NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2021 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) A line charge of uniform charge density ρ_l is located in free space along the z axis from $z = a$ to $z = b > a$.

    (i) Using Coulomb's law, determine the electric field intensity $\vec{E}$ at the point $(x, y, 0)$ due to the line charge.

    (ii) Simplify your expression of $\vec{E}$ above for the case of $a = -\infty$ and $b = \infty$. Give your answers for both Cartesian and cylindrical coordinate systems.

    Note: $$\int \frac{1}{\left(x^2+u^2\right)^{3/2}}dx = \frac{x}{u^2\sqrt{x^2+u^2}}$$

(15 Marks)

Note: Question No. 1 continues on page 2.

(b) Assume that the charges in part (a)(ii) are moving in the +z direction to form an infinitely long direct current I. Determine the magnetic field intensity $\vec{H}$ at the point $(x, y, 0)$ due to the line current. Give your answers for both Cartesian and cylindrical coordinate systems.

(10 Marks)

2. (a) Consider a square loop of area a^2 in the xy-plane in free space. At time $t = 0$, the loop has its center position at the origin, and is moving at constant velocity v along the +x axis. The loop region is subjected to a spatially uniform but time-varying magnetic flux density of the form (for time $t \geq 0$)

$$\vec{B} = (C_1 t^2 + C_2 t)\ \vec{a}_z \text{ T},$$

where C_1 and C_2 are arbitrary constants.

(i) Find the magnetic flux Φ_m passing through the loop and the induced voltage V_{emf} at time $t \geq 0$. State any assumption made.

(ii) Assume that the loop has a uniform per-unit-length resistance of R_l (in Ω/m), determine the induced current I_{ind} at time $t \geq 0$.

(11 Marks)

(b) A lossy medium is characterized by dielectric constant $\varepsilon_r = 10$, loss tangent $\tan\delta = 8$, and relative permeability $\mu_r = 1$ at 500 MHz.

(i) Comment whether the medium is a good conductor. Find the conductivity of the medium.

(ii) Assume that a 500 MHz plane wave is propagating along +z direction in the medium, calculate the complex intrinsic impedance η_c, the attenuation constant α and the phase constant β.

(14 Marks)

3. (a) The magnetic field of a uniform plane wave (UPW) travelling in a lossless non-magnetic medium occupying the region $z \le 0$ is given as

$$\tilde{H}_i(z,t) = -\vec{a}_y 50 \cos\left(6\times10^9 t - 24.5z + 40°\right) \text{ mA/m.}$$

The UPW is incident normally on a plane interface at $z = 0$ with a lossy medium having complex intrinsic impedance $\eta_c = 10\angle 45°\ \Omega$ and occupying the region $z \ge 0$.

Determine the following and state any assumption(s) made:

(i) The phase velocity u_p of the UPW in the lossless medium.

(ii) The permittivity of the lossless medium.

(iii) The time-domain expression of the incident electric field $\tilde{E}_i(z,t)$.

(iv) The percentage of average incident power reflected at the planar interface at $z = 0$.

(12 Marks)

(b) A uniform plane wave (UPW) in free space occupying the region $z \le 0$ is incident at a plane interface with a lossless dielectric medium having $\mu_r = 1$ and $\varepsilon_r = 2.25$, occupying the region $z \ge 0$. The incident electric field of the UPW is given by

$$\vec{E}_i(x,z) = (20\vec{a}_x - 40\vec{a}_z)\, e^{-j(8x+4z)} \text{ V/m.}$$

Find the following:

(i) The angle of incidence θ_i and the angle of transmission θ_t. Give both angles in degrees.

(ii) The amplitude of the transmitted electric field at $z = 0$, i.e., E_{ot}.

(iii) The time-average power transmitted through a 2-m^2 area at $z = 0$.

(13 Marks)

4. (a) A generator having an open-circuit voltage $V_g(t) = 96\cos(2.5\pi \times 10^8 t)$ V and an internal impedance $Z_g = 100\ \Omega$ is connected to a $Z_o = 100\ \Omega$ lossless air-filled transmission line of length $\ell = 0.64$ m. The phase velocity on the line is $u_p = 3 \times 10^8$ m/s and the line is terminated in a complex load $Z_L = 140 - j64\ \Omega$. Assuming that the load end is at $z = 0$ and the source end is at $z = -\ell$, find the following and state any assumption(s) made:

(i) The electrical length $\dfrac{\ell}{\lambda}$ of the transmission line.

(ii) The reflection coefficient $\Gamma(z)$ in polar form, i.e., $|\Gamma|\angle\theta_\Gamma$ at $z = 0$.

(iii) The input impedance $Z_{in}(z)$ in polar form at $z = -\ell$.

(iv) The amplitude of the incident voltage wave at $z = 0$, i.e., V_o^+.

(v) The time-domain expression for the voltage at $z = 0$, i.e., $V(z = 0, t)$.

(20 Marks)

(b) Find the position z of maximum voltage for the transmission line in part (a).

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \dfrac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r \partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (r A_r)}{r \partial r} + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\| = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\| = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+\ e^{-j\beta z} + V_o^-\ e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+\ e^{-j\beta z} - V_o^-\ e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L\ e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

EE3001 ENGINEERING ELECTROMAGNETICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.