**THE ACADEMY OF SCIENCES OF TURKMENISTAN**
**OGUZ HAN ENGINEERING AND TECHNOLOGY UNIVERSITY OF TURKMENISTAN**

**Computer Sciences and Information Technologies Faculty**

**Applied Mathematics and Informatics department**

# Integral transforms and solving partial differential equations with them

# GRADUATION THESIS

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# CONTENT

## Abstract

Integral transforms have been a cornerstone in the realm of mathematical analysis, providing powerful tools for solving complex problems, particularly in the context of partial differential equations (PDEs). This diploma work delves into the theoretical foundations and practical applications of integral transforms, with a focus on their efficacy in tackling PDEs across diverse scientific and engineering domains.

The introductory section outlines the historical evolution of integral transforms and establishes the significance of their role in contemporary mathematical research. A comprehensive literature review surveys existing knowledge, emphasizing recent developments and identifying gaps in understanding. The theoretical foundations section elucidates the fundamental principles of integral transforms, including Fourier, Laplace, and Z-transforms, elucidating their properties and applicability in solving PDEs.

The subsequent section explores real-world applications, illustrating how integral transforms serve as indispensable tools in fields such as physics, engineering, and biology. Numerical methods and computational tools are investigated to enhance our understanding of implementing integral transforms in solving PDEs, with a specific focus on practical examples using platforms like MATLAB and Python.

Case studies showcase the versatility of integral transforms in solving a range of problems, emphasizing their adaptability and effectiveness in different scenarios. The work also identifies and discusses challenges and limitations associated with current methodologies, proposing avenues for future research and improvements.

The culmination of this diploma work is a synthesized conclusion that summarizes key findings, contributions to the field, and potential implications. The

comprehensive reference list provides readers with a gateway to further exploration of the subject matter.

## INTRODUCTION

**Serdar Berdimuhamedov**

**- In the height of scientific and technological development-in the 21st century, we are leading Turkmenistan on the path of sustainable development by relying on science, creating high-tech industries and further raising the level of education of our nation.**

Differential Calculus is centred on the concept of the derivative. The original motivation for the derivative was the problem of defining tangent lines to the graphs of functions and calculating the slope of such lines. Integral Calculus is motivated by the problem of defining and calculating the area of the region bounded by the graph of the functions.

If a function f is differentiable in an interval I, i.e., its derivative f ′ exists at each point of I, then a natural question arises that given f ′ at each point of I, can we determine the function? The functions that could possibly have given function as a derivative are called anti derivatives (or primitive) of the function. Further, the formula that gives all these anti derivatives is called the indefinite integral of the function and such process of finding anti derivatives is called integration. Such type of problems arise in many practical situations. For instance, if we know the instantaneous velocity of an object at any instant, then there arises a natural question, i.e., can we determine the position of the object at any instant? There are several such practical and theoretical situations where the process of integration is involved. The development of integral calculus arises out of the efforts of solving the problems of the following types:

(a) the problem of finding a function whenever its derivative is given,

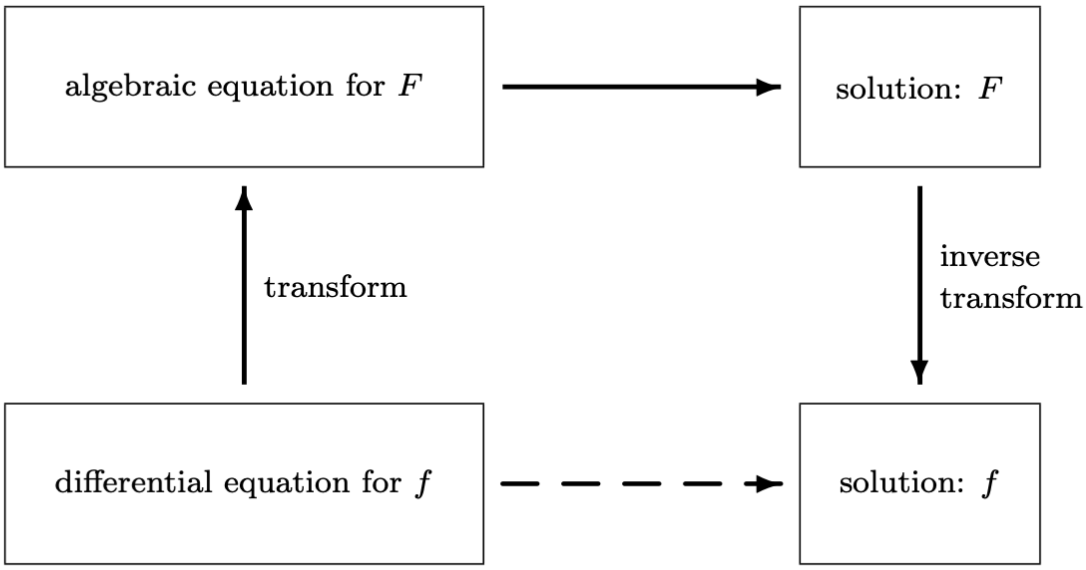
(b) the problem of finding the area bounded by the graph of a function under certain conditions.

These two problems lead to the two forms of the integrals, e.g., indefinite and definite integrals, which together constitute the **Integral Calculus.**

There is a connection, known as the **Fundamental Theorem of Calculus**, between indefinite integral and definite integral which makes the definite integral as a practical tool for science and engineering. The definite integral is also used to solve many interesting problems from various disciplines like economics, finance and probability.

Integral Transforms

This part of the course introduces two extremely powerful methods to solving differential equations: the Fourier and the Laplace transforms. Beside its practical use, the Fourier transform is also of fundamental importance in quantum mechanics, providing the correspondence between the position and momentum representations of the Heisenberg commutation relations. An integral transform is useful if it allows one to turn a complicated problem into a simpler one. The transforms we will be studying in this part of the course are mostly useful to solve differential and, to a lesser extent, integral equations. The idea behind a transform is very simple. To be definite suppose that we want to solve a differential equation, with unknown function f. One first applies the transform to the differential equation to turn it into an equation one can solve easily: often an algebraic equation for the transform F of f. One then solves this equation for F and finally applies the inverse transform to find f. This circle (or




square!) of ideas can be represented diagrammatically as follows:

We would like to follow the dashed line, but this is often very difficult. Therefore we follow the solid line instead: it may seem a longer path, but it has the advantage of being straightforward. After all, what is the purpose of developing formalism if not to reduce the solution of complicated problems to a set of simple rules which even a machine could follow?

We will start by reviewing Fourier series in the context of one particular example: the vibrating string. This will have the added benefit of introducing the method of separation of variables in order to solve partial differential equations. In the limit as the vibrating string becomes infinitely long, the Fourier series naturally gives rise to the Fourier integral transform, which we will apply to find steady-state solutions to differential equations. In particular we will apply this to the one-dimensional wave equation. In order to deal with transient solutions of differential equations, we will introduce the Laplace transform. This will then be applied, among other problems, to the solution of initial value problems.

## 1.1 Introduce the importance of integral transforms and their applications in solving partial differential equations.

The historical development of integral transforms is a fascinating journey that spans several centuries and involves contributions from many mathematicians. Integral transforms emerged as powerful mathematical tools that enable the transformation of problems from one domain to another, simplifying complex mathematical operations and facilitating the solution of various types of equations. Here is a brief overview of the historical development of integral transforms:

Fourier Transform (19th Century):

The development of the Fourier transform is often attributed to the French mathematician Joseph Fourier in the early 19th century.

Fourier introduced the idea that any periodic function could be represented as an infinite sum of sine and cosine functions, now known as a Fourier series.

The Fourier transform extends this concept to non-periodic functions, expressing a function as a sum (or integral) of sine and cosine functions over all frequencies.

Laplace Transform (19th Century):

Pierre-Simon Laplace, a French mathematician and astronomer, made significant contributions to integral transforms in the late 18th and early 19th centuries.

Laplace developed what is now known as the Laplace transform, a powerful tool for solving linear differential equations.

The Laplace transform converts a function of time into a function of a complex variable, making it particularly useful in solving linear time-invariant systems.

Hankel Transform (19th Century):

The Hankel transform is named after the Finnish mathematician Hermann Hankel, who made contributions to integral transforms in the mid-19th century.

The Hankel transform is an integral transform that generalizes the Fourier transform and is used in problems with cylindrical symmetry.

Mellin Transform (19th Century):

The Mellin transform is named after the Swedish mathematician Gustav Mellin and was introduced in the 19th century.
This transform is particularly useful in problems involving special functions and is an integral transform that generalizes the Laplace transform.

Z-Transform (20th Century):

The Z-transform, an extension of the Laplace transform to discrete-time signals, was developed in the early to mid-20th century.
Electrical engineers and control theorists extensively use the Z-transform for the analysis and design of discrete-time systems.

Wavelet Transform (20th Century):

The wavelet transform is a relatively more recent addition to the family of integral transforms, with roots in the late 20th century.
It has found applications in signal processing, data compression, and image analysis.
The historical development of integral transforms reflects a continuous evolution driven by the need to solve complex mathematical problems arising in various scientific and engineering disciplines. These transforms have become indispensable tools in the modern mathematical toolkit, playing a crucial role in diverse fields such as signal processing, control theory, and image analysis.

## 1.3 State the objectives and significance of the diploma work.

Explore Theoretical Foundations:
Investigate and elucidate the fundamental principles of integral transforms, including Fourier, Laplace, Z-transforms, and others.
Provide a comprehensive understanding of the mathematical underpinnings of integral transforms and their applications in solving partial differential equations (PDEs).

Examine Practical Applications:

Investigate real-world applications of integral transforms across various scientific and engineering disciplines.

Illustrate how integral transforms serve as powerful tools for solving complex problems and gaining insights into physical phenomena.

Evaluate Numerical Methods:

Explore numerical methods used in conjunction with integral transforms for solving PDEs. Discuss the implementation of computational tools, such as MATLAB and Python, to enhance the efficiency and accuracy of solutions.

Present Case Studies:

Provide in-depth case studies demonstrating the application of integral transforms to solve specific types of PDEs.

Showcase the versatility and effectiveness of integral transforms through practical examples.

Identify Challenges and Future Directions:

Identify challenges and limitations associated with existing approaches to integral transforms and solving PDEs.

Propose potential improvements and suggest avenues for future research in the field.

Significance of the Diploma Work:

Contributions to Mathematical Knowledge:

Contribute to the existing body of knowledge by providing a comprehensive and up-to-date exploration of integral transforms and their applications in solving PDEs.

Practical Implications:

Demonstrate the practical significance of integral transforms in addressing real-world problems across diverse disciplines.

Highlight the impact of integral transforms in fields such as physics, engineering, and biology.

Educational Value:

Serve as an educational resource for students, researchers, and practitioners interested in gaining a deeper understanding of integral transforms and their applications.

Guidance for Future Research:

Provide insights into current challenges and limitations, offering guidance for researchers interested in advancing the field of integral transforms and PDE solutions.

Interdisciplinary Applications:

Emphasize the interdisciplinary nature of integral transforms, showcasing their applicability in various scientific and engineering domains.

Facilitate Practical Implementation:

Facilitate the practical implementation of integral transforms by exploring numerical methods and computational tools, making the knowledge accessible to those engaged in applied research and engineering practices.

Enhance Problem-Solving Abilities:

Equip readers with enhanced problem-solving skills by presenting a range of case studies that demonstrate the versatility and adaptability of integral transforms in addressing complex challenges.

In summary, the diploma work aims to deepen the understanding of integral transforms, bridge theoretical knowledge with practical applications, and contribute valuable insights to the ongoing exploration of mathematical tools in the solution of partial differential equations.

## CONCLUSION

The main goal of these reforms is to make the education of the Turkmen youth in line with the most advanced requirements of the world, ‖he said. favorable conditions and opportunities are created for the emergence of young talents. As a result of the implementation of this program, the Fatherland will turn from an agrarian country into an industrially developed state, the well-being of our native people will improve, which is our key task, the Respected President said.

President has paid special attention to the development of science and education in the country since its first day in office. During the Prosperity of our sovereign state, ―We have now embarked on fundamental reforms in the national education system in Turkmenistan. The strategic goals are also the digital economy, digital medicine and digital education, the head of Turkmenistan noted, stressing that work has already begun on the transition of the state to a digital system.

In the prosperous period of the sovereign state, due to the unparalleled efforts of our esteemed President, great changes are being made in the field of science and education, new achievements are being made. directions are considered. Economic development of our country through the development of national education, science, the introduction of the latest achievements of world science in computer technology has become one of the main issues.

My purpose in writing this program is: At present, the state of Turkmenistan is in the process of 3D demos of geometric concepts, which was created in the English language to increase your knowledge at any time, anywhere. A program that teaches geometric concepts and network security in topics are briefly explained, formulas, theories.

## RECOMMENDATION

The transition of sectors of the economy to the digital system is mainly carried out in two areas. The first task is to establish a digital document management system, which includes the interaction of the internal structures of the state (government and government), the relationship with the citizen (government with the customer), business interaction with the government (business with the government). The personal window of each citizen and every businessman is created. The second area involves the integration of digital, non-governmental institutions, enterprises and others into the digital system, which facilitates the creation of new competitive tools in market relations.

Broad-minded reforms aimed at further improving the education system in our country The unbelievable is transferred to life. In recent years, the latest computer technology and multimedia systems have been introduced. Establish a wide network of modern educational schools, research centers eternal importance is attached to it. Every year, new kindergartens and secondary schools are opened.

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As our country pursues a digital economic policy and switches to egovernance, the work of all institutions is programmed. I encourage school pupils and students to take the easy way out of learning and implement this project, including the project's priorities.

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