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1(a)(i) For Line1 lying along x-axis,

$\vec{r}_1 = z\vec{a}_z, \quad \vec{s}_1 = x\vec{a}_x$

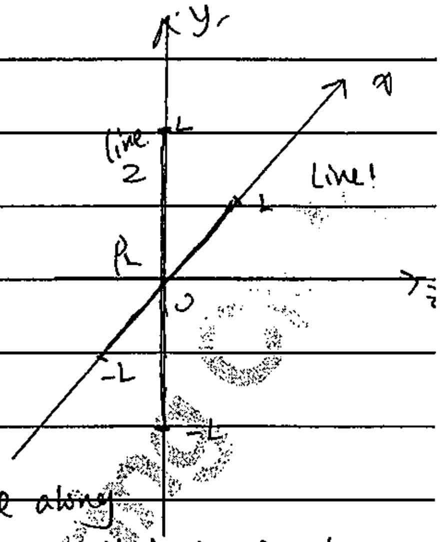
$\vec{R}_1 = \vec{r}_1 - \vec{s}_1 = -x\vec{a}_x + z\vec{a}_z, \quad R_1 = \sqrt{x^2+z^2}$

$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\int_c \frac{\rho_L \cdot \vec{R}_1}{R_1^3} dL$

$= \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{\rho_L(-x\vec{a}_x + z\vec{a}_z)}{(x^2+z^2)^{\frac{3}{2}}} dx$

$= \frac{\rho_L}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{z\vec{a}_z}{(x^2+z^2)^{\frac{3}{2}}} dx$, ~~magnitude along~~ x-axis is cancelled due to the symmetrical location of charge.

$= \frac{\rho_L L}{2\pi\varepsilon_0 z\sqrt{z^2+L^2}}\vec{a}_z$



for Line2, same principle applies

$\vec{E}_1 = \vec{E}_2 = \frac{\rho_L L}{2\pi\varepsilon_0 z\sqrt{z^2+L^2}}\vec{a}_z$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho_L \cdot L}{\pi\varepsilon_0 z\sqrt{z^2+L^2}}\vec{a}_z$

(ii) $\rho_L = 1\,nC/m, \quad L = 4, \quad z = 3$

$\vec{E} = \frac{10^{-9}\times 4}{\pi\times\frac{1}{36\pi}\times 10^{-9}\times 3\times\sqrt{3^2+4^2}} = 9.6\,V/m$

(b)(i) For line 1, $I = \rho_L \cdot V_0, \quad \vec{R}_1 = z\vec{a}_z - x\vec{a}_x \quad R_1 = \sqrt{x^2+z^2}$

$\vec{H}_1 = \frac{1}{4\pi}\int_c \frac{I\,d\vec{l}_1\times\vec{R}_1}{R_1^3} = \frac{1}{4\pi}\int_{-L}^{L}\frac{\rho_L V_0(dx\,\vec{a}_x)\times(z\vec{a}_z - x\vec{a}_x)}{(x^2+z^2)^{\frac{3}{2}}}$

$= \frac{1}{4\pi}\int_{-L}^{L}\frac{\rho_L V_0(-z\vec{a}_y)}{(x^2+z^2)^{\frac{3}{2}}}dx$

$= \frac{\rho_L V_0 z}{4\pi}(-\vec{a}_y)\int_{-L}^{L}\frac{dx}{(x^2+z^2)^{\frac{3}{2}}}$

$= \frac{\rho_L V_0 L}{2\pi z\sqrt{L^2+z^2}}(-\vec{a}_y)$

$\vec{a}_z \leftarrow \vec{a}_y$, $\vec{a}_x$

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For line 2, $I = \rho_L v_0$, $\vec{R_2} = z\vec{a_z} - y\vec{a_y}$, $R_2 = \sqrt{y^2+z^2}$

$$\vec{H_2} = \frac{1}{4\pi}\int_C \frac{I\, d\vec{l_2} \times \vec{R_2}}{R_2^3} = \frac{1}{4\pi}\int_{-L}^{L} \frac{\rho_L v_0 (dy\vec{a_y}) \times (z\vec{a_z} - y\vec{a_y})}{(y^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{1}{4\pi}\int_{-L}^{L} \frac{\rho_L v_0 z \vec{a_x}}{(y^2+z^2)^{\frac{3}{2}}} dy$$

$$= \frac{\rho_L v_0 z}{4\pi} \vec{a_x} \int_{-L}^{L} \frac{dy}{(y^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{\rho_L v_0 L}{2\pi z\sqrt{L^2+z^2}} \vec{a_x}$$

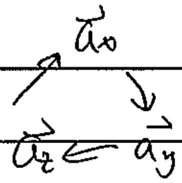
$$\vec{H} = \vec{H_1} + \vec{H_2} = \frac{\rho_L v_0 L}{2\pi z\sqrt{L^2+z^2}} (\vec{a_x} - \vec{a_y})$$

(ii) Now $\vec{r_1} = z\vec{a_z}$, $\vec{s_1} = x(\vec{a_x} + \vec{a_z})$

$$\vec{R_1} = \vec{r_1} - \vec{s_1} = z\vec{a_z} - x\vec{a_x} - x\vec{a_z}$$

$$= (z-x)\vec{a_z} - x\vec{a_x}$$

$$d\vec{l_1} = \vec{a_x} + \vec{a_z}$$



$$d\vec{l_1} \times \vec{R_1} = (\vec{a_x} + \vec{a_z}) \times [(z-x)\vec{a_z} - x\vec{a_x}]$$

$$= (z-x)(-\vec{a_y}) - x\vec{a_y}$$

$$= -z\vec{a_y}$$

$\vec{r_2} = z\vec{a_z}$, $\vec{s_2} = y(\vec{a_y} + \vec{a_z})$

$$\vec{R_2} = \vec{r_2} - \vec{s_1} = (z-y)\vec{a_z} - y\vec{a_y}$$

$$d\vec{l_2} = \vec{a_y} + \vec{a_z}$$

$$d\vec{l_2} \times \vec{R_2} = (\vec{a_y} + \vec{a_z}) \times ((z-y)\vec{a_z} - y\vec{a_y})$$

$$= (z-y)\vec{a_x} + y\vec{a_x}$$

$$= z\vec{a_x}$$

$$d\vec{l_1} \times \vec{R_1} + d\vec{l_2} \times \vec{R_2} = z(\vec{a_x} - \vec{a_y})$$

In overall, the direction of $\vec{H}$ does not change since

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\, d\vec{l} \times \vec{R}}{R^3}$$

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2(a)(i) $\vec{B} = 0.1\vec{a_z}$

Let $|OC| = |OH| = a$

$\vec{S} = \frac{1}{2}(a - x_0 - V_0 t)^2 \vec{a_z}$

$\phi_m = \vec{B}\cdot\vec{S} = 0.05(a - x_0 - V_0 t)^2$ Wb

$emf = -\frac{d\phi_m}{dt} = -0.05\times 2\times(a - x_0 - V_0 t)\cdot(-V_0)$

$= 0.1V_0(a - x_0 - V_0 t)$ V

(ii) $V_0 = 5$ m/s, $x_0 = 1$ m, $a = 4$ m

$a - x_0 = 3$

$emf = 0.5(3 - 5t)$ V

$emf|_{t=0} = 0.5\times 3 = 1.5$V

$emf|_{t=1} = 0.5\times(3-5) = -1$V

variation is $1.5V - (-1V) = 2.5V$

(b) $f = 1.575\times 10^9$ Hz

$\vec{H} = 10^{-6}(\vec{a_x} + jm\vec{a_y})e^{-jk_0 z}$ A/m

(i) Circularly polarized

$$\begin{cases} |H_{0x}| = |H_{0y}| = 1 = m \Rightarrow m = 1 \\ |\phi_y - \phi_x| = 90^\circ - 0^\circ = 90^\circ \end{cases}$$

(ii) $\omega = 2\pi f = 9.896\times 10^9$ rad/s

In free space $\lambda_0 = \frac{c}{f} = \frac{3\times 10^8}{1.575\times 10^9} = 0.19$ m

$k_0 = \frac{2\pi}{\lambda} = 32.99$ rad/m

(iii) $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega = 377\Omega$

$\vec{a_k} = \vec{a_z}$

$\vec{E}(z) = \vec{H}(z)\times\vec{a_k}\cdot\eta$

$= 10^{-6}\times 120\pi\times(\vec{a_x} + jm\vec{a_y})\times\vec{a_z}\, e^{-jk_0 z}$

$= 3.77\times 10^{-4}(-\vec{a_y} + jm\vec{a_x})\, e^{-j32.99z}$ V/m

$\hat{E}(t) = 3.77\times 10^{-4}(-\vec{a_y} + jm\vec{a_x})\cos(9.896\times 10^9 t - 32.99z)$ V/m

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(iv) $\varepsilon_r = 80, \mu_r = 1, \sigma = 0$

$\cancel{\eta' = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 120\pi\sqrt{\frac{1}{80}} = 42.15\Omega} = \frac{\eta}{\sqrt{80}}$

$\cancel{\beta' = \omega\sqrt{\mu_0\varepsilon_0}\cdot\sqrt{\mu_r\varepsilon_r} = 295.1 \text{ rad/m}} = \sqrt{80}\cdot k_0$

$\tilde{E}'(t) = 0.422\times10^{-4}(\vec{a_y} + jm\vec{a_x})\cos(9.896\times10^{9}t - 295.1z)$

Since the magnitude of the signal decreases and the phase of signal changes, so the operation of the GPS will be affected.

3 (a) (i) $f = 15\times10^{6} = 1.5\times10^{7}$ Hz

$\omega = 2\pi f = 9.425\times10^{7}$ rad/s

$k_i = \frac{\omega}{u_p} = \frac{9.425\times10^{7}}{3\times10^{8}} = 0.314$ rad/m

(ii) For lossy medium

Loss tangent $\frac{\sigma}{\omega\varepsilon} = \frac{0.1}{9.425\times10^{7}\times2.25\times\frac{1}{36\pi}\times10^{-9}} = 53.33 \gg 1$

It is lossy material, good conductor.

$\eta_c = e^{j\frac{\pi}{4}}\sqrt{\frac{\omega\mu}{\sigma}} = e^{j\frac{\pi}{4}}\sqrt{\frac{9.425\times10^{7}\times4\pi\times10^{-7}}{0.1}}$

$= 34.4e^{j\frac{\pi}{4}}$

(iii) $\alpha = \beta = \sqrt{\pi f\mu\sigma} = \sqrt{\pi\times1.5\times10^{7}\times4\pi\times10^{-7}\times0.1} = 2.43$

$\alpha = 2.43$ Np/m

(iv) $\Gamma = \frac{\eta_c - \eta_1}{\eta_c + \eta_1} = \frac{34.4\angle\frac{\pi}{4} - 120\pi}{34.4\angle\frac{\pi}{4} + 120\pi} = 0.8793\angle172.6°$

$|\Gamma| = 0.8793$

(v) $P_t(0) = (1 - |\Gamma|^2)P_i = (1 - 0.8793^2)P_i$

$= 0.227P_i$

$P_t(z) = P_t(0)e^{-2\alpha z} = 0.227e^{-4.86z}\cdot P_i$

$0.227e^{-4.86z} = 0.005$

$z = 0.79$ m

(b) (i) $k_i = 5\vec{a}_x + 2\vec{a}_z \quad |k_i| = \sqrt{29}$

$\vec{a}_{ki} = k_i / |k_i| = 0.9285\,\vec{a}_x + 0.3714\,\vec{a}_z$

$\theta_i = \tan^{-1}(\frac{5}{2}) = 68.2°$

$$\frac{\sin(\theta_t)}{\sin(\theta_i)} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \frac{1}{\sqrt{1.5}} = 0.8165$$

$\theta_t = 49.30°$

$\vec{a}_{kt} = \sin\theta_t\,\vec{a}_x + \cos\theta_t\,\vec{a}_z$

$= 0.7581\,\vec{a}_x + 0.6521\,\vec{a}_z$

(ii) $\vec{E}_i(z) = \eta\,\vec{H}_i(z) \times \vec{a}_k$

$$= \frac{120\pi}{\sqrt{29}}(-0.1\vec{a}_x + 0.25\vec{a}_z) \times (5\vec{a}_x + 2\vec{a}_z)\, e^{-j(5x+2z)}$$

$= 101.51\,\vec{a}_y\, e^{-j(5x+2z)}$

$\vec{a}_{Et} = \vec{a}_y \qquad \vec{a}_{kt} = 0.7581\,\vec{a}_x + 0.6521\,\vec{a}_z$

$\vec{a}_{Ht} = \vec{a}_k \times \vec{a}_E = -0.6521\,\vec{a}_x + 0.7581\,\vec{a}_z$

(iii) $$\frac{|k_i|}{|k_t|} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \Rightarrow |k_t| = \sqrt{1.5}\,|k_i| = 6.5955$$

$k_t = \vec{a}_{kt}\,|k_t| = 6.5955(\sin 49.3°\,\vec{a}_x + \cos 49.3°\,\vec{a}_z)$

$= 5\vec{a}_x + 4.3\vec{a}_z$

For perpendicular polarization

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\sqrt{\frac{1}{1.5}} = 120\pi\cdot\frac{1}{\sqrt{1.5}}$

$$= \frac{\frac{2}{\sqrt{1.5}}\cos 68.2°}{\frac{1}{\sqrt{1.5}}\cos 68.2° + \cos 49.30°} = 0.6348$$

$|H_{0t}| = |H_{0i}|\cdot\tau_\perp = \sqrt{0.1^2+0.25^2}\cdot 0.6348 = 0.171$

$\vec{H}_t(z) = |H_{0t}|\cdot\vec{a}_{Ht}\, e^{-jk_t}$

$= 0.171 \times (-0.6521\,\vec{a}_x + 0.7581\,\vec{a}_z)\, e^{-j(5x+4.3z)}$

$= (-0.11\vec{a}_x + 0.13\vec{a}_z)\, e^{-j(5x+4.3z)}$

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4(a)(i) $\frac{\lambda}{2} = 0.35m \quad \lambda = 0.7m$

$f = \frac{u_p}{\lambda} = 4.286 \times 10^8 Hz = 428.6 MHz$

(ii) $SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 2.8 \quad |\Gamma_L| = 0.474$

$\theta_r = \theta_L + 2\beta z_{min} = -\pi$

$= \theta_L + \frac{4\pi}{\lambda} z_{min} = \theta_L + \frac{4\pi}{\lambda}(-0.1) = -\pi$

$\theta_L = -77.14°$

$\Gamma_L = |\Gamma_L| \angle \theta_L = 0.474 \angle -77.14°$

(iii) $Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L} = 59.5 \angle -50°$

(iv) $Z_{min} = Z_0 \frac{1-|\Gamma_L|}{1+|\Gamma_L|} = Z_0 / SWR = 17.86 \Omega$

(v) $P = \frac{|V_L|^2}{|Z_L|} = \frac{|V_{min}|^2}{|Z_{min}|}$

$|V_{min}| = |V_L| \cdot \sqrt{\frac{|Z_{min}|}{|Z_L|}} = 20 \times \sqrt{\frac{17.86}{59.5}} = 10.96V$

(b) $Z_{in}(-l) = -j30 = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0$

$-j0.6 = \frac{j75 + j50 \tan\beta l}{50 - 75 \tan\beta l}$

$\Rightarrow \tan\beta l = -21$

$\frac{2\pi}{\lambda} l = -82.274° + n \times 180°, \quad n = 0, 1, 2 \ldots$

Let n = 1 $\quad l = \frac{(180° - 82.274°) \times 0.7}{360°}$

$= \cancel{0.17} \; 0.19 \, m$

Hope all the best for your final exams !!

Sorry for the mistakes I might have made.