Circuit Analysis

EE2001

NANYANG TECHNOLOGICAL UNIVERSITY

AC Power Analysis

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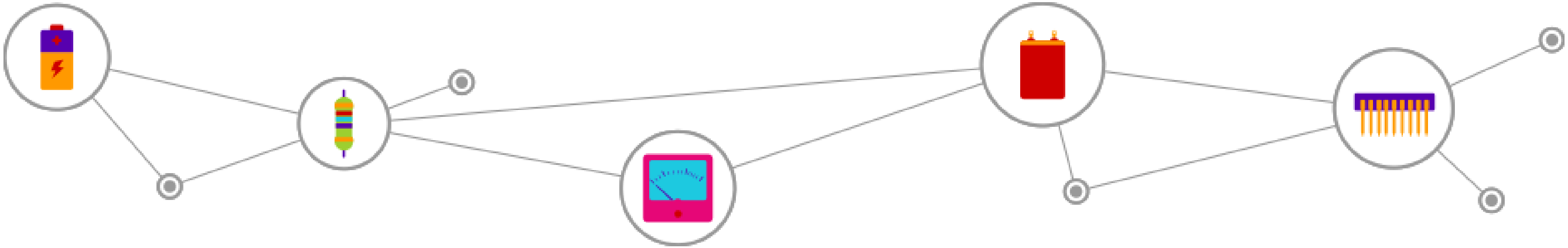
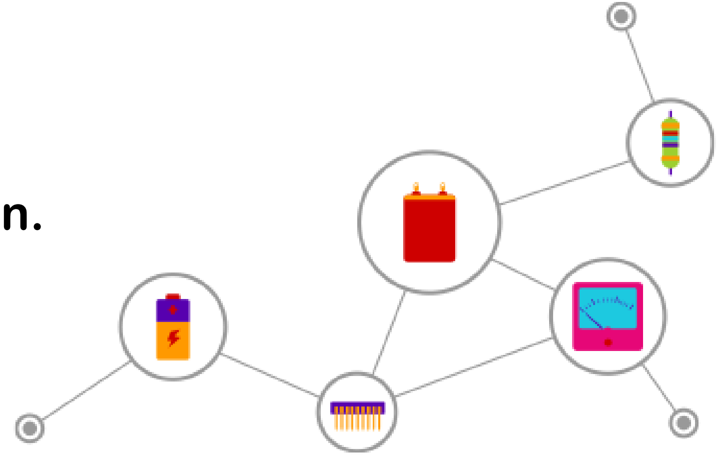
Overview

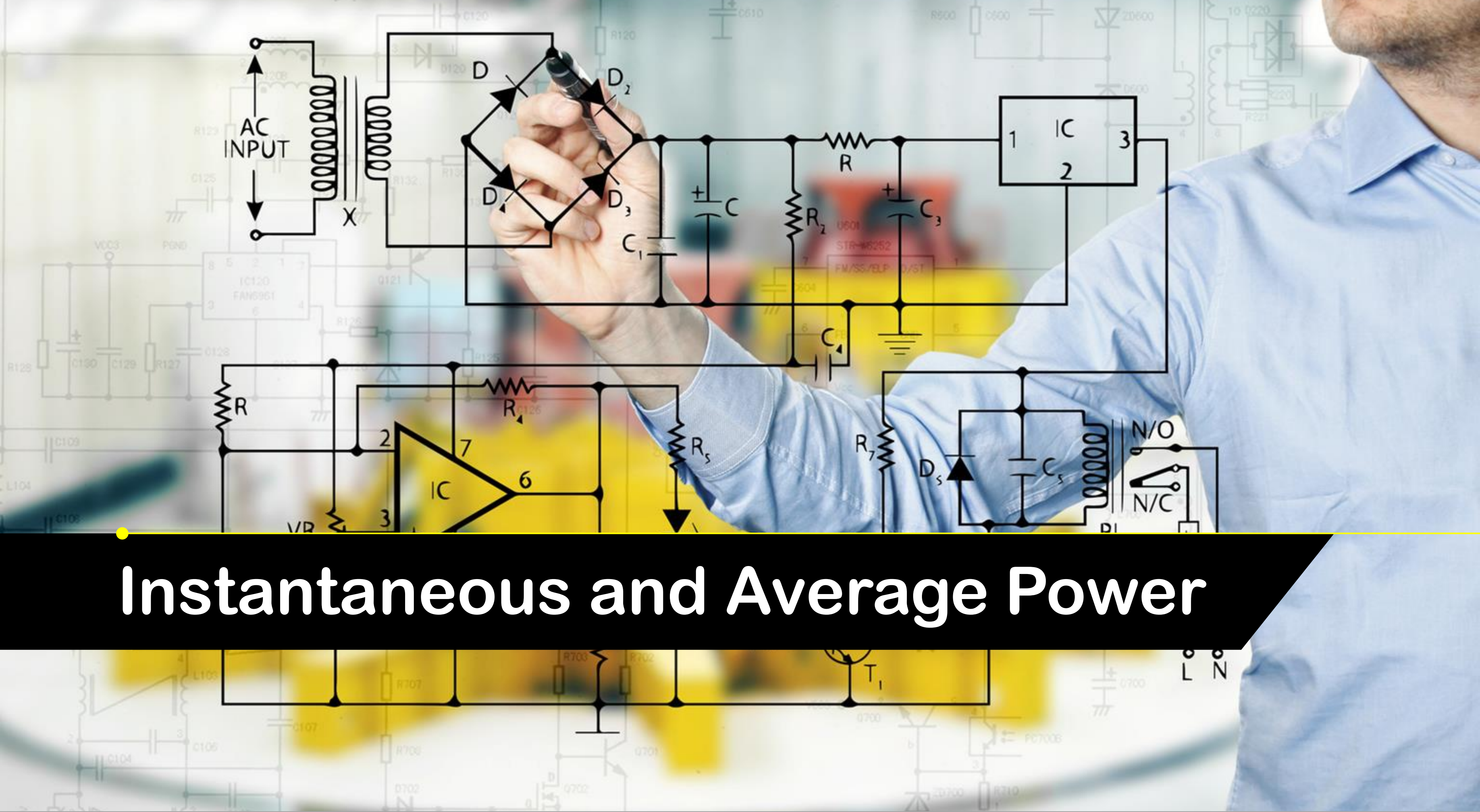
- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Conservation of AC Power
- Power Factor Correction
- Power Measurement



By the end of this lesson, you should be able to…

- Calculate instantaneous power and average power.
- Calculate maximum average power.
- Calculate RMS values.
- Calculate apparent power and power factor.
- Calculate complex power.
- Explain how an AC power can be conserved.
- Explain the process of power factor correction.
- Explain how power is measured.





Instantaneous and Average Power

Instantaneous and Average Power

Power is a very important quantity in electric utilities, electronics and communication systems as such systems involve the transmission of power from one point to another.

Every industrial and household device, such as fans, motors, lamps, televisions and personal computers, has a power rating that indicates how much power the equipment requires.

The most common form of electric power is the 50 Hz AC power in Singapore.

Instantaneous and Average Power

Power

The instantaneous power, $p(t)$, absorbed by an element, is the product of the instantaneous voltage, $v(t)$, across the element and the instantaneous current $i(t)$ through it.

$$p(t) = v(t)i(t)$$

$p(t)$ is the power (in watts) at any instant of time.

It is the rate at which an element absorbs energy.

Instantaneous and Average Power

Let

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Using this equation, it can be shown that the instantaneous power

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$p(t) = v(t)i(t)$$

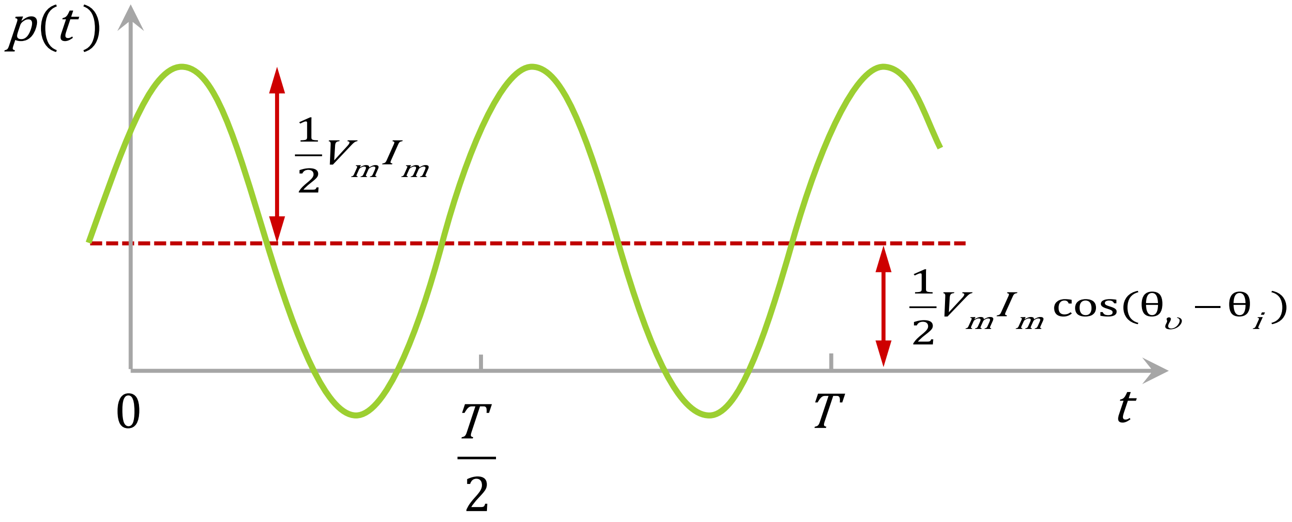
$$= V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

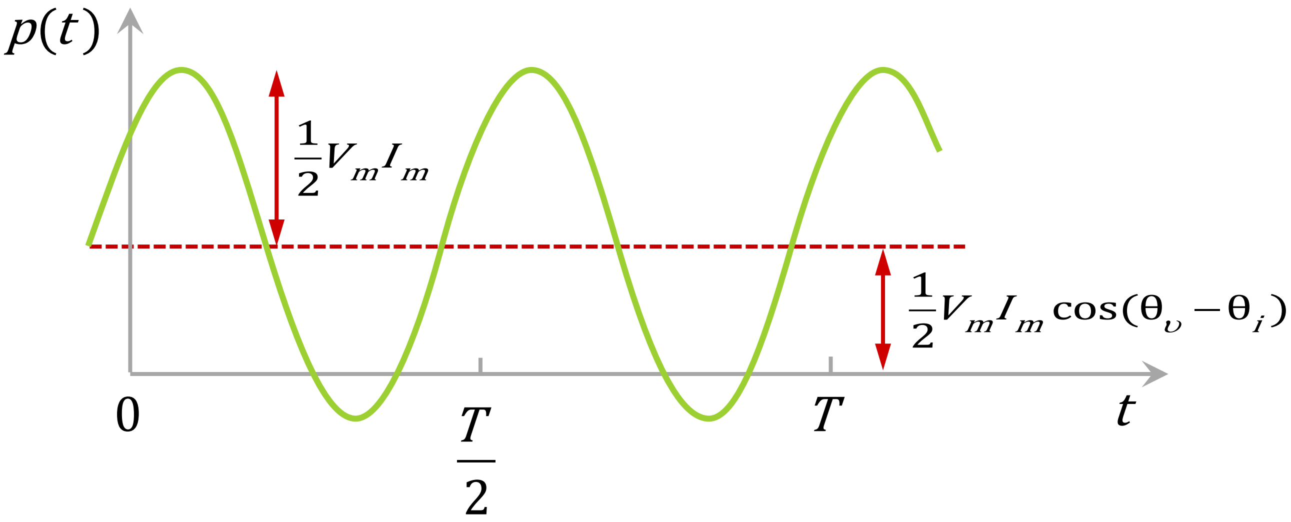
Instantaneous and Average Power

The instantaneous power

$$p(t) = \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant Power}} + \underbrace{\frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal Power at } 2\omega t}$$



Instantaneous and Average Power



$p(t) > 0$: **power is absorbed by the circuit**

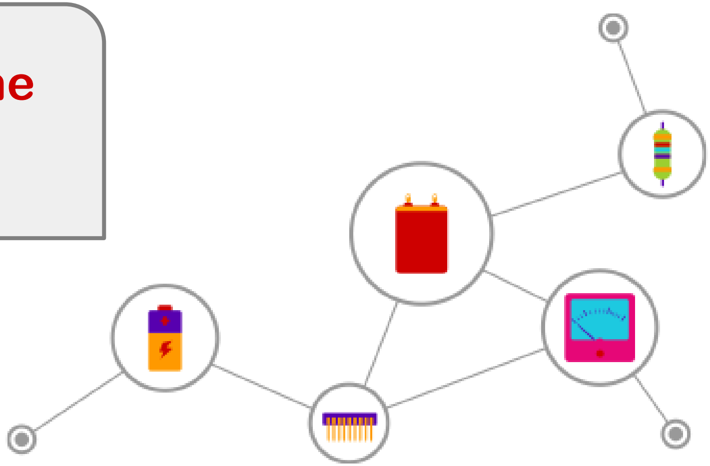
$p(t) < 0$: **power is absorbed by the source**

Instantaneous and Average Power

Instantaneous power changes with time. Hence, it is difficult to measure.

Average power is more convenient to measure. The wattmeter measures average power.

The average power, P, **is the average of the instantaneous power over one period.**



Instantaneous and Average Power

Using this equation

$$p(t) = \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant Power}} + \underbrace{\frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal Power at } 2\omega t}$$

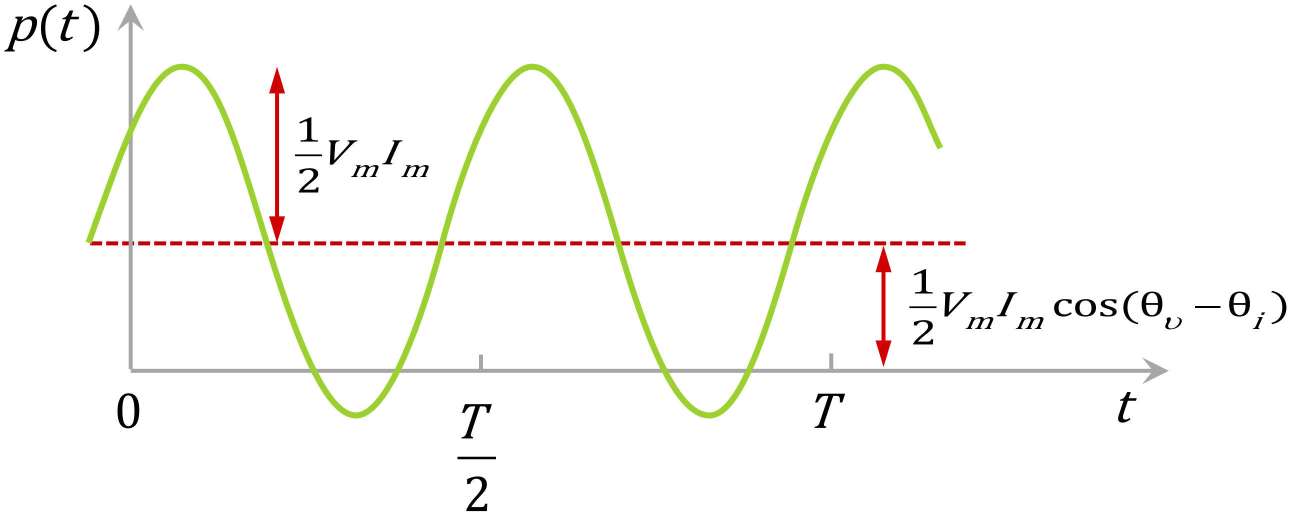
$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{T}\int_0^T \left(\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\right)dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Instantaneous and Average Power

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$



Instantaneous and Average Power

We can also find the average power using phasors (frequency domain).

The phasor forms of *v(t)* and *i(t)* are

$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

Notice that

$$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m \angle(\theta_v - \theta_i) \Rightarrow \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m \left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

The average power

$$P = \frac{1}{2}\mathrm{Re}(\mathbf{V}\mathbf{I}^*) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Instantaneous and Average Power

Special Cases

(1) $\theta_v = \theta_i$

Voltage and current are in phase. Purely resistive load R.

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m{}^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

$$|\mathbf{I}|^2 = \mathbf{I} \times \mathbf{I}^*$$

(2) $\theta_v - \theta_i = \pm 90^o$

Purely reactive circuit.

$$P = \frac{1}{2}V_m I_m \cos 90^o = 0$$

A resistive load R absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

Instantaneous and Average Power: Example 1

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 120\cos(377t + 45°)\,\text{V} \qquad i(t) = 10\cos(377t - 10°)\,\text{A}$$

$$p(t) = vi = 1200\cos(377t + 45°)\cos(377t - 10°)$$

$$p(t) = 600\left[\cos(754t + 35^o) + \cos 55^o\right]$$

$$p(t) = 344.2 + 600\cos(754t + 35^o)$$

Average power = 344.2 W

Or

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2}120(10)\cos\left[45^o - (-10^o)\right]$$

$$\boxed{P = 344.2\ \text{W}}$$

Instantaneous and Average Power: Example 2

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\angle 0^o = V_m \angle \theta_v$ is applied across it.

Current through the impedance

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\angle 0^o}{30 - j70} = 1.576\angle 66.8^o \text{ A}$$

The average power

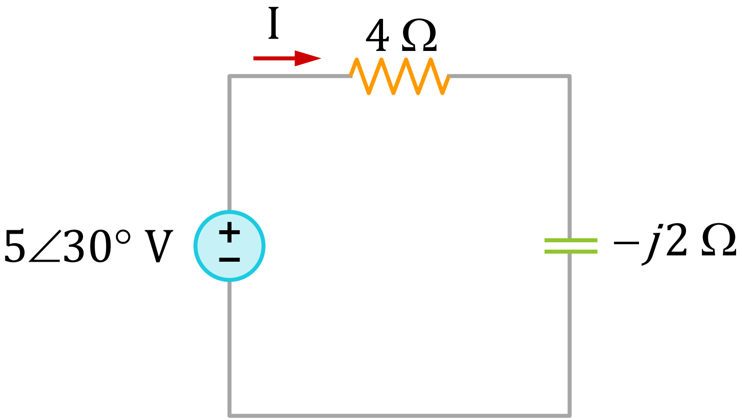
$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2} 120(1.576)\cos(0^o - 66.8^o)$$

$$\boxed{P = 37.24 \text{ W}}$$

Instantaneous and Average Power: Example 3

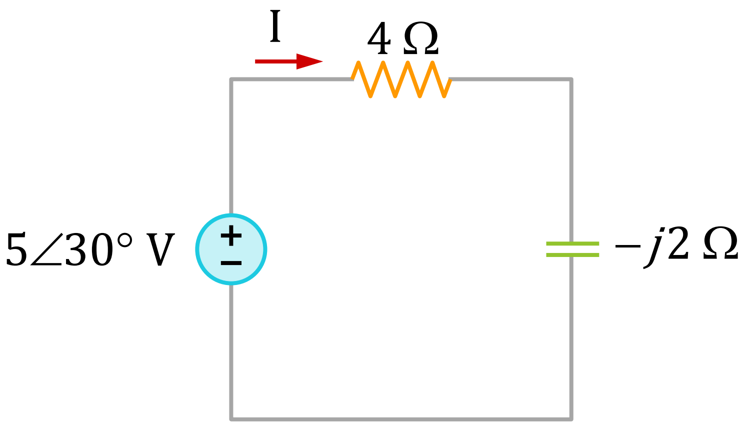
Calculate the average power supplied by the source and the average power absorbed by the resistor.



Instantaneous and Average Power: Example 3

The current

$$\mathbf{I} = \frac{5\angle 30^\circ}{4 - j2} = 1.118\angle 56.57^\circ$$



For the source, the average power is

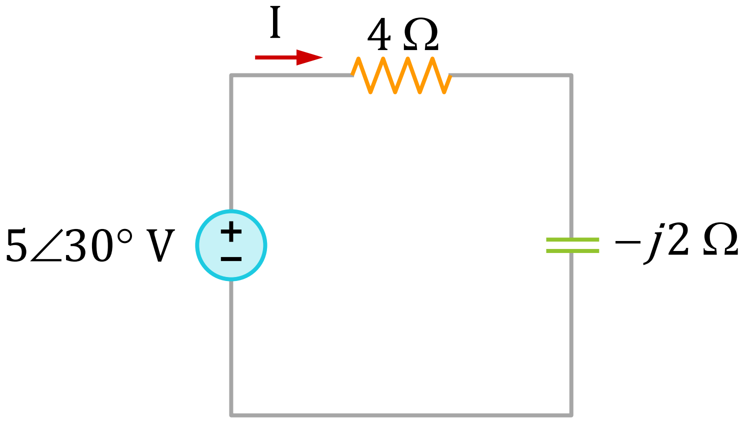
$$P = -\frac{1}{2}5(1.118)\cos(30^o - 56.57^o) = -2.5\text{ W}$$

The negative sign indicates that the source is supplying power to the circuit, based on the passive sign convention.

Instantaneous and Average Power: Example 3

Voltage across resistor

$$\mathbf{V}_R = 4\mathbf{I} = 4.472\angle 56.57°$$



Average power absorbed by resistor

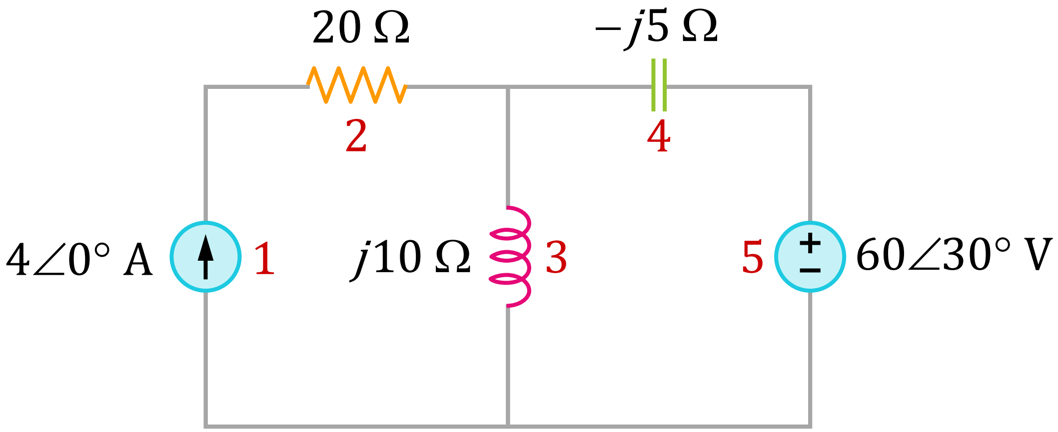
$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

which is the same as average power supplied by the source.

Zero average power is absorbed by capacitor.

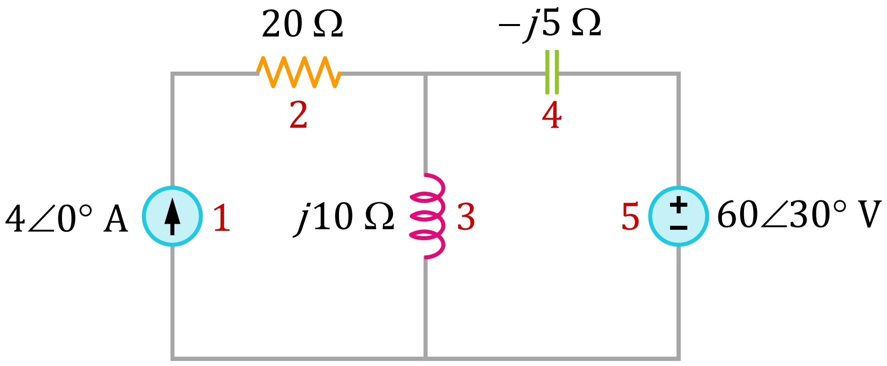
Instantaneous and Average Power: Example 4

Calculate the average power supplied by each source and the average power absorbed by each passive element.

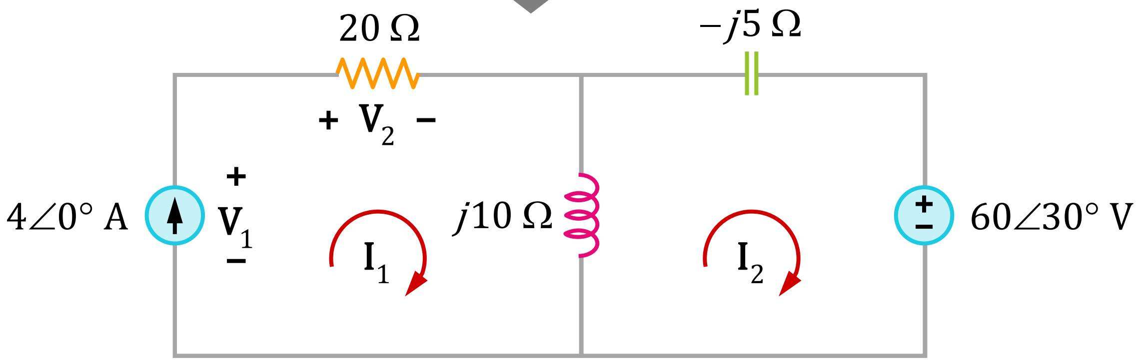


(a)

Instantaneous and Average Power: Example 4



(a)



(b)

Instantaneous and Average Power: Example 4

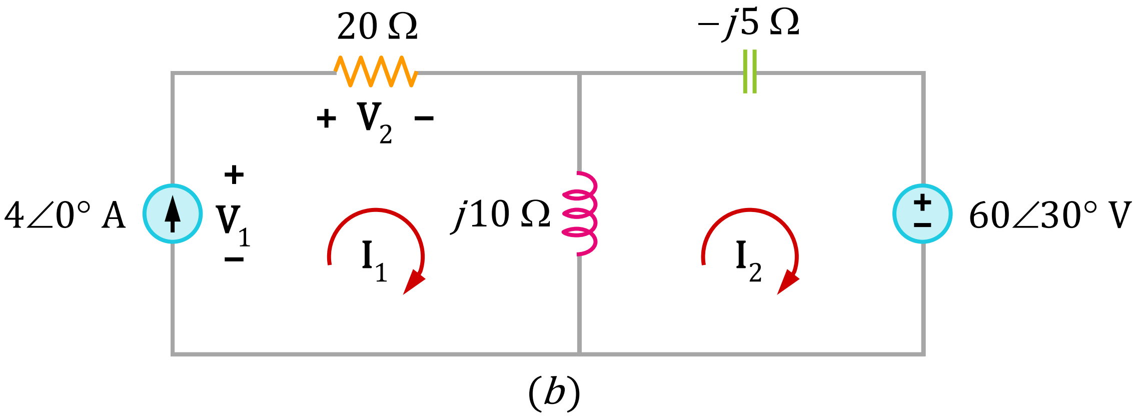
Applying mesh analysis, for mesh 1

$$\boxed{\mathbf{I}_1 = 4\ \text{A}}$$

For mesh 2

$$-j5\mathbf{I}_2 + 60\angle 30^o + j10(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$\boxed{\mathbf{I}_2 = 10.58\angle 79.1^o\ \text{A}}$$



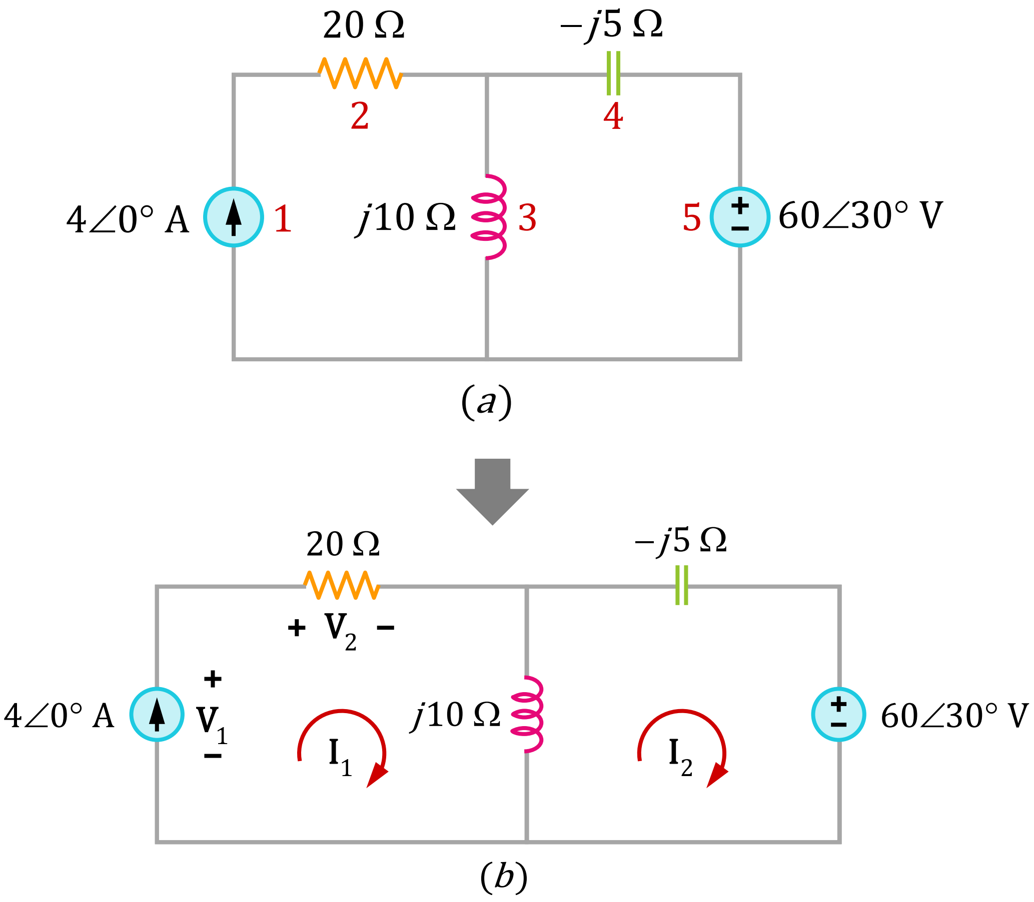
Instantaneous and Average Power: Example 4

For the voltage source, following the passive sign convention, the average power is

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^o - 79.1^o)$$

$$\boxed{P_5 = 207.8 \text{ W}}$$

The voltage source is absorbing average power. The circuit is delivering average power to the voltage source.



Instantaneous and Average Power: Example 4

For the current source, the voltage across it is

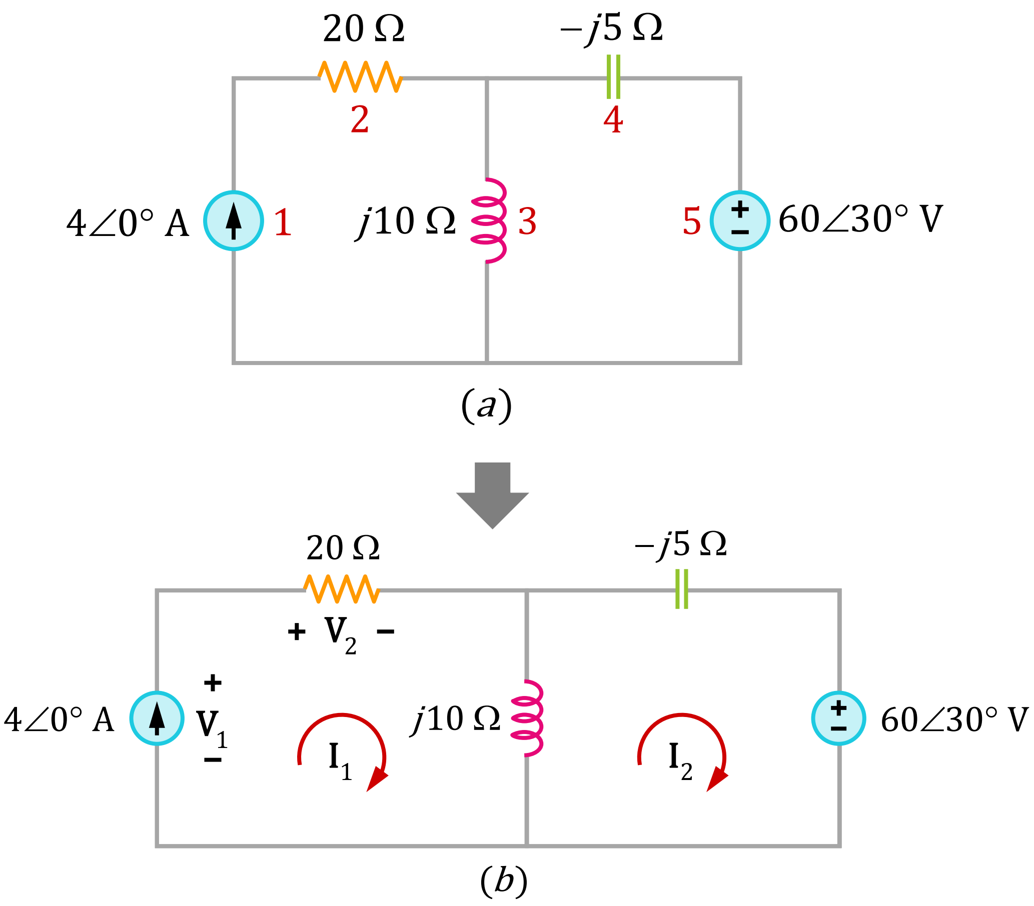
$$\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\mathbf{V}_1 = 184.984\angle 6.21^o\,\mathrm{V}$$

The associated average power is

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^o - 0^o)$$

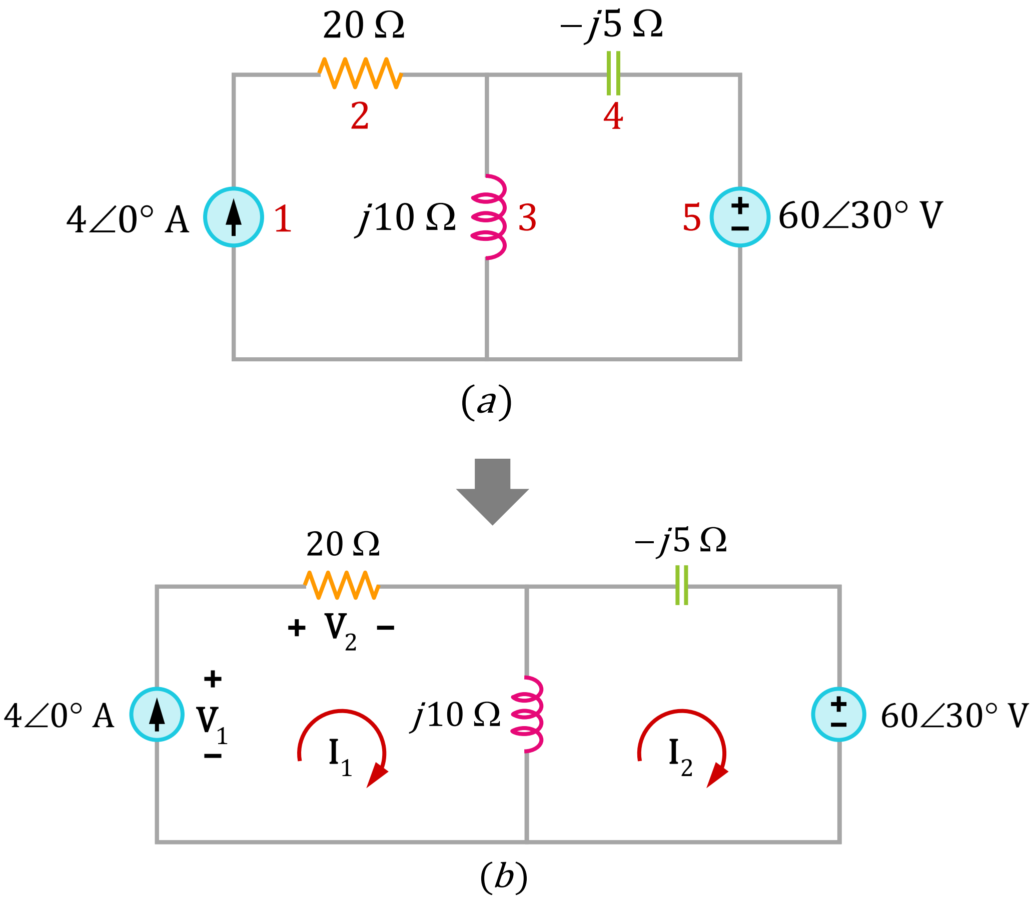
$$\boxed{P_1 = -367.8\ \mathrm{W}}$$



Instantaneous and Average Power: Example 4

$$\boxed{P_1 = -367.8 \text{ W}}$$

It is negative according to the passive sign convention, meaning that the current source is supplying average power to the circuit.



Instantaneous and Average Power: Example 4

For the resistor, the current through it is

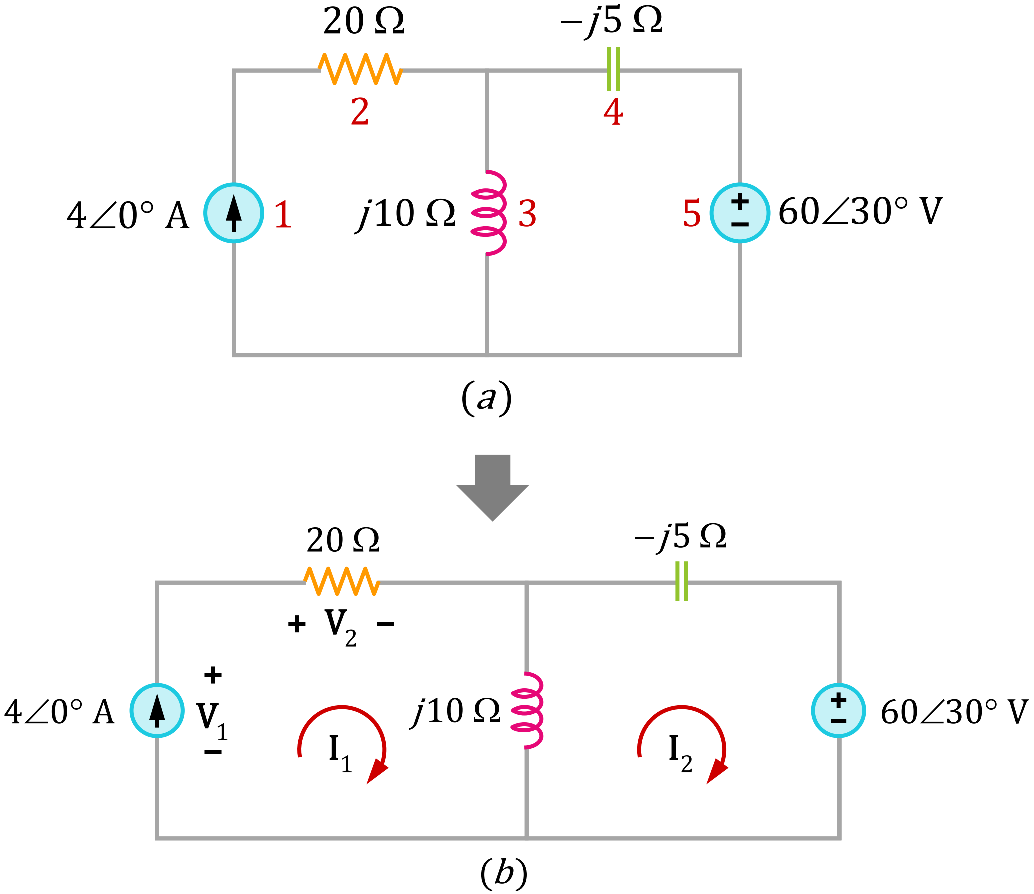
$$\boxed{\mathrm{I}_1 = 4\ \mathrm{A}}$$

The voltage across it is

$$\boxed{20\mathrm{I}_1 = 80\ \mathrm{V}}$$

The average power absorbed is

$$\boxed{P_2 = \frac{1}{2}(80)(4) = 160\ \mathrm{W}}$$



Instantaneous and Average Power: Example 4

For the capacitor, the current through it is

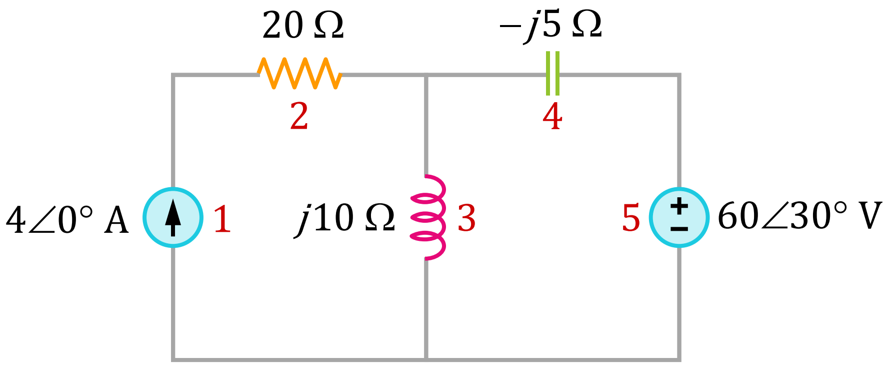
$$\mathbf{I}_2 = 10.58\angle 79.1^o \text{ A}$$

The voltage across it is

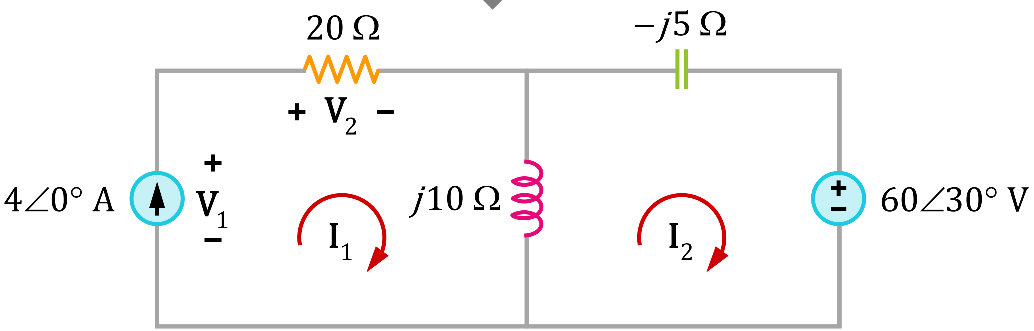
$$-j5\mathbf{I}_2 = 52.9\angle -10.9^o \text{ V}$$

The average power absorbed is

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^o) = 0 \text{ W}$$



(a)



(b)

Instantaneous and Average Power: Example 4

For the inductor, the current through it is

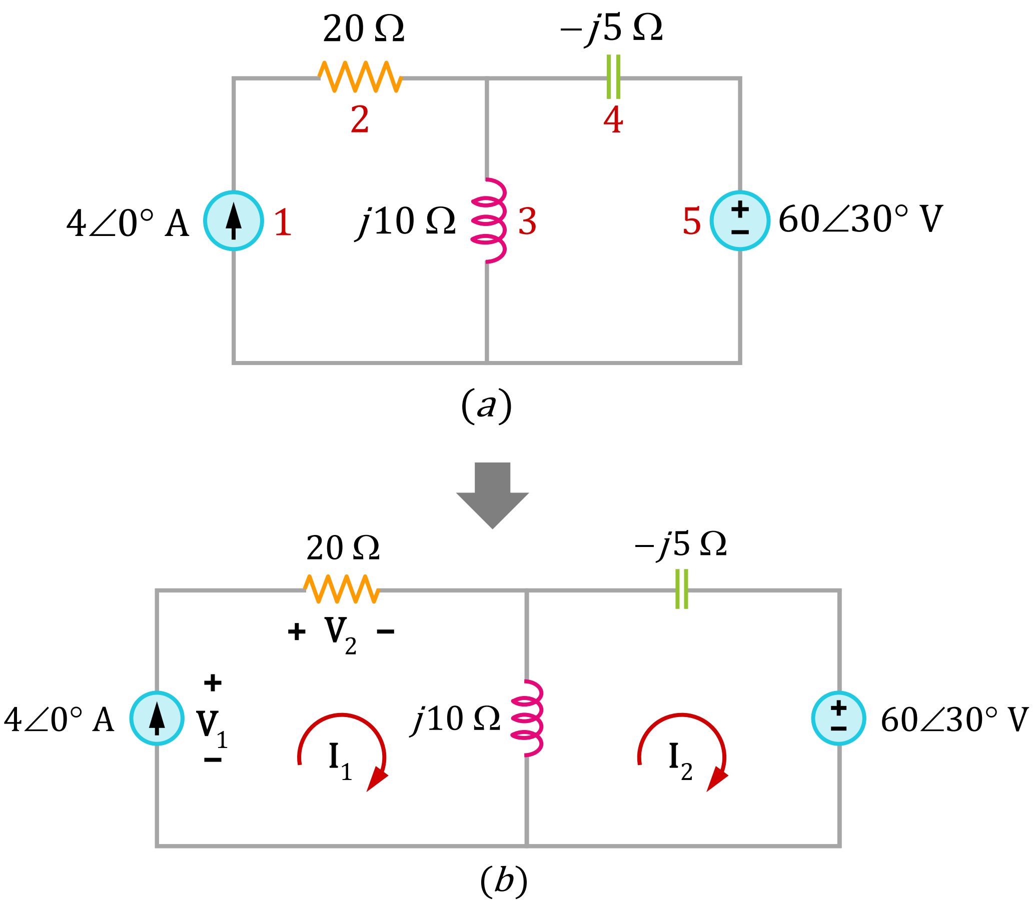
$$\boxed{\mathbf{I}_1 - \mathbf{I}_2 = 10.58\angle -79.1^\circ \text{ A}}$$

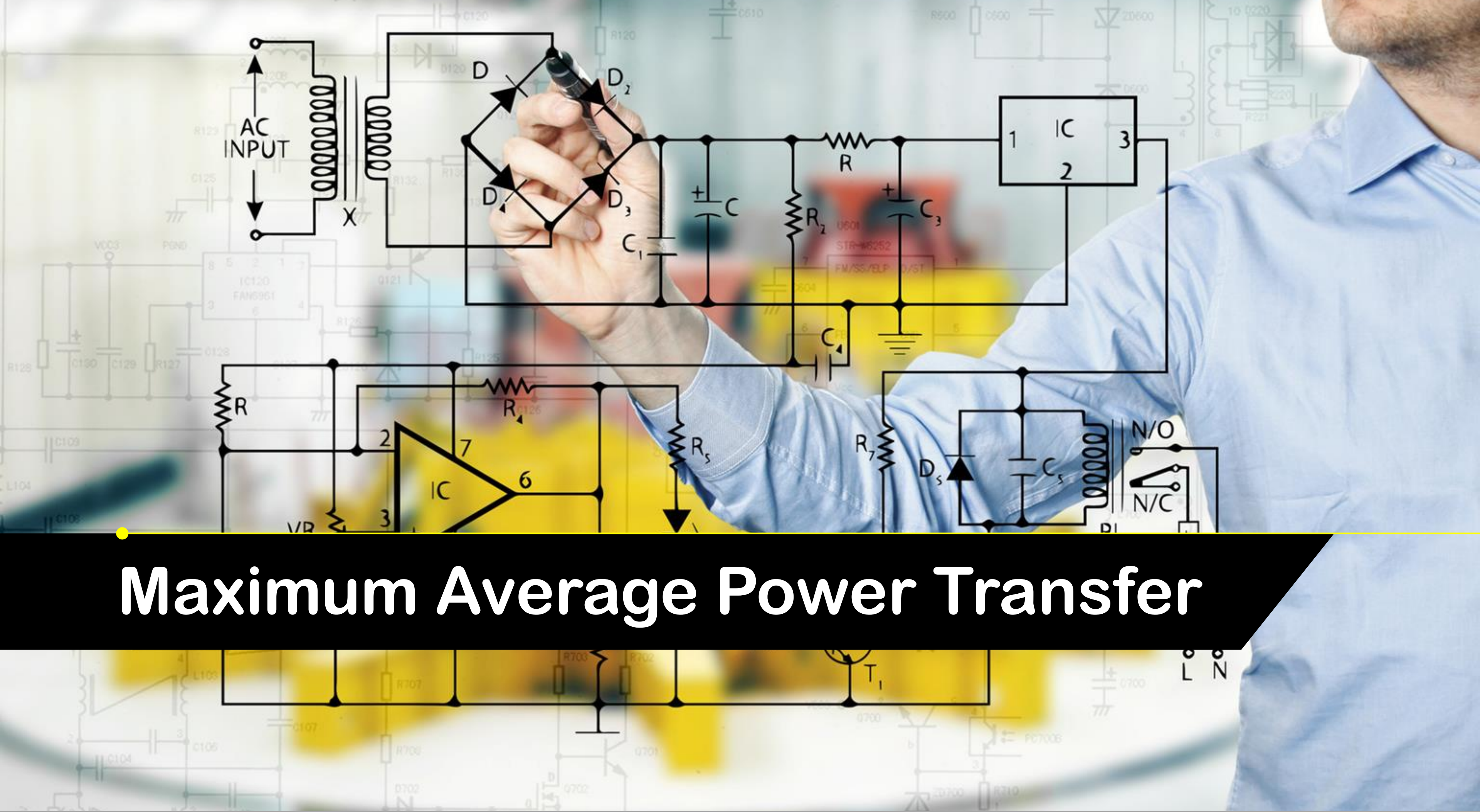
The voltage across it is

$$\boxed{j10\left(\mathbf{I}_1 - \mathbf{I}_2\right) = 105.8\angle 10.9^o \text{ V}}$$

The average power absorbed is

$$\boxed{P_3 = \frac{1}{2}(105.8)(10.58)\cos(90^o) = 0 \text{ W}}$$





Maximum Average Power Transfer

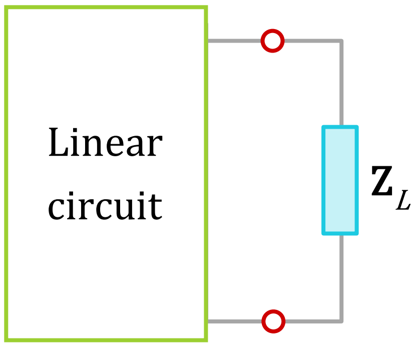
Maximum Average Power Transfer

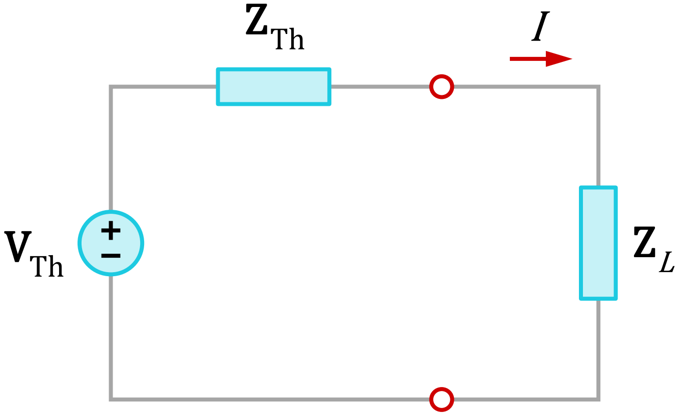
- The problem of maximising the power delivered by a power-supplying resistive network to a load R_L has been discussed.

- Representing the circuit by its Thevenin equivalent, it has been proven that the maximum power would be delivered to the load if the load resistance is equal to the Thevenin resistance. $\boxed{R_L = R_{\text{Th}}}$

- The extension to AC circuits is now discussed.

Maximum Average Power Transfer





$$\mathbf{Z}_{\mathrm{Th}} = R_{\mathrm{Th}} + jX_{\mathrm{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L$$

$$\mathbf{I} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{Z}_{\mathrm{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\mathrm{Th}}}{(R_{\mathrm{Th}} + jX_{\mathrm{Th}}) + (R_L + jX_L)}$$

The average power delivered to the load is

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L$$

$$P = \frac{|\mathbf{V}_{\mathrm{Th}}|^2 R_L / 2}{(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2}$$

Maximum Average Power Transfer

$$\frac{\partial P}{\partial X_L} = -\frac{\left|\mathbf{V}_{\mathrm{Th}}\right|^2 R_L (X_{\mathrm{Th}} + X_L)}{\left[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2\right]^2}$$

$$\frac{\partial P}{\partial X_L} = 0$$

$$X_L = -X_{\mathrm{Th}}$$

$$\frac{\partial P}{\partial R_L} = \frac{\left|\mathbf{V}_{\mathrm{Th}}\right|^2 \left[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2 - 2R_L(R_{\mathrm{Th}} + R_L)\right]}{2\left[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2\right]^2}$$

$$\frac{\partial P}{\partial R_L} = 0$$

$$R_L = \sqrt{R_{\mathrm{Th}}^2 + \left(X_{\mathrm{Th}} + X_L\right)^2}$$

Maximum Average Power Transfer

- For maximum average power transfer, select $\mathbf{Z}_L$ such that

$$X_L = -X_{\mathrm{Th}}$$

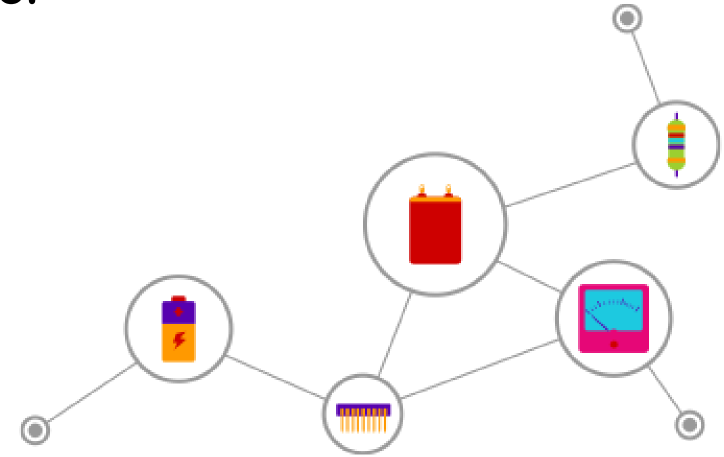
$$R_L = R_{\mathrm{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L = R_{\mathrm{Th}} - jX_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{Th}}^*$$

- Maximum average power transfer theorem: For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\mathrm{Th}}$.

Then,

$$P_{\max} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2}{8\,R_{\mathrm{Th}}}$$



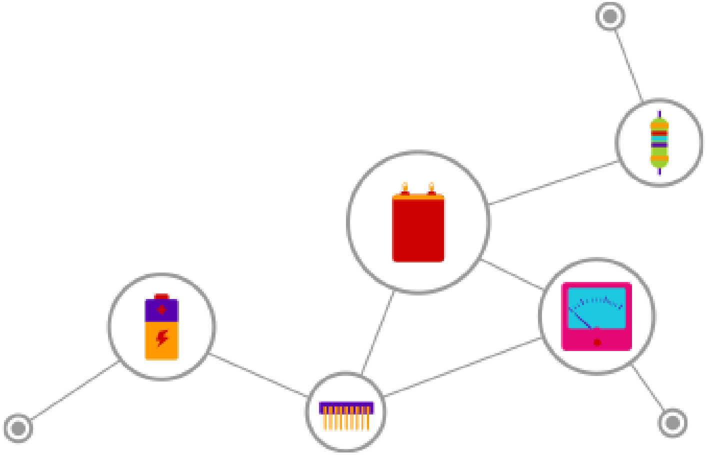
Maximum Average Power Transfer

- If the load is purely real

$$\boxed{X_L = 0}$$

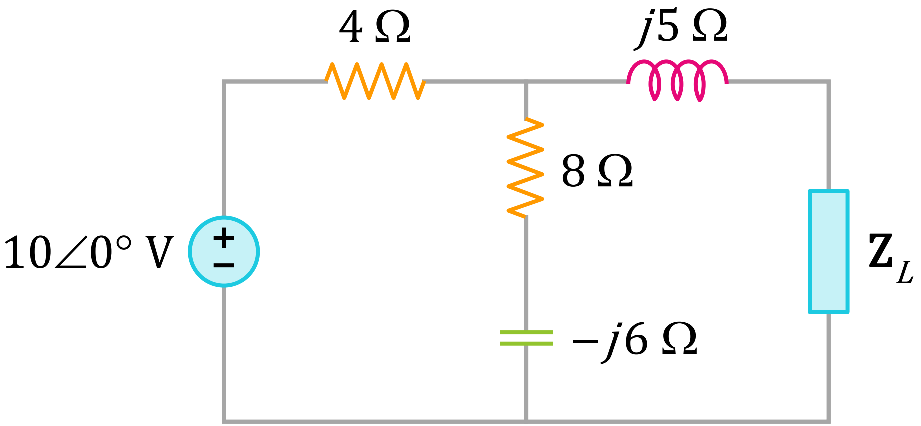
$$\boxed{R_L = \sqrt{R_{\mathrm{Th}}^2 + X_{\mathrm{Th}}^2} = |\mathbf{Z}_{\mathrm{Th}}|}$$

- This means that for maximum power transfer to a purely resistive load, the load resistance is equal to the magnitude of the Thevenin impedance.



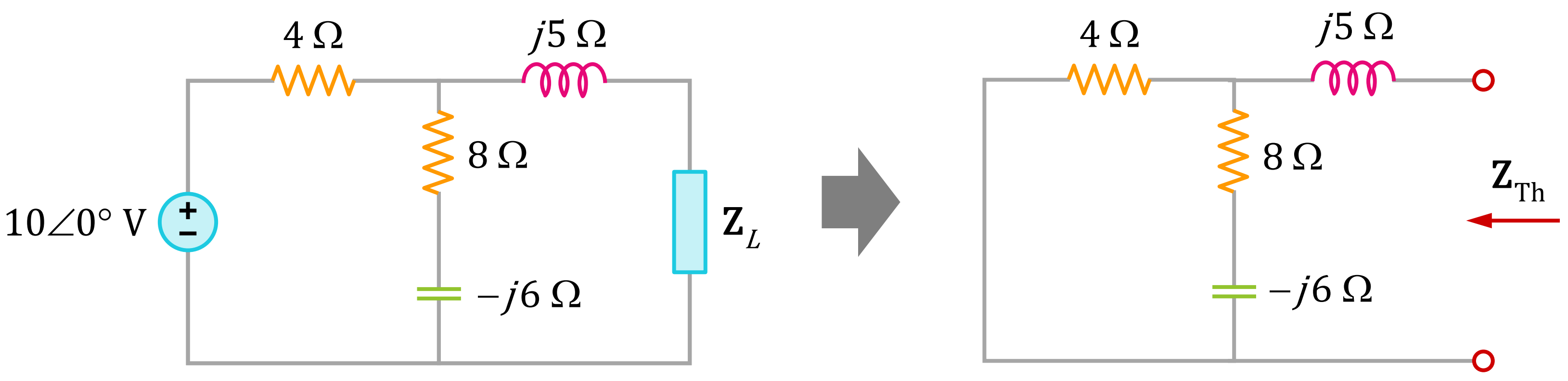
Maximum Average Power Transfer: Example 1

Find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate the maximum average power.



Maximum Average Power Transfer: Example 1

First, obtain the Thevenin equivalent at the load terminals.

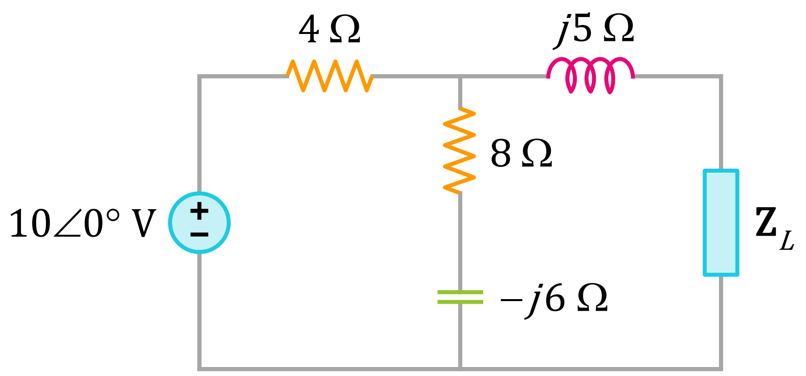


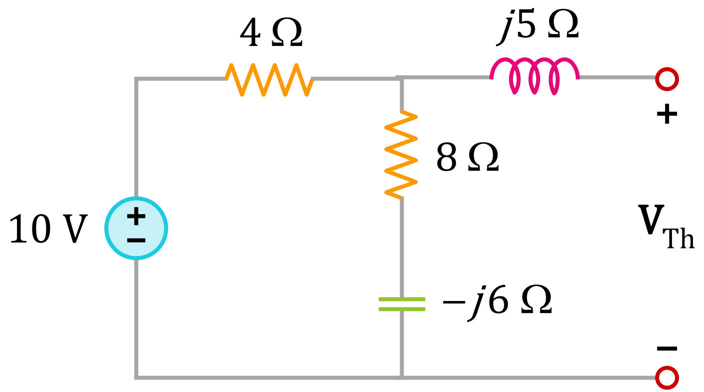
$$\mathbf{Z}_{\mathrm{Th}} = j5 + \left(4 \,||\, (8 - j6)\right)$$

$$\mathbf{Z}_{\mathrm{Th}} = 2.933 + j4.467\ \Omega$$

Maximum Average Power Transfer: Example 1

First, obtain the Thevenin equivalent at the load terminals.





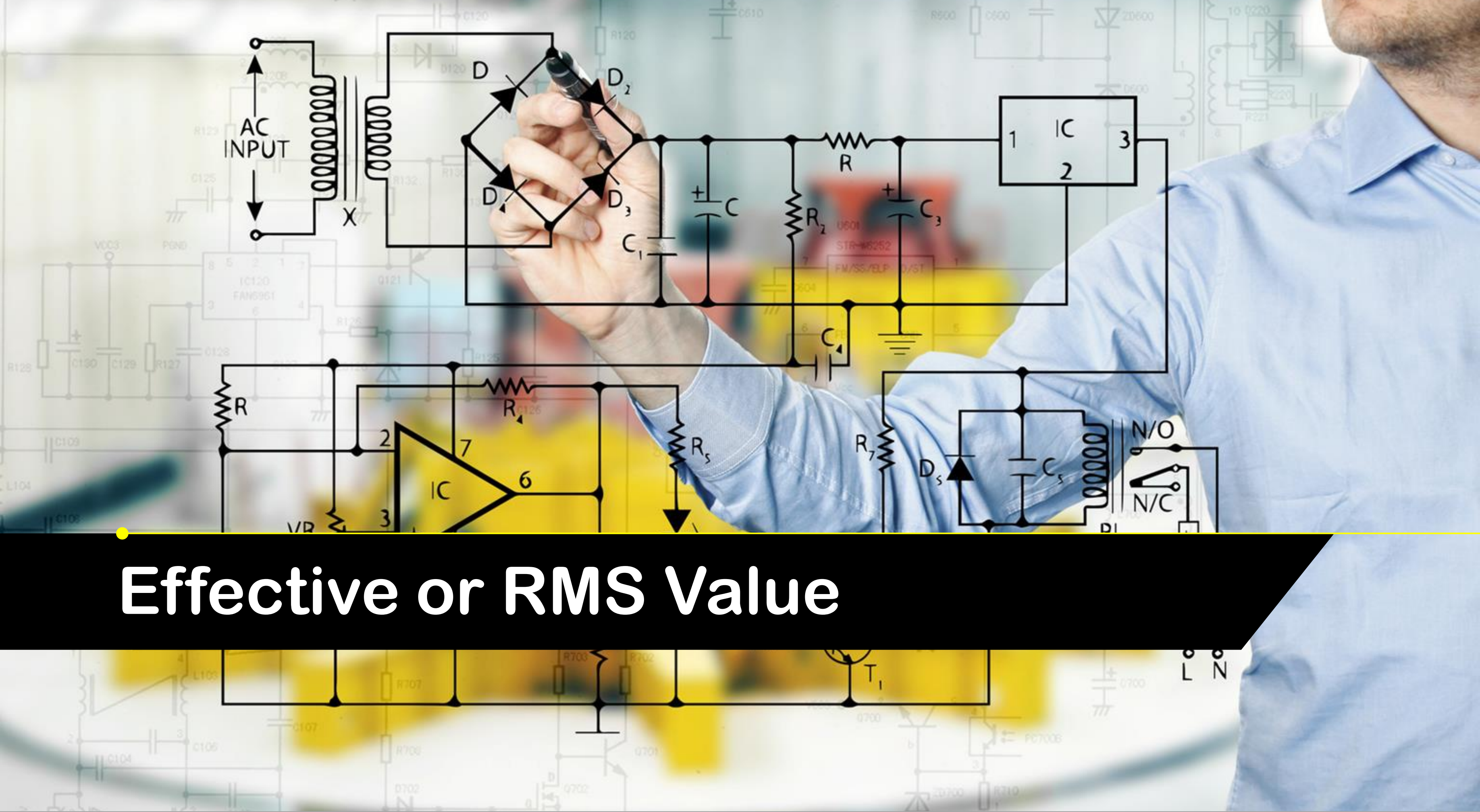
$$\mathbf{V}_{\text{Th}} = \frac{8-j6}{4+8-j6}(10)$$

$$\mathbf{V}_{\text{Th}} = 7.454\angle -10.3^{o}$$

For maximum average power transfer

$$\mathbf{Z}_L = \mathbf{Z}_{\text{Th}}^{*} = 2.933 - j4.467\,\Omega$$

$$P_{\max} = \frac{\left|\mathbf{V}_{\text{Th}}\right|^2}{8\,R_{\text{Th}}} = \frac{(7.454)^2}{8(2.933)} = 2.368\text{ W}$$

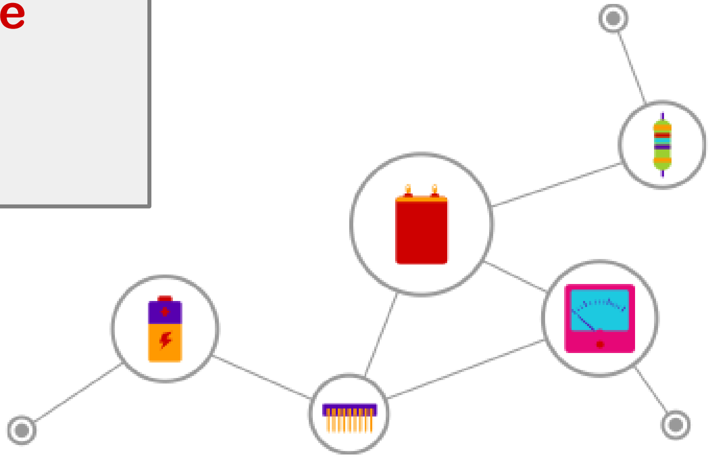


Effective or RMS Value

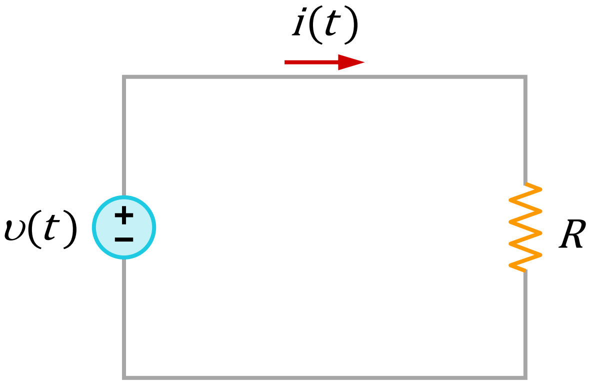
Effective or RMS Value

- The idea of effective value arises from the need to measure the effectiveness of a voltage or current source in delivering power to a resistive load.

- The effective value of a periodic current is the DC current that delivers the same average power to a resistor as the periodic current.

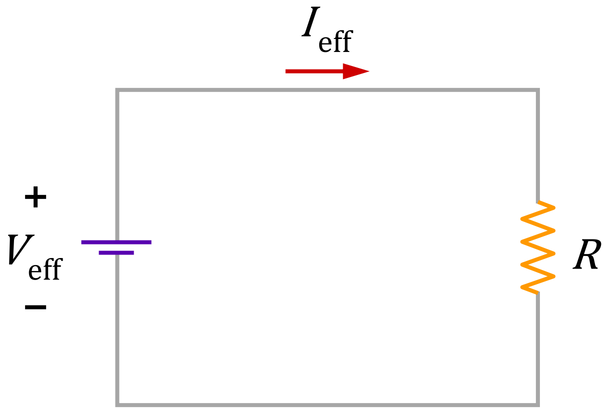


Effective or RMS Value



The average power absorbed by the resistor in the AC circuit is

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt$$



The average power absorbed by the resistor in the DC circuit is

$$P = I_{eff}^2 R$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 \, dt}$$

Similarly,

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2 \, dt}$$

Effective or RMS Value

- **The effective value is the (square) root of the mean (or average) of the square of the periodic signal.**

- **The effective value is also known as the root-mean-square, or *rms* value.**

$$I_{eff} = I_{rms}$$

$$V_{eff} = V_{rms}$$

- **The *rms* value of a constant is a constant itself.**

Effective or RMS Value

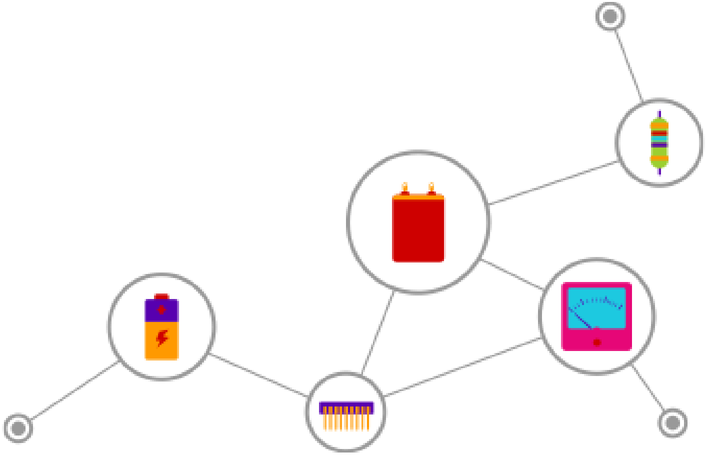
- For a sinusoid, $i(t) = I_m \cos \omega t$,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \; dt} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)\; dt} = \frac{I_m}{\sqrt{2}}$$

Similarly,

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$



Effective or RMS Value

- The average power can be written in terms of the *rms* values:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

- The average power absorbed by a resistor

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

Effective or RMS Value

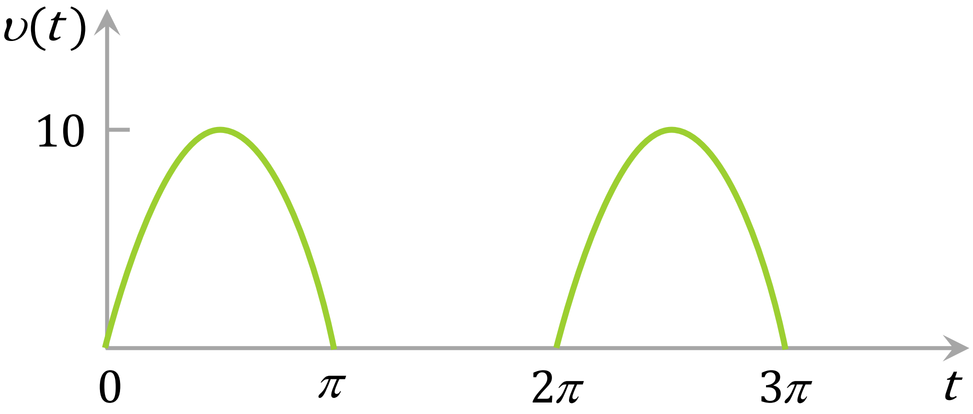
Note:

- If you express amplitude of a phasor source(s) in rms, then all the answers, as a result of this phasor source(s), must also be in rms value.

- The power industries specify phasor magnitudes in terms of their rms values rather than peak values (amplitudes). E.g., the $220\ \mathrm{V}$ in households is the rms value.

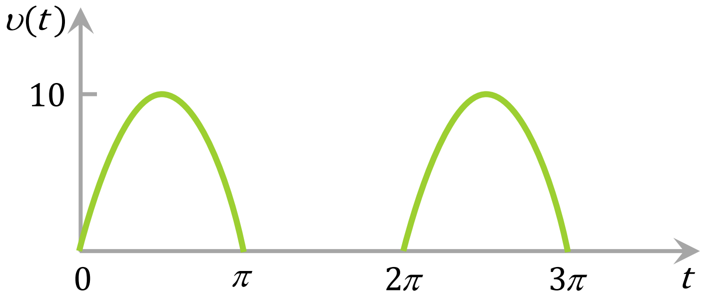
Effective or RMS Value: Example 1

The waveform shown is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a $10\ \Omega$ resistor.



Effective or RMS Value: Example 1

The period of the waveform is $T = 2\pi$ and



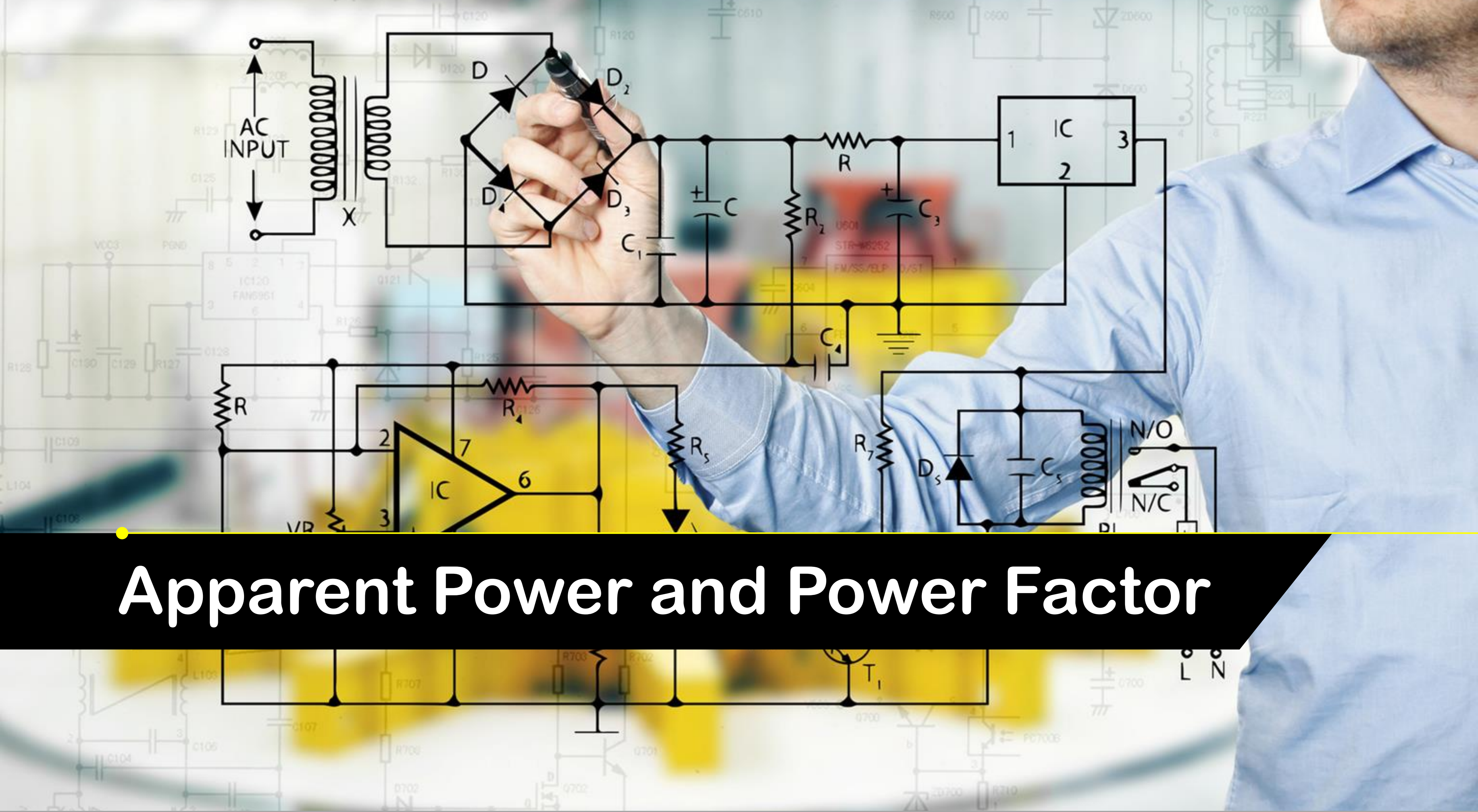
$$v(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi}\left(10\sin t\right)^2(t)dt + \int_0^{2\pi}\left(0\right)^2 dt\right] = 25\ \text{V}$$

$$\boxed{V_{rms} = 5\ \text{V}}$$

The average power absorbed is

$$\boxed{P = \frac{V_{rms}^2}{R} = 2.5\ \text{W}}$$



Apparent Power and Power Factor

Apparent Power and Power Factor

Average Power $$P = \left(V_{rms}\ I_{rms}\right)\cos(\theta_v - \theta_i)$$

$$P = \underbrace{\left(S\right)}_{\text{Apparent Power } S}\underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor } pf}$$

- **Apparent Power**, S, is the product of the rms values of voltage and current.

$$S = V_{rms} I_{rms}$$

- It is measured in **volt-amperes** or VA to distinguish it from the average or real power which is measured in watts.

Apparent **P**ower

Apparent Power and Power Factor

- Power factor pf is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Power Factor

- Power factor angle

$$\theta = \theta_v - \theta_i$$

- It is also the angle of the load impedance.

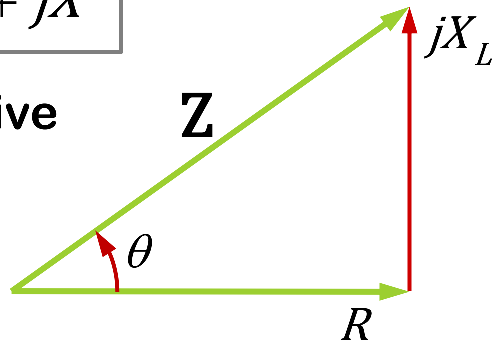
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle(\theta_v - \theta_i) \Rightarrow \mathbf{Z} = \frac{V_{rms}}{I_{rms}} \angle(\theta_v - \theta_i)$$

Apparent Power and Power Factor

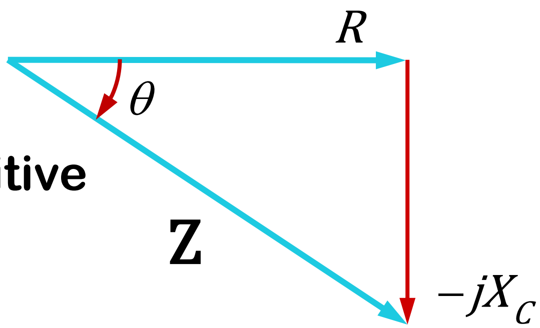
Impedance Triangle

$$\boxed{\mathbf{Z} = R + jX}$$

Inductive

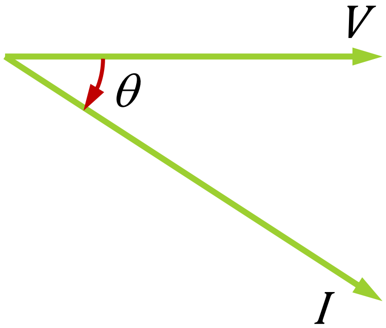


Capacitive

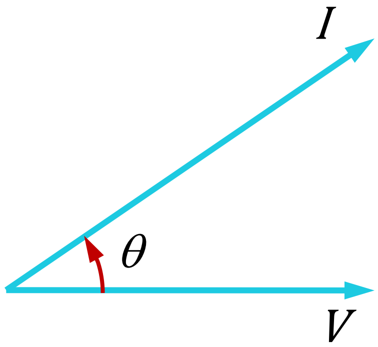


V-I Relationship

Lagging pf



Leading pf



Apparent Power and Power Factor

Impedance	*V-I* Relationship	Power Relationship
Purely resistive load (R)	$\theta_v - \theta_i = 0$, $pf = 1$	$P/S = 1$, **all power is consumed**
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, $pf = 0$	$P = 0$, **no real power consumption**
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• **Lagging-inductive load** • **Leading-capacitive load**

Apparent Power and Power Factor: Example 1

A series connected load draws a current $i(t) = 4\cos(100\pi t + 10^o)\text{A}$ when the applied voltage is $v(t) = 120\cos(100\pi t - 20^o)\text{V}$. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Apparent power

$$S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240\ \text{VA}$$

Power factor

$$pf = \cos(-20^o - 10^o)$$

$$\boxed{pf = 0.866}$$ leading

Apparent Power and Power Factor: Example 1

$$i(t) = 4\cos(100\pi t + 10^o)\,\mathrm{A}$$

$$v(t) = 120\cos(100\pi t - 20^o)\,\mathrm{V}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle -20^o}{4\angle 10^o} = 30\angle -30^o$$

$$\mathbf{Z} = 25.98 - j15\,\Omega$$

capacitive

Load impedance is a $25.98\ \Omega$ resistor in series with a capacitor.

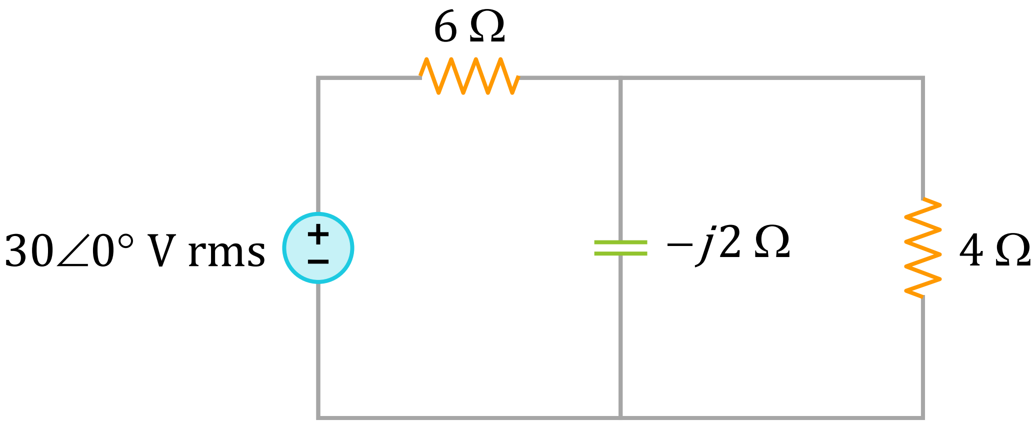
$$X_C = 15 = \frac{1}{\omega C}$$

$$\omega = 100\pi$$

$$C = 212.2\ \mu\mathrm{F}$$

Apparent Power and Power Factor: Example 2

Determine the power factor of the entire circuit. Calculate the average power delivered by the source.



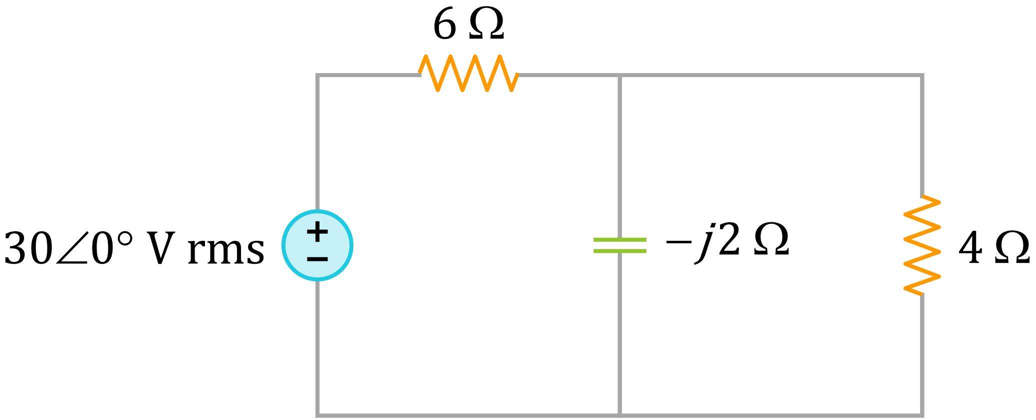
Apparent Power and Power Factor: Example 2

Load impedance

$$\mathbf{Z} = 6 + \left(4 \,||\, (-j2)\right)$$

$$\mathbf{Z} = 6.8 - j1.6$$

$$\boxed{\mathbf{Z} = 7\angle -13.24^o\ \Omega}$$



Power factor

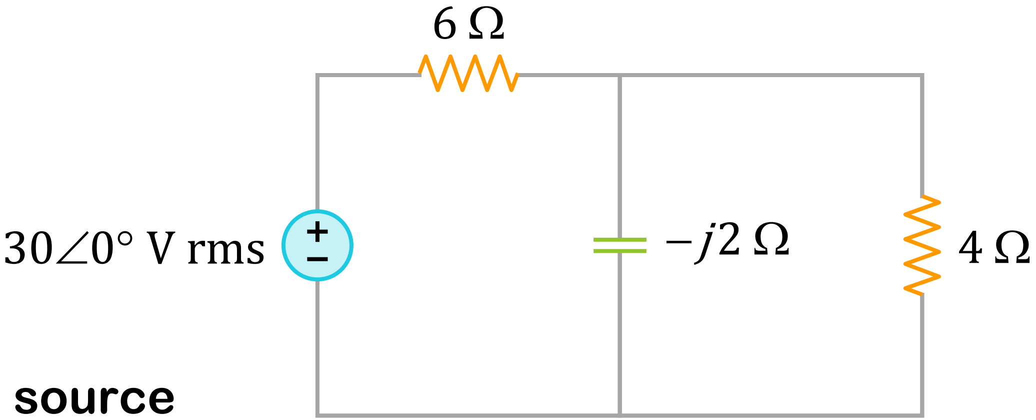
$$pf = \cos(-13.24^o)$$

$$\boxed{pf = 0.9734}$$ leading

Apparent Power and Power Factor: Example 2

Current supplied by voltage source

$$\mathbf{I}_{rms} = \frac{\mathbf{V}_{rms}}{\mathbf{Z}} = 4.286\angle 13.24^{\circ}\ \mathrm{A}$$



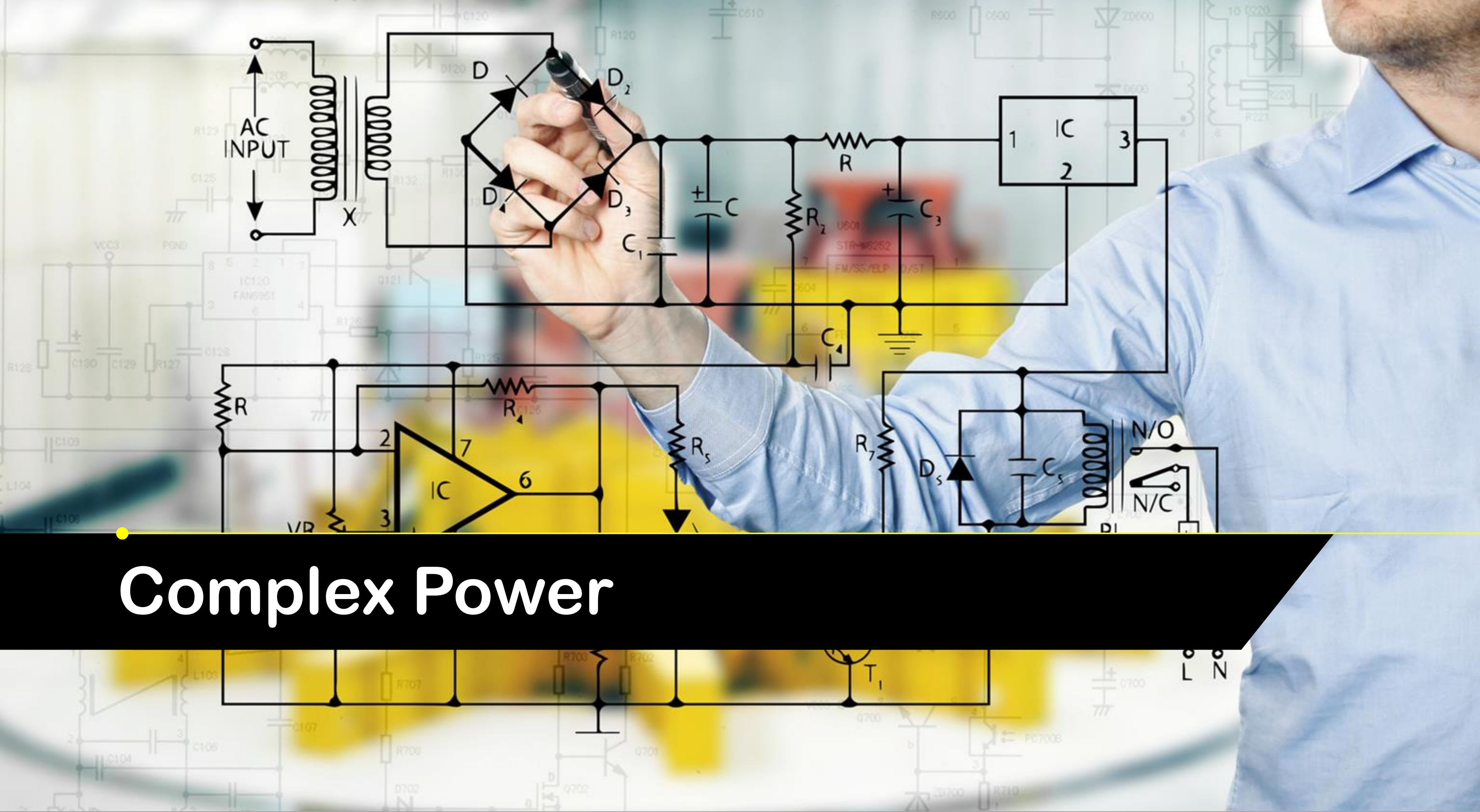
The average power supplied by source

$$P = V_{rms} I_{rms}\, pf = 125\ \mathrm{W}$$

Or

$$P = I_{rms}^{2} R = \left(4.286\right)^{2} 6.8 = 125\ \mathrm{W}$$

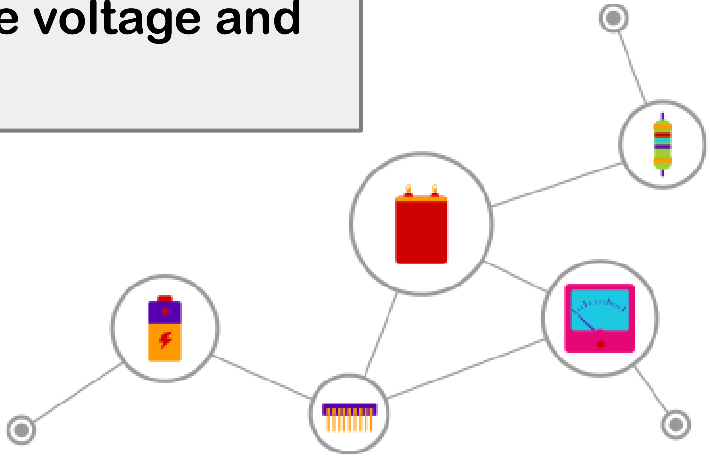
Where R is the resistive part of $\mathbf{Z}$.



Complex Power

Complex Power

- Complex power is useful when finding the total effects of parallel loads.
- It contains all the information pertaining to the power absorbed by a given load.
- Complex power S is the product of the voltage and the complex conjugate of the current.



Complex Power

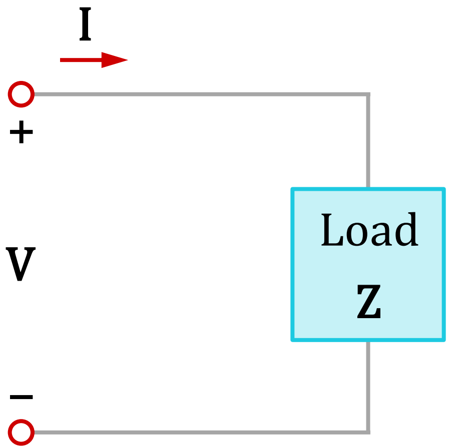
$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = \mathbf{V}_{rms}\,\mathbf{I}_{rms}^{\ *}$$

$$\mathbf{S} = V_{rms}\,I_{rms}\,\angle\left(\theta_v - \theta_i\right)$$

$$\mathbf{S} = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

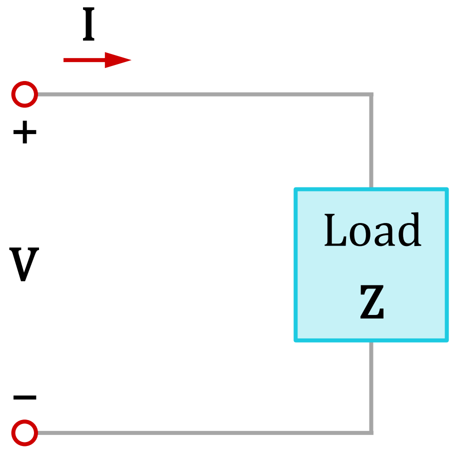
$$\mathbf{S} = P + jQ$$



Complex Power

$$\mathbf{S} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + jV_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = P + j\,Q$$

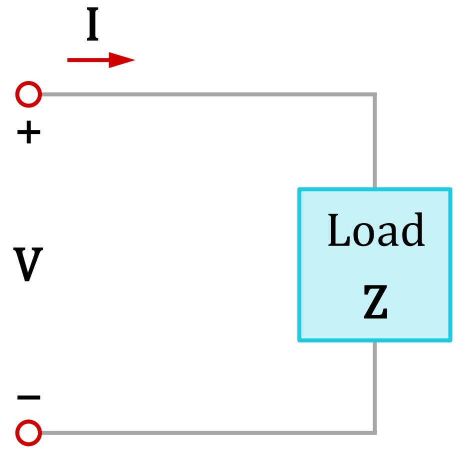


P: **is the average power delivered to a load and it is the only useful power in watts.**

Complex Power

$$\mathbf{S} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + jV_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = P + jQ$$



Q : is the **reactive power**. It is a measure of the exchange between the source and the reactive part of the load. It is measured in volt-ampere reactive (VAR).

- $Q = 0$ for **resistive loads** (unity pf)
- $Q < 0$ for **capacitive loads** (leading pf)
- $Q > 0$ for **inductive loads** (lagging pf)

Complex Power

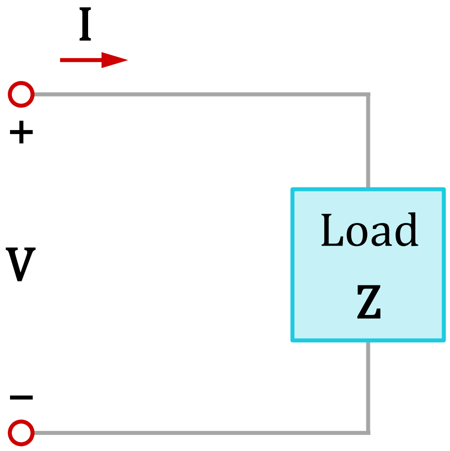
The complex power may be expressed by the load impedance

$$\mathbf{Z} = R + jX = \frac{V_{rms}}{I_{rms}} \angle(\theta_v - \theta_i)$$

Since $\mathbf{V}_{rms} = \mathbf{Z}\mathbf{I}_{rms}$

$$\mathbf{S} = \mathbf{V}_{rms}\,\mathbf{I}_{rms}{}^{*} = I^2_{rms}\mathbf{Z} = \frac{V^2_{rms}}{\mathbf{Z}^*}$$

$$\mathbf{S} = I^2_{rms}\mathbf{Z} = I^2_{rms}(R + jX) \Rightarrow \mathbf{S} = I^2_{rms}R + jI^2_{rms}X \Rightarrow \mathbf{S} = P + jQ$$



Complex Power

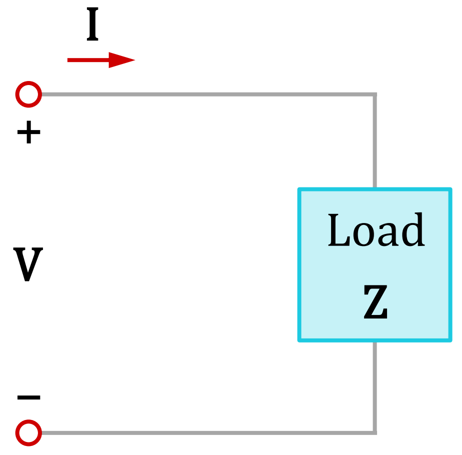
Introducing complex power enables us to obtain the real and reactive powers directly from voltage and current phasors.

Complex power

$$\boxed{\mathbf{S} = P + jQ}$$

$$\mathbf{S} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = V_{rms} I_{rms} \angle(\theta_v - \theta_i)$$



Complex Power

Complex power

$$\mathbf{S} = V_{rms} I_{rms} \angle(\theta_v - \theta_i)$$

Apparent power

$$S = |\mathbf{S}| = V_{rms}\ I_{rms} = \sqrt{P^2 + Q^2}$$

Real power

$$P = \mathrm{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

Reactive power

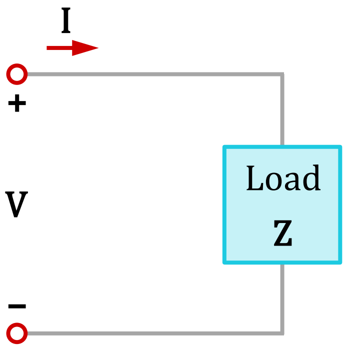
$$Q = \mathrm{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

Power factor

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

It shows how complex power contains all the relevant power information in a given load.

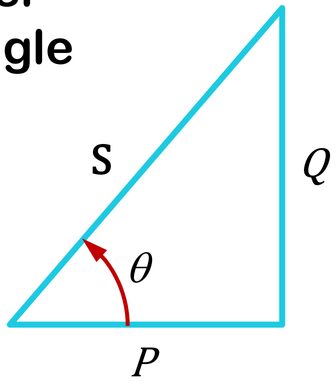
Complex Power



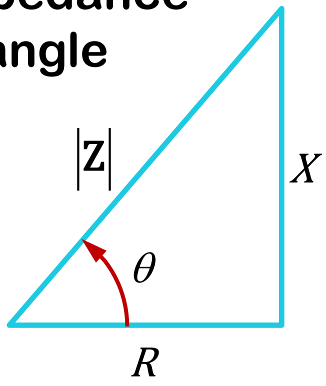
$$\mathbf{S} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + jV_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = P + jQ$$

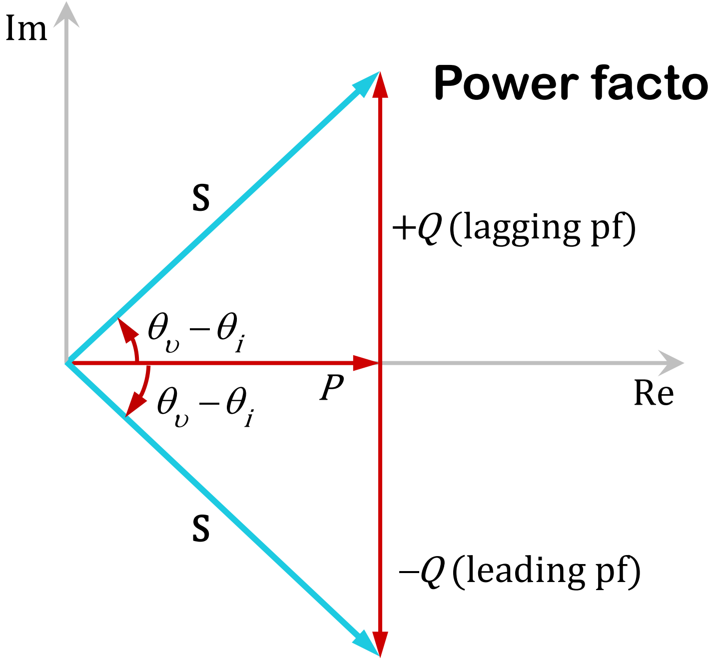
Power triangle



Impedance triangle

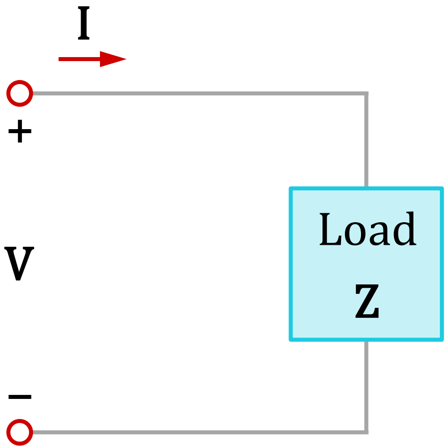


Power factor



Complex Power: Example 1

Given that $v(t) = 60\cos(\omega t - 10^o)\,\mathrm{V}$ **and** $i(t) = 1.5\cos(\omega t + 50^o)\,\mathrm{A}$**, find: (a) the complex power and apparent powers, (b) the real and reactive powers, (c) the** pf **and load impedance.**



$$\mathbf{V}_{rms} = \frac{60}{\sqrt{2}}\angle -10^o$$

$$\mathbf{I}_{rms} = \frac{1.5}{\sqrt{2}}\angle 50^o$$

Complex Power: Example 1

(a)

Complex power

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^{*}_{rms}$$

$$\mathbf{S} = 45\angle -60^{o}\ \text{VA}$$

$$\mathbf{S} = 22.5 - j38.97$$

Apparent power

$$S = |\mathbf{S}| = 45\ \text{VA}$$

(b)

Real power

$$P = \text{Re}(\mathbf{S}) = 22.5\ \text{W}$$

Reactive power

$$Q = \text{Im}(\mathbf{S}) = -38.97\ \text{VAR}$$

$$\mathbf{V}_{rms} = \frac{60}{\sqrt{2}}\angle -10^{o}$$

(c)

Power factor

$$pf = \frac{P}{S} = \cos(-60^{o}) = 0.5$$

Load impedance

$$\mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = 40\angle -60^{o}\ \Omega$$

which is a capacitive impedance

$$\mathbf{I}_{rms} = \frac{1.5}{\sqrt{2}}\angle 50^{o}$$

Complex Power: Example 2

A load $\mathbf{Z}$ draws $12\ \mathrm{kVA}$ at a power factor of 0.856 lagging from a $120\ \mathrm{Vrms}$ sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance.

Given

$$pf = \cos\theta = 0.856$$

$$S = 12000$$

$$\mathbf{V}_{rms} = 120\angle 0^o$$

(a)

Average power

$$\boxed{P = S\cos\theta = 10.272\ \mathrm{kW}}$$

Reactive power

$$\boxed{Q = S\sin\theta = 6.204\ \mathrm{kVAR}}$$ **lagging**

Complex Power: Example 2

(b)

Complex power

$$\mathbf{S} = P + jQ$$

$$\mathbf{S} = 10.272 + j6.204\ \text{kVA}$$

Peak current

$$\mathbf{I}_{rms}^{*} = \frac{\mathbf{S}}{\mathbf{V}_{rms}} = 100\angle 31.13^{o}\ \text{A}$$

$$\mathbf{I}_{rms} = 100\angle -31.13^{o}\ \text{A}$$

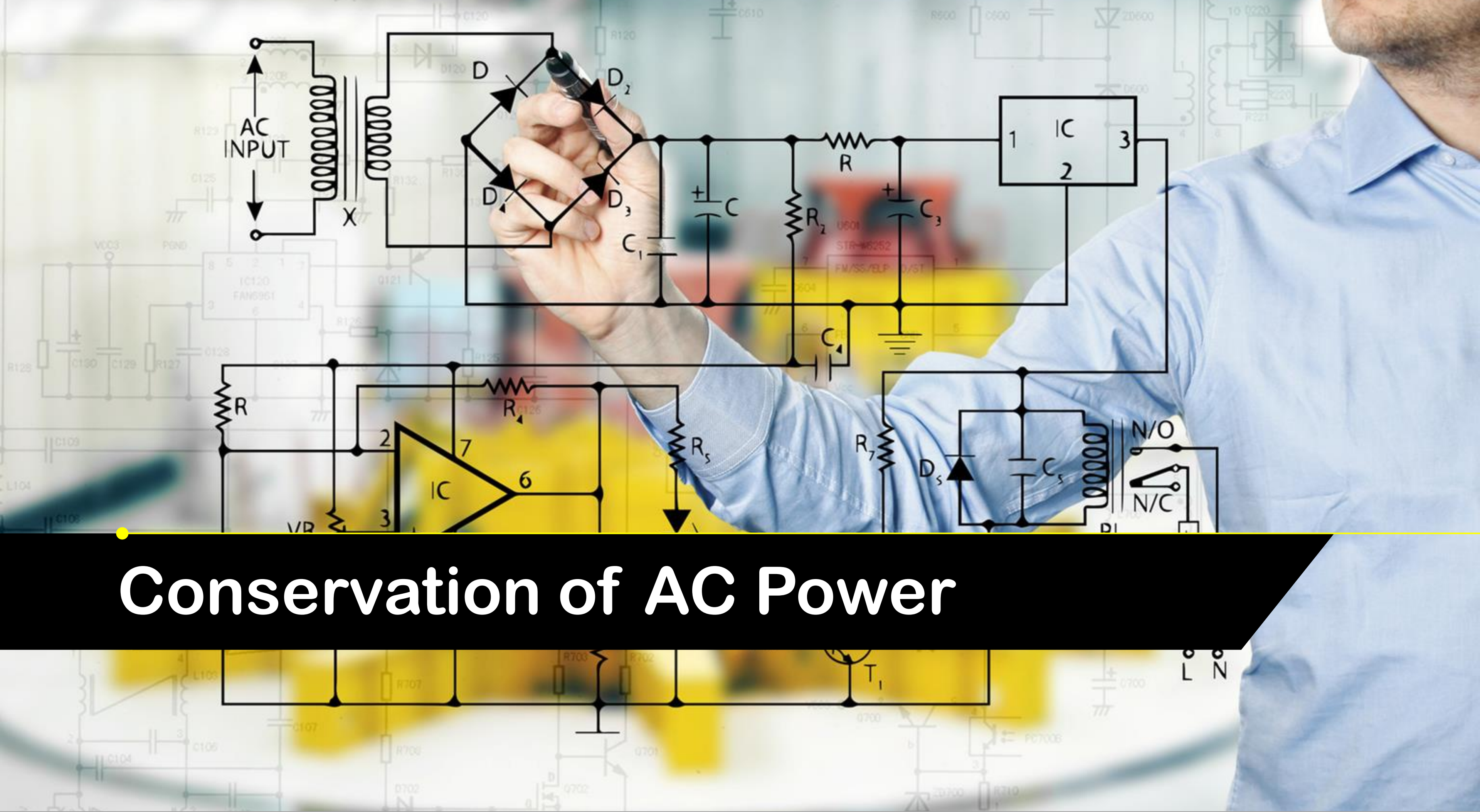
$$I_m = \sqrt{2} I_{rms} = 141.4\ \text{A}$$

(c)

Load impedance

$$\mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = 1.2\angle 31.13^{o}\ \Omega$$

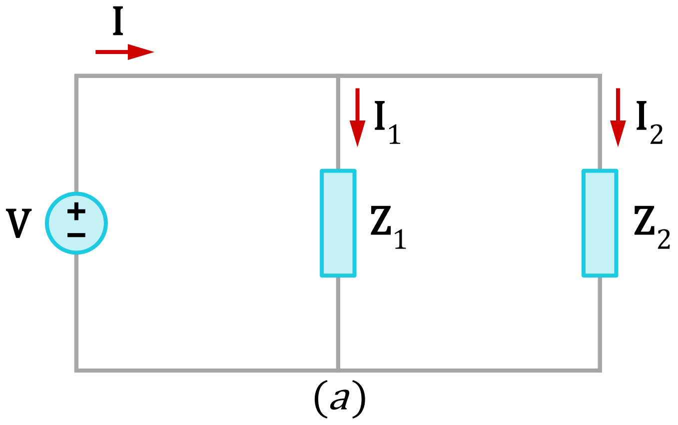
inductive



Conservation of AC Power

Conservation of AC Power

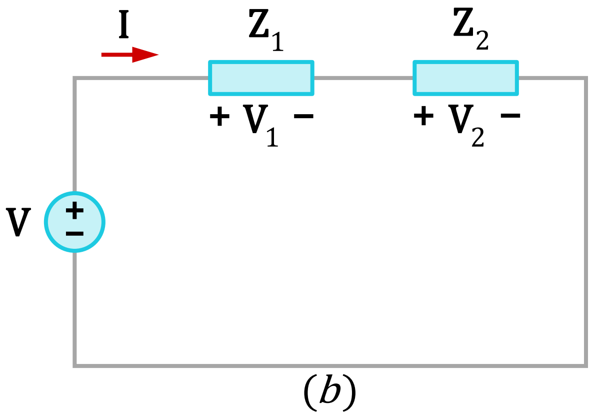
Parallel Connection



For the **parallel connection**, the total complex power absorbed by the two loads is the total complex power supplied by the source.

$$\mathbf{S}_1 + \mathbf{S}_2 = \mathbf{V}_{rms}\left(\mathbf{I}^*_{1rms} + \mathbf{I}^*_{2rms}\right) = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \mathbf{S}$$

Series Connection



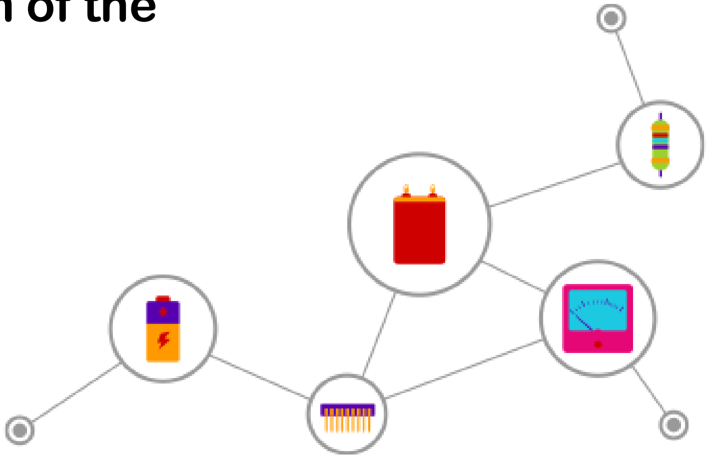
The **same result** can be obtained for the **series connection**.

Conservation of AC Power

In general, for a source connected to N loads, $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2 + \cdots + \boldsymbol{S}_N$.

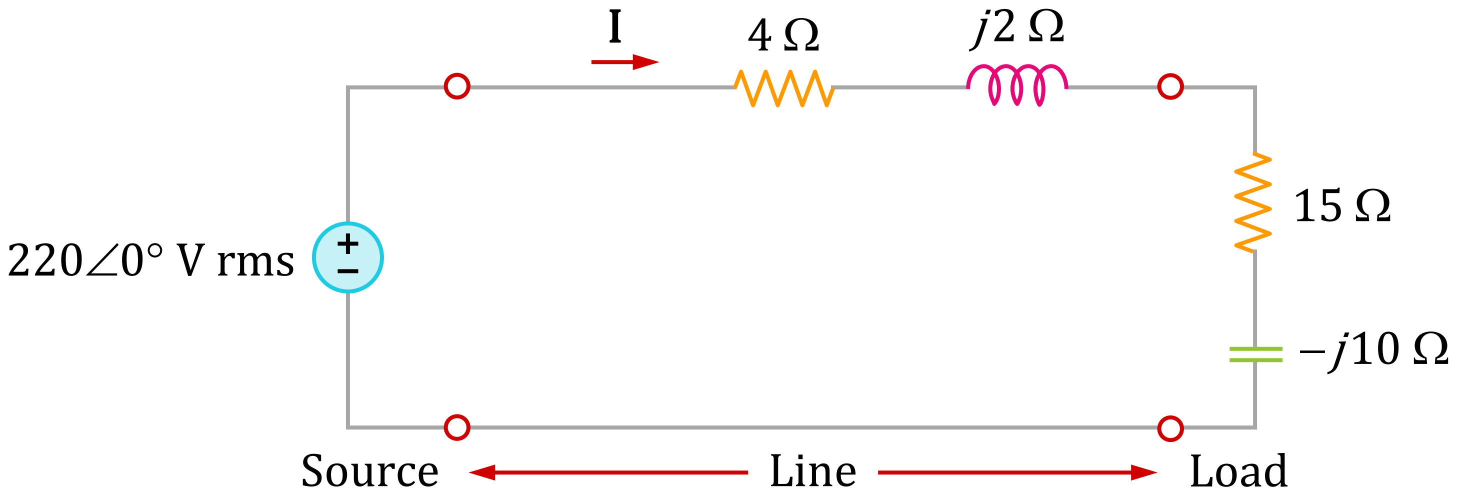
The total complex power in a network is the sum of the complex powers of the individual components.

In fact, all forms of AC power are conserved: instantaneous, real, reactive and complex.



Conservation of AC Power: Example 1

A load is fed by a voltage source through a transmission line. The impedance of the line is represented by $(4 + j2)\ \Omega$ and a return path. Find the real power and reactive power absorbed/ delivered by the (a) line, (b) load and (c) source.

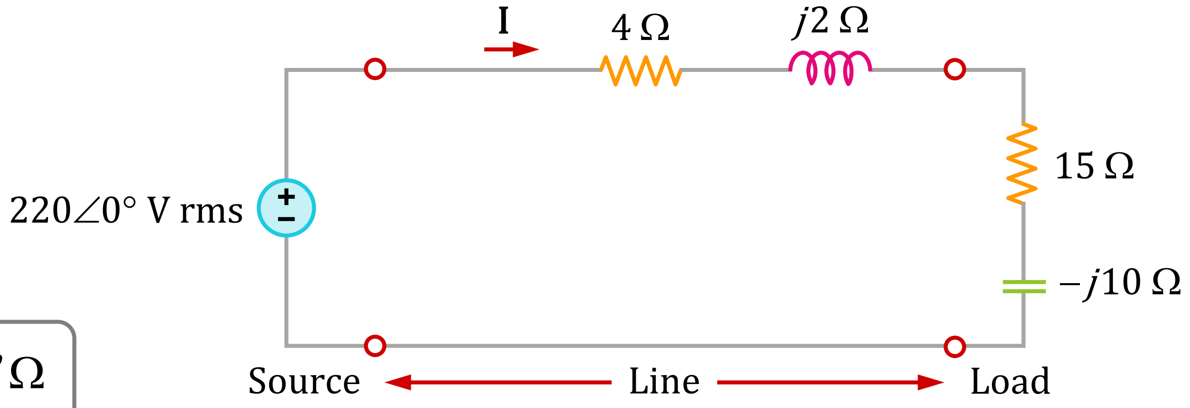


Conservation of AC Power: Example 1

All the voltages and currents indicated are *rms* values. The total impedance is

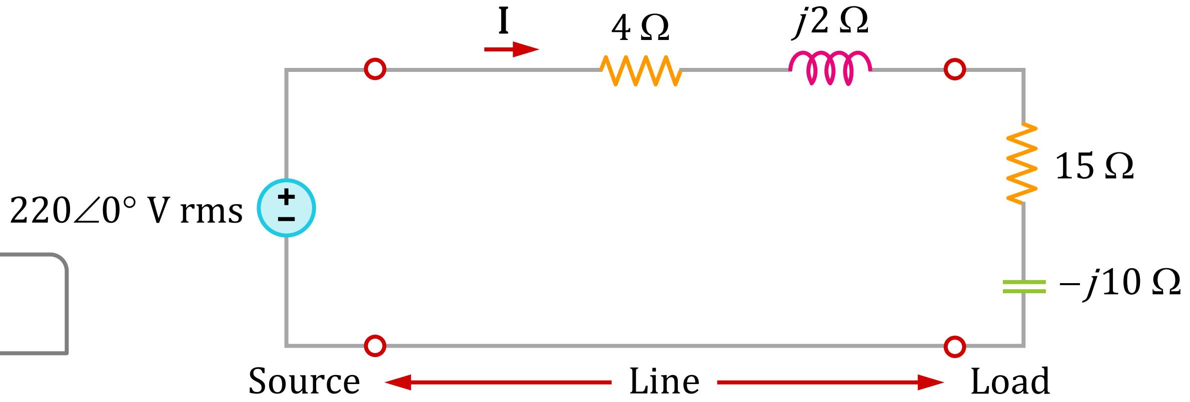
$$\mathbf{Z} = 19 - j8 = 20.62\angle -22.83^o\,\Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = 10.672\angle 22.83^o$$



Conservation of AC Power: Example 1

(a)
For the line, complex power



$$\mathbf{S}_{line} = \mathbf{V}_{line}\mathbf{I}^* = \big((4+j2)\mathbf{I}\big)\mathbf{I}^*$$

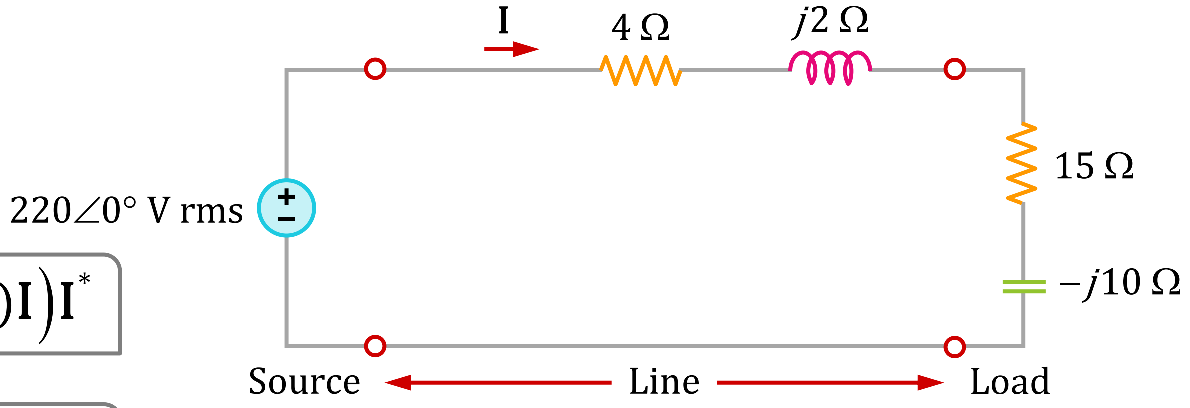
$$\mathbf{S}_{line} = 455.57 + j227.78\ \text{VA}$$

$$\mathbf{S}_{line} = P_{line} + jQ_{line}$$

The line is absorbing an average power of 455.57 W and a reactive power of 227.78 VAR.

Conservation of AC Power: Example 1

(b)
For the load, complex power



$$\mathbf{S}_{load} = \mathbf{V}_{load}\mathbf{I}^* = \left((15 - j10)\mathbf{I}\right)\mathbf{I}^*$$

$$\mathbf{S}_{load} = 1708.37 - j1138.92 \text{ VA}$$

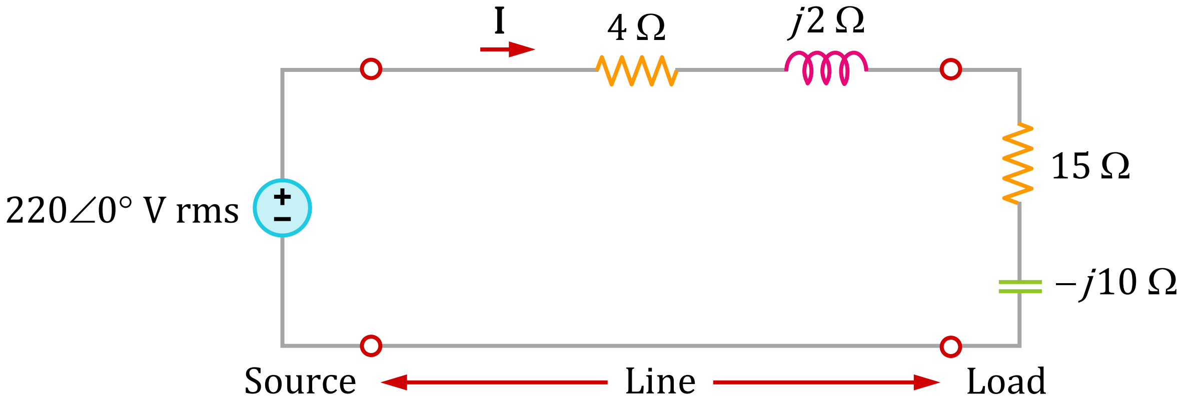
$$\mathbf{S}_{load} = P_{load} + jQ_{load}$$

The load is absorbing an average power of 1708.37 W and delivering a reactive power of 1138.92 VAR.

Conservation of AC Power: Example 1

(c)

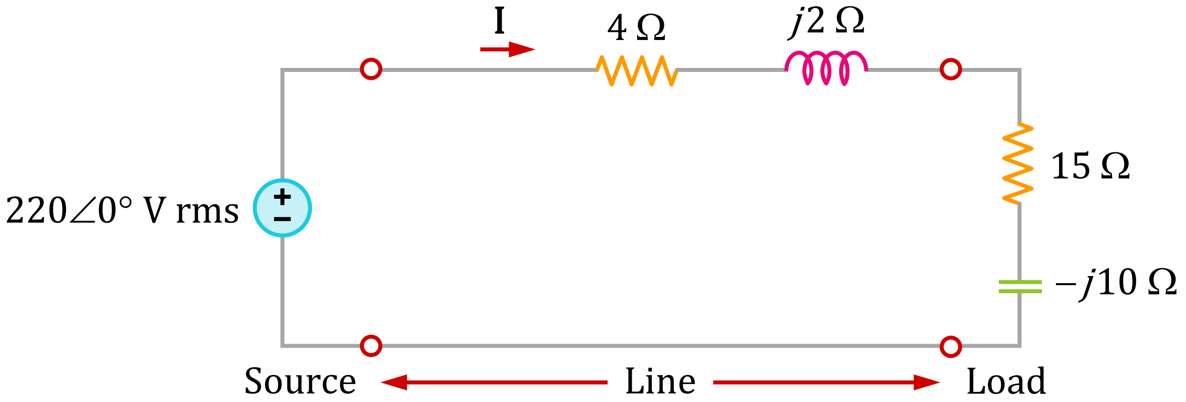
One way to calculate the complex power **supplied by** the source is to add the complex powers delivered to the individual components.



$$\mathbf{S} = \mathbf{S}_{load} + \mathbf{S}_{line} = 2163.94 - j911.14 \text{ VA}$$

Conservation of AC Power: Example 1

(c)
The associated complex power for the source, based on passive sign convention, can also be calculated as



$$\mathbf{S}_s = -\mathbf{V}_s\mathbf{I}^* = -220\angle 0^o\left(10.672\angle -22.83^o\right)$$

$$\mathbf{S}_s = -2163.91 + j910.96 \text{ VA}$$

$$\mathbf{S}_s = P_s + jQ_s$$

The load is delivering an average power of 2163.91 W **and absorbing** 910.96 VAR**.**

Conservation of AC Power: Example 1

Note that average power delivered,

P_s **= average power absorbed =** P_{line} **+** P_{load}

$$-(-2163.94) = 455.57 + 1708.37$$

Note that VARs **delivered,**

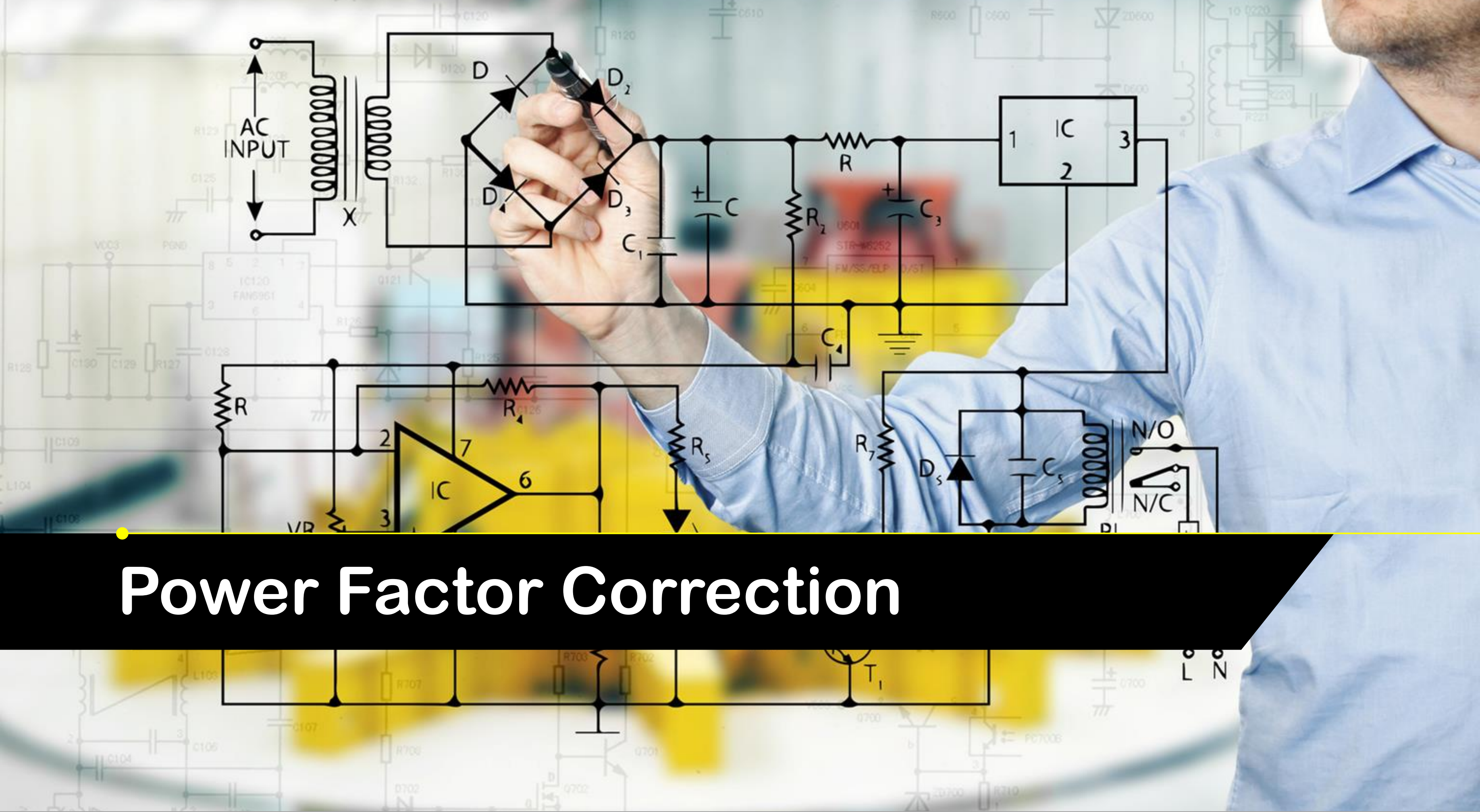
Q_{load} **=** VARs **absorbed =** Q_{ine} **+** Q_s

$$-(-1138.92) = 227.78 + 911.14$$

$$\mathbf{S}_s = -2163.94 + j911.14 \text{ VA}$$

$$\mathbf{S}_{line} = 455.57 + j227.78 \text{ VA}$$

$$\mathbf{S}_{load} = 1708.37 - j1138.92 \text{ VA}$$

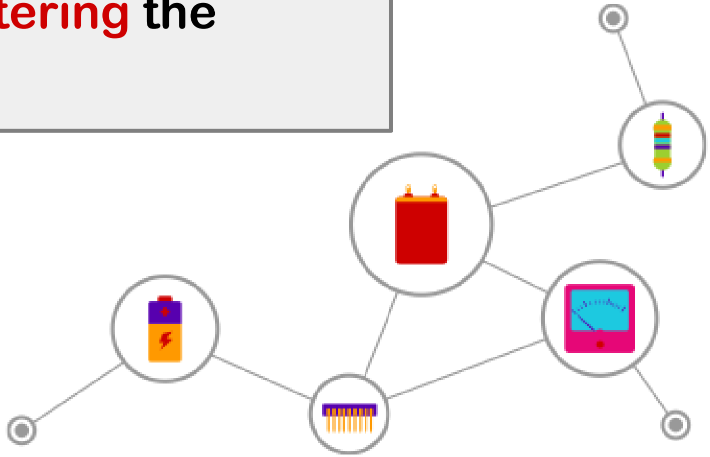


Power Factor Correction

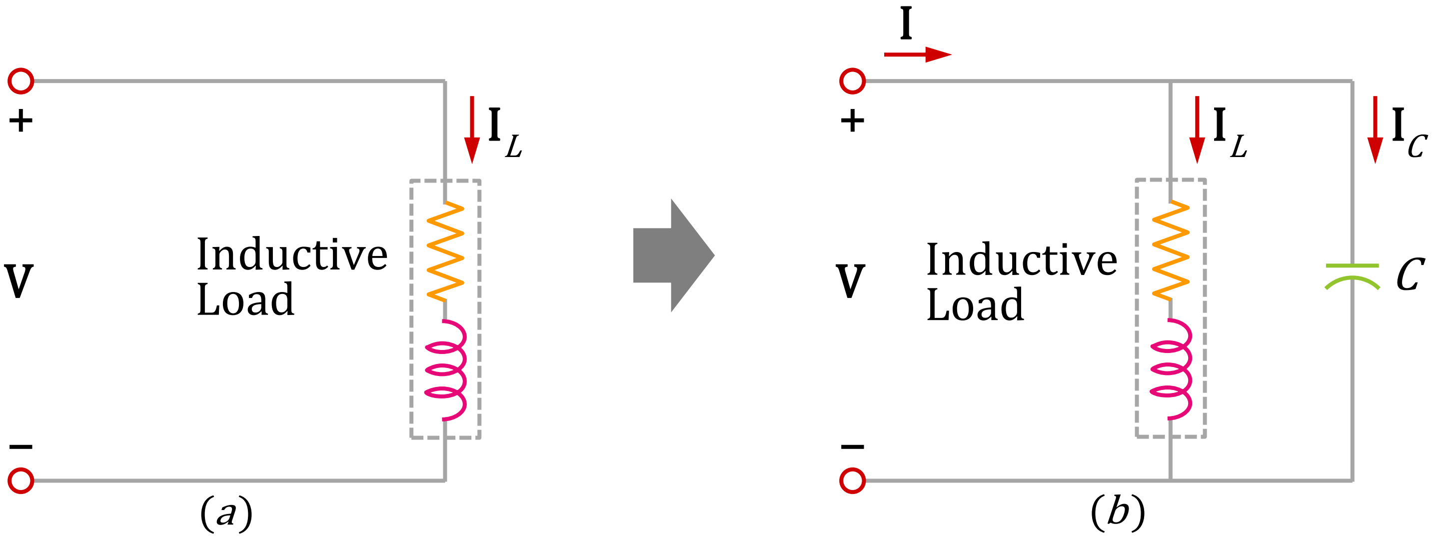
Power Factor Correction

- Most domestic loads such as washing machines, air conditioners and refrigerators, and industrial loads such as induction motors, are inductive and operate at a low lagging power fact.

- Power factor correction is the process of increasing the power factor without altering the voltage or current to the original load.



Power Factor Correction

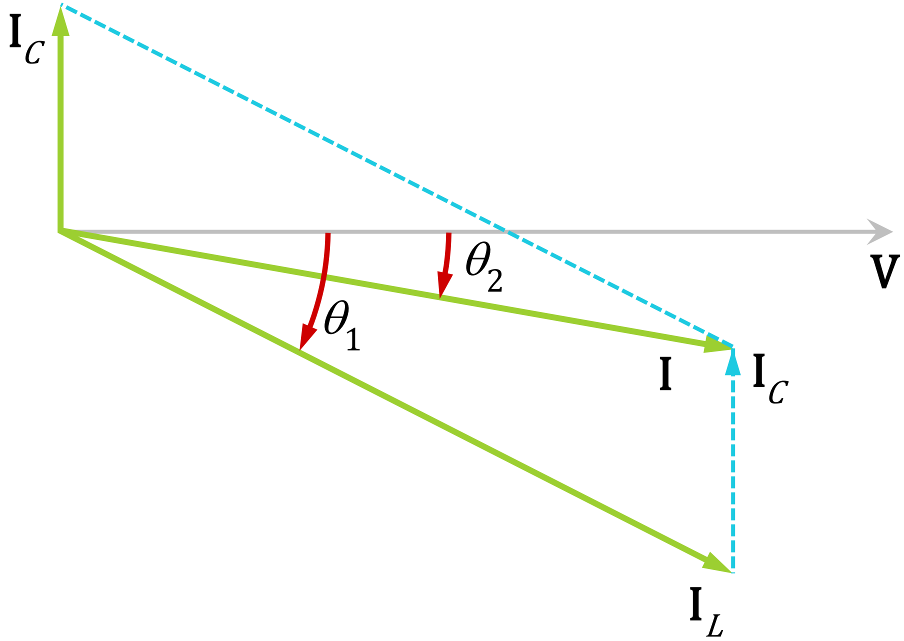


The pf of the inductive load can be improved or corrected by deliberately installing a capacitor in parallel with the load.

The original circuit (a) has a pf of $\cos\,\theta_1$, while circuit (b) has a pf of $\cos\,\theta_2$.

Power Factor Correction

Adding the capacitor causes the phase angle between supplied voltage and current to reduce from θ_1 to θ_2, thereby increasing the pf.

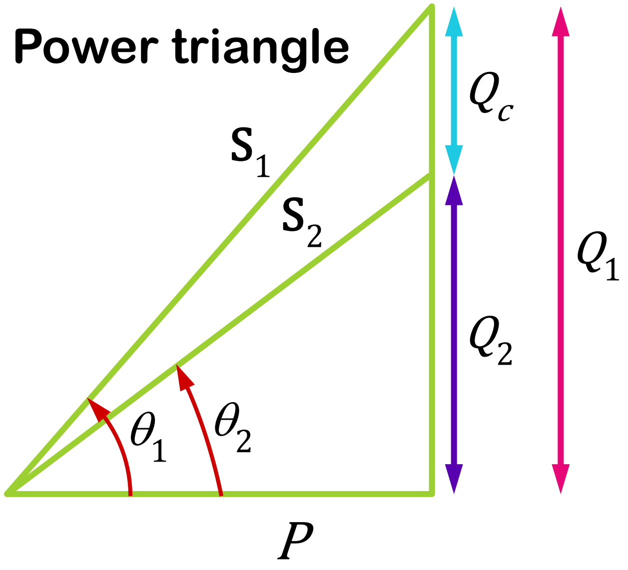


$$\boxed{|\mathbf{I}| < |\mathbf{I}|_L}$$

Power factor correction is necessary for economic reason.

Power Factor Correction

Power triangle



$$P = S_1 \cos\theta_1$$

$$Q_c = Q_1 - Q_2 = P(\tan\ \theta_1 - \tan\ \theta_2) = \frac{V_{rms}^2}{X_C} = \omega C V_{rms}^2$$

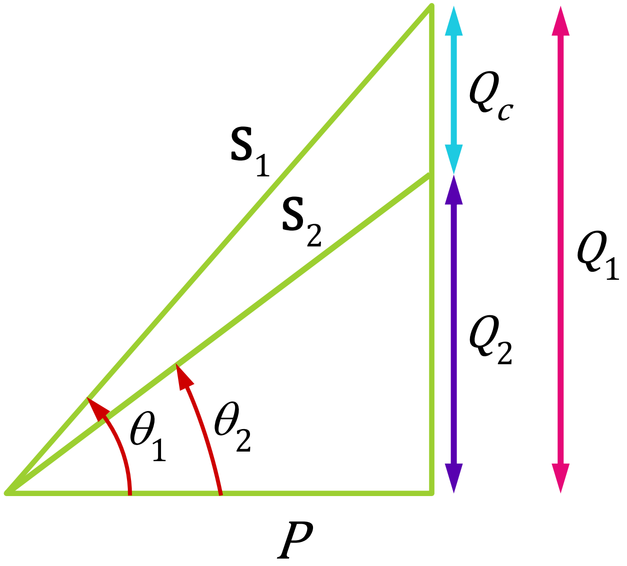
$$Q_1 = S_1 \sin\theta_1 = P\tan\theta_1$$

$$Q_2 = P\tan\theta_2$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\ \theta_1 - \tan\ \theta_2)}{\omega V_{rms}^2}$$

Power Factor Correction: Example 1

When connected to a 120 Vrms, 60 Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.



$P = 4\ \text{kW}$

$\cos(\theta_1) = 0.8 \rightarrow \theta_1 = 36.87^o$

$\cos(\theta_2) = 0.95 \rightarrow \theta_2 = 18.19^o$

$Q_1 = 3000\ \text{VAR}$

$Q_2 = 1314.4\ \text{VAR}$

$Q_c = Q_1 - Q_2 = 1685.6\ \text{VAR}$

$$C = \frac{Q_c}{\omega V_{rms}^2} = 310.5\ \mu\text{F}$$

Power Factor Correction: Example 1

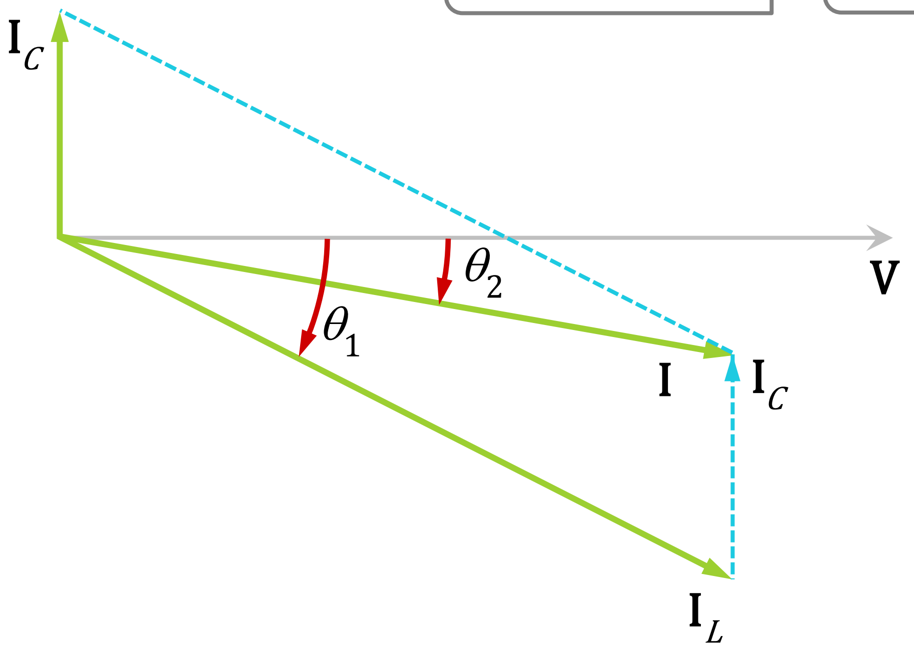
Alternate approach:

$$P = 4\,\text{kW} = V_{rms} I_L \cos\theta_1$$

$$I_L = 41.67\,\text{A}$$

$$\theta_1 = 36.87^o$$

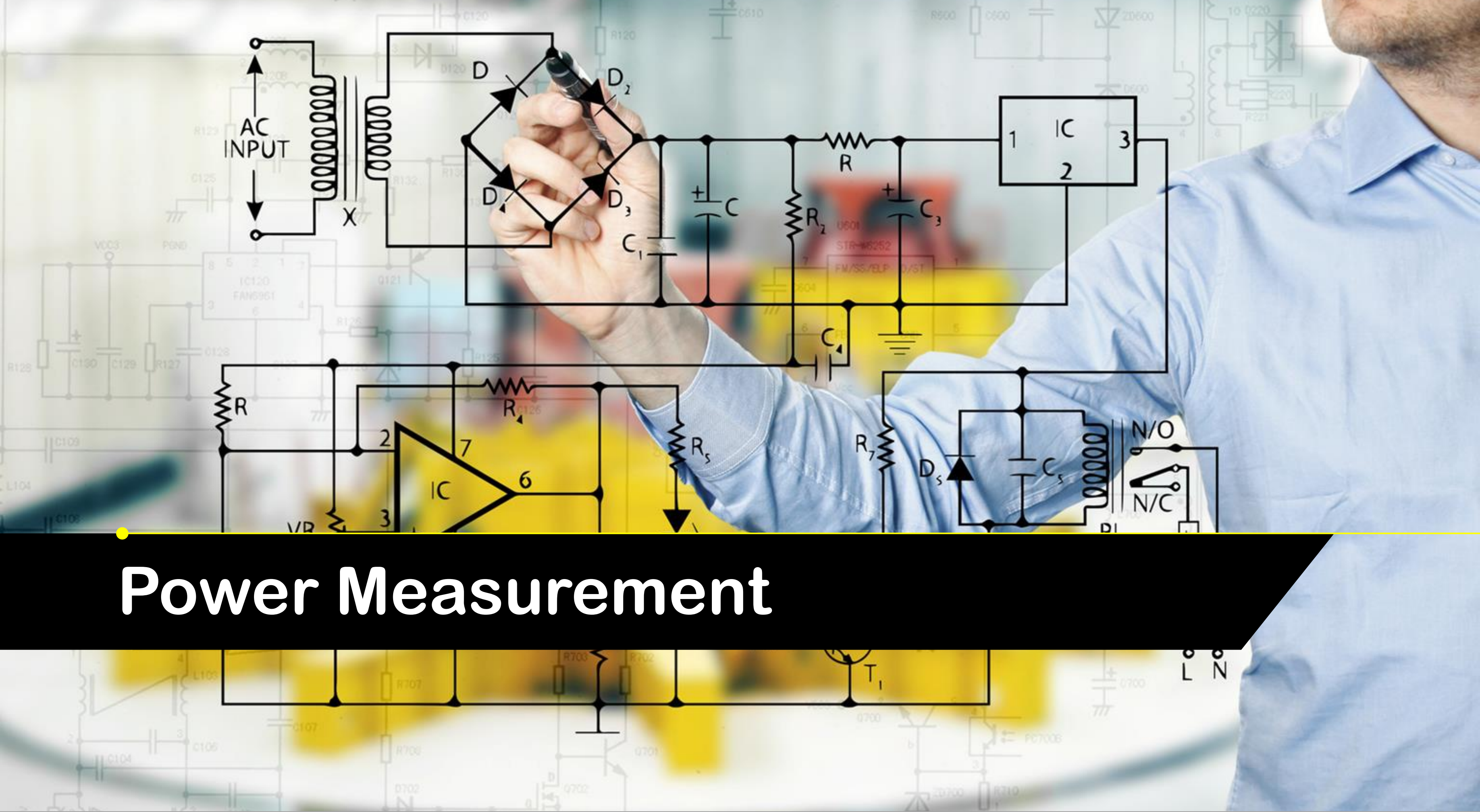
$$\theta_2 = 18.19^o$$



Solving the trigonometric problem

$$I_C = 14.05\text{ A} = \omega C V_{rms}$$

$$\boxed{C = 310.5\,\mu\text{F}}$$



Power Measurement

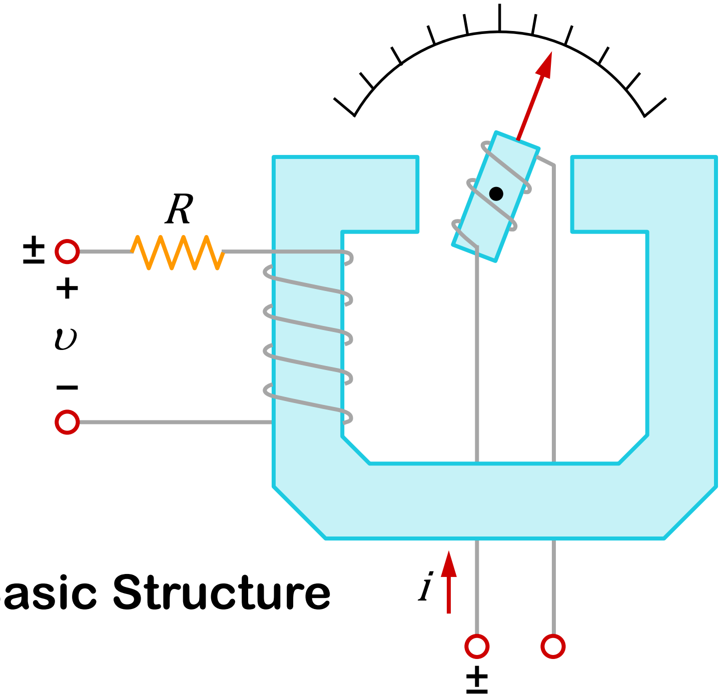
Power Measurement

The wattmeter is the instrument for measuring the average power.

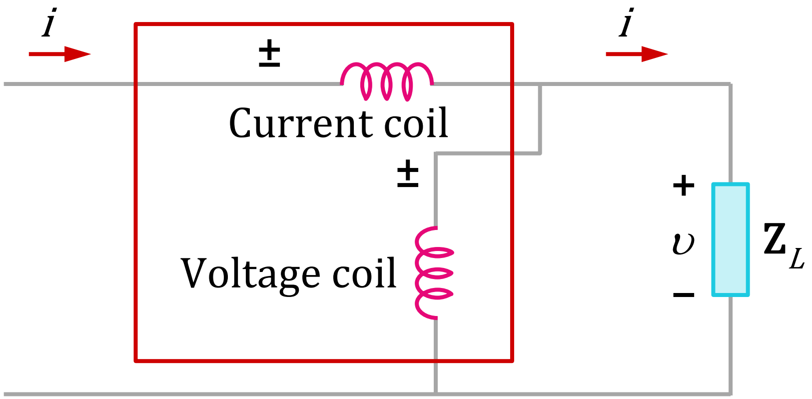
- It consists of a voltage coil and a current coil.
- The current coil of very low impedance is connected in series with the load and responds to load current.
- The voltage coil with very high impedance is connected in parallel with the load and responds to load voltage.

When the two coils are energised, the mechanical inertia of the moving system produces a deflection angle that is proportional to the average value of the product $v(t)i(t)$.

Power Measurement



Basic Structure



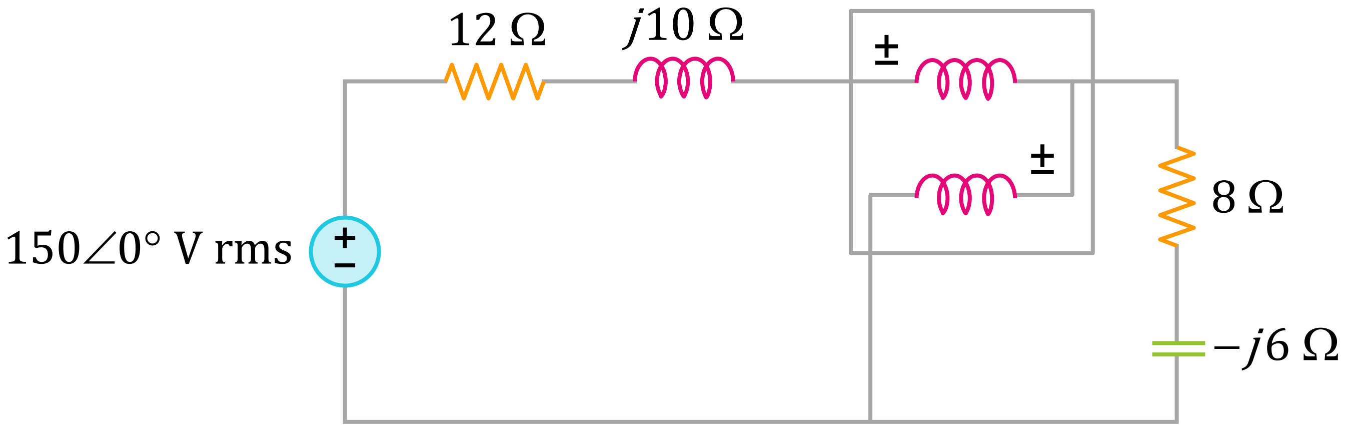
Equivalent Circuit with Load

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$\boxed{P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

Power Measurement: Example 1

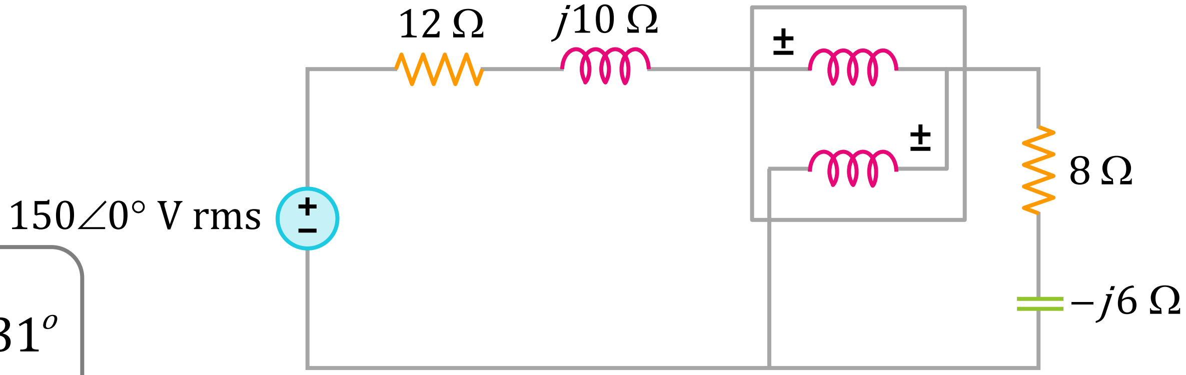
Find the wattmeter reading.



Power Measurement: Example 1

$$\mathrm{I}_{rms} = \frac{150\angle 0^o}{(12+j10)+(8-j6)}$$

$$\mathrm{I}_{rms} = \frac{150\angle 0^o}{20+j4} = 7.354\angle -11.31^o$$

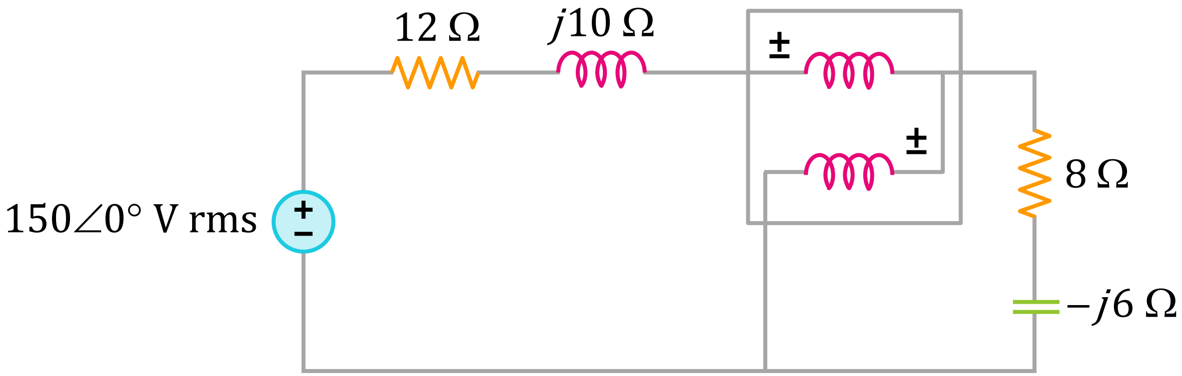


The wattmeter reads $= 7.354^2(8) = 432.7\text{ W}$

Power Measurement: Example 1

Alternatively,

$$\mathbf{I}_{rms} = 7.354\angle -11.31^{o}$$



The voltage across the $(8 - j6)\ \Omega$ **resistor**

$$\mathbf{V}_{rms} = \mathbf{I}_{rms}(8 - j6) = \frac{150(8 - j6)}{20 + j4}$$

Complex power $\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^{*}_{rms} = 432.7 - j324.6$

The wattmeter reads 432.7 W

Summary

Summary

- **The instantaneous power absorbed by an element**

$$p(t) = v(t)i(t)$$

- **Average or real power (watts) is the average of the instantaneous power**

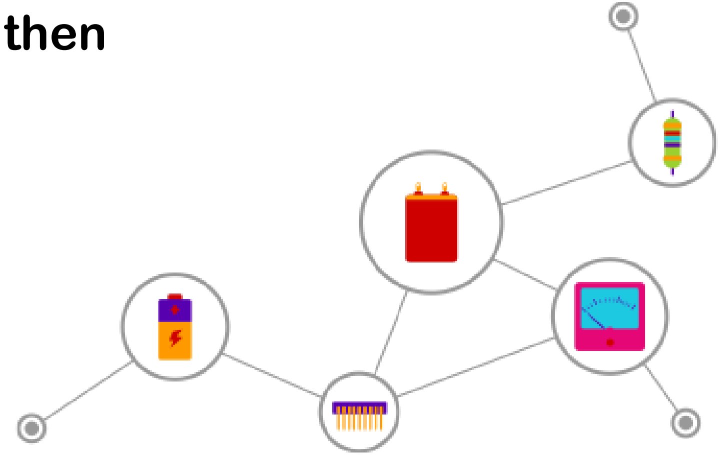
$$P = \frac{1}{T}\int_0^T p(t)dt$$

If $v(t) = V_m \cos(\omega t + \theta_v)$ **and** $i(t) = I_m \cos(\omega t + \theta_i)$**, then**

$$V_{rms} = V_m / \sqrt{2}$$

$$I_{rms} = I_m / \sqrt{2}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

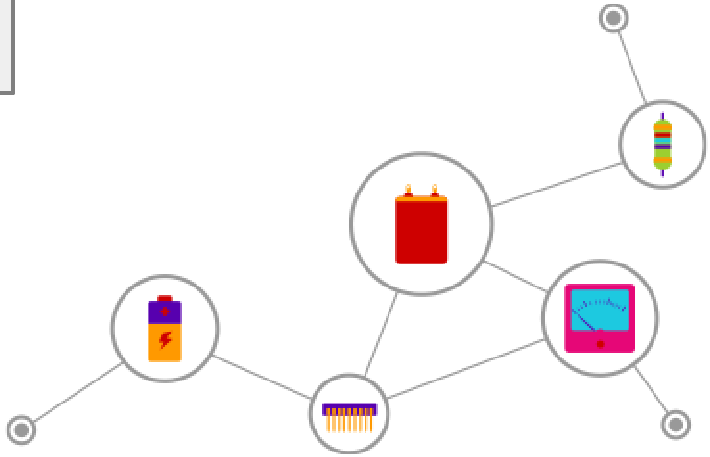


Summary

- Inductors and capacitors absorb no average power, while the average power absorbed by a resistor is $$(1/2)I_m^2 R = I_{rms}^2 R$$

- Maximum average power is transferred to a load when the load impedance is the complex conjugate of the Thevenin impedance as seen from the load terminals $$\mathbf{Z}_L = \mathbf{Z}_{\mathrm{Th}}^*$$

- The power factor $$pf = \cos(\theta_v - \theta_i)$$



Summary

- The pf is also the cosine of the angle of the load impedance or the ratio of the real power to apparent power.
- The pf is lagging if the current lags the voltage-inductive load.
- The pf is leading if the current leads the voltage-capacitive load.

Complex power
$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}_{rms}^{*} = V_{rms}I_{rms}\angle\left(\theta_v - \theta_i\right) = I_{rms}^{2}\mathbf{Z} = \frac{V_{rms}^{2}}{\mathbf{Z}^{*}} = P + jQ$$

Apparent power
$$S = V_{rms}I_{rms} = \left|\mathbf{S}\right| = \sqrt{P^2 + Q^2}$$

Reactive power
$$Q = \frac{1}{2}V_m I_m \sin\left(\theta_v - \theta_i\right) = V_{rms}I_{rms}\sin\left(\theta_v - \theta_i\right)$$

Summary

- The total complex power in a network is the sum of the complex powers of the individual components.

- Total real power and reactive power are also, respectively, the sums of the individual real powers and the reactive powers, but the total apparent power is not calculated by the process.

- Power factor correction is necessary for economic process.

- The wattmeter is the instrument for measuring the average power.

- Energy consumed is measured by a kilowatt-hour meter.