NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2020-2021

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of useful formulae is given in the Appendix A.

---

1. (a) Find the row echelon form of the matrix

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}.$$

(3 Marks)

(b) Find the span of

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}.$$

(5 Marks)

(c) If $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 0 \\ a & 0 & 1 \end{bmatrix}$, where a, b are unknown non-zero constants, what are E^2, E^8 and $8E$? Show your working clearly.

(5 Marks)

(d) Let v_1, v_2, v_3 be unknown constants, and assume $v_2 \neq 0$. Perform a LU factorisation of the matrix $A = \begin{bmatrix} 1 & v_1 & 0 \\ 0 & v_2 & 0 \\ 0 & v_3 & 1 \end{bmatrix}$, and find A^{-1}. Show the key steps in obtaining matrices L, U and A^{-1} clearly.

(12 Marks)

2. Let

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

(a) Find the eigenvalues and eigenvectors of A.

(10 Marks)

(b) Solve the following differential equation using the result of 2(a).

$$\frac{d\boldsymbol{u}}{dt} = A\boldsymbol{u}$$

where $\boldsymbol{u}$ is a vector of appropriate dimensions.

(10 Marks)

(c) Find a time T at which the solution $\boldsymbol{u}(t)$ is equal to the initial value $\boldsymbol{u}(0)$. Justify your answer.

(5 Marks)

3. (a) (i) Determine whether the following function is continuous at the origin.

$$f(x) = \begin{cases} \frac{xy}{x^2+y^2}, & z \neq 0, \\ 0, & z = 0. \end{cases}$$

(ii) Discuss the differentiability and analyticity of the function $f(z) = xy^2 + ix^2y$ and find $f'(z)$.

(11 Marks)

(b) Evaluate the following integrals

(i) $\oint_C [z \sin^2(z - 0.3) + \frac{(z^2+0.5)^2 \sin z}{z^2}]\, dz$, $C : |z| = 1$, counterclockwise.

(ii) $\oint_C \frac{\bar{z}}{|z|}\, dz$, $C : |z| = 4$, counterclockwise.

(10 Marks)

(c) Determine the real and imaginary parts of the complex number i^n where n is a positive integer.

(4 Marks)

4. (a) Consider the function $f(x, y, z) = x^2 + 2y^2 + 3z^2 + xy + 3x - 2y - 6z$. Find the directional derivative at the origin $(0, 0, 0)$ in the direction $-5\mathbf{i}$. Determine the point at which its gradient is a zero vector.

(9 Marks)

(b) Show that the force field $\mathbf{F}(x, y, z) = 2xyz^3\mathbf{i} + x^2z^3\mathbf{j} + 3x^2yz^2\mathbf{k}$ is a conservative force field. Find the work done in moving an object in this field from $(1, 4, 1)$ to $(2, 3, 1)$.

(10 Marks)

(c) Find the outward flux of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ across the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ $(z \geq 0)$.

(6 Marks)

END OF PAPER

Appendix A

Some Useful Formulae for Complex Analysis

1. Complex Power: $z^c = e^{c \ln z}$

2. Euler's Formula: $e^{ix} = \cos x + i \sin x$

3. De Moivre's Formula: $(\cos x + i \sin x)^n = \cos nx + i \sin nx$

4. Cauchy-Riemann equations:

$$u_x = v_y, \quad v_x = -u_y, \quad \text{or} \quad u_r = \frac{1}{r} v_\theta, \quad v_r = \frac{-1}{r} u_\theta$$

5. Derivative, if exists: $f'(z) = u_x + i v_x = e^{-i\theta}(u_r + i v_r)$

6. Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z - z_o)^m} \, dz = \frac{2\pi i}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} f(z)|_{z=z_o}$$

Some Useful Formulae for Vector Calculus

Let $\mathbf{F} = F_1 \, \mathbf{i} + F_2 \, \mathbf{j} + F_3 \, \mathbf{k}$.

1. Scalar Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$

2. Gradient: $\nabla f = \frac{\partial f}{\partial x} \, \mathbf{i} + \frac{\partial f}{\partial y} \, \mathbf{j} + \frac{\partial f}{\partial z} \, \mathbf{k}$

3. Divergence: $\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

4. Curl: $\nabla \times \mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

5. Gauss Theorem: $\iiint_T \nabla \cdot \mathbf{F} \, dv = \iint_S \mathbf{F} \cdot \mathbf{n} \, dA$

6. Stokes Theorem: $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA = \int_C \mathbf{F} \cdot \mathbf{dr}$

EE2007 ENGINEERING MATHEMATICS II
IM2007 ENGINEERING MATHEMATICS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

EE2007 Exam Solution

1.(a)
$$\begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix} \underset{\sim}{\overset{R_2 \leftarrow R_2+R_1}{R_3 \leftarrow R_3-R_1}} \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -9 & -3 \end{bmatrix}$$

$$\underset{\sim}{R_3 \leftarrow R_3+3R_1} \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.(b)
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & w \\ 1 & 2 & 3 & x \\ 1 & 3 & 4 & y \\ 1 & 4 & 5 & z \end{array}\right] \underset{\sim}{\begin{matrix} R_2 \leftarrow R_2-R_1 \\ R_3 \leftarrow R_3-R_1 \\ R_4 \leftarrow R_4-R_1 \end{matrix}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & w \\ 0 & 1 & 1 & x-w \\ 0 & 2 & 2 & y-w \\ 0 & 3 & 3 & z-w \end{array}\right] \underset{\sim}{\begin{matrix} R_3 \leftarrow R_3-2R_2 \\ R_4 \leftarrow R_4-3R_2 \end{matrix}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & w \\ 0 & 1 & 1 & x-w \\ 0 & 0 & 0 & y-w-2(x-w) \\ 0 & 0 & 0 & z-w-3(x-w) \end{array}\right]$$

This have solution when $w-2x+y=0$ and $2w-3x+z=0$

1.(c)
$$E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 0 \\ a & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 0 \\ a & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b^2 & 0 \\ 2a & 0 & 1 \end{bmatrix}$$

$$E^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b^2 & 0 \\ 2a & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & b^2 & 0 \\ 2a & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b^4 & 0 \\ 4a & 0 & 1 \end{bmatrix}$$

$$E^8 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b^4 & 0 \\ 4a & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & b^4 & 0 \\ 4a & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b^8 & 0 \\ 8a & 0 & 1 \end{bmatrix}$$

$$8E = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8b & 0 \\ 8a & 0 & 8 \end{bmatrix}$$

1.(d)
$$\begin{bmatrix} 1 & V_1 & 0 \\ 0 & V_2 & 0 \\ 0 & V_3 & 1 \end{bmatrix} \underset{\sim}{E_1 : R_3 \leftarrow R_3 - \frac{V_3}{V_2} R_2} \begin{bmatrix} 1 & V_1 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-V_3}{V_2} & 1 \end{bmatrix} \longrightarrow E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{V_3}{V_2} & 1 \end{bmatrix}$$

$$L = E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{V_3}{V_2} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & V_1 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1.(d) Inverse:

$$\left[\begin{array}{ccc|ccc} 1 & v_1 & 0 & 1 & 0 & 0 \\ 0 & v_2 & 0 & 0 & 1 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_1 \leftarrow R_1 - \frac{v_1}{v_2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{v_1}{v_2} & 0 \\ 0 & v_2 & 0 & 0 & 1 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\sim}{R_2 \leftarrow \frac{1}{v_2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{v_1}{v_2} & 0 \\ 0 & 1 & 0 & 0 & 1/v_2 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{R_3 \leftarrow R_3 - v_3 R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{v_1}{v_2} & 0 \\ 0 & 1 & 0 & 0 & 1/v_2 & 0 \\ 0 & 0 & 1 & 0 & -v_3/v_2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -v_1/v_2 & 0 \\ 0 & 1/v_2 & 0 \\ 0 & -v_3/v_2 & 1 \end{bmatrix}$$

2.(a) $\lambda I - A = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \lambda & 1 & 0 \\ -1 & \lambda & 1 \\ 0 & -1 & \lambda \end{bmatrix}$

$\det(\lambda I - A) = \lambda(\lambda^2 + 1) - (-\lambda) = \lambda^3 + 2\lambda$

$\det(\lambda I - A) = 0 \longrightarrow \lambda^3 + 2\lambda = 0 \longrightarrow \lambda(\lambda^2 + 2) = 0$

$\lambda_1 = 0, \quad \lambda_2 = \sqrt{2}i, \quad \lambda_3 = -\sqrt{2}i$

When $\lambda = 0$,

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \underset{\sim}{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \underset{\sim}{R_3 \leftrightarrow R_3 + R_2} \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $x_2 = 0$, $-x_1 + x_3 = 0 \rightarrow x_1 = x_3 = t$

When $\lambda = \sqrt{2}i$,

$$\left[\begin{array}{ccc|c} \sqrt{2}i & 1 & 0 & 0 \\ -1 & \sqrt{2}i & 1 & 0 \\ 0 & -1 & \sqrt{2}i & 0 \end{array}\right] \underset{\sim}{R_2 \leftarrow R_2 + \frac{1}{\sqrt{2}i}R_1} \left[\begin{array}{ccc|c} \sqrt{2}i & 1 & 0 & 0 \\ 0 & \sqrt{2}i/2 & 1 & 0 \\ 0 & -1 & \sqrt{2}i & 0 \end{array}\right] \underset{\sim}{R_3 \leftarrow R_3 - \sqrt{2}i R_2} \left[\begin{array}{ccc|c} \sqrt{2}i & 1 & 0 & 0 \\ 0 & \sqrt{2}i/2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $\frac{\sqrt{2}}{2}i x_2 + x_3 = 0 \rightarrow x_2 = \sqrt{2}it$, $\sqrt{2}i x_1 + x_2 = 0 \longrightarrow x_1 = -t$

When $\lambda = -\sqrt{2}i$,

$$\left[\begin{array}{ccc|c} -\sqrt{2}i & 1 & 0 & 0 \\ -1 & -\sqrt{2}i & 1 & 0 \\ 0 & -1 & -\sqrt{2}i & 0 \end{array}\right] \underset{\sim}{R_2 \leftarrow R_2 - \frac{1}{\sqrt{2}i}R_1} \left[\begin{array}{ccc|c} -\sqrt{2}i & 1 & 0 & 0 \\ 0 & -\sqrt{2}i/2 & 1 & 0 \\ 0 & -1 & -\sqrt{2}i & 0 \end{array}\right] \underset{\sim}{R_3 \leftarrow R_3 + \sqrt{2}i R_2} \left[\begin{array}{ccc|c} -\sqrt{2}i & 1 & 0 & 0 \\ 0 & -\sqrt{2}i/2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$, $-\frac{\sqrt{2}}{2}i x_2 + x_3 = 0 \rightarrow x_2 = -\sqrt{2}it$, $-\sqrt{2}i x_1 + x_2 = 0 \rightarrow x_1 = -t$

Eigen vector: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ \sqrt{2}i \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -\sqrt{2}i \\ 1 \end{bmatrix}$

2(b) $\frac{du}{dt} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} u(t)$

$\lambda_1 = 0,\ v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \lambda_2 = \sqrt{2}i,\ v_2 = \begin{bmatrix} -1 \\ \sqrt{2}i \\ 1 \end{bmatrix}, \quad \lambda_3 = -\sqrt{2}i,\ v_3 = \begin{bmatrix} -1 \\ -\sqrt{2}i \\ 1 \end{bmatrix}$

$P = [v_1 \ v_2 \ v_3] \qquad D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

$\dot{u} = Au = PDP^{-1}u$

$P^{-1}\dot{u} = DP^{-1}u \rightarrow \dot{w} = Dw$

$w(t) = e^{Dt}w(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\sqrt{2}it} & 0 \\ 0 & 0 & e^{-\sqrt{2}it} \end{bmatrix} w(0) \rightarrow w(0) = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \rightarrow w(t) = \begin{bmatrix} c_1 \\ c_2 e^{\sqrt{2}it} \\ c_3 e^{-\sqrt{2}it} \end{bmatrix}$

$u(t) = Pw(t) = Pe^{Dt}w(0)$

$= [v_1 \ v_2 \ v_3] \begin{bmatrix} c_1 \\ c_2 e^{\sqrt{2}it} \\ c_3 e^{-\sqrt{2}it} \end{bmatrix} = c_1 v_1 + c_2 e^{\sqrt{2}it} v_2 + c_3 e^{-\sqrt{2}it} v_3$

$= c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 e^{\sqrt{2}it} \begin{bmatrix} -1 \\ \sqrt{2}i \\ 1 \end{bmatrix} + c_3 e^{-\sqrt{2}it} \begin{bmatrix} -1 \\ -\sqrt{2}i \\ 1 \end{bmatrix}$

2(c) $u(t) = u(0)$

$\begin{bmatrix} c_1 - c_2 e^{\sqrt{2}it} - c_3 e^{-\sqrt{2}it} \\ c_2 e^{\sqrt{2}it}\sqrt{2}i - c_3 e^{-\sqrt{2}it}\sqrt{2}i \\ c_1 + c_2 e^{\sqrt{2}it} + c_3 e^{-\sqrt{2}it} \end{bmatrix} = \begin{bmatrix} c_1 - c_2 - c_3 \\ c_2\sqrt{2}i - c_3\sqrt{2}i \\ c_1 + c_2 + c_3 \end{bmatrix}$

$c_2 e^{\sqrt{2}it} + c_3 e^{-\sqrt{2}it} = c_2 + c_3$

$T = 0$

3(a)(i) Let $y = kx$,

$$\lim_{z \to 0} f(z) = \lim_{x=0, y=kx} f(z) = \lim_{x \to 0} \frac{kx^2}{x^2 + kx^2} = \lim_{x \to 0} \frac{k}{1+k}$$

Since it depends on k for direction, the limit does not exist, and it is not continuous at the origin

3(a)(ii) $f(z) = xy^2 + ix^2y$

$$\left.\begin{array}{ll} u_x = y^2 & v_x = 2xy \\ u_y = 2xy & v_y = x^2 \end{array}\right\} u_x = v_y \ \& \ v_x = -u_y \text{ only at } x=0, y=0$$

It is differentiable only at $x=0, y=0$, and it is not analytic anywhere

$f'(z) = u_x + iv_x = y^2 + ix^2$ (for $x=0, y=0$)

3.(b)(i) $\oint_c z\sin^2(z-0.3)dz = 0$

$$\oint_c \frac{(z^2+0.5)^2 \sin z}{z^2} dz = 2\pi i \frac{d}{dz}\left((z^2+0.5)^2 \sin z\right)_{z=0} = 2\pi i \left(\frac{1}{720}\pi\right) = \frac{1}{360}\pi^2 i$$

3(b)(ii) $z = 4e^{i\theta}$ $\quad \frac{dz}{d\theta} = 4ie^{i\theta}$ $\quad dz = 4ie^{i\theta}d\theta$

$$\oint_c \frac{\bar{z}}{|z|} dz = \int_0^{2\pi} \frac{4e^{-i\theta}}{4} 4ie^{i\theta} d\theta = \int_0^{2\pi} 4i\, d\theta = 8\pi i$$

3(c) $i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \longrightarrow i^n = \cos\left(n\frac{\pi}{2}\right) + i\sin\left(n\frac{\pi}{2}\right)$

When $n = 1, 5, 9, 13, \ldots\ldots$

$$i^n = i \begin{cases} \text{real part} = 0 \\ \text{imaginary part} = 1 \end{cases}$$

When $n = 2, 6, 10, 14, \ldots\ldots$

$$i^n = -1 \begin{cases} \text{real part} = -1 \\ \text{imaginary part} = 0 \end{cases}$$

When $n = 3, 7, 11, 15, \ldots\ldots$

$$i^n = -i \begin{cases} \text{real part} = 0 \\ \text{imaginary part} = -1 \end{cases}$$

When $n = 4, 8, 12, 16, \ldots\ldots$

$$i^n = 1 \begin{cases} \text{real part} = 1 \\ \text{imaginary part} = 0 \end{cases}$$

4.(a)

$$\nabla f = \begin{pmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{pmatrix} = \begin{pmatrix} 2x+y+3 \\ 4y+x-2 \\ 6z-6 \end{pmatrix} \longrightarrow \nabla f \text{ at } (0,0,0) = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$$

$$\nabla f . \hat{u} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix} . \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = -3$$

$$\nabla f = \begin{pmatrix} 2x+y+3 \\ 4y+x-2 \\ 6z-6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} 2x+y+3=0 &\rightarrow 2x+y=-3 \\ 4y+x-2=0 &\rightarrow x+4y=2 \end{aligned}\right\} 7y=7 \rightarrow y=1,\ x=-2$$

$$6z-6=0 \rightarrow z=1$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \longrightarrow f(-2,1,1) = -7$$

4.(b)

$$\nabla \times F = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} \times \begin{pmatrix} 2xyz^3 \\ x^2z^3 \\ 3x^2yz^2 \end{pmatrix} = \begin{pmatrix} 3x^2z^2 - 3x^2z^2 \\ 6xyz^2 - 6xyz^2 \\ 2xz^3 - 2xz^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{(shown)}$$

$$F(x,y,z) = \begin{pmatrix} 2xyz^3 \\ x^2z^3 \\ 3x^2yz^2 \end{pmatrix} = \begin{pmatrix} \partial V/\partial x \\ \partial V/\partial y \\ \partial V/\partial z \end{pmatrix}$$

$$\left.\begin{aligned} \frac{\partial V}{\partial x} = 2xyz^3 &\rightarrow V = x^2yz^3 + f_1(y,z) \\ \frac{\partial V}{\partial y} = x^2z^3 &\rightarrow V = x^2yz^3 + f_2(x,z) \\ \frac{\partial V}{\partial z} = 3x^2yz^2 &\rightarrow V = x^2yz^3 + f_3(x,y) \end{aligned}\right\} V = x^2yz^3 + C$$

$$WD = V(2,3,1) - V(1,4,1) = 4(3)(1) - 4 = 12$$

4.(c)

$$\oiint_S F.n\, dA = \iiint_V \nabla . F\, dV$$

$$\nabla . F = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 1+1+1 = 3$$

$$3\iiint_V dV = 3\left(\frac{2}{3}\pi a^3\right) = 2\pi a^3$$