Digital Image Processing

Morphological Image Processing

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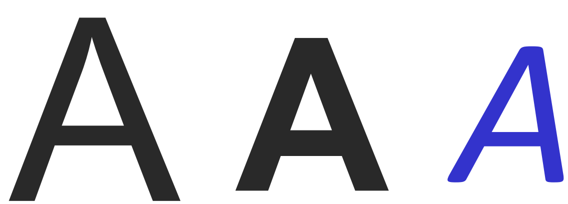
Lecture 04

Morphological Processing

- Morphology
  - Morpho-: shape/form/structure
  - -ology: study
- Morphological image processing
  - Post-processing
  - Binary images → gray-level image

Morphological Processing

- For some applications
  - Structures of objects composed by lines or arcs
  - Care about the pattern connectivity
  - Independent of width

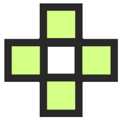


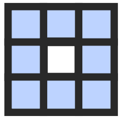
Hand-written characters

Fingerprint patterns

Morphological Processing

- Binary image connectivity
  - Pixel bond
    - Specify the connectivity of a pixel with its neighbors
    - Four-connected neighbor → bond = 2
    - Eight-connected neighbor → bond = 1





  - Minimally connected
    - Elimination of any black (object) pixel (except boundary pixels) results in disconnection of the remaining black (object) pixels

Morphological Processing

- **Binary image connectivity**
  - **Example**

$$\begin{matrix}0&0&0\\0&1&1\\0&1&0\end{matrix}$$
Four-connected
$B = 4$

$$\begin{matrix}0&0&0\\0&1&1\\0&0&1\end{matrix}$$
Eight-connected
$B = 3$

$$\begin{matrix}0&0&0\\0&1&0\\0&0&0\end{matrix}$$
Isolated
$B = 0$

$$\begin{matrix}0&0&0\\0&1&1\\0&1&1\end{matrix}$$
Corner
$B = 5$

$$\begin{matrix}0&1&1\\1&1&1\\1&1&1\end{matrix}$$
Interior
$B = 11$

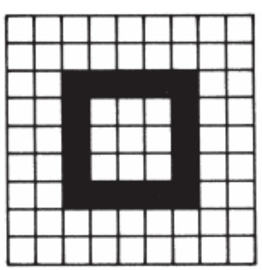
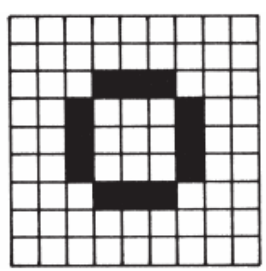
$$\begin{matrix}0&0&0\\0&1&0\\0&0&1\end{matrix}$$
Spur
$B = 1$

$$\begin{matrix}1&0&0\\1&1&1\\1&0&1\end{matrix}$$
Bridge
$B = 7$

$$\begin{matrix}1&1&1\\0&1&0\\1&1&1\end{matrix}$$
H-connected
$B = 8$

$$\begin{matrix}0&1&1\\0&1&1\\0&1&1\end{matrix}$$
Exterior
$B = 8$

  - **Another example**

Four-connectivity
v.s.
Eight-connectivity

Morphological Processing

- Binary hit or miss transformations
  - Select a nxn hit pattern (odd-sized mask)
  - Compare with a nxn image window
    - Match → hit → change the central pixel value
    - Otherwise → miss → do nothing
  - Example
    - To clean the isolated binary noise

0	0	0	
0	1	0	Hit or miss?
0	0	0	

Morphological Processing

- Binary hit or miss transformations
  - 0 → background
  - 1 → object (black)

  - Logical expression

$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix} \qquad G(j,k) = X \cap (X_0 \cup X_1 \cup \cdots \cup X_7)$$

  - Example
    - If $G(j,k) = X \cap 1$ → do nothing
    - If $G(j,k) = X \cap 0$
      - If $X = 0 \Longrightarrow G(j,k) = 0$ → do nothing
      - If $X = 1$ → hit → $G(j,k) = 0$

0	0	0
0	1	0
0	0	0

Hit or miss?

Morphological Processing

- **Binary hit or miss transformations**

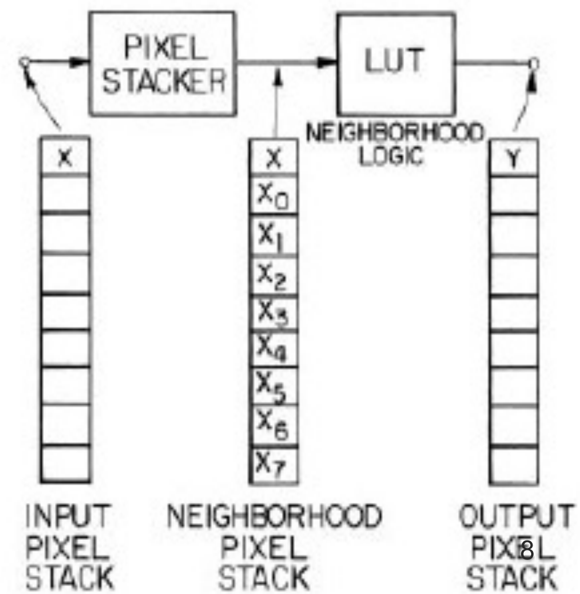
$$G(j,k) = X \cap (X_0 \cup X_1 \cup \cdots \cup X_7)$$

$\Longrightarrow 2^9$ possible mask patterns

- **Implementation**
  - **Pixel stack**
    - **Treat the 8 neighboring pixels as a "byte"**

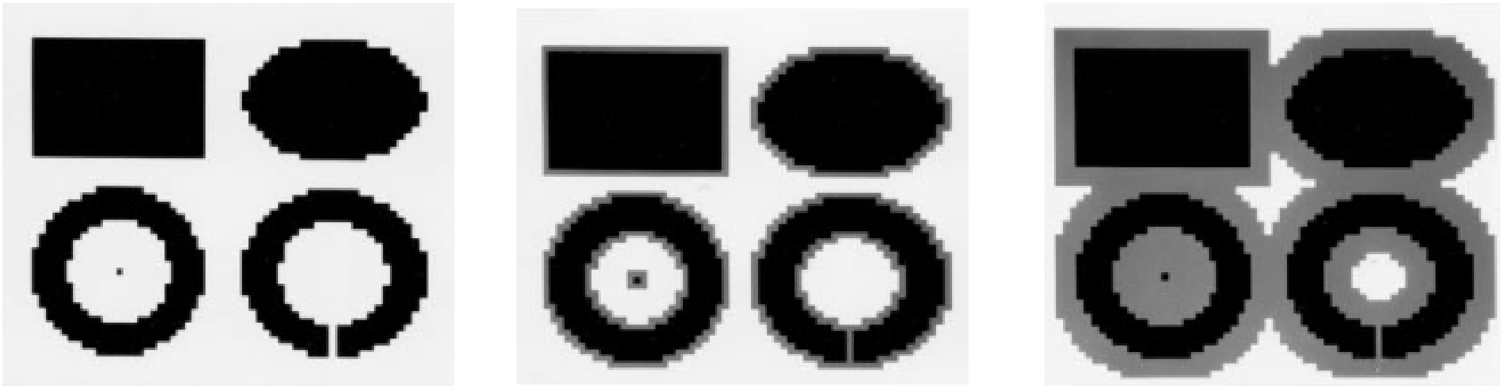
$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix} \otimes \begin{bmatrix} 2^{-4} & 2^{-3} & 2^{-2} \\ 2^{-5} & 2^{0} & 2^{-1} \\ 2^{-6} & 2^{-7} & 2^{-8} \end{bmatrix}$$

  - **Look-Up-Table (LUT)**



Morphological Processing

- Simple morphological processing based on binary hit or miss rules
  - Additive operators (0→1)
    - Interior fill
    - Diagonal fill
    - Bridge
    - 8-neighbor dilate



Morphological Processing

- **Interior fill**
  - Create a black pixel if all four-connected neighbor pixels are black

	1	
1	0	1
	1	

	1	
1	1	1
	1	

0 → background
1 → object (black)

- **Diagonal fill**
  - Create a black pixel if creation eliminates the eight-connectivity of the background

	1	0
	0	1

	1	0
	1	1

Morphological Processing

- **Bridge**
  - Create a black pixel if creation results in connectivity of previously unconnected neighboring black pixels

1	0	0
0	0	1
0	0	0

1	0	0
0	1	1
0	0	0

0 → background
1 → object (black)

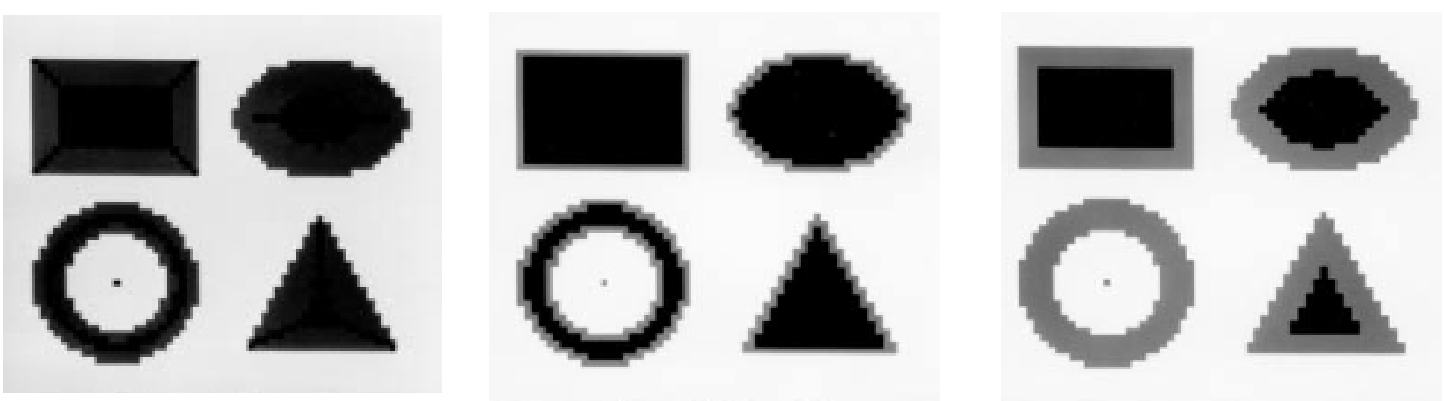
- **8-neighbor dilate**
  - Create a black pixel if at least one eight-connected neighbor pixel is black

0	0	0
0	0	0
1	0	0

0	0	0
0	1	0
1	0	0

Morphological Processing

- Simple morphological processing based on binary hit or miss rules
  - Subtractive operators (1→0)
    - Isolated pixel removal
    - Spur removal
    - Interior pixel removal
    - H-break / Eight-neighbor erode



Morphological Processing

- **Isolated pixel removal**
  - Erase a black pixel with eight white neighbors

0	0	0
0	1	0
0	0	0

→

0	0	0
0	0	0
0	0	0

0 → background
1 → object (black)

- **Spur removal**
  - Erase a black pixel with a single eight-connected neighbor

1	0	0
0	1	0
0	0	0

→

1	0	0
0	0	0
0	0	0

Morphological Processing

- **Interior pixel removal**
  - Erase a black pixel if all four-connected neighbors are black

	1	
1	1	1
	1	

→

0	1	0
1	0	1
0	1	0

0 → background
1 → object (black)

- **H-break**
  - Erase a black pixel that is H-connected

1	1	1
0	1	0
1	1	1

→

1	1	1
0	0	0
1	1	1

Morphological Processing

- **Eight-neighbor erode**
  - Erase a black pixel if at least one eight-connected neighbor pixel is white

0		
	1	

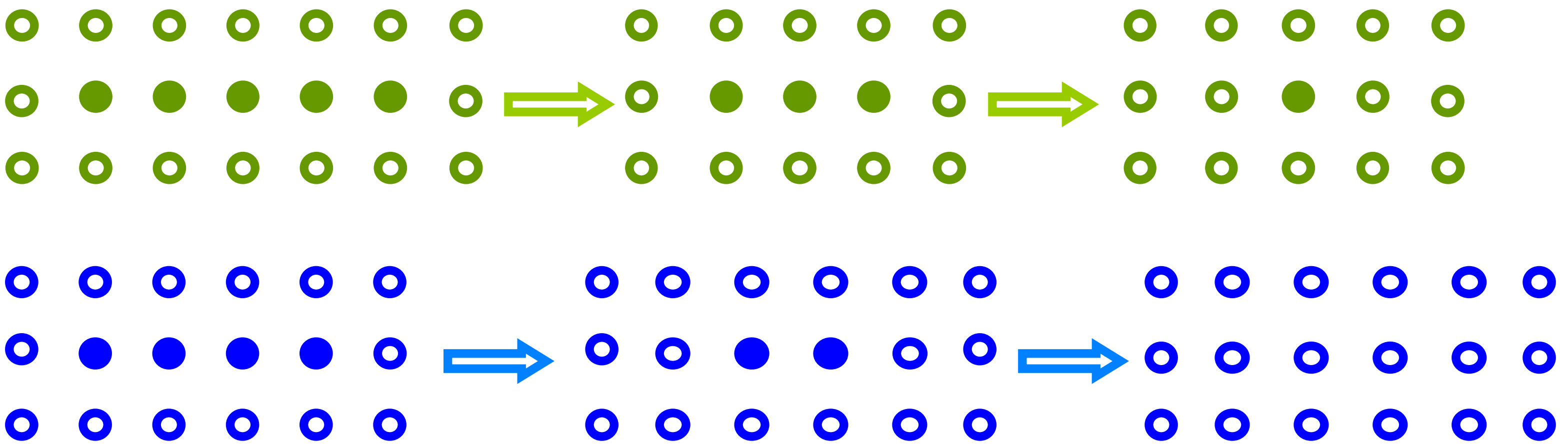
→

0		
	0	

0 → background
1 → object (black)

Morphological Processing

- Example
  - Subtractive operator



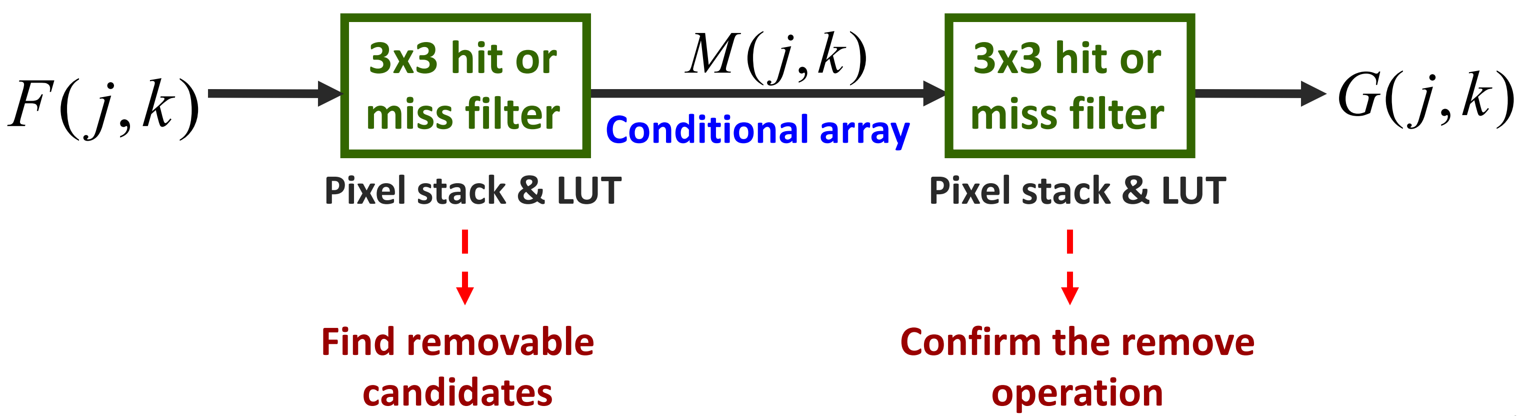
- doesn’t prevent total erasure and ensure connectivity
- In this case, only a 3x3 window does not sufficient to tell whether the final stage of iteration is reached or not

Morphological Processing

- Solutions
  - Approach I
    - Apply a filter with larger size
      - “fairly complicated patterns”, “many combinations”
  - Approach II
    - Consider a structural (composite) design with 3x3 filters: two-stage approach
      - Application dependent
      - Thinning, shrinking, skeletonizing
      - Share the same structure but vary in some modular details

Morphological Processing

- Advanced morphological processing
  - Shrinking/Thinning/Skeletonizing
    - Conditional erosion
    - Prevent total erasure & Ensure connectivity

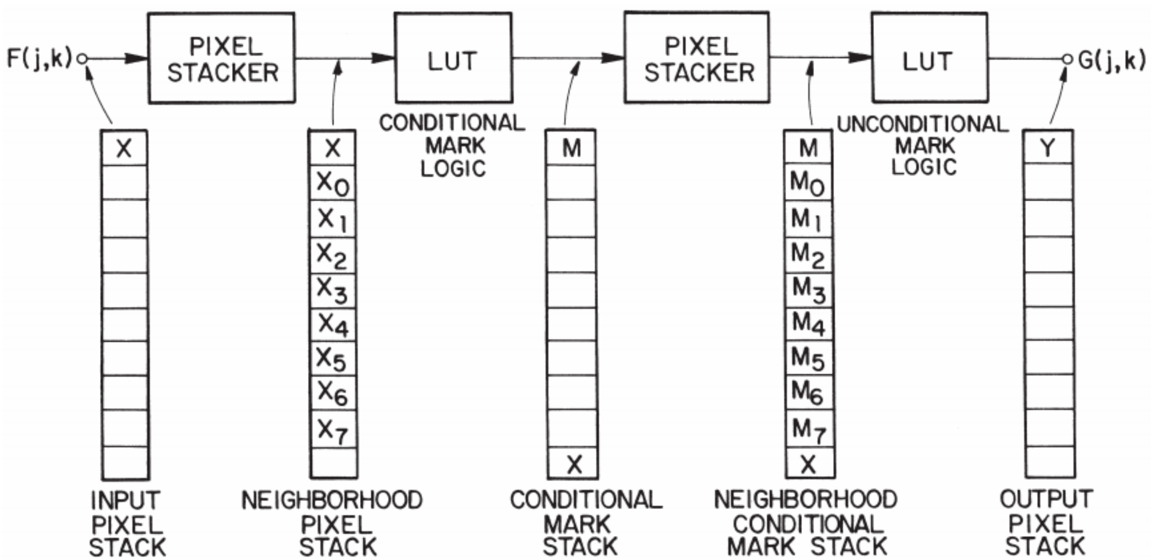


Morphological Processing

- Advanced morphological processing
  - Shrinking/Thinning/Skeletonizing
    - Conditional erosion
    - Prevent total erasure & Ensure connectivity

$$\begin{bmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{bmatrix}$$

$$\begin{bmatrix} M_3 & M_2 & M_1 \\ M_4 & M & M_0 \\ M_5 & M_6 & M_7 \end{bmatrix}$$



Morphological Processing

- Shrinking/Thinning/Skeletonizing
  - Stage I
    - Generate a binary image M(j,k) called the conditional array (or mask)
      - If M(j,k)=1, it means (j,k) is a candidate for erasure
      - If M(j,k)=0, it means no further operation is needed on (j,k)
  - Stage II
    - Based on the center pixel, X, and M(j,k) pattern, we decide whether to erase X or not in the output G(j,k)
      - If there's a hit → do nothing
      - If there's a miss → erase the center pixel

Morphological Processing

- **Stage I → Part of Table 14.3-1**

TABLE 14.3-1. Shrink, Thin and Skeletonize Conditional Mark Patterns [$M = 1$ if hit]

Table	Bond	Pattern
S	1	$\begin{matrix}0&0&1\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}1&0&0\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\1&0&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\0&0&1\end{matrix}$
S	2	$\begin{matrix}0&0&0\\0&1&1\\0&0&0\end{matrix}$ $\begin{matrix}0&1&0\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}0&0&0\\1&1&0\\0&0&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\0&1&0\end{matrix}$
S	3	$\begin{matrix}0&0&1\\0&1&1\\0&0&0\end{matrix}$ $\begin{matrix}0&1&1\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}1&1&0\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}1&0&0\\1&1&0\\0&0&0\end{matrix}$ $\begin{matrix}0&0&0\\1&1&0\\1&0&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\1&1&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\0&1&1\end{matrix}$ $\begin{matrix}0&0&0\\0&1&1\\0&0&1\end{matrix}$
TK	4	$\begin{matrix}0&1&0\\0&1&1\\0&0&0\end{matrix}$ $\begin{matrix}0&1&0\\1&1&0\\0&0&0\end{matrix}$ $\begin{matrix}0&0&0\\1&1&0\\0&1&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&1\\0&1&0\end{matrix}$
STK	4	$\begin{matrix}0&0&1\\0&1&1\\0&0&1\end{matrix}$ $\begin{matrix}1&1&1\\0&1&0\\0&0&0\end{matrix}$ $\begin{matrix}1&0&0\\1&1&0\\1&0&0\end{matrix}$ $\begin{matrix}0&0&0\\0&1&0\\1&1&1\end{matrix}$

Table: Shrink (S), Thin (T), Skeletonize (K)
Bond: classification, narrow down the search space
Pattern: coded as an 8-bit symbol for a filter

$$\begin{bmatrix}X_3 & X_2 & X_1\\ X_4 & X & X_0\\ X_5 & X_6 & X_7\end{bmatrix} \otimes \begin{bmatrix}2^{-4} & 2^{-3} & 2^{-2}\\ 2^{-5} & 2^{0} & 2^{-1}\\ 2^{-6} & 2^{-7} & 2^{-8}\end{bmatrix}$$

Morphological Processing

- **Stage II → Part of Table 14.3-2**

TABLE 14.3-2. Shrink and Thin Unconditional Mark Patterns [$P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit][a]

Pattern

Spur

$\begin{matrix}0&0&M\\0&M&0\\0&0&0\end{matrix}\quad\begin{matrix}M&0&0\\0&M&0\\0&0&0\end{matrix}$

Single 4-connection

$\begin{matrix}0&0&0\\0&M&0\\0&M&0\end{matrix}\quad\begin{matrix}0&0&0\\0&M&M\\0&0&0\end{matrix}$

L Cluster

$\begin{matrix}0&0&M\\0&M&M\\0&0&0\end{matrix}\quad\begin{matrix}0&M&M\\0&M&0\\0&0&0\end{matrix}\quad\begin{matrix}M&M&0\\0&M&0\\0&0&0\end{matrix}\quad\begin{matrix}M&0&0\\M&M&0\\0&0&0\end{matrix}\quad\begin{matrix}0&0&0\\M&M&0\\M&0&0\end{matrix}\quad\begin{matrix}0&0&0\\0&M&0\\M&M&0\end{matrix}\quad\begin{matrix}0&0&0\\0&M&0\\0&M&M\end{matrix}\quad\begin{matrix}0&0&0\\0&M&M\\0&0&M\end{matrix}$

4-Connected offset

$\begin{matrix}0&M&M\\M&M&0\\0&0&0\end{matrix}\quad\begin{matrix}M&M&0\\0&M&M\\0&0&0\end{matrix}\quad\begin{matrix}0&M&0\\0&M&M\\0&0&M\end{matrix}\quad\begin{matrix}0&0&M\\0&M&M\\0&M&0\end{matrix}$

$$G(j,k) = X \cap [\overline{M} \cup P(M, M_1, \ldots, M_7)]$$

where $P(M, M_1, \ldots, M_7)$ **is an erasure inhibiting logical variable**

$$\begin{bmatrix} M_3 & M_2 & M_1 \\ M_4 & M & M_0 \\ M_5 & M_6 & M_7 \end{bmatrix} \otimes \begin{bmatrix} 2^{-4} & 2^{-3} & 2^{-2} \\ 2^{-5} & 2^{0} & 2^{-1} \\ 2^{-6} & 2^{-7} & 2^{-8} \end{bmatrix}$$

Morphological Processing

- **Stage II → Part of Table 14.3-2 (cont'd)**

Spur corner cluster

$$\begin{matrix} 0 & A & M \\ 0 & M & B \\ M & 0 & 0 \end{matrix} \qquad \begin{matrix} M & B & 0 \\ A & M & 0 \\ 0 & 0 & M \end{matrix} \qquad \begin{matrix} 0 & 0 & M \\ A & M & 0 \\ M & B & 0 \end{matrix} \qquad \begin{matrix} M & 0 & 0 \\ 0 & M & B \\ 0 & A & M \end{matrix}$$

Corner cluster

$$\begin{matrix} M & M & D \\ M & M & D \\ D & D & D \end{matrix}$$

Tee branch

$$\begin{matrix} D & M & 0 \\ M & M & M \\ D & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & M & D \\ M & M & M \\ 0 & 0 & D \end{matrix} \qquad \begin{matrix} 0 & 0 & D \\ M & M & M \\ 0 & M & D \end{matrix} \qquad \begin{matrix} D & 0 & 0 \\ M & M & M \\ D & M & 0 \end{matrix} \qquad \begin{matrix} D & M & D \\ M & M & 0 \\ 0 & M & 0 \end{matrix} \qquad \begin{matrix} 0 & M & 0 \\ M & M & 0 \\ D & M & D \end{matrix} \qquad \begin{matrix} 0 & M & 0 \\ 0 & M & M \\ D & M & D \end{matrix} \qquad \begin{matrix} D & M & D \\ 0 & M & M \\ 0 & M & 0 \end{matrix}$$

$$A \cup B \cup C = 1, \quad D = 0 \cup 1, \quad A \cup B = 1$$

Morphological Processing

- **Stage II → Part of Table 14.3-3**

TABLE 14.3-3. Skeletonize Unconditional Mark Patterns
$[P(M, M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7) = 1$ if hit$]^a$

$A \cup B \cup C = 1, \quad D = 0 \cup 1$

Pattern											
Spur											
0	0	0	0	0	0	0	0	M	M	0	0
0	M	0	0	M	0	0	M	0	0	M	0
0	0	M	M	0	0	0	0	0	0	0	0
Single 4-connection											
0	0	0	0	0	0	0	0	0	0	M	0
0	M	0	0	M	M	M	M	0	0	M	0
0	M	0	0	0	0	0	0	0	0	0	0
L corner											
0	M	0	0	M	0	0	0	0	0	0	0
0	M	M	M	M	0	0	M	M	M	M	0
0	0	0	0	0	0	0	M	0	0	M	0

Morphological Processing

- **Example - shrinking**

0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	0	0

$$F(j,k)$$

0	0	0	0	0	0
0	0	M	M	0	0
0	0	0	0	0	0

$$M(j,k)$$

0	0	0	0	0	0
0	0	P	0	0	0
0	0	0	0	0	0

$$P(j,k)$$

0	0	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$$G(j,k)$$

Morphological Processing

- **Example - shrinking**

0	0	0	0
0	1	1	0
0	1	1	0
0	0	0	0

$F(j,k)$

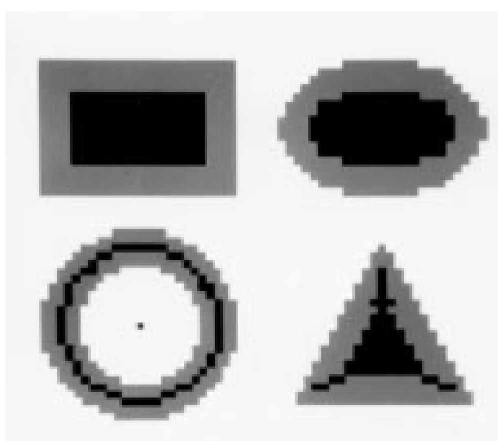
$M(j,k)$

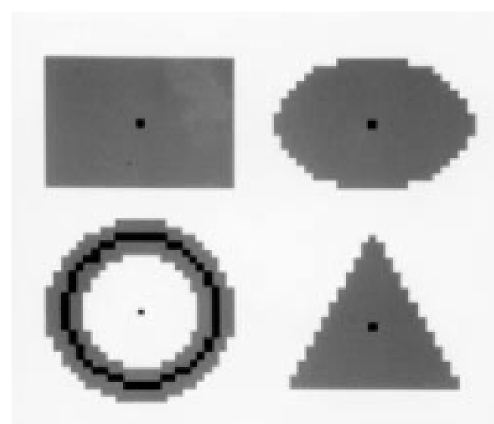
$P(j,k)$

$G(j,k)$

Morphological Processing

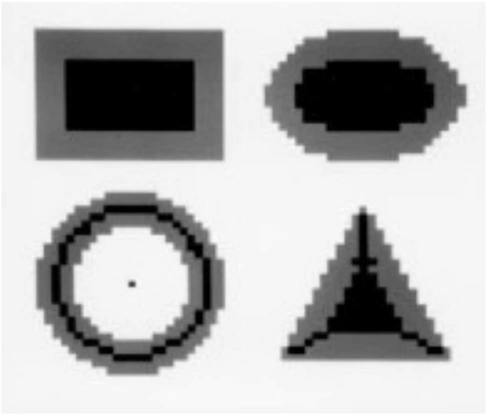
- **Shrinking**
  - Erase black pixels such that an object without holes erodes to a single pixel at or near its center of mass, and an object with holes erodes to a connected ring lying midway between each hole and its nearest outer boundary

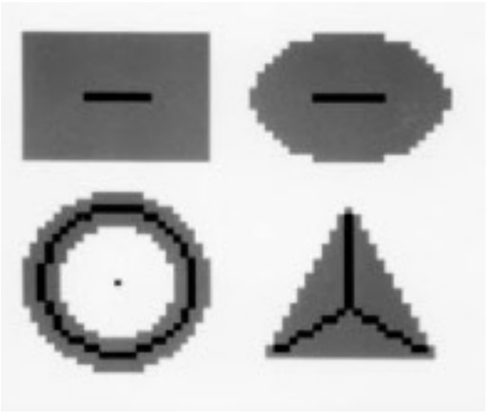




Morphological Processing

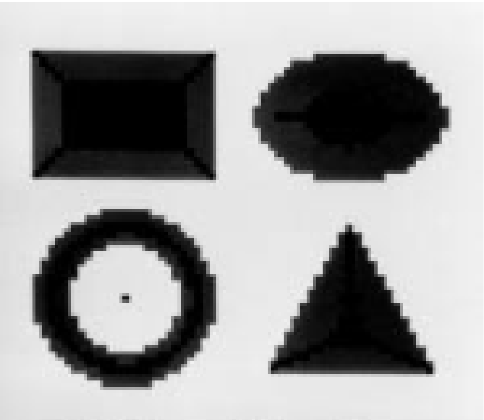
- **Thinning**
  - Erase black pixels such that an object without holes erodes to a minimally connected stroke located equidistant from its nearest outer boundaries, and an object with holes erodes to a minimally connected ring midway between each hole and its nearest outer boundary

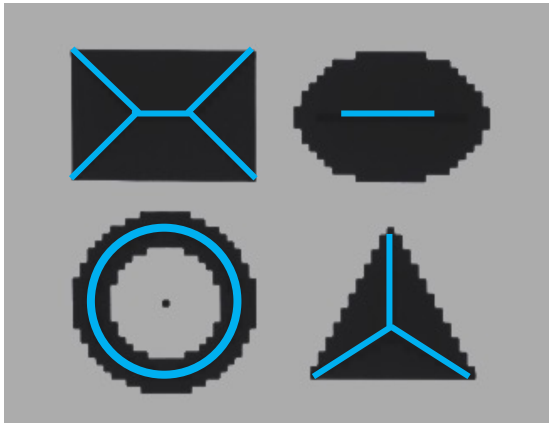




Morphological Processing

- **Skeletonizing**
  - The medial axis skeleton consists of the set of points that are **equally distant** from **two closest** points of an object boundary





Morphological Processing

- **Algebraic operations on binary arrays**

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0

A

0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0
0	0	0	0	0	0

B

1	1	1	1	1	1
1	1	0	0	1	1
1	1	0	0	1	1
1	1	0	0	1	1
1	1	0	0	1	1
1	1	1	1	1	1

$\bar{A}$

complement

0	0	0	0	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	1	1	0	0
0	0	0	0	0	0

$A \cup B$

union

OR

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$A \cap B$

intersection

AND

0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	0	1	0
0	0	1	1	0	0
0	0	0	0	0	0

A XOR B

exclusive-OR

XOR

Morphological Processing

- **Generalized dilation and erosion**
  - **Reflection and translation of a binary image**

$$\begin{matrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&1&1&1&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{matrix}$$

$F(j,k)$

$$\begin{matrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&1&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{matrix}$$

$\tilde{F}(j,k)$

$$\begin{matrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&1&1&1&0\\ 0&0&0&0&0&0&0&0 \end{matrix}$$

$T_{1,2}\{F(j,k)\}$

  - **Dilation**

$$G(j,k) = F(j,k) \oplus \underbrace{H(j,k)}_{\text{Structuring element}}$$

  - **Erosion**

$$G(j,k) = F(j,k) \ominus H(j,k)$$

Morphological Processing

- **Dilation** $G(j,k) = F(j,k) \oplus H(j,k)$
  - **Can be implemented in several ways**
  - **Minkowski addition definition**

$$\boxed{G(j,k) = \bigcup_{(r,c)\in H}\bigcup T_{r,c}\{F(j,k)\}}$$

$$\underset{F(j,k)}{\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}} \qquad \underset{H(j,k)}{\begin{array}{ccc} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{array}}$$

$$G(j,k) = T_{0,0}\{F(j,k)\} \cup T_{0,1}\{F(j,k)\} \cup T_{1,0}\{F(j,k)\}$$
$$\cup\, T_{1,1}\{F(j,k)\} \cup T_{2,0}\{F(j,k)\}$$

$$\underset{T_{0,0}\{F(j,k)\}}{\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}} \quad \underset{T_{0,1}\{F(j,k)\}}{\begin{array}{cccccc} \cdot & 0 & 0 & 0 & 0 & 0 \\ \cdot & 0 & 0 & 1 & 0 & 0 \\ \cdot & 0 & 1 & 1 & 0 & 0 \\ \cdot & 0 & 0 & 1 & 1 & 0 \\ \cdot & 0 & 0 & 0 & 0 & 0 \end{array}} \quad \underset{T_{1,0}\{F(j,k)\}}{\begin{array}{ccccc} \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}} \quad \underset{T_{1,1}\{F(j,k)\}}{\begin{array}{cccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & 0 & 0 & 0 \\ \cdot & 0 & 0 & 1 & 0 & 0 \\ \cdot & 0 & 1 & 1 & 0 & 0 \\ \cdot & 0 & 0 & 1 & 1 & 0 \\ \cdot & 0 & 0 & 0 & 0 & 0 \end{array}} \quad \underset{T_{2,0}\{F(j,k)\}}{\begin{array}{ccccc} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}} \Rightarrow \underset{G(j,k)}{\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}}$$

Morphological Processing

- **Erosion** $G(j,k) = F(j,k) \Theta H(j,k)$
  - **Can be implemented in several ways**
  - **Dual relationship of Minkowski addition**

$$\boxed{G(j,k) = \bigcap_{(r,c)\in H}\bigcap T_{r,c}\{F(j,k)\}}$$

//Sternberg definition//

$$G(j,k) = \bigcap \bigcap_{(r,c)\in H} T_{r,c}\{F(j,k)\}$$

$$\underset{F(j,k)}{\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}} \ominus \underset{H(j,k)}{\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}} = \underset{G(j,k)}{\begin{matrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}}$$

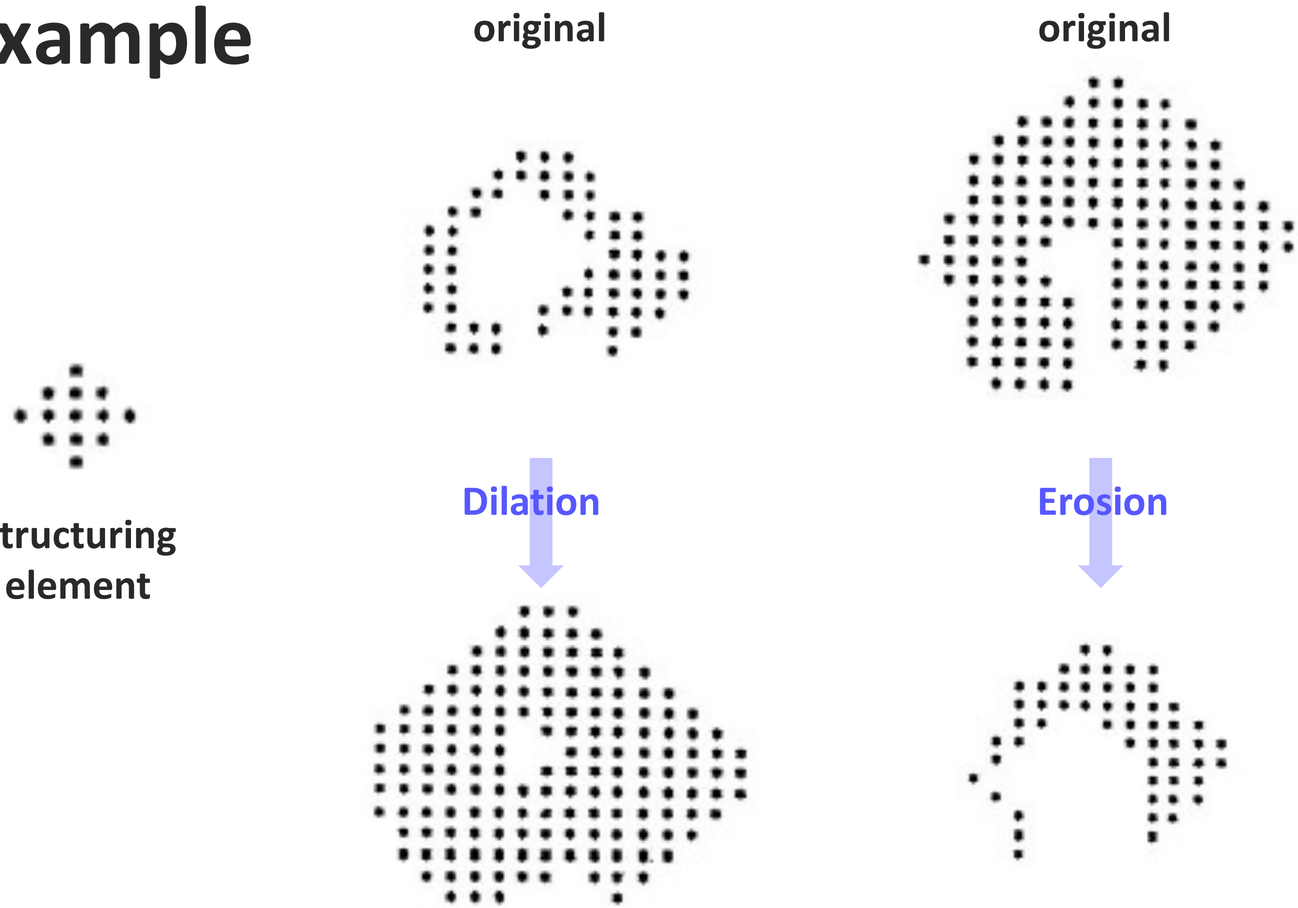
//Serra definition//

$$G(j,k) = \bigcap \bigcap_{(r,c)\in \tilde{H}} T_{r,c}\{F(j,k)\}$$

$$\underset{F(j,k)}{\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}} \ominus \underset{H(j,k)}{\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}} = \underset{G(j,k)}{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}$$

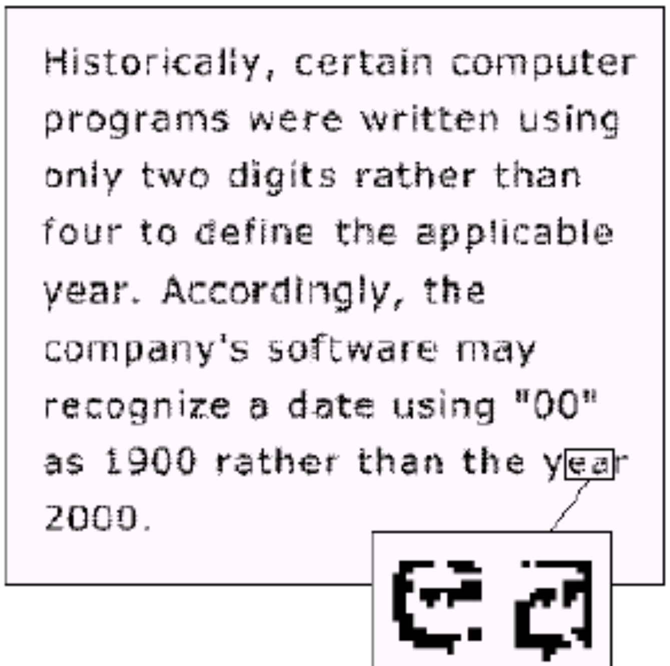
Morphological Processing

- **Example**



Morphological Processing

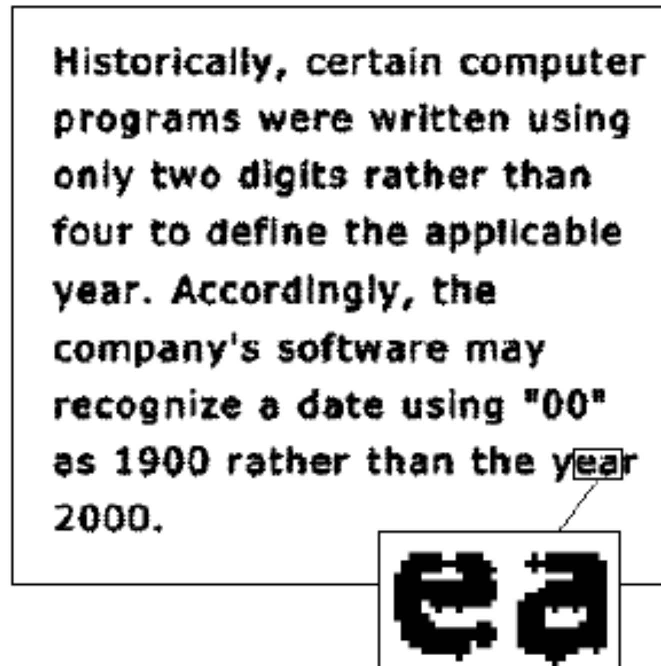
- **Example**



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Structuring element

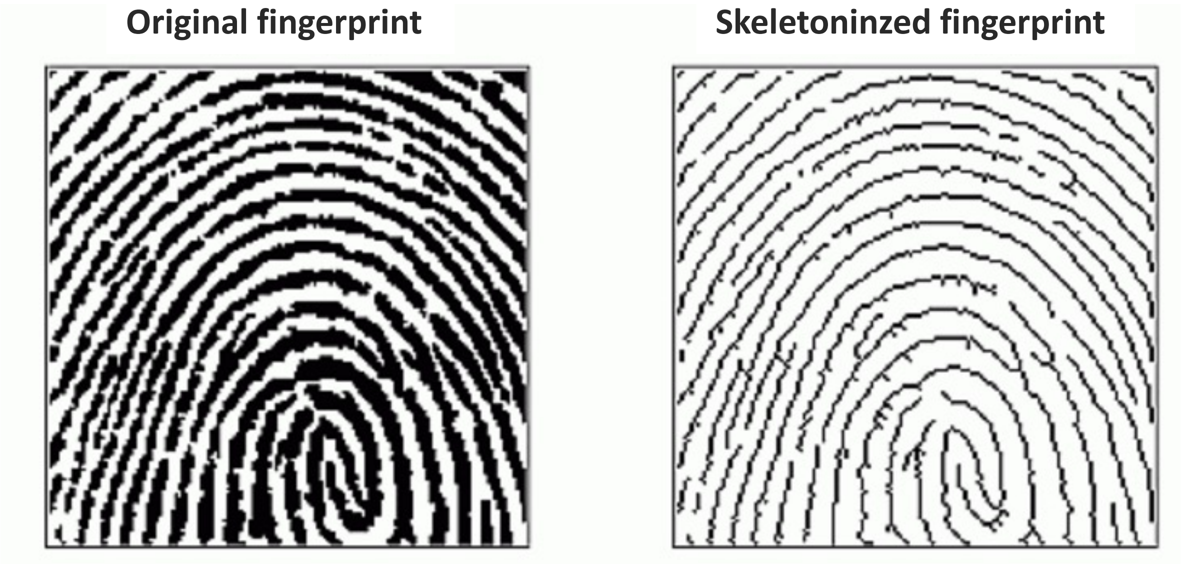
0	1	0
1	1	1
0	1	0



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Morphological Processing

- **Example**



The original fingerprint contains ridges with width of several pixels. The skeletonized fingerprint contains ridges only a single pixel wide.

http://www.dspguide.com/ch25/4.htm

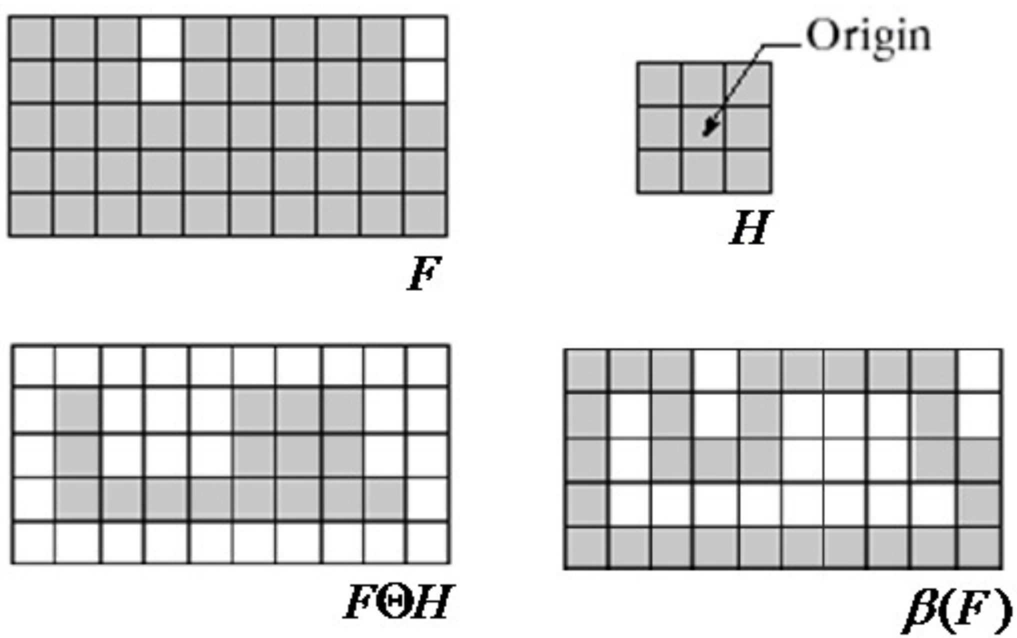
Morphological Processing

- Applications
  - Boundary Extraction
    - Extract the boundary (or outline) of an object
  - Hole Filling
    - Given a pixel inside a boundary, hole filling attempts to fill that boundary with object pixels
  - Connected Component Labeling
    - Scan an image and groups its pixels into components based on pixel connectivity

Morphological Processing

- **Boundary Extraction**

$$\beta(F(j,k)) = F(j,k) - (F(j,k)\,\Theta\,H(j,k))$$



Morphological Processing

- **Example**

Original Image

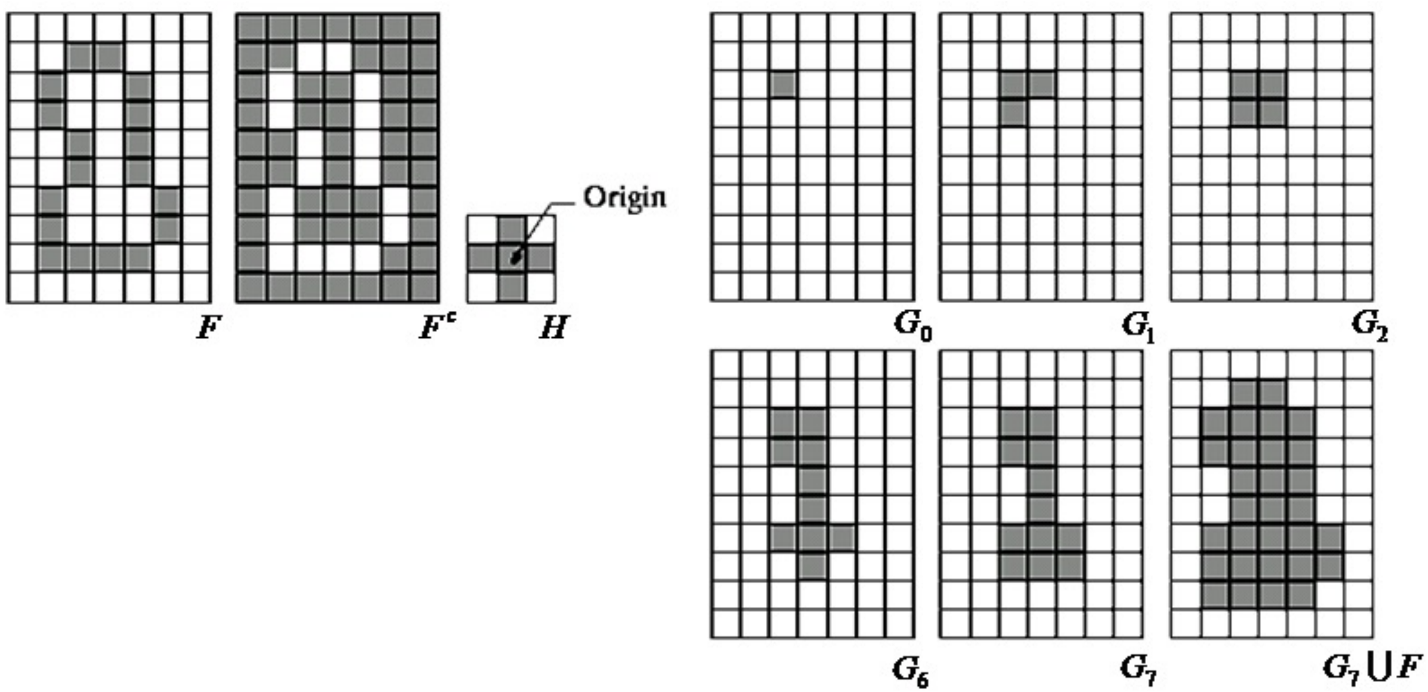
Extracted Boundary

Morphological Processing

- **Hole Filling**

$$G_i(j,k) = (G_{i-1}(j,k) \oplus H(j,k)) \cap F^c(\mathrm{j},\mathrm{k}) \qquad i = 1,2,3\ldots$$

$$G(j,k) = G_i(j,k) \cup F(\mathrm{j},\mathrm{k})$$



Morphological Processing

- **Example**

Original Image

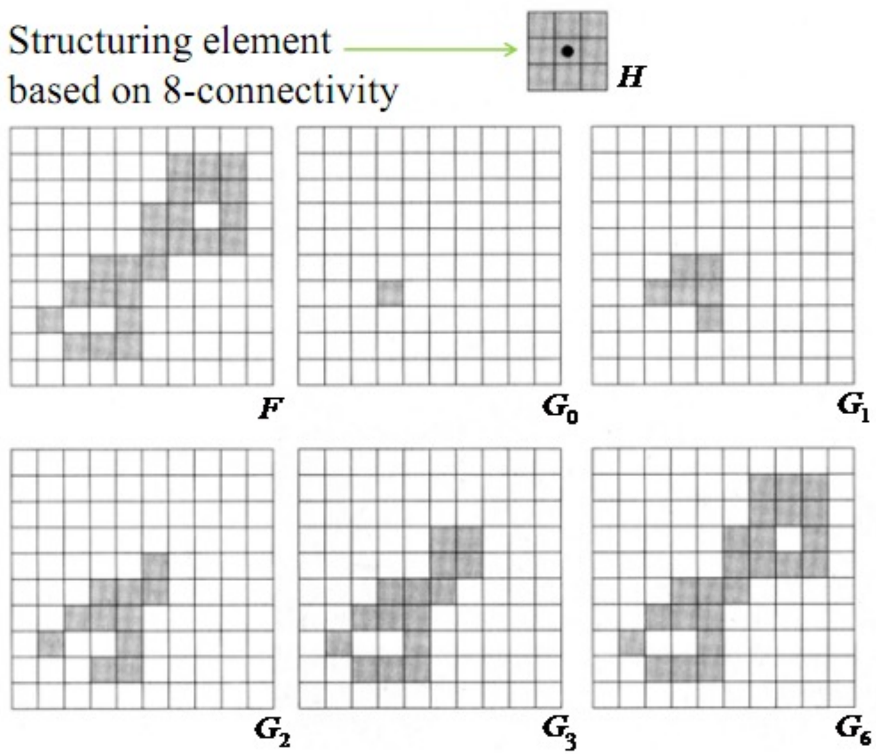
One Hole Filled

All Holes Filled

Morphological Processing

- **Connected Component Labeling**

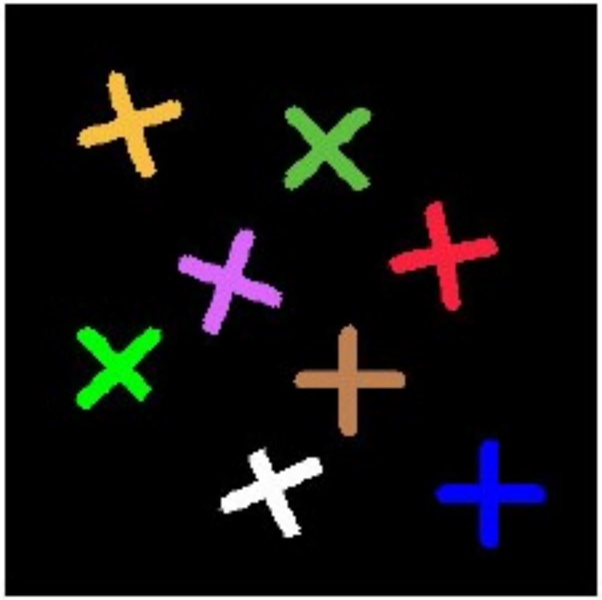
$$G_i(j,k) = (G_{i-1}(j,k) \oplus H(j,k)) \cap F(j,k) \quad \mathrm{i} = 1,2,3,\ldots$$



Morphological Processing

- **Example**

Original Image



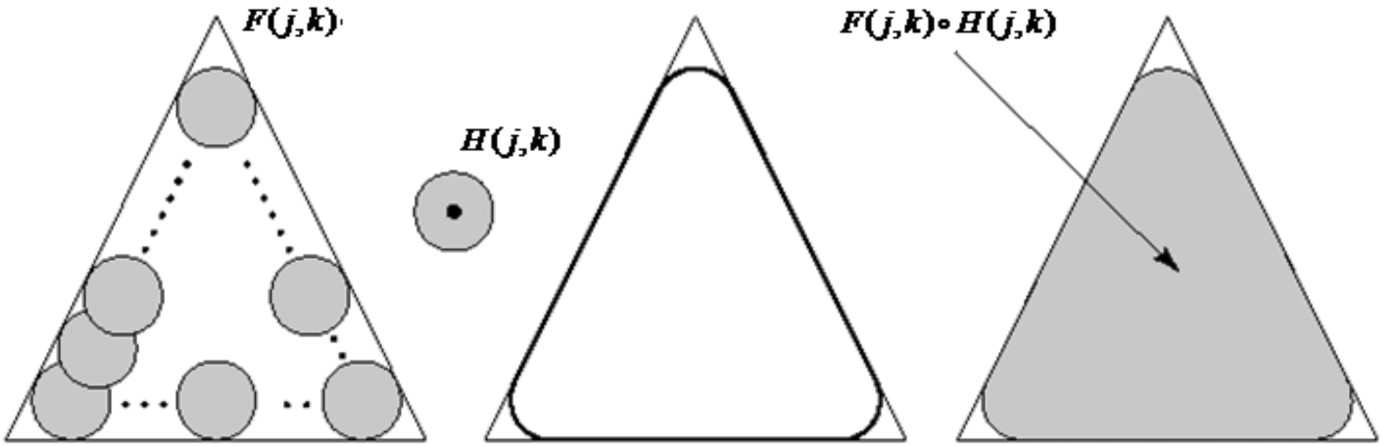
Labelled Components

Morphological Processing

- **Applications**
  - **Open operator**

$$G(j,k) = F(j,k) \circ H(j,k) = [F(j,k) \Theta \tilde{H}(j,k)] \oplus H(j,k)$$

- **With a compact structuring element**
  - Smoothes contours of objects
  - Eliminates small objects
  - Breaks narrow strokes

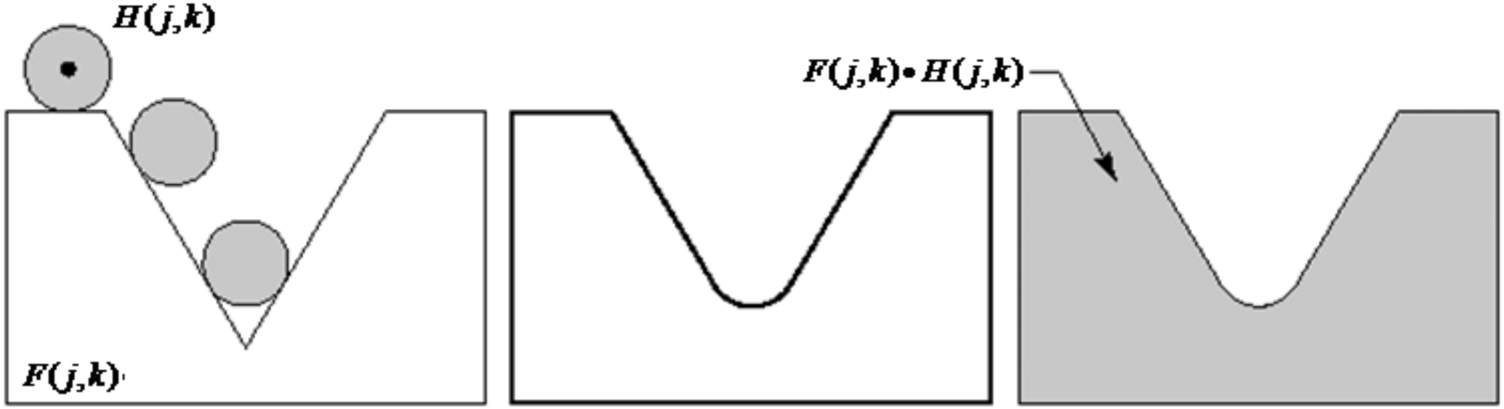


Morphological Processing

- **Applications**
  - **Close operator**

$$G(j,k) = F(j,k) \bullet H(j,k) = [F(j,k) \oplus H(j,k)] \Theta \tilde{H}(j,k)$$

- **With a compact structuring element**
  - Smoothes contours of objects
  - Eliminate small holes
  - Fuses short gaps between objects



Morphological Processing

- Example

original

(a) close

(b) open

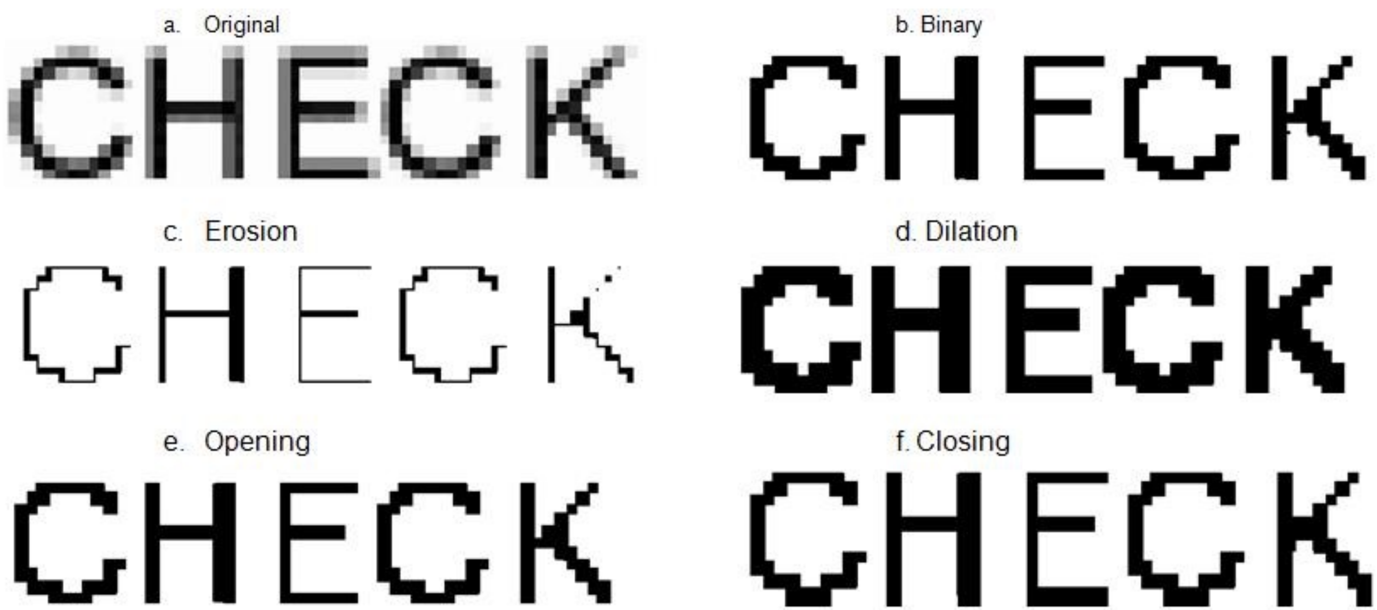
Q: repeated openings/closings?

Compare (a) with the original image

Compare (b) with the original image

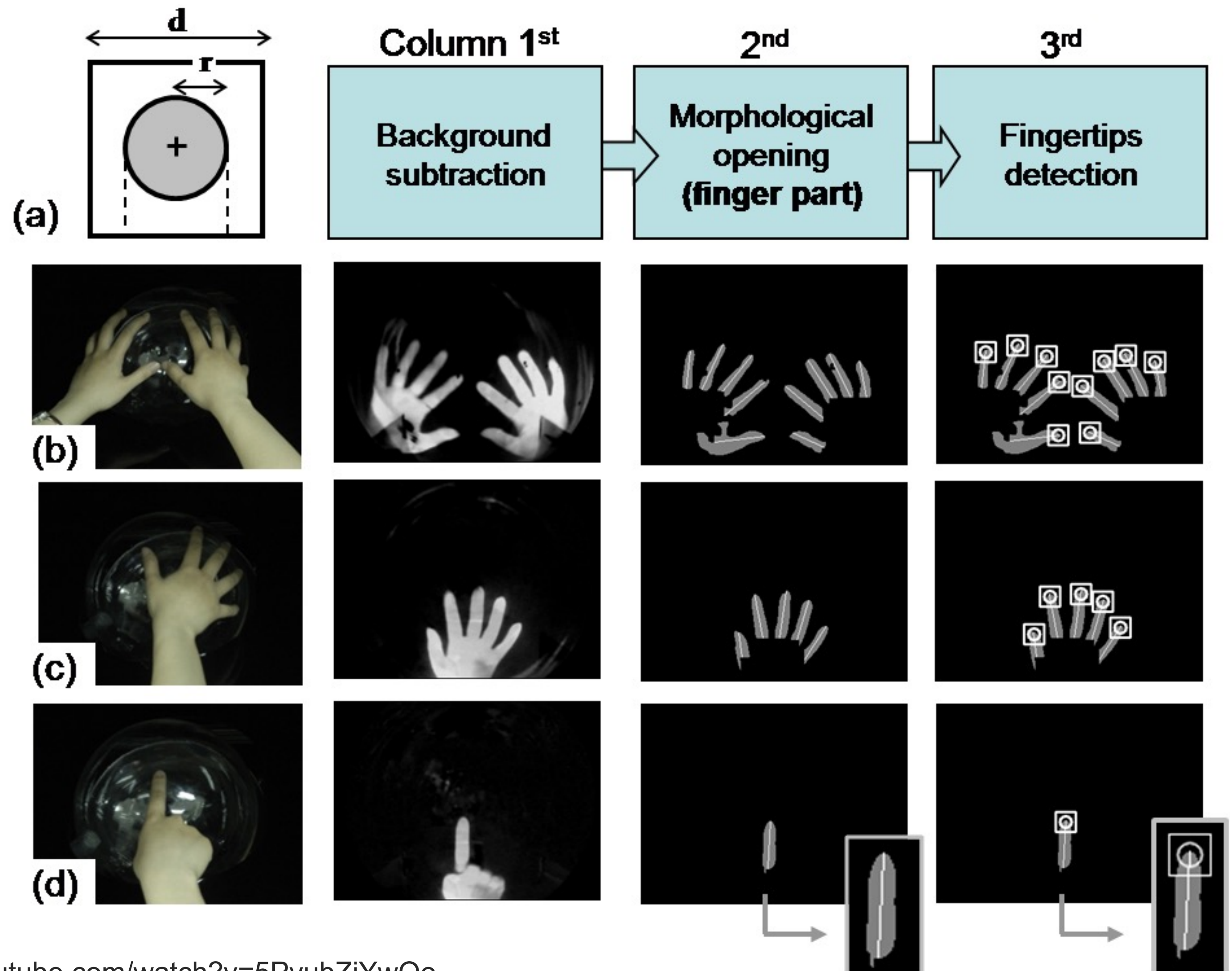
Morphological Processing

- **Example**



http://xplorimaging.com/DocumentCleanUp.html

MCBall



http://www.youtube.com/watch?v=5PvubZiYwOo

Some videos

- Morphing
  - https://www.youtube.com/watch?v=-rnVUzA8yMY

- SIGGRAPH
  - **2013** https://www.youtube.com/watch?v=JAFhkdGtHck
  - **2015** https://www.youtube.com/watch?v=XrYkEhs2FdA
  - **2017** https://www.youtube.com/watch?v=5YvIHREdVX4
  - **2018** https://www.youtube.com/watch?v=t952yS8tcg8
  - **2019** https://www.youtube.com/watch?v=EhDr3Rs5fTU
  - **2020** https://www.youtube.com/watch?v=jYdMKdRUq_8
  - **2021** https://www.youtube.com/watch?v=Ros7ZXqLbFg