Lecture 10: Transformers

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Evaluate the transformer performance using the voltage regulation.
- ❖ Calculate the core losses and copper losses in a practical transformer.
- ❖ Apply the concepts learnt in maximum efficiency and full-load efficiency to evaluate the transformer performance.

❖ **Voltage Regulation**:

- Consider a transformer supplying a **full load** at **rated voltage** as shown in Fig. 27.
- To maintain rated voltage V_2 at the load terminals, the input voltage V_1' has to be maintained at a level different from V_2 by the amount of voltage drop in the series impedance ($Z_{e2} = R_{e2} + jX_{e2}$).

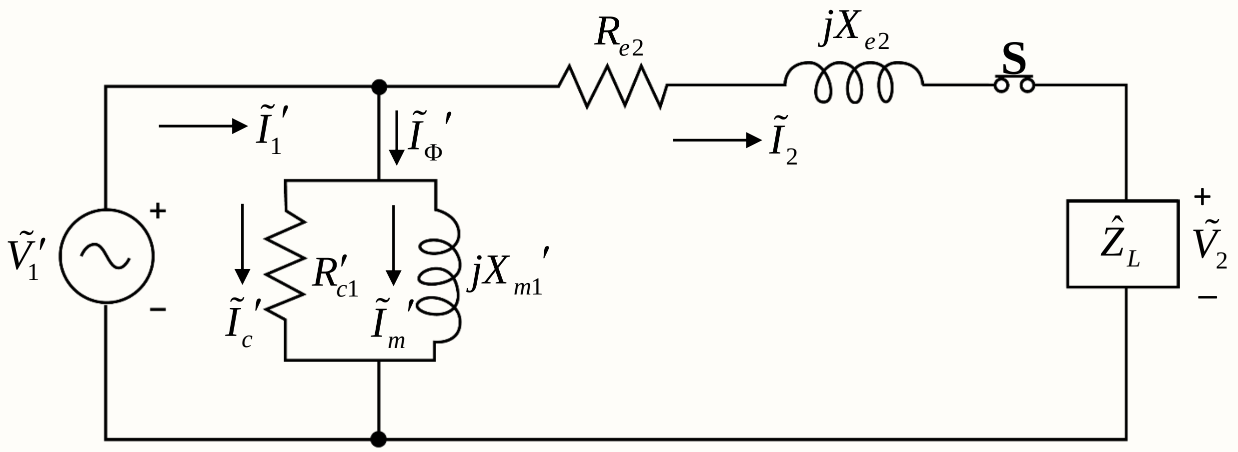

Fig. 27. Approximate equivalent circuit referred to the secondary.

Transformer Performance

- If the load switch is now opened, there would be no current and so no voltage drop in Z_{e2}. Hence, the voltage across the load terminals will change from V_2 to V_1'. The amount of voltage change at the load terminals when the full load operating at rated voltage is disconnected expressed as a percentage of the rated voltage is called the **Voltage Regulation**.

- Ideal transformers will have zero voltage regulation. Smaller voltage regulation is better for transformer operation. Therefore, the voltage regulation is used as a '**figure of merit**' for transformers.

Transformer Performance

- Voltage regulation (VR) can be quantitatively defined as:

$$VR\% = \frac{V_{2NL} - V_{2FL}}{V_{2FL}} \times 100 = \frac{V'_1 - V_2}{V_2} \times 100$$ (**referred to secondary** as shown in Fig. 28)

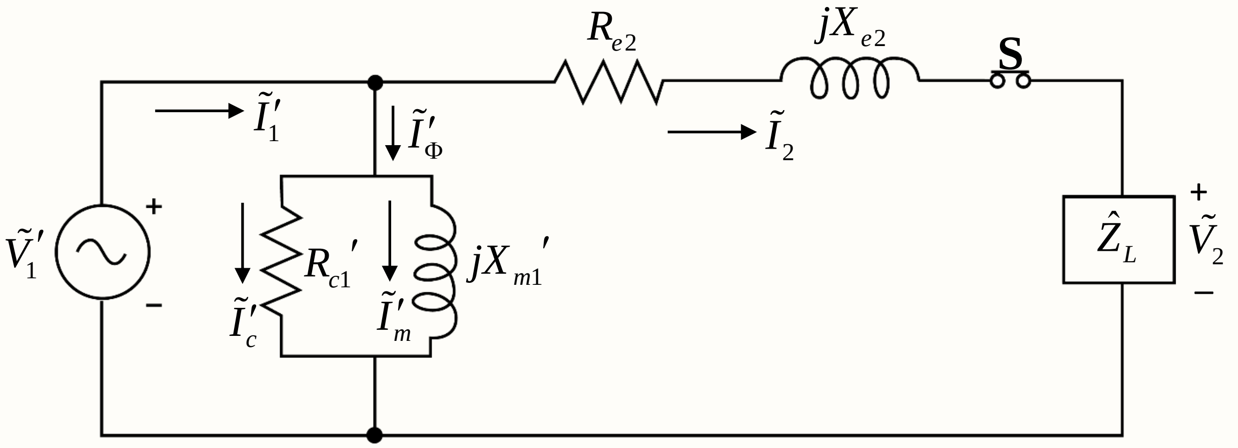

Fig. 28. Approximate equivalent circuit referred to the secondary.

Transformer Performance

- VR can also be referred to the primary as

$$VR\% = \frac{V_1' - V_2}{V_2} \times 100 = \frac{V_1/a - V_2}{V_2} \times 100 = \frac{V_1 - aV_2}{aV_2} \times 100 = \frac{V_1 - V_2'}{V_2'} \times 100$$

- This is equivalent to the use of the approximate equivalent circuit referred to the primary side, as shown in Fig. 29.

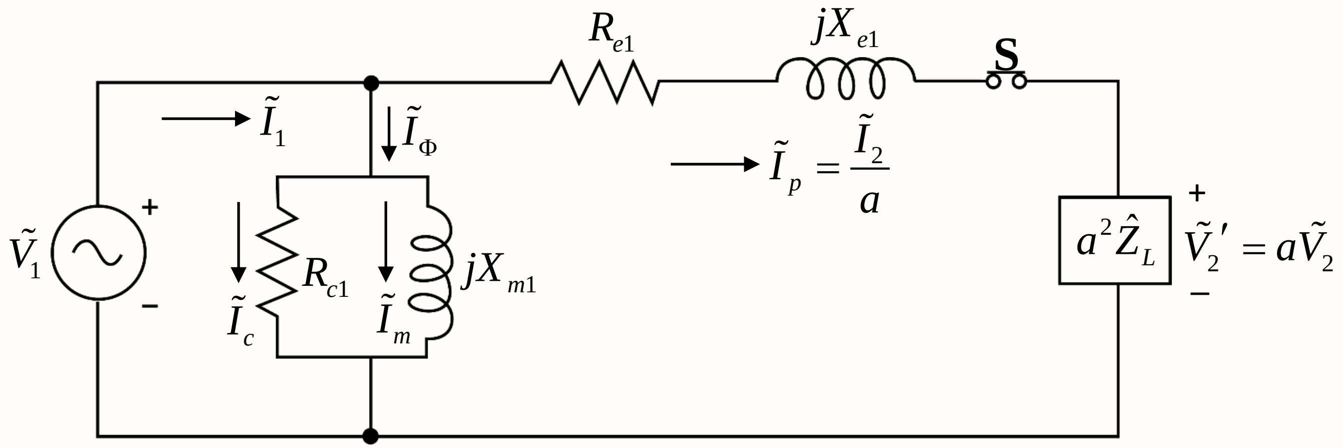

Fig. 29. Approximate equivalent circuit referred to the primary.

Transformer Performance

- It should be noted that while using the approximate equivalent circuit to calculate the voltage regulation, the shunt branch of X_m and R_c does not affect the calculations in any way and can be totally ignored.

Example 4

A 23-kVA, 2300/230-V, 60-Hz transformer has the following parameters:

$R_1 = 4\ \Omega,\ R_2 = 0.04\ \Omega,\ X_1 = 12\ \Omega,\ X_2 = 0.12\ \Omega,$

$R_{c1} = 20\ \mathrm{k}\Omega,$ and $X_{m1} = 15\ \mathrm{k}\Omega$

Calculate the voltage regulation of the transformer referred to the secondary side when it delivers full load at 0.8 pf (lag) at rated voltage.

(Solutions →)

Example 4 – Solutions

$$a = \frac{2300}{230} = 10$$

The circuit is shown in Fig. 30, where

$$R_{e2} = \frac{4}{10^2} + 0.04 = 0.08\ \Omega, \text{ and } X_{e2} = \frac{12}{10^2} + 0.12 = 0.24\ \Omega$$

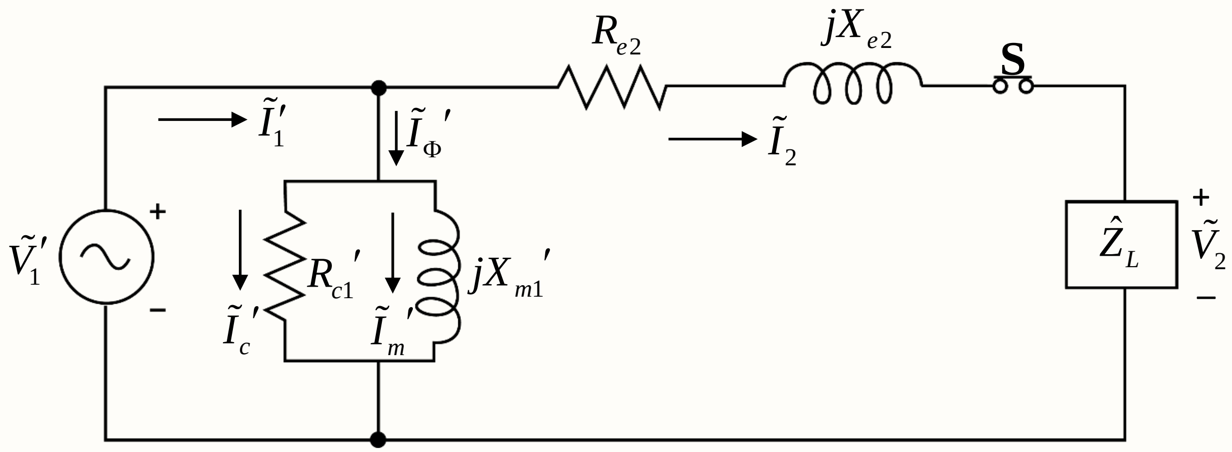

Fig. 30. Approximate equivalent circuit referred to the secondary.

Example 4 – Solutions

Let $V_2 = 230\angle 0^\circ$ V (**Reference**)

At full load, $I_2 = 23 \times \dfrac{1000}{230} = 100$ A, and

for 0.8 pf, $\varphi = \cos^{-1} 0.8 = 36.87^\circ$

For lagging pf, $I_2 = 100\angle -36.87^\circ$ A, so that,

$$V_1' = V_2 + I_2(R_{e2} + jX_{e2}) = 230\angle 0^\circ + 100\angle -36.87^\circ(0.08 + j0.24) = 251.2\angle 3.29^\circ \text{ V}$$

Therefore, $VR\% = \dfrac{251.2 - 230}{230} \times 100 = 9.22$ % (rejected)

Phasor Diagrams

- A sketch of the phasor diagram can be very helpful in understanding the nature of the voltage regulation.
  - For the above example with **lagging** power factor, it can be drawn as shown in Fig. 31.
  - It is clearly seen that V_1' is larger than V_2, and therefore the voltage regulation will be positive.

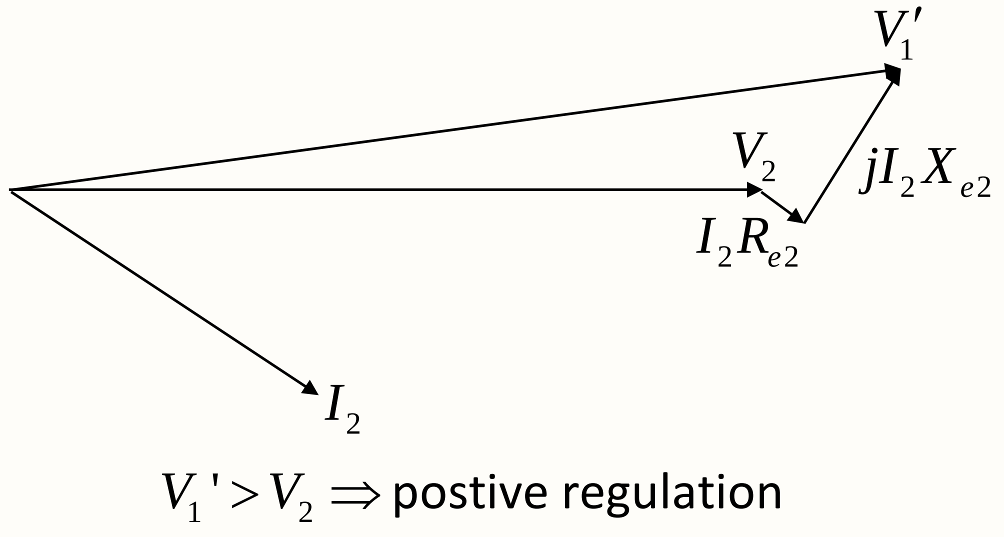

Fig. 31. Phasor diagram.

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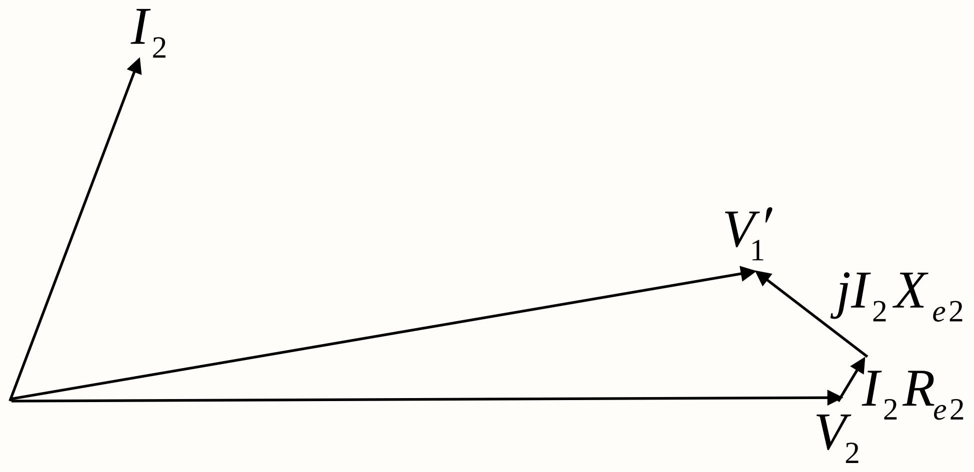
Phasor Diagrams

- For a sufficiently **leading** power factor, the phasor diagram will look as shown Fig. 32.

- It can be clearly seen that V_1' is smaller than V_2, and therefore the voltage regulation will be negative. This implies that the voltage at the load terminals will decrease when the load is disconnected.

❖ Repeat Example 4 for a pf of 0.8 lead to verify that the voltage regulation is negative in this situation.



$V_1' < V_2 \Rightarrow$ negative regulation

Fig. 32. Phasor diagram.

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Losses and Efficiency

- The losses in transformers consist of:
  - The magnetic losses – core losses
  - The copper losses
- The magnetic core losses P_m consist of two components:
  - Hysteresis loss
  - Eddy current loss

As the flux and the flux density remain practically constant in a transformer, both these components of the magnetic losses remain constant. Therefore, these losses are also called fixed losses.

Losses and Efficiency

- The copper losses P_{cu} (also known as I^2R loss) occur in the primary and the secondary windings. These depend on the currents in the windings and therefore on the load supplied by the transformer. These losses are also called variable losses.

- The power flow diagram of a transformer is shown in Fig. 33, and the input power can be expressed as $P_{in} = P_o + P_{cu} + P_m$.

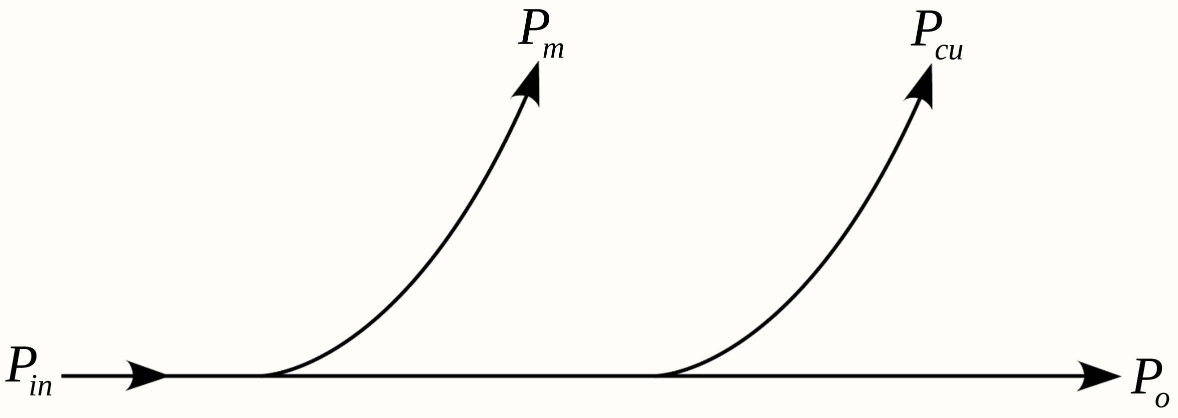

Fig. 33. Power flow diagram of a transformer.

Losses and Efficiency

- Consider the approximate equivalent circuit referred to the secondary side as shown in Fig. 34, the copper loss is

$$P_{cu} = I_2^2 R_{e2}$$

It is useful to note that the copper loss at a fraction ***x*** of the rated current, i.e., $I_2 = xI_{2fl}$, can be written as

$$P_{cu} = I_2^2 R_{e2} = \frac{I_2^2}{I_{2\,fl}^2} I_{2\,fl}^2 R_{e2} = x^2 P_{cu\,fl} \quad (\text{since } x = \frac{I_2}{I_{2\,fl}})$$

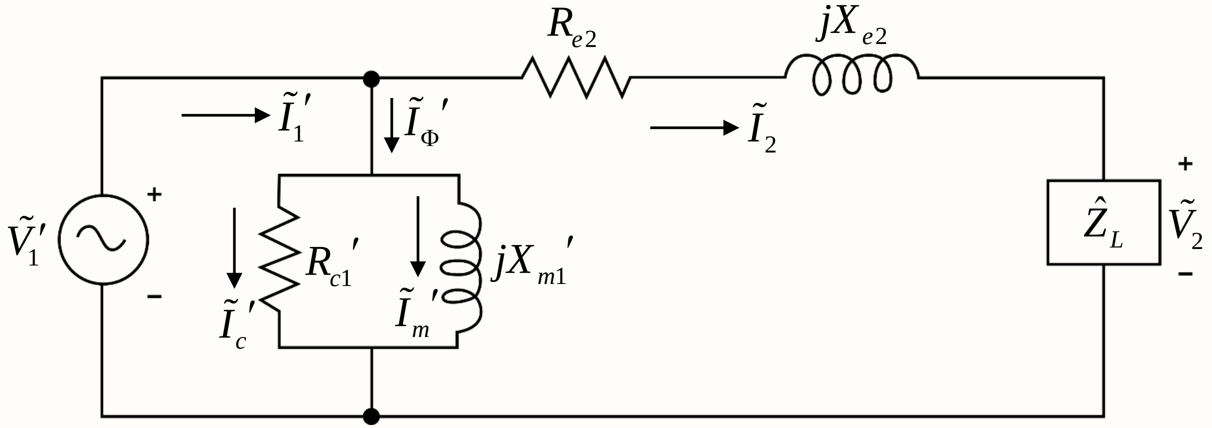

Fig. 34. Approximate equivalent circuit referred to the secondary.

Losses and Efficiency

- Noting that output $P_o = V_2 I_2 \cos\theta$, the efficiency can be expressed as

$$\eta = \frac{P_o}{P_i} = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + I_2^2 R_{e2} + P_m}$$

- The variation of efficiency η with load current I_2 is shown in Fig. 35.

- Clearly, at no load: $P_o = 0 \;\Rightarrow\; \eta = 0$

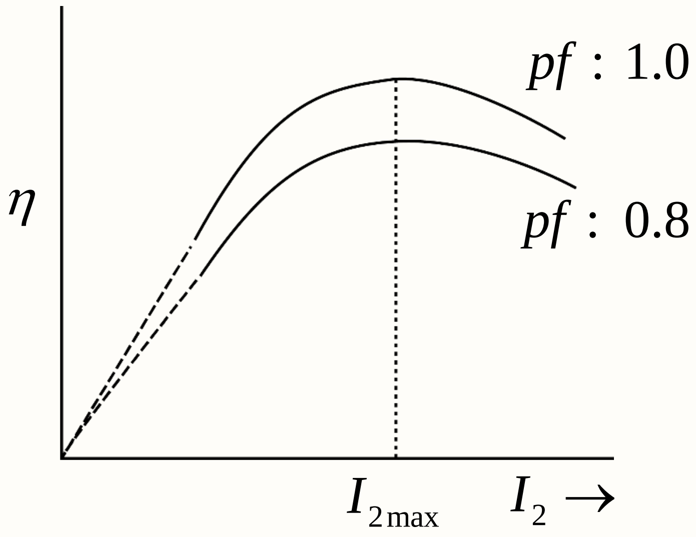

Fig. 35. Variation of efficiency with load current.

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Losses and Efficiency

- For a given pf, the efficiency increases with increasing I_2 (i.e., increasing load) and reaches a maximum value before it starts to drop.

- By equating: $\dfrac{d\eta}{dI_2} = 0,$

  the condition for maximum efficiency $\eta_{\max}$ is obtained as

  $$I_{2\max}^2 R_{e2} = P_m$$

  i.e., the maximum efficiency occurs at a current level, at which **the copper losses are equal to the core losses**, as depicted in Fig. 36.

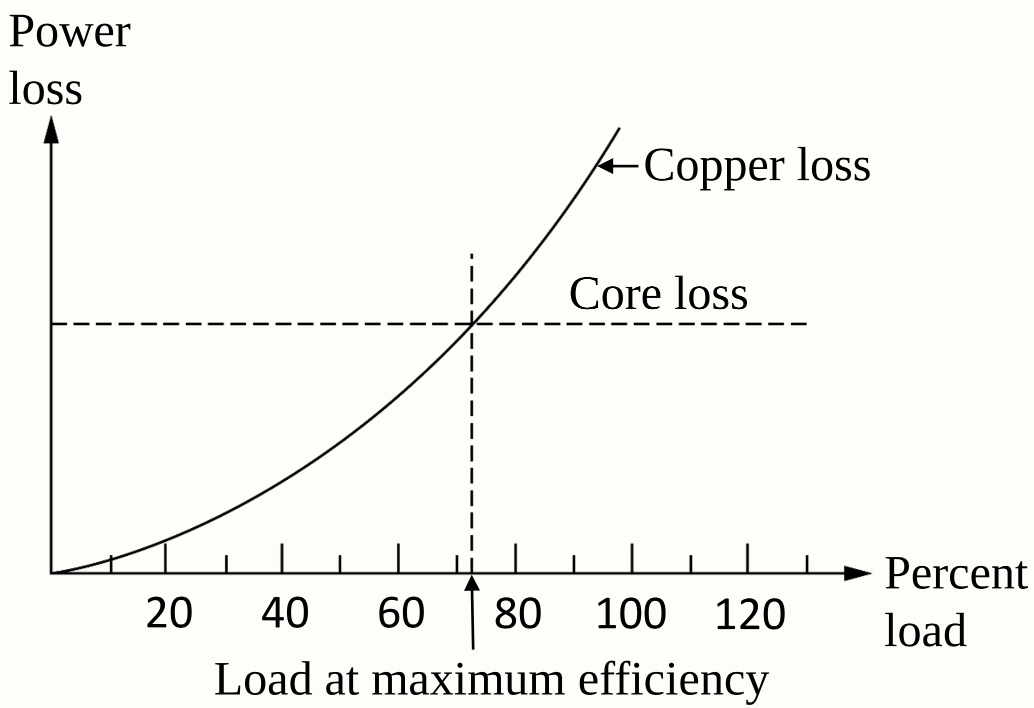


Fig. 36. Losses in a transformer.

Example 5

A 120-kVA, 2400/240-V transformer has the following parameters:

$$R_1 = 0.75\ \Omega,\ X_1 = 0.8\ \Omega,\ R_2 = 0.01\ \Omega,\ X_2 = 0.02\ \Omega$$

The transformer is designed to have maximum efficiency at 70 % of its rated load. Determine

a) the kVA loading of the transformer at maximum efficiency,

b) the maximum efficiency at 0.8 pf lagging,

c) the full-load efficiency at 0.8 pf lagging, and

d) the half-load efficiency at 0.8 pf lagging.

(Solutions →)

Example 5 – Solutions

a) kVA loading at maximum efficiency: $S_{\max} = 0.7 \times 120 \text{ kVA} = 84 \text{ kVA}$

At full load:

$$I_{1fl} = \frac{S_{fl}}{V_1} = \frac{120000}{2400} = 50 \text{ A}$$

$$I_{2fl} = \frac{S_{fl}}{V_2} = \frac{120000}{240} = 500 \text{ A}$$

Full load copper loss:

$$P_{cu-fl} = I_{1fl}^2 R_1 + I_{2fl}^2 R_2 = 50^2 \times 0.75 + 500^2 \times 0.01 = 4375 \text{ W}$$

Copper loss at maximum efficiency:

$$P_{cu-\max} = 0.7^2 \times 4375 = 2143.75 \text{ W (since } P_{cu} = x^2 P_{cu-fl})$$

Therefore, magnetic core losses: $P_m = 2143.75$ W

Example 5 – Solutions

b) Maximum efficiency at 0.8 pf lagging:

Fraction of load for maximum efficiency $= x = 0.7$

Power output $= VIx\cos\theta = 120000 \times 0.7 \times 0.8 = 67200 \text{ W}$

Copper loss $= 0.7^2 \times 4375 = 2143.75 \text{ W (since } P_{cu} = x^2 P_{cu\text{-}fl})$

Core loss $= 2143.75 \text{ W}$

$$\eta_{\max} = \frac{P_{o-\max}}{P_{o-\max} + P_{cu-\max} + P_m} \times 100 = \frac{67200}{67200 + 2143.75 + 2143.75} \times 100 = 94.00\ \%$$

Example 5 – Solutions

c) Efficiency at full load and 0.8 pf lagging:

Fraction of full load $= x = 1$

Power output $= 120000 \times 1 \times 0.8 = 96000 \text{ W}$

Copper loss $= 4375 \text{ W}$

Core loss $= 2143.75 \text{ W}$

Efficiency $\eta_{fl} = \dfrac{P_{o-fl}}{P_{o-fl} + P_{cu-fl} + P_m} \times 100 = \dfrac{96000}{96000 + 4375 + 2143.75} \times 100 = 93.64\ \%$

Example 5 – Solutions

d) Efficiency at half load and 0.8 pf:

Fraction of half load $= x = 0.5$

Power output $= 120000 \times 0.5 \times 0.8 = 48000\ \text{W}$

Copper loss $= 0.5^2 \times 4375 = 1093.75\ \text{W}$

Core loss $= 2143.75\ \text{W}$

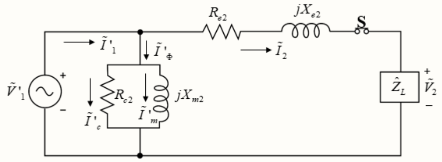
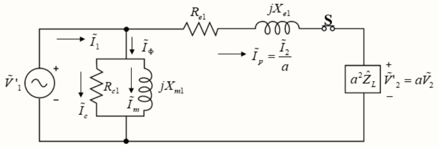
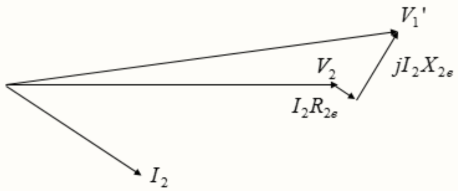
$$\eta_{hl} = \frac{P_{o-hl}}{P_{o-hl} + P_{cu-hl} + P_m} \times 100 = \frac{48000}{48000 + 1093.75 + 2143.75} \times 100 = 93.68\ \%$$

Summary

In this lecture, you have learnt:

- The evaluation of the transformer performance using the voltage regulation.
- The calculation of the core losses and copper losses in a practical transformer.
- The concepts in maximum efficiency and full-load efficiency to evaluate the transformer performance.

References

No.	Slide No.	Image	Reference
1	4, 6, 10, and 16		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 223), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
2	7		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 222), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
3	12		Reprinted from *AC Circuits and Machines*, (p. 159), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.

References

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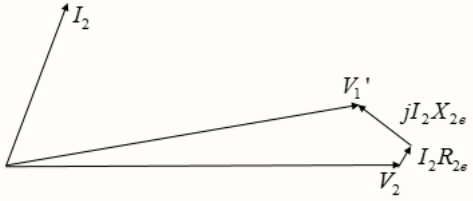
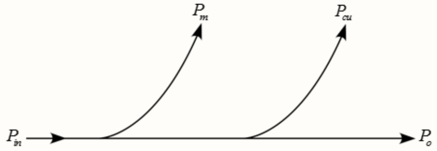
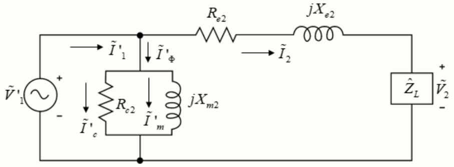
4	13		Reprinted from *AC Circuits and Machines*, (p. 159), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
5	15		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 228), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
6	16		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 223), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.

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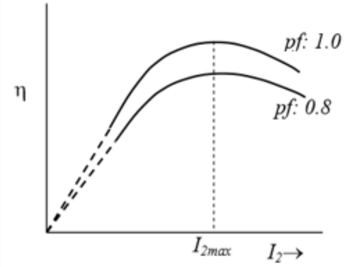
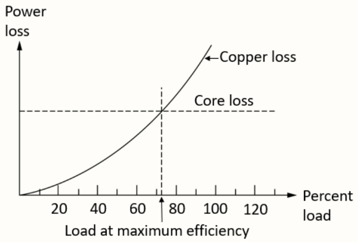
7	17		Reprinted from *AC Circuits and Machines*, (p. 163), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
8	18		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 229), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.