AC Analysis of BJT and MOSFET Inverting Amplifiers

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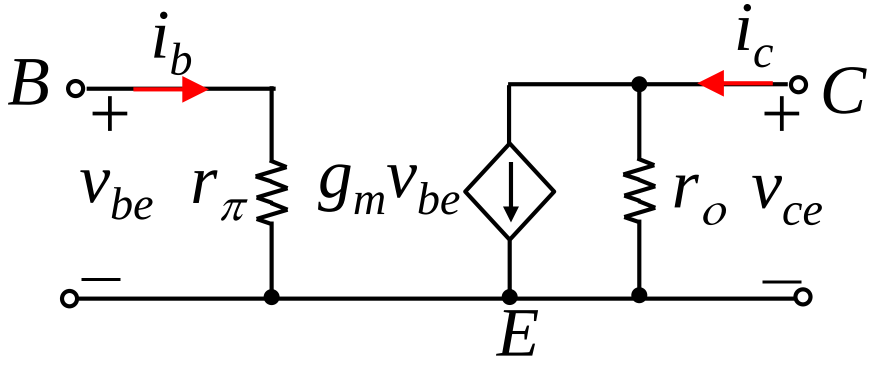
EE2002 Analog Electronics

Lesson Objectives

At the end of this lesson, you should be able to:

- Draw small signal model for BJT and MOSFET
- Calculate the small signal parameters of BJT and MOSFET
- Construct AC equivalent circuit of BJT and MOSFET inverting amplifiers
- Calculate the following performance characteristics of C-E and C-S amplifiers
  - Voltage gain
  - Input resistance
  - Output resistance

Hybrid-Pi Model of BJT



- This hybrid-pi small-signal model is the intrinsic low-frequency representation of the BJT.
- Small-signal parameters are controlled by the Q-point and are independent of geometry of BJT.

Transconductance:

$$g_m = \frac{I_C}{V_T} \approx 40 I_C$$

where $V_T = \dfrac{kT}{q} \approx 25 \text{ mV@}25°\text{C}$

Input resistance:

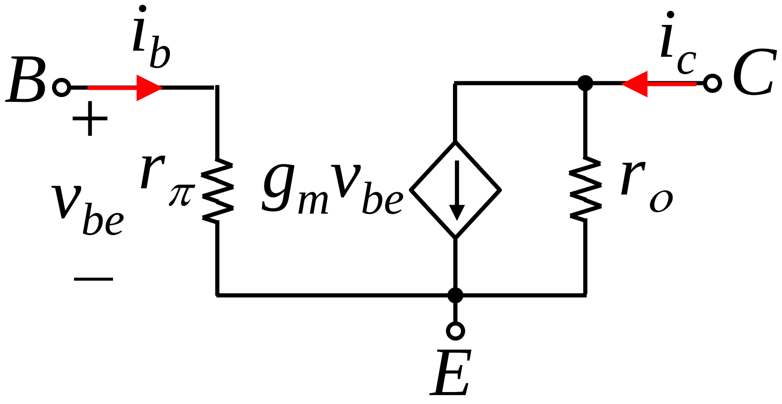
$$r_\pi = \frac{\beta}{g_m}$$

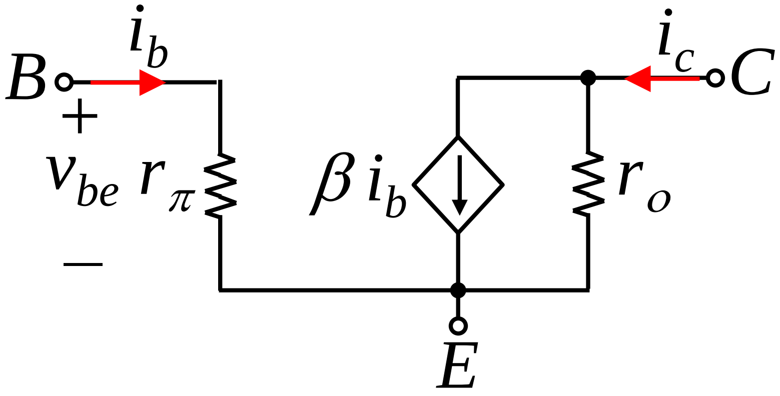
Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} \quad \text{if } V_A \gg V_{CE}$$

Equivalent Forms of Small-Signal Model for BJT

Voltage-controlled current source $g_m v_{be}$ can be transformed into current-controlled current source.





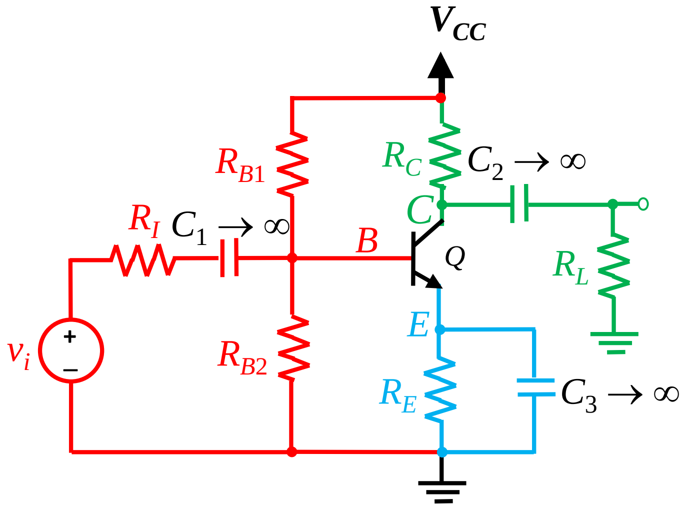
$$v_{be} = i_b r_\pi$$

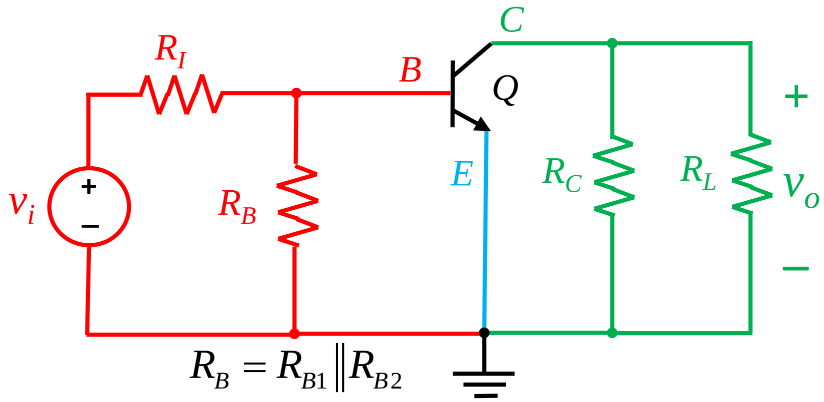
$$g_m v_{be} = g_m i_b r_\pi = \beta i_b$$

$$i_c = \beta i_b + \frac{v_{ce}}{r_o} \approx \beta i_b$$

Basic relationship $i_c = \beta\, i_b$ is useful for both dc and ac analysis when BJT is in forward-active region.

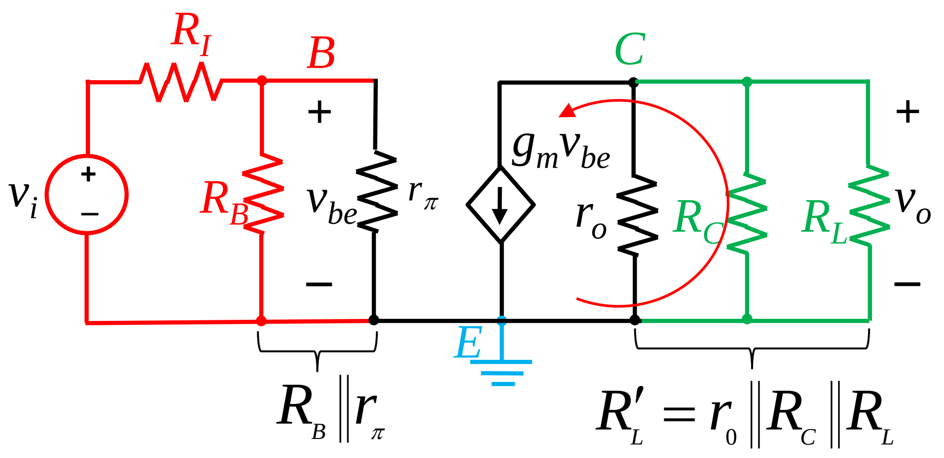
Small Signal Analysis of C-E Amplifier with Fully Bypass R_E





- The ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage source is ac ground.
- Assume that Q-point has already been calculated from DC analysis. Hence, g_m, r_π, and r_o of BJT can be calculated.

C-E Amplifier with Fully Bypass R_E: Voltage Gain



Overall voltage gain from source v_i to output voltage v_o across R_L is:

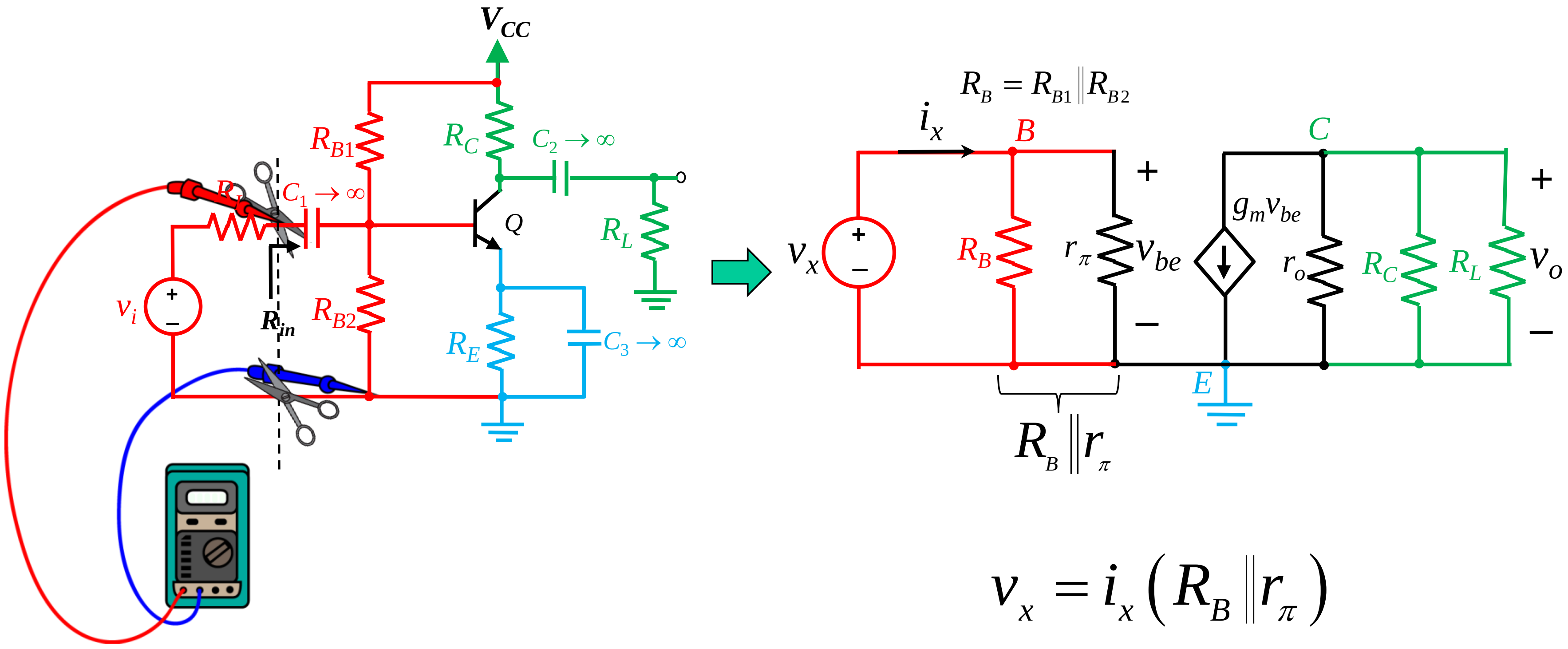
$$A_v = \frac{v_o}{v_i} = \frac{v_c}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{v_b}{v_i}$$

$$\therefore A_v = -g_m R_L' \left(\frac{R_B \| r_\pi}{R_I + R_B \| r_\pi} \right)$$

Terminal voltage gain between base and collector is:

$$A_{vt} = \frac{v_c}{v_b} = \frac{v_o}{v_{be}} = \frac{-g_m v_{be} R_L'}{v_{be}} = -g_m R_L'$$

C-E Amplifier with Fully Bypass R_E: Input Resistance

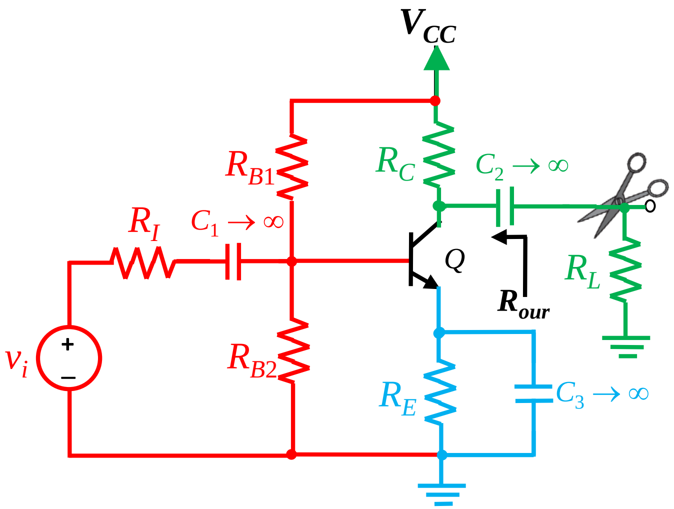


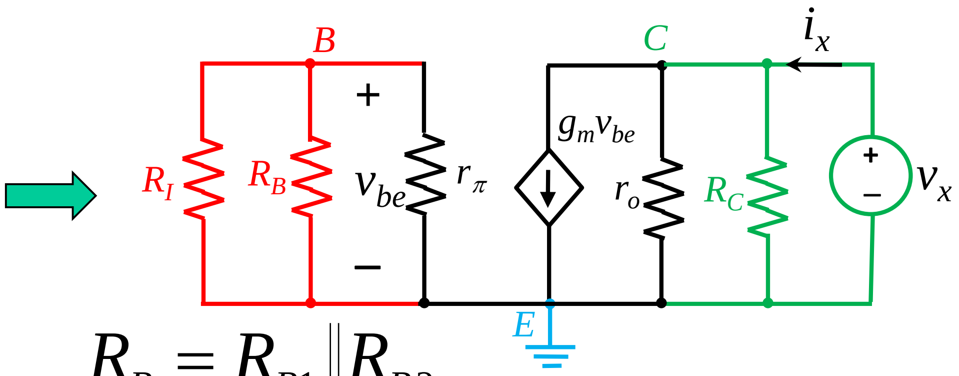
$$v_x = i_x \left(R_B \| r_\pi \right)$$

$$R_{in} = \frac{v_x}{i_x}$$

$$= R_B \| r_\pi$$

C-E Amplifier with Fully Bypass R_E: Output Resistance





$$R_B = R_{B1} \| R_{B2}$$

$$i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o} + g_m v_{be}$$

$$v_{be} = 0 \Rightarrow i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o}$$

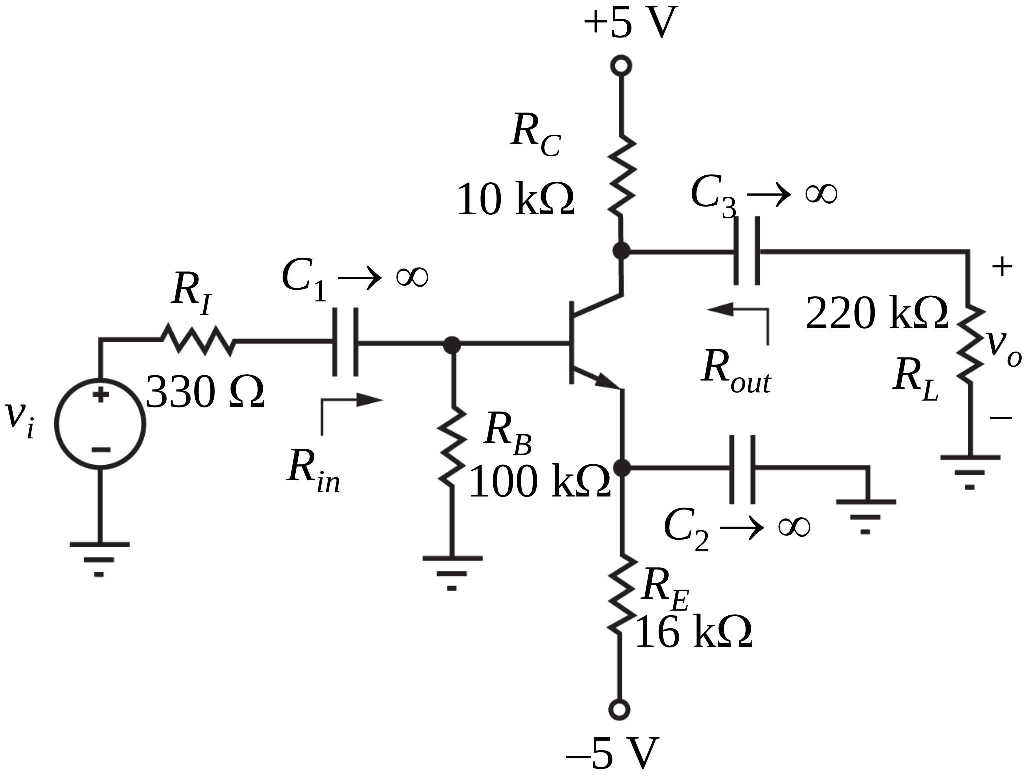
$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_C} + \frac{1}{r_o} \right)^{-1} = R_C \| r_o$$

$$R_{out} \approx R_C \text{ if } r_o \gg R_C$$

C-E Amplifier with Fully Bypass R_E: Example

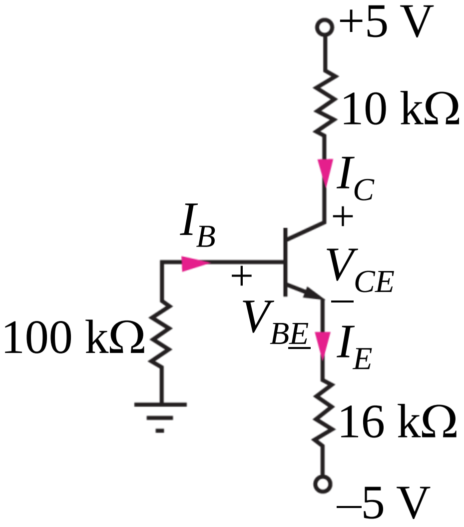
Problem: Find voltage gain, input and output resistances.

Given: $\beta = 65$, $V_A = 50$ V



Find the Q-point from dc equivalent circuit:

$$100\times10^3 I_B + 0.7 + 66 I_B\left(16\times10^3\right) + (-5) = 0$$



$$I_B = 3.71\ \mu\text{A}$$

$$\boxed{I_C = 65 I_B = 241\ \mu\text{A}}$$

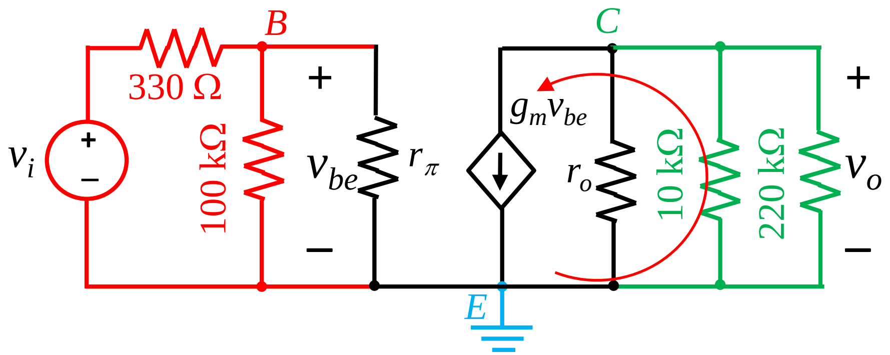
$$I_E = 66 I_B = 245\ \mu\text{A}$$

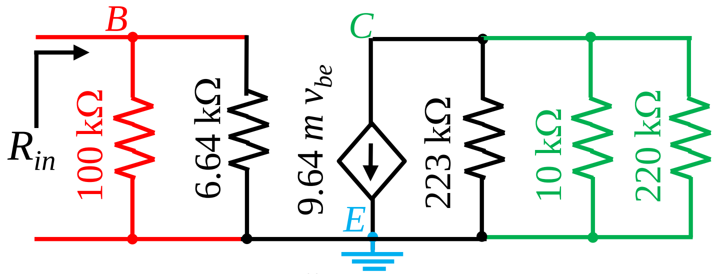
$$5 - 10\text{ k}\times 241\ \mu - V_{CE} - 16\text{ k}\times 245\ \mu - (-5) = 0$$

$$\boxed{V_{CE} = 3.67\text{ V}}$$

AC Analysis of C-E Amplifier with Fully Bypass R_E

Small signal equivalent



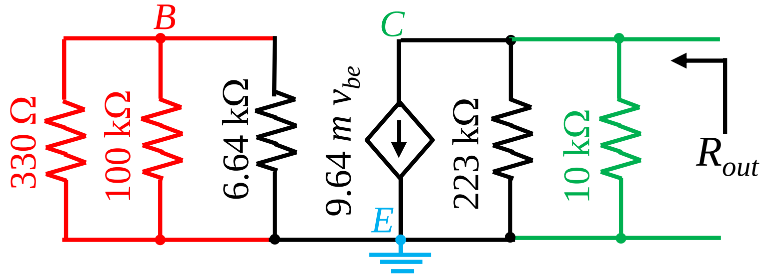


$$R_{in} = 100\text{ k}\|6.64\text{ k} = 6.23\text{ k}\Omega$$

$$g_m \approx 40 \times 241\ \mu = 9.64\text{ mS}$$

$$r_\pi = \frac{65}{9.64\text{ m}} = 6.64\text{ k}\Omega$$

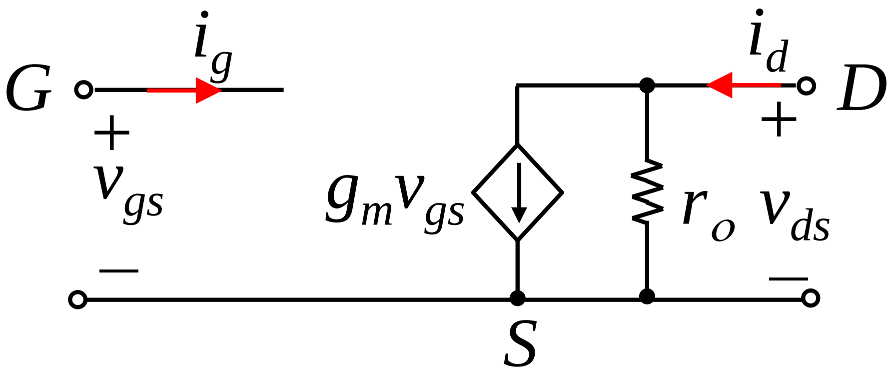
$$r_o = \frac{50 + 3.67}{241\ \mu} = 223\text{ k}\Omega$$



$$R_{out} = 223\text{ k}\|10\text{ k} = 9.57\text{ k}\Omega$$

$$A_v = A_{vt}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \left(\frac{-9.64 \times v_{be} \times (223\|10\|220)}{v_{be}}\right)\left(\frac{6.23\text{ k}}{330 + 6.23\text{ k}}\right) = -84.0$$

Small Signal Parameters of MOSFET



- Since gate is insulated from channel by gate-oxide, input resistance = ∞.
- Small-signal parameters are controlled by the Q-point.
- MOSFET transconductance is geometry dependent.

$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q-pt} = K_n\left(V_{GS} - V_{TN}\right)\left(1 + \lambda V_{DS}\right)$$

where $K_n = \mu_n C_{OX}\left(\frac{W}{L}\right)$

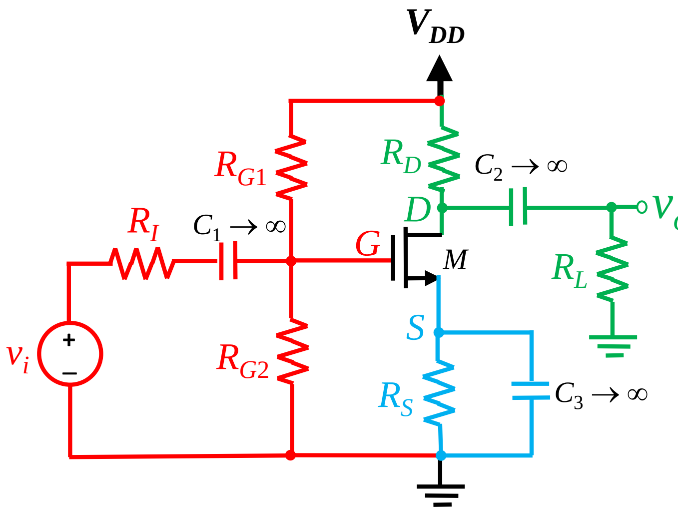
$$g_m = \frac{I_D}{\frac{V_{GS} - V_{TN}}{2}} \approx \sqrt{2K_n I_D}$$

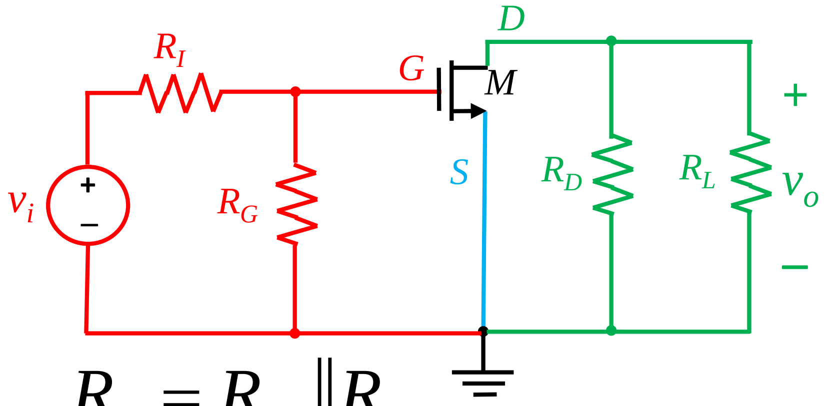
$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \approx \frac{1}{\lambda I_D} \text{ if } \frac{1}{\lambda} \gg V_{DS}$$

Small Signal Analysis of C-S Amplifier with Fully Bypass R_S

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent ac grounds.

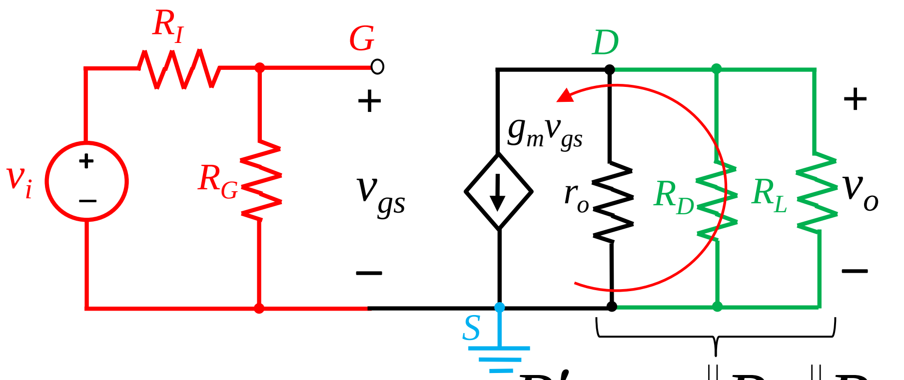
- The small signal parameters, g_m and r_o of the MOSFET is calculated at the Q-point, I_D and V_{DS}.





$$R_G = R_{G1} \| R_{G2}$$

C-S Amplifier with Fully Bypass R_S: Voltage Gain



$$R_L' = r_0 \| R_D \| R_L$$

Terminal voltage gain between gate and drain is:

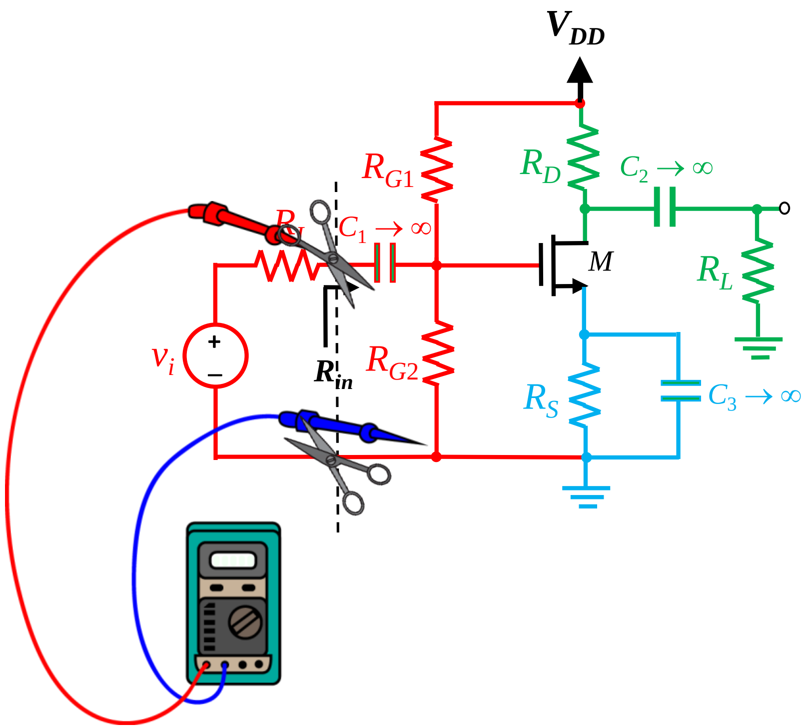
$$A_{vt} = \frac{v_d}{v_g} = \frac{-g_m v_{gs} R_L'}{v_{gs}} = -g_m R_L'$$

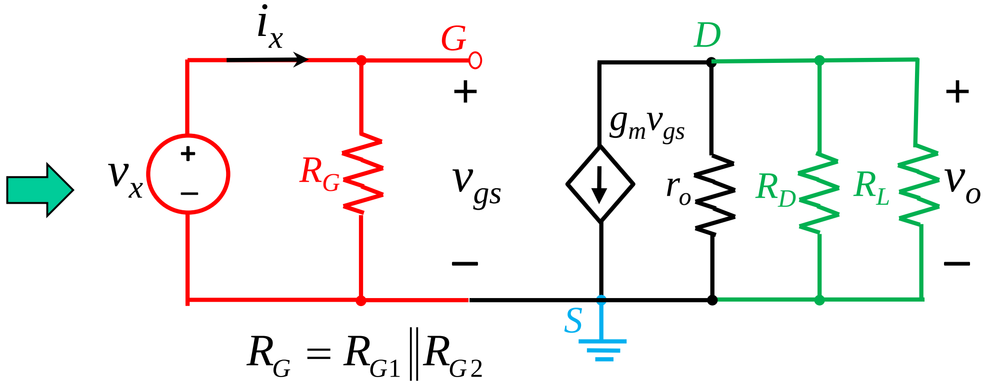
Overall voltage gain from source v_i to output voltage v_o across R_L is:

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{v_g}{v_i}$$

$$= -g_m R_L' \left(\frac{R_G}{R_I + R_G} \right)$$

C-S Amplifier with Fully Bypass R_S: Input Resistance





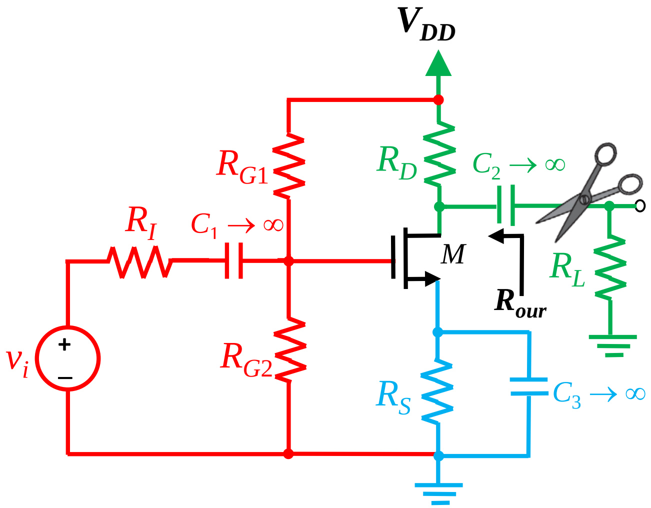
$$R_G = R_{G1} \| R_{G2}$$

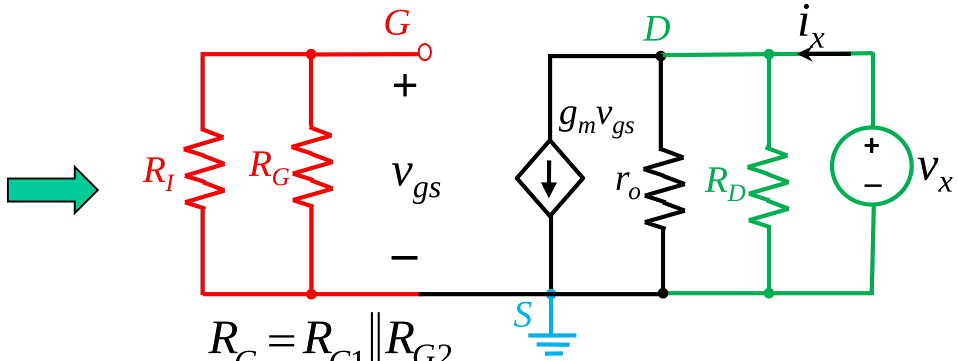
$$v_x = i_x R_G$$

$$R_{in} = \frac{v_x}{i_x}$$

$$= R_G$$

C-S Amplifier with Fully Bypass R_S: Output Resistance





$R_G = R_{G1} \| R_{G2}$

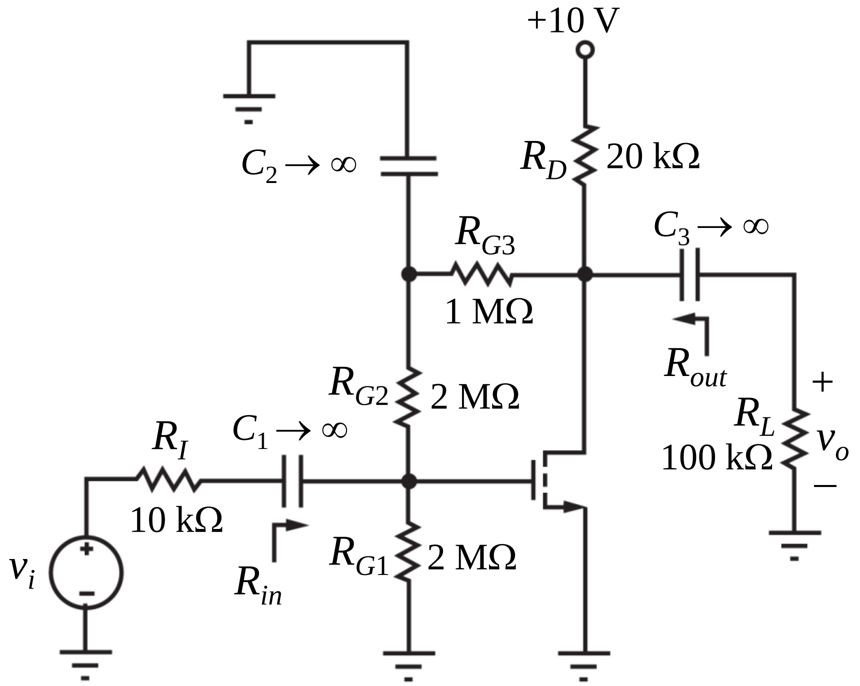
Since $v_{gs} = 0$, $g_m v_{gs} = 0$.

$$v_x = i_x \left(R_D \| r_o \right)$$

$$R_{out} = \frac{v_x}{i_x} = R_D \| r_o$$

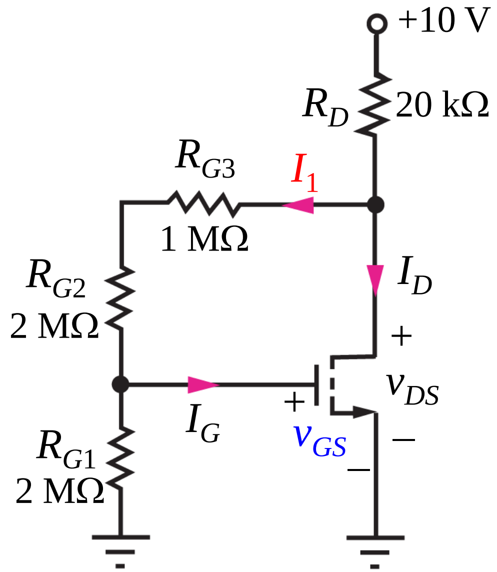
$$R_{out} \approx R_D \text{ if } r_o \gg R_D$$

C-S Amplifier with Fully Bypass R_S: Example



Problem: Find voltage gain, input and output resistances.

Given: $K_n = 500\ \mu A/V^2,\ V_{TN} = 1V,$
$\lambda = 0.0167\ V^{-1}$



DC Analysis:

$$I_1 = \frac{V_{DS}}{5\times 10^6}$$

$$V_{GS} = I_1 \times 2\times 10^6 = 0.4V_{DS}$$

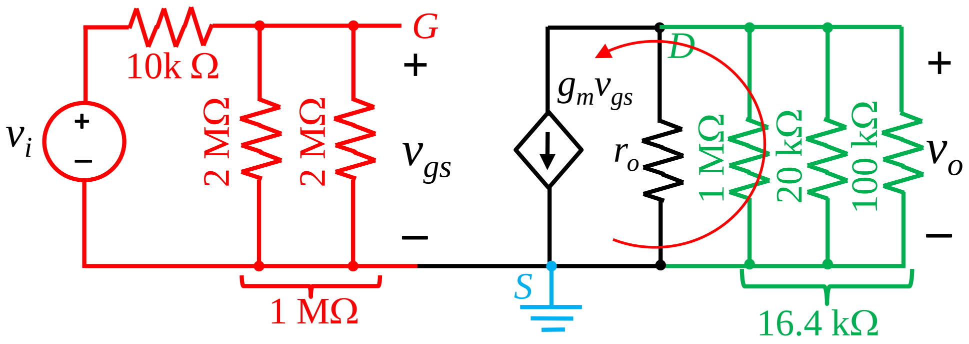
$$I_D = \frac{500\ \mu}{2}(V_{GS}-1)^2 \text{ —— } (1)$$

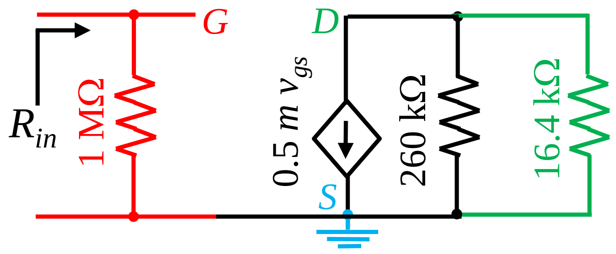
$$V_{DS} = 10 - 20\times 10^3 (I_D + I_1) \text{ —— } (2)$$

$$\therefore\ V_{DS} = 5V, V_{GS} = 2V, I_D = 250\ \mu A$$

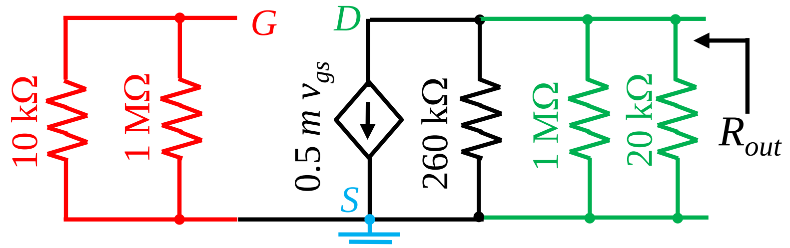
C-S Amplifier with Fully Bypass R_S: Example

Small Signal Equivalent Circuit:





$$R_{in} = 2M \| 2M = 1M\Omega$$



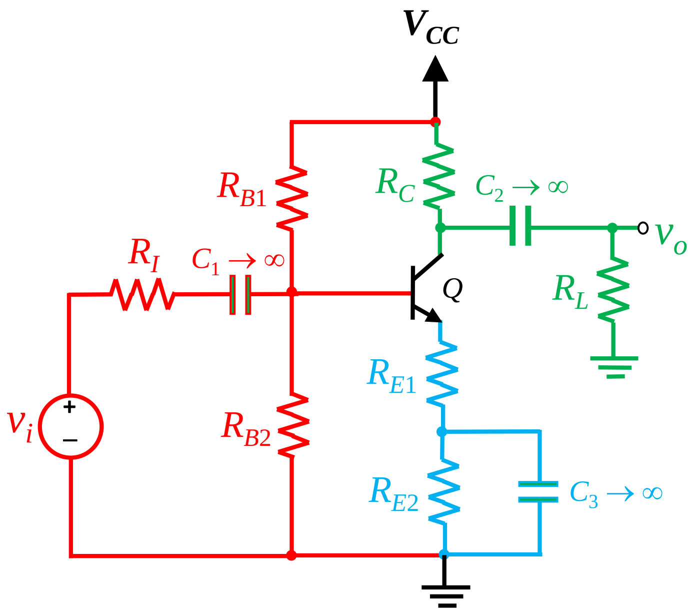
$$g_m \approx \sqrt{2 \times 500\ \mu \times 250\ \mu} = 0.5\ \text{mS}$$

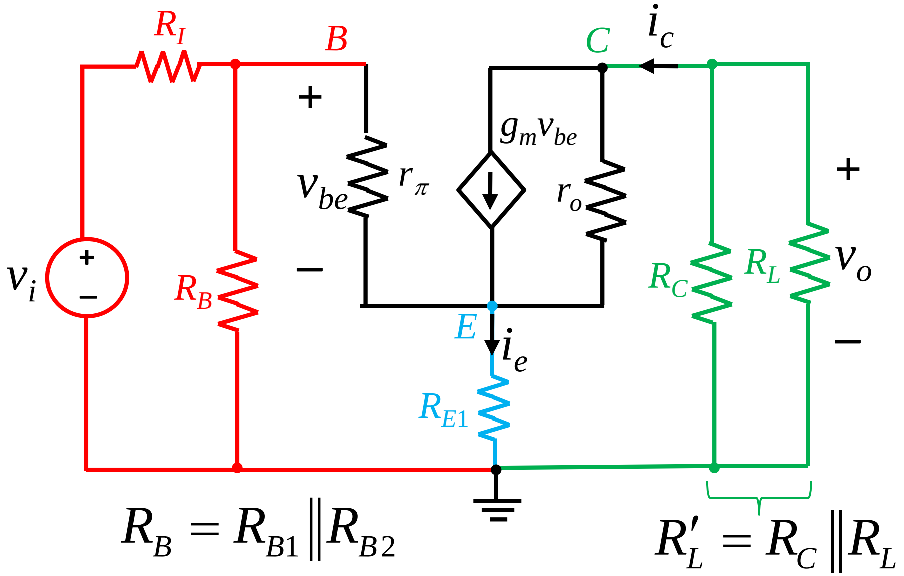
$$r_o = \frac{\frac{1}{0.0167} + 5}{250\ \mu} = 260\ \text{k}\Omega$$

$$R_{out} = 260k \| 1M \| 20k = 18.2k\Omega$$

$$A_v = A_{vt}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{-(0.5m)v_{gs}\left(260\ \text{k} \| 1\ \text{M} \| 20\ \text{k} \| 100\ \text{k}\right)}{v_{gs}}\left(\frac{1\ \text{M}}{10\ \text{k} + 1\ \text{M}}\right) = -7.93$$

C-E Amplifier with Unbypass R_E: Terminal Voltage Gain

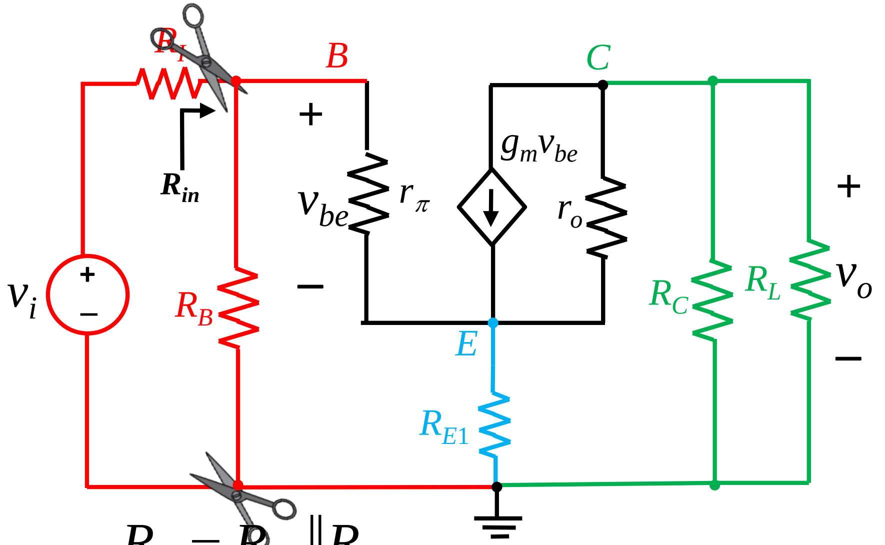




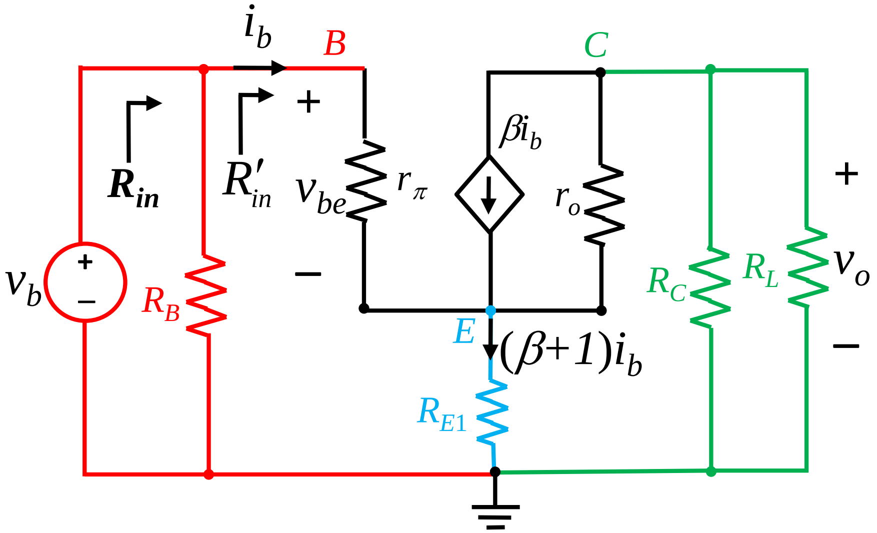
$$\because i_{r_o} \ll g_m v_{be}, i_c \approx g_m v_{be} \approx i_e$$

$$A_{vt} = \frac{v_c}{v_b} = \frac{-i_c R_L'}{v_{be} + i_e R_{E1}} \approx \frac{-g_m v_{be} R_L'}{v_{be} + g_m v_{be} R_{E1}} = \frac{-g_m R_L'}{1 + g_m R_{E1}}$$

C-E Amplifier with Unbypass R_E: Input Resistance



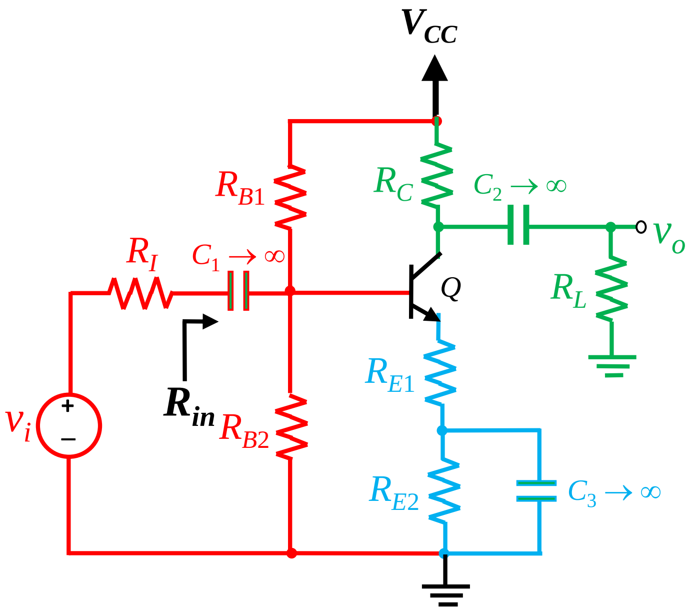
$R_B = R_{B1} \| R_{B2}$

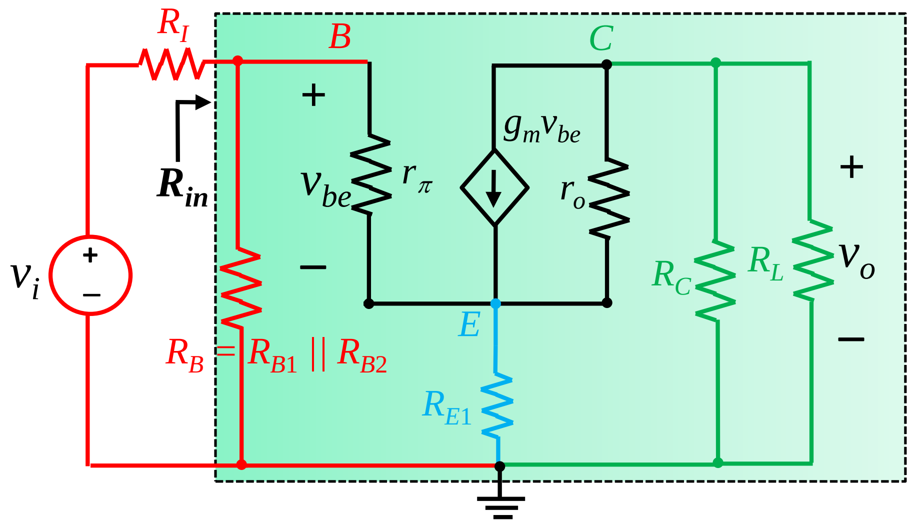


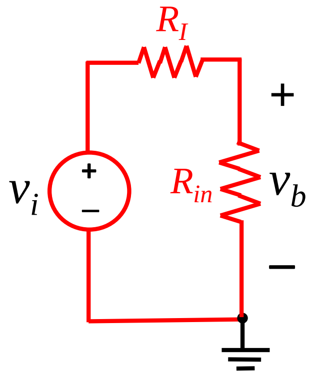
$$v_b = i_b r_\pi + (\beta+1) i_b R_{E1}$$

$$R'_{in} = \frac{v_b}{i_b} = r_\pi + (\beta+1) R_{E1} \Rightarrow R_{in} = R'_{in} \| R_B$$

C-E Amplifier with Unbypass R_E: Overall Voltage Gain

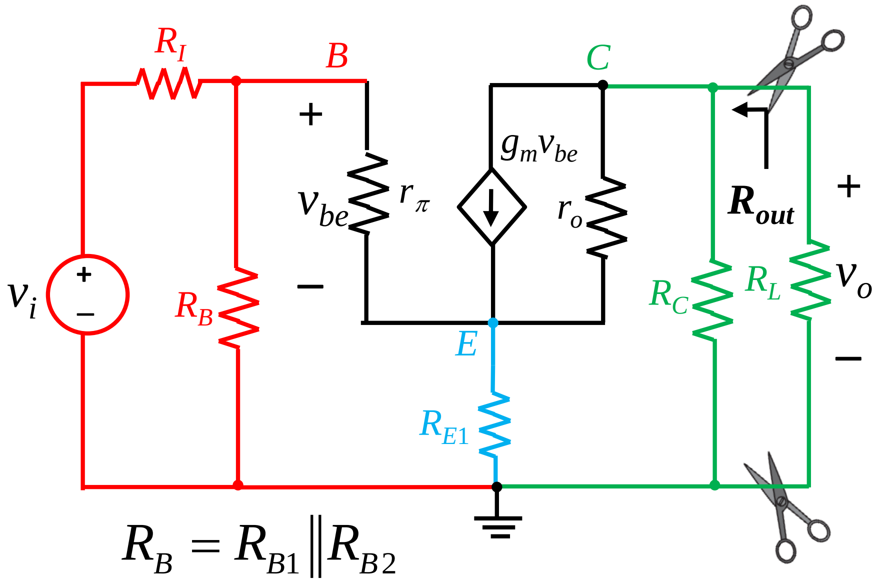




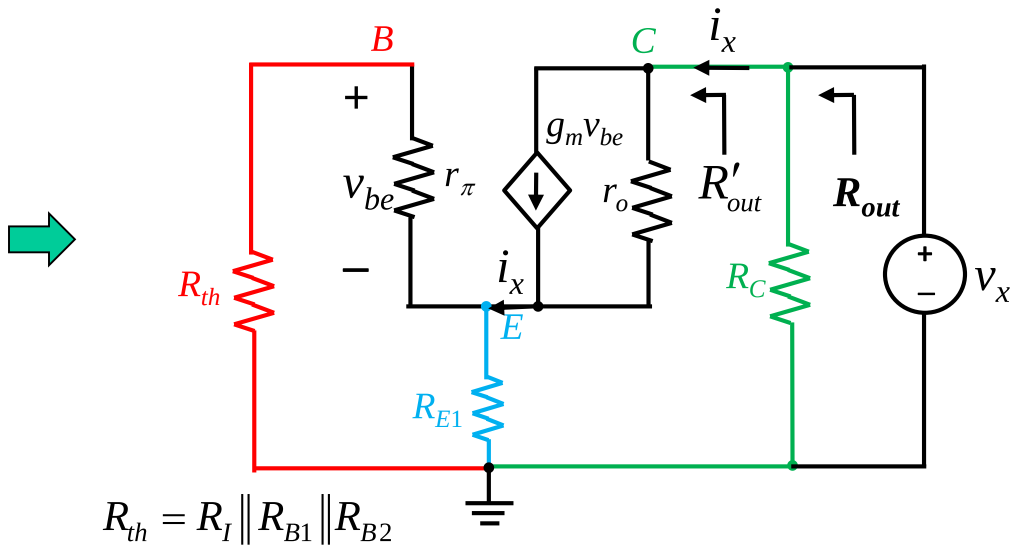


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-E Amplifier with Unbypass R_E: Output Resistance



$$R_B = R_{B1} \| R_{B2}$$



$$R_{th} = R_I \| R_{B1} \| R_{B2}$$

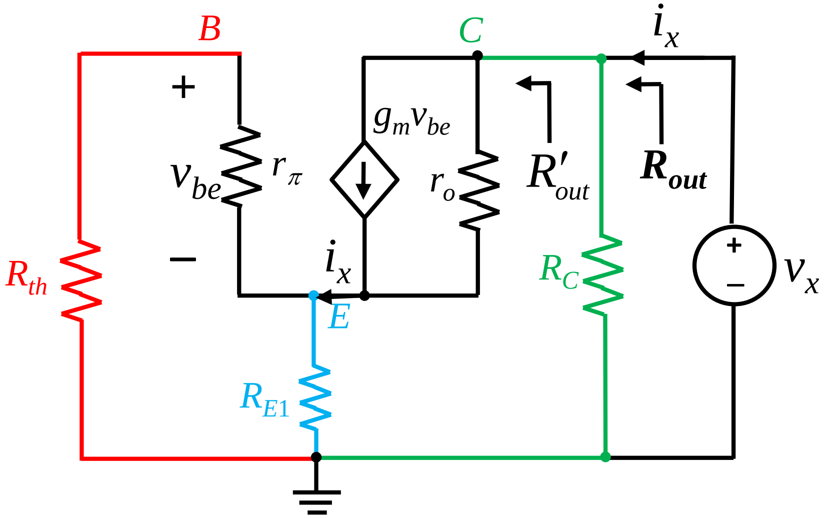
$$v_x = \underbrace{\left(i_x - g_m v_{be}\right)}_{\text{current through } r_o} r_o + v_e$$

$$v_e = i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\}$$

$$v_{be} = -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) v_e$$

$$= -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\}$$

C-E Amplifier with Unbypass R_E: Output Resistance



$$v_x = \left(i_x - g_m v_{be}\right) r_o + v_e$$

$$= \left(i_x + g_m \left(\frac{r_\pi}{r_\pi + R_{th}} \right) i_x \left\{ \left(r_\pi + R_{th} \right) \| R_{E1} \right\} \right) r_o$$

$$+ i_x \left\{ \left(r_\pi + R_{th} \right) \| R_{E1} \right\}$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + g_m \left(\frac{r_\pi}{r_\pi + R_{th}} \right) \left\{ \left(r_\pi + R_{th} \right) \| R_{E1} \right\} \right) r_o$$

$$R_{out} = R'_{out} \| R_C$$

C-S Amplifier with Unbypass R_S: Terminal Voltage Gain

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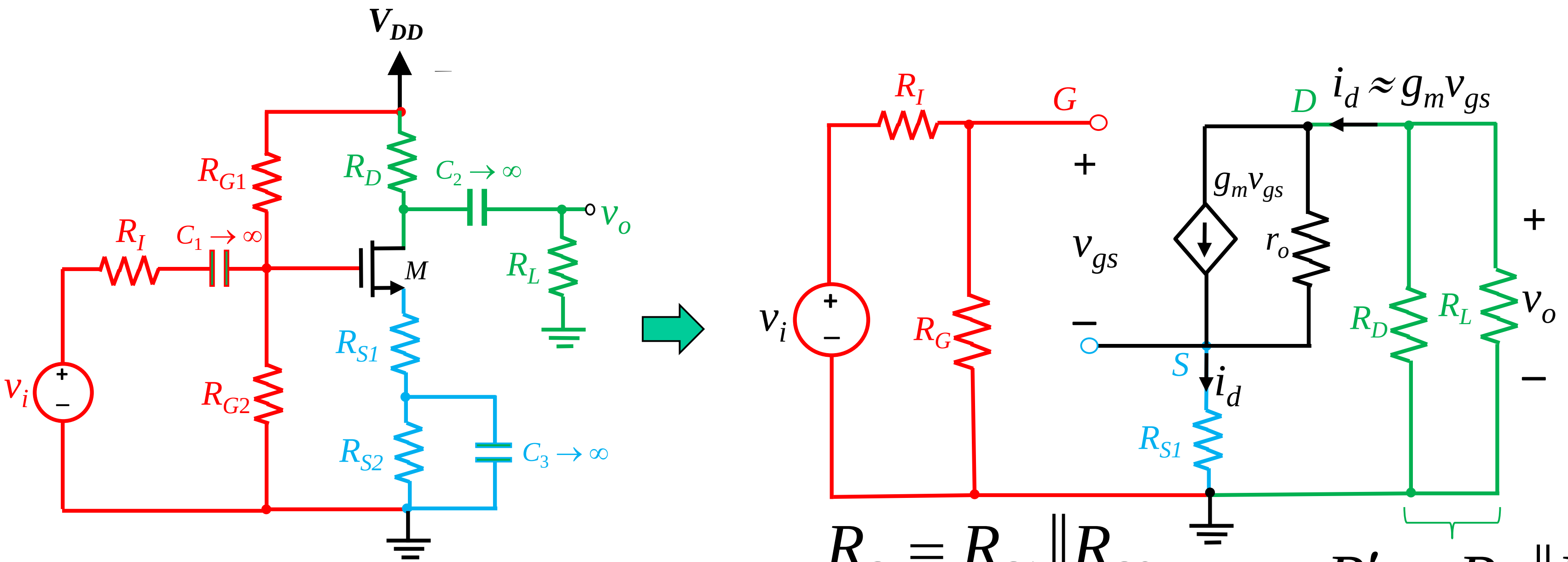


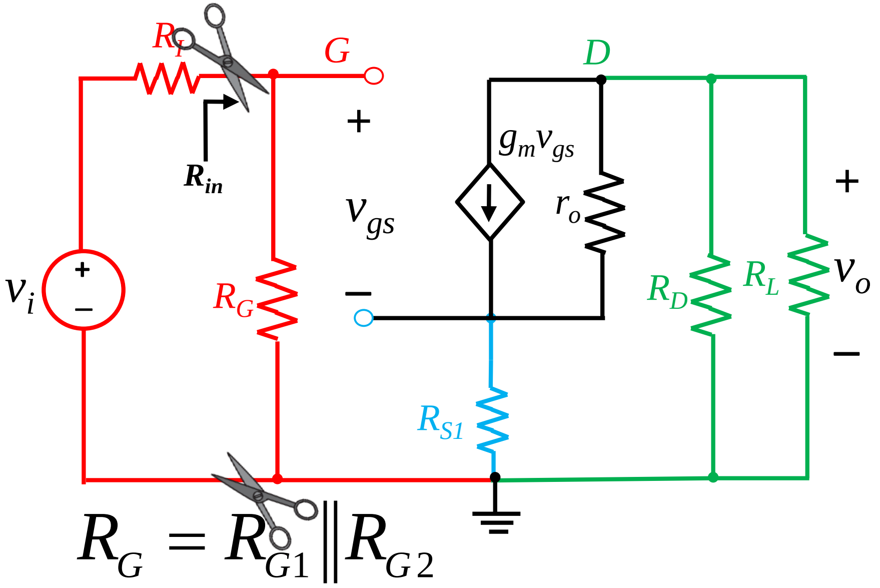
$$R_G = R_{G1} \| R_{G2}$$

$$R_L' = R_D \| R_L$$

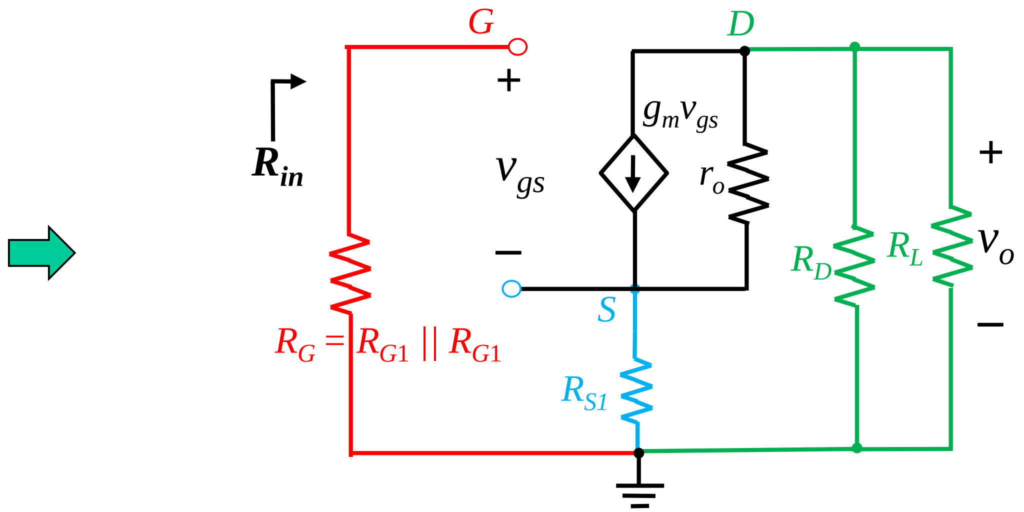
$$\because i_{r_o} \ll g_m v_{gs}, i_d \approx g_m v_{gs}$$

$$A_{vt} = \frac{v_d}{v_g} = \frac{-i_d R_L'}{v_{gs} + i_d R_{S1}} \approx \frac{-g_m v_{gs} R_L'}{v_{gs} + g_m v_{gs} R_{S1}} = \frac{-g_m R_L'}{1 + g_m R_{S1}}$$

C-S Amplifier with Unbypass R_S: Input Resistance

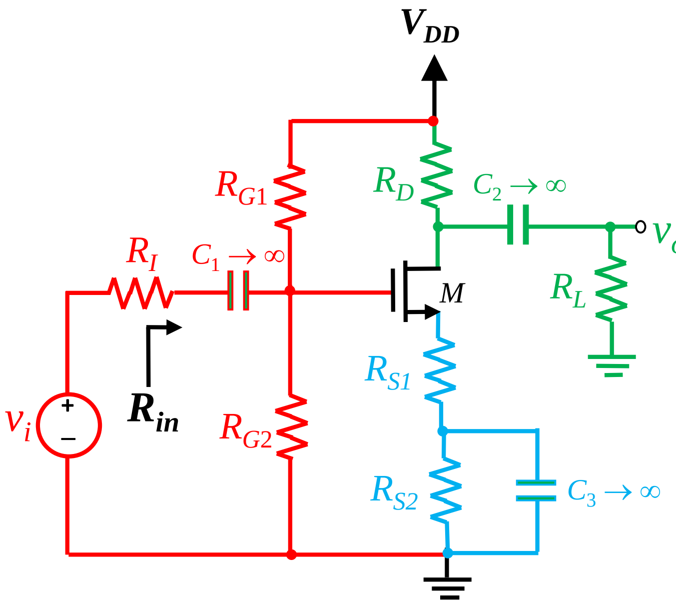


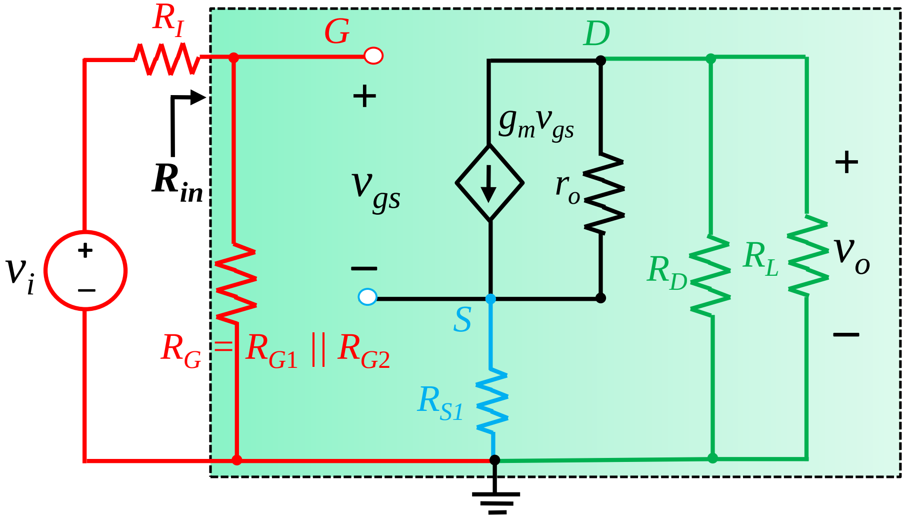
$$R_G = R_{G1} \| R_{G2}$$

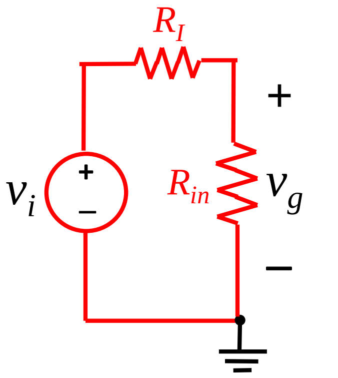


$$R_{in} = R_G$$

C-S Amplifier with Unbypass R_S: Overall Voltage Gain

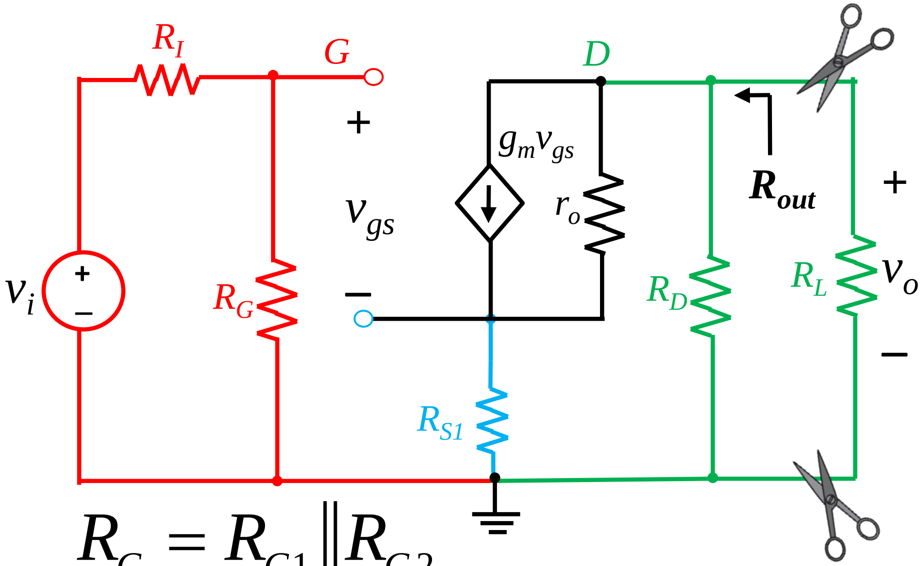




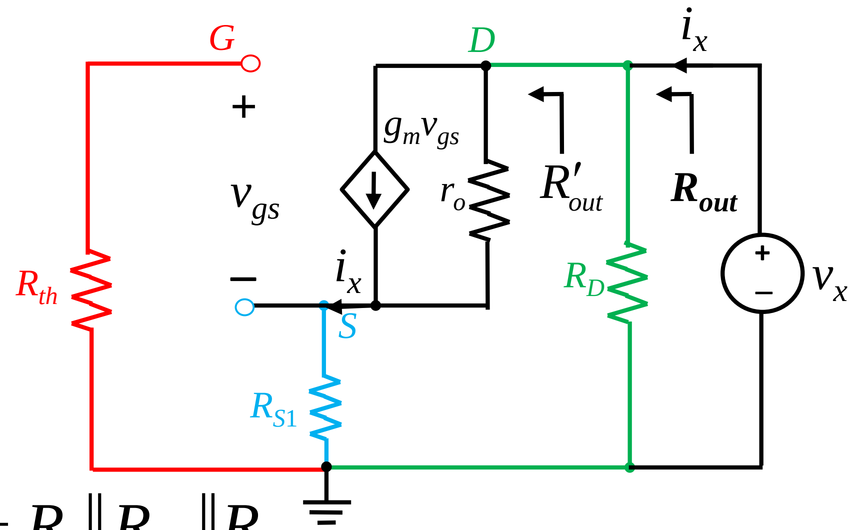


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-S Amplifier with Unbypass R_S: Output Resistance



$$R_G = R_{G1} \| R_{G2}$$



$$R_{th} = R_I \| R_{G1} \| R_{G2}$$

$$v_x = \underbrace{\left(i_x - g_m v_{gs}\right)}_{\text{current through } r_o} r_o + v_s$$

$$v_s = i_x R_{S1}$$

$$\because v_g = 0, v_{gs} = -v_s = -i_x R_{S1}$$

$$v_x = \left(i_x + g_m i_x R_{S1}\right) r_o + i_x R_{S1}$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + g_m R_{S1}\right) r_o$$

$$R_{out} = R'_{out} \| R_D$$