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EE3001 Engineering Electromagnetics

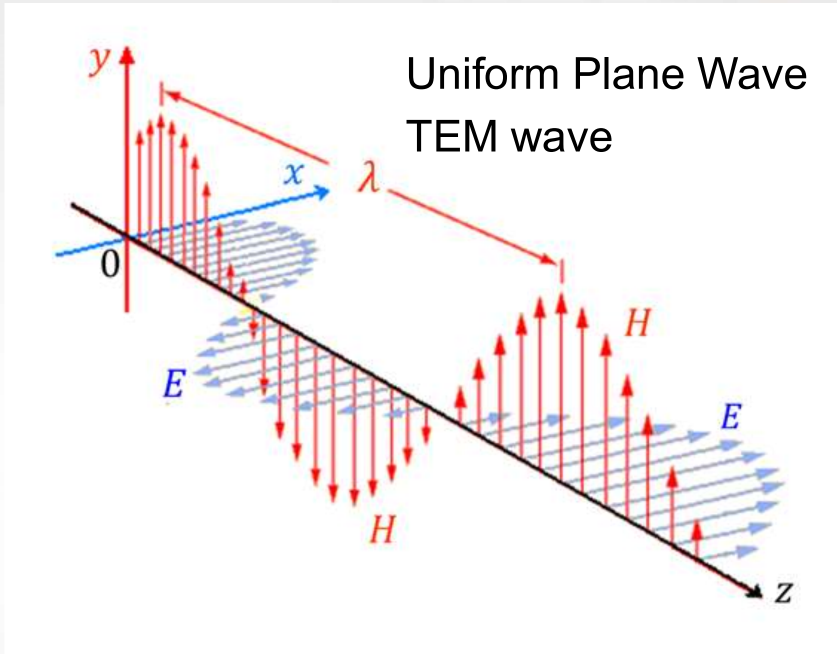
Session 16-1a

Plane Waves: General Expressions and Application in RADAR

Learning Objectives

- Obtain general expressions for plane waves in a lossless medium; and
- Explain the use and principle of operation of RADARs (Radio Detection And Ranging).

Propagation of Uniform Plane EM Waves



- $\vec{a}_E \times \vec{a}_H = \vec{a}_k =$ direction of propagation

Example: Plane Wave in a Lossless Medium

A $3\ \mathrm{GHz}$ uniform plane wave is propagating in a lossless medium along the $+z$ direction. The electric and magnetic fields of this plane wave are given by

$$\tilde{\vec{E}}(z,t) = \vec{a}_x 60\pi \cos(\omega t - 40\pi z)\ \ \mathrm{V/m}$$

$$\tilde{\vec{H}}(z,t) = \vec{a}_y \qquad \cos(\omega t - 40\pi z)\ \ \mathrm{A/m}$$

Determine:

(i) The intrinsic impedance, wavelength, and the phase velocity

(ii) The relative permittivity and the relative permeability.

Plane Wave in a Lossless Medium: Solution

(i) From the magnitudes of the electric and magnetic fields

$$|\tilde{\vec{E}}(z,t)| = 60\pi \ \mathrm{V/m} \qquad\qquad |\tilde{\vec{H}}(z,t)| = 1 \ \mathrm{A/m}$$

we know,

$$\eta = \frac{|\tilde{\vec{E}}(z,t)|}{|\tilde{\vec{H}}(z,t)|} = \frac{60\pi}{1} = 60\pi\ \Omega$$

Since the wave-number k of the plane wave is

$$k = 40\pi \ \mathrm{rad/m}$$

the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{40\pi} = 0.05\ \mathrm{m}$$

Plane Wave in a Lossless Medium: Solution

The phase velocity of the plane wave is

$$u_p = \frac{\omega}{k} = \frac{2\pi f}{k} = \frac{2\pi \times 3 \times 10^9}{40\pi} = 1.5 \times 10^8 \,\mathrm{m/s} \qquad \frac{1}{\sqrt{\mu\varepsilon}}$$

(ii) From the following two equations:

$$k = \omega\sqrt{\mu\varepsilon} = 40\pi \ \mathrm{rad/m}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi \ \Omega$$

Plane Wave in a Lossless Medium: Solution

$$k = \omega\sqrt{\mu\varepsilon} = 40\pi = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = (\omega/c)\sqrt{\mu_r\varepsilon_r}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

we obtain

$$\sqrt{\mu_r\varepsilon_r} = \frac{k \times c}{\omega} = \frac{40\pi \times 3 \times 10^8}{2\pi \times 3 \times 10^9} = 2$$

$\times$

$$\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta}{\eta_0} = \frac{60\pi}{120\pi} = \frac{1}{2}$$

which in turn lead to the following answer:

$$\mu_r = 1$$

$$\varepsilon_r = 4$$

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Session 16-1b

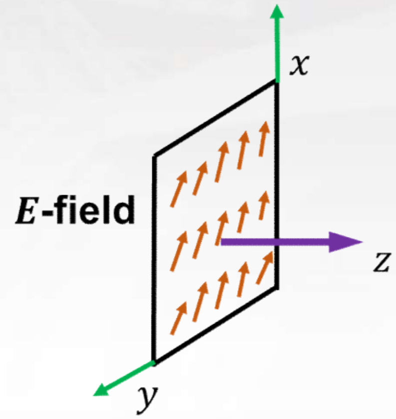
Plane Waves: General Expressions and Application in RADAR

General Expressions

- For a plane wave traveling in the $+z-\text{direction}$, the fields are of the general form

$$\vec{E} = (\vec{a}_x E_{x0} - \vec{a}_y E_{y0})e^{-jkz}$$

$$\vec{H} = \vec{a}_z \times \frac{\vec{E}}{\eta} = \frac{1}{\eta}(\vec{a}_y E_{x0} + \vec{a}_x E_{y0})e^{-jkz}$$



- **Notes:** For a plane wave:
  - $\vec{E}$, $\vec{H}$ and the propagation direction are mutually perpendicular (**TEM wave**).
  - $\vec{a}_E \times \vec{a}_H = \vec{a}_k = \text{direction of propagation}$

Example 2: Plane Wave in a Lossless Medium

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- A uniform plane wave is propagating in a lossless medium ($\boldsymbol{\varepsilon = 6\varepsilon_0},\ \boldsymbol{\mu = 1.5\mu_0}$). Its electric field is given as

$$\tilde{\vec{E}}(z,t) = 60\pi\left(\vec{a}_x - \vec{a}_y\right)\cos(2\pi \times 10^7 \times t - kz)\ \mathrm{V/m}$$

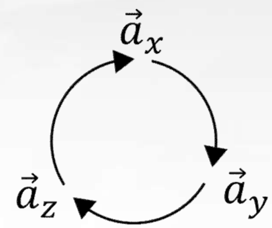
Determine the corresponding magnetic field.

Plane Wave in a Lossless Medium: Solution

$$\tilde{\vec{E}}(z,t) = 60\pi(\vec{a}_x - \vec{a}_y)\cos(2\pi \times 10^7 \times t - kz)$$

- The intrinsic impedance of the lossless medium is

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1.5}{6}} = 60\pi\ \Omega$$



$$\vec{a}_z \times \vec{a}_x = +\vec{a}_y$$

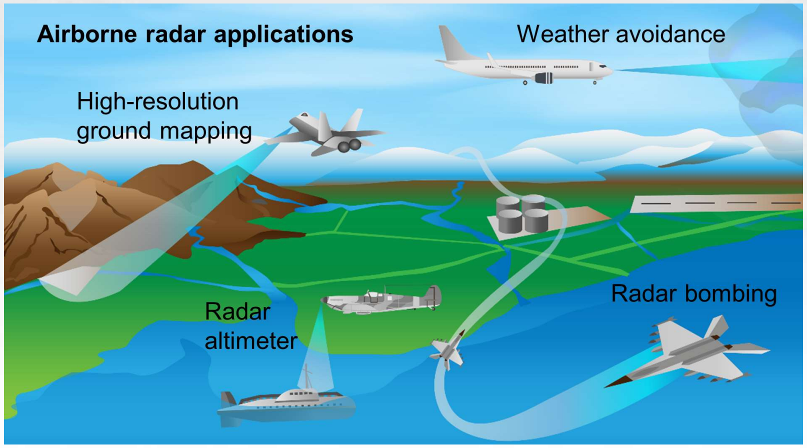
The magnetic field is

$$\vec{a}_z \times (-\vec{a}_y) = +\vec{a}_x$$

$$\tilde{\vec{H}} = \vec{a}_z \times \frac{\tilde{\vec{E}}}{\eta} = \frac{60\pi}{60\pi}\vec{a}_z \times (\vec{a}_x - \vec{a}_y)\cos(2\pi \times 10^7 t - kz)$$

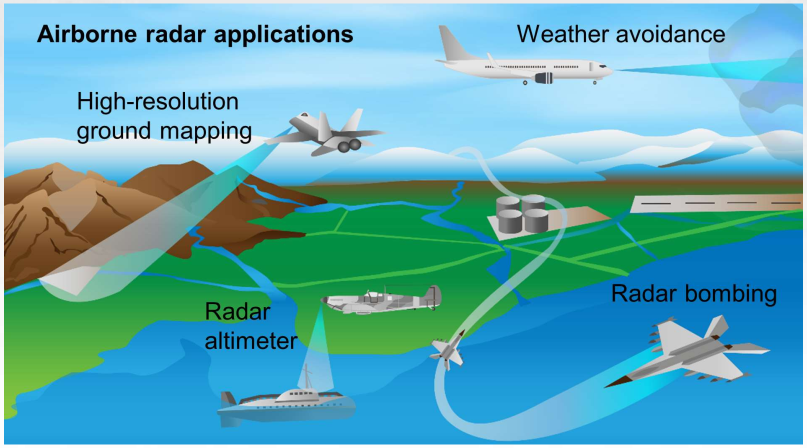
$$= (\vec{a}_x + \vec{a}_y)\cos(2\pi \times 10^7 t - kz)\ \ \mathrm{A/m}$$

RADAR: RAdio Detection And Ranging



- Variety of civilian and military applications:
  - Air-surveillance; control & guidance of weapon systems
  - Air-traffic control; enforcement of speed-limits; Google cars
  - Weather forecast; remote sensing of environment and Earth-resources

RADAR: RAdio Detection And Ranging



- Detects the presence of a reflecting object
- Can also determine range and radial velocity
- Ultrasonic (SONAR) and light-based radars (LIDARs) also possible

RADAR: Principle of Operation

- http://www.youtube.com/watch?v=BuK46aJMKeA (4:13 minutes)
- Uploaded by odzadze123

- http://www.youtube.com/watch?v=eo_owZ2UK7E (Doppler effect; 1:43 minutes)
- Uploaded by Khyar

Summary

- ❑ General expressions for plane waves in a lossless medium;
- ❑ Solution of more examples on plane wave propagation in lossless media; and
- ❑ Applications and principle of operation of RADARs (Radio Detection And Ranging).
- ❑ RADAR is used to:
  - Detect the presence of a reflecting object; and
  - Determine range and radial velocity.

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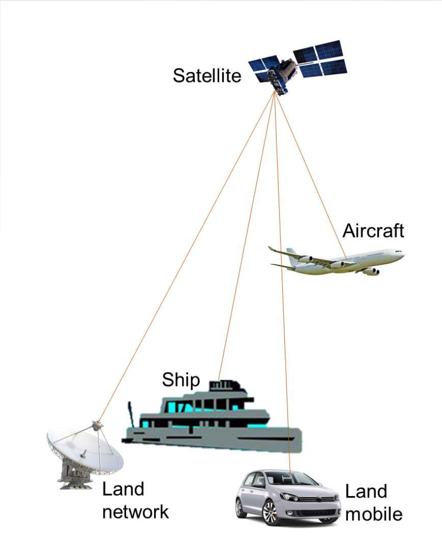
Session 16-2a

Plane Waves: Application in Satellite Communication and Example

Learning Objectives

- State the features of satellite communications; and
- Solve one more example on propagation of plane waves in a lossless medium.

Satellite Communication - 1

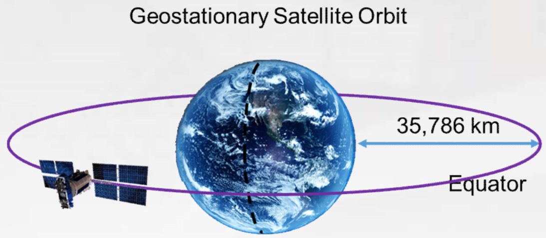


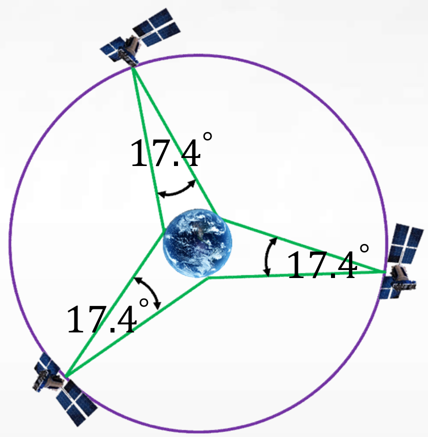
Elements of a satellite communications network

- ❑ **Sputnik 1**: First satellite, launched by Soviet Union in Oct. 1957, one-way telemetry
- ❑ **Score**: Launched by US in Dec. 1958, 2-way voice communication
- ❑ **Today**: A vast satellite communication network providing voice, data, video services to fixed **and** mobile users
- ❑ **Features**:
  - Large area of coverage;
  - Broadband;
  - Reconfigurable;
  - Minimal disruption to infrastructure

Satellite Communication - 2

- ❑ Many types of orbits for satellites
  - Low-earth orbit (elliptical)
  - Medium-earth orbit (elliptical)
  - Geostationary orbit (circular)
- ❑ 3 satellites in geostationary orbit can cover the entire Earth!
- ❑ Antenna beamwidth
  - 17.4 degrees for global coverage
  - Multi-spot beams also possible





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Session 16-2b

Plane Waves: Application in Satellite Communication and Example

Example (Sem 1, AY2009-10)

A uniform plane wave travels in the $-\vec{a}_z$ direction in free space with a wave number of $10\pi\ \mathrm{rad/m}$. The sinusoidally time-varying magnetic field intensity of the wave has a maximum value of $\frac{1}{3\pi}\ \mathrm{A/m}$ and is oriented along $-\vec{a}_y$ when $t = 0$ and $z = 0$.

(i) Determine the wavelength, frequency, and the amplitude of the electric field intensity.

(ii) Write the time-domain expressions for the magnetic field intensity and the electric field intensity.

Example (i) Solution

$$k = 10\pi \ \text{rad/m}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2 \text{ m}$$

$$f = \frac{u_p}{\lambda} = \frac{3 \times 10^8}{0.2} = 1.5 \times 10^9 \text{ Hz}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$|E| = \eta|H| = 120\pi \times \frac{1}{3\pi} = 40 \ \text{V/m}$$

$$E/H = \eta$$

Example (ii) Solution

$$\tilde{\vec{H}} = -\frac{1}{3\pi}\cos(\omega t + kz)\,\vec{a}_y \text{ A/m}$$

which satisfies the given conditions at $t = 0$ and $z = 0$

$$\tilde{\vec{E}} = -\eta\vec{a}_k \times \tilde{\vec{H}}$$

$$= \frac{120\pi}{3\pi}\cos(\omega t + kz)\vec{a}_x$$

$$= 40\cos(\omega t + kz)\vec{a}_x \text{ V/m}$$

$$= 40\cos(3\pi \times 10^9 t + 10\pi z)\vec{a}_x \text{ V/m}$$

Summary

- The features of satellite communications are:
  - Each satellite can cover a very large area of surface of the Earth;
  - Provides broadband communication;
  - It is reconfigurable; and
  - Minimal disruption to infrastructure.

- Solution of one more example on propagation of plane waves in a lossless medium.

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Session 17-1a

Plane Waves in Lossy Media - Introduction

Learning Objectives

- ❑ Obtain the solution for plane waves in lossy media; and
- ❑ State the parameters used for a plane wave in lossy media.

Lossy Medium

- **A lossy medium has non-zero conductivity. Therefore, there are 3 constitutive parameters for a lossy medium**
  - Conductivity, $\sigma \neq 0$ ($\sigma = 0$ for lossless media)
  - Permittivity, $\varepsilon = \varepsilon_r\,\varepsilon_0$
  - Permeability, $\mu = \mu_r\,\mu_0$

- **There are 2 types of currents in a lossy medium**
  - Conduction current density, $\vec{J}_c = \sigma\vec{E}$
  - Displacement current density,

$$\tilde{\vec{J}}_d = \frac{\partial\tilde{\vec{D}}}{\partial t} = \varepsilon\frac{\partial\tilde{\vec{E}}}{\partial t} \Rightarrow \vec{J}_d = j\omega\varepsilon\vec{E}$$

Complex Permittivity

- Because of the existence of both conduction current and displacement current, Maxwell's second curl equation (Ampere's law) is:

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \vec{J}_c = j\omega\varepsilon\vec{E} + \sigma\vec{E}$$
$$= j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\vec{E}$$
$$= j\omega\varepsilon_c\vec{E}$$

- where $\boxed{\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}}$ is the **complex permittivity** of the medium

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

with $\varepsilon' = \varepsilon$ and $\varepsilon'' = \frac{\sigma}{\omega}$.

Loss Tangent

- The **loss tangent** is the ratio of the imaginary and real parts of the complex permittivity,

$$\boxed{\text{loss tangent} = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}}$$

- The value of the loss tangent describes how lossy a material is.
- If the loss tangent is small, it represents a low-loss material.
- If $\sigma >> \omega\varepsilon$, it represents a **very lossy material**.

Note: a very good conductor is a very lossy material in this context!

Plane Wave Solution in a Lossy Medium

Let us assume that the plane wave is propagating along the $+z$ direction and has only an x-directed electric field component.

Recall: The electric field of a plane wave propagating in a **lossless** medium is

$$\vec{E} = \vec{a}_x E_0^+ e^{-jk}$$

where $k = \omega\sqrt{\mu\varepsilon}$ is a **real** wave number.

Plane Wave Solution in a Lossy Medium

By comparison, the electric field of a plane wave propagating in a **lossy** medium is

$$\vec{E} = \vec{a}_x E_0^+ e^{-jk_c z}$$

where $k_c = \omega\sqrt{\mu\varepsilon_c}$ is a **complex** wave number.

We also define a **complex propagation constant** γ as follows:

$$\gamma = jk_c = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\,\sigma/\omega)}$$

Complex Propagation Constant

$$\gamma = j\omega\sqrt{\mu\left(\varepsilon - j\frac{\sigma}{\omega}\right)} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$
$$= \alpha + j\beta$$

- In general, the propagation constant is a complex number having a real and an imaginary part.
- The real part α is the **attenuation constant**. The SI units for α are $\mathrm{Np/m}$ (Nepers per meter).
- The imaginary part β is the **phase constant**. The SI units for β are $\mathrm{rad/m}$.

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Session 17-1b

Plane Waves in Lossy Media - Introduction

Plane Wave Propagation in a Lossy Medium

The electric field of a plane wave propagating in a lossy medium has the form:

$$\vec{E} = \vec{a}_x E_0^+ e^{-jk_c z} = \vec{a}_x E_0^+ e^{-\gamma z} = \vec{a}_x E_0^+ e^{-\alpha} \; e^{-j\beta z}$$

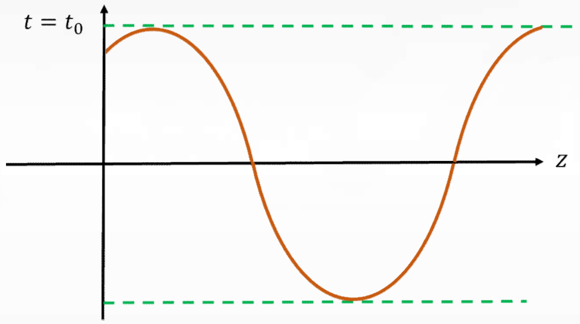
$$= \vec{a}_x |E_0^+| e^{-\alpha z} e^{-j\beta z} e^{+j\varphi}$$

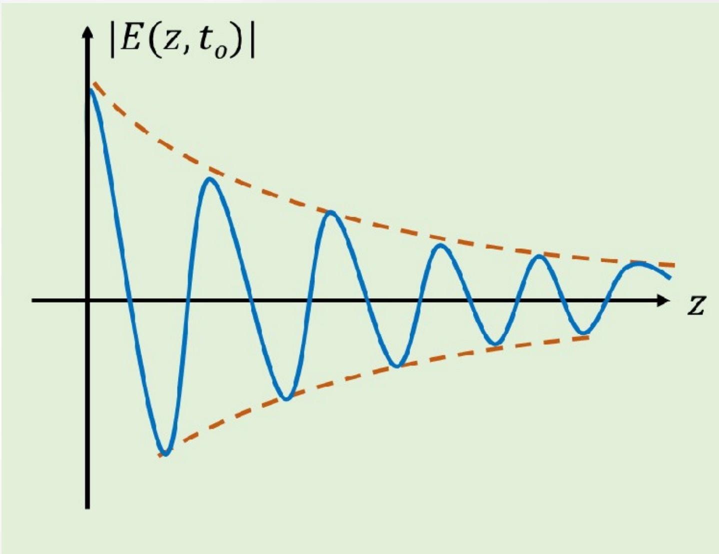
The time varying expression for the electric field is

$$\tilde{\vec{E}}(z,t) = \vec{a}_x |E_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \varphi)$$

Plane Wave Propagation in a Lossy Medium

$$\tilde{\vec{E}}(z,t) = \vec{a}_x |E_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \varphi)$$





Parameters for Plane Wave in a Lossy Medium

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From conductivity σ, permittivity ε, permeability μ, and the angular frequency ω, we can get:

- Complex permittivity $\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}$
- Complex propagation constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

- Complex intrinsic impedance $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\quad/\omega}}$
- Phase velocity $u_{\mathrm{p}} = \frac{\omega}{\beta}$
- Wavelength $\lambda = \frac{2\pi}{\beta}$

Summary

- The solution for plane waves in lossy media; and
- The parameters that are used to describe a plane wave propagating in lossy media.

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Session 17-2

Plane Waves in Lossy Media – Good Conductors

Learning Objectives

- ❑ Describe plane wave propagation in a good conductor;
- ❑ Describe the skin effect in good conductors; and
- ❑ Calculate the skin depth for a good conductor.

A Useful Result

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sqrt{\mathrm{j}} = \sqrt{e^{j\pi/2}}$$
$$= e^{j\pi/4}$$
$$= \frac{1+j}{\sqrt{2}}$$

Plane Wave Propagation in a Good Conductor

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Good conductor $\sigma \gg \omega\varepsilon \ \left(say, \quad \dfrac{\sigma}{\omega\varepsilon} > 20\right)$

The complex propagation constant has the form of

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \approx \sqrt{j\omega\mu\sigma} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

Recall that $\sqrt{j} = \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \frac{1+j}{\sqrt{2}}$

Then $$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}}$$

Intrinsic Impedance of a Good Conductor

The intrinsic impedance of a good conductor is

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} = e^{j\pi/4}\sqrt{\frac{\omega\mu}{\sigma}}$$

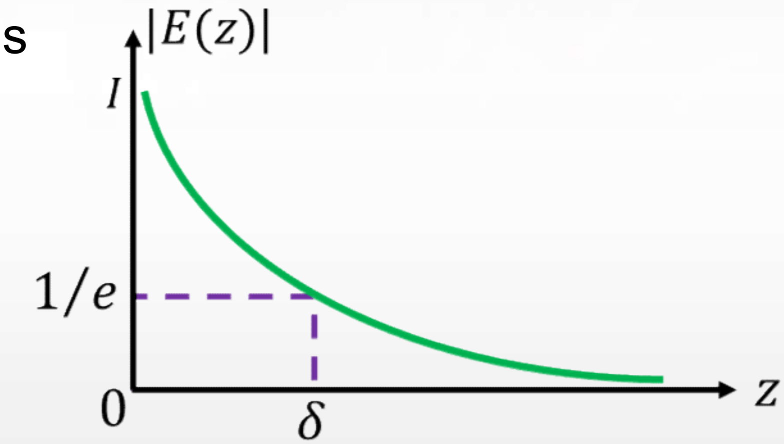
Two indicators for a good conductor:

- ❑ $\alpha = \beta$
- ❑ $\angle(\eta_c) = \frac{\pi}{4}$ or $45°$

Skin Depth or Depth of Penetration

- The magnitude of $\vec{E}$ diminishes rapidly with distance, as $e^{-\alpha z}$, in a good conductor.
- A large current density $\vec{J}_c = \sigma\vec{E}$ is created close to the conductor's surface → high loss.
- The wave penetrates only a small distance below the surface of a good conductor.
- The skin depth or depth of penetration is defined as the distance over which $\vec{E}$ decreases by a factor of e.

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$



Example: Skin Depth

Example:
Determine the skin depth for copper at $1.8\ \mathrm{GHz}$

Solution:
For copper, $\mu = \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$

$$\sigma = 5.7 \times 10^{7}\ \mathrm{S/m}$$

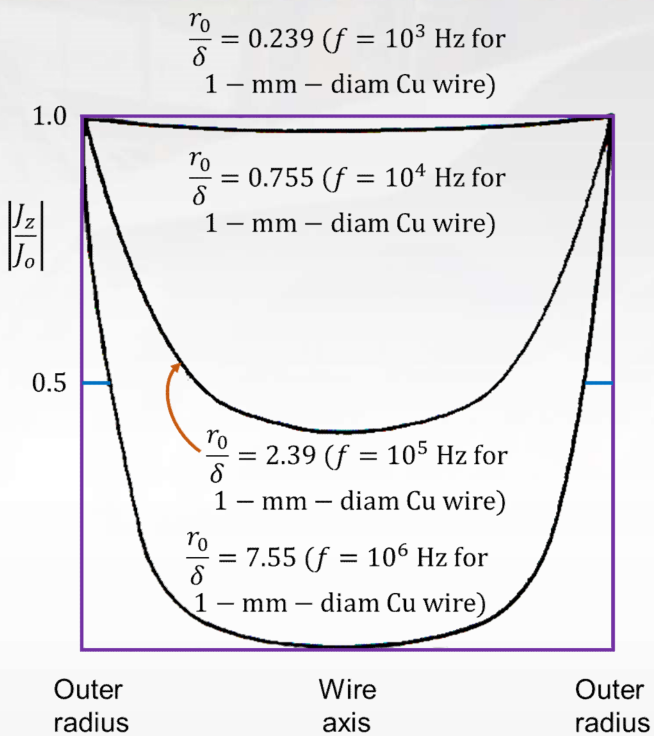
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 1.8 \times 10^{9} \times 4\pi \times 10^{-7} \times 5.7 \times 10^{7}}}$$

$$= 1.57 \times 10^{-6}\ \mathrm{m}$$

Good conductors can be used for electromagnetic shielding

Skin Effect in Practical Conductors

Current distribution in a cylindrical wire at several frequencies[1]



1. 'Fields and Waves in Communication Electronics' by S. Ramo, J. R. Whinnery, and T. Van Duzer, p. 183, 3rd Edition, 1994, John Wiley & Sons

Cellphone Signals

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Cellphone signal

Why are cellphone signals weak

- ❑ Inside a lift?
- ❑ In basements of buildings?

Summary

- A plane wave gets attenuated rapidly as it propagates in a good conductor;
- The depth of penetration in good conductors is very small; and
- The skin depth for a good conductor can be calculated using this expression:

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

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Session 17-3

Plane Waves in Lossy Media-Seawater

Learning Objectives

- ❑ Apply the concepts for plane waves in lossy media such as seawater.

Submarines

Submarines

How do submarines communicate / navigate underwater?

Plane Wave in a Lossy Medium (Seawater)

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Example:

The electric field intensity of a plane wave propagating in the $+z$ direction in **seawater** is

$$\tilde{\vec{E}}(z,t) = \vec{a}_x 100 \cos(10^7 \pi t) \text{ V/m}$$

at $z = 0$. The constitutive parameters of seawater are $\varepsilon_r = 72$, $\mu_r = 1$, $\sigma = 4 \text{ S/m}$.

a. Determine α, β, η_c, u_p, λ and δ;
b. Find the distance at which $|\vec{E}|$ is 1% of $|\vec{E}|$ at $z = 0$;
c. Write $\tilde{\vec{E}}(z,t)$ and $\tilde{\vec{H}}(z,t)$, at $z = 0.8 \text{ m}$.

Plane Wave in a Lossy Medium: Solution

The general expression for the electric field of a plane wave traveling in a lossy medium is

$$\tilde{\vec{E}}(z,t) = \vec{a}_x E_0 e^{-\alpha z}\cos(\omega t - \beta z + \phi) \qquad \phi = 0$$

$$\omega = 10^7\pi \ \text{rad/s} \qquad f = \frac{\omega}{2\pi} = 5\times 10^6\ \text{Hz}$$

Since $$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r} = \frac{4\times 36\pi\times 10^9}{10^7\pi\times 72} = 200 \gg 1$$

Therefore, ***seawater is a good conductor*** **at this frequency**.

Plane Wave in a Lossy Medium: Solution

(a) The attenuation constant is

$$\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 5 \times 10^6 (4\pi \times 10^{-7}) \times 4} = 8.89 \text{ Np/m}$$

and the phase constant is

$$\alpha = \beta = 8.89 \text{ rad/m}$$

The intrinsic impedance is

$$\eta_c = e^{j\frac{\pi}{4}} \sqrt{\frac{\omega\mu}{\sigma}} = e^{j\frac{\pi}{4}} \sqrt{\frac{\pi \times 10^7 \times 4\pi \times 10^{-7}}{4}} = \pi e^{j\frac{\pi}{4}} \ \Omega$$

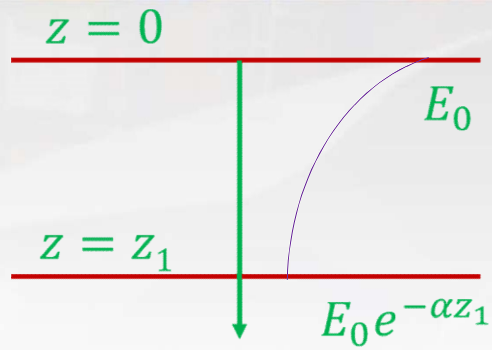
The phase velocity is

$$u_p = \frac{\omega}{\beta} = \frac{10^7 \pi}{8.89} = 3.53 \times 10^6 \text{ m/s}$$

Plane Wave in a Lossy Medium: Solution

The wavelength is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{8.89} = 0.707 \text{ m}$$



The skin depth is

$$\delta = \frac{1}{\alpha} = \frac{1}{8.89} = 0.112 \text{ m}$$

(b) Since $e^{-\alpha z_1} = 0.01$

then $$z_1 = \frac{\ln 0.01}{-\alpha} = 0.518 \text{ m} \quad !$$

Plane Wave in a Lossy Medium: Solution

(c) The general expression for the electric field is

$$\tilde{\vec{E}}(z,t) = \vec{a}_x 100 e^{-\alpha z} \cos(10^7 \pi t - \beta z) \ \text{V/m}$$

Then at $z = 0.8$ m, $\beta z = 8.89 \times 0.8 = 7.112$ rad or $407.5°$, $e^{-\alpha z} = 8.2 \times 10^{-4}$, we have

$$\tilde{\vec{E}}(0.8,t) = \vec{a}_x 0.082 \cos(10^7 \pi t - 407.5°) \ \text{V/m}$$

Plane Wave in a Lossy Medium: Solution

$$\tilde{\vec{E}}(z,t) = \vec{a}_x 100 e^{-\alpha z}\cos(10^7 \pi t - \beta z)\ \text{V/m}$$

In phasor notation: $\vec{E}(z) = \vec{a}_x 100 e^{-\alpha z} e^{-j\beta}$

$$\vec{H}(z) = \vec{a}_y \frac{100}{\eta_c} e^{-\alpha z} e^{-j\beta} \qquad \eta_c = \pi e^{j\frac{\pi}{4}} \qquad |\eta_c| = \pi$$

$$= \vec{a}_y \frac{100}{|\eta_c|} e^{-\alpha z} e^{-j\beta z} e^{-j\frac{\pi}{4}} = \vec{a}_y \frac{100}{|\eta_c|} e^{-\alpha z} e^{-j\left(\beta z + \frac{\pi}{4}\right)}$$

and at $z = 0.8\ \text{m}$, the magnetic field becomes

$$\vec{H}(0.8) = \vec{a}_y 0.026 e^{-j7.9^c} = \vec{a}_y 0.026 e^{-j452.5^\circ}$$

Then the time-varying expression for H is

$$\tilde{\vec{H}}(0.8,t) = \vec{a}_y 0.026 \cos(10^7 \pi t - 452.5^\circ)\ \text{A/m}$$

Conversion Between $\vec{E}$ and $\vec{H}$

- **Lossless Medium**

$$\vec{H} = \vec{a}_k \times \frac{\vec{E}}{\eta} \qquad \tilde{\vec{H}}(x,y,z,t) = \vec{a}_k \times \frac{\tilde{\vec{E}}(x,y,z,t)}{\eta}$$

- **Lossy Medium**

$$\vec{H} = \vec{a}_k \times \frac{\vec{E}}{\eta_c}$$

Summary

- Solve problems on propagation of plane waves in lossy media such as seawater.

EE3001 Engineering Electromagnetics

Session 18-1

Plane Waves in Lossy Media – Lift Carriage

Learning Objectives

- ❑ Compare the wave propagation parameters for lossless and lossy media; and
- ❑ Apply the concepts of wave propagation to solve an example on cellphone signals in a lift carriage.

Summary of Plane Wave Parameters

Medium	Lossless	Lossy
Conductivity	$\sigma = 0$	$\sigma \neq 0$
Ampere's law	$\nabla \times H = j\omega\varepsilon E$	$\nabla \times H = J_c + j\omega\varepsilon E$ $= (\sigma + j\omega\varepsilon)E = j\omega\varepsilon_c E$
Permittivity	ε	$\varepsilon_c = \varepsilon - j\,\sigma/\omega$
Wave number	$k^2 = \omega^2\mu\varepsilon$	${k_c}^2 = \omega^2\mu\varepsilon_c$
Wave equation	$d^2E_x/dz^2 + k^2E_x = 0$	$d^2E_x/dz^2 + {k_c}^2E_x = 0$

Summary of Plane Wave Parameters (cont'd)

Medium	**Lossless**	**Lossy**
Wave solution	$E_x = E_o e^{-jkz}$	$E_x = E_o e^{-\alpha z} e^{-j\beta z}$
Attenuation constant	$\alpha = 0$	$\alpha \neq 0$
Phase constant	$\beta = k$	$\beta > k$
Intrinsic impedance	$\eta^2 = \mu/\varepsilon$	${\eta_c}^2 = \mu/\varepsilon_c$

Example 1 (Sem 2, AY 2012-13)

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

A cellphone signal at $1800\ \mathrm{MHz}$ propagates through the walls of a lift carriage made of sheet-steel which is $2\ \mathrm{mm}$ thick. The constitutive parameters of sheet-steel are $\varepsilon_r = 1$, $\sigma = 2 \times 10^6\ \mathrm{S/m}$, and $\mu_r = 10$.

(i) Determine α, β, δ, u_p and η_c in sheet-steel.

(ii) By what factor does the cellphone signal attenuate as it propagates through the walls of the lift carriage (ignore the effect of reflections)?

Example 1 (i)

(i) Determine α, β, δ, u_p and η_c.

$$\frac{\sigma}{\omega\varepsilon} = \frac{2 \times 10^6}{2\pi \times 1800 \times 10^6} \frac{36\pi \times 10^9}{1} = 2 \times 10^7$$ a very good conductor

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi \times 1800 \times 10^6 \times 10 \times 4\pi \times 10^{-7} \times 2 \times 10^6}}$$

$$= 2.65 \times 10^{-6} \text{ m} = 2.65\ \mu\text{m}$$

$$\alpha = \frac{1}{\delta} = 3.77 \times 10^5 \text{ Np/m} \qquad \beta = \frac{1}{\delta} = 3.77 \times 10^5 \text{ rad/m}$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 1800 \times 10^6}{3.77 \times 10^5} = 3 \times 10^4 \text{ m/s}$$

Example 1 (i) and (ii)

$$\eta_c = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$= \sqrt{j}\sqrt{\frac{2\pi\times1800\times10^6\times10\times4\pi\times10^{-7}}{2\times10^6}}$$

$$= e^{j45^\circ} \times 0.267\ \Omega$$

$$= 0.189(1 + j)\ \Omega$$

(ii) Attenuation factor $e^{-\alpha z} = e^{-3.77\times10^5\times2\times10^{-3}}$

$$= e^{-7.54\times10^2}$$

$$= 10^{-327}$$

$$20\log_{10}(10^{-32}\ ) = 20(-327) = -6540\ \text{dB}$$

Summary

- Comparison of the wave propagation parameters for lossless and lossy media; and
- Application of the concepts of wave propagation to solve an example on cellphone signals in a lift carriage.

EE3001 Engineering Electromagnetics

Session 18-2a

Plane Waves in Lossy Media - Applications

Learning Objectives

- ❑ Describe the principle of operation of microwave oven;
- ❑ Apply the concepts for propagation of plane waves in lossy media such as grey matter of the brain; and
- ❑ Describe the principle of detection of tumors using microwaves.

Applications

- How does food get heated up in a microwave oven?

- How are microwaves used for detection and treatment of tumours / cancers?

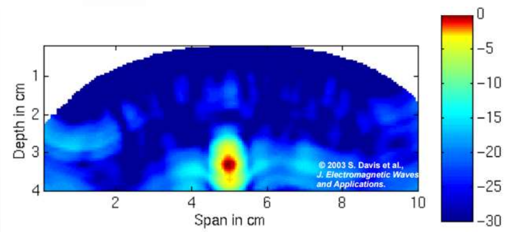


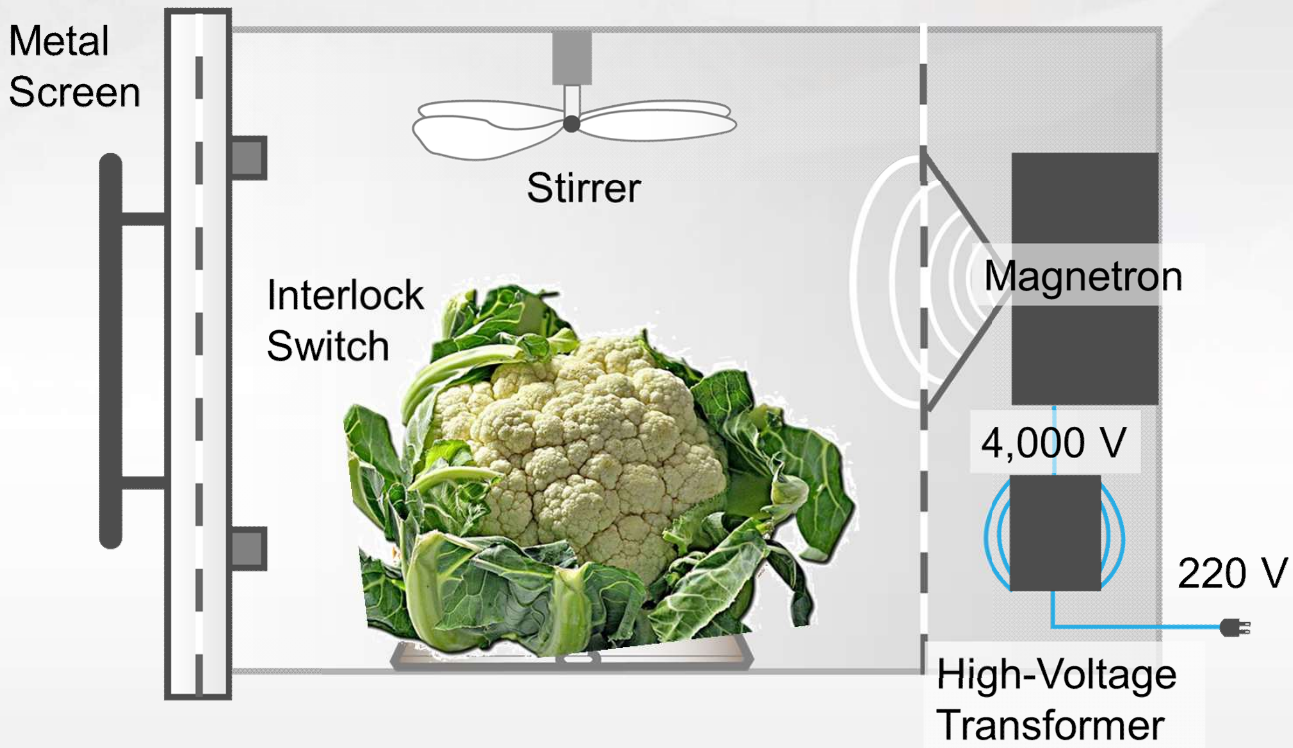
Fig. 9. Top: MRI-derived breast model shows the location of a 2-mm diameter malignant tumor at a depth of 3 cm. Bottom: Image reconstructed from backscattered waveforms obtained by solving Maxwell's equations. The color scale is in decibels. Note that the tumor signature is 15–30 dB (30–1,000 times) stronger than the backscattering clutter due to the surrounding normal tissues. *Source:* S. Davis et al., *J. Electromagnetic Waves and Applications*, in press [9].

A. Taflove, "Why study electromagnetics: The first unit in an undergraduate electromagnetics course," IEEE Antennas & Propagation Magazine, vol. 44, no. 2, pp. 132-139, April 2002.

Microwave Oven - 1

- ❑ Microwaves: 300 MHz – 300 GHz
- ❑ Water molecules: similar to electric dipole
- ❑ When exposed to microwaves, water molecules tend to align with the oscillating electric field and rotate rapidly
- ❑ The resulting rotation generates heat
- ❑ The amount of heat depends upon
  - Frequency of the microwave signal (typically 2.45 GHz)
  - Concentration of fats/ salts dissolved in water
  - Power of the microwave signal
  - Temperature of water

Microwave Oven - 2



Constitutive Parameters for Muscle Tissue

TABLE 1. The parameters for electromagnetic wave propagation in muscle tissue.

	400 MHz	900 MHz	2.45 GHz	5.8 GHz
$\varepsilon' - j\varepsilon''$	$57.1 - j35.8$	$55.0 - j18.8$	$52.7 - j12.8$	$48.5 - j15.4$
σ (S/m)	0.80	0.94	1.74	4.96
δ (mm)	52.6	42.4	22.3	7.5
λ (mm)	95.0	44.3	16.7	7.3
λ_0 (mm)	750.0	333.3	122.4	51.7
λ_0/λ	7.9	7.5	7.3	7.1

EE3001 Engineering Electromagnetics

Session 18-2b

Plane Waves in Lossy Media - Applications

Example (Sem 2, AY 2010-11)

A cellphone signal at 900 MHz propagates in the grey matter of the brain which has constitutive parameters $\varepsilon_r = 50$, $\sigma = 0.8\ \mathrm{S/m}$, and $\mu_r = 1$.

(i) Determine α, β, δ, and λ.

(ii) Find the distance inside the brain at which the magnitude of the fields is 10% of that at the surface.

Example (i)

(i) Determine α, β, δ and λ.

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.8}{2\pi \times 900 \times 10^6} \frac{36\pi \times 10^9}{50} = 0.32$$

not a good conductor

$$\varepsilon_c = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) = 50\varepsilon_o(1 - j0.32)$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu_o\varepsilon_o}\sqrt{50(1 - j0.32)}$$

$$= j6\pi(7.16 - j1.116) = 21.34 + j134.95$$

$$\alpha = 21.34 \text{ Np/m} \qquad \beta = 134.95 \text{ rad/m}$$

$$\delta = \frac{1}{\alpha} = 0.0469 \text{ m} \qquad \lambda = \frac{2\pi}{\beta} = 0.0466 \text{ m}$$

Example (ii)

(ii) Find the distance inside the brain at which the magnitude of the fields is 10% of that at the surface.

$$e^{-\alpha z_1} = 10\% = \frac{1}{10} = 0.1$$

$$z_1 = -\frac{\ln 0.1}{\alpha}$$

$$= \frac{2.3025}{21.34} = 0.108\ \mathrm{m}$$

Appendix

Imaging of the Human Body - 1

- Detection of malignant breast tumors at the earliest possible stage
- Use of narrow microwave pulses: safer than X-rays, higher resolution

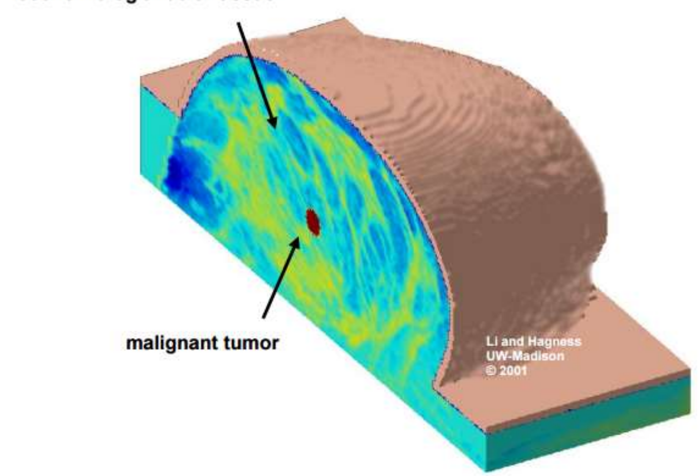
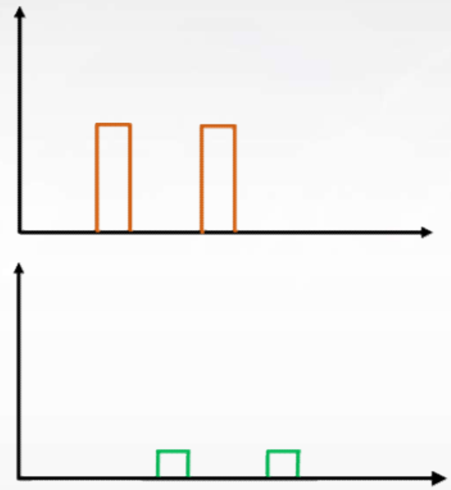
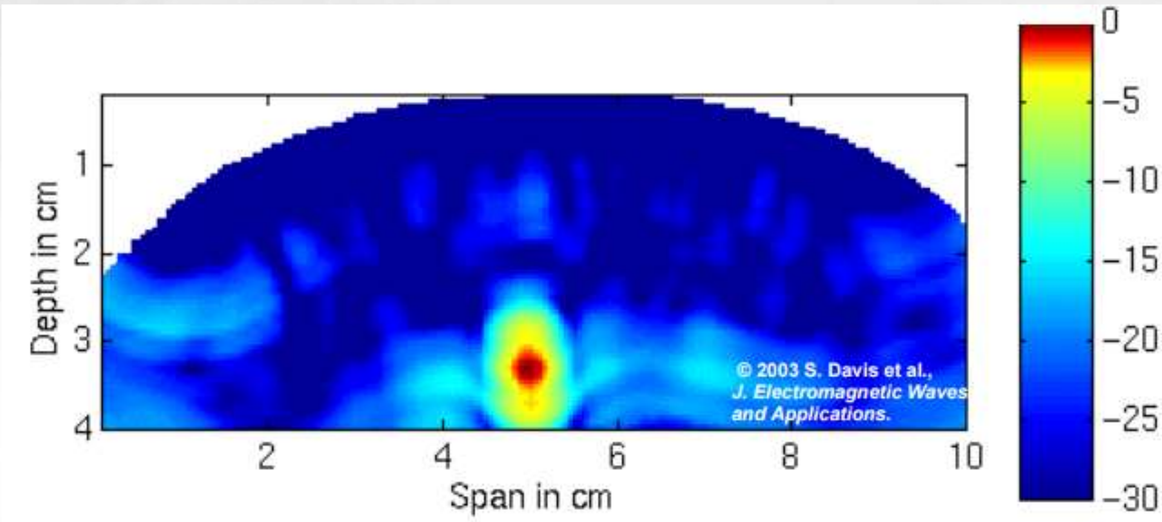


Fig. 8. Realistic breast-tissue model derived from high-resolution magnetic resonance imaging used in the ultrawideband pulse tumor-imaging study [8]. Source: X. Li and S. C. Hagness, University of Wisconsin–Madison.



A. Taflove, "Why study electromagnetics: The first unit in an undergraduate electromagnetics course," IEEE Antennas & Propagation Magazine, vol. 44, no. 2, pp. 132-139, April 2002.

Imaging of the Human Body - 2



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- Image reconstructed from backscattered waveforms obtained by solving Maxwell's equations. The color scale is in decibels. Note that the tumor signature is 15–30 dB (30–1,000 times) stronger than the backscattering clutter due to the surrounding normal tissues.

Summary

- Principle of operation of microwave oven;
- Application of the concepts for propagation of plane waves in lossy media such as grey matter of the brain; and
- Principle of detection of tumors using microwaves.

Health Effects of EM fields

- EM fields can have health effects on human beings.
- These effects may be adverse or beneficial.
- The extent of effects depends upon frequency, intensity, duration of fields, and part of the body exposed.
- Some recent articles:
  - James C. Lin, "*Reexamining Biological Studies of Effect of Low-Frequency Electromagnetic Field Exposure on Cells in Culture*," IEEE Microwave Magazine, June 2014.
  - James C. Lin, "*Current Activities on High-Frequency Exposure Guidelines for Humans,*" IEEE Microwave Magazine, May 2015.
  - Ting Wu et al, "*Safe for generations to come: Considerations of safety for millimetre waves*" IEEE Microwave Magazine, March 2015.

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