NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

EE2002 - ANALOG ELECTRONICS

April / May 2017 | Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 9 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A List of Formulae is provided in Appendix A on pages 7 to 9.

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1. (a) A negative feedback non-ideal op-amp circuit is shown in Figure 1 on page 2.

(i) Derive the expression for the output voltage v_{OUT} in terms of V_{IO}, I_+, I_-, v_{in}, R_1, R_2, R_3 and R_4.

Note: *Parallel resistance of R_a and R_b can be written as $R_a//R_b$ without expanding it.*

(4 Marks)

(ii) Assume that v_{in} = 510 mV$_{peak}$ sinusoid, V_{IO} = 10 mV, I_+ = 10 μA, I_- = 10 μA, and $R_1 = R_2 = R_3 = 2$ kΩ. Find the value of R_4 that would give the maximum gain within the output linear range no closer than 1 V to either power supply voltage. (Hint: $v_{OUT}|_{DC}$ is positive.)

(8 Marks)

Note: Question No. 1 continues on page 2.

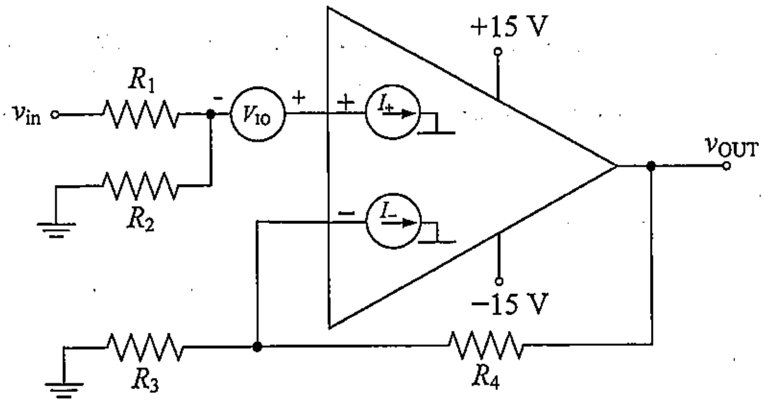

Figure 1

(b) The empirical diode junction equations expressed in voltage and current are as follows:

$$v_D \cong nV_T \ln\frac{i_D}{I_S} \text{ and } i_D \cong I_S e^{\frac{v_D}{nV_T}}.$$

Assume that the total current $i_D = i_d + I_D$ and total voltage $v_D = v_d + V_D$ in the diode, where i_d and v_d are the ac current and voltage components, respectively, and I_D and V_D are the dc current and voltage components, respectively.

(i) Prove that i_d and v_d are linearly related for small values of $\frac{v_d}{nV_T}$.

(ii) Hence, within the linear validity range, calculate the numerical value of v_d in mV, given that the thermal voltage $V_T = 26$ mV at 27 °C, and the forward emission coefficient $n = 1$.

(Note: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, and $e^x \cong 1 + x$ for small values of x where $\frac{x^2}{2!} << x$.)

(8 Marks)

2. For the amplifier shown in Figure 2, the MOSFET transistor M has $V_{TN} = 1$ V, $K_n = 0.2$ mA/V^2 and $\lambda = 0.01$ V^{-1}. Assume that $R_{G1} = 50$ kΩ, $R_{G2} = 100$ kΩ, $R_D = R_S = 10$ kΩ and $R_L = 100$ kΩ. The DC blocking capacitors are assumed to have infinite capacitance.

(a) Find the Q-point of the amplifier.

(6 Marks)

(b) Draw the small signal equivalent circuit. Determine the voltage gain $A_v = \frac{v_o}{v_i}$, input resistance R_{in} and output resistance R_{out} of the amplifier.

(10 Marks)

(c) Determine the input signal range of v_i for small-signal linear amplification. If the signal source v_i is not ideal and has an internal resistance of 10 kΩ, what will be the input signal range to maintain the small-signal operation?

(4 Marks)

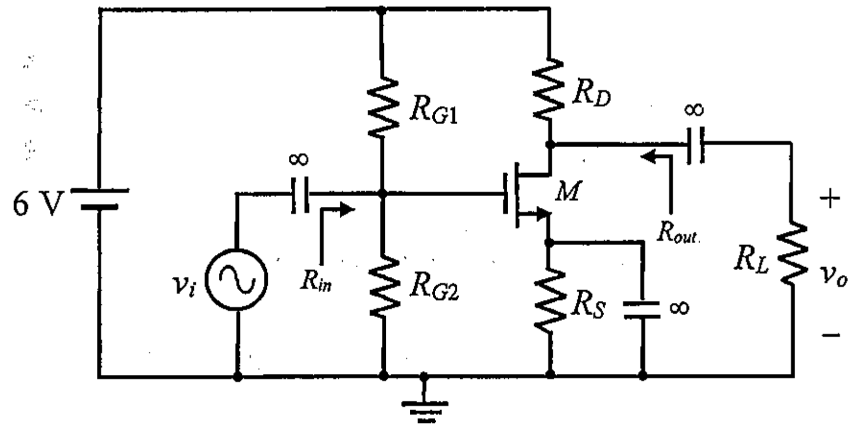

Figure 2

3. In Figure 3 on page 4, the DC operating point for MOSFET M_1 is at $I_D = 83.6$ µA and $V_{DS} = 5.97$ V, and the DC operating point for BJT Q_2 is at $I_C = 1.27$ mA and $V_{CE} = 1.65$ V. M_1 has $K_n = 0.5$ mA/V^2 and $\lambda = 0.02$ V^{-1}, and Q_2 has $\beta = 100$, $V_T = 25$ mV and $V_A = 70$ V at room temperature. Assume that the capacitors have infinite values, and resistors have the values as indicated in Figure 3 on page 4.

Note: Question No. 3 continues on page 4.

(a) Determine the voltage gain $A_v = \frac{v_o}{v_i}$.

(9 Marks)

(b) Determine the input resistance R_{in} and output resistance R_{out} of the amplifier.

(6 Marks)

(c) Determine the input small signal range for this amplifier to remain in the linear output range.

(5 Marks)

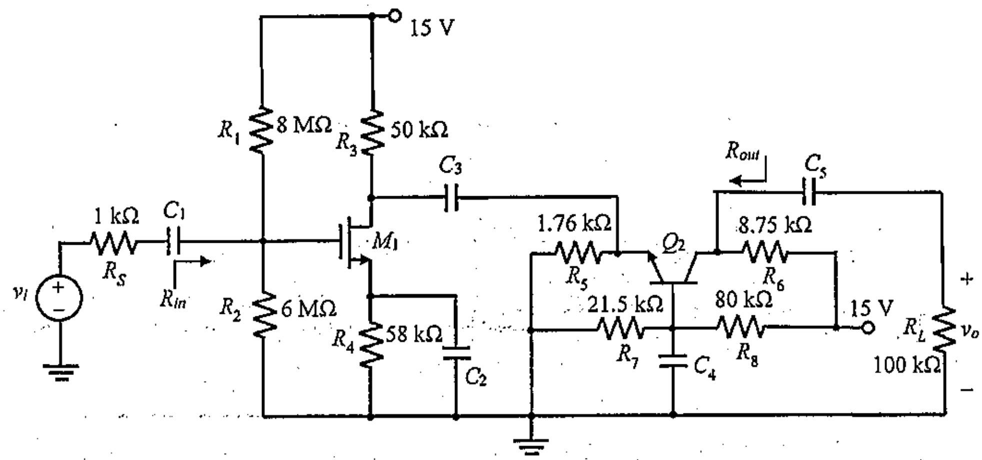

Figure 3

4. Consider the cascode current mirror circuit shown in Figure 4 on page 5. Assume all transistors are identical and in saturation. In the following small-signal analysis, indicate clearly the g_m and r_o of the i-th transistor as $g_{m,i}$ and $r_{o,i}$. For example, the g_m and r_o for transistor M_2 should be written as $g_{m,2}$ and $r_{o,2}$, respectively.

(a) Draw the AC small signal model of the circuit and <u>derive</u> the expression for the output resistance R_{out} in terms of the small signal parameters g_m and r_o of the MOSFET.

(8 Marks)

(b) Draw the AC small signal model of the circuit and <u>derive</u> the expression for the input resistance R_{in} in terms of the small signal parameters g_m and r_o of the MOSFET.

(7 Marks)

Note: Question No. 4 continues on page 5.

(c) Given that $I_{REF} = 100\ \mu A$, K_n for the MOSFET is $50\ \mu A/V^2$, $V_{TN} = 0.5V$ and $\lambda = 0.01\ V^{-1}$, find the value of the input resistance R_{in}.

(5 Marks)

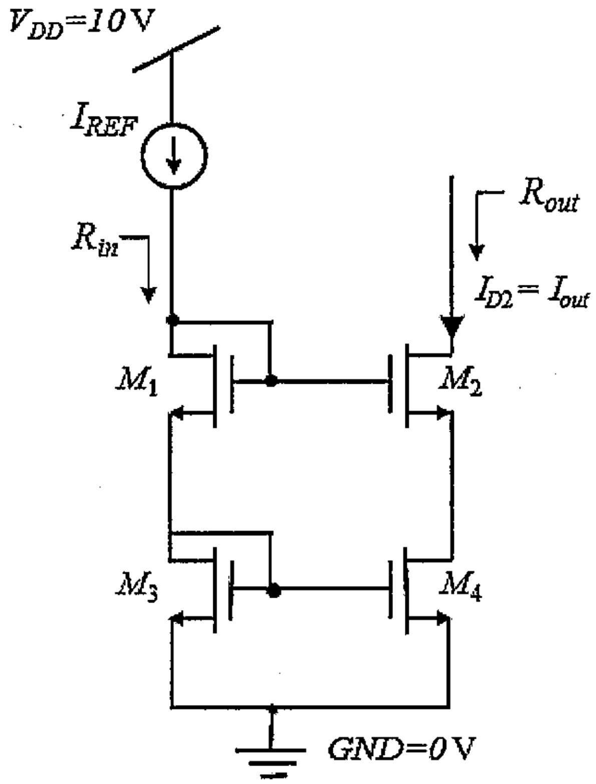

Figure 4

5. (a) A voltage amplifier has the transfer function:

$$A = \frac{200\omega}{(1+j\frac{\omega}{20^3})(1+j\frac{\omega}{10^6})(1+j\frac{\omega}{10^9})}$$

(i) Determine all the pole and zero frequencies.

(4 Marks)

(ii) What is the frequency (to a good approximation) at which the phase shift is −90° ?

(2 Marks)

Note: Question No. 5 continues on page 6.

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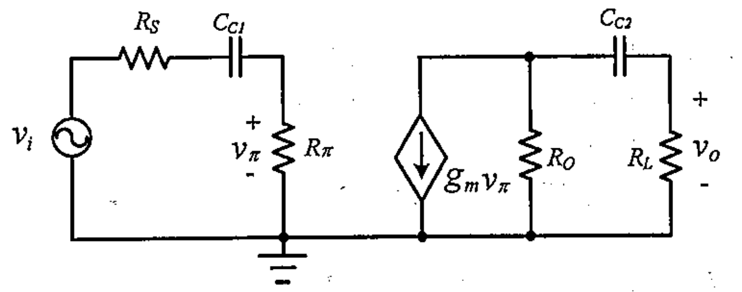
(iii) What is the magnitude of gain at the frequency determined in part (ii)? Express it in dB.

(2 Marks)

(b) Figure 5 shows the small-signal model of a single stage amplifier. All the design and operation parameters are given as follows.



$R_S = 100\ \Omega$
$R_O = 50\ \text{k}\Omega$
$R_L = 20\ \text{k}\Omega$
$R_\pi = 5\ \text{k}\Omega$
$g_m = 0.04$ A/V
$C_{C1} = 100$ pF
$C_{C2} = 100$ pF

Figure 5

(i) Derive the transfer function of the amplifier, and determine the number of poles and zeros.

(6 Marks)

(ii) In order to find out the contribution of capacitor C_{C1} to the **lower** roll-off frequency, what should you do with the capacitor C_{C2}? In other words, should it be short-circuited, or open-circuited? Determine the equivalent resistance seen by the capacitor C_{C1}.

(6 Marks)

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* The solution is only for your references

1a i. $V_1 = I_- R_4$

$V_2 = -I_+ (R_1 // R_2)(1 + \frac{R_4}{R_3})$

$V_3 = \frac{R_2}{R_1 + R_2} V_{in} (1 + \frac{R_4}{R_3})$

$V_4 = V_{IO} (1 + \frac{R_4}{R_3})$

$V_{out} = V_1 + V_2 + V_3 + V_4$

$= I_- R_4 + (\frac{R_2}{R_1+R_2} V_{in} + V_{IO} - I_+(R_1//R_2))(1 + \frac{R_4}{R_3})$

ii. $14 = 10\mu R_4 + (\frac{2k}{2k+2k} \times 510m + 10m - 10m)(1 + \frac{R_4}{2k})$

$= 10\mu R_4 + 255m (1 + \frac{R_4}{2k})$

$= 137.5\mu R_4 + 0.255$

$R_4 \approx 100k\Omega$

* The solution is only for your references

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61.答 Assume $I_D = I_s e^{\frac{V_D}{nV_T}}$, $V_D = 0.7V$

$$i_d = I_D \frac{V_d}{nV_T}$$

$$\frac{I_D}{I_s} = 4.9266 \times 10^{11}$$

$$i_d = I_s e^{\frac{V_d}{nV_T}} = I_s\left[1 + \frac{V_d}{nV_T}\right] = I_D \frac{V_d}{nV_T}$$

$$\frac{1 + \frac{V_d}{nV_T}}{\frac{V_d}{nV_T}} = \frac{I_D}{I_s} = 4.9266 \times 10^{11}$$

$$V_d = 7.81 \times 10^{-11}\ V$$

*The solution is only for your references

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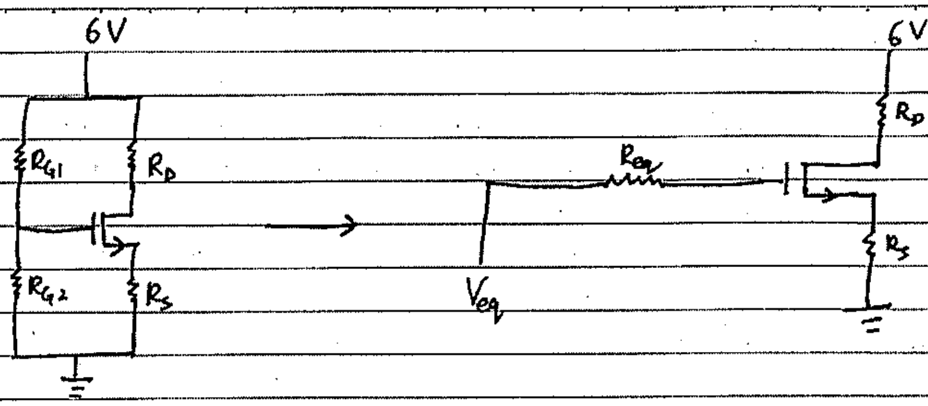
2a.



$$V_{eq} = \frac{R_{G2}}{R_{G1}+R_{G2}} \times 6 = \frac{2}{3} \times 6 = 4V$$

$$R_{eq} = R_{G1} // R_{G2} = 33.33k\Omega$$

$$V_{eq} = V_{GS} + I_D R_S \Rightarrow 4 = V_{GS} + \frac{0.2m}{2}(V_{GS}-1)^2 \times 10k$$

$$4 = V_{GS} + (V_{GS}-1)^2$$

$$V_{GS} = 3.79V$$

$$I_D = \frac{4-3.79}{10k} = 21\mu A$$

$$V_{DS} = 6 - I_D R_D - I_D R_S$$

$$= 6 - 21\mu(10k + 10k)$$

$$= 5.58V$$

$\because V_{DS} > V_{GS} - V_{TN}$

$\therefore$ M is in saturation region

Q (21μA, 3.79V)

* The solution is only for your references

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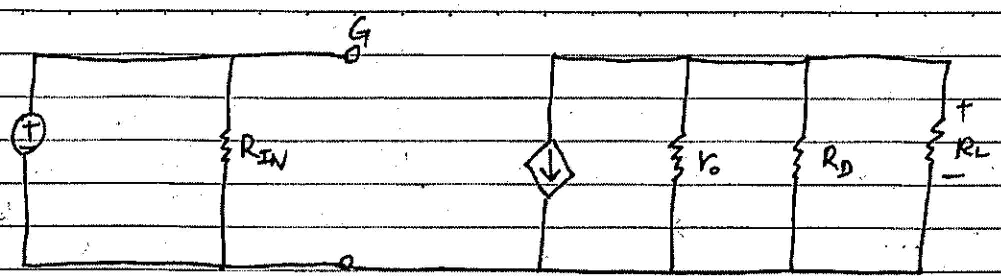
b.



$A_{vt} = -g_m R_L'$

$= -91.65\mu(R_D // R_L)$

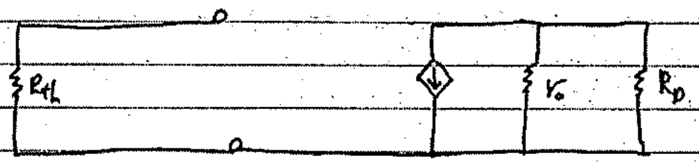
$= -91.65\mu(10k // 100k)$

$A_v = -0.833$

$g_m = \sqrt{2 \times 0.2m \times 21\mu}$

$= 91.65\,\mu S$

$R_{in} = R_{G1} // R_{G2} = 33.33k\,\Omega$



$R_{th} = R_{G1} // R_{G2} = 33.33k\,\Omega$

$R_{out} = r_o // R_D$

$= 4.762M // 10k$

$= 9.98k\,\Omega$

$r_o = \frac{1}{\lambda I_D}$

$= \frac{1}{0.01 \times 21\mu}$

$= 4.762M\Omega$

c. $|V_i| \leq 0.2(V_{GS} - V_{TN})\left(\frac{10k + R_{in}}{R_{in}}\right)$

$= 0.2(3.79 - 1)\left(\frac{10k + 33.33k}{33.33k}\right)$

$= 0.725\,V$

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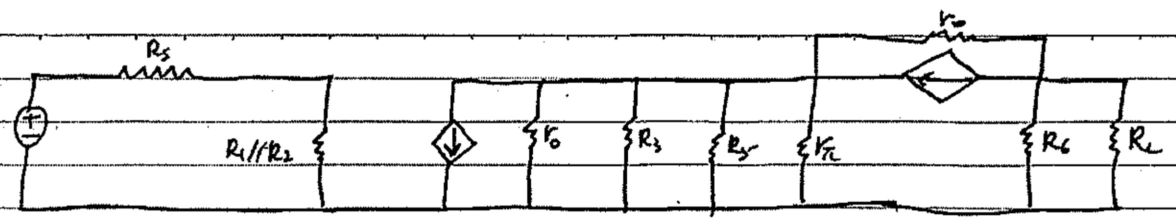
3a.



$A_{vt1} = -g_{m1}(R_3 // R_5)$ $\qquad g_{m1} = \sqrt{2K_nI_D}$

$= -g_m(50k // 1.76k)$ $\qquad = \sqrt{2 \times 0.5m \times 83.6\mu}$

$= -0.4913$ $\qquad = 0.289mS$

$A_{vt2} = g_{m2}(R_6 // R_L)$ $\qquad g_{m2} = \frac{I_c}{V_T}$

$= g_{m2}(8.75k // 100k)$ $\qquad = \frac{1.27m}{25m}$

$= 410.3$ $\qquad = 0.051mS$

$A_{vt} = A_{vt1} \times A_{vt2} = -201.6$

$A_v = A_{vt} \times \frac{R_{in}}{R_s + R_{in}}$ $\qquad R_{in} = R_1 // R_2$

$= 201.6 \times \frac{3.429M}{1k + 3.429M}$ $\qquad = 8M // 6M$

$= \cancel{201.6}\ 201.5$ $\qquad = 3.429M\Omega$

b. $R_{in} = 3.429M\Omega$

$R_{out} = \left[1 + g_{m1}\left(\frac{r_o // R_3}{R_5}\right)\right] r_o // R_6$ $\qquad r_o = \frac{1}{\lambda I_D}$

$= \left[1 + 0.289m\left(\frac{598k // 50k}{1.76k}\right)\right] \times 598k \ // 8.75k$ $\qquad = \frac{1}{0.02 \times 83.6\mu}$

$= 8.625k\Omega$ $\qquad \simeq \cancel{600}\ 598 k\Omega$

* The solution is only for your references

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c. $V_2 = \frac{R_{in}}{R_I + R_{in}} \times A_{vnl} V_i$; $|V_2| \leq 0.005$

$|V_2| \leq \frac{0.005(R_s + R_{in})}{R_{in} A_{vnl}}$

$= \frac{0.005(1k + 3.429M)}{3.429M \times 0.4913}$

$= 0.0102V$

* The solution is only for your references

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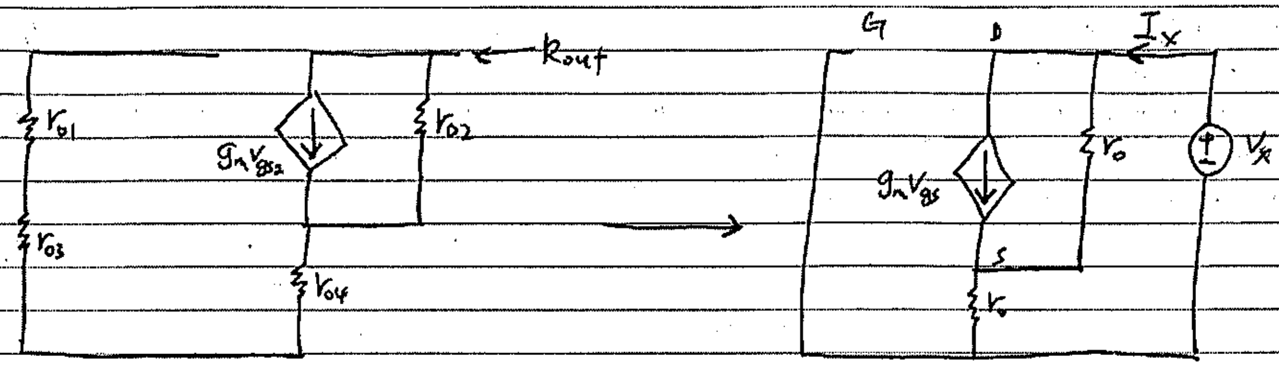
4a.



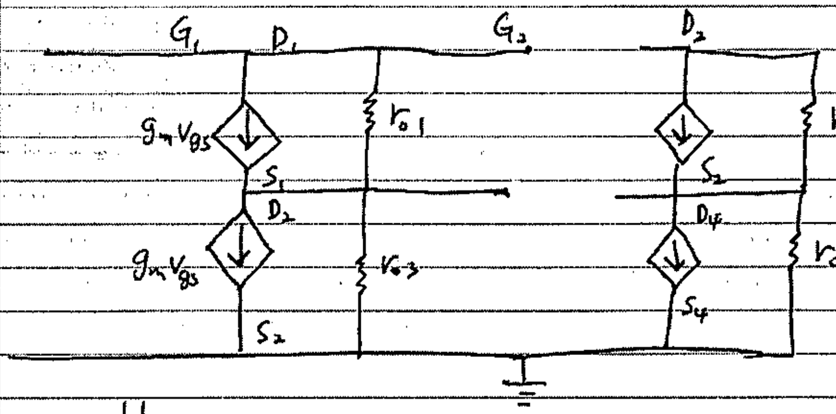
$$I_x = g_m V_{gs} + \frac{V_x - V_{gs}}{r_o}$$

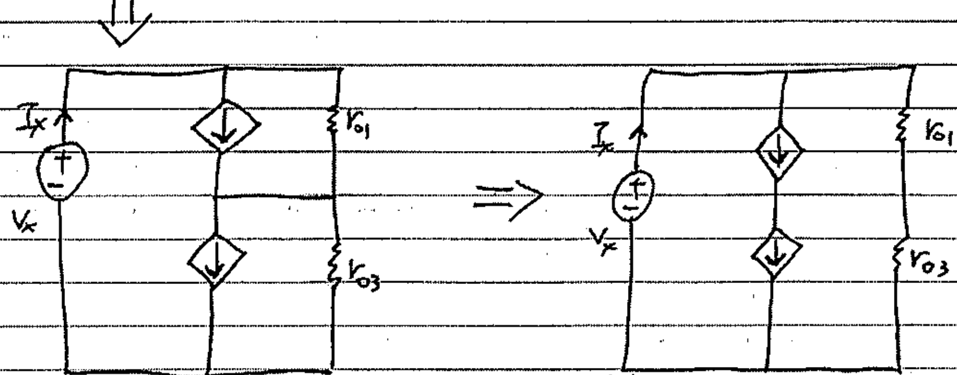
$$V_{gs} = -V_s = -I_x r_o$$

$$I_x = -g_m r_o I_x + \frac{V_x}{r_o} + I_x$$

$$\frac{V_x}{I_x} = g_m r_o^2 = R_{out}$$

b.





$\because$ Identical, $\therefore g_m V_{gs1} = g_m V_{gs3}$, $r_{o1} = r_{o3}$, $2V_{gs} = V_x$

$$I_x = g_m V_{gs} + \frac{V_x}{2r_o} \Rightarrow I_x = \frac{g_m V_x}{2} + \frac{V_x}{2r_o}$$

$$\frac{V_x}{I_x} = \frac{2r_o}{g_m r_o + 1} = R_{in}$$

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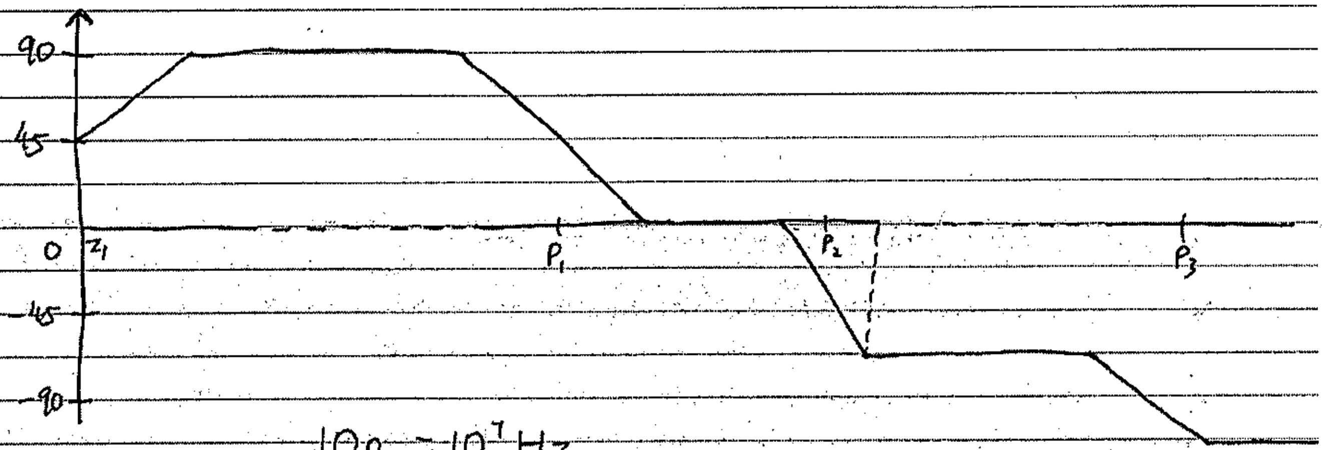
5ai $\dfrac{200\omega}{(1+j\cdot\frac{\omega}{20^3})(1+j\cdot\frac{\omega}{10^6})(1+\frac{\omega}{10^9})}$

~~$z_1=0$~~ $z_1=0$

$P_1=20^3$, $P_2=10^6$, $P_3=10^9$

ii.



$10P_2=10^7\,Hz$

iii. gain (dB) $=20\left[\log 200+\log 10^7-\log\frac{10^7}{20^3}-\log\frac{10^7}{10^6}\right]$

$=104\,dB$

*The solution is only for your references

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~~bi. Answer~~ $\cancel{= \left(\frac{R_L}{R_L + R_{cc2}}\right)\left(\frac{R_\pi}{R_s + R_{cc1}}\right)\{-g_m[R_o//(R_L + R_{cc2})]\}}$

~~4 zeros, 4 poles~~

b ii. Short C_{c2}

$\cancel{R_{cc1} = R_s + R_\pi}$

$\cancel{= 5100\,\Omega}$

i. $\cancel{V_\pi = \frac{R_\pi}{R_s + R_\pi + \frac{1}{sC_{c1}}} V_i \quad (1)}$

$\cancel{V_o = -(g_m V_\pi + I_{R0}) R_L \quad (2)}$

~~Sub (1) into (2),~~ $\cancel{V_o = -\left[g_m\left(\frac{R_\pi}{R_s+R_\pi+\frac{1}{sC_{c1}}}\right)V_i + I_{R0}\right]R_L}$

$\cancel{\approx -g_m R_L\left(\frac{R_\pi}{R_s+R_\pi+\frac{1}{sC_{c1}}}\right)V_i}$

$\cancel{\frac{V_o}{V_i} \approx -g_m R_L\left(\frac{R_\pi}{R_s+R_\pi+\frac{1}{sC_{c1}}}\right)}$

$\cancel{= \frac{-g_m R_L R_\pi s C_{c1}}{s(C_1 R_s + C_1 R_\pi + \frac{1}{s})}}$

~~Zero: 1~~

~~Pole: 1~~

$\cancel{-I(R_L + \frac{1}{sC_{c2}}) = V_x \Rightarrow V_x = \frac{-V_o}{R_L}\left(R_L + \frac{1}{sC_{c2}}\right) \quad (1)}$

$\cancel{iR_\pi = V_\pi}$

$\cancel{V_i = i[R_s + sC_{c1} + R_\pi] \Rightarrow V_i = \frac{V_\pi}{R_\pi}[R_s + \frac{1}{sC_{c1}} + R_\pi]}$

$\cancel{V_\pi = \frac{sR_\pi V_i C_{c1}}{[R_s C_{c1} + 1 + R_\pi s C_{c1}]} \quad (2)}$

$\cancel{g_m V_\pi = \left(\frac{V_x}{R_o} + \frac{V_x}{\frac{1}{sC_{c2}} + R_L}\right) \quad (3)}$

~~Sub (1) & (2) into (3),~~ $g_m \cdot \frac{sR_\pi V_i C_{c1}}{R_s C_{c1} + 1 + R_\pi s C_{c1}} = \frac{V_o}{R_L}\left(R_L + \frac{1}{sC_{c2}}\right)\left(\frac{1}{R_o} + \frac{1}{\frac{1}{sC_{c2}} + R_L}\right)$

$\frac{V_o}{V_i} = -\frac{g_m sR_\pi C_1 R_L}{sRC_{c1} + 1 + sR_\pi C_{c1}} \cdot \frac{sC_{c2}}{1 + sR_L C_{c2}} \cdot \frac{R_o \cdot (R_L + \frac{1}{sC_{c2}})}{R_o + \frac{1}{sC_{c2}} + R_L}$

Zeros: 4

Poles: 4