Regions of Operation of BJT and MOSFET

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EE2002 Analog Electronics

Lesson Objectives

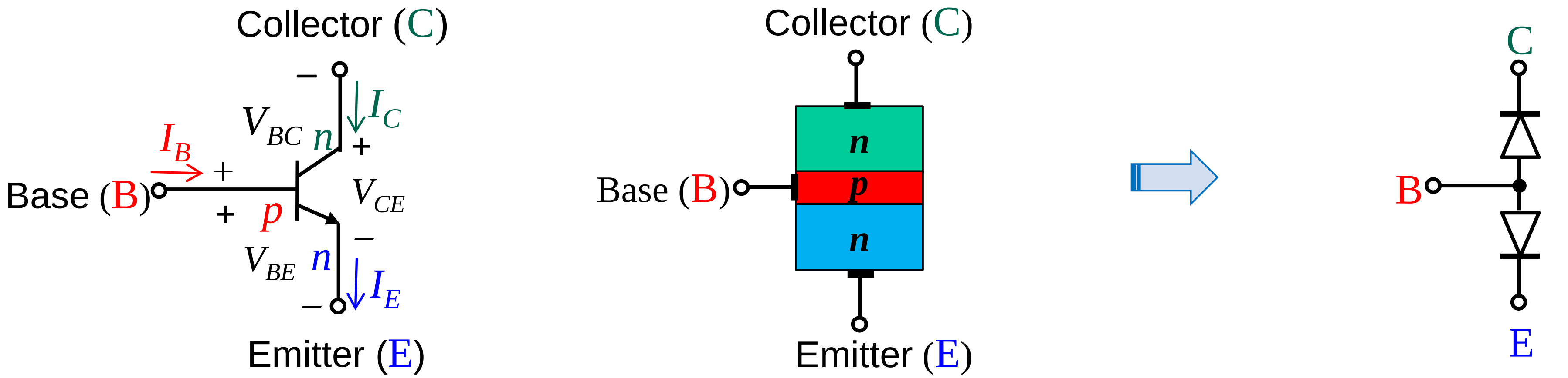
At the end of this lesson, you should be able to:

- Identify the symbols used to represent transistors in circuit schematics
- Identify the three different operation regions of BJT and MOSFET
- Describe the criteria for the different operation regions of BJT and MOSFET
- Discuss the i-v characteristics of MOSFET
- Analyse circuits used to bias BJT and MOSFET transistors into various regions of operation

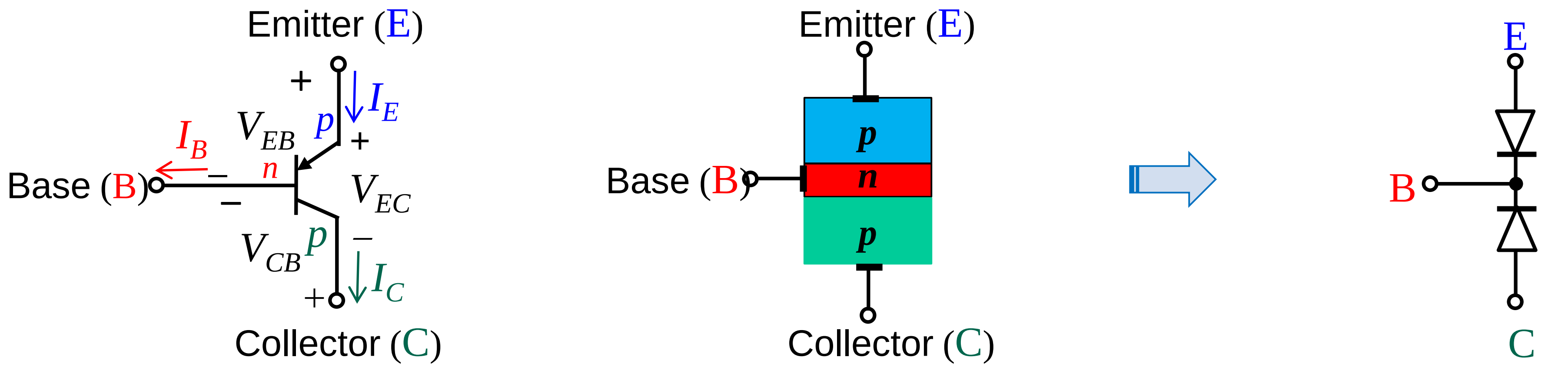
NPN and PNP BJTs

Don’t get it

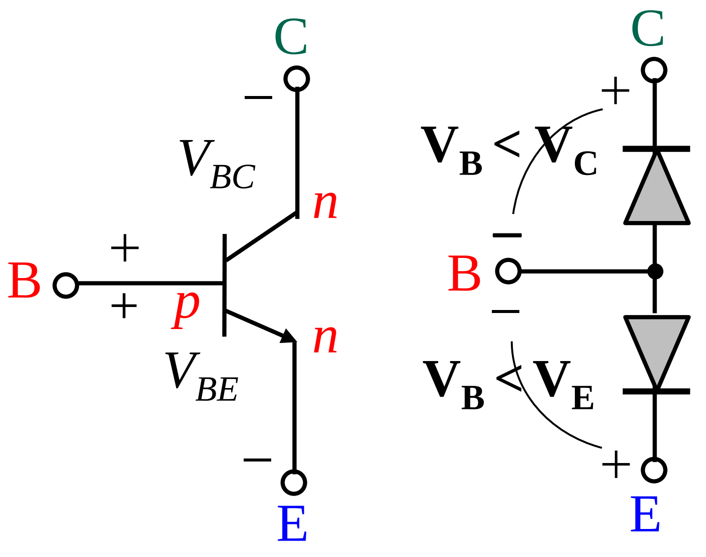
npn



pnp



Operation Regions of BJT

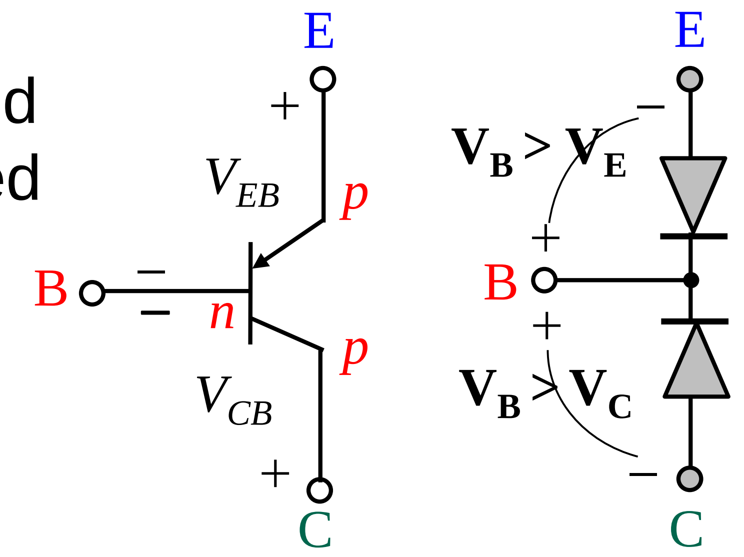


Cutoff region

BEJ (npn) reverse biased
BCJ (npn) reverse biased

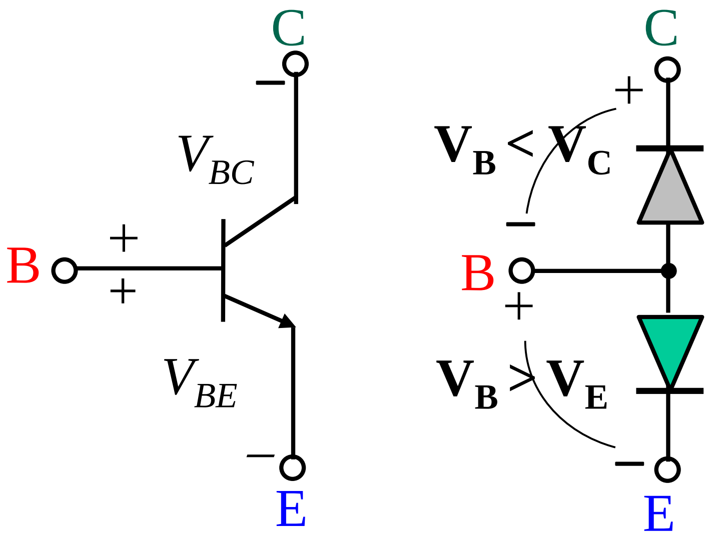
$I_C = 0$

$\Rightarrow$ **Open Switch**



Note: The junctions refer to EBJ and CBJ for pnp transistor.

Operation Regions of BJT



Forward-active region

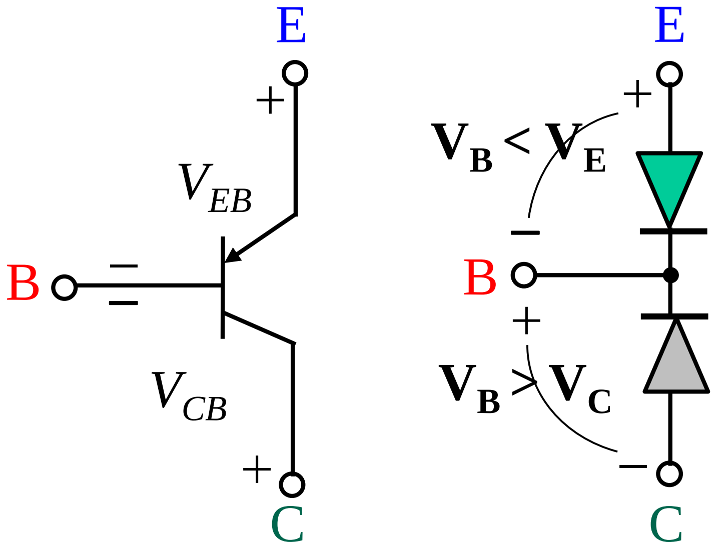
BEJ (npn) forward biased
BCJ (npn) reversed biased

$V_{BE} \approx 0.7\ \text{V}$

$I_C = \beta I_B = \alpha I_E$

$\Rightarrow$ **Good amplifier**

$\alpha = \beta/(\beta + 1)$



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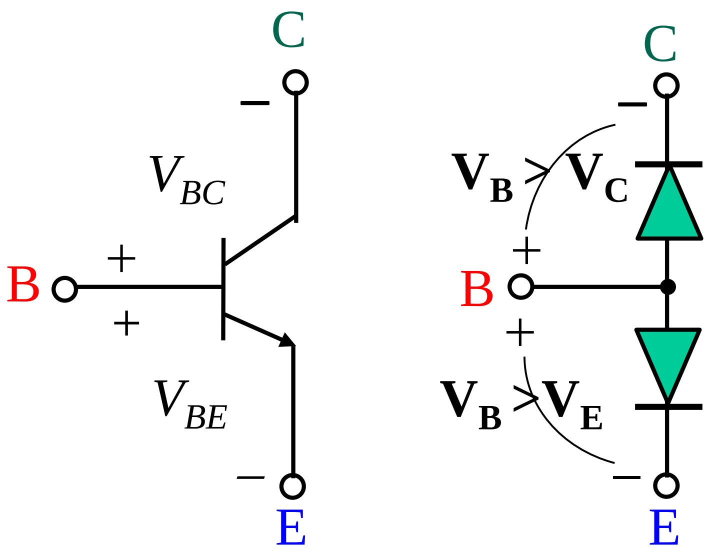
Operation Regions of BJT



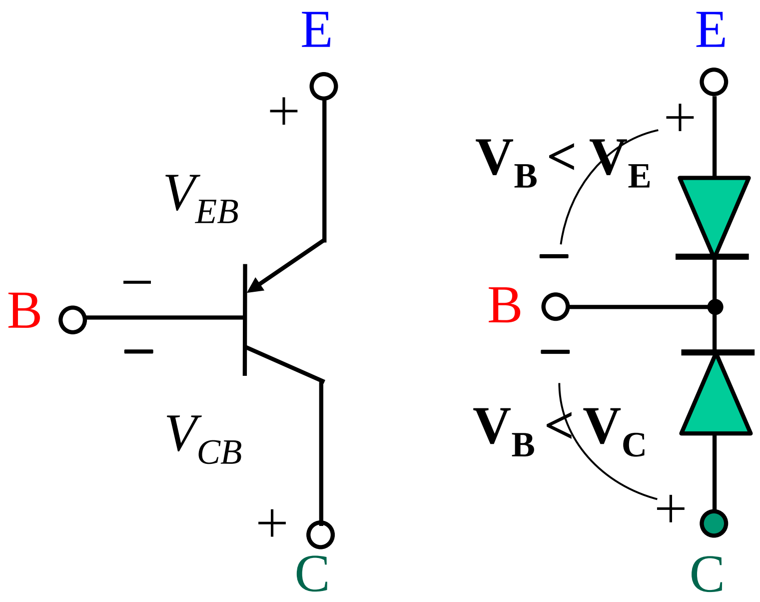
Saturation region

BEJ (npn) forward biased
BCJ (npn) forward biased

$\Rightarrow$ **Closed switch**

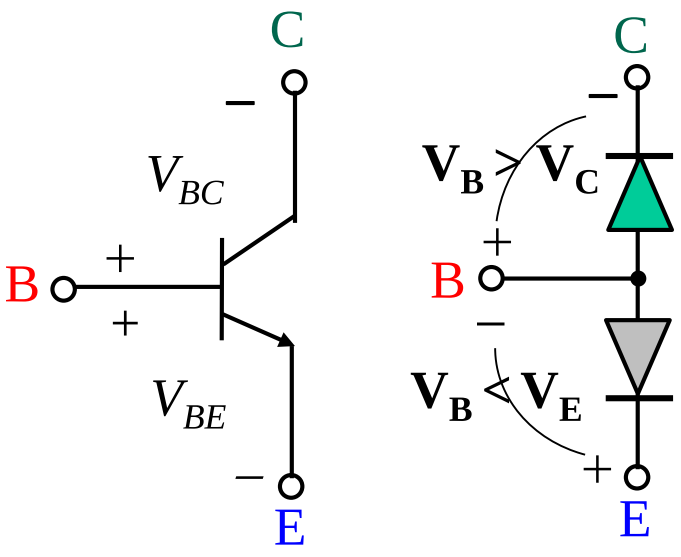
$\Rightarrow V_{BE} \approx 0.7$ V
$V_{BC} = 0.4 \sim 0.5$ V

$\Rightarrow V_{CE(SAT)} = 0.2 \sim 0.3$ V



Note: The junctions refer to EBJ and CBJ for pnp transistor.

Operation Regions of BJT

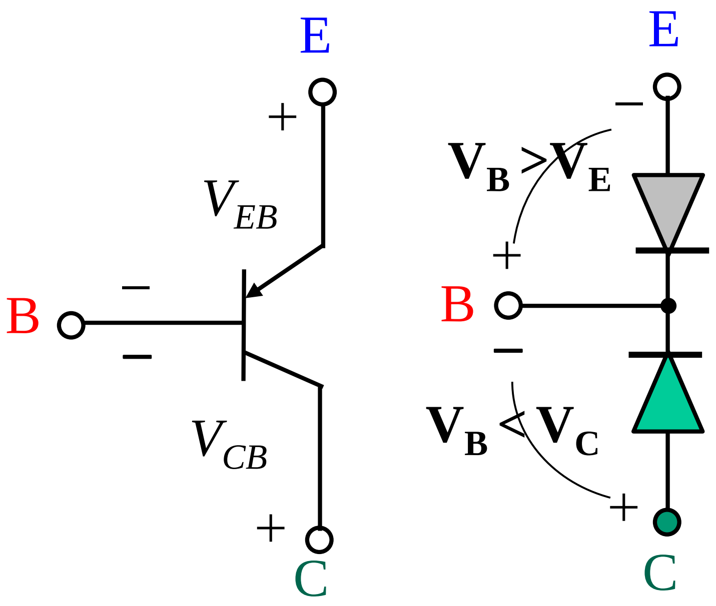


Reverse-active region

BEJ (npn) reverse biased
BCJ (npn) forward biased

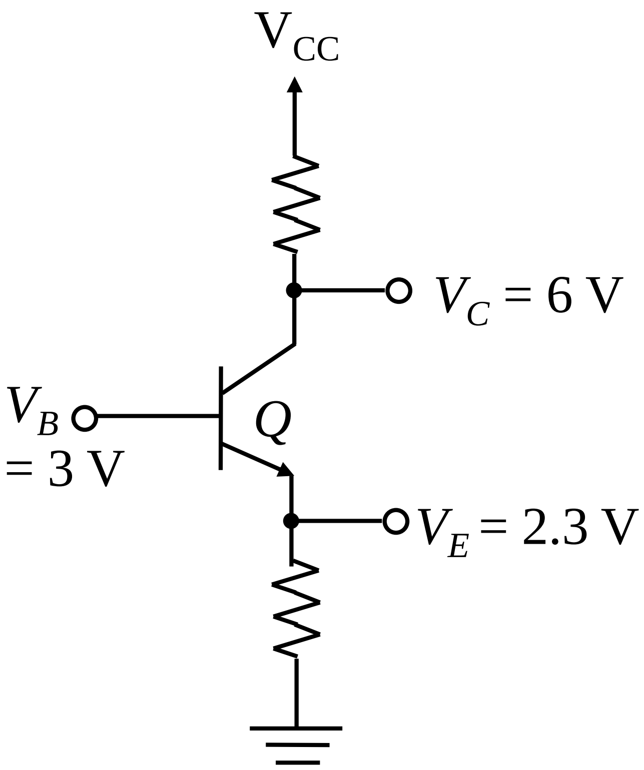
⇒ **Weak amplifier**

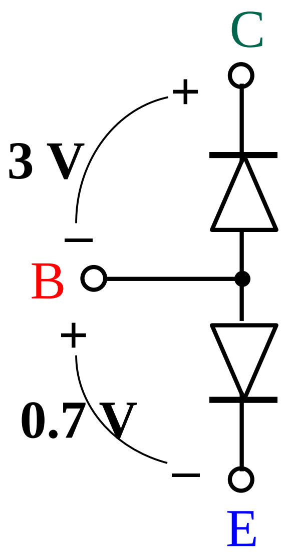
⇒ **Normally not use**



Note: The junctions refer to EBJ and CBJ for pnp transistor.

BJT Bias Analysis: Active Mode





$$\begin{aligned} V_{BE} &= V_B - V_E \\ &= 3 - 2.3 \\ &= 0.7\ \text{V} \end{aligned}$$

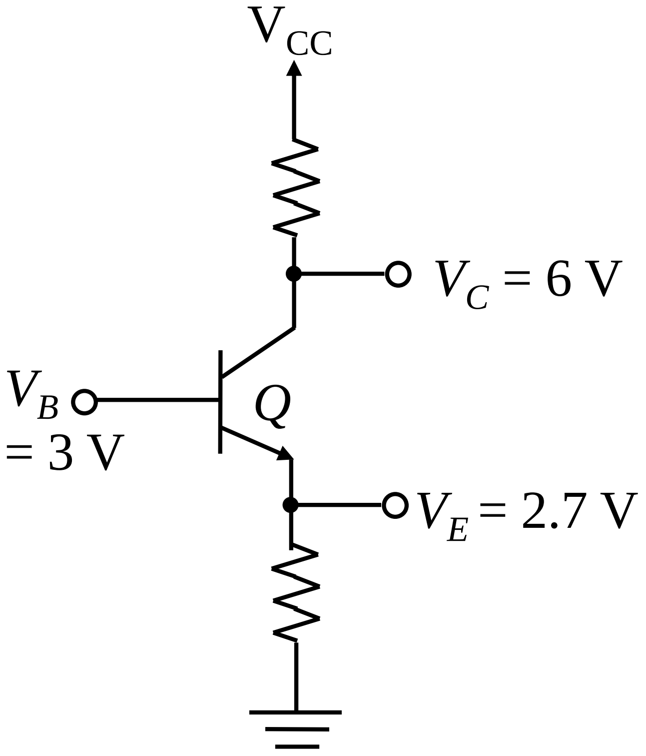
$$\begin{aligned} V_{BC} &= V_B - V_C \\ &= 3 - 6 \\ &= -3\ \text{V} \end{aligned}$$

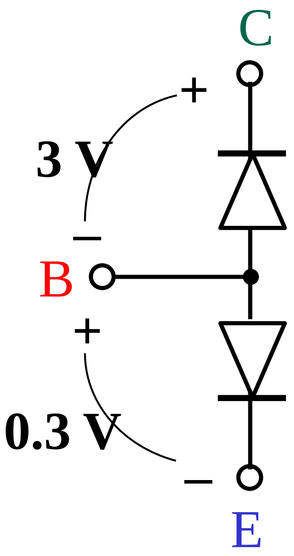
BCJ is reversed biased by 3 V

BEJ is forward biased by 0.7 V

Q in active mode and can be used as linear amplifier.

BJT Bias Analysis: Cutoff Mode





$$V_{BE} = V_B - V_E = 3 - 2.7 = 0.3\text{ V}$$

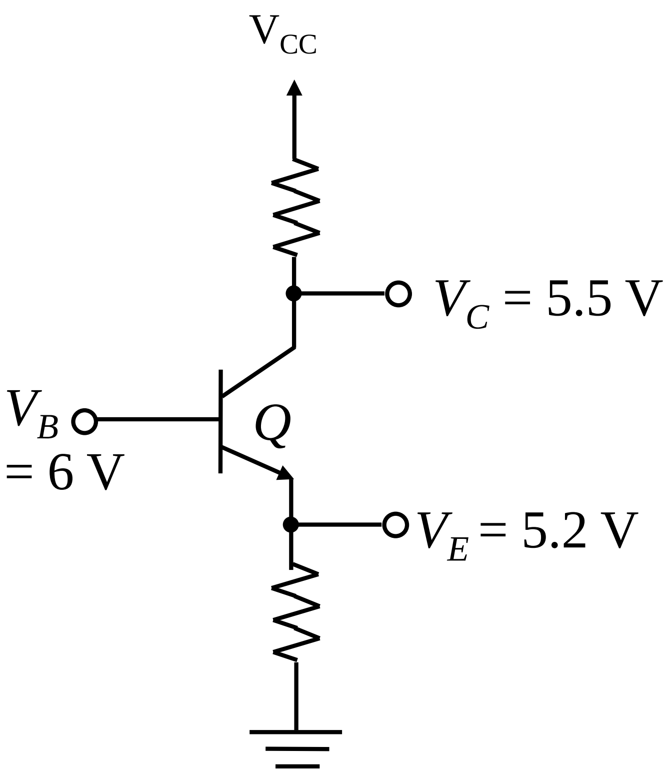
$$V_{BC} = V_B - V_C = 3 - 6 = -3\text{ V}$$

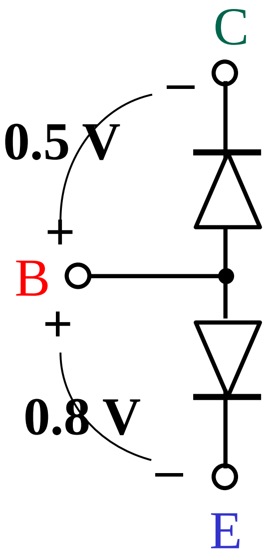
BCJ is reversed biased by 3 V

BEJ is forward biased by 0.3 V but inadequate to turn on BEJ

Q is cutoff.

BJT Bias Analysis: Saturation Mode





$$V_{BE} = V_B - V_E = 6 - 5.2 = 0.8 \text{ V}$$

$$V_{BC} = V_B - V_C = 6 - 5.5 = 0.5 \text{ V}$$

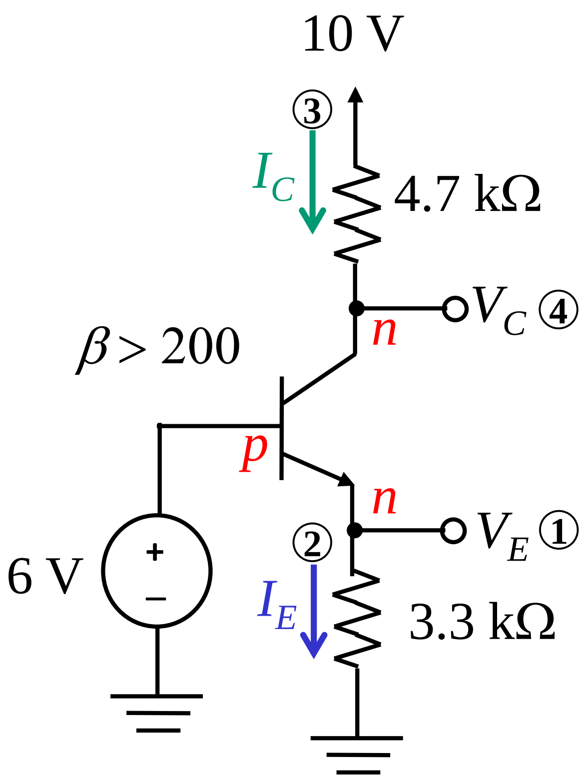
BCJ is forward biased by 0.5 V

BEJ is forward biased by 0.8 V

$$V_{CE} = V_{CB} + V_{BE} = -0.5 + 0.8 = 0.3 \text{ V}$$

→ Q is in Saturation.

BJT Bias Analysis: Determine DC Node Voltages and Branch Currents



Assume active-mode operation,

$$V_E = V_B - V_{BE} = 6 - 0.7 = 5.3\text{ V}$$

$$I_E = \frac{5.3}{3.3} = 1.6\text{ mA}$$

Since β is very large,

$$\alpha \approx 1 \Rightarrow I_C \approx I_E = 1.6\text{ mA}$$

$$V_C = 10 - 1.6 \times 4.7 = 2.48\text{ V}$$

$$V_{BC} = V_B - V_C = 6 - 2.48 = 3.52\text{ V}$$

=> Wrong assumption! *Q* is in saturation mode.

BJT Bias Analysis: Determine DC Node Voltages and Branch Currents

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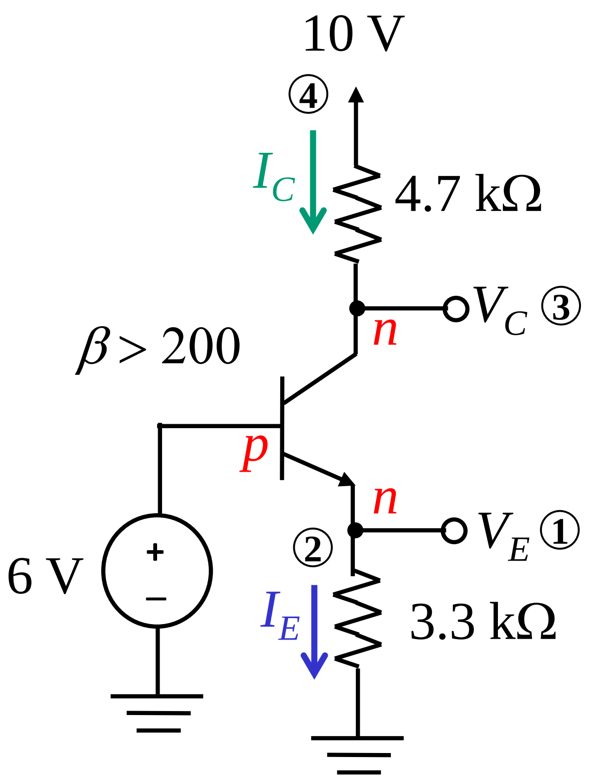


In saturation region, $V_{CE} \approx 0.2$ to 0.3.

Assume $V_{CE(SAT)} = 0.2$ V,

$$V_E = 6 - 0.7 = 5.3\text{ V}$$

and

$$I_E = \frac{5.3}{3.3} = 1.6\text{ mA}$$

$$V_C = V_E + V_{CE(SAT)} = 5.3 + 0.2 = 5.5\text{ V}$$

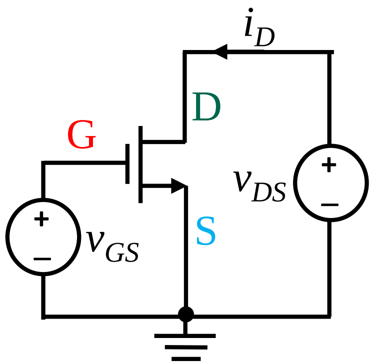
$$I_C = \frac{10 - 5.5}{4.7} = 0.96\text{ mA}$$

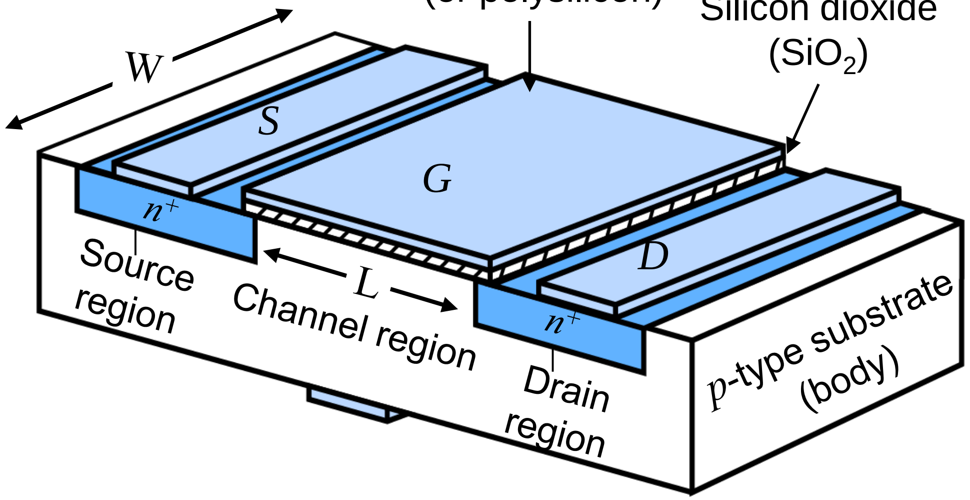
$$I_B = I_E - I_C = 1.6 - 0.96 = 0.64\text{ mA}$$

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{0.96}{0.64} = 1.5$$

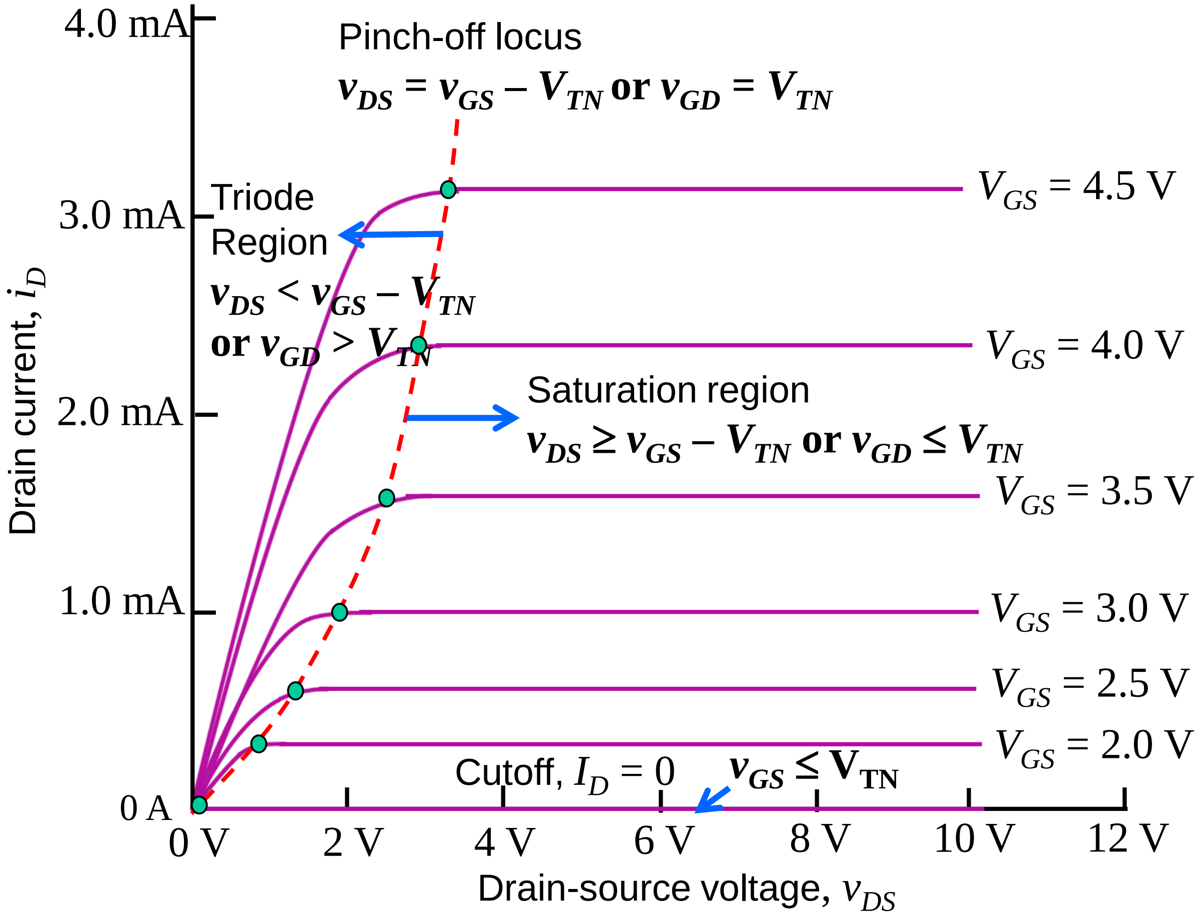
=> In saturation, $I_c \neq \beta I_B$.

nMOS Transistor Structure and I-V Characteristics

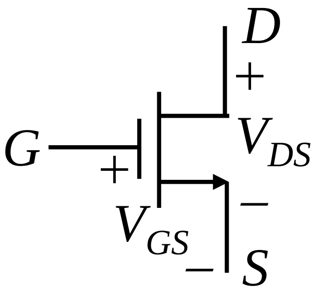




V_{TN} : Threshold Voltage of NMOS; V_{TN} = 1 V for this graph

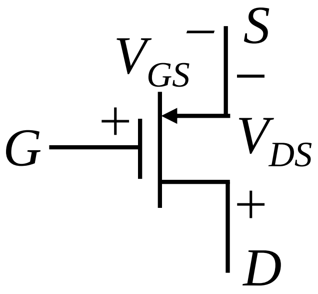


MOSFET Biasing for Different Regions of Operation



$V_{TN} > 0$

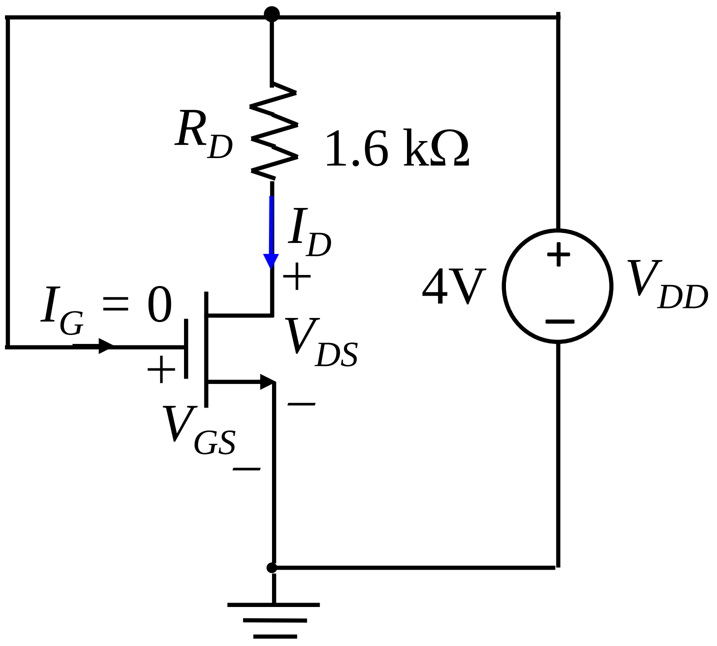
What is V(tn)



$V_{TP} < 0$

Region	NMOS	PMOS
Cutoff	$V_{GS} < V_{TN}$ $I_D = 0$	$\lvert V_{GS}\rvert < \lvert V_{TP}\rvert$ $I_D = 0$
Triode	$V_{GS} \geq V_{TN}$ *and* $V_{DS} < V_{GS} - V_{TN}$ $I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$	$\lvert V_{GS}\rvert \geq \lvert V_{TP}\rvert$ *and* $\lvert V_{DS}\rvert < \lvert V_{GS}\rvert - \lvert V_{TP}\rvert$ $I_D = K_p\left(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert - \frac{\lvert V_{DS}\rvert}{2}\right)\lvert V_{DS}\rvert$
Saturation	$V_{GS} \geq V_{TN}$ *and* $V_{DS} \geq V_{GS} - V_{TN}$ $I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2$	$\lvert V_{GS}\rvert \geq \lvert V_{TP}\rvert$ *and* $\lvert V_{DS}\rvert \geq \lvert V_{GS}\rvert - \lvert V_{TP}\rvert$ $I_D = \frac{K_p}{2}\left(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert\right)^2$

MOSFET Bias Analysis: Triode Region



$K_n = 250\ \mu A/V^2$

$V_{TN} = 1\ V$

$V_{GS} = V_{DD} = 4$ V.

Assume transistor is saturated,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$= \frac{250\ \mu}{2}(4-1)^2$$

$$= 1.13\ \text{mA}$$

$$V_{DD} = I_D R_D + V_{DS}$$

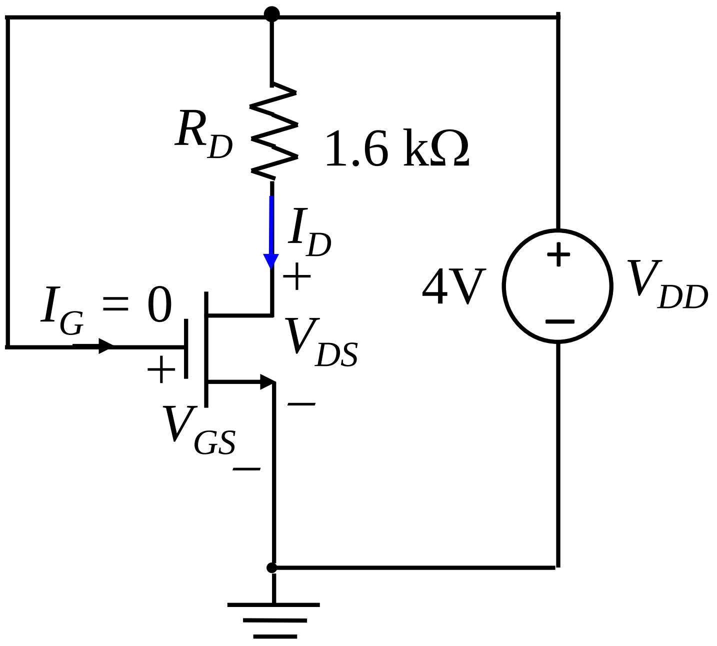
$$V_{DS} = 4 - 1.6 \times 1.13$$

$$= 2.19\ \text{V}$$

But $V_{DS} = 2.19 < V_{GS} - V_{TN} = 3$

Saturation region assumption is incorrect.

MOSFET Bias Analysis: Triode Region



$K_n = 250\ \mu A/V^2$

$V_{TN} = 1$ V

(Cont…)

Using triode region equation,

$$I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$$

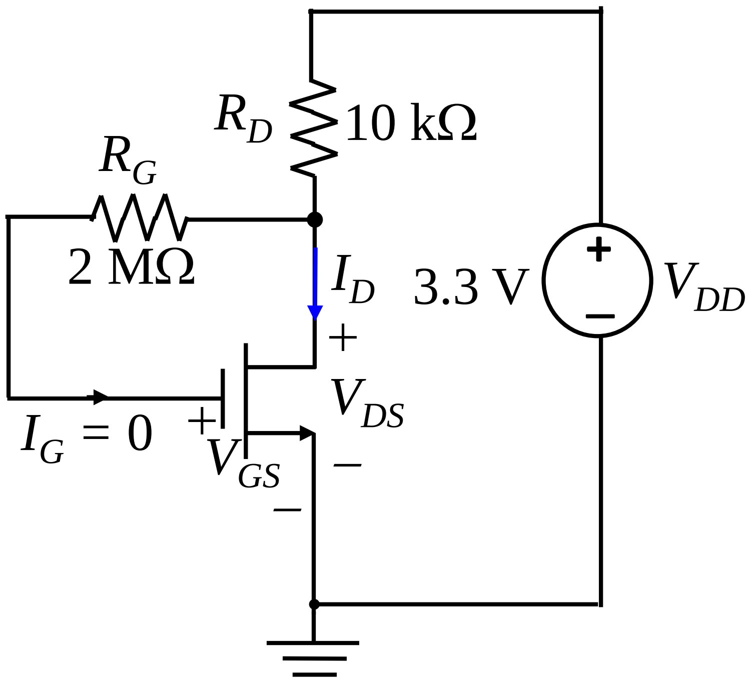
$$V_{DS} = 4 - 1600 \times 250\mu\left(4 - 1 - \frac{V_{DS}}{2}\right)V_{DS}$$

$$0.2V_{DS}^2 - 2.2V_{DS} + 4 = 0$$

$V_{DS} = 8.7$ (infeasible) or **2.3 V** (< $V_{GS} - V_{TN} = 3$)

Hence, $V_{DS} = \mathbf{2.3\ V}$ and $I_D = \mathbf{1.06\ mA}$

MOSFET Bias Analysis: nMOS Two-Resistor Biasing



$K_n = 260\ \mu\text{A/V}^2$
$V_{TN} = 1$ V

Since $I_G = 0$, $V_{DS} = V_{GS}$.

Transistor is saturated because $V_{DS} > V_{GS} - 1$

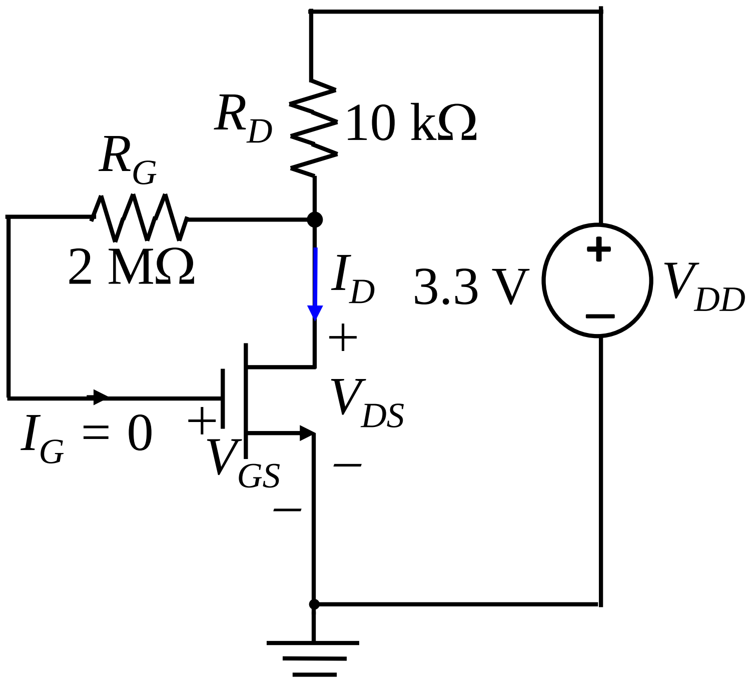
$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$= \frac{260\ \mu}{2}(V_{GS} - 1)^2$$

$$V_{DS} = V_{DD} - I_D R_D$$

$$V_{GS} = 3.3 - \frac{260\ \mu}{2}(V_{GS} - 1)^2 \times 10000$$

MOSFET Bias Analysis: nMOS Two-Resistor Biasing



$K_n = 260\ \mu A/V^2$
$V_{TN} = 1\ V$

(Cont.)

$$1.3V_{GS}^2 - 1.6V_{GS} - 2 = 0$$

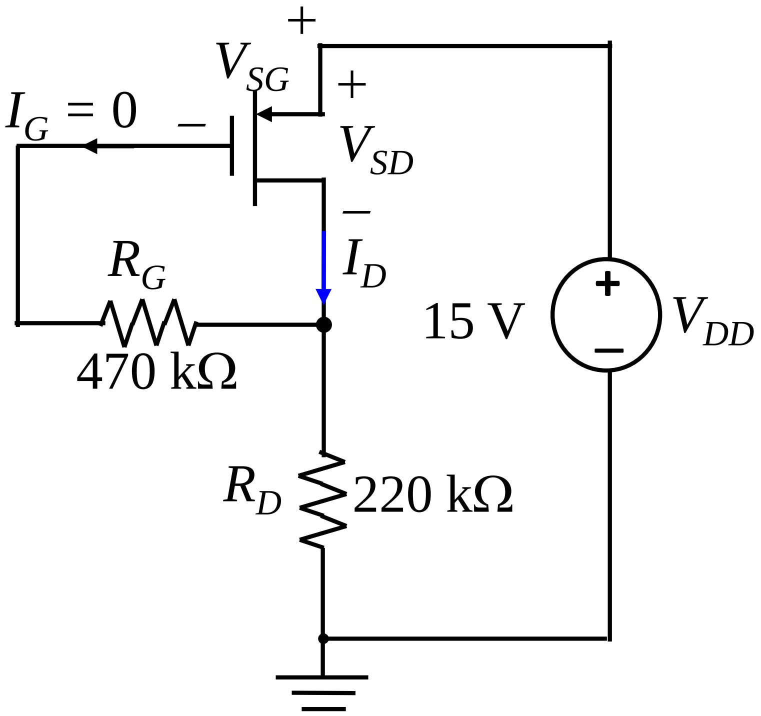
$$V_{GS} = \frac{1.6 \pm \sqrt{1.6^2 - 4 \times 1.3 \times (-2)}}{2 \times 1.3}$$
$$= -0.77\ \text{V or } 2\ \text{V}$$

$V_{GS} = -0.77$ V implies MOSFET is cutoff and contradicts the observation.

$V_{GS} = 2$ V and $V_{DS} = V_{GS} = 2$ V.

$I_D = 130\ \mu \times (2 - 1)^2 = 130\ \mu A$

MOSFET Bias Analysis: pMOS Two-Resistor Biasing



$K_p = 50\ \mu A/V^2$
$V_{TP} = -2$ V

Since $I_G = 0$, $V_{SG} = V_{SD}$.

Transistor is saturated because $|V_{DS}| > |V_{GS}| - |-2|$

$$I_D = \frac{K_p}{2}\left(|V_{GS}| - |V_{TP}|\right)^2$$

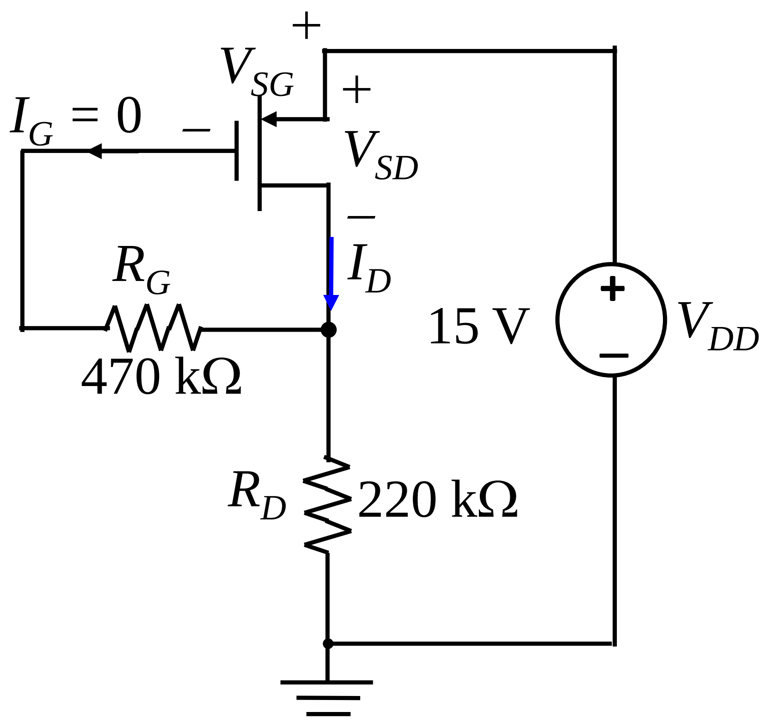
$$= \frac{50\ \mu}{2}\left(|V_{GS}| - 2\right)^2$$

$$|V_{DS}| = V_{DD} - I_D R_D$$

$$|V_{GS}| = 15 - 25\times10^{-6}\left(|V_{GS}| - 2\right)^2 \times 220\times10^3$$

$$= 15 - 5.5\left(|V_{GS}| - 2\right)^2$$

MOSFET Bias Analysis: pMOS Two-Resistor Biasing



$K_p = 50\ \mu\text{A/V}^2$

$V_{TP} = -2\text{ V}$

(Cont.)

$$5.5|V_{GS}|^2 - 21|V_{GS}| + 7 = 0$$

$$|V_{GS}| = \frac{21 \pm \sqrt{21^2 - 4\times5.5\times7}}{2\times5.5}$$

$$= 0.37\text{ V or } 3.45\text{ V}$$

$|V_{GS}| = 0.37\text{ V} < |V_{TP}| = 2\text{ V},$

$|V_{GS}| = 3.45\text{ V or } V_{SG} = 3.45\text{ V}.$

$V_{DS} = V_{GS} = -3.45\text{ V}.$

$I_D = 25\ \mu \times (3.45 - 2)^2 = 52.5\ \mu\text{A}$

DC Analysis of Four-Resistor Biasing Circuit

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Lesson Objectives

At the end of this lesson, you should be able to:

- Draw DC equivalent circuits for four-resistor biasing BJT and MOSFET amplifiers
- Calculate the Q-points from the DC equivalent circuits of BJT and MOSFET amplifiers by using appropriate circuit analysis techniques

DC and AC Analyses

DC Analysis

- Obtain DC equivalent circuit by replacing all capacitors by open circuits, inductors by short circuit, AC voltage sources by ground connections, and AC current sources by open circuits.
- Find Q-point from DC equivalent circuit by using appropriate circuit analysis techniques, such as Thevenin equivalent circuit, KVL, and KCL.

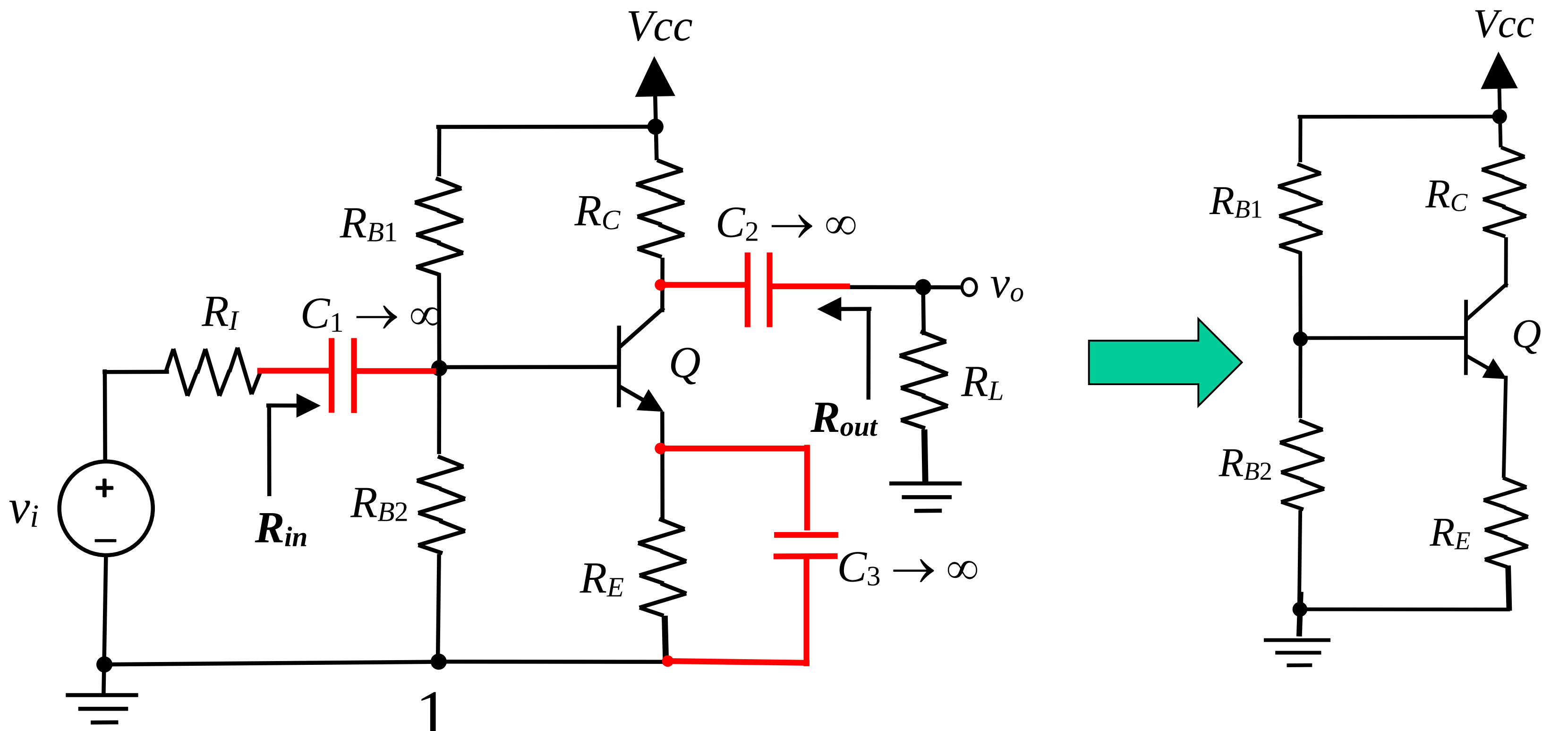
DC and AC Analyses

AC Analysis

- Obtain AC equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, DC voltage sources by ground connections, and DC current sources by open circuits.
- Replace transistor by its small signal model.
- Use small signal AC equivalent to analyse AC characteristics of amplifier.

Combine end results of DC and AC analyses to yield total voltages and currents in the network.

DC Equivalent Circuit for BJT Amplifier

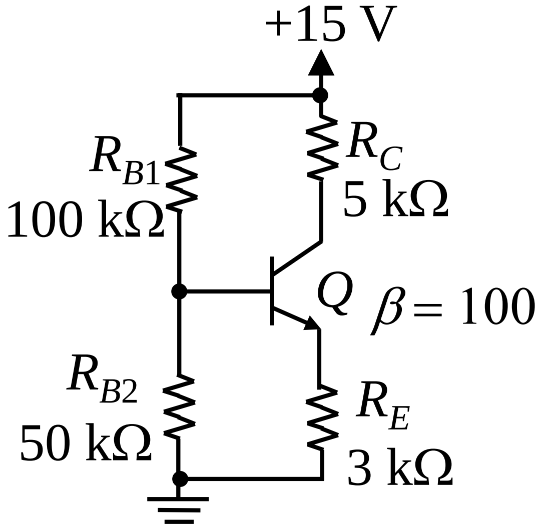


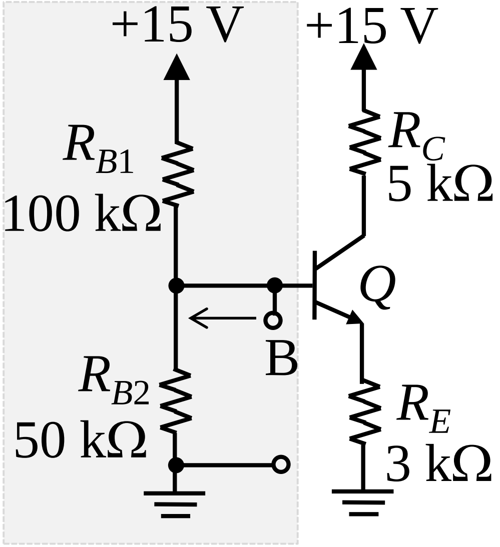
$$X_C = \frac{1}{\omega C} \qquad \omega = 0 \text{ rad/s (DC)}, X_C = \infty \text{(open circuit)}$$

All capacitors in original amplifier circuits are replaced by open circuits, disconnecting v_i, R_I, and R_L from circuit.

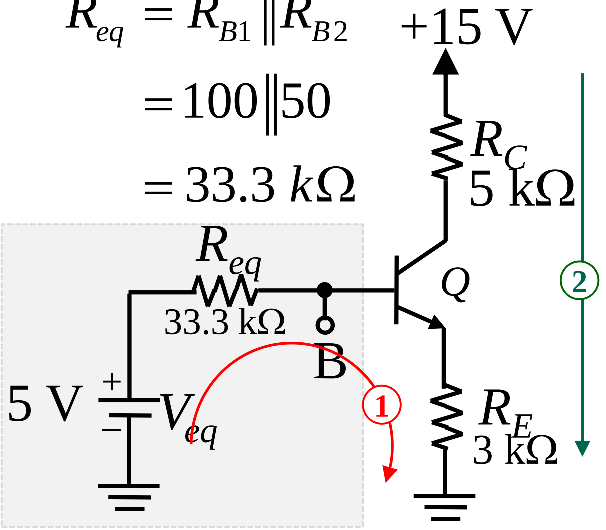
DC Analysis Example: Four-Resistor BJT Biasing Circuit

Equi Resistance how???





$$R_{eq} = R_{B1} \| R_{B2} = 100 \| 50 = 33.3\ k\Omega$$



KVL 1: $V_{eq} = I_B R_{eq} + V_{BE} + I_E R_E$

$$5 = 33.3 I_B + 0.7 + 101 \times I_B \times 3$$

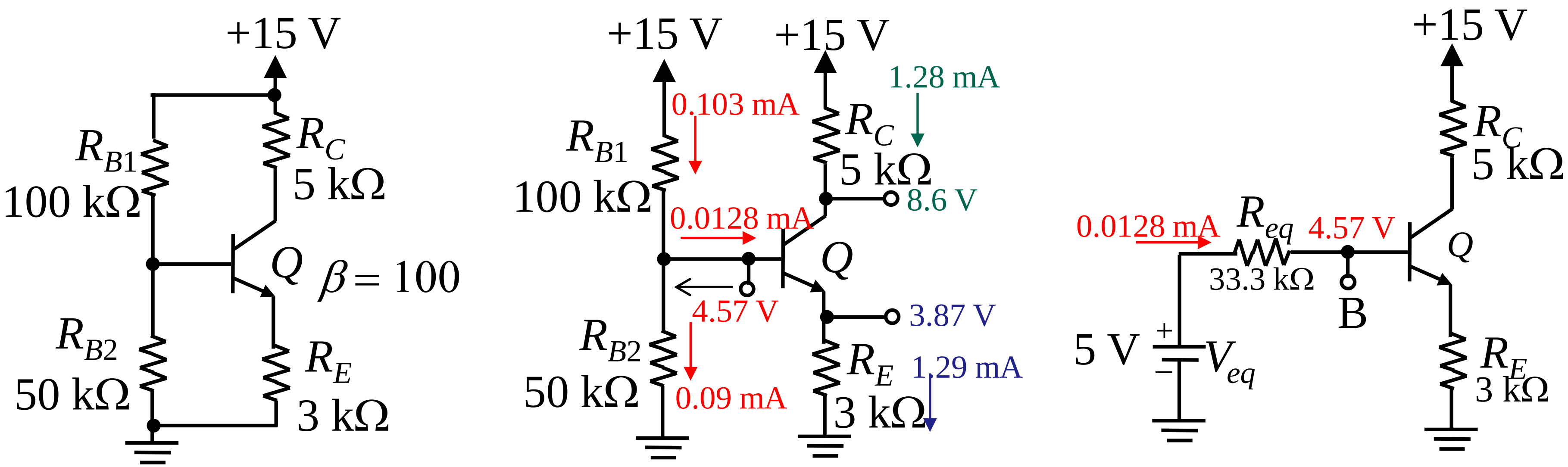
$$I_B = 0.0128 \text{ mA}$$

$$V_{eq} = \left(\frac{R_{B2}}{R_{B1} + R_{B2}} \right) V_{CC} = \frac{50}{100+50} \times 15 = 5 \text{ V}$$

$I_C = \beta I_B = \mathbf{1.28\ mA}$, $I_E = (\beta+1)\, I_B = 1.29$ mA

KVL 2: $V_{CE} = 15 - I_C R_C - I_E R_E = 15 - 1.28 \times 5 - 1.29 \times 3 = \mathbf{4.73\ V}$

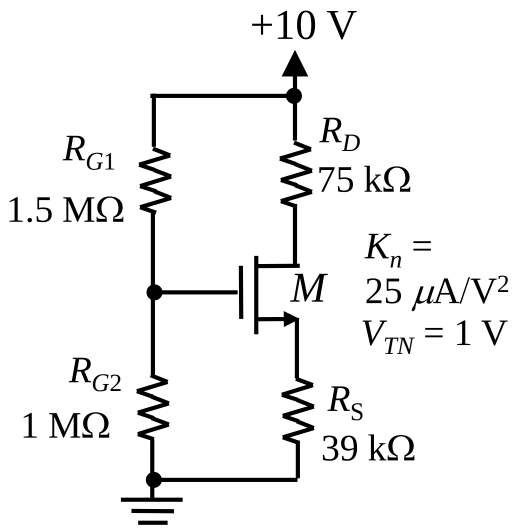
DC Analysis Example: Four-Resistor BJT Biasing Circuit



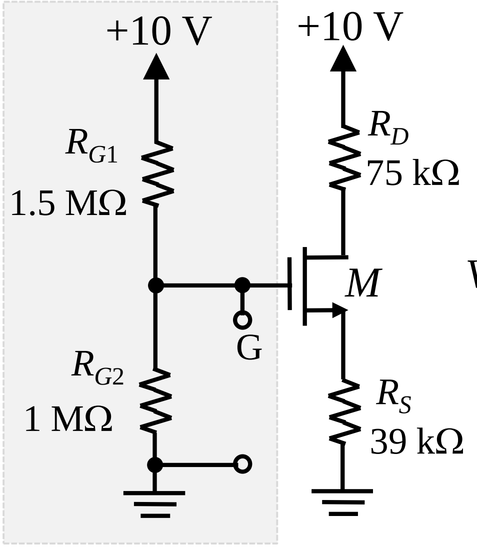
$$V_B = V_{BE} + I_E R_E = 0.7 + 3.87 = 4.57\text{ V}$$
$$V_C = V_{CC} - I_C R_C = 15 - 1.28 \times 5 = 8.6\text{ V}$$
$$V_{BC} = V_B - V_C = 4.57 - 8.6 = -4.03\text{ V}$$

BCJ is reverse biased, *Q* is indeed in active mode as had been assumed.

DC Analysis Example: Four-Resistor MOSFET Biasing Circuit

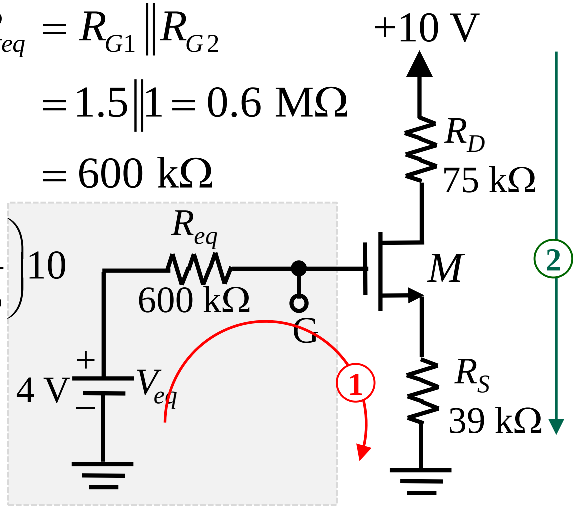


$K_n = 25\ \mu\text{A/V}^2$
$V_{TN} = 1$ V



$$R_{eq} = R_{G1} \| R_{G2} = 1.5 \| 1 = 0.6\ \text{M}\Omega = 600\ \text{k}\Omega$$

$$V_{eq} = \left(\frac{1}{1+1.5}\right) 10 = 4\ \text{V}$$



KVL 1: Since $I_G = 0$, $V_{eq} = V_{GS} + I_D R_S$

$$4 = V_{GS} + 0.5 \times 25\mu\,(V_{GS} - 1)^2 \times 39\ \text{k}$$
$$V_{GS}^2 + 0.05\ V_{GS} - 7.21 = 0$$
$$V_{GS} = -2.71 \text{ or } 2.66\ \text{V}$$

Since $V_{GS} = -2.71 < V_{TN} = 1$, $V_{GS} = 2.66$ V.

$\boldsymbol{I_D} = (4 - 2.66)/39\text{k} = \mathbf{34.4\ \mu A}$

KVL 2: $V_{DS} = 10 - I_D R_D - I_D R_S$

$\boldsymbol{V_{DS}} = 10 - 0.0344 \times (75 + 39) = \mathbf{6.08\ V}$

Since $V_{DS} > V_{GS} - V_{TN} = 1.66$,

***M* is in saturation region.**

AC Analysis of BJT and MOSFET Inverting Amplifiers

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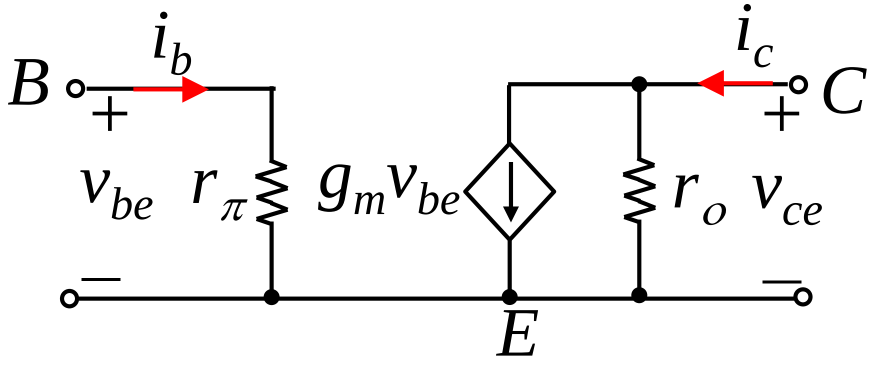
EE2002 Analog Electronics

Lesson Objectives

At the end of this lesson, you should be able to:

- Draw small signal model for BJT and MOSFET
- Calculate the small signal parameters of BJT and MOSFET
- Construct AC equivalent circuit of BJT and MOSFET inverting amplifiers
- Calculate the following performance characteristics of C-E and C-S amplifiers
  - Voltage gain
  - Input resistance
  - Output resistance

Hybrid-Pi Model of BJT



- This hybrid-pi small-signal model is the intrinsic low-frequency representation of the BJT.
- Small-signal parameters are controlled by the Q-point and are independent of geometry of BJT.

Transconductance:

$$g_m = \frac{I_C}{V_T} \approx 40 I_C$$

where $V_T = \dfrac{kT}{q} \approx 25\ \text{mV@}25^\circ\text{C}$

Input resistance:

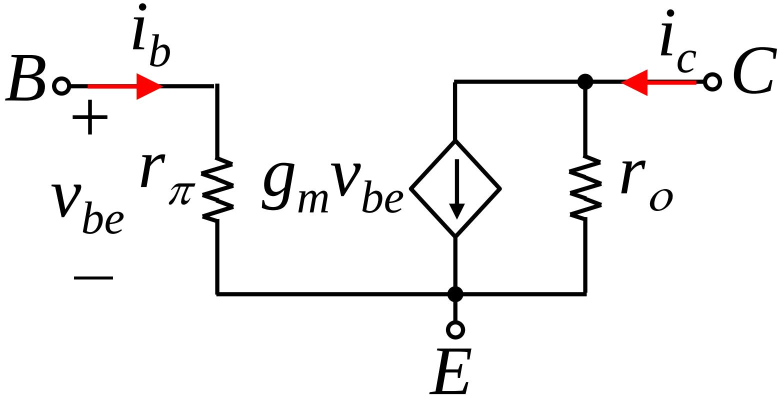
$$r_\pi = \frac{\beta}{g_m}$$

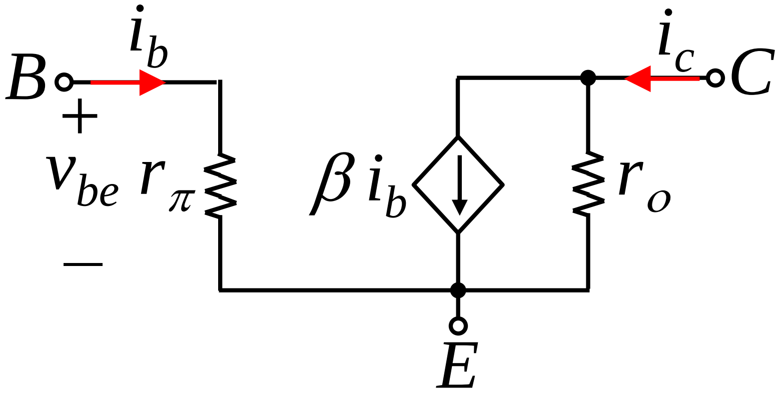
Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} \quad \text{if } V_A \gg V_{CE}$$

Equivalent Forms of Small-Signal Model for BJT

Voltage-controlled current source $g_m v_{be}$ can be transformed into current-controlled current source.





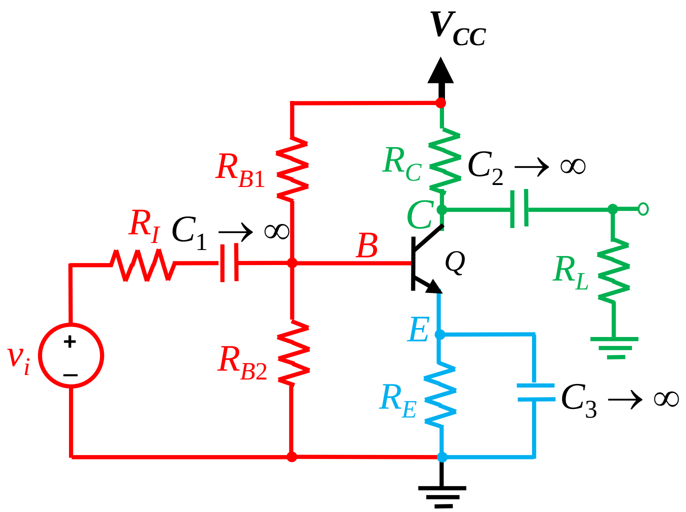
$$v_{be} = i_b r_\pi$$

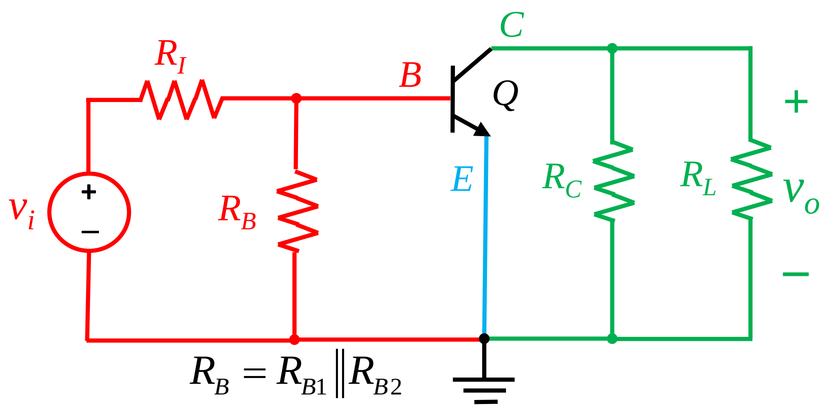
$$g_m v_{be} = g_m i_b r_\pi = \beta i_b$$

$$i_c = \beta i_b + \frac{v_{ce}}{r_o} \approx \beta i_b$$

Basic relationship $i_c = \beta\, i_b$ is useful for both dc and ac analysis when BJT is in forward-active region.

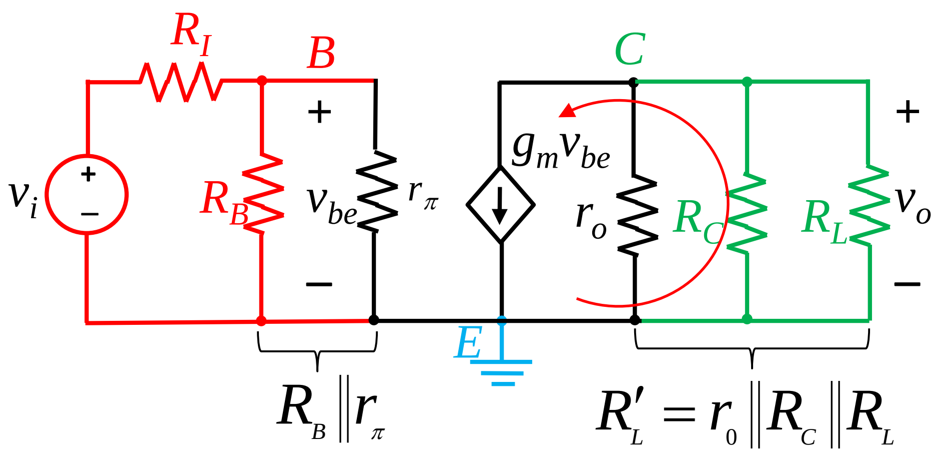
Small Signal Analysis of C-E Amplifier with Fully Bypass R_E





- The ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage source is ac ground.
- Assume that Q-point has already been calculated from DC analysis. Hence, g_m, r_π, and r_o of BJT can be calculated.

C-E Amplifier with Fully Bypass R_E: Voltage Gain



Overall voltage gain from source v_i to output voltage v_o across R_L is:

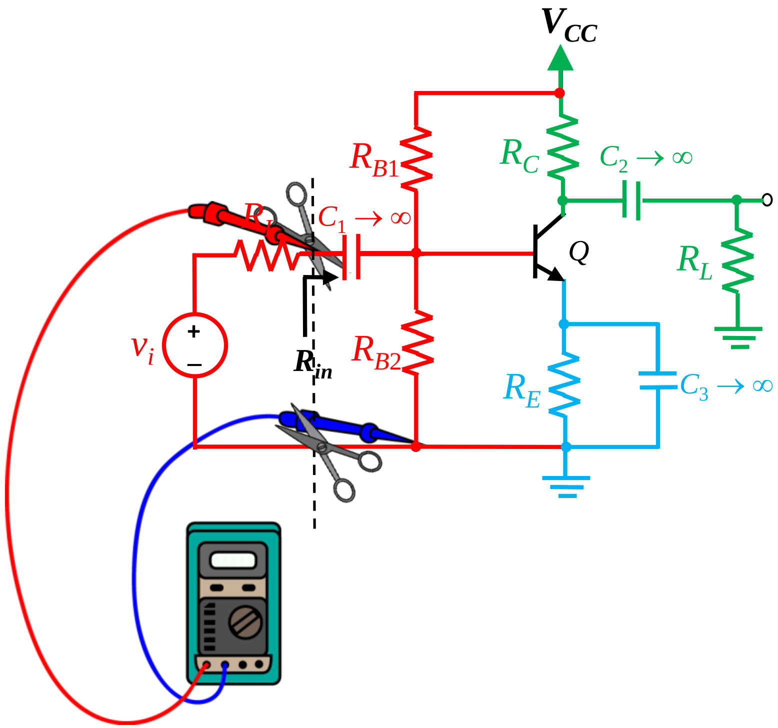
$$A_v = \frac{v_o}{v_i} = \frac{v_c}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{v_b}{v_i}$$

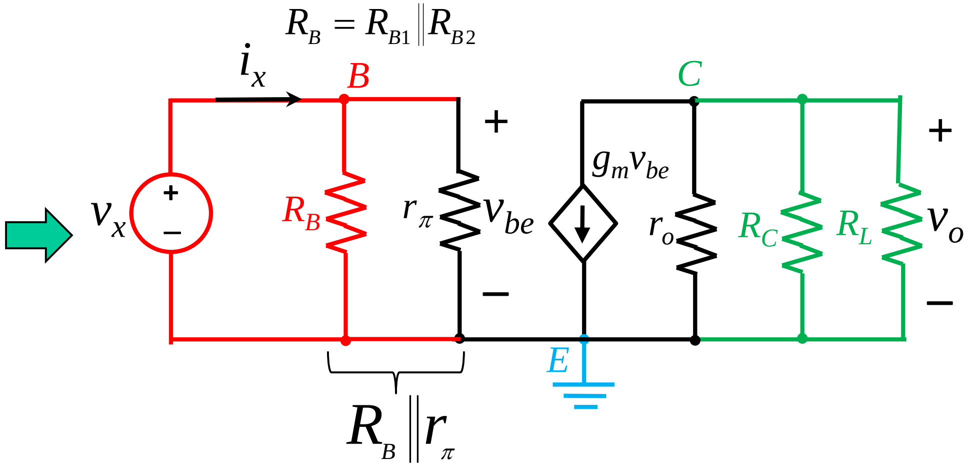
$$\therefore A_v = -g_m R_L' \left(\frac{R_B \| r_\pi}{R_I + R_B \| r_\pi} \right)$$

Terminal voltage gain between base and collector is:

$$A_{vt} = \frac{v_c}{v_b} = \frac{v_o}{v_{be}} = \frac{-g_m v_{be} R_L'}{v_{be}} = -g_m R_L'$$

C-E Amplifier with Fully Bypass R_E: Input Resistance



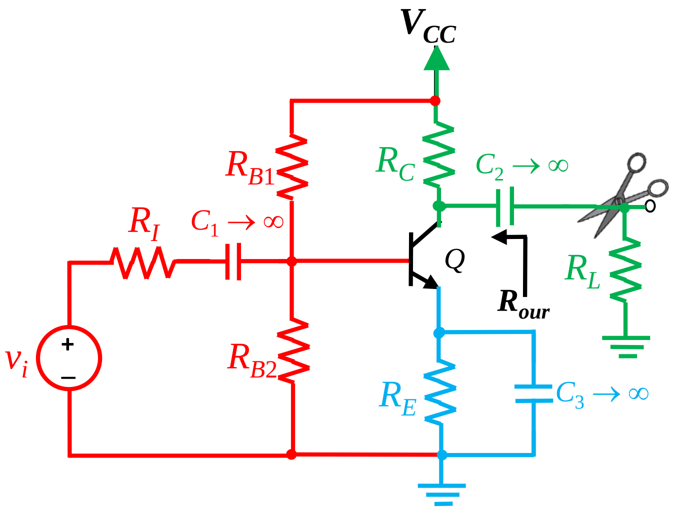


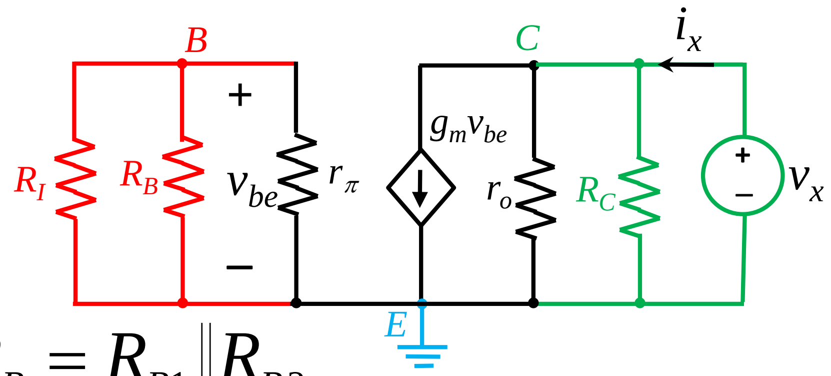
$$v_x = i_x \left(R_B \| r_\pi \right)$$

$$R_{in} = \frac{v_x}{i_x}$$

$$= R_B \| r_\pi$$

C-E Amplifier with Fully Bypass R_E: Output Resistance





$$R_B = R_{B1} \| R_{B2}$$

$$i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o} + g_m v_{be}$$

$$v_{be} = 0 \Rightarrow i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o}$$

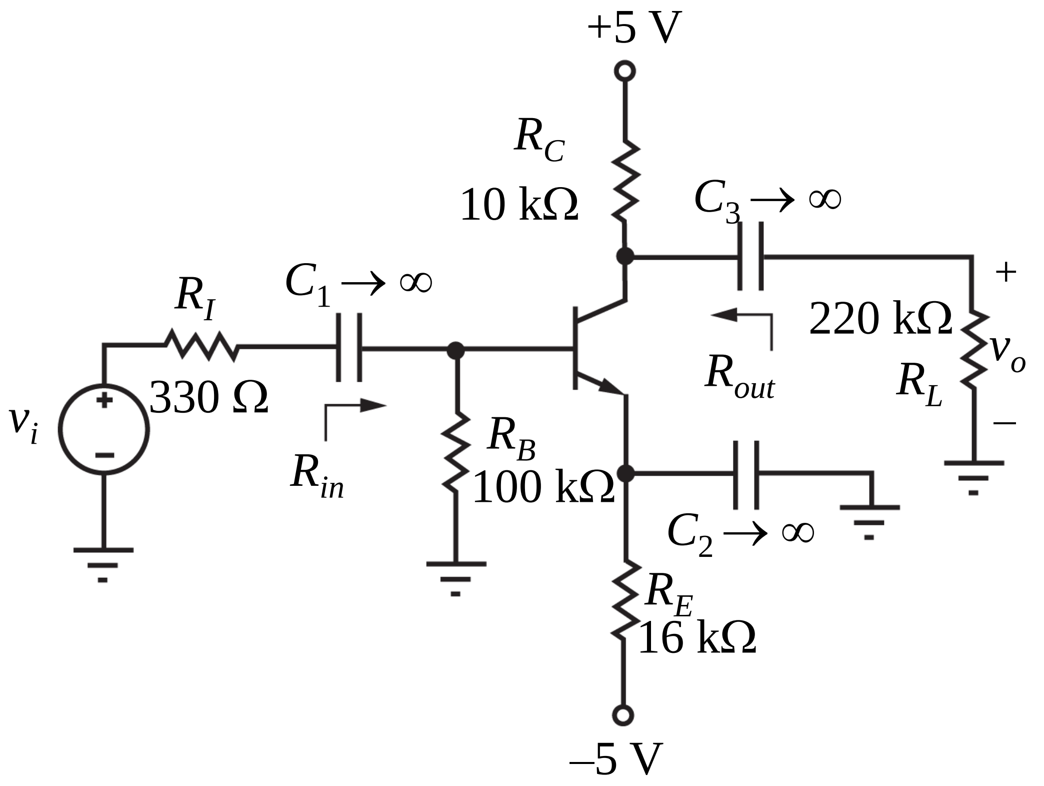
$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_C} + \frac{1}{r_o} \right)^{-1} = R_C \| r_o$$

$$R_{out} \approx R_C \text{ if } r_o \gg R_C$$

C-E Amplifier with Fully Bypass R_E: Example

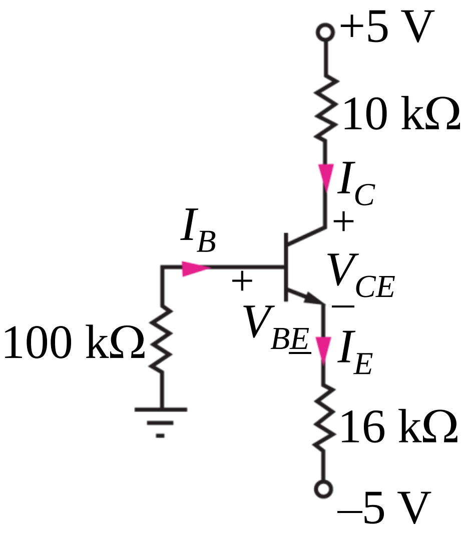
Problem: Find voltage gain, input and output resistances.

Given: $\beta = 65$, $V_A = 50$ V



Find the Q-point from dc equivalent circuit:

$$100\times10^3 I_B + 0.7 + 66 I_B\left(16\times10^3\right) + (-5) = 0$$



$$I_B = 3.71\ \mu\text{A}$$

$$\boxed{I_C = 65 I_B = 241\ \mu\text{A}}$$

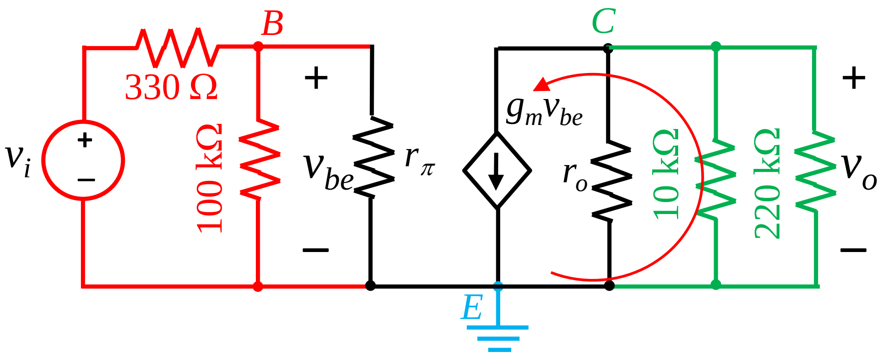
$$I_E = 66 I_B = 245\ \mu\text{A}$$

$$5 - 10\text{ k}\times 241\ \mu - V_{CE} - 16\text{ k}\times 245\ \mu - (-5) = 0$$

$$\boxed{V_{CE} = 3.67\text{ V}}$$

AC Analysis of C-E Amplifier with Fully Bypass R_E

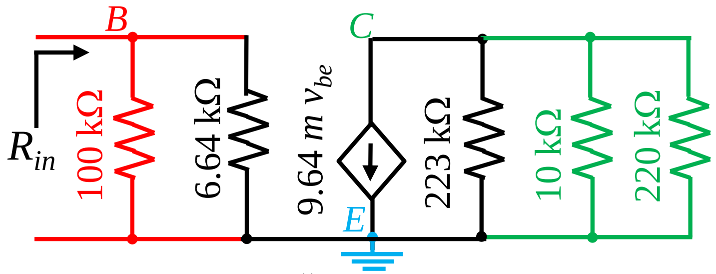
Small signal equivalent



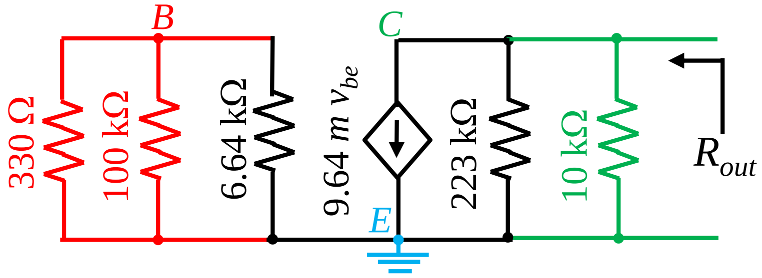
$$g_m \approx 40 \times 241\ \mu = 9.64\ \text{mS}$$

$$r_\pi = \frac{65}{9.64\ \text{m}} = 6.64\ \text{k}\Omega$$

$$r_o = \frac{50 + 3.67}{241\ \mu} = 223\ \text{k}\Omega$$



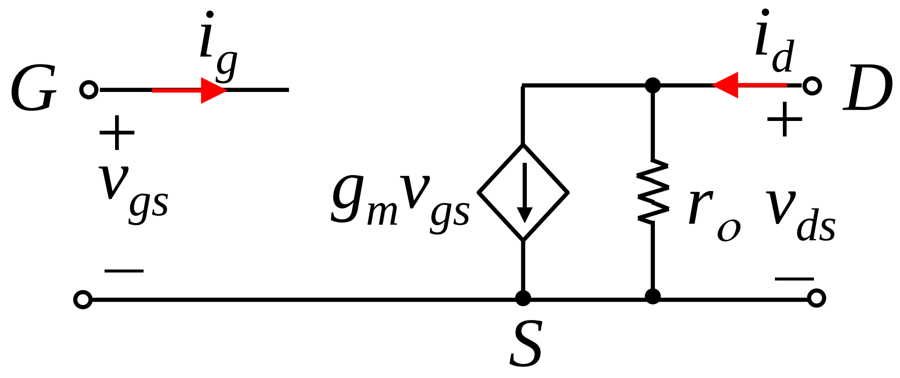
$$R_{in} = 100\ \text{k} \| 6.64\ \text{k} = 6.23\ \text{k}\Omega$$



$$R_{out} = 223\ \text{k} \| 10\ \text{k} = 9.57\ \text{k}\Omega$$

$$A_v = A_{vt}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \left(\frac{-9.64 \times v_{be} \times (223 \| 10 \| 220)}{v_{be}}\right)\left(\frac{6.23\ \text{k}}{330 + 6.23\ \text{k}}\right) = -84.0$$

Small Signal Parameters of MOSFET



- Since gate is insulated from channel by gate-oxide, input resistance = ∞.
- Small-signal parameters are controlled by the Q-point.
- MOSFET transconductance is geometry dependent.

$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q-pt} = K_n\left(V_{GS} - V_{TN}\right)\left(1 + \lambda V_{DS}\right)$$

where $K_n = \mu_n C_{OX}\left(\frac{W}{L}\right)$

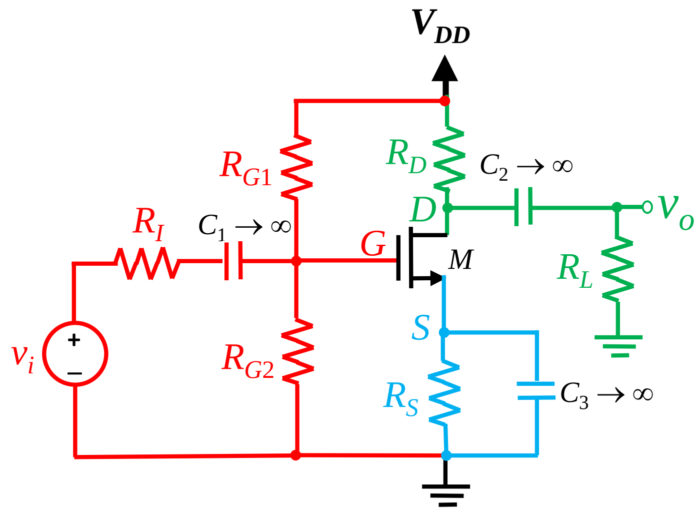
$$g_m = \frac{I_D}{\frac{V_{GS} - V_{TN}}{2}} \approx \sqrt{2K_n I_D}$$

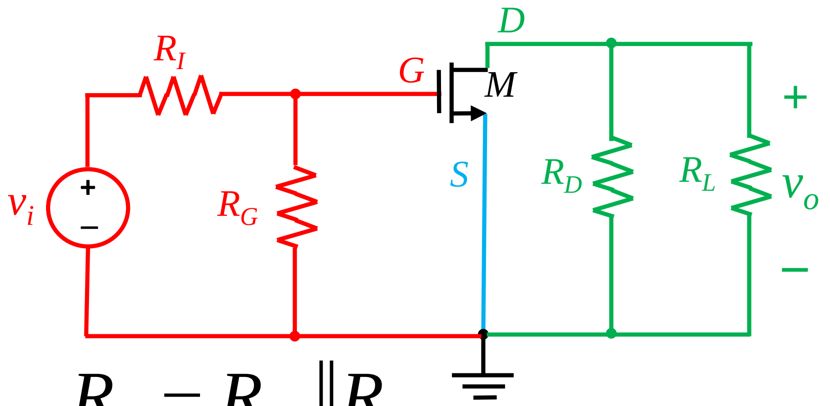
$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \approx \frac{1}{\lambda I_D} \text{ if } \frac{1}{\lambda} \gg V_{DS}$$

Small Signal Analysis of C-S Amplifier with Fully Bypass R_S

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent ac grounds.

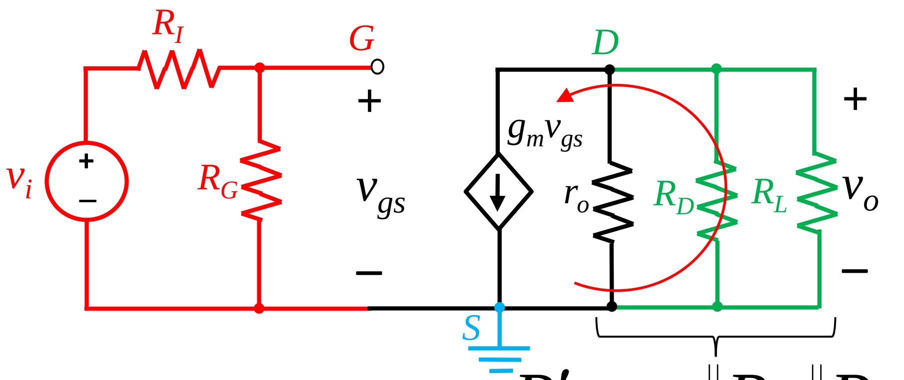
- The small signal parameters, g_m and r_o of the MOSFET is calculated at the Q-point, I_D and V_{DS}.





$$R_G = R_{G1} \| R_{G2}$$

C-S Amplifier with Fully Bypass R_S: Voltage Gain



$$R_L' = r_0 \| R_D \| R_L$$

Terminal voltage gain between gate and drain is:

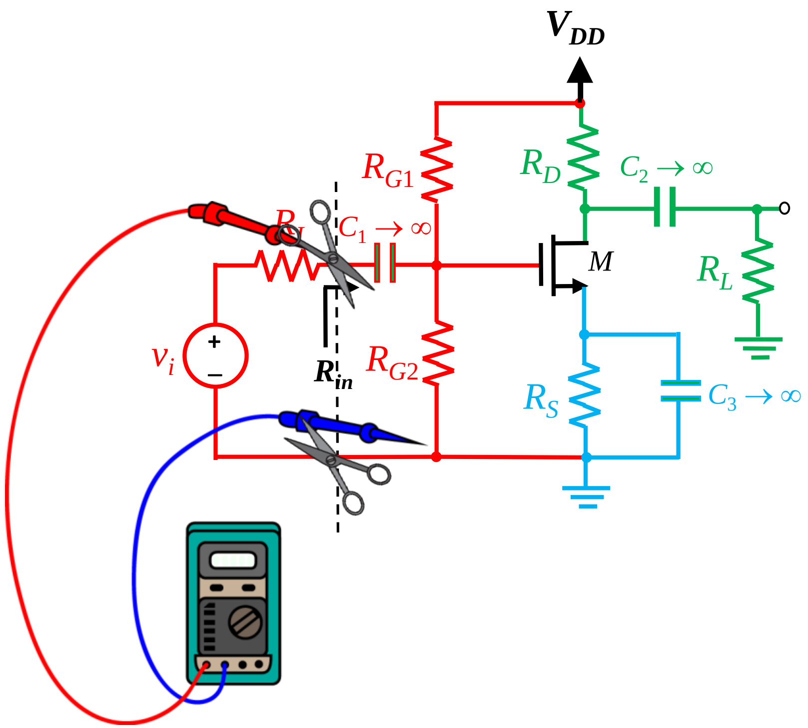
$$A_{vt} = \frac{v_d}{v_g} = \frac{-g_m v_{gs} R_L'}{v_{gs}} = -g_m R_L'$$

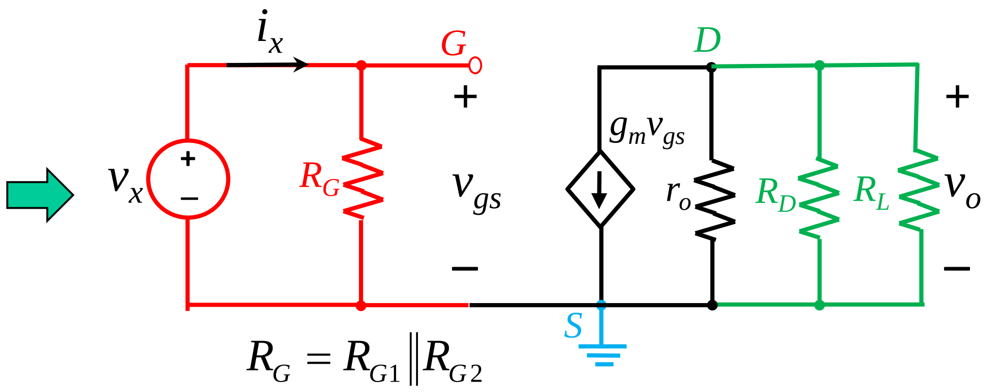
Overall voltage gain from source v_i to output voltage v_o across R_L is:

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{v_g}{v_i}$$

$$= -g_m R_L' \left(\frac{R_G}{R_I + R_G} \right)$$

C-S Amplifier with Fully Bypass R_S: Input Resistance



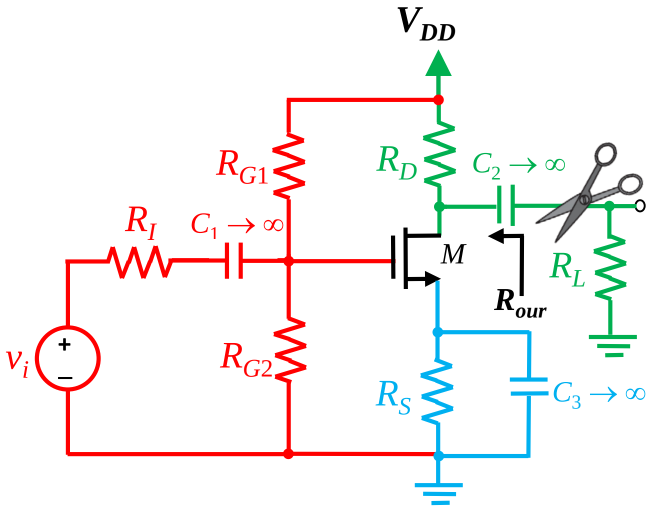


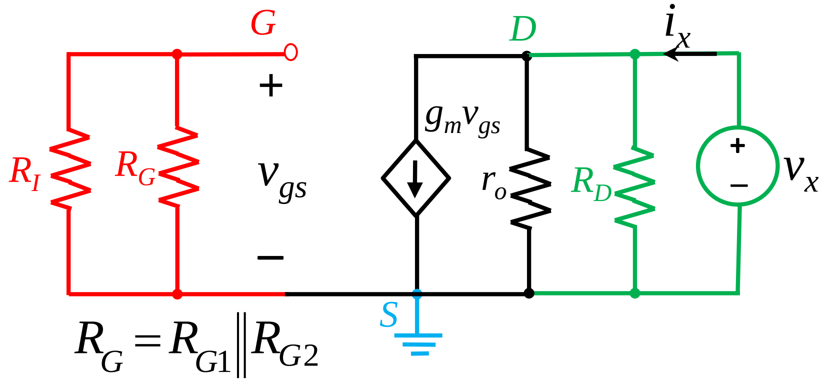
$$v_x = i_x R_G$$

$$R_{in} = \frac{v_x}{i_x}$$

$$= R_G$$

C-S Amplifier with Fully Bypass R_S: Output Resistance





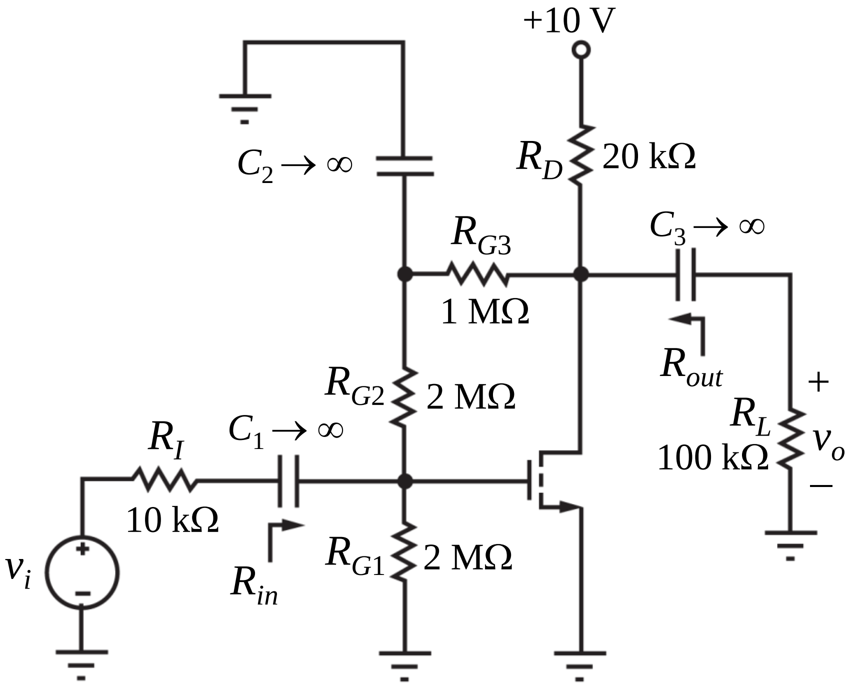
Since $v_{gs} = 0$, $g_m v_{gs} = 0$.

$$v_x = i_x \left(R_D \| r_o \right)$$

$$R_{out} = \frac{v_x}{i_x} = R_D \| r_o$$

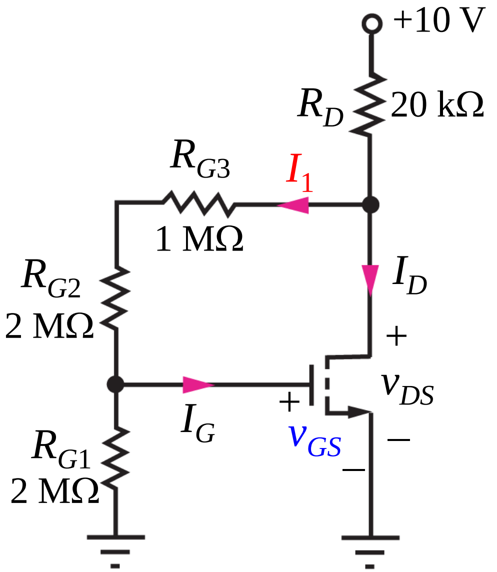
$R_{out} \approx R_D$ if $r_o \gg R_D$

C-S Amplifier with Fully Bypass R_S: Example



Problem: Find voltage gain, input and output resistances.

Given: $K_n = 500\ \mu A/V^2$, $V_{TN} = 1V$, $\lambda = 0.0167\ V^{-1}$



DC Analysis:

$$I_1 = \frac{V_{DS}}{5\times10^6}$$

$$V_{GS} = I_1 \times 2\times10^6 = 0.4V_{DS}$$

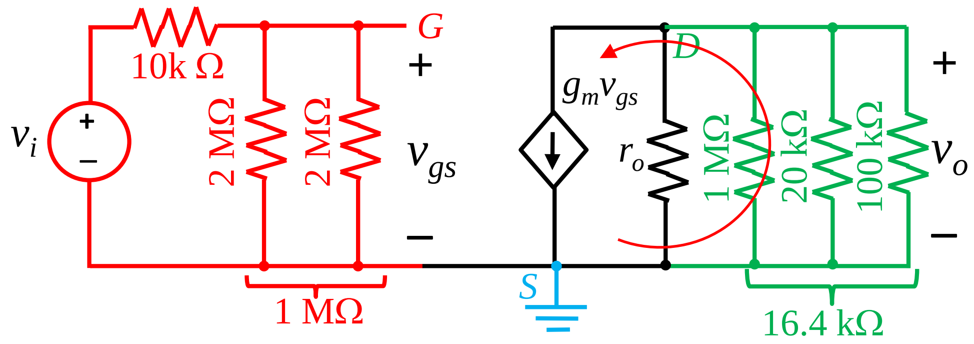
$$I_D = \frac{500\ \mu}{2}(V_{GS}-1)^2 \text{ —— (1)}$$

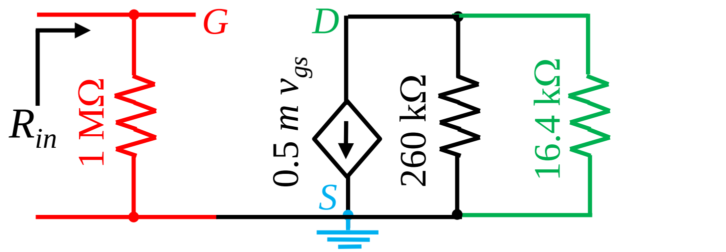
$$V_{DS} = 10 - 20\times10^3(I_D + I_1) \text{ —— (2)}$$

$$\therefore\ V_{DS} = 5V, V_{GS} = 2V, I_D = 250\ \mu A$$

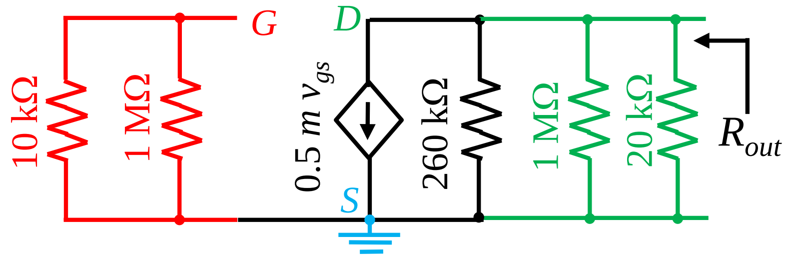
C-S Amplifier with Fully Bypass R_S: Example

Small Signal Equivalent Circuit:





$$R_{in} = 2M \| 2M = 1M\Omega$$



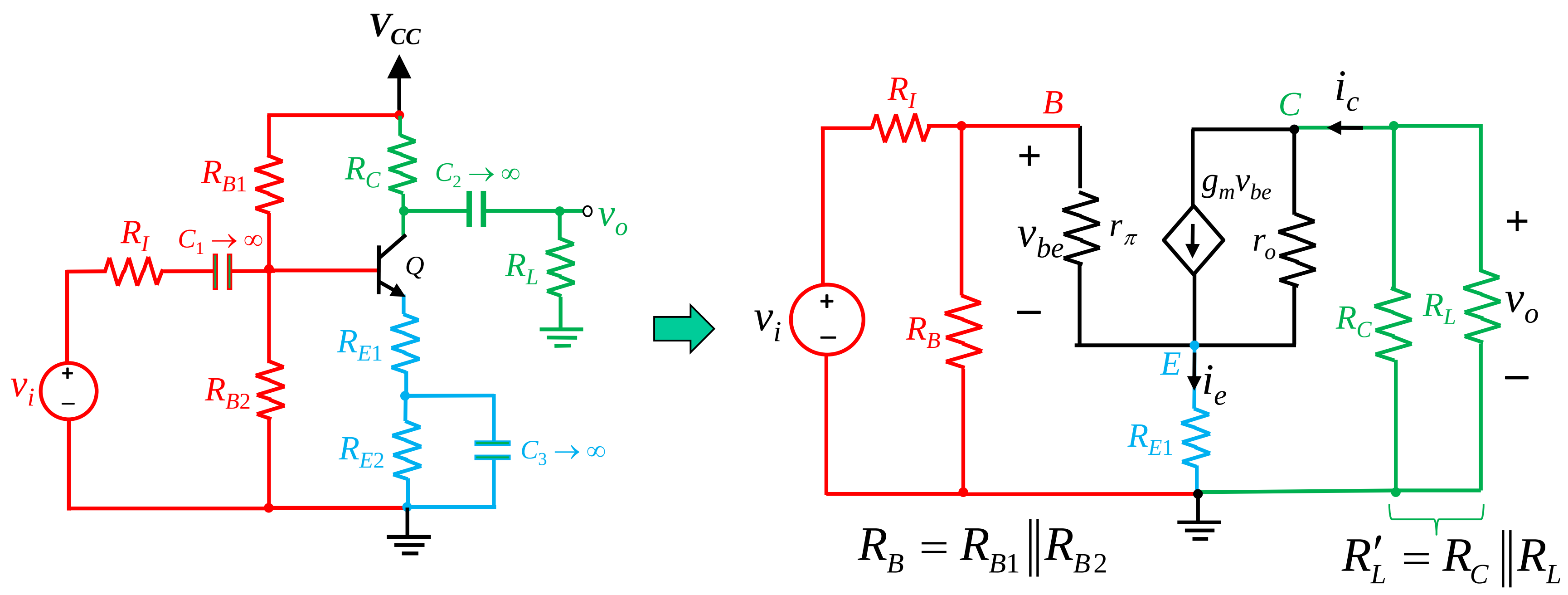
$$R_{out} = 260k \| 1M \| 20k = 18.2k\Omega$$

$$g_m \approx \sqrt{2\times 500\ \mu \times 250\ \mu} = 0.5\ \text{mS}$$

$$r_o = \frac{\dfrac{1}{0.0167} + 5}{250\ \mu} = 260\ \text{k}\Omega$$

$$A_v = A_{vt}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{-(0.5m)v_{gs}\left(260\ \text{k} \| 1\ \text{M} \| 20\ \text{k} \| 100\ \text{k}\right)}{v_{gs}}\left(\frac{1\ \text{M}}{10\ \text{k} + 1\ \text{M}}\right) = -7.93$$

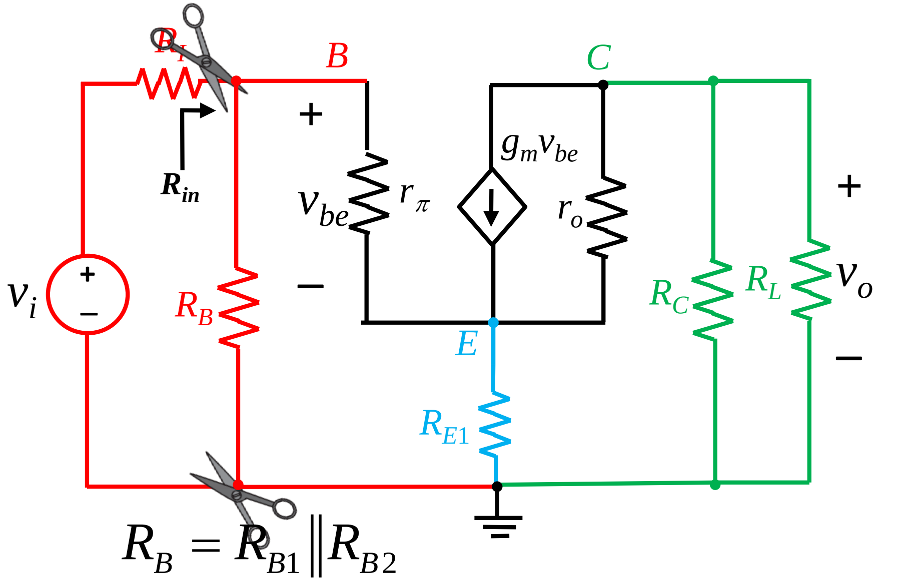
C-E Amplifier with Unbypass R_E: Terminal Voltage Gain

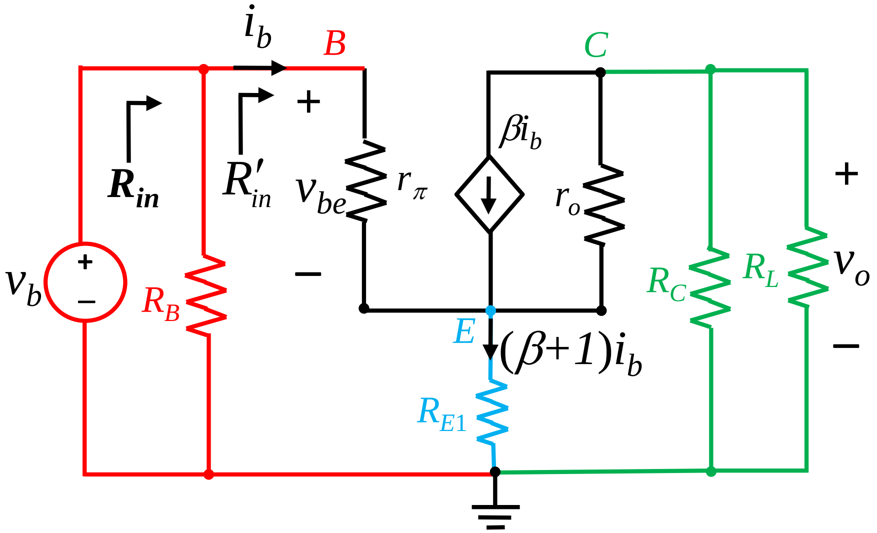


$$\because i_{r_o} \ll g_m v_{be}, i_c \approx g_m v_{be} \approx i_e$$

$$A_{vt} = \frac{v_c}{v_b} = \frac{-i_c R_L'}{v_{be} + i_e R_{E1}} \approx \frac{-g_m v_{be} R_L'}{v_{be} + g_m v_{be} R_{E1}} = \frac{-g_m R_L'}{1 + g_m R_{E1}}$$

C-E Amplifier with Unbypass R_E: Input Resistance

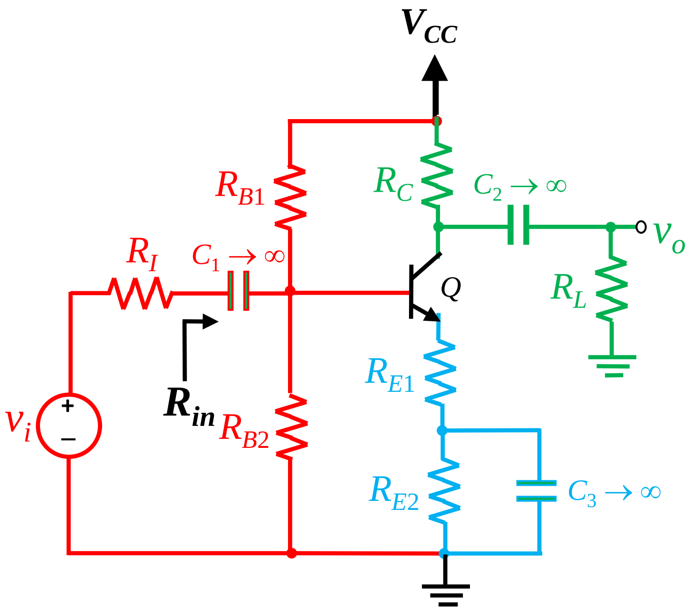


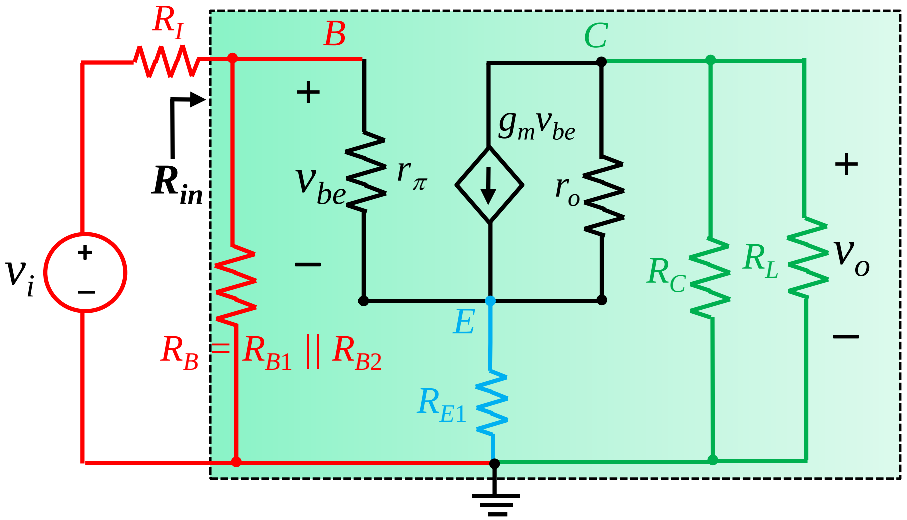


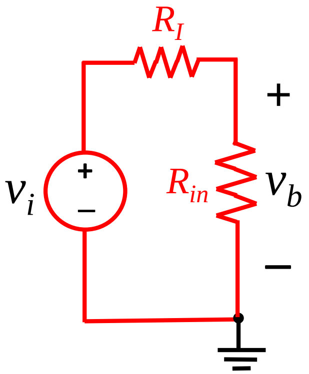
$$v_b = i_b r_\pi + (\beta+1) i_b R_{E1}$$

$$R'_{in} = \frac{v_b}{i_b} = r_\pi + (\beta+1) R_{E1} \Rightarrow R_{in} = R'_{in} \| R_B$$

C-E Amplifier with Unbypass R_E: Overall Voltage Gain

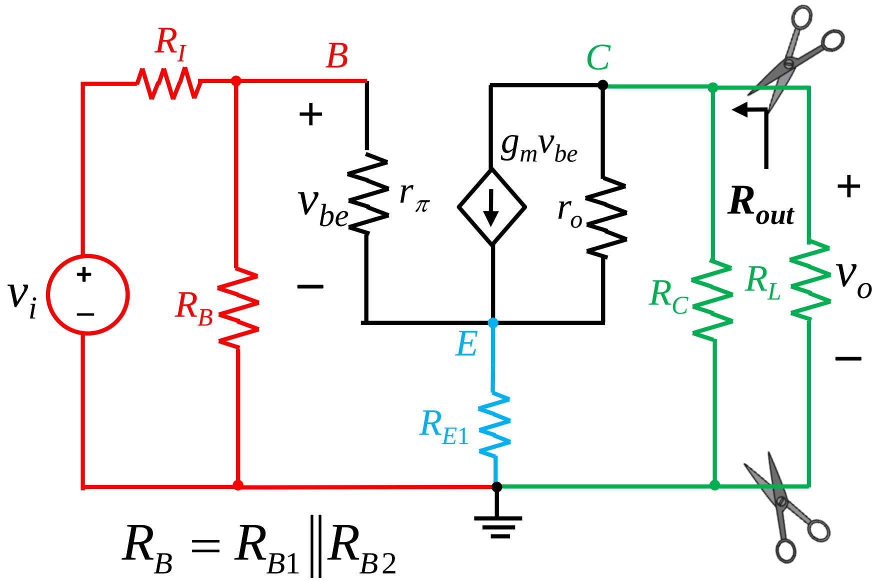




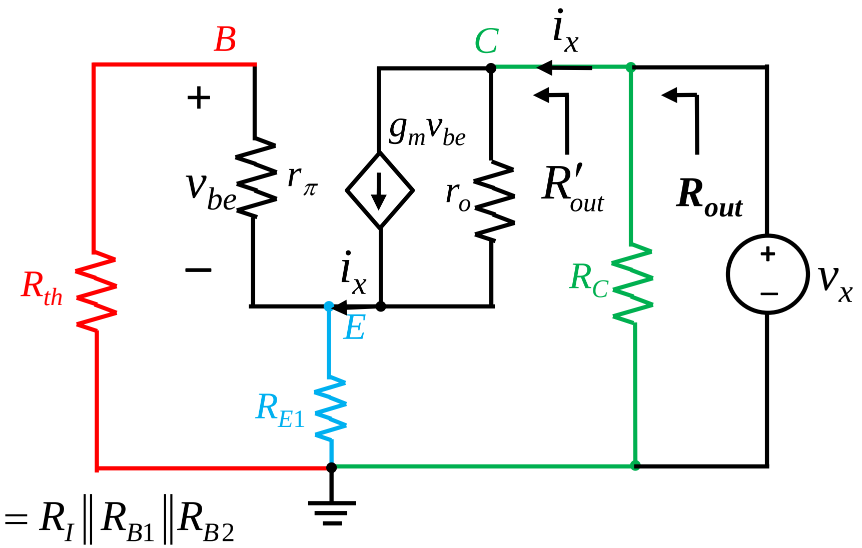


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-E Amplifier with Unbypass R_E: Output Resistance



$$R_B = R_{B1} \| R_{B2}$$



$$R_{th} = R_I \| R_{B1} \| R_{B2}$$

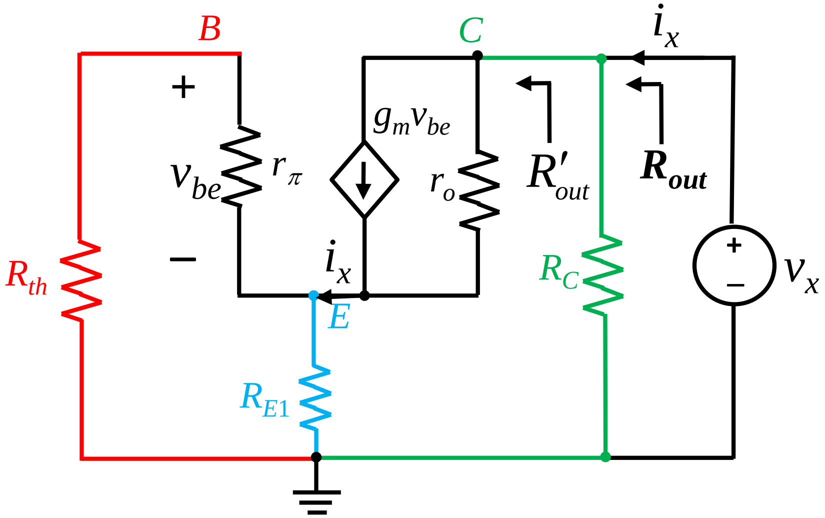
$$v_x = \underbrace{\left(i_x - g_m v_{be}\right)}_{\text{current through } r_o} r_o + v_e$$

$$v_e = i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\}$$

$$v_{be} = -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) v_e$$

$$= -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\}$$

C-E Amplifier with Unbypass R_E: Output Resistance



$$v_x = (i_x - g_m v_{be}) r_o + v_e$$

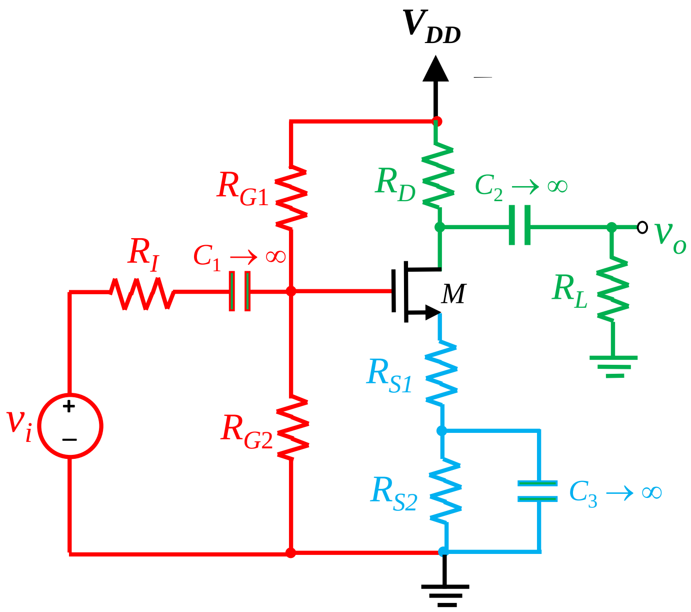
$$= \left(i_x + g_m \left(\frac{r_\pi}{r_\pi + R_{th}} \right) i_x \left\{ (r_\pi + R_{th}) \| R_{E1} \right\} \right) r_o$$

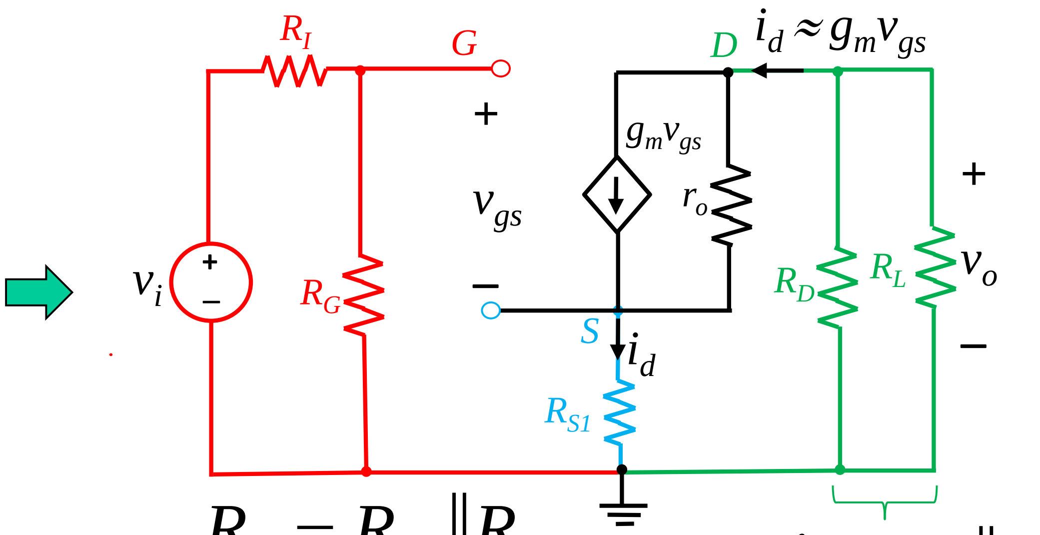
$$+ i_x \left\{ (r_\pi + R_{th}) \| R_{E1} \right\}$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + g_m \left(\frac{r_\pi}{r_\pi + R_{th}} \right) \left\{ (r_\pi + R_{th}) \| R_{E1} \right\} \right) r_o$$

$$R_{out} = R'_{out} \| R_C$$

C-S Amplifier with Unbypass R_S: Terminal Voltage Gain





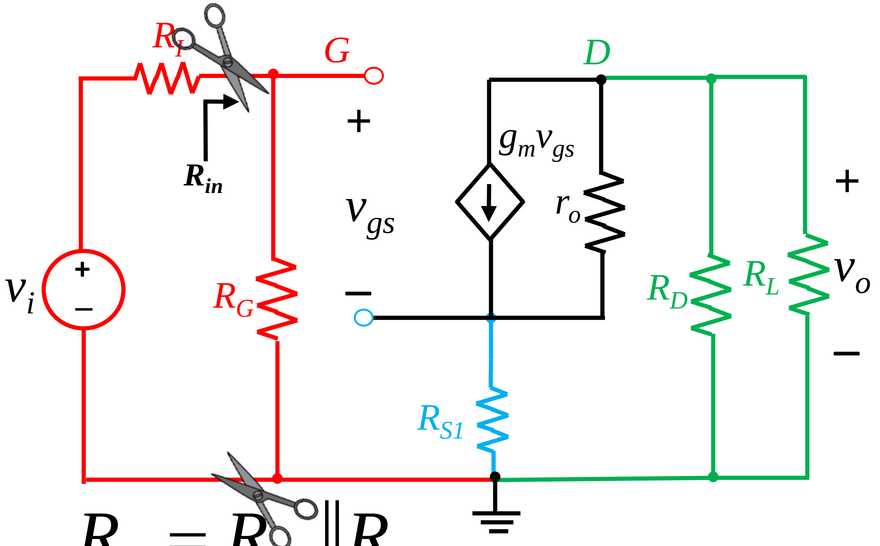
$$R_G = R_{G1} \| R_{G2}$$

$$R_L' = R_D \| R_L$$

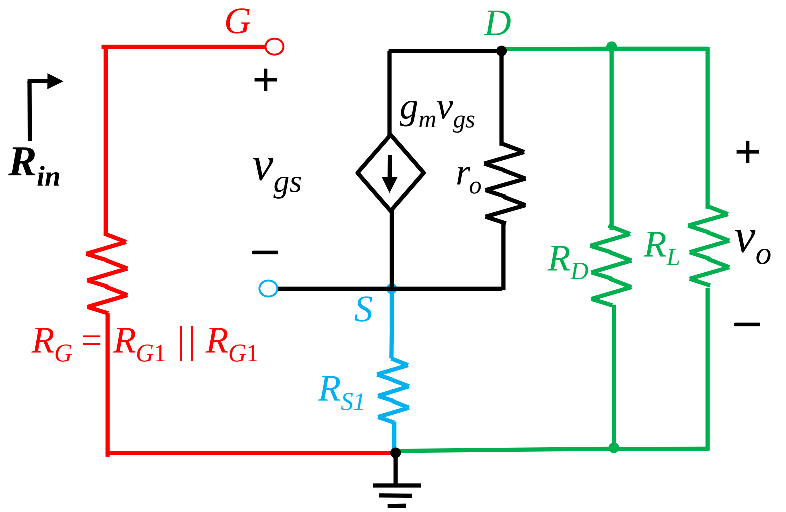
$$\because i_{r_o} \ll g_m v_{gs}, i_d \approx g_m v_{gs}$$

$$A_{vt} = \frac{v_d}{v_g} = \frac{-i_d R_L'}{v_{gs} + i_d R_{S1}} \approx \frac{-g_m v_{gs} R_L'}{v_{gs} + g_m v_{gs} R_{S1}} = \frac{-g_m R_L'}{1 + g_m R_{S1}}$$

C-S Amplifier with Unbypass R_S: Input Resistance

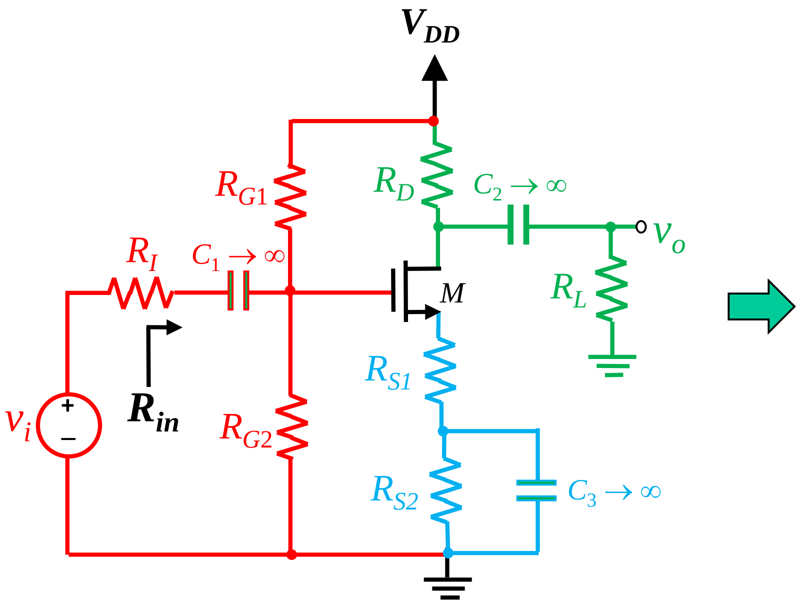


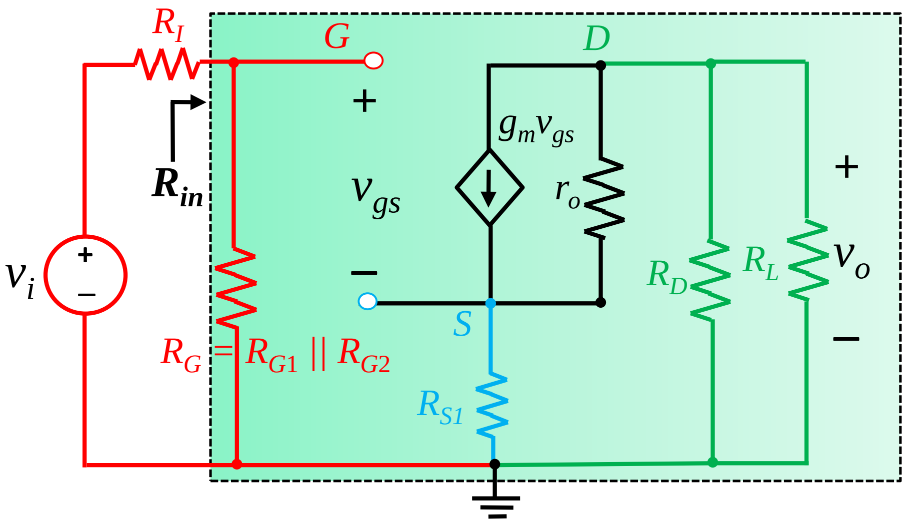
$$R_G = R_{G1} \| R_{G2}$$

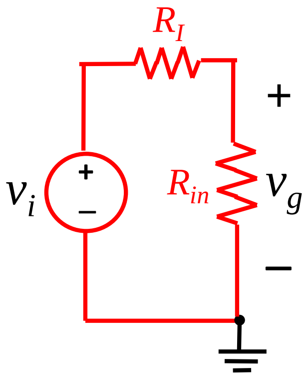


$$R_{in} = R_G$$

C-S Amplifier with Unbypass R_S: Overall Voltage Gain

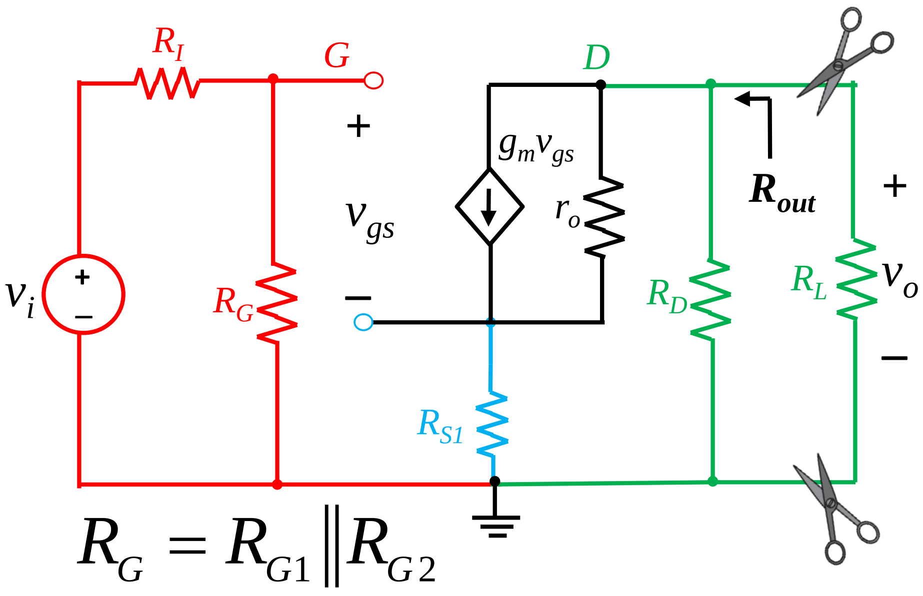




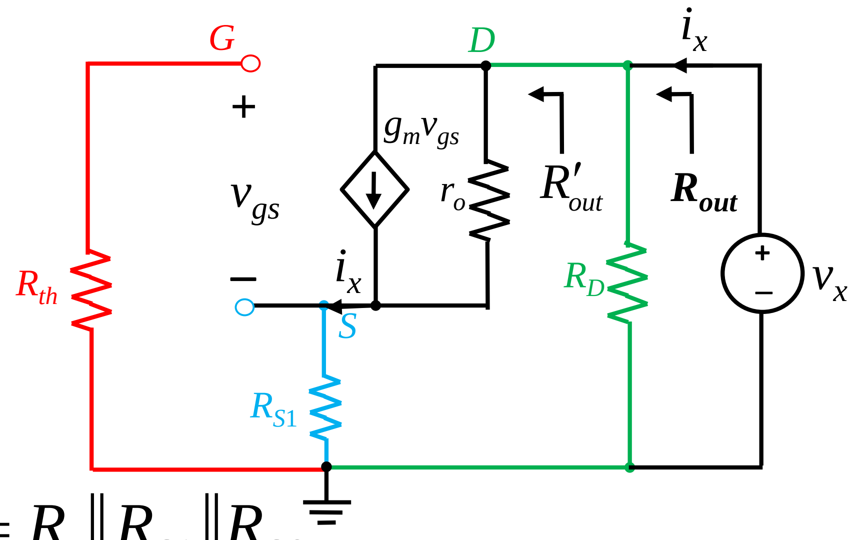


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-S Amplifier with Unbypass R_S: Output Resistance



$$R_G = R_{G1} \| R_{G2}$$



$$R_{th} = R_I \| R_{G1} \| R_{G2}$$

$$v_x = \underbrace{\left(i_x - g_m v_{gs}\right)}_{\text{current through } r_o} r_o + v_s$$

$$v_s = i_x R_{S1}$$

$$\because v_g = 0, v_{gs} = -v_s = -i_x R_{S1}$$

$$v_x = \left(i_x + g_m i_x R_{S1}\right) r_o + i_x R_{S1}$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + g_m R_{S1}\right) r_o$$

$$R_{out} = R'_{out} \| R_D$$

AC Analysis of BJT and MOSFET Voltage Followers

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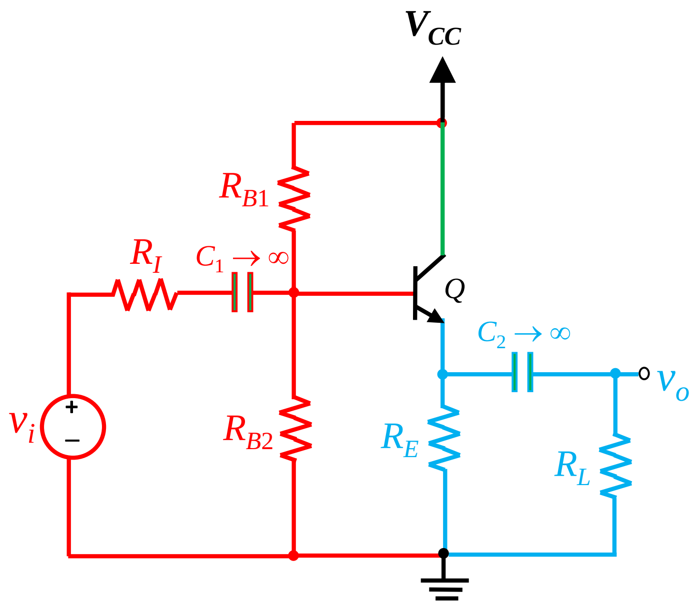
EE2002 Analog Electronics

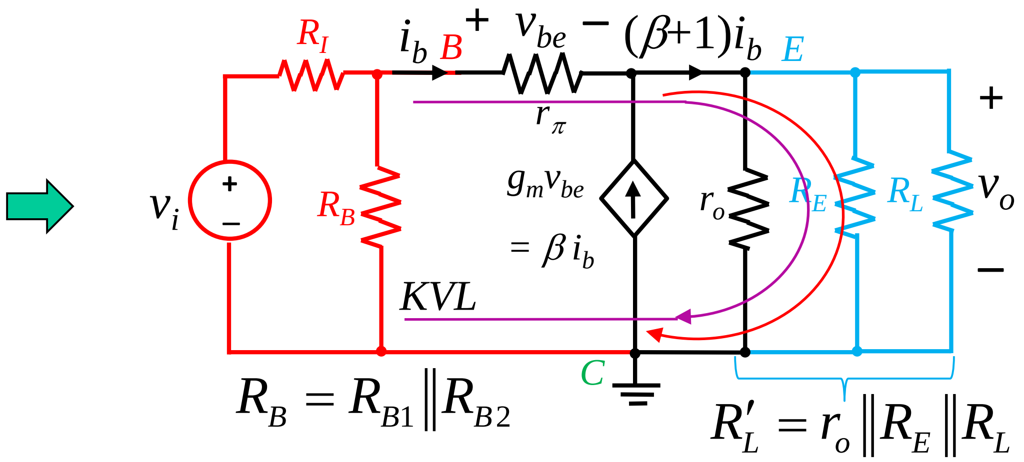
Lesson Objectives

At the end of this lesson, you should be able to:

- Identify BJT and MOSFET voltage follower circuits
- Draw small-signal AC equivalent circuits of C-C and C-D amplifiers
- Calculate the following performance characteristics of C-C and C-D amplifiers
  - Voltage gain
  - Input resistance
  - Output resistance

C-C Amplifier (Voltage Follower): Terminal Voltage Gain

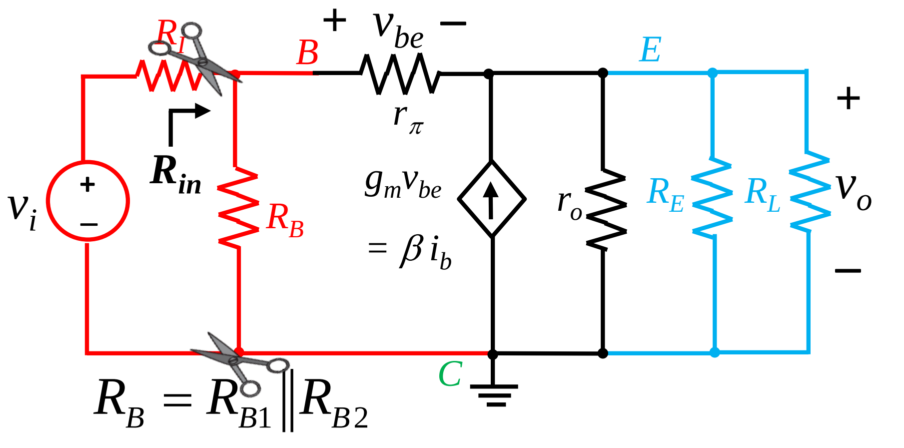


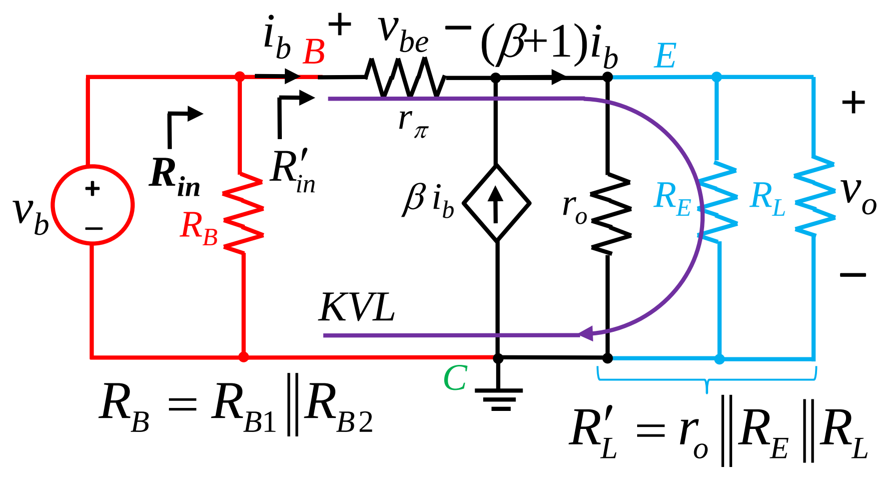


$$A_{vt} = \frac{v_e}{v_b} = \frac{(\beta+1) i_b R'_L}{i_b r_\pi + (\beta+1) i_b R'_L} = \frac{(\beta+1) R'_L}{r_\pi + (\beta+1) R'_L} \approx \frac{g_m R'_L}{1 + g_m R'_L}$$

If $g_m R'_L \gg 1,\ A_{vt} \approx 1 \Rightarrow v_o \approx v_b$.

C-C Amplifier (Voltage Follower): Input Resistance



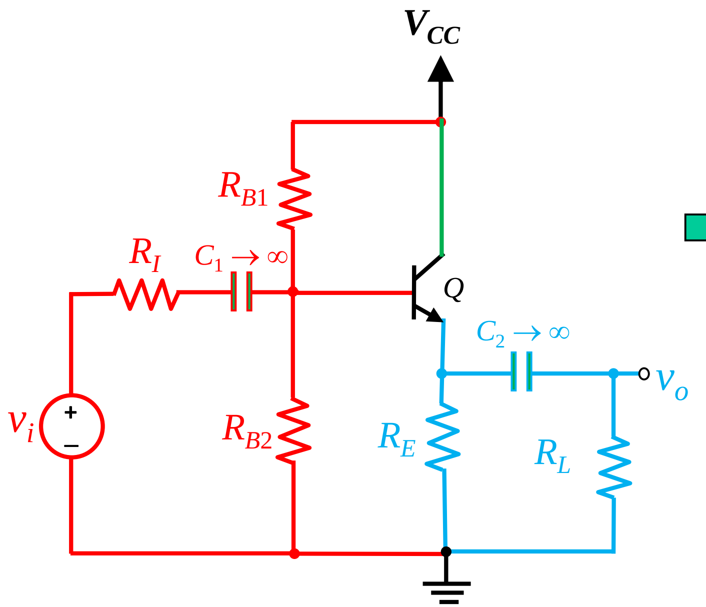


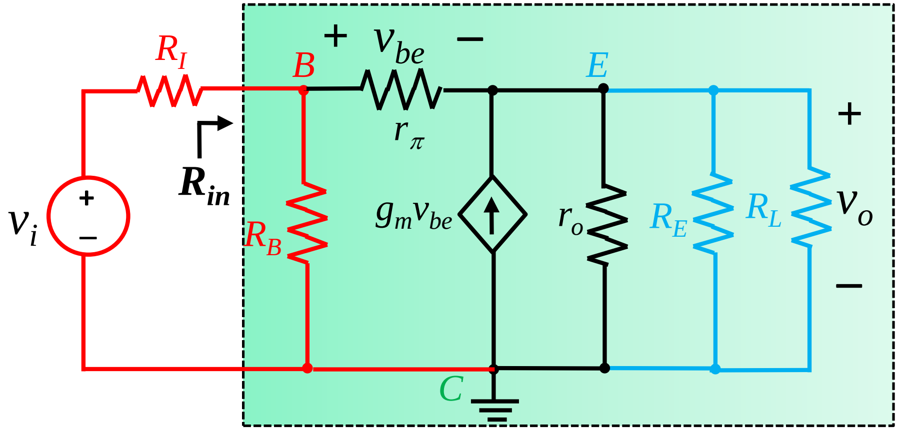
$$v_b = i_b r_\pi + (\beta + 1) i_b R'_L$$

$$R'_{in} = \frac{v_b}{i_b} = r_\pi + (\beta + 1) R'_L$$

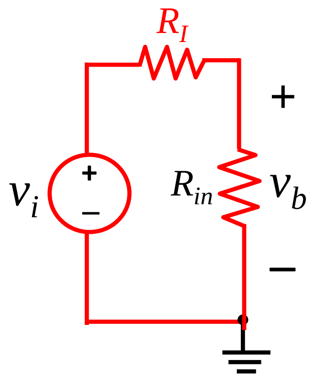
$$R_{in} = R'_{in} \| R_B$$

C-C Amplifier (Voltage Follower): Overall Voltage Gain



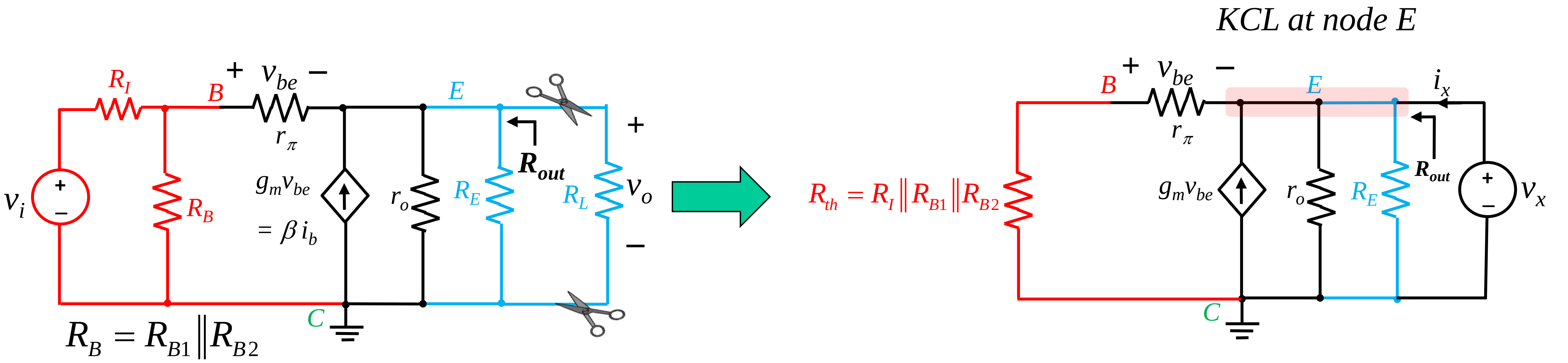


$$R_B = R_{B1} \| R_{B2}$$



$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-C Amplifier (Voltage Follower): Output Resistance



$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_o} - g_m v_{be} + \frac{v_x}{r_\pi + R_{th}}$$

$$v_{be} = -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) v_x$$

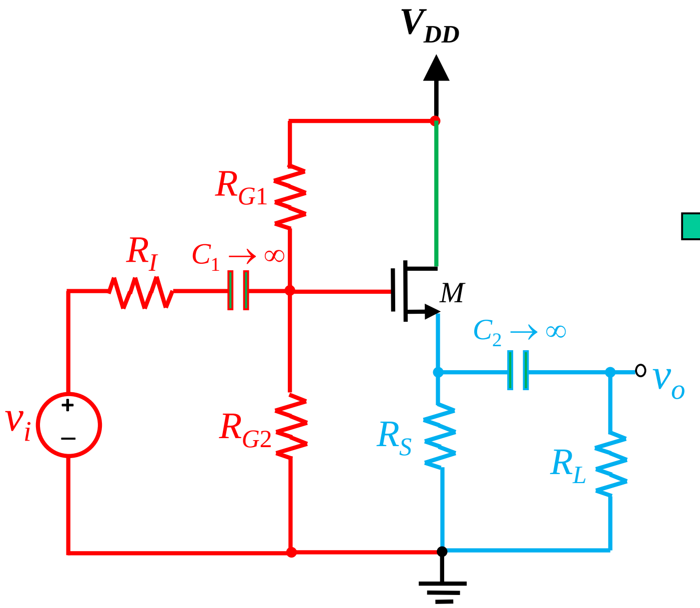
$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_o} + \frac{g_m r_\pi v_x}{r_\pi + R_{th}} + \frac{v_x}{r_\pi + R_{th}}$$

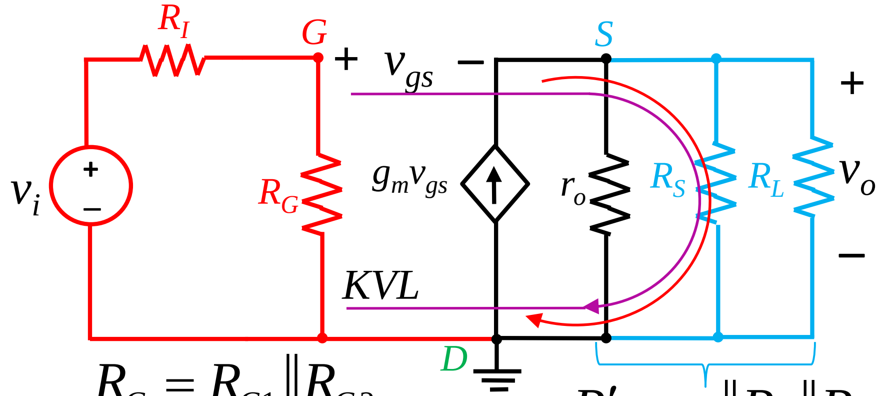
$$= \left(\frac{1}{R_E} + \frac{1}{r_o} + \frac{\beta}{r_\pi + R_{th}} + \frac{1}{r_\pi + R_{th}}\right) v_x$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_E} + \frac{1}{r_o} + \frac{\beta + 1}{r_\pi + R_{th}}\right)^{-1}$$

$$= R_E \left\| r_o \right\| \left(\frac{r_\pi + R_{th}}{\beta + 1}\right)$$

C-D Amplifier (Voltage Follower): Terminal Voltage Gain





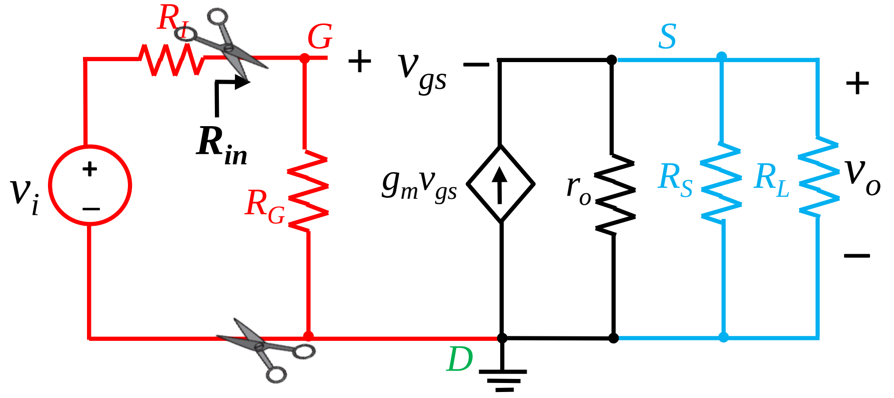
$$R_G = R_{G1} \| R_{G2}$$

$$R'_L = r_o \| R_S \| R_L$$

$$A_{vt} = \frac{v_s}{v_g} = \frac{g_m v_{gs} R'_L}{v_{gs} + g_m v_{gs} R'_L} = \frac{g_m R'_L}{1 + g_m R'_L}$$

If $g_m R'_L \gg 1,\ A_{vt} \approx 1 \Rightarrow v_o \approx v_g$.

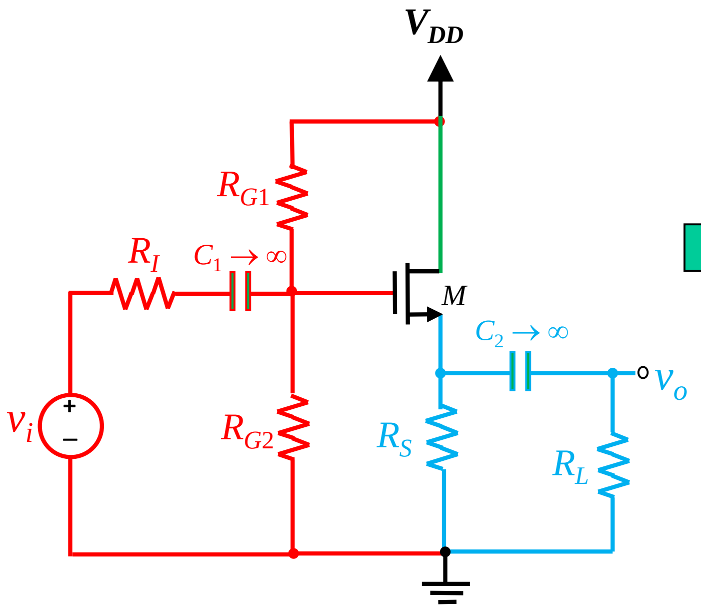
C-D Amplifier (Voltage Follower): Input Resistance

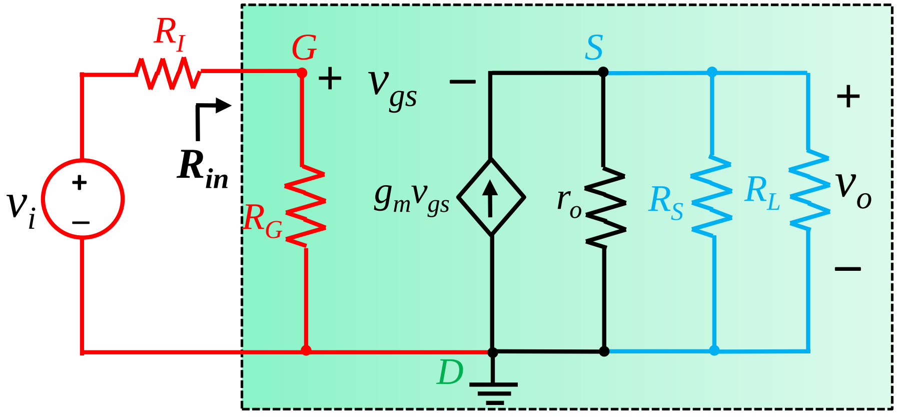


$$R_G = R_{G1} \| R_{G2}$$

$$R_{in} = R_G$$

C-D Amplifier (Voltage Follower): Overall Voltage Gain

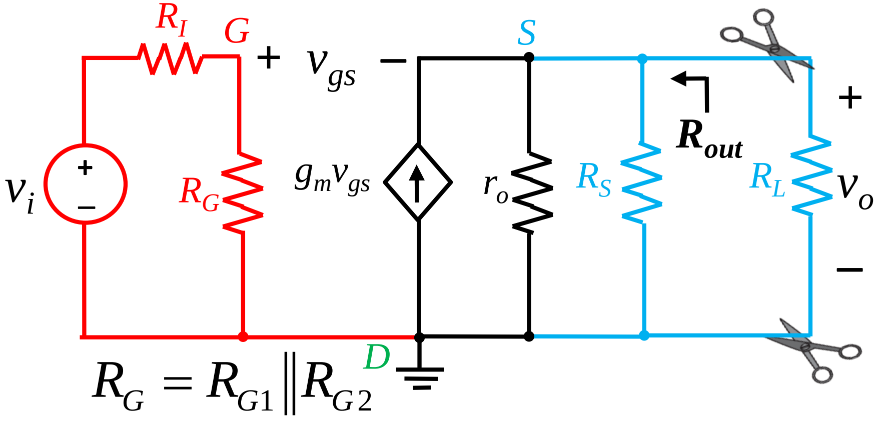


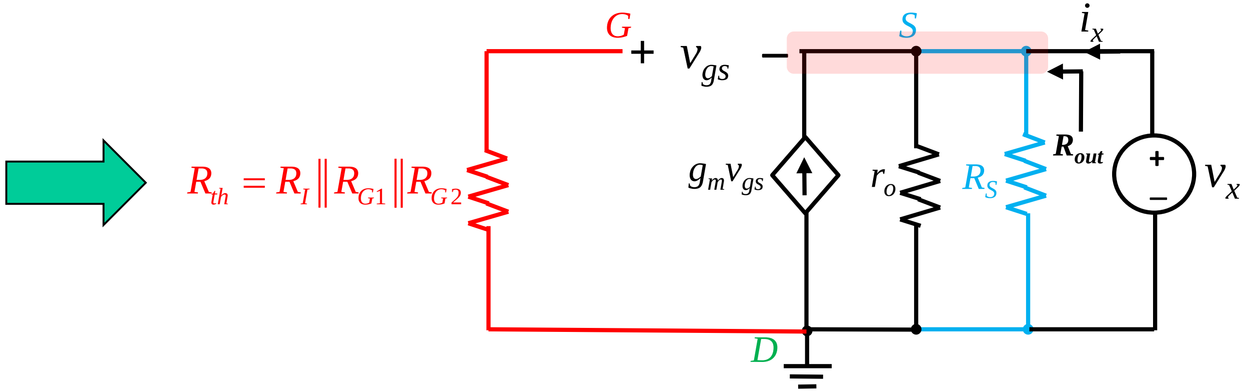


$$R_G = R_{G1} \| R_{G2}$$

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_G}{R_I + R_G}$$

C-D Amplifier (Voltage Follower): Output Resistance





$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} - g_m v_{gs}$$

$$v_{gs} = -v_x$$

$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} + g_m v_x = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) v_x$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)^{-1}$$

$$= R_S \| r_o \| \frac{1}{g_m}$$

AC Analysis of BJT and MOSFET Non-inverting Amplifiers

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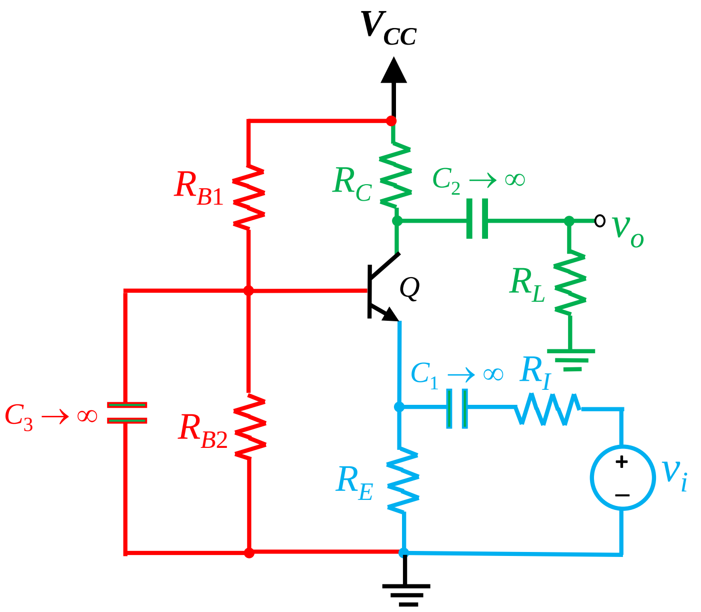
EE2002 Analog Electronics

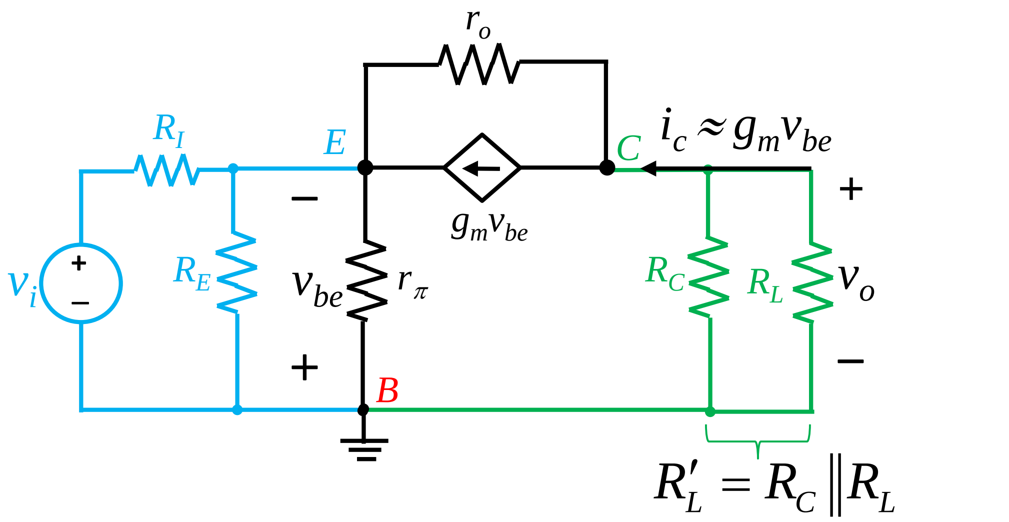
Lesson Objectives

At the end of this lesson, you should be able to:

- Recognise BJT and MOSFET non-inverting amplifiers
- Draw small-signal AC equivalent circuits of C-B and C-G amplifiers
- Calculate the following performance characteristics of C-B and C-G amplifiers
  - Voltage gain
  - Input resistance
  - Output resistance

C-B Amplifier (Non-inverting Amplifier): Terminal Voltage Gain

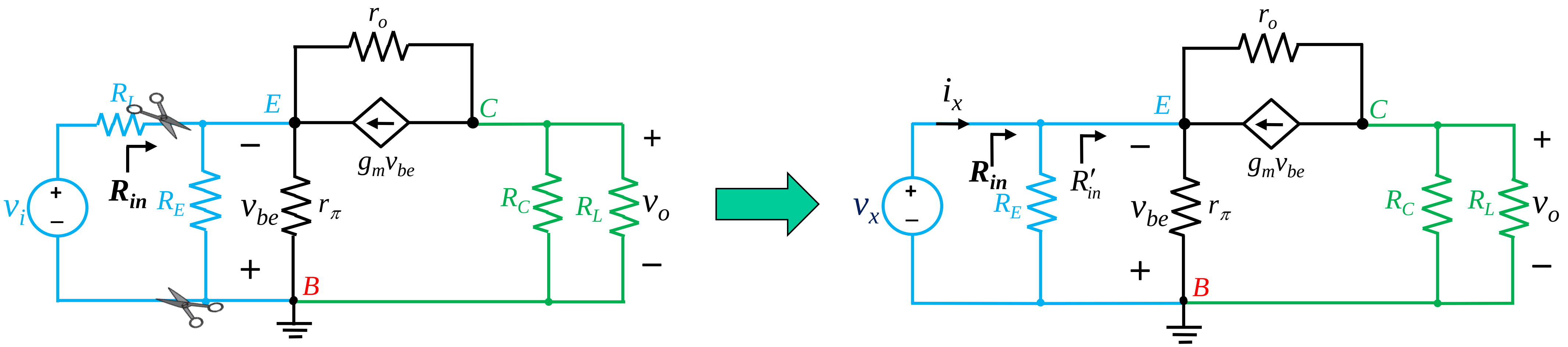




$$\because \; i_{r_o} \ll g_m v_{be}, i_c \approx g_m v_{be}$$

$$A_{vt} = \frac{v_c}{v_e} = \frac{-i_c R_L'}{-v_{be}} \approx \frac{-g_m v_{be} R_L'}{-v_{be}} = g_m R_L'$$

C-B Amplifier (Non-inverting Amplifier): Input Resistance



$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be} + \cancel{\frac{v_{ec}}{r_o}}$$

This current is small because of large r_o

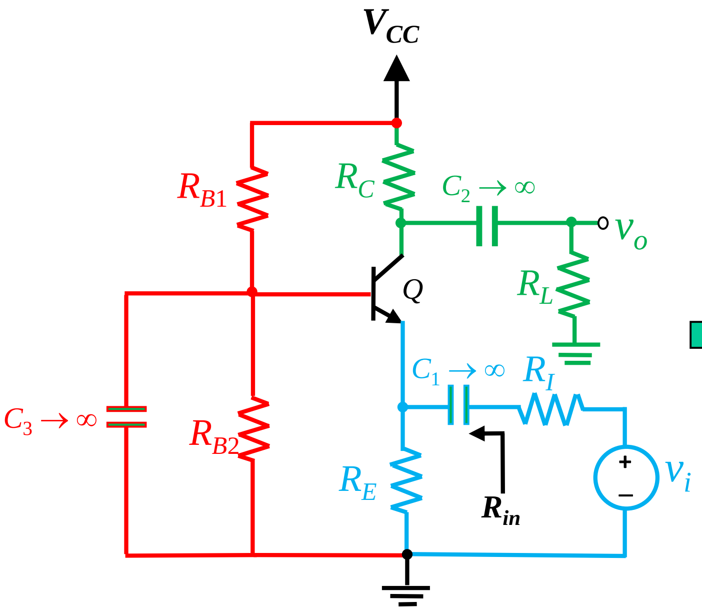
$$\because \; v_{be} = -v_x, \; i_x = \frac{v_x}{r_\pi} + g_m v_x$$

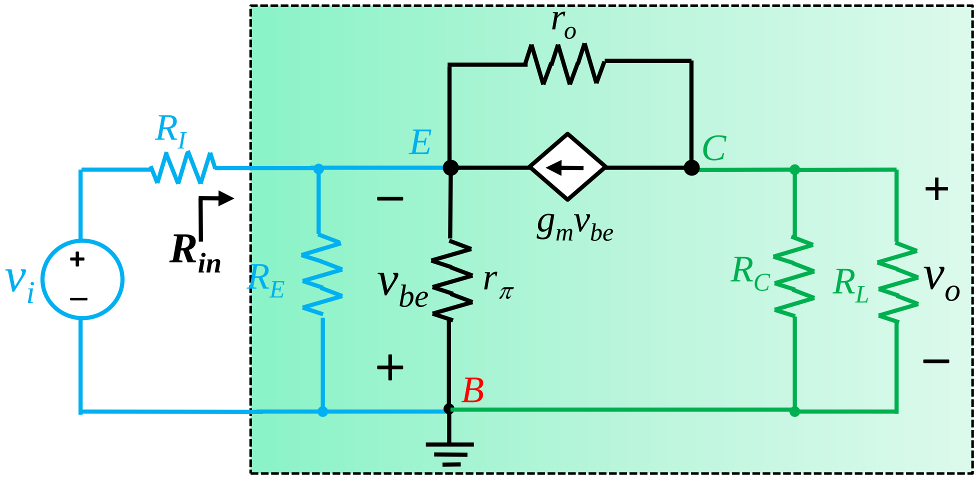
$$i_x = \left(\frac{1 + g_m r_\pi}{r_\pi}\right) v_x = \left(\frac{1+\beta}{r_\pi}\right) v_x$$

$$R'_{in} = \frac{v_x}{i_x} = \left(\frac{r_\pi}{\beta + 1}\right) \approx \frac{r_\pi}{\beta} = \frac{1}{g_m}$$

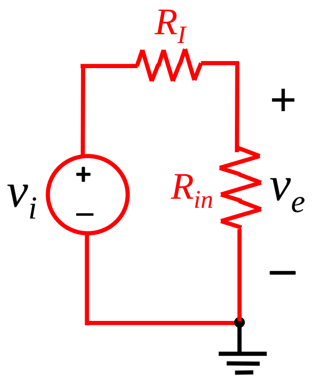
$$R_{in} = R'_{in} \| R_E$$

C-B Amplifier (Non-inverting Amplifier): Overall Voltage Gain

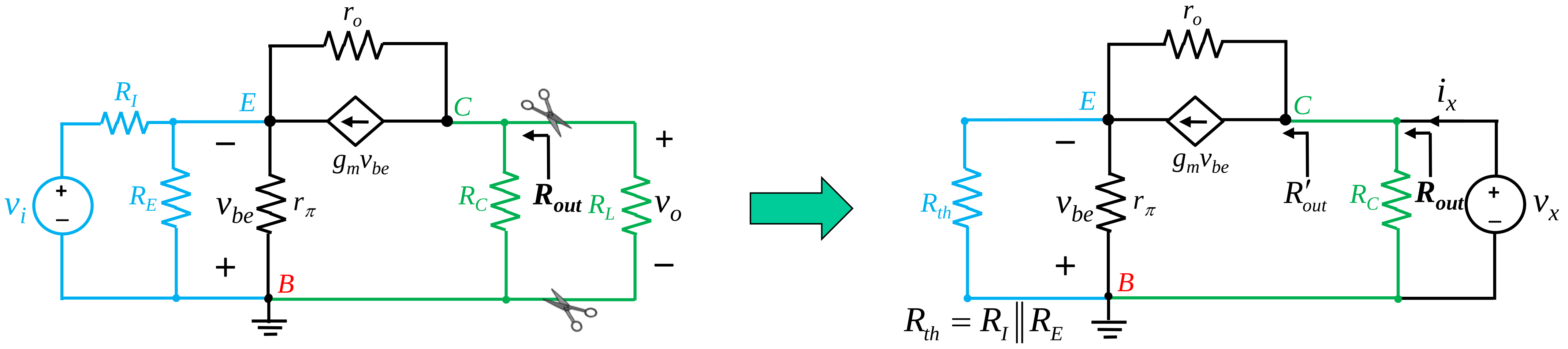




$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_e} \times \frac{v_e}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$



C-B Amplifier (Non-inverting Amplifier): Output Resistance



$$v_x = \underbrace{\left(i_x - g_m v_{be}\right)}_{\text{current through } r_o} r_o + v_e$$

$$v_e = i_x \left(r_\pi \| R_{th}\right)$$

$$v_{be} = -v_e = -i_x \left(r_\pi \| R_{th}\right)$$

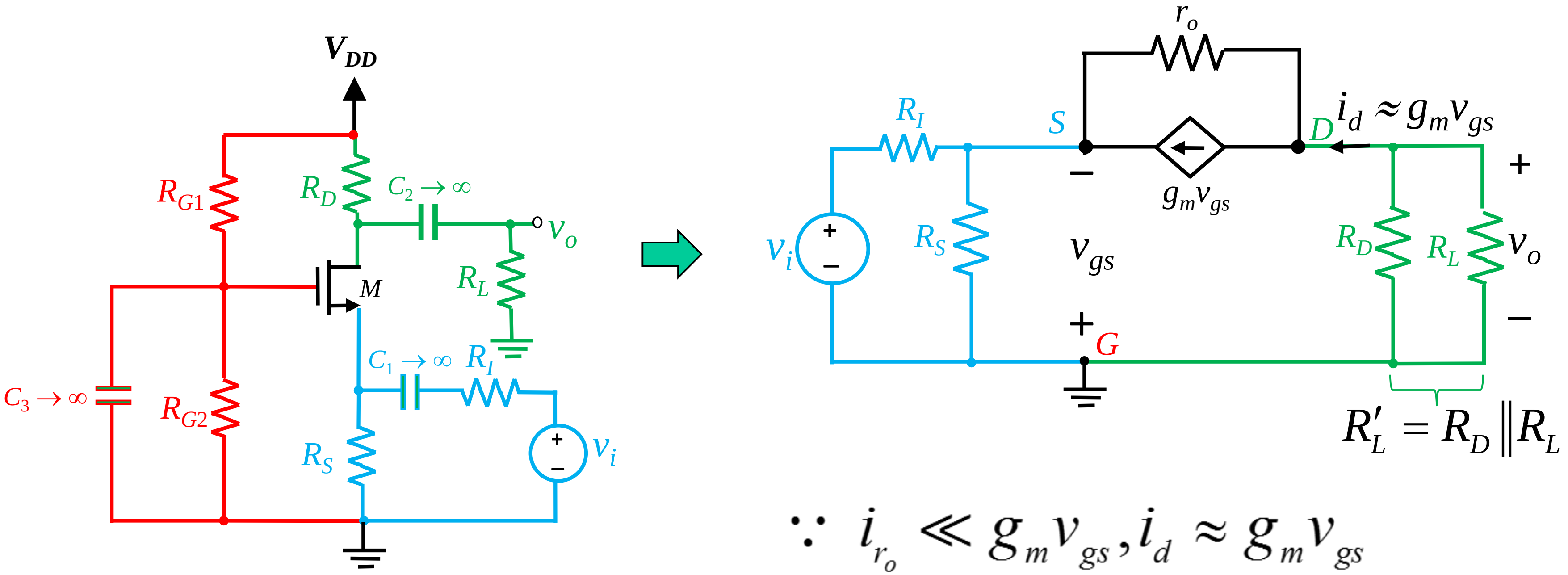
$$v_x = \left[i_x + g_m i_x \left(r_\pi \| R_{th}\right)\right] r_o + i_x \left(r_\pi \| R_{th}\right)$$

$$R'_{out} = \frac{v_x}{i_x} = \left[1 + g_m \left(r_\pi \| R_{th}\right)\right] r_o + r_\pi \| R_{th}$$

$$\approx \left[1 + g_m \left(r_\pi \| R_{th}\right)\right] r_o$$

$$R_{out} = R'_{out} \| R_C$$

C-G Amplifier (Non-inverting Amplifier): Terminal Voltage Gain

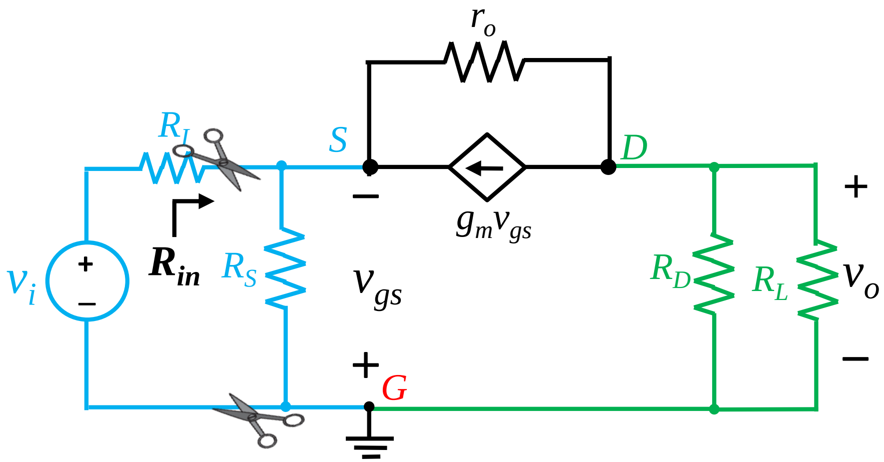


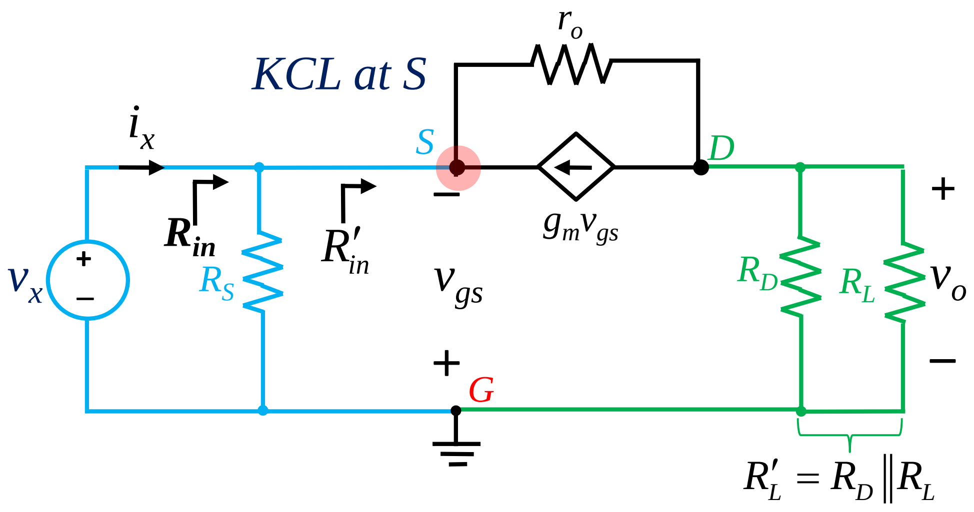
$$\because \; i_{r_o} \ll g_m v_{gs}, i_d \approx g_m v_{gs}$$

$$v_s = -v_{gs}$$

$$A_{vt} = \frac{v_d}{v_s} = \frac{-i_d R'_L}{-v_{gs}} \approx \frac{-g_m v_{gs} R'_L}{-v_{gs}} = g_m R'_L$$

C-G Amplifier (Non-inverting Amplifier): Input Resistance





$$\because\ v_{gs} = -v_x,\ i_x \approx g_m v_x$$

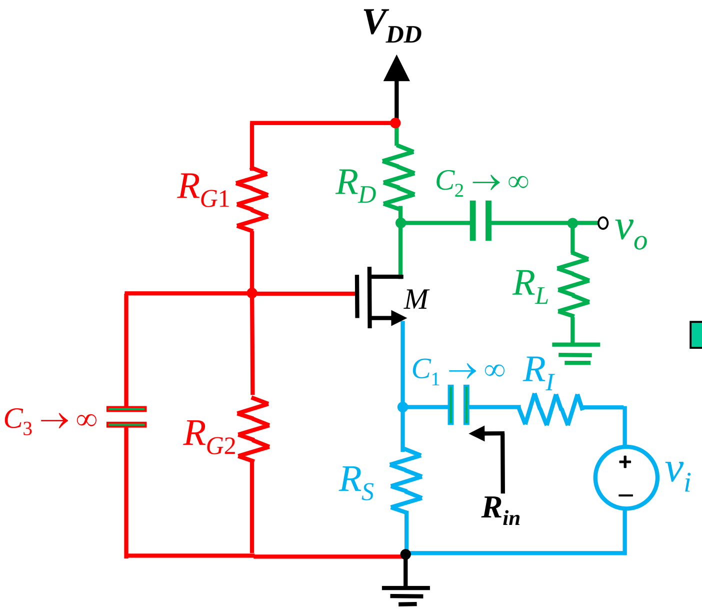
$$i_x = -g_m v_{gs} + \cancel{\frac{v_{sd}}{r_o}}$$

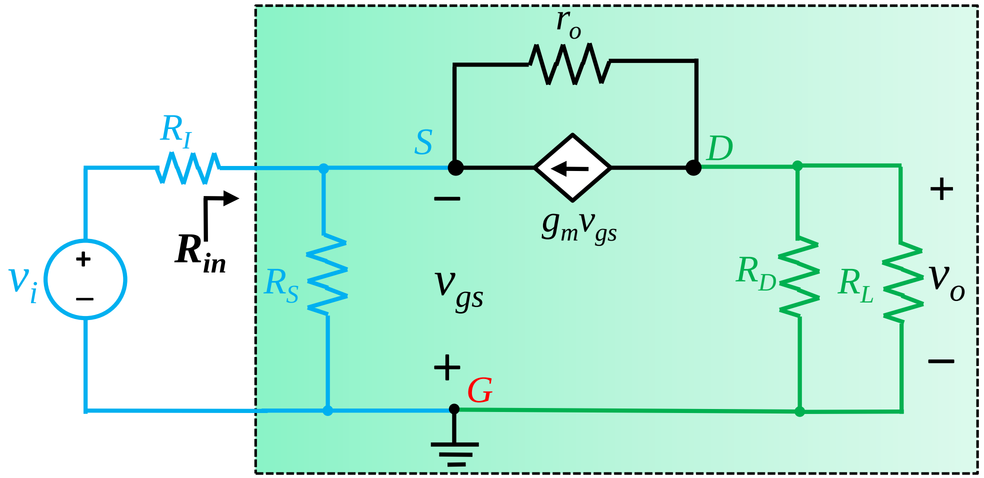
This current can be ignored due to large r_o

$$R'_{in} = \frac{v_x}{i_x} \approx \frac{1}{g_m}$$

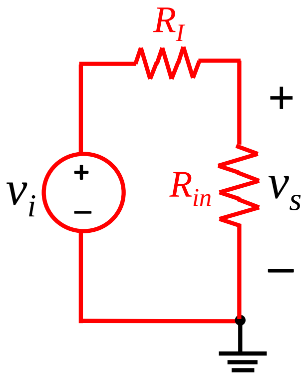
$$R_{in} = R'_{in} \| R_S$$

C-G Amplifier (Non-inverting Amplifier): Overall Voltage Gain

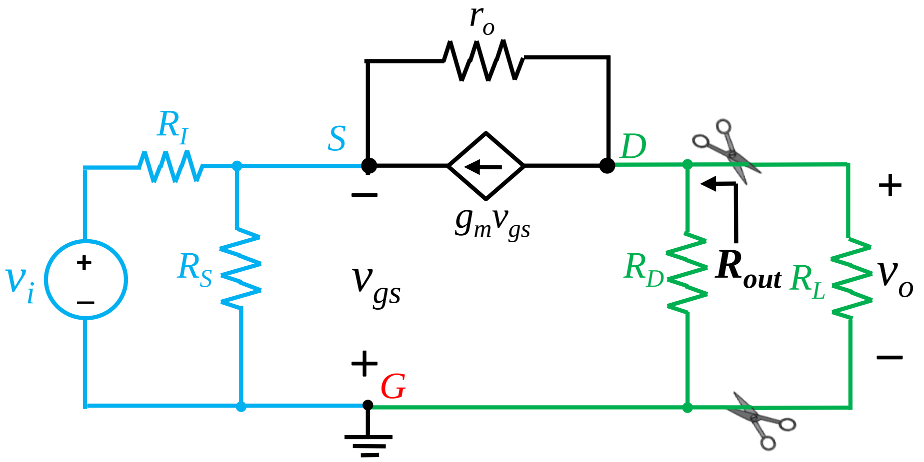


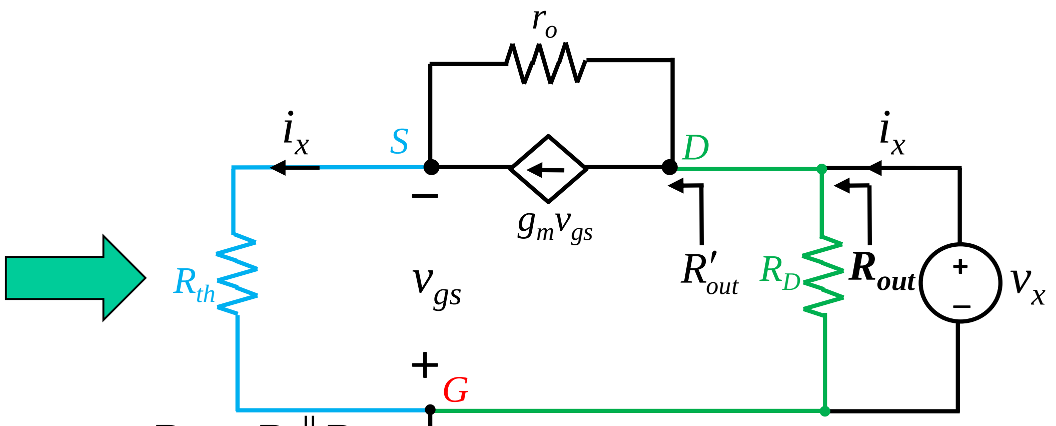


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_s} \times \frac{v_s}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$



C-G Amplifier (Non-inverting Amplifier): Output Resistance





$$R_{th} = R_I \| R_S$$

$$v_x = \left(i_x - g_m v_{gs}\right) r_o + v_s$$

$$v_s = i_x R_{th}$$

$$v_{gs} = -v_s = -i_x R_{th}$$

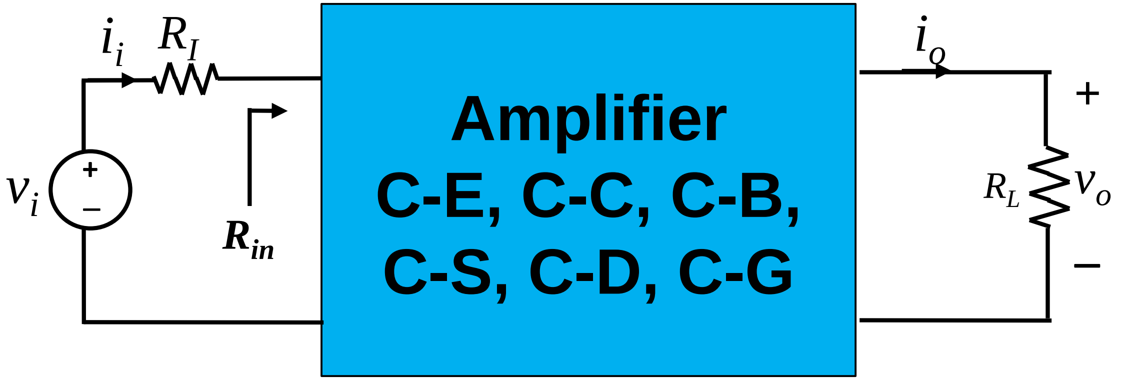
$$v_x = \left(i_x + g_m i_x R_{th}\right) r_o + i_x R_{th}$$

$$R'_{out} = \frac{v_x}{i_x} = \left(1 + g_m R_{th}\right) r_o + R_{th}$$

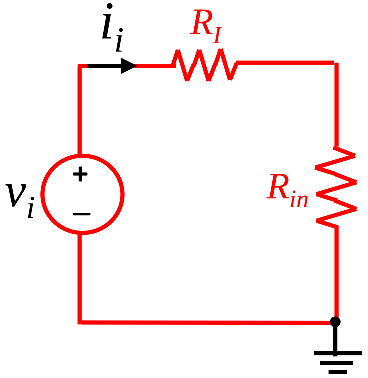
$$\approx \left(1 + g_m R_{th}\right) r_o$$

$$R_{out} = R'_{out} \| R_D$$

Current Gain



$$A_i = \frac{i_o}{i_i} = \frac{\dfrac{v_o}{R_L}}{\dfrac{v_i}{R_I + R_{in}}} = \frac{v_o}{v_i} \times \frac{R_I + R_{in}}{R_L} = A_v \times \frac{R_I + R_{in}}{R_L}$$



Input Signal Range of BJT and MOSFET

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EE2002 Analog Electronics

Lesson Objectives

At the end of this lesson, you should be able to:

- Derive the small signal operation criterion for BJT and MOSFET
- Analyse the input signal range of BJT and MOSFET

Small-Signal Operation of BJT

$$i_C \approx I_S \exp\left(\frac{v_{BE}}{V_T}\right) = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_T}\right)$$

$$\therefore i_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\exp\left(\frac{v_{be}}{V_T}\right) = I_C\left[1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_{be}}{V_T}\right)^3 + \cdots\right]$$

$$i_c = i_C - I_C = I_C\left[\frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \cdots\right]$$

For linearity, i_c should be proportional to v_{be}.

$$\frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 \ll \frac{v_{be}}{V_T} \Rightarrow |v_{be}| \ll 2V_T = 0.05\,V \Rightarrow |v_{be}| \le 0.005\,V$$

C-E Amplifier Input Signal Range

For BJT small-signal operation, $|v_{be}| \le 5\text{ mV}$.

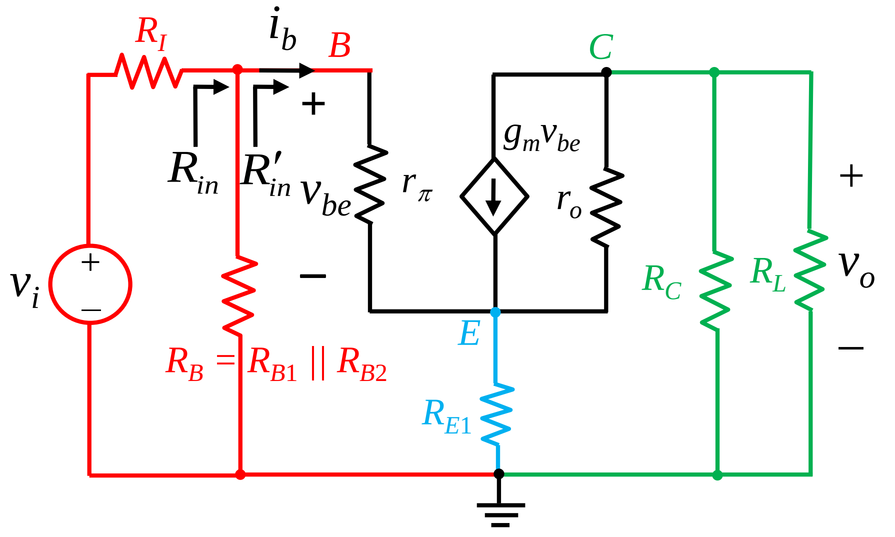
$$|v_{be}| = i_b r_\pi = \frac{|v_b| r_\pi}{R'_{in}} = \frac{|v_b| r_\pi}{r_\pi + (\beta+1) R_{E1}}$$

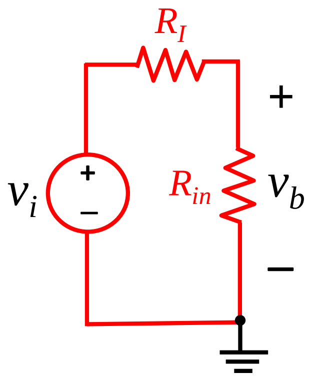
$$|v_{be}| \le 0.005$$

$$\Rightarrow |v_b| \le 0.005 \left(\frac{r_\pi + (\beta+1) R_{E1}}{r_\pi} \right)$$

$$\because \beta + 1 \approx \beta = g_m r_\pi, \quad |v_b| \le 0.005 (1 + g_m R_{E1})$$

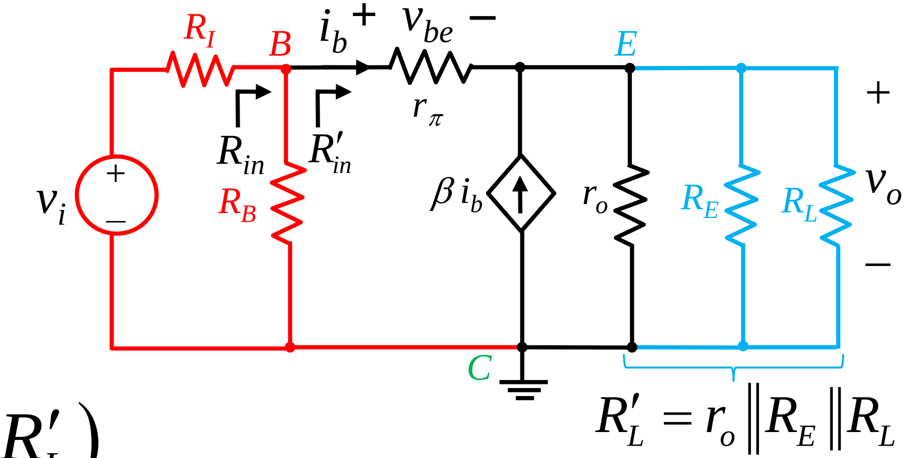
$$\because v_b = \left(\frac{R_{in}}{R_I + R_{in}} \right) v_i \Rightarrow |v_i| \le 0.005 (1 + g_m R_{E1}) \left(\frac{R_I + R_{in}}{R_{in}} \right)$$





C-C Amplifier Input Signal Range

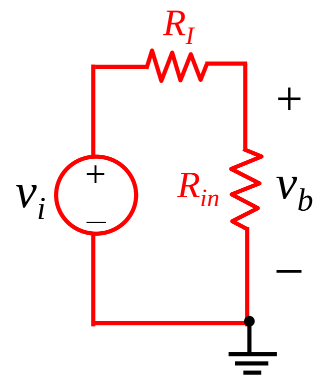
For BJT small-signal operation, $|v_{be}| \le 5\ \text{mV}$.



$$|v_{be}| = i_b r_\pi = \frac{|v_b| r_\pi}{R'_{in}} = \frac{|v_b| r_\pi}{r_\pi + (\beta+1) R'_L}$$

$$\Rightarrow |v_b| \le 0.005\left(\frac{r_\pi + (\beta+1)R'_L}{r_\pi}\right) \approx 0.005(1+g_m R'_L)$$

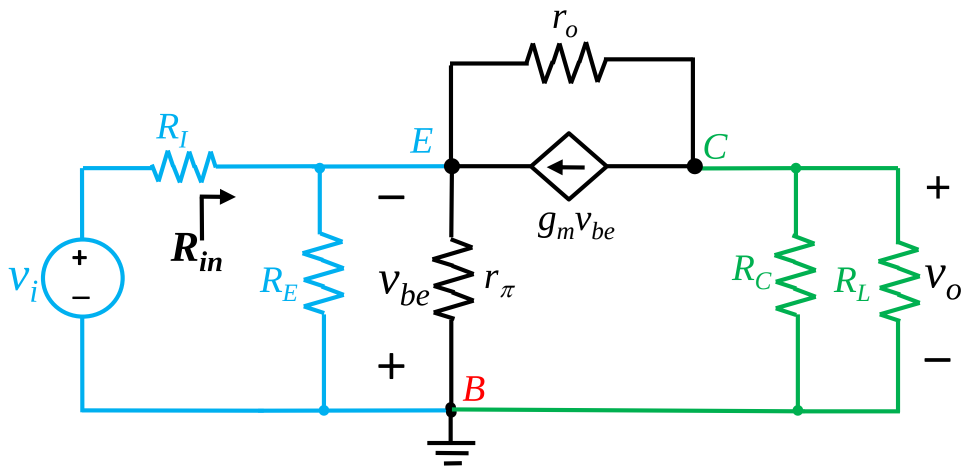
$$\because v_b = \left(\frac{R_{in}}{R_I + R_{in}}\right) v_i \Rightarrow |v_i| \le 0.005(1+g_m R'_L)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$



C-B Amplifier Input Signal Range

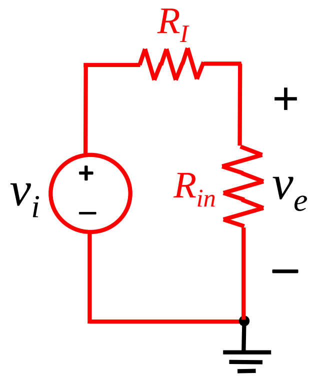
For BJT small-signal operation, $|v_{be}| \leq 5\ \mathrm{mV}$.

$$v_{be} = -v_e = -\left(\frac{R_{in}}{R_I + R_{in}}\right) v_i$$



$$|v_{be}| \leq 0.005$$

$$\Rightarrow |v_i| \leq 0.005\left(\frac{R_I + R_{in}}{R_{in}}\right)$$



Small-Signal Operation of MOSFET

$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2 \text{ for } v_{DS} \geq v_{GS} - V_{TN}$$

$$i_D = \frac{K_n}{2}\left(V_{GS} + v_{gs} - V_{TN}\right)^2 = \frac{K_n}{2}\left[\left(V_{GS} - V_{TN}\right)^2 + 2v_{gs}\left(V_{GS} - V_{TN}\right) + v_{gs}^2\right]$$

$$i_d = i_D - I_D = \frac{K_n}{2}\left[2v_{gs}\left(V_{GS} - V_{TN}\right) + v_{gs}^2\right]$$

For linearity, i_d should be proportional to v_{gs}.

$$\left|v_{gs}\right| \ll 2\left(V_{GS} - V_{TN}\right) \Rightarrow \left|v_{gs}\right| \leq 0.2\left(V_{GS} - V_{TN}\right)$$

C-S Amplifier Input Signal Range

For MOSFET small-signal operation, $|v_{gs}| \le 0.2(V_{GS} - V_{TN})$.

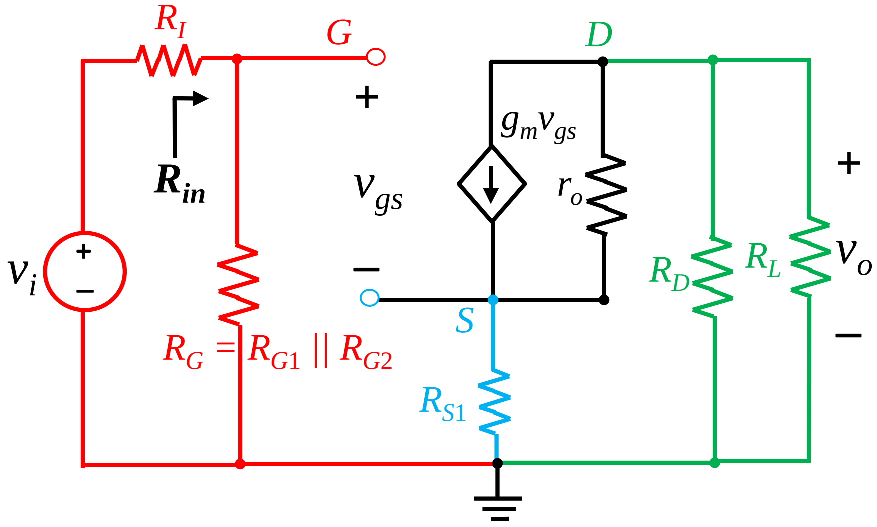
$$v_g \approx v_{gs} + g_m v_{gs} R_{S1} = v_{gs}\left(1 + g_m R_{S1}\right)$$

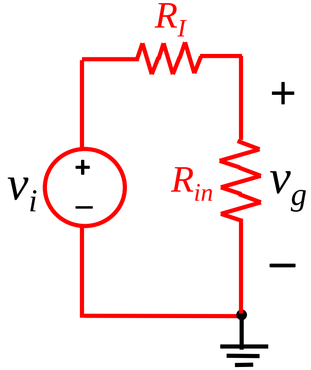
$$v_{gs} = \frac{v_g}{1 + g_m R_{S1}}$$

$$\frac{|v_g|}{1 + g_m R_{S1}} \le 0.2\left(V_{GS} - V_{TN}\right)$$

$$\Rightarrow |v_g| \le 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_{S1}\right)$$

$$\because v_g = \left(\frac{R_{in}}{R_I + R_{in}}\right) v_i \Rightarrow |v_i| \le 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_{S1}\right)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$

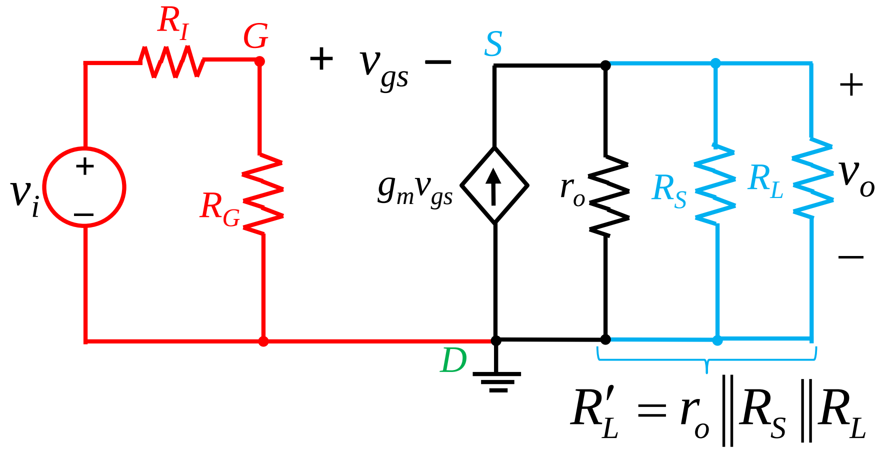




C-D Amplifier Input Signal Range

For MOSFET small-signal operation, $|v_{gs}| \le 0.2(V_{GS} - V_{TN})$.

$$v_g \approx v_{gs} + g_m v_{gs} R_L'$$



$$v_{gs} = \frac{v_g}{1 + g_m R_L'}$$

$$\frac{|v_g|}{1 + g_m R_L'} \le 0.2(V_{GS} - V_{TN})$$

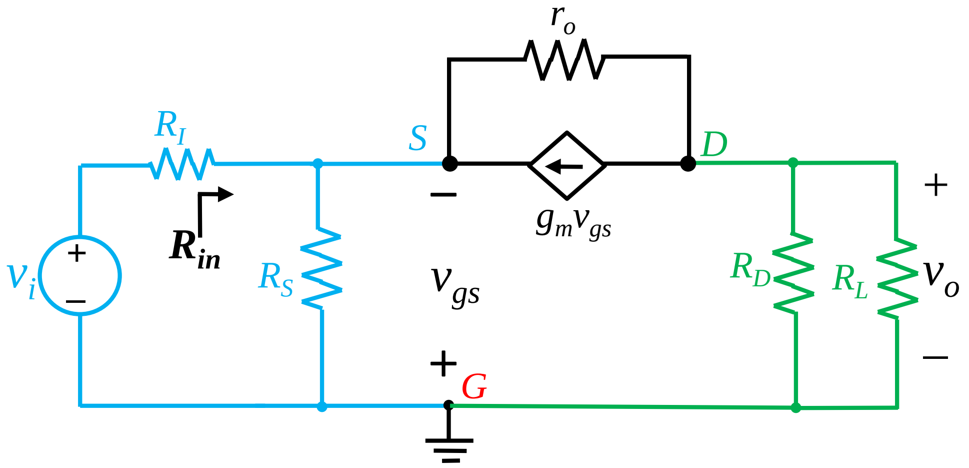
$$\Rightarrow |v_g| \le 0.2(V_{GS} - V_{TN})(1 + g_m R_L')$$

$$\because v_g = \left(\frac{R_G}{R_I + R_G}\right) v_i \Rightarrow |v_i| \le 0.2(V_{GS} - V_{TN})(1 + g_m R_L')\left(\frac{R_I + R_G}{R_G}\right)$$

C-G Amplifier Input Signal Range

For MOSFET small-signal operation, $|v_{gs}| \le 0.2(V_{GS} - V_{TN})$.

$$v_{gs} = -v_s = -\left(\frac{R_{in}}{R_I + R_{in}}\right) v_i$$



$$|v_{gs}| \le 0.2\left(V_{GS} - V_{TN}\right)$$

$$\Rightarrow |v_i| \le 0.2\left(V_{GS} - V_{TN}\right)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$