Chapter 10: Model comparison and Hierachical Modelling

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1 Introduction

- When we have multiple models describing the same data, we need to assign credibilities to each model.
- Bayesian model comparison reallocates credibility across models given the data.
- Model comparison $\implies$ bayesian estimatation of hierachical models where the top-level is the index of the models.

2 Bayes Factor

2.1 General Formula

- Assume we have data D with parameters θ.
- Prior distribution is $p(\theta)$
- Parameter m to specify the index of the model.
- Hence, we will get

$$likelihood = p_m(y|\theta_m, m).$$

$$prior = p(\theta_m|m).$$

- Priors have different subscripts because they might have different distributions for each model.
- Assume each model is given a prior probability of $p(\theta)$. Then, for all possible models $\theta_1, \theta_2 \dots m$, we have:

$$p(\theta_1, \theta_2 \dots |D) = \frac{P(D|\theta_1, \theta_2 \dots m) * p(\theta_1, \theta_2 \dots m)}{\sum_m \int d\theta_m p(D|\theta_1, \theta_2 \dots m) p(\theta_1, \theta_2 \dots m)}.$$

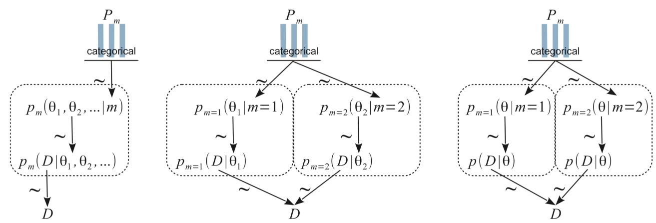

Figure 1: Model comparisons as a hierachical model

- To get the relative probabilities of the models, we will divide their posterior outputs.

$$\frac{p(m=1|D)}{p(m=2|D)} = \frac{p(D|m=1) * p(m=1) / \sum_m P(D|m) * p(m)}{p(D|m=2) * p(m=2) / \sum_m P(D|m) * p(m)}.$$

- The above equation is called the Bayes Factor
- We can use the below table for reference on figuring out when to report a model is better than the alternative model.

K	dHart	bits	Strength of evidence
$< 10^0$	0	—	Negative (supports M_2)
10^0 to $10^{1/2}$	0 to 5	0 to 1.6	Barely worth mentioning
$10^{1/2}$ to 10^1	5 to 10	1.6 to 3.3	Substantial
10^1 to $10^{3/2}$	10 to 15	3.3 to 5.0	Strong
$10^{3/2}$ to 10^2	15 to 20	5.0 to 6.6	Very strong
$> 10^2$	> 20	> 6.6	Decisive

Figure 2: bayes factor