NANYANG TECHNOLOGICAL UNIVERSITY

Lecture 4

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

Associate Professor Lee Peng Hin
Tel: +65 6790 4474 | Email: ephlee@ntu.edu.sg

Learning Objectives

By the end of this lecture, you should be able to:

- Describe the different extended operating regions of a typical torque-speed characteristic curves of an induction motor.
- Calculate the maximum or pullout torque and the speed at this torque of an induction motor, using the Thevenin equivalent circuit approach.
- Analyse the performance of an induction motor with varying rotor resistance and how it affects the torque-speed characteristics.

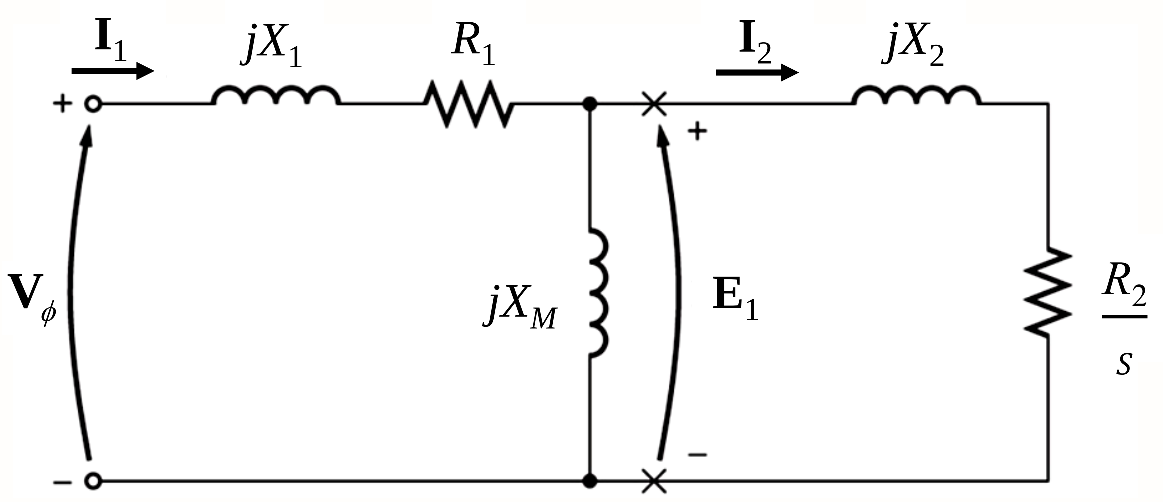
Induction Motor Torque-speed Characteristics

- It is possible to use the equivalent circuit of an induction motor and the power flow diagram for the motor to derive a general expression for induced torque (developed torque) as a function of speed.

$$T_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$P_{AG} = 3I_2^2\left(\frac{R_2}{s}\right)$$

$$\Rightarrow T_{ind} = \left(\frac{1}{\omega_{sync}}\right)3I_2^2\left(\frac{R_2}{s}\right)$$



The per phase equivalent circuit of an induction motor.

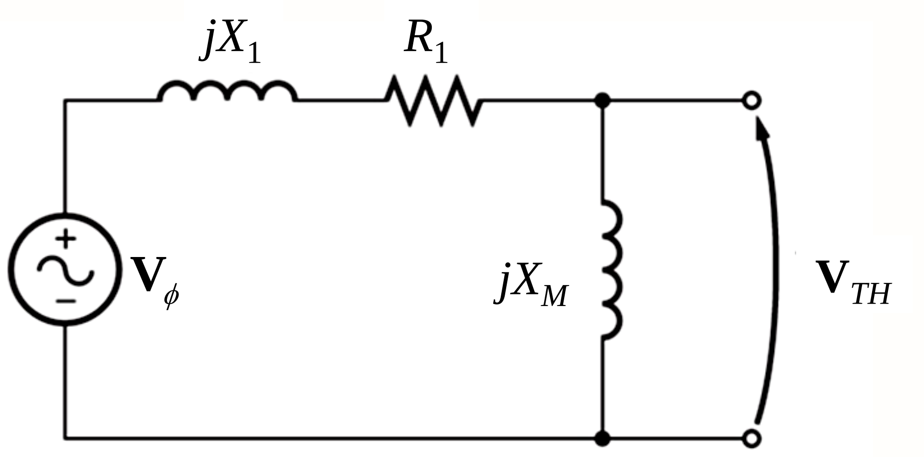
Induction Motor Torque-speed Characteristics

- We want to find an expression for the current $\mathbf{I}_2$.
- One approach is to find the Thevenin equivalent to the left of the points marked “x” in the figure.

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

$$\left|\mathbf{V}_{TH}\right| = V_{TH}$$

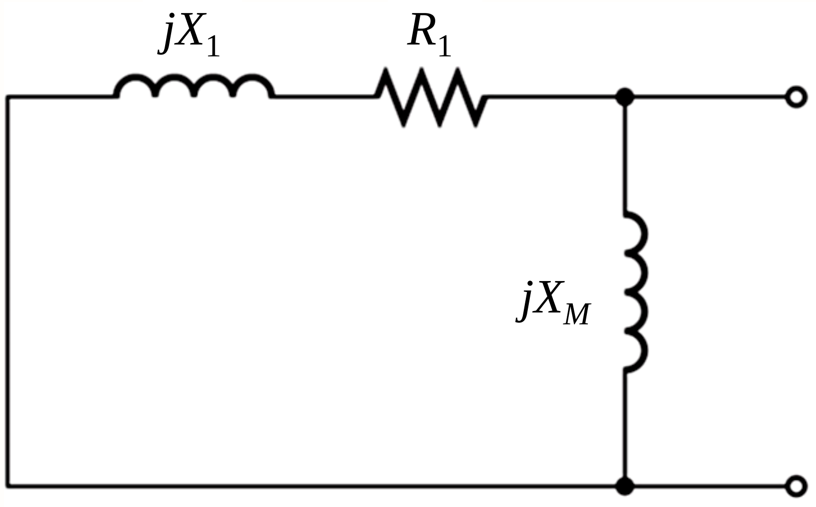
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}V_\phi$$



Induction Motor Torque-speed Characteristics

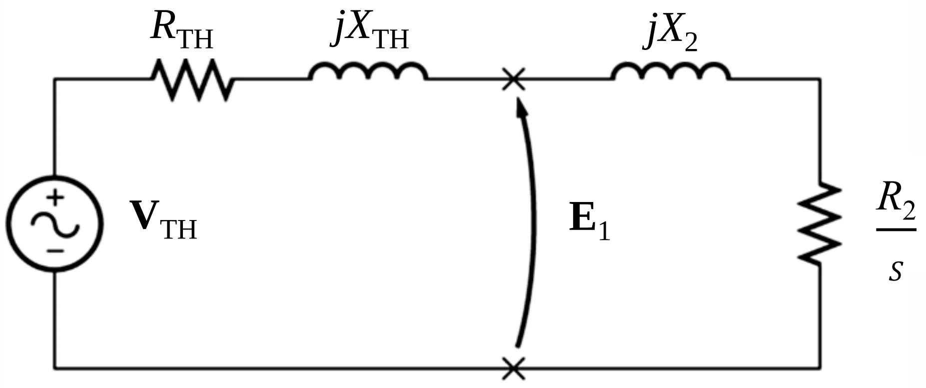
$$\mathbf{Z}_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + jX_1 + jX_M}$$

$$= R_{TH} + jX_{TH}$$



Induction Motor Torque-speed Characteristics

$$\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{TH} + (R_2/s) + jX_2} = \frac{\mathbf{V}_{TH}}{(R_{TH} + R_2/s) + j(X_{TH} + X_2)}$$

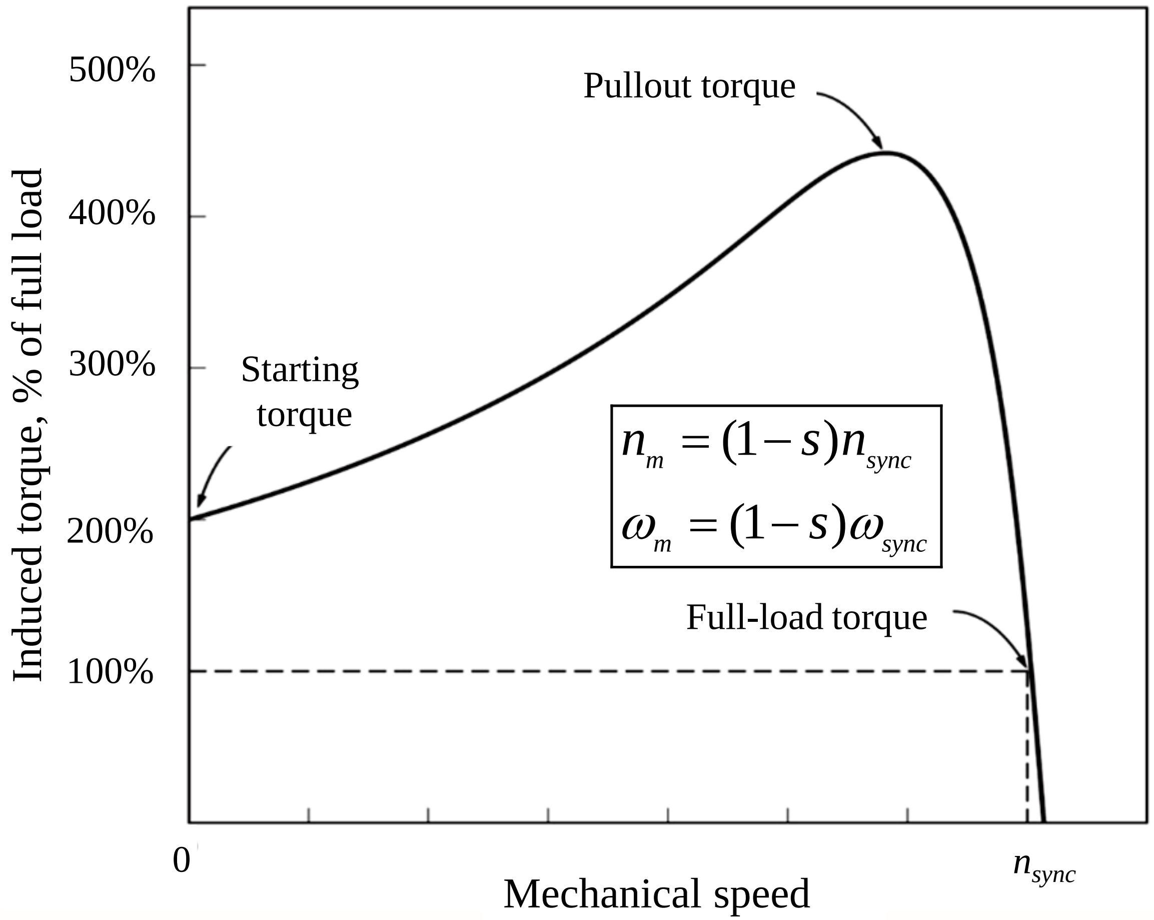


The resultant simplified equivalent circuit of an induction motor.

$$|\mathbf{I}_2| = I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}}$$

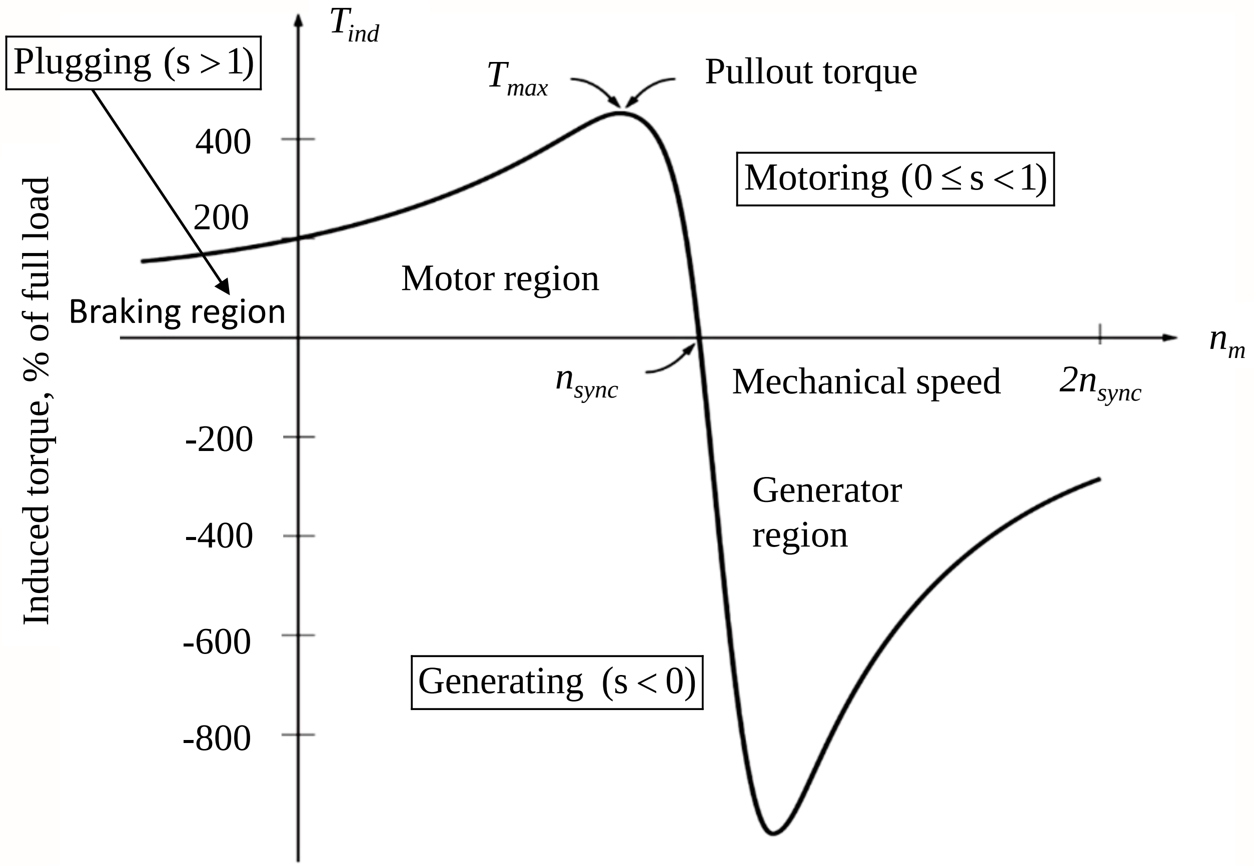
$$T_{ind} = \left(\frac{1}{\omega_{sync}}\right) 3I_2^2 \left(\frac{R_2}{s}\right) = \left(\frac{1}{\omega_{sync}}\right) 3 \left(\frac{R_2}{s}\right) \frac{V_{TH}^2}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$$

Induction Motor Torque-speed Characteristics Curve



A typical induction motor torque-speed characteristics curve.

Induction Motor Torque-speed Characteristics Curve



Induction motor torque-speed characteristics curve showing the extended operating regions (braking and generator regions).

Induction Motor Torque-speed Characteristics Curve

- Motoring region
  - The motor rotates in same direction but slower than the rotating magnetic field.
  - The induced torque is zero at synchronous speed.
  - The starting torque is slightly higher than the full-load torque.
  - There is a maximum possible developed torque, called the pullout torque.

Induction Motor Torque-speed Characteristics Curve

❖ Generating region

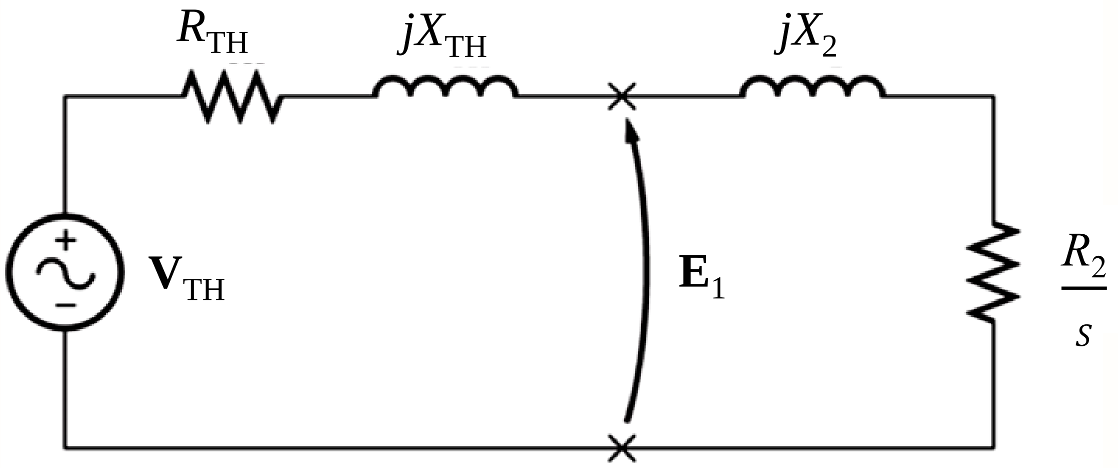
- If the rotor of the induction motor is driven faster than synchronous speed by an external prime mover, the direction of the induced torque reverses and the machine becomes a generator.
- Induction generators in wind turbines are operated in this manner. As the torque applied to its shaft increases, the amount of power produced by the induction generator increases.

Induction Motor Torque-speed Characteristics Curve

❖ Plugging region

- If the motor is turning backwards relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction.
- This is done by reversing the phase sequence of the 3-phase stator voltage supply, simply by switching any two stator phases.
- The act of switching two phases in order to stop the motor very rapidly is called plugging.

Maximum (Pullout) Torque in Induction Motor



- Maximum possible torque occurs when air-gap power is maximum.

- From $T_{ind} = \frac{P_{AG}}{\omega_{sync}}$ and since $P_{AG} = 3I_2^2\left(\frac{R_2}{s}\right)$, then max T_{ind} occurs when the power consumed by $\frac{R_2}{s}$ is a maximum.

Maximum (Pullout) Torque in Induction Motor

- From the maximum power transfer theorem, $\dfrac{R_2}{s} = \left| R_{TH} + jX_{TH} + jX_2 \right|$

- The slip at pullout torque, $\Rightarrow s_{\max} = \dfrac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$

- Maximum or pullout torque

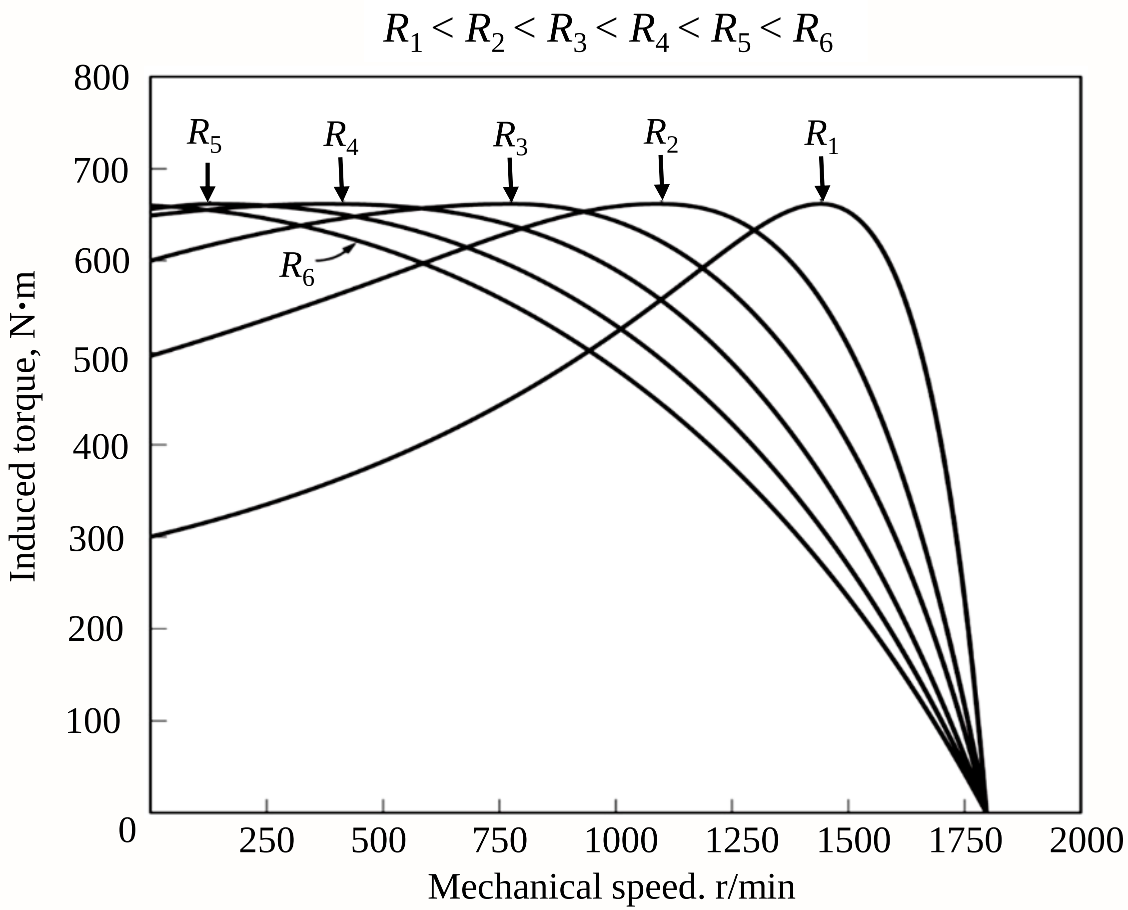
$$T_{\max} = \left(\frac{1}{\omega_{sync}} \right) \left(\frac{R_2}{s_{\max}} \right) \frac{3V_{TH}^2}{\left(R_{TH} + R_2 / s_{\max}\right)^2 + \left(X_{TH} + X_2\right)^2}$$

$$= \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2} \right]}$$

Maximum (Pullout) Torque in Induction Motor

- The slip at maximum torque can be varied by changing the rotor resistance, while the maximum torque is independent of the rotor resistance.
- The max torque is proportional to the square of the supply voltage and inversely related to the size of the stator impedance and the rotor reactance.
- The torque-speed characteristics of a wound-rotor induction motor is as shown. For this type of motor, external resistance can be inserted to the rotor because the rotor circuit is brought out through slip rings.
- As rotor resistance increases, the pullout speed decreases, but the max torque remains constant.

Maximum (Pullout) Torque in Induction Motor



Effect of varying rotor resistance on torque-speed characteristics of a wound-rotor induction motor.

Example 1

A 460 V, 60 Hz, 25-hp, four-pole induction motor is wye-connected. The impedances per phase referred to the stator circuit are:

$$R_1 = 0.641\Omega, \qquad R_2 = 0.332\Omega, \qquad X_1 = 1.106\Omega,$$

$$X_2 = 0.464\Omega, \qquad X_M = 26.3\Omega$$

a) What is the maximum torque of this motor?
   At what speed and slip does it occur?

b) What is the starting torque of this motor?

c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs?
   What is the new starting torque of the motor?

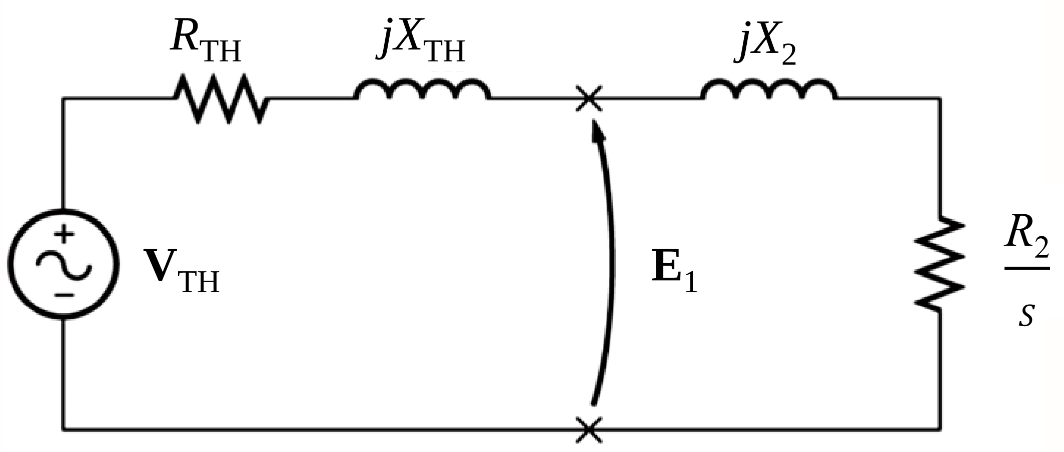
(Solutions →)

Example 1 – Solutions

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi = 254.8\angle 1.34°\text{V}$$

$$\mathbf{Z}_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + jX_1 + jX_M} = 0.59 + j1.08\Omega$$

$$= R_{TH} + jX_{TH}$$



Example 1 – Solutions

a) Slip at maximum torque,

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = 0.2 \Rightarrow n_{sync} = \frac{120 f_s}{p} = \frac{120(60)}{4} = 1800 \text{ r/min}$$

$$\Rightarrow n_m = (1 - s_{\max})\, n_{sync} = 1440 \text{ r/min}$$

$$\mathrm{I}_2 = \frac{\mathbf{V}_{TH}}{(R_{TH} + R_2 / s_{\max}) + j(X_{TH} + X_2)} = 93.37\angle -33.12^\circ \text{ A}$$

$$P_{AG} = 3I_2^2\left(\frac{R_2}{s_{\max}}\right) = 43415 \text{ W} \Rightarrow T_{\max} = \frac{P_{AG}}{\omega_{sync}} = 230.3 \text{ N.m}$$

Example 1 – Solutions

b) Starting torque, $s = 1$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{\left(R_{TH} + R_2 / s\right) + j(X_{TH} + X_2)} = 141.69\angle -57.82° \text{ A}$$

$$P_{AG} = 3I_2^2\left(\frac{R_2}{s}\right) = 19995.75 \text{ W} \Rightarrow T_{start} = \frac{P_{AG}}{\omega_{sync}} = 106 \text{ N.m}$$

Example 1 – Solutions

c) If the rotor resistance is doubled, the slip at maximum torque is doubled too.

$$s_{\max} = 0.4 \Rightarrow n_m = (1 - s_{\max})\, n_{sync} = 1080 \text{ r/min}$$

$$\text{But } T_{\max} = 230.3 \text{ N.m. Same.}$$

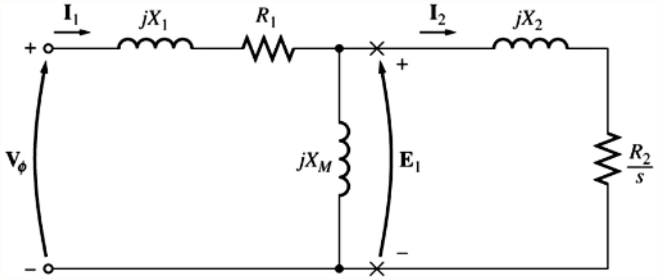
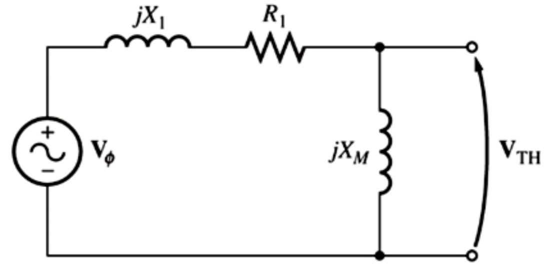
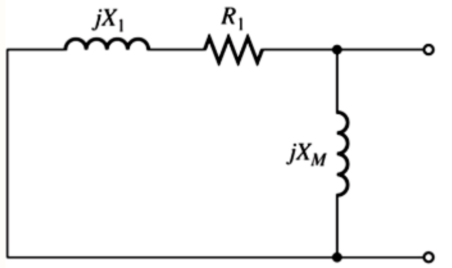
Similarly as in (b), the starting torque can be calculated as 170 N.m.

Summary

In this lecture, you have learnt:

- The analysis of an induction motor using the Thevenin equivalent circuit.
- A typical induction motor torque-speed characteristics curve and its extended operating regions.
- The computations of the maximum torque developed and the speed of the motor at this torque.
- The slip or speed at which maximum torque occurs can be controlled by varying the rotor resistances and the value of the maximum torque is independent of the rotor resistance.

References

No.	Slide No.	Image	Reference
1	3		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 333), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
2	4		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 334), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
3	5		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 334), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

References

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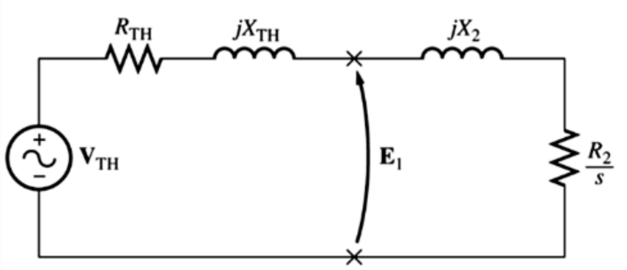
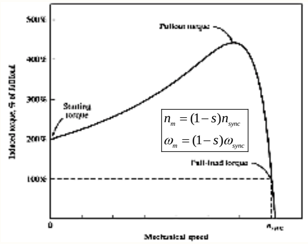
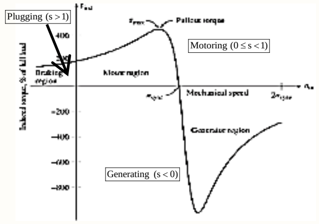
4	6, 12 and 17		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 334), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	7		Adapted from *Electric Machinery Fundamentals, 5th ed.*, (p. 336), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Adapted with permission.
6	8		Adapted from *Electric Machinery Fundamentals, 5th ed.*, (p. 337), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Adapted with permission.

References

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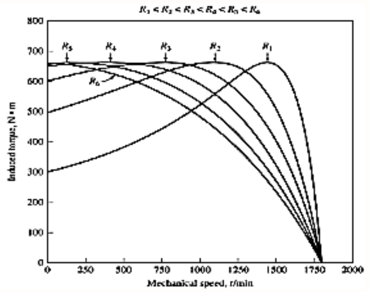
7	15		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 340), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.