Exercises for Chapter 5

Exercise 42. A set menu proposes 2 choices of starters, 3 choices of main dishes, and 2 choices of desserts. How many possible set menus are available?

Solution. You have 2 choices of starters, then for any choice, you get 3 choices of main dishes, or for each of them you get 2 choices of desserts. Therefore the total is

$$2 \cdot 3 \cdot 2 = 12.$$

Exercise 43.
- In a race with 30 runners where 8 trophies will be given to the top 8 runners (the trophies are distinct, there is a specific trophy for each place),in how many ways can this be done?
- In how many ways can you solve the above problem if a certain person, say Jackson, must be one of the top 3 winners?

Solution.
- For the 1st trophy, there are 30 possible choices, for the second trophy, since one person got given the 1st one, there are only 29 choices, so this gives

  $$30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 = P(30, 8) = \frac{30!}{22!}$$

  choices.
- Next we need Jackson to be of of the top 3 winners. If Jackson is the 1st winner, then we are left with 29 choices for the second trophy, 28 for the third trophy, etc. But it could be that Jackson is the 2nd winner. Now we are left with 29 choices for the 1st trophy, 28 for the 3rd trophy, etc. Or maybe Jackon is the 3rd winner, now we have 29 choices for the 1st trophy, 28 for the 2nd trophy, 27 for the 4rth trophy, etc. Thus we have 3 times $29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 = P(29, 7)$ choices, that is

  $$3 \cdot P(29, 7) = 3\frac{29!}{22!}.$$

Exercise 44. In how many ways can you pair up 8 boys and 8 girls?

Solution. Boy 1 can be paired with any of the 8 girls. Then boy 2 can be paired with any of the 7 girls left. Then boy 3 can be paired with any of the 6 girls left, etc. So there are $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$ ways.

Exercise 45. How many ternary strings of length 4 have zero ones?

Solution. We are looking at strings of length 4, and ternary means that the symbols are 0,1 and 2. How if 1 is forbidden, in the first position of the string, we have only 2 choices, and 2 choices for the 2nd, 3rd, and 4rth position. Then a total of 2^4 choices.

Exercise 46. How many permutations are there of the word "repetition"?

Solution. It is a word of length 10. Suppose we want to permute R, we have 10 choices. Now that R is fixed, we are left with 9 slots to fill. Let us try to put the E. There are two E. Thus we have $C(9,2)$ ways to put them, since we do not distinguish between the two of them. Then we have $C(7,1)=7$ for P, $C(6,2)$ for T, $C(4,2)$ for I, 2 choices for O, and 1 spot left for N. The total is thus

$$10 \cdot C(9,2) \cdot 7 \cdot C(6,2) \cdot C(4,2) \cdot 2 = 10 \cdot \frac{9!}{7!2} \cdot 7 \cdot \frac{6!}{4!2} \cdot \frac{4!}{4} \cdot 2.$$

We can simplify this expression to get

$$10 \cdot 36 \cdot 7 \cdot 15 \cdot 6 \cdot 2.$$

Alternatively, we can use the formula

$$\frac{10!}{2!2!2!} = 5 \cdot 9 \cdot 4 \cdot 7 \cdot 3 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

and both of them give the same solution!

Exercises for Chapter 6

Exercise 47. Consider the linear recurrence $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_1 = 3$, $a_0 = 0$.

- Solve it using the backtracking method.
- Solve it using the characteristic equation.