EE3001 Tutorial #1

Q1. A point charge Q_1 is situated at (a, 0, 0), as shown in Figure 1. In addition, there is a line charge of length 2L which is situated at $x = -a$ and carries a total charge Q_2 which is uniformly distributed along the line.

(i) Find the electric field intensity $\vec{\mathbf{E}}_1$ at (0, 0, 0) due to the point charge Q_1 only.

(ii) Find the electric field intensity $\vec{\mathbf{E}}_2$ at (0, 0, 0) due to the line charge only. Given

$$\left[\int \frac{dx}{\left(a^2+x^2\right)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{a^2+x^2}}\right]$$

(iii) Find the relation between Q_1 and Q_2 such that the total electric field at the origin is zero.

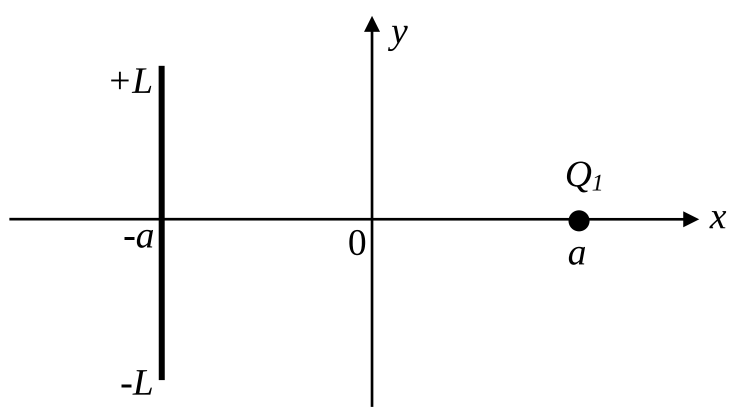


Figure 1

Q2. A square conducting plate of side length 2a, positioned in the x-y plane and centred at the origin, is charged with a uniform surface charge density ρ_0. Determine the electric field intensity $\vec{E}$ at (0, 0, z) due to this surface charge distribution.

[Given $\int_0^a\int_0^a \frac{dxdy}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} = \frac{1}{z}\tan^{-1}\frac{a^2}{z\sqrt{2a^2+z^2}}$].

Answers:

Q1. $\vec{\mathbf{E}}_1 = \frac{-Q_1}{4\pi\varepsilon_0 a^2}\vec{\mathbf{a}}_x, \quad \vec{\mathbf{E}}_2 = \frac{Q_2}{4\pi\varepsilon_0 a\left(a^2+L^2\right)^{\frac{1}{2}}}\vec{\mathbf{a}}_x, \quad Q_1 = \frac{aQ_2}{\left(a^2+L^2\right)^{\frac{1}{2}}}$

Q2. $\vec{\mathrm{E}} = \frac{\rho_0}{\pi\varepsilon_0}\tan^{-1}\frac{a^2}{z\sqrt{2a^2+z^2}}\vec{a}_z \qquad \mathrm{V}/m$

EE3001 Tutorial #2

Q1. By expansion in Cartesian system of coordinates, verify the null identities:

$$\text{(i) } \nabla \times \nabla f = 0; \quad \text{(ii) } \nabla \bullet \left(\nabla \times \vec{\mathbf{A}}\right) = 0$$

Q2. Verify the divergence theorem for a vector field

$$\vec{\mathrm{A}}(x, y, z) = xy\vec{\mathbf{a}}_x + yz\vec{\mathbf{a}}_y + xz\vec{\mathbf{a}}_z$$

for a closed surface (cube) defined by

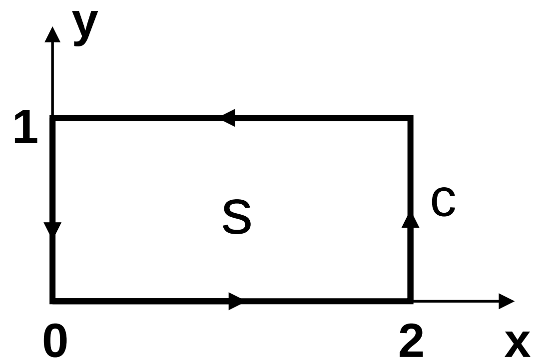
$$0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1.$$

Q3. Assume a vector field $\vec{\mathbf{A}} = \vec{\mathbf{a}}_x\left(2x^2 + y^2\right) + \vec{\mathbf{a}}_y\left(2xy - y^2\right)$.

(a) Find $\oint_C \vec{\mathbf{A}} \bullet \vec{\mathbf{dl}}$ around the rectangular contour shown in the figure below.

(b) Find $\int_S \left(\nabla \times \vec{\mathbf{A}}\right) \bullet \vec{\mathbf{ds}}$ over the rectangular area.

(c) Is $\vec{\mathbf{A}}$ irrotational?



Answers:

Q2. $\oiint_s \vec{\mathrm{A}} \bullet \vec{\mathbf{ds}} = \frac{3}{2}; \quad \nabla \bullet \vec{\mathrm{A}} = x + y + z$ Q3. $\oint_C \vec{\mathbf{A}} \bullet \vec{\mathbf{dl}} = 0; \ \nabla \times \vec{\mathbf{A}} = 0$

EE3001 Tutorial #3

Q1. Two parallel circular disks are both of radius a. One is positioned in the x-y plane and centered at the origin, and the other is parallel to the x-y plane and is centered at (0, 0, d). Assume that d is positive. The top disk is uniformly charged with a charge density ρ_s, and the bottom disk is uniformly charged with a charge density $-\rho_s$ .Determine the electric potential $V(z)$ and the field intensity $\vec{\mathbf{E}}(z)$ at a point (0, 0, z) along the z-axis.

$$\left[Given \quad \int \frac{xdx}{\sqrt{x^2+b}} = \sqrt{x^2+b} \right]$$

Q2. A wire loop has the shape of an equilateral triangle of side length a. The loop is positioned in the x-y plane and centred at the origin, as shown in Figure 1. The loop is uniformly charged with a line charge density ρ_l.

(i) Determine the electric potential V at a point (0, 0, z). It is given that $\int \frac{dy}{\sqrt{y^2+b}} = \ln\left(y+\sqrt{y^2+b}\right)$

(ii) Using the above result, determine the electric field intensity $\vec{\mathbf{E}}$ at a point (0, 0, z).

(iii) What major change, if any, is expected in the value of $\vec{\mathbf{E}}$ if it is evaluated at points other than the z-axis? Justify your answer.

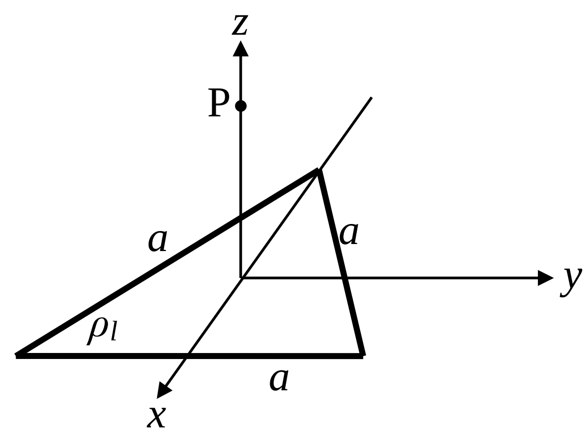


Figure 1

Answers:

Q1. $$V(z)=\frac{\rho_S}{2\varepsilon}\left[\sqrt{(z-d)^2+a^2}-|z-d|\right]-\frac{\rho_S}{2\varepsilon}\left[\sqrt{z^2+a^2}-|z|\right]$$

$$\vec{\mathbf{E}}=\vec{\mathbf{a}}_Z\frac{\rho_S}{2\varepsilon_0}\left[\frac{z-d}{|z-d|}-\frac{z-d}{\sqrt{(z-d)^2+a^2}}\right]-\vec{a}_Z\frac{\rho_S}{2\varepsilon_0}\left[\frac{z}{|z|}-\frac{z}{\sqrt{z^2+a^2}}\right]$$

Q2.

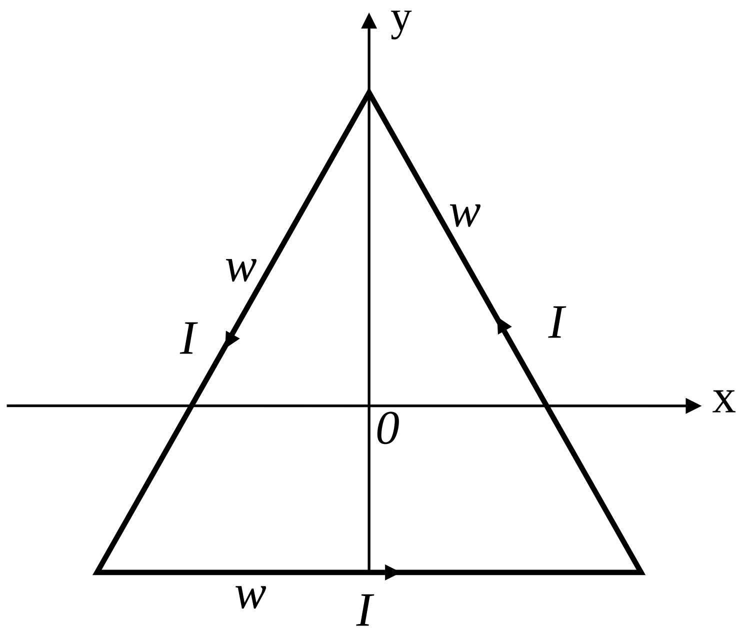
(i) $$V=\frac{3\rho_l}{4\pi\varepsilon_0}\left[\ln\left(\frac{a}{2}+\sqrt{\frac{a^2}{4}+h^2+z^2}\right)-\ln\left(-\frac{a}{2}+\sqrt{\frac{a^2}{4}+h^2+z^2}\right)\right]\quad \mathrm{V}$$

(ii)

$$\vec{\mathbf{E}}(z)=-\vec{\mathbf{a}}_z\frac{3\rho_l z}{4\pi\varepsilon_0\sqrt{\frac{a^2}{4}+h^2+z^2}}\left[\frac{1}{\frac{a}{2}+\sqrt{\frac{a^2}{4}+h^2+z^2}}-\frac{1}{-\frac{a}{2}+\sqrt{\frac{a^2}{4}+h^2+z^2}}\right]\quad \mathrm{V}/m$$

EE3001 Tutorial #4

Q1. A thin conducting wire of length 3*w* forms an equilateral triangle in the *x*-*y* plane. A direct current *I* flows in the wire along the counter-clockwise direction. Find the magnetic field intensity at the center of the triangle.



$$\left[Given \quad \int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{x}{a^2\left(x^2+a^2\right)^{1/2}} \right]$$

Q2. A long, straight, solid, conducting cylinder, oriented with its axis coinciding with the *z*-direction, carries a current whose current density is $\vec{\mathbf{J}}$. The current density, although symmetrical about the cylinder axis, is not constant but varies according to the relation:

$$\vec{\mathbf{J}} = \begin{bmatrix} \dfrac{2I_0}{\pi a^2}\left[1-\left(\dfrac{r}{a}\right)^2\right]\vec{\mathbf{a}}_z & for \;\; r \le a, \\ 0 & for \;\; r \ge a, \end{bmatrix}$$

where a is the radius of the cylinder, r is the radial distance from the cylinder axis, and I_0 is a constant having units of Amperes.

(i) Show that I_0 is the total current passing through the entire cross section of the cylinder.

(ii) Using Ampere's law, determine the magnetic field intensities at points ($\frac{a}{2}$, 0, 0) and (2a, $\frac{\pi}{2}$, 0) in the cylindrical coordinate system.

Answers:

Q1. $\vec{\mathbf{H}} = \vec{\mathbf{a}}_z \dfrac{9I}{2\pi w}$

Q2. $\vec{\mathbf{H}}_1 = \dfrac{7I_0}{16\pi a}\vec{\mathbf{a}}_y$ $\qquad \vec{\mathbf{H}}_2 = \dfrac{-I_0\vec{\mathbf{a}}_x}{4\pi a}$

EE3001 Tutorial #5

Q1. The loop of area $4m^2$ in the $x - y$ plane, shown in Fig. 1 below, is situated in a magnetic field with the flux density $\vec{\mathbf{B}} = -\vec{\mathbf{a}}_z 0.3t\ T$. Determine the voltage V_1 and V_2 across the 2-Ω and 4-Ω resistors assuming that the internal resistance of the wire may be ignored.

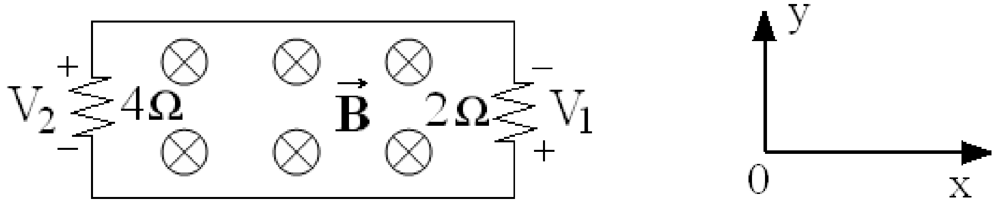


Figure 1

Q2. An infinitely long two-wire line carrying a direct current I_0 is arranged in air as shown in Fig. 2. A square loop of side length a is moving along the x-axis at a constant velocity v_0 starting at $t = 0$ from the position shown in the figure. Determine the:

(i) total magnetic flux density $\vec{\mathbf{B}}$ due to the two-wire line at a point (x,0,0);

(ii) magnetic flux Φ_m passing through the square loop at time t, and,

(iii) induced EMF in the loop at time t.

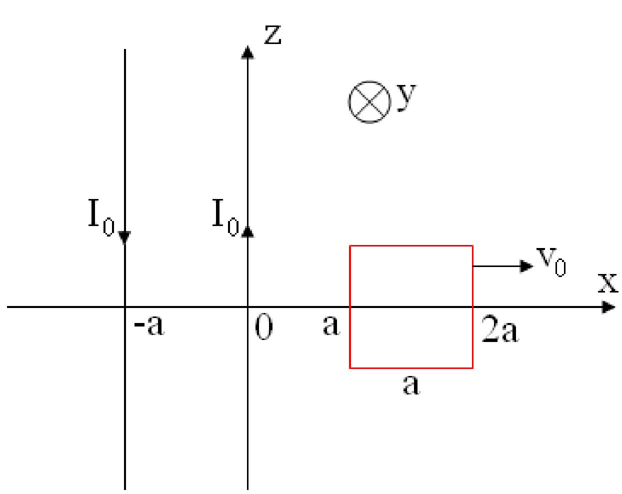


Figure 2

Q3. A bar conductor shown in Fig. 3 is parallel to the y-axis and it completes a loop through sliding contacts with the conductors at $y = 0$ and $y = 0.1\ m$. The loop is under the illumination of a magnetic flux density $\vec{\mathbf{B}} = 2\sin(20\,t)\vec{\mathbf{a}}_Z\ T$. Find the induced voltage when the bar conductor is moving at a velocity of $\vec{\mathbf{u}} = 25\vec{\mathbf{a}}_x\ m/s$, starting from the position shown at $t = 0$.

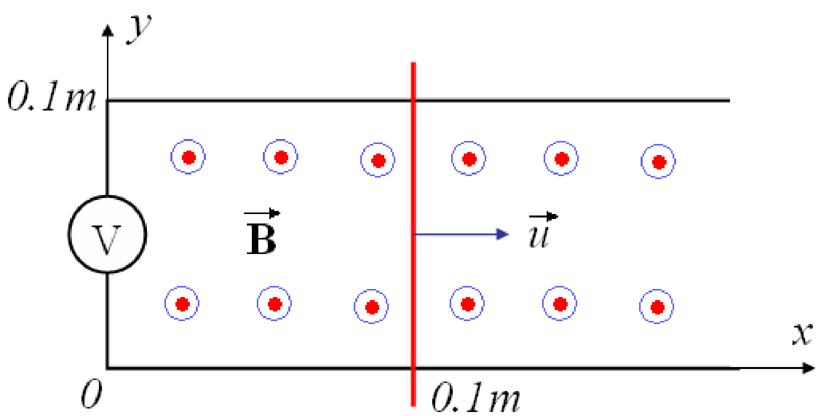


Figure 3

Answers:

Q1. $V_1 = IR_1 = 0.2\times2=0.4\ V$, $V_2 = IR_2 = 0.2\times4=0.8\ V$

Q2.
$$\vec{\mathbf{B}} = \frac{\mu_0 I_0}{2\pi}\left(\frac{1}{x} - \frac{1}{x+a}\right)\vec{\mathbf{a}}_y$$

$$\Phi_m = \frac{\mu_0 I_0 a}{2\pi}\ln\left[\frac{(x_0+a)^2}{x_0(x_0+2a)}\right] \quad , \quad x_0 = a + v_0 t$$

$$emf = -\frac{\mu_0 I_0 a v_0}{\pi}\left[\frac{1}{x_0+a} - \frac{x_0+a}{x_0(x_0+2a)}\right]$$

Q3. $emf = -5\sin(20\,t) - (0.4+100\,t)\cos(20\,t)\ \ V$

EE3001 Tutorial #6

Q1. A 5-GHz uniform plane wave is propagating in a dielectric medium that is characterized by $\epsilon_r = 2.53$ and $\mu_r = 1$. If the electric field intensity is given by $\tilde{\mathbf{E}}(z,t) = \vec{\mathbf{a}}_x 10\cos(\omega t - kz)$ *V/m*, determine:

(i) the wavenumber k, the wavelength λ, and the phase velocity u_p;

(ii) a time-varying expression for the magnetic field intensity.

Q2. The magnetic field intensity of a uniform plane wave propagating in a lossless dielectric medium having $\mu_r = 1$ is given by $\tilde{\mathbf{H}} = \vec{\mathbf{a}}_x 10\cos(6\pi\times10^7 t + 0.4\pi z)$ *A/m*. Determine the direction of propagation of the wave, frequency, wavelength, phase velocity, permittivity of the medium and the corresponding electric field intensity $\tilde{\mathbf{E}}$.

Q3. A 1-MHz plane wave is propagating along the positive z-axis in a conducting medium with $\epsilon_r = 8$, $\sigma = 4.8\times10^{-2}$ *S/m* and $\mu_r = 1$. Determine:

(i) The ratio between the magnitudes of the conduction current and the displacement current.

(ii) A time-varying expression for $\tilde{\mathbf{E}} = \vec{\mathbf{a}}_x E_x$, if the maximum magnitude of the sinusoidal variation of the x-directed electric field is 100 *V/m* at $t = 0$ and $z = 0.3\pi$ *m*.

(iii) The corresponding magnetic field intensity $\tilde{\mathbf{H}}(z,t)$.

Answers:

Q1. $k = 166.567\ rad/m$, $\lambda = 0.0377\ m$, $u_p = 1.89 \times 10^8\ m/s$

$$\tilde{\mathbf{H}}(z,t) = \vec{\mathbf{a}}_y 0.042\ \cos(10\pi \times 10^9 t - 166.567\ z)\ A/m.$$

Q2. The wave is propagating along the –z axis.

$f = 3 \times 10^7\ Hz$, $\lambda = 5\text{m}$, $u_p = 1.5 \times 10^8\ m/s$, $\epsilon_r = 4$

$$\tilde{\mathbf{E}}(z,t) = \vec{\mathbf{a}}_y 600\pi \cos(6\pi \times 10^7 t + 0.4\pi\ z)\,V/m$$

Q3. $\left|\dfrac{\vec{\mathbf{J}}_c}{\vec{\mathbf{J}}_d}\right| = 108,$

$$\tilde{\mathbf{E}} = \vec{\mathbf{a}}_x 150.7 e^{-0.435z}\ \cos(2\pi \times 10^6 t - 0.435z + 0.41)\ V/m$$

$$\tilde{\mathbf{H}}(z,t) = \vec{\mathbf{a}}_y 11.75 e^{-0.435z}\ \cos(2\pi \times 10^6 t - 0.435z - 0.38)\ A/m$$