Lecture 13: Transformers

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Analyse the performance of three-phase transformer banks and three-phase transformers using the one phase star equivalent circuit or one phase Y-Y equivalent circuit.
- Calculate the power rating and voltage rating of three-phase transformer banks.
- Apply the principles and concepts learnt to solve three-phase transformer bank problems and three-phase transformer problems.

Analysis of Three-Phase Transformers

❖ Both three-phase transformer banks and three-phase transformers can be analysed by considering **one phase star equivalent circuit** of the transformer.

❖ **Transformer Banks:**

- Consider the three-phase transformer bank as shown in Fig. 61 . The rating is known for single-phase transformer, e.g., $S_{1\text{-}ph}$, E_1 / E_2.

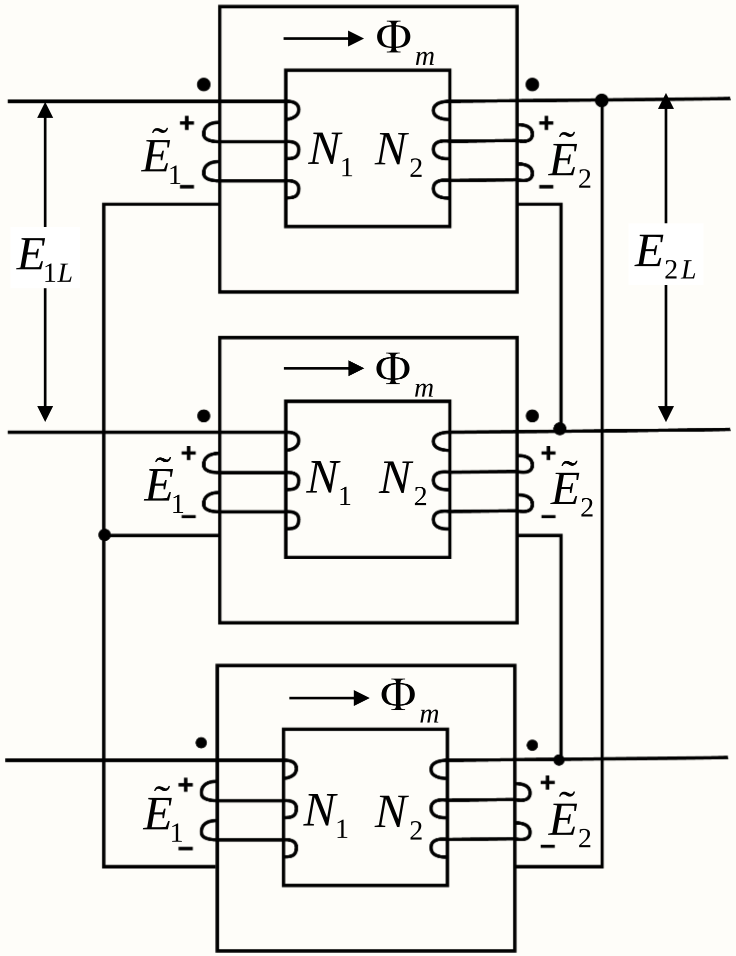

Fig. 61. Three-phase transformer bank.

Analysis of Three-Phase Transformers

- **To obtain the rating of the three-phase transformer bank**:
  - The power rating of the three-phase bank will be three times larger:

    $$S_{3-ph\ bank} = 3 \times S_{1-ph}$$

  - The line voltage will be the same if connected in delta, but larger by a factor of $\sqrt{3}$ if connected in star. For this example:

    $E_{1L} = \sqrt{3} \times E_1$, and $E_{2L} = E_2$

- **To form the one phase star equivalent circuit**:

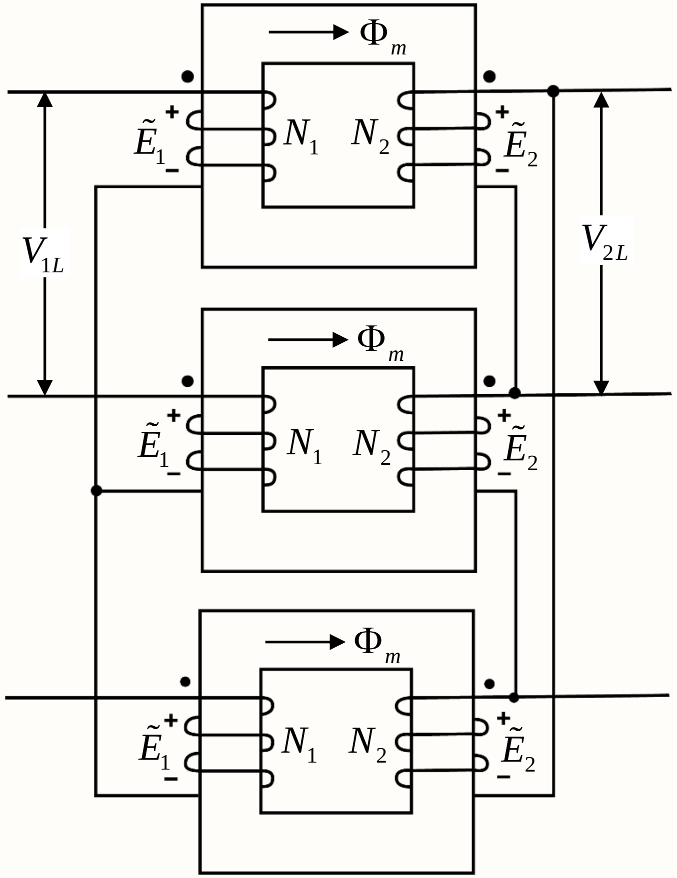

Fig. 62. Three-phase transformer bank.

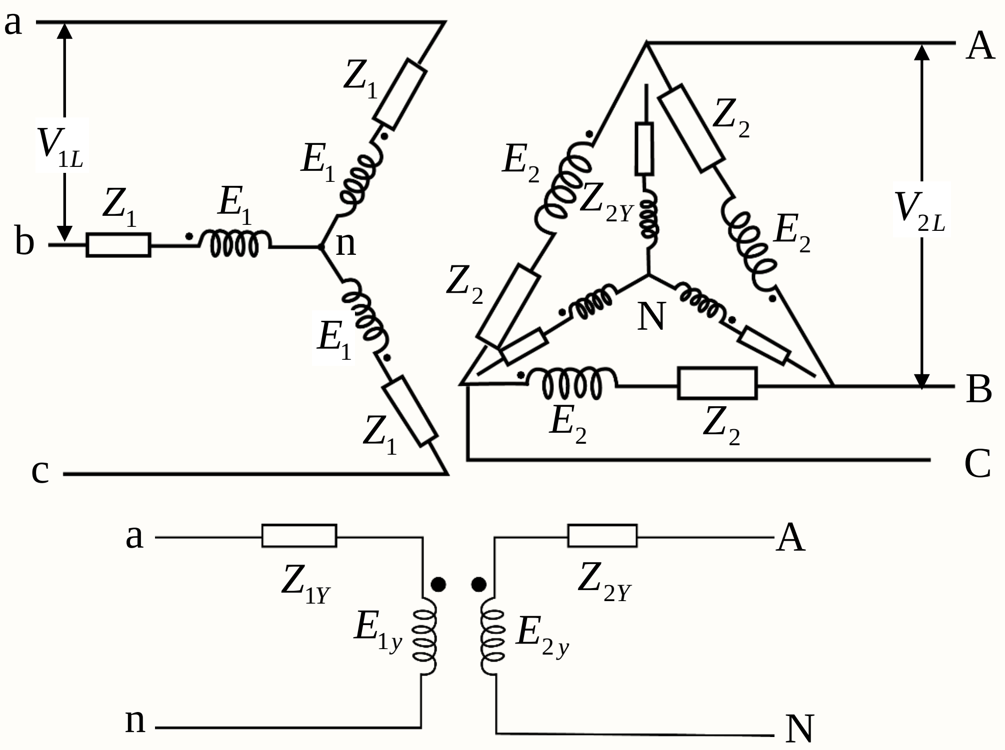

Fig. 63. One phase star equivalent circuit or one phase Y-Y equivalent circuit.

Analysis of Three-Phase Transformers

- **To form the one phase star equivalent circuit**:
  - The star equivalent phase voltages are obtained from the line voltages:

$$V_{1Y} = V_{an} = \frac{V_{1L}}{\sqrt{3}}, \quad V_{2Y} = V_{AN} = \frac{V_{2L}}{\sqrt{3}}$$

    Turns ratio $a = V_{1Y}/V_{2Y}$ will depend on whether the windings are connected in star or in delta.

  - The impedance per phase remains the same if it was star connected, but should be reduced by a factor of three if it is delta connected $(Z_Y = Z_\Delta/3)$.

$Z_{1Y} = Z_1$, and $Z_{2Y} = \dfrac{Z_2}{3}$

Example 8

Three identical single-phase transformers, each single-phase transformer rated at 12 kVA, 120/240 V, 60 Hz, are connected to form a three-phase step-up Y/Δ transformer bank. The parameters of the transformer are

$$R_1 = 0.0395\,\Omega,\ X_1 = 0.0615\,\Omega,\ R_2 = 0.1335\,\Omega,\ X_2 = 0.201\,\Omega.$$

a) What are the ratings of the three-phase transformer bank?

b) Find the input current and the input voltage of the transformer bank when it delivers the rated load at rated voltage and 0.8 pf lagging. Ignore no load current.

(Solutions →)

Example 8 – Solutions

a) Power rating of the bank: $S_3 = 3 \times 12 \text{ kVA} = 36 \text{ kVA}$

Line voltage on the primary $= \sqrt{3} \times 120 = 207.85 \text{ V}$ (Y connected)

Line voltage on the secondary $= 240 \text{ V}$ (Δ connected)

Ratings of the three-phase transformer bank: **36 kVA, 207.85/240 V, 60 Hz**

b) Consider a one phase star equivalent circuit:

Primary star equivalent phase voltage $= \dfrac{207.85}{\sqrt{3}} = 120 \text{ V} = V_{an} = V_{1Y}$

Secondary star equivalent phase voltage $= \dfrac{240}{\sqrt{3}} = 138.56 \text{ V} = V_{AN} = V_{2Y}$

Turns ratio $a = \dfrac{120}{138.56} = 0.866$

Example 8 – Solutions

Primary star phase impedance:

$$Z_{1Y} = 0.0395 + j0.0615\ \Omega$$

Secondary star phase impedance:

$$Z_{2Y} = \frac{0.1335 + j0.201}{3} = 0.0445 + j0.067\ \Omega$$

Referring to the secondary side:

$$Z_{e2} = R_{e2} + jX_{e2} = Z_{2Y} + \frac{Z_{1Y}}{a^2} = 0.0972 + j0.1590\ \Omega$$

Let $V_2 = 138.56\angle 0^\circ\ \text{V}$ **(rated voltage as reference)**

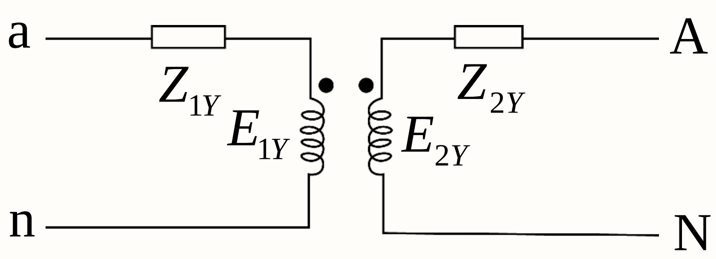

Fig. 64. One phase Y-Y equivalent circuit.

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Example 8 – Solutions

$$\frac{\text{Rated load}}{\text{phase}} = \frac{36}{3} = 12\ \text{kVA}$$

$$\text{pf angle} = 36.87^\circ\ (\mathbf{cos^{-1} 0.8})$$

$$\therefore S_2 = 12000\angle 36.87^\circ\ \text{VA}$$

Rated load current

$$I_2 = \frac{S_2{}^*}{V_2{}^*} = \frac{12000\angle -36.87^\circ}{138.56\angle 0^\circ} = 86.61\angle -36.87^\circ\ \text{A}$$

$$V_1' = V_2 + I_2(R_{e2} + jX_{e2})$$

$$= 138.56\angle 0^\circ + 86.61\angle -36.87^\circ \times (0.0972 + j0.1590) = 153.67\angle 2.23^\circ\ \text{V}$$

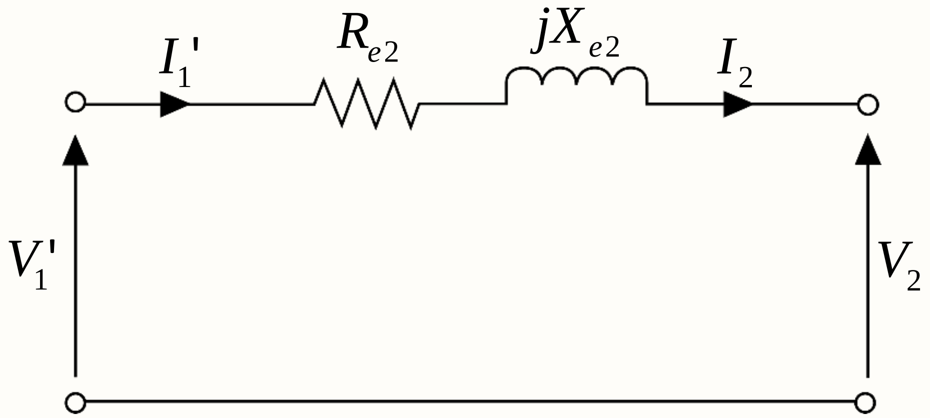

Fig. 65. Approximate equivalent circuit referred to the secondary side.

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Example 8 – Solutions

Input star phase current $= I_1 = \dfrac{I_1'}{a} = \dfrac{I_2}{a} = \dfrac{86.61}{0.866} = 100 \text{ A}$ (**line current**)

Input star phase voltage $= V_1 = a \times V_1' = 0.866 \times 153.67 = 133.1 \text{ V}$

Input line voltage $= \sqrt{3} V_1 = \sqrt{3} \times 133.1 = 230.5 \text{ V}$

Three-Phase Transformers

❖ The ratings of the three-phase transformer are given in terms of total three-phase power and line voltages. To find the equivalent circuit for one phase star equivalent of a three-phase transformer:

- The power rating of each phase will be only one third of the three-phase rating:

$$S_{1-ph} = \frac{S_{3-ph}}{3}$$

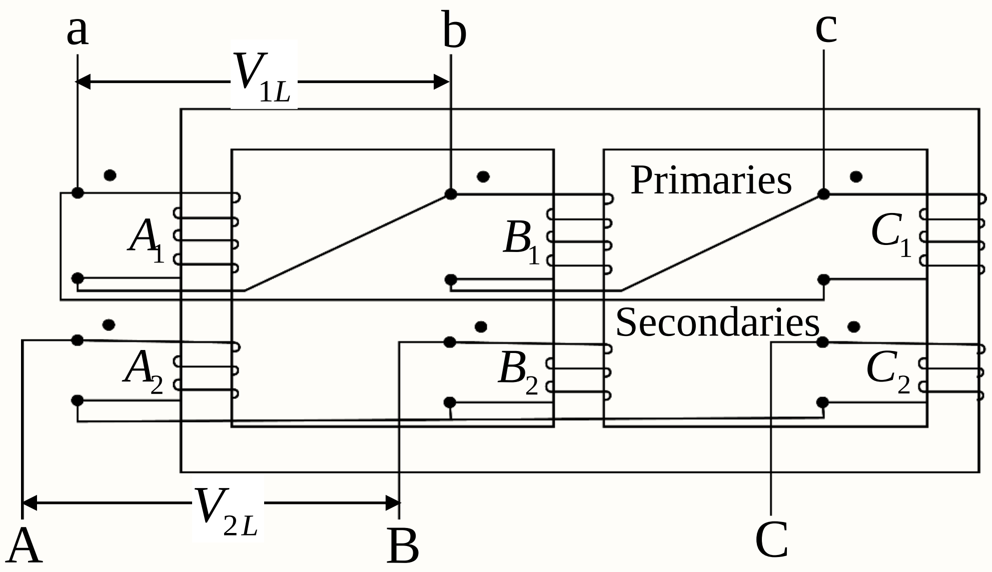

Fig. 66. Three-phase transformer.

Three-Phase Transformers

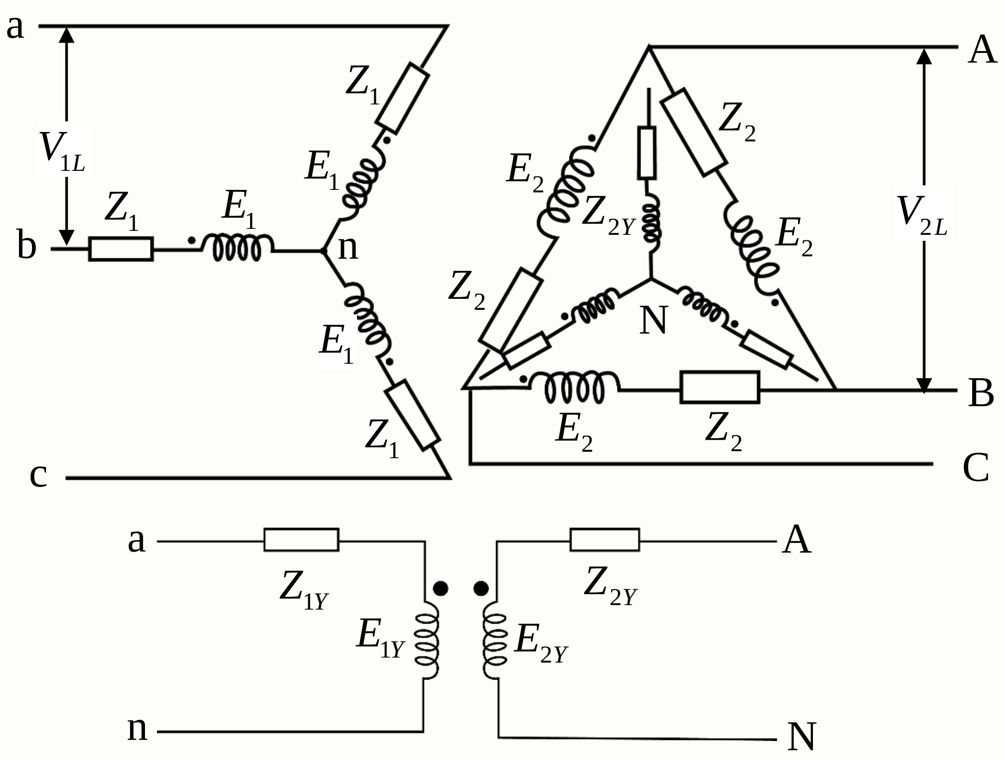

Fig. 67. One phase star equivalent circuit or one phase Y-Y equivalent circuit.

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Three-Phase Transformers

- The star equivalent phase voltage will be smaller than the respective line voltage by a factor of ($1/\sqrt{3}$). This will be the voltage rating of the actual star phase voltage if the winding was connected in star. It will be the voltage rating of the star equivalent phase voltage if the winding was connected in delta.

$$V_{1Y} = V_{an} = \frac{V_{1L}}{\sqrt{3}}$$

$$V_{2Y} = V_{AN} = \frac{V_{2L}}{\sqrt{3}}$$

Turns ratio $a = \dfrac{V_{1Y}}{V_{2Y}} = \dfrac{V_{1L}/\sqrt{3}}{V_{2L}/\sqrt{3}} = \dfrac{V_{1L}}{V_{2L}}$

which does not depend on whether the windings are connected in star or in delta.

Three-Phase Transformers

- The impedance per phase remains the same if the winding is star connected, but it should be reduced by a factor of three if it is delta connected.

$$Z_Y = \frac{Z_\Delta}{3}$$

$$Z_{1Y} = \frac{Z_1}{3}, \text{ and } Z_{2Y} = Z_2$$

Example 9

The resistance and the leakage reactance of a three-phase, 300-kVA, 2400/208-V, Δ/Y, 50-Hz transformer are:

Primary winding	$R_1 = 0.4\ \Omega$	$X_1 = 1.6\ \Omega$
Secondary winding	$R_2 = 0.001\ \Omega$	$X_2 = 0.004\ \Omega$

Determine the input line voltage when the transformer delivers 300 kVA at 0.9 pf lagging with rated voltage across the load. Ignore no load current.

(Solutions →)

Example 9 – Solutions

Primary is Δ connected: $Z_{1\Delta} = (0.4 + j1.6)\ \Omega$

Secondary is Y connected: $Z_{2Y} = (0.001 + j0.004)\ \Omega$

For one phase Y-Y equivalent circuit:

$$Z_{1Y} = \frac{0.4 + j1.6}{3}\ \Omega$$

$$Z_{2Y} = (0.001 + j0.004)\ \Omega$$

$$V_{1Y} = V_{an} = \frac{2400}{\sqrt{3}}\ \text{V}$$

$$V_{2Y} = V_{AN} = \frac{208}{\sqrt{3}}\ \text{V}$$

Turns ratio $a = \dfrac{V_{1Y}}{V_{2Y}} = 11.5385$

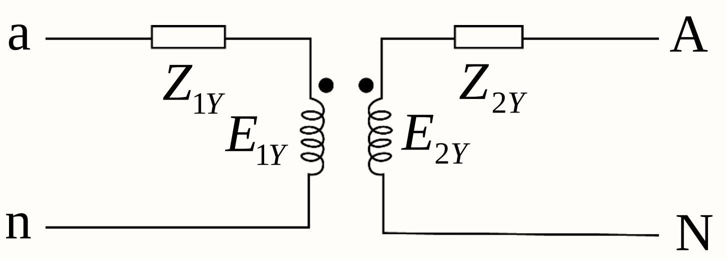

Fig. 68. One phase Y-Y equivalent circuit.

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Example 9 – Solutions

Therefore, for the equivalent circuit, referred to the secondary side:

$$Z_{e2} = R_{e2} + jX_{e2} = Z_{2Y} + \left(\frac{1}{a^2}\right) Z_{1Y}$$

$$= 0.001 + j0.004 + \left(\frac{1}{a^2}\right)\left(\frac{0.4 + j1.6}{3}\right)$$

$$= 0.002 + j0.008 = 0.0082462\angle 75.96° \ \Omega$$

Rated load per phase:

$$S_2 = \frac{300}{3} = 100 \text{ kVA at } 0.9 \text{ pf lagging}$$

$$= 100 \times 1000 \angle 25.84° \text{ VA } (\mathbf{\cos^{-1} 0.9 = 25.84°})$$

Example 9 – Solutions

Let $V_2 = \dfrac{208}{\sqrt{3}} \angle 0° \text{ V}$ **(Rated voltage as reference)**

$$I_2 = \left(\frac{S_2}{V_2} \right)^*$$

$$= \frac{100 \times 1000 \angle -25.84°}{208 / \sqrt{3} \angle 0°}$$

$$= 832.7 \angle -25.84° \text{ A}$$

Example 9 – Solutions

Therefore,

$$V_1' = V_2 + I_2(R_{e2} + jX_{e2})$$

$$= \frac{208}{\sqrt{3}}\angle 0° + 832.7\angle -25.84° \times 0.0082462\angle 75.96°$$

$$= 124.52\angle 2.43°\ \text{V}$$

Therefore, $V_1 = a \times V_1' = 1436.77\ \text{V}$

Input line voltage $= \sqrt{3}V_1 = \sqrt{3} \times 1436.77 = 2488.6\ \text{V}$

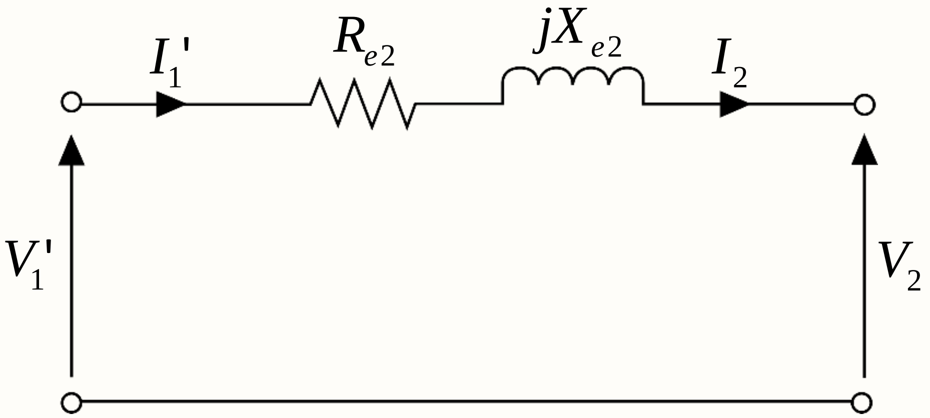

Fig. 69. Approximate equivalent circuit referred to the secondary side.

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Summary

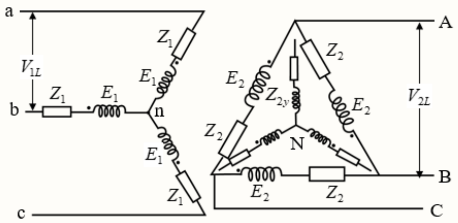
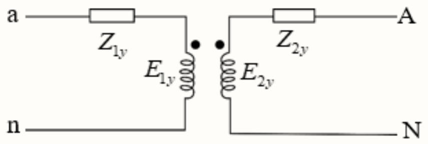
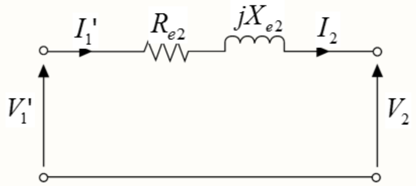
In this lecture, you have learnt:

- Analysis of the performance of three-phase transformer banks and three-phase transformers using the one phase star equivalent circuit or one phase Y-Y equivalent circuit.

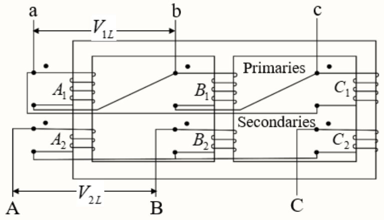
- The calculation of the power rating and voltage rating of three-phase transformer banks.

- The principles and concepts to solve three-phase transformer bank problems and three-phase transformer problems.

References

No.	Slide No.	Image	Reference
3	6 and 14		Reprinted from *AC Circuits and Machines*, (p. 182), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
4	6, 10, 14, and 18		Reprinted from *AC Circuits and Machines*, (p. 182), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.
5	11 and 21		Reprinted from *AC Circuits and Machines*, (p. 185), by G. B. Shrestha, & M. H. Haque, 2006, Singapore: Pearson Education South Asia Pte Ltd. Copyright 2006 by Pearson Education South Asia Pte Ltd. Reprinted with permission.

References

No.	Slide No.	Image	Reference
6	13		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 254), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.