TUTORIAL 8 PN Junction under an External Voltage Bias

1. An abrupt Si pn junction is doped uniformly with 1×10^{16} cm^{-3} impurity on the n-side and 1×10^{17} cm^{-3} impurity on the p-side. The cross-sectional area of the pn junction is 1×10^{-4} cm^{2}. Calculate the following parameters under (i) a forward bias of 0.65 V and (ii) a reverse of 2 V. Draw the energy band diagram under both biasing conditions, indicating clearly the extent of the band bending.

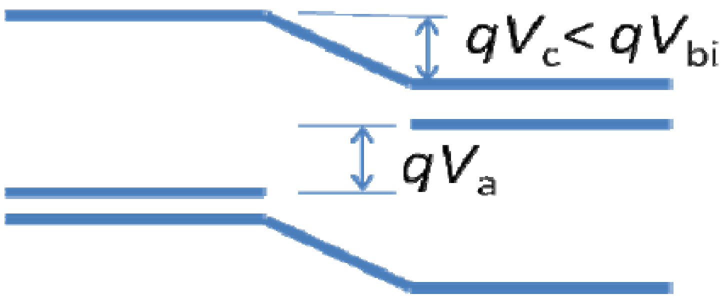
   a. contact potential;
   b. the total depletion width;
   c. the depletion width in the n- and p-side;
   d. the space charge in each side of the depletion region;
   e. peak electric field.

[0.104 V; 2.75 V; 0.122 μm; 0.628 μm; 0.111 μm; 0.011 μm; 1.76×10^{-12} C; -1.69×10^{4} V/cm; 0.571 μm; 0.057 μm; 9.12×10^{-12} C; -8.73×10^{4} V/cm]

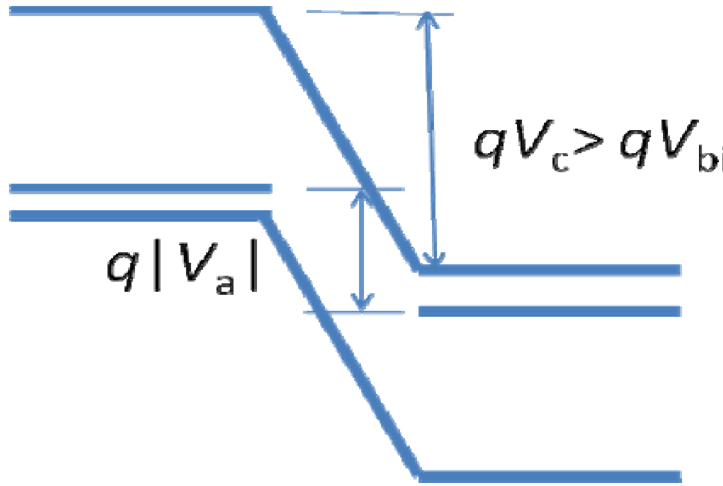
a. contact potential

Energy Band Diagrams

Forward Bias



Reverse Bias



Built-in potential:

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259 \ln\left[\frac{1\times10^{17}\times1\times10^{16}}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.754\ \mathrm{V}$$

Under forward bias, $V_c = 0.754 - 0.65 = 0.104$ V

Under reverse bias, $V_c = 0.754 + 2 = 2.75$ V

b. the total depletion width

Under forward bias,

$$W_{\mathrm{FB}} = \left[\frac{2\varepsilon_{\mathrm{r}}\varepsilon_0}{q} \left(V_{\mathrm{bi}} - V_{\mathrm{a}} \right) \left(\frac{1}{N_{\mathrm{a}}} + \frac{1}{N_{\mathrm{d}}} \right) \right]^{1/2}$$

$$= \left[\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 0.104}{1.6 \times 10^{-19}} \left(\frac{1}{1 \times 10^{17}} + \frac{1}{1 \times 10^{16}} \right) \right]^{1/2}$$

$$= 1.22 \times 10^{-5} \text{ cm} \quad \text{or } 0.122\ \mu\text{m}$$

Under reverse bias,

$$W_{\mathrm{RB}} = \left[\frac{2\varepsilon_{\mathrm{r}}\varepsilon_0}{q} \left(V_{\mathrm{bi}} + |V_{\mathrm{a}}| \right) \left(\frac{1}{N_{\mathrm{a}}} + \frac{1}{N_{\mathrm{d}}} \right) \right]^{1/2}$$

$$= \left[\frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 2.75}{1.6 \times 10^{-19}} \left(\frac{1}{1 \times 10^{17}} + \frac{1}{1 \times 10^{16}} \right) \right]^{1/2}$$

$$= 6.28 \times 10^{-5} \text{ cm} \quad \text{or } 0.628\ \mu\text{m}$$

c. Find x_n and x_p

Charge neutrality: $x_p N_a = x_n N_d$ (1)

Geometrically, $x_p + x_n = W$ (2)

From (1) and (2),

$$x_n = \left(\frac{N_a}{N_a + N_d}\right) W$$

$$x_p = \left(\frac{N_d}{N_a + N_d}\right) W$$

d. Space charge: $Q_{sc} = qN_A x_p A = qN_D x_n A$

e. Max. electric field: $\xi_{max} = -\frac{qN_A x_p}{\varepsilon_r \varepsilon_0} = -\frac{qN_D x_n}{\varepsilon_r \varepsilon_0}$

(negative sign means electric field is pointing in the negative x-direction)

Under forward bias,

$$x_n = \left(\frac{1 \times 10^{17}}{1 \times 10^{17} + 1 \times 10^{16}} \right) \times 0.122$$

$$= 0.111\ \mu\text{m}$$

$$x_p = W - x_n = 0.011\ \mu\text{m}$$

Space charge on either side of the junction,

$$Q = qN_a x_p A = qN_d x_n A = 1.76 \times 10^{-12}\ \text{C}$$

Peak electric field (at metallurgical junction),

$$\xi_m = -\frac{qN_a x_p}{\varepsilon_r \varepsilon_0} = -\frac{qN_d x_n}{\varepsilon_r \varepsilon_0} = -1.69 \times 10^4\ \text{V/cm}$$

Under reverse bias,

$$x_{\mathrm{n}} = \left(\frac{1 \times 10^{17}}{1 \times 10^{17} + 1 \times 10^{16}} \right) \times 0.628$$

$$= 0.571\ \mu\mathrm{m}$$

$$x_{\mathrm{p}} = W - x_{\mathrm{n}} = 0.057\ \mu\mathrm{m}$$

Space charge on either side of the junction,

$$Q = qN_{\mathrm{a}} x_{\mathrm{p}} A = qN_{\mathrm{d}} x_{\mathrm{n}} A = 9.12 \times 10^{-12}\ \mathrm{C}$$

Peak electric field (at metallurgical junction),

$$\xi_{\mathrm{m}} = -\frac{qN_{\mathrm{a}} x_{\mathrm{p}}}{\varepsilon_{\mathrm{r}} \varepsilon_0} = -\frac{qN_{\mathrm{d}} x_{\mathrm{n}}}{\varepsilon_{\mathrm{r}} \varepsilon_0} = -8.73 \times 10^{4}\ \mathrm{V/cm}$$

2. A silicon p-n junction diode has the following parameters at 300 K, $D_n = 25\ \text{cm}^2/\text{sec}$, $D_p = 10\ \text{cm}^2/\text{sec}$, $\tau_n = \tau_p = 0.5\ \mu\text{s}$, $\varepsilon_r = 11.7$, $n_i = 1.5\times10^{10}\ /\text{cm}^3$. Given that the electron diffusion current density is 20 A/cm² and the hole diffusion current density is 5 A/cm² at a forward bias of 0.65 V, determine the doping densities of the p- and the n-regions of the diode.

($N_a = 10^{15}\ /\text{cm}^3$ and $N_d = 2.5\times10^{15}/\text{cm}^3$)

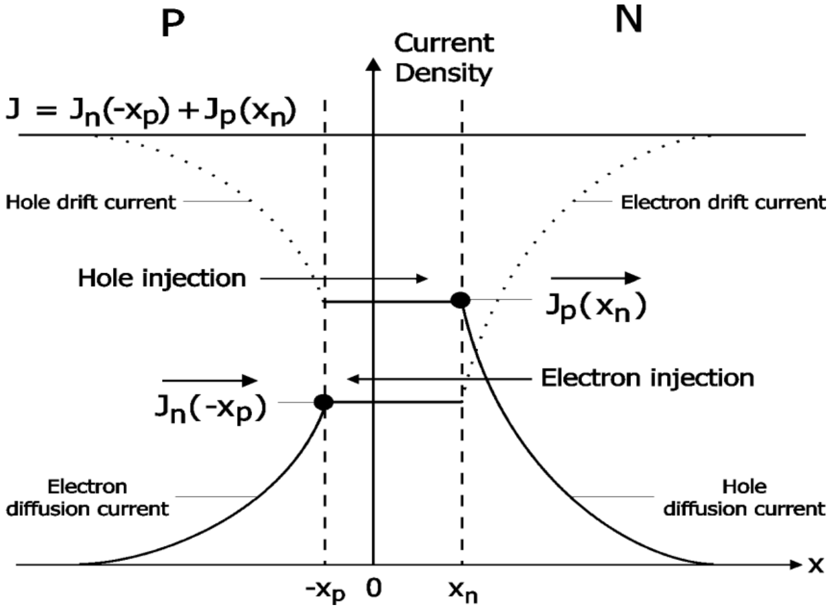
$$J_n(-x_p) = 20\ \text{A/cm}^2$$

$$J_p(x_n) = 5\ \text{A/cm}^2$$

Determine N_a

Use J_n to find N_a as Jn is the minority carrier current density in p-side

$$J_n(-x_p) = q\frac{D_n}{L_n}\Delta n_p(-x_p)$$



$$\therefore \mathrm{J}_{\mathrm{n}} = q\frac{D_{\mathrm{n}}}{L_{\mathrm{n}}} n_{\mathrm{po}}\left(\exp\frac{qV}{kT} - 1\right); \qquad L_{\mathrm{n}} = \sqrt{D_{\mathrm{n}}\tau_{\mathrm{n}}}$$

$$= q\sqrt{\frac{D_{\mathrm{n}}}{\tau_{\mathrm{n}}}}\frac{n_i^2}{N_a}\left(\exp\frac{qV}{kT} - 1\right) \qquad (2)$$

$$20 = 1.6\times10^{-19}\sqrt{\frac{25}{5x10^{-7}}}\frac{2.25\times10^{20}}{N_a}\left(\exp\frac{0.65}{0.026} - 1\right)$$

$$\Rightarrow N_a = 1.0\times10^{15}\,cm^{-3}$$

Determine N_d

Similarly,

$$J_p = q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_d}\left(\exp\frac{qV}{kT} - 1\right)$$

$$5 = 1.6\times10^{-19}\sqrt{\frac{10}{5x10^{-7}}}\frac{2.25\times10^{20}}{N_d}\left(\exp\frac{0.65}{0.026} - 1\right)$$

$$\Rightarrow N_d = 2.55\times10^{15}\,cm^{-3}$$

3. Show that the ratio of the hole and electron currents injected across an ideal pn junction is given by

$$\frac{J_p(x = x_n)}{J_n(x = -x_p)} = \frac{L_n \sigma_p}{L_p \sigma_n}$$

where σ_n and σ_p are the conductivities of the n and p regions, respectively; You may assume the base lengths to be much longer than the respective minority carrier diffusion lengths.

What is the implication if the **p-**region is doped much more heavily than the **n-**region?

Minority carrier (diffusion) current densities in a long-base pn junction:

$$J_\mathrm{p}(x = x_\mathrm{n}) = \frac{qD_\mathrm{p} p_\mathrm{no}}{L_\mathrm{p}}\left[e^{qV_\mathrm{a}/(k_\mathrm{B}T)} - 1\right] \quad \text{(for holes)} \qquad \text{(1a)}$$

$$J_\mathrm{n}(x = -x_\mathrm{p}) = \frac{qD_\mathrm{n} n_\mathrm{po}}{L_\mathrm{n}}\left[e^{qV_\mathrm{a}/(k_\mathrm{B}T)} - 1\right] \quad \text{(for electrons)} \qquad \text{(1b)}$$

x_n and x_p are the space charge region widths on the n- and p-type sides.

Dividing (1a) by (1b),

$$\frac{J_\mathrm{p}(x = x_\mathrm{n})}{J_\mathrm{n}(x = -x_\mathrm{p})} = \left(\frac{D_\mathrm{p} p_\mathrm{no}}{L_\mathrm{p}}\right)\left(\frac{L_\mathrm{n}}{D_\mathrm{n} n_\mathrm{po}}\right) \qquad (2)$$

Assuming complete ionization of dopants and noting that N_D , N_A are >> n_i, in the n and p regions, respectively

$$p_\mathrm{no} = \frac{n_\mathrm{i}^2}{n_\mathrm{no}} \approx \frac{n_\mathrm{i}^2}{N_\mathrm{D}} \quad \text{for the n region} \qquad \text{(3a)}$$

$$n_\mathrm{po} = \frac{n_\mathrm{i}^2}{p_\mathrm{po}} \approx \frac{n_\mathrm{i}^2}{N_\mathrm{A}} \quad \text{for the p region} \qquad \text{(3b)}$$

Substituting (3a) and (3b) into (2),

$$\frac{J_\mathrm{p}(x = x_\mathrm{n})}{J_\mathrm{n}(x = -x_\mathrm{p})} = \frac{D_\mathrm{p} L_\mathrm{n} N_\mathrm{A}}{D_\mathrm{n} L_\mathrm{p} N_\mathrm{D}}$$

Recall that $D_\mathrm{p} = \mu_\mathrm{p} k_\mathrm{B} T / q$ and $D_\mathrm{n} = \mu_\mathrm{n} k_\mathrm{B} T / q$ (Einstein's relations),

$$\frac{J_\mathrm{p}\left(x = x_\mathrm{n}\right)}{J_\mathrm{n}\left(x = -x_\mathrm{p}\right)} = \frac{\mu_\mathrm{p} L_\mathrm{n} N_\mathrm{A}}{\mu_\mathrm{n} L_\mathrm{p} N_\mathrm{D}} = \frac{L_\mathrm{n}\left(q\mu_\mathrm{p} N_\mathrm{A}\right)}{L_\mathrm{p}\left(q\mu_\mathrm{n} N_\mathrm{D}\right)}$$

$$\therefore \quad \frac{J_\mathrm{p}\left(x = x_\mathrm{n}\right)}{J_\mathrm{n}\left(x = -x_\mathrm{p}\right)} = \frac{L_\mathrm{n}\sigma_\mathrm{p}}{L_\mathrm{p}\sigma_\mathrm{n}} \qquad (4)$$

If $N_\mathrm{A} >> N_\mathrm{D}$, $\sigma_\mathrm{p} >> \sigma_\mathrm{n}$. We may conclude from (4) that

$$J_\mathrm{p}\left(x = x_\mathrm{n}\right) >> J_\mathrm{n}\left(x = -x_\mathrm{p}\right)$$

since L_p and L_n are typically comparable.

Total current density:

$$J = J_\mathrm{p}\left(x = x_\mathrm{n}\right) + J_\mathrm{n}\left(x = -x_\mathrm{p}\right) \approx J_\mathrm{p}\left(x = x_\mathrm{n}\right)$$

The current across the p$^+$n junction is largely made up of the hole diffusion current in the quasi-neutral n region. Conversely, for a n$^+$p junction, the current is mainly made up of the electron diffusion current in the quasi-neutral p region.

Q4 Consider two ideal p-n junction diodes A and B at 300 K with the same diode current of 15 mA.

- Calculate the forward bias voltages applied if the reverse saturation current I_s is (a) 5 μA for diode A and (b) 8 pA for diode B.
- If the diodes are made of Si and GaAs, which one corresponds to diode A? Explain briefly.

(0.2 V, 0.53 V)

Diode current is given as:

$$I = I_s\left(\exp\frac{qV_F}{kT} - 1\right) \Rightarrow V_F = \frac{kT}{q}\ln\left(1 + \frac{I}{I_s}\right)$$

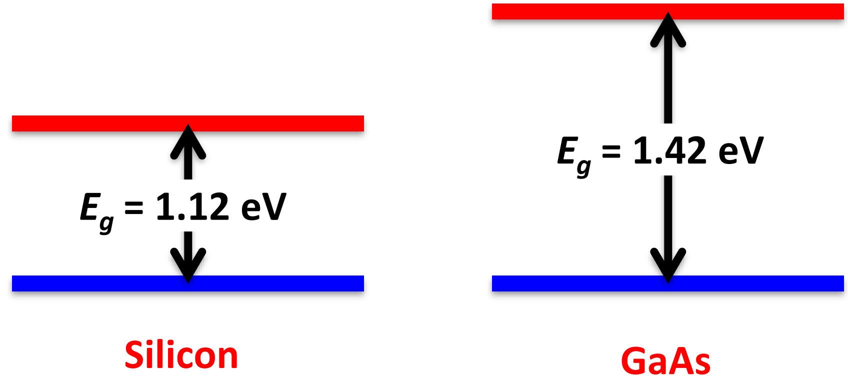
$$\text{For } I_s = 5\ \mu\text{A},\ V_F = 0.0259.\ln\left(1 + \frac{15\times10^{-3}}{5\times10^{-6}}\right) = 0.2\ \text{V}$$

$$\text{For } I_s = 8\ \text{pA},\ V_F = 0.0259.\ln\left(1 + \frac{15\times10^{-3}}{8\times10^{-12}}\right) = 0.53\ \text{V}$$

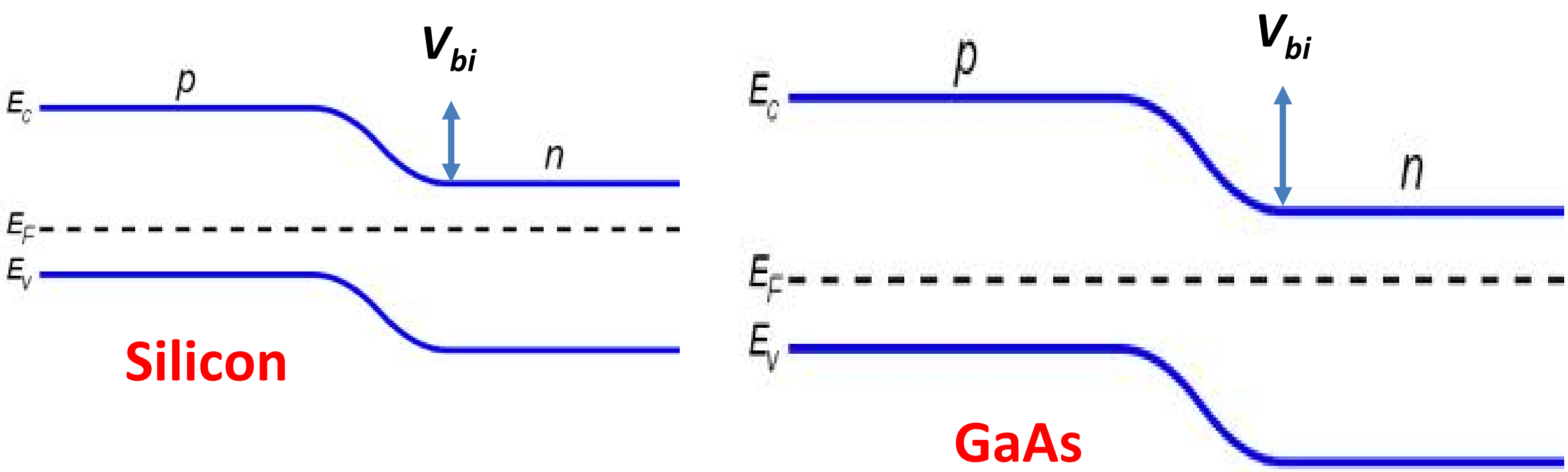
$Note: V_F \downarrow as\ I_s \uparrow$ for the same current.

WHY? What's the physics?

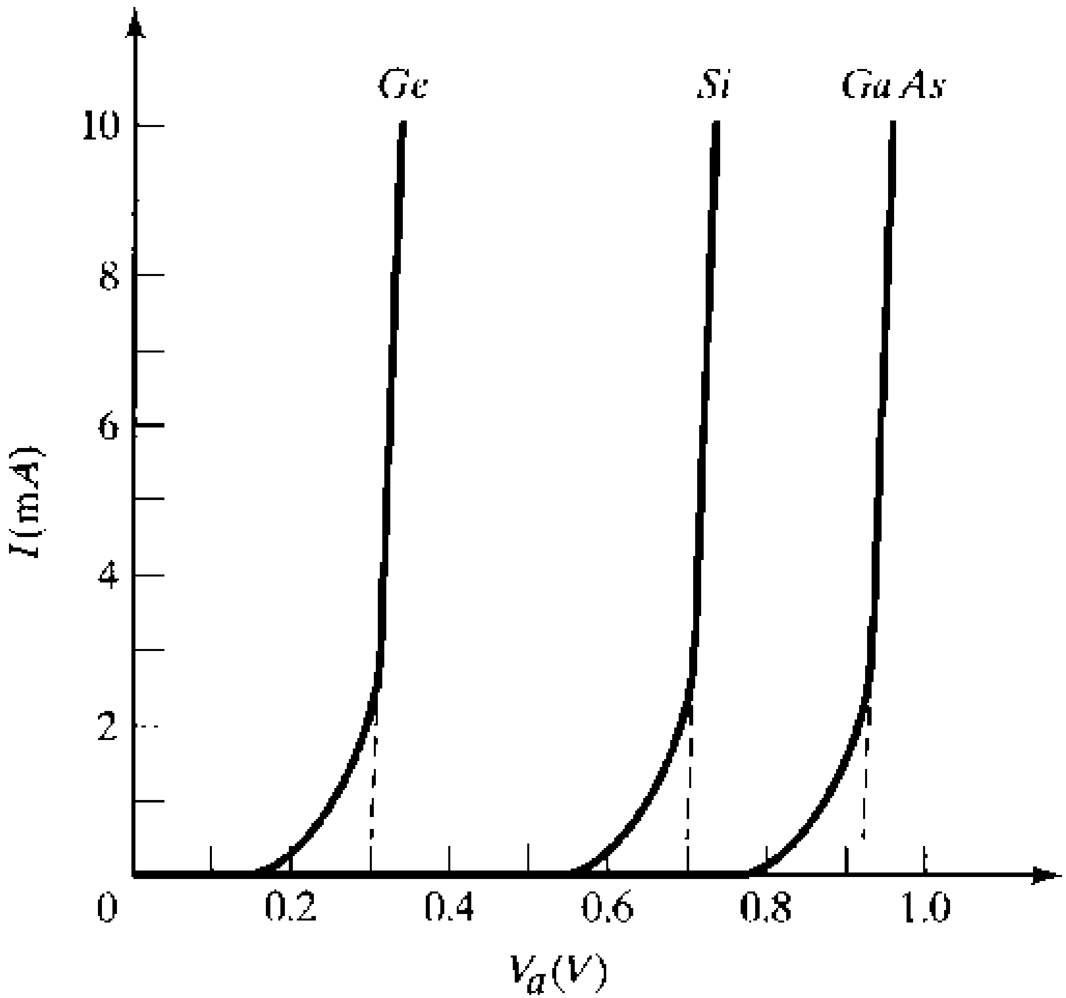
$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^{\,2}} \right)$$



- For higher band-gap materials, intrinsic carrier density is lower.
- $\therefore$ V_{bi} is higher, which is the potential barrier
- $\therefore$ more biasing voltage is required to achieve the same current as that in a lower band-gap material diode.



V_{bi} of Si < V_{bi} of GaAs



E_g of Ge < E_g of Si < E_g of GaAs

V_{bi} of Ge < V_{bi} of Si < V_{bi} of GaAs