NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001
Engineering Electromagnetics

Information

Lecturer's Information (Part II)

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Text Books

- Sadiku, M. N. O. (2014). *Elements of electromagnetics* (6th ed.). Oxford, OX: Oxford University Press.
- Hayt, Jr. W. H., & Buck, J. A. (2012). *Engineering electromagnetics* (8th ed.). New York, NY: McGraw Hill.

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EE3001 Lecture Hours

Part I (18 Hours)

- ❑ Static Electric and Magnetic Fields
- ❑ Time-Varying Fields and Maxwell's Equations
- ❑ Uniform Plane Wave (UPW) in Unbounded Medium

Part II (21 Hours)

- ❑ Review of UPW, RHR; Wave Polarisation; Poynting Theorem
- ❑ Reflection and Transmission of UPW at plane boundary
- ❑ Transmission Line; Smith Chart

Information

Common Greek Letters and Vector Differential Operators							
α	**alpha**	β	**beta**	γ	**gamma**	δ	**delta**
μ	**miu**	ε	**epsilon**	σ	**sigma**	λ	**lambda**
η	**eta**	ω	omega	ϕ	phi	θ	theta
ℓ	ell	ρ	rho	τ	**tau**	Λ	Lambda
Δ	Delta	Γ	**Gamma**	Ω	Omega	∇	Del
∇f	Gradient of f	$\nabla \cdot \bar{A}$	Divergent of $\bar{A}$	$\nabla \times \bar{A}$	Curl of $\bar{A}$	$\nabla^2 f$	Laplacian of f

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This session is about

2. Review of Uniform Plane Wave (UPW) and Right-Hand Rule (RHR)

Learning Objectives

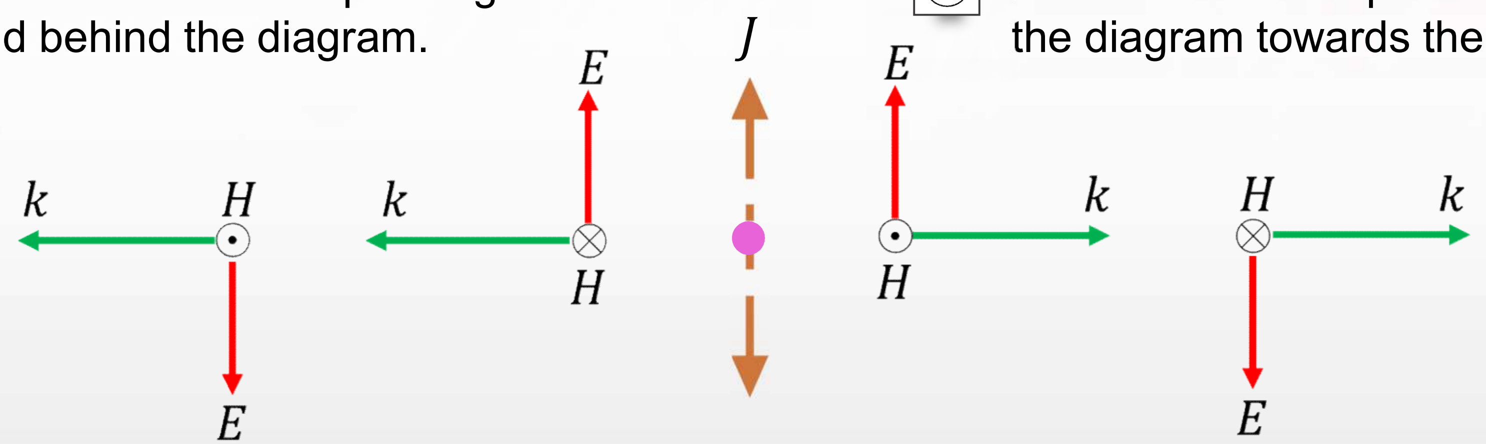
- ❑ List the properties of uniform plane wave; and
- ❑ Apply Right-Hand Rule (RHR).

Review of Electromagnetic (EM) Wave

- ❑ Sources of EM Wave: Time varying currents (Accelerated Charges)
- ❑ EM Wave propagates away from the source.
- ❑ Once EM waves leave the source, they are independent from it. They are no longer in any way connected to the source.

⊗ A circle with a cross inscribed in it indicates a vector pointing into and behind the diagram.

⊙ A circle with a dot at its centre indicates a vector pointing out of the diagram towards the viewer.



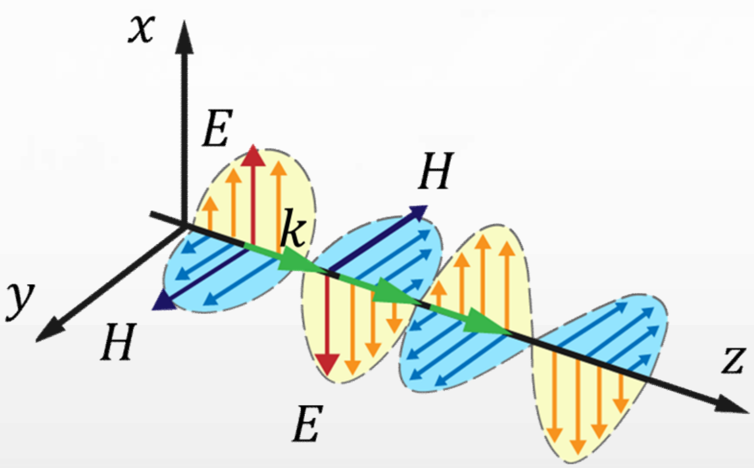
Review of Uniform Plane Wave (UPW)

Given **any UPW**, it may be shown that:

$$\vec{a}_E \perp \vec{a}_H \perp \vec{a}_k; \quad \vec{a}_E \times \vec{a}_H = \vec{a}_k; \quad \frac{E}{H} = \eta = \sqrt{\frac{\mu}{\varepsilon}}\ \Omega$$

$$[E] = \mathrm{V/m}$$
$$[H] = \mathrm{A/m}$$

where $\vec{a}_E$, $\vec{a}_H$ and $\vec{a}_k$ are the unit vectors that denote the direction of $\vec{E}$, $\vec{H}$ and $\vec{k}$ (direction of propagation of the UPW).

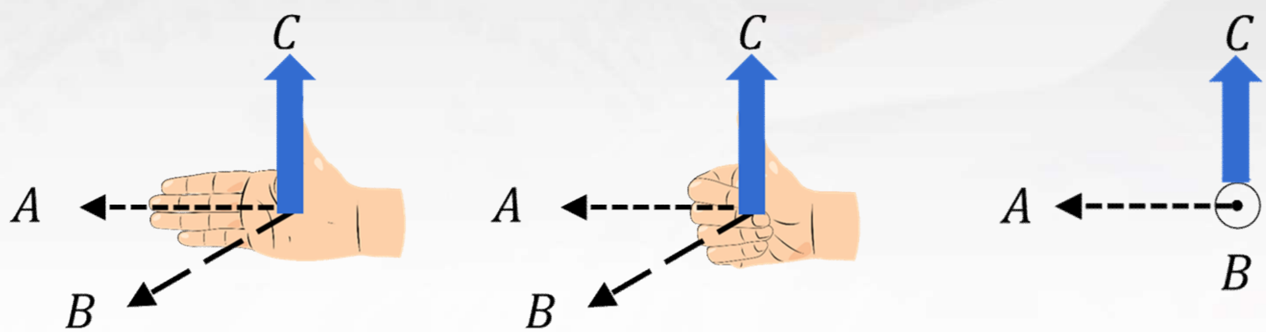


In Free Space ($\mu = \mu_o;\ \varepsilon = \varepsilon_o;\ \sigma = 0$):

$$\eta = 120\pi \simeq 377\ \Omega$$
$$u_p = c = 3 \times 10^8\ \mathrm{m/s}$$

Review of Right-Hand Rule (RHR)

Consider a vector cross product: $\vec{A} \times \vec{B} = \vec{C}$ e.g. $\vec{a}_x \times \vec{a}_y = \vec{a}_z$; $\vec{a}_E \times \vec{a}_H = \vec{a}_k$

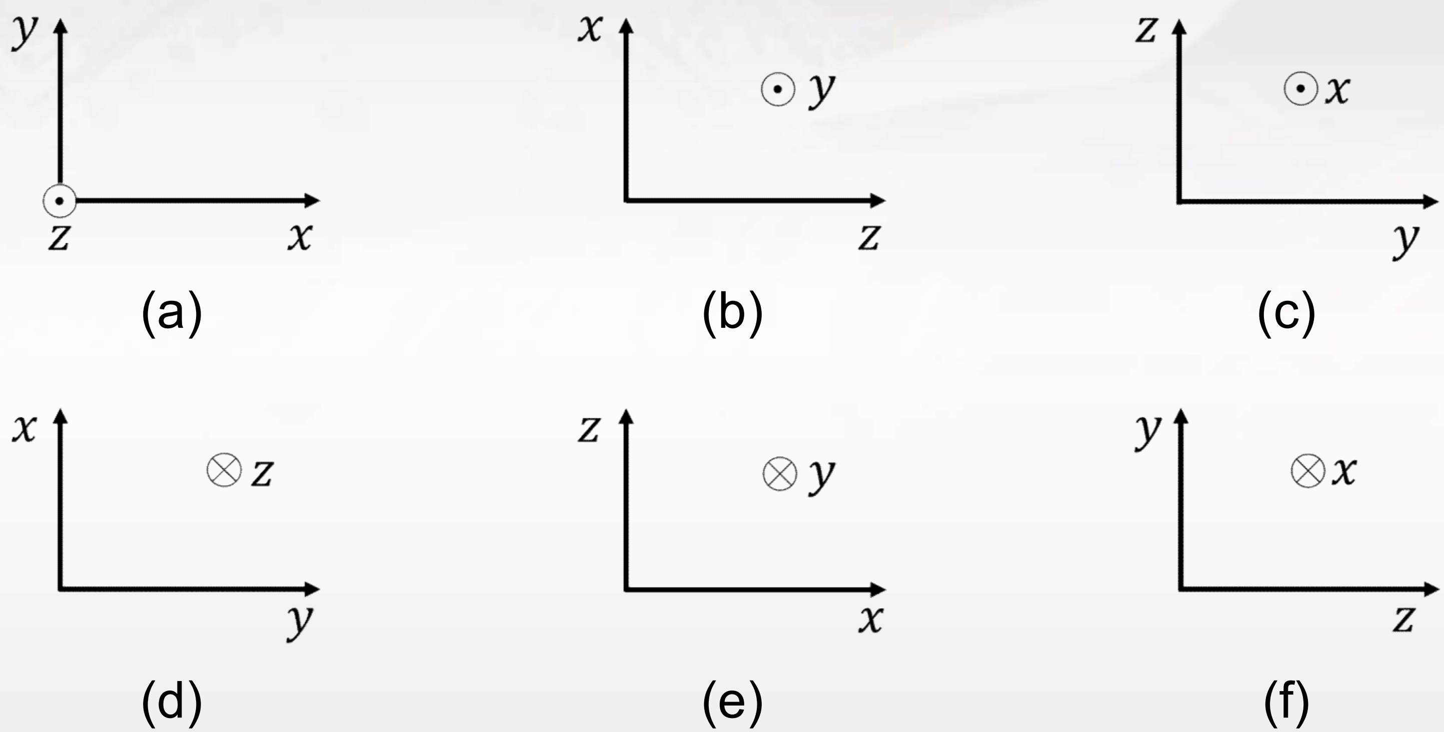


Right-Hand Rule (RHR)

Fingers point in the direction of $\vec{A}$ (1st vector) with $\vec{B}$ (2nd vector) coming out of your palm. The direction of $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ and follows the thumb of your right hand when the fingers rotate from $\vec{A}$ to $\vec{B}$.

Right-Handed Coordinate System

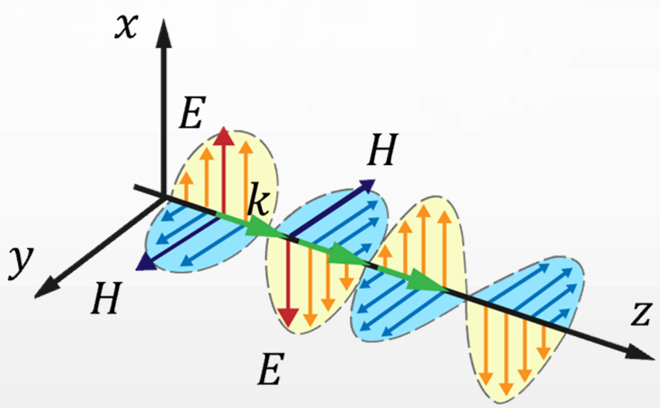
Right-Handed Coordinate System: $\vec{a}_x \times \vec{a}_y = \vec{a}_z$



Summary

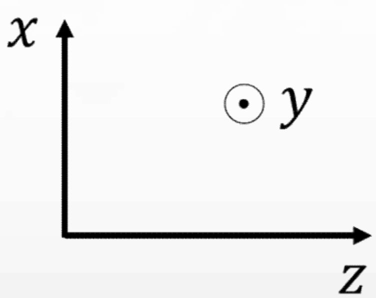
- The properties of EM waves are:
  - EM waves are produced by accelerated charges (time-varying current);
  - EM wave always travel away from the source that produces it;
  - Once EM waves leave the source, they are independent from it. They are no longer in any way connected to the source; and
  - All EM waves travel at constant speed in lossless medium.

- Given any UPW:
  - $\vec{a}_E \perp \vec{a}_H \perp \vec{a}_k$;
  - $\vec{a}_E \times \vec{a}_H = \vec{a}_k$; and
  - $\frac{E}{H} = \eta = \sqrt{\frac{\mu}{\varepsilon}}\ \Omega$.

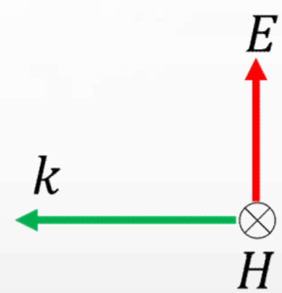


Summary

- Given any vector cross product $\vec{A} \times \vec{B} = \vec{C}$, to apply RHR:
  - You have to place your right hand such that your fingers are pointing in the direction of $\vec{A}$ (1st vector) and with $\vec{B}$ (2nd vector) coming out of your palm;
  - You will be able to curl your fingers from $\vec{A}$ to $\vec{B}$ using the smallest possible angle; and
  - The direction of $\vec{C}$ follows the thumb of your right hand.



$$\vec{a}_x \times \vec{a}_y = \vec{a}_z$$



$$\vec{a}_E \times \vec{a}_H = \vec{a}_k$$

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This session is about

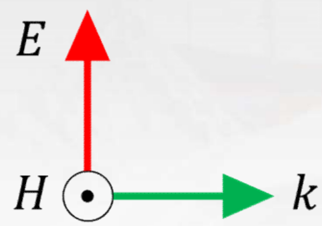
3. Examples of UPW and RHR

Learning Objectives

- Given H field of a UPW propagating in a lossless medium, determine the corresponding E field and vice versa.

Review of UPW

Consider the UPW as shown below:



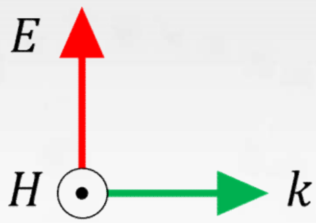
- ❑ The UPW is propagating from left to right (→) with E pointing up (↑) and H points out of the page $\odot$.

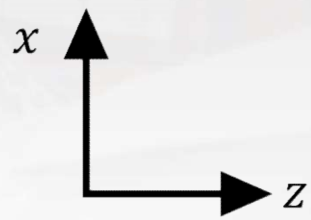
- ❑ To describe the UPW, we need to define the coordinates i.e. $x-$, $y-$ and $z-$ axis.

- ❑ **NOTE:** Two axes can be chosen arbitrarily and the 3rd must be determined using RHR such that $\vec{a}_x \times \vec{a}_y = \vec{a}_z$.

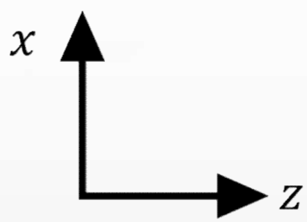
Example 1

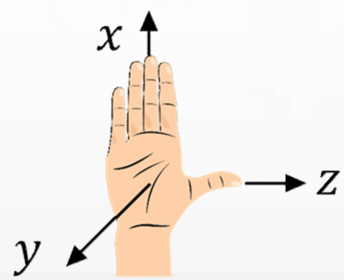
We may choose $\rightarrow \equiv z -$ axis and $\uparrow$ as $x -$ axis:

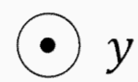




- The 3rd axis, y, must be determined by using the RHR: in this case, y points out of the page.







- Since direction of H is out of the page: $\vec{a}_H = \vec{a}_y$.

The E and H field of UPW

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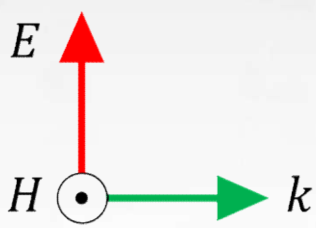
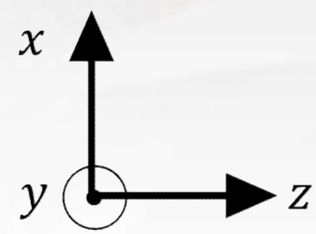
Once the coordinate system is established, the E and H fields of the UPW (in a lossless medium) can be written as:





$$\tilde{\vec{E}} = |E_o| \cos(\omega t - kz + \phi)\ \vec{a}_E \quad \text{V/m} \qquad (1)$$

$$\tilde{\vec{H}} = |H_o| \cos(\omega t - kz + \phi)\ \vec{a}_H \quad \text{A/m} \qquad (2)$$

1. Direction of propagation: $\vec{a}_k = +\vec{a}_z$; $\cos(\omega t - kz + \phi)$
2. Direction of E and H fields: $\vec{a}_E = \vec{a}_x$; $\vec{a}_H = \vec{a}_y$
3. Amplitude of E and H fields: $|E_o|$; $|H_o|$; $\frac{|E_o|}{|H_o|} = \eta$

Example 2

- Given $\tilde{\vec{E}}$ field of any UPW, $\vec{a}_k$, $\vec{a}_E$ and $|E_o|$ can be determined. The same applies to the $\tilde{\vec{H}}$ field.

Example: Determine the direction of propagation $\vec{a}_k$, direction of E field $\vec{a}_E$ and amplitude $\tilde{\vec{E}}$ of a UPW given below:

$$\tilde{\vec{E}} = \vec{a}_z\, 10 \cos(\omega t + kx + 30°)\ \mathrm{V/m}$$

ANS: $\vec{a}_k = -\vec{a}_x$; $\vec{a}_E = +\vec{a}_z$; $|E_o| = 10\ \mathrm{V/m}$

- Given $\tilde{\vec{E}}$ field of a UPW, the corresponding $\tilde{\vec{H}}$ can be determined: $\tilde{\vec{E}}$ & $\tilde{\vec{H}}$ **propagate in the same direction**; $\vec{a}_E$, $\vec{a}_H$ & $\vec{a}_k$ follow RHR and $\frac{|E_o|}{|H_o|} = \eta$.

Example 3

Given $\tilde{\vec{E}} = \vec{a}_z 10\cos\left(\omega t + kx + 30^\circ\right)$ V/m in **free space**, find the corresponding $\tilde{\vec{H}}$.

ANS: [$\vec{a}_k = -\vec{a}_x$; $\vec{a}_E = +\vec{a}_z$; $|E_o| = 10$ V/m] and $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\ \Omega$

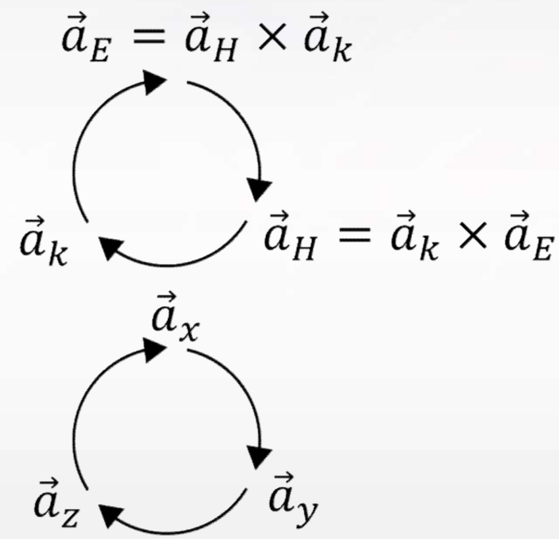
- General expression of $\tilde{\vec{H}}$:

$$\tilde{\vec{H}} = \vec{a}_H\ |H_o|\ \cos(\omega t + kx + 30^\circ)\ \text{A/m}$$

$$\vec{a}_H = \vec{a}_k \times \vec{a}_E = -\vec{a}_x \times \vec{a}_z = -(-\vec{a}_y) = +\vec{a}_y$$

$$\frac{|E_o|}{|H_o|} = \eta = 120\pi \rightarrow |H_o| = \frac{|E_o|}{\eta} = \frac{1}{12\pi}\ \text{A/m}$$

$$\therefore\ \tilde{\vec{H}} = \vec{a}_y \frac{1}{12\pi}\cos(\omega t + kx + 30^\circ)\ \text{A/m}$$



Example 4

Given $\tilde{\vec{H}} = \vec{a}_y \frac{1}{12\pi} \cos(\omega t + kx + 30°)$ A/m in a **lossless and non-magnetic** medium with relative permittivity $\varepsilon_r = 4$, find the corresponding $\tilde{\vec{E}}$.

ANS: [$\vec{a}_k = -\vec{a}_x$; $\vec{a}_H = +\vec{a}_y$; $|H_o| = \frac{1}{12}$ and $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi\ \Omega$]

$$\tilde{\vec{E}} = \vec{a}_z\ 5 \cos(\omega t + kx + 30°)\ \text{V/m}$$

Exercise:

$$\tilde{\vec{E}} = \vec{a}_y\ 240\pi \cos\left(\omega t + kz - 20^\circ\right)\ \text{V/m} \xleftrightarrow{\text{AIR}} \tilde{\vec{H}} = \vec{a}_x\ 2 \cos\left(\omega t + kz - 20^\circ\right)\ \text{A/m}$$

$$\tilde{\vec{E}} = \vec{a}_x\ 120\pi \cos\left(\omega t - ky + 30^\circ\right)\ \text{V/m} \xleftrightarrow{\text{AIR}} \tilde{\vec{H}} = -\vec{a}_z \cos\left(\omega t - ky + 30^\circ\right)\ \text{A/m}$$

- **Conclusion:** Given E field of any UPW, the corresponding H can be determined and vice versa.

Summary

- ❑ The general equations used for E and H fields of the UPW (in a lossless medium) are:
  - $\tilde{\vec{E}} = |E_o| \ \cos(\omega t - kz + \phi) \ \vec{a}_E \ \text{V/m}$
  - $\tilde{\vec{H}} = |H_o| \ \cos(\omega t - kz + \phi) \ \vec{a}_H \ \text{A/m}$
  - These are the components for the equations:
    1. Direction of propagation: $\vec{a}_k = +\vec{a}_z; \quad \cos(\omega t - kz + \phi)$
    2. Direction of E and H fields: $\vec{a}_E \times \vec{a}_H = \vec{a}_k$
    3. Amplitude of E and H fields: $|E_o|$; $|H_o|$

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This session is about

4. Wave Polarisation

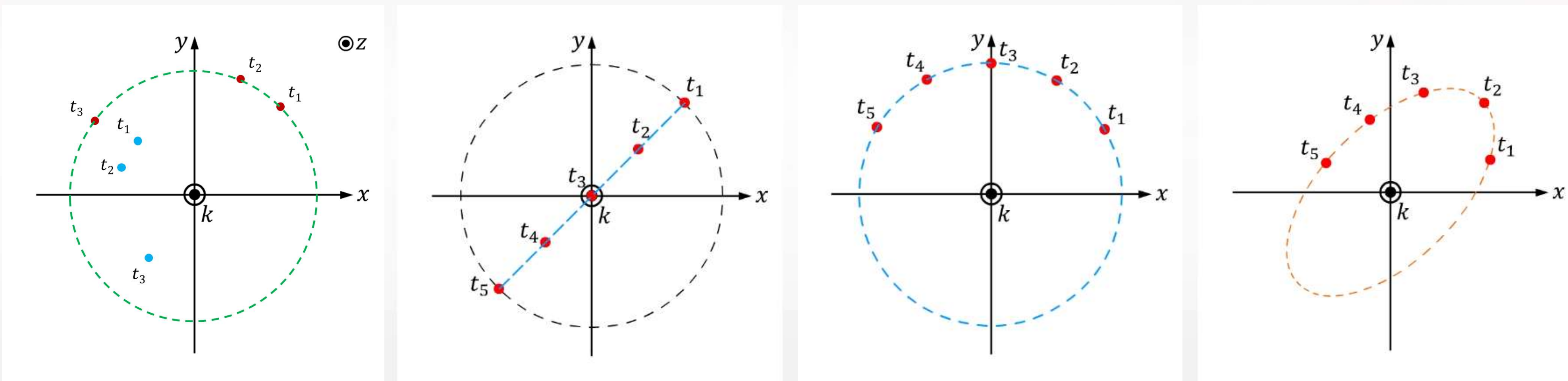
Learning Objectives

- ❑ Define the wave polarisation; and
- ❑ Given the E or H field of a UPW in time domain, identify whether the wave is linearly, circularly, or elliptically polarised.

Different Types of Wave Polarisation

- **Wave Polarisation** refers to the **locus of the tip** of $\tilde{\vec{E}}$ or $\tilde{\vec{H}}$ **at any point in space**, **as function of time**.

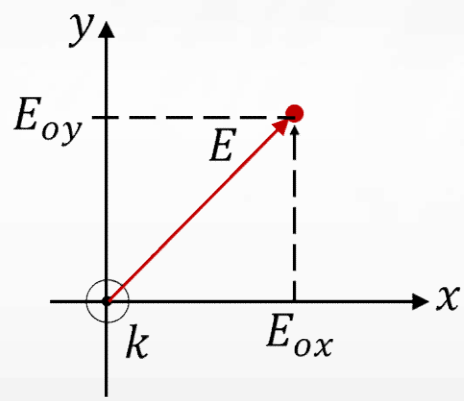
- For a UPW traveling in the $+z$ direction, $\vec{a}_E \perp \vec{a}_k$ i.e. $\vec{E}$ must lie on the xy −plane.



Different Types of Wave Polarisation

- The **locus of the tip** of $\tilde{\vec{E}}$ may be a <u>**straight line**</u> **(Linearly polarised),** <u>**Circular**</u> or <u>**Elliptical**</u>.

- For a UPW propagating in the $+z$ direction, the E field may have x and y components given by:



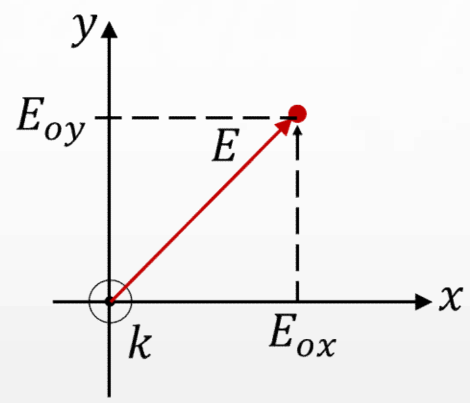
$$\tilde{\vec{E}} = \vec{a}_x \left|E_{ox}\right| \cos(\omega t - kz + \phi_x) + \vec{a}_y \left|E_{oy}\right| \cos\left(\omega t - kz + \phi_y\right) \qquad (3)$$

Wave Polarisation (Fixed Point in Space)

- At a **fixed point in space** e.g. $z = 0$, (3) becomes

$$\tilde{\vec{E}} = \vec{a}_x \, |E_{ox}| \, \cos(\omega t + \phi_x) + \vec{a}_y \, |E_{oy}| \, \cos(\omega t + \phi_y) \qquad (4)$$

- It should be noted that the **overall magnitude** and **phase** of a UPW **does not affect the polarisation**.

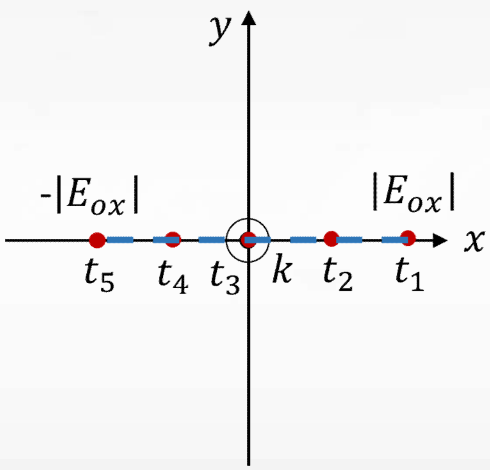


Case 1 and 2 (Linearly Polarised)

Case 1: $|E_{oy}| = 0;$

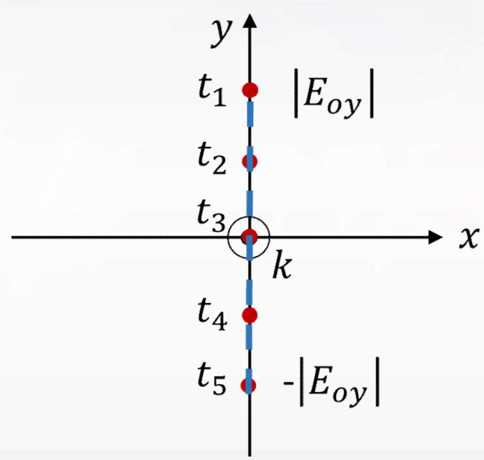
$\tilde{\vec{E}} = \vec{a}_x \; |E_{ox}| \; \cos(\omega t + \phi_x)$

$(\omega t + \phi_x) = 0, \; \frac{\pi}{3}, \; \frac{\pi}{2}, \; \frac{2\pi}{3}, \; \pi$



Case 2: $|E_{ox}| = 0;$

$\tilde{\vec{E}} = \vec{a}_y \; |E_{oy}| \; \cos(\omega t + \phi_y)$



- The UPW is said to be **linearly polarised** because the **locus of the tip** of $\tilde{\vec{E}}$ traced out a **straight line**.

Case 3a and 3b (Linearly Polarised)

- **Case 3a:** $(\phi_y - \phi_x) = 0$ $\quad [\phi_y = \phi_x \rightarrow (4)]$

$$\tilde{\vec{E}} = |E_{ox}| \cos(\omega t + \phi_x)\ \vec{a}_x + |E_{oy}| \cos(\omega t + \phi_x)\ \vec{a}_y$$

- **Case 3b:** $(\phi_y - \phi_x) = \pm\pi$ $\quad [\phi_y = \phi_x \pm \pi \rightarrow (4)]$

$$\tilde{\vec{E}} = |E_{ox}| \cos(\omega t + \phi_x)\ \vec{a}_x + |E_{oy}| \cos(\omega t + \phi_x \pm \pi)\ \vec{a}_y$$

$$= |E_{ox}| \cos(\omega t + \phi_x)\ \vec{a}_x - |E_{oy}| \cos(\omega t + \phi_x)\ \vec{a}_y$$

Case 3a:

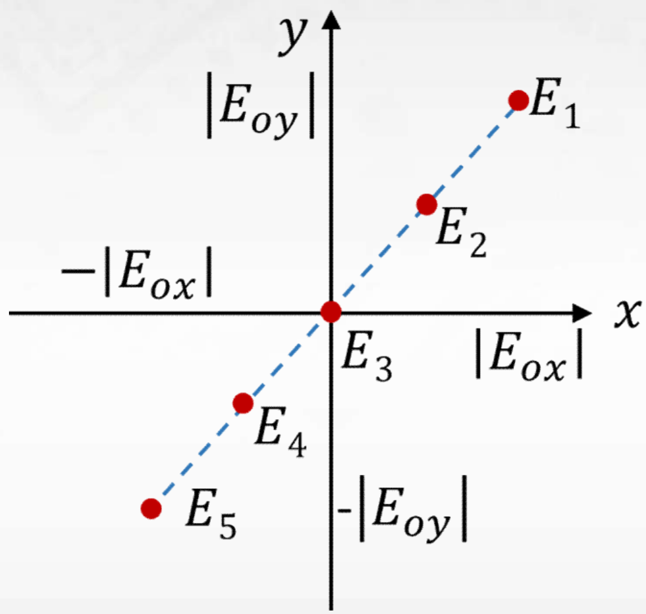
$$\theta_1 = (\omega t_1 + \phi_x) = 0: \qquad \tilde{\vec{E}}_1 = \vec{a}_x\ |E_{ox}| + \vec{a}_y\ |E_{oy}|$$

$$\theta_2 = (\omega t_2 + \phi_x) = \frac{\pi}{3}: \qquad \tilde{\vec{E}}_2 = \vec{a}_x\ 0.5\ |E_{ox}| + \vec{a}_y\ 0.5\ |E_{oy}|$$

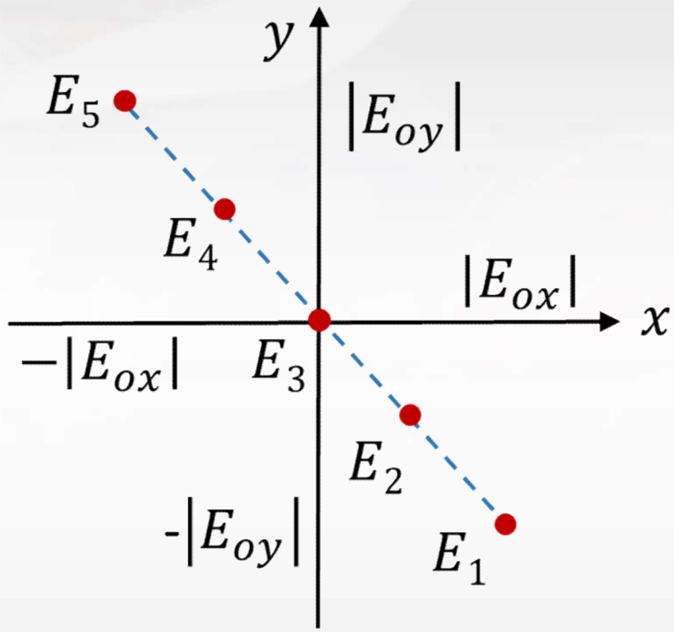
$$\theta_3 = (\omega t_3 + \phi_x) = \frac{\pi}{2}: \qquad \tilde{\vec{E}}_3 = 0$$

Case 3a and 3b (Linearly Polarised)

- $\theta_1 = 0; \quad \theta_2 = \frac{\pi}{3}; \quad \theta_3 = \frac{\pi}{2}; \quad \theta_4 = \frac{2\pi}{3}; \quad \theta_5 = \pi$



Case 3a



Case 3b

Linearly Polarised UPW

Conclusion:

- A UPW is **linearly polarised** if $|E_{ox}| = 0$ or $|E_{oy}| = 0$ or phase difference $|\phi_y - \phi_x| = 0, 180^{\circ}$

Example of linearly polarised UPW:

- $\tilde{\vec{E}}_1 = 5\ \cos(\omega t - 30^{\circ})\ \vec{a}_x + 20\ \cos(\omega t + 150^{\circ})\ \vec{a}_y$ $\qquad [|\phi_y - \phi_x| = 180^{\circ}]$
- $\tilde{\vec{E}}_2 = 5\ \cos(\omega t - 30^{\circ})\ \vec{a}_x + 20\ \sin(\omega t + 60^{\circ})\ \vec{a}_y$

$$= 5\ \cos(\omega t - 30^{\circ})\ \vec{a}_x + 20 \cos(\omega t - 30^{\circ})\ \vec{a}_y \qquad [|\phi_y - \phi_x| = 0]$$

Note: $\boxed{\sin(\theta) = \cos(\theta - 90^{\circ});\ \ \cos(\theta) = -\sin(\theta - 90^{\circ})}$

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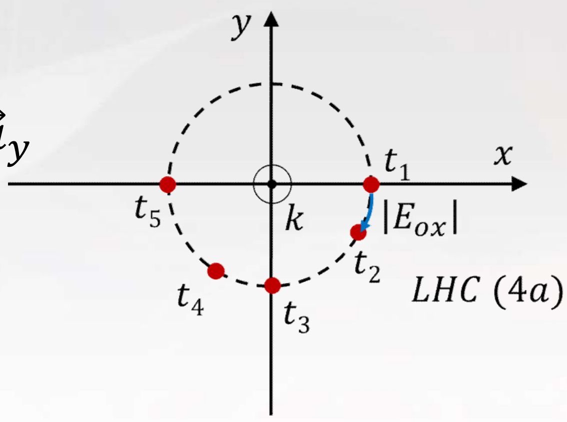
4. Wave Polarisation (2)

Case 4a and 4b (Circularly Polarised)

- **Case 4a:** $(\phi_y - \phi_x) = \frac{\pi}{2}$ AND $|E_{oy}| = |E_{ox}|$ $\rightarrow$ (4)

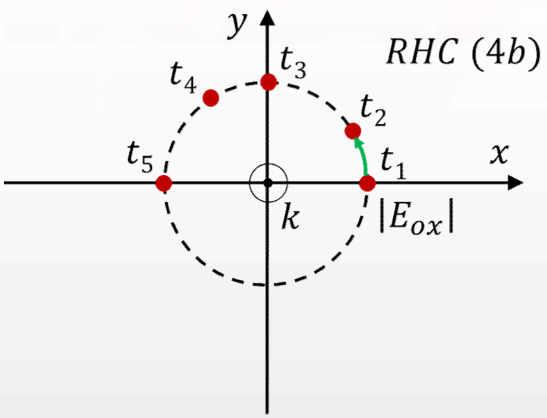
$$\tilde{E} = |E_{ox}|\ \cos(\omega t + \phi_x)\,\vec{a}_x + |E_{ox}|\ \cos\left(\omega t + \phi_x + 90^\circ\right)\vec{a}_y$$
$$= |E_{ox}|\ \cos(\omega t + \phi_x)\,\vec{a}_x - |E_{ox}|\ \sin(\omega t + \phi_x)\,\vec{a}_y$$



- **Case 4b:** $(\phi_y - \phi_x) = -\frac{\pi}{2}$ AND $|E_{oy}| = |E_{ox}|$ $\rightarrow$ (4)

$$\tilde{E} = |E_{ox}|\ \cos(\omega t + \phi_x)\,\vec{a}_x + |E_{ox}|\ \sin(\omega t + \phi_x)\,\vec{a}_y$$

- $(\omega t_1 + \phi_x) = 0 : \tilde{E}_1 = \vec{a}_x\,|E_{ox}|$
- $(\omega t_2 + \phi_x) = \frac{\pi}{6} : \tilde{E}_2 = \vec{a}_x\,0.866\,|E_{ox}| + \vec{a}_y\,0.5\,|E_{ox}|$
- $(\omega t_3 + \phi_x) = \frac{\pi}{2} : \tilde{E}_3 = \vec{a}_y\,|E_{ox}|$



Circularly Polarised UPW

Conclusion:

- A UPW is **circularly polarised** if $\left|\phi_y - \phi_x\right| = 90^{\circ}$ AND $\left|E_{oy}\right| = \left|E_{ox}\right|$

<u>Example of circularly polarised UPW:</u>

- $\tilde{\vec{E}} = 5\ \cos\left(\omega t + 20^{\circ}\right)\ \vec{a}_x + 5\ \cos\left(\omega t - 70^{\circ}\right)\ \vec{a}_y$

The Institute of Electrical and Electronics Engineers (IEEE) defined the following **handedness convention** for polarisation:

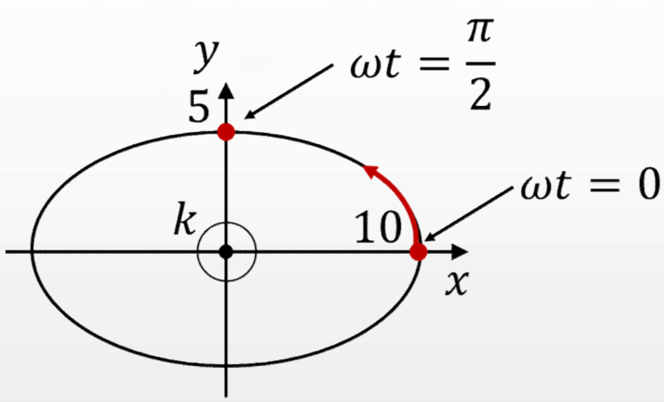
- Right-Hand Circularly Polarised (**RHC, 4b**): Orient your **right hand** so that your fingers curl in the direction of E as time increases. If your **thumb is pointing in the direction of propagation,** then the field is RHC. If your thumb is pointing the other way, the wave is Left-Hand Circularly Polarised (**LHC, 4a**).

Elliptically Polarised UPW

- **Elliptical Polarisation**: It may be shown that all other cases are elliptically polarised.

<u>Examples of elliptically polarised UPW:</u>

- $\tilde{\vec{E}}_1 = 5\ \cos(\omega t)\ \vec{a}_x + 5\ \cos\left(\omega t - 60^\circ\right)\ \vec{a}_y$ $\left[\left|\phi_y - \phi_x\right| = 60^\circ\right]$
- $\tilde{\vec{E}}_2 = 10\ \cos(\omega t)\ \vec{a}_x + 5\ \cos\left(\omega t - 90^\circ\right)\ \vec{a}_y$ $\left|\phi_y - \phi_x\right| = 90^\circ$ BUT $\left|E_{oy}\right| \neq \left|E_{ox}\right|$



Summary

- Assume wave propagating in the $+z$ direction:
  - $\tilde{\vec{E}} = \vec{a}_x \, |E_{ox}| \, \cos(\omega t - kz + \phi_x) + \vec{a}_y \, |E_{oy}| \, \cos(\omega t - kz + \phi_y)$ (Time)
  - $\vec{E} = \vec{a}_x \, |E_{ox}| \angle\phi_x \, e^{-jkz} + \vec{a}_y \, |E_{oy}| \angle\phi_y \, e^{-jkz}$ (Phasor)

- $|E_{ox}| = 0$ **or** $|E_{oy}| = 0 \rightarrow$ **Linear**

- $|\phi_y - \phi_x| = 0, 180^{\circ} \rightarrow$ **Linear**

- $|\phi_y - \phi_x| = 90^{\circ}$ **AND** $|E_{ox}| = |E_{oy}| \rightarrow$ **Circular**

- **Others → Elliptical**

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This session is about

5. Time and Phasor Domain

Learning Objectives

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- ❑ Convert time domain to phasor domain and vice versa; and
- ❑ Determine the direction of propagation and polarisation (linear, circular, elliptical) of a UPW in phasor domain.

Time Domain → Phasor Domain

From **time domain to phasor domain**:

$$\boxed{\tilde{\vec{E}}(z,t) = \text{Re}\left(\vec{E}(z)\, e^{j\omega t}\right)}$$

- Given $\tilde{\vec{E}}(z,t)$, find $\vec{E}(z)$
  - $\tilde{\vec{E}} = 10\ \cos\left(\omega t - kz + 30^{\circ}\right)\ \vec{a}_x$

    $\left[e^{j\theta} = \cos\theta + j\sin\theta\right]$

    $\tilde{\vec{E}} = 10\,\text{Re}\left(e^{+j\omega t}\ e^{-jkz}\ e^{+j30^{\circ}}\right)\ \vec{a}_x$

    $\left[\cos(\theta) = \text{Re}\left(e^{j\theta}\right)\right]$

    $\vec{E} = 10\ \left(e^{-jkz}\ e^{+j30^{\circ}}\right)\ \vec{a}_x = \vec{a}_x\ 10\,\angle 30^{\circ}\ e^{-jkz}$

  - $\tilde{\vec{E}} = 10\ \sin\left(\omega t + kz - 30^{\circ}\right)\ \vec{a}_x$

    $\boxed{\sin(\theta) = \cos\left(\theta - 90^{\circ}\right)}$

    $\tilde{\vec{E}} = 10\ \cos\left(\omega t + kz - 120^{\circ}\right)\ \vec{a}_x$

    $\vec{E} = 10\ e^{+jkz}\ e^{-j120^{\circ}}\ \vec{a}_x = \vec{a}_x\ 10\,\angle -120^{\circ}\ e^{+jkz}$

Phasor Domain → Time Domain

From **phasor domain to time domain**:

$$\boxed{\tilde{\vec{E}}(z,t) = \mathrm{Re}\left(\vec{E}(z)e^{j\omega t}\right)}$$

- ❑ Given $\vec{E}(z)$, find $\tilde{\vec{E}}(z,t)$
  - $\vec{E} = \left(-j\,10\,e^{-jkz}\,e^{-j3\ ^\circ}\right)\vec{a}_x = \left(10\,e^{-jk}\quad e^{-j12\ ^\circ}\right)\vec{a}_x \quad \left[-j = e^{-j90^\circ}\right]$

$$\tilde{\vec{E}} = \mathrm{Re}\left(\vec{E}\,e^{j\omega t}\right) = \mathrm{Re}\left(10\,e^{j(\omega t - kz - 120^\circ)}\right)\vec{a}_x \qquad \left[e^{j\theta} = \cos\theta + j\sin\theta\right]$$

$$= \mathrm{Re}(10\ \cos\theta + j\,10\ \sin\theta)\,\vec{a}_x \qquad \left[\theta = \omega t - kz - 120^\circ\right]$$

$$\therefore \tilde{\vec{E}} = 10\ \cos\left(\omega t - kz - 120^\circ\right)\,\vec{a}_x$$

Exercise:

❑ $\vec{H} = \left(j\,3\,\vec{a}_x + 4\,\vec{a}_y\right)e^{-jkz} \underset{\text{Phasor}}{\overset{\text{Time}}{\rightleftharpoons}} \tilde{\vec{H}} = -\,\vec{a}_x\,3\ \sin(\omega t - kz) + \vec{a}_y\,4\ \cos(\omega t - kz)$

Example

Determine the direction of propagation $\vec{a}_k$ and the polarisation (linear, circular or elliptical) of the following UPW.

- ❑ $\vec{E}_1 = \left(3\angle 30^\circ\ \vec{a}_x + 3\angle -150^\circ\ \vec{a}_y\right) e^{+jkz}$

  $\phi_x = 30^\circ;\ \phi_y = -150^\circ \quad \left[\left|\phi_y - \phi_x\right| = 180^\circ\right]$

  The UPW is linearly polarised $\left[e^{+jk} : \vec{a}_k = -\vec{a}_z\right]$

- ❑ $\vec{E}_2 = \left(3\angle 30^\circ\ \vec{a}_x + 3\angle -60^\circ\ \vec{a}_y\right) e^{-jkz}$

  $\phi_x = 30^\circ;\ \phi_y = -60^\circ;\ |E_{ox}| = 3;\ \left|E_{oy}\right| = 3$

  $\left|\phi_y - \phi_x\right| = 90^\circ$ AND $|E_{ox}| = \left|E_{oy}\right|$

  The UPW is circularly polarised $\left[e^{-jkz}: \vec{a}_k = \vec{a}_z\right]$

Example (cont.)

- $\vec{E}_3 = \left(3\angle 30^\circ\, \vec{a}_y + 4\angle - 60^\circ\, \vec{a}_z\right)\, e^{+jkx}$

  $\phi_y = 30^\circ;\ \ \phi_z = -60^\circ;\ \ \left|E_{oy}\right| = 3;\ \ |E_{oz}| = 4$

  $\left|\phi_z - \phi_y\right| = 90^\circ$ BUT $\left|E_{oy}\right| \neq |E_{oz}|$

  The UPW is elliptically polarised $\left[e^{+jkx}\text{: } \vec{a}_k = -\vec{a}_x\right]$

Exercise: Determine direction of propagation $\vec{a}_k$ and the polarisation (linear, circular or elliptical) of the following UPW.

- $\vec{E} = \left(5\angle 60^\circ\, \vec{a}_x + 10\angle - 120^\circ\, \vec{a}_y\right) e^{-j(kz-\pi/3)}$
- $\tilde{\vec{E}} = \vec{a}_x\, 5\, \sin\left(\omega t + kz + 90^\circ\right) + \vec{a}_y\, 5\, \cos\left(\omega t + kz + 90^\circ\right)$
- $\tilde{\vec{H}} = \vec{a}_x\, 5\, \sin\left(\omega t - kz + 90^\circ\right) + \vec{a}_y\, 10\, \sin(\omega t - kz)$

Summary

- The equation that is used to convert time domain equation to phasor domain equation and vice versa is $\tilde{\vec{E}}(z,t) = \mathrm{Re}\left(\vec{E}(z)\, e^{j\omega t}\right)$.

- Time Domain ↔ Phasor Domain
  - $\tilde{\vec{E}} = |E_{ox}|\ \cos(\omega t - kz + \phi_x)\,\vec{a}_x + |E_{oy}|\ \cos(\omega t - kz + \phi_y)\,\vec{a}_y$ (Time)
  - $\vec{E} = \vec{a}_x\,|E_{ox}|\,\angle\phi_x\, e^{-jkz} + \vec{a}_y\,|E_{oy}|\,\angle\phi_y\, e^{-jkz}$ (Phasor)

Summary

- Assume wave propagating in the $+z$ direction:
  - $\tilde{\vec{E}} = \vec{a}_x \, |E_{ox}| \, \cos(\omega t - kz + \phi_x) + \vec{a}_y \, |E_{oy}| \, \cos(\omega t - kz + \phi_y)$ (Time)
  - $\vec{E} = \vec{a}_x \, |E_{ox}| \, \angle\phi_x \, e^{-jk} + \vec{a}_y \, |E_{oy}| \, \angle\phi_y \, e^{-jkz}$ (Phasor)
- $|E_{ox}| = 0$ **or** $|E_{oy}| = 0 \rightarrow$ **Linear**
- $|\phi_y - \phi_x| = 0, 180^\circ \rightarrow$ **Linear**
- $|\phi_y - \phi_x| = 90^\circ$ **AND** $|E_{ox}| = |E_{oy}| \rightarrow$ **Circular**
- **Others → Elliptical**

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6. Poynting Theorem

Learning Objectives

- ❑ Interpret Poynting theorem in terms of power density vector, energy stored and power dissipation; and

- ❑ Determine the instantaneous Poynting vector from E and H field.

Poynting Theorem

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- This theorem relates the power flow between a source and a receiver to the amplitudes of the (local) electric and magnetic fields.
- Using the vector identity: $\nabla \cdot (\tilde{\tilde{E}} \times \tilde{\tilde{H}}) = \tilde{\tilde{H}} \cdot (\nabla \times \tilde{\tilde{E}}) - \tilde{\tilde{E}} \cdot (\nabla \times \tilde{\tilde{H}})$

$$\nabla \cdot (\tilde{\tilde{E}} \times \tilde{\tilde{H}}) = \tilde{\tilde{H}} \cdot \left(-\frac{\partial \tilde{\tilde{B}}}{\partial t}\right) - \tilde{\tilde{E}} \cdot \left(\tilde{\tilde{J}} + \frac{\partial \tilde{\tilde{D}}}{\partial t}\right)$$

$$\nabla \cdot (\tilde{\tilde{E}} \times \tilde{\tilde{H}}) = -\left(\mu \tilde{\tilde{H}} \cdot \frac{\partial \tilde{\tilde{H}}}{\partial t} + \varepsilon \tilde{\tilde{E}} \cdot \frac{\partial \tilde{\tilde{E}}}{\partial t}\right) - \tilde{\tilde{E}} \cdot \tilde{\tilde{J}}$$

- Note: $\tilde{\tilde{H}} \cdot \frac{\partial \tilde{\tilde{H}}}{\partial t} = \frac{1}{2}\frac{\partial \tilde{\tilde{H}}^2}{\partial t}, \quad \tilde{\tilde{E}} \cdot \frac{\partial \tilde{\tilde{E}}}{\partial t} = \frac{1}{2}\frac{\partial \tilde{\tilde{E}}^2}{\partial t}, \quad \tilde{\tilde{J}} = \sigma \tilde{\tilde{E}}$

$$\therefore \boxed{\nabla \cdot (\tilde{\tilde{E}} \times \tilde{\tilde{H}}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon \tilde{\tilde{E}}^2 + \frac{1}{2}\mu \tilde{\tilde{H}}^2\right) - \sigma \tilde{\tilde{E}}^2}$$

Poynting Theorem

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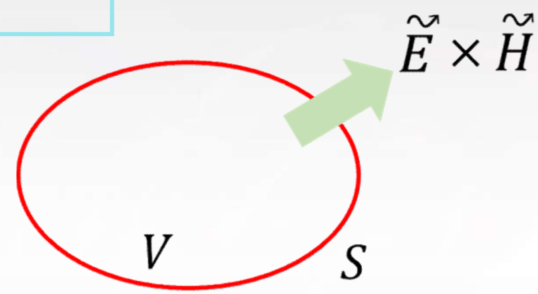
Integrating over a volume V and using the **DIVERGENCE THEOREM:**

$$\boxed{\oint_S \left(\tilde{\vec{E}} \times \tilde{\vec{H}}\right) \cdot d\vec{S} = -\frac{d}{dt}\int_V \left(\frac{1}{2}\varepsilon \tilde{\vec{E}}^2 + \frac{1}{2}\mu \tilde{\vec{H}}^2\right) dV - \int_V \sigma \tilde{\vec{E}}^2 \, dV}$$



- $U_e(t) = \frac{1}{2}\varepsilon \tilde{\vec{E}}^2$ ≡ Electric energy density $[J/\mathrm{m}^3]$

  $U_m(t) = \frac{1}{2}\mu \tilde{\vec{H}}^2$ ≡ Magnetic energy density $[J/\mathrm{m}^3]$

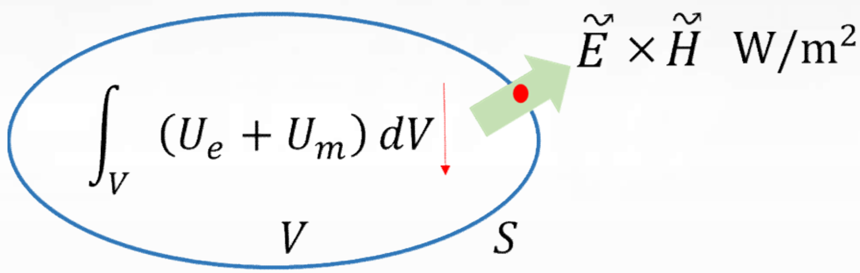
  $P_d(t) = \sigma \tilde{\vec{E}}^2$ ≡ Ohmic power dissipation density $[\mathrm{W}/\mathrm{m}^3]$
- $\int_V (U_e + U_m)\, dV$ ≡ Total Electromagnetic Energy Stored in V $[J]$
- $\int_V \sigma \tilde{\vec{E}}^2 \, dV$ ≡ Total power dissipation [W]

Lossless Medium

- In a Lossless Medium ($\sigma = 0$), $P_d = 0$

$$\boxed{\oint_S (\tilde{\vec{E}} \times \tilde{\vec{H}}) \cdot d\vec{S} = -\frac{d}{dt}\int_V (U_e + U_m) dV}$$

Net outflow of $(\tilde{\vec{E}} \times \tilde{\vec{H}})$ = Rate of decrease in energy stored in V



Using **energy-conservation arguments:**

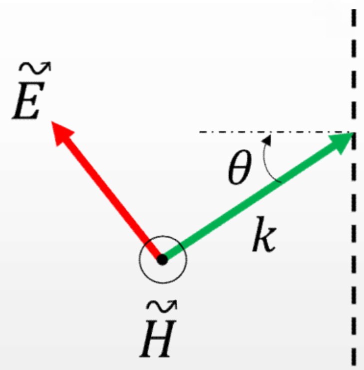
- $\boxed{P(t) = \oint_S (\tilde{\vec{E}} \times \tilde{\vec{H}}) \cdot d\vec{S}}$ ≡ Instantaneous power [W] flowing out of a given volume V through a closed surface S.

Lossless Medium

- $\tilde{\vec{S}} \equiv \tilde{\vec{E}} \times \tilde{\vec{H}}$ $\mathrm{W/m^2}$ may be interpreted as instantaneous power density vector, known as **"Poynting vector"**.

- It gives the **magnitude and direction** of power flow per unit area [$\mathrm{W/m^2}$].

- It should be noted that the direction of $\tilde{\vec{S}}$ is given by $(\vec{a}_E \times \vec{a}_H)$ and is therefore along the direction of wave propagation $(\vec{a}_k)$ i.e. $\tilde{\vec{S}} = \vec{a}_k\, S$ $\mathrm{W/m^2}$.

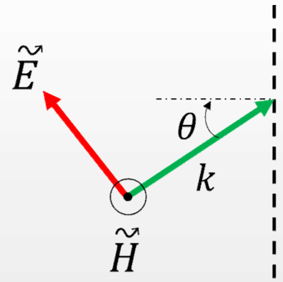


Summary

- Poynting theorem relates the power flow between a source and a receiver to the amplitudes of the (local) electric and magnetic fields.

$$\oint_S (\tilde{\vec{E}} \times \tilde{\vec{H}}) \cdot d\vec{S} = -\frac{d}{dt}\int_V \left(\frac{1}{2}\varepsilon\tilde{\vec{E}}^2 + \frac{1}{2}\mu\tilde{\vec{H}}^2\right) dV - \int_V \sigma\tilde{\vec{E}}^2\, dV$$

- Instantaneous "Poynting vector", $\tilde{\vec{S}} \equiv \tilde{\vec{E}} \times \tilde{\vec{H}}$ $\mathrm{W/m^2}$, gives the magnitude and direction of power flow per unit area $[\mathrm{W/m^2}]$. It should be noted that the direction of $\tilde{\vec{S}}$ is given by $(\vec{a}_E \times \vec{a}_H)$ and is therefore along the direction of wave propagation $(\vec{a}_k)$ i.e. $\tilde{\vec{S}} = \vec{a}_k\, S$ $\mathrm{W/m^2}$.



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7. Time-Average Poynting Vector

Learning Objectives

- ❑ Determine the time-average Poynting vector for UPW propagating in lossless and lossy medium; and
- ❑ Apply time-average Poynting vector to determine the power dissipation in lossy medium.

Example 1

The E and H fields of a UPW is given by

$$\tilde{\vec{E}} = \vec{a}_x \; 61.4 \; \cos(2\pi \times 10^9 t - 6.67\pi z + 30^\circ) \quad \text{V/m}$$

$$\tilde{\vec{H}} = \vec{a}_y \; 0.164 \; \cos(2\pi \times 10^9 t - 6.67\pi z + 30^\circ) \quad \text{A/m}$$

Find the instantaneous Poynting vector ($\tilde{\vec{S}} \equiv \tilde{\vec{E}} \times \tilde{\vec{H}}$)

Solution:

$$\tilde{\vec{S}} = \tilde{\vec{E}} \times \tilde{\vec{H}} = (\vec{a}_x \times \vec{a}_y)\,(61.4 \times 0.164)\; \cos^2(\omega t - kz + 30^\circ) \quad \text{W/m}^2$$

$$= \vec{a}_z \; 10 \; \cos^2(2\pi \times 10^9 t - 6.67\pi z + 30^\circ) \quad \text{W/m}^2$$

Question: What is the **time-average value** of $\tilde{\vec{S}}$?

$$\tilde{\vec{S}} = \vec{a}_z \; 10 \; \cos^2(2\pi \times 10^9 t - 6.67\pi z + 30^\circ) \quad \text{W/m}^2$$

Example 1 Part (b)

Time-average value: Time Domain Approach

- The time average value of $\tilde{\tilde{S}}$, $<\tilde{\tilde{S}}>$, is obtained by integrating $\tilde{\tilde{S}}$ over one period, $T = \frac{2\pi}{\omega}$, and takes the average value. Thus:

$$<\tilde{\tilde{S}}> = \frac{1}{T}\int_0^T \tilde{\tilde{S}}(t)\,dt = \frac{1}{T}\int_0^T \vec{a}_z\ 10\ \cos^2\left(\omega t - kz + 30^\circ\right)\ dt$$

$$= \vec{a}_z\ \frac{10}{T}\int_0^T \frac{1}{2}\ (1+\cos(2\theta))\,dt \qquad \left[\theta = \omega t - kz + 30^\circ\right]$$

$$= \vec{a}_z\, 5\ \ \mathrm{W/m^2}$$

- **NOTE:** $\cos^2(\theta) = \frac{1}{2}\ (1+\cos(2\theta))$ and $\int_0^T \cos(2\theta) = 0$

Time-Average Value: Phasor Domain Approach

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- The direct approach to calculate the time average value is to apply the formula:

$$<\tilde{\vec{S}}> \equiv \vec{S} = \frac{1}{2}\,\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right)\ \mathrm{W/m^2} \qquad [\ \tilde{\vec{S}} = \tilde{\vec{E}} \times \tilde{\vec{H}}\ ]$$

where $\vec{E}$ and $\vec{H}$ are phasors of $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$ respectively;

$\vec{H}^*$ is the complex conjugate of $\vec{H}$.

- This is analogue to the **circuit theory** where the average power is given by

$$P = \frac{1}{2}\,\mathrm{Re}(VI^*)\ \ \mathrm{W}.$$

Example 2

UPW in Lossless Medium, $\sigma = 0$:

- ❑ $\tilde{\vec{E}} = \vec{a}_x\ 61.4\ \cos(\omega t - kz + 30^\circ)$ $\rightarrow \vec{E} = \vec{a}_x\, 61.4\, \angle 30^\circ\ e^{-jkz}$
- ❑ $\tilde{\vec{H}} = \vec{a}_y\ 0.164\ \cos(\omega t - kz + 30^\circ)$ $\rightarrow \vec{H} = \vec{a}_y\, 0.164\, \angle 30^\circ\ e^{-jkz}$

$$\vec{H}^* = \vec{a}_y\, 0.164\, \angle -30^\circ\ e^{+jkz}$$

- ❑ $\vec{S} = \frac{1}{2}\ \mathrm{Re}(\vec{E} \times \vec{H}^*)$

$$[\vec{a}_x \times \vec{a}_y = \vec{a}_z]$$

$$[61.4\, \angle 30^\circ \times 0.164\, \angle{-30^\circ} = 10\, \angle 0^\circ]$$

$$[e^{-jkz} \times e^{+jkz} = 1]$$

- ❑ $\vec{S} = \frac{1}{2}\ \mathrm{Re}[\vec{a}_x\ 10\, \angle 0^\circ] = \vec{a}_z\ 5\ \ \mathrm{W/m^2}$
- ❑ **NOTE:** $z = 1\, \angle\theta = 1\ e^{j\theta}$ $\rightarrow$ $z^* = 1\, \angle - \theta = 1\ e^{-j}$; $z_1\, z_2 = |z_1|\, |z_2|\, \angle(\theta_1 + \theta_2)$

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7. Time-Average Poynting Vector (2)

Example 3

UPW in Lossy or Conducting Medium, $\sigma \neq 0$:

The electric and magnetic fields of a 100 MHz UPW propagating in a lossy medium ($\mu = \mu_o,\ \ \varepsilon = 8\varepsilon_o,\ \sigma = 0.048\ \text{S/m}$) are given by:

- ❑ $\vec{E}(z) = \vec{a}_x\, 100\, e^{-2.88z}\, e^{-j\ \ .59z}\ \ \text{V/m}$ $\qquad [\alpha = 2.88,\ \ \beta = 6.59]$

- ❑ $\vec{H}(z) = \vec{a}_y\, 0.91\angle -23.6^{\circ}\, e^{-2.88z}\, e^{-j6.59z}\ \ \text{A/m}$

Find the time-average Poynting vector $\vec{S}$. $\qquad \left[\vec{S} = \frac{1}{2}\,\text{Re}\left(\vec{E} \times \vec{H}^*\right)\right]$

- ❑ $\vec{H}^*(z) = \vec{a}_y\, 0.91 \angle +23.6^{\circ}\ \ e^{-\alpha z}\ \ e^{+j\beta z}$ $\qquad [\vec{a}_x \times \vec{a}_y = \vec{a}_z]$

$$[100 \angle 0^{\circ} \times 0.91 \angle 23.6^{\circ} = 91 \angle 23.6^{\circ}]$$

$$[e^{-\alpha z} \times e^{-\alpha z} = e^{-2\alpha z}]\ [e^{-j\beta z} \times e^{+j\beta z} = 1]$$

Solution to Example 3

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$\vec{E}(z) = \vec{a}_x\, 100\,\angle 0 \qquad e^{-\alpha z}\;\; e^{-j\beta z} \qquad\qquad [\alpha = 2.88,\;\beta = 6.59]$

$\vec{H}^*(z) = \vec{a}_y\, 0.91\,\angle 23.6^\circ\, e^{-\alpha z}\;\; e^{+j\beta z}$

- $\vec{S}(z) = \frac{1}{2}\,\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) = \frac{1}{2}\,\mathrm{Re}\left(\vec{a}_z\;\, 91\,\angle 23.6^\circ\, e^{-2\alpha z}\right)$
- $\vec{S}(z) = \vec{a}_z\, 45.5\;\, e^{-2\alpha z}\,\mathrm{Re}\left(e^{j23.6^\circ}\right) = \vec{a}_z\, 41.7\;\, e^{-5.76 z}\quad \mathrm{W/m^2}$

- **NOTE:**
  1. $e^{j\theta} = \cos\theta + j\sin\theta \quad \rightarrow \mathrm{Re}\left(e^{j\theta}\right) = \cos\theta$
  2. $S(z) \propto e^{-2\alpha z} \quad \rightarrow S(z) = S(0)\, e^{-2\alpha} \qquad [\alpha = \text{attenuation constant}]$

Example 3 Part (b)

Find the average power dissipated $< P_d >$ in the volume shown below as the UPW travels from $z = 0$ to $z = 0.4$ m.

- $\vec{S}(z) = \vec{a}_z \, 41.7 \, e^{-2\alpha z} = \vec{a}_z \, 41.7 \, e^{-5.76z}$ W/m^2 $[\alpha = 2.88]$
- $S(z=0) = 41.7$ W/m^2

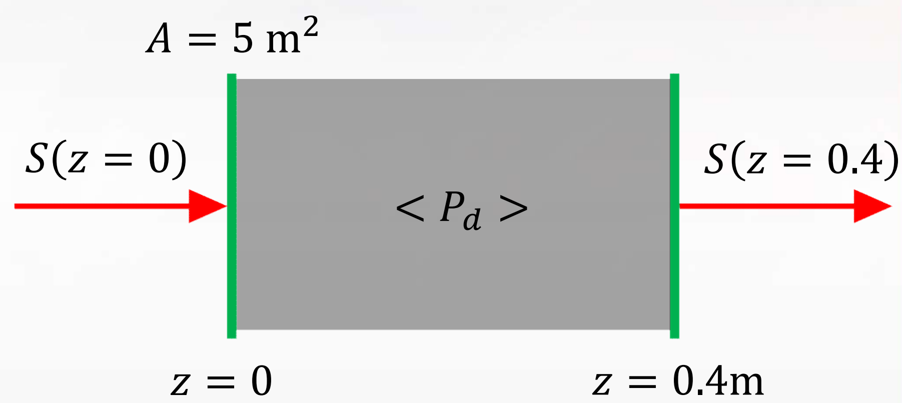
  $P_i = 41.7 \text{ W/m}^2 \times 5 \text{ m}^2 = 209 \text{ W}$

  $S(z=0.4) = 41.7 \, e^{-5.76(0.4)} = 4.2 \text{ W/m}^2$

  $P_o = 4.2 \text{ W/m}^2 \times 5 \text{ m}^2 = 21 \text{ W}$

- $< P_d >= P_i - P_o = 209 - 21 = 188$ W
- Proportion of power dissipated:

$$\Delta P_d = \frac{P_i - P_o}{P_i} = \frac{S(0) - S(0.4)}{S(0)} = 0.9 \qquad [90\%]$$



Example 3 Part (b) (cont.)

Note: $S(z) \propto e^{-2\alpha z} \rightarrow S(z) = S(0)\, e^{-2\alpha z}\ \ \mathrm{W/m^2}$

- The **proportion** of average power dissipated as the UPW propagates from $z = 0$ to $z = \ell$:

$$\Delta P_d = \frac{S(0) - S(\ell)}{S(0)} = \frac{S(0) - S(0)e^{-2\alpha\ell}}{S(0)} = \left(1 - e^{-2\alpha\ell}\right)$$

Question: Find the percentage of average power dissipated as a UPW with $\vec{E}(z) = \vec{a}_x\, 100\, e^{-2.88z}\, e^{-j6.59z}\ \ \mathrm{V/m}$ travels from $z = 0$ to $z = 0.4\ \ \mathrm{m}$.

ANS: $\Delta P_d = \left(1 - e^{-2\alpha}\ \right) = \left(1 - e^{-2(2.88)(0.4)}\right) = 0.9$ $[90\%]$

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7. Time-Average Poynting Vector (3)

Example 4

The magnetic field of a $3-\mathrm{MHz}$ uniform plane wave (UPW) travelling in a lossy medium characterized by $\mu_r = 1,\ \varepsilon_r = 2$ and $\sigma = 0.04\ S/\mathrm{m}$ has the form:

$$\vec{H}(z) = \left(j0.3\vec{a}_x + 0.3\vec{a}_y\right)e^{-\gamma z}\ \mathrm{A/m}$$

Find the following:

(i) The propagation constant γ.

(ii) The distance travelled by the UPW such that the average power density of the UPW drops to 2% of its value at $z = 0$.

Solution to Example 4

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ANS: $f = 3\ \mathrm{MHz}$

- **Lossy Medium:** $\varepsilon_c = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$, $\boxed{\gamma = \alpha + j\beta = j\,\omega\sqrt{\mu\,\varepsilon_c}}$
- $\frac{\sigma}{\omega\varepsilon} = \frac{0.04}{2\pi \times 3 \times 10^6 \times 2\,\varepsilon_o} = 120 > 20$ **(Good Conducting Medium)**

(i) $\delta = \frac{1}{\sqrt{\pi\, f\, \mu\, \sigma}} = 1.45\ \mathrm{m},$ $\gamma = j\,\omega\sqrt{\mu\,\varepsilon_c} \simeq \frac{1+j}{\delta} = 0.69 + j\,0.69 = \alpha + j\beta$

(ii) $\Delta P_d = \left(1 - e^{-2\alpha\ell}\right) = 0.98 \rightarrow e^{-2\alpha\ell} = 0.02 \rightarrow \boxed{\ell = -\frac{\ln(0.02)}{2\alpha}}$

- $\ell = -\frac{\ln(0.02)}{2(0.69)} = 2.84\ \mathrm{m}$ $[\alpha = 0.69]$

Example 4 Part (b)

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Repeat the question if the frequency of the UPW is 300 MHz.

ANS: $f = 300\ \mathrm{MHz}$

- $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{0.04}{2\pi \times 3 \times 10^{8} \times 2\,\varepsilon_o} = 1.2 < 20$
- $\varepsilon_c = \varepsilon\left(1 - j\dfrac{\sigma}{\omega\,\varepsilon}\right) = 2\,\varepsilon_o(1 - j\,1.2)$

$$\boxed{\begin{gathered} 1 - j1.2 = 1.56\angle -50.2^{\circ} \\ \sqrt{1 - j1.2} = 1.25\angle -25.1^{\circ} \end{gathered}}$$

(i) $\gamma = j\,\omega\sqrt{\mu\,\varepsilon_c} = j\,\omega\sqrt{\mu_o\,2\,\varepsilon_o}\sqrt{1 - j\,1.2} = j\,8.89 \times 1.25\angle -25.1^{\circ} = 4.71 + j\,10.1$

(ii) $\boxed{\ell = -\dfrac{\ln(0.02)}{2\,\alpha}} = -\dfrac{\ln(0.02)}{2\,(4.71)} = 0.415\ \mathrm{m}$ $[\alpha = 4.71]$

Appendix

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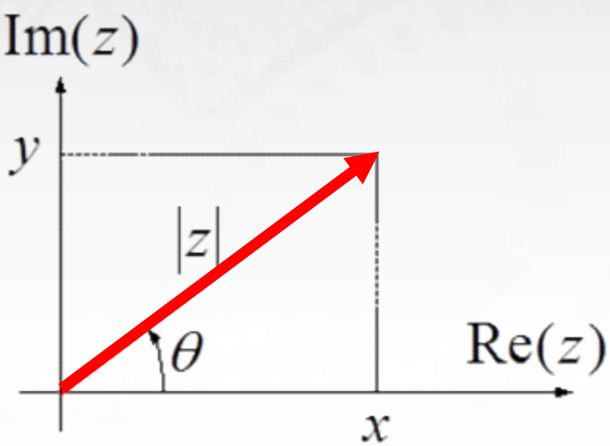
COMPLEX NUMBERS



$$z = x + j\,y = |z|\ e^{j\theta} = |z|\ \angle\theta$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

- ❑ $e^{j\theta} = \cos\theta + j\sin\theta = 1\angle\theta$
- ❑ $\frac{1}{z} = \frac{1}{|z|\ \angle\theta} = \frac{1}{|z|}\angle-\theta$

$$z = x + j\,y = |z|\ e^{j\theta} = |z|\ \angle\theta$$

$$z^* = x - j\,y = |z|\ e^{-j\theta} = |z|\ \angle-\theta$$

$$\sqrt{z} = \sqrt{|z|}\ \angle\frac{\theta}{2}$$

Summary

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- **Lossless Medium** $(\sigma = 0)$
  - $\boxed{\vec{S} = \frac{1}{2}\,\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) = \vec{a}_k \frac{|E_o|^2}{2\,\eta} \;\; \mathrm{W/m^2}}$ independent of z

- **Lossy Medium** $(\sigma \neq 0)$
  - $\boxed{\vec{S}(z) = \frac{1}{2}\,\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right)} = \vec{S}(0)\, e^{-2\alpha z} \;\; \mathrm{W/m^2}$