NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

EE3010 – ELECTRICAL DEVICES AND MACHINES

November / December 2019 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A magnetic circuit is shown in Figure 1. The mean length of the magnetic core is $l_c = 79.99$ cm and the air gap length is $l_g = 0.01$ cm. The core has a uniform cross-sectional area of 4 cm^2 and is wound with an N-turn coil.

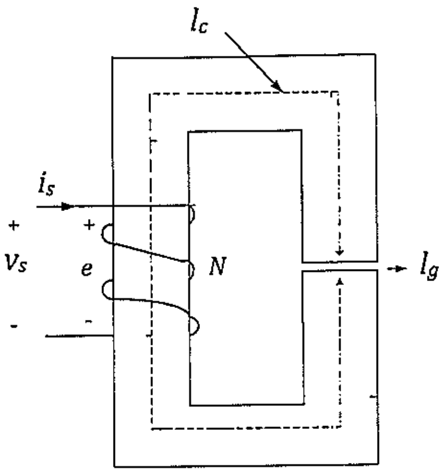

Figure 1

Note: Question No. 1 continues on page 2.

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The relative permeability μ_r of the core is 5000 and the permeability of free space μ_o is $4\pi \times 10^{-7}$ H/m. Neglect leakage and fringing in your calculations.

(a) Determine the expression of the flux density in the core if the voltage applied to the coil is $v_s = 78.54cos100\pi t$ V and $N = 500$. Comment on the phase relationship between the applied voltage and the core flux.

(Hint: Induced emf $e = N\dfrac{d\phi}{dt}$, where ϕ is the flux.)

(6 Marks)

(b) Determine the inductance of the coil and hence obtain the expression of the exciting current i_s. Comment on the phase relationship between the applied voltage and the exciting current.

(11 Marks)

(c) Using the information and answer found in part (a), and given that the eddy current loss in the core is $P_e = 250$ W, determine the eddy current loss in the core if the applied voltage is now $v_s = 75.4cos120\pi t$ V. The eddy current loss is given by $P_e = K_e B_m^2 f^2$ W, where K_e is a constant, B_m is the maximum flux density in the magnetic core and f is the frequency of the source.

(8 Marks)

2. Consider a 20-kVA, 230/2300-V, 50-Hz single-phase transformer. Open-circuit and short-circuit tests on the transformer gave the following results:

Open-circuit test, with the high-voltage side opened:

$V_{oc} = 230$ V, $I_{oc} = 0.75$ A, $P_{oc} = 100$ W

Short-circuit test, with the low-voltage side short circuited:

$V_{sc} = 50$ V, $I_{sc} = 8.7$ A, $P_{sc} = 125$ W

(a) Determine the approximate equivalent circuit referred to the high-voltage side.

(8 Marks)

(b) Using the equivalent circuit obtained in part (a), determine the secondary terminal voltage when the input voltage is 230 V and the transformer is delivering rated load current at 0.85 power factor lagging. (Hint: Use the secondary terminal voltage as the reference, i.e., $V_2\angle 0^o$ and the input voltage as $230\angle\theta_1{}^o$ to solve for θ_1 and V_2.)

(10 Marks)

(c) Using the data given in part (a) or otherwise, find the efficiency of the transformer at 80% of full load and 0.85 power factor lagging, with the secondary terminal voltage maintained at 2300 V.

(7 Marks)

3. (a) A three-phase, 2300-V, 60-Hz, 6-pole, wye-connected induction motor is tested with the following results:

No-load test : $V_{line} = 2300$ V, $I_{line} = 7.8$ A, $P_{in} = 2.88$ kW

Locked-rotor test : $V_{line} = 268$ V, $I_{line} = 48.8$ A, $P_{in} = 18.2$ kW

DC test : $V_{DC} = 13.2$ V, $I_{DC} = 2.96$ A

(i) Obtain the parameters of the single-phase equivalent circuit of the induction motor referred to the stator. Assume that the stator leakage reactance and the rotor leakage reactance referred to the stator are equal in magnitude.
(8 Marks)

(ii) Determine the rotational losses of the motor.
(2 Marks)

(b) A three-phase, 230-V, 60-Hz, 4-pole, wye-connected induction motor has the following per phase parameters referred to the stator:

$$R_1 = 0\ \Omega,\quad X_M = 20\ \Omega,\quad R_2 = X_1 = X_2 = 1\ \Omega$$

The rotational losses can be neglected.

(i) Determine the slip at which maximum torque is developed by the motor.
(4 Marks)

(ii) Determine the value of the maximum torque developed by the motor.
(4 Marks)

(iii) If the maximum torque developed by the motor is 200% of its rated load torque, determine the slip at rated load torque.
(7 Marks)

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4. The magnetization curve of a dc machine obtained at a speed of 1400 rpm is shown in Figure 2 on page 5. The armature and field winding resistances are 0.3 Ω and 100 Ω, respectively. The constant rotational losses of the machine are 500 W. The effects of armature reaction can be neglected.

Consider the following cases:

(a) The machine is operated as a separately-excited dc generator. An external variable resistor R_{ext} is connected in series with the field winding and the field circuit is fed by a constant 230-V dc supply.

(i) The generator is driven at 1400 rpm and is supplying power to a load. If R_{ext} is adjusted such that the internal generated voltage E_A is 180 V and the terminal voltage is 165 V, determine the efficiency of the generator.

(8 Marks)

(ii) If the generator is driven at 1800 rpm, determine the value of R_{ext} required to give a no-load terminal voltage of 180 V.

(4 Marks)

(b) The machine is operated as a self-excited dc generator and an external variable resistor R_{ext} is connected in series with the field winding. If the generator is driven at 1400 rpm and R_{ext} is adjusted to 20 Ω, determine the no-load terminal voltage.

(3 Marks)

(c) A dc machine operates as a separately-excited motor with an armature terminal voltage supply of 250 V. The armature winding resistance is 0.18 Ω. Ignoring saturation effects, the magnetization curve of the internal generated voltage E_A versus the field current I_F of the machine, obtained at a speed of 1200 rpm, is assumed to be a straight line with a constant slope of 150 Volts per Ampere. The field current is adjusted to be 1.67 A for the following cases:

(i) If the no-load speed is 1188 rpm, determine the rotational losses of the motor.

(4 Marks)

(ii) If the power developed by the motor is 28 kW, determine the speed of the motor.

(6 Marks)

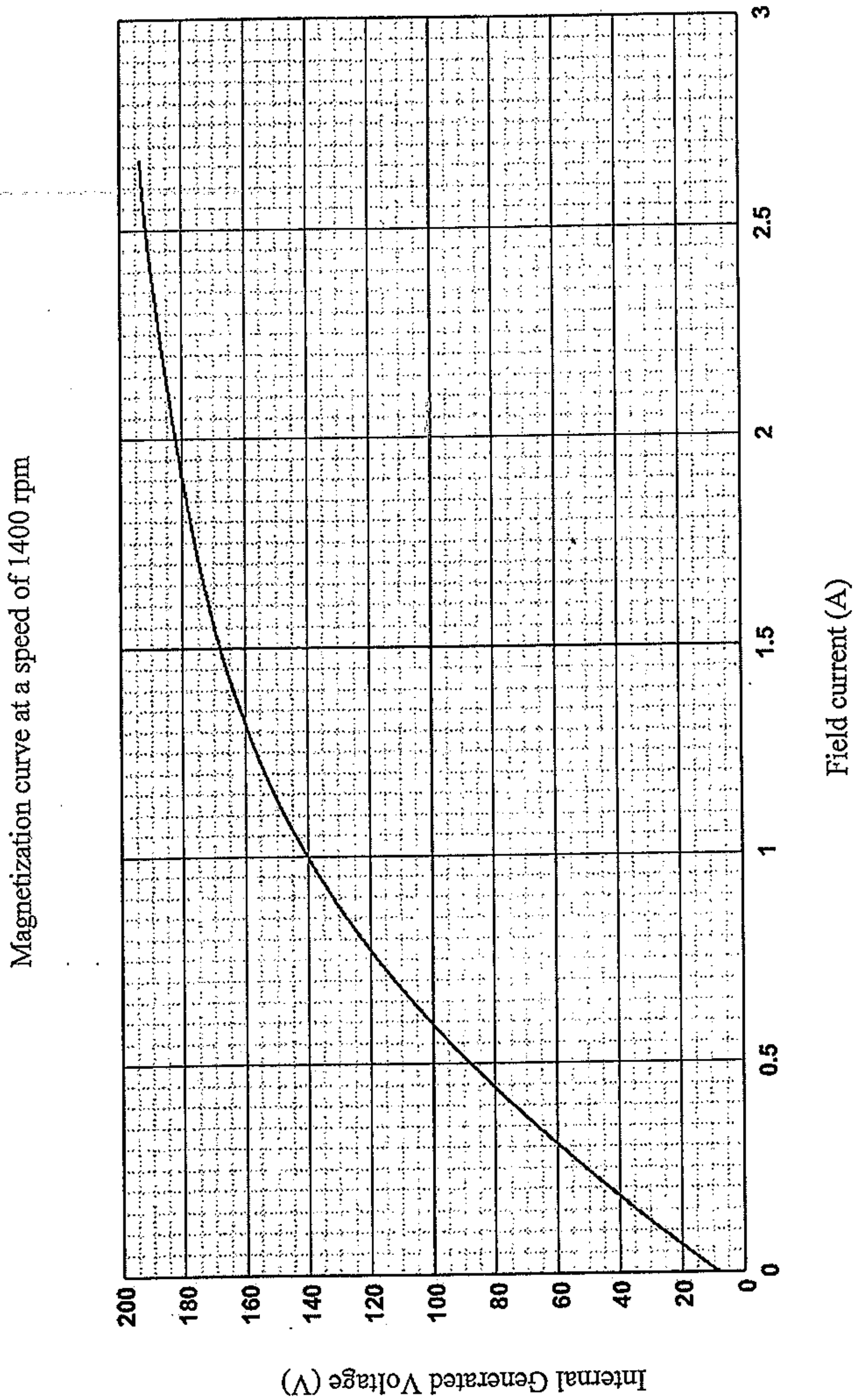
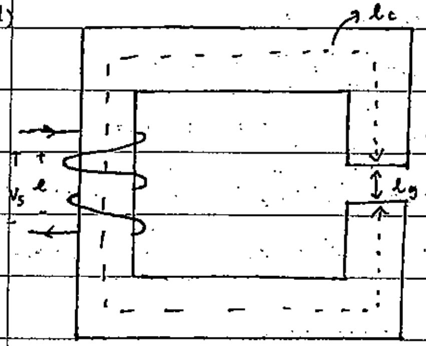


Figure 2

END OF PAPER

1)



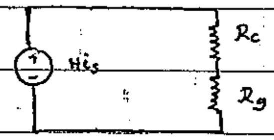
$l_c = 0.7999\,m$

$l_g = 1\times10^{-4}\,m$

$A = 4\times10^{-4}\,m^2$

$\mu_0 = 4\pi\times10^{-7}$

magnetic equivalent circuit



$$R_c = \frac{l_c}{\mu_r\mu_0 A} = \frac{0.7999}{5000(4\pi\times10^{-7})(4\times10^{-4})} = 318270.0974\ H^{-1}$$

$$R_g = \frac{l_g}{\mu_0 A} = \frac{1\times10^{-4}}{4\pi\times10^{-7}(4\times10^{-4})} = 198943.6789\ H^{-1}$$

1 a) since resistance coil negligible

$e = v_s \Rightarrow v_s = N\frac{d\phi}{dt}$

$\cancel{500\left(\frac{d\phi}{dt}\right) = 78.54\cos(100\pi t)}$

$d\phi = 0.15708\cos(100\pi t)\,dt \Rightarrow$ integrate both sides.

$\phi = 5\times10^{-4}\sin(100\pi t) \Rightarrow$ since ϕ is sinosuidal and v_s is cosine graph, v_s lags by $90°$ compared to ϕ

1 b) $R_{total} = R_c + R_g = 517213.7763\ H^{-1}$

$$L = \frac{d\lambda}{di} = \frac{d}{di}\left(\frac{N^2 i}{R_{total}}\right) = \frac{N^2}{R_{total}} = \frac{500^2}{517213.7763} = \cancel{0.48336\ H}$$

$$v_s = L\frac{di_s}{dt} \Rightarrow i_s = \int\frac{V_s}{L}\,dt = \int\frac{78.54\cos(100\pi t)}{0.48336}\,dt = 0.5172\sin(100\pi t)$$

$\Rightarrow$ exciting current lags by $90°$ compared to applied voltage

1 c) $P_e = k_e B_m^2 f$

$P_e = 250\ W$

$B_m = 5\times10^{-4}/4\times10^{-4} = 1.25\ T$ (part a)

$f_m = 100\pi/2\pi = 50\ Hz$ (part a)

$$k_e = \frac{P_e}{B_m^2 f} = \frac{250}{(1.25)^2(50)} = 3.2$$

$f_m = 120\pi/2\pi = 60\ Hz \Rightarrow v_s = 4.44NAB_m f \Rightarrow 4.44NA$ is ~~same~~ equal

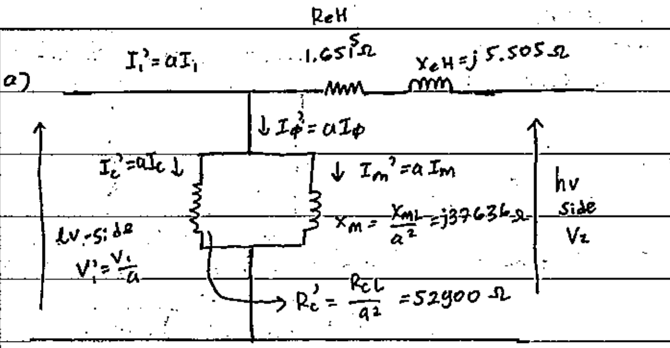
$$\frac{v_{s1}}{v_{s2}} = \frac{78.54}{75.4} = \frac{1.25(50)}{B_m(60)}$$

$B_m = 1.000\ T$

$P_e = 3.2(1)^2(60)$

$= 192\ W$

2 a)



from O.C test =

$R_{cL} = \frac{V_{oc}^2}{P_{oc}} = \frac{230^2}{100} = 529\,\Omega$

$I_c = \frac{V_{oc}}{R_{cL}} = \frac{230}{529} = 0.4348\,A$

$I_m = \sqrt{I_{oc}^2 - I_c^2} = \sqrt{0.75^2 - 0.4348^2} = 0.6111\,A$

$X_{mL} = \frac{V_{oc}}{I_m} = 376.3600\,\Omega$ => refered to low voltage

$a = \frac{1}{10}$

from S.C test =

$R_{eH} = \frac{P_{sc}}{I_{sc}^2} = \frac{12.5}{(8.7)^2} = 1.6515\,\Omega$

$Z_{eH} = \frac{V_{sc}}{I_{sc}} = \frac{50}{8.7} = 5.7471\,\Omega$

$X_{eH} = \sqrt{Z_{eH}^2 - R_{eH}^2} = \sqrt{(5.7471)^2 - (1.6515)^2} = 5.505\,\Omega$

=> refered to high voltage side

2 b) $V_2 \angle 0^\circ + I_2 \angle \theta_2^\circ \times Z_{eH} = V_1 \angle \theta_1^\circ$

$I_2 = \frac{20 \times 10^3 \angle \theta_2^\circ}{V_2}$ ~~$\theta_2 = -\cos^{-1}(-0.85) = -31.78^\circ$~~

$Z_{eH} = 5.9797 \angle 73.9674^\circ$

$I_2 \times Z_{eH} = \frac{110594}{V_2} \angle 42.1874^\circ$ => phase add, magnitude multiplied

$\Downarrow$ a+jb form

$$V_2 + \left(\frac{88613.44906}{V_2}\right) + j\frac{80314.26699}{V_2} = 2300 \angle \theta_1^\circ$$

=> complex magnitude $2300^2 = a^2 + b^2$ => $a + bj$ = complex number

$$\left(\left(\left(V_2 + \frac{88613.44906}{V_2}\right)^2 + \left(\frac{80314.26699}{V_2}\right)^2\right) = 2300^2\right) \times V_2^2$$

$V_2^4 + 177226.8981 V_2^2 + 1.430272484 \times 10^{10} = 2300^2 V_2^2$

$V_2^4 - 5112773.102 V_2^2 + 1.430272484 \times 10^{10} = 0$

=> use calc to solve or $V_2^2 = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$V_{2_1} = 2260.53$ (correct) $V_{2_2} = 2798.8155$ => invalid, since V_2 is always < 2300

$\theta_1 = \tan^{-1}\left(\frac{35.529}{2260.53 + 39.200}\right) = 0.885^\circ$ => from $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, b is imaginary, a is real

2 c) $P_{cu} = P_e$ at max efficiency

=> ~~maximum~~ 80% at full load

$\theta_2 = -\cos^{-1}(0.85) = -31.78^\circ$

Secondary side is $2300 \angle 0^\circ$ V

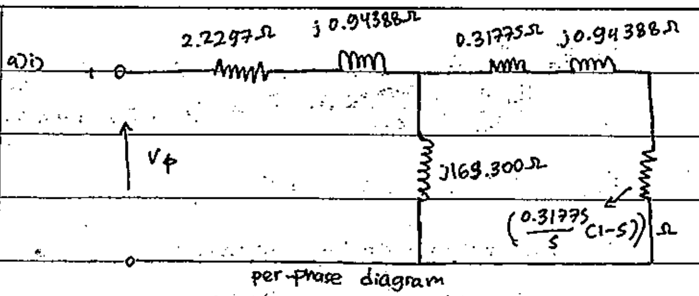
$I_2 = \frac{(0.8)(0.85)(20 \times 10^3)}{2300} = 5.913\,A$

$P = VI^* = VI\cos(\theta_2) \times p.f = 13600\,W$

$n = \frac{P_{out}}{P_{in} + P_{cu} + P_m}$ $P_{cu} = (0.8)^2 P_{cu-fe} = (0.8)^2 (5.913)^2 (1.6515) = 36.96\,w$ ($\to x^2$) $P_{cu} = P_e$

~~$n = \frac{13600}{13600 + 2(36.96)} = 99.5\%$~~

3 a)i)



DC test: $R_1 = \frac{V_1}{2I_{DC}} = \frac{13.2}{2(2.96)} = 2.2297\,\Omega$

locked-rotor test (s=1)

$P_{in} = 3I^2(R_1+R_2)$ $\quad R_2 = \frac{P_{in}}{3I_{line}^2} - R_1 = \frac{18.2\times10^3}{3(48.8)^2} - 2.2297$

$R_2 = 0.31775\,\Omega$ $\quad Z_{LR} = \frac{V_\phi}{I_1} = \sqrt{(R_1+R_2)^2+(x_1+x_2)^2}$

$x_1 = x_2$

$\left(\frac{268}{\sqrt{3}} \times \frac{1}{48.8}\right)^2 = (2.2297 + 0.31775)^2 + (2x_1)^2$

$10.05330 = 6.48848 + 4x_1^2$

$x_1^2 = 0.890914$ $\quad x_1 = x_2 = 0.94388\,\Omega$

~~no-load~~ no-load test (s=0)

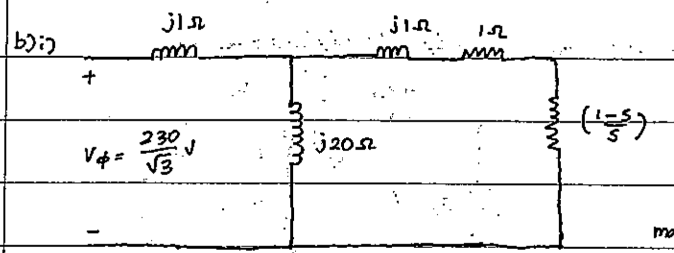
$\frac{V_\phi}{I_m} \approx X_1 + X_m$ $\quad \cancel{\frac{2300}{\sqrt{3}\times 7.8} = 0.94388 + X_m}$

$X_m = 169.300\,\Omega$

3 a)ii) $P_{rot} = P_{in} - P_{SCL}$ $\quad P_{SCL} = 3I_1^2R = 3(7.8)^2(2.2297) = 406.97\,\Omega$

$P_{rot} = 2.88\times10^3 - 406.97 = 2.473$ kW

3 b)i)



$V_{TH} = \frac{jX_M V_\phi}{R_1 + jX_1 + jX_M}$ $\quad Z_{TH} = \frac{jX_M(R_1+jX_1)}{R_1+jX_1+jX_M} = \frac{j(20)(1j)}{j20+j1}$

$= \frac{j20}{j1+j20}\left(\frac{230}{\sqrt{3}}\angle 0^\circ\right)$ $\quad = \frac{20}{21}j$

$= 126.4672\angle 0^\circ$

max power transfer when $\frac{R_2}{s_{max}} = |Z_{TH} + jX_2|$

$|Z_{TH} + jX_2| = \left|\frac{20}{21}j + j\right| = \frac{41}{21}$

$\frac{1}{s_{max}} = \frac{41}{21}$

$s_{max} = \frac{21}{41} = 0.51220$

3 b)ii) $T_{ind} = \frac{P_{AG}}{w_{sync}}$ $\quad P_{AG} = 3I_2^2\left(\frac{R_2}{s}\right)$

$\cancel{I_2 = \frac{126.4672\angle 0^\circ}{\frac{20}{21}j + j + 1.9524} = 45.8032\angle -44.999^\circ} \Rightarrow I_2 = \frac{V_{TH}}{Z_{eq}}$

$|I_2|^2 = 2097.94$ $\quad P_{AG} = 3(2097.94)(1.9524) = 12288.03681$ W

$n_{sync} = \frac{120f_s}{P} = \frac{120(60)}{4} = 1800$ rpm $\quad T_{max} = \frac{12288.03681}{1800(2\pi/60)} = 65.19$ N·m

3 b)iii) max torque 200% of rated voltage

$T_{rat} = \frac{65.19}{2} = 32.60$ N·m

$n_{sync} = \frac{120f_s}{P} = \frac{120(60)}{4} = 1800$ rpm

$w_{sync} = 1800\left(\frac{2\pi}{60}\right) = 188.50$ rad s^{-1}

$P_{AG} = T_{ind}\, w_{sync} = 32.60(188.50) = 6144.9523$ W

$I = \frac{126.4672\angle 0^\circ}{j41/21 + R_2/s}$ $\quad R_2/s \Rightarrow x$

$|I|^2 = \frac{(126.4672)^2}{x^2 + (41/21)^2}$

$6144.95523 = 3\left(\frac{126.4672}{x^2+(41/21)^2}\right)^2 x$

continue here ⇒

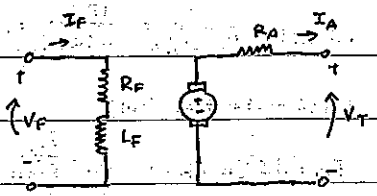
$\frac{x}{(x^2+(41/21)^2)^2} = 0.128068305$

$0.128068305x^2 - x + \left(\frac{41}{21}\right)^2(0.128068305) = 0$

$x_1 = 7.2851$ $\quad x_2 = 0.5232$ ⇒ slip > 1 ⇒ invalid

$s = \frac{1}{7.2851} = 0.1373$

4 a)i)



$V_T = E_A - I_L R_A$

$I_L = \frac{E_A - V_T}{R_A} = \frac{180-165}{0.3} = 50A \Rightarrow I_F = 1.9\,A$ from magnetization curve

$P_{dev} = E_A I_A = 180(50) = 9\,kW$

$E_A = K\phi w_n \quad T_{ind} = K\phi I_A$

$P_{in} = P_{dev} + P_{rot} = 9kW + 0.5kW = 9.5kW$

$\eta = \frac{P_{dev}}{P_{in}} \times 100\% = \frac{9}{9.5} \times 100\% = 94.737\%$

$R_A = 0.3\,\Omega \quad R_F = 100\,\Omega$

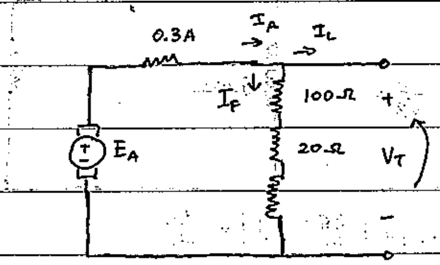
4 a)ii) $\frac{E_A}{E_{A0}} = \frac{n_m}{n_o} \Rightarrow \cancel{E = k\phi n}$

$\frac{180}{E_{A0}} = \frac{1800}{1400}$

$E_{A0} = 140V \Rightarrow I_F = 1\,A$ from magnetization curve

$I_F = \frac{230}{R_{ext} + R_F} \Rightarrow R_{ext} + R_F = 230 \Rightarrow R_{ext} = 230 - 100 = 130\,\Omega$

4 b) at no load terminal voltage of shunt motor $I_L = 0$ $I_A = I_F$ $V_T = I_F(R_F + R_{ext})$



$V_T = E_A - I_F(R_A)$

$I_F(120) = E_A - I_F(0.3)$

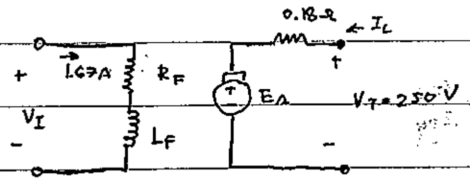
$I_F(120.3) = E_A$

$I_F = \frac{E_A}{120.3} \Rightarrow$ draw at magnetization curve

intersect at $I_F = 1.3A \quad E_A = 160A$

$V_T = 160 - 1.3(0.3) = 159.61\,V$

4 c)i)



$E_A/I_F = 150\,V/A \Rightarrow 1200$ rpm

$\frac{E_A}{E_{A0}} = \frac{n_0}{n_m} \quad E_A = \frac{1188}{1200} \times 250 = 247.5\,V$

$E_A + I_L(R) = V_T \Rightarrow \cancel{E_A \times 250}\ I_L(R) = 250 - 247.5$

$I_L = 13.889\,A \Rightarrow P_{rot} = E_A I_A = 3437.5\,W$

4 c)ii) $P_{dev} = 28 \times 10^3\,W$

$P_{dev} = E_A I_A$

$E_A = 250 - I_A(0.18)$

$28 \times 10^3 = (250 - I_A(0.18))\,I_A$

$I_A^2(0.18) - 250 I_A + 28 \times 10^3 = 0$

$I_{A_1} = 1266.019A \quad I_{A_2} = 122.8698\,A \Rightarrow$ both cases are true

$E_{A_1} = 22.11658 \quad E_{A_2} = 227.8834$

$\frac{22.11658}{250} = \frac{n_{sync}}{n_{speed}} \Rightarrow 1200$ rp.m. $\qquad \frac{227.8834}{250} = \frac{n_{sync}}{n_{speed}}$

$n_{sync} = 106.16$ rpm $\qquad n_{sync} = 1094$ rpm