Lecture 6

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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NANYANG TECHNOLOGICAL UNIVERSITY

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the basic structure of a DC machine.
- Explain how voltage is induced in a rotating loop of wire and how the equation for induced voltage is derived in a DC machine.
- Explain the purpose of commutation.
- Explain how torque is produced when current is supplied to the loop, and how the equation for the induced torque is derived in a machine.

Introduction to DC Machinery Fundamentals

- DC machines are generators that convert mechanical energy to DC electric energy and motors that convert DC electric energy to mechanical energy.

- DC motors are used in a wide variety of industrial drives such as robots, machine tools, paper and steel mills.

- DC generators are quite rare in modern power systems. They are being replaced by solid-state devices that convert available AC power to DC power for DC drive systems and other DC applications.

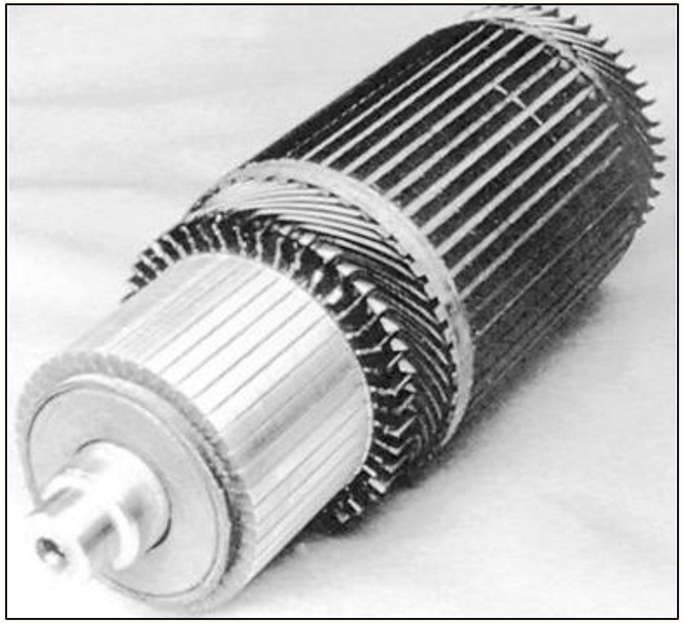
Introduction to DC Machinery Fundamentals

- However, DC generators have had a limited renaissance in last few years, as power sources for stand-alone cellular telephone towers.

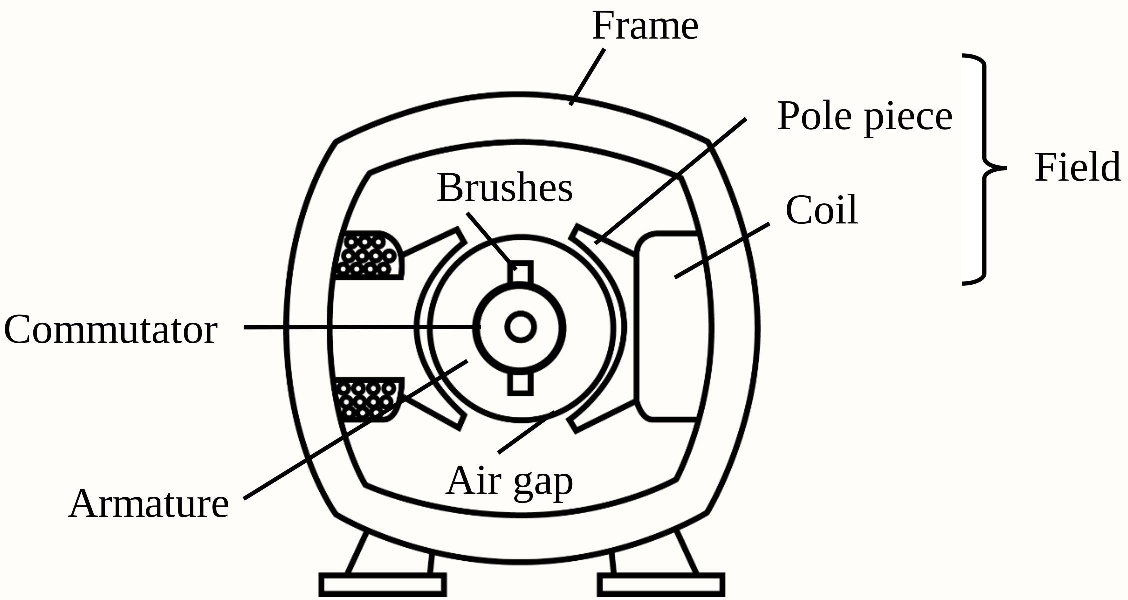
- Most DC machines are like AC machines in that they have AC voltages and currents within them. A mechanism (commutators) exists, that converts the internal AC voltages to DC voltages at the terminals.

- Main parts of a DC machine are:
  - Stator – stationary part, consisting of the frame and pole pieces
  - Rotor – rotating part

Introduction to DC Machinery Fundamentals

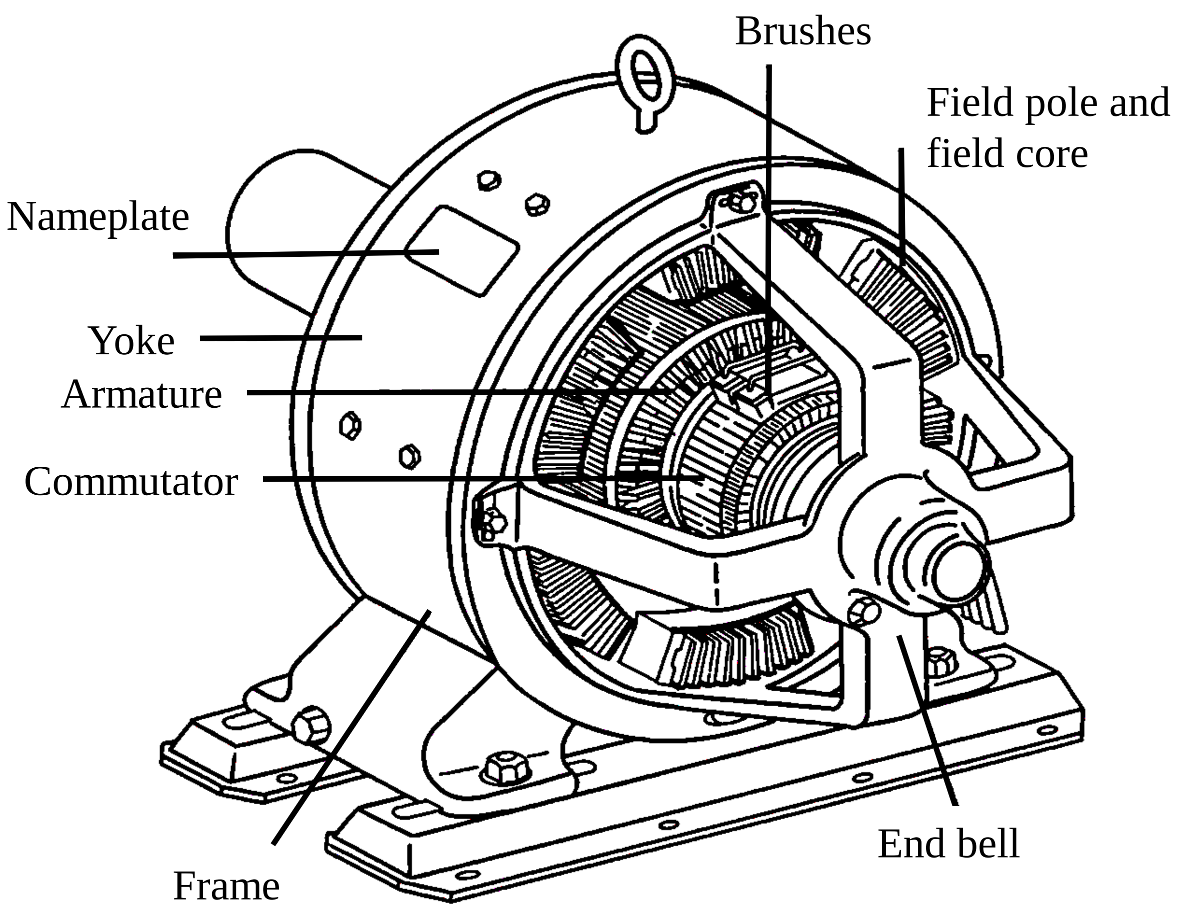


Armature of a DC generator. (*Used with courtesy of General Electric Company*) *WILDI, THEODORE, ELECTRICAL MACHINES, DRIVES AND POWER SYSTEMS, 6th, 2006*. Reprinted by permission of Pearson Education, Inc., New York, New York.



Cross section of a two-pole generator.
WILDI, THEODORE, ELECTRICAL MACHINES, DRIVES AND POWER SYSTEMS, 6th, 2006. Reprinted by permission of Pearson Education, Inc., New York, New York.

Introduction to DC Machinery Fundamentals



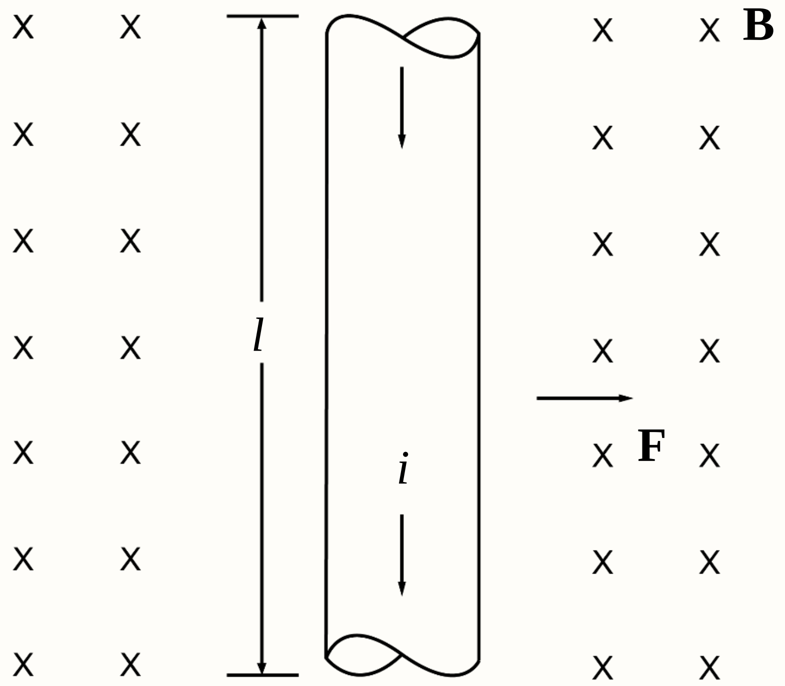
A simplified diagram of a DC machine.

Introduction to DC Machinery Fundamentals

- Two principal windings on a DC machine:
  - Armature windings - located on the rotor, where voltages are induced. Hence, a DC machine's rotor itself is sometimes called an armature.
  - Field windings - located on the stator, excited by DC currents to produce main magnetic flux in the machine.
- The fundamental principles involved in the operation of DC machines are very simple. Unfortunately, they are usually somewhat obscured by the complicated construction of real machines.

Fleming's Left Hand Rule (Motor)

❖ Whenever a current carrying conductor comes under a magnetic field, there is a force (Lorentz force) acting on the conductor. The force induced on the conductor is:



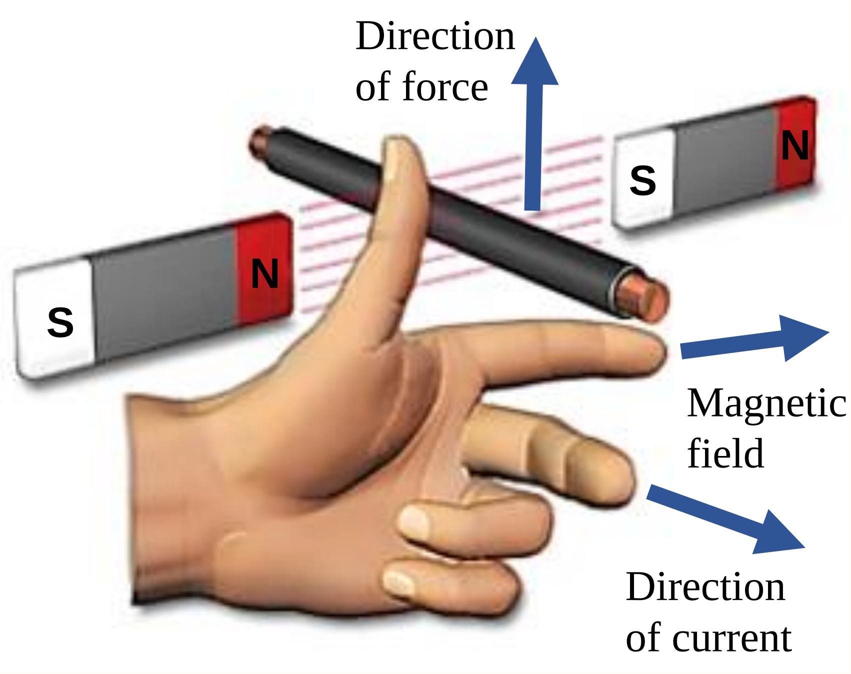
Force on a current-carrying wire in a magnetic field.

$$\boxed{F = Bli}$$

where

i = magnitude of the current
l = length of wire
B = magnetic flux density

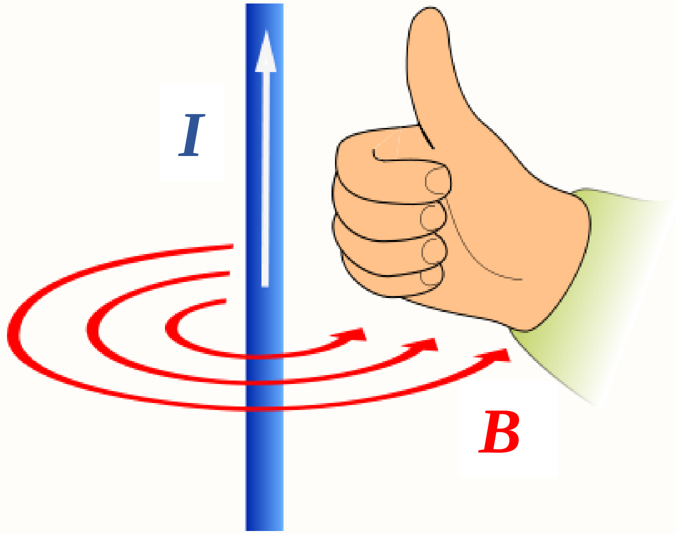
Fleming's Left Hand Rule (Motor)



Fleming's left hand rule.

- Hold out your left hand with forefinger, second finger and thumb at right angle to one another.
- If the forefinger represents the direction of the field and the second finger that of the current, then the thumb gives the direction of the force.

Maxwell’s Right Hand Grip Rule

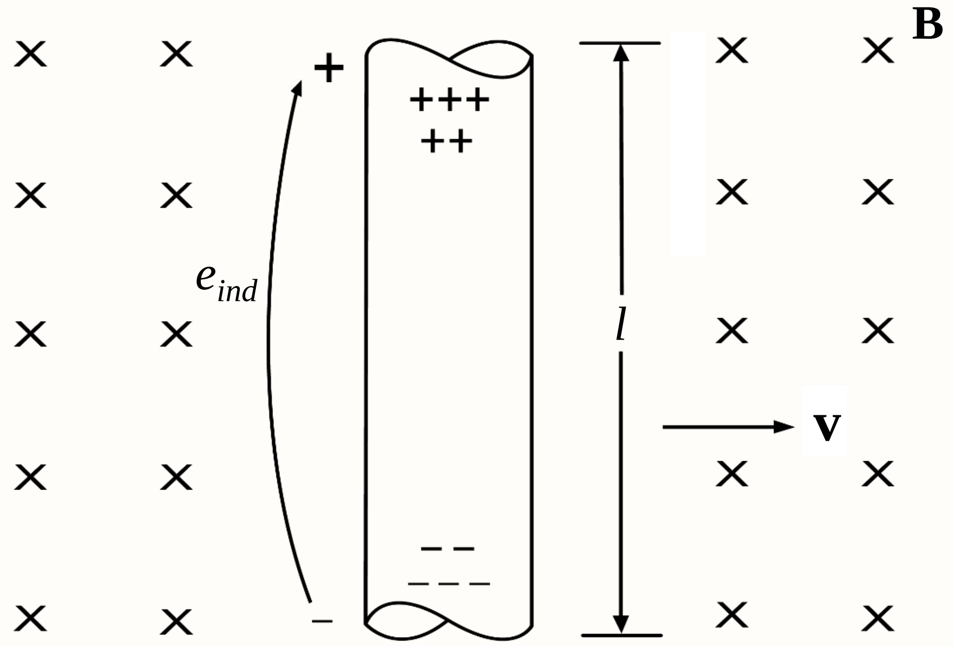


Maxwell’s right hand grip rule.

- When electric current flows through a conductor, a magnetic field is induced around it.
- The direction of magnetic lines of force can be determined by Maxwell's right-hand grip rule.

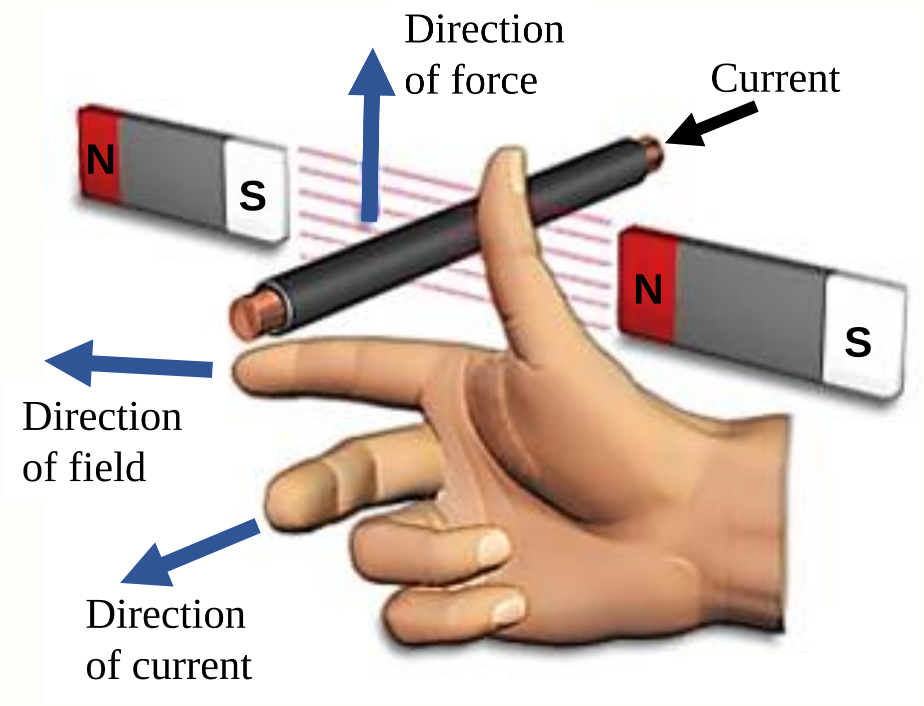
Fleming's Right Hand Rule (Generator)

- When a conductor of length l is moving at a velocity v to the right in a magnetic field of density B, according to Faraday's Law of Electromagnetic Induction, an emf e_{ind} will be induced.



$$\boxed{e_{ind} = Blv}$$

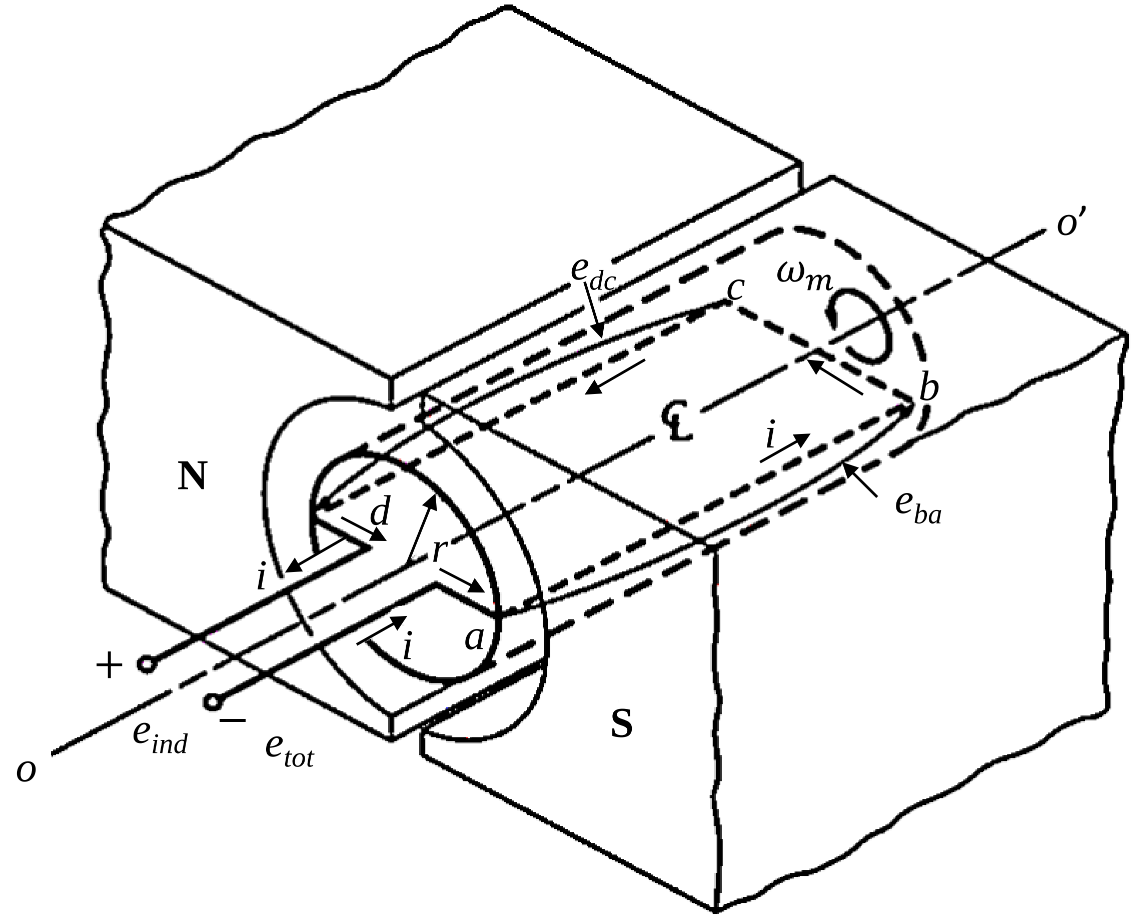
Fleming's Right Hand Rule (Generator)



Fleming's right hand rule.

- Hold out the right hand with the forefinger, second finger and thumb at right angle to each other.
- If forefinger represents the direction of the line of field, the thumb points in the direction of motion or applied force, then the second finger points in the direction of the induced current.

Principles of DC Machine Operations



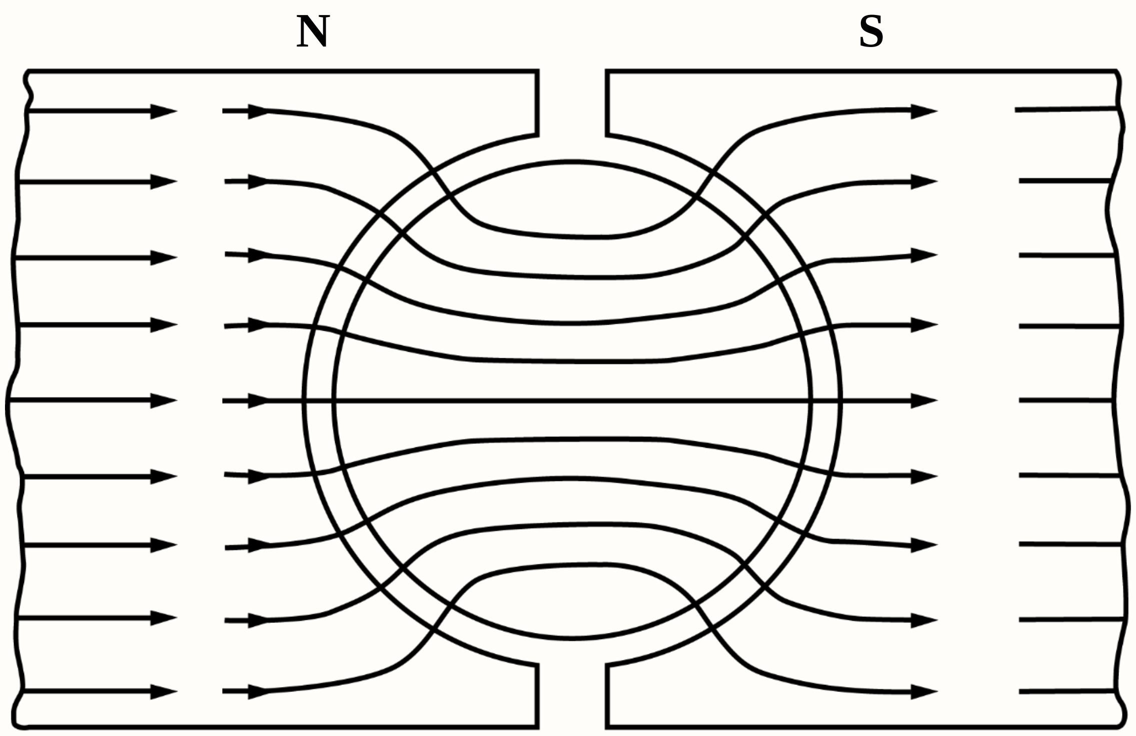
Simplest possible rotating DC machine: a simple rotating loop between curved pole faces.

Principles of DC Machine Operations

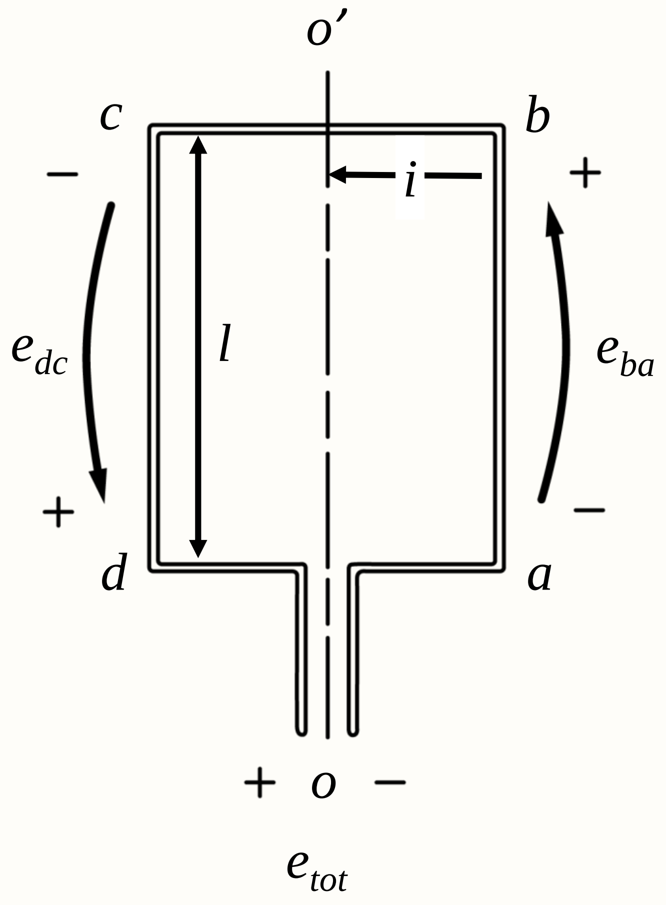
- The simplest possible rotating DC machine consists of a single loop of wire which lies in a slot carved in a ferromagnetic core and rotating about a fixed axis. The magnetic field is supplied by the magnetic north and south poles as shown.

- Assumptions:
  - Constant-width air gap between rotor and stator.
  - Air gap flux density is radial.
  - The flux density is constant everywhere under the poles faces and rapidly falls to zero beyond the edges of the poles.

The Voltage Induced in a Rotating Loop



View of field lines.



Top view.

The Voltage Induced in a Rotating Loop

- As the rotor is rotated, a voltage will be induced in the wire loop.
- To determine the total voltage induced on the loop, examine each segment of the loop separately and sum all the resulting voltages. The velocity of the wire is tangential to the path of rotation.

a) Segment *ab.*

$$e_{ba} = \begin{cases} Blv & \text{positive into page, under pole face} \\ 0, & \text{beyond the pole edges} \end{cases}$$

The Voltage Induced in a Rotating Loop

b) Segments *bc* and *da*.

They are the end connections used to connect the conductor segments *ab* and *cd* in series. The voltages induced are zero.

$$\underline{e_{cb} = 0}, \ \underline{e_{ad} = 0}$$

c) Segment *cd.*

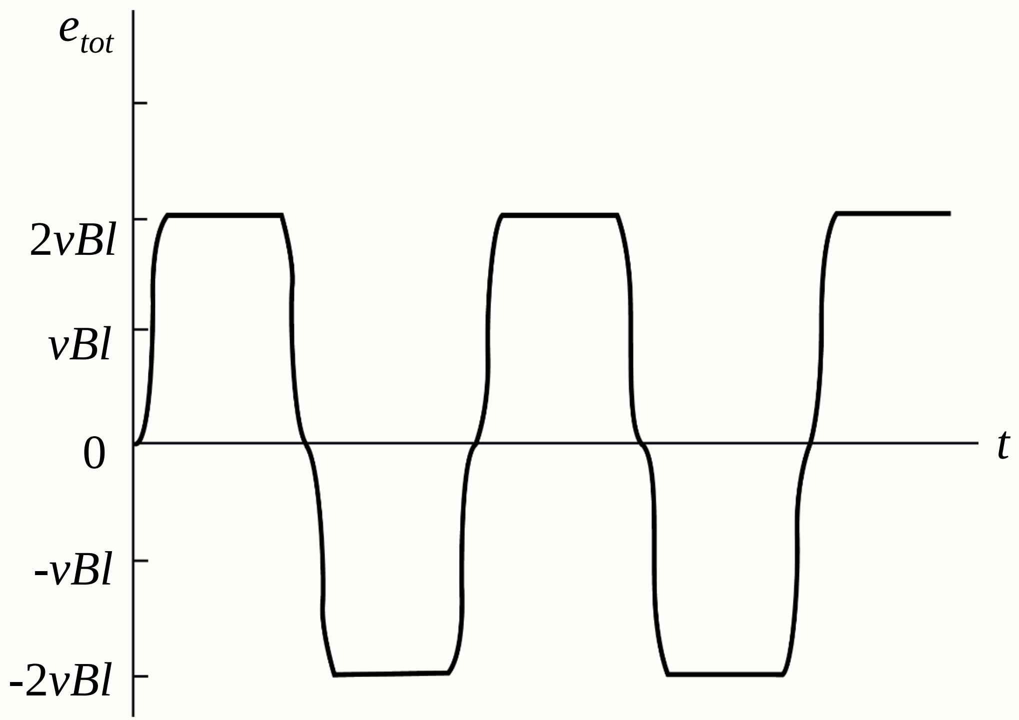
$$e_{dc} = \begin{cases} Blv & \text{positive out of page, under pole face} \\ 0, & \text{beyond the pole edges} \end{cases}$$

<u>The total induced voltage in the loop is given by</u>:

$$e_{ind} = e_{ba} + e_{ad} + e_{dc} + e_{cb} = \begin{cases} 2Blv, & \text{under the pole faces} \\ 0, & \text{beyond the pole edges} \end{cases}$$

The Voltage Induced in a Rotating Loop

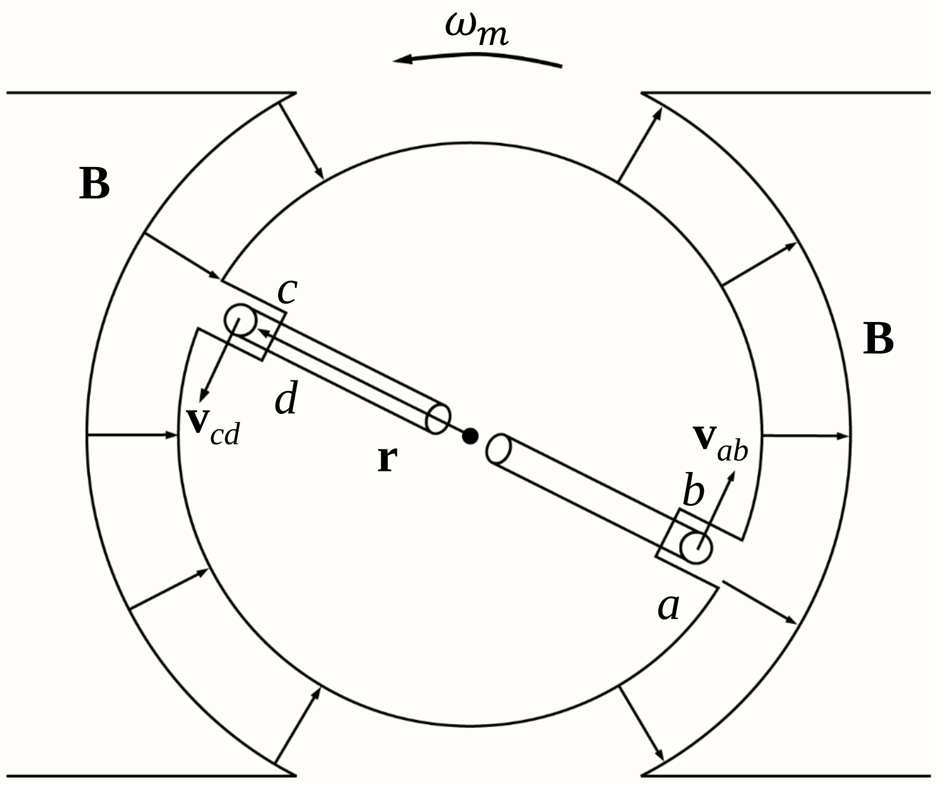
- When the loop rotates through 180 degrees, segment *ab* is under the north pole face instead of the south pole face. At that time, the direction of the voltage on the segment reverses, but its magnitude remains constant. The resulting total output voltage is as shown.



The total output voltage waveform of the loop.

Alternative Way to Express Total Induced Voltage

- To clearly relate the behaviour of the single loop to the behaviour of the larger, real DC machines, the tangential velocity v of the loop can be expressed as:



$$v = r\omega_m$$

where

r = radius from axis of rotation out to the edge of loop

ω_m = angular velocity of loop

Alternative Way to Express Total Induced Voltage

$$e_{ind} = \begin{cases} 2Blr\omega_m = \dfrac{2}{\pi} A_p B \omega_m = \dfrac{2}{\pi} \phi \omega_m, & \text{under the pole faces} \\ 0, & \text{beyond the pole edges} \end{cases}$$

Where $\begin{cases} \phi = A_p B \\ A_p = \pi r l, \ \text{is the rotor surface area under each pole} \end{cases}$

Thus, the voltage generated in a DC machine is directly proportional to the field inside the machine and the speed of rotation of the machine. In general,

$$e_{ind} = \begin{cases} k\phi\omega_m & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

k is a constant representing the mechanical construction of the machine.

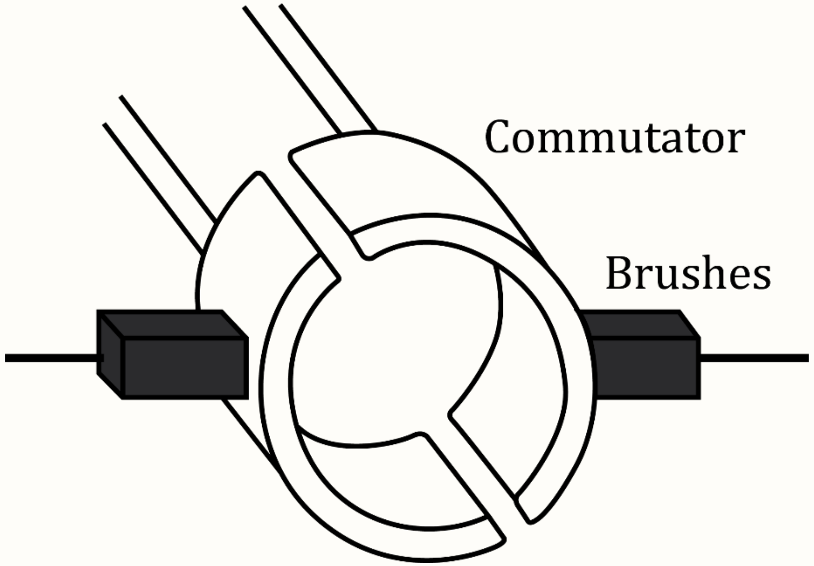
Getting DC Voltage out of Rotating Loop: Commutation

- Two semi-circular conducting segments are added to the end of the loop, and two fixed contacts are set up, such that at the instant when the voltage in the loop is zero, the contacts short-circuit the two segments. Hence, every time the voltage of the loop switches directions, the contacts also switch connections, and the output of the contacts is always built up in the same way (DC).

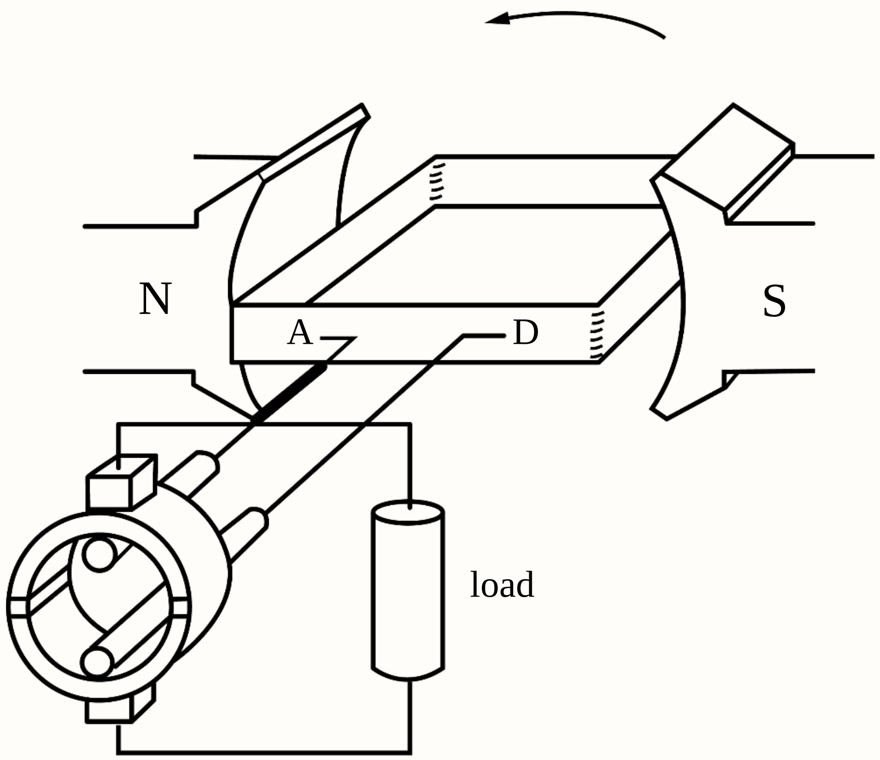
- This connection-switching process is known as <u>commutation</u>.

Getting DC Voltage out of Rotating Loop: Commutation

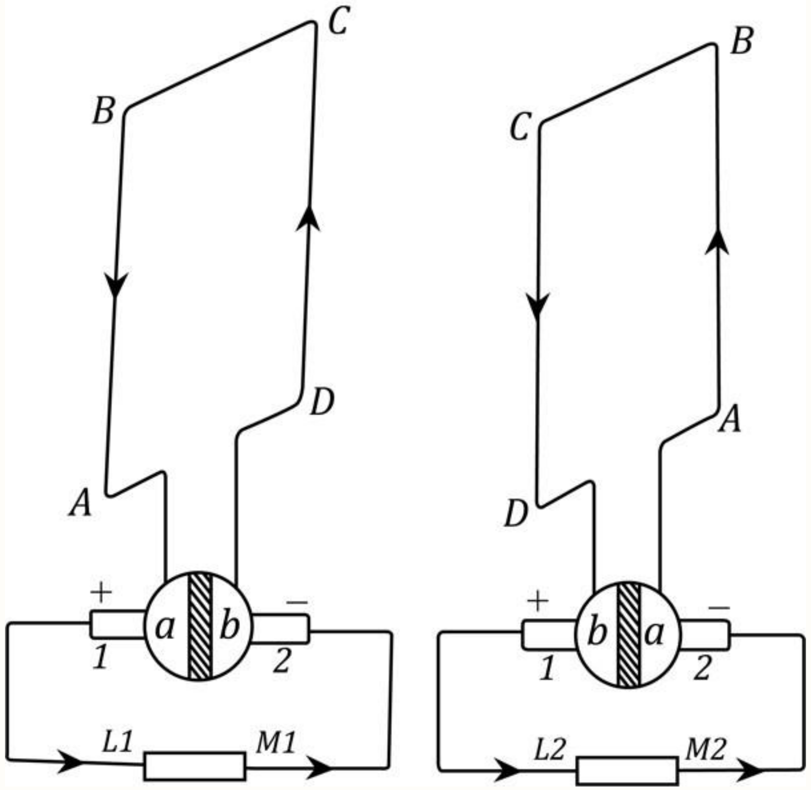
❖ The rotating semi-circular segments are called commutator segments, and the fixed contacts are called brushes. The brushes are pressed against the commutator by springs.



Getting DC Voltage out of Rotating Loop: Commutation



Elementary DC generator is simply an AC generator equipped with a mechanical rectifier called a commutator. *WILDI, THEODORE, ELECTRICAL MACHINES, DRIVES AND POWER SYSTEMS, 6th, 2006*. Reprinted by permission of Pearson Education, Inc., New York, New York.

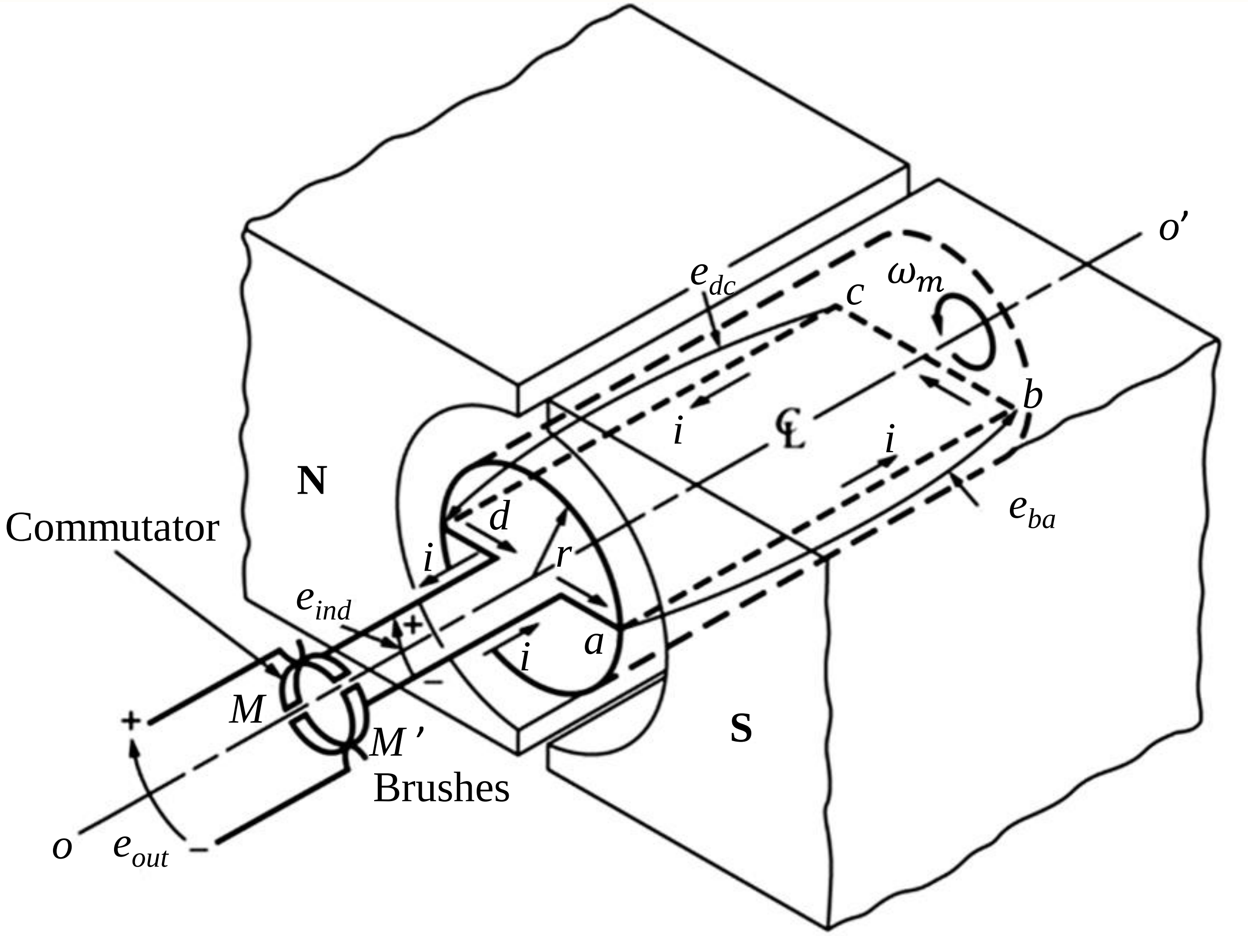


Principle of commutation.

Getting DC Voltage out of Rotating Loop: Commutation

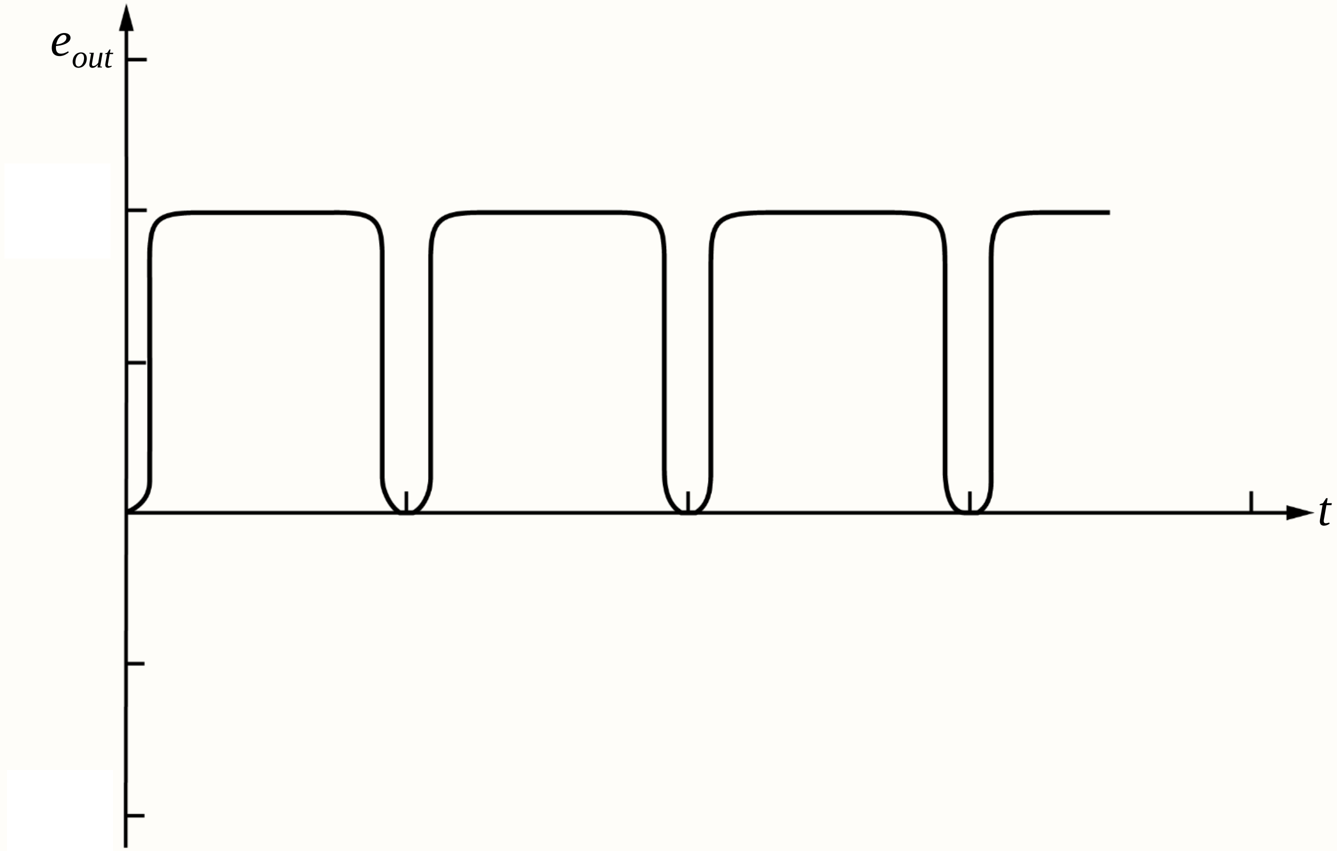
- A commutator in its simplest form is composed of a slip ring that is cut in half, with each segment insulated from one another as well as from the shaft. One segment is connected to each end of the loop.
- The commutator revolves with the loop and the voltage between the segments is picked up by two stationary brushes.
- The voltage between the brushes never changes polarity.
- The AC voltage in the loop is rectified by the commutator, which actually acts as a mechanical rectifier.

Getting DC Voltage out of Rotating Loop: Commutation



Producing a DC output with commutator and brushes.

Getting DC Voltage out of Rotating Loop: Commutation

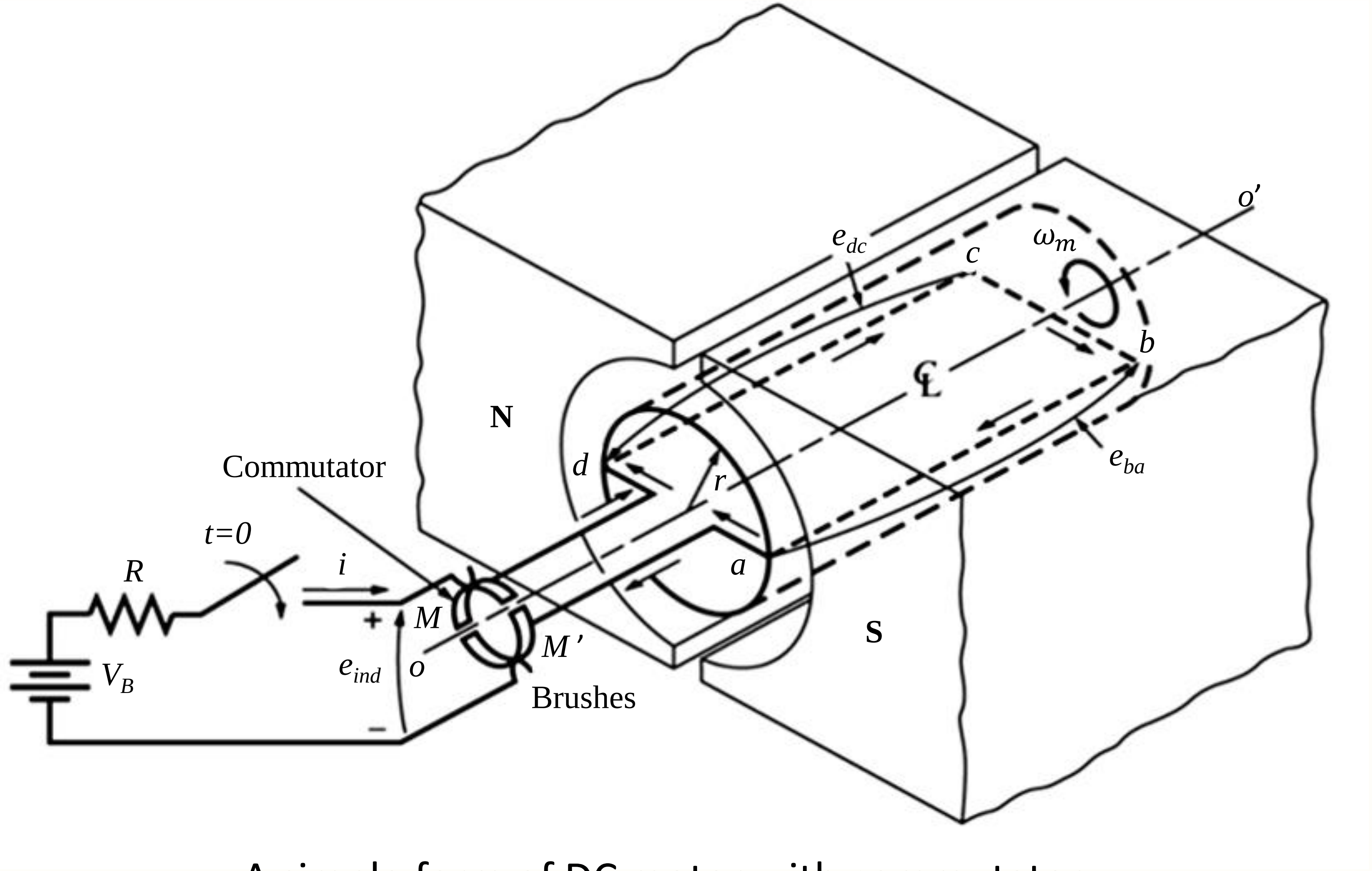


Resulting output voltage after commutation.

The Induced Torque in the Rotating Loop

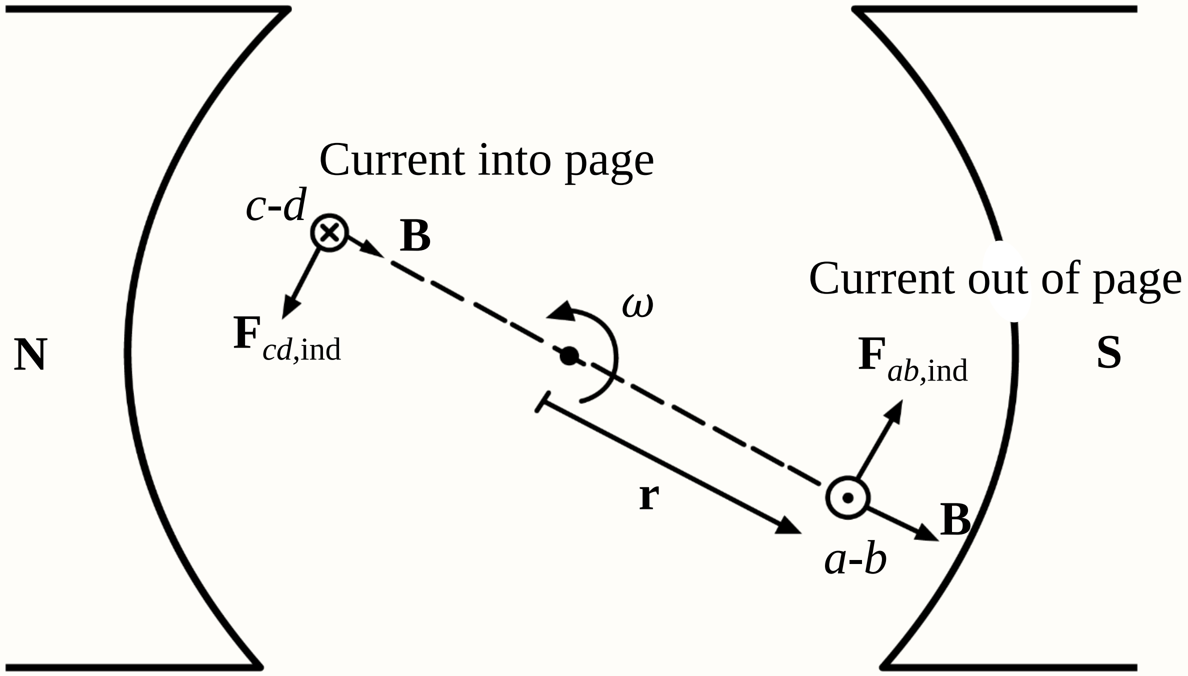
- Suppose a battery is connected to the machine as shown.
- The current that flows in the loop will result in a torque which rotates the loop.
- As the loop turns, the current in its windings is commutated to produce a continuous torque output.
- How much torque will be produced in the loop when the switch is closed and a current is flowing into it?
- The approach is to look at one segment of the loop at a time, and then sum the effects of all the individual segments.

The Induced Torque in Rotating Loop



A simple form of DC motor with commutator.

The Induced Torque in Rotating Loop



a) Segment *ab*.

$F_{ab} = Bli,$ tangent to the direction of motion

$T_{ab} = F_{ab}r = Blir,$ counter clockwise

The Induced Torque in Rotating Loop

b) Segments *bc* and *da*.

They are the end connections used to connect the conductor segments *ab* and *cd* in series. $T_{bc} = T_{da} = 0$

c) Segment *cd.*

$$F_{cd} = Bli, \qquad \text{tangent to the direction of motion}$$

$$T_{cd} = F_{cd} r = Blir, \text{ counter clockwise}$$

The total induced torque on the loop is given by:

$$T_{ind} = T_{ab} + T_{bc} + T_{cd} + T_{da} = \begin{cases} 2Blir, & \text{under the pole faces} \\ 0, & \text{beyond the pole edges} \end{cases}$$

The Induced Torque in Rotating Loop

$$T_{ind} = \begin{cases} \dfrac{2}{\pi}\phi i, & \text{under the pole faces} \\ 0, & \text{beyond the pole edges} \end{cases}$$

$$\phi = A_p B, \quad A_p = \pi r l$$

Thus, the torque produced in the machine is directly proportional to the field (flux) in the machine and the rotor (armature) current. In general,

$$T_{ind} = \begin{cases} k\phi i, & \text{under the pole faces} \\ 0, & \text{beyond the pole edges} \end{cases}$$

k is a constant representing the construction of the machine.

Summary

In this lecture, you have learnt:

- An internal voltage is generated when the rotor is rotated in the magnetic field created by the stator.
- The internal generated emf equation that shows that it is directly proportional to the field in the machine and the speed of rotation of the machine.
- A torque is produced when the rotor is supplied with a DC current and under the influence of the magnetic field set up by the stator circuit.
- The torque equation that shows that it is directly proportional to the field in the machine and the rotor current.
- The concept of commutation that converts AC voltages in the rotor of a DC machine to DC voltage at its terminals.

References

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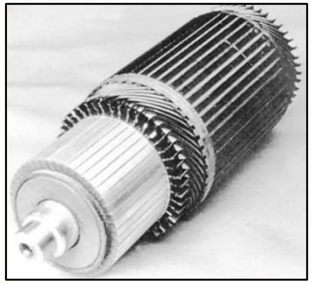
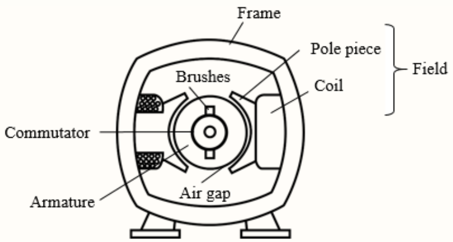
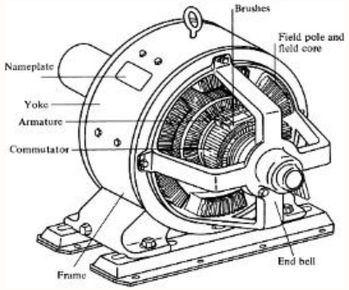
1	6		Reprinted from *Electrical Machines, Drives, and Power Systems, 6th ed.*, (p. 86), by T. Wildi, 2006, Upper Saddle River, NJ: Pearson/ Prentice Hall. Copyright 2006 by Pearson Education, Inc., New York. Reprinted with permission.
2	6		Reprinted from *Electrical Machines, Drives, and Power Systems, 6th ed.*, (p. 85), by T. Wildi, 2006, Upper Saddle River, NJ: Pearson/ Prentice Hall. Copyright 2006 by Pearson Education, Inc., New York. Reprinted with permission.
3	7		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 450), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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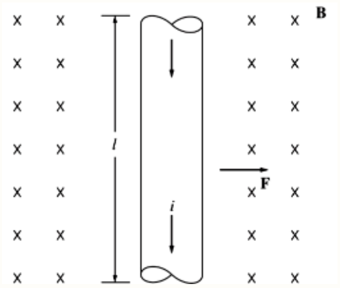
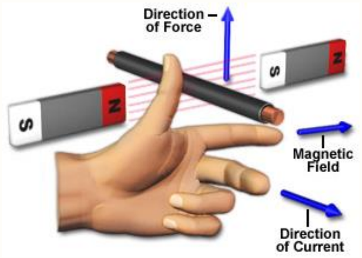
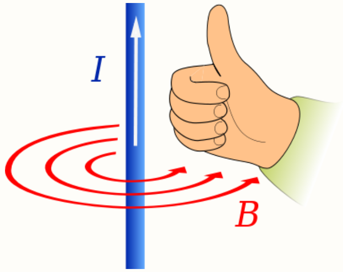
4	9		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 33), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	10		From *Fleming left hand rule and Fleming right hand rule*, by electrical4u (https://www.electrical4u.com/fleming-left-hand-rule-and-fleming-right-hand-rule/). Copyright 2017 by electrical4u. Reprinted with permission. Retrieved on 7 June 2017.
6	11		By Jfmelero (Own work) [CC BY-SA 4.0 (https://creativecommons.org/licenses/by-sa/4.0/)], Wikimedia Commons. Retrieved on 7 June 2017.

References

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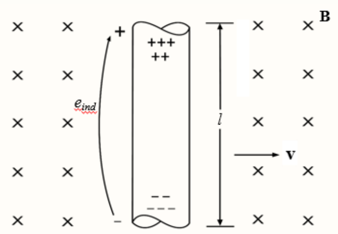
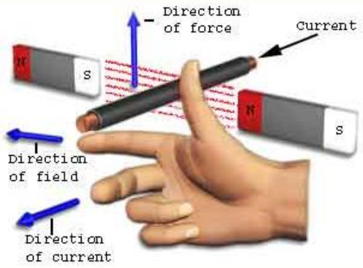
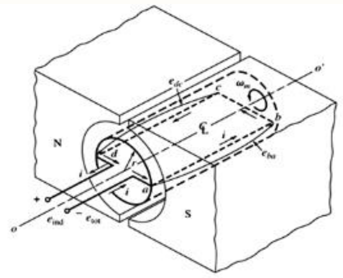
7	12		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 35), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
8	13		From *Fleming left hand rule and Fleming right hand rule*, by electrical4u (https://www.electrical4u.com/fleming-left-hand-rule-and-fleming-right-hand-rule/). Copyright 2017 by electrical4u. Reprinted with permission. Retrieved on 7 June 2017.
9	14		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 406), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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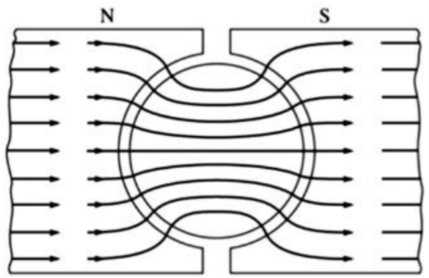
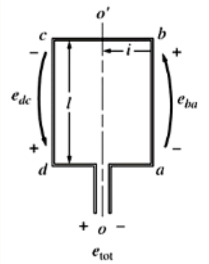
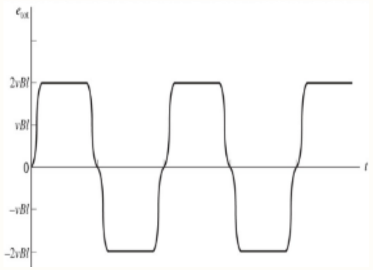
10	16		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 406), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
11	16		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 406), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
12	19		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 408), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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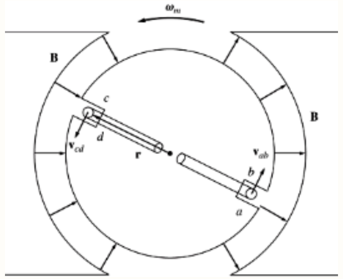
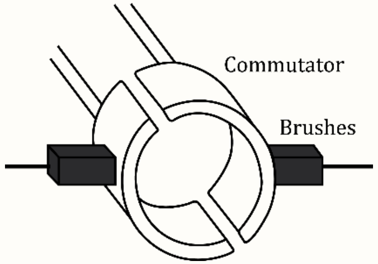
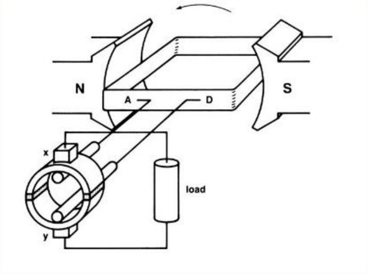
13	20		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 407), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
14	23		From *Commutator and Brushes on DC Motor*, by C. R. Nave, 2012 (http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/comtat.html). Copyright 2012 by C. R. Nave. Retrieved on 7 June 2017.
15	24		Reprinted from *Electrical Machines, Drives, and Power Systems, 6th ed.*, (p. 73), by T. Wildi, 2006, Upper Saddle River, NJ: Pearson/ Prentice Hall. Copyright 2006 by Pearson Education, Inc., New York. Reprinted with permission.

References

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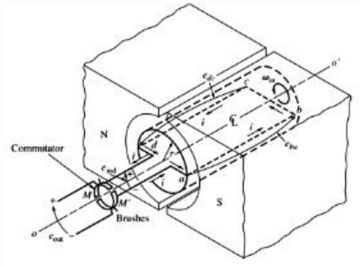
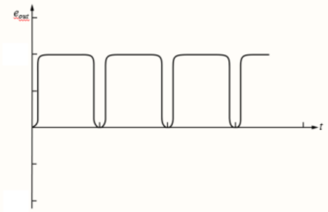
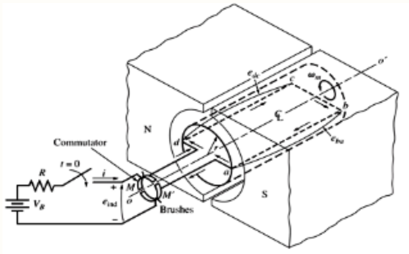
16	26		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 410), by S.J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
17	27		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 411), by S.J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
18	29		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 411), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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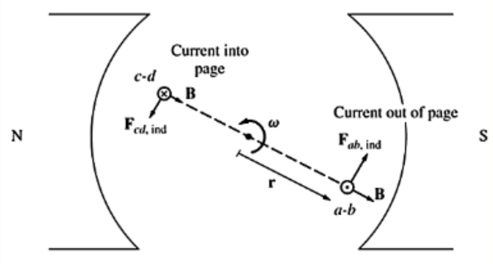
19	30		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 411), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

Lecture 7

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the problems associated with the commutation process in real DC machines.
- Explain the problems caused by armature reaction and the inductive voltage kick.
- Identify the solutions that may be used to overcome the problems associated with the commutation process.
- Describe the power flows and the power flow diagram of a DC machine.

Problems with Commutation in Real Machines

- Two major effects occur in the real world to disturb the commutation process:
  - Armature reaction
  - The $L\frac{di}{dt}$ voltages

Armature Reaction

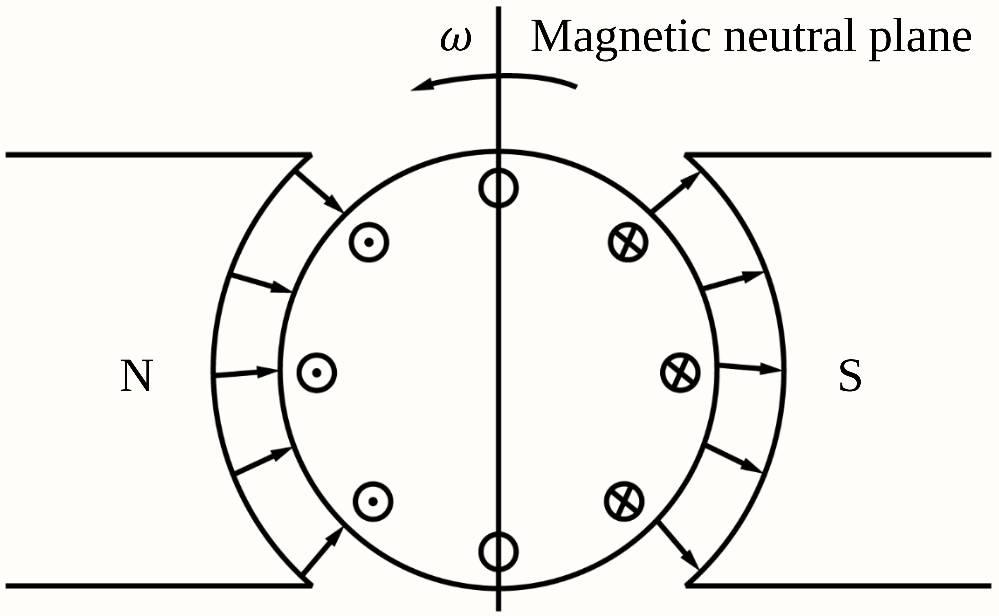
❖ The current flowing in the armature windings will produce a magnetic field (armature mmf) of its own, which will distort the original magnetic field from the machine's poles. The effect produced by the armature mmf is called armature reaction.

❖ Problems caused by armature reaction:

- Magnetic neutral plane shift
- Flux weakening effect

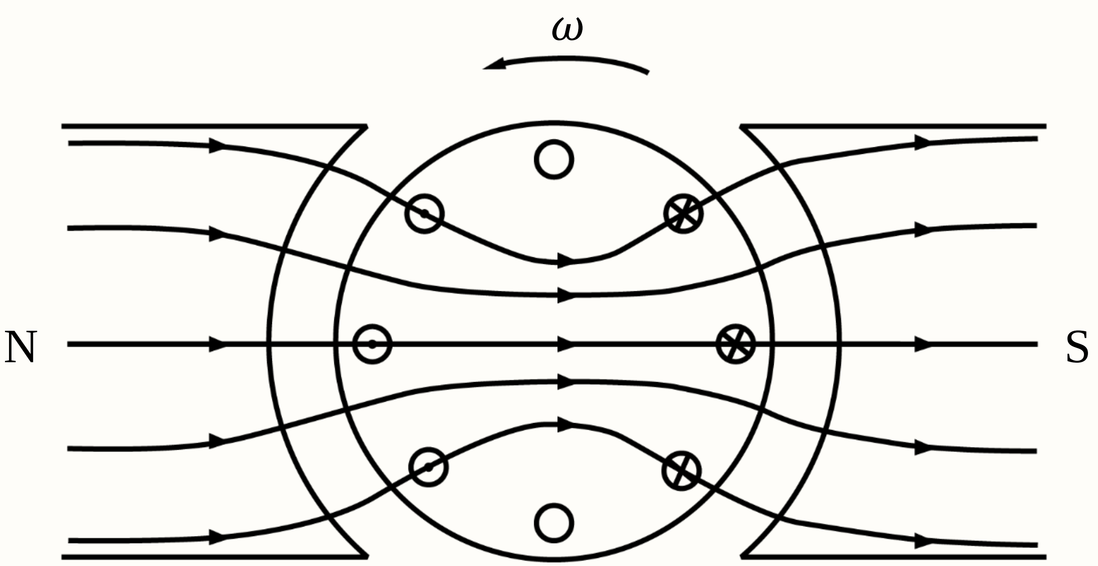
Magnetic Neutral Plane Shift

- The magnetic neutral plane is the position where the armature windings are moving parallel to the magnetic flux lines. They do not cut any lines of flux and there are no induced voltages.

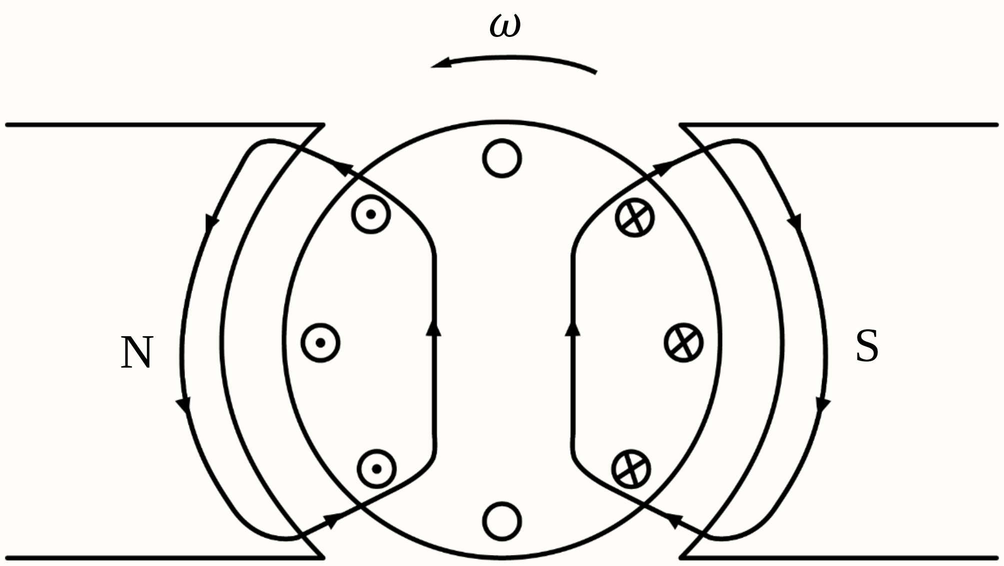


Initially, the pole flux is uniformly distributed in a DC generator, and the magnetic neutral plane is vertical.

Development of Armature Reaction in a DC Generator

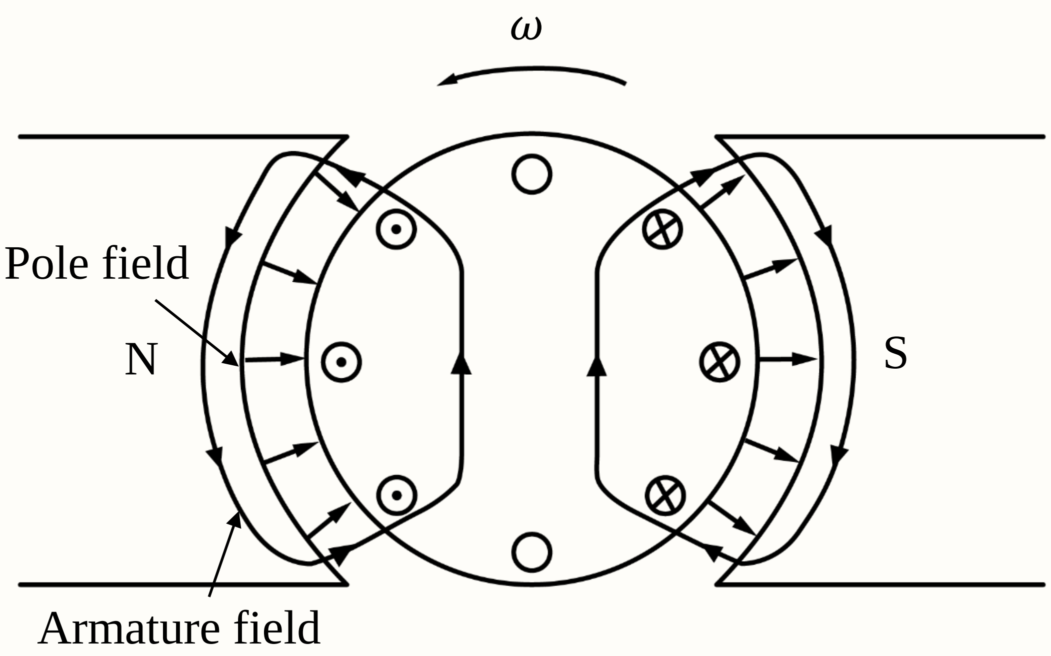


Effect of air gap on the pole flux distribution.

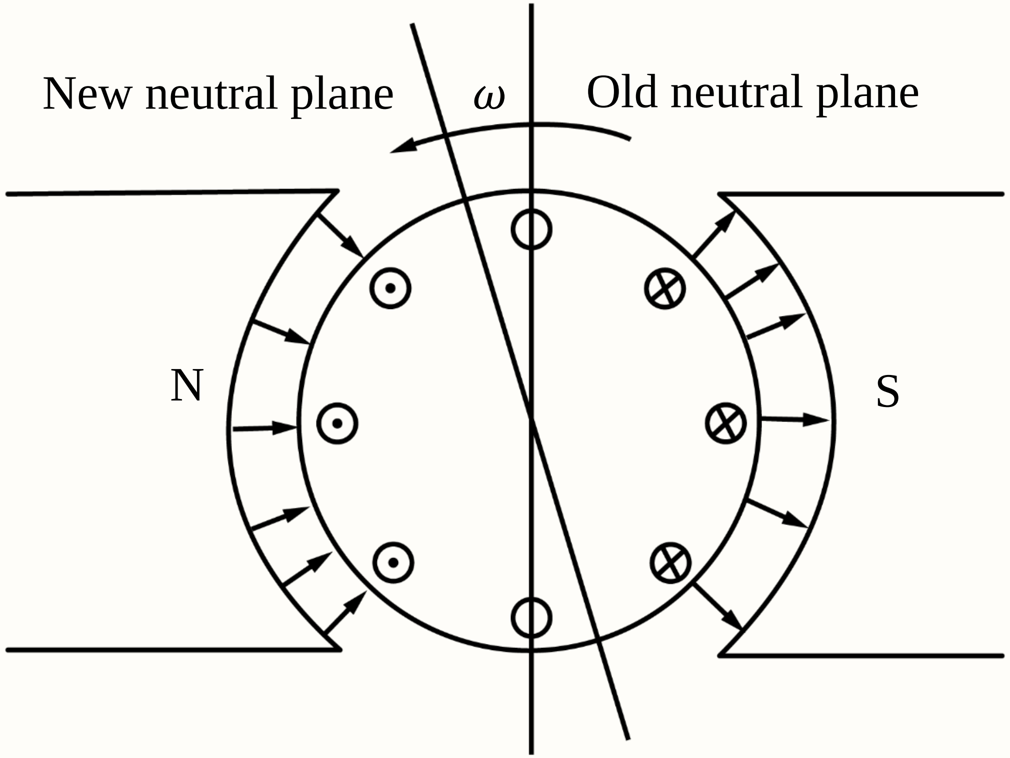


Magnetic field produced by currents flowing in armature conductors.

Development of Armature Reaction in a DC Generator



Rotor and pole fluxes adding and subtracting.



Resulting flux under the poles.
Neutral plane has shifted.

Magnetic Neutral Plane Shift

- The process of commutation requires the brushes shorting out the commutator segments just at the moment when the voltage across the segments is equal to zero.
- When the neutral plane shifts, the brushes short out commutator segments with a finite voltage across them. The result is a current flow circulating between the shorted segments.
- Severe arcing and sparking at the brushes may occur.

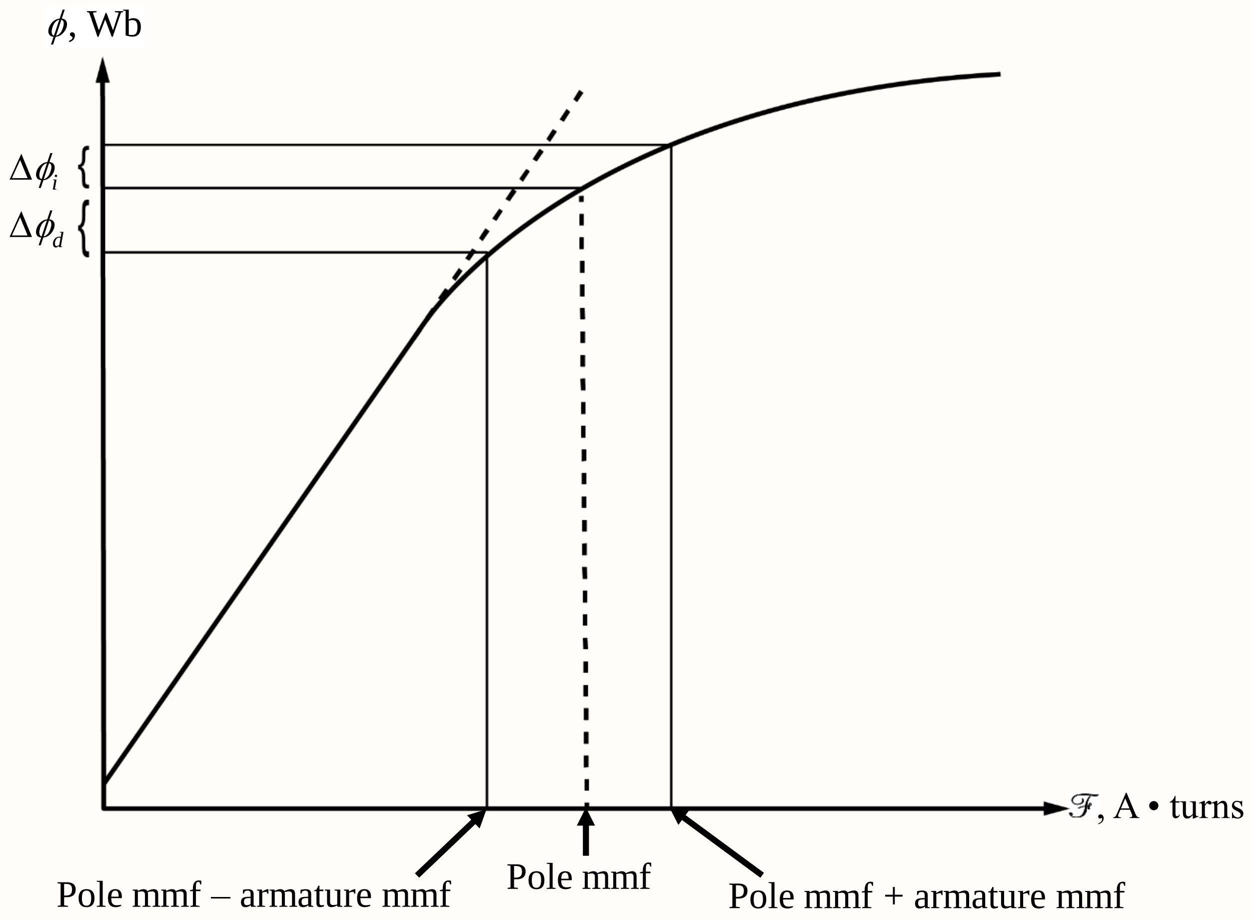
Flux Weakening Effect

❖ Most machines operate at flux densities near the saturation point.

- At locations where rotor and pole mmfs add, only a small increase in flux occurs.
- At locations where rotor mmf subtracts from the pole mmf, there is a larger decrease in flux.

❖ As a result, the total average flux under the entire pole face is decreased.

Flux Weakening Effect



$\Delta\phi_i \equiv$ flux increase under reinforced sections of poles

$\Delta\phi_d \equiv$ flux decrease under subtracting sections of poles

Flux Weakening Effect

- In generators, flux weakening reduces the voltage supplied by the generator for any given load.
- In motors, when flux is decreased, the speed of the motor increases.

$L\frac{di}{dt}$ Voltages

- They occur in commutator segments being shorted out by the brushes, sometimes called inductive kick.

- When a commutator segment is shorted out, the current flow through that commutator segment must reverse.

- The rate of change in current with respect to time in the shorted loop can be very high and a very significant inductive voltage kick will be induced in the shorted commutator segment.

- This high voltage naturally causes sparking at the brushes of the machine.

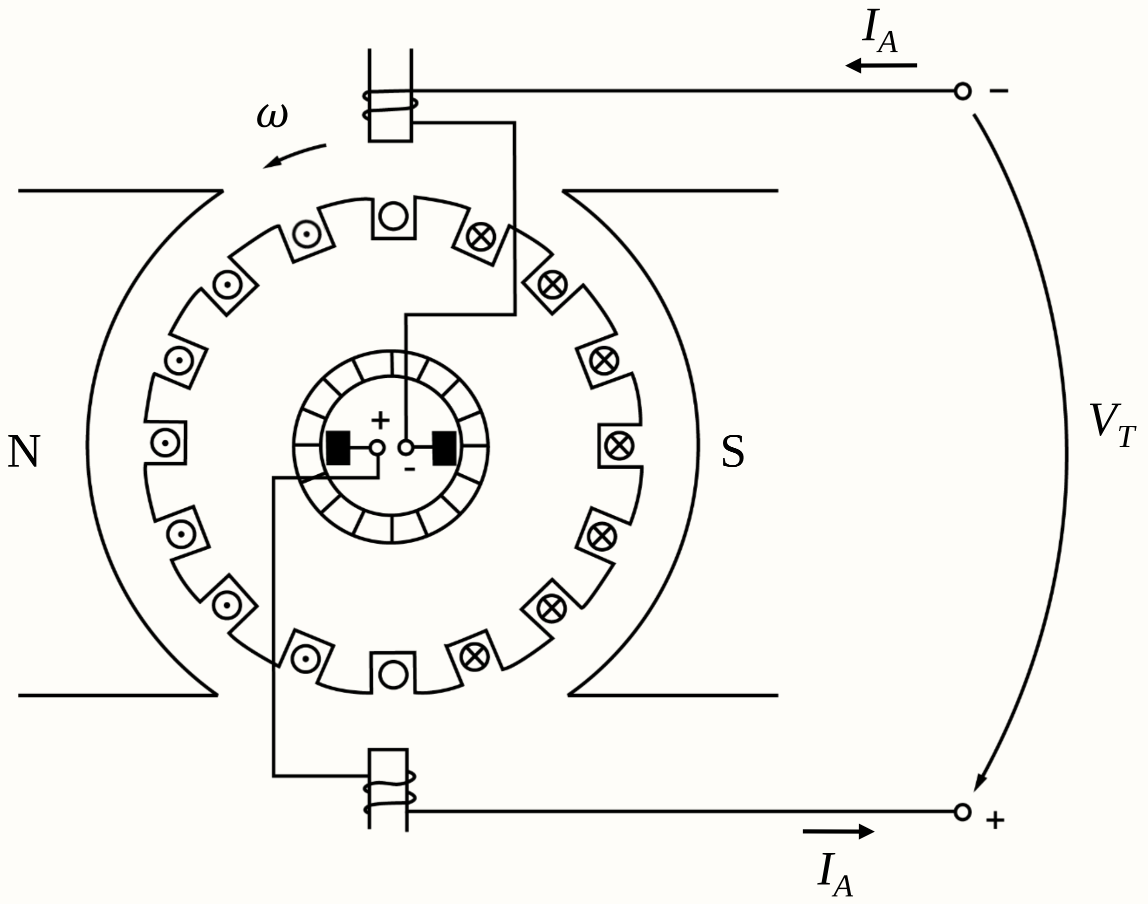
Solutions to the Problems with Commutation

- Brush shifting
  - Shifting the brushes to the new neutral plane to reduce sparking.
  - But if the load fluctuates, the armature mmfs fluctuate, and so the neutral plane shifts back and forth between no load and load conditions.
  - Someone has to adjust the brushes every time the load on the machine changes. This is not practical.

Solutions to the Problems with Commutation

- Commutating poles or interpoles
  - These small poles carry windings that are connected in series with the armature and are located directly over the conductors being commutated.
  - The number of turns on the windings is such that the interpoles developed an mmf that is equal and opposite to the armature mmf.
  - As the load current varies, the two mmfs rise and fall together, exactly bucking each other at all times. The voltage in the coils undergoing commutation can be exactly cancelled, leading to no sparking at the brushes.

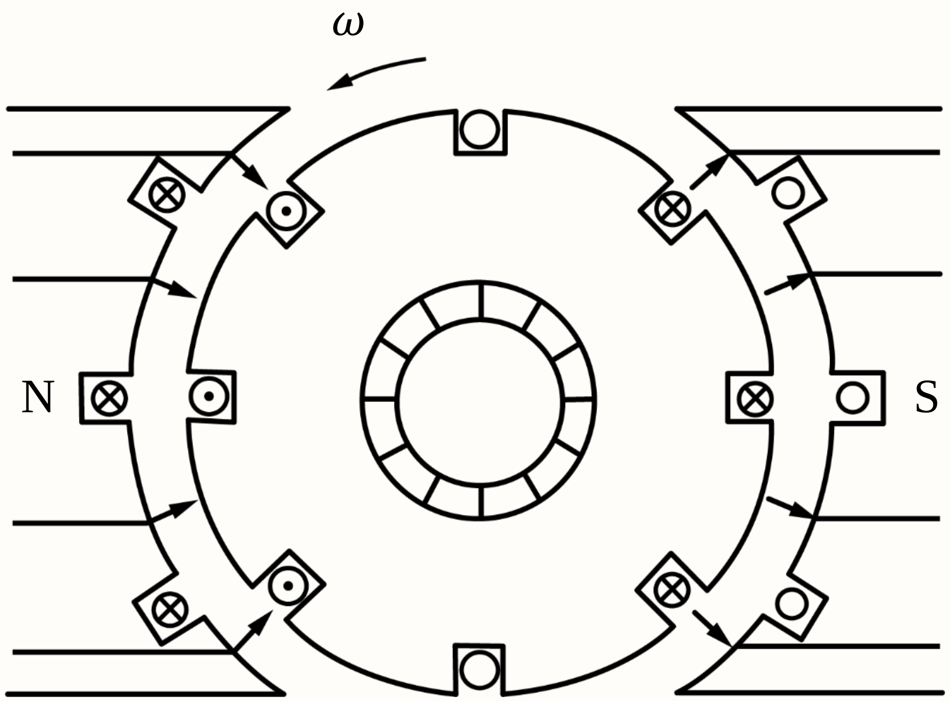
Solutions to the Problems with Commutation



A DC machine with interpoles, placed midway between the main poles.

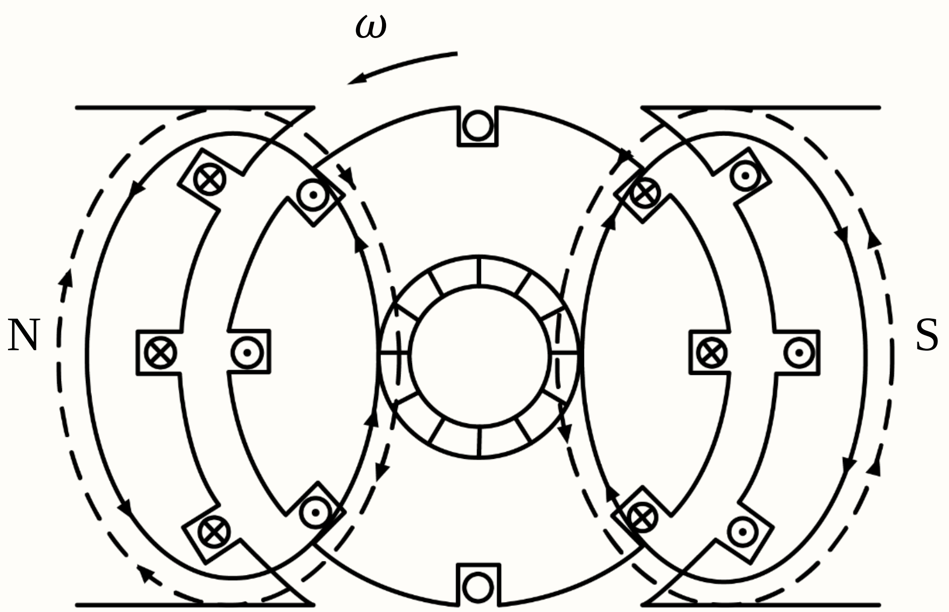
Solutions to the Problems with Commutation

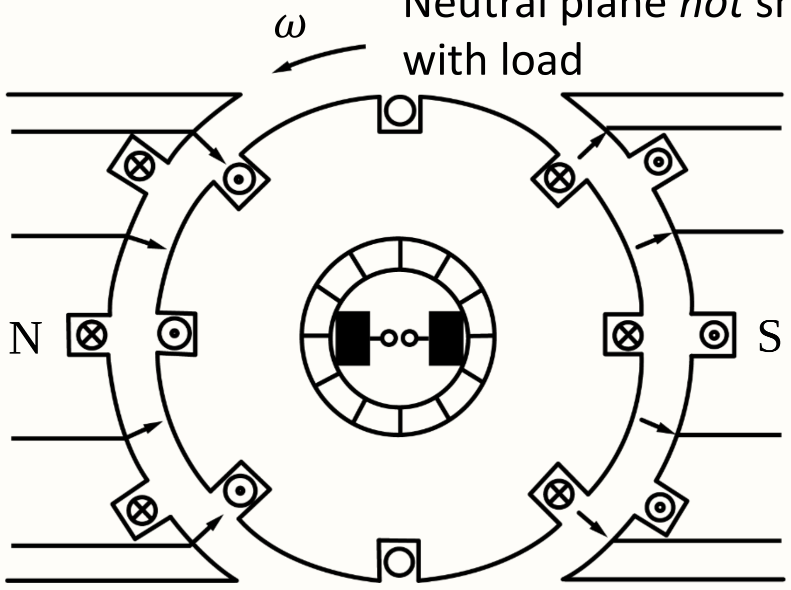
- Compensating windings
  - The windings are distributed in slots carved in the faces of the poles parallel to the rotor conductors and connected in series with rotor windings.



The pole flux in the machine.

Solutions to the Problems with Commutation





Rotor (armature) flux - - - - Flux from compensating windings

Fluxes from armature and compensating windings. They are equal and opposite and cancel each other out.

The net flux in the machine, which is just the original pole flux.

Power Flow and Losses in DC Machines

- DC generators take in mechanical power and produce electric power.
- DC motors take in electric power and produce mechanical power.
- Efficiency of a DC machine is defined as $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{in} - P_{losses}}{P_{in}} \times 100\%$
- Losses in a DC Machine:

a) Electrical or copper losses

b) Brush losses

c) Core losses

d) Mechanical losses

e) Stray load losses

Power Flow and Losses in DC Machines

a) Electrical or copper losses

- Losses that occur in the armature (rotor) and field (stator) windings of machine:

Armature copper loss: $P_A = I_A^2 R_A$

Field copper loss: $P_F = I_F^2 R_F$

Power Flow and Losses in DC Machines

b) Brush losses

- The power lost across the contact potential at the brushes of the machine. Relatively small.

$$P_{BD} = V_{BD} I_A$$

$$V_{BD} = \text{brush voltage drop, usually assumed to be 2 V for graphite brushes}$$

$$I_A = \text{armature current}$$

c) Core losses

- The hysteresis losses and eddy current losses occurring in the metal.

Power Flow and Losses in DC Machines

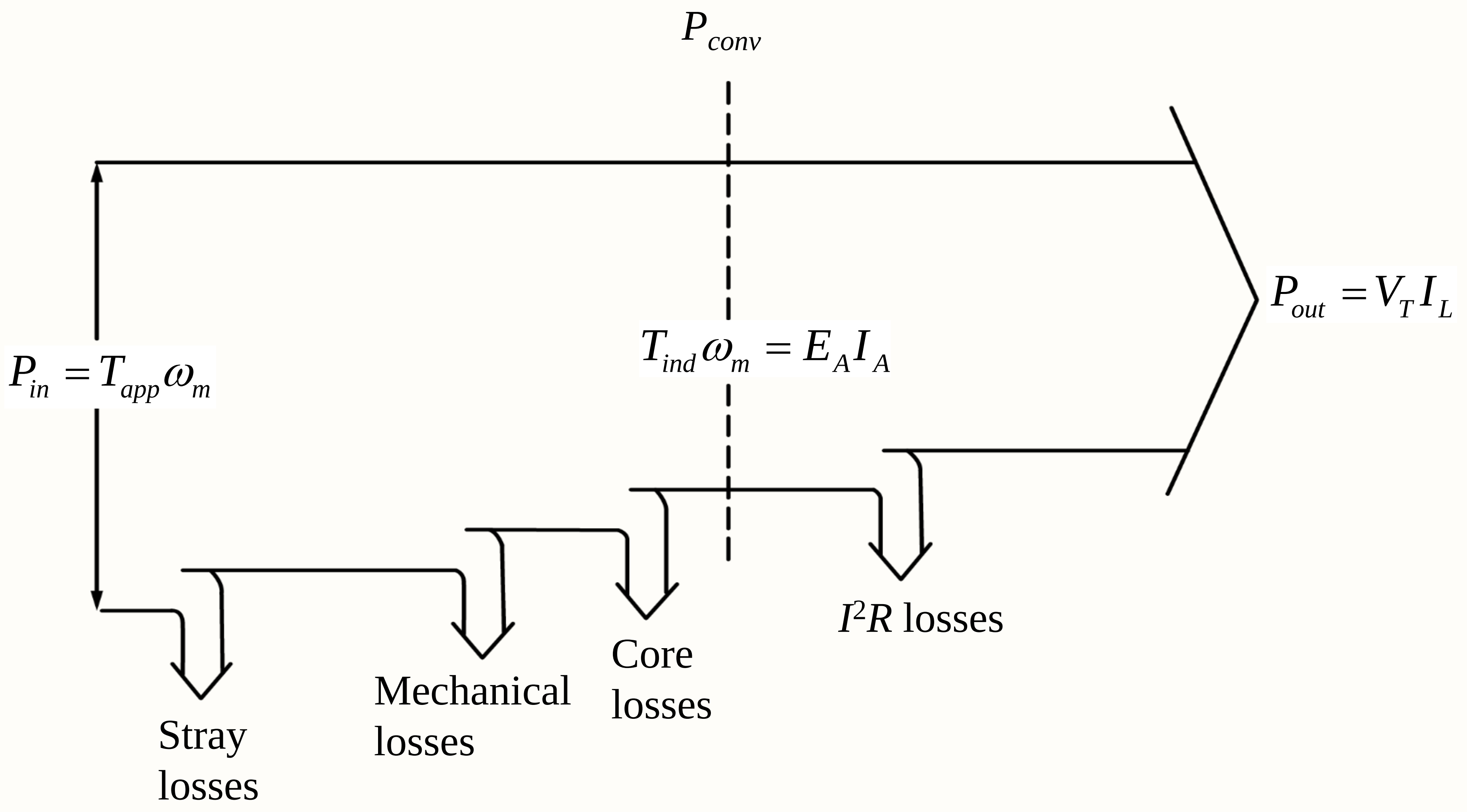
d) Mechanical losses

- Friction loss: caused by friction of the bearings in the machine and friction between brushes and commutator.
- Windage loss: caused by friction between the moving parts of the machine and the air inside the motor's casing.

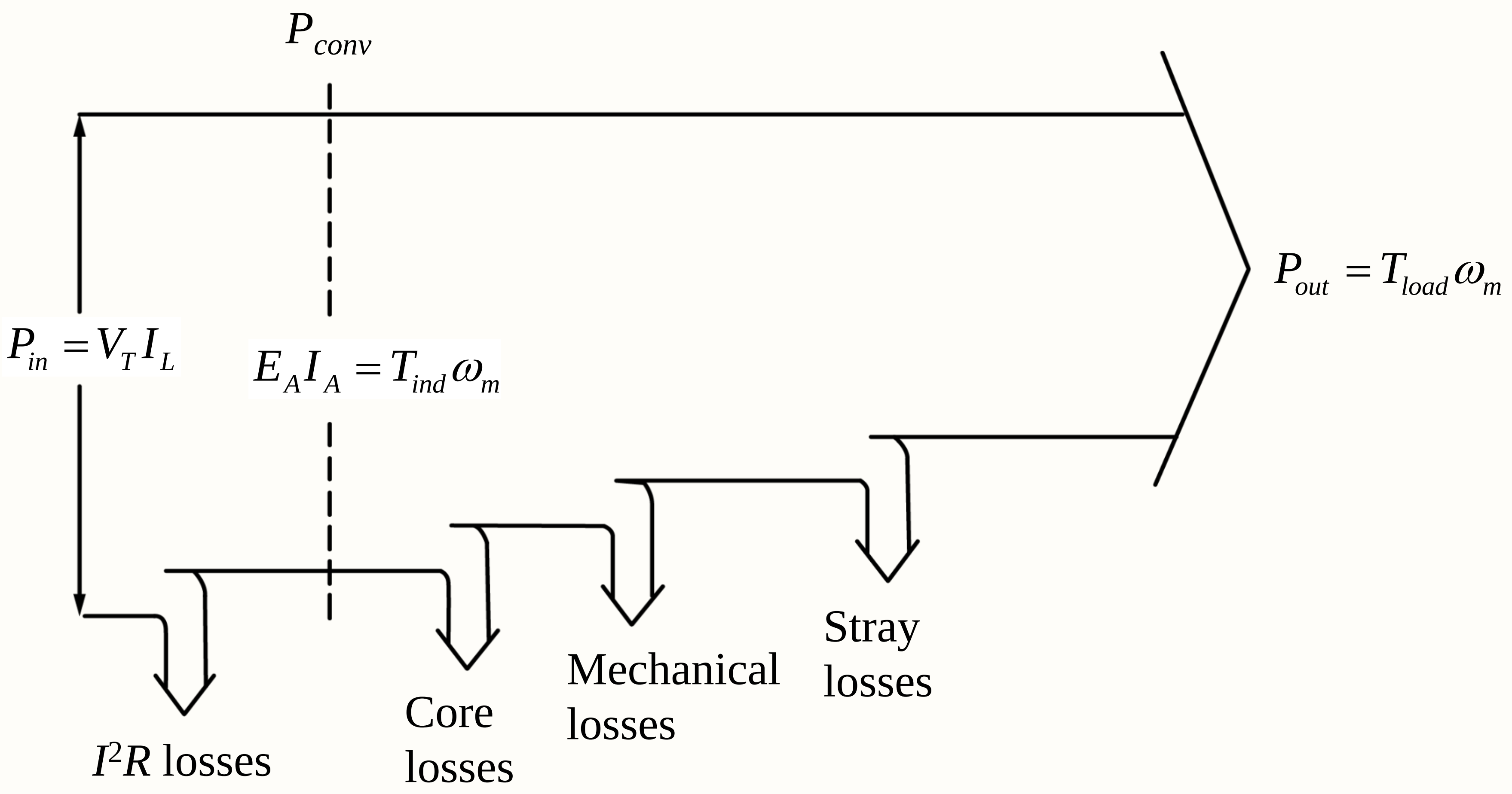
e) Stray (or miscellaneous) losses

- Any losses that cannot be placed in the above categories, e.g., losses in coils undergoing commutation. Approx 1% of full load (IEEE-Std 113-1985). In this course, if it is not mentioned, will assumed to be neglected.

Power Flow Diagram for DC Generator



Power Flow Diagram for DC Motor



Summary

In this lecture, you have learnt:

- The problems caused by armature reaction and the inductive voltage kick in real DC machines.
- The solutions to the problems with commutation, such as utilising brush shifting, interpoles and compensating windings.
- The power flow diagram and losses of a DC machine.

References

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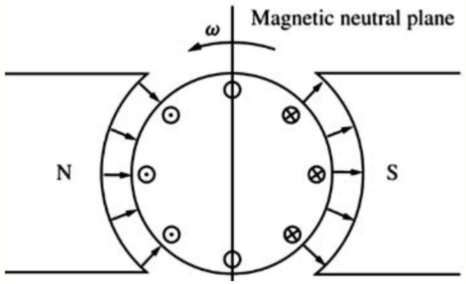
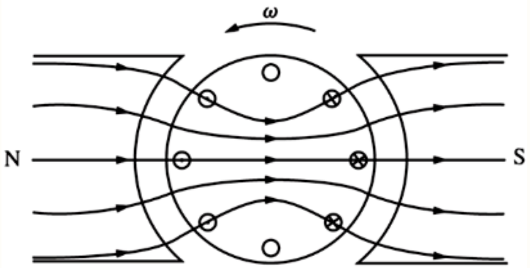
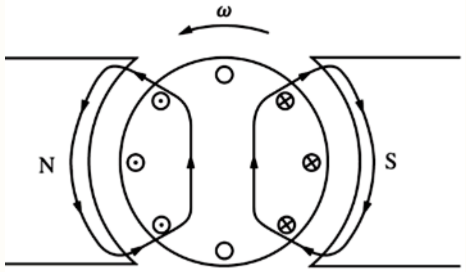
1	5		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 434), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
2	6		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 434), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
3	6		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 434), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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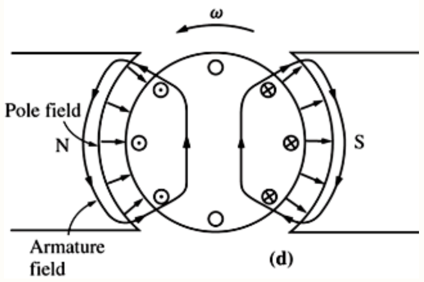
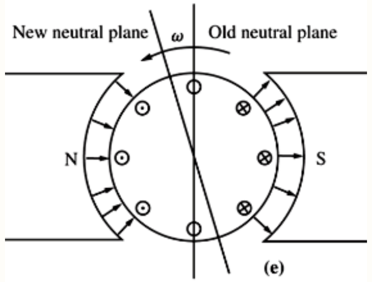
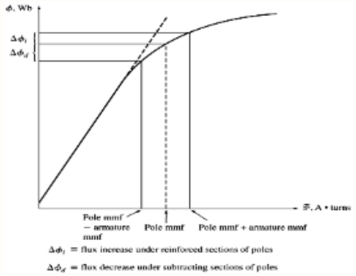
4	7		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 434), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	7		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 434), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
6	10		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 436), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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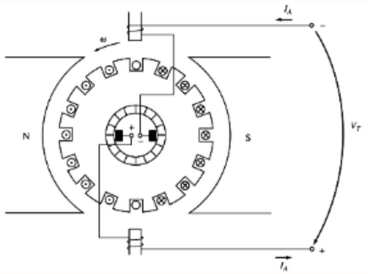
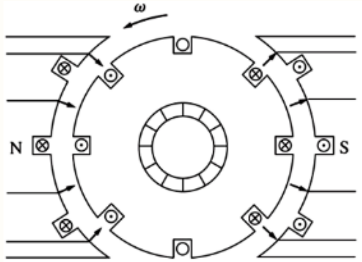
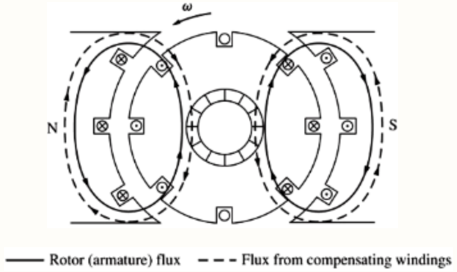
7	15		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 441), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
8	16		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 443), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
9	17		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 443), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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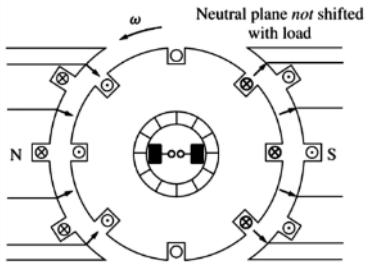
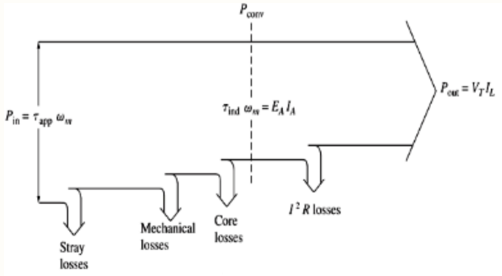
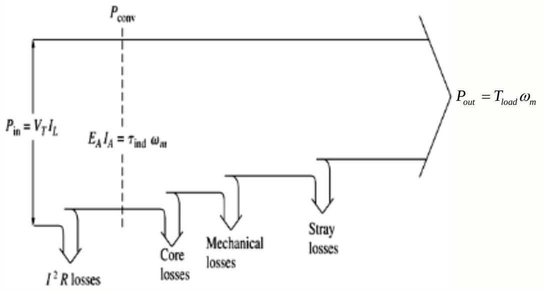
10	17		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 443), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
11	22		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 457), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
12	23		Adapted from *Electric Machinery Fundamentals, 5th ed.*, (p. 457), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Adapted with permission.

Lecture 8

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the equivalent circuit of a DC motor.
- Identify the types of DC motors.
- Derive the torque-speed characteristics of a DC motor.
- Describe the losses and calculate the motor efficiency with given operating condition from the motor equivalent circuit.

Introduction to DC Motors

- DC motors are DC machines used as motors.
- The same DC machine can operate as either a motor or a generator.
- DC power systems are still common in cars, trucks and aircrafts. When a vehicle has a DC power system, it makes sense to consider using DC motors.
- Even if no DC power sources are available, solid state rectifier and chopper circuits are used to create the necessary DC power for the DC motors.
- In areas where wide variations in torque-speed characteristics are required, such as in steel mills, DC motors are needed.

Speed Regulation of DC Motors

- DC motors are compared by their speed regulations which is defined by:

$$\mathrm{SR} = \frac{\omega_{m,nl} - \omega_{m,fl}}{\omega_{m,fl}} \times 100\% = \frac{n_{m,nl} - n_{m,fl}}{n_{m,fl}} \times 100\%$$

- It is the change in speed from no load to full load, expressed as a percentage of full-load speed.

- The magnitude of the SR gives an indication of how steep the slope of the torque-speed curve is.

Example 1

A 120-V, 1750 r/min, 5-hp motor, operating at rated conditions, has a speed regulation of 4.0 percent. Determine the no-load speed.

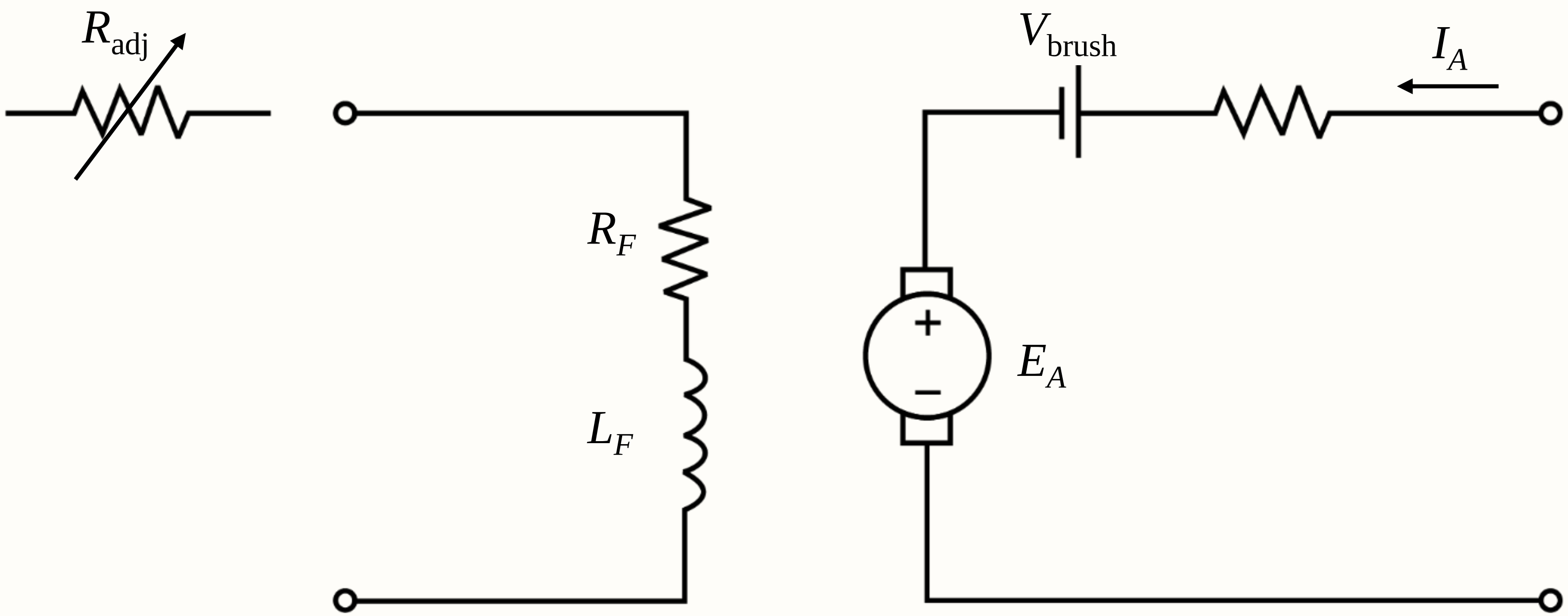
$$\text{SR} = \frac{\omega_{m,nl} - \omega_{m,fl}}{\omega_{m,fl}} \times 100\%$$

$$\Rightarrow 0.04 = \frac{n_{m,nl} - 1750}{1750} \Rightarrow n_{m,nl} = 1820 \text{ r/min}$$

In this course, the following types of DC motors will be studied in detail:

- The separately excited DC motor
- The shunt DC motor

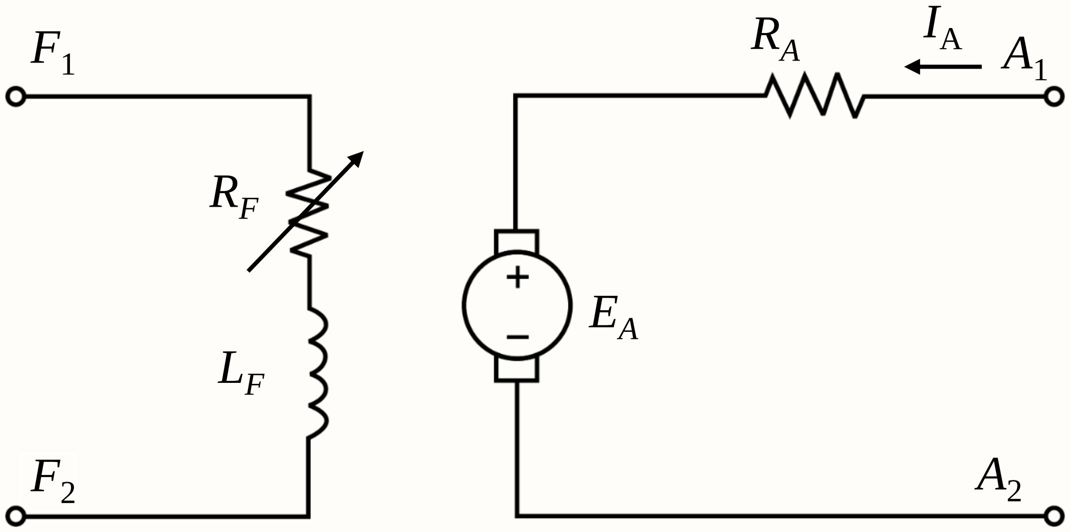
The Equivalent Circuit of a DC Motor



Equivalent circuit of a DC motor.

❖ The armature (rotor) circuit is represented on the right and the field circuit on the left. The brush voltage is represented by a small battery opposing the current flow in the machine.

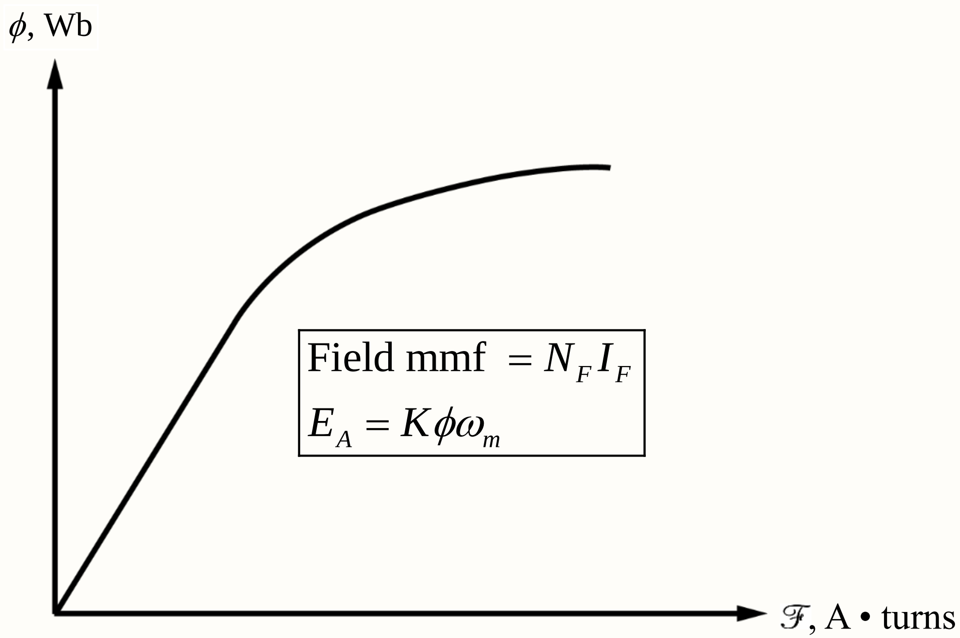
The Equivalent Circuit of a DC Motor



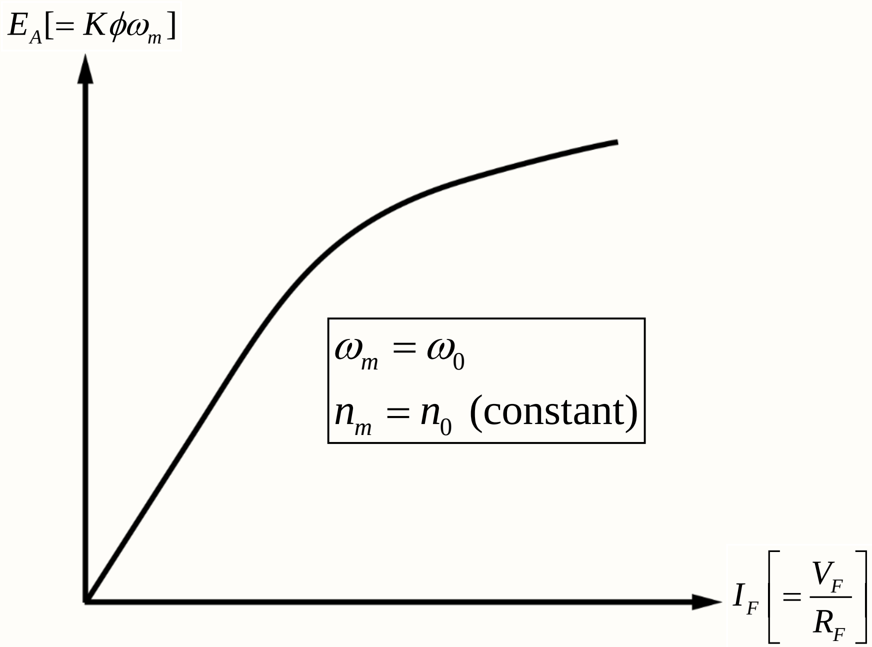
A simplified equivalent circuit of a DC motor.

- Internal generated voltage, $E_A = K\phi\omega_m$
- Induced torque, $T_{ind} = K\phi I_A$
- The induced torque T_{ind} is also referred to as the developed torque T_{dev}.

The Magnetisation Curve of a DC Machine



Magnetisation curve of a ferromagnetic material.



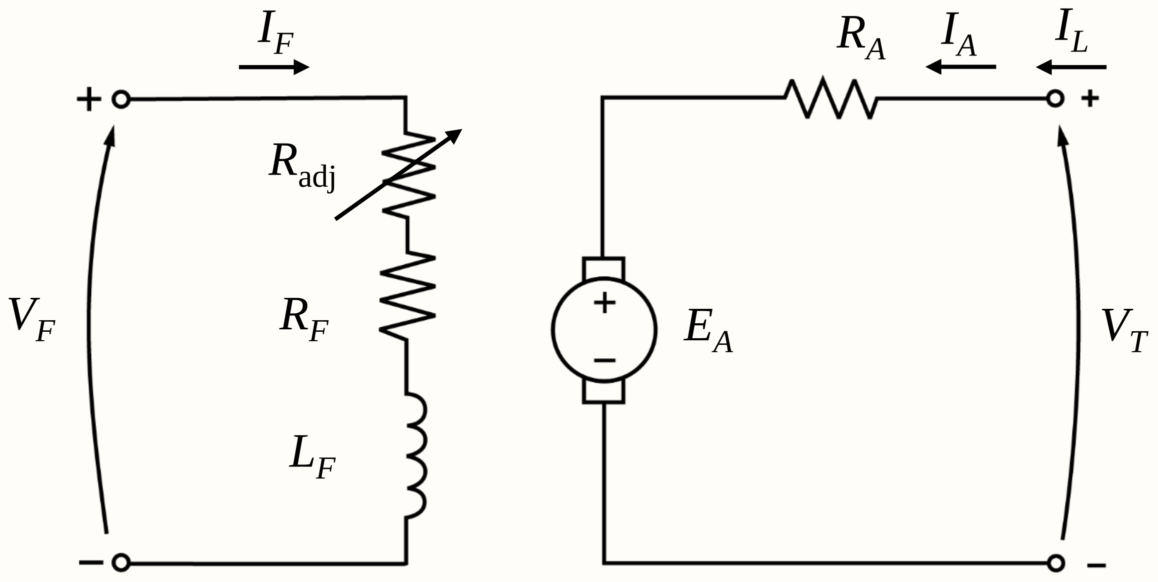
Magnetisation curve expressed as E_A vs I_F for fixed rotor speed ω_o.

- Since I_F is proportional to mmf and E_A is proportional to ϕ, we can represent the magnetisation curve as E_A versus I_F for a given speed ω_o.

Separately Excited DC Motor

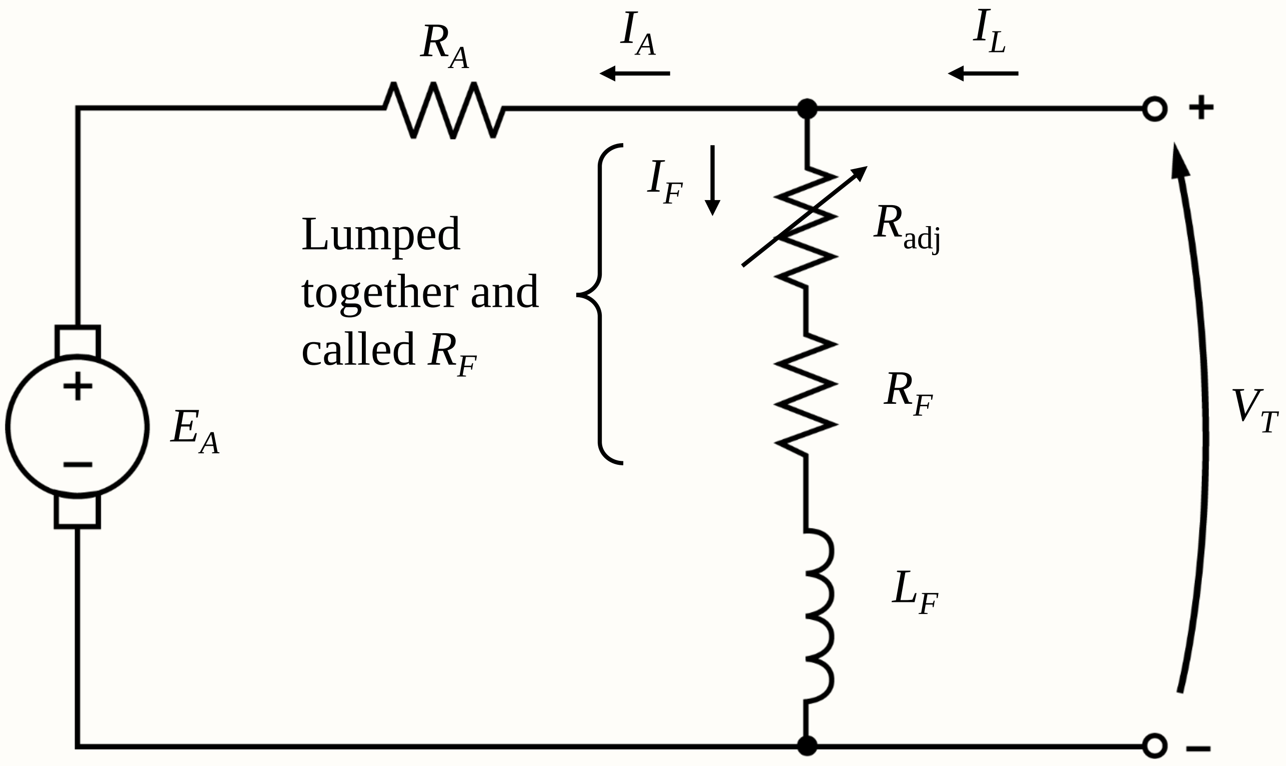
$$E_A = K\phi\omega_m,\ T_{ind} = K\phi I_A$$

$$I_F = \frac{V_F}{R_F},\ V_T = E_A + R_A I_A,\ I_L = I_A$$



Equivalent circuit.

Shunt DC Motor



Equivalent circuit.

$$I_A = I_L - I_F$$

$$I_F = \frac{V_T}{R_F}$$

$$V_T = E_A + R_A I_A$$

$$E_A = K\phi\omega_m$$

$$T_{ind} = K\phi I_A$$

Shunt DC Motor

- Separately excited DC motor – the field circuit is supplied from a separate constant-voltage power supply.
- Shunt DC motor – the field circuit gets its power supply directly across the armature terminals of the motor.
- When the supply voltage is constant, there is no practical difference in behaviour between the two machines.
- Unless otherwise stated, whenever the behaviour of a shunt motor is described, the separately excited motor is included too.

The Terminal Characteristics of a Shunt DC Motor

- It is a plot of its output torque versus speed.

  From $E_A = K\phi\omega_m$ and $I_A = \dfrac{T_{ind}}{K\phi}$,

  and then substituting into $V_T = E_A + R_A I_A$

  $$\Rightarrow \omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} T_{ind}$$

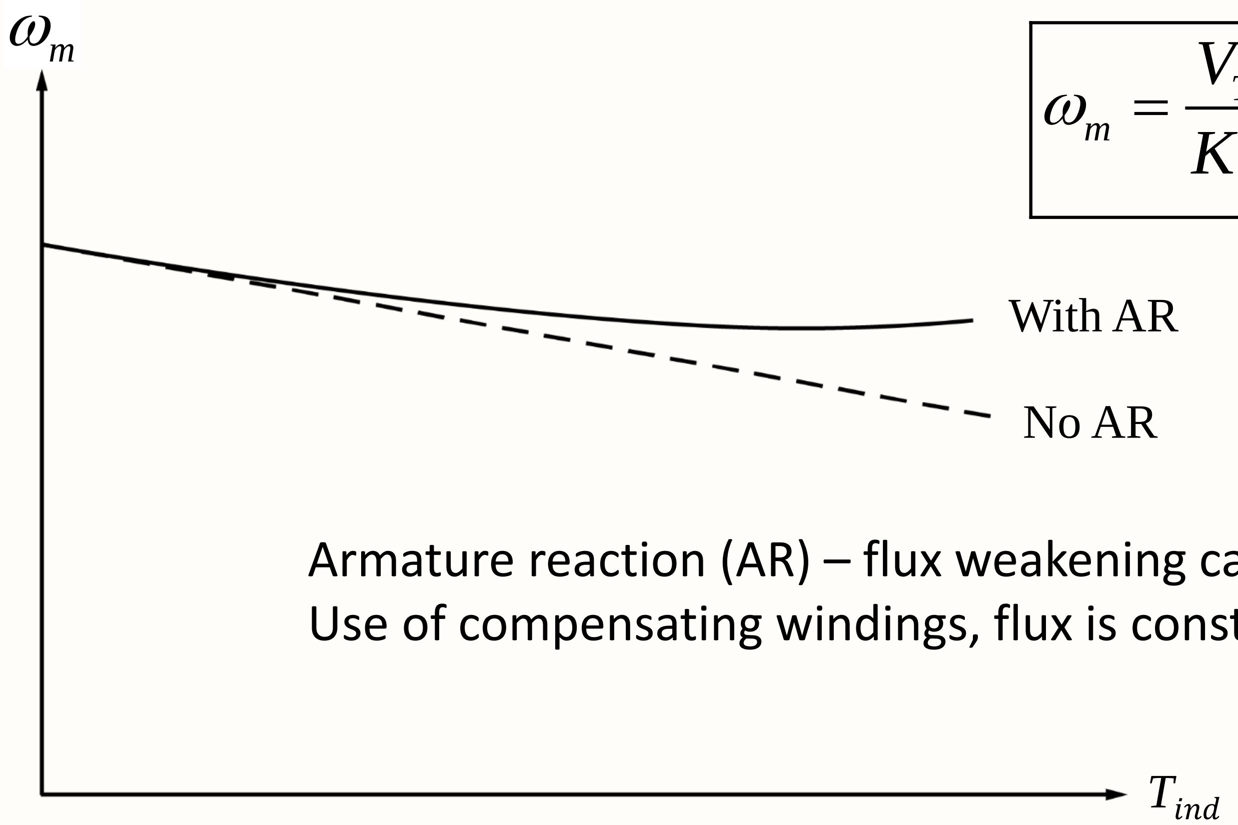
  This equation is just a straight line with a negative slope.

The Terminal Characteristics of a Shunt DC Motor

- How does a shunt DC motor respond to a load?
  - Suppose the load on motor shaft ↑.
  - Then $T_{load} > T_{ind}$ in the machine.
  - Motor will start to slow down.
  - When the motor slows down, $E_A = K\phi\omega_m \downarrow$, and so $I_A = (V_T - E_A)/R_A \uparrow$.
  - As $I_A \uparrow$, the induced torque in the motor $T_{ind} \uparrow$.
  - Finally, $T_{load} = T_{ind}$ at a lower ω_m .

The Terminal Characteristics of a Shunt DC Motor

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} T_{ind}$$



Armature reaction (AR) – flux weakening causes speed to increase.
Use of compensating windings, flux is constant.

Torque-speed characteristics of a shunt or separately excited DC motor with no AR, and with AR.

Example 2

A 30-hp, 500-V, 850 r/min DC shunt motor draws a line current of 51.0 A. The resistance of the armature windings is 0.602 ohms and the combined resistance of the compensating and interpoles windings is 0.201 ohm. Its field resistance is 408.5 ohms. Calculate the internal generated voltage.

$$I_F = \frac{V_T}{R_F} = \frac{500}{408.5} = 1.224\,\mathrm{A}$$

$$I_A = I_L - I_F = 49.776\,\mathrm{A}$$

$$V_T = E_A + R_A I_A$$

$$E_A = V_T - I_A(0.602 + 0.201) = 460\,\mathrm{V}$$

Losses and Efficiency of a DC Motor

- Consider the DC shunt motor as an example:

Power input, $P_{in} = V_T I_L$

Power output, $P_{out} = T_{load}\omega_m$

$P_{dev} = E_A I_A$

$P_{out} = P_{dev} - P_{rot}$

where P_{rot} are the rotational losses representing the core and mechanical losses lumped together.

Efficiency, $\eta = \dfrac{P_{out}}{P_{in}} \times 100\%$

Note: For separately excited motor, $P_{in} = V_T I_L + V_F I_F$

Determination of DC Machine Parameters

It is imperative to have access to the magnetisation curve as well as the machine parameters to analyse the performance of the machine. Run the machine with a separately excited connection and carry out the following tests:

❖ Open-circuit test – magnetisation curve

- Run the DC machine as a generator at a certain speed ω_m and its terminal voltage V_T is measured as the field current I_F is varied.
- Since armature circuit is open-circuited, $E_A = V_T$.
- As I_F is varied, the generated emf E_A also varies.
- Plot E_A versus I_F to obtain the magnetisation curve.

Determination of DC Machine Parameters

- Blocked-rotor test – armature and field resistances
  - The rotor is locked to inhibit rotation so that $\omega_m = 0$ and thus, $E_A = 0$.
  - Apply separate DC supplies V_T and V_F to the armature and field circuits, respectively.
  - Adjust till corresponding rated I_A and I_F flow.
  - Then $R_A = \dfrac{V_T}{I_A}$ and $R_F = \dfrac{V_F}{I_F}$

Determination of DC Machine Parameters

❖ No load test – rotational losses

- The machine is allowed to run freely at no load, without any mechanical load attached to it.
- Both V_T and I_A are measured.
- There is no output power to the machine at no-load and $E_A = V_T - I_A R_A$ where R_A is found from the blocked-rotor test.
- Then $P_{rot} = P_{dev} = E_A I_A$ can be found.

Example 3

A shunt motor rotating at 1500 rpm is fed by a 120-V source. The line current is 51 A and the field resistance is 120 ohms. If the armature resistance is 0.1 ohm, calculate:

a) The current in the armature.

b) The internal generated voltage.

c) Mechanical power developed by motor.

d) Motor efficiency, if the rotational losses are negligible.

(Solutions →)

Example 3 – Solutions

a) $$I_F = \frac{V_T}{R_F} = \frac{120}{120} = 1 \text{ A}$$

$$I_A = I_L - I_F = 50 \text{ A}$$

b) $$E_A = 120 - 50(0.1) = 115 \text{ V}$$

c) $$P_{dev} = E_A I_A = 5750 \text{ W}$$

d) $$P_{out} = P_{dev} - (P_{rot} = 0) = 5750 \text{ W}$$

$$\text{Eff} = \frac{P_{out}}{(P_{in} = 120\ (51))}(100\%) = 93.95\%$$

Example 4

A 240-V DC shunt motor has rotational losses amounting to 790 W. The field resistance is 24 ohms and the armature resistance is 0.1 ohm. If the line current is 100 A, calculate the motor efficiency.

$$P_{in} = V_T I_L = 240(100) = 24 \text{ kW}$$

$$I_F = \frac{V_T}{R_F} = \frac{240}{24} = 10 \text{ A}; \; I_A = I_L - I_F = 90 \text{ A}$$

$$E_A = 240 - 90(0.1) = 231 \text{ V}$$

$$P_{dev} = E_A I_A = 20790 \text{ W}; \; P_{out} = P_{dev} - P_{rot} = 20 \text{ kW}$$

$$\text{Eff} = \frac{P_{out}}{P_{in}}(100\%) = 83.33\%$$

Example 5

A 50-hp, 250-V DC shunt motor has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 ohms. Its total field circuit resistance is 50 ohms. Assume that the no-load speed is 1200 r/min and, at no load, the armature current is negligible, so that $E_A \simeq V_T = 250\ \text{V}$. There are 1200-turns per pole on the field winding.

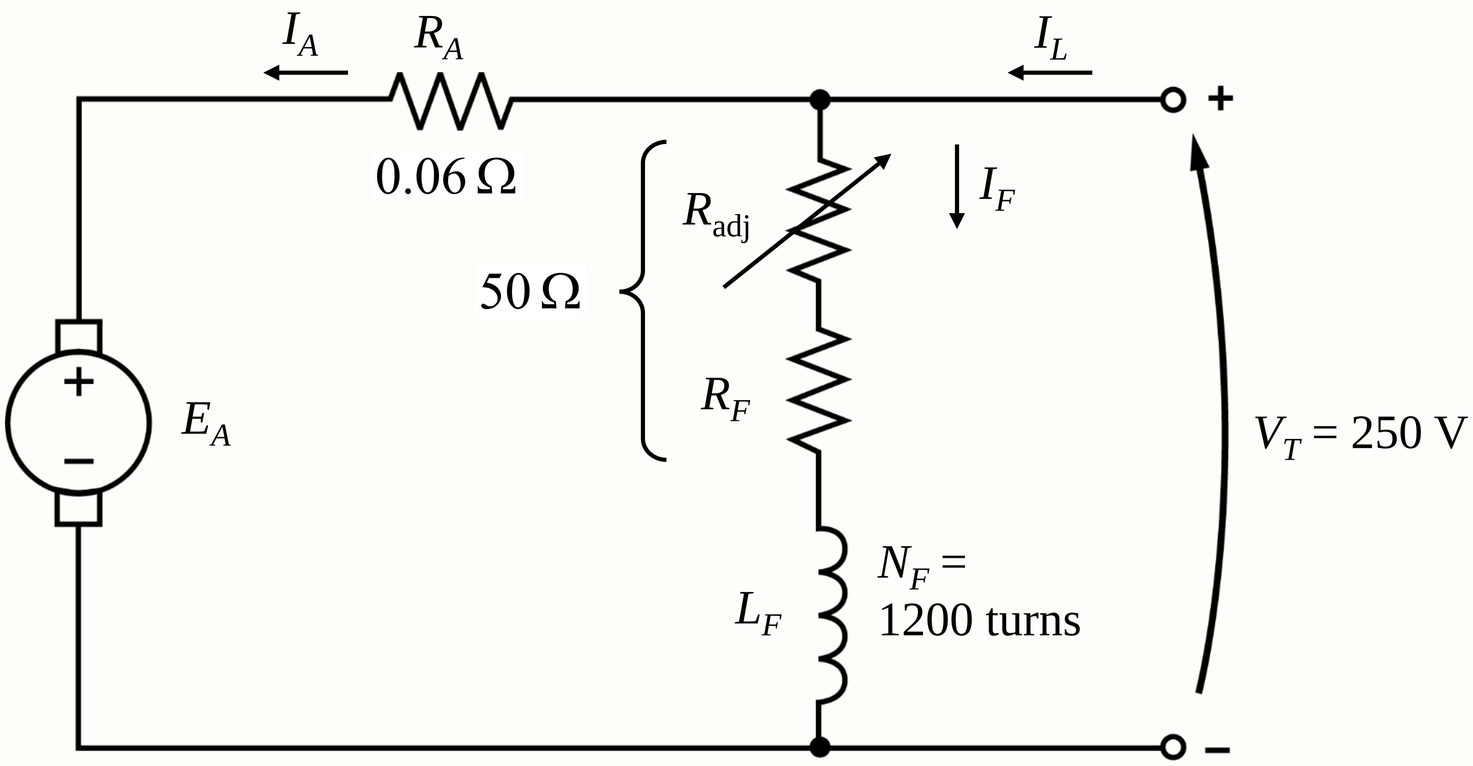
a) Find the speed of this motor when its input current is 100A.

b) Find the speed of this motor when its input current is 200A.

c) Find the speed of this motor when its input current is 300A.

Example 5

d) Plot the torque-speed characteristics of this motor.



(Solutions →)

Example 5 – Solutions

Since the field current is constant, and since there are no armature reaction, the flux is constant.

From $E_A = K\phi\omega_m = \hat{K}\phi n_m,\ n_m$ in rev/min

$$\Rightarrow \frac{E_{A2}}{E_{A1}} = \frac{\hat{K}\phi n_{m2}}{\hat{K}\phi n_{m1}} = \frac{n_{m2}}{n_{m1}}$$

a) At no load, $E_{A1} \simeq V_T = 250\ \text{V},\ n_{m1} = 1200\ \text{rev/min}$

If $I_L = 100\ \text{A},\ I_A = I_L - I_F = 95\ \text{A}$

$$E_{A2} = V_T - I_A R_A = 244.3\ \text{V}$$

$$\Rightarrow n_{m2} = \frac{E_{A2}}{E_{A1}}(1200) = 1173\ \text{rev/min}$$

Example 5 – Solutions

b) If $I_L = 200 \text{ A}$, $I_A = I_L - I_F = 195 \text{ A}$

$$E_{A3} = V_T - I_A R_A = 238.3 \text{ V},$$

$$\Rightarrow n_{m3} = \frac{E_{A3}}{E_{A1}}(1200) = 1144 \text{ rev/min}$$

c) If $I_L = 300 \text{ A}$, $I_A = I_L - I_F = 295 \text{ A}$

$$E_{A4} = V_T - I_A R_A = 232.3 \text{ V},$$

$$\Rightarrow n_{m4} = \frac{E_{A4}}{E_{A1}}(1200) = 1115 \text{ rev/min}$$

Example 5 – Solutions

d) The electrical power that is converted to mechanical power,

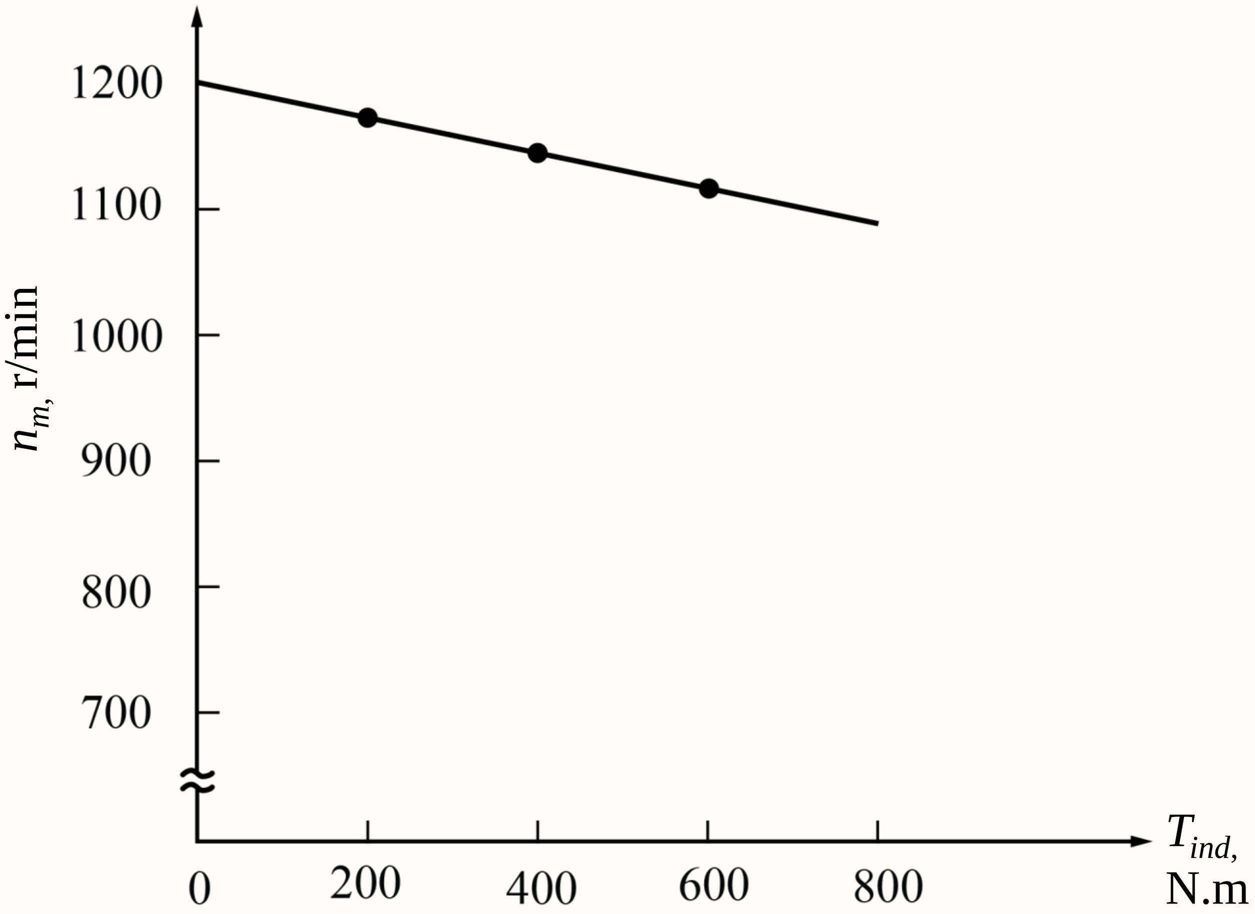
$$P_{conv} = P_{dev} = E_A I_A = T_{ind}\omega_m \Rightarrow T_{ind} = \frac{E_A I_A}{\omega_m}$$

The torque developed or torque induced when $I_L = 100$ A is

$$T_{ind} = \frac{244.3(95)}{(1173)(2\pi / 60)} = 189 \text{ N.m.}$$

Similarly, the induced torque when $I_L = 200$ A is 388.2 N.m, and when $I_L = 300$ A is 586.9 N.m.

Determination of DC Machine Parameters



Torque-speed characteristics of the motor in the example.

Summary

In this lecture, you have learnt:

- The equivalent circuit of a DC motor.
- The torque-speed characteristics of a shunt or separately excited DC motor.
- The power flows and analysis of the motor from the equivalent circuit.

Lecture 9

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

Associate Professor Lee Peng Hin
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Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Analyse the DC motor using the magnetisation curve, taking into account the armature reaction effects.
- ❖ Describe the various methods of speed control and the speed control limits of a DC motor.
- ❖ Explain the techniques that may be used for starting a DC motor.

Nonlinear Analysis of a Shunt DC Motor

- The flux, and hence, the internal generated voltage of a DC machine is a non-linear function of its mmf.
- Anything that changes mmf will have a nonlinear effect on the internal generated voltage.
- Hence, an accurate calculation of the motor torque, the internal generated voltage and speed of the motor for various operating conditions require the use of the magnetisation curve.

Nonlinear Analysis of a Shunt DC Motor

- The two principal contributions to the mmf are its field current and armature reaction, if present.

- The magnetisation curve of the machine is used to determine the generated voltage for a given mmf or equivalently, the field current.

- If the machine has armature reaction, the total net mmf:

$$mmf_{net} = N_F I_F - mmf_{AR}$$

Letting, $mmf_{net} = N_F I_F^*$

where I_F^* = equivalent field current that will produce the same output voltage as the combination of all the mmfs in the machine. Then $I_F^* = I_F - \dfrac{mmf_{AR}}{N_F}$.

Nonlinear Analysis of a Shunt DC Motor

- The resulting E_A can then be determined by locating I_F^* on the magnetisation curve.

- The magnetisation curve for a machine is drawn for a particular speed, usually the rated speed. How can the effects of a given field current be determined, if the motor is turning at other than rated speed?

- For a given effective field current, the flux is fixed. Then,

$$E_A = \hat{K}\phi n_m \Rightarrow \frac{E_A}{E_{A0}} = \frac{n_m}{n_0}$$

where E_{A0} and n_0 represent the reference values of voltages and speed respectively.
n_m is the actual speed that the motor is turning.

Example 1

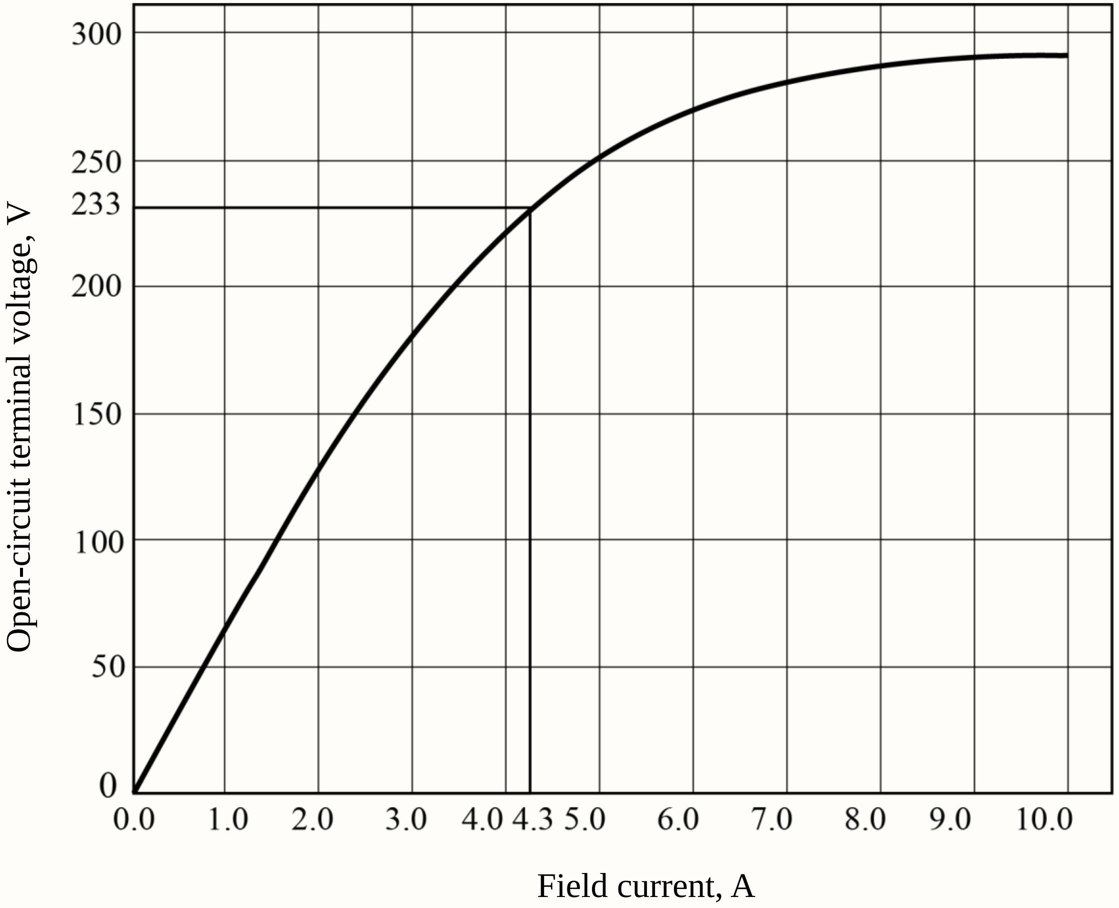
A 50-hp, 250-V DC shunt motor without compensating windings has an armature resistance (including the brushes and interpoles) of 0.06 ohms. Its field circuit has a total resistance of 50 ohms. Assume that the no-load speed is 1200 r/min. There are 1200 turns per pole on the field winding, and the armature reaction produces a demagnetising mmf of 840 A-turns when the input current is 200 A. The magnetisation curve of this machine is as shown.

a) Find the speed of the motor when its input current is 200 A.

b) How does its speed compare to that of the motor in the previous example at an input current of 200 A?

(Solutions →)

Example 1



The magnetisation curve of a typical 250-V DC motor, at 1200 r/min.

Example 1 – Solutions

a) If $I_L = 200$ A, $I_A = I_L - I_F = 195$ A, $E_A = V_T - I_A R_A = 238.3$ V.

At $I_L = 200$ A, the demagnetising mmf due to armature reaction is 840 A-turns.

The effective field current: $I_F^* = I_F - \dfrac{mmf_{AR}}{N_F} = 5 - \dfrac{840}{1200} = 4.3 \text{ A}$

From the magnetisation curve, it produces an E_{A0} of 233 V at n_0 of 1200 r/min.

The actual generated voltage is 238.3 V. Therefore,

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0} \Rightarrow \frac{238.3}{233} = \frac{n_m}{1200} \Rightarrow n_m = 1227 \text{ r/min}$$

Example 1 – Solutions

b) The motor speed at an input current of 200 A in the previous example was 1144 r/min. In this example, the motor's speed is 1227 r/min. Notice that the speed of the motor with armature reaction is higher than the speed of the motor with no armature reaction. This relative increase in speed is due to the flux weakening effect in the machine with armature reaction.

Speed Control of a Shunt DC Motor

❖ The torque-speed equation is:

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} T_{ind}$$

❖ The speed of a DC shunt motor can be controlled by:

- Adjusting the field resistance (and thus the field flux).
- Adjusting the terminal voltage applied to the armature.
- Inserting a resistor in series with the armature circuit.

Speed Control of a Shunt DC Motor

- Adjusting the field resistance
  - Increasing R_F leads to,

$$I_F = \frac{V_T}{R_F} \downarrow \Rightarrow \phi \downarrow \Rightarrow E_A = K\phi\omega_m \downarrow \Rightarrow I_A = \frac{V_T - E_A}{R_A} \uparrow$$

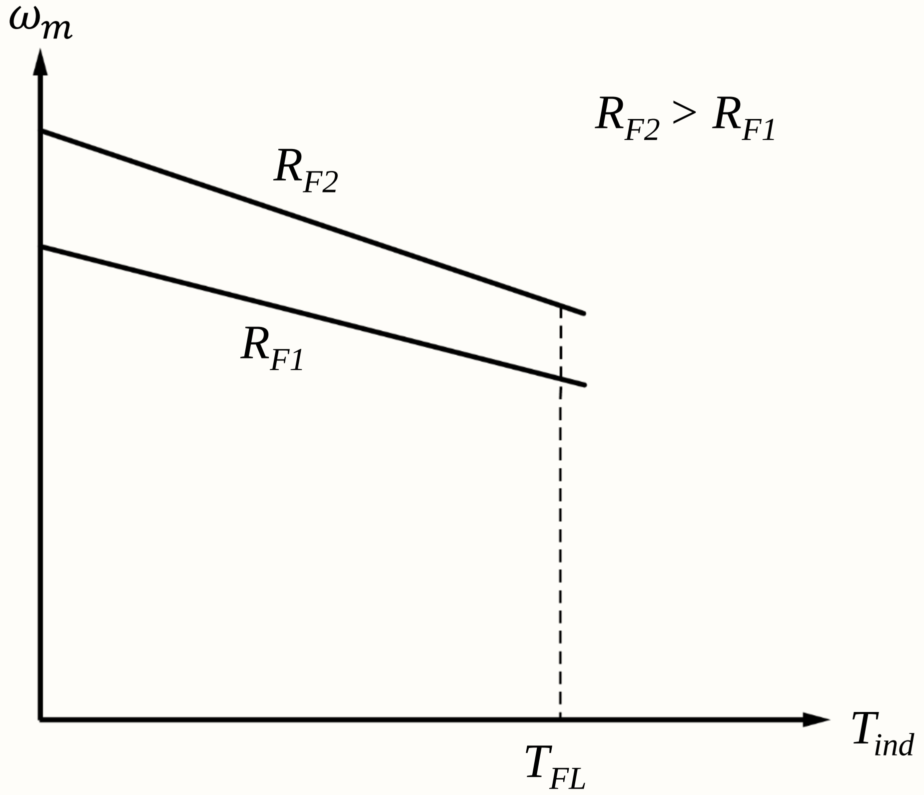
$\Rightarrow T_{ind} = K\left(\phi \downarrow I_A \Uparrow\right)$ increases, with the change in I_A dominant over the change in flux.

$$\Rightarrow T_{ind} > T_{load} \Rightarrow \omega_m \uparrow \Rightarrow E_A = K\phi\omega_m \uparrow \text{again}$$

$\Rightarrow I_A \downarrow \Rightarrow T_{ind} \downarrow$ until $T_{ind} = T_{load}$ at a higher steady-state speed than originally.

Speed Control of a Shunt DC Motor

- The effect of field resistance speed control on a shunt motor's torque-speed characteristics.

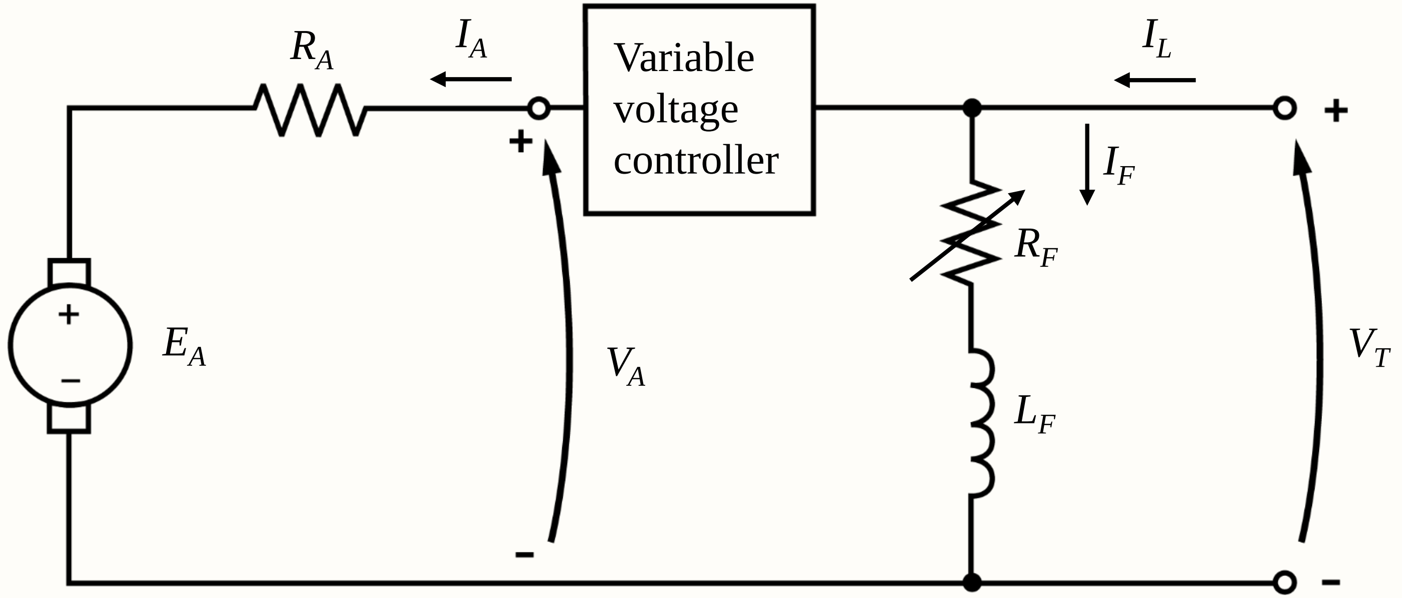


Over the normal motor's operating range.

Speed Control of a Shunt DC Motor

❖ Adjusting the armature terminal voltage

- Change the voltage applied to the armature, without changing the voltage applied to the field.
- The motor must be separately excited to use armature voltage control.



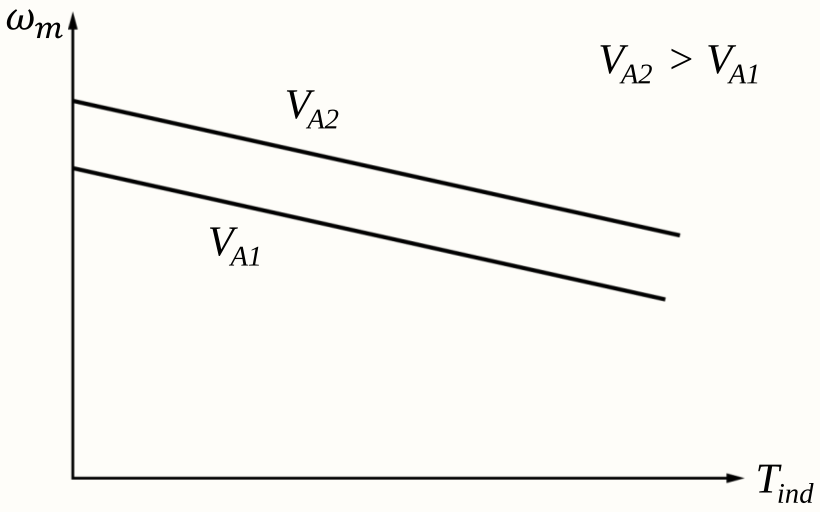
V_T is constant
V_A is variable

Speed Control of a Shunt DC Motor

- Increasing V_A leads to,

$$I_A = \frac{V_A - E_A}{R_A} \uparrow \Rightarrow T_{ind} = K\phi I_A \uparrow \Rightarrow T_{ind} > T_{load} \Rightarrow \omega_m \uparrow \Rightarrow E_A = K\phi\omega_m \uparrow$$

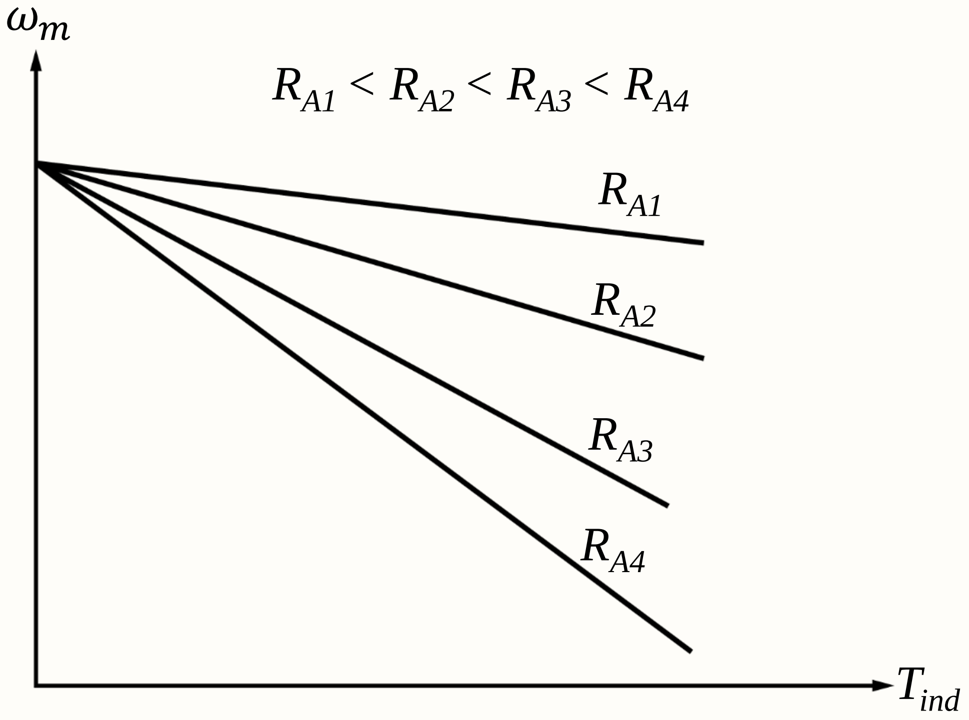
$$\Rightarrow I_A \downarrow \Rightarrow T_{ind} \downarrow \text{ until } T_{ind} = T_{load} \text{ at a higher speed } \omega_m$$



The no-load speed is shifted, but the slope of the curve remains constant.

Speed Control of a Shunt DC Motor

- Inserting a resistor in series with the armature circuit (armature resistance speed control).
  - This method will incur large losses in the inserted resistor.



$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2} T_{ind}$$

Example 2

A 25-hp, 240-V shunt motor operating at 850 r/min draws a line current of 91 A when operating at rated conditions. A 2.14 ohms resistor inserted in series with the armature causes the speed to drop to 634 r/min. The respective armature-circuit resistance and field-circuit resistance are 0.221 ohm and 120 ohms, respectively. Determine the new armature current.

(Solutions →)

Example 2 – Solutions

Before the addition of the resistor,

$$n_{m_1} = 850 \text{ rpm},$$

$$E_{A_1} = 240 - \left(91 - 2\right)(0.221) = 220.331 \text{ V}$$

After the addition of the resistor,

$$n_{m_2} = 634 \text{ rpm}, I_{F_1} = I_{F_2} \Rightarrow \text{ flux is constant.}$$

$$\frac{E_{A_1}}{E_{A_2}} = \frac{n_{m_1}}{n_{m_2}} \Rightarrow \frac{220.331}{E_{A_2}} = \frac{850}{634}$$

$$\Rightarrow E_{A_2} = 164.3 = 240 - I_{A_2}(0.221 + 2.14)$$

$$\Rightarrow I_{A_2} = 32.06 \text{ A}$$

Speed Control Limits

❖ Field resistance control

- The lower the field current in shunt (or separately excited) motors, the faster they turn and vice versa.
- An increase in field current causes a decrease in speed, and hence, there is a minimum achievable speed by field circuit control. (Limited by the maximum permissible field current flowing through it, to avoid burning up the field windings.)
- It is used to control for speeds above base speed or rated speed.

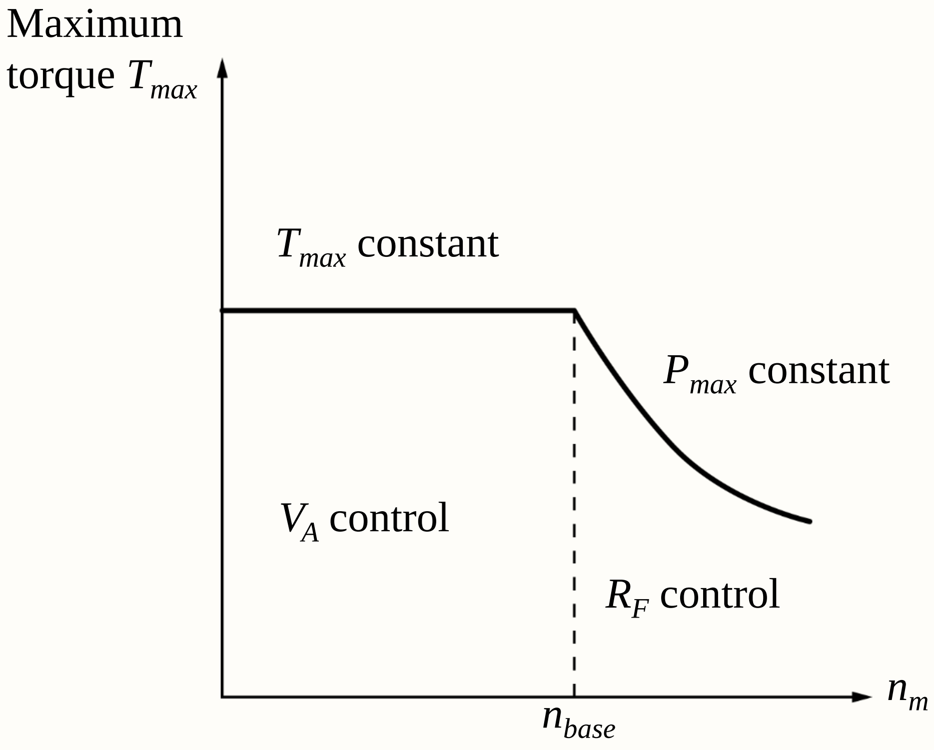
Speed Control Limits

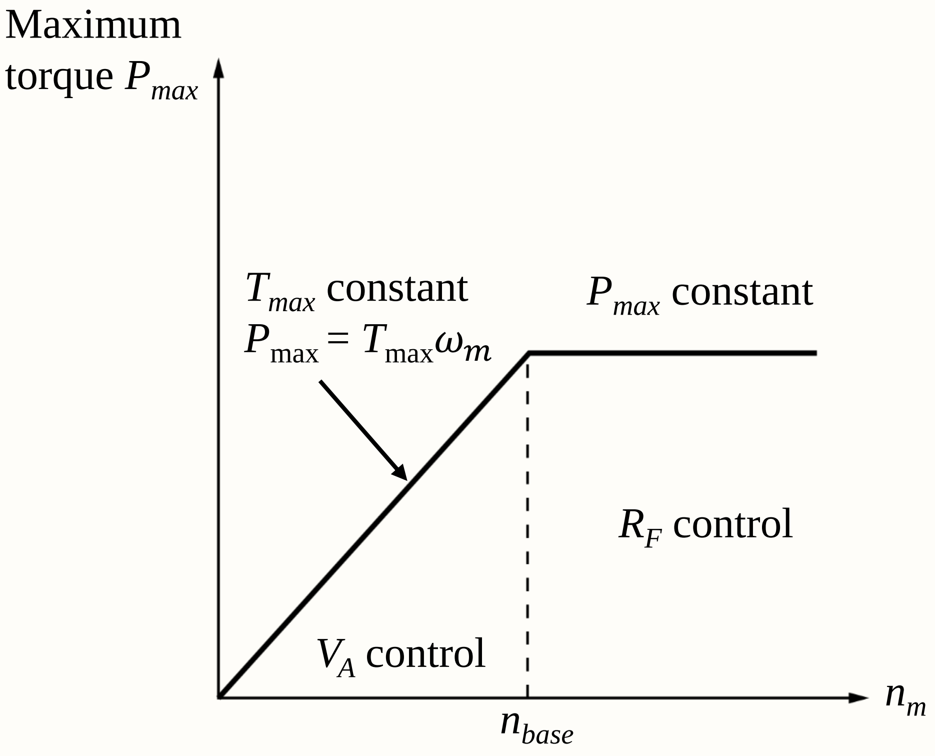
❖ Armature voltage control

- The lower the armature voltage (in separately excited motors), the slower they turn and vice versa.
- An increase in armature voltage causes an increase in speed, and hence, there is a maximum achievable speed by armature voltage control. (Limited by the maximum permissible value of the armature voltage.)
- It is used to control for speeds below base speed or rated speed. To control speeds above base speed would require excessive armature voltage, possibly damaging the armature circuit.

Speed Control Limits

- The two methods are complementary. Combining the two can get a range of speed variations of up to 40 to 1. Shunt and separately-excited DC motors have excellent speed control characteristics.





Armature Voltage Control

- The flux is constant at rated value as the terminal voltage is adjusted. The maximum torque achievable is independent of the speed of rotation and is given by:

$$T_{\max} = K\phi_{rated} I_{A,\max}$$

- T_{max} is a constant since ϕ_{rated} and the max allowable $I_{A,max}$, without damaging the armature windings, are constants. Conversely, the maximum developed power out of the motor under armature voltage control below rated speed

$$P_{\max} = T_{\max}\omega_m$$

is directly proportional to the operating speed.

Field Resistance Control

- Speed increase is caused by decrease in flux. For speeds beyond the base speeds, the terminal voltage is kept constant at its maximum value, while the field current is adjusted.

- Since $E_A = K\phi\omega_m$ and is relatively constant due to fixed V_T, then ϕ will be directly proportional to $\left(\frac{1}{\omega_m}\right)$. Thus, the maximum torque $T_{max} = K\phi I_{A,max}$ also becomes directly proportional to $\left(\frac{1}{\omega_m}\right)$.

  On the other hand, $P_{max} = T_{max}\omega_m$ remains constant.

Field Resistance Control

- Operation below rated or base speed is known as the constant torque region because the max induced torque is constant.

- Operation above rated speed is known as constant power region.

DC Motor Problems on Starting

- The motor must be protected from physical damage during the starting period.
- For example, a 50-hp, 250-V motor DC motor has an armature resistance of 0.06 ohms. The full-load current is less than 200 A. At starting conditions, the motor is not turning and thus there is no internal voltage generated. The current at starting is:

$$I_A = \frac{V_T - E_A}{R_A} = \frac{250 - 0}{0.06} = 4167\ \text{A}$$

which is more than 20 times the motor's rated current, and this can severely damage the motor.

DC Motor Problems on Starting

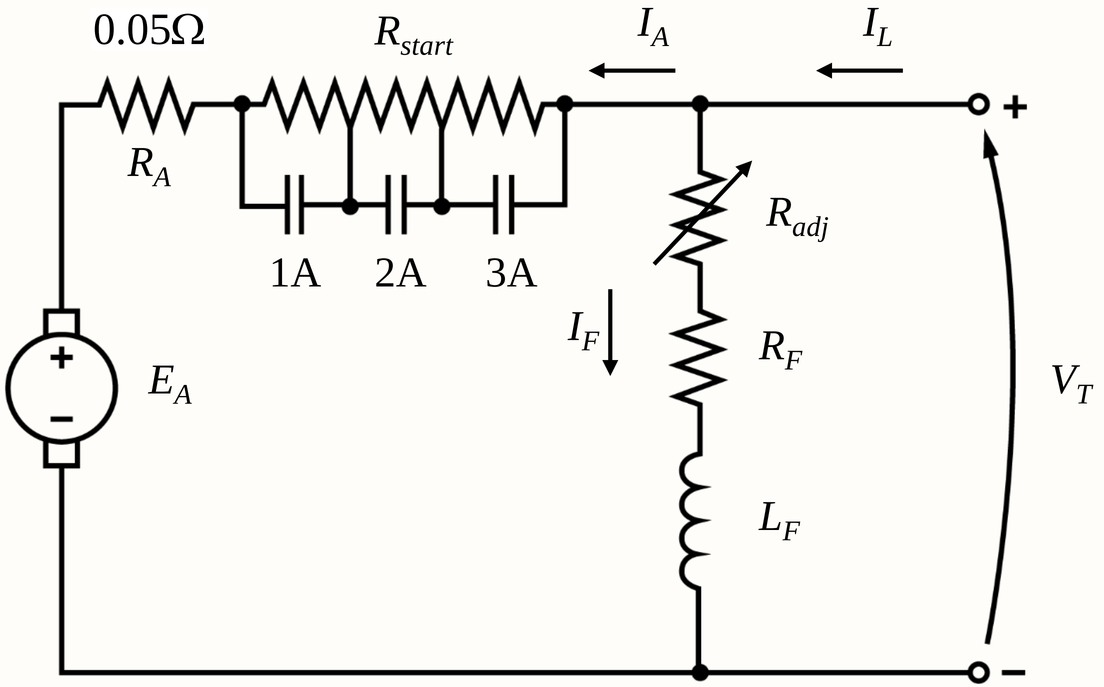
- A solution is to insert a starting resistor in series with the armature to limit the current flow, until the internal generated voltage can be build up to do the limiting.

- The resistor must not be in the circuit permanently, because it will result in excessive losses.

- In practice, a starting resistor is made up of a series of pieces, each of which is removed from the motor circuit in succession as the motor speeds up.

Speed Control Limits

- A shunt motor with a starting resistor in series with its armature.
- Contacts 1A, 2A and 3A short-circuit portions of the starting resistor, when they close.



Summary of DC Motors

You have learnt:

- There are several types of DC motors, differing in the manner in which their field fluxes are derived.

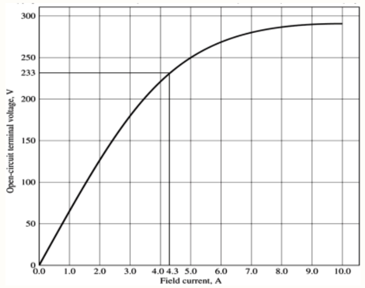
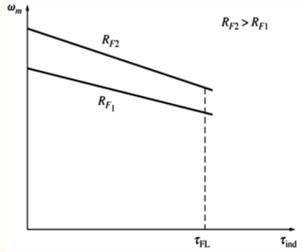
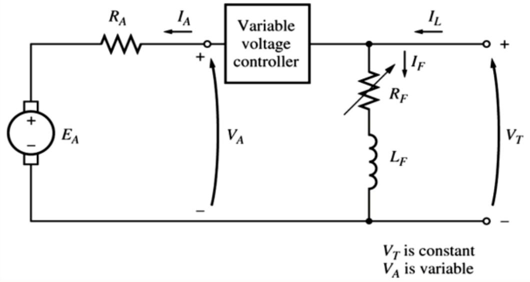
- A shunt or separately excited DC motor has a torque-speed characteristics whose speed drops linearly with increasing speed. Its speed can be controlled by changing its field current, armature voltage, or armature circuit resistance.

- Combinations of armature voltage and field resistance speed controls allow a wide range of speed variations.

Summary of DC Motors

- The nonlinear analysis of a DC motor.
- The applications of the armature voltage and field resistance speed control methods to allow a wide range of speed variations.
- Starting circuits to limit the starting current to a safe level.

References

No.	Slide No.	Image	Reference
1	7		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 477), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
2	12		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 482), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
3	13		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 483), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

References

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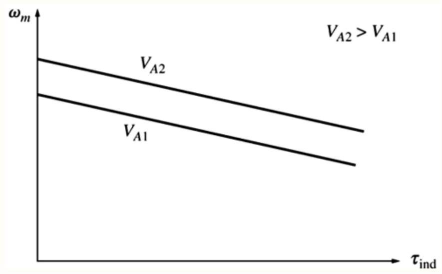
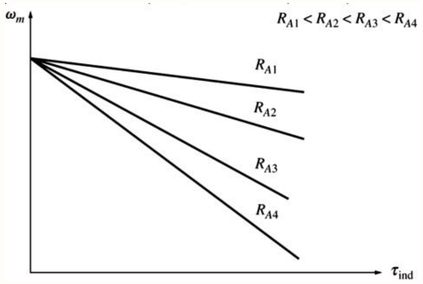
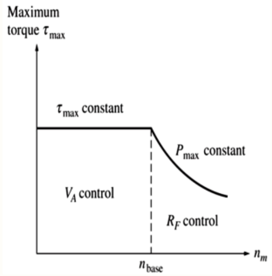
4	14		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 484), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	15		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 485), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
6	20		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 486), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

References

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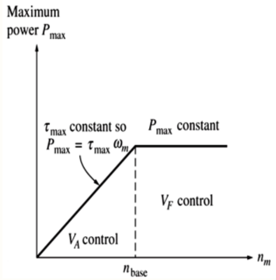
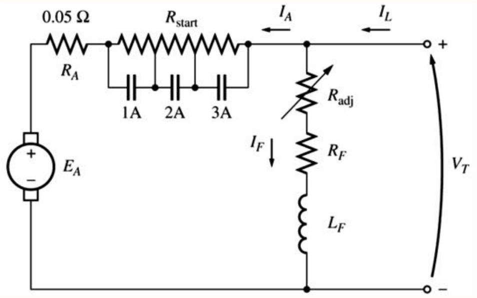
7	20		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 486), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
8	26		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 506), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

Lecture 10

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- ❖ Describe the equivalent circuit of a DC generator.
- ❖ Derive and explain the voltage-current characteristics of a separately excited DC generator.
- ❖ Explain the methods used to control the terminal voltage of a separately excited DC generator.
- ❖ Analyse the separately excited DC generator, using the magnetisation curve and the equivalent circuit.

Introduction to DC Generators

- DC generators are DC machines used as generators.
- There is no real difference between a DC generator or motor, except for the direction of power flow.
- The various types of DC generators available differ in their terminal (voltage-current) characteristics, and therefore, in the applications to which they are suited.

Introduction to DC Generators

- In this course, the following types DC generators will be discussed in detail:
  - Separately excited DC generator – the field flux is derived from a separate power source and independent of the generator itself.
  - Shunt generator – the field flux is derived by connecting the field circuit directly across the terminals of the generator.

Introduction to DC Generators

- DC generators are compared by their voltages, power ratings, efficiencies, and voltage regulations defined as:

$$\text{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

$$V_{nl} = \text{no-load terminal voltage}$$

$$V_{fl} = \text{full-load terminal voltage}$$

- VR is the percentage change in terminal voltage, when the load is removed from the generator.

- DC generators are driven by a source of mechanical power called the prime mover of the generator. It can be a steam engine, diesel engine, or motor.

Example 1

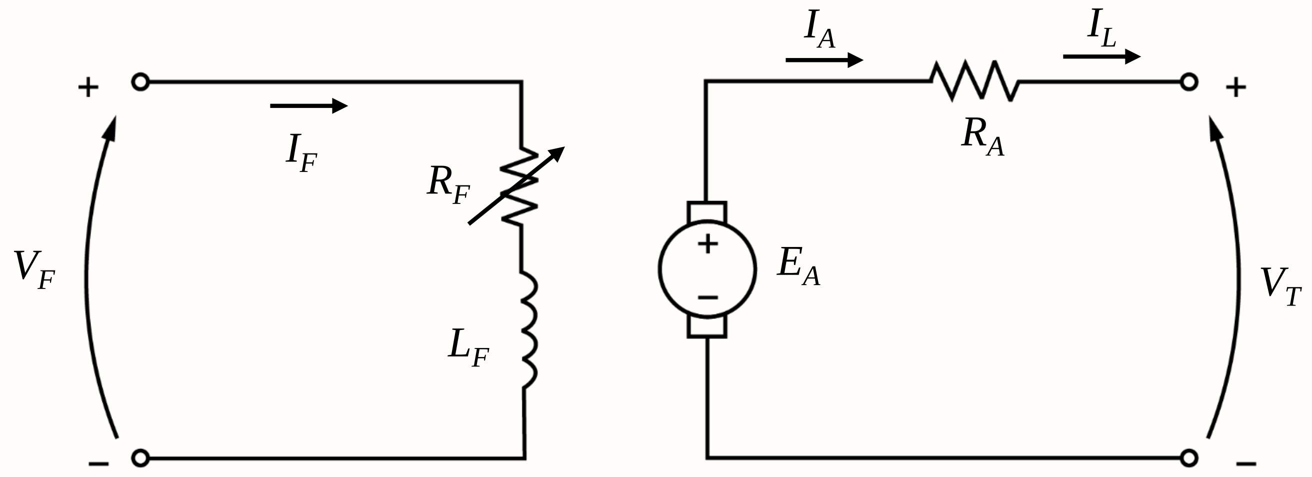
A 100 kW, 1800 r/min DC generator operating at rated load has a terminal voltage of 240 V. If the voltage regulation is 2.3 percent, determine the no-load voltage.

$$\mathrm{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

$$\Rightarrow 0.023 = \frac{V_{nl} - 240}{240}$$

$$\Rightarrow V_{nl} = 245.52\ \mathrm{V}$$

Separately Excited DC Generator



Equivalent circuit of a separately excited generator.

R_F - combination of the resistance of the field windings and the external variable resistor, if any.

I_L - the current flowing in the lines connected to the terminals of the generator.

$$\boxed{I_A = I_L,\ V_T = E_A - I_A R_A,\ I_F = \frac{V_F}{R_F}}$$

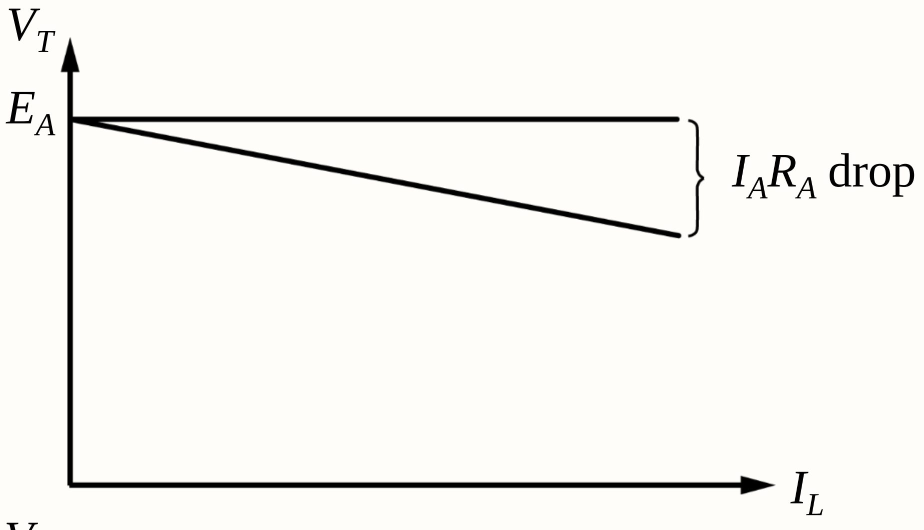
Example 2

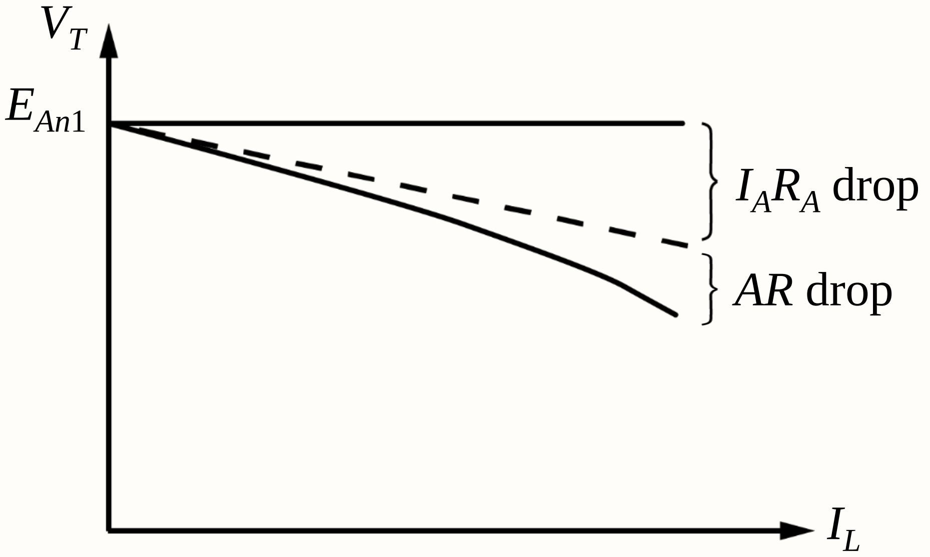
Determine the internal generated voltage, at rated conditions, of a separately excited generator which is rated at 25 kW, 250 V and 1450 r/min and has the following parameters.

$$R_A = 0.1053\Omega,\ R_{\text{interpoles}} = 0.0306\Omega,$$
$$R_{\text{comp windings}} = 0.0141\Omega,\ R_F = 96.3\Omega$$

$$I_A = \frac{P}{V_T} = \frac{25000}{250} = 100\ \text{A}$$
$$E_A = 250 + 100\left(0.1053 + 0.0306 + 0.0141\right) = 265\ \text{V}$$

V-I Characteristics of a Separately Excited DC Generator





- When load supplied by generator ↑

  $\Rightarrow I_L$ (hence I_A) $\uparrow$

  $\Rightarrow I_A R_A \uparrow \Rightarrow V_T \downarrow$

- Armature reaction present (no compensating windings)

  $\Rightarrow$ flux weakening

  $\Rightarrow E_A = K\left(\phi \downarrow\right)\omega_m \downarrow$

  $\Rightarrow \mathrm{V}_T \downarrow$ further

Control of Terminal Voltage of a Separately Excited DC Generator

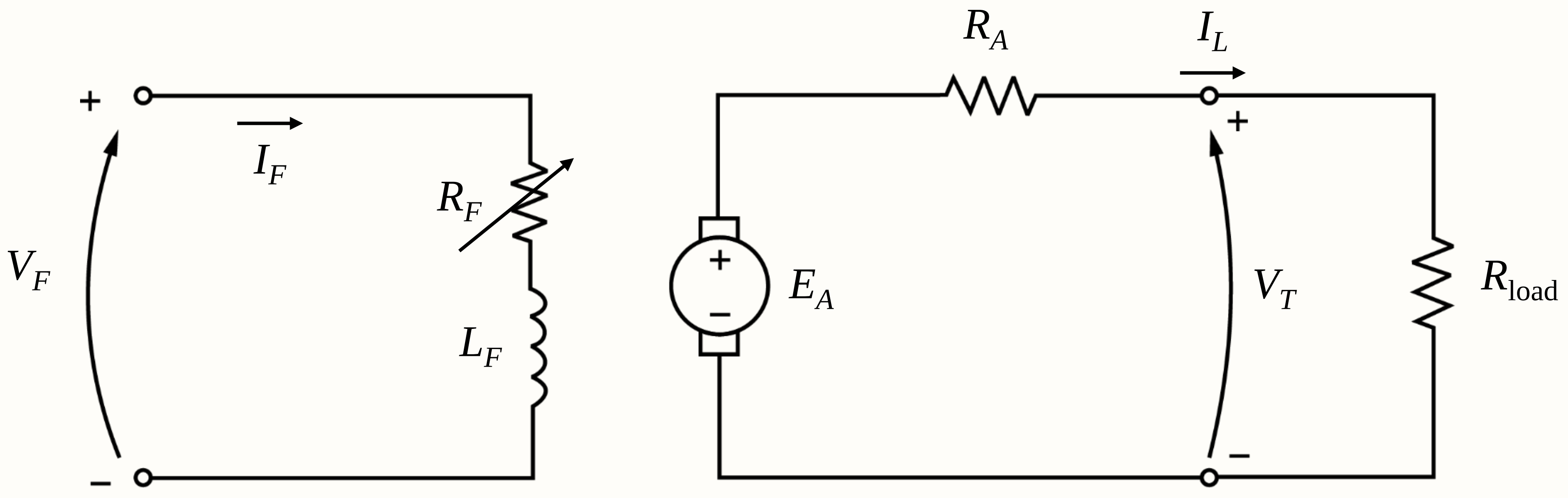
- The terminal voltage can be controlled by changing the internal generated voltage of the machine, as:

$$V_T = E_A - I_A R_A \Rightarrow \text{If } E_A \uparrow \Rightarrow V_T \uparrow$$

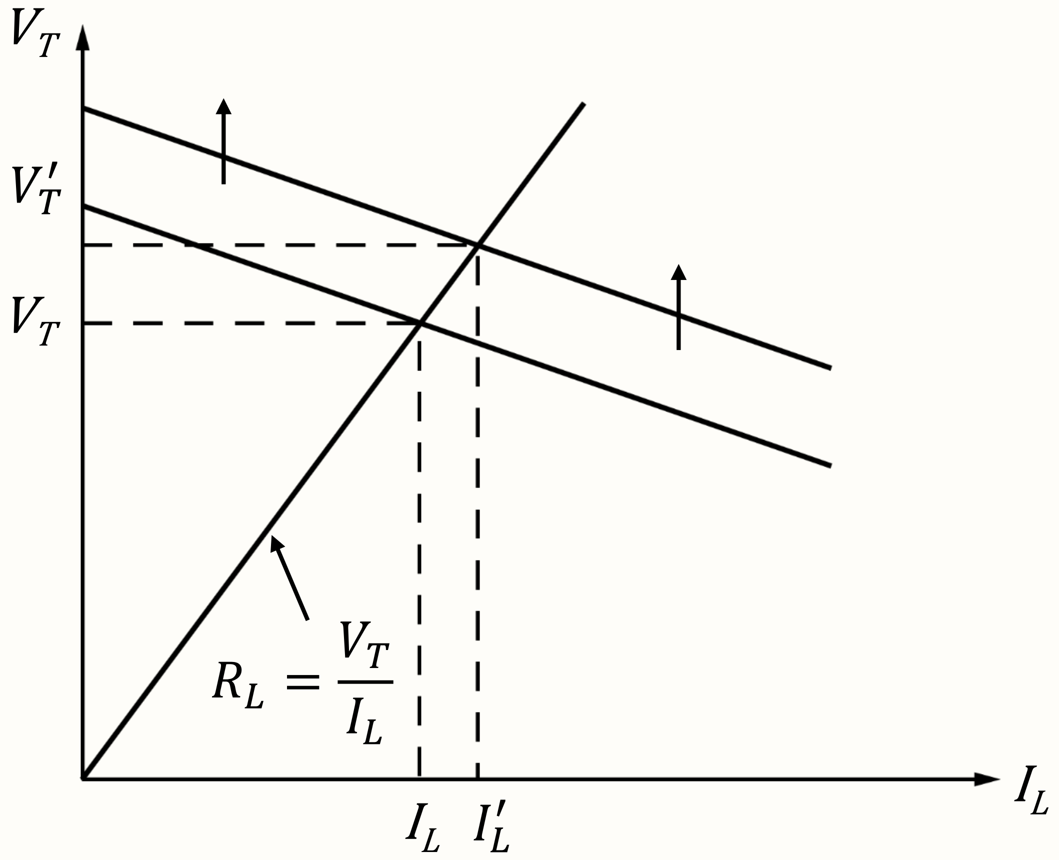
- Since $E_A = K\phi\omega_m$, there are two ways to control E_A:
  - Change ω_m : if $\omega_m \uparrow \Rightarrow E_A \uparrow \Rightarrow V_T \uparrow$
  - Change I_F : if $R_F \downarrow \Rightarrow I_F \uparrow \Rightarrow \phi \uparrow \Rightarrow E_A \uparrow \Rightarrow V_T \uparrow$

- In many applications, the speed range of the prime mover is quite limited, so the terminal voltage is most commonly controlled by changing the field current.

Control of Terminal Voltage of a Separately Excited DC Generator



A separately excited DC generator with a resistive load.



Effect of a decrease in field resistance on the output voltage of the DC generator.

Non-linear Analysis of a Separately Excited DC Generator

- If a machine has armature reaction AR, its flux will be reduced with each increase in load, causing the internal generated voltage to decrease.
- The output voltage of the generator with AR is determined using the magnetisation curve of the generator.
- The total mmf in the generator is the field circuit mmf less the mmf due to AR.

Nonlinear Analysis of a Separately Excited DC Generator

- $mmf_{net} = N_F I_F^* \quad = N_F I_F - mmf_{AR}$

  Then, $I_F^* \quad = I_F - \dfrac{mmf_{AR}}{N_F}$

  where I_F^*= equivalent field current that will produce the same output voltage as the combination of all the mmfs in the machine.

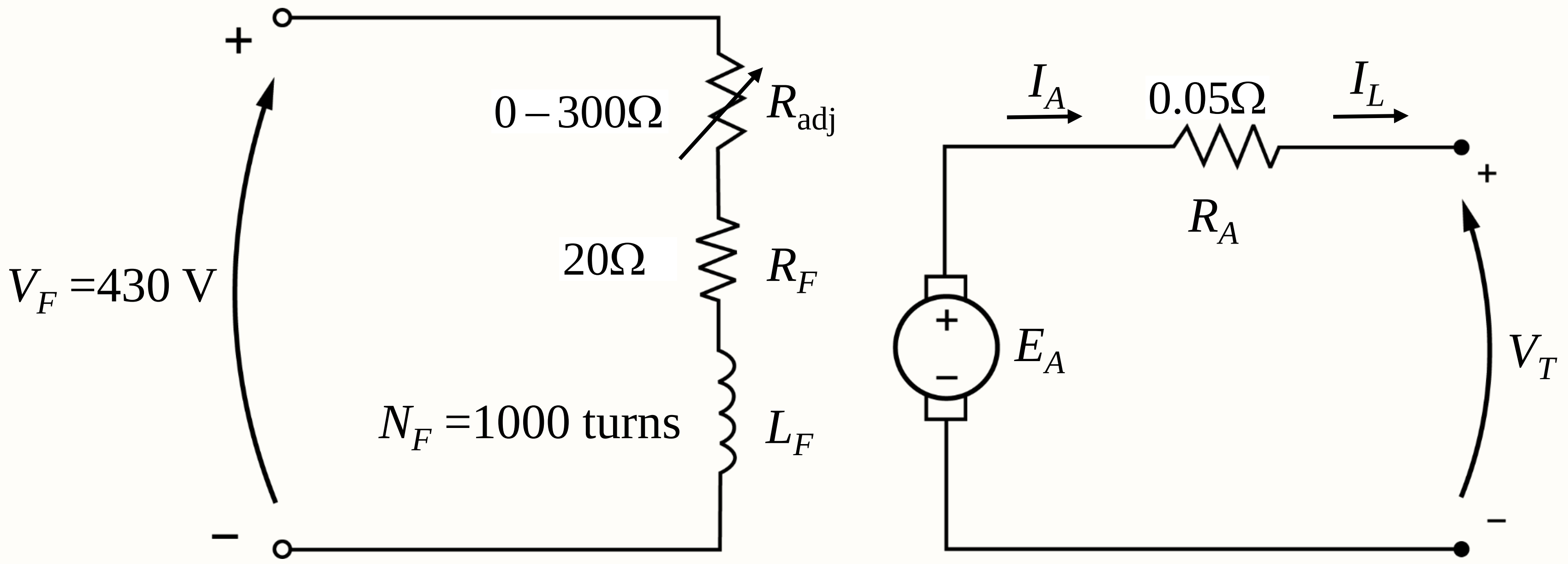
- The resulting voltage E_{A0} can then be determined by locating that equivalent field current on the magnetisation curve.

- The difference between the speed n_0 , at which the magnetisation curve was obtained, and the actual speed n_m of the generator must be taken into account using $\dfrac{E_A}{E_{A0}} = \dfrac{n_m}{n_0}$.

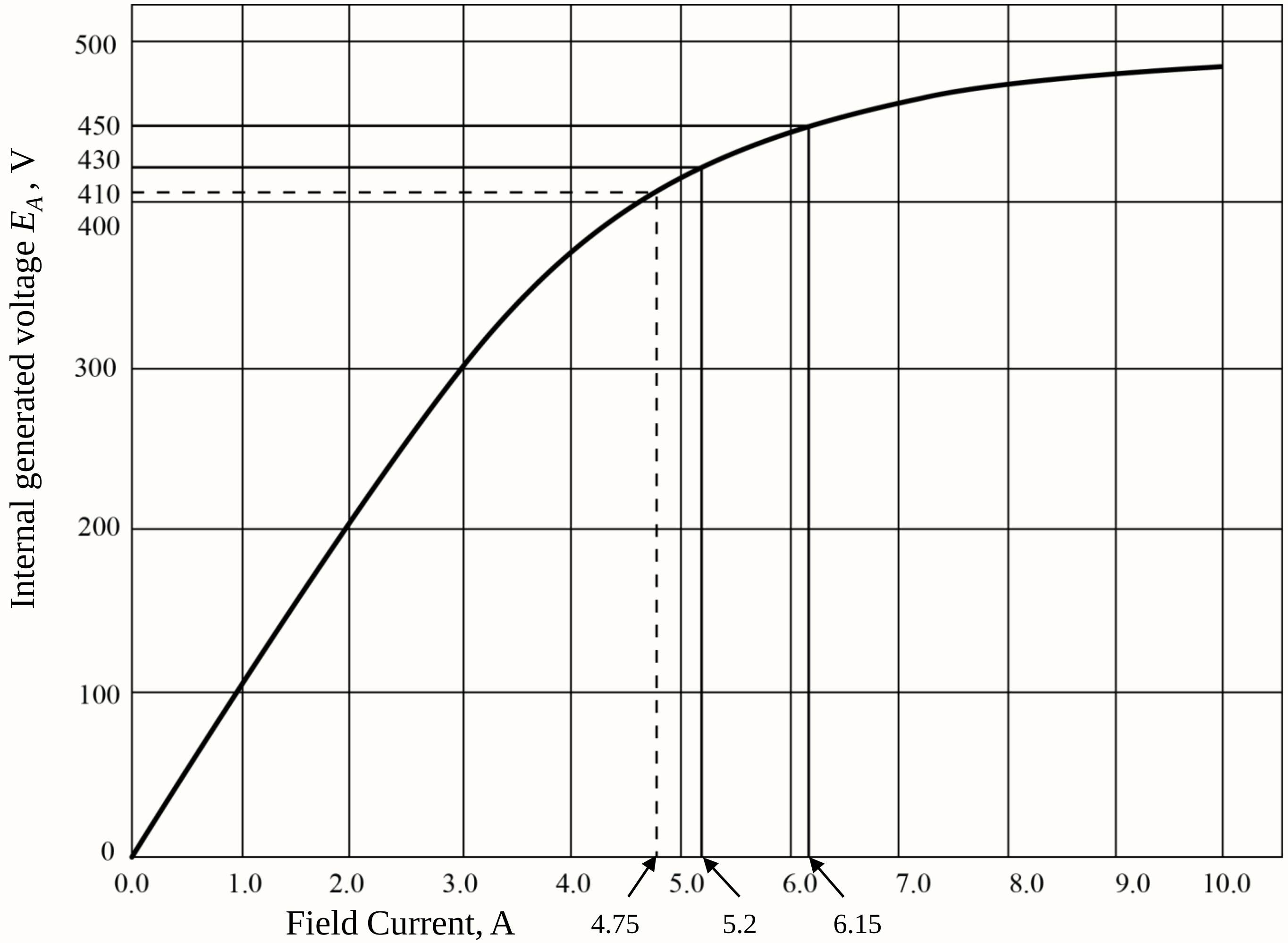
Example 3

- A separately excited DC generator is rated at 172 kW, 430 V, 400 A and 1800 r/min. The magnetisation curve is as shown and the machine characteristics are:

$$R_A = 0.05\Omega,\ V_F = 430\ \text{V},\ R_F = 20\Omega,\ R_{adj} = 0\ \text{to}\ 300\Omega,\ N_F = 1000\ \text{turns per pole}$$



Example 3



Note: When the field current is zero, E_A is about 3 V.

Example 3

a) If the variable resistor in the generator's field circuit is adjusted to 63 ohms, and the generator's prime mover is driving it at 1600 r/min, what is the generator's no-load terminal voltage?

b) What would its terminal voltage be if a 360 A load were connected to its terminals? Assume that the generator has compensating windings.

c) What would its terminal voltage be if a 360 A load were connected to its terminals, but the generator does not have compensating windings? Assume that its armature reaction at this load is 450 A-turns.

Example 3

d) For the conditions in (b), what adjustment could be made to the generator to restore its terminal voltage to the value found in part (a)?

e) How much field current would be needed to restore the terminal voltage to its no-load value? Assume that the machine has compensating windings. What is the required value of the variable resistor to accomplish this?

(Solutions →)

Example 3 – Solutions

a) $$I_F = \frac{V_T}{R_F} = \frac{430}{(20+63)} = 5.2\ \text{A}$$

From the magnetisation curve, this much of current would produce an E_{A0} = 430 V at a speed of n_0 = 1800 rpm. Since the generator is rotating at n_m = 1600 rpm, its internal generated voltage will be:

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0} \Rightarrow E_A = 382\ \text{V}$$

Since $V_T = E_A$ at no-load conditions, the generator's no-load terminal voltage is $V_{T_{NL}}$ = 382 V.

Example 3 – Solutions

b) There is no AR in the machine. If a 360 A load were connected to the generator's terminals, the terminal voltage of the generator would be:

$$V_T = E_A - R_A I_A = 382 - (360)(0.05) = 364 \text{ V}$$

Example 3 – Solutions

c) If a 360 A load were connected to the generator's terminals and the generator has 450 A-turns of AR, then

$$I_F^* = I_F - \frac{mmf_{AR}}{N_F} = 5.2 - \frac{450}{1000} = 4.75 \text{ A}$$

From the magnetisation curve, this much current would produce an E_{A0} = 410 V at 1800 rpm, so the internal generated voltage at 1600 rpm would be:

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0} \Rightarrow E_A = 364 \text{ V}$$

Therefore, the terminal voltage of the generator would be:

$$V_T = E_A - R_A I_A = 364 - (360)(0.05) = 346 \text{ V}$$

It is lower than before due to the armature reaction.

Example 3 – Solutions

d) Increase E_A by decreasing R_{adj} so as to increase I_F.

e) For V_T = 382 V, the required E_A is:

$$E_A = V_T + R_A I_A = 382 + (360)(0.05) = 400 \text{ V}$$

To get E_A = 400 V at n_m = 1600 rpm, the internal generated equivalent voltage at 1800 rpm would be:

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0} \Rightarrow E_{A0} = 450 \text{ V}$$

From the magnetisation curve, this voltage would require a field current I_F = 6.15 A, so that

$$20 + R_{adj} = \frac{430}{6.15} \Rightarrow R_{adj} = 50\Omega$$

Summary

In this lecture, you have learnt:

- The equivalent circuit of a separately excited DC generator.
- The methods used to control the terminal voltage of a separately excited DC generator.
- The analysis of a separately excited DC generator.

References

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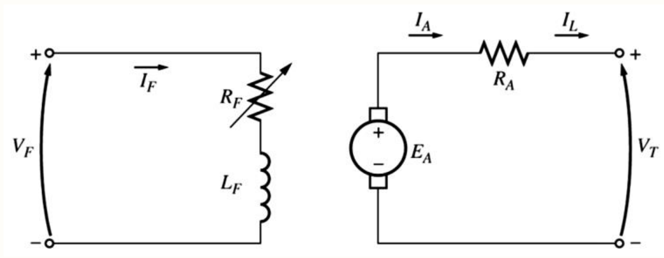
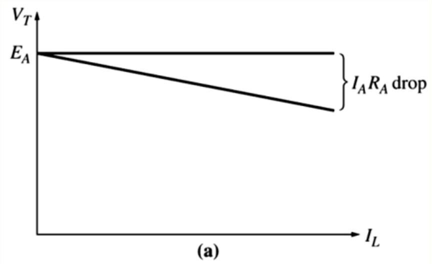
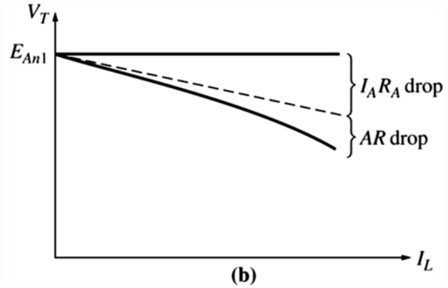
1	7		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 529), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
2	9		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 530), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
3	9		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 530), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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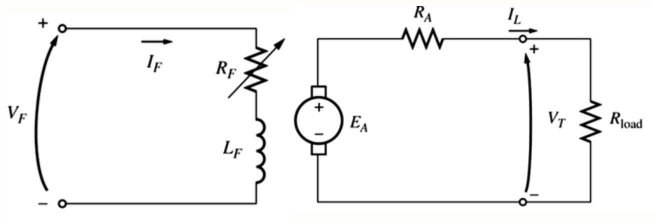
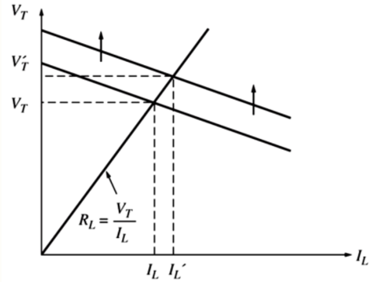
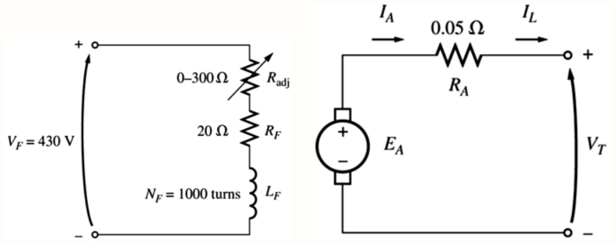
4	11		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 531), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	12		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 531), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
6	15		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 532), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

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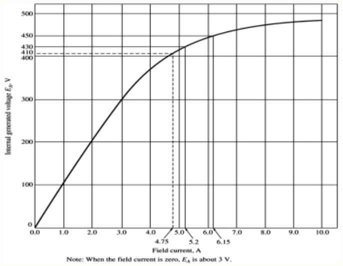
7	16		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 533), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

NANYANG TECHNOLOGICAL UNIVERSITY

Lecture 11

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Tel: +65 6790 4474 | Email: ephlee@ntu.edu.sg

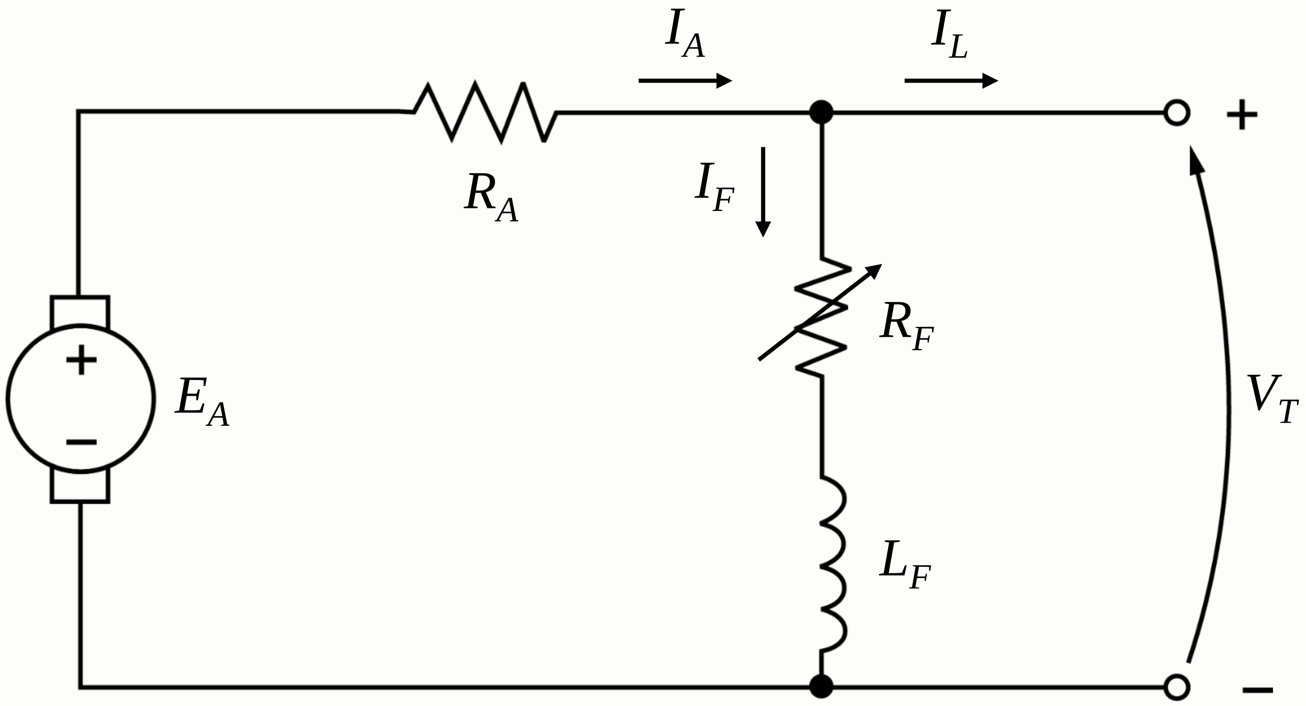
Learning Objectives

By the end of this lecture, you should be able to:

- Describe the voltage build-up in a shunt DC generator.
- Interpret the meaning of critical resistance and how to obtain it.
- Describe the voltage-current characteristics of a DC shunt generator.
- Carry out graphical analysis of a DC shunt generator.
- Describe the losses and power flows, and calculate the generator efficiency at given operating condition.

The Shunt DC Generator

- The generator supplies its own field current, by having its field winding connected directly across the terminals of the machine.



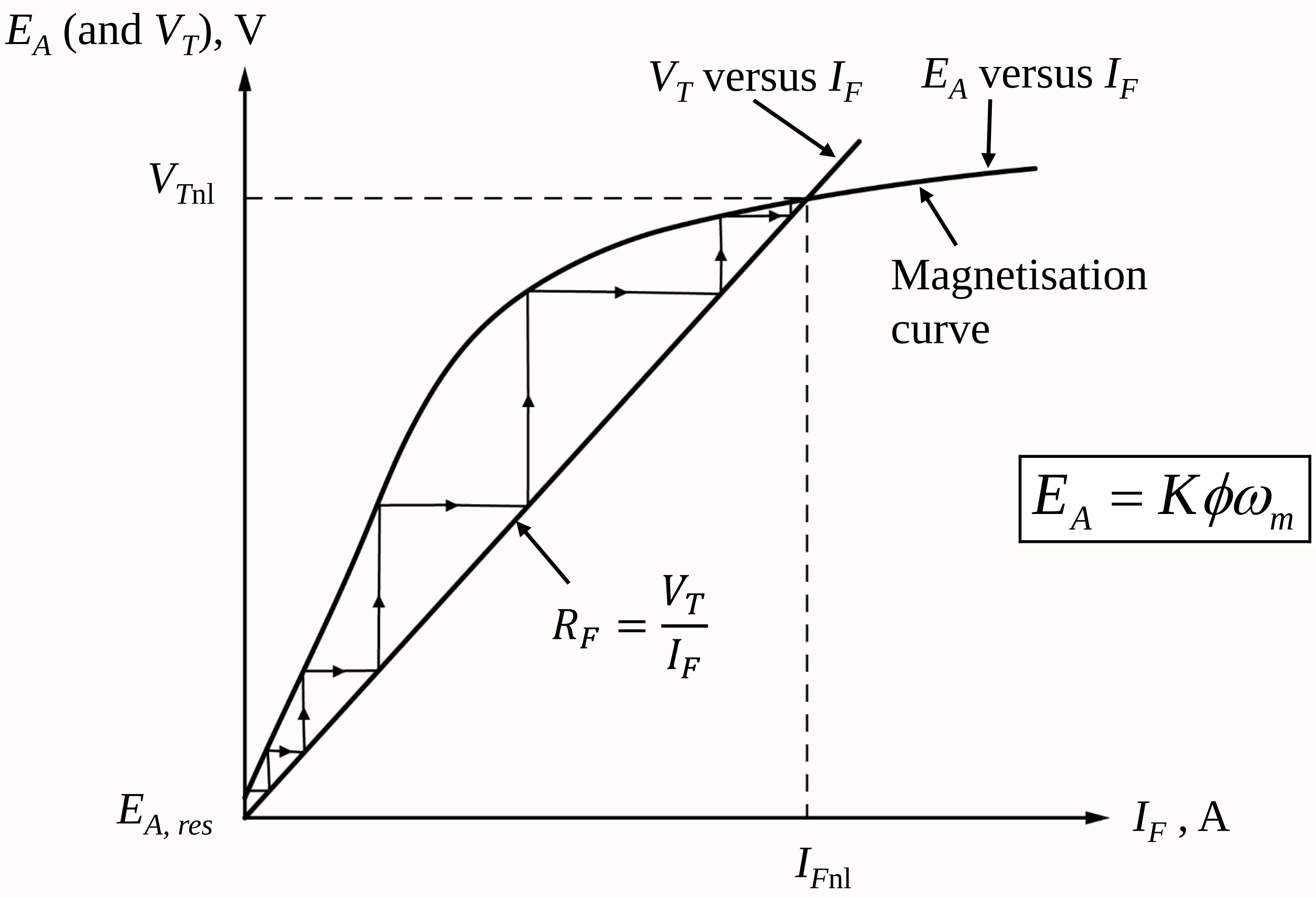
Equivalent circuit of a shunt DC generator.

$$
\boxed{\begin{aligned} I_A &= I_F + I_L, \\ V_T &= E_A - I_A R_A, \\ I_F &= \frac{V_T}{R_F} \end{aligned}}
$$

The Shunt DC Generator

- The armature current of the machine supplies both the field circuit and the load attached to the machine.
- This type of generator has an advantage over the separately excited generator, in that no external power supply is required for the field circuit.
- If the generator supplies its own field current, how does it get the initial field flux when it is first turned on?

Voltage Build-up in a Shunt DC Generator



Voltage build-up on starting in a shunt DC generator.

Voltage Build-up in a Shunt DC Generator

- The voltage build-up depends on the presence of a residual flux ϕ_{res} in the machine. When the generator first starts to turn with no load connected to it, an internal voltage given by $E_A = K\phi_{res}\omega_m$ will be generated. A small V_T appears across the field windings and I_F flows which produces an mmf, and the flux increases which in turn increases E_A, etc.

- It is the effect of the magnetic saturation in the machine which eventually limits the terminal voltage of the generator.

- The field resistance should be less than $R_{critical}$ for voltage build-up to occur.

Voltage Build-up in a Shunt DC Generator

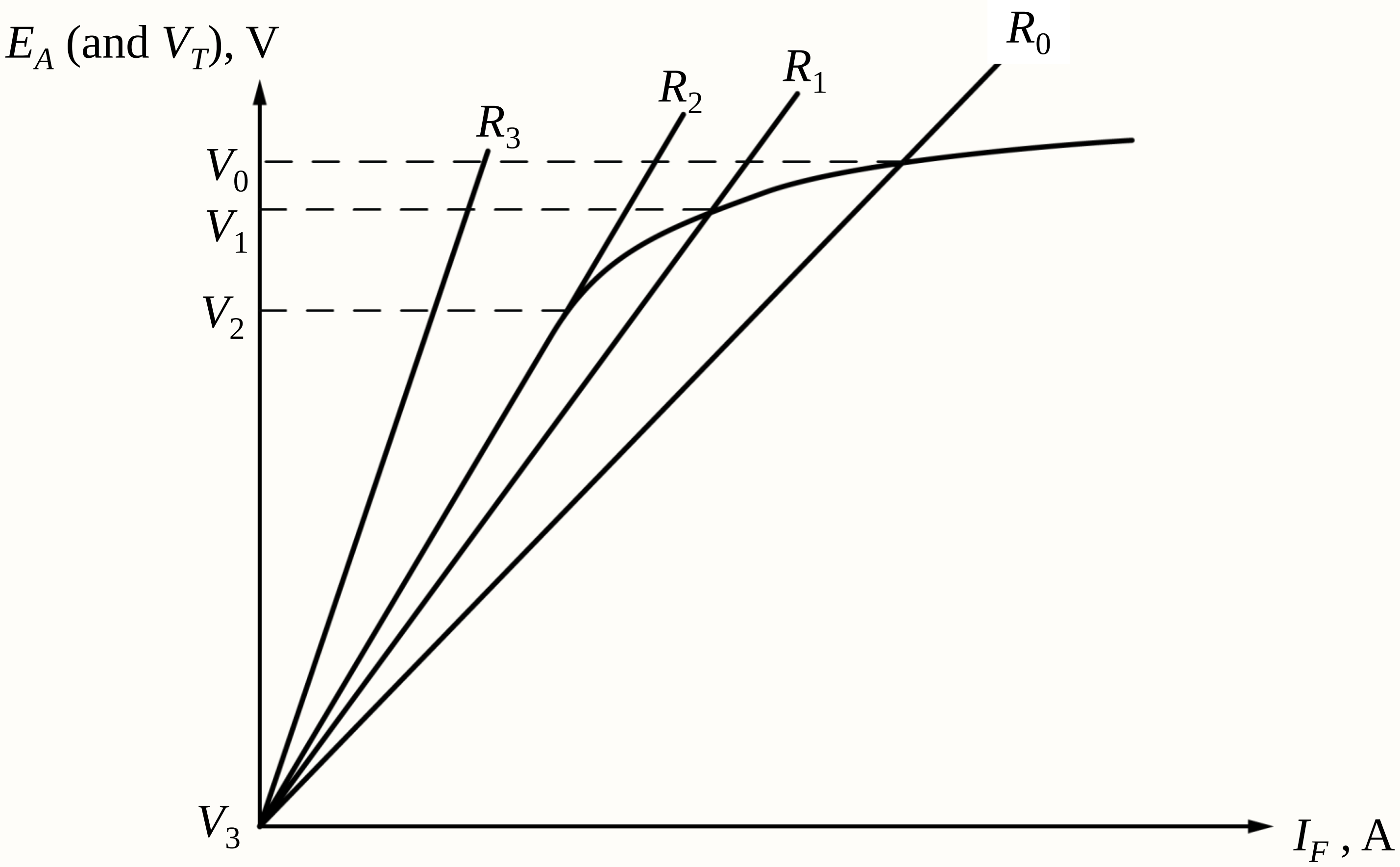
- Normally, the shunt DC generator's voltage will build up to the point where the magnetisation curve intersects the field resistance line, giving the no-load terminal voltage.

- The field resistance should be less than $R_{critical}$ for voltage build-up to occur.

- If R_F exceeds $R_{critical}$, the steady-state operating voltage is essentially at the residual level, and it never builds up.

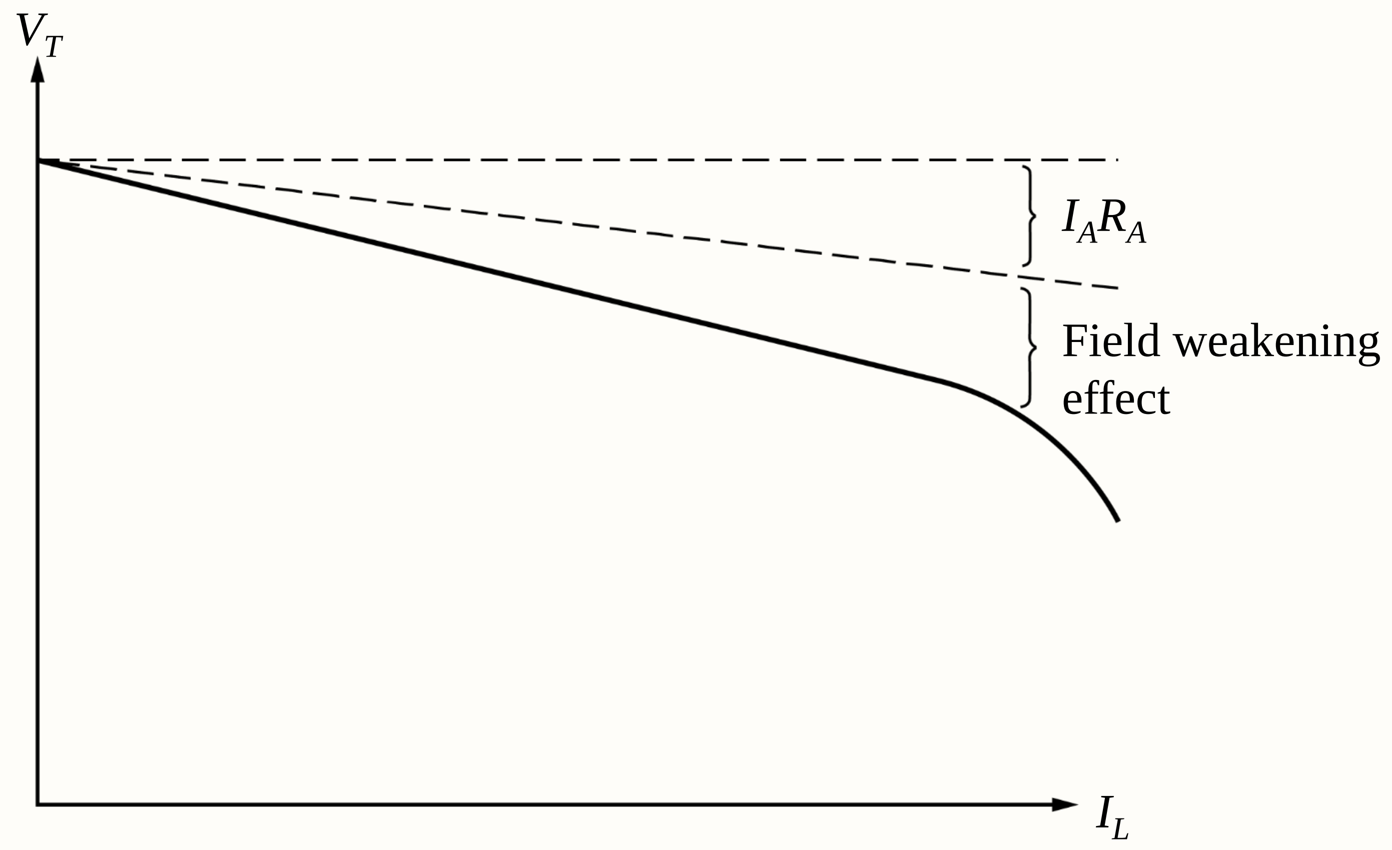
- The solution to this problem is to reduce R_F.

Voltage Build-up in a Shunt DC Generator



The effect of the field resistance on the no-load terminal voltage.
If $R_F > R_2 = R_{critical}$, the generator's voltage will never build up.

V-I Characteristics of a Shunt DC Generator



When load on the generator $\uparrow \Rightarrow I_L$ (hence I_A) $\uparrow \Rightarrow I_A R_A \uparrow \Rightarrow V_T \downarrow$.

When $V_T \downarrow \Rightarrow I_F \downarrow \Rightarrow \phi \downarrow \Rightarrow E_A \downarrow \Rightarrow V_T \downarrow$ further.

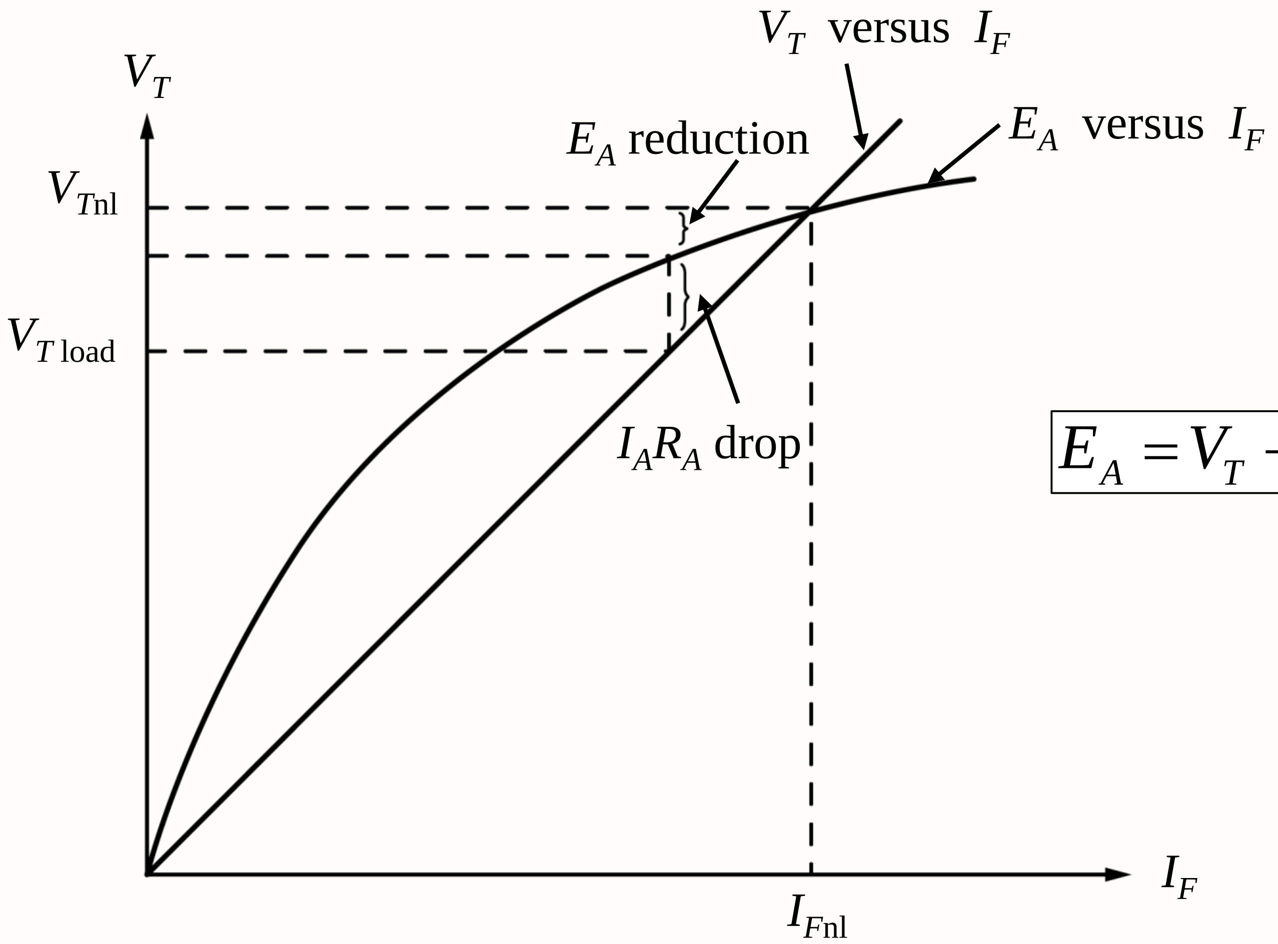
Voltage Control of a Shunt DC Generator

- As with the separately excited DC generator, there are two ways to control the voltage of a shunt DC generator:
  - Change the shaft speed of the generator.
  - Change the total field resistance of the generator, thus changing the field current – this method is widely used.

Analysis of Shunt DC Generators

- The analysis of a shunt DC generator is more complicated than the separately excited generator, because the field current in the machine depends directly on the machine's own output voltage.
- Shunt DC generators with armature reaction will further complicate the analysis, and will not be discussed in this course.

Analysis of Shunt DC Generators



$$\boxed{E_A = V_T + I_A R_A}$$

Graphical analysis of a shunt DC generator with compensating windings.

Analysis of Shunt DC Generators

- The field resistance $R_F = \frac{V_T}{I_F}$ is shown by the straight line laid over the magnetisation curve.
- At no load, $V_T \simeq E_A$ and the generator operates at the voltage where R_F line intersects the magnetisation curve.
- As $E_A = V_T + I_A R_A$, to find the terminal voltage for a given load, just determine the $I_A R_A$ drop and locate the place on the graph, where that drop fits exactly between the E_A line and the V_T line.
- If there are two possible places on the curve where the $I_A R_A$ drop will fit exactly, the one nearer the no-load voltage will represent the normal operating point.

Example 1

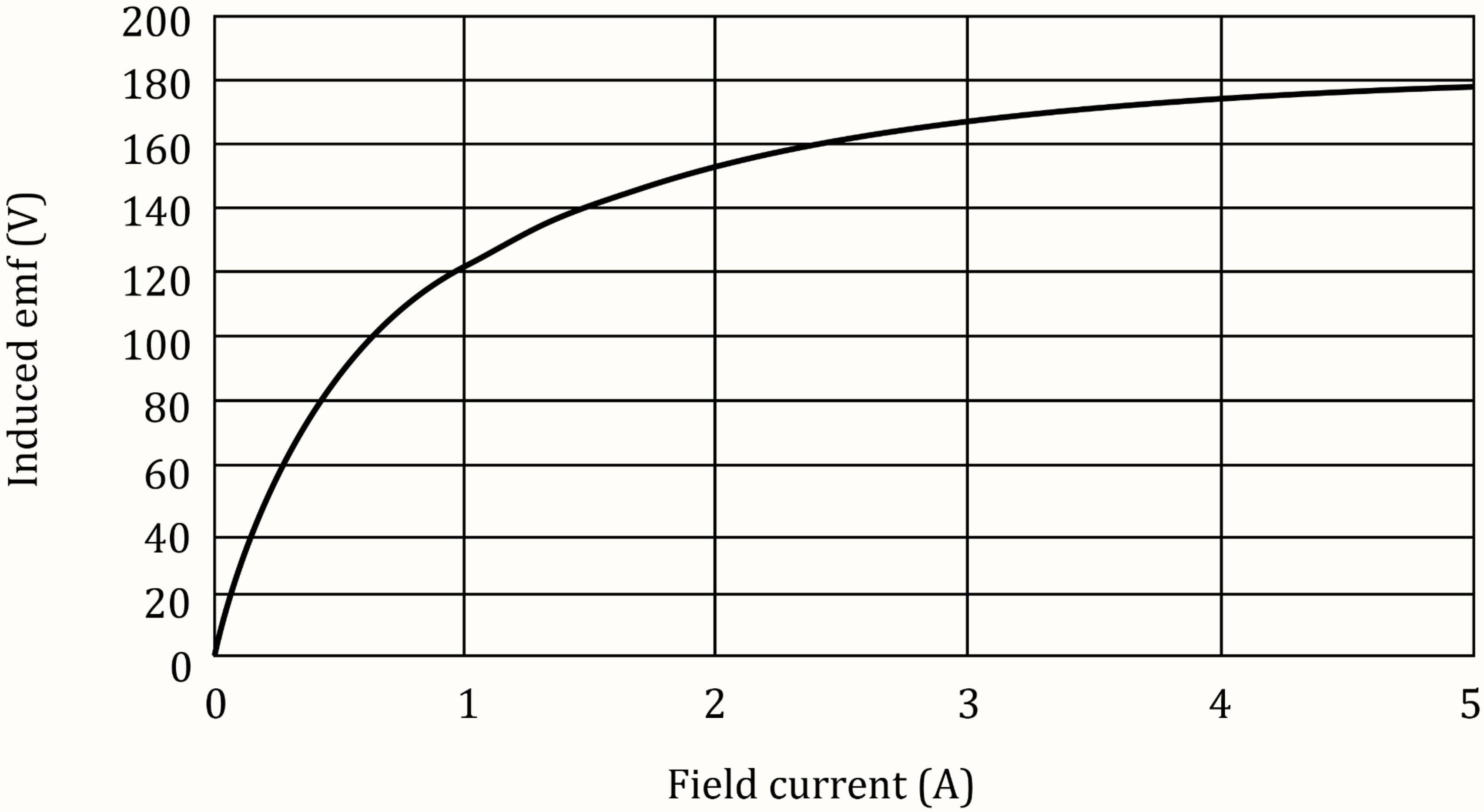
A DC shunt generator is driven at 1200 rpm. The magnetisation curve at 1200 rpm is shown, and the armature and field resistances are 0.2 ohm and 30 ohms, respectively. Neglect armature reaction.

a) An external rheostat is inserted in series with the field winding, and adjusted to give a no-load voltage of 170 V. What is the value of the rheostat setting?

b) If the generator supplies an armature current of 103.5 A, determine the armature terminal voltage with the rheostat setting obtained in (a).

(Solutions →)

Example 1



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Example 1 – Solutions

a) From the magnetisation curve, the field current corresponding to a no-load terminal voltage of 170 V is 3.5 A. The total field-circuit resistance is:

$$R_F = \frac{170}{3.5} = 48.57\Omega$$

Thus, the external rheostat setting in the field-winding circuit is:

$$R_{ext} = 48.57 - 30 = 18.57\Omega$$

Example 1 – Solutions

b) For $I_A = 103.5$ A,

$$V_T = I_F(48.57) = E_A - (103.5)(0.2)$$

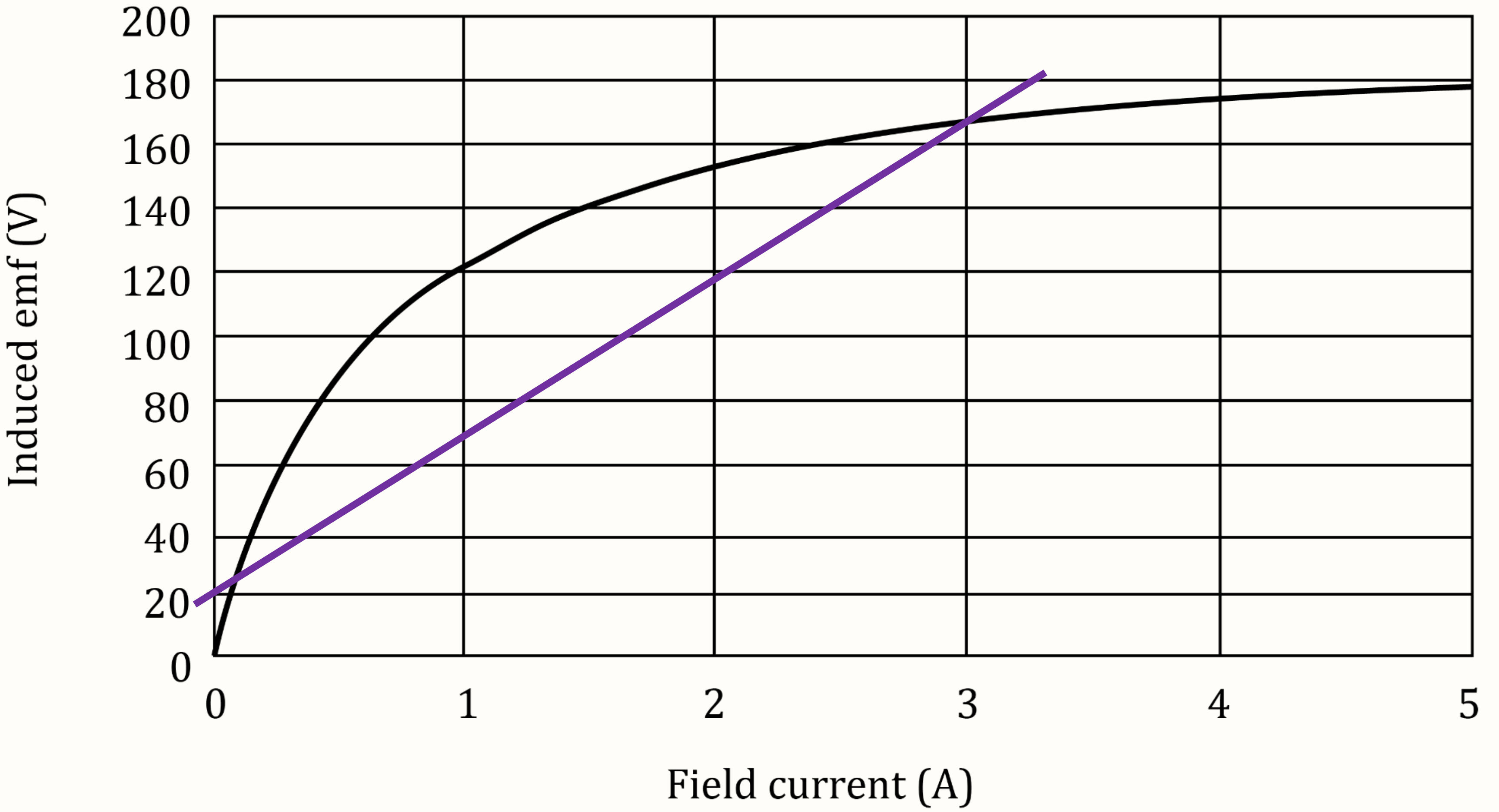
$$\Rightarrow E_A = 48.57 I_F + 20.7$$

Superimposing the above equation onto the magnetisation curve yields the intersection point.

$$I_F = 3 \text{ A}, \; E_A = 166 \text{ V}$$

$$\Rightarrow V_T = 48.57 I_F = 145.71 \text{ V}$$

Example 1 – Solutions



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Losses and Efficiency of a DC Generator

❖ Consider a shunt DC generator as an example:

Power output, $P_{out} = V_T I_L$

Power input $P_{in}(mech) = T_{in}\omega_m$ where T_{in} is the prime mover's torque.

$$P_{dev} = E_A I_A$$
$$P_{in}(mech) = P_{dev} + P_{rot}$$

Generator efficiency, $\eta = \dfrac{P_{out}}{P_{in}} \times 100\%$

Note: For separately excited DC generator, $P_{in} = T_{in}\omega_m + V_F I_F$

Summary of DC Generators

You have learnt:

- DC generators are DC machines used as generators. The different types differ in the manner in which their field fluxes are derived.
- Voltage build-up in self-excited DC generators.
- The voltage-current characteristics of a shunt DC generator.
- The graphical approach to the analysis of shunt DC generator, using the magnetisation curve and the equivalent circuit.

References

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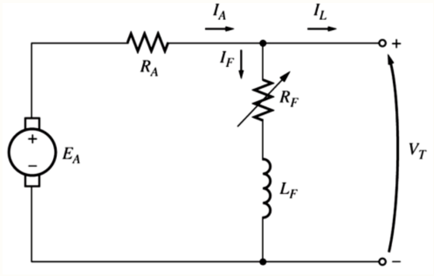
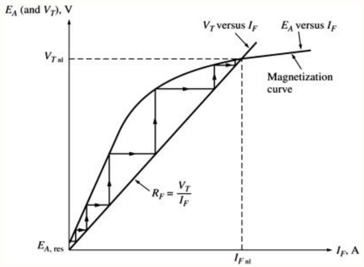
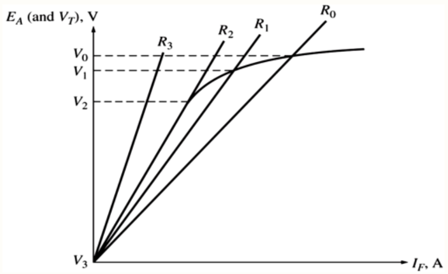
1	3		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 535), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
2	5		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 536), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
3	8		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 537), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.

References

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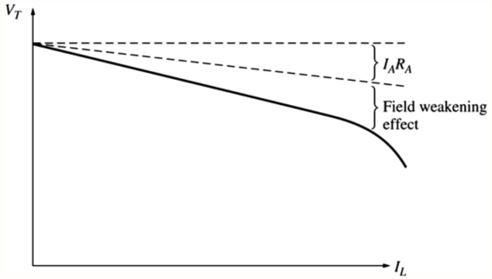
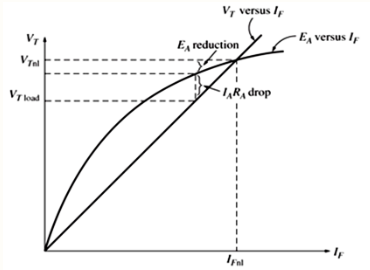
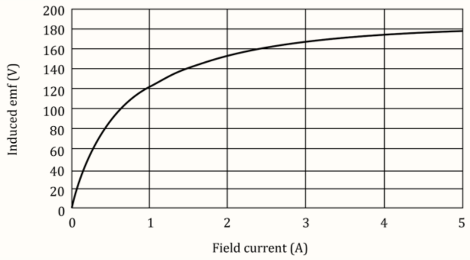
4	9		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 538), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
5	12		Reprinted from *Electric Machinery Fundamentals, 5th ed.*, (p. 539), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Reprinted with permission.
6	15		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 327), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press. Reprinted with permission.

References

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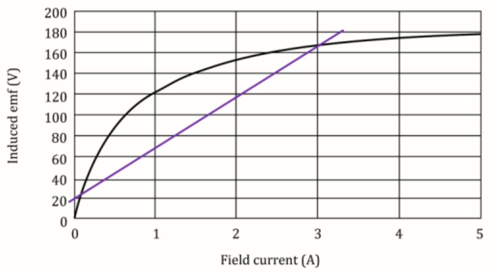
7	18		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 327), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press. Reprinted with permission.