NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) An equilateral triangle loop of side length a is centered at the origin in the xy plane. The triangle loop carries a uniform charge distribution with line charge density ρ_l in free space.

(i) Express the distance h between the triangle center and its base in terms of a.

(ii) Using Coulomb's law, determine the electric field intensity $\vec{E}(z)$ along the z axis due to the triangle loop.

Note: $$\int \frac{1}{\left(x^2+u^2\right)^{3/2}}dx = \frac{x}{u^2\sqrt{x^2+u^2}}$$

(14 Marks)

Note: Question No. 1 continues on page 2.

(b) Let the charges on the triangle loop of part (a) be moving to form a steady current I in the counter-clockwise direction (as viewed from $z > 0$). Determine the magnetic field intensity $\vec{H}(z)$ along the z axis due to the triangle loop current.

(11 Marks)

2. (a) A circular loop of radius a containing two series resistors with resistance values of R_1 and R_2 is centered at the origin in the xy plane in free space. The loop is subjected to a time-varying magnetic field intensity of the form (for time $t \geq 0$)

$$\vec{H} = \exp(-t)\vec{a}_x + \cos(t)\vec{a}_y + \sin(t)\vec{a}_z \ \ \text{A/m}.$$

(i) Derive the magnetic flux Φ_m passing through the loop and the induced voltage V_{emf} at time $t \geq 0$.

(ii) Derive the induced current I as well as the voltages V_1 and V_2 across the resistors at time $t \geq 0$. Sketch a diagram to indicate the current direction and label the voltage polarities assumed in your answers.

(12 Marks)

(b) A 100 MHz plane wave is propagating in a lossy medium with relative permittivity $\varepsilon_r = 10$, conductivity $\sigma = 2$ S/m and relative permeability $\mu_r = 1$.

(i) Using the good conductor approximation, determine the skin depth δ and loss tangent $\tan\delta$. Discuss the physical significance of both δ's.

(ii) Without using the good conductor approximation, calculate the propagation constant γ and the values of both δ's.

(13 Marks)

3. (a) A 60 MHz uniform plane wave (UPW) in free space occupying the region $z \leq 0$ is given by:

$$\vec{E}_i(z,t) = \vec{a}_y \; 0.3 \cos\left(\omega t - k_i z + \frac{\pi}{3}\right) \; \text{V/m}.$$

The UPW is incident normally on a planar interface with a lossy medium having $\mu = \mu_o$, $\varepsilon = 4.5\varepsilon_o$ and $\sigma = 0.9$ S/m occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) ω and k_i.

(ii) The attenuation constant α for the lossy medium.

(iii) The position z at which the average power density of the transmitted wave drops to 5% of its value at $z = 0$.

(13 Marks)

(b) The magnetic field intensity of a uniform plane wave (UPW) propagating in free space $(z \leq 0)$ is given by:

$$\vec{H}_i(x,z) = \left(-5\vec{a}_x + 12.5\vec{a}_z\right) e^{-j(10x+4z)} \; \text{mA/m}.$$

The UPW is obliquely incident on a second medium made of lossless dielectric having $\mu = \mu_o$ and $\varepsilon = 1.4\varepsilon_o$ at $z = 0$, and occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) The electric field intensity of the incident UPW, i.e., $\vec{E}_i(x,z)$.

(ii) The magnitude of transmitted magnetic field intensity in the second medium, i.e., H_{ot}.

(12 Marks)

4. (a) A 12.5-cm long lossless transmission line operating at a frequency of 1.2 GHz has a characteristic impedance $Z_0 = 100\ \Omega$ and a phase velocity $u_p = 2.25 \times 10^8$ m/s. The line is terminated in a load $Z_L = 70 + j60\ \Omega$.

Assume that the load end is located at $z = 0$ and the source end at $z = -\ell$, where ℓ is the length of the transmission line.

Find the following and state any assumption(s) made:

(i) The wavelength λ on the transmission line.

(ii) The reflection coefficient $\Gamma(z)$ in polar form at $z = 0$ and $z = -\ell$.

(iii) The position z at which the magnitude of current on the line is maximum.

(iv) The average power delivered to the load if the magnitude of maximum current on the line is $|I|_{\max} = 5$ A.

(20 Marks)

(b) An unknown load Z_L' is connected to the transmission line in part (a) and it causes a standing wave ratio (SWR) of 3.3 and a 180° phase shift on the reflected voltage wave. Determine the unknown load Z_L'.

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}\ \mathrm{F/m}$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{Id\vec{l} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{Id\vec{l} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\| = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\| = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \, e^{-j\beta z} + V_o^- \, e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \, e^{-j\beta z} - V_o^- \, e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

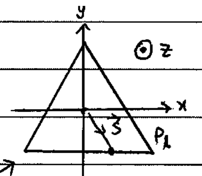
$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

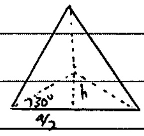
1) a)



(not to scale)

Analyse this portion only!

i)



An equilateral triangle has an interior angle of 60°

$h = \frac{a}{2}\tan 30^\circ = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{\sqrt{3}}{6}a$ #

ii) $\vec{r} = z\vec{a}_z$; $\vec{s} = x\vec{a}_x - h\vec{a}_y$ (for base portion only)

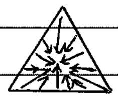
$\vec{R} = \vec{r} - \vec{s} = -x\vec{a}_x + h\vec{a}_y + z\vec{a}_z \Rightarrow R = \sqrt{x^2+h^2+z^2}$

$$\vec{E}_{base}(z) = \frac{1}{4\pi\varepsilon_0}\int_{-a/2}^{a/2} \frac{\rho_l \vec{R}}{R^3}\,dl \quad \leftarrow dQ = \rho_l\,dl$$

ε_0 as it is in free space.

$$= \frac{\rho_l}{4\pi\varepsilon_0}\int_{-a/2}^{a/2} \frac{-x\vec{a}_x + h\vec{a}_y + z\vec{a}_z}{(x^2+h^2+z^2)^{3/2}}\,dl \text{ V/m, where } h = \frac{\sqrt{3}}{6}a$$

For the whole triangle, the x- and y- components of the electric field cancel out as shown below.

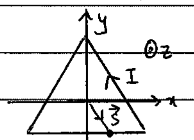


$$\therefore \vec{E}_{triangle}(z) = \frac{3\rho_l z}{4\pi\varepsilon_0}\vec{a}_z \int_{-a/2}^{a/2} \frac{1}{(x^2+h^2+z^2)^{3/2}}\,dl$$

$$= \frac{3\rho_l z}{4\pi\varepsilon_0}\vec{a}_z \left[\frac{x}{(h^2+z^2)\sqrt{x^2+(h^2+z^2)}}\right]_{-a/2}^{a/2}, \text{ where } h = \frac{\sqrt{3}}{6}a$$

$$= \frac{3\rho_l z}{4\pi\varepsilon_0}\vec{a}_z \cdot \frac{a}{(z^2+\frac{1}{12}a^2)\sqrt{z^2+\frac{1}{3}a^2}} \text{ V/m} \;\#$$

1) b)



Analyse base only

$\vec{r} = z\vec{a}_z$; $\vec{s} = x\vec{a}_x - h\vec{a}_y$ (for base portion only)

$\vec{R} = \vec{r} - \vec{s} = -x\vec{a}_x + h\vec{a}_y + z\vec{a}_z \Rightarrow R = \sqrt{x^2+h^2+z^2}$

} same as (a) $\quad h = \frac{\sqrt{3}}{6}a$ from (a)

$$\vec{H}_{base}(z) = \frac{1}{4\pi}\int_{-a/2}^{a/2} \frac{I(\vec{a}_x\,dl)\times\vec{R}}{R^3} \quad \leftarrow d\vec{l} = \vec{a}_x\,dl$$

$$= \frac{I}{4\pi}\int_{-a/2}^{a/2} \frac{\vec{a}_x \times (-x\vec{a}_x + h\vec{a}_y + z\vec{a}_z)}{(x^2+h^2+z^2)^{3/2}}\,dl$$

$$= \frac{I}{4\pi}\int_{-a/2}^{a/2} \frac{h\vec{a}_z + z\vec{a}_y}{(x^2+h^2+z^2)^{3/2}}\,dl \text{ A/m}$$

For the whole triangle, the x- and y-component of the magnetic field will cancel out as shown earlier.

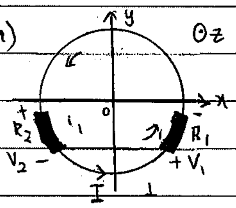
$$\therefore \vec{H}_{triangle}(z) = \frac{3Ih}{4\pi}\vec{a}_z \int_{-a/2}^{a/2} \frac{1}{(x^2+h^2+z^2)^{3/2}}\,dl$$

$$= \frac{3Ih}{4\pi}\vec{a}_z \left[\frac{x}{(h^2+z^2)\sqrt{x^2+(h^2+z^2)}}\right]_{-a/2}^{a/2}, \text{ where } h = \frac{\sqrt{3}}{6}a$$

$$= \frac{3Ih}{4\pi}\vec{a}_z \cdot \frac{a}{(z^2+\frac{1}{12}a^2)\sqrt{z^2+\frac{1}{3}a^2}}$$

$$= \frac{\sqrt{3}\,Ia^2}{8\pi(z^2+\frac{1}{12}a^2)\sqrt{z^2+\frac{1}{3}a^2}} \text{ A/m}^2 \;\#$$

2) a) $\odot z$ Take $\vec{a}_N = \vec{a}_z$ and the contour direction to be counter-clockwise (CCW).



i) $\Phi_m = \iint_S \vec{B}\cdot d\vec{S}$ [μ_0 − free space]

$= \int_0^{2\pi}\int_0^a \mu_0 (e^{-t}\vec{a}_x + \cos t\,\vec{a}_y + \sin t\,\vec{a}_z)\cdot dr(a d\phi)\vec{a}_z$

$= \int_0^{2\pi}\int_0^a \mu_0 a \sin t\, dr\, d\phi$

$= \int_0^{2\pi} \frac{1}{2}\mu_0 a^2 \sin t\, d\phi$

$= \mu_0 \pi a^2 \sin t$ Wb #

emf $= -\frac{d}{dt}\Phi_m = -\mu_0\pi a^2 \cos t$ V #

ii) $I = \frac{emf}{R_1+R_2} = -\frac{\mu_0 \pi a^2}{R_1+R_2}\cos t$ A #

$V_1 = \frac{R_1}{R_1+R_2}\times emf = -\frac{\mu_0\pi a^2 R_1}{R_1+R_2}\cos t$ V # Diagram see above ...

$V_2 = \frac{R_2}{R_1+R_2}\times emf = -\frac{\mu_0\pi a^2 R_2}{R_1+R_2}\cos t$ V #

2) b) i) loss tangent $= \frac{\sigma}{\varepsilon\omega} = \frac{2}{10\varepsilon_0(2\pi\times100\times10^6)} = 35.95 > 20$ (good conductor approximation can be used!)

$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f\mu\sigma}} = \frac{1}{\sqrt{\pi(100\times10^6)(\mu_0)(2)}} = 0.03559$ m #

This δ is the depth at which the 100MHz can penetrate before, the $\vec{E}$ field attenuates ~~attenuating~~ to a magnitude of $\frac{1}{e}$ of its original value.

$\tan\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{J_c}{J_d}$ { The argument of the loss tangent is δ. Loss tangent is the ratio of imaginary and real parts of complex permittivity ε_c, or the ratio of conduction current density and displacement current density, or the dissipation loss when the EM wave is propagating.

* Not sure for this

ii) $\gamma = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)} = j(2\pi\times100\times10^6)\sqrt{\mu_0[10\varepsilon_0 - j2/(2\pi\times100\times10^6)]} = j(200\pi\times10^6)\sqrt{(1.11265-j40)\times10^{-16}}$

$= j(200\pi\times10^6)(4.53476-j4.410375)\times10^{-8} = 27.71 + j28.49$ #

* Use calc $\sqrt{Abs(Ans)}\angle(Arg(Ans)\div2)$

Skin depth, $\delta = \frac{1}{\alpha} = \frac{1}{27.71} = 0.03609$ m #

loss tangent, $\tan\delta = \frac{\sigma}{\varepsilon\omega} = 35.95$ (from b(i)) $\Rightarrow \delta = 1.543$ rad # (not sure)

3) a) i) For free space, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\eta_1 = 120\pi\,\Omega$, $u_p = c$

$\omega = 2\pi\times60\times10^6 = 3.77\times10^8$ rad s^{-1} #

$u_p = c = \frac{\omega}{k} \Rightarrow k_1 = \frac{\omega}{c} = 0.4\pi = 1.2566$ rad/m #

ii) loss tangent $= \frac{\sigma}{\varepsilon\omega} = \frac{0.9}{4.5\varepsilon_0(3.77\times10^8)} = 59.916 > 20 \Rightarrow$ Assume good conductor

$\therefore \alpha \approx \sqrt{\pi f\mu\sigma} = \sqrt{\pi(100\times10^6)(\mu_0)(0.9)} = 18.85$ Np/m #

iii) $P(z) = P(0)e^{-2\alpha z}$

$0.05 = e^{-2\alpha z}$

$z = -\frac{1}{2}\ln 0.05 = 1.498$ m #

3) b) i) $\vec{H}_i(x,z)$ only has x- and z-components (ie x-z plane) $\Leftrightarrow$ $\vec{E}_i(x,z)$ will have y-components $\Rightarrow$ perpendicularly polarised

$\eta_1 = 120\pi\,\Omega$ (free space) $\qquad \vec{E}_i(x,z) = \eta_1 \vec{H}_i(x,z) \times \vec{a}_k \leftarrow$ Find this!

$\vec{a}_k = \frac{\vec{\beta}}{k} = \frac{1}{\sqrt{10^2+4^2}}(10\vec{a}_x + 4\vec{a}_z) = \frac{1}{\sqrt{29}}(5\vec{a}_x + 2\vec{a}_z)$ rad/m

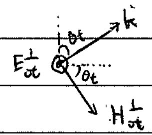
$\therefore \vec{E}_i(x,z) = 120\pi(-5\vec{a}_x + 12.5\vec{a}_z)e^{-j(10x+4z)} \times \frac{1}{\sqrt{29}}(5\vec{a}_x + 2\vec{a}_z) \times 10^{-3}$

$= 5.0754\,\vec{a}_y\, e^{-j(10x+4z)}$ V/m #

Alternatively, $\vec{E}_i(x,z) = \eta_1 \vec{a}_y |H_{0i}^{\perp}| e^{-j(10x+4z)}$ ← $\vec{a}_y$ because perpendicularly polarised w.r.t. incidence plane.

$= (120\pi)\vec{a}_y \sqrt{5^2 + 12.5^2}\,(10^{-3})e^{-j(10x+4z)} = 5.0754\,\vec{a}_y\, e^{-j(10x+4z)}$ V/m #

ii) $\eta_1 = 120\pi\,\Omega$, $\eta_2 = \sqrt{\frac{\mu_r}{\varepsilon_r}}(120\pi) = 120\pi\sqrt{\frac{1}{1.4}} = 318.616\,\Omega$



$\tan\theta_i = \frac{k_{ix}}{k_{iz}} \Rightarrow \theta_i = \tan^{-1}\left(\frac{10}{4}\right) = 68.2^\circ$ $\qquad \vec{a}_H = -\sin\theta_t\,\vec{a}_x + \cos\theta_t\,\vec{a}_z$

$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \Rightarrow \theta_t = \sin^{-1}\left(\sqrt{\frac{(1)(1)}{(1)(1.4)}}\sin 68.2^\circ\right) = 51.7^\circ$

$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = 0.67234$ $\qquad E_{t0}^{\perp} = \tau_\perp E_{i0}^{\perp} = 0.67234(5.0754) = 3.4124$ V/m

$\vec{k}_t = k_{xi}\vec{a}_x + \frac{k_{xi}}{\tan\theta_t}\vec{a}_z = 10\vec{a}_x + 7.898\vec{a}_z$ rad/m

extra! $\therefore \vec{H}_t(x,z) = \frac{1}{\eta_2}(3.4124)(-\sin\theta_t\,\vec{a}_x + \cos\theta_t\,\vec{a}_z)e^{-j(10x+7.898z)}$

$= (-8.405\,\vec{a}_x + 6.638\,\vec{a}_z)e^{-j(10x+7.898z)} \times 10^{-3}$ A/m

$H_{ot} = \frac{3.4124}{\eta_2}(-\sin\theta_t\,\vec{a}_x + \cos\theta_t\,\vec{a}_z)$

$= (-8.405\,\vec{a}_x + 6.638\,\vec{a}_z) \times 10^{-3}$ A/m #

[only perpendicular component!]

4) a) i) $\omega = 2\pi \times 1.2 \times 10^9 = 2.4\pi \times 10^9$ rad s^{-1}

$u_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\omega}{u_p} = \frac{2.4\pi \times 10^9}{2.25 \times 10^8} = 33.51$ rad m^{-1} $\quad \therefore \lambda = \frac{2\pi}{\beta} = 0.1875$ m #

ii) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(70+j60)-100}{(70+j60)+100} = 0.3721\angle 97.1^\circ$

$\Gamma_L(0) = \Gamma_L = 0.3721\angle 97.1^\circ$ # ; $\Gamma_L(-\ell) = \Gamma_L e^{+j2(33.51)(-0.125)} = 0.3721\angle -22.9^\circ$ #

iii) $z_{min} \Rightarrow |V|_{min}$ & $|I|_{max}$ $\qquad \theta_0 = 97.1^\circ = 1.6947$ rad

$\theta_0 + 2\beta z_{min} = -\pi \Rightarrow z_{min} = -7.22$ cm #

iv) $Z_{in}(-0.0722) = \frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)} Z_0 = 45.76 - j0.0178\,\Omega \approx 45.76\,\Omega$

* $Z_{in}(-z_{min})$ should be purely resistive, but due to rounding error, it is not – hence ignore the imaginary part

$P_{\text{transmission line}} = \frac{1}{2}|I|^2 R = \frac{1}{2}(5)^2(45.76) = 572$ W (constant throughout the transmission line)

$P_L = P_{\text{transmission line}} = 572$ W #

4) b) $\frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 3.3 \Rightarrow |\Gamma_L| = \frac{23}{43}$ $\qquad$ phase shift $180^\circ \Rightarrow \angle|\Gamma_L| = 180^\circ$

$\Gamma_L = \frac{23}{43}\angle 180^\circ = -\frac{23}{43}$

$\frac{Z_L' - 100}{Z_L' + 100} = -\frac{23}{43} \Rightarrow Z_L' = 30.3\,\Omega$ #

Good luck!