| $\frac{D_{j} \vdash D_{j} \qquad \frac{\bot \vdash I}{\bot \vdash \bot} \bot_{R}}{D_{j} \rightarrow \bot \vdash D_{j} \rightarrow \bot} \rightarrow_{L} \\ \frac{D_{j} \rightarrow \bot \vdash D_{j} \rightarrow \bot}{(D_{j} \rightarrow \bot \vdash D_{j} \rightarrow \bot} \rightarrow_{R} \\ \frac{(father_{n})[no_{n}]^{k} \Box_{j'} D_{j'} \vdash [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'}}{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'}} \rightarrow_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \vdash D_{j} \rightarrow_{T} \rightarrow_{R} \\ (D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \vdash D_{j} \rightarrow_{T} \rightarrow_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \vdash (D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \rightarrow_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \land (father_{n})(no_{n})^{k} no(n) \vdash (father_{n})(no_{n})^{k} no(n)}{(father_{n})(no_{n})^{k} no(n) \vdash (D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \land (father_{n})(no_{n})^{k} no(n) \vdash (D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow [father_{n}][no_{n}]^{k} \Box_{j'} D_{j'} \land (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)}{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(no_{n})^{k} no(n)} \land_{R} \\ \frac{(D_{j} \rightarrow \bot) \rightarrow (father_{n})(n$ | $ \begin{array}{c} \frac{L \vdash I}{D_j \rightarrow \bot \vdash D_j > \bot} \bot_R \\ \frac{D_j \vdash D_j}{D_j \rightarrow \bot \vdash D_j > \bot} \to R \\ \hline D_j \rightarrow \bot \vdash D_j \rightarrow \bot \\ \hline D_j \rightarrow \bot \\ \hline D_j \rightarrow \bot \vdash D_j \rightarrow \bot \\ \hline $ | $\frac{D_{j'} \vdash D_{j'} \stackrel{\bot \vdash I}{\bot \vdash \bot} \perp_{R}}{D_{j'} \to \bot \vdash D_{j'} > \bot} \to L} {\frac{D_{j'} \vdash D_{j'} \stackrel{\bot \vdash I}{\bot \vdash \bot} \perp_{R}}{D_{j'} \to \bot \vdash D_{j'} > \bot}}{D_{j'} \to \bot \vdash D_{j'} \to \bot} \to R} \\ \frac{D_{j'} \vdash D_{j'} \to \bot \vdash D_{j'} \to \bot}{D_{j'} \to \bot \vdash D_{j'} (D_{j'} \to \bot)} F diam K_{R}} {\frac{D_{j'}D_{j'} \vdash D_{j'}D_{j'} \to \bot}{D_{j'}D_{j'}D_{j'} \to \bot}}{D_{j'}D_$ | $\frac{[\operatorname{non}]^k\Box_j\cdot D_j\cdot \vdash [\operatorname{non}]^k\Box_j\cdot D_j\cdot}{[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot}FboxA_L}{[\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \vdash [\operatorname{non}]^k\Box_j\cdot D_j\cdot}Back_jorw_A} \\ = \frac{\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \vdash [\operatorname{non}]^k\Box_j\cdot D_j\cdot}{[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \vdash [\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot ; (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash (\operatorname{father}_n) ([\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot ; (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash (\operatorname{father}_n) ([\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot ; (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash (\operatorname{father}_n) ([\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot ; (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash (\operatorname{father}_n) ([\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash \operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash \operatorname{father}_n[\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(father}_n)(\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash [\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash [\operatorname{father}_n][\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(father}_n)(\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot) \vdash [\operatorname{father}_n](\operatorname{non}]^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Box_j\cdot D_j\cdot \land (\operatorname{father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Box_j\cdot D_j\cdot \land (\operatorname{father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Box_j\cdot D_j\cdot \land (\operatorname{father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}}{\operatorname{(father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Diamond_j\cdot (D_j\cdot \to \bot)}} \\ = \frac{\operatorname{hon}_n(\operatorname{hon})^k\Box_j\cdot D_j\cdot \land (\operatorname{father}_n)(\operatorname{non})^k\Box_j\cdot D_j\cdot \land (\operatorname{non})^k\Box_j$ | $\frac{D_{j'} + D_{j'}}{\square_{j'}D_{j'} + [j']D_{j'}} Fbox K_L \\ Back forw_K \\ \hline D_{j'} \rightarrow \bot \vdash \widehat{j'} \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} IW_R \\ Back forw_K \\ \hline D_{j'} \rightarrow \bot \vdash \widehat{j'} \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} IW_R \\ \hline D_{j'} \rightarrow \bot \vdash \widehat{j'} \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'} \rightarrow \bot \vdash \widehat{j'} \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'} \rightarrow \bot \vdash \widehat{j'} \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'} \rightarrow \bot \vdash \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'} \rightarrow \bot \vdash \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \rightarrow \bot \vdash \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \rightarrow \bot \vdash \square_{j'}D_{j'} > \widehat{j'} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot) \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot) \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot) \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \Diamond_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ \hline D_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} \end{bmatrix} V_R \\ D_{j'}D_{j'} \wedge \partial_{j'}D_{j'} \wedge \partial_{j'}(D_{j'} \rightarrow \bot \vdash \bot} V_R $ $\hline D_{j'}D_{j'} \wedge \partial_{j'}D_{j'} \wedge \partial_{j'}D_{j'$ |
|--|---|---|--|---|
| $\frac{(D_j \to \bot) \to [lattler_n][lio_n] \cap \sqcup_{j'} D_{j'} \vdash D_j \to \bot >> [rattler_n][loo_n] \cap \sqcup_{j'} D_{j'}}{(D_! \to \bot) \to [father_n][loo_n] \cap \sqcup_{j'} D_{j'}} \to_R} \to \frac{(lattler_n)(loo_n) \cap loo_n) \cap loo_n}{(father_n)(loo_n) \cap loo_n} \cap loo_n) \cap loo_n}{(father_n)(loo_n) \cap loo_n} \cap loo_n} \to_R}$ | $\frac{(D_j \to \bot) \to [lattier_n][no_n]^* \sqcup_{j'} D_{j'}; (D_j \to \bot) \to (father_n)(no_n)^* no(n) \vdash D_j \to \bot >> [rattier_n][no_n]^* \sqcup_{j'} D_{j'} \land (father_n)(no_n)^* no(n)}{(D_i \to \bot) \to [father_n][no_n]^k \sqcup_{j'} D_{j'} \land (father_n)(no_n)^* no(n)} \to_R$ | $\frac{(D_j \to \bot) \to [lattler_n][lio_n]^{m} \sqcup_{j'} D_{j'} \wedge (ratner_n) \langle no_n \rangle^{m} no(n) \vdash D_j \to \bot >> [tatner_n][no_n]^{m} \sqcup_{j'} D_{j'} \wedge (tatner_n) \langle no_n \rangle^{m} \vee_{j'} (D_{j'} \to \bot)}{(D_{+} \to \bot) \to [father_n][no_n]^{k} \sqcup_{j'} D_{j'} \wedge (father_n) \langle no_n \rangle^{k} \vee_{j'} (D_{j'} \to \bot)} \to_R$ | $\frac{(D_j \to \bot) \to [father_n][no_n]^k \Box_{j'} D_{j'} \wedge \langle father_n \rangle \langle no_n \rangle^k \Diamond_{j'} (D_{j'} \to \bot) \vdash D_j \to \bot >> \bot}{(D_j \to \bot) \to [father_n][no_n]^k \Box_{j'} D_{j'} \wedge \langle father_n \rangle \langle no_n \rangle^k \Diamond_{j'} (D_{j'} \to \bot) \vdash (D_j \to \bot) \to \bot} \xrightarrow{Cut} Cut$ | |
| $\frac{(D_j \land \bot) \land [lattler_n][liO_n] \ \Box_j \land D_j \land (D_j \to \bot) \to [lattler_n][liO_n] \ \Box_j \land D_j \land (D_j \to \bot) \to (lattler_n)(liO_n) \ liO(n)}{(D_i \to \bot) \to [father\][no\]^k \Box_i D_i \land (D_j \to \bot) \to (father\)(no\)^k no(n)} \land_R$ | $\frac{(D_j \to \bot) \to [father_n][no_n] \ \Box_{j'} D_{j'}, (D_j \to \bot) \to (father_n/no_n) \ no(n) + (D_j \to \bot) \to [father_n][no_n] \ \Box_{j'} D_{j'} \land (father_n/no_n) \ \land L}{((D_j \to \bot) \to [father_n][no_n]^k \Box_{j'} D_{j'}) \land ((D_j \to \bot) \to (father_n/no_n)^k no(n)) \vdash (D_j \to \bot) \to [father_n][no_n]^k \Box_{j'} D_{j'} \land (father_n/no_n)^k no(n)} \land L}$ | $(D_j \rightarrow \bot) \rightarrow [\text{lattice}_n][\text{lio}_n] \ \Box_{j'} D_{j'} \land (\text{lattice}_n/(\text{lio}_n) \mid \text{lio}(n)) \vdash (D_j \rightarrow \bot) \rightarrow [\text{lattice}_n][\text{lio}_n] \ \Box_{j'} D_{j'} \land (\text{lattice}_n/(\text{lio}_n) \lor J') \land (D_j \rightarrow \bot) $ $(D_i \rightarrow \bot) \rightarrow [\text{father}][\text{no} \]^k \Box \cup D \cup \land / \text{father} \ \backslash \text{no} \ \setminus^k \text{no}(n) \vdash (D_i \rightarrow \bot) $ | $\frac{(\mathcal{D}_{j} \wedge \pm) \neg [rather_{n}][no_{n}] \ \Box_{j'} \mathcal{D}_{j'} \wedge (rather_{n}/(no_{n}/\ \lor_{j'}(\mathcal{D}_{j'} \neg \pm) + (\mathcal{D}_{j} \neg \pm) \neg \pm}{Cut}}{}$ | |
| $\frac{(D_j \to \pm) \to [rather_n][no_n] \sqcup j'D_j', rather_n/no_n/no_n/no_n/no_n] \cup (D_j \to \pm) \to [rather_n][no_n] \sqcup j'D_j') \land ((D_j \to \pm) \to (rather_n/no_n/no_n/no_n) \cup (D_j \to \pm) \to (rather_n/no_n/no_n/no_n/no_n)}{((D_j \to \pm) \to (rather_n/no_n/no_n/no_n) \cup (D_j \to \pm) \to (rather_n/no_n/no_n/no_n/no_n)} \land L$ | $\frac{((D_j \land \bot) \land (Indition) \sqcup j \land D_j \land ((D_j \land \bot) \land (Indition) \land (Ind)) \land (D_j \land \bot) \land (Indition) \sqcup j \land D_j \land (Indition) \land (Ind)}{(Ind) \sqcup j \land Ind} = \frac{((D_j \land \bot) \land (Ind) \sqcup j \land Ind) \land (Ind)}{(Ind) \sqcup j \land Ind} = \frac{((D_j \land \bot) \land (Ind) \sqcup j \land Ind) \land (Ind)}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \land Ind}{(Ind) \sqcup j \land Ind} = \frac{(Ind) \sqcup j \lor Ind}{(Ind) \sqcup j \lor Ind} = \frac{(Ind) \sqcup j \sqcup j \lor Ind}{(Ind) \sqcup j \sqcup $ | $(D_j \to \bot) \to [father_n][no_n]^k \square_{j'} D_{j'} \land \langle father_n \rangle \langle no_n \rangle^k no(n) \vdash (D_j \to \bot) \wedge \langle no_n \rangle^k no(n) \wedge \langle no_n \rangle$ | $\frac{\perp_{f}}{}$ Cut | |
| $\frac{((D_j \to \pm) \to [lattler_n][llo_n] \ \Box_{j'} D_{j'}) \land (lattler_n/(llo_n) \ llo(n) \vdash ((D_j \to \pm) \to [lattler_n][llo_n] \ \Box_{j'} D_{j'}) \land ((D_j \to \pm) \to (lattler_n/(llo_n) \ llo(n))}{}$ | $((D_j \to \bot) \to [father\][no\]^k \Box \cup D \cup \wedge / father\ \setminus no\ \setminus^k no(n) \vdash (D_i \to \bot) \to \bot$ | $ \stackrel{\text{$^k\square_{j'}D_{j'}$}}{\longrightarrow} \wedge ((D_j \to \bot) \to \langle father_n \rangle \langle no_n \rangle^k no(n)) \vdash (D_j \to \bot) \to \bot \\ Cut $ | | |

 $((D_j \to \bot) \to [\mathsf{father}_n][\mathsf{no}_n]^k \square_{j'} D_{j'}) \wedge \langle \mathsf{father}_n \rangle \langle \mathsf{no}_n \rangle^k \mathsf{no}(n) \vdash (D_j \to \bot) \to \bot$