BART Webinar #3

Subspace-constrained reconstruction

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and
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2. University Medical Center Göttingen
3. German Center for Cardiovascular Research (DZHK)
4. The University of Texas at Austin

BART

```
bart phantom -kB bart
```

March 24 2021

Schedule

- BART updates

- Interactive demos
  1. Frequency-modulated SSFP (Volkert Roeloffs)
  2. Analytical multi-contrast phantom simulation (Nick Scholand)
  3. Inversion recovery FLASH (Xiaoqing Wang)

- Hands-on exercise [**Breakout rooms]**

https://github.com/mrirecon/bart-webinars

BART updates

- Version 0.7.00 released (doi: 10.5281/zenodo.4570601)
  - Linear and non-linear model-based reconstruction
  - Improvements to MR signal simulation (phantom and signal tools)
  - Improvements to backend, reproducibility, continuous integration

- ISMRM 2021 Educational session: software demo
  - **Non-linear operators and deep learning with BART**

- Join our mailing list! `mrirecon@lists.eecs.berkeley.edu`

Subspace-constrained reconstructions

An incomplete history

Early development: partial separability for dynamic imaging

- Z. Liang, *Spatiotemporal imaging with partially separable functions*. IEEE ISBI 2007.
- Pedersen et al., MRM 62:706–716 (2009).

An incomplete history

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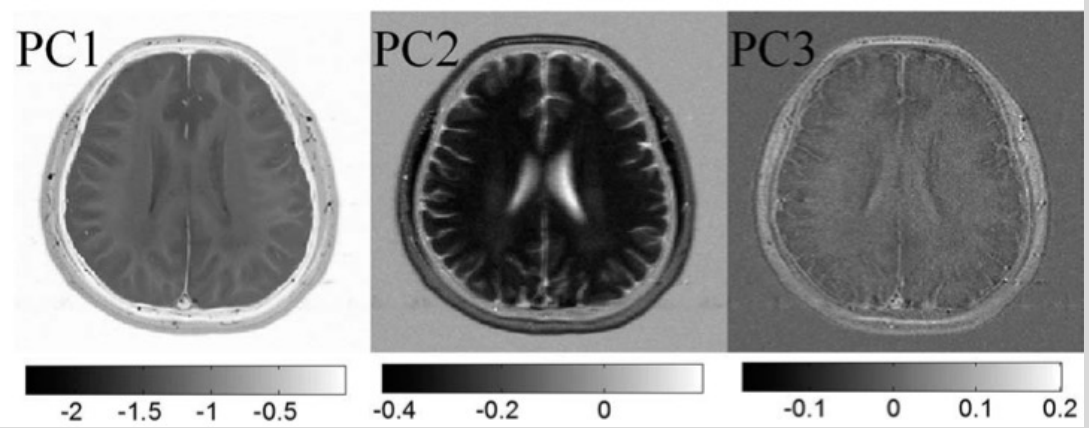
Initial application to parameter mapping

- Petzchner et al., MRM 66:706–716 (2011).
- Huang et al., MRM 67:1355–1366 (2012).



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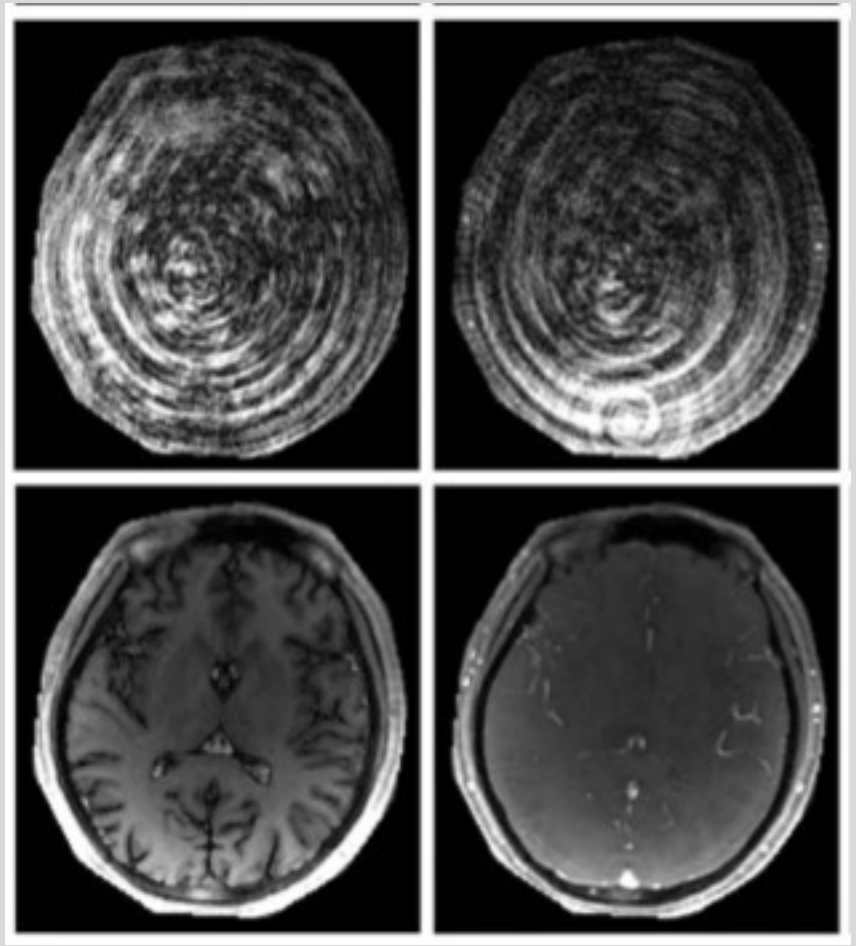
Additional applications

- FSE deblurring: Tamir et al., MRM 2017
- MR Fingerprinting: Zhao et al., MRM 2018.

MR Fingerprinting

Gridding

Subspace



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Additional applications

- FSE deblurring: Tamir et al., MRM 2017
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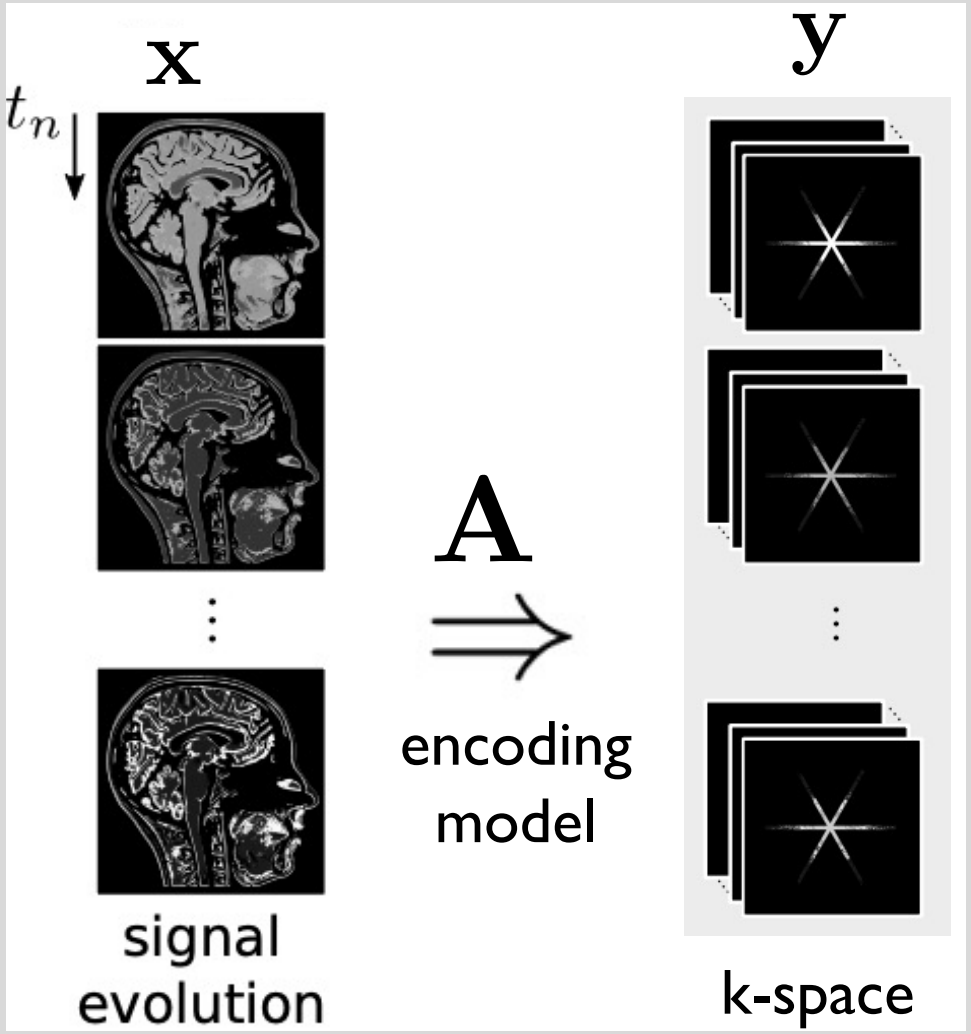
Review papers:

- Wang, et al. "Physics-based Reconstruction Methods for Magnetic Resonance Imaging." arXiv:2010.01403 (2020).
- Tamir et al. "Computational MRI with physics-based constraints: Application to multicontrast and quantitative imaging." IEEE Signal Processing Magazine 37.1 (2020): 94-104.

Multi-contrast MRI reconstruction

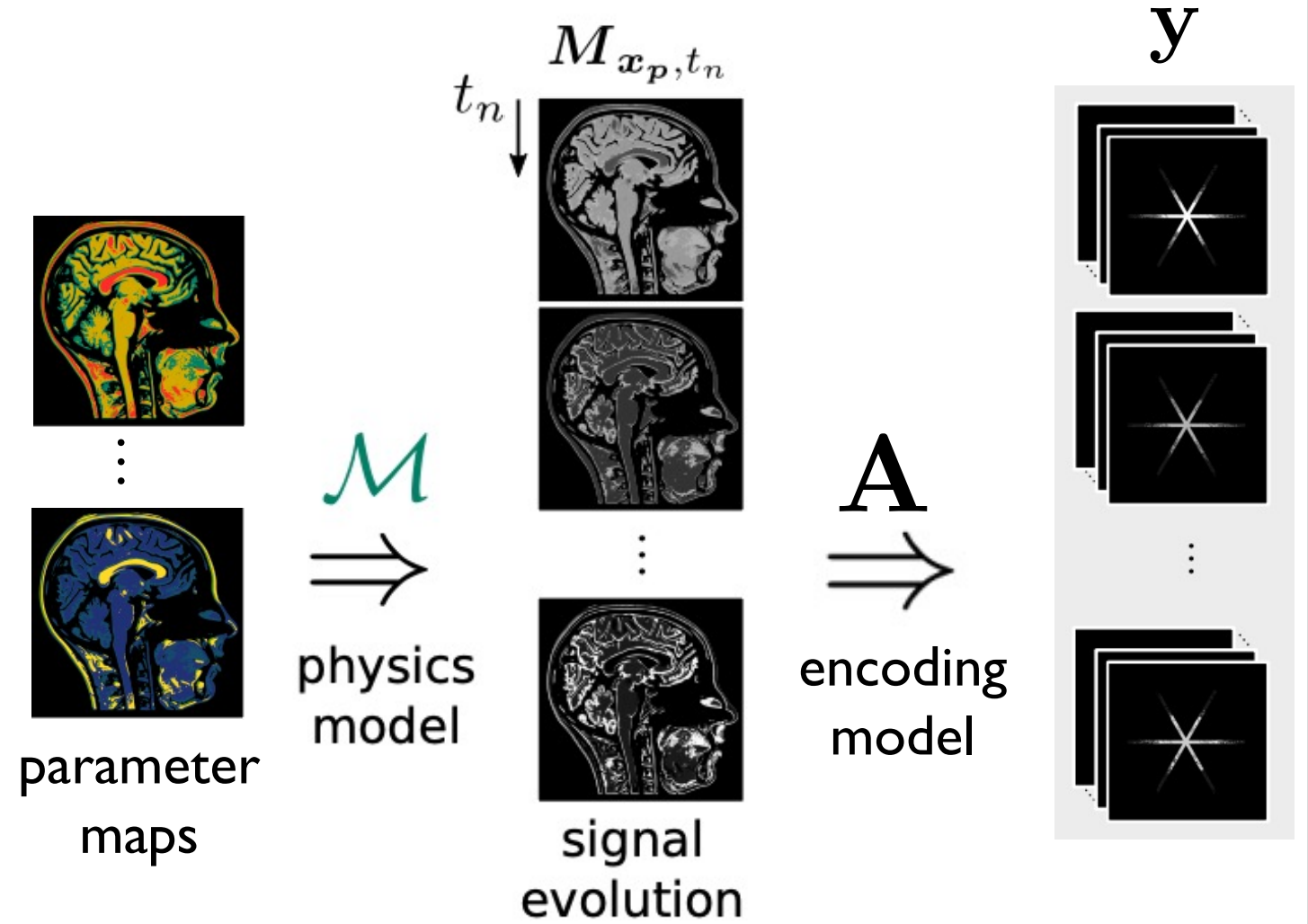


- Measure k-space at multiple time points along the signal evolution
- Reconstruction: **Parallel Imaging and Compressed Sensing (PICS)**

$$\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 + \sum_k f_k(\mathbf{x})$$

- **Solving for many images → highly under-sampled!**

Model-based MRI reconstruction



- Solve directly for parameter maps
- Significant dimensionality reduction
- Non-linear model
  → **cannot use PICS**

$$\mathbf{y} = \mathbf{A}\mathcal{M}(\mathbf{x})$$

Subspace-constrained MRI reconstruction

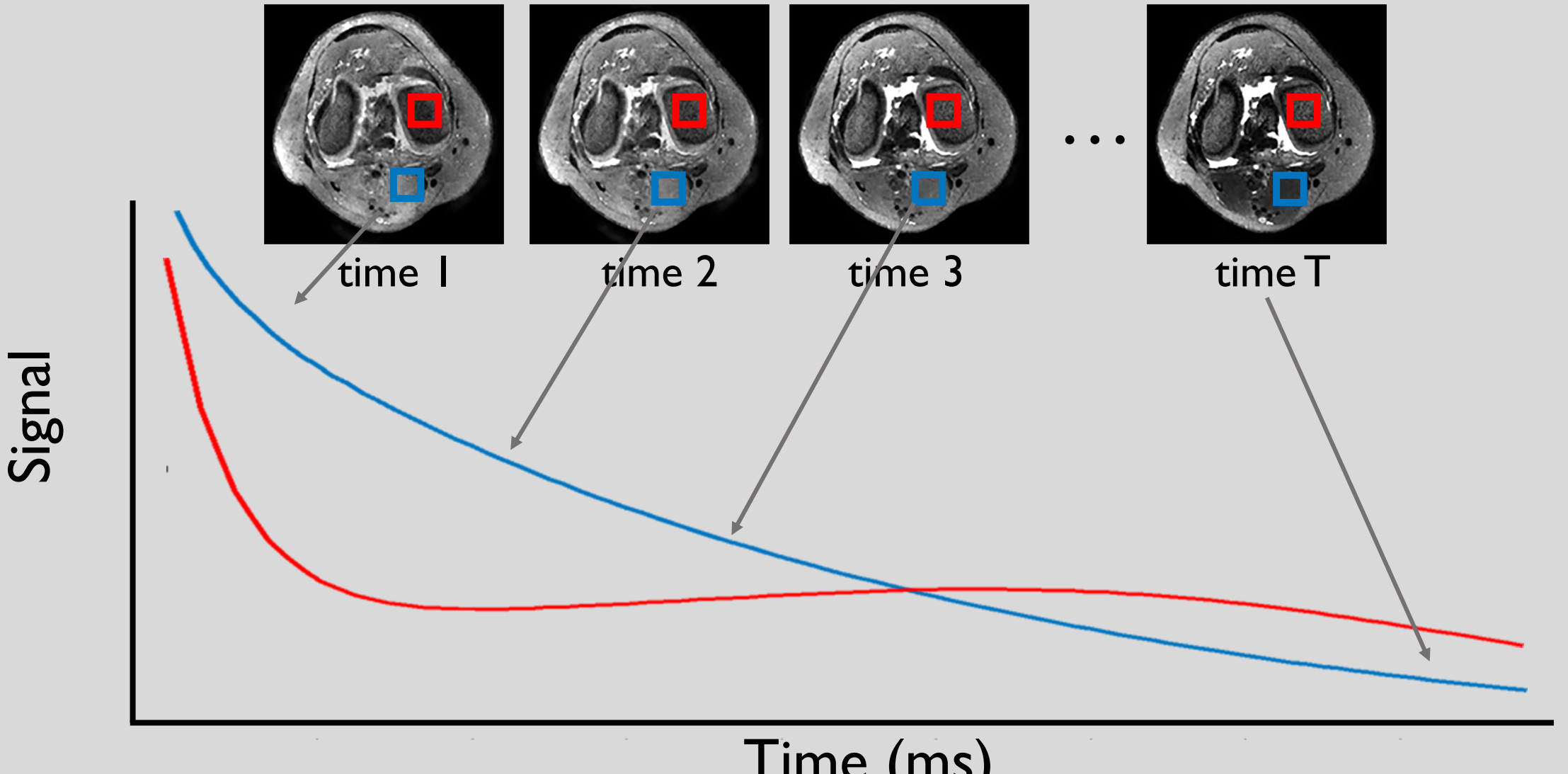
Goal: reduce dimensionality and maintain a linear model

Approach:

- Represent signal evolutions in a linear subspace
- Reconstruct with PICS
- Back-project to get time series of images

Subspace-constrained MRI reconstruction

- Signal evolutions are (often) highly correlated



Subspace-constrained MRI reconstruction

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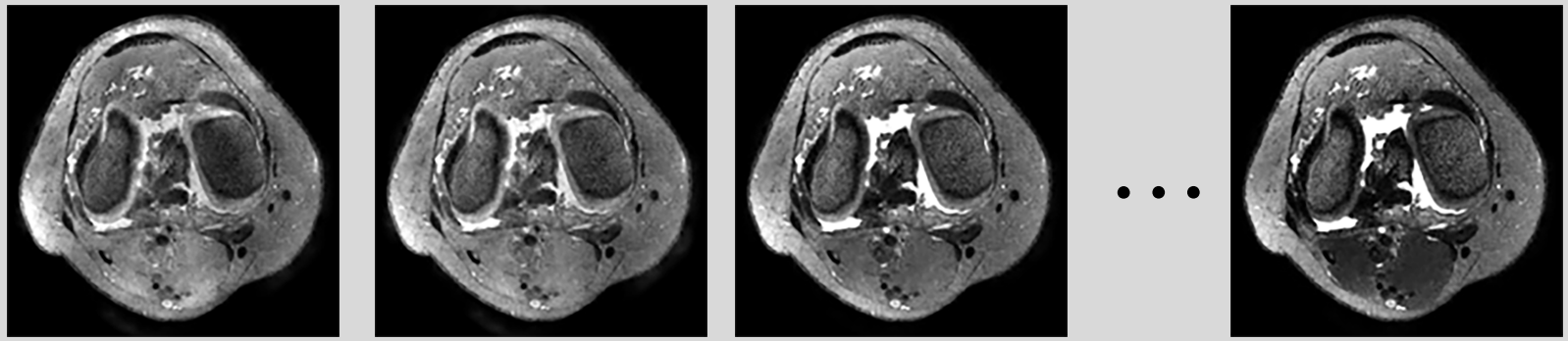
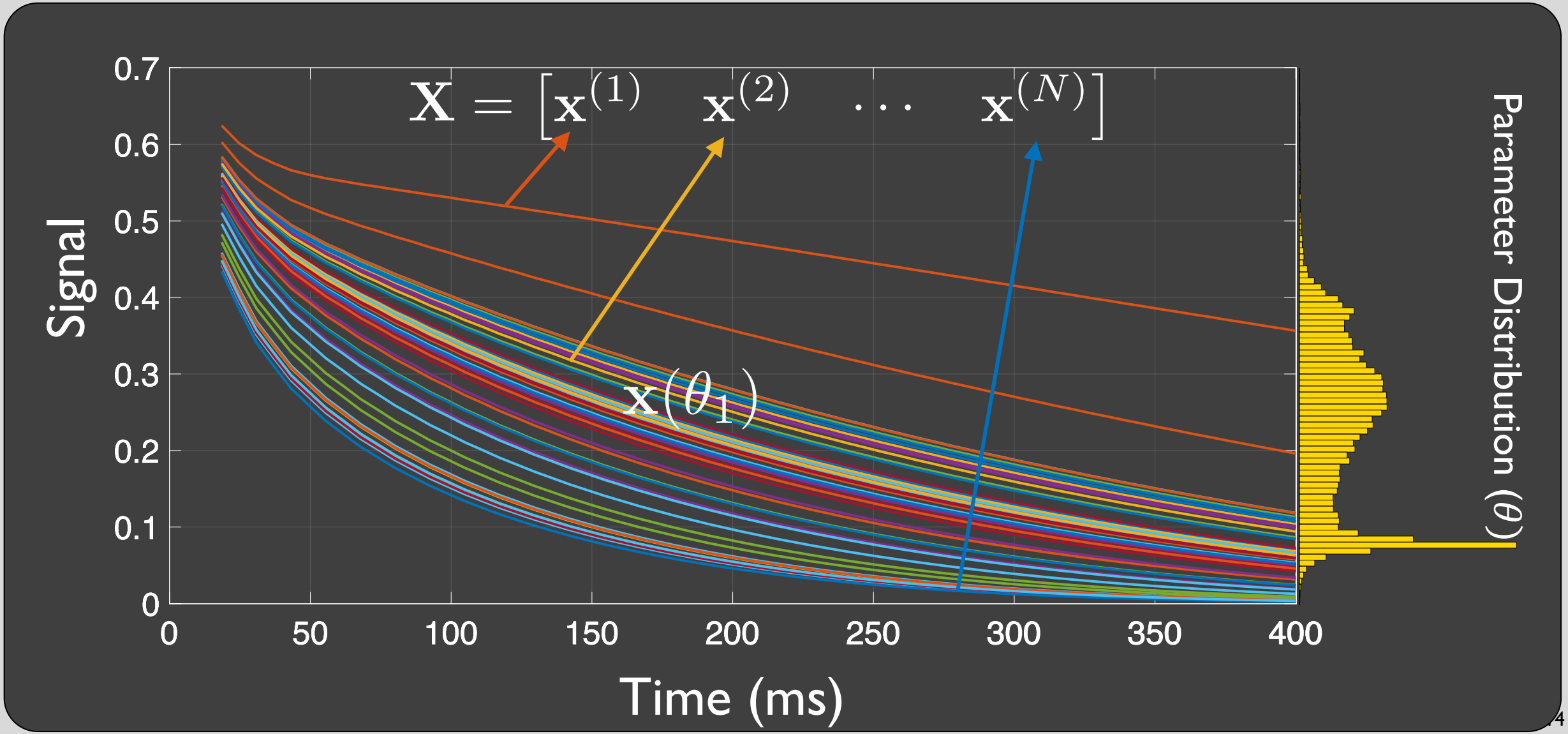
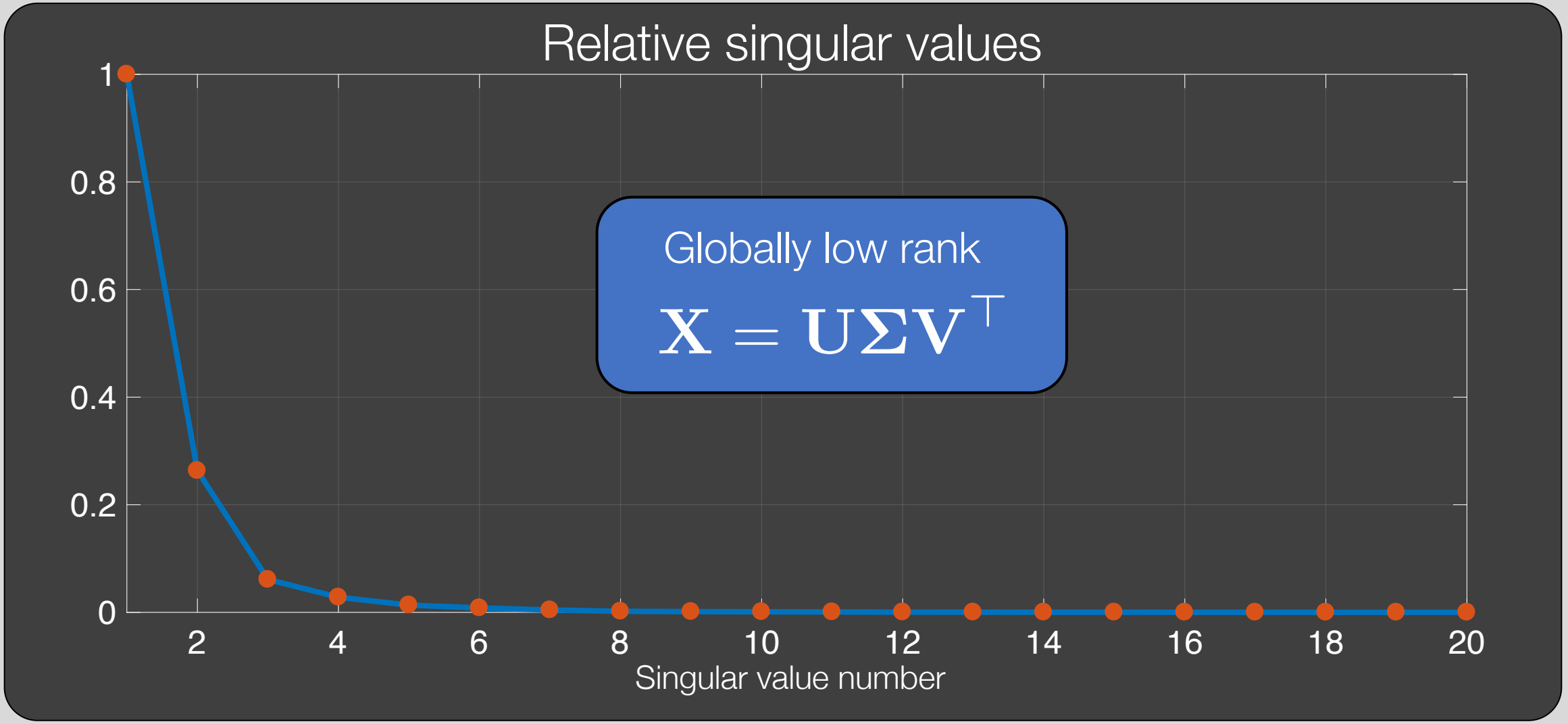


- We can represent time series by a small basis (linear subspace)
  - Fourier basis
  - Polynomial basis
  - Principal component analysis (PCA)

Signal modeling

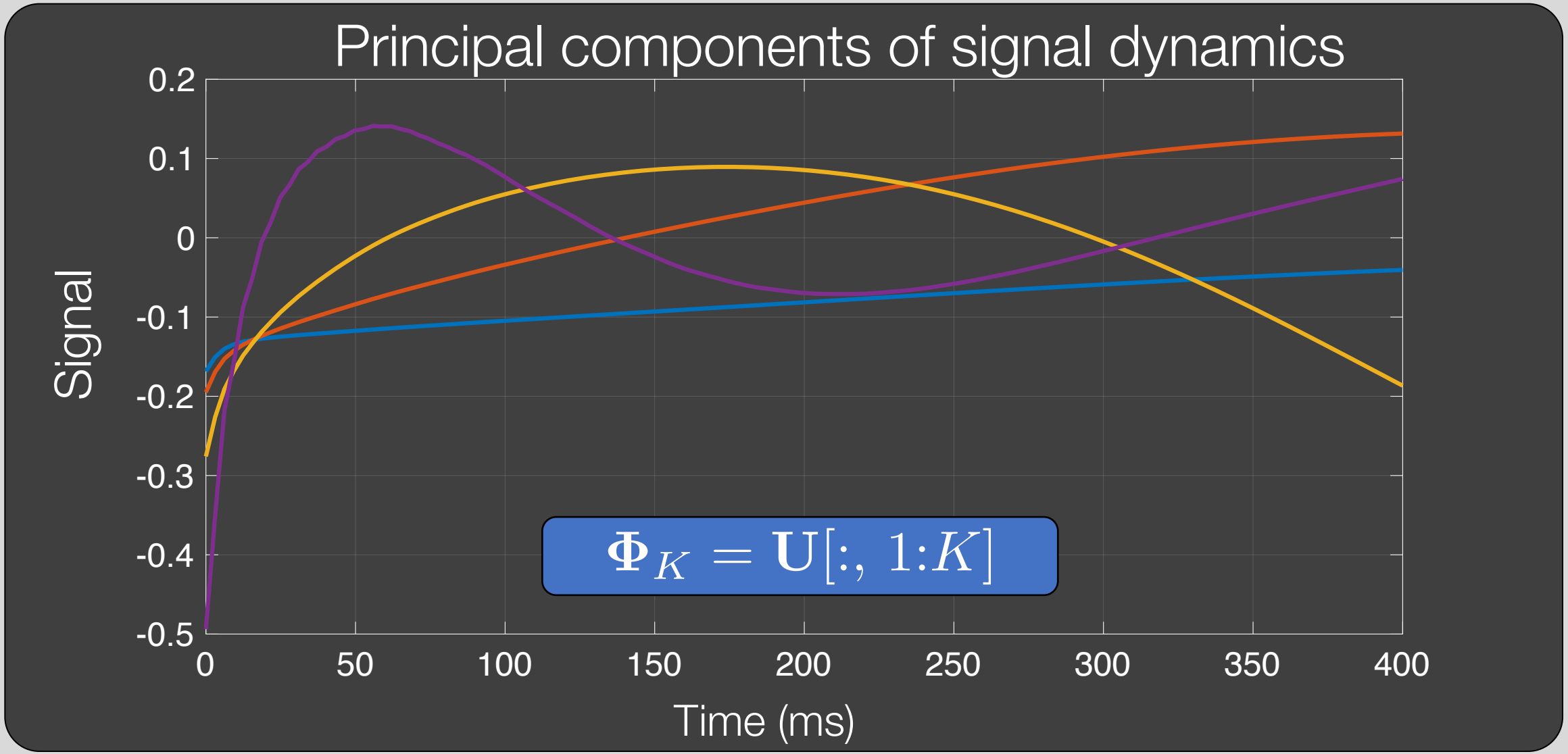


Subspace modeling



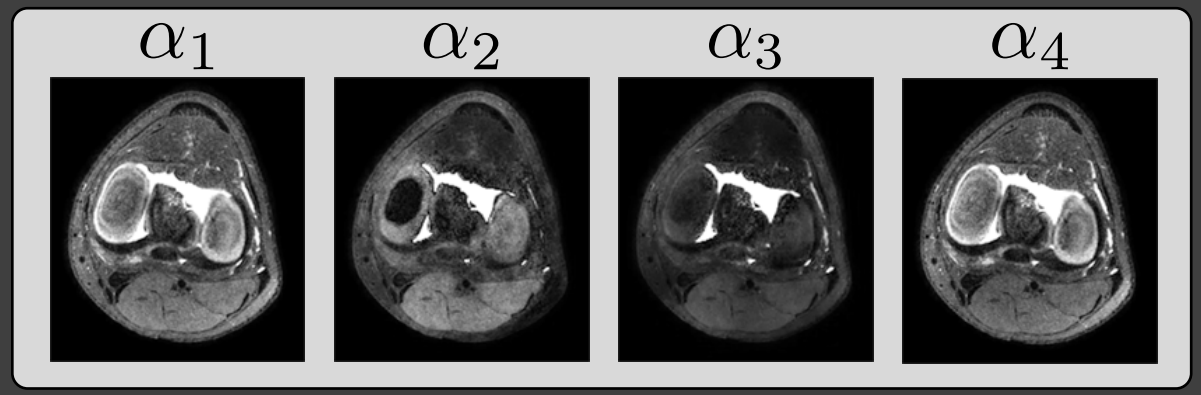
*C. Huang, et al., MRM 2012. doi:10.1002/mrm.23128

Subspace modeling

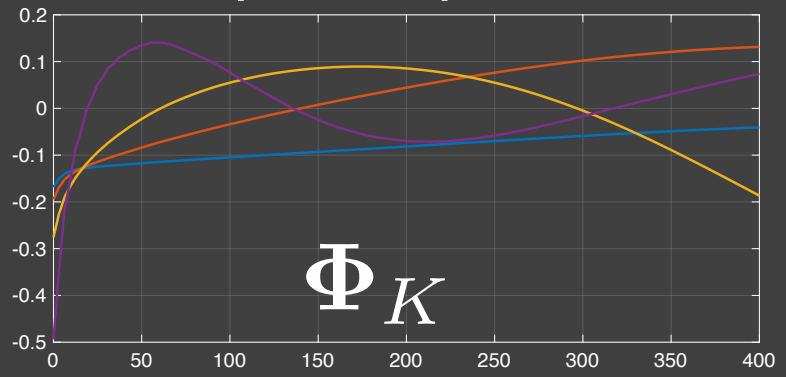


Subspace modeling

Principal component images

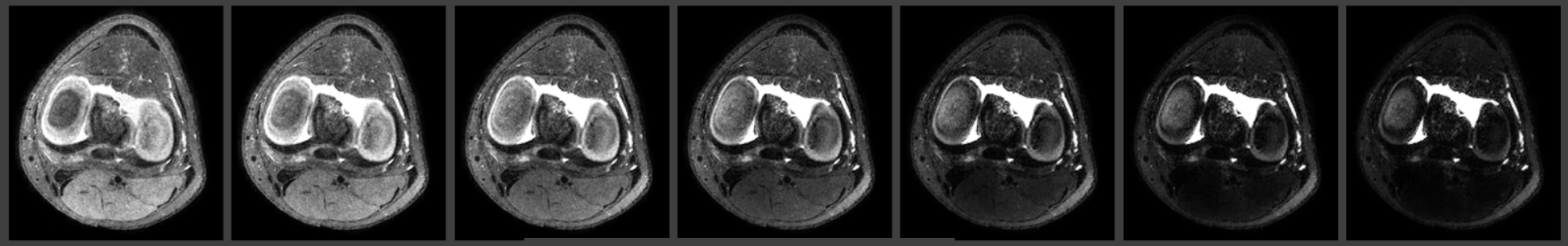


Principal components



$$\mathbf{x} = \Phi_K \alpha$$

Multi-contrast reconstruction

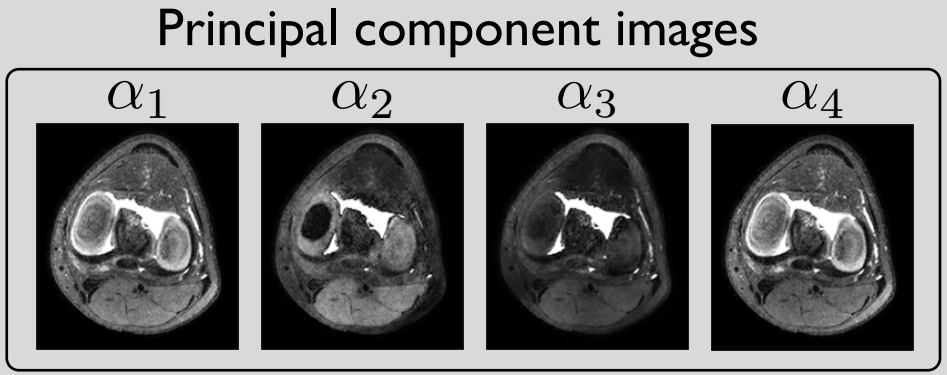


Temporal dynamics

BART: PICS with basis

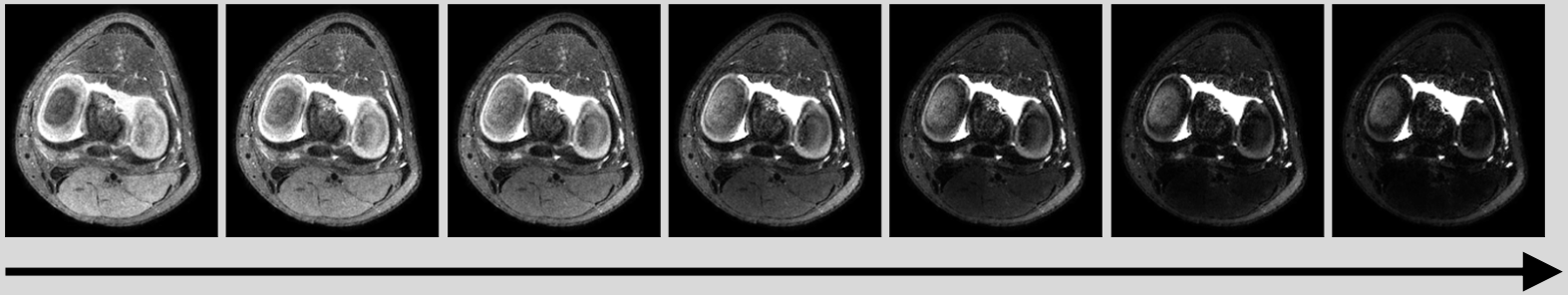
- Same interface as other PICS recons
  - Only difference: **specify the data array containing the basis**

```
bart pics -B basis [...] ksp sens subspace_reco
```




```
bart fmac ...
```

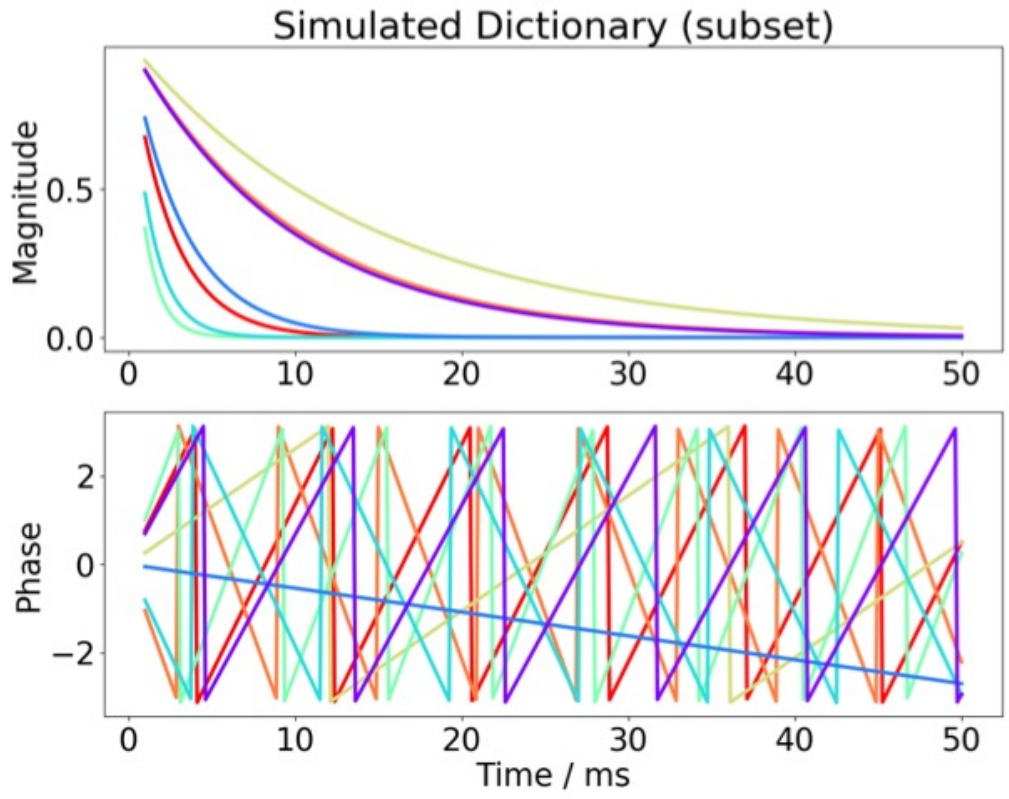
- Back-project:

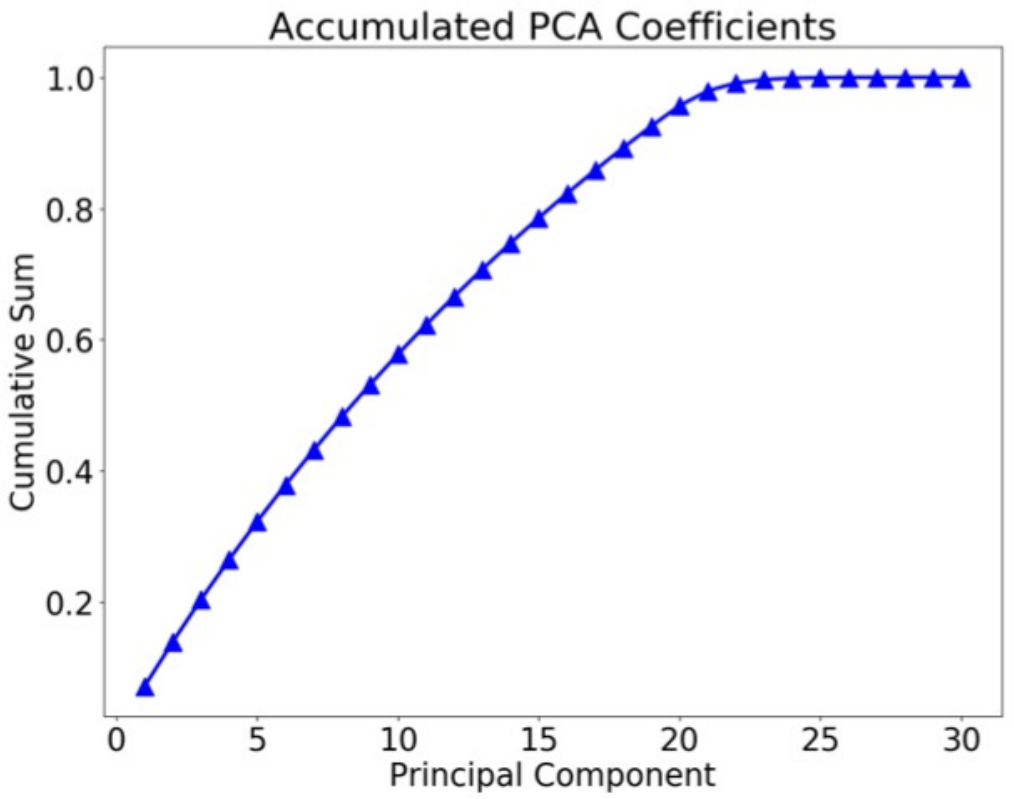


Word of caution

- Not all signal models are low-dimensional

Multi-echo gradient-echo simulation





Subspace constrained reconstruction recipe

1. Evaluate signal model over range of parameters of interest
2. Check and fit to a low-dimensional subspace (basis)
3. Reconstruct with PICS and known basis
4. Back-project to get time-series
5. [Optional] Fit signal to physical model

Today's examples

- Interactive demos
  1. Frequency-modulated SSFP[1] (Volkert Roeloffs)
  2. Analytical multi-contrast phantom simulation (Nick Scholand)
  3. Inversion recovery FLASH[2] (Xiaoqing Wang)

- Hands-on exercise [**Breakout rooms]**

[1]Roeloffs V et al., Magn. Reson. Med. 81:1566–1579 (2019). doi:10.1002/mrm.27505
[2]Pfister J et al., Magn. Reson. Med. 81:3488–3502 (2019). doi:10.1002/mrm.27657