NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

EE2002 – ANALOG ELECTRONICS

April / May 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 10 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A list of Formulae is provided in Appendix A on pages 8 to 10.

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1. (a) For the non-ideal Op-Amp in negative feedback shown in Figure 1(a) on page 2, derive the expression for the output voltage v_{OUT} in terms of all or some of the following variables v_1 , v_2 , V_{IO}, I_+, I_-, R_1, R_2, R_3, R_4, R_5, R_6 and R_7 assuming that the output is in the linear range of operation.

   Note: *Parallel resistance of R_a and R_b can be written as $R_a // R_b$ without expanding it.*

   (10 Marks)

Note: Question No. 1 continues on page 2

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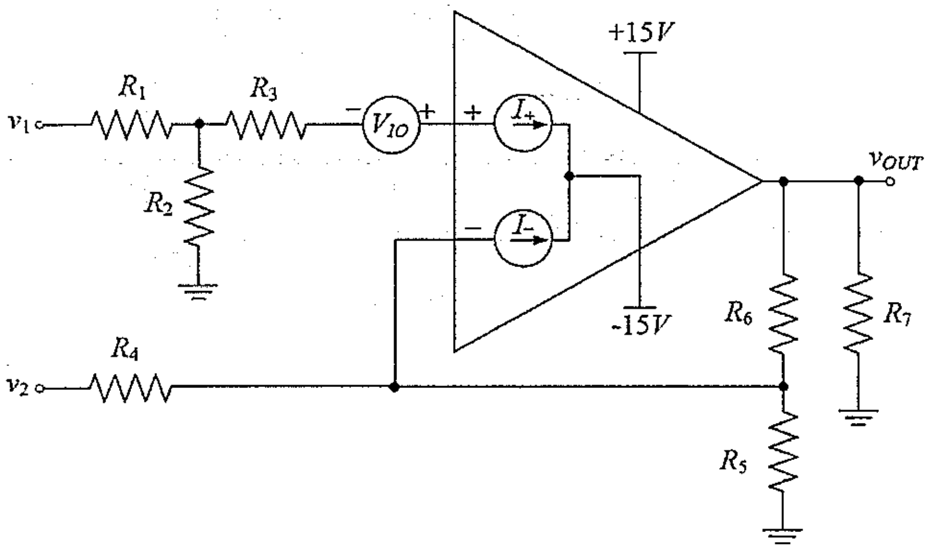

Figure 1(a)

(b) In Figure 1(b) on page 3, the empirical junction diode equation is:

$$V_D = nV_T \ln [I_D / I_S]$$

for diodes D_1, D_2, D_3 and D_4 where $n = 1$, $V_T = 26$ mV at room temperature, $I_S = 5 \times 10^{-17}$ A, the resistors $R_1 = R_2 = R_4 = 30$ kΩ , $R_3 = R_5 = 15$ kΩ and the DC voltage source $V_S = 8$ V. Find the DC quiescent operating point or Q-point (I_D ; V_D) of the diodes D_1, D_2, D_3 and D_4 (to 3 decimal places in μA and V, respectively).

Note: *The diodes are all identical where* $V_{D1} = V_{D2} = V_{D3} = V_{D4}$ *and* $I_{D1} = I_{D2} = I_{D3} = I_{D4}$.

(10 Marks)

Note: Question No. 1 continues on page 3

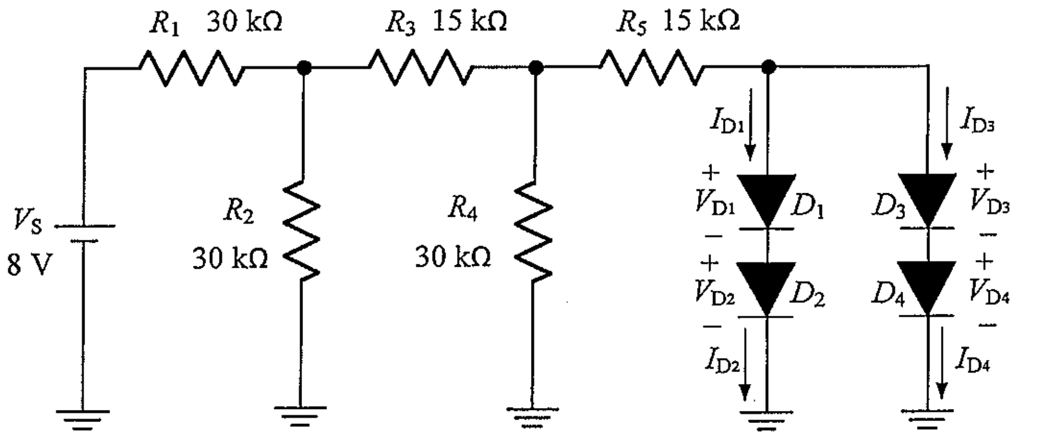

Figure 1(b)

2. In Figure 2 on page 4, R_{B1} = 10 kΩ, R_{B2} = 1.2 kΩ, R_E = 1.2 kΩ, R_C = 10 kΩ and R_L = 10 kΩ. The transistor Q has $\beta = 100$ and V_A = 80 V. v_i and v_o are the input and output signal voltages, respectively. Assume that all the capacitors are ideal and have infinite capacitance.

(a) Calculate the Q-point of transistor Q.

(7 Marks)

(b) Determine the voltage gain $A_v = \frac{v_o}{v_i}$, input resistance R_{in} and the resistance R'_{out} looking directly into the collector terminal of the amplifier, excluding R_C and R_L.

(10 Marks)

(c) What is the current gain $A_i = \frac{i_o}{i_i}$ of the amplifier?

(3 Marks)

Note: Question No. 2 continues on page 4

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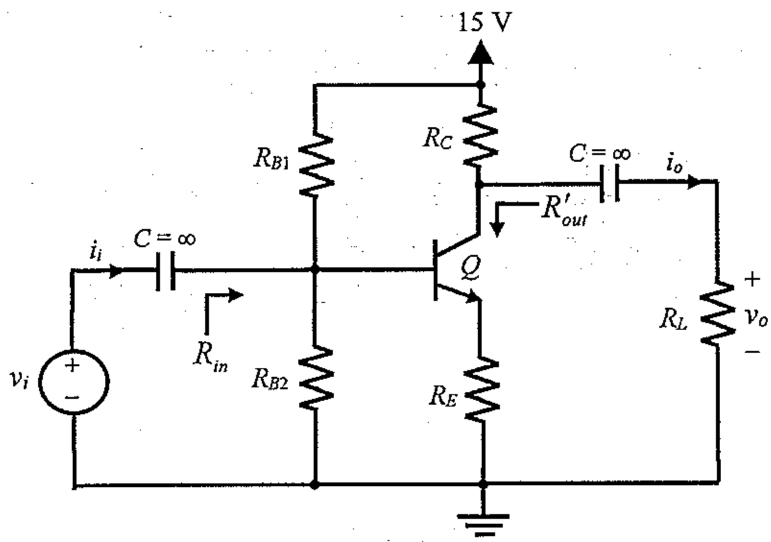

Figure 2

3. In Figure 3 on page 5, the DC operating point for the PNP transistor Q_1 is $I_C = 0.33$ mA and $V_{EC} = 2.4$ V, and the DC operating point for the NMOS transistor M_1 is $I_D = 2.81$ mA and $V_{DS} = 2.92$ V. Q_1 has $\beta = 100$, $V_A = 85$ V at room temperature and M_1 has $K_n = 1$ mA/V^2, $V_{TN} = 1$ V and $\lambda = 0.02$ V^{-1}. Assume that the capacitors have infinite values, and the resistors have the values as indicated in Figure 3.

(a) Determine the voltage gain $A_v = \dfrac{v_o}{v_i}$.

(9 Marks)

(b) Determine the input resistance R_{in} and output resistance R_{out} of the amplifier.

(6 Marks)

(c) Determine the small signal input range of v_i for this amplifier.

(5 Marks)

Note: Question No. 3 continues on page 5

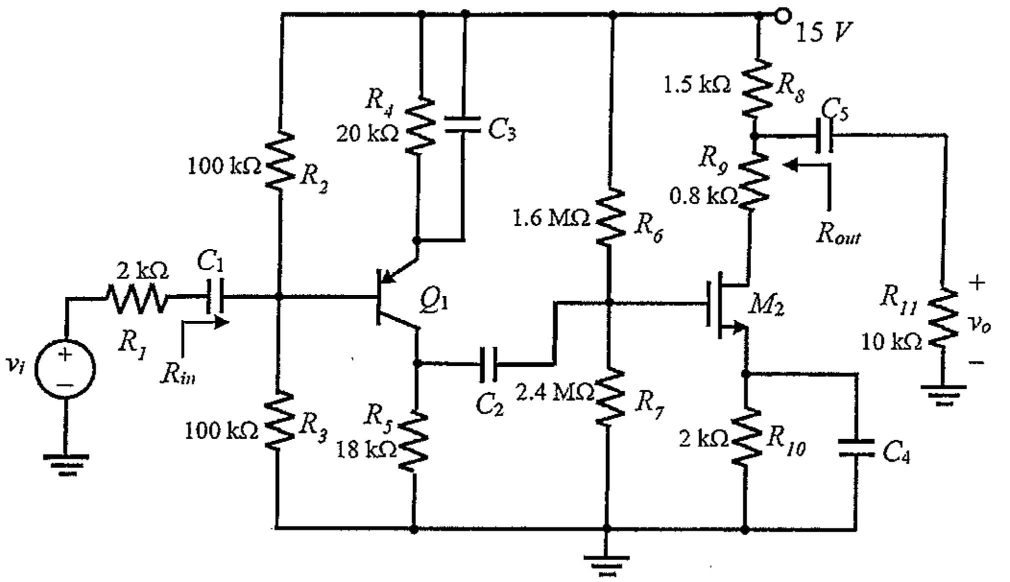

Figure 3

4. (a) Consider the circuit in Figure 4(a). Assume all transistors are biased in the Forward Active region and have the same current gain β and Early Voltage V_A.

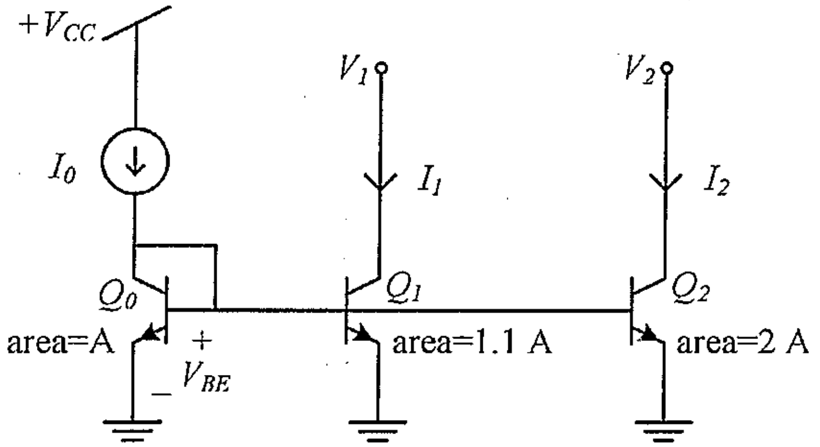

Figure 4(a)

Note: Question No. 4 continues on page 6

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(i) Find the algebraic expressions for the current mirror ratios I_1/I_0 and I_2/I_0 including the Early Effect (HINT: include the terms β, V_A and V_{BE} in your answer).

(5 Marks)

(ii) Using your answer to part (a), choose EITHER transistor Q_1 or Q_2 and calculate its output resistance, i.e. r_{o1} OR r_{o2}, respectively (you need not do both). Use the following values: $\beta = 100$, $V_A = 50$ V, $V_{BE} = 0.7$ V, $I_0 = 10$ µA, $V_{CC} = 5$ V, $V_1 = 4$ V, $V_2 = 3$ V.

(5 Marks)

(b) Consider the current mirror in Figure 4(b). Both transistors are biased in the Saturation region and have the same values for K'_n, V_{TN} and λ. Find the algebraic expression for the current mirror ratio I_1/I_0 including the Early Effect (HINT: include λ, V_1 and V_G in your answer).

(4 Marks)

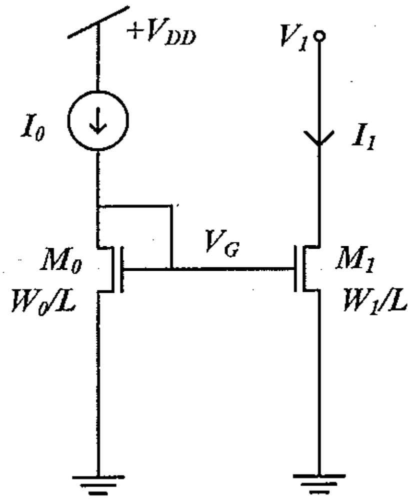

Figure 4(b)

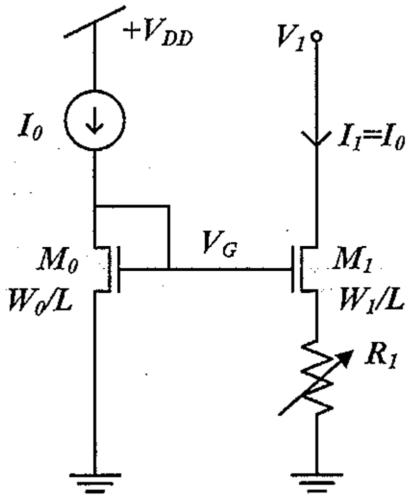

Figure 4(c)

(c) In Figure 4(c), a potentiometer R_1 is added into the source of M_1 to make $I_1 = I_0$. Find the value of R_1 given the following: $W_0 = 10$ µm, $W_1 = 11$ µm, $L = 1$ µm, $I_0 = 10$ µA, $V_{DD} = 5$ V, $V_1 = 4$ V, $K'_n = 50$ µA/V^2, $V_{TN} = 0.75$ V, $\lambda = 0.01$ V^{-1} (HINT: start by calculating the value of V_G).

(6 Marks)

5. Consider the circuit shown in Figure 5 and assume that the Op-amp is ideal.

(a) Derive the transfer function V_{o1}/V_s. How many poles and zeros are there in this transfer function?

(5 Marks)

(b) Derive the transfer function V_{out}/V_s. How many poles and zeros are there in this transfer function?

(5 Marks)

(c) Find the algebraic expressions of the poles and zeros in the transfer function found in part (b) of this question.

(3 Marks)

(d) Given that $R_f = 5$ kΩ, $R_L = 0.2$ kΩ, $C_s = C_f = 2$ μF and $C_L = 5$ μF, draw the Bode plot for the transfer function derived in part (b) of this question. Clearly mark the slopes for all the regions in the plot and indicate the gain at frequency $\omega = 100$ rad/sec.

(7 Marks)

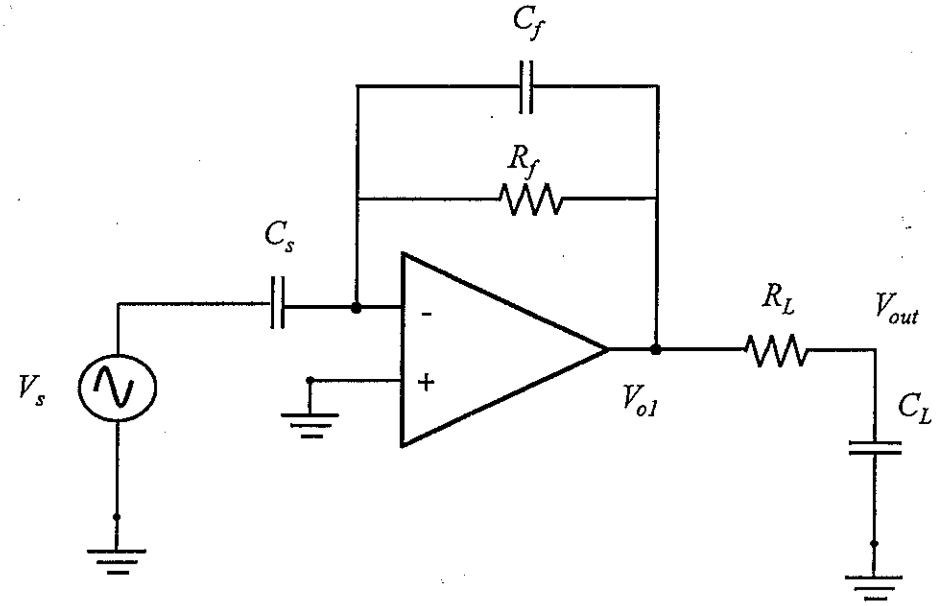

Figure 5

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Appendix A
List of Selected Formulae (with the usual notations)

Op-Amps:

Closed-Loop Negative Feedback Inverting Gain, $A_{VCL} = \frac{v_o}{v_i} = -\frac{R_f}{R_i}$ Figure (a)

Closed-Loop Negative Feedback Non-Inverting Gain, $A_{VCL} = \frac{v_o}{v_i} = \left(1 + \frac{R_f}{R_i}\right)$ Figure (b)

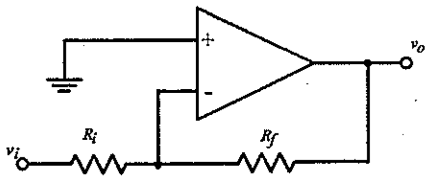


Figure (a)

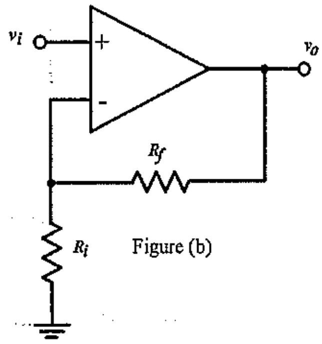


Figure (b)

Op-Amp's Slew Rate, $SR \geq \left|\frac{dv_o}{dt}\right|_{\max} = A_{VCL}\omega a_m = A_{VCL} a_m 2\pi f$,

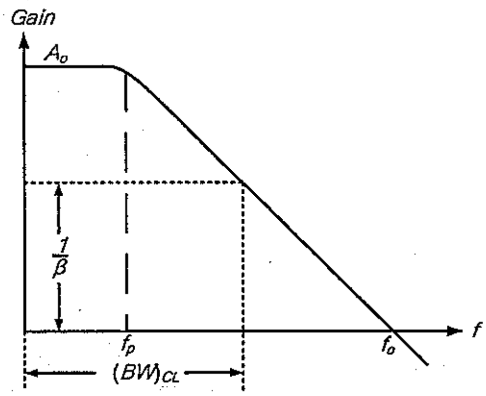
where $v_i = a_m \sin(\omega t)$, $v_o = A_{VCL} v_i$, $v_o = A_{VCL} a_m \sin(\omega t)$ and $\left|\frac{dv_o}{dt}\right| = A_{VCL}\omega\, a_m \cos(\omega t)$

Op-Amp's frequency response: $A_{VOL}(jf) = \frac{A_o}{\left(1 + \frac{jf}{f_p}\right)}$

Gain-Bandwidth Product: $A_o f_p = f_o = \frac{1}{\beta}(BW)_{CL}$

where $\frac{1}{\beta} = \frac{R_f + R_i}{R_i}$

$t_r = \frac{0.35}{(BW)_{CL}}$



Diodes:

$v_D \approx nV_T \ln\left(\frac{i_D}{I_S}\right)$ or $i_D \approx I_S e^{\left(\frac{v_D}{nV_T}\right)}$

where $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$

Diode conductance: $g_D = \frac{1}{r_D} = \frac{I_D}{nV_T}$

BJT in Forward Active Region:

Ignore early effect: $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right)$

With early effect: $i_C = I_S \exp\left(\frac{|v_{BE}|}{V_T}\right)\left(1+\frac{|v_{CE}|}{V_A}\right)$

where I_S: Saturation current,
V_T: Thermal voltage, assume 25 mV at room temperature,
V_A: Early voltage.
For npn transistor, $|v_{BE}| = v_{BE}$ and $|v_{CE}| = v_{CE}$;
For pnp transistor, $|v_{BE}| = v_{EB}$ and $|v_{CE}| = v_{EC}$.

Small-signal model parameters of BJT:

$g_m = \frac{I_C}{V_T}$, $r_\pi = \frac{\beta}{g_m}$ and $r_o = \frac{V_A + |V_{CE}|}{I_C} \approx \frac{V_A}{I_C}$

where I_C : DC collector current at Q-point
V_{CE}: DC collector-emitter voltage at Q-point

Criterion for small-signal operation of BJT: $|v_{be}| \le 0.2V_T$

MOSFET in Saturation Region:

Criterion: $V_{DS} \ge V_{GS} - V_{TN}$ for NMOS;
$|V_{DS}| \ge |V_{GS}| - |V_{TP}|$ for PMOS

where V_{TN}, V_{TP}: Threshold voltage,
V_{DS}: DC drain-source voltage,
V_{GS}: DC gate-source voltage.

Ignore channel-length modulation effect: $i_D = \frac{K_n}{2}(v_{GS} - V_{TN})^2$ for NMOS,

$i_D = \frac{K_p}{2}(|v_{GS}| - |V_{TP}|)^2$ for PMOS.

With channel-length modulation effect: $i_D = \frac{K_n}{2}(v_{GS} - V_{TN})^2(1+\lambda v_{DS})$ for NMOS,

$i_D = \frac{K_p}{2}(|v_{GS}| - |V_{TP}|)^2(1+\lambda |v_{DS}|)$ for PMOS.

where λ: channel length modulation parameter,

For NMOS $K_n = K_n'\left(\frac{W}{L}\right)$ and $K_n' = \mu_n C_{ox}$; For PMOS $K_p = K_p'\left(\frac{W}{L}\right)$ and $K_p' = \mu_p C_{ox}$.

MOSFET in Triode Region:

Criterion: $V_{DS} < V_{GS} - V_{TN}$ for NMOS;
$|V_{DS}| < |V_{GS}| - |V_{TP}|$ for PMOS

Ignore channel-length modulation effect: $i_D = K_n\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right)v_{DS}$ for NMOS,

$i_D = K_p\left(|v_{GS}| - |V_{TP}| - \frac{|v_{DS}|}{2}\right)|v_{DS}|$ for PMOS.

With channel-length modulation effect: $i_D = K_n\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right)v_{DS}\left(1+\lambda v_{DS}\right)$ for NMOS,

$$i_D = K_p\left(\left|v_{GS}\right| - \left|V_{TP}\right| - \frac{\left|v_{DS}\right|}{2}\right)\left|v_{DS}\right|\left(1+\lambda\left|v_{DS}\right|\right) \text{ for PMOS.}$$

Small-signal model parameters of MOSFET

For NMOS: $g_m = \sqrt{2K_n I_D\left(1+\lambda V_{DS}\right)} \approx \sqrt{2K_n I_D}$ and $r_o = \dfrac{\frac{1}{\lambda}+V_{DS}}{I_D} \approx \dfrac{1}{\lambda I_D}$

For PMOS: $g_m = \sqrt{2K_p I_D\left(1+\lambda\left|V_{DS}\right|\right)} \approx \sqrt{2K_p I_D}$ and $r_o = \dfrac{\frac{1}{\lambda}+\left|V_{DS}\right|}{I_D} \approx \dfrac{1}{\lambda I_D}$

where I_D : DC drain current at Q-point
V_{DS}: DC drain-source voltage at Q-point

Criterion for small-signal operation:
For NMOS: $\left|v_{gs}\right| \le 0.2\left(V_{GS} - V_{TN}\right)$
For PMOS: $\left|v_{gs}\right| \le 0.2\left(\left|V_{GS}\right| - \left|V_{TP}\right|\right)$
where V_{GS} : DC gate-source voltage at Q-point.

Miller Effect

The equivalent shunt input capacitance: $C_X = C\times(1+A_V)$

The equivalent shunt output capacitance: $C_Y = C\times(1+\frac{1}{A_V})$

where $-A_V$: the gain of the voltage amplifier
C: the original capacitance between the input and output terminals of the voltage amplifier

Frequency Response

By using OCTC and SCTC method, the upper cut-off frequency is estimated by $\omega_{H_{-3dB}} \approx \dfrac{1}{\sum_i C_i R_i}$

and the lower cut-off frequency is estimated by $\omega_{L_{-3dB}} \approx \sum_i \dfrac{1}{C_i R_i}$

where C_i : the contributing capacitor for the cut-off frequency
R_i : the equivalent resistance seen by the capacitor C_i

END OF PAPER

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1.(a) V_1: $V_{+1} = V_1 \cdot \frac{R_2}{R_1+R_2} = V_{-1}$

$V_{out_1} = V_- \cdot \frac{R_4 \| R_5 + R_6}{R_4 \| R_5} = V_1 \cdot \frac{R_2}{R_1+R_2} \cdot \frac{R_4 \| R_5 + R_6}{R_4 \| R_5}$

V_2: $V_{+2} = V_{+2} = 0$

$V_{out_2} = -V_2 \cdot \frac{R_6}{R_4 \| R_5}$

I_+ : $V_{+3} = V_{-3} = -I_+ \cdot (R_1 \| R_2 + R_3)$

$V_{out_3} = -I_+ (R_1 \| R_2 + R_3) \cdot \frac{R_4 \| R_5 + R_6}{R_4 \| R_5}$

I_- : $V_{+4} = V_{-4} = 0$

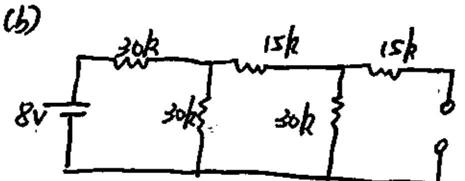
$V_{out_4} = I_- R_6$

V_{IO} : $V_{+5} = V_{-5} = V_{IO}$

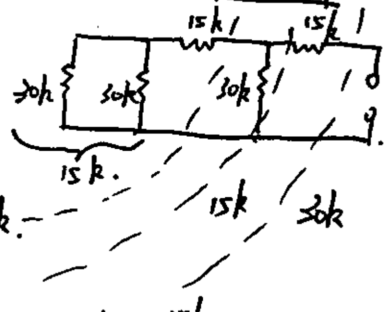
$V_{out_5} = V_{IO} \cdot \frac{R_4 \| R_5 + R_6}{R_4 \| R_5}$

$\Rightarrow V_{out} = \sum_{i=1}^{5} V_{out_i} = \left(V_1 \cdot \frac{R_2}{R_1+R_2} - I_+ (R_1 \| R_2 + R_3) + V_{IO}\right) \cdot \frac{R_4 \| R_5 + R_6}{R_4 \| R_5} - V_2 \cdot \frac{R_6}{R_4 \| R_5} + I_- R_6$

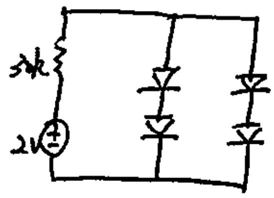
(b)



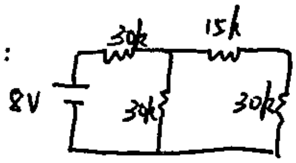
Rth:



$\Rightarrow R_{th} = 30k\Omega$

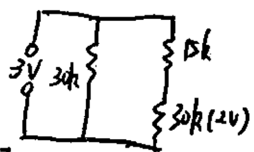


Vth:



$\Rightarrow$

$\Rightarrow$



$\Rightarrow V_{th} = 2V$

$I_S = 5 \times 10^{-17} A = 5 \times 10^{-14} mA$, $n=1$, $V_T = 26mV$, $V_D = nV_T \ln(I_D/I_S)$

① $V_D = 0.7V$ $2I_D = 0.02mA$ $I_D = 0.01mA$ $V_{D_1} = 0.67656V$

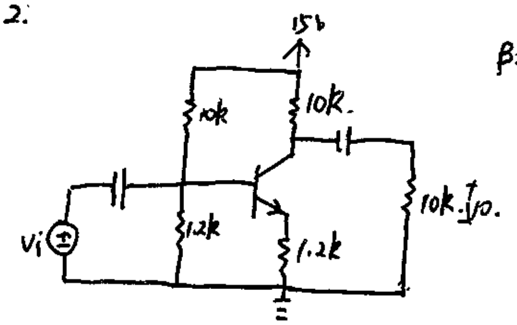
② $V_{D_1} = 0.67656V$ $2I_D = 0.02156mA$ $I_D = 0.01078mA$ $V_{D_2} = 0.67851V$

③ $V_{D_2} = 0.67851V$ $2I_D = 0.02143mA$ $I_D = 0.01072mA$ $V_{D_3} = 0.67837V$

④ $V_{D_3} = 0.67837V$ $2I_D = 0.02144mA$ $I_D = 0.01072mA$ $V_{D_4} = 0.67837V$

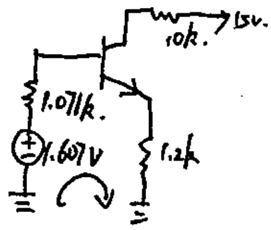
Q(10.7μA, 0.678V) for D_1 to D_4

2.



$\beta = 100$ $V_A = 80V$.

(a)



$1.607 - 1.071 I_E - 1.2 \times 101 I_E + 0.7 = 0$

$I_E = 7.418 \times 10^{-3} mA.$

$I_C = 0.7418 mA$

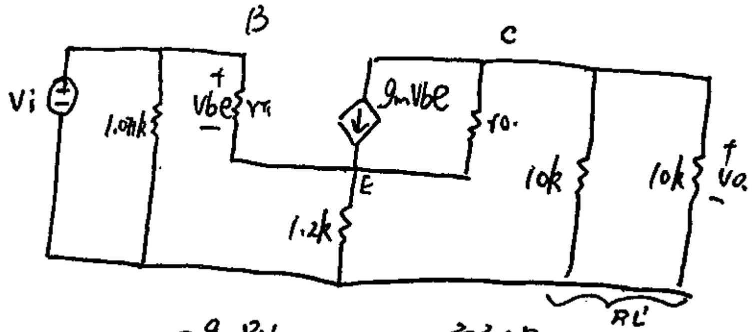
$V_C = 7.582V$

$V_E = 0.899V$

$V_{CE} = 6.683V$

$\Rightarrow Q(0.7418mA, 6.683V)$

(b)



$g_m = \frac{I_C}{V_T} = 303.28 mS$

$r_\pi = \frac{\beta}{g_m} = 0.333 k\Omega.$

$r_o = \frac{V_A + |V_{CE}|}{I_C} = 116.855 k\Omega.$

$Av_t = \frac{-g_m R_L'}{1 + g_m R_{E1}} = \frac{-303.28 \times 5}{1 + 303.28 \times 1.2} = -4.155$

$\Rightarrow AV = -4.155$

$R_{in} = [r_\pi + (\beta+1) R_{E1}] \| R_B = (0.333 + 101 \times 1.2) \| 1.071 = 121.533 \| 1.071 = 1.0616 k\Omega.$

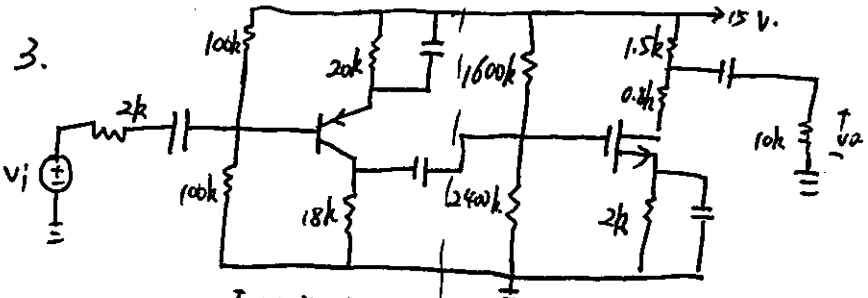
$R_{out} = \left(1 + g_m\left(\frac{r_\pi}{r_\pi + R_{th}}\right)\{(r_\pi + R_{th}) \| R_{E1}\}\right) r_o$, $R_{th} = 1.071k$

$= \left(1 + 303.28 \times \frac{0.333}{0.333 + 1.071} \times ((0.333 + 1.071) \| 1.2)\right) \times 116.855$

$= (1 + 71.932 \times 0.647) \times 116.855 = 5555.29 k\Omega.$

(c) $A_i = Av \times \frac{R_2 + R_{in}}{R_L} = -4.155 \times \frac{0 + 1.0616}{10} = -0.441$

3.



$I_C = 0.33mA$ $\quad$ $I_D = 2.81mA$

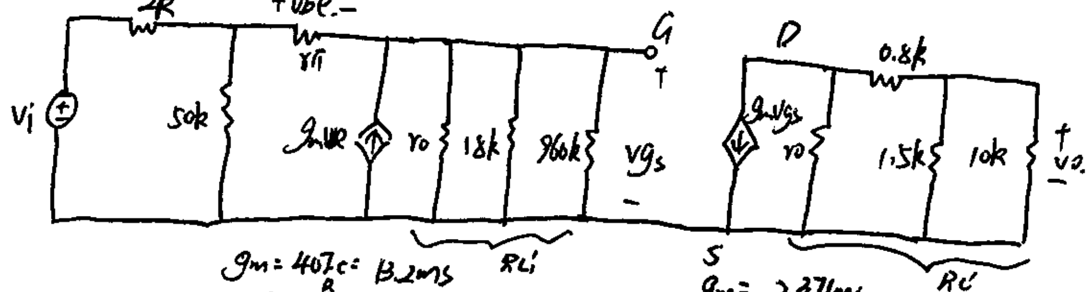
$V_{EC} = 2.4V$ $\quad$ $V_{DS} = 2.92V$

$\beta = 100$ $\quad$ $k_n = 1mA/V^2$

$V_A = 85V$ $\quad$ $V_{TN} = 1V$

$\lambda = 0.02 V^{-1}$

(a)



$g_m = 40 I_C = 13.2mS$ $\quad$ $g_m = 2.371mS$

$r_\pi = \frac{\beta}{g_m} = 7.576k\Omega$ $\quad$ $r_o = 18.833k\Omega$

$r_o = 264.85k\Omega$

$A_{vt2} = -g_m R_L' = -2.371 \times (18.833 \,//\, (0.8 + 1.5//10)) = -2.371 \times (18.833 \,//\, 2.104) = -4.487$

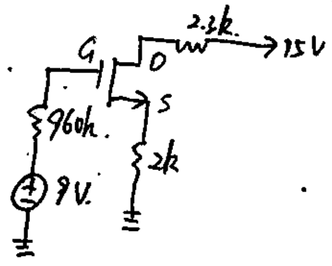
$A_{vt1} = \frac{g_m R_{L1}'}{1+g_m R_{L1}'} = \frac{13.2 \times (264.85 \,//\, 18 \,//\, 960)}{1+13.2 \times (264.85 \,//\, 18 \,//\, 960)} = \frac{218.641}{219.641} = 0.9954$

$R_{in} = (r_\pi + (\beta+1) R_L') \,//\, R_B = (7.576 + 101 \times 16.564) \,//\, 50 = 1680.54 \,//\, 50 = 48.56k\Omega$

$\Rightarrow A_v = -4.2897$

(b) $R_{in} = 48.56k\Omega$ $\qquad$ $R_{out} = 1.5 \,//\, (0.8 + 18.833) = 1.394k\Omega$

(c) $|v_{gs}| \le 0.2 (V_{GS} - V_{TN})$



$V_{DS} = 2.92V$

$V_G = 9V$

$15 - 2.3I - 2.92 - 2I = 0$

$I = 2.809mA$

$V_S = 5.619V$

$V_{GS} = 3.381V$

$|v_{gs}| \le 0.2 (3.381 - 1) = 0.4762V$

$|v_{gs}| = |v_i| \times \frac{48.56}{2+48.56} \times 0.9954 \quad \Rightarrow |v_i| \le 0.4981V$

4.(a)

(i) $\frac{I_1}{I_0} = \frac{n(1+V_{CE_2}/V_A)}{1+\frac{V_{BE}}{V_A}+\frac{n+1}{\beta}} = \frac{(1+V_1/V_A)1.1}{1+\frac{V_{BE}}{V_A}+\frac{2.1}{\beta}}$

$\frac{I_2}{I_0} = \frac{2(1+\frac{V_2}{V_A})}{1+\frac{V_{BE}}{V_A}+\frac{3}{\beta}}$

(ii) choose $Q_1 \Rightarrow r_{o1}$

$r_{o1} = \frac{V_A+|V_{CE}|}{I_C} = \frac{50+0.7}{I_1}$

$I_1 = I_0 \cdot \frac{1.1(1+4/50)}{1+\frac{0.7}{50}+\frac{2.1}{100}} = 11.478\mu A$.

$r_{o1} = 4.417M\Omega$

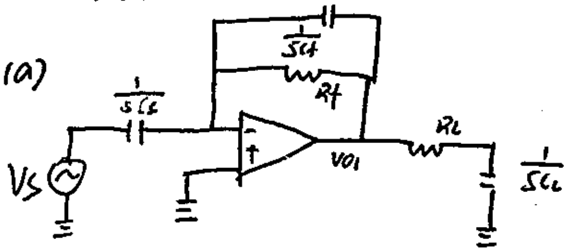
(b) $I_1/I_0 = \frac{(\frac{W}{L})_2(1+\lambda V_1)}{(\frac{W}{L})_1(1+\lambda V_G)}$

(c) $R_1 = \frac{1}{I_0}\sqrt{\frac{2I_{REF}}{K_{n1}}}\left(1-\sqrt{\frac{I_0(W/L)_0}{I_{REF}(W/L)_1}}\right)$

$= \frac{1}{10\mu}\sqrt{\frac{2\times10\mu}{50\mu}}\left(1-\sqrt{\frac{10}{11}}\right)$

$= 2.943k\Omega$.

5.(a)



$\frac{V_{o1}}{V_s} = \frac{\frac{1}{sC_s}+\frac{1}{sC_f}\|R_f}{\frac{1}{sC_s}} = \frac{\frac{1}{sC_s}+\frac{\frac{R_f}{sC_f}}{\frac{1}{sC_f}+R_f}}{\frac{1}{sC_s}} = \frac{\frac{1}{C_f}+(R_f+\frac{R_fC_s}{C_f})s}{\frac{1}{C_f}+R_f s} \Rightarrow$ 1 pole, 1 zero

(b) $\frac{V_{out}}{V_{o1}} = \frac{\frac{1}{sC_L}}{\frac{1}{sC_L}+R_L} = \frac{\frac{1}{C_L}}{\frac{1}{C_L}+R_L s}$, 1 pole. $\Rightarrow \frac{V_{out}}{V_s} = \frac{\frac{1}{C_L}(\frac{1}{C_f}+(R_f+\frac{R_fC_s}{C_f})s)}{(\frac{1}{C_L}+R_L s)(\frac{1}{C_f}+R_f s)}$

(c) poles: $-\frac{1}{R_LC_L}$ $\quad -\frac{1}{R_fC_f}$

zero: $\frac{-\frac{1}{C_f}}{R_f+\frac{R_fC_s}{C_f}}$

(d) poles: -10^3 $\quad -10^2$