$$
\begin{prooftree}
\AxiomC{$\mathsf{E}(n)\mathsf{vision}(n) \vdash \mathsf{vision}_n$}
\AxiomC{$\mathsf{dirty}(n,J); \mathsf{vision}_n \vdash \Box_j(D_h \to \bot)$}
\RightLabel{$(;,>)$}
\UnaryInfC{$\mathsf{vision}_n \vdash \mathsf{dirty}(n,J) >> \Box_j(D_h \to \bot)$}
\RightLabel{Cut}
\BinaryInfC{$\mathsf{E}(n)\mathsf{vision}(n) \vdash \mathsf{dirty}(n,J) >> \Box_j(D_h \to \bot)$}
\RightLabel{$(;,>)$}
\UnaryInfC{$\mathsf{dirty}(n,J); \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \to \bot)$}
\RightLabel{$\wedge_L$}
\UnaryInfC{$\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) \vdash \Box_j(D_h \to \bot)$}
\RightLabel{$W >_R$}
\UnaryInfC{$\mathsf{father}_n \vdash \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n) >> \Box_j(D_h \to \bot)$}
\RightLabel{$(;,>)$}
\UnaryInfC{$\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \to \bot)$}
\AxiomC{$D_h \vdash D_h$}
\RightLabel{$FdiamK_R$}
\UnaryInfC{$\{j\}D_h \vdash \langle j\rangle D_h$}
\RightLabel{$\wedge_R$}
\BinaryInfC{$(\mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n); \{j\}D_h \vdash \Box_j(D_h \to \bot) \wedge \langle j\rangle D_h$}
\RightLabel{E_L}
\UnaryInfC{$\{j\}D_h; \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \Box_j(D_h \to \bot) \wedge \langle j\rangle D_h$}
\AxiomC{$D_h \vdash D_h$}
\AxiomC{$\bot \vdash I$}
\RightLabel{$\bot_R$}
\UnaryInfC{$\bot \vdash \bot$}
\RightLabel{$\to_L$}
\BinaryInfC{$D_h \to \bot \vdash D_h >> \bot$}
\RightLabel{$\to_R$}
\UnaryInfC{$D_h \to \bot \vdash D_h \to \bot$}
\AxiomC{$D_h \vdash D_h$}
\RightLabel{$\wedge_R$}
\BinaryInfC{$D_h \to \bot; D_h \vdash (D_h \to \bot) \wedge D_h$}
\RightLabel{$FdiamK_R$}
\UnaryInfC{$\{j\}(D_h \to \bot; D_h) \vdash \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$Forw\_back\_K$}
\UnaryInfC{$D_h \to \bot; D_h \vdash \overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$(;,<)$}
\UnaryInfC{$D_h \to \bot \vdash \overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h) << D_h$}
\RightLabel{$FboxK_L$}
\UnaryInfC{$\Box_j(D_h \to \bot) \vdash \{j\}(\overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h) << D_h)$}
\RightLabel{$Back\_forw\_K$}
\UnaryInfC{$\overbrace{\underbrace{j}} \Box_j(D_h \to \bot) \vdash \overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h) << D_h$}
\RightLabel{$(;,<)$}
\UnaryInfC{$\overbrace{\underbrace{j}} \Box_j(D_h \to \bot); D_h \vdash \overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$(;,>)$}
\UnaryInfC{$D_h \vdash \overbrace{\underbrace{j}} \Box_j(D_h \to \bot) >> \overbrace{\underbrace{j}} \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$K_F S_R$}
\UnaryInfC{$D_h \vdash \overbrace{\underbrace{j}} (\Box_j(D_h \to \bot) >> \langle j\rangle((D_h \to \bot) \wedge D_h))$}
\RightLabel{$Forw\_back\_K2$}
\UnaryInfC{$\{j\}D_h \vdash \Box_j(D_h \to \bot) >> \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$FdiamK_L$}
\UnaryInfC{$\langle j\rangle D_h \vdash \Box_j(D_h \to \bot) >> \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$(;,>)$}
\UnaryInfC{$\Box_j(D_h \to \bot); \langle j\rangle D_h \vdash \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{$\wedge_L$}
\UnaryInfC{$\Box_j(D_h \to \bot) \wedge \langle j\rangle D_h \vdash \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\RightLabel{Cut}
\BinaryInfC{$\{j\}D_h; \mathsf{dirty}(n,J) \wedge \mathsf{E}(n)\mathsf{vision}(n); \mathsf{father}_n \vdash \langle j\rangle((D_h \to \bot) \wedge D_h)$}
\end{prooftree}
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