Dependent Random Weighting

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Introduction

We were interested in learning about resampling methods for irregularly spaced time series data. This led us to read the paper

> *"The Dependent Random Weighting" (2015) by Srijan Sengupta, Xiaofeng Shao, and Yingchuan Wang.*

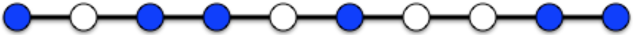
The paper:

- Introduces a method that assigns random weights to the irregular time series data
- Weights are created using a dependence structure that mimics that of the observed data

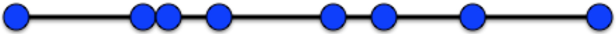
Irregular Time Series Data

Irregular time series data can occur in two ways.

1. **Missing Values**: Time series occurs at equally space intervals but not all data points are observed



2. **Unequal Intervals**: Times when the data are observed are generated from a 1-D point process



Dependent Random Weighting (the process)

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Dependent Random Weighting (theorems)

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Dependent Random Weighting (their simulations)

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Our Simulations: Overview

We wanted to apply and compare DRW to methods learned in STAT 651. We decided to compare the following situations.

- **Methods**: DRW versus MBB
- **Data**: MA versus AR time series
- **Estimators**: mean versus median
- **Bandwidth**: blocksize versus l-dependence

Note on irregular data type:

- Paper used unequal time intervals (type 2)
- We used equal time intervals with missing values (type 1)

Our Simulations: The Procedure

We used the following procedure for our simulations.

1. **Generate irregular time series of size $n = 400$.**

   (i) Simulate y_t for $t = 1, ..., n$ from
   - an MA process with $\mu = 0$, $\theta_1 = -1$, and $\theta_2 = 0.7$ or
   - an AR process with $\mu = 0$, $\phi_1 = -0.1$, or $\phi_2 = 0.6$.

   (ii) Assign a weight ω_t to y_t where

   $$\omega_t = \sin\left(\frac{\pi \cdot t}{n}\right).$$

   (iii) Generate $z_t \sim \mathit{binomial}(\omega_t)$ for $t = 1, ..., n$.

   (iv) Let

   $$x_t = \begin{cases} y_t & \text{if } z_t = 1 \\ \text{missing} & \text{if } z_t = 0 \end{cases}$$

   for $t = 1, ..., n$.

   (v) Reindex the non-missing x_t as x_i for i from 1 to n_j and use as the observed sample.

Our Simulations: The Procedure

2. **Let $\ell = 1$, and apply the resampling method to $K = 1000$ samples.**

    - MBB: Draw block bootstrap samples from $x_1, ..., x_{n_j}$ with blocks of size $b = \ell$. (ignores missing values)
    - DRW: Randomly assign weights to $x_1, ..., x_{n_j}$ using the method from the paper assuming m-dependence with $m = \ell$.

3. **Compute the mean and median from the K samples.**

4. **Use the distributions of means and medians to compute evaluative measures.**

    - Determine if the 95% confidence interval contains the true value. (True process medians were approximated using 100,000 Monte Carlo simulations.)
    - Compute the standard deviation of the distribution. (Denote this as $\sigma_{n_j}^{(j)} / \sqrt{n_j}$.)

Our Simulations: The Procedure

5. **Repeat steps 1 to 4 for $M = 500$ times.**
6. **Compute final evaluative measures.**
   - Coverage rate for both the mean and median
   - MSE:
   $$\frac{1}{M}\sum_{j=1}^{M}\left(\sigma_{n_j}^{(j)} - \sigma_n\right)^2$$
   - Normalized MSE:
   $$\frac{1}{M}\sum_{j=1}^{M}\left(\frac{n_j\sigma_{n_j}^{(j)}}{n\sigma_n} - 1\right)^2$$
   - Note: $\sigma_n = \sqrt{n}Var(\hat{\theta}_n)$, where $\hat{\theta}_n$ is the estimator of interest, was approximated using 100,000 Monte Carlo simulations for both the mean and median.
7. **Repeat steps 1 to 6 for $\ell = 2, ..., 10$.**

Our Simulations: Results for Means

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Our Simulations: Results for Medians

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Our Simulations: Results for Computing Time

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Conclusion

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