Tutorial 5

Question 1

Under equilibrium conditions at room temperature, a certain region of a Si device of length L has non-uniform acceptor doping as follows:

$$p(x)=N_A\,(x) = n_i \exp\left((a-x)/b\right) \;\ldots\ldots\ldots\; 0 \le x \le L$$

where a = 1.8 μm, b = 0.1 μm, and L = 0.8 μm.

(a) Draw the energy band diagram for the $0 \le x \le L$ region by showing E_c, E_f, E_i, and E_v on your diagram. Explain your steps

(b) Make a sketch of the field inside the region as a function of position, and compute the value of x at $x = L/2$.

Solution 1

(a) (i) We know E_F = constant on the diagram since equilibrium conditions prevail.

(ii)........ $p(x) = n_i e^{(a-x)/b} = n_i e^{(E_i - E_F)/kT}$

Therefore

$$E_i - E_F = kT(a-x)/b \quad \text{...a linear function of } x$$

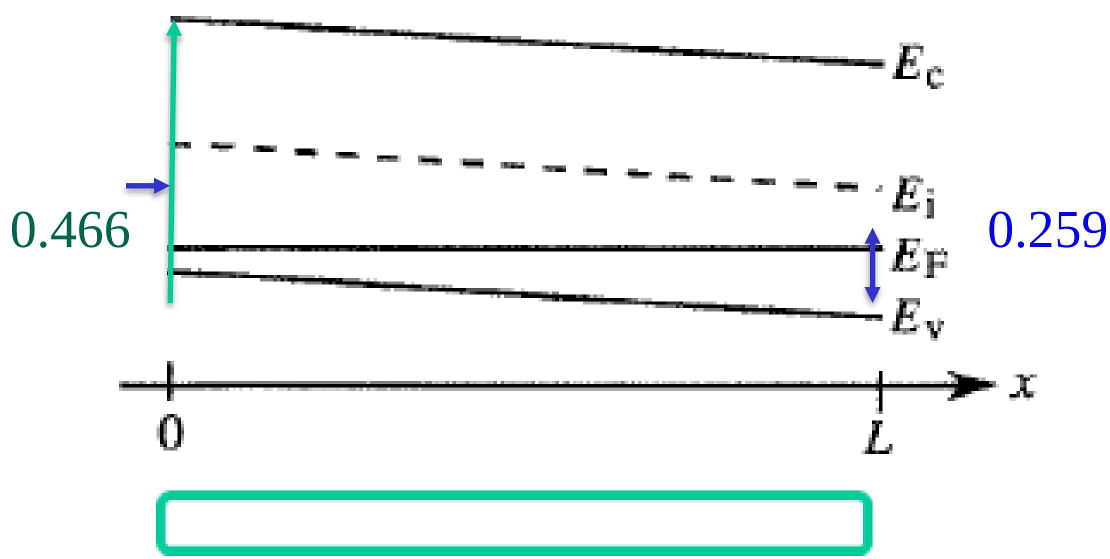
Moreover

at $x = 0$, $E_i - E_F = kTa/b = 18kT = 0.466\text{ eV}$

at $x = L$, $E_i - E_F = kT(a-L)/b = 10kT = 0.259\text{ eV}$

(iii) $E_G(\text{Si}) = 1.12\text{ eV}$ at room temperature and $E_i \cong (E_c + E_v)/2$.

Using the above information one concludes...



(b) Sketch of the ε field inside the region as a function of position, and compute the value of x at x = L/2.

(b)

The above $\mathcal{E}$ versus x plot can be deduced by inspection from the slope of the energy band diagram. Quantitatively,

$$\mathcal{E} = \frac{1}{q}\frac{dE_i}{dx} = -\frac{kT/q}{b} = -\frac{0.0259}{10^{-5}} = -\mathbf{2.59 \times 10^3\ V/cm}$$

$E_i - E_F = kT(a-x)/b$

kT = eV ➔ kT/q = V

Question2

A pure silicon sample maintained at room temperature has an intrinsic carrier concentration of 1.5×10^{10} cm^{-3}. It is first doped with donors of concentration 2×10^{14} cm^{-3}, followed by acceptors of concentration 4×10^{14} cm^{-3}. Assuming that the carrier mobilities are μ_n = 1350 cm^2/Vs and μ_p = 480 cm^2/Vs,

a) calculate the majority and minority carrier concentrations,

b) what is the resistivity of the pure sample, prior to the two types of dopings?

c) how will the resistivity change after the dopings?

[2×10^{14} cm-^{3}, 1.125×10^{6} cm^{-3}, $2.\times10^{5}$ Ω-cm, 65 Ω-cm]

Solution 2

2a) The majority and minority carrier concentrations

$$N_d + p_o = n_o + N_a$$

$$p_o \, n_o = n_i^2$$

$$\Longrightarrow N_d + p_o = \frac{n_i^2}{p_o} + N_a$$

$$p_o^2 - p_o \left(N_a - N_d\right) - n_i^2 = 0 \Longrightarrow p_o = \frac{\left(N_a - N_d\right) + \sqrt{\left(N_a - N_d\right)^2 + 4\,n_i^2}}{2}$$

$$\left|N_a - N_d\right| >> n_i$$

$$p_o \approx \left(N_a - N_d\right) \approx 2\times10^{14}\,\text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10}\ \text{cm}^{-3})^2}{2\times10^{14}\ \text{cm}^{-3}} = 1.125\times10^{6}\ \text{cm}^{-3}$$

Solution 2 (continued)

2b) The resistivity of the pure sample

$$n = p = n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q\left(\mu_n\, n + \mu_p\, p\right)}$$

$$\rho = \frac{1}{1.6 \times 10^{-19}\left(1350 \times 1.5 \times 10^{10} + 480 \times 1.5 \times 10^{10}\right)}$$

$$\rho = \frac{1}{4.392 \times 10^{-6} \text{ C cm}^2/\text{Vs cm}^{-3}} = 2.28 \times 10^{5}\ \Omega - \text{cm}$$

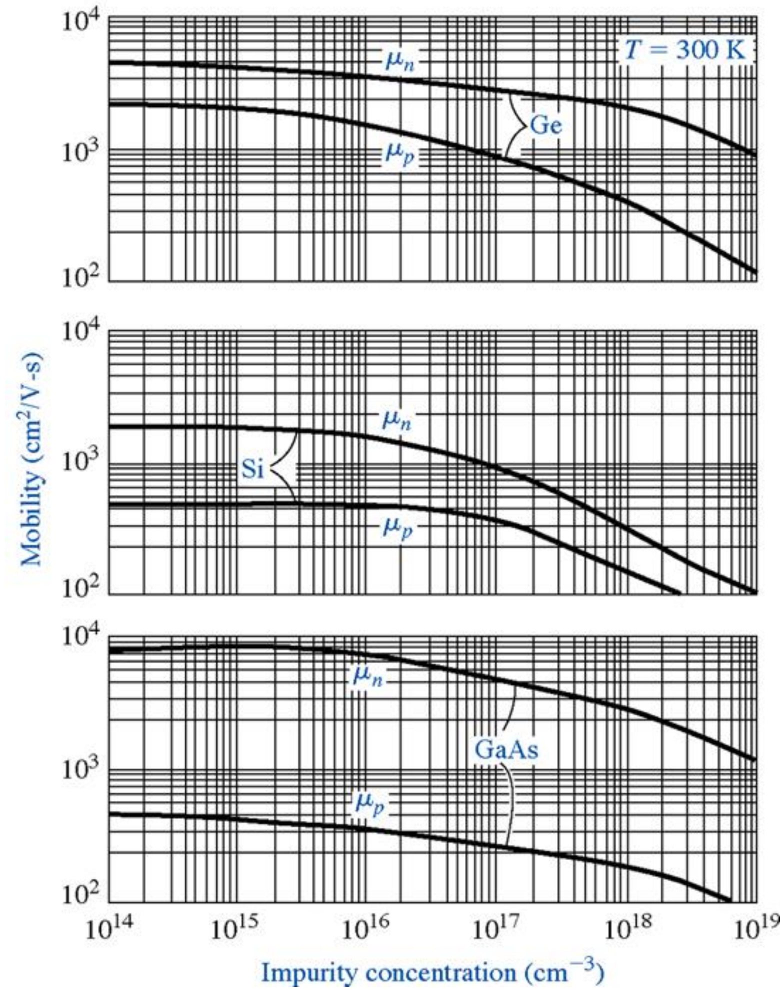
Solution 2 (continues)

2c) After doping

$$n = 1.125\times10^{6}\ \mathrm{cm}^{-3}$$

$$p = 2\times10^{14}\ \mathrm{cm}^{-3}$$

Assuming the mobilities stay the same with dopings



$$\rho = \frac{1}{\sigma} = \frac{1}{q\left(\mu_n\, n + \mu_p\ p\right)}$$

$$\rho = \frac{1}{1.6\times10^{-19}\left(1350\times1.125\times10^{6} + 480\times2\times10^{14}\right)}$$

$$\rho = \frac{1}{1.536\times10^{-2}\ \mathrm{C\ cm^{2}/Vs\ cm^{-3}}} = 65\ \Omega - \mathrm{cm}$$

Question 3

The electron concentration in silicon at 300K is given by

$$n(x) = 10^{16} \exp\left(-\frac{x}{a}\right) \quad \text{cm}^{-3}$$

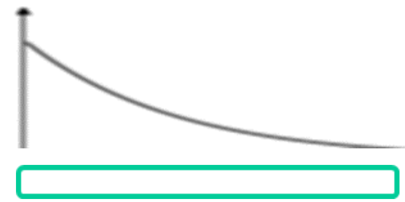
where a = 18 μm and x is valid for $0 \le x \le 25$ μm. The electron diffusion coefficient is 25 cm^2/s and the electron mobility is 960 cm^2/Vs. The total electron current density through the semiconductor is constant and equal to –40 A/cm^2. The electron current has both diffusion and drift current components.

Determine the electric field as a function of x which must exist in the semiconductor.

[14.5 – 26.0 exp(x/18) V/cm]

Solution 3

$$J_{n\,drift} = n\,q\,\mu_n\,\xi \quad \text{and} \quad J_{n\,diffu} = q\,D_n\frac{dn}{dx}$$



(1) $$n(x) = 10^{16}\exp\left(-\frac{x}{a}\right)\ /\text{cm}^3$$

$$\Rightarrow \frac{dn}{dx} = -\frac{10^{16}}{18\times10^{-4}}\exp\left(-\frac{x}{18}\right)\ /\text{cm}^4$$

(2) $$J_{n\,drift} = 1.6\times10^{-19}\times10^{16}\exp\left(-\frac{x}{18}\right)\times960\,\xi$$

$$(\text{C cm}^{-3}\ \text{cm}^2/\text{Vs V/cm}) = \text{A/cm}^2$$

$$J_{n\,drift} = 1.536\ \exp\left(-\frac{x}{18}\right)\xi \quad (\text{A/cm}^2)$$

(3) $$J_{n\,diffu} = 1.6\times10^{-19}\times25\times-\frac{10^{16}}{18\times10^{-4}}\exp\left(-\frac{x}{18}\right)$$

$$(\text{C cm}^2/\text{s}\ \ \text{cm}^{-4}) = \text{A/cm}^2$$

$$J_{n\ diffu} = -22.22\ \exp\left(-\frac{x}{18}\right)\ \ \text{A/cm}^2$$

Solution 3 (continued)

$$J_n = J_{n\ drift} + J_{n\,diff}$$

$$-40 = 1.536\ \exp\left(-\frac{x}{18}\right)\xi - 22.22\ \exp\left(-\frac{x}{18}\right)$$

$$\xi = \frac{22.22\ \exp\left(-\frac{x}{18}\right) - 40}{1.536\ \exp\left(-\frac{x}{18}\right)}\ \text{V/cm}$$

$$E = 14.48 - 26.04\ \exp\left(\frac{x}{18}\right)\ \text{V/cm}$$

Question 4

A semiconductor material (Si at T=300K) is doped such that electron concentration varies linearly across the sample, which is 0.5 μm thick. Donor concentration varies from 0 (at x = 0) to 10^{16} cm^{-3} (at x = 0.5 μm).

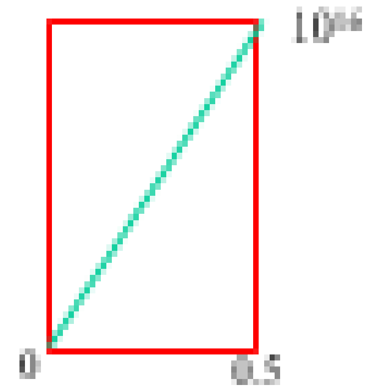
(a) Write equations for total electron and hole concentrations as a function of distance x.

(a) Determine electron and hole diffusion current densities if the diffusion coefficients are D_n = 30 cm^2/V.s and D_p = 12 cm^2/V.s, respectively.

(a) At x = 0.5 μm, determine the hole diffusion current density and the position of Fermi level E_F with respect to conduction band edge E_C. Assume temperature = 300K.

Solution 4

Since at x = 0, the donor concentration is 0, thus the equilibrium electron concentration becomes,



(a) $n(x) = \dfrac{\Delta N_D}{\Delta x} x = \dfrac{1 * 10^{16}}{0.5 * 10^{-4}} = 2 * 10^{20} x$ /cm^4.

Equilibrium hole concentration, $p(x) = \dfrac{n_i^{\,2}}{n(x)} = \dfrac{n_i^{\,2}}{2 * 10^{20} . x}$.

(b) Now the diffusion current densities can be expressed as,

$$J_n(diff) = q.D_n.\frac{dn(x)}{dx} = 1.6 * 10^{-19} * 30 * 2 * 10^{20} = 960 \text{ Amp/cm}^2.$$

$$J_p(diff) = -q.D_p.\frac{dp(x)}{dx} = 1.6 * 10^{-19} * 12 * \frac{n_i^{\,2}}{2 * 10^{20}} * (x^{-2})$$

(c) At x = 0.5 µm, $J_p(diff) = 4.5 * 10^{-10}$ Amp/cm^2.

For Fermi level, $E_C(x) - E_F = -KT \ln\left[\dfrac{n(x)}{N_C}\right]$

NC = $2.8 * 10^{19}$ /cm3, then at x = 0.5 µm, $E_C - E_F = 0.326$ eV.