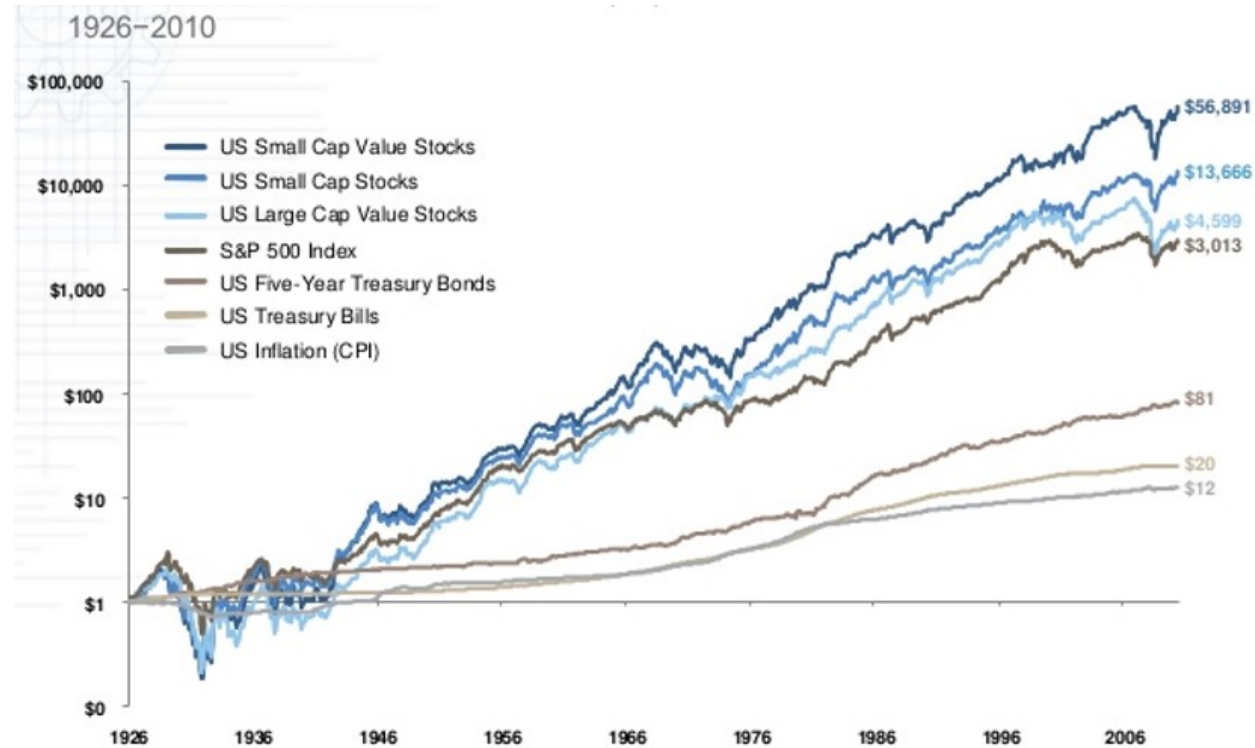


MBAS 821: Topic 5

Risk and Return

Ryan Riordan

\$1 in 1926 is now worth:



The return really depends on what you were investing in. But so does the risk...

Overview



- Return of a Security
- Risk of a Security
- Risk and Return of a Portfolio
- Diversification

Return of a Security

Holding Period Return (HPR)

Recall the formula for HPR:

$$HPR = \frac{P_n - P_0 + \sum_{t=1}^n \text{Income}_t}{P_0}$$

Note: we changed CPN to Income so that the formula can be used for all securities, not just bonds.

- For a stock, the income generated from stocks is in the form of dividend.

HPR: Example



Assume 10 shares of stock "Get Rich Quick" is bought immediately after issuing a dividend and sold immediately after issuing a dividend one year later. The stock was bought for \$22, sold for \$27 and the dividend was \$2. What was the holding period return?

HPR: Example

Assume 10 shares of stock "Get Rich Quick" is bought immediately after issuing a dividend and sold immediately after issuing a dividend one year later. The stock was bought for \$22, sold for \$27 and the dividend was \$2. What was the holding period return?

Answer:

For buying one share:

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} = \frac{27 - 22 + 2}{22} = 31.8\%$$

For 10 shares:

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} = \frac{270 - 220 + 20}{220} = 31.8\%$$

They are the same - return is a percentage and is not affected by the amount of initial investment.

Historical Return

There are two ways of calculating the historical returns:

- Arithmetic Average Return:

$$\bar{r} = \frac{1}{n} \sum_{t=1}^n r_t$$

- Geometric Average Return:

$$r_{geo} = \left[\prod_{t=1}^n (1 + r_t) \right]^{\frac{1}{n}} - 1$$

Historical Return: Example

You bought a stock at \$100 in year 0. The returns are 5%, -10%, +15% in the next 3 years.

What is the arithmetic average return?

What is the geometric average return?

Historical Return: Example

You bought a stock at \$100 in year 0. The returns are +10%, -30%, +25% in the next 3 years.

What is the arithmetic average return?

What is the geometric average return?

Answer:

$$\bar{r} = \frac{10\% - 30\% + 25\%}{3} = 1.67\%$$

The arithmetic average return is 1.67%

$$r_{geo} = \left[(1 + 0.1) \times (1 - 0.3) \times (1 + 0.25) \right]^{\frac{1}{3}} - 1 = -1.27\%$$

The geometric average return is -1.27%

Historical Return: Discussion

- The arithmetic return is the return assuming you invest the same amount every year.
- The geometric return is the return assuming you make the invest at the beginning and hold until the end date.
- The geometric average return is always lower than the arithmetic average return.

Which one to use?

- Use geometric return when calculating historical return of holding an investment over a period of time. (e.g. \$1 of investment from 1926 to 2020.)
- Use arithmetic return when estimating the expected return of securities for a year. (e.g. What is the expected return of a random year?)

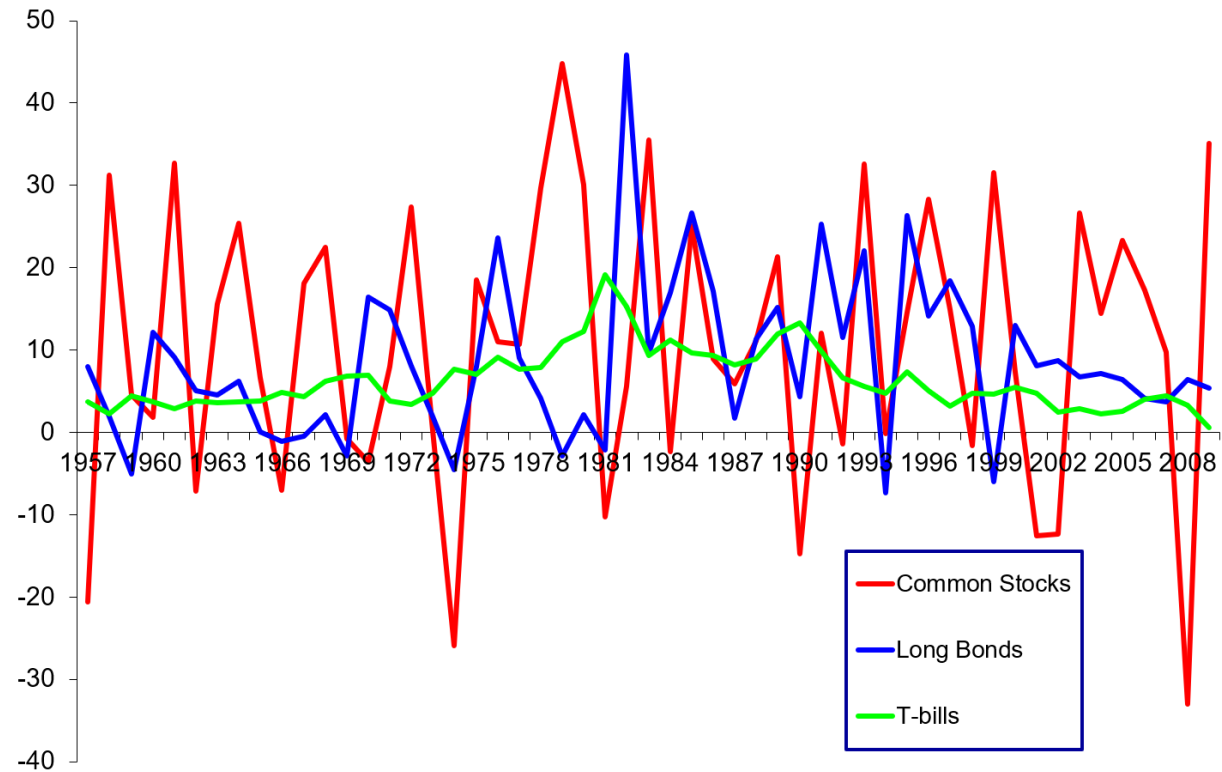
Return of a Security

Risk

Different asset types have different risks:

- Cash equivalents
 - Debt securities that are very liquid, very low-risk and very short-term (e.g. T-bills)
- Bonds
 - Debt instruments that have a longer maturity than cash (e.g. Long term bonds)
- Stocks
 - Ownership of shares in publicly held corporations (e.g. Firms listed on TSX or NYSE)

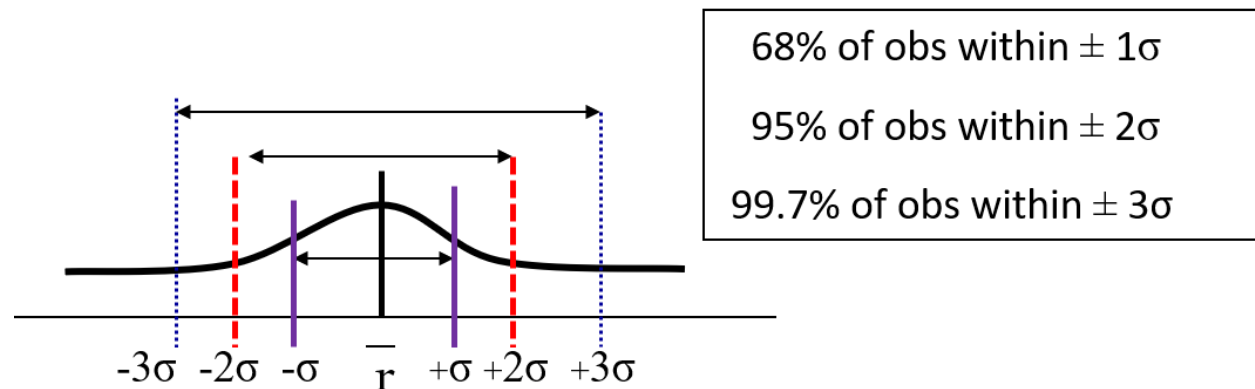
Risk



Rates of Return (1957 - 2009)

Risk

- The spread of a return distribution is a measure of how much an observed return may deviate from its mean value.
- We usually use the standard deviation (σ) to represent the spread of a normal distribution, often referred to as its “risk”.
 - Standard deviation is the square root of variance.



Risk

Computing Variance and Standard Deviation

$$\sigma^2 = \frac{\sum_{t=1}^n (r_t - \bar{r})^2}{n - 1}$$

Note: σ is the **standard deviation** of the return. It can be computed by taking the square root of the variance σ^2 .

Risk: Example

Calculate the average return for Berkshire Hathaway (BRK.B) shares and their standard deviation from data in the following table. No dividends were paid.

Date	Price
Dec 31 2013	118.56
Dec 31 2014	150.15
Dec 31 2015	132.04
Dec 31 2016	162.98
Dec 31 2017	198.22
Dec 31 2018	204.18

What are the variance and standard deviation of the returns?

Note: If dividends were paid you would need to include them when calculating the annual returns.

Risk: Example

Answer:

Date	Price	Annual_Return	Return_Diff	Return_Diff_Sq
Dec 31 2013	118.56	NA	NA	NA
Dec 31 2014	150.15	0.2664474	0.1411578	0.0199255
Dec 31 2015	132.04	-0.1206127	-0.2459023	0.0604680
Dec 31 2016	162.98	0.2343229	0.1090333	0.0118883
Dec 31 2017	198.22	0.2162228	0.0909332	0.0082689
Dec 31 2018	204.18	0.0300676	-0.0952220	0.0090672

$$\sigma^2 = \frac{0.0199 + 0.0605 + 0.0119 + 0.0083 + 0.0091}{5 - 1} = 0.0274045$$

$$\sigma = 16.55\%$$

Risk Premium

- Risk Premium = Return – r_f
 - Excess return above the risk-free rate which stems from holding risky assets rather than risk-free ones (e.g. T-bills)
- A key empirical observation from stock & bond data is the existence of long-run excess returns:
 - Average excess return (risk-premium) to Canadian common stocks between 1957 and 2016 was: **4.33%** = 10.3% – 5.97%
 - Average excess return (risk premium) to Canadian long-term bonds between 1957 and 2016 was: **2.19%** = 8.16% – 5.97%

Equity Market Risk Premium

The **Equity Market Risk Premium** is the additional return (over and above the risk-free rate) resulting from bearing risk.

Average Canadian Investment Returns, 1938 - 2011

	<u>Annual Arithmetic Mean (%)</u>	<u>Annual Geometric Mean (%)</u>
Government of Canada Bonds	6.44	6.08
Canadian Stocks	11.35	10.03
<i>Implied Equity Market Risk Premium:</i>		
Canadian Stocks - Canadian Government Bonds	4.91	3.95

- The lower the Equity Market Risk Premium, the lower the discount rate.
- The lower the discount rate, the higher the valuation!

Expected Return and Risk of a Security

Expected Return:

$$E(R) = \sum_{i=1}^N R_i Pr(i)$$

$$\sigma^2 = \sum_{i=1}^N \left[(R_i - E(R))^2 Pr(i) \right]$$

Where:

$E(R)$ = Expected Return

N = Number of possible states

R_i = Return when in state i

$Pr(i)$ = Probability of state i happening.

Expected Return and Risk: Example



Return estimates for Company Corp. Bust: $r = 10\%$ with probability 0.20 Normal: $r = 15\%$ with probability 0.50 Boom: $r = 20\%$ with probability 0.30 Compute the expected return and standard deviation for Company Corp.

Expected Return and Risk: Example

Return estimates for Company Corp. Bust: $r = 10\%$ with probability 0.20 Normal: $r = 15\%$ with probability 0.50 Boom: $r = 20\%$ with probability 0.30 Compute the expected return and standard deviation for Company Corp.

Answer:

$$E(R) = (0.1 \times 0.2) + (0.15 \times 0.5) + (0.2 \times 0.3) = 15.5\%$$

$$\sigma^2 = (0.2 \times (0.1 - 0.155)^2) + (0.5 \times (0.15 - 0.155)^2) + (0.3 \times (0.2 - 0.155)^2) = 0.001225$$

$$\sigma = \sqrt{0.001225} = 0.035 = 3.5\%$$

Breakout Activity: Historical return

Go to **Yahoo! Finance** Maybe the only thing Yahoo is doing better than Google.

Pick a stock with > 10 years available historical prices.

Compute the average (arithmetic and geometric) monthly return.

Compute the standard deviation of the historical return.

Report to the class about your findings.

Risk and Return of a Portfolio

Portfolio Expected Return (R_p)

A portfolio's expected return R_p is a **weighted average** of the individual assets' expected returns.

$$E(R_p) = \sum_{i=1}^N W_i E(R_i)$$

Where:

- W_i : Weight of the i^{th} asset in the portfolio.
 - Note: $\sum_{i=1}^N W_i = 1$
- $E(R_i)$: Expected return of the i^{th} asset in the portfolio.
- N : Total number of assets in the portfolio

Portfolio Expected Return: Example

Calculate the *expected annual return* of the following portfolio:

Stock	Invested_Amount	Expected_Annual_Return
A	\$100	10%
B	\$300	20%

Portfolio Expected Return: Example

Calculate the *expected annual return* of the following portfolio:

Stock	Invested_Amount	Expected_Annual_Return
A	\$100	10%
B	\$300	20%

Answer:

$$W_A = \frac{100}{100 + 300} = 25\% \quad W_B = \frac{300}{100 + 300} = 75\%$$

$$\begin{aligned} E(R_p) &= W_A E(R_A) + W_B E(R_B) \\ &= (0.25 \times 0.1) + (0.75 \times 0.2) = 17.5\% \end{aligned}$$

The expected annual return of the portfolio is 17.5%.

Portfolio Variance

The variance of a portfolio is slightly more complicated to compute:

For a portfolio with 2 assets, A and B:

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B \text{cov}(R_A, R_B)$$

Where:

- σ_P^2 is the variance of the portfolio.
- W_A, W_B are weights of asset A and B.
- σ_A^2, σ_B^2 are variance of returns of asset A and B.
- $\text{cov}(R_A, R_B)$ is the **covariance** between the returns of asset A and B.

How to calculate covariance?

Covariance

Covariance is a way to measure whether two random variables are related, and how.

- Positive when returns of two assets tend to move in the same direction, and vice versa.

For two assets A and B:

$$\text{Covariance}(cov(R_A, R_B)) = \sum_{i=1}^N Pr(i)(R_{A,i} - E(R_A))(R_{B,i} - E(R_B))$$

Where:

- N is the total number of possible states.
- $Pr(i)$ is the probability of state i .
- $R_{A,i}$ and $R_{B,i}$ are the returns of asset A and B at state i .
- $E(R_A)$ and $E(R_B)$ are the expected returns of asset A and B.

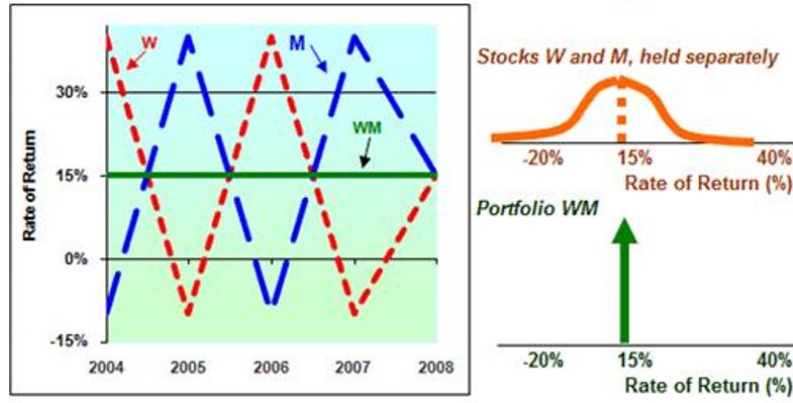
Correlation

Correlation normalizes covariance so that the parameter is bounded between -1 and +1.

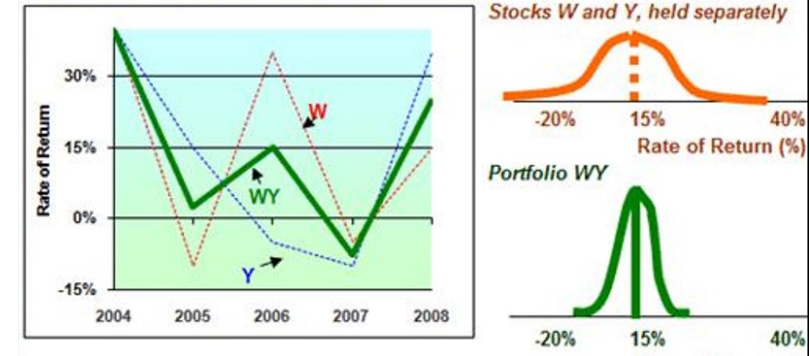
$$\text{Correlation}(\text{corr}(R_A, R_B)) = \rho_{A,B} = \frac{\text{Cov}(R_A, R_B)}{\sigma_A \sigma_B}$$

- A correlation of -1 means that the two asset prices move at completely opposite directions.
- A correlation of +1 means that the two asset prices move together in the same direction.
- A correlation of 0 means that the two asset price movements are unrelated.

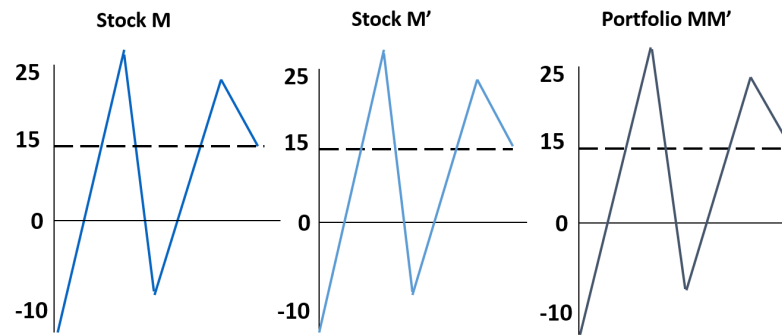
Correlation: Graphs



Correlation = -1



Correlation = 0.35



Correlation = +1

Portfolio Risk: Example

State	Probability	Return_A	Return_B
Recession	30%	0%	-5%
Normal	40%	10%	20%
Boom	30%	20%	35%

What is the correlation of A and B?

What is the variance and the standard deviation of a portfolio consisting of 25% A and 75% B?

Portfolio Risk: Example

State	Probability	Return_A	Return_B
Recession	30%	0%	-5%
Normal	40%	10%	20%
Boom	30%	20%	35%

Find the expected returns and risks of asset A and B:

$$E(R_A) = 0.3 \times 0\% + 0.4 \times 10\% + 0.3 \times 20\% = 10\%$$

$$E(R_B) = 0.3 \times (-5\%) + 0.4 \times 20\% + 0.3 \times 35\% = 17\%$$

$$\sigma_A = \sqrt{0.3 \times (0 - 10\%)^2 + 0.4 \times (10\% - 10\%)^2 + 0.3 \times (20\% - 10\%)^2} = 7.75\%$$

$$\sigma_B = \sqrt{0.3 \times (-5\% - 17\%)^2 + 0.4 \times (20\% - 17\%)^2 + 0.3 \times (35\% - 17\%)^2} = 15.68\%$$

Portfolio Risk: Example

State	Probability	Return_A	Return_B
Recession	30%	0%	-5%
Normal	40%	10%	20%
Boom	30%	20%	35%

Find covariance and correlation between returns of asset A and B:

$$\begin{aligned} Cov(R_A, R_B) &= \sum_{i=1}^N Pr(i)(R_{A,i} - E(R_A))(R_{B,i} - E(R_B)) \\ &= 0.3 \times (0 - 10\%) \times (-5\% - 17\%) + 0.4 \times (10\% - 10\%) \times (20\% - 17\%) \\ &\quad + 0.3 \times (20\% - 10\%) \times (35\% - 17\%) \\ &= 0.012 \end{aligned}$$

$$\text{Correlation}(\rho_{A,B}) = \frac{Corr(R_A, R_B)}{\sigma_A \sigma_B} = \frac{0.012}{0.0775 \times 0.1568} = 0.9872$$

The correlation between returns of asset A and B is 0.9872.

Portfolio Risk: Example

State	Probability	Return_A	Return_B
Recession	30%	0%	-5%
Normal	40%	10%	20%
Boom	30%	20%	35%

Find the variance and standard deviation of a portfolio containing 25% A and 75% B:

$$\begin{aligned}\sigma_P^2 &= W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_BCov(R_A, R_B) \\ &= 0.25^2 \times 0.0775^2 + 0.75^2 \times 0.1568^2 + 2 \times 0.25 \times 0.75 \times 0.012 \\ &= 0.0187\end{aligned}$$

The variance of the portfolio is 0.0187.

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.0187} = 13.68\%$$

The standard deviation of the portfolio is 13.68%.

Diversification

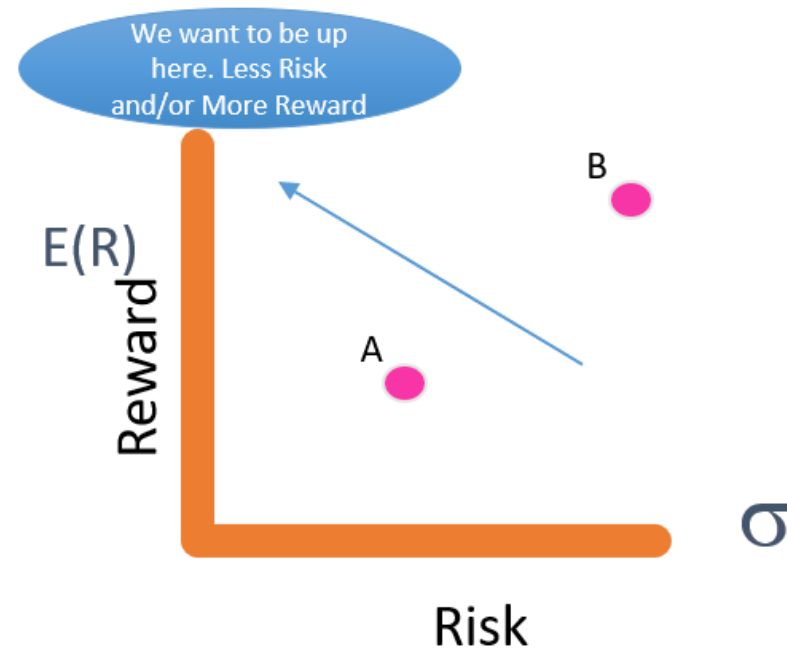


Diversification

Consider a 2 asset portfolio, with assets A and B:

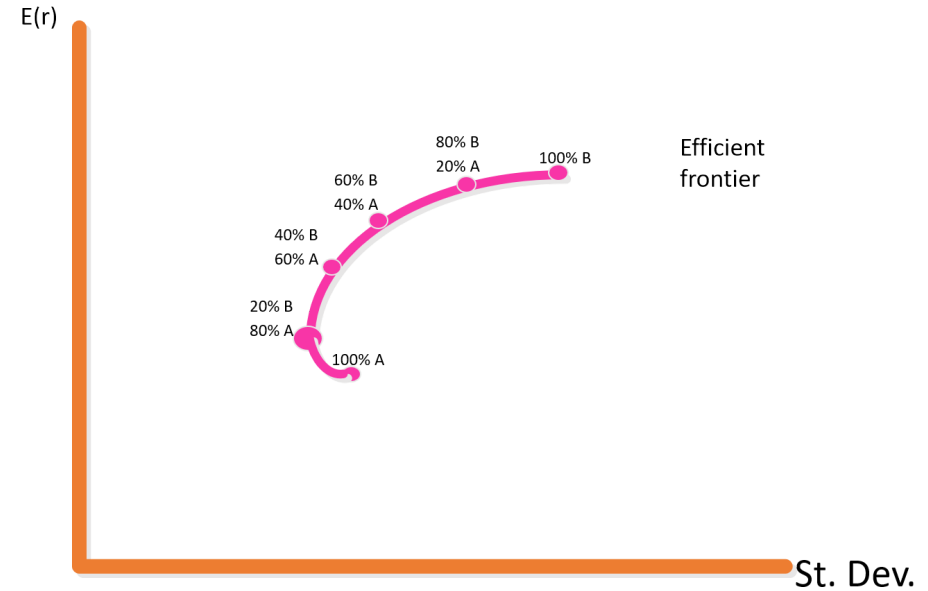
- $E(R_A) = 8\%$, and $\sigma_A = 9\%$
- $E(R_B) = 13\%$, and $\sigma_B = 20\%$
- $Cov(R_A, R_B) = 0.0054$, $\rho_{A, B} = 0.3$

What happens to the portfolio risk and return as we change the weights of A and B? Lets graph this.

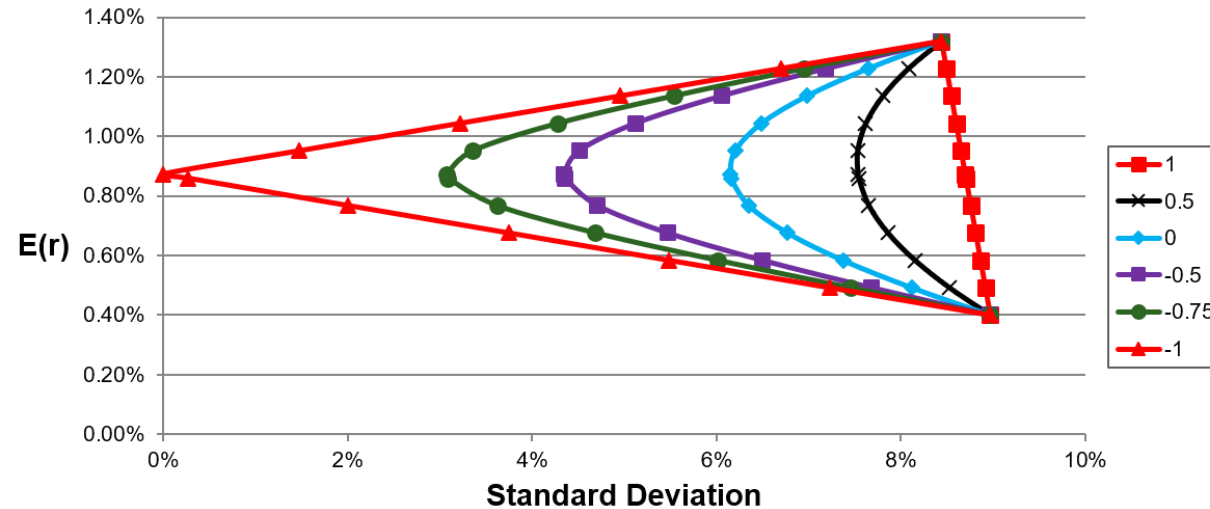


Diversification

Portfolio Weighting			Portfolio	Portfolio
A	B		Return	Std Dev
100%	0%		8.00%	9.00%
90%	10%		8.50%	8.91%
80%	20%		9.00%	9.23%
70%	30%		9.50%	9.92%
60%	40%		10.00%	10.91%
50%	50%		10.50%	12.13%
40%	60%		11.00%	13.52%
30%	70%		11.50%	15.03%
20%	80%		12.00%	16.63%
10%	90%		12.50%	18.29%
0%	100%		13.00%	20.00%



Diversification: Effects

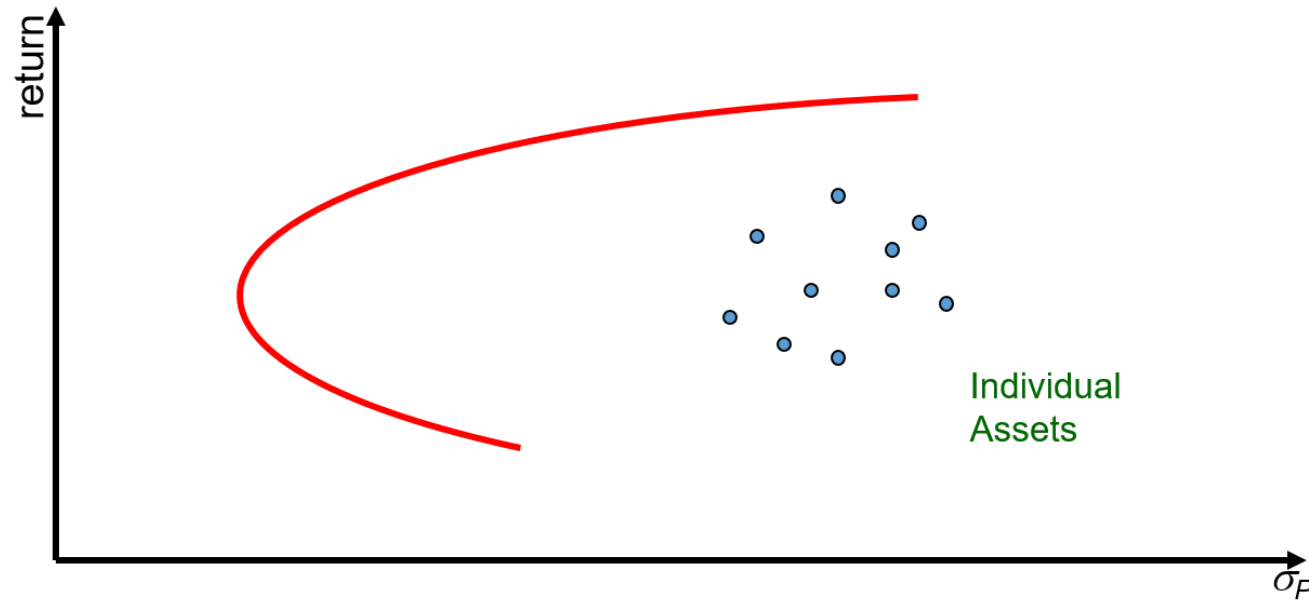


- The standard deviation of a portfolio is **less than** the weighted average of the individual (asset) standard deviations as long as the assets' returns are imperfectly correlated. (i.e. $\rho < 1$)
- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns.
- This risk reduction arises because worse than expected returns from one asset are offset by better than expected returns from another asset. (*Think about owning an Umbrella and Sunglass firms*)

Extension to > 2 Securities



Father of Modern Portfolio
Theory (and Nobel Laureate)
Harry Markowitz

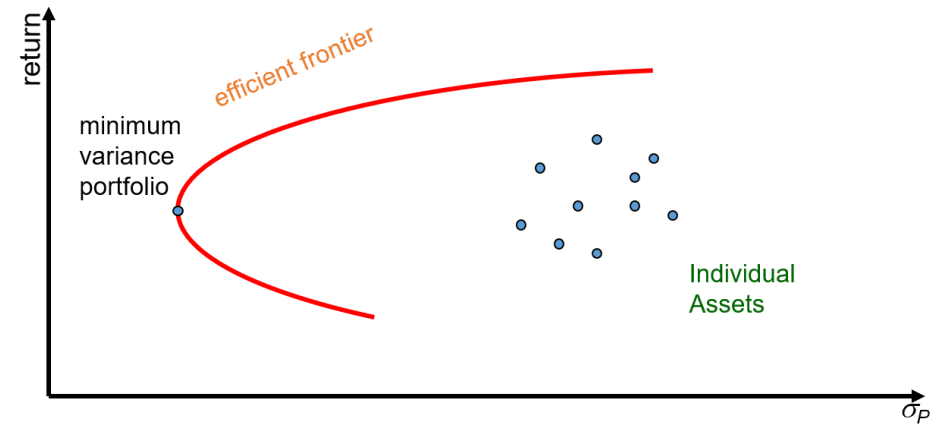


Consider a world with many risky assets; we can still identify the opportunity set of risk-return combinations of various portfolios

Extension to > 2 Securities

Portfolio Optimization Step by Step:

1. Define the feasible set of investments.
2. Locate the minimum variance portfolio.
3. Derive the efficient set. The risk return characteristics of efficient portfolios dominate inefficient portfolios.
4. Choose location on efficient portfolio depending on risk preferences.



The section of the opportunity set above the minimum variance portfolio is the efficient frontier

Risk: A Breakdown

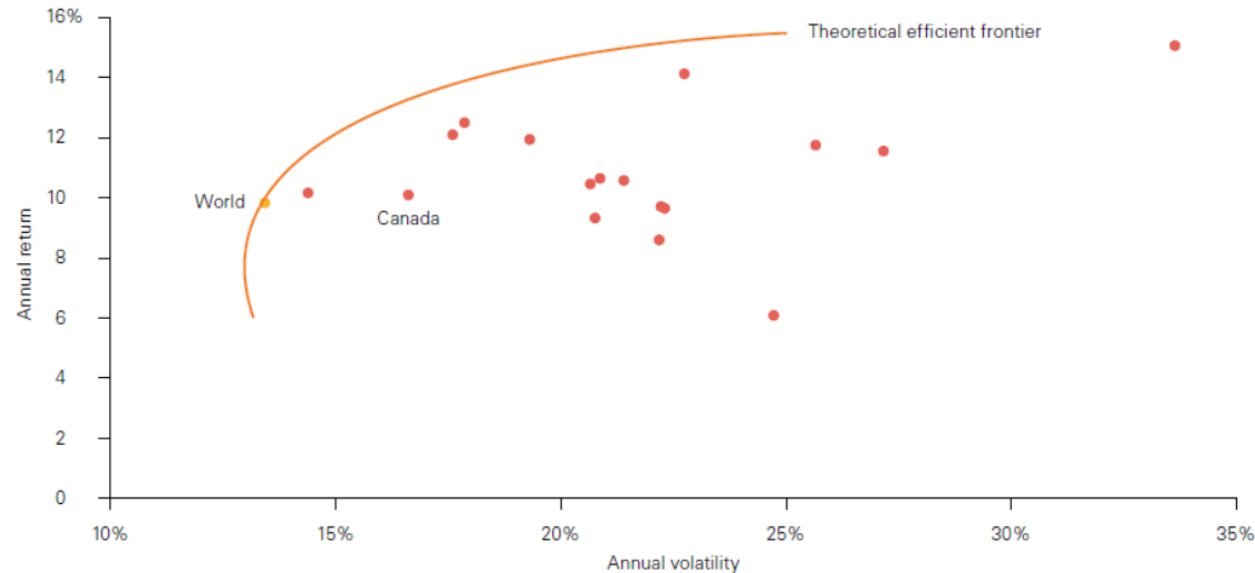
- Variance (risk) of an asset's return can be decomposed
 - **Systematic (Market) Risk**
 - Economy-wide random events that affect almost all assets to some degree
 - **Idiosyncratic (diversifiable) Risk**
 - Risks that is caused by a single security or small groups of securities (e.g. specific industry, country, etc.)
- Diversification Effects
 - **Idiosyncratic risk** is significantly reduced in “large” portfolios
 - **Systematic risk** is not affected by diversification because (almost) all securities in the portfolio are affected by macro-events.

International Diversification

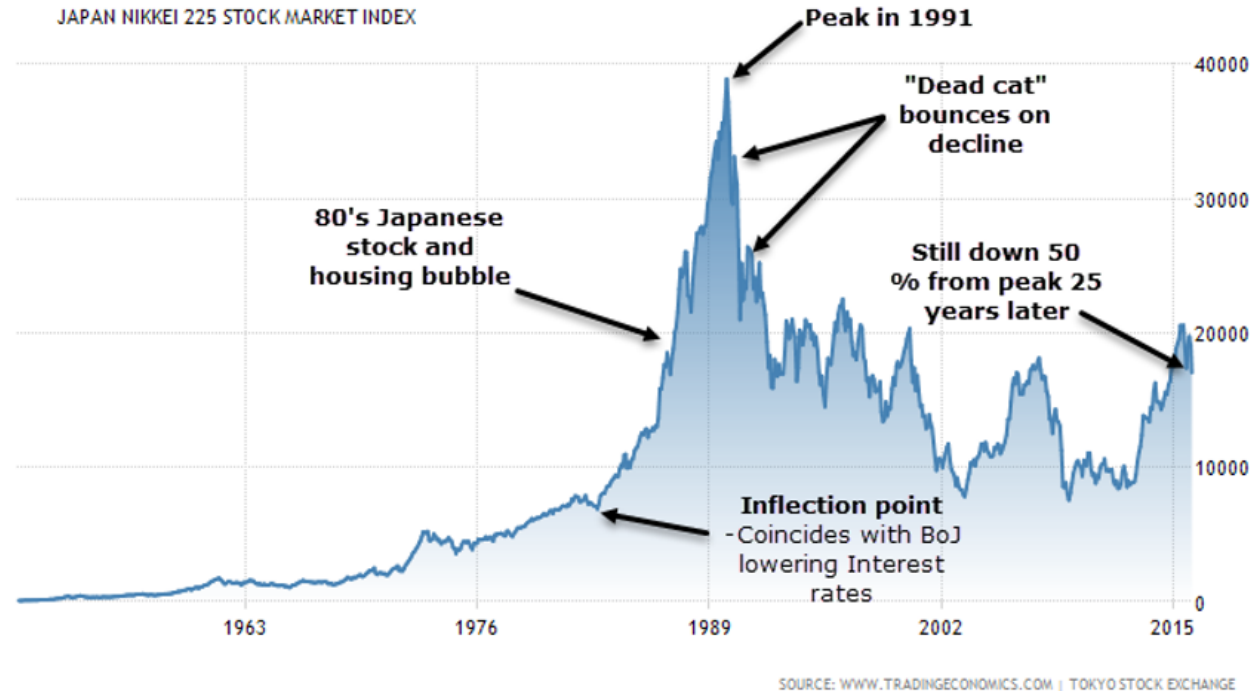
Historically, correlations between countries' stock exchanges have been low.

- Markets have been relatively “segmented”

Figure 4: Portfolios concentrated in Canadian stocks (or any single country) are inefficient



International Diversification



Moral of the Story:
Avoid “Home Bias”. Diversifying across country factors has been more important than across industry factors. (Especially if you are Japanese.)

Asset Class Diversification

- Combining equity portfolios with Bonds and Commodities have also provided great diversification benefits.
- Notice that correlations have been time varying.

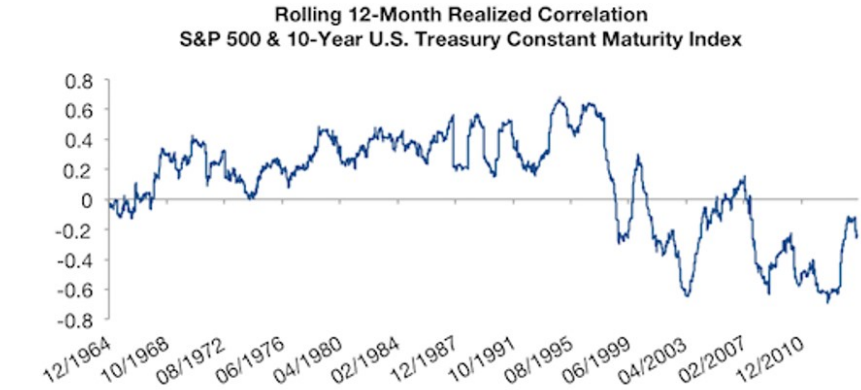


Figure 2. Monthly correlations of S&P GSCI Total Return Index with stocks and bonds: 1988–2006



Diversification: Considerations

- Some risks look big and dangerous but really are diversifiable.
- Market risks are macro risks and cannot be completely diversified. (can diversify some through across asset class investments, i.e. bonds and commodities)
- Always diversify. Some risks can be measured, but the big risks we don't see coming, and diversification (both within asset class, across asset class, and across country) will never hurt you.
- ALWAYS DIVERSIFY, it's the only “free lunch” out there.

In Summary

We learned:

- How to calculate historical returns.
- How to calculate expected return, variance and standard deviation of a stock, or a portfolio of stocks.
- How to calculate the covariance and correlation between two asset returns.
- How to diversify a portfolio and its risk-reducing effects.

Breakout Activity: Minimum Variance Portfolio

Go to [Yahoo! Finance](#)

Pick two stocks.

Compute the expected return, standard deviation of both stocks. (Based on monthly return)

Compute the covariance between the 2 stock returns.

Generate a table of possible portfolios and calculate the portfolio expected return and standard deviation.

Plot the results, find the weights that make up the minimum variance portfolio.

Report the results to the class.

Formula Sheet

$$HPR = \frac{P_n - P_0 + \sum_{t=1}^n \text{Income}_t}{P_0}$$

$$\bar{r} = \frac{1}{n} \sum_{t=1}^n r_t$$

$$r_{geo} = \left[\prod_{t=1}^n (1 + r_t) \right]^{\frac{1}{n}} - 1$$

$$\sigma^2 = \frac{\sum_{t=1}^n (r_t - \bar{r})^2}{n - 1}$$

Formula Sheet cont'd

$$E(R) = \sum_{i=1}^N R_i Pr(i)$$

$$\sigma^2 = \sum_{i=1}^N \left[(R_i - E(R))^2 Pr(i) \right]$$

$$E(R_p) = \sum_{i=1}^N W_i E(R_i)$$

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B cov(R_A, R_B)$$

$$cov(R_A, R_B) = \sum_{i=1}^N Pr(i) (R_{A,i} - E(R_A))(R_{B,i} - E(R_B))$$

$$corr(R_A, R_B) = \rho_{A,B} = \frac{Cov(R_A, R_B)}{\sigma_A \sigma_B}$$