



数学分析 B 第一学期期末试题(B)解答(2008.1)

- 一. 1. $-\frac{f'(\frac{1}{x})}{x^2 f(\frac{1}{x})} dx$ (没有 dx 扣 1 分)
2. 2
3. $y = 3ex - 2e^2$
4. -16
5. 4
6. $y'' + 2y' + y = 0$
7. $\frac{3\pi}{8}$
8. 1, -2, 4 (1 分, 1 分, 1 分)
9. $\frac{128}{5}\pi$
10. $y = Ce^{-2x^2} + \frac{1}{2}$ (没写 y 扣 1 分) (只写出通解公式没算出积分给 1 分)
- 二. $\int_0^{\frac{\pi}{2}} \left| x - \frac{\pi}{2} \right| \sin x dx = \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) \sin x dx + \int_{\frac{\pi}{2}}^{\pi} \left(x - \frac{\pi}{2} \right) \sin x dx \dots\dots\dots(2 \text{ 分})$
- $= -\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) d \cos x - \int_{\frac{\pi}{2}}^{\pi} \left(x - \frac{\pi}{2} \right) d \cos x \dots\dots\dots(3 \text{ 分})$
- $= -\left(\frac{\pi}{2} - x \right) \cos x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos x dx - \left(x - \frac{\pi}{2} \right) \cos x \Big|_{\frac{\pi}{2}}^{\pi} + \int_{\frac{\pi}{2}}^{\pi} \cos x dx \dots\dots\dots(6 \text{ 分})$
- $= \frac{\pi}{2} - 1 + \frac{\pi}{2} - 1 = \pi - 2 \dots\dots\dots(8 \text{ 分})$
- 三. $f'(x) = \frac{2(2x-2)}{3(x^2-2x)^{\frac{1}{3}}} = \frac{4(x-1)}{3(x^2-2x)^{\frac{1}{3}}} \dots\dots\dots(2 \text{ 分})$
- 令 $f'(x) = 0$, 得 $x = 1$
- 当 $x = 0$, $x = 2$ 时, $f'(x)$ 不存在 $\dots\dots\dots(5 \text{ 分})$
- $f(0) = 0 \quad f(2) = 0 \quad f(1) = 1$
- $f(3) = \sqrt[3]{9} \quad f(-2) = 4$
- $M = 4 \quad m = 0 \dots\dots\dots(8 \text{ 分})$

四. $f'(x) = ax^2 - 4x$ (1 分)

$$f''(x) = 2ax - 4 \quad \dots\dots\dots(2 \text{ 分})$$

$$f''(-1) = -2a - 4 = 0 \quad a = -2 \quad \dots\dots\dots(4 \text{ 分})$$

$$f(x) = \int f'(x)dx = \int (-2x^2 - 4x)dx = -\frac{2}{3}x^3 - 2x^2 + C \quad \dots\dots\dots(6 \text{ 分})$$

$$\text{由 } f(-1) = \frac{2}{3} - 2 + C = \frac{8}{3} \quad \text{得 } C = 4$$

$$f(x) = -\frac{2}{3}x^3 - 2x^2 + 4 \quad \dots\dots\dots(8 \text{ 分})$$

五. 设 t 时刻物体表面温度为 $T = T(t)$, 则

$$\frac{dT}{dt} = -k(T - 20) \quad \dots\dots\dots(2 \text{ 分})$$

$$\frac{dT}{T - 20} = -kdt \quad \dots\dots\dots(3 \text{ 分})$$

$$\ln|T - 20| = -kt + C_1$$

$$T = 20 + Ce^{-kt} \quad \dots\dots\dots(4 \text{ 分})$$

$$\text{由 } T(0) = 100 \quad \text{得 } C = 80$$

$$T = 20 + 80e^{-kt} \quad \dots\dots\dots(6 \text{ 分})$$

$$\text{由 } T(20) = 60 \quad \text{得 } e^{-k} = \left(\frac{1}{2}\right)^{\frac{1}{20}}$$

$$T = 20 + \frac{80}{2^{\frac{t}{20}}} \quad \dots\dots\dots(8 \text{ 分})$$

六. $x \int_0^x f(t) dt - \int_0^x t f(t) dt = x e^x - f(x)$ (1 分)

$$\int_0^x f(t) dt = e^x + x e^x - f'(x)$$
(2 分)
$$f(x) = e^x + e^x + x e^x - f''(x)$$

$$f''(x) + f(x) = (2+x)e^x$$
(3 分)
$$f(0) = 0 \quad f'(0) = 1$$
(5 分)
$$r^2 + 1 = 0 \quad r = \pm i$$
(7 分)
$$\bar{f}(x) = C_1 \cos x + C_2 \sin x$$
(8 分)

设 $f^*(x) = (Ax + B)e^x$ (9 分)

代入微分方程得 $A = \frac{1}{2} \quad B = \frac{1}{2}$

$$f^*(x) = \frac{1}{2}(x+1)e^x$$
(11 分)

通解为 $f(x) = C_1 \cos x + C_2 \sin x + \frac{1}{2}(x+1)e^x$ (12 分)

由初始条件得 $C_1 = -\frac{1}{2} \quad C_2 = 0$

$$f(x) = -\frac{1}{2} \cos x + \frac{1}{2}(x+1)e^x$$
(14 分)

七. 对任意 $x \in (-\infty, +\infty)$

由于 $f(x)$ 在 $x=0$ 处连续, $\lim_{\Delta x \rightarrow 0} f(\Delta x) = f(0)$ (2 分)

$$\lim_{\Delta x \rightarrow 0} f(x + \Delta x) = \lim_{\Delta x \rightarrow 0} [f(x) + f(\Delta x)]$$
(4 分)
$$= f(x) + \lim_{\Delta x \rightarrow 0} f(\Delta x) = f(x) + f(0)$$
(6 分)
$$= f(x+0) = f(x)$$
(7 分)

故 $f(x)$ 在 x 处连续, 因此在 $(-\infty, +\infty)$ 连续(8 分)

八. 由题设 $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 1$ (1 分)

又 $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$
 $= \lim_{x \rightarrow 0} \frac{\int_0^{x^2} \frac{\ln(1+t^{2k})}{t} dt}{a(-\frac{1}{2}x^2) \cdot \frac{1}{2}x^2} = \lim_{x \rightarrow 0} \frac{\int_0^{x^2} \frac{\ln(1+t^{2k})}{t} dt}{-\frac{a}{4}x^4}$ (3 分)

$= \lim_{x \rightarrow 0} \frac{\frac{\ln(1+x^{4k})}{x^2} 2x}{-ax^3} = \lim_{x \rightarrow 0} \frac{2\ln(1+x^{4k})}{-ax^4}$ (5 分)

$= \lim_{x \rightarrow 0} \frac{2x^{4k}}{-ax^4}$ (6 分)

故 $2 = -a$ $4k = 4$
 得 $a = -2$ $k = 1$ (8 分)

九. $\left| \int_0^a f(x) dx - af(a) \right|$

$= \left| \int_0^a f(x) dx - \int_0^a f(a) dx \right| = \left| \int_0^a (f(x) - f(a)) dx \right|$ (2 分)

$= \left| \int_0^a f'(\xi)(x-a) dx \right|$ ($\xi \in (0, a)$)(4 分)

$\leq \int_0^a |f'(\xi)(x-a)| dx$ (5 分)

$\leq M \int_0^a |x-a| dx$ (6 分)

$= M \int_0^a (a-x) dx$ (7 分)

$= \frac{Ma^2}{2}$ (8 分)