NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE2002 – ANALOG ELECTRONICS

April / May 2021 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 10 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulae is provided in Appendix A on pages 7-10.

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1. (a) A non-ideal Op-Amp configured with resistors is shown in Figure 1(a) on page 2. The Op-Amp is powered by $\pm V_{CC}$ power supplies. It has 2 AC input sources, v_1 and v_2, and 3 non-ideal DC sources, I_+, I_- and V_{IO}.

Derive the expression for the output voltage v_{OUT}, in terms of all or some of the followings: v_1, v_2, R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11}, I_+, I_- and V_{IO}.

Note: *Parallel resistance of R_x and R_y can be written as $R_x//R_y$ without expanding it.*

(12 Marks)

Note: Question No. 1 continues on page 2.

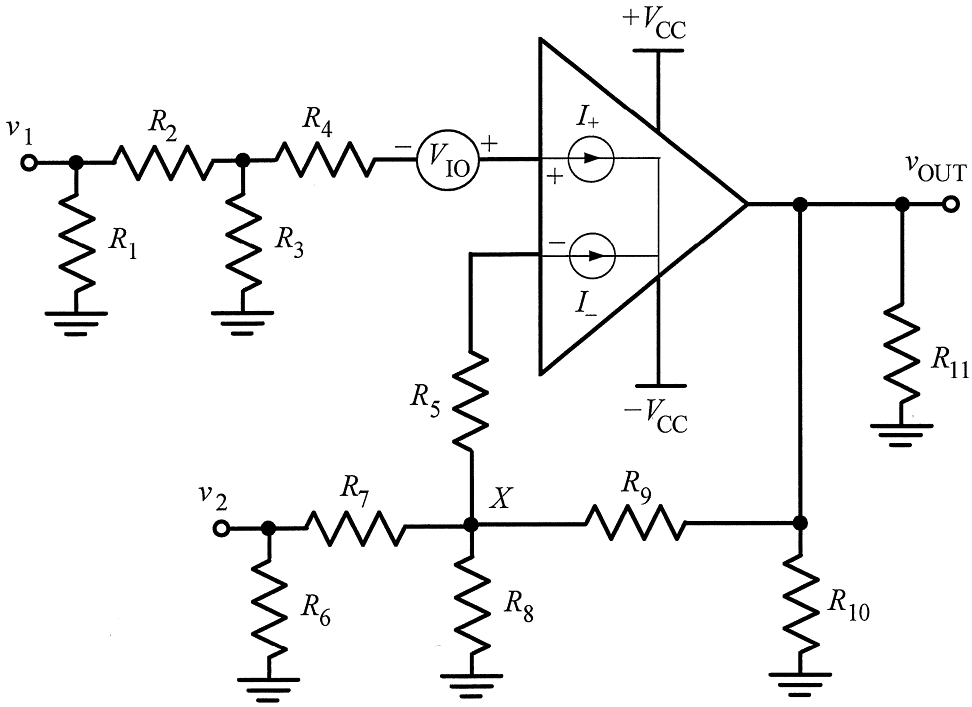


Figure 1(a)

(b) In Figure 1(b) on page 3, the empirical junction diode equation is:

$$V_D = nV_T \, ln[I_D/I_S]$$

for the identical diodes D_1, D_2, D_3 and D_4, given that 2 points on the diode *I-V* characteristic curve are:

$$V_{Dx} = 0.680 \text{ V at } I_{Dx} = 600 \ \mu\text{A and}$$
$$V_{Dy} = 0.800 \text{ V at } I_{Dy} = 6 \text{ mA.}$$

i) Find nV_T and I_S to 3 decimal places in V and nA, respectively.

Also given is $V_{SQ} = 10 \ V_{pp}$ square wave source operating at a very low frequency and $R_L = 10 \text{ k}\Omega$.

ii) Find the DC operating point or quiescent (Q) - point (I_D, V_D) for the diodes D_1, D_2, D_3 and D_4 (to 3 decimal places in mA and V, respectively).

Note: Question No. 1 continues on page 3.

iii) Hence find the voltage, V_{AB} and current in R_L.

Note: D_1, D_2, D_3 and D_4 have the same nV_T and I_S. Also note that the low frequency square wave can be treated as toggling of ±5V DC with respect to ground.

(13 Marks)

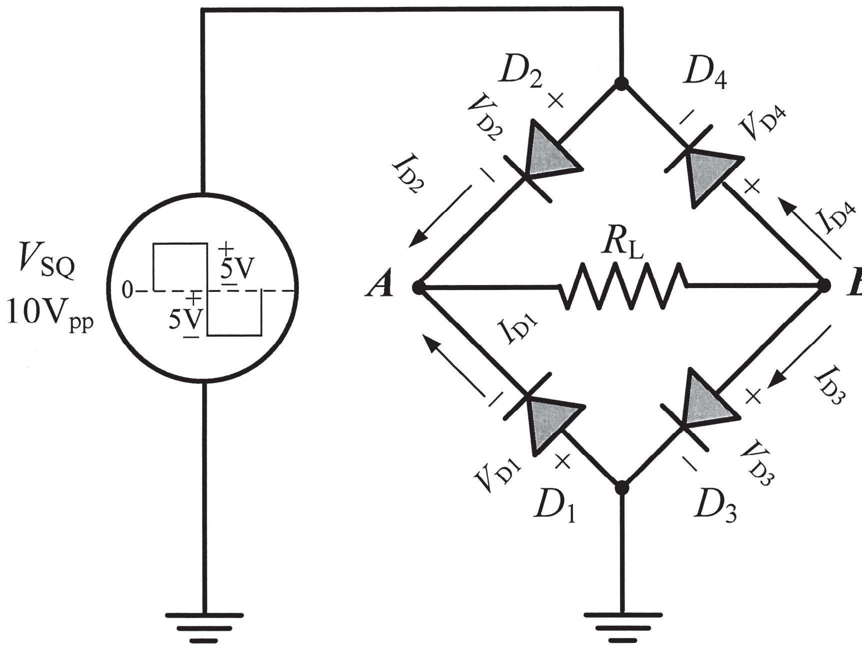


Figure 1(b)

2. A single-stage amplifier biased using dual supplies with V_{CC} = 3V is shown in Figure 2 on page 4. Assume that the transistor Q_1 has β = 100 and V_A = 60 V. The resistor values are as follows: R_1 = 1 kΩ, R_2 = 30 kΩ, R_E = 2 kΩ, R_C = 3 kΩ, R_3 = 270 Ω, R_L = 30 kΩ. Assume that all capacitors have infinite capacitance value.

Note: Question No. 2 continues on page 4

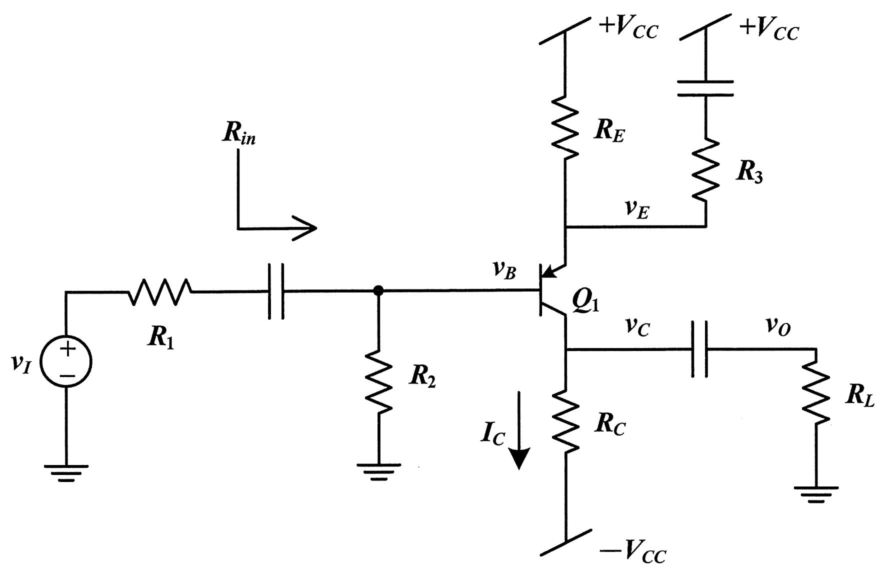


Figure 2

(a) Calculate the Q-point of the transistor, and verify its region of operation.

(6 Marks)

(b) Determine the input resistance R_{in}.

(6 Marks)

(c) Determine the voltage gain $A_v = v_O / v_I$.

(7 Marks)

(d) Determine the maximum amplitude v_I for small signal operation.

(6 Marks)

3. For the differential circuit in Figure 3 on page 5, assume V_{CC} = 12 V, V_{EE} = 12 V, V_{DC} = 1 V, I_{EE} = 500 μA, R_D = 30 kΩ, β = 100, $V_A = \infty$. Here $v_{i1} = V_{DC} + v_{AC}/2$ and $v_{i2} = V_{DC} - v_{AC}/2$ represent the differential inputs (v_{AC} is the small AC signal inputs), and v_{o1} and v_{o2} represent the differential outputs. Assume all the capacitors have infinite values.

(a) Determine the Q-points for the transistors and R_{EE}.

(5 Marks)

(b) Determine the differential-mode gains, common-mode gains for the cases of single-ended and differential outputs, respectively.

(10 Marks)

Note: Question No. 3 continues on page 5.

(c) Determine the common-mode rejection ratios (CMRRs) for the cases of single-ended and differential outputs, respectively. In addition, discuss the effect of variation of R_{EE} to the CMRRs.

(6 Marks)

(d) Determine the input resistances and output resistances for the cases of single-ended and differential outputs.

(4 Marks)

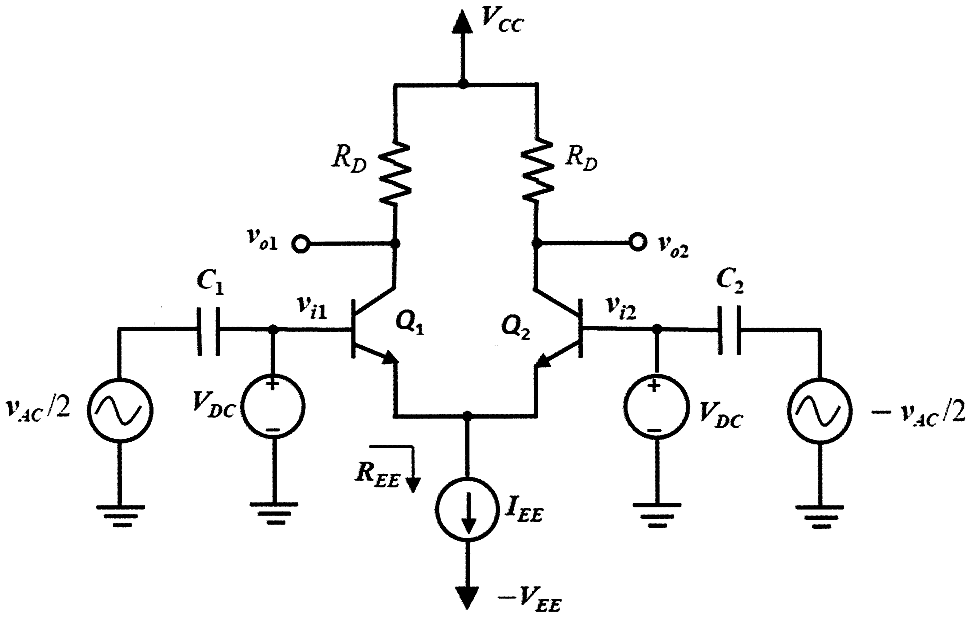


Figure 3

4. (a) Figure 4(a) on page 6 depicts a MOS Widlar current mirror comprising transistors M_1, M_2 and resistor R. Given the transconductance parameter $K_n' = 100\ \mu A/V^2$, $(W/L)_1 = 5$, $(W/L)_2 = 20$, $I_{REF} = 20\ \mu A$ and $I_O = 10\ \mu A$, find the value of the resistor R. You may ignore Early effect and assume both transistors to have the same threshold voltage V_{TN}.

(10 Marks)

(b) Figure 4(b) on page 6 depicts an Opamp based filter circuit. You may assume the Opamp to be ideal.

(i) For both capacitors C_S and C_1, determine if they contribute to lower cut-off (ω_L) or higher cut-off (ω_H) frequency.

(3 Marks)

Note: Question No. 4 continues on page 6.

(ii) Using OCTC methods, determine the higher cut-off (ω_H) frequency.

(4 Marks)

(iii) Using SCTC methods, determine the lower cut-off (ω_L) frequency.

(4 Marks)

(iv) Now derive the transfer function V_{out}/V_{in} for this circuit and match the poles/zeros with the cut-off frequencies derived in the earlier parts.

(4 marks)

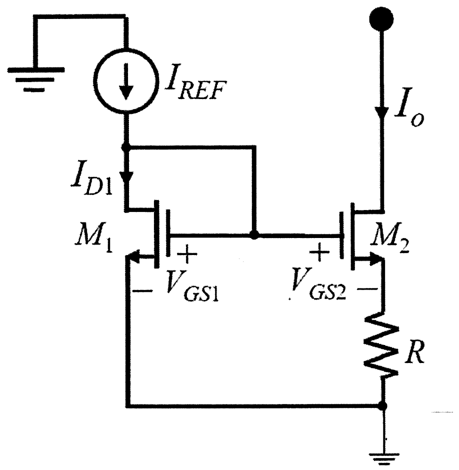


Figure 4(a)

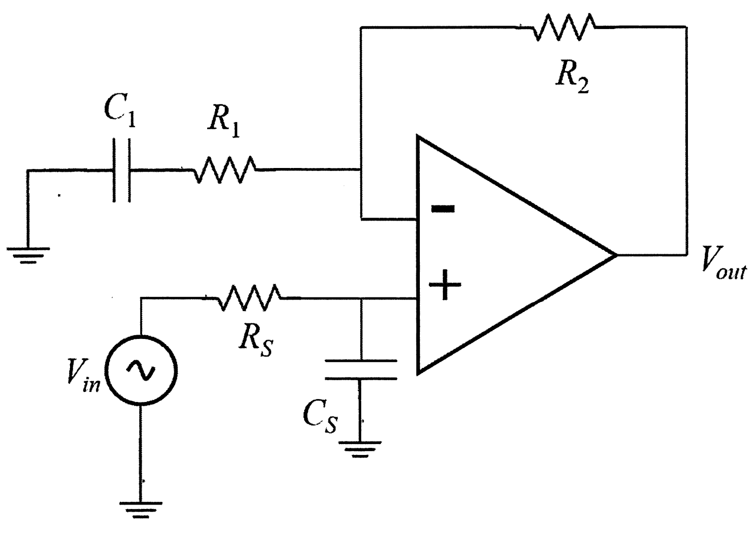


Figure 4(b)

Appendix A

List of Formulae (with the usual notations)

Op-Amps:

Closed-Loop Negative Feedback Inverting Gain, $A_{VCL} = \frac{v_o}{v_i} = -\frac{R_f}{R_i}$ Figure (a)

Closed-Loop Negative Feedback Non-Inverting Gain, $A_{VCL} = \frac{v_o}{v_i} = \left(1 + \frac{R_f}{R_i}\right)$ Figure (b)

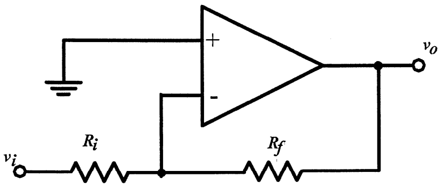


Figure (a)

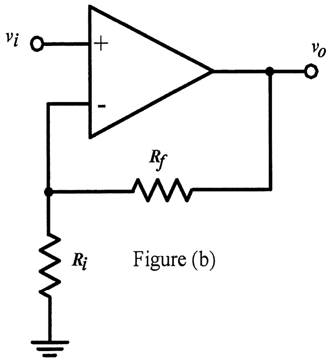


Figure (b)

Op-Amp's Slew Rate, $SR \geq \left|\frac{dv_o}{dt}\right|_{\max} = A_{VCL}\omega a_m = A_{VCL} a_m 2\pi f$,

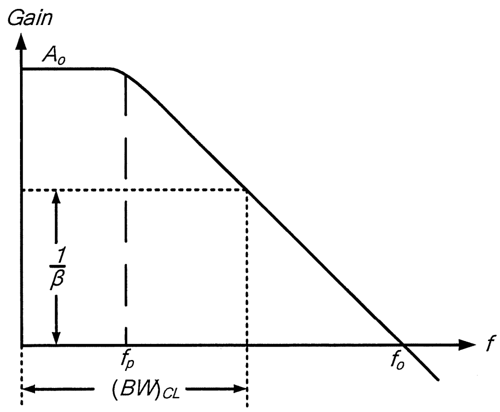
where $v_i = a_m \sin(\omega t)$, $v_o = A_{VCL} v_i$, $v_o = A_{VCL} a_m \sin(\omega t)$ and $\left|\frac{dv_o}{dt}\right| = A_{VCL}\omega\, a_m \cos(\omega t)$

Op-Amp's frequency response: $A_{VOL}(jf) = \frac{A_o}{\left(1 + \frac{jf}{f_p}\right)}$

Gain-Bandwidth Product: $A_o f_p = f_o = \frac{1}{\beta}(BW)_{CL}$

where $\frac{1}{\beta} = \frac{R_f + R_i}{R_i}$

$t_r = \frac{0.35}{(BW)_{CL}}$



Appendix A (continued)

Diodes:

$v_D \approx nV_T \ln\left(\frac{i_D}{I_S}\right)$ or $i_D \approx I_S e^{\left(\frac{v_D}{nV_T}\right)}$

where $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$

Diode conductance: $g_D = \frac{1}{r_D} = \frac{I_D}{nV_T}$

BJT in Forward Active Region:

Ignore early effect: $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right)$

With early effect: $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right)\left(1+\frac{|v_{CE}|}{V_A}\right)$

where I_s: Saturation current,
V_T: Thermal voltage, assume 25 mV at room temperature,
V_A: Early voltage.
For npn transistor, $|v_{BE}| = v_{BE}$ and $|v_{CE}| = v_{CE}$;
For pnp transistor, $|v_{BE}| = v_{EB}$ and $|v_{CE}| = v_{EC}$.

Small-signal model parameters of BJT:

$g_m = \frac{I_C}{V_T}$, $r_\pi = \frac{\beta}{g_m}$ and $r_o = \frac{V_A + |V_{CE}|}{I_C} \approx \frac{V_A}{I_C}$

where I_C : DC collector current at Q-point
V_{CE}: DC collector-emitter voltage at Q-point

Criterion for small-signal operation of BJT: $|v_{be}| \le 0.2V_T$

MOSFET in Saturation Region:

Criterion: $V_{DS} \ge V_{GS} - V_{TN}$ for NMOS;
$|V_{DS}| \ge |V_{GS}| - |V_{TP}|$ for PMOS

where V_{TN}, V_{TP}: Threshold voltage,
V_{DS}: DC drain-source voltage,
V_{GS}: DC gate-source voltage.

Ignore channel-length modulation effect: $i_D = \frac{K_n}{2}(v_{GS} - V_{TN})^2$ for NMOS,

$i_D = \frac{K_p}{2}(|v_{GS}| - |V_{TP}|)^2$ for PMOS.

Appendix A (continued)

With channel-length modulation effect: $i_D = \frac{K_n}{2}(v_{GS} - V_{TN})^2(1+\lambda v_{DS})$ for NMOS,

$i_D = \frac{K_p}{2}(|v_{GS}| - |V_{TP}|)^2(1+\lambda|v_{DS}|)$ for PMOS.

where λ: channel length modulation parameter,

For NMOS $K_n = K_n'\left(\frac{W}{L}\right)$ and $K_n' = \mu_n C_{ox}$; For PMOS $K_p = K_p'\left(\frac{W}{L}\right)$ and $K_p' = \mu_p C_{ox}$.

MOSFET in Triode Region:

Criterion: $V_{DS} < V_{GS} - V_{TN}$ for NMOS;
$|V_{DS}| < |V_{GS}| - |V_{TP}|$ for PMOS

Ignore channel-length modulation effect: $i_D = K_n\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right)v_{DS}$ for NMOS,

$i_D = K_p\left(|v_{GS}| - |V_{TP}| - \frac{|v_{DS}|}{2}\right)|v_{DS}|$ for PMOS.

With channel-length modulation effect: $i_D = K_n\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right)v_{DS}(1+\lambda v_{DS})$ for NMOS,

$i_D = K_p\left(|v_{GS}| - |V_{TP}| - \frac{|v_{DS}|}{2}\right)|v_{DS}|(1+\lambda|v_{DS}|)$ for PMOS.

Small-signal model parameters of MOSFET

For NMOS: $g_m = \sqrt{2K_n I_D(1+\lambda V_{DS})} \approx \sqrt{2K_n I_D}$ and $r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \approx \frac{1}{\lambda I_D}$

For PMOS: $g_m = \sqrt{2K_p I_D(1+\lambda|V_{DS}|)} \approx \sqrt{2K_p I_D}$ and $r_o = \frac{\frac{1}{\lambda} + |V_{DS}|}{I_D} \approx \frac{1}{\lambda I_D}$

where I_D : DC drain current at Q-point
V_{DS}: DC drain-source voltage at Q-point

Criterion for small-signal operation:
For NMOS: $|v_{gs}| \le 0.2(V_{GS} - V_{TN})$
For PMOS: $|v_{gs}| \le 0.2(|V_{GS}| - |V_{TP}|)$
where V_{GS} : DC gate-source voltage at Q-point.

Appendix A (continued)

Frequency Response: OCTC and SCTC

0) DISABLE DC sources…
voltage sources -> SHORT CIRCUIT, current sources -> OPEN CIRCUIT

1) Identify capacitors contributing (reducing V_o or causing trouble) to the frequency of interest (i.e. lower or higher cut-off).

2) DISABLE all independent AC sources...
voltage sources -> SHORT CIRCUIT, current sources -> OPEN CIRCUIT
DO NOT remove or "disable" dependent sources!

⇓ higher cut-off (OCTC)

3) Idealize irrelevant capacitors by SHORT CIRCUIT (because at high f, cap → short)

Next step to find time constant

4) For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) removed (i.e. OPEN CIRCUITS) and determine the resistance, R_i seen by C_i

5) Higher cut-off frequency is estimated as:

$$\omega_{H-3dB} \approx \frac{1}{\sum_i C_i R_i} = \frac{1}{C_1 R_1 + C_2 R_2 + \ldots}$$

⇓ lower cut-off (SCTC)

3) Idealize irrelevant capacitors by OPEN CIRCUIT (because at low f, cap → open)

Next step to find time constant

4) For each contributing capacitor C_i, set all other capacitors (other than the one you are looking at) removed (i.e. SHORT CIRCUITS) and determine the resistance, R_i seen by C_i

5) Lower cut-off frequency is estimated as:

$$\omega_{L-3dB} \approx \sum_i \frac{1}{C_i R_i} = \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + ..$$

END OF PAPER

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1. (a).

$$V_{V_1} = V_1 \times \frac{R_3}{R_2+R_3} \times \left(\frac{R_7 // R_8 + R_9}{R_7 // R_8}\right)$$

$$V_{V_{IO}} = V_{IO} \times \left(\frac{R_7 // R_8 + R_9}{R_7 // R_8}\right)$$

$$V_{I_+} = -I_+ \times [R_2 // R_3 + R_4] \times \left(\frac{R_7 // R_8 + R_9}{R_7 // R_8}\right)$$

$$V_{V_2} = -V_2 \times \frac{R_9}{R_7}$$

$$V_{I_-} = I_- \times R_5 \times \frac{R_5 // R_7 // R_8 + R_9}{R_5 // R_7 // R_8}$$

$$V_{out} = V_{V_1} + V_{V_{IO}} + V_{I_+} + V_{V_2} + V_{I_-}$$

1. (b). (i)

$$0.68 = nV_T \ln\left(\frac{6\times10^{-4}}{I_s}\right) \quad \ldots (1)$$

$$0.8 = nV_T \ln\left(\frac{6\times10^{-3}}{I_s}\right) \quad \ldots (2)$$

$$0.8 - 0.68 = nV_T \ln\left(\frac{6\times10^{-3}}{6\times10^{-4}}\right)$$

$$0.12 = nV_T \ln(10)$$

$$nV_T = 0.052\,V$$

$$I_S = 1.256\,nA$$

(ii) When $V_{SQ} = 5V$

$$5 = V_{D2} + R_L \cdot I_D + V_{D3}$$

$$5 = 2V_D + 10k\, I_D \quad \ldots (1)$$

$$V_D = 0.052 \ln\left(\frac{I_D}{1.256\times10^{-9}}\right) \quad \ldots (2)$$

Take $V_D = 0.7$ for iteration

use eq (2)

V_D (V)	0.7	0.653	0.655	0.655
I_D (mA)	0.36	0.369	0.369	

use eq (1)

$$V_D = 0.655V$$

$$I_D = 0.369\,mA$$

(iii)

$$I_{R_L} = I_D = 0.369\,mA$$

$$V_{AB} = R_L \times I_{R_L} = 3.69\,V$$

2.

(a) Assume forward active

$30k\, I_B + V_{BE} + 2k\, I_E = 3$

$30k\, I_B + 0.7 + 2k\,(101)\, I_B = 3$

$232k\, I_B = 2.3$

$I_B = 9.9\ \mu A$

$I_C = 100 . I_B = 0.99\ mA$ ⫽

$V_C = -3 + 0.99 \times 3$
$= -0.03$

$V_E = 3 - 2 \times 0.9999$
$= 1.0002$

$V_B = 30 \times 9.9 \times 10^{-3}$
$= 0.297$

$30k\, I_B = V_{CB} + 3k\, I_C + (-3)$

$V_{CB} = 0.327$

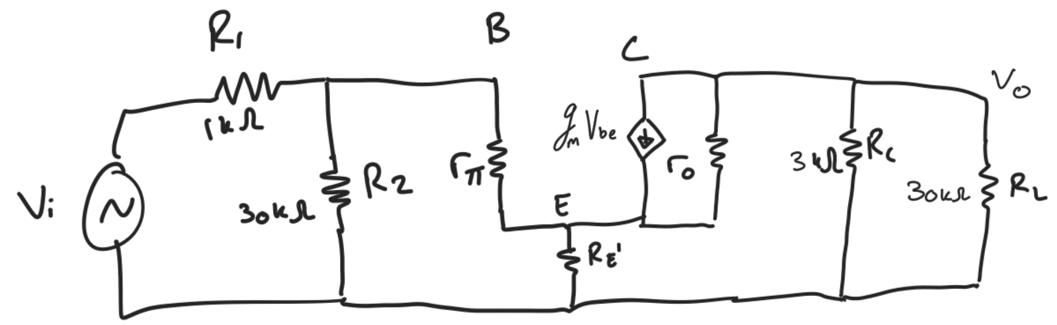
$V_{CE} = V_{CB} + V_{BE}$
$= 0.327 + 0.7$
$= 1.027\ V$
$\simeq 1.03V$ ⫽

$V_{CE} = V_E - V_C$
$= 1.0302$
$\simeq 1.03V$ ⫽

$V_E > V_B > V_C$

Forward Active ⫽

(b)



$g_m = \frac{I_C}{V_T} = \frac{0.99}{25} = 39.6\,mS$

$r_\pi = \frac{\beta}{g_m} = 2525.25\,\Omega$

$r_o = \frac{V_A}{I_C} = \frac{60}{9.9 \times 10^{-4}} = 60606\,\Omega$

$R_E' = R_E \,//\, R_3 = 237.88\,\Omega$

$R_{in} = R_2 \,//\, [r_\pi + (\beta+1) R_E']$
$= 14.085\ k\Omega$

(c) $V_o = V_c = -i_c . (R_C // R_L)$
$= 100\, i_b\, (2.727)$
$= -272.7k\, i_b$

$V_b = r_\pi\, i_b + R_E'\, i_e$
$= i_b\, (r_\pi + 101\, R_E')$
$= 26551.13\, i_b$

$V_i = \frac{R_{in}}{R_{in} + R_1} \times V_b = 24.79k\, i_b$

$A_V = \frac{V_o}{V_i} = \frac{-272.7k\, i_b}{24.79\ k\ \ i_b}$

$= -11$ ⫽

(d) $V_{be} = r_\pi \cdot i_b$

$V_{be} = 2.525k \; i_b$

$V_i = 25.666k \; i_b$

$V_i = \frac{25.666k}{2.525k} V_{be}$

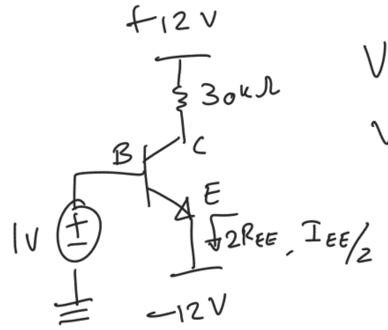
$V_i = 10.165 \, V_{be}$

$V_i \le 50.825 \, mV$ #

$V_{be} \le 0.2 \, V_T$

$V_{be} \le 5 \, mV$

3(a).



$V_B = 1 \, V$

$V_E = 0.3 \, V$ → Assume forward active

$I_{EE}/2 = \frac{0.3 - (-12)}{R_{EE}}$

$2R_{EE} = \frac{12.3V}{250 \mu A}$

$R_{EE} = 24.6k\Omega$ #

$\frac{12 - V_C}{30k\Omega} = \frac{I_{EE}}{2} \times \frac{\beta}{\beta + 1}$

$12 - V_C = 7.43$

$V_C = 4.57$

$V_{CE} = 4.27 \, V$ #

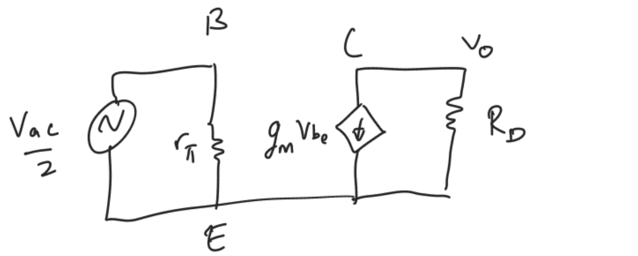
$I_C = \frac{I_{EE}}{2} \times \frac{\beta}{\beta+1} = 247.5 \, \mu A$ #

(b). $g_m = \frac{I_C}{V_T} = 9.9 \, mS$

$r_\pi = \frac{\beta}{g_m} = 10.1 \, k\Omega$

$r_o = \frac{V_A}{I_C} = \infty$ → Open

diff - mode



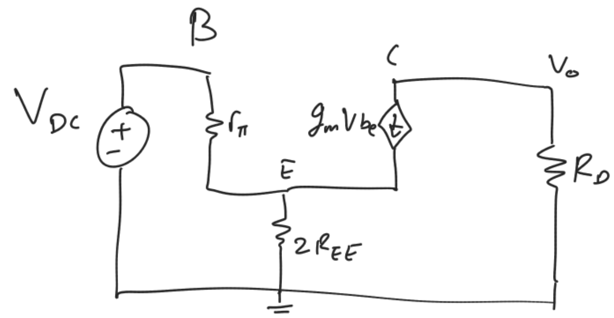
$V_c = -g_m V_{be} \cdot R_D = -g_m \frac{V_{ac}}{2} \cdot R_D$

$A_{dm-se1} = \frac{-g_m \frac{V_{ac}}{2} \cdot R_D}{V_{ac}} = 148.5$ #

$A_{dm-se2} = \frac{g_m \frac{V_{ac}}{2} \cdot R_D}{V_{ac}} = -148.5$ #

$A_{dm-diff} = A_{dm-se_1} - A_{dm-se_2} = -297$ #

Common mode



$V_{be} = r_\pi \times i_b = 10.1k \; i_b$

$V_{DC} = r_\pi \times i_b + 2R_{EE} \times 101 i_b$

$= 4.979 \, M \, i_b$

$V_{DC} = 493 \, V_{be}$

$V_o = -g_m V_{be} \cdot R_D$

$= -99.99 \, V_{be}$

$A_{cm-se1} = \frac{V_o}{V_{DC}} = \frac{-99.99 V_{be}}{493 \, V_{be}} = -0.2028$ #

$A_{cm-se2} = A_{cm-se1} = -0.2028$ #

$A_{cm-diff} = 0$ #

(c). $CMRR_{se} = \dfrac{A_{dm-se_1}}{A_{cm-se_1}} = 732.25$

$CMRR_{diff} = \dfrac{A_{dm-diff}}{A_{cm-diff}} = \infty$

$A_{cm-se_1} = \dfrac{V_o}{V_{DC}} = \dfrac{-g_m V_{be} R_D}{r_\pi \times i_b + 2R_{EE} \times 101\, i_b}$

↳ Increase in R_{EE} will reduce A_{cm-se_1} and increase $CMRR_{se}$.

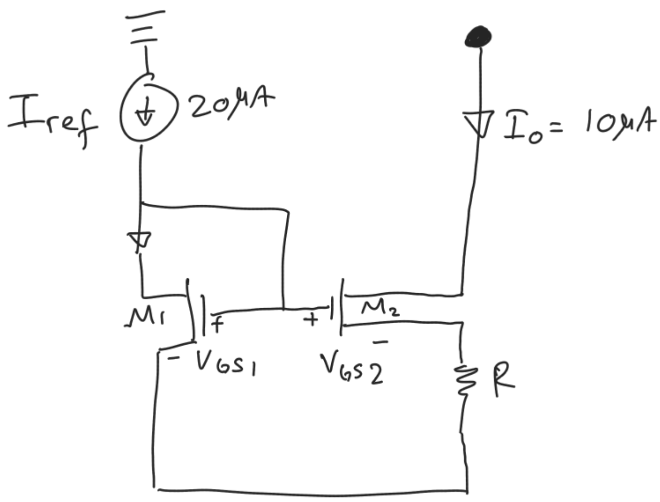
(d) $R_{in-cmse} = \dfrac{V_{DC}}{i_b} = r_\pi + (\beta+1)\, 2 \cdot R_{EE} = 4.979\ M\Omega$ #

$R_{in-cm\,diff} = R_{in-cmse_1} \,//\, R_{in-cmse_2} = 2.4895\ M\Omega$ #

$R_{out-dm\,se} = R_D // r_o = R_D = 30k\Omega$ #

$R_{out-dm\,diff} = 2\, R_{out-dm\,se} = 60k\Omega$ #

4(a)



$I_{ref} = \dfrac{k_n'}{2}\left(\dfrac{W}{L}\right)_1 (V_{GS_1} - V_{TN})^2$

$2\cancel{0}\mu A = 5\cancel{0}\mu A \cdot 5\, (V_{GS_1} - V_{TN})^2$

$0.08 = (V_{GS_1} - V_{TN})^2 \ldots ①$

$I_o = \dfrac{k_n'}{2}\left(\dfrac{W}{L}\right)_2 (V_{GS_2} - V_{TN})^2$

$1\cancel{0}\mu A = 5\cancel{0}\mu A \cdot 20\, (V_{GS_2} - V_{TN})^2$

$0.01 = (V_{GS_2} - V_{TN})^2 \ldots ②$

$V_{GS_1} = V_{GS_2} + I_o R$

$0.2\sqrt{2} + \cancel{V_{TN}} = 0.1 + \cancel{V_{TN}} + I_o \cdot R$

$I_o R = 0.183$

$R = 18.3\ k\Omega$ #

(b).

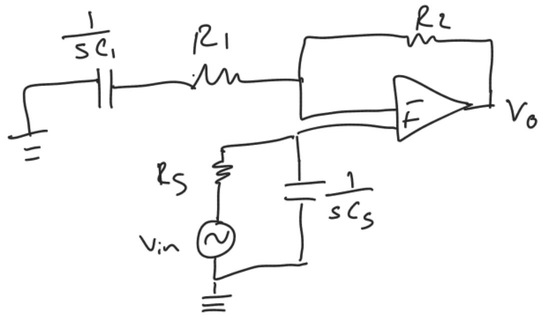
(i) Short $C_S \rightarrow V_+ = 0$
Open $C_S \rightarrow V_+ = V_{in}$
} Troublemaker for high freq.

Short $C_1 \rightarrow V_{out} = \left(1 + \dfrac{R_2}{R_1}\right) V_+$
Open $C_1 \rightarrow V_{out} = V_+$
} Troublemaker for low freq.

(ii) $\omega_{H-3dB} = \dfrac{1}{C_S R_S}$

(iii) $\omega_{S-3dB} = \dfrac{1}{C_1 R_1}$

(iv)



$$V_o = \frac{V_{in}}{R_s + \frac{1}{sC_s}} \times \frac{1}{sC_s} \times \frac{R_1 + \frac{1}{sC_1} + R_2}{R_1 + \frac{1}{sC_1}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{1+sR_sC_s} \times \frac{1+sC_1R_1+sC_1R_2}{1+sC_1R_1}$$

Zero: $\left(\frac{1}{C_1R_1 + C_1R_2}\right)$

Poles: $\left(\frac{1}{C_1R_1}\right)$, $\left(\frac{1}{C_sR_s}\right)$

Low freq. cut-off

High freq. cut-off