CBSE – X – 2016 EXAMINATION

MATHEMATICS

Paper & Solution

SET-1

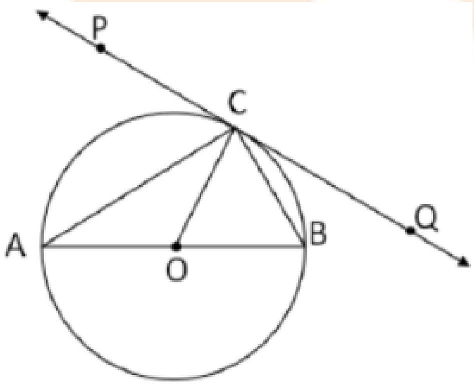
Time: 3 Hrs. Max. Marks: 90

General Instructions:

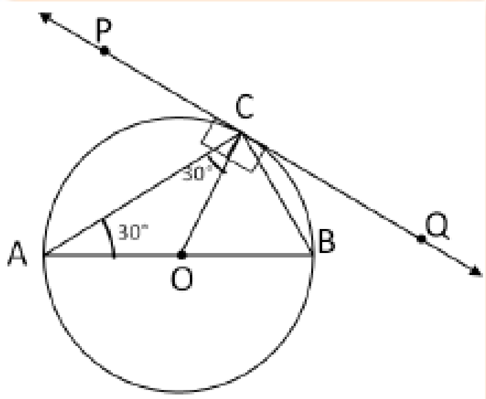
1. All questions are compulsory.
2. The question paper consists of 31 questions divided into four sections – A, B, C and D.
3. Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
4. Use of calculators is not permitted.

SECTION A

Question 1. In the figure, PQ is a tangent at a point C to a circle with centre O. if AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$.



Solution: In the given figure,



In ΔACO,

OA = OC …(Radii of the same circle)

$\therefore$ ΔACO is an isosceles triangle.

$\angle CAB = 30°$ …(Given)

$\therefore$ $\angle CAO = \angle ACO = 30°$ …. (angles opposite to equal sides of an isosceles triangle are equal)

$\angle PCO = 90°$ …(radius drawn at the point of contact is perpendicular to the tangent)

Now $\angle PCA = \angle PCO - \angle CAO$

$\therefore$ $\angle PCA = 90° - 30° = 60°$

Marks: 1

Question 2. For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P?

Solution: If k + 9, 2k – 1 and 2k + 7 are the consecutive terms of A.P., then the common difference will be the same.

$\therefore (2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$

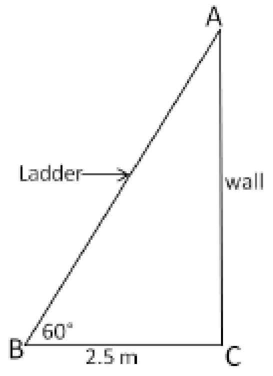
$\therefore k - 10 = 8$

$\therefore k = 18$

Marks: 1

Question 3. A ladder leaning against a wall makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Solution:



Let AB be the ladder and CA be the wall.

The ladder makes an angle of 60° with the horizontal.

$\therefore \Delta ABC$ is a 30°-60°-90°, right triangle.

Given: BC = 2.5 m, $\angle ABC = 60°$

$\therefore AB = 5$ m

Hence, length of the ladder is AB = 5 m.

Marks: 1

Question 4. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

Solution: There are 26 red cards including 2 red queens.

Two more queens along with 26 red cards will be 26 + 2 = 28

$\therefore$ P(getting a red card or a queen) = $\frac{28}{52}$

$\therefore$ P(getting neither a red card nor a queen) = $1 - \frac{28}{52} = \frac{24}{52} = \frac{6}{13}$

Marks: 1

SECTION B

Question 5. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x)k = 0$ has equal roots, find the value of k.

Solution: Given –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$.

$\therefore$ –5 satisfies the given equation.

$\therefore 2(-5)^2 + p(-5) - 15 = 0$

$\therefore 50 - 5p - 15 = 0$

$\therefore 35 - 5p = 0$

$\therefore 5p = 35 \Rightarrow p = 7$

Substituting p = 7 in $p(x^2 + x) + k = 0$, we get

$7(x^2 + x) + k = 0$

$\therefore 7x^2 + 7x + k = 0$

The roots of the equation are equal.

$\therefore$ Discriminant = $b^2 - 4ac = 0$

Here, a = 7, b = 7, c = k

$b^2 - 4ac = 0$

$\therefore (7)^2 - 4(7)(k) = 0$

$\therefore 49 - 28k = 0$

$\therefore 28k = 49$

$$\therefore\ k = \frac{49}{28} = \frac{7}{4}$$

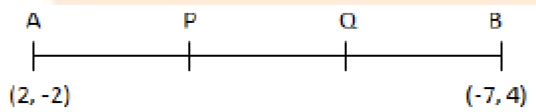
Marks: 2

Question 6. Let P and Q be the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Solution: Since P and Q are the points of trisection of AB, AP = PQ = QB

Thus, P divides AB internally in the ratio 1 : 2

and Q divides AB internally in the ratio 2 : 1.



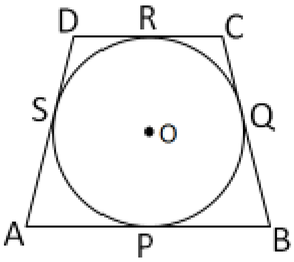
$\therefore$ By section formula,

$$P = \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = \left(\frac{-3}{3}, 0\right) = (-1, 0)$$

$$Q = \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

Marks: 2

Question 7. In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



Solution: Since tangents drawn from an exterior point to a circle are equal in length,
AP = AS ….(1)
BP = BQ ….(2)
CR = CQ ….(3)
DR = DS ….(4)
Adding equations (1), (2), (3) and (4), we get
AP + BP + CR + DS = AS + BQ + CQ + DS
∴ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
∴ AB + CD = AD + BC
∴ AB + CD = BC + DA …..(proved)
Marks: 2

Question 8. Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.
Solution: Let A(3, 0), B(6, 4) and C(–1, 3) be the given points.
Now,

$$AB = \sqrt{(6-3)^2+(4-0)^2} = \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25}$$

$$BC = \sqrt{(-1-6)^2+(3-4)^2} = \sqrt{(-7)^2+(-1)^2} = \sqrt{49+1} = \sqrt{50}$$

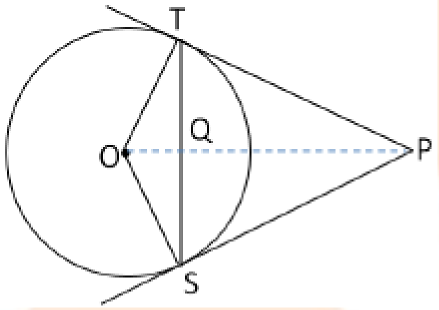
$$AC = \sqrt{(-1-3)^2+(3-0)^2} = \sqrt{(-4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25}$$

∴ AB = AC
$AB^2 = (\sqrt{25})^2 = 25$
$BC^2 = (\sqrt{50})^2 = 50$
$AC^2 = (\sqrt{25})^2 = 25$
∴ $AB^2 = AC^2 = BC^2$
Thus, ΔABC is a right-angled isosceles triangle.
Marks: 2

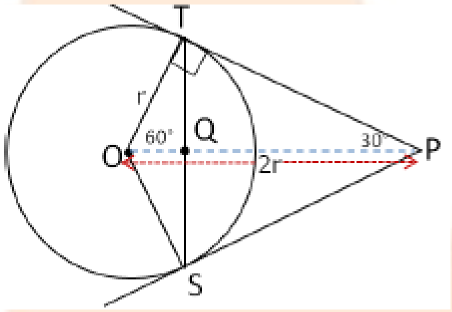
Question 9. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.
Solution: 4th term of an A.P. = a_4 = 0
∴ a + (4 – 1)d = 0
∴ a + 3d = 0
∴ a = –3d ….(1)
25^{th} term of an A.P. = a_{25}
= a + (25 – 1)d
= –3d + 24d ….[From (1)]
= 21d
3 times 11^{th} term of an A.P. = $3a_{11}$
= 3[a + (11 – 1)d]

= 3[a + 10d]
= 3[–3d + 10d]
= 3 × 7d
= 21d
∴ $a_{25} = 3a_{11}$
i.e., the 25th term of the A.P. is three times its 11th term.
Marks: 2

Question 10. In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that ∠ OTS = ∠ OST = 30°.



Solution:

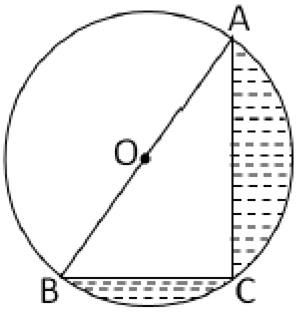


In the given figure,
OP = 2r … (Given)
∠OTP = 90° … (radius drawn at the point of contact is perpendicular to the tangent)
In ΔOTP,
$$\sin \angle OPT = \frac{OT}{OP} = \frac{1}{2} = \sin 30°$$
∠OPT = 30°
∴ ∠TOP = 60°
∴ ΔOTP is a 30°-60°-90°, right triangle.
In ΔOTS,
OT = OS … (Radii of the same circle)
∴ ΔOTS is an isosceles triangle.
∴ ∠OTS = ∠OST … (Angles opposite to equal sides of an isosceles triangle are equal)
In ΔOTQ and ΔOSQ
OS = OT … (Radii of the same circle)
OQ = OQ ... (side common to both triangles)
∠OTQ = ∠OSQ … (angles opposite to equal sides of an isosceles triangle are equal)
∴ ΔOTQ ≅ ΔOSQ … (By S.A.S)
∴ ∠TOQ = ∠SOQ = 60° … (C.A.C.T)

∴ ∠TOS = 120°   … (∠TOS = ∠TOQ + ∠SOQ = 60° + 60° = 120°)
∴ ∠OTS + ∠OST = 180° – 120° = 60°
∴ ∠OTS = ∠OST = 60° ÷ 2 = 30°
Marks: 2

SECTION C

Question 11. In figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14)



Solution: Diameter, AB = 13 cm

∴ Radius of the circle, r = $\frac{13}{2}$ = 6.5 cm

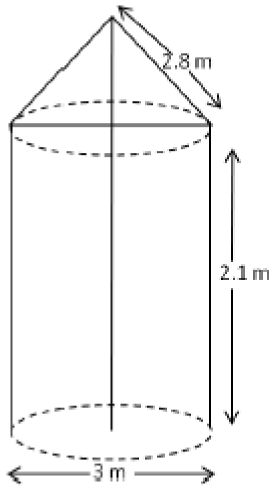
∠ACB is the angle in the semi-circle.
∴ ∠ACB = 90°
Now, in ΔACB, using Pythagoras theorem, we have
$AB^2 = AC^2 + BC^2$
$(13)^2 = (12)^2 + (BC)^2$
$(BC)^2 = (13)^2 – (12)^2 = 169 – 144 = 25$
BC = $\sqrt{25}$ = 5 cm
Now, area of shaded region = Area of semi-circle – Area of ACB

$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{BC} \times \text{AC}$

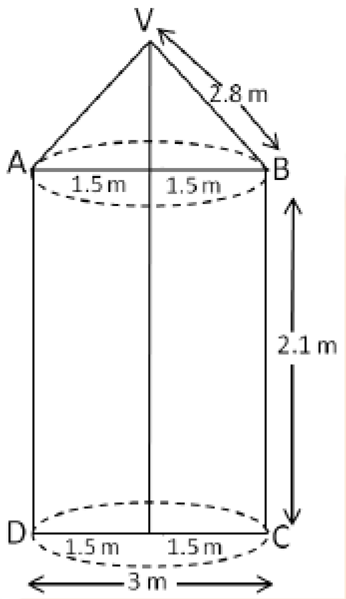
$= \frac{1}{2} \times 3.14 \times (6.5)^2 - \frac{1}{2} \times 5 \times 12$

= 66.33 – 30
= 36.33 cm^2
Thus, the area of the shaded region is 36.33 cm^2.
Marks: 3

Question 12. In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs. 500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Solution:



For conical portion, we have
$r = 1.5$ m and $l = 2.8$ m
$\therefore S_1$ = Curved surface area of conical portion
$\therefore S_1 = \pi rl$
$= \pi \times 1.5 \times 2.8$
$= 4.2\pi \text{ m}^2$
For cylindrical portion, we have
$r = 1.5$ m and $h = 2.1$ m
$\therefore S_2$ = Curved surface area of cylindrical portion
$\therefore S_2 = 2\pi rh$
$= 2 \times \pi \times 1.5 \times 2.1$
$= 6.3\pi \text{ m}^2$
Area of canvas used for making the tent $= S_1 + S_2$
$= 4.2\pi + 6.3\pi$
$= 10.5\pi$
$= 10.5 \times \dfrac{22}{7}$
$= 33 \text{ m}^2$

Total cost of the canvas at the rate of Rs. 500 per m^2 = Rs. (500 × 33) = Rs. 16500.
Marks: 3

Question 13. If the point P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b). Prove that bx = ay.
Solution: P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b).
∴ AP = BP

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2}=\sqrt{[x-(a-b)]^2+[y-(a+b)]^2}$$

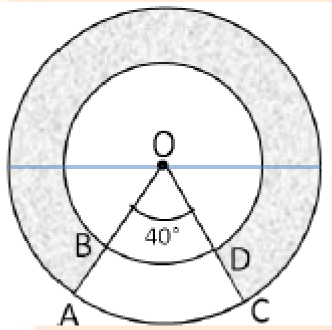
$$[x-(a+b)]^2+[y-(b-a)]^2=[x-(a-b)]^2+[y-(a+b)]^2$$

$$x^2-2x(a+b)+(a+b)^2+y^2-2y(b-a)+(b-a)^2=x^2-2x(a-b)+(a-b)^2+y^2-2y(a+b)+(a+b)^2$$

$$-2x(a+b)-2y(b-a)=-2x(a-b)-2y(a+b)$$

ax + bx + by – ay = ax – bx + ay + by
2bx = 2ay
∴ bx = ay ....(proved)
Marks: 3

Question 14. In figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where ∠AOC = 40°. $\left(\text{Use } \pi=\frac{22}{7}\right)$



Solution: Area of the region ABDC = Area of sector AOC – Area of sector BOD

$$=\frac{40°}{360°}\times\frac{22}{7}\times14\times14-\frac{40°}{360°}\times\frac{22}{7}\times7\times7$$

$$=\frac{1}{9}\times22\times14\times2-\frac{1}{9}\times22\times7\times1$$

$$=\frac{22}{9}\times(28-7)$$

$$=\frac{22}{9}\times21$$

$$=\frac{154}{3}$$

= 51.33 cm^2

Area of circular ring = $\frac{22}{7}\times14\times14-\frac{22}{7}\times7\times7$

= 22 × 14 × 2 – 22 × 7 × 1
= 22 × (28 – 7)
= 22 × 21

$= 462\ cm^2$

∴ Required shaded region = Area of circular ring – Area of region ABDC

= 462 – 51.33

$= 410.67\ cm^2$

Thus, the area of shaded region is $410.67\ cm^2$

Marks: 3

Question 15. If the ratio of the sum of first n terms of two A.P's is (7n +1): (4n + 27), find the ratio of their m^{th} terms.

Solution: Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P's.

Thus, we have $S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and $S_n' = \frac{n}{2}[2a_2 + (n-1)d_2]$

$$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$$

It is given that $\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$$\therefore \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27} \quad ....(1)$$

To find the ratio of the m^{th} terms of the two given A.P's, replace n by (2m – 1) in equation (1).

$$\therefore \frac{2a_1 + (2m-1-1)d_1}{2a_2 + (2m-1-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\therefore \frac{2a_1 + (2m-2)d_1}{2a_2 + (2m-2)d_2} = \frac{14m-7+1}{8m-4+27}$$

$$\therefore \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{14m-6}{8m+23}$$

Hence, the ratio of the m^{th} terms of the two A.P's is 14m – 6 : 8m 23.

Marks: 3

Question 16. Solve for x: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3},\ x \neq 1, 2, 3$

Solution:

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{x-3+x-1}{(x^2-3x+2)(x-3)} = \frac{2}{3}$$

$$\frac{2x-4}{x^3-3x^2-3x^2+9x+2x-6} = \frac{2}{3}$$

$$\frac{2x-4}{x^3-6x^2+11x-6} = \frac{2}{3}$$

$6x-12=2x^3-12x^2+22x-12$

$2x^3-12x^2+16x=0$

$2x(x^2-6x+8)=0$

$x^2-6x+8=0$

$x^2-4x-2x+8=0$

$x(x-4)-2(x-4)=0$

$(x-4)(x-2)=0$

$x-4=0$ or $x-2=0$

$x=4$ or $x=2$

Marks: 3

Question 17. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left(\text{Use } \pi=\frac{22}{7}\right)$

Solution: Let the radius of the conical vessel = r_1 = 5 cm

Height of the conical vessel = h_1 = 24 cm

Radius of the cylindrical vessel = r_2

Let the water rise upto the height of h_2 cm in the cylindrical vessel.

Now, volume of water in conical vessel = volume of water in cylindrical vessel

$\frac{1}{3}\pi r_1^2 h_1 = r_2^2 h_2$

$r_1^2 h_1 = 3r_2^2 h_2$

$5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$

$h_2 = \frac{5\times5\times24}{3\times10\times10} = 2$ cm

Thus, the water will rise upto the height of 2 cm in the cylindrical vessel.

Marks: 3

Question 18. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Solution: Radius of sphere = r = 6 cm

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 = 288\pi$ cm^3

Let R be the radius of cylindrical vessel.

Rise in the water level of cylindrical vessel = h = $3\frac{5}{9}$ cm = $\frac{32}{9}$ cm

Increase in volume of cylindrical vessel = $\pi R^2 h = \pi R^2 \times \frac{32}{9} = \frac{32}{9}\pi R^2$

Now, volume of water displaced by the sphere is equal to volume of sphere.

$\therefore \frac{32}{9}\pi R^2 = 288\pi$

$\therefore R^2 = \frac{288\times 9}{32} = 81$

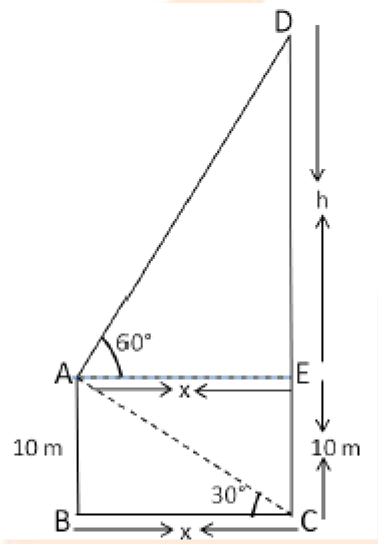
$\therefore$ R = 9 cm

$\therefore$ Diameter of the cylindrical vessel = 2 × R = 2 × 9 = 18 cm

Marks: 3

Question 19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of a hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Solution:



Let CD be the hill and suppose the man is standing on the deck of a ship at point A.

The angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°.

$\therefore \angle EAD = 60°$ and $\angle BCA = 30°$

In ΔAED,

$\tan 60° = \frac{DE}{EA}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}x$ ....(1)

In ΔABC,

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{10}{x}$

$x = 10\sqrt{3}$ ....(2)

Substituting $x = 10\sqrt{3}$ in equation (1), we get

$h = \sqrt{3} \times 10\sqrt{3} = 10 \times 3 = 30$

DE = 30 m

CD = CE + ED = 10 + 30 = 40 m

Thus, the distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is 40 m.

Marks: 3

Question 20. Three different coins are tossed together. Find the probability of getting

(i) exactly two heads

(ii) at least two heads

(iii) at least two tails.

Solution: When three coins are tossed together, the possible outcomes are

HHH, HTH, HHT, THH, THT, TTH, HTT, TTT

$\therefore$ Total number of possible outcomes = 8

(i) Favourable outcomes of exactly two heads are HTH, HHT, THH

$\therefore$ Total number of favourable outcomes = 3

$\therefore$ P(exactly two heads) = $\frac{3}{8}$

(ii) Favourable outcomes of at least two heads are HHH, HTH, HHT, THH

$\therefore$ Total number of favourable outcomes = 4

$\therefore$ P(at least two heads) = $\frac{4}{8} = \frac{1}{2}$

(iii)Favourable outcomes of at least two tails are THT, TTH, HTT, TTT

$\therefore$ Total number of favourable outcomes = 4

$\therefore$ P(at least two tails)= $\frac{4}{8} = \frac{1}{2}$

Marks: 3

SECTION D

Question 21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the governments and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 cm and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs. 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem? $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution: Height of conical upper part = 3.5 m, and radius = 2.8 m

(Slant height of cone)^2 = $2.1^2 + 2.8^2$ = 4.41 + 7.84

Slant height of cone = $\sqrt{12.25}$ = 3.5 m

The canvas used for each tent

= curved surface area of cylindrical base + curved surface area of conical upper part

= $2\pi rh + \pi rl$

= $\pi r(2h + l)$

= $\frac{22}{7} \times 2.8(7 + 3.5)$

= $\frac{22}{7} \times 2.8 \times 10.5$

$= 92.4\ m^2$

So, the canvas used for one tent is $92.4\ m^2$.

Thus, the canvas used for 1500 tents $= (92.4 \times 1500)\ m^2$.

Canvas used to make the tents cost Rs. 120 per sq. m.

So, canvas used to make 1500 tents will cost Rs. $92.4 \times 1500 \times 120$.

The amount shared by each school to set up the tents

$= \dfrac{92.4 \times 1500 \times 120}{50}$ = Rs. 332640

The amount shared by each school to set up the tents is Rs. 332640.

The value to help others in times of troubles is generated from the problem.

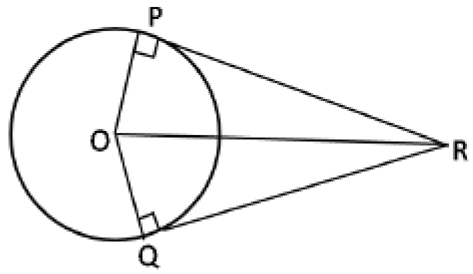
Marks: 4

Question 22. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Solution: Consider a circle centered at O.

Let PR and QR are tangents drawn from an external point R to the circle touching at points P and Q respectively.

Join OR.



Proof:

In ΔOPR and ΔOQR,

OP = OQ ... (Radii of the same circle)

∠OPR = ∠OQR .... (Since PR and QR are tangents to the circle)

OR = OR ... (Common side)

∴ ΔOPR ≅ ΔOQ R ....(By R.H.S)

∴ PR = QR ....(c.p.c.t)

Thus, tangents drawn from an external point to a circle are equal.

Marks: 4

Question 23. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Solution: Steps of construction:

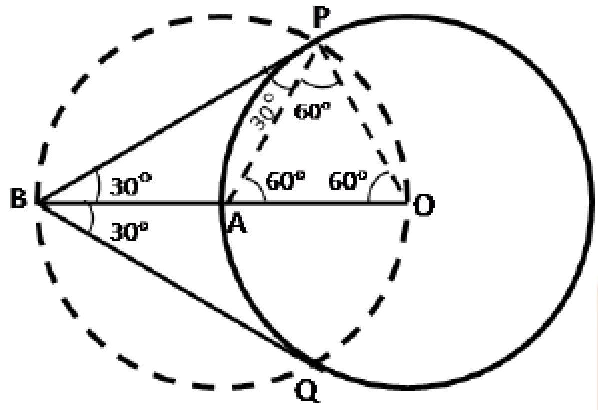
(i) Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm.

(ii) Produce OA to B such that OA = AB = 4 cm.

(iii) Draw a circle with centre at A and radius AB.

(iv) Suppose it cuts the circle drawn in step (i) at P and Q.

(v) Join BP and BQ to get the desired tangents.



Justification:

In ΔOAP, OA = OP = 4 cm ... (radii of the same circle)

Also, AP = 4 cm ….(Radius of the circle with centre A)

$\therefore$ ΔOAP is equilateral.

$\therefore \angle PAO = 60°$

$\therefore \angle BAP = 120°$

In ΔBAP, we have BA = AP and $\angle$ BAP = 120°

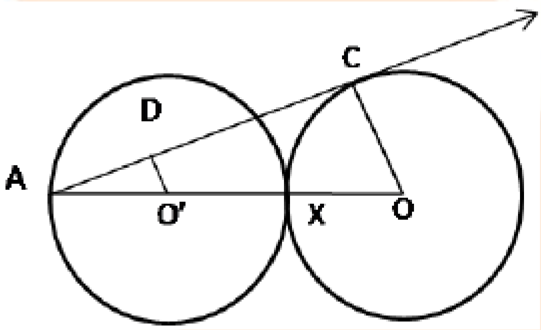
$\therefore \angle ABP = \angle APB = 30°$

Similarly we can get $\angle ABQ = 30°$

$\therefore \angle PBQ = 60°$

Marks: 4

Question 24. In figure, two equal circles, with centres O and O’, touch each other at X. OO’ produced meets the circle with centre O’ at A. AC is tangent to the circle with centre O, at the point C. O’D is perpendicular to AC. Find the value of $\dfrac{DO'}{CO}$.



Solution: AO’ = O’X = XO = OC …..(Since the two circles are equal.)

So, OA = AO’ + O’X + XO …..(A-O’-X-O)

$\therefore$ OA = 3O’A

In $\Delta AO'D$ and ΔAOC,

$\angle DAO' = \angle CAO$ .... (Common angle)

$\angle ADO' = \angle ACO$ .... (both measure $90°$)

$\Delta ADO' \sim \Delta ACO$ .... (By AA test of similarity)

$$\frac{DO'}{CO} = \frac{O'A}{OA} = \frac{O'A}{3O'A} = \frac{1}{3}$$

Marks: 4

Question 25. Solve for x: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$; $X \neq -1,-2,-4$

Solution: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$

L.C.M. of all the denominators is (x + 1)(x + 2)(x + 4)

Multiply throughout by the L.C.M., we get

(x + 2)(x + 4) + 2(x + 1)(x + 4) = 4(x + 1)(x + 2)

(x + 4)(x + 2 + 2x +2) = 4(x^2 + 3x + 2)

(x + 4)(3x + 4) = 4x^2 + 12x + 8

3x^2 + 16x + 16 = 4x^2 + 12x + 8

∴ $x^2 - 4x - 8 = 0$

Now, a = 1, b = –4, c = –8

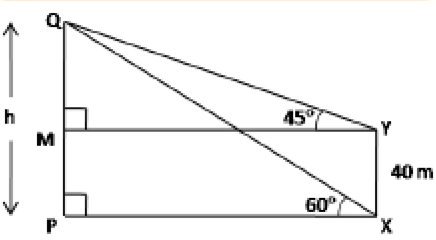
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm4\sqrt{3}}{2}$$

∴ x = $2\pm2\sqrt{3}$

Marks: 4

Question 26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$ = 1.73)

Solution:



MP = YX = 40 m

∴ QM = h – 40

In right angled ΔQMY,

$$\tan 45^\circ=\frac{\text{QM}}{\text{MY}}\Rightarrow 1=\frac{h-40}{\text{PX}} \quad ....(\text{MY} = \text{PX})$$

∴ PX = h – 40 ....(1)

In right angled ΔQPX,

$$\tan 60^\circ=\frac{QP}{PX}\Rightarrow\sqrt{3}=\frac{QP}{PX}$$

PX = $\frac{h}{\sqrt{3}}$ ....(2)

From (1) and (2), h – 40 = $\frac{h}{\sqrt{3}}$

∴ $\sqrt{3}$ h – 40$\sqrt{3}$ = h

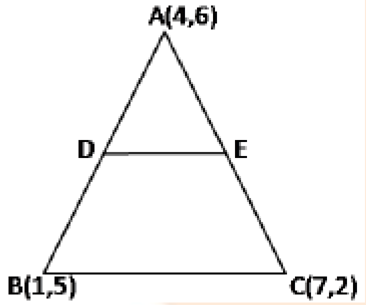
∴ $\sqrt{3}$ h – h = 40$\sqrt{3}$

∴ 1.73h – h = 40(1.73) ⇒ h = 94.79 m

Thus, PQ is 94.79 m.
Marks: 4

Question 27. The houses in a row numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X.
Solution: Let there be a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it.
That is, $1 + 2 + 3 + \ldots + (x - 1) = (x + 1) + (x + 2) + \ldots + 49$
$\therefore 1 + 2 + 3 + \ldots + (x - 1)$
$= [1 + 2 + \ldots + x + (x - 1) + \ldots + 49] - (1 + 2 + 3 + \ldots + x)$
$\therefore \frac{x-1}{2}[1+x-1] = \frac{49}{2}[1+49] - \frac{x}{2}[1+x]$
$\therefore x(x - 1) = 49 \times 50 - x(1 + x)$
$\therefore x(x - 1) + x(1 + x) = 49 \times 50$
$\therefore x^2 - x + x + x^2 = 49 \times 50$
$\therefore x^2 = 49 \times 50$
$\therefore x^2 = 49 \times 25$
$\therefore x = 7 \times 5 = 35$
Since x is not a fraction, the value of x satisfying the given condition exists and is equal to 35.
Marks: 4

Question 28. In figure, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



Solution:
$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$
$\therefore \frac{AB}{AD} = \frac{AC}{AE} = 3$
$\therefore \frac{AD+DB}{AD} = \frac{AE+EC}{AE} = 3$
$\therefore 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} = 3$

$$\therefore \frac{DB}{AD} = \frac{EC}{AE} = 2$$

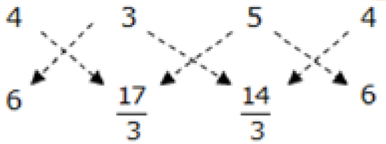
$$\therefore \frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{2}$$

$$\therefore AD : DB = AE : EC = 1:2$$

So, D and E divide AB and AC respectively in the ratio 1:2.

So the coordinates of D and E are

$$\left(\frac{1+8}{1+2}, \frac{5+12}{1+2}\right) \equiv \left(3, \frac{17}{3}\right) \text{ and } \left(\frac{7+8}{1+2}, \frac{2+12}{1+2}\right) \equiv \left(5, \frac{14}{3}\right) \text{ respectively.}$$



Area of ΔADE

$$= \frac{1}{2}\left|\left(4\times\frac{17}{3}+3\times\frac{14}{3}+5\times 6\right)-\left(3\times 6+5\times\frac{17}{3}+4\times\frac{14}{3}\right)\right|$$

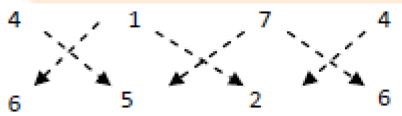
$$= \frac{1}{2}\left|\left(\frac{68}{3}+14+30\right)-\left(18+\frac{85}{3}+\frac{56}{3}\right)\right|$$

$$= \frac{1}{2}\left|\left(\frac{68+42+90}{3}\right)-\left(\frac{54+85+56}{3}\right)\right|$$

$$= \frac{1}{2}\left|\left(\frac{200}{3}\right)-\left(\frac{195}{3}\right)\right|$$

$$= \frac{1}{2}\times\frac{5}{3}$$

$$= \frac{5}{6} \text{ sq. units}$$



Area of ΔABC

$$= \frac{1}{2}\left|(4\times 5+1\times 2+7\times 6)-(1\times 6+7\times 5+4\times 2)\right|$$

$$= \frac{1}{2}\left|(20+2+42)-(6+35+8)\right|$$

$$= \frac{1}{2}\left|(64)-(49)\right|$$

$$= \frac{15}{2} \text{ sq. units}$$

$$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{5/6}{15/2} = \frac{1}{9}$$

Marks: 4

Question 29. A number x is selected at random from the numbers 1, 2, 3, and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Solution: x is selected from 1, 2, 3 and 4

1, 2, 3, 4

y is selected from 1, 4, 9 and 16

Let A = {1, 4, 9, 16, 2, 8, 18, 32, 3, 12, 27, 48, 36, 64} which consists of elements that are product of x and y

P(product of x and y is less than 16) = $\dfrac{\text{Number of outcomes less than 16}}{\text{Total number of outcomes}}$

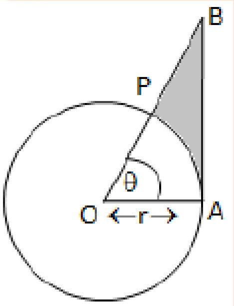
$= \dfrac{7}{14}$

$= \dfrac{1}{2}$

Marks: 4

Question 30. In figure, is shown a sector OAP of a circle with centre O, containing ∠θ. AB is perpendicular to the radius OQ and meets OP produced at B. Prove that the perimeter of shaded region is

$r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180} - 1\right]$.



Solution: Perimeter of shaded region = AB + PB + arc length AP ...(1)

Arc length AP = $\dfrac{\theta}{360} \times 2\pi r = \dfrac{\pi\theta r}{180}$ ....(2)

In right angled ΔOAB,

tan θ = $\dfrac{AB}{r} \Rightarrow$ AB = r tan θ ....(3)

sec θ = $\dfrac{OB}{r} \Rightarrow$ OB = r sec θ

OB = OP + PB

∴ r sec θ = r + PB

∴ PB = r sec θ – r .....(4)

Substitute (2), (3) and (4) in (1), we get

Perimeter of shaded region = AB + PB + arc length AP

$= r\tan\theta + r\sec\theta - r + \dfrac{\pi\theta r}{180}$

$= r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180} - 1\right]$

Marks: 4

Question 31. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution: Let the speed of the stream be s km/h.

Speed of the motor boat 24 km/h

Speed of the motor boat upstream 24 – s

Speed of the motor boat downstream 24 + s

According to the given condition,

$$\frac{32}{24-s}-\frac{32}{24+s}=1$$

$$\therefore 32\left(\frac{1}{24-s}-\frac{1}{24+s}\right)=1$$

$$\therefore 32\left(\frac{24+s-24+s}{576-s^2}\right)=1$$

$\therefore 32 \times 2s = 576 - s^2$

$\therefore s^2 + 64s - 576 = 0$

$\therefore (s + 72)(s - 8) = 0$

$\therefore s = -72$ or $s = 8$

Since, speed of the stream cannot be negative, the speed of the stream is 8 km/h.

Marks: 4