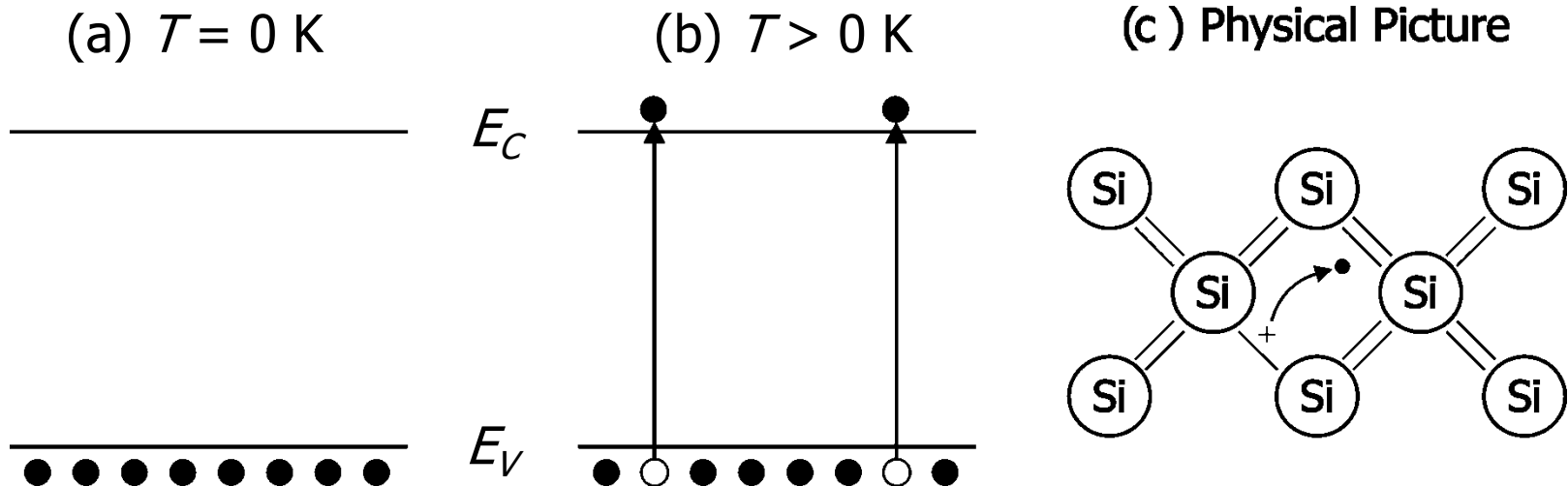




EE2003 Semiconductor Fundamentals

Carrier Generation/Recombination

Electron-Hole Pair Generation



Schematic illustration of band-to-band or direct electron-hole pair (EHP) generation. (a) At $T = 0 \text{ K}$, the conduction band is entirely empty and the valence band is fully filled with electrons. (b) When $T > 0 \text{ K}$, electrons gaining enough thermal energy are excited onto the conduction band, leaving behind positively charged holes in the valence band. (c) Physically, the generation of an electron-hole pair is a bond breaking process.



Electron-Hole Pair Generation

- **Generation** refers to the process whereby an electron-hole pair (EHP) is created.
 - Physically, this means a covalent bond between a Si atom and its neighbor is broken.
 - From an energy point of view, this refers to the excitation of an electron from the valence band to the conduction band.
- The generation process shown is known as ***band-to-band*** or ***direct generation***, i.e. the electron makes a direct transition from the valence band to the conduction band.
- Band-to-band generation always involves the creation of an **equal number** of electrons and holes.

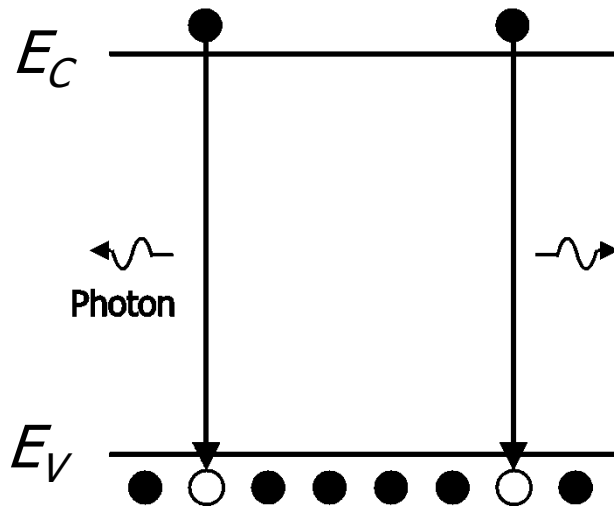


Electron-Hole Pair Generation

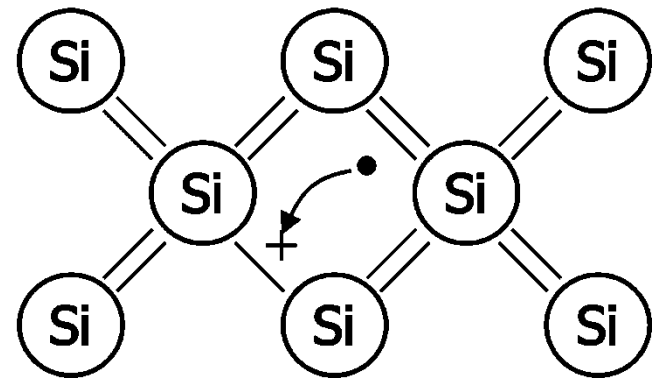
- Energy must be provided in order for generation to occur. Two means by which energy can be provided:
 - Changing the temperature – **Thermal generation**
 - Exposing the semiconductor to light – **Photo generation**
- In order for photo generation to occur, the photons must have energy at least equal or greater than the semiconductor band gap.
 - Recall: Photon energy, $E_{ph} = hf$, where h - Planck's constant; f ($= c/\lambda$) – frequency; c - speed of light; λ - wavelength

Electron-Hole Pair Recombination

(a) Energy Band Perspective



(b) Physical Picture



Schematic illustration of band-to-band or direct recombination. (a) Energy band perspective and (b) physical picture showing the reformation of a broken bond.



Electron-Hole Pair Recombination

- **Recombination** refers to the process whereby an electron-hole pair is annihilated.
 - Physically, this means the covalent bond between a Si atom and its neighbour is reformed.
 - From an energy point of view, it refers to the case where an electron in the conduction band falls back onto an empty electron state in the valence band.
- The recombination process shown is called **band-to-band** or **direct recombination**, i.e. the electron makes a direct transition from the conduction to the valence band.
- Band-to-band recombination always involves the annihilation or removal of an **equal number** of electrons and holes.



Electron-Hole Pair Recombination

- Energy is released during recombination.
- Depending on the type of semiconductor bandgap (direct/indirect), energy can be released in the form of electromagnetic radiation or heat.



Recombination Rate

- For band-to-band or direct recombination, one expects the rate at which electrons and holes recombine to be directly proportional to the electron (n) and hole (p) concentration in the respective conduction and valence band.
- A general expression for the **recombination rate R** may be expressed as

$$R = \alpha_r \cdot n \cdot p$$

– α_r (cm^3s^{-1}) is some constant of proportionality



Thermal Equilibrium

- Under thermal equilibrium (i.e. absence of an external excitation), the recombination rate equals the **thermal generation rate** G_{th} such that the electron and hole concentrations are constant and independent of time.

$$G_{th} = R = \alpha_r \cdot n \cdot p$$

$$G_{th} = \alpha_r \cdot n_0 \cdot p_0 = \alpha_r n_i^2$$

$$\sim \exp\left(-\frac{E_g}{k_B T}\right)$$

– Recall: $n_0 p_0 = n_i^2$ under thermal equilibrium

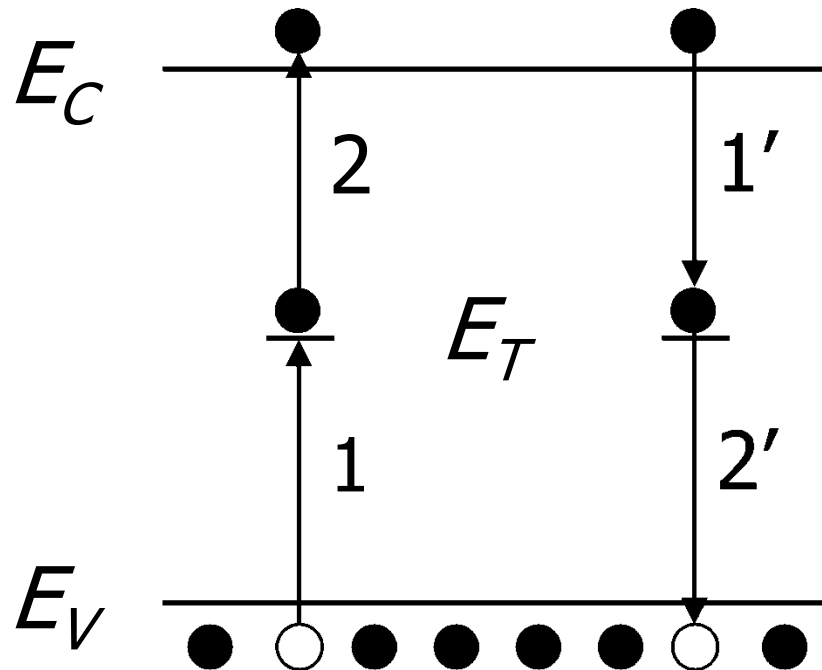


Non-Equilibrium

- When the **generation rate G** is **greater** than the **recombination rate R** (e.g. exposing the semiconductor to light), the concentrations of electrons and holes in the system increase with time, i.e. $\partial n/\partial t, \partial p/\partial t > 0$.
- On the other hand, when **$R > G_{th}$** (e.g. switching off the light after some time), the concentrations of electrons and holes in the system **decrease** with time, i.e. $\partial n/\partial t, \partial p/\partial t < 0$.

More on this later!

Indirect Processes



An energy band diagram illustrating the indirect transitions between the conduction and valence bands via an intermediate defect or trap level E_T .



Indirect Processes

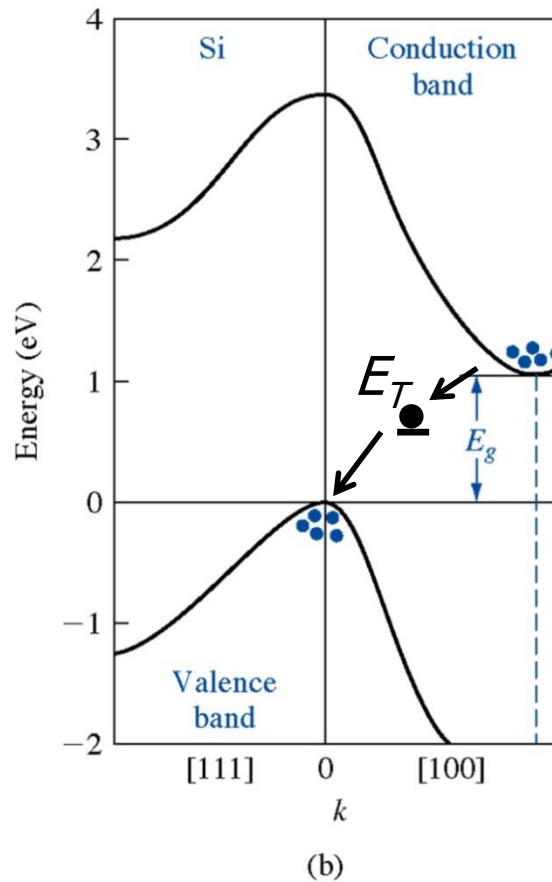
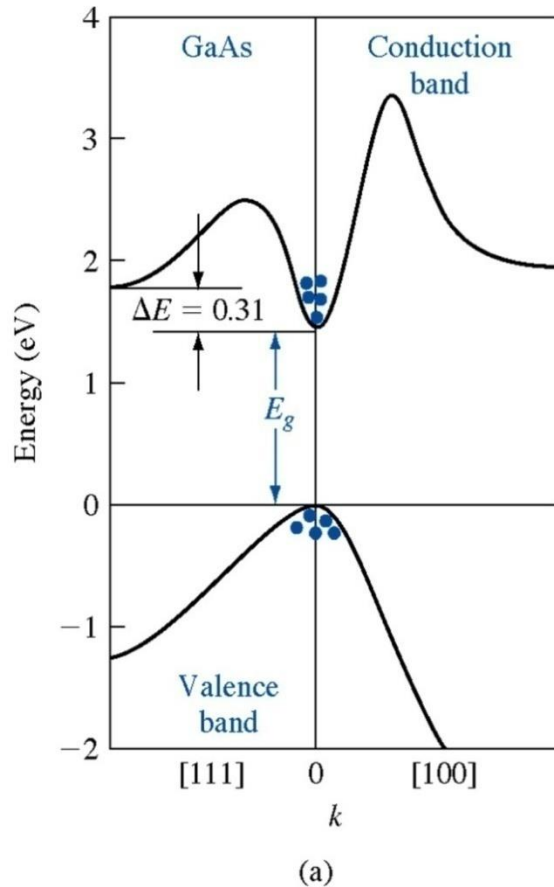
- When a valence/conduction band electron makes a transition to an **intermediate energy level E_T** within the bandgap, before proceeding to the conduction/valence band, the process is known as **indirect transition**.
- The energy level E_T is commonly known as the **generation-recombination (G–R) center**. It arises from crystallographic defects or impurity atoms present in a semiconductor crystal.
- Only energy levels situated near the **middle** of the band gap are **efficient** generation-recombination centers.



Indirect Processes

- The transition rate increases with the defect or trap (present in all practical semiconductors) density.
- Indirect transitions are the main mechanism for electron-hole pair recombination in an indirect bandgap semiconductor such as **silicon** (Si).

Direct/Indirect Bandgap Semiconductor



The requirement for momentum conservation makes direct transition between the conduction and valence bands **difficult** in an indirect bandgap semiconductor (e.g. Si).

In an indirect bandgap semiconductor, transitions take place via an intermediate defect state E_T , such as that shown in the diagram.

Energy band structure of (a) GaAs & (b) Si



Summary

- You should now be quite familiar with the following concepts:
 - Meanings of carrier generation and recombination in the context of semiconductors.
 - What do they mean physically and how to represent them in an energy band diagram.
 - Thermal equilibrium is a special case where the carrier generation rate equals the recombination rate, in the absence of an external excitation.
 - Differences between direct and indirect recombination.



EE2003 Semiconductor Fundamentals

Semiconductor in Non-Equilibrium



Non-Equilibrium

- When the concentration of electrons (n) and holes (p) in a semiconductor **deviate** from the thermal equilibrium values (n_0 and p_0 , respectively), the semiconductor is said to be in non-equilibrium.
- Such deviations occur when an external excitation is applied to the semiconductor.
- Ways by which an external excitation may be applied:
 - Applying an external voltage
 - Exposing the semiconductor to light
 - Changing the temperature (less common)
- When a semiconductor deviates from thermal equilibrium, the law of mass action ($n_0 p_0 = n_i^2$) **no longer** holds true, i.e. $np \neq n_i^2$.



Non-Equilibrium

- **Case 1 - Excess carriers: $np > n_i^2$**

- Also referred to as carrier injection. The corresponding changes in the carrier concentrations, defined as $\Delta n = n - n_0$ and $\Delta p = p - p_0$, are greater than zero.
- The recombination rate increases. i.e. $R (= \alpha_r np) > G_{th} (= \alpha_r n_0 p_0)$ and the semiconductor tries to restore thermal equilibrium by eliminating the excess carriers.

- **Case 2 - Deficit of carriers: $np < n_i^2$**

- Also referred to as carrier extraction. Δn and Δp are less than zero (i.e. negative).
- The recombination rate decrease, i.e. $R < G_{th}$ and the semiconductor tries to restore thermal equilibrium by generating more carriers.



Non-Equilibrium

- **Degree of deviation:**

- ***Low-level injection/extraction***: The change in the minority carrier concentration is **much smaller** compared to the majority carrier concentration at thermal equilibrium.
 - For example, in a p-type semiconductor, if $|\Delta n| \ll p_0$, then low-level injection/extraction is said to prevail.
 - Where to draw the line? Rule-of-thumb would be for the change in the minority carrier concentration **not to exceed 10%** of the majority carrier concentration at thermal equilibrium.
- ***High-level injection***: The change in minority carrier concentration is **comparable to or greater than** the majority carrier concentration at thermal equilibrium (usually not of interest as secondary effects dominate the characteristics of the semiconductor device).



Minority Carrier Lifetime

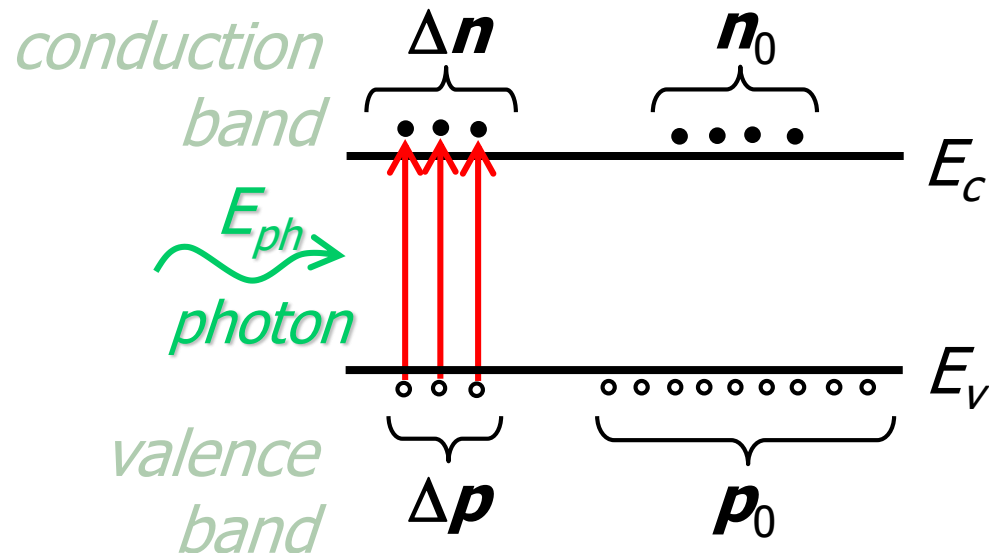
- Whenever there is a departure from thermal equilibrium, a “restoring force” would act to revert the semiconductor to thermal equilibrium.
- Under **low-level injection¹ condition**, the length of time it takes the semiconductor to revert to thermal equilibrium is determined by the **excess² minority carrier concentration** and **lifetime**.
 - The average time that excess minority carriers exist in the system before they recombine with the majority carriers is termed **recombination lifetime**.
 - The average time it takes for a minority carrier to be created is termed **generation lifetime**.

¹ The term “carrier injection” is inclusive of the case of carrier extraction.

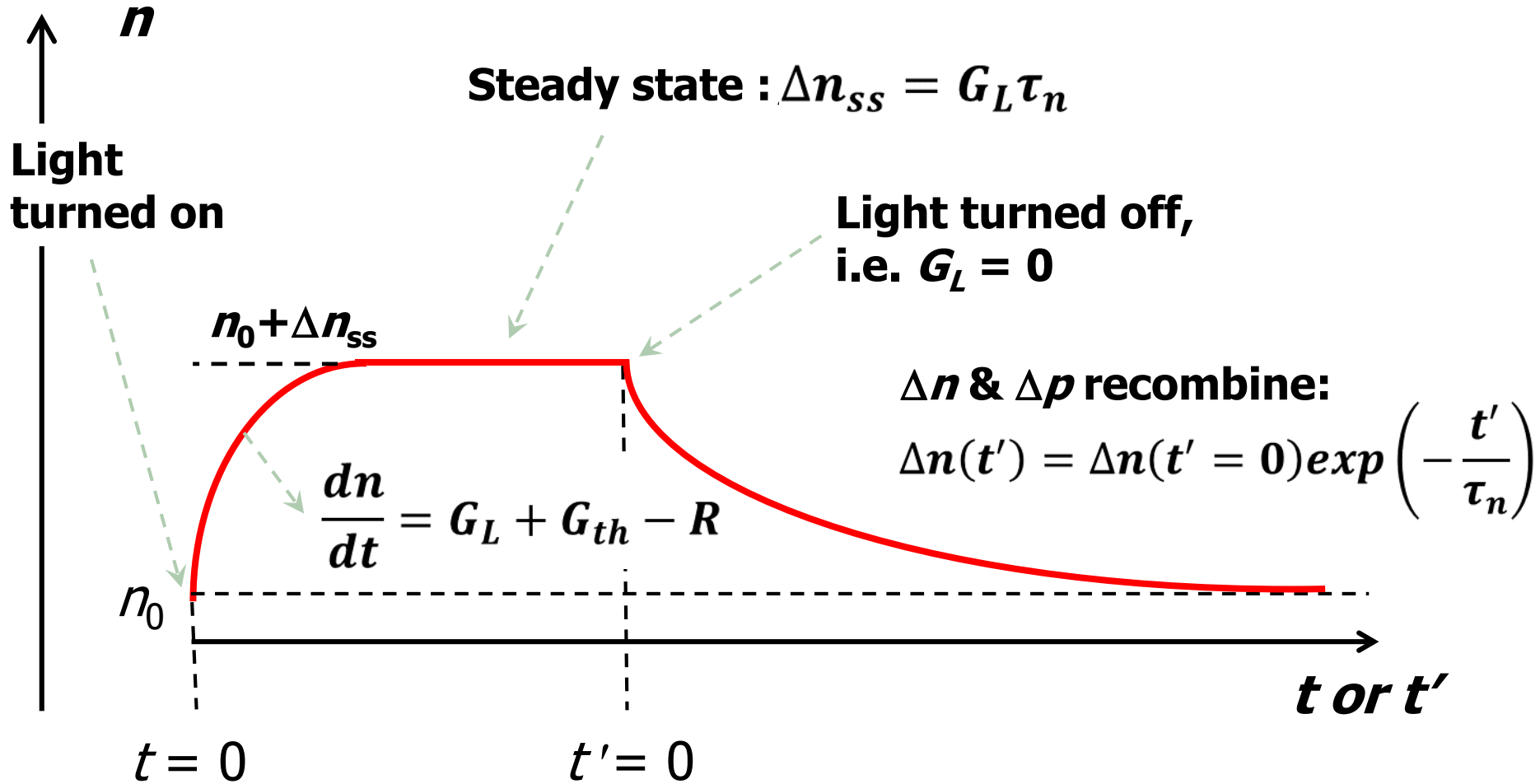
² A negative excess refers to a deficit of minority carriers.

Example: Optical Illumination

- Consider a p-type semiconductor (minority carrier is electron) exposed to light of a constant intensity at a given temperature. The light generates electron-hole pairs at a rate G_L ($\text{cm}^{-3}\text{s}^{-1}$).
- With photon energy $E_{ph} > E_{gr}$, additional electrons (Δn) are generated in the conduction band, in addition to those arising from thermal generation (n_0).



p-type semiconductor where: $p_0 \gg n_0$ & low injection: $\Delta n \ll p_0$





Example: Optical Illumination

- The new electron and hole concentration are, respectively, $n = n_0 + \Delta n$, $p = p_0 + \Delta p$. Obviously, the product np ($= n_0 p_0 + n_0 \Delta p + p_0 \Delta n + \Delta n \Delta p$) $> n_0 p_0$.
- The recombination rate R ($= \alpha_r np$) $> G_{th}$ ($= \alpha_r n_0 p_0 = \alpha_r n_i^2$) and the semiconductor tries to restore thermal equilibrium by getting rid of the excess carriers.
- The rate of change of the electron concentration may be expressed as the difference between the total generation rate ($G_L + G_{th}$) and the recombination rate R :

$$\frac{dn}{dt} = G_L + G_{th} - R$$



Low-Level Injection (Example)

- Consider a piece of p-type semiconductor (Si), of uniform doping $1 \times 10^{16} \text{ cm}^{-3}$. If the electron concentration changes to $1 \times 10^{14} \text{ cm}^{-3}$, is low-level injection condition valid? Given $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$.
 - The majority carrier concentration at thermal equilibrium, $p_0 \approx 1 \times 10^{16} \text{ cm}^{-3}$. Note: $n_0 = n_i^2/p_0 = 2.25 \times 10^4 \text{ cm}^{-3}$.
 - Since $\Delta n (= n - n_0 = 1 \times 10^{14} \text{ cm}^{-3})$ is much less than 10% of p_0 , low-level injection condition is valid in this case.
 - Under low level-injection, the minority carrier concentration can change by many orders of magnitude whereas the majority carrier concentration remains almost unchanged.
 - Note that if $\Delta n > 1 \times 10^{15} \text{ cm}^{-3}$ ($> 0.1 p_0$), then low-level injection would no longer be valid.



Example: Optical Illumination

$$\frac{dn}{dt} = G_L + G_{th} - R$$

- With a constant light intensity and temperature, $G_L + G_{th}$ is fixed and time invariant.
- In the initial stage, Δn (and Δp) are small and R is only marginally larger than G_{th} ; $(G_L + G_{th}) \gg R$ and thus n or Δn increases with time.
- With Δn (and Δp) and hence R increasing but $(G_L + G_{th})$ fixed, the increase of n or Δn will eventually slow down.

Recall: $R = \alpha_r np = \alpha_r (n_0 + \Delta n)(p_0 + \Delta p)$
 $= G_{th} + \alpha_r (n_0 \Delta p + p_0 \Delta n + \Delta n \Delta p)$



Example: Optical Illumination

- After some time, R will eventually become equal to $(G_L + G_{th})$. This **balance** will be maintained thereafter as any minute difference will be offset by either excess carrier generation or recombination.
- This special condition is called **steady state**. Under steady state, $dn/dt = 0$, i.e. n as well as p stop increasing and become a constant.
- Under steady state,

$$\begin{aligned} G_L &= -G_{th} + R = -\alpha_r(n_i^2 - np) \\ &= \alpha_r(n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \end{aligned}$$

- The term on the right hand side, i.e. $-G_{th} + R$ may be interpreted as the **net recombination rate**.

Example: Optical Illumination

- For a p-type semiconductor, $p_0 \gg n_0$. Assuming band-to-band transitions dominate, i.e. $\Delta n = \Delta p$, and since $\Delta n \ll p_0$ under low-level injection, the above eqn. simplifies to

$$\begin{array}{l} \boxed{\text{cm}^3\text{s}^{-1}} \\ \boxed{\text{cm}^{-3}} \end{array} \quad G_L = \alpha_r p_0 \Delta n_{ss} = \frac{\Delta n_{ss}}{\tau_n}; \quad \tau_n = \frac{1}{\alpha_r p_0}$$

\swarrow steady state
 \uparrow

where τ_n is the minority carrier (electron) lifetime.

- Note: **Under low-level injection, the net recombination rate is determined by the excess minority carrier concentration Δn and lifetime τ_n .**



Example: Optical Illumination

- Now consider the case when the light is turned off (i.e. $G_L = 0$) at an instant $t' = 0$, after steady state is reached.

$$\frac{dn}{dt'} = G_L - (-G_{th} + R) = -\frac{\Delta n}{\tau_n}$$

- This means that the electron concentration will start to decrease with time due to recombination of the excess carriers. Again, the recombination is determined by the excess minority carrier concentration Δn and lifetime τ_n .



Example: Optical Illumination

- Noting that $dn/dt = d(n_0 + \Delta n)/dt = d(\Delta n)/dt$, a solution to the above first-order differential equation is given as

$$\Delta n(t') = \Delta n(t' = 0) \exp\left(-\frac{t'}{\tau_n}\right)$$

where $\Delta n(t' = 0)$ is excess electron concentration at the instant when the light is turned off. This is equal to the steady state Δn or Δn_{ss} .

- **The excess electron concentration decays exponentially with time, characterized by a time constant τ_n . That is why τ_n is called the minority carrier lifetime.**



Example: Optical Illumination

- Similarly, for an n-type semiconductor with $n_0 \gg p_0$ and under low-level injection (i.e. $\Delta p \ll n_0$), we have

for steady state: $G_L = \alpha_r n_0 \Delta p_{ss} = \frac{\Delta p_{ss}}{\tau_p}; \quad \tau_p = \frac{1}{\alpha_r n_0}$

after turning off the light:

$$\Delta p(t') = \Delta p(t' = 0) \exp\left(-\frac{t'}{\tau_p}\right)$$

where τ_p is the minority carrier (hole) lifetime.



A Numerical Example

Consider a **p-type** semiconductor sample at 300 K doped to a concentration $N_a = 1 \times 10^{15} \text{ cm}^{-3}$. The intrinsic carrier concentration is $1.5 \times 10^{10} \text{ cm}^{-3}$. Assume that 1×10^{14} electron-hole pairs per cm^{-3} have been created and existed in the sample for $t < 0$. The minority carrier lifetime is 10 ns. With the external excitation removed for $t \geq 0$, calculate the **excess carrier concentration** for $t = 10 \text{ ns}$, 20 ns , 30 ns & ∞ .

Plot the carrier concentrations versus time showing the decay of the excess carrier concentration.

Comment on the percentage change of the minority & majority carrier concentration.



A Numerical Example

Assume complete ionization at 300 K and since $N_a \gg n_i$, $p_0 = N_a = 1 \times 10^{15} \text{ cm}^{-3}$.

The initial excess minority carrier concentration $\Delta n = 1 \times 10^{14} \text{ cm}^{-3} = 0.1 p_0$; low-level injection is valid.

After switching off the light at $t = 0$, the excess minority carrier concentration decays exponentially according to

$$\Delta n(t) = \Delta n(t = 0) \exp\left(-\frac{t}{\tau_n}\right)$$

with $\Delta n(t = 0) = 10^{14} \text{ cm}^{-3}$ and $\tau_n = 10 \text{ ns}$.



A Numerical Example

Assuming band-to-band recombination, $\Delta n(t) = \Delta p(t)$.

At $t = 10$ ns,

$$\Delta n(t = 10 \text{ ns}) = 10^{14} \times \exp\left(-\frac{10 \text{ ns}}{10 \text{ ns}}\right) = 3.7 \times 10^{13} \text{ cm}^{-3}$$

At $t = 20$ ns,

$$\Delta n(t = 20 \text{ ns}) = 10^{14} \times \exp\left(-\frac{20 \text{ ns}}{10 \text{ ns}}\right) = 1.35 \times 10^{13} \text{ cm}^{-3}$$

At $t = 30$ ns,

$$\Delta n(t = 30 \text{ ns}) = 10^{14} \times \exp\left(-\frac{30 \text{ ns}}{10 \text{ ns}}\right) = 5.0 \times 10^{12} \text{ cm}^{-3}$$

At $t = \infty$,

$$\Delta n(t = \infty) = 10^{14} \times \exp\left(-\frac{\infty}{10 \text{ ns}}\right) = 0$$

i.e. the semiconductor has completely revert to thermal equilibrium.

A Numerical Example

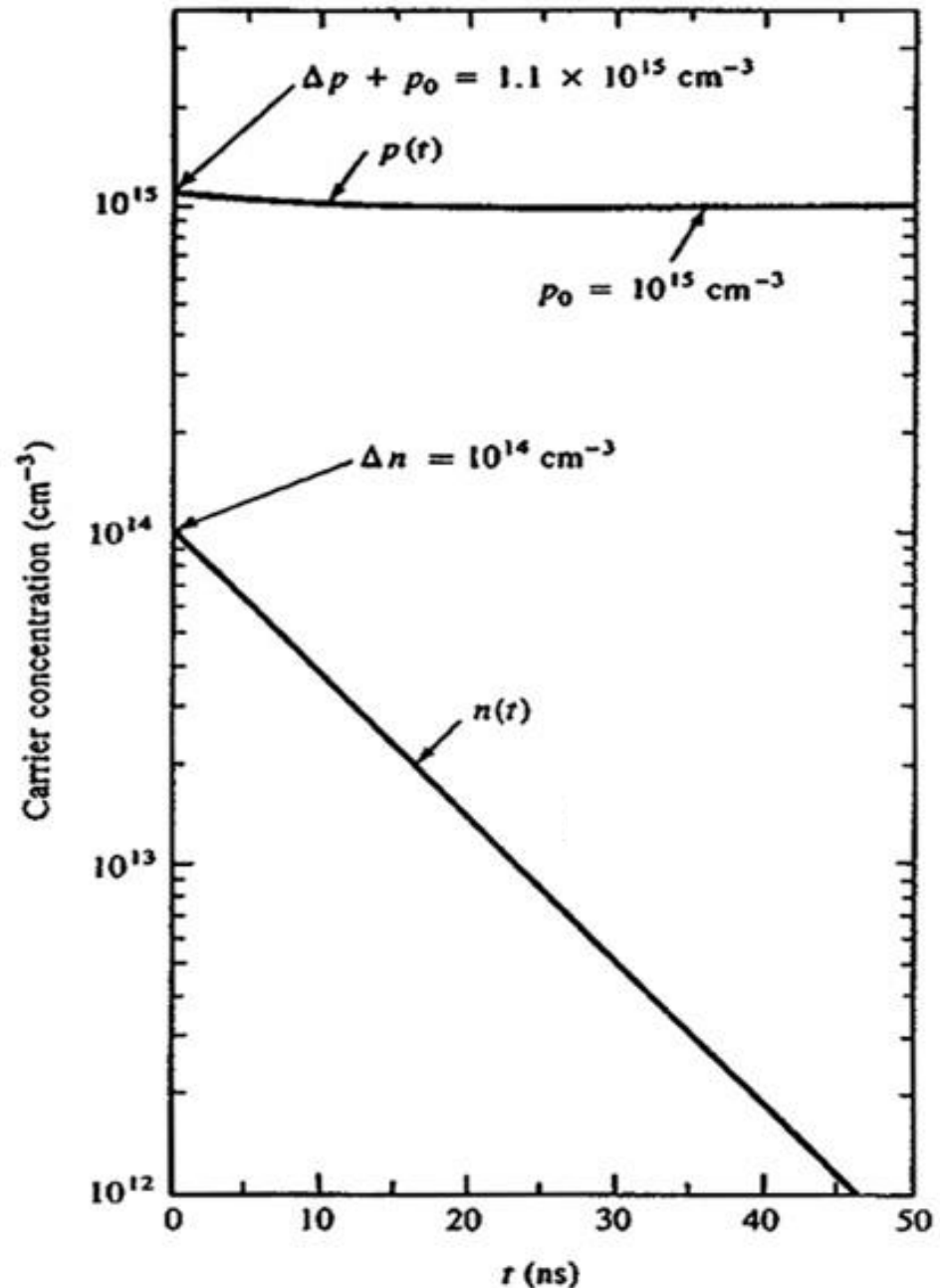
On a log-linear plot, the decay of Δn varies linearly with time.

$$\Delta n(t) = \Delta n(t = 0) \exp\left(-\frac{t}{\tau_n}\right)$$

$$\log_{10} \Delta n = \log_{10} \Delta n(0) - \frac{t}{2.3\tau_n}$$

Minority carrier lifetime τ_n can be determined from the slope of the log-linear plot.

Note that $p \approx p_0$ (due to low-level injection) whereas n is changed significantly ($n_0 \sim 10^5 \text{ cm}^{-3}$).





Summary

- Important concepts discussed in this section include:
 - Non-equilibrium
 - Low-level injection
 - Minority carrier lifetime

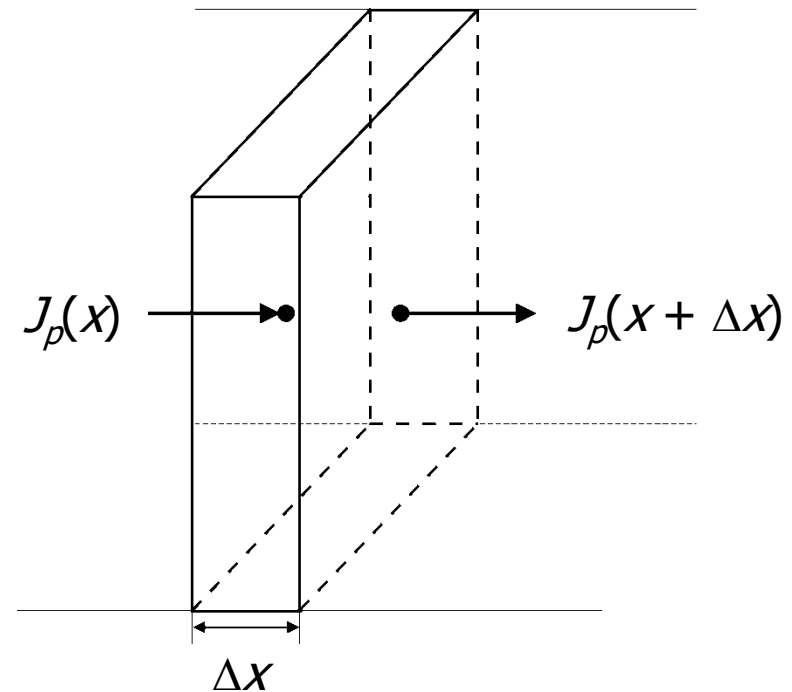


EE2003 Semiconductor Fundamentals

Continuity Equations

Continuity Equation

- The **continuity equation** is a mathematical expression for the **conservation of charges** at any point in a semiconductor.
- One dimensional analysis:
 - Consider an elemental volume of unit cross-sectional area and length Δx .
 - $J_p(x)$ and $J_p(x + \Delta x)$ are the hole current densities at the cross-sectional plane x and $x + \Delta x$ respectively.



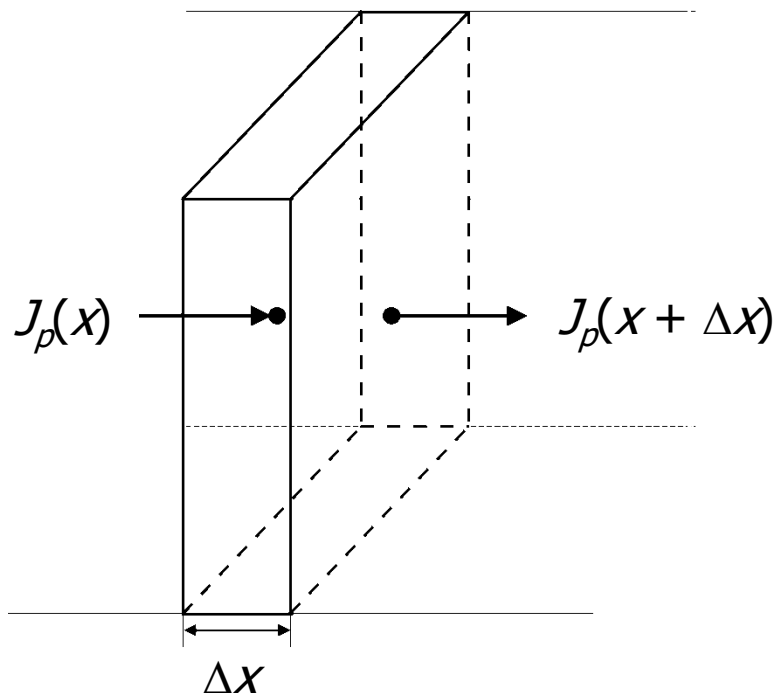
An elemental volume of a piece of semiconductor, showing the flow of holes across the two cross-sectional planes.



Continuity Equation

- The hole concentration within the elemental volume could be changed if:
 - $J_p(x)$ is not equal to $J_p(x+\Delta x)$, i.e. the number of holes flowing into the elemental volume is not equal to that flowing out of it.
 - There is **net** recombination inside the elemental volume.
 - Additional holes are produced by an external light source (i.e. generation due to an external excitation).

Continuity Equation



$$\begin{aligned} \text{No. of holes entering on the left} \\ = \frac{J_p(x) \cdot A}{q} \end{aligned}$$

$$\begin{aligned} \text{No. of holes leaving on the right} \\ = \frac{J_p(x + \Delta x) \cdot A}{q} \end{aligned}$$

Text

$$\begin{aligned} \text{Change in number of holes} \\ = \frac{J_p(x) \cdot A - J_p(x + \Delta x) \cdot A}{q} \end{aligned}$$

Recall the definition of first-order differential:

$$\frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Continuity Equation

- **Continuity equation for holes:**

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{1}{q} \left[\frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} \right] + G_L + G_{th} - R \\ &= -\frac{1}{q} \frac{\partial J_p(x)}{\partial x} + G_L - \frac{\Delta p}{\tau_p} \end{aligned}$$

Recall: Net recombination rate, $R - G_{th} = \Delta p / \tau_p$

- The difference in the hole flux entering and leaving the volume.
- The rate at which holes are generated by an external source.
- The net recombination rate within the volume.



Continuity Equation

- **Continuity equation for electrons:**

$$\begin{aligned}\frac{\partial n}{\partial t} &= \frac{1}{-q} \left[\frac{J_n(x) - J_n(x + \Delta x)}{\Delta x} \right] + G_L + G_{th} - R \\ &= \frac{1}{q} \frac{\partial J_n(x)}{\partial x} + G_L - \frac{\Delta n}{\tau_n}\end{aligned}$$

- Recall:

$$\begin{aligned}J_p(x) &= qp\mu_p\xi - qD_p \frac{\partial p}{\partial x} \\ J_n(x) &= qn\mu_n\xi + qD_n \frac{\partial n}{\partial x}\end{aligned}$$



Continuity Equation

- **Continuity equations:**

$$\frac{\partial p}{\partial t} = -\mu_p \left(p \frac{\partial \xi}{\partial x} + \xi \frac{\partial p}{\partial x} \right) + D_p \frac{\partial^2 p}{\partial x^2} + G_L - \frac{\Delta p}{\tau_p}$$

$$\frac{\partial n}{\partial t} = \mu_n \left(n \frac{\partial \xi}{\partial x} + \xi \frac{\partial n}{\partial x} \right) + D_n \frac{\partial^2 n}{\partial x^2} + G_L - \frac{\Delta n}{\tau_n}$$



Continuity Equation

- In general, the continuity equations are complex functions of space and time. Numerical methods are needed to solve them.
- However, certain simplifying assumptions, depending on the conditions of the experiment, may be applied to obtain analytical solutions.
 - No electric field ($\xi = 0$).
 - No external generation source, i.e. $G_L = 0$.
 - Steady state, i.e. $\partial p / \partial t, \partial n / \partial t = 0$.



Continuity Equation

- The above assumptions reduce the continuity equations to second-order differential equations:

- N-type semiconductor:
$$\frac{\partial^2 p_n}{\partial x^2} = \frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{D_p \tau_p}$$

- P-type semiconductor:
$$\frac{\partial^2 n_p}{\partial x^2} = \frac{\partial^2 \Delta n_p}{\partial x^2} = \frac{\Delta n_p}{D_n \tau_n}$$

- In uniformly doped semiconductors,

$$\frac{\partial p_n}{\partial x} = \frac{\partial}{\partial x} (p_{n0} + \Delta p_n) = \frac{\partial \Delta p_n}{\partial x}, \quad \frac{\partial n_p}{\partial x} = \frac{\partial}{\partial x} (n_{p0} + \Delta n_p) = \frac{\partial \Delta n_p}{\partial x}$$

(Note: Subscript 'n' or 'p' denotes the doping type)




Example 1

- **Steady-state** excess carrier concentration in a **uniformly doped** n-type semiconductor subjected to **uniform photo-generation**.
 - Incident light induces uniform electron-hole pair generation at a rate of G_L ($\text{cm}^{-3}\text{s}^{-1}$) \Rightarrow zero carrier-concentration gradient.
 - No electric field present.
 - Steady-state $\Rightarrow \partial p / \partial t, \partial n / \partial t = 0$.
 - Continuity equation for holes (assuming low-level injection):

$$G_L - \frac{\Delta p_n}{\tau_p} = 0$$

- Steady-state excess hole concentration:

$$\Delta p_{n,ss} = G_L \tau_p$$



$$\frac{\partial p_n}{\partial t} = -\mu_p \left(p_n \frac{\partial \xi}{\partial x} + \xi \frac{\partial p_n}{\partial x} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

Example 1

- **Steady-state** excess carrier concentration in a **uniformly** doped n-type semiconductor subjected to **uniform photo-generation**.
 - Incident light induces uniform electron-hole pair generation at a rate of G_L ($\text{cm}^{-3}\text{s}^{-1}$) \Rightarrow zero carrier-concentration gradient.
 - No electric field present.
 - Steady-state $\Rightarrow \partial p / \partial t, \partial n / \partial t = 0$.
 - Continuity equation for holes (assuming low-level injection):

$$G_L - \frac{\Delta p_n}{\tau_p} = 0$$

- Steady-state excess hole concentration:

$$\Delta p_{n,ss} = G_L \tau_p$$

$$\frac{\partial p_n}{\partial t} = -\mu_p \left(p_n \frac{\partial \xi}{\partial x} + \xi \frac{\partial p_n}{\partial x} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

Example 1


$$\frac{\partial p_{n0}}{\partial x} = 0; \frac{\partial \Delta p_n}{\partial x} = 0 \Rightarrow \frac{\partial p_n}{\partial x} = 0$$

- **Steady-state** excess carrier concentration in a **uniformly doped** n-type semiconductor subjected to **uniform photo-generation**.
 - Incident light induces uniform electron-hole pair generation at a rate of G_L ($\text{cm}^{-3}\text{s}^{-1}$) \Rightarrow zero carrier-concentration gradient.
 - No electric field present.
 - Steady-state $\Rightarrow \partial p / \partial t, \partial n / \partial t = 0$.
 - Continuity equation for holes (assuming low-level injection):

$$G_L - \frac{\Delta p_n}{\tau_p} = 0$$

- Steady-state excess hole concentration:

$$\Delta p_{n,ss} = G_L \tau_p$$



$$\frac{\partial p_n}{\partial t} = -\mu_p \left(p_n \frac{\partial \xi}{\partial x} + \xi \frac{\partial p_n}{\partial x} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

Example 1

- **Steady-state** excess carrier concentration in a **uniformly doped** n-type semiconductor subjected to **uniform photo-generation**.
 - Incident light induces uniform electron-hole pair generation at a rate of G_L ($\text{cm}^{-3}\text{s}^{-1}$) \Rightarrow zero carrier-concentration gradient.
 - **No electric field present.**
 - Steady-state $\Rightarrow \partial p / \partial t, \partial n / \partial t = 0$.
 - Continuity equation for holes (assuming low-level injection):

$$G_L - \frac{\Delta p_n}{\tau_p} = 0$$

- Steady-state excess hole concentration:

$$\Delta p_{n,ss} = G_L \tau_p$$



Example 2


- Continue from Example 1: Decay of photo-generated carriers in an n-type semiconductor with time.
 - Assume steady-state has been reached before the **light source is removed at $t = 0$ s.**
 - Note that $G_L = 0$ for $t \geq 0$ s. The continuity equation for holes (assuming low-level injection) becomes

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(t) = C e^{-t/\tau_p}$$

- The initial condition: $\Delta p_n(t=0) = G_L \tau_p$ (from Example 1) enables the integration constant C to be determined.

$$\Delta p_n(t) = G_L \tau_p e^{-t/\tau_p}$$



$$\frac{\partial p_n}{\partial t} = -\mu_p \left(p_n \frac{\partial \xi}{\partial x} + \xi \frac{\partial p_n}{\partial x} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

0
0
0
0

Example 2

- Continue from Example 1: Decay of photo-generated carriers in an n-type semiconductor with time.
 - Assume steady-state has been reached before the **light source is removed at $t = 0$ s**.
 - Note that $G_L = 0$ for $t \geq 0$ s. The continuity equation for holes (assuming low-level injection) becomes

$$\frac{\partial \Delta p_n}{\partial t} = - \frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(t) = C e^{-t/\tau_p}$$

- The initial condition: $\Delta p_n(t=0) = G_L \tau_p$ (from Example 1) enables the integration constant C to be determined.

$$\Delta p_n(t) = G_L \tau_p e^{-t/\tau_p}$$



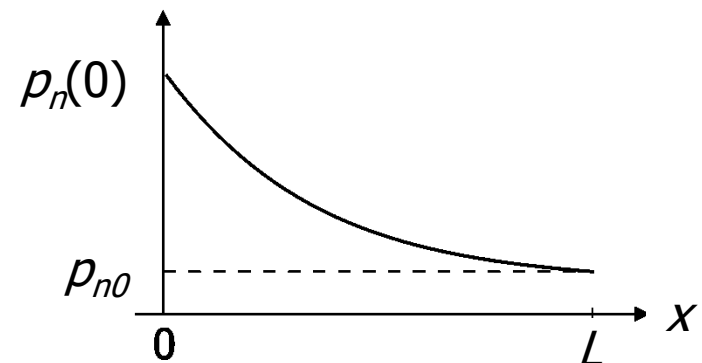
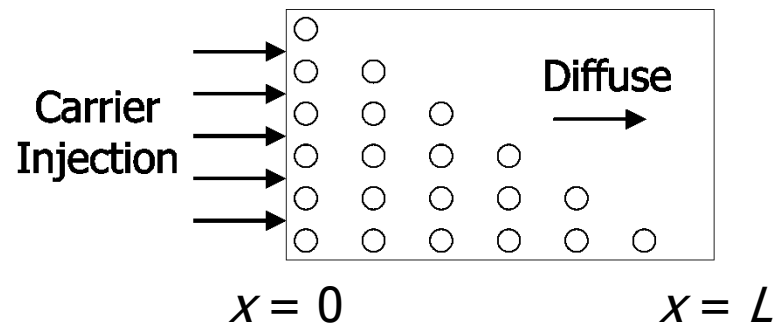
Example 2

- Recall that the conductivity or conductance of the sample is proportional to the carrier concentrations. If the decay in conductance can be monitored as a function of time (in the form of a current decay), the minority carrier lifetime is easily determined.

Example 3

- **Steady-state** carrier concentration distribution with carrier injection happening at one end of a **uniformly doped** n-type semiconductor sample.

- Assumptions:
 - Hole generation occurs only at the plane $x = 0$.
 - Thermal equilibrium prevails at the other end of the sample.
 - The electric field is very small and may be neglected, $\xi = 0$.

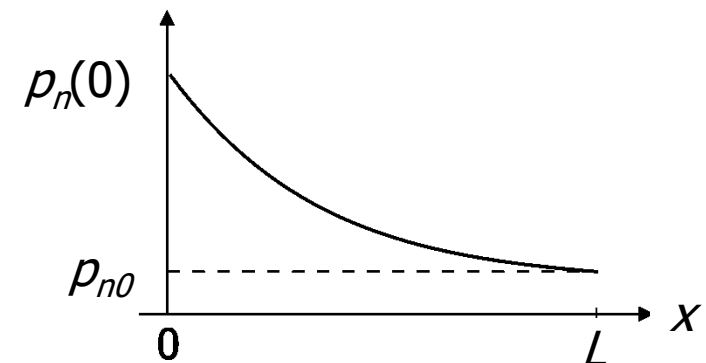
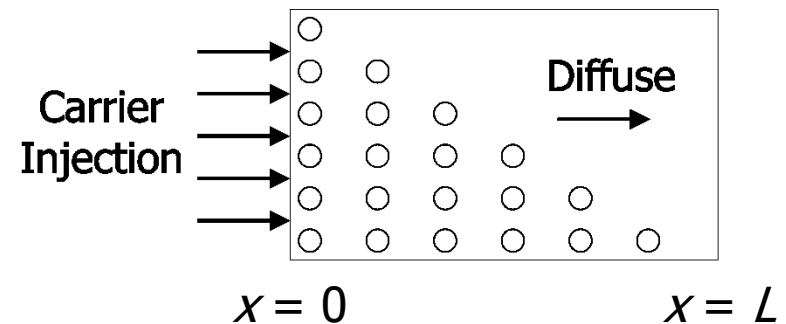


$$\frac{\partial p_n}{\partial t} = -\mu_p \left(p_n \frac{\partial \xi}{\partial x} + \xi \frac{\partial p_n}{\partial x} \right) + D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{\Delta p_n}{\tau_p}$$

Example 3

- **Steady-state** carrier concentration distribution with carrier injection happening at one end of a **uniformly doped** n-type semiconductor sample.

- Assumptions:
 - Hole generation occurs only at the plane $x = 0$.
 - Thermal equilibrium prevails at the other end of the sample.
 - The electric field is very small and may be neglected, $\xi = 0$.





Example 3

Recall:

$$D_p \rightarrow \text{cm}^2 \text{s}^{-1}$$

$$\tau_p \rightarrow \text{s}$$

$$L_p = (D_p \tau_p)^{1/2} \rightarrow \text{cm}$$

- Under steady state:

$$D_p \frac{\partial^2 p_n}{\partial x^2} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{\tau_p}$$

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{L_p^2}, L_p = (D_p \tau_p)^{1/2}$$

- This is a second-order differential equation, whose solution is of the form:

$$\Delta p_n(x) = C_1 e^{-x/L_p} + C_2 e^{x/L_p}$$



Example 3

- Two **boundary conditions** are needed to determine integration constants C_1 and C_2 .
 - At $x = 0$, $\Delta p_n(x = 0) = \Delta p_n(0)$.
 - At $x = L$, $\Delta p_n(x = L) = 0$.
- Assuming a long-base sample, i.e. $L \rightarrow \infty$,
$$C_1 = \Delta p_n(0), C_2 = 0$$
- The lateral hole distribution is an exponentially decaying function:
$$\Delta p_n(x) = \Delta p_n(0)e^{-x/L_p}$$
- L_p is the **minority carrier diffusion length**. At $x = L_p$, 1/e of the initial amount remains. Physically, L_p refers to the average distance a hole (minority carrier) diffuses before recombining.