

EE3001

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 1 EXAMINATION 2019-2020
EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2019

Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

1. (a) Three parallel circular loops of radius a are located at $z = 0, +d, -d$ respectively in free space with their centers aligned along the z axis. Each circular loop has uniform charge distribution with line charge density ρ_l .
 - (i) Find the electric potential $V_0(z), V_{+d}(z), V_{-d}(z)$ along the z axis due to each circular loop located at $z = 0, +d, -d$ respectively.
 - (ii) Determine the total electric potential $V(z)$ and the electric field intensity $\vec{E}(z)$ along the z axis due to all three circular loops.

(13 Marks)

Note: Question No. 1 continues on page 2.

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- (b) Assume that a steady current I flows in each circular loop of part (a) in the counter clockwise direction (as viewed from $z > d$).
- (i) Determine the magnetic field intensity $\vec{H}_0(z)$, $\vec{H}_{+d}(z)$, $\vec{H}_{-d}(z)$ along the z axis due to each current loop located at $z = 0, +d, -d$ respectively.
- (ii) Find the total magnetic field intensity at the origin.

(12 Marks)

2. (a) Figure 1 shows a square loop of side length L which has its left side lying at rest along the y axis in free space at time $t = 0$. The whole loop is accelerated with constant acceleration a_0 toward $+x$ direction while being subjected to magnetic flux density of the form $\vec{B} = \frac{-\mu_0}{2\pi x} \vec{a}_z$ T.

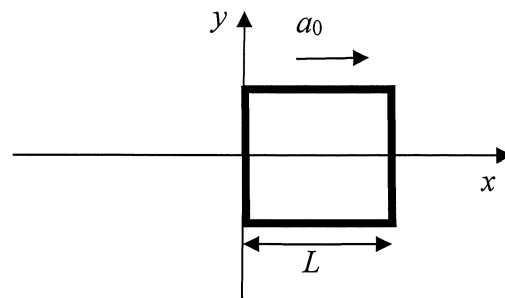


Figure 1

- (i) Derive the expression of magnetic flux Φ_m passing through the loop at time t .
- (ii) Determine the induced voltage V_{emf} in the loop at time t .
- (b) A 1-MHz plane wave is propagating along the $+z$ direction in a lossy nonmagnetic medium with relative permittivity ϵ_r and conductivity σ .
- (i) Let us assume that the medium is a very good conductor. What can you say about the relationship between the real and imaginary parts of propagation constant γ ? What is the phase angle of γ ? Also, what can you say about the relationship between the real and imaginary parts of intrinsic impedance η_c ? What is the phase angle of η_c ?

Note: Question No. 2 continues on page 3.

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- (ii) Given that $\epsilon_r = 1$ and $\sigma = 0.01$ S/m, calculate the real and imaginary parts, as well as the magnitude and phase angle of γ and η_c . Are the results consistent with your answers in part (i)? How can you tell whether the assumption of very good conductor is valid or not?

(15 Marks)

3. (a) The electric field of a uniform plane wave (UPW) in air occupying the region $z \leq 0$ is given by:

$$\vec{E}_i(z) = (50\vec{a}_x - j140\vec{a}_y) e^{-j6\pi z} \text{ V/m}$$

The UPW is incident normally on a planar interface with a lossy medium having intrinsic impedance $\eta_c = 150 \angle 0.2^\circ \Omega$ occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

- (i) The frequency of the UPW.
- (ii) The polarization (Linear, Circular or Elliptical) of the UPW. Briefly explain your answer.
- (iii) The reflection coefficient Γ (in polar form) at the planar interface.
- (iv) The position z at which the total electric field in the air medium is minimum i.e., z_{\min} .

(12 Marks)

- (b) The magnetic field of a uniform plane wave (UPW) propagating in free space ($z \leq 0$) is given by:

$$\vec{H}_i = \vec{a}_y 75 e^{-j(0.35x + 0.25z)} \text{ mA/m}$$

The UPW is obliquely incident on a lossless dielectric having $\mu = \mu_0$, $\epsilon = 1.6\epsilon_0$ at $z = 0$, and occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

- (i) The angle of incidence θ_i (in degrees) and angle of transmission θ_t (in degrees) of the incident UPW.
- (ii) The time-average Poynting vector of the incident and reflected waves, i.e., \vec{S}_i and \vec{S}_r .

(13 Marks)

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4. (a) A $50\ \Omega$ air-filled transmission line is terminated in a load $Z_L = 68 - j27\ \Omega$. The line is operating at 900 MHz.

Find the following:

- (i) The reflection coefficient Γ_L (in polar form) and the standing wave ratio (SWR) due to this load.
- (ii) The shortest length of the line to make the impedance at the input end purely resistive and as large as possible.
- (iii) A $75\ \Omega$ generator with maximum power available $P_{av} = 5\ \text{W}$ is connected to the transmission line in part (ii). Find the average power delivered to the load.

(13 Marks)

- (b) A 50-cm long lossless transmission line having characteristic impedance $Z_0 = 75\ \Omega$ and a phase velocity $u_p = 3 \times 10^8\ \text{m/s}$ is open-circuited at the load end. A generator having an open-circuit voltage $V_g(t) = 100 \cos(4\pi \times 10^8 t - \pi/6)\ \text{V}$ and an internal impedance $Z_g = 75\ \Omega$ is connected to the transmission line.

Assume that the load is located at $z = 0$ and the generator is at $z = -\ell$ where ℓ is the length of the transmission line.

Find the following and state any assumption(s) made:

- (i) The input impedance at the source end of the transmission line, i.e., $Z_{in}(-\ell)$.
- (ii) The time-domain expression for the voltage at $z = -\ell$, i.e., $V(-\ell, t)$.

(12 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r \partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial(rA_r)}{r \partial r} + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi} \int_C \frac{I \vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi} \int_C \frac{I \vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \vec{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \iint_S \vec{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon_c} = j\omega\sqrt{\mu(\epsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon - j\sigma/\omega}}$$

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin \theta_t}{\sin \theta_i} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} \quad \tan \theta_{B\parallel} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad \sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cos(\theta_t)}{\eta_2 \cos(\theta_i) + \eta_1 \cos(\theta_t)} \quad \tau_{\perp} = \frac{2\eta_2 \cos(\theta_i)}{\eta_2 \cos(\theta_i) + \eta_1 \cos(\theta_t)}$$

$$\Gamma_{\parallel} = \frac{\eta_2 \cos(\theta_t) - \eta_1 \cos(\theta_i)}{\eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)} \quad \tau_{\parallel} = \frac{2\eta_2 \cos(\theta_i)}{\eta_2 \cos(\theta_t) + \eta_1 \cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o} \{ V_o^+ e^{-j\beta z} - V_o^- e^{+j\beta z} \}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L e^{+j2\beta z} \quad -\ell \leq z \leq 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} Z_o$$

END OF PAPER

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Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.