NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2015 Time Allowed: $2\frac{1}{2}$ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 8 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings and all coordinates given are Cartesian coordinates.
6. A list of physical constants and useful formulae is provided in Appendix A (on pages 6 - 8).
7. The Smith chart may be used in the solution of Question 4.

---

1. (a) Figure 1 (on page 2) shows the geometry (not to scale) of a quarter-circular arc of radius a, which lies in free-space in the $z = 0$ plane and carries a uniform charge density ρ_l.

    (i) Determine the electric field intensity $\vec{E}_o$ at the origin.

    (ii) Determine the electric field intensity $\vec{E}_c$ at the centroid C $\left(\frac{2}{\pi}a, \frac{2}{\pi}a, 0\right)$ of the quarter-circular arc.

Note: Question No. 1 continues on page 2.

Note:

$$\int_0^{\pi/2} \vec{a}_r \, d\phi = \vec{a}_x + \vec{a}_y \, ; \qquad \vec{a}_r = \cos\phi \, \vec{a}_x + \sin\phi \, \vec{a}_y$$

$$\int_0^{\pi/2} \frac{\frac{2}{\pi} - \cos\phi}{\left[\left(\frac{2}{\pi} - \cos\phi\right)^2 + \left(\frac{2}{\pi} - \sin\phi\right)^2\right]^{3/2}} d\phi \simeq -13.42$$

$$\int_0^{\pi/2} \frac{\frac{2}{\pi} - \sin\phi}{\left[\left(\frac{2}{\pi} - \cos\phi\right)^2 + \left(\frac{2}{\pi} - \sin\phi\right)^2\right]^{3/2}} d\phi \simeq -13.42$$

(13 Marks)

(b) Consider the quarter-circular arc of Figure 1 forming one quadrant of a whole circular loop (centred at origin) with steady current I flowing in the counter-clockwise direction.

(i) Determine the magnetic field intensity $\vec{H}_q$ at a point (0, 0, z) on the z axis due to the current on the quarter-circular arc only.

(ii) Determine the magnetic field intensity $\vec{H}_t$ at a point (0, 0, z) on the z axis due to the current on the whole circular loop.

(12 Marks)

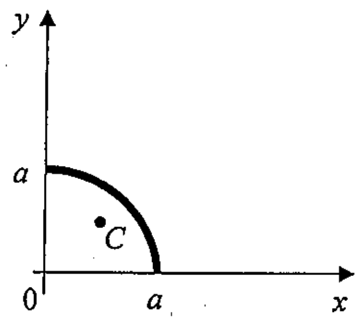

Figure 1

2. (a) Figure 2 shows an infinitely long line carrying a direct current I along the y axis in free space. A set of conducting wires form a closed rectangular loop ABCD with the left wire AB fixed at $x = a$ and the right wire CD movable along the long horizontal conducting wires at $y = -a$ and $y = a$. Assume that the wire CD is moving at constant velocity v_0 away from the current starting at time $t = 0$ and initial position $x = 2a$.

(i) Determine the magnetic flux density $\vec{B}$ due to the current I at point $(x,0,0)$.

(ii) Determine the magnetic flux Φ_m passing through the loop at time t.

(iii) Determine the induced voltage V_{emf} in the loop at time t.

(10 Marks)

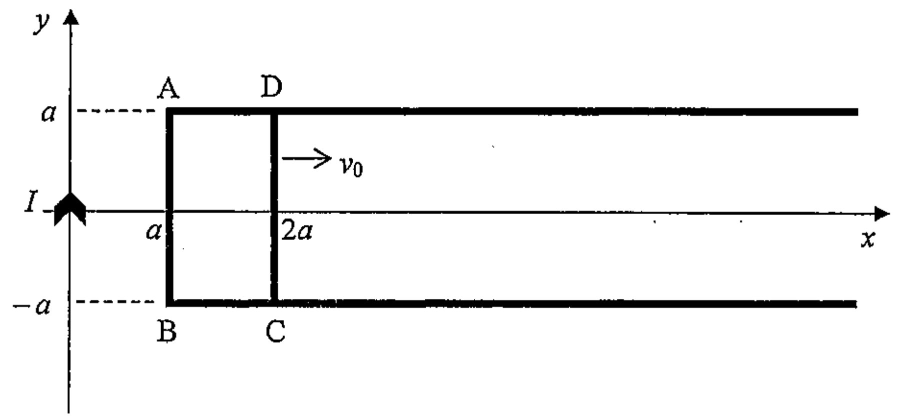

Figure 2

(b) A plane wave travelling in an unknown medium has the electric and magnetic fields expressed as

$$\tilde{E} = \vec{a}_x 1505 e^{-2.878z} \cos(2\pi \times 10^8 t - 6.586z + 6.207) \text{ V/m}$$
$$\tilde{H} = \vec{a}_y 13.7 e^{-2.878z} \cos(2\pi \times 10^8 t - 6.586z + 5.795) \text{ A/m}$$

(i) Calculate the propagation constant $\gamma = \alpha + j\beta$, wavenumber k_c and intrinsic impedance η_c.

(ii) Determine the dielectric constant ε_r and conductivity σ of the medium.

(15 Marks)

3. (a) The incident electric field of a uniform plane wave (UPW) in air occupying the region $z \leq 0$ is given by

$$\vec{E}_i(z) = (j30\vec{a}_x - 40\vec{a}_y)e^{-j4\pi z} \text{ V/m}$$

The UPW is incident normally on a planar interface at $z = 0$ with a lossy medium having $\mu_r = 1$, $\varepsilon_r = 2$ and $\sigma =$ 0.08 S/m, occupying the region $z > 0$.

Find the following:

(i) The frequency of the incident UPW.

(ii) The polarization (linear, circular or elliptical) of the incident UPW. Briefly explain your answer.

(iii) The intrinsic impedance in air and in the lossy medium, i.e., η_1 and η_{2c}.

(iv) The positions z at which the magnitude of the total electric field in the air medium is maximum, i.e., $z_{\max}$.

(v) The maximum magnitude of the total electric field in the air medium, i.e., $|E_1|_{\max}$.

State any assumption made.

(13 Marks)

(b) The magnetic field of a uniform plane wave (UPW) travelling in a lossless dielectric medium with $\varepsilon = 1.5\varepsilon_o$, $\mu = \mu_o$ and occupying the region $z \leq 0$ is given by

$$\vec{H}_i(x,z) = \vec{a}_y 0.4e^{-j(6x+5z)} \text{ A/m}$$

The UPW is incident on an air boundary at $z = 0$, occupying the region $z > 0$.

Determine the following:

(i) The angles of incidence θ_i and transmission θ_t.

Note: Question No. 3 continues on page 5.

(ii) The direction of incident electric field vector $\vec{a}_{E_i}$ and the polarization of the incident UPW with respect to the plane of incidence.

(iii) The percentage of average power reflected from the interface at $z = 0$.

State any assumption made.

(12 Marks)

4. (a) A 75-Ω air-filled transmission line is terminated in a load $Z_L = 85 - j40\ \Omega$. The line operates at 1 GHz.

(i) Determine the load reflection coefficient Γ_L and the standing wave ratio (SWR) due to this load.

(ii) Find the input impedance Z_{in} when the line is 0.4 m long.

(iii) Find the shortest length $\ell_{\min}$ of the line in order to make the input impedance both real and as large as possible.

State any assumption made.

(13 Marks)

(b) A 50-Ω air-filled transmission line of length ℓ is connected to a source with an open-circuit voltage $V_g = 35\angle 0°$ V and internal impedance $Z_g = 50 - j40\ \Omega$ at $z = -\ell$. The line is terminated in a load Z_L at $z = 0$.

If $Z_L = 50 - j60\ \Omega$, $\ell = 50$ m and $\beta = 0.025$ rad/m, find the following:

(i) The input impedance Z_{in} at $z = -\ell$.

(ii) The input voltage V_{in} at $z = -\ell$.

(iii) The average power delivered to the load Z_L.

State any assumption made.

(12 Marks)

The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

3

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r \partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (r A_r)}{r \partial r} + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi} \int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \vec{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \iint_S \vec{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_{//} = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_{//} = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \, e^{-j\beta z} + V_o^- \, e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \, e^{-j\beta z} - V_o^- \, e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

EE3001 - Engineering Electromagnetics

1. a) i) $\vec{E}_0 = \frac{1}{4\pi\varepsilon_0}\int_c \frac{\rho_l\, dl}{R^3}\vec{R}$

$= \frac{1}{4\pi\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{\rho_l\, a\, d\phi}{a^3}\left(-a\cos\phi\,\vec{a}_x - a\sin\phi\,\vec{a}_y\right)$

$= \frac{\rho_l}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}} -\cos\phi\,\vec{a}_x - \sin\phi\,\vec{a}_y\, d\phi$

$= \frac{\rho_l}{4\pi\varepsilon_0 a}\left[-\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y\right]_0^{\frac{\pi}{2}}$

$= \frac{\rho_l}{4\pi\varepsilon_0 a}\left(-\vec{a}_x + 0 - 0 - \vec{a}_y\right)$

$\therefore \vec{E}_0 = \frac{\rho_l(-\vec{a}_x - \vec{a}_y)}{4\pi\varepsilon_0 a}$ V/m #

$\vec{r} = 0\,\vec{a}_x + 0\,\vec{a}_y + 0\,\vec{a}_z$

$\vec{s} = a\cos\phi\,\vec{a}_x + a\sin\phi\,\vec{a}_y + 0\,\vec{a}_z$

$\vec{R} = \vec{r} - \vec{s}$

$\vec{R} = -a\cos\phi\,\vec{a}_x - a\sin\phi\,\vec{a}_y + 0\,\vec{a}_z$

$R = |\vec{R}| = \sqrt{(-a\cos\phi)^2 + (-a\sin\phi)^2}$

$R = a$

$dl = a\, d\phi$

1 a) ii) $\vec{E}_c = \frac{1}{4\pi\varepsilon_0}\int_c \frac{\rho_l\, dl}{R^3}\vec{R}$

$= \frac{1}{4\pi\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{\rho_l\, a\, d\phi\left[a(\frac{2}{\pi} - \cos\phi)\vec{a}_x + a(\frac{2}{\pi} - \sin\phi)\vec{a}_y\right]}{a^3\left[(\frac{2}{\pi} - \cos\phi)^2 + (\frac{2}{\pi} - \sin\phi)^2\right]^{3/2}}$

$= \frac{\rho_l}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}} \frac{(\frac{2}{\pi} - \cos\phi)\vec{a}_x + (\frac{2}{\pi} - \sin\phi)\vec{a}_y}{\left[(\frac{2}{\pi} - \cos\phi)^2 + (\frac{2}{\pi} - \sin\phi)^2\right]^{3/2}}\, d\phi$

$= \frac{\rho_l}{4\pi\varepsilon_0 a}\left[\int_0^{\frac{\pi}{2}} \frac{\frac{2}{\pi} - \cos\phi}{\left[(\frac{2}{\pi} - \cos\phi)^2 + (\frac{2}{\pi} - \sin\phi)^2\right]^{3/2}}\, d\phi\,\vec{a}_x\right.$

$\left. + \int_0^{\frac{\pi}{2}} \frac{\frac{2}{\pi} - \sin\phi}{\left[(\frac{2}{\pi} - \cos\phi)^2 + (\frac{2}{\pi} - \sin\phi)^2\right]^{3/2}}\, d\phi\,\vec{a}_y\right]$

$\simeq \frac{\rho_l}{4\pi\varepsilon_0 a}\left[-13.42\,\vec{a}_x - 13.42\,\vec{a}_y\right]$

$\therefore \vec{E}_c = \frac{\rho_l(-13.42\,\vec{a}_x - 13.42\,\vec{a}_y)}{4\pi\varepsilon_0 a}$ V/m #

$\vec{r} = \frac{2}{\pi}a\,\vec{a}_x + \frac{2}{\pi}a\,\vec{a}_y + 0\,\vec{a}_z$

$\vec{s} = a\cos\phi\,\vec{a}_x + a\sin\phi\,\vec{a}_y + 0\,\vec{a}_z$

$\vec{R} = \vec{r} - \vec{s}$

$\vec{R} = a(\frac{2}{\pi} - \cos\phi)\vec{a}_x + a(\frac{2}{\pi} - \sin\phi)\vec{a}_y + 0\,\vec{a}_z$

$R = |\vec{R}| = \sqrt{a^2(\frac{2}{\pi} - \cos\phi)^2 + a^2(\frac{2}{\pi} - \sin\phi)^2}$

$R = a\left[(\frac{2}{\pi} - \cos\phi)^2 + (\frac{2}{\pi} - \sin\phi)^2\right]^{\frac{1}{2}}$

$dl = a\, d\phi$

1. b) i) $\vec{H}_q = \frac{1}{4\pi}\int \frac{I\, d\vec{l} \times \vec{R}}{R^3}$

$= \frac{1}{4\pi}\int_0^{\frac{\pi}{2}} \frac{I(a\, d\phi\,\vec{a}_\phi) \times (-a\,\vec{a}_r + z\,\vec{a}_z)}{(a^2 + z^2)^{3/2}}$

$= \frac{1}{4\pi}\int_0^{\frac{\pi}{2}} \frac{I(az\, d\phi\,\vec{a}_r + a^2\, d\phi\,\vec{a}_z)}{(a^2 + z^2)^{3/2}}$

$= \frac{Ia(z\,\vec{a}_r + a\,\vec{a}_z)}{4\pi(a^2 + z^2)^{3/2}}\int_0^{\frac{\pi}{2}} d\phi$

$\therefore \vec{H}_q = \frac{Ia(z\,\vec{a}_r + a\,\vec{a}_z)}{8(a^2 + z^2)^{3/2}}$ A/m #

$\vec{r} = 0\,\vec{a}_r + 0\,\vec{a}_\phi + z\,\vec{a}_z$

$\vec{s} = a\,\vec{a}_r + 0\,\vec{a}_\phi + 0\,\vec{a}_z$

$\vec{R} = \vec{r} - \vec{s} = -a\,\vec{a}_r + 0\,\vec{a}_\phi + z\,\vec{a}_z$

$R = \sqrt{(-a)^2 + z^2}$

$R = \sqrt{a^2 + z^2}$

$d\vec{l} = a\, d\phi\,\vec{a}_\phi$

1. b) ii) $\vec{H}_t = 4 \times \vec{H}_q$

$$= 4 \times \frac{Ia\,(z\vec{a}_r + a\vec{a}_z)}{8\,(a^2+z^2)^{3/2}}$$

$$\vec{H}_t = \frac{Ia\,(z\vec{a}_r + a\vec{a}_z)}{2\,(a^2+z^2)^{3/2}} \ \text{A/m} \ \#$$

2 a) i) $\oint_c \vec{H}\cdot d\vec{l} = I$ $\qquad \vec{H} = \vec{a}_\phi H_\phi$

$$\int_0^{2\pi} \vec{a}_\phi H_\phi \cdot \vec{a}_\phi x\, d\phi = I \qquad d\vec{l} = \vec{a}_\phi x\, d\phi$$

$$H_\phi\, 2\pi x = I$$

$$H_\phi = \frac{I}{2\pi x}$$

at point $(x,0,0)$, $\vec{H} = \frac{I}{2\pi x}(-\vec{a}_z)$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi x}(-\vec{a}_z)\ \text{T} \ \#$$

2 a) ii) $\phi_m = \iint_s \vec{B}\cdot d\vec{S} = \int_a^{2a+v_0t}\int_{-a}^{a} \frac{\mu_0 I}{2\pi x}(-\vec{a}_z)\cdot(\vec{a}_z\, dy\, dx)$

$$= \int_a^{2a+v_0t}\int_{-a}^{a} -\frac{\mu_0 I}{2\pi x}\, dy\, dx$$

$$= \int_a^{2a+v_0t} -\frac{\mu_0 I}{2\pi x}(2a)\, dx$$

$$= -\frac{\mu_0 I a}{\pi}\int_a^{2a+v_0t} \frac{1}{x}\, dx$$

$$= -\frac{\mu_0 I a}{\pi}\left[\ln x\right]_a^{2a+v_0t}$$

$$\therefore \phi_m = -\frac{\mu_0 I a}{\pi} \ln \frac{2a+v_0t}{a}\ \text{Wb}$$

2 a) iii) $V_{emf} = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\left(-\frac{\mu_0 I a}{\pi}\ln\frac{2a+v_0t}{a}\right)$

$$= \frac{\mu_0 I a}{\pi}\,\frac{a}{2a+v_0t}\left(\frac{v_0}{a}\right)$$

$$V_{emf} = \frac{\mu_0 I a v_0}{\pi(2a+v_0t)}\ \text{V}$$ ~~(anti clockwise in the loop)~~

2 b) i) $\tilde{E} = \vec{a}_E\,|E_0|\,e^{-\alpha z}\cos(\omega t - \beta z + \phi)$

$$\tilde{E} = \vec{a}_x\, 1505\, e^{-2.878z}\cos(2\pi\times10^8 t - 6.586z + 6.207)$$

$\vec{a}_k = \vec{a}_z$ $\quad \omega = 2\pi\times10^8$ rad/s $\quad \alpha = 2.878$ Np/m $\quad \beta = 6.586$ rad/m

$$\gamma = \alpha + j\beta = 2.878 + j6.586 \ \#$$

$$k_c = -j\gamma = 6.586 - j2.878 \ \#$$

$$\eta_c = \frac{|E_0|}{|H_0|}\angle\phi_E - \phi_H = \frac{1505}{13.7}\angle(6.207-5.795) = 109.85\angle 0.412\ \Omega \ \#$$

$$= 109.85\angle 23.6^\circ\ \Omega$$

2 b) ii) $k_c = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu(\varepsilon - j\frac{\sigma}{\omega})}$

$$\varepsilon - j\frac{\sigma}{\omega} = \left(\frac{k_c}{\omega}\right)^2 \frac{1}{\mu} \qquad \mu = \mu_0 = 4\pi\times10^{-7}\ H/m$$

$$= \left(\frac{6.586 - j2.878}{2\pi\times10^8}\right)^2 \frac{1}{4\pi\times10^{-7}}$$

$$\varepsilon - j\frac{\sigma}{\omega} = 7.074\times10^{-11} - j7.641\times10^{-11}$$

$\varepsilon = \varepsilon_r\varepsilon_0 = 7.074\times10^{-11}\ F/m$ $\qquad \frac{\sigma}{\omega} = 7.641\times10^{-11}$

$\varepsilon_r = \frac{7.074\times10^{-11}}{\frac{1}{36\pi}\times10^{-9}} = 8$ # $\qquad \sigma = 7.641\times10^{-11}\times2\pi\times10^8$

$\sigma = 0.0480\ S/m$ #

3 a) i) $k_1 = \omega\sqrt{\mu\varepsilon_1} \Rightarrow k_1 = 2\pi f\sqrt{\mu\varepsilon_1}$ Assume $\mu_1 = \mu_0$, $\varepsilon_1 = \varepsilon_0$

$$f = \frac{k_1}{2\pi\sqrt{\mu\varepsilon_1}} = \frac{4\pi}{2\pi\sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi}\times10^{-9}}} = 600\ MHz$$ #

3 a) ii) $\vec{E}_i(z) = (j30\vec{a}_x - 40\vec{a}_y)\, e^{-j4\pi z}\ V/m$

$$= \vec{a}_x\, 30\angle 90^\circ\, e^{-4\pi z} + \vec{a}_y\, 40\angle 180^\circ\, e^{-4\pi z}\ V/m$$

The incident UPW is elliptically polarized. #

$\because |E_{0x}| = 30 \neq 0$, $|E_{0y}| = 40 \neq 0$, $|\phi_y - \phi_x| = 90^\circ \neq 0^\circ, 180^\circ \Rightarrow$ not linear

$|\phi_y - \phi_x| = 90^\circ$ but $|E_{0x}| \neq |E_{0y}| \Rightarrow$ not circular

3 a) iii) $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = 120\pi\ \Omega$ # $\qquad \frac{\sigma_2}{\omega\varepsilon_{2c}} = 1.2 < 20$

$$\eta_{2c} = \sqrt{\frac{\mu_2}{\varepsilon_{2c}}} = \sqrt{\frac{\mu_2}{\varepsilon_2 - j\frac{\sigma_2}{\omega}}}$$

$$= \sqrt{\frac{4\pi\times10^{-7}}{2\times\frac{1}{36\pi}\times10^{-9} - j\frac{0.08}{2\pi\times600\times10^6}}} = \sqrt{29123.42 + j34948.12} = \sqrt{45492\angle 50.19^\circ}$$

$\therefore \eta_{2c} = 213.3\angle 25.1^\circ\ \Omega$ #

3 a) iv) $\Gamma = \frac{\eta_{2c} - \eta_1}{\eta_{2c} + \eta_1} = \frac{213.3\angle 25.1 - 120\pi}{213.3\angle 25.1 + 120\pi} = 0.3549\angle 144.8^\circ = |\Gamma|\angle\theta_\Gamma$

$= 0.3549\angle 2.527\text{rad}$

$$\theta_\Gamma = \theta_\Gamma + 2k_1 z_{max} = 0, -2\pi, -4\pi, \ldots$$

$$2.527 + 8\pi z_{max} = 0, -2\pi, -4\pi, \ldots$$

$$z_{max} = \frac{0 - 2.527}{8\pi}, \frac{-2\pi - 2.527}{8\pi}, \frac{-4\pi - 2.527}{8\pi}, \ldots$$

$z_{max} = -0.1005\ m, -0.3505\ m, -0.6005\ m, \ldots$ #

3 a) v) $|E_1|_{max} = |E_{0i}|(1 + |\Gamma|) = \sqrt{30^2 + 40^2}(1 + 0.3549)$

$|E_1|_{max} = 67.75\ V/m$ #

3 b) i) $\tan\theta_i = \frac{k_{xi}}{k_{zi}} = \frac{6}{5} \Rightarrow \theta_i = 50.2^\circ$ #

$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \sqrt{\frac{1.5}{1}} \Rightarrow \theta_t = 70.2^\circ$ #

3 b) ii) $\vec{H}_i(x,z) = \vec{a}_y\, 0.4 e^{-j(6x+5z)}$ A/m

$\vec{a}_{H_i} = \vec{a}_y \quad \vec{a}_{k_i} = \frac{6\vec{a}_x + 5\vec{a}_z}{\sqrt{6^2+5^2}} = \frac{6}{\sqrt{61}}\vec{a}_x + \frac{5}{\sqrt{61}}\vec{a}_z$

$\vec{a}_{E_i} = \vec{a}_{H_i} \times \vec{a}_{k_i} = \vec{a}_y \times \left(\frac{6}{\sqrt{61}}\vec{a}_x + \frac{5}{\sqrt{61}}\vec{a}_z\right) = \frac{5}{\sqrt{61}}\vec{a}_x - \frac{6}{\sqrt{61}}\vec{a}_z$ #

The incident UPW is parallel polarized with respect to the plane of incidence #

3 b) iii) $\vec{S}_i'' = \vec{a}_{k_i} \frac{1}{2}|H_{0i}''|^2 \eta_1$ $\qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{1}{1.5}}\,120\pi = 307.8\ \Omega$

$= \left(\frac{6}{\sqrt{61}}\vec{a}_x + \frac{5}{\sqrt{61}}\vec{a}_z\right)\frac{1}{2}(0.4)^2\ 307.8$

$= 18.92\vec{a}_x + 15.76\vec{a}_z$ W/m^2 $\qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = 120\pi\ \Omega$

$P_i'' = 15.76$ W/m^2

$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = -0.2135$

$P_r'' = |\Gamma_{\parallel}|^2 P_i''$

$\% P_r'' = \frac{P_r''}{P_i''} = |\Gamma_{\parallel}|^2 = 0.2135^2 = 4.56\%$ #

4 a) i) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{85-j40-75}{85-j40+75} = 0.25\angle -61.9^\circ$ # $= |\Gamma_L|\angle\theta_0$

$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.25}{1-0.25} = 1.667$ #

4 a) ii) $\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{u_p} = \frac{2\pi\times 1\times10^9}{3\times10^8} = 20.94$ rad/m

$Z_{in}(-0.4) = \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}Z_0 = \frac{85-j40+j(75)\tan(20.94\times0.4)}{75+j(85-j40)\tan(20.94\times0.4)}(75)$

$Z_{in}(-0.4) = 87.95 + j39.89\ \Omega$ #

4 a) iii) $\theta_n = \theta_0 + 2\beta z_{max} = -2\pi$

$\theta_0 = -61.9^\circ = -1.08$ rad.

$z_{max} = \frac{-2\pi - (-1.08)}{2(20.94)} = -0.1242$ m

$\therefore l_{min} = 0.1242$ m #

4 b) i) $Z_{in}(-\ell) = \dfrac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} Z_0$

$= \dfrac{50 - j60 + j50 \tan(0.025 \times 50)}{50 + j(50 - j60)\tan(0.025 \times 50)} (50)$

$Z_{in}(-\ell) = 16.58 + j8.797\ \Omega$ #

4 b) ii) $V_{in} = \dfrac{Z_{in}}{Z_{in} + Z_g} V_g = \dfrac{16.58 + j8.797}{16.58 + j8.797 + 50 - j40} (35\angle 0^\circ) = 8.935\angle 53.1^\circ$ #

4 b) iii) $P_L = \frac{1}{2} \times \left|\dfrac{V_{in}}{Z_{in}}\right|^2 R_e(Z_{in})$

$= \frac{1}{2} \times \left|\dfrac{8.935\angle 53.1^\circ}{16.58 + j8.797}\right|^2 \times 16.58$

$P_L = 1.878\,W$ #