Digital Image Processing

Digital Halftoning

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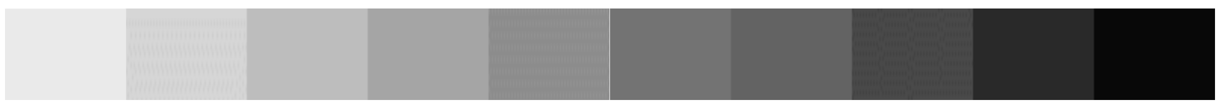
Lecture 06

Digital Halftoning

- Goal
  - Render the illusion of a continuous-tone image based on two-tone (half-tone) display



  - Applications
    - Computer hardcopies
      - Laser printers/dot-matrix printers/color printers
      - Fax machine
  - Implementation
    - Thresholding at 1/2 ?

Digital Halftoning

Gray-level image

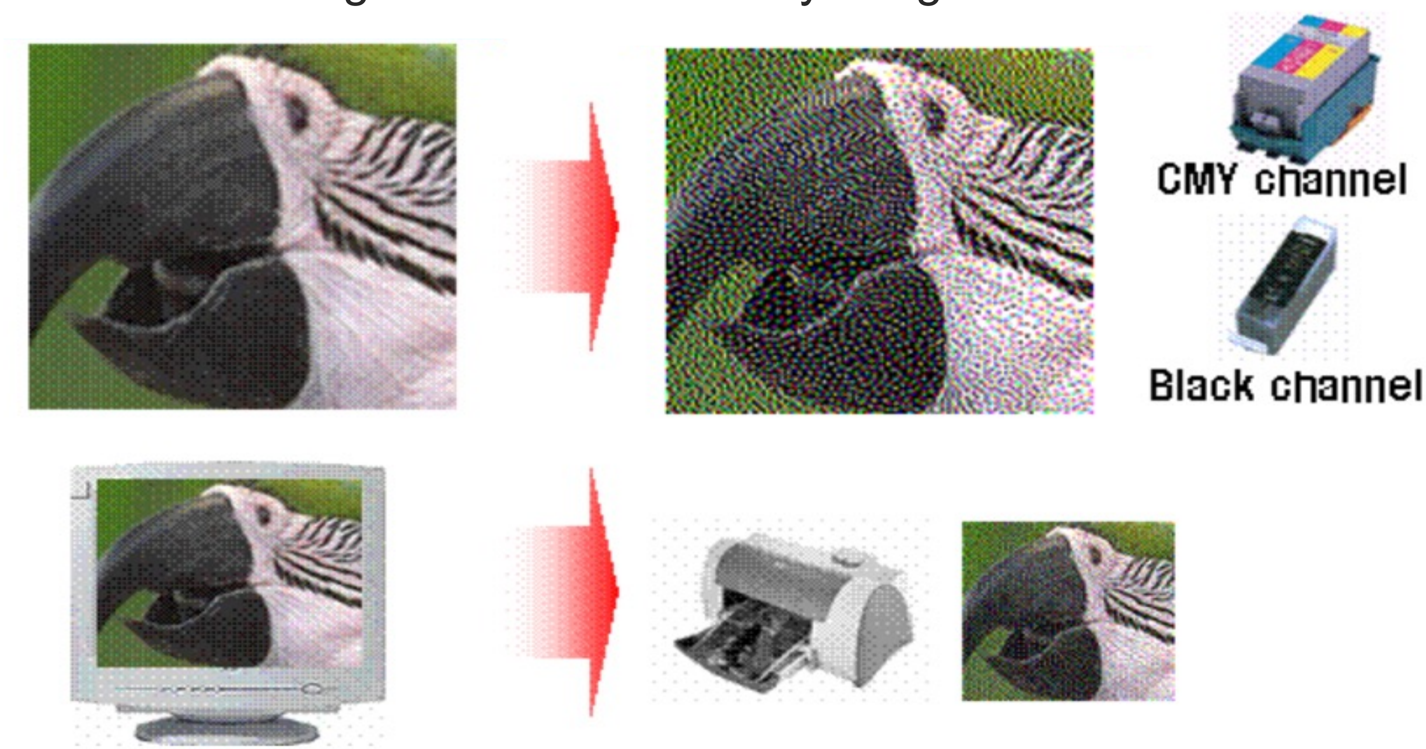
Half-toned images

Digital Halftoning

- **Color Printer**

Continuous Image

Binary Image



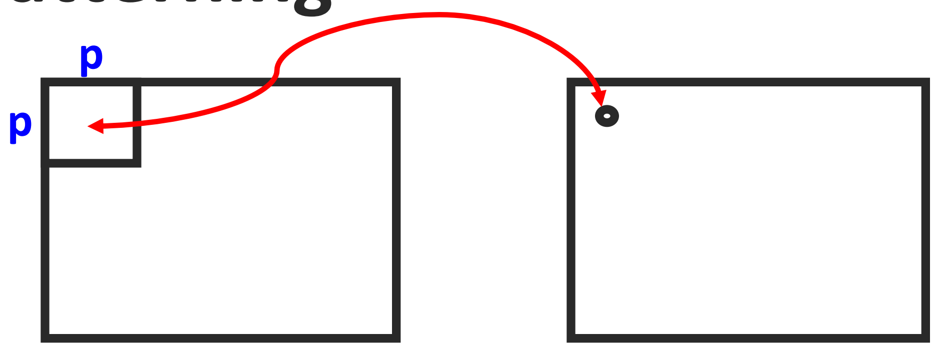
http://cilab.knu.ac.kr/English/research/Color/printer.htm

Digital Halftoning

- Basic idea
  - Spatial modulation
    - Gray-level ⟷ black/white
    - Darker area ⟷ denser black points per area
    - Whiter area ⟷ sparser black points per area
  - Three approaches
    - Patterning
    - Dithering
    - Error Diffusion

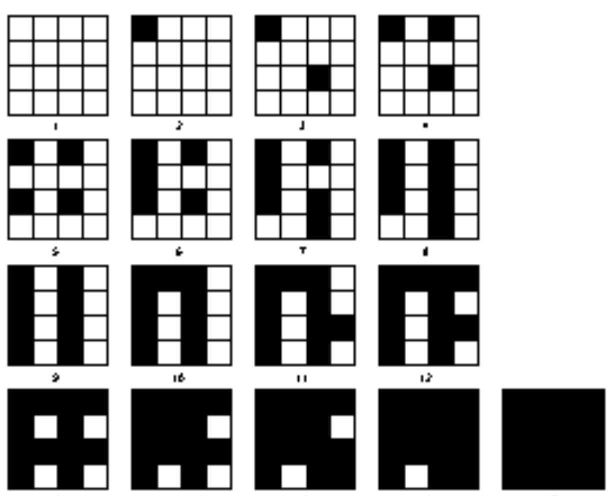
Digital Halftoning

- **Patterning**



1 Dot pattern ⟷ 1 Gray-level pixel

If p=4
→ 16 binary pixels
→ 17 levels (0~16)
→ 256 gray levels
→ Quantization



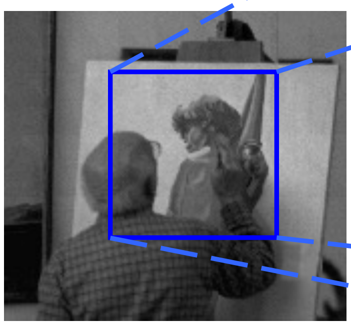
Rylander's recursive patterning matrices

Digital Halftoning

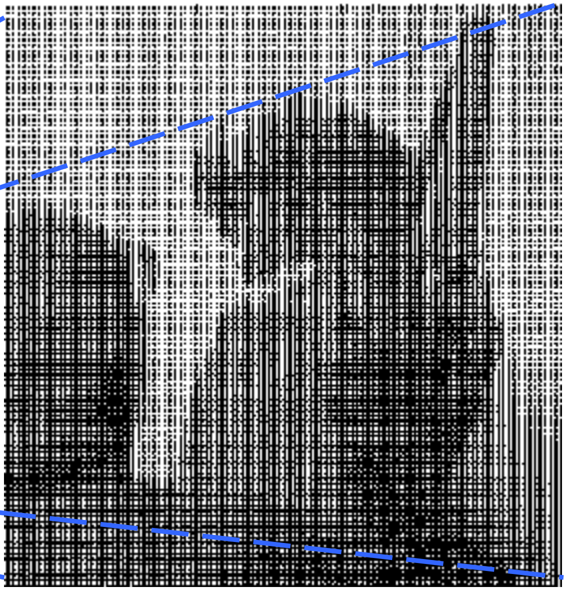
- Patterning
  - Four steps
    - Read in the given grey-level image
    - Quantization
    - Design the patterning table
    - Map each pixel to its corresponding pattern
  - Simplest way
  - Generates image with higher spatial resolution than the source image

Digital Halftoning

- **Patterning**
  - **Example**



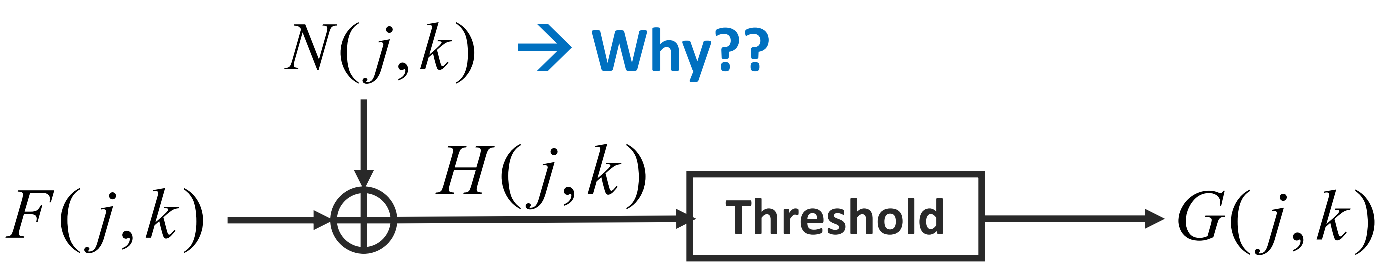
Original gray-level image



Half-toned image: patterning

Digital Halftoning

- **Dithering**
  - **Create an image with the same number of dots as the number of pixels in the source image**
  - **Idea**



Digital Halftoning

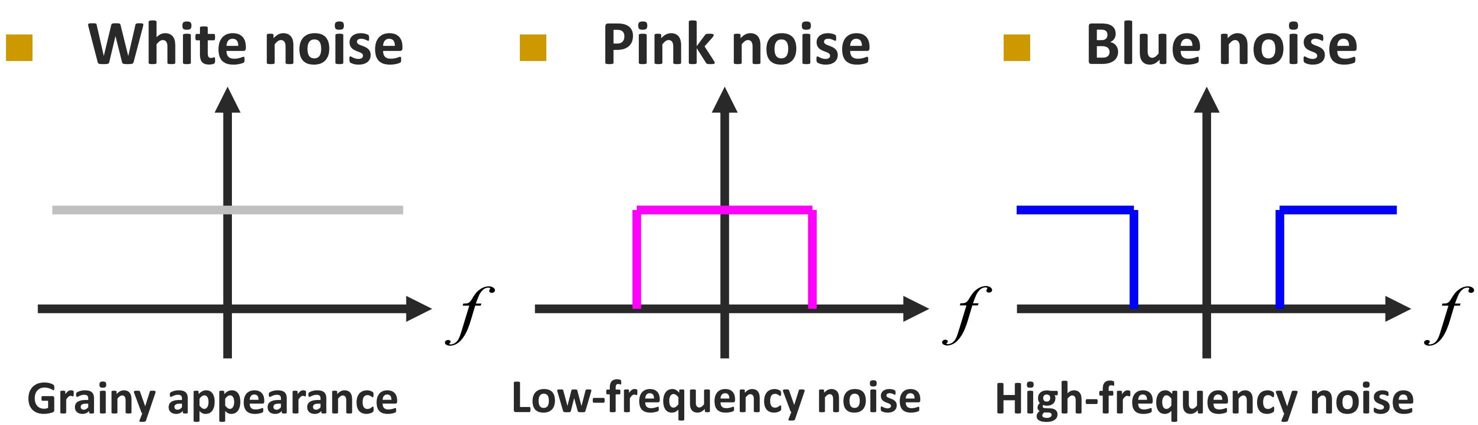
- Dithering
  - Why adding noise?
    - Under fixed thresholding → taking MSB
      - E.g. before and after adding noise

$\boxed{\geq 128}$ ⇨ **1** ; $\boxed{<127}$ ⇨ **0**

    - To break the monotonicity of accumulated error in the area of constant (nearly constant) gray level
    - Noise type
      - White noise, pink noise, blue noise and green noise

Digital Halftoning

- Dithering
  - Noise Type
    - Power spectral density



  - Robert Ulichney, "Digital Halftoning"
    - http://www.hpl.hp.com/people/u/

Digital Halftoning

- Dithering
  - Adaptive thresholding
    - Generate a threshold matrix according to a dither matrix
    - Whenever the pixel value of the image is greater than the value in the threshold matrix, the pixel is turned on
  - Notes
    - No randomness
    - Region-to-region mapping
    - Recursive definition allowed

Digital Halftoning

- **Dithering**
  - **Dither matrix**

$$I_2(i,j)=\begin{bmatrix}1 & 2\\ 3 & 0\end{bmatrix}; \quad I_2(i,j)=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$$

    - **0 → lowest threshold**
    - **3 → highest threshold**

Digital Halftoning

- **Dithering**
  - **The general form of the NxN dither matrix**
    - **2x2 → 4x4 → 8x8 → 16x16…**

$$I_{2n}(i,j) = \begin{bmatrix} 4I_n(i,j)+1 & 4I_n(i,j)+2 \\ 4I_n(i,j)+3 & 4I_n(i,j)+0 \end{bmatrix}$$

    - **Eg. What is** $I_4(i,j)$ **if** $I_2(i,j) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$**?**

Digital Halftoning

- **Dithering**
  - **Determine the threshold matrix**

$$T(i,j) = 255 \cdot \frac{I(i,j)+0.5}{N^2}$$

- **Eg. N=4**

$$I_4(i,j) = \begin{bmatrix} 5 & 9 & 6 & 10 \\ 13 & 1 & 14 & 2 \\ 7 & 11 & 4 & 8 \\ 15 & 3 & 12 & 0 \end{bmatrix}, \qquad T_4(i,j) = ?$$

Digital Halftoning

- **Dithering**

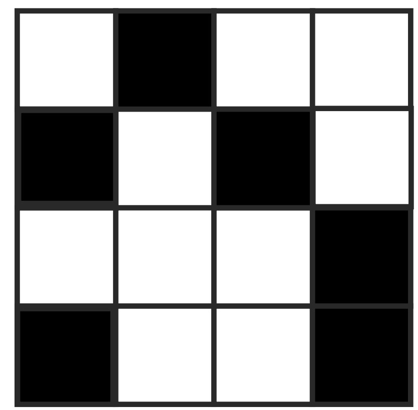
Input image

12	51	34	121
78	254	10	97
45	113	110	16
90	200	206	34

Repeated threshold matrix

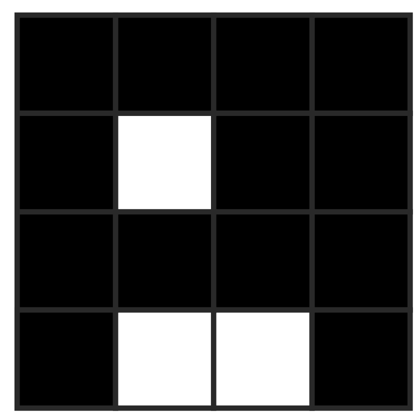
0	60	0	60
110	110	110	45
0	60	0	60
110	45	110	45

Output image



Another repeated threshold matrix

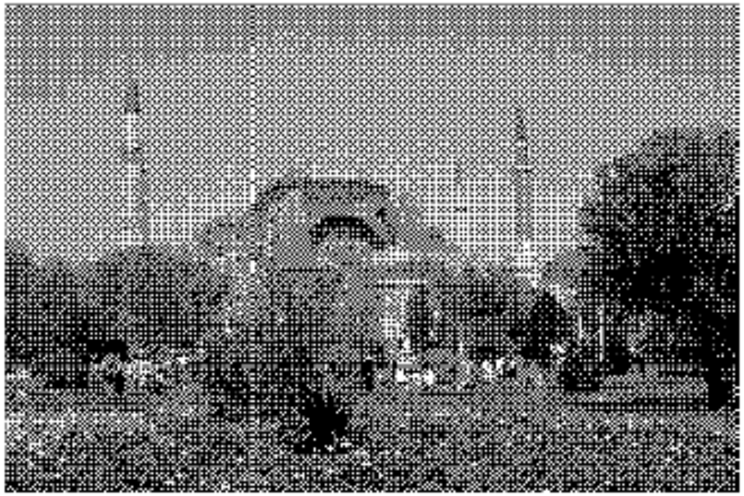
128	128	128	128
128	128	128	128
128	128	128	128
128	128	128	128



Digital Halftoning

- **Experimental results**

Original Image



Dithering

Digital Halftoning

- **Experimental results**

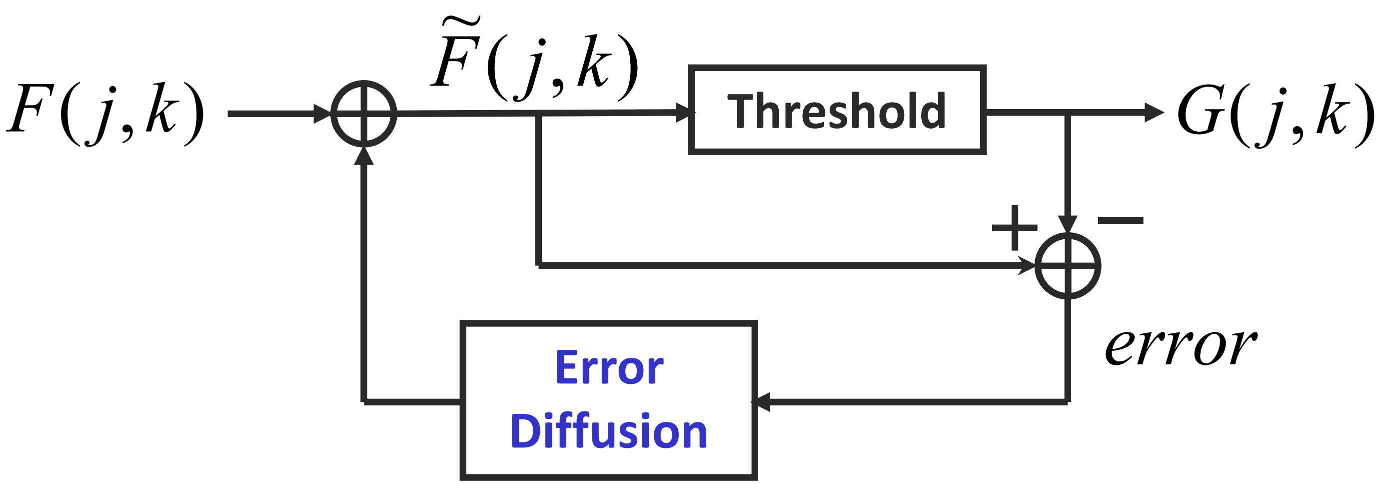
Original Image

Dithering

http://www.wikiwand.com/de/Dithering_(Bildbearbeitung)

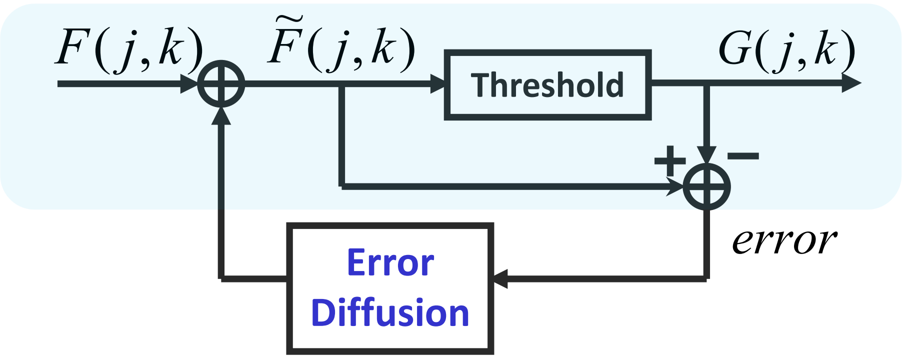
Digital Halftoning

- Error diffusion
  - 1975 Floyd & Steinberg
    - A practical algorithm to implement blue noise dithering
    - Framework



Digital Halftoning

- Error diffusion
  - Normalize *F(j,k)* to lie between [0,1]
  - Set threshold=0.5
  - Output image: 0 or 1

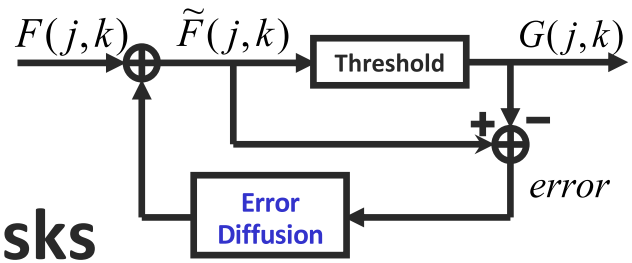


$$\text{if} \quad \widetilde{F}(j,k) \geq 0.5 \quad \rightarrow \quad G(j,k) = 1$$

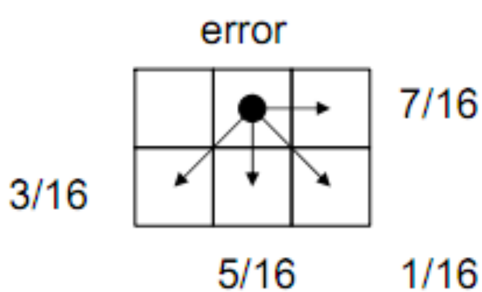
$$\text{if} \quad \widetilde{F}(j,k) < 0.5 \quad \rightarrow \quad G(j,k) = 0$$

$$\text{Define} \quad E(j,k) = \widetilde{F}(j,k) - G(j,k)$$

Digital Halftoning



- Error diffusion
  - Error diffusion filter masks
    - 1975 Floyd Steinberg: 1976 Jarvis et al:



error

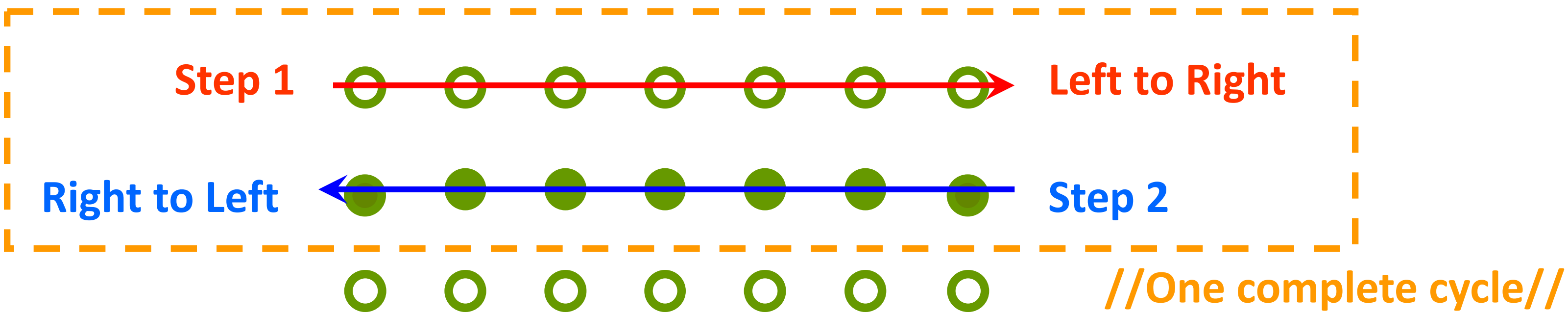
		●	7	5
3	5	7	5	3
1	3	5	3	1

(1/48)

- Other diffusion matrices
  - http://www.tannerhelland.com/4660/dithering-eleven-algorithms-source-code/

Digital Halftoning

- Error diffusion
  - Error diffusion + serpentine scanning



$$\frac{1}{16}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7 \\ 3 & 5 & 1 \end{pmatrix} \qquad \frac{1}{16}\begin{pmatrix} 0 & 0 & 0 \\ 7 & 0 & 0 \\ 1 & 5 & 3 \end{pmatrix}$$

Left to Right　　Right to Left

Digital Halftoning

- **Experimental results**

Original Image

Error Diffusion

Digital Halftoning

- **Experimental results**

Original Image

Floyd-Steinberg

Jarvis

http://www.wikiwand.com/de/Dithering_(Bildbearbeitung)

Digital Halftoning

- Multi-scale Error diffusion
  - Several issues
    - Region-to-region mapping
      - Multi-resolution
    - Time series/causal error diffusion process
      - Easy to implement
      - Causality appears to be artificial in images
      - Is non-causal error diffusion possible?
    - Quality metrics of half-toned images

Digital Halftoning

- ## Multi-scale Error diffusion

  “A multiscale error diffusion technique for digital halftoning”

  Ioannis Katsavounidis and C. –C. Jay Kuo

  - ### Problem set-up
    - **Input image** → $X(i,j) \in [0,1]$
    - **Output image** → $B(i,j) \in \{0,1\}$
    - **Error image** → $E(i,j) = X(i,j) - B(i,j)$
    - **Intermediate stage** →

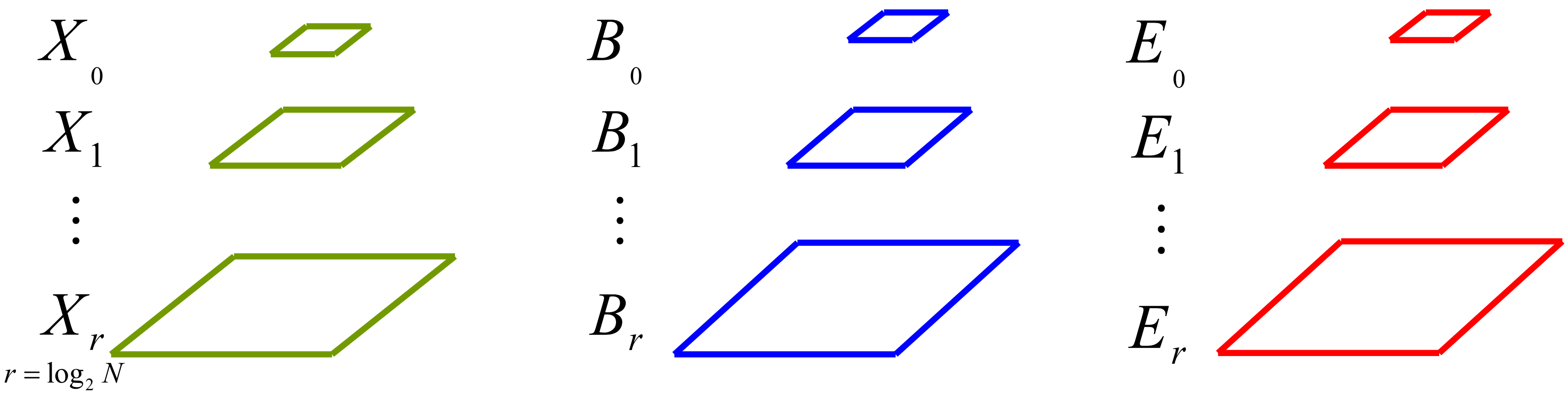
      $X_k(i_k, j_k), \quad 0 \le k \le r, \quad r = \log_2 N$

      $$X_k(i_k, j_k) = \sum_{i=0}^{1}\sum_{j=0}^{1} X_{k+1}(2i_k + i, 2j_k + j)$$

Digital Halftoning

- **Multi-scale Error diffusion**

input $X(i,j) \in [0,1]$ **output** $B(i,j) \in \{0,1\}$ **error** $E(i,j) = X(i,j) - B(i,j)$



$$X_k(i_k, j_k) = \sum_{i=0}^{1}\sum_{j=0}^{1} X_{k+1}(2i_k + i, 2j_k + j), \quad 0 \le k \le r$$

$$E_k(i_k, j_k) = X_k(i_k, j_k) - B_k(i_k, j_k), \quad 0 \le k \le r$$

Goal: minimize the error pyramid in a certain way!

Digital Halftoning

- **Multi-scale Error diffusion**
  - **//Step 1// Initialization**
    - Set the entire output image pyramid to “0”
  - **//Step 2// Dot assignment**
    - Find the largest error from top to bottom level
    - 1 parent node distributes its dots (integer numbers) to 4 children
  - **//Step 3// Error diffusion process**

$$\frac{1}{12}\begin{pmatrix} 1 & 2 & 1 \\ 2 & -12 & 2 \\ 1 & 2 & 1 \end{pmatrix} \qquad \frac{1}{8}\begin{pmatrix} 0 & 0 & 0 \\ 2 & -8 & 2 \\ 1 & 2 & 1 \end{pmatrix} \qquad \frac{1}{5}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

center **side** **corner**

Digital Halftoning

- **Multi-scale Error diffusion**
  - **Quality management**
    - **MSE vector**

$$MSEV = \begin{pmatrix} MSE_0 \\ MSE_1 \\ \vdots \\ MSE_r \end{pmatrix} \qquad MSE_k = \frac{1}{N^2} \sum_{i=0}^{2^k-1} \sum_{j=0}^{2^k-1} E_k^2(i,j)$$

  - **Notes**
    - **Preserve contrast of the original image**
    - **Does not over-smooth the image**

Digital Halftoning

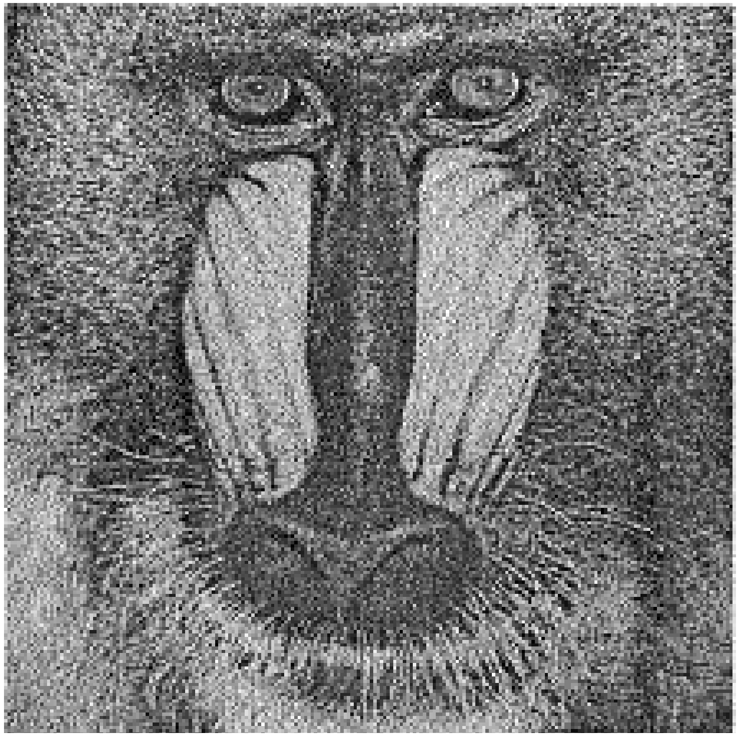
- **Experimental results**

Error Diffusion

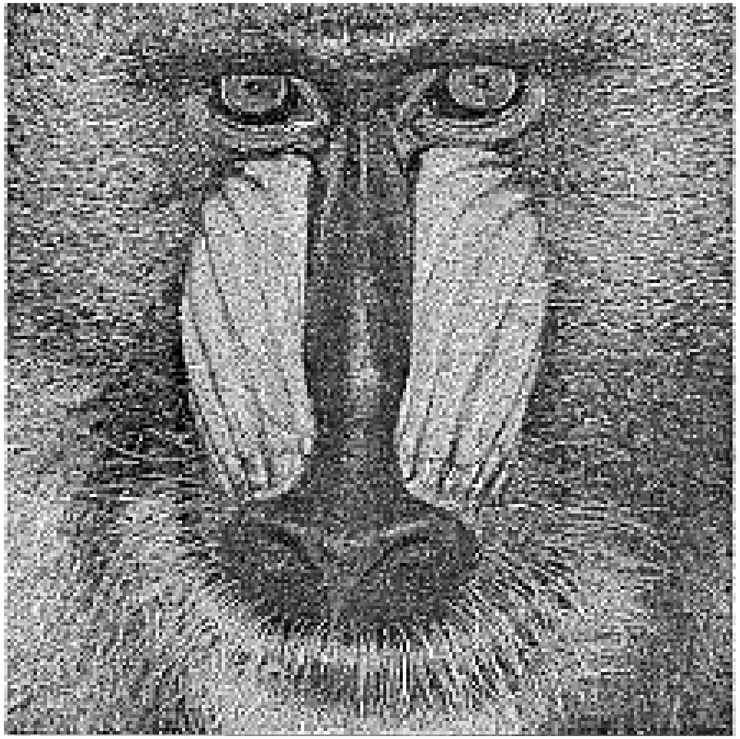
Multi-Scale Error Diffusion

Digital Halftoning

- **Experimental results**



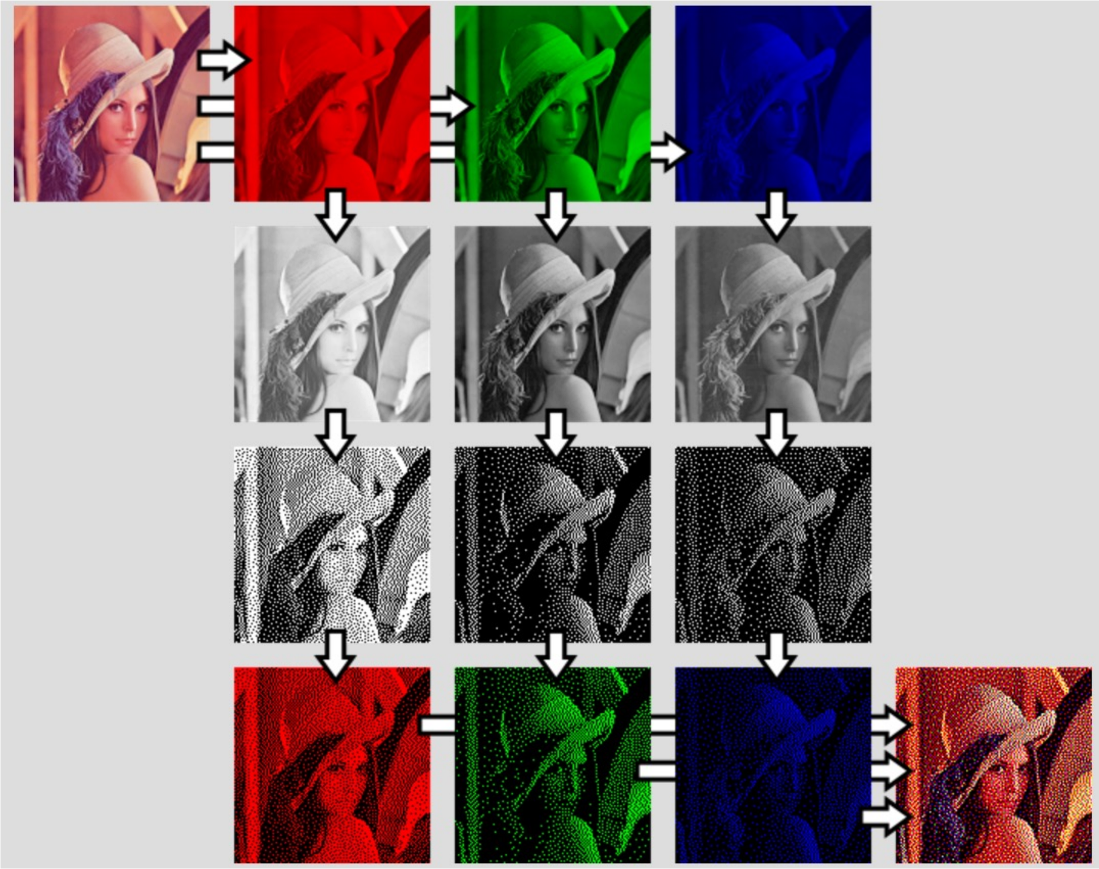
Error Diffusion



Multi-Scale Error Diffusion

Digital Halftoning

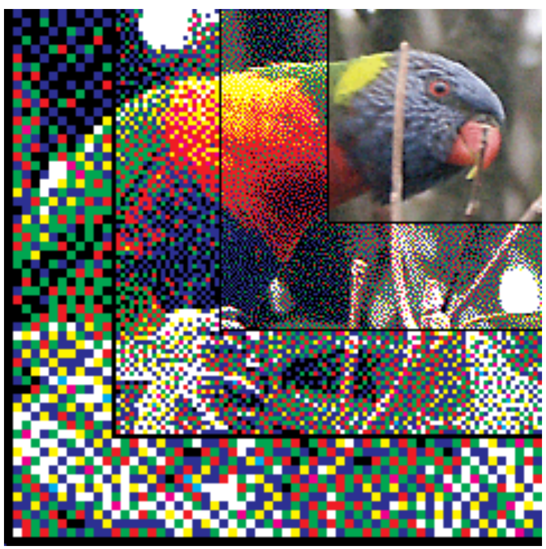
- Color image

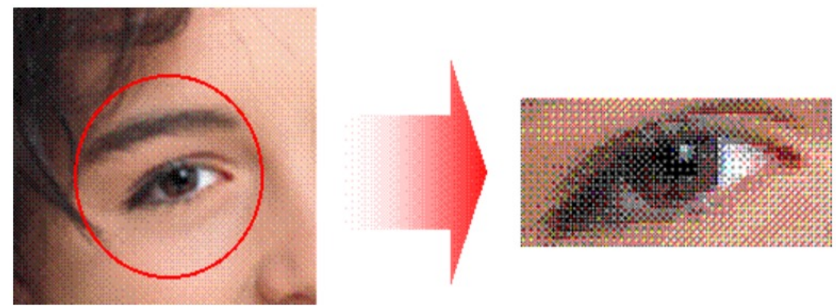


http://caca.zoy.org/study/part6.html

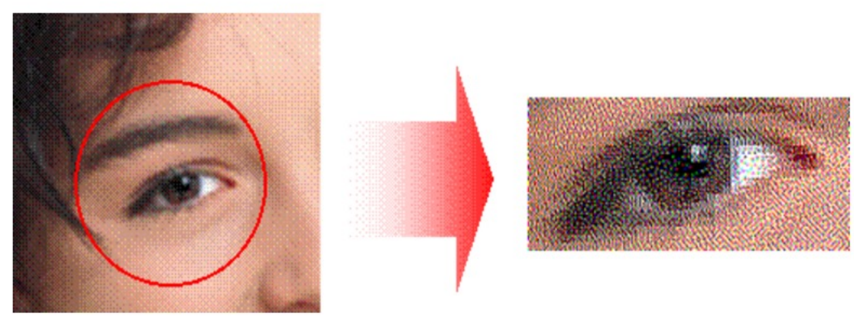
Digital Halftoning

- **Examples**





Dithering



Error Diffusion

http://www.fieggen.com/graphics/scanning.htm

Digital Halftoning

- **Application**
  - **Visual cryptography**

    "visual cryptography based on void-and-cluster halftoning technique" E. Myodo, S. Sakazawa and Y. Takishima