(·)+, 303 1-norm, 59, 66 2-norm, 59, 66, 225, 252, 255 best approximation, 256 ∞-norm, 59, 65, 225 best approximation, 228 ∼, 43 ≥, 99	derivative boundary condition, 367 eigenvalue problem, 373 error bound, 365, 368 finite element approximation, 391 global error, 364 Maximum Principle, 365, 369, 372 self-adjoint problem, 370 truncation error, 364, 368, 371 weak formulation, 390
A posteriori error analysis adaptivity, 410 dual problem, 405	weak solution, 390 Brouwer's Fixed Point Theorem, 4, 125
$A \setminus B$ , 64 Absolutely continuous function, 295 Adaptive finite element algorithm refinement criterion, 410	C[a, b], 225 $C^{k}[a, b], 293$ $\mathbb{C}^{n}, 62$ $\mathbb{C}^{n \times n}, 64, 145$
stopping criterion, 410 Agmon's inequality, 415 Asymptotic convergence, 16 convergence rate, 13, 21	$\mathbb{C}_*^n$ , 64 Céa's Lemma, 397 Cauchy sequence, 105
$B_{\varepsilon}(\boldsymbol{\xi})$ , 104 Backward Differentiation Formulae, 349	Cauchy-Schwarz inequality, 59, 254 Central difference, 362 Characteristic polynomial, 136, 137 Chebyshev polynomials, 241, 263
Band matrix, 98 Bauer–Fike Theorem, 173, 174 Bernoulli numbers, 214	Cholesky factorisation, 91 Closed ball, 63 Closed set, 105
Bernstein polynomials, 227 Bessel's inequality, 266 Best approximation in 2-norm, 256	Cofactor, 40 Comparison functions, 366, 372 Completeness, 105 Composite integration formulae,
in ∞-norm, 228 Bidiagonal matrix, 164 Bilinear functional, 388 symmetric, 388	209 Condition number, 58, 70 ill-conditioned matrix, 70 ill-conditioned problem, 68
Binet–Cauchy Theorem, 51 Bisection method, 28 Boundary value problems, 361	Consistent linear multistep method, 337 Consistent one-step method, 321
central difference approximation, 363	Continuous function, 106 Contraction, 6

Contraction Mapping Theorem, 7, 110	Fixed point
Convergence	definition, 4, 108
asymptotic, 16	simple iteration, 6
asymptotic rate, 13	simultaneous iteration, 108
linear, 12	stable, 12
of linear multistep method, 340	unstable, 12
of one-step method, 322	Frobenius norm, 141
quadratic, 16, 22, 119	
sublinear, 13	Galerkin approximation, 393
superlinear, 13	Galerkin basis functions, 393
Cramer's rule, 41	Galerkin finite element method, 394
Cubic splines, 298	Galerkin method, 392, 393
	Galerkin orthogonality, 397, 405
Dahlquist's Theorems	Galerkin principle, 389
Barrier Theorem, 340	Gauss quadrature, 277
Equivalence Theorem, 340	composite, 285
Second Barrier Theorem, 348	convergence, 283
de la Vallée Poussin's Theorem, 232	error estimate, 282
$\det(A)$ , 40	quadrature points, 279
Determinant, 40	quadrature weights, 279
Diagonal dominance, 96, 117, 367,	Gaussian elimination, 44
371	pivoting, 52
Differential equations	Gerschgorin discs, 145
boundary value problems, 361, 385	Gerschgorin similarity transformation,
initial value problems, 310	149
	Gerschgorin theorems, 145
Eigenfunctions, 373	Global convergence, 29
Eigenvalues, 133, 373	Newton's method, 31, 123
characteristic polynomial, 136, 137	Global error
definition, 66	boundary value problem, 364
Jacobi's method, 137, 149	Euler's method, 318
QR algorithm, 162	initial value problem, 317
Rayleigh quotient, 170	Gram-Schmidt orthogonalisation, 261
tridiagonal matrix, 156	
Eigenvectors, 136	$H^{k+1}(a,b), 296$
definition, 66	Hat function, 297, 394
inverse iteration, 166	Hermite cubic spline, 300
Jacobi's method, 144	Hermite interpolation, 187, 277
orthogonal, 136	error, 190
Energy norm, 399	Hilbert matrix, 72, 259
Euler's method, 317, 323	Hölder's inequality, 61
global error, 318	Householder matrix, 150
truncation error, 318	Householder reflector, 151
Euler–Maclaurin formula, 211	Householder's method, 155
Finite element method, 385	Implicit methods
a posteriori error analysis, 402	linear multistep methods, 330
a priori error analysis, 397	one-step methods, 324
adaptive algorithm, 409	Runge–Kutta methods, 351
basis functions, 394	Improved Euler method, 328
Galerkin method, 394	Infinity norm, 59, 65, 225
Galerkin principle, 386	best approximation, 228
interpolant, 399	Initial value problems, 310
Rayleigh–Ritz principle, 386	linear multistep methods, 329
residual, 406	one-step methods, 317
Ritz method, 394	Inner product, 252, 388
subdivision, 394	inner product space, 253
Subdivision, our	miler product space, 200

orthogonality, 253	characteristic polynomials, 332
weight function, 255	consistency, 337
Integral Mean Value Theorem, 421	error constant, 338
Integration, 200	explicit, 330
composite Simpson's rule, 210	implicit, 330
composite trapezium rule, 209	order of accuracy, 338
Euler-Maclaurin formula, 211	region of absolute stability, 348
Gauss quadrature, 277	Root Condition, 332, 335
Lobatto rule, 287	Simpson's rule method, 330
midpoint rule, 286	truncation error, 337
Newton–Cotes quadrature, 201	zero-stability, 331
quadrature points, 202, 279	Lipschitz condition, 7, 11, 109, 318
quadrature weights, 202	Lipschitz constant, 109
Radau rule, 287	$\ln = \log_e, 5, 315$
Richardson extrapolation, 216	Lobatto quadrature, 287
Romberg integration, 217	Logistic equation, 30
Simpson's rule, 203	Lower triangular matrix, 46
trapezium rule, 202	inverse, 47
Interchanges, 52	LU factorisation
interchange matrix, 53	existence, 50
Interlace Theorem, 157	of matrix, 48
Intermediate Value Theorem, 419	with pivoting, 53
Interpolation, 179, 244, 292	
at Chebyshev points, 244	M-matrix, 101
cubic spline, 298	Mass matrix, 396
Hermite, 187, 292	Matrix
Lagrange, 180, 292	band, 98
linear spline, 293	bidiagonal, 164
Interpolation points, 182	condition number, 58, 70
Inverse	diagonally dominant, 96
of a lower triangular matrix, 47	Hilbert, 72
of a matrix, 40	lower triangular, 46
Inverse iteration, 166	M-matrix, 101
Iteration, simple, 2	monotone, 99
Jacobi's method 140	orthogonal, 138
Jacobi's method, 149	permutation, 53
classical, 140	positive definite, 87, 88, 97
convergence, 142	principal submatrix, 50
eigenvalues, 137	strictly diagonally dominant, 96
eigenvectors, 144 serial, 143	symmetric, 87 tridiagonal, see Tridiagonal
Jacobian matrix, 113, 346	matrix
Jacobian maurix, 119, 940	unit lower triangular, 46
Kronecker delta, 267	upper triangular, 47
Thomseller derica, 201	Matrix factorisation
$L_w^2(a,b), 225$	Cholesky, 90
Lagrange interpolation, 180, 201	LU, 48
error, 183	QR, 76, 78, 163
Laguerre polynomials, 290	Matrix norm, 58
Least squares solution of linear	1-norm, 66
equations, 74	2-norm, 66
Lebesgue integral, 256	∞-norm, 65
Legendre polynomials, 263	Frobenius norm, 141
Linear convergence, 12	subordinate norm, 64
Linear multistep methods, 329	Maximum norm, see Infinity norm
A-stable, 348	Maximum Principle, 365, 369, 372
absolutely stable, 348	comparison function, 366, 372

Mean Value Theorem, 8, 10, 11, 26, 113, Orthogonal matrix, 138 Orthogonal polynomials, 259, 260, 277 Midpoint rule, 286 construction, 260 Minimax approximation, 230 zeros, 269, 279 Minkowski's inequality, 62 Orthogonal transformation Modified Euler method, 328 eigenvalues, 137 Monic polynomial, 243 invariance of sum of squares, 140 Monotone matrix, 99 plane rotation, 138 tridiagonal, 100 Orthonormal polynomials, 265 Moore—Penrose generalised inverse, 70, Oscillation Theorem, 232, 233, 243 critical point, 233  $P_n, 180$ Natural cubic spline, 298 Near-minimax polynomial, 245, 270 Permutation matrices, 53 Neighbourhood, 105 Piecewise polynomials, 292 Newton's method, 19, 21, 116 Pivoting, 52, 92, 95 convergence, 23, 116 Plane rotations, 138, 163 global behaviour, 31, 123 Poincaré-Friedrichs inequality, 414 simultaneous equations, 118 Positive definite matrix, 87, 97 Newton-Cotes quadrature, 201 properties, 88 convergence, 208 Principal submatrix, 50 error estimate, 204 Simpson's rule, 203 QR algorithm, 162 trapezium rule, 202 shift, 164 Norm, 58, 224 QR factorisation, 76, 78, 163 1-norm, 59, 66 Quadratic convergence, 16, 22, 119 2-norm, 59, 66, 225, 252, 255 Quadrature, see Integration  $\infty$ -norm, 59, 65, 225  $\mathbb{R}^{m \times n}$ ,  $\mathbb{R}^{n \times n}$ , 40 energy norm, 399  $\mathbb{R}_{\mathrm{sym}}^{n \times n}$ , 87 Frobenius norm, 141  $\mathbb{R}^{n}_{\star}$ , 64 induced norm, 254 normed linear space, 58, 224 Radau quadrature, 287 p-norm, 60 Rayleigh quotient, 170 Sobolev norm, 387 Rayleigh-Ritz principle, 388 vector and matrix norm, 58 Relaxation, 19 Normal equations, 76 convergence, 20, 117 simultaneous equations, 116 One-step methods, 317 Richardson extrapolation, 216 consistent, 321 Ritz approximation, 393 convergence, 322 Ritz method, 392, 393 Euler's method, 317, 323 Ritz projector, 398 Rolle's Theorem, 184, 191, 419 general form, 317 Romberg integration, 217 implicit methods, 324 improved Euler method, 328 Row operations, 46 modified Euler method, 328 Runge phenomenon, 208 order of accuracy, 323 Runge-Kutta methods, 323, 325 Runge-Kutta methods, 323, 325 algebraically stable, 354 trapezium rule method, 324 Butcher tableau, 352 truncation error, 317 classical fourth order, 328 Open ball, 63 diagonally implicit (DIRK), 353 Open set, 104 implicit, 349 Operation count, 92 improved Euler method, 328 Order of accuracy modified Euler method, 328 linear multistep methods, 338 one-step methods, 323 Secant method, 25

convergence, 26

Self-adjoint problem, 370

Orthogonal, inner product space, 252

Orthogonal eigenvectors, 136

G	G 1 11 1 1 20 4
Set of measure zero, 295	Subdivision, 394
Shift, QR algorithm, 165	Sublinear convergence, 13
Simple iteration, 2	Subordinate matrix norm, 64
convergence, 11	Superlinear convergence, 13
divergence, 15	Support, 394
global behaviour, 29	Symmetric bilinear functional, 388
Simpson's rule, 203	Symmetric matrix, 87
composite, 210	
error estimate, 205	Taylor's Theorem, 420
Simultaneous iteration, 106	several variables, 422
convergence, 110, 113	with integral remainder, 420
Simultaneous nonlinear equations, 104	Thomas algorithm, 95, 363
Newton's method, 118	Trace, 136
Simultaneous relaxation, 116	Trapezium rule, 202
Singular value	composite, 209
decomposition, 82	Tridiagonal matrix, 93, 363, 367, 371,
definition, 67	373, 395
Sobolev norm, 387	eigenvalues, 156
Sobolev space, 296, 387	factorisation, 94
Solution of linear equations, 44, 55	monotone, 100
computational work, 56	reduction of real symmetric matrix,
least squares, 74	150
sensitivity, 71	Truncation error
Spline, 292, 394	Euler's method, 318
cubic, 298	linear multistep method, 337
end conditions, 298	one-step method, 317
Hermite cubic, 300	one step method, or
error bound, 301	Unit lower triangular matrix, 46
interpolating cubic, 298	Upper triangular matrix, 47
knot, 292	oppor triangular matrix, 17
linear, 293	Variational problem, 385
basis functions, 297	Vector norm, 58
error bound, 293	1-norm, 59
optimum property, 296	2-norm, 59
natural cubic, 298	∞-norm, 59
construction, 299	<i>p</i> -norm, 60
end conditions, 298	p-norm, oo
optimum property, 300	Weak formulation, 390
Stability polynomial, 347	Weak solution, 390
Stable fixed point, 12	Weierstrass Theorem, 227, 283
	the state of the s
Stiff linear ODE system, 345	Weight function, 255, 260, 277
Stiffness matrix, 396	Voung's inequality 61
Strictly diagonally dominant matrix, 96	Young's inequality, 61
	Zoro stability 331
Sturm Liquille problem 373	Zero-stability, 331
Sturm-Liouville problem, 373	Root Condition, 335