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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 4.

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1. (a) Consider the following system of equations:

$$(1+\lambda)x_1 + x_2 + x_3 = 0$$
$$x_1 + (1+\lambda)x_2 + x_3 = 3$$
$$x_1 + x_2 + (1+\lambda)x_3 = \lambda.$$

Determine the values of λ such that the system has

(i) a unique solution,

(ii) no solution,

(iii) many solutions. Also, find these solutions in this case.

(12 Marks)

(b) Suppose that matrices A, B and C satisfy $[I_3 - C^{-1}B]^T C^T A = I_3$, where

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Note: Question 1 continues on page 2.

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and I_3 stands for a 3×3 identity matrix. Without calculating the inverse of C, show how A can be determined. Hence determine A.

(8 Marks)

(c) Given that

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \quad \text{and} \quad A = \mathbf{a}\mathbf{b}^T,$$

determine A^{2000}.

(5 Marks)

2. A linear system is given as

$$A\mathbf{x} = \mathbf{b}$$

where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ a^2 & 4 & 3a \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) Use elementary row operations to determine the rank of the matrix A if

(i) $a = 2\sqrt{2}$

(ii) $a = 4$

Hence, determine the condition imposed on a so that a unique solution can be obtained for any vector $\mathbf{b}$.

(10 Marks)

(b) (i) Determine the eigenvalues of matrix A in terms of a.

(ii) Determine the eigenvectors for the case of $a = \sqrt{3}$.

(11 Marks)

(c) Consider the linear system described by $B\mathbf{x} = \mathbf{b}$, where

$$B = \begin{bmatrix} -5 & 1 & 0 \\ a^2 & -3 & 3a \\ 0 & 0 & -5 \end{bmatrix}.$$

Note: Question 2 continues on page 3.

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By using the results in part (b)(i), determine the values of a so that a unique solution can be obtained.

Hint: First determine the relationship between A and B by subtracting the matrices.

(4 Marks)

3. (a) (i) Using the Cauchy-Riemann equations, comment on the differentiability and analyticity of $f(z) = \cos^2 z$.

(ii) Hence, or otherwise, evaluate $\int_{i}^{2\pi} \cos^2 z\, dz$ along the straight-line path from $z = i$ to $z = 2\pi$.

(12 Marks)

(b) Evaluate $\int_{0}^{\pi} \frac{\sin 2\theta}{2+\cos 2\theta} d\theta$.

(13 Marks)

4. (a) Evaluate the line integral $\int_{C} \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + \cos y\,\mathbf{j} + e^{z}\,\mathbf{k}$, along the straight-line path C from (3, 0, -2) to (4, 2, -1).

(8 Marks)

(b) Using a suitable spherical surface parameterization, evaluate $\iint_{S} curl\,\mathbf{F} \cdot d\mathbf{A}$ for surface $S: x^2 + y^2 + z^2 = a^2$, $x \geq 0$, and $\mathbf{F}(x, y, z) = -z\,\mathbf{j} + y\,\mathbf{k}$.

(10 Marks)

(c) Using Stokes' Theorem, evaluate $\iint_{S} curl\,\mathbf{F} \cdot d\mathbf{A}$ for surface $S: x^2 + y^2 + z^2 = a^2$, $x \geq 0$, and $\mathbf{F}(x, y, z) = -z\,\mathbf{j} + y\,\mathbf{k}$.

(7 Marks)

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Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c \ln z}$

(b) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(c) Cauchy-Riemann equations:

$u_x = v_y$, $v_x = -u_y$, or $u_r = \frac{1}{r}v_\theta$, $v_r = \frac{-1}{r}u_\theta$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\Big|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Divergence Theorem: $\iiint_T \nabla\cdot\mathbf{F}\,dv = \oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.