$$\dfrac{\mathsf{E}(n)\mathsf{vision}(n) \vdash \mathsf{vision}_n \qquad \dfrac{\mathsf{dirty}(n,J); \mathsf{vision}_n \vdash \Box_j(D_h \rightarrow \bot)}{\mathsf{vision}_n \vdash \mathsf{dirty}(n,J) >> \Box_j(D_h \rightarrow \bot)}\,(;,>)}{\mathsf{E}(n)\mathsf{vision}(n) \vdash \mathsf{dirty}(n,J) >> \Box_j(D_h \rightarrow \bot)}\,Cut$$

$$\dfrac{\mathsf{E}(n)\mathsf{vision}(n) \vdash \mathsf{dirty}(n,J) >> \Box_j(D_h \rightarrow \bot)}{\mathsf{dirty}(n,J); \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \rightarrow \bot)}\,(;,>)$$

$$\dfrac{\mathsf{dirty}(n,J); \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \rightarrow \bot)}{\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \rightarrow \bot)}\,\wedge_L$$

$$\dfrac{\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \rightarrow \bot)}{\mathsf{father}_n \vdash \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) >> \Box_j(D_h \rightarrow \bot)}\,W >_R$$

$$\dfrac{\mathsf{father}_n \vdash \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) >> \Box_j(D_h \rightarrow \bot)}{\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \rightarrow \bot)}\,(;,>)$$

$$\dfrac{\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \rightarrow \bot) \qquad \dfrac{D_h \vdash D_h}{\{j\}D_h \vdash \langle j \rangle D_h}\,FdiamK_R}{(\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n); \{j\}D_h \vdash \Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h}\,\wedge_R$$

$$\dfrac{(\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n); \{j\}D_h \vdash \Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h}{\{j\}D_h; \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h}\,E_L$$

$$\dfrac{D_h \vdash D_h \qquad \dfrac{\bot \vdash I}{\bot \vdash \bot}\,\bot_R}{D_h \rightarrow \bot \vdash D_h >> \bot}\,\rightarrow_L$$

$$\dfrac{D_h \rightarrow \bot \vdash D_h >> \bot}{D_h \rightarrow \bot \vdash D_h \rightarrow \bot}\,\rightarrow_R$$

$$\dfrac{D_h \rightarrow \bot \vdash D_h \rightarrow \bot \qquad D_h \vdash D_h}{D_h \rightarrow \bot; D_h \vdash (D_h \rightarrow \bot) \wedge D_h}\,\wedge_R$$

$$\dfrac{D_h \rightarrow \bot; D_h \vdash (D_h \rightarrow \bot) \wedge D_h}{\{j\}(D_h \rightarrow \bot; D_h) \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,FdiamK_R$$

$$\dfrac{\{j\}(D_h \rightarrow \bot; D_h) \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{D_h \rightarrow \bot; D_h \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,Forw_back_K$$

$$\dfrac{D_h \rightarrow \bot; D_h \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{D_h \rightarrow \bot \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h}\,(;,<)$$

$$\dfrac{D_h \rightarrow \bot \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h}{\Box_j(D_h \rightarrow \bot) \vdash \{j\}(\overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h)}\,FboxK_L$$

$$\dfrac{\Box_j(D_h \rightarrow \bot) \vdash \{j\}(\overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h)}{\overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot) \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h}\,Back_forw_K$$

$$\dfrac{\overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot) \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h) << D_h}{\overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot); D_h \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,(;,<)$$

$$\dfrac{\overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot); D_h \vdash \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{D_h \vdash \overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot) >> \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,(;,>)$$

$$\dfrac{D_h \vdash \overbrace{\underbrace{j}}\Box_j(D_h \rightarrow \bot) >> \overbrace{\underbrace{j}}\langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{D_h \vdash \overbrace{\underbrace{j}}(\Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h))}\,K_F S_R$$

$$\dfrac{D_h \vdash \overbrace{\underbrace{j}}(\Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h))}{\{j\}D_h \vdash \Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,Forw_back_K 2$$

$$\dfrac{\{j\}D_h \vdash \Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{\langle j \rangle D_h \vdash \Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,FdiamK_L$$

$$\dfrac{\langle j \rangle D_h \vdash \Box_j(D_h \rightarrow \bot) >> \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{\Box_j(D_h \rightarrow \bot); \langle j \rangle D_h \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,(;,>)$$

$$\dfrac{\Box_j(D_h \rightarrow \bot); \langle j \rangle D_h \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{\Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,\wedge_L$$

$$\dfrac{\{j\}D_h; \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h \qquad \Box_j(D_h \rightarrow \bot) \wedge \langle j \rangle D_h \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}{\{j\}D_h; \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \langle j \rangle((D_h \rightarrow \bot) \wedge D_h)}\,Cut$$