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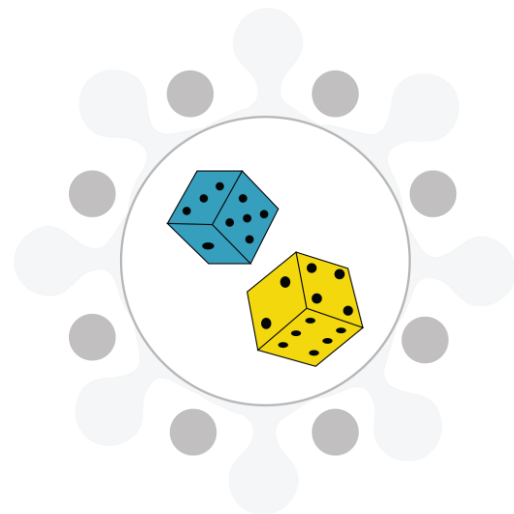
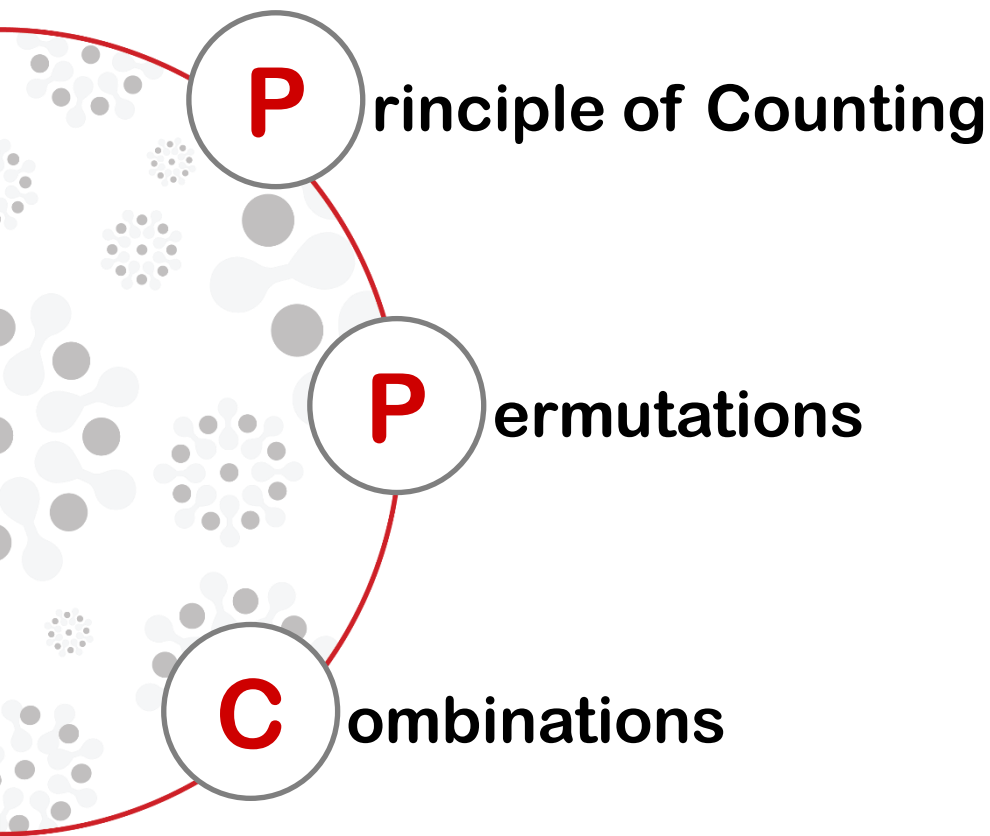
# Discrete Mathematics

## MH1812

**Topic 5.1 – Combinatorics**  
**Dr. Guo Jian**

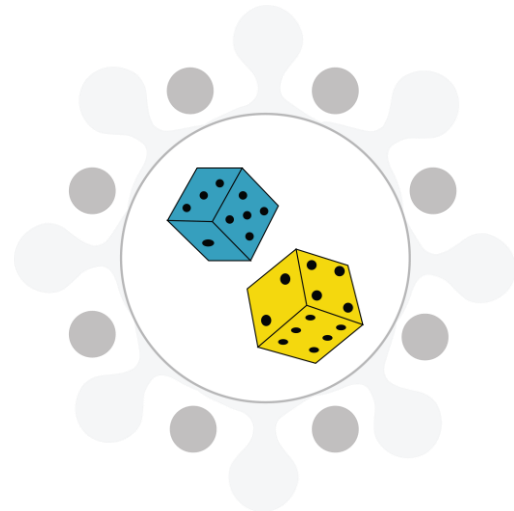
# Topic Overview

# What's in store...



# By the end of this lesson, you should be able to...

- Explain the concepts of the principle of counting.
- Determine the number of possible permutations using the counting principle, where order does matter.
- Determine the number of possible combinations using the counting principle, where order does not matter.





# Principle of Counting

# Principle of Counting: Choices and Slots

There are two slots to be filled, there are  $n_1$  choices for slot 1 and  $n_2$  choices for slot 2.



Example

Slot 1:  
Main Course

Slot 2:  
Dessert

The total number of unique choices to fill the slots is  $n_1 n_2$ .

3 Main Course Choices



2 Dessert Choices



# Principle of Counting: Choices and Slots

The total number of unique choices for this example is  $3 * 2 = 6$ .

3 Main Course Choices



2 Dessert Choices



# Principle of Counting: Choices and Slots

- In general:  $n_1, n_2, \dots, n_k$  choices for  $k$ -slots
- $n_1 * n_2 * \dots * n_k$  **ways**
  - Cardinality of the Cartesian product of sets





# Principle of Counting: Choices and Slots



## Example

Create a yoghurt dessert with *1* fruit, *1* crunch, and *1* sauce.

- *11* fruits
- *16* crunches
- *15* sauces

Slot 1:  
Fruit

Slot 2:  
Crunch

Slot 3:  
Sauce



# Principle of Counting: Cardinality of Power Set

- Consider a set  $A = \{a_1, \dots, a_n\}$  with  $n$  elements.
- List all subsets of  $A$ . Create a table.

All subsets of $A$	Binary vectors
$\{a_1\}$	$10\dots 0$

- Each of these  $n$  elements are **either in a subset of  $A$  or not**: 2 choices.
- Such a choice needs to be made for **each of the  $n$  elements**.
- Thus  **$2*2*\dots*2 = 2^n$  choices**.

# Principle of Counting: Filling $r$ Slots With $n$ Choices

There are  $n$  elements, with which to fill  $r$  slots.

## Can Be Repeated

When elements **can be** repeated using the principle of counting:  
 $n * n * ... * n = n^r$  **choices.**

## Cannot Be Repeated

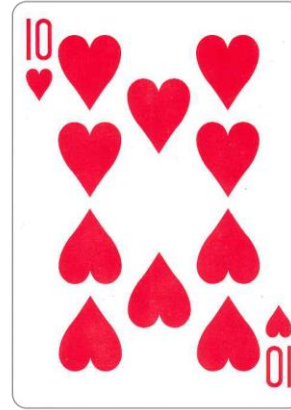
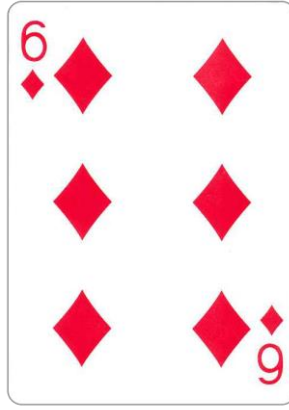
When elements **cannot be** repeated:

- $n$  choices for first slot
- $n - 1$  choices for second slot
- $n - (r - 1)$  choices for last slot
- In total:  $n(n - 1)(n - 2) ... (n - r + 1)$  choices

# Principle of Counting: Example 1



Sequence of choice of cards  
from a deck of cards.

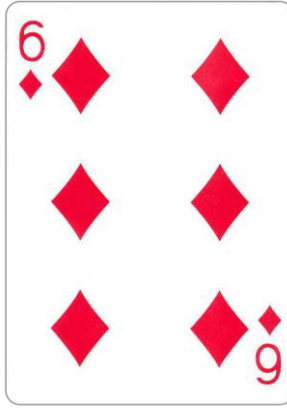




# Principle of Counting: Example 2

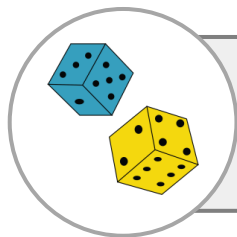


Sequence of choice of cards  
from a deck of cards.



# Permutations

# Permutations: $n!$



A **permutation** is an arrangement of all or part of a set of objects, **with** regard to the order of the arrangement.



1 <sup>st</sup> position	3 choices
2 <sup>nd</sup> position	2 choices
3 <sup>rd</sup> position	1 choice

Number of permutations of  $n$  objects

$$n * (n - 1) * (n - 2) \dots * 2 * 1 = n!$$

where  $n!$  is called  **$n$  factorial**.

# Permutations: $P(n,r)$

Filling  $r$  slots with  $n$  choices and no repetition:

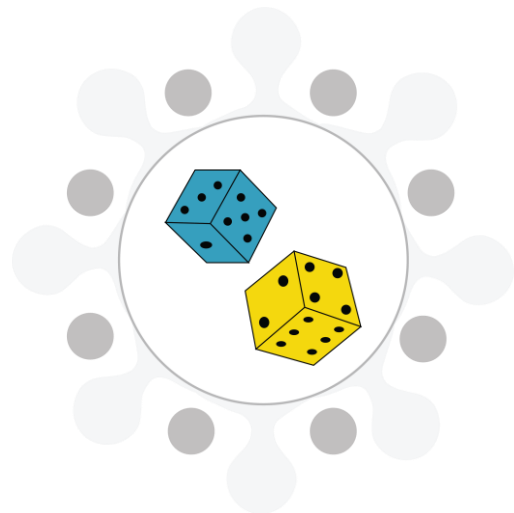
$$n(n-1)(n-2)\dots(n-r+1)$$

Permutations of  $n$  objects:  $n!$

Number of permutations of  $n$  objects taken  $r$  at a time ( $n$  objects, the number of ways in which  $r$  items can be ordered):

$$P(n,r) = n(n-1)(n-2)\dots(n-r+1) = n!/(n-r)!$$

where  $n! = n*(n-1)*(n-2)*\dots*2*1$  (called  $n$  factorial).



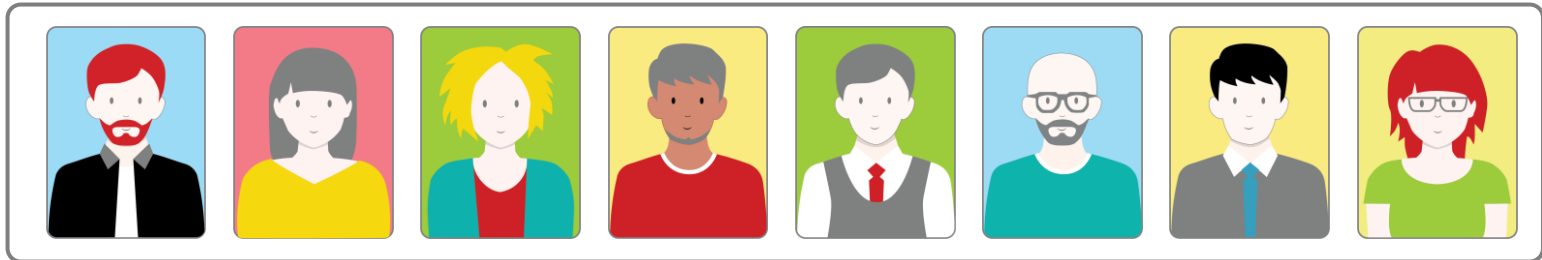
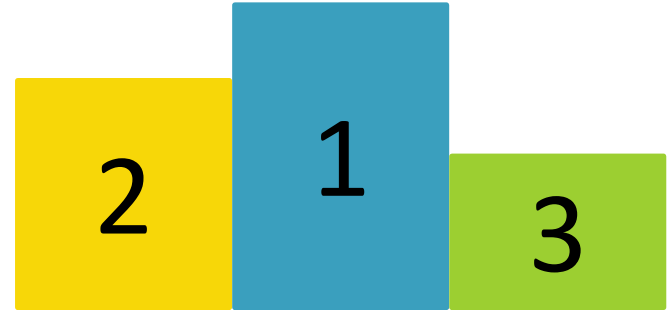


# Permutations: Example



In how many ways can we award a 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> prize to 8 contestants?

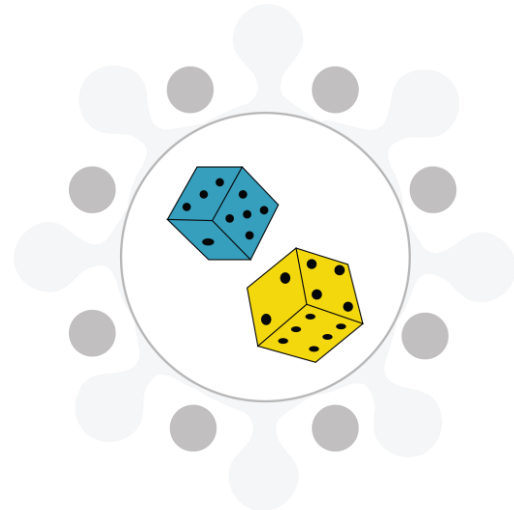
- For the 1<sup>st</sup> prize, any of the 8.
- Then for the 2<sup>nd</sup> prize, any of the 7 left.
- And for the 3<sup>rd</sup> prize, any of the 6 left.
- Hence  $8 * 7 * 6 = 336$ .



# Permutations: Distinguishable Permutations

In general, the number of distinguishable permutations from a collection of objects, where the first object appears (repeats)  $k_1$  times, the second object  $k_2$  times, ... for  $r$  distinct objects:

$$n!/(k_1! k_2! \dots k_r!)$$



# Permutations: Distinguishable Permutations



## Example

How many permutations are there of “MISSISSIPPI”?

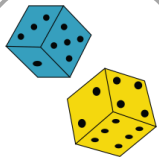
M	Appears $k_1 = 1$
I	Appears $k_2 = 4$
S	Appears $k_3 = 4$
P	Appears $k_4 = 2$

$$11!/(1! 4! 4! 2!)$$

# Combinations



# Combinations: $C(n,r)$ or $\binom{n}{r}$



A **combination** is a selection of all or part of a set of objects, **without** regard to the order in which objects are selected.



## Example

Team of 4 people from a group of 10.

Number of combinations of  $n$  objects taken  $r$  at a time:

$$\binom{n}{r} = C(n,r) = n!/r!(n-r)!$$

There are  $r!$  possible orderings within each combination.

So  $r! C(n,r) = P(n,r)$  by definition of permutation.

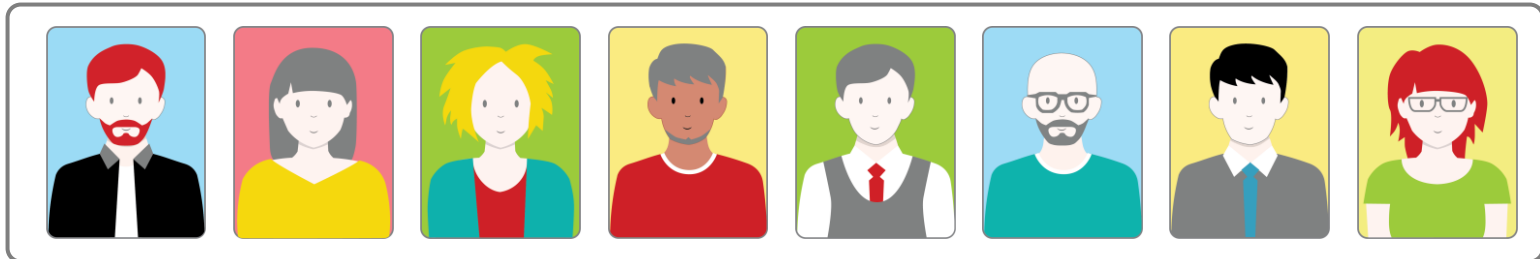
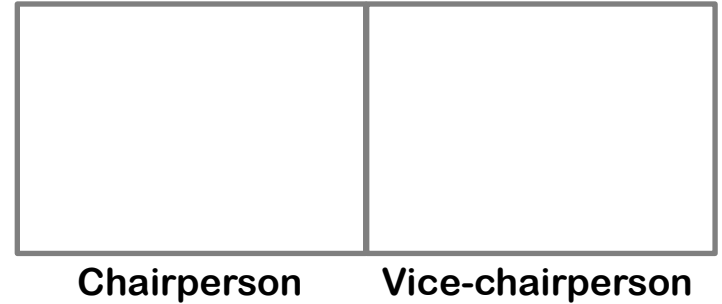
# Combinations: Example



From a committee of 8 people, in how many ways can you choose:

- A chairperson and vice-chairperson (one person cannot hold more than one position)?

$$P(8,2)$$



# Combinations: Example



From a committee of 8 people, in how many ways can you choose:

- A subcommittee of 2 people?

$$C(8,2)$$



Subcommittee



# Topic Summary



# Let's recap...

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- Principle of counting
- Permutations (with order)
- Combinations (without order)

