Lecture 9: Transformers

EE3010: Electrical Devices and Machines

School of Electrical and Electronic Engineering

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Learning Objectives

By the end of this lecture, you should be able to:

- Describe the methods to develop the exact equivalent circuits and approximate equivalent circuits of a practical transformer.
- Calculate the input voltage and input current using the exact equivalent circuits and approximate equivalent circuits.
- Compare the results obtained by the approximate method with those obtained by the exact method.

Exact Equivalent Circuits

❖ Fig. 17 shows the exact equivalent circuit of a transformer. The calculations involved in transformer problems can be significantly reduced if the transformer circuit is simplified by referring all the circuit parameters to only one side. The circuits thus obtained are called the exact equivalent circuits referred to that side.

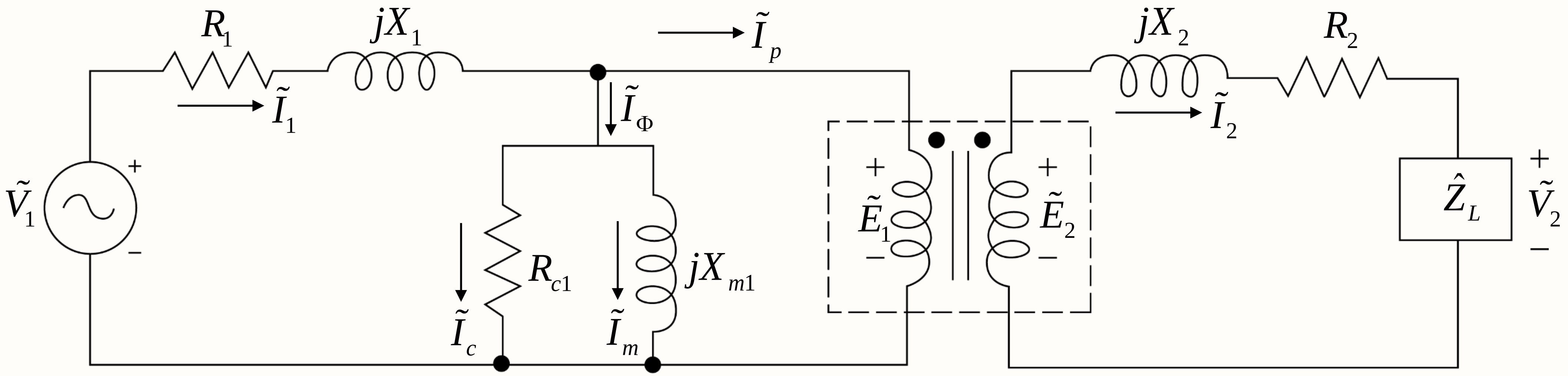


Fig. 17. Exact equivalent circuit of a transformer.

Exact Equivalent Circuits

- The equivalent circuit can be referred to the primary side (see Fig. 18) by making the following transformations:

$$Z_2 \rightarrow Z_2' = a^2 Z_2 \;\; (R_2' = a^2 R_2,\; X_2' = a^2 X_2,\; Z_L' = a^2 Z_L)$$

$$V_2 \rightarrow V_2' = aV_2,\; E_2 \rightarrow E_2' = aE_2 = E_1$$

$$I_2 \rightarrow I_2' = \frac{1}{a} I_2$$

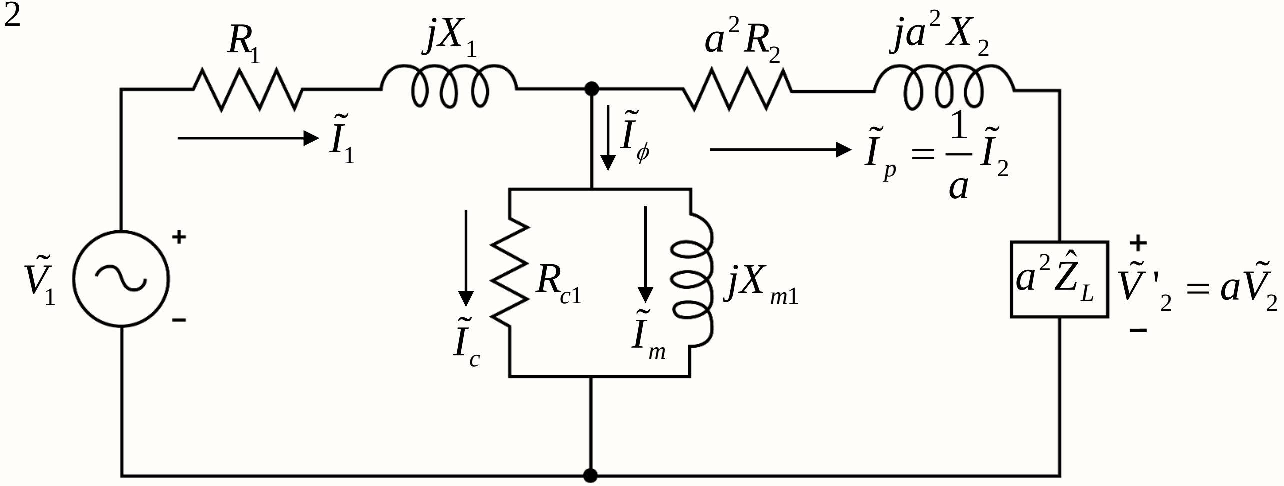


Fig. 18. Exact equivalent circuit referred to the primary.

Exact Equivalent Circuits

❖ The equivalent circuit can be referred to the secondary side (see Fig. 19) by making the following transformations:

$$Z_1 \rightarrow Z_1' = \frac{1}{a^2} Z_1$$

$$R_1 \rightarrow R_1' = \frac{R_1}{a^2}, \quad X_1 \rightarrow X_1' = \rightarrow \frac{X_1}{a^2}$$

$$R_{c1} \rightarrow R_{c1}' = \frac{R_{c1}}{a^2}, \quad X_{m1} \rightarrow X_{m1}' = \frac{X_{m1}}{a^2}$$

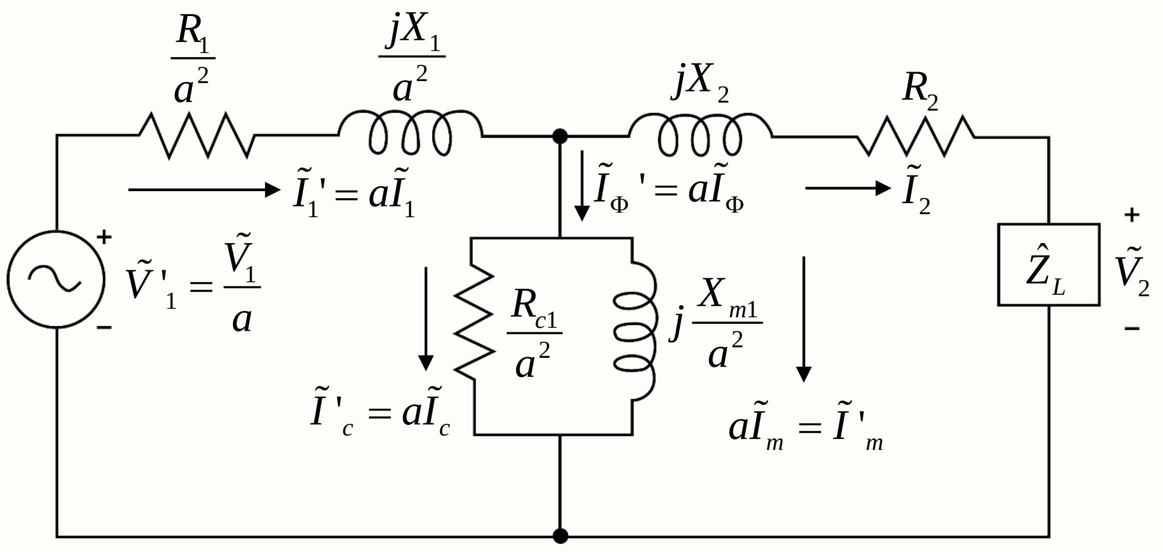


Fig. 19. Exact equivalent circuit referred to the secondary.

Exact Equivalent Circuits

$$I_1 \to I_1' = aI_1$$

$$I_\Phi \to I_\Phi' = aI_\Phi$$

$$I_c \to I_c' = aI_c$$

$$I_m \to I_m' = aI_m$$

$$V_1 \to V_1' = \frac{V_1}{a}$$

$$E_1 \to E_1' = \frac{E_1}{a} = E_2$$

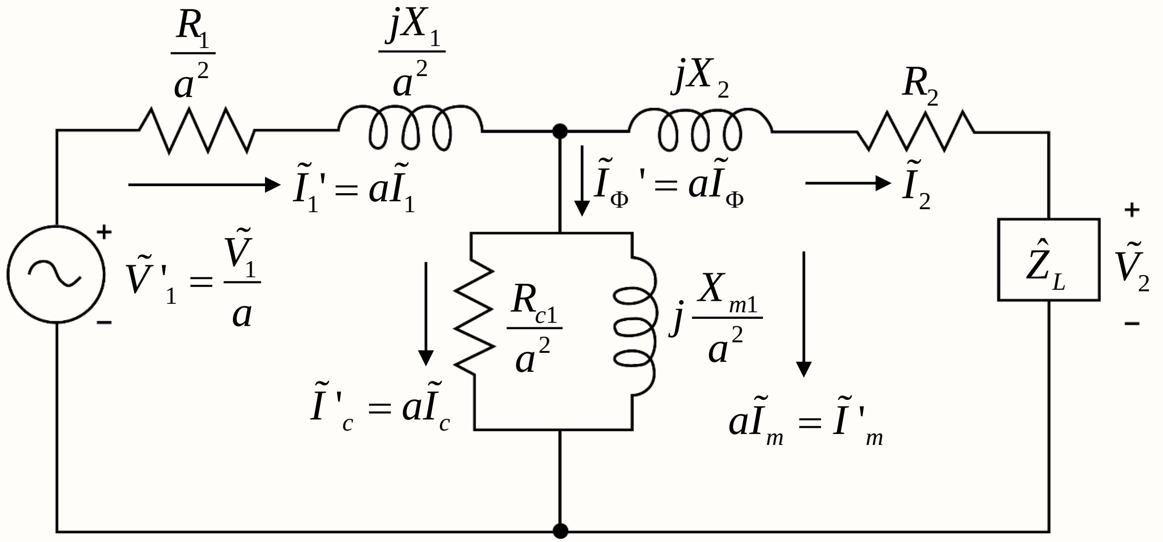


Fig. 19. Exact equivalent circuit referred to the secondary.

- These equivalent circuits can be used to analyse transformer circuits to obtain any desired results. The referred values can be converted back to the actual values as required.
- Solve Example 2 in Lecture 8 using these exact equivalent circuits.

Transformer Parameter Values

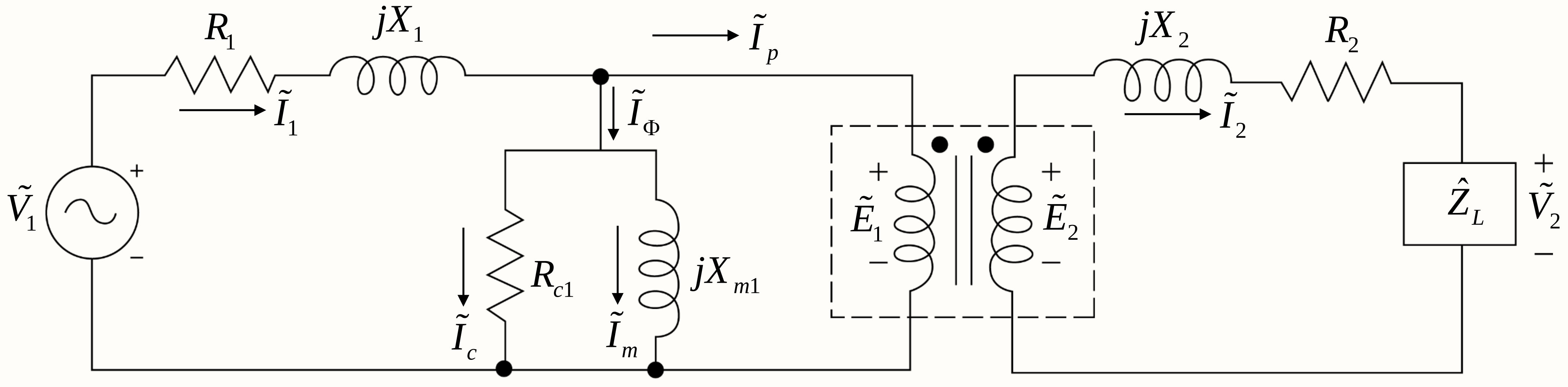


Fig. 20. Exact equivalent circuit of a transformer.

Transformer Parameter Values

❖ **Series Components**:

- R_1 and R_2 represent the resistances of the copper coils, and therefore these resistances are usually very small.
- X_1 and X_2 represent the reactances arising from the leakage fluxes, which are very small. Therefore their magnitudes are also very small.
- The losses and the voltage drops in these series parameters are usually very small, and often ignorable.

Transformer Parameter Values

- **Shunt Components**:
  - I_m represents the currents drawn to set up the magnetic flux in the core, and I_c represents the current that accounts for the core loss.
  - Since very good magnetic material is used in transformers, both these currents are very small, often ignorable.
  - The corresponding shunt impedances R_{c1} and X_{m1} are very large, often taken as infinity and ignored from the circuit.

Approximate Equivalent Circuits

❖ More practically, because of the relative values of the series and the shunt components, it introduces very small error if we move the shunt branch from the middle position to the input side, as indicated in Fig. 21.

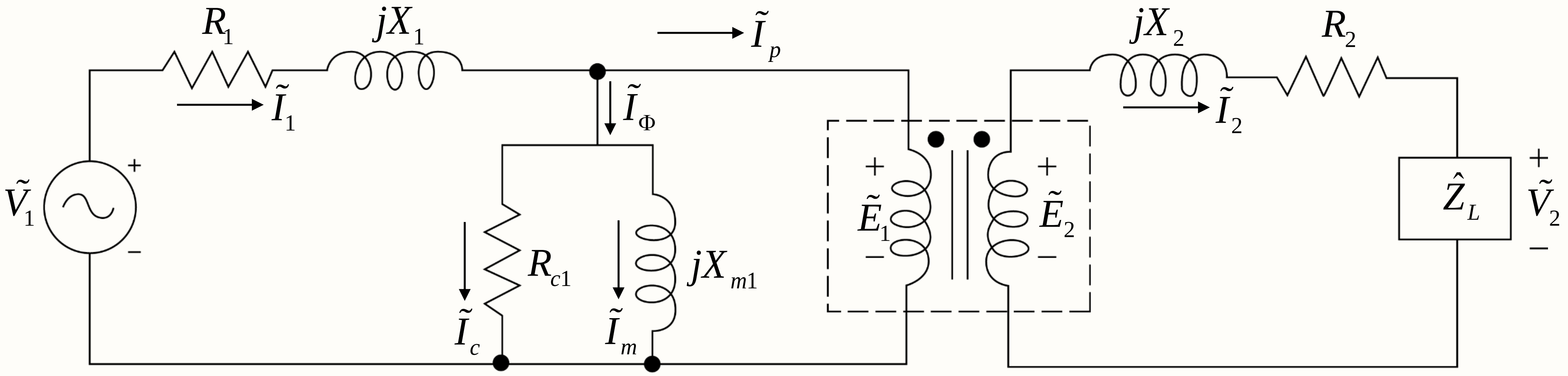


Fig. 21. Exact equivalent circuit of a transformer.

Approximate Equivalent Circuits

- Such a simplified circuit is called the **approximate equivalent circuit** as shown in Fig. 22, and commonly used to simplify the computations. This, however, will introduce a small amount of error.

- The circuit may be further simplified by referring all the secondary quantities to the primary side.

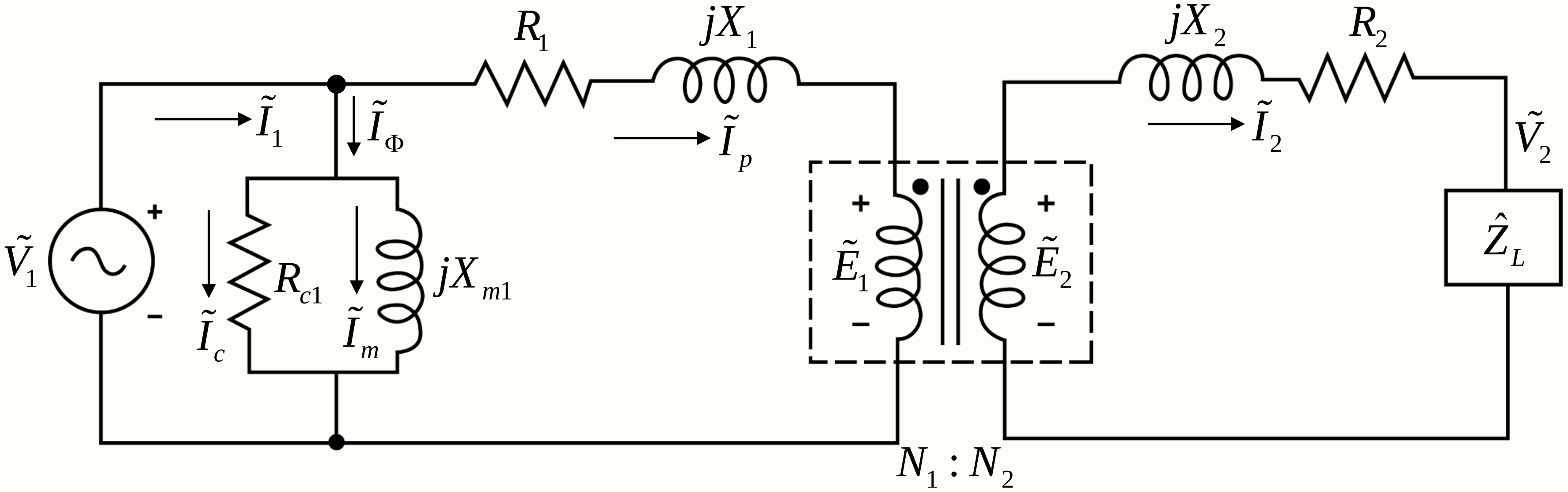


Fig. 22. Approximate equivalent circuit of a transformer.

Approximate Equivalent Circuit Referred to the Primary

❖ The series components in the approximate equivalent circuit are in series, and can be combined as

$$R_{e1} = R_1 + a^2 R_2, \text{ and } X_{e1} = X_1 + a^2 X_2$$

❖ They are called the total effective resistance and the total effective leakage reactance referred to the primary side (side 1) respectively, as shown in Fig. 23.

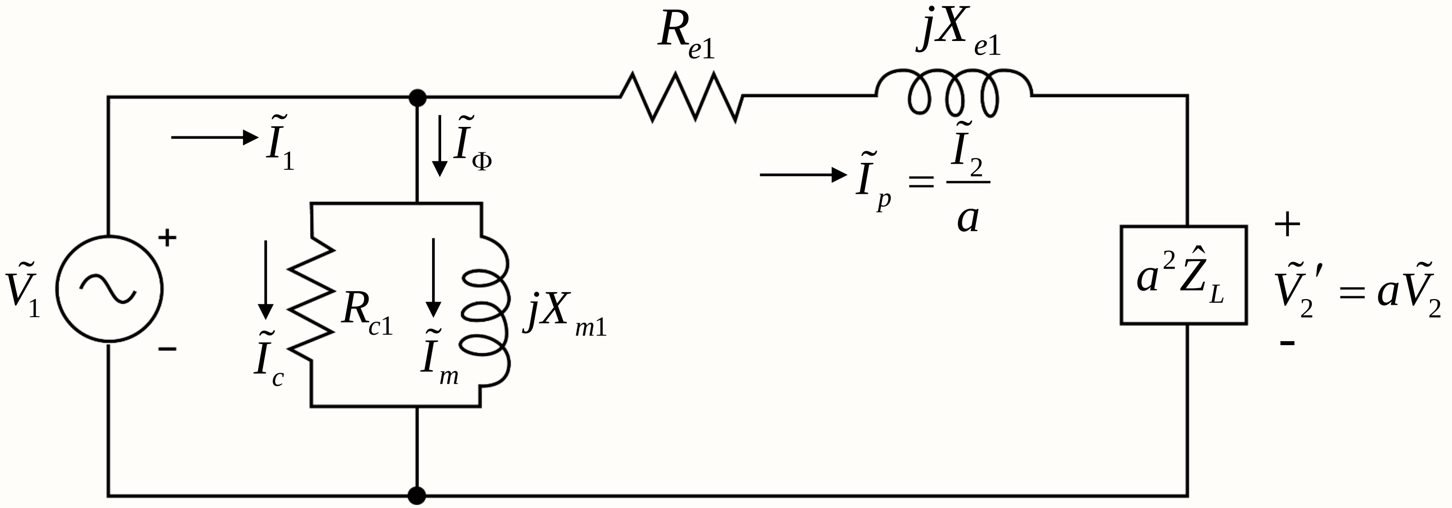


Fig. 23. Approximate equivalent circuit referred to the primary.

Approximate Equivalent Circuit Referred to the Secondary

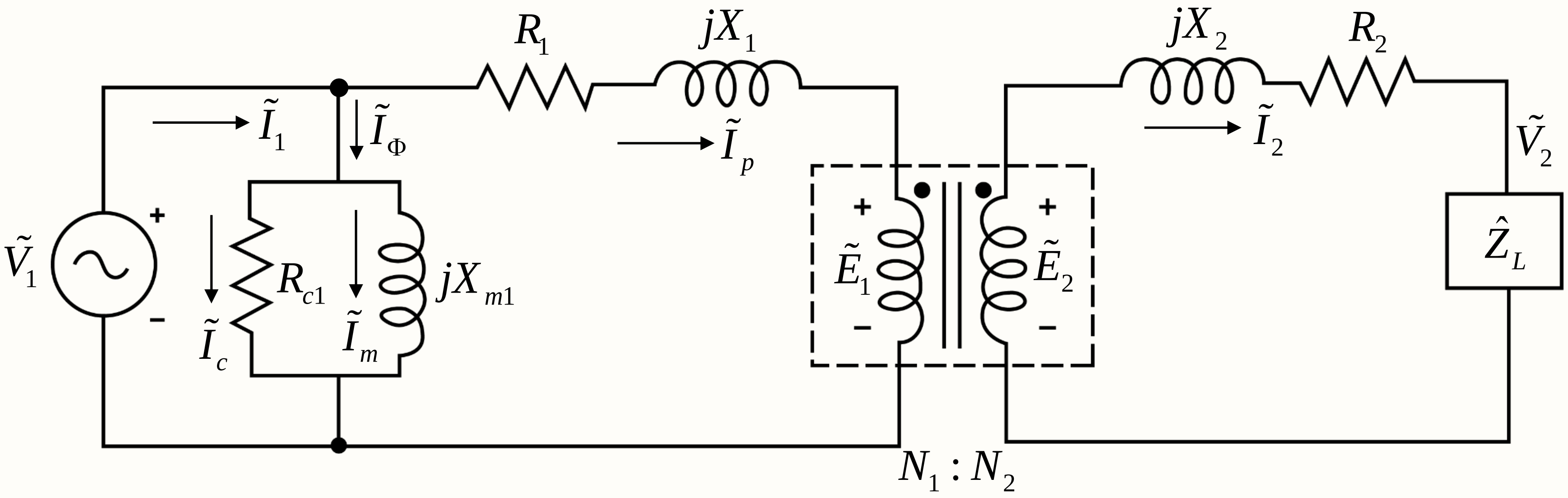


Fig. 24. Approximate equivalent circuit of a transformer.

Approximate Equivalent Circuit Referred to the Secondary

- Similarly, the circuit may be referred to the secondary side as shown Fig. 25, where

  $$R_{e2} = \frac{R_1}{a^2} + R_2, \text{ and } X_{e2} = \frac{X_1}{a^2} + X_2$$

  are the total effective resistance and the total effective leakage reactance referred to the secondary side (side 2) respectively.

- It should further be noted that the shunt components are also referred to the secondary side.

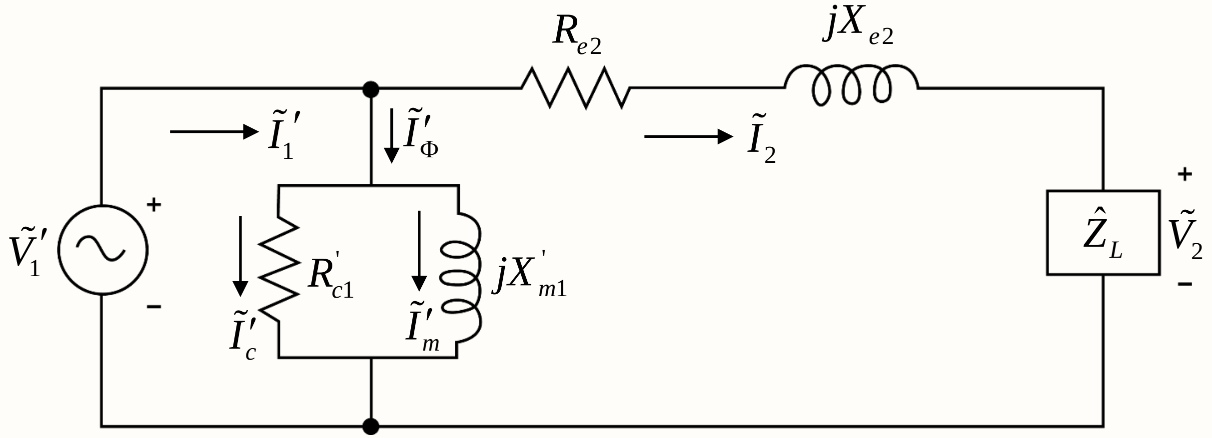


Fig. 25. Approximate equivalent circuit referred to the secondary.

Example 3

A 23-kVA, 2300/230-V, 60-Hz transformer has the following parameters:

$$R_1 = 4\ \Omega,\ R_2 = 0.04\ \Omega,\ X_1 = 12\ \Omega,\ X_2 = 0.12\ \Omega$$

$$R_{c1} = 20\ \mathrm{k\Omega},\ \text{and}\ \ X_{m1} = 15\ \mathrm{k\Omega}.$$

If the transformer delivers 75% of its rated load at 0.866 pf (lag) at rated voltage, calculate the input voltage using the approximate equivalent circuit referred to the primary.

(Solutions →)

Example 3 – Solutions

$$a = \frac{2300}{230} = 10,$$ and the circuit is shown in Fig. 26, where

$$R_{e1} = 4 + 10^2 \times 0.04 = 8\ \Omega,$$ and

$$X_{e1} = 12 + 10^2 \times 0.12 = 24\ \Omega$$

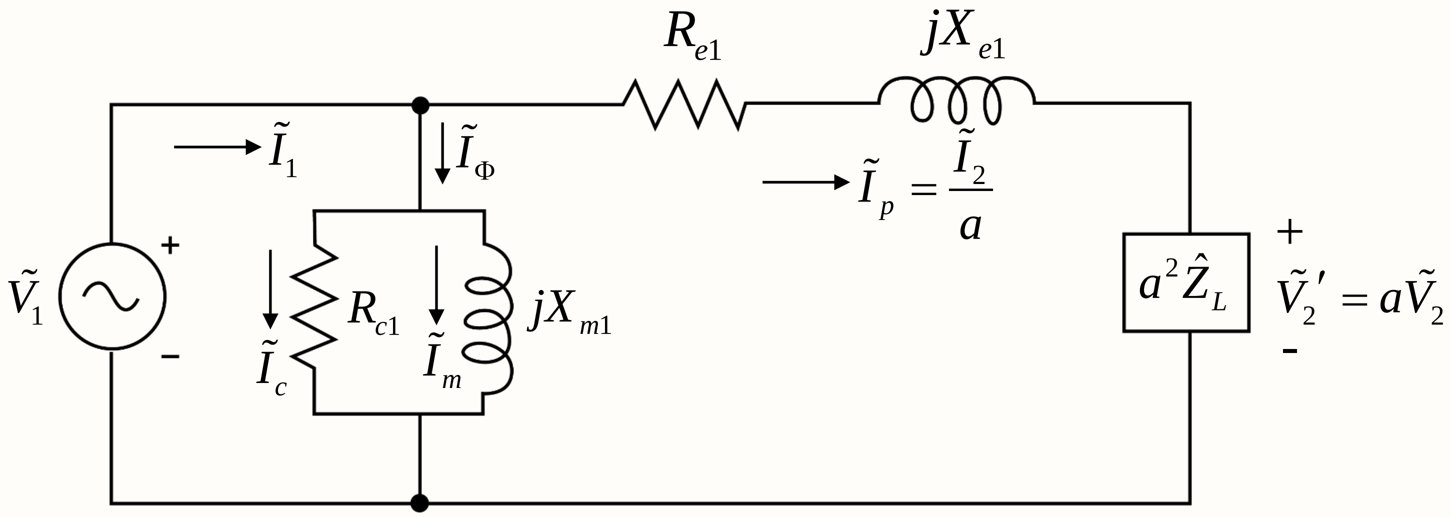


Fig. 26. Approximate equivalent circuit referred to the primary.

Example 3 – Solutions

As before,

$V_2 = 230\angle 0^\circ$ V (**Reference**)

$I_2 = 75\angle -30^\circ$ A, so that

$$V_2' = aV_2 = 10\times 230\angle 0^\circ = 2300\angle 0^\circ \text{ V}$$

$$I_p = I_2' = \frac{I_2}{a} = \frac{75\angle -30^\circ}{10} = 7.5\angle -30^\circ \text{ A}$$

$$V_1 = V_2' + I_2' \times (R_{e1} + jX_{e1}) = 2300\angle 0^\circ + 7.5\angle -30^\circ \times (8 + j24) = 2445.21\angle 2.95^\circ$$

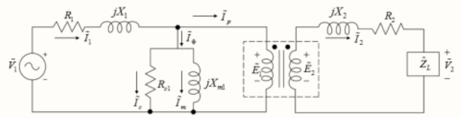
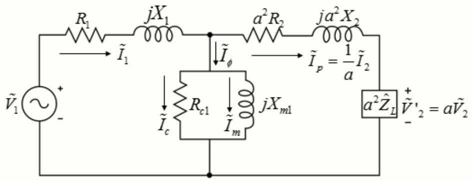
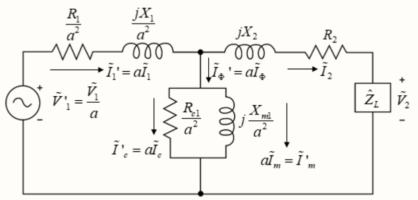
- ❖ Compare this with the value obtained by the exact method ($2447.6\angle 2.97^\circ$ V).
- ❖ Calculate other values as before and compare the answers.

Summary

In this lecture, you have learnt:

- The methods to develop the exact equivalent circuits and approximate equivalent circuits of a practical transformer.
- The calculation of input voltage and input current using the exact equivalent circuits and approximate equivalent circuits.
- The comparison of the results obtained by the approximate method with those obtained by the exact method.

References

No.	Slide No.	Image	Reference
1	4, 8, and 11		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 218), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
2	5		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 219), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
3	6 and 7		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 219), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.

References

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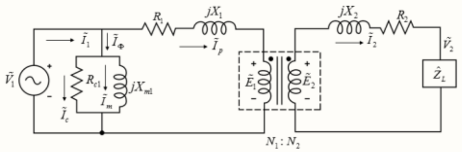
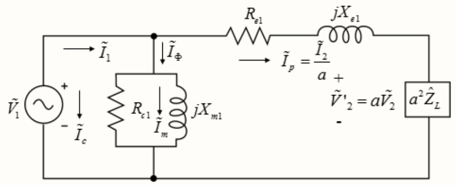
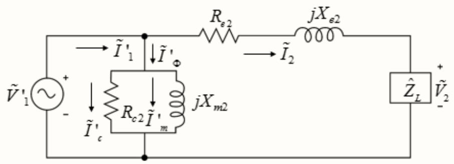
4	12 and 14		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 222), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
5	13 and 17		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 222), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.
6	15		Reprinted from *Electric Machinery and Transformers, 3rd ed.*, (p. 223), by B. S. Guru, & H. R. Hiziroglu, 2001, New York, NY: Oxford University Press. Copyright 2001 by Oxford University Press.