E3001 Engineering Electromagnetics: Tutorial 7

1. Determine the polarization state (Linear, Circular or elliptical) and time-average Poynting vector for each of the following plane wave.

   (i) $\tilde{E}_1(z,t) = \vec{a}_x 10\cos(\omega t - kz - 120°) + \vec{a}_y 10\sin(\omega t - kz + 150°)$ V/m

   (ii) $\tilde{E}_2(z,t) = \vec{a}_x 10\cos(\omega t + kz - 120°) + \vec{a}_y 10\cos(\omega t + kz - 30°)$ V/m

   (iii) $\vec{E}_3(z) = (30\angle 20°\vec{a}_x + 40\angle -70°\vec{a}_y)e^{-jkz}$ V/m

   Assume Free Space.

[Ans: Linear, $\vec{S}_1 = +\vec{a}_z 0.266$ W/m^2, Circular, $\vec{S}_2 = -\vec{a}_z 0.266$ W/m^2, Elliptical, $\vec{S}_3 = \vec{a}_z 3.31$ W/m^2]

2. A 10 kHz uniform plane wave is traveling along the +z direction in seawater with the $x-y$ plane denoting the sea surface at $z=0$. The constitutive parameters of seawater are $\mu_r=1$, $\varepsilon_r=81$ and $\sigma=4$ S/m. The phasor of the electric field intensity is given by

$$\overline{E}(z)=(4.45\overline{a}_x - j4.45\overline{a}_y)e^{-\gamma z} \text{ V/m}$$

Determine the following:

(i) The polarization (linear, circular or elliptical) of the uniform plane wave.

(ii) The propagation constant γ and the intrinsic impedance η_c

(iii) The phasor vector of magnetic field intensity $\overline{H}(z)$

(iv) The time average Poynting vector at $z=0$ and $z=5\delta$ where δ is the skin depth in seawater.

(v) The percentage of average power dissipated as the wave travels from $z=0$ to $z=5\delta$.

[Ans: circular, $0.397+j0.397$; $0.14\angle 45^\circ\,\Omega$; $31.8(\overline{a}_y + j\overline{a}_x)e^{-j\pi/4}e^{-0.397z}e^{-j0.397z}$; $\overline{a}_z 100$ W/m^2; $\overline{a}_z 0.0045$ W/m^2; 99.996%]

E3001 Engineering Electromagnetics: Tutorial 8

1. The electric field of a uniform plane wave (UPW) in air occupying the region $z \le 0$ is given by

$$\bar{E}_i(z) = \bar{a}_x 10 e^{-j(6\pi z + \pi/3)} \text{ V/m}$$

The UPW is incident normally on a plane interface at $z = 0$ with a lossy medium e.g. a concrete wall, having $\mu = \mu_o$, $\varepsilon = 6\varepsilon_o$ and $\sigma = 0.09$ S/m, occupying the region $z > 0$. Find the following:

(a) The frequency of the wave

(b) The phasor expressions for $\bar{E}_r(z)$, $\bar{H}_r(z)$, $\bar{E}_t(z)$, $\bar{H}_t(z)$

(c) The percentage of average power reflected and transmitted at the interface.

(d) The expressions for the time-average Poynting vector in air and in the lossy medium.

(e) $|E|_{\max}$, $z_{\max}$, $|E|_{\min}$ and $z_{\min}$ in the region $z \le 0$

Ans: (a) $f = 900\ \text{MHz}$

(b) $\overline{E}_i(z) = \overline{a}_x E_{oi}\, e^{-j6\pi z}$ V/m $\qquad E_{oi} = 10\angle -60°$

$\overline{E}_r(z) = \overline{a}_x 4.35\angle 112°\, e^{+j6\pi z}$ V/m $\qquad \Gamma = 0.435\angle 172°$

$\overline{H}_r(z) = -\overline{a}_y 0.0115\angle 112°\, e^{+j6\pi z}$ A/m

$\overline{E}_t(z) = \overline{a}_x 5.72\angle -54°\, e^{-6.85z}\, e^{-j46.7z}$ V/m $\qquad \tau = 0.572\angle 6.0°$

$\overline{H}_t(z) = \overline{a}_y 0.038\angle -62.4°\, e^{-6.85z}\, e^{-j46.7z}$ A/m $\qquad \eta_{2c} = 150.5\angle 8.4°$

(c) 18.9%; 81.1%

(d) $\overline{a}_z 0.108\ \text{W/m}^2$, $\overline{a}_z 0.108 \exp(-13.7z)\ \text{W/m}^2$

(e) $|E|_{\max} = 14.4$ V/m @ $z_{\max} = -7.9$ cm;

$|E|_{\min} = 5.6$ V/m @ $z_{\min} = -16.3$ cm

E3001 Engineering Electromagnetics: Tutorial 9

1. The magnetic field of a uniform plane wave traveling in a lossless dielectric medium ($\mu_r = 1, \varepsilon_r = 4$) has the form of

   $$\overline{H} = \left(-4\overline{a}_x + 5m\overline{a}_z\right)e^{-j(60x+80z)} \text{ A/m}$$

   Find the following:

   (i) The direction of propagation, the frequency of the wave and the value of m

   (ii) The corresponding electric field $\vec{E}(z)$

   (iii) The time-average Poynting vector $\vec{S}$

   [Ans: $\overline{a}_k = \overline{a}_x 0.6 + \overline{a}_z 0.8$; $f = 2.39$ GHz; $m = 0.6$; $\overline{E} = \overline{a}_y 300\pi\, e^{-j(60x+80z)}$ V/m; $\overline{S} = \overline{a}_x 1.4 + \overline{a}_z 1.9$ kW/m^2]

2. A uniform plane wave in free space occupying the region $z < 0$ is incident at a plane interface with a lossless dielectric medium ($\varepsilon_r = 2.1$, $\mu_r = 1$), occupying the region $z > 0$. The incident electric field is given by

$$\tilde{E}_i = (\bar{a}_x\sqrt{3} - \bar{a}_z)\cos\left(\omega t - \frac{\pi}{2}x - \frac{\sqrt{3}\pi}{2}z\right)\text{ V/m}$$

(a) Determine the frequency of the wave, the angle of incidence, and the angle of transmission.

(b) Obtain a time domain expression for the transmitted electric field $\tilde{E}_t$.

[Ans: $f = 150$ MHz; $\theta_i = 30°$; $\theta_t = 20.2°$;

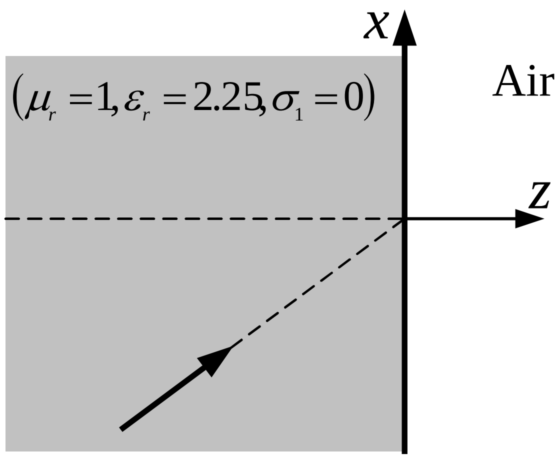
$\tilde{E}_t = (\bar{a}_x 1.48 - \bar{a}_z 0.54)\cos\left(3\pi\times 10^8 t - \frac{\pi}{2}x - 4.27z\right)$ V/m]

E3001 Engineering Electromagnetics: Tutorial 10

1. The electric field of a uniform plane wave traveling (UPW) in a lossless dielectric medium with $\mu_r = 1, \varepsilon_r = 2.25$ and occupying the region $z \le 0$ is given by

$$\overline{E} = \left(3\overline{a}_x - j5\overline{a}_y - 4\overline{a}_z\right)e^{-j(8x+6z)} \text{ V/m}$$

The wave is incident on an air boundary at $z = 0$, occupying the region $z \ge 0$.



Find the following:

(i) The frequency f and direction of propagation $\overline{a}_k$ of the UPW

(ii) The amplitude of the perpendicular- and parallel- polarized component of the incident electric field with respect to the plane of incidence

(iii) The polarization (linear, circular or elliptical) of the incident wave

(iv) The time-average Poynting vector $\overline{S}_i$

(v) The angle of incidence θ_i

(vi) The Brewster angle $\theta_{B\|}$

(vii) The Critical angle θ_c

(viii) The percentage of average incident power reflected from the interface.

[Ans: 318 MHz, $\overline{a}_k = 0.8\overline{a}_x + 0.6\overline{a}_z$, $E_{oi}^{\|} = 5$; $E_{oi}^{\perp} = 5\angle -90°$; circular; $\overline{S}_i = 0.08\overline{a}_x + 0.06\overline{a}_z$, $\theta_i = 53.1°$; $\theta_{B\|} = 33.7°$; $\theta_c = 41.8°$; 100%]

2. A 385 MHz uniform plane wave (UPW) travelling in a lossless dielectric medium $(\mu_r = 1,\ \varepsilon_r = 2.6)$ occupying the region $z \le 0$ has an electric field in the form of

$$\vec{E}_i = -j78\vec{a}_y e^{-j(k_i \sin(\theta_i)x + k_i \cos(\theta_i)z)} \ \text{V/m}$$

The wave is incident on an air boundary at $z = 0$, occupying the region $z \ge 0$.

Assume angle of incident $\theta_i = \theta_{B\|}$, find the following:

(a) Direction of propagation of incident, reflected and transmitted waves, $\vec{a}_{k_i}$, $\vec{a}_{kr}$ and $\vec{a}_{kt}$

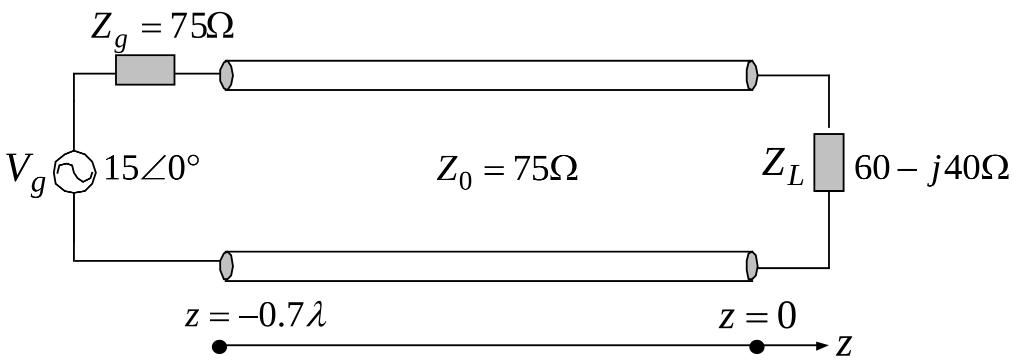
(b) Time-average Poynting vector of the incident, reflected and transmitted waves i.e. $\vec{S}_i$, $\vec{S}_r$ and $\vec{S}_t$

(c) The average power transmitted through 0.5m^2 area at $z = 0$.

[Ans: $\theta_{B\|} = 31.8°$, $\vec{a}_{k_i} = 0.53\vec{a}_x + 0.85\vec{a}_z$, $\vec{a}_{kr} = 0.53\vec{a}_x - 0.85\vec{a}_z$, $\vec{a}_{kt} = 0.85\vec{a}_x + 0.53\vec{a}_z$; $\vec{S}_i = 6.85\vec{a}_x + 11.1\vec{a}_z$, $\vec{S}_r = 1.35\vec{a}_x - 2.2\vec{a}_z$, $\vec{S}_t = 14.3\vec{a}_x + 8.9\vec{a}_z$, 4.45 W]

E3001 Engineering Electromagnetics: Tutorial 11

1. A transmission line has $V_g = 15\,\text{V}$, $Z_g = 75\,\Omega$, $Z_o = 75\,\Omega$, $Z_L = 60 - j40\,\Omega$ and $\ell = 0.7\lambda$. Compute the power delivered to the load using two different techniques:



a) Find Γ_L and compute $P_L = \frac{V_g^2}{8Z_g}\left(1 - |\Gamma_L|^2\right)$ and

b) Find Z_{in} and compute $P_L = \frac{1}{2} \times \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}(Z_{in})$;

Discuss the rationale for each of these methods. [Ans: P_L = 0.34]

2. A lossless transmission line of length $\ell = 0.45\text{m}$ has a characteristic impedance $Z_o = 50\Omega$ and a phase velocity $u_p = 1.5\times10^8$ m/s.

The power supply at $z = -\ell$ has an open-circuited voltage $V_g = 100\cos\left(240\pi\times10^6 t\right)$ V and an internal impedance $Z_g = 25\Omega$. The line is terminated in an impedance $Z_L = 40 + j70\Omega$ at $z = 0$.

Find the following:

(a) The reflection coefficient Γ at $z = 0$.

(b) The input impedance Z_{in} at $z = -\ell$.

(c) The voltage at $z = -\ell$ i.e. $V\left(-\ell\right)$

(d) The amplitude of incident voltage wave at $z = 0$ i.e. V_o^+

(e) The position of maximum voltage $z_{\max}$ on the line.

(f) The magnitude of voltage maximum $|V|_{\max}$ on the line.

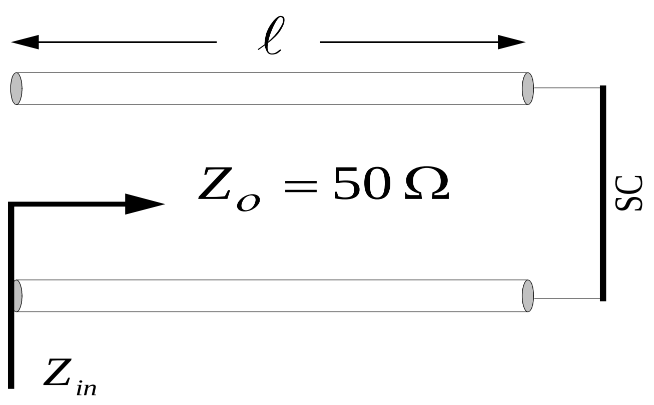
ANS: 133.8 V $\Gamma_L = 0.62\angle 60°$, $Z_{in} = 12 + j7.9\Omega$, $38\angle 21.2°$, V_o^+=$82.6\angle -134.4°$, $z_{\max} = -0.10\text{m}$

E3001 Engineering Electromagnetics: Tutorial 12

1. A section of a lossless 50 Ω coaxial cable having $u_p = 2 \times 10^8$ m/s is terminated in a short circuit and operates at a frequency 100 MHz. Determine the shortest length of the line ℓ such that at the input terminal, the line appears to be a

   (a) 100 nH Inductor.

   (b) 31.8 pF Capacitor



[Ans: 28.6 cm (0.143λ); 75 cm (0.375λ)]

2. In a laboratory experiment conducted on a 50 Ω transmission line terminated in an unknown load impedance. It is found that the SWR = 2. The successive voltage minima are 25 cm apart and the first voltage minimum occurs at 5 cm from the load. Find

(a) the unknown load impedance

(b) the reflection coefficient at the load end

(c) If the load end is replaced by a short circuit, where would be the first voltage maximum be located?

[Ans: $Z_L = 33.8 - j23.8\Omega$; $\Gamma_L = 0.33\angle -108°$; 12.5 cm]