מטלה 1:

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$$X = \begin{pmatrix} 1 & -1 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad X^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 2 \end{pmatrix}$$

$$X^T \cdot X = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 7 \end{pmatrix}$$

$$(X^T \cdot X)^{-1} = \left(\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 1 & 7 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\left(\begin{array}{cc|cc} 1 & 7 & 0 & 1 \\ 4 & 1 & 1 & 0 \end{array}\right) \xrightarrow{R_2 = R_2 - 4R_1} \left(\begin{array}{cc|cc} 1 & 7 & 0 & 1 \\ 0 & -27 & 1 & -4 \end{array}\right) \xrightarrow{R_1 = R_1 + \frac{7}{27} R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{7}{27} & -\frac{1}{27} \\ 0 & -27 & 1 & -4 \end{array}\right)$$

$$\xrightarrow{R_2 = R_2 \cdot (-\frac{1}{27})} \left(\begin{array}{cc|cc} 1 & 0 & \frac{7}{27} & -\frac{1}{27} \\ 0 & 1 & -\frac{1}{27} & \frac{4}{27} \end{array}\right) = \begin{pmatrix} \frac{7}{27} & \frac{-1}{27} \\ -\frac{1}{27} & \frac{4}{27} \end{pmatrix}$$

$$(X^T \cdot X)^{-1} \cdot X^T = \begin{pmatrix} \frac{7}{27} & \frac{-1}{27} \\ -\frac{1}{27} & \frac{4}{27} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{8}{27} & \frac{8}{27} & \frac{6}{27} & \frac{5}{27} \\ \frac{-5}{27} & \frac{-5}{27} & \frac{3}{27} & \frac{7}{27} \end{pmatrix}$$

$$(X^T \cdot X)^{-1} \cdot X^T \cdot y = \begin{pmatrix} \frac{8}{27} & \frac{8}{27} & \frac{6}{27} & \frac{5}{27} \\ \frac{-5}{27} & \frac{-5}{27} & \frac{3}{27} & \frac{7}{27} \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

לכן, $\theta^* = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$J(\theta^*) = \sum_{i=1}^{4} \left((\theta^*)^T (1, x^i) - y^i\right)^2 = 1 + 1 + 0 + 0 = 2 \Rightarrow J(\theta^*) = 2$$

$i=3 \to \left((1,1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2\right)^2 = 0$ $\qquad i=1 \to \left((1,1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} - (-1)\right)^2 = 1$

$i=4 \to \left((1,1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 3\right)^2 = 0$ $\qquad i=2 \to \left((1,1), \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 3\right)^2 = 1$

$$\sum_{i=1}^{4}\left((\theta_0,\theta_1)\begin{pmatrix}1\\x^i\end{pmatrix}-y^{(i)}\right)^2=\sum_i\left(\theta_0+\theta_1x^i-y^{(i)}\right)^2$$ ②

$$=(\theta_0+\theta_1\cdot(-1)+1)^2+(\theta_0+\theta_1(-1)-1)^2+$$
$$(\theta_0+\theta_1(1)-2)^2+(\theta_0+\theta_1(2)-3)^2$$

(*) $(\theta_0-\theta_1+1)^2=(\theta_0+\theta_1(-1)+1)(\theta_0-\theta_1+1)=\theta_0^2-\theta_0\theta_1+\theta_0-\theta_0\theta_1-\theta_1+\theta_0-\theta_1+1=\theta_0^2-2\theta_0\theta_1-2\theta_0-2\theta_1+1+\theta_1^2$

(*) $(\theta_0+\theta_1\cdot(-1)-1)^2=(\theta_0-\theta_1-1)^2=(\theta_0-\theta_1-1)(\theta_0-\theta_1-1)=\theta_0^2-\theta_0\theta_1-\theta_0-\theta_1\theta_0+\theta_1^2+\theta_1-\theta_0+\theta_1+1=$
$\theta_0^2-2\theta_0\theta_1+2\theta_0+\theta_1^2+2\theta_1+1$

(*) + (*) $=\theta_0^2-2\theta_0\theta_1+2\theta_0-2\theta_1+1+\theta_1^2+\theta_0^2-2\theta_0\theta_1-2\theta_0+\theta_1^2+2\theta_1+1=2\theta_0^2-4\theta_0\theta_1+2\theta_1^2+2$

(*) $(\theta_0+\theta_1-2)(\theta_0+\theta_1-2)=\theta_0^2+\theta_0\theta_1-2\theta_0$
$+\theta_0\theta_1+\theta_1^2-2\theta_1$
$-2\theta_0-2\theta_1+4$
$\theta_0^2+2\theta_0\theta_1+\theta_1^2-4\theta_0-4\theta_1+4$

(*) $(\theta_0+2\theta_1-3)(\theta_0+2\theta_1-3)=\theta_0^2+2\theta_0\theta_1$
$-3\theta_0+2\theta_0\theta_1$
$+4\theta_1^2-6\theta_1$
$-3\theta_0-6\theta_1+9$
$\theta_0^2+4\theta_0\theta_1+4\theta_1^2-6\theta_0-12\theta_1+9$

$2\cancel{\theta_0^2}+6\theta_0\theta_1+\cancel{5\theta_1^2}-10\cancel{\theta_0}\cancel{-16\theta_1}+\cancel{13}$

$\cancel{2\theta_0^2}-4\cancel{\theta_0}\theta_1\cancel{-4\theta_1}+\cancel{2\theta_1^2}+\cancel{2}$

$J(\theta)=4\theta_0^2+2\theta_0\theta_1+7\theta_1^2-10\theta_0-16\theta_1+15$ תשובה סופית:

$\frac{\partial J}{\partial\theta_0}=8\theta_0+2\theta_1-10$

$\frac{\partial J}{\partial\theta_1}=2\theta_0+14\theta_1-16$

$$\nabla J(\theta)=\begin{bmatrix}8\theta_0+2\theta_1-10\\2\theta_0+14\theta_1-16\end{bmatrix}$$

$$\theta^*=\begin{bmatrix}1\\1\end{bmatrix}$$

$$\nabla J(\theta^*)=\begin{bmatrix}8+2-10\\2+14-16\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

א $\hat{y} = \theta_0 + \theta_1 x = 1 + x = 1 + 1.5 = 2.5$ 3

$\theta^* = \binom{1}{1}$

ב $x_1 = 1 \longrightarrow y_1 = 2$ נבחר שני ערכי x הקרובים ביותר ל- $x = 1.5$

$x_2 = 2 \longrightarrow y_2 = 3$

$\hat{y} = \frac{2+3}{2} = 2.5$ ולכן