TUTORIAL 9

PN Junction Capacitances; Metal-Semiconductor Contacts

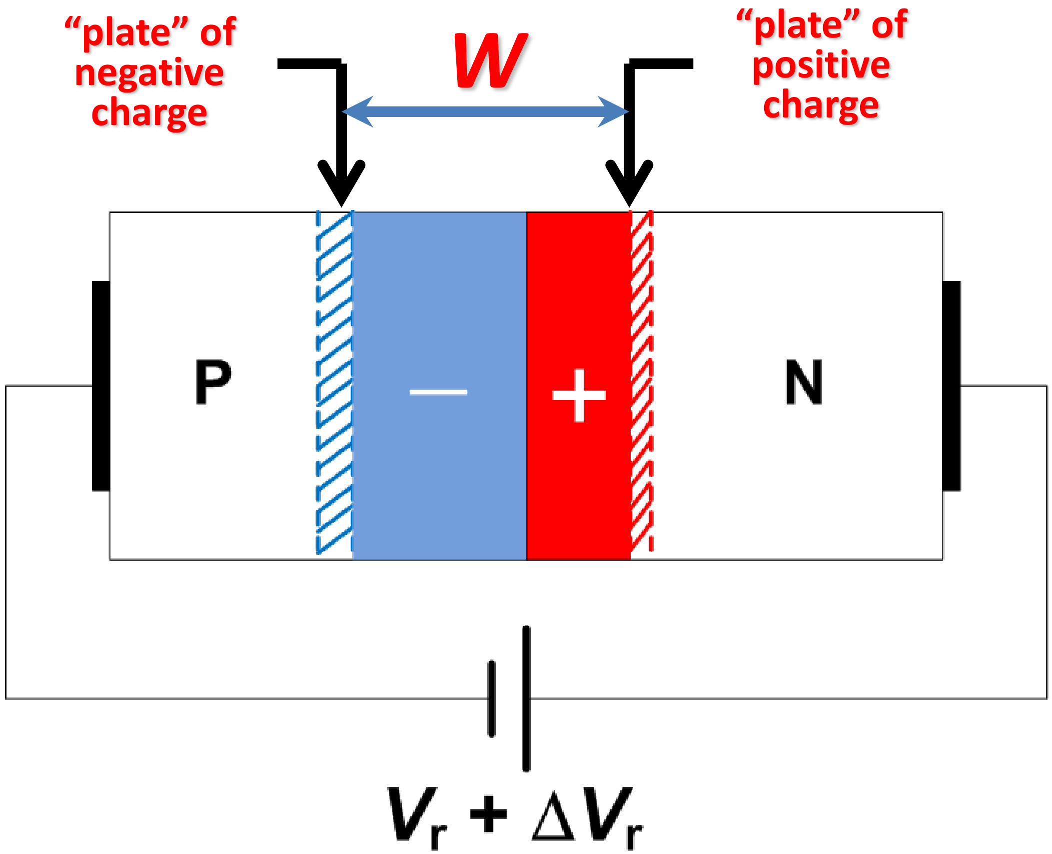
1. Room temperature (300 K) measurements on an abrupt Si p^+-n junction yield the following results: With a reverse bias of 4.2 V, the junction capacitance is 20 pF. When the reverse bias is changed to 0.43 V, the junction capacitance is 40 pF. If the area of the junction is 2×10^{-3} cm^2,
determine the built-in potential V_o and the doping density of the n-side.

[0.83 V; 6.02 x 10^{15} cm^{-3}]

Junction Capacitance C_j

Origin: A change in the voltage applied causes a change in the amount of space charge

$$\boxed{C_j = \frac{\varepsilon_r \varepsilon_0}{W} \cdot A}$$



$$C_j = \left| \frac{dQ_{sc}}{dV_a} \right|$$

$$Q_{sc} = qN_a x_p = qN_d x_n$$

$$x_n = \left(\frac{N_a}{N_a + N_d} \right) W ; x_p = \left(\frac{N_d}{N_a + N_d} \right) W$$

$$W = \left[\frac{2\varepsilon_r \varepsilon_0}{q} (V_{bi} - V_a) \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

Junction capacitance: $C_j = \frac{\varepsilon A}{W} = A\left\{\frac{q\varepsilon}{2(V_o - V_a)}\left(\frac{N_a N_d}{N_a + N_d}\right)\right\}^{1/2}$

The junction is p$^+$n, $\Rightarrow N_a >> N_d$

$$\therefore C_j = A\left[\frac{q\varepsilon N_d}{2(V_o - V_a)}\right]^{1/2}$$

$$C_j^{\,2} = \frac{A^2 q\varepsilon}{2}\frac{N_d}{(V_o - V_a)} = \frac{K\ N_d}{(V_o - V_a)} \qquad (1) \quad where\ \ K = \frac{A^2 q\varepsilon}{2}$$

Given: $V_a = -4.2$ V, $C_j = 20$ pF

$V_a = -0.43$ V, $C_j = 40$ pF

$$\therefore 400 \times 10^{-24} = \frac{K\ N_d}{(V_o + 4.2)} \qquad (2)$$

$$1600 \times 10^{-24} = \frac{K\ N_d}{(V_o + 0.43)} \qquad (3)$$

Solving (2) & (3), $V_o = 0.827$ V

From eqn (1), $N_d = \dfrac{2C_j^{\,2}(V_o - V_a)}{A^2 q\varepsilon}$

$$= \frac{2\left(20\times10^{-12}\right)^2 \times (0.827+4.2)}{\left(2\times10^{-3}\right)^2 \times 1.6\times10^{-19} \times 11.8\times 8.85\times10^{-14}}$$

$$= 6\times10^{15}\ \text{cm}^{-3}$$

2. Consider an abrupt Si p^+n junction with $N_d = 1\times10^{16}$ cm^{-3}. The area of the junction is 1×10^{-4} cm^2. Holes in the n-region have a lifetime of 0.5 μs and a diffusion coefficient of 10 cm^2/s. Calculate the storage capacitance of the p^+n junction under a forward bias of 0.65 V.

Storage Capacitance

$$\boxed{C_s = \frac{q}{kT} \cdot Q_s}$$

Origin: A change in the forward-bias voltage causes a change in the amount of excess minority carrier charge stored in the quasi-neutral regions (see shaded regions)

$$Q_s = Q_p + Q_n$$

For p⁺n junction,

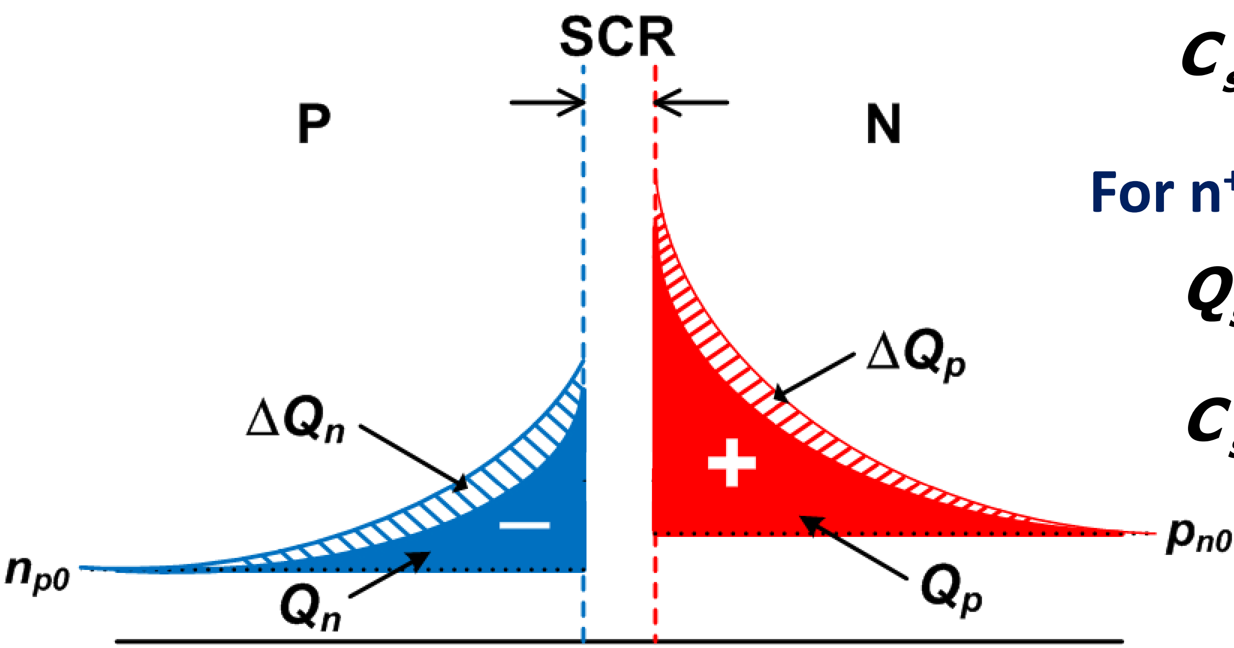
$$Q_s \approx Q_p$$

$$C_s = \frac{q}{kT} \cdot Q_p = \frac{q}{kT} \cdot I\tau_p$$

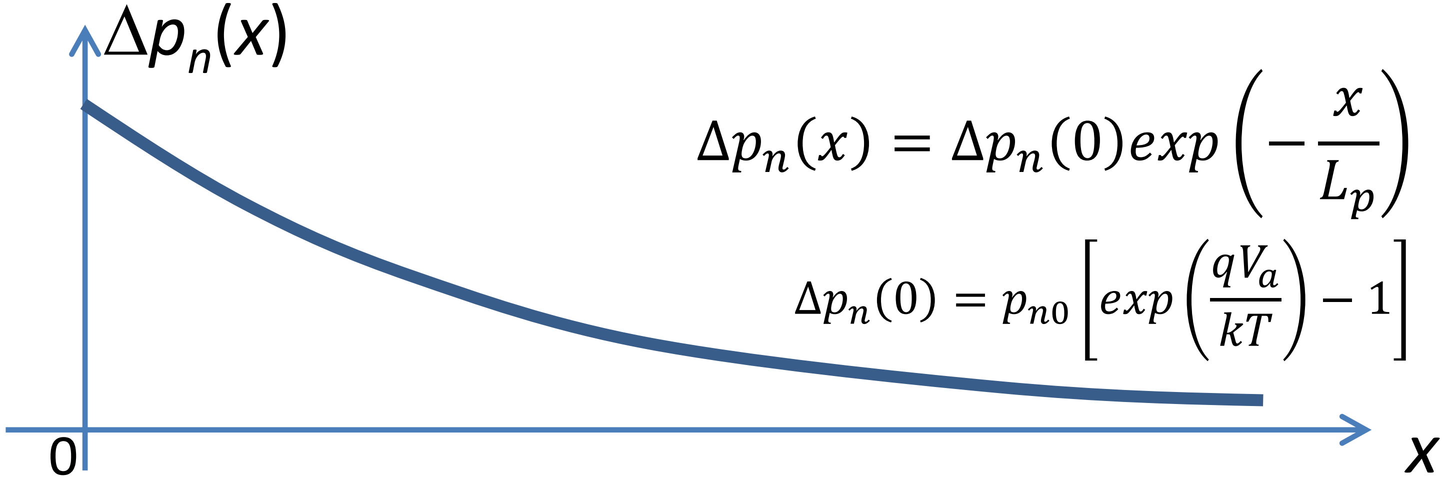
For n⁺p junction,

$$Q_s \approx Q_n$$

$$C_s = \frac{q}{kT} \cdot Q_n = \frac{q}{kT} \cdot I\tau_n$$



In a forward biased p⁺n junction, hole injection dominates.



$$\Delta p_n(x) = \Delta p_n(0) exp\left(-\frac{x}{L_p}\right)$$

$$\Delta p_n(0) = p_{n0}\left[exp\left(\frac{qV_a}{kT}\right) - 1\right]$$

On the n side, the excess holes distribution is given as

$$\Delta p_n(x) = p_{n0}\left[\exp\left(\frac{qV_a}{k_B T}\right) - 1\right]\exp\left(-\frac{x}{L_p}\right)$$

The total excess positive charge stored per unit area,

$$Q_p = q\int_0^\infty \Delta p_n(x)dx = qp_{n0}\left[\exp\left(\frac{qV_a}{k_B T}\right) - 1\right]\int_0^\infty \exp\left(-\frac{x}{L_p}\right)dx$$

$$= qL_p p_{n0}\left[\exp\left(\frac{qV_a}{k_B T}\right) - 1\right]$$

Storage capacitance: $C_s = A \cdot \frac{dQ_p}{dV_a} = \frac{Aq^2 L_p p_{n0}}{k_B T}\exp\left(\frac{qV_a}{k_B T}\right)$

Given minority carrier (hole) lifetime $\tau_p = 0.5$ μs, donor impurity concentration $N_d = 1 \times 10^{16}$ cm^{-3},

$$L_p = \sqrt{D_p \tau_p} = \sqrt{10 \times 0.5 \times 10^{-6}} = 2.24 \times 10^{-3} \text{ cm}$$

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{\left(1.5 \times 10^{10}\right)^2}{1 \times 10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

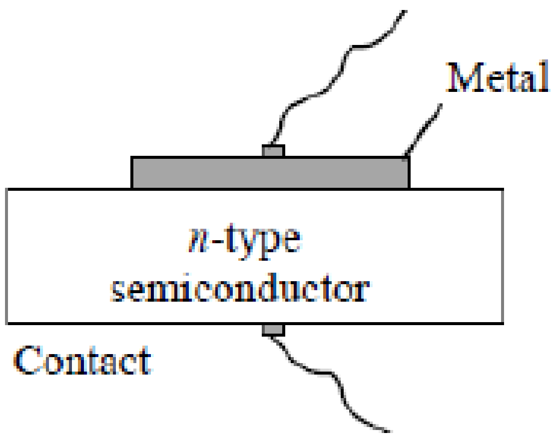
$$C_s = \frac{1 \times 10^{-4} \times \left(1.6 \times 10^{-19}\right)^2 \times 2.24 \times 10^{-3} \times 2.25 \times 10^4}{1.38 \times 10^{-23} \times 300} \times$$

$$\exp\left(\frac{0.65}{0.0259}\right)$$

$$= 2.47 \times 10^{-9} \text{ F} \quad \text{or } 2.47 \text{ nF}$$

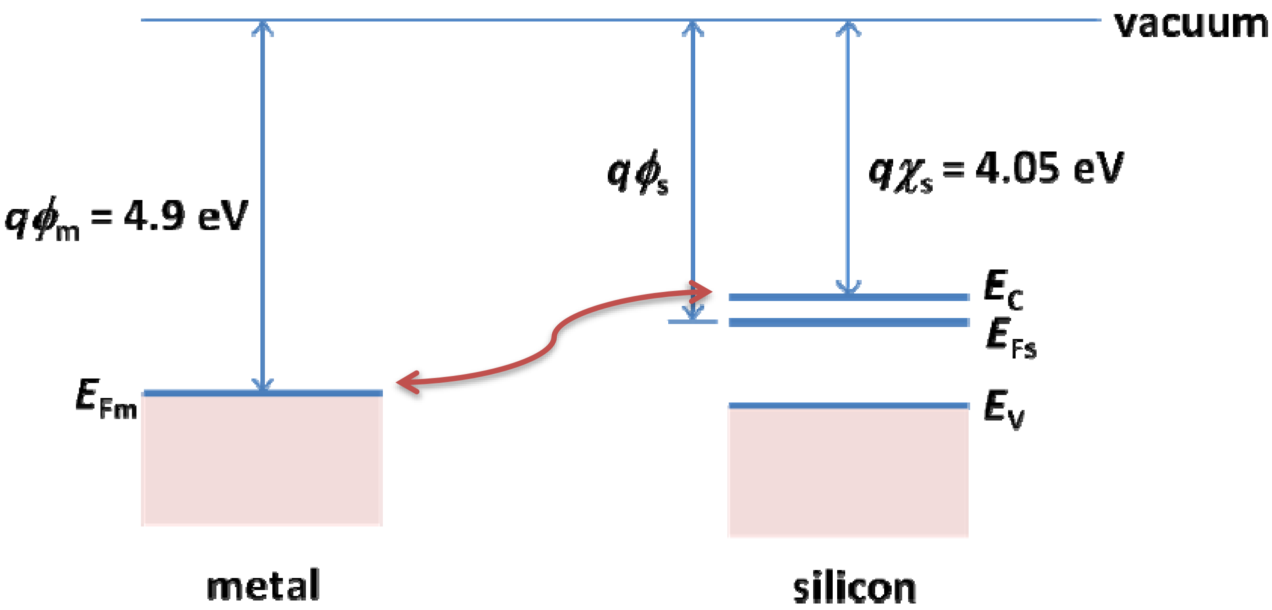
3. An ideal Schottky contact is formed on n-type Si with a donor impurity concentration of 1×10^{17} cm^{-3}. The metal work function $q\phi_m$ is 4.9 eV and the electron affinity $q\chi_s$ of Si is 4.05 eV at 300 K.

- Calculate the Si work function $q\phi_s$.
- Draw the energy band diagrams before and after contact formation under (a) thermal equilibrium; (b) a forward bias of 0.4 V and (c) a reverse bias of 3.2 V.
- What is the significance of the Schottky barrier height?



- Calculate the Si work function $q\phi_s$.

→ Energy band diagrams before contact:



From Maxwell-Boltzmann eqn.

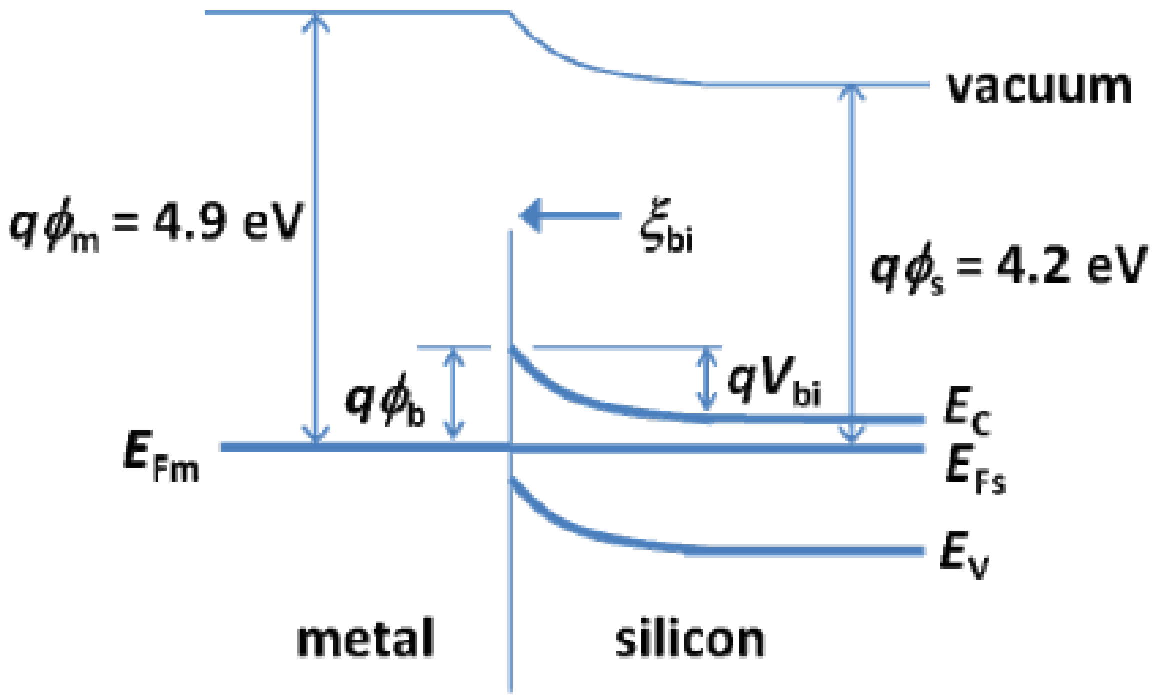
$$E_C - E_{Fs} = k_B T \ln \frac{N_C}{N_d} \quad \left(\text{recall: } n_0 = N_C \exp\left(-\frac{E_C - E_{Fs}}{k_B T} \right) \right)$$

$$= 0.0259 \ln \frac{2.8\times 10^{19}}{1\times 10^{17}} = 0.146 \text{ eV}$$

Semiconductor work function: $q\Phi_S = q\chi + (E_C - E_{Fs})$

$$= 4.05 + 0.146 = 4.2 \text{ eV}$$

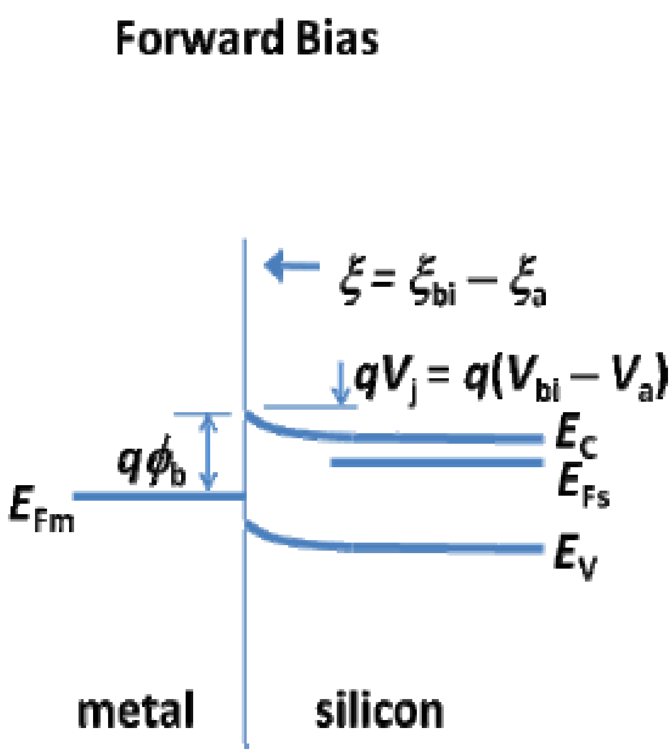
Energy band diagram after contact (thermal equilibrium):

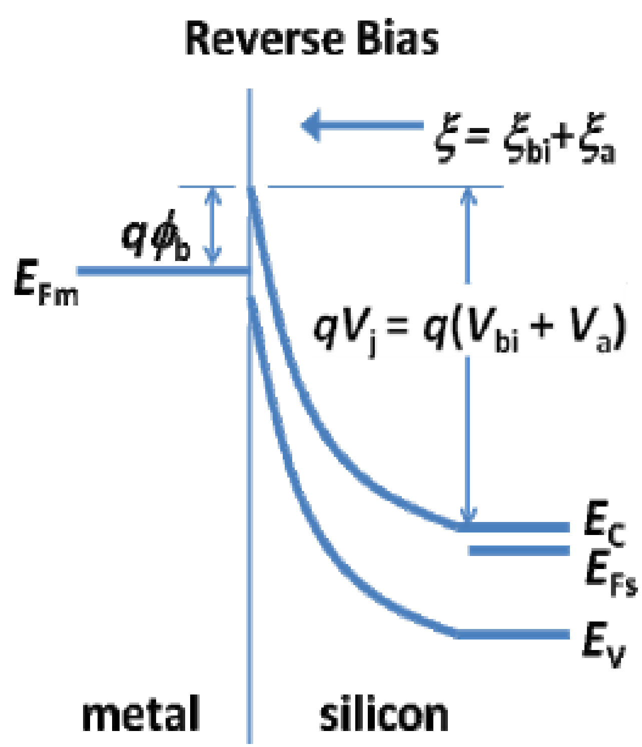


Built-in voltage: $V_{bi} = \phi_m - \phi_s =$ 0.7 V

Schottky barrier: $\phi_{bi} = \phi_m - \chi_s = 0.85$ eV

Energy band diagram (external bias):





- **Significance of the Schottky barrier height:**

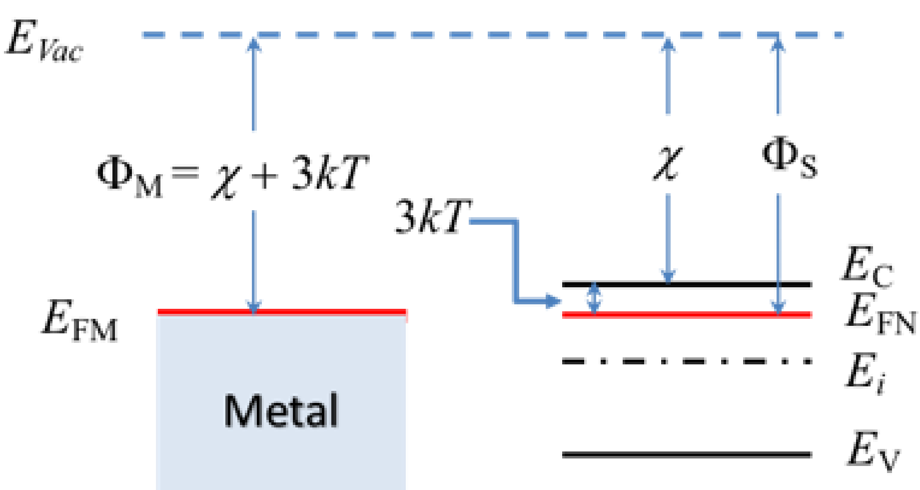
Does not permit electron flow from metal into semiconductor; current is unidirectional due to electrons flowing from semiconductor into metal; contact is rectifying.

Q4 Consider a metal-semiconductor junction at room temperature. Assume that the work function of the metal Φ_M is equal to the electron affinity of the semiconductor χ plus $3kT$. Also, assume that the Fermi level in the semiconductor is located $3kT$ below the conduction band edge E_C. The bandgap of the semiconductor is 1 eV. Determine:

a) Whether the semiconductor is intrinsic, *n*-type or *p*-type.

b) The value of the built in voltage V_{bi}.

c) Whether the metal-semiconductor forms a rectifying or ohmic contact. Justify your answer using a band diagram with proper labeling.



Q4

a) Since the Fermi level is near the conduction band ($3kT$), the semiconductor is ***n*-type**.

b) The band diagram before the contact is shown. When the contact is made, there is no charge transfer since the semiconductor work function is equal to the metal work function, so $\boldsymbol{V_{bi} = 0}$.

$$qV_{bi} = \Phi_M - \Phi_S = 0$$

c) Since there is no barrier for the electrons to flow from the semiconductor to metal and only a small barrier for electrons to flow from the metal to semiconductor, the junction is **ohmic**.