COMP4033 Fuzzy Logic and Fuzzy Systems - CW1

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| Index	Target Quality	Plot
1	When the temperature is within ± 1 SD of the mean, the urgency of the headache is directly proportional to the pain level. This assures that normal temperature has no effect on the result, relying only on the pain level.	——
2	When the temperature is at hypothermic or hyperthermic temperatures, the urgency is high. At these temperatures medical assistance would be necessary regardless of pain level.	—— and ——
3	At all temperatures, the urgency increases monotonically with the pain level. This is to say a more painful headache should never be less urgent than a less painful headache.	
4	At all temperatures, if the pain level is 10, the urgency is high.	——

Table 1: Target qualities of the FIS model
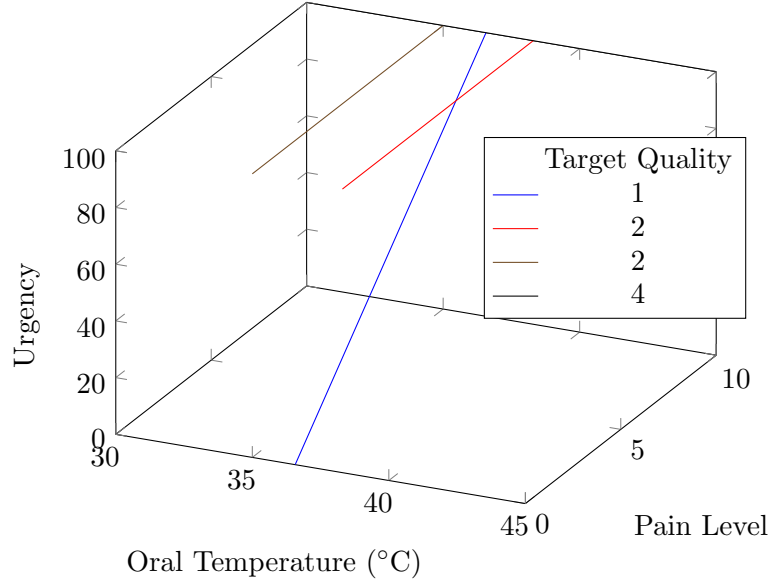
Figure 1: Plot of the target model qualities in the model space

1 Data Gathering

1.1 Temperature

For this exercise, "temperature" is taken as oral temperature. [2] is a systemic review of many studies documenting body temperate measurements. Over 5257 measurements across 33 studies using both male and female subjects of all ages determine the mean oral temperature to be 36.57°C with a standard deviation of 0.42°C.

Hypothermia is considered to define body temperatures less than 35°C [4]. Hyperthermia is considered to be body temperatures above 38.3°C [3]. The range for the temperature input is hence given as a range from 30°C to 45°C.

2 Target Qualities

In order to assess the quality of a given model, we must first determine a set of desired qualities of the FIS model. A good model will satisfy all of these qualities. Table 1 lists these qualities, as well as their plots in Figure 1.

Note that Plot —— is invisible as it follows the upper Oral Temperature axis.

3 Fuzzy Operation Methods

Unless otherwise stated, all models use the methods listed in Table 2 for their fuzzy operations.

Fuzzy Operation	Method
and	min
or	max
implication	min
aggregation	max

Table 2: Table of the models' fuzzy operations and the methods used

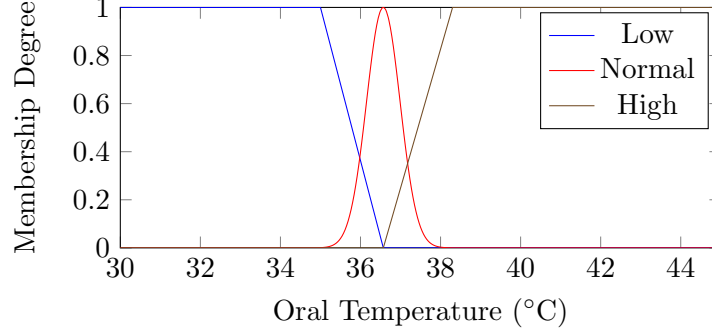
Figure 2: The temperature membership functions for model 1

4 Candidate Models

4.1 Candidate Model 1

The first candidate model aims to be a baseline model designed to be as simple as possible.

4.1.1 Membership Functions

4.1.1.1 Oral Temperature

For this model, there are only three temperature membership functions:

- **Low (——):** a shoulder trapezoidal membership function extending from 30°C with its right shoulder at 35°C (hypothermia) and ending at the mean temperature: 36.57°C.

- **Normal (——):** a Gaussian membership function with mean of 36.57°C and standard deviation of 0.42°C.

- **High (——):** a shoulder trapezoidal membership function starting at 36.57°C, a left shoulder at 38.3°C (hyperthermia) and extending to 45°C.

These membership functions are plotted in Figure 2.

4.1.1.2 Pain Level

For this input we use three triangular membership functions distributed evenly over the range of levels. These functions are named Low, Medium and High. The membership functions are plotted in Figure 3.

4.1.1.3 Urgency

Similar to the membership functions for the pain level input, for the output we use three triangular membership functions distributed evenly over the range of outputs. These functions are named Low, Medium and High. The membership functions are plotted in Figure 4.

4.1.2 Rules

Table 3 lists the rules for model 1 and their corresponding target qualities as mentioned in Section 2. For this model, all rules have equal weighting.

4.1.3 Inference and Defuzzification

This model uses Mamdami inference and centroid defuzzification.

4.1.4 Evaluation

Figure 5 shows the output surface of model 1; additional views are available in Appendix A.1. Table 4 shows the target qualities achieved by model 1.
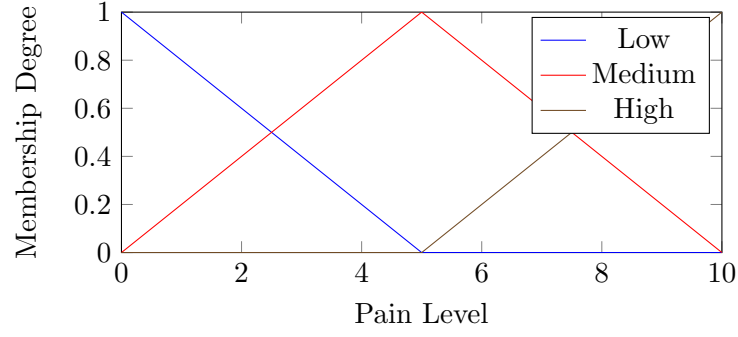
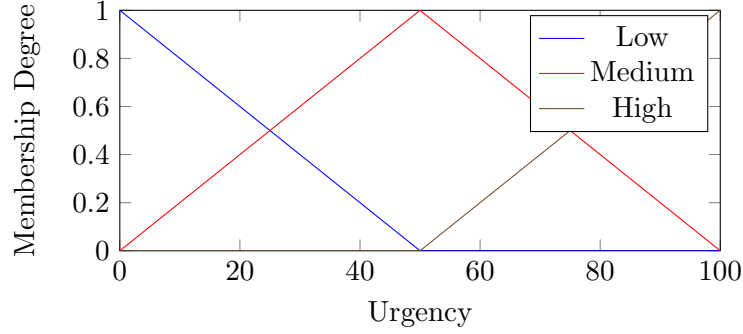Figure 3: The pain level membership functions for model 1

Figure 4: The urgency membership functions for model 1

Index	Rule	Target Qualities
1	If temperature is low then urgency is high	2
2	If temperature is high then urgency is high	2
3	If temperature is normal and pain is low then urgency is low	1 and 3
4	If temperature is normal and pain is medium then urgency is medium	1 and 3
5	If pain is high then urgency is high	1, 3 and 4

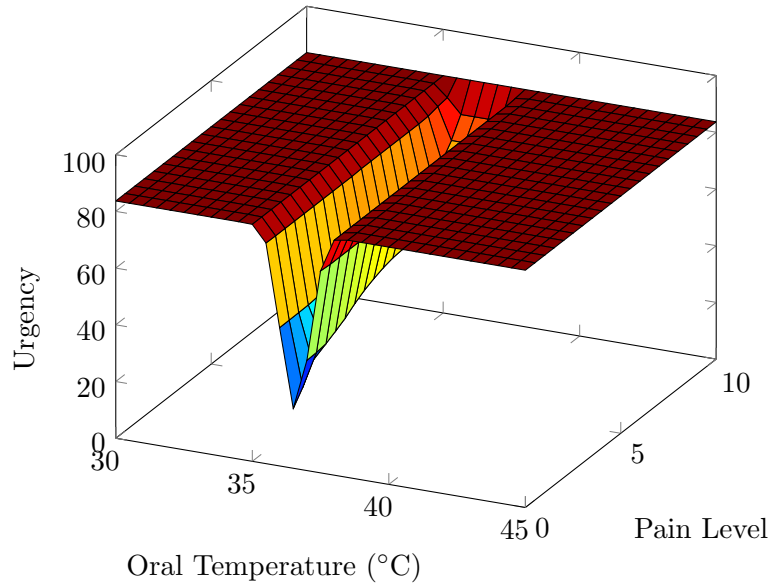Table 3: Table of the rules for model 1 with their corresponding target qualities

Figure 5: Plot of the output surface of model 1

Target Quality Index	Model has Quality?	Notes
1	No	At normal temperatures, the relationship between pain and urgency is not linear.
2	Yes	
3	Yes	
4	Yes	

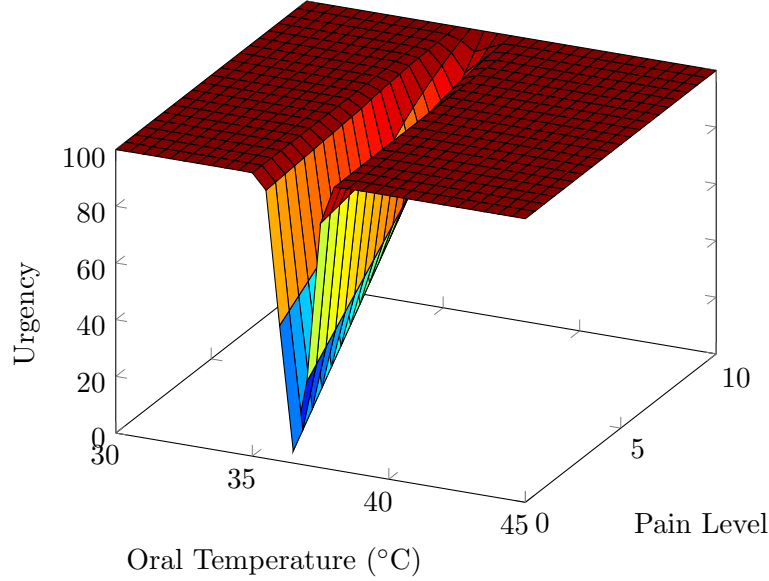Table 4: Table of the achieved target qualities of model 1

Figure 6: Plot of the output surface of model 1a

4.2 Candidate Model 1a

Unless otherwise stated, model 1a is identical to model 1. This is a variation of model 1 designed to rectify unachieved target quality 1.

4.2.1 Inference, Defuzzification and Fuzzy Operations

This model uses Sugeno inference with weighted average defuzzification. Additionally, this model uses the product method for the fuzzy `and` operation.

4.2.2 Urgency Membership Functions

The urgency output now has slightly modified membership functions:

- **Low:** a constant of value 0

- **Medium** a constant of value 50

- **High:** a constant of value 100

4.2.3 Evaluation

Figure 6 shows the output surface of model 1a; additional views are available in Appendix A.2. Table 5 shows the target qualities achieved by model 1a. This model is superior to model 1 as it fulfils all of the target qualities.

4.3 Candidate Model 2

The second candidate model aims to be even simpler than the first, whilst still having all of the target qualities. A simpler model would be preferable as it would be faster to evaluate and more suitable for extension in the future.

Target Quality Index	Model has Quality?	Notes
1	Yes	
2	Yes	
3	Yes	
4	Yes	

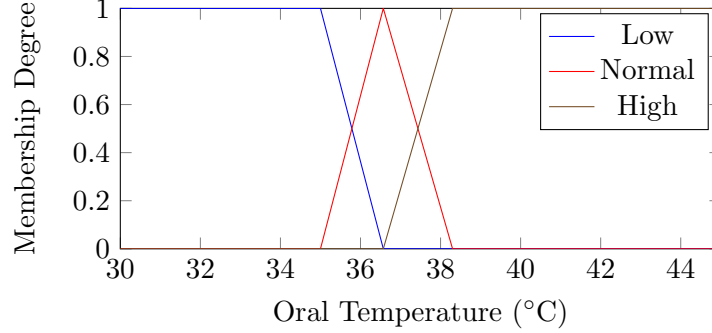Table 5: Table of the achieved target qualities of model 1a

Figure 7: The temperature membership functions for model 2

4.3.1 Membership Functions

4.3.1.1 Oral Temperature

For this model, like model 1, there are three membership functions. However, this model makes use of a triangular membership function for normal temperature ranges.

- **Low (——):** a shoulder trapezoidal membership function extending from 30°C with its right shoulder at 35°C (hypothermia) and ending at the mean temperature: 36.57°C.

- **Normal (——):** a triangular membership function starting at 35°C, its peak at 36.57°C and ending at 38.3°C.

- **High (——):** a shoulder trapezoidal membership function starting at 36.57°C, a left shoulder at 38.3°C (hyperthermia) and extending to 45°C.

These membership functions are plotted in Figure 7.

4.3.1.2 Pain Level

For this input we use only one, linear, membership function: Painful. This is an attempt to more accurately describe the intended linear relationship between pain level and headache urgency. The membership function is plotted in Figure 8.
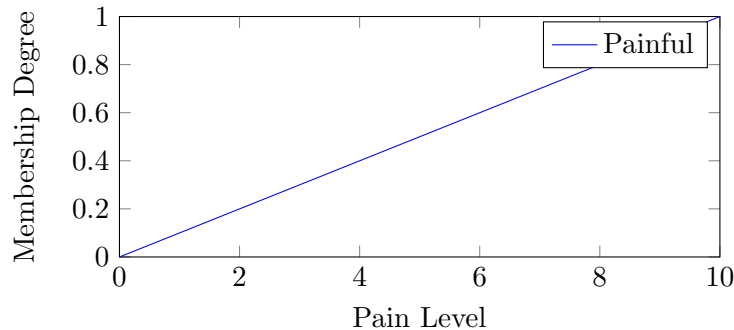

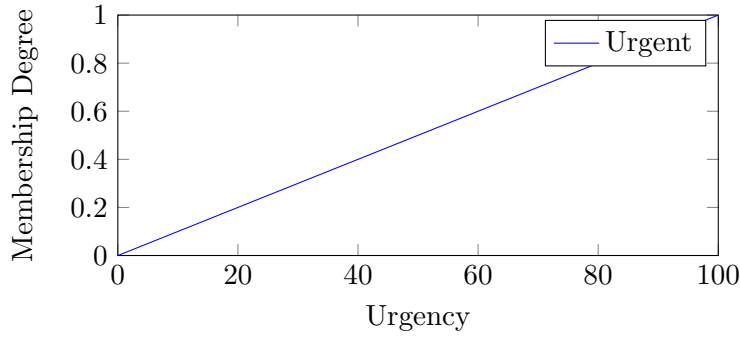Figure 8: The pain level membership function for model 2

Figure 9: The urgency membership function for model 2

Index	Rule	Target Qualities
1	If temperature is low then urgency is urgent	2
2	If temperature is high then urgency is urgent	2
3	If pain level is painful then urgency is urgent	4 and 1

Table 6: Table of the rules for model 2 with their corresponding target qualities

4.3.1.3 Urgency

Like for the pain level input, for the output we use only one membership function: Urgent. It also represents a simple linear relationship between membership degree and urgency. The membership function is plotted in Figure 9.

4.3.2 Rules

Table 6 lists the rules for model 2 and their corresponding target qualities as mentioned in Section 2. For this model, all rules have equal weighting.

4.3.3 Inference and Defuzzification

This model uses Mandami inference with centroid defuzzification.

4.3.4 Evaluation

Figure 10 shows the output surface of model 2; additional views are available in Appendix A.3. Table 7 shows the target qualities achieved by model 2. This model fails as it does not allow for low urgency at low pain levels and normal temperatures. This hence violates target quality 1: direct proportionality between pain levels and urgency at normal temperatures.

4.4 Candidate Model 2a

Unless otherwise stated, model 2a is identical to model 2. This is a variation of model 2 designed to modify the output range, as this is the main failing of model 2.

4.4.1 Rules

In this variant, we add a single additional rule. This is to ensure that when the pain level is low and the temperature is low, the urgency is low. Table 8 lists these updated rules.

Target Quality Index	Model has Quality?	Notes
1	No	At normal temperatures, when the pain level is low, the urgency is still high.
2	Yes	
3	Yes	
4	Yes	

Table 7: Table of the achieved target qualities of model 2
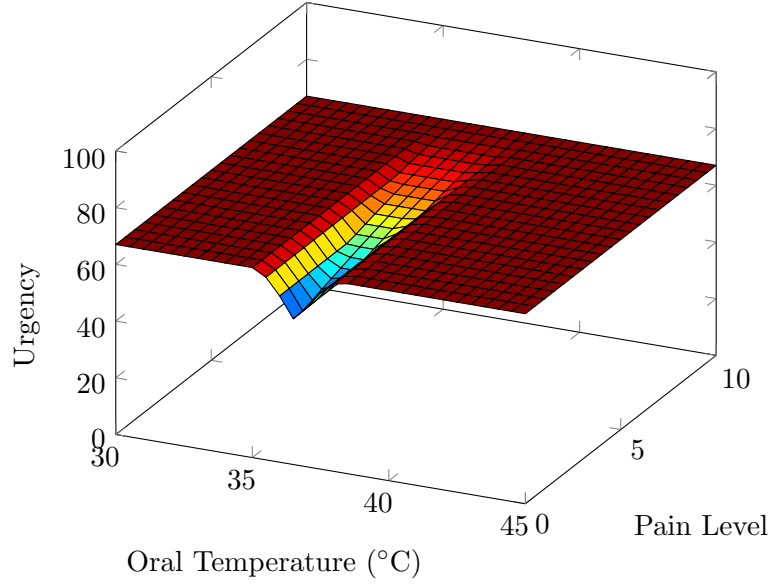
Figure 10: Plot of the output surface of model 2

Index	Rule	Target Qualities
1	If temperature is low then urgency is urgent	2
2	If temperature is high then urgency is urgent	2
3	If pain level is painful then urgency is urgent	4 and 1
4	If pain level is not painful and temperature is normal then urgency is not urgent	1 and 3

Table 8: Table of the rules for model 2a with their corresponding target qualities

4.4.2 Defuzzification

For this model, we decide between three defuzzification methods with the goal of more accurately achieving target quality 1. For illustration, we view a slice of the output surface where the temperature input is set to the normal temperature: 36.57°C. Figure 11 plots this slice using varying defuzzification methods. For reference, Plot —— plots the use of centroid defuzzification as is used by model 2.

4.4.2.1 Middle of Maximum

Plot —— shows a slice of the output surface when using middle of maximum defuzzification. Whilst the range of the urgency output is far superior than using centroid defuzzification, the linear relationship is not present.

4.4.2.2 Largest of Maximum

Plot —— shows a slice of the output surface when using largest of maximum defuzzification. This method still
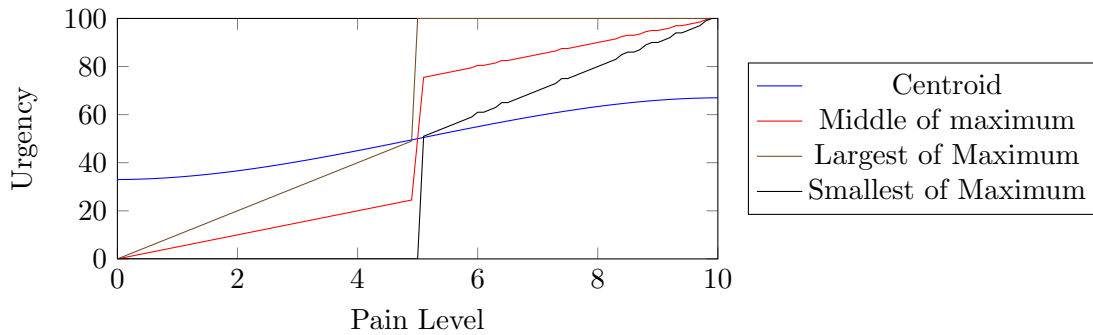

Figure 11: Plot of urgency as pain level increases where oral temperature is 36.57°C when using varying defuzzification methods

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provides the superior range of urgency levels but still does not provide linearity.

4.4.2.3 Smallest of Maximum

Plot —— shows a slice of the output surface when using smallest of maximum defuzzification. This method is also inappropriate as it allows for headaches of a pain level of up to 5 to go unnoticed.

4.4.3 Evaluation

None of model 2a's variants satisfy all of the target qualities. This shows that a model aiming to be simpler than models 1 or 1a may not provide the flexibility to handle the full range of inputs.

5 Chosen Model

The final model chosen is model 1a. This is due to it's fulfilling of all of the target qualities. Additionally, this model preserves the gaussian nature of the distribution of oral temperatures whilst still preserving the linearity of the relationship between pain level and urgency.

The model's shortcomings may arise from the nonlinear relationship between how one would assign a level of pain and the actual level of pain experienced. [1] studies this relationship; they find that NRS (Numeric Rating Scale), the method used for the pain input of the FIS, is a reliable indicator of a patient's pain level, albeit a subjective one. Hence, this indicates a linear relationship between pain level and headache urgency would be most suitable.

A   Additional Views of Candidate Model Surfaces
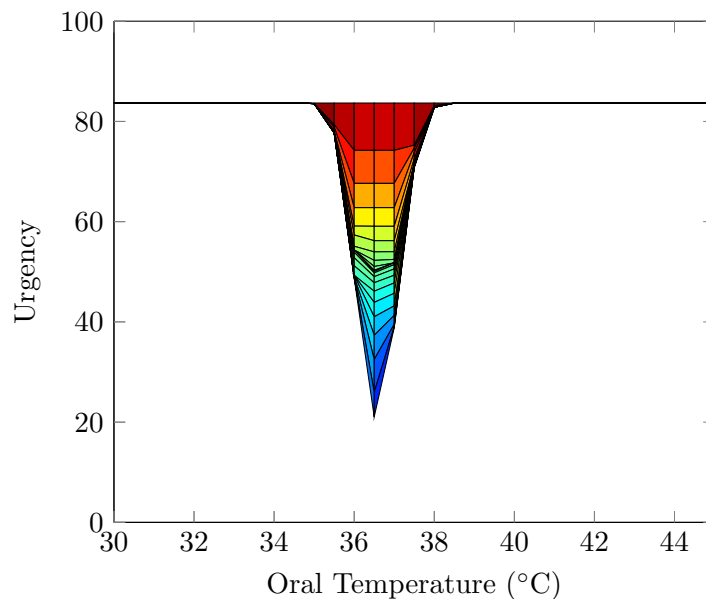
A.1   Model 1

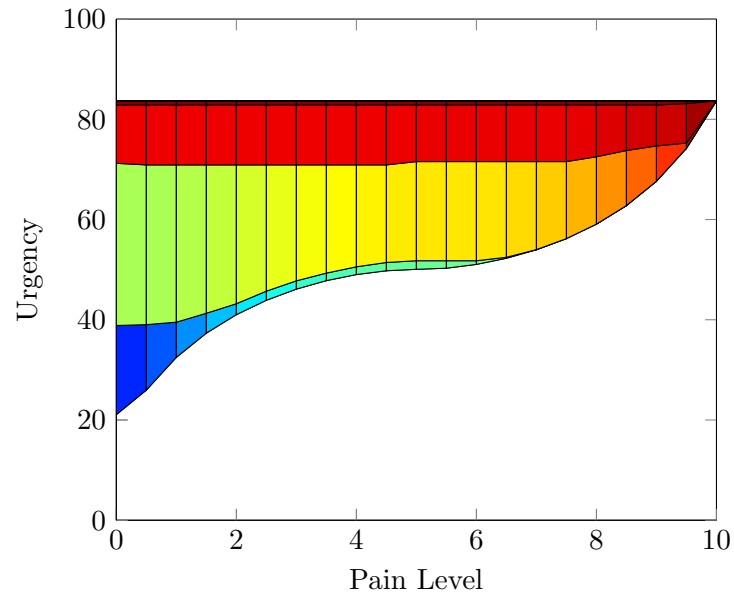Figure 12: Front-on view of the output surface of model 1

Figure 13: Side-on view of the output surface of model 1
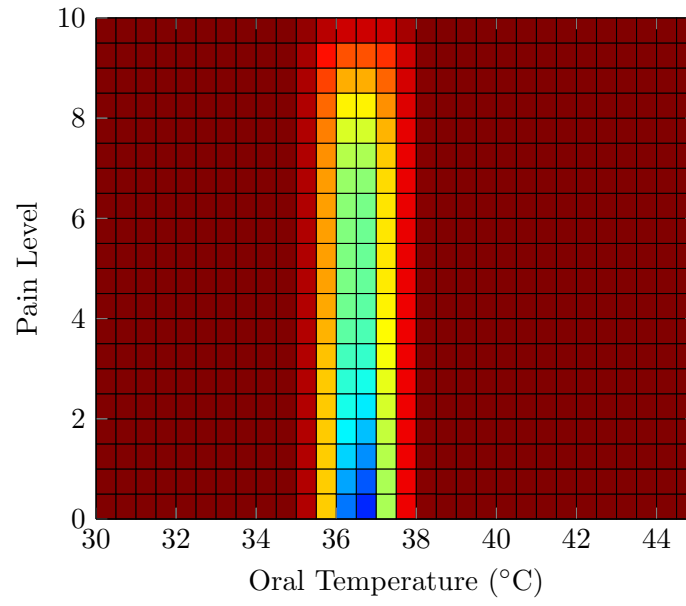

Figure 14: Top-down view of the output surface of model 1
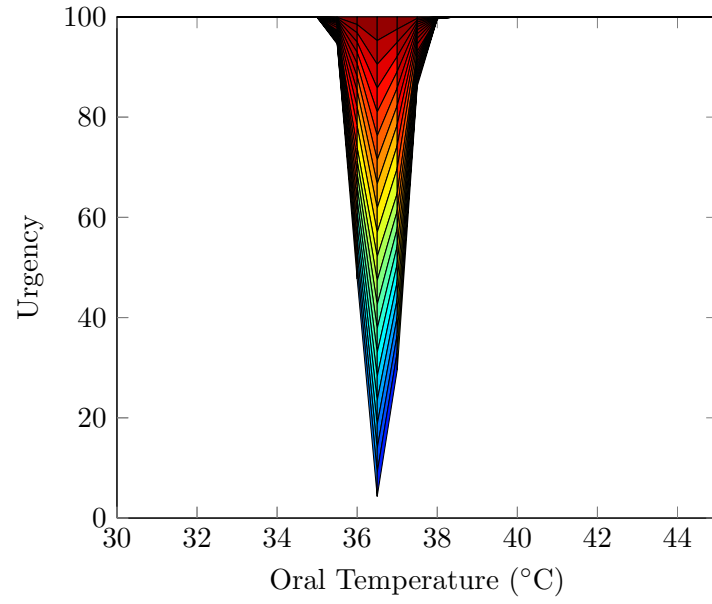
A.2   Model 1a

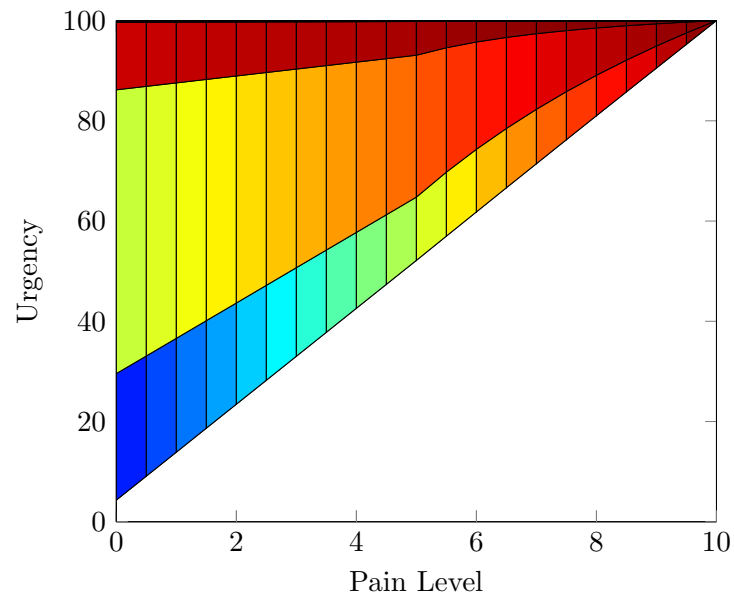Figure 15: Front-on view of the output surface of model 1a

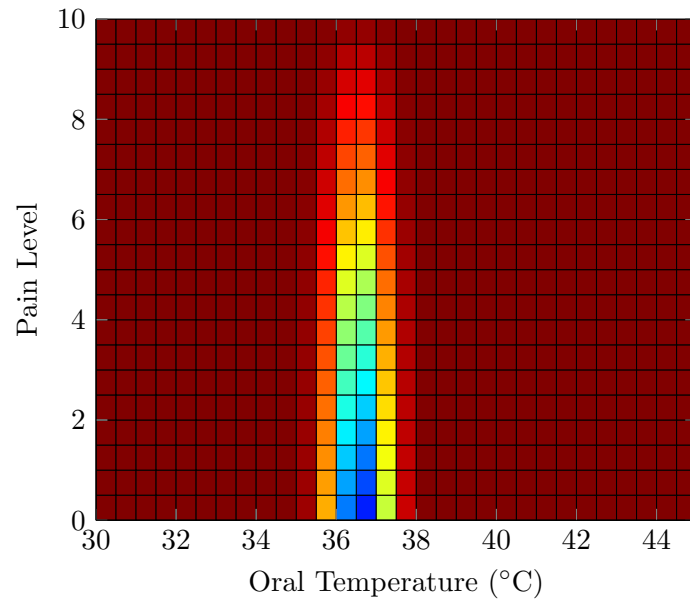Figure 16: Side-on view of the output surface of model 1a

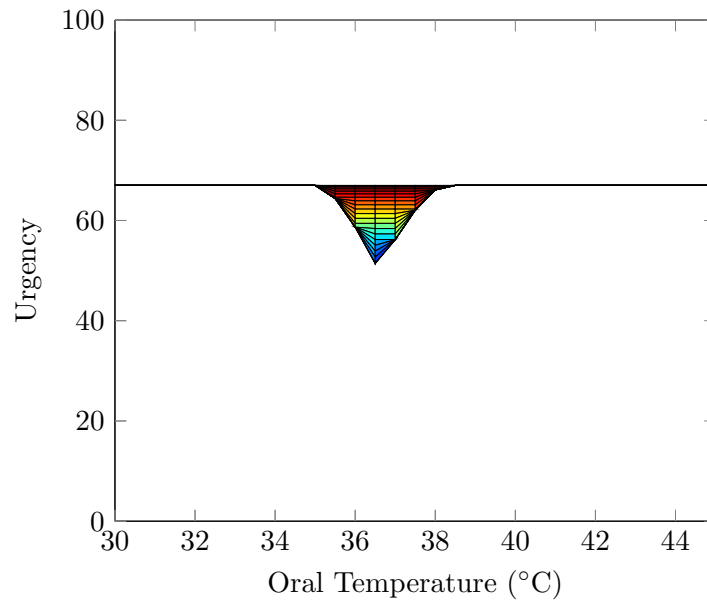Figure 17: Top-down view of the output surface of model 1a

A.3   Model 2

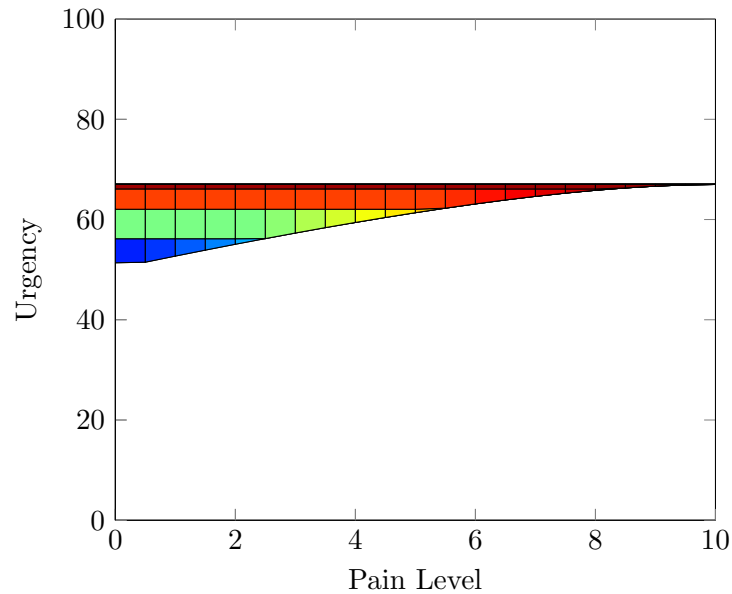

Figure 18: Front-on view of the output surface of model 2

Figure 19: Side-on view of the output surface of model 2
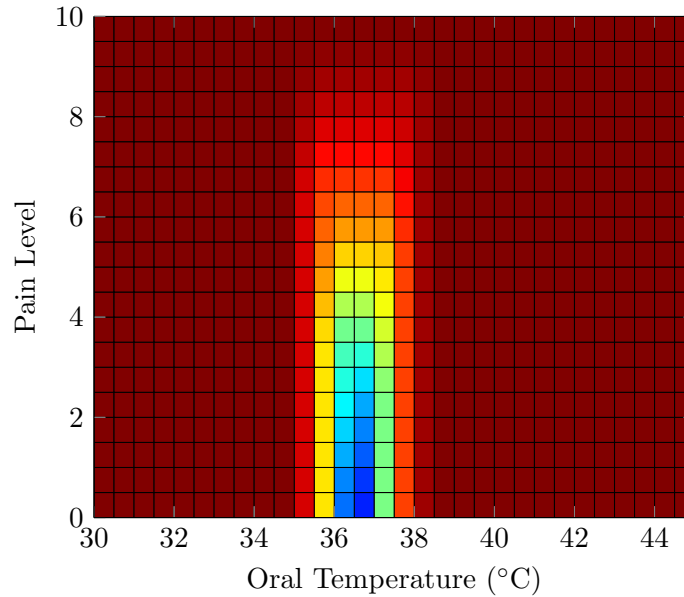

Figure 20: Top-down view of the output surface of model 2

References

[1] H. Breivik et al. "Assessment of pain". In: *British Journal of Anaesthesia* 101.1 (July 2008), pp. 17–24. DOI: 10.1093/bja/aen103. URL: https://doi.org/10.1093/bja/aen103.

[2] Ivayla I Geneva et al. "Normal Body Temperature: A Systematic Review". In: *Open Forum Infectious Diseases* 6.4 (Apr. 2019). ofz032. ISSN: 2328-8957. DOI: 10.1093/ofid/ofz032. eprint: https://academic.oup.com/ofid/article-pdf/6/4/ofz032/28311638/ofz032.pdf. URL: https://doi.org/10.1093/ofid/ofz032.

[3] Kevin B. Laupland. "Fever in the critically ill medical patient". In: *Critical Care Medicine* 37.Supplement (July 2009), S273–S278. DOI: 10.1097/ccm.0b013e3181aa6117. URL: https://doi.org/10.1097/ccm.0b013e3181aa6117.

[4] John Marx. *Rosen's emergency medicine : concepts and clinical practice*. Philadelphia: Mosby/Elsevier, 2006. ISBN: 978-0-323-02845-5.