NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

EE3001 – ENGINEERING ELECTROMAGNETICS

April / May 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 8 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of physical constants and useful formulae is given in Appendix A, pages 6-8.
7. The Smith chart may be used in the solution of Question 4. Please put the Smith Chart inside the answer script and tie it with a thread.

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1. (a) Figure 1 (on page 2) shows a square of side length a centered at the origin in free space. The square carries a total charge Q which is uniformly distributed along its sides.

   (i) Find the line charge density ρ_l.

   (ii) Using Coulomb's law, determine the electric field intensity $\vec{E}$ along the z axis.

Note: $$\int \frac{1}{\left(x^2+u^2\right)^{3/2}}dx = \frac{x}{u^2\sqrt{x^2+u^2}}$$

(14 Marks)

Note: Question No. 1 continues on page 2.

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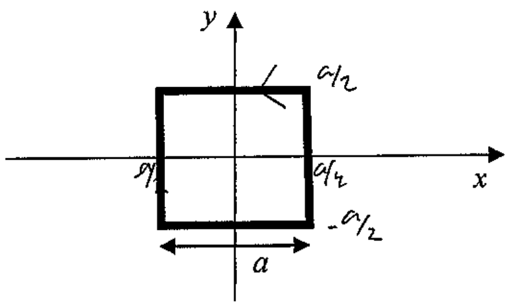

Figure 1

(b) Consider that a steady current I flows in the counter clockwise direction along the square sides in Figure 1.

(i) Determine the magnetic field intensity $\vec{H}$ along the z axis.

(ii) Comment on the applicability of Ampere's law for part (i).

(11 Marks)

2. (a) An circular loop of radius a is centered at the origin in the xy plane in free space. The loop is subjected to a magnetic flux density of the form

$$\vec{B} = \vec{a}_r + \cos(\omega t)\vec{a}_\phi + \sin(\omega t)\vec{a}_z \ \text{ T}$$

(i) Give the expression of magnetic flux Φ_m passing through the loop at time t.

(ii) What is the induced emf (electromotive force) in the loop?

(iii) If the loop contains a small series capacitor of capacitance C, find the induced current that will flow in the loop.

(10 Marks)

Note: Question No. 2 continues on page 3.

(b) A plane wave propagating in a lossy nonmagnetic medium has its magnetic field expression given by

$$\tilde{H} = \vec{a}_y\, 11.8e^{-0.435z}\cos(2\pi \times 10^6 t - 0.435z - 21.8^\circ)\ \text{A/m}$$

(i) Based on the expression, describe how you could tell whether the medium is a good conductor or not.

(ii) Given that $\varepsilon_r = 1$, calculate the wavenumber k_c, conductivity σ and intrinsic impedance η_c.

(iii) Derive the corresponding electric field expression.

(15 Marks)

3. (a) A uniform plane wave (UPW) in free space $(z \leq 0)$ is incident normally at the planar interface with a lossy medium having $\mu = \mu_o$, $\varepsilon = 2.25\varepsilon_o$ and $\sigma = 1.3$ S/m, occupying the region $z \geq 0$. The incident electric field intensity is given by

$$\tilde{E}_i(z,t) = \vec{a}_y 100\cos(\omega t - 4z + 1.28)\ \text{V/m}$$

Find the following and state any assumption(s) made:

(i) The angular frequency ω of the UPW.

(ii) The propagation constant γ and intrinsic wave impedance η_c of the UPW in the lossy medium.

(iii) The transmission coefficient τ at the planar interface.

(iv) The time-domain expression for the transmitted electric field, i.e. $\tilde{E}_t(z,t)$.

(15 Marks)

Note: Question No. 3 continues on page 4.

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(b) A 10 MHz uniform plane wave (UPW) travelling in a lossless and nonmagnetic dielectric medium ($z \le 0$) is given by

$$\vec{H}_i(x,z) = \vec{a}_y 0.05 e^{-j(0.15x+0.53z)} \ \text{A/m}$$

The UPW is obliquely incident on an air boundary at $z = 0$ occupying the region $z \ge 0$. Find the following and state any assumption(s) made:

(i) The vector wave number $\vec{k}_i$ and the phase velocity u_p of the incident UPW.

(ii) The relative permittivity of the dielectric medium, i.e. ε_r.

(iii) The percentage of average power reflected from the air boundary at $z = 0$.

(10 Marks)

4. (a) An unknown load Z_L connected to a $Z_o = 100\ \Omega$ lossless transmission line produces a standing wave ratio (SWR) of 3.2. Measurements on the transmission line show that successive voltage minima are 15 cm apart and the first voltage minimum occurs at 12.5 cm from the load.

Find the following:

(i) The unknown load impedance Z_L.

(ii) The reflection coefficient at the load end.

(iii) If Z_L is replaced by an impedance of $j130\ \Omega$, the impedance at the input end becomes $-j40\ \Omega$. Obtain a possible value for the length of the transmission line.

(15 Marks)

Note: Question No. 4 continues on page 5.

(b) A generator having an open-circuit voltage $V_g(t) = 125\cos(2\pi \times 10^8 t)$ V and an internal impedance $Z_g = 50\ \Omega$ is connected to a 80-cm long lossless transmission line having characteristic impedance $Z_o = 50\ \Omega$ and phase velocity $u_p = 2.5\times10^8$ m/s . The transmission line is terminated with a load impedance $Z_L = 63 - j32\ \Omega$..

Assume that the load is located at $z = 0$ and the generator is at $z = -\ell$, where ℓ is the length of the transmission line. Find the following and state any assumption(s) made:

(i) The input impedance $Z_{in}(z)$ at $z = -\ell$.

(ii) The time-domain expression for the input current at $z = -\ell$.

(10 Marks)

The Smith chart may be used in the solution of one or both parts of this question. Please put the Smith chart inside the answer script and tie it with a thread.

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Appendix A

Physical Constants

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m

∇ Operator

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{\partial V}{r\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial (rA_r)}{r\partial r} + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Electric and Magnetic Fields

$$\vec{E} = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{a}_R}{R^2} dv = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v \vec{R}}{R^3} dv$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho_v}{R} dv$$

$$\vec{H} = \frac{1}{4\pi}\int_C \frac{Id\vec{l} \times \vec{a}_R}{R^2} = \frac{1}{4\pi}\int_C \frac{Id\vec{l} \times \vec{R}}{R^3}$$

$$\oint_C \vec{H} \cdot d\vec{l} = I = \iint_S \vec{J} \cdot d\vec{s}$$

$$emf = \oint_C \tilde{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\iint_S \tilde{B} \cdot d\vec{s}$$

Maxwell's Equations

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}$$

$$\nabla \cdot \tilde{D} = \rho$$

$$\nabla \cdot \tilde{B} = 0$$

Complex Propagation Constant

$u_p = \frac{\omega}{\beta}$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu(\varepsilon - j\sigma/\omega)}$$

Complex Intrinsic Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

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Reflection and Transmission of Electromagnetic Wave

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \qquad \tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\Gamma_\perp = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)} \qquad \tau_\perp = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_i) + \eta_1\cos(\theta_t)}$$

$$\Gamma_\parallel = \frac{\eta_2\cos(\theta_t) - \eta_1\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)} \qquad \tau_\parallel = \frac{2\eta_2\cos(\theta_i)}{\eta_2\cos(\theta_t) + \eta_1\cos(\theta_i)}$$

Transmission Line

$$V(z) = V_o^+ \; e^{-j\beta z} + V_o^- \; e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\{V_o^+ \; e^{-j\beta z} - V_o^- \; e^{+j\beta z}\}$$

$$Z_{in}(z) = Z_o \frac{1 + \Gamma_L e^{+2\beta z}}{1 - \Gamma_L e^{+2\beta z}}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Gamma_L = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma(z) = \Gamma_L e^{+j2\beta z} \qquad -\ell \le z \le 0$$

$$Z_{in}(-\ell) = \frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} Z_o$$

END OF PAPER

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EE3001 PYP Solution

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1a. i) $\rho_\ell = \dfrac{Q}{4a}$

ii) $\vec{r} = (0, 0, z)$; $\vec{s} = \left(\frac{a}{2}, y, 0\right)$ for $-\frac{a}{2} \leq y \leq \frac{a}{2}$ [consider the right side]

$\vec{R} = -\frac{a}{2}\vec{a}_x - y\vec{a}_y + z\vec{a}_z$ for $-\frac{a}{2} \leq y \leq \frac{a}{2}$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon}\int_\ell \frac{\rho_\ell \vec{R}}{R^3}\, dl$$

$$= \frac{1}{4\pi\varepsilon}\cdot\frac{Q}{4a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{-\frac{a}{2}\vec{a}_x - y\vec{a}_y + z\vec{a}_z}{\left(\left(\frac{a}{2}\right)^2 + y^2 + z^2\right)^{3/2}}\, dy$$

Considering all sides, $\vec{a}_x$ and $\vec{a}_y$ components are eliminated due to symmetry.

$$\vec{E} = 4\cdot\frac{1}{4\pi\varepsilon}\cdot\frac{Q}{4a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{z\vec{a}_z}{\left(\left(\frac{a}{2}\right)^2 + y^2 + z^2\right)^{3/2}}\, dy$$

$$= \frac{Qz}{4\pi\varepsilon a}\left[\frac{y}{\left(z^2 + \frac{a^2}{4}\right)\sqrt{y^2 + z^2 + \frac{a^2}{4}}}\right]_{-a/2}^{a/2}$$

$$= \frac{Qz}{4\pi\varepsilon a}\cdot a\left(\frac{1}{\left(z^2 + \frac{a^2}{4}\right)\sqrt{z^2 + \frac{a^2}{2}}}\right)$$

$$= \frac{Qz}{\pi\varepsilon\left(4z^2 + a^2\right)\sqrt{z^2 + \frac{a^2}{2}}}$$

1b. i) Consider the right side of the loop,

$\vec{dl} = \vec{a}_y\, dy$

$\vec{R} = -\frac{a}{2}\vec{a}_x - y\vec{a}_y + z\vec{a}_z$ for $-\frac{a}{2} \leq y \leq \frac{a}{2}$

$\vec{dl} \times \vec{R} = \left(z\vec{a}_x + \frac{a}{2}\vec{a}_z\right) dy$

$$\vec{H}_1 = \frac{I}{4\pi}\int_c \frac{\vec{dl} \times \vec{R}}{R^3}$$

Considering all sides, $\vec{a}_x$ component is eliminated due to symmetry.

$$\vec{H} = 4 \times \frac{I}{4\pi}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{\frac{a}{2}\vec{a}_z}{\left(\left(\frac{a}{2}\right)^2 + y^2 + z^2\right)^{1/2}}\, dy$$

$$= \frac{I}{\pi}\cdot\frac{a}{2}\vec{a}_z\left[y\left(\left(\frac{a}{2}\right)^2 + y^2 + z^2\right)^{-3/2}\right]_{-a/2}^{a/2}$$

$$= \frac{Ia^2}{2\pi}\cdot\frac{1}{\left(z^2 + \frac{a^2}{2}\right)^{3/2}}\vec{a}_z$$

ii) Ampere's Law is not useful in this case since the line is finite and the current is not symmetrical on z-axis.

2a. i) Only consider $\vec{B}$ that is perpendicular to the plane.

$$\Phi_m = \vec{B} \cdot \vec{A}$$
$$= \sin(\omega t)\ (\pi a^2)$$
$$= \pi a^2 \sin(\omega t)$$

ii) $$emf = -\frac{d\Phi_m}{dt} = -\pi a^2 \omega \cos(\omega t)$$

iii) $$i = C\frac{dv}{dt}$$
$$= C\pi a^2 \omega^2 \sin(\omega t)$$

2b. i) $$\vec{H} = \vec{a}_y\, 11.8\, e^{-0.435z} \cos(2\pi \times 10^6 t - 0.435z - 21.8^\circ)$$

$\omega = 2\pi \times 10^6$ $\quad \beta = 0.435$ $\quad \vec{a}_H = +\vec{a}_y$

$\alpha = 0.435$ $\quad \varphi = -21.8^\circ$ $\quad \vec{a}_k = +\vec{a}_z$

Since $\alpha = \beta = 0.435$, the medium is a good conductor.

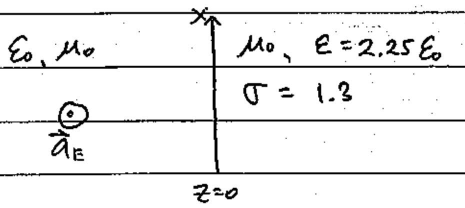
ii) $$\sigma = \frac{\alpha^2}{\pi f \mu_0} = \frac{0.435^2}{\pi(10^6)(4\pi \times 10^{-7})} = 0.048$$

$$\eta_c = e^{j\frac{\pi}{4}}\sqrt{\frac{\omega\mu_0}{\sigma}} = 12.825 \angle 45^\circ$$

$$\gamma = \alpha + j\beta = jk_c$$
$$k_c = \beta - j\alpha = 0.435 - 0.435j = 0.615 \angle -45^\circ$$

iii) $$\vec{a}_E = \vec{a}_H \times \vec{a}_k = \vec{a}_x$$
$$\vec{E} = \eta_c \vec{H} \times \vec{a}_k = \vec{a}_x\, 151.335\, e^{-0.435z} \cos(2\pi \times 10^6 t - 0.435z + 23.2^\circ)$$

3a.



$$\vec{E}_i(z,t) = \vec{a}_y\, 100\cos(\omega t - 4z + 1.28)\ \text{V/m}$$

i) $$k_1 = 4 = \omega\sqrt{\mu_0 \varepsilon_0}$$
$$\omega = \frac{k}{\sqrt{\mu_0\varepsilon_0}} = \frac{4}{\sqrt{(4\pi \times 10^{-7})\left(\frac{10^{-9}}{36\pi}\right)}} = 1.2 \times 10^9$$

ii) $$k_2 = \omega\sqrt{\mu_0 \varepsilon_r \varepsilon_0\left(1 - j\frac{\sigma}{\omega\varepsilon_r\varepsilon_0}\right)}$$
$$= 1.2 \times 10^9 \sqrt{4\pi \times 10^{-7} \times 2.25 \times \frac{10^{-9}}{36\pi}\left(1 - j\frac{1.3}{1.2 \times 10^9 \times 2.25 \times \frac{10^{-9}}{36\pi}}\right)}$$
$$= 31.60 - 31.02j$$

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$\gamma_2 = jk_2 = 31.02 + 31.60j$

$\eta_2 = \eta_c = \sqrt{\dfrac{\mu_0}{\varepsilon_c}} = 24.30 + 23.86j = 34.06 \angle 44.5^\circ$

$\varepsilon_c = \varepsilon - \dfrac{j\sigma}{\omega} = 2.25\,\varepsilon_0 - \dfrac{j(1.3)}{1.2\times10^9}$

iii) $\tau = \dfrac{2\eta_2}{\eta_1+\eta_2} = \dfrac{2\,(34.06\angle 44.5^\circ)}{120\pi + 34.06\angle 44.5^\circ} = 0.17 \angle 41.07^\circ$

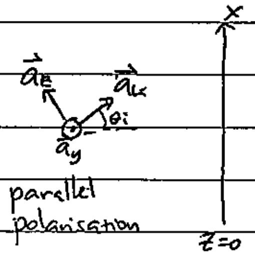
iv) $E_{0t} = \tau E_{0i} = 16.94 \angle 41.07^\circ = 16.94 \angle 0.72\ \text{rad}$

$\alpha_2 = 31.02, \quad \beta_2 = 31.60$

$\vec{E}_t(z,t) = \vec{a}_y\, 16.94\, e^{-31.02z} \cos\left(1.2\times10^9 t - 31.6z + 2\right)$

b. $f = 10\ \text{MHz}$

$\vec{H}_i(x,z) = \vec{a}_y\, 0.05\, e^{-j(0.15x + 0.53z)}$



$\theta_i = \tan^{-1}\left(\dfrac{k_x}{k_z}\right) = 0.28 = 15.8^\circ$

$\theta_t = \sin^{-1}\left(\sin\theta_i \sqrt{\dfrac{\mu_0\varepsilon_r\varepsilon_0}{\mu_0\varepsilon_0}}\right)$

$\theta_t = 45.64^\circ$

i) $\vec{k}_i = 0.15\,\vec{a}_x + 0.53\,\vec{a}_z$

$|k_i| = \sqrt{(0.15)^2 + (0.53)^2} = 0.55$

$u_p = \dfrac{\omega}{|k_i|} = \dfrac{2\pi f}{|k_i|} = 1.14\times10^8\ \text{m/s}$

ii) $k = \omega\sqrt{\mu_0\varepsilon_r\varepsilon_0}$

$\varepsilon_r = \left(\dfrac{k}{\omega}\right)^2 \dfrac{1}{\mu_0\varepsilon_0} = \left(\dfrac{0.55}{2\pi\times10\times10^6}\right)^2 \dfrac{1}{4\pi\times10^{-7}\times\frac{1}{36\pi}\times10^{-9}} = 6.896$

iii) $\eta_1 = \sqrt{\dfrac{\mu_0}{\varepsilon_r\varepsilon_0}} = \dfrac{120\pi}{\sqrt{6.896}} = 143.56 \qquad \eta_2 = 120\pi$

$\Gamma_{\parallel} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \dfrac{120\pi\cos 45.64^\circ - 143.56\cos 15.8^\circ}{120\pi\cos 45.64^\circ + 143.56\cos 15.8^\circ} = 0.31$

$\dfrac{P_r^{\parallel}}{P_i^{\parallel}} = |\Gamma_{\parallel}|^2 = 9.75\%$

a. i) $SWR = 3.2 = \dfrac{|V|_{max}}{|V|_{min}} = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|}$ | $\lambda/2 = 15\ \text{cm} \rightarrow \lambda = 30\ \text{cm}$

$|\Gamma_L| = \dfrac{2.2}{4.2} = 0.524$

$\beta = \dfrac{2\pi}{\lambda} = 20.94$

$\theta_0 = \pi - 2\beta z_{min} = \pi - 2(20.94)(-0.125) = 8.38\ \text{rad} = 2.09\ \text{rad}$

$\Gamma_L = |\Gamma_L| \angle\theta_0 = 0.524 \angle 2.09$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow Z_L = \frac{Z_0(1+\Gamma_L)}{(1-\Gamma_L)} = \frac{100(1+0.524\angle 2.09)}{1-0.524\angle 2.09}$$

$$Z_L = 64.84 \angle 0.90 \ \Omega$$

ii) $\Gamma_L = 0.524 \angle 2.09$ (found in (i))

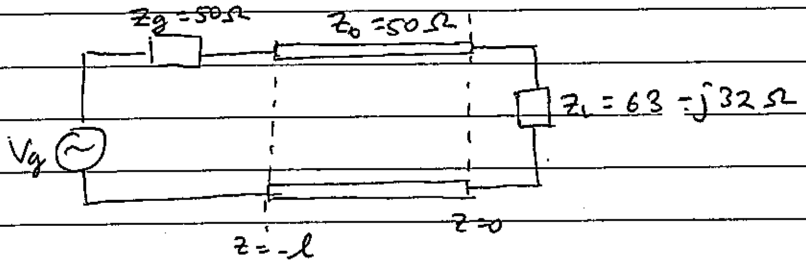
iii) $Z_L = j130\Omega \rightarrow Z_{in} = -j40\Omega$

$$Z_{in}(-l) = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0 \rightarrow \tan\beta l = \frac{Z_0(Z_L - Z_{in})}{j(Z_{in}Z_L - Z_0^2)}$$

$$\tan\beta l = \frac{100(j130 + j40)}{j((-j40)(j130) - 100^2)} = -3.54$$

$$l = \frac{1}{\beta}\tan^{-1}(-3.54) = -6.19 \text{ cm}$$

4b.



$V_g = 125\cos(2\pi \times 10^8 t)$

$l = 80$ cm

$u_p = 2.5 \times 10^8$

i) $$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 10^8}{2.5 \times 10^8} = 2.51$$

$$Z_{in}(-l) = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} Z_0 = \frac{63 - j32 + j50(\tan(2.51 \times 0.8))}{50 + j(63 - j32)\tan(2.51 \times 0.8)} \cdot 50$$

$$Z_{in}(-l) = -42.90 + j17.49 = 46.33 \angle 2.75$$

ii) $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{63 - j32 - 50}{63 - j32 + 50} = 0.29 \angle -0.91$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma_L e^{-j\beta z}\right]$$

$$I(-l) = \frac{125}{50}\left[e^{-j(2.51)(-0.8)} - (0.29\angle -0.91) e^{-j(2.51)(-0.8)}\right]$$

$$= 2.13 \angle 2.28$$

$$I(z = -l) = 2.13\cos(2\pi \times 10^8 t + 2.28) = 2.13\cos(2\pi \times 10^8 t + 130.63^\circ)$$