70

UNIVERSITAS SEBELAS MARET
FAKULTAS TEKNIK

Nama : Izzuddin 'Ulwan
NIM : I0719061
Mata Kuliah : Kalkulus
Dosen Penguji : Agus Ramelan

1. a. $f'(x) = x^4 \ln x$

$f'(x) = \frac{3}{x^4} \cdot \frac{1}{x}$

maka $\frac{dy}{dx}$ $f'(x) = x^3$

$f'' = 3x^2$

$f'''(x) = 6x$

b.

2. a. $\int 3e^x + 5\cos x - 10\sec^2 x\, dx$

$= 3e^x + 5\sin x - 10\tan x + C$

$= 3e^x + 5\sin x - 10\tan x + C$

b. $\int_0^1 4x + 6\sqrt[3]{x^2}\, dx$

$= \left[2x^2 + \frac{6}{5/3} \cdot \sqrt[3]{x^5} \right]_0^1$

$= \left(2 \cdot 1^2 + \frac{6}{5/3} \sqrt[3]{1^5}\right) - \left(2 \cdot 0^2 + \frac{6}{5/3} \cdot \sqrt[3]{0^5}\right)$

$= \left(2 + \frac{18}{5}\right) = \frac{28}{5}$

c.) Hitung $\frac{dy}{dx}$ jika $y = \int_2^x \frac{2t+5}{\sqrt{t^2+16}}$

= ingat $\frac{d}{dx}\left[\int_a^x f(t)\right] = f(x)$

maka $\frac{dy}{dx} = \frac{d}{dx}\left[\int_2^x \frac{2t+5}{\sqrt{t^2+16}}\right] = f(x)$

$\Rightarrow f(x) = \frac{2x+5}{\sqrt{x^2+16}}$

$= \frac{2x+5}{\sqrt{x^2+16}}$

3. a. Nilai ekstrimnya:

① titik ujung selang $[-5, -1]$

② titik stasioner $f(c) = 0$

$Q'(y) = 0$

tidak punya titik stasioner

$Q(y) = 3y\sqrt[3]{(y+4)^2}$

$= \sqrt[3]{(y+4)^2 (3y)^3}$

$= \sqrt[3]{(27y^5 + 202y^4 + 432y^3)}$

$Q'(y) = \frac{1}{3}(27y^5 + 202y^4 + 432y^3)^{-\frac{2}{3}} \cdot (135y^4 + 808y^3 + 1296y^2)$

$Q'(y) = \frac{1}{3}(27y^5 + 202y^4 + 432y^3)^{-\frac{2}{3}} \cdot (135y^4 + 808y^3 + 1296y^2)$

$Q'(y) = (45y^4 + 269{,}33y^3 + 432y^2)(27y^5 + 202y^4 + 432y^3)^{-2/3}$

$Q'(y) = \frac{45y^4 + 269{,}33y^3 + 432y^2}{\sqrt[3]{(27y^5 + 202y^4 + 432y^3)^2}}$

$Q(0) = 3 \cdot 0\sqrt[3]{(0+4)^2}$

$Q(0) = 0 \rightarrow$ nilai maksimum

$Q(-5) = 3 \cdot (-5)\sqrt[3]{((-5)+4)^2}$

$= -15\sqrt[3]{(-1)^2}$

$= -15 \rightarrow$ nilai minimum

$Q(-1) = 3 \cdot (-1)\sqrt[3]{((-1)+4)^2}$

$= -3\sqrt[3]{9}$

=

③ titik singular : $y = 0$

3 a. ~~... = ... -2x~~
~~... -2x + ... = -2x -2x~~
~~... = -4x~~

c.) $f(x) = 3x^5 - 5x^3 + 3$

$f'(x) = 15x^4 - 15x^2$

$f'(2) = 15 \cdot 2^4 - 15 \cdot 2^2$

$= 240 - 60 = 300 \rightarrow$ monoton naik

$f''(x) = 60x^3 - 30x$

$f''(2) = 480 - 60$

$= 420 \rightarrow$ cekung ke atas

4 a $f(x) = 3x^2 - 9x + 6$ pd interva $[1,4]$

$f(c) = \frac{1}{4-1} \int_1^4 3x^2 - 9x + 6 \; dx$

$= \frac{1}{3} \left(x^3 - \frac{9}{2}x^2 + 6x \Big]_1^4 \right)$

$= \frac{1}{3} \left[\left(4^3 - \frac{9}{2} \cdot 4^2 + 6 \cdot 4\right) - \left(1^3 - \frac{9}{2} \cdot 1^2 + 6 \cdot 1\right) \right]$

$= \frac{1}{3} \left[(64 - 72 + 24) - \left(1 - \frac{9}{2} + 6\right) \right]$

$= \frac{1}{4} \left(8 - 7/12 \right)$

$= \frac{1}{3} \cdot \frac{89}{12} = \frac{89}{36}$

$\int_1^4 f(x)\,dx = f(c) \cdot (b-a)$

$\frac{89}{12} = f(c) \cdot 3$

$f(c) = \frac{89}{16}$

b $A = \int_{-1}^{4} x_2 - x_1 \; dy$

$= \int_{-1}^{4} \left(\frac{4+3y}{4}\right) - \left(\frac{y}{4}\right) dy$

$= \int_{-1}^{4} \left(\frac{4 + 3y - y}{4} \right) dy$

$= \int_{-1}^{4} \left(\frac{4 + 2y}{4} \right) dy$

$= \int_{-1}^{4} 1 + \frac{2y}{4} \; dy$

$= y + \frac{2}{4} \cdot \frac{1}{2} y^2 \Big]_{-1}^{4}$

$= y + \frac{1}{4} y^2 \Big]_{-1}^{4}$

$\left(4 + \frac{1}{4} \cdot 4^2\right) - \left(-1 + \frac{1}{4}(-1)^2\right)$

$= 8 - \left(-\frac{5}{4}\right)$

$= \frac{37}{4}$ satuan kuadrat

c. $y_1 = y_2$

$(4\sqrt{x-1})^2 = (2x-2)^2$

$16(x-1) = 4x^2 - 8x + 4$

$16x - 16 = 4x^2 - 8x + 4$

$0 = 4x^2 - 24x + 20$

$0 = x^2 - 6x + 5$

$(x-5)(x-1)$

$x = 5 \lor x = 1$

~~$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-6 + \sqrt{32}}{2a}$~~

~~$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-6 - \sqrt{32}}{2a}$~~

$V = \pi \int_1^5 (a - x_1^2) - (a - x_2^2) \; dy$

$= \pi \int_1^5 (-2 - x_1^2) - (-2 - x_2^2) \; dy$

$$V = \pi \int_1^5 \left(-2-\left(\frac{4+3y}{4}\right)^2 - \left(-2-\left(\frac{y}{4}\right)^2\right)\right) dy$$

$$= \pi \int_1^5 \left(-2 - \frac{(16+24y+9y^2)}{16}\right) - \left(-2 - \frac{y^2}{16}\right)$$

$$= \pi \int_1^5 \left(\frac{-32-16-24y-9y^2}{16}\right) - \left(\frac{-32-y^2}{16}\right)$$

$$= \pi \int_1^5 \frac{-48-24y-9y^2}{16} + \frac{32+y^2}{16}\, dy$$

$$= \pi \int_1^5 \frac{\cancel{-48}-8y^2-24y-16}{16}\, dy$$

$$= \pi \int_1^5$$