Circuit Analysis

EE2001

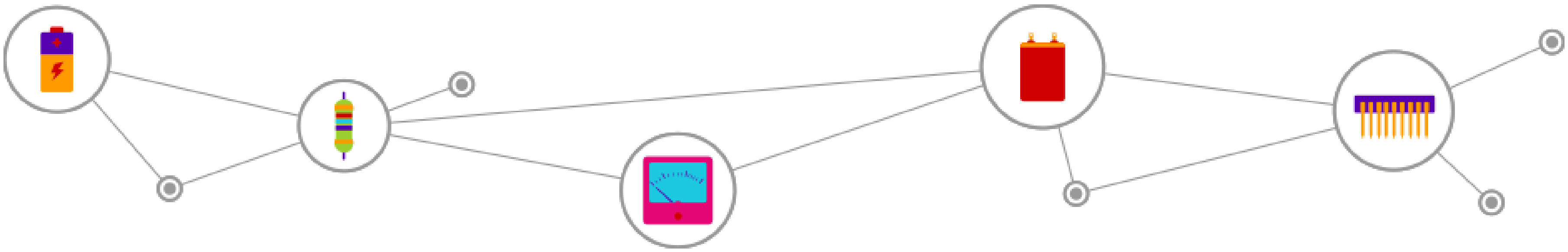
NANYANG TECHNOLOGICAL UNIVERSITY

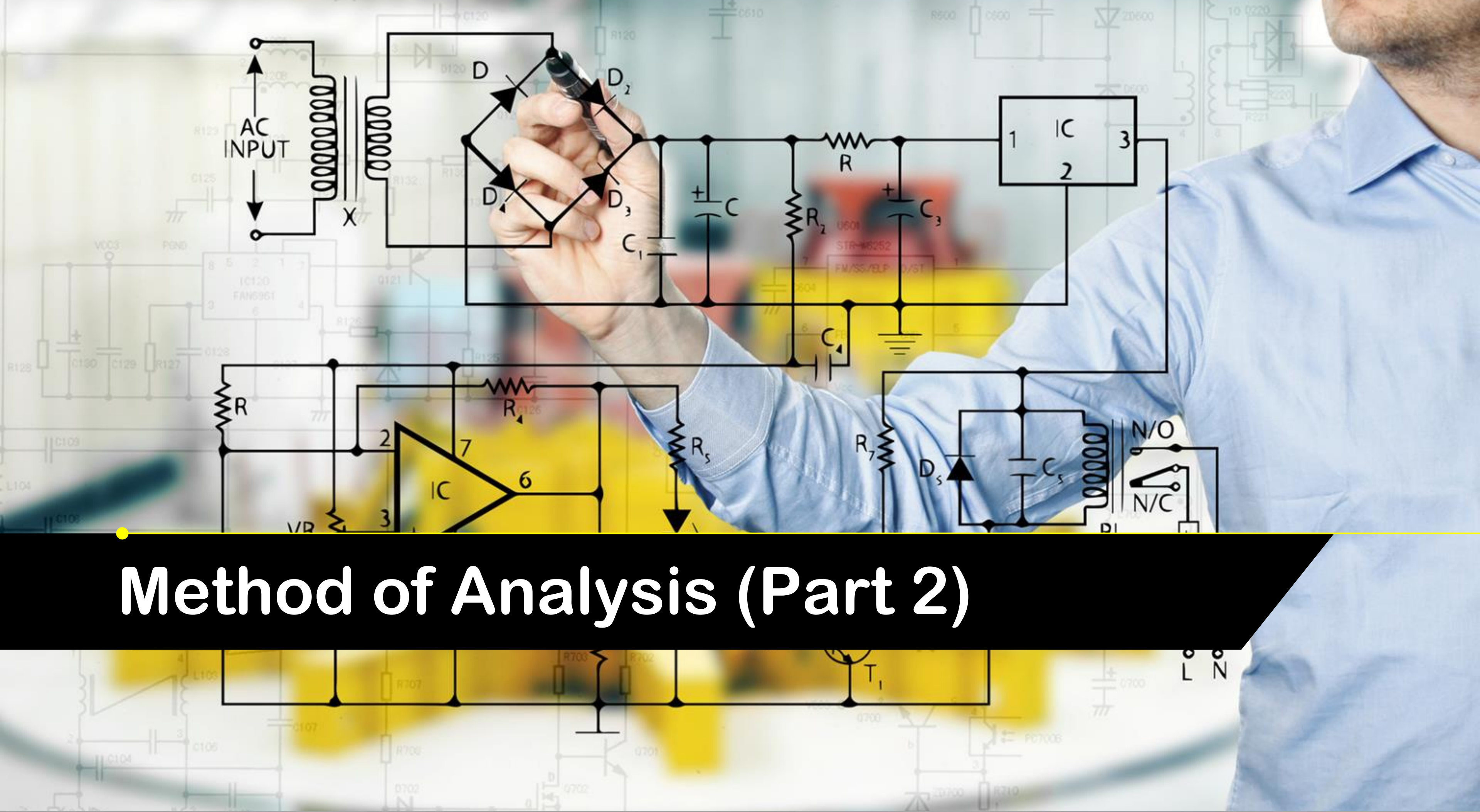
Method of Analysis (Part 2), Circuit Theorems and Operational Amplifiers

Professor Er Meng Joo

By the end of this lesson, you should be able to…

- Analyse circuits using mesh analysis.
- Describe the key characteristics of the different types of circuit theorems.
- Analyse circuits using the different types of circuit theorems.
- Describe the key characteristics of operational amplifiers.





Method of Analysis (Part 2)

Mesh Analysis

Recall that a mesh is a loop that does not contain any other loops within it.

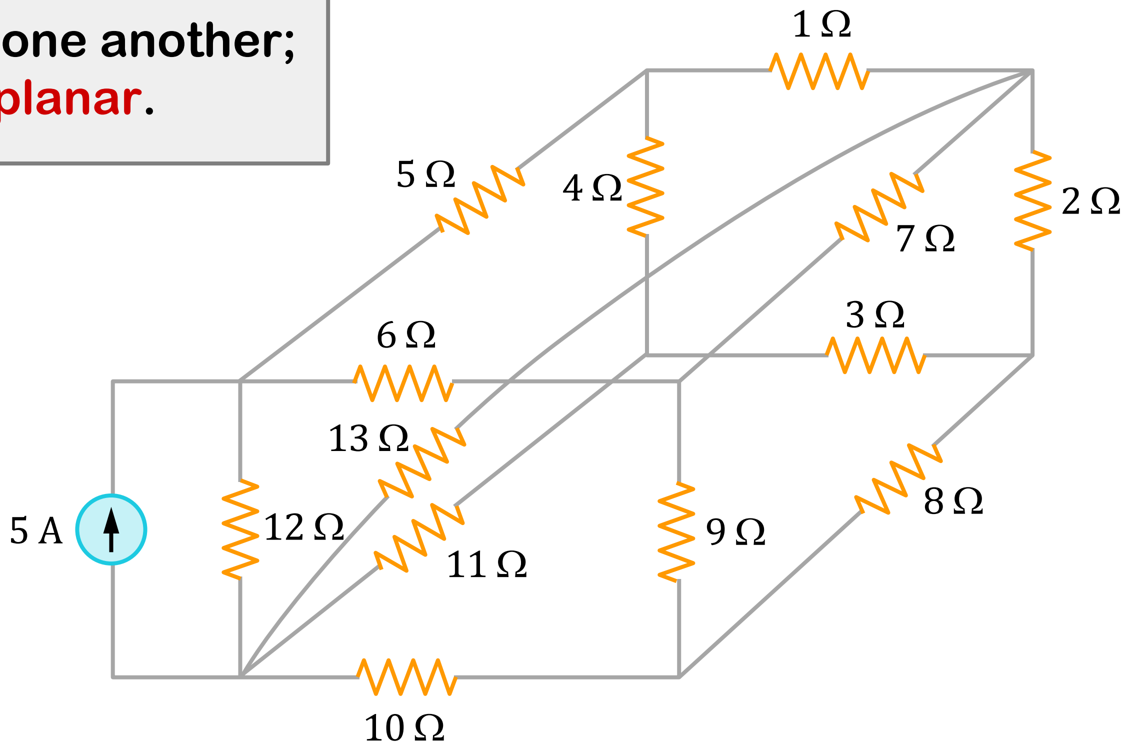
The current through the mesh is known as the **mesh current**.

Mesh analysis provides another general procedure for analysing circuits using mesh currents as the circuit variables.

Mesh analysis applies **KVL** to find unknown currents and is only applicable to **planar circuits**.

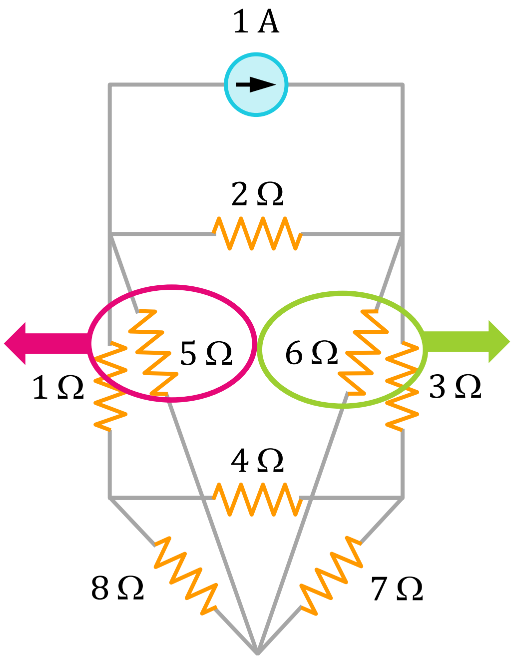
Planar Circuit

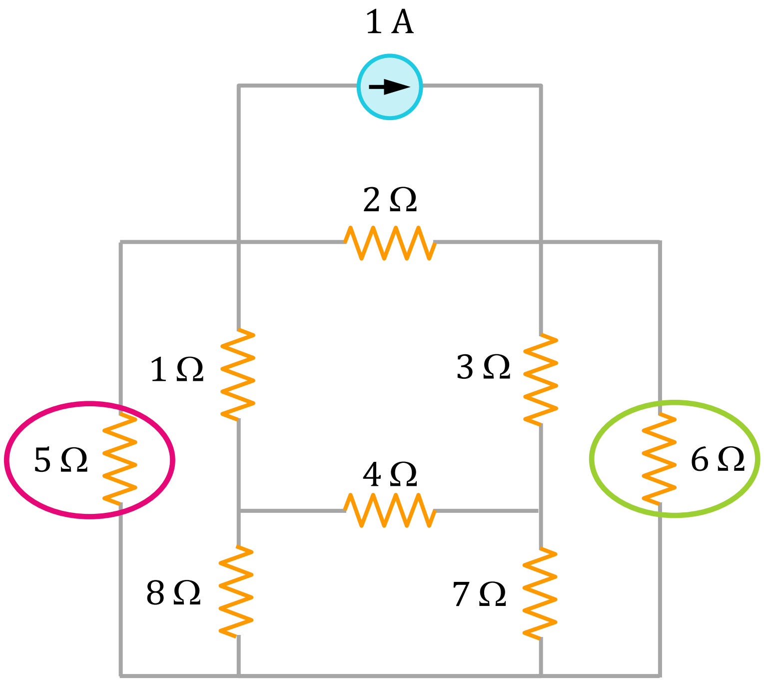
A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise, it is **nonplanar**.



Planar Circuit

A circuit that is drawn with crossing branches is still considered planar if it can be redrawn with no crossover branches.





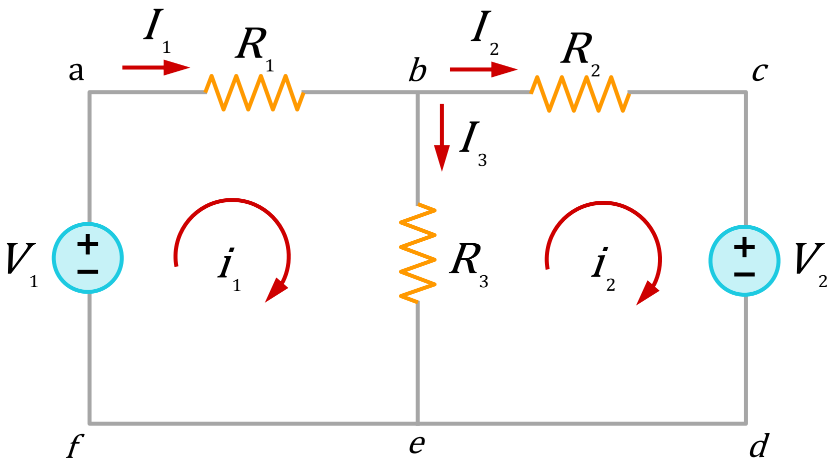
Mesh Analysis Without Current Sources

For mesh analysis with m meshes, we take the following steps:

1. Assign mesh currents $i_1, i_2, \dots, i_m$ to the m meshes.
2. Apply **KVL** to each of the m meshes. Use **Ohm's law** to express the voltages in terms of the mesh currents.
3. Solve the resulting simultaneous equations to obtain the mesh currents.

Example 14

Consider the following circuit.

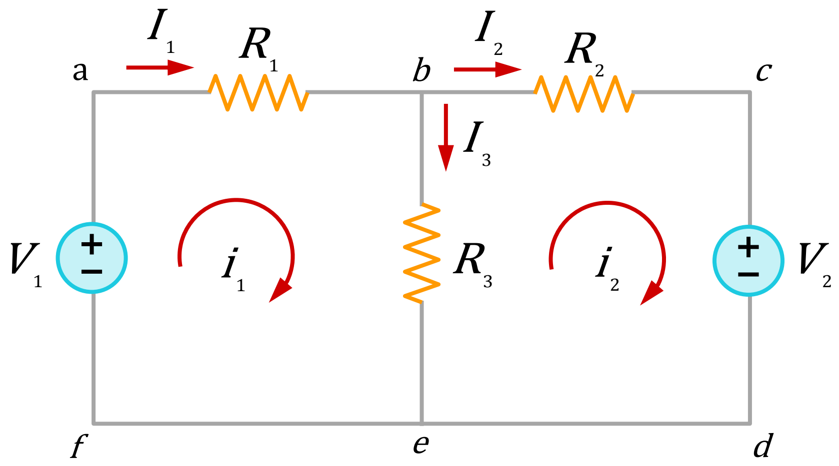


Assign mesh currents i_1 and i_2 to meshes 1 and 2.

Note that no branches can appear in more than two meshes. For example, R_3 appears in both meshes and the current flowing downward through it is $i_1 - i_2$.

Example 14

The R_1 resistor appears only in mesh 1 and the current flowing to the right in that branch (I_1) is equal to the mesh current i_1.



Also, the R_2 resistor appears only in mesh 2 and the current flowing to the right in that branch (I_2) is equal to the mesh current i_2.

Example 14

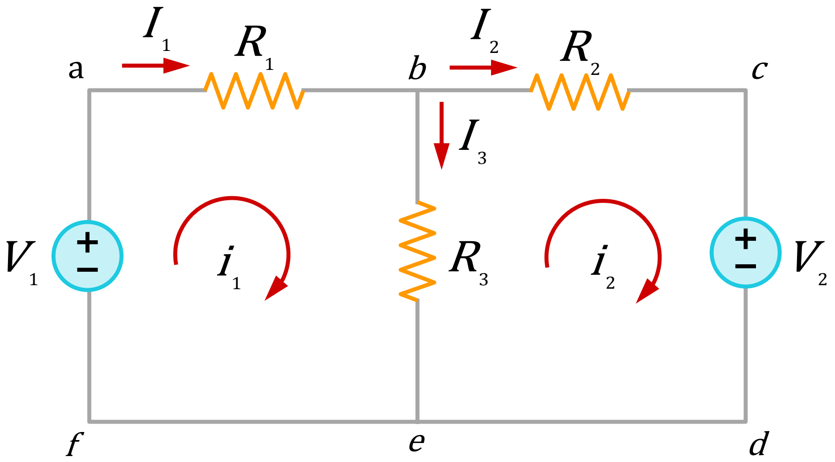
Applying **KVL** to each mesh,

Mesh 1: $-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$ **(1)** ➡ $(R_1 + R_3)i_1 - R_3 i_2 = V_1$

Mesh 2: $R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$ **(2)** ➡ $-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$

In Matrix form, we have

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

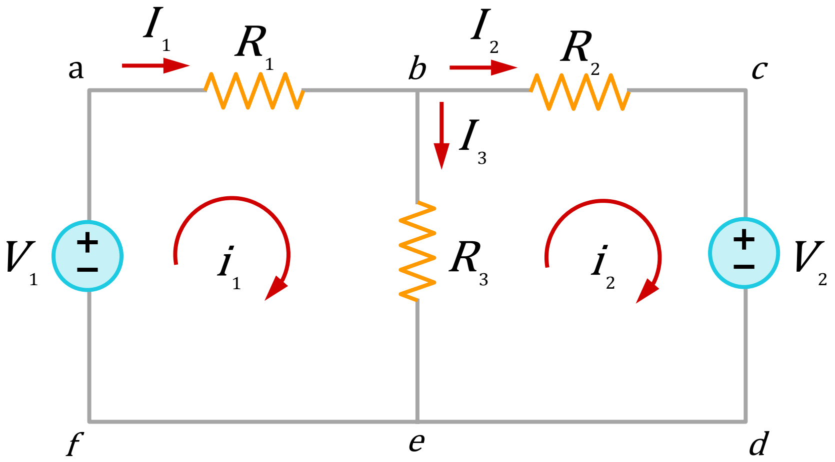


Using Cramer's rule, the branch currents can be found: $I_1 = i_1$, $I_2 = i_2$, $I_3 = i_1 - i_2$

Example 14

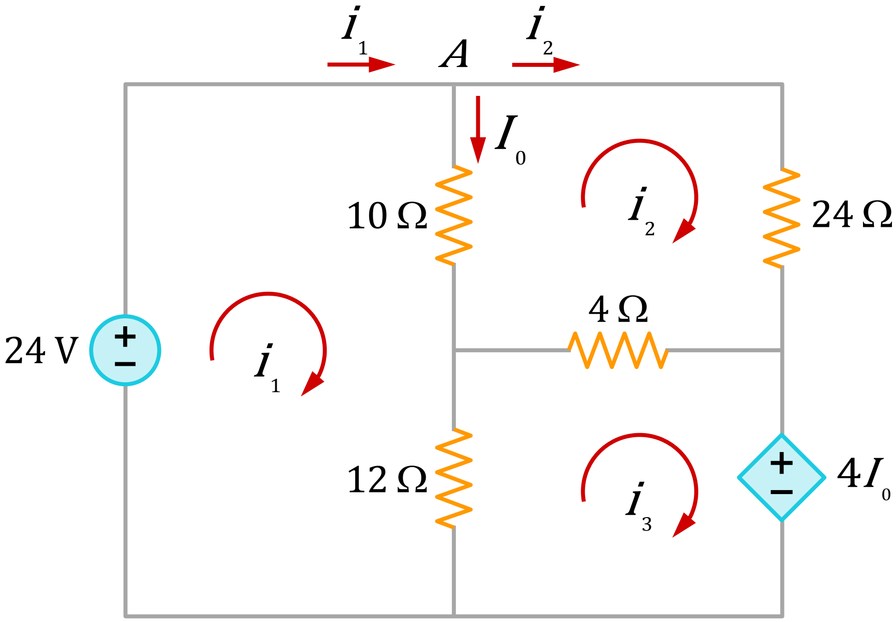
The direction of the mesh currents is arbitrary; it can be clockwise (CW) or counter clockwise (CCW), and the choice will not affect the validity of the solution.

We normally assume the mesh current flows CW.



Example 15

Use mesh analysis to find the current I_o in the following circuit.



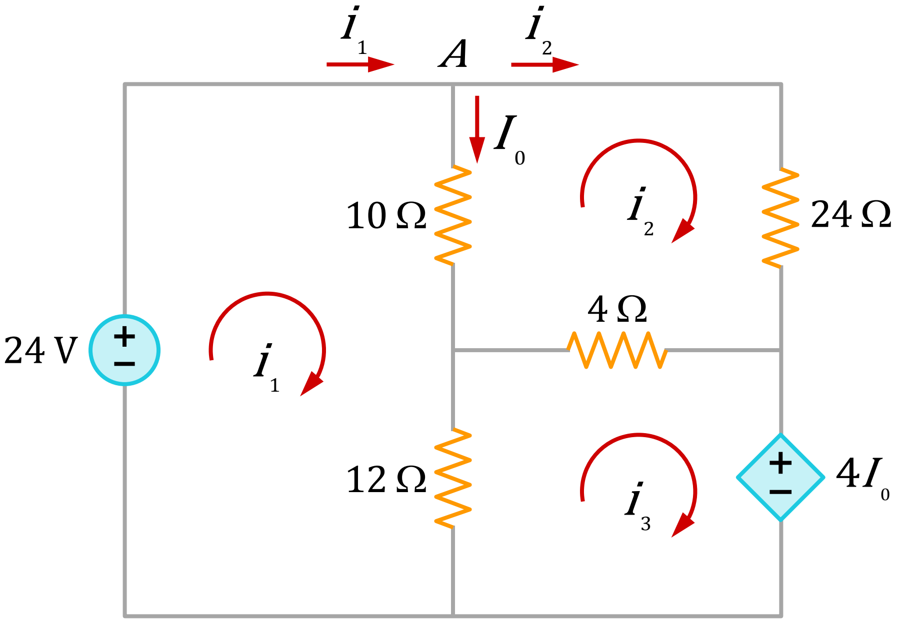
Need to find i_1, i_2 and i_3 first.
Apply KVL to each mesh.

Mesh 1:

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$11i_1 - 5i_2 - 6i_3 = 12$$

Example 15

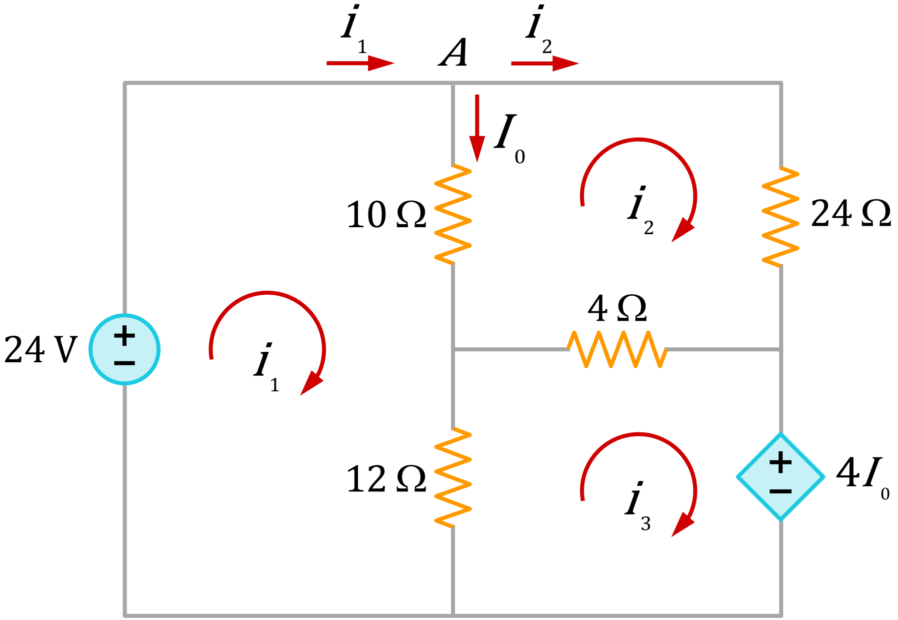


Mesh 2:

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$-5i_1 + 19i_2 - 2i_3 = 0$$

Example 15



Mesh 3:

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

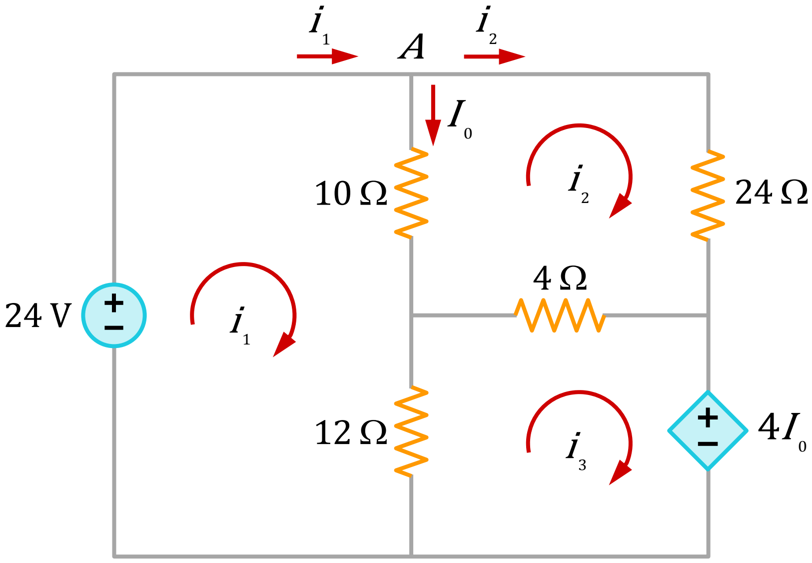
$$-i_1 - i_2 + 2i_3 = 0$$

Example 15

Mesh 1: $11i_1 - 5i_2 - 6i_3 = 12$

Mesh 2: $-5i_1 + 19i_2 - 2i_3 = 0$

Mesh 3: $-i_1 - i_2 + 2i_3 = 0$



In Matrix form, we have…

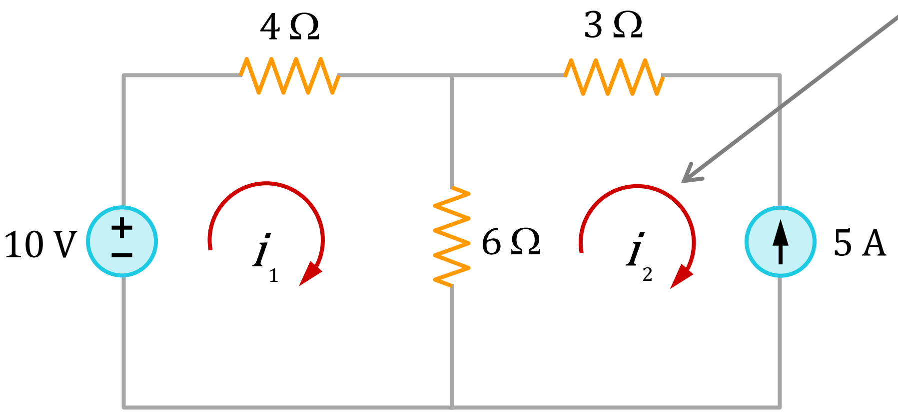
$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's rule to solve for the mesh currents, we have
$i_1 = 2.25\ A,\ i_2 = 0.75$ **and** $i_3 = 1.5\ A.$

Hence, $I_o = i_1 - i_2 = 2.25 - 0.75 = 1.5\ A$

Mesh Current With Current Source

Case One: When a current source exists **only in one mesh**.



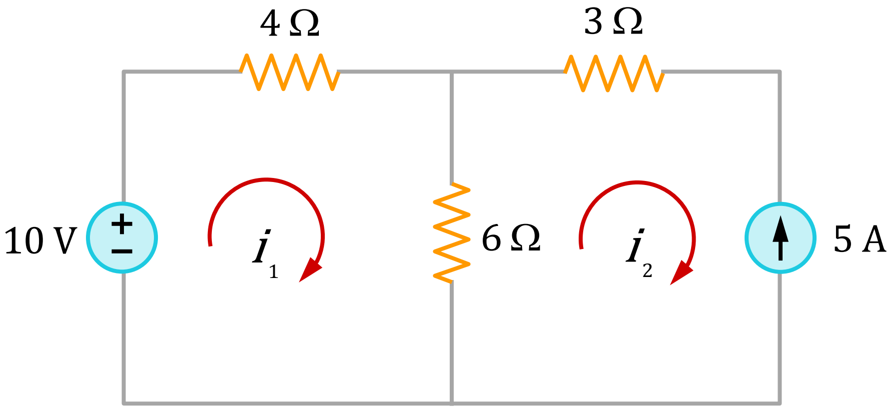
It is clear that $i_2 = -5\ \mathrm{A}$ (thus we eliminate mesh 2 from consideration).

The mesh equation for mesh 1 is

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$i_1 = -2\ A$$

Mesh Current With Current Source

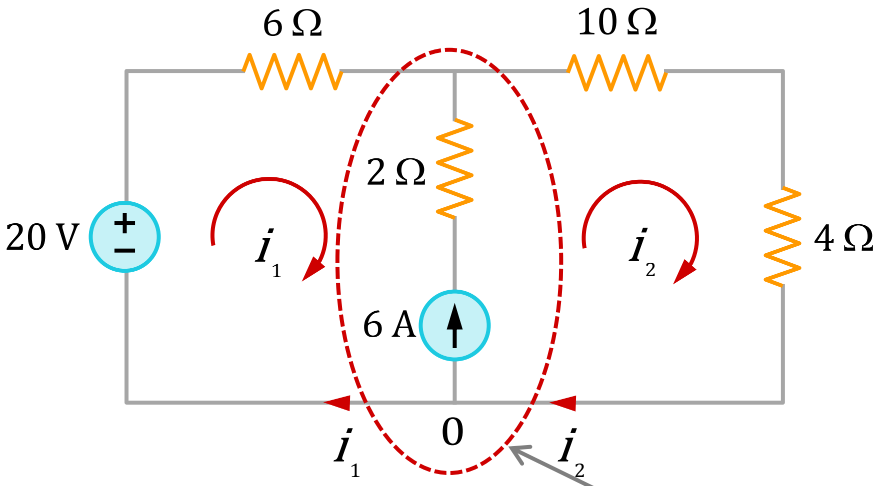


Note: we could have calculated the current i_1 using nodal analysis, but we have to find the node voltage v_n first (using $\frac{10-v_n}{4} + 5 - \frac{v_n - 0}{6} = 0$) and then find the current using $i_1 = (10 - v_n)/4$.

In this case, mesh analysis is simpler.

Mesh Current With Current Source

Case Two : When a current source exists **between two meshes**.



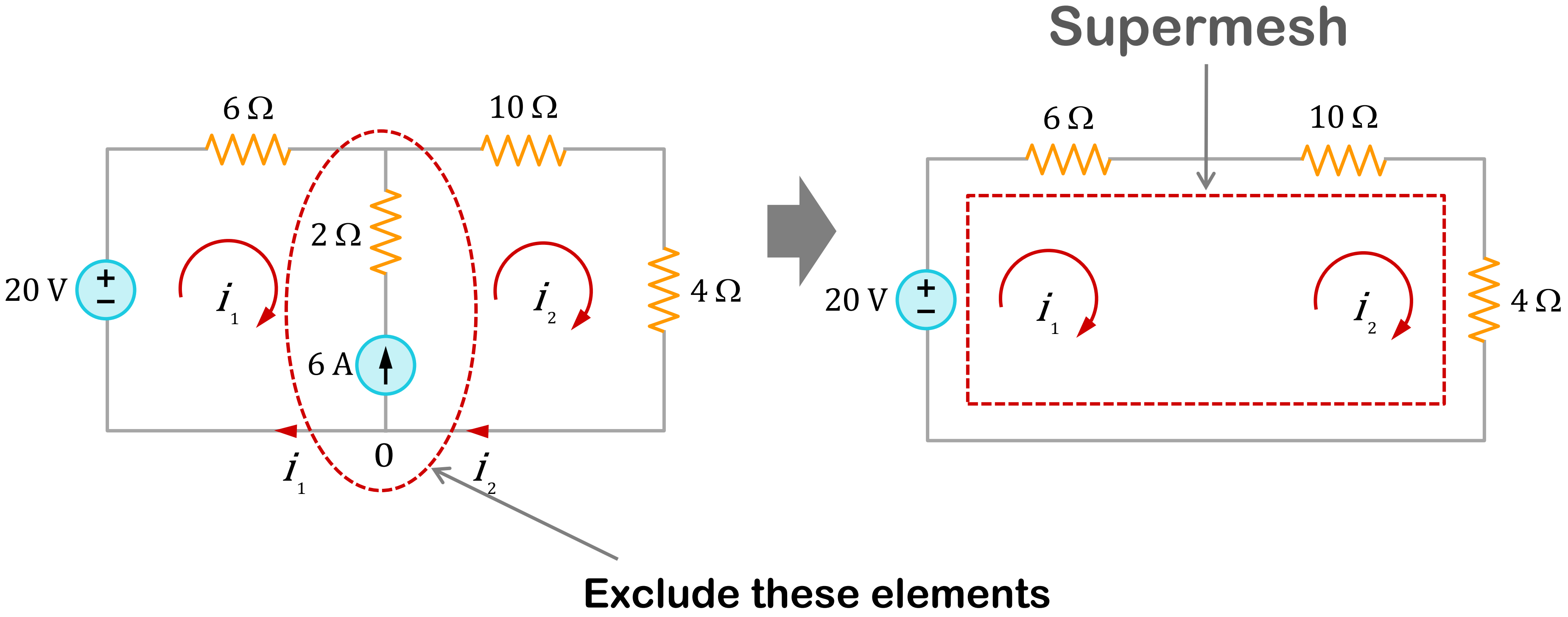
We create a supermesh by excluding the current source and **any elements connected in series with it** as shown.

A supermesh results when two meshes have a (dependent or independent) current source in common.

Exclude these elements

Mesh Current With Current Source

The current source and the element connected in series with it is in the interior of the supermesh.

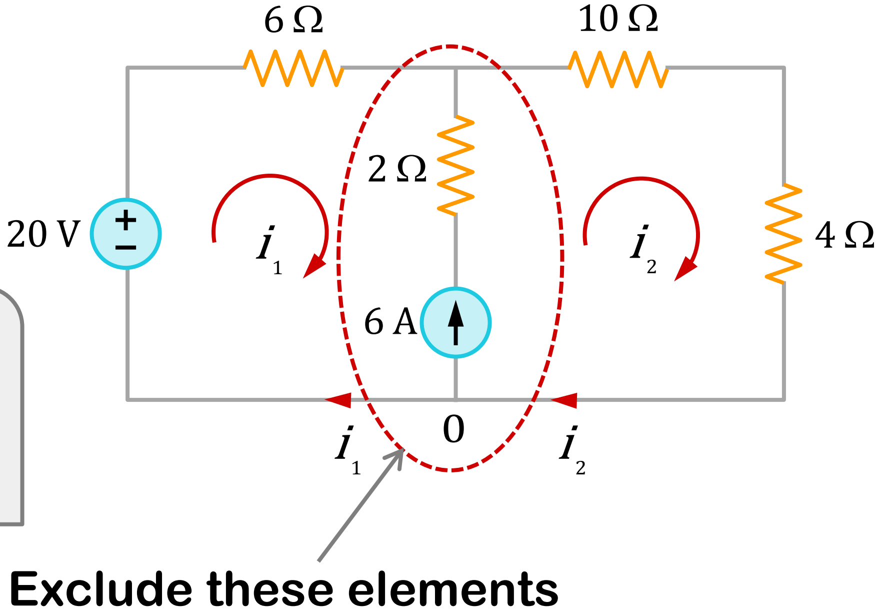


Mesh Current With Current Source

Applying **KVL** to the supermesh gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \qquad (1)$$

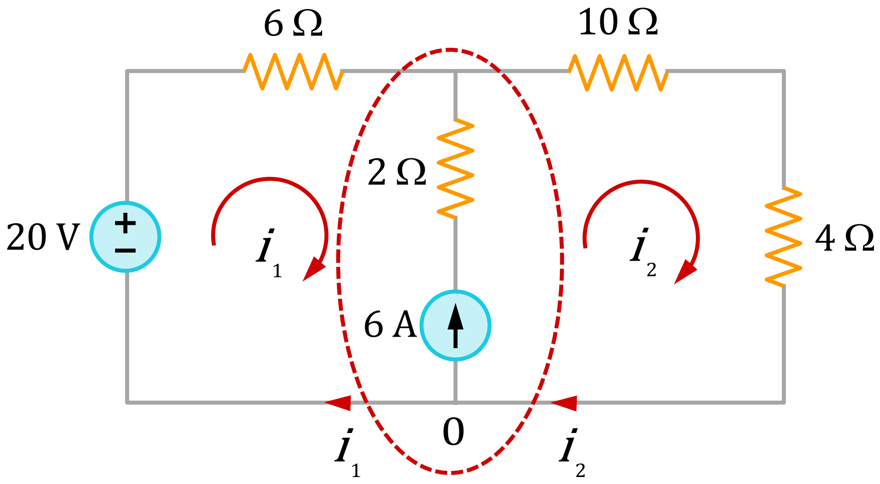


Note: the voltages around the supermesh are in terms of the original mesh currents.

Mesh Current With Current Source

Applying **KCL** to a node in the branch where the two meshes intersect. At node 0 (or at the top node),

$$\boxed{i_2 = i_1 + 6} \quad (2)$$



The current source in the supermesh provides the constraint equation (2).

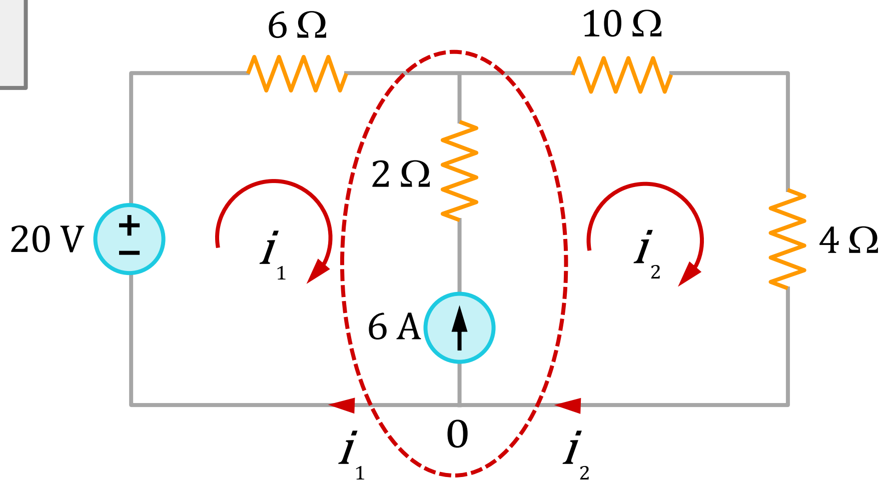
Using (2) in (1) gives

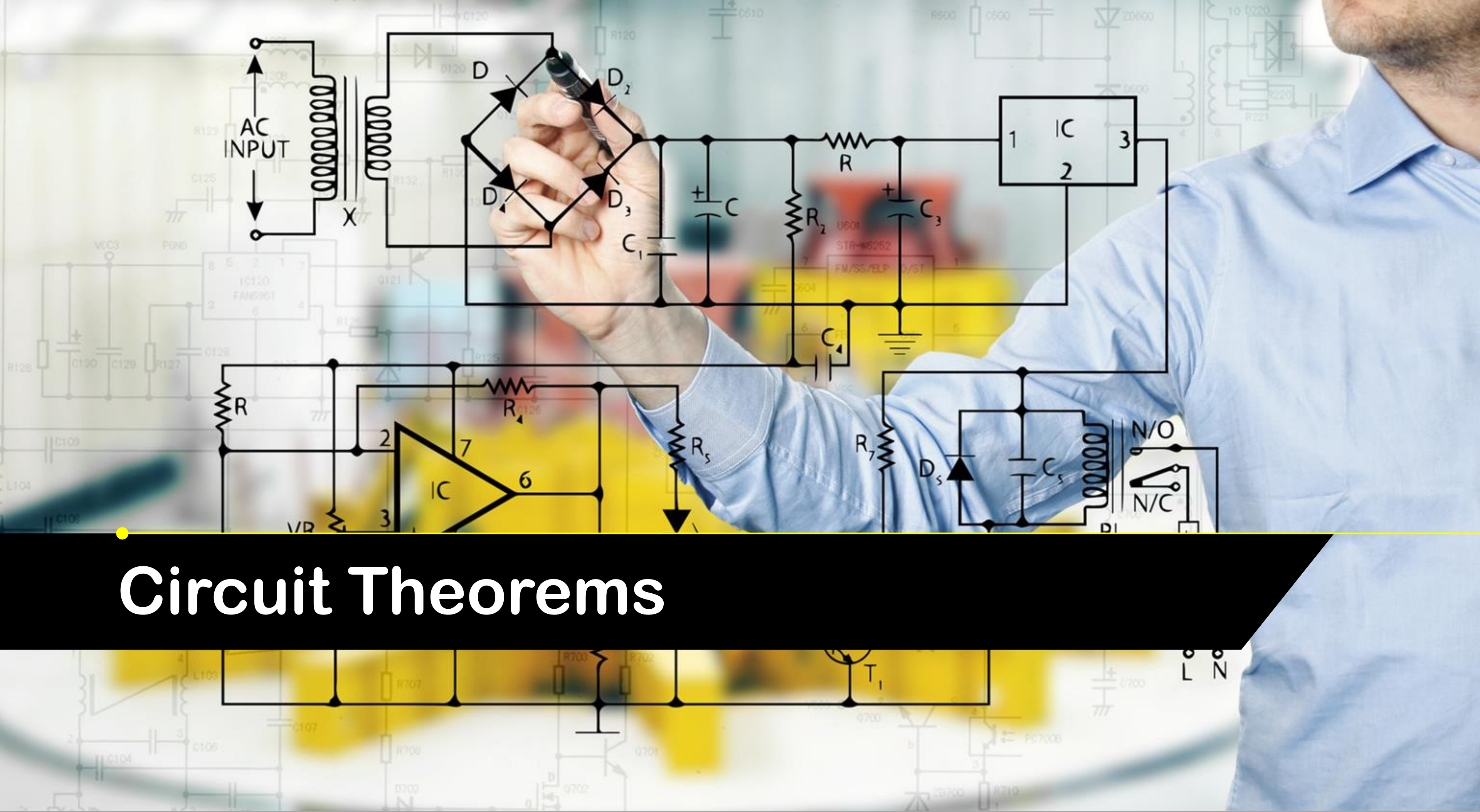
$$\boxed{6i_1 + 14i_2 = 20} \quad (1)$$

$$\boxed{i_1 = -3.2\ A} \qquad \boxed{i_2 = 2.8\ A}$$

Mesh Current With Current Source

Note: again, we could have used nodal analysis to find the currents i_1 and i_2.





Circuit Theorems

Circuit Theorems

Linearity Property

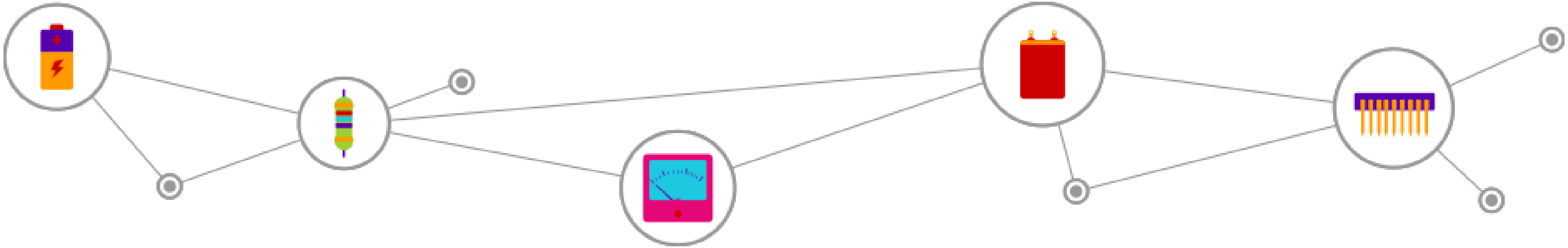
Superposition

Source Transformation

Thevenin’s Theorem

Norton’s Theorem

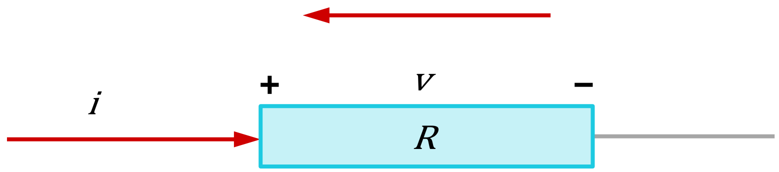
Maximum Power Transfer



Linearity Property

Consider a resistor R.

The v-i relationship is $v = Ri$.



If i is increased by a constant k, then the voltage increases correspondingly by k, i.e., $kv = kRi$ (it has the **scaling property**).

If $v_1 = Ri_1$, $v_2 = Ri_2$, then applying $i_1 + i_2$ gives

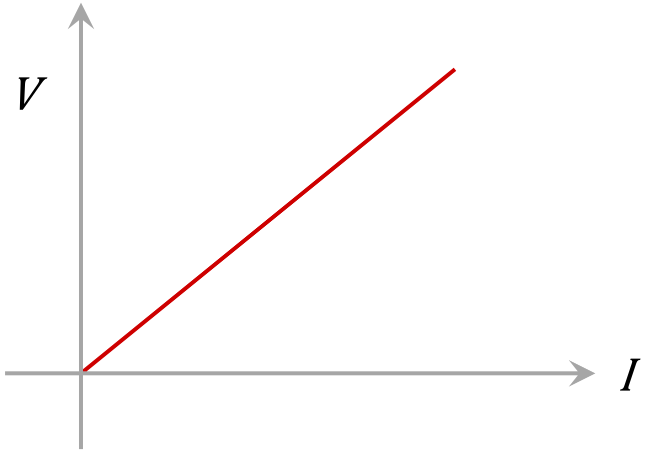
$$v = R(i_1+i_2) = Ri_1 + Ri_2 = v_1 + v_2$$

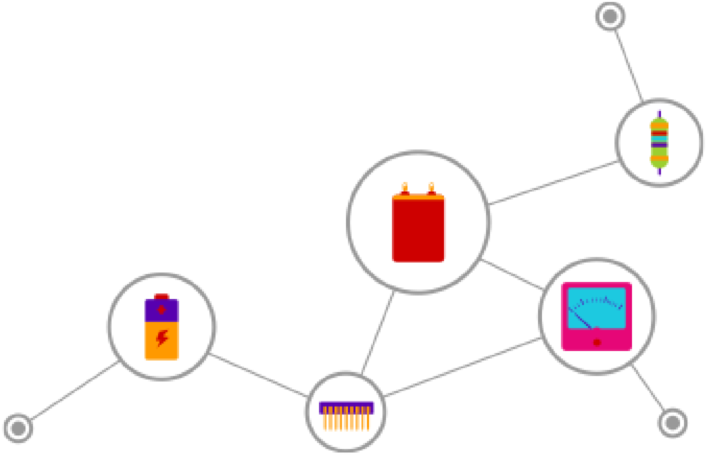
(it has the **additivity property**).

Linearity Property

The resistor is a linear element because it satisfies both the scaling and the additivity properties.

For $v = Ri$, if v is plotted as a function of i, the result is a straight line, i.e., the v-i relationship is linear.



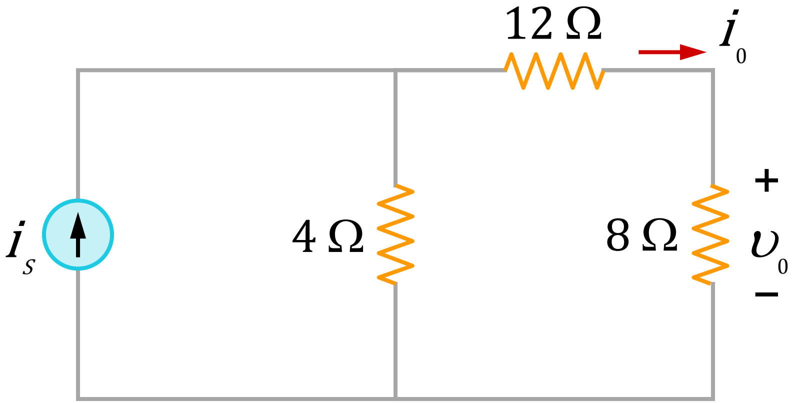


Linearity Property

- In general, a circuit is linear if it has both the additivity and scaling properties.
- A linear circuit is one whose output is directly proportional to its input.
- A linear circuit consists of linear elements, linear dependent sources and independent sources.
- A linear dependent source is a source whose output current or voltage is proportional only to the first power of a specified current or voltage variable in the circuit (i.e., $\propto i$ or $\propto v$).

Example 16

For the circuit shown, find v_o when $i_s = 15$ and $i_s = 30$ A.



By current division,

$$i_o = \frac{4}{4+12+8} i_s = \frac{1}{6} i_s$$

$$v_o = 8i_o = \frac{4}{3} i_s$$

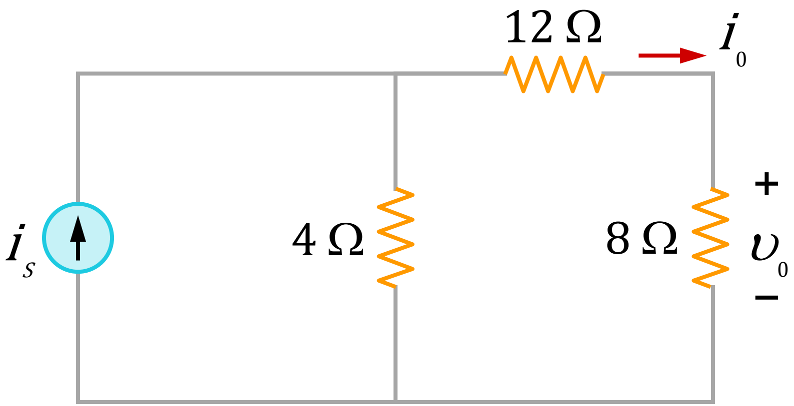
Example 16

When $i_s = 15$ A

$$\boxed{v_o = \frac{4}{3} i_s = \frac{4}{3}(15) = 20\ V}$$

When $i_s = 30$ A

$$\boxed{v_o = \frac{4}{3} i_s = \frac{4}{3}(30) = 40\ V}$$

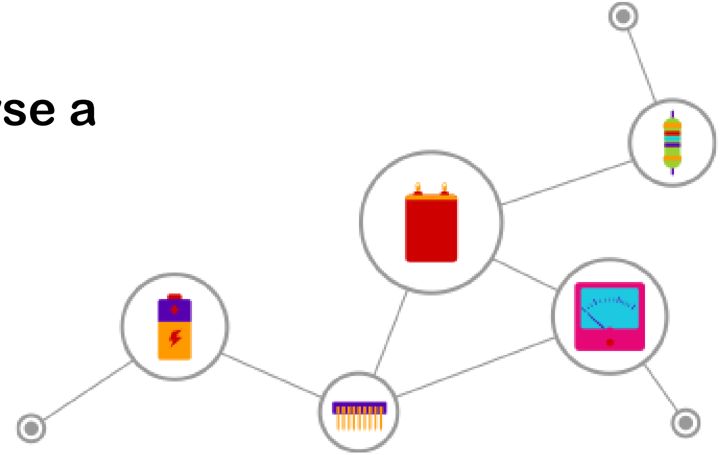


Note: this shows that when the input source value i_s is doubled, the output v_o will be doubled.

Superposition

The **superposition principle** states that the voltage across (or current through) an element in a linear circuit is the **algebraic sum** of the voltages across (or currents through) that element due **to each independent source acting alone**.

The superposition principle enables us to analyse a linear circuit with more than one independent source by calculating the contribution of each independent source separately.



Superposition

Steps to apply superposition principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques learned previously.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Superposition

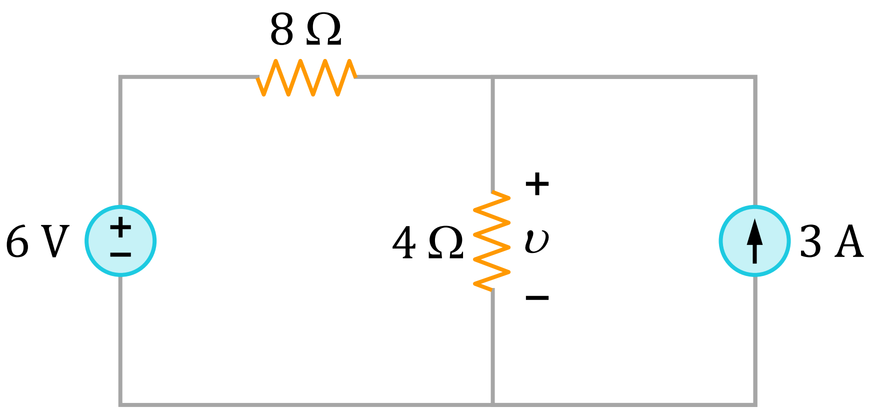
- We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by $0\ \mathrm{V}$ (or a short-circuit), and every current source by $0\ \mathrm{A}$ (or an open-circuit).
- One major disadvantage of using superposition is that it involves more work, although it helps to reduce a complex circuit to a simpler circuit.

Voltage Source → Short-circuit

Current Source → Open-circuit

Example 17

Use the superposition to find v in the following circuit.



Since there are two independent sources, let $v = v_1 + v_2$.

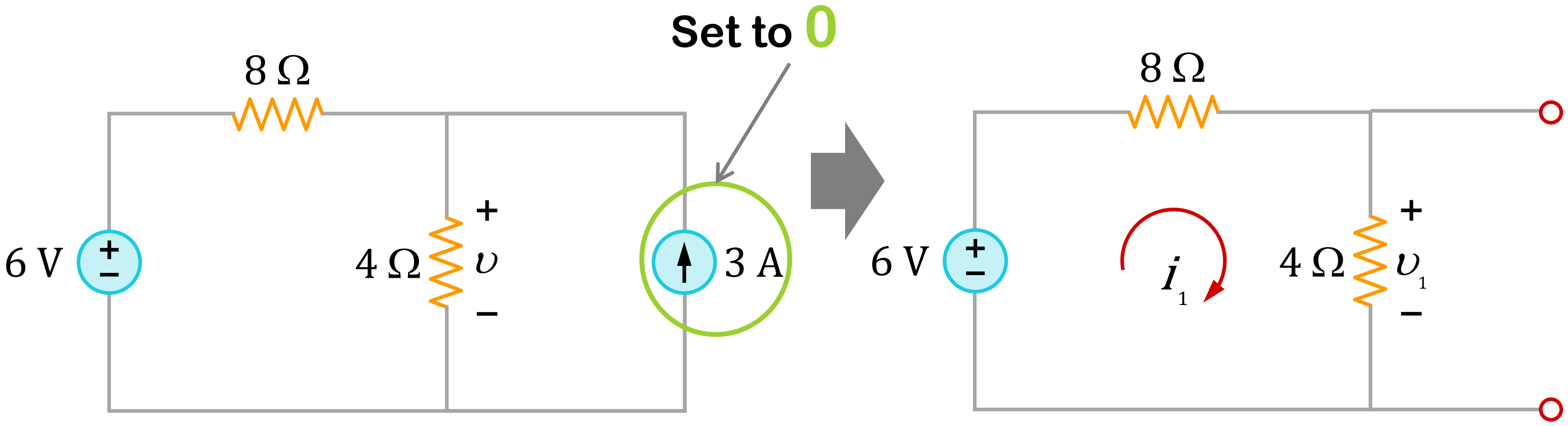
v_1 is the contribution due to the $6\ \mathrm{V}$ voltage source.

v_2 is the contribution due to the $3\ \mathrm{A}$ current source.

Example 17

To obtain v_1, we set the current source to zero as shown.

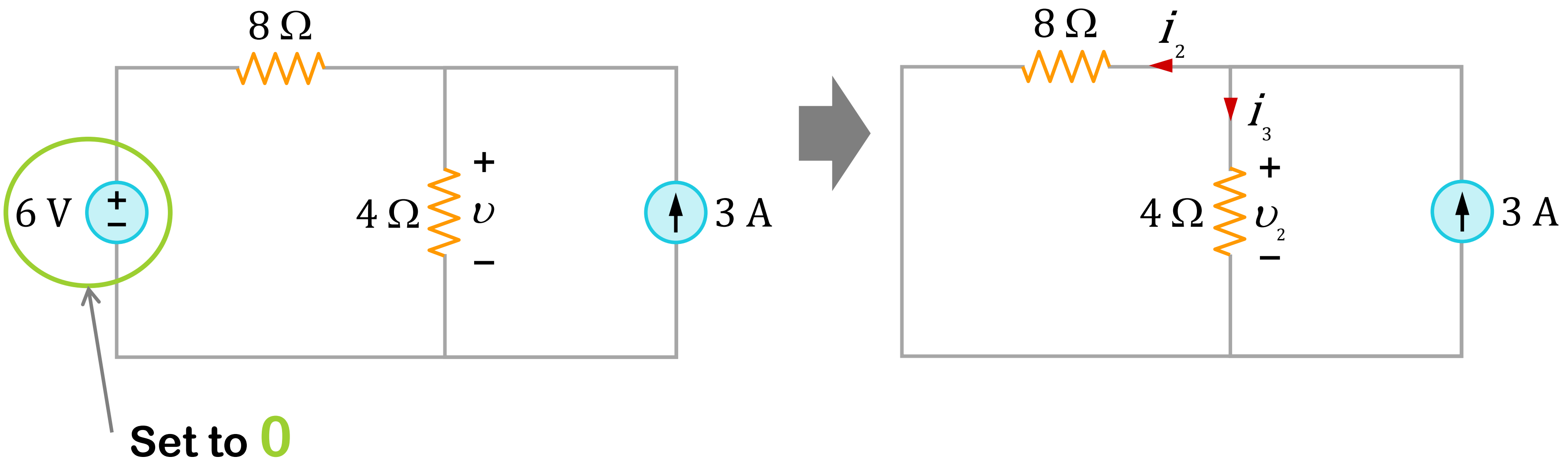
Using voltage division principle, $\boxed{v_1 = \frac{4}{4+8} 6 = 2\ V}$



Example 17

To obtain v_2, we set the voltage source to zero as shown.

Using current division principle, $\boxed{i_3 = \dfrac{8}{4+8}(3) = 2\ A}$



Example 17

With,

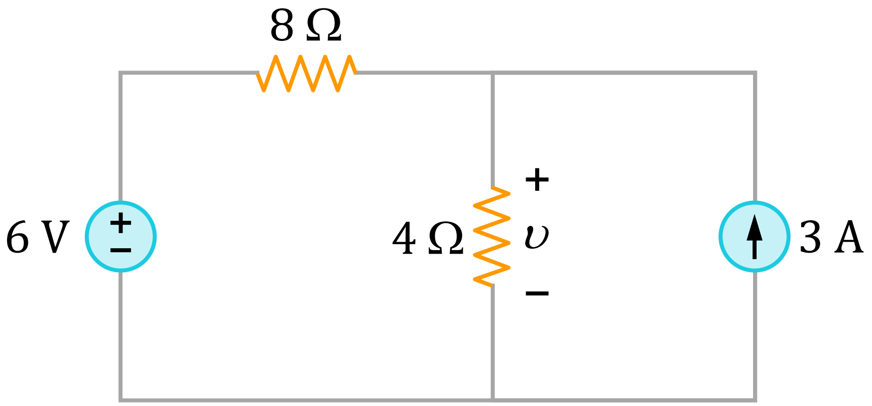
$$i_3 = \frac{8}{4+8}(3) = 2\ A$$

$$v_2 = 4i_3 = 8\ V$$

$$v_1 = \frac{4}{4+8}6 = 2\ V$$

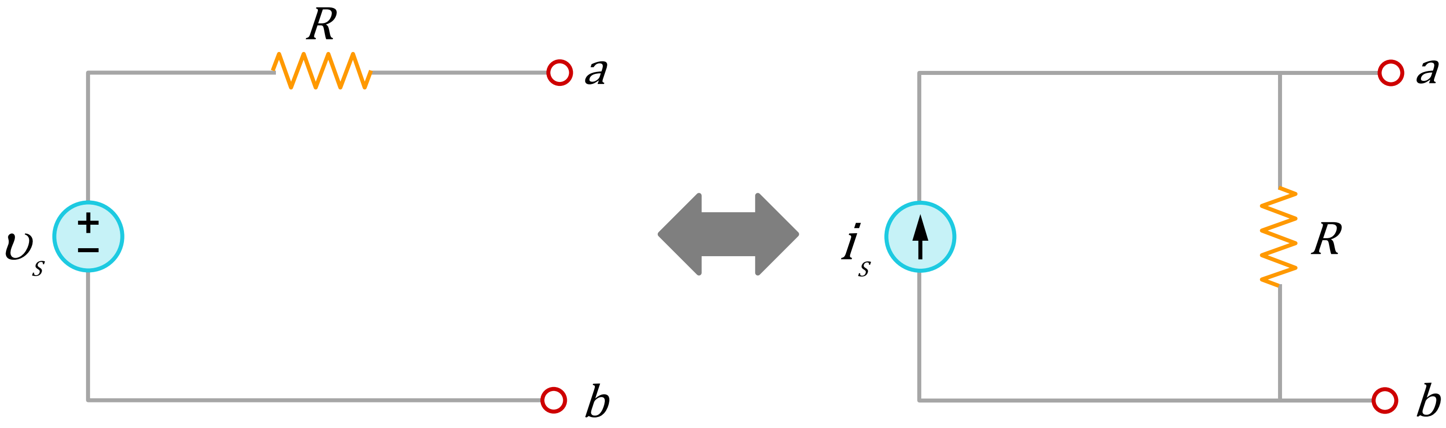
We have

$$v = v_1 + v_2 = 2 + 8 = 10\ V$$



Source Transformation

A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice-versa.

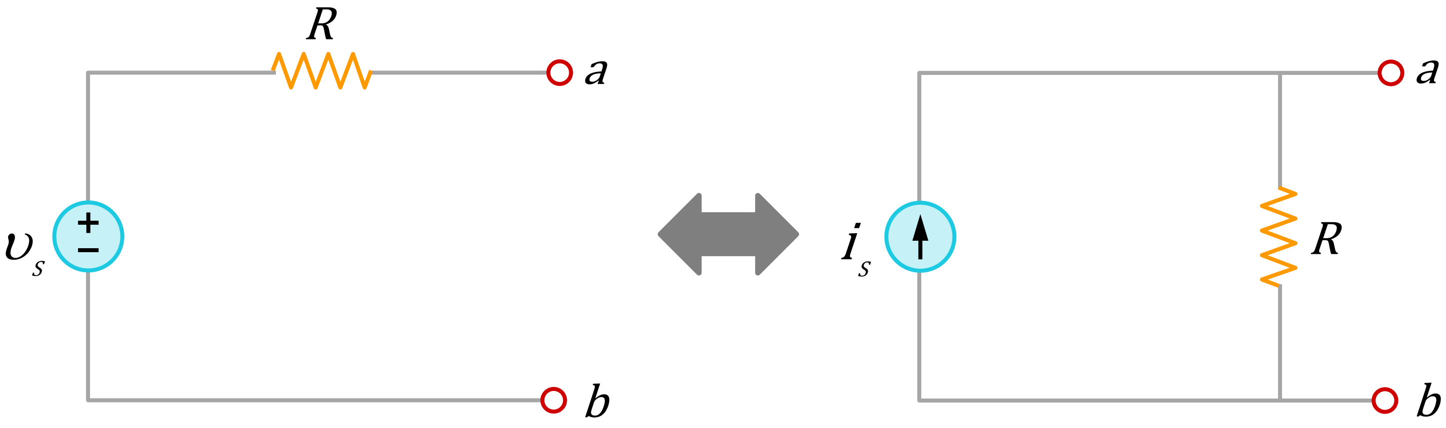


Transformation of Independent Sources

Source Transformation

The two circuits are equivalent since they have the same v-i relationship at terminals a-b.

If the sources are turned off, the equivalent resistance at terminals a-b in both circuits is R.



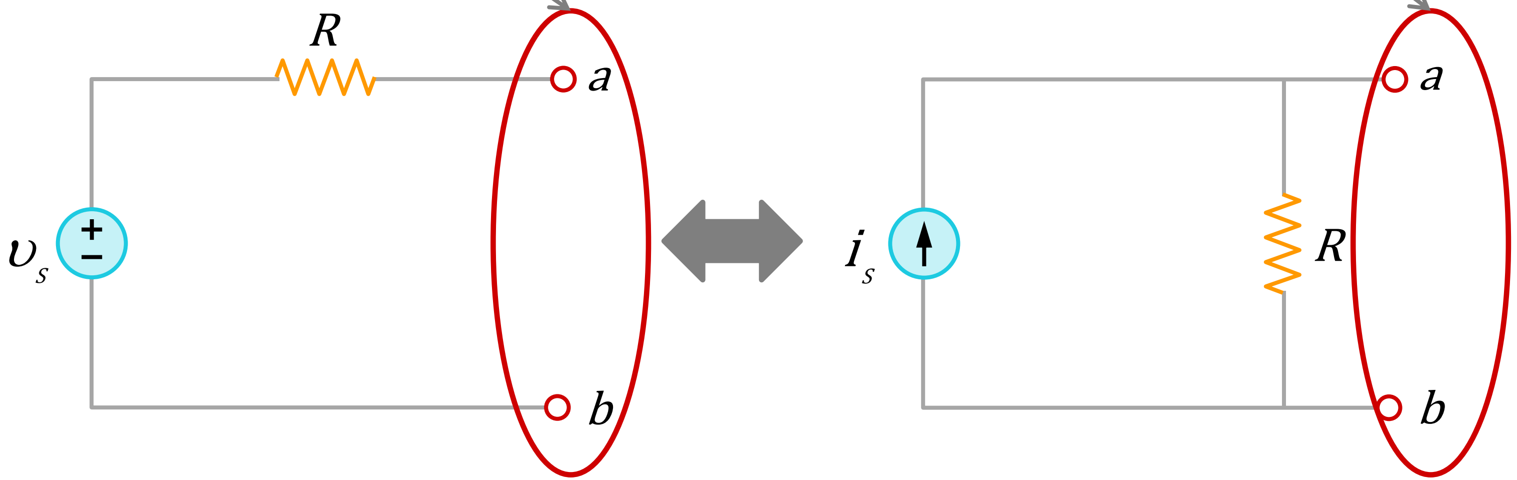
Transformation of Independent Sources

Source Transformation

Also, when terminals a-b are short-circuited, the short-circuit current i_{sc} flowing from a-b is

$i_{sc} = v_s/R$ in the first circuit

$i_{sc} = i_s$ for the second circuit



Transformation of Independent Sources

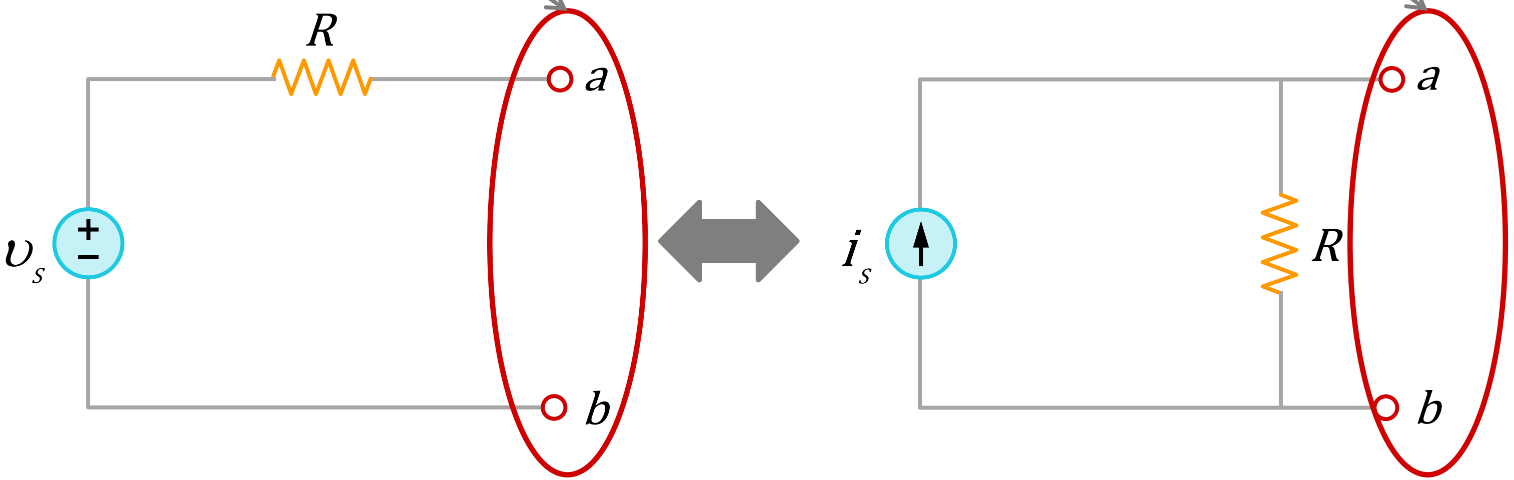
Source Transformation

Thus, in order for the two circuits to be equivalent, we must have

$$\boxed{\frac{v_s}{R} = i_s}$$

$i_{sc} = v_s/R$ **in the first circuit**

$i_{sc} = i_s$ **for the second circuit**

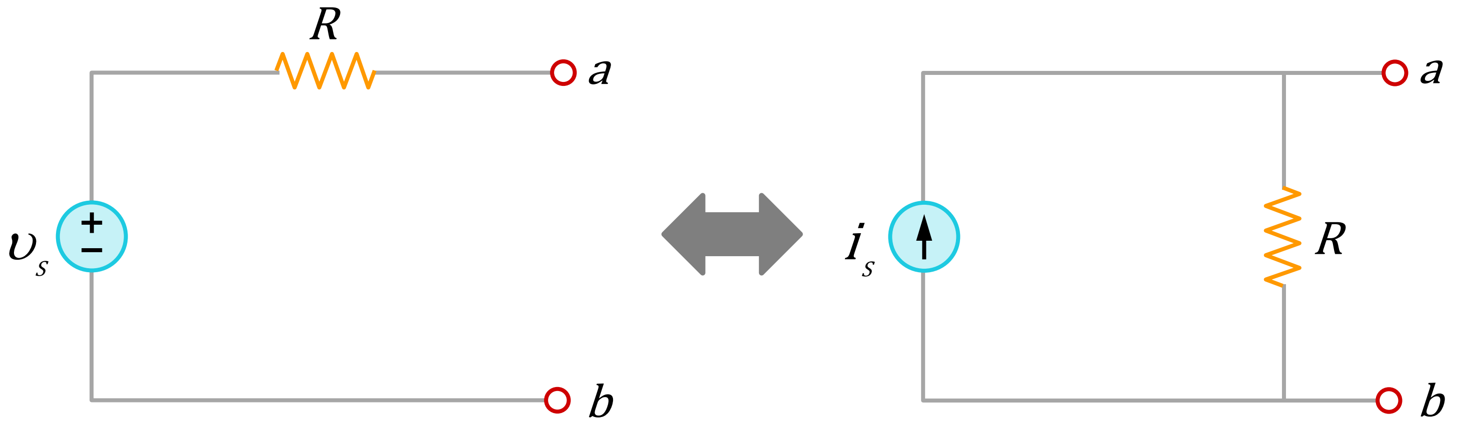


Transformation of Independent Sources

Source Transformation

Source transformation requires that $\boxed{\dfrac{v_s}{R} = i_s}$

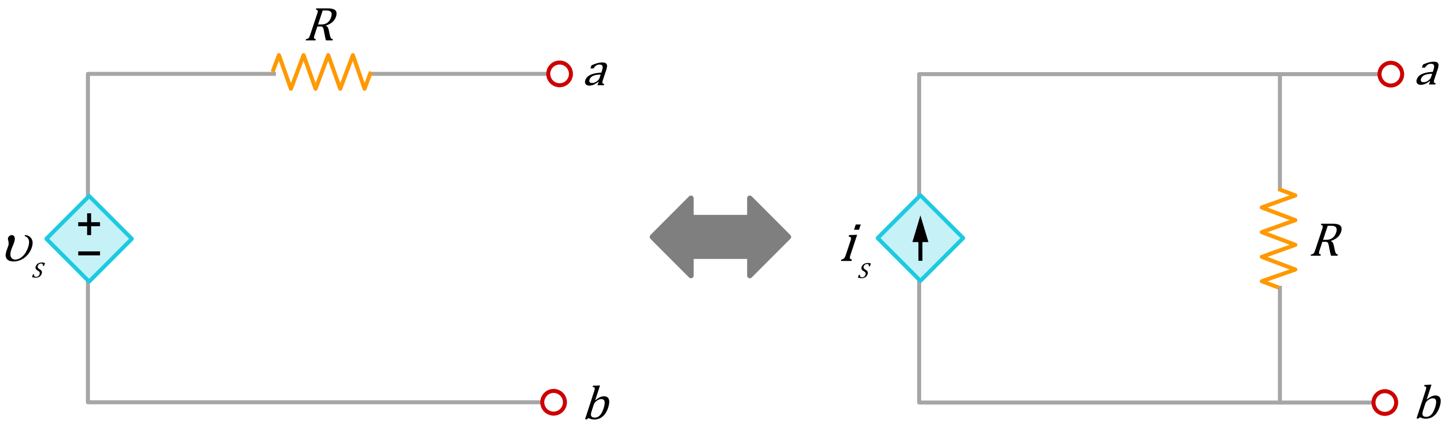
Note: the arrow of the current source is directed toward the positive terminal of the voltage source.



Transformation of Independent Sources

Source Transformation

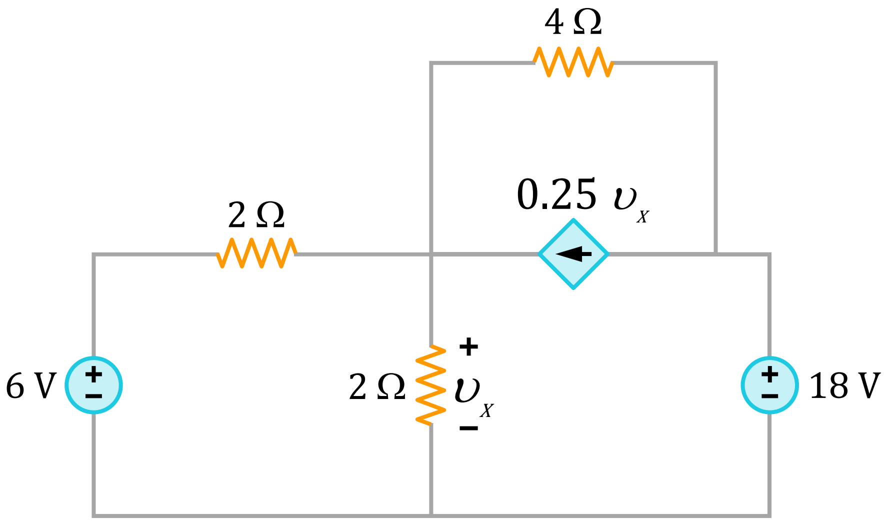
Source transformation also applies to dependent sources, provided we carefully handle the dependent variable, i.e., $i_s = \frac{v_s}{R}$.



Transformation of Dependent Sources

Example 18

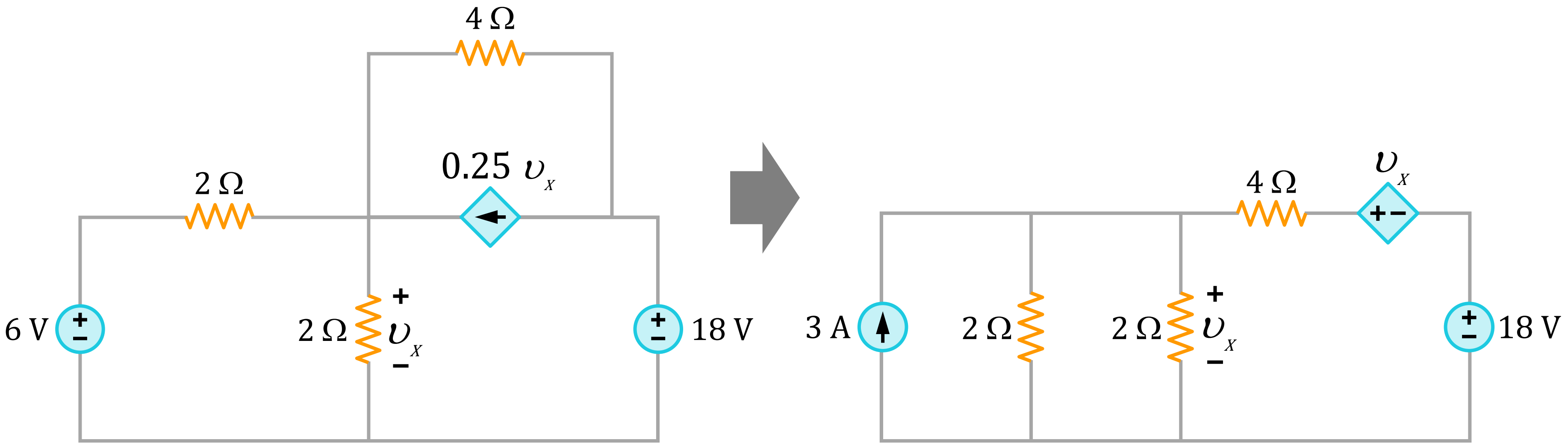
Find v_x in the following circuit using source transformation.



Example 18

Transform the dependent current source and the $6\ \mathrm{V}$ independent source as shown.

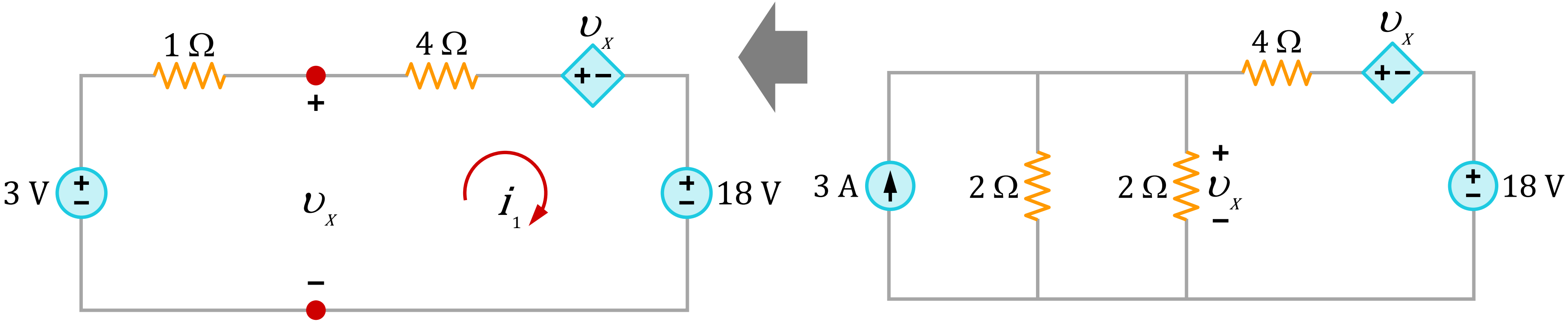
Note that the $18\ \mathrm{V}$ voltage source is not transformed because it is not connected in series with any resistors.



Example 18

The two $2\,\Omega$ resistors in parallel are combined to give a $1\,\Omega$ resistor, which is in parallel with the $3\,\mathrm{A}$ current source.

The current source is transformed to a voltage source as shown.



Example 18

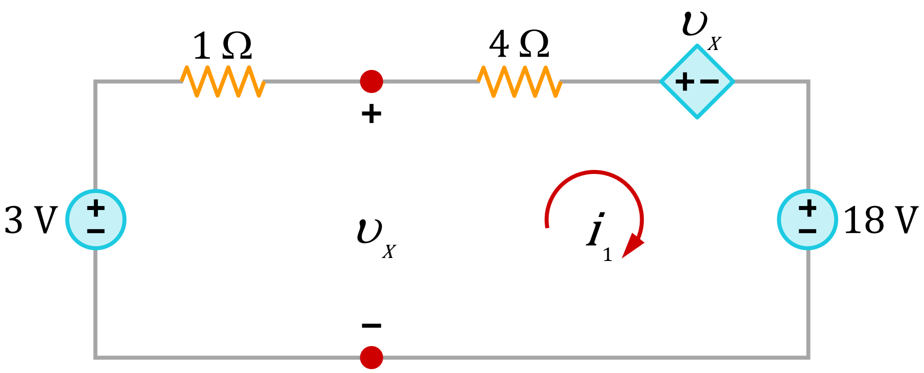
Applying KVL around the loop gives

$$\boxed{-3 + (1 + 4)i + v_x + 18 = 0} \quad \textbf{(1)}$$

Applying KVL to the loop containing only the $3\ \mathrm{V}$ voltage source, the $1\ \Omega$ resistor, and v_x yields,

$$\boxed{-3 + 1i + v_x = 0}$$

$$\boxed{v_x = 3 - i} \quad \textbf{(2)}$$



Using (2) in (1) gives

$$\boxed{-15 + 5i + 3 - i = 0}$$

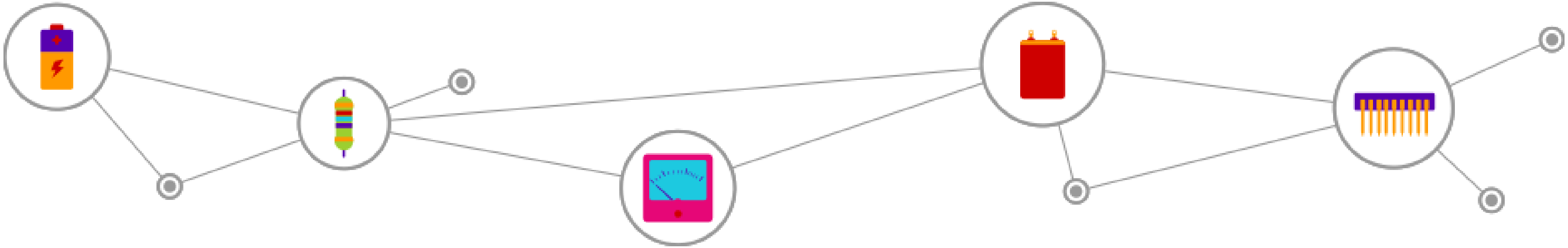
$$\boxed{i = -4.5\ A} \rightarrow \boxed{v_x = 7.5\ V}$$

Thevenin’s Theorem

In practice, only the load of a circuit is variable while the rest of the circuits are fixed.

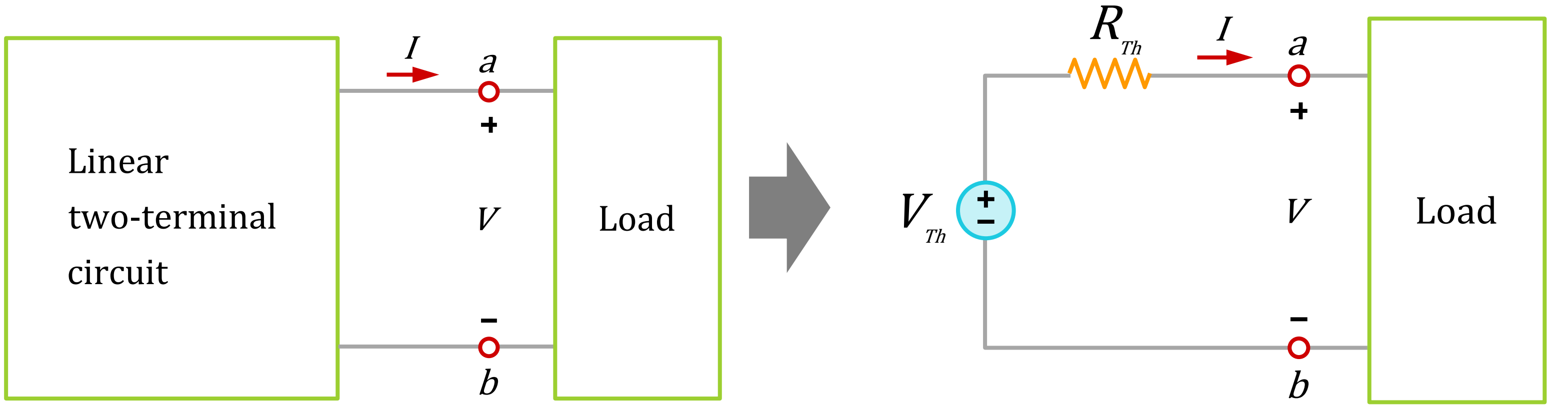
Each time the load is changed, the entire circuit needs to be analysed all over again.

To avoid this problem, the Thevenin’s theorem provides a technique whereby the fixed part of the circuit is replaced by an equivalent circuit.

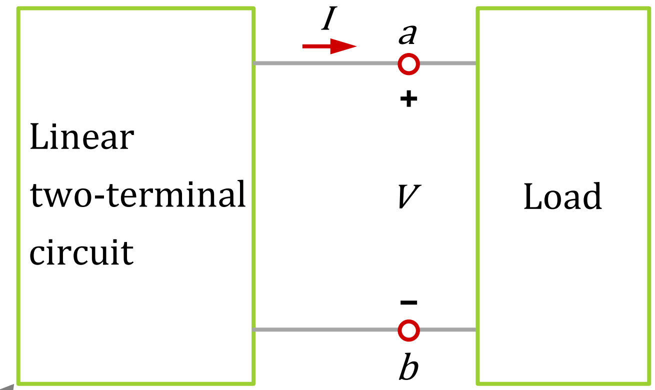


Thevenin's Theorem

Thevenin's Theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the **independent sources** are turned off.

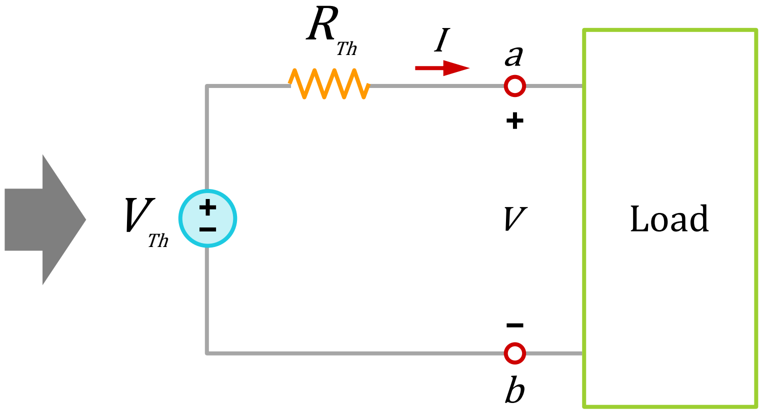


How to Find V_{Th} and R_{Th}?



1(a) Original Circuit

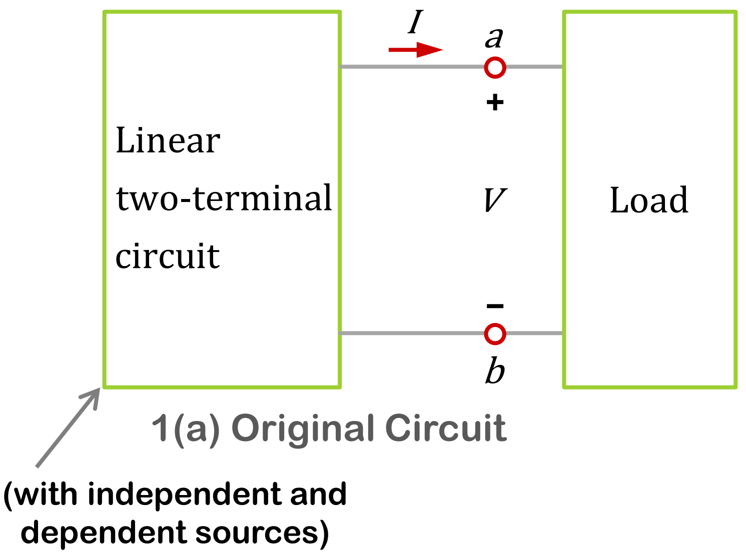
(with independent and dependent sources)



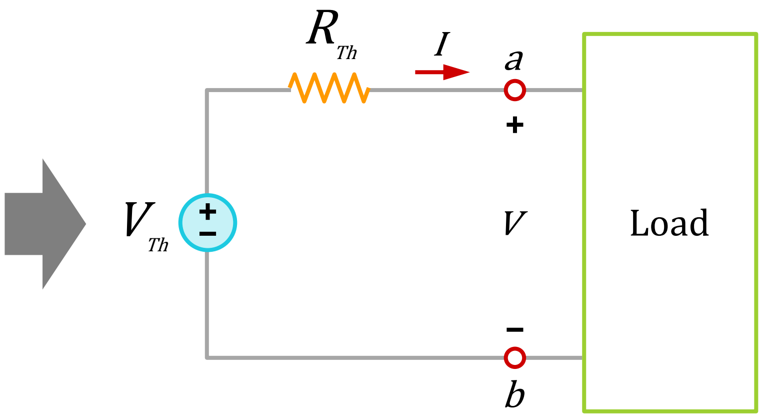
1(b) Thevenin Equivalent Circuit

Suppose Figs. 1(a) and 1(b) are equivalent, i.e., they have the same v-i relationship at their terminals.

How to Find V_{Th} and R_{Th}?



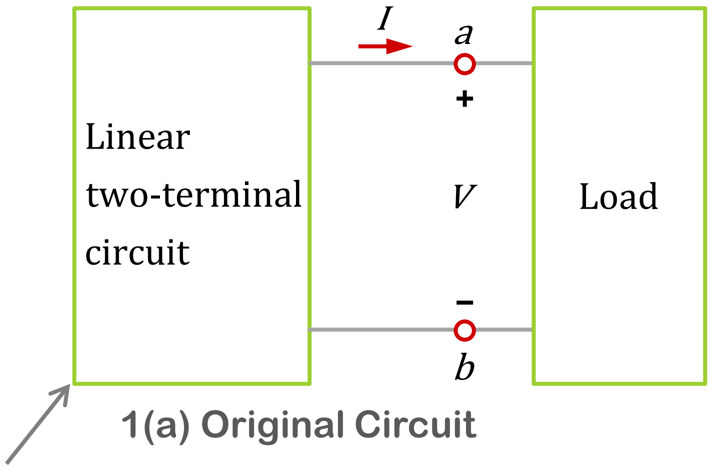
1(a) Original Circuit



1(b) Thevenin Equivalent Circuit

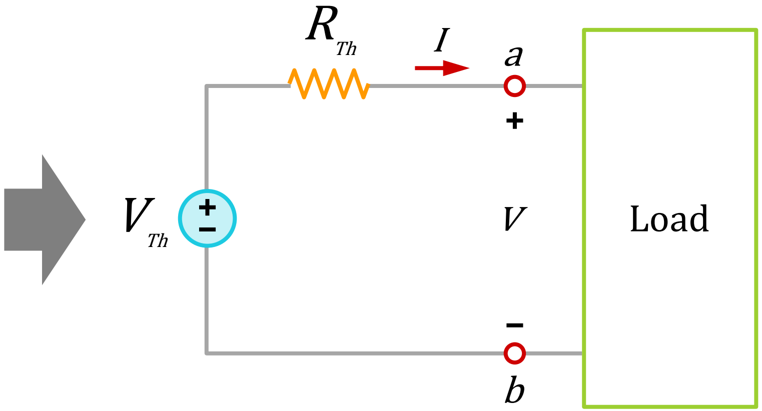
When the terminals a-b are made open-circuited (by removing the load), no current flows so that the open-circuit voltage across the terminals a-b in Fig. 1(a) must be equal to the voltage source in Fig. 1(b) since the two circuits are equivalent.

How to Find V_{Th} and R_{Th}?



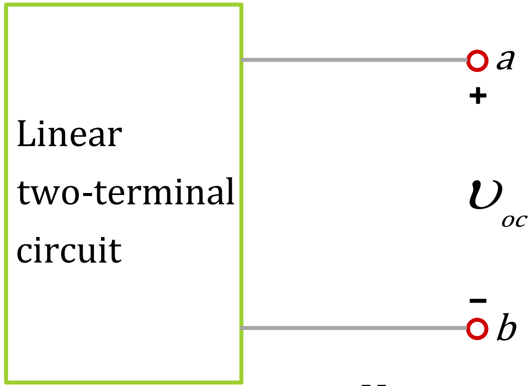
1(a) Original Circuit

(with independent and dependent sources)



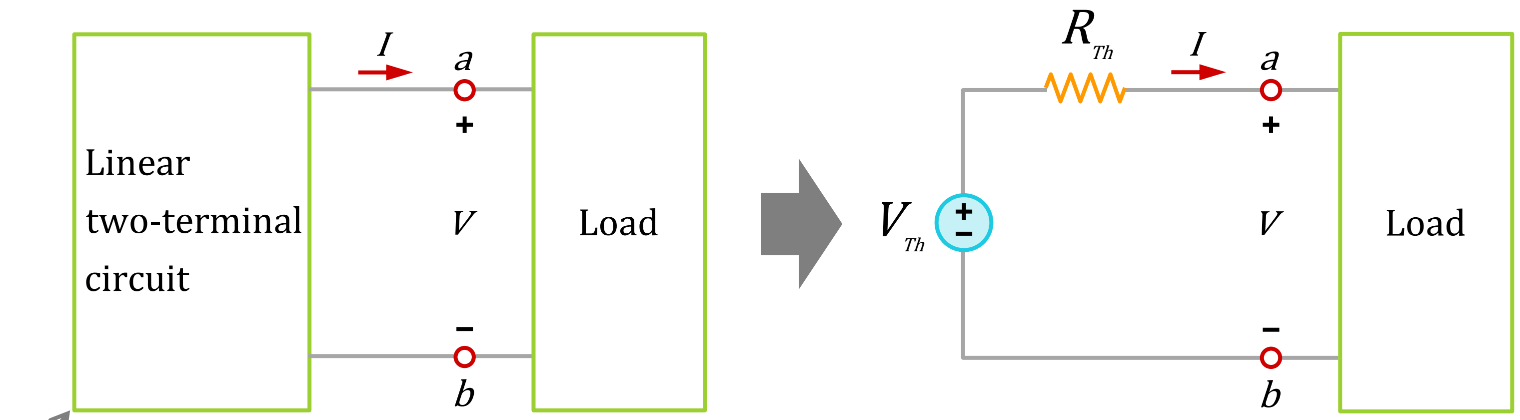
1(b) Thevenin Equivalent Circuit

Thus, V_{Th} is the open-circuit voltage across the terminals as shown on Fig. 2(a), i.e., $V_{Th} = v_{oc}$.



2(a) $V_{Th} = v_{oc}$

How to Find V_{Th} and R_{Th}?



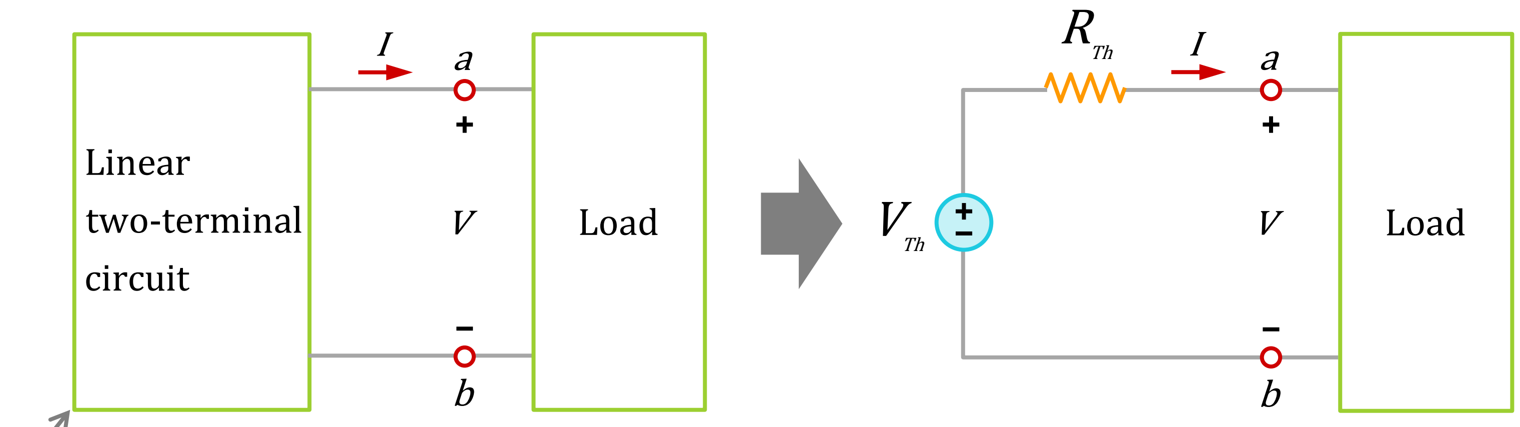
1(a) Original Circuit

1(b) Thevenin Equivalent Circuit

(with independent and dependent sources)

Again, with the load removed and terminals a-b open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals a-b of Fig. 1(a) must be equal to R_{Th} in Fig. 1(b) because the circuits are equivalent.

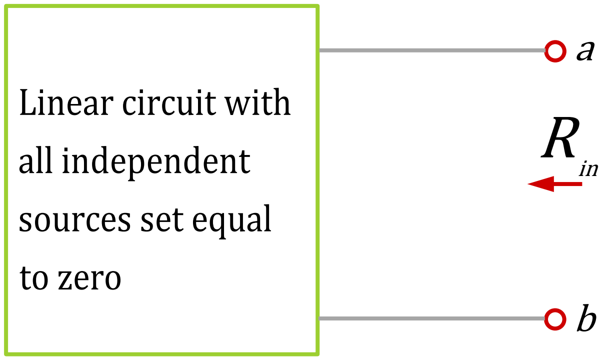
How to Find V_{Th} and R_{Th}?



1(a) Original Circuit

1(b) Thevenin Equivalent Circuit

(with independent and dependent sources)



2(b) $R_{Th} = R_{in}$

Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off as shown in Fig. 2(b), i.e., $R_{Th} = R_{in}$.

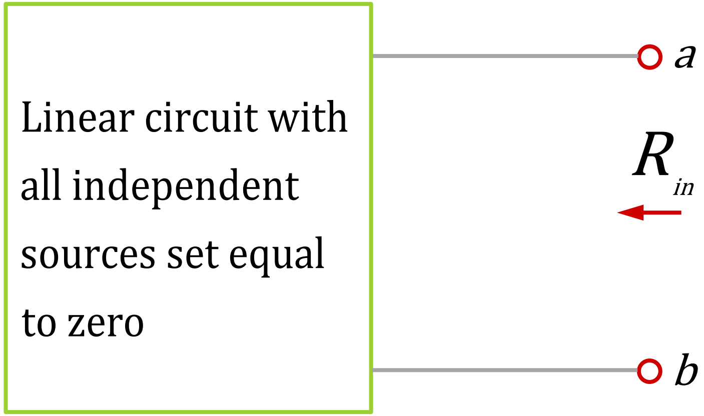
How to Find R_{Th}?

To find the Thevenin resistance R_{Th}, we need to consider two cases.

Case One

If the circuit has no dependent sources, we turn off all independent sources.

R_{Th} is the input resistance of the circuit seen between terminals a-b as shown in Fig. 2(b).



2(b) $R_{Th} = R_{in}$

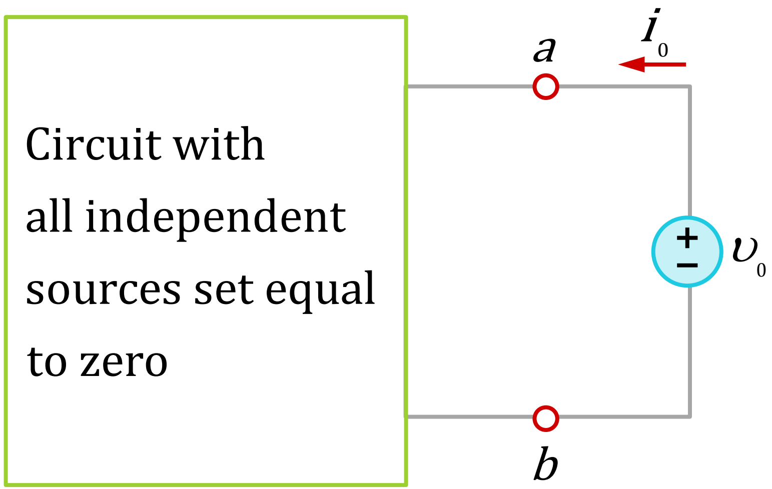
How to Find R_{Th}?

Case Two

If the circuit has dependent sources, we turn off all independent sources. Dependent sources are not turned off because they are controlled by circuit variables.

We apply a known voltage source v_o (say, $1\ \mathrm{V}$) at terminals a-b and determine the resulting current i_o.

Then, $R_{Th} = v_o / i_o$ as shown.

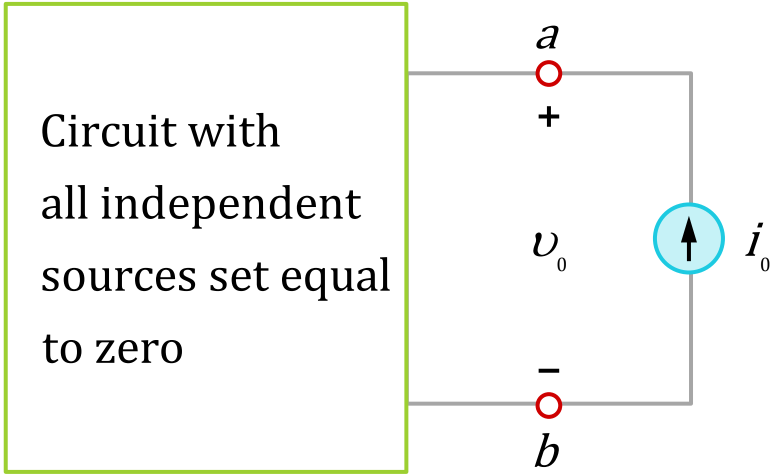


$$R_{Th} = \frac{v_0}{i_0}$$

How to Find R_{Th}?

Case Two

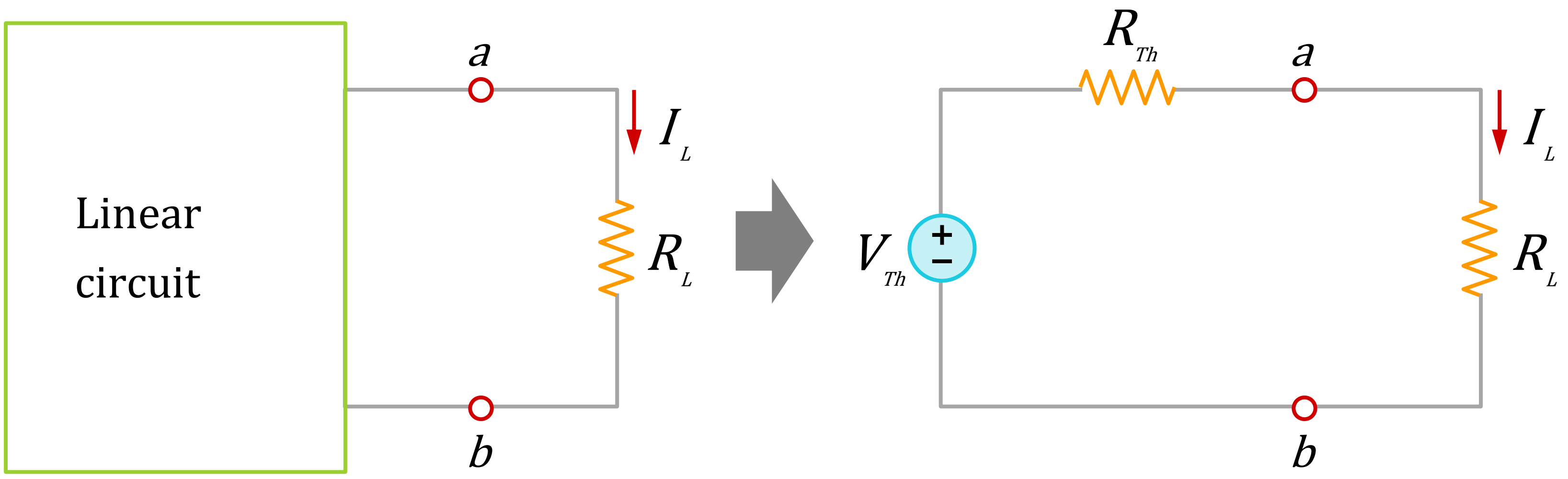
Alternatively, we may insert a known current source i_o (say, $1\ \mathrm{A}$) at terminals a-b as shown and find the terminal voltage v_o and $R_{Th} = v_o/i_o$.



$$R_{Th} = \frac{v_0}{i_0}$$

Use of Thevenin Equivalent Circuit

Consider a linear circuit terminated by a load R_L as shown.

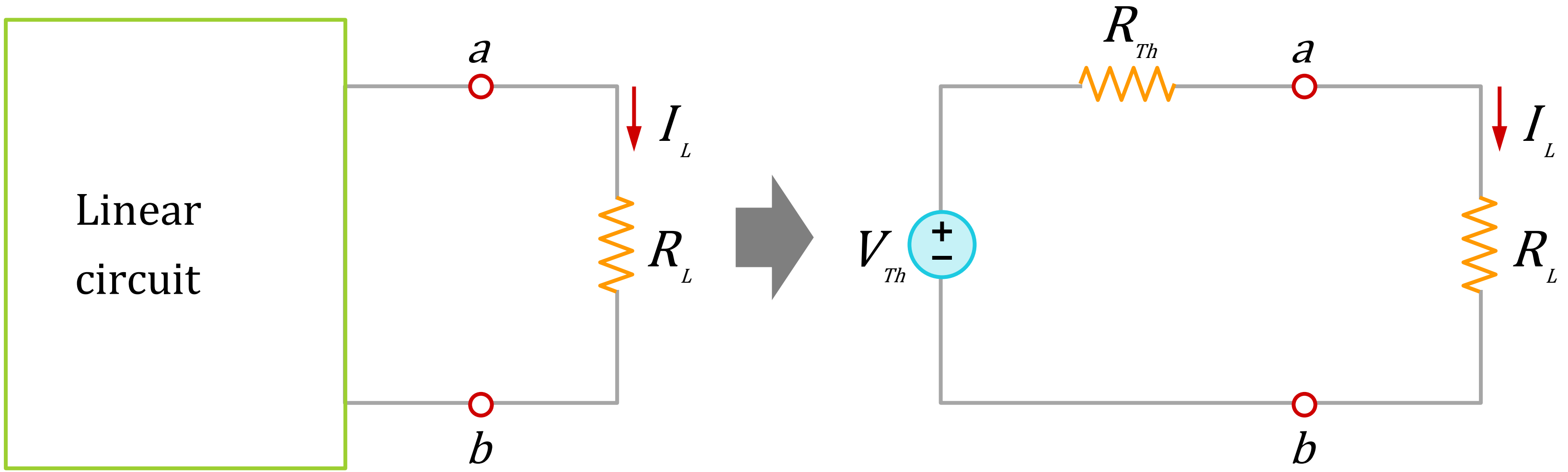


Use of Thevenin Equivalent Circuit

The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent circuit at the load's terminals is obtained as follows:

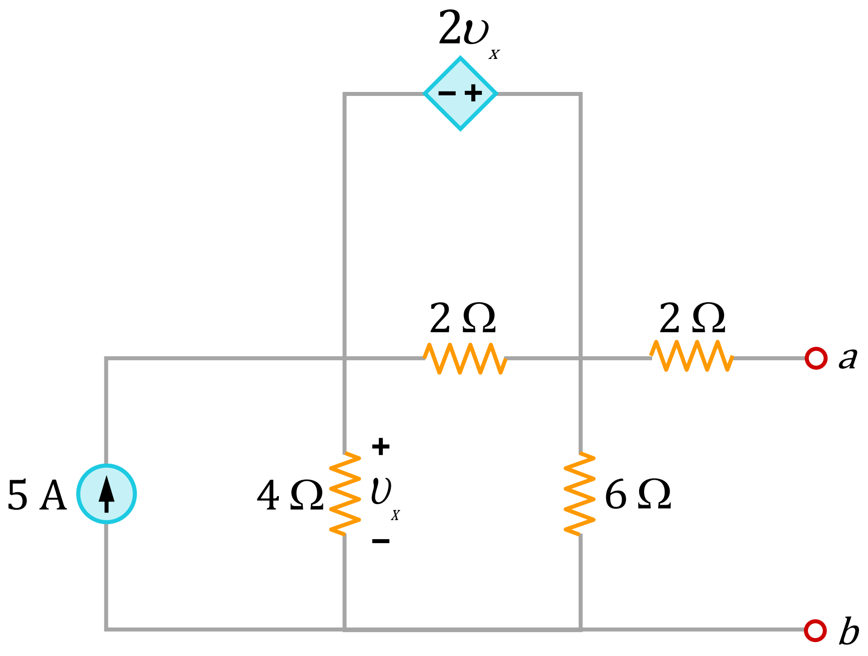
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$



Example 19

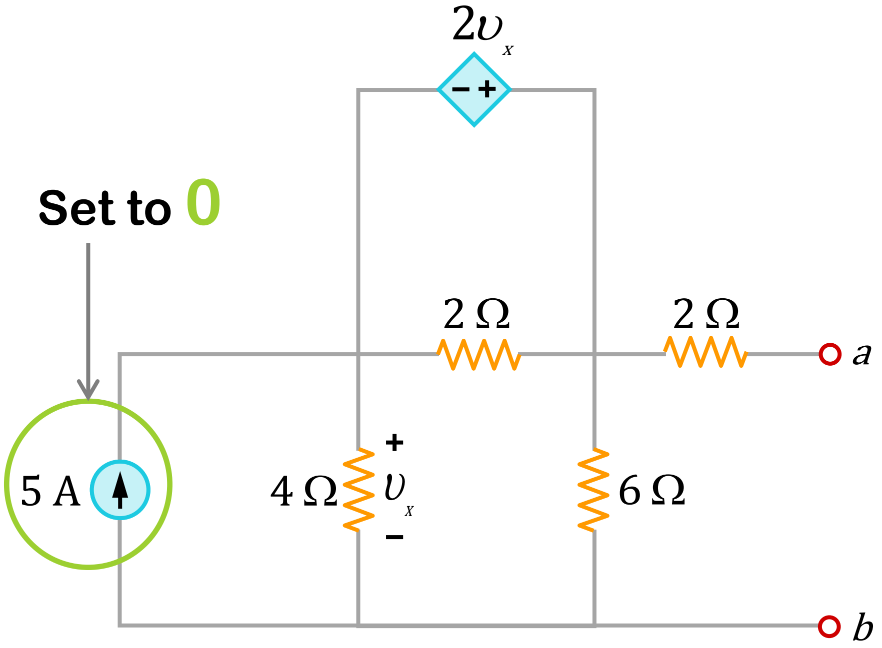
Find the Thevenin equivalent circuit as shown in the terminals a-b.

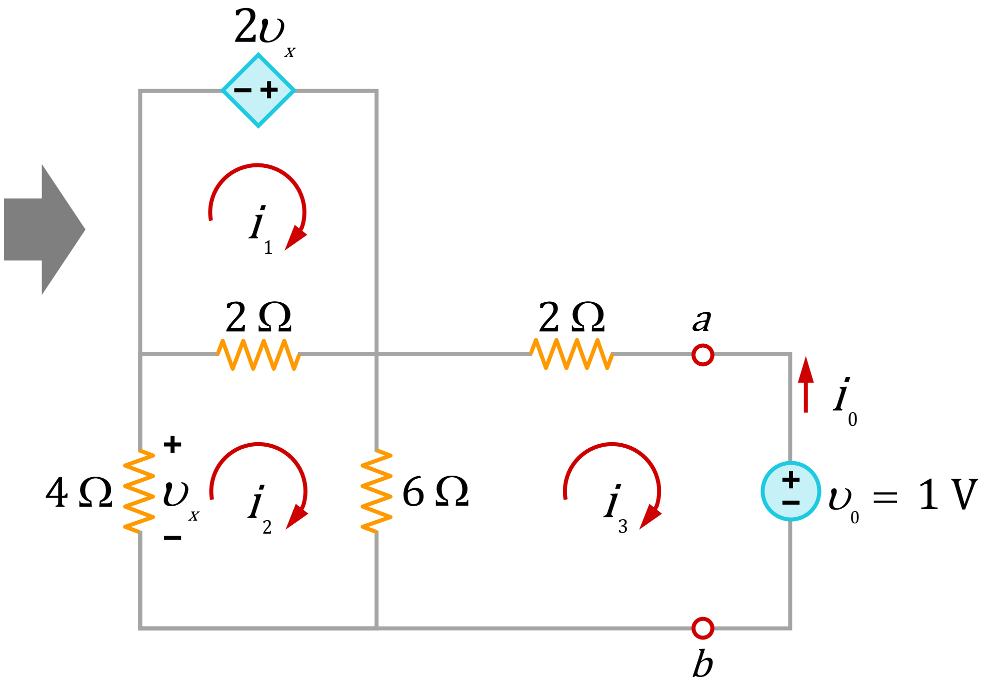


Example 19

The circuit contains a dependent voltage source.

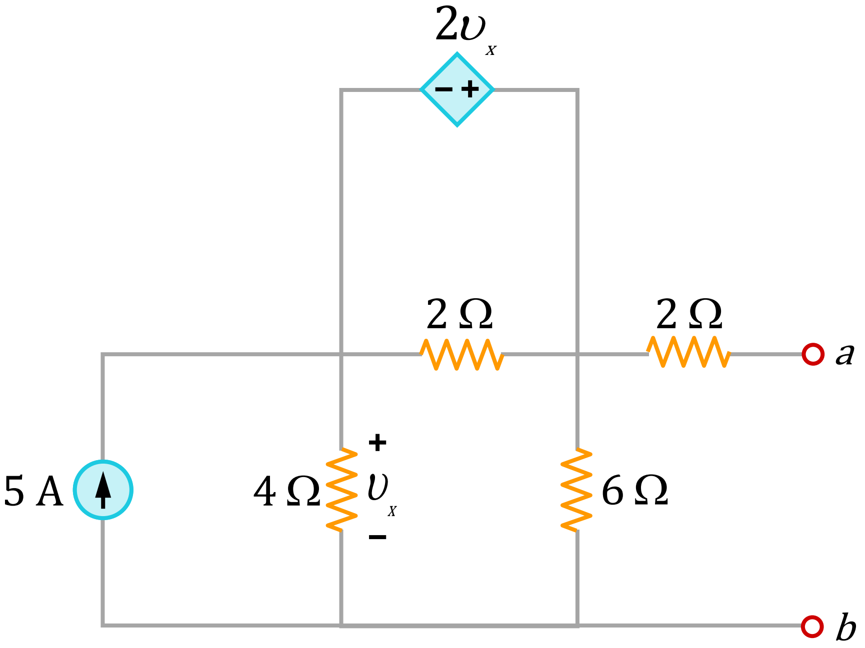
To find R_{Th}, set the independent current source equal to zero but leave the dependent voltage source alone.

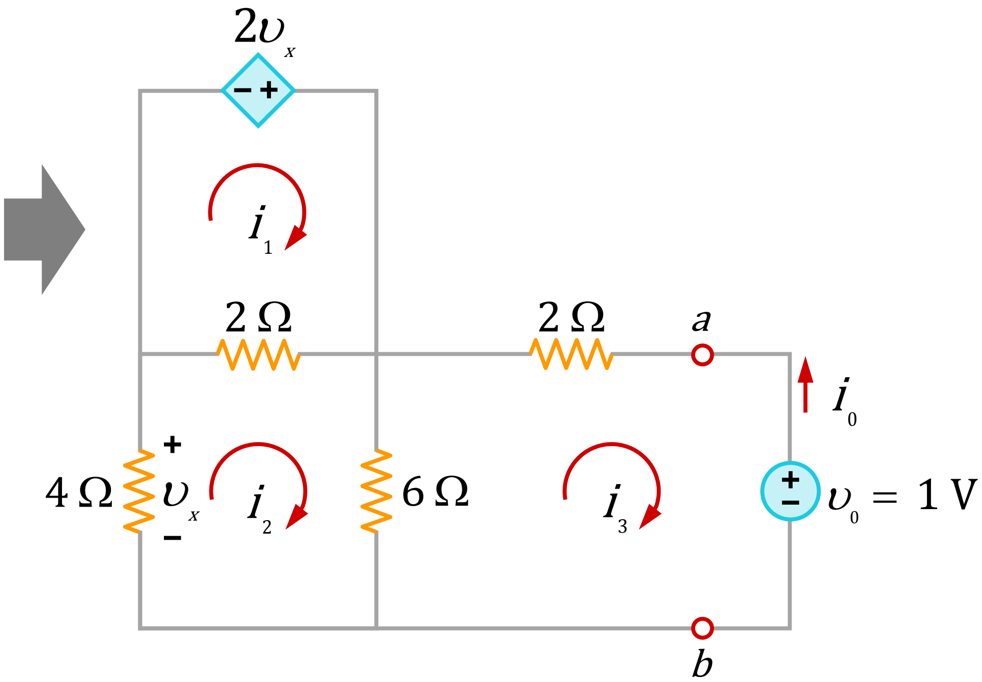




Example 19

Furthermore, we excite the circuit with a voltage source $v_o = 1\ \mathrm{V}$ across the terminals a-b as shown.





Example 19

The aim is to find the current i_o through the terminals a-b and then obtain $R_{Th} = v_o/i_o$.

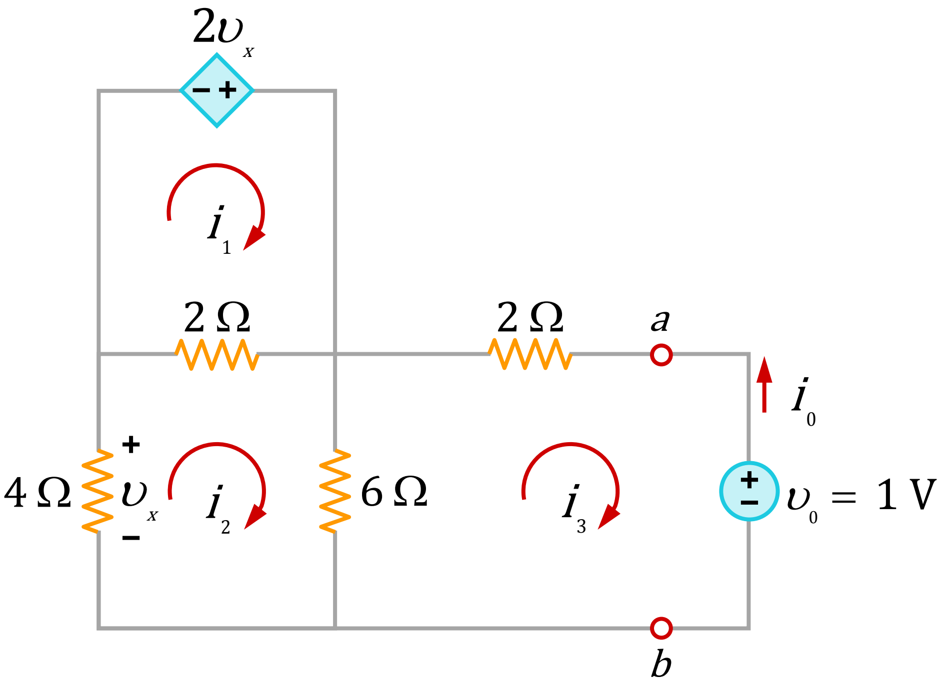
Applying mesh analysis to the circuit:

Mesh 1: $-2v_x + 2(i_1 - i_2) = 0$

$$v_x = i_1 - i_2$$

But, $v_x = -4i_2 = i_1 - i_2$

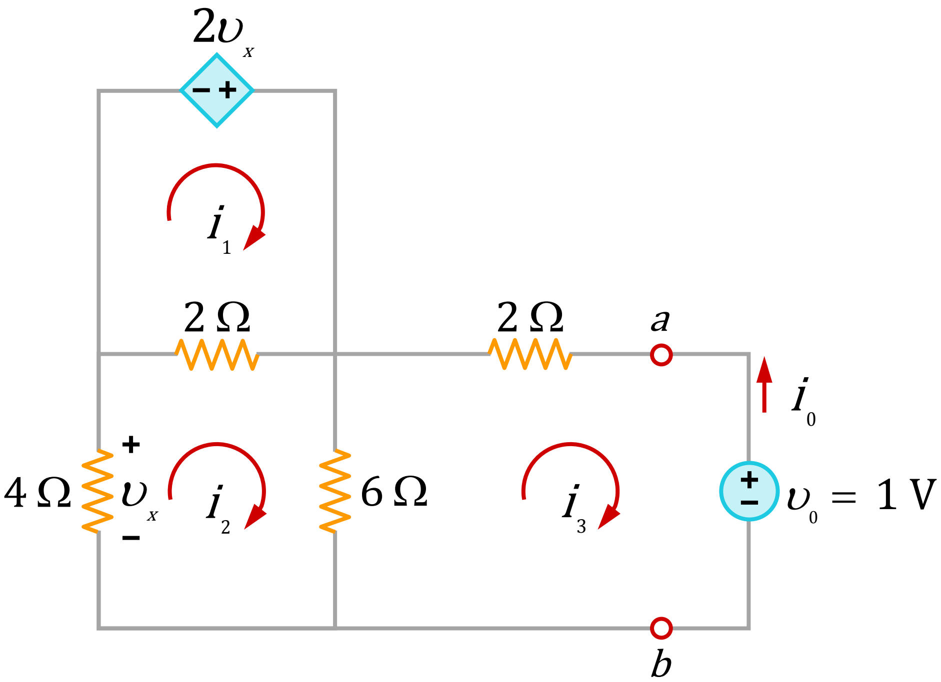
$$i_1 = -3i_2$$



Example 19

Mesh 2: $$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

Mesh 3: $$6(i_3 - i_2) + 2i_3 + 1 = 0$$



Example 19

These equations can be written as follows:

Mesh 1: $i_1 = -3i_2$ $\Rightarrow$ $i_1 + 3i_2 = 0$

Mesh 2: $4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$ $\Rightarrow$ $-2i_1 + 12i_2 - 6i_3 = 0$

Mesh 3: $6(i_3 - i_2) + 2i_3 + 1 = 0$ $\Rightarrow$ $-6i_2 + 8i_3 = -1$

But, we only need to solve for $i_3 = -i_o$ **and solving it gives**

$$i_3 = -\frac{1}{6}\ \mathrm{A} \quad \Rightarrow \quad i_o = \frac{1}{6}\ \mathrm{A}$$

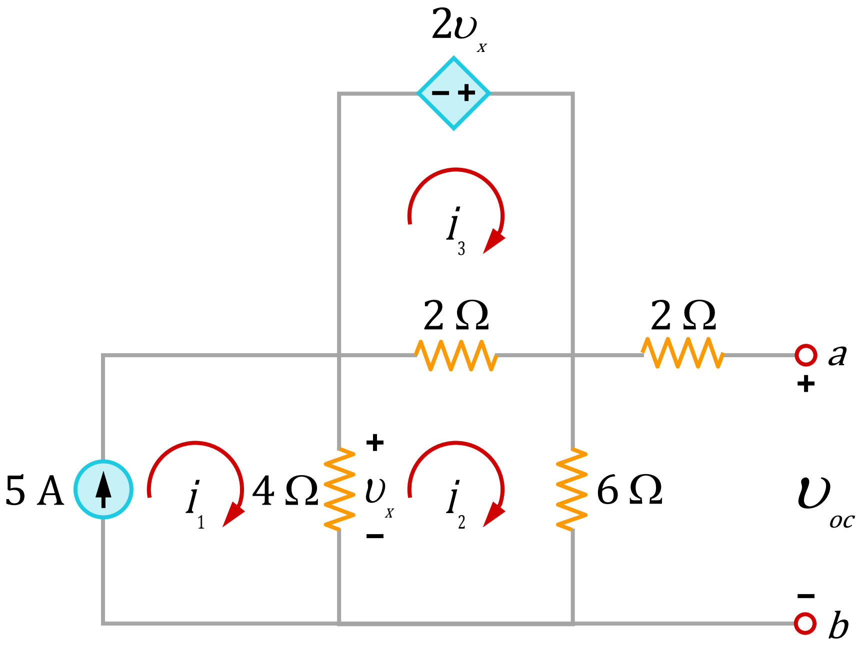
Hence,

$$\boxed{R_{Th} = \frac{v_o}{i_o} = \frac{1}{1/6} = 6\ \Omega}$$

Example 19

To get V_{Th}, we find v_{oc} in the circuit as shown.

Note that no current flows through the $2\ \Omega$ resistor on the right (with load removed).



Example 19

Applying mesh analysis :

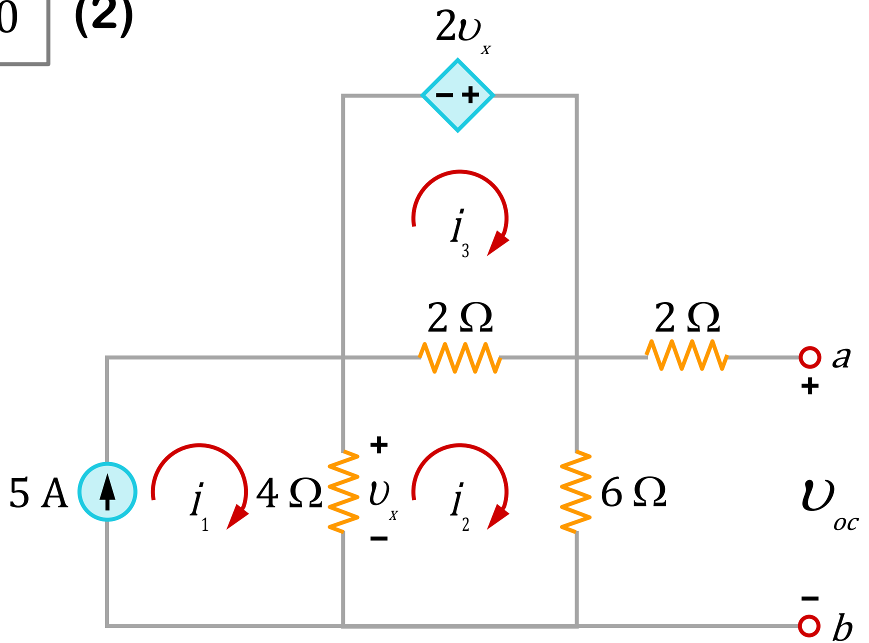
Mesh 1: $$i_1 = 5\text{ A}$$ **(1)**

Mesh 2: $$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$ **(2)**

Mesh 3: $$-2v_x + 2(i_3 - i_2) = 0$$

$$v_x = i_3 - i_2$$

But, $$v_x = 4(i_1 - i_2) = i_3 - i_2$$ **(3)**



Example 19

From (1) – (3) we get,

Mesh 1: $\boxed{i_1 = 5\ \mathrm{A}}$ **(1)**

Mesh 2: $\boxed{4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0}$ **(2)** $\Rightarrow$ $\boxed{12i_2 - 2i_3 = 20}$

Mesh 3: $\boxed{v_x = 4(i_1 - i_2) = i_3 - i_2}$ **(3)** $\Rightarrow$ $\boxed{3i_2 + i_3 = 20}$

Example 19

Solving for i_2 from the equations gives

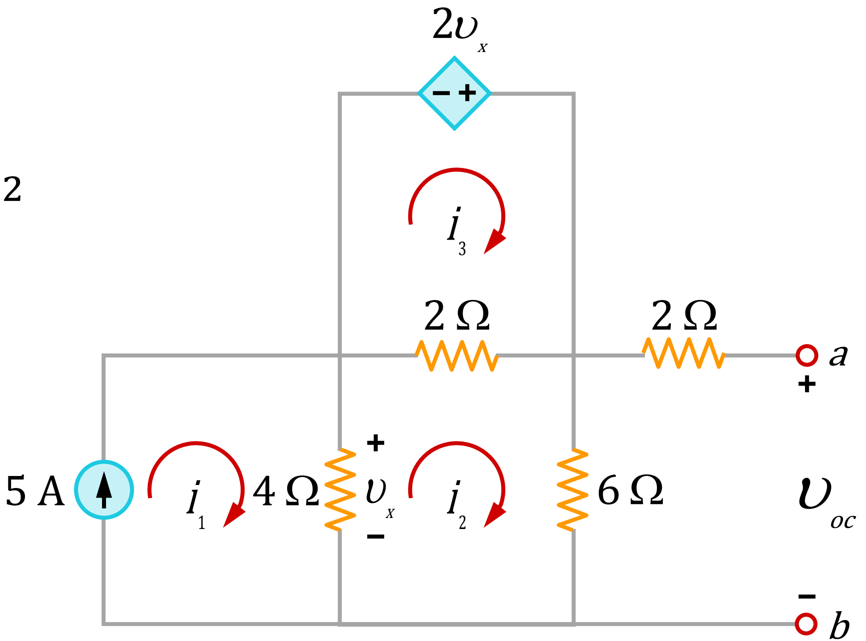
$$12i_2 - 2i_3 = 20$$

$$3i_2 + i_3 = 20$$

$$\Rightarrow \quad i_2 = \frac{10}{3}\,\text{A}$$

Note that we only need i_2 as $v_{oc} = 6i_2$

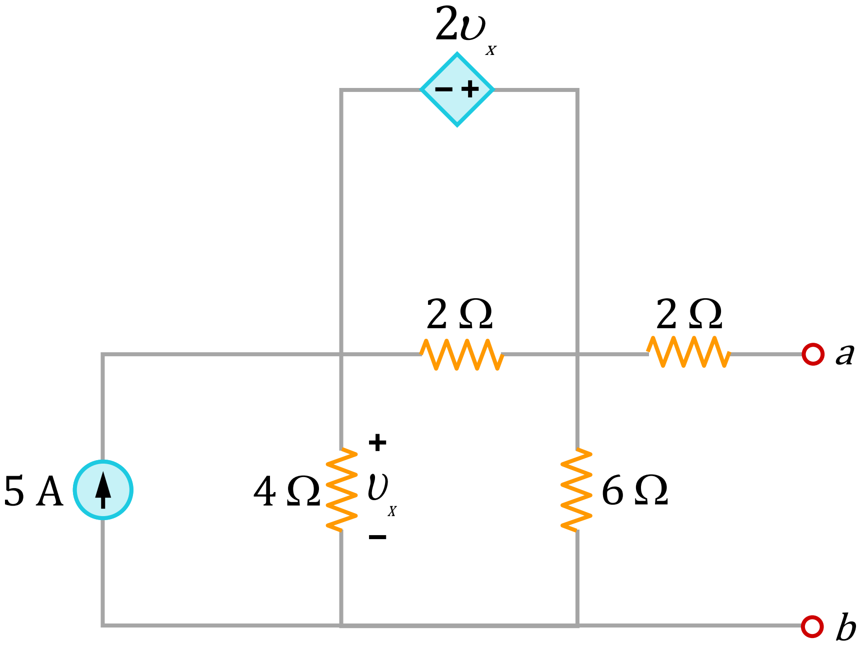
Hence, $$\boxed{V_{Th} = v_{oc} = 6i_2 = 20\ \text{V}}$$



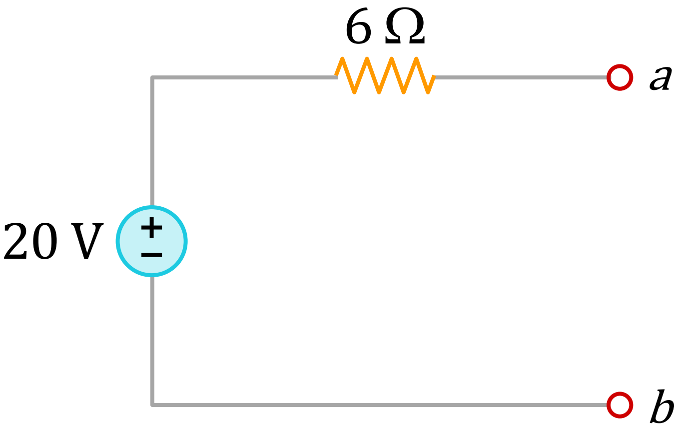
Example 19

$$R_{Th} = \frac{v_o}{i_o} = \frac{1}{1/6} = 6\ \Omega$$

$$V_{Th} = v_{oc} = 6i_2 = 20\ \text{V}$$

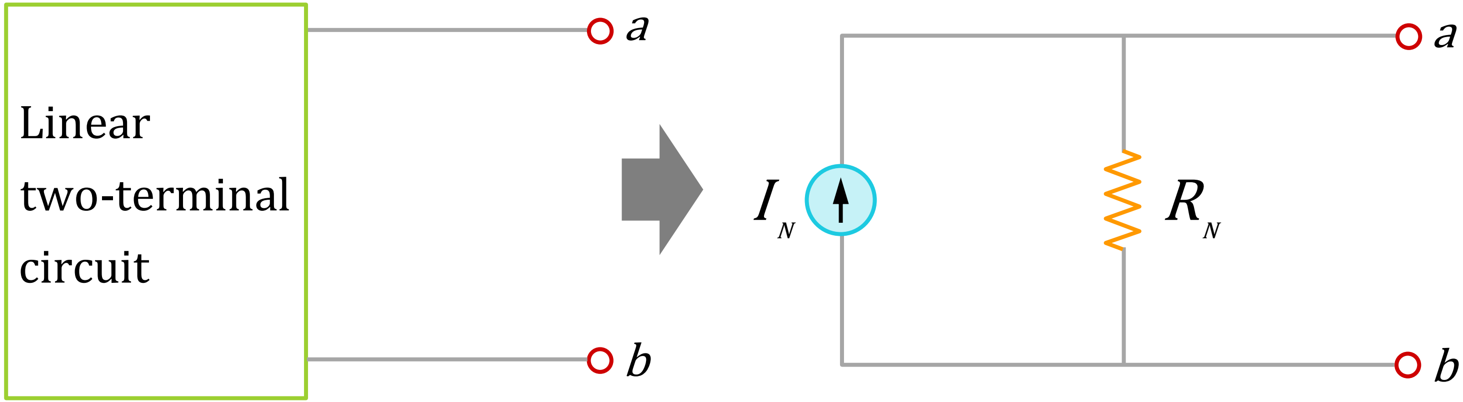


The Thevenin equivalent circuit is as shown.

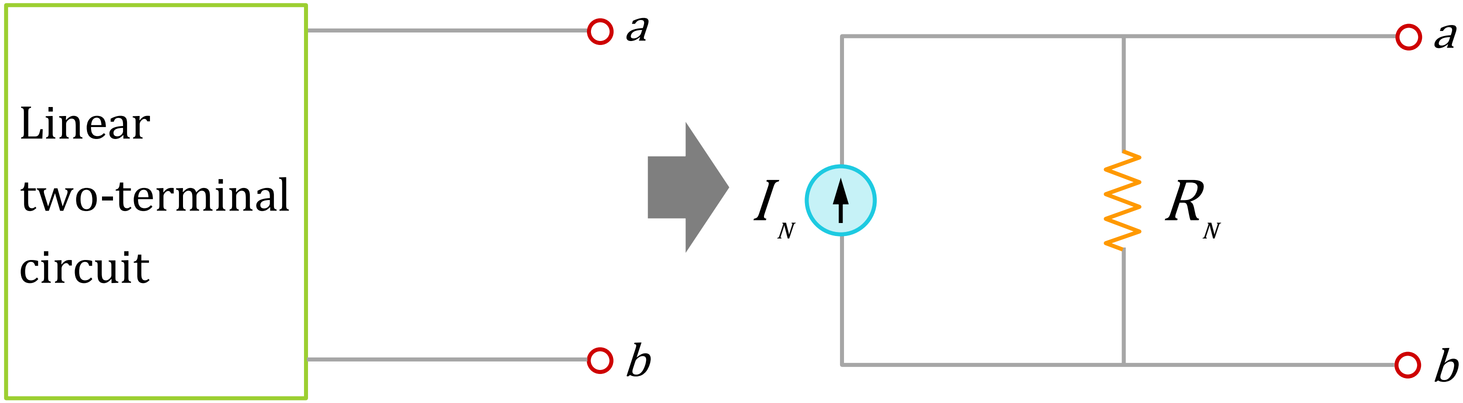


Norton’s Theorem

Norton’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the **independent sources** are turned off.



How to Find I_N and R_N?



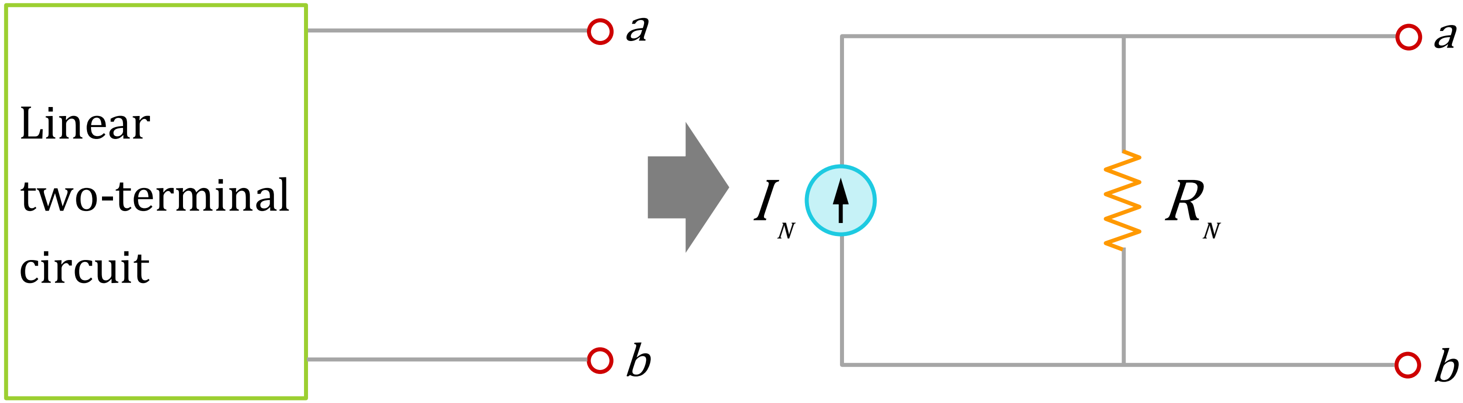
3(a) Original Circuit

3(b) Norton Equivalent Circuit

We find R_N in the same way as we find R_{Th}.

Note that by using source transformation, the Thevenin and Norton resistances are equal, i.e., $R_N = R_{Th}$.

How to Find I_N and R_N?



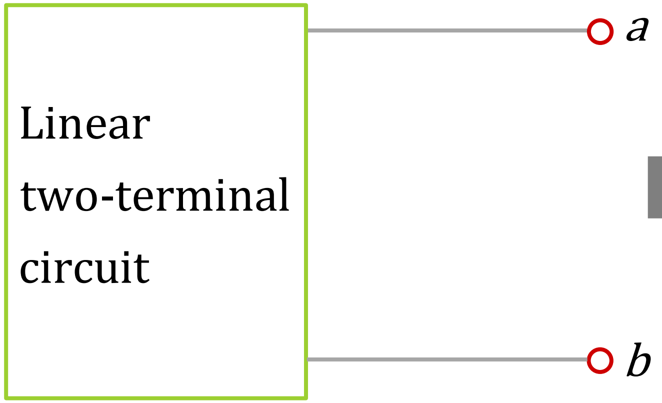
3(a) Original Circuit

3(b) Norton Equivalent Circuit

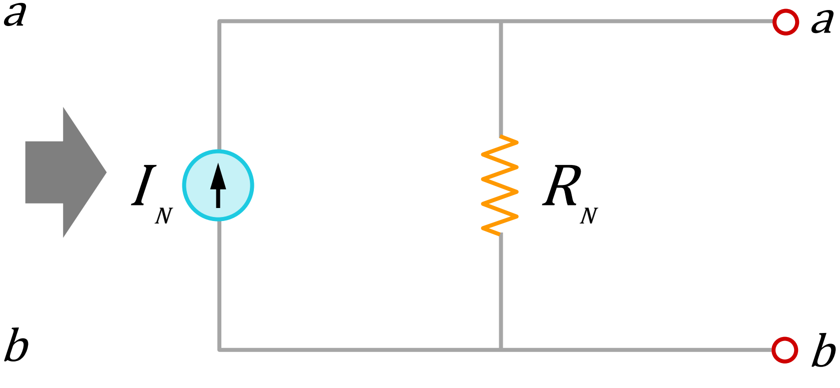
To find the Norton current I_N, we determine the short-circuit current flowing from terminals a-b in Figs. 3(a) and 3(b).

From Fig. 3(b), the short-circuit current is I_N, which is equal to the short-circuit current from terminals a-b in Fig. 3(a).

How to Find I_N and R_N?



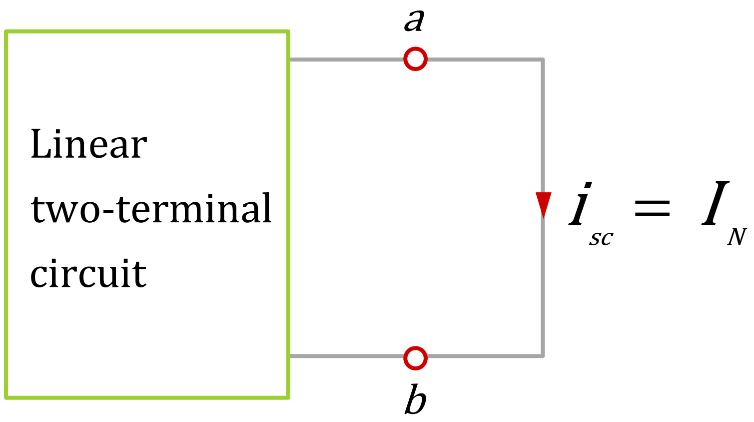
3(a) Original Circuit



3(b) Norton Equivalent Circuit

Thus, $I_N = i_{sc}$ as shown.

Dependent and independent sources are treated the same way as in Thevenin's Theorem.



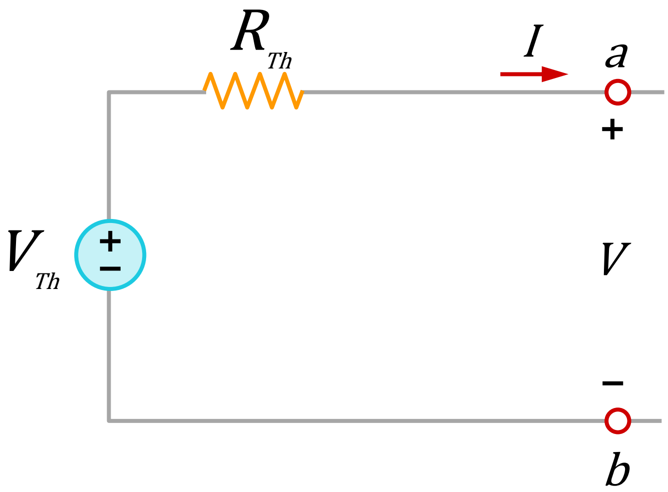
Thevenin vs. Norton Equivalent

The Thevenin and Norton equivalent circuits are related by a source transformation.

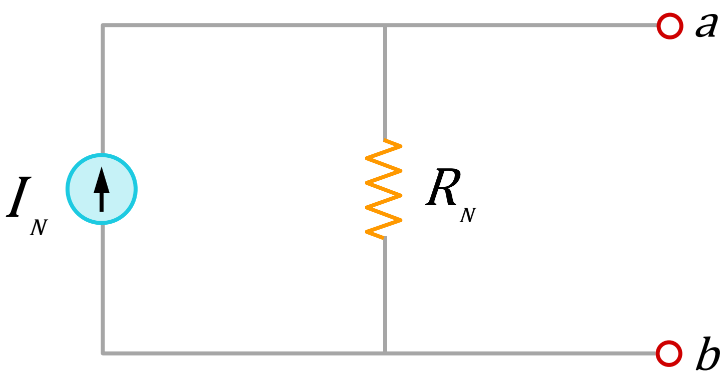
$$R_N = R_{Th}$$

$$V_{Th} = v_{oc}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}}$$



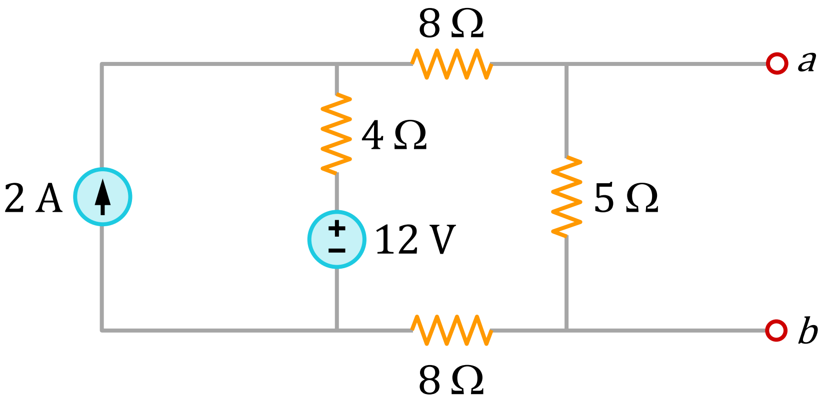
Thevenin Equivalent Circuit



Norton Equivalent Circuit

Example 20

Find the Norton equivalent circuit of the circuit as shown at the terminals a-b.



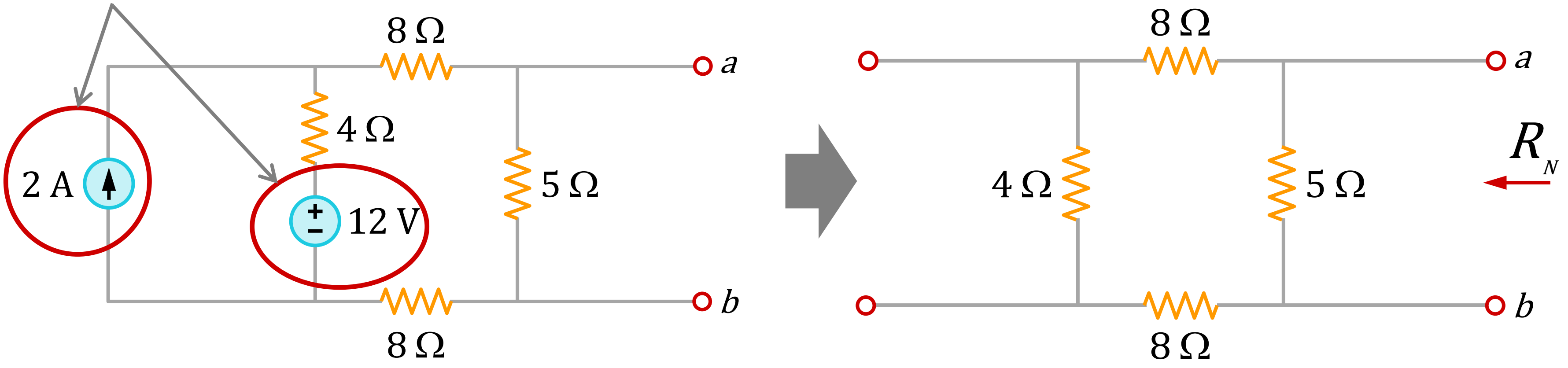
Example 20

We may find R_N the same way as we find R_{Th} in the Thevenin equivalent circuit.

Setting the independent sources to zero leads to the following circuit.

$$\boxed{R_N = 5||(8+4+8) = 5||20 = \frac{20\ x\ 5}{25} = 4\ \Omega}$$

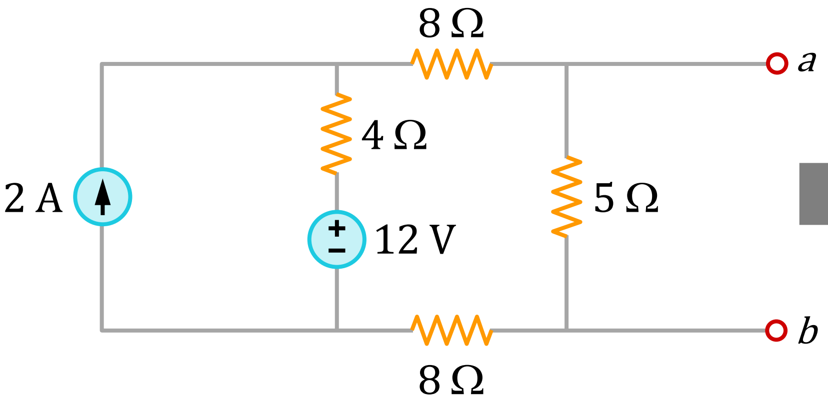
Set to 0

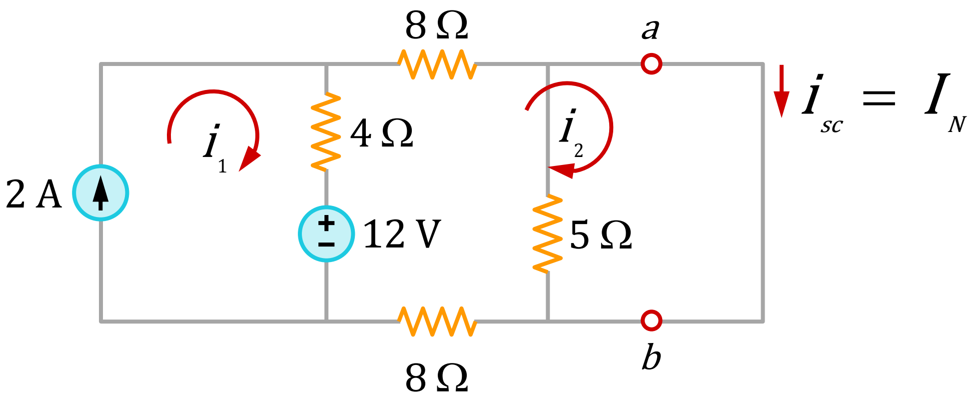


Example 20

To find I_N, we short-circuit terminals a-b as shown.

The $5\ \Omega$ resistor can be ignored because it has been short-circuited.





Example 20

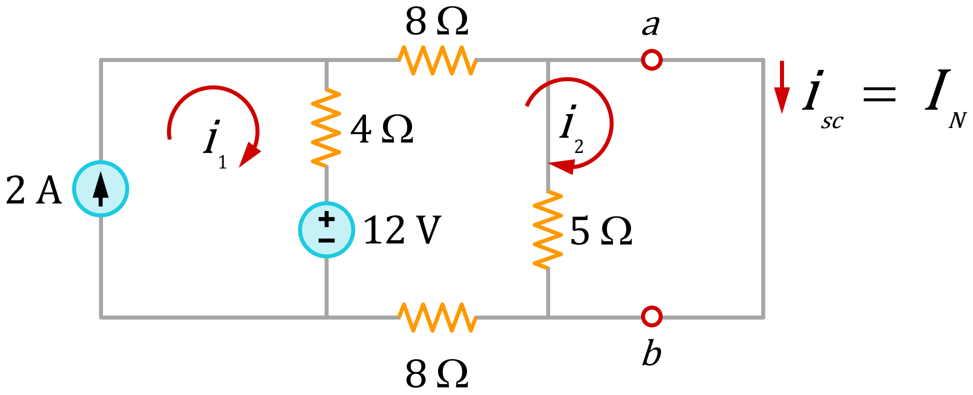
Applying mesh analysis:

Mesh 1: $\boxed{i_1 = 2\text{ A}}$

Mesh 2: $\boxed{-12 + 4(i_2 - i_1) + (8+8)i_2 = 0}$

$$\boxed{i_2 = 1\text{ A}}$$

$$\boxed{i_{sc} = I_N = 1\text{ A}}$$

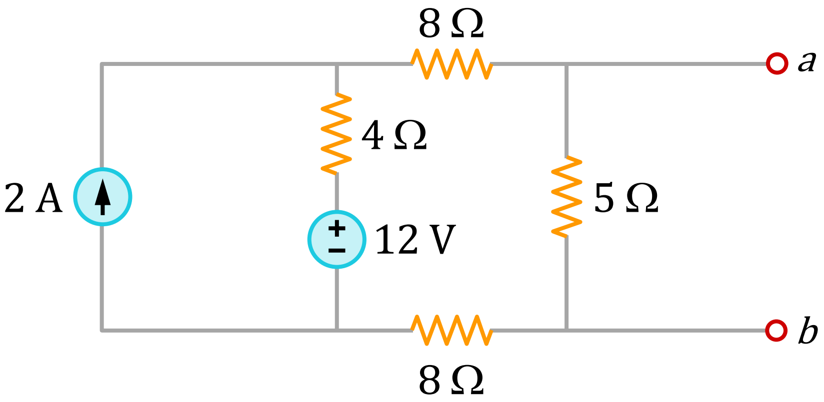


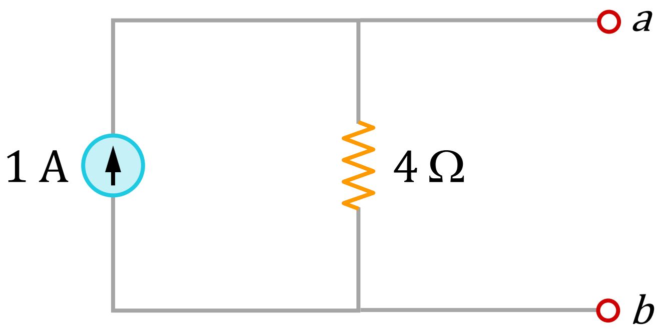
Example 20

$$R_N = 5||(8+4+8) = 5||20 = \frac{20x5}{25} = 4\ \Omega$$

$$i_{sc} = I_N = 1\ \text{A}$$

The Norton's equivalent circuit is as shown.



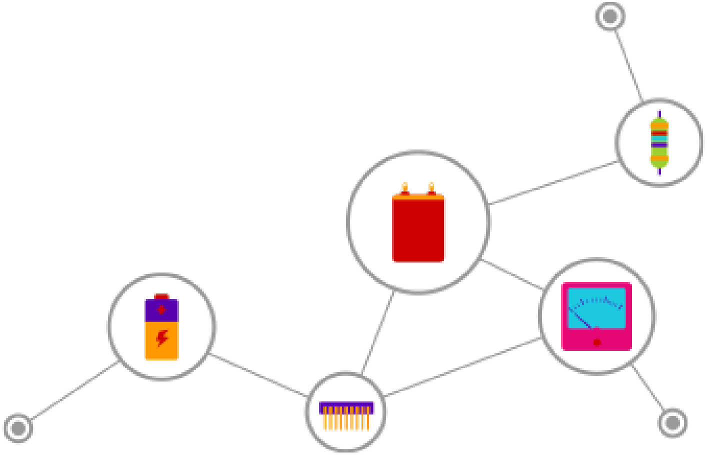


Open-circuit and Short-circuit Tests

The open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent circuit of a given circuit which contains at least one independent source.

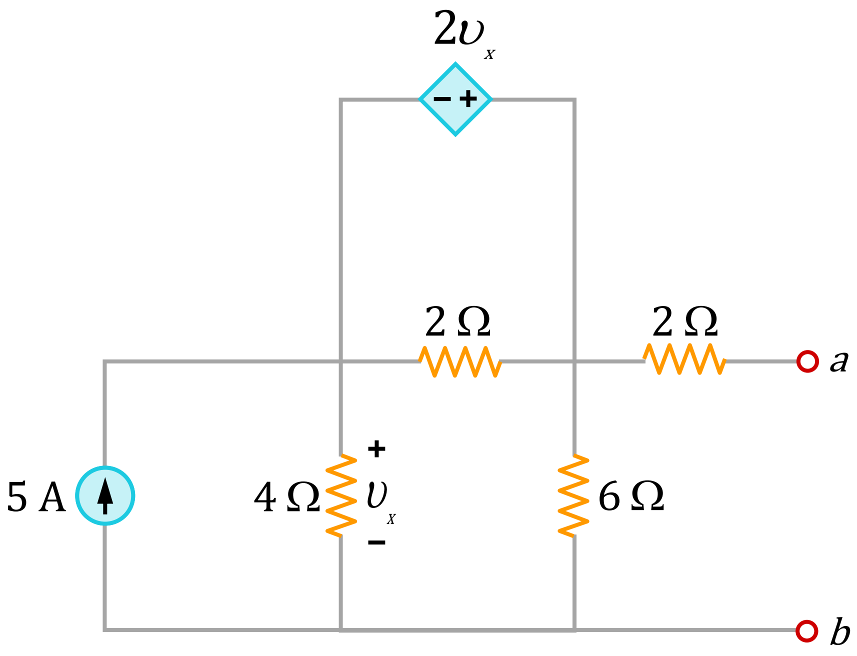
We have

$$v_{oc} = V_{Th},\ \ i_{sc} = I_N,\ \ R_{Th} = v_{oc}/i_{sc}$$



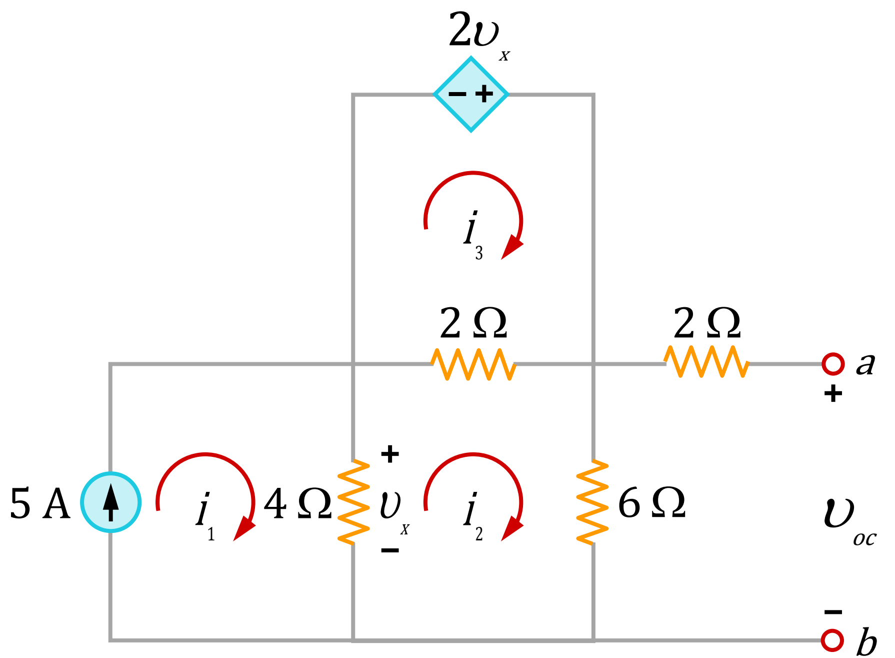
Example 21

Use the open-circuit and short-circuit tests to find the Thevenin equivalent circuit of the following circuit across terminals a-b.



Example 21: Open-circuit Test

Find V_{Th} as the open-circuit voltage v_{oc} across terminals a-b as shown.



Example 21: Open-circuit Test

Applying mesh analysis:

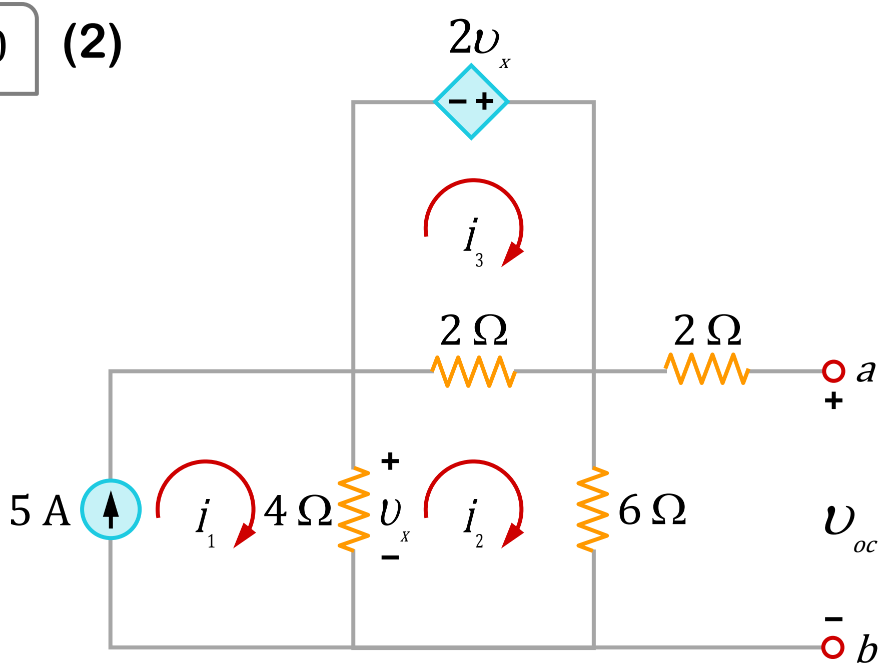
Mesh 1: $i_1 = 5\text{ A}$ **(1)**

Mesh 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$ **(2)**

Mesh 3: $-2v_x + 2(i_3 - i_2) = 0$

$v_x = i_3 - i_2$

But, $v_x = 4(i_1 - i_2) = i_3 - i_2$ **(3)**



Example 21: Open-circuit Test

From (1) – (3) we get,

Mesh 1: $i_1 = 5\ \mathrm{A}$ **(1)**

Mesh 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$ **(2)** $\Rightarrow$ $12i_2 - 2i_3 = 20$

Mesh 3: $v_x = 4(i_1 - i_2) = i_3 - i_2$ **(3)** $\Rightarrow$ $3i_2 + i_3 = 20$

Example 21: Open-circuit Test

Solving for i_2 from the equations gives

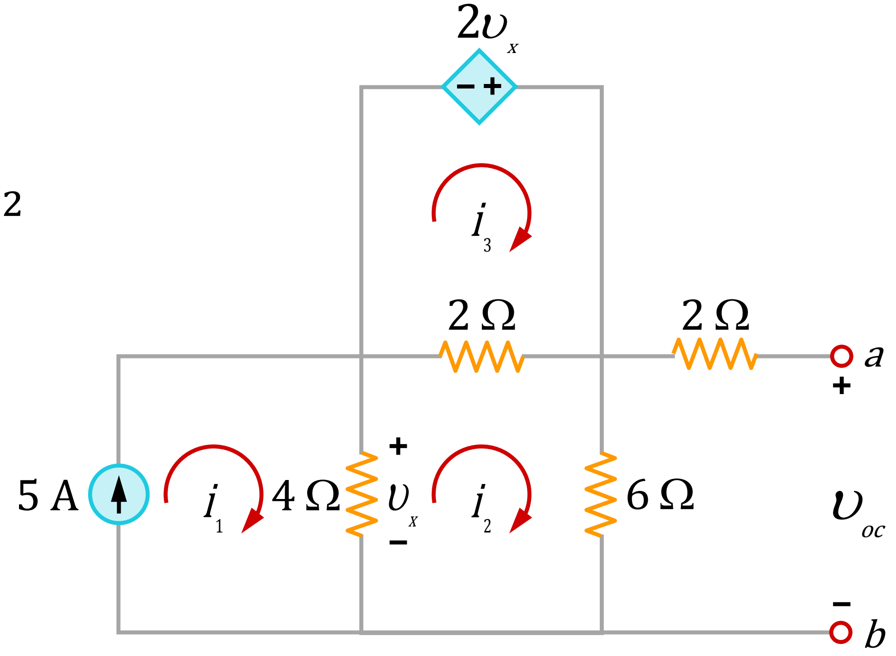
$$12i_2 - 2i_3 = 20$$

$$3i_2 + i_3 = 20$$

$$\Rightarrow \quad i_2 = \frac{10}{3}\text{ A}$$

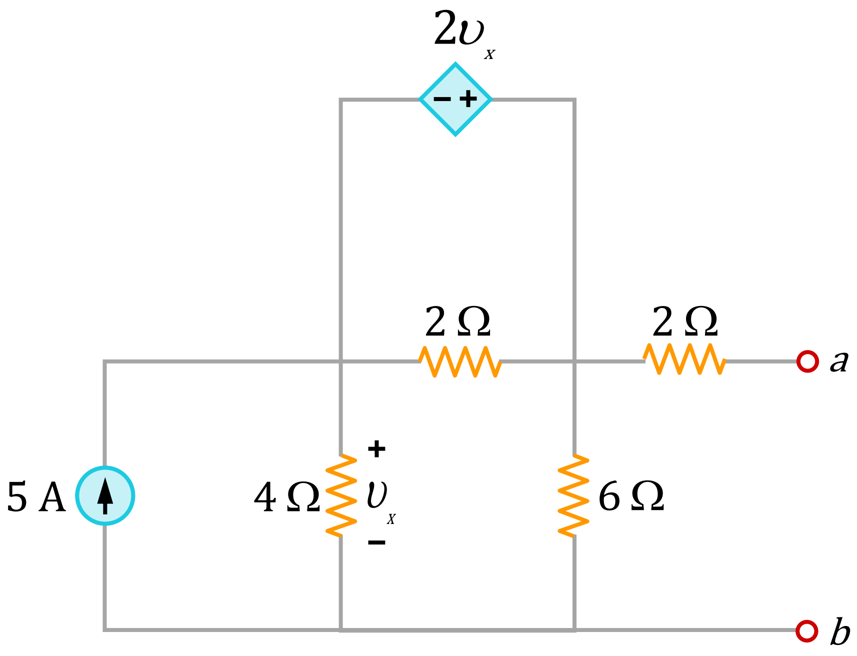
Note that we only need i_2 as $v_{oc} = 6i_2$

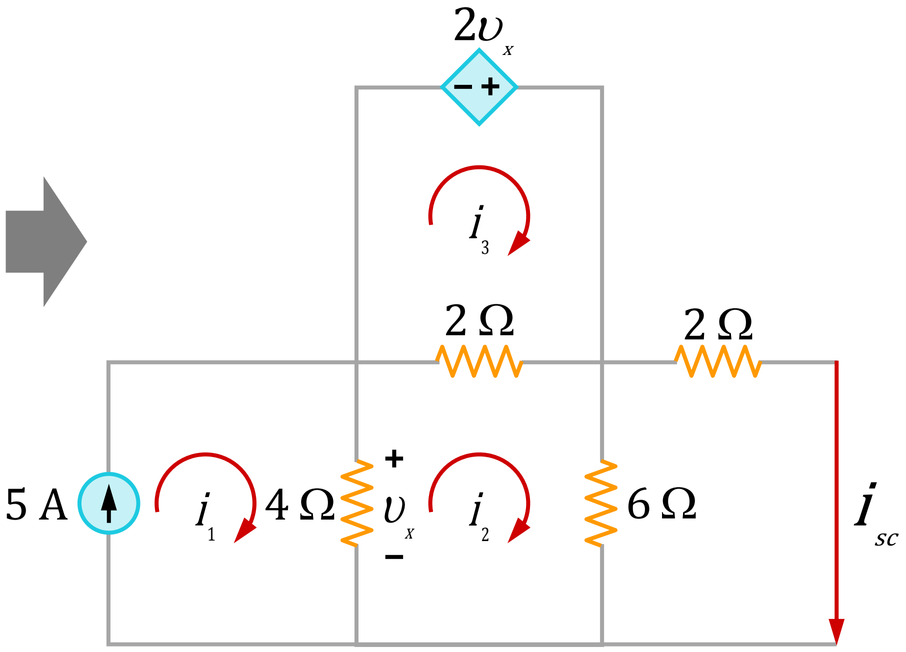
Hence, $\boxed{V_{Th} = v_{oc} = 6i_2 = 20\text{ V}}$



Example 21: Short-circuit Test

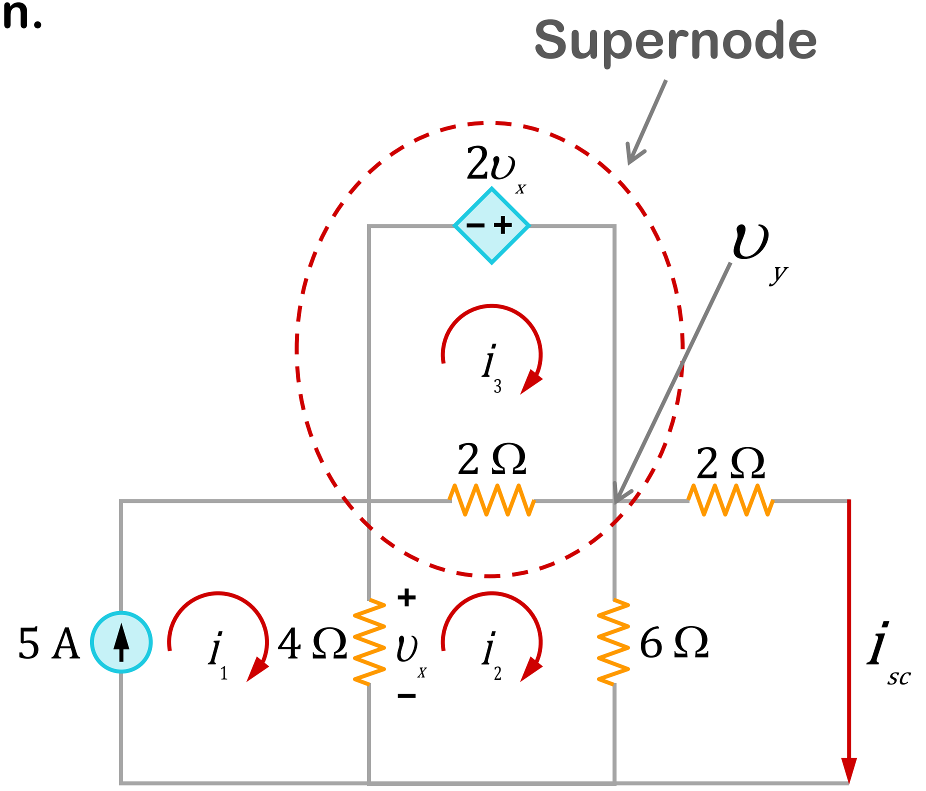
To find i_{sc}, we short-circuit terminals a-b as shown.





Example 21: Short-circuit Test

The supernode contains the top two nodes, the dependent source and the $2\ \Omega$ resistor as shown.



Example 21: Short-circuit Test

At the supernode, applying KCL gives

$$\boxed{5 - \frac{v_x}{4} - \frac{v_y}{6} - \frac{v_y}{2} = 0} \quad (1)$$

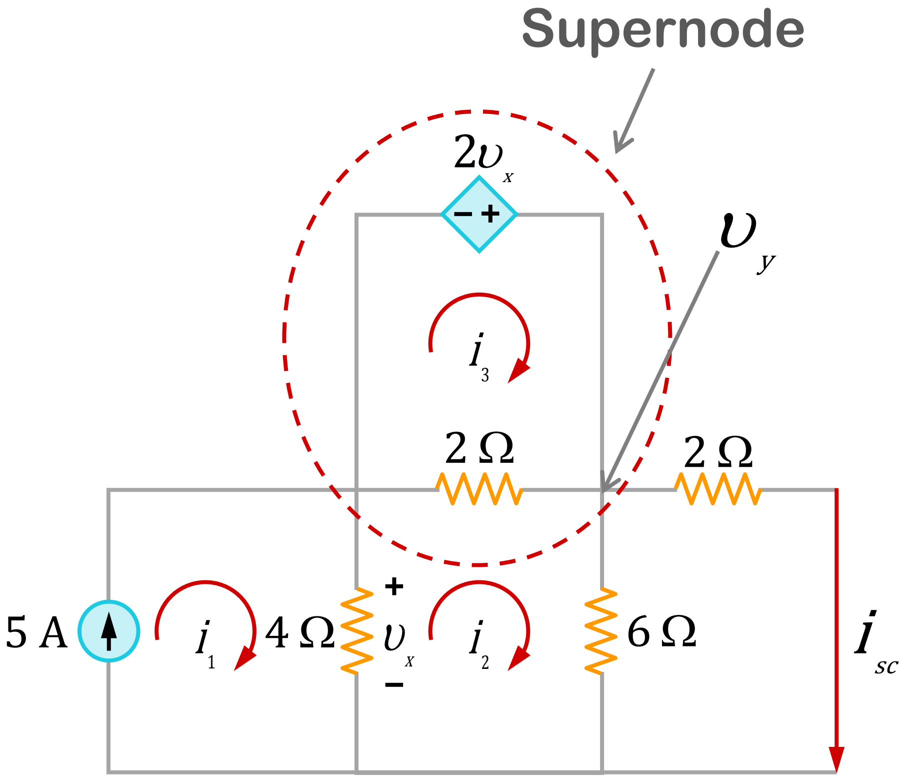
The constraint equation is

$$\boxed{v_y - v_x = 2v_x} \quad (2)$$

Solving for v_y from (1) and (2) gives

$$\boxed{v_y = \frac{20}{3}\,\mathrm{V}} \qquad \boxed{i_{sc} = \frac{v_y}{2} = \frac{10}{3}\,\mathrm{A}}$$

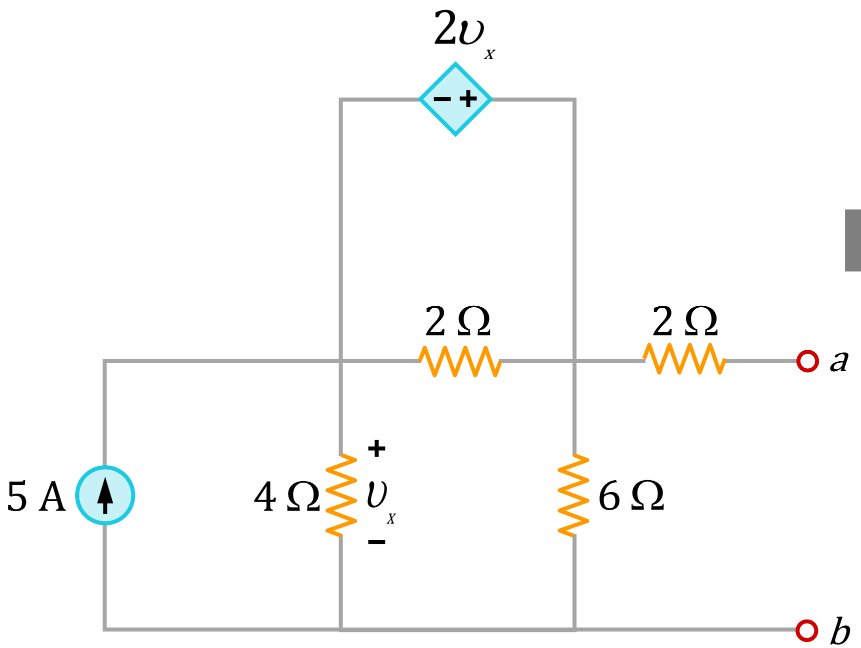
Thus, $$\boxed{R_{Th} = \frac{v_{oc}}{i_{sc}} = 6\ \Omega}$$



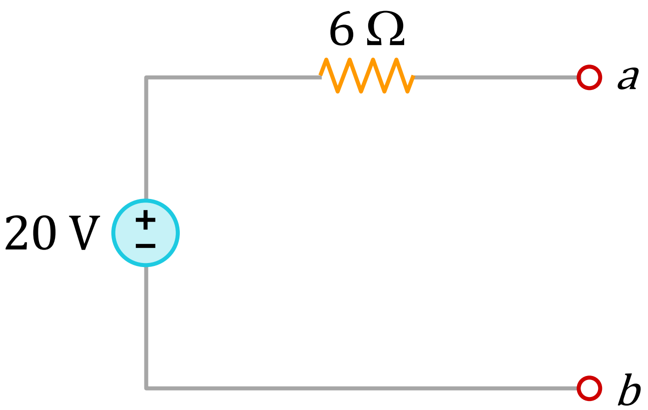
Example 21

$$V_{Th} = v_{oc} = 6i_2 = 20\text{ V}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = 6\ \Omega$$



The Thevenin equivalent circuit is as shown.



Maximum Power Transfer

Often we deal with small amount of power in electronics and we want to make full use of the power available.

Obtaining the maximum power out of a circuit is very important.

The Thevenin equivalent circuit is useful in finding the maximum power a linear circuit can deliver to a load.

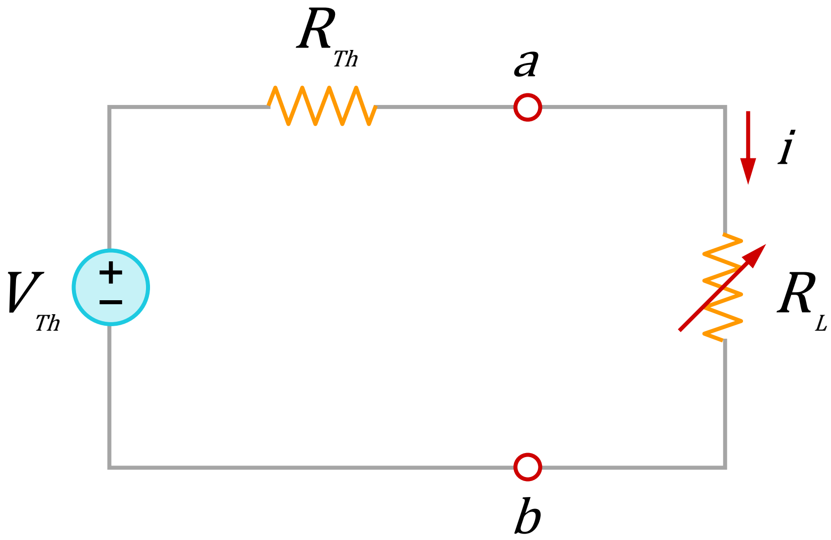
We assume that we can adjust the load resistance R_L.

Condition for Maximum Power Transfer

Suppose that the entire circuit is replaced by its Thevenin equivalent circuit except for the load as shown.

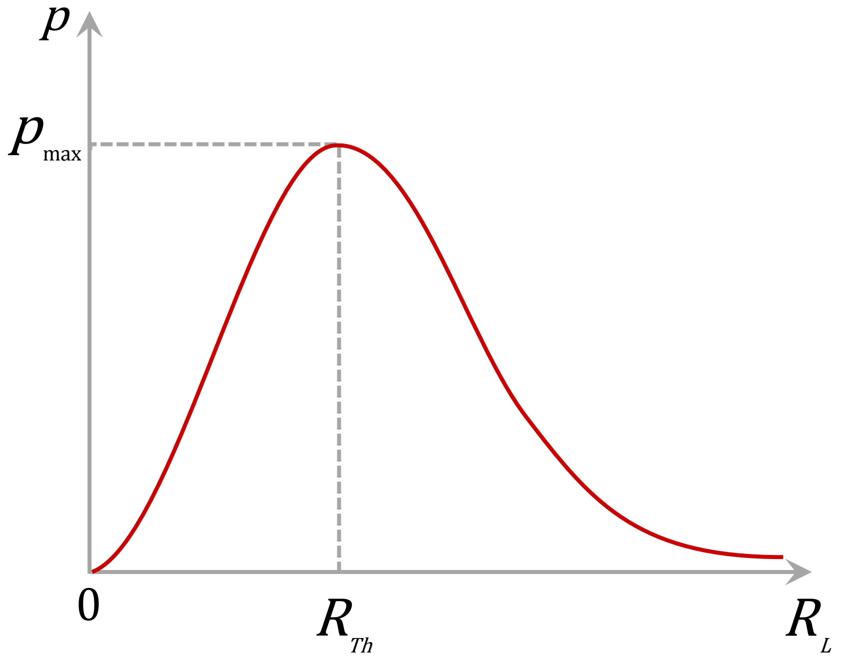
The power delivered to the load is,

$$\boxed{p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L} \quad \textbf{(1)}$$



Condition for Maximum Power Transfer

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as shown.



Condition for Maximum Power Transfer

Differentiate p in (1) w.r.t. R_L and set the differentiation result to zero gives

$$p = i^2 R_L = (\frac{V_{Th}}{R_{Th} + R_L})^2 R_L \quad \textbf{(1)}$$

$$\frac{dP}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

$$(R_{Th} + R_L - 2R_L) = R_{Th} - R_L = 0$$

This yields, $$\boxed{R_L = R_{Th}} \quad \textbf{(2)}$$

Condition for Maximum Power Transfer

The maximum power delivered by a source to the load R_L occurs when R_L is equal to R_{Th}, i.e., the Thevenin resistance at the terminals of the load.

Using (2) in (1) yields the maximum power transferred (for $R_L = R_{Th}$):

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \quad (1)$$

$$R_L = R_{Th} \quad (2)$$

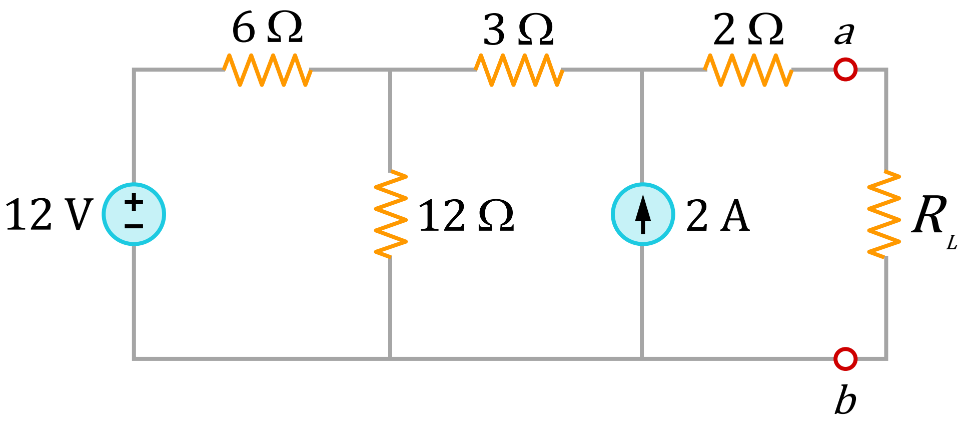
$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

When $R_L \neq R_{Th}$, the power delivered to the load is given by (1), i.e.,

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

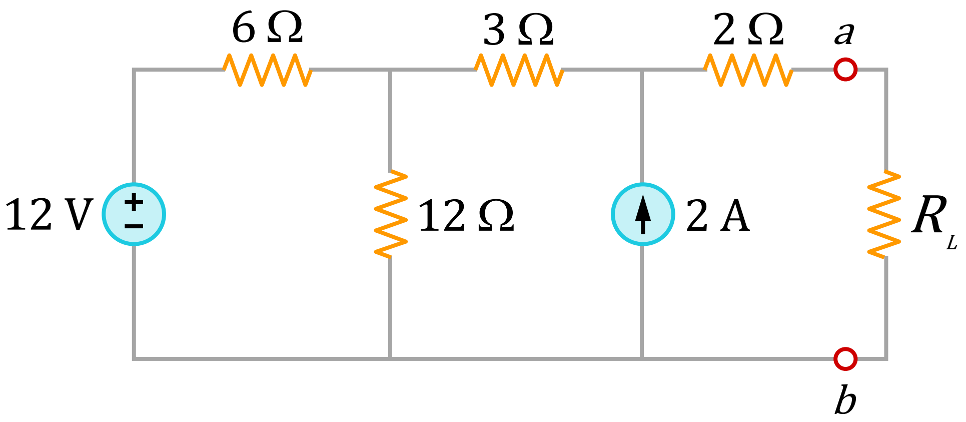
Example 22

Find the value of R_L for maximum power transfer and the maximum power in the following circuit.



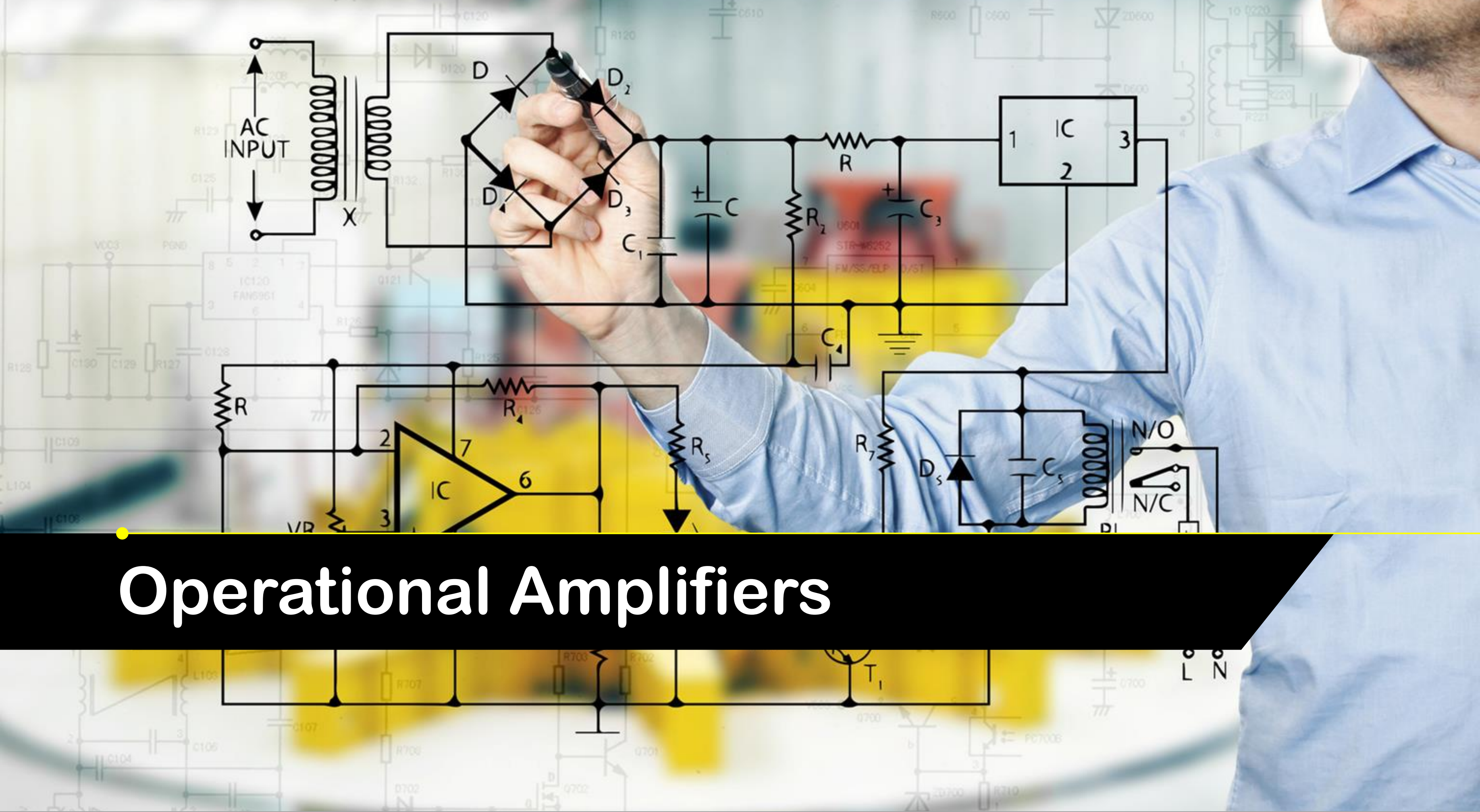
Example 22

Using the techniques of finding the Thevenin resistance and the Thevenin voltage, we find that $R_{Th} = 9\ \Omega$ and $V_{Th} = 22\ \text{V}$ (exercise).



For maximum power transfer, $R_L = R_{Th} = 9\ \Omega$ and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{22^2}{4\ x\ 9} = 13.44\ \text{W}$$



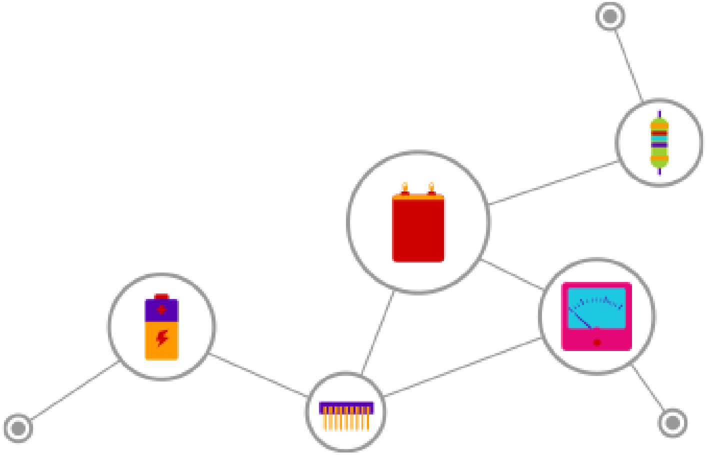
Operational Amplifiers

Operational Amplifiers (Op-amps)

Operational amplifiers are commonly used in a large variety of electronic applications (AC and DC signal amplification, active filters, oscillators, comparators and regulators).

An op-amp is an active element of an electric circuit. It acts like a voltage controlled voltage source.

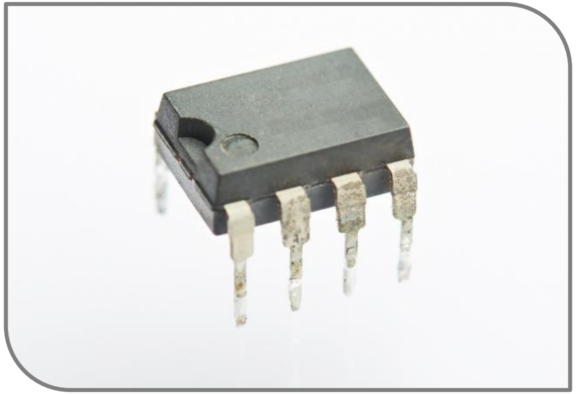
Our focus is on the terminal behavior of the op-amp, i.e., we are not interested in the internal structure of the op-amp and the currents and voltages that exist in this structure.

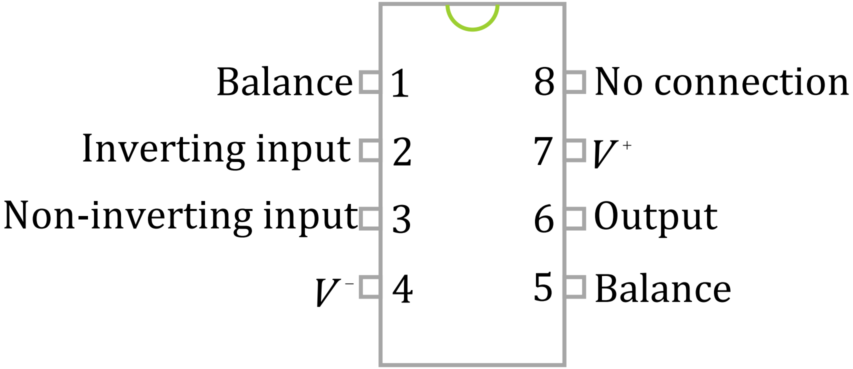


Operational Amplifiers (Op-amps)

Op-amps are commercially available in IC packages in several forms.

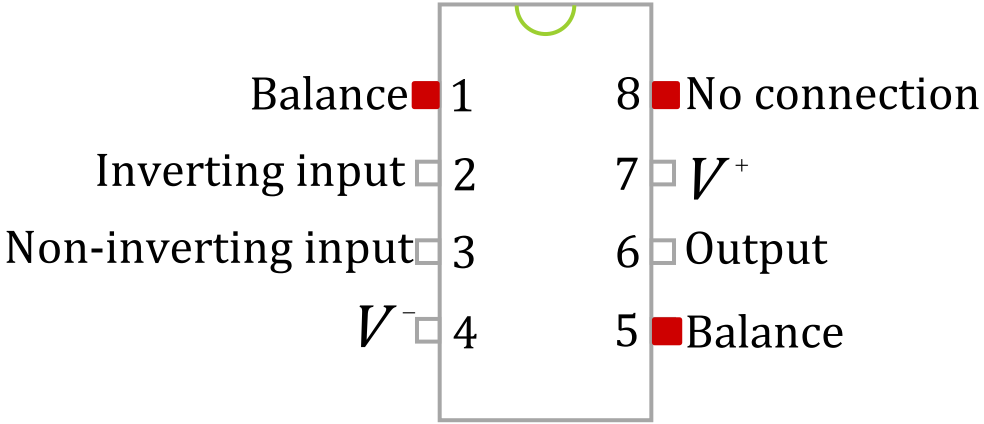
The following figure shows a typical op-amp package which is an eight-pin dual in-line package (DIP) as shown.





Pin Configuration

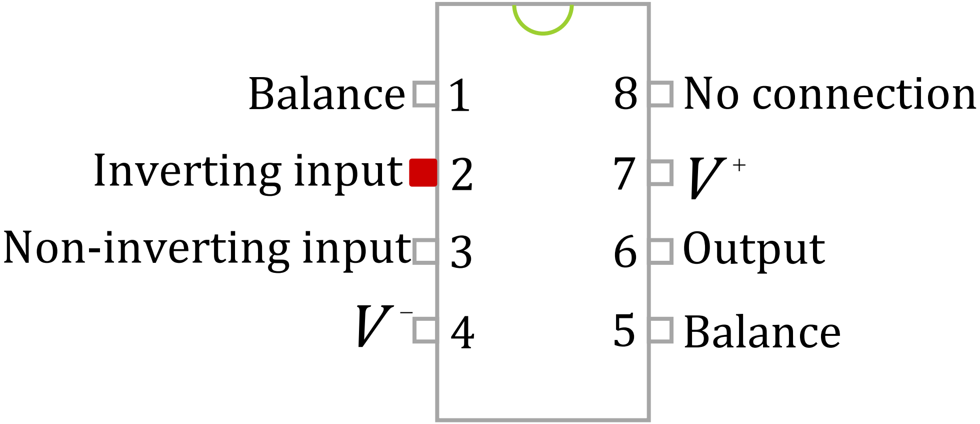
Pin Configuration of an Op-amp



Pin or terminal 8 is unused and terminals 1 and 5 are of little concern to us.

Pin Configuration of an Op-amp

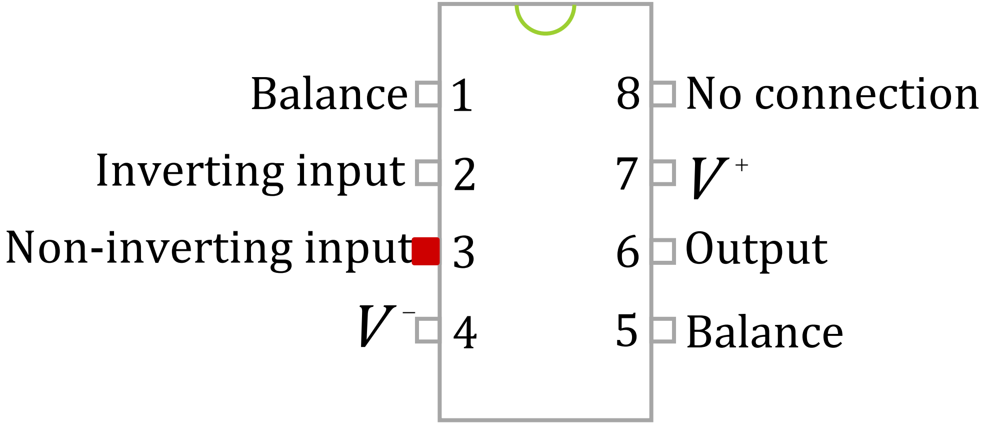
The five important terminals are:



1. The inverting input, pin 2.

Pin Configuration of an Op-amp

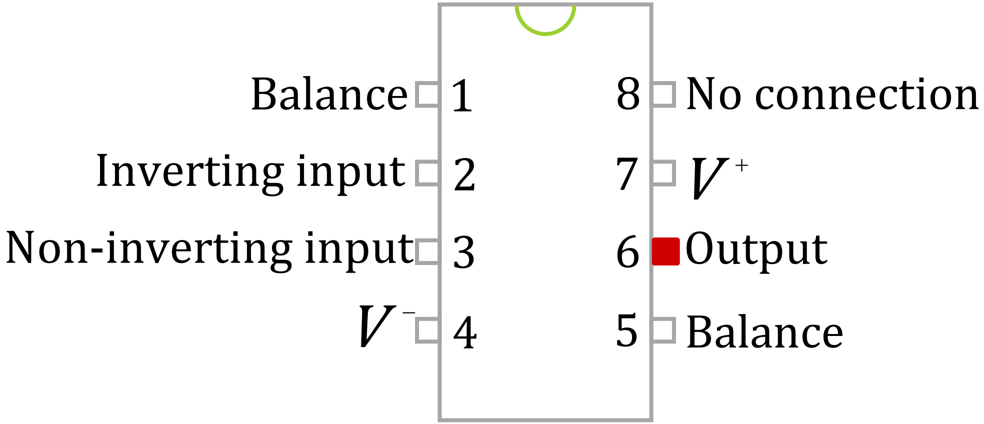
The five important terminals are:



2. The non-inverting input, pin 3.

Pin Configuration of an Op-amp

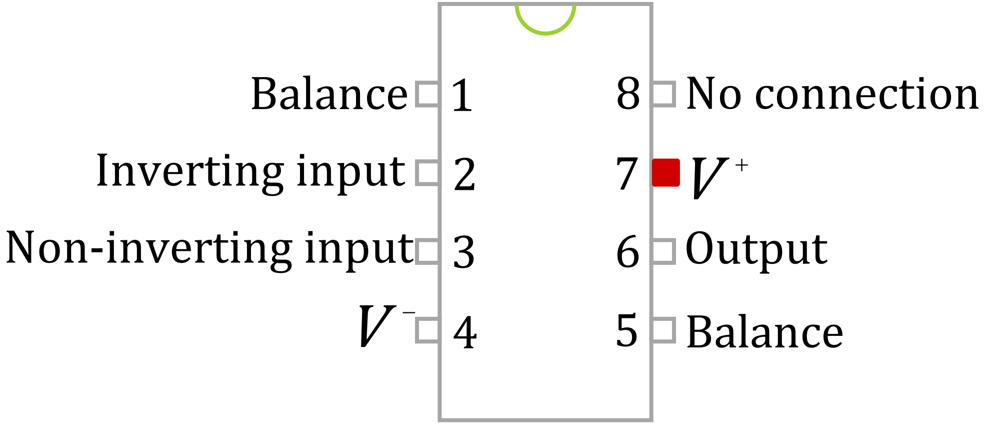
The five important terminals are:

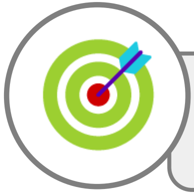


3. The output, pin 6.

Pin Configuration of an Op-amp

The five important terminals are:

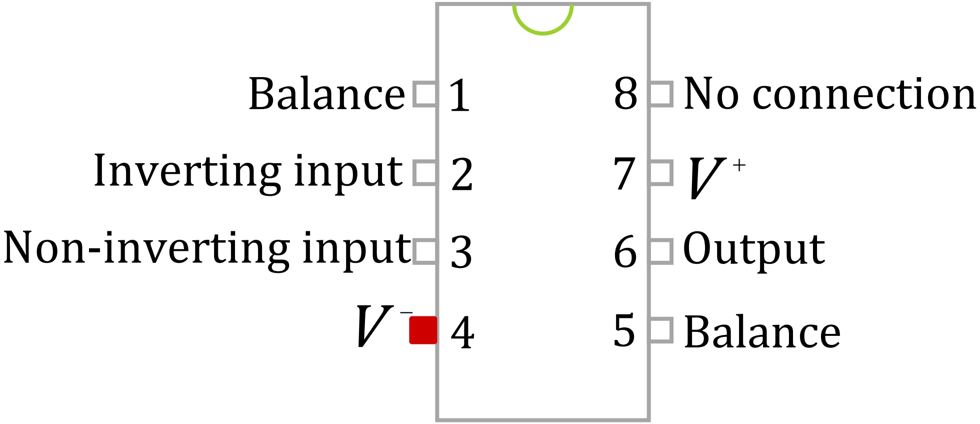




4. The positive power supply V^+, pin 7.

Pin Configuration of an Op-amp

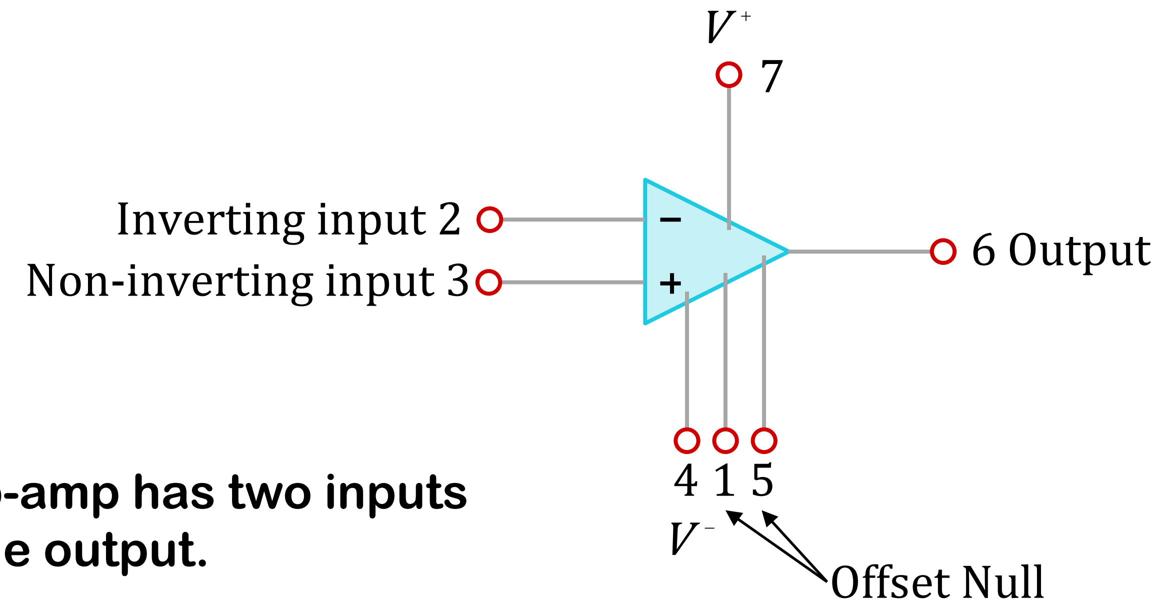
The five important terminals are:



5. The negative power supply V^-, pin 4.

Circuit Symbol of an Op-amp

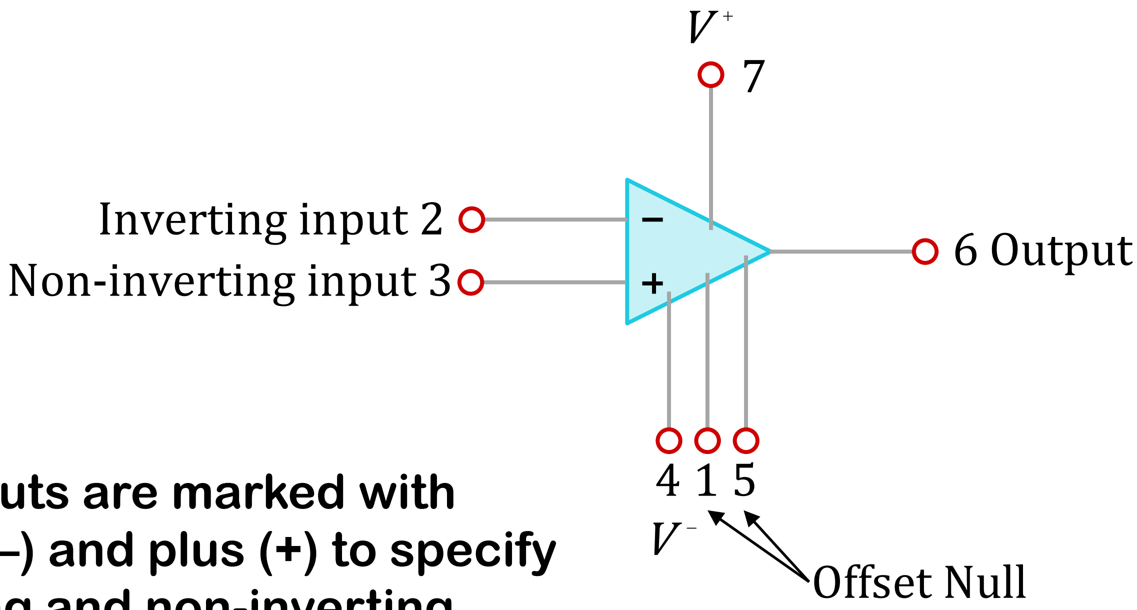
The circuit symbol commonly used for an op-amp is as shown.



The op-amp has two inputs and one output.

Circuit Symbol of an Op-amp

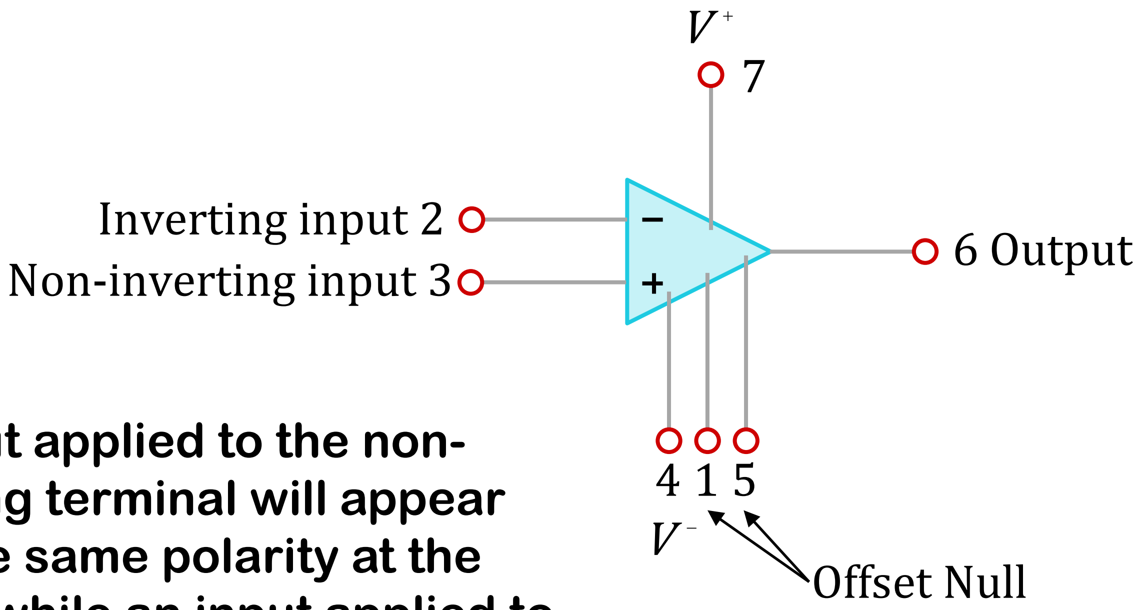
The circuit symbol commonly used for an op-amp is as shown.



The inputs are marked with minus (–) and plus (+) to specify inverting and non-inverting inputs respectively.

Circuit Symbol of an Op-amp

The circuit symbol commonly used for an op-amp is as shown.

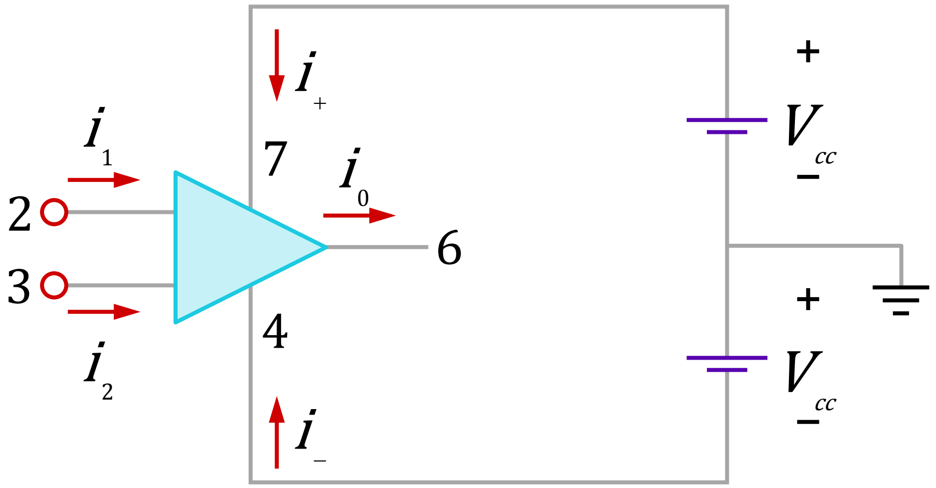


An input applied to the non-inverting terminal will appear with the same polarity at the output while an input applied to the inverting terminal will appear inverted at the output.

Operation of an Op-amp

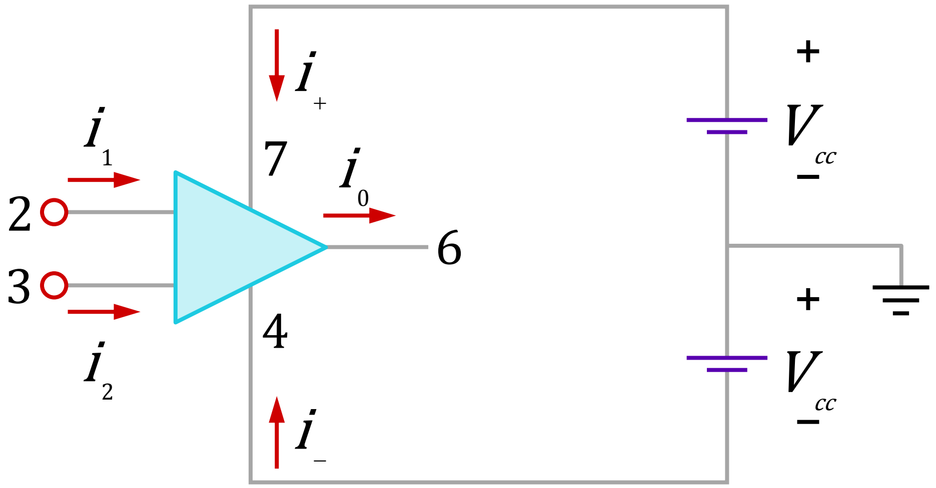
The op-amp is typically powered by a voltage supply as shown.

The power supplies are often ignored in an op-amp circuit diagram for the sake of simplicity. However, note that $i_o = i_1 + i_2 + i_+ + i_-$.



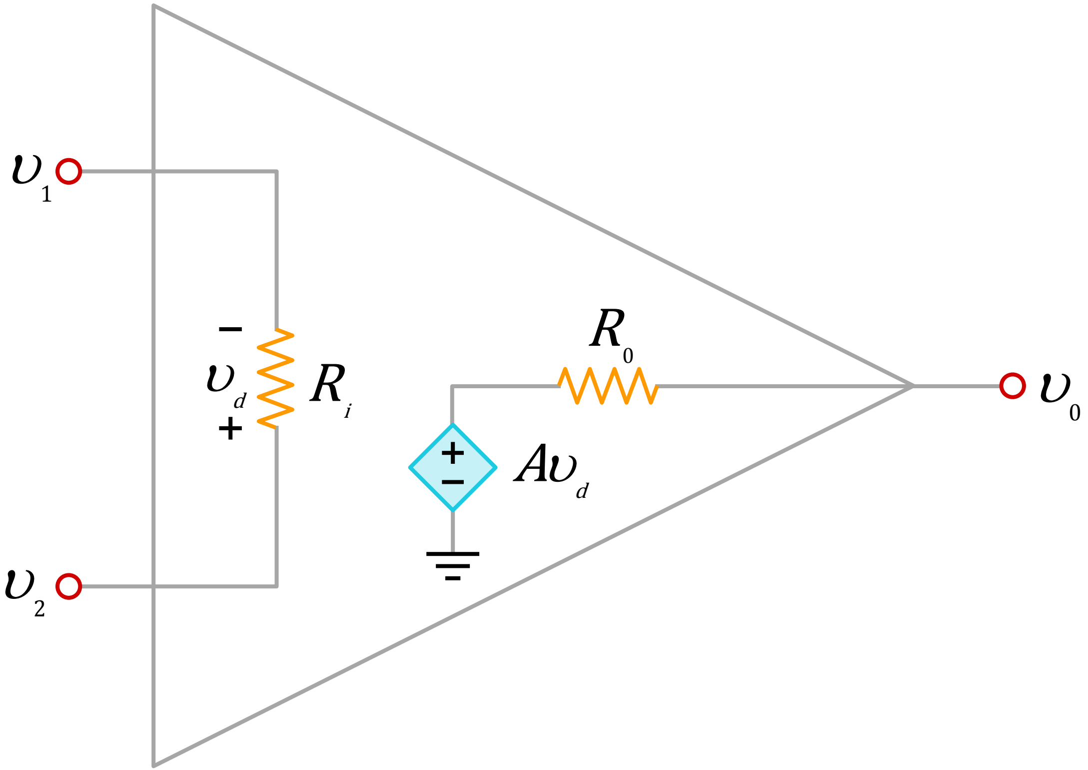
Operation of an Op-amp

The output voltage is limited by the values of the voltage sources, i.e., $-V_{CC} \leq v_o \leq V_{CC}$.



Equivalent Circuit Model

The equivalent circuit model of an op-amp is:

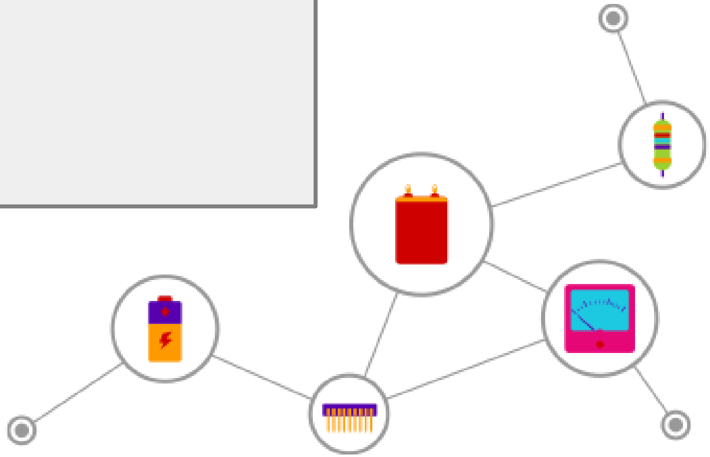


Ideal Op-amps

To facilitate the understanding and analysis of op-amp circuits, we consider only ideal op-amps.

An ideal op-amp is an amplifier with infinite open-loop gain, infinite input resistance and zero output resistance, i.e., it has the following characteristics:

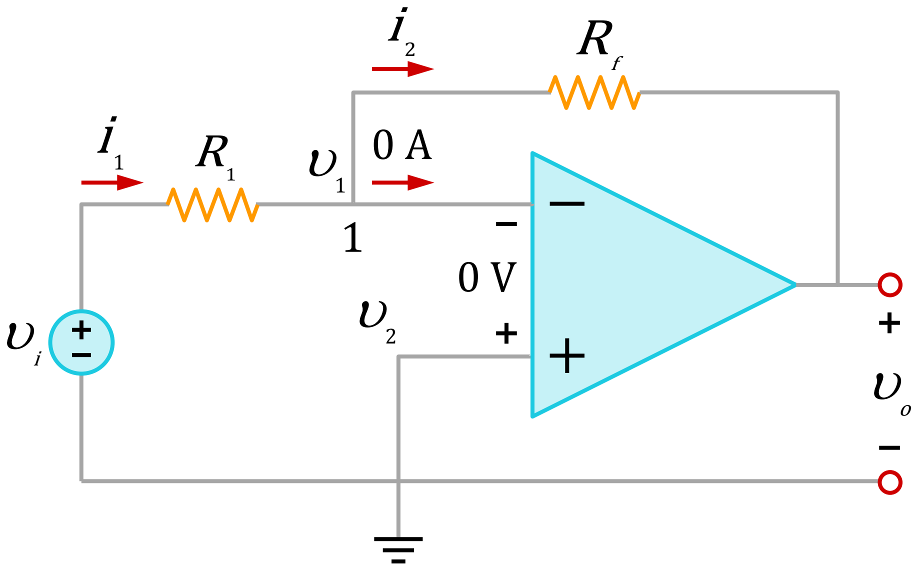
- Infinite open-loop gain, $A = \infty$.
- Infinite input resistance, $R_i = \infty$.
- Zero output resistance, $R_o = 0$.



Ideal Op-amps

As an op-amp has a very high open-loop voltage gain, A, negative feedback is usually considered to control the output voltage and to limit the voltage gain.

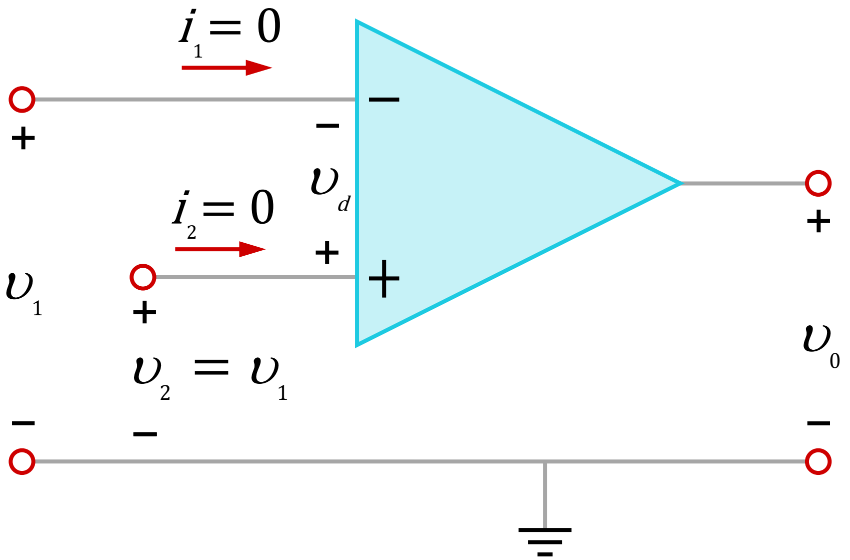
A negative feedback is achieved when the output is fed back to the inverting terminal of the op-amp as shown.



The voltage gain of the op-amp with negative feedback is called closed-loop gain.

Analysis of the Ideal Op-amp

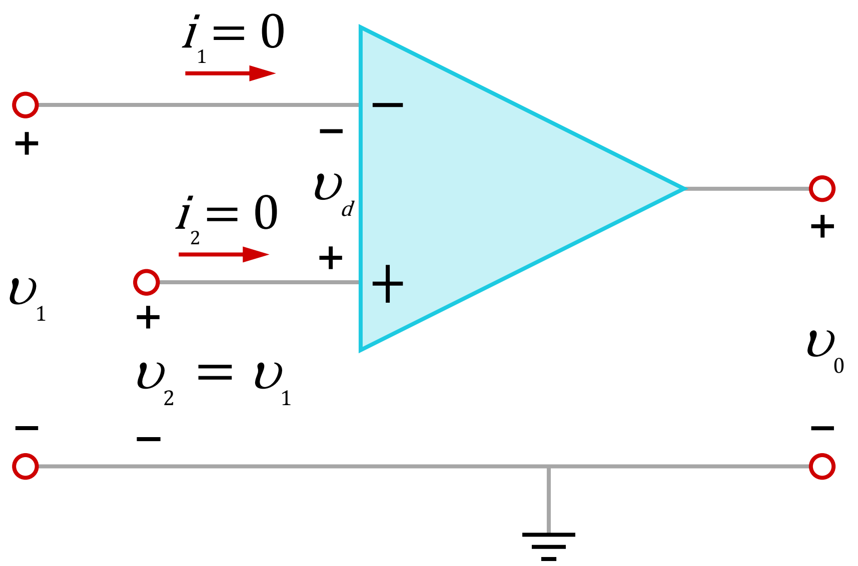
For circuit analysis, the ideal op-amp is as shown.



Characteristics of the Ideal Op-amp

Two important characteristics of the ideal op-amp:

1. **The currents into both the input terminals are zero, i.e., $i_1 = 0$, $i_2 = 0$. This is due to the infinite input resistance. This implies that there is an open-circuit between the input terminals and current cannot enter the op-amp.**

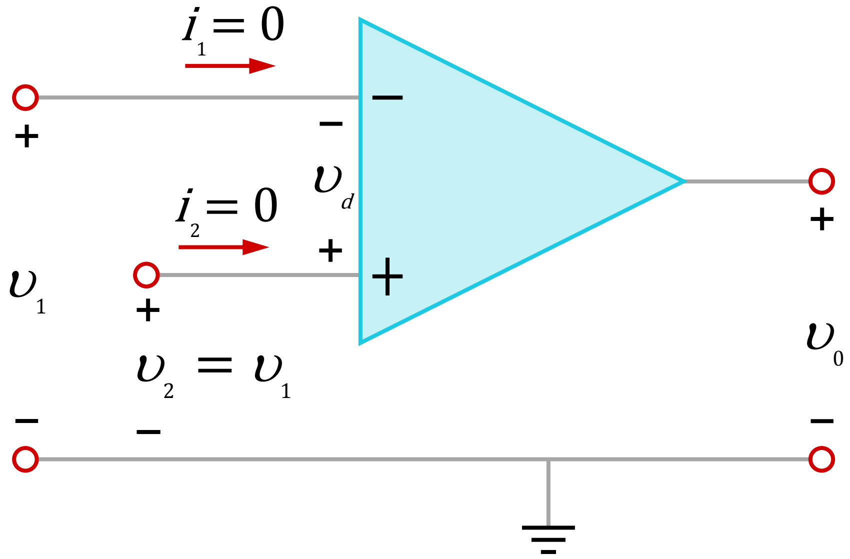


But, the output current is not necessarily zero, according to $i_o = i_1 + i_2 + i_+ + i_-$.

Characteristics of the Ideal Op-amp

Two important characteristics of the ideal op-amp:

2. **The voltage across the input terminals is equal to zero, i.e.,**
   $v_d = v_2 - v_1 = 0 \Longrightarrow v_1 = v_2.$

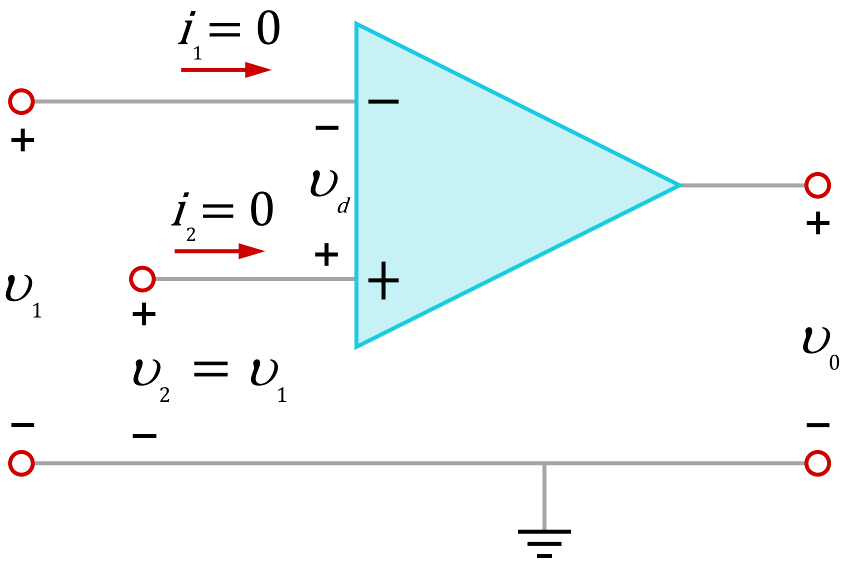


Characteristics of the Ideal Op-amp

These two equations:

1. $\boxed{i_1 = 0}$ and $\boxed{i_2 = 0}$

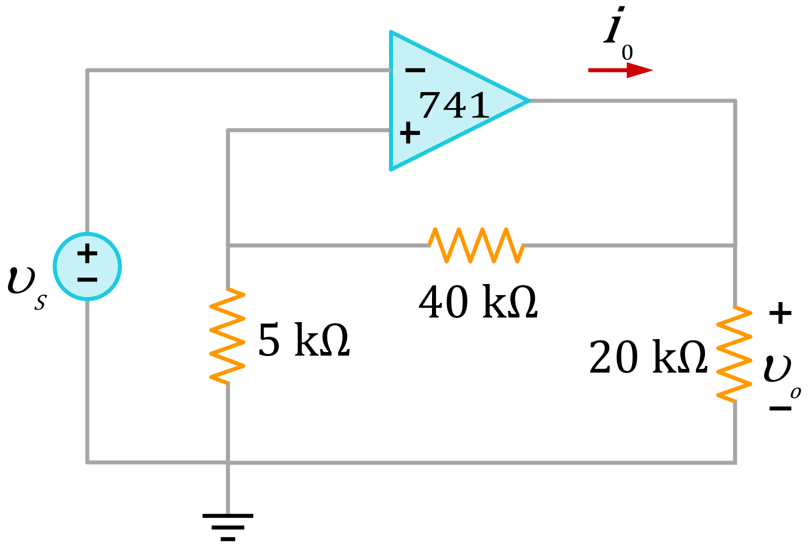
2. $\boxed{v_1 = v_2}$



are extremely important and should be regarded as the key information to analysing op-amp circuits.

Example 23

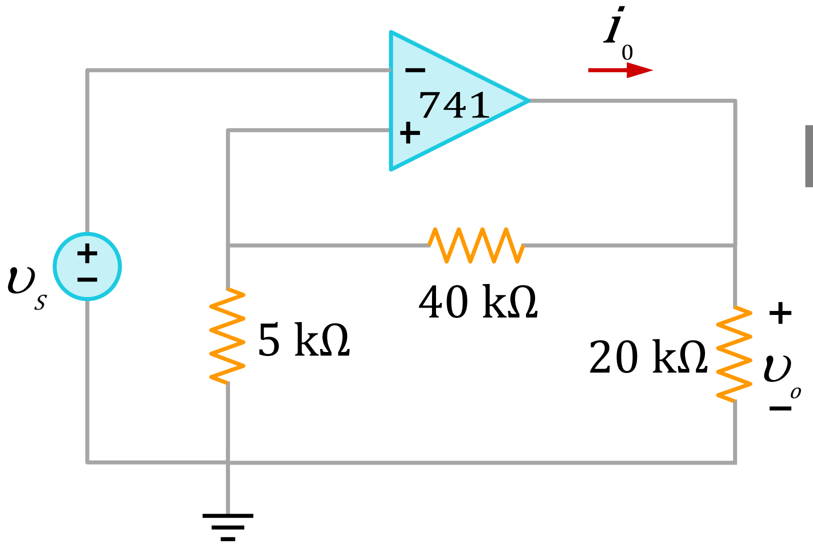
For the ideal op-amp circuit as shown, calculate the closed-loop gain v_o/v_s. Find i_o when $v_s = 1\text{ V}$.

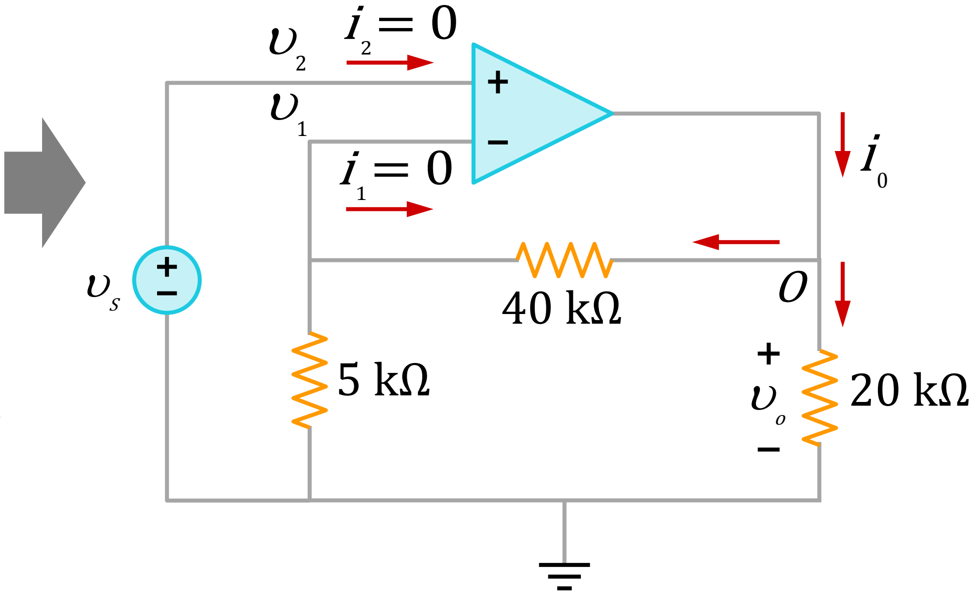


Example 23

The figure is redrawn as shown.

Note that, $\boxed{v_2 = v_s}$



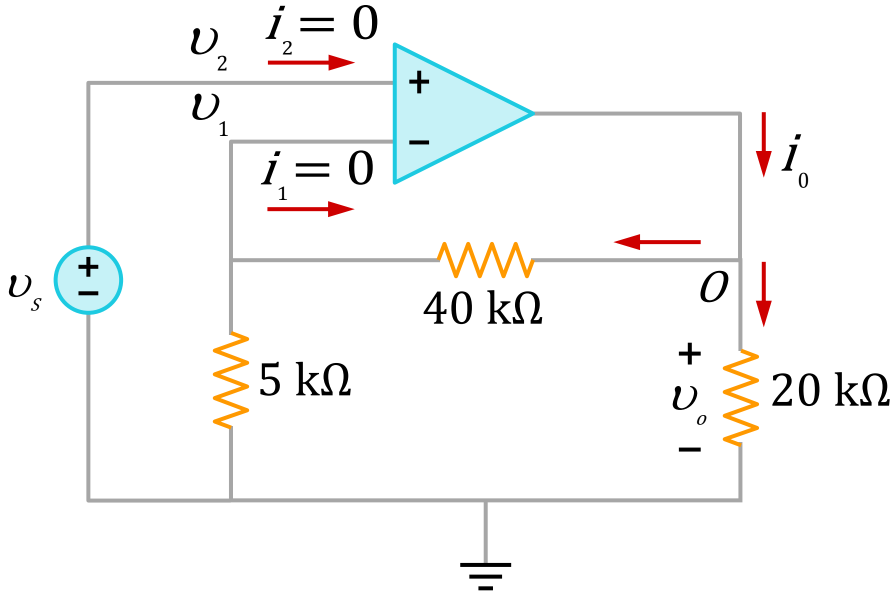


Example 23

Since $i_1 = 0$, the $40\ \mathrm{k\Omega}$ and $5\ \mathrm{k\Omega}$ resistors are in series, the same current flows through them.

Since v_1 is the voltage across the $5\ \mathrm{k\Omega}$ resistor, using the voltage division principle,

$$v_1 = \frac{5}{5+40} v_o = \frac{v_o}{9}$$



Example 23

But, $v_2 = v_1$ **for an ideal op-amp.**

Since $v_2 = v_s$**, we have**

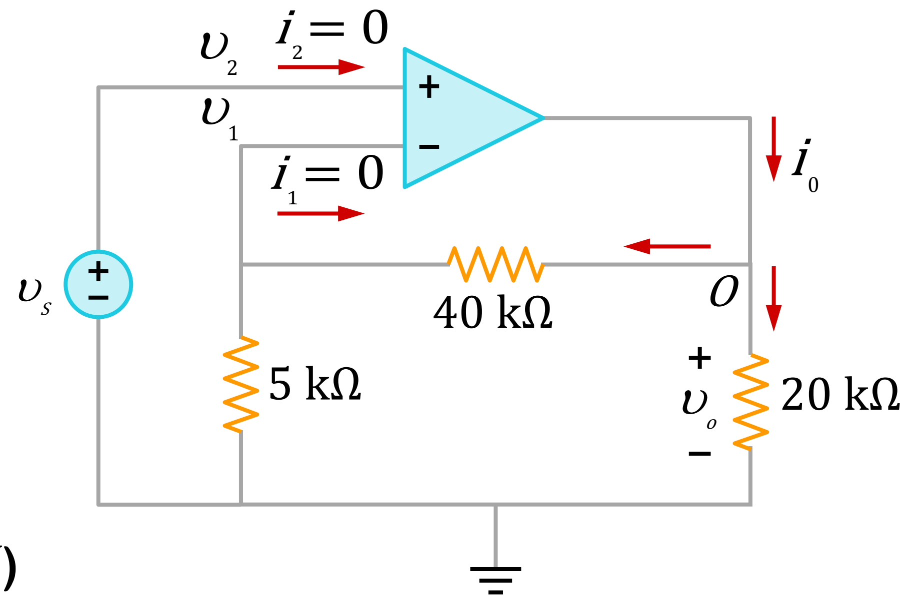
$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9$$

At node 0 at the output,

$$i_o = \frac{v_o}{40+5} + \frac{v_o}{20} \ \text{mA}$$

Substituting for $v_o = 9\,V$ **(as** $v_s = 1\text{ V})$ **gives**

$$\boxed{i_o = \frac{9}{40+5} + \frac{9}{20} = 0.2 + 0.45 = 0.65 \ \text{mA}}$$



Thank You!