

$$\begin{array}{c}
\frac{\frac{\frac{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \text{father}_n >> \{j\}(\text{father}_n >> D_j)}{\text{father}_n; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}(\text{father}_n >> D_j)} (;, >)}{\frac{\text{father}_n \vdash \{j\}(\text{father}_n >> D_j) << \text{dirty}(n, J) \wedge E(n)\text{vision}(n)}{1_{\text{father}_n} \vdash \{j\}(\text{father}_n >> D_j) << \text{dirty}(n, J) \wedge E(n)\text{vision}(n)} \text{Pre}_L} \text{Cut} \\
\frac{\Phi_{\text{father}_n} \vdash 1_{\text{father}_n}}{\Phi_{\text{father}_n} \vdash \{j\}(\text{father}_n >> D_j) << \text{dirty}(n, J) \wedge E(n)\text{vision}(n)} (;, <) \\
\frac{\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}(\text{father}_n >> D_j)}{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash \text{father}_n >> D_j} \text{Back_forw_K} \\
\frac{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash \text{father}_n >> D_j}{\text{father}_n; \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash D_j} (;, >) \\
\frac{\text{father}_n; \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash D_j}{\text{father}_n \vdash D_j << \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n))} (;, <) \\
\frac{\text{father}_n \vdash D_j << \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n))}{1_{\text{father}_n} \vdash D_j << \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n))} \text{Pre}_L \\
\frac{1_{\text{father}_n} \vdash D_j << \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n))}{1_{\text{father}_n}; \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash D_j} (;, <) \\
\frac{1_{\text{father}_n}; \widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash D_j}{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash 1_{\text{father}_n} >> D_j} (;, >) \\
\frac{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash 1_{\text{father}_n} >> D_j}{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash 1_{\text{father}_n} \rightarrow D_j} \rightarrow_R \\
\frac{\Phi_{\text{father}_n} \vdash 1_{\text{father}_n} \quad D_j \vdash \{\text{father}_n\}D_j}{1_{\text{father}_n} \rightarrow D_j \vdash \Phi_{\text{father}_n} >> \{\text{father}_n\}D_j} \rightarrow_L \\
\frac{1_{\text{father}_n} \rightarrow D_j \vdash \Phi_{\text{father}_n} >> \{\text{father}_n\}D_j}{1_{\text{father}_n} \rightarrow D_j \vdash \{\text{father}_n\}D_j} \text{Reduce}'R \\
\frac{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash 1_{\text{father}_n} \rightarrow D_j}{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash \{\text{father}_n\}D_j} \text{Cut} \\
\frac{\widehat{j}(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)) \vdash \{\text{father}_n\}D_j}{\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}\{\text{father}_n\}D_j} \text{Back_forw_K2} \\
\frac{\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}\{\text{father}_n\}D_j}{\text{;}[\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)] \vdash \{\text{father}_n\}\{j\}D_j} \text{Swapout}'R \\
\frac{\text{;}[\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)] \vdash \{\text{father}_n\}\{j\}D_j}{(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)); \text{;} \vdash \{\text{father}_n\}\{j\}D_j} \text{Bigcomma_Cons}_L \\
\frac{(\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)); \text{;} \vdash \{\text{father}_n\}\{j\}D_j}{\text{;} \vdash \{\text{father}_n\}\{j\}D_j} E_L \\
\frac{\text{;} \vdash \{\text{father}_n\}\{j\}D_j}{\text{;} \vdash \{\text{father}_n\}\{j\}D_j << \Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)} (;, <) \\
\frac{\text{;} \vdash \{\text{father}_n\}\{j\}D_j << \Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)}{I \vdash \{\text{father}_n\}\{j\}D_j << \Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)} \text{Bigcomma_Nil}_L \\
\frac{I \vdash \{\text{father}_n\}\{j\}D_j << \Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n)}{\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\{j\}D_j} I <_R \\
\frac{\Phi_{\text{father}_n}; \text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\{j\}D_j}{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \Phi_{\text{father}_n} >> \{\text{father}_n\}\{j\}D_j} (;, >) \\
\frac{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \Phi_{\text{father}_n} >> \{\text{father}_n\}\{j\}D_j}{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\{j\}D_j} \text{Reduce}'R \\
\frac{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\{j\}D_j}{\text{father}_n \text{ dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}D_j} \text{Back_forw_A} \\
\frac{\text{father}_n \text{ dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{j\}D_j}{\text{father}_n \text{ dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \square_j D_j} \text{Fbox}K_R \\
\frac{\text{father}_n \text{ dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \square_j D_j}{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\square_j D_j} \text{Back_forw_A2} \\
\frac{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash \{\text{father}_n\}\square_j D_j}{\text{dirty}(n, J) \wedge E(n)\text{vision}(n) \vdash [\text{father}_n]\square_j D_j} \text{Fbox}A_R
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