Circuit Analysis
EE2001

NANYANG TECHNOLOGICAL UNIVERSITY

Basic Concepts, Basic Laws and Method of Analysis (Part 1)

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Welcome

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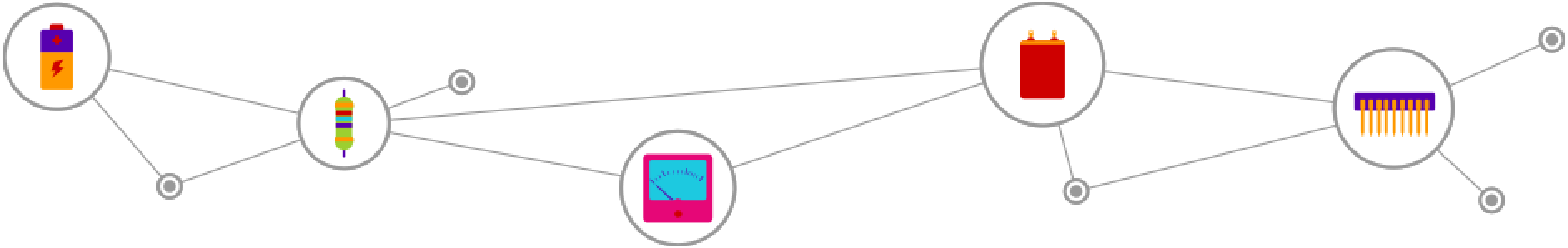
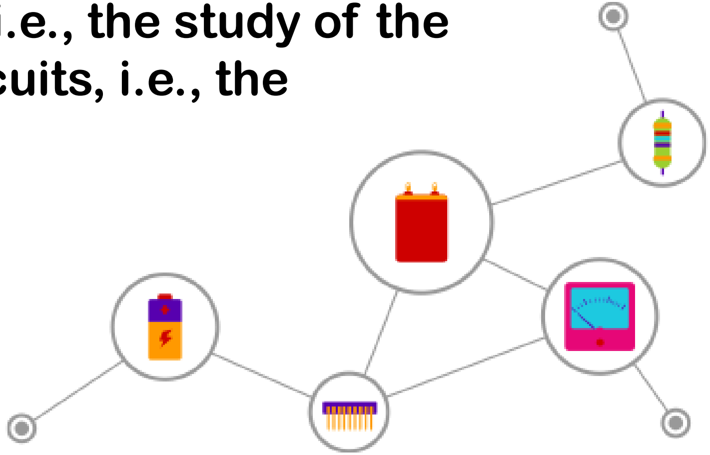


Course Learning Objectives

- This course focuses on the fundamental principles of circuit theorems and circuit elements, DC/AC and three-phase circuits, transient and steady-state responses, and circuit analysis using Laplace Transform.

- In this course, we will learn various techniques (“tools”) to analyse the operation of real circuits.

- Our major concern is the analysis of circuits, i.e., the study of the behavior of the circuit, not the creation of circuits, i.e., the engineering design of the circuit.

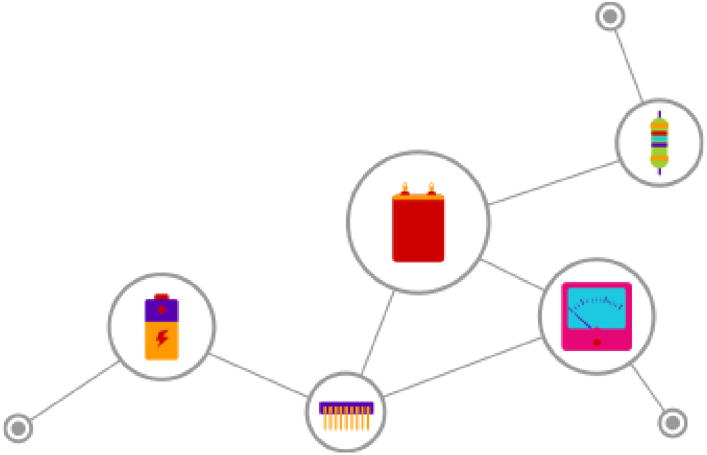


Course Learning Outcomes

- Apply the various techniques learnt to strengthen analytical skills of DC and AC linear circuits.

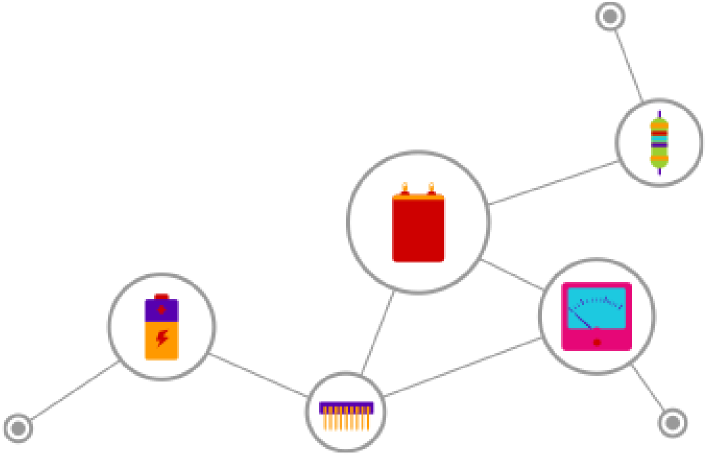
- Set-up independent equations of linear circuits and solve them using the techniques and skills acquired.

- Apply the knowledge learnt in this course for the study of linear control systems, power networks, electronics and communications systems in later years.



By the end of this lesson, you should be able to…

- Describe the key characteristics of voltage, current and power.
- Describe the key characteristics of passive and active elements.
- Explain the basic laws of circuit analysis: ohm’s law, Kirchhoff’s current law and Kirchhoff’s voltage law.
- Analyse circuits using the basic laws.
- Analyse circuits using nodal analysis.



Introduction

Electric circuit theory is one of the fundamental theories upon which all branches of electrical engineering are built.

Many branches of electrical engineering, such as power, electric machines, control, electronics, communications and instrumentation are based on electric circuit theory.

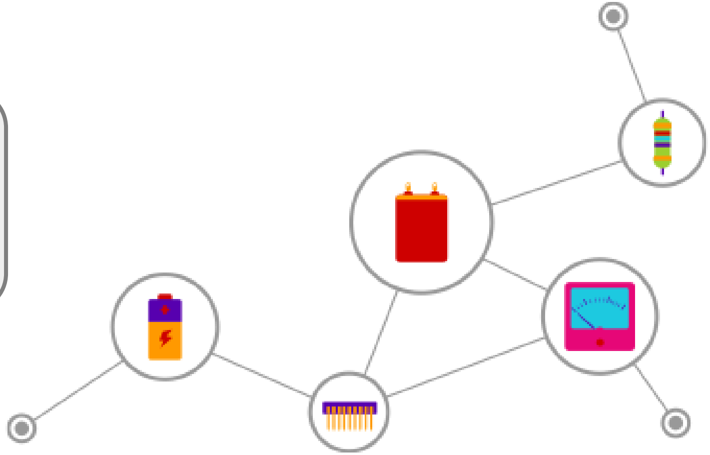
In electrical engineering, we are often interested in transferring energy from one point to another.

Introduction

An interconnection of electrical devices is required for **transfer of energy**.

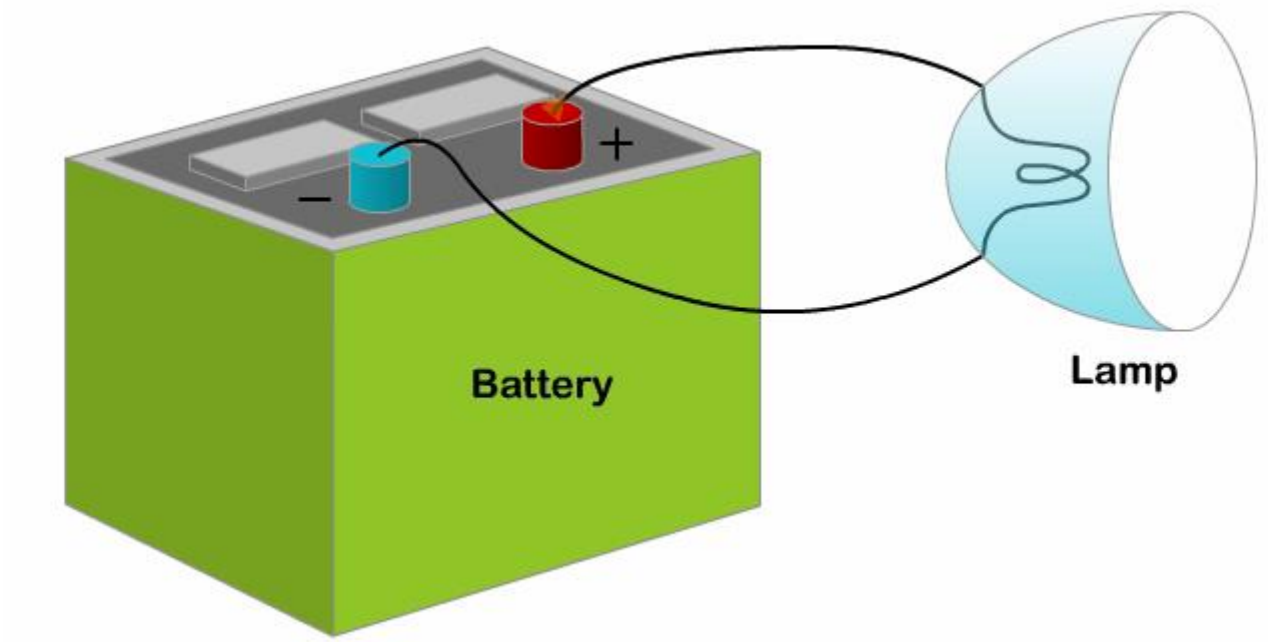
Such interconnection is referred to as an **electric circuit**. And each component of the circuit is known as an **element**.

An electric circuit is an interconnection of electric elements.

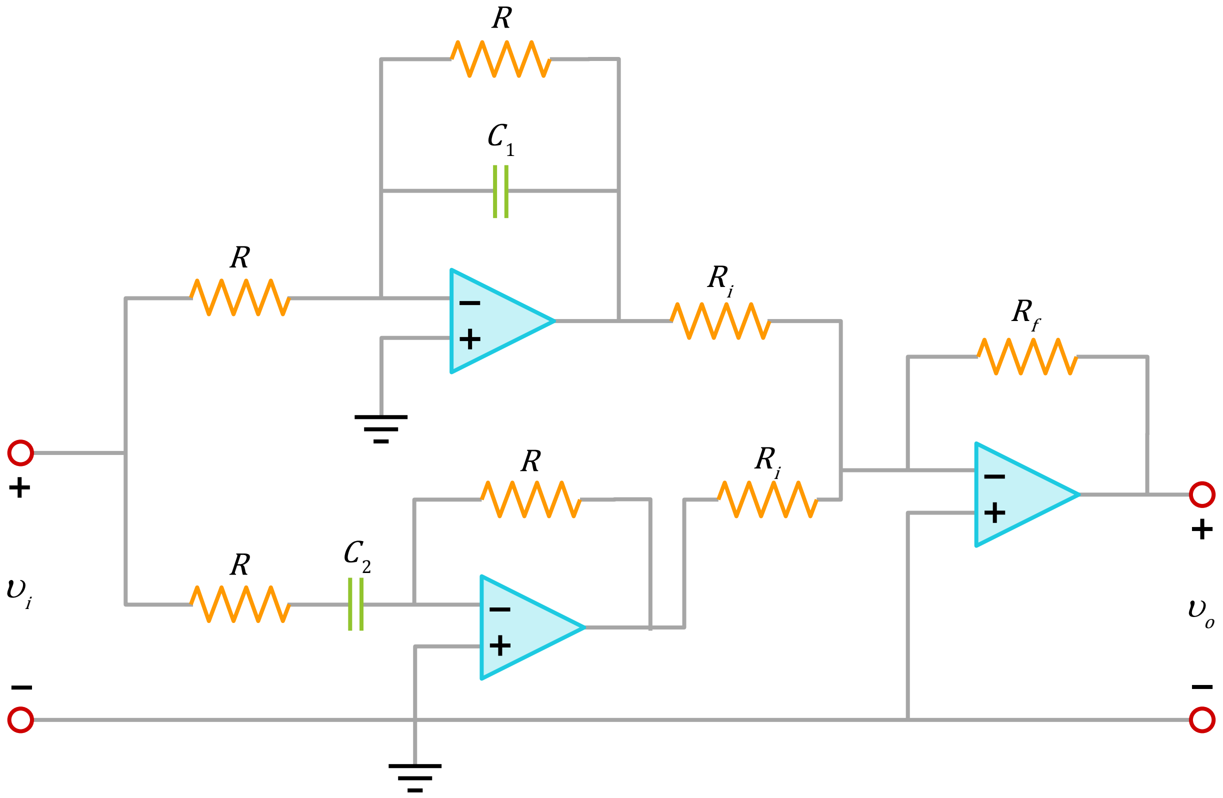


A Simple Circuit Diagram

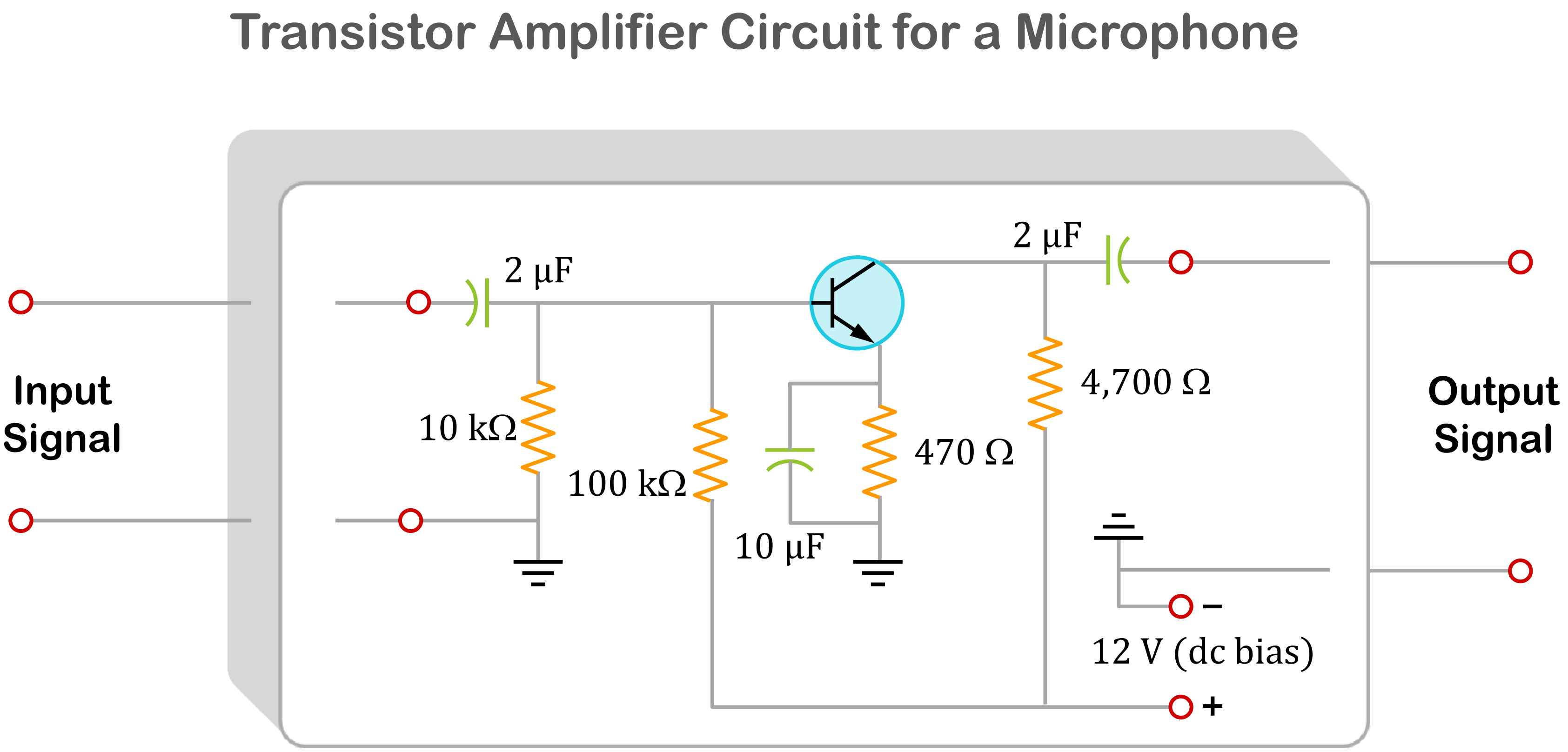
An electric circuit consists of three basic elements: a battery, a lamp and some connecting wires.

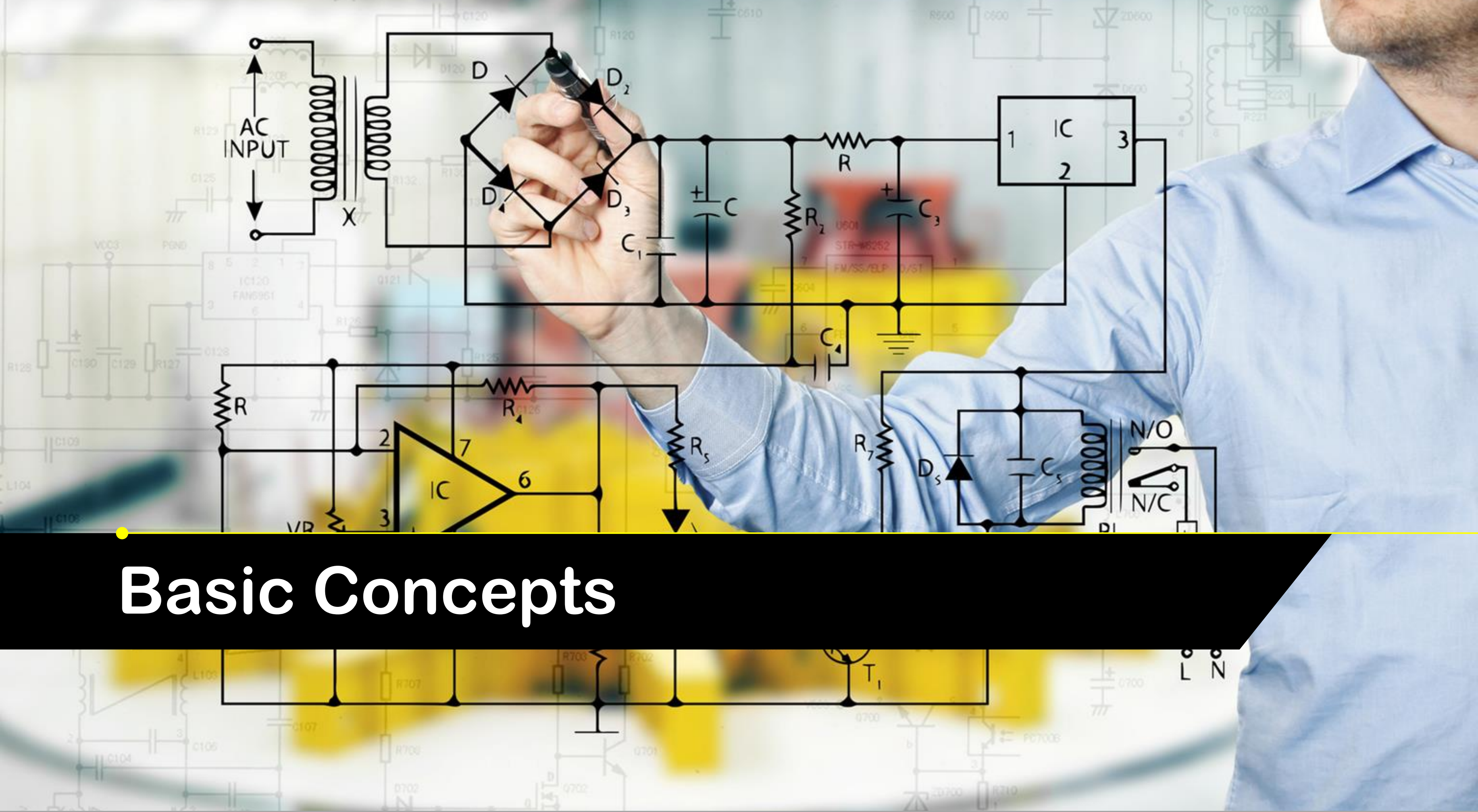


A Notch Filter with Op-amps



An Amplifier Circuit for a Microphone





Basic Concepts

Introduction

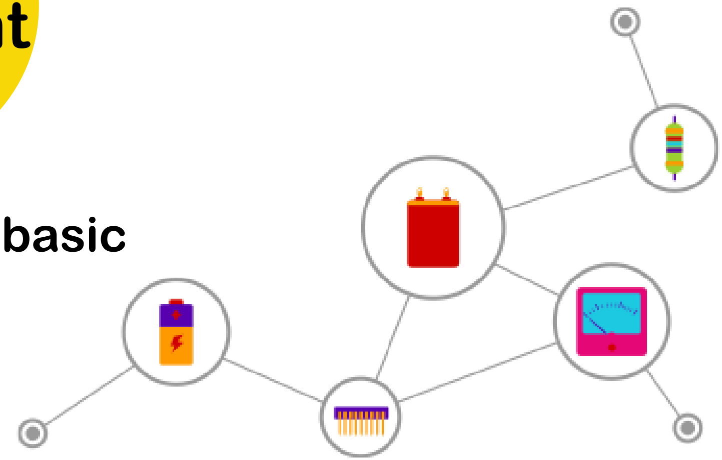
In carrying out circuit analysis, we often deal with

Power

Voltage

Current

Electric charge and its movement are the most basic items of interest in electrical engineering.



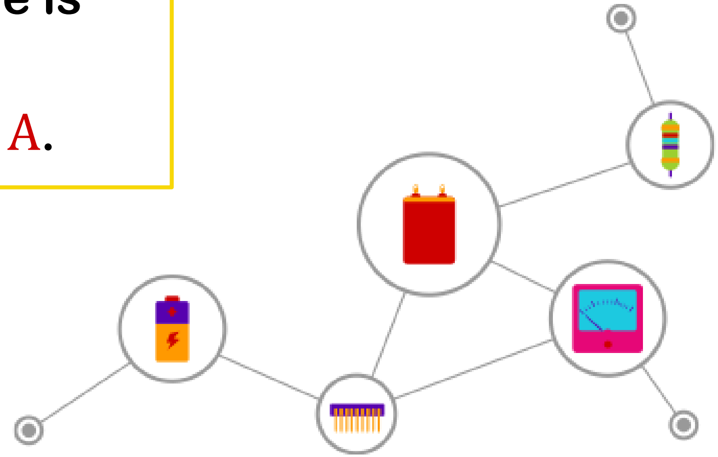
Concept of Electric Charge and Current

Electric Charge: An electric property of matter, measured in coulombs. Like charges repel and unlike charges attract each other. The magnitude of the electron's charge is 1.602×10^{-19} coulomb (unit is C).

Current

When there is a net flow of charge across any area, we say that there is a current across the area.

Current is measured in amperes, A.



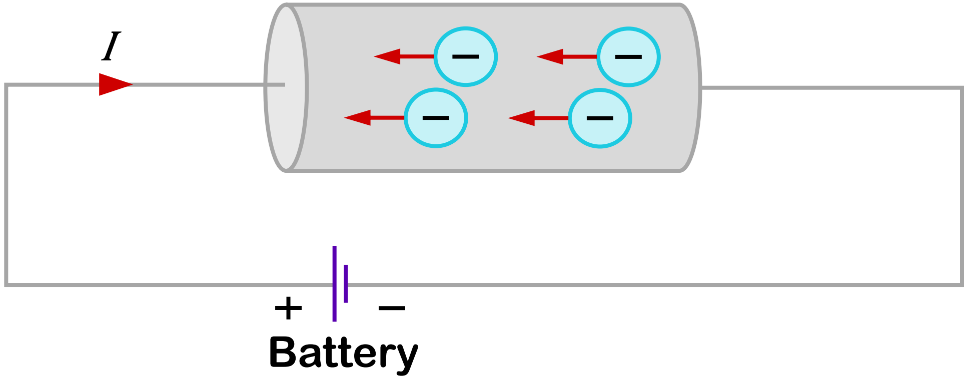
Relationship Between Charge and Current

One ampere ($1\ \mathrm{A}$) is the movement of 1 C of charge through a cross section of a conductor in 1 sec.

The relationship between current i, charge q and time t is

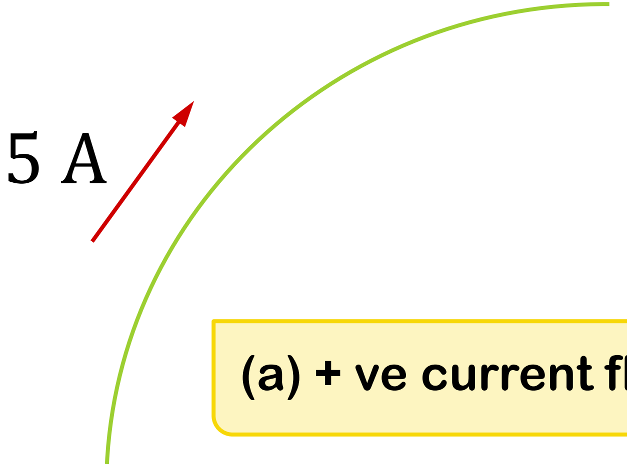
$$\boxed{i = \frac{dq}{dt}}$$

The direction of current flow is taken by convention as opposite to the direction of electron flow.

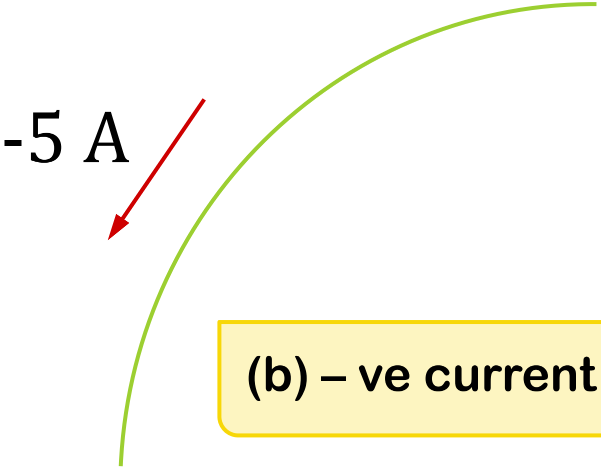


Two Methods of Representing a Current

Conventional current flow:



(a) + ve current flow



(b) – ve current flow

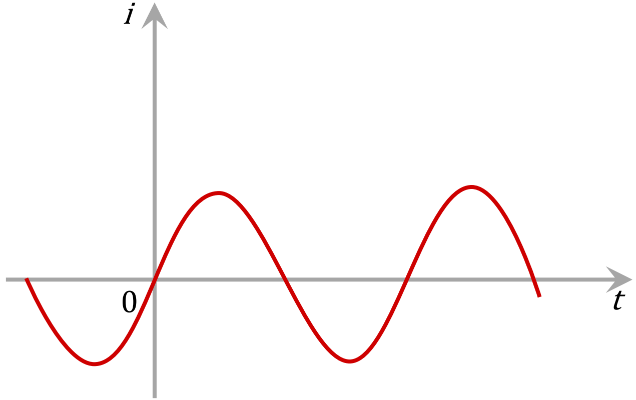
Type of Current

Direct Current (DC)

A **Direct Current** (DC) is a current that remains constant with time and is denoted by I.

A common source of DC is a battery.

Alternating Current (AC)



An **Alternating Current** (AC) is a current that varies sinusoidally with time and is denoted by i.

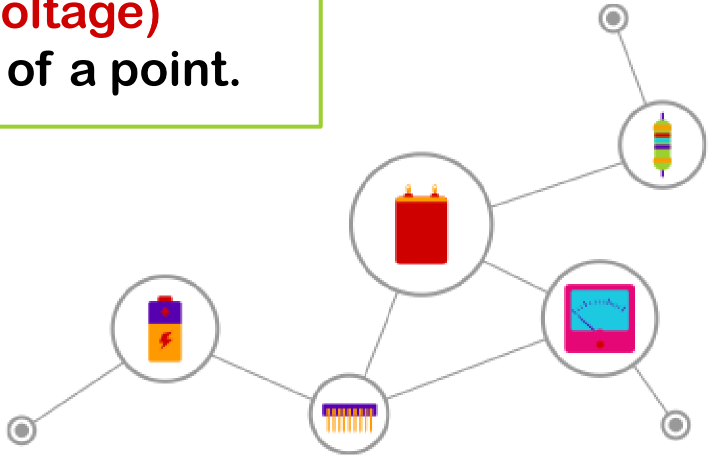
The mains power is an example of AC.

Concept of Voltage

To move an electron in a conductor in a particular direction requires some work or energy transfer.

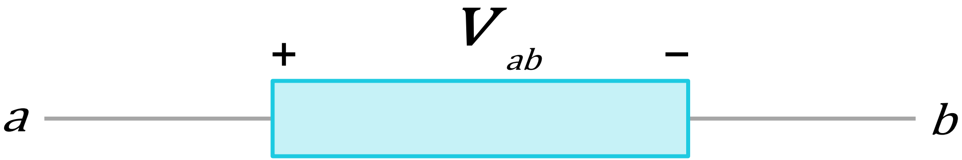
This work is performed by an external electromotive force (emf) which is known as a potential difference or voltage (measured in volts, V).

We are interested in the potential difference (voltage) between two points, not the absolute potential of a point.



Concept of Voltage

The voltage v_{ab} between two points, a and b, in a circuit is the energy (or work) needed to move a unit charge from a to b.



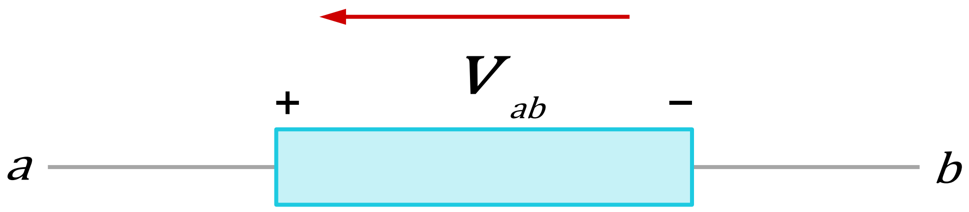
The relationship between the energy w (in joules, J) and the charge q (in C) is

$$v_{ab} = \frac{dw}{dq}$$

One V = One J/C

Concept of Voltage

The + and – signs are used to designate which point is at the assumed higher potential (the + point).

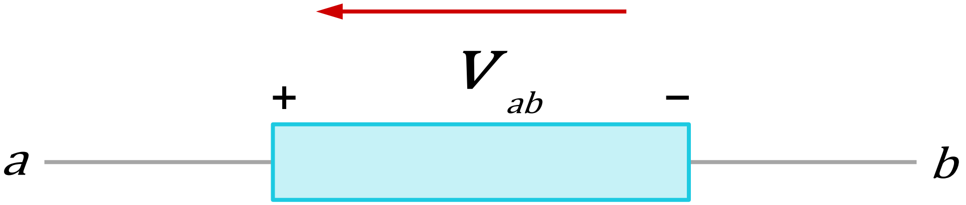


The v_{ab} can be interpreted as follows: the potential is at a higher potential of v_{ab} volts higher than point b.

An arrow is used to point to the terminal of assumed higher potential (the + point).

Concept of Voltage

Suppose $v_{ab} = 9\ V$



We may say that there is a $9\ \mathrm{V}$ **voltage rise** from b to a.

Or equivalently a $9\ \mathrm{V}$ **voltage drop** from a to b.

Note that $v_{ab} = -\,v_{ba}$

Type of Voltage

DC Voltage

A constant voltage is called a DC voltage and is represented by V.

A DC voltage is commonly produced by a battery.

AC Voltage

A sinusoidally time-varying voltage is called an AC voltage and is represented by v.

An AC voltage is produced by an electric generator.

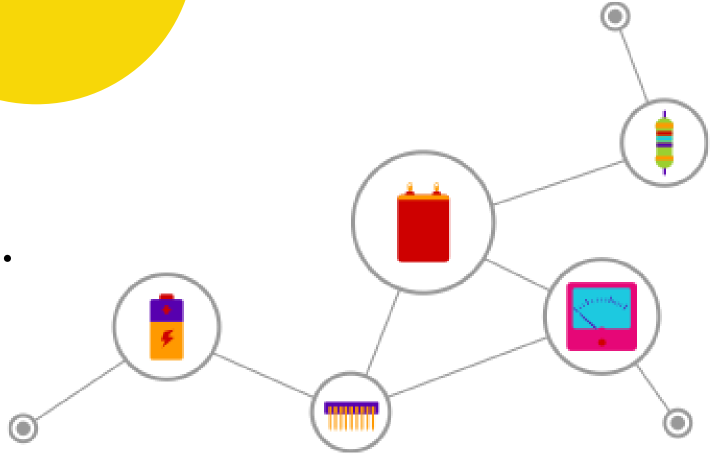
Note: electric voltage is always across the element or between two points.

Concept of Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient for circuit analysis.

Power **V**oltage **C**urrent

Power and energy calculations are also needed.



Concept of Power and Energy

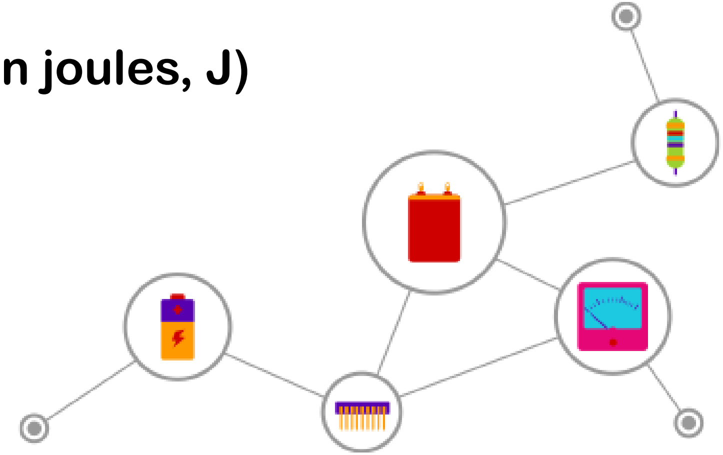
Power

Power is the rate of absorbing or supplying energy.

It is measured in Watts (W).

The relationship between power p, energy w (in joules, J) and time t (in sec) is

$$\boxed{p = \frac{dw}{dt}}$$



Relationship Between Power, Voltage and Current

The relationship between power p, voltage v and current i is given by,

$$\boxed{p = vi}$$

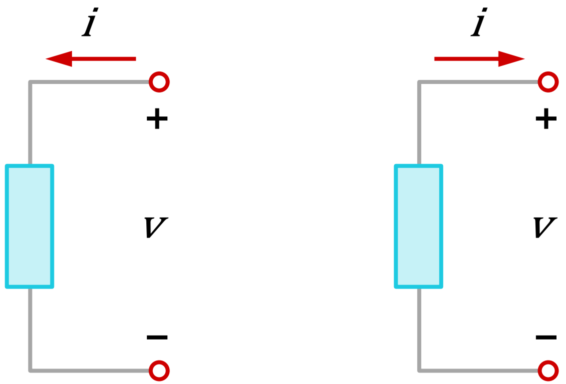
The power **absorbed** or **supplied** by an element is the product of voltage across the element and the current passing through it.

Passive Sign Convention

By convention, we say that an element being supplied power has positive power.

A power source such as a battery has negative power.

Passive sign convention is satisfied if the direction of current is selected such that current enters through the +ve terminal that is more positively biased.



$p = +vi$

(a)

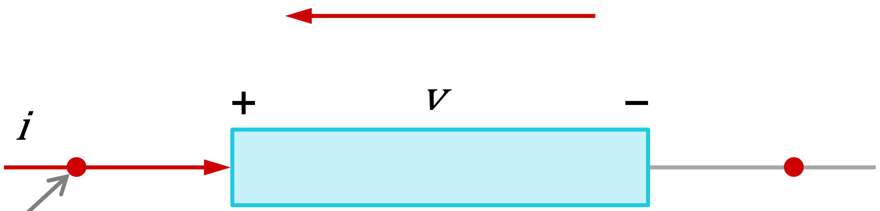
$p = v(-)i \Rightarrow p = -vi$

(b)

Passive Sign Convention

Consider the element (represented by a block) as shown.

It has two terminals (also called nodes).

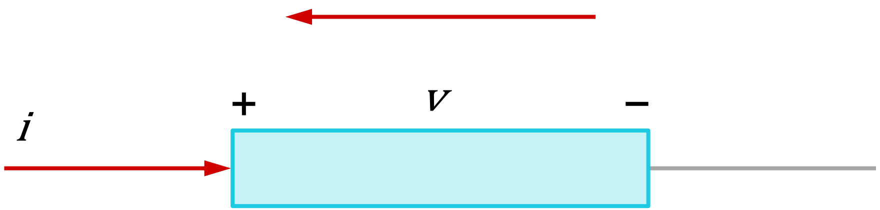


It conducts current from one node to the other and in the process, voltage drop occurs across the element in the direction of current flow (shown by arrow).

Passive Sign Convention

The terminal at which the current enters acquires +ve polarity with respect to the terminal at which the current exits.

We assume that the current enters the terminal of higher potential.

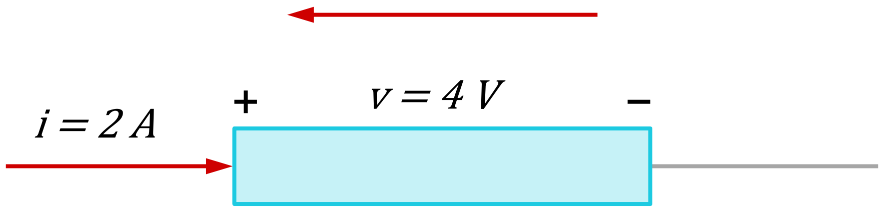


In the passive sign convention, the word "passive" means that the element is assumed to absorb power.

It is assumed that the power has a +ve sign when the current enters the +ve polarity of the voltage across an element.

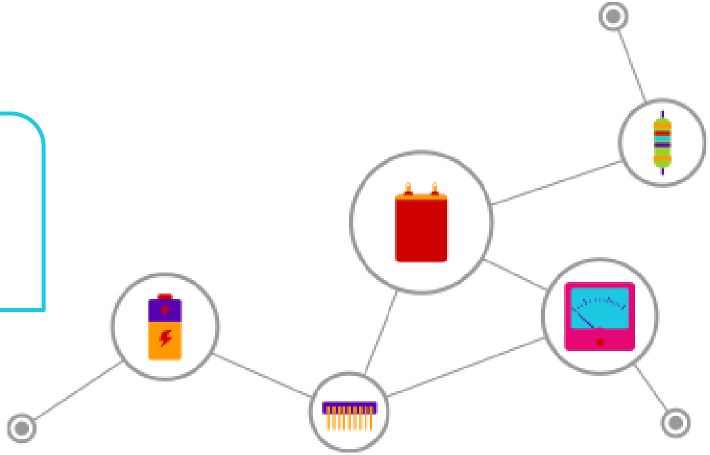
Element Absorbing Power

The (actual) current flows into the + ve terminal of the element:



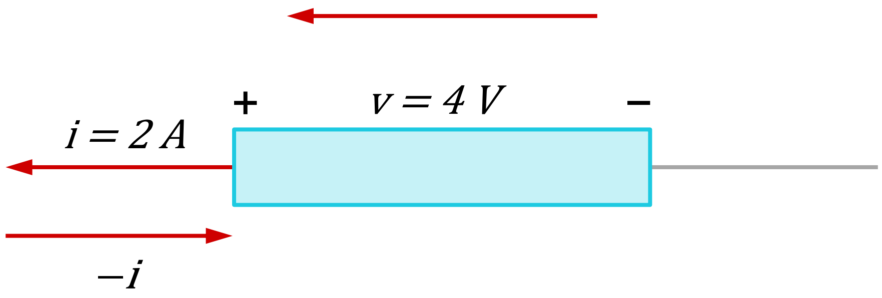
$$p = vi = (4)(2) = +8\,W$$

The element is **absorbing** $8\,W$ of power.



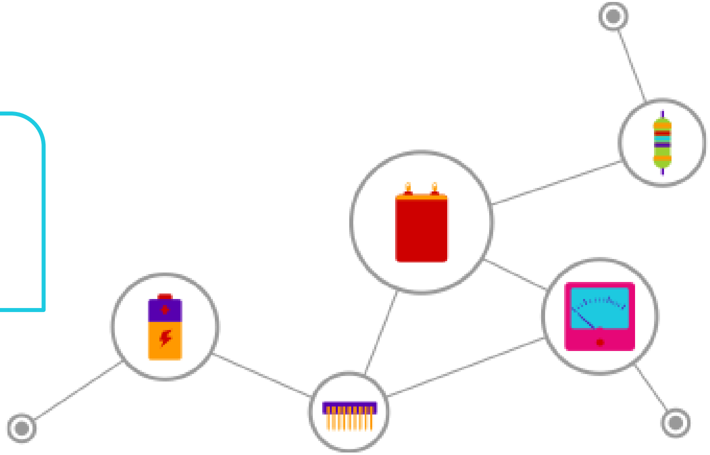
Element Delivering Power

The (actual) current flows out from the + ve terminal of the element:



$$p = v(-i) = (4)(-2) = -8\ W$$

The element is **delivering** $8\ W$ of power.



Basic Circuit Elements

An element is the basic building block of a circuit.

An electric circuit is simply an interconnection of the elements.

Circuit analysis is the process of determining voltages across (or the current through) the elements of the circuit.

Passive and Active Elements

There are two types of elements found in electric circuits, passive and active elements.

Passive

A passive element is not capable of generating energy.

Examples of passive elements: resistors, capacitors and inductors.

Active

An active element is capable of generating energy.

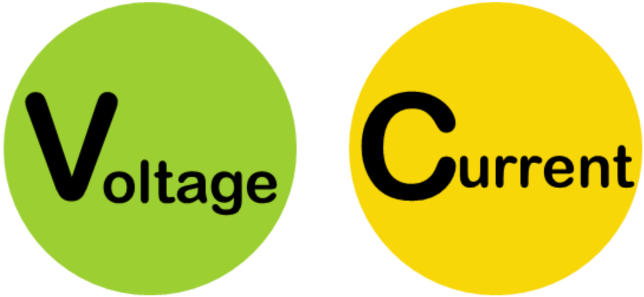
Typically, active elements include generators, batteries and operational amplifiers.

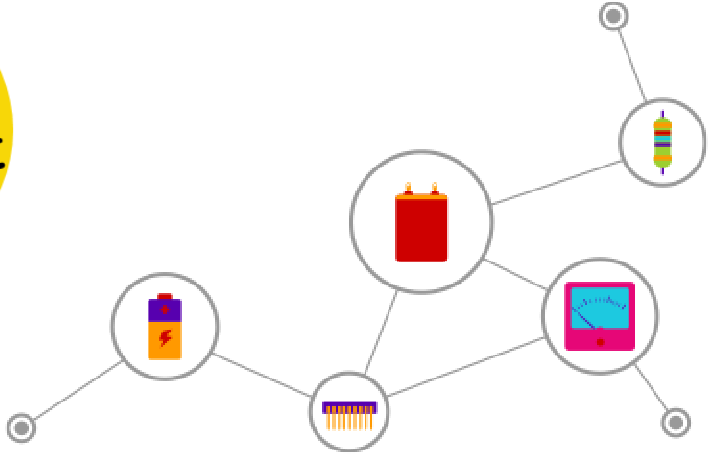
The most important active elements are voltage or current sources that deliver power to the circuit.

There are two kinds of sources: independent and dependent sources.

Active Element: Independent Source

An **ideal independent source** is an active element that provides a specified **voltage** or **current** that is completely independent of other circuit elements.





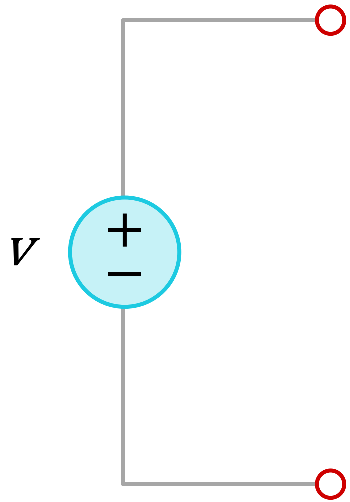
Independent Voltage Source

An ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage.

Practical sources, such as batteries and generators, are regarded as approximations to ideal voltage sources.

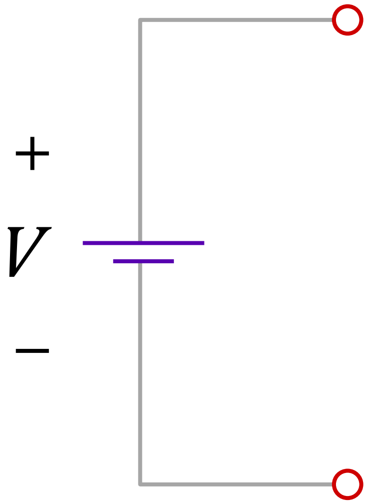
A voltage function and a polarity must be specified to describe a voltage source.

Symbols for Independent Voltage Source



(a)

Used for constant or time-varying voltage



(b)

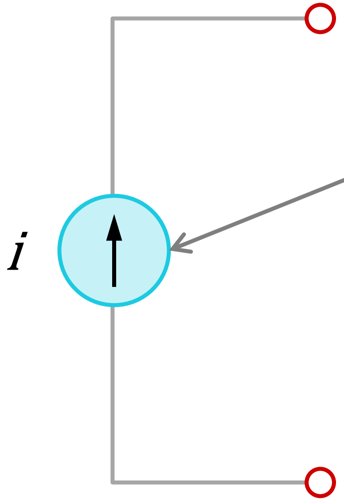
Used for constant voltage (DC)

Independent Current Source

An ideal independent current source is an active element that provides a specified current, completely independent of the voltage across the source.

The current source delivers to the circuit whatever voltage is necessary to maintain the designated current.

Symbols for Independent Current Source

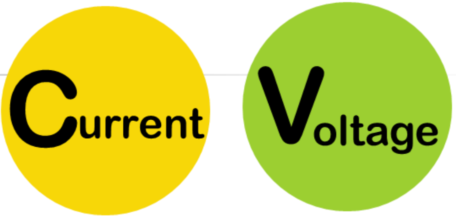


Note that the arrow indicates the direction of current flow.

It is mandatory to specify the current direction.

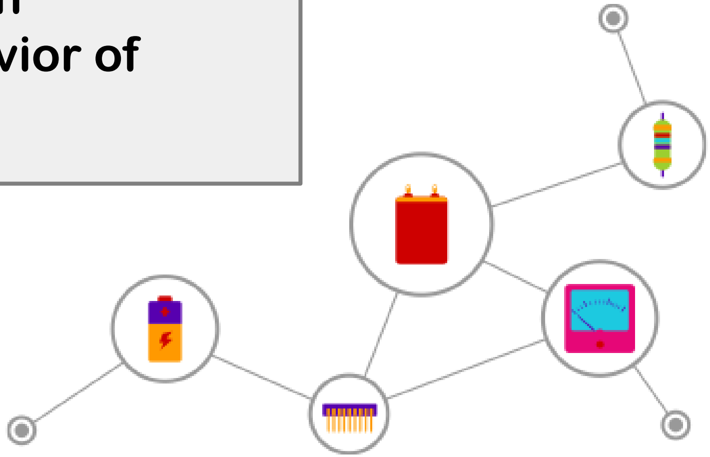
The terminal voltage is determined by the condition of the circuit to which it is connected.

Active Element: Dependent Source

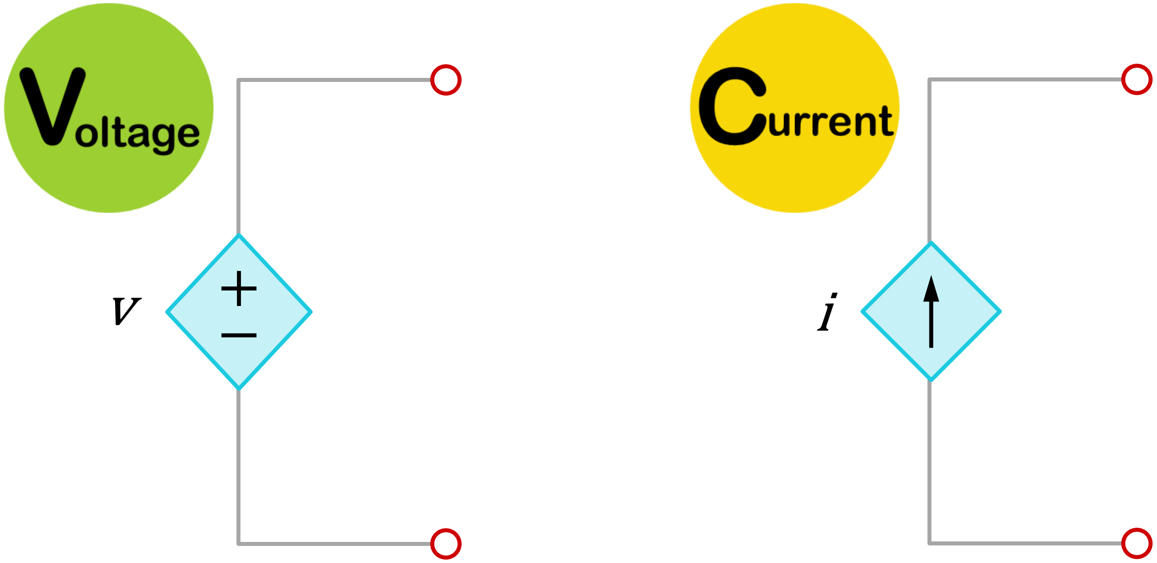


An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current elsewhere in the circuit.

Dependent sources are used a great deal in electronics to model both DC and AC behavior of transistors, especially in amplifier circuits.



Type of Dependent Source



(a) (b)

A dependent source has its output controlled by an input value.
Symbolically represented by diamond-shaped symbols as shown.

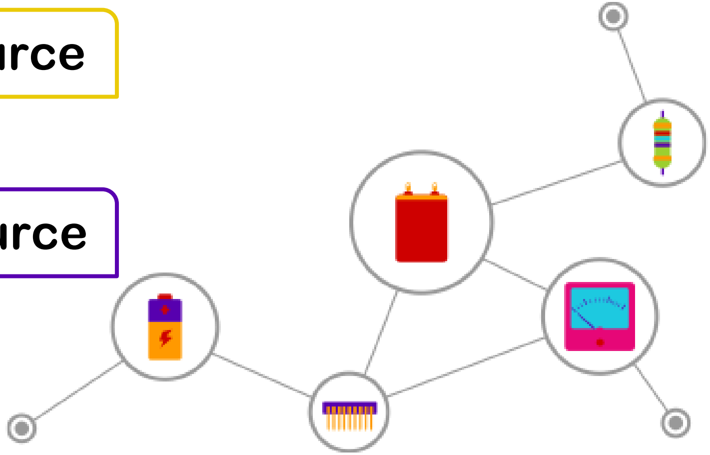
Type of Dependent Source: Four Types

VCVS — A **V**oltage-**C**ontrolled **V**oltage **S**ource

CCVS — A **C**urrent-**C**ontrolled **V**oltage **S**ource

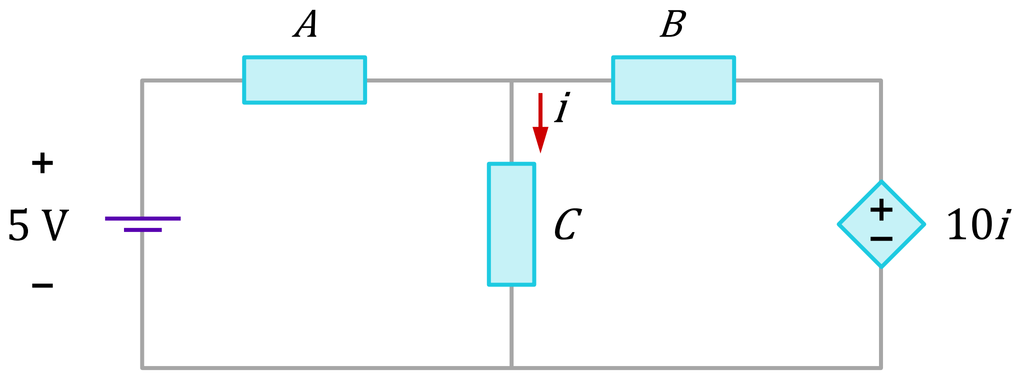
VCCS — A **V**oltage-**C**ontrolled **C**urrent **S**ource

CCCS — A **C**urrent-**C**ontrolled **C**urrent **S**ource



Example 1

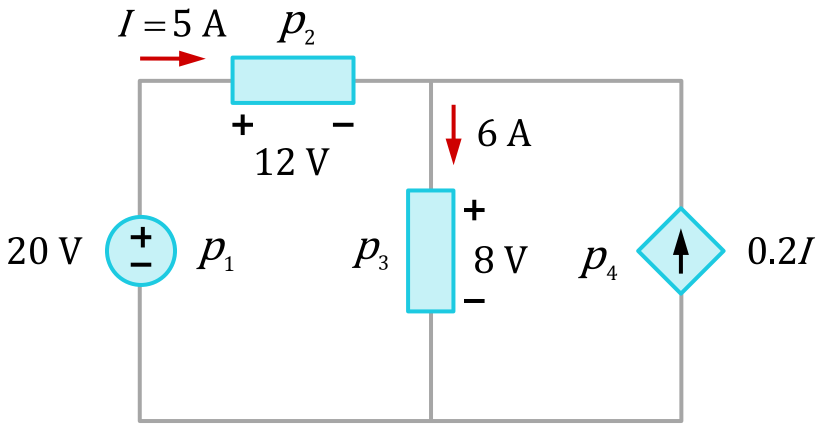
The source on the right-hand side is a current-controlled voltage source.



The value of the voltage supplied is $10i$ V (not $10i$ A), as it depends on the current i through element C.

Example 2

The source on the right-hand side is a current-controlled current source.

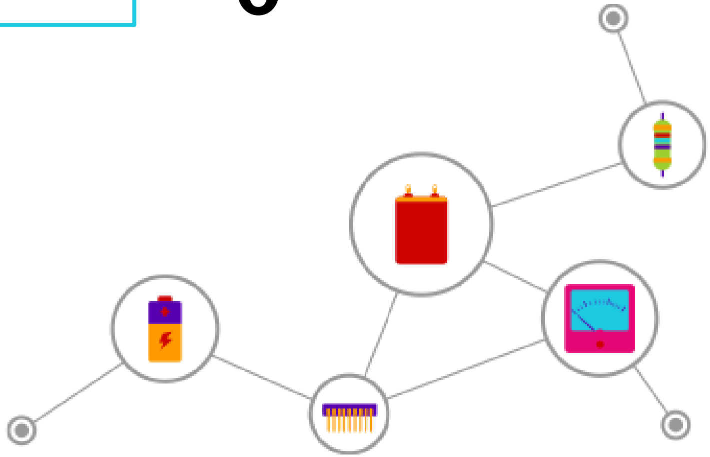


The value of the current supplied is $0.2I$ A. Since current $I = 5$ A the current-controlled current source will deliver 1 A of current.

Power Absorbed or Delivered in a Circuit

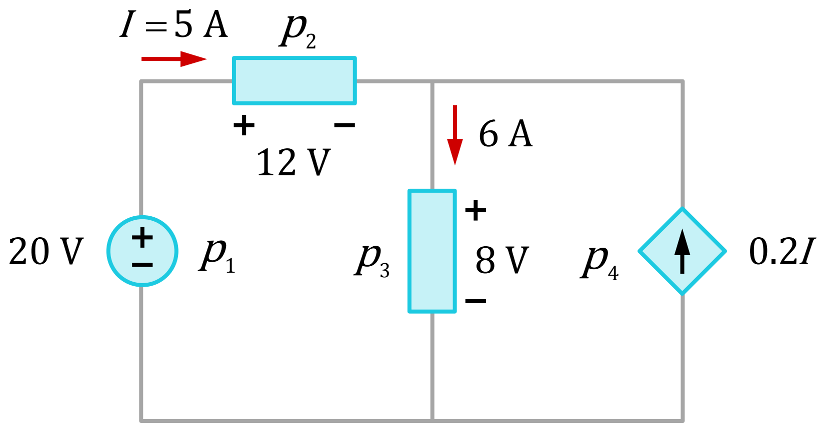
The algebraic sum of power in a circuit is zero.

$$\boxed{\text{Total Power Supplied}} + \boxed{\text{Total Power Absorbed}} = 0$$



Example 3

Calculate the power supplied or absorbed by each element in the following circuit.



Power Supplied

$$p_1 = 20(-5) = -100 \text{ W}$$

Power Absorbed

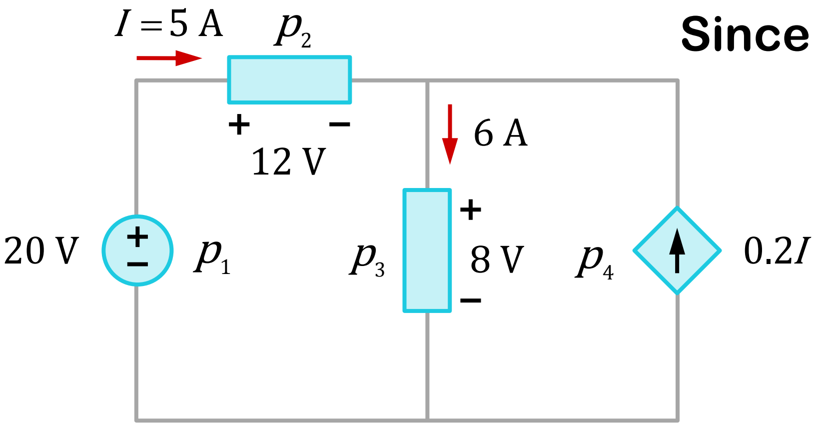
$$p_2 = 12(5) = 60 \text{ W}$$

Power Absorbed

$$p_3 = 8(6) = 48 \text{ W}$$

Example 3

For p_4, note that the voltage is 8 V (+ ve at the top), the same as the voltage for p_3, since both the passive element and the dependent source are connected to the same terminals.



Since the current flows out of the + ve terminal,

Power Supplied

$$p_4 = 8(-0.2I) = 8(-0.2\ x\ 5) = -8\text{ W}$$

Note:

Total Power Supplied + Total Power Absorbed = 0

– 100 – 8 + 60 + 48 = 0

Basic Laws

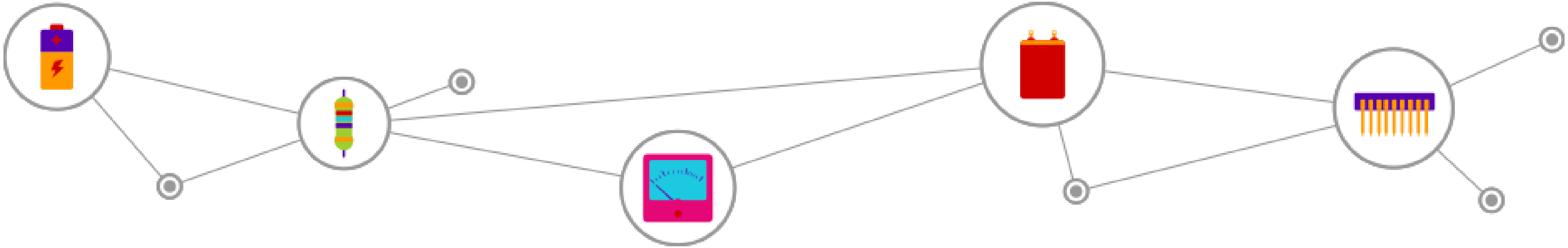
Basic Laws

Ohm's Law

Kirchhoff's Current Law

Kirchhoff's Voltage Law

Some Commonly Used Techniques



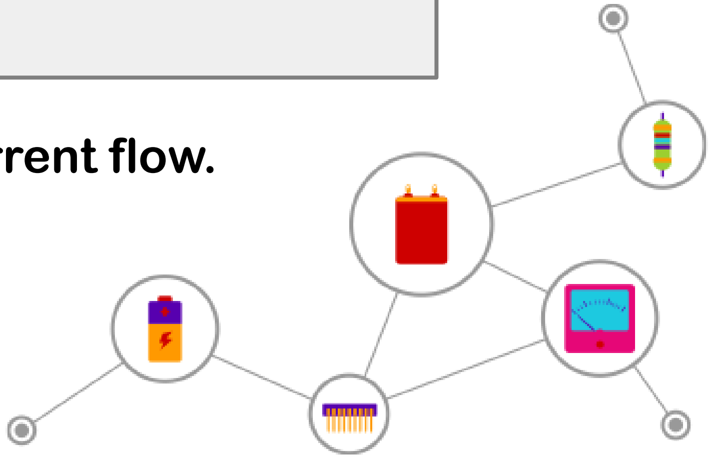
Concept of a Resistor

In general, materials tend to resist the flow of electric current through them.

The physical property or ability to resist electric current is known as **resistance** (represented by the symbol R). It is measured in ohms (designated as Ω).

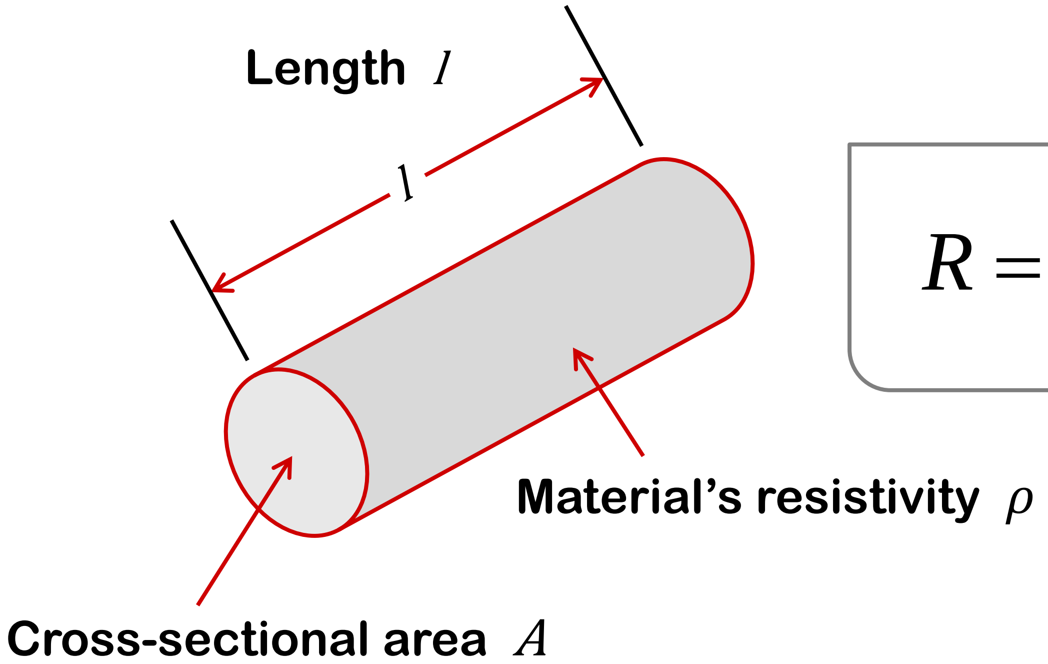
A **resistor** is a device that resists the flow of current flow. It is the simplest passive element.

Resistors can be used to control current flow in a circuit.



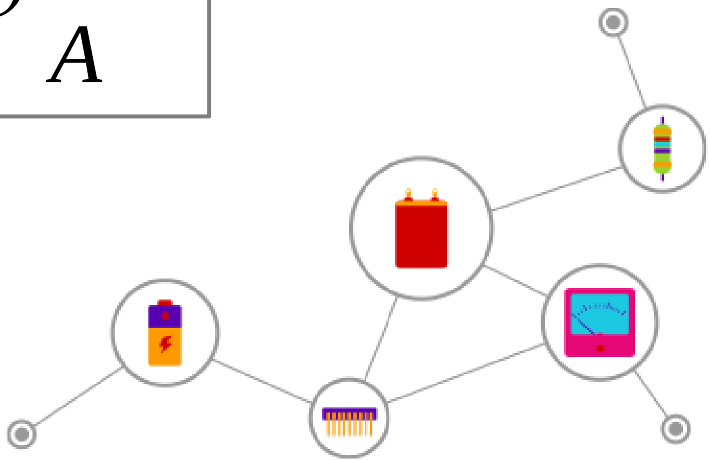
Concept of Resistance

The resistance of any material is a function of its

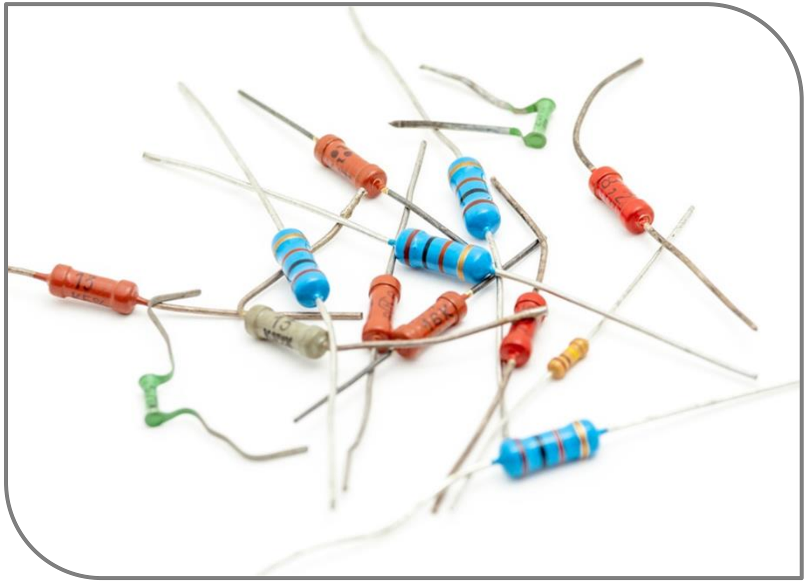


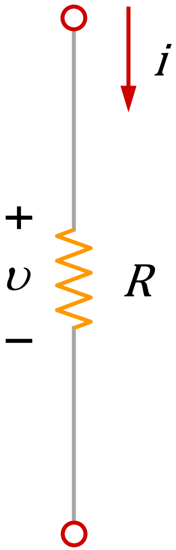
Resistance

$$R = \rho \frac{l}{A}$$



Circuit Symbol of Resistor





Resistor and its Circuit Symbol

Resistivity of Common Materials

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Ohm's Law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$\boxed{v \propto i}$$

Voltage = **C**urrent **R**esistance

$$\boxed{v = iR}$$

G.S. Ohm (1787-1854), a German Physicist, defines the constant of proportionality for a resistor to be the resistance, R.

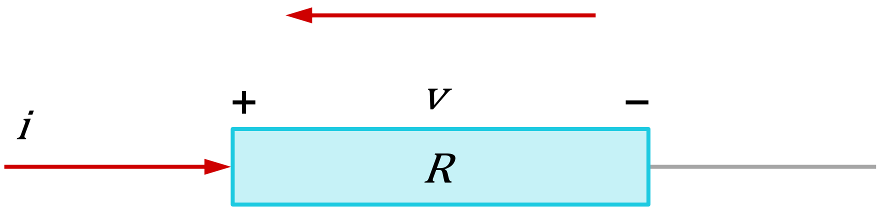
Application of Ohm's Law

To apply Ohm's law, we must pay careful attention to the

Current Direction

Voltage Polarity

The direction of current i and the polarity of voltage must conform with the **passive sign convention.**



That is, the current flows from a higher potential to a lower potential so that $v = iR$.

Short-circuit and Open-circuit

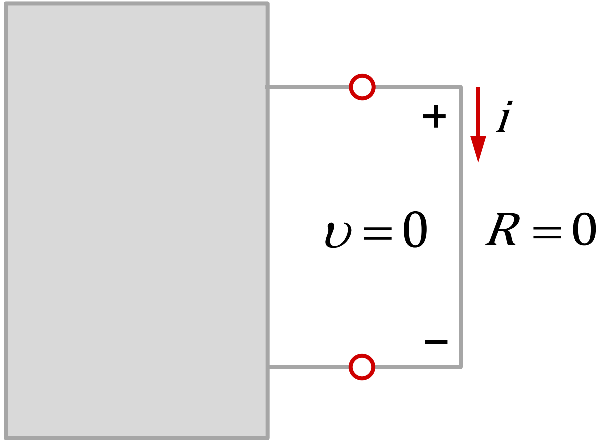
Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R.

A short-circuit is a circuit element with resistance approaching zero, $R = 0$.

An open-circuit is a circuit element with resistance approaching infinity, $R = \infty$.

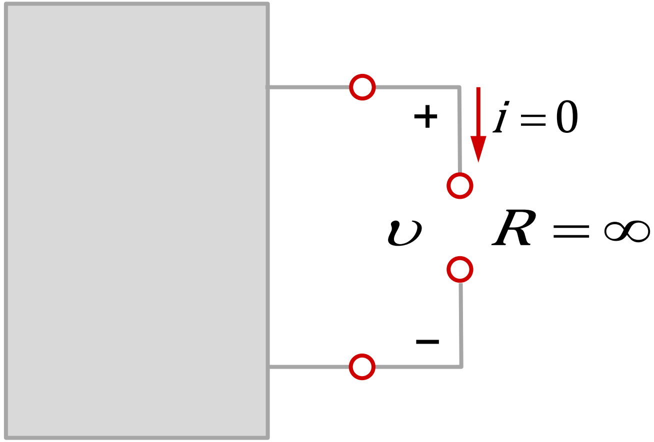
Short-circuit and Open-circuit

Short-circuit



For a short-circuit, $v = iR = 0$,
i.e., the voltage is zero but the current could be anything.

Open-circuit



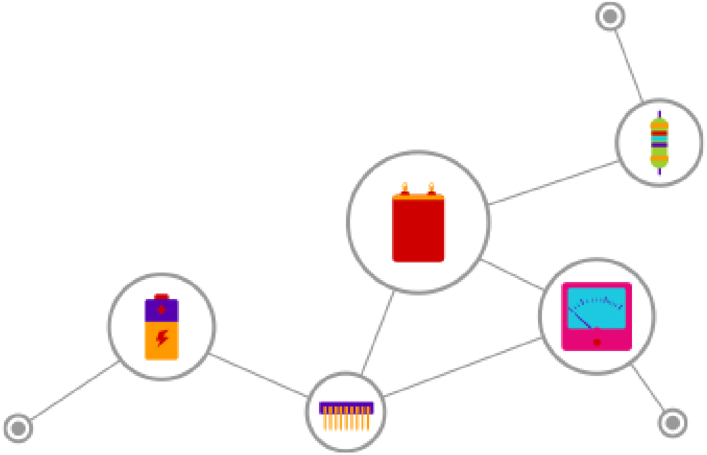
For an open-circuit, $\lim_{R \to \infty} \frac{v}{R} = 0$,
i.e., the current is zero though the voltage could be anything.

Conductance

A useful quantity in circuit analysis is the reciprocal of resistance R, known as conductance, G.

$$G = \frac{1}{R} = \frac{i}{v}$$

Conductance is the ability of an element to conduct electric current and is measured in Siemens (**S**).

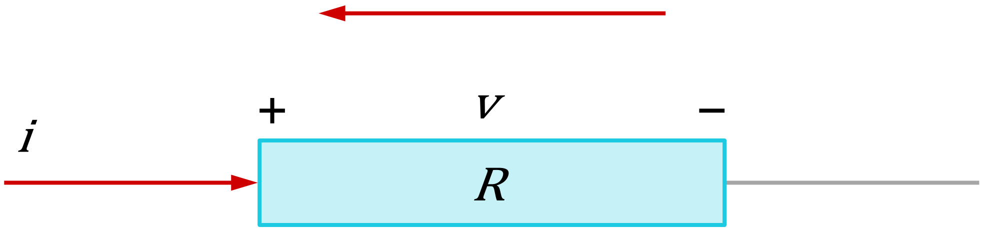


Power Dissipated by a Resistor

The power dissipated by a resistor can be expressed as,

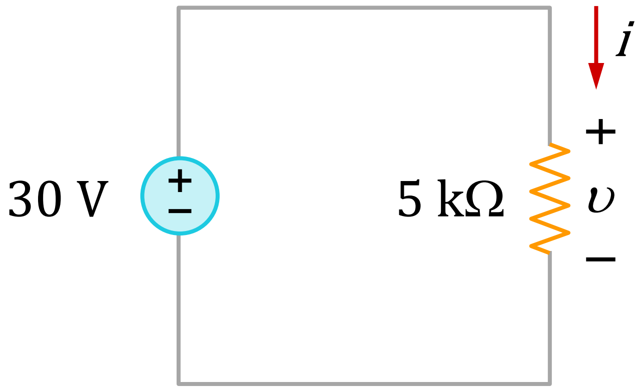
$$p = vi = i^2R = \frac{v^2}{R}$$

The power dissipated in a resistor is always + ve, i.e., a resistor always absorbs power from the circuit.



Example 4

In the following circuit, calculate the current i, the conductance G, and the power p.



Current

$$i = \frac{v}{R} = \frac{30}{5\ x\ 10^3} = 6\ x\ 10^{-3} = 6\ mA$$

Conductance

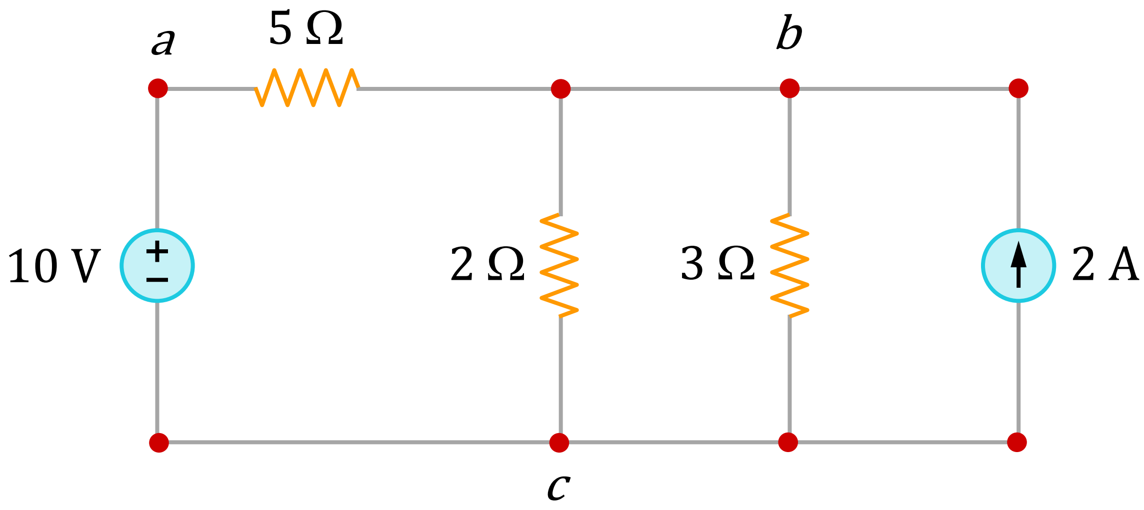
$$G = \frac{1}{R} = \frac{1}{5\ x\ 10^3} = 0.2\ x\ 10^{-3} = 0.2\ mS$$

Power

$$p = \frac{v^2}{R} = \frac{(30)^2}{5\ x\ 10^3} = 180\ mW$$

Branches

A **branch** represents a single two-terminal element like a voltage source or a resistor.

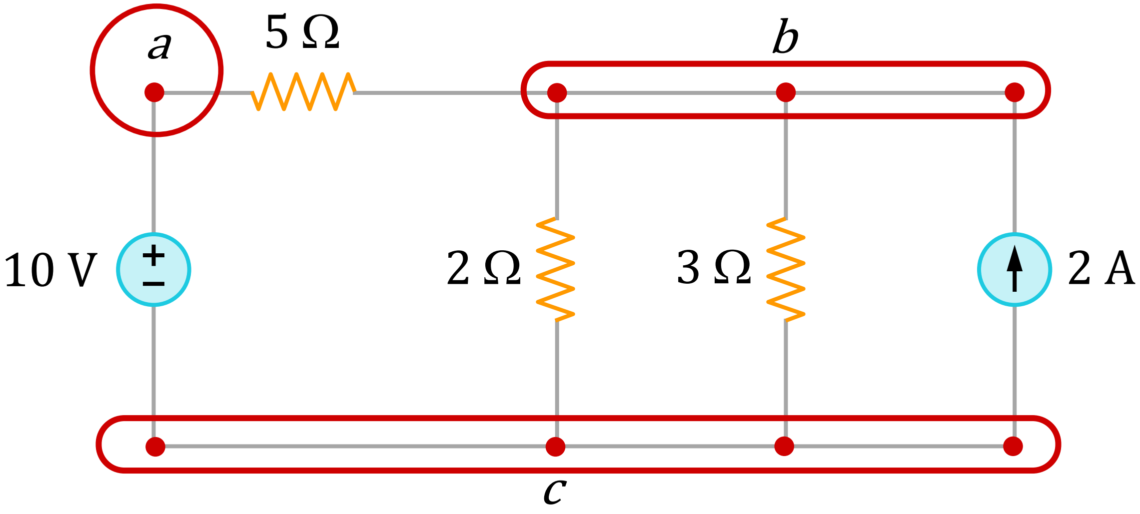


The circuit has 5 branches: the $10\ \mathrm{V}$ voltage source, the $2\ \mathrm{A}$ current source and the 3 resistors.

Nodes

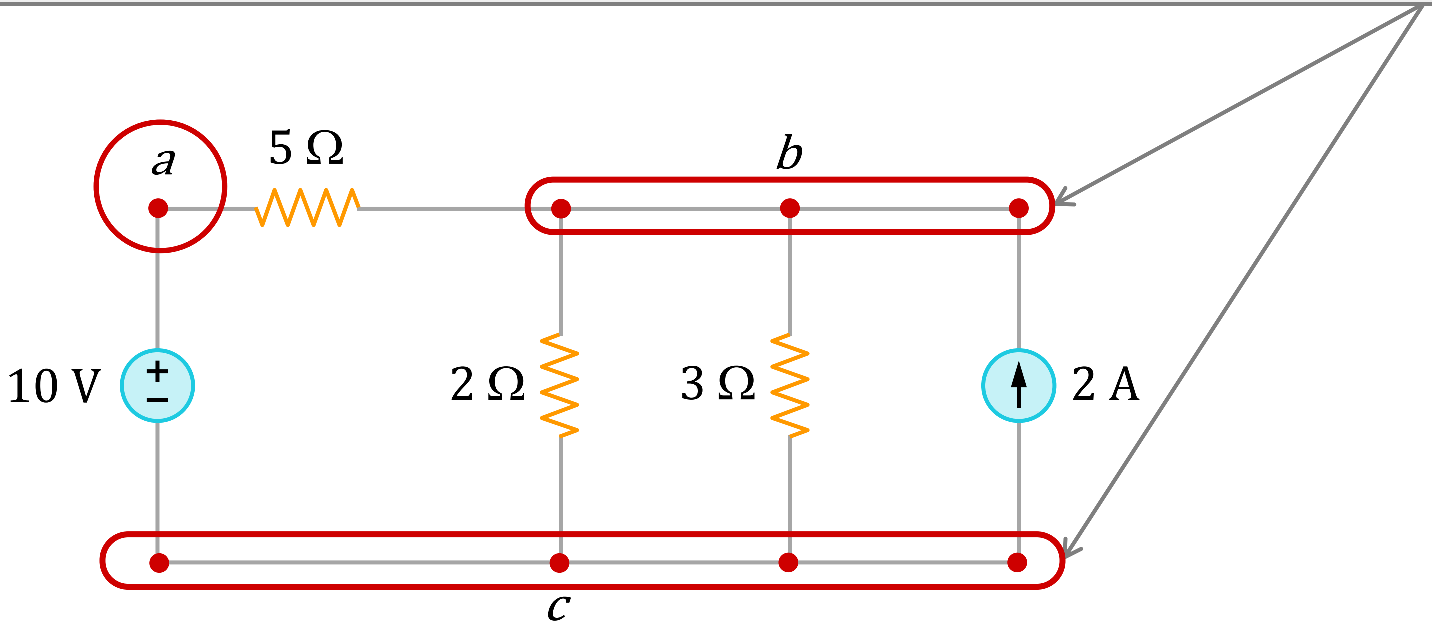
A **node** is the point of connection between two or more branches. A node is indicated by a dot in a circuit.

The circuit has 3 nodes: a, b and c.



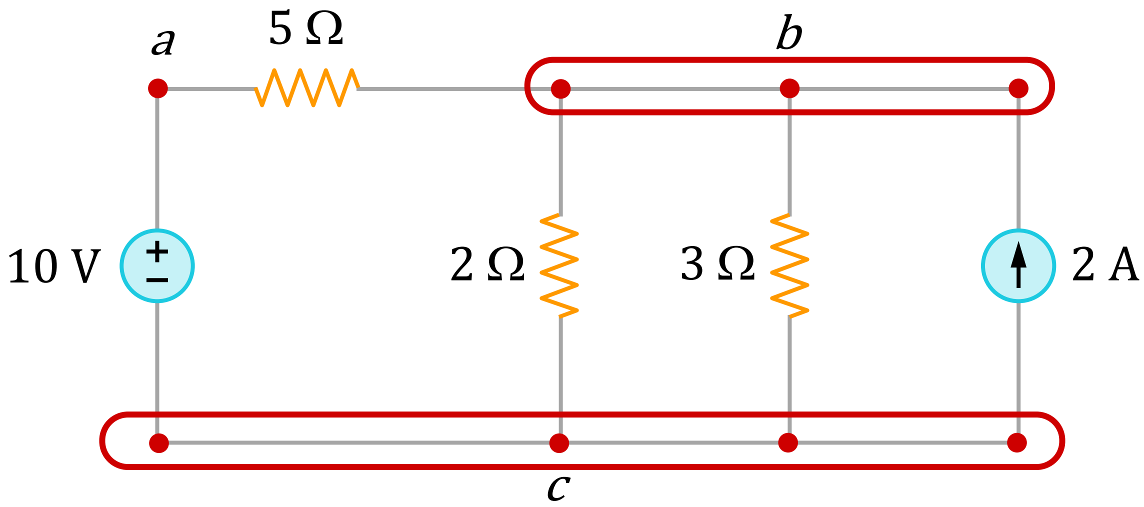
Nodes

Note: the 3 points that form node b are connected by wires and therefore constitute a single point. The same is true for the 4 points that form node c.



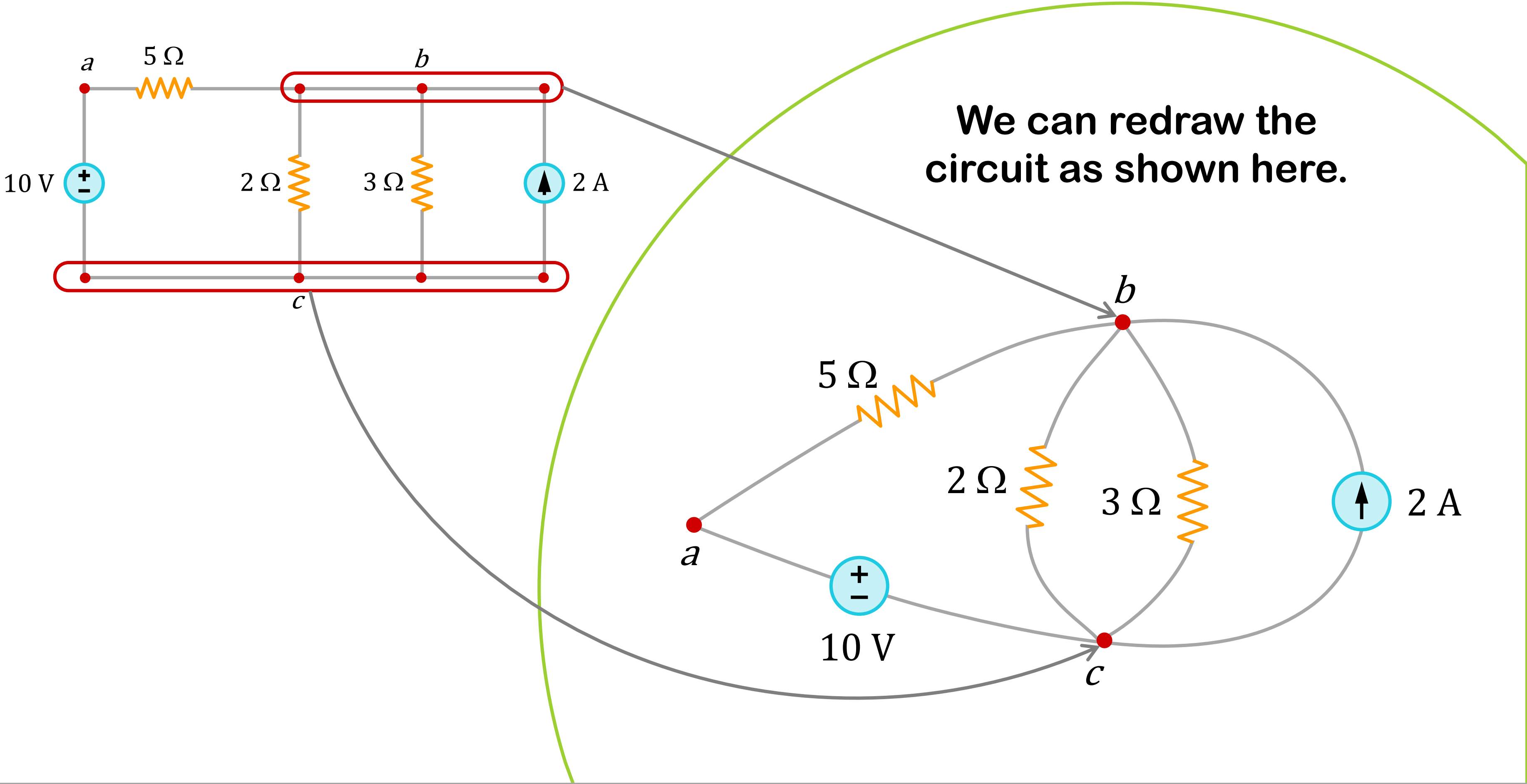
Loops

A **loop** is a closed path formed by starting at a node, passing through a set of nodes and returning to the starting node without passing through any node more than once.

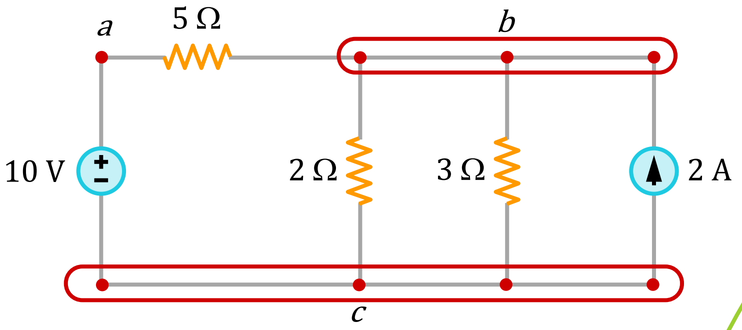


Loops

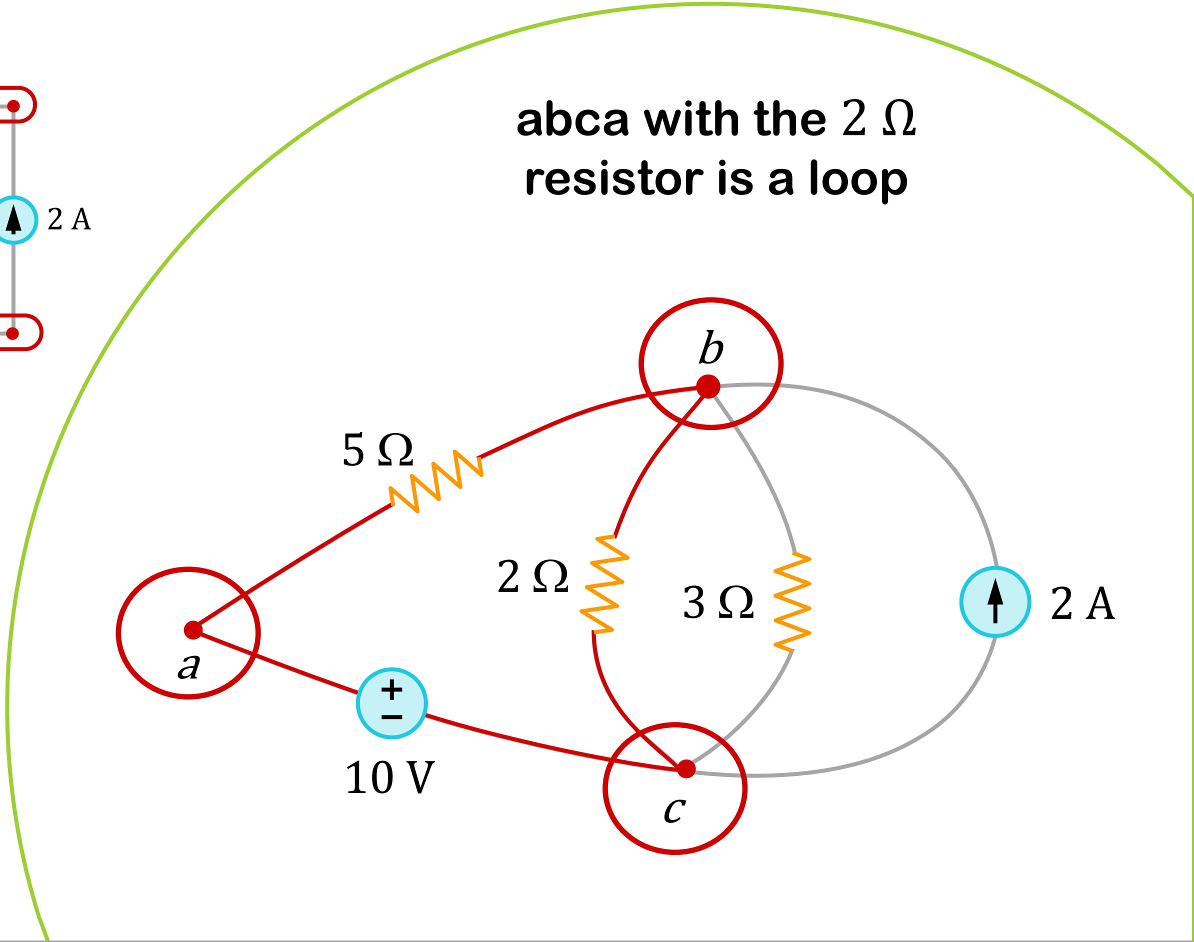
We can redraw the circuit as shown here.



Loops

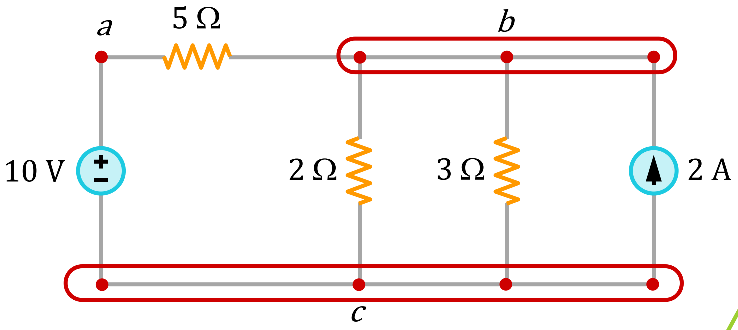


a
5 Ω
b
10 V
2 Ω
3 Ω
2 A
c

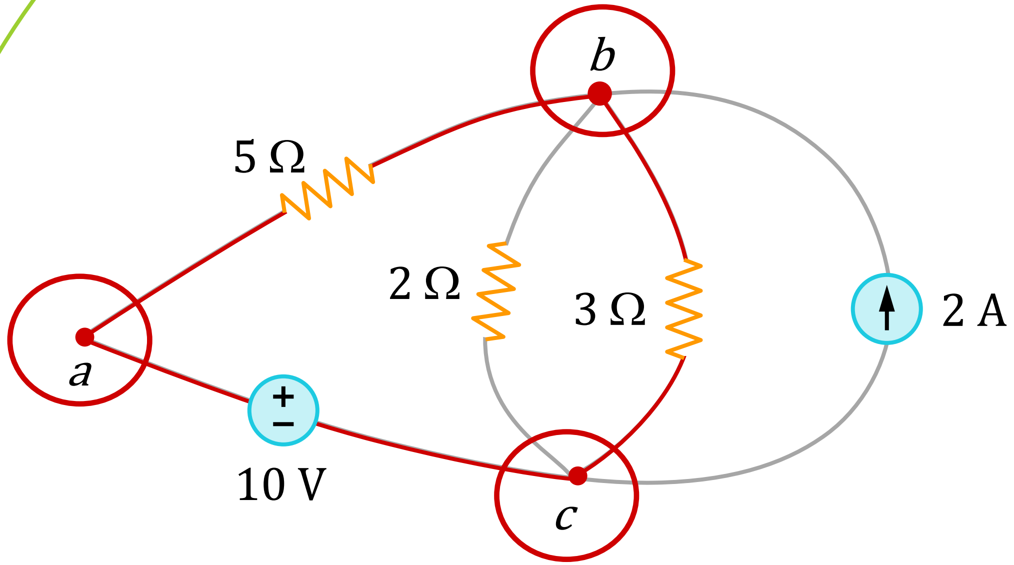


abca with the 2 Ω
resistor is a loop
b
5 Ω
2 Ω
3 Ω
2 A
a
10 V
c

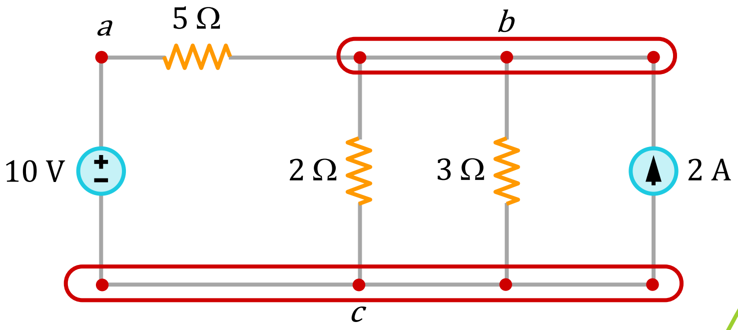
Loops



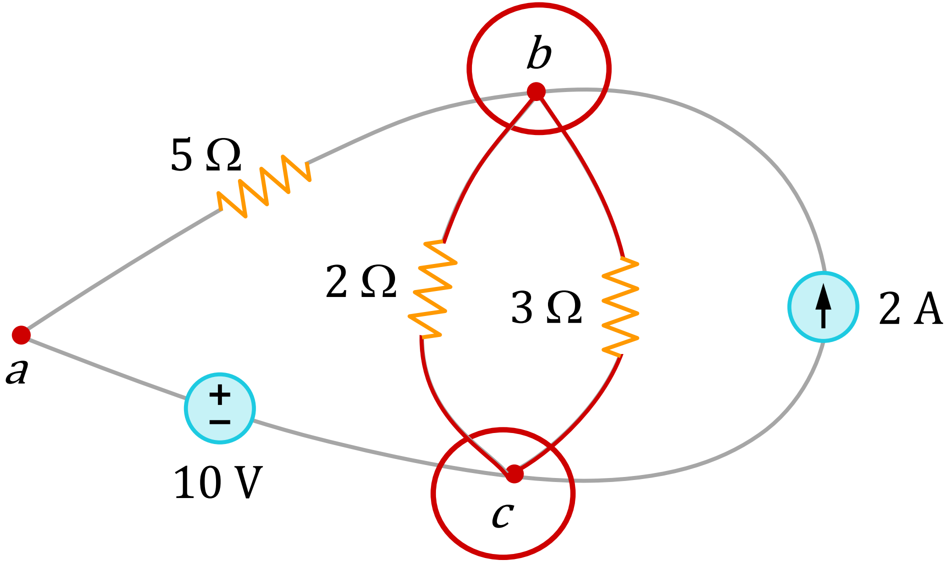
abca with the 3 Ω resistor is a loop



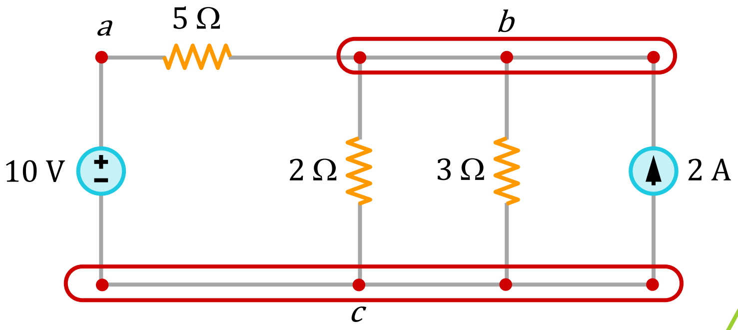
Loops



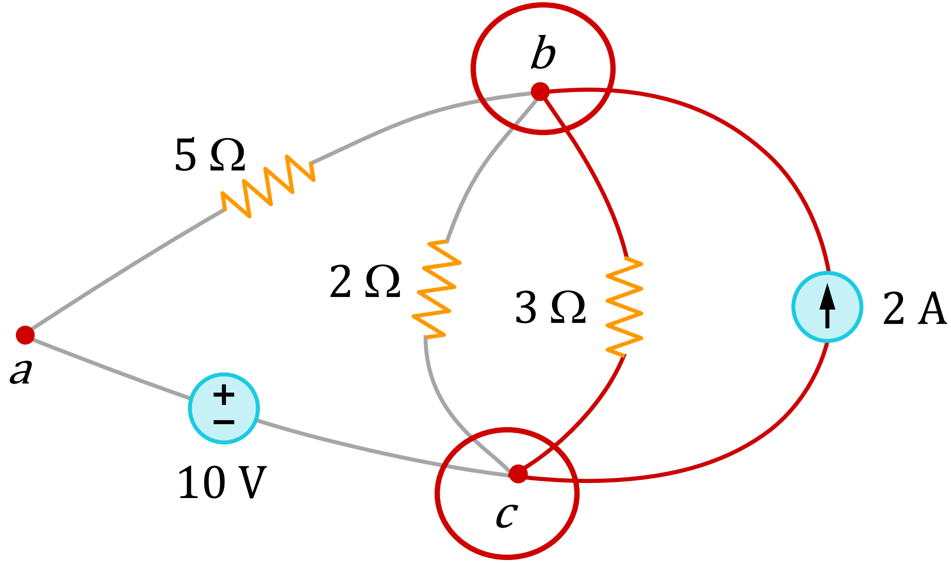
bcb with the 2 Ω and 3 Ω resistors is a loop



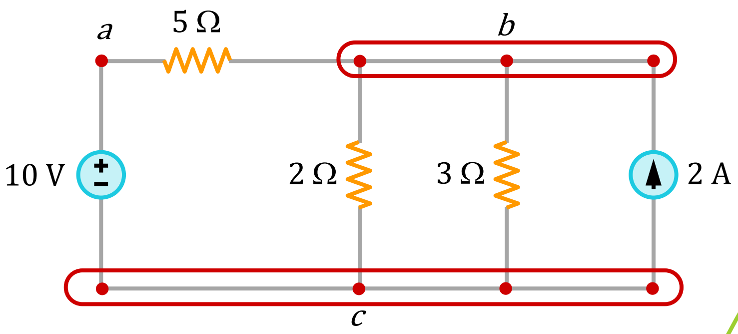
Loops



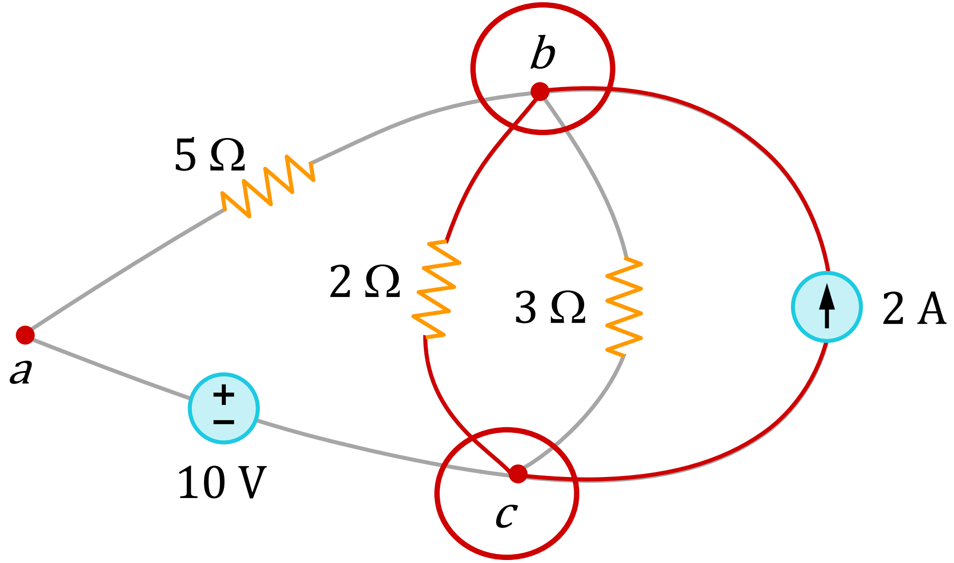
bcb with the $3\ \Omega$ resistor and the $2\ \mathrm{A}$ current source is a loop



Loops

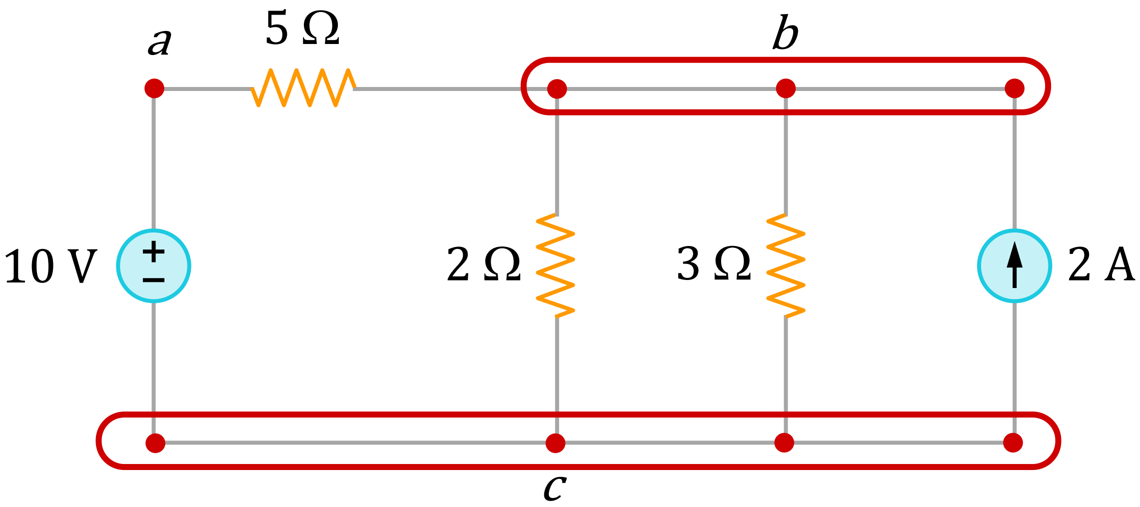


bcb with the $2\ \Omega$ resistor and the $2\ \mathrm{A}$ current source is a loop

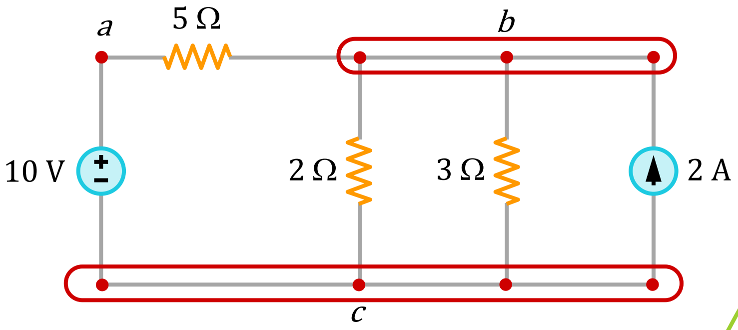


Mesh

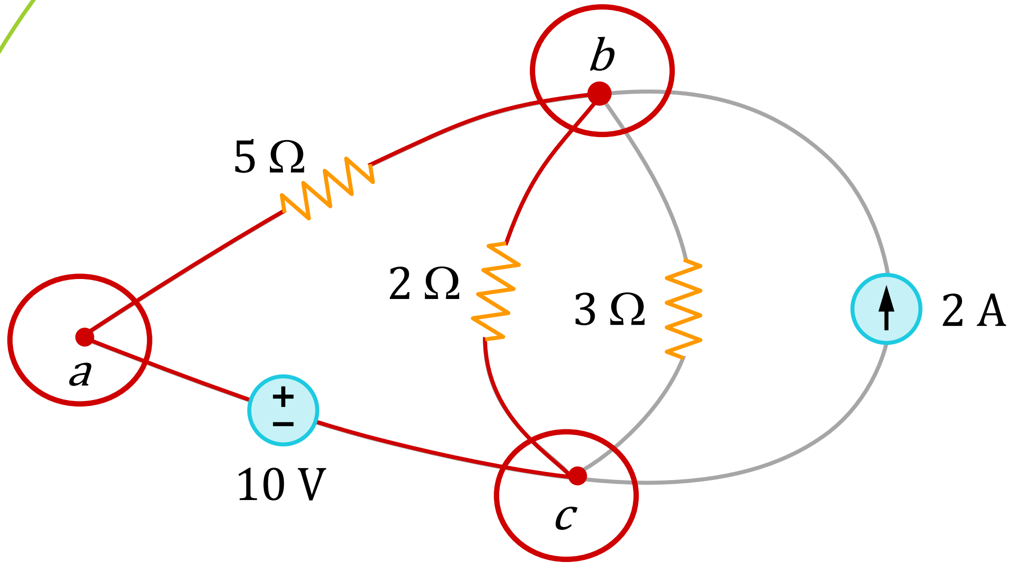
A **mesh** is a loop which does not contain any other loops within it (it is an independent loop).



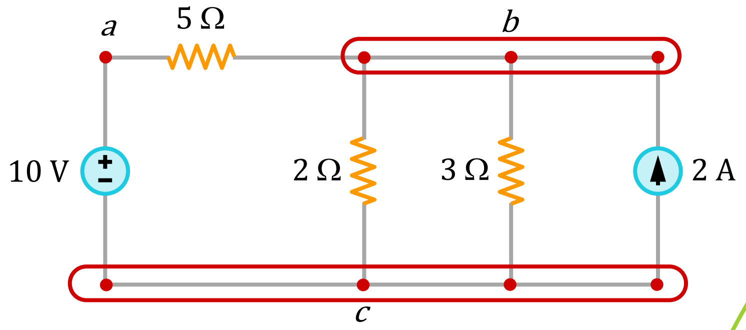
Mesh



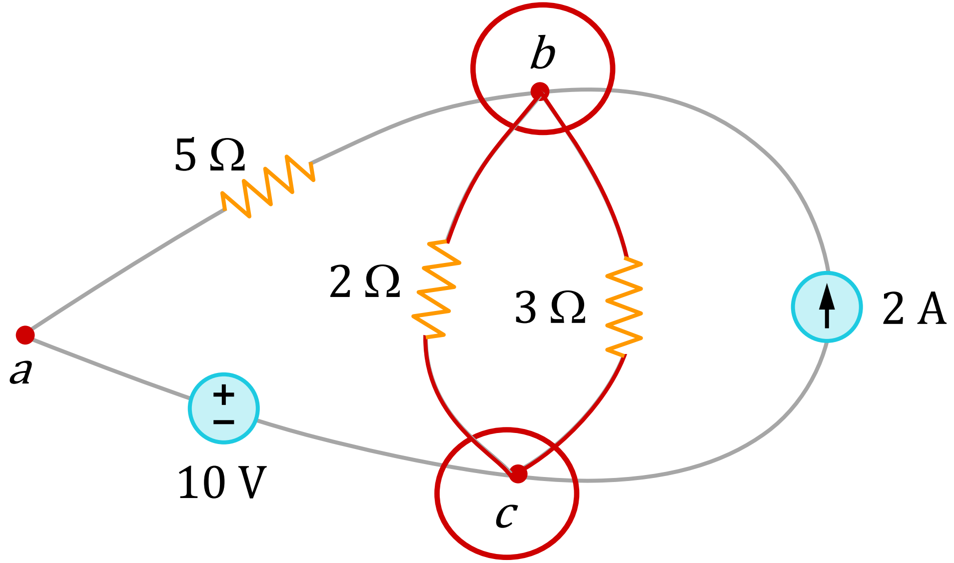
abca with the 2 Ω resistor is mesh



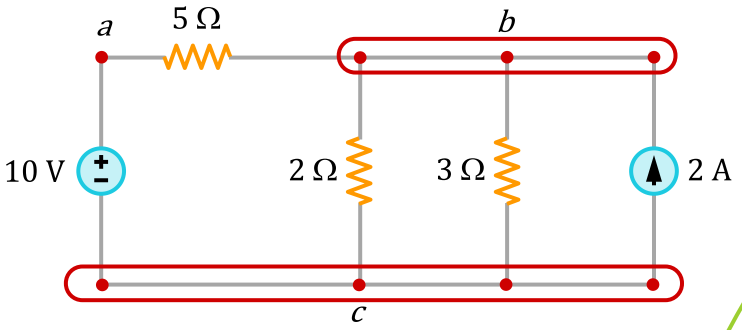
Mesh



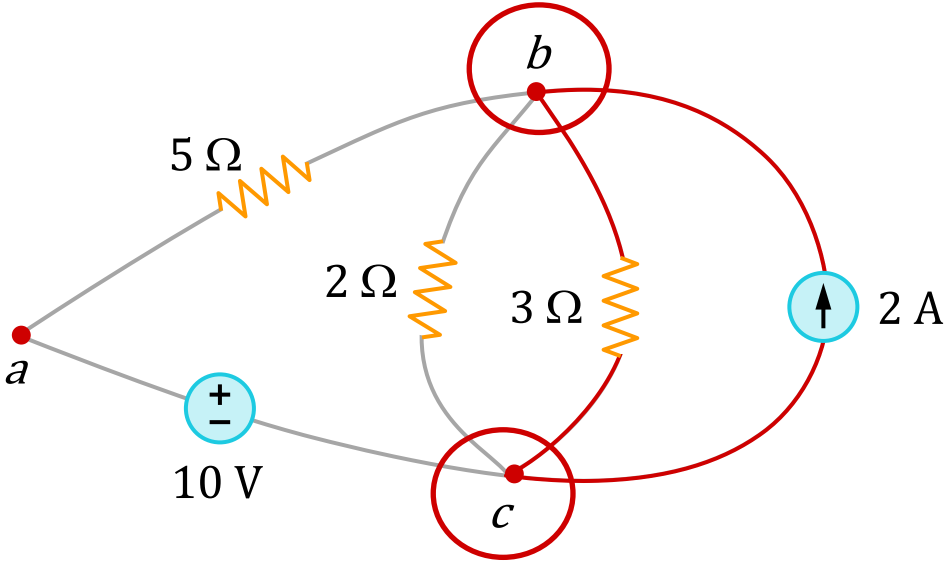
bcb with the 2 Ω and 3 Ω resistors is a mesh



Mesh



bcb with the $3\ \Omega$ resistor and the $2\ \text{A}$ current source is a mesh



Mesh

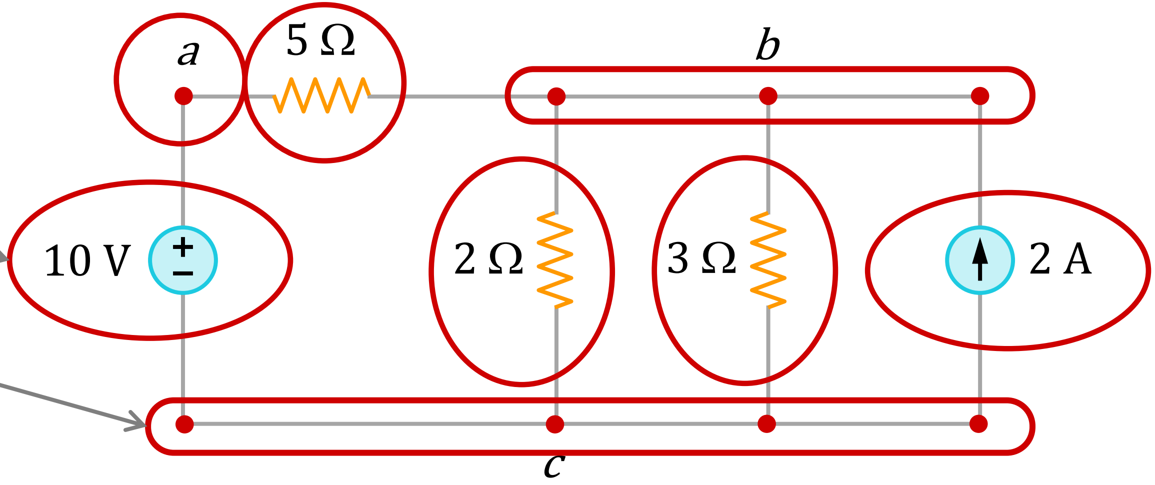
In a circuit with **b** branches and **n** nodes, the number of meshes is $\boldsymbol{m = b - n} + 1$.

For this example, $m = 5 - 3 + 1 = 3$.

Independent loops result in independent sets of equations (to be used later).

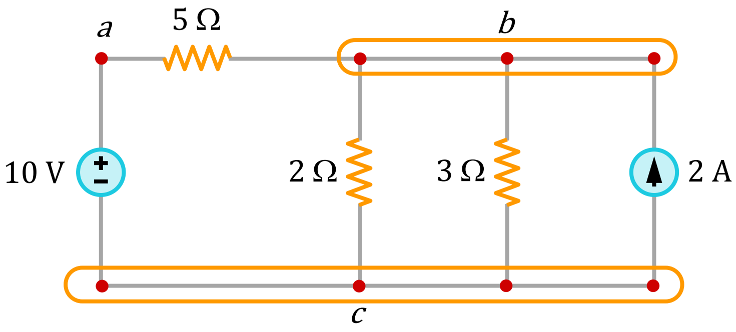
5 Branches

3 Nodes



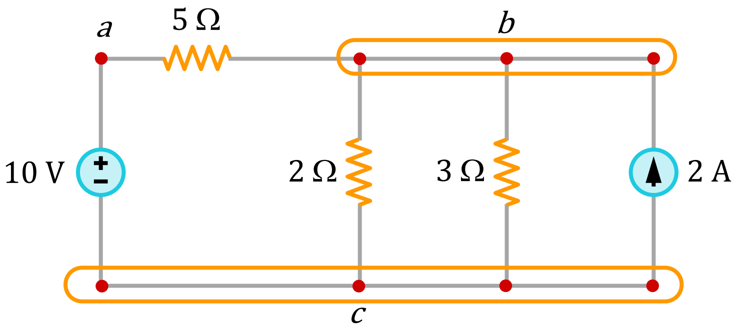
Series and Parallel Connection

Series Connection



Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.

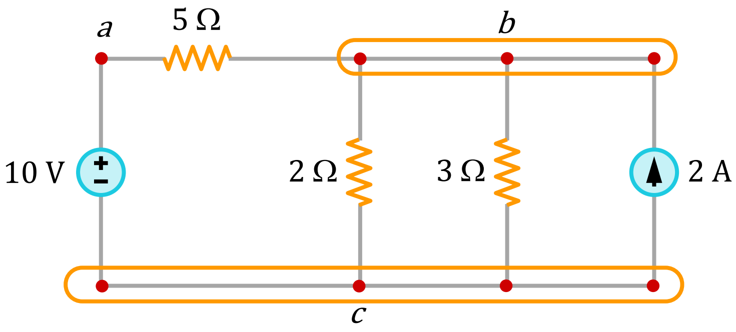
Parallel Connection



Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

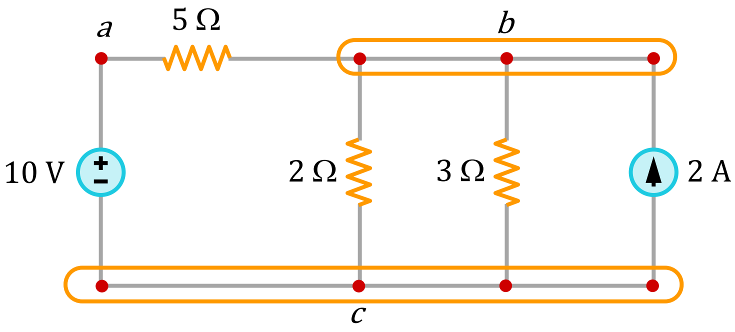
Series and Parallel Connection

Series Connection



The $10\ \mathrm{V}$ source and the $5\ \Omega$ resistor are in series as the same current flows through them.

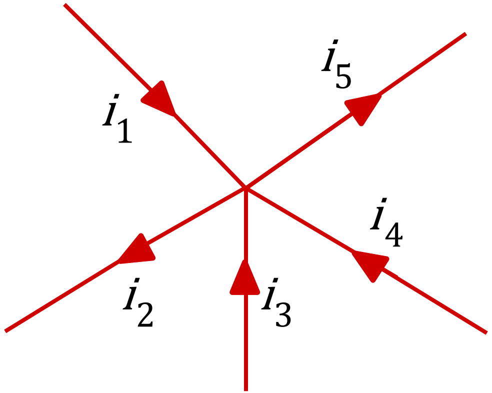
Parallel Connection



The $2\ \Omega$ and $3\ \Omega$ resistors and the $2\ \mathrm{A}$ current source are in parallel because they are connected to the same two nodes, b and c, and consequently have the same voltage across them.

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law (KCL) states that the algebraic sum of the currents entering any node is zero.



Kirchhoff's Current Law (KCL)

Consider the node in the figure.

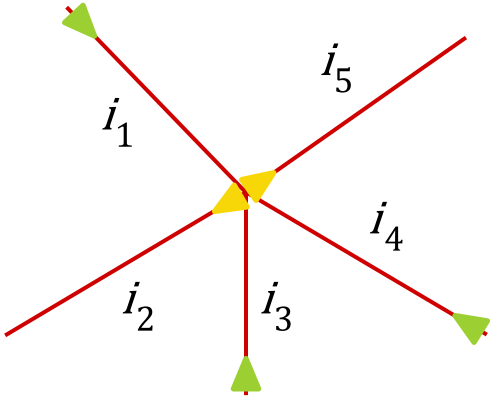
If we take current entering the node as + ve, applying KCL yields gives the following.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

in out in in out

$$i_1 + i_3 + i_4 = i_2 + i_5$$

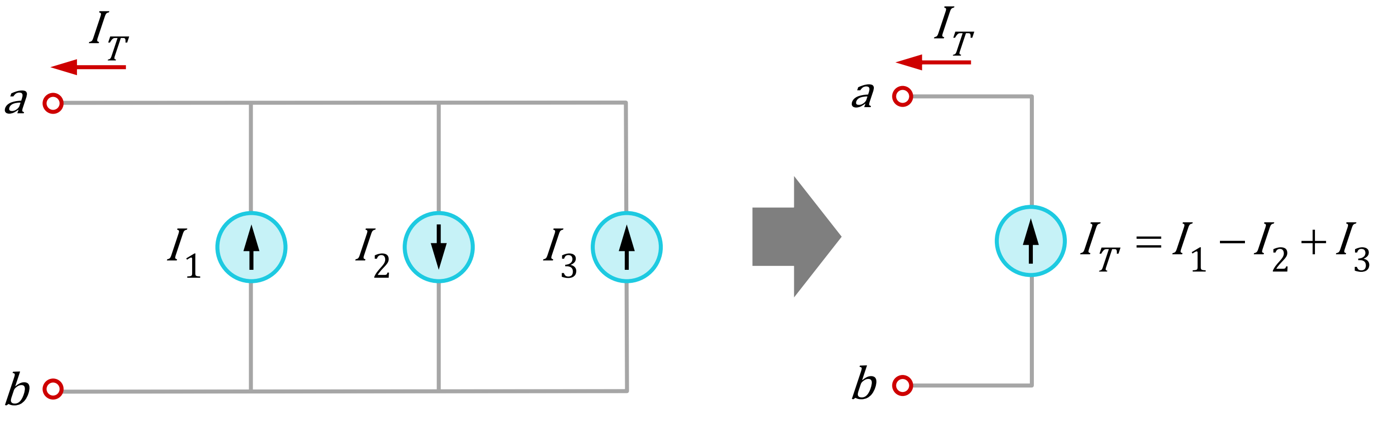
in in in out out



An alternative form of KCL: The sum of the currents entering a node is equal to the sum of the currents leaving the node. **KCL is based on conservation of charge.**

Example 5

Consider the current sources as shown. Applying KCL to node a:



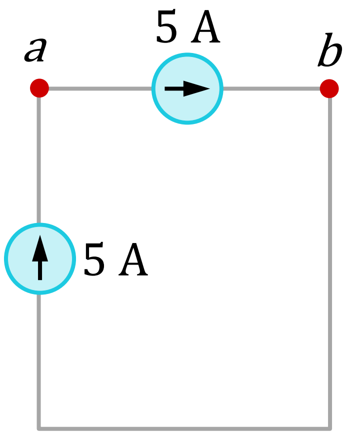
$$-I_T + I_1 - I_2 + I_3 = 0$$

$$I_T = I_1 + I_3 - I_2$$

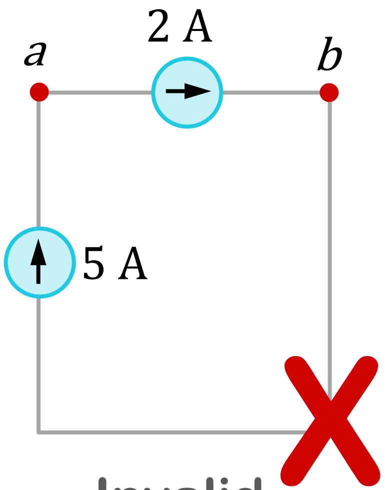
Violation of KCL Law

A circuit cannot contain two different current sources in series unless each has the same current, including sign.

Otherwise, KCL would be violated.



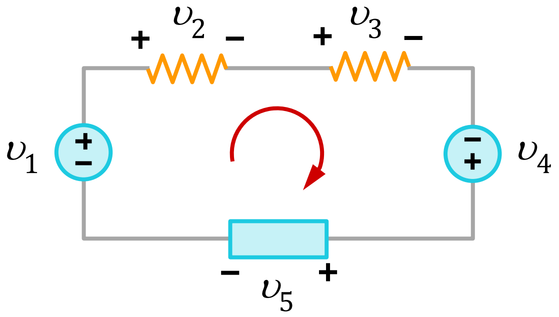
Valid



Invalid

Kirchhoff’s Voltage Law (KVL)

Kirchhoff’s Voltage Law (KVL) states that the algebraic sum of all voltages around a loop in a specified direction is zero.

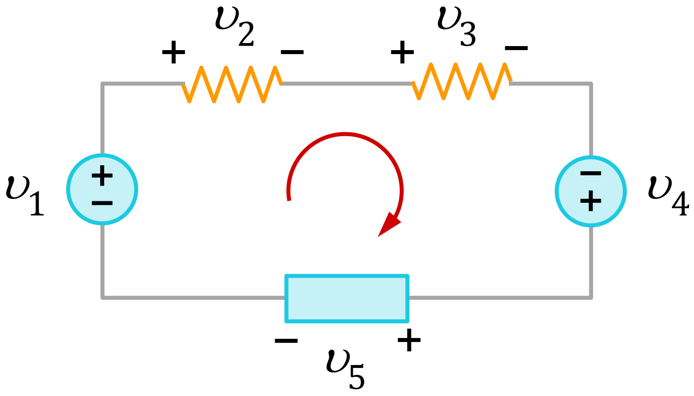


Kirchhoff's Voltage Law (KVL)

Express the loop current in the clockwise (CW) direction (preferred direction).

We take voltage rise as –ve and voltage drop as +ve.

KVL is based on conservation of energy.



Kirchhoff's Voltage Law (KVL)

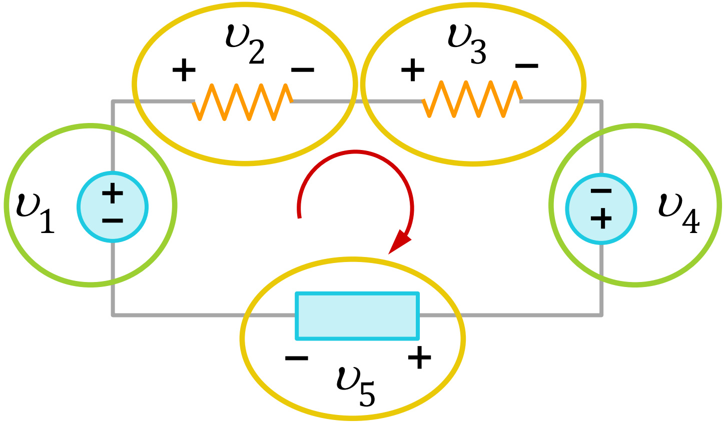
Begin at the v_1 source and go CW around the loop applying KVL:

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

vr vd vd vr vd

$$v_1 + v_4 = v_2 + v_3 + v_5$$

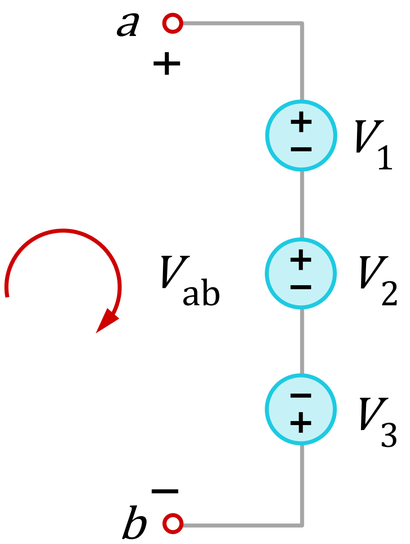
vr vr vd vd vd

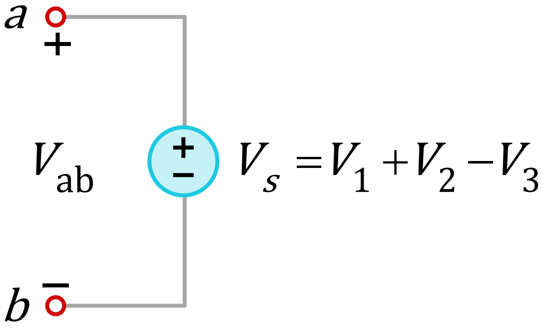


An alternative form of KVL: Around a loop in a clockwise direction, the sum of voltage rises equals to the sum of voltage drops.

Example 6

Consider the voltage sources as shown.
Applying KVL (in the CW direction):



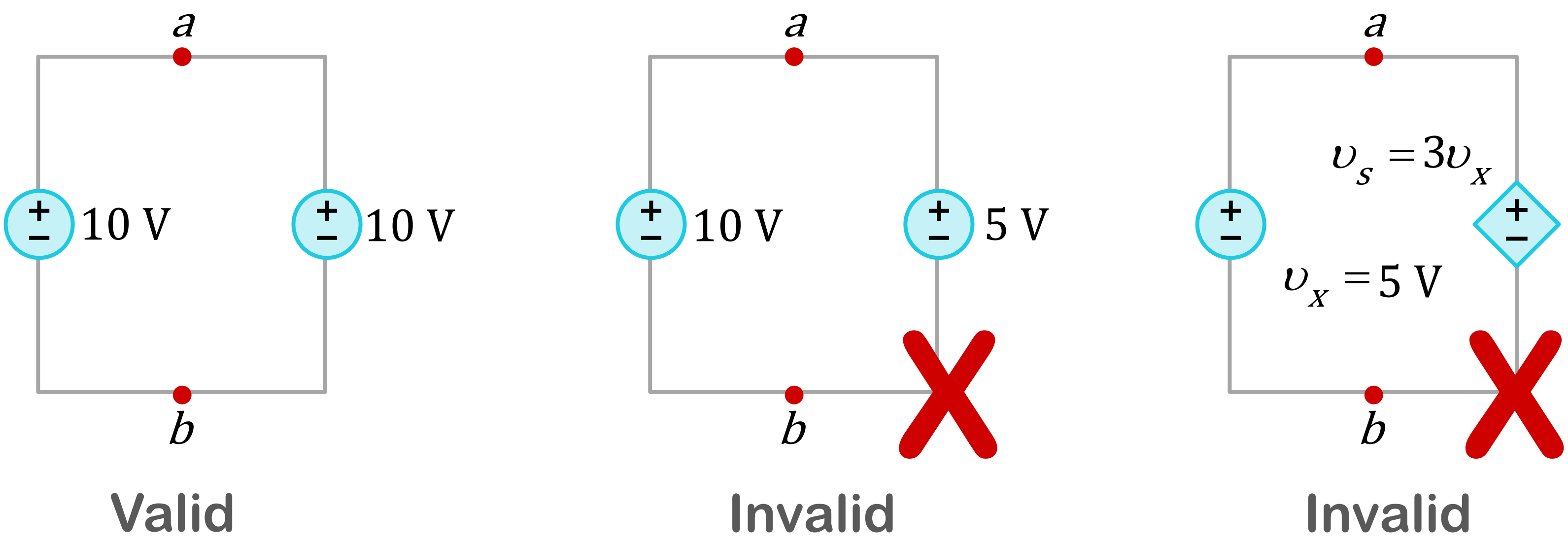


$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

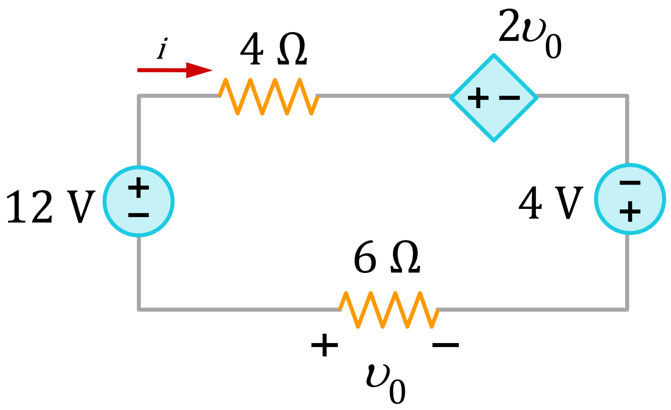
Violation of KVL Law

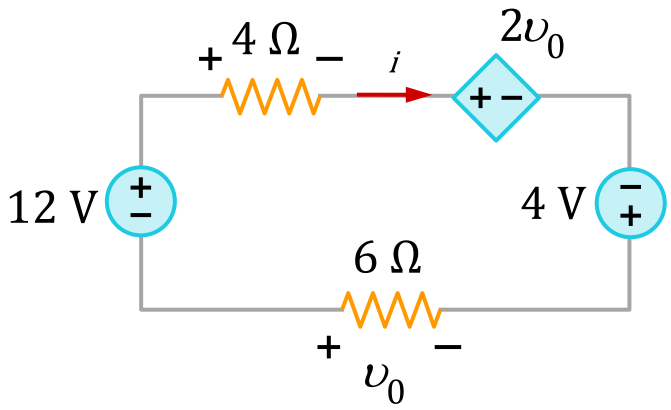
To avoid violating KVL, a circuit cannot contain two different voltages sources in parallel unless their terminal voltages are the same.



Example 7

Determine v_o and i in the following circuit.



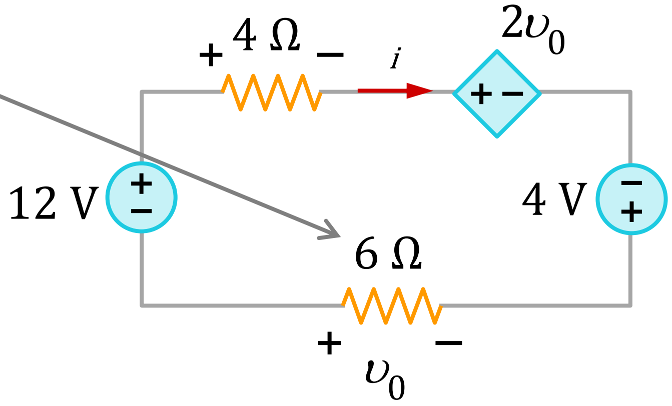


Example 7

Applying **KVL** and using **Ohm's Law**,

$$-12 + 4i + 2v_o - 4 - v_o = 0 \quad (1)$$

$$v_{6\Omega} = 6i = -v_o \quad (2)$$



Using (2) in (1) gives

$$-12 + 4i - 12i - 4 + 6i = 0$$

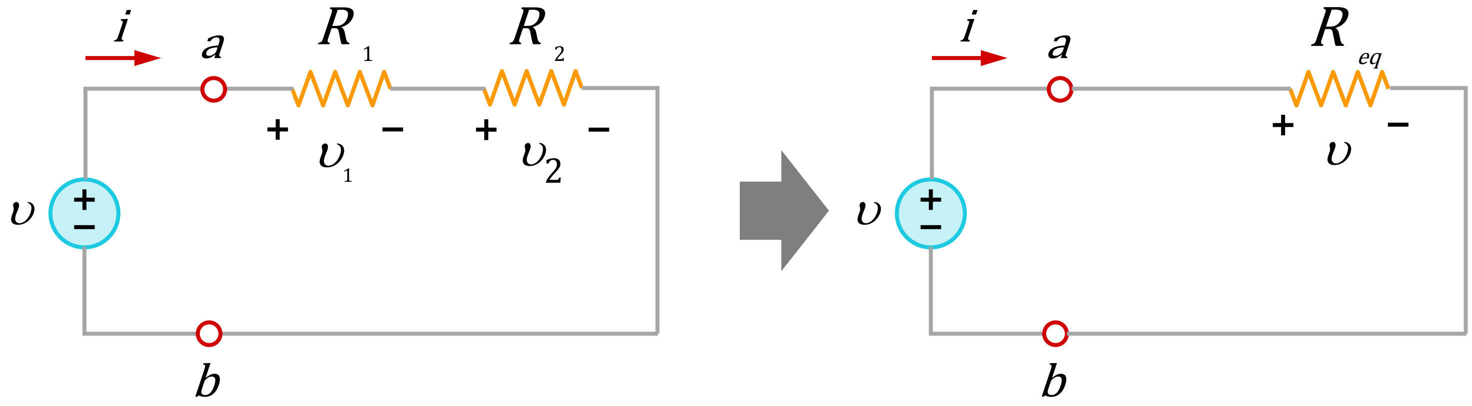
$$-16 - 2i = 0$$

Hence, $i = -8\ A$ and $v_o = -6i = 48\ V$

Series Resistor

For two resistors connected in series, the equivalent resistance of these two resistors is the sum of the individual resistances, i.e.,

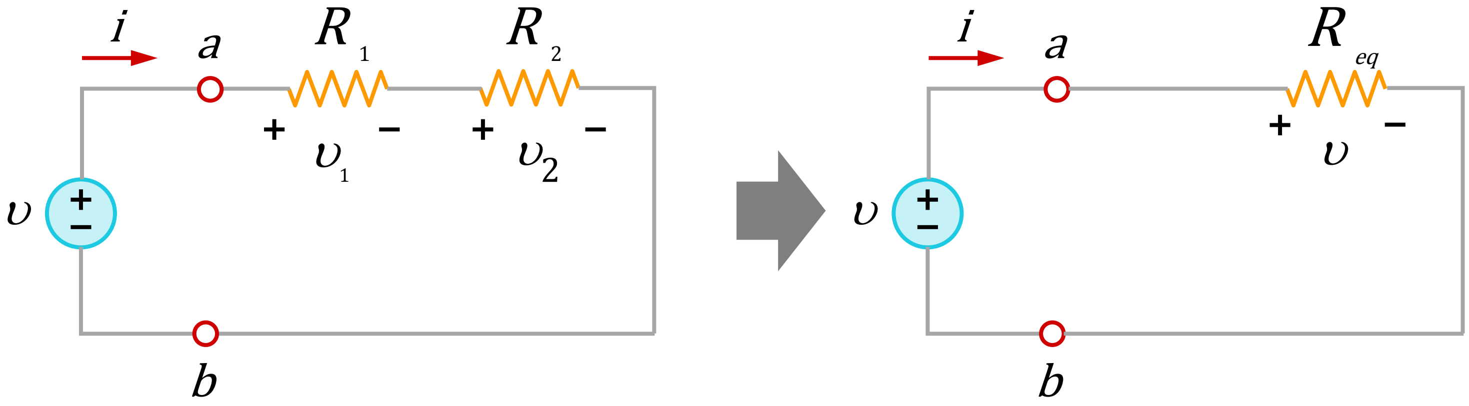
$$R_{eq} = R_1 + R_2$$



Equivalent Resistance

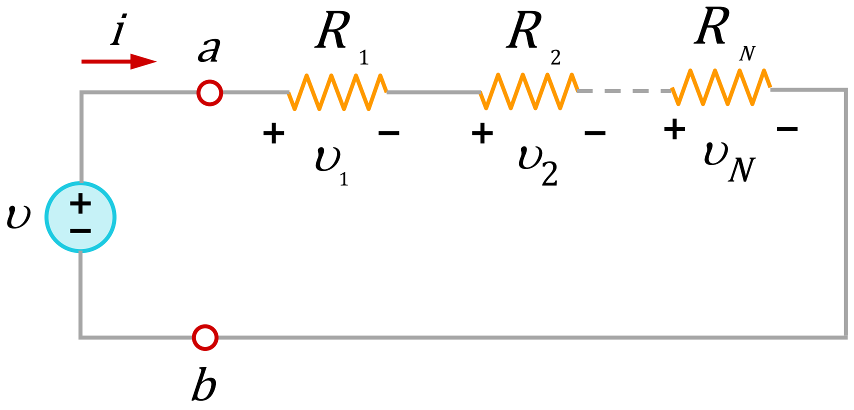
The two circuits are equivalent as they have the same $v - i$ relationship at the terminals a-b, i.e., $\boxed{i = \dfrac{v}{R_1 + R_2} = \dfrac{v}{R_{eq}}}$ **because** $\boxed{R_{eq} = R_1 + R_2}$

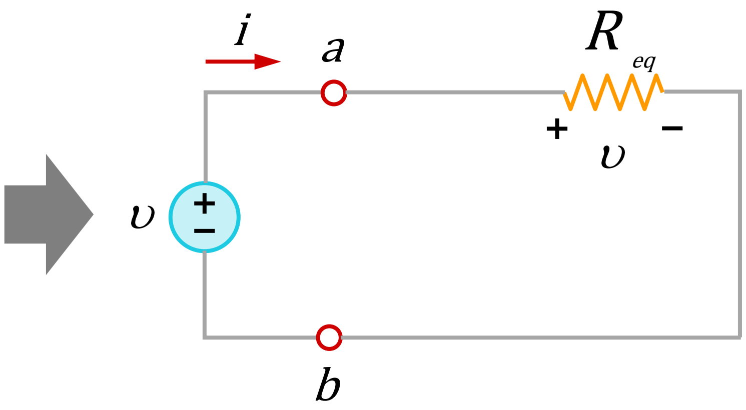
An equivalent circuit is useful in simplifying the analysis of a circuit.



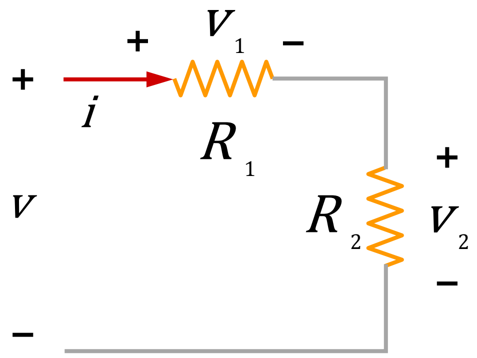
Equivalent Resistance

For **N** resistors connected in series, we have $R_{eq} = R_1 + R_2 + \cdots + R_N$.





Voltage Divider



For this circuit with two resistors connected in series, we have

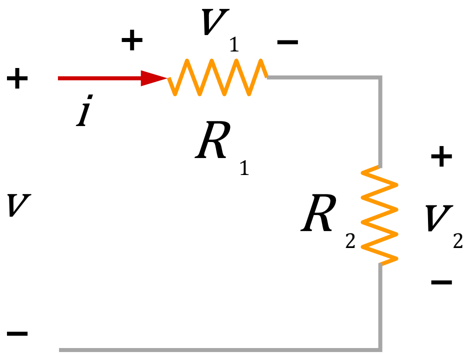
$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

The method is known as the voltage divider.

Example 8

Using the voltage divider, calculate R_2 so that $v_2 = 0.75\ v$ when $R_1 = 100\ \Omega$.

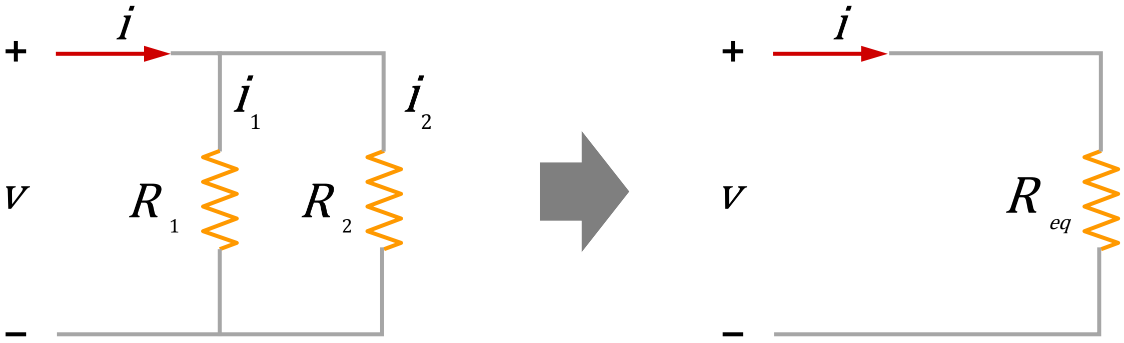


$$v_2 = \frac{R_2}{R_1 + R_2} v = 0.75v$$

$$\frac{R_2}{100 + R_2} = 0.75$$

$$R_2 = 300\ \Omega$$

Parallel Resistor



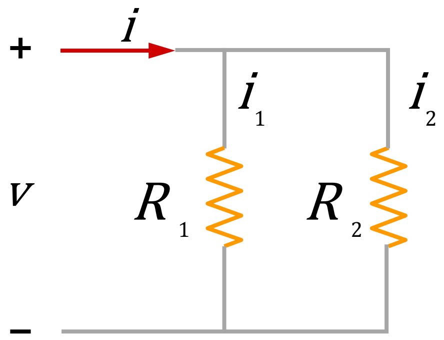
The symbol || is used to indicate a parallel combination of resistors.

For the above circuit, we have $R_{eq} = R_1||R_2$.

For N resistors connected in parallel, we have $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}$.

If $R_1 = R_2 = \cdots = R_N = R,\ R_{eq} = R/N$

Current Divider



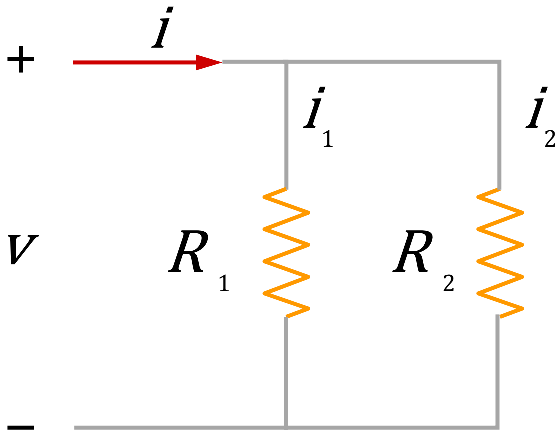
For this circuit with two resistors connected in parallel, we have

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$

The circuit is known as the current divider.

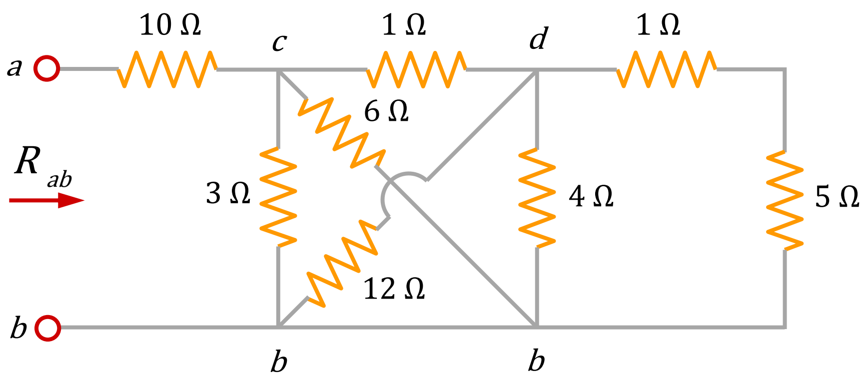
Current Divider



Note that when $R_2 = 0$ (a short-circuit has occurred), $i_1 = 0$ and $i_2 = i$, i.e., the input current i will bypass R_1 and flow through the short-circuited path.

Example 9

Calculate the equivalent resistance R_{ab} in the following circuit.



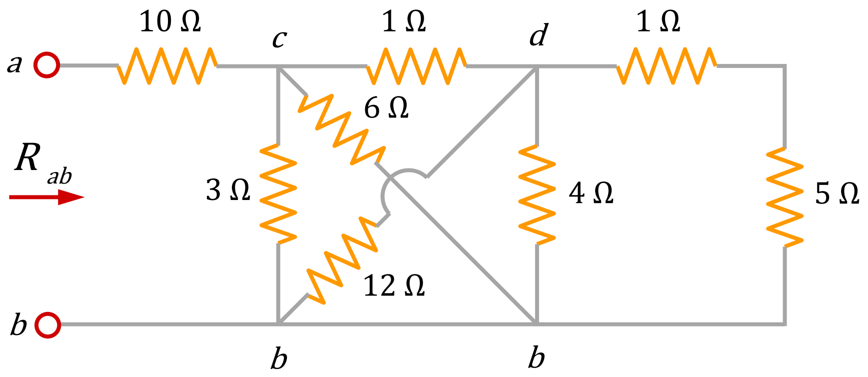
The $3\ \Omega$ and $6\ \Omega$ resistors are in parallel as they are connected to the same two nodes, c and b.

Their combined resistance is

$$3\ \Omega || 6\ \Omega = \frac{3 \times 6}{3+6} = 2\ \Omega$$

Example 9

Also, the $12\ \Omega$ and $4\ \Omega$ resistors are in parallel as they are connected to the same two nodes d and b.

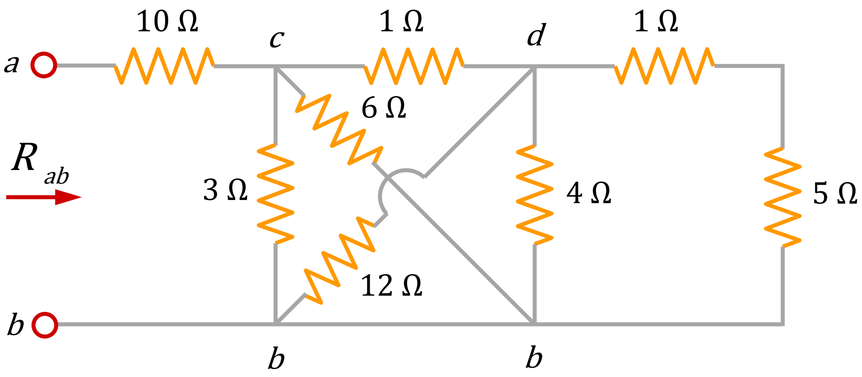


Hence,

$$12\ \Omega || 4\ \Omega = \frac{12 \text{ x } 4}{12 + 4} = 3\ \Omega$$

Example 9

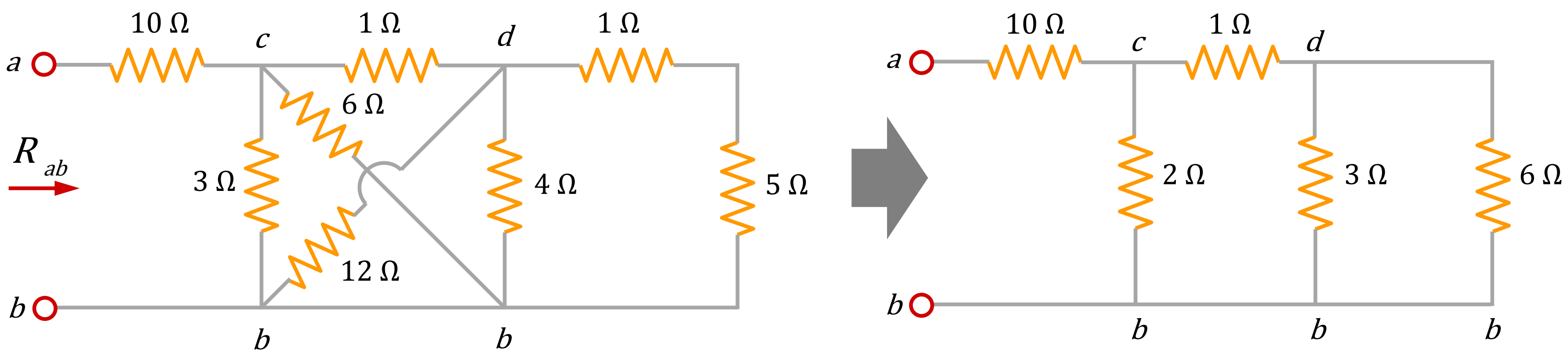
The $1\ \Omega$ and $5\ \Omega$ resistors are in series.



Hence, their equivalent resistance is $\boxed{1 + 5 = 6\ \Omega}$

Example 9

With these combinations, we have the following circuit.

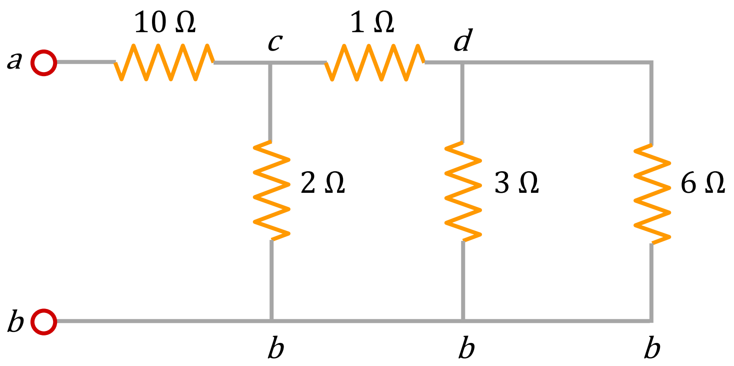


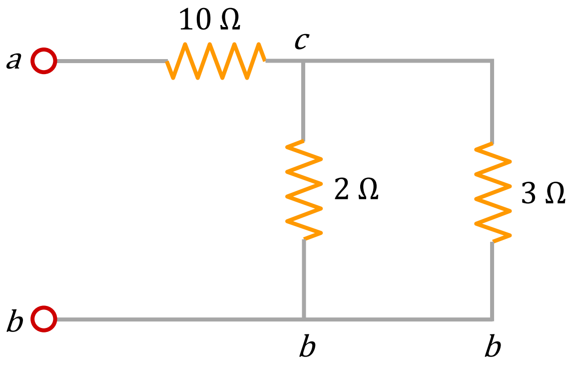
Example 9

The parallel combination of the $3\ \Omega$ and $6\ \Omega$ resistors gives $2\ \Omega$.

This $2\ \Omega$ equivalent resistance is now in series with the $1\ \Omega$ resistance to give a combined resistance of $\boxed{1 + 2 = 3\ \Omega}$

With these combinations, we have the following circuit.



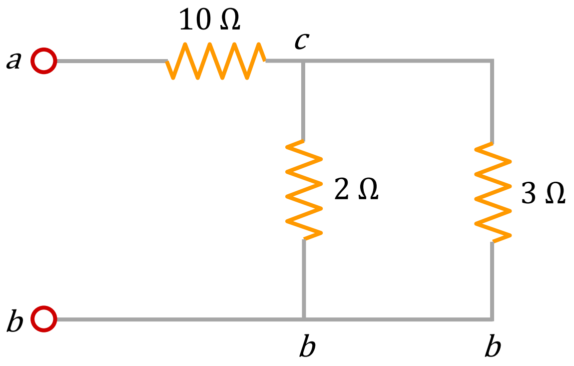


Example 9

The $2\ \Omega$ and $3\ \Omega$ resistors in parallel gives $1.2\ \Omega$.

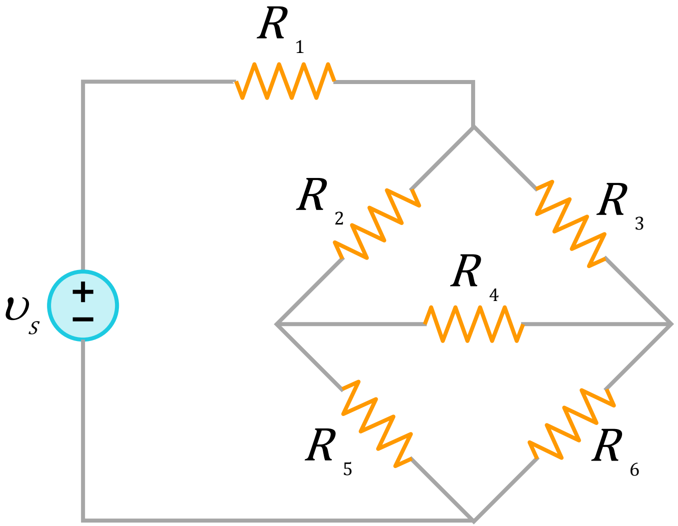
This $1.2\ \Omega$ resistor is in series with the $10\ \Omega$ resistor so that

$$\boxed{R_{ab} = 10 + 1.2 = 11.2\ \Omega}$$



Y-Δ (Wye-Delta) Transformation

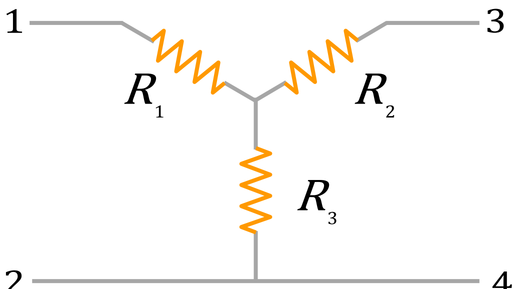
Situations often arise in circuit analysis where the resistors are neither in parallel nor in series (see the following bridge circuit).



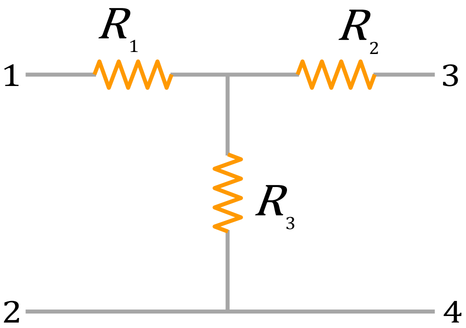
Many circuits of the type shown can be simplified to a three-terminal equivalent network shown next.

Y-Δ (Wye-Delta) Transformation

Y or Tee (T) Network

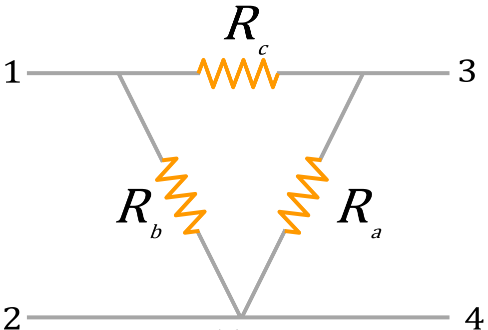


(a) Y

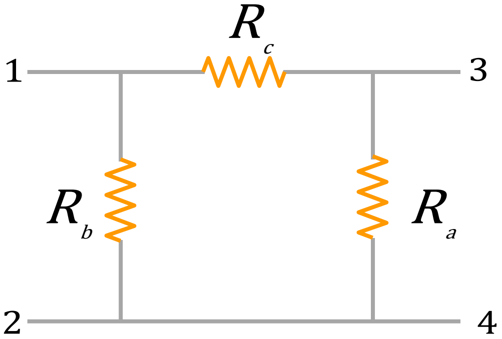


(b) T

Δ or pi (Π) Network



(a) Δ



(b) pi

Δ to Y conversion (Δ known)

If it is more convenient to work with a Y network in a place where the circuit contains a Δ configuration.

We superimpose a Y network on the existing Δ network and find the equivalent resistances in the Y network.

To obtain the equivalent resistances in the Y network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ network is the same as the resistance between the same pair of nodes in the Y network.

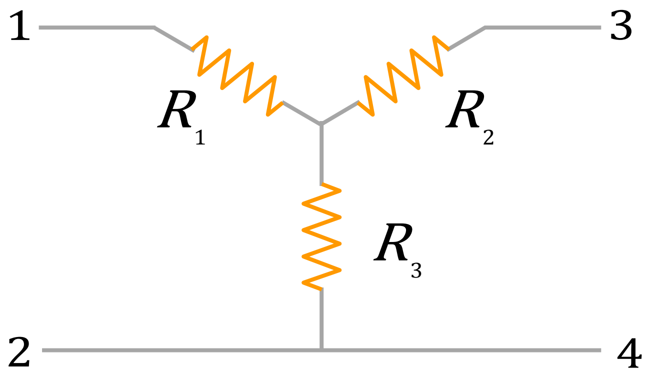
Δ to Y conversion (Δ known)

For example, for terminals 1 and 2, $R_{12}(\mathrm{Y}) = R_1 + R_3$.

And $R_{12}(\Delta) = R_b || (R_a + R_c)$.

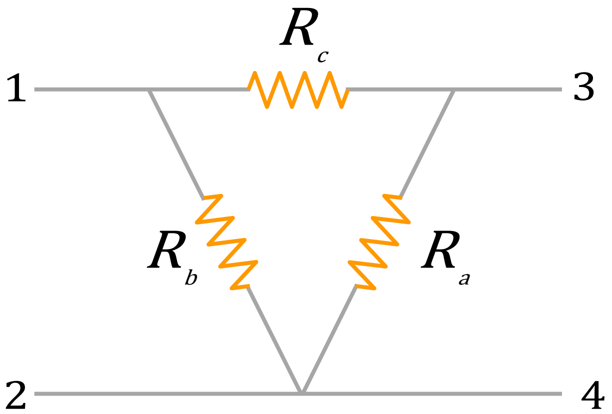
Then, we set $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$.

Y Network



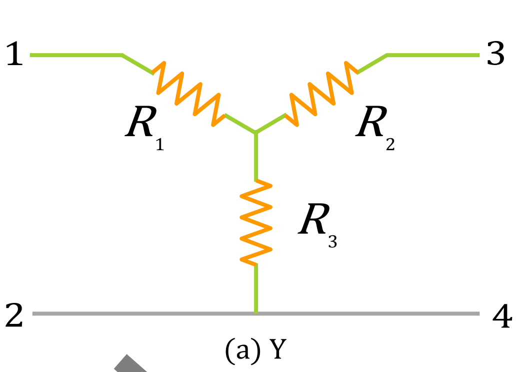
(a) Y

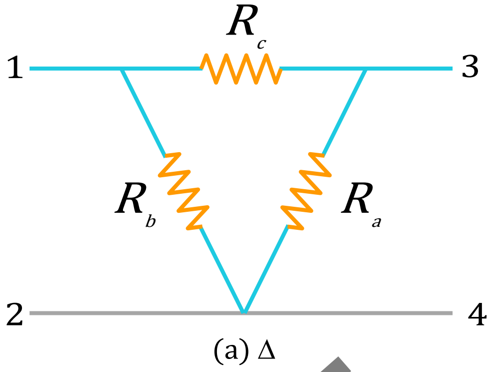
Δ Network

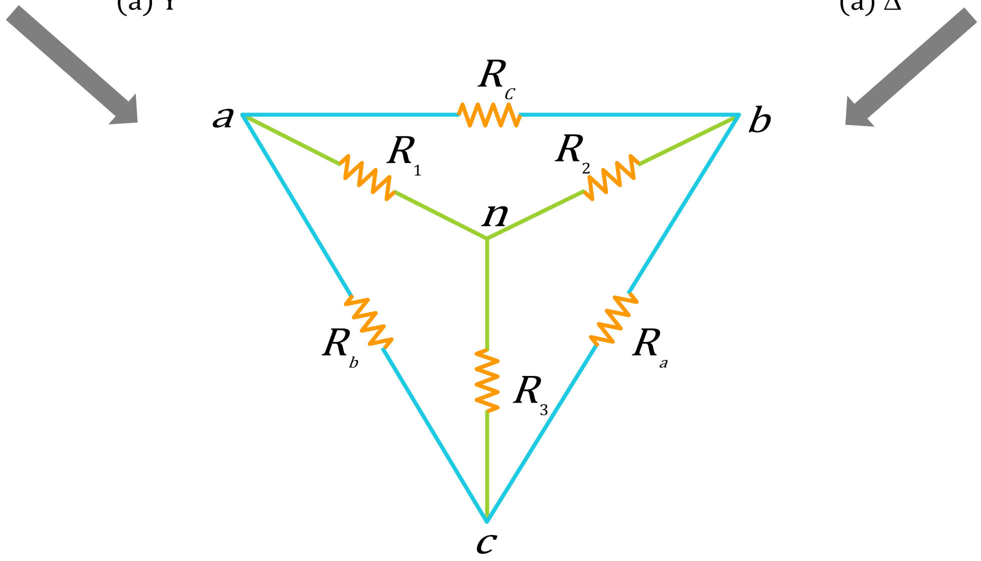


(a) Δ

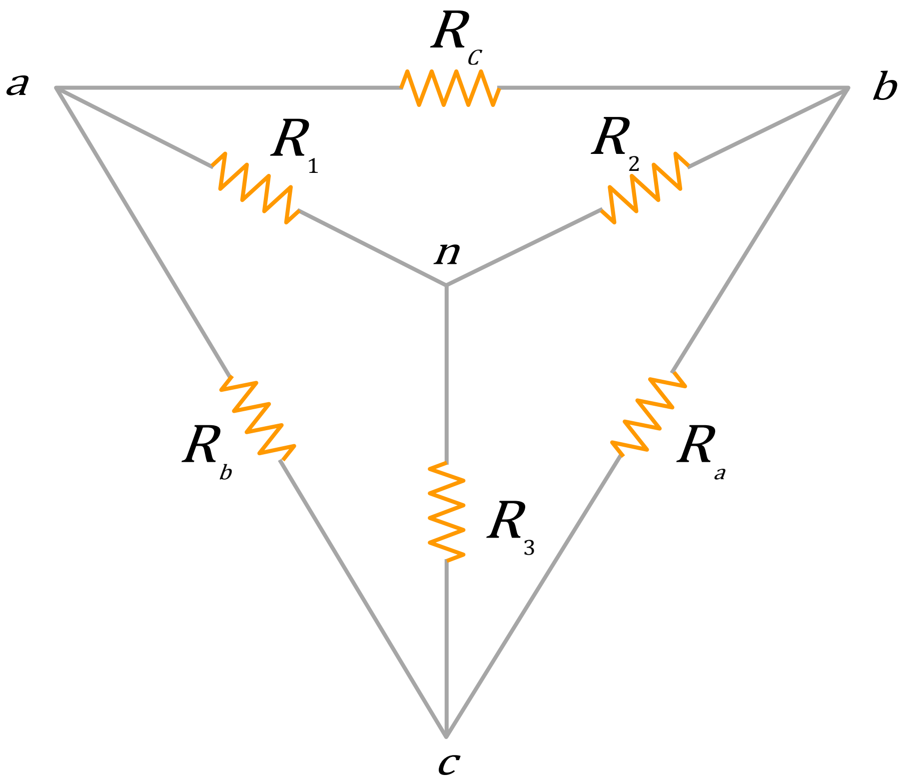
Δ to Y conversion (Δ known)







Δ to Y conversion (Δ known)



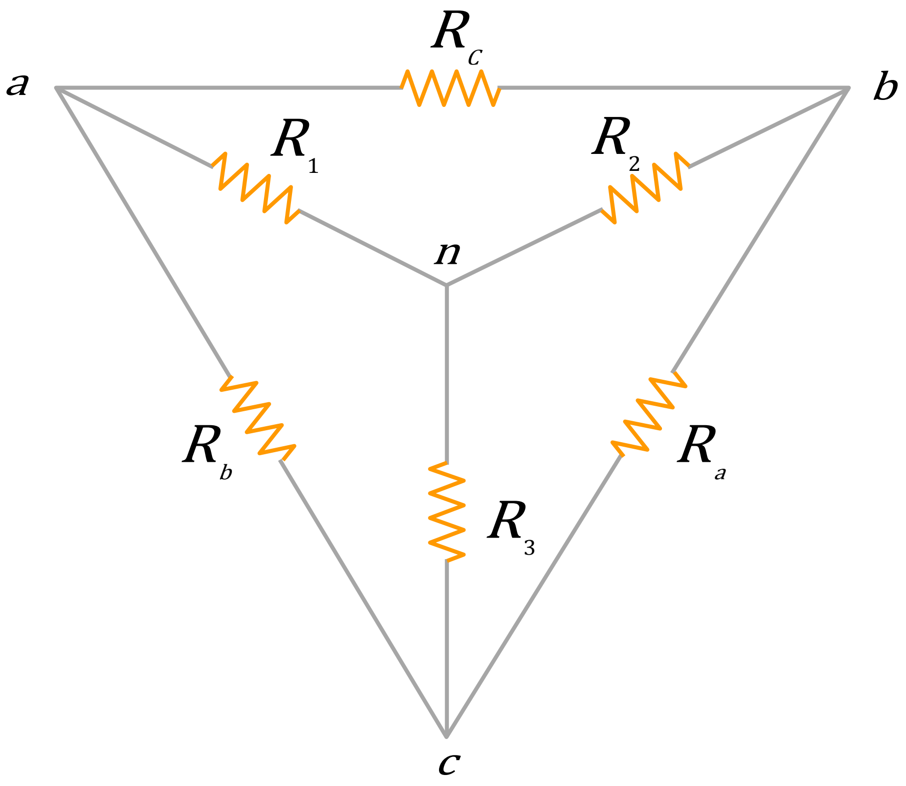
The conversion formula for a **delta to wye** transformation is:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y to Δ Conversion (Y known)



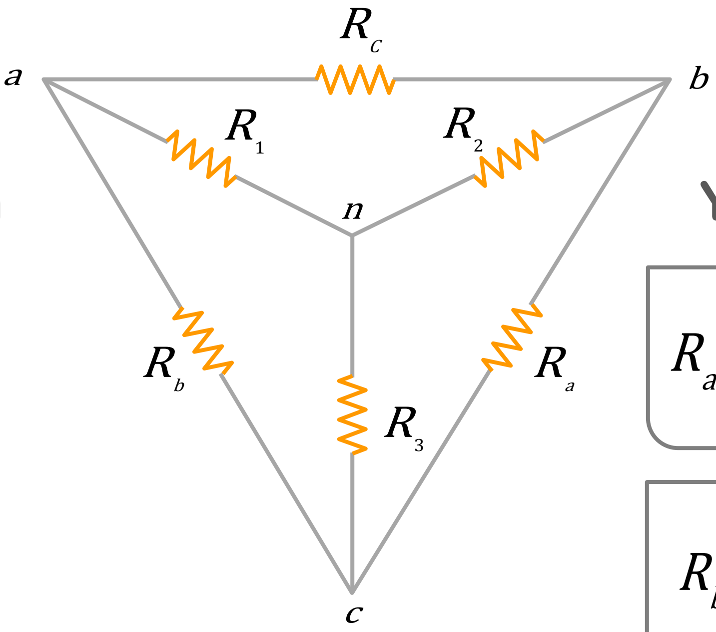
The conversion formula for a **wye to delta** transformation is:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$\Delta \leftrightarrow$ Y Conversion



Δ to Y Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y to Δ Conversion

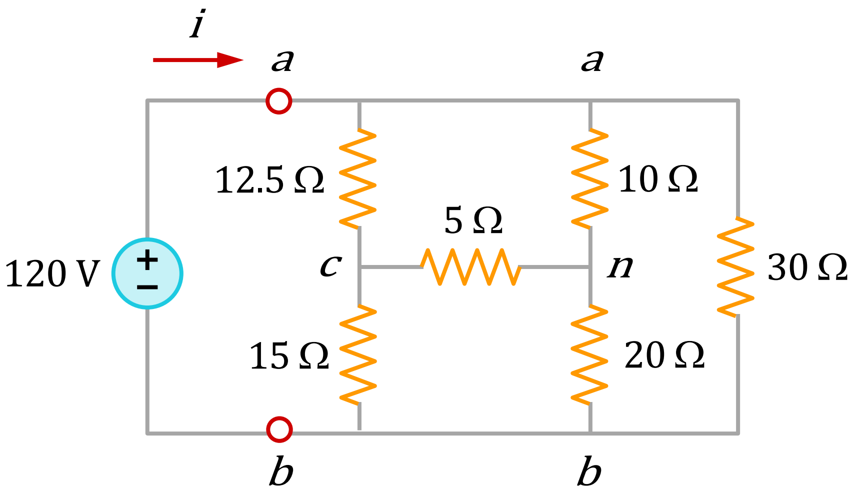
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Example 10

Obtain the equivalent resistance R_{ab} for the circuit shown and use it to find current i.

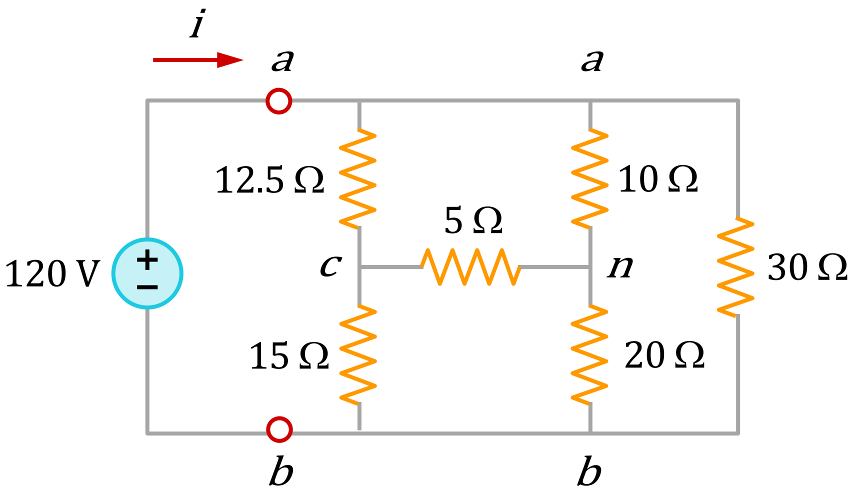


There are 2 Y networks (one at n and one at c), and 3 delta networks (can, abn, cnb).

We only need to transform one Y network comprising the $5\ \Omega$, $10\ \Omega$ and $20\ \Omega$ resistors to a Δ network.

Example 10

We let $R_1 = 10\ \Omega,\ R_2 = 20\ \Omega\ , R_3 = 5\ \Omega$ **and we obtain the following using the conversion formulae:**



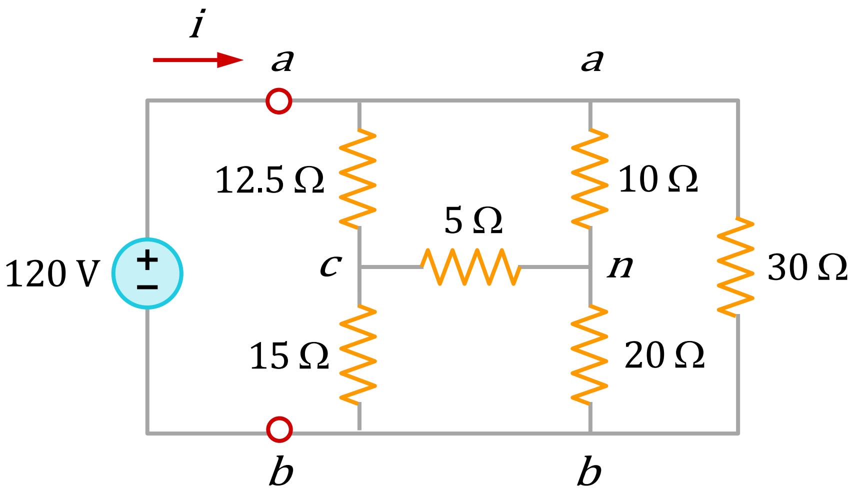
$$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{350}{10} = 35\ \Omega$$

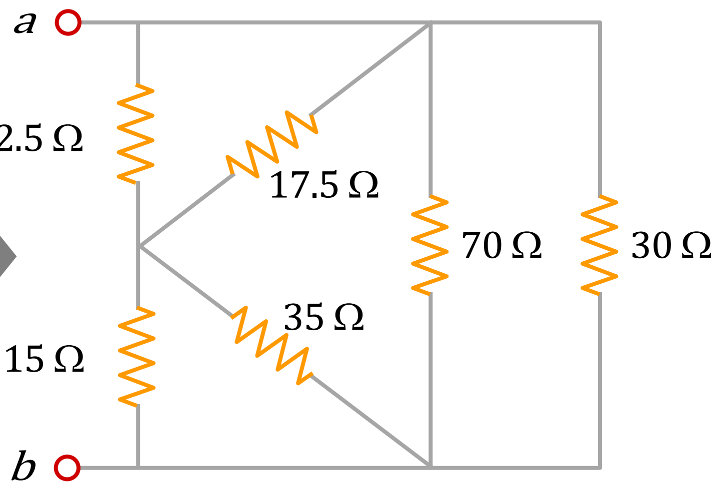
$$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

Example 10

With the Y converted to Δ, the equivalent circuit (with the voltages source removed for now) is shown as follows:





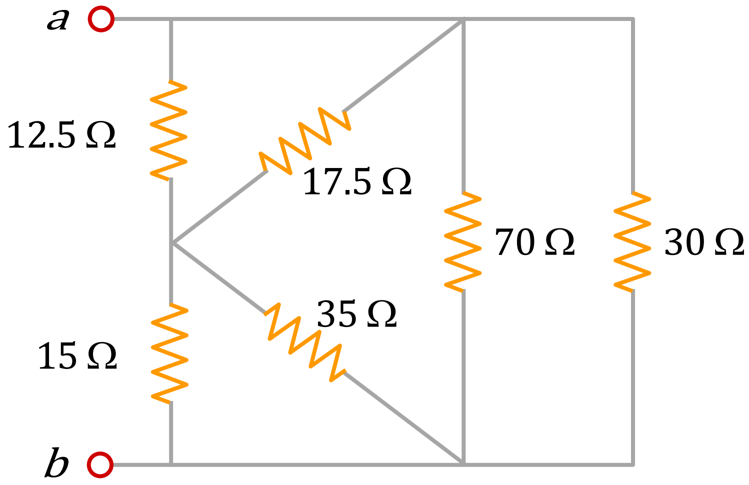
Example 10

Combining the 3 pairs of resistors in parallel we obtain,

$$70||30 = 21\ \Omega$$

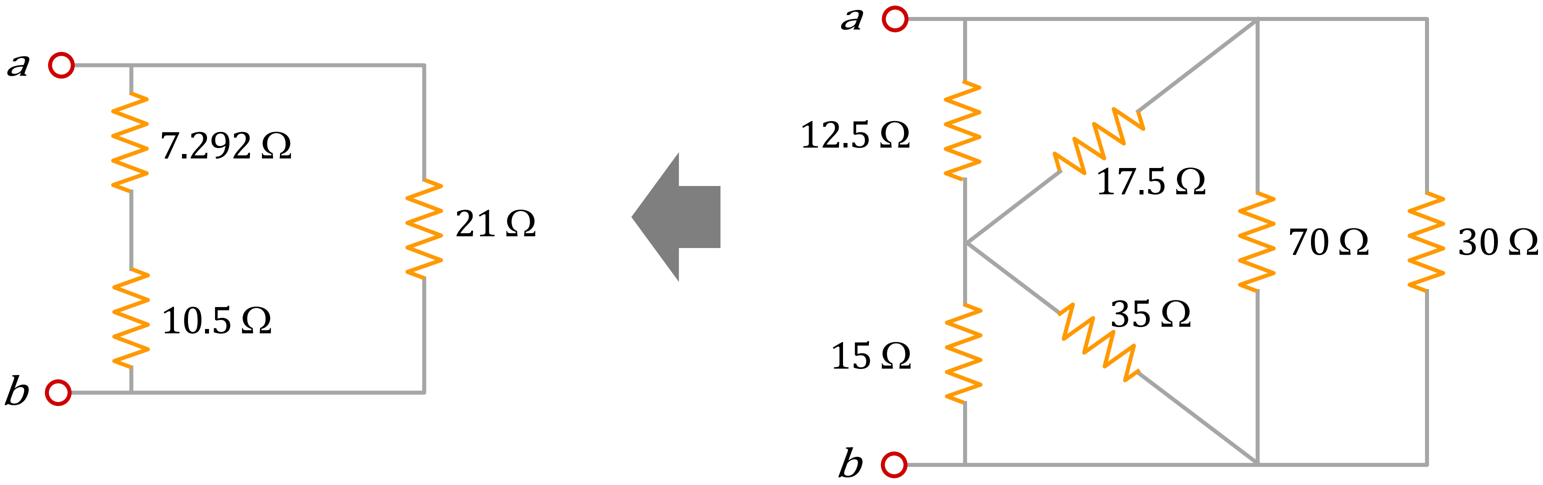
$$12.5||17.5 = 7.292\ \Omega$$

$$15||35 = 10.5\ \Omega$$



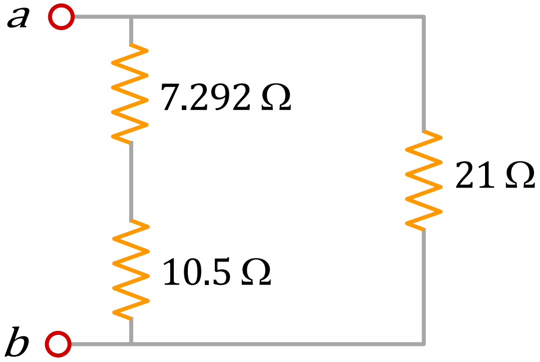
Example 10

So that the equivalent circuit is as shown.



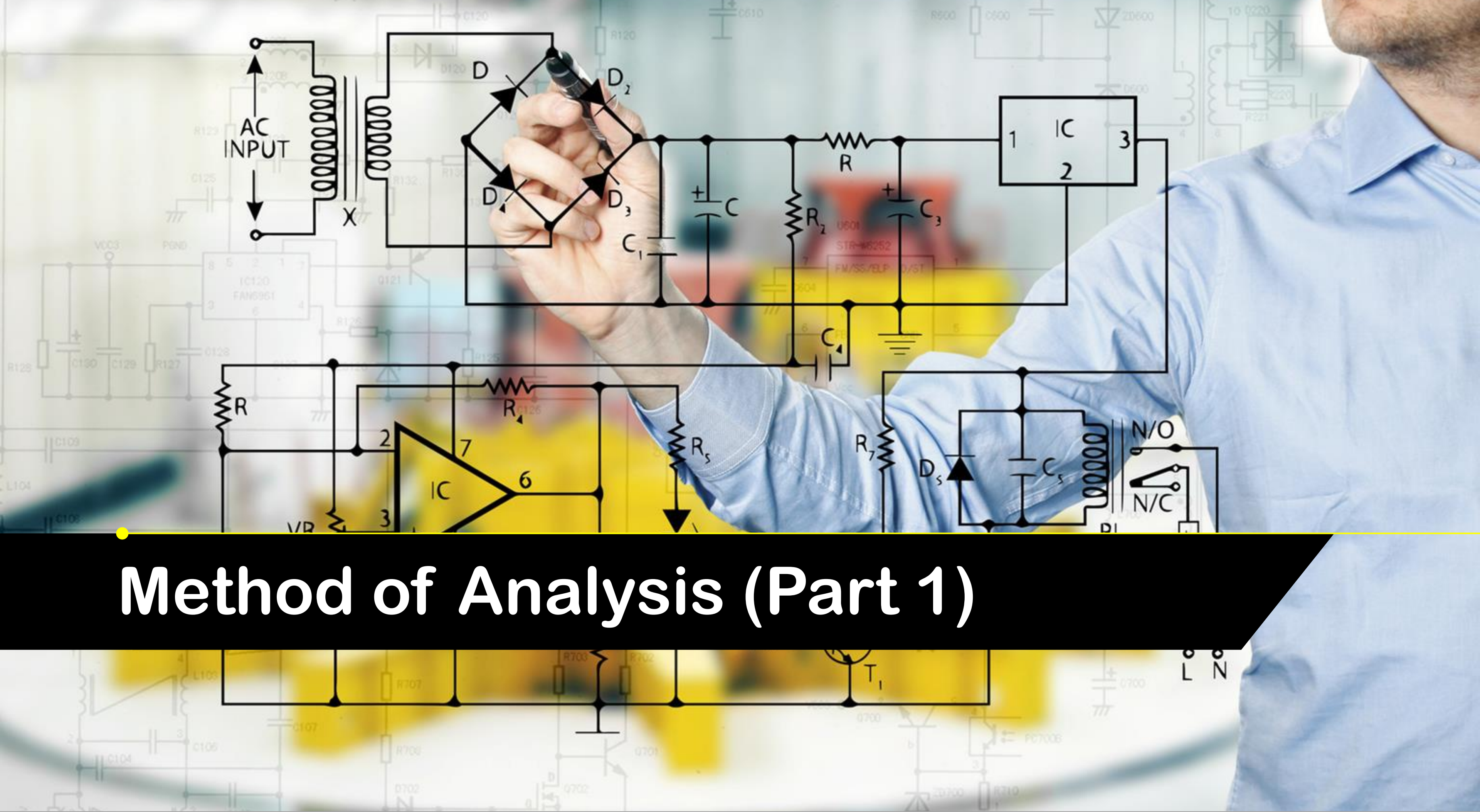
Example 10

So that the equivalent circuit is as shown.



Hence, $$\boxed{R_{ab} = (7.292 + 10.5)||21 = 9.632\ \Omega}$$

Then, $$\boxed{i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ \text{A}}$$

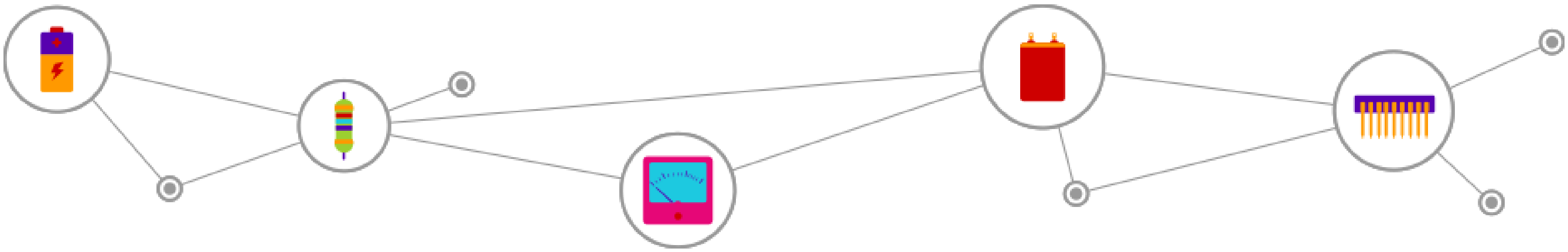


Method of Analysis (Part 1)

Method of Analysis

Nodal Analysis

Mesh Analysis



Overview

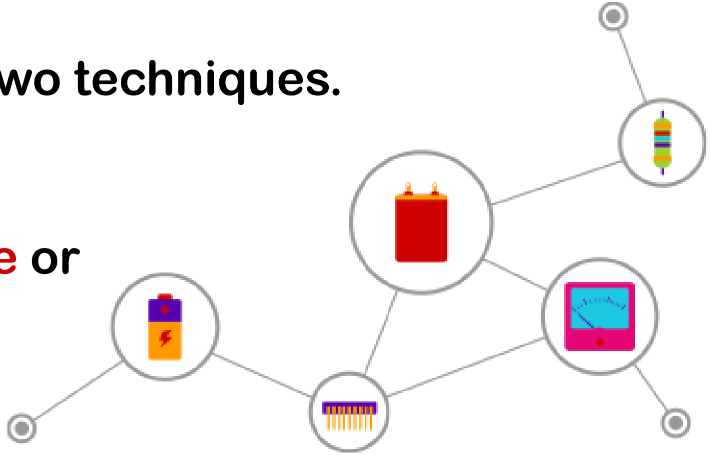
With Ohm’s and Kirchhoff’s law established, we are now ready to apply them in circuit analysis.

Two powerful techniques:

- Nodal analysis, which is based on KCL.
- Mesh analysis, which is based on KVL.

Any linear circuit can be analysed using these two techniques.

The analysis will result in a set of simultaneous equations which may be solved by Cramer’s rule or a commercial software such as MATLAB.



Nodal Analysis

It is based on the application of **KCL**.

It uses node voltages as the circuit variables.

In nodal analysis, we are interested in finding the **node voltages**.

Nodal Analysis Without Voltage Sources

To simplify matters, we begin with a circuit with n nodes without voltage sources.

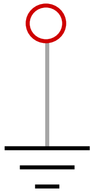
The nodal analysis of the circuit is as follows:

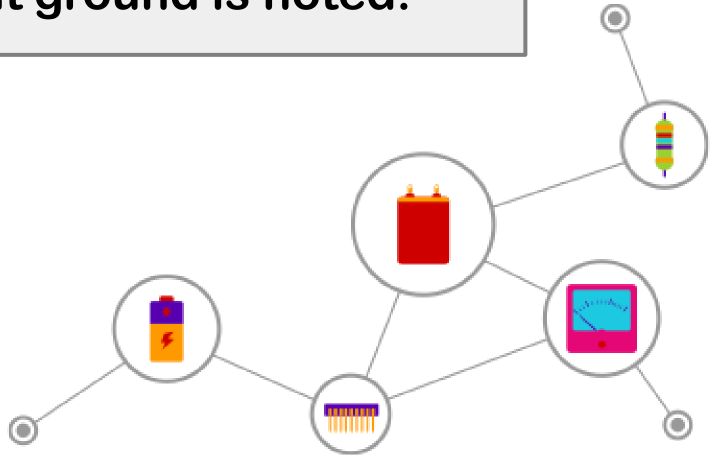
1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the (n-1) non-reference nodes. Use Ohm's law to express the branch currents in terms of the node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Points to Note

- The reference node is commonly called the ground since it is assumed to have zero potential.
- If there is a ground node, it is usually most convenient to select it as the reference node.
- Often, the bottom node of a circuit is selected as the reference node, especially if no explicit ground is noted.

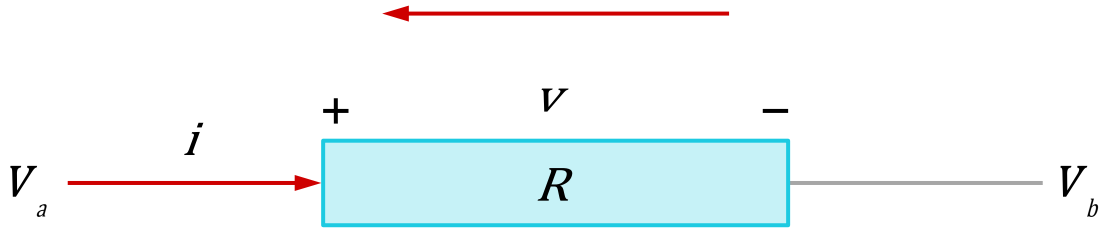
The symbol of the reference node used is as follows:





Points to Note

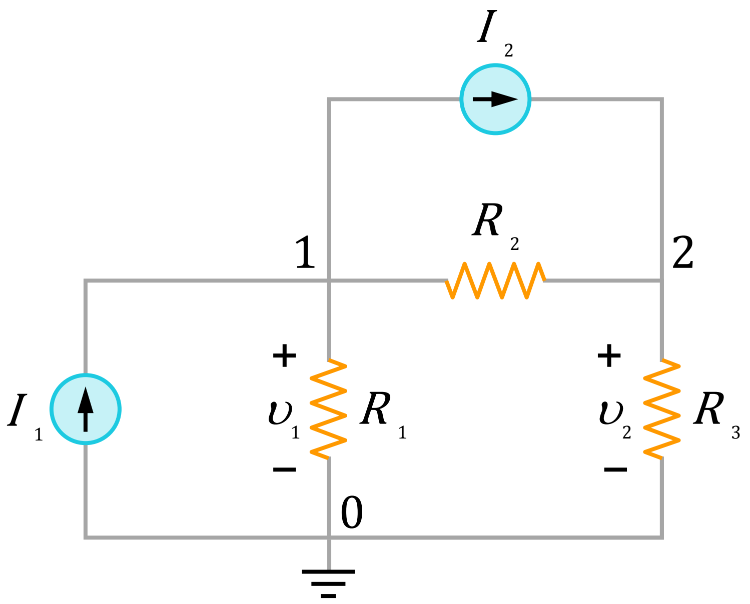
Recall that by the passive sign convention, current must always flow from a higher potential to a lower potential in a resistor, as seen below.



$$\boxed{i = \frac{v_a - v_b}{R}}$$

Example 11

Calculate the node voltages in the circuit as shown.

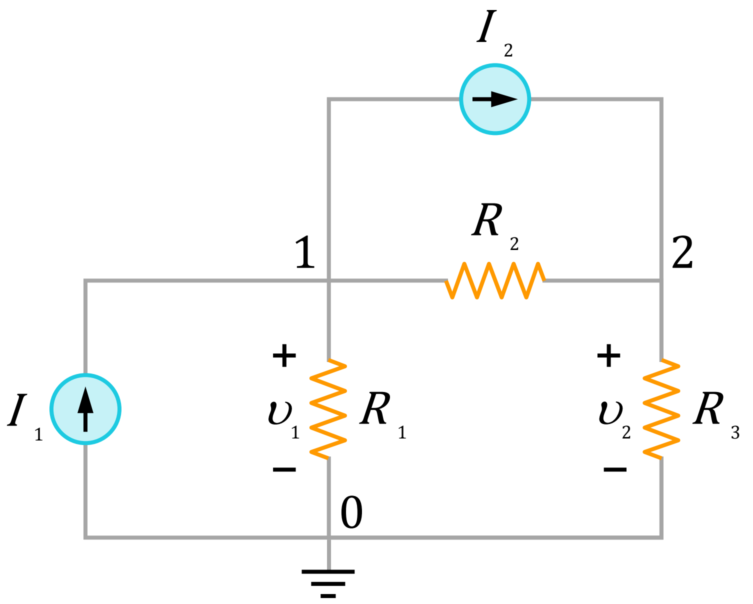


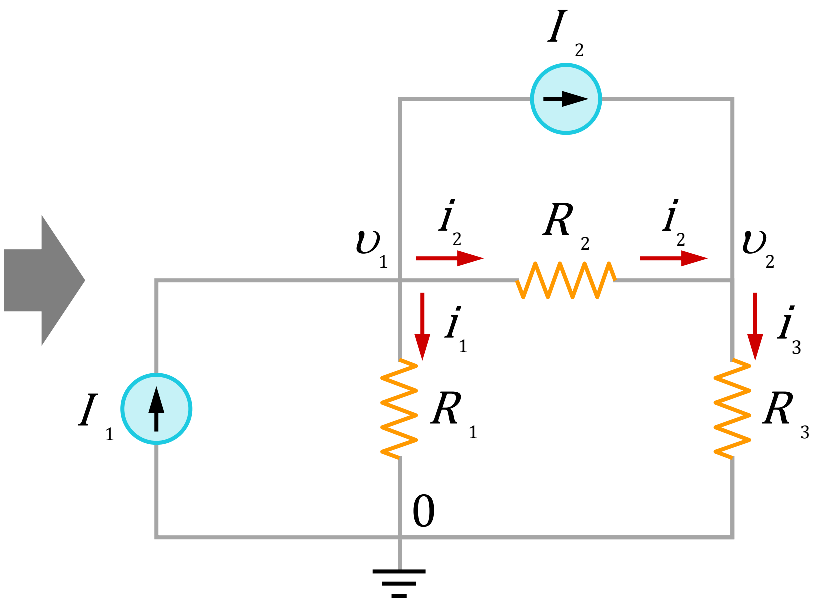
Node 0 is the reference node ($v = 0$), while nodes 1 and 2 are assigned voltages v_1 and v_2 respectively.

These node voltages are defined with respect to the reference node.

Example 11

Redraw the circuit by adding i_1, i_2 and i_3 as the currents through resistors R_1, R_2 and R_3 respectively.



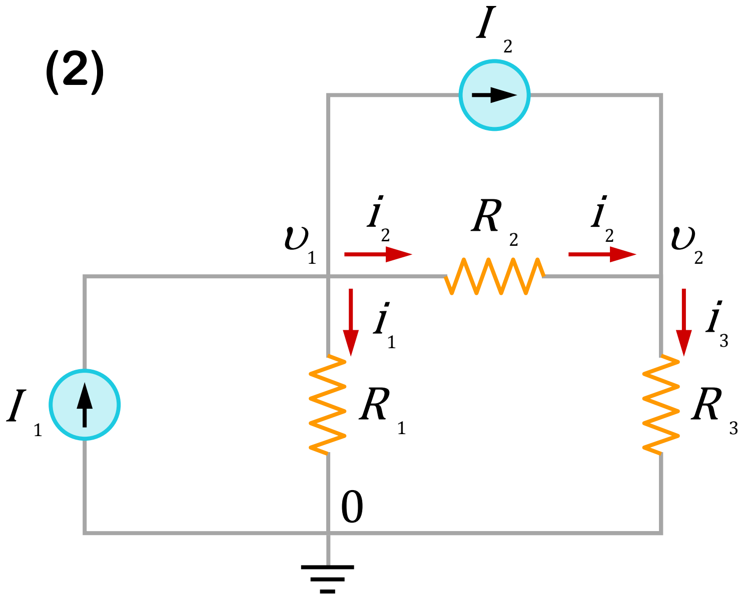


Example 11

Applying the **KCL** to each non-reference node,

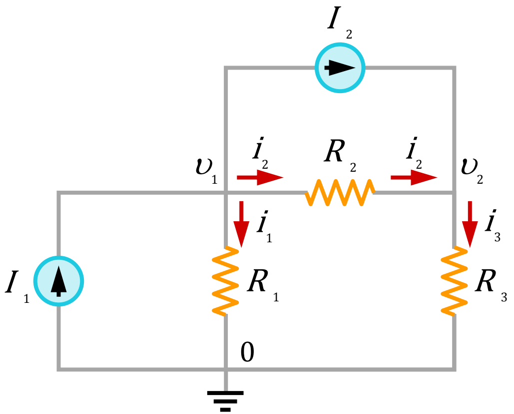
Node 1: $$I_1 - I_2 - i_1 - i_2 = 0 \quad (1)$$

Node 2: $$I_2 + i_2 - i_3 = 0 \quad (2)$$



Example 11

Now, apply **Ohm's Law** to express the unknown currents i_1, i_2 and i_3 in terms of the node voltages:



$$i_1 = \frac{v_1 - 0}{R_1}$$

$$i_2 = \frac{v_1 - v_2}{R_2}$$

$$i_3 = \frac{v_2 - 0}{R_3}$$

Using these in (1) and (2) gives

$$I_1 - I_2 - i_1 - i_2 = 0 \quad \text{(1)} \quad \Rightarrow \quad I_1 - I_2 - \frac{v_1}{R_1} - \frac{v_1 - v_2}{R_2} = 0 \quad \text{(1)}$$

$$I_2 + i_2 - i_3 = 0 \quad \text{(2)} \quad \Rightarrow \quad I_2 + \frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} = 0 \quad \text{(2)}$$

Example 11

Rearranging these yields,

$$I_1 - I_2 - \frac{v_1}{R_1} - \frac{v_1 - v_2}{R_2} = 0 \quad \textbf{(1)} \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{1}{R_2} v_2 = I_1 - I_2 \quad \textbf{(1)}$$

$$I_2 + \frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} = 0 \quad \textbf{(2)} \Rightarrow -\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2 = I_2 \quad \textbf{(2)}$$

Example 11

We can now solve for the node voltages v_1 and v_2 using the elimination method or Cramer's rule.

The equations can be rewritten into the matrix form:

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_1-\frac{1}{R_2}v_2=I_1-I_2 \quad \textbf{(1)}$$

$$-\frac{1}{R_2}v_1+\left(\frac{1}{R_2}+\frac{1}{R_3}\right)v_2=I_2 \quad \textbf{(2)}$$

$$\begin{bmatrix}\frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2}\\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}I_1-I_2\\ I_2\end{bmatrix}$$

Example 11

Here, **A** is a 2 x 2 matrix.

x and **b** are 2 x 1 column vectors.

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\underbrace{\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}}_{\mathbf{b}}$$

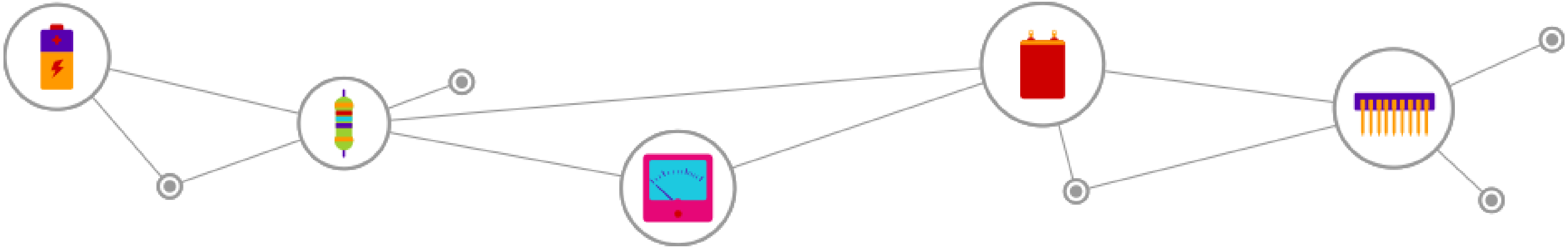
We can then use the Cramer's rule to solve for the node voltages v_1 and v_2.

Solution Using Cramer's Rule

If **A** is square and non-singular, then the solution to **Ax = b** can be computed using the Cramer's rule as follows:

$$\boxed{x_i = D_i / D}$$

Where D = det(A), D_i = det(A_i), and A_i is a matrix obtained from A by replacing the ith column of A with b.



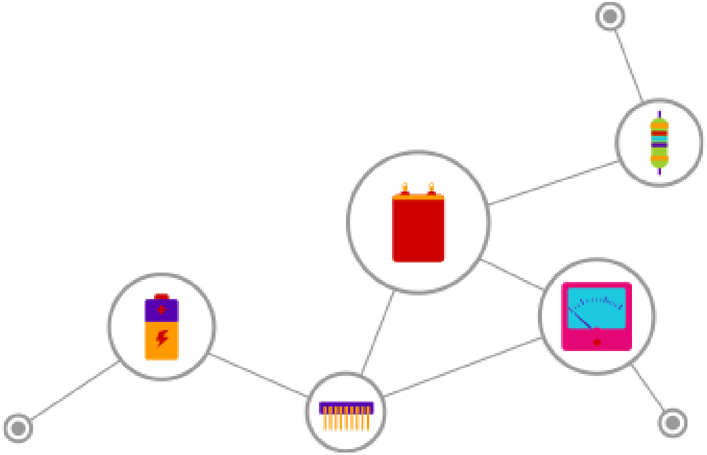
Computation of Determinants

The determinant of an n × n matrix, denoted by |A| or det(A), is a scalar.

Let A = $[aij]$

For n = 2

$$\det(\mathbf{A}) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$



Computation of Determinants

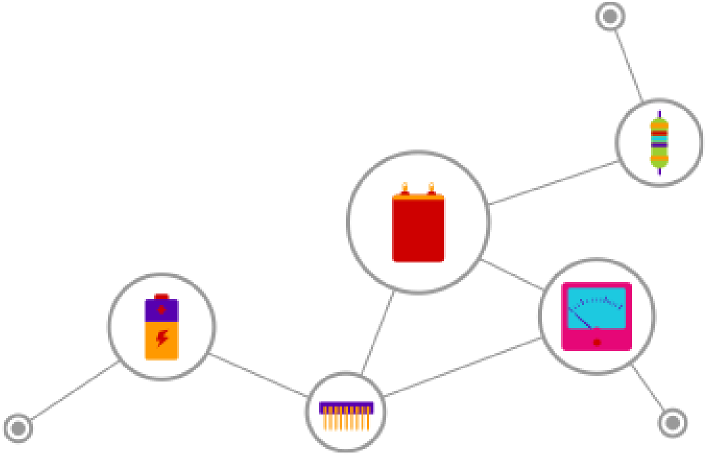
For n = 3

$$\det(\mathbf{A}) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{aligned} &= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ &\quad - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{aligned}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

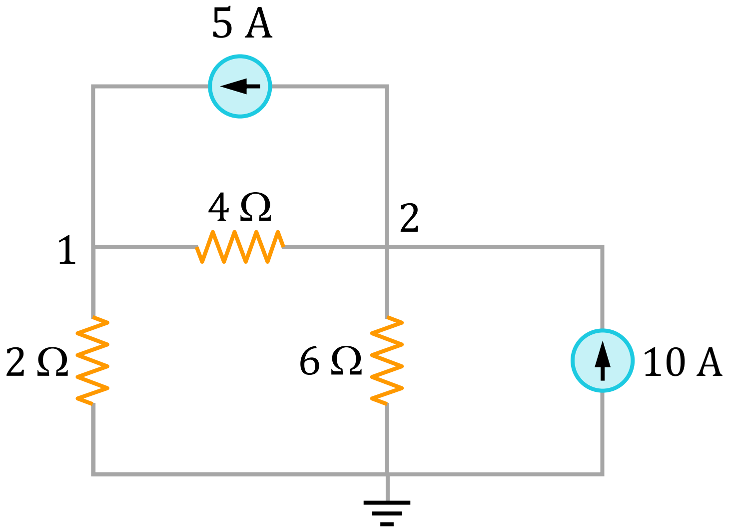
$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

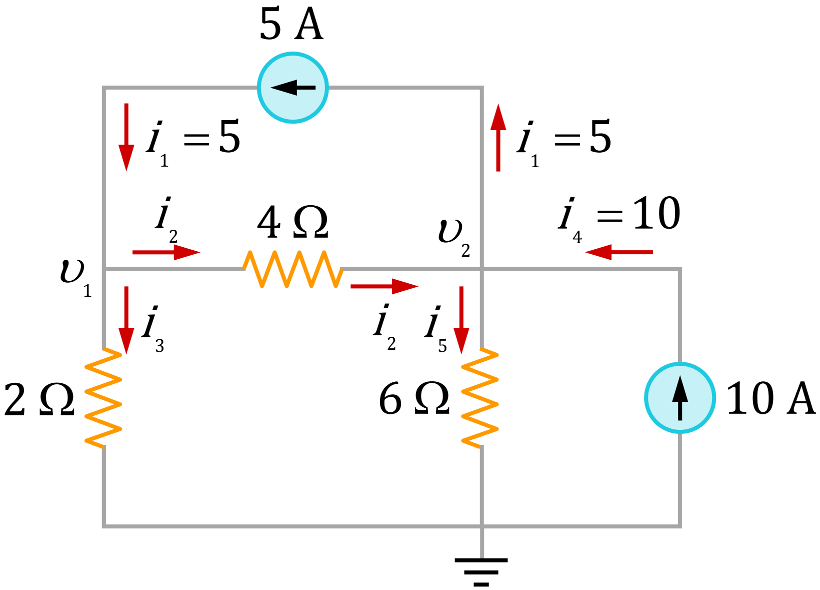


Example 12

Calculate the node voltages of the following circuit.

Redraw the circuit as shown. Note that the labeling of currents is arbitrary.



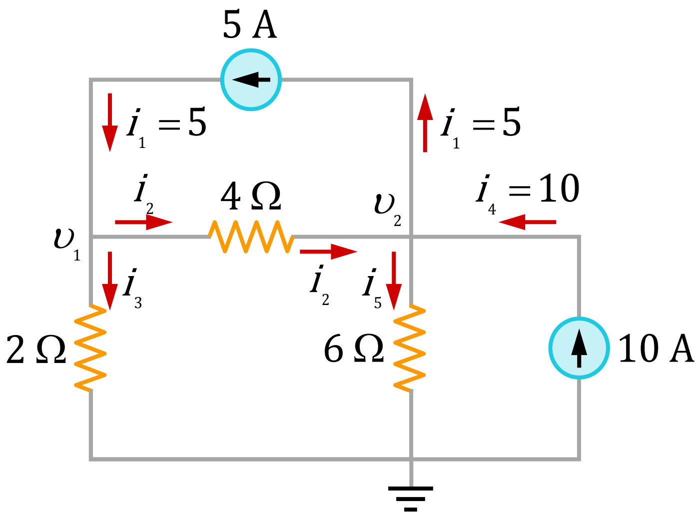


Example 12

Applying KCL and Ohm's Law,

Node 1: $i_1 - i_2 - i_3 = 0$ ➡ $5 - \frac{v_1 - v_2}{4} - \frac{v_1 - 0}{2} = 0$

Node 2: $i_2 + i_4 - i_1 - i_5 = 0$ ➡ $\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$

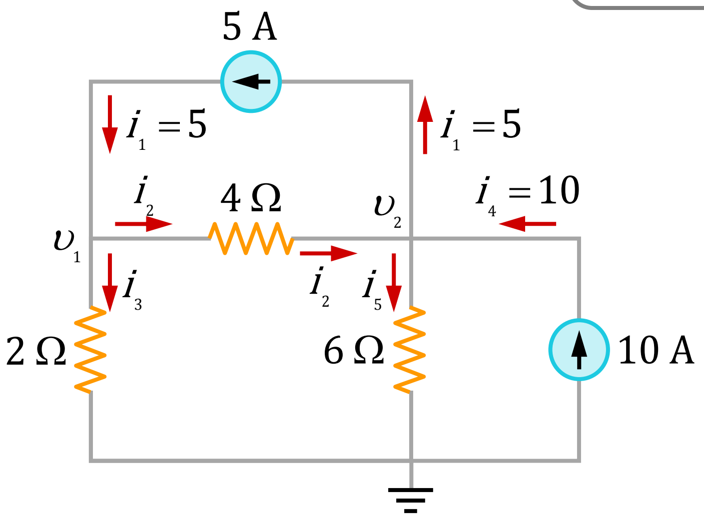


Example 12

The two equations can be written as follows:

$$5 - \frac{v_1 - v_2}{4} - \frac{v_1 - 0}{2} = 0 \quad \Rightarrow \quad \frac{3}{4}v_1 - \frac{1}{4}v_2 = 5$$

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6} \quad \Rightarrow \quad \frac{1}{4}v_1 - \frac{5}{12}v_2 = -5$$



Example 12

$$\frac{3}{4}v_1 - \frac{1}{4}v_2 = 5$$

$$\frac{1}{4}v_1 - \frac{5}{12}v_2 = -5$$

$$D = \begin{vmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{5}{12} \end{vmatrix} = \left(\frac{3}{4}\right)\left(-\frac{5}{12}\right) - \left(\frac{1}{4}\right)\left(-\frac{1}{4}\right) = -\frac{1}{4}$$

$$D_1 = \begin{vmatrix} 5 & -\frac{1}{4} \\ -5 & -\frac{5}{12} \end{vmatrix} = 5\left(-\frac{5}{12}\right) - (-5)\left(-\frac{1}{4}\right) = -\frac{10}{3}$$

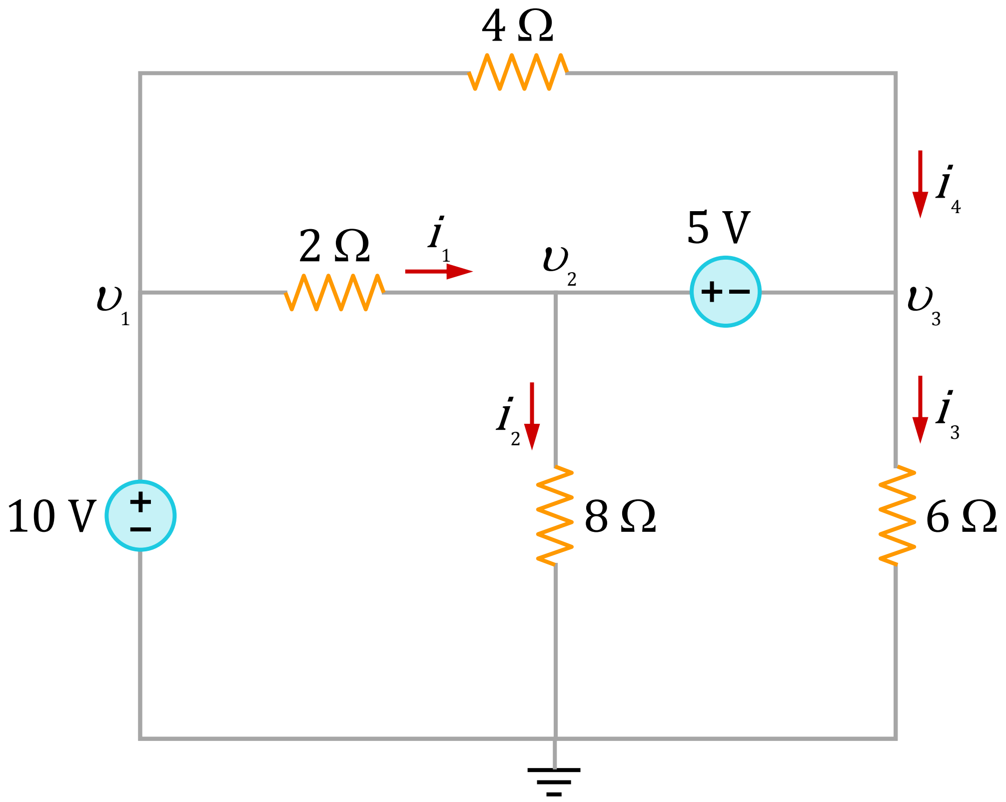
$$D_2 = \begin{vmatrix} \frac{3}{4} & 5 \\ \frac{1}{4} & -5 \end{vmatrix} = \left(\frac{3}{4}\right)(-5) - \left(\frac{1}{4}\right)(5) = -5$$

$$v_1 = \frac{D_1}{D} = \frac{40}{3} = 13.33$$

$$v_2 = \frac{D_2}{D} = 20$$

Nodal Analysis With Voltage Source

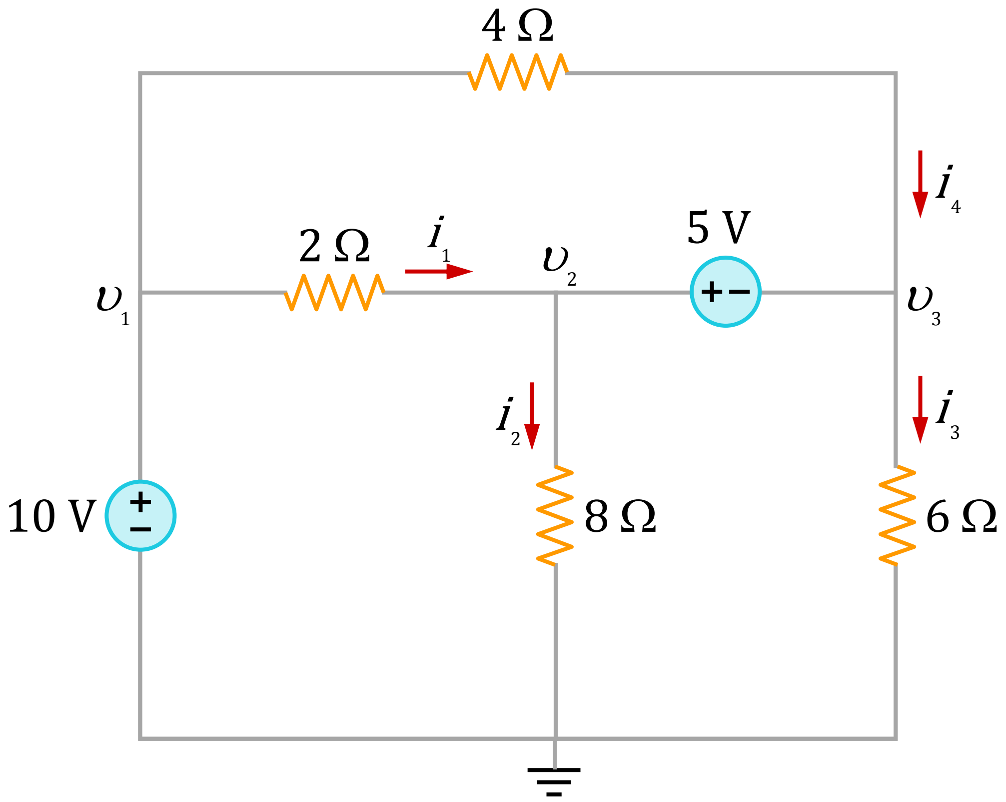
We now consider how voltage sources affect nodal analysis.



If a voltage source is connected between the reference node and a non-reference node, the voltage at the non-reference node is equal to that of the voltage source.

That is, $\boxed{v_1 = 10\ \mathrm{V}}$ **(1)**

Nodal Analysis With Voltage Source

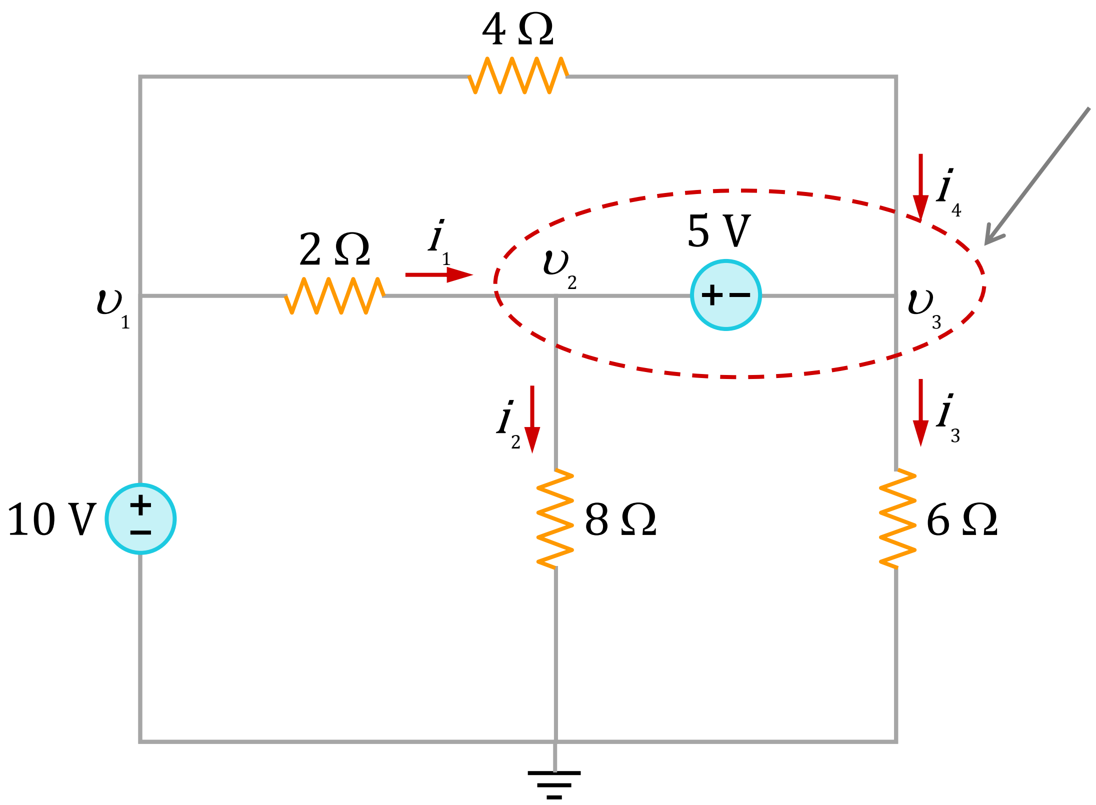


Supernode

If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a supernode.

We apply both KCL and Ohm’s law to determine the node voltages.

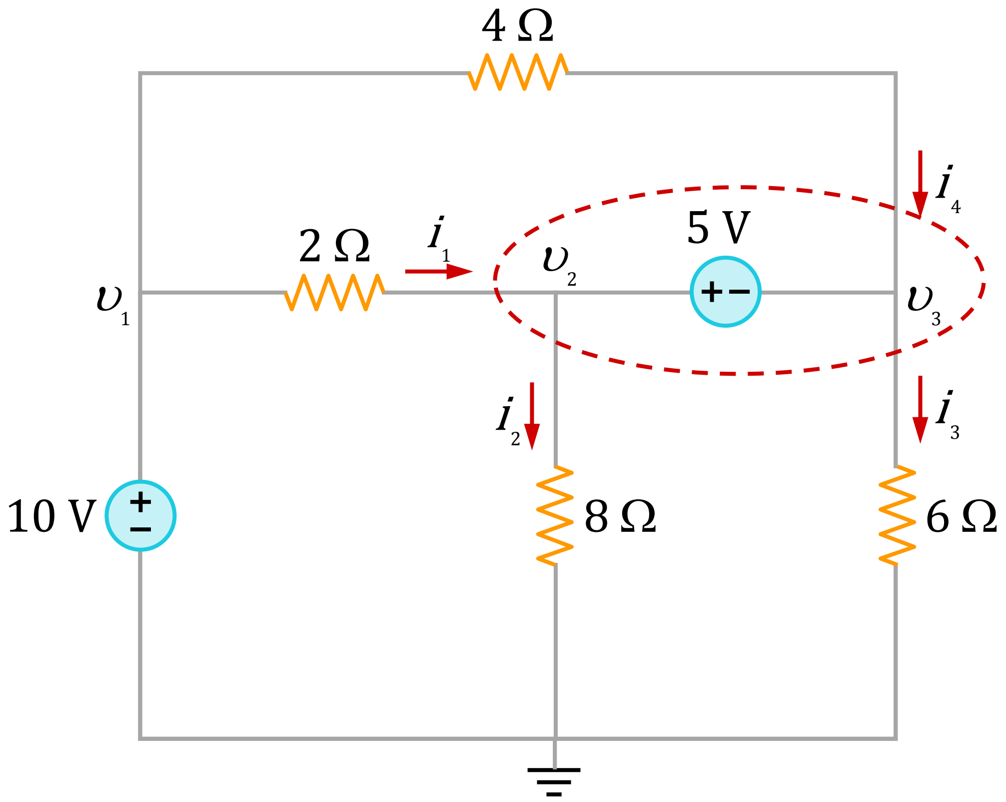
Nodal Analysis With Voltage Source



Supernode

In the circuit, nodes 2 and 3 form a supernode (indicated by the region enclosed by the broken line).

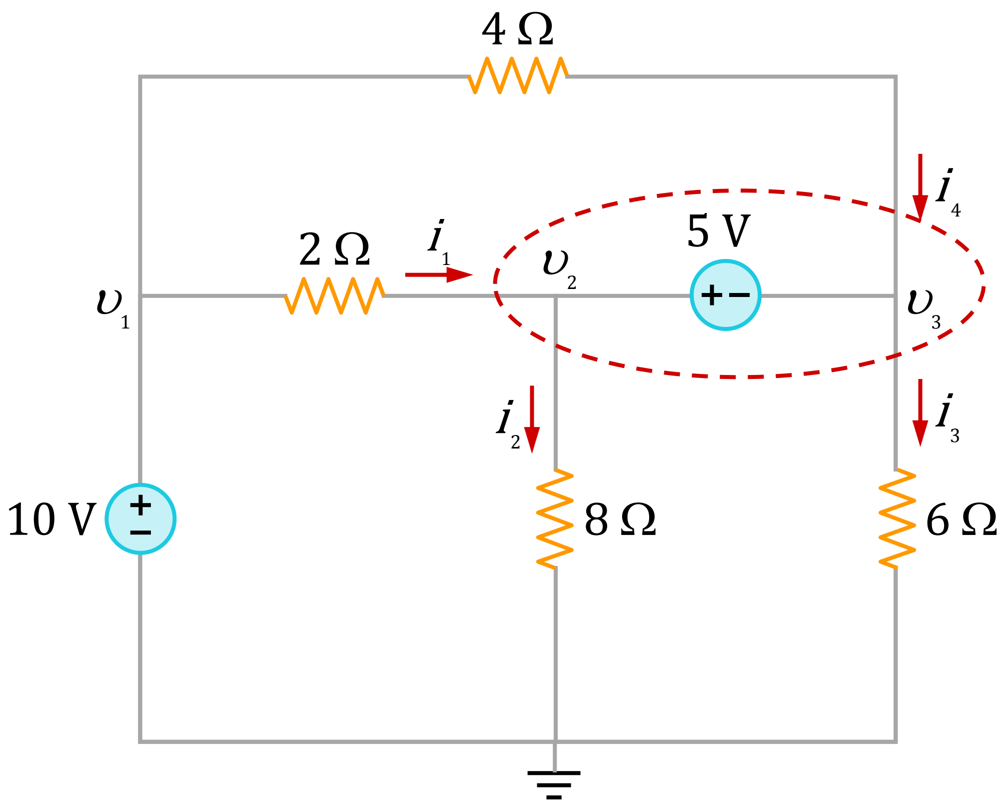
Nodal Analysis With Voltage Source



We analyse the circuit with a supernode using the same 3 steps mentioned before, except that the supernode is treated differently.

We apply KCL to both the nodes by noting that all currents flowing into the region sum to zero.

Nodal Analysis With Voltage Source

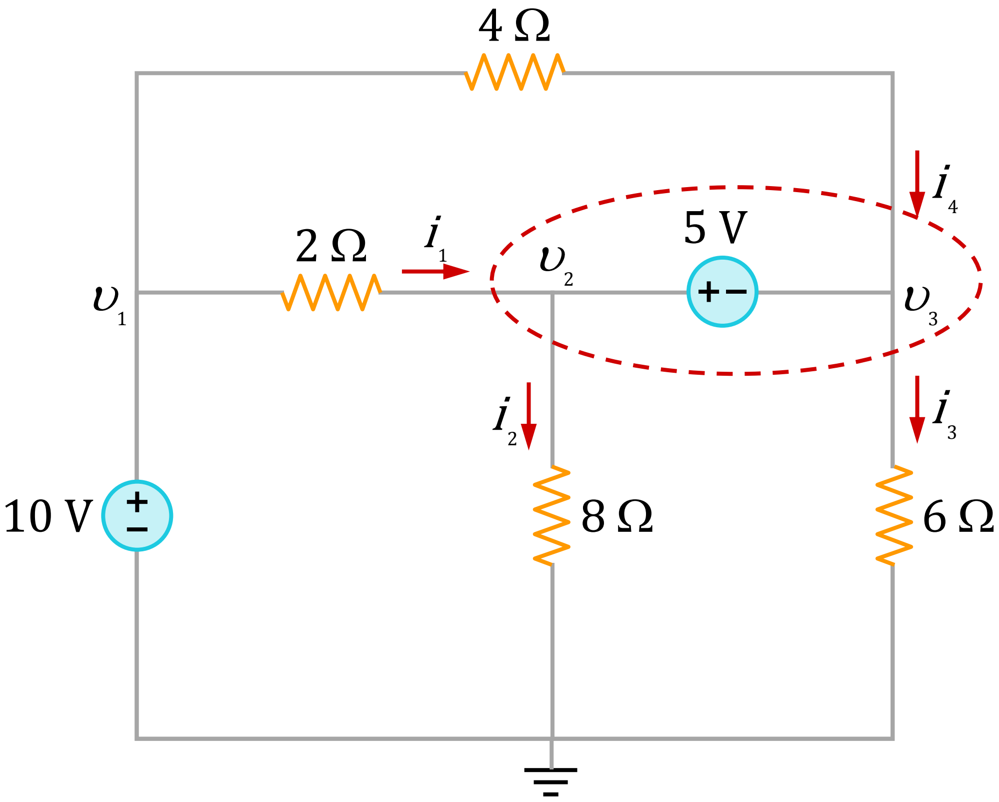


At the supernode of the circuit shown,

$$i_1 + i_4 - i_2 - i_3 = 0$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} - \frac{v_2 - 0}{8} - \frac{v_3 - 0}{6} = 0 \quad \textbf{(2)}$$

Nodal Analysis With Voltage Source

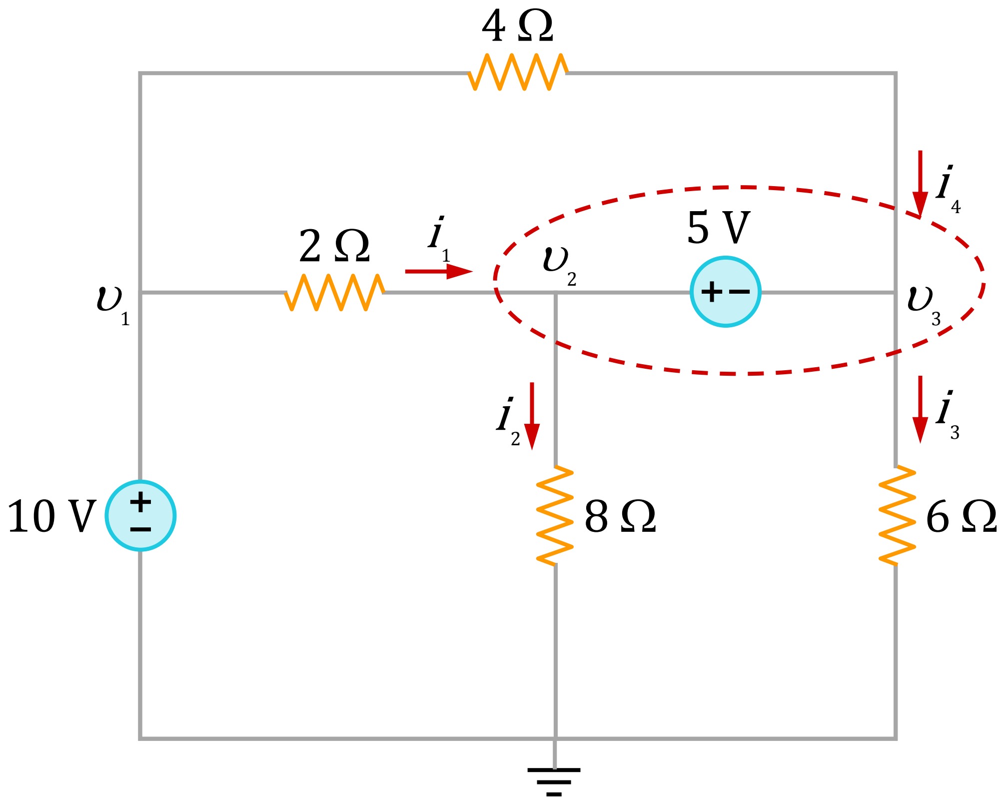


The voltage source inside the supernode provides a constraint equation, i.e., it constrains the difference between the node voltages at these two nodes to be equal to the voltage of the source.

That is, $\boxed{v_2 - v_3 = 5}$ (3)

The constraint equation is needed to solve for the unknown node voltages.

Nodal Analysis With Voltage Source



Equation (2) can be simplified to,

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} - \frac{v_2 - 0}{8} - \frac{v_3 - 0}{6} = 0 \quad \textbf{(2)}$$

$$\frac{3}{4}v_1 = \frac{5}{8}v_2 + \frac{5}{12}v_3 \quad \textbf{(4)}$$

Nodal Analysis With Voltage Source

Using (1) and (3) gives

$$\frac{3}{4}v_1 = \frac{5}{8}v_2 + \frac{5}{12}v_3 \quad \textbf{(4)}$$

$$v_1 = 10\text{ V} \quad \textbf{(1)}$$

$$v_2 - v_3 = 5 \quad \textbf{(3)}$$

$$\frac{30}{4} = \frac{5}{8}v_2 + \frac{5}{12}(v_2 - 5)$$

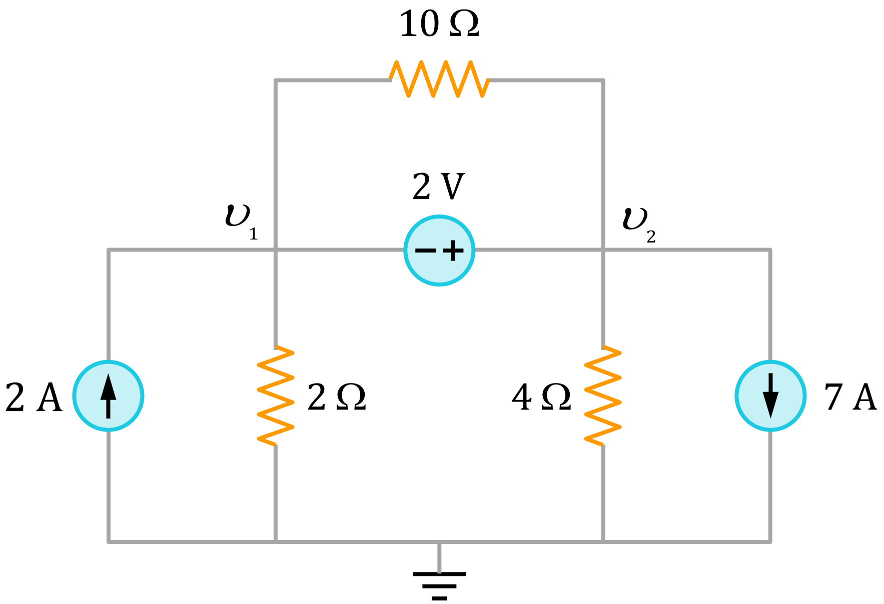
$$\frac{30}{4} = \frac{25}{24}v_2 - \frac{25}{12}$$

$$v_2 = 9.2\text{ V}$$

$$v_3 = v_2 - 5 = 4.2\text{ V}$$

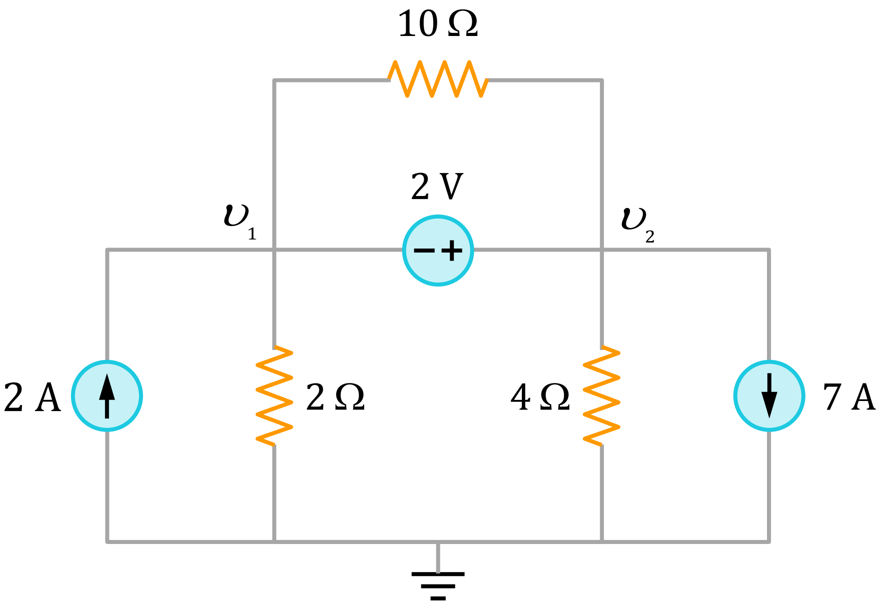
Example 13

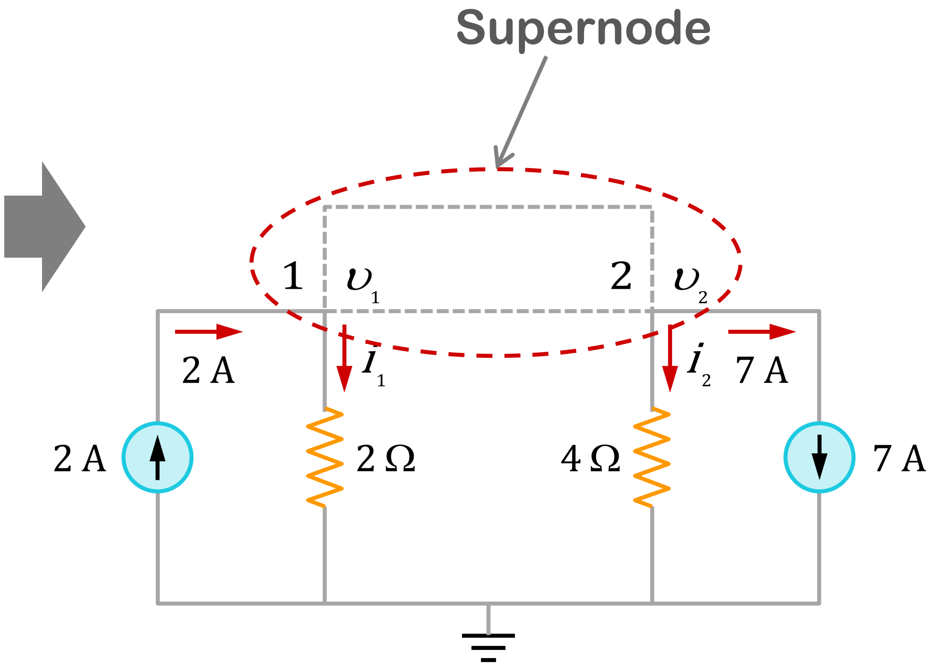
Consider the following circuit, find the node voltages.



Example 13

The supernode contains the $2\ \mathrm{V}$ source, nodes 1 and 2 and the $10\ \Omega$ resistor as shown in the following circuit.





Example 13

Applying **KCL** to the supernode gives

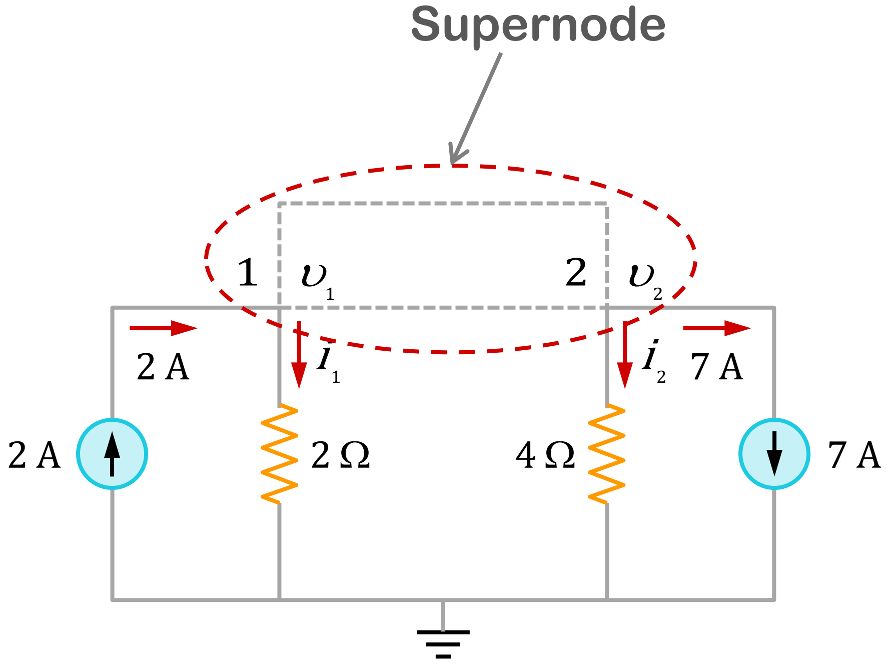
$$2 - i_1 - i_2 - 7 = 0$$

Expressing i_1 and i_2 in terms of the node voltages gives

$$2 - \frac{v_1 - 0}{2} - \frac{v_2 - 0}{4} - 7 = 0$$

$$-\frac{1}{2}v_1 - \frac{1}{4}v_2 = 5$$

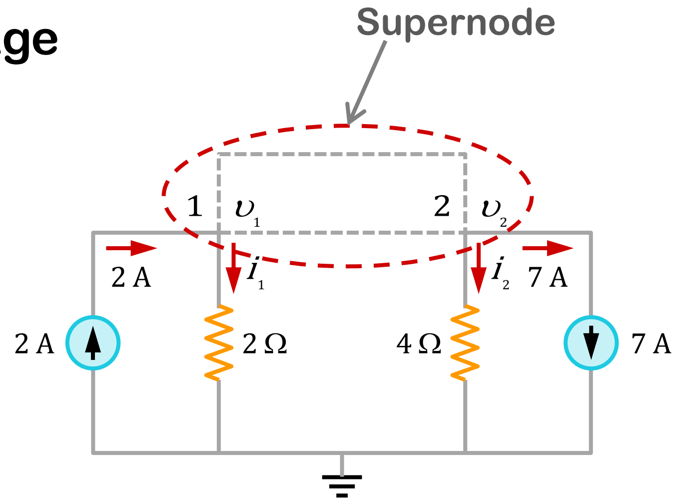
$$v_2 = -20 - 2v_1 \quad (1)$$



Example 13

The constraint equation provided by the voltage source in the supernode is

$$v_2 - v_1 = 2 \quad \textbf{(2)}$$



Using (2) in (1) gives

$$v_2 = -20 - 2v_1 \quad \textbf{(1)}$$

$$v_2 - v_1 = 2 \quad \textbf{(2)}$$

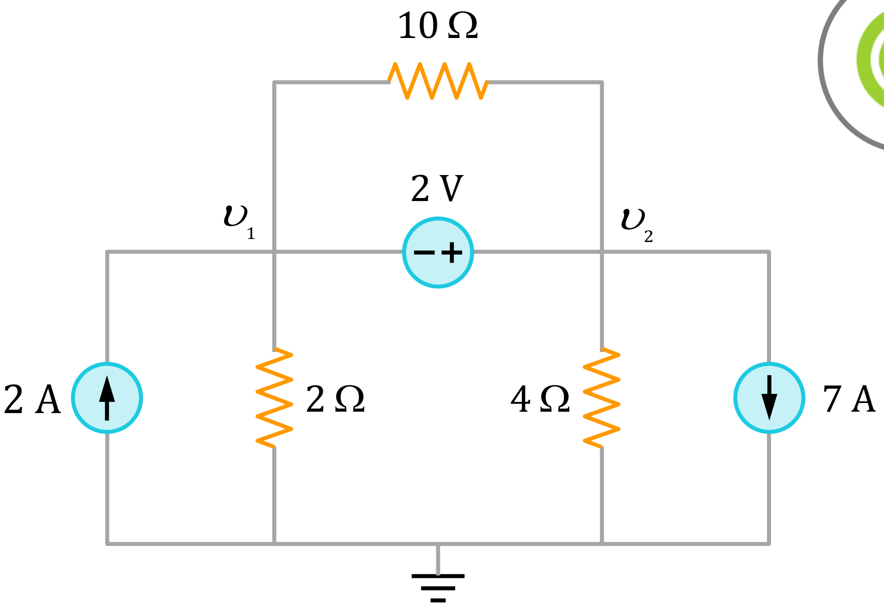
$$3v_1 = -22$$

$$v_1 = -7.33\ V$$

$$v_2 = v_1 + 2 = -5.33\ V$$

Example 13

Note that the $10\ \Omega$ resistor does not make any difference because it is connected across the supernode.



Note: nodal analysis is a straightforward analysis technique when only current sources are present and voltage sources are easily accommodated with the supernode concept.

Thank You!