TUTORIAL 7 PN Junction in Thermal Equilibrium

You may assume a temperature of 300 K for all your calculations unless stated otherwise. The following parameters apply to silicon (Si) at 300 K :

Effective density of states in the conduction band, N_C	2.80×10^{19} cm^{-3}
Effective density of states in the valence band, N_V	1.04×10^{19} cm^{-3}
Intrinsic carrier concentration, n_i	1.5×10^{10} cm^{-3}
Band gap energy, E_g	1.12 eV
Relative permittivity, e_r	11.8
Electron mobility, $\propto_n$	1450 cm^2/V-s
Hole mobility, $\propto_p$	450 cm^2/V-s

1. An abrupt Si pn junction is doped uniformly with 1×10^{17} cm^{-3} acceptor impurity on one side and 1×10^{16} cm^{-3} donor impurity on the other side.

a) Determine the position of the Fermi level with respect to the valence band edge in the p region and with respect to the conduction band edge in the n region.

b) Draw the energy band diagram of the pn junction under thermal equilibrium and determine the contact or built-in potential V_{bi} from the diagram.

c) Using the depletion approximation, calculate the following parameters of the pn junction under thermal equilibrium:

- Width of the space charge region $\boldsymbol{x_{no}}$ on the n-type side;
- Width of the space charge region x_{po} on the p-type side;
- Total width $\boldsymbol{W_o}$ of the space charge region;
- Maximum electric field ξ_m in the space charge region.

Comment on the values of x_{n0}, x_{p0} and W_0.

d) Sketch the electric field distribution across the pn junction. With reference to the neutral p region, determine the potential V at $x = 0$ (metallurgical junction) and compare it to the built-in potential found in (b).

[0.120 eV; 0.206 eV; 0.794 V; 0.307 µm; 0.0307 µm; 0.338 µm; –4.70x10^4 V/cm; 72.1 mV]

1(a) Determine the position of the Fermi level in p and n regions

$N_A = 1\times10^{17}\ \text{cm}^{-3} >> n_i$ and $N_D = 1\times10^{16}\ \text{cm}^{-3} >> n_i$

Assume complete ionization of dopants at 300 K,

The hole concentration in the p region, $p_{po} \approx N_A = 1\times10^{17}\ \text{cm}^{-3}$

The electron concentration in the n region, $n_{no} \approx N_D = 1\times10^{16}\ \text{cm}^{-3}$.

From Maxwell-Boltzmann equation,

$$p_{po} = N_V \exp\left[-\frac{(E_{Fp} - E_V)}{k_B T}\right]$$

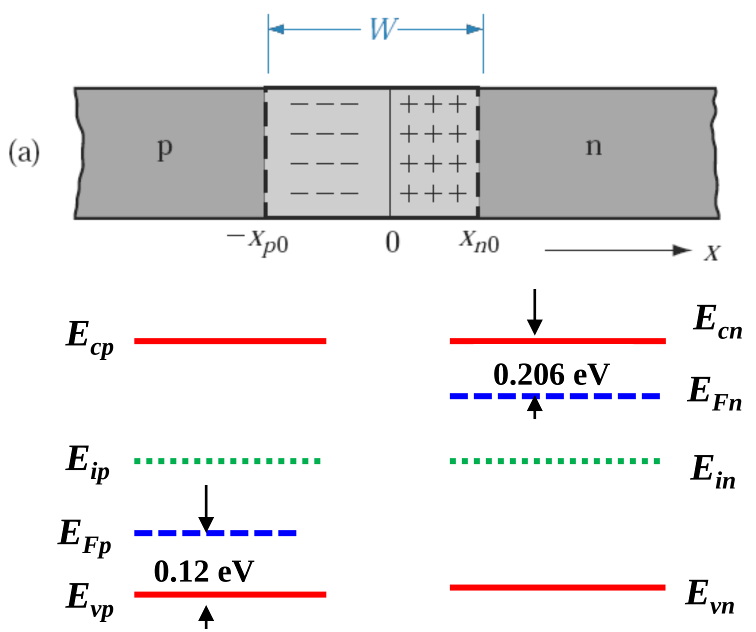
$$1\times10^{17} = 1.04\times10^{19} \exp\left[-\frac{(E_{Fp} - E_V)}{0.0259}\right]$$

$$E_{Fp} - E_V = 0.120\ \text{eV}$$

$$n_{no} = N_C \exp\left[-\frac{(E_C - E_{Fn})}{k_B T}\right]$$

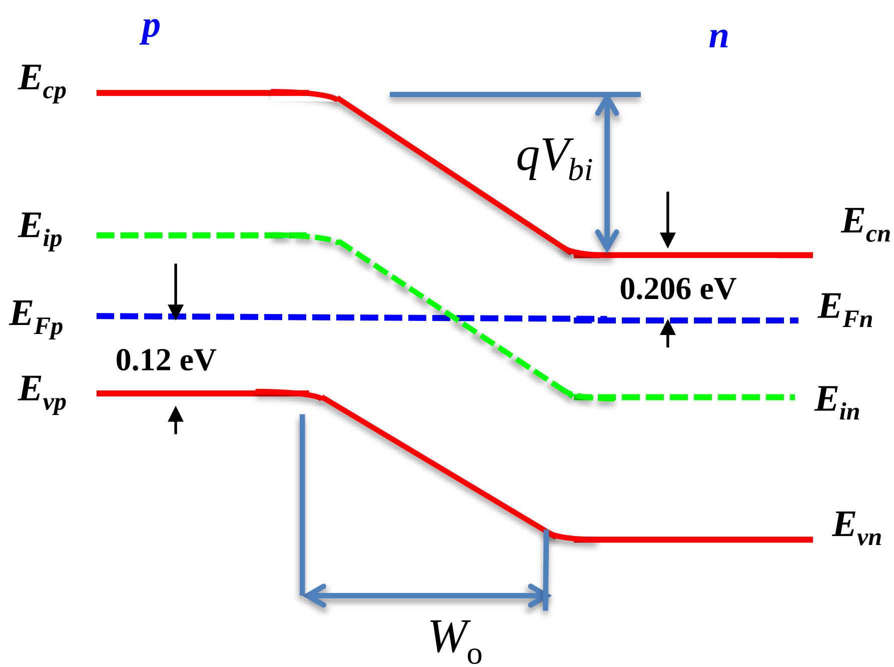
$$10^{16} = 2.80\times10^{19} \exp\left[-\frac{(E_C - E_{Fn})}{0.0259}\right]$$

$$E_C - E_{Fn} = 0.206\ \text{eV}$$



1(b) Draw the energy band diagram of the pn junction and determine the contact or built-in potential Vbi from the diagram

Under thermal equilibrium, the Fermi levels in the n and p regions are aligned with one another, i.e. the Fermi level is independent of position.



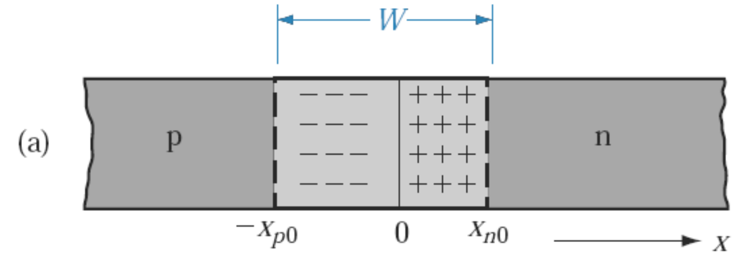
$$E_g(\mathrm{Si}) = 1.12\ \mathrm{eV}$$

Built-in potential,

$$qV_{bi} = E_g - (E_{Fp} - E_V) - (E_C - E_{fn})$$

$$\therefore \quad V_{bi} = 1/q\ (1.12 - 0.120 - 0.206)\,\mathrm{eV} = 0.794\ \mathrm{V}$$

1(c) Find x_{no}, x_{po}, W_o, ξ_m



(i) $$x_{no} = \left\{ \frac{2\varepsilon}{q} V_{bi} \left(\frac{N_a}{N_d\left(N_a + N_d\right)} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2\times 11.8\times 8.85\times 10^{-14}}{1.6x10^{-19}} 0.794 \left(\frac{10^{17}}{10^{16}(10^{17}+10^{16})} \right) \right\}^{\frac{1}{2}} = 3.07\times 10^{-5}\,cm = 0.307\,\mu m$$

(ii) From charge neutrality condition: $x_{no}N_d = x_{po}N_a$

$$x_{po} = \frac{N_d}{N_a} x_{no} = 0.1\left(3.07\times 10^{-5}\right) = 3.07\times 10^{-6}\,cm = 0.0307\,\mu m$$

(iii) Total depletion region width, W_o

$$\therefore W_o = x_{no} + x_{po} = 0.307 + 0.0307 = 0.337\,\mu m$$

(iv) Peak or maximum electrical fied occurs at the metallurgical junction

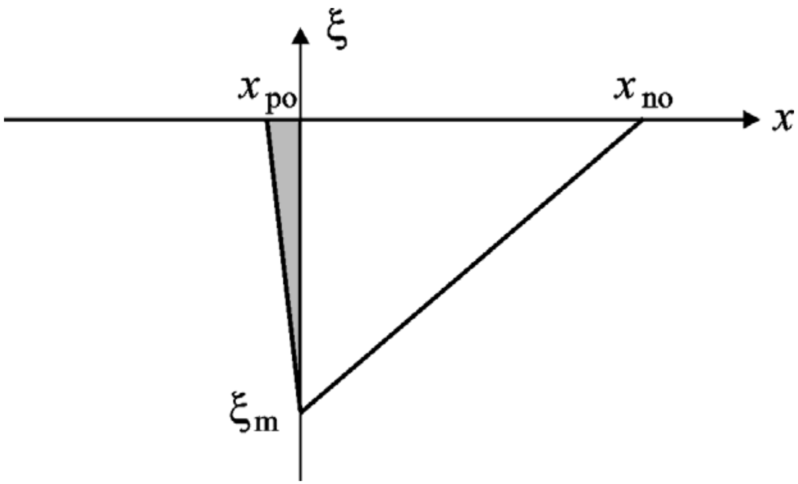
$$\xi_m = -q\frac{N_a x_{po}}{\varepsilon} = -q\frac{N_d x_{no}}{\varepsilon}$$

$$= -\frac{1.6\times 10^{-19}\times 10^{16}\times 3.07\times 10^{-5}}{11.8\times 8.85\times 10^{-14}} = -4.7\times 10^{4}\ V/cm$$

Comment on the values of x_{n0}, x_{p0} and W_0.

From the results, $W_o \approx x_{no} >> x_{po}$. $\therefore$ when one side of the pn junction is doped more heavily than the other (known as a one-sided pn junction), the space charge region lies almost entirely in the lightly doped region. In our case, we have a p^+n junction, where the sign '+' denotes the much higher doping concentration in the p-region compared to n-region.

1(d) Sketch the electric field distribution



$$V(x=0) = -\int_{-\infty}^{0} \xi \, \partial x = -\frac{x_{po}\xi_m}{2}$$

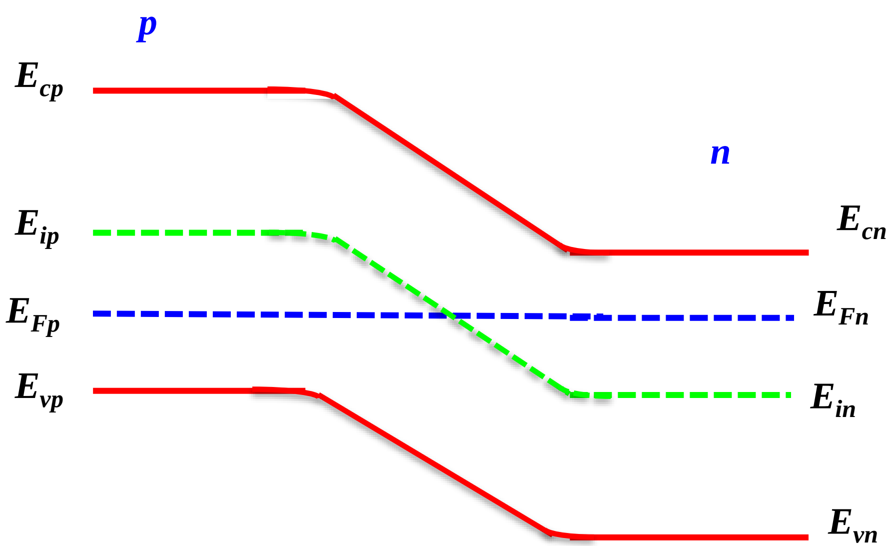
$$= \frac{3.07\times10^{-6}\times4.7\times10^{4}}{2}$$

$$= 0.0721\ V = 72.1\ \mathrm{m}V$$

Note that $V(x=0) << V_{bi}$ ($=0.794$ V). This implies that in a one-sided pn junction, most of the built-in voltage is dropped across the lightly doped side.

2. Show that the Fermi levels on the n side and p side of a pn junction **at thermal equilibrium** are constant with respect to distance throughout the junction (i.e. a single horizontal line on the energy band diagram).

(Hint: Make use of the fact that the electron and hole current density across the junction are each equal to zero under thermal equilibrium and show $dE_F/dx = 0$; E_F is the Fermi level.



Show:

$$\frac{dE_F}{dx} = 0$$

We make use of the fact that under thermal equilibrium, the hole and electron current density are each equal to zero across the pn junction.

Using the hole current density,

$$J_{\mathrm{p}}(x) = q\left[\mu_{\mathrm{p}} p(x)\xi(x) - D_{\mathrm{p}}\frac{dp(x)}{dx}\right] = 0 \qquad (1)$$

$$p = N_{\mathrm{V}} \exp\left(\frac{E_{\mathrm{V}} - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right)$$

$$\begin{aligned} \frac{dp}{dx} &= N_{\mathrm{V}} \exp\left(\frac{E_{\mathrm{V}} - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right)\cdot\left(\frac{dE_{\mathrm{V}}}{dx} - \frac{dE_{\mathrm{F}}}{dx}\right)\frac{1}{k_B T} \\ &= \frac{p}{k_{\mathrm{B}}T}\left(\frac{dE_{\mathrm{V}}}{dx} - \frac{dE_{\mathrm{F}}}{dx}\right) \end{aligned} \qquad (2)$$

Substituting (2) in (1), Eqn. (1) becomes:

$$\mu_{\mathrm{p}} p \xi - D_{\mathrm{p}} \frac{p}{k_{\mathrm{B}} T} \left(\frac{dE_{\mathrm{V}}}{dx} - \frac{dE_{\mathrm{F}}}{dx} \right) = 0$$

Since $\xi = \dfrac{1}{q} \dfrac{dE_{\mathrm{V}}}{dx}$ and $\dfrac{D_{\mathrm{p}}}{\mu_{\mathrm{p}}} = \dfrac{k_{\mathrm{B}} T}{q}$, we have

$$\mu_{\mathrm{p}} p \xi - \left(\frac{q}{k_{\mathrm{B}} T} D_{\mathrm{p}} \right) p \left(\frac{1}{q} \right) \left(\frac{dE_{\mathrm{V}}}{dx} - \frac{dE_{\mathrm{F}}}{dx} \right) = 0$$

$$\mu_{\mathrm{p}} p \xi - \mu_{\mathrm{p}} p \xi + \frac{\mu_{\mathrm{p}} p}{q} \left(\frac{dE_{\mathrm{F}}}{dx} \right) = 0$$

$$\Rightarrow \quad \frac{dE_{\mathrm{F}}}{dx} = 0, \text{ i.e. } E_{\mathrm{F}} \text{ is a horizontal line}$$

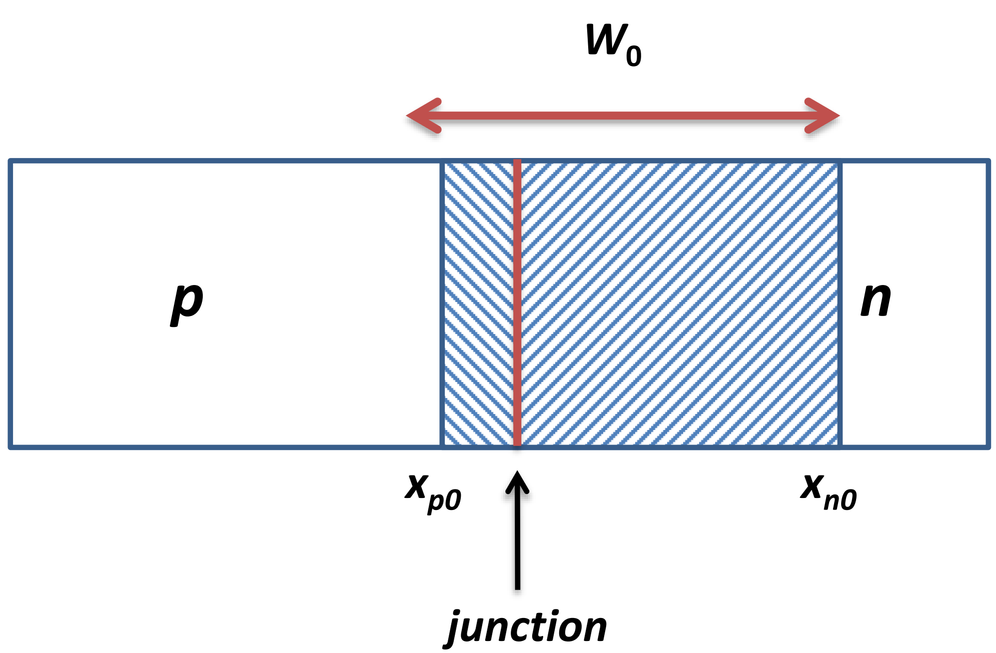
The same result, i.e. $dE_F/dx = 0$ can be obtained using the electron current density.

Thus, we can conclude that E_F must be independent of distance under thermal equilibrium.

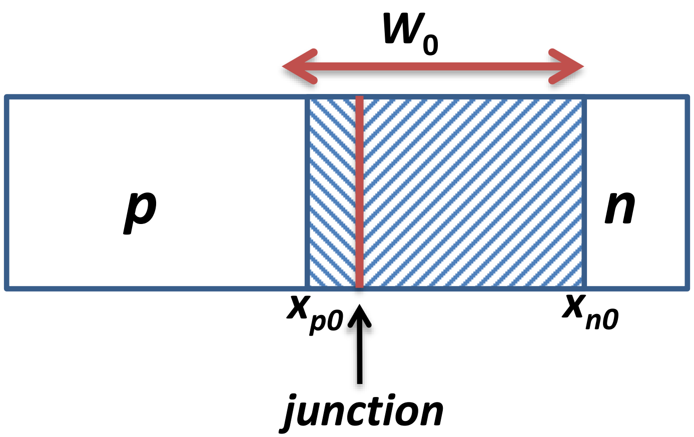
3. Consider a uniformly doped abrupt pn junction at 300 K. At thermal equilibrium, it is designed such that 10 % of the total depletion width region lies in the p region. You are given that the built in potential is 0.8 V.

Determine the doping concentration N_a and N_d of the p and n region, respectively, and the total depletion width.

[2.59×10^{16} cm^{-3}; 2.33×10^{17} cm^{-3}; 0.212 μm]



$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.8\ V \qquad (1)$$



Space charge neutrality:

$x_{po} N_a = x_{no} N_d$

Given: $x_{po} = 0.1\ W \Rightarrow x_{no} = 0.9\ W$

Thus, $N_a = 9 N_d \qquad (2)$

$$0.8 = 0.026\ \ln\left(\frac{9 N_d^{\,2}}{n_i^2}\right)$$

$$\therefore N_d = 2.55\times 10^{16} / cm^3 \quad \text{and} \quad N_a = 9 N_d = 2.29\times 10^{17} / cm^3$$

$$W_0 = \left\{ \frac{2\varepsilon}{q} V_{bi} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right\}^{1/2}$$

$$= \left\{ \frac{2\times 8.85\times 10^{-14} \times 11.8}{1.6\times 10^{-19}} 0.8 \left(\frac{1}{2.29\times 10^{17}} + \frac{1}{2.55\times 10^{16}} \right) \right\}^{\frac{1}{2}} \approx 0.213 \mu m$$

Q4 The magnitude of the peak electric field in a Si p-n junction is 20 kV/cm under thermal equilibrium condition. If the built-in potential is 0.8 V, what is the donor doping if $N_a >> N_d$?

Assume $\varepsilon_r = 11.8$, $\varepsilon_0 = 8.85\times10^{-14}$ F/cm, $q = 1.6\times10^{-19}$ C.

$$V_{bi} = -\frac{1}{2}\xi_m W_0$$

$$W_0 = -\frac{2V_{bi}}{\xi_m} = \frac{2\times0.8\,V}{20000\ V/cm} = 0.8\ \mu m$$

$$W = \left[\frac{2\varepsilon V_{bi}}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\right]^{1/2}$$

Since $N_a >> N_d$, $W_0 = \left[\frac{2\varepsilon V_{bi}}{q}\frac{1}{N_d}\right]^{1/2}$

Solving gives $N_d = 1.63\times10^{15} cm^{-3}$