Lecture 3:
Electromagnetic Principles and Actuators

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Interpret the basic concepts of magnetic circuits with air gaps.
- Simplify the magnetic equivalent circuit using the laws of circuit analysis, e.g., series parallel combinations.
- Determine the magnetic flux and flux density in the air gaps using the simplified magnetic equivalent circuit.
- Analyse the concepts of magnetic circuits excited by sinusoidal sources.
- Explain how voltage is induced in a magnetic circuit using the Faraday's Law of Electromagnetic Induction.
- Derive equations to calculate the inductances of magnetic circuits.

Magnetic Circuits with Air Gaps

- Air gaps are integral part of magnetic circuits in various electric machines (see Fig. 27).
- Fringing of flux occurs in air gaps, but will be ignored in this course.
- The effects of air gaps in magnetic circuits will be illustrated with an example.

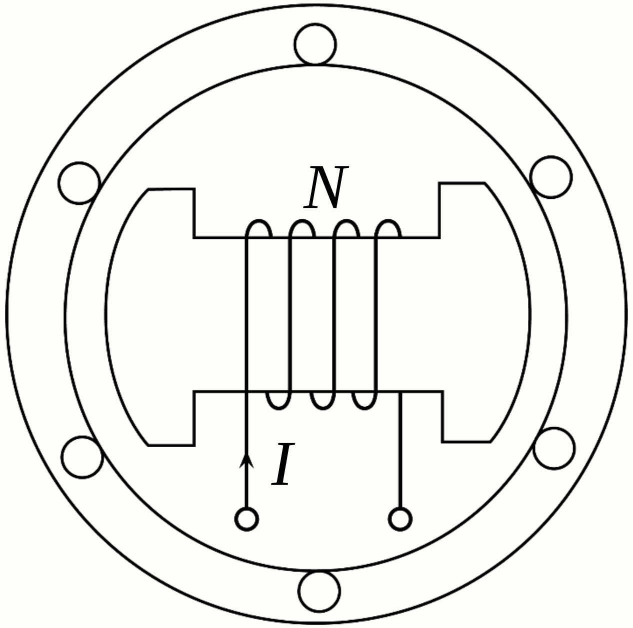

Fig. 27. Magnetic circuit with air gap.

Example 4

An electromagnet of square cross section, shown in Fig. 28, has a coil of 1500 turns. The inner and outer radii of the core are 10 cm and 12 cm respectively, and the air gap is 1 cm. If the current in the coil is 4 A and the relative permeability of the core material is 1200, determine the flux density in the circuit.

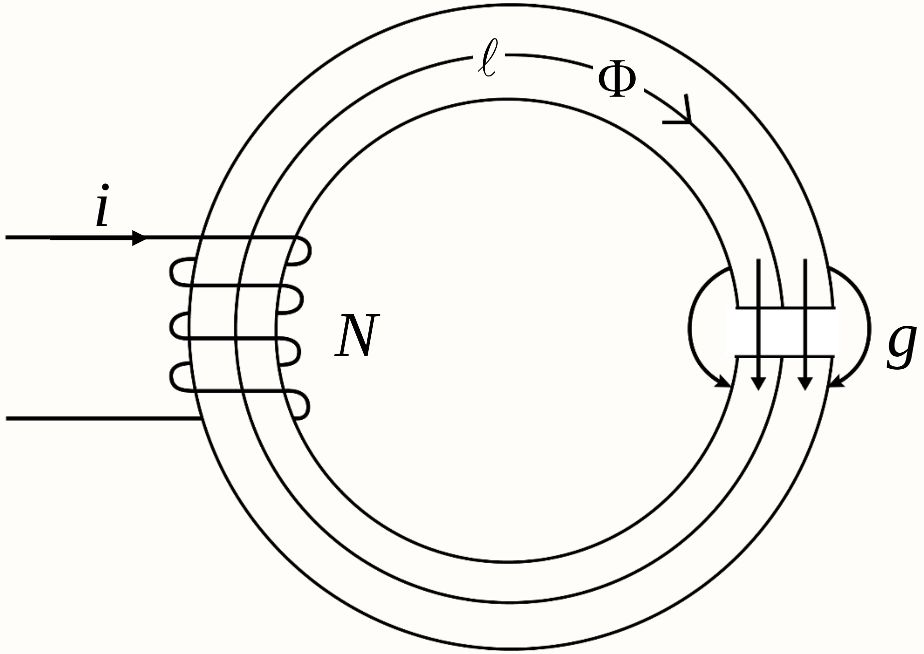

Fig. 28. Electromagnet with air gap.

(Solutions →)

Example 4 – Solutions

The magnetic equivalent circuit is drawn as shown in Fig. 29 using the approach described earlier.

Cross-section area $A_c = A_g = 2\ \text{cm} \times 2\ \text{cm} = 4 \times 10^{-4}\ \text{m}^2$

Mean radius $r = (10 + 12)/2 = 11\ \text{cm}$

Length of core $\ell = 2\pi r - 1 = 68.12\ \text{cm}$

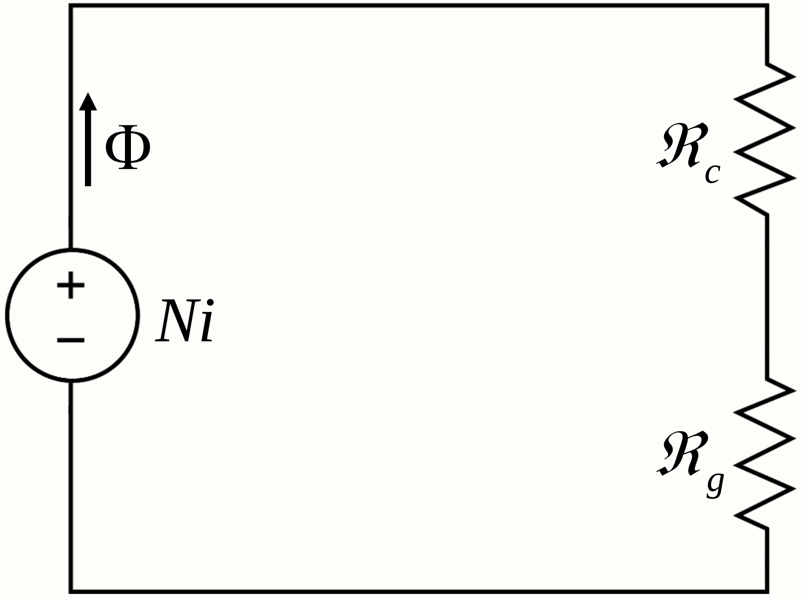

Fig. 29. Magnetic equivalent circuit.

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Example 4 – Solutions

$$\mathfrak{R}_c = \frac{68.12\times10^{-2}}{1200\times4\pi\times10^{-7}\times4\times10^{-4}} = 1.129\times10^{6}\ \text{H}^{-1}$$

$$\mathfrak{R}_g = \frac{1\times10^{-2}}{4\pi\times10^{-7}\times4\times10^{-4}} = 19.894\times10^{6}\ \text{H}^{-1}$$

$$\mathfrak{R}_{eq} = \mathfrak{R}_c + \mathfrak{R}_g = 21.023\times10^{6}\ \text{H}^{-1}$$

$$\varphi = (1500\times4)/21.023\times10^{6} = 2.85\times10^{-4}\ \text{Wb}$$

$$B_c = B_g = (2.85\times10^{-4})/(4\times10^{-4}) = 0.713\ \text{T}$$

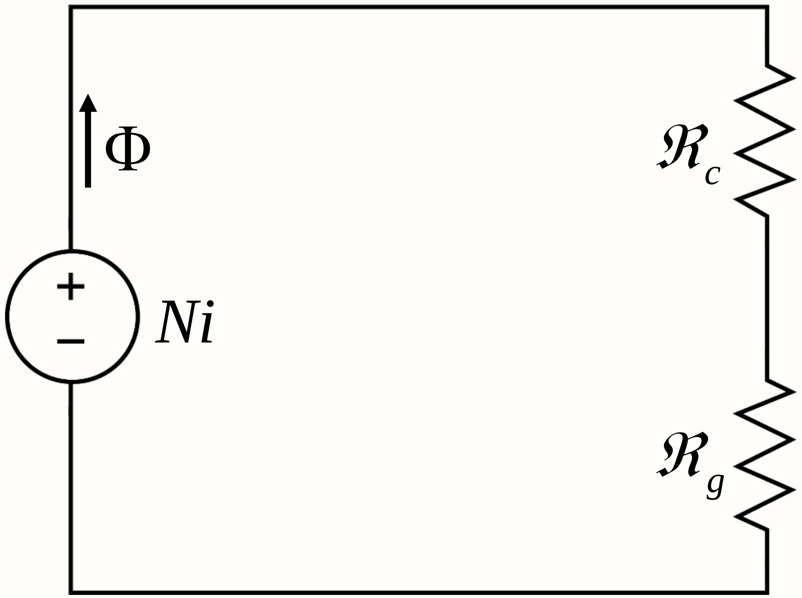

Fig. 29. Magnetic equivalent circuit.

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Example 4 – Solutions

- It should be noted that even if we ignore the core reluctance in the above calculation, the flux density will not change significantly.

- Core sections in magnetic circuits like conductors in electric circuits. Air gap reluctances are like load resistances in electric circuits. The core reluctances can be ignored in many approximate analyses.

Example 5

The magnetic circuit shown in Fig. 30 has a core area of 2×2 cm^2 and a relative permeability of 1000. The coil has 500 turns and carries a current of 2 A. Find the flux density in each gap.

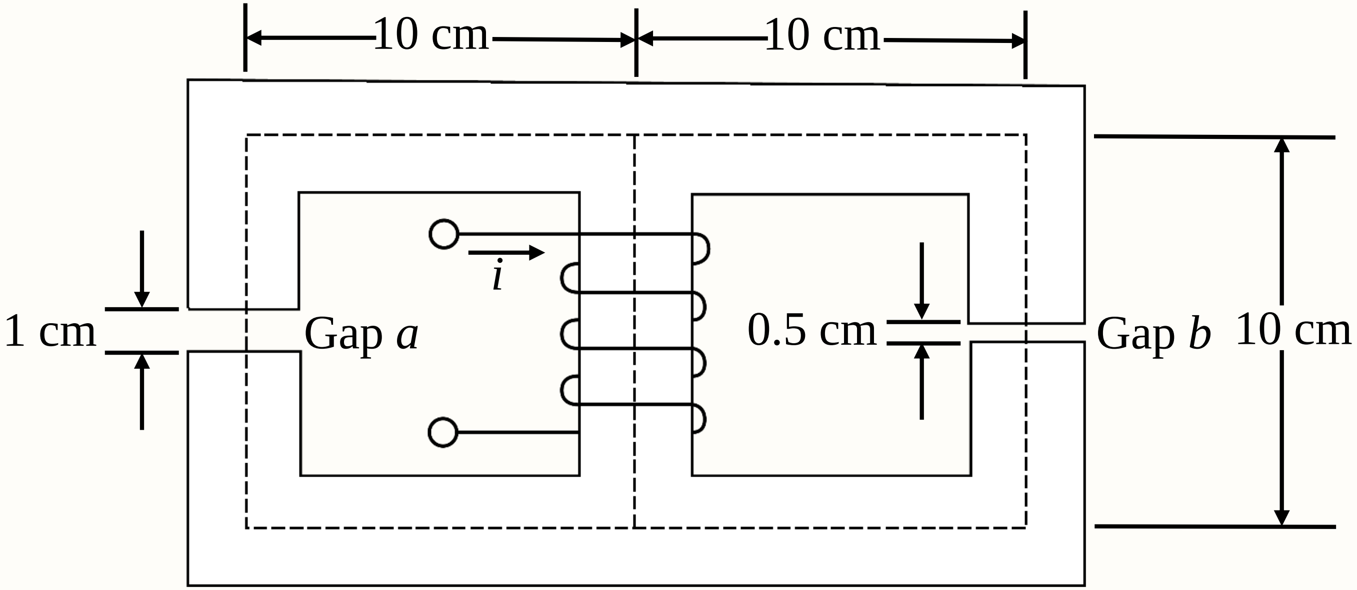

Fig. 30. Magnetic circuit.

(Solutions →)

Example 5 – Solutions

The magnetic equivalent circuit can be drawn as shown in Fig. 31 using the approach discussed earlier. The reluctances are computed as follows:

For the centre core:

$$\mathfrak{R}_c = \frac{10\times10^{-2}}{1000\times4\pi\times10^{-7}\times4\ \times10^{-4}}\,\mathrm{H}^{-1}$$

$$=1.989\times10^{5}\ \mathrm{H}^{-1}$$

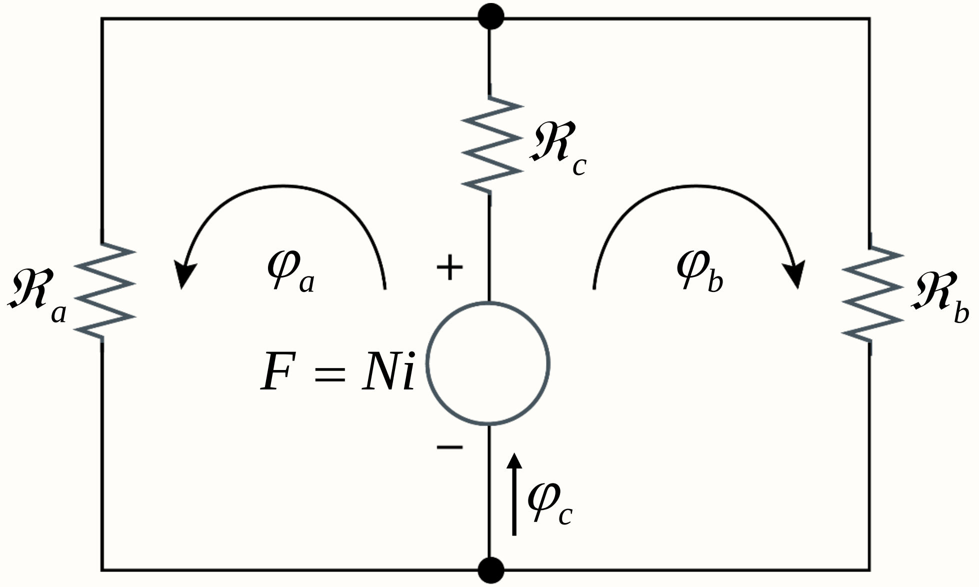

Fig. 31. Magnetic equivalent circuit.

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Example 5 – Solutions

For the left-hand path:

$$\mathfrak{R}_a = \frac{1\times10^{-2}}{4\pi\times10^{-7}\times4\times10^{-4}} + \frac{29\times10^{-2}}{1000\times4\pi\times10^{-7}\times4\times10^{-4}}$$
$$= 204.7\times10^{5}\ \mathrm{H}^{-1}$$

For the right-hand path:

$$\mathfrak{R}_b = \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times4\times10^{-4}} + \frac{29.5\times10^{-2}}{1000\times4\pi\times10^{-7}\times4\times10^{-4}}$$
$$= 104.34\times10^{5}\ \mathrm{H}^{-1}$$

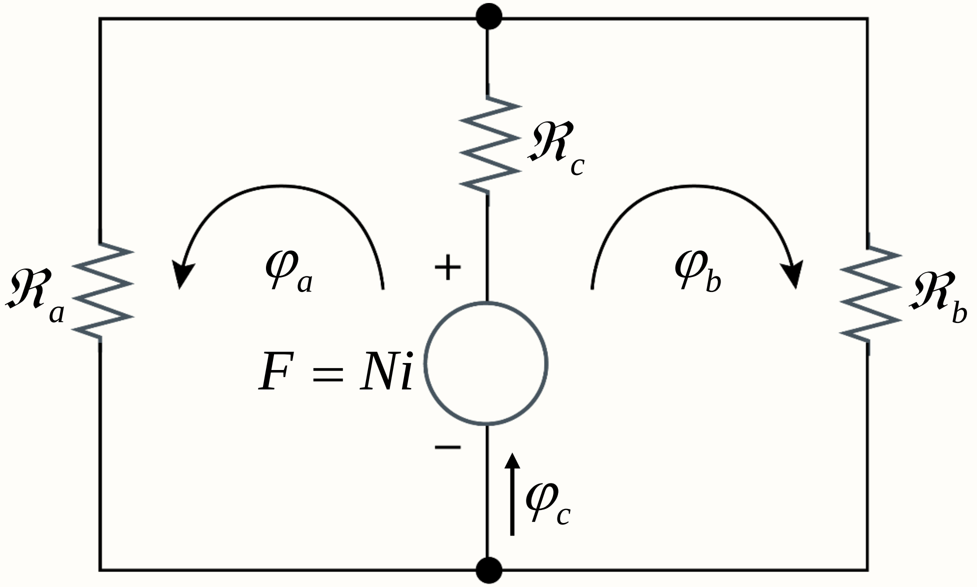

Fig. 31. Magnetic equivalent circuit.

Example 5 – Solutions

The magnetic equivalent circuit can be solved like electric circuits. Two common methods to solve the magnetic equivalent circuit are:

Method 1:

a) Write the loop equations:

i. $Ni = \mathfrak{R}_a \varphi_a + \mathfrak{R}_c \varphi_c$ (Apply KVL)

ii. $Ni = \mathfrak{R}_b \varphi_b + \mathfrak{R}_c \varphi_c$ (Apply KVL)

iii. $\varphi_c = \varphi_a + \varphi_b$ (Apply KCL)

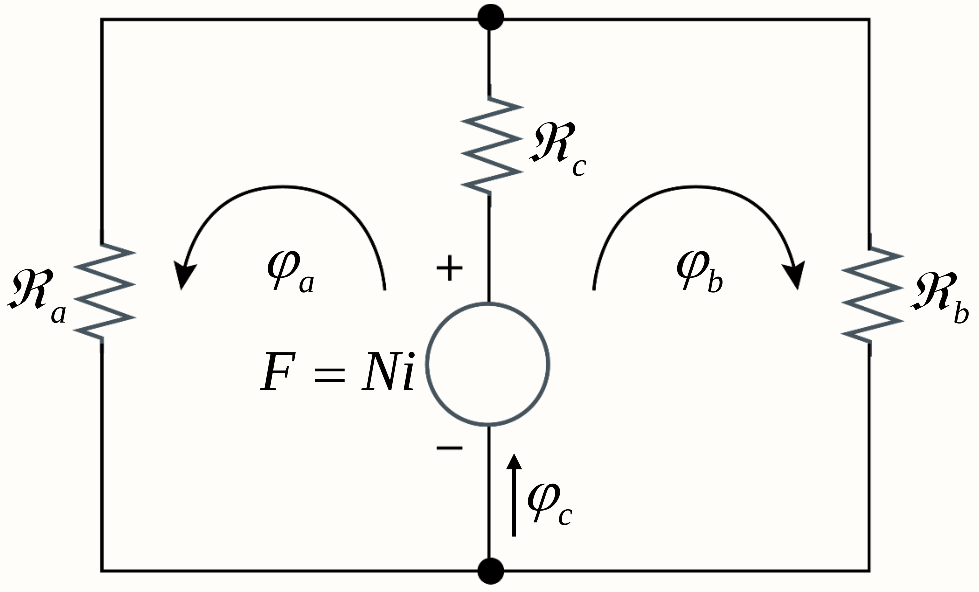

Fig. 31. Magnetic equivalent circuit.

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Example 5 – Solutions

b) Solve these equations to get the values of $\varphi_a, \varphi_b, \varphi_c$, and then any other quantities of interest.

$$B_a = \frac{\varphi_a}{A_a}, \text{and } B_b = \frac{\varphi_b}{A_b}$$

c) Complete the rest of the problem.

Answers :

$$\varphi_a = 48.4\ \mu\text{Wb},\ \varphi_b = 91.4\ \mu\text{Wb}$$

$$B_a = 0.121\,\text{T}, B_b = 0.228\,\text{T}$$

Example 5 – Solutions

Method 2:
Simplify the magnetic equivalent circuit utilising the laws of circuit analysis, e.g., series parallel combinations, etc. In this circuit, it is clear that $\Re_a$ and $\Re_b$ are in parallel which is then in series with $\Re_c$. Therefore, the circuit can be reduced as shown in Fig. 32, where

$$\Re_{eq} = (\Re_a // \Re_b) + \Re_c$$

Then,

$$\varphi_c = \frac{Ni}{\Re_{eq}}, \varphi_a = \frac{\Re_b}{\Re_a + \Re_b}\varphi_c, \text{and } \varphi_b = \frac{\Re_a}{\Re_a + \Re_b}\varphi_c$$

$$B_a = \varphi_a / A_a, \text{and } B_b = \varphi_b / A_b$$

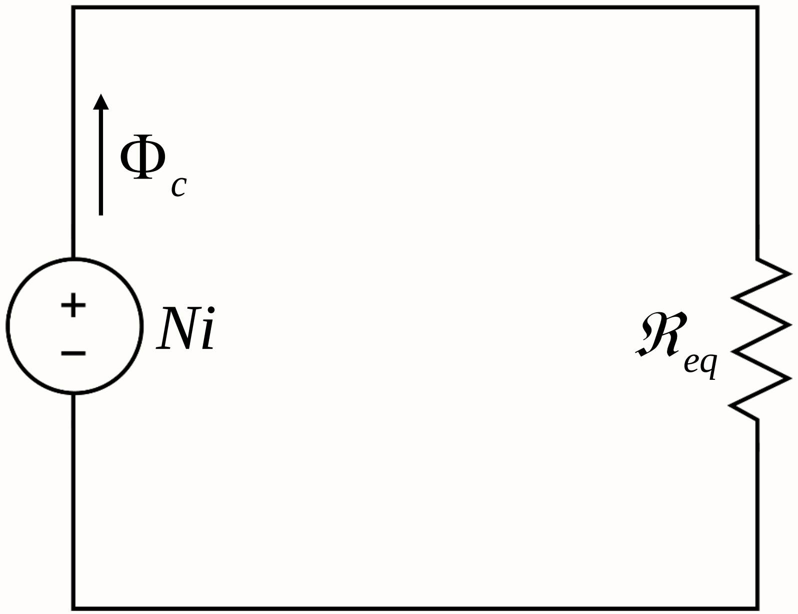

Fig. 32. Magnetic equivalent circuit.

Example 5 – Solutions

- A Thevenin equivalent like this can be formed for any magnetic circuit.
- Discussion on further concepts in magnetic circuits will be based on such a simplified circuit, which can easily be applied to more elaborated circuits by reducing these elaborated circuits to their Thevenin equivalents.

Faraday's Law (Induced EMF)

❖ Consider a magnetic circuit as shown in Fig. 33. Input current i in the coil of N turns establishes a flux φ in the core.

- **Flux Linkage** (λ):
  When a flux φ passes through a coil of N turns, the flux φ is said to link the coil, and $\lambda = N\varphi$ is called the flux linkage of the coil.

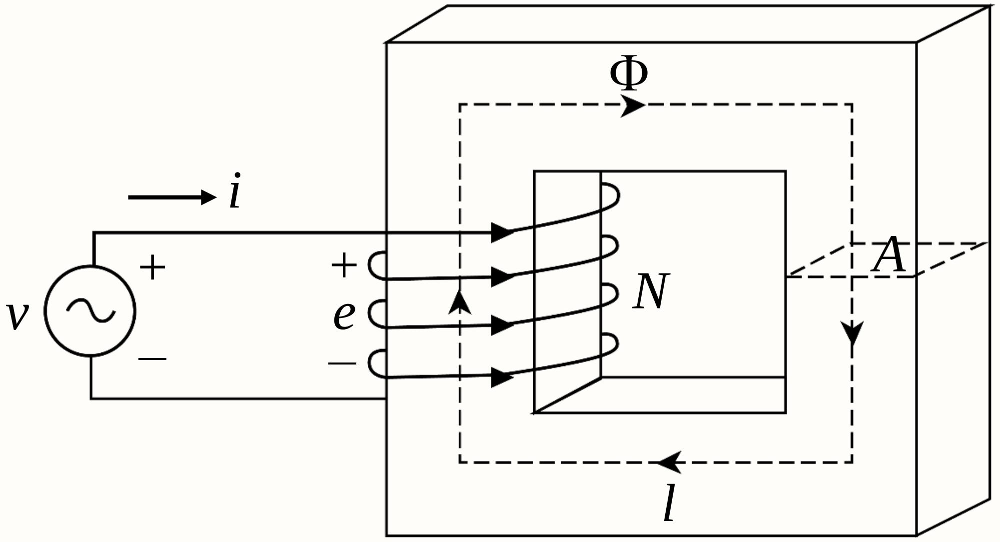

Fig. 33. Magnetic circuit.

Faraday's Law (Induced EMF)

- **Faraday's Law**, which is embedded in the Maxwell's Equations, states that a coil with flux linkage λ will have an induced voltage $\boldsymbol{e}$ in it, given by

$$e = -\frac{d\lambda}{dt} = -N\frac{d\varphi}{dt}$$

The negative sign indicates that the direction of the induced voltage will be such that it will tend to oppose the current i/voltage v creating the flux.

Sinusoidal Excitation of Magnetic Circuits

- When a sinusoidal input current $\boldsymbol{i}$ (due to a sinusoidal voltage v) is provided to a linear magnetic circuit as shown in Fig. 34, the *mmf* ($\boldsymbol{Ni}$) and therefore the flux ($\boldsymbol{Ni}/\mathfrak{R}$) produced in the core will also be sinusoidal.

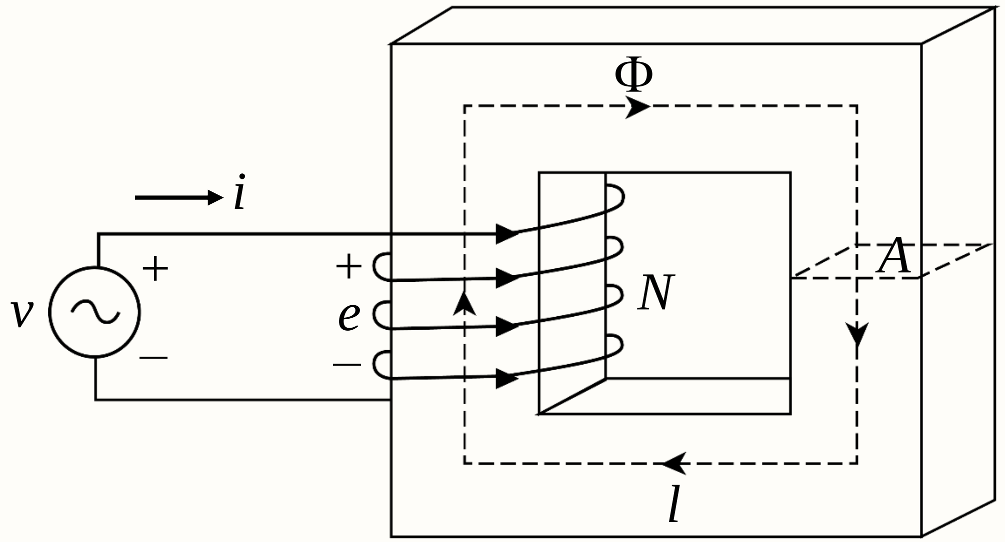

Fig. 34. Magnetic circuit.

Sinusoidal Excitation of Magnetic Circuits

Let $\varphi = \Phi_m \sin \omega t$, then

$$v = e = N\frac{d\Phi}{dt} = N\Phi_m \omega \cos \omega t$$

$$= N(B_m A)(2\pi f)\cos \omega t$$

$$= 2\pi NA(B_m f)\cos \omega t$$

Therefore, the root mean square (rms) value of the voltage is given by

$$V = V_m / \sqrt{2} = 2\pi NA(B_m f)/\sqrt{2} = 4.44 NAB_m f$$

- Thus, it is seen that the product $B_m f$ is related to the input voltage, and these two variables are required to be treated jointly in the analysis of many magnetic circuits excited by sinusoidal sources.

Inductance (Self-inductance)

- Consider the magnetic circuit shown in Fig. 35. A current ***i*** through the coil will produce a flux φ, and therefore a flux linkage $\lambda = N\varphi$ of the coil.

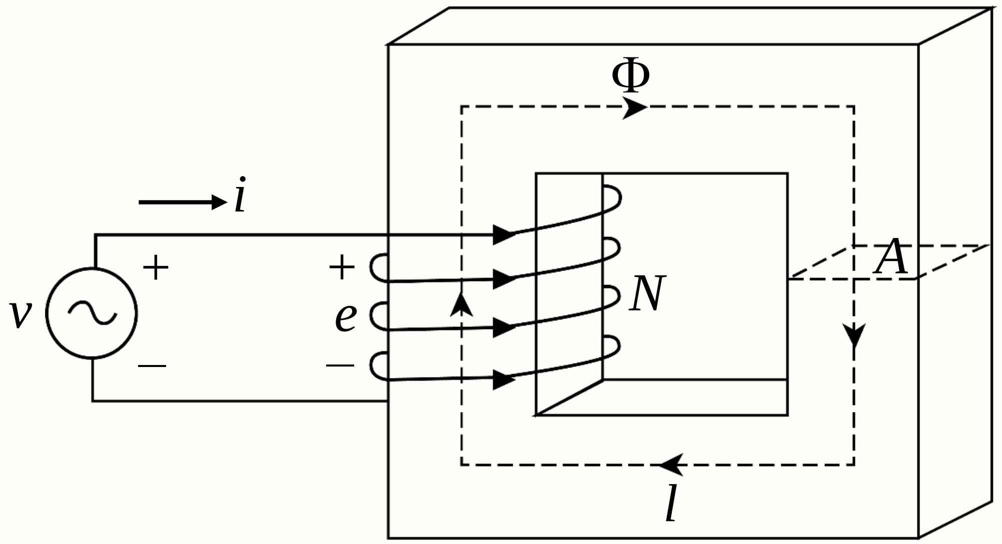

Fig. 35. Magnetic circuit.

Inductance (Self-inductance)

- The inductance of the coil is defined as

$$L = \frac{d\lambda}{dt}, (L = \frac{\lambda}{i} \text{ for linear systems})$$

Typo

Since $\lambda = N\varphi = N\dfrac{Ni}{\Re}$, then $L = \dfrac{\lambda}{i} = \dfrac{N^2 i}{i\Re} = \dfrac{N^2}{\Re}$ H

Thus, the inductance $\boldsymbol{L}$ depends on the coil properties and the physical properties (dimensions) of the magnetic material ($\Re = \ell / \mu A$).

- For linear magnetic circuits, reluctance $\boldsymbol{\Re}$ is constant, and therefore the inductance $\boldsymbol{L}$ is constant for a given coil.

Inductance (Self-inductance)

- If the magnetic circuit consists entirely of ferro-magnetic material, the B-H curve is hardly linear. Saturation often occurs, $\boldsymbol{\mu}$ and $\mathfrak{R}$ do not remain constant, so that $\boldsymbol{L}$ does not remain constant. If constant inductance $\boldsymbol{L}$ is desired, air gaps are often introduced in the magnetic circuit.

- For a magnetic circuit excited by AC source as shown in Fig. 35, the induced *emf* $\boldsymbol{e}$ can be expressed as

$$e = -\frac{d\lambda}{dt} = -\frac{d\lambda}{di}\frac{di}{dt} = -L\frac{di}{dt}, \text{i.e., } e = -L\frac{di}{dt}$$

This is the common form of Faraday's Law used in electric circuit analysis. Often, the negative sign is dropped by considering the direction of $\boldsymbol{e}$ separately.

Inductance (Self-inductance)

- If the resistance of the coil is negligibly small, then the input voltage ***v*** is completely balanced by the induced emf ***e***, so that

$$v = e = L\frac{di}{dt}$$

- If the resistance of the coil is considered, then the input voltage is balanced by the drop in the resistance and the induced voltage in the inductor, so that

$$v = ri + L\frac{di}{dt} \;\Rightarrow\; V = Ir + IjX = Ir + Ij\omega L = I(r + j\omega L) = IZ$$

Summary

In this lecture, you have learnt:

- The concepts of magnetic circuits with air gaps in various electric machines.
- Simplified magnetic equivalent circuit using the laws of circuit analysis, e.g., series parallel combinations.
- The basic principles of magnetic circuits excited by sinusoidal sources.
- Voltage induced in a magnetic circuit using the Faraday's Law of Electromagnetic Induction.

References

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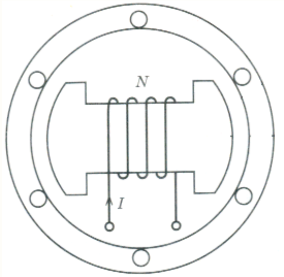
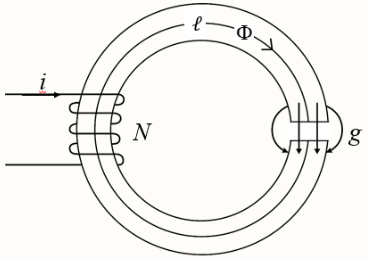
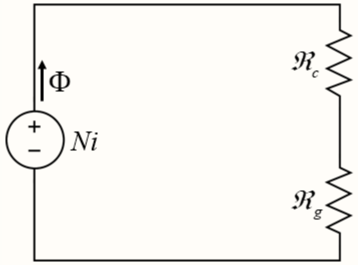
1	4		Reprinted from *AC Circuits and Machines*, (p. 90), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
2	5		Reprinted from *AC Circuits and Machines*, (p. 101), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
3	6 and 7		Reprinted from *AC Circuits and Machines*, (p. 101), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.

References

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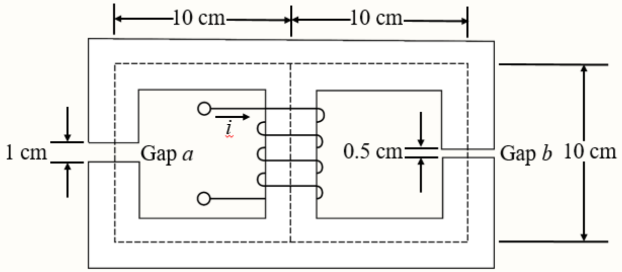
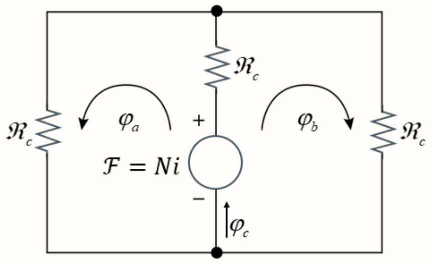
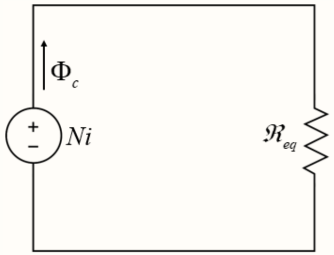
4	9		Reprinted from *AC Circuits and Machines*, (p. 103), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
5	10, 11 and 12		Reprinted from *AC Circuits and Machines*, (p. 103), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.
6	14		Reprinted from *AC Circuits and Machines*, (p. 99), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte. Ltd. Copyright 2006 by Pearson Education South Asia Pte. Ltd. Reprinted with permission.

References

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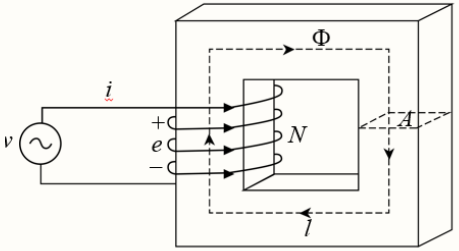
7	16, 18 and 20		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 147), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.