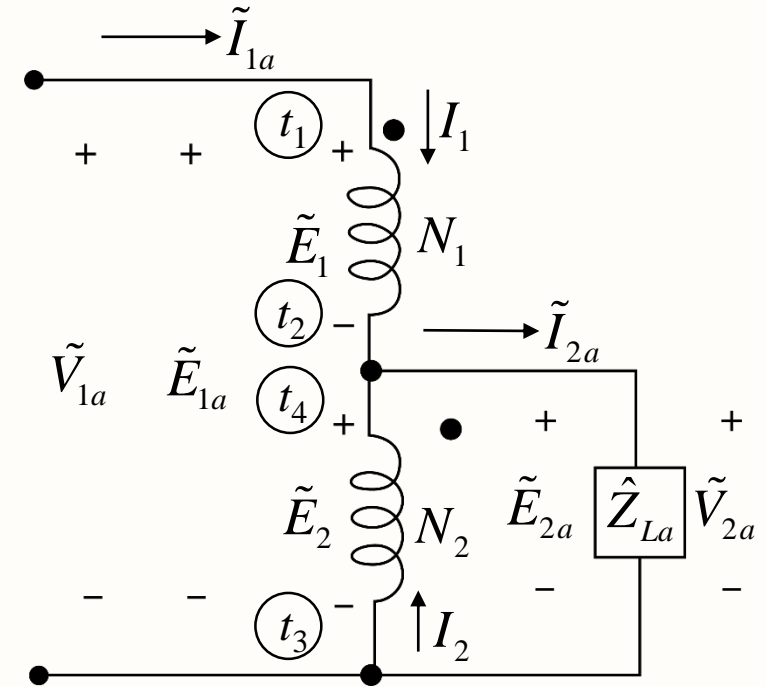


Fig. 57. Configuration 1:  
( $V_1 + V_2$ )/ $V_2$  step-down connection.

## Example 7 – Solutions

For the chosen configuration of the autotransformer:

- a) Input voltage =  $E_1 + E_2 = 2400 + 240 = 2640 \text{ V}$
- b) Output voltage =  $E_2 = 240 \text{ V}$
- c) Rated value of the output current  $I_{2a} = I_1 + I_2 = (10 + 100) = 110 \text{ A}$   
Rated output power  $E_2 I_{2a} = 240 \times 110 = 26.4 \text{ kVA}$
- d) Power conducted directly  $S_c = E_2 I_1 = 240 \times 10 = 2.4 \text{ kVA}$
- e) Power transformed magnetically  $S_m = E_1 I_1 = 2400 \times 10 = 24 \text{ kVA}$

$$\text{Also, } S_m = E_2 I_2 = 240 \times 100 = 24 \text{ kVA}$$

## Example 7 – Solutions

- ❖ Note: Power transferable through induction will always be the transformer rating, while the power transferred through conduction will vary with the autotransformer configuration.
- ❖ Complete the solutions for Figs. 47, 49, and 50. Refer to the textbook.

# Three-Phase Transformer Banks

- ❖ Most power systems utilise three-phase systems. A number of single-phase transformers may be interconnected and used in three-phase systems. Such systems are called **Transformer Banks**.
- ❖ The simplest transformer bank utilises three identical single-phase transformers as shown in Fig. 58. The primary and secondary windings can be connected either in **Star** or **Delta** configuration as necessary.

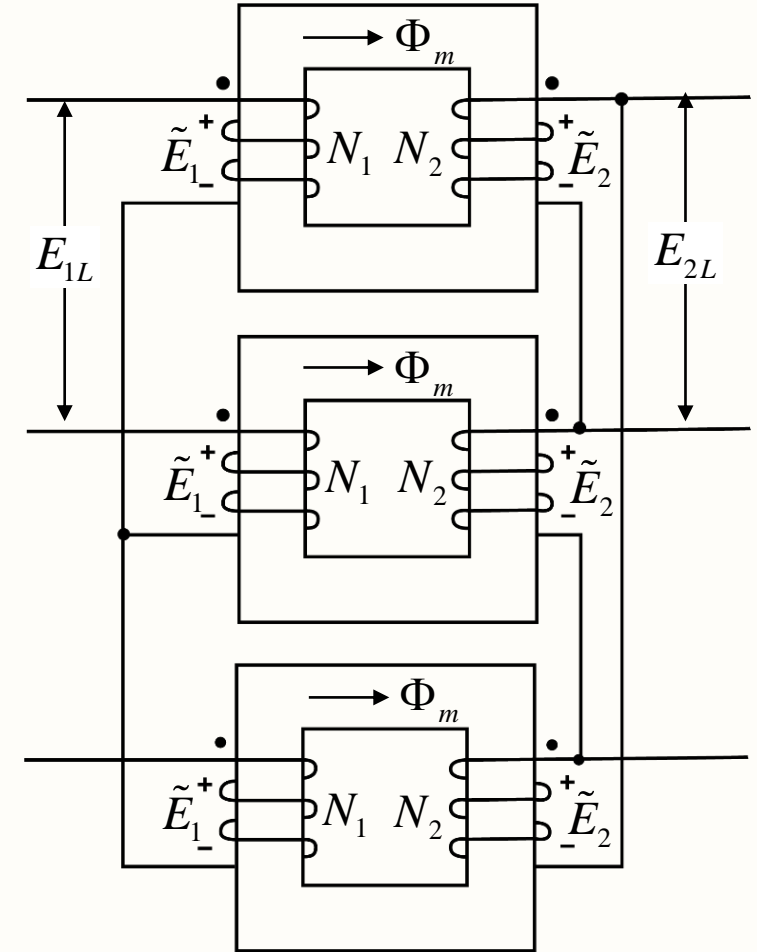


Fig. 58. Three-phase transformer bank.

# Three-Phase Transformer Banks

- ❖ Note that a transformer bank consists of three separate one phase transformers. There is no common magnetic core. Electrical connections are made externally.
- ❖ The ratings of the transformer bank have to be specified by stating the rating of one of the three single-phase transformers.
  - E.g., a transformer bank of 3 x 1-phase, 10 kVA, 440/220 V transformers.

- ❖ Three-phase transformer has a common magnetic core with three primary and three secondary windings so that they can be connected to three-phase systems, both at the input as well as the output terminals. The magnetic core may be of two types as shown in Figs. 59 and 60.
  - Shell Type
  - Core Type



# Three-Phase Transformers

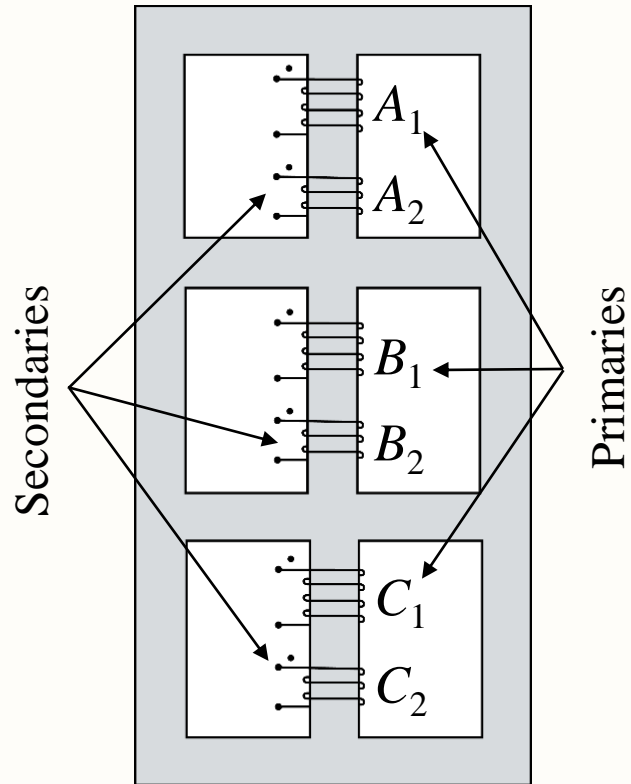


Fig. 59. Shell type three-phase transformer construction.

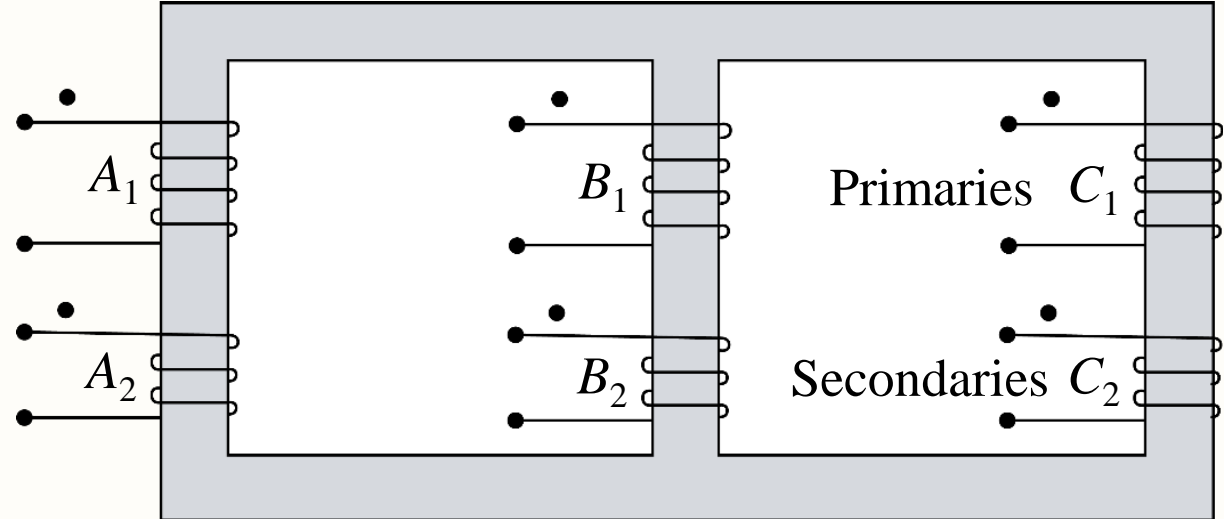
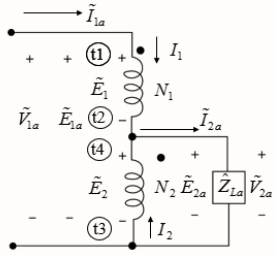
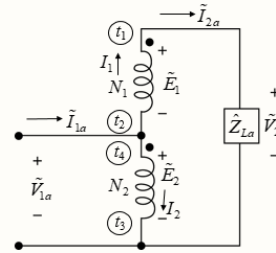
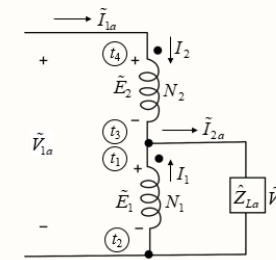


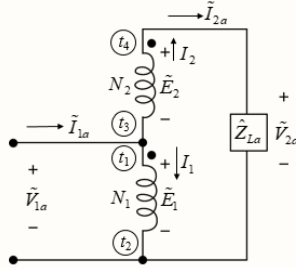
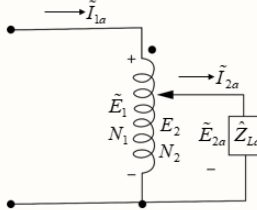
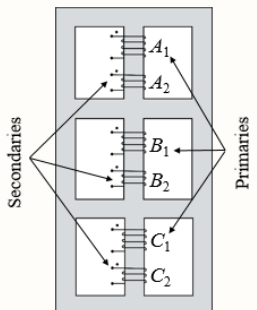
Fig. 60. Core type three-phase transformer construction.

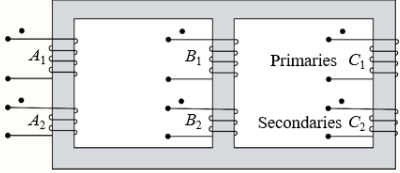
In this lecture, you have learnt:

- ❖ The design and operating principles of autotransformers.
- ❖ Analysis of autotransformers to determine the power transferred by induction and the power delivered by conduction.
- ❖ The more power transfer capacity of autotransformers than the conventional two-winding transformers.
- ❖ The construction of three-phase transformer banks and three-phase transformers.

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