Lecture 4: Electromagnetic Principles and Actuators

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Perform design and analysis of magnetic circuits excited by AC and DC sources.
- ❖ Describe the use of magnetic field as a medium to store electrical energy.
- ❖ Evaluate the magnetic field energy using different approaches.

Example 6

The circular magnetic core shown in Fig. 36 has a relative permeability of 2200. The dimensions of the core are: r_1 = 25 cm, r_2 = 20 cm, and the cross section A is circular. The coil has 102 turns and a resistance of 4 Ω.

Calculate the inductance of the coil. What will be the flux density in the core and the flux linkage of the coil if it is connected to (a) 10 V dc source, and (b) 10 V, 50 Hz ac source?

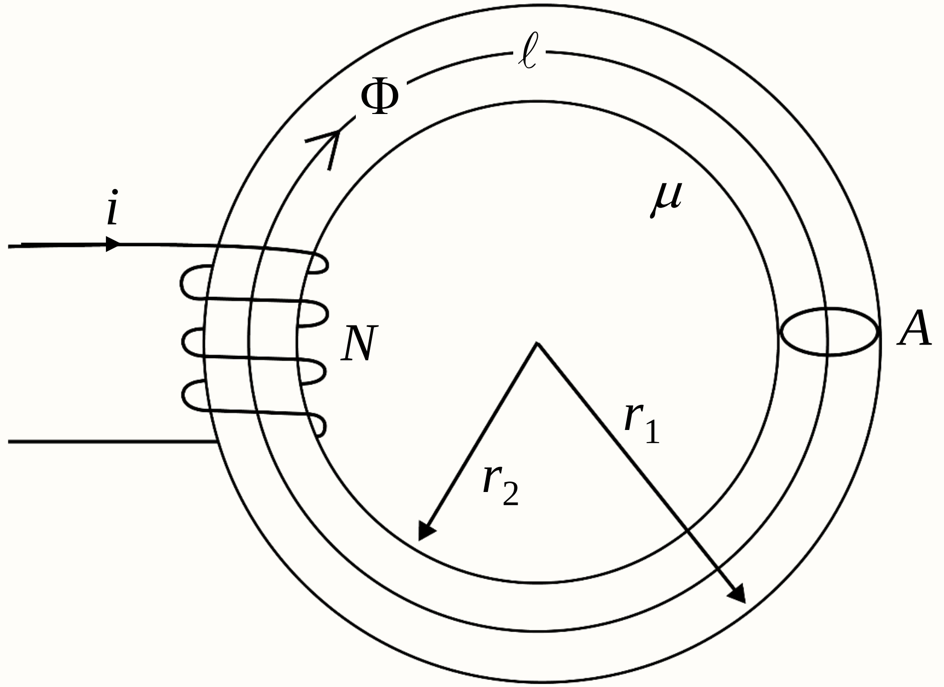

Fig. 36. Circular magnetic core.

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(Solutions →)

Example 6 – Solutions

Mean radius $r = \left(\dfrac{25+20}{2}\right)\times 10^{-2} = 0.225 \text{ m}$

Diameter of core section = $r_1 - r_2 = 5 \text{ cm} = 0.05 \text{ m}$

$\Rightarrow$ Cross section area $A = \pi r^2 = \pi 0.025^2 \text{ m}^2$

Reluctance of core:

$$\Re = \frac{\ell}{\mu A} = \frac{2\pi \times 0.225}{2200 \times 4\pi \times 10^{-7} \times \pi 0.025^2} = 260435 \text{ H}^{-1}$$

The magnetic equivalent circuit is shown in Fig. 37.

Inductance $L = \dfrac{N^2}{\Re} = \dfrac{102^2}{260435} = 0.04 \text{ H}$

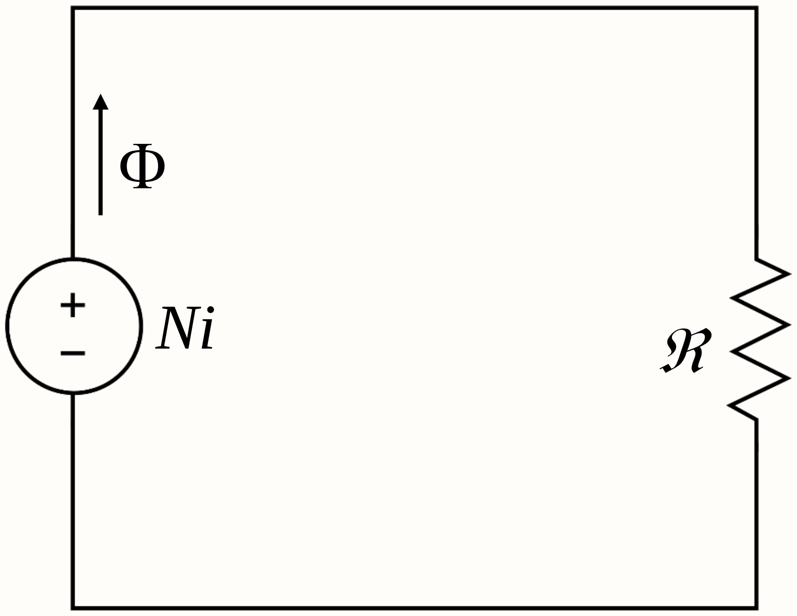

Fig. 37. Magnetic equivalent circuit.

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Example 6 – Solutions

a) When connected to 10 V dc, the electric circuit is shown in Fig. 38, where

$$i = V / R = 10/4 = 2.5 \text{ A}$$

Then,

flux $\varphi = Ni / \Re = 102 \times 2.5 / 260435 = 9.79 \times 10^{-4}$ Wb

$$B = \varphi / A = 9.79 \times 10^{-4} / (\pi 0.025^2) = 0.50 \text{ T}$$

$$\lambda = N\varphi = 102 \times 9.79 \times 10^{-4} = 0.10 \text{ Wb t}$$

(Check: $L = \lambda / i = 0.10 / 2.5 = 0.04$ H)

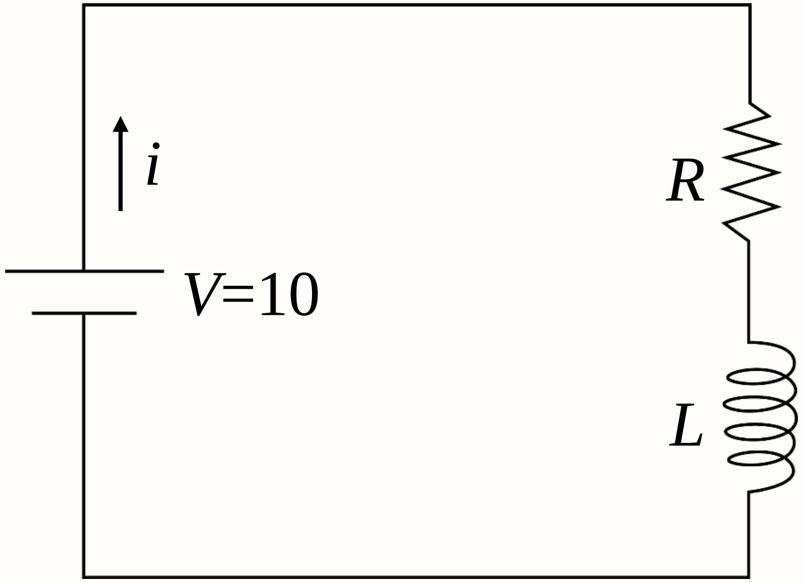

Fig. 38. Electric equivalent circuit.

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Example 6 – Solutions

b) When connected to 10 V ac, 50 Hz, the electric circuit is shown in Fig. 39, where

$$Z = R + j\omega L = 4 + j2\pi fL = 4 + j2\pi 50 \times 0.04$$

$$= 4 + j12.57 = 13.19\angle 72.3^\circ\ \Omega$$

so that $I = V / Z = 10 / 13.19 = 0.758\ \text{A}$

Then,

flux $\varphi = NI / \mathfrak{R} = 102 \times 0.758 / 260435 = 2.97 \times 10^{-4}\ \text{Wb}$

$B = \varphi / A = 2.97 \times 10^{-4} / (\pi 0.025^2) = 0.15\ \text{T}$

$\lambda = N\varphi = 102 \times 2.97 \times 10^{-4} = 0.0303\ \text{Wb t}$

(Check: $L = \lambda / i = 0.0303 / 0.758 = 0.04\ \text{H}$)

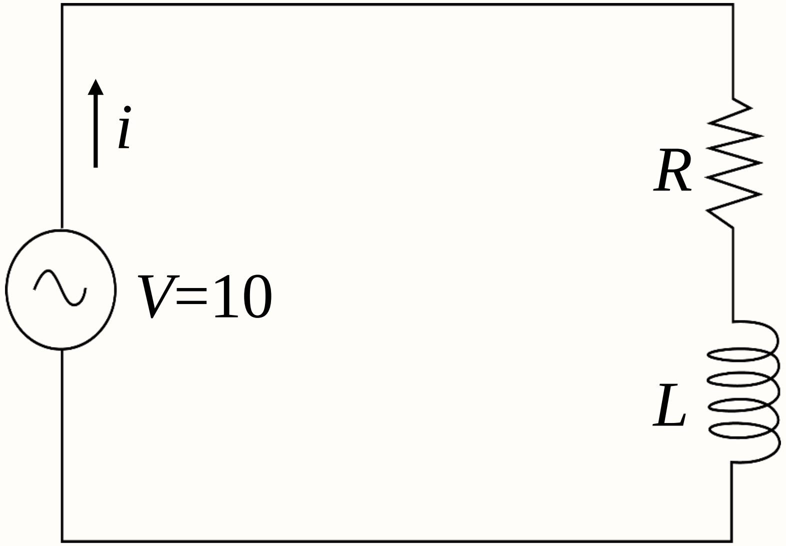

Fig. 39. Electric equivalent circuit.

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Magnetic Field Energy (Magnetic Stored Energy)

- Consider a lossless magnetic circuit as shown in Fig. 40. If the current input is i at a voltage of v, electrical energy input in time interval $\boldsymbol{dt}$ is

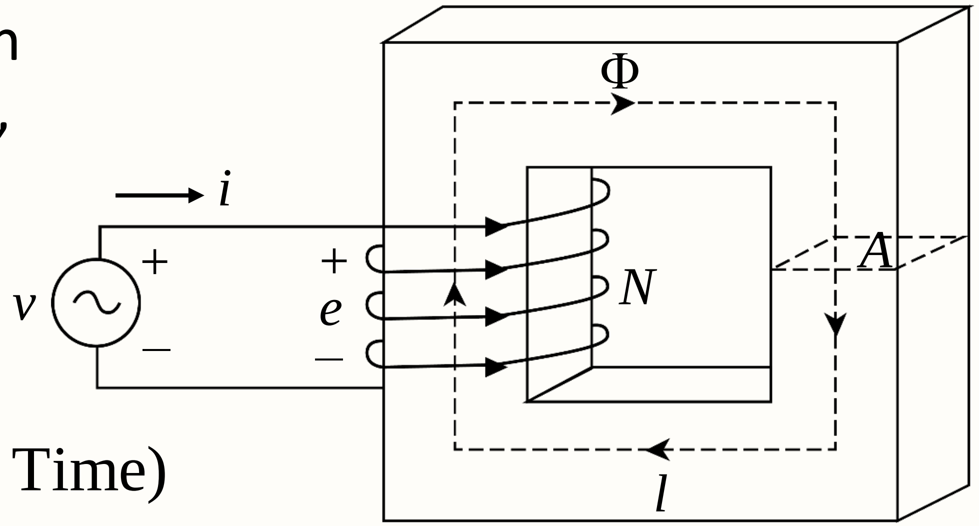

Fig. 40. Lossless magnetic circuit.

$$dW_i = vi\,dt = -ei\,dt \quad \text{(Energy = kWh = Power x Time)}$$

But $e = -\dfrac{d\lambda}{dt}$, $\Rightarrow -e\,dt = d\lambda$

and $\lambda = N\varphi \;\Rightarrow d\lambda = Nd\varphi$

Hence, $dW_i = -ei\,dt = -ie\,dt = i\,d\,\lambda = Ni\,d\varphi$

Magnetic Field Energy (Magnetic Stored Energy)

Therefore, energy input to establish flux φ in N-turn coil is

$$W_i = \int i \, d\lambda = \int Ni \, d\varphi$$

This expression can be used to calculate the total electrical energy input to a magnetic system.

Magnetic Field Energy (Magnetic Stored Energy)

- In a lossless system, since there is no output, the input energy will be stored as magnetic field energy (W_f or W_m). Therefore,

$$W_f = \int i \, d\lambda = \int Ni \, d\varphi$$

- Noting that $Ni = \varphi \Re$ for linear circuits, the field energy may be evaluated as

$$W_f = \int Ni \, d\varphi = \int \varphi \Re \, d\varphi = \frac{1}{2}\varphi^2 \Re \;\left(= \frac{1}{2} Ni\varphi = \frac{1}{2}\lambda i\right)$$

Magnetic Field Energy (Magnetic Stored Energy)

❖ Further, if we note that $L = \dfrac{\lambda}{i} = \dfrac{N\varphi}{i} \Rightarrow N\varphi = iL$, the stored energy can be written as

$$W_f = \frac{1}{2} N\varphi i = \frac{1}{2} i^2 L$$

- It should be noted that these expressions for stored magnetic field energy are for linear systems only, since we assume constant reluctance $\mathfrak{R}$.
- For nonlinear magnetic circuits, proper integration must be carried out to evaluate the stored energy.

Energy Density

- The stored magnetic field energy (W_f or W_m) has been expressed as

$$W_f = \int i\, d\lambda = \int Ni\, d\varphi$$

Noting that $Ni = H\ell$ and $\varphi = BA$, so that $d\varphi = A\,dB$, the field energy can also be written as

$$W_f = A\ell \int H\,dB$$

- Note that $A\ell$ is the volume of the magnetic material and $W_f/(A\ell)$ is the energy per unit volume. Therefore,

$\dfrac{W_f}{A\ell} = \int H\,dB$ is called the energy density, usually denoted as $\boldsymbol{w}$.

Energy Density

- Noting further that $B = \mu H \Rightarrow H = B/\mu$, the energy density is evaluated as

$$w_i = \int H\,dB = \int (B/\mu)\,dB = \frac{B^2}{2\mu}\ \ \mathrm{J/m^3} = \frac{1}{2}HB\ \ \mathrm{J/m^3}$$

for a linear magnetic circuit, where μ remains constant.

Example 7

For the magnetic circuit of Example 6, calculate the magnetic field energy stored when connected to 10 V dc using different approaches.

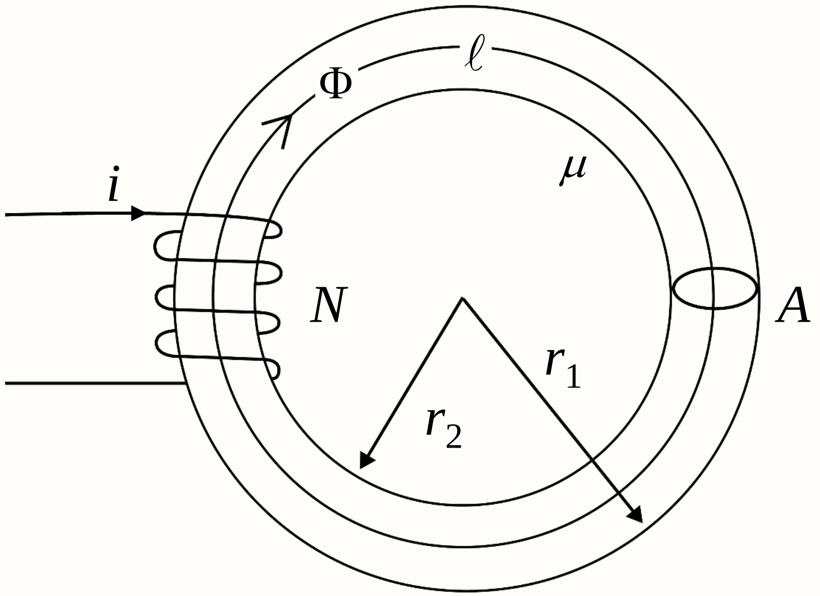

Fig. 41. Circular magnetic core.

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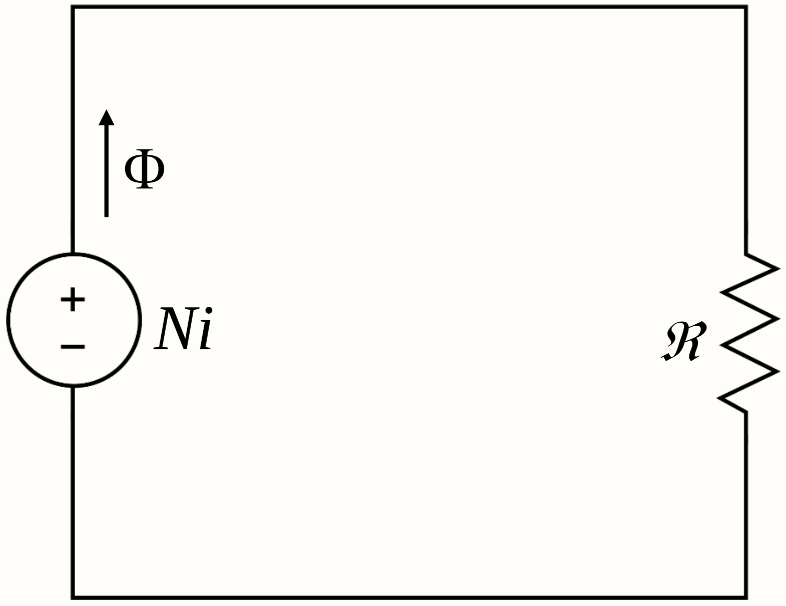

Fig. 42. Magnetic equivalent circuit.

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(Solutions →)

Example 7 – Solutions

From Example 6,

$\Re = 260435\ \mathrm{H}^{-1}$, and $L = 0.04\ \mathrm{H}$

With 10 V dc source,

$i = 2.5\ \mathrm{A}, \varphi = 9.79 \times 10^{-4}\ \mathrm{Wb}$

$B = 0.50\ \mathrm{T}, \lambda = 0.10\ \ \mathrm{Wb\ t}$

Then, $W_f = \frac{1}{2}\varphi^2 \Re = \frac{1}{2}(9.79 \times 10^{-4})^2 \times 260435 = 0.125\ \mathrm{J}$, or

$$W_f = \frac{1}{2} i^2 L = \frac{1}{2}(2.5)^2 \times 0.04 = 0.125\ \mathrm{J}$$

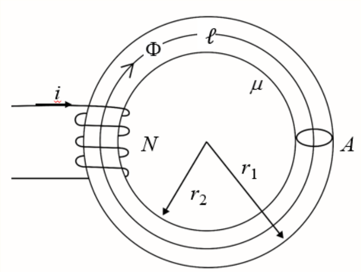
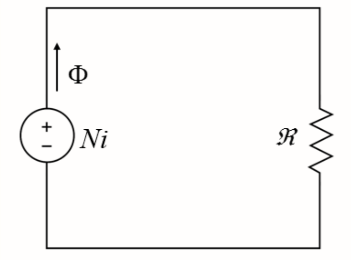
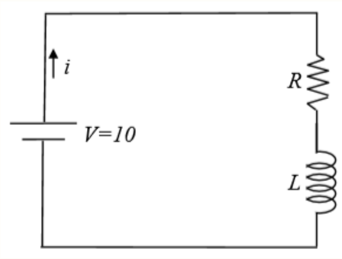
(You can also try other energy equations discussed above, say $w_i = \frac{B^2}{2\mu}$, etc.)

Summary

In this lecture, you have learnt:

- ❖ Design and analysis of magnetic circuits excited by AC and DC sources.
- ❖ Use of magnetic field as a medium to store electrical energy.
- ❖ Evaluation of magnetic field energy using different approaches.

References

No.	Slide No.	Image	Reference
1	4 and 14		Reprinted from *AC Circuits and Machines*, (p. 108), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
2	5 and 14		Reprinted from *AC Circuits and Machines*, (p. 105), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
3	6		Reprinted from *AC Circuits and Machines*, (p. 109), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.

References

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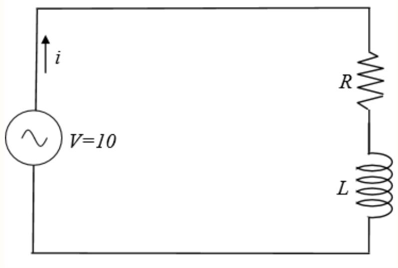
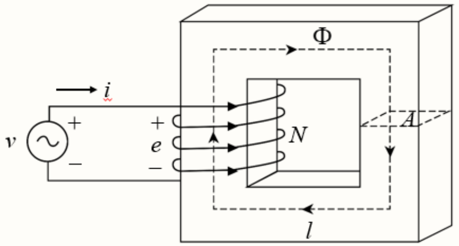
4	7		Reprinted from *AC Circuits and Machines*, (p. 109), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
5	8		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 147), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.