Lecture 6: Electromagnetic Principles and Actuators

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the principles of electromechanical energy conversion and the operations of electromechanical devices, i.e., convert electrical energy into mechanical energy and vice versa.
- Apply the Law of Conservation of Energy to establish the energy balance equation.
- Employ differential calculus to derive the incremental analysis equation.
- Apply Constant Flux and Constant Current methods to evaluate the magnetic force with respect to the magnetic field energy.

Principles of Electromechanical Energy Conversion

- Electromechanical devices (e.g., machines) convert electrical energy into mechanical energy and vice versa. Most of these devices utilise magnetic field as a medium.

- Conservation of energy has to be satisfied by all these processes.

$$W_i = W_o + W_f + W_\ell$$

where
W_i is the input (electrical) energy,
W_o is the output (mechanical) energy,
W_f is the field or stored energy, and
W_ℓ is the energy lost in the system.

Principles of Electromechanical Energy Conversion

- The flow of energy in the process is shown in Fig. 51. This process is reversible except for the losses.

- Ignoring losses, the energy balance equation is reduced to

$$W_i = W_o + W_f$$

- Incremental analysis between two states gives

$$dW_i = dW_o + dW_f$$

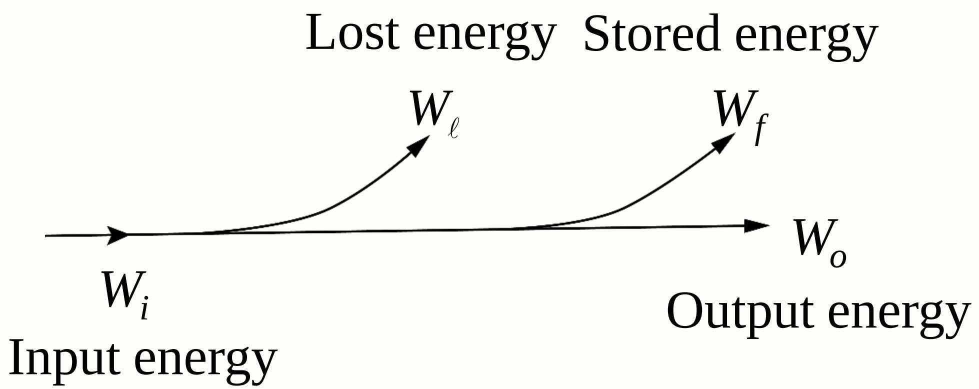

Fig. 51. Energy flow diagram.

Production of Force

- Consider an electromagnetic system with one fixed and one movable part separated by a gap x as shown in Fig. 52.

- If the total reluctance is $\Re_1$, the flux in the structure for an input current i is given by

$$\varphi = \frac{Ni}{\Re_1}$$

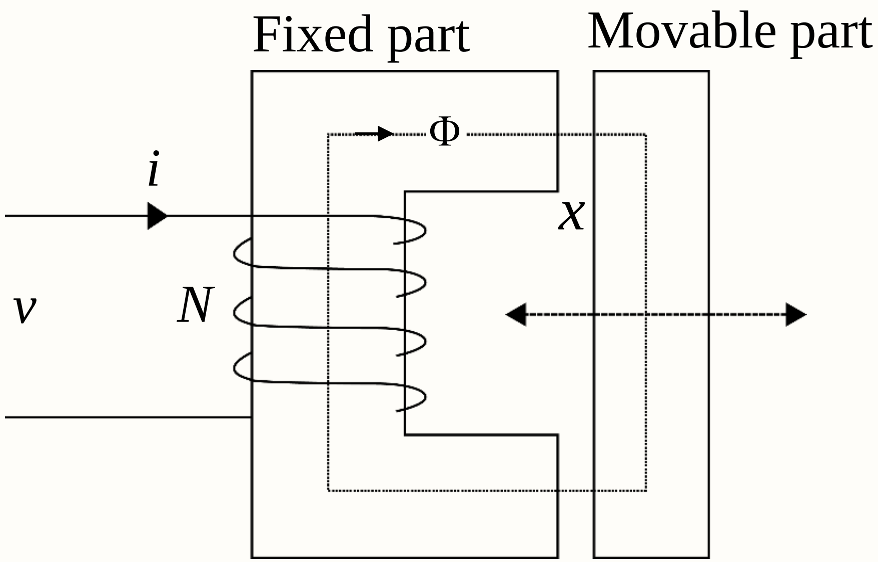

Fig. 52. Electromagnetic system.

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Production of Force

- The relationship between Ni and φ is a straight line (1) with slope $1/\mathfrak{R}_1$ as shown in Fig. 53. If the system is operating at point $\boldsymbol{p_1}$ with input current i_1 and flux φ_1, the field energy is

$$W_f = \frac{1}{2} N i_1 \varphi_1$$

which is the area indicated in the diagram.

- Let the force experienced by the movable part at a gap distance of $\boldsymbol{x}$ be F_m, and it moves towards the fixed part by an incremental distance of $\boldsymbol{dx}$. The incremental energy output is $dW_o = F_m dx$.

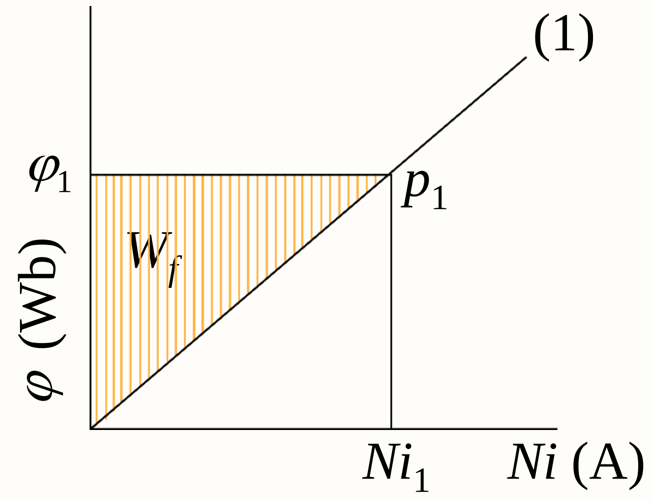

Fig. 53. Field energy diagram.

Production of Force

- In this process, since the air gap has decreased, the reluctance should be reduced to, say $\mathfrak{R}_2$, ($\mathfrak{R}_2 < \mathfrak{R}_1$) and the relationship between the flux and the mmf is given by

$$\varphi = \frac{Ni}{\mathfrak{R}_2}$$

which is a straight line with slope $1/\mathfrak{R}_2$ and can be represented by the line (2) in Fig. 54.

- The operating point has to move from p_1 to somewhere in line (2). Consequently, there will be changes in various energy components: input energy and stored energy.

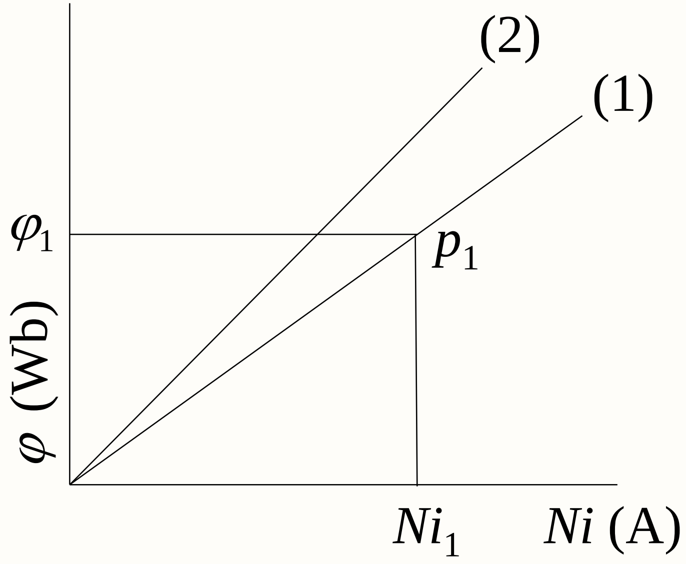

Fig. 54. Field energy diagram.

Production of Force

- Incremental analysis of the various energy components in this process gives

$$dW_i = dW_o + dW_f \Rightarrow dW_0 = dW_i - dW_f$$
$$\Rightarrow F_m dx = dW_i - dW_f$$

- This relationship can be used to derive the expression for the force F_m by evaluating the energy components dW_i and dW_f corresponding to the new operating point in line (2). This will be done for two different processes.

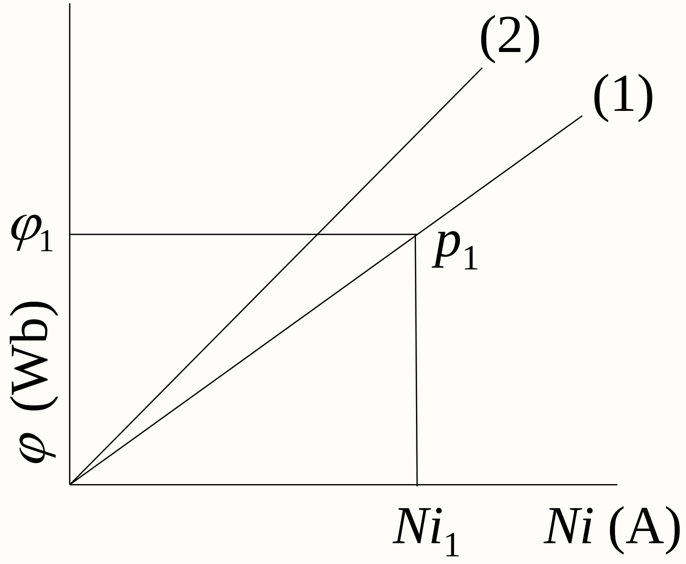

Fig. 54. Field energy diagram.

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Method 1 – Constant Flux

- If the flux φ is held constant while the movable part moves a distance dx under the force F_m, the operating point changes from p_1 to p_2 (see Fig. 55) Then,

  $$dW_i = id\lambda = Nid\varphi = Ni(\varphi_2 - \varphi_1) = 0$$

  ($\varphi_2 = \varphi_1$ since the flux is held constant.)

  Therefore, $F_m dx = dW_i - dW_f = -dW_f$

  $$\Rightarrow F_m = -\frac{dW_f}{dx}$$

- The force developed is proportional to the rate of decrement of the stored energy.

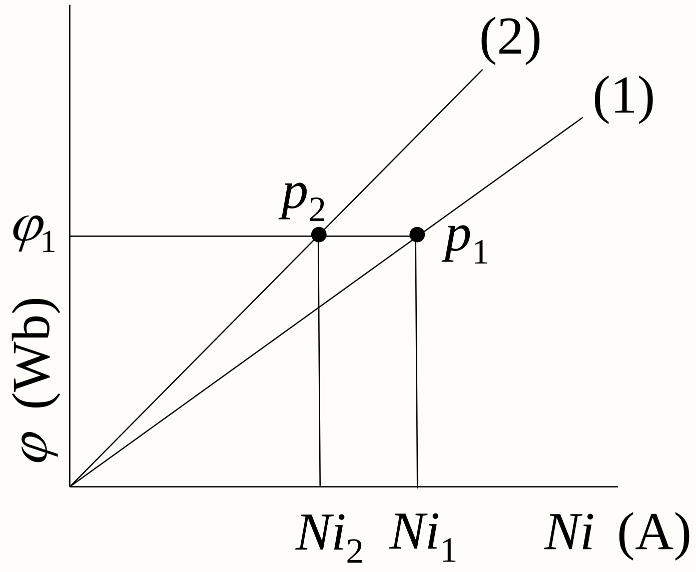

Fig. 55. Field energy diagram.

Method 2 – Constant Current

❖ If the input current is held constant during the process, the operating point moves from p_1 to p_3 and the flux changes from φ_1 to φ_2 in Fig. 56, so that

$$d\varphi = \varphi_2 - \varphi_1$$

Under this condition,

a) $dW_i = id\lambda = Ni_1 d\varphi = Ni_1(\varphi_2 - \varphi_1)$

b) $dW_f = W_{f2} - W_{f1} = \frac{1}{2}Ni_1\varphi_2 - \frac{1}{2}Ni_1\varphi_1$

$$= \frac{1}{2}Ni_1(\varphi_2 - \varphi_1) = \frac{1}{2}dW_i$$

Thus, $dW_i = 2dW_f$

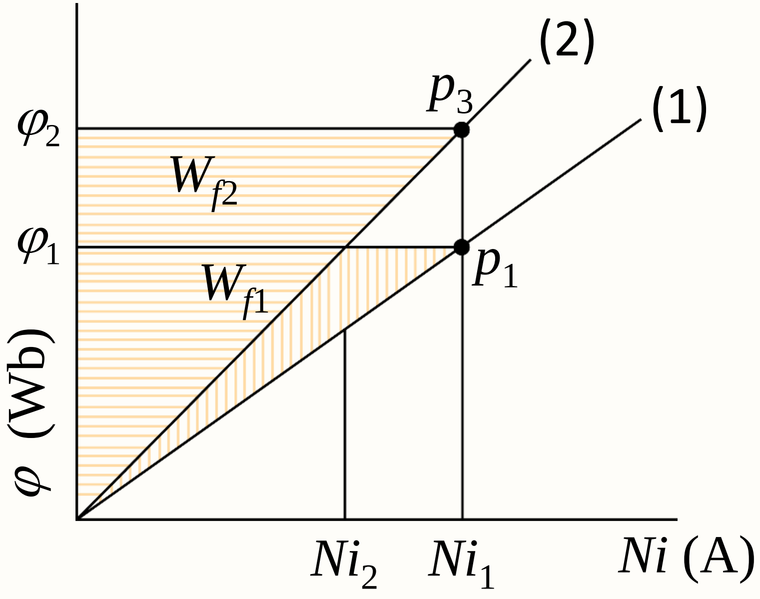

Fig. 56. Field energy diagram.

Method 2 – Constant Current

Therefore,

$$F_m dx = dW_i - dW_f = 2dW_f - dW_f = dW_f$$

and, $F_m = \dfrac{dW_f}{dx}$

Thus, the force developed is proportional to the rate of increment of the stored energy.

Constant Flux and Constant Current

- In both the cases, the evaluation of force F_m requires the expression for the magnetic field energy W_f , which may be a function of either
  - flux $\boldsymbol{\varphi}$ and gap distance $\boldsymbol{x}$, or
  - current $\boldsymbol{i}$ and gap distance $\boldsymbol{x}$.
- When the flux is held constant, $F_m = -\dfrac{dW_f}{dx} = -\dfrac{\partial W_f(\varphi, x)}{\partial x}$

  Since $W_f = \dfrac{1}{2}\varphi^2 \Re$, this approach usually takes the form,

$$F_m = -\frac{dW_f}{dx} = -\frac{\partial W_f(\varphi, x)}{\partial x} = -\frac{\partial\left[\frac{1}{2}\varphi^2 \Re(x)\right]}{\partial x} = -\frac{1}{2}\varphi^2 \frac{d\Re(x)}{dx}$$

Constant Flux and Constant Current

❖ When the current is held constant,

$$F_m = \frac{dW_f}{dx} = \frac{\partial W_f(i,x)}{\partial x}$$

Since $W_f = \frac{1}{2} i^2 L$, this approach usually takes the form,

$$F_m = \frac{dW_f}{dx} = \frac{\partial W_f(i,x)}{\partial x} = \frac{\partial \left[\frac{1}{2} i^2 L(x) \right]}{\partial x} = -\frac{1}{2} i^2 \frac{dL(x)}{dx}$$

Example 9

Determine the minimum amount of current required to keep the magnetic plate at a distance of 1 mm from the pole faces of the fixed electromagnet having 1000 turns when the force exerted by the spring is 100 N as shown in Fig. 57. Each pole face cross-sectional area is 9 cm^2. Ignore the reluctances of the core material and magnetic plate.

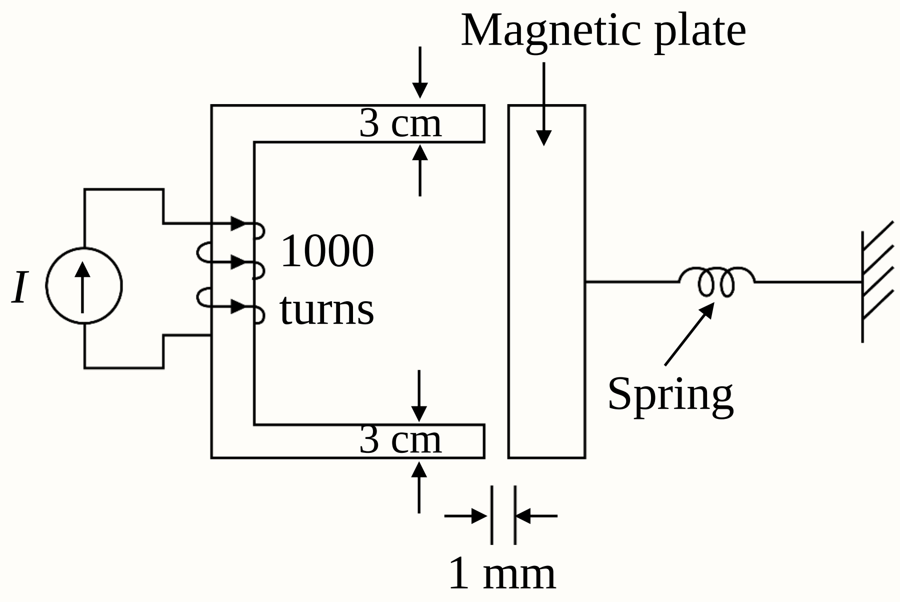

Fig. 57. Electromagnet System.

(Solutions →)

Example 9 – Solutions

The magnetic equivalent circuit can be drawn as shown in Fig. 58. For a gap x, the total reluctance is

$$\mathcal{R}_{eq} = \frac{2x}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 1.768 \times 10^{9} x \text{ H}^{-1}$$

$$L = \frac{N^2}{\mathcal{R}_{eq}} = \frac{1000^2}{1.768 \times 10^{9} x} = \frac{565.5 \times 10^{-6}}{x} \text{ H}$$

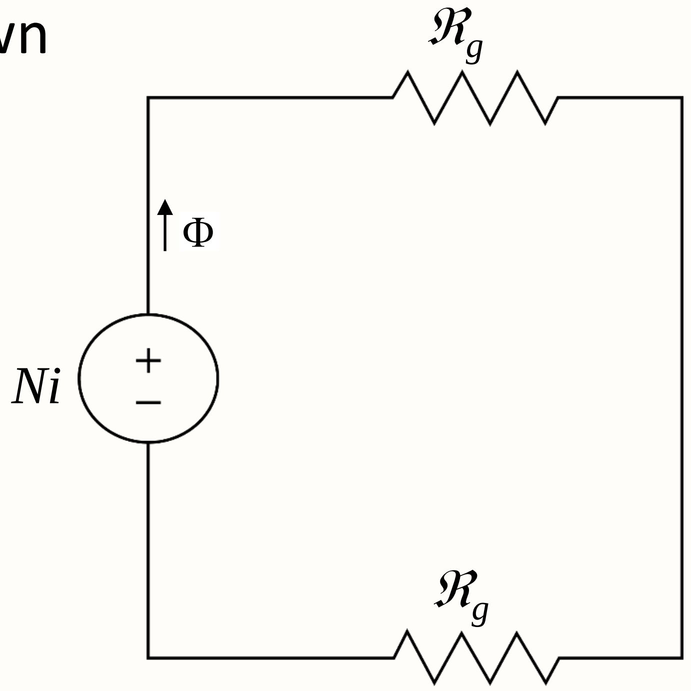

Fig. 58. Magnetic equivalent circuit.

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Example 9 – Solutions

a) If the flux is held constant, the field energy can be expressed as

$$W_f(\varphi, x) = \frac{1}{2}\varphi^2 \Re(x)$$

Then, $F_m = -\frac{dW_f}{dx} = -\frac{\partial\left[\frac{1}{2}\varphi^2 \Re(x)\right]}{\partial x} = -\frac{1}{2}\varphi^2 \frac{d\Re(x)}{dx} = -\frac{1}{2}\varphi^2 \times 1.768\times 10^9$

$$\Rightarrow \frac{1}{2}\varphi^2 \times 1.768\times 10^9 = 100 \text{ N}, \text{ at } x = 1 \text{ mm}$$

$$\Rightarrow \varphi = 0.3363\times 10^{-3} \text{ Wb}, \text{ at } x = 1 \text{ mm}$$

Also, $\varphi = \frac{NI}{\Re} = \frac{1000I}{1.768\times 10^9 \times 10^{-3}}$, at $x = 1$ mm

Therefore, $\frac{1000I}{1.768\times 10^9 \times 10^{-3}} = 0.3363\times 10^{-3} \Rightarrow I = 0.595 \text{ A}$

Example 9 – Solutions

b) If the current is held constant, then

$$W_f(I,x) = \frac{1}{2} I^2 L(x)$$

so that

$$F_m = \frac{dW_f}{dx} = \frac{\partial\left[\frac{1}{2} I^2 L(x)\right]}{\partial x} = \frac{1}{2} I^2 \frac{dL(x)}{dx} = -\frac{1}{2} I^2 \times \frac{565.5\times 10^{-6}}{x^2} \ \mathrm{N}$$

$$\Rightarrow \frac{1}{2} I^2 \times \frac{565.5\times 10^{-6}}{x^2} = 100\ \mathrm{N}, \text{at } x = 1\ \mathrm{mm}$$

$$\Rightarrow \frac{1}{2} I^2 \times \frac{565.5\times 10^{-6}}{10^{-6}} = 100\ \mathrm{N} \Rightarrow I = 0.595\ \mathrm{A}$$

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Summary

In this lecture, you have learnt:

- The principles of electromechanical energy conversion and the operations of electromechanical devices, e.g., convert electrical energy into mechanical energy.
- The Law of Conservation of Energy to establish the energy balance equation.
- Constant Flux and Constant Current methods to evaluate the magnetic force with respect to the magnetic field energy.

References

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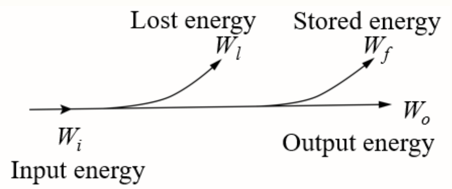
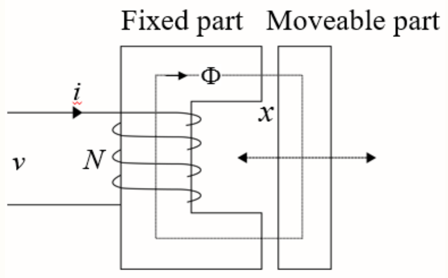
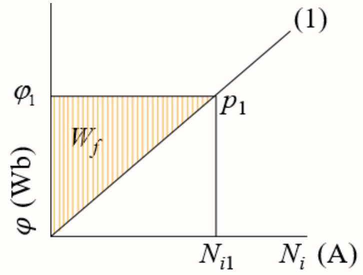
1	5		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 140), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
2	6		Reprinted from *AC Circuits and Machines*, (p. 121), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
3	7		Reprinted from *AC Circuits and Machines*, (p. 121), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.

References

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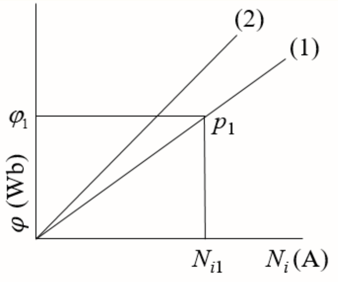
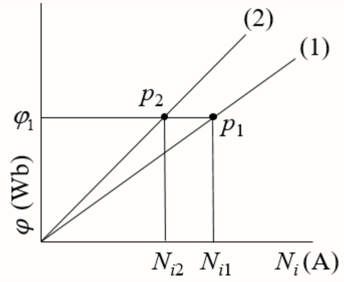
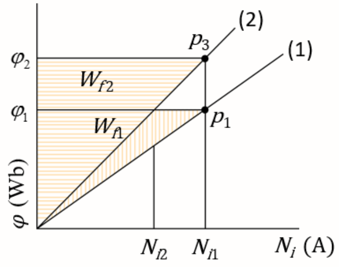
4	8 and 9		Reprinted from *AC Circuits and Machines*, (p. 122), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
5	10		Reprinted from *AC Circuits and Machines*, (p. 123), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
6	11		Reprinted from *AC Circuits and Machines*, (p. 124), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.

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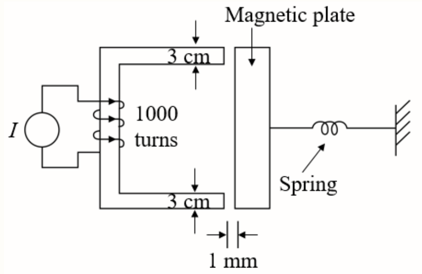
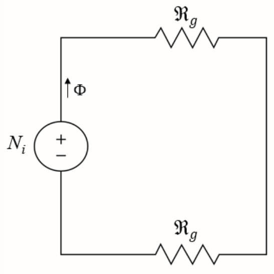
7	15		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 140), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
8	16		Reprinted from *AC Circuits and Machines*, (p. 126), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.