$$
\begin{prooftree}
\AxiomC{$D_j \vdash D_j$}
\AxiomC{$\bot \vdash I$}
\RightLabel{$\bot_R$}
\UnaryInfC{$\bot \vdash \bot$}
\RightLabel{$\to_L$}
\BinaryInfC{$D_j \to \bot \vdash D_j >> \bot$}
\RightLabel{$\to_R$}
\UnaryInfC{$D_j \to \bot \vdash D_j \to \bot$}
\AxiomC{$\mathsf{father}_n \vdash \mathsf{father}_n$}
\RightLabel{$\wedge_R$}
\BinaryInfC{$D_j \to \bot ; \mathsf{father}_n \vdash (D_j \to \bot) \wedge \mathsf{father}_n$}
\AxiomC{$[\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$\to_L$}
\BinaryInfC{$(D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j \to \bot ; \mathsf{father}_n >> [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$(;,>)$}
\UnaryInfC{$(D_j \to \bot ; \mathsf{father}_n) ; (D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{A_L}
\UnaryInfC{$D_j \to \bot ; \mathsf{father}_n ; (D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$(;,>)$}
\UnaryInfC{$\mathsf{father}_n ; (D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j \to \bot >> [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$\to_R$}
\UnaryInfC{$\mathsf{father}_n ; (D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$(;,>)$}
\UnaryInfC{$(D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \mathsf{father}_n >> (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$\to_R$}
\UnaryInfC{$(D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \mathsf{father}_n \to (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\AxiomC{$\mathsf{father}_n \vdash \mathsf{father}_n$}
\AxiomC{$D_j \vdash D_j$}
\AxiomC{$\bot \vdash I$}
\RightLabel{$\bot_R$}
\UnaryInfC{$\bot \vdash \bot$}
\RightLabel{$\to_L$}
\BinaryInfC{$D_j \to \bot \vdash D_j >> \bot$}
\RightLabel{$\to_R$}
\UnaryInfC{$D_j \to \bot \vdash D_j \to \bot$}
\AxiomC{$[\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$\to_L$}
\BinaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j \to \bot >> [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\RightLabel{$\to_R$}
\UnaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}$}
\AxiomC{$\mathsf{no}_n \vdash \mathsf{no}_n$}
\RightLabel{Pre_L}
\UnaryInfC{$1_{\mathsf{no}_n} \vdash \mathsf{no}_n$}
\RightLabel{$FdiamA_R$}
\UnaryInfC{$\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} \vdash \langle \mathsf{no}_n \rangle^{k-1}\mathsf{no}_n$}
\RightLabel{$FdiamA_R$}
\UnaryInfC{$\{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} \vdash \langle \mathsf{father}_n \rangle \langle \mathsf{no}_n \rangle^{k-1}\mathsf{no}_n$}
\RightLabel{$\wedge_R$}
\BinaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} ; \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} \vdash ((D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle \mathsf{father}_n \rangle \langle \mathsf{no}_n \rangle^{k-1}\mathsf{no}_n$}
\AxiomC{$((D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle \mathsf{father}_n \rangle \langle \mathsf{no}_n \rangle^{k-1}\mathsf{no}_n \vdash (D_j \to \bot) \to \bot$}
\AxiomC{$D_j \vdash D_j$}
\RightLabel{$W <_R$}
\UnaryInfC{$D_j << D_j \vdash \bot$}
\RightLabel{$(<,;)$}
\UnaryInfC{$D_j \vdash \bot ; D_j$}
\RightLabel{$W >_R$}
\UnaryInfC{$I \vdash D_j >> \bot ; D_j$}
\RightLabel{$Grishin_R$}
\UnaryInfC{$I \vdash (D_j >> \bot) ; D_j$}
\RightLabel{E_R}
\UnaryInfC{$I \vdash D_j ; (D_j >> \bot)$}
\RightLabel{$(>,;)$}
\UnaryInfC{$D_j >> I \vdash D_j >> \bot$}
\RightLabel{$\to_R$}
\UnaryInfC{$D_j >> I \vdash D_j \to \bot$}
\AxiomC{$\bot \vdash I$}
\RightLabel{$\mathrm{I}W_R$}
\UnaryInfC{$\bot \vdash D_j$}
\RightLabel{$\to_L$}
\BinaryInfC{$(D_j \to \bot) \to \bot \vdash (D_j >> I) >> D_j$}
\RightLabel{$(;,>)$}
\UnaryInfC{$(D_j >> I) ; (D_j \to \bot) \to \bot \vdash D_j$}
\RightLabel{E_L}
\UnaryInfC{$(D_j \to \bot) \to \bot ; (D_j >> I) \vdash D_j$}
\RightLabel{$(;,>)$}
\UnaryInfC{$D_j >> I \vdash (D_j \to \bot) \to \bot >> D_j$}
\RightLabel{$(>,;)$}
\UnaryInfC{$I \vdash D_j ; ((D_j \to \bot) \to \bot >> D_j)$}
\RightLabel{E_R}
\UnaryInfC{$I \vdash ((D_j \to \bot) \to \bot >> D_j) ; D_j$}
\RightLabel{$Grishin_R$}
\UnaryInfC{$I \vdash (D_j \to \bot) \to \bot >> D_j ; D_j$}
\RightLabel{$I >_R$}
\UnaryInfC{$(D_j \to \bot) \to \bot \vdash D_j ; D_j$}
\RightLabel{C_R}
\UnaryInfC{$(D_j \to \bot) \to \bot \vdash D_j$}
\AxiomC{$D_j \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{Cut}
\BinaryInfC{$(D_j \to \bot) \to \bot \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{Cut}
\BinaryInfC{$((D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle \mathsf{father}_n \rangle \langle \mathsf{no}_n \rangle^{k-1}\mathsf{no}_n \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{Cut}
\BinaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} ; \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{E_L}
\UnaryInfC{$\{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} ; (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{$(;,>)$}
\UnaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} >> \{\mathsf{father}_n\}\{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{FS_A_R}
\UnaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{father}_n\}(\{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} >> \{\mathsf{no}_n\}^{k-1}D_j)$}
\RightLabel{$Back_forw_A$}
\UnaryInfC{$\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{no}_n\}^{k-1}1_{\mathsf{no}_n} >> \{\mathsf{no}_n\}^{k-1}D_j$}
\RightLabel{FS_A_R}
\UnaryInfC{$\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{no}_n\}^{k-1}(1_{\mathsf{no}_n} >> D_j)$}
\RightLabel{$Back_forw_A$}
\UnaryInfC{${\overbrace{\underbrace{\mathsf{no}}}}_n{}^{k-1}\,\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash 1_{\mathsf{no}_n} >> D_j$}
\RightLabel{$\to_R$}
\UnaryInfC{${\overbrace{\underbrace{\mathsf{no}}}}_n{}^{k-1}\,\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash 1_{\mathsf{no}_n} \to D_j$}
\AxiomC{$\Phi_{\mathsf{no}_n} \vdash 1_{\mathsf{no}_n}$}
\AxiomC{$D_j \vdash \{\mathsf{no}_n\}D_j$}
\RightLabel{$\to_L$}
\BinaryInfC{$1_{\mathsf{no}_n} \to D_j \vdash \Phi_{\mathsf{no}_n} >> \{\mathsf{no}_n\}D_j$}
\RightLabel{$Reduce'R$}
\UnaryInfC{$1_{\mathsf{no}_n} \to D_j \vdash \{\mathsf{no}_n\}D_j$}
\RightLabel{Cut}
\BinaryInfC{${\overbrace{\underbrace{\mathsf{no}}}}_n{}^{k-1}\,\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{no}_n\}D_j$}
\RightLabel{$FboxA_R$}
\UnaryInfC{${\overbrace{\underbrace{\mathsf{no}}}}_n{}^{k-1}\,\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{no}_n]D_j$}
\RightLabel{$Back_forw_A2$}
\UnaryInfC{$\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{no}_n\}^{k-1}[\mathsf{no}_n]D_j$}
\RightLabel{$FboxA_R$}
\UnaryInfC{$\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{no}_n]^{k-1}[\mathsf{no}_n]D_j$}
\AxiomC{$[\mathsf{no}_n]^{k-1}[\mathsf{no}_n]D_j \vdash [\mathsf{no}_n]^k D_j$}
\RightLabel{Cut}
\BinaryInfC{$\mathsf{fat}\overbrace{\underbrace{\mathsf{he}}}\mathsf{r}_n\,(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{no}_n]^k D_j$}
\RightLabel{$Back_forw_A2$}
\UnaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \{\mathsf{father}_n\}[\mathsf{no}_n]^k D_j$}
\RightLabel{$FboxA_R$}
\UnaryInfC{$(D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash [\mathsf{father}_n][\mathsf{no}_n]^k D_j$}
\RightLabel{$\to_L$}
\BinaryInfC{$\mathsf{father}_n \to (D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \mathsf{father}_n >> [\mathsf{father}_n][\mathsf{no}_n]^k D_j$}
\RightLabel{Cut}
\BinaryInfC{$(D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \mathsf{father}_n >> [\mathsf{father}_n][\mathsf{no}_n]^k D_j$}
\RightLabel{$\to_R$}
\UnaryInfC{$(D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'} \vdash \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^k D_j$}
\RightLabel{$FboxK_L$}
\UnaryInfC{$\Box_j((D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}) \vdash \{j\}\mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^k D_j$}
\RightLabel{$FboxK_R$}
\UnaryInfC{$\Box_j((D_j \to \bot) \wedge \mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^{k-1}\Box_{j'}D_{j'}) \vdash \Box_j(\mathsf{father}_n \to [\mathsf{father}_n][\mathsf{no}_n]^k D_j)$}
\end{prooftree}
$$