NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2021-2022

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

November / December 2021 | Time Allowed: 2.5 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A.

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1. (a) Matrix B is obtained by performing the following Elementary Row Operations (EROs) on matrix A: First, $R_1 \leftrightarrow R_2$, then $R_2 \leftarrow R_2 + \beta R_3$, followed by $R_4 \leftarrow \alpha R_4$, where α and β are non-zero constants.

(i) How is the determinant of B related to the determinant of A?

(ii) In general, matrix B can be expressed as $B = EA$. If A is a 4×7 matrix, write down the matrix E as well as its inverse.

(iii) Let C be a matrix of 4 columns, i.e., $C = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$. If $D = CE^T$ where E is the matrix obtained from part (ii). Express the columns of D in terms of the columns of C.

(15 Marks)

(b) Show that

$$\det\left(\begin{bmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{bmatrix}\right) = 0$$

represents the equation of the line passing through the points (a_1, b_1) and (a_2, b_2).

(5 Marks)

(c) If $B = M^{-1}AM$, how is $\det(A)$ related to $\det(B)$? Hence, compute $\det(A^{-1}B)$. Show your working clearly and justify your answer.

(5 Marks)

2. (a) Consider the following system

$$\begin{aligned} x + 4y - 2z &= 1 \\ 2x + 7y - 6z &= 6 \\ 3y + qz &= t \end{aligned}$$

where q and t are unknown real numbers. For what values of q and t will this system has (i) unique solution, (ii) many solutions, and (iii) no solution? Hence, find the solution that has $z = 1$.

(10 Marks)

(b) What is the maximum number of vectors that can be taken from the following to form a set of linearly independent vectors? Give one example of this set of linearly independent vectors.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix},$$

Note: Question No. 2 continues on page 3.

$$\mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

(5 Marks)

(c) For what values of a will the following matrix be non-singular?

$$\begin{bmatrix} a & 2 & 3 & 4 \\ a & a & 5 & 6 \\ a & a & a & 7 \\ a & a & a & a \end{bmatrix}$$

(5 Marks)

(d) Consider the matrix $A = \begin{bmatrix} -16 & 2 & 24 \\ 11 & -1 & 12 \\ -16 & 2 & 23 \end{bmatrix}$. Determine which of the following vectors is (are) eigenvector(s) of A, and if so, what is (are) the corresponding eigenvalue(s)?

$$\mathbf{v}_1 = \begin{bmatrix} 8 \\ 4 \\ 6 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ -3 \\ -2 \end{bmatrix}.$$

(5 Marks)

3. (a) For the following function, does its limit at the origin exist?

$$f(z) = \frac{5x^3y^2}{2x^5 + 2y^5} + i\,6xy^2$$

(5 Marks)

(b) Using the Cauchy-Riemann equations, determine the analyticity of the following function and find its derivatives at the points where they exist.

$$f(z) = \frac{z^{100} + i}{z^{100}}$$

(8 Marks)

(c) Evaluate the integral $\oint_C \frac{1}{z(z^2+1)}\, dz$, along each of the following counter-clockwise paths: (i) $C : |z| = \frac{1}{2}$, (ii) $C : |z| = \frac{3}{2}$, and (iii) $C : |z - i| = \frac{3}{2}$.

(7 Marks)

(d) Determine the real and imaginary parts of $(1 + \cos\theta + i\sin\theta)^n$.

(5 Marks)

4. (a) Consider the function $f(x, y, z) = xyz$. Find its derivative along the downward normal direction of the surface $2z - xy = 0$ at the point $(2, 3, 3)$.

(8 Marks)

(b) Find the work done in moving a particle in the force field given by $\mathbf{F} = 2xyz^2\mathbf{i} + (x^2z^2 + \cos y)\mathbf{j} + 2x^2yz\mathbf{k}$ from $(1, \pi/2, 0)$ to $(7, \pi/6, 6)$ along an arbitrary path.

(11 Marks)

(c) Given that $u = zx^2y$ and $v = x^2 + y^2 - z^2$, determine $\nabla\cdot(\nabla u \times \nabla v)$ and $\nabla\times(\nabla u \times \nabla v)$.

(6 Marks)

END OF PAPER

Appendix A

Some Useful Formulae for Complex Analysis

1. Complex Power: $z^c = e^{c \ln z}$

2. Euler's Formula: $e^{ix} = \cos x + i \sin x$

3. De Moivre's Formula: $(\cos x + i \sin x)^n = \cos nx + i \sin nx$

4. Cauchy-Riemann equations:

$$u_x = v_y, \quad v_x = -u_y, \quad \text{or} \quad u_r = \frac{1}{r} v_\theta, \quad v_r = \frac{-1}{r} u_\theta$$

5. Derivative, if exists: $f'(z) = u_x + i v_x = e^{-i\theta}(u_r + i v_r)$

6. Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m} dz = \frac{2\pi i}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} f(z)|_{z=z_o}$$

Some Useful Formulae for Vector Calculus

Let $\mathbf{F} = F_1\,\mathbf{i} + F_2\,\mathbf{j} + F_3\,\mathbf{k}$.

1. Scalar Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$

2. Gradient: $\nabla f = \frac{\partial f}{\partial x}\,\mathbf{i} + \frac{\partial f}{\partial y}\,\mathbf{j} + \frac{\partial f}{\partial z}\,\mathbf{k}$

3. Divergence: $\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

4. Curl: $\nabla \times \mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

5. Gauss Theorem: $\iiint_T \nabla \cdot \mathbf{F}\, dv = \iint_S \mathbf{F} \cdot \mathbf{n}\, dA$

6. Stokes Theorem: $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dA = \int_C \mathbf{F} \cdot \mathbf{dr}$

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.