NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE3010 – ELECTRICAL DEVICES AND MACHINES

April / May 2021 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. Consider the magnetic frame shown in Figure 1 whose core has a cross-sectional area of 9 cm^2. The first, second and third sections of the frame have mean lengths of $l_1 = 100\ cm$, $l_2 = 40\ cm$ and $l_3 = 80\ cm$, respectively. A 200-turn coil is wound uniformly on the second section. The relative permeability of the magnetic material is $\mu_r = 1500$ and the permeability of free space is $4\pi x10^{-7}$ H/m. Neglect the leakage and fringing effects.

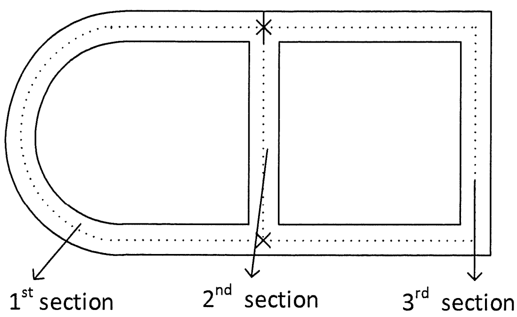


Figure 1

(a) If the current in the coil is $I = 2.5$ A, determine the flux and the flux density in each of the three sections of the magnetic frame.

(13 Marks)

Note: Question No. 1 continues on page 2.

(b) Determine the total energy stored in the magnetic system and the energy stored in each of the three sections of the magnetic frame. What is the inductance of the coil?

(6 Marks)

(c) Suppose the energy stored in the first section is now 0.1504 J. Using the information given in part (a) and the answers found in part (b) or otherwise, determine the current in the coil and the energies stored in the magnetic system, in the second and in third sections, respectively.

(6 Marks)

2. A 20-kVA, 2400/240-V, 50-Hz single-phase transformer has the following parameters:

$$R_1 = 0.25\ \Omega,\quad X_1 = 2.25\ \Omega,\qquad R_2 = 0.025\ \Omega,\qquad X_2 = 0.08\ \Omega,$$

$$R_{c1} = 5000\ \Omega,\qquad X_{m1} = 2500\ \Omega$$

(a) Draw the approximate equivalent circuit of the transformer referred to the primary side and use it to determine the secondary terminal voltage V_2 at rated secondary-side current and at a power factor of 1 if the primary supply voltage is 2400 V. You may use the low voltage as the reference, i.e., $\vec{V}_2 = V_2\angle 0^\circ\ V$ and $\vec{V}_1 = 2400\angle\theta^\circ$, where θ and V_2 are to be determined.

(12 Marks)

(b) Determine the input power factor and efficiency of the transformer at the specified condition in part (a).

(7 Marks)

(c) A two-winding transformer is connected as an autotransformer to give 720/240 V. Sketch the connection diagram of the autotransformer. If a load of 9 kW at a power factor of 0.75 is connected to the output terminals of the autotransformer, determine its kVA rating and the currents in its windings. What is the kVA of the two-winding transformer?

(6 Marks)

3. (a) A 6-pole, 50-Hz, three-phase induction motor has an efficiency of 88% when delivering an output power of 18 kW. Assume that the stator copper loss and the constant rotational losses are 800 W and 1200 W, respectively. Determine

(i) the slip,
(ii) the torque developed, and
(iii) the rotor copper loss.

(7 Marks)

Note: Question No. 3 continues on page 3.

(b) A three-phase, 230-V, 60-Hz, 4-pole, wye-connected induction motor has the following per phase parameters referred to the stator:

$$R_1 = 0.2\ \Omega,\ R_2 = 0.2\ \Omega,\ X_M = 50\ \Omega,\ X_1 = 1\ \Omega,\ X_2 = 1\ \Omega$$

Determine

(i) the input current drawn by the motor at starting, and
(ii) the starting torque developed.

(10 Marks)

(c) A three-phase, 208-V, 50-Hz, wye-connected induction motor operates at rated voltage and frequency. Assume that the stator winding resistance and rotational losses can be neglected. The rotor copper loss at maximum developed torque is 10 times the rotor copper loss when the motor is running at full load. If the slip at full load is 0.02, determine the slip at which maximum torque is developed.
(Hint: Use Thevenin equivalent circuit of the motor.)

(8 Marks)

4. (a) The armature terminals of a dc shunt motor are connected to a dc supply of 200 V. The motor has an armature winding resistance of 0.1 Ω and a field winding resistance of 250 Ω. The rotational losses of the motor are 220 W. The motor takes an input current of 30 A from the supply. Determine

(i) the power developed, and
(ii) the efficiency of the motor.

(8 Marks)

(b) A dc shunt motor has an armature winding resistance of 0.1 Ω and a field winding resistance of 400 Ω. The armature terminals are connected to a dc supply of 240 V and the motor is running at a speed of 1000 rpm when the armature current is 50 A. Determine the additional resistance to be inserted in series with the field winding to increase the speed to 1200 rpm. Assume that the shaft load is adjusted such that the armature current remains the same and the machine is operating in the linear part of the magnetization curve.

(7 Marks)

(c) The armature and field winding resistances of a separately excited dc generator are 0.04 Ω and 45 Ω, respectively. The field winding is connected to a dc supply of 120 V. The rotational losses of the machine are 1500 W. The effects of armature reaction can be neglected. Consider the following two cases:

Note: Question No. 4 continues on page 4.

(i) The generator is driven at a speed of 1000 rpm by an external prime mover and the no-load terminal voltage is 150 V. Determine the efficiency of the generator when the generator is supplying a load at a terminal voltage of 140 V.

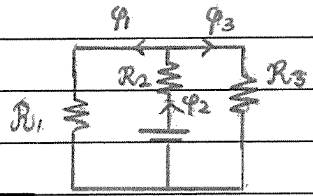
(ii) The speed of the external prime mover is increased to 1200 rpm and the generator is delivering the same armature current as that in part (c)(i). Determine the mechanical power supplied by the external prime mover to the armature. Assume that the field current is unchanged.

(10 Marks)

END OF PAPER

1. $l_1 = 100\ cm$ $l_2 = 40\ cm$ $l_3 = 80\ cm$ $A = 9\ cm^2$

$N = 200$ turns $\mu_r = 1500$ $\mu_o = 4\pi \times 10^{-7}\ H/m$



a) $I = 2.5A$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_o A}$$

$$= \frac{100 \times 10^{-2}}{1500 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$= 589462.75\ H^{-1}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_r \mu_o A}$$

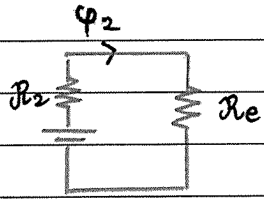
$$= \frac{40 \times 10^{-2}}{1500 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$= 235785.10\ H^{-1}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_o A}$$

$$= \frac{80 \times 10^{-2}}{1500 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}}$$

$$= 471570.20\ H^{-1}$$



$$\mathcal{R}_e = \mathcal{R}_1 \parallel \mathcal{R}_3 = \frac{\mathcal{R}_1 \mathcal{R}_3}{\mathcal{R}_1 + \mathcal{R}_3}$$

$$= \frac{(589462.75)(471570.20)}{589462.75 + 471570.20}$$

$$= 261983.44\ H^{-1}$$

$$\mathcal{F} = NI = (\mathcal{R}_e + \mathcal{R}_2)\varphi_2$$

$$200 \times 2.5 = (261983.44 + 235785.10) \times \varphi_2$$

$$\varphi_2 = \frac{200 \times 2.5}{261983.44 + 235785.10}$$

$$= 1\ mWb$$

$$\varphi_1 = \frac{\mathcal{R}_3}{\mathcal{R}_1 + \mathcal{R}_3}\varphi_2$$

$$= \frac{471570.20}{589462.75 + 471570.20} \times 1m$$

$$= 4.44 \times 10^{-4}\ Wb$$

$$\varphi_3 = \varphi_2 - \varphi_1 = 5.56 \times 10^{-4}\ Wb$$

$$B_1 = \frac{\varphi_1}{A}$$

$$= \frac{4.44 \times 10^{-4}}{9 \times 10^{-4}} = 0.49\ T$$

$$B_2 = \frac{\varphi_2}{A}$$

$$= \frac{1 \times 10^{-3}}{9 \times 10^{-4}} = 1.11\ T$$

$$B_3 = \frac{\varphi_3}{A}$$

$$= \frac{5.56 \times 10^{-4}}{9 \times 10^{-4}} = 0.62\ T$$

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b) $L = \dfrac{N^2}{\mathcal{R}_T} = \dfrac{200^2}{261983.44 + 235785.1}$ $\quad * \mathcal{R}_T = \mathcal{R}_e + \mathcal{R}_2$

$= 80.359 \times 10^{-3}$ H

Energy stored in the coil:

$W = \frac{1}{2} L i^2 = \frac{1}{2} \times 80.359 \times 10^{-3} \times (2.5)^2$

$= 0.2511$ J

Energy stored in the core:

$W_c = \frac{1}{2} \mathcal{R}_2 \varphi_2^2 = \frac{1}{2} \times 235785.1 \times (1 \times 10^{-3})^2$

$= 0.1179$ J

Energy stored in 1st section:

$W_1 = \frac{1}{2} \mathcal{R}_1 \varphi_1^2 = \frac{1}{2} \times 589462.75 \times (4.44 \times 10^{-4})^2$

$= 0.0581$ J

Energy stored in 3rd section:

$W_3 = \frac{1}{2} \mathcal{R}_3 \varphi_3^2 = \frac{1}{2} \times 471570.2 \times (5.56 \times 10^{-4})^2$

$= 0.0729$ J

$W_T = 0.2511 + 0.1179 + 0.0581 + 0.0729$

$= 0.5$ J

c) $W_1 = 0.1504$ J

$\frac{1}{2} \mathcal{R}_1 \varphi_1^2 = 0.1504$

$\varphi_1^2 = \dfrac{2 \times 0.1504}{589462.75}$

$\varphi_1 = 7.1435 \times 10^{-4}$ Wb

$\varphi_2 = \dfrac{\mathcal{R}_3 + \mathcal{R}_1}{\mathcal{R}_3} \times \varphi_1$

$= \dfrac{471570.2 + 589462.75}{471570.2} \times 7.1435 \times 10^{-4}$

$= 1.6073 \times 10^{-3}$ Wb

$\varphi_3 = \varphi_2 - \varphi_1 = 1.6073 \times 10^{-3} - 7.1435 \times 10^{-4}$

$= 8.9295 \times 10^{-4}$ Wb

$\mathcal{F} = NI = \varphi_1 \mathcal{R}_1 + \varphi_2 \mathcal{R}_2 + \varphi_3 \mathcal{R}_3$

$200 I = (7.1435 \times 10^{-4})(589462.75) + (1.6073 \times 10^{-3})(235785.10)$

$+ (8.9295 \times 10^{-4})(471570.20)$

$I = 6.1057 = 6.11$ A

Energy stored in the coil:

$W = \frac{1}{2} L i^2 = \frac{1}{2} \times 80.359 \times 10^{-3} \times (6.1057)^2$

$= 1.4979 \text{ J}$

$W_2 = \frac{1}{2} \mathcal{R}_2 \varphi_2^2 = \frac{1}{2} \times 235785.1 \times (1.6073 \times 10^{-3})^2$

$= 0.3046 \text{ J}$

$W_3 = \frac{1}{2} \mathcal{R}_3 \varphi_3^2 = \frac{1}{2} \times 471570.2 \times (8.9295 \times 10^{-4})^2$

$= 0.1880 \text{ J}$

$W_T = 0.1504 + 1.4979 + 0.3046 + 0.1880$

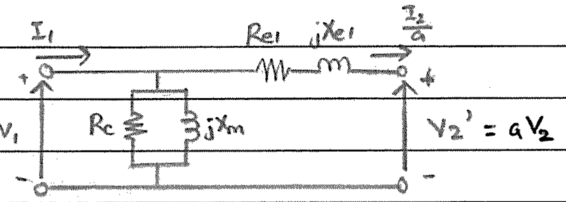
$= 2.1409 \text{ J}$

2. 20 KVA , 2400/240 V , 50 Hz

$R_1 = 0.25\,\Omega$ $X_1 = 2.25\,\Omega$ $R_2 = 0.025\,\Omega$ $X_2 = 0.08\,\Omega$

$R_{c1} = 5000\,\Omega$ $X_{m1} = 2500\,\Omega$

a)



$a = \frac{V_1}{V_2} = \frac{2400}{240} = 10$

$R_{e1} = R_1 + a^2 R_2$

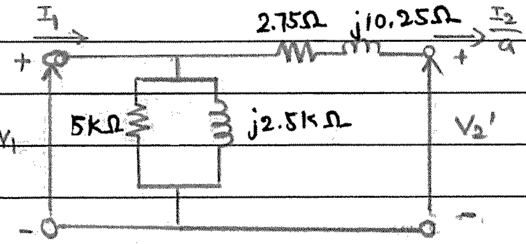
$= 0.25 + (100)(0.025)$

$= 2.75\,\Omega$

$X_{e1} = X_1 + a^2 X_2$

$= 2.25 + (100)(0.08)$

$= 10.25\,\Omega$



$R_c = R_{c1} = 5000\,\Omega$, $jX_m = jX_{m1} = j2500\,\Omega$

At rated secondary current I_2

Rated $I_2 = \frac{20k}{240} = 83.33 \text{ A}$

p.f = 1 $\cos^{-1}(1) = 0°$

$I_2 = 83.33\angle 0°$, $I_2' = 8.33\angle 0°$

Using V_2 as reference , $\vec{V_2} = V_2\angle 0° \text{ V}$ and $\vec{V_1} = 2400\angle\theta°$, $V_2' = aV_2\angle 0° \text{ V}$

$V_1 = aV_2\angle 0° + I_2'(R_{e1} + jX_{e1})$

$2400\angle\theta° = aV_2\angle 0° + (8.33\angle 0°)(2.75 + j10.25)$ Note: aV_2 is real value

$2400\angle\theta° = aV_2 + 22.91 + j85.38$

rect form

$x + jy = aV_2 + 22.91 + j85.38$

As we have value of V_1 and value of y, we can guess θ and aV_2 using calculator

$2400\angle 1° = 2399.63 + j41.89$

$2400\angle 2° = 2398.54 + j83.76$

$2400\angle 2.03° = 2398.49 + j85.01$

$2400\angle 2.0388° = 2398.48 + j85.38$

$V_2 = \frac{2398.48}{10} = 239.848 = 239.85 \text{ V}$

$\theta = 2.0388° = 2.039°$

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b) $pf = \cos 2.039^\circ = 0.99937$

$P_{in} = 0.99937 \times 20K = 19987.4\ W$

$\text{Copper Loss} = (I_2')^2 \times R_{e1} = 8.33^2 \times 2.75 = 190.82\ W$

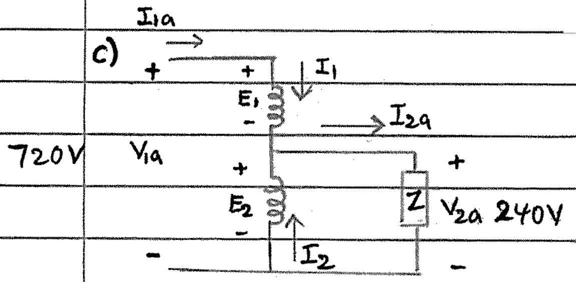
$\text{Core Loss} = \frac{V_1^2}{R_{c1}} = \frac{2400^2}{5000} = 1152\ W$

$P_{out} = 19987.4 - 190.82 - 1152 = 18644.58\ W$

$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{18644.58}{19987.4} \times 100\%$

$= 93.282\%$

c)



$P_{out} = 9\ kW \qquad p.f = 0.75$

$S_{2a} = \frac{9k}{0.75} = 12\ kVA$

$I_{2a} = \frac{12k}{240} = 50\ A$

$I_1 = I_{1a} = \frac{12k}{720} = \frac{50}{3}$

$I_2 = I_{2a} - I_1 = 50 - \frac{50}{3} = \frac{100}{3}$

$S = I_2 E_2 = \frac{100}{3} \times 240$

$= 8000\ VA$

3. a) $p = 6 \quad f_s = 50\ Hz \quad \eta = 88\% \quad P_{out} = 18\ kW$

$P_{SCL} = 800\ W \qquad P_{rot} = 1200\ W$

i) $n_{sync} = \frac{120 f_s}{P} = \frac{120 \times 50}{6} = 1000\ rpm$

$P_{in} = \frac{100}{88} \times 18\ kW = 20454.55\ W$

$P_{dev} = P_{out} + P_{rot} = 18K + 1200 = 19200\ W = (1-s) P_{AG}$

$P_{AG} = P_{in} - P_{SCL} = 20454.55 - 800 = 19654.55\ W$

$1 - s = \frac{P_{dev}}{P_{AG}}$

$= \frac{19200}{19654.55}$

$= 0.9769$

$s = 0.0231 = 2.31\%$

ii) $n_m = (1 - s)\, n_{sync}$

$= (1 - 0.0231) \times 1000$

$= 976.9\ rpm$

$\omega_m = 976.9 \times \frac{2\pi}{60} = 102.30\ rad$

$P_{dev} = T_{dev}\, \omega_m$

$19200 = T_{dev} \times 102.3$

$T_{dev} = 187.68\ N\text{-}m$

iii) $P_{RCL} = s P_{AG} = (0.0231)(19654.55) = 454.02\ W$

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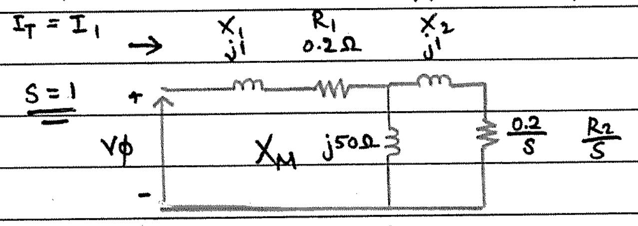
b) $V_L = 230\text{ V}$ $f_s = 60\text{ Hz}$ $p=4$ Y-connected

$R_1 = 0.2\Omega$ $R_2 = 0.2\Omega$ $X_M = 50\Omega$ $X_1 = 1\Omega$ $X_2 = 1\Omega$

i) $\underline{\underline{s=1}}$



Per phase equivalent circuit of an induction motor referred to the stator

The motor is wye-connected. Thus, link current is the same as the phase current.

$$V\phi = \frac{230}{\sqrt{3}}\angle 0^\circ = 132.79\angle 0^\circ\text{ V}$$

Total impedance seen by source

$$Z_T = (R_1 + jX_1) + (jX_M \,//\, (\tfrac{R_2}{s} + jX_2))$$

$$= 0.2 + \cancel{j0}\, j1 + (j50 \,//\, (\tfrac{0.2}{s} + j1))$$

$$= 0.2 + j1 + (j50 \,//\, (0.2 + j1))$$

$$= 0.2 + j1 + 0.19223 + j0.98115$$

$$= 0.39223 + j1.98115 = 2.020\angle 78.8^\circ$$

$$I_T = \frac{132.79\angle 0^\circ}{2.020\angle 78.8^\circ} = 65.738\angle -78.8^\circ\text{ A} //$$

ii) $n_{sync} = \frac{120 f_s}{P} = \frac{120\times 60}{4}$

$$= 1800\text{ rpm}$$

$$\omega_{sync} = 1800 \times (\tfrac{2\pi}{60}) = 60\pi$$

$$P_{in} = 3V\phi I_1 \cos\theta$$

$$= 3\times 132.79\times 65.738 \times \cos 78.8^\circ$$

$$= 5086.62\text{ W}$$

$$P_{SCL} = 3 I_1^2 R_1$$

$$= 3\times (65.738)^2 \times 0.2$$

$$= 2592.89\text{ W}$$

$$P_{AG} = P_{in} - P_{SCL}$$

$$= 5086.62 - 2592.89$$

$$= 2493.73\text{ W}$$

$$T_{start} = \frac{P_{AG}}{\omega_{sync}}$$

$$= \frac{2493.73}{60\pi}$$

$$= 13.23\text{ N-m} //$$

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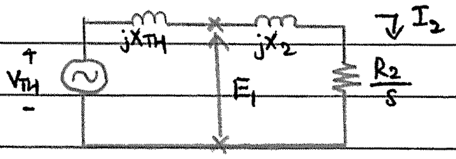
I am not sure about this part, but here are my steps

c) $V_L = 208\text{ V}$ $f = 50\text{ Hz}$ $P_{scL} = 0$ $R_1 = 0$ $P_{rot} = 0$

$P_{max\ RCL} = 10\, P_{fl\ RCL}$ $s_{fl} = 0.02$



$P_{in} = P_{AG}$

$P_{RCL} = s\,P_{AG}$

$$\frac{P_{max\ RCL}}{P_{fl\ RCL}} = 10 = \frac{s_{max}\,P_{AG}}{s_{fl}\,P_{AG}}$$

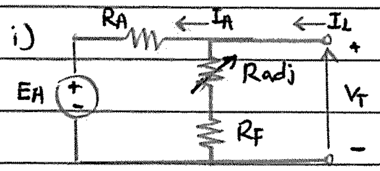
$s_{max} = 10 \times s_{fl}$

$= 10 \times 0.02$

$= 0.2$

4. a) $V_T = 200\text{ V}$ $R_A = 0.1\,\Omega$ $R_F = 250\,\Omega$ $P_{rot} = 220\text{ W}$ $I_L = 30\text{ A}$

i)



$P_{in} = V_T I_L = 200 \times 30 = 6000\text{ W}$

$I_F = \frac{V_T}{R_F} = \frac{200}{250} = 0.8\text{ A}$

$I_A = I_L - I_F = 30 - 0.8$

$= 29.2\text{ A}$

$V_T = E_A + R_A I_A$

$E_A = V_T - R_A I_A$

$= 200 - (0.1 \times 29.2)$

$= 197.08\text{ V}$

$P_{dev} = E_A I_A = 197.08 \times 29.2 = 5754.74\text{ W}$

ii) $P_{out} = P_{dev} - P_{rot} = 5754.74 - 220 = 5534.74\text{ W}$

$\eta = \frac{5534.74}{6000} \times 100\%$

$= 92.25\%$

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b) $R_A = 0.1\Omega$ $R_F = 400\Omega$ $V_T = 240V$ $n_m = 1000$ rpm $I_A = 50A$

$$I_F = \frac{V_T}{R_F} = \frac{240}{400} = 0.6\ A$$

$$E_A = V_T - I_A R_A$$

$$= 240 - (50 \times 0.1)$$

$$= 235V$$

E_A is fixed

In linear part of the magnetization curve

$$E_A \propto I_F\, n_m$$

$$\frac{\cancel{E_A}}{\cancel{E_A}} = \frac{I_{F\,old}}{I_{F\,new}} \frac{n_{old}}{n_{new}}$$

$$\frac{I_{F\,old}}{I_{F\,new}} = \frac{n_{new}}{n_{old}}$$

$$\frac{0.6}{I_{F\,new}} = \frac{1200}{1000}$$

$$I_{F\,new} = 0.5\ A$$

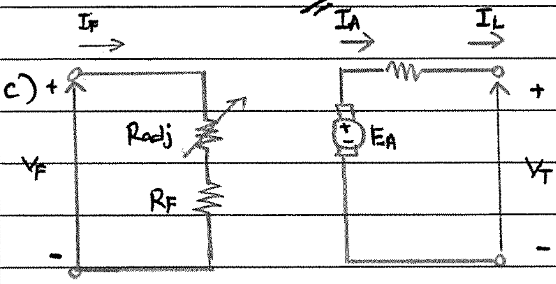
$$I_{F\,new} = \frac{V_T}{R_F + R_{ext}}$$

$$0.5 = \frac{240}{400 + R_{ext}}$$

$$R_{ext} = \frac{240}{0.5} - 400$$

$$= 80\Omega$$

c)



$R_A = 0.04\Omega$ Armature reaction can be neglected

$R_F = 45\Omega$

$V_F = 120V$

$P_{rot} = 1500W$

i) $V_{T(no\,load)} = E_A = 150V$

No armature reaction, flux is constant

$V_T = 140V$ $n = 1000$ rpm

$$I_F = \frac{V_F}{R_F} = \frac{120}{45} = \frac{8}{3}$$

$$E_A = V_T + I_A R_A$$

$$150 = 140 + I_A(0.04)$$

$$I_A = 250\ A = I_L$$

$$P_{out} = V_T I_L = 140 \times 250 = 35\ kW$$

$$P_{in} = E_A I_A + V_F I_F + P_{rot}$$

$$= 150(250) + 120\left(\tfrac{8}{3}\right) + 1500$$

$$= 39320\ W$$

$$\eta = \frac{35k}{39320} \times 100\% = 89.01\%$$

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ii) $n = 1200$ rpm $\quad I_A = 250$ A $\quad I_F = \frac{8}{3}$

Field current is fixed, flux is fixed

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0}$$

$$\frac{E_A}{150} = \frac{1200}{1800}$$

$$E_A = 180\text{ V}$$

$$P_{dev} = E_A I_A = 180 \times 250$$

$$= 45\text{ kW}$$