1.1 Introduction to Polynomials

1.1 –Introduction to Polynomial Functions

A ***polynomial function*** is a series of terms where each term is the product of a constant and power of x. It has the form: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ...a_3x^3 + a_2x^2 + a_1x + a_0$ where 'a' is a constant and 'n' is a nonnegative integer.

Example:

EX 1 – Identify the following as polynomial functions or not.

a) $f(x) = 2x^3$ Polynomial	b) $g(x) = \sin x + 5$ no -> Sinusodial	c) $h(x) = 4^x + x + 1$ no
d) $y = x^4 + 6x^2 - x$ polynomial	e) $j(x) = \frac{-6}{x} + 5x + 2$ no	f) $k(x) = \frac{x^2}{3} + 1$ Polynonial

A ***power function*** (simplest form of a polynomial function) has the form $y = ax^n$ Where 'a' is a constant and 'n' is a nonnegative integer.

EX 2 – State the type of each of the following power functions:

a) $y = a$ Constant power	b) $y = ax$ linear power	c) $y = ax^2$ quadratic
d) $y = ax^3$ Cubic	e) $y = ax^4$ Quartic	f) $y = ax^5$ quintic

Polynomial functions have the following characteristics:

Degree: n – exponent of the greatest power of x

Leading Coefficient: a_n - the coefficient of the greatest power of x

**** Polynomial functions are typically written in descending powers of x***

EX 3 – State the degree and leading coefficient of the following polynomials:

Polynomial Functions	Degree	Leading Coefficient
a) $f(x) = 13x^6 - 2x^2 + 14$	6	13
b) $f(x) = -\frac{1}{2}x^3 + 8x + 1$	3	$-\frac{1}{2}$
c) $f(x) = 2x^3 - 5x^8 - 14$	8	-5
d) $f(x) = \frac{3x}{2} + 9$	1	$\frac{3}{2}$

End behaviour describes what happens to the y-values of a function as the x-values get very large or

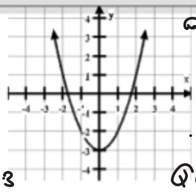
very small for a polynomial function.

End behaviour can be described in two ways:

1) as, $x \to \infty$, $y \to \infty$

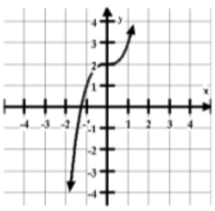
as, $x \to -\infty$ $y \to \infty$

2) graph extends from: Q3 to Q1



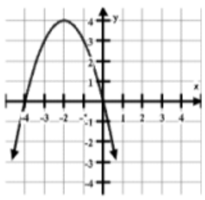
EX 4 – State the end behaviour (in two ways) of the following functions:

a)



- Q3 to Q1
- $x \to \infty$, $y \to \infty$
- $x \to -\infty$, $y \to -\infty$

b)



- Q3 to Q4
- $x \to \infty$, $y \to -\infty$
- $x \to -\infty$, $y \to -\infty$

Interval Notation

Recall:
The *domain* of a function is the set of all values of the independent variable (ie:)

The *range* of a function is the set of all values of the dependent variable (ie: ,)

Notes

- Polynomial function : An expression containing a variable (x, a, y) raised to a series of positive whole number exponents.

↳ General form: $a_n x^n + a_{(n-1)} x^{(n-1)} + a_{(n-2)} x^{(n-2)} ..a_1 x + a_0$

exponents decrease →

↳ Degrees: The highest power present in a polynomial is called a degree.

↳ Ex: $5x^2 - 2x + 7 = 0$
2nd degree polynomial
" Quadratic polynomial"

Ex: $3x^3 + 2x^2 - 7 = 0$
3rd degree polynomial
" Cubic polynomial"

4th degree = quartic
5th degree = quintic

↳ Side Note: not all terms must be present

Ex: $4x^2 + 7 = 0$ = 2nd degree polynomial

↳ End behaviour:

Interval Notation

Recall:
The *domain* of a function is the set of all values of the independent variable (ie:)

The *range* of a function is the set of all values of the dependent variable (ie: ,)

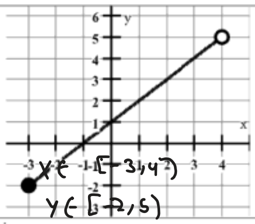
These sets can be described in:

- Set notation: **OR** interval notation:

Interval Notation

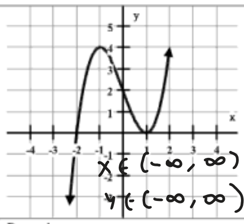
- Square brackets show the end value <u>**is**</u> in the interval
- Round brackets show the end value <u>**is not**</u> in the interval
- Intervals that are infinite use the symbols: (infinity) or (negative infinity)
- Round brackets are <u>always</u> used at infinity

EX 6 - For the following functions, state the domain and range using interval notation



Domain:

Range:



Domain:

Range:

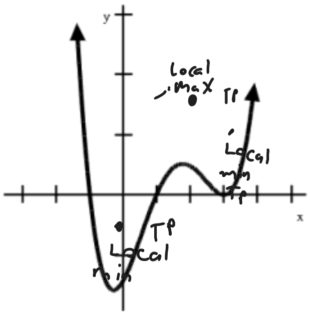
Definitions:

A ***Local Minimum*** is a point with the smallest y-value on some interval close to that point

A ***Local Maximum*** is a point with the largest y-value on some interval close to that point

Global Maxima/Minima are the <u>absolute</u> max or min points of the function *(and are also considered local maxima/minima)*

Turning Points are <u>all</u> local maxima/minima points



EX 7 - Label the above key features on the graph to the right:

<u>Practice</u> <u>Questions</u>

1. Identify whether each of the following are polynomials or not:

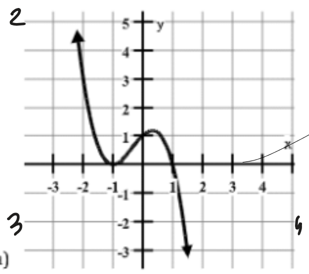
a) $f(x)=\cos x$ No	b) $g(x)=8^x+2$ No	c) $h(x)=\frac{1}{2}x^3+x^2+1$ Polynomial
d) $y=2x^6$ polynomial	e) $j(x)=x^{-3}$ no	f) $k(x)=x^2+\frac{4}{x}$ no

2. State the degree and leading coefficient of the following polynomials:

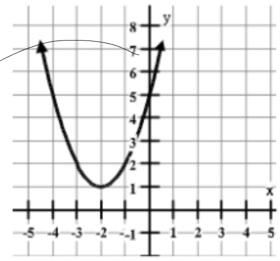
Polynomial Functions	Degree	Leading Coefficient
$f(x)=5x^4-3x^3+4$	4	5
$f(x)=-\frac{x}{4}+1$	1	$-\frac{1}{4}$
$f(x)=x+2$	1	1
$f(x)=3x^2+15x^8+7$	8	15
$f(x)=1$	0	1

4. State the end behaviour (in two ways) for the following functions:

a)



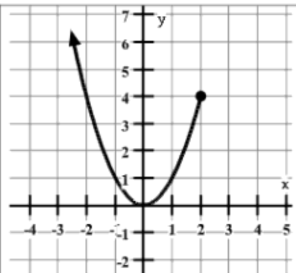
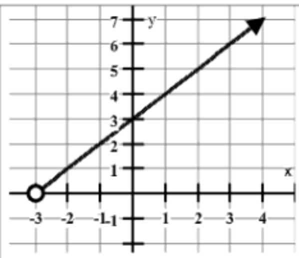
b)



- Q_2 to Q_4
- $x \to \infty,\ y \to -\infty$
  $x \to -\infty,\ y \to \infty$

- Q_2 to Q_1
- $x \to \infty,\ y \to \infty$
  $x \to -\infty,\ y \to \infty$

5. State the domain and range in set notation and interval notation

Graph:		
Set Notation	Domain: $\{x \in \mathbb{R} \mid x \le 2\}$ Range: $\{y \in \mathbb{R} \mid y \ge 0\}$	Domain: $\{x \in \mathbb{R} \mid x > -3\}$ Range: $\{y \in \mathbb{R} \mid y > 0\}$
Interval Notation	Domain: $x \in (-\infty, 2]$ Range: $y \in [0, \infty]$	Domain: $x \in [3, \infty)$ Range: $y \in (0, \infty)$
Equation:	?	?
Set Notation	Domain: Range:	Domain: Range:
Interval Notation	Domain: Range:	Domain: Range: