EE2003 – SEMICONDUCTOR FUNDAMENTALS

Tutorial 4

Donor & acceptor impurities, Thermal equilibrium (Lectures 7, 8, 9, 10, 11)

Prof Zhang Dao Hua

Office: S2-B2a-10

E-mail: edhzhang@ntu.edu.sg

Tel: 6790 4841

Question 1

Refer to the bonding model of GaAs in Fig. 3.1.

(a) Draw the bonding model for GaAs depicting the removal of the shaded Ga and As atoms. (*Hint: Ga and As take their bonding electrons with them when they are removed from the lattice*).

(b) Redraw the bonding model for GaAs showing the insertion of Si atoms into the missing Ga and As atom sites.

(c) Is the GaAs *p*- or *n*-type when Si atoms replace the Ga atoms? Explain.

(d) Is the GaAs *p*- or *n*-type when Si atoms replace the As atoms? Explain.

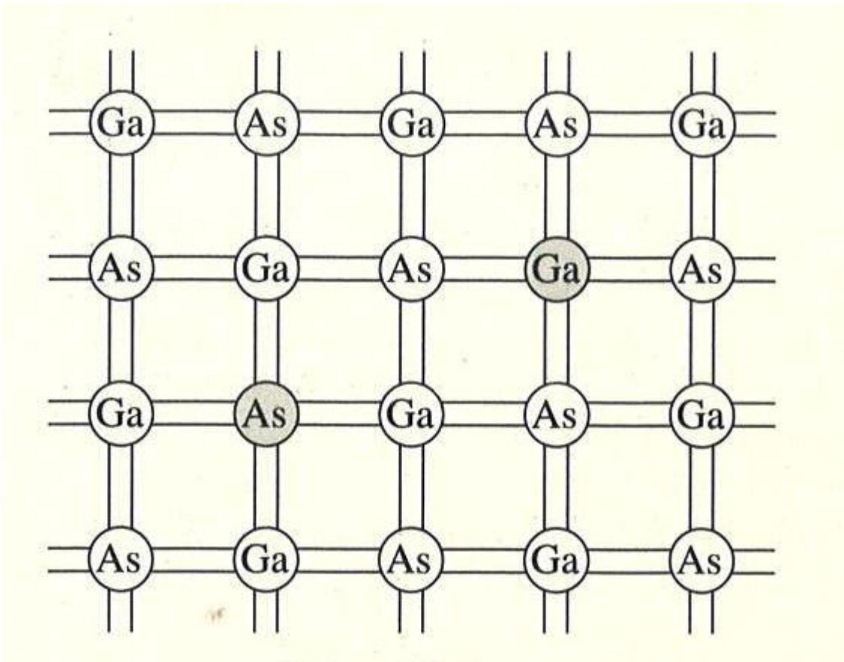
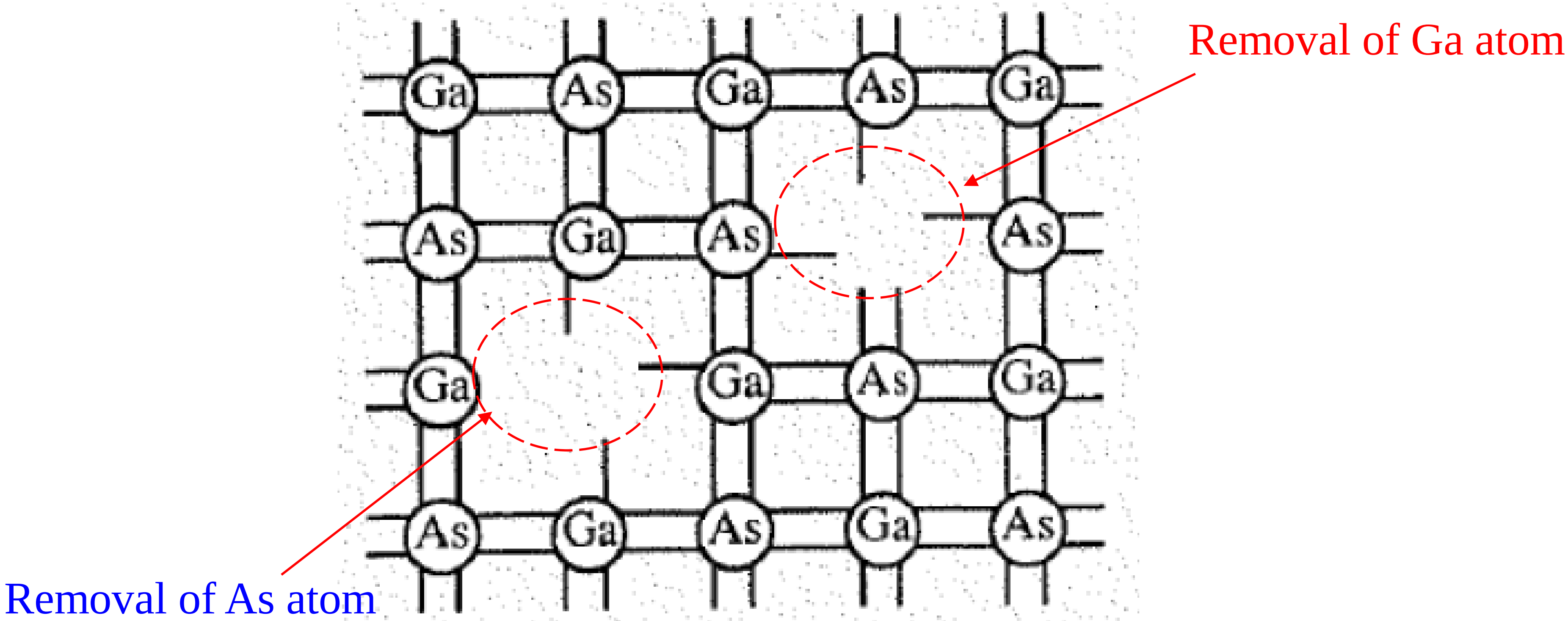


Fig. 3.1 The 2D bonding model for GaAs.

Solution 1

(a) The removal of column III Ga atom with three valence electrons leaves five dangling bonds in the vicinity of the vacancy.

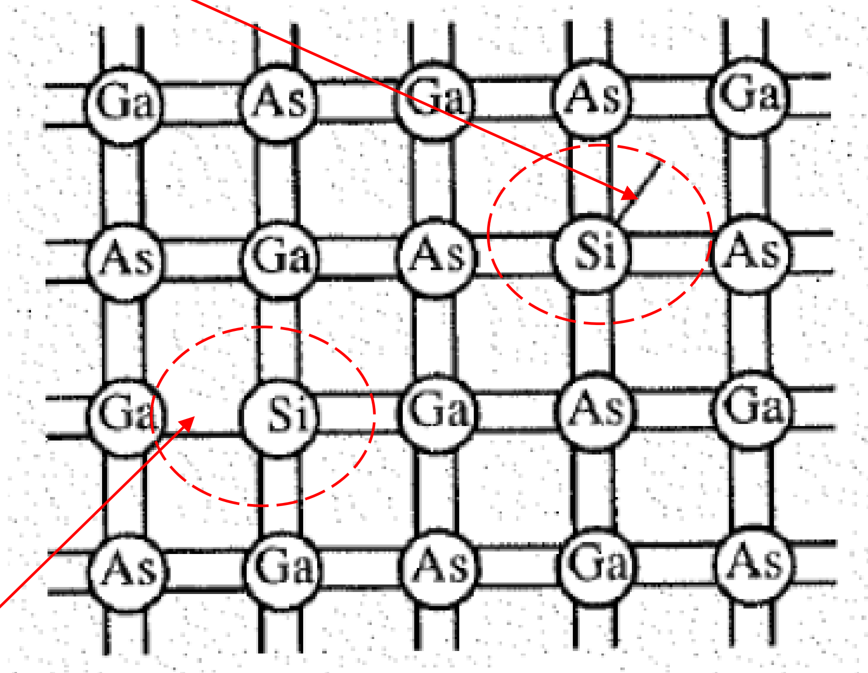
The removal of column V As atom with five valence electrons leaves three dangling bonds in the vicinity of the vacancy



Answer-(a)

(b) Bonding model for GaAs showing the insertion of Si atoms into the missing Ga and As atom sites

When a Si atom with four valence electrons is inserted into the missing Ga site, there is one extra electron that does not fit snugly into the bonding pattern



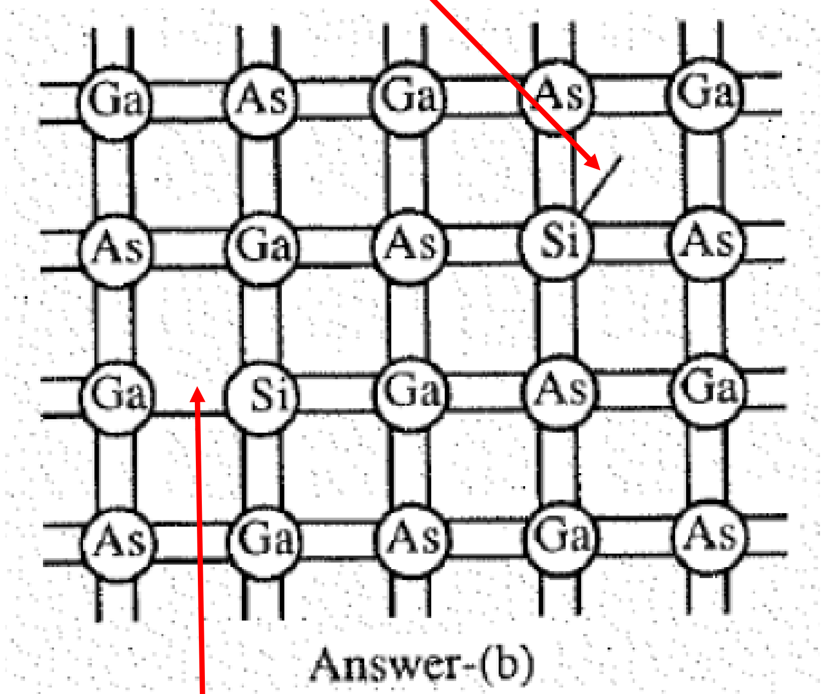
Answer-(b)

Conversely, when a Si atom is inserted into the missing As site, there are one too few bonds to complete the bonding scheme – there is a hole in the bonding scheme.

Solution 1 (continued)

(c) Is the GaAs p- or n-type when Si atoms replace the Ga atoms?

n-type: the extra electron noted in part (b) is readily released yielding an increase in the electron concentration.



(d) Is the GaAs p- or n-type when Si atoms replace the As atoms?
p-type: the missing bond noted in part (b) is readily filled at room temperature yielding an increase in the electron concentration.

Question 2

Assuming that the electrons in a particular material follow the Fermi-Dirac distribution function,

a) show that the probability of finding a hole with energy E is given by

$$\frac{1}{1+\exp[(E_F - E)/k_B T]}$$

b) calculate the temperature at which there is a 1% probability that a state 0.30 eV below the Fermi energy level will contain a hole.

2a) The probability of finding an electron at energy ε is

$$f(E) = \frac{1}{1 + \exp\left[\dfrac{E - E_F}{k_B T}\right]}$$

The probability of finding hole at energy ε is

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left[\dfrac{E - E_F}{k_B T}\right]}$$

$$1 - f(E) = \frac{\exp\left[\dfrac{E - E_F}{k_B T}\right]}{1 + \exp\left[\dfrac{E - E_F}{k_B T}\right]}$$

$$1 - f(E) = \frac{1}{1 + \exp\left[\dfrac{E_F - E}{k_B T}\right]}$$

b) Calculate the temperature at which there is a 1% probability that a state 0.30 eV below the Fermi energy level will contain a hole

The energy E is 0.30 eV below E_F, therefore $E_F - E = 0.30\ \text{eV}$

The probability of finding a hole with energy E is given by: $\dfrac{1}{1+\exp[(E_F - E)/k_B T]}$

$$0.01 = \frac{1}{1+\exp\left[\dfrac{0.30\ \text{eV}}{k_B\ T}\right]}$$

$$\exp\left[\frac{0.30\ \text{eV}}{k_B\ T}\right] = \frac{1-0.01}{0.01}, \qquad \text{and} \quad \frac{0.30\ \text{eV}}{k_B\ T} = \ln 99$$

$$T = \frac{0.30\ \times 1.6\times 10^{-19}\ \text{J}}{\ln 99\ \times 1.38\times 10^{-23}\ \ \text{J/K}} = 757\ \ \text{K}$$

Question 3

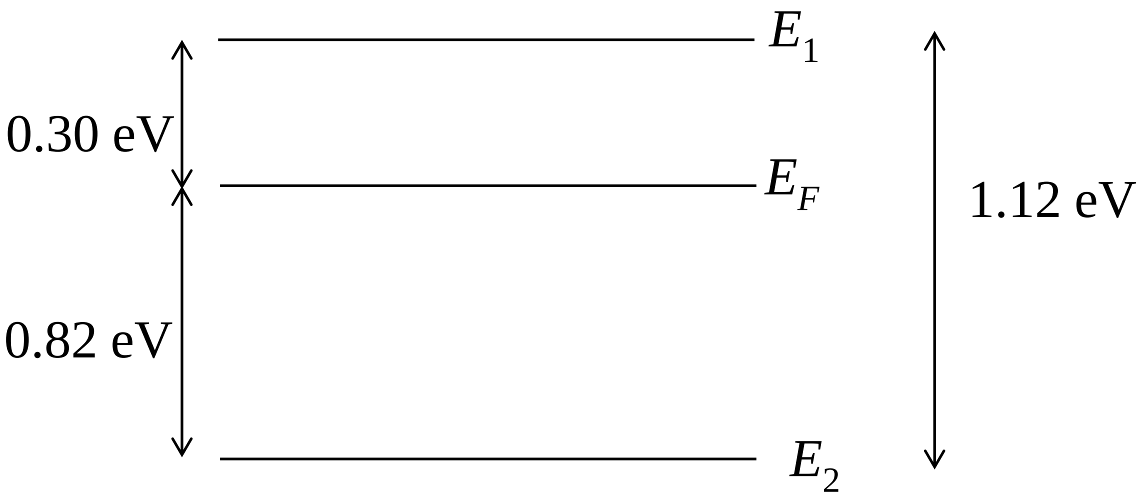
Consider the two energy levels E_1 and E_2 with $E_1 > E_2$ and an energy separation of 1.12 eV. Assume that the Fermi level E_F is in between the two levels and that $T = 300$ K.

If $E_1 - E_F = 0.30$ eV,

- determine the probability that an energy state at $E = E_1$ is occupied by an electron and

- the probability that an energy state at $E = E_2$ is empty.

Solution 3



The probability that E_1 is occupied by an electron is

$$f\left(E_1\right) = \frac{1}{1 + \exp\left[\dfrac{E_1 - E_F}{k_B \ T}\right]}$$

$$f\left(E_1\right) = \frac{1}{1 + \exp\left[\dfrac{0.3 \times 1.6 \times 10^{-19}\,\mathrm{J}}{1.38 \times 10^{-23}\,\mathrm{J/K} \times 300\mathrm{K}}\right]} = 9.2 \times 10^{-6}$$

The probability that E_2 is empty is

$$1-f\left(E_2\right)=1-\frac{1}{1+\exp\left[\frac{E_2-E_F}{k_B\ T}\right]}$$

$$1-f\left(E_2\right)=\frac{1}{1+\exp\left[\frac{E_F-E_2}{k_B\ T}\right]}$$

$$1-f\left(E_2\right)=\frac{1}{1+\exp\left[\frac{(1.12-0.3)\times 1.6\times 10^{-19}\,\mathrm{J}}{1.38\times 10^{-23}\,\mathrm{J/K}\times 300\mathrm{K}}\right]}=1.73\times 10^{-14}$$

Question 4

Consider a silicon crystal doped with boron atoms to a concentration of 5×10^{17} cm^{-3} at 300K,

a) determine the majority and minority carrier concentrations,

b) determine the position of the Fermi energy level inside the bandgap.

Take n_i to be 1.5×10^{10} cm^{-3} .

Solution 4

4a) Majority carriers are hole and the concentration is

$$p_o = N_a = 5\times10^{17}\,\mathrm{cm}^{-3}$$

Since: • we assume complete ionization at 300 K

• $N_a >> n_i$

Justification using the charge neutrality for complete ionization

$$N_d + p_0 = N_a + n_0$$

$$N_d = 0$$

$$p_o = \frac{n_i^2}{n_o}$$

$$p_o = N_a + \frac{n_i^2}{p_o}$$

$$p_o^2 - \left(N_a\right)p_0 - n_i^2 = 0$$

$$p_0 = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$p_o \neq \left(N_a\right) + n_i$$

$$N_a >> n_i \qquad \therefore p_o \approx N_a$$

The minority carriers are electron and the concentration is

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2\ \mathrm{cm}^{-6}}{5\times10^{17}\,\mathrm{cm}^{-3}} = 450\ \mathrm{cm}^{-3}$$

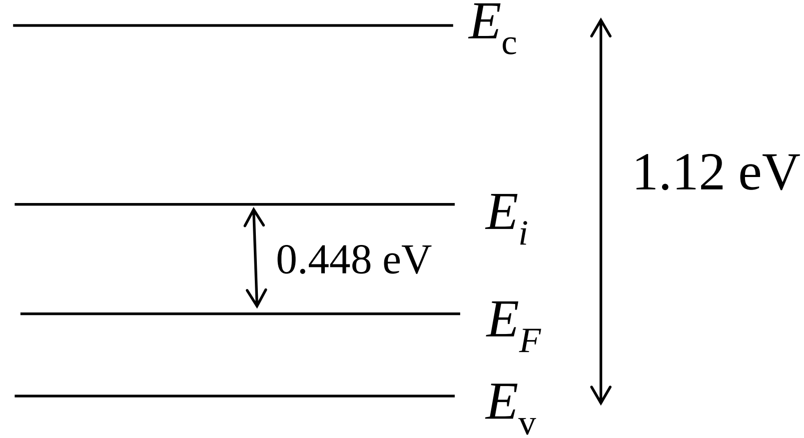
4b) The position of the Fermi energy level inside the bandgap.

$$p_o = n_i \exp\left[\frac{E_i - E_F}{k_B T}\right]$$

$$5\times10^{17} = 1.5\times10^{10} \exp\left[\frac{E_i - E_F}{1.38\times10^{-23}\,\text{J/K}\times300\text{K}}\right]$$

$$\left[\frac{E_i - E_F}{1.38\times10^{-23}\,\text{J/K}\times300\text{K}}\right] = \ln\left(\frac{5\times10^{17}}{1.5\times10^{10}}\right)$$

$$E_i - E_F = 7.17\times10^{-20}\,\text{J} = 0.448\ \text{eV}$$



So, the Fermi level lies 0.448 eV below the intrinsic Fermi level E_i

Question 5

Consider a germanium sample at 350 K which has been doped with donor impurities to a concentration of 6.0×10^{13} cm^{-3}. Taking the intrinsic carrier concentration as 2×10^{13} cm^{-3},

a) calculate the thermal equilibrium electron and hole concentrations.

b) determine the position of the Fermi energy level inside the bandgap

Solution 5

5a) Calculate the thermal equilibrium electron and hole concentrations.

$$N_d = 6\times10^{13}\text{cm}^{-3}$$

$$n_i = 2\times10^{13}\ \text{cm}^{-3}$$

Since N_d is of the same order as the n_i, we need to use the charge neutrality

$$N_d + p_0 = n_0$$

$$N_d + \frac{n_i^{\ 2}}{n_0} = n_0$$

$$n_0^{\ 2} - n_0 \ \ N_d - n_i^{\ 2} = 0$$

$$n_0 = \frac{N_d \pm \sqrt{N_d^{\ 2} + 4\ \ n_i^{\ 2}}}{2}$$

$$n_0 = \frac{6\times10^{13}\ \ + \sqrt{(6\times10^{13})^2 + 4\ \ (2\times10^{13})^2}}{2}\ \ \text{cm}^{-3}$$

$$n_0 = 6.6\times10^{13}\ \ \text{cm}^{-3}$$

$$p_0 = \frac{n_i^{\ 2}}{n_0} = \frac{(2\times10^{13})^2}{6.6\times10^{13}} = 6.05\times10^{12}\ \ \text{cm}^{-3}$$

5b) The position of the Fermi energy level inside the bandgap

$$n_0 = n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]$$

$$E_F - E_i = k_B T \ln\left(\frac{n_0}{n_i}\right)$$

$$E_F - E_i = \cancel{0.0259}\ 1.38\times10^{-23}\times350\times \ln\left(\frac{6.6\times10^{13}}{2\times10^{13}}\right) = 0.036\ \text{eV}$$

$$p_0 = n_i \exp\left[\frac{E_i - E_F}{k_B T}\right]$$

$$E_i - E_F = k_B T \ln\left(\frac{p_0}{n_i}\right)$$

$$E_i - E_F = \cancel{0.0259}\ 1.38\times10^{-23}\times350\times\ln\left(\frac{6.05\times10^{12}}{2\times10^{13}}\right) = -0.036\ \text{eV}$$

For thermal equilibrium, the Fermi level can be obtained from either the expressions of n_0 or p_0 regardless the material is n-type or p-type

Question 6

A hypothetical semiconductor has an intrinsic carrier concentration of 1.0×10^{10} cm^{-3} at 300K, it has conduction and valence band effective densities of states N_c and N_v, both equal to 10^{19} cm^{-3}.

(a) What is the band gap E_g?

(b) If the semiconductor is doped with $N_d = 1.0 \times 10^{16}$ donors/cm^3, what are the equilibrium electron and hole concentrations at 300K?

(c) If the same piece of semiconductor, already having $N_d = 1.0 \times 10^{16}$ donors/cm^3, is now doped with $N_a = 2 \times 10^{16}$ acceptors/cm^3, what are the new equilibrium electron and hole concentrations at 300K?

(d) Consistent with your answer to part (c), what is the Fermi level position with respect to the intrinsic Fermi level, $E_F - E_i$?

Solution 6

(a) What is the bandgap, E_g?

$$n_i = \sqrt{N_c N_v} \cdot e^{\frac{-E_g}{2kT}}$$

$$10^{10}\,\frac{1}{cm^3} = \sqrt{10^{19}\,\frac{1}{cm^3} \cdot 10^{19}\,\frac{1}{cm^3}} \cdot e^{\frac{-E_g}{2 \cdot 0.026eV}}$$

$$E_g = 0.052eV \cdot \ln\frac{10^{10}\,\frac{1}{cm^3}}{10^{19}\,\frac{1}{cm^3}} = 1.08eV$$

(b) If the semiconductor is doped with $N_d = 1 \times 10^{16}$ donors/cm^3, what are the equilibrium electron and hole concentrations at 300K?

$$n_o = 10^{16}\,\frac{1}{cm^3} \qquad p_o = \frac{n_i^2}{n_o} = \frac{10^{20}\,\frac{1}{cm^6}}{10^{16}\,\frac{1}{cm^3}} = 10^4\,\frac{1}{cm^3}$$

Solution 6

(c) If the same piece of semiconductor, already having $N_d = 1 \text{ x } 10^{16}$ donors/cm^3, is also doped with $N_a = 2 \text{ x } 10^{16}$ acceptors/cm^3, what are the new equilibrium electron and hole concentrations at 300K?

$$p_o = 10^{16} \frac{1}{cm^3} \qquad\qquad n_o = \frac{n_i^2}{p_o} = \frac{10^{20} \frac{1}{cm^6}}{10^{16} \frac{1}{cm^3}} = 10^4 \frac{1}{cm^3}$$

(d) Consistent with your answer to Part (c), what is the Fermi level position with respect to the intrinsic Fermi level, $E_F - E_i$?

$$E_F - E_i = kT \cdot \ln\left(\frac{p_o}{n_i}\right) = 0.026eV \cdot \ln\left(\frac{10^{16} \frac{1}{cm^3}}{10^{10} \frac{1}{cm^3}}\right) = 0.36eV$$