NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

EE3001 Engineering Electromagnetics

This session is about

31. Design of Low Loss Inductor and Capacitor Using Open- and Short-Circuited Stubs

Learning Objectives

- To design the low loss inductor and capacitor using open- and short-circuited stubs.

Design of Low Loss Inductor and Capacitor

- To transform a short- or open-circuited transmission line to a desirable Inductor and Capacitor, it is cumbersome to solve ℓ using (6).

- **Example:** Find the length ℓ of a $50\ \Omega$ short-circuited **air-filled** T-line $(Z_L = 0)$ such that the impedance at the input end Z_{in} is equivalent to an inductor $L = 80\text{nH}$ at $f = 100\text{MHz}$.

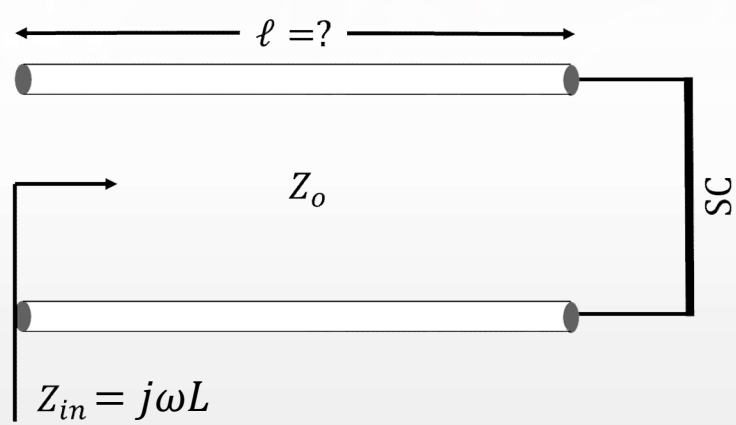
$[Z_o = 50\ \Omega, \beta = 2\pi/3]$

- $\boxed{Z_{in}(-\ell) = \frac{1+\Gamma_L e^{-j2}}{1-\Gamma_L e^{-j2\beta\ell}} Z_o}$ (6)

- $Z_L = 0 \longrightarrow \Gamma_L = -1$ (5)

- $Z_{in} = j\omega L = j50\ \Omega$

- $Z_{in}(-\ell) = j50 = \frac{1-e^{-j4\pi\ell/3}}{1+e^{-j4\pi\ell/3}} Z_o \rightarrow \ell = ?$



Design of Low Loss Inductor and Capacitor

- To simplify the design of low loss inductor and capacitor using short- or open-circuited load, we need to re-write (6) into another form:

- Substitute $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$ into $Z_{in}(-\ell) = \frac{1+\Gamma_L e^{-j2\beta\ell}}{1-\Gamma_L e^{-j2}} Z_o$

$$Z_{in}(-\ell) = \frac{(Z_L+Z_o)e^{j\beta\ell}+(Z_L-Z_o)e^{-j\beta\ell}}{(Z_L+Z_o)e^{j\beta\ell}-(Z_L-Z_o)e^{-j\beta\ell}} Z_o$$

$$Z_{in}(-\ell) = \frac{Z_L(e^{j\beta\ell}+e^{-j\beta\ell})+Z_o(e^{j\beta\ell}-e^{-j\beta\ell})}{Z_o(e^{j\beta\ell}+e^{-j\beta\ell})+Z_L(e^{j\beta\ell}-e^{-j\beta\ell})} Z_o$$

$$\begin{bmatrix} e^{j\beta\ell} = \cos(\beta\ell) + j\sin(\beta\ell) \\ e^{-j\beta} = \cos(\beta\ell) - j\sin(\beta\ell) \end{bmatrix}$$

Design of Low Loss Inductor and Capacitor

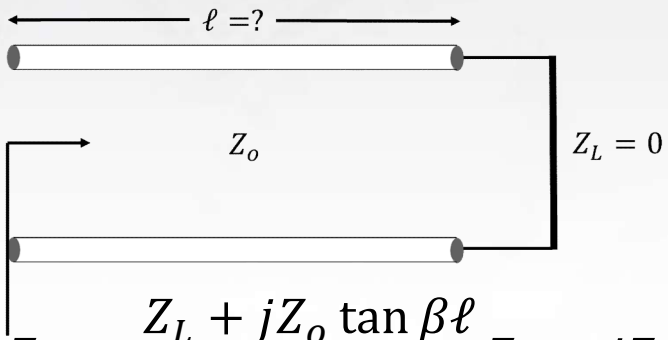
$$Z_{in}(-\ell) = \frac{2Z_L\cos(\beta\ell) + j2Z_o\sin(\beta\ell)}{2Z_o\cos(\beta\ell) + j2Z_L\sin(\beta\ell)} Z_o$$

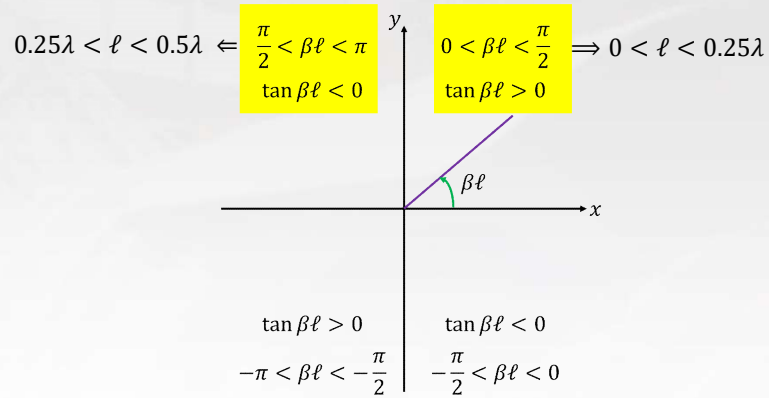
$$\boxed{Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o} \qquad (\ell > 0)\ (18) \equiv (6)$$

- **REMARKS:**
- Equation (18) is used extensively in the design of **low loss Inductor L and Capacitor C** at **Microwave frequencies.**
- L and C can be designed using short or open circuited transmission lines–referred to as short- or open- circuited stubs.

Short-Circuited Stubs

- **Short-circuited Stubs** ($Z_L = 0$)





$$Z_{sc} = \frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell} Z_o = jZ_o \tan\beta\ell \qquad (19)$$

$$\left[\beta\ell = 2\pi\frac{\ell}{\lambda} < \frac{\pi}{2} \rightarrow \ell < 0.25\lambda\right]$$

- $0 < \ell < 0.25\lambda \rightarrow \tan\beta\ell > 0$

$\tan\beta\ell > 0$: $\boxed{Z_{sc} = jZ_o \tan(\beta\ell) = j\omega L}$ $\qquad \rightarrow \ \ell = \frac{1}{\beta}\tan^{-1}\left(\frac{\omega L}{Z_o}\right)$

$$\tan\beta\ell = \frac{\omega L}{Z_o}$$

Short-Circuited Stubs

- All possible values of L can be design using a short-circuited transmission line with length $0 < \ell < 0.25\lambda$ $[0 < \tan\beta\ell < \infty]$

- $0.25\lambda < \ell < 0.5\lambda \rightarrow \tan\beta\ell < 0$

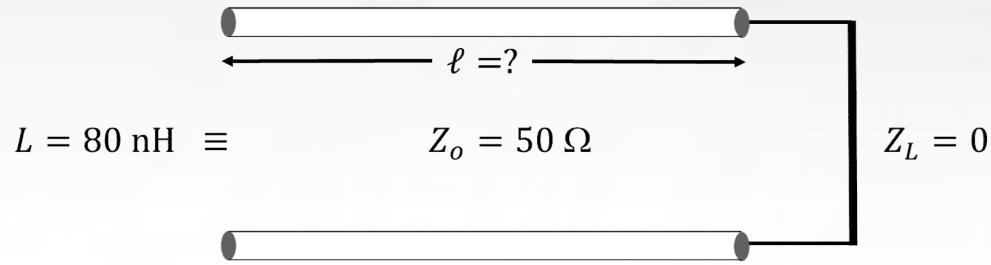
$$\tan\beta\ell < 0: Z_{sc} = jZ_o\tan(\beta\ell) = -\frac{j}{\omega C} \rightarrow \ell = \frac{1}{\beta}\tan^{-1}\left(\frac{-1}{\omega C Z_o}\right)$$

$$\tan\beta\ell = -\frac{1}{\omega C Z_o}$$

- All possible values of C can be design using a short-circuited transmission line with length $0.25\lambda < \ell < 0.5\lambda$ $[-\infty < \tan\beta\ell < 0]$

Example

- Find the shortest length ℓ_{min} of a short-circuited transmission line with $Z_o = 50\ \Omega$ and $u_p = 3 \times 10^8\ \mathrm{m/s}$ such that it is equivalent to a $\boldsymbol{L = 80\mathrm{nH}}$ inductor operating at $\boldsymbol{f = 100\ \mathrm{MHz}}$.



- **Solution:** $\left[Z_o = 50\ \Omega,\ \beta = \frac{2\pi}{3}\right]$ $\qquad \left[\beta = \frac{\omega}{u_p}\right]$
- $Z_{in} = j\omega L = j2\pi \times 10^8 \times 80 \times 10^{-9} = j50\ \Omega$
- $Z_{sc} = Z_{in} = j50 = jZ_o \tan(\beta\ell)$ $\qquad (19)$

Solution

- $\tan(\beta\ell) = 1 \Rightarrow \beta\ell = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots$

  $\beta\ell_{\min} = \frac{\pi}{4}$ $\qquad \left[\beta = \frac{2\pi}{\lambda}\right]$

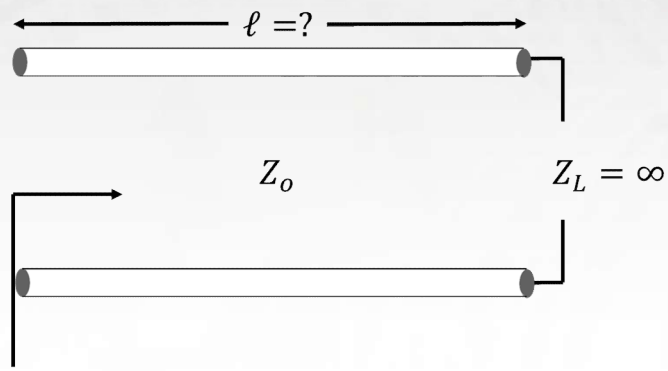
- $\ell_{\min} = \frac{\pi}{4}\frac{\lambda}{2\pi} = 0.125\lambda$ $\qquad \ell = \ell_{\min} + n\frac{\lambda}{2}$

- $f\lambda = u_p \rightarrow \lambda = 3\ \mathrm{m}$ $\qquad \left[f = 100\ \mathrm{MHz},\ u_p = 3 \times 10^8\ \mathrm{m/s}\right]$

- $\ell_{\min} = 0.375\ \mathrm{m}$

Open-circuited Stubs

- **Open-circuited Stubs** $(Z_L = \infty)$



$$Z_{oc} = \frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell} Z_o = \frac{\frac{Z_L}{Z_L} + \frac{jZ_o \tan\beta\ell}{Z_L}}{\frac{Z_o}{Z_L} + \frac{jZ_L \tan\beta\ell}{Z_L}} Z_o = \frac{Z_o}{j\tan\beta\ell} \qquad (20)$$

- $0 < \ell < 0.25\lambda$ $\quad \tan\beta\ell > 0,\ Z_{oc} = \frac{1}{j\omega C}$ $\quad [0 < C < \infty]$
- $0.25\lambda < \ell < 0.5\,\lambda$ $\quad \tan\beta\ell < 0,\ Z_{oc} = j\omega L$ $\quad [0 < L < \infty]$

April/ May 2014: 3(b)

A $100\ \Omega$ lossless transmission line having phase velocity $u_p = 2 \times 10^8\ \mathrm{m/s}$ is terminated with an **open-circuit**. Find the shortest length of the line for its input impedance Z_{in} to be equivalent to a **20-pF capacitor** when the line is operating at a frequency of **50 MHz.**

ANS: $Z_o = 100\ \Omega,\ f = 50 \times 10^6\ \mathrm{Hz},\ f\lambda = u_p \rightarrow\ \lambda = 4\ \mathrm{m}, \beta = 1.57\ \mathrm{rad/m}$

$[\lambda = 4\ \mathrm{m}]$

$$Z_{in} = \frac{1}{j\omega} = -j159\ \Omega$$

$$Z_{oc} = \frac{Z_o}{j\tan\beta\ell} = -j159 \quad \rightarrow\ \tan\beta\,\ell = 0.629$$

$$\beta\ell_{\min} = 0.561\ \mathrm{rad} \rightarrow\ \ell_{\min} = 0.0893\lambda = 0.357\ \mathrm{m}$$

Summary

- To design the low loss inductor and capacitor using open- and short-circuited stubs.

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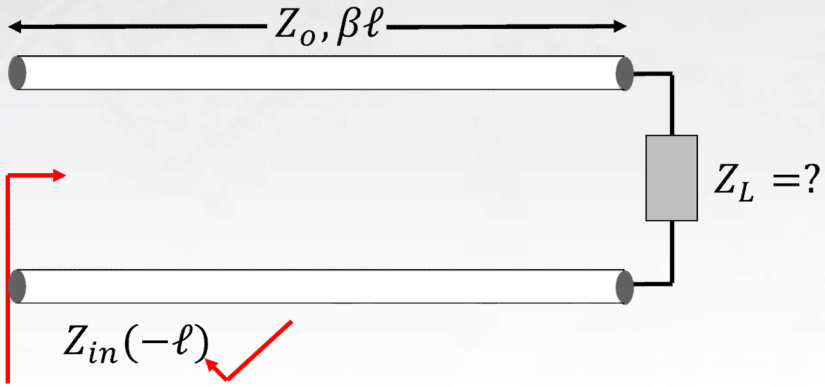
32. Determine Unknown Load Impedance and Electrical Length of a Transmission Line

Learning Objectives

- Given $Z_{in}(-\ell)$ and ℓ/λ, determine unknown load impedance Z_L; and
- Given $Z_{in}(-\ell)$ and Z_L, determine ℓ/λ of a transmission line.

Determine Unknown Load Impedance



$$Z_{in}(-\ell) = \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} Z_o \quad (18)$$

$$\Downarrow$$

$$Z_L = \frac{Z_{in}(-\ell) - jZ_o \tan(\beta\ell)}{Z_o - jZ_{in}(-\ell)\tan(\beta\ell)} Z_o \quad (18a)$$

- Determination of Z_L from $Z_{in}(-\ell)$ requires knowledge of Z_o and $\tan(\beta\ell)$ or electrical length $\frac{\ell}{\lambda}$ of the transmission line.

Determine Unknown Load Impedance Z_L

- Z_o and $\tan(\beta\ell)$ can be determined by measuring input impedance with the line terminated with short and open circuit.

$$Z_{sc} = jZ_o \tan \beta\ell \qquad (19)$$

$$Z_{oc} = \frac{Z_o}{j \tan \beta\ell} \qquad (20)$$

(19) x (20): $Z_{sc}Z_{oc} = Z_o^2$

$$Z_o = \sqrt{Z_{sc}Z_{oc}} \qquad (21)$$

$$\tan(\beta\ell) = \frac{Z_{sc}}{jZ_o} \qquad (22)$$

Determine Unknown Load Impedance Z_L

Example: Find Z_L using the following information:

$$Z_{sc} = j103\ \Omega,\ Z_{oc} = -j54.6\ \Omega \text{ and } Z_{in} = 50 + j60\ \Omega.$$

(21): $Z_o = \sqrt{Z_{sc}Z_{oc}} = 75\ \Omega$

(22): $\tan(\beta\ell) = \frac{Z_{sc}}{jZ_o} = 1.373$

$$\boxed{Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)}Z_o} \qquad (18)$$

$$\boxed{Z_L = \frac{Z_{in}(-\ell) - jZ_o\tan(\beta\ell)}{Z_o - jZ_{in}(-\ell)\tan(\beta\ell)}Z_o} = 27.5 - j8.6\ \Omega \qquad (18a)$$

Determine Electrical Length ℓ/λ or $\beta\ell$

- $Z_{in}(-\ell) = \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} Z_o$ (18)
- $(R_{in} + jX_{in})[Z_o + j(R_L + jX_L)\tan(\beta\ell)] = [R_L + jX_L + jZ_o \tan(\beta\ell)]Z_o$
- $\tan(\beta\ell) = \frac{Z_o R_{in} - Z_o R_L}{(R_{in}X_L + X_{in}R_L)}$ (18b)
- Example: Given $Z_{in} = 50 + j60\ \Omega$, $Z_L = 27.5 - j8.6\ \Omega$ and $Z_o = 75\ \Omega$ find ℓ/λ.

$$\tan(\beta\ell) = \frac{Z_o R_{in} - Z_o R_L}{(R_{in}X_L + X_{in}R_L)} = 1.38$$

$$\beta\ell_{min} = 0.945 \rightarrow \ell_{min} = 0.15\lambda$$

Summary

- Given $Z_{in}(-\ell)$ and ℓ/λ, determine unknown load impedance Z_L; and
- Given $Z_{in}(-\ell)$ and Z_L, determine ℓ/λ of a transmission line.

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33. Recap of Transmission Line Equations

Transmission Line Equations

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Transmission Line Equations:

(1) $V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$ $= V_o^+ e^{-j\beta z}[1 + \Gamma_L e^{+j2\beta z}]$ (3)

(2) $I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{+j\beta z}$ $= \frac{V_o^+}{Z_o} e^{-j\beta z}\ [1 - \Gamma_L e^{+j2}\ \ ]$ (4)

(5) $\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$ $= |\Gamma_L| \angle \theta_o$

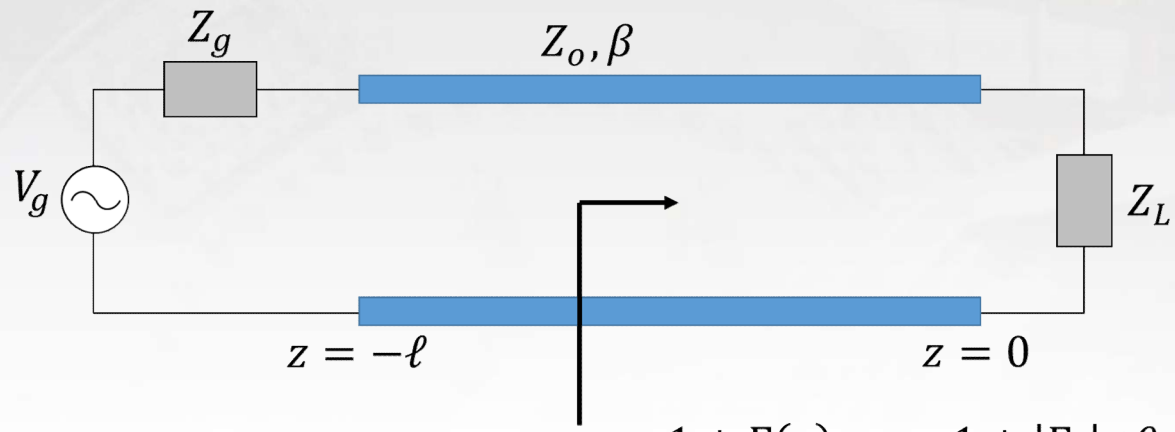
(11) $\Gamma(z) = \frac{V_o^- e^{+j\beta z}}{V_o^+ e^{-j\beta z}} = \Gamma_L e^{+j2\beta z}$ $= |\Gamma_L| \angle \theta_\Gamma$

$\theta_\Gamma = \theta_o + 2\beta z \quad -\ell \le z \le 0$

(18) $Z_{in}(-\ell) = \frac{Z_L + jZ_o \tan \beta\ell}{Z_o + jZ_L \tan \beta\ell} Z_o$ $\Longrightarrow$ $Z_L = \frac{Z_{in}(-\ell) - jZ_o \tan(\beta\ell)}{Z_o - jZ_{in}(-\ell)\tan(\beta\ell)} Z_o$ (18a)

$\tan(\beta\ell) = \frac{Z_o R_{in} - Z_o R_L}{(R_{in} X_L + X_{in} R_L)}$ (18b)

Transmission Line Equations



$$Z_{in}(z) = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = Z_o \frac{1 + |\Gamma_L| \angle \theta_\Gamma}{1 - |\Gamma_L| \angle \theta_\Gamma} \qquad (6) \equiv (18)$$

$@z_{\max}, \theta_\Gamma = 0$: $\quad Z_{in}(z_{\max}) = Z_o \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = R_{\max} + j0$

$@z_{\min}, \theta_\Gamma = -\pi$: $\quad Z_{in}(z_{\min}) = Z_o \frac{1-|\Gamma_L|}{1+|\Gamma_L|} = R_{\min} + j0$

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This session is about

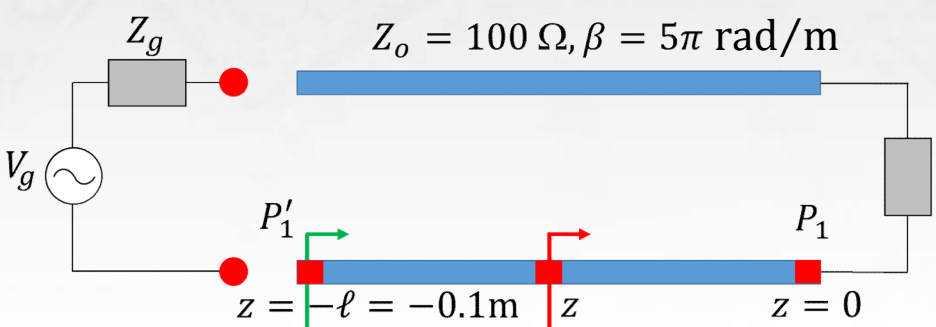
34. Introduction to Smith Chart

Learning Objectives

- Identify the $\hat{Z}_{in}(z)$ –plane (constant–r and constant–x circle) and the Γ –plane on the Smith Chart;
- Given any point on the Smith Chart, obtain values of normalised input impedance $\hat{Z}_{in}(z) = r + jx$ and the reflection coefficient $\Gamma(z) = |\Gamma|\angle\theta_\Gamma$; and
- Given $\hat{Z}_{in}(z)$ or $\Gamma(z)$, plot it on the Smith Chart.

Basis of Smith Chart

Basis for the Smith Chart:



$$f = 750\text{ MHz}, u_p = 3 \times 10^8\text{ m/s}$$

$$\lambda = 0.4\text{m}, \ell = 0.1\text{m} = 0.25\lambda$$

$$Z_L = Z_{in}(0) = 38 - j36\,\Omega$$

$$\Gamma_L = \Gamma(0) = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.5\angle -135^\circ$$

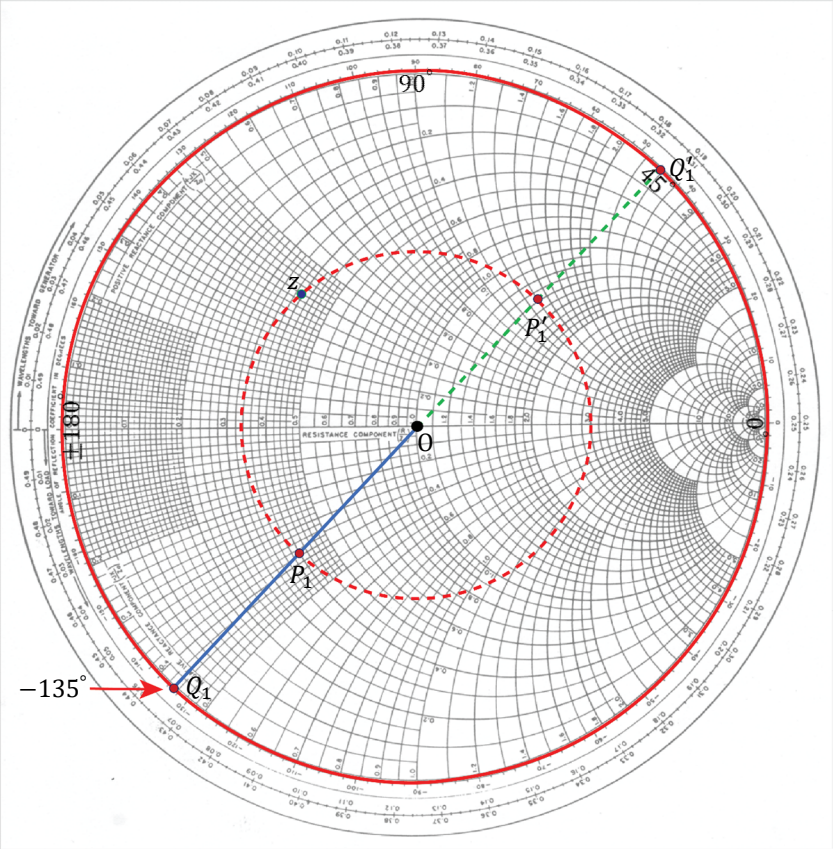
$$Z_{in}(z) = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \qquad (6)$$

$$\Gamma(z) = |\Gamma_L|\angle(\theta_o + 2\beta z) \qquad (11)$$

$$(11) \quad \Gamma(-\ell) = |\Gamma_L|\angle(\theta_o + 2\beta z) = 0.5\angle -315^\circ = 0.5\angle 45^\circ$$

$$(6) \quad Z_{in}(-\ell) = Z_o \frac{1+\Gamma(-\ell)}{1-\Gamma(-\ell)} = 140 + j130\,\Omega$$

Basis of Smith Chart



1 circle $\equiv 0.5\lambda$

$P_1P_1' \equiv 0.25\lambda$

$$|\Gamma| = \frac{OP_1}{OQ_1} = \frac{OP_1'}{OQ_1'}$$

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34. Introduction to Smith Chart (2)

Basis of Smith Chart

(6): $Z_{in}(z) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)} \rightleftarrows \Gamma(z) = \frac{Z_{in}(z)-Z_o}{Z_{in}(z)+Z_o}$ $\qquad -\ell \leq z \leq 0$

- Given $\Gamma(z)$ at any position z, the corresponding $Z_{in}(z)$ can be determined using (6) and vice versa:
- $\Gamma(0) = 0.5\angle -135° \rightleftharpoons Z_{in}(0) = 38 - j36\ \Omega = (0.38 - j0.36)Z_o$ $\qquad [\mathrm{P}_1]$
- $\Gamma(-\ell) = 0.5\angle 45° \rightleftharpoons Z_{in}(-\ell) = 140 + j130\ \Omega = (1.4 + j1.3)Z_o$ $\qquad [\mathrm{P}_1']$
- Instead of calculating $\Gamma(z) \rightleftharpoons Z_{in}(z)$ using (6), Smith Chart solves this equation graphically. $\qquad [Z_o = 100\ \Omega]$

Basis of Smith Chart

- To provide a **universal chart** independent of Z_o, Smith Chart uses **normalised input impedance:**

$$\boxed{\hat{Z}_{in}(z) = \frac{Z_{in}(z)}{Z_o}} = \frac{1+\Gamma(z)}{1-\Gamma(z)} = r + jx \qquad \text{(6a)}$$

$$\boxed{\begin{array}{c} 0 \le r \le \infty \\ -\infty < x < \infty \end{array}} \quad \hat{Z}_{in}(z)\text{–plane}$$

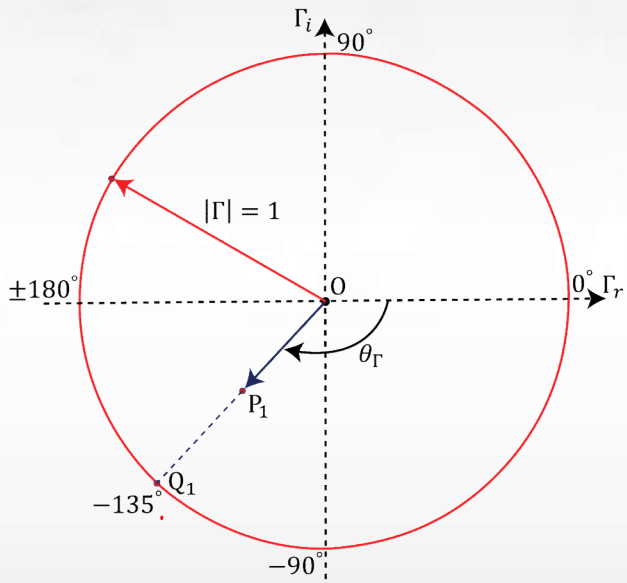
$$\boxed{\begin{array}{c} |\Gamma(z)| \le 1 \\ -180^\circ \le \theta_\Gamma \le 180^\circ \end{array}} \quad \Gamma\text{–plane}$$

- Given $\Gamma(z)$, the corresponding $\hat{Z}_{in}(z)$ can be calculated using (6a) and vice versa:
- $\Gamma(z) = 0.5\angle -135^\circ \quad \rightleftharpoons \hat{Z}_{in}(z) = (0.38 - j0.36)$
- $\Gamma(z) = 0.5\angle 45^\circ \rightleftharpoons \hat{Z}_{in}(z) = (1.4 + j1.3)$

$$|\Gamma|\angle\theta_\Gamma \rightleftharpoons r + jx; \qquad Z_{in}(z) = \hat{Z}_{in}(z)\, Z_o$$

Complex Γ –Plane

- $|\Gamma(z)| \leq 1$: All possible values of $\Gamma(z)$ lie within a **Unit Circle** on the complex $\Gamma -$ plane [Polar Plot: $\Gamma(z) = |\Gamma|\angle\theta_\Gamma$]. The **radius** of the unit circle: $R \equiv |\Gamma| = 1$.

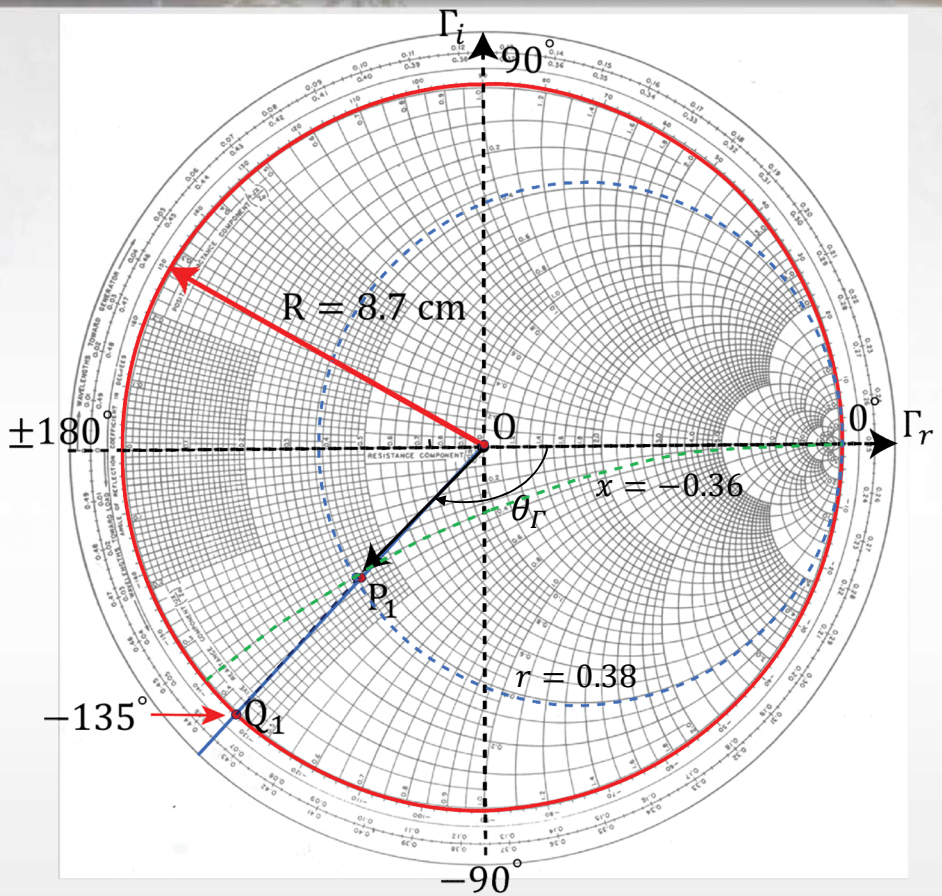


- $|\Gamma| = \dfrac{OP_1}{OQ_1} = 0.5$

  $\theta_\Gamma = -135°$

  $\Gamma(P_1) = 0.5\angle -135^\circ$

Complex Γ –Plane



- $OQ_1 = 8.7 \text{ cm} = R \equiv |\Gamma| = 1$
- $OP_1 = 4.35 \text{ cm}$

$$|\Gamma| = \frac{OP_1}{OQ_1} = 0.5$$

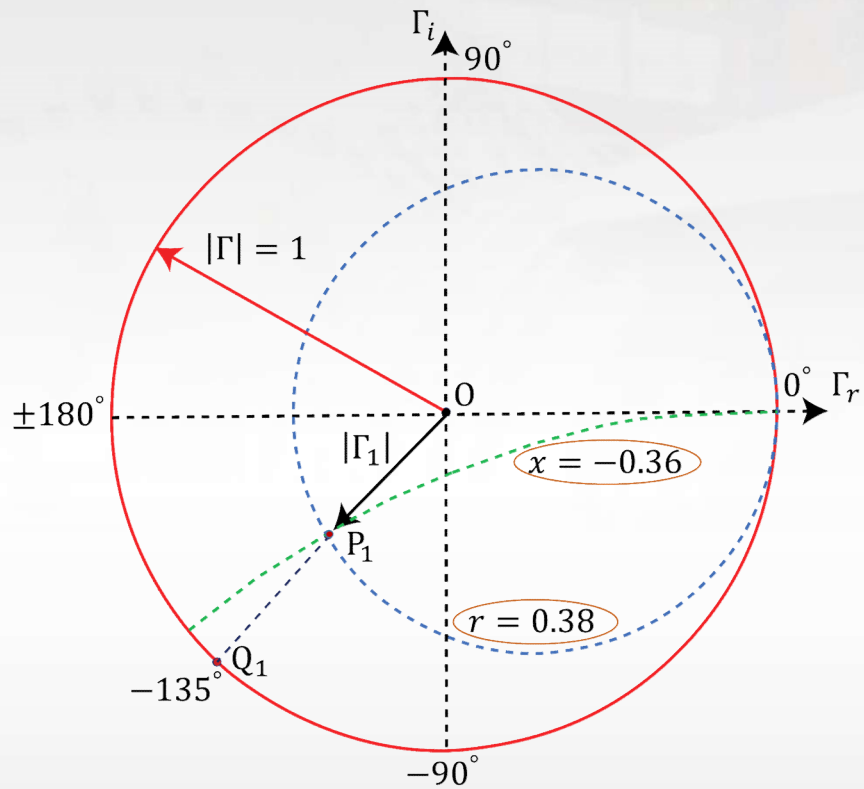
$$\theta_\Gamma = -135°$$

$$\Gamma(P_1) = 0.5\angle -135^\circ$$

- $\hat{Z}_{in}(P_1) = \frac{1+\Gamma(P_1)}{1-\Gamma(P_1)} = 0.38 - j0.36$

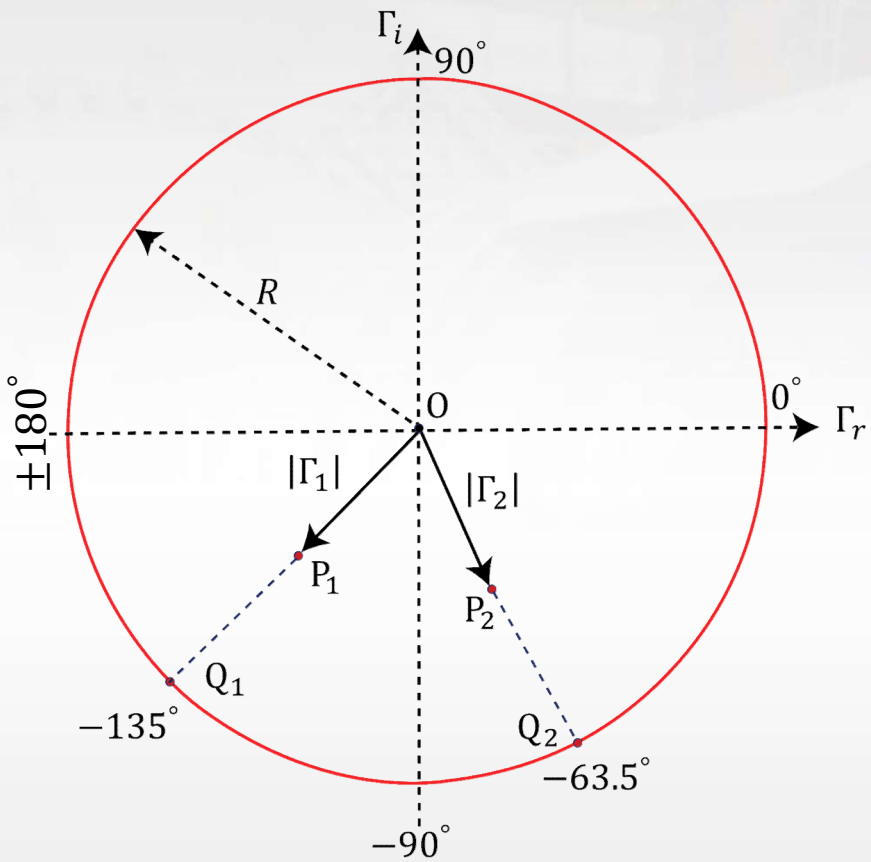
$$Z_{in} = \hat{Z}_{in}(z)\, Z_o$$

Complex Γ –Plane



$$\Gamma(z) = 0.5\angle -135^\circ \;\rightleftharpoons\; \hat{Z}_{in}(z) = (0.38 - j0.36)$$

Complex Γ –Plane



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34. Introduction to Smith Chart (3)

Complex Γ –Plane

- $|\Gamma(z)| \leq 1$: All possible values of $\Gamma(z)$ lie within a **unit circle on** complex Γ –plane

  $\Gamma(z) = |\Gamma|\angle\theta_\Gamma$.

- The **radius** of the unit circle: $R \equiv |\Gamma| = 1$.

- The scale closest to the $|\Gamma| = 1$ circle is θ_Γ.

  $\theta_\Gamma = 0°$ is on the right and $\theta_\Gamma = \pm 180°$ is on the left.

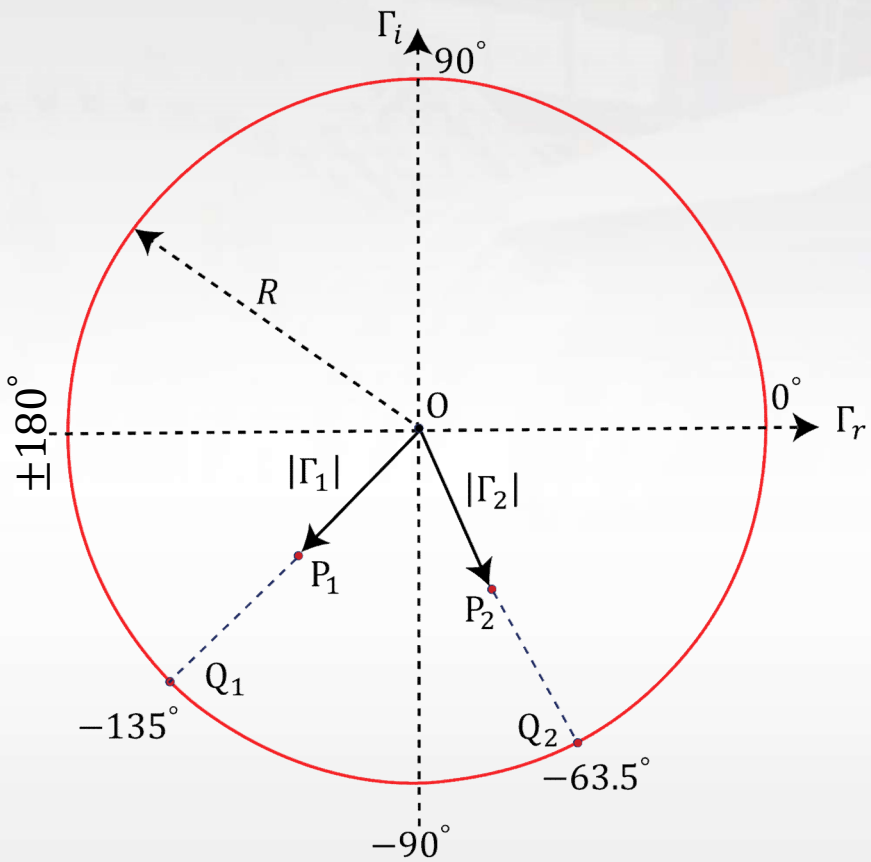
  $\theta_\Gamma > 0$ on the upper half and $\theta_\Gamma < 0$ on the lower half.

- Given any point e.g. P_1 on the Smith Chart,

  $$|\Gamma_1| = \frac{\mathrm{OP}_1}{R} = \frac{\mathrm{OP}_1}{\mathrm{OQ}_1} = 0.5 \;\&\; \theta_{\Gamma_1} = -135° \quad [R = 8.7\mathrm{cm}, \mathrm{OP}_1 = 4.35\mathrm{cm}]$$

  $\Gamma_1 = 0.5\angle -135°$

Complex Γ –Plane



$\hat{Z}_{in}(z)$–Plane

- Given $\Gamma_2(z) = 0.45\angle -63.5^\circ$, P_2 can be plotted onto the Smith Chart

$\theta_{\Gamma_2} = -63.5^\circ \rightarrow \mathrm{P}_2$ along OQ_2

$|\Gamma_2| = 0.45 \quad \rightarrow \mathrm{OP}_2 = 0.45 \times R$ $\qquad [R = 8.7\mathrm{cm} \rightarrow \mathrm{OP}_2 = 3.92\mathrm{cm}]$

- Given any point on the Smith Chart e.g. P_2, instead of calculate $\hat{Z}_{in}$ from (6a), we would like to know $\hat{Z}_{in}$ @ P_2 directly from the chart.

In other words, we would like to plot all possible $\hat{Z}_{in}(z) = r + jx\ [0 \le r \le \infty; -\infty \le x \le \infty]$ onto the chart such that given any point on the Chart e.g. P_2, $\hat{Z}_{in}\ (\mathrm{P}_2) = r + jx$ can be obtained directly from the chart.

Constant$-r$ and Constant$-x$ Circle

- Equating the real & imaginary parts of (6a) gives

$$\hat{Z}_{in}(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)} = r + jx \qquad (6a) \qquad [\Gamma(z) = \Gamma_r + j\Gamma_i]$$

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + {\Gamma_i}^2 = \left(\frac{1}{1+r}\right)^2 \qquad (5.1)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (5.2)$$

- Constant$-r$ circle: All possible values of $0 \leq r < \infty$ can be plotted on the Smith Chart using (5.1).
- Constant$-x$ circle: Can be used to plot all possible $-\infty < x < \infty$ on the Smith Chart using (5.2).

Constant–r Circle

- NOTE: The center-radius form of a circle equation is in the format:

$$(x - x_o)^2 + (y - y_o)^2 = r_o^2$$

Center at point (x_o, y_o) and the radius= r_o

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \qquad (5.1)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (5.2)$$

(5.1)
- $r = 0$: $(\Gamma_r - 0)^2 + (\Gamma_i - 0)^2 = 1.0^2$ center at (0,0), $r_o = 1.0$
- $r = 1$: $(\Gamma_r - 0.5)^2 + (\Gamma_i - 0)^2 = 0.5^2$ center at (0.5,0), $r_o = 0.5$

(5.2)
- $x = 1$: $(\Gamma_r - 1)^2 + (\Gamma_i - 1)^2 = 1.0^2$ center at (1,1), $r_o = 1.0$
- $x = -1$: $(\Gamma_r - 1)^2 + (\Gamma_i + 1)^2 = 1.0^2$ center at $(1, -1)$, $r_o = 1.0$

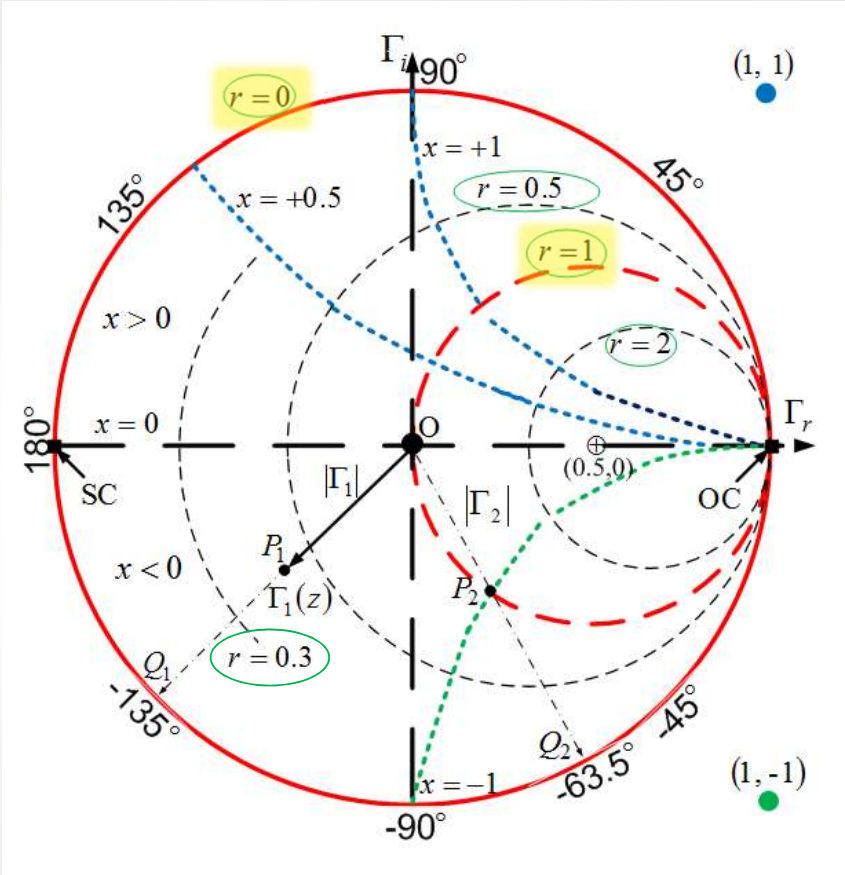
Constant–r Circle

$$@P_2: r = 1$$
$$x = -1$$
$$\hat{Z}_{in} = 1 - j1$$

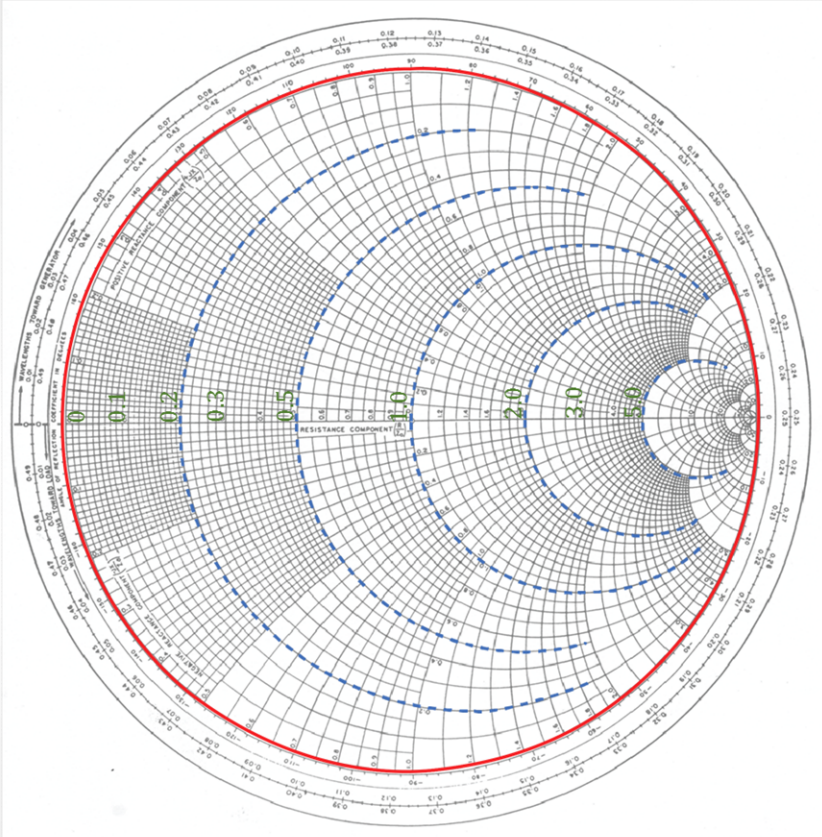
$$Z_{in} = \hat{Z}_{in} Z_o$$

$$@P_2: |\Gamma_2| = \frac{OP_2}{OQ_2}$$

$$\theta_\Gamma = -63.5^\circ$$



Constant–r Circle



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EE3001 Engineering Electromagnetics

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34. Introduction to Smith Chart (4)

Constant–x Circle

- NOTE: The center-radius form of a circle equation is in the format:

$(x - x_o)^2 + (y - y_o)^2 = r_o^2$ Center at point (x_o, y_o) and the radius= r_o

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \qquad (5.1)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (5.2)$$

(5.1)
$$\begin{cases} r = 0: & (\Gamma_r - 0)^2 + (\Gamma_i - 0)^2 = 1.0^2 \quad \text{center at } (0,0), \quad r_o = 1.0 \\ r = 1: & (\Gamma_r - 0.5)^2 + (\Gamma_i - 0)^2 = 0.5^2 \quad \text{center at } (0.5,0), \quad r_o = 0.5 \end{cases}$$

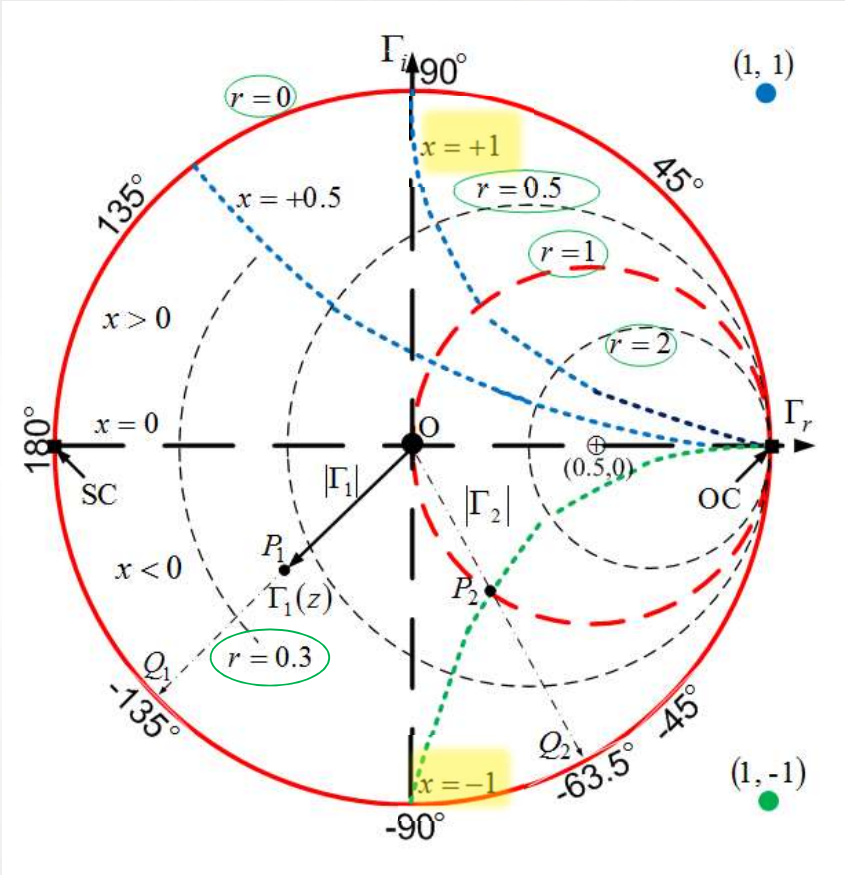
(5.2)
$$\begin{cases} x = 1: & (\Gamma_r - 1)^2 + (\Gamma_i - 1)^2 = 1.0^2 \quad \text{center at } (1,1), \quad r_o = 1.0 \\ x = -1: & (\Gamma_r - 1)^2 + (\Gamma_i + 1)^2 = 1.0^2 \quad \text{center at } (1,-1), \quad r_o = 1.0 \end{cases}$$

Constant–x Circle

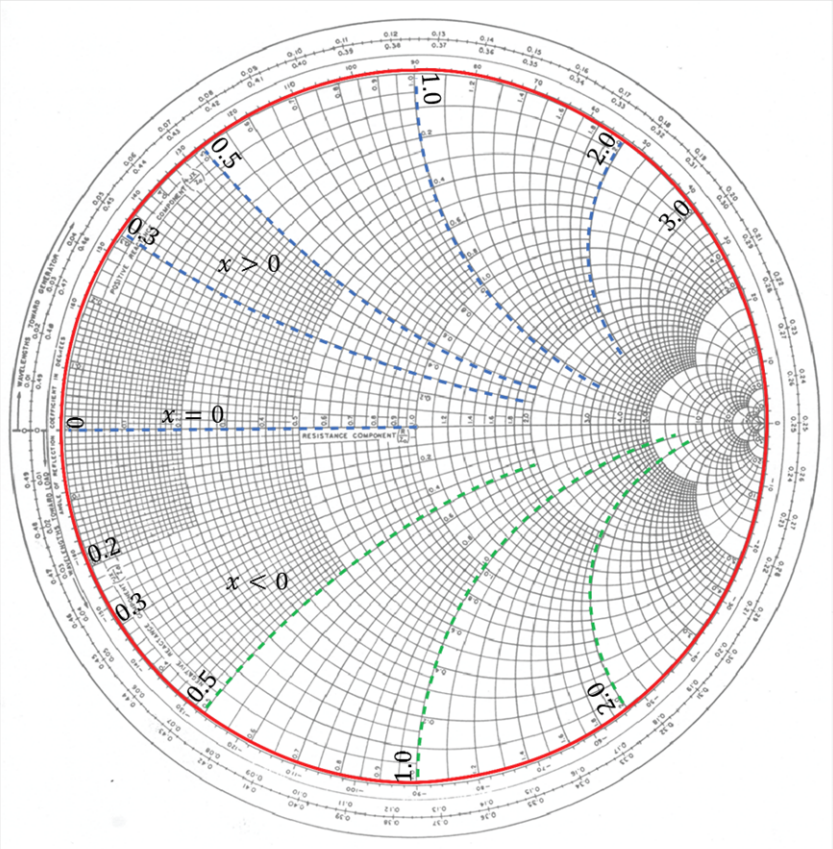
$$@P_2: r = 1$$
$$x = -1$$
$$\hat{Z}_{in} = 1 - j1$$

$$Z_{in} = \hat{Z}_{in} Z_o$$

$$@P_2: |\Gamma_2| = \frac{OP_2}{OQ_2}$$
$$\theta_\Gamma = -63.5^\circ$$

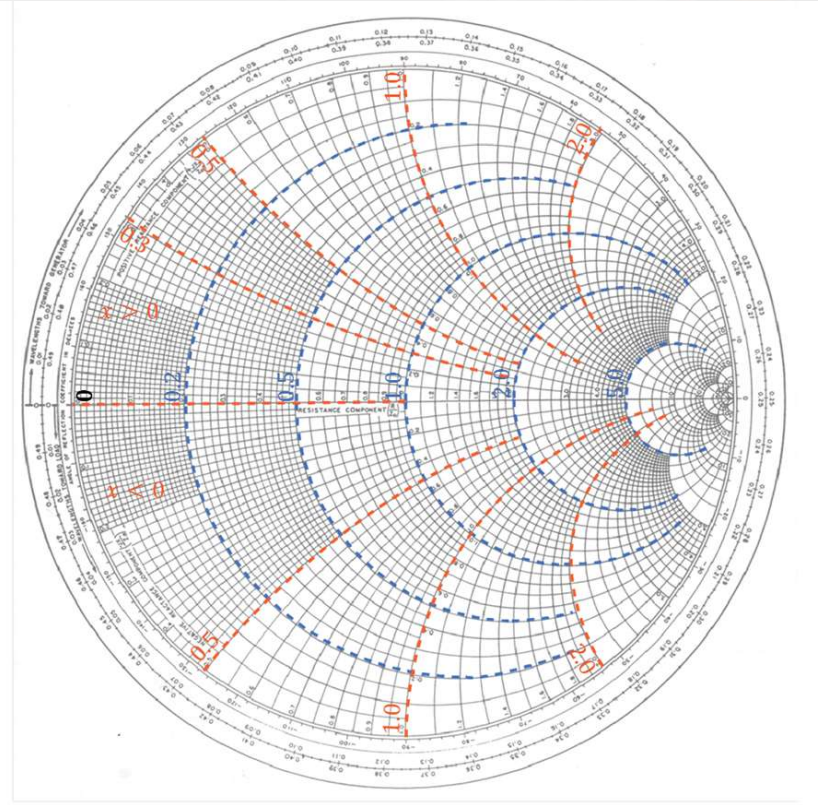


Constant–x Circle



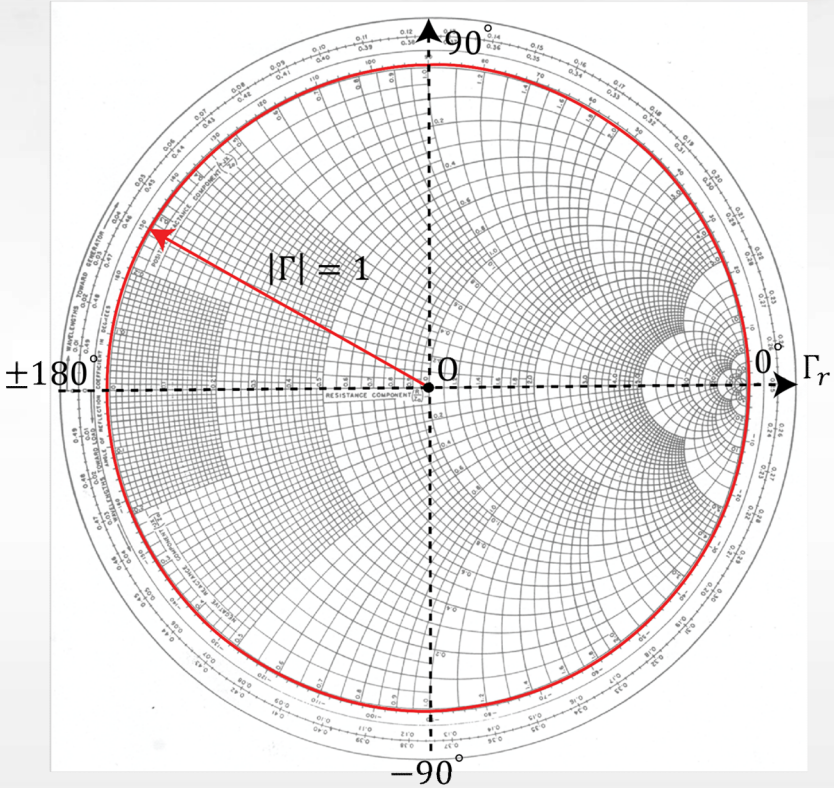
$\hat{Z}_{in}(z)$–Plane

$$\hat{Z}_{in}(z) = r + jx$$



Γ –Plane

$$\Gamma(z) = |\Gamma|\angle\theta_\Gamma$$

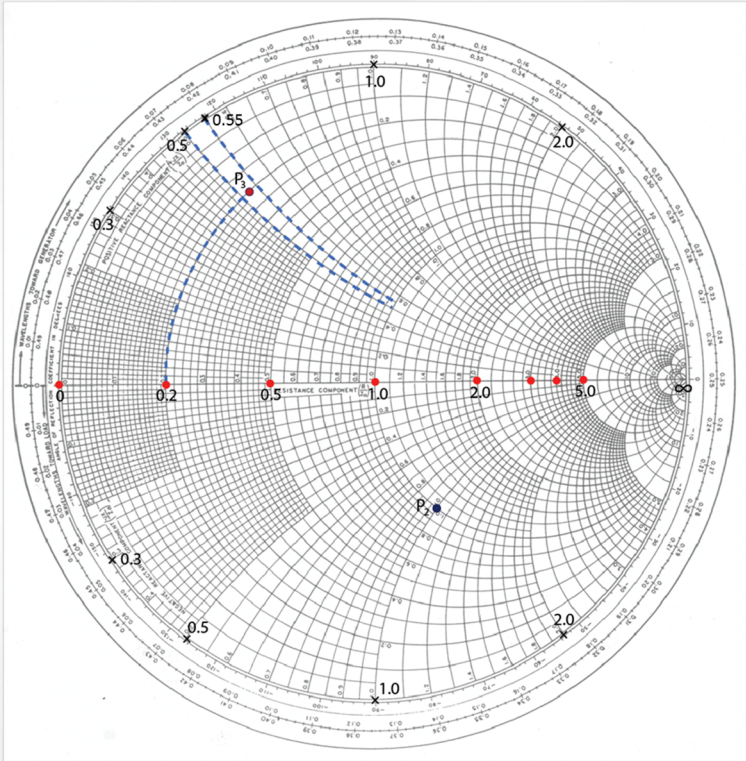


Example

Given Point P_3 on Smith Chart, find $\hat{Z}_{in}(z)$ and $\Gamma(z)$.

$\hat{Z}_{in}(z)$−plane

$$@\mathrm{P}_3: r = 0.2$$
$$x = 0.525$$
$$\hat{Z}_{in}(z) = 0.2 + j0.525$$



Example

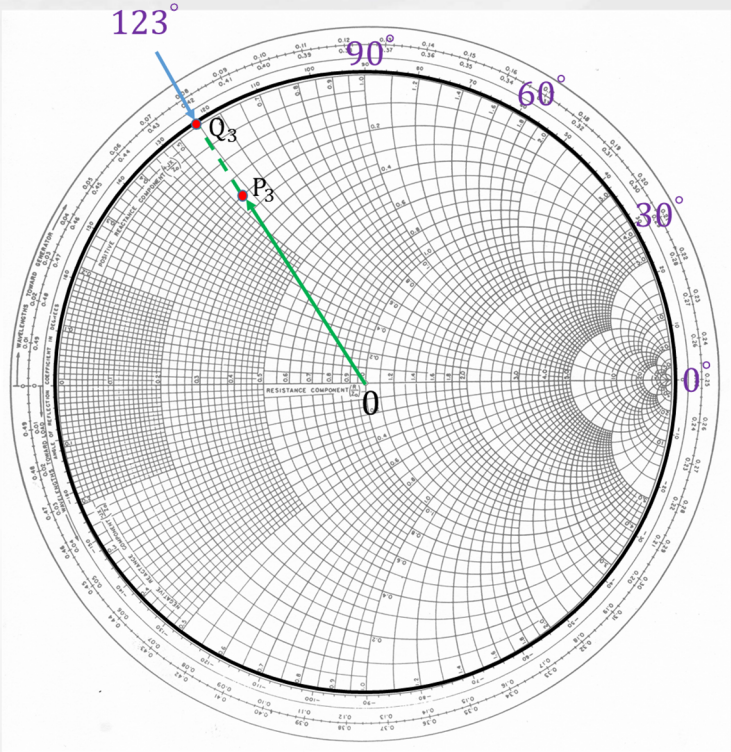
Given Point P_3 on Smith Chart, find $\hat{Z}_{in}(z)$ and $\Gamma(z)$.

Γ –plane

$$@P_3: |\Gamma_3| = \frac{OP_3}{OQ_3} = 0.73$$

$$\theta_\Gamma = 123^\circ$$

$$\Gamma(z) = 0.73\angle 123^\circ$$



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34. Introduction to Smith Chart (5)

Summary

- Given any point on the Smith Chart, $\hat{Z}_{in}(z) = r + jx$ can be obtained and vice versa.

  Eg. @ $\mathrm{P}_2 : r = 1$ and $x = -1$

  $\hat{Z}_{in}(z) = 1 - j1 \quad \leftrightarrows \quad \mathrm{P}_2$ on Smith Chart $\leftrightarrows \ \Gamma_2 = 0.45\angle -63.5^{\circ}$

  $\hat{Z}_{in}(z) = (1 - j1)Z_o$

- Imaginary impedance, $r = 0 : \hat{Z}_{in}(z) = 0 + jx$, map onto the unit circle. Imaginary impedance $\leftrightarrow |\Gamma| = 1$.

- Inductive impedances $\hat{Z}_{in} = r + jx \ [x > 0]$ map onto the upper half of the chart, while

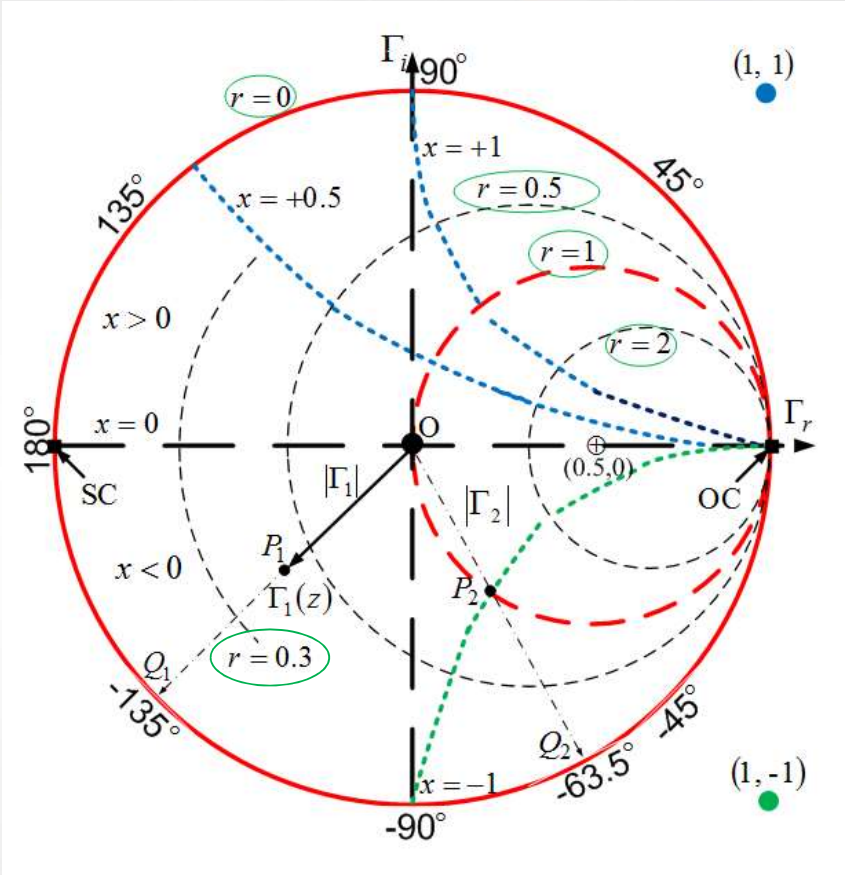
- Capacitive impedances $[x < 0]$ appear in the lower half.

Summary

$$@P_2: r = 1$$
$$x = -1$$
$$\hat{Z}_{in} = 1 - j1$$

$$Z_{in} = \hat{Z}_{in} Z_o$$

$$@P_2: |\Gamma_2| = \frac{OP_2}{OQ_2}$$
$$\theta_\Gamma = -63.5^\circ$$



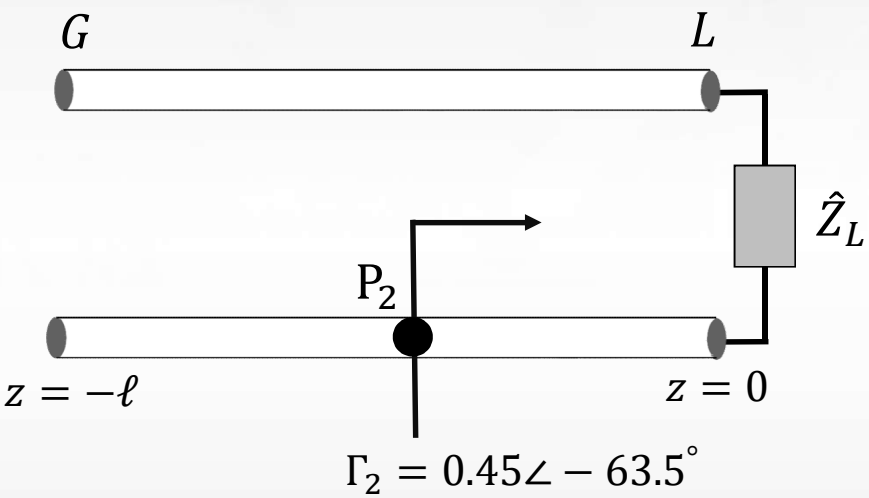
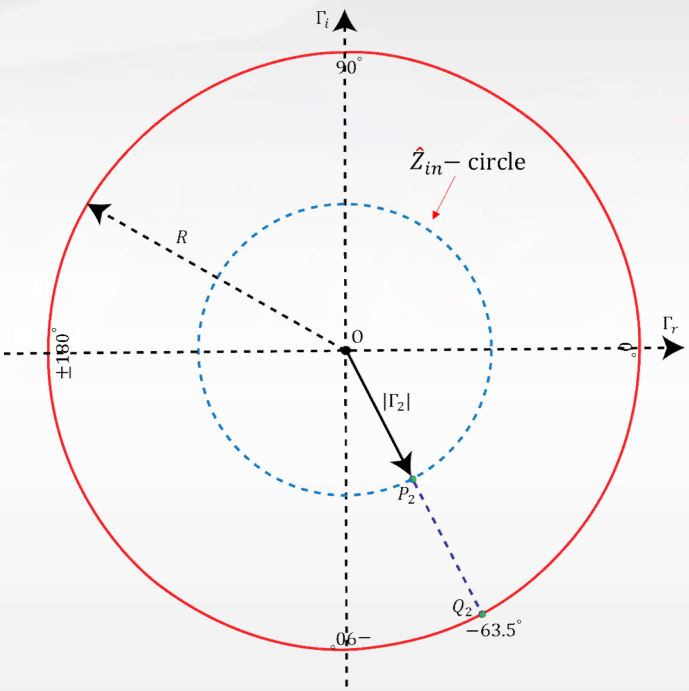
Summary

- Real impedance. $x = 0$: $\hat{Z}_{in} = r + j0$ map onto a horizontal line that divides the **top (Inductive)** from the **bottom (Capacitive)** of the unit circle.

- Matched Load $(Z_{in}(0) = Z_o)$, Short- and Open- Circuit

$\hat{Z}_{in} = 1 \leftrightarrow \Gamma(z) = 0$:	at the center $[r = 1, x = 0]$
$\hat{Z}_{in} = 0 \leftrightarrow \Gamma(z) = 1\angle 180^{\circ}$:	extreme left $[r = 0, x = 0]$
$\hat{Z}_{in} = \infty \leftrightarrow \Gamma(z) = 1\angle 0^{\circ}$:	extreme right $[r = \infty]$

- Once we know the value of $\Gamma(z) = |\Gamma|\angle\theta_{\Gamma}$ at any position z e.g. $\mathrm{P}_2 \rightleftarrows \Gamma_2 = 0.45\angle -63.5^{\circ}$, we can infer all possible values of $\Gamma(z)$ and the corresponding $\hat{Z}_{in}(z)$ along the line $[-\ell \leq z \leq 0]$.

Summary

This is because $|\Gamma(z)| = |\Gamma| \equiv$ constant along the line, the locus is a **<u>circle centered at the origin</u>** --- referred to as " $\hat{Z}_{in}$-circle".