NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

November / December 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 4.

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1. (a) Let E and F be elementary matrices corresponding to the elementary row operations. If E adds row 1 to row 2, and F adds row 2 to row 1, does EF equal FE? Justify your answer.

(5 Marks)

(b) Let

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad |A|=5$$

and

$$B=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^n A \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^n$$

where n is a positive integer.

(i) With $n=1$, explain, in terms of row and column operations, the operations that were performed on matrix A.

(ii) Write down the resulting matrix B when n is odd, i.e., $n=1,3,5,...$ and the determinant of matrix B. Justify your answer.

Note: Question 1 continues on page 2

(iii) Write down the resulting matrix B when n is even, i.e., $n = 2,4,6,...$ and the determinant of matrix B. Justify your answer.

(10 Marks)

(c) Find a LU decomposition of the matrix

$$\begin{bmatrix} a & r & r & r & r \\ a & b & s & s & s \\ a & b & c & t & t \\ a & b & c & d & u \\ a & b & c & d & e \end{bmatrix}$$

You may assume that the variables a,b,c,d,e,r,s,t,u are unique, non-zero real numbers.

(10 Marks)

2. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be vectors in vector space $\mathbf{V}$.

(a) Explain how you would determine whether $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent or dependent. Hence determine whether the following are linearly independent or dependent. Justify your answers.

(i) The vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$;

(ii) The matrices $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$;

(iii) The polynomials $p(x) = 1 + x$, $q(x) = 1 - x$ and $h(x) = 1 - x^2$.

(15 Marks)

(b) Consider the following matrix A and vector $\mathbf{v}$:

$$A = \begin{bmatrix} 1 & -1 & 4 & 0 & -3 \\ 0 & 1 & -5 & -1 & 2 \\ 0 & 1 & 3 & -1 & 0 \\ 1 & 0 & -1 & -1 & 0 \end{bmatrix}; \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Determine whether $\mathbf{v}$ is in the column space of A. Hence, or otherwise, find a basis for the column space of A.

It is said that the set of basis vectors for a vector space is not unique. Therefore, from the five columns of A, one can form many bases. Which two columns of A must be included in all the bases that will span the column space of A?

Justify all your answers.

(10 Marks)

3. (a) By considering only the principal value of the natural logarithm, and given

$$\left(z^3-4\right)\ln 5 = 4\sqrt{3}\ln\left[\cos\left(\ln 5\right)+i\sin\left(\ln 5\right)\right],$$

find z in the exponential form, leaving your answer in terms of π.

(8 Marks)

(b) Express $f(z)=\exp(iz)-\exp(-iz)$, $z=x+iy$, in the form $f(z)=u(x,y)+i\,v(x,y)$, where x, y, $u(x, y)$ and $v(x, y)$ are real. Hence, discuss the differentiability and analyticity of $f(z)=\exp(iz)-\exp(-iz)$ by using the Cauchy-Riemann equations.

(8 Marks)

(c) Evaluate $\displaystyle\int_{-\infty}^{+\infty}\frac{1}{\left(x^2-2x+3\right)^2}dx.$

(9 Marks)

4. (a) Show that

$$\nabla\times\left(f\,\mathbf{v}\right)=f\left(\nabla\times\mathbf{v}\right)+\left(\nabla f\right)\times\mathbf{v}$$

for scalar function $f\left(x,y,z\right)$ and vector function
$\mathbf{v}\left(x,y,z\right)=v_1\left(x,y,z\right)\mathbf{i}+v_2\left(x,y,z\right)\mathbf{j}+v_3\left(x,y,z\right)\mathbf{k}.$

(8 Marks)

(b) Find the work done in moving a particle along the path $x=\cos y$, $z=0$ from $y=0$ to $y=2\pi$, in the field

$$\mathbf{F}\left(x,y,z\right)=e^{2z}\cos y\,\mathbf{i}-x\,e^{2z}\sin y\,\mathbf{j}+2xe^{2z}\cos y\,\mathbf{k}.$$

(10 Marks)

(c) Evaluate $\iint_S \mathbf{F}\cdot d\mathbf{A}$ for surface S: $x=z^2, 0\le y\le 2, -1\le z\le 1$ and
$\mathbf{F}\left(x,y,z\right)=3y^2\mathbf{i}+ze^x\cos y\mathbf{j}+3xz^2\mathbf{k}.$

(7 Marks)

25

Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c\ln z}$

(b) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(c) Cauchy-Riemann equations:

$$u_x = v_y,\ v_x = -u_y,\ \text{or } u_r = \frac{1}{r}v_\theta,\ v_r = \frac{-1}{r}u_\theta$$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\ \frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\Big|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Gauss Theorem: $\iiint_T \nabla\cdot\mathbf{F}dv = \oiint_S \mathbf{F}\cdot\mathbf{n}dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

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1. (a) EF is not equal to FE.

A counterexample:

Let $E=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ $F=\begin{bmatrix}1&1\\0&1\end{bmatrix}$. Then E and F are elementary matrices.

$E'=\begin{bmatrix}0&1\\1&1\end{bmatrix}$ $F'=\begin{bmatrix}1&2\\0&1\end{bmatrix}$

$E'F'=\begin{bmatrix}0&1\\1&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}0&1\\1&3\end{bmatrix}$

$F'E'=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}0&1\\1&1\end{bmatrix}=\begin{bmatrix}2&3\\1&1\end{bmatrix}$ $\Rightarrow E'F'\neq F'E'$

(b) (i) When $n=1$.

$$B=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}a_{31}&a_{32}&a_{33}\\a_{21}&a_{22}&a_{23}\\a_{11}&a_{12}&a_{13}\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}a_{33}&a_{32}&a_{31}\\a_{23}&a_{22}&a_{21}\\a_{13}&a_{12}&a_{11}\end{bmatrix}$$

The operations are interchanging rows 1 and 3 and interchanging coloumns 1 and 3.

(ii) When $n=2$.

$$B=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}^2A\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}^2=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}a_{33}&a_{32}&a_{31}\\a_{23}&a_{22}&a_{21}\\a_{13}&a_{12}&a_{11}\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}a_{13}&a_{12}&a_{11}\\a_{23}&a_{22}&a_{21}\\a_{33}&a_{32}&a_{31}\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}=A$$

Thus, when n is odd.

$B=\begin{bmatrix}a_{33}&a_{32}&a_{31}\\a_{23}&a_{22}&a_{21}\\a_{13}&a_{12}&a_{11}\end{bmatrix}$ $\det(B)=(-1)(-1)|A|=5.$

(iii) When n is even, $B=A$.

$\det(B)=|A|=5.$

(c) $\begin{bmatrix}a&r&r&r&r\\a&b&s&s&s\\a&b&c&t&t\\a&b&c&d&u\\a&b&c&d&e\end{bmatrix}\sim\begin{bmatrix}a&r&r&r&r\\a&b-r&s-r&s-r&s-r\\0&0&c-s&t-s&t-s\\0&0&0&d-t&u-t\\0&0&0&0&e-u\end{bmatrix}$

where the element operations are

$E_1: R_5\leftarrow R_5-R_4$

$E_2: R_4\leftarrow R_4-R_3$

$E_3: R_3\leftarrow R_3-R_2$

$E_4: R_2\leftarrow R_2-R_1$

$E_4E_3E_2E_1A=U \Rightarrow A=(E_4E_3E_2E_1)^{-1}U$

$L=(E_4E_3E_2E_1)^{-1}=E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}=\begin{bmatrix}1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\\1&1&1&1&1\end{bmatrix}$

Thus, $\begin{bmatrix}a&r&r&r&r\\a&b&s&s&s\\a&b&c&t&t\\a&b&c&d&u\\a&b&c&d&e\end{bmatrix}=\begin{bmatrix}1&0&0&0&0\\1&1&0&0&0\\1&1&1&0&0\\1&1&1&1&0\\1&1&1&1&1\end{bmatrix}\begin{bmatrix}a&r&r&r&r\\a&b-r&s-r&s-r&s-r\\0&0&c-s&t-s&t-s\\0&0&0&d-t&u-t\\0&0&0&0&e-u\end{bmatrix}$

2. (a) $v_1, v_2, \ldots, v_n$ are linearly independent if and only if the only solution to the equation $c_1v_1+c_2v_2+\ldots+c_nv_n=0$ is the trivial solution $c_1=c_2=\cdots=c_n=0$.

(i) $c_1\begin{bmatrix}1\\1\\1\end{bmatrix}+c_2\begin{bmatrix}0\\1\\1\end{bmatrix}+c_3\begin{bmatrix}1\\0\\-1\end{bmatrix}=0$

$\left[\begin{array}{ccc|c}1&0&1&0\\1&1&0&0\\1&1&-1&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&1&0\\0&1&-1&0\\0&1&-2&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&1&0\\0&1&-1&0\\0&0&-1&0\end{array}\right]$

Since the solution is $c_1=c_2=c_3=0$,

The vectors are linearly independent.

(ii) $c_1\begin{bmatrix}1&2\\2&1\end{bmatrix}+c_2\begin{bmatrix}-1&1\\1&-1\end{bmatrix}+c_3\begin{bmatrix}2&2\\1&1\end{bmatrix}=0.$

$$\begin{bmatrix}c_1-c_2+2c_3 & 2c_1+c_2+2c_3\\ 2c_1+c_2+c_3 & c_1-c_2+c_3\end{bmatrix}=0 \Rightarrow \left[\begin{array}{ccc|c}1&-1&2&0\\2&1&2&0\\2&1&1&0\\1&-1&1&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&-1&2&0\\0&3&-2&0\\0&3&-3&0\\0&0&-1&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&-1&2&0\\0&3&-2&0\\0&0&-1&0\\0&0&-1&0\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&-1&2&0\\0&3&2&0\\0&0&-1&0\\0&0&0&0\end{array}\right]$$

Since the solution is $c_1=c_2=c_3=$

The matrices are linearly independe

(iii) $c_1p(x)+c_2q(x)+c_3h(x)=0.$

$c_1(1+x)+c_2(1-x)+c_3(1-x^2)=0$

$-c_3x^2+(c_1-c_2)x+c_1+c_2+c_3=0.$

$$\Rightarrow\begin{cases}-c_3=0\\c_1-c_2=0\\c_1+c_2+c_3=0\end{cases}$$

$\Rightarrow$ The solution is $c_1=c_2=c_3=0$

The polynomials are linearly independent

(b) $c_1\begin{bmatrix}1\\0\\0\\1\end{bmatrix}+c_2\begin{bmatrix}-1\\1\\1\\0\end{bmatrix}+c_3\begin{bmatrix}4\\-5\\3\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\-1\\-1\\-1\end{bmatrix}+c_5\begin{bmatrix}-3\\2\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}.$

$$\Rightarrow\left[\begin{array}{ccccc|c}1&-1&4&0&-3&1\\0&1&-5&-1&2&2\\0&1&3&-1&0&3\\1&0&-1&-1&0&4\end{array}\right]\sim\left[\begin{array}{ccccc|c}1&-1&4&0&-3&1\\0&1&-5&-1&2&2\\0&0&8&0&-2&1\\0&1&-5&-1&3&3\end{array}\right]\sim\left[\begin{array}{ccccc|c}1&-1&4&0&-3&1\\0&1&-5&-1&2&2\\0&0&8&0&-2&1\\0&0&0&0&1&1\end{array}\right]$$

The equation always has solution (multiple).

Thus, v is in the column space of A.

A basis for the column space of A can be $\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}-1\\1\\1\\0\end{bmatrix},\begin{bmatrix}4\\-5\\3\\-1\end{bmatrix},\begin{bmatrix}-3\\2\\0\\0\end{bmatrix}\right\}.$

We can find that $\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=(-1)\begin{bmatrix}-1\\1\\1\\0\end{bmatrix}+(-1)\begin{bmatrix}0\\-1\\-1\\-1\end{bmatrix}$. We can choose any two of them as bases.

Thus, the remaining column $\begin{bmatrix}-3\\2\\0\\0\end{bmatrix},\begin{bmatrix}4\\-5\\3\\-1\end{bmatrix}$ must be included.

3. (a). $(z^3-4)\ln 5=4\sqrt{3}\ln[\cos(\ln 5)+i\sin(\ln 5)]$

$(z^3-4)\ln 5=4\sqrt{3}\ln e^{i\ln 5}$

$(z^3-4)\ln 5=4\sqrt{3}\cdot i\ln 5$

$z^3=4+i4\sqrt{3}=8e^{i(\frac{\pi}{3}+2k\pi)} \Rightarrow z=2e^{i(\frac{\pi}{9}+\frac{2}{3}k\pi)}$, $k=0,\pm1,\pm2\ldots$

Thus, $z=2e^{i\frac{\pi}{9}}$, $2e^{i\frac{7}{9}\pi}$, $2e^{i-\frac{5}{9}\pi}$.

(b). $f(z)=e^{iz}-e^{-iz}=e^{i(x+yi)}-e^{-i(x+yi)}=e^{-y+xi}-e^{y-xi}=e^{-y}(\cos x+i\sin x)-e^{y}(\cos x-i\sin x)$

$=(e^{-y}-e^{y})\cos x+(e^{-y}+e^{y})\sin x$

$u(x,y)=(e^{-y}-e^{y})\cos x$

$v(x,y)=(e^{-y}+e^{y})\sin x$

$u_x=(e^{y}-e^{-y})\sin x$ $\quad v_x=(e^{-y}+e^{y})\cos x$

$u_y=-(e^{-y}+e^{y})\cos x$ $\quad v_y=(e^{y}-e^{-y})\sin x$ $\quad\Rightarrow u_x=v_y.\ v_x=-u_y.$

Thus, $f(z)$ is differentiable and analytic everywhere.

(c) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{(x^2-2x+3)^2}\,dx$

$x^2-2x+3=0$

$\Delta = 4-4\times 3 = -8$

$x = \dfrac{2 \pm i2\sqrt{2}}{2} = 1 \pm \sqrt{2}i$

$$= \oint_{UHP} \frac{1}{(x-1-\sqrt{2}i)^2(x-1+\sqrt{2}i)^2}\,dz$$

$$= \oint_{C_1} \frac{\frac{1}{(x-1+\sqrt{2}i)^2}}{(x-1-\sqrt{2}i)^2}\,dz$$

$$= 2\pi i \frac{d}{dz}\frac{1}{(x-1+\sqrt{2}i)^2}\Big|_{z=1+\sqrt{2}i} = 2\pi i \frac{-2}{(x-1+\sqrt{2}i)^3}\Big|_{z=1+\sqrt{2}i} = \frac{\pi}{4\sqrt{2}}.$$

4. (a) $$\nabla\times(f\underset{\sim}{v}) = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ fv_1 & fv_2 & fv_3 \end{vmatrix} = \begin{pmatrix} \frac{\partial}{\partial y}fv_3 - \frac{\partial}{\partial z}fv_2 \\ \frac{\partial}{\partial z}fv_1 - \frac{\partial}{\partial x}fv_3 \\ \frac{\partial}{\partial x}fv_2 - \frac{\partial}{\partial y}fv_1 \end{pmatrix}$$

$$f(\nabla\times\underset{\sim}{v}) + (\nabla f)\times\underset{\sim}{v} = f\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix} + \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ \frac{\partial}{\partial x}f & \frac{\partial}{\partial y}f & \frac{\partial}{\partial z}f \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{pmatrix} f\frac{\partial}{\partial y}v_3 - f\frac{\partial}{\partial z}v_2 + v_3\frac{\partial}{\partial y}f - v_2\frac{\partial}{\partial z}f \\ f\frac{\partial}{\partial z}v_1 - f\frac{\partial}{\partial x}v_3 + v_1\frac{\partial}{\partial z}f - v_3\frac{\partial}{\partial x}f \\ f\frac{\partial}{\partial x}v_2 - f\frac{\partial}{\partial y}v_1 + v_2\frac{\partial}{\partial x}f - v_1\frac{\partial}{\partial y}f \end{pmatrix}$$

$$= \begin{pmatrix} (f\frac{\partial}{\partial y}v_3 + v_3\frac{\partial}{\partial y}f) - (f\frac{\partial}{\partial z}v_2 + v_2\frac{\partial}{\partial z}f) \\ (f\frac{\partial}{\partial z}v_1 + v_1\frac{\partial}{\partial z}f) - (f\frac{\partial}{\partial x}v_3 + v_3\frac{\partial}{\partial x}f) \\ (f\frac{\partial}{\partial x}v_2 + v_2\frac{\partial}{\partial x}f) - (f\frac{\partial}{\partial y}v_1 + v_1\frac{\partial}{\partial y}f) \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial y}(fv_3) - \frac{\partial}{\partial z}(fv_2) \\ \frac{\partial}{\partial z}(fv_1) - \frac{\partial}{\partial x}(fv_3) \\ \frac{\partial}{\partial x}(fv_2) - \frac{\partial}{\partial y}(fv_1) \end{pmatrix}$$ by chain rule.

Thus, $\nabla\times(f\underset{\sim}{v}) = f(\nabla\times\underset{\sim}{v}) + (\nabla f)\times\underset{\sim}{v}$.

(b). $$\nabla\times\vec{F} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{2z}\cos y & -xe^{2z}\sin y & 2xe^{2z}\cos y \end{vmatrix} = \begin{pmatrix} -2xe^{2z}\sin y - (-2xe^{2z}\sin y) \\ 2e^{2z}\cos y - 2e^{2z}\cos y \\ -e^{2z}\sin y - (-e^{2z}\sin y) \end{pmatrix} = 0.$$

Hence, $\vec{F}$ is conservative, $\vec{F} = \nabla V$.

$\vec{F} = F_1\underset{\sim}{i} + F_2\underset{\sim}{j} + F_3\underset{\sim}{k} = \frac{\partial V}{\partial x}\underset{\sim}{i} + \frac{\partial V}{\partial y}\underset{\sim}{j} + \frac{\partial V}{\partial z}\underset{\sim}{k}$

$\frac{\partial V}{\partial x} = F_1 = e^{2z}\cos y \Rightarrow V = xe^{2z}\cos y + f(y,z)$

$\frac{\partial V}{\partial y} = F_2 = -xe^{2z}\sin y \Rightarrow V = xe^{2z}\cos y + g(x,z)$

$\frac{\partial V}{\partial y} = F_3 = 2xe^{2z}\cos y \Rightarrow V = xe^{2z}\cos y + h(x,y)$.

Thus, $V = xe^{2z}\cos y + c$.

$\int_C \vec{F}\cdot dr = V(1,2\pi,0) - V(1,0,0)$

$= (1+c) - (1+c)$

$= 0$.

(c) The surface S: $x = z^2$, $0 \le y \le 2$, $-1 \le z \le 1$

Let $z = u$, $y = v$, then $x = u^2$, $-1 \le u \le 1$, $0 \le v \le 2$.

Then $r = u^2\underset{\sim}{i} + v\underset{\sim}{j} + u\underset{\sim}{k}$.

$\underset{\sim}{r_u} = \frac{\partial r}{\partial u} = 2u\underset{\sim}{i} + \underset{\sim}{k}$

$\underset{\sim}{r_v} = \frac{\partial r}{\partial v} = \underset{\sim}{j}$

$\vec{N} = \underset{\sim}{r_u}\times\underset{\sim}{r_v} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 2u & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\underset{\sim}{i} + 2u\underset{\sim}{k}$

$\vec{F} = 3y^2\underset{\sim}{i} + ze^x\cos y\underset{\sim}{j} + 3xz^2\underset{\sim}{k} = 3v^2\underset{\sim}{i} + ue^{u^2}\cos v\underset{\sim}{j} + 3u^4\underset{\sim}{k}$

$$\iint_S \vec{F}\cdot d\vec{A} = \int_0^2\int_{-1}^1 \begin{pmatrix} 3v^2 \\ ue^{u^2}\cos v \\ 3u^4 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 2u \end{pmatrix} du\,dv$$

$= \int_0^2\int_{-1}^1 -3v^2 + 6u^5\,du\,dv$

$= \int_0^2 -6v^2\,dv$

$= -16$