$$\frac{D_j \vdash D_j}{\frac{D_j << D_j \vdash \bot}{D_j \vdash \bot; D_j}\,(<,;)}\,W<_R$$

$$\frac{D_j \vdash \bot; D_j}{\frac{I \vdash D_j >> \bot; D_j}{\frac{I \vdash (D_j >> \bot); D_j}{\frac{I \vdash D_j; (D_j >> \bot)}{\frac{D_j >> I \vdash D_j >> \bot}{D_j >> I \vdash D_j \to \bot}\to_R}\,(>,;)}\,E_R}\,Grishin_R}\,W>_R$$

$$\frac{\dfrac{\bot \vdash I}{\bot \vdash D_j}\,IW_R}{\frac{(D_j \to \bot) \to \bot \vdash (D_j >> I) >> D_j}{\frac{(D_j >> I); (D_j \to \bot) \to \bot \vdash D_j}{\frac{(D_j \to \bot) \to \bot; (D_j >> I) \vdash D_j}{\frac{D_j >> I \vdash (D_j \to \bot) \to \bot >> D_j}{\frac{I \vdash D_j; ((D_j \to \bot) \to \bot >> D_j)}{\frac{I \vdash ((D_j \to \bot) \to \bot >> D_j); D_j}{\frac{I \vdash (D_j \to \bot) \to \bot >> D_j; D_j}{(D_j \to \bot) \to \bot \vdash D_j}\,I>_R}\,Grishin_R}\,E_R}\,(;,>)}\,(;,>)}\,E_L}\,(;,>)}\,\to_L}$$

$$\frac{(D_j \to \bot) \to \bot \vdash D_j \quad D_j \vdash \{father_n\}\{no_n\}^{k-1}D_j}{(D_j \to \bot) \to \bot \vdash \{father_n\}\{no_n\}^{k-1}D_j}\,Cut$$

$$\frac{D_j \vdash D_j}{\frac{\dfrac{\bot \vdash I}{\bot \vdash \bot}\,\bot_R}{\frac{D_j \to \bot \vdash D_j >> \bot}{D_j \to \bot \vdash D_j \to \bot}\to_R}\to_L}$$

$$\frac{[father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j >> [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}\to_R}\to_L$$

$$\frac{no_n \vdash no_n}{\frac{1no_n \vdash no_n}{\{no_n\}^{k-1}1no_n \vdash \langle no_n\rangle^{k-1}no_n}\,Fdiam A_R}\,Pre_L$$

$$\frac{\{father_n\}\{no_n\}^{k-1} \vdash \langle father_n\rangle\langle no_n\rangle^{k-1}no_n}{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}; \{father_n\}\{no_n\}^{k-1}1no_n \vdash ((D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle father_n\rangle\langle no_n\rangle^{k-1}no_n}\,FdiamA_R \wedge_R$$

$$\frac{((D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle father_n\rangle\langle no_n\rangle^{k-1}no_n \vdash (D_j \to \bot) \to \bot}{((D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}) \wedge \langle father_n\rangle\langle no_n\rangle^{k-1}1no_n \vdash \{father_n\}\{no_n\}^{k-1}D_j}\,Cut$$

$$\frac{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}; \{father_n\}\{no_n\}^{k-1}1no_n \vdash \{father_n\}\{no_n\}^{k-1}D_j}{\frac{\{father_n\}\{no_n\}^{k-1}1no_n; (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{father_n\}\{no_n\}^{k-1}D_j}{\frac{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{father_n\}\{no_n\}^{k-1}1no_n >> \{father_n\}\{no_n\}^{k-1}D_j}{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{father_n\}(\{no_n\}^{k-1}1no_n >> \{no_n\}^{k-1}D_j)}\,FS\_A\_R}\,(;,>)}\,E_L$$

$$\frac{\widetilde{father}_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{no_n\}^{k-1}1no_n >> \{no_n\}^{k-1}D_j}{\frac{\widetilde{no_n}^{k-1}father_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash 1no_n >> D_j}{\widetilde{no_n}^{k-1}father_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash 1no_n \to D_j}\to_R}\,FS\_A\_R}\,Back\_forw\_A$$

$$\frac{\Phi no_n \vdash 1no_n \quad D_j \vdash \{no_n\}D_j}{\frac{1no_n \to D_j \vdash \Phi no_n >> \{no_n\}D_j}{1no_n \to D_j \vdash \{no_n\}D_j}\,Reduce'R}\to_L$$

$$\frac{\widetilde{no_n}^{k-1}father_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{no_n\}D_j}{\frac{\widetilde{no_n}^{k-1}father_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [no_n]D_j}{\frac{\widetilde{father}_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [no_n]^{k-1}[no_n]D_j}{\widetilde{father}_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [no_n]^{k-1}[no_n]D_j}\,FboxA_R}\,Back\_forw\_A2}\,Cut$$

$$\frac{[no_n]^{k-1}[no_n]D_j \vdash [no_n]^kD_j}{\frac{\widetilde{father}_n(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [no_n]^kD_j}{\frac{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash \{father_n\}[no_n]^kD_j}{(D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [father_n][no_n]^kD_j}\,FboxA_R}\,Back\_forw\_A2}\,Cut$$

$$\frac{D_j \vdash D_j}{\frac{\dfrac{\bot \vdash I}{\bot \vdash \bot}\,\bot_R}{\frac{D_j \to \bot \vdash D_j >> \bot}{D_j \to \bot \vdash D_j \to \bot}\to_R}\to_L}$$

$$\frac{father_n \vdash father_n}{D_j \to \bot; father_n \vdash (D_j \to \bot) \wedge father_n}\wedge_R$$

$$\frac{[father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{(D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j \to \bot; father_n >> [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{(D_j \to \bot); father_n; (D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{D_j \to \bot; father_n; (D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{father_n; (D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash D_j \to \bot >> [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{father_n; (D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{\frac{(D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash father_n >> (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}{(D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash father_n \to (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}}\to_R}\,(;,>)}\to_R}\,(;,>)}\,A_L}\,(;,>)}\to_L$$

$$\frac{father_n \vdash father_n}{father_n \to (D_j \to \bot) \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash father_n >> [father_n][no_n]^kD_j}\,Cut$$

$$\frac{(D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash father_n >> [father_n][no_n]^kD_j}{\frac{(D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'} \vdash father_n \to [father_n][no_n]^kD_j}{\frac{\Box_j((D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}) \vdash \{j\}father_n \to [father_n][no_n]^kD_j}{\Box_j((D_j \to \bot) \wedge father_n \to [father_n][no_n]^{k-1}\Box_{j'}D_{j'}) \vdash \Box_j(father_n \to [father_n][no_n]^kD_j)}\,FboxK_R}\,FboxK_L}\to_R$$