

EE2007 / IM2007

**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER 1 EXAMINATION 2018-2019**

**EE2007 / IM2007 – ENGINEERING MATHEMATICS II**

November / December 2018

Time Allowed: 2½ hours

**INSTRUCTIONS**

1. This paper contains 4 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A on page 4.

1. (a) Let  $E$  and  $F$  be elementary matrices corresponding to the elementary row operations. If  $E$  adds row 1 to row 2, and  $F$  adds row 2 to row 1, does  $EF$  equal  $FE$ ? Justify your answer.

(5 Marks)

- (b) Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad |A| = 5$$

and

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^n A \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^n$$

where  $n$  is a positive integer.

- (i) With  $n=1$ , explain, in terms of row and column operations, the operations that were performed on matrix  $A$ .
- (ii) Write down the resulting matrix  $B$  when  $n$  is odd, i.e.,  $n=1,3,5,\dots$  and the determinant of matrix  $B$ . Justify your answer.

Note: Question 1 continues on page 2

EE2007 / IM2007

- (iii) Write down the resulting matrix  $B$  when  $n$  is even, i.e.,  $n = 2, 4, 6, \dots$  and the determinant of matrix  $B$ . Justify your answer.

(10 Marks)

- (c) Find a LU decomposition of the matrix

$$\begin{bmatrix} a & r & r & r & r \\ a & b & s & s & s \\ a & b & c & t & t \\ a & b & c & d & u \\ a & b & c & d & e \end{bmatrix}$$

You may assume that the variables  $a, b, c, d, e, r, s, t, u$  are unique, non-zero real numbers.

(10 Marks)

2. Let  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$  be vectors in vector space  $\mathbf{V}$ .

- (a) Explain how you would determine whether  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$  are linearly independent or dependent. Hence determine whether the following are linearly independent or dependent. Justify your answers.

- (i) The vectors  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ ;

- (ii) The matrices  $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$  and  $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ ;

- (iii) The polynomials  $p(x) = 1 + x$ ,  $q(x) = 1 - x$  and  $h(x) = 1 - x^2$ .

(15 Marks)

- (b) Consider the following matrix  $A$  and vector  $\mathbf{v}$ :

$$A = \begin{bmatrix} 1 & -1 & 4 & 0 & -3 \\ 0 & 1 & -5 & -1 & 2 \\ 0 & 1 & 3 & -1 & 0 \\ 1 & 0 & -1 & -1 & 0 \end{bmatrix}; \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Determine whether  $\mathbf{v}$  is in the column space of  $A$ . Hence, or otherwise, find a basis for the column space of  $A$ .

It is said that the set of basis vectors for a vector space is not unique. Therefore, from the five columns of  $A$ , one can form many bases. Which two columns of  $A$  must be included in all the bases that will span the column space of  $A$ ?

Justify all your answers.

(10 Marks)

EE2007 / IM2007

3. (a) By considering only the principal value of the natural logarithm, and given

$$(z^3 - 4) \ln 5 = 4\sqrt{3} \ln [\cos(\ln 5) + i \sin(\ln 5)],$$

find  $z$  in the exponential form, leaving your answer in terms of  $\pi$ .

(8 Marks)

- (b) Express  $f(z) = \exp(iz) - \exp(-iz)$ ,  $z = x + iy$ , in the form  $f(z) = u(x, y) + i v(x, y)$ , where  $x, y, u(x, y)$  and  $v(x, y)$  are real. Hence, discuss the differentiability and analyticity of  $f(z) = \exp(iz) - \exp(-iz)$  by using the Cauchy-Riemann equations.

(8 Marks)

- (c) Evaluate  $\int_{-\infty}^{+\infty} \frac{1}{(x^2 - 2x + 3)^2} dx$ .

(9 Marks)

4. (a) Show that

$$\nabla \times (f \mathbf{v}) = f(\nabla \times \mathbf{v}) + (\nabla f) \times \mathbf{v}$$

for scalar function  $f(x, y, z)$  and vector function

$$\mathbf{v}(x, y, z) = v_1(x, y, z) \mathbf{i} + v_2(x, y, z) \mathbf{j} + v_3(x, y, z) \mathbf{k}.$$

(8 Marks)

- (b) Find the work done in moving a particle along the path  $x = \cos y, z = 0$  from  $y = 0$  to  $y = 2\pi$ , in the field

$$\mathbf{F}(x, y, z) = e^{2z} \cos y \mathbf{i} - x e^{2z} \sin y \mathbf{j} + 2x e^{2z} \cos y \mathbf{k}.$$

(10 Marks)

- (c) Evaluate  $\iint_S \mathbf{F} \cdot d\mathbf{A}$  for surface  $S: x = z^2, 0 \leq y \leq 2, -1 \leq z \leq 1$  and

$$\mathbf{F}(x, y, z) = 3y^2 \mathbf{i} + z e^x \cos y \mathbf{j} + 3xz^2 \mathbf{k}.$$

(7 Marks)

## Appendix A

### 1. Complex Analysis

(a) Complex Power:  $z^c = e^{c \ln z}$

(b) De Moivre's Formula:  $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

(c) Cauchy-Riemann equations:

$$u_x = v_y, \quad v_x = -u_y, \quad \text{or} \quad u_r = \frac{1}{r} v_\theta, \quad v_r = -\frac{1}{r} u_\theta$$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m} dz = \frac{2\pi i}{(m-1)!} \left. \frac{d^{(m-1)}}{dz^{(m-1)}} f(z) \right|_{z=z_o}$$

### 2. Vector Analysis. Let $\mathbf{F} = F_1 \mathbf{i} + F_2 \mathbf{j} + F_3 \mathbf{k}$ .

(a) Scalar Triple Product:  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$

(b) Gradient:  $\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$

(c) Divergence:  $\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl:  $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Gauss Theorem:  $\iiint_T \nabla \cdot \mathbf{F} dv = \oiint_S \mathbf{F} \cdot \mathbf{n} dA$

(f) Stokes Theorem:  $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dA = \oint_C \mathbf{F} \cdot d\mathbf{r}$

END OF PAPER







**EE2007 ENGINEERING MATHEMATICS II**  
**IM2007 ENGINEERING MATHEMATICS II**

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.