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Design of digital IIR filter: A research survey

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ABSTRACT

This paper presents an overview on advancement made in designing of a digital infinite impulse response (IIR) filter. The design problem of an IIR filter was found to be challenging due to presence of poles in its transfer function. This makes the phase response of an IIR transfer function nonlinear and its magnitude response is also drifted due to quantization of coefficient values of denominator polynomials, which leads towards instability. Therefore, numerous efforts were made in order to acquire an optimal filter response using several optimization methods. Design problem of an IIR filter with various constraints was developed and solved using gradient based techniques, which resulted in optimal passband response with nearly linear phase or in some cases, absolute linearity was also achieved. However, the obtained solutions were sub-optimal in many cases due to transferring the multimodal design problem of an IIR filter into convex optimization. The solution was also affected due to the quantization of filter coefficients and in case of absolute linear phase response; a strong hick in magnitude response at beginning of transition edge frequency was obtained. To overcome the sub-optimality, researchers used evolutionary algorithms (EAs) for designing of an IIR filter. In time domain, system identification (SI) was adopted, whereas various error functions were developed in frequency domain for obtained magnitude responses close to desired response. This approach resulted in an optimal IIR filter response, however phase response linearity was not improved. Thus, EA approach was appropriated for lower order IIR filters. The design of various IIR filters like lowpass filter (LPF), highpass filter (HPF), bandpass filter (BPF) and bandstop filter (BSP) using an all-pass infinite impulse response (APF-IIR) was also reported. This approach is most appropriate, because the filter response is stable, nearly linear and magnitude response was also accurate. However, there is high error at the band edges of passband. Therefore, literature reveals that an APF based approach is most appropriate for various magnitude response filters.

1. Introduction

Digital signal processing (DSP) grew tremendously in past few decades and was extensively practiced for numerous engineering application, which includes biomedical signal processing, radar signal processing, adaptive antenna design, intelligent control etc. [1–5]. In DSP, filtering of recorded signal is essential, which may be contaminated by the artifact and noise, thus digital filters are the integral part of digital systems. Digital IIR filter owing to their resemblance to analog filter behavior have sharp frequency response and would be applied for applications for linear phase insensitive applications like internet of things (IoT) applications, portable devices, bio-signal processing, and audio applications [6]. Digital filters are used for the decomposition of signals for computations of short time Fourier transform (STFT). Digital filter are broadly classified as a finite impulse response (FIR) filter and an infinite impulse response (IIR) filter, which are classified on the basis of their response obtained when excited by unit impulse. It is well established phenomena that FIR base system is linear and always stable [7], while, the computational requirement is on higher side, because FIR system usually realized with higher number of filter taps for prescribed response [1]. Besides this an IIR filter is a counter part of analog filter in digital domain, which consists of feedback loop; and therefore, it perform efficiently when measured on the basis of computational complexity, transition width and roll-off factor. However, an IIR filter lacks in linear phase response and potentially unstable towards quantization and truncation of filter tap coefficients.

In conventional designing of an IIR filter, first step is the prototype filter design in analog domain using classical techniques like: Butterworth, Chebyshev or Elliptical approach [8]. Now this analog

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prototype filter is transformed into digital filter using suitable approximation like impulse invariant or bilateral transform of analog transfer function into digital transfer function. This approach is simple, fast and provides appropriate transfer function with acceptable frequency response. However, the design system suffers from degradation due quantization and truncation of filter tap coefficients, nonlinear phase response and sub-optimal solution.

To overcome the limitations of conventional method, the other methods, which are optimization based technique, were developed for an IIR filter design. These methods are based on the minimization or maximization of an objective function, which must also satisfy the constrained environment. This aids in designing of an IIR filter, which is stable, optimal and robust in sense of practical realization. Designing of an IIR using optimization based techniques includes adaptive filter designing by system identification [9–36] optimal filter by gradient based [6,37–65] and designing using meta-heuristic (evolutionary techniques) [66–86]. System identification based methodology is practiced to design an IIR filter with lower order and design is conducted by tuning the system response for Gaussian white random noise that should be similar to that obtained from *unknown* system.

Designing of an IIR filter using gradient based method was developed for getting linear-phase response with optimal passband region and stopband region magnitude response. Chebyshev approximation was the initial step followed by minimum *p*-error criterion [37]. Later numerous methods were proposed using linear programing (LP) [41,48], weighted least squares (WLS) [52], peak constrain least squares [57,58], least squares (LS) [59], semidefinite programming (SP) [60], Minimax design using iterative linear programming (ILP) [61], conic-quadratic-programming (CQP) [62], and many more [6,38,39,42–47,49,50,63,65]. In these methods, solution obtained was improved, however making design problem convex leaded to suboptimal solution. Moreover the obtained filter performance also suffers from quantization effect of filter tap values. Recently, realization of an IIR filter using a sparse normal-form IIR filter realization with minimal zero/*L*2 resulted in overcome of limit cycle effect [6]. Digital IIR filter suffers for truncation and quantization error when realized in direct form, and to overcome this issue, authors in [51] proposed technique for designing of imperfect but stable IIR filter in direct form.

Evolutionary Techniques (ETs) were developed for solving non-differentiable, multimodal, and multi-objective problems [87–89]. Therefore, researchers were inspired to develop methodologies for designing of FIR filters [90–99], filterbanks [100–103], and an IIR filter [9–36] using ETs. Recently, ETs based design of FIR filter have acquired less hardware resource [104–108]. In past few decades, ETs were used in system identification based design of an IIR filter [9–36], and error minimization approach for an IIR filter design in frequency domain [66–81,84–86]. The initial step was using adaptive genetic algorithm (AGA) [9]. Then onwards many such experiments were conducted, where improved GA based techniques were formed and tested [10,13,23,26]. The existing literature revealed that GA suffers of getting trapped in local minima, because of its poor local search capability and premature convergence.

Later simulated annealing (SA) technique was adopted for design of an IIR filter. However, its initial version was quite slow and requires too many functions evaluation. Thus, an enhanced and modified variant known as adaptive simulated annealing (ASA) was developed and used for design of an adaptive IIR filter [23,24]. Tabu search technique was exploited for design [27]. In SA and Tabu search, only single solution was developed for throughout search, therefore the quality of solution was highly depended on initial guess; and thus, resulted a sub-optimal solution. Thus, new dimensions were explored by utilizing new method like artificial immune algorithm (AIA) [29], particle swarm optimization (PSO) [25], artificial bee colony (ABC) algorithm [33], cat swarm optimization (CSO) [14], seeker optimization algorithm (SOA) [35], harmony search algorithm (HSA) and modified imperialist competitive (MIC) for the designing purpose. All these techniques belong to the class of evolutionary based global search methods and are able to search in multi direction. These techniques were found suitable for lower order IIR filter. The phase response of an IIR filter designed using ETs were not linear, and the initially designed IIR filters do suffer from quantization effect.

An all-pass filter (APF) IIR filter was used to design lowpass and highpass IIR filter [109]. In this approach, an APF was either connected with a pure delay function having order less than APF or two APFs were connected in parallel [110,111]. In all of the mentioned case, the phase of an APF is altered such that, in passband region, the phase response of parallel connected system must match and should be out of phase in stopband region. An IIR filter designed using an APF was found nearly linear; however the magnitude response was not degraded due to quantization of filter taps.

2. An IIR filter overview

Digital IIR filter is characterized by linear difference equation defined as [8]:

$$y(k) + \sum_{n=1}^{N} b_n \cdot y(k-n) = \sum_{n=0}^{M} a_n \cdot x(k-n) \tag{1}$$

where, $y(k)$ is the response sequence, when excited by the input sequence of $x(n)$, a_n and b_n are the associated coefficients. The relation shows that current output of IIR filter consist of both previous output and current input of the system. The transfer function of this system is expressed as [8]:

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\sum_{n=0}^{M} a_n \cdot z^{-n}}{1 + \sum_{n=1}^{N} b_n \cdot z^{-n}} \; . \tag{2}$$

In Eq. (2) z is a complex plan given by $z = r \cdot e^{j\omega}$, where ω is digital frequency whose range is from 0 to 2π, and r is the radius of circle which will be formed by angular rotation of ω. in digital system, a circle formed with $r = 1$ known as unit circle is considered, as it represents the transformation of analog frequency axis of s plan to circle on z plan. therefore for computation of frequency response of any digital system $z = e^{j\omega}$ is considered. the position of the roots of the denominator polynomials of Eq. (2), which are knows as poles describes the stability of the digital system; such as if all poles are inside unit circle then digital system is stable for causal input. On the other hand if any pole is outside the unit circle then digital system is not stable for causal input, which emphasis that such system are not applicable for real time applications. The magnitude response of an IIR filter is evaluated by solving the following Eq. for ω for the range of 0 to π for specific grid points as [8]:

$$\left|H(e^{j\omega})\right| = \frac{\left|\sum_{n=0}^{M} a_n \cdot e^{-j\omega n}\right|}{\left|1 + \sum_{n=1}^{N} b_n \cdot e^{-j\omega n}\right|} \tag{3}$$

whereas, the phase response of the filter is computed as [8,73]:

$$\theta_{Phase} = \arg\{H(e^{j\omega})\} \tag{4}$$

The presence of the poles in transfer function makes the phase response slightly nonlinear. Also, the design of IIR filter is quite challenging as the constrained is required, that the poles should be kept inside unit circle on z-plane. Designing of an IIR using optimization based approach includes the search of appropriate filter coefficients of Eq. (2) such that it satisfies the magnitude and phase response.

3. Designing of IIR filter using optimization techniques

Linear and global search methodologies brought revolution in the field of engineering optimization. There are two distinguished approaches being used for solving the multimodal error surface based designing of an IIR filter. In first method, the design problem is formulated in time domain and formulated as system identification. In another approach, digital filters are designed such that it gains the desired response in frequency domain via evaluation of some objective function in the frequency domain.

3.1. System identification (SI) approach

The design approach using a mathematical model for an unknown system by observing its input–output data pairs is known as system identification (SI) [26]. An IIR system design using SI is carried out by recursively updating the system coefficients until its response gets close enough to that of the unknown system. Since gradient based techniques produce sub-optimal solution [112], therefore various ETs are employed in SI based approach for IIR system filter as shown in Fig. 1 [9–36]. In SI based approach, minimization of mean squared error between desired and output signal was adopted [12,35].

An IIR filter response is estimated using Eq. (1), and numerous ETs have been used for designing an adaptive IIR filter with least possible order [23,29]. In this, an unknown plant transfer function was modeled with either a same order or smaller order transfer function. If smaller order transfer function is considered and then local minima problem could be encountered and create multimodel environment. Thus, ETs were employed to explore an optimal solution, which is set of filter coefficients for numerator and denominator polynomials. In initial stage of development, GA was considered in numerous forms for experiments [9,10,13]. In GA, too many functions were evaluated and a local minimum trapping was observed. Therefore, other ET like; PSO, ABC, CSA, CSO, HAS ICA, immune optimization algorithm and several other techniques have been used for an IIR filter design using SI [14,15,17,18,25,29,33,36,113]. The stability of filter was taken in account either by imposing some limit on coefficients of denominator polynomial such that entire denominator polynomial was split in first and second order polynomial, and each coefficient of these smaller polynomial terms had a prescribed limit [9]. While in another approach, instead of taking denominator polynomial in direct form, equivalent lattice coefficients for denominator coefficients during search operation were taken and converted into direct form coefficients during evaluation [36]. This method is suitable for filter designing of lower order.

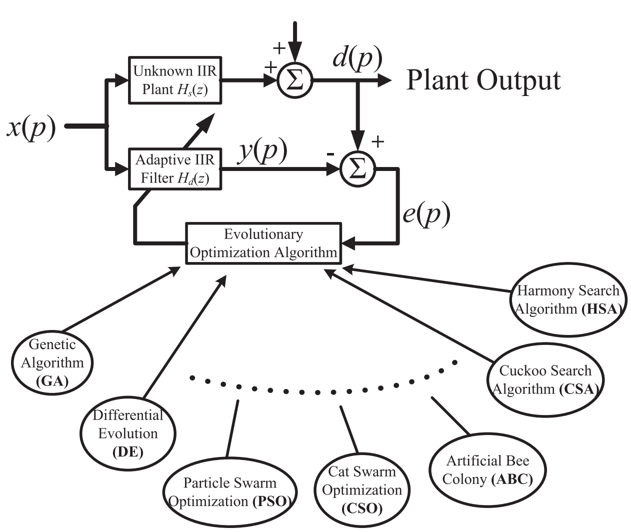

Fig. 1. Digital IIR filter designing approach using evolutionary techniques constructed as minimization of error function in time domain and known as system identification.

3.2. Designing of an IIR filter using gradient optimization based methods

In linear optimization technique, gradient of an error function with respect to filter coefficients is minimized, which ensures the optimal possible solution. The minimization of L_p error by using Fletcher-Powell algorithm was the initial step for obtaining the prescribed magnitude response [37]. Linear programming based approach was proposed for designing an IIR filter with linear phase response [41,48]. Then weighted least squares (WLS) algorithm for quasi-equiripple FIR and IIR filter was developed [52]. In [52], authors had developed a new enhanced method, which was based on Lawson's algorithm, for getting suitable solution. Another method based on the Eigen vector corresponding to least Eigen values of positive, real, and symmetric matrix was developed for an IIR lowpass filter (LPF) and bandpass filter (BPF) [53,54]. In [54], the design problem was constructed in WLS sense and weighting function was chosen as envelop of the absolute error between the desired and designed response of an IIR filter. This approach was utilized in designing of IIR allpass filter (APF) for Hilbert Transformer (HT) [55]. Later this technique was further elaborated with other weighting strategy, in which reciprocal of phase response of denominator polynomial was taken as weighting function [56]. Eigenvalue based design approach reported in [53–56] were iterative in nature.

Peak constraint least-squares (PCLS) technique was introduced to meet the requirements on magnitude response and group delay in frequency domain [57]. Later on, PCLS in stopband and with equiripple passband band design of an IIR filter with stability was proposed [58]. In design method of [58], three constraints were imposed along with an error function. Least-squares design method with constraint on poles radius was developed using Gauss-Newton method [59]. This aided ease for implementation and avoided the problem of instability. Inspired from the concept of semidefinite programming (SDP) used in FIR filter, authors had used same SDP for an IIR filter design using minimax objective function, and improvement group delay in passband with less number of poles was achieved [60]. Minimax based objective function with iterative LP was proposed, and improvement in form of peak magnitude error was reported in [61]. The minimax was also clubbed with conic-quadratic-programming (CQP) using required constraints, which are necessary and least required [62] and improvement over SDP was reported. Authors in [63], put a concept with simplified procedure for quasi-equiripple using iterative reweighting mechanism. In [64], new constraint that is the radius of pole distance based on argument principle (AP) was proposed. WLS was used to design the digital lowpass filter (LPF). In most of these techniques, an irrational transfer function was considered, while in [65], authors were come with an approach to design an IIR filter with rational transfer function using minimization of weighted absolute error. Authors in [65], used an established optimization algorithm for obtaining the optimal solution. In [38], second-order cone programming (SOCP) was considered in designing of an IIR filter using minimax as objective function, and improvement in magnitude response and GD was reported over previously used minimax based approach in [61]. Minimax design method using SDP was reported in [40]. Authors in [39], utilized the design concept of an IIR filter in WLS sense, which was proposed in [64] and also using second-order cone programing (SOCP) with AP for stability constraint [39].

Authors in [40], used the monitoring strategy, in which the energy of an impulse response of the denominator is suppressed that caused poles to move towards unit cricle. Also authors used the incorporated bisection search so that the feasible minimax solution was achieved. Constrained Newton method based design approach for nearly linear phase IIR filter was proposed in [43]. Authors in [43], used the cascade connected lower order IIR filter, and constraints are deployed for stability, step-size and pole migration problem. In order to overcome the problem of non-convex issue occurred and to explore the possibility for better results, authors in [44] used the sequential constrained least-squares (SCLS) technique using minimax design approach. SOCP based minimax IIR filter design was proposed where a linear constraint was developed to guarantee the convergence [42]. For effectively utilization of linearity, necessity and with the required triangle based stability, a new concept for the design of an IIR filter by converting the minimax design into sequence minimax sub problem was proposed [45]. In this method, only second order denominator was updated beside numerator term [45] and improvement was observed when compared with SCLS method. Later authors developed an improved technique based on minimax phase error as an objective function subjected to minimization of maximum magnitude error [46]. Authors used the Levy-Sanathanan-Koerner strategy for obtaining the convex constraints from non-convex. A new method for nearly linear-phase IIR filter was proposed by controlling the group delay in constraint environment, while for stability, complete filter transfer function was represented as cascade of first and second order filter [47]. Authors in [49] developed a new method, in which the Steiglitz–McBride (SM) scheme with modification was adopted for simplification. Recently, a new method was reported, in which two different weight functions were used with modified Gauss-Newton method [50].

3.3. Designing of an IIR filter using ETs in frequency domain

ETs were extensively used in filter design which was deployed in several signal processing applications like; audio equalization, compensating nonlinear distortions, active noise control and many more [114–116]. In this approach, the various techniques were used to explore the filter coefficients, which resulted in close approximation to desired response of an IIR filter. Several objective functions were used with various methods to solve the nonlinear design problem of an IIR filter.

3.3.1. Genetic algorithm (GA)

GA was the first method proposed and developed by the inspiration of genetic behavior in biological organism. This method can be divided into four stage operations, which include: initialization of a population of solution vectors, then recursive evaluation and execution of an objective function for each vector solution, acceptance of the best solution vector and genetic operation to reproduce a new solution vector. Crossover and mutation are two operation executed during fourth stage of GA, which helped in global exploration of solution [88]. Crossover produces new solution, and mutation avoids algorithm to get trapped in local minima. The quality of solution is analyzed by the objective function value, which is either to be minimized or maximized. Initially, a modified version GA known as Hierarchical-GA (HGA) was used for an IIR filter designing, where multi-objective was constructed as minimization of ripple based error with permissible limit in passband region, stopband region, and minimum required order as described in Table 1 [66]. This method was found to be suitable for filter designing for lower order, and phase response was also not taken in account during the design procedure. Moving forward standard GA was tested for an IIR filter design, which was aimed toward minimization of multi-objective error function which includes the weighted sum of all three bands mentioned in Table 1 [67].

Taguchi GA was next attempted using variant of GA for an IIR filter design with a multi-objective error that includes the weight sum of L_p-norm, maximum passband and stopband ripple [72]. The complete objective function aim is summarized in Table 1. The improvement was found when compared with previous methodology used in [66]. Cooperative co-evolutionary genetic algorithm (CCGA) was an advancement made for continuous improvisation in exploration abilities of global search abilities [73]. The objective function was formulated as minimization of multi-objective criteria, which involves the minimization of ripple content in passband region, stopband region and phase error with least passible order elaborated in Table 1. In this work, the frequency sample, which are less than permissible limit are taken in account, whereas the values at frequency sample which are above the permissible limit are discarded. The phase error is evaluated by the summation of deviation for phase response in the region of passband and transition band.

Real structured genetic algorithm (RSGA) was used in optimal design of an IIR filter which is formulated such that it included the merits of real genetic algorithm (RGA) and structured genetic algorithm (SGA) simultaneously during solving the optimization based problems [75]. The objective function was same, which was used in [73]. The relevant work was further carried out, in which local search of GA is modified and the entire method is entitled as local search operator enhanced multi-objective evolutionary algorithm (LS-MOEA) [77]. The local search is empowered by deploying Gaussian based local search to reproduce five new individuals. In all these approaches for designing of digital IIR filter using GA, the stability was considered by formulating the search space for the filter such that the entire transfer function was split in suitable number of first and second order system function cascaded together. The coefficient values of poles for first and second order were restricted to stay in some prescribed limit. Another approach for designing of digital IIR filter was conducted using GA in which constrained was imposed such that the position of poles should be inside circle, which is sub region of the unit circle. This smaller circle range denotes the most appropriated location for the poles for specific type. The objective function used is minimization of MSE evaluated in frequency domain between the ideal and designed response. In Table 2, a comparative study has been shown, where the fidelity parameters obtained by various GA is mentioned.

3.3.2. Particle swarm optimization (PSO)

Particle swarm optimization (PSO) was found computationally fast when compared to GA, and initially used for the designing of an adaptive IIR filter using system identification approach [25]. In PSO, the search space is formulated as position matrix of swarm. Now the best solution is computed as position for which least value of error function is obtained. The new position is evaluated after the update of velocity as shown [117].

The inertia weight used in the principle update equation leads to the variants of PSO, which are also employed in numerous application including designing of digital FIR and IIR filter [78,80–83,96,102,118–121]. These variants were tested using minimization of MSE in frequency domain. Similarly, another variant of PSO family were employed using a different objective function, in which the absolute error with permissible ripples in passband and stopband was minimized in frequency domain. PSO was applied in several optimization problem; however, premature convergence was observed, and hence tried to

Table 1
Objective function used for evaluation of cost function for the optimized design of digital IIR filter using evolutionary techniques.

Description	Objective function	
This objective function is formulated to evaluate passband and stopband absolute error with minimum possible order. $H(e^{j\omega})$ is the response obtained and p_a and q_j represents the value of control bit for first and second order filter transfer function inclusion [66].	$\Delta H_{\omega}^{(p)} = \begin{cases} \lvert H(e^{j\omega})\rvert - 1 \text{ if} \lvert H(e^{j\omega})\rvert > 1 \\ (1-\delta_1) - \lvert H(e^{j\omega})\rvert \text{ if} \lvert H(e^{j\omega})\rvert < (1-\delta_1) \end{cases}$ $\Delta H_{\omega}^{(p)} = \lvert H(e^{j\omega})\rvert - \delta_2 \text{ if} \lvert H(e^{j\omega})\rvert > \delta_2$	$f_3 = \sum_{i=1}^{m} p_i + 2\sum_{j=1}^{n} q_j$ $obj = \min\left(\Delta H_{\omega}^{(p)}(e^{j\omega}), \Delta H_{\omega}^{(s)}(e^{j\omega})\right)$, subjected to, $\min(f_3)$
This is multi-objective error function with weight which are defined explicitly by user accordingly to the application and used in the designing of low pass in [67].	$q_1 = \max\{\lvert H(e^{i\omega})\rvert - \lvert H_d(e^{i\omega})\rvert\},\ \lvert\omega\rvert \leqslant \omega_c$ $q_2 = \lvert\omega_s - \omega_c\rvert$ $q_1 = \max\{\lvert H(e^{i\omega})\rvert - \lvert H_d(e^{i\omega})\rvert\},\ \omega_c \leqslant \omega \leqslant \pi$	$\min(g(s)) = \frac{\sum_{k=1}^{3} w_k q_k}{\sum_{k=1}^{3} w_k}$
In this objective function the suitable *P-NORM* along with the maximum ripple in passband and stopband is added and the entire summed error function is minimized used in [72].	$E_M^P(m) = \left\{\sum_{i=1}^{K} \lvert T_M(\omega_i) - \lvert H(\omega_i, x)\rvert\rvert^p\right\}^{\frac{1}{p}}$, $\omega \in \Omega;\ p = 1 \text{ and } 2$, $\delta_1(x) = \max_{\omega_i}\{\lvert H(\omega_i, x)\rvert\} - \min_{\omega_i}\{\lvert H(\omega_i, x)\rvert\}, \omega_i \in \text{pb}$	$\delta_2(x) = \max_{\omega_i}\{\lvert H(\omega_i, x)\rvert\},\ \text{for } \omega_i \in \text{sb}$, $\min_x f(x) = \min_x v_1 E_M^1(x) + v_1 E_M^2(x) + v_3\delta_1(x) + v_4\delta_2(x)$
This is also a multi-objective minimization error problem formulated as the minimization of summed ripple contained with least possible aggregated phase deviation also subjected to minimum possible order as suggested in [66] and deployed for same problem in [73].	$eH_p(\omega) = \begin{cases} 1 - \delta_1 - \lvert H(e^{j\omega})\rvert,\ \lvert H(e^{j\omega})\rvert < (1-\delta_1) \\ 0,\ \lvert H(e^{j\omega})\rvert \geqslant (1-\delta_1) \end{cases}$ $eH_s(\omega) = \begin{cases} \lvert H(e^{j\omega})\rvert \delta_1 -,\ \lvert H(e^{j\omega})\rvert > \delta_2 \\ 0,\ \lvert H(e^{j\omega})\rvert \leqslant \delta_2 \end{cases}$	$\Delta phase = \{\Delta\theta_1, \Delta\theta_2, \ldots, \Delta\theta_{n-1}\}$, where $\Delta\theta_i = \theta_{i+1} - \theta_i$ $f_1 = \sum_{\omega_i} eH_p(\omega) + \sum_{\omega_j} eH_s(\omega)$ $obj = \min(f_1, \Delta phase), \textit{subjected to } \min(f_3)$
This is simple mean squared error function which is the difference of desired response ($H_D(\Omega)$) and obtained response ($H_p(\Omega)$) whereas Ω represents the normalized digital frequency. In most case either E_p was minimized [74] or its reciprocal has to be maximized [68].	$E_p = \frac{\sum_{\omega}\lvert H_p(\omega) - H_D(\omega)\rvert^2}{N_s}, 0 \leqslant \omega \leqslant \pi$	$obj = fitness = \max\left(\frac{1}{E_p^2 + 1}\right)$ subjected to, $\left\lvert\sum_{k=1}^{n} a_{k0}(re^{-j\theta} + \alpha) - k\right\rvert < 1, \forall\theta \in [0, 2\pi]$
This is also a multi-objective minimization error problem formulated as the minimization of summed ripple contained with minimum possible denominator order which is represented by J_s with completed description included [75] in and deployed for optimal design of IIR filter in same.	$J_{pp} = \begin{cases} \sum_{pb=1}^{r_p} \lvert\lvert H(e^{j\omega_{pb}})\rvert - 1\rvert,\ \text{if} \lvert H(e^{j\omega_{pb}})\rvert > 1 \\ \sum_{pb=1}^{r_p} \lvert 1 - \delta_p - \lvert H(e^{j\omega_{pb}})\rvert\rvert,\ \text{if} \lvert H(e^{j\omega_{pb}})\rvert < 1 - \delta_p \end{cases}$, $\forall\omega_{pb} \in passband$, and $J_{pp} = \{\sum_{sb=1}^{r_s} \lvert H(e^{j\omega_{sb}})\rvert - \delta_s,\ \text{if} \lvert H(e^{j\omega_{sb}})\rvert > \delta_s$, $\forall\omega_{sb} \in stopband$	$obj = \min(J_{tot}(\cdot)) = \rho J_s(\cdot) + (1-\rho)\left[J_{pp}(\cdot) + J_{ps}(\cdot)\right]$, where $J_s = \frac{1}{\delta} Log(N_{\max} - N + 1)$, $\delta = \max\{\varepsilon, sgn(N - M)\}$
This is simple absolute error based objective function with permissible passband ripple (δ_p) and stopband ripple (δ_s) whose value has to be minimized for optimal design of IIR filter and used in for the same in [84].	$J(\omega) = \sum_{\omega_i \in passband} abs\left[abs(\lvert H_d(\omega_i)\rvert - 1) - \delta_p\right] + \sum_{\omega_j \in stopband} abs\left[abs(\lvert H_d(\omega_j)\rvert - 1) - \delta_s\right]$	$obj = \min(J(\omega))$

overcome via a modification reported in [76]. In this work, the search space is redistributed upon the detection of premature convergence and redistribution based on sorting of the rank of fitness was performed.

3.3.3. Quantum PSO

There is also a variant of PSO exhibits, which is probabilistic based approach known as Quantum PSO (QPSO). Since the position and velocity cannot be determined simultaneously; therefore, the exploration mechanism of PSO is then modified by the wave function of quantum mechanism, and a detailed study has been reported in [122,123]. QPSO, because of its stable search strategy, was used in digital filter design [124], and found to be very effective when compared with GA and PSO.

3.3.4. Artificial bee colony (ABC) algorithm

ABC algorithm is evolutionary based global search technique, which was developed by the intelligent foraging behavior of honey bees [125]. This method was found to be robust and has shown ability to handle the non-linear optimization problem quite effectively; therefore, exhaustively practiced in adaptive filtering [2,3], multi-rate signal processing [103], and IIR filter designing [33]. In this method, a probabilistic random search was executed by the formulation of two search spaces, and the entire method in each cycle of iteration conducts three operations. Thus, this method needs too many function evaluations as compared to PSO. However, ABC algorithm has been clubbed with PSO to form an improved hybrid technique and tested for filter bank design [126].

3.3.5. Immune algorithm (IA)

This technique also belongs to evolutionary based computation inspired by the biological immune system. This method was used in the optimal design of an IIR filter [70] and same objective function was employed for the evaluation of objective/cost function as proposed in [72].

3.3.6. Gravitational search algorithm (GSA)

GSA is the recently proposed metaheuristic method, in which the search mechanism is governed by the analogy of the law of gravitational and mass attraction forces [127]. In GSA, solution vectors are considered as objects and their enactments are measured by their masses. The heavier masses (solution vector related to better solutions) travel quite slowly than lighter ones, and one could found the detailed study in [84]. GSA was formulated for the designing of an IIR filter with eight order, and objective function adopted was the ripple base as in Table 1 [84]. The filter response obtained was quit optimal in sense of error and maximum ripple in passband, stopband, and stopband attenuation. However, the filter coefficients obtained are still susceptible to quantization and truncation. The fidelity parameters obtained by various GSA has been summarized in Table 2.

3.3.7. Cuckoo search algorithm (CSA)

CSA method was newly proposed global search technique, which adopts the behavior of laying eggs of some bird species for exploration [128,129]. In CSA, the search for optimal solution is constituted as in first stage placing of new elements in possible

Table 2
Comparative performance evaluation of LPF and HPF designed using various variants of GA and Swarm based techniques.

Algorithm	Type	Passband response	Stopband response	Max δ_p	Max δ_s	e_p	e_s	A_s	Remark
HGA [66]	LPF	$0.89125 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.17783$	–	–	4.60×10^{-3}	1.90×10^{-3}	15.0007	In this approach the minimization of ripple in *pb* and *sb* was used with constrained on filter coefficients for stability. Filter designed is stable and withstands upon quantization of filter coefficients
	HPF	$0.8912 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.17783$	–	–	1.16×10^{-4}	1.50×10^{-2}	23.4429	
HTGA [72]	LPF	$0.8994 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1695$	0.1006	0.1695	4.50×10^{-3}	1.60×10^{-3}	15.5381	In this method three distinguish approaches were used for optimal design of IIR filter. In case 1 for which same design objective methodology was used with HTGA as used in case of HGA. In case 2 the sum of the ripple in *pb* and *sb* was imposed for the optimal design of IIR filter In case 3, for which the constructed objective function as shown in Table 1 for HTGA was employed for IIR filter design. In all the cases case 2 and 3 have performed better and designed filter were stable
	HPF	$0.9403 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1668$	0.0597	0.1668	3.43×10^{-4}	1.60×10^{-3}	15.6963	
	LPF	$0.9586 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.8$	0.0414	0.0800	4.20×10^{-3}	3.90×10^{-3}	22.2405	
	HPF	$0.9612 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.833$	0.0388	0.0833	2.24×10^{-4}	4.50×10^{-3}	21.8984	
	LPF	$0.9004 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1247$	0.0996	0.1247	4.50×10^{-3}	1.00×10^{-3}	17.9597	
	HPF	$0.9460 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1457$	0.0540	0.1457	3.13×10^{-4}	1.60×10^{-3}	16.8948	
CCGA [73]	LPF	$0.9034 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1669$	–	–	4.30×10^{-3}	1.90×10^{-2}	16.1378	In this approach the ripple based function was redefined with modification in GA. The results obtained had shown in reduction in passband response
	HPF	$0.9044 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1749$	–	–	1.16×10^{-4}	1.50×10^{-2}	15.0086	
RSGA [75]	LPF	–	–	0.0960	0.1387	5.30×10^{-3}	1.30×10^{-3}	17.2935	GA was used with similar type of objective function as in previous method (CCGA)
	HPF	–	–	0.0736	0.1228	5.10×10^{-3}	3.60×10^{-3}	17.4885	
LS-MOEA [77]	LPF	$0.9083 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 1586$	–	–	4.10×10^{-3}	1.54×10^{-2}	16.1956	In this approach the multi-objective method including reduction of phased error along with order was constructed.
	HPF	$0.9004 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1746$	–	–	2.09×10^{-4}	2.15×10^{-2}	15.3783	
Hybrid heuristic search method [71]	LPF	$0.9202 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1476$	–	–	4.00×10^{-3}	2.16×10^{-2}	16.8553	In this approach DE method for global search was clubbed with binary GA for local search with multi-objective error function. For all the method
	HPF	$0.9004 \leq H(e^{j\omega}) \leq 1$	$H(e^{j\omega}) \leq 0.1746$	–	–	1.20×10^{-4}	2.39×10^{-2}	15.6584	
GSA [84]	LPF	–	–	0.0028	3.406×10^{-3}	7.80×10^{-3}	2.19×10^{-6}	49.3552	Simple absolute error function was imposed for higher order IIR filter design. The filter response was excellent; however the performance degrades upon quantization.
	HPF	–	–	0.0207	2.4628×10^{-3}	6.63×10^{-6}	1.96×10^{-6}	52.1714	

solution vector, which are analogues to a random walk tracked via a cuckoo bird for the laying of eggs. For these, various mathematical methodologies have been adopted; however, literature shows that *Lévy Flight* has been practiced deliberately, and is also efficient. In second stage, the search space is updated and the new eggs, which haven't contributed in improvement, are discarded during course of iterative computation.

The step size and probability for dropping of the solution are very important parameters and required the tuning carefully otherwise algorithm may get unstable. Because of the empowered search abilities, CSA has been extensively used in various multi-model optimization problems including digital filter design [15,96,130–133], and has shown tremendous improvement in the fidelity parameters obtained.

3.3.8. Hybrid techniques

In order to improvise the performance of existing evolutionary techniques, some modifications have been made. These were the clubbing of two distinguished methods together to form a new enhanced method. Hybrid-PSO was one of the method developed by the hybridization of search mechanism of PSO followed by the check and replacement concept of scout bee of ABC algorithm [126]. As PSO is quite fast in convergence and includes less function Evaluation, but sometimes stuck in local minima. Whereas, ABC is slow in convergence but the check and replacement mechanism helped in better exploration. Now the proposed method has shown significant enhancement and robust to handle higher order design problem. Intensive work has been carried out for the investigation and development of hybrid method, which leads to the development of methods, in which the search is conducted in two stages. The method has been proposed in which the local search is executed using binary successive approximation based evolutionary search method, whereas DE is used for global search. This method was tested for the designing of lower order filter with minimization of multi-objective error function as reported in [71]. Similarly, two methods known as Two-stage ensemble memetic algorithm (TSEM) and two-stage ensemble evolutionary algorithm (TEEA) are also developed with same principle with different method [68]. While using TSEM, only the magnitude error function is considered of multi-objective error function evaluated. Whereas in TEEA, the same objective function was used. Same stability criteria were used as mentioned in [73]. TEEA was tested only for the lower order, whereas TSEM was intensively tested for various filters with different order. The comparative performance is summarized in Table 3. Recently, researchers have used to club the wavelet concept with evolutionary technique which is GSA in this case. GSA is used for exploration of the solution, whereas the wavelet mutation is used for the random selection of particles as deployed and described in [69,134]. For stability of the designed filter, coefficients are restricted to stay in certain value, but it doesn't ensure the stability. Also, the effect of quantization and truncation was not addressed.

3.4. Designing of an IIR filter using APF

An APF can be used in two configurations for obtaining desired response of overall transfer function. In first approach, an APF was connected with a pure delay function in parallel such that the difference of order of delay and APF was one [109,110]. An error was formulated as minimization of phase error of APF such that it should be in phase with phase of pure delay in passband region, while the phase difference of APF and pure delay must be $\pi/2$ in stopband region. When the output of APFs was added with the output of pure delay, then LPF response was obtained, while difference operation resulted in HPF response. While for bandpass filter (BPF) and bandstop filter (BSF), the only difference between pure delay

Table 3
Comparative analysis of Hybrid meta-heuristic techniques on the basis of statistical parameters mean, standard deviation (Std), and NFE.

Algorithm	Problem	Type	Pass-band	Stop-band	Desired passband attenuation (δ_1) in dB	Desired stopband attenuation (δ_2) in dB	Order
a) Filter specifications used for performance evaluation based on statically parameters and scalability test							
TEEA	1	LPF	[0 0.5π]	[0.6π π]	1	80	9
	2	LPF	[0 0.5π]	[0.6π π]	1	110	11
	3	LPF	[0 0.5π]	[0.6π π]	1	140	13
	4	HPF	[0.6π π]	[0 0.5π]	1	80	8
TSMA	1	LPF	[0 0.5π]	[0.6π π]	1	220	20
	2	LPF	[0 0.5π]	[0.6π π]	1	300	27

Algorithm	Problem-1				Problem-2				Problem-3				Problem-4			
	Mean	Std	Srun	NFE	mean	std	srun	NFE	mean	std	srun	NFE	mean	std	srun	NFE
b) Comparative performance evaluation based on statically parameters and scalability test																
TEEA	–	–	30	16,419	–	–	30	3.352×10^4	–	–	30	5.57×10^4	–	30	30	1.004×10^5
TSMA	00	00	30	2.53×10^{05}	00	00	30	1.996×10^5	–	–	–	–	–	–	–	–

and an APF was set to two else other requirements were kept as same. Eigenvalue based techniques gives closed form solution [110].

While in second case, two APFs are connected in parallel and the order of APFs might be kept as same or can have difference of one [111]. The use of an IIR-APF resulted in magnitude response, which was not degraded due to quantization of filter taps. The phase response was nearly linear; however, the magnitude response was equiripple with less magnitude ripple. Parallel connection of two APF for design of an IIR filter using minor component connection was put forward, where phase response of both filters were designed by finding minor components in iterative manner [111]. This connect was used to produce multirate filterbank using an IIR filter with less order. Therefore, it can be stated that the parallel connection based approach results in nearly linear phase response with acceptable magnitude response, which do not deteriorate due to quantization of filter coefficients.

4. Results and future scope

The literature survey revealed that an exhaustive work has been performed for an optimal design of an IIR filter, which has improved magnitude response and phase response in passband region. The use of gradient based methods was resulted in solution which fulfils the imposed constraint environment and resulted in acceptable solution. On the other hand, researchers also explored the possibilities of global solution using ETs. However, the gradient based method resulted in sub-optimal solution, whereas ETs were suitable for lower order IIR filters. APF based approach was found to be effective, since it has nearly linear phase and accurate magnitude response. However, the phase error at the band edge frequency was quite high, and that gap may be overcome in future.

5. Conclusion

An IIR filter is a highly recognized topic of interest because of its wide area of application in signal processing. The filters designed using evolutionary/swarm based method yields the superiority of performing when coefficients of filter tap are realized by finite number of bit. The optimized based approach also helps in design of unconventional filters like fractional delay filter having optimal performance in term of the resources employed. Therefore, this paper presents the experimentally work performed in optimal design of IIR filter by exploiting various objective functions using various evolutionary/swarm optimization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Dutta Roy SC. A state of the art survey on linear phase digital filter design. IETE J Res 1976;22:1–4. https://doi.org/10.1080/03772063.1976.11450887.
[2] Karaboga N, Latifoglu F. Elimination of noise on transcranial Doppler signal using IIR filters designed with artificial bee colony – ABC-algorithm. Digit Signal Process 2013;23:1051–8. https://doi.org/10.1016/j.dsp.2012.09.015.
[3] Ahirwal MK, Kumar A, Singh GK. Study of ABC and PSO algorithms as optimized adaptive noise canceller for EEG/ERP. Int J Bio-Inspired Comput 2015.
[4] Lai E. Practical digital signal processing for engineers and technicians. 1st ed. Burlington: Elsevier; 2003.
[5] Cao L, Zhang D, Tang S, Deng F. A practical parameter determination strategy based on improved hybrid PSO algorithm for higher-order sliding mode control of air-breathing hypersonic vehicles. Aerosp Sci Technol 2016;59:1–10. https://doi.org/10.1016/j.ast.2016.10.001.
[6] Ko HJ, Tsai JJP. Robust and computationally efficient digital IIR filter synthesis and stability analysis under finite precision implementations. IEEE Trans Signal Process 2020;68:1807–22. https://doi.org/10.1109/TSP.2020.2977848.
[7] Dutta Roy SC, Jain SB, Kumar B. Design of digital FIR notch filters from second order IIR prototype. IETE J Res 1997;43:275–9. https://doi.org/10.1080/03772063.1997.11415989.
[8] Proakis JG, Manolakis DG. Digital signal processing. 4th ed. Prentice Hall; 2006.
[9] Etter DM, Hicks M, Cho K. Recursive adaptive filter design using an adaptive genetic algorithm. In: IEEE Int. Conf. Acoust. Speech, Signal Process. ICASSP '82. p. 635–8. https://doi.org/10.1109/ICASSP.1982.1171777.
[10] Nambiar R, Mars P. Genetic and annealing approaches to adaptive digital filtering. In: [1992] Conf. Rec. Twenty-Sixth Asilomar Conf. Signals, Syst. Comput.. IEEE Comput Soc. Press; 1992. p. 871–5. https://doi.org/10.1109/ACSSC.1992.269148.
[11] Fang W, Sun J, Xu W. A new mutated quantum-behaved particle swarm optimizer for digital IIR filter design. EURASIP J Adv Signal Process 2010;2009:. https://doi.org/10.1155/2009/367465367465.
[12] Luitel B, Venayagamoorthy GK. Particle swarm optimization with quantum infusion for system identification. Eng Appl Artif Intell 2010;23:635–49. https://doi.org/10.1016/J.ENGAPPAI.2010.01.022.
[13] Ma Q, Cowan CFN. Genetic algorithms applied to the adaptation of IIR filters. Signal Process 1996;48:155–63. https://doi.org/10.1016/0165-1684(95)00131-X.
[14] Panda G, Pradhan PM, Majhi B. IIR system identification using cat swarm optimization. Expert Syst Appl 2011;38:12671–83. https://doi.org/10.1016/j.eswa.2011.04.054.
[15] Patwardhan AP, Patidar R, George NV. On a cuckoo search optimization approach towards feedback system identification. Digit Signal Process 2014;32:156–63. https://doi.org/10.1016/j.dsp.2014.05.008.
[16] Upadhyay P, Kar R, Mandal D, Ghoshal SP. Craziness based particle swarm optimization algorithm for IIR system identification problem. AEU - Int J Electron Commun 2014;68:369–78. https://doi.org/10.1016/j.aeue.2013.10.003.
[17] Saha SK, Kar R, Mandal D, Ghoshal SP. Harmony search algorithm for infinite impulse response system identification. Comput Electr Eng 2014;40:1265–85. https://doi.org/10.1016/j.compeleceng.2013.12.016.
[18] Sarangi A, Sarangi SK, Padhy SK, Panigrahi SP, Panigrahi BK. Swarm intelligence based techniques for digital filter design. Appl Soft Comput 2014;25:530–4. https://doi.org/10.1016/j.asoc.2013.06.001.
[19] Sharifi MA, Mojallali H. A modified imperialist competitive algorithm for digital IIR filter design. In: 2015 Int Symp. Artif. Intell. Signal Process.. IEEE; 2015. p. 7–12. https://doi.org/10.1109/AISP.2015.7123526.
[20] Gotmare A, Patidar R, George NV. Nonlinear system identification using a cuckoo search optimized adaptive Hammerstein model. Expert Syst Appl 2015;42:2538–46. https://doi.org/10.1016/j.eswa.2014.10.040.
[21] Chang W-D. Coefficient estimation of IIR filter by a multiple crossover genetic algorithm. Comput Math with Appl 2006;51:1437–44. https://doi.org/10.1016/j.camwa.2006.01.003.
[22] Wang J, Shi P, Peng H. Membrane computing model for IIR filter design. Inf Sci (Ny) 2016;329:164–76. https://doi.org/10.1016/j.ins.2015.09.011.
[23] Chen S, Luk BL. Adaptive simulated annealing for optimization in signal processing applications. Signal Process 1999;79:117–28. https://doi.org/10.1016/S0165-1684(99)00084-5.
[24] Chen S, Istepanian R, Luk BL. Digital IIR filter design using adaptive simulated annealing. Digit Signal Process 2001;11:241–51. https://doi.org/10.1006/dspr.2000.0384.
[25] Krusienski DJ, Jenkins WK. Adaptive filtering via particle swarm optimization. In: Thrity-Seventh Asilomar Conf. Signals, Syst. Comput. 2003.. IEEE; 2003. p. 571–5. https://doi.org/10.1109/ACSSC.2003.1291975.
[26] Montiel O, Castillo O, Melin P, Sepulveda R. The evolutionary learning rule for system identification. Appl Soft Comput 2003;3:343–52. https://doi.org/10.1016/j.asoc.2003.05.005.
[27] Kalinli A, Karaboga N. A new method for adaptive IIR filter design based on tabu search algorithm. AEU - Int J Electron Commun 2005;59:111–7. https://doi.org/10.1016/j.aeue.2004.11.003.
[28] Karaboga N. Digital IIR filter design using differential evolution algorithm. EURASIP J Adv Signal Process 2005;2005:1269–76. https://doi.org/10.1155/ASP.2005.1269.
[29] Kalinli A, Karaboga N. Artificial immune algorithm for IIR filter design. Eng Appl Artif Intell 2005;18:919–29. https://doi.org/10.1016/j.engappai.2005.03.009.
[30] Haseyama M, Matsuura D. A filter coefficient quantization method with genetic algorithm, including simulated annealing. IEEE Signal Process Lett 2006;13:189–92. https://doi.org/10.1109/LSP.2005.863695.
[31] Fang W, Sun J, Xu W. Analysis of adaptive IIR filter design based on quantum-behaved particle swarm optimization. In: 2006 6th World Congr. Intell. Control Autom. IEEE; 2006. p. 3396–400. https://doi.org/10.1109/WCICA.2006.1712998.
[32] Theofilatos K, Beligiannis G, Likothanassis S. Combining evolutionary and stochastic gradient techniques for system identification. J Comput Appl Math 2009;227:147–60. https://doi.org/10.1016/j.cam.2008.07.014.
[33] Karaboga N. A new design method based on artificial bee colony algorithm for digital IIR filters. J Franklin Inst 2009;346:328–48. https://doi.org/10.1016/j.jfranklin.2008.11.003.

[34] Yu X, Liu J, Li H. An adaptive inertia weight particle swarm optimization algorithm for IIR digital filter. In: 2009 Int. Conf. Artif. Intell. Comput. Intell. IEEE; 2009. p. 114–8. https://doi.org/10.1109/AICI.2009.28.
[35] Dai C, Chen W, Zhu Y. Seeker optimization algorithm for digital IIR filter design. IEEE Trans Ind Electron 2010;57:1710–8. https://doi.org/10.1109/TIE.2009.2031194.
[36] Sheng C, Bing LL. Digital IIR filter design using particle swarm optimisation. Int. J. Model. Identif. Control 2010;9:327–35. https://doi.org/10.1504/IJMIC.2010.033208.
[37] Deczky A. Synthesis of recursive digital filters using the minimum p-error criterion. IEEE Trans Audio Electroacoust 1972;20:257–63. https://doi.org/10.1109/TAU.1972.1162392.
[38] Jiang A, Kwan H. Minimax IIR digital filter design using SOCP. In: 2008 IEEE Int. Symp. Circuits Syst.. IEEE; 2008. p. 2454–7. https://doi.org/10.1109/ISCAS.2008.4541952.
[39] Jiang A, Kwan HK. IIR digital filter design with new stability constraint based on argument principle. IEEE Trans Circuits Syst I Regul Pap 2009;56:583–93. https://doi.org/10.1109/TCSI.2008.2002116.
[40] Jiang A, Kwan HK. Minimax design of IIR digital filters using SDP relaxation technique. In: 2009 IEEE Int Symp. Circuits Syst.. p. 477–80. https://doi.org/10.1109/ISCAS.2009.5117789.
[41] Chottera A, Jullien G. A linear programming approach to recursive digital filter design with linear phase. IEEE Trans Circuits Syst 1982;29:139–49. https://doi.org/10.1109/TCS.1982.1085123.
[42] Jiang A, Kwan HK. Minimax design of IIR digital filters using iterative SOCP. IEEE Trans Circuits Syst I Regul Pap 2010;57:1326–37. https://doi.org/10.1109/TCSI.2009.2028749.
[43] Guindon D, Shpak DJ, Antoniou A. Design methodology for nearly linear-phase recursive digital filters by constrained optimization. IEEE Trans Circuits Syst I Regul Pap 2010;57:1719–31. https://doi.org/10.1109/TCSI.2009.2035412.
[44] Lai X, Lin Z. Minimax design of IIR digital filters using a sequential constrained least-squares method. IEEE Trans Signal Process 2010;58:3901–6. https://doi.org/10.1109/TSP.2010.2046899.
[45] Lai X, Lin Z, Kwan HK. A sequential minimization procedure for minimax design of IIR filters based on second-order factor updates. IEEE Trans Circuits Syst II Express Briefs 2011;58:51–5. https://doi.org/10.1109/TCSII.2010.2092820.
[46] Lai X, Lin Z. Minimax phase error design of IIR digital filters With prescribed magnitude and phase responses. IEEE Trans Signal Process 2012;60:980–6. https://doi.org/10.1109/TSP.2011.2175389.
[47] Nongpiur RC, Shpak DJ, Antoniou A. Improved design method for nearly linear-phase IIR filters using constrained optimization. IEEE Trans Signal Process 2013;61:895–906. https://doi.org/10.1109/TSP.2012.2231678.
[48] Rabiner L, Graham N, Helms H. Linear programming design of IIR digital filters with arbitrary magnitude function. IEEE Trans Acoust 1974;22:117–23. https://doi.org/10.1109/TASSP.1974.1162558.
[49] Jiang A, Kwan HK, Zhu Y, Xu N, Liu X. Minimax design of IIR digital filters using partial second-order factor updates. In: 2015 IEEE Int. Conf. Digit. Signal Process. IEEE; 2015. p. 1–5. https://doi.org/10.1109/ICDSP.2015.7445518.
[50] Lai X, Lin Z. Iterative reweighted minimax phase error designs of IIR digital filters with nearly linear phases. IEEE Trans Signal Process 2016;64:2416–28. https://doi.org/10.1109/TSP.2016.2521610.
[51] Volkova A, Istoan M, De Dinechin F, Hilaire T. Towards Hardware IIR filters computing just right: direct Form i case study. IEEE Trans Comput 2019;68:597–608. https://doi.org/10.1109/TC.2018.2879432.
[52] Lim Y-C, Lee J-H, Chen CK, Yang R-H. A weighted least squares algorithm for quasi-equiripple FIR and IIR digital filter design. IEEE Trans Signal Process 1992;40:551–8. https://doi.org/10.1109/78.120798.
[53] Zhang X, Iwakura H. Design of IIR digital filters based on eigenvalue problem. IEEE Trans Signal Process 1996;44:1325–33. https://doi.org/10.1109/78.506600.
[54] Argenti F, Del Re E. Design of IIR eigenfilters in the frequency domain. In: Proc. 13th Int. Conf. Digit. Signal Process.. Santorini: IEEE; 1997. p. 629–32. https://doi.org/10.1109/ICDSP.1997.628428.
[55] Kidambi SS. Weighted least-squares design of recursive allpass filters. IEEE Trans Signal Process 1996;44:1553–7. https://doi.org/10.1109/78.506621.
[56] Argenti F, Del Re E. Design of IIR eigenfilters in the frequency domain. IEEE Trans Signal Process 1998;46:1694–8. https://doi.org/10.1109/78.678495.
[57] Sullivan JL, Adams JW. PCLS IIR digital filters with simultaneous frequency response magnitude and group delay specifications. IEEE Trans Signal Process 1998;46:2853–61. https://doi.org/10.1109/78.726800.
[58] Lu W-S. Design of stable IIR digital filters with equiripple passbands and peak-constrained least-squares stopbands. IEEE Trans Circuits Syst II Analog Digit Signal Process 1999;46:1421–6. https://doi.org/10.1109/82.803482.
[59] Lang MC. Least-squares design of IIR filters with prescribed magnitude and phase responses and a pole radius constraint. Signal Process IEEE Trans 2000;48:3109–21. https://doi.org/10.1109/78.875468.
[60] Lu W-S. Design of stable minimax IIR digital filters using semidefinite programming. In: IEEE Int. Symp. Circuits Syst., Geneva. p. 355–8.
[61] Tseng Chien-Cheng, Lee Su-Ling. Minimax design of stable IIR digital filter with prescribed magnitude and phase responses. IEEE Trans Circuits Syst I Fundam Theory Appl 2002;49:547–51. https://doi.org/10.1109/81.995676.
[62] Hinamoto T. Optimal design of IIR digital filters with robust stability using conic-quadraticprogramming updates. IEEE Trans Signal Process 2003;51:1581–92. https://doi.org/10.1109/TSP.2003.811229.
[63] Niemistö R, Dumitrescu B. Simplified procedures for quasi-equiripple IIR filter design. IEEE Signal Process Lett 2004;11:308–11.
[64] Lu W-S. An argument-principle based stability criterion and application to the design of IIR digital filters. In: 2006 IEEE Int. Symp. Circuits Syst.. IEEE; 2006. p. 4431–4. https://doi.org/10.1109/ISCAS.2006.1693612.
[65] Ho CYF, Ling BWK, Liu YQ, Tam PKS, Teo KL. Optimal design of magnitude responses of rational infinite impulse response filters. IEEE Trans Signal Process 2006;54:4039–46. https://doi.org/10.1109/TSP.2006.880317.
[66] Tang KS, Man KF, Kwong S, Liu ZF. Design and optimization of IIR filter structure using hierarchical genetic algorithms. Ind Electron IEEE Trans 1998;45:481–7. https://doi.org/10.1109/41.679006.
[67] Zhang G, Jin W, Jin F. Multi-criterion satisfactory optimization method for designing IIR digital filters. In: Int. Conf. Commun. Technol.. Beijing: IEEE; 2003. p. 1484–90. https://doi.org/10.1109/ICCT.2003.1209809.
[68] Wang Y, Li B, Weise T. Two-stage ensemble memetic algorithm: function optimization and digital IIR filter design. Inf Sci (Ny) 2013;220:408–24. https://doi.org/10.1016/j.ins.2012.07.041.
[69] Saha SK, Kar R, Mandal D, Ghoshal SP. Optimal IIR filter design using Gravitational Search Algorithm with Wavelet Mutation. J King Saud Univ - Comput Inf Sci 2015;27:25–39. https://doi.org/10.1016/j.jksuci.2014.03.002.
[70] Tsai J-T, Chou J-H. Design of optimal digital IIR filters by using an improved immune algorithm. IEEE Trans Signal Process 2006;54:4582–96. https://doi.org/10.1109/TSP.2006.881248.
[71] Sidhu DS, Dhillon JS, Kaur D. Hybrid heuristic search method for design of digital IIR filter with conflicting objectives. Soft Comput 2016. https://doi.org/10.1007/s00500-015-2023-8.
[72] Tsai J-T, Chou J-H, Liu T-K. Optimal design of digital IIR filters by using hybrid taguchi genetic algorithm. IEEE Trans Ind Electron 2006;53:867–79. https://doi.org/10.1109/TIE.2006.874280.
[73] Yu Y, Xinjie Y. Cooperative coevolutionary genetic algorithm for digital IIR filter design. IEEE Trans Ind Electron Electron 2007;54:1311–8. https://doi.org/10.1109/TIE.2007.893063.
[74] Pan ST. Design of robust D-Stable IIR filters using genetic algorithms with embedded stability criterion. IEEE Trans Signal Process 2009;57:3008–16. https://doi.org/10.1109/TSP.2009.2017003.
[75] Tsai C-W, Huang C-H, Lin C-L. Structure-specified IIR filter and control design using real structured genetic algorithm. Appl Soft Comput 2009;9:1285–95. https://doi.org/10.1016/j.asoc.2009.04.001.
[76] Zhang X, Jia P, Guo J. n improved particle swarm optimizer for IIR digital filter design. In: 2010 IEEE Int. Conf. Intell. Syst. Knowl. Eng.. IEEE; 2010. p. 191–6. https://doi.org/10.1109/ISKE.2010.5680821.
[77] Wang Y, Li B, Chen Y. Digital IIR filter design using multi-objective optimization evolutionary algorithm. Appl Soft Comput 2011;11:1851–7. https://doi.org/10.1016/j.asoc.2010.05.034.
[78] Saha SK, Sarkar S, Kar R, Mandal D, Ghoshal SP. Digital stable IIR low pass filter optimization using particle swarm optimization with improved inertia weight. In: 2012 Ninth Int. Conf. Comput. Sci. Softw. Eng.. IEEE; 2012. p. 147–52. https://doi.org/10.1109/JCSSE.2012.6261942.
[79] Pan S-T. Evolutionary computation on programmable robust IIR filter pole-placement design. IEEE Trans Instrum Meas 2011;60:1469–79. https://doi.org/10.1109/TIM.2010.2086850.
[80] Saha SK, Kar R, Mandal D, Ghoshal SP. Digital stable IIR low pass filter optimization using PSO-CFIWA. In: 2012 1st Int. Conf. Recent Adv. Inf. Technol.. IEEE; 2012. p. 196–201. https://doi.org/10.1109/RAIT.2012.6194505.
[81] Saha SK, Kar R, Mandal D, Ghoshal SP. Optimal IIR filter design using novel particle swarm optimization technique. In: Int. J. Circuits, Syst. Signal Process.. p. 152–62.
[82] Saha S, Chaudhuri A, Mandal D, Kar R, Ghoshal SP. Optimization of IIR high pass filter using craziness based particle swarm optimization technique. In: Humanit. Sci. Eng. Res. (SHUSER), 2012 IEEE Symp.. p. 401–6. https://doi.org/10.1109/SHUSER.2012.6268873.
[83] Saha SK, Kar R, Mandal D, Ghoshal SP. An efficient craziness based particle swarm optimization technique for optimal IIR filter design. Trans. Comput. Sci. 2013;8160:230–52. https://doi.org/10.1007/978-3-642-45318-2_10.
[84] Saha SK, Kar R, Mandal D, Ghoshal SP. Gravitation search algorithm: application to the optimal IIR filter design. J King Saud Univ - Eng Sci 2014;26:69–81. https://doi.org/10.1016/j.jksues.2012.12.003.
[85] Singh R, Arya SK. Optimization of IIR digital filters using particle swarm optimization. In: 2012 Int. Conf. Commun. Inf. Comput. Technol.. IEEE; 2012. p. 1–7. https://doi.org/10.1109/ICCICT.2012.6398172.
[86] Li B, Wang Y, Weise T, Long L. Fixed-point digital IIR filter design using two-stage ensemble evolutionary algorithm. Appl Soft Comput 2013;13:329–38. https://doi.org/10.1016/j.asoc.2012.09.004.
[87] Poli R, Kennedy J, Blackwell T. Particle swarm optimization an overview. Swarm Intell 2007;1:33–57. https://doi.org/10.1007/s11721-007-0002-0.
[88] Man KF, Tang KS, Kwong S. Genetic algorithms: concepts and applications in engineering design. IEEE Trans Ind Electron 1996;43:519–34. https://doi.org/10.1109/41.538609.
[89] Yang X-S. In: Cuckoo search and firefly algorithm - theory and applications. Springer International Publishing; 2014. https://doi.org/10.1007/978-3-319-02141-6.
[90] Karaboga N, Cetinkaya B. Performance comparison of genetic and differential evolution algorithms for digital FIR filter design. In: Lect. NOTES Comput. Sci.. Berlin, Heidelberg: Springer; 2004. p. 482–8. https://doi.org/10.1007/978-3-540-30198-1_49.

[91] Karaboğa N, Çetinkaya B. Efficient design of fixed point digital FIR filters by using differential evolution algorithm. In: Lect. NOTES Comput. Sci.. Berlin, Heidelberg: Springer; 2005. p. 812–9. https://doi.org/10.1007/11494669_99.
[92] Karaboga N, Cetinkaya B. Design of digital FIR Filters using differential evolution algorithm. Circuits, Syst Signal Process 2006;25:649–60.
[93] Ghoshal SP, Saha SK, Kar R, Mandal D. Seeker optimisation algorithm: application to the design of linear phase finite impulse response filter. IET Signal Process 2012;6:763–71. https://doi.org/10.1049/iet-spr.2011.0353.
[94] Saha SK, Ghoshal SP, Kar R, Mandal D. Cat Swarm Optimization algorithm for optimal linear phase FIR filter design. ISA Trans 2013;52:781–94. https://doi.org/10.1016/j.isatra.2013.07.009.
[95] Saha SK, Kar R, Mandal D, Ghoshal SP. Design and simulation of FIR band pass and band stop filters using gravitational search algorithm. Memetic Comput 2013;5:311–21. https://doi.org/10.1007/s12293-013-0122-6.
[96] Aggarwal A, Rawat TK, Upadhyay DK. Design of optimal digital FIR filters using evolutionary and swarm optimization techniques. AEU - Int J Electron Commun 2016;70:373–85. https://doi.org/10.1016/j.aeue.2015.12.012.
[97] Sharma I, Kuldeep B, Kumar A, Singh VK. Performance of swarm based optimization techniques for designing digital FIR filter: a comparative study. Eng Sci Technol Int J 2016. https://doi.org/10.1016/j.jestch.2016.05.013.
[98] Baderia K, Kumar A, Kumar Singh G. Hybrid method for designing digital FIR filters based on fractional derivative constraints. ISA Trans 2015;58:493–508. https://doi.org/10.1016/j.isatra.2015.05.015.
[99] San-José-Revuelta LM, Arribas JI. A new approach for the design of digital frequency selective FIR filters using an FPA-based algorithm. Expert Syst Appl 2018;106:92–106. https://doi.org/10.1016/J.ESWA.2018.03.045.
[100] Kuldeep B, Kumar A, Singh GK. Design of quadrature mirror filter bank using Lagrange multiplier method based on fractional derivative constraints. Eng Sci Technol Int J 2015;18:235–43.
[101] Baderia K, Kumar A, Singh GK. Design of quadrature mirror filter bank using polyphase components based on optimal fractional derivative constraints. AEU - Int J Electron Commun 2015;69:1254–64.
[102] Kuldeep B, Singh VK, Kumar A, Singh GK. Design of two-channel filter bank using nature inspired optimization based fractional derivative constraints. ISA Trans 2015;54:101–16. https://doi.org/10.1016/j.isatra.2014.06.005.
[103] Agrawal SK, Sahu OP. Artificial bee colony algorithm to design two-channel quadrature mirror filter banks. Swarm Evol Comput 2015;21:24–31. https://doi.org/10.1016/j.swevo.2014.12.001.
[104] Reddy KS, Sahoo SK. An approach for FIR filter coefficient optimization using differential evolution algorithm. AEU - Int J Electron Commun 2015;69:101–8. https://doi.org/10.1016/j.aeue.2014.07.019.
[105] Sharma I, Kumar A, Singh GK. An efficient method for designing multiplier-less non-uniform filter bank based on hybrid method using CSE technique. Circuits, Syst Signal Process 2017;36:1169–91. https://doi.org/10.1007/s00034-016-0351-2.
[106] Dwivedi AK, Ghosh S, Londhe ND. Modified artificial bee colony optimisation based FIR filter design with experimental validation using field-programmable gate array. IET Signal Process 2016;10:955–64. https://doi.org/10.1049/iet-spr.2015.0214.
[107] Sharma I, Kumar A, Balyan L, Singh GKA. new hybrid CSE technique for multiplier-less FIR filter. In: 2017 22nd Int. Conf. Digit. Signal Process. IEEE; 2017. p. 1–5. https://doi.org/10.1109/ICDSP.2017.8096084.
[108] Dwivedi AK, Ghosh S, Londhe ND. Low-power FIR filter design using hybrid artificial bee colony algorithm with experimental validation over FPGA. Circuits, Syst Signal Process 2017;36:156–80. https://doi.org/10.1007/s00034-016-0297-4.
[109] Selesnick IW. Low-pass filters realizable as all-pass sums: design via a new flat delay filter. IEEE Trans Circuits Syst II Analog Digit Signal Process 1999;46:40–50. https://doi.org/10.1109/82.749080.
[110] Djebbari A, Rouvaen JM, Djebbari A, Belbachir MF, Elahmar SA. A new approach to the design of limit cycle-free IIR digital filters using eigenfilter method. Signal Process 1999;72:193–8. https://doi.org/10.1016/S0165-1684(98)00180-7.
[111] Chen L-W, Jou Y-D, Hao S-S. Design of two-channel quadrature mirror filter banks using minor component analysis algorithm. Circuits, Syst Signal Process 2015;34:1549–69. https://doi.org/10.1007/s00034-014-9914-2.
[112] Gotmare A, Bhattacharjee SS, Patidar R, George NV. Swarm and evolutionary computing algorithms for system identification and filter design: a comprehensive review. Swarm Evol Comput 2017;32:68–84. https://doi.org/10.1016/j.swevo.2016.06.007.
[113] Hong LA. A multi-modal immune optimization algorithm for IIR filter design. In: 2008 Int. Conf. Intell. Comput. Technol. Autom.. IEEE; 2008. p. 73–7. https://doi.org/10.1109/ICICTA.2008.187.
[114] Iwai K, Kajikawa Y. Modified second-order nonlinear infinite impulse response (IIR) filter for equalizing frequency response and compensating nonlinear distortions of electrodynamic loudspeaker. Appl Acoust 2018;132:202–9. https://doi.org/10.1016/j.apacoust.2017.11.014.
[115] Ho CY, Shyu KK, Chang CY, Kuo SM. Development of equation-error adaptive IIR-filter-based active noise control system. Appl Acoust 2020;163:107226. https://doi.org/10.1016/j.apacoust.2020.107226.
[116] Pepe G, Gabrielli L, Squartini S, Cattani L. Evolutionary tuning of filters coefficients for binaural audio equalization. Appl Acoust 2020;163:107204. https://doi.org/10.1016/j.apacoust.2019.107204.
[117] Kennedy J, Eberhart R. Particle swarm optimization. Neural Networks. In: 1995. Proceedings., IEEE Int. Conf.. p. 1942–8. https://doi.org/10.1109/ICNN.1995.488968.
[118] Kumar A, Ahirwal MK, Singh GK. Sub-band adaptive filtering method for electroencephalography/event related potential signal using nature inspired optimisation techniques. IET Sci Meas Technol 2015;9:987–97. https://doi.org/10.1049/iet-smt.2015.0048.
[119] Dash PK, Panigrahi BK, Hasan S. Hybrid particle swarm optimization and unscented filtering technique for estimation of non-stationary signal parameters. IETE J Res 2014.
[120] Ahirwal MK, Kumar A, Singh GK. Analysis and testing of PSO variants through application in EEG/ERP adaptive filtering approach. Biomed Eng Lett 2012;2:186–97. https://doi.org/10.1007/s13534-012-0071-x.
[121] Sharma I, Kumar A, Singh GK. Adjustable window based design of multiplier-less cosine modulated filter bank using swarm optimization algorithms. AEU - Int J Electron Commun 2016;70:85–94. https://doi.org/10.1016/j.aeue.2015.10.008.
[122] Sun J, Feng B, Xu W. Particle swam optimization with particles having quantum behavior. In: Proc. 2004 Congr. Evol. Comput.. IEEE; 2004. p. 325–31.
[123] Fang W, Sun J, Ding Y, Wu X, Xu W. A review of quantum-behaved particle swarm optimization. IETE Tech Rev 2014.
[124] Wei F, Jun S, Wenbo X. Design IIR digital filters using quantum-behaved particle swarm optimization. Adv Nat Comput 2006;4222:637–40. https://doi.org/10.1007/11881223.
[125] Dervis K, Bahriye B. A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. J. Glob. Optim. 2007;39:459–71. https://doi.org/10.1007/s10898-007-9149-x.
[126] Rafi SM, Kumar A, Singh GK. An improved particle swarm optimization method for multirate filter bank design. J Franklin Inst 2013;350:757–69.
[127] Rashedi E, Nezamabadi-pour H, Saryazdi SGSA. A gravitational search algorithm. Inf Sci (Ny) 2009;179:2232–48. https://doi.org/10.1016/j.ins.2009.03.004.
[128] Yang X-S, Deb S. Multiobjective cuckoo search for design optimization. Comput Oper Res 2013;40:1616–24. https://doi.org/10.1016/j.cor.2011.09.026.
[129] Yang X-S, Deb S. Engineering optimisation by Cuckoo search. Int J Math Model Numer Optim 2010;1:330–43 [math.OC].
[130] Mallick A, Dutta S, Roy S, Kumar U, Chattopadhyay A, Chakraborty D, et al. Cuckoo Search Optimization based design of linear phase FIR filters: a comparison approach. In: Green Comput. Commun. Electr. Eng. (ICGCCEE), 2014 Int. Conf.. p. 1–3. https://doi.org/10.1109/ICGCCEE.2014.6922313.
[131] Kumar M, Rawat TK. Optimal fractional delay-IIR filter design using cuckoo search algorithm. ISA Trans 2015;59:39–54. https://doi.org/10.1016/j.isatra.2015.08.007.
[132] Ahirwal MK, Kumar A, Singh GK. EEG/ERP adaptive noise canceller design with controlled search space (CSS) approach in Cuckoo and other Optimization Algorithms. IEEE/ACM Trans Comput Biol Bioinform 2013. https://doi.org/0767B928-2780-414C-BC49-070F8A683EB6.
[133] Kumar M, Rawat TK. Optimal design of FIR fractional order differentiator using cuckoo search algorithm. Expert Syst Appl 2015;42:3433–49. https://doi.org/10.1016/j.eswa.2014.12.020.
[134] Saha SK, Kar R, Mandal D, Ghoshal SP. Gravitational Search Algorithm with Wavelet Mutation applied for optimal IIR band pass filter design. In: Commun. Signal Process. (ICCSP), 2013 Int. Conf.. p. 14–8. https://doi.org/10.1109/iccsp.2013.6577005.