NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2021 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) A line charge of uniform charge density ρ_l is located in free space along the z axis from $z = a$ to $z = b > a$.

   (i) Using Coulomb's law, determine the electric field intensity $\vec{E}$ at the point $(x, y, 0)$ due to the line charge.

   (ii) Simplify your expression of $\vec{E}$ above for the case of $a = -\infty$ and $b = \infty$. Give your answers for both Cartesian and cylindrical coordinate systems.

   Note: $$\int \frac{1}{\left(x^2 + u^2\right)^{3/2}} dx = \frac{x}{u^2\sqrt{x^2 + u^2}}$$

(15 Marks)

Note: Question No. 1 continues on page 2.

(b) Assume that the charges in part (a)(ii) are moving in the $+z$ direction to form an infinitely long direct current I. Determine the magnetic field intensity $\vec{H}$ at the point $(x, y, 0)$ due to the line current. Give your answers for both Cartesian and cylindrical coordinate systems.

(10 Marks)

2. (a) Consider a square loop of area a^2 in the xy-plane in free space. At time $t = 0$, the loop has its center position at the origin, and is moving at constant velocity v along the $+x$ axis. The loop region is subjected to a spatially uniform but time-varying magnetic flux density of the form (for time $t \geq 0$)

$$\vec{B} = (C_1 t^2 + C_2 t)\, \vec{a}_z \ \text{T},$$

where C_1 and C_2 are arbitrary constants.

(i) Find the magnetic flux Φ_m passing through the loop and the induced voltage V_{emf} at time $t \geq 0$. State any assumption made.

(ii) Assume that the loop has a uniform per-unit-length resistance of R_l (in Ω/m), determine the induced current I_{ind} at time $t \geq 0$.

(11 Marks)

(b) A lossy medium is characterized by dielectric constant $\varepsilon_r = 10$, loss tangent $\tan\delta = 8$, and relative permeability $\mu_r = 1$ at 500 MHz.

(i) Comment whether the medium is a good conductor. Find the conductivity of the medium.

(ii) Assume that a 500 MHz plane wave is propagating along $+z$ direction in the medium, calculate the complex intrinsic impedance η_c, the attenuation constant α and the phase constant β.

(14 Marks)

3. (a) The magnetic field of a uniform plane wave (UPW) travelling in a lossless non-magnetic medium occupying the region $z \leq 0$ is given as

$$\tilde{H}_i(z,t) = -\vec{a}_y 50 \cos\left(6\times10^9 t - 24.5z + 40°\right) \text{ mA/m.}$$

The UPW is incident normally on a plane interface at $z = 0$ with a lossy medium having complex intrinsic impedance $\eta_c = 10\angle 45°\ \Omega$ and occupying the region $z \geq 0$.

Determine the following and state any assumption(s) made:

(i) The phase velocity u_p of the UPW in the lossless medium.

(ii) The permittivity of the lossless medium.

(iii) The time-domain expression of the incident electric field $\tilde{E}_i(z,t)$.

(iv) The percentage of average incident power reflected at the planar interface at $z = 0$.

(12 Marks)

(b) A uniform plane wave (UPW) in free space occupying the region $z \leq 0$ is incident at a plane interface with a lossless dielectric medium having $\mu_r = 1$ and $\varepsilon_r = 2.25$, occupying the region $z \geq 0$. The incident electric field of the UPW is given by

$$\vec{E}_i(x,z) = (20\vec{a}_x - 40\vec{a}_z)\, e^{-j(8x+4z)} \text{ V/m.}$$

Find the following:

(i) The angle of incidence θ_i and the angle of transmission θ_t. Give both angles in degrees.

(ii) The amplitude of the transmitted electric field at $z = 0$, i.e., E_{ot}.

(iii) The time-average power transmitted through a 2-m^2 area at $z = 0$.

(13 Marks)

4. (a) A generator having an open-circuit voltage $V_g(t) = 96\cos(2.5\pi \times 10^8 t)$ V and an internal impedance $Z_g = 100\ \Omega$ is connected to a $Z_o = 100\ \Omega$ lossless air-filled transmission line of length $\ell = 0.64$ m. The phase velocity on the line is $u_p = 3 \times 10^8$ m/s and the line is terminated in a complex load $Z_L = 140 - j64\ \Omega$. Assuming that the load end is at $z = 0$ and the source end is at $z = -\ell$, find the following and state any assumption(s) made:

(i) The electrical length $\dfrac{\ell}{\lambda}$ of the transmission line.

(ii) The reflection coefficient $\Gamma(z)$ in polar form, i.e., $|\Gamma|\angle\theta_\Gamma$ at $z = 0$.

(iii) The input impedance $Z_{in}(z)$ in polar form at $z = -\ell$.

(iv) The amplitude of the incident voltage wave at $z = 0$, i.e., V_o^+.

(v) The time-domain expression for the voltage at $z = 0$, i.e., $V(z = 0, t)$.

(20 Marks)

(b) Find the position z of maximum voltage for the transmission line in part (a).

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\| = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\| = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+\, e^{-j\beta z} + V_o^-\, e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+\, e^{-j\beta z} - V_o^-\, e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

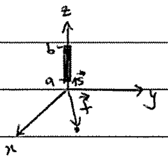
$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L\, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

1) a)



i) $\vec{r} = x\vec{a}_x + y\vec{a}_y \quad , \quad \vec{r}' = z\vec{a}_z \text{ for } a \le z \le b \quad , \quad dl = dz$

$\vec{R} = \vec{r} - \vec{r}' = x\vec{a}_x + y\vec{a}_y - z\vec{a}_z \quad , \quad R = \sqrt{(x^2+y^2)+z^2}$

$$\vec{E}(x,y) = \frac{1}{4\pi\varepsilon_0}\int_c \frac{\rho_l \vec{R}}{R^3}\,dl$$

$$= \frac{1}{4\pi\varepsilon_0}\int_a^b \frac{\rho_l(x\vec{a}_x + y\vec{a}_y - z\vec{a}_z)}{[(x^2+y^2)+z^2]^{3/2}}\,dz$$

Note: $\int \frac{1}{(x^2+b)^{3/2}}\,dx = \frac{x}{b\sqrt{x^2+b}}$

$\int \frac{x}{(x^2+b)^{3/2}}\,dx = \frac{-1}{\sqrt{x^2+b}}$

$$= \frac{\rho_l}{4\pi\varepsilon_0}\left[\frac{z(x\vec{a}_x + y\vec{a}_y)}{(x^2+y^2)\sqrt{(x^2+y^2)+z^2}} + \frac{\vec{a}_z}{\sqrt{(x^2+y^2)+z^2}}\right]_a^b$$

$$= \frac{\rho_l}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{(x^2+y^2)+b^2}}\left(\frac{b(x\vec{a}_x+y\vec{a}_y)}{x^2+y^2} + \vec{a}_z\right) - \frac{1}{\sqrt{(x^2+y^2)+a^2}}\left(\frac{a(x\vec{a}_x+y\vec{a}_y)}{x^2+y^2} + \vec{a}_z\right)\right] \text{ V/m}$$ #

ii) Note: For the next part, $\frac{\alpha}{\sqrt{(x^2+y^2)+\alpha^2}} = \text{sign}(\alpha)\frac{1}{\sqrt{(x/\alpha)^2+(y/\alpha)^2+1^2}}$ where $\text{sign}(\alpha) = \begin{cases} 1 & \text{for } \alpha > 0 \\ -1 & \text{for } \alpha < 0 \end{cases}$.

Hence, for $a = -\infty$, $b = \infty$,

$$\vec{E}(x,y) = \frac{\rho_l}{4\pi\varepsilon_0}\left\{\lim_{a\to-\infty,\, b\to\infty}\left[\left(\frac{\text{sign}(b)}{\sqrt{(x/b)^2+(y/b)^2+1^2}} - \frac{\text{sign}(a)}{\sqrt{(x/a)^2+(y/a)^2+1^2}}\right)\left(\frac{x\vec{a}_x+y\vec{a}_y}{x^2+y^2}\right) + \left(\frac{1}{\sqrt{x^2+y^2+b^2}} - \frac{1}{\sqrt{x^2+y^2+a^2}}\right)\vec{a}_z\right]\right\}$$

$$= \frac{\rho_l}{4\pi\varepsilon_0}\left[\left(\frac{(1)}{\sqrt{0^2+0^2+1^2}} - \frac{(-1)}{\sqrt{0^2+0^2+1^2}}\right)\left(\frac{x\vec{a}_x+y\vec{a}_y}{x^2+y^2}\right) + (0-0)\vec{a}_z\right]$$

$$= \frac{\rho_l}{2\pi\varepsilon_0}\left(\frac{x\vec{a}_x+y\vec{a}_y}{x^2+y^2}\right) \text{ V/m}$$

For cylindrical coordinate system,

$x = r\cos\phi \quad , \quad y = r\sin\phi \quad , \quad x^2+y^2 = r^2 \quad , \quad \vec{a}_x = \cos\phi\,\vec{a}_r - \sin\phi\,\vec{a}_\phi \quad , \quad \vec{a}_y = \sin\phi\,\vec{a}_r + \cos\phi\,\vec{a}_\phi$

$$\vec{E}(r,\phi) = \frac{\rho_l}{2\pi\varepsilon_0 r^2}\left[(r\cos\phi)(\cos\phi\,\vec{a}_r - \sin\phi\,\vec{a}_\phi) + (r\sin\phi)(\sin\phi\,\vec{a}_r + \cos\phi\,\vec{a}_\phi)\right]$$

$$= \frac{\rho_l}{2\pi\varepsilon_0 r^2}\left[(r\cos^2\phi + r\sin^2\phi)\,\vec{a}_r + (-r\sin\phi\cos\phi + r\sin\phi\cos\phi)\,\vec{a}_\phi\right]$$

$$= \frac{\rho_l}{2\pi\varepsilon_0 r^2}(r\vec{a}_r) = \frac{\rho_l}{2\pi\varepsilon_0 r}\vec{a}_r \text{ V/m}$$

$$\therefore \vec{E}(x,y) = \frac{\rho_l}{2\pi\varepsilon_0}\left(\frac{x\vec{a}_x+y\vec{a}_y}{x^2+y^2}\right) \text{ V/m} \quad \text{and} \quad \vec{E}(r,\phi) = \frac{\rho_l}{2\pi\varepsilon_0 r}\vec{a}_r \text{ V/m}$$

#

1) b) Method 1: As this question deals with current of infinite extent, Ampere's law can be used.

Using RHR, $\vec{H} = H_\phi\vec{a}_\phi$ only has an $\vec{a}_\phi$ component.

$$\oint_c \vec{H}\cdot d\vec{l} = I$$

$$\int_0^{2\pi} H_\phi\vec{a}_\phi \cdot r\,d\phi\,\vec{a}_\phi = I$$

$\int_0^{2\pi} H_\phi\, r\,d\phi = I \quad \Leftarrow$ As the Ampere loop is symmetrical / circular, H_ϕ is same at every part of the loop.

$H_\phi\int_0^{2\pi} r\,d\phi = I \qquad$ Hence it is a constant and can be taken out of the integral.

$$H_\phi(2\pi r) = I$$

$$H_\phi = \frac{I}{2\pi r} \text{ A/m} \Rightarrow \vec{H}(r,\phi) = H_\phi\vec{a}_\phi = \frac{I}{2\pi r}\vec{a}_\phi \text{ A/m}$$

For cartesian coordinate system,

$x = r\cos\phi$, $y = r\sin\phi$, $x^2+y^2 = r^2$, $\vec{a}_\phi = -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y$

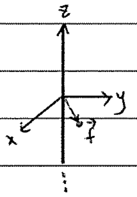
$$\vec{H}(x,y) = \frac{I}{2\pi r^2} r\vec{a}_\phi$$
$$= \frac{I}{2\pi(x^2+y^2)} r(-\sin\phi\,\vec{a}_x + r\cos\phi\,\vec{a}_y)$$
$$= \frac{I}{2\pi}\left(\frac{-y\vec{a}_x + x\vec{a}_y}{x^2+y^2}\right) \text{ A/m}$$

Method 2: Integration in cartesian coordinate system

$\vec{r} = x\vec{a}_x + y\vec{a}_y$, $\vec{s} = z\vec{a}_z$ for $-\infty < z < \infty$, $d\vec{l} = \vec{a}_z\,dz$

$\vec{R} = \vec{r} - \vec{s} = x\vec{a}_x + y\vec{a}_y - z\vec{a}_z$, $R = \sqrt{x^2+y^2+z^2}$

$$\vec{H}(x,y) = \frac{1}{4\pi}\int_C \frac{I\,d\vec{l} \times \vec{R}}{R^3}$$



$$= \frac{1}{4\pi}\int_{-\infty}^{\infty} \frac{I(\vec{a}_z\,dz) \times (x\vec{a}_x + y\vec{a}_y - z\vec{a}_z)}{[(x^2+y^2)+z^2]^{3/2}}$$

Note: $\int \frac{1}{(x^2+b)^{3/2}}\,dx = \frac{x}{b\sqrt{x^2+b}}$

$$= \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{x\vec{a}_y - y\vec{a}_x}{[(x^2+y^2)+z^2]^{3/2}}\,dz$$
$$= \frac{I}{4\pi}\left[\frac{z(x\vec{a}_y - y\vec{a}_x)}{(x^2+y^2)\sqrt{(x^2+y^2)+z^2}}\right]_{-\infty}^{\infty}$$
$$= \frac{I}{4\pi}\left[\frac{\operatorname{sign}(z)(x\vec{a}_y - y\vec{a}_x)}{(x^2+y^2)\sqrt{(x/z)^2+(y/z)^2+1^2}}\right]_{-\infty}^{\infty}$$
$$= \frac{I}{4\pi}\left(\frac{(1)(x\vec{a}_y - y\vec{a}_x)}{(x^2+y^2)\sqrt{0^2+0^2+1^2}} - \frac{(-1)(x\vec{a}_y - y\vec{a}_x)}{(x^2+y^2)\sqrt{0^2+0^2+1^2}}\right)$$
$$= \frac{I}{2\pi}\left(\frac{x\vec{a}_y - y\vec{a}_x}{x^2+y^2}\right) \text{ A/m}$$

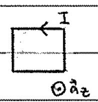
For cylindrical coordinate system,

$x = r\cos\phi$, $y = r\sin\phi$, $x^2+y^2 = r^2$, $\vec{a}_x = \cos\phi\,\vec{a}_r - \sin\phi\,\vec{a}_\phi$, $\vec{a}_y = \sin\phi\,\vec{a}_r + \cos\phi\,\vec{a}_\phi$

$$\vec{H}(r,\phi) = \frac{I}{2\pi r^2}[(r\cos\phi)(\sin\phi\,\vec{a}_r + \cos\phi\,\vec{a}_\phi) - (r\sin\phi)(\cos\phi\,\vec{a}_r - \sin\phi\,\vec{a}_\phi)]$$
$$= \frac{I}{2\pi r^2}[(r\sin\phi\cos\phi - r\sin\phi\cos\phi)\vec{a}_r + (r\cos^2\phi + r\sin^2\phi)\vec{a}_\phi]$$
$$= \frac{I}{2\pi r^2}(r\vec{a}_\phi) = \frac{I}{2\pi r}\vec{a}_\phi \text{ A/m}$$

$$\vec{H}(x,y) = \frac{I}{2\pi}\left(\frac{x\vec{a}_y - y\vec{a}_x}{x^2+y^2}\right) \text{ A/m} \quad \text{and} \quad \vec{H}(r,\phi) = \frac{I}{2\pi r}\vec{a}_\phi \text{ A/m} \quad \#$$

2) a) i) $\iint_S dx\,dy = a^2$ = total area of open surface



Assume surface normal is $\vec{a}_z$ and the corresponding closed contour in the CCW direction.

$$\Phi_m = \iint_S \vec{B}\cdot d\vec{s}$$
$$= \iint_S (C_1 t^2 + C_2 t)\vec{a}_z \cdot \vec{a}_z\,dx\,dy$$
$$= \iint_S (C_1 t^2 + C_2 t)\,dx\,dy$$

Note: $C_1 t^2 + C_2 t$ is not dependent on x and y $\Rightarrow$ can be taken out of integral

$$= (C_1 t^2 + C_2 t)\iint_S dx\,dy$$
$$= (C_1 t^2 + C_2 t)a^2 \text{ Wb} \quad \#$$

$V_{emf} = -\frac{d}{dt}(\Phi_m) = -a^2(2C_1 t + C_2)$ V # (Assuming CCW loop)

ii) Length of square $= \sqrt{a^2} = a$ m $\Rightarrow$ Perimeter / Total length $= 4a$ m $\Rightarrow$ Resistance, $R = 4aR_\ell$ Ω

$I_{ind} = \frac{V_{emf}}{R} = -\frac{a}{4R_\ell}(2C_1t + C_2)$ A (in CCW direction) #

OR $\frac{a}{4R_\ell}(2C_1t + C_2)$ A (in CW direction) #

2) b) i) loss tangent $= \frac{\sigma}{\varepsilon\omega} = 8 < 20 \Rightarrow$ so it is not a good conductor #

$\sigma = 8\varepsilon\omega = 8(\varepsilon_r\varepsilon_0)(2\pi f)$

$\sigma = 8(10)(\frac{1}{36\pi}\times10^{-9})(2\pi)(500\times10^6)$

$\sigma = \frac{22}{9} \approx 2.222$ S/m #

ii) * Note: To calculate the square root of a complex number in a scientific calculator, we can use this:

Let the complex number $= A$, $\sqrt{A} = \sqrt{\text{Abs}(A)} \angle (\text{Arg}(A) \div 2)$

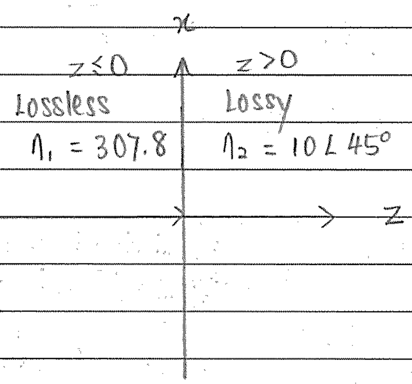
$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} = \varepsilon_r\varepsilon_0 - j\frac{\sigma}{2\pi f} = (8.8419 - j77.8091)\times10^{-11}$ F/m

$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu_0}{\varepsilon_c}} = 41.986\angle 0.7232$ Ω #

$\gamma = j\omega\sqrt{\mu\varepsilon_c} = j(2\pi f)\sqrt{\mu_0\varepsilon_c} = 62.228 + j70.4907$

$\therefore \alpha = 62.228$ Np/m #, $\beta = 70.4907$ rad/m #

3) a) Normally incident wave



(i)

$k = \frac{w}{u_p}$

$\therefore u_p = \frac{w}{k} = \frac{6\times10^9}{24.5} = 2.45\times10^8$ m/s #

(ii) $u_p = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$

$\mu_r\varepsilon_r = \frac{c^2}{u_p^2}$, $\mu_r = 1$ [Non-magnetic medium]

$\varepsilon_r = \frac{c^2}{\mu_r u_p^2} = \frac{(3\times10^8)^2}{(2.45\times10^8)^2} = 1.5$

$\therefore$ Permittivity, $\varepsilon = \varepsilon_0\varepsilon_r = 1.5\times\frac{1}{36\pi}\times10^{-9} = 1.327\times10^{-11}$ F/m #

(iii) $\eta_1 = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{1.5}} = 307.8$ Ω

$\vec{a}_E = \vec{a}_H \times \vec{a}_k = -\vec{a}_y \times \vec{a}_z = -\vec{a}_x$

$\vec{H}_i(z) = -\vec{a}_y\, 50\angle40°\, e^{-24.5z}\times10^{-3}$ A/m

$\vec{E}_i(z) = \vec{a}_E\, |E_{0i}|\angle \cancel{\phi}(E_{0i})\, e^{-24.5z}\times10^{-3}$, $|E_{0i}| = \eta_1|H_{0i}| = 15.4$

$\vec{E}_i(z) = (-\vec{a}_x)\,15.4\angle40°\, e^{-24.5z}$

$\therefore \vec{E}_i(z,t) = (-\vec{a}_x)\,15.4\cos(6\times10^9t - 24.5z + 40°)$ V/m #

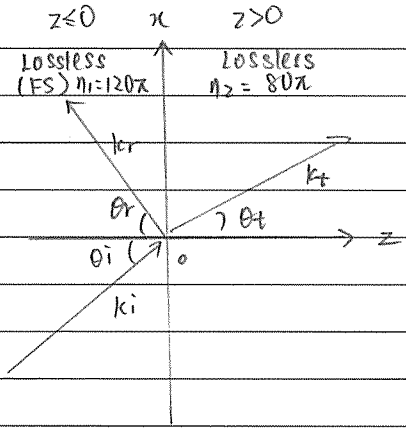
(iv) $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{10\angle45° - 308}{10\angle45° + 308} = 0.9551\angle177.4°$

$\therefore$ Percentage of average incident power reflected, $\frac{P_r}{P_i} = |\Gamma|^2$

$= 0.9551^2$

$= 91.22\%$ #

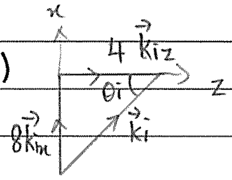
Q3.b) Oblique incident wave



$\because \vec{a}_E = \alpha \vec{a}_x + \beta \vec{a}_y$, $\alpha, \beta \in \mathbb{R}$

The UPW is parallely polarized.

(i)



$\tan \theta_i = \frac{8}{4}$

$\therefore \theta_i = 63.43°$ #

Using Snell's Law :

$$\sin\theta_t = \sin\theta_i \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} = \sin 63.43° \times \sqrt{\frac{1\times1}{1\times2.25}}$$

$\therefore \theta_t = 36.60°$ #

(ii) $\theta_i = 63.43°$, $\theta_t = 36.60°$, $\eta_1 = 120\pi$ (Free space) , $\eta_2 = 120\pi\sqrt{\frac{1}{2.25}} = 80\pi$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = 0.607$$

For E_{0i} : $|E_{0i}| = \sqrt{20^2 + (-40)^2} = 44.72$

$\therefore |E_{0t}| = |E_{0i}|\,\tau_{\parallel} = 27.15$ V #

(iii) For $\vec{a}_{kt}$: $\vec{k}_t = |k_t| \cos\theta_t \,\vec{a}_z + |k_t| \sin\theta_t \,\vec{a}_x$

$\vec{a}_{kt} = \frac{|k_t|}{|k_t|}\cos\theta_t \,\vec{a}_z + \frac{|k_t|}{|k_t|}\sin\theta_t \,\vec{a}_x$

$$\vec{S}_t = \vec{a}_{kt}\frac{|E_{0t}|^2}{2\eta_2} = \left[(\cos\theta_t)\vec{a}_z + \sin\theta_t\,\vec{a}_x\right] \times \frac{27.15^2}{2(80\pi)}$$

$$= (0.803\vec{a}_z + 0.596\vec{a}_x) \times 1.466 \text{ W/m}^2$$

At z=0, time-average power is transmitted only normal to the boundary, i.e. $\vec{a}_z$:

$\therefore$ Power $= 0.803 \times 1.466 \times 2$

$= 2.3539$ W #

Q4.a) (i) $\beta = \frac{w}{u_p} = \frac{2.5\pi \times 10^8}{3\times 10^8} = \frac{5}{6}\pi$

$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\frac{5}{6}\pi} = 2.4\,m$

$\therefore \frac{l}{\lambda} = \frac{0.64}{2.4}$

$= 0.2667$ #

(ii) At $z=0$,

$\therefore \Gamma(z) = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{140-64j-100}{140-64j+100}$

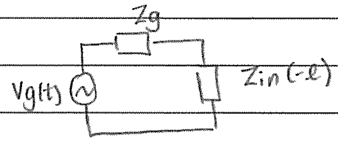
$= 0.304 \angle -0.752$ #

(iii) At $z=-l$, $\tan(\beta l) = \tan(\frac{8\pi}{15}) = 1.676$

$Z_{in}(-l) = \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} Z_0$

$= \frac{140-64j + j(100)(1.676)}{100 + j(140-64j)(1.676)}(100) = 71.89 \angle 0.502\ \Omega$ #

(iv) For V_0^+, first we find $V(-l)$:



$V(-l) = V_g(t) \times \frac{Z_{in}(-l)}{Z_{in}(-l) + Z_g}$

$= 96 \times \frac{71.89 \angle 0.502}{71.89\angle 0.502 + 100}$

$= 41.41 \angle 0.293$ V

$V(-l) = 40.15 \angle 0.292 = V_0^+ e^{-j\beta(-l)} + V_0^- e^{j\beta(-l)}$

$41.41\angle 0.292 = V_0^+ \left[e^{j\times\frac{5\pi}{6}\times 0.64} + \Gamma_L e^{j\times\frac{5\pi}{6}\times -0.64} \right]$, $\Gamma_L = 0.304\angle -0.752$

$= V_0^+ (0.863 \angle 1.969)$

$\therefore V_0^+ = 47.98 \angle -1.68$,

$\therefore |V_0^+| = 48$ V #

(v) $V(z=0) = V_0^+ \cancelto{1}{e^{-j\beta(0)}} + V_0^- \cancelto{1}{e^{j\beta(0)}}$

$= V_0^+ + V_0^- = 48\angle -1.675 + 48\angle -1.675 \times 0.304\angle -0.752$

$\therefore V(z=0,t) = 48\cos(2.5\pi\times 10^8 t - 1.675) + 14.58 \cos(2.5\pi \times 10^8 t - 2.43)$ V #

b) For max V: $\theta_r = \theta_0 + 2\beta z = 0, 2\pi \ldots$

$Z_{max} = \frac{0+0.752}{2\beta} = 0.144m > 0$, $Z_{max} = \frac{-2\pi + 0.752}{2\beta} = -1.056m < -0.64m$

There are no global maximum. However there are local maximum voltage:

$|V(-l)| = 41.41$ V, $|V(0)| = 59.46$

$\because V(0) > V(-l)$, $\therefore$ The maximum voltage is at $z=0$ #