

INTEGRATED ELECTRONICS

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Topics

- 1. Power Supplies
- 2. Bias Circuits
- 3. Operational Amplifiers
- 4. Applications of Operational Amplifiers





Reference Textbooks

- 1. Sedra and Smith, *Microelectronic Circuits*, 5th Edition, Oxford University Press, 2004.
- 2. Gray, Hurst, Lewis and Meyer, Analysis and Design of Analogue Integrated Circuits, 4th Edition, John Wiley & Sons, 2001.
- 3. Franco S, Design with Operational Amplifiers and Analog Integrated Circuits, 3rd Edition, McGraw-Hill, 2002.





Bias Circuits

- 1. Introduction
- 2. Current Mirrors
- 3. Bias circuits based on V_{BE} or V_{GS} of a transistor, Thermal voltage V_T , Breakdown voltage of a reversebiased pn junction
- 4. Voltage Level Shifters





Bias Circuits should provide a *constant* bias current and/or voltage so that the circuit parameters do not change over varying supply voltages, temperature and other variations.

The Figure-of-Merit to qualify how well a desired parameter remains unchanged to the variation of a given parameter is Sensitivity.

The Sensitivity of a desired parameter (e.g. output current) to the variation of a given parameter (e.g. supply voltage) is:

$$S_{V_{CC}}^{I_o} = \frac{\Delta I_o}{\Delta V_{CC}} = \frac{V_{CC}}{I_o} \frac{\partial I_o}{\partial V_{CC}}$$

In general, the sensitivity of currents $S_{V_{CC}}^{I_o} = \frac{\Delta I_o}{\Delta V_{CC}} = \frac{V_{CC}}{I_o} \frac{\partial I_o}{\partial V_{CC}}$ In general, the sensitivity of current or voltages in a bias circuit to the variation of other parameters should be kept small.



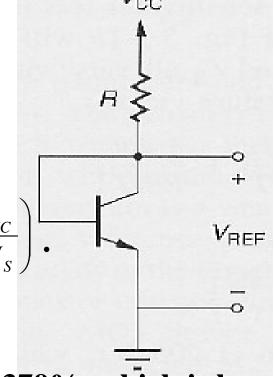


Example 1: Determine the sensitivity of V_{REF} to V_{CC} for the circuit shown in right. Assume $V_{CC} >> V_{REF}$, $V_{CC} = 5$ V, R = 43k Ω and $I_S = 0.4$ fA.

Solution: Since $I_C = I_S \exp(V_{BE}/V_T)$,

$$V_{REF} = V_{BE} = V_{T} \ln(I_{C}/I_{S}) = V_{T} \ln\left(\frac{V_{CC}-V_{REF}}{RI_{S}}\right) \approx V_{T} \ln\left(\frac{V_{CC}}{RI_{S}}\right)$$

$$\mathbf{Hence} \quad S_{V_{CC}}^{V_{REF}} = \frac{V_{CC}}{V_{REF}} \frac{\partial V_{REF}}{\partial V_{CC}} \cong \frac{1}{\ln(V_{CC}/RI_{S})} \cong 0.0379.$$



If V_{CC} changes by 10%, V_{REF} will change by 0.379%, which is large for a voltage bias circuit.





The Fractional Temperature Coefficient of a desired parameter (e.g. output current) is the fractional variation of the desired parameter to the variation of temperature. Denoted as TC_F , it is defined as:

$$TC_F = \frac{1}{I_O} \frac{\partial I_O}{\partial T}$$

Here the TC_F of an output current is used, other parameters can similarly be qualified.





Example 2: Determine the $TC_F(V_{REF})$ for V_{CC} for the circuit shown. Assume $V_{CC} >> V_{BE}$, $I_S = KT^3 \exp(-V_{GO}/V_T)$ where $V_{GO} = 1.205 \mathrm{V}$ is the bandgap of silicon and is independent of temperature, $TC_F(R) = 1500 \mathrm{ppm}/^{\circ}\mathrm{C}$, $V_{REF} = 0.7 \mathrm{V}$, and $T = 300 \mathrm{^{\circ}K}$.

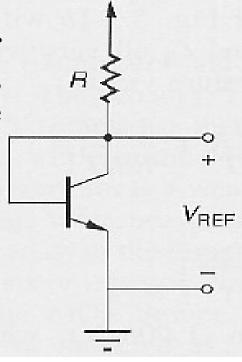
Solution:

$$V_{REF} \approx V_T \ln \left(\frac{V_{CC}}{RI_S} \right)$$

where

$$V_T = \frac{kT}{q}$$





 $V_{
m CC}$



$$\begin{split} \frac{\partial V_{REF}}{\partial T} &= \frac{\partial V_{T}}{\partial T} \ln \frac{V_{CC}}{RI_{S}} + V_{T} \frac{\partial}{\partial T} (\ln \frac{V_{CC}}{RI_{S}}) = \frac{k}{q} \ln \frac{V_{CC}}{RI_{S}} + V_{T} \frac{\partial}{\partial T} (\ln \frac{V_{CC}}{RI_{S}}) \\ &= \frac{kT}{qT} \ln \frac{V_{CC}}{RI_{S}} + V_{T} \frac{\partial}{\partial T} (\ln \frac{V_{CC}}{RI_{S}}) = \frac{V_{REF}}{T} + V_{T} \frac{\partial}{\partial T} (\ln \frac{V_{CC}}{RI_{S}}) \\ &= \frac{V_{REF}}{T} + V_{T} \frac{RI_{S}}{V_{CC}} \frac{\partial}{\partial T} (\frac{V_{CC}}{RI_{S}}) = \frac{V_{REF}}{T} + V_{T} \frac{RI_{S}}{V_{CC}} [\frac{RI_{S} \cdot 0 - V_{CC} \cdot \frac{\partial}{\partial T} (RI_{S})}{(RI_{S})^{2}}] \\ &= \frac{V_{REF}}{T} - \frac{V_{T}}{RI_{S}} \frac{\partial}{\partial T} (RI_{S}) = \frac{V_{REF}}{T} - \frac{V_{T}}{RI_{S}} [I_{S} \frac{\partial R}{\partial T} + R \frac{\partial I_{S}}{\partial T}] \\ &= \frac{V_{REF}}{T} - V_{T} \left(\frac{1}{R} \frac{\partial R}{\partial T} + \frac{1}{I_{S}} \frac{\partial I_{S}}{\partial T}\right). \end{split}$$





Since $I_S = kT^3 \exp(-V_{GO}/V_T)$, we can get

$$\frac{\partial I_{S}}{\partial T} = 3kT^{2} \exp(-\frac{V_{GO}}{V_{T}}) + kT^{3} \frac{\partial}{\partial T} \left[\exp(-\frac{V_{GO}}{V_{T}})\right]$$

$$= \frac{3I_{S}}{T} + kT^{3} \exp(-\frac{V_{GO}}{V_{T}}) \frac{\partial}{\partial T} \left(-\frac{V_{GO}}{V_{T}}\right) = \frac{3I_{S}}{T} - I_{S} \frac{\partial}{\partial T} \left(\frac{V_{GO}}{V_{T}}\right)$$

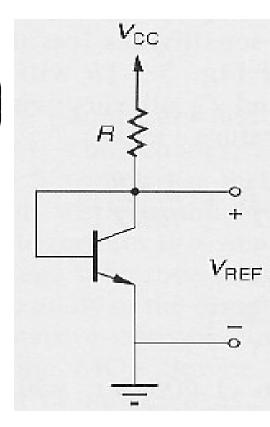
$$= \frac{3I_{S}}{T} - I_{S} \left(\frac{V_{T} \cdot 0 - V_{GO}}{V_{T}^{2}} \frac{\partial V_{T}}{\partial T}\right) = \frac{3I_{S}}{T} + I_{S} \left(\frac{V_{GO} \frac{k}{q}}{V_{T}^{2}}\right) = \frac{3I_{S}}{T} + \frac{I_{S} V_{GO}}{V_{T}^{2}}.$$





$$\begin{split} \frac{\partial V_{REF}}{\partial T} &= \frac{V_{REF}}{T} - V_T \Bigg(\frac{1}{I_S} \frac{\partial I_S}{\partial T} + \frac{1}{R} \frac{\partial R}{\partial T} \Bigg) = \frac{V_{REF}}{T} - V_T \Bigg(\frac{1}{I_S} (\frac{3I_S}{T} + \frac{I_S}{T} \frac{V_{GO}}{V_T}) + \frac{1}{R} \frac{\partial R}{\partial T} \Bigg) \\ &= \frac{V_{REF}}{T} - V_T \Bigg(\frac{3}{T} + \frac{V_{GO}}{TV_T} + \frac{1}{R} \frac{\partial R}{\partial T} \Bigg) = \frac{V_{REF}}{T} - \left(\frac{3V_T}{T} + \frac{V_{GO}}{T} + \frac{V_T}{R} \frac{\partial R}{\partial T} \right) . \end{split}$$

$$\begin{split} TC_F(V_{REF}) &= \frac{1}{V_{REF}} \frac{\partial V_{REF}}{\partial T} \\ &= \frac{V_{REF} - 3V_T - V_{GO}}{V_{REF}T} - \frac{V_T}{V_{REF}} \frac{1}{R} \frac{\partial R}{\partial T} \\ &= \frac{700 - 78 - 1205}{700 \cdot 300} - \frac{26}{700} 1500 \approx -2832 \, ppm/^{\circ}C \quad . \end{split}$$



This voltage bias circuit has a large temperature coefficient.



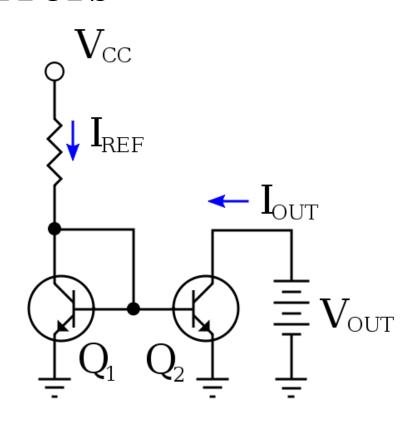


For BJT in active mode,

$$I_{C1} = I_{S1} \exp\left(\frac{V_{BE1}}{V_T}\right) => V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right)$$
.

Since
$$V_{BE1} = V_{BE2}$$
, So $I_{C1} = \frac{I_{S1}}{I_{C2}} = \frac{I_{S1}}{I_{S2}}$.

As $I_S \propto$ emitter area, the ratio of the emitter areas to a first order determines the current ratio of the collectors.



Basic BJT current mirror





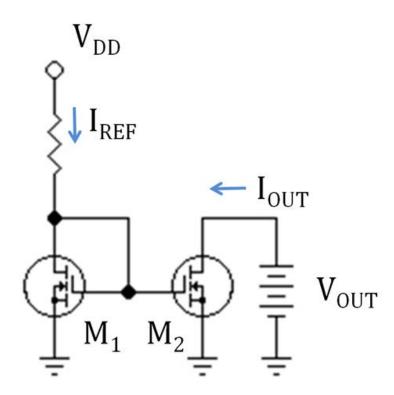
For CMOS in saturation mode,

$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2 .$$

Since $V_{GS1}=V_{GS2}$,

$$\frac{I_{D1}}{I_{D2}} = \frac{\left(W/L\right)_1}{\left(W/L\right)_2} .$$

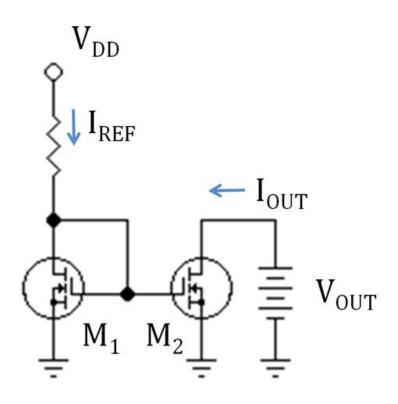
The ratio of the (W/L) or aspect ratio of the CMOS transistors to a first order determines the drain current ratio.







Example 3: Given two MOSFETs of channe lengths of 10 μm, channel widths of 100 μm threshold voltage of 1 V, early voltage of 100 V and $\mu_n C_{ox} = 20 \text{mA/V}^2$, it is required t design the circuit for a basic MOSFET constant current source to obtain an output of 100 μ A with a power supply $V_{DD} = 5$ V. What is the lowest possible value of output voltage? Find the change in output current resulting from a 3-V change in output voltage.





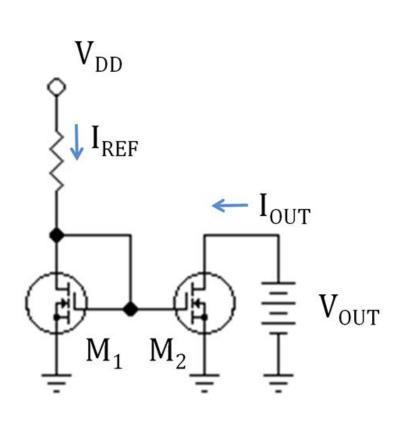


Solution:
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \frac{V_{DS}}{V_A})$$
.

$$100 \times 10^{-6} = \frac{20 \times 10^{-3}}{2} \frac{100}{10} (V_{GS} - 1)^{2} (1 + \frac{V_{GS}}{100})$$
$$\approx \frac{20 \times 10^{-3}}{2} \frac{100}{10} (V_{GS} - 1)^{2}.$$

$$V_{GS} \approx 1.03 \text{ V}$$
.

$$R = \frac{V_{DD} - V_{GS}}{I_o} \approx \frac{5 - 1.03}{0.0001} = 39.7 \ k\Omega$$





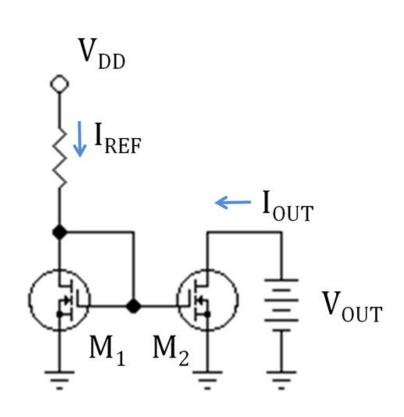


Solution: $V_o = V_{DS2} \ge V_{GS2} - V_t$.

$$V_{o \min} = V_{DS2} = V_{GS2} - V_t = V_{GS1} - V_t \approx 1.03 - 1 = 0.03 \ V$$
 .

$$r_o = \frac{V_A}{I_o} = \frac{100}{0.0001} = 1 M\Omega$$
.

$$\Delta I_o = \frac{\Delta V_o}{r_o} = 3 \quad \mu A \quad \bullet$$

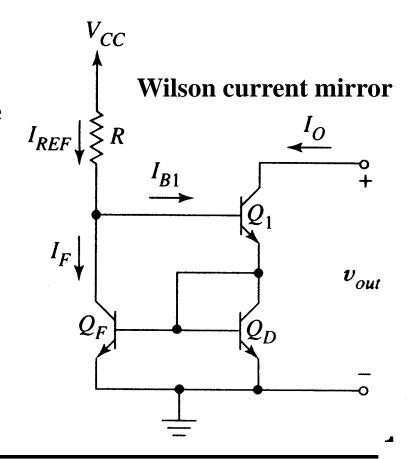






The Wilson current mirror has the advantage of virtually eliminating the base current mis-match of the basic current mirror thereby ensuring that the output current I_O is almost equal to the reference or input current I_{REF} . It also has a very high output impedance.

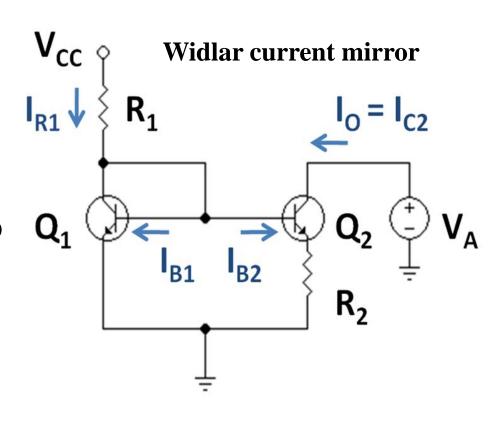
$$\frac{I_o}{I_{REF}} = \frac{\beta^2 + 2\beta}{\beta^2 + 2\beta + 2} \quad .$$







Widlar current mirror uses bipolar transistors, where the emitter resistor R₂ is connected to the output transistor Q_2 , and has the effect of reducing the current in Q_2 relative to Q_1 . The key to this circuit is that the voltage drop across the resistor R_2 subtracts from the base-emitter voltage of transistor Q_2 , thereby turning this transistor off compared to transistor Q_1 .







Example 4: Design a Widlar current mirror.

Select the desired output current I₀.

Select the reference current I_{R1} .

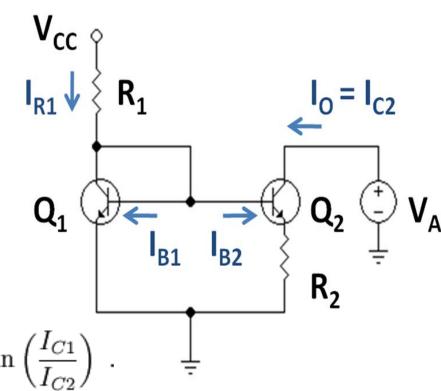
Find the collector current of Q₁

$$I_{C1} = \frac{\beta_1}{\beta_1 + 1} \left(I_{R1} - I_{C2} / \beta_2 \right) .$$

Find the base voltage of Q_1

$$V_{BE1} = V_T \ln \left(\frac{I_{C1}}{I_S} \right) = V_A .$$

Find
$$R_1 = \frac{V_{CC} - V_A}{I_{R1}}$$
 and $R_2 = \frac{V_T}{(1 + 1/\beta_2) I_{C2}} \ln \left(\frac{I_{C1}}{I_{C2}}\right)$.





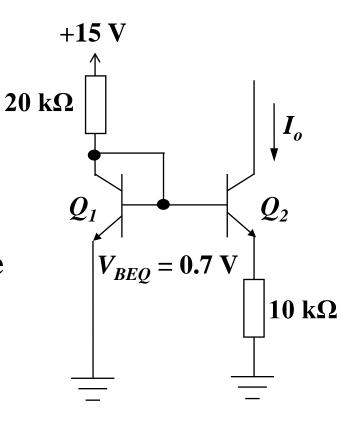


Example 5: For the Widlar current source given, assuming that the two transistors are identical, the early voltage is infinite, and the base currents are negligible, determine the output current and the sensitivity of the output current to the power–supply voltage.

Solution: Based on the given assumptions, we have

$$V_{BE1} - V_{BE2} - I_o R_2 = 0 .$$

$$V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} - I_o R_2 = 0 .$$





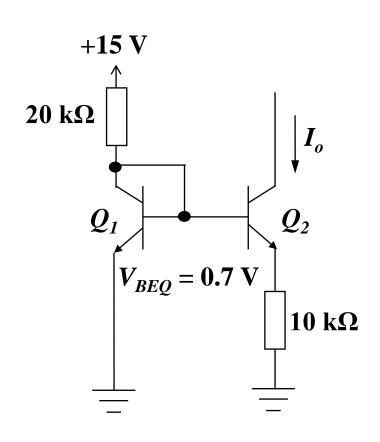


$$I_o R_2 = V_T \ln \frac{I_{C1}}{I_{C2}} = V_T \ln \frac{I_{C1}}{I_o}$$
.

$$I_{C1} = \frac{V_{CC} - V_{BEQ}}{R_1} = \frac{15 - 0.7}{20} = 0.715 \text{mA}$$

$$I_o \times 10000 = 0.026 \ln \frac{0.000715}{I_o}$$
.

$$I_o \approx 11 \mu A$$



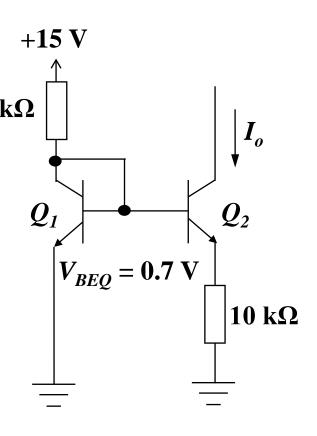




$$I_o R_2 = V_T \ln \frac{I_{C1}}{I_o} .$$

Differentiating the above equation, we have:

$$\begin{split} R_2 \frac{\partial I_o}{\partial V_{CC}} &= V_T \frac{\partial}{\partial V_{CC}} (\ln \frac{I_{C1}}{I_o}) \\ &= V_T \frac{I_o}{I_{C1}} [\frac{I_o \frac{\partial I_{C1}}{\partial V_{CC}} - I_{C1} \frac{\partial I_o}{\partial V_{CC}}}{I_o^2}] \quad . \end{split}$$



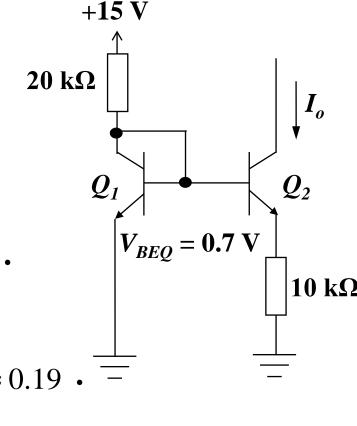




$$\frac{\partial I_o}{\partial V_{CC}} = \frac{I_o}{I_{C1}} \left(\frac{\frac{\partial I_{C1}}{\partial V_{CC}}}{1 + \frac{I_o R_2}{V_T}} \right).$$

$$S_{V_{CC}}^{I_o} = \frac{V_{CC}}{I_o} \frac{\partial I_o}{\partial V_{CC}} = \frac{V_{CC}}{I_{C1}} \left(\frac{\partial I_{C1}}{\partial V_{CC}} - \frac{\partial I_{C1}}{\partial V_{CC}} \right) = \frac{S_{V_{CC}}^{I_{C1}}}{1 + \frac{I_o R_2}{V_T}} \cdot V_{BEQ} = 0.7 \text{ V}$$

$$I_{C1} \approx \frac{V_{CC}}{R_1}, S_{V_{CC}}^{I_{C1}} \approx 1, S_{V_{CC}}^{I_o} = \frac{1}{1 + \frac{I_o R_2}{V_T}} = \frac{1}{1 + \frac{110}{26}} \approx 0.19$$





 V_{CC}



V_{BE}-Based Bias Circuit

 I_{ref} is forced through Q_1 , so Q_2 must supply enough current into R_2 so that $V_{BEQ1}=V_{R2}$.

Neglecting base currents we have $I_{out}=I_{EQ2}=I_{R2}$.

Hence we have

$$I_{out} = \frac{V_{BE1}}{R_2} = \frac{V_T}{R_2} \ln \left(\frac{I_{ref}}{I_{S1}} \right).$$

This bias circuit is <u>NOT</u> supply independent since V_{BEQ1} will change slightly with the supply voltage because I_{CQ1} is approximately proportional to V_{CC} . This is often a problem for bias circuits whose I_{out} is derived from a resistor connected to the supply terminal.





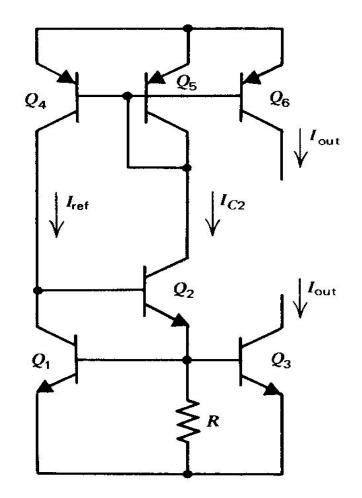
V_{BE}-Based Bias Circuit with Bootstrap

The idea is instead of developing the reference current through a resistor, the reference circuit is made to depend on the current source itself.

Assuming $V_A = \infty$ and neglecting base currents: Q_1 , Q_2 and R are same as before.

 Q_3 is the current mirror of Q_1 .

 Q_4, Q_5 and Q_6 are part of a simple current mirror, hence $I_{ref} = I_{C2}$.



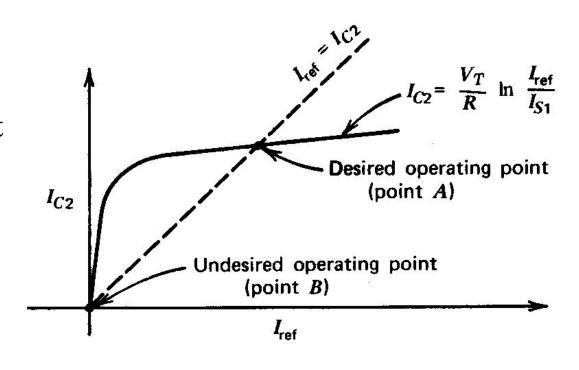




V_{BE}-Based Bias Circuit with Bootstrap

The operating point must satisfy both the upper current mirror and lower bias reference circuit.

The desired operating point is point A as shown in the figure. Need a start-up circuit to push the circuit out of the other possible operating point B. Here I_{out} is independent of the supply voltage.







V_{BE}-Based Bias Circuit with Bootstrap

However, the TC_F of this circuit is an issue.

Since
$$I_{out} = \frac{V_{BE1}}{R}$$
,

$$\frac{\partial I_{out}}{\partial T} = \frac{1}{R} \frac{\partial V_{BE1}}{\partial T} - \frac{V_{BE1}}{R^2} \frac{\partial R}{\partial T} = I_{out} \left(\frac{1}{V_{BE1}} \frac{\partial V_{BE1}}{\partial T} - \frac{1}{R} \frac{\partial R}{\partial T} \right) \cdot$$

Hence
$$TC_F = \frac{1}{V_{BE1}} \frac{\partial V_{BE1}}{\partial T} - \frac{1}{R} \frac{\partial R}{\partial T}$$
.

As $TC_F(V_{BE1})$ is —ve and $TC_F(R)$ is +ve, the net TC_F is large => sensitive to temperature with a negative gradient, i.e. as $T \uparrow$, $I_{out} \downarrow$.



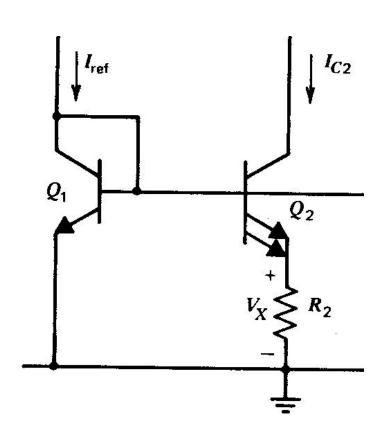


V_T-Based Bias Circuit

The difference in the junction potential between two junctions operated at different current densities can be shown to be ∞ to V_T . From the Widlar current source shown on the right,

$$V_X = I_{C2}R_2 = V_T \ln \left(\frac{I_{C1}I_{S2}}{I_{C2}I_{S1}} \right)$$
.

If the ratio of the collector currents is constant, then $V_X \propto V_T$. If the emitter area of $Q_2 = 2 \times$ that of Q_1 , then $I_{S2} = 2 \times I_{S1}$.







V_T-Based Bias Circuit with Bootstrap

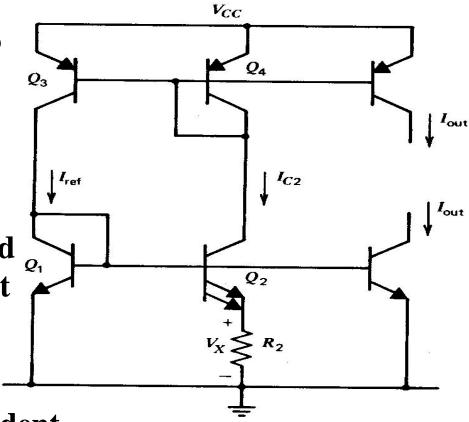
The reference circuit is again made to depend on the current source itself.

Due to the current mirror formed by Q_3 and Q_4 , $I_{C1} = I_{C2}$. For Widlar current source: $V_X = V_T \ln 2$.

Satisfying both the current mirror and current source equations requires that

$$I_{C2} = \frac{V_T}{R_2} \ln 2 .$$

The output current is supply independent.





V_T-Based Bias Circuit with Bootstrap

Since
$$TC_F = \frac{1}{I_{C2}} \frac{\partial I_{C2}}{\partial T} = \frac{1}{I_{C2}} \frac{\partial}{\partial T} \left(\frac{V_T}{R_2} \ln 2 \right) = \frac{1}{I_{C2}} \left(\frac{1}{V_T} \frac{\partial V_T}{\partial T} - \frac{1}{R_2} \frac{\partial R_2}{\partial T} \right) \left(\frac{V_T}{R_2} \ln 2 \right)$$

$$= \frac{1}{V_T} \frac{\partial V_T}{\partial T} - \frac{1}{R_2} \frac{\partial R_2}{\partial T} .$$

As $TC_F(V_T)$ is +ve and $TC_F(R)$ is +ve, they tend to cancel out! The net TC_F is therefore small => only slightly sensitive to temperature with a positive gradient, i.e. as $T \uparrow$, $I_{out} \uparrow$.

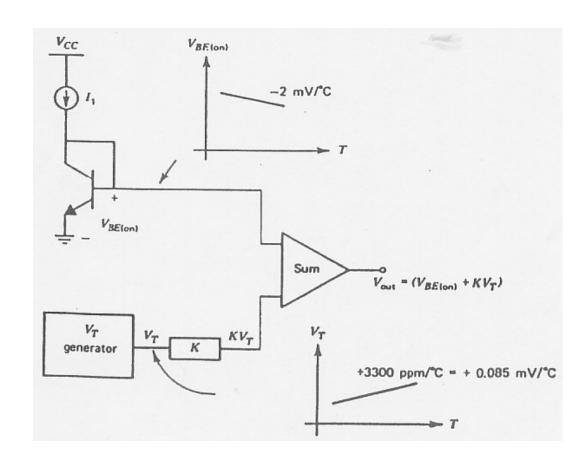




Bandgap Bias Circuit

Notice that the V_{RE} -based and V_T -based reference circuits have opposite TC_F s, we can obtain a reference current that is a weighted sum of V_{RE} based and V_T -based reference currents. The idea is that by proper weighting of the currents, we can achieve a reference source with $TC_F=0$.

 $V_{out} = V_{BE} + KV_T$ (K=constant)







Bandgap Bias Circuit

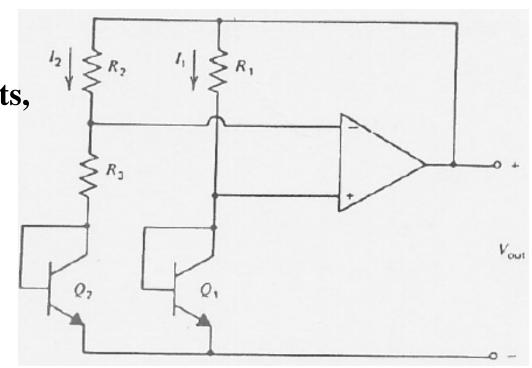
In an op amp circuit with negative feedback, $V_{+}=V_{-}$.

Hence
$$V_{R1} = V_{R2}$$
 and $\frac{I_1}{I_2} = \frac{R_2}{R_1}$.

Assuming negligible base currents, the difference between V_{BE1} and V_{BE2} is the voltage across R_3 .

$$\Delta V_{BE} = V_{BE1} - V_{BE2} = V_T \ln \left(\frac{I_1}{I_2} \frac{I_{S2}}{I_{S1}} \right)$$

$$=V_T \ln \left(\frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}}\right) .$$







Bandgap Bias Circuit

The voltage across
$$R_2$$
 is $V_{R2} = \frac{R_2}{R_3} \Delta V_{BE} = \frac{R_2}{R_3} V_T \ln \left(\frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}} \right)$.

This means that both I_1 and I_2 are proportional to temperature if the resistors have zero TC_F . The output voltage is the sum of voltages across R_1 and Q_1 :

$$V_{out} = V_{BE1} + \frac{R_2}{R_3} V_T \ln \left(\frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}} \right) = V_{BE1} + KV_T \text{ where } K = \frac{R_2}{R_3} \ln \left(\frac{R_2}{R_1} \frac{I_{S2}}{I_{S1}} \right)$$
.

By proper choice of K, V_{out} can be made virtually independent of temperature and supply voltage.





Voltage Level Shifters

Functions of a voltage level shifter: Adjust to specific levels the quiescent voltages at various points of a dc coupled circuit and adjust the output voltage to appropriate quiescent level for optimum swing. The quiescent dc levels in each subcircuit have to be managed as all subcircuits are dc coupled.

By inspection: $V_o = V_i - V_{BEQ} \approx V_i - 0.7 \text{V}$.

Two problems: Voltage gain is slightly less than 1. The voltage level shift is limited to only 0.7V.

May use a voltage divider to increase the voltage shift.





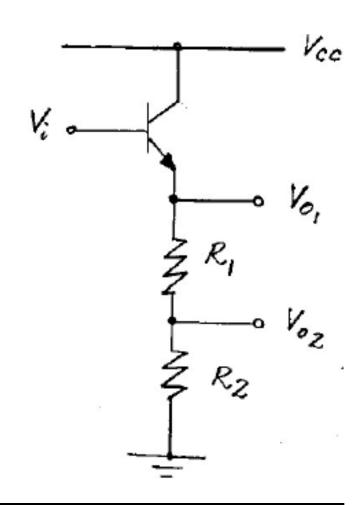
Voltage Level Shifters

By inspection:

$$V_{o2} = \frac{R_2}{R_1 + R_2} (V_i - V_{BEQ})$$
.

The voltage level shift has increased (by how much?). However, the input signal is attenuated => unacceptable!

Solution: replace R_2 with a current source.







Voltage Level Shifters

By inspection:

$$V_o = V_i - V_{BEQ} - I \times R_1 .$$

The voltage level shift increases by $I \times R_1$ where I is the biasing current of the current source.

 R_1 can be varied to achieve different dc voltage shift.

The ideal impedance of a current source is ∞ , hence for any input signal, R_2 is $\infty =>$ no signal attenuation.

