**Computer Vision, Homework 4**
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# 1 Parametrizing Curves

## 1.1 Find $\alpha(s)$

$$\alpha(s) = (\frac{1}{k}cos(s), \frac{1}{k}sin(s))$$

$$s \in [-\pi, \pi]$$

$$\alpha'(s) = (-\frac{1}{k}sin(s), \frac{1}{k}cos(s))$$

$$\|\alpha'(s)\| = \sqrt{sin^2(s) + cos^2(s)} = \frac{1}{k}$$

## 1.2 Find Frenet approximation to the circle

$$\alpha''(s) = (-\frac{1}{k}cos(s), -\frac{1}{k}sin(s))$$

$$\alpha(s_0 + s) = \alpha(s_0) + s(\alpha'(s_0)) + \frac{s^2}{2}\alpha''(s_0)$$

From here we can sub in $\alpha$, $\alpha'$, and $\alpha''$.

## 1.3 Plot magnitude error

Before plotting the magnitude, I wants to plot the scatter graph to get a better scene of what's going on, see figure below.

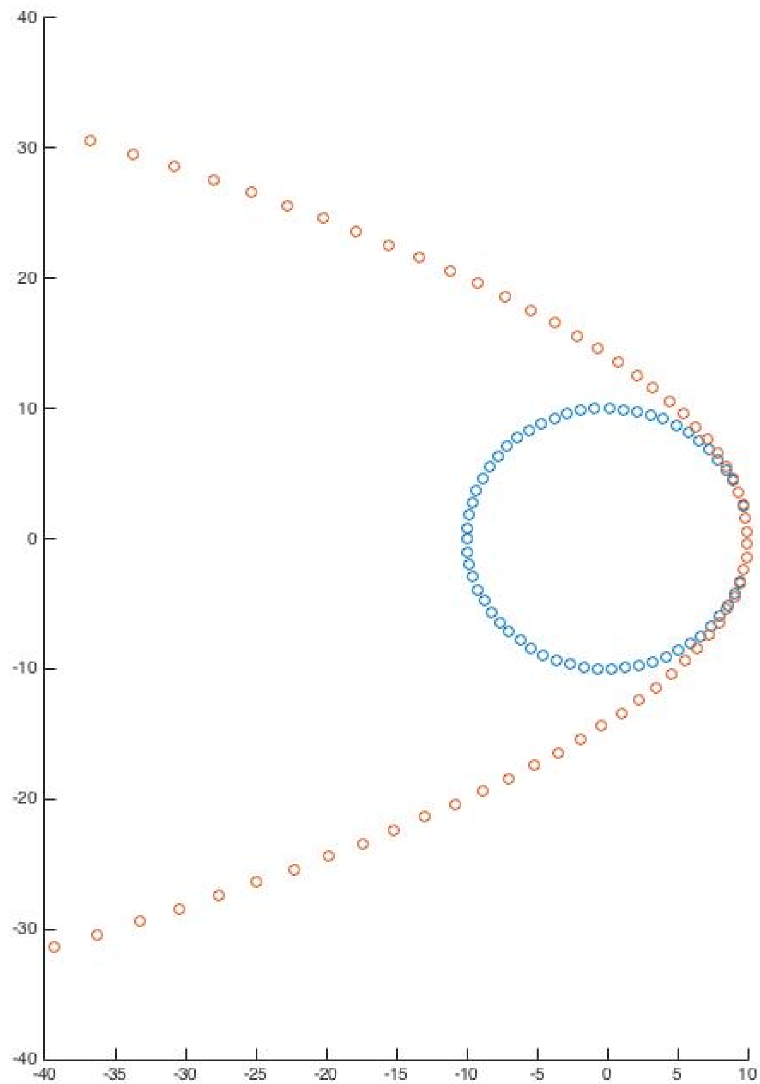


Figure 1: The blue circle is our scatter of $\alpha(s)$, and the red parabola is our scatter of frenet approximation. $s \in [-\pi, \pi]$ In the picture shown, the parabola lies in the plane through $\alpha(s_0)$ orthogonal to $\alpha(s_0)$

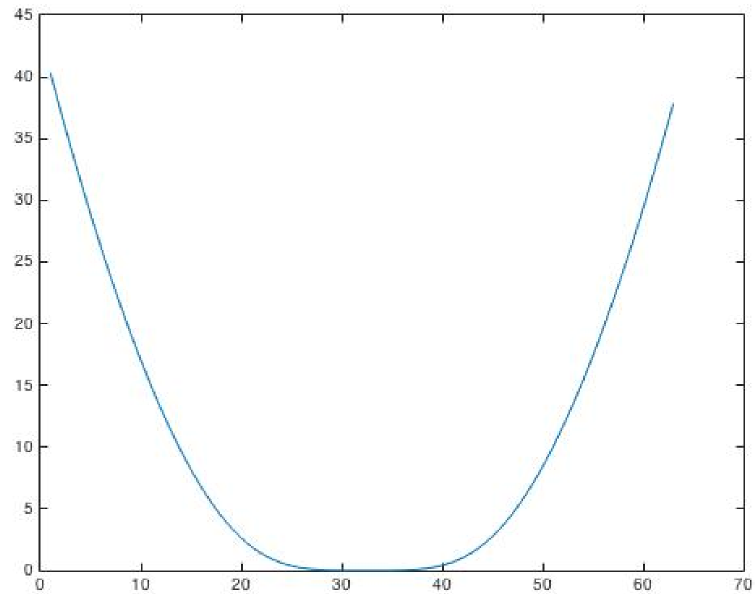


Figure 2: The x-axis is the number of data points, and y-axis is the magnitude of error, we see that the peak error is around 40 when curvature is 0.1, meaning radius is 10

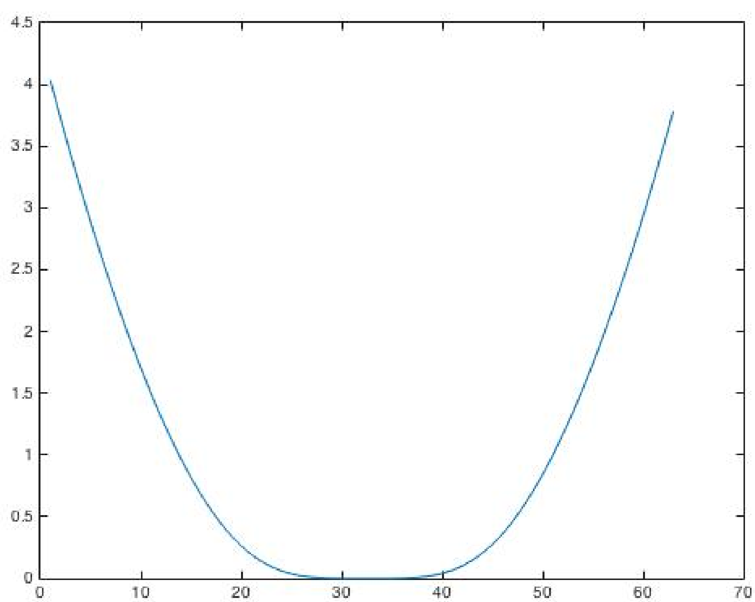


Figure 3: The x-axis is the number of data points, and y-axis is the magnitude of error, we see that the peak error is around 4 when curvature is 1, meaning radius is 1

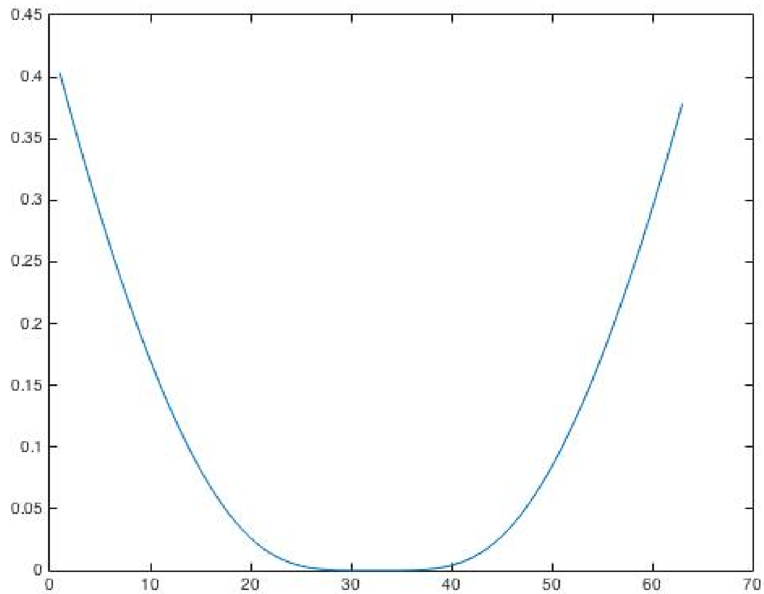


Figure 4: The x-axis is the number of data points, and y-axis is the magnitude of error, we see that the peak error is around 0.4 when curvature is 10, meaning radius is 0.1

The results show that the error is inverse-proportional to the curvature. However, I think the magnitude does not change the fact that the frenet approximation is the best quadratic approximation to $\alpha$ at $s_0$.

# 2 Geometric Insights into Shape from Texture

## 2.1 calculate $F_{*p}$

We have:

$$F_{*p}(\boldsymbol{v}) = \frac{d}{du}(F(\alpha(u))), u = 0$$

Given:

$$F(\boldsymbol{p}) = r(\boldsymbol{p})\boldsymbol{p}$$

$$\Rightarrow \frac{d}{du}(F(\alpha(u))) = \frac{d}{du}(r(\alpha(u))\alpha(u))$$

$$= r'((\alpha(u))\alpha(u) + r(\alpha(u))\alpha'(u)$$

We also know that:

$$\alpha(u) = \boldsymbol{p}, \alpha'(u) = \boldsymbol{v}, u = 0$$

Therefore, we have:

$$\Rightarrow F_{*p}(\boldsymbol{v}) = \nabla r\ \boldsymbol{p} + r\ \boldsymbol{v}$$

### 2.2 Prove argmax

If $\boldsymbol{v} = \nabla r / \|\nabla r\|$, we know that $\boldsymbol{v}$ is tangential to $r$, so the angle between $r$ and $\boldsymbol{v}$ is 0. Plug it back to the equation above:

$$\|F_{*p}(\boldsymbol{v})\| = \|\nabla r\, \boldsymbol{p} + r(\nabla r / \|\nabla r\|) cos(0)\|$$

This form will clearly reach optima as $cos(0)$ equal to 1.

If $v$ is tangential to $r$, we can just move $v$ from viewsphere at $p$ to our surface $s$.