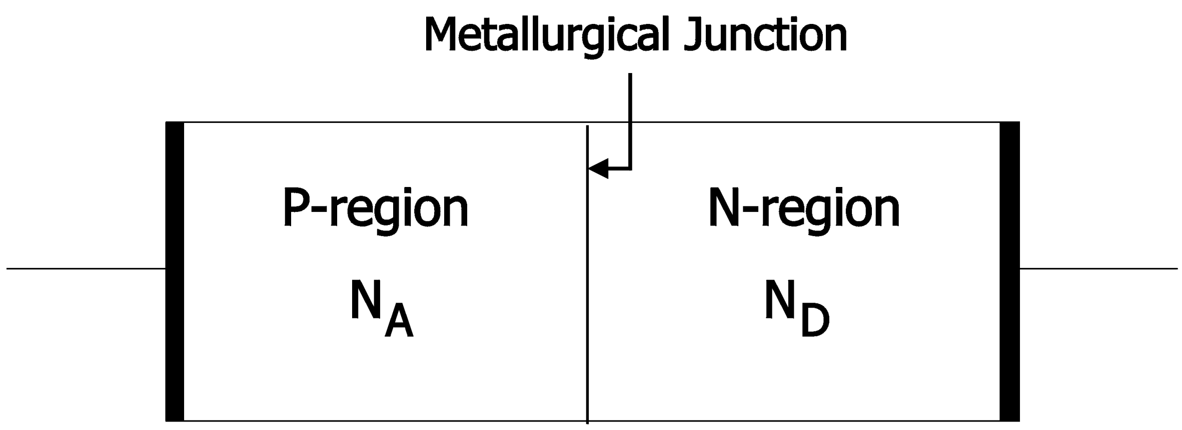
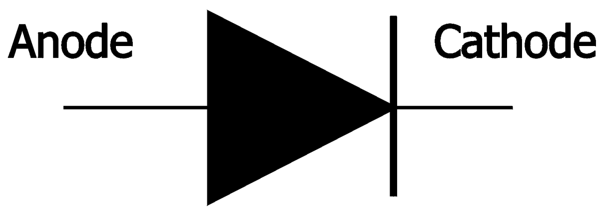
EE2003 Semiconductor Fundamentals

Electrostatics of the P-N Junction

Basic Structure

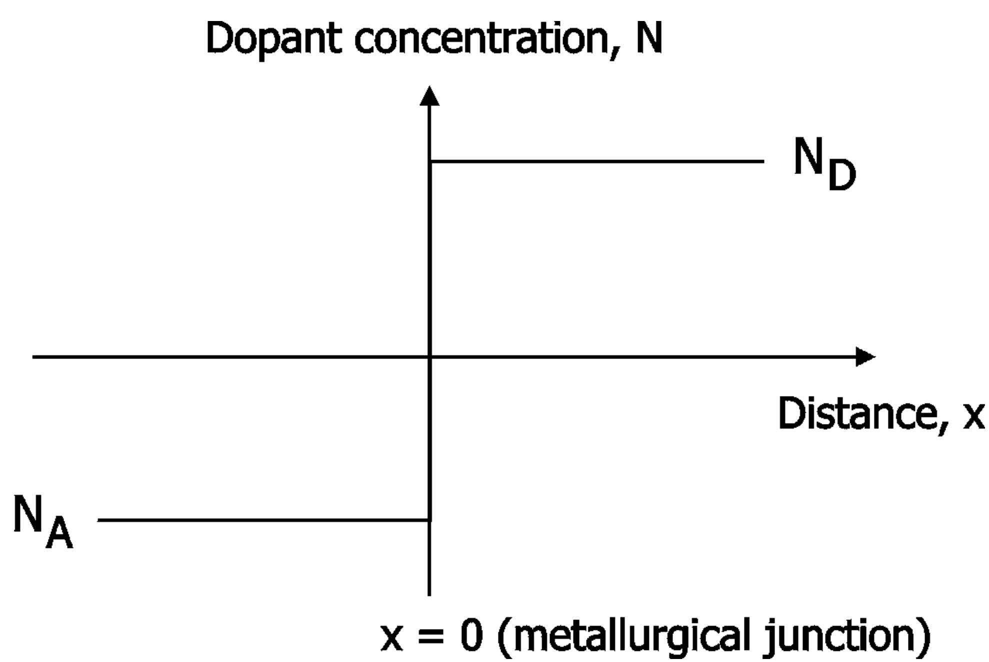
- A P-N junction is formed when an **n-type** semiconductor region is brought into close contact with a **p-type** semiconductor region.





Basic Structure

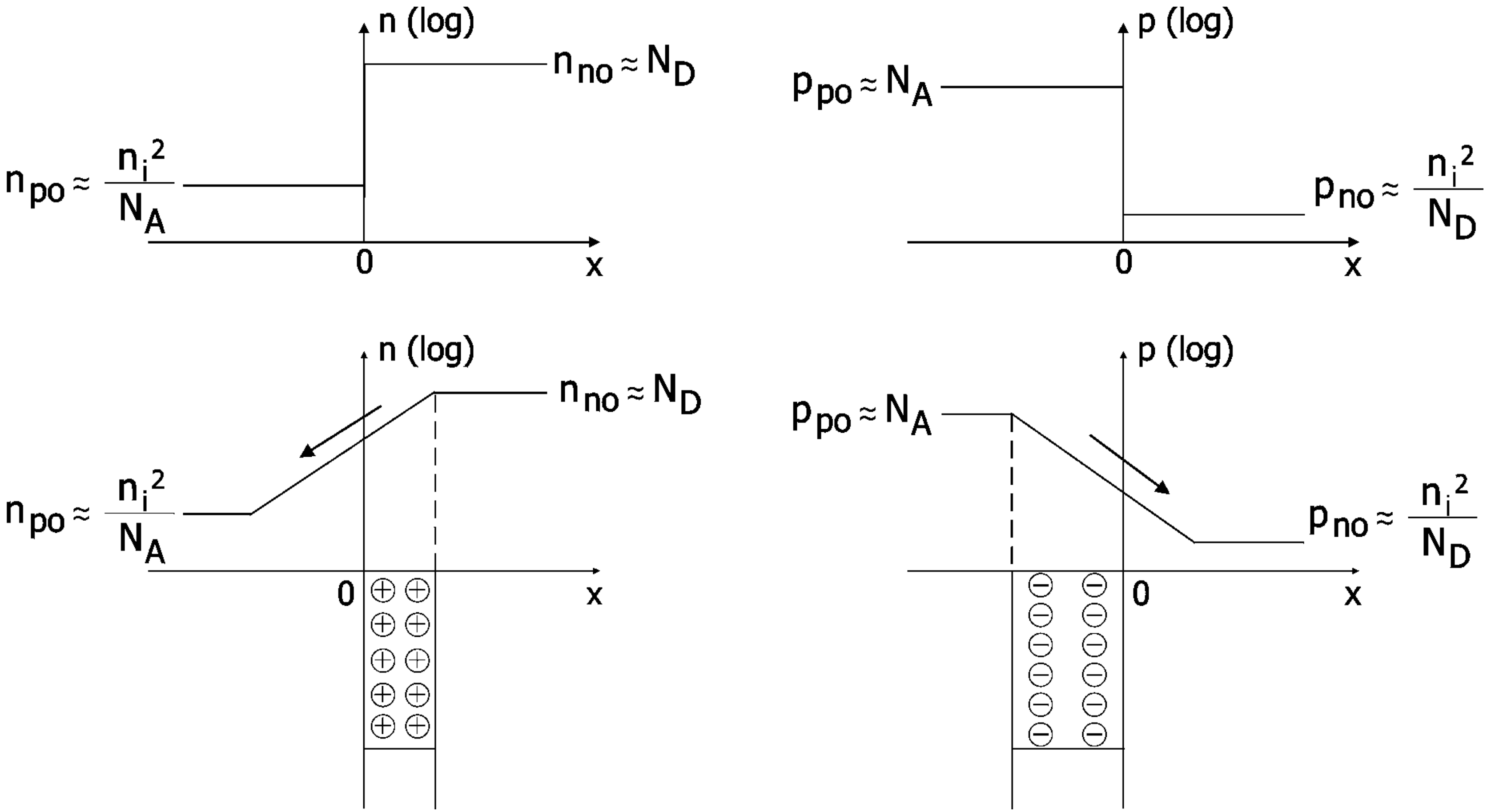
- **Assumptions:**
  - Uniformly doped n and p regions
  - Abrupt or step junction



Important Concepts

- Space charge (depletion) region
- Built-in electric field
- Built-in potential (barrier height)

Formation of the Space Charge Region



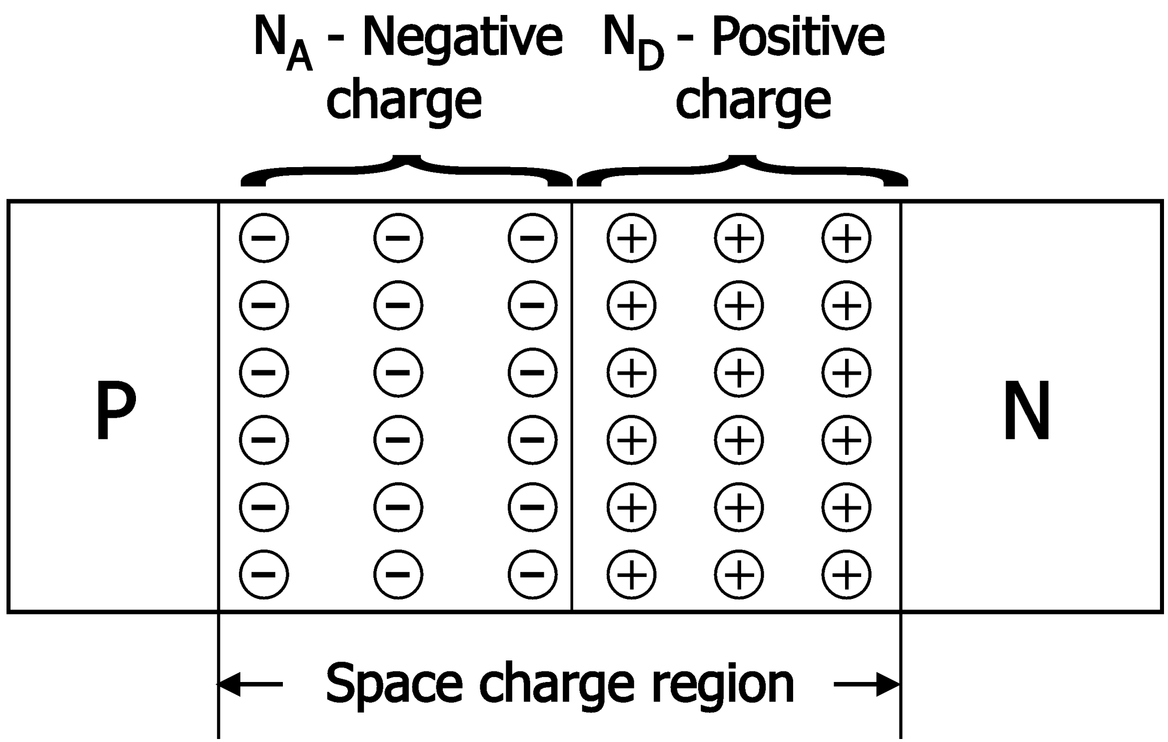
Formation of the Space Charge Region

- N region – High concentration of electrons (majority carriers)
- P region – Low concentration of electrons (minority carriers)
- An electron concentration gradient exists across a PN junction:
  - Electrons from the n region diffuse to the p region.
  - The departure of electrons leaves behind the immobile donor ions (positively charged) in the n region.

Formation of the Space Charge Region

- P region – High concentration of holes (majority carriers)
- N region – Low concentration of holes (minority carriers)
- A hole concentration gradient exists across a PN junction:
  - Holes from the p region diffuse to the n region.
  - The departure of holes leaves behind the immobile acceptor ions (negatively charged) in the p region.

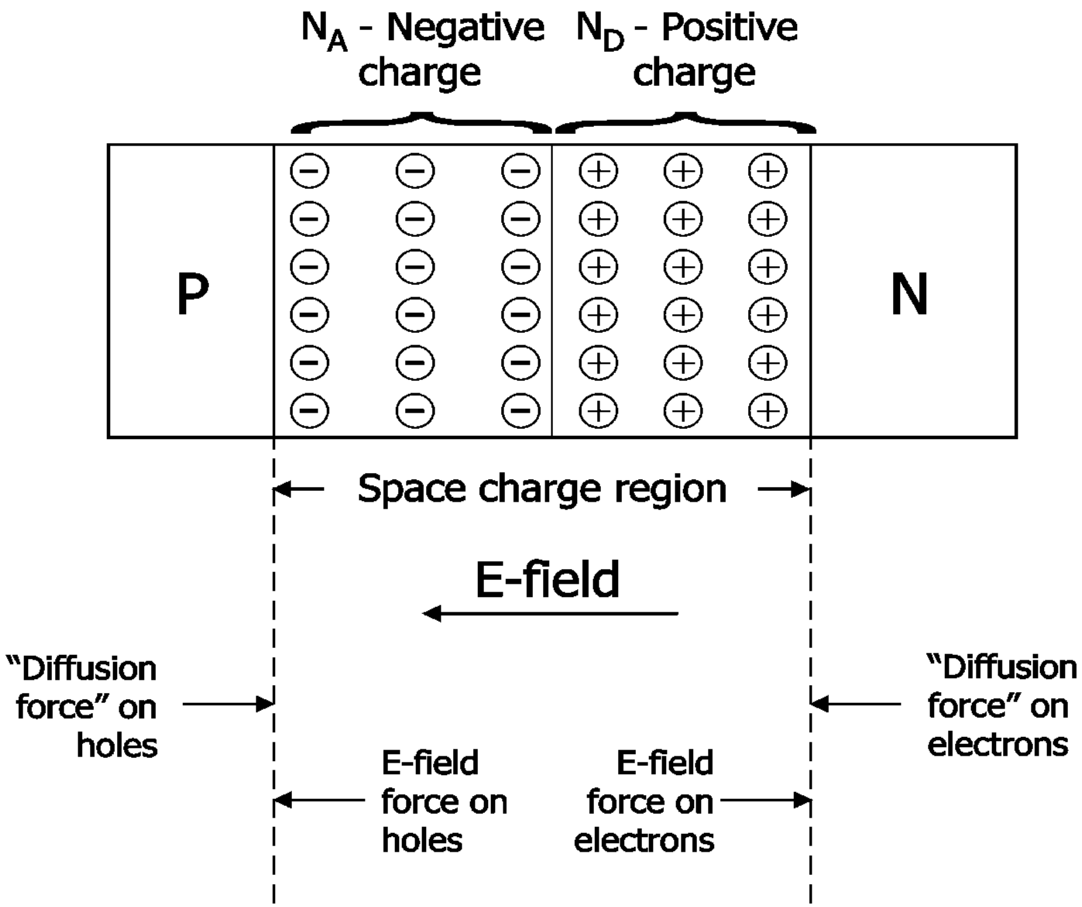
Formation of the Space Charge Region



Formation of the Space Charge Region

- The net positively and negatively charged regions are known as the **space charge regions**.
- Because the concentrations of electrons and holes in this region are lower than that in the respective neutral n and p regions, the space charge region is also known as the **depletion region**.

Built-In Electric Field



Built-In Electric Field

- The net positive and negative space charges in the n and p regions induce an electric field.
- Since this electric field is automatically created when a PN junction is formed, it is known as the **built-in electric field**.
- This electric field points from the positive to the negative charge, i.e. from the n to the p region.

Built-In Electric Field

- The built-in electric field counteracts the electron and hole diffusion processes.
- At **thermal equilibrium**, this counteracting force exactly balances the “diffusion force” exerted by the concentration gradient.
- There is **no net movement** of mobile charges across the PN junction under thermal equilibrium.

Built-In Electric Field

- Under thermal equilibrium,

$$\underbrace{qn\mu_{\mathrm{n}}\xi}_{\text{Electron drift current}} + \underbrace{qD_{\mathrm{n}}\frac{\partial n}{\partial x}}_{\text{Electron diffusion current}} = 0$$

$$\underbrace{qp\mu_{\mathrm{p}}\xi}_{\text{Hole drift current}} - \underbrace{qD_{\mathrm{p}}\frac{\partial p}{\partial x}}_{\text{Hole diffusion current}} = 0$$

Built-In Electric Field

- A mathematical expression for the built-in electric field $\xi(x)$ can be derived by solving the **Poisson's equation**.

$$\frac{\partial^2 V}{\partial x^2} = -\frac{\rho(x)}{\varepsilon_r \varepsilon_0} = -\frac{\partial \xi}{\partial x}$$

  - ρ is the volume charge density/concentration
  - ε_r is the relative permittivity of the semiconductor
  - ε_0 is the permittivity of free space

Solving the Poisson's Equation

- Volume charge density, ρ is given by the sum of **all** positive and negative charges in the space charge region.

$$\rho(x) = \underbrace{qN_D + qp(x)}_{\textbf{positive charges}} - \underbrace{qN_A - qn(x)}_{\textbf{negative charges}}$$

  - Since we have assumed uniform doping concentration, N_A and N_D are independent of distance. However, it is imporant to realise that N_A and N_D are in general functions of distance in practical diodes.
  - p and n are functions of distance, x. The Poisson's equation is thus difficult to solve since we do not exactly know $p(x)$ and $n(x)$.

Solving the Poisson's Equation

- **Assumptions:**
  - **Depletion approximation** – The concentrations of mobile charges in the space charge region are negligible compared to the concentration of the immobile ionic (space) charge.
  - The space charge region ends abruptly at $x = +x_{no}$ and $x = -x_{po}$.

Solving the Poisson's Equation

- Applying the depletion approximation, we arrive at a simplified expression for the volume charge density.

$$\rho \approx qN_\mathbf{D} - qN_\mathbf{A}$$

- Considering the p-region,

$$\rho \approx -qN_\mathbf{A}$$

- Simplified Poisson's equation:

$$\frac{\partial \xi}{\partial x} = -\frac{qN_\mathbf{A}}{\varepsilon_\mathbf{r}\varepsilon_\mathbf{o}}$$

Solving the Poisson's Equation

- Integrating w.r.t. x,

$$\xi(x) = -\frac{qN_A}{\varepsilon_r \varepsilon_0} x + C$$

- C is an integration constant that can be determined by applying an appropriate boundary condition.
- There are two possible boundary conditions: $\xi\,(x = 0)$ and $\xi\,(x = -x_{p0})$.
- Which one should we use?

Solving the Poisson's Equation

- Since we do not yet know what ξ $(x = 0)$ is, the relevant boundary condition to use is:

$$\xi\left(x = -x_{p0}\right) = 0$$

- Integration constant:

$$\xi\left(-x_{p0}\right) = -\frac{qN_A}{\varepsilon_r \varepsilon_0}\left(-x_{p0}\right) + C = 0$$

$$\therefore \quad C = -\frac{qN_A x_{p0}}{\varepsilon_r \varepsilon_0}$$

Solving the Poisson's Equation

- Built-in electric field (p region):

$$\xi(\boldsymbol{x}) = -\frac{q N_A}{\varepsilon_r \varepsilon_0}\left(\boldsymbol{x} + \boldsymbol{x}_{p0}\right), \quad -\boldsymbol{x}_{p0} \le \boldsymbol{x} \le 0$$

  - For an abrupt or step junction, the electric field is a linear function of distance.
  - Note that x is negative. But since $x \geq -x_{p0}$, the electric field ξ is negative.
  - The negative sign denotes the direction of the electric field, from **the n to the p region** or in the negative x-axis direction.

Exercise

- Show that the built-in electric field in the n region can be expressed as:

$$\xi(x) = \frac{qN_D}{\varepsilon_r \varepsilon_0}(x - x_{n0}), \quad 0 \le x \le x_{n0}$$

  - Note that since $x > 0$ and $x \le x_{n0}$, ξ is also negative in this case.

Built-In Electric Field

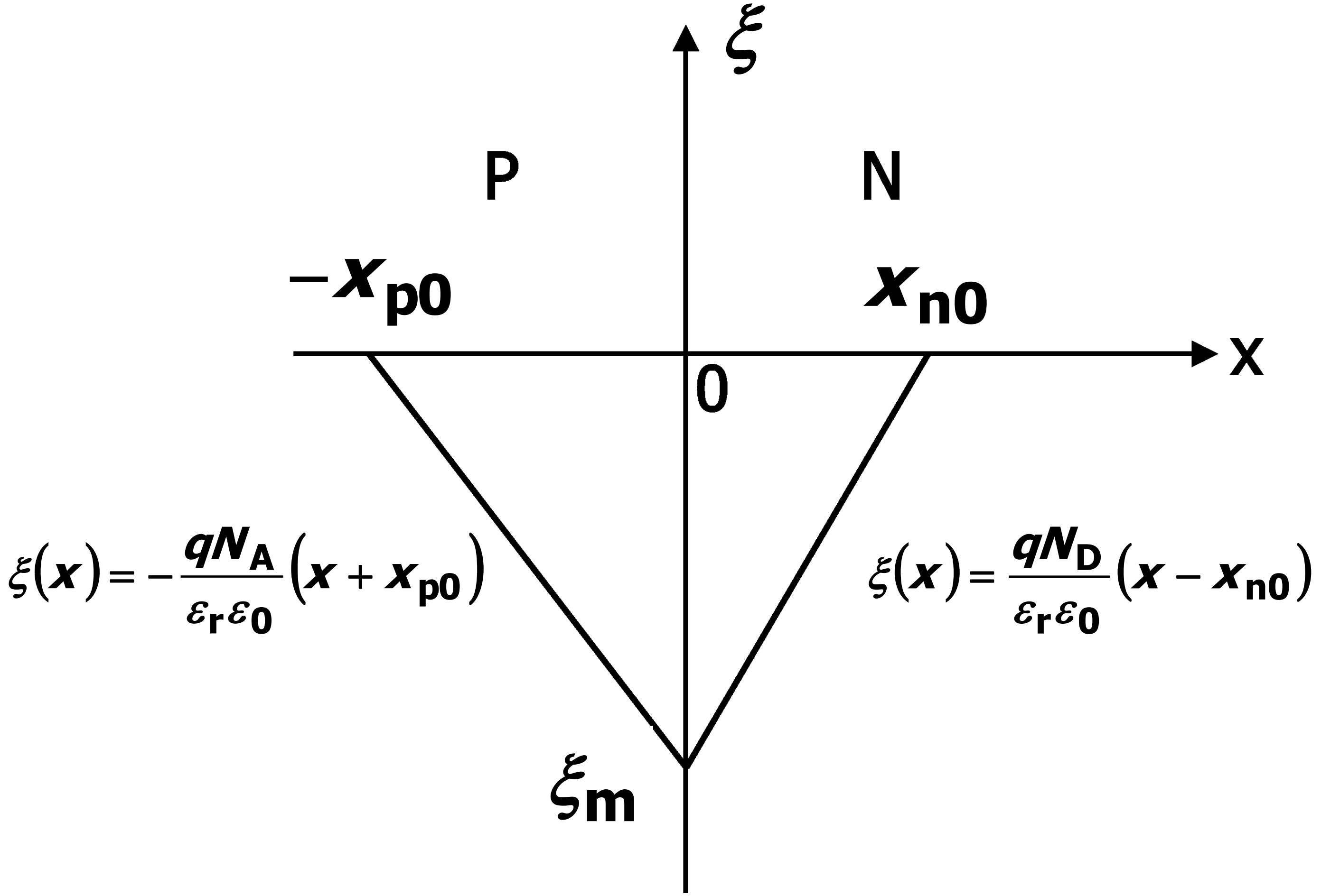
- **Charge neutrality**

$$\underbrace{qN_A x_{p0}}_{\substack{\text{net negative charge} \\ \text{on p-type side}}} = \underbrace{qN_D x_{n0}}_{\substack{\text{net positive charge} \\ \text{on n-type side}}}$$

$$\underbrace{\frac{qN_A x_{p0}}{\varepsilon_r \varepsilon_0}}_{\substack{|\xi(x=0)| \text{ using electric field} \\ \text{expression on p-type side}}} = \underbrace{\frac{qN_D x_{n0}}{\varepsilon_r \varepsilon_0}}_{\substack{|\xi(x=0)| \text{ using electric field} \\ \text{expression on n-type side}}}$$

- $\therefore$ the electric field is **continuous** at $x = 0$ (metallurgical junction).

Built-In Electric Field



Maximum Built-In Field

- The **maximum** built-in field occurs at the **metallurgical junction** ($x = 0$).
- Remarks:
  - Although we arrive at this conclusion by assuming a step junction, the same conclusion applies to PN junction with arbitrary junction doping profile.
  - The maximum built-in field is important as it determines the breakdown voltage of the PN junction.

Built-In Voltage/Potential

- The separation of positive and negative charges in the space charge region of a PN junction induces a built-in electric field.
- The built-in electric field in turn causes a potential difference between the n and p regions. This potential difference is called the built-in voltage of a PN junction.
- **Question**:
  - Which side (n or p) is at a higher potential?

Built-In Voltage/Potential

- The built-in voltage can be evaluated using the following fundamental relationship:

$$V_{bi} = -\int \xi \ \partial x$$

- Geometrically, the above integral gives the **area** under the electric field versus distance plot.
- In the case of the step junction,

$$V_{bi} = \frac{1}{2} \cdot |\xi_m| \cdot \left(x_{n0} + x_{p0}\right) = \frac{|\xi_m| \cdot W_0}{2}$$

Built-In Voltage/Potential

- Recall that the maximum electric field,

$$\left|\xi_{\mathbf{m}}\right| = \left|\xi(\boldsymbol{x} = \mathbf{0})\right| = \frac{qN_{\mathrm{A}}x_{\mathrm{p0}}}{\varepsilon_{\mathrm{r}}\varepsilon_{\mathbf{0}}} = \frac{qN_{\mathrm{D}}x_{\mathrm{n0}}}{\varepsilon_{\mathrm{r}}\varepsilon_{\mathbf{0}}}$$

- Therefore, the built-in voltage,

$$V_{\mathrm{bi}} = \frac{qN_{\mathrm{A}}x_{\mathrm{p0}}}{2\varepsilon_{\mathrm{r}}\varepsilon_{\mathbf{0}}}\left(x_{\mathrm{n0}} + x_{\mathrm{p0}}\right) = \frac{qN_{\mathrm{D}}x_{\mathrm{n0}}}{2\varepsilon_{\mathrm{r}}\varepsilon_{\mathbf{0}}}\left(x_{\mathrm{n0}} + x_{\mathrm{p0}}\right)$$

Space Charge Width

- The distance that the space charge region extends into the p and n regions can be determined by solving the following two equations:

$$V_{bi} = \frac{qN_A x_{p0}}{2\varepsilon_r \varepsilon_0}\left(x_{n0} + x_{p0}\right) \qquad (1)$$

$$x_{n0} N_D = x_{p0} N_A \qquad (2)$$

Space Charge Width

- Width of the depletion region extending into the n region:

$$x_{n0} = \left\{ \frac{2\varepsilon_r \varepsilon_0 V_{bi}}{q} \left[\frac{N_A}{N_D (N_A + N_D)} \right] \right\}^{1/2}$$

- Width of the depletion region extending into the p region:

$$x_{p0} = \left\{ \frac{2\varepsilon_r \varepsilon_0 V_{bi}}{q} \left[\frac{N_D}{N_A (N_A + N_D)} \right] \right\}^{1/2}$$

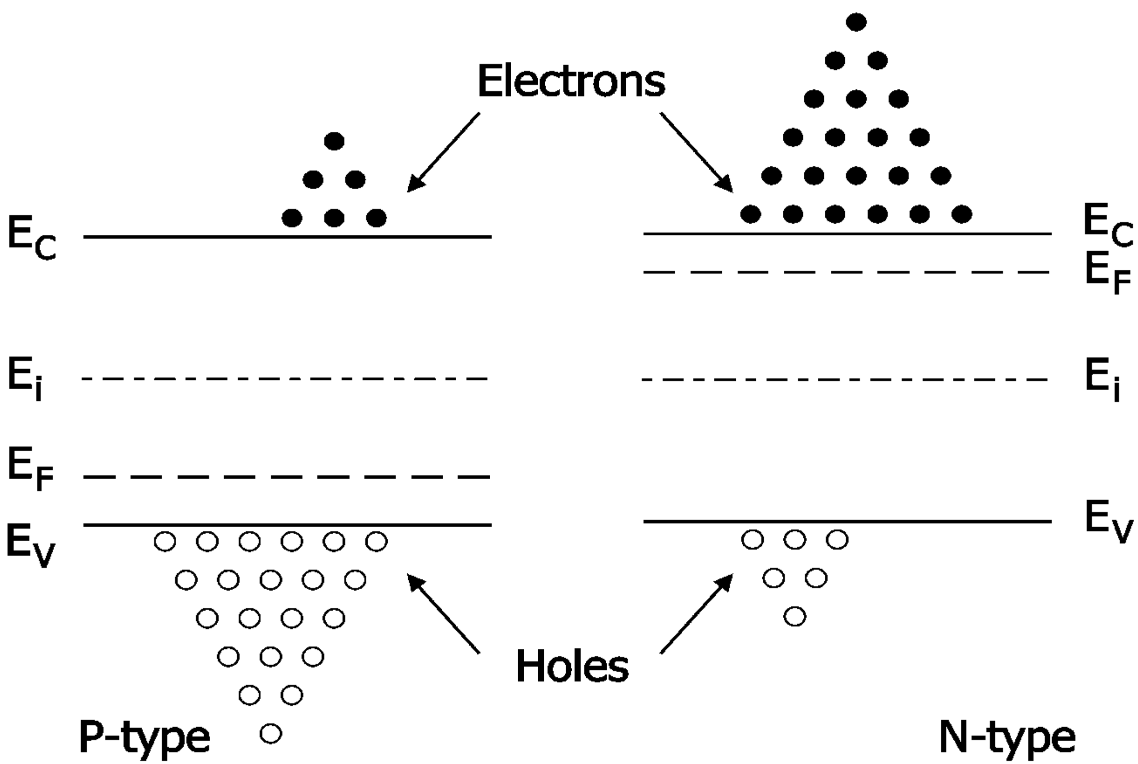
Space Charge Width

- Total space charge width:

$$\boldsymbol{W_0 = x_{n0} + x_{p0}}$$

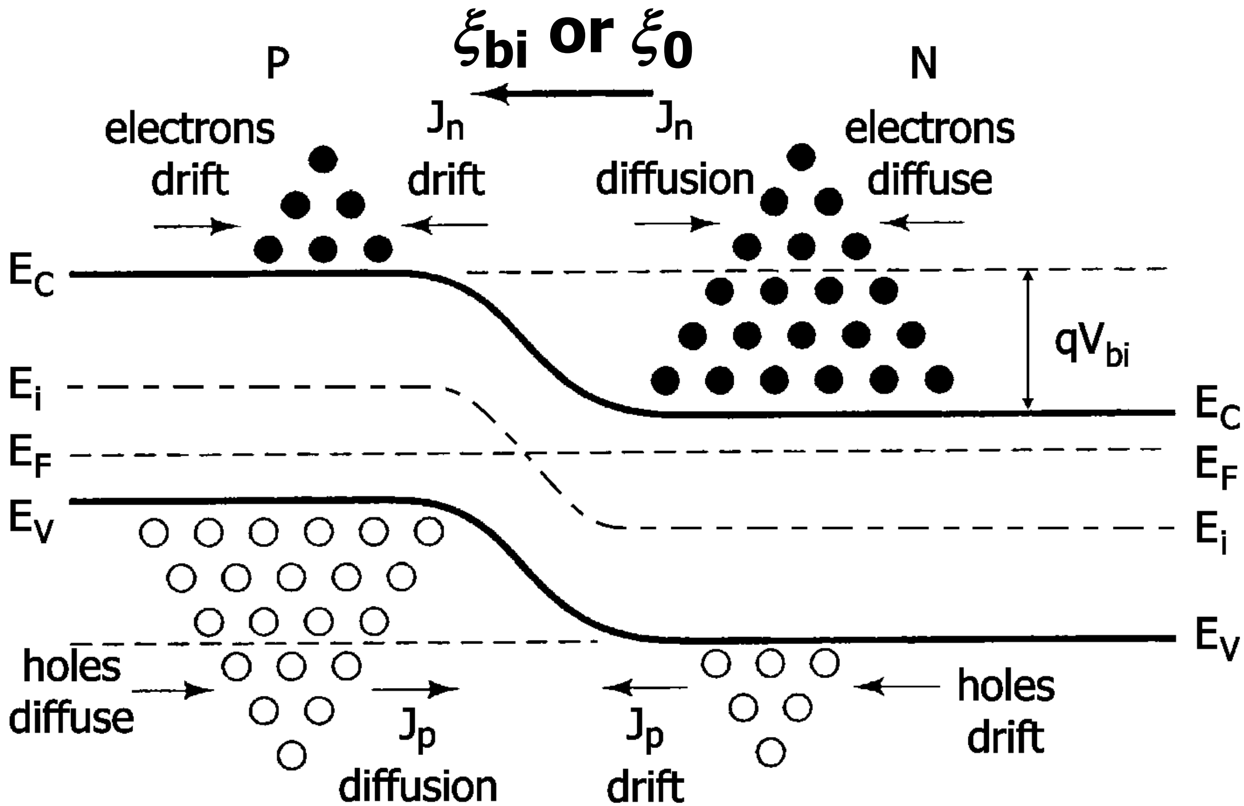
$$= \left\{ \frac{2\varepsilon_r \varepsilon_0 V_{bi}}{q} \left[\frac{N_A + N_D}{N_A N_D} \right] \right\}^{1/2}$$

Energy Band Diagram



Energy band diagrams of the n and p regions before contact

Energy Band Diagram



Energy band diagram of a pn junction under thermal equilibrium

Energy Band Diagram

- Built-in voltage:

$$V_{bi} = \frac{(E_i - E_F)_p + (E_F - E_i)_n}{q}$$

- Recall Maxwell-Boltzmann equations:

$$p = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right], \quad n = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

Energy Band Diagram

- Rewriting:

$$p = N_V \exp\left[\frac{-\left(E_F - E_i + E_i - E_V\right)}{kT}\right]$$

$$= N_V \exp\left[\frac{-\left(E_i - E_V\right)}{kT}\right] \exp\left[\frac{E_i - E_F}{kT}\right]$$

$$= n_i \exp\left[\frac{E_i - E_F}{kT}\right]$$

$$n = n_i \exp\left[\frac{E_F - E_i}{kT}\right]$$

Energy Band Diagram

- For complete ionization of dopants,

$$N_A = n_i \exp\left[\frac{E_i - E_F}{kT}\right], \quad N_D = n_i \exp\left[\frac{E_F - E_i}{kT}\right]$$

- Taking the natural log on both sides, and re-arranging:

$$(E_i - E_F)_p = kT \ln\left(\frac{N_A}{n_i}\right), \quad (E_F - E_i)_n = kT \ln\left(\frac{N_D}{n_i}\right)$$

Energy Band Diagram

- Built-in voltage:

$$V_{\mathrm{bi}} = \frac{kT\ln\left(N_{\mathrm{A}}/n_{\mathrm{i}}\right) + kT\ln\left(N_{\mathrm{D}}/n_{\mathrm{i}}\right)}{q}$$

$$= \frac{kT}{q}\ln\left(\frac{N_{\mathrm{A}}N_{\mathrm{D}}}{n_{\mathrm{i}}^{2}}\right)$$

Example 1

- Consider a silicon p-n junction at T=300 K with doping densities $N_A=10^{18}$ cm^{-3} and $N_D=10^{15}$ cm^{-3}. Given that $n_i=1.5\text{x}10^{10}$ cm^{-3} at T=300 K.
  - Calculate the built-in potential barrier.
  - If we change the acceptor doping from 10^{18} cm^{-3} to 10^{16} cm^{-3}, what is the new built-in potential barrier?

Example 1

- The built-in potential barrier is determined using the following relation:

$$V_{bi,1} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5\times 10^{10})^2}\right]$$

$$= 0.754\ \text{V}$$

Example 1

- If the acceptor doping is 10^{16} cm^{-3} instead of 10^{18} cm^{-3}, the built-in potential barrier is

$$V_{bi,2} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5\times 10^{10})^2}\right]$$

$$= 0.635 \text{ V}$$

  - Note that $V_{bi,1}$-$V_{bi,2}$=119mV for a 100 times decrease in acceptor doping concentration.
  - This weak dependence is because of the logarithmic function.

Example 2

- Consider a silicon p-n junction at T=300 K with doping concentrations of $N_A=10^{16}$cm^{-3} and $N_D=10^{15}$ cm^{-3}.
  - Calculate the space charge width
  - Calculate the maximum electric field

- In example 1, we have already determined the built-in voltage as V_{bi}=0.635 V.

Example 2

- Since we know the built-in voltage and the doping concentrations of the n and p regions, the space charge width can be easily calculated:

$$W_0 = \left\{ \frac{2\varepsilon_r \varepsilon_0 V_{bi}}{q} \left[\frac{N_A + N_D}{N_A N_D} \right] \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 0.951 \times 10^{-4}\ \text{cm} = 0.951\ \mu\text{m}$$

Example 2

- We can also calculate the respective space charge widths in the n and p regions:

$$x_{n0} = 0.864\ \mu\text{m}, \qquad x_{p0} = 0.086\ \mu\text{m}$$

- Note that $x_{n0} >> x_{p0}$.

- **Why?** Because of space charge neutrality:

$$\underbrace{qN_A x_{p0}}_{\text{Net negative charge on p-type side}} = \underbrace{qN_D x_{n0}}_{\text{Net positive charge on n-type side}}$$

Example 2

- The maximum electric field occurs at the metallurgical junction, i.e. $x = 0$.

$$\xi_{\mathrm{m}} \text{ or } \xi_{\mathrm{max}} = \xi(x = 0)$$

$$= -\frac{qN_{\mathrm{D}}x_{\mathrm{n0}}}{\varepsilon_{\mathrm{r}}\varepsilon_0}$$

$$= \frac{(1.6\times10^{-19})(10^{15})(0.864\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

$$= -1.34\times10^{4} \text{ V/cm}$$

Summary

- Electrostatics of the pn junction:
  - Space charge/depletion region
  - Built-in electric field
    - Maximum electric field @ metallurgical junction
  - Built-in potential barrier
    - Area under the electric field distribution plot
    - Energy band diagram
  - Space charge width