Part 3.2

Frequency Response

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EE2002 Analog Electronics

References

Textbook:

Jaeger, Richard C. & Blalock, Travis N., “Microelectronic Circuit Design”, 4th Edition, McGraw Hill, 2011 (Chapters 13 and 14)

References

- Hambley, Allan R. “Electronics”, 2nd Edition, Prentice Hall, 2000
- Neamen, Donald A. “Electronic Circuit Analysis and Design”, 2nd Edition, McGraw-Hill, 2002

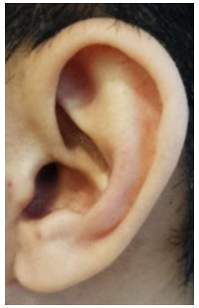
Lesson Objectives

At the end of this lesson, you should be able to:

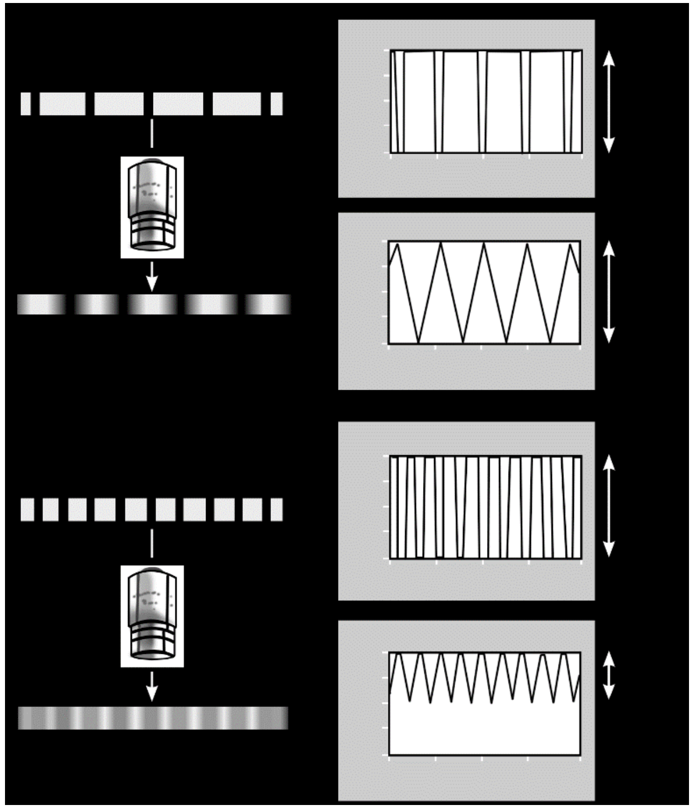
- Explain the concepts of transfer functions, poles and zeros, bode plots, and –3 dB frequency
- Draw bode plot diagrams using hand drawing
- Explain the typical frequency response of an amplifier
- Identify parasitic capacitors
- Explain short circuit analysis and open circuit analysis

Frequency Response: Real Life Experience

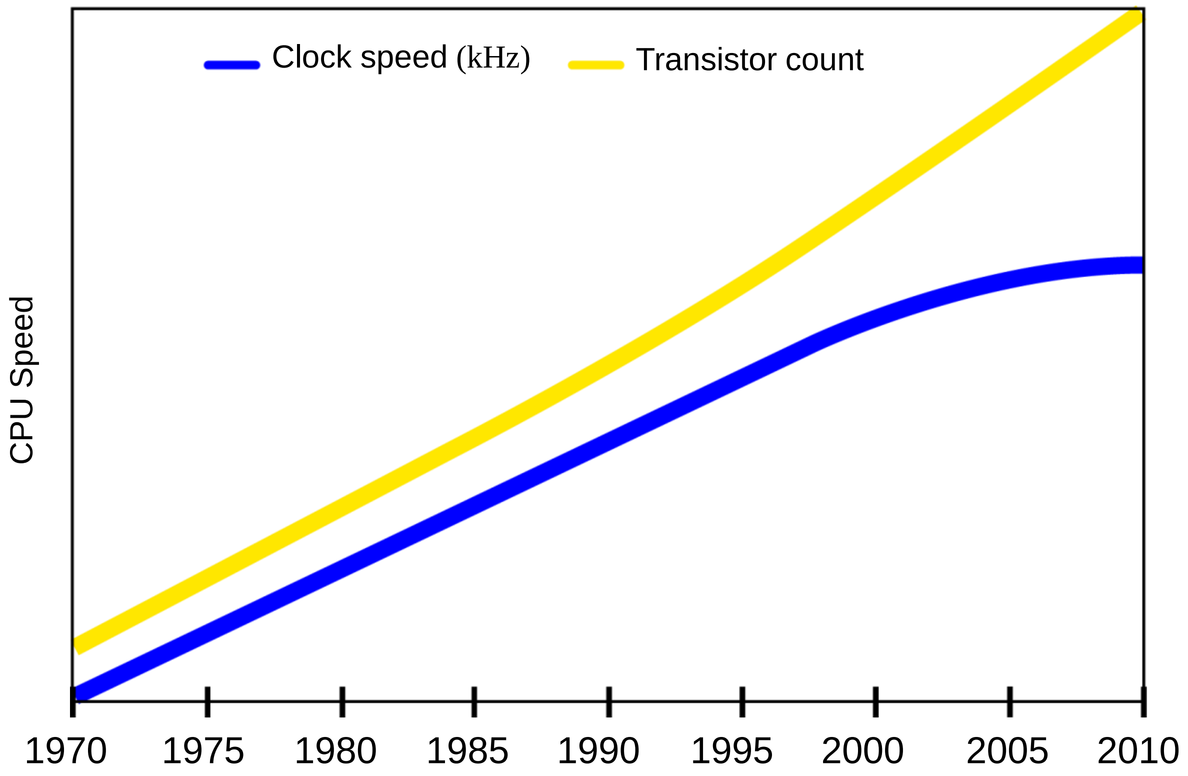
Audio



Video

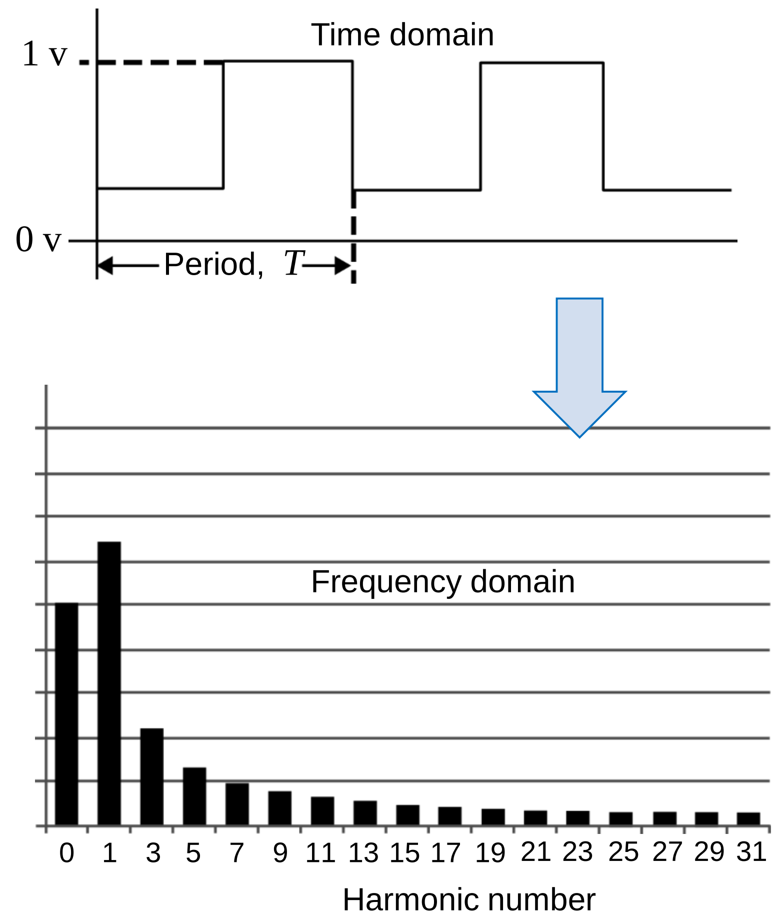


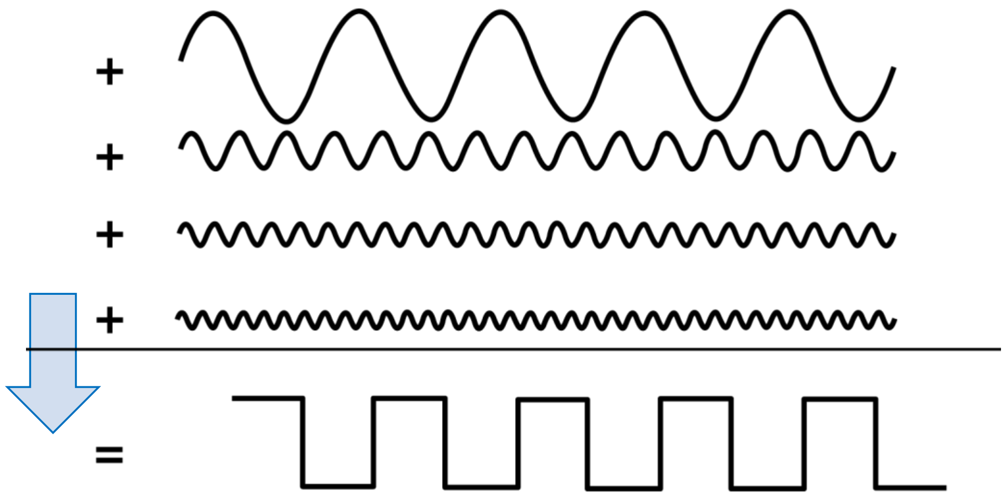
Frequency Response: Real Life Experience



Why don’t we have a 100 GHz CPU for cellphone?

Frequency Response





Real world signals can be decomposed into many single frequency primitives, namely 0th harmonic, 1st harmonic, etc.

Recap: Sine Wave Features

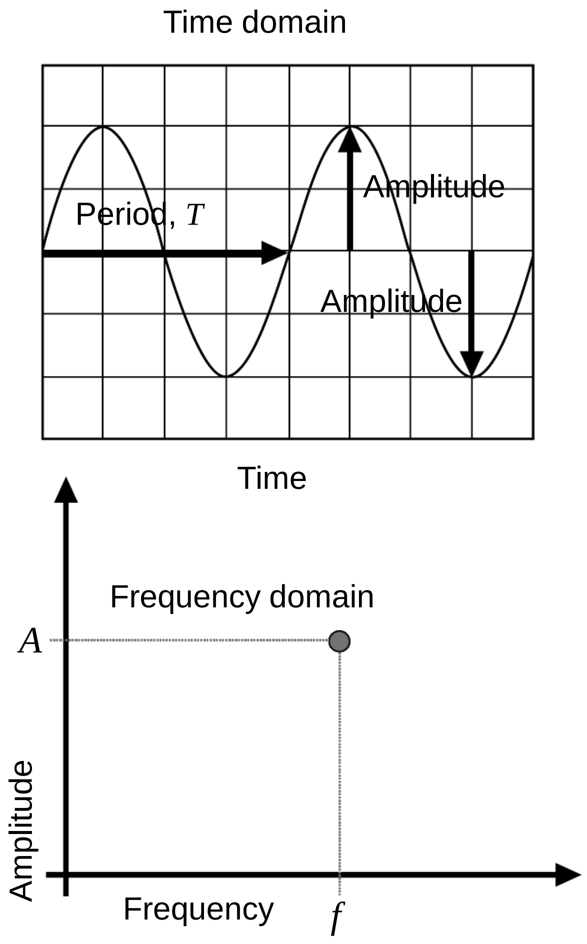
Three terms fully describe a sine wave:

- Frequency
- Amplitude
- Phase

$$\omega = 2\pi \times f = \frac{2\pi}{T}$$

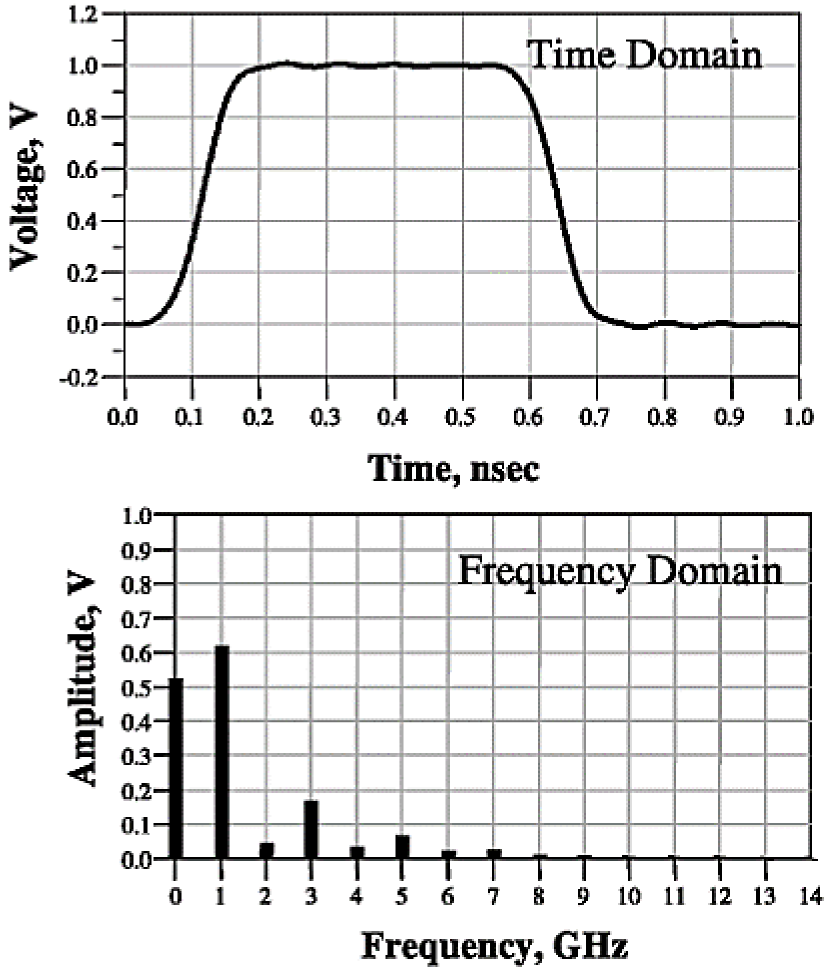
Angular frequency, in radians/sec

Sine-wave frequency, in Hz



Recap: Fourier Transform (1)
Signal Decomposition

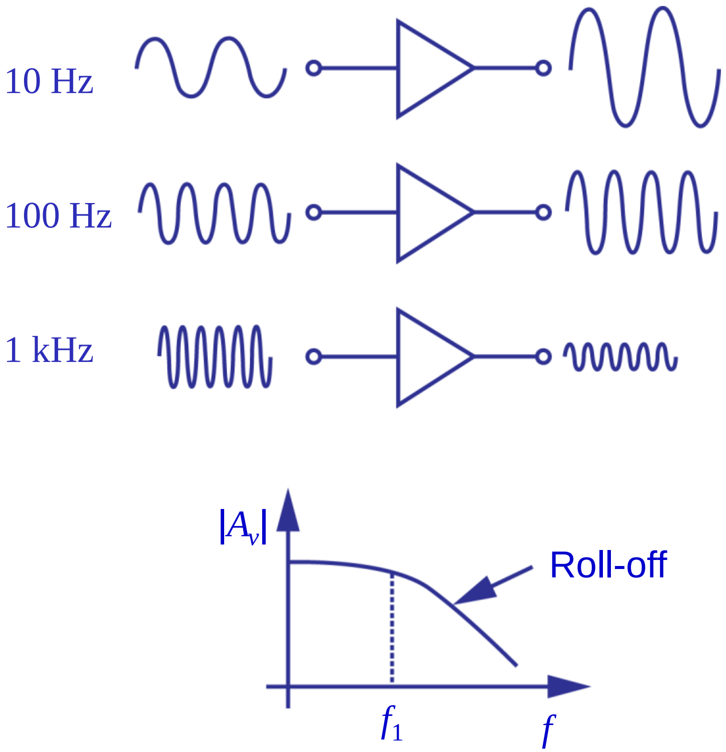
Transforms is able to convert a waveform from the time domain into a waveform in the frequency domain.

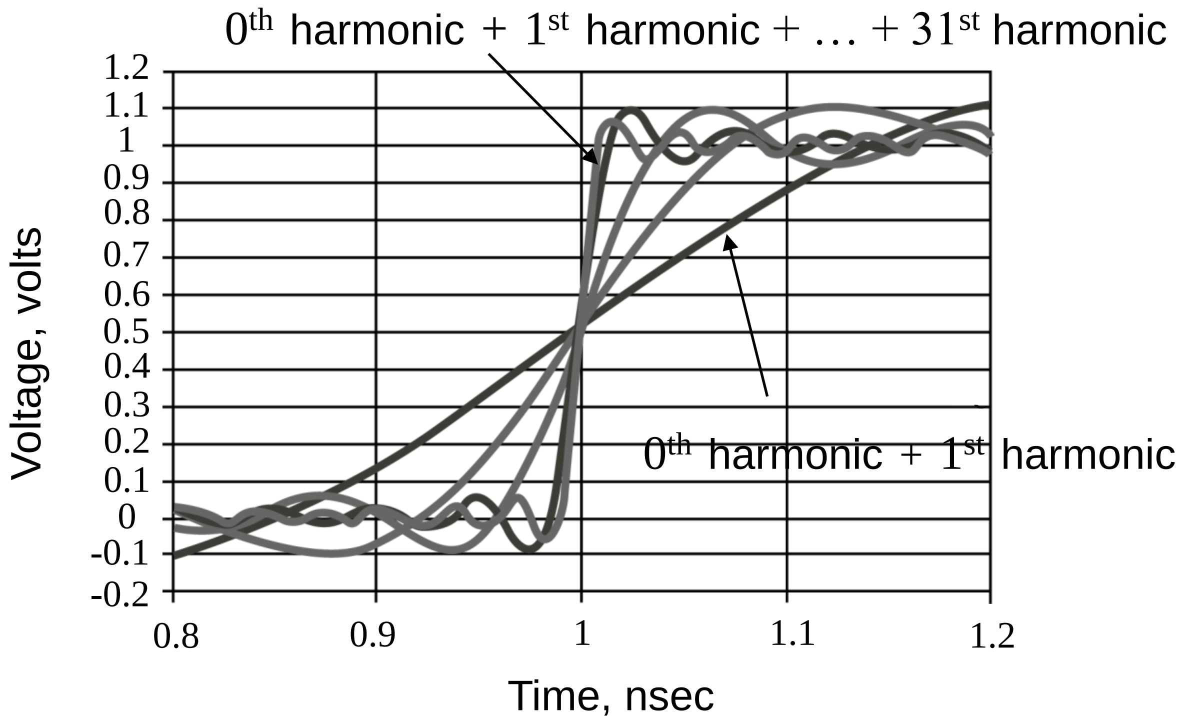


Three types of Fourier Transforms:

- Fourier Integral (FI)
- Discrete Fourier Transform (DFT)
- Fast Fourier Transform (FFT)

Frequency Response: Why Do We Care?





Each harmonic signal has different gain when it passes through an electronic element/ system.
$\Rightarrow$ The output signal may get distorted.

Conclusion: Different frequency response results in different reconstructed signal at the output.

Introduction

- An ideal amplifier would have an infinite bandwidth.

- The gain would remain constant at all frequencies.

- However, **the bandwidth is finite in practice, and the gain changes with frequency.**

- This is due to the fact that capacitive/ inductive elements in the circuit becomes prominent when the signal frequency changes.

- This effect occurs in all circuits including amplifier design.

Introduction

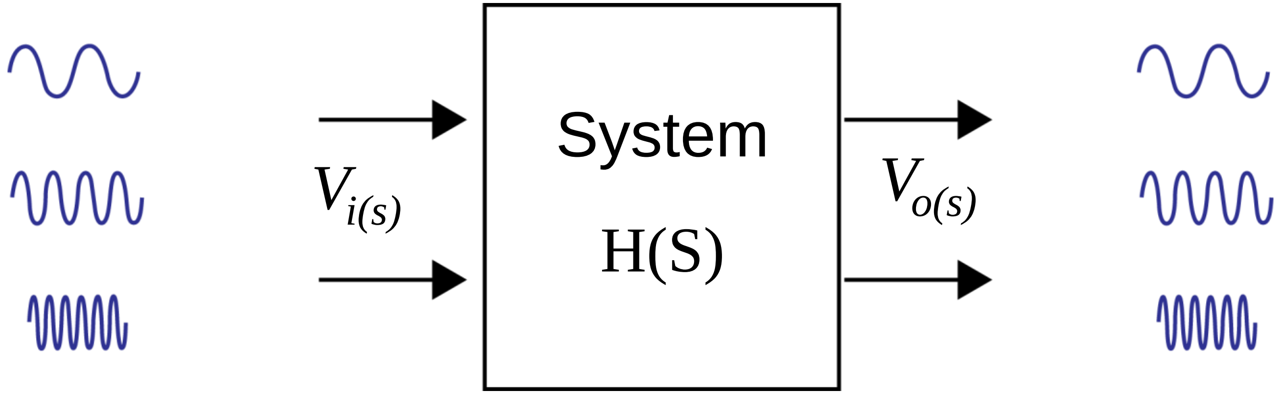
(Cont.)

- The basics of signal processing and analysis will be covered.
- Study frequency response of a few examples in order to account the behavior of a circuit operating at different frequencies.
- Study time constants for approximating the response of amplifiers.

$\Rightarrow$ Understand bode plot, pole, zero, and the method to identify the pole/zero elements in real circuits.

Transfer Function

Transfer function, also known as the **system function** or **network function** is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a linear time-invariant system.



Transfer function is defined by, $\boxed{H(S)=\dfrac{V_O(S)}{V_i(S)}}$.

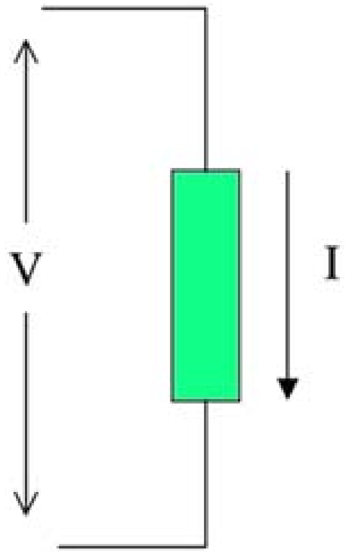
Note: We evaluate the system's behavior using a **single** frequency signal.

Transfer Functions and Pole-Zero Plots

Powerful tool: **AC impedance**

$$R_{DC} : Z = \frac{V}{I}$$

$$R_{AC} : Z = \frac{dV}{dI}$$

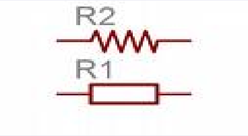
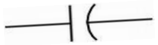


- Definition is always true.
- Don’t forget to **disable** (independent current/ voltage) **sources** in the design under test.

Review of Transfer Functions and Pole-Zero Plots

The transfer function is a property of the circuit and does not depend on the input or the output.

Translation from the circuit domain to the Laplace ($s = j\omega$) domain is made by making the following substitutions of circuit elements:

Element		Definition	Impedance
		$V = I \times R$	$Z = R$
		$I = \frac{dQ}{dt} = C\frac{dV}{dt}$	$Z = \frac{1}{j\omega C} = \frac{1}{sC}$
		$I = L\frac{dI}{dt}$	$Z = j\omega L = sL$

Complex Number Recap (1): Format

- A complex number has a real part and an imaginary part.

- The standard format is:

$$z = 4 + 5i$$

or

$$z = \underbrace{4}_{\text{Real part}} + \underbrace{5j}_{\text{Imaginary part}}$$

Complex Number Recap (2): Graphing in the Complex Plane

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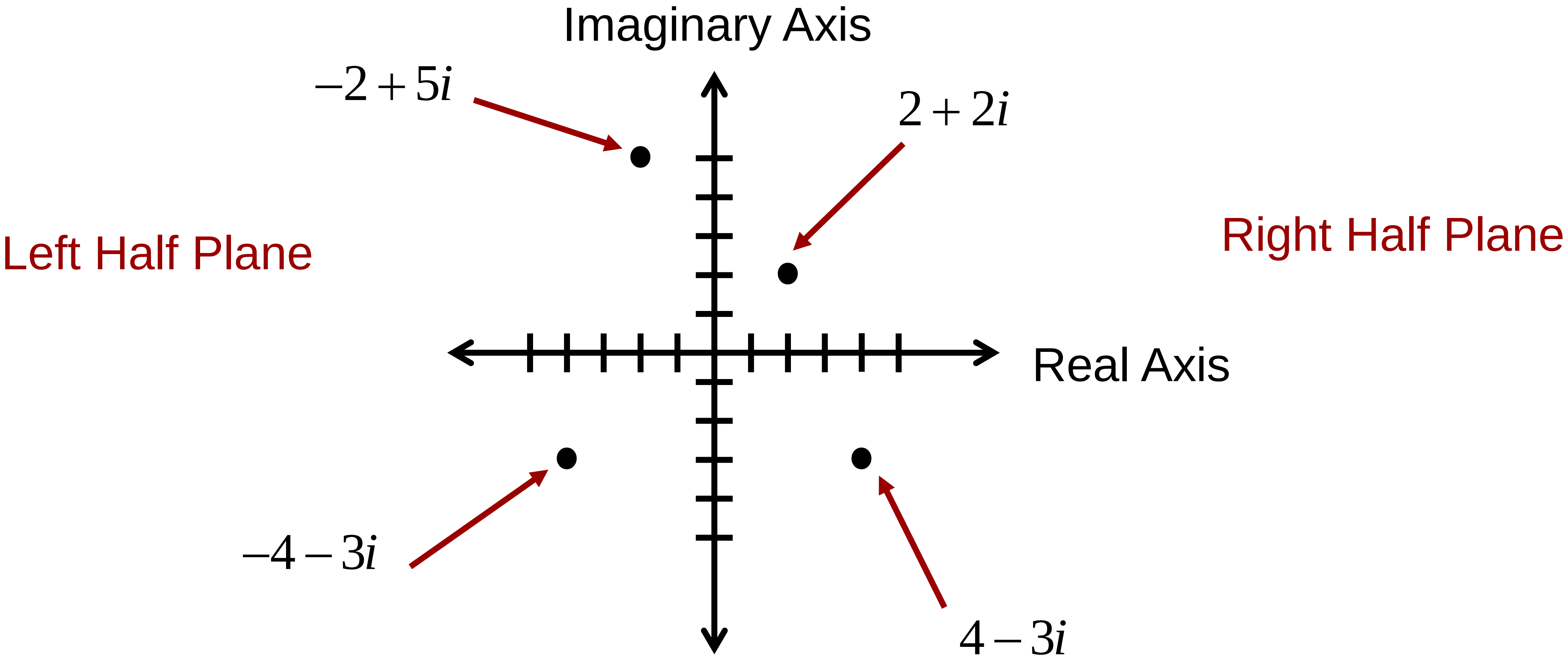


Complex Number Recap (3): Addition, Subtraction, and Multiplication

Addition:

$$(-1+2i)+(3+3i)$$
$$=(-1+3)+(2+3)i$$
$$=2+5i$$

Subtraction:

$$(-1+2i)-(3+3i)$$
$$=(-1-3)+(2-3)i$$
$$=-4-i$$

Multiplication:

$$i(3+i)=i\times 3+i\times i$$
$$=3i+(-1)$$
$$=3i-1$$

$$(2+3i)(5+6i)=2\times(5+6i)+3i\times(5+6i)$$
$$=10+12i+15i+18i^2$$
$$=10+27i+(-18)$$
$$=-8+27i$$

Complex Number Recap (4): Division and Absolute Value

Division:

$$\frac{3+11i}{-1-2i} = \frac{3+11i}{-1-2i} \times \frac{-1+2i}{-1+2i}$$

$$= \frac{(3+11i)\times(-1+2i)}{(-1-2i)\times(-1+2i)}$$

$$= \frac{-3+6i-11i+22i^2}{(-1)^2-(2i)^2}$$

$$= \frac{-25-5i}{1-(-4)}$$

$$= -5-i$$

Absolute Value:

$$\left|-2+5i\right| = \sqrt{(-2)^2+5^2}$$

$$= \sqrt{29}$$

$$\left|a+bi\right| = \sqrt{a^2+b^2}$$

$$\left|a+b\omega i\right| = \sqrt{a^2+(b\omega)^2}$$

Complex Number Recap (5): Root of Polynomials

Solve: $x^2 - 6x + 13 = 0$

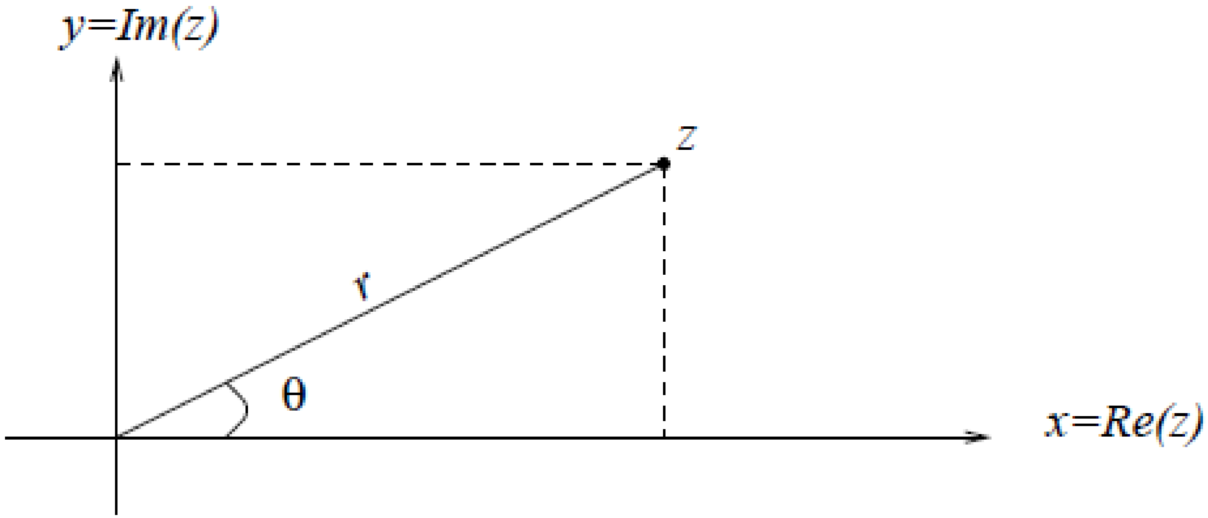
$$x = \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm \sqrt{16}\sqrt{-1}}{2}$$

$x = 3 \pm 2i$ (Two roots, a pair of complex conjugates)

Complex Number Recap (6): Polar Form for Complex Numbers

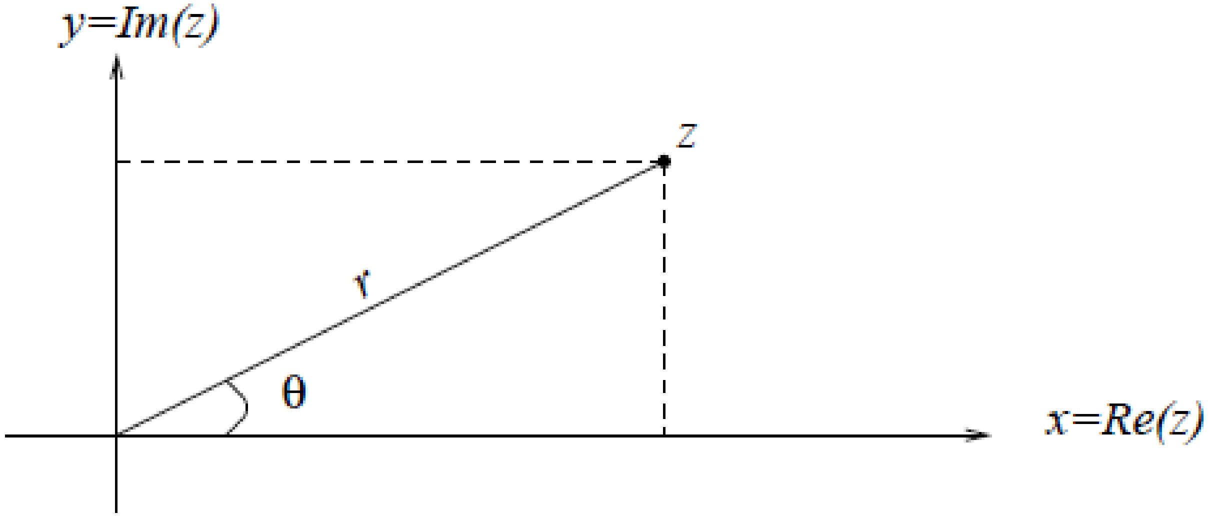


$$z = x + iy$$
$$= r\cos(\theta) + ir\sin(\theta)$$
$$= r(\cos(\theta) + i\sin(\theta))$$

$$|z| = \sqrt{r^2\cos(\theta)^2 + r^2\sin(\theta)^2}$$
$$= \sqrt{r^2}$$
$$= r$$

$$\angle(z) = \theta$$
$$= \arctan\left(\frac{y}{x}\right)$$

Complex Number Recap (7): Exponential Form



$$z = x + iy = r\cos(\theta) + ir\sin(\theta) = re^{i\theta}$$

$$|z| = r$$

$$\angle(z) = \theta$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Complex Number Recap (8): Important Examples

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Given: $Z_1 = a + bi,\ Z_2 = c + di,\ Z_3 = e + fi$

$$\left|Z_1 \times Z_2 \times Z_3\right| = \left|Z_1\right| \times \left|Z_2\right| \times \left|Z_3\right|$$

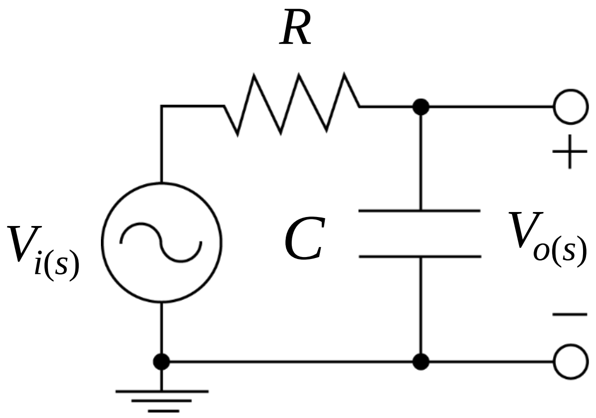
$$\log\left(\left|Z_1 \times Z_2 \times Z_3\right|\right) = \log\left|Z_1\right| + \log\left|Z_2\right| + \log\left|Z_3\right|$$

$$\log\left(\left|\frac{Z_1}{Z_2}\right|\right) = \log\left|Z_1\right| - \log\left|Z_2\right|$$

$$\angle\left(Z_1 \times Z_2 \times Z_3\right) = \angle\left(Z_1\right) + \angle\left(Z_2\right) + \angle\left(Z_3\right)$$

$$\angle\left(\frac{Z_1}{Z_2}\right) = \angle\left(Z_1\right) - \angle\left(Z_2\right)$$

Frequency Response Analysis: First Example



$$H(s) = \frac{V_O(s)}{V_i(s)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|H(s)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Tasks:

- $H(s)$ describes the relationship between the output to the input: gain.
- $H(s)$ varies with frequency, due to the impedance of the capacitor changes with frequency (like a variable resistor).
- This circuit is a low pass filter: $|H(s)|$ drops when ω increases, so only low frequency signals can pass.

This is so called frequency response analysis!

General Transfer Function

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{m\text{-}1} s^{m\text{-}1} + b_m s^m}{a_0 + a_1 s + a_2 s_2 + \ldots + a_{n\text{-}1} s^{n\text{-}1} + a_n s^n} = \frac{H_o\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

H_0 : Low-frequency gain

Zeros: Roots of numerator, $-Z_1,\ -Z_2,\ \ldots,\ -Z_m$

Poles: Roots of denominator, $-p_1, -p_2, \ldots, -p_n$

Poles and Zeros: Examples

1) $S^2 + 5S + 6 = (S+2)(S+3)$

roots: $r_1 = -2, r_2 = -3$

2) $aS^2 + bS + c = (S - r_1)(S - r_2)$

roots: $r_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

$r_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$$H(S) = \frac{S^2 - 5S + 6}{aS^2 + bS + c}$$

zeros: $z_1 = -2, z_2 = -3$

poles: $p_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

$p_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Poles and Zeros

- The values of poles and zeros can be either real or imaginary.
- For a **physical system**, all the coefficients are real, hence, the poles and zeros can only have two forms:

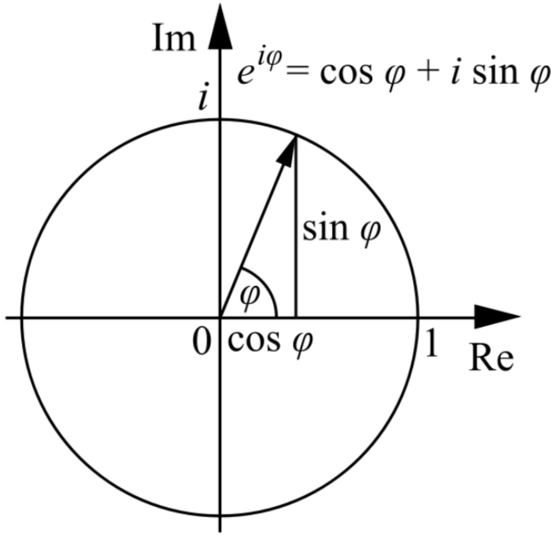
1. The roots are real.
2. If one root is complex, its complex conjugate has to be another root, i.e., if $-p = a + jb$ is a root, $-z^* = a - jb$ must be another root.

- Both poles and zeros can be located in either left half-plane (LHP) or right half-plane (RHP).

Bode Plots

To study the frequency response of a system, the transfer function $H(s)$ is re-written in the polar form.

$$H(s) = |H(s)| \cdot \angle H(s)$$



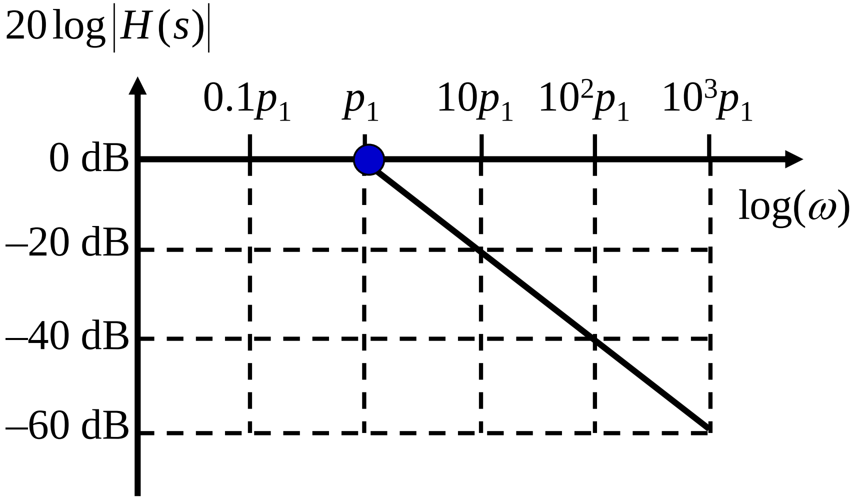
Bode plots are a very useful way to represent the gain and phase of a system as a function of frequency.

Bode Plots of Single Pole System: Amplitude Response

$$H(s)=\frac{1}{1+\dfrac{s}{p_1}} \Rightarrow |H(s)|=\frac{1}{\sqrt{1+\left(\dfrac{\omega}{p_1}\right)^2}}$$

$$dB(H)=20\log\left(\frac{1}{\sqrt{1+\left(\dfrac{\omega}{p_1}\right)^2}}\right)=-10\log\left[1+\left(\frac{\omega}{p_1}\right)^2\right]$$

$$dB(H)=\begin{cases}0 & \text{when } \omega << p_1\\ -20\log\omega+20\log p_1 & \text{when } \omega >> p_1\end{cases}$$



Low pass

Try some frequency $\omega_1 >> p_1$, dB[$H(\omega_1)$],
$= -20\log\omega_1 + 20\log p_1$

Try another frequency $\omega_2 = 10^* \omega_1$, dB[$H(\omega_2)$],
$= -20\log\omega_1 - 20 + 20\log p_1$

When we hit a pole, p_1, the Bode magnitude falls with a slope of –20 dB/dec.

Bode Plots of Single Pole System: Phase Response

$$H(s) = \frac{1}{1+\dfrac{s}{p_1}}$$

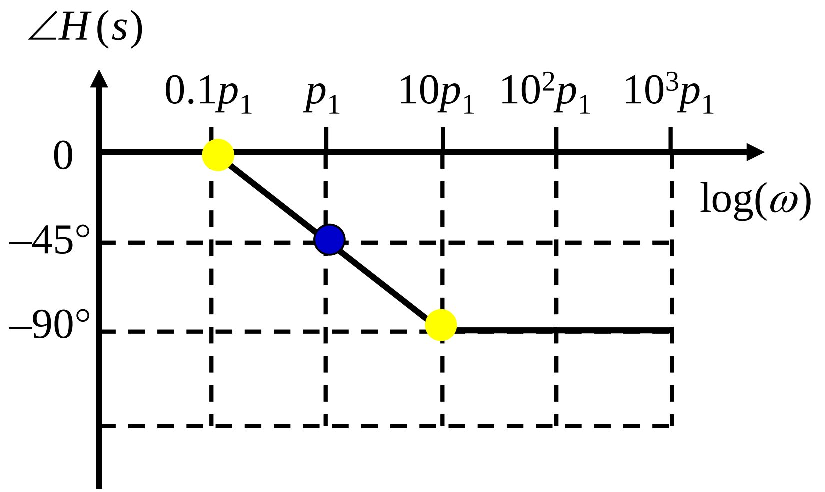
$$\angle H(j\omega) = \angle(\text{Numerator}) - \angle(\text{Denominator})$$

$$= \angle(1) - \angle\left(1+\frac{j\omega}{p_1}\right) = 0 - \arctan\left(\frac{\omega}{p_1}\right)$$

At $\omega_1 = p_1,\ \angle H(j\omega_1) = -\arctan\left(\frac{p_1}{p_1}\right) = -45^o$

At $\omega_2 = 10\,p_1,\ \angle H(j\omega_2) = -\arctan\left(\frac{10\,p_1}{p_1}\right) = -84.3^o \approx -90^o$

At $\omega_3 = 100\,p_1,\ \angle H(j\omega_3) = -\arctan\left(\frac{100\,p_1}{p_1}\right) = -89.4^o \approx -90^o$

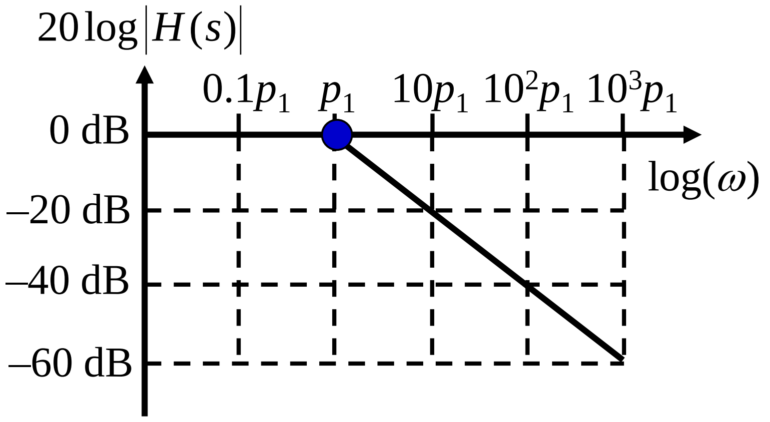


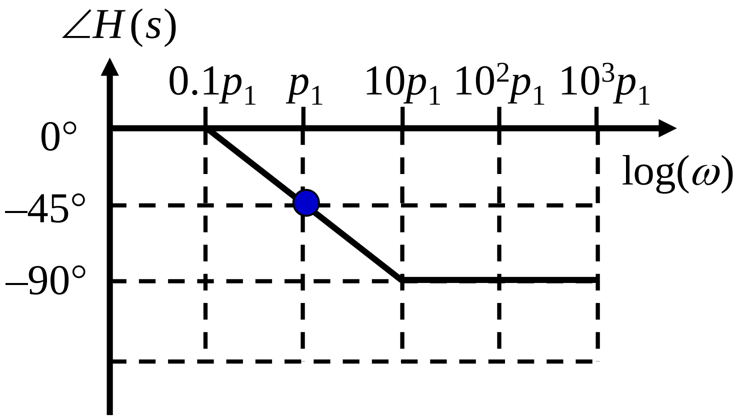
At p_1, phase shift is –45°. Phase shit is –90° when frequency is very high.

Summary of Single Pole System

Low pass, LHP pole

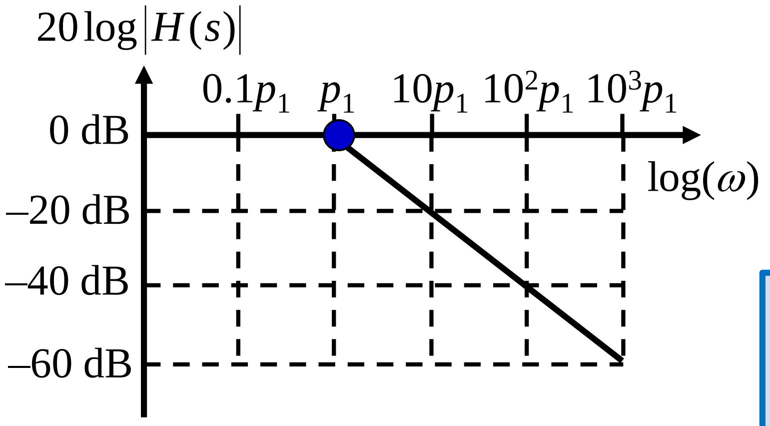
$$H(s) = \frac{1}{1+\dfrac{s}{p_1}}$$

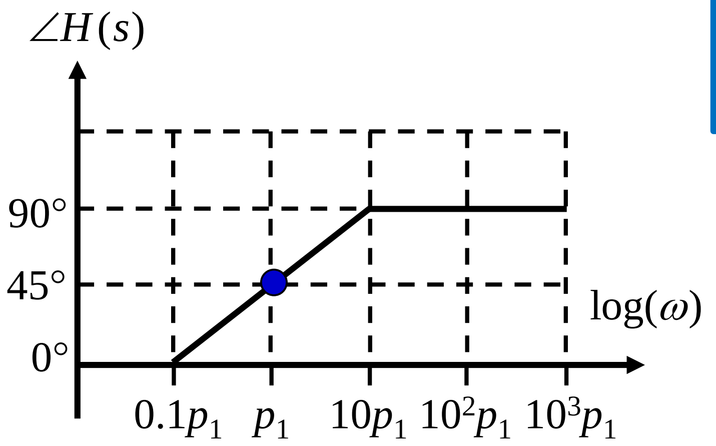




Low pass, RHP pole

$$H(s) = \frac{1}{1-\dfrac{s}{p_1}}$$





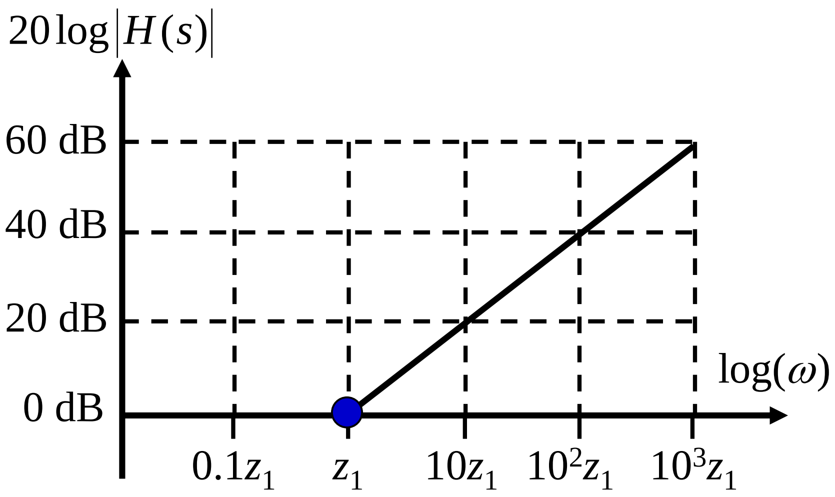
When we hit a pole, p_1, the Bode magnitude falls with a slope of -20 dB/dec.

Bode Plots of Single Zero System: Amplitude Response

$$H(s) = 1 + \frac{s}{z_1} \Rightarrow |H(s)| = |H(j\omega)| = \sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}$$

$$dB(H) = 20\log\left(\sqrt{1 + \left(\frac{\omega}{z_1}\right)^2}\right) = 10\log\left[1 + \left(\frac{\omega}{z_1}\right)^2\right]$$

$$dB(H) = \begin{cases} 0 & \text{when } \omega << z_1 \\ 20\log\omega - 20\log z_1 & \text{when } \omega >> z_1 \end{cases}$$



High pass

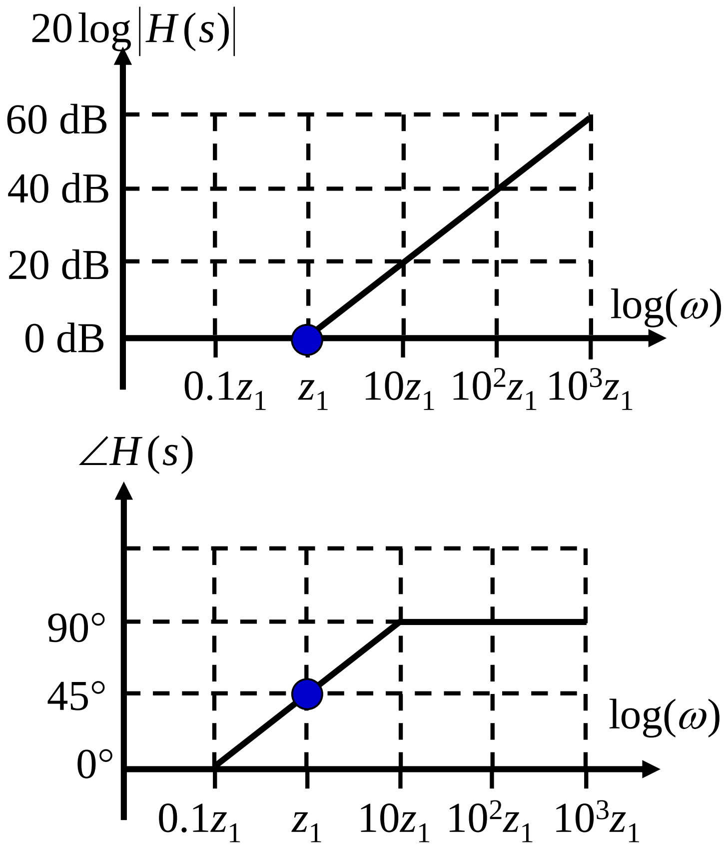
Try some frequency $\omega_1 >> z_1$, dB[$H(\omega_1)$], = 20 log ω_1 – 20 log z_1

Try another frequency $\omega_2 = 10^* \omega_1$, dB[$H(\omega_2)$], = –20 log ω_1 + 20 + 20 log z_1

When we hit a zero, z_1, the Bode magnitude rises with a slope of +20 dB/dec.

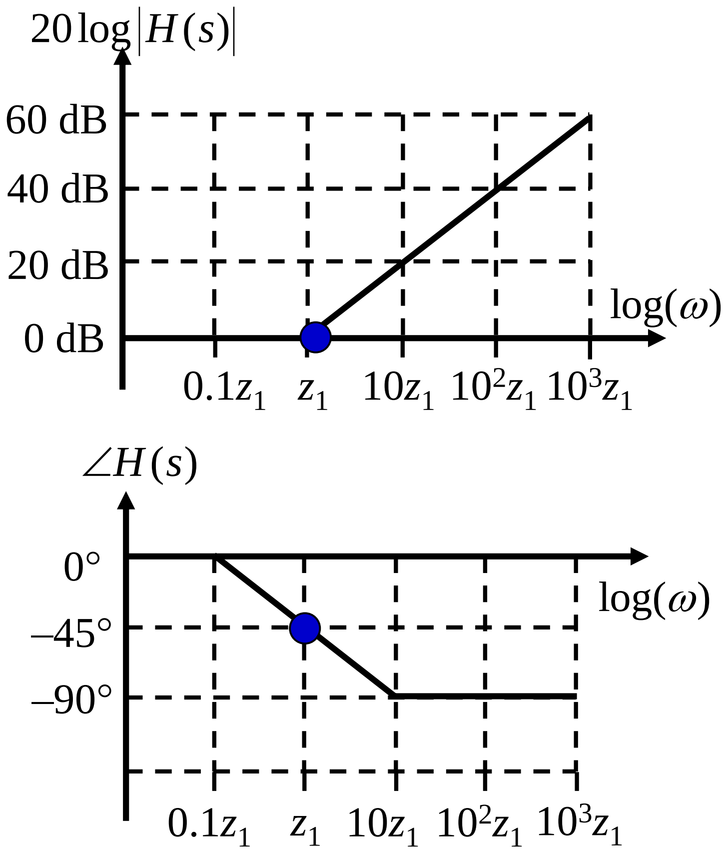
Summary of Single Zero System

High pass, LHP zero



$$H(s) = 1 + \frac{s}{z_1}$$

High pass, RHP zero



$$H(s) = 1 - \frac{s}{z_1}$$

When we hit a zero, z_1, the Bode magnitude rises with a slope of +20 dB/dec.

Many-Pole and Many-Zero System's Bode Plots

Now you can deal with a system with many poles and/ or many zeros using the complex number's knowledge:

$$H(s)=\frac{H_0\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

Amplitude: $\mathrm{dB}(H)=20\log(H_0)+dB\left(1+\frac{s}{z_1}\right)+\ldots dB\left(1+\frac{s}{z_m}\right)+dB\left(\frac{1}{1+\frac{s}{p_1}}\right)+..dB\left(\frac{1}{1+\frac{s}{p_n}}\right)$

Phase: $\angle(H)=\angle\left(1+\frac{s}{z_1}\right)+\ldots\angle\left(1+\frac{s}{z_m}\right)+\angle\left(\frac{1}{1+\frac{s}{p_1}}\right)+\ldots\angle\left(\frac{1}{1+\frac{s}{p_n}}\right)$

Linear addition of many single pole and single zero system.

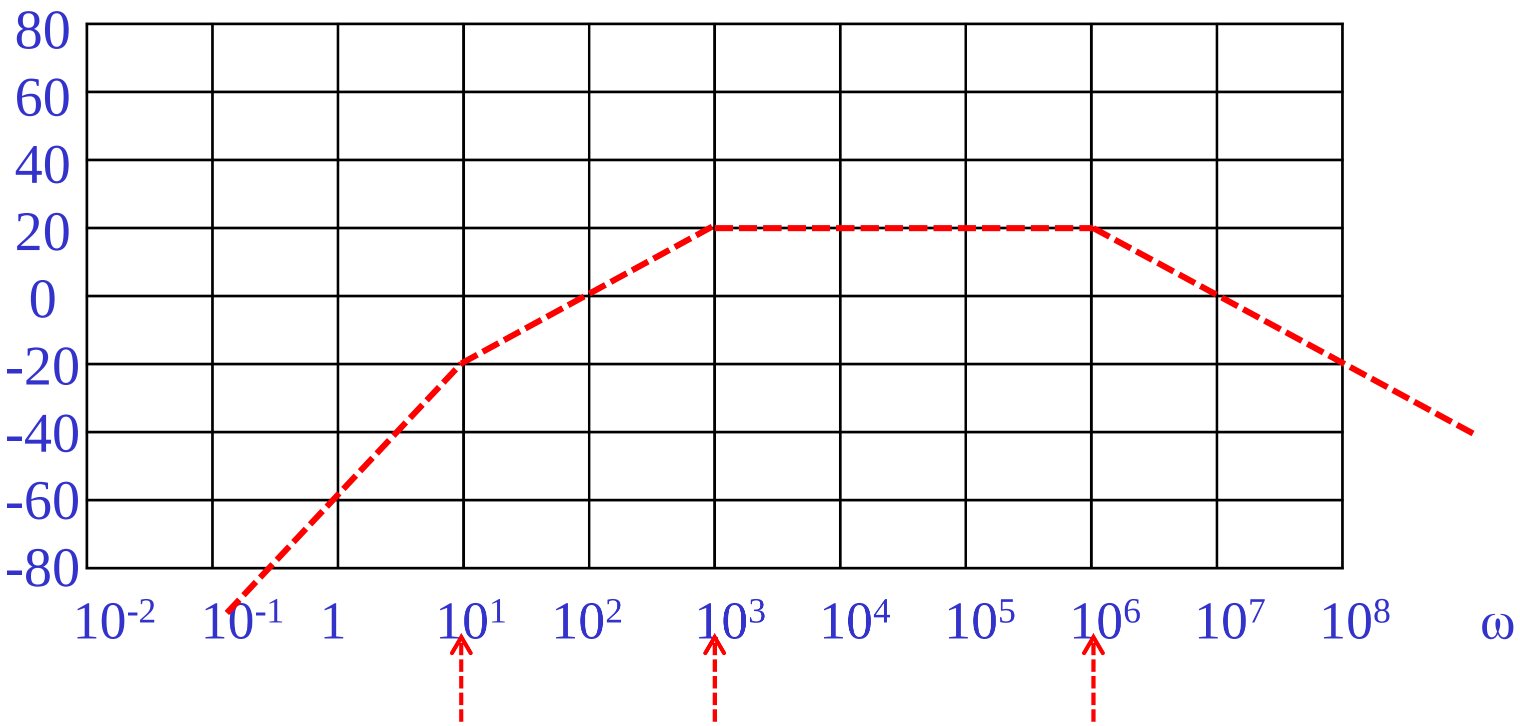
Many-pole and Many-zero System: Example

Given: $H(s) = \dfrac{10^8 s^2}{(s-10)(s-10^3)(s-10^6)}$

2 Zeros: $z_1 = z_2 = 0$

3 Poles: $p_1 = 10,\ p_2 = 10^3,\ p_3 = 10^6$

Step 1: **Shape**

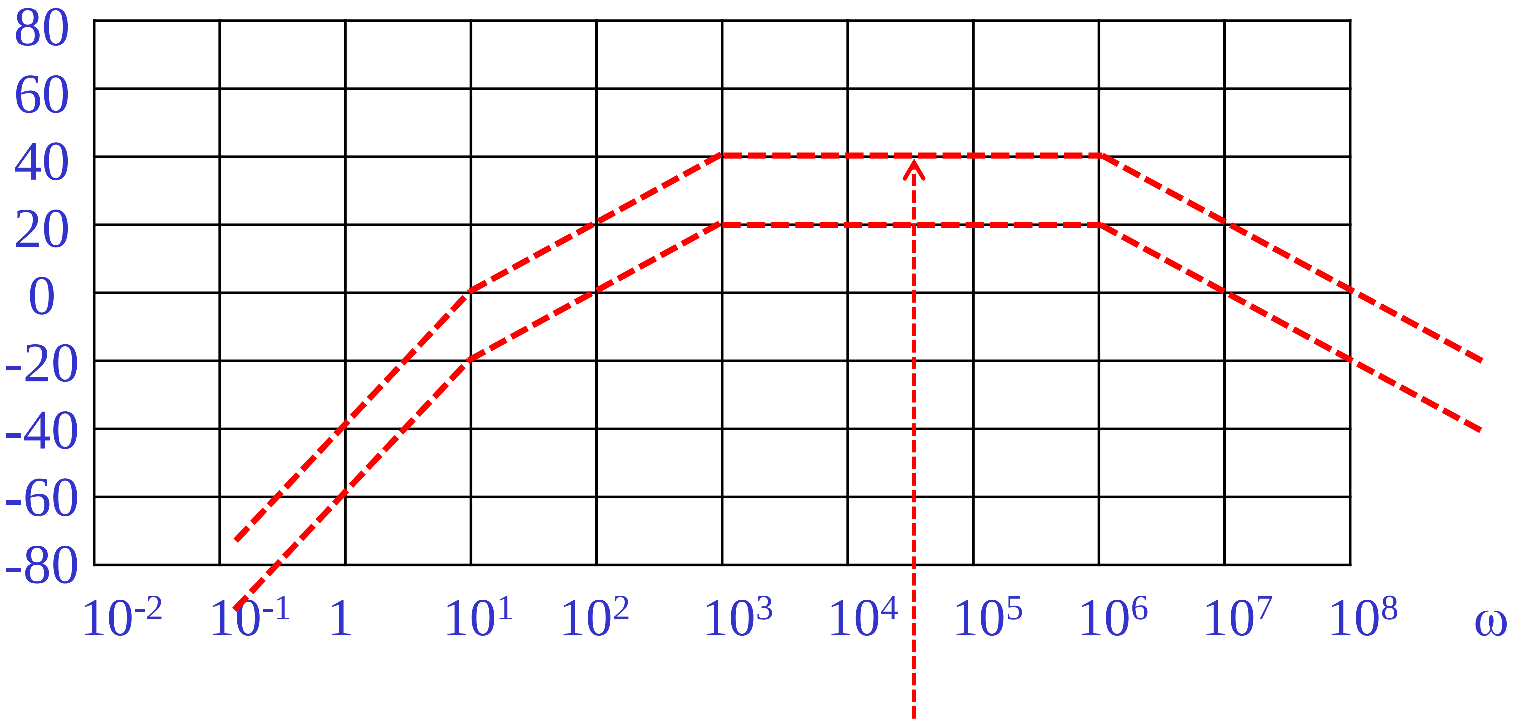


Many-pole and Many-zero System: Example

Try any specific frequency you prefer, for example, 10^4:

$$20\log|H(s=j10^4)| = 20\log\left|\frac{10^8(j10^4)^2}{(j10^4-10)(j10^4-10^3)(j10^4-10^6)}\right| = 40$$

Step 2:
Height



Hand Drawing Steps (dB)

1. Replace "s" by "$j\omega$" to get $H(j\omega)$. Then take magnitude of the resulting complex number $|H(j\omega)|$.

2. Label the frequency of poles (p_1, p_2 etc.) and zeros (z_1, z_2 etc.) on the "ω" axis.

3. Find the slope of each segment between each pole/ zero frequency: suppose, there are n_p poles and n_z zeros at frequencies **lower** than ω_0. Then, the slope at ω_0 is $n_z \times (20\ \mathrm{dB/\ dec}) + n_p \times (-20\ \mathrm{dB/\ dec})$, i.e. it is the sum of contributions of all poles and zeros at frequencies lower than ω_0.

4. Pick a frequency other than any pole/ zero frequency: usually at least one decade away from p_i or z_i. Using this frequency value to find out the exact complex number of the $H(j\omega)$. Calculate its amplitude and convert to dB.

5. Now you should have the complete Bode plot drawing.

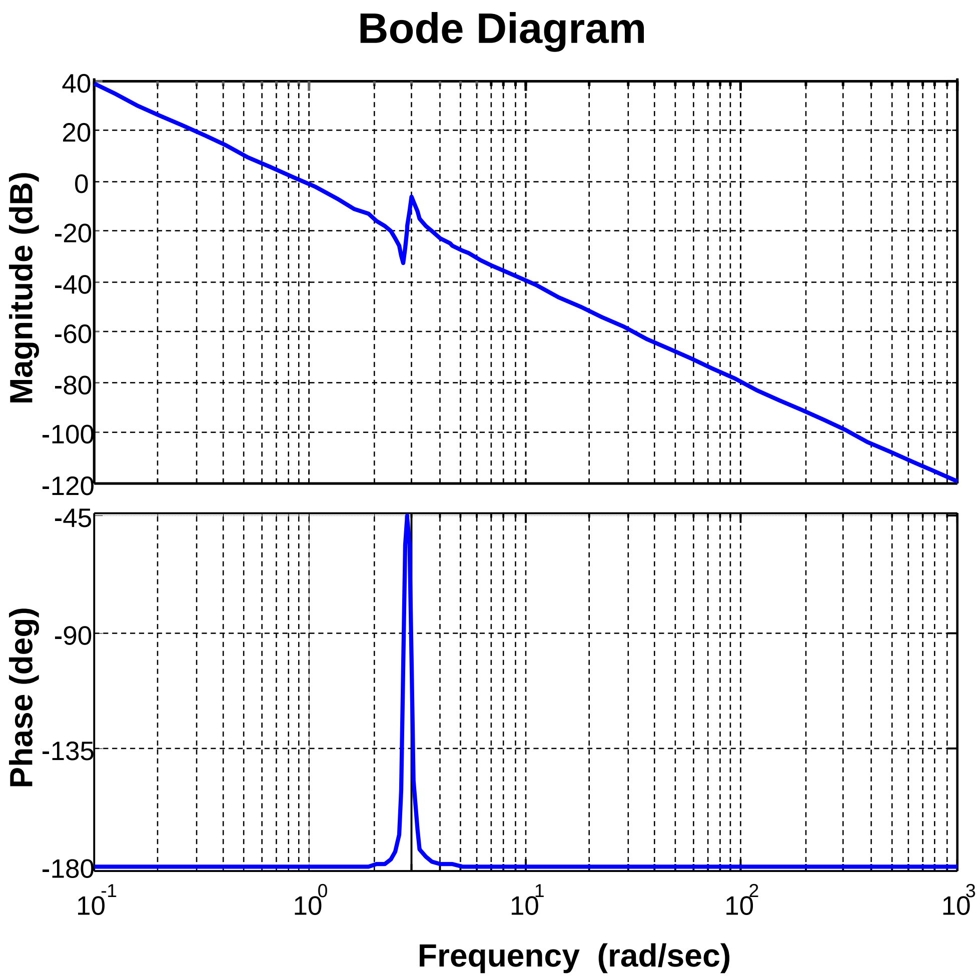
Play by Yourself: Bode Plots Using Matlab (1)

How can we display the following transfer function?

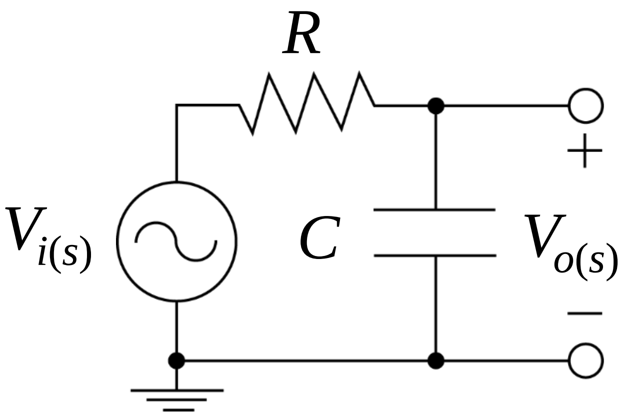
$$H(s) = \frac{s^2 + 0.1s + 7.5}{s^4 + 0.12s^3 + 9s^2}$$

Matlab code:

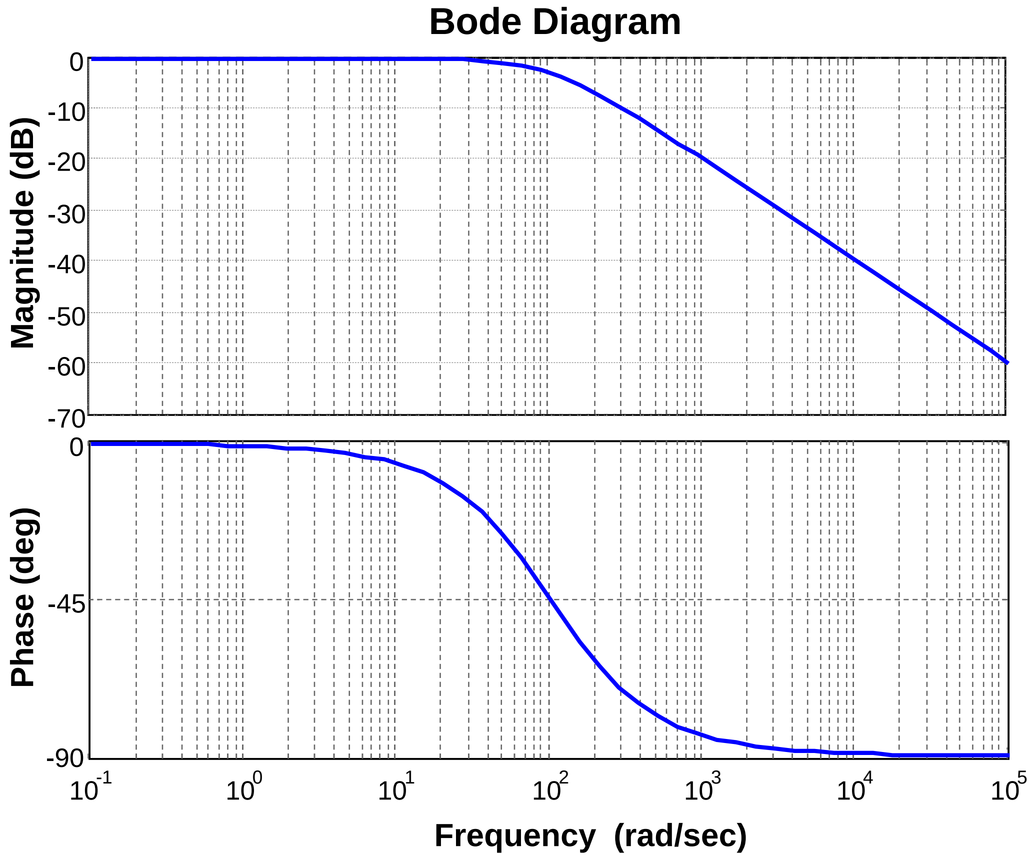
```
H = tf([1  0.1   7.5], [1  0.12  9  0  0]);
bode(H,{0.1,1e3});
set(findall(gcf,'type','line'),'linewidth',2)
grid on;
```



Play by Yourself: Bode Plots Using Matlab (2)



$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{\frac{1}{sC}}{R+\frac{1}{sC}}=\frac{1}{1+sRC}$$

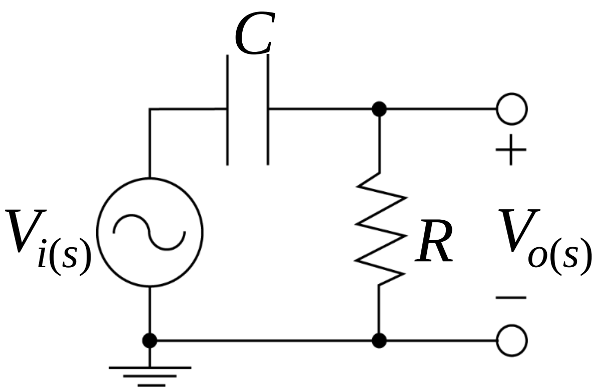


Given some real numbers: $R = 10\ \mathrm{K},\ C = 1\ \mu\mathrm{F}$, LHP pole $= -100$ rad/s

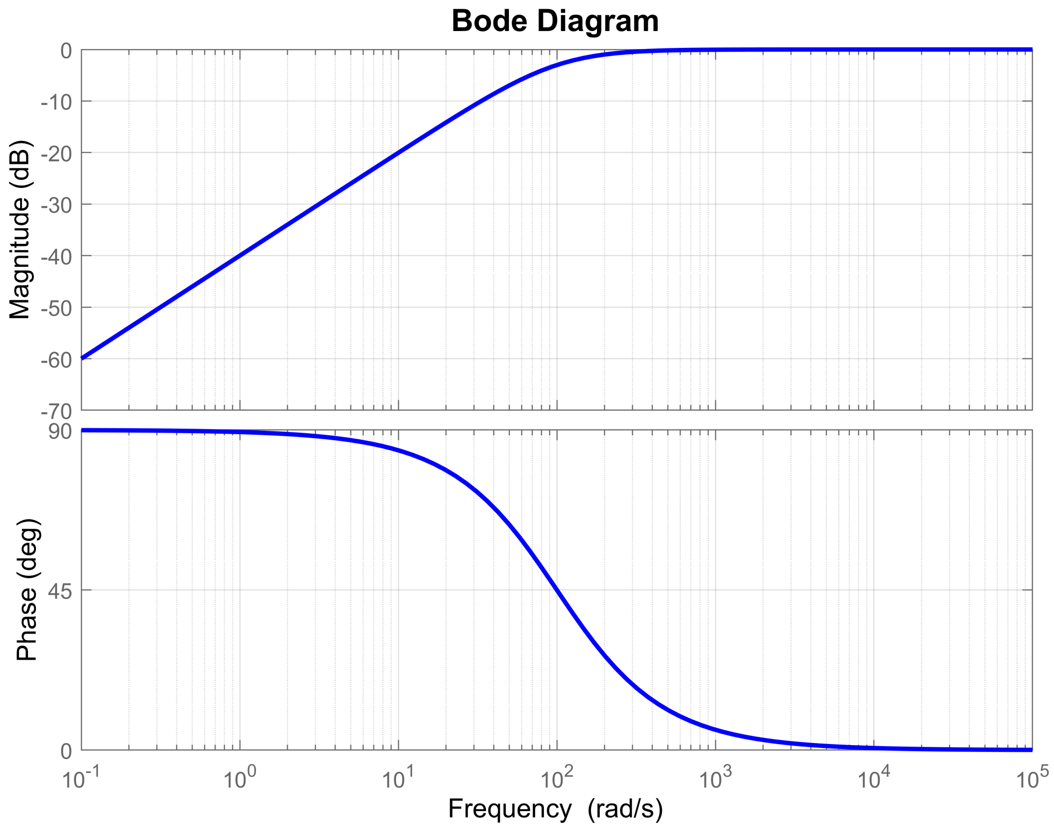
Note the capacitor's contribution to the roll-off.

Play by Yourself: Bode Plots Using Matlab (3)

Swap the capacitor with the resistor:



$$H(s)=\frac{V_O(s)}{V_i(s)}=\frac{R}{R+\frac{1}{sC}}=\frac{sRC}{1+sRC}$$



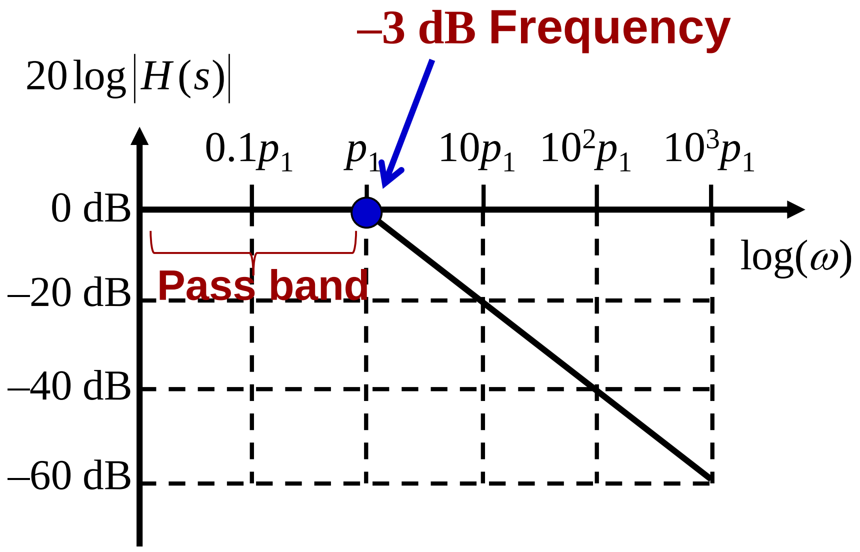
Note the capacitor’s contribution to the roll-off.

–3 dB Frequency

$$H(s) = 1 + \frac{s}{p_1} \Rightarrow |H(s)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{p_1}\right)^2}}$$

$$dB(H) = 20\log\left(\sqrt{1+\left(\frac{\omega}{p_1}\right)^2}\right) = -10\log\left[1+\left(\frac{\omega}{p_1}\right)^2\right]$$

$$dB(H) = \begin{cases} 0 & \text{when } \omega << p_1 \\ -20\log\omega + 20\log p_1 & \text{when } \omega >> p_1 \end{cases}$$



How about exactly at frequency p_1?

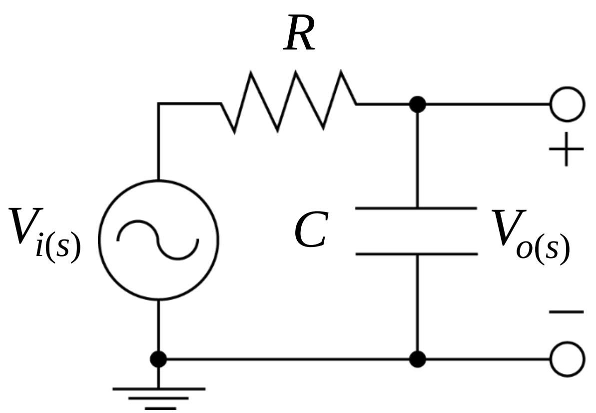
$$dB\left[H\left(p_1\right)\right] = -10\log 2 \simeq -3 \text{ when } \omega = p_1$$

Lecture Milestones

- The basics of signal processing and analysis are covered.
- Math ←→ Circuits, toolkit: impedance, KVL, KCL
- Study frequency response of a few examples in order to account the behavior of a circuit operating at different frequencies.
- Study time constants for approximating the response of amplifiers.

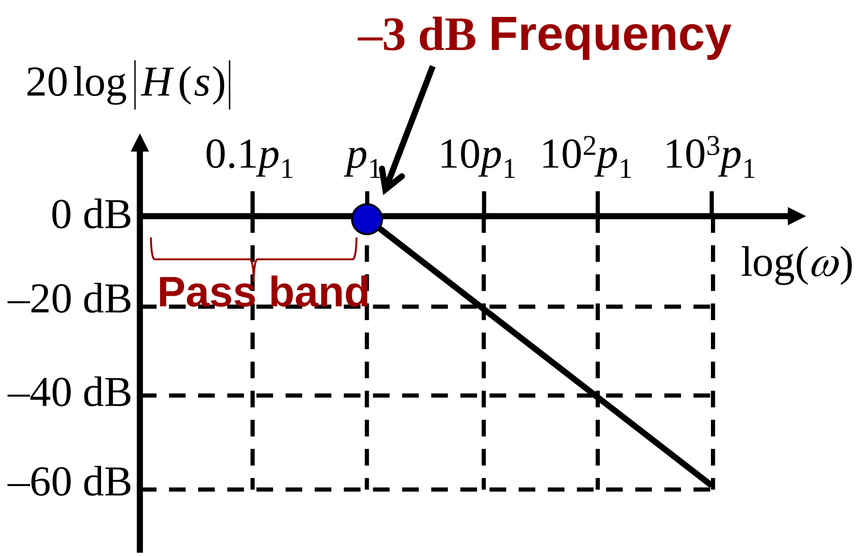
Understand bode plot, pole, zero, the method to identify the pole/zero elements in real circuits.

Real Circuit Frequency Analysis



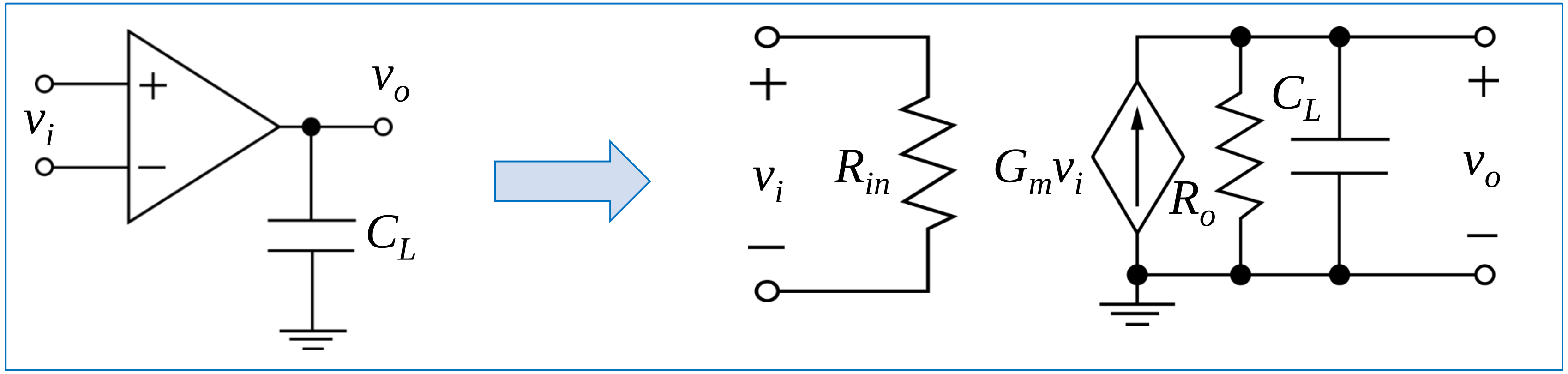
$$H(s)=\frac{V_O(s)}{V_i(s)}=\frac{\frac{1}{sC}}{R+\frac{1}{sC}}=\frac{1}{1+sRC},\quad p_1=\frac{1}{RC}$$

$$20\log|H(\omega\to 0)|=0,\ \ 20\log|H(\omega=p_1)|=-3$$



- At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease.
- At a special frequency, the gain drops by 3 dB; This gives **idea** of frequency beyond which $|H|$ starts rolling off quickly → **pass band**

Frequency Response of General Single Stage Amplifier (1)

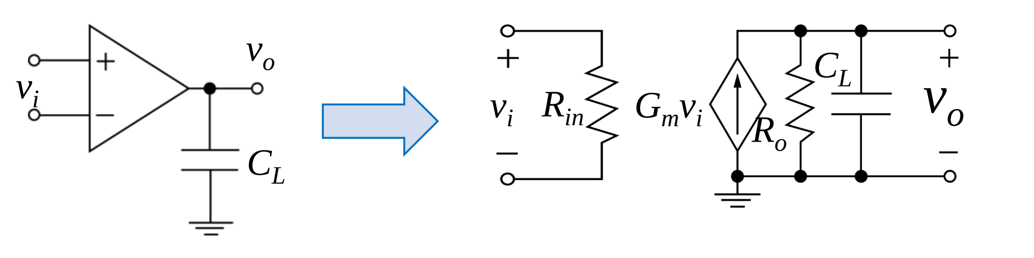


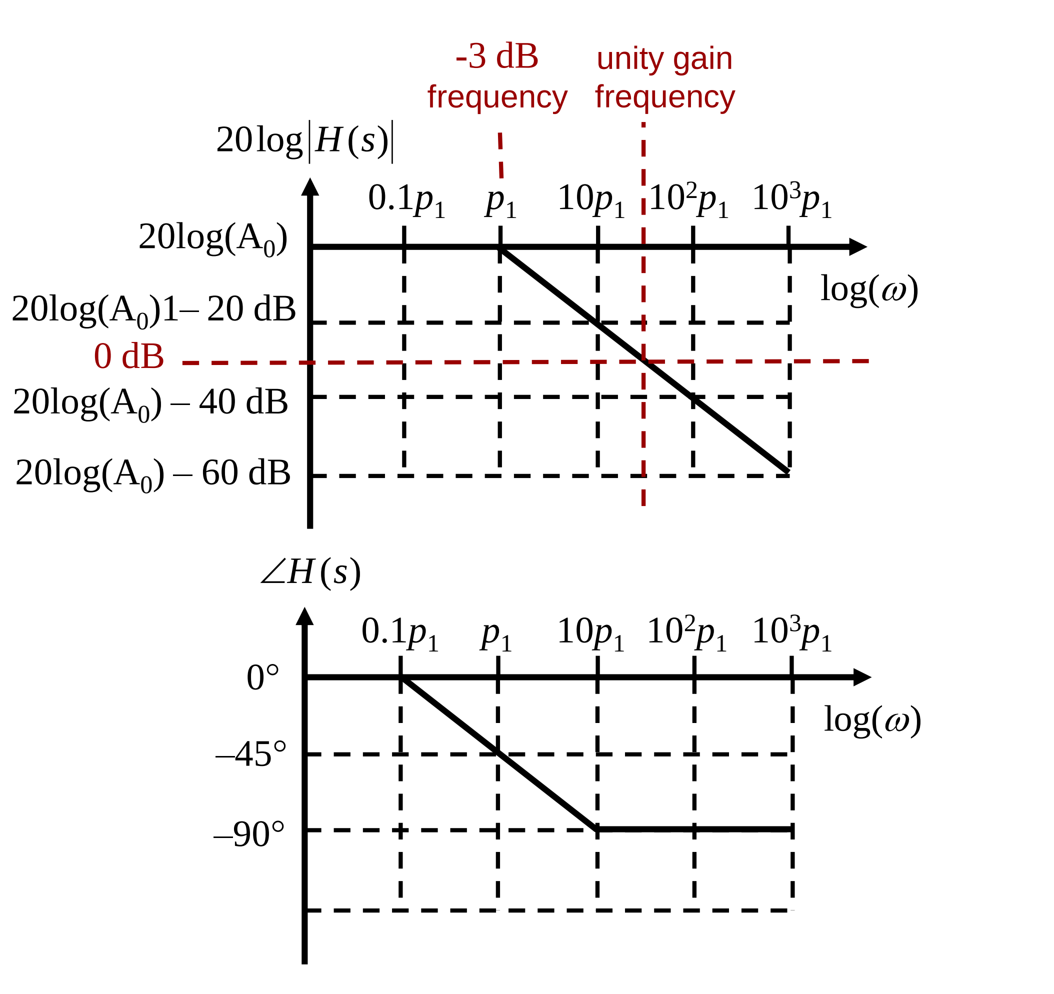
$$V_O(s) = [V_i(s) \times G_m] \times \left[R_O // \frac{1}{sC_L} \right]$$

$$H(s) = \frac{V_O(s)}{V_i(s)} = G_m \times \left(R_O // \frac{1}{sC_L} \right) = \frac{G_m R_O}{1 + sR_O C_L} = \frac{G_m R_O}{1 + \frac{s}{p_1}} \quad \text{where } p_1 = \frac{1}{R_O C}$$

A single pole low pass system, its low frequency gain: $\left| \mathrm{H}(\omega \to 0) \right| = \left| \frac{G_m R_O}{1 + j\omega R_O C_L} \right| \approx G_m R_O$

Frequency Response of General Single Stage Amplifier (1)





We can find its unity gain frequency:

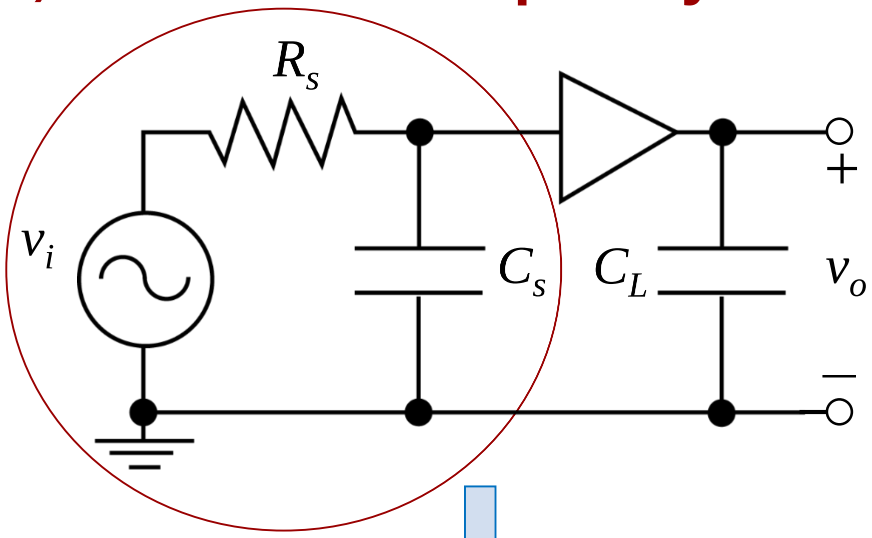
Let $|H(s)|=1, \Rightarrow \left|\dfrac{G_m R_O}{1+\dfrac{j\omega}{p_1}}\right|=1$

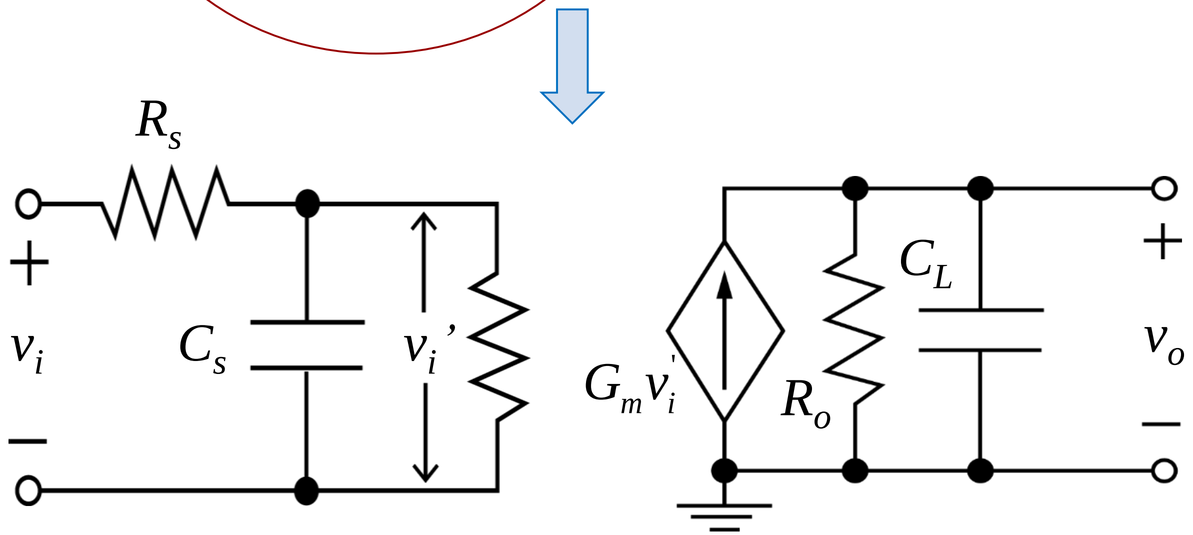
$$\Rightarrow G_m R_O = \left|1+\frac{j\omega}{p_1}\right| \simeq \frac{\omega}{p_1}$$

$$\Rightarrow \omega = (G_m R_O) \times p_1$$

Frequency Response of Single Stage Amplifier (2)

Now, add some complexity at the input:





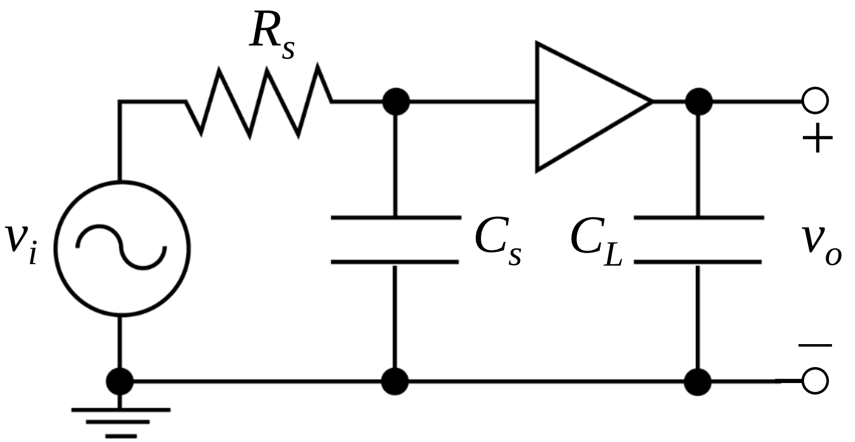
$$V_i' = V_i \frac{\frac{1}{sC_s} // R_{in}}{R_s + \frac{1}{sC_s} // R_{in}}$$

$$H(s) = \frac{V_O(s)}{V_i(s)} = \frac{\left[G_m V_i'\right] \times \left[R_o // \frac{1}{sC}\right]}{V_i}$$

$$= \frac{R_{in}}{R_s + R_{in} + sR_s R_{in} C_s} \times \frac{G_m R_o}{1 + sR_o C_L}$$

This is a 2-pole system

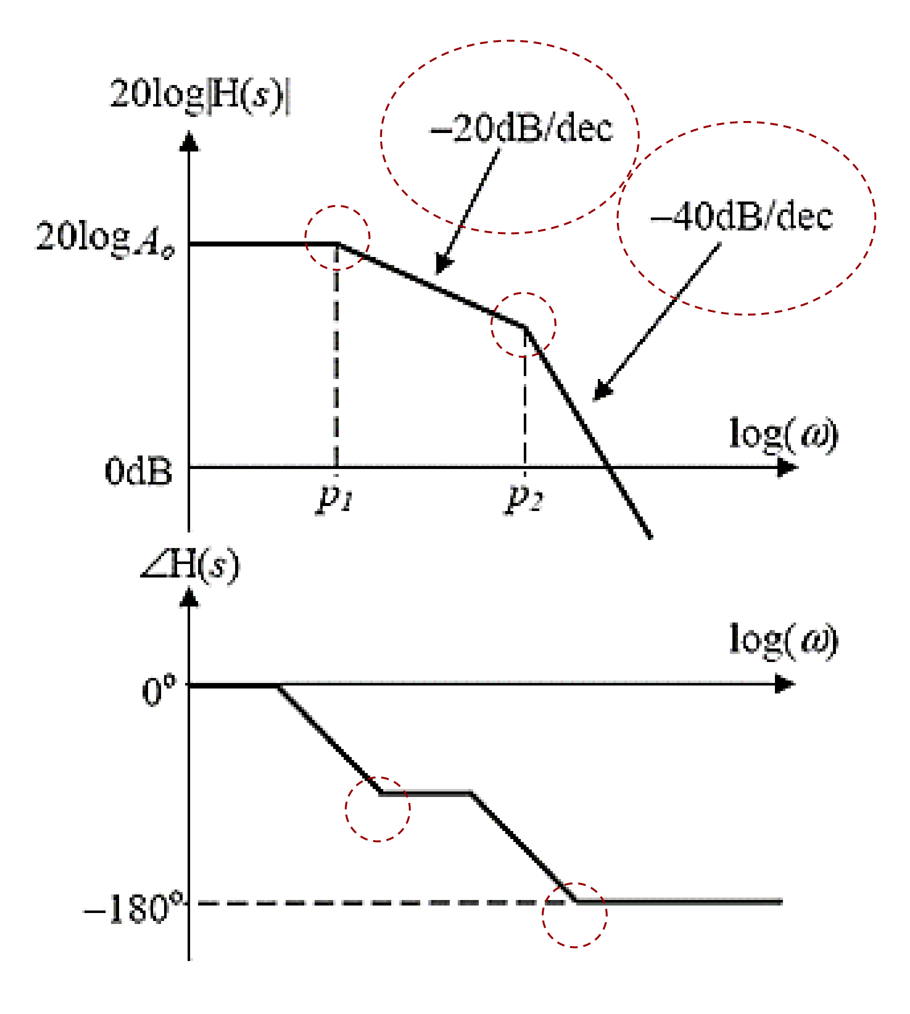
Frequency Response of Single Stage Amplifier (2)



$$H(s) = \frac{R_{in}}{R_s + R_{in} + sR_sR_{in}C_s} \times \frac{G_mR_o}{1 + sR_oC_L}$$

$$p_1 = \frac{1}{R_oC_L}$$

$$p_2 = \frac{R_s + R_{in}}{R_sR_{in}C_S} = \frac{1}{\left(R_s // R_{in}\right)C_S}$$



How About a Very Complicated Circuit?

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$$H(s)=\frac{b_0+b_1s+b_2s^2+\ldots+b_{m\text{-}1}s^{m\text{-}1}+b_ms^m}{a_0+a_1s+a_2s^2+\ldots+a_{n\text{-}1}s^{n\text{-}1}+a_ns^n}=\frac{H_0\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)}$$

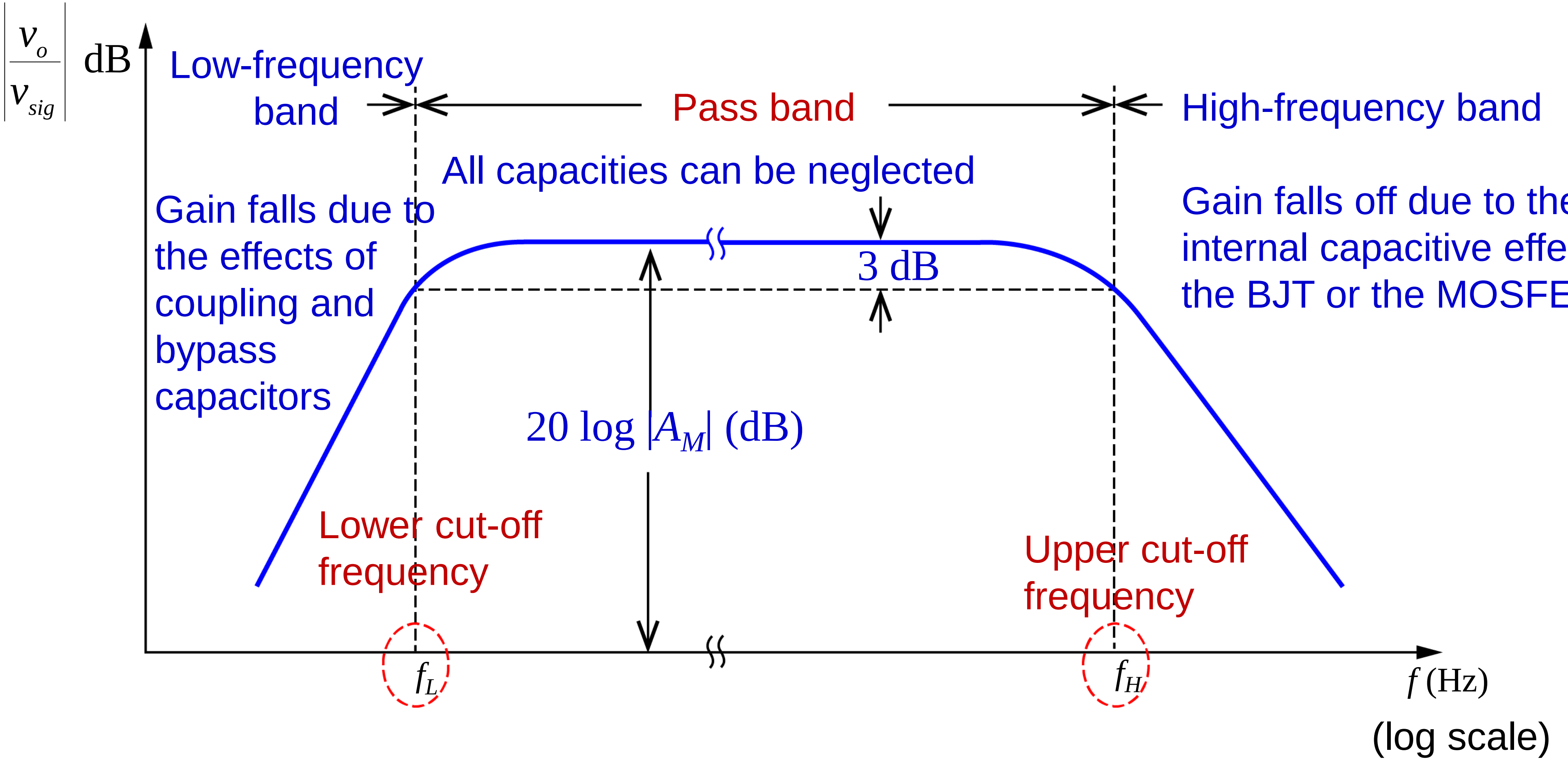
- However, for multistage amplifier with many capacitive elements, explicit computation (by hand) of the frequency response (i.e. transfer function) is generally **impractical**.
- Machine computation is cheap and getting cheaper all the time, so perhaps the analysis of networks doesn't present much of a problem. However, we are interested in developing design insight so that if a simulator tells us that there is a problem, we have some idea of what to do about it.
- In fact, accurate calculation on the frequency response may not be required but only a very rough estimation to predict the performance is sufficient. In that case, the -3 dB frequency is the most important parameter.

Lecture Milestones

- The basics of signal processing and analysis are covered.
- Math ←→ Circuits, toolkit: impedance, KVL, KCL
- Study frequency response of a few examples in order to account the behavior of a circuit operating at different frequencies.
- Study time constants for approximating the response of amplifiers.

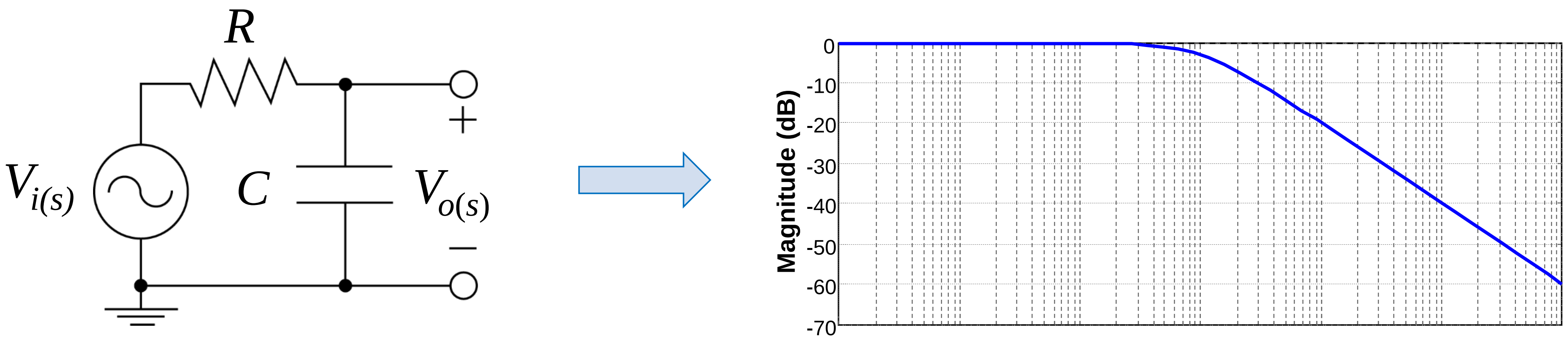
Understand bode plot, pole, zero, the method to identify the pole/zero elements in real circuits.

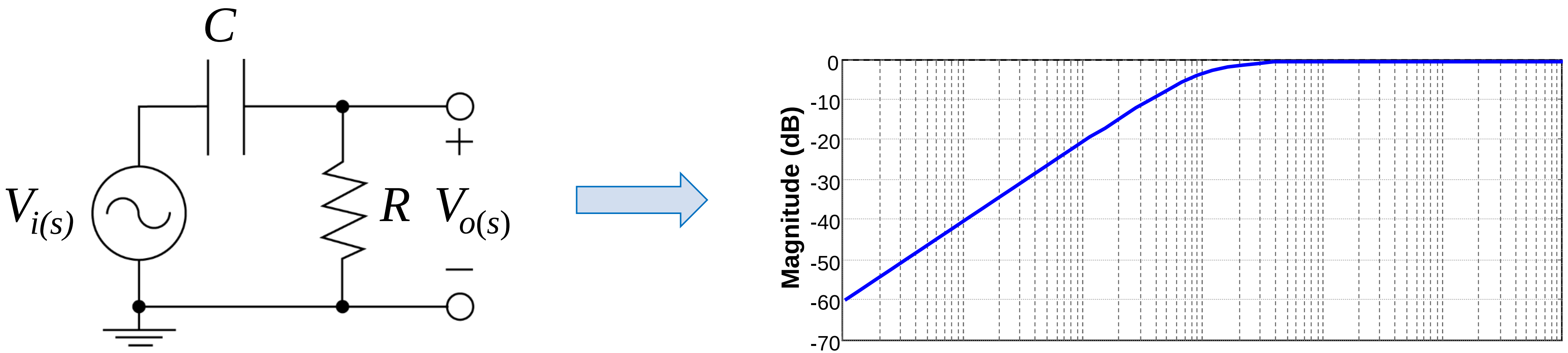
Find the Sources of the Lower and Higher Cut-off Frequencies



Identify capacitors that contribute to the cut-off frequency.

Root Cause of Frequency Response

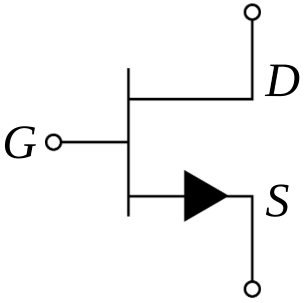


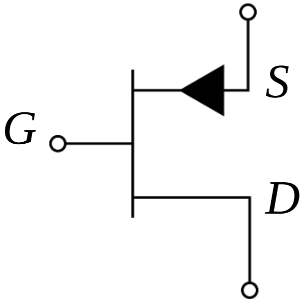


Capacitor: impedance changes with frequency (like a variable resistor).

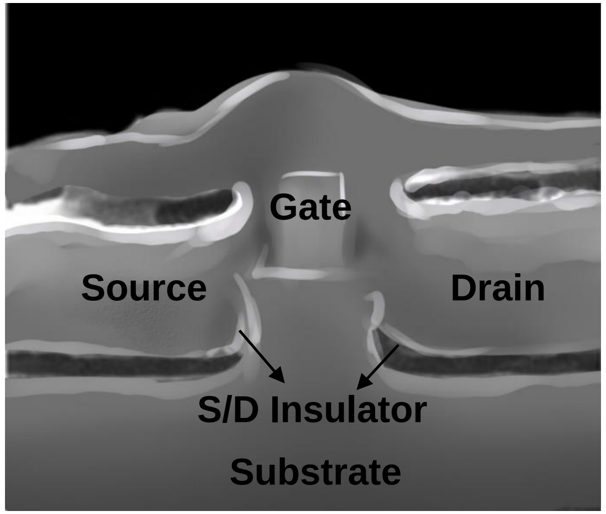
Where do Capacitors come from?

Clean Schematic → **Complicated Real Silicon** → **Interconnection**

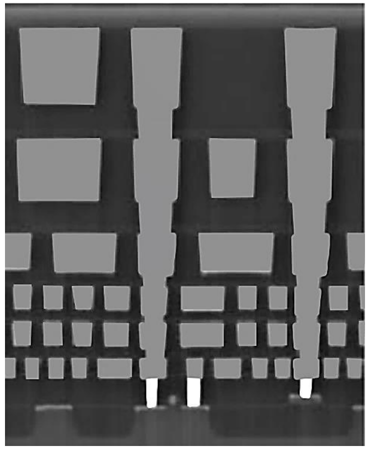


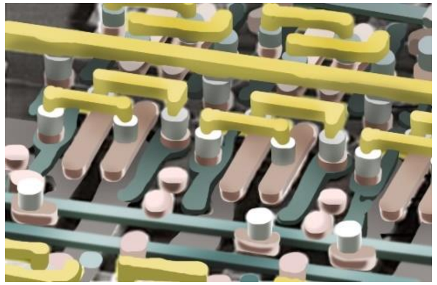


Transistor symbol



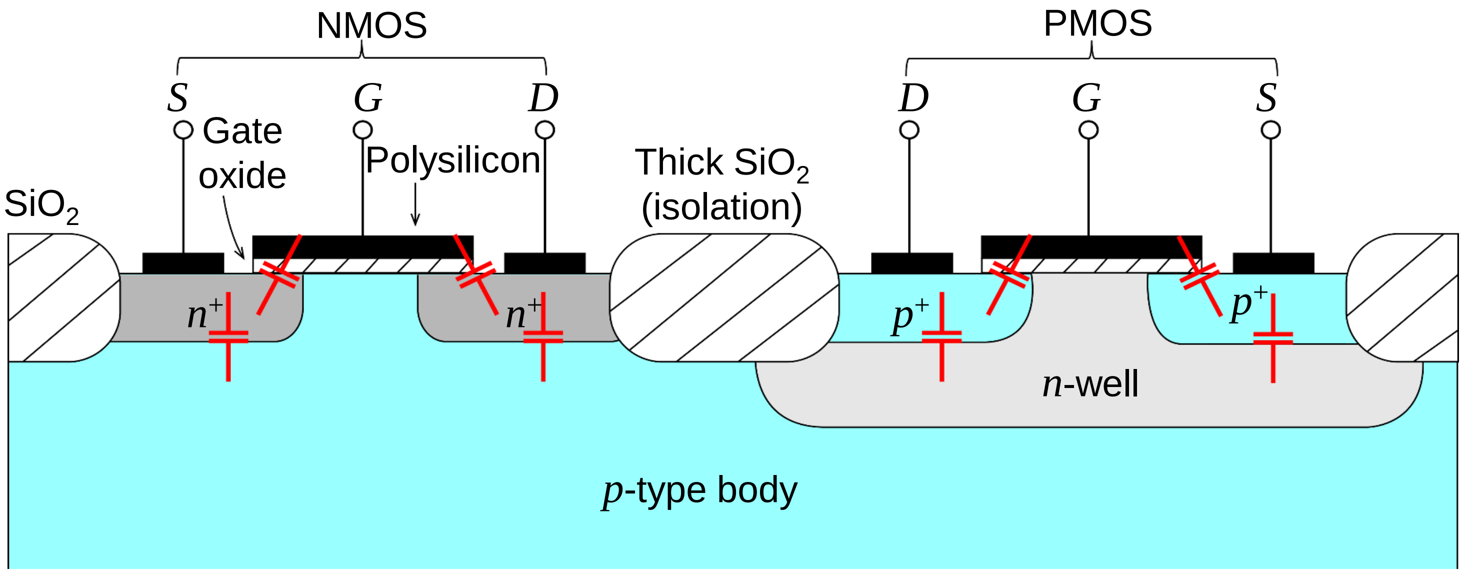
Cross section





Some capacitors come from the capacitive structures from active devices (transistors), some come from the coupling effect between interconnections.

Identify Parasitic Capacitors

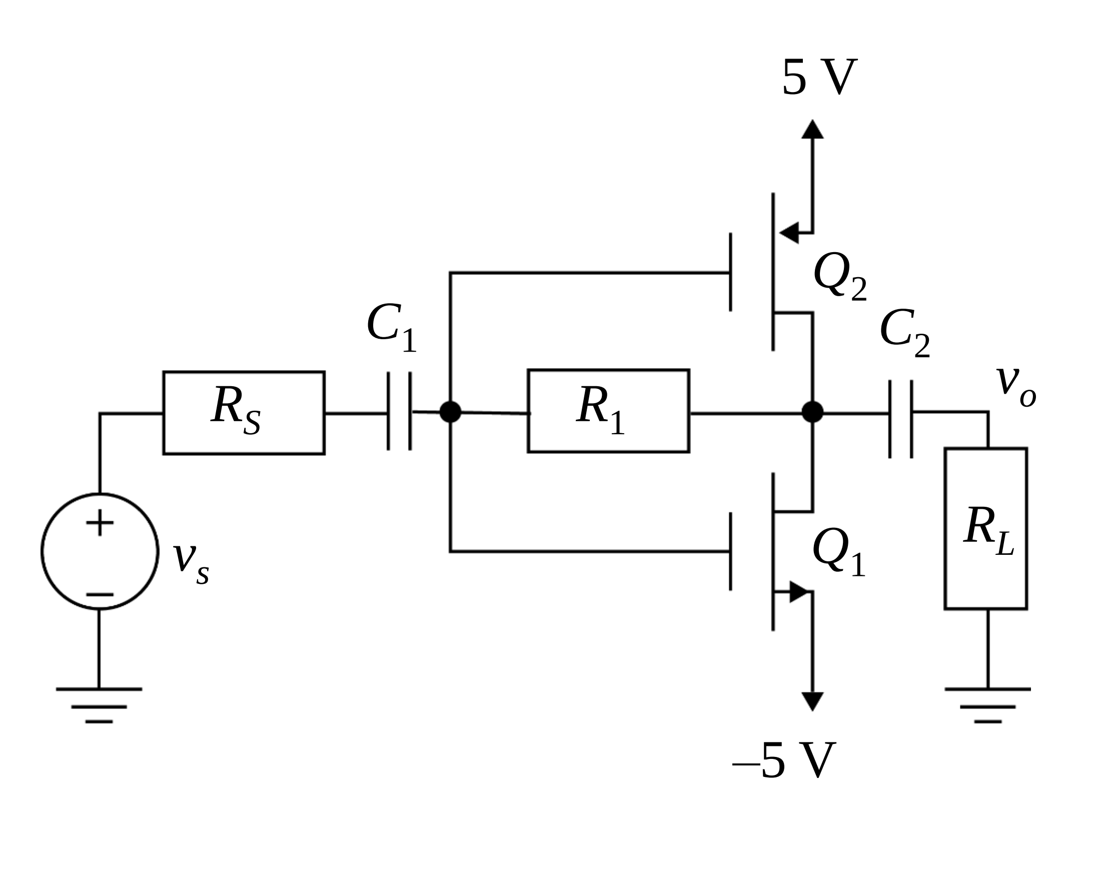


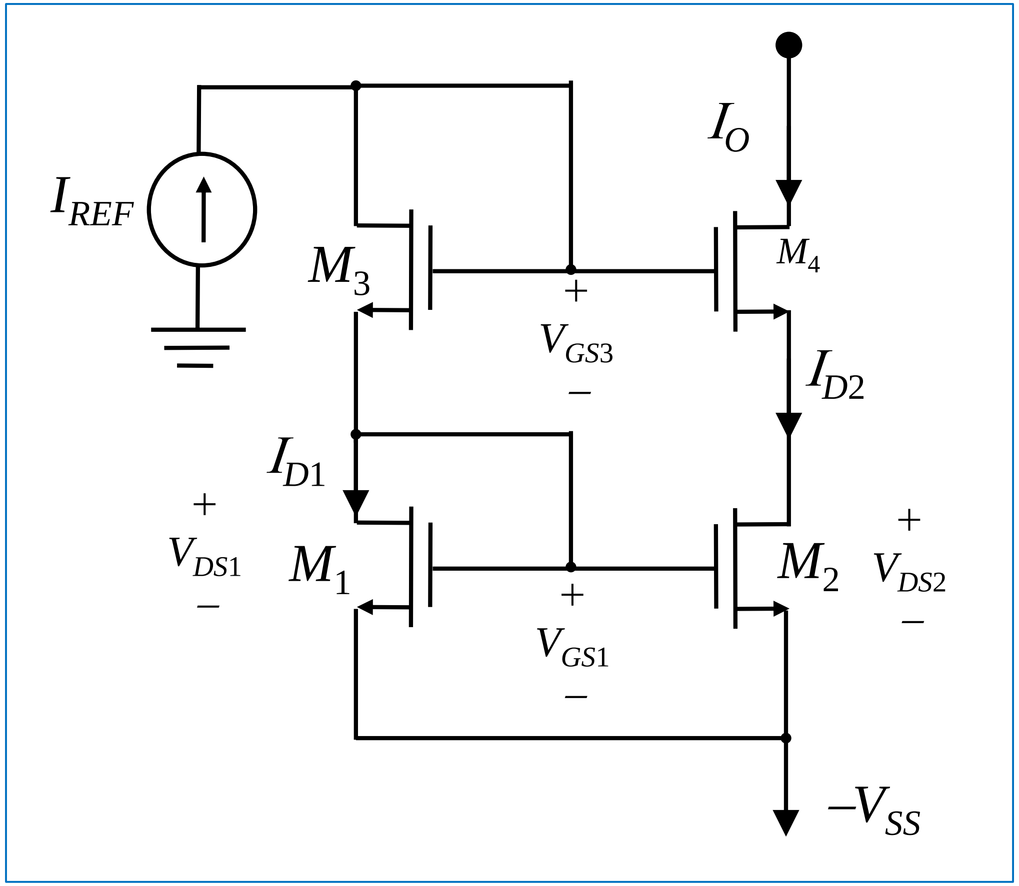
1. Capacitance between **Gate/Source** and **Gate/Drain** due to the overlap of gate electrode (Parallel-plate capacitor)

2. Junction capacitance between **Source/Body** and **Drain/Body** (Reverse-bias junction)

Note: The body of NMOS is automatically connected to the lowest voltage in the system; the body of PMOS is automatically connected to the highest voltage in the system.

Identify Parasitic Capacitors: Examples





SCTC and OCTC Methods

Developed in the mid-1960s at MIT. Procedure is as follows:

1. **Disable** all independent sources (voltage sources $\rightarrow$ Short Circuit; current sources $\rightarrow$ Open Circuit); **Do not** remove or "disable" dependent sources!
2. **Identify** capacitors contributing to the frequency of interest, i.e., lower of higher cut-off.

higher cut-off

3. **Idealise** irrelevant capacitors by **short circuit** (because at high f, cap → short)
4. For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) **removed** (i.e. **Open Circuits**) and determine the resistance, R_i seen by C_i
5. Higher cut-off frequency is estimated as:

$$\omega_{H-3\mathrm{dB}} \approx \frac{1}{\sum_i C_i R_i}$$

lower cut-off

3. **Idealise** irrelevant capacitors by **open circuit** (because at low f, cap → open)
4. For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) **removed** (i.e. **Short Circuits**) and determine the resistance, R_i seen by C_i
5. Lower cut-off frequency is estimated as:

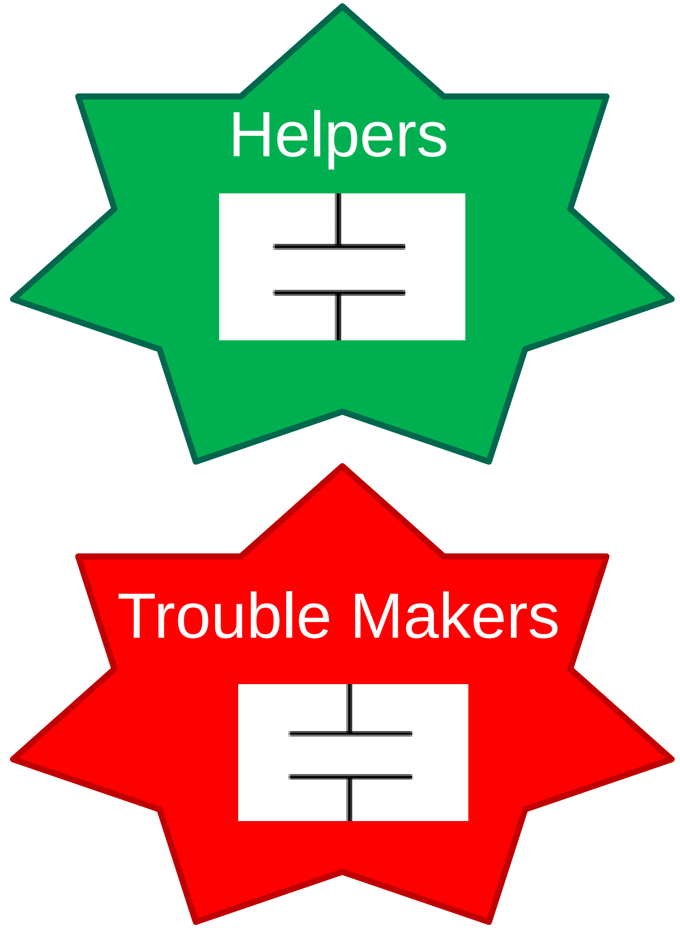
$$\omega_{L-3\mathrm{dB}} \approx \sum_i \frac{1}{C_i R_i}$$

SCTC and OCTC Methods

4-step Standard Procedure

- **Helpers = Irrelevant Caps**
- **Trouble Makers = Contributing Caps**

Disable Batteries → Group Capacitors → Idealise Helpers → Find out the effect of the trouble makers one by one



Q1: How to Classify Capacitors?

A Game of Short/ Open

- Short a cap ➔ Higher Gain ➔ **Helper** for **high frequency**
  ⟺ **Trouble maker** for **low frequency**

- Open a cap ➔ Higher Gain ➔ **Helper** for **low frequency**
  ⟺ **Trouble maker** for **high frequency**

Q2: How to Idealise the Helpers

Helpers are those who don't contribute to the drop of the gain.

- For **low frequency** helpers: open them

- For **high frequency** helpers: short them

Q3: How to find out the Effect of the Trouble Makers One by One?

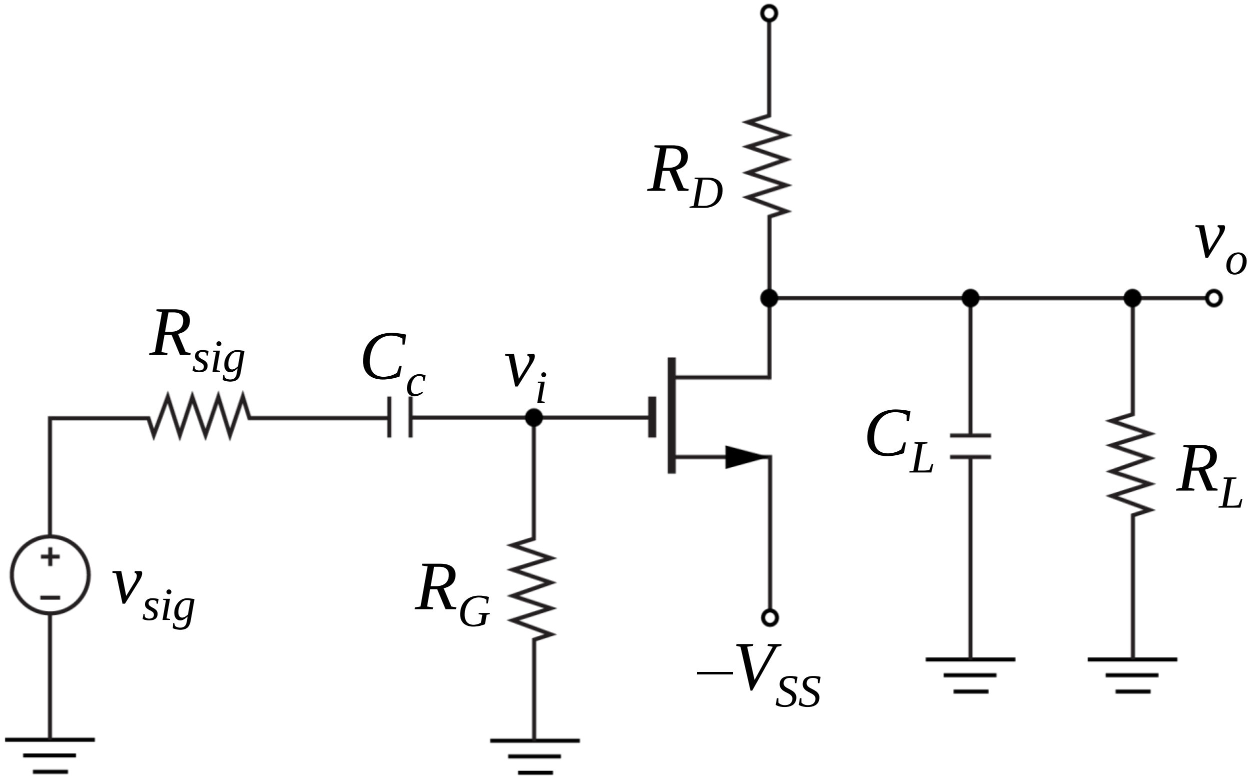
Trouble makers are those who cause the drop of the gain.

- For low frequency trouble makers: when we calculate one cap’s bad effect (i.e., open circuit effect), we have to temporally remove the bad effect of the others by shortening them.

- For high frequency trouble makers : when we calculate one cap’s bad effect (i.e., short wire effect), we have to temporally remove the bad effect of the others by opening them.

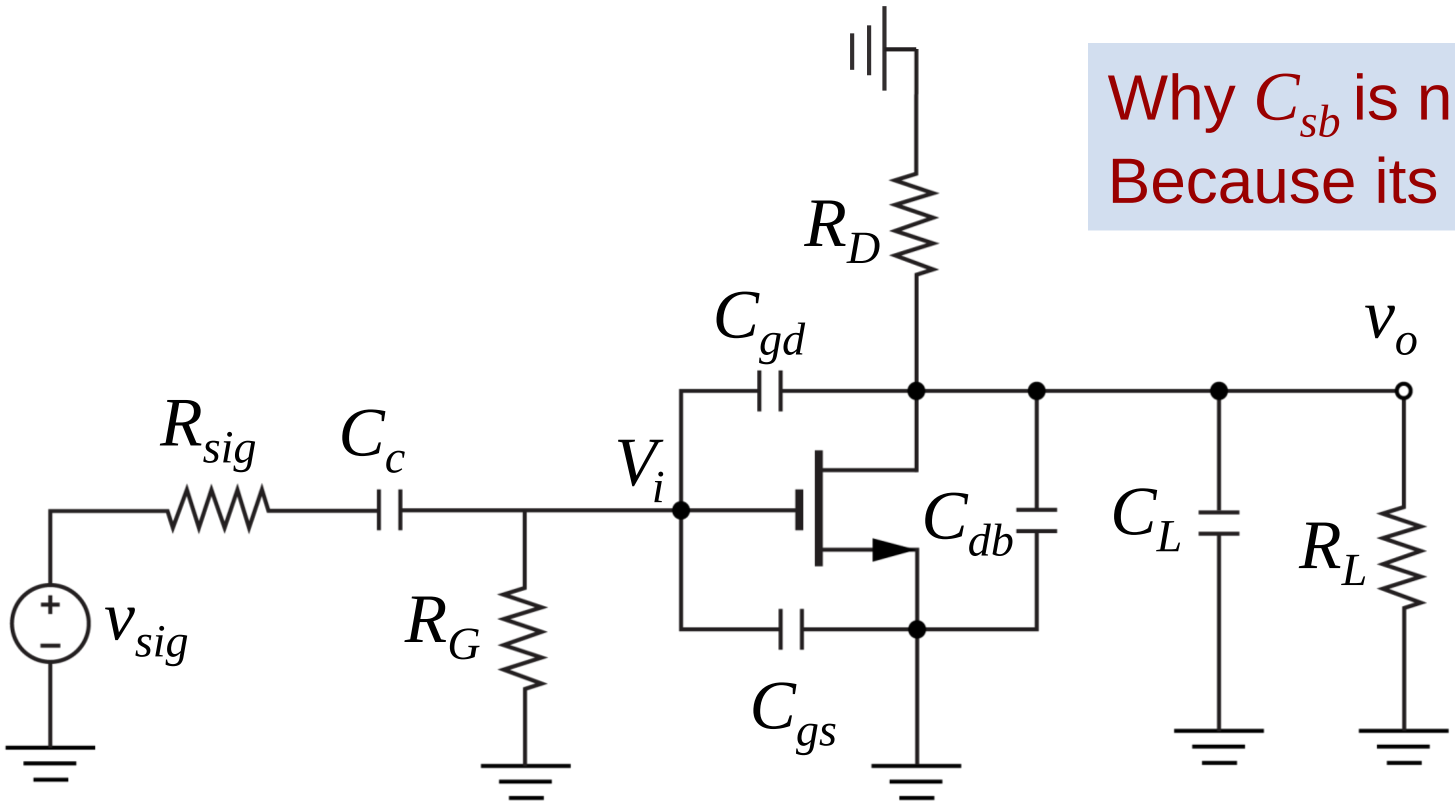
Learn by Case Study: Frequency Analysis of Common-Source Amplifier

Common Source Amp:



Learn by Case Study: Frequency Analysis of Common-Source Amplifier

Step 1: Label parasitic capacitors



Why C_{sb} is not labeled?
Because its body → $-V_{ss}$.

Note that body of the transistor is connected to $-V_{SS}$ (AC ground).

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

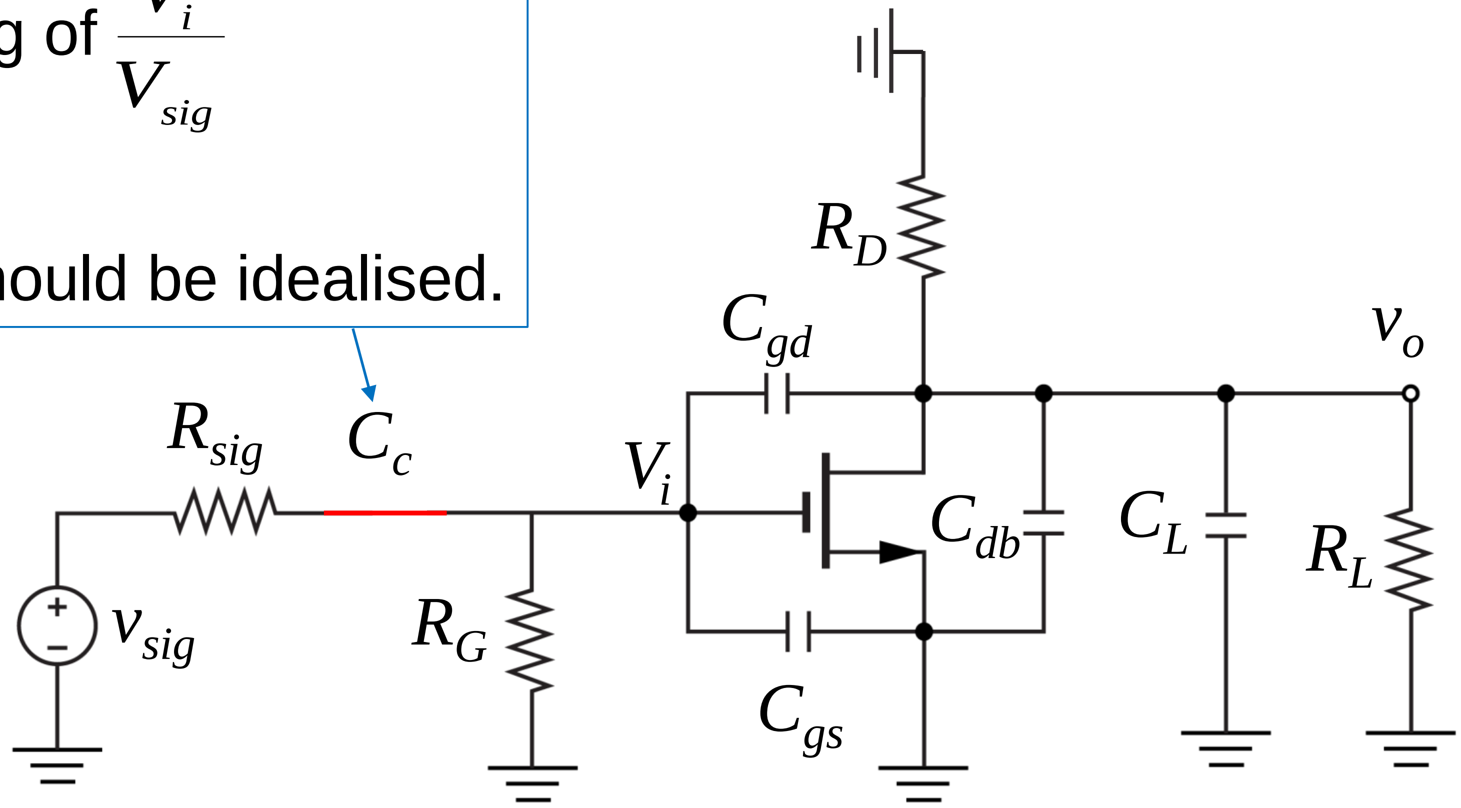
Learn by Case Study: Frequency Analysis of Common-Source Amplifier

At High Frequency

Short C_c → better sharing of $\frac{V_i}{V_{sig}}$

→ higher gain

→ C_c helper for high f, should be idealised.

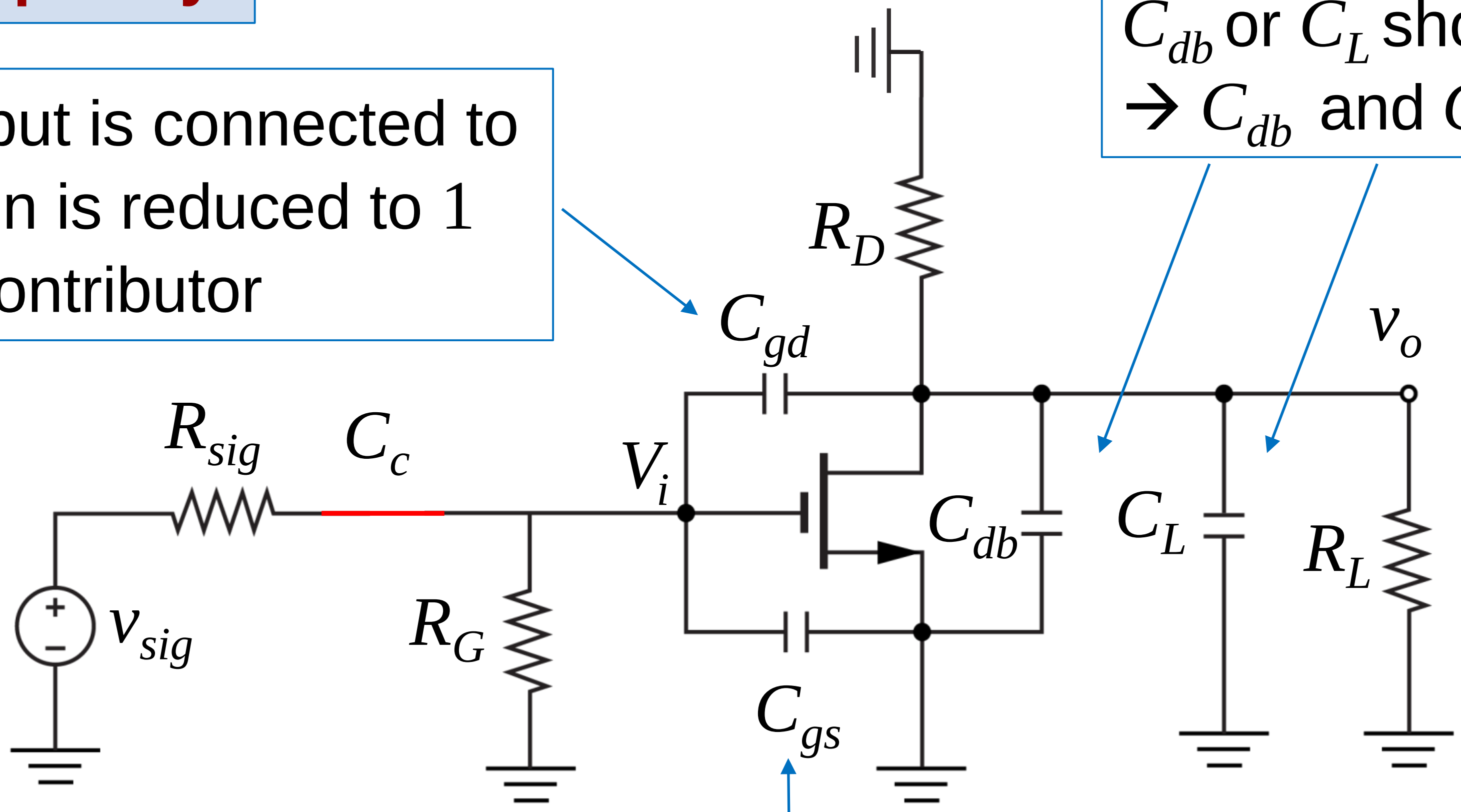


Learn by Case Study: Frequency Analysis of Common-Source Amplifier

At High Frequency

C_{gd} short: Input is connected to output → gain is reduced to 1
→ C_{gd} is a contributor

C_{db} or C_L short: $V_o = 0$,
→ C_{db} and C_L are contributors



One helper and four trouble makers

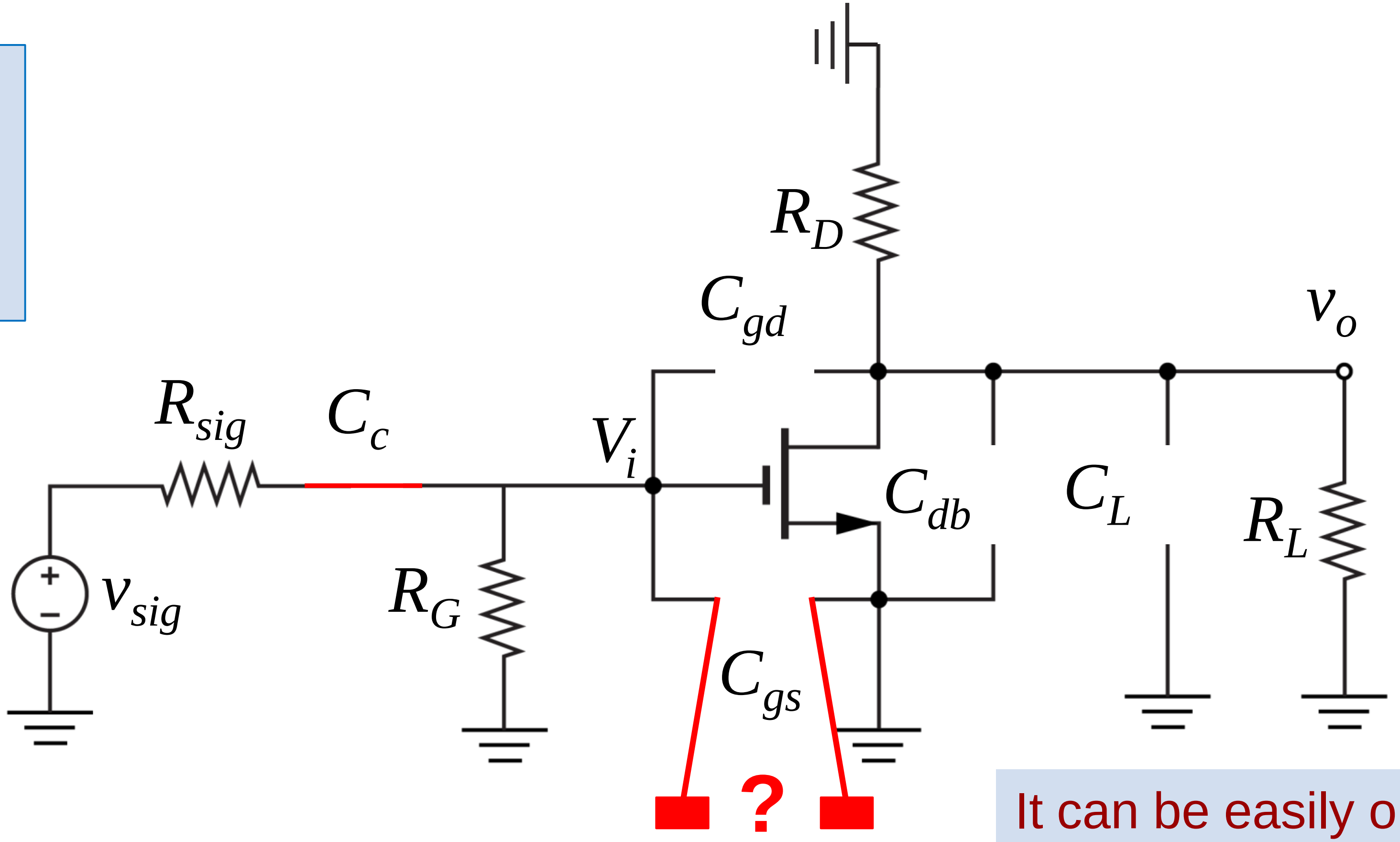
C_{gs} short: $V_i = 0$, → $V_o = 0$, → C_{gs} is a contributor

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

At High Frequency	Step 3: Find out *Ri* seen by C_i

Seen by C_{gs}:

$$R_{C_{gs}} = R_{sig} // R_G$$



It can be easily obtained by placing a test voltage.

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

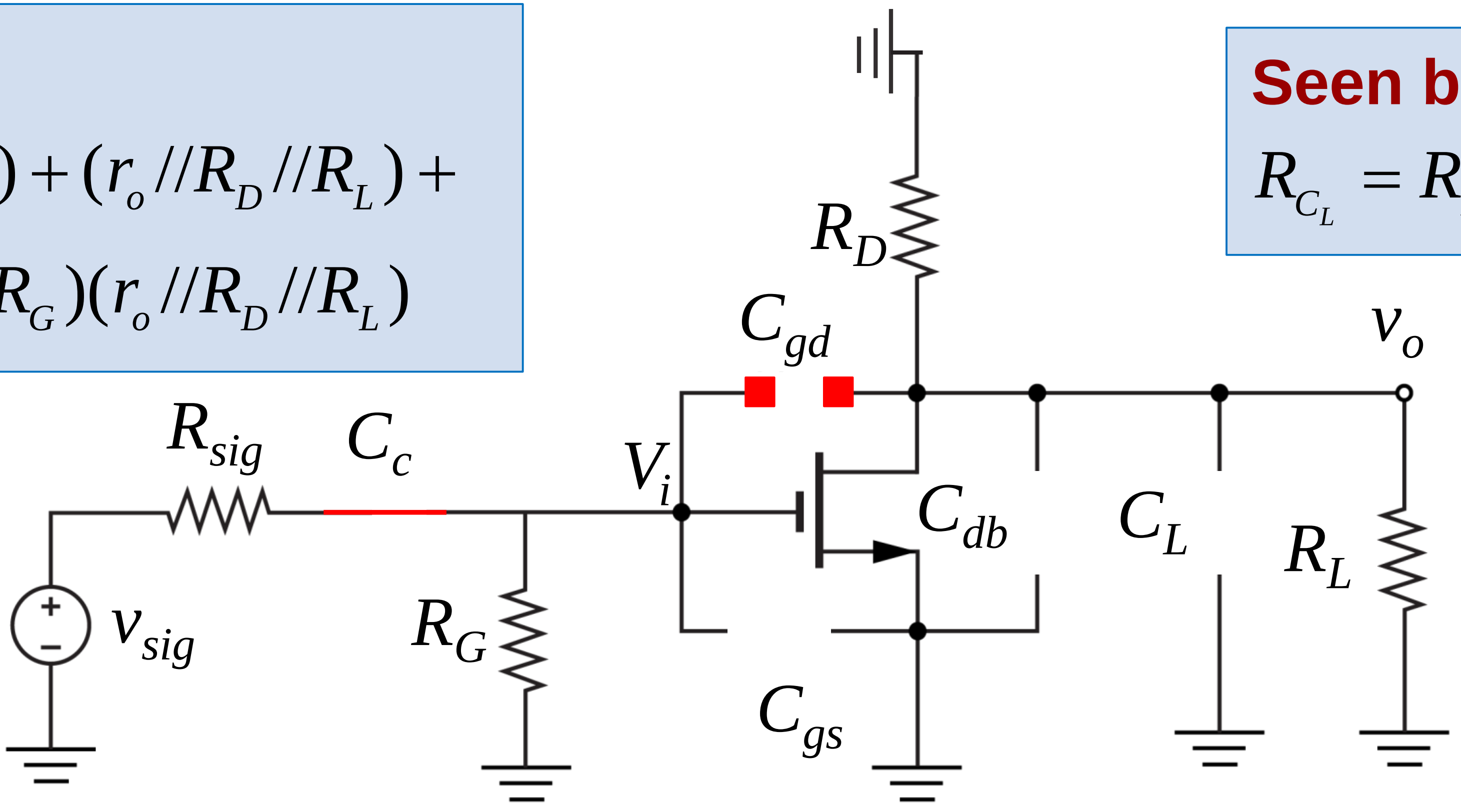
At High Frequency	Step 3: Find out R_i seen by C_i

Seen by C_{gd}:

$$R_{C_{gd}} = (R_{sig}//R_G) + (r_o//R_D//R_L) + g_m(R_{sig}//R_G)(r_o//R_D//R_L)$$

Seen by C_L (and C_{db}):

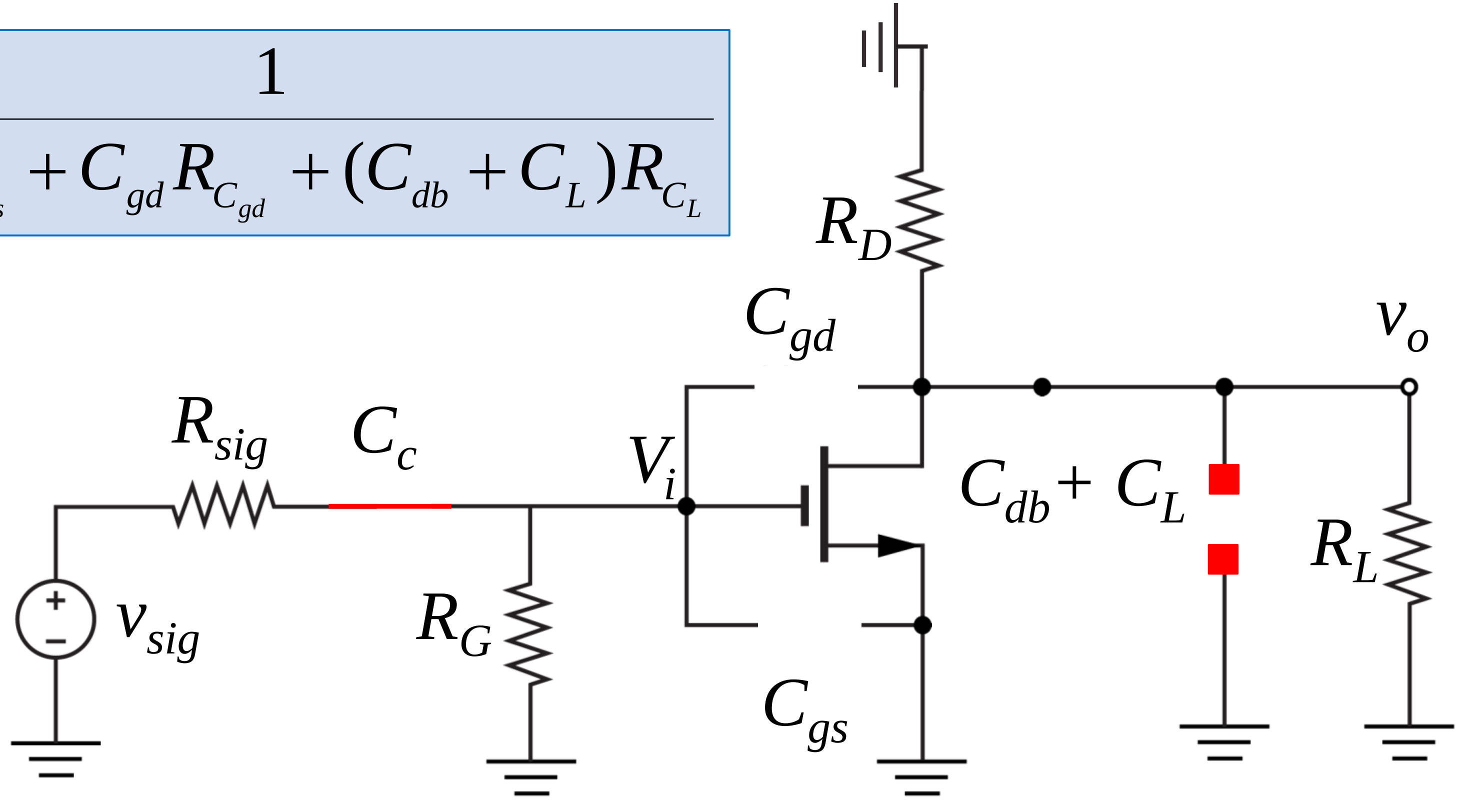
$$R_{C_L} = R_L//R_D//r_o$$



Learn by Case Study: Frequency Analysis of Common-Source Amplifier

At High Frequency	Step 4: Higher cut-off frequency

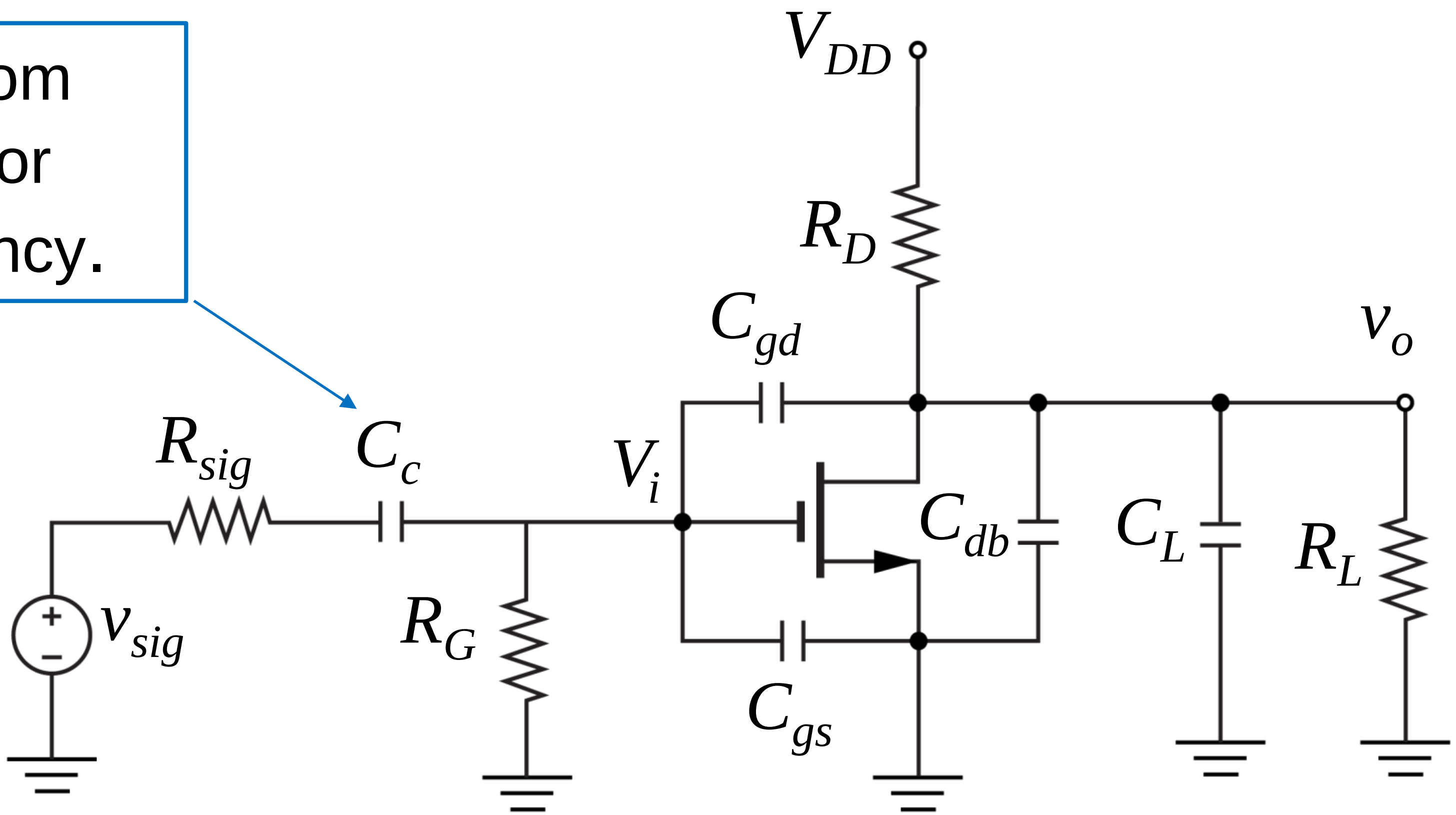
$$\omega_{H-3dB} = \frac{1}{C_{gs}R_{C_{gs}} + C_{gd}R_{C_{gd}} + (C_{db} + C_L)R_{C_L}}$$



Learn by Case Study: Frequency Analysis of Common-Source Amplifier

At Low Frequency	Step 2: Find out contributing capacitors

Open C_c → input is isolated from the circuits → C_c is a contributor (trouble maker) for low frequency.



Learn by Case Study: Frequency Analysis of Common-Source Amplifier

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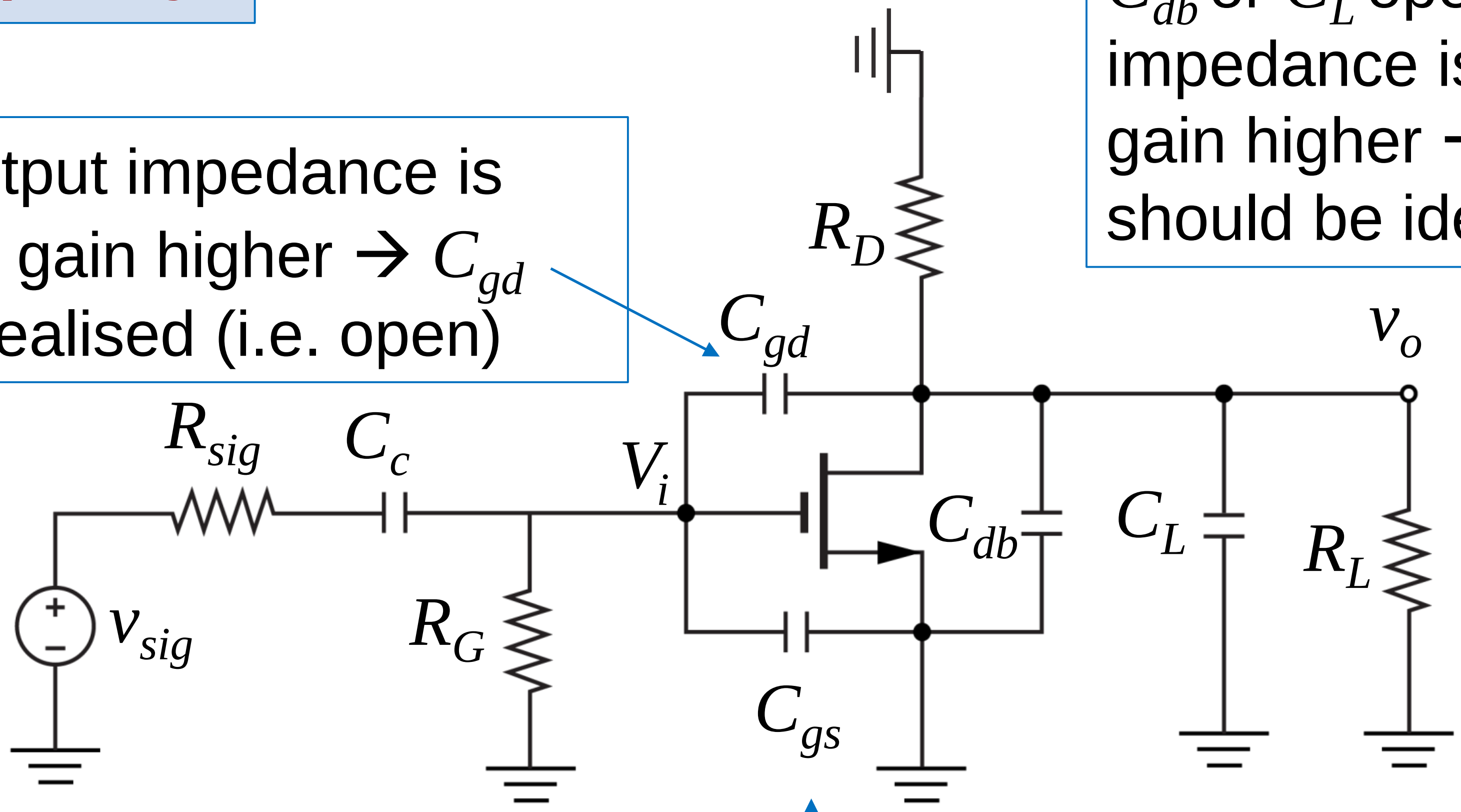
At Low Frequency

C_{gd} open: output impedance is increased → gain higher → C_{gd} should be idealised (i.e. open)

C_{db} or C_L open: output impedance is increased → gain higher → both of them should be idealized (open)



four helpers and one trouble makers

C_{gs} open: V_i better sharing of V_{sig}, → C_{gs} should be idealised (open.)

Learn by Case Study: Frequency Analysis of Common-Source Amplifier

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At Low Frequency

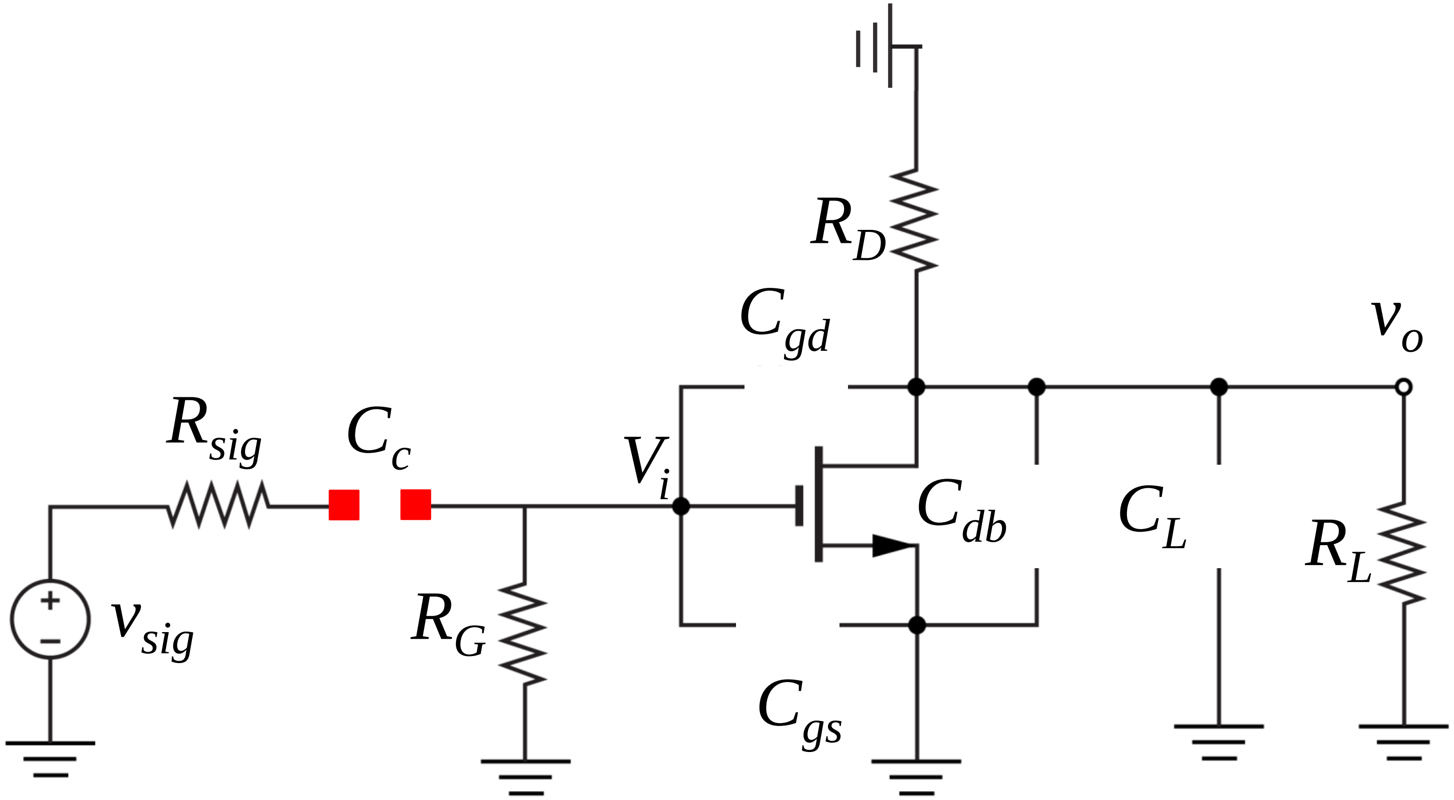
Step 3: Find out Ri seen by C_i

Seen by C_c:

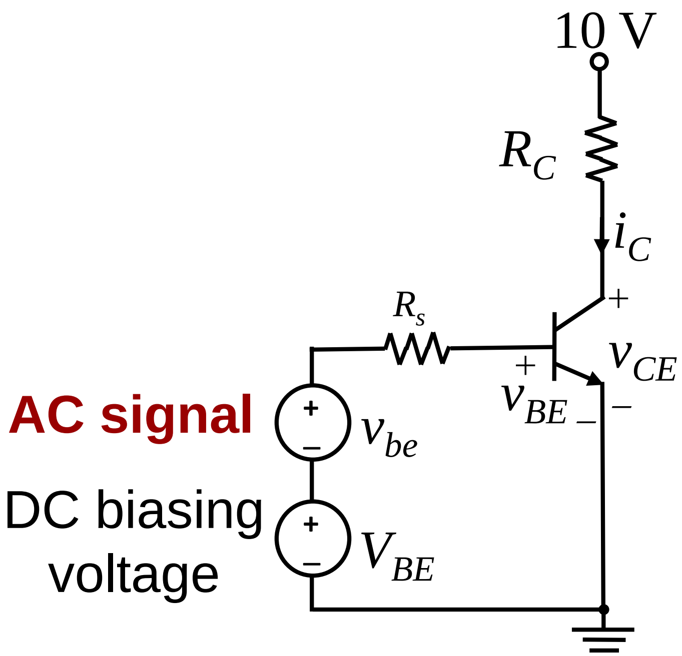
$$R_{C_c} = R_{sig} + R_G$$

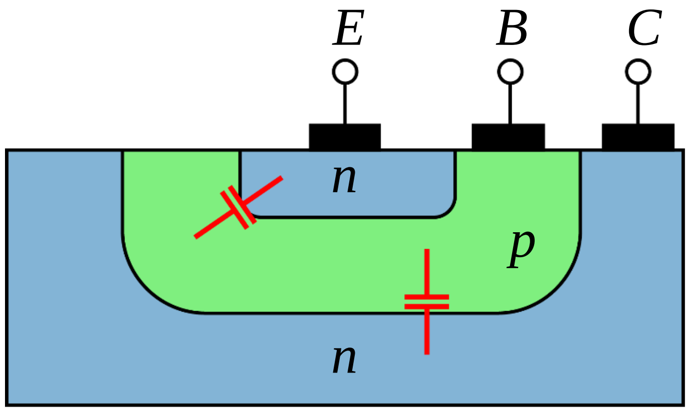
Step 4: Lower cut-off frequency

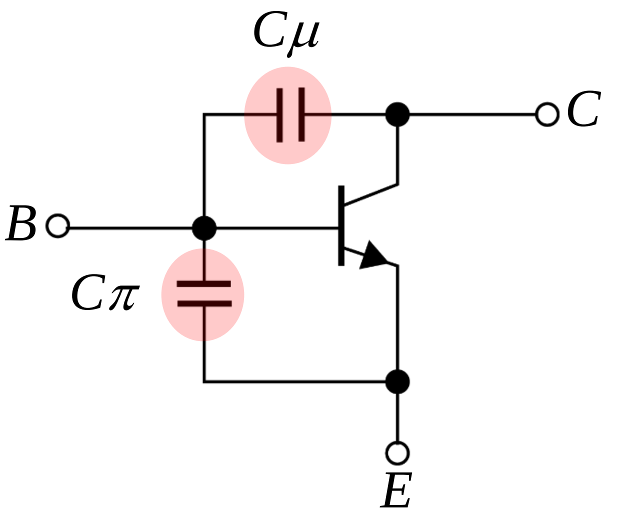
$$\omega_{L-3dB} = \frac{1}{C_c R_{C_c}}$$



Apply What We Learnt to BJT Circuits





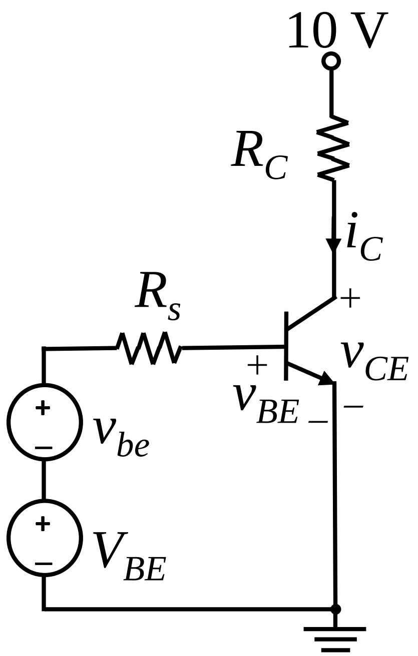


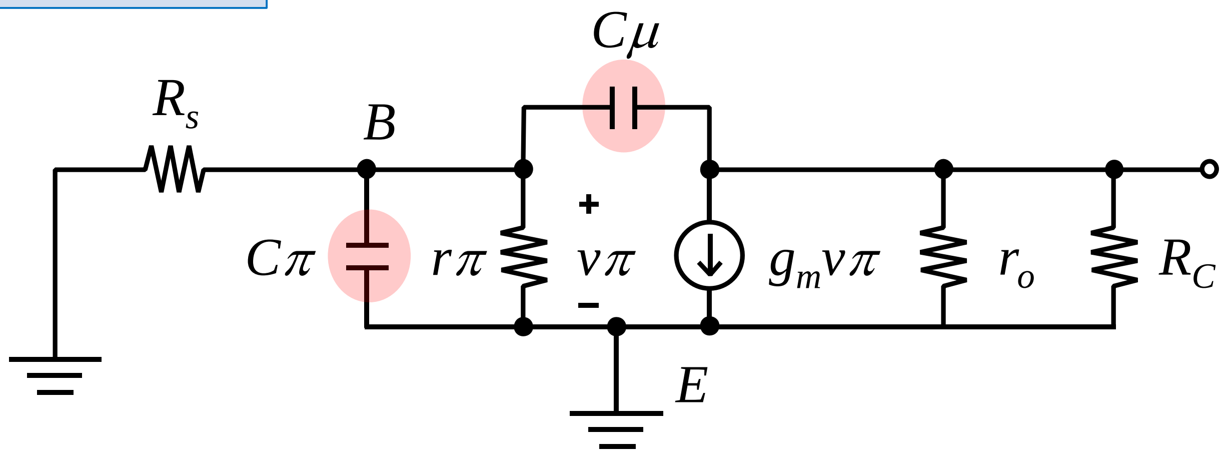
Similar to MOS transistors,

- At low frequency, no capacitor plays a role.
- At high frequency, capacitive effects come into play.

Apply What We Learnt to BJT Circuits: Example 1

Common emitter at high frequency





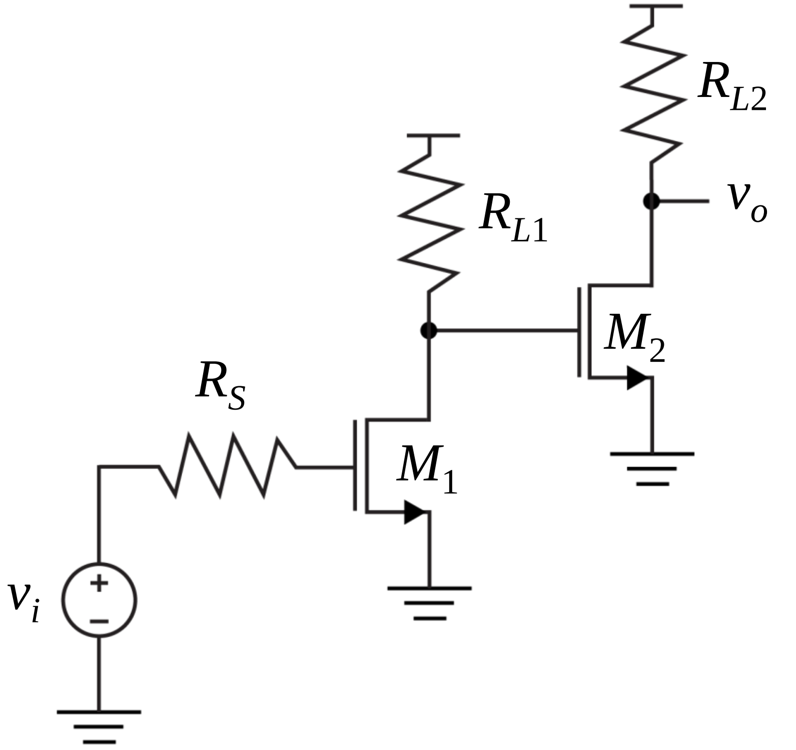
$$C_\pi : \; r_\pi // R_S$$

$$C_\mu : \; (r_\pi // R_S) + (r_O // R_C) + g_m (r_\pi // R_S)(r_O // R_C)$$

$$\boxed{\omega_{H-3\text{dB}} \approx \frac{1}{\sum_i C_i R_i}}$$

Apply What We Learnt to BJT Circuits: Example 2

Two stage at high frequency



C_{gs1}, C_{gd1}, C_{db1}, C_{gs2}, C_{gd2}, C_{db2}

For C_{gs1}, $R_{gs1} = R_s$
For C_{gs2}, $R_{gs2} = R_{L1} // r_{o1}$
For C_{db1}, $R_{db1} = R_{L1} // r_{o1}$
For C_{db2} , $R_{db2} = R_{L2} // r_{o2}$

$$C_{gd1} : R_S + [1 + g_{m1} R_S](r_{O1} // R_{L1})$$

$$C_{gd2} : (r_{O1} // R_{L1}) + [1 + g_{m2}(r_{O1} // R_{L1})](r_{O2} // R_{L2})$$

$$\boxed{\omega_{H-3dB} \approx \frac{1}{\sum_i C_i R_i}}$$

Summary

- Complex Numbers
- Transfer function and **Bode plots**
- -3 dB frequency, bandwidth
- **Capacitor's contribution** to the roll-off frequency
- Short Circuit and Open Circuit Analysis to identify roll-off frequency

Part 3.2

Frequency Response

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-End of Lecture-