

[illegible]

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$$\begin{array}{c}
\frac{\frac{[no_n]^k \Box_j D_j \vdash [no_n]^k \Box_j D_j}{[father_n][no_n]^k \Box_j D_j \vdash \{father_n\}[no_n]^k \Box_j D_j} FboxA_L}{\widehat{father_n}[father_n][no_n]^k \Box_j D_j \vdash [no_n]^k \Box_j D_j} Backjorw_A \quad (no_n)^k \Diamond_j (D_j \rightarrow \perp) \vdash (no_n)^k \Diamond_j (D_j \rightarrow \perp) \\
\frac{\widehat{father_n}[father_n][no_n]^k \Box_j D_j; \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash [no_n]^k \Box_j D_j \wedge (no_n)^k \Diamond_j (D_j \rightarrow \perp)}{\{father_n\}(\widehat{father_n}[father_n][no_n]^k \Box_j D_j; \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp)) \vdash (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))} FdiamA_R \\
\frac{\widehat{father_n}[father_n][no_n]^k \Box_j D_j; \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash \widehat{father_n}(father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))}{\widehat{father_n}[father_n][no_n]^k \Box_j D_j; \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash \widehat{father_n}(father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))} ForwAck_A \\
\frac{\langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash \widehat{father_n}[father_n][no_n]^k \Box_j D_j >> \widehat{father_n}(father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))}{\langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash \widehat{father_n}([father_n][no_n]^k \Box_j D_j >> (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp)))} A_F S_R \\
\frac{\widehat{father_n}([father_n][no_n]^k \Box_j D_j >> (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp)))}{\{father_n\} \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash [father_n][no_n]^k \Box_j D_j >> (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))} ForwAck_A2 \\
\frac{\{father_n\} \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash [father_n][no_n]^k \Box_j D_j >> (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))}{\{father_n\} \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash [father_n][no_n]^k \Box_j D_j >> (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))} FdiamA_L \\
\frac{\frac{D_j \vdash D_j \quad \frac{\perp \vdash I}{\perp \vdash \perp} \perp_R}{D_j \rightarrow \perp \vdash D_j >> \perp} \rightarrow_L}{\frac{D_j \rightarrow \perp \vdash D_j >> \perp}{D_j \rightarrow \perp \vdash D_j \rightarrow \perp} \rightarrow_R} \quad \frac{\widehat{father_n}[father_n][no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash (father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))}{\widehat{father_n}[father_n][no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash \widehat{father_n}(father_n)([no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp))} \wedge_L \\
\frac{\frac{D_j \rightarrow \perp \vdash [father_n][no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash D_j \rightarrow \perp >> \perp}{(D_j \rightarrow \perp) \vdash [father_n][no_n]^k \Box_j D_j \wedge \langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash (D_j \rightarrow \perp) \vdash \perp} \rightarrow_R}{\frac{\langle no_n \rangle^k \Diamond_j (D_j \rightarrow \perp) \vdash (D_j \rightarrow \perp) \vdash \perp}{Cut} Cut}
\end{array}$$

$$\frac{\frac{\frac{\{ \langle \text{no}_n \rangle^k \square_{j'} D_{j'} \wedge \langle \text{no}_n \rangle^k \Diamond_{j'} (D_{j'} \rightarrow \perp) \} \vdash \perp}{\{ \text{father}_n \} \{ \text{no}_n \}^k \square_{j'} D_{j'} \wedge \langle \text{no}_n \rangle^k \Diamond_{j'} (D_{j'} \rightarrow \perp) \vdash \perp} \text{Forwack}_{A^2}}{\frac{\langle \text{father}_n \rangle \{ \text{no}_n \}^k \square_{j'} D_{j'} \wedge \langle \text{no}_n \rangle^k \Diamond_{j'} (D_{j'} \rightarrow \perp) \vdash \perp}{j' \rightarrow \perp \vdash \perp} \rightarrow_L} \text{Fdiam}_{A_L} \text{Cut}$$