$$\left[\begin{array}{cccc|c} 1 & 3 & 6 & 1 & 5 \\ 0 & -2 & -4 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 5.

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1. (a) Consider the following system of equations:

$$x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 8x_3 + 2x_4 = 4$$

$-8 + 12 - 8t + 8t$ $x_1 = -4$

$$x_1 + 2x_2 + 4x_3 + 2x_4 = 2$$

$-4 + 6 - 4t + 4t = 2$ $x_2 = 3$

$$x_1 + 3x_2 + 6x_3 + x_4 = 5$$

$-4 + 9 - 6t + 6t$ $x_3 = t$ $x_4 =$

Obtain the reduced row echelon form of its augmented matrix and solve the system of equations.

$-4 + 9 - 6t + 6t$

(13 Marks)

(b) Suppose that matrices A, B, C and D have appropriate dimensions so that the operation $A + BCD$ is defined. If $A, C, DA^{-1}B + C^{-1}$ and $A + BCD$ are invertible, then the following result holds:

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B[DA^{-1}B + C^{-1}]^{-1}DA^{-1}.$$

$x_1 + 9 - 6t + 6t + 0 = 5$

Note: Question 1 continues on page 2.

Based on this result and by choosing $A = I_{100}$, $B = v$, $C = 1$ and $D = v^{\mathrm{T}}$, determine the sum of the first row of $[I_{100} + vv^{\mathrm{T}}]^{-1}$, where I_{100} stands for a 100×100 identity matrix and v is a 100 dimensional column vector.
with all its elements be 1

(7 Marks)

(c) Prove the result in part (b) by showing that

$$\left[A + BCD\right]\left[A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}\right] = I.$$

(5 Marks)

2. Consider the following difference equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad y = \lambda y. \qquad (\lambda - 0.5)(\lambda - 0.8). \quad \lambda^2 - 1.3\lambda + 0.3.$$

where

$$A = \begin{bmatrix} a & 0.2 \\ b & 0.8 \end{bmatrix} \qquad \begin{bmatrix} 1.2 & 0.2 \\ -0.2 & 0.8 \end{bmatrix}. \qquad \lambda^2 - 2\lambda + 1.2 \times 0.8 + 0.04. \quad \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}$$

(a) Find the eigenvalues and the corresponding eigenvectors of matrix A with

(i) $a = 1.2$ and $b = -0.2$

(ii) $a = b = 0.5$

Determine which of the two matrices is diagonalizable. Justify your answer.

(10 Marks)

(b) Consider the matrix A in part (a) that is diagonalizable.

(i) Find A^n where n is a positive integer. Hence, determine the solution of the difference equations if $x_1(0)$ and $x_2(0)$ denote the initial conditions. Show that the steady state solution is independent of the initial conditions if $[x_1(k),\ x_2(k)]^{\mathrm{T}}$ denotes a probability vector whose entries add up to one.

(10 Marks)

(ii) Given that

$$B = 6I - 20A^{-1}$$
$$C = (6I - 20A)^{-1}$$
$$x = \lambda_1 v_1 + \lambda_2 v_2 = PD$$

PD $\quad [v_1\ v_2]\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0.3 \end{bmatrix}$

Note: Question 2 continues on page 3.

$[v_1\ v_2]$

where I is an identity matrix, λ_1, λ_2 are the eigenvalues and v_1, v_2 are the corresponding eigenvectors of A. Find the values of Bx and Cx without calculating the inverse matrices.

(5 Marks)

3. (a) Given that $2y^4 e^{i\frac{\pi}{3}} - \ln\left[\cos\sqrt{3} + i\sin\sqrt{3}\right] = 1$ and considering only principal values, find the value(s) of y without the use of a calculator.

(6 Marks)

(b) Using the Cauchy-Riemann equations, discuss the differentiability and analyticity of $f(z) = \mathrm{Re}\left[e^{-iz}\right] + i\,\mathrm{Im}\left[-e^{i\bar{z}}\right]$.

(9 Marks)

(c) Evaluate $\int_C \frac{(z+1)(z^2-4z+8)-z}{z^2-4z+7}dz$, where path C is the locus of z described by $|z-2-2i| = 2$, counter-clockwise.

(10 Marks)

$(z-2)^2+3=0$

$(z-2)^2=$

4. (a) Evaluate the following:

(i) ∇f for $f(x,y,z) = x^3y^2 + 2e^z \cos y$

(ii) $\nabla \cdot \mathbf{F}$ for $\mathbf{F}(x,y,z) = 3x^2y^2\,\mathbf{i} + (2x^3y - 2e^z \sin y)\,\mathbf{j} + 2e^z \cos y\,\mathbf{k}$

(iii) $\nabla \times \mathbf{F}$ for $\mathbf{F}(x,y,z) = 3x^2y^2\,\mathbf{i} + (2x^3y - 2e^z \sin y)\,\mathbf{j} + 2e^z \cos y\,\mathbf{k}$

(6 Marks)

(b) Path C is a straight line from (1,2,3) to (2,3,4). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for each of the following:

(i) $\mathbf{F} = x\,\mathbf{i} - e^y\,\mathbf{j} + \sin z\,\mathbf{k}$

(ii) $\mathbf{F} = y\,\mathbf{i} - e^z\,\mathbf{j} + \sin x\,\mathbf{k}$

$i \quad j \quad k.$

$\frac{d}{dx} \quad \frac{d}{dy} \quad \frac{d}{dz}.$

$y \quad -e^z \quad \sin x$

(10 Marks)

Note: Question 4 continues on page 4.

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(c) An open box (open at the top) is bounded by sides at $x = 0$, $x = 1$, $y = 0$, $y = 1$ and $z = 0$. The base of the box is at $z = 0$, and the top (open) is at $z = 1$. The open box is placed in a vector field defined by

$$\mathbf{F}(x, y, z) = xy^2\ \mathbf{i} + \left(x^2 y + 3x\right)\mathbf{j} + e^{2y}\cos 5z\ \mathbf{k}.$$

(i) Evaluate $\iint\limits_S curl\ \mathbf{F} \cdot d\mathbf{A}$, where S is the combined surface of all the five sides of the open box.

(ii) The box is now closed by placing a cover at $z = 1$. Evaluate $\iint\limits_{S'} curl\ \mathbf{F} \cdot d\mathbf{A}$, where S' is now the combined surface of all the six sides of the closed box.

(9 marks)

Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c \ln z}$

(b) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(c) Cauchy-Riemann equations:

$$u_x = v_y,\ v_x = -u_y,\ \text{or } u_r = \frac{1}{r}v_\theta,\ v_r = \frac{-1}{r}u_\theta$$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\ \frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\Big|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Gauss Theorem: $\iiint_T \nabla\cdot\mathbf{F}dv = \oiint_S \mathbf{F}\cdot\mathbf{n}dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

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1. (a)

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 0 & 3 \\ 2 & 4 & 8 & 2 & 4 \\ 1 & 2 & 4 & 2 & 2 \\ 1 & 3 & 6 & 1 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 6 & 1 & 5 \\ 1 & 2 & 4 & 2 & 2 \\ 2 & 4 & 8 & 2 & 4 \\ 0 & 1 & 2 & 0 & 3 \end{array}\right] \begin{array}{l} \\ R_2 \leftarrow R_2 - R_1 \\ R_3 - 2R_1 \\ \\ \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 6 & 1 & 5 \\ 0 & -1 & -2 & 1 & -3 \\ 0 & -2 & -4 & 0 & -6 \\ 0 & 1 & 2 & 0 & 3 \end{array}\right] \begin{array}{l} \\ \\ R_3 \leftarrow R_3 - 2R_2 \\ R_4 \leftarrow R_4 + R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 6 & 1 & 5 \\ 0 & -1 & -2 & 1 & -3 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ R_4 \leftarrow R_4 + R_3 \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 6 & 1 & 5 \\ 0 & -1 & -2 & 1 & -3 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 \leftarrow R_1 + R_3 \\ R_2 \leftarrow R_2 + R_3 \\ \\ \\ \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 6 & 0 & 5 \\ 0 & -1 & -2 & 0 & -3 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_1 \leftarrow R_1 + 3R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & -1 & -2 & 0 & -3 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $x_2 = 3-2t$

$$\begin{cases} x_1 = -4 \\ x_2 = 3-2t \\ x_3 = t \\ x_4 = 0 \end{cases} \#$$

(b) Since $[A+BCD]^{-1} = A^{-1} - A^{-1}B[DA^{-1}B + C^{-1}]^{-1}DA^{-1}$

$$[I_{100} + vv^T]^{-1}$$
$$= I_{100}^{-1} - I_{100}^{-1} v [v^T I_{100} v + 1^{-1}]^{-1} v^T I_{100}$$
$$= I_{100}^{-1} - v[100+1]^{-1} v^T I_{100}$$
$$= I_{100}^{-1} - \frac{1}{101} v \cdot v^T I_{100}$$
$$= I_{100}^{-1} - \frac{100}{101} I_{100}^{-1} = \frac{1}{101} I_{100}^{-1}$$

Sum of first row $= \frac{1}{101}$ #

(c) I don't know QAQ Sorry

2. (a)

(i) $A = \begin{bmatrix} 1.2 & 0.2 \\ -0.2 & 0.8 \end{bmatrix}$

$\det(A - \lambda I) = 0$ $A - \lambda I = \begin{bmatrix} 1.2-\lambda & 0.2 \\ -0.2 & 0.8-\lambda \end{bmatrix}$ $(1.2-\lambda)(0.8-\lambda) + 0.04 = 0$

$\lambda = 1$

eigenvector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ #

not diagonalizable

(ii) $A = \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\begin{bmatrix} 0.5-\lambda & 0.2 \\ 0.5 & 0.8-\lambda \end{bmatrix}$ $\quad (0.5-\lambda)(0.8-\lambda) - 0.1 = 0$

$\lambda_1 = 1$

$\lambda_2 = 0.3$

eigenvectors

$\lambda = 1$ $\quad v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $\qquad \lambda = 0.3$ $\quad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

It is diagonalizable. and

$D = \begin{bmatrix} 1 & 0 \\ 0 & 0.3 \end{bmatrix} \quad P = \begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}$

(b)(i) $A^n = \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}^n = [PDP^{-1}]^n = PD^nP^{-1}$

$= P \begin{bmatrix} 1 & 0 \\ 0 & 0.3^n \end{bmatrix} P^{-1}$

$= \begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.3^n \end{bmatrix} P^{-1}$

$= \frac{1}{7} \begin{bmatrix} 2 & 0.3^n \\ 5 & -0.3^n \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 5 & -2 \end{bmatrix}$

$= \frac{1}{7} \begin{bmatrix} 2 + 5\times 0.3^n & 2 - 2\times 0.3^n \\ 5 - 5\times 0.3^n & 5 + 2\times 0.3^n \end{bmatrix}$

$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$

$= A^2 \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix}$

$\vdots$

$= A^{k+1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$

Steady state : $k+1 \to \infty \quad A^\infty = \frac{1}{7} \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix}$

$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$

$= \frac{1}{7} \begin{bmatrix} 2(x_1(0) + x_2(0)) \\ 5(x_1(0) + x_2(0)) \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 2 \\ \end{bmatrix}$

It is independent of initial conditions

(ii) $B = 6I - 20A^{-1}$

$x = \lambda_1 V_1 + \lambda_2 V_2$

$A = \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix} \quad V_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\because AV_1 = \lambda_1 V_1$

$AV_2 = \lambda_2 V_2$

$\therefore x = AV_1 + AV_2 = A(V_1 + V_2)$

$\therefore Bx = (6I - 20A^{-1}) A (V_1 + V_2)$

$= (6A - 20I)(V_1 + V_2)$

$= \begin{bmatrix} 3-20 & 1.2 \\ 3 & 4.8-20 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -17 & 1.2 \\ 3 & -15.2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -46.2 \\ -51.8 \end{bmatrix}$

$C = (6I - 20A)^{-1}$ (Sorry !!)

3. (a). $2y^4 e^{i\frac{\pi}{3}} - \ln[\cos\sqrt{3} + i\sin\sqrt{3}] = 1$

$\ln[\cos\sqrt{3} + i\sin\sqrt{3}]$

$= \ln[e^{i\sqrt{3}}]$

$= \sqrt{3}\, i$

$e^{i\frac{\pi}{3}} = (\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\therefore 2y^4 e^{i\frac{\pi}{3}} - \ln[\cos\sqrt{3} + i\sin\sqrt{3}] = 1$

$\Rightarrow 2y^4 (\frac{1}{2} + \frac{\sqrt{3}}{2} i) - \sqrt{3} i = 1$

$(1+\sqrt{3}i) y^4 = 1 + \sqrt{3} i \qquad y^4 = 1$

$y = \sqrt[4]{1} \qquad 1 = e^{0i}$

$= \sqrt[4]{1} \not\ \frac{0+2k\pi}{4} \qquad k = -2, -1, 0, 1$

$\therefore y = e^{-\pi i},\ e^{-\frac{\pi}{2}i},\ 1,\ e^{\frac{\pi}{2}i}$ #

(b) $f(z) = Re[e^{-iz}] + i\,Im[-e^{i\bar{z}}]$

Let $z = x + iy$ then $\bar{z} = x - iy$

$Re[e^{-iz}]$
$= Re[e^{-i(x+iy)}]$
$= Re[e^{-ix+y}]$
$= Re[e^{y}(\cos x - i\sin x)]$
$= e^{y}\cos x$

$Im[-e^{i\bar{z}}]$
$= Im[-e^{i(x-iy)}]$
$= Im[-e^{ix+y}]$
$= Im[-e^{y}(\cos x + i\sin x)]$
$= -e^{y}\sin x$

$\therefore f(z) = e^{y}\cos x + i(-e^{y}\sin x)$

$u(x,y) = e^{y}\cos x \qquad v(x,y) = -e^{y}\sin x$

$u_x = -e^{y}\sin x \qquad v_x = -e^{y}\cos x$

$-u_y = -e^{y}\cos x \qquad v_y = -e^{y}\sin x$

Since $u_x = v_y$

$-u_y = v_x$ for every x and y

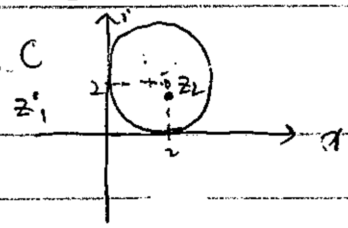
$\therefore$ It's differentiable and analytic everywhere.

(c) $\int_C \frac{(z+1)(z^2-4z+8)-z}{z^2-4z+7}\,dz = \int_C \frac{(z+1)(z^2-4z+8)-z}{(z-\sqrt{3}i+2)(z-\sqrt{3}i-2)}\,dz$

$z_1 = \sqrt{3}i - 2$

$z_2 = \sqrt{3}i + 2$

path C



z_2 is within the locus.

$\int_C \frac{\overbrace{[(z+1)(z^2-4z+8)-z]/(z-\sqrt{3}i+2)}^{f(z)}}{(z-\sqrt{3}i-2)} = 2\pi i\, f(z_0)$, $z_0 = \sqrt{3}i + 2$

$= 2\pi i \times \frac{1}{4} = \frac{\pi}{2} i$ #

4\. (a)

(i) $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

$= (3x^2y^2) i + (2x^3y - 2e^z \sin y) j + (2e^z \cos y) k$ #

(ii) $\nabla \cdot F = (\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k) \cdot (F_1 i + F_2 j + F_3 k)$

$= 6xy^2 + 2x^3 - 2e^z \cos y + 2e^z \cos y$

$= 6xy^2 + 2x^3$ #

(iii) $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^2 & 2x^3y - 2e^z \sin y & 2e^z \cos y \end{vmatrix}$

$= (6x^2y - 6x^2y) i - (0 - 0) j + (6x^2y - 6x^2y) k$

$= 0$ #

(b) (i) ~~$\vec{F}$ is a conservative field since $\nabla \times \vec{F} = 0$~~

$\vec{F} = \nabla V = (\frac{\partial V}{\partial x} i + \frac{\partial V}{\partial y} j + \frac{\partial V}{\partial z} k)$

$\frac{\partial F_1}{\partial x} = x \quad \frac{\partial F_2}{\partial y} = -e^y \quad \frac{\partial F_3}{\partial z} = \sin z$

$\therefore F_1 = \frac{1}{2}x^2 \quad F_2 = -e^y \quad F_3 = -\cos z$

$\therefore V = \frac{1}{2}x^2 - e^y - \cos z$

$\therefore \int_C \vec{F} = V(B) - V(A) = 2 - e^3 - \cos 4 - (\frac{1}{2} - e^2 - \cos 3)$

$= \frac{3}{2} - (e^3 - e^2) - (\cos 4 - \cos 3)$ #

(ii) $A = (1,2,3) \quad B = (2,3,4)$

$\vec{AB} = (1,1,1)$

line $\Rightarrow (x,y,z) = (1,2,3) + t(1,1,1)$

$$\begin{cases} x_1(t) = 1+t \\ y(t) = 2+t \\ z(t) = 3+t \end{cases}$$

$\vec{F} = (2+t)i - e^{3+t}j + \sin(1+t)k \qquad \vec{r}(t) = (1+t)i + (2+t)j + (3+t)k$

$\frac{d\vec{r}}{dt} = i + j + k$

$\int_C \vec{F}\cdot d\vec{r}$

$= \int_0^1 [(2+t)i - e^{3+t}j + \sin(1+t)k][i+j+k]\,dt$

$= \int_0^1 (2+t) - e^{3+t} + \sin(1+t)\,dt$

$= 2t + \frac{1}{2}t^2 - e^{3+t} - \cos(1+t)\Big|_0^1$

$= 2 + \frac{1}{2} - e^4 - \cos 2 - e^3 - \cos 1$

$= \frac{5}{2} - (e^4 + e^3) - (\cos 1 + \cos 2)$ #

(c) (i) According to Stokes Theorem

$$\iint_S (\nabla\times\vec{F})\cdot\hat{n}\,dA = \oint_C \vec{F}\cdot d\vec{r}$$

$r: (0,0,1) \xrightarrow{①} (0,1,1) \xrightarrow{②} (1,1,1) \xrightarrow{③} (1,0,1) \xrightarrow{④} (0,0,1)$

For Path ①

$x=0 \quad y=t \quad z=1$

$\vec{F} = e^{2t}\cos 5\,k \qquad \vec{r}(t) = tj + k \qquad 0 \le t \le 1$

$d\vec{r} = j$

$\int \vec{F}\cdot d\vec{r} = \int 0\,dt = 0$

For Path ②

$x = t \quad y = 1 \quad z = 1 \quad 0 < t \le 1$

$\vec{F} = ti + (t^2+3t)j + e^2\cos 5\,k \qquad \vec{r} = ti + j + k \qquad d\vec{r} = i$

$\int \vec{F}\cdot d\vec{r} = \int_0^1 (t)\,dt = \frac{1}{2}t^2\Big|_1^1 = \frac{1}{2}$

For Path (3)

$x = 1$

$y = t$ t from 1 to 0

$z = 1$

$\vec{r} = i + tj + k \qquad d\vec{r} = j$

$\vec{F} = t^2 i + (t+3)j + e^{2t}\cos 5\, k$

$$\int_1^0 \vec{F}\, d\vec{r} = \int_1^0 (t+3)\, dt = \frac{1}{2}t^2 + 3t \Big|_1^0 = -\frac{1}{2} - 3 = -\frac{7}{2}$$

For Path (4)

x from 1 to 0 $x = t$

$y = 0$

$z = 1$

$\vec{r} = ti + k \qquad d\vec{r} = i$

$$\int \vec{F} \cdot dr = \int_1^0 x^2 y\, dt = 0$$

$$\therefore \oint_C \vec{F} \cdot d\vec{r} = \frac{1}{2} - \frac{7}{2} = -3 \ \#$$

(ii)

$$\iint_{S'} \text{curl}\, \vec{F} \cdot d\vec{A} = 0 \ \#$$