Chapter 9: Hierachical Models

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1 Introduction

- They involve multiple parameters.
- When the value of one parameter θ depends on another variable ω, the hierachical structure of these variables can be represented by a hierachical model.
- To infer these parameters, we apply the joint probability rule for the parameters.
$$P(\theta, \omega|D) \propto P(D|\theta, \omega)p(\theta, \omega).$$
$$= P(D|\theta) * P(\theta|\omega) * P(\omega).$$
- The above equation implies that value of D is dependent only on θ and independant of other variables. Similarly, the value of θ is dependant only on the value of ω and is conditionally independant of all other parameters.
- The dependencies between parameters are useful because:
  - They are meaningful for the given application
  - Because of dependencies across parameters, they can jointly inform all parameter estimates.
  - Easier convergence with smart algorithms that exploit this joint probability.

1.1 Coin flipping from a single mint

- We will use bernoulli distribution for the data and beta distribution for the prior.
$$y_i \approx dbern(\theta).$$
$$\theta \approx dbeta(a, b).$$
- We know that a and b can be represented as using mode ω and concertration κ as:
$$a = \omega(\kappa - 2) + 1.$$
$$b = (1 - \omega)(\kappa - 2) + 1.$$

- Hence, we can write θ as:

$$\theta \approx dbeta(\omega(\kappa - 2) + 1, (1 - \omega)(\kappa - 2) + 1).$$

  - The value κ controls how close the value of ω is θ.
  - Higher value of κ = Closer to value of θ

- Let us assume ω is another parameter to be estimated. Assume this to be a beta distribution: $\omega \approx beta(\omega|A_\omega, B_\omega)$

- We know the value of ω is closer to the mode of the distribution in this case i.e: $\frac{A_\omega - 1}{A_\omega + B_\omega - 2}$

- Substituting bayes rule, we get:

$$p(\theta, \omega|y) = \frac{p(y|\theta, \omega)p(\theta, \omega)}{p(y)} = \frac{p(y|\theta)p(\theta|\omega)p(\omega)}{p(y)}.$$

- We have the equations for all of the above components. We can get the posterior probability by solving the above equation.

- We can solve them using grid approximation as well as the parameters are finite.

1.2 Multiple coins from single mint

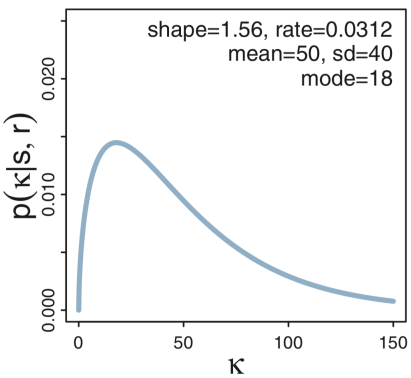
- Assume we have multiple coins for a single mint. Each coin will now have it's own parameter θ_s and we will estimate this using all the data for ω.

1.3 Real example

- For the multiple coins problem, we do not know the value for ω in advance. We will have to estimate it from the data available.

- We will assume ω follows a gamma distribution. The gamma distribution has the following formula: $gamma(\kappa|s, r)$. Here, s is the shape parameter and r is the rate parameter.

- We will use the parameters s =1.56 and r =0.0312 because these values have a boundary at 0 and infinite possible positive values.

- 

- Mean: $\mu = \frac{s}{r}$
- Mode: $\omega = \frac{s-1}{r}$
- SDev: $\sigma = \frac{\sqrt{s}}{r}$
- We can derive s and r from the above as:

$$s = \frac{\mu^2}{\sigma^2}.$$

$$r = \frac{\mu}{\sigma^2}.$$

  when the mean $\mu > 0$

- It can also be written as:

$$s = 1 + \omega r.$$

$$r = \frac{\omega + \sqrt{\omega^2 + 4\sigma^2}}{2\sigma^2}.$$

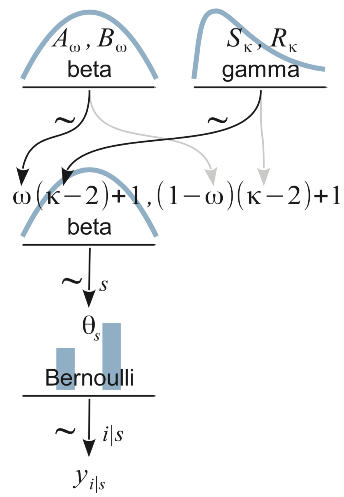


Figure 1: Hierachical Model

1.4 Theraupeutic Touch

- Relieve congestion and improve balance by manipulating "energy field" without touching the patient.
- Experiment
  - Practitioner should be able to tell if hand is near their hand without touching the hand.
  - Experimenter flips a coin. Depending on the outcome, places hand above or below practitioner hand.
  - Practitioner guesses if hand is above or below.
  - Chance performance for guessing the result is 0.5
- Questions:
  - How much did group differ from chance performance?
  - How much did each individual differ from chance performance?

2 Shrinkage

- Estimates of low-level params are pulled together than they would if they were higher-level params. This pulling is called **shrinkage**.
- It occurs because:
  - Subset of data is directly dependant on the low-level parameter.

- The higher-level params that depend on the low-level params.

- Shrikage occurs because of hierachical models, not bayesian estimation.
- Intuitively, shrinkage occurs because data from all individuals influence the higher-order params, and these params in-turn influence the estimates for each individual.

3 Extending the hierachy

- Didn't really understand this. I'll work through the problems and come back to update the notes.