NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

EE2003 – SEMICONDUCTOR FUNDAMENTALS

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 9 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae and Table of Physical Constants are provided in Appendices A and B on pages 5 - 8. A Table of Material Properties is provided in Appendix C on page 9.

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1. (a) Figure 1 shows a plane (shaded) in a cubic crystal of lattice constant *a*. Write down the Miller indices of the plane. Determine the number of atom(s) laying on the plane if the crystal is simple cubic with one atom at each lattice point. Briefly explain your answer.

(5 Marks)

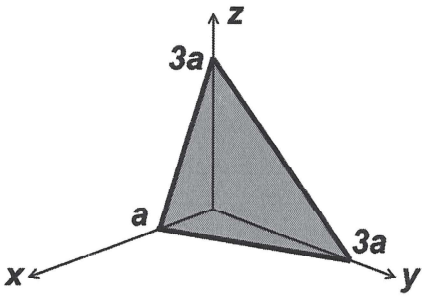

Figure 1

Note: Question No. 1 continues on page 2.

(b) A hypothetical semiconductor has a unit cell shown in Figure 2. The top and bottom surfaces are each a regular hexagon (all sides equal) as indicated. Each solid circle represents a lattice point (i.e. the presence of an atom). Determine the volume density of atoms for this semiconductor.

(5 Marks)

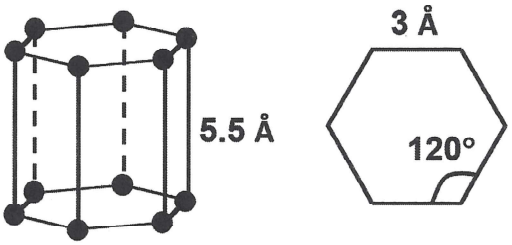

Figure 2

(c) A semiconductor with a bandgap energy of 1.2 eV is to be doped with p-type impurity. The intrinsic carrier concentration is 6.5×10^{9} cm^{-3} at 300 K.

(i) Determine the maximum allowable doping concentration such that the assumption of non-degenerate doping remains valid at 300 K.

(ii) What type of bandgap must the semiconductor have such that it would emit light during electron-hole recombination? Briefly explain. If the semiconductor has the required bandgap type, determine the wavelength of the light emitted.

(iii) While doping helps increase the conductivity of a semiconductor, it has an undesirable effect on carrier transport. Briefly discuss this effect.

(15 Marks)

2. (a) Excess minority carriers, of concentration 2.2×10^{12} cm^{-3} and of lifetime 1 μs, are injected into one end ($x = 0$) of a semi-infinitely long p-type silicon bar ($x \geq 0$). The silicon bar is uniformly doped to a concentration of 5×10^{14} cm^{-3} and no voltage is applied to it. Briefly explain your answer to the following questions.

(i) Is the low-level injection assumption valid in this situation?

(ii) What is the current conduction mechanism in the silicon bar? Calculate the value of the current density at $x = 10$ μm.

(iii) Would the magnitude of the current in part (a)(ii) increase or decrease if the doping concentration of the silicon bar is increased by 10 times, with the rest of the parameters remaining unchanged?

(15 Marks)

Note: Question No. 2 continues on page 3.

(b) Draw the minority carrier concentration in a PNP transistor in the forward active regime. Indicate the electrical field directions in the emitter-base and the base-collector space charge region.

(5 Marks)

(c) In a bipolar junction transistor the width of base region is very critical for the operation of the device, and it generally should be made possibly narrow. Briefly explain why the base region should be made possibly narrow? What would happen if the base region is too large?

(5 Marks)

3. (a) Two oppositely doped regions A and B of a silicon sample form an abrupt p-n junction with a depletion width W_0 of 0.74 μm and a built-in voltage V_{bi} of 0.64 V at 300 K. Figure 3 shows the positive electric field profile across the depletion region, where $x = 0$ corresponds to the metallurgical junction.

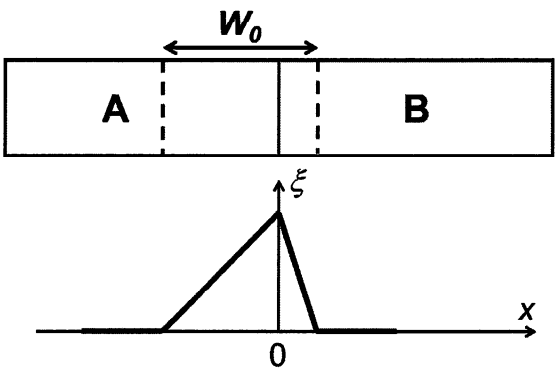

Figure 3

(i) State the doping type of region A and B. Which region has a higher doping concentration? Briefly justify your answers.

(ii) Using the value of W_0, write down an equation relating the acceptor and donor doping concentrations, N_A and N_D, respectively. Hence, determine the value of N_A and N_D.

(iii) The p-n junction breaks down electrically when the maximum electric field is equal to or exceeds 3.5×10^5 V/cm. Calculate the depletion width W at the onset of breakdown. Hence, determine the breakdown voltage.

(15 Marks)

Note: Question No. 3 continues on page 4.

(b) A metal forms a contact with a semiconductor doped uniformly with donor impurity to a concentration of 6×10^{15} cm^{-3} at 300 K. The semiconductor has an electron affinity of 3.75 eV, a bandgap energy of 1 eV and an intrinsic carrier concentration of 2×10^{10} cm^{-3}.

(i) State the range of the metal work function that will form a Schottky contact.

(ii) The metal used to form a contact in part (b)(i) has a work function of 4.5 eV. Sketch a *labelled* energy band diagram of the contact at thermal equilibrium, clearly indicating the value of the Schottky barrier and the built-in voltage.

(10 Marks)

4. (a) GaAs is a compound semiconductor material that has an energy bandgap of 1.43eV. If GaAs is used as a photodetector, what is the cut-off wavelength of the detector? Can a GaAs photodetector be used to detect the near-infrared light at wavelength 1.0 μm? Briefly justify your answer.

(6 Marks)

(b) Consider a GaAs photodiode with an absorption layer thickness of 1.0 μm and a surface area of 10^{-4} cm^2. The detector is illuminated by light at a wavelength of 775 nm with an intensity of 0.1 W/cm^2. The absorption coefficient of GaAs is 10^4 cm^{-1} at the wavelength of interest.

(i) Calculate the number of photons absorbed per second by the photodiode.

(ii) Assuming the responsivity of the photodiode is 0.5A/W, calculate the induced photocurrent.

(iii) What is the quantum efficiency of the photodiode?

(12 Marks)

(c) Explain how a PN-junction diode can emit light under forward bias? What property of the semiconductor primarily determines the frequency of the light emitted. Briefly explain why under forward bias a silicon-based PN-junction cannot emit light.

(7 Marks)

List of Selected Formulae

$$\xi = \frac{1}{q}\frac{dE}{dx}, \quad E_{ph} = h\nu = \frac{hc}{\lambda}, \quad \frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}, \quad E_n = -\frac{q^4}{2(4\pi\hbar)^2}\left(\frac{m_n^*}{\varepsilon_r^2\varepsilon_0^2}\right)\frac{1}{n^2},$$

$$f(E) = \frac{1}{1+\exp\left[\frac{E-E_F}{k_BT}\right]}, \quad g_c(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E-E_c}, \quad g_v(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v-E},$$

$$n_0 = N_c\exp\left[-\frac{E_c-E_F}{k_BT}\right], \quad N_c = 2\left(\frac{2\pi m_n^* k_BT}{h^2}\right)^{3/2},$$

$$p_0 = N_v\exp\left[-\frac{E_F-E_v}{k_BT}\right], \quad N_v = 2\left(\frac{2\pi m_p^* k_BT}{h^2}\right)^{3/2},$$

$$p_0 + N_d = n_0 + N_a, \quad E_{thermal\,(3-D)} = \frac{3}{2}k_BT, \quad v_{dp} = \mu_p\,\xi, \quad \mu_p = \frac{q\,\tau_{cp}}{m_p^*},$$

$$v_{dn} = -\mu_n\,\xi, \quad \mu_n = \frac{q\,\tau_{cn}}{m_n^*}, \quad J_{p\,drift} = q\,p\,\mu_p\,\xi, \quad J_{n\,drift} = q\,n\,\mu_n\,\xi,$$

$$J_{drift} = J_{n\,drift} + J_{p\,drift} = \sigma\,\xi, \quad \sigma = q\,\mu_n\,n + q\,\mu_p\,p, \quad \rho = \frac{1}{\sigma}, \quad J = \frac{I}{A}, \quad \xi = \frac{V}{l},$$

$$R_R = \rho\frac{l}{A}, \quad l = v_{th}\,\tau_{cn}, \quad v_{th}\,l = D_n, \quad J_{n\,diff} = q\,D_n\frac{dn}{dx}, \quad J_{p\,diff} = -q\,D_p\frac{dp}{dx},$$

$$J_n = J_{n\,drift} + J_{n\,diff}, \quad J_p = J_{p\,drift} + J_{p\,diff}, \quad J_{total} = J_n + J_p,$$

$$D_n = \frac{k_B\,T}{q}\mu_n, \quad D_p = \frac{k_B\,T}{q}\mu_p$$

$$n_0 = n_i\exp\left(\frac{E_F-E_i}{k_BT}\right), \quad p_0 = n_i\exp\left(\frac{E_i-E_F}{k_BT}\right)$$

$$n_0 p_0 = n_i^2$$

List of Selected Formulae (cont'd)

$$R = \alpha_r np,\ G_{th} = \alpha_r n_i^2,\ \tau_n = \frac{1}{\alpha_r p_0},\ \tau_p = \frac{1}{\alpha_r n_0}$$

$$\frac{dn}{dt} = \frac{d\Delta n}{dt} = G_L + G_{th} - R = G_L - \frac{\Delta n}{\tau_n},\ \Delta n_{ss} = G_L \tau_n,\ \Delta n(t) = \Delta n(t=0)\exp\left(-\frac{t}{\tau_n}\right)$$

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{q}\frac{\partial J_n(x,t)}{\partial x} + G_L - \frac{\Delta n}{\tau_n},\ \Delta n(x) = \Delta n(x=0)\exp\left(-\frac{x}{L_n}\right),\ L_n = \sqrt{D_n \tau_n}$$

$$\frac{dp}{dt} = \frac{d\Delta p}{dt} = G_L + G_{th} - R = G_L - \frac{\Delta p}{\tau_p},\ \Delta p_{ss} = G_L \tau_p,\ \Delta p(t) = \Delta p(t=0)\exp\left(-\frac{t}{\tau_p}\right)$$

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x,t)}{\partial x} + G_L - \frac{\Delta p}{\tau_p},\ \Delta p(x) = \Delta p(x=0)\exp\left(-\frac{x}{L_p}\right),\ L_p = \sqrt{D_p \tau_p}$$

$$\frac{d^2V(x)}{dx^2} = -\frac{d\xi(x)}{dx} = -\frac{\rho_c}{\varepsilon_r \varepsilon_0} = -\frac{q}{\varepsilon_r \varepsilon_0}(p - n + N_d - N_a)$$

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{p_{p0}}{p_{n0}}\right) = \frac{kT}{q}\ln\left(\frac{N_a N_d}{n_i^2}\right),\ \frac{p_{p0}}{p_{n0}} = \frac{n_{n0}}{n_{p0}} = \exp\left(\frac{qV_{bi}}{kT}\right)$$

$$N_d x_n = N_a x_p,\ \xi_{max} = -\frac{qN_d x_n}{\varepsilon_r \varepsilon_0} = -\frac{qN_a x_p}{\varepsilon_r \varepsilon_0},\ W = \left[\frac{2\varepsilon_r \varepsilon_0 (V_{bi} - V_a)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

$$\frac{p_{p0}}{p_n(x_n)} = \frac{n_{n0}}{n_p(-x_p)} = \exp\left[\frac{q}{kT}(V_{bi} - V_a)\right],\ \frac{p_n(x_n)}{p_{n0}} = \frac{n_p(-x_p)}{n_{p0}} = \exp\left(\frac{qV_a}{kT}\right)$$

$$\Delta n_p(x) = \Delta n_p(-x_p)\exp\left(-\frac{x}{L_n}\right) = n_{p0}\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right]\exp\left(-\frac{x}{L_n}\right)$$

$$\Delta p_n(x) = \Delta p_n(x_n)\exp\left(-\frac{x}{L_p}\right) = p_{n0}\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right]\exp\left(-\frac{x}{L_p}\right)$$

$$I = I_0\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right],\ I_0 = qA\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right),\ C_j = \left|\frac{dQ_j}{dV_a}\right| = \frac{\varepsilon_r \varepsilon_0 A}{W}$$

$$C_s = \left|\frac{dQ_n}{dV_a}\right| = \frac{q}{kT}|Q_n| = \frac{q}{kT}I\tau_n \text{ (n}^+\text{p diode)},\ C_s = \frac{dQ_p}{dV_a} = \frac{q}{kT}Q_p = \frac{q}{kT}I\tau_p \text{ (p}^+\text{n diode)}$$

$$Q_n = -qAL_n \Delta n_p,\ Q_p = qAL_p \Delta p_n$$

List of Selected Formulae (cont'd)

$$I(x) = I_0 \exp(-\alpha x), \quad G = R_1 R_2 \exp(2(k-\gamma)L), \quad k_{th} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$\frac{n\lambda}{2} = L, \quad f = \frac{nc}{2L}, \quad \Delta f = \frac{\Delta nc}{2L}, \quad \frac{hc}{\lambda} = E_{ph}$$

$$\text{Reflectivity, r} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2, \quad I_t = (1-r)I_0, \quad I = RP, \quad R = \eta\frac{e}{E_{ph}}, \quad \eta = \frac{N_e}{N_p}$$

$$i_C = \frac{-eD_n A_{BE}}{x_B} \times n_{B0} \exp\left(\frac{ev_{BE}}{kT}\right), \quad \frac{i_C}{i_E} \equiv \alpha, \quad \frac{i_C}{i_B} \equiv \beta, \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1,$$

$$V_B + I \times R_L + V = 0$$

Table of Physical Constants

	Symbol	Value	Unit
Planck's constant	h	6.626×10^{-34}	J-s
Speed of light	c	3.0×10^{8}	m/s
Electronic charge	e (or q)	1.6×10^{-19}	C
Boltzmann's constant	k_B (or k)	1.38×10^{-23}	J/K
Free electron rest mass	m_0	9.1×10^{-31}	kg
Proton rest mass	m_p	1.67×10^{-27}	kg
Avogadro's number	N_A	6.02×10^{23}	mol^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	H/m
Permittivity of free space	ε_0	8.85×10^{-12}	F/m
Rydberg constant	R_d	1.097×10^{7}	m^{-1}
Bohr radius	a_0	5.292×10^{-11}	m
Gas constant	R	8.31	$Jmol^{-1}K^{-1}$
Electron-volt	1 eV	1.6×10^{-19}	J
Thermal voltage (T = 300 K)	k_BT/q	0.0259	V

APPENDIX C

Properties of Silicon, Gallium Arsenide, and Germanium (T = 300 K)

Property	**Si**	**GaAs**	**Ge**
Atomic density (cm^{-3})	5.00×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	Si: 11.7 SiO_2: 3.8	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s) Electron, μ_n Hole, μ_p	 1350 480	 8500 400	 3900 1900

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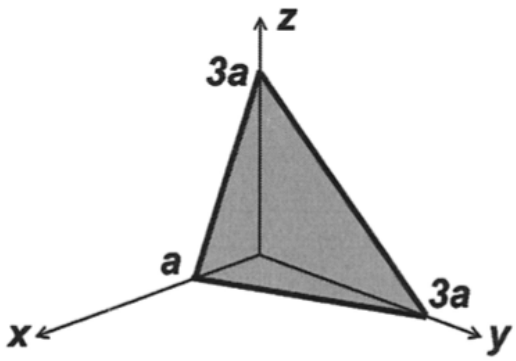
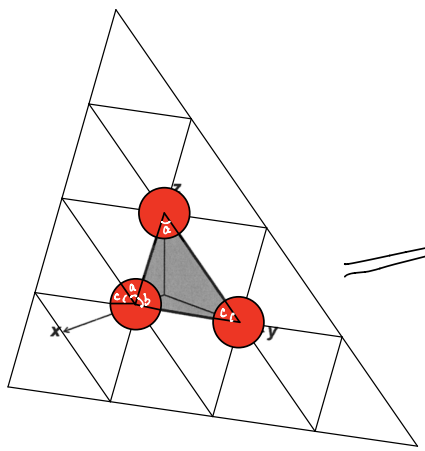
1.(a)



	x	y	z
Intercepts	a	3a	3a
Intercepts (numbers only)	1	3	3
Reciprocal	1	$\frac{1}{3}$	$\frac{1}{3}$
Reduction	3	1	1
Enclosure	[3 1 1]		

∴ Miller indices of the plane: [3 1 1] #

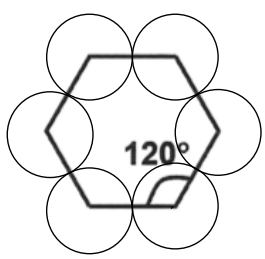


∴ Number of atoms $= 3\times\left(\frac{a}{360^\circ}+\frac{b}{360^\circ}+\frac{c}{360^\circ}\right)$

∵ $a+b+c=180^\circ$ (Actually no need to draw, the three interior angles in a triangle will always add up to 180°)

∴ Number of atoms $= 3\times\frac{1}{2}=\frac{3}{2}$ #

(b) ① Get the no. of atoms in the unit cell



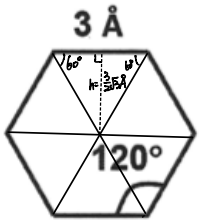
∴ 1 atom occupied:

$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ in the structure

In total we have 12 atoms.

∴ The number of atoms $=\frac{1}{6}\times 12=2$ atoms.

② Caculate the total volume



Base area:

$6\times\left(3\times\frac{3}{2}\sqrt{3}\times\frac{1}{2}\right)Å^2$

$=\frac{27}{2}\sqrt{3}\,Å^2$

Total volume:

$\frac{27}{2}\sqrt{3}\times 5.5\,Å^3$

$=128.6\,Å^3$

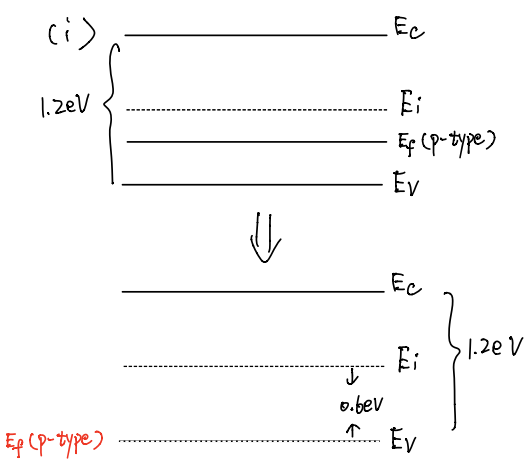
③ Calculate the no. of atoms per unit volume (volume density)

$$\frac{2}{128.6 \times (10^{-8})^3} = 1.555 \times 10^{22} \text{ atoms}/cm^3 \quad \#$$

△ Note: Volume density & the percentage of unit cell volume occupied by the atoms are different.

(C) $E_g = 1.2eV$, p-type, $n_i = 6.5 \times 10^9 cm^{-3}$, $T = 300K$

(i)



The difficulty lies in comprehending the term: "non-degenerate doping"

① below the conduction band edge (n-type)

② top of the valence band edge (p-type) (✓)

Using $p_0 = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right)$ $\quad k_B T\ (T = 300K) \approx 0.0259eV$

$$\therefore p_0 = 6.5 \times 10^9 \times \exp\left(\frac{0.6eV}{0.0259eV}\right)$$

$$= 7.478 \times 10^{19} \quad \#$$

(ii) Direct bandgap will emit light, in direct bandgap semiconductor, the transitions do not require changes in the crystal momentum. Therefore, the transitions are very efficient, the energy is given off as photon (light).

$$E_{ph} = h\nu = \frac{hc}{\lambda}$$

$\because E_{ph} = E_g = 1.2eV$

$$\therefore \lambda = \frac{hc}{E_{ph}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{10}}{1.2 \times 1.6 \times 10^{-19}}$$

$$= 1.035 \times 10^{-4} cm$$

$$= 1.035 \mu m \quad \#$$

(iii) With high doping concentration, the mobility decrease due to ionized impurity scattering with the ionized doping atoms. The actual mobility also depends on the type of dopant.

2. (a) p-type Si, uniformly doped $\Rightarrow N_a = 5\times10^{14}\,cm^{-3}$, No voltage applied $\Rightarrow V_a = 0$, $\tau_{no} = 1\times10^{-6}\,s$

(i) low level injection : minority < 10% × majority (Thermal equilibrium)

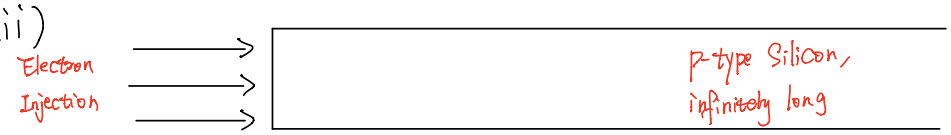
∵ Only inject electrons, so only Δn present. (if optical, $\Delta n = \Delta p$, they come as pairs, so $p_0 = N_a - \Delta p$)

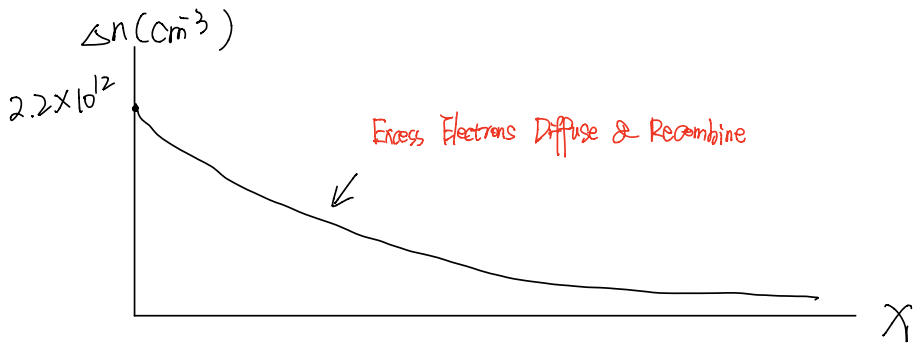
$\Delta n = 2.2\times10^{12}\,cm^{-3}$, $N_a = 5\times10^{14} \approx P_0$

∵ $\Delta n = 2.2\times10^{12} < 10\% \times 5\times10^{14} = 5\times10^{13}$ (valid)

∴ low level injection valid. #

(ii)





∵ No external voltage applied, $V_a = 0$

- Drift : movement of charge due to electric fields
- Diffusion : flow of charge due to concentration gradients

∴ There's no drift current

When we have electron injection, the bar is no longer uniformly doped, so diffusion current will present.

∴ The current conduction mechanism in the silicon bar is diffusion current. #

$\therefore \Delta n(x) = \Delta n(x=0)\exp\left(-\frac{x}{L_n}\right) cm^{-3}$ (Equation given)

Minority electron carrier diffusion length L_n,

$L_n = \sqrt{D_n \tau_n} = \sqrt{\frac{k_B T}{q}\mu_n \tau_n}$ (Never indicate Temperature, we only can assume $T = 300K$ and $k_B T = 0.0259 eV$)

$= \sqrt{\frac{0.0259}{q} \times 1350 \times 10^{-6}} = 5.913 \times 10^{-3} cm$

$\therefore \Delta n(x) = 2.2 \times 10^{12} \times \exp\left(-\frac{x}{5.913 \times 10^{-3}}\right) cm^{-3}$

The minority diffusion current density:

$J_{n\,diff} = q D_n \frac{dn}{dx} = q D_n \frac{d(n_0 + \Delta n)}{dx} = k_B T \mu_n \frac{d\Delta n}{dx}$

$= -\frac{1.38 \times 10^{-23}}{5.913 \times 10^{-3}} \times 300 \times 1350 \times 2.2 \times 10^{12} \times \exp\left(-\frac{x}{5.913 \times 10^{-3}}\right) cm^{-3}$

$= -2.079 \times 10^{-3} \times \exp\left(-\frac{x}{5.913 \times 10^{-3}}\right) cm^{-3}$

At $x = 10 \mu m$, $|J_{n\,diff}| = -2.079 \times 10^{-3} \times \exp\left(-\frac{10}{5.913 \times 10^{-3}}\right) = 1.7556 \times 10^{-3} A/cm^2$

$= 1.7556 mA/cm^2$ #

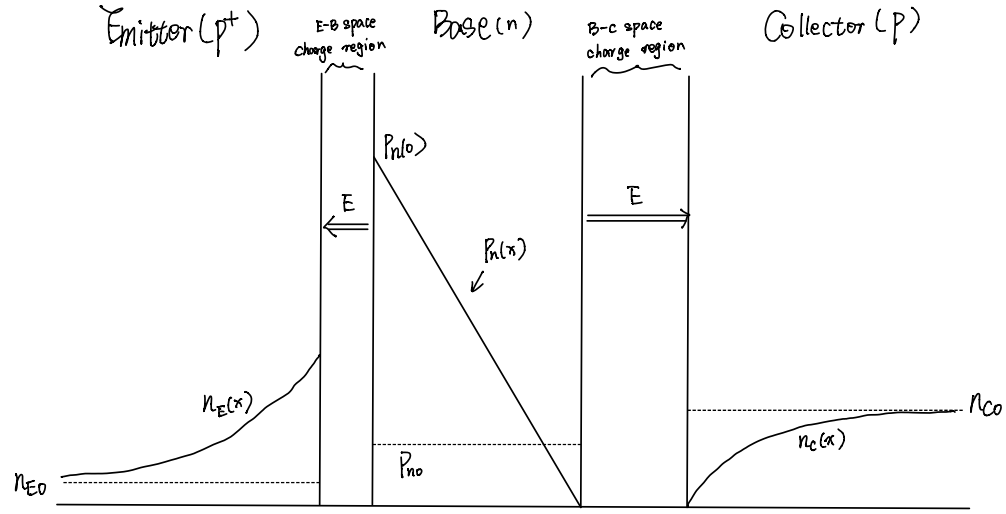
(iii) (Not sure about the answer)

When the doping is increased, the impurity scattering will degrades the carrier mobility. Since $L_n = \sqrt{\frac{k_B T}{q}\mu_n \tau_n}$, $\mu_n \downarrow$, $L_n \downarrow$, $\exp\left(-\frac{x}{L_n}\right) \downarrow$

and $J_{n\,diff} = k_B T \mu_n \downarrow \frac{d\Delta n}{dx} \downarrow$, both decrease.

$\therefore |J_{n\,diff}|$ will decrease.

(b)

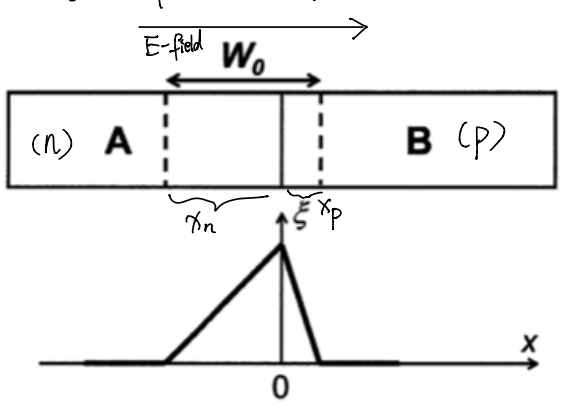


☆ Remember the e-field is always pointing from n to p, and from left to right, the minority concentration indicates the doping density from high to low.

(c) Narrow: to prevent the electrons recombine with holes in the base region. When the current is passing through the base region to minimize the opposing force, to which we can achieve maximum Drift current.

Large: If it's too large, recombination will happen and we cannot achieve maximum Drift current. The efficiency is low.

3.(a) $W_0 = 0.74\,\mu m = 0.74\times10^{-4}\,cm$, $V_{bi} = 0.64\,V$, $T = 300\,K$, $KT \approx 0.0259\,eV$



(i) We can tell from the graph that the electric field is positive, means it points from left to right.

∵ E-field always from n-side points to p-side,

∴ A is n region & B is p region.

from equation : $N_d x_n = N_a x_p$ $\because x_n > x_p$

$\therefore$ $N_d < N_a$, p region has a higher doping concentration.

(ii)

① $V_{bi} = \frac{kT}{q}\ln\left(\frac{p_{p0}}{p_{n0}}\right) = \frac{kT}{q}\ln\left(\frac{N_a N_d}{n_i^2}\right)$ $n_i = 1.5\times10^{10}$ (from table at 300K)

$0.64 = 0.0259 \cdot \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$\therefore N_a \cdot N_d = 1.2128\times10^{31}$ ------- ①

② $W = \left[\frac{2\varepsilon_r\varepsilon_0(V_{bi}-V_a)}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{1/2}$ no external voltage : $V_a = 0\,V$

$0.74\times10^{-4} = \left[\frac{2\times11.8\times8.85\times10^{-14}\times0.64}{1.6\times10^{-19}}\cdot\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{\frac{1}{2}}$

$\therefore \frac{1}{N_a}+\frac{1}{N_d} = 6.5546\times10^{-16}$ ------- ②

$$\begin{cases} N_a \cdot N_d = 1.2128\times10^{31} & \text{------ ①} \\ \frac{1}{N_a}+\frac{1}{N_d} = 6.5546\times10^{-16} & \text{------ ②} \end{cases}$$

$$\begin{cases} N_a = 5.891\times10^{15} \\ N_d = 2.059\times10^{15} \end{cases}$$

$N_a = \frac{1.2128\times10^{31}}{N_d} \Rightarrow \frac{N_d}{1.2128\times10^{31}} + \frac{1}{N_d} = 6.5546\times10^{-16} \Rightarrow N_d^2 - 7.9495\times10^{15} N_d + 1.2128\times10^{31} = 0$

(iii) $E_c = 3.5\times10^5\,V/cm$

$\xi_{max} = -\frac{qN_d x_n}{\varepsilon_r\varepsilon_0} = -\frac{qN_a x_p}{\varepsilon_r\varepsilon_0}$

$$\begin{cases} x_n = \frac{3.5\times10^5\times11.8\times8.85\times10^{-14}}{1.6\times10^{-19}\times2.059\times10^{15}} = 1.109\times10^{-3}\,cm \\ x_p = \frac{3.5\times10^5\times11.8\times8.85\times10^{-14}}{1.6\times10^{-19}\times5.891\times10^{15}} = 3.8778\times10^{-4}\,cm \end{cases}$$

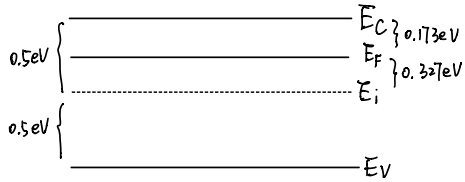
$\therefore W = x_n + x_p = 1.49678\times10^{-3}\,cm$

$V_{br} \approx \frac{\varepsilon_r\varepsilon_0(N_A+N_D)}{2qN_AN_D}E_c^2 = \frac{11.8\times8.85\times10^{-14}\times(2.059+5.891)\times10^{15}}{2\times1.6\times10^{-19}\times2.059\times5.891\times10^{30}}\times(3.5\times10^5)^2$

$= 262.02\,V$

(b) $N_d = 6\times10^{15}$, $T = 300K$, $q\chi = 3.75eV$, $E_g = 1eV$, $n_i = 2\times10^{10}$

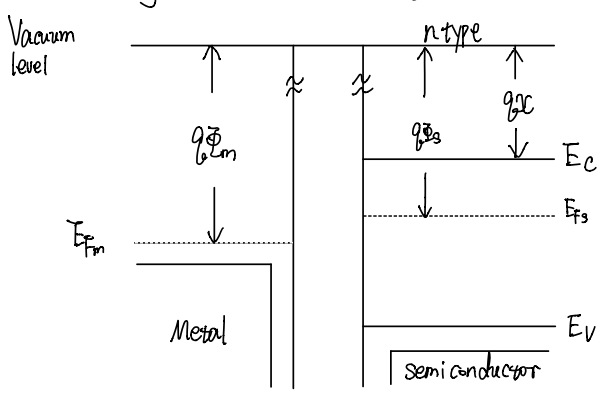
(i) Find the fermi level first: $n_0 \approx N_d = 6\times10^{15}$, $k_BT \approx 0.0259eV$



$$n_0 = n_i \exp\left(\frac{E_F - E_i}{k_BT}\right) \qquad 6\times10^{15} = 2\times10^{10} \times \exp\left(\frac{E_F - E_i}{0.0259}\right)$$

$$\therefore \begin{cases} E_F - E_i = 0.327eV \\ E_c - E_F = 0.173eV \end{cases}$$

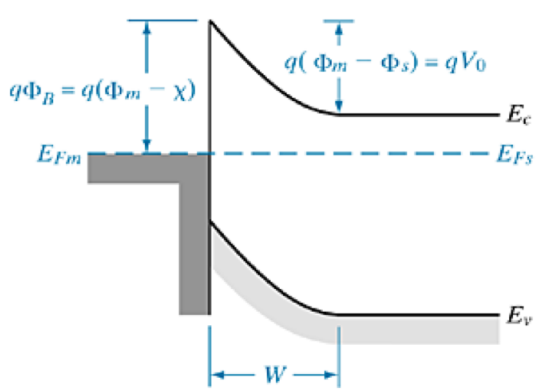
Schottky Contact: $q\phi_m > q\phi_s$



$$\therefore q\Phi_s = q\chi + E_c - E_{Fs}$$
$$= 3.75eV + 0.173eV$$
$$= 3.923eV$$

$$\therefore q\Phi_m > 3.923eV$$

(ii)



$q\Phi_B = 4.5 - 3.75$ (barrier)
$= 0.75eV$

$qV_0 = 4.5 - 3.923$ (built-in Voltage)
$= 0.577eV$

4. (a) $E_g = 1.43\,eV$ $\quad E_{ph} = h\nu = \frac{hc}{\lambda}$

$$\therefore \lambda = \frac{hc}{E_{ph}} = \frac{6.626\times10^{-34}\times3\times10^{10}}{1.43\times1.6\times10^{-19}} = 8.688\times10^{-5}\,cm \quad \#$$

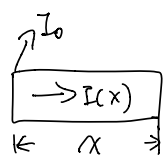
No, GaAs photodetector cannot be used to detect wavelength $1.0\,\mu m$, the maximum wavelength can be detected is $0.8688\,\mu m$.

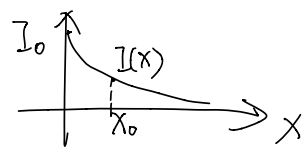
Not sure if (b) is correct or not.

(b) $x = 1\,\mu m$, $A = 10^{-4}\,cm^2$, $\lambda = 775\,nm$, $I = 0.1\,W/cm^2$, $\alpha = 10^4\,cm^{-1}$

(i) $I(x) = I_0 \exp(-\alpha x)$

$= 0.1\,W/cm^2 \cdot \exp(-10^4 x)$





$\because$ thickness $= 1\,\mu m = 10^{-4}\,cm$

$\therefore x = 10^{-4} \Rightarrow I(10^{-4}) = 0.1 \times \exp(-10^4 \cdot 10^{-4}) = 0.0368\,W/cm^2$

$\therefore I_{absorb} = I_0 - I(10^{-4}) = 0.1 - 0.0368 = 0.0632\,W/cm^2$

$$\#\ of\ e/h = I_{absorb}/e = \frac{0.0632}{1.6\times10^{-19}} = 3.95\times10^{18} \quad \#$$

(ii) $R = 0.5\,A/W$, $R = \frac{I_{ph}}{P_0}$

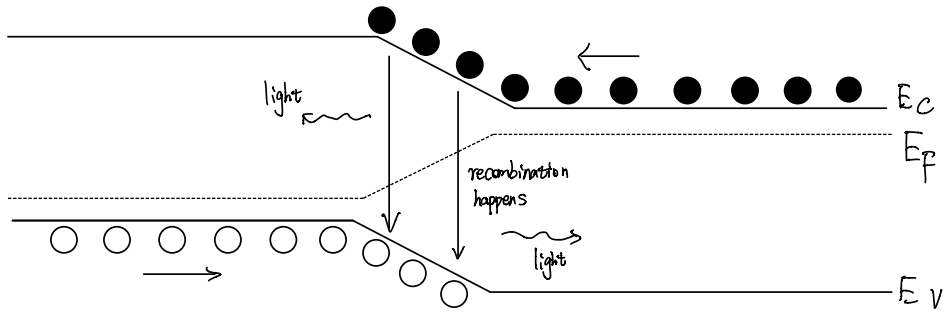
$$\therefore I_{ph} = R\cdot P_0 = R\cdot A\cdot I_{absorb} = 0.5\,A/W \cdot 10^{-4}\,cm^2 \cdot 0.0632\,W/cm^2$$

$$= 3.16\times10^{-6}\,A \quad \#$$

(iii) $$\eta = R\times\frac{hc}{e\lambda} = 0.5\times\frac{6.626\times10^{-34}\times3\times10^{10}}{1.6\times10^{-19}\times775\times10^{-7}}$$

$$\approx 0.8015 = 80.15\% \quad \#$$

(c) ① PN-junction diode emit light by electron-hole pairs recombination.



Under forward-bias, the electron and hole meet in the space so they can re-combine, the barrier between p and n juction is decrease due to forward-bias, as it's direct band-gap so the energy is given out as light.

② The energy bandgap will determine the frequency of the light emitted. also can know from the equation: $E_{ph} = \frac{hc}{\lambda}$.

③ Because a silicon-based PN-junction is in-direct bandgap, they will give out the energy as heat.

△ Note: III-IV are direct bandgap, like GaAs.