**ASSIGNMENT COVER SHEET**

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| Date of submission | / / 2020 |

Please complete your details above and include this cover sheet as the first page on your assignment submission.

Please also remember to upload your source file (Matlab or otherwise), in a format that can be executed to confirm functionality, i.e. ensure all support files are included, and files are logically names and commented. A ‘zip’ file with subdirectories for each question and/or a ‘README’ file is highly recommended.

The allocation of marks will be:

|  |  |  |
| --- | --- | --- |
| Question | Max [%] | Mark [%] |
| **Transmitter/20** | |  |
| a | 5 |  |
| b | 5 |  |
| c | 5 |  |
| d | 5 |  |
| **Receiver/40** | |  |
| e | 10 |  |
| f | 10 |  |
| g | 20 |  |
| **Equaliser/40** | |  |
| h | 20 |  |
| i | 10 |  |
| j | 10 |  |
| **TOTAL** | **100** |  |

a) In this case, a throughput of 1Mb/s is required with a bandwidth of 500kHz. Three modulation types are available:

* Bipolar Signalling, or 2 PAM sends a stream of raised cosine shaped pulse of amplitude -1 or 1, representing the state of each individual bit. Hence, Rs = Rb. The bandwidth of the signal is therefore 1Mb/s. However, the transmitted signal of this method has only two voltage levels, making resilient to Inter-Symbol Interference (ISI).
* 4 PAM, like Bipolar Signalling, sends a series of shaped pulses. These are of amplitude -3, -1, 1 or 3, representing the value of each set of two bits. Hence, two bits are transmitted for each pulse, so Rs=2Rb, giving a bandwidth of 500kHz for this case.
* 8 PAM is similar to 4 PAM, but with a symbol size of 8, giving a data rate of 1.5Mb/s for a symbol rate of 500kHz. Because of the decreased margins between levels in the transmitted signal, this is significantly more susceptible to ISI than 4 PAM.

Considering these characteristics, 4 PAM is considered the most suitable method of modulation as its bandwidth of 500kHz for a bit rate of 1Mb/s meets requirement. If 8 PAM were used, it would also meet these requirements, and would occupy less bandwidth than 4 PAM. However, its increased susceptibility to channel noise makes it less reliable.

b) In the time domain 4 PAM is defined:



Where defines the pulse shape, which is then convolved with a series of impulses of height ak, representing the data to be modulated and sent over the channel.

To find the Power Spectral Density (PSD) function:



Where |YT(f)|2 is the energy density function of the Fourier transform of y(t). By convolution theory:

Where is the Fourier transform of:



Thus, the PSD becomes:



Rearranging gives:



Where is the PSD function of, and is the Fourier transform of . In this case, no pulse shaping is used. is therefore a square sided rectangle function:

In graphical form:

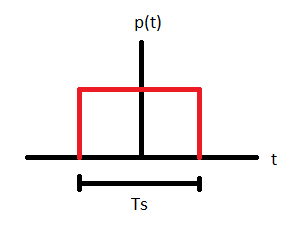


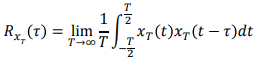
Figure 1; time domain visualisation of a rectangular pulse

Therefore:

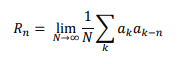
)

To find the energy spectral density, this is squared:

Using the autocorrelation properties of , , the autocorrelation of x(t), where:



Because x(t) is a discrete function, the integral will be zero for all values of tau that do not cause the xT functions to align. This will occur at tau = 0 and every other multiple of the signal period, Ts. For the purpose of explanation, these two cases are defined separately as:





Where N represents the number of symbols transmitted in the time T, given a constant symbol rate of Ts. Given that an infinite number of symbols are transmitted, and that a symbol values of -3, 3, 1, and -1 are equally likely to occur, R0 becomes:

Therefore:

To calculate Rn, all 16 possible combinations of must be considered:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | | | |
|  | -3 | -1 | 1 | 3 |
| -3 | 9 | 3 | -3 | -9 |
| -1 | 3 | 1 | -1 | -3 |
| 1 | -3 | -1 | 1 | 3 |
| 3 | -9 | -3 | 3 | 9 |

Table 1; All possible combinations of an autocorrelated 4 PAM signal

From table 1, it is clear that the autocorrelated values of -3 and 3 each occur ¼ of the time, whereby -1, 1, -9 and 9 occur 1/8 of the time. As Rn assumes an infinitely long symbol stream, Rn becomes:

Therefore:

,

As is an even function:



Substituting for Ro and Rn gives:

Correlating this with in the time domain gives the overall PSD:

Given this, a PSD was generated using Matlab:

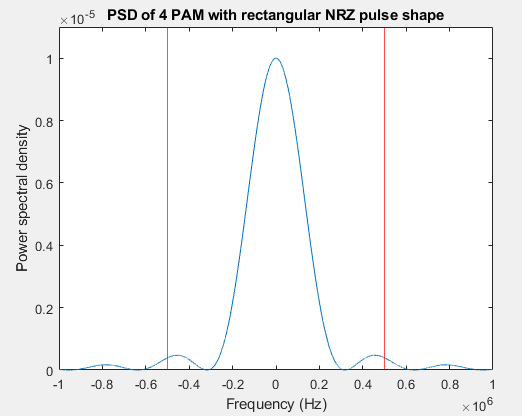


Figure 2

From this, it is clear that the signal has an essential bandwidth well within the 500kHz limitation, as its first crossing of zero amplitude is ~300kHz. However, the signal does have ‘lobes’ outside the bandwidth limit. These will be suppressed when sent over the channel, causing distortion.

c) Nyquist pulse shaping is used to overcome the physical limitations of a baseband communication system. Excess bandwidth is added to a signal to allow for some error in sampling synchronisation in a decoder. For this simulation, a raised cosine pulse shape was chosen.

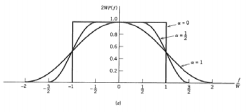


Figure 3; raised cosine pulse in the frequency domain. Taken from lecture notes part 2

With a rectangular pulse shape, the signal has a bandwidth of less than 500kHz, therefore, excess bandwidth of up to 500kHz can be added to the signal. From the frequency domain representation of a raised cosine pulse as shown above, an alpha of 1 was used. This value was chosen as it meets bandwidth requirements, and maximises signal round off.

Given the alpha value chosen, the PSD function was then calculated.

Because the folding frequency, f1 = 0Hz when alpha = 1, the frequency domain pulse shape becomes:

As in question b, is found:

Using the PSD function from question b:

for |f| <= 1/Ts

Using Matlab, this was plotted:

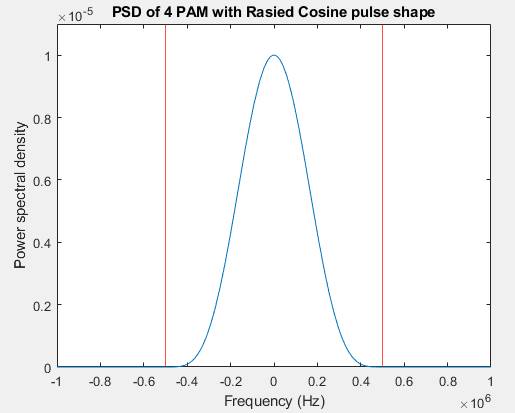


Figure 4

Comparing figures 2&4, it is clear that the bandwidth occupied is greater when using raised cosine pulse shaping. The ‘lobes’ present in figure 2 are no longer an issue, as the rounding off of pulses removes these from the channel.

Bandwidth limits, shown in red, clearly show that the signal meets bandwidth requirements when utilising a raised cosine pulse shape.

d) Given an input bit sequence:

These are then encoded into 4 PAM symbols

Modulating these as a stream using a root raised cosine pulse shaping gives the time domain output below:

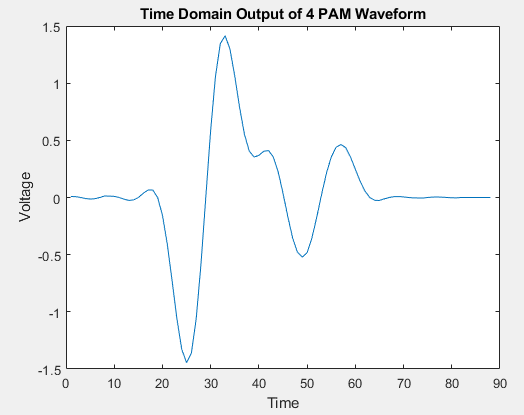


Figure 5

Figure 5 clearly shows each symbol in the stream. However, variations in amplitude between each occurrence of -1 indicate the presence of ISI.

Because a root raised cosine filter is used, the amplitude of each value is the square root of its original. This is used in a transmitter to ensure that a raised cosine filter can be used on the receiving end, resulting in the signal having correct amplitude values after being decoded. For

e) Using an alpha value of 1, and a random set of data, a signal was modulated using a root raised cosine filter, sent over an ideal ‘channel’, then passed through a raised cosine filter. The resultant decoded signal was then displayed in eye diagram form over the period of two symbols:

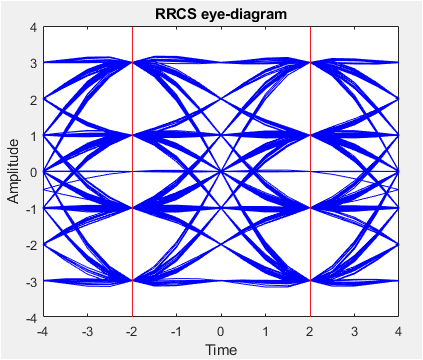
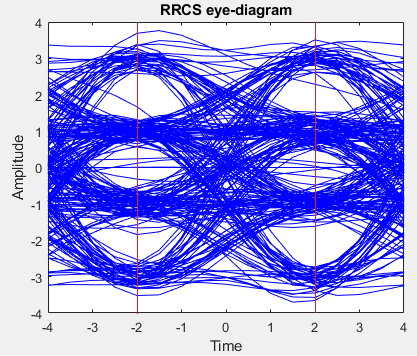


Figure 6

Ideal sampling points were chosen to be at the widest opening of the eyes and are shown with red lines. These are the points of least uncertainty. It is clear that in this case, there is negligible variation in symbol amplitude at the sampling points. Therefore, no ISI is present.

f) When affected by an Eb/N0 or Signal to Noise Ratio (SNR) of 10dB, the eye diagram of the receiver matched filter output will have the following characteristics when compared with its noise free counterpart:

* The eye opening will be slightly closed, meaning the ISI will have increased.
* At the optimum sampling point, the individual signals will vary in value, introducing some uncertainty in the decoding process.
* Despite the effect of noise, the maximum eye opening will remain at the same point during the symbol period.



g) A hard decider was implemented in Matlab. This sampled the incoming signal at the optimum point, and characterised every incoming signal based on amplitude. This was then used on varying levels of Signal to Noise Ratio (SNR), its accuracy calculated for each case. The results of which are represented in figure 7:

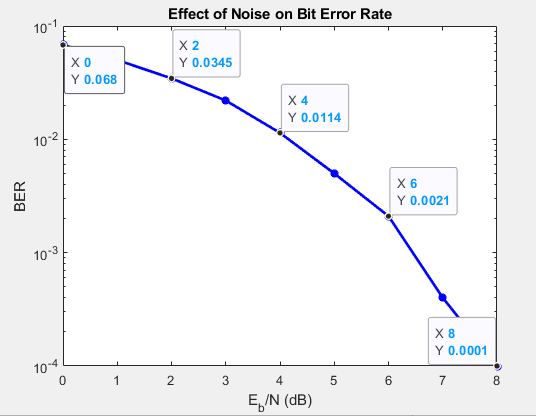


Figure 7

Using a data set of 10000 bytes, or 5000 symbols, the minimum detectable BER corresponds to a single bit being erroneously decoded:

As the BER is an exponentially decreasing function of SNR, a significantly large array of symbols would need to be transmitted through the simulated system to accurately measure the BER corresponding to an SNR of >10dBm.

h) To compensate for ISI caused by the channel, an equalizer can be used.

Taken from assignment specification:

The received signal can be represented in discrete time as

Where w[i] is a sample of AWGN noise. Using the definition of the autocorrelation coefficients

We can derive the autocorrelation expression using the following steps:

The conjugate signal of , is found by substituting the imaginary ‘j’ for ‘k’:

Substituting and into the autocorrelation coefficients definition for gives:

(1)

Expanding the multiplied summations gives:

(2)

Because Noise is added onto a signal at random, the signal data and noise are statistically independent. Therefore, by the principal of orthogonality the expected value of their multiplication is zero:

(3)

As j is the complex counterpart of k, is the conjugate of, meaning that they are independent. However, when j=k, the data sets they represent are the same, meaning that their expected value is, denoting the variance of the data energy, or.

Extending equation 3 gives:

(2)

The noise signals and are independent when two different data signals are considered. However, when two samples of noise contain the same value, the expected value of their product is the noise variance on the channel, or.

 (4)

Using equations 2, 3 and 4, equation 2 can be simplified with the following steps:

Given that the expected value for multiples of two noise or data samples = zero, we are only concerned about the case where j=k We therefore constrain the time instant to j-k. Using equation 3, terms 2 and 3 in equation 7 cancel out, giving:

Because j=k, combining the summations, and using equation 2, this can then be simplified to:

Miltiplying the second term by an impulse function, this is further simplified:

Using equation 4, we get step 5:

Assigning new variable j=i-k, we get the final step:



1. An MMSE equalizer was used in an attempt to correct ISI caused by the effect of multipath behaviour on a 4 PAM signal. Figure 8 shows the eye diagram visualisation of the 4 PAM signal after being transmitted over the channel.

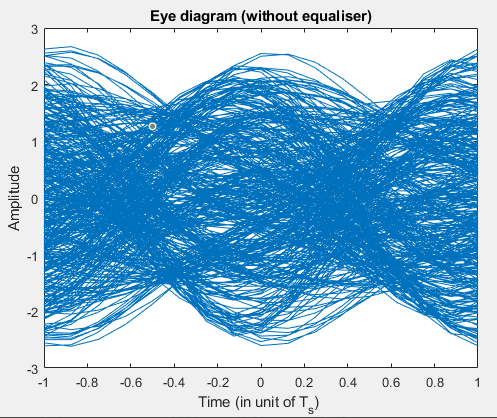


Figure 8

Figure 9 compares the symbol error probability of a decoder with and without the equaliser.

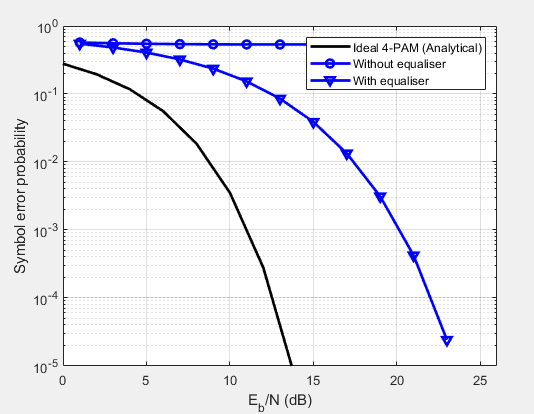


Figure 9

Figure 10 shows the signal scatter before and after equalisation.

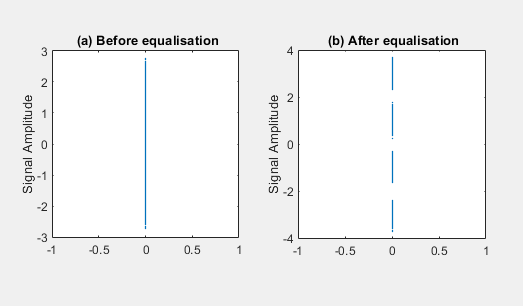


Figure 10

The equaliser in use is of the MMSE or Minimum Mean Square Error type, which will attempt to minimise the effect of noise and ISI. Therefore, it is expected that the symbol error probability continues to decrease with the SNR.

From figure 10 it is clear that the equalizer corrects symbols that have uncertain values. Before equalisation, all incoming signals are evenly distributed between the minimum and maximum amplitude. After equalisation, these symbols are concentrated around the values -3, -1, 1 and 3. This would decrease the uncertainty in demodulation, causing the eyes in figure 8 to open. This is clearly shown in the equalized performance data set shown in figure 9 through the decrease in symbol error probability with SNR.

From figure 9, it is clear that the non-equalised performance of the receiver does not improve with reduced SNR. This is an expected behaviour, and shows the presence of strong ISI. Increasing signal power, or increasing SNR does serve to reduce AWGN, but does not reduce the effect of ISI on the accuracy of deciding. This shows that the equaliser is required in this situation to achieve a symbol error probability of less than ¼.

j)

First, the signal output must be fed into the oscilloscope. This is done by using a BNC lead to connect the output if the PAM generator to input 1 on the scope.

To display the signal, input 1 is tuned on. Creating an eye diagram requires the input signal to be triggered at the start of each symbol. Because of the variable level of PAM at the start of each symbol, an external trigger source must be used. A signal clock output is used for this to remove the need for clock recovery.

Using a BNC lead, the clock signal is plugged into the external trigger input of the oscilloscope, which is then set to source its trigger signal from the external trigger input.

At this point, each symbol would be shown on the screen independently. The generator box is then set to ‘random pattern’. This is important as all possible symbol transitions must be generated and observed to firm an eye diagram. In sequential mode, this does not occur, as the signal will follow a set pattern, and will not show transitions such as ‘01’ to ‘10’. To attain an eye diagram the persistence of an analogue oscilloscope must be simulated. To achieve this, the persistence setting is set to ‘high’ to show multiple modulated symbols overlayed, creating an eye diagram.