NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

EE2007 / IM2007 – ENGINEERING MATHEMATICS II

April / May 2017 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of formulas is provided in Appendix A on page 5.

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1. (a) Given the matrix

$$\mathbf{A} = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

where a, b, c are unknown real numbers.

(i) Find the determinant of $\mathbf{A}$ and state the condition(s) for a, b, c such that $\mathbf{A}$ is invertible.

(ii) Assuming that $\mathbf{A}$ is invertible, find a LU factorization of $\mathbf{A}$.

(10 Marks)

Note: Question No. 1 continues on page 2.

6

(b) Find the matrix $\mathbf{B}$ if its eigenvalues are -2 and 5 and the corresponding eigenvectors are $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, respectively. How can you tell in advance, without computing $\mathbf{B}$, whether $\mathbf{B}$ is symmetric? Explain clearly your reasoning. [Hint: A symmetric matrix is one such that $\mathbf{B} = \mathbf{B}^T$.]

(8 Marks)

(c) Suppose $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ are linearly independent vectors. Show that the vectors $\mathbf{v}_1 = \mathbf{w}_2 + \mathbf{w}_3$, $\mathbf{v}_2 = \mathbf{w}_1 + \mathbf{w}_3$ and $\mathbf{v}_3 = \mathbf{w}_1 + \mathbf{w}_2$ are linearly independent. [Hint: Write $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = 0$ in terms of $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ and solve for c_1, c_2, c_3.]

(7 Marks)

2. The augmented matrix

$$\left[\begin{array}{cccc|c} k & 1 & 1 & 1 & a \\ 1 & k & 1 & 1 & 1 \\ 1 & 1 & k & 1 & 1 \\ 1 & 1 & 1 & k & 1 \end{array}\right]$$

represents a system of linear equations with 4 unknowns, where k and a are parameters of the linear system.

(a) Determine the values of k and a such that the system has:

(i) no solution,

(ii) unique solution, and

(iii) many solutions.

(15 Marks)

Note: Question No. 2 continues on page 3.

(b) Hence or otherwise, answer the following and justify your answers:

(i) Are the following vectors linearly independent?

$$\begin{bmatrix}4\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\4\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\4\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\4\end{bmatrix}$$

(ii) Are the following vectors linearly independent?

$$\begin{bmatrix}-3\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-3\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-3\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-3\end{bmatrix}.$$

(iii) Is the vector $\begin{bmatrix}4\\1\\1\\1\end{bmatrix}$ in the column space of $\begin{bmatrix}7&1&1&1\\1&7&1&1\\1&1&7&1\\1&1&1&7\end{bmatrix}$?

(10 Marks)

3. (a) Given that $e^{i\frac{3\pi}{2}}\left[\ln y-\ln\left(8e^{-\frac{\pi}{2}}\right)\right]=\frac{\pi}{3}$, show that $y = 4e^{-\frac{\pi}{2}}\left[1+i\sqrt{3}\right]$.

(6 Marks)

(b) Discuss the differentiability and analyticity of $f(z)=ze^{\bar{z}}$. Use the Cauchy-Riemann equations to support your answer.

(9 Marks)

(c) Using the parameterization $z = e^{i2\theta}$, evaluate $\int_0^{2\pi}\frac{1}{2+\cos 2\theta}\,d\theta$, where θ is real.

(10 Marks)

4. (a) "The closed path integral of the gradient of any scalar function is always equal to zero." Justify the truth of this statement with proof(s).

(6 Marks)

(b) Is the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = 2xye^z\ \mathbf{i} + x^2e^z\ \mathbf{j} + x^2ye^z\ \mathbf{k}$, independent of path C? Justify your answer. Hence or otherwise, evaluate the line integral from (1, 2, 0) to (2, 1, 0).

(10 Marks)

(c) Suppose a particle moves in straight-line segments from (0, 0, 2) to (2, 0, 2) to (2, 2, 2) to (0, 2, 2) and back to (0, 0, 2) in a field given by

$$\mathbf{F}(x, y, z) = xy^2z^3\ \mathbf{i} + x^2y^2z^2\ \mathbf{j} + x^3y^2z\ \mathbf{k}.$$

Find the amount of work done in the course of moving the particle.

(9 Marks)

Appendix A

1. Complex Analysis

(a) Complex Power: $z^c = e^{c\ln z}$

(b) De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(c) Cauchy-Riemann equations:

$$u_x = v_y,\ v_x = -u_y,\ \text{or}\ u_r = \frac{1}{r}v_\theta,\ v_r = \frac{-1}{r}u_\theta$$

(d) Cauchy Integral Formula:

$$\int_C \frac{f(z)}{(z-z_o)^m}dz = \frac{2\pi i}{(m-1)!}\left.\frac{d^{(m-1)}}{dz^{(m-1)}}f(z)\right|_{z=z_o}$$

2. Vector Analysis. Let $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$.

(a) Scalar Triple Product: $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$

(b) Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

(c) Divergence: $\nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

(d) Curl: $\nabla\times\mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

(e) Gauss Theorem: $\iiint_T \nabla\cdot\mathbf{F}dv = \oiint_S \mathbf{F}\cdot\mathbf{n}dA$

(f) Stokes Theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$

END OF PAPER

8

*The solution is only for your reference

EE 2007 April/May 2017

Date No.

1. (a) (i) $\begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix} \xrightarrow[E_2: R_3 \leftarrow R_3 - R_1]{E_1: R_2 \leftarrow R_2 - R_1} \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & b-a & c-a \end{bmatrix} \xrightarrow{E_3: R_3 \leftarrow R_3 - R_2} \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & 0 & c-b \end{bmatrix}$

$$\det(A) = \det \begin{vmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & 0 & c-b \end{vmatrix} = a(b-a)(c-b)$$

If A is invertible. $\det(A) \neq 0 \Rightarrow \begin{cases} a \neq 0 \\ a \neq b \\ b \neq c \end{cases}$

(ii) $E_3 E_2 E_1 A = \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & 0 & c-b \end{bmatrix} = U$

$A = (E_3 E_2 E_1)^{-1} U = E_1^{-1} E_2^{-1} E_3^{-1} U$

$L = E_1^{-1} E_2^{-1} E_3^{-1}$

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & 0 & c-b \end{bmatrix}$

(b) $P = \begin{bmatrix} -1 & 3 \\ 1 & 4 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} -\frac{4}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{1}{7} \end{bmatrix}$

$B = PDP^{-1} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

For different eigenvalues. B is symmetric if the two eigenvectors are orthonormal.

Assume B is a real symmetric matrix. Let $Bv_1 = \lambda_1 v_1$ and $Bv_2 = \lambda_2 v_2$, then

$$v_2^T B v_1 = v_2^T \lambda_1 v_1$$

$$\Rightarrow \lambda_1 v_2^T v_1 = v_2^T (Bv_1) = (v_2^T B) v_1 = (B^T v_2)^T v_1 = (Bv_2)^T v_1 = \lambda_2 v_2^T v_1$$

$$\Rightarrow (\lambda_1 - \lambda_2) v_2^T v_1 = 0$$

As $\lambda_1 \neq \lambda_2$, $v_2^T v_1 = 0$.

For this case $[-1 \ 1] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 1 \neq 0 \Rightarrow$ B is not symmetric

1.(c) As w_1, w_2, w_3 are linearly independent vectors, $a_1w_1+a_2w_2+a_3w_3=0$ only has the trivial solution $a_1=a_2=a_3=0$

To show that v_1, v_2, v_3 are linearly independent, we only need to show that $c_1v_1+c_2v_2+c_3v_3=0$ only has the trivial solution $c_1=c_2=c_3=0$

$$c_1(w_2+w_3)+c_2(w_1+w_3)+c_3(w_1+w_2)=0$$

$$\Rightarrow (c_2+c_3)w_1+(c_1+c_3)w_2+(c_1+c_2)w_3=0$$

$$\begin{cases} c_2+c_3=0 \\ c_1+c_3=0 \\ c_1+c_2=0 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow \begin{cases} c_1=0 \\ c_2=0 \\ c_3=0 \end{cases}$$

Therefore, v_1, v_2, v_3 are linearly independent.

2. $$\left[\begin{array}{cccc|c} k & 1 & 1 & 1 & a \\ 1 & k & 1 & 1 & 1 \\ 1 & 1 & k & 1 & 1 \\ 1 & 1 & 1 & k & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & k & 1 \\ 1 & k & 1 & 1 & 1 \\ 1 & 1 & k & 1 & 1 \\ k & 1 & 1 & 1 & a \end{array}\right] \xrightarrow[R_4 \leftarrow R_4 - kR_1]{R_2 \leftarrow R_2 - R_1,\ R_3 \leftarrow R_3 - R_1} \left[\begin{array}{cccc|c} 1 & 1 & 1 & k & 1 \\ 0 & k-1 & 0 & 1-k & 0 \\ 0 & 0 & k-1 & 1-k & 0 \\ 0 & 1-k & 1-k & 1-k^2 & a-k \end{array}\right]$$

$$\xrightarrow{R_4 \leftarrow R_4 + R_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & k & 1 \\ 0 & k-1 & 0 & 1-k & 0 \\ 0 & 0 & k-1 & 1-k & 0 \\ 0 & 0 & 1-k & 2-k-k^2 & a-k \end{array}\right] \xrightarrow{R_4 \leftarrow R_4 + R_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & k & 1 \\ 0 & k-1 & 0 & 1-k & 0 \\ 0 & 0 & k-1 & 1-k & 0 \\ 0 & 0 & 0 & 3-2k-k^2 & a-k \end{array}\right]$$

(a) determinant of the original matrix is $(-1)(k-1)^2(3-2k-k^2)$

$$= (k-1)^2(k^2+2k-3) = (k-1)^2(k-1)(k+3)$$

(i) no solution: $\text{rank}(A|b) > \text{rank}(A) \Rightarrow \begin{cases} k=1 \text{ or } k=-3 \\ a \neq k \end{cases}$

(ii) unique solution: $\text{rank}(A|b) = \text{rank}(A) = n \Rightarrow k \neq 1$ and $k \neq -3$, $a \in R$

(iii) many solutions: $\text{rank}(A|b) = \text{rank}(A) < n \Rightarrow \begin{cases} k=1 \text{ or } k=-3 \\ a=k \end{cases}$

2.(b) (i) Form the equation $c_1v_1+c_2v_2+c_3v_3+c_4v_4=0$ and solve for c_1, c_2, c_3 and c_4.

According to 2(a), let $k=4$, then we can get

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 0 \\ 0 & 4-1 & 0 & 1-4 & 0 \\ 0 & 0 & 4-1 & 1-4 & 0 \\ 0 & 0 & 0 & 3-8-16 & 0 \end{array}\right] \Rightarrow \begin{cases} c_1=0 \\ c_2=0 \\ c_3=0 \\ c_4=0 \end{cases}$$

Therefore, these vectors are linearly independent.

(ii) Form the equation $c_1v_1+c_2v_2+c_3v_3+c_4v_4=0$ and solve for c_1, c_2, c_3 and c_4

According to 2(a), let $k=-3$, then we can see that it has no solution or many solutions. Therefore, these vectors are not linearly independent.

(iii) We need to check whether the following equations has solution or not.

$$\begin{bmatrix} 7 & 1 & 1 & 1 \\ 1 & 7 & 1 & 1 \\ 1 & 1 & 7 & 1 \\ 1 & 1 & 1 & 7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

According to 2(a), we can see that when $k=7$ and $a=4$, this equation has unique solution. Therefore, the vector $\begin{bmatrix} 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is in the column space of $\begin{bmatrix} 7 & 1 & 1 & 1 \\ 1 & 7 & 1 & 1 \\ 1 & 1 & 7 & 1 \\ 1 & 1 & 1 & 7 \end{bmatrix}$

3. (a) $e^{i\frac{3\pi}{2}}\left[\ln y - \ln(8e^{-\frac{\pi}{2}})\right] = \frac{\pi}{3}$

$\Rightarrow \ln y - \ln(8e^{-\frac{\pi}{2}}) = \frac{\pi}{3}e^{-i\frac{3\pi}{2}}$

$\Rightarrow \frac{y}{8e^{-\frac{\pi}{2}}} = e^{\frac{\pi}{3}e^{-i\frac{3\pi}{2}}}$

$\Rightarrow y = 8e^{-\frac{\pi}{2}}e^{\frac{\pi}{3}e^{-i\frac{3\pi}{2}}}$

$= 8e^{-\frac{\pi}{2}}e^{\frac{\pi}{3}(\cos\frac{3\pi}{2} - i\sin\frac{3\pi}{2})}$

$= 8e^{-\frac{\pi}{2}}e^{\frac{\pi}{3}(0+i)}$

$= 8e^{-\frac{\pi}{2}}e^{\frac{\pi}{3}i}$

$= 8e^{-\frac{\pi}{2}}(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$= 4e^{-\frac{\pi}{2}}(1+i\sqrt{3})$

Date No.

3.(b) $f(z) = ze^{z}$

$$= (x+iy)e^{x+iy}$$
$$= (x+iy)\,e^{x}(\cos y + i\sin y)$$
$$= e^{x}\left[x\cos y - y\sin y + i(y\cos y + x\sin y)\right]$$

$u(x,y) = e^{x}(x\cos y - y\sin y)$ $\qquad$ $v(x,y) = e^{x}(y\cos y + x\sin y)$

$u_x = xe^{x}\cos y + e^{x}\cos y - e^{x}y\sin y$ $\qquad$ $v_x = e^{x}y\cos y + e^{x}\sin y + xe^{x}\sin y$

$u_y = -xe^{x}\sin y - e^{x}\sin y - e^{x}y\cos y$ $\qquad$ $v_y = e^{x}\cos y - ye^{x}\sin y + xe^{x}\cos y$

$\begin{cases} u_x = v_y \\ v_x = -u_y \end{cases} \Rightarrow f(z)$ is differentiable and analytic

(c) $z = e^{i2\theta} \Rightarrow dz = 2ie^{i2\theta}d\theta \Rightarrow \frac{1}{2iz}dz = d\theta$, $\cos 2\theta = \frac{1}{2}(e^{i2\theta} + e^{-i2\theta}) = \frac{1}{2}(z + \frac{1}{z})$

$$\int_0^{2\pi} \frac{1}{2+\cos 2\theta}d\theta = 2\int_0^{\pi}\frac{1}{2+\cos 2\theta}d\theta$$
$$= 2\oint_c \frac{1}{2+\frac{1}{2}(z+\frac{1}{z})}\cdot\frac{1}{2iz}dz$$
$$= \frac{2}{i}\oint_c \frac{1}{4z+(z^2+1)}dz$$
$$= \frac{2}{i}\oint_c \frac{1}{[z-(-2+\sqrt{3})][z-(-2-\sqrt{3})]}dz$$
$$= \frac{2}{i}\oint_c \frac{\frac{1}{[z-(-2-\sqrt{3})]}}{[z-(-2+\sqrt{3})]}dz$$
$$= \frac{2}{i}\cdot 2\pi i\cdot\left[\frac{1}{z-(-2-\sqrt{3})}\Big|_{z=-2+\sqrt{3}}\right]$$
$$= \frac{2\pi}{\sqrt{3}}$$
$$= \frac{2\sqrt{3}\pi}{3}$$

4.(a) $\nabla f = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k$

Let $\nabla f = \vec{F}$, then $\oint_c \nabla f\, d\vec{r} = \oint_c \vec{F}\, d\vec{r} = \iint_s \nabla\times\vec{F}\, d\vec{A}$

$$\nabla\times\vec{F} = \nabla\times\nabla f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z}\end{vmatrix} = \left(\frac{\partial^2 f}{\partial y\partial z} - \frac{\partial^2 f}{\partial z\partial y}\right)i - \left(\frac{\partial^2 f}{\partial x\partial z} - \frac{\partial^2 f}{\partial z\partial x}\right)j + \left(\frac{\partial^2 f}{\partial x\partial y} - \frac{\partial^2 f}{\partial y\partial x}\right)k = 0$$

$\therefore \oint_c \nabla f\, d\vec{r} = 0$

4. (b) $\nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xye^z & x^2e^z & x^2ye^z \end{vmatrix}$

$= (x^2e^z - x^2e^z)i - (2xye^z - 2xye^z)j + (2xe^z - 2xe^z)k$

$= 0$

Therefore, $\vec{F}$ is a conservative field so the line integral $\int_C \vec{F}\cdot d\vec{r}$ is independent of path C.

$\frac{\partial V}{\partial x} = 2xye^z \Rightarrow V(x,y,z) = x^2ye^z + V(y,z)$

$\frac{\partial V}{\partial y} = x^2e^z \Rightarrow V(x,y,z) = x^2ye^z + V(x,z)$

$\frac{\partial V}{\partial z} = x^2ye^z \Rightarrow V(x,y,z) = x^2ye^z + V(x,y)$

$\therefore V(x,y,z) = x^2ye^z + C$

$\int_C \vec{F}\cdot d\vec{r} = V(2,1,0) - V(1,2,0)$

$= 2^2\times1\times1 - 1^2\times2\times1$

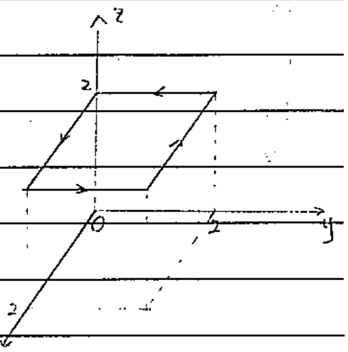
$= 2$

(c) solution 1: C_1: $\vec{r} = ti + 2k$, $d\vec{r} = i\,dt$ $\quad 0\le t\le 2$

C_2: $\vec{r} = 2i + tj + 2k$, $d\vec{r} = j\,dt$, $0\le t\le 2$

C_3: $\vec{r} = ti + 2j + 2k$, $d\vec{r} = i\,dt$, $2\ge t\ge 0$

C_4: $\vec{r} = tj + 2k$, $d\vec{r} = j\,dt$, $2\ge t\ge 0$



$\oint_C \vec{F}\cdot d\vec{r} = \int_{C_1}\vec{F}\cdot d\vec{r} + \int_{C_2}\vec{F}\cdot d\vec{r} + \int_{C_3}\vec{F}\cdot d\vec{r} + \int_{C_4}\vec{F}\cdot d\vec{r}$

$= \int_0^2 xy^2z^3\,dt + \int_0^2 x^2y^2z^2\,dt + \int_2^0 xy^2z^3\,dt + \int_2^0 x^2y^2z^2\,dt$

$= \int_0^2 0\,dt + \int_0^2 2^2\times t^2\times 2^2\,dt + \int_2^0 t\times 2^2\times 2^3\,dt + \int_2^0 0\,dt$

$= \int_0^2 16t^2\,dt + \int_2^0 32t\,dt$

$= \left[\frac{16t^3}{3}\right]_0^2 + \left[\frac{32t^2}{2}\right]_2^0$

$= \frac{16\times2^3}{3} - 0 + 0 - \frac{32\times2^2}{2}$

$= -\frac{64}{3}$

Date No.

Solution 2: $\oint_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F}\, d\vec{A}$

$= \iint_S \nabla \times \vec{F} \cdot \hat{n}\, dA$

$\hat{n} = k$

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2z^3 & x^2y^2z^2 & x^3y^2z \end{vmatrix}$$

$\therefore \iint_S \nabla \times \vec{F} \cdot \hat{n}\, dA$

$= \int_0^2 \int_0^2 2xy^2z^2 - 2xyz^3\, dx\, dy$

$= \int_0^2 (8y^2 - 16y)\left[\frac{x^2}{2}\right]_0^2 dy$

$= \int_0^2 16y^2 - 32y\, dy$

$= \left[\frac{16y^3}{3} - \frac{32y^2}{2}\right]_0^2$

$= -\frac{64}{3}$