NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2021-2022

EE3001 – ENGINEERING ELECTROMAGNETICS

November / December 2021 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae and physical constants is provided in Appendix A on pages 5 to 7.
7. The Smith Chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) A circular loop of radius a is centered at the origin in the xy-plane. The loop carries a line charge of uniform density ρ_l in free space.

(i) Using Coulomb's law, determine the electric field intensity $\vec{E}(z)$ at the point (0, 0, z) due to the line charge.

(ii) Given that the potential at the origin is $V(0) = \dfrac{\rho_l}{2\varepsilon_0}$, find the potential $V(z)$ at the point (0, 0, z) by considering the work done in moving a unit positive test charge against $\vec{E}(z)$ in part (i) above.

Note: $\displaystyle\int \frac{x}{\left(x^2+u^2\right)^{3/2}}\,dx = \frac{-1}{\sqrt{x^2+u^2}}$

(13 Marks)

Note: Question No. 1 continues on page 2.

(b) Let the charges in part (a) be moving to form a direct current I along the circular loop.

(i) Make an assumption about the current direction and determine the corresponding magnetic field intensity $\vec{H}(z)$ at the point (0, 0, z).

(ii) What is the strongest $\vec{H}(z)$ and where is its position?

(12 Marks)

2. (a) A rectangular loop circuit has width W and length L in the xy-plane in free space. While the width is fixed, the length is changing with time $t \geq 0$ in the form

$$L(t) = L_0 + L_1 t + L_2 t^2 \quad \text{m}$$

where L_0, L_1 and L_2 are some constants. The circuit is subjected to a spatially uniform and time-varying magnetic flux density of the form

$$\vec{B} = \sin(t)\, \vec{a}_z \quad \text{T.}$$

(i) Determine the magnetic flux Φ_m through the circuit and the induced voltage V_{emf} at time $t \geq 0$. State any assumption made.

(ii) If the circuit contains a capacitor of capacitance C, determine the induced current I_{ind} through the capacitor at time $t \geq 0$.

(11 Marks)

(b) A plane wave propagating in a nonmagnetic medium has its electric field expression given by

$$\tilde{E} = \vec{a}_x\, 150 e^{-\alpha z} \cos(2\pi \times 10^6 t - 0.4z + 25°) \quad \text{V/m}$$

(i) What is the value of α if the medium is a good conductor?

(ii) Using your α value in part (i) and $\varepsilon_r = 1$, calculate the wavenumber k_c, conductivity σ and intrinsic impedance η_c.

(iii) Derive the corresponding magnetic field expression.

(14 Marks)

3. (a) The electric field of a uniform plane wave (UPW) travelling in a lossless and non-magnetic medium occupying the region $z \leq 0$ with permittivity $\varepsilon = 5\varepsilon_o$ is given by:

$$\vec{E}_i(z) = \vec{a}_y 3e^{-j4z} \text{ kV/m}$$

The UPW is incident normally on a planar interface with a lossy medium having $\mu = \mu_o$, $\varepsilon = 2\varepsilon_o$ and $\sigma = 0.5$ S/m occupying the region $z \geq 0$.

Determine the following and state any assumption(s) made:

(i) The frequency of the UPW.

(ii) The propagation constants of the UPW in the lossless and lossy media i.e., γ_1 and γ_2.

(iii) The percentage of the average power dissipated as the transmitted wave travels from $z = 0$ to $z = 0.1$ m.

(10 Marks)

(b) The magnetic field intensity of a uniform plane wave (UPW) propagating in free space $(z \leq 0)$ is given by:

$$\tilde{H}_i = (0.2\vec{a}_x - 0.4\vec{a}_z)\cos(\omega t - 8x - 4z) \text{ A/m}$$

The UPW is obliquely incident on a second medium made of lossless dielectric having $\mu = \mu_o$, $\varepsilon = 2.5\varepsilon_o$ and occupying the region $z \geq 0$.

Find the following and state any assumption(s) made:

(i) The angular frequency ω and the direction of propagation of the incident UPW $\vec{a}_{k_i}$.

(ii) The time-domain expression of the incident electric field $\tilde{E}_i$.

(iii) The polarization of the incident UPW with respect to the plane of incidence. Briefly explain your answer.

(iv) The percentage of average incident power reflected from the interface at $z = 0$.

(15 Marks)

4. (a) A generator having an open-circuited voltage $V_g(t) = 50\cos\left(4\pi\times10^8 t\right)$ V and an internal impedance $Z_g = 75\ \Omega$ is connected to a 1.25-m long lossless transmission line having characteristic impedance $Z_0 = 75\ \Omega$ and phase velocity $u_p = 3\times10^8$ m/s. The line is terminated in a load $Z_L = 100 + j220\ \Omega$.

Assume that the load end is located at $z = 0$ and the source end at $z = -\ell$, where ℓ is the length of the transmission line.

Find the following and state any assumption(s) made:

(i) The input impedance $Z_{in}(z)$ in polar form at $z = -\ell$.

(ii) The average power delivered to the load.

(iii) The position z at which the real part of the input impedance $\mathrm{Re}\{Z_{in}(z)\}$ is minimum.

(iv) The magnitude of minimum voltage i.e., $|V|_{\min}$, on the line.

(20 Marks)

(b) The load in part(a) is subsequently removed and the transmission line is left open-circuited. Find all the positions of voltage minimum $z_{\min}$ on the transmission line.

(5 Marks)

Note: The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Appendix A (continued)

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{I\vec{dl} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot \vec{dl} = I = \iint_S \vec{J} \cdot \vec{ds}$$

$$emf = \oint_C \tilde{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot \vec{ds}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Appendix A (continued)

Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\| = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\| = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \; e^{-j\beta z} + V_o^- \; e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \; e^{-j\beta z} - V_o^- \; e^{+j\beta z}\}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L \, e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

EE3001 ENGINEERING ELECTROMAGNETICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.