NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

EE3010 – ELECTRICAL DEVICES AND MACHINES

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

---

1. Consider the magnetic circuit with two air gaps shown in Figure 1, where a 500-turn coil is wound on the left limb. Each side limb has a cross-sectional area of 4 cm^2 and the centre limb has a cross-sectional area of 8 cm^2.

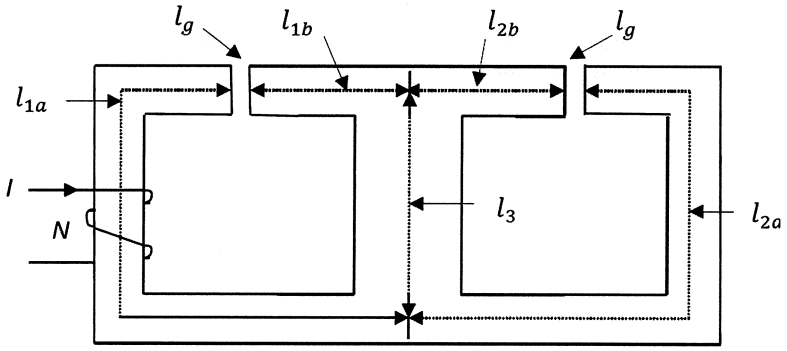


Figure 1

The mean lengths of the various sections and air gap length are as follows:

$$l_{1a} = l_{2a} = 34 \text{ cm}, \quad l_{1b} = l_{2b} = 11.8 \text{ cm}, \quad l_3 = 10 \text{ cm}, \quad l_g = 0.2 \text{ cm}$$

Note: Question No. 1 continues on page 2.

The relative permeability of the magnetic material is $\mu_r = 1200$ and the permeability of free space μ_o is $4\pi x10^{-7}$ H/m. Ignore leakage and fringing in your calculations.

(a) It is required to produce a flux density of 1.00 T in the centre limb. Calculate the current *I* needed to produce this flux density.

(15 Marks)

(b) Calculate the inductance of the coil and the percentage of the total energy stored in the two air gaps.

(6 Marks)

(c) Comment on the effect of the current *I* if the flux density in the centre limb is now 0.75 T. Justify your answer.

(4 Marks)

2. Tests were conducted on a 3-kVA, 120/240-V, 50-Hz single-phase transformer and the following results were obtained:

Open-circuit test (with 240 V side opened):

$V_{oc} = 120$ V, $I_{oc} = 0.505$ A, $P_{oc} = 26$ W

Short-circuit test (with 120 V side shorted):

$V_{sc} = 11$ V, $I_{sc} = 8$ A, $P_{sc} = 30$ W

(a) Determine all the parameters of the approximate equivalent circuit of the transformer referred to the 120 V side.

(8 Marks)

(b) Using the approximate equivalent circuit obtained in part 2(a), determine the voltage regulation when the transformer is delivering full load at 0.9272 power factor leading and rated secondary voltage. Use the secondary terminal voltage as the reference. Comment on the performance of the transformer.

(7 Marks)

(c) Find the efficiency of the transformer operated at 75% of full load and 0.8 power factor lagging, with rated secondary terminal voltage. Determine the output KVA corresponding to maximum efficiency as well as the maximum efficiency at 0.8 power factor lagging, with rated secondary voltage. State any assumption made.

(10 Marks)

3. (a) A three-phase, 208-V, 60-Hz, 4-pole, wye-connected induction motor has the following per phase parameters referred to the stator:

$$R_1 = 1.0\ \Omega,\ R_2 = 0.5\ \Omega,\ X_M = 100\ \Omega,\ X_1 = 2.0\ \Omega,\ X_2 = 2.0\ \Omega$$

The rotational losses of the motor are 80 W. If the motor is running at a speed of 1692 rpm, determine

(i) The input current drawn by the motor.

(ii) The efficiency of the motor.

(10 Marks)

(b) A three-phase, 120-V, 60-Hz, 6-pole, wye-connected induction motor has the following per phase parameters referred to the stator:

$$R_1 = 0\ \Omega,\ R_2 = 0.2\ \Omega,\ X_M = 100\ \Omega,\ X_1 = 0.3\ \Omega,\ X_2 = 0.3\ \Omega$$

Determine

(i) The slip at which maximum torque is developed by the motor.

(ii) The torque developed by the motor when it is running at a speed of 1164 rpm.

(iii) The resistance to be added to each phase of the rotor circuit, referred to the stator, such that the torque developed is 75% of the value obtained in part 3(b)(ii) and the motor is running at a speed of 1140 rpm.

(15 Marks)

4. (a) A dc shunt motor has an armature winding resistance of $0.18\ \Omega$ and a field winding resistance of $70.2\ \Omega$. The rotational losses of the motor are 800 W.

The armature terminals are connected to a dc supply of 240 V.

(i) If the motor's armature current is 170 A, determine the efficiency of the motor.

(ii) If the motor has its flux reduced to 96 percent of the flux value in part 4(a)(i) by the insertion of a resistor in series with the field winding, determine the new armature current. Assume that the shaft load is adjusted such that the speed of the motor is the same as in part 4(a)(i).

(12 Marks)

Note: Question No. 4 continues on page 4.

(b) The armature and field winding resistances of a separately-excited dc generator are $0.125\ \Omega$ and $50\ \Omega$, respectively. The rotational losses of the machine are 500 W. The generator is driven at a constant speed and an external variable resistor R_{ext} is connected in series with the field winding to a constant 240 V dc supply. The effects of armature reaction can be neglected. Consider the following two cases:

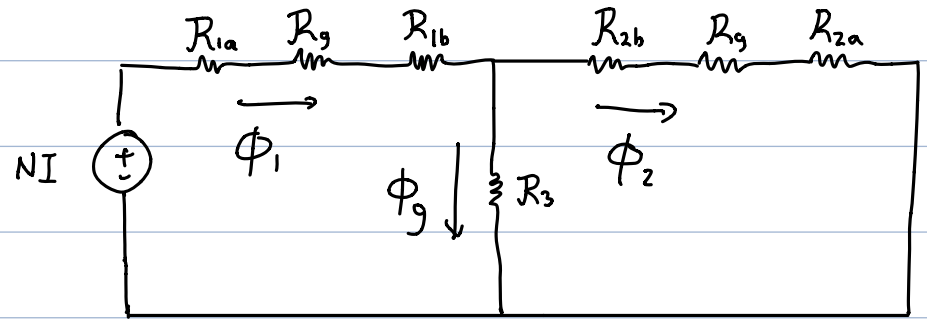
(i) The external variable resistor R_{ext} is adjusted to $28\ \Omega$ to give a no-load terminal voltage of 250 V. Determine the efficiency of the generator when it is supplying an armature current of 80 A.

(ii) The external variable resistor R_{ext} is adjusted to $8\ \Omega$. Determine the efficiency of the generator when it is supplying an armature current of 90 A. Assume that the machine is operating in the linear part of the magnetization curve.

(13 Marks)

END OF PAPER

① Magnetic equivalent circuit:



$N = 500$

$A_1 = A_2 = A_g = 4.10^{-4} m^2$

$A_3 = 8.10^{-4} m^2$

$l_{1a} = l_{2a} = 0.34 m$

$l_{1b} = l_{2b} = 0.118 m$

$l_3 = 0.1 m$

$l_g = 0.002 m$

$\mu_r = 1200$

$\mu_0 = 4\pi . 10^{-7} H/m$

(a.) $B_3 = 1 T$ $\phi_3 = B_3 . A_3 = 1 . 8.10^{-4} = 0.8 mWb$

calculate the reluctance of each segments:

$$\therefore R = \frac{l}{\mu_r \mu_0 A}$$

$$R_{1a} = R_{2a} = \frac{0.34}{1200 . 4\pi . 10^{-7} . 4.10^{-4}} = 563673.76 \quad H^{-1}$$

$$R_{1b} = R_{2b} = \frac{0.118}{1200 . 4\pi . 10^{-7} . 4.10^{-4}} = 195627.95 \quad H^{-1}$$

$$R_g = \frac{0.002}{4\pi . 10^{-7} . 4.10^{-4}} = 3978873.58 \quad H^{-1}$$

$$R_3 = \frac{0.1}{1200 . 4\pi . 10^{-7} . 8.10^{-4}} = 82893.2 \quad H^{-1}$$

$$R_{right} = (R_{2b} + R_g + R_{2a}) = 4738175.29$$

$$\phi_2 = \frac{R_3}{R_{right}} \times \phi_3 = \frac{82893.2}{4738175.29} \times 0.8 \cdot 10^{-3} = 1.4 \times 10^{-5} Wb$$

$$\phi_1 = \phi_2 + \phi_3 = 8.14 \times 10^{-4}$$

parallel with

$$R_t = \left[(R_{2b} + R_g + R_{2a}) \,//\, R_3 \right] + (R_{1a} + R_g + R_{1b})$$

$$= \left[\frac{4738175.29 \times 82893.2}{(4738175.29 + 82893.2)} \right] + 4738175.29$$

$$= 4819643.23 \quad H^{-1}$$

$$NI = \phi_1 \times R_t = 8.14 \times 10^{-4} \times 4819643.23$$

$$= 3923.19$$

$$I = \frac{3923.19}{500} = 7.846 \quad A$$

(b.) Inductance of the coil

$$L = \frac{N^2}{R_t} = \frac{(500)^2}{4819643.23} = 0.052 \; H$$

Total energy stored in the systems :

$$E = \frac{1}{2} L I^2 = \frac{1}{2} \times 0.052 \times (7.846)^2 = 1.597 \text{ J}$$

Energy store in airgaps :

$$E_g = \frac{1}{2} \phi_1^2 R_g + \frac{1}{2} \phi_2^2 R_g$$

$$= \frac{1}{2} (8.14 \times 10^{-4})^2 \cdot 3978873.58 +$$

$$\frac{1}{2} (0.14 \times 10^{-4})^2 \cdot 3978873.58$$

$$= 1.319 \text{ J}$$

% of energy stored in airgaps :

$$\frac{E_g}{E} = \frac{1.319}{1.597} \times 100\% = 82.57 \%$$

(c.) $B_3' = 0.75T$ $\quad \frac{B_3'}{B_3} = \frac{0.75}{1}$ $\quad B_3' = \frac{3}{4} B_3$

$$\phi_3' = B_3' \cdot A_3$$

since A_3 is still the same then

$$\phi_3' = \frac{3}{4} B_3 A_3 = \frac{3}{4} \phi_3$$

$$\phi_2' = \frac{R_3}{R_{right}} \times \phi_3' \quad , \quad R_3 \text{ \& } R_{right} \text{ don't change}$$

$$\phi_2' = \frac{3}{4} \phi_2$$

$$\phi_1' = \phi_2' + \phi_3' = \frac{3}{4}(\phi_2 + \phi_3) = \frac{3}{4}\phi_1$$

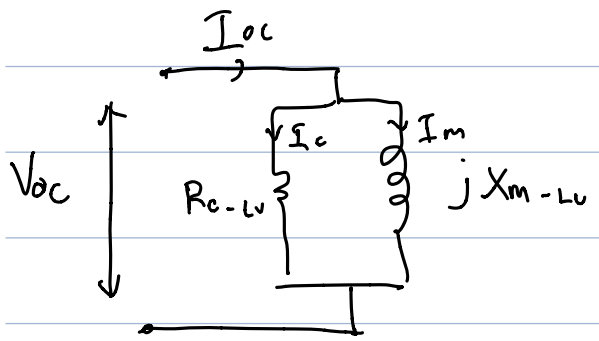
with N & Rf doesn't change

$$I' = \frac{3}{4} I$$

The change of I will be directly proportional to the change of the central limb flux (ϕ_3)

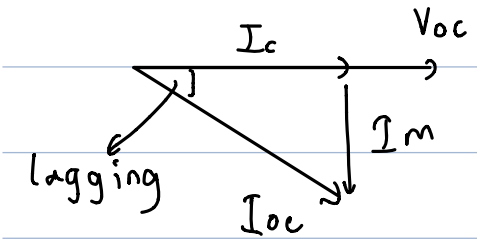
(2) (a.) i. open circuit test

↳ R_c, X_m (referred to Low Voltage side)



$$R_{c-LV} = \frac{V_{oc}^2}{P_{oc}} = \frac{120^2}{26} = 553.85\ \Omega$$

$$I_c = \frac{V_{oc}}{R_c} = \frac{120}{553.85} = 0.217\ A$$



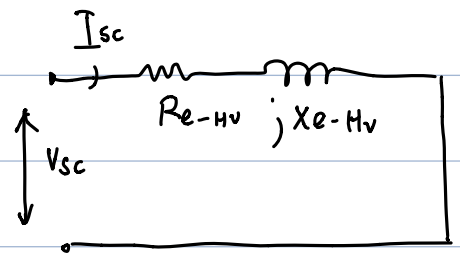
$$I_m = \sqrt{I_{oc}^2 - I_c^2} = \sqrt{0.505^2 - 0.217^2}$$

$$= 0.456\ A$$

$$X_m = \frac{V_{oc}}{I_m} = \frac{120}{0.456} = 263.066\ \Omega$$

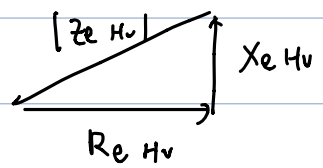
ii short circuit test

↳ R_e, X_e (referred to High Voltage side)



$$R_{e\text{-}Hv} = \frac{P_{sc}}{I_{sc}^2} = \frac{30}{8^2} = 0.469 \ \Omega$$

$$|Z| = \frac{V_{sc}}{I_{sc}} = \frac{11}{8} = 1.375$$



$$X_{e\text{-}Hv} = \sqrt{Z^2 - R_e^2} = \sqrt{1.375^2 - 0.469^2}$$

$$= 1.293 \ \Omega$$

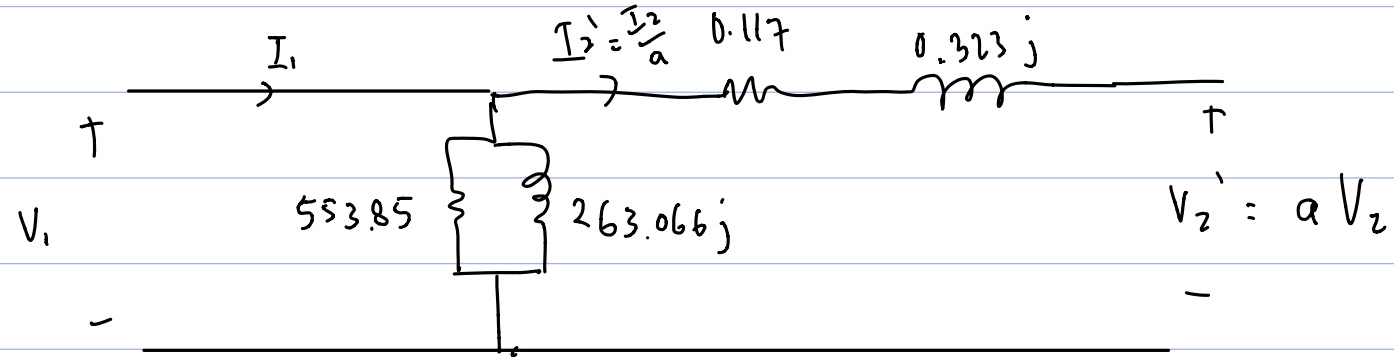
$\Rightarrow$ referred to the low voltage (120V) side

$$a = \frac{120}{240} = \frac{1}{2}$$

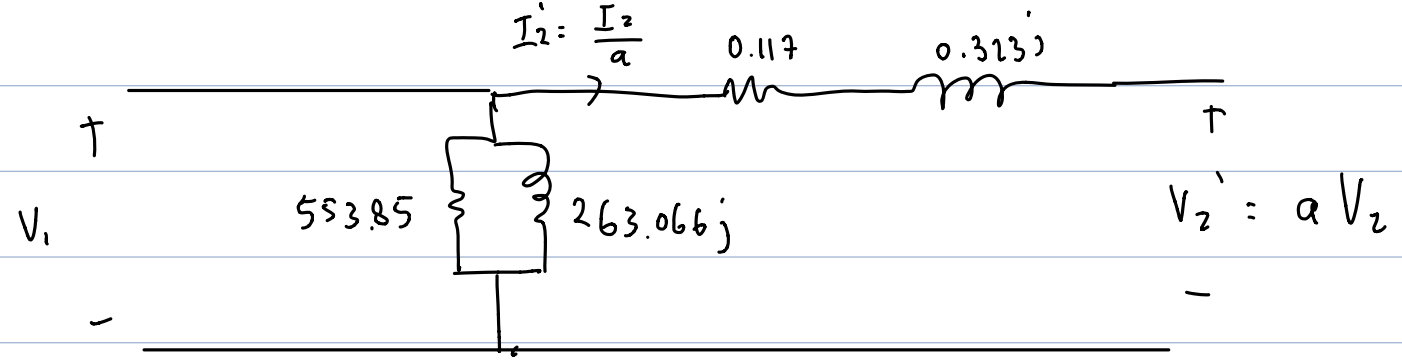
$$R_{e\text{-}Lv} = 0.469 \cdot \left(\frac{1}{2}\right)^2 = 0.117 \ \Omega$$

$$X_{e\text{-}Lv} = 1.293 \cdot \left(\frac{1}{2}\right)^2 = 0.323 \ \Omega$$

approximate equivalent circuit referred to the 120V side



(b.) voltage regulation at full load, p.f. = 0.9272 leading, rated secondary voltage, $V_2 = 240V$



$|V_2'| = 120\ V$ $\qquad \theta = \cos^{-1} 0.9272 = 21.998^\circ$

$$I_2' = \left(\frac{3 \cdot 10^3 \angle -21.998}{120} \right)^* = 25 \angle 21.998^\circ$$

$$V_1 = 120 \angle 0^\circ + 25 \angle 21.998^\circ \times (0.117 + 0.323j)$$
$$= 119.995 \angle 4.102^\circ$$

Voltage regulation: $\dfrac{|V_1| - |V_2'|}{|V_2'|} \times 100\% = \dfrac{119.99873 - 120}{120} \times 100\%$

$$= -0.00106\ \%$$

Voltage regulation is the 'figure of merit' for transformers' performance. The smaller the voltage regulation, the better the operation. Ideal transformer will have zero voltage regulation. Hence, this transformer is very good and the performance is close to the ideal one.

(c.) Efficiency @ 75% load, 0.8 p.f. lagging, rated V_2

$$P_{out} = 0.75 \times 0.8 \times 3000 = 1800\ W$$

@ full load:

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$

$$I_2' = \left(\frac{3 \cdot 10^3 \angle 36.87}{120} \right)^* = 25 \angle -36.87$$

$$V_1 = 120 \angle 0^\circ + 25 \angle -36.87 \ (0.117 + 0.323j)$$

$$= 127.272 \angle 2.119^\circ$$

$$P_m = \frac{|V_1|^2}{R_c} = \frac{(127.272)^2}{553.85} = 29.25\ W$$

$\Rightarrow$ assuming to use calculated value of core-loss (P_m) instead of from the given P_{oc} (26 W)

$$P_{cu-fl} = (25)^2 \times 0.117 = 73.125\ W$$

@ 75 % load, (P_m fixed value)

$$P_{cu-0.75} = (0.75)^2 \times 73.125 = 41.133\ W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_m + P_{cu-0.75}} \times 100\%$$

$$= \frac{1800}{1800 + 29.25 + 41.133} \times 100\% = 96.237\%$$

max efficiency (η_{max}) occurs when $P_m = P_{cu-x}$

$$29.25 = x^2 \times 73.125$$

$$x = 0.6325 \rightarrow \eta_{max} @ 63.25\% \text{ load}$$

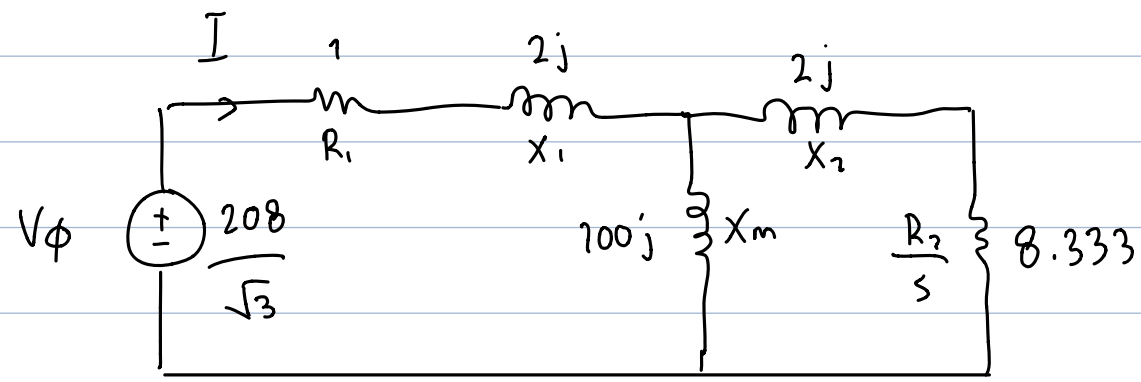
Output KVA = $0.6325 \times 3000 = 1.8975$ KVA

$$\eta_{max} = \frac{0.6325 \times 3000 \times 0.8}{0.6325 \times 0.8 \times 3000 + 29.25 + 29.25} \times 100\%$$

$$= 96.289\ \%$$

(3) (a.) $n_{sync} = \frac{120 f_s}{P} = \frac{120 \cdot 60}{4} = 1800$ rpm

$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{1800 - 1692}{1800} = 0.06$$

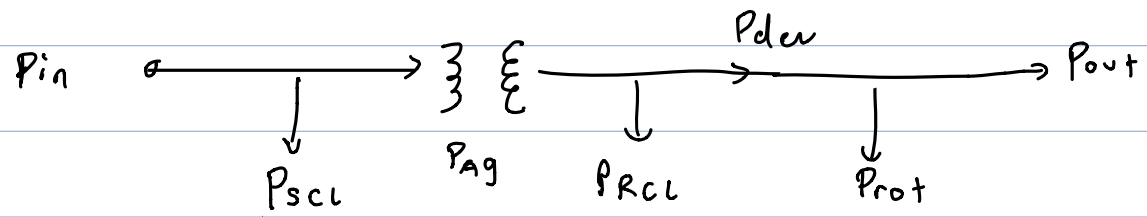


10.25 − 1.61j

$$Z_T = \left(\frac{(8.333 + 2j) \times 100j}{(8.333 + 2j) + 100j} \right) + 1 + 2j = 10.074 \angle 27.24^\circ$$

(i) $I = \frac{V_\phi}{Z_t} = \frac{208/\sqrt{3}}{10.074 \angle 27.24} = 11.921 \angle -27.24^\circ$

(ii)



$$P_{in} = \sqrt{3}\ V_L\ I_L \cos\theta$$

$$= \sqrt{3} \cdot 208 \cdot 10.074 \cdot \cos 27.24 = 3226.82\ W$$

$$P_{SCL} = 3\ (I)^2\ R_1$$

$$= 3\ (10.074)^2 \cdot 1 = 304.46\ W$$

$$P_{AG} = P_{in} - P_{SCL}$$

$$= 3226.82 - 304.46 = 2922.36\ W$$

$$P_{dev} = P_{AG} - P_{SCL} = (1-s)\ P_{AG}$$

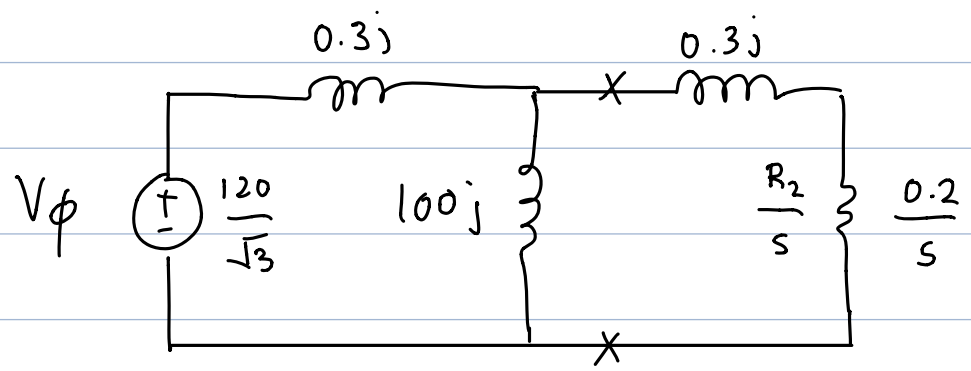
$$= (1-0.06) \cdot 2922.36 = 2747.021$$

$$P_{out} = P_{dev} - P_{rot}$$

$$= 2747.021 - 80 = 2667.02\ W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{2667.02}{3226.82} \times 100\% = 82.65\%$$

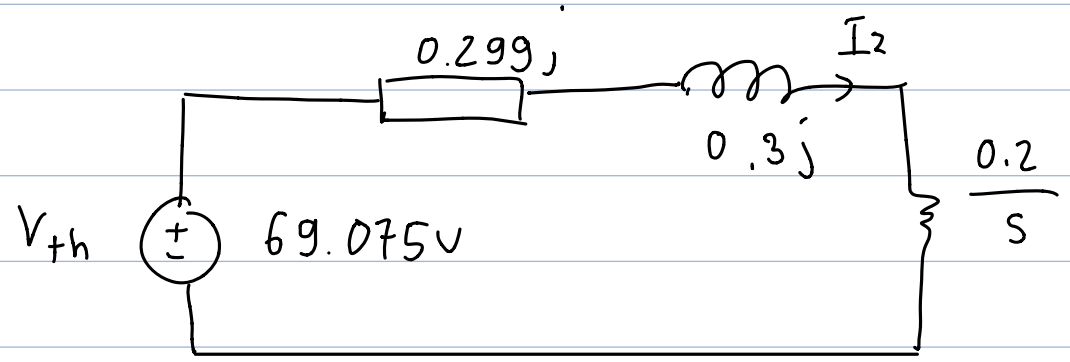
(b.) $$n_{sync} = \frac{120\ f_s}{P} = \frac{120 \cdot 60}{6} = 1200\ rpm$$



$$Z_{th} = 0.3j \,//\, 100j = 0.299j$$

$$V_{th} = \frac{100j}{100j + 0.3j} \times \frac{120}{\sqrt{3}} = 69.075\ V$$

Thevenin equivalent circuit



(i) $$S_{max} = \frac{R_2}{|Z_{th} + X_2|} = \frac{0.2}{|0.299j + 0.3j|} = 0.334$$

(ii) $n_m = 1164$ rpm

$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{1200 - 1164}{1200} = 0.03$$

$$|I_2| = \frac{69.075}{\left|\left(0.299j + 0.3j + \frac{0.2}{0.03}\right)\right|} = 10.32 \text{ A}$$

$$P_{dev} = T_{dev} \cdot \omega_m \Rightarrow P_{AG} \cancel{(1-s)} = T_{dev} \cancel{(1-s)} \omega_{sync}$$

$$T_{dev} = \frac{P_{AG}}{\omega_{sync}} = \frac{3\,|I_2|^2 \cdot \frac{R_2}{s}}{\omega_{sync}}$$

$$T_{dev} = \frac{3 \cdot (10.32)^2 \cdot \frac{0.2}{0.03}}{1200 \cdot \frac{\pi}{30}} = 16.95 \text{ Nm}$$

(iii) $n_m' = 1140$ rpm.

$$s' = \frac{1200 - 1140}{1200} = 0.05$$

$$|I_2| = \frac{V_{th}}{\sqrt{(X_{th}+X_2)^2 + \left(\frac{R_2}{s}\right)^2}}$$

$$|I_2|^2 = \frac{V_{th}^2}{(X_{th}+X_2)^2 + \left(\frac{R_2}{s}\right)^2}$$

$$\frac{T_{dev}'}{T_{dev}} = 0.75 = \frac{\dfrac{\cancel{3}\,|I_2'|^2 \dfrac{R_2'}{s'}}{\cancel{\omega_{sync}}}}{\dfrac{\cancel{3}\,|I_2|^2 \dfrac{R_2}{s}}{\cancel{\omega_{sync}}}}$$

$$\frac{|I_2'|^2}{|I_2|^2} = \frac{(X_{th}+X_2)^2 + \left(\frac{R_2}{s}\right)^2}{(X_{th}+X_2)^2 + \left(\frac{R_2'}{s'}\right)^2} = \frac{0.599^2 + 6.667^2}{0.599^2 + (20R_2')^2}$$

$$\frac{T_{dev}'}{T_{dev}} = 0.75 = \frac{44.808}{[0.359 + 400(R_2')^2]} \times \frac{20R_2'}{6.667}$$

$$5(0.359 + 400(R_2')^2) = 896.16\ R_2'$$

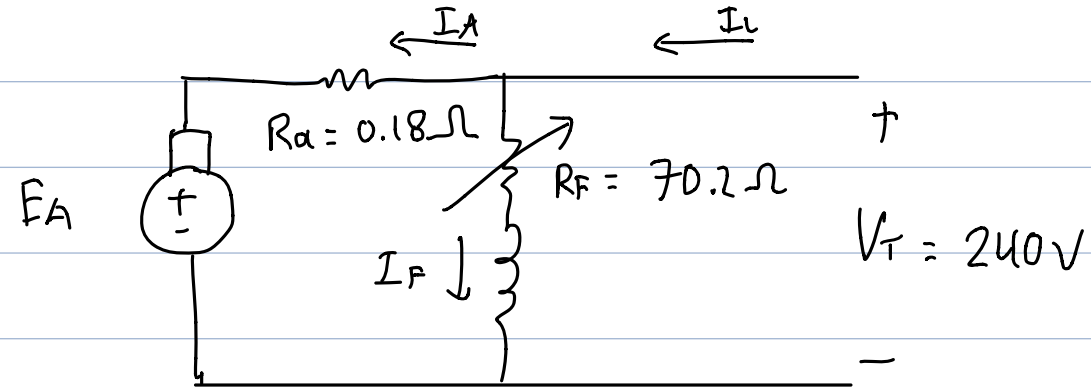
$$2000\ (R_2')^2 - 896.16\ R_2' + 1.795 = 0$$

$$R_2' = 0.446\ ,\ \cancel{0.002}$$

rejected (too small)

Added resistance $= 0.446 - 0.2 = 0.246\ \Omega$

(4) (a.) DC shunt motor equivalent circuit



(i) $I_A = 170\ A$

$E_A = V_T - I_A R_a = 240 - 170 . 0.18 = 209.4\ V$

$P_{dev} = E_A . I_A = 209.4 \times 170 = 35598\ W$

$P_{out} = P_{dev} - P_{rot} = 35598 - 800 = 34798\ W$

$I_F = \frac{V_T}{R_F} = \frac{240}{70.2} = 3.419\ A$

$I_L = I_F + I_A = 170 + 3.419 = 173.419 A$

$P_{in} = V_T . I_F = 240 . 173.419 = 41620.513\ W$

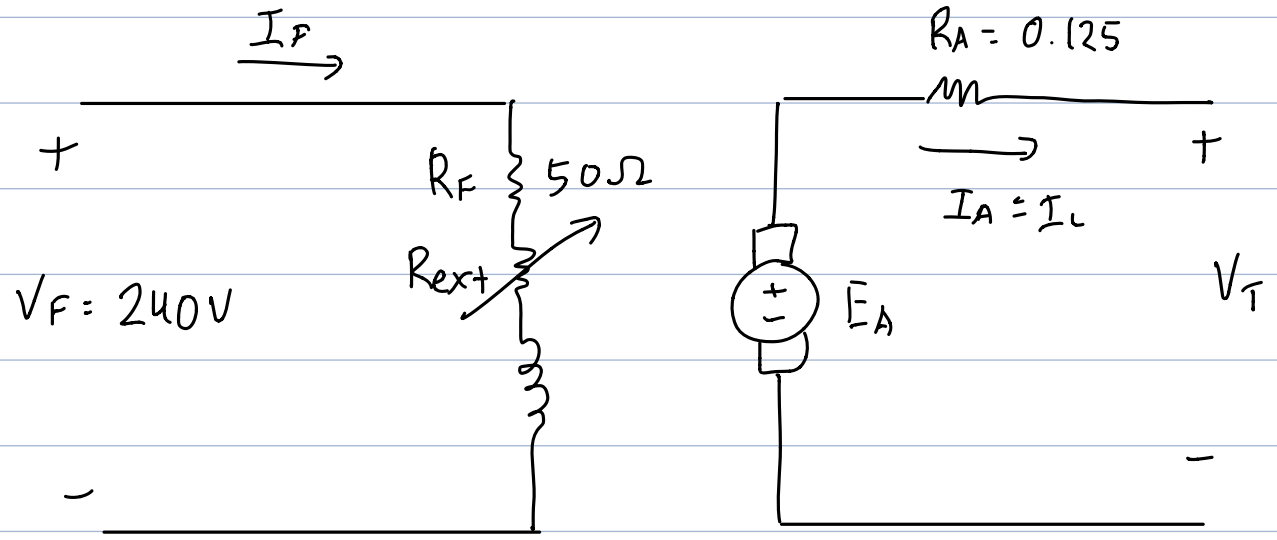
$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{34798}{41620.5} \times 100\% = 83.61\%$

(ii) $E_A = K_E \phi n_m$, constant n_m

$\frac{E_A'}{E_A} = \frac{\phi'}{\phi}$ $E_A' = 0.96 \times 209.4 = 201.024\ V$

$I_a' = \frac{V_T - E_A'}{R_a} = \frac{240 - 201.024}{0.18} = 216.53\ A$

(b.) Seperately-excited DC generator.



(i) $R_{ext} = 28\,\Omega$ $\quad V_{TnL} = 250V = E_A$

$$I_F = \frac{240}{50+28} = 3.077\ A$$

@ constant speed (n_m constant)

I_F constant (same config in the field circuit)

$I_F \propto \phi$ (constant flux)

$E_A = k_E \phi\, n_m$ (E_A constant)

$I_A = 80\ A$

$V_T = E_A - I_A R_A = 250 - 80 \cdot 0.125 = 240V$

$P_{out} = V_T I_L = 240 \cdot 80 = 19200\ W$

$P_{in} = E_A I_A + V_F I_F + P_{rot}$

$= 250 \cdot 80 + 240 \cdot 3.077 + 500 = 21238.5\,w$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{19200}{21238.5} \times 100\% = 90.4\%$$

(ii) $R_{ext} = 8\,\Omega$, $I_a = 90\ A$

$$I_F' = \frac{240}{50+8} = 4.138\ A$$

$E_a = k_e \phi n_m$, $\phi \propto I_F$

@ constant speed & linear part of the magnetization curve

$$\frac{E_a'}{E_a} = \frac{I_F'}{I_F} \Rightarrow E_a' = 250 \times \frac{4.138}{3.077} = 336.2\ V$$

$$V_T' = 336.2 - 90 \times 0.125 = 324.95\ V$$

$$P_{out} = V_T I_L = 324.95 \times 90 = 29245.87\ W$$

$$P_{in} = E_A I_A + V_F I_F + P_{rot}$$

$$= 336.2 \times 90 + 240 \times 4.138 + 500$$

$$= 31751.12\ W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{29245.87}{31751.12} \times 100\% = 92.12\%$$

~ GOOD LUCK ~