NANYANG TECHNOLOGICAL UNIVERSITY
SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
ACADEMIC YEAR 2022-2023
SEMESTER 1

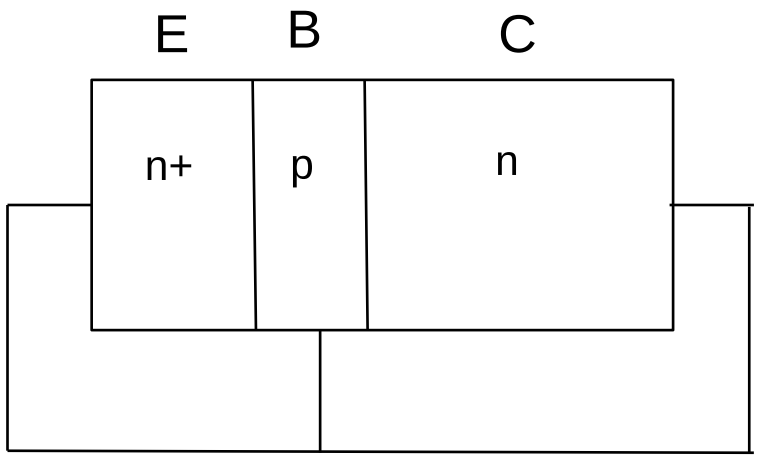
EE3013 SEMINCONDUCTOR DEVICES AND PROCESSING

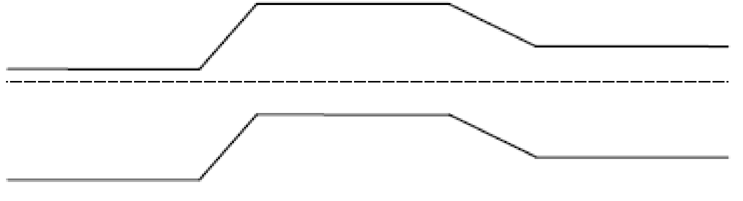
Bipolar Junction Transistors (BJTs)

Instructor: Prof Ng Geok Ing

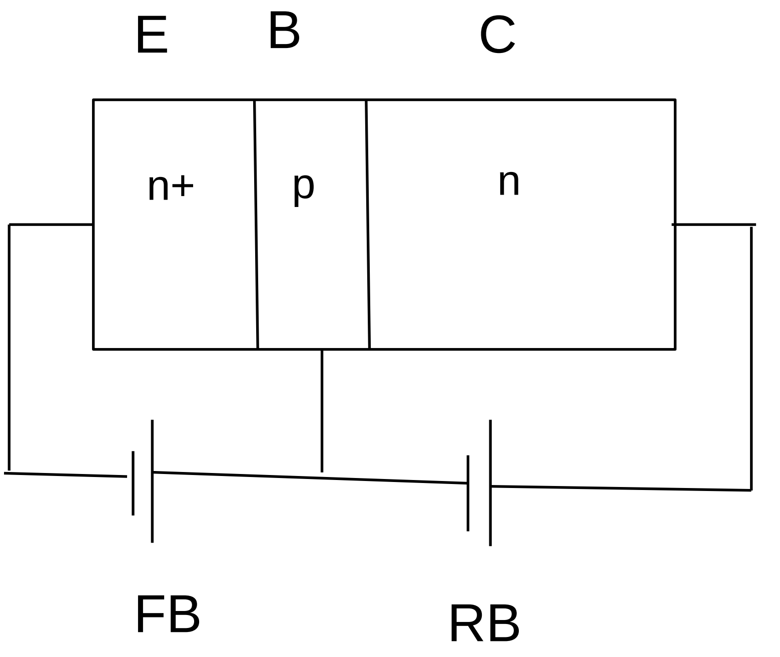
1. Draw the energy band diagram for an n^+pn transistor for the following cases: (a) thermal equilibrium, (b) active mode, (c) cut off mode, (d) saturation mode.

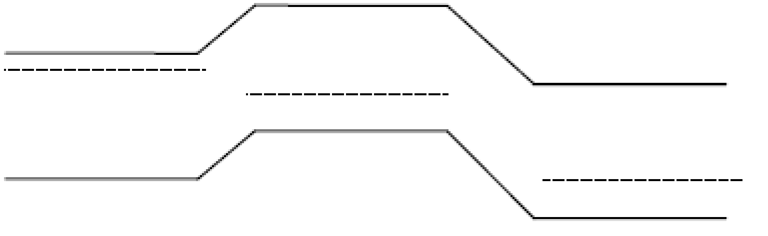
(a) Zero Bias **(V_{EB}=0 and V_{CB}=0)**





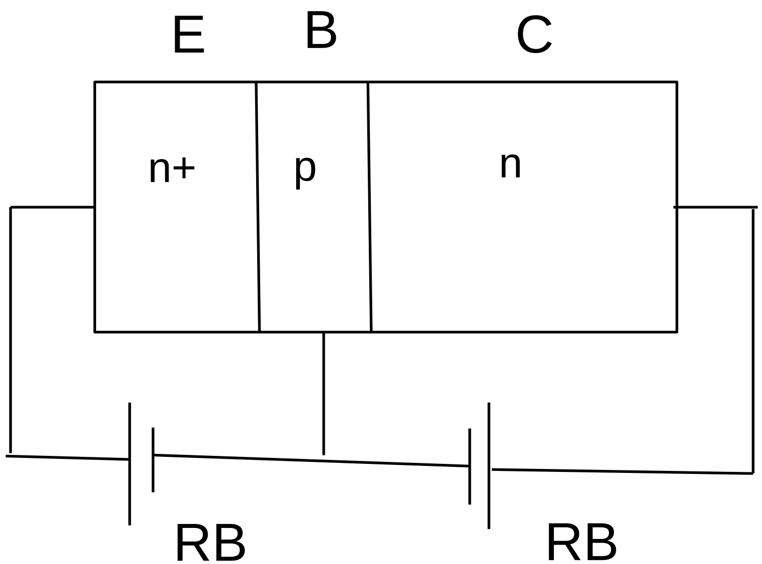
(b) Active Mode **(V_{EB}<0 and V_{CB}>0)**

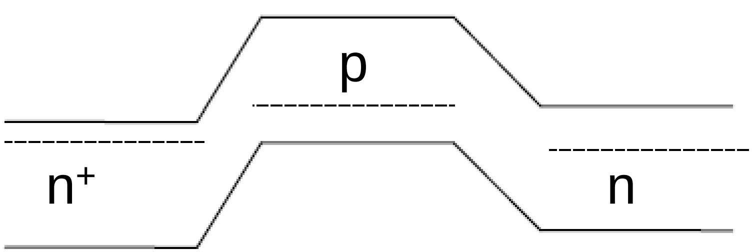




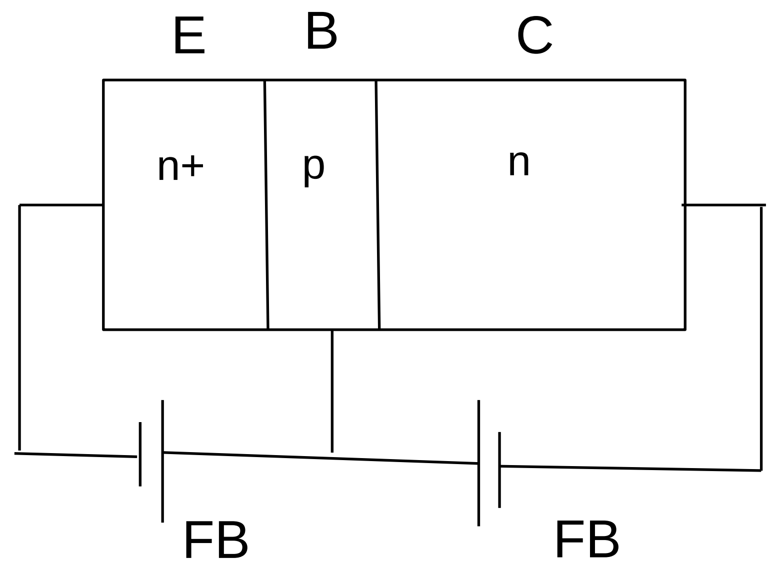
1. Draw the energy band diagram for an $n^{+}pn$ transistor for the following cases: (a) thermal equilibrium, (b) active mode, (c) cut off mode, (d) saturation mode.

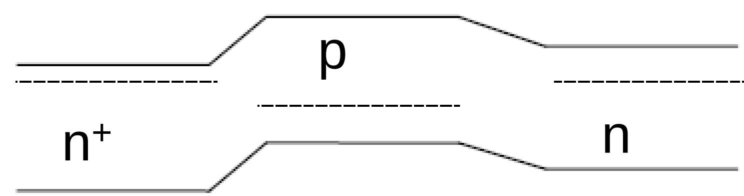
(c) Cutoff Mode **($V_{EB}>0$ and $V_{CB}>0$)**





(d) Saturation Mode **($V_{EB}<0$ and $V_{CB}<0$)**





2. A p-*n*-*p* transistor has a base transport factor α_T of 0.998, and emitter efficiency of 0.997, and an I_{Cn} of 10 nA.

(a) Calculate α_0 and β_0 for the device.

(b) If I_E = 1 mA, what is the total collector current?

2 (*a*) The common-base and common-emitter current gains is given by

$$\alpha_0 = \gamma\alpha_T = 0.997 \times 0.998 = 0.995$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = \frac{0.995}{1-0.995}$$

$$= 199 \; .$$

(*b*) I_{Cn} refers to the leakage current at the collector for the p-n-p transistor

$$I_C = \alpha_o I_E + I_{Cn}$$

Since $I_E = 1$ mA and $I_{Cn} = 10\times10^{-9}$ A, then I_C is 0.995 mA + 10×10^{-9} A. $= 0.99501$ mA

3. A silicon p-*n*-*p* transistor has impurity concentrations of 5×10^{18}, 2×10^{17}, and 10^{16} cm^{-3} in the emitter, base, and collector, respectively. The base width is 1.0μm, and the device cross-sectional area is 0.2 mm^2. When the emitter-base junction is forward biased to 0.5 V and the base-collector junction is reverse biased to 5 V, calculate:

(a) the neutral base width

(b) the minority carrier concentration at the emitter-base junction.

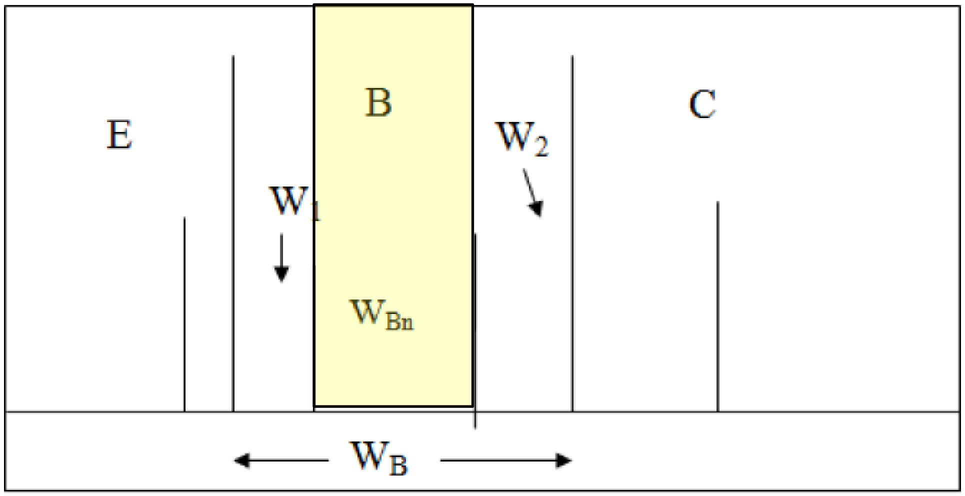
(c) if the diffusion constants of minority carriers in the emitter, base, and collector are 52, 40, and 115cm^2/s, respectively; and the corresponding lifetimes are 10^{-8}, 10^{-7}, 10^{-6} s. find the current components I_{Ep}, I_{Cp}, I_{En}, I_{Cn}, and I_{BB} .

(d) hence find the terminal currents I_E, I_C, and I_B of the transistor and calculate emitter efficiency, base transport factor, common-base current gain, and common-emitter current gain.

(e) comment on how the emitter efficiency and base transport factor can be improved.

Solution:

(a) W_{Bn} = neutral base width



The emitter-base junction is forward biased.

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= 0.0259\ln\left[\frac{5\times10^{18}\cdot2\times10^{17}}{\left(9.65\times10^{9}\right)^2}\right] = 0.956\text{ V}$$

$$x_n = \frac{N_A W}{N_A + N_D}, x_p = \frac{N_D W}{N_A + N_D} \qquad W = \sqrt{\frac{2\varepsilon_s}{q}\left(V_{bi} - V\right)\left(\frac{N_A + N_D}{N_D N_A}\right)}$$

The depletion-layer width in the base is

$$W_1 = \left(\frac{N_A}{N_A + N_D}\right) \text{(Total depletion width of E - B junction)}$$

$$= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A + N_D}\right)\left(V_{bi} - V\right)}$$

$$= \sqrt{\frac{2 \cdot 1.05 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{5 \times 10^{18}}{2 \times 10^{17}}\right)\left(\frac{1}{5 \times 10^{18} + 2 \times 10^{17}}\right)(0.956 - 0.5)}$$

$$= 5.364 \times 10^{-6} \text{ cm} = 5.364 \times 10^{-2} \ \mu\text{m} \ .$$

Similarly we obtain for the base-collector junction

$$V_{bi} = 0.0259 \ln\left[\frac{2\times10^{17}\cdot10^{16}}{\left(9.65\times10^{9}\right)^{2}}\right] = 0.795\ \text{V} \ .$$

$$W_2 = \sqrt{\frac{2\cdot1.05\times10^{-12}}{1.6\times10^{-19}}\left(\frac{10^{16}}{2\times10^{17}}\right)\left(\frac{1}{10^{16}+2\times10^{17}}\right)(0.795+5)}$$

$$= 4.254\times10^{-6}\ \text{cm} = 4.254\times10^{-2}\ \mu\text{m} \ .$$

Therefore the neutral base width is

$$W_{Bn} = W_{\text{B}} - W_1 - W_2$$

$$= 1 - 5.364\times10^{-2} - 4.254\times10^{-2} = 0.904\ \mu\text{m}$$

(b) Injected hole concentration at x=0

$$p_n(0) = p_{no} e^{qV_{EB}/kT} = \frac{n_i^2}{N_D} e^{qV_{EB}/kT}$$

$$= \frac{\left(9.65\times10^9\right)^2}{2\times10^{17}} e^{0.5/0.0259} = 1.13\times10^{11}\ \text{cm}^{-3}$$

(c) In the emitter region

$$D_E = 52 \ \text{cm}^2/\text{s} \qquad L_E = \sqrt{52 \cdot 10^{-8}} = 0.721 \times 10^{-3} \ \text{cm}$$

$$n_{EO} = \frac{\left(9.65 \times 10^9\right)^2}{5 \times 10^{18}} = 18.625 \ .$$

In the base region

$$D_p = 40 \ \text{cm}^2/\text{s} \qquad L_p = \sqrt{D_p \tau_p} = \sqrt{40 \cdot 10^{-7}} = 2 \times 10^{-3} \ \text{cm}$$

$$p_{no} = \frac{n_i^{\,2}}{N_D} = \frac{\left(9.65 \times 10^9\right)^2}{2 \times 10^{17}} = 465.613 \ .$$

In the collector region

$$D_C = 115 \ \text{cm}^2/\text{s} \qquad L_C = \sqrt{115 \cdot 10^{-6}} = 10.724 \times 10^{-3} \ \text{cm}$$

$$n_{CO} = \frac{\left(9.65 \times 10^9\right)^2}{10^{16}} = 9.312 \times 10^3 \ .$$

The current components are given by

$$I_{Ep} = A\left(-qD_p \left.\frac{dp}{dx}\right|_{x=0}\right) \equiv qA\frac{D_p p_{no}}{W} e^{qV_{EB}/kT} \cong I_{Cp}$$

$$I_{En} = A\left[qD_{En} \left.\frac{dn_E}{dx}\right|_{x=-x_E}\right] = \frac{qAD_{En} n_{EO}}{L_E}\left(e^{qV_{EB}/kT} - 1\right)$$

$$I_{Cn} = A\left(qD_{Cn} \left.\frac{dn_C}{dx}\right|_{x=x_C}\right) = \frac{qAD_{Cn} n_{CO}}{L_C}$$

$$I_{Ep} = \frac{1.6\times10^{-19}\times0.2\times10^{-2}\times40\times465.613}{0.904\times10^{-4}} e^{0.5/0.0259} = 1.596\times10^{-5}\ \text{A}$$

$$I_{Cp} \cong I_{Ep} = 1.596\times10^{-5}\ \text{A}$$

$$I_{En} = \frac{1.6\times10^{-19}\times0.2\times10^{-2}\times52\times18.625}{0.721\times10^{-3}}\left(e^{0.5/0.0259}-1\right) = 1.041\times10^{-7}\ \text{A}$$

$$I_{Cn} = \frac{1.6\times10^{-19}\times0.2\times10^{-2}\times115\times9.312\times10^{3}}{10.724\times10^{-3}} = 3.196\times10^{-14}\ \text{A}$$

$$I_{BB} = I_{Ep} - I_{Cp} \cong 0\ .$$

(d) The emitter, collector, and base currents are given by

$$I_E = I_{Ep} + I_{En} = 1.606 \times 10^{-5} \text{ A}$$

$$I_C = I_{Cp} + I_{Cn} = 1.596 \times 10^{-5} \text{ A}$$

$$I_B = I_{En} + I_{BB} - I_{Cn} = 1.041 \times 10^{-7} \text{ A} \quad .$$

We can obtain the emitter efficiency and the base transport factor

$$\gamma = \frac{I_{Ep}}{I_E} = \frac{1.596 \times 10^{-5}}{1.606 \times 10^{-5}} = 0.9938$$

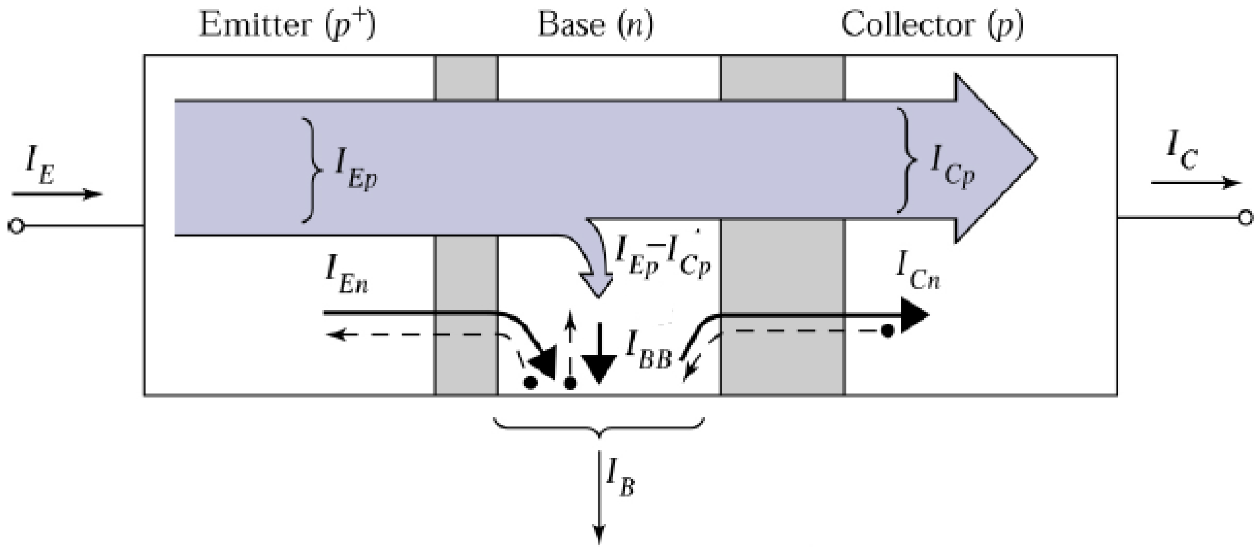
$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1.596 \times 10^{-5}}{1.596 \times 10^{-5}} = 1 \quad .$$

Hence, the common-base and common-emitter current gains are

$$\alpha_0 = \gamma \alpha_T = 0.9938$$

$$\beta_0 = \frac{\alpha_0}{1 - \alpha_0} = 160.3 \quad .$$

(e) To improve γ, the emitter has to be doped much heavier than the base.
To improve α_T, we can make the base narrower.



$$\alpha_0 \equiv \frac{I_{Cp}}{I_E} = \left[\frac{I_{Ep}}{I_{Ep}+I_{En}}\right]\left[\frac{I_{Cp}}{I_{Ep}}\right]$$

Emitter efficiency γ

Base transport factor α_T,

4. A Si *pnp* bipolar transistor has impurity concentrations of $3\text{x}10^{18}$, $2\text{x}10^{16}$ and $5\text{x}10^{15}$ cm^{-3} in the emitter, base and collector regions. The mobilities of electrons and holes are assumed to be expressed by

$$\mu_n = 88 + \frac{1252}{\left(1 + 0.698 \times 10^{-17} N\right)} \quad \text{cm}^2/\text{V.s and}$$

$$\mu_p = 54.3 + \frac{407}{\left(1 + 0.374 \times 10^{-17} N\right)} \quad \text{cm}^2/\text{V.s}$$

Assuming T = 300K, determine the Diffusion coefficients of minority carriers in the three regions respectively.

(a) In the emitter region, N=3×10^{18} cm^{-3}. The minority carriers are electrons

$$\mu_{nE} = 88 + \frac{1252}{\left(1 + 0.698\times10^{-17}\times3\times10^{18}\right)} = 145 \text{ cm}^2/\text{Vs}.$$

The diffusion coefficient of the minority carriers

$$D_{nE} = \tfrac{KT}{q}\mu_{nE} = 0.0259\times145 = 3.756 \text{ cm}^2/\text{s}.$$

(b) In the base region, N=2×10^{16} cm^{-3}. The minority carriers are holes

$$\mu_{pB} = 54.3 + \frac{407}{\left(1 + 0.374\times10^{-17}\times2\times10^{16}\right)} = 433 \text{ cm}^2/\text{Vs}.$$

The diffusion coefficient of the minority carriers

$$D_{pB} = \tfrac{KT}{q}\mu_{pB} = 0.0259\times433 = 11.2 \text{ cm}^2/\text{s}.$$

(c) In the collector region, $N=N_C=\times10^{15}$ cm^{-3}. The minority carriers are electrons

$$\mu_{nE} = 88 + \frac{1252}{\left(1 + 0.698\times10^{-17}\times5\times10^{15}\right)} = 1297 \text{ cm}^2/\text{Vs}.$$

The diffusion coefficient of the minority carriers

$$D_{nC} = \frac{KT}{q}\mu_{nC} = 0.0259\times1297 = 33.6 \text{ cm}^2/\text{s}.$$

5. In a Si npn transistor at 300 K, the impurity concentrations are 10^{18}, 3×10^{16} and 5×10^{15} cm^{-3} in the emitter, base, and collector, respectively. Assume D_B = 20 cm^2/s, τ_{BO} = 5×10^{-7} s, V_{BE} =0.7 V and the total base width is 1 μm. Assume V_{CB} = 5 V and 10 V as two data points.

(a) Find collector current density as a function of neutral base (ignore J_{Cp}).

(b) Find neutral base at 5 V and 10 V, respectively.

(c) Estimate the early voltage.

(a) J_C vs. W

$$n(x) = n_B(0)\left(1-\frac{x}{W}\right) = n_{BO}\exp\left(\frac{qV_{BE}}{kT}\right)\left(1-\frac{x}{W}\right)$$

$$J_C = J_{Cn} = \left(qD_B \frac{dn_B}{dx}\bigg|_{x=W}\right) \equiv q\frac{D_B n_{BO}}{W} e^{qV_{BE}/kT}$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(9.65\times10^9\right)^2}{3\times10^{16}} = 3.1\times10^3 \text{ cm}^{-3}$$

and

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right) = \left(3.1\times10^3\right)\exp\left(\frac{0.7}{0.0259}\right) = 1.7\times10^{15} \text{ cm}^{-3}$$

We have

$$J_C = \frac{qD_B n_B(0)}{W} = \frac{\left(1.6\times10^{-19}\right)(20)\left(1.7\times10^{15}\right)}{W}$$

So, $J_C = \dfrac{5.44\times10^{-3}}{W}$

(b) Neutral base width at 5V and 10 V

Neglecting the space charge width at the B-E junction,

$$W_{Bn}(\text{neutral base}) = W_B(\text{base width}) - W_{db}(\text{depletion of C-B in base})$$

$$V_{bi} = (0.0259)\ln\left[\frac{(3\times10^{16})(5\times10^{15})}{(9.65\times10^{9})^2}\right] = 0.728\ V$$

and $$W_{db} = \left\{\frac{2\varepsilon(V_{bi}+V_{CB})}{q}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(V_{bi}+V_{CB})}{1.6\times10^{-19}}\times\left(\frac{5\times10^{15}}{3\times10^{16}}\right)\left(\frac{1}{5\times10^{15}+3\times10^{16}}\right)\right\}^{1/2}$$

$$= \left\{(6.163\times10^{-11})(V_{bi}+V_{CB})\right\}^{1/2}$$

Now, for $V_{CB} = 5V$, $W_{db} = 0.188\ \mu\text{m}$ and

for $V_{CB} = 10V$, $W_{db} = 0.257\ \mu\text{m}$

$$W_{Bn}(5V) = 1 - 0.188 = 0.812\,\mu m$$

$$W_{Bn}(10V) = 1 - 0.257 = 0.743\,\mu m.$$

(c) Find Early voltage

$$J_C(5V) = \frac{5.44\times10^{-3}}{0.812\times10^{-4}} = 67\text{A}/\text{cm}^2$$

$$J_C(10V) = \frac{5.44\times10^{-3}}{0.743\times10^{-4}} = 73.2\text{A}/\text{cm}^2$$

We can write $$J_C = \frac{\Delta J_C}{\Delta V_{CE}}(V_{CE}+V_A)$$

where

$$\frac{\Delta J_C}{\Delta V_{CE}} = \frac{\Delta J_C}{\Delta V_{CB}} = \frac{73.2-67}{5} = 1.24\ \text{A}/\text{cm}^2/\text{V}$$

Then

$$67 = 1.24(5.7+V_A) \Rightarrow V_A = 48.3\,V$$

or

$$73.2 = 1.24(10.7+V_A)$$

6. (a) A Si transistor has D_p of 10 cm^2/s and W of 0.5 μm. Find the cut-off frequencies for the transistor with a common base current gain of 0.998. Neglect the emitter and collector delays.

(b) To design a bipolar transistor with 5 GHz cutoff frequency. What the neutral base width *W* will be?

(a) The base transit time is given by

$$\tau_B = W_{Bn}{}^2 / 2D_p = \frac{\left(0.5\times10^{-4}\right)^2}{2\times10} = 1.25\times10^{-10} \text{ s}$$

We can obtain the cutoff frequency:

$$f_T \cong 1/2\pi\tau_B = 1.27\times10^9 \text{ Hz} = 1.27 \text{ GHz.}$$

(b) Neglect the time delays of emitter and collector. the base transit time is given by

$$\tau_B = \frac{1}{2\pi f_T} = \frac{1}{2\pi \times 5 \times 10^9} = 31.83 \times 10^{-12}\,\mathrm{s}\,.$$

W can be expressed by

$$W_{Bn} = \sqrt{2D_p \tau_B}\,.$$

Therefore,

$$W_{Bn} = \sqrt{2 \times 10 \times 31.83 \times 10^{-12}}$$

$$= 2.52 \times 10^{-5}\mathrm{cm} \;\; = 0.252\mu\mathrm{m}\,.$$

The neutral base width should be 0.252 μm.