Lecture 3

EE3010: Electrical Devices and Machines

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the respective powers, derive their relationships and equations for an induction motor.
- Explain the notions of developed torque and output torque and how to compute them.
- Use the induction motor equivalent circuit to study and predict the performance of the motor for a given operating condition.

Power and Torque in Induction Motors

From the per-phase equivalent circuit,

- The stator copper losses $P_{SCL} = 3I_1^2 R_1$
- The air-gap power $P_{AG} = P_{in} - P_{SCL}$
- Since X_2 does not consume real power, $P_{AG} = 3I_2^2 \left(\dfrac{R_2}{s} \right)$
- The rotor copper losses $P_{RCL} = 3I_2^2 R_2$
- The converted power, also called developed power,

$$P_{dev} = P_{conv} = P_{AG} - P_{RCL} = 3I_2^2 \left(R_2 \left(\frac{1-s}{s} \right) \right)$$

Power and Torque in Induction Motors

- Also, $P_{RCL} = sP_{AG}$, $P_{conv} = (1-s)P_{AG}$, $P_{out} = P_{conv} - P_{rot}$

- The induced torque T_{ind} is the torque generated by the internal electrical to mechanical power conversion. Also called developed torque T_{dev}.

$$T_{ind} = T_{dev} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

Separating Rotor Cu Losses and the Power Converted

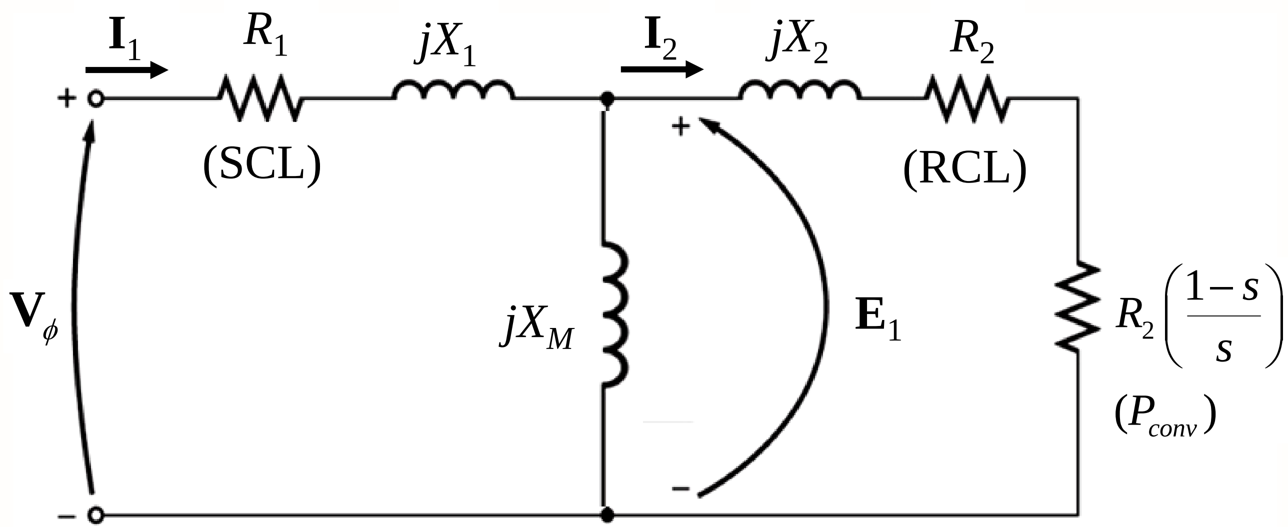
- The air-gap power $P_{AG} = 3I_2^2\left(\frac{R_2}{s}\right)$

- Rotor copper losses $P_{RCL} = 3I_2^2 R_2$

- The converted power

$$P_{conv} = P_{AG} - P_{RCL} = 3I_2^2\left\{R_2\left(\frac{1-s}{s}\right)\right\}$$

- Thus, $R_{conv} = R_2\left(\frac{1-s}{s}\right)$



The per phase equivalent circuit with rotor copper losses and power converted separated.

Example 1

A 460 V, 60 Hz, four-pole induction motor is wye-connected. The core losses are lumped together with the friction and windage losses and the stray losses to give the total rotational losses of 1100 W and they are assumed to be constant. The impedances per phase referred to the stator circuit are:

$$R_1 = 0.641\Omega\ , \quad R_2 = 0.332\Omega\ , \quad X_1 = 1.106\Omega\ ,$$
$$X_2 = 0.464\Omega\ , \quad X_M = 26.3\Omega$$

For a rotor slip of 2.2% at the rated voltage and rated frequency, find the motor’s

a) Speed

b) Input stator current

c) Input power factor

d) P_{conv} and P_{out}

e) T_{ind} and T_{load}

f) Efficiency

(Solutions →)

Example 1 – Solutions

a) $$n_{sync} = \frac{120 f_s}{p} = \frac{120(60)}{4} = 1800 \text{ r/min}$$

$$\Rightarrow n_m = (1-s)n_{sync} = 1760 \text{ r/min}$$

b) The total impedance per phase is calculated as

$$\mathbf{Z}_{total} = 11.72 + j7.79 = 14.07\angle 33.6° \Omega$$

$$\Rightarrow \mathbf{I}_1 = \frac{\mathbf{V}_\phi}{\mathbf{Z}_{total}} = \frac{266\angle 0°}{14.07\angle 33.6°} = 18.88\angle -33.6° \text{ A}$$

c) The pf is $\cos(33.6°) = 0.833$ lagging.

Example 1 – Solutions

d) $P_{AG} = P_{in} - P_{SCL} = \sqrt{3}V_{line}I_{line}\cos\theta - 3I_1^2R_1$

$$= \sqrt{3}(460)(18.88)0.833 - 3(18.88)^2 0.641 = 11845 \text{ W}$$

$$P_{conv} = (1-s)P_{AG} = 11585 \text{ W} \Rightarrow P_{out} = P_{conv} - P_{rot} = 10485 \text{ W}$$

e) $T_{ind} = \dfrac{P_{conv}}{\omega_m} = \dfrac{P_{AG}}{\omega_{sync}} = 62.8 \text{ N.m.} \quad T_{load} = \dfrac{P_{out}}{\omega_m} = 56.9 \text{ N.m.}$

f) The efficiency of the motor $\eta = \dfrac{P_{out}}{P_{in}} \times 100\% = 83.7\%$

Example 2

A three-phase 230 V, 60 Hz, 100-hp (1 hp = 746 W), six-pole induction motor operating at rated conditions has an efficiency of 91 % and draws a line current of 248 A. The stator copper and rotor copper losses are 2803 W and 1549 W, respectively. Determine:

a) Power input
b) Total losses
c) Air-gap power
d) Rotor speed
e) Input power factor
f) Rotational losses

(Solutions →)

Example 2 – Solutions

a) $$\text{Eff } \eta = \frac{(P_{out} = 100 \text{ x } 746)}{P_{in}} \Rightarrow P_{in} = 81978 \text{ W}$$

b) $$\text{Total losses} = P_{in} - P_{out} = 7378 \text{ W}$$

c) $$P_{AG} = P_{in} - P_{SCL} = 79175 \text{ W}$$

Example 2 – Solutions

d) From $P_{RCL} = sP_{AG} \Rightarrow s = 0.01956$

$$n_{sync} = 120\frac{f_s}{p} = 1200 \text{ rpm}$$

$$n_m = (1-s)\ n_{sync} = 1176 \text{ rpm}$$

e) From $P_{in} = \sqrt{3}V_{line}I_{line}\cos\theta \Rightarrow \cos\theta = 0.83$

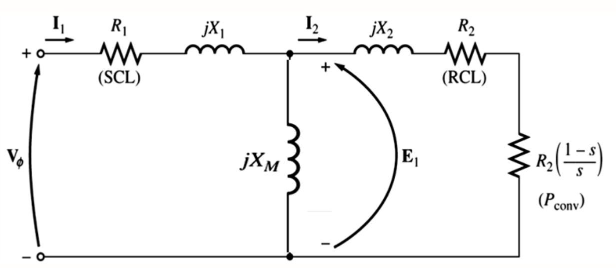
f) $P_{rot} = \text{Total losses} - P_{SCL} - P_{RCL} = 3026\ \text{W}$

Summary

In this lecture, you have learnt:

- The equations that relate the respective powers and torques in an induction motor.
- The separation of air-gap power into the rotor copper loss and the power converted in an induction motor, and representing them as distinct elements in the equivalent circuit.
- The calculations of the various powers, torques and motor efficiency from the induction motor equivalent circuit for a given operating condition.

References

No.	Slide No.	Image	Reference
1	5		Adapted from *Electric Machinery Fundamentals, 5th ed.*, (p. 326), by S. J. Chapman, 2012, New York, NY: McGraw-Hill. Copyright 2012 by The McGraw-Hill Companies, Inc. Adapted with permission.