

Dependent Random Weighting

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Introduction

We were interested in learning about resampling methods for irregularly spaced time series data. This led us to read the paper

"The Dependent Random Weighting" (2015) by Srijan Sengupta, Xiaofeng Shao, and Yingchuan Wang.

The paper:

- Introduces a method that assigns random weights to the irregular time series data
- Weights are created using a dependence structure that mimics that of the observed data

Irregular Time Series Data

Irregular time series data can occur in two ways.

1. **Missing Values:** Time series occurs at equally space intervals but not all data points are observed



2. **Unequal Intervals:** Times when the data are observed are generated from a 1-D point process



Dependent Random Weighting (the process)

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Dependent Random Weighting (theorems)

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Dependent Random Weighting (their simulations)

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Our Simulations: Overview

We wanted to apply and compare DRW to methods learned in STAT 651. We decided to compare the following situations.

- **Methods:** DRW versus MBB
- **Data:** MA versus AR time series
- **Estimators:** mean versus median
- **Bandwidth:** blocksize versus l -dependence

Note on irregular data type:

- Paper used unequal time intervals (type 2)
- We used equal time intervals with missing values (type 1)

Our Simulations: The Procedure

We used the following procedure for our simulations.

1. **Generate irregular time series of size $n = 200$.**

- (i) Simulate y_t for $t = 1, \dots, n$ from
 - an MA process with $\mu = 0$, $\theta_1 = -1$, and $\theta_2 = 0.7$ or
 - an AR process with $\mu = 0$, $\phi_1 = -0.1$, or $\phi_2 = 0.6$).
- (ii) Assign a weight ω_t to y_t where

$$\omega_t = \sin\left(\frac{\pi \cdot t}{n}\right).$$

- (iii) Generate $z_t \sim \text{binomial}(\omega_t)$ for $t = 1, \dots, n$.
- (iv) Let

$$x_t = \begin{cases} y_t & \text{if } z_t = 1 \\ \text{missing} & \text{if } z_t = 0 \end{cases}$$

for $t = 1, \dots, n$.

- (v) Reindex the non-missing x_t as x_i for i from 1 to n_j and use as the observed sample.

Our Simulations: The Procedure

2. **Let $\ell = 1$, and apply the resampling method to $K = 1000$ samples.**
 - MBB: Draw block bootstrap samples from x_1, \dots, x_{n_j} with blocks of size $b = \ell$. (ignores missing values)
 - DRW: Randomly assign weights to x_1, \dots, x_{n_j} using the method from the paper assuming m -dependence with $m = \ell$.
3. **Compute the mean and median from the K samples.**
4. **Use the distributions of means and medians to compute evaluative measures.**
 - Determine if the 95% confidence interval contains the true value. (Note: True process medians were approximated using 100,000 Monte Carlo simulations.)
 - Compute the standard deviation of the distribution. (Denote this as $\sigma_{n_j}^{(j)} / \sqrt{n_j}$.)

Our Simulations: The Procedure

5. Repeat steps 1 to 4 for $M = 500$ times.

6. Compute final evaluative measures.

- Coverage rate for both the mean and median
- MSE:

$$\frac{1}{M} \sum_{j=1}^M \left(\frac{\sigma_{n_j}^{(j)}}{\sqrt{n_j}} - \frac{\sigma_n}{\sqrt{n}} \right)^2$$

- Normalized MSE:

$$\frac{1}{M} \sum_{j=1}^M \left(\frac{n_j \sigma_{n_j}^{(j)}}{n \sigma_n} - 1 \right)^2$$

- Note: $\sigma_n = \sqrt{n} \text{Var}(\hat{\theta}_n)$ where $\hat{\theta}_n$ is the estimator of interest. σ_n was approximated using 100,000 Monte Carlo simulations for both the mean and median.

7. Repeat steps 1 to 6 for $\ell = 2, \dots, 10$.

Our Simulations: Results for Means

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Our Simulations: Results for Medians

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Our Simulations: Results for Computing Time

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Conclusion and Ideas for Future Work

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