Information Processing Letters 115 (2015) 786–790

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

On Secure Domination in Graphs

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ARTICLE INFO

Article history:
Received 28 January 2015
Received in revised form 13 May 2015
Accepted 15 May 2015
Available online 21 May 2015
Communicated by Ł. Kowalik

Keywords:
Combinatorial problems
Computational complexity
Secure domination
Independence
Domination

ABSTRACT

A set $D \subset V$ of a graph $G = (V, E)$ is a dominating set of G if every vertex not in D is adjacent to at least one vertex in D. A secure dominating set S of a graph G is a dominating set with the property that each vertex $u \in V \setminus S$ is adjacent to a vertex $v \in S$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The secure domination number $\gamma_s(G)$ equals the minimum cardinality of a secure dominating set of G. We first show that the problem of computing the secure domination number is NP-complete for bipartite and split graphs. Then we present bounds relating the secure domination number to the domination number $\gamma(G)$, the independence number $\beta_0(G)$ and the independent domination number $i(G)$. In particular, we prove that $\gamma_s(G) \le \gamma(G) + \beta_0(G) - 1$ if G is an arbitrary graph, $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$ if G is triangle-free, and $\gamma_s(G) \le \beta_0(G)$ if G has girth at least six. Finally, we show for the class of trees T that $\gamma_s(T) \ge i(T)$ and $\gamma_s(T) > \beta_0(T)/2$. The last result answers the question posed by Mynhardt at the 22nd Clemson mini-Conference, 2007

1. Introduction

Let $G = (V, E)$ be a simple graph. The *open neighborhood* of a vertex $v \in V$ is the set $N(v) = \{u \in V : uv \in E\}$ and the *closed neighborhood* of v is the set $N[v] = \{v\} \cup N(v)$. For a set $X \subseteq V$, we denote by $G[X]$ the subgraph of G induced by X. The open and closed neighborhoods of X are $N(X) = \cup_{x \in X} N(x)$ and $N[X] = X \cup N(X)$, respectively. An S-*external private neighbor* of a vertex $v \in S$ is a vertex $u \in V \setminus S$ which is adjacent to v but to no other vertex of S. The set of all S-external private neighbors of $v \in S$ is called the S-*external private neighbor set* of v and is denoted $\mathrm{epn}(v, S)$. The *degree* of v is the cardinality of its open neighborhood. A vertex of degree one is called a *leaf* and its neighbor is called a *stem*.

A vertex in a graph G is said *to dominate* itself and every vertex adjacent to it. A subset D of V is said to be a *dominating set* of G if every vertex not in D is adjacent to at least one vertex in D. The *domination number* $\gamma(G)$ equals the minimum cardinality of a dominating set in G. A set $S \subset V$ is *independent* if no two vertices in S are adjacent. The minimum and maximum cardinalities, respectively, of a maximal independent set in G equal the *independent domination number* $i(G)$ and the *independence number* $\beta_0(G)$, respectively.

A subset $S \subseteq V$ is a 2-*dominating set* if every vertex of $V \setminus S$ has at least two neighbors in S and a *double dominating set* if S is a 2-dominating set and the subgraph induced by S has no isolated vertex. The 2-*domination number* $\gamma_2(G)$ and the *double domination number* $\gamma_{\times 2}(G)$ represent the cardinality of a minimum 2-dominating set and, respectively, of a minimum double dominating set of G. Clearly, $\gamma_{\times 2}(G) \ge \gamma_2(G)$ holds for every graph G with no isolated vertices.

A *secure dominating set* (SDS) S of a graph G is a dominating set with the property that each vertex $u \in V \setminus S$ is adjacent to a vertex $v \in S$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The *secure domination number* $\gamma_s(G)$ equals

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http://dx.doi.org/10.1016/j.ipl.2015.05.006
0020-0190/© 2015 Published by Elsevier B.V.

the minimum cardinality of a secure dominating set of G. Secure domination was introduced by Cockayne et al. [5], and is studied, for example, in [5,7,9]. For any parameter $\mu(G)$ associated with a graph property $\mathcal{P}$, we refer to a set of vertices with Property $\mathcal{P}$ and cardinality $\mu(G)$ as a $\mu(G)$-set.

A *tree* is a connected graph that contains no cycle. We denote the *star* consisting of one central vertex and k leaves as $K_{1,k}$. The *girth* of a graph G is the length of a shortest cycle (if any) in G. If the graph does not contain any cycles, then its girth is defined to be infinity. If a graph G does not contain an induced subgraph that is isomorphic to some graph F, then we say that G is F-*free*. In particular, if $F = K_{1,3}$, we say that G is *claw-free*, and if $F = K_3$, we say that G is *triangle-free*. A *clique* of a graph G is a complete subgraph of G. A graph is *split* if its vertex set can be partitioned into a clique and an independent set.

In this paper, we show that the problem of computing the secure domination number is in the NP-complete class, even when restricted to bipartite graphs and split graphs. Then we prove that for every graph G, $\gamma_s(G) \le \gamma(G) + \beta_0(G) - 1$, which improves a previous bound of Klostermeyer and Mynhardt [7]. Bounds relating the secure domination to the independence number are also presented for triangle-free graphs. More precisely, we show that if G is a triangle-free graph, then $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$ and if G has girth at least six, then $\gamma_s(G) \le \beta_0(G)$. For the class of trees T, we show that $\gamma_s(T) \ge i(T)$ and $\gamma_s(T) > \beta_0(T)/2$.

Before presenting our results, we need to recall some results that are important for our investigations. We begin by giving a fundamental property of secure dominating sets due to Cockayne et al. [5].

Proposition 1.1. *(See Cockayne et al. [5].) If D is a secure dominating set of a graph G, then for every vertex $v \in D$, the subgraph induced by* $\mathrm{epn}(v, D)$ *is complete.*

Theorem 1.2. *For every nontrivial tree T,*

i) *(See Blidia et al. [3].) $\gamma_{\times 2}(T) \ge 2i(T)$.*
ii) *(See Blidia et al. [2].) $\gamma_2(T) \ge \beta_0(T)$, with equality if and only if T has a unique $\gamma_2(T)$-set that also is a $\beta_0(T)$-set.*

2. Complexity results

Our aim in this section is to establish the NP-complete results for the secure domination problem in split graphs and in bipartite graphs. For this purpose, we use a transformation from the domination problem.

DOMINATION PROBLEM (DOM).
Instance: Graph G and a positive integer k.
Question: Does G have a dominating set of cardinality at most k?

Garey and Johnson [6] were the first to show that the domination problem is NP-Complete for arbitrary graphs. Thereafter Bertossi [1] showed that the problem remains NP-Complete even when restricted to bipartite graphs or split graphs.

Now let us consider the following decision problem, to which we shall refer as SDOM.

SECURE DOMINATION PROBLEM (SDOM)
Instance: Graph G^* and a positive integer k^*.
Question: Does G^* have a secure dominating set of cardinality at most k^*?

Theorem 2.1. *Problem SDOM is NP-Complete for split graphs.*

Proof. Clearly, SDOM is a member of $\mathcal{NP}$, since we can check in polynomial time whether or not a set of vertices is a secure dominating set of G. Now let us show how a polynomial time algorithm for SDOM could be used to solve DOM in polynomial time. Given a positive integer k and a split graph G whose vertex set is partitioned into a clique Q and an independent set I, we construct a split graph G^* with clique Q^* and independent set I^* by adding a path $P_2 : x\text{-}y$ such that x is adjacent to all vertices of G. Clearly y is a leaf in G^* and x its stem. It is worth pointing out that G^* is a split graph, where $V(Q^*) = V(Q) \cup \{x\}$ and $I^* = I \cup \{y\}$. We also note that $|V(G^*)| = |V(G)| + 2$ and $|E(G^*)| = |E(G)| + |V(G)| + 1$, and so G^* can be constructed from G in polynomial time.

Next, we shall show that G has a dominating set D with $|D| \le k$ if and only if G^* has a secure dominating set D^* with $|D^*| \le k^* = k + 1$. Let D be a dominating set of size at most k. Then it is evident that $D^* = D \cup \{x\}$ is a secure dominating set of G^* of cardinality $|D| + 1 \le k + 1$. Conversely, suppose that G^* has a secure dominating set D^* with $|D^*| \le k^* = k + 1$. Let $D' = D^* \cap V(G)$. It is clear that $|D^* \cap \{x, y\}| \ge 1$ and hence $\left|D'\right| \le k$. If D' dominates G, then $D = D'$ and we are finished. Suppose now that D' is not a dominating set of G. Let A be the set of vertices of G having no neighbor in D' and let $w \in A$. Since A is securely dominated by D^* in G^*, we deduce that $x \in D^*$ and $A \subset \mathrm{epn}(x, D^*)$. Since $\mathrm{epn}(x, D^*)$ induces a complete graph, by Proposition 1.1, it follows that $y \in D^*$, and so $\left|D'\right| = |D^*| - 2$. Now it is clear that $D = D' \cup \{w\}$ is a dominating set of G with $|D| = \left|D' \cup \{w\}\right| \le (k^* - 2) + 1 = k$. □

Theorem 2.2. *Problem SDOM is NP-Complete for bipartite graphs.*

Proof. Clearly, SDOM is a member of $\mathcal{NP}$. We next show how a polynomial time algorithm for SDOM could be used to solve DOM in polynomial time. Given a bipartite graph G with partite sets X and Y, and a positive integer k, we construct a bipartite graph G^* with partite sets X^* and Y^* by adding two disjoint paths P_2, say x_1-x_2 and y_1-y_2 such that x_1 is adjacent to all X and y_1 is adjacent to all Y. Note that x_2 and y_2 are leaves in G^*. Clearly G^* is a bipartite graph, where $|V(G^*)| = |V(G)| + 4$ and $|E(G^*)| = |E(G)| + |V(G)| + 2$, and so G^* can be constructed from G in polynomial time.

We will show that G has a dominating set D with $|D| \le k$ if and only if G^* has a secure dominating set D^* with $|D^*| \le k^* = k + 2$. Let D be a dominating set of G of size at most k. Then clearly $D^* = D \cup \{x_1, y_1\}$ is a secure dominating set of G^* of cardinality $|D|+2 \le k+2$. Conversely, suppose that G^* has a secure dominating set D^* with $|D^*| \le k^* = k+2$. Let $D' = D^* \cap V(G)$. It is clear that $|D^* \cap \{x_1, x_2\}| \ge 1$ and likewise $|D^* \cap \{y_1, y_2\}| \ge 1$. Hence $\left|D'\right| \le k$. Now if D' dominates G, then $D = D'$ and we are finished. Thus, we assume that D' is not a dominating set of G. Let $B \subset V(G)$ be the set of vertices not dominated by D', and let $B_X = B \cap X$ and $B_Y = B \cap Y$. Without loss of generality, we assume that $B_X \neq \emptyset$. Since B_X is securely dominated by D^* in G^*, we obtain $x_1 \in D^*$. By Proposition 1.1, $x_2 \in D^*$ and so $|B_X| = 1$. Likewise, if $B_Y \neq \emptyset$, then $y_1, y_2 \in D^*$ and $|B_Y| = 1$. Therefore $\left|D'\right| = |D^*| - |B| - 2$. Now it is evident that $D = D' \cup B$ is a dominating set of G with $|D| = \left|D'\right| + |B| = |D^*| - 2 \le k^* - 2 = k$. □

3. Secure domination and independence

In this section we present some bounds relating the secure domination number to the independence number in a graph G. It is worth mentioning that relationships between these two parameters have been already established. Indeed, it was shown by Cockayne et al. [4] that if G is claw-free, then $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$, and if further, G is also C_5-free, then $\gamma_s(G) \le \beta_0(G)$. As we shall show, we extend these two bounds to triangle-free graphs and graphs with girth at least six, respectively. Moreover, Klostermeyer and Mynhardt [7] observed that for every graph G, $\gamma_s(G) \le 2\beta_0(G)$, and posed the problem of finding graphs for which the bound is sharp. In the next, we show that it is possible to improve Klostermeyer and Mynhardt's bound which will explain in some sense why there has been a difficulty finding a graph reaching their bound.

Theorem 3.1. *For every graph G, $\gamma_s(G) \le \gamma(G) + \beta_0(G) - 1$.*

Proof. Let A be a $\gamma(G)$-set, B a maximal independent set of $G[V(G) \setminus A]$ and $C = V \setminus (A \cup B)$. Observe that every vertex in C has at least two neighbors in $A \cup B$, one neighbor in A and another in B. Hence $A \cup B$ is a secure dominating set of G. If $|B| \le \beta_0(G) - 1$, then we are finished, since $\gamma_s(G) \le |A \cup B| \le \gamma(G) + \beta_0(G) - 1$. Hence, we may assume that $|B| = \beta_0(G)$, which implies that all maximal independent sets of $G[V(G) \setminus A]$ have the same size. Since $|B| = \beta_0(G)$, B is a maximum independent set of G. If some vertex $x \in A$ has no neighbor in B, then $B \cup \{x\}$ is an independent set larger than B, a contradiction. We deduce that every vertex of A has a neighbor B. Now let y be a vertex of B. We shall show that $A \cup (B \setminus \{y\}) = F$ is a secure dominating set of G.

Let C_y be the set of vertices of C having y as a unique neighbor in B. Note that if $C_y \neq \emptyset$, then C_y induces a complete graph for otherwise any two non-adjacent vertices of C_y together with $B - \{y\}$ is an independent set larger than B, a contradiction. Since A dominates $V(G)$, let z be a neighbor of y in A. Assume now that F is not a secure dominating set. Then there is a vertex $u \in C \cup \{y\}$ such that for every vertex $w \in N(u) \cap F$, the set $\{u\} \cup F \setminus \{w\}$ is not a dominating set of G. Let R be the set of vertices not dominated by $\{u\} \cup F \setminus \{w\}$. Since every vertex of $C \setminus C_y$ has two neighbors in F, one in A and another in $(B \setminus \{y\})$, we obtain that $u \in C_y \cup \{y\}$. Now if $u = y$, then $w = z$, but then all $C \cup \{z\}$ is dominated by $B \subset \{u\} \cup F \setminus \{w\}$, a contradiction. Thus $u \in C_y$, and so $w \in A$. Recall that since $C_y \neq \emptyset$, C_y induces a complete graph and so u dominates all $C_y \cup \{y\}$. Since every vertex of $C \setminus C_y$ has a neighbor in $B \setminus \{y\}$, we obtain that $R \subset C_y$. But then R is dominated by u, a contradiction. Consequently, $F = A \cup (B \setminus \{y\})$ is a secure dominating set of G, implying that $\gamma_s(G) \le |A \cup (B - \{y\})| = \gamma(G) + \beta_0(G) - 1$. □

Note that the bound of Theorem 3.1 is sharp for the cycle C_5 and for complete graphs of order at least two. Since $\gamma(G) \le \beta_0(G)$ for every graph G, we obtain the following.

Corollary 3.2. *(See Klostermeyer et al. [7].) For every graph G, $\gamma_s(G) < 2\beta_0(G)$.*

3.1. Triangle-free graphs

Theorem 3.3. *For every connected triangle-free graph G, $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$.*

Proof. Let X be a $\beta_0(G)$-set, $X_1 = \{v \in X : \mathrm{epn}(v, X) \neq \emptyset\}$ and $X_2 = X \setminus X_1$. Note that for every $v \in X_1$, $G[\mathrm{epn}(v, X)]$ is complete for otherwise any two non-adjacent vertices of $\mathrm{epn}(v, X)$ union $X \setminus \{v\}$ is an independent set of G larger than X, a contradiction. Moreover, since G is triangle-free, we deduce that $|\mathrm{epn}(v, X)| = 1$ for every $v \in X_1$. Let $A = \bigcup_{v \in X_1} \mathrm{epn}(v, X)$. Clearly, $|A| = |X_1|$.

If X is a secure dominating set for G, then $\gamma_s(G) \le |X| = \beta_0(G) < \frac{3}{2}\beta_0(G)$. Hence we assume that X is not a secure dominating set of G. Thus there is a non-empty set C in $V \setminus X$ such that for every $x \in C$ and for every $x' \in N(x) \cap X$, the set $\{x\} \cup X - \{x'\}$ is not a dominating set. Clearly, $C \cap A = \emptyset$ and so every vertex of C has at least two neighbors in X. Observe that if a vertex $x \in C$ has a neighbor $y \in X_2$, then $\{x\} \cup X \setminus \{y\}$ is a dominating set of G, a contradiction. Thus $N(x) \subset X_1$ for every $x \in C$.

Now consider a vertex $u \in C$ and let $X_{1,u} = N(u) \cap X_1$. Note that $\left|X_{1,u}\right| \ge 2$ and u is not adjacent to the neighbors of $X_{1,u}$, since G is triangle-free. If $\bigcup_{v \in X_{1,u}} \mathrm{epn}(v, X) = A_u$ is independent, then $\left(X \setminus X_{1,u}\right) \cup A_u \cup \{u\}$ is an independent set of G larger than X, a contradiction. Therefore for every $u \in C$, the subgraph induce by A_u contains at least one edge. Now let H be the set of all adjacent vertices in $\bigcup_{u \in C} A_u$, and let D be a minimum dominating set of the subgraph induced by H. Observe that $|H| \le |A| = |X_1|$, and hence $|D| \le |A|/2 = |X_1|/2$. Now it is evident that every $u \in C$ has a neighbor in $X_{1,u}$, say y, where $\mathrm{epn}(y, X)$ is dominated by D. Consequently, $X \cup D$ is a secure dominating set of G, implying that $\gamma_s(G) \le |X| + |D| \le |X| + |X_1|/2 \le \frac{3}{2}|X|$. Therefore $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$. □

The following corollary immediately follows from Theorem 3.3.

Corollary 3.4. *For every bipartite graph G, $\gamma_s(G) \le \frac{3}{2}\beta_0(G)$.*

Theorem 3.5. *If G is a connected graph with girth $g(G) \geq 6$, then $\gamma_s(G) \leq \beta_0(G)$.*

Proof. Let X be a maximum independent set of G. Clearly $V \setminus X \neq \emptyset$. Recall that for any $v \in X$, if $\text{epn}(v, X) \neq \emptyset$, then $\text{epn}(v, X)$ induces a complete graph. Also, since $g(G) \geq 6$, $\text{epn}(v, X)$ is independent and so $|\text{epn}(v, X)| = 1$. We shall show that X is a secure dominating set of G. Let x be any vertex not in X. If $x \in \text{epn}(y, X)$ for some vertex $y \in X$, then clearly $\{x\} \cup X \setminus \{y\}$ is a dominating set of G. Hence we can assume that x has at least two neighbors in X. If x has a neighbor $z \in X$ such that $\text{epn}(z, X) = \emptyset$, then $\{x\} \cup X \setminus \{z\}$ is a dominating set of G. Hence we assume that each neighbor of x in X has a non-empty X-external private neighbor set in $V \setminus X$. Let $X_x = N(x) \cap X$. Note that since $g(G) \geq 6$, x is adjacent to no vertex of $\text{epn}(u, X)$ for every $u \in X_x$. Also, no edge exists between $\text{epn}(u, X)$ and $\text{epn}(w, X)$ for every pair $u, w \in X_x$ (for otherwise G has a cycle C_5). Let $A = \bigcup_{v \in X_x} \text{epn}(v, X)$. According to the previous facts, $A \cup \{x\}$ is independent. But then using the fact that $|X_x| = |A|$, we obtain that $(X \setminus X_x) \cup A \cup \{x\}$ is an independent set of G larger than X, a contradiction. Hence X is a secure dominating set of G, and so $\gamma_s(G) \leq \beta_0(G)$. $\square$

As an immediate consequence, we obtain the following.

Corollary 3.6. *For every tree T, $\gamma_s(T) \leq \beta_0(T)$.*

We note that the difference $\beta_0(G) - \gamma_s(G)$ may be arbitrary large for graphs with girth at least 6. This can be seen by the graph G_k obtained from k ($k \geq 2$) cycles C_6 sharing the same vertex. One can easily see that $\gamma_s(G_k) = 2k + 1$ while $\beta_0(G_k) = 3k$.

3.2. Trees

In what follows, we answer in the affirmative an open question posed by Mynhardt [8] in her talk presented at the 22nd Clemson mini-Conference, 2007. The content of the presentation can be found in http://people.cs.clemson.edu/~goddard/MINI/2007/.

Question 3.7. *Is it true that $\gamma_s(T) > \beta_0(T)/2$ for all trees T?*

We first need to prove the following useful result.

Proposition 3.8. *For every graph G without isolated vertices, $\gamma_{\times 2}(G) \leq 2\gamma_s(G)$.*

Proof. Let D be a $\gamma_s(G)$-set. Let $D' = \{x \in D : \text{epn}(x, D) \neq \emptyset\}$ and $D'' = D \setminus D'$. Recall that by Proposition 1.1, for every $x \in D'$, $\text{epn}(x, D)$ induces a complete graph. Let $A = \bigcup_{x \in D'} \text{epn}(x, D)$ and $B = (V \setminus D) \setminus A$. Clearly every vertex of B has at least two neighbors in D. Let A' be a subset of A containing exactly one vertex of each $\text{epn}(x, D)$ for every $x \in D'$. Thus $|A'| = |D'|$. Note that every vertex of $A \setminus A'$ has at least two neighbors in $D' \cup A'$. Let B' be the smallest subset of $(V \setminus D) \setminus A'$ that dominates D''. Since every vertex of D'' has no private neighbor in $V \setminus D$, we obtain $|B'| \leq |D''|$. From above, $D \cup A' \cup B'$ is a double dominating set of G. It follows that $\gamma_{\times 2}(G) \leq |D| + |B'| + |A'| \leq |D| + |D''| + |D'| = 2\gamma_s(G)$. $\square$

Now we are ready to answer Question 3.7.

Proposition 3.9. *For every tree T, $\gamma_s(T) > \beta_0(T)/2$.*

Proof. Obviously, the result is valid if T has order $n = 1$. Hence we assume that $n \geq 2$. By Proposition 3.8 and Theorem 1.2-(ii), we have

$$2\gamma_s(T) \geq \gamma_{\times 2}(T) \geq \gamma_2(T) \geq \beta_0(T).$$

Now if $2\gamma_s(T) > \beta_0(T)$, then we are finished. Hence assume that $2\gamma_s(T) = \beta_0(T)$. Then we have equality throughout this inequality chain. In particular, $\gamma_{\times 2}(T) = \gamma_2(T) = \beta_0(T)$. By Theorem 1.2-(ii), T has a unique $\gamma_2(T)$-set, say D, that also is a $\beta_0(T)$-set. Since D is independent, D is not a double dominating set of T. But then any $\gamma_{\times 2}(T)$-set is also a $\gamma_2(T)$-set which contradicts the unicity of D. $\square$

We note that the bound in Proposition 3.9 is not valid for every graph. This may be seen by considering the graph G obtained from a star $K_{1,t}$ ($t \geq 5$) by introducing a new vertex, which is connected to every leaf of the star. Clearly $\gamma_s(G) = 2 < \beta_0(G)/2 = t/2$.

We finish by showing that the independent domination number is a lower bound for the secure domination number in the class of trees. Then we list two open problems.

Proposition 3.10. *For every tree T, $\gamma_s(T) \geq i(T)$, and this bound is sharp.*

Proof. Since the result holds if T is trivial, we assume that T has order at least two. By Proposition 3.8 and Theorem 1.2-(i), we have $2\gamma_s(T) \geq \gamma_{\times 2}(T) \geq 2i(T)$, and so $\gamma_s(T) \geq i(T)$.

That this bound is sharp may be seen by a tree T, where every vertex of T is a leaf or a stem and every stem is adjacent to exactly one leaf. $\square$

Problem 3.11. *Characterize the trees T with $\gamma_s(T) = i(T)$.*

Problem 3.12. *Is it true that $\gamma_s(G) \leq \frac{3}{2}\beta_0(G)$ for graphs G having triangles?*

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