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EE3001 Engineering Electromagnetics

*This session is about*

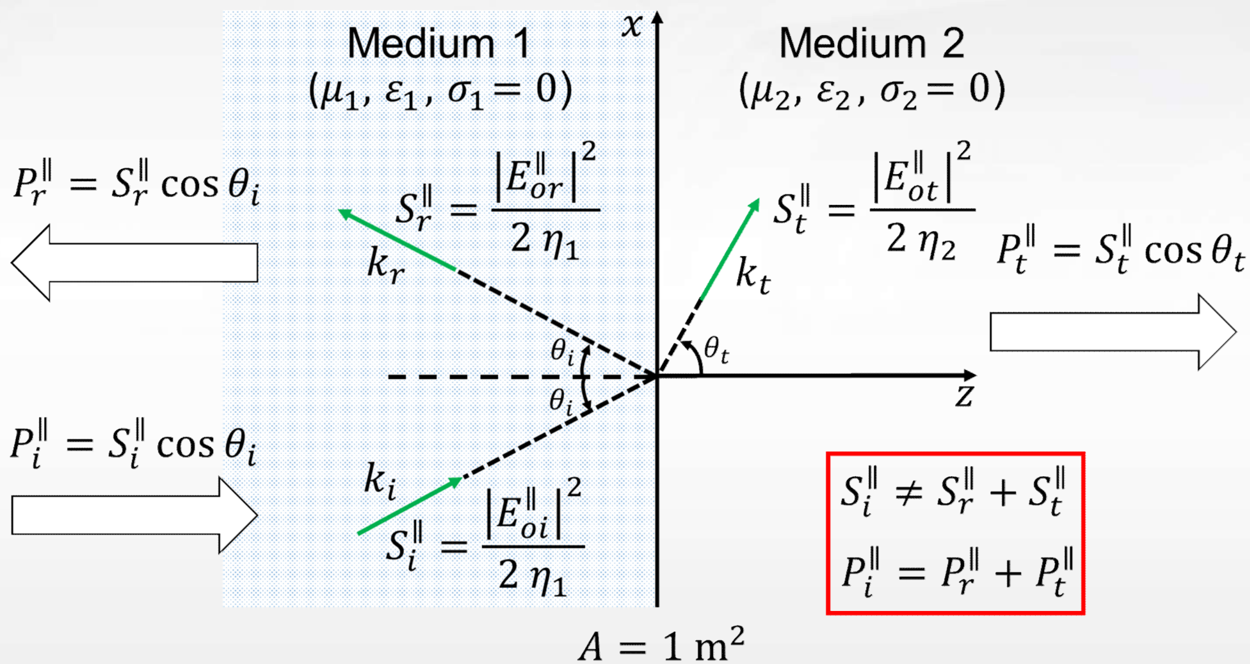
# 21. Power Incident, Power Reflected and Power Transmitted at the Interface

# Learning Objectives

- Determine power incident, power reflected and power transmitted at the interface; and
- Apply power conservation to simplify the calculations of power reflected and power transmitted at the interface.

# Power Incident, Reflected and Transmitted at the Interface

**Power Conservation for Incident, Reflected and Transmitted Waves at the Interface between two Lossless Dielectric Media**




$$S_i^{\parallel} \neq S_r^{\parallel} + S_t^{\parallel}$$

$$P_i^{\parallel} = P_r^{\parallel} + P_t^{\parallel}$$

# Power Incident, Reflected and Transmitted at the Interface

The proportion of average incident wave power reflected and transmitted at the interface is:

- $$\boxed{\frac{P_r^{\parallel}}{P_i^{\parallel}} = \frac{S_r^{\parallel}\cos\theta_i}{S_i^{\parallel}\cos\theta_i} = \frac{\left|E_{or}^{\parallel}\right|^2}{\left|E_{oi}^{\parallel}\right|^2} = \left|\Gamma_{\parallel}\right|^2}, \qquad \boxed{\frac{P_r^{\perp}}{P_i^{\perp}} = \frac{S_r^{\perp}\cos\theta_i}{S_i^{\perp}\cos\theta_i} = \frac{\left|E_{or}^{\perp}\right|^2}{\left|E_{oi}^{\perp}\right|^2} = \left|\Gamma_{\perp}\right|^2} \qquad (44)$$

- $$\frac{P_t^{\parallel}}{P_i^{\parallel}} = \frac{S_t^{\parallel}\cos\theta_t}{S_i^{\parallel}\cos\theta_i} = \frac{\eta_1}{\eta_2}\left|\tau_{\parallel}\right|^2\frac{\cos\theta_t}{\cos\theta_i}, \quad \frac{P_t^{\perp}}{P_i^{\perp}} = \frac{S_t^{\perp}\cos\theta_t}{S_i^{\perp}\cos\theta_i} = \frac{\eta_1}{\eta_2}\left|\tau_{\perp}\right|^2\frac{\cos\theta_t}{\cos\theta_i} \qquad (45)$$

Power conservation at the interface gives

- $$\boxed{\frac{P_t^{\parallel}}{P_i^{\parallel}} = 1 - \left|\Gamma_{\parallel}\right|^2}, \qquad \boxed{\frac{P_t^{\perp}}{P_i^{\perp}} = 1 - \left|\Gamma_{\perp}\right|^2}, \qquad (46)$$

# Power Incident, Reflected and Transmitted at the Interface

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For any UPW, total power Reflected and Transmitted at an interface can be determined using $P_i = S_i \cos\theta_i$, $|\Gamma_\parallel|$ and $|\Gamma_\perp|$:

- $\vec{E}_i = \vec{E}_i^{\parallel} + \vec{E}_i^{\perp}$
- $P_i = P_i^{\parallel} + P_i^{\perp}$
- $P_r = P_r^{\parallel} + P_r^{\perp} = |\Gamma_\parallel|^2 P_i^{\parallel} + |\Gamma_\perp|^2 P_i^{\perp}$
- $P_t = P_t^{\parallel} + P_t^{\perp} = \left(1 - |\Gamma_\parallel|^2\right) P_i^{\parallel} + (1 - |\Gamma_\perp|^2) P_i^{\perp}$ or

  $P_t = P_i - P_r$

# Example

A UPW travelling in **air** $(\mu_o, \varepsilon_o)$ is obliquely incident at a **lossless** dielectric interface $(\mu_o, 9\varepsilon_o)$ at $z = 0$. The incident electric field is given by

$$\vec{E}_i = \left(30e^{j\pi/3}\vec{a}_x + 100e^{j\pi/2}\vec{a}_y - 40e^{j\pi/3}\vec{a}_z\right)e^{-j(80x+60z)} \text{ V/m}$$

Find the average power incident, reflected and transmitted at the interface, i.e. $P_i$, $P_r$ and $P_t$

- $\vec{E}_i = \vec{E}_i^{\parallel} + \vec{E}_i^{\perp}$
- $\vec{E}_i^{\parallel} = \left(30\, e^{j\pi/3}\, \vec{a}_x - 40\, e^{j\pi/3}\, \vec{a}_z\right) e^{-j(80x+6\quad)}$
- $\vec{E}_i^{\perp} = 100\, e^{j\pi/2}\, \vec{a}_y\, e^{-j(80x+60z)}$

$\boxed{E_{oi}^{\parallel} = 50\,\angle 60^{\circ}}$

$\boxed{E_{oi}^{\perp} = 100\,\angle 90^{\circ}}$

# Example

- $\vec{k}_i = \vec{a}_x\, 80 + \vec{a}_z\, 60 \rightarrow \tan\theta_i = \dfrac{k_{xi}}{k_{zi}} = \dfrac{80}{60} \rightarrow \theta_i = 53.1^\circ$
- $P_i^{\parallel} = \dfrac{\left|E_{oi}^{\parallel}\right|^2}{2\eta_1}\cos\theta_i = 2\ \mathrm{W/m^2}$ , $\left[\vec{S}_i^{\parallel} = \vec{a}_{k_i}\dfrac{\left|E_{oi}^{\parallel}\right|^2}{2\eta_1} = \vec{a}_x\, 2.66 + \vec{a}_z\, 2.0\ \mathrm{W/m^2}\right]$
- $P_i^{\perp} = \dfrac{\left|E_{oi}^{\perp}\right|^2}{2\eta_1}\cos\theta_i = 8\ \mathrm{W/m^2}$ , $\left[\vec{S}_i^{\perp} = \vec{a}_{k_i}\dfrac{\left|E_{oi}^{\perp}\right|^2}{2\eta_1} = \vec{a}_x\, 10.65 + \vec{a}_z\, 8.0\ \mathrm{W/m^2}\right]$
- $\eta_1 = 120\pi\ \Omega$, $\eta_2 = 40\pi\ \Omega$; $\theta_i = 53.1^\circ$
- Snell's Law: $\theta_t = 15.5^\circ$
- (29) and (34): $\boxed{\Gamma_{\parallel} = -0.303 \text{ and } \Gamma_{\perp} = -0.656}$

# Example

- $\left[P_i^{\parallel} = 2,\ P_i^{\perp} = 8,\ \Gamma_{\parallel} = -0.303 \text{ and } \Gamma_{\perp} = -0.656\right]$
- $P_i = P_i^{\parallel} + P_i^{\perp} = 2 + 8 = 10\ \mathrm{W/m^2}$
- $P_r^{\parallel} = \left|\Gamma_{\parallel}\right|^2 P_i^{\parallel} = 0.184\ \mathrm{W/m^2};\quad P_r^{\perp} = |\Gamma_{\perp}|^2 P_i^{\perp} = 3.44\ \mathrm{W/m^2}$

  $P_r = P_r^{\parallel} + P_r^{\perp} = 3.62\ \mathrm{W/m^2}$
- $P_t = P_i - P_r = 6.38\ \mathrm{W/m^2}$ [Power Conservation] or

  $P_t = P_t^{\parallel} + P_t^{\perp} = \left(1 - \left|\Gamma_{\parallel}\right|^2\right) P_i^{\parallel} + (1 - |\Gamma_{\perp}|^2)\, P_i^{\perp} = 1.82 + 4.56 = 6.38\ \mathrm{W/m^2}$

# Summary

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- The power incident at the interface for parallel and perpendicular polarisation are given by:

$$P_i^{\parallel} = S_i^{\parallel} \cos\theta_i \ \ \mathrm{W/m^2} \ ; \qquad P_i^{\perp} = S_i^{\perp} \cos\theta_i \ \ \mathrm{W/m^2}$$

- The proportion of average power reflected at the interface are given by:

$$P_r^{\parallel} = \left|\Gamma_{\parallel}\right|^2 P_i^{\parallel} \quad ; \quad P_r^{\perp} = |\Gamma_{\perp}|^2 P_i^{\perp}$$

- The proportion of average power transmitted at the interface are given by:

$$P_t^{\parallel} = \left(1 - \left|\Gamma_{\parallel}\right|^2\right) P_i^{\parallel} \quad ; \quad P_t^{\perp} = (1 - |\Gamma_{\perp}|^2) P_i^{\perp}$$

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# 22. The Brewster Angle

# Learning Objectives

- ❑ Define the Brewster angle for UPW at oblique incident; and
- ❑ Determine the Brewster angle at the interface between two lossless dielectric media.

# The Brewster Angle

- **The Brewster Angle** $\left[P_r^{\parallel} = \left|\Gamma_{\parallel}\right|^2 P_i^{\parallel},\ P_r^{\perp} = |\Gamma_{\perp}|^2 P_i^{\perp}\right]$
- Brewster angle is defined as the **incident angle** $\theta_i$ at which reflection coefficient becomes zero. That is $P_r = 0$ or $P_t = 100\%$.

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \quad (29) \qquad \Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \quad (34)$$

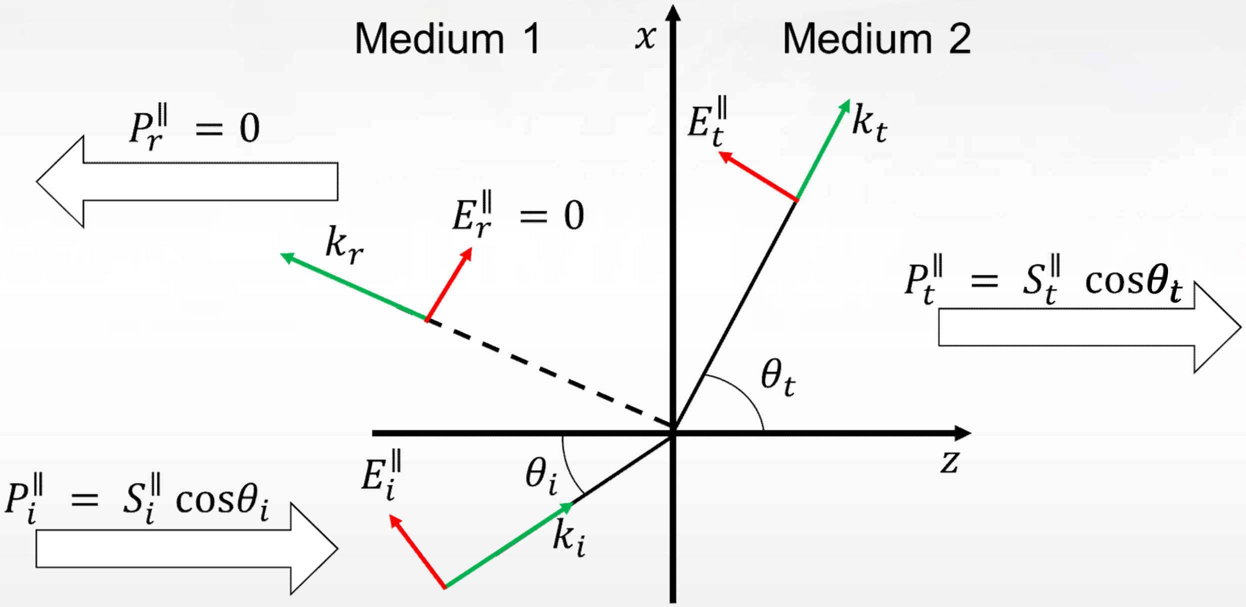
[Page 19-12]

- Assume **Medium 1** and **Medium 2** are both **lossless** $[\sigma_1 = \sigma_2 = 0]$ and **non-magnetic** $[\mu_1 = \mu_2 = \mu_o]$
- It may be shown that $|\Gamma_{\perp}| > 0\ \forall\ \theta_i$ i.e. there is no Brewster angle for perpendicular polarised wave.

# The Brewster Angle

For **Parallel Polarised Wave,** $\Gamma_{\parallel} = 0$ at $\theta_i = \theta_{B\parallel}$

$$\theta_i = \theta_{B\parallel} \rightarrow \Gamma_{\parallel} = 0,\ E_r^{\parallel} = 0,\ P_r^{\parallel} = 0,\ P_t^{\parallel} = P_i^{\parallel}$$




$$S_i^{\parallel} \cos\theta_i = S_t^{\parallel} \cos\theta_t$$

# The Brewster Angle

For **Parallel Polarised Wave**

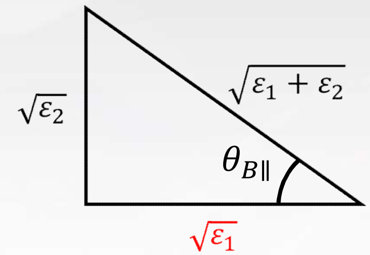
- $\Gamma_{\parallel} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$ (29)
- $\Gamma_{\parallel} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_{B\parallel}}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_{B\parallel}} = 0$ $[\Gamma_{\parallel} = 0 \text{ at } \theta_i = \theta_{B\parallel}]$
- $\boxed{\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}}$ $\boxed{\left[\dfrac{\sin\theta_t}{\sin\theta_{B\parallel}} = \sqrt{\dfrac{\varepsilon_1}{\varepsilon_2}}\right]}$
- $\cos^2\theta_t = \left(\dfrac{\eta_1}{\eta_2}\right)^2 \cos^2\theta_{B\parallel}$
- $1 - \sin^2\theta_t = \dfrac{\varepsilon_2}{\varepsilon_1} \cos^2\theta_{B\parallel}$

# The Brewster Angle



- Snell's Law: $\frac{\sin\theta_t}{\sin\theta_{B\parallel}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \rightarrow \sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin\theta_{B\parallel}$

- $1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_{B\parallel} = \frac{\varepsilon_2}{\varepsilon_1}\cos^2\theta_{B\parallel} \rightarrow \sin\theta_{B\parallel} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1+\varepsilon_2}}$




- $\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ (47)

- Since $0 < \tan\theta_{B\parallel} < \infty$, (47) shows that there is a Brewster angle [parallel polarisation] for any combination of $\varepsilon_1$ and $\varepsilon_2$.

# Example



Medium 1 $(\mu_o,\ \varepsilon_o,\ \sigma = 0)$ —— Medium 2 $(\mu_o,\ 3\varepsilon_o,\ \sigma = 0)$

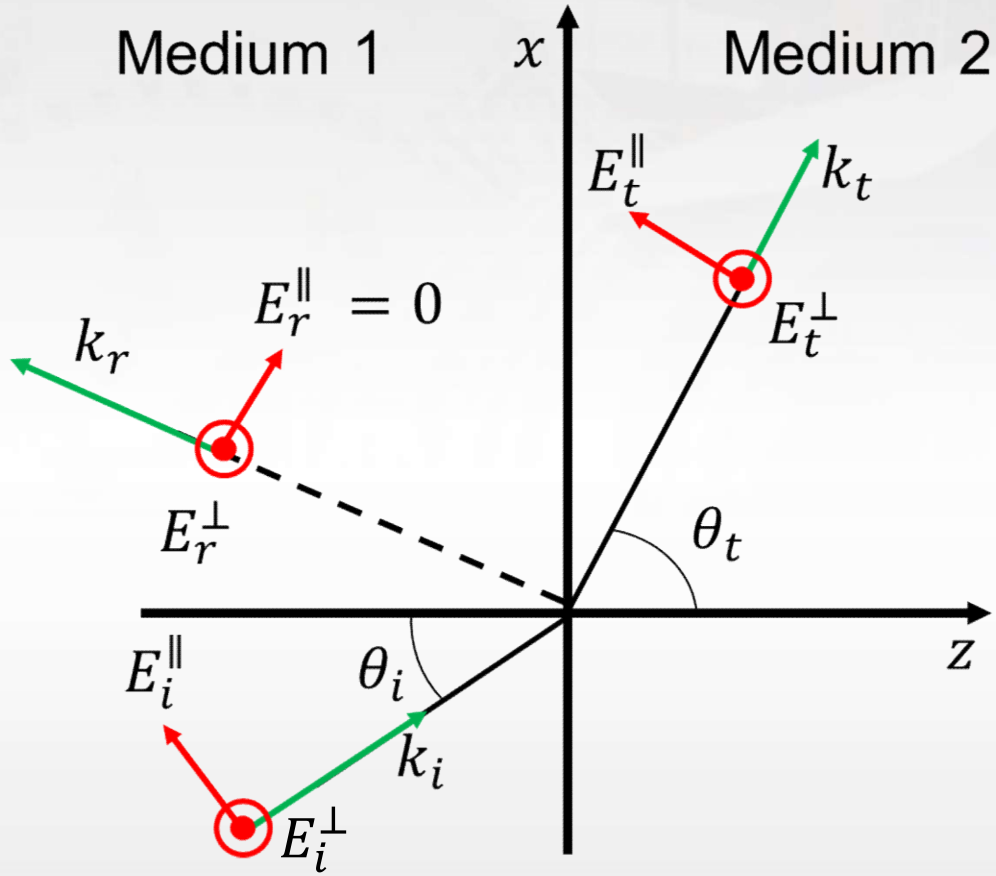
- $\eta_1 = 120\pi$; $\eta_2 = 120\pi/\sqrt{3}$
- $\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{3} \quad \rightarrow \theta_{B\parallel} = 60° = \theta_i = \theta_r$
- $\frac{\sin\theta_t}{\sin\ \ _i} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{1}{\sqrt{3}} \quad \rightarrow \theta_t = 30°$
- $\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\ \ _i} \rightarrow \Gamma_\parallel = 0$
- $\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \rightarrow \Gamma_\perp = 0.5\ \angle 180°$

# Example

- A UPW with $E_{oi}^{\parallel} = 5\angle 60°$ and $E_{oi}^{\perp} = 5\angle 90°$ incident at $\theta_{i=}\theta_{B\parallel}$, find $\vec{E}_r$.
- $E_{oi}^{\parallel} = 5\angle 60° \rightarrow E_{or}^{\parallel} = \Gamma_{\parallel} E_{oi}^{\parallel} = 0$

  $E_{oi}^{\perp} = 5\angle 90° \rightarrow E_{or}^{\perp} = \Gamma_{\perp} E_{oi}^{\perp} = 2.5\angle 270°$
- $\vec{E}_r = \vec{a}_y 2.5\angle 270° e^{-jk_1 \sin 60° x} e^{+jk_1 \cos 60° z}$ $\perp xz - \text{plane}$
- **Brewster angle** is also called the **polarising angle** because any UPW incident on an interface at this angle will be reflected with $\vec{E}_r$ **polarised** in the direction **perpendicular to the plane of Incidence**.

# Polarising Angle



# Summary

- Brewster angle is defined as the **incident angle** $\theta_i$ at which reflection coefficient becomes zero.

- Calculate the Brewster angle for parallel polarised wave at the interface between two lossless dielectric media.

- Brewster angle is also called the polarising angle because any UPW incident at this angle will be reflected with $\vec{E}_r$ polarised in the direction perpendicular to the plane of Incidence.

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# 23. Total Internal Reflection

# Learning Objectives

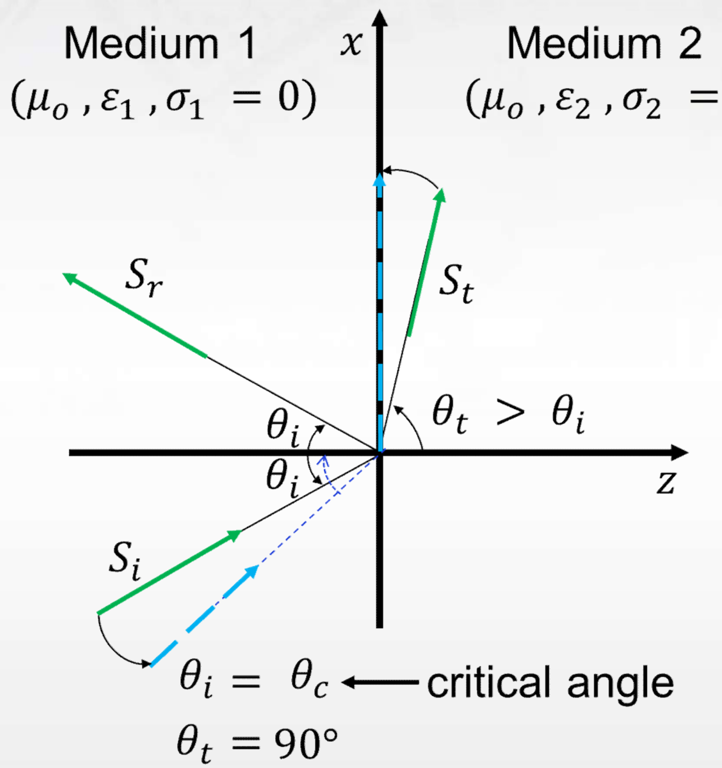
- Define Total Internal Reflection at the interface between two lossless dielectric media.

- Calculate the critical angle, $\theta_c$, at the interface between two lossless dielectric media.

- As long as $\varepsilon_1 > \varepsilon_2$, $\theta_c$ exists for both parallel and perpendicular polarisation.

# Total Internal Reflection

- Total Internal Reflection: $[\varepsilon_1 > \varepsilon_2]$




$$\text{Snell's Law: } \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$\varepsilon_1 > \varepsilon_2 \rightarrow \theta_t > \theta_i$$

$$P_t = S_t \cos\theta_t$$

$$\theta_t = 90^\circ:\ P_t = 0$$

# Total Internal Reflection



- $\theta_t \to 90^\circ$: $\theta_i = \theta_c$
- $\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{\sin(90^\circ)}{\sin(\theta_c)} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$

$$\boxed{\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \qquad (48)$$

- Since $\sin\theta_c < 1 \to \varepsilon_1 > \varepsilon_2$. (48) shows that $\theta_c$ exists if $\varepsilon_1 > \varepsilon_2$ independent of polarisation.

# Example

- Show that when $\theta_i = \theta_c$, $100\%$ of average incident power reflected from the interface.
- $\theta_i = \theta_c \rightarrow \theta_t = \ 90^{\circ} \ [\cos\theta_t = 0]$
- $\Gamma_{\parallel} = \frac{\eta_2 \cancel{\cos(\theta_t)} - \eta_1 \cos(\theta_i)}{\eta_2 \cancel{\cos(\theta_t)} + \eta_1 \cos(\theta_i)} = -1 \ \rightarrow \ P_r^{\parallel} = \left|\Gamma_{\parallel}\right|^2 P_i^{\parallel} = P_i^{\parallel}$
- $\Gamma_{\perp} = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cancel{\cos(\theta_t)}}{\eta_2 \cos(\theta_i) + \eta_1 \cancel{\cos(\theta_t)}} = +1 \ \rightarrow \ P_r^{\perp} = |\Gamma_{\perp}|^2 P_i^{\perp} = P_i^{\perp}$
- **REMARKS:** It may be shown that $\left|\Gamma_{\parallel}\right| = |\Gamma_{\perp}| = 1$ for all $\theta_i \geq \theta_c$.

$\therefore$ Total Internal Reflection occurs for all $\theta_i \geq \theta_c$

# Summary

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- Calculate the critical angle, $\theta_c$, at the interface between two lossless dielectric media using $\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$.
- $\theta_c$ exists if $\varepsilon_1 > \varepsilon_2$ for both parallel and perpendicular polarisation.
- Total Internal Reflection occurs for all $\theta_i \geq \theta_c$, i.e. $\left|\Gamma_{\parallel}\right| = |\Gamma_{\perp}| = 1$ for all $\theta_i \geq \theta_c$.

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# 24. Appendix

# Appendix



**Fresnel's Equations:** $\Gamma_{\parallel}, \Gamma_{\perp}, \tau_{\parallel}, \tau_{\perp} = f(\eta_1, \eta_2, \theta_i, \theta_t)$

- $\Gamma_{\parallel} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos \quad {}_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \dfrac{E_{or}^{\parallel}}{E_{oi}^{\parallel}}$ (29)

- $\tau_{\parallel} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \dfrac{E_{ot}^{\parallel}}{E_{oi}^{\parallel}}$ (30)

- $\Gamma_{\perp} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \dfrac{E_{or}^{\perp}}{E_{oi}^{\perp}}$ (34)

- $\tau_{\perp} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \dfrac{E_{ot}^{\perp}}{E_{oi}^{\perp}}$ (35)

# Appendix

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## Snell's Law:

- $\dfrac{\sin\theta_t}{\sin\ \ {}_{i}} = \dfrac{k_i}{k_t} = \sqrt{\dfrac{\mu_1\,\varepsilon_1}{\mu_2\,\varepsilon_2}}$ (26)

## Brewster Angle:

- $\tan\theta_{B\parallel} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}$ **[Parallel Polarisation]** (47)

## Critical Angle:

- $\sin\theta_c = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}$ $[\varepsilon_1 > \varepsilon_2]$ (48)

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# 25. Guided Electromagnetic Wave: Transmission Line

# Learning Objectives

- Define incident and reflected voltage waves on the transmission line, i.e. $V_o^+$ and $V_o^-$;
- Define incident and reflected current waves on the transmission line, i.e. $I_o^+$ and $I_o^-$;
- Define characteristic impedance, i.e. $Z_o = \frac{V_o^+}{I_o^+}$; and
- Define distributed voltage and current, i.e. $V(z)$ and $\mathrm{I}(z)$, and input impedance, i.e. $Z_{in}(z) = \frac{V(z)}{I(z)}$, of a transmission line.

# Recap

- So far we have considered **Reflection and Transmission of UPW** at a plane boundary **without using any guiding structure.**

- Electromagnetic energy associated with the waves spread over a wide area. Such **unguided electromagnetic wave** is useful, for example, in Broadcast TV System and Mobile Communications.

- For **point-to-point transmission**, the **energy from the source** must be **guided to the load**.

- **Guiding Structures** are used to **direct power from the source to the load efficiently**. For example, **from a microwave source to an antenna on the rooftop.**

# Electromagnetic Wave guided by Transmission Line

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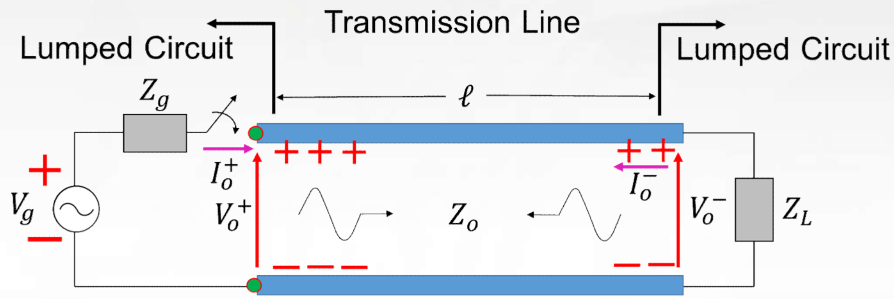
- In this course, we will study Electromagnetic Wave **guided by two parallel conductors** (**Transmission Line**) under **Time-Harmonic conditions** (**single frequency at steady state**).



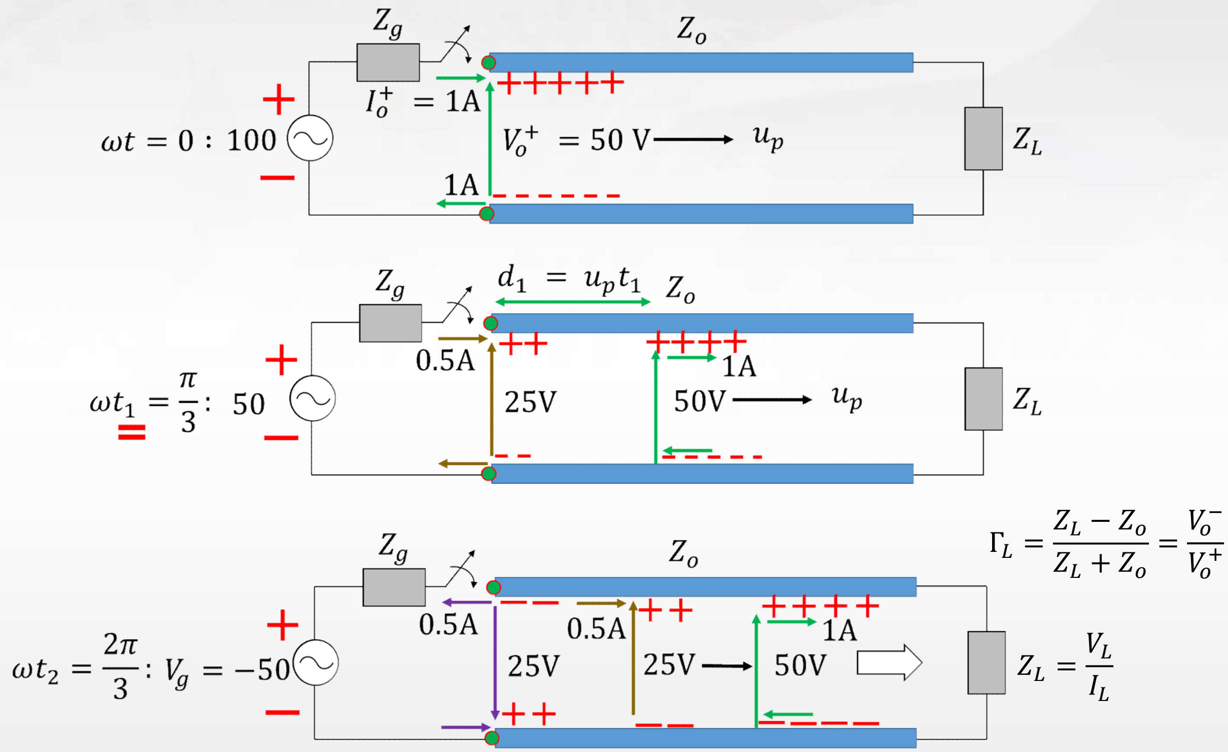

- Consider the moment when the generator is turn on, say at $t = 0$.
- The generator action can be thought of as leaking charges onto the lines: positive on the upper conductor and negative on the lower.
- The **Characteristic Impedance** $Z_o$ of the transmission line is defined as

$$Z_o = \frac{V_o^+}{I_o^+} = f(\text{geometry}, \eta).$$

# Voltage and Current Waves on Transmission Line



$$V_g = 100\cos(\omega t), \quad Z_g = 50\,\Omega, \quad Z_o = \frac{V_o^+}{I_o^+} = 50\,\Omega$$



# Voltage and Current Waves on Transmission Line

- When the incident waves ($V_o^+ = 50\,V, I_o^+ = 1\,A$) reach the load:

  (i) If $Z_L = Z_o = 50\,\Omega$: $\frac{V_0^+}{I_o^+} = Z_L$

  Ohm's Law is satisfied; there will be no reflected wave.

  Steady state solution: $V_L = V_o^+ = 50\text{V}$ and $I_L = I_o^+ = 1\text{A}$

  (ii) If $Z_L \neq Z_o$: Incident wave will be reflected at the load end such that

  $V_L = (V_o^+ + V_o^-)$ and $I_L = (I_o^+ + I_o^-)$ satisfied Ohm's Law $\frac{V_L}{I_L} = Z_L$.

The reflected wave ($V_o^- = \Gamma_L\, V_o^+$) will then propagates toward the source and may be re-reflected at the source end. The process will continue until steady state is reached (Time-Harmonic Conditions).

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# 25. Guided Electromagnetic Wave: Transmission Line (2)

# Circuit Viewpoint and EM Viewpoint

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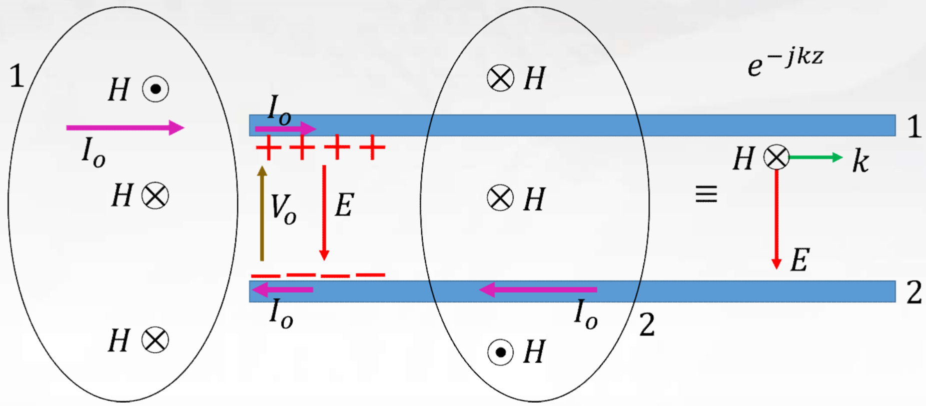
- Power flow on Transmission Line:

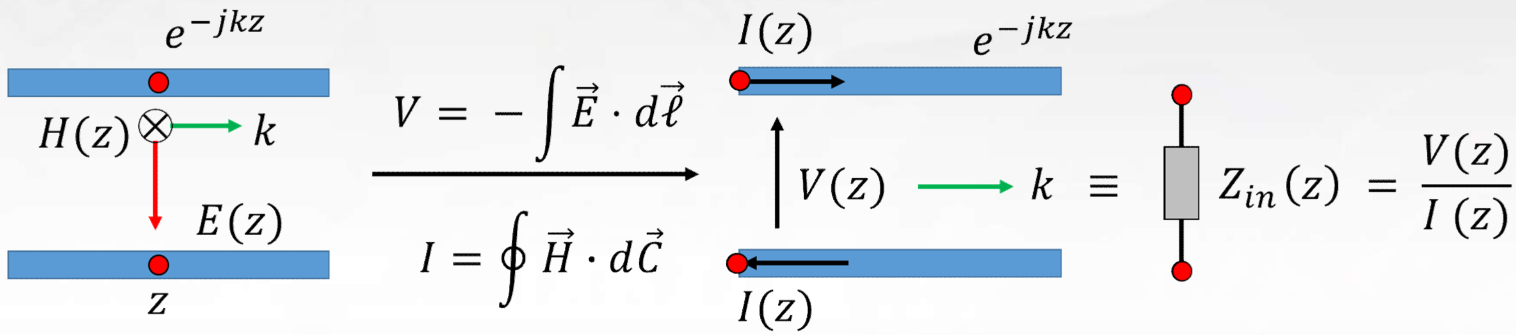



- **Circuit viewpoint:** $V_o$ ↑ and $I_o$ → on conductor 1 and to the left on conductor 2; Power is flowing to the right.

- **EM viewpoint:** E ↓ and H ⊗ between the 2 parallel conductors. **Power flow** to the right at velocity $u_p = 1/\sqrt{\mu\varepsilon}$.

# Distributed Circuit Model

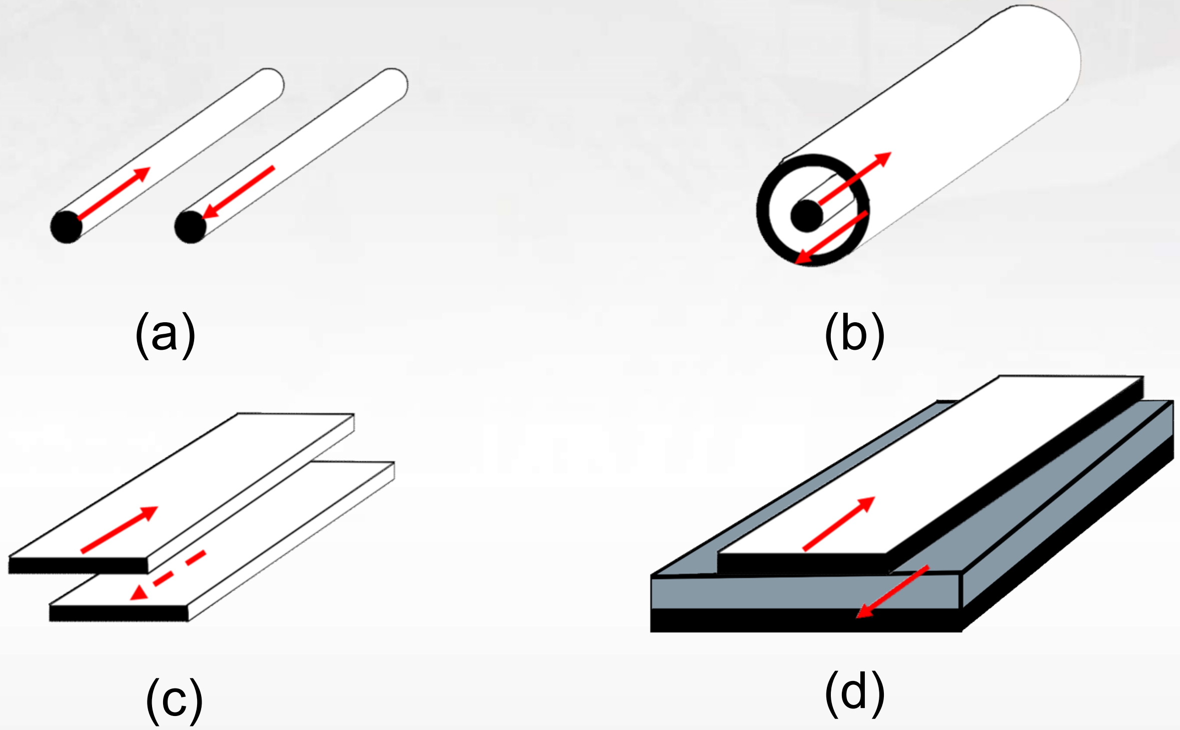
- **Distributed circuit model:** $V(z)$ and $I(z)$ at any position $z$, is directly proportional to $\vec{E}(z)$ and $\vec{H}(z)$ respectively.




- Such a "**circuit model**" provides practical results for the Voltage, Current, Impedance and Power flow along a transmission line **without having resort to field equations**.

- $Z_{in}(z) = \frac{V(z)}{I(z)} \equiv R_{in} + jX_{in},\ P = \frac{1}{2}|I(z)|^2\ R_{in} = \frac{1}{2}\ Re\,(V(z)I(z)^*)$

# Commonly used Transmission Line




- (a) Parallel-Wire, (b) Coaxial, (c) Parallel Plate, (d) Microstrip

# Summary

- Define incident and reflected voltage waves on the transmission line, i.e. $V_o^+$ and $V_o^-$;
- Define incident and reflected current waves on the transmission line, i.e. $I_o^+$ and $I_o^-$;
- Define characteristic impedance, i.e. $Z_o = \frac{V_o^+}{I_o^+}$; and
- Define distributed voltage and current, i.e. $V(z)$ and $\mathrm{I}(z)$, and input impedance, i.e. $Z_{in}(z) = \frac{V(z)}{I(z)}$, of a transmission line.

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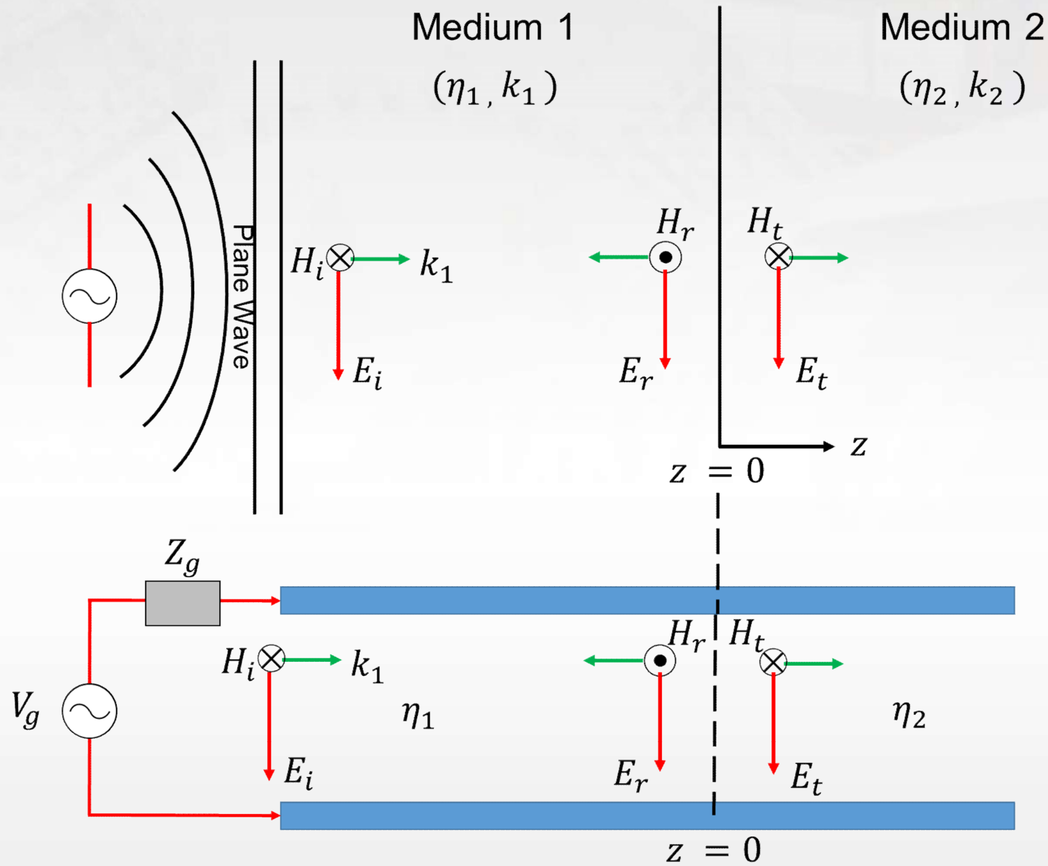
# 26. Analogy Between Plane Wave and Transmission Line

# Learning Objectives

- Draw the analogy between $E$ and $H$ fields for plane wave at normal incidence and voltage and current waves on transmission line;
- Describe standing wave on transmission line; and
- Obtain transmission line equations from plane wave at normal incidence.

# Plane Wave Propagation VS Transmission Line



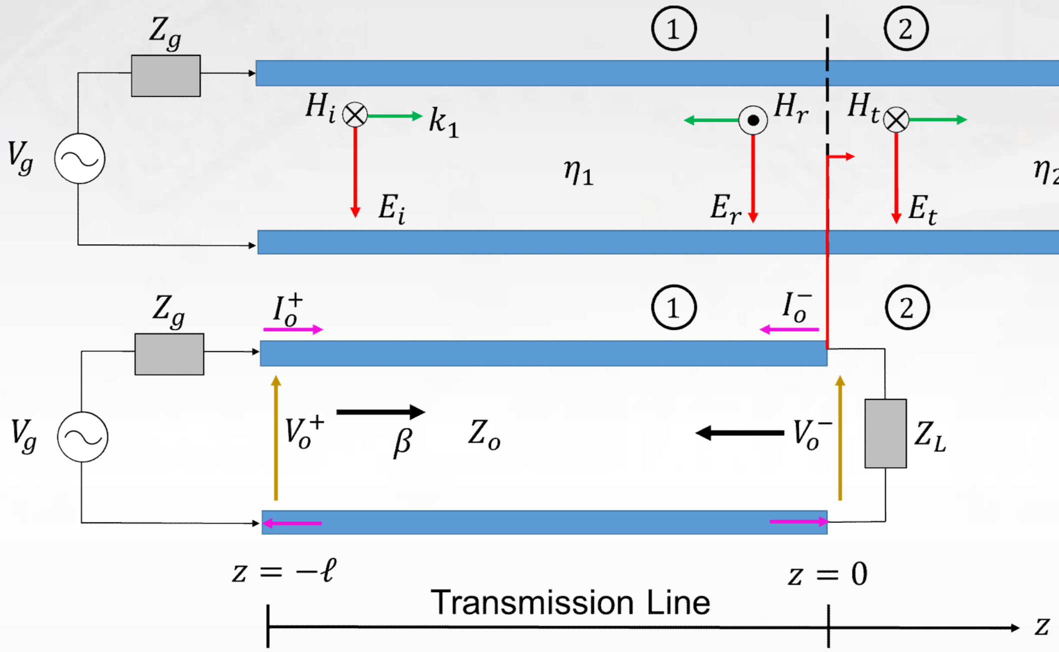

$$\frac{E_{oi}}{H_{oi}} = \eta_1$$

$$\Gamma = \frac{E_{or}}{E_{oi}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{E_{ot}}{E_{oi}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

# Plane Wave Propagation VS Transmission Line






$$\frac{E_{oi}}{H_{oi}} = \eta_1$$

$$\Gamma = \frac{E_{or}}{E_{oi}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Downarrow$$

$$\frac{V_o^+}{I_o^+} = Z_o$$

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z} \qquad (1)$$

$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{+j\beta z} \qquad (2) \qquad -\ell \le z \le 0$$

# Transmission Line As Impedance Transformer

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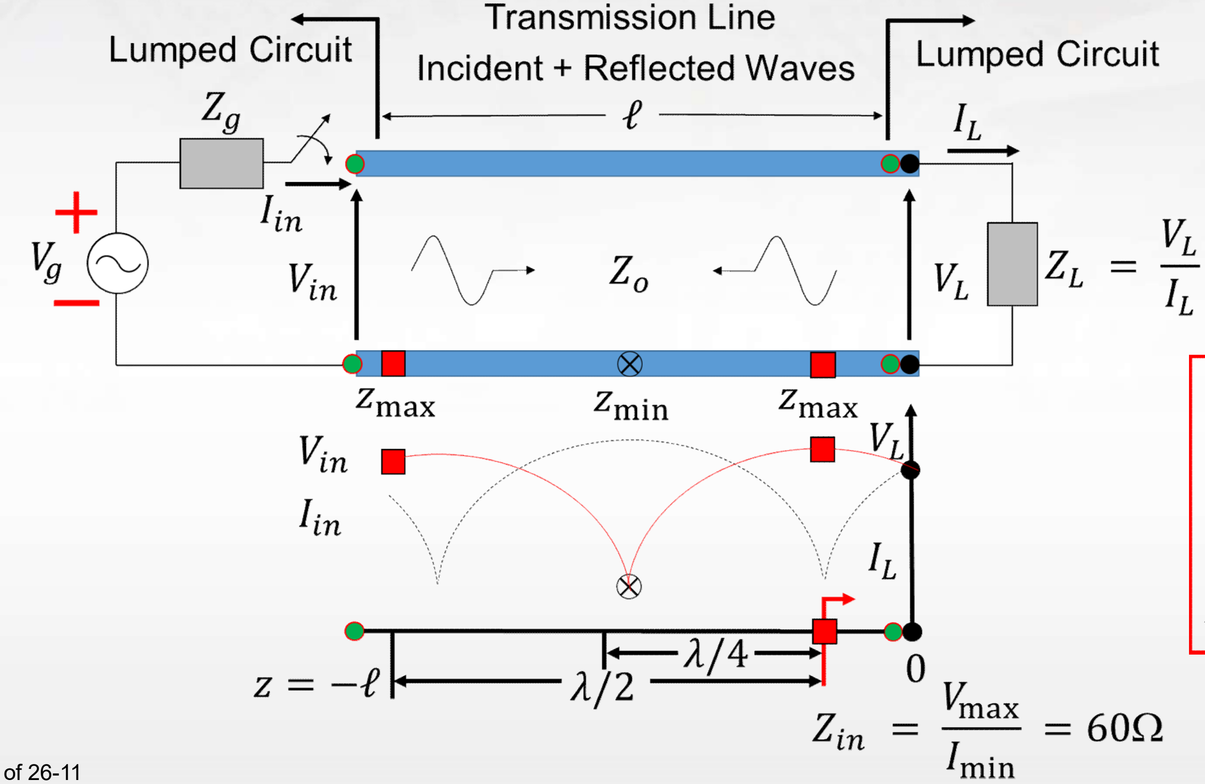
- Transmission Line (**Steady State**) ≡ Impedance Transformer




[Page 12-7]

[Page 12-8]

$$\left.\begin{aligned} V_L &= 40\angle 30^\circ \text{ V} \\ I_L &= 2\angle -10^\circ \text{ A} \\ Z_L &= 20\angle 40^\circ\ \Omega \end{aligned}\right\} @\ z = 0$$

$$\left.\begin{aligned} V_{max} &= 60\angle 80^\circ \text{V} \\ I_{min} &= 1\angle 80^\circ \text{ A} \end{aligned}\right\} @\ z_{max}$$

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# 26. Analogy Between Plane Wave and Transmission Line (2)

# Transmission Line As Impedance Transformer

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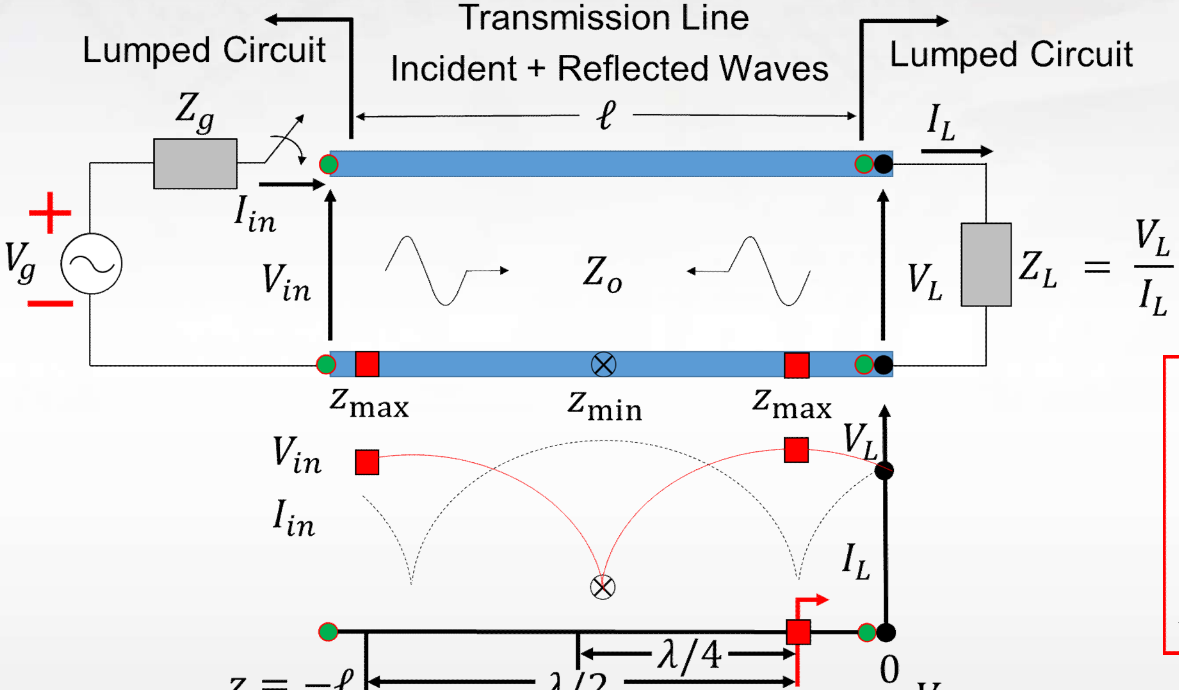
- Transmission Line (**Steady State**) ≡ Impedance Transformer




$$Z_{in} = \frac{V_{max}}{I_{min}} = 60\Omega$$

[Page 12-7]

[Page 12-8]

$$\left.\begin{aligned} V_L &= 40\angle 30^\circ \text{ V} \\ I_L &= 2\angle -10^\circ \text{ A} \\ Z_L &= 20\angle 40^\circ\ \Omega \end{aligned}\right\} @\ z = 0$$

$$\left.\begin{aligned} V_{max} &= 60\angle 80^\circ \text{V} \\ I_{min} &= 1\angle 80^\circ \text{ A} \end{aligned}\right\} @\ z_{max}$$

# Transmission Line As Impedance Transformer

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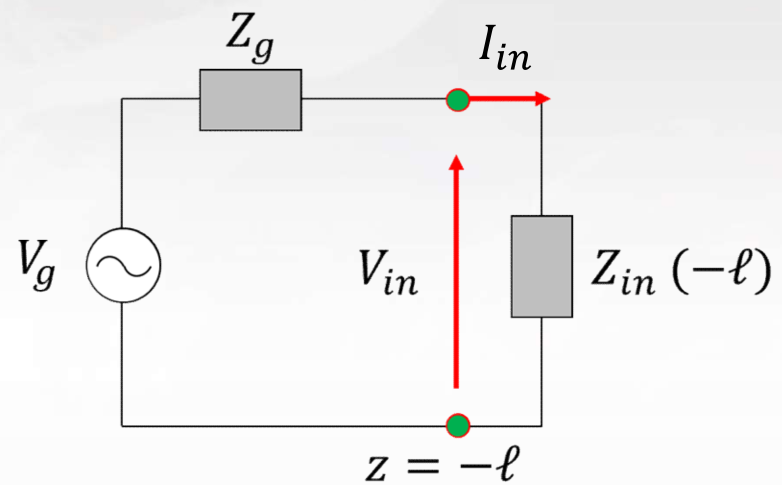
- In general: $V_{in} \neq V_L;\ \ I_{in} \neq I_L;\ \ Z_{in}(-\ell) \neq Z_L = Z_{in}(0)$

- $Z_{in}(z) = \dfrac{V(z)}{I(z)} \qquad -\ell \leq z \leq 0$

- $Z_{in}(z_{max}) = \dfrac{V_{max}}{I_{min}} = R_{max} + j0$

- $Z_{in}(z_{min}) = \dfrac{V_{min}}{I_{max}} = R_{min} + j0$

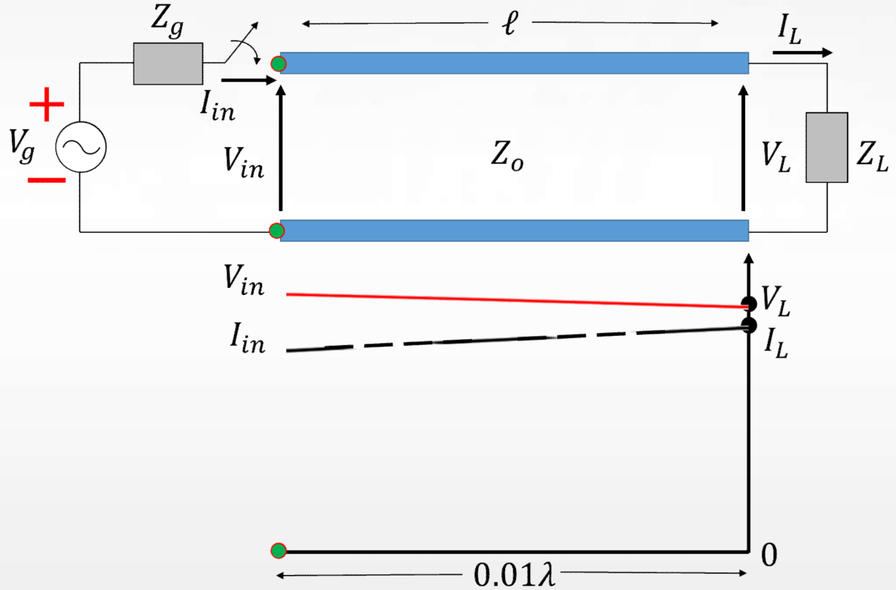



- At the source end of the line, $z = -\ell$, the source $(V_g, Z_g)$ looking into the lines sees an **input impedance** $Z_{in}(-\ell) = \dfrac{V_{in}}{I_{in}} \neq Z_L$.

# Transmission Line As Impedance Transformer

- If $\ell \ll \lambda$ say, $\ell < 0.01\lambda$: $V_{in} \simeq V_L$;  $I_{in} \simeq I_L$;  $Z_{in}(-\ell) \simeq Z_L$.

- **Transmission line equations** (1) and (2) → **Circuit Theory (KVL, KCL, Ohm's Law).**




$f = 50$ Hz

$f\lambda = c = 3 \times 10^8$ m/s

$\lambda = 6000$ km

# Transmission Line As Impedance Transformer

**Remarks:**

- The **source and load** are lumped circuit elements. **Circuit theory** such as **Ohm's law and Kirchhoff's Law** are satisfied at the source and the load ends.

- The **thick lines** represent the **transmission line** where the **wave nature** of the fields must be considered e.g. incident and reflected wave [**Standing Wave**].

- **Lossless** Transmission Line: $\boxed{P_{in} = P_L = P(z)}$

# Summary of UPW VS Transmission Line

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- The **analogy** between **plane wave** and **voltage and current waves** in a lossless transmission line is in fact an **exact and complete** one.
- The algebraic steps worked out for the solutions of one system [e.g. plane wave] need not be repeated in analysing the other [e.g. Transmission Line].

| **Plane Wave** $z \leq 0$ | **Transmission Line** $-\ell \leq z \leq 0$ |
|---|---|
| $E(z) = E_{oi} e^{-jk} + E_{or} e^{+jkz}$ | $V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$ |
| $H(z) = \frac{1}{\eta}[E_{oi} e^{-jkz} - E_{or} e^{+jkz}]$ | $I(z) = \frac{1}{Z_o}[V_o^+ e^{-j\beta z} - V_o^- e^{+j\beta z}]$ |
| $k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$ | $\beta = k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$ |

# Summary of UPW VS Transmission Line

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| | |
|---|---|
| $u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$ | $u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$ |
| $\eta = \frac{E_{oi}}{H_{oi}} = \sqrt{\frac{\mu}{\varepsilon}}$ | $Z_o = \frac{V_o^+}{I_o^+} = f(\eta, geometry)$ |
| $\Gamma = \frac{E_{or}}{E_{oi}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ | $\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$ $\boxed{\frac{P_r}{P_i} = \lvert\Gamma_L\rvert^2}$ |
| $\bar{S} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*)\ \mathrm{W/m^2}$ | $P = \frac{1}{2} Re(VI^*)\ \mathrm{W}$ |
| $SWR = \frac{\lvert E\rvert_{max}}{\lvert E\rvert_{min}} = \frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}$ | $SWR = \frac{\lvert V\rvert_{max}}{\lvert V\rvert_{min}} = \frac{1 + \lvert\Gamma_L\rvert}{1 - \lvert\Gamma_L\rvert}$ |

# Summary



- ❑ Draw the analogy between $E$ and $H$ fields for plane wave at normal incidence and voltage and current waves on transmission line;
- ❑ Describe standing wave on transmission line; and
- ❑ Obtain transmission line equations from plane wave at normal incidence.

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# 27. Distributed Circuit Model of a Lossless Transmission Line

# Learning Objectives

- Derive transmission line equations from distributed circuit model.

# Lossless Transmission Line

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- A lossless line is defined as a transmission line that has no line resistance ($\sigma = \infty$) and no dielectric loss ($\sigma = 0$).

- $L$ and $C$ are the per-unit length ($\Delta z = 1\ \mathrm{m}$) inductance [H/m] and capacitance [F/m] of a given Transmission Line.

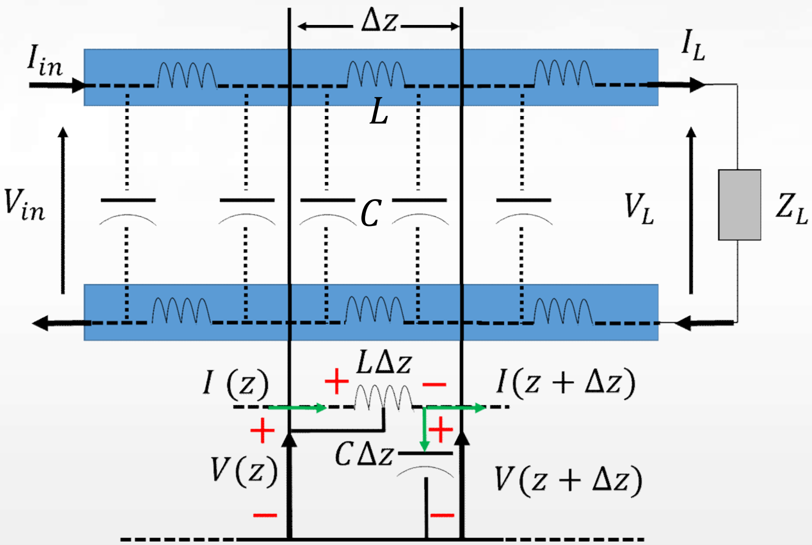
- Applying KVL:

$$V(z) = I(z)\, j\omega L\, \Delta z + V(z + \Delta z)$$

$$\frac{V(z+\Delta z) - V(z)}{\Delta z} = -j\omega L\, I(z)$$

As $\Delta z \to 0$,

$$\frac{dV(z)}{dz} = -j\omega L\, I(z) \quad \text{(A.1)}$$



# Lossless Transmission Line



- Similarly, applying KCL:

$$I(z) = I\,(z + \Delta z) + V\,(z + \Delta z)\, j\omega C \Delta\, z$$

$$\frac{dI(z)}{dz} = -j\omega C\, V(z) \qquad \text{(A.2)}$$

- Differentiate (A.1) and substituting (A.2):

$$\frac{d^2V(z)}{dz^2} + \beta^2\, V(z) = 0 \;\; [\beta = \omega\sqrt{LC}] \qquad \text{(A.3)}$$

- Differentiate (A.2) and substituting (A.1):

$$\frac{d^2I(z)}{dz^2} + \beta^2\, I(z) = 0 \qquad \text{(A.4)}$$

# Lossless Transmission Line

- The solutions to (A.3) and (A.4) are

$$V(z) = V_o^+ \, e^{-j\beta z} + V_o^- \, e^{+j\beta z} \qquad (1)$$

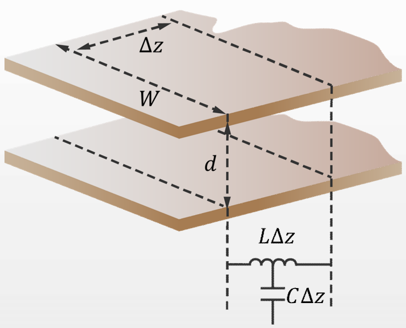
$$I(z) = I_o^+ \, e^{-j\beta z} + I_o^- \, e^{+j\beta z} \qquad (2)$$

where $\beta = \omega \sqrt{LC} = \omega\sqrt{\mu\varepsilon}$

$$\boxed{LC = \mu\varepsilon}$$

- Characteristic impedance $Z_o$ of a lossless transmission line is determined by the geometric and the material $(\mu, \varepsilon)$, and is not depend on its length, $\ell$.

- Example:




$$\boxed{C = \frac{\varepsilon W}{d} \;\; \mathrm{F/m}}$$

$$\boxed{L = \frac{\mu d}{W} \;\; \mathrm{H/m}}$$

$$\boxed{Z_o = \sqrt{\frac{L}{C}} = \eta \frac{d}{W} \, \Omega}$$

# Summary

- ☐ Derive transmission line equations from distributed circuit model.