NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2022 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) A finite-length line carrying a uniform charge density ρ_l is situated in free space along the z axis from $z = 0$ to $z = L$.

    (i) Determine the potential V at the point $(r, \phi, 0)$ in cylindrical coordinate system.

    (ii) Based on the potential in (i), determine the radial component of the electric field intensity $\vec{E}$ at the point $(r, \phi, 0)$.

    (iii) Discuss whether the z-component of $\vec{E}$ is zero or non-zero at the point $(r, \phi, 0)$, and whether it can or cannot be determined from your answer in (i) directly.

Note: $\int \frac{1}{(x^2+u^2)^{1/2}} dx = \ln\left(x + \sqrt{x^2 + u^2}\right)$

(15 Marks)

Note: Question No. 1 continues on page 2.

(b) Assume that the line in part (a) extends from $z = 0$ to $z = \infty$ and carries a direct current I. Determine the magnetic field intensity $\vec{H}$ at the point $(r, \phi, 0)$ due to the line current.

Note: $\int \frac{1}{(x^2+u^2)^{3/2}} dx = \frac{x}{u^2\sqrt{x^2+u^2}}$

(10 Marks)

2. (a) An AC voltage generator comprises a square loop of area a^2 with N turns rotating around the $+x$ axis in free space. The angular velocity of the rotation is ω rad/s, and at time $t = 0$, the loop lies in the xy-plane with its center position at the origin. The loop is subjected to a uniform magnetic flux density $\vec{B} = B_0\, \vec{a}_z$.

(i) Give the expressions for the unit vector normal to the loop surface $\vec{a}_n$, the magnetic flux Φ_m passing through the loop and the induced voltage V_{emf} at time $t \geq 0$.

(ii) Assume that $a^2 = 100 \text{ cm}^2$, $N = 200$, $B_0 = 1.2$ T, and the rotation is 1000 RPM (revolutions per minute). Calculate the angular velocity ω and the peak amplitude of the AC voltage generated.

(13 Marks)

(b) A microwave oven operating at 2.45 GHz has a wall of conductivity $\sigma = 1.2 \times 10^6$ S/m, relative permittivity $\varepsilon_r = 1$, and relative permeability $\mu_r = 500$.

(i) Justify if the wall medium is a good conductor at the operating frequency.

(ii) Calculate the complex intrinsic impedance η_c, the attenuation constant α, the phase constant β, and the skin depth δ of the wall medium at the operating frequency.

(12 Marks)

3. (a) The total magnetic field in air can be described by the following expression that consists of incident and reflected waves as

$$\vec{H} = \left(0.3\angle 45°\vec{a}_x + 0.3\angle - 45°\vec{a}_y\right)e^{-j0.4z}$$
$$+(0.2\angle 45°\,\vec{a}_x + 0.2\angle - 45°\,\vec{a}_y)e^{+j0.4z}\quad \text{A/m}$$

(i) Derive the corresponding total electric field and write down separately the expressions of incident and reflected waves, i.e., $\vec{E}_i$ and $\vec{E}_r$.

(ii) Determine the polarization type (linear, circular or elliptical) for each of the incident and reflected waves.

(iii) Find the time-average Poynting vectors for the incident and reflected waves, i.e., $\vec{S}_i$ and $\vec{S}_r$.

(14 Marks)

(b) A uniform plane wave in a lossless medium ($z \leq 0$) with $\varepsilon_r = 2.3$ and $\mu_r = 1$ is obliquely incident on an air medium ($z \geq 0$).

(i) Determine the Brewster angle of incidence $\theta_i = \theta_B$ and the corresponding angle of transmission θ_t. Give your angles in degree. Calculate the reflection coefficient $\Gamma_{\perp}$.

(ii) Determine the critical angle for $\mu_r = 1$.

(iii) If the medium relative permeability is changed from $\mu_r = 1$ to $\mu_r = 10$, what would be the critical angle?

(11 Marks)

4. (a) A 50-Ω transmission line of length $l = 0.36\lambda$ (with λ being the wavelength) and propagation constant $\beta = 5$ rad/m has an input impedance $Z_{in} = 12 + j8\ \Omega$ and a voltage $V_{in} = 38\angle 20°$ V at $z = -l$. The line is terminated with an unknown load at $z = 0$.

(i) Find the input reflection coefficient Γ_{in} at $z = -l$.

(ii) Determine the load reflection coefficient Γ_L and load impedance Z_L at $z = 0$.

(iii) Find the first positions (from the load end) of maximum and minimum voltage magnitudes on the line, i.e., z_{max} and z_{min}.

(iv) Determine the maximum and minimum voltage magnitudes as well as the corresponding input impedances at the positions in (iii).

(20 Marks)

(b) Assume that the source end $z = -l$ in part (a) is connected to a power supply with an open-circuit voltage $V_g = 100\angle 0°$ V and an internal impedance $Z_g = 25\ \Omega$.

(i) Find the incident voltage wave V_o^+ at $z = 0$.

(ii) Find the reflected voltage wave V_o^- at $z = 0$.

(5 Marks)

Note: The Smith chart may be used in the solutions for this question. Please put the Smith chart inside (not outside) the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \dfrac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E}=\frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2}dv=\frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3}dv$$

$$V=\frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R}dv$$

$$\vec{H}=\frac{1}{4\pi}\int_C \frac{I\vec{dl}\times\vec{a}_R}{R^2}=\frac{1}{4\pi}\int_C \frac{I\vec{dl}\times\vec{R}}{R^3}$$

$$\oint_C \vec{H}\cdot\vec{dl}=I=\iint_S \vec{J}\cdot\vec{ds}$$

$$emf=\oint_C \tilde{E}\cdot\vec{dl}=-\frac{d\Phi_m}{dt}=-\frac{d}{dt}\iint_S \tilde{B}\cdot\vec{ds}$$

Maxwell's Equations

$$\nabla\times\tilde{E}=-\frac{\partial\tilde{B}}{\partial t}$$

$$\nabla\times\tilde{H}=\frac{\partial\tilde{D}}{\partial t}+\tilde{J}$$

$$\nabla\cdot\tilde{D}=\rho$$

$$\nabla\cdot\tilde{B}=0$$

Complex Propagation Constant

$$\gamma=\alpha+j\beta=j\omega\sqrt{\mu\varepsilon_c}=j\omega\sqrt{\mu(\varepsilon-j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c=\sqrt{\frac{\mu}{\varepsilon_c}}=\sqrt{\frac{\mu}{\varepsilon-j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\| = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\| = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \; e^{-j\beta z} + V_o^- \; e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \; e^{-j\beta z} - V_o^- \; e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

EE3001 ENGINEERING ELECTROMAGNETICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.