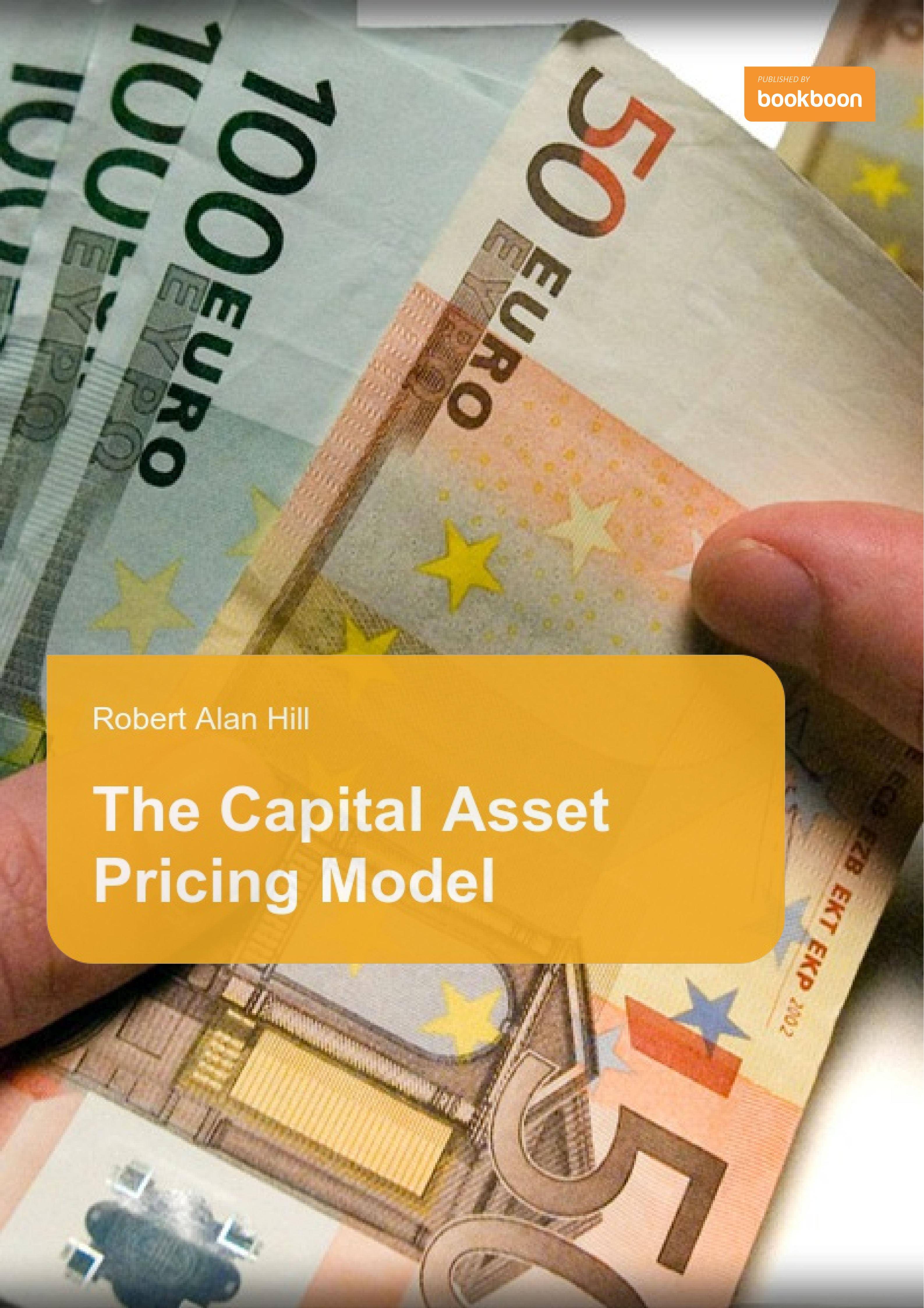

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Robert Alan Hill

The Capital Asset Pricing Model

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About the Author

With an eclectic record of University teaching, research, publication, consultancy and curricula development, underpinned by running a successful business, Alan has been a member of national academic validation bodies and held senior external examinerships and lectureships at both undergraduate and postgraduate level in the UK and abroad.

With increasing demand for global e-learning, his attention is now focussed on the free provision of a financial textbook series, underpinned by a critique of contemporary capital market theory in volatile markets, published by bookboon.com.

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1 The Beta Factor

Introduction

In an ideal world, the portfolio theory of Markowitz (1952) should provide management with a practical model for measuring the extent to which the pattern of returns from a new project affects the risk of a firm's existing operations. For those playing the stock market, portfolio analysis should also reveal the effects of adding new securities to an existing spread. The objective of efficient portfolio diversification is to achieve an overall standard deviation lower than that of its component parts without compromising overall return.

However, if you've already read "Portfolio Theory and Investment Analysis" (*PTIA*) 2. edition, 2014, by the author, the calculation of the covariance terms in the risk (variance) equation becomes unwieldy as the number of portfolio constituents increase. So much so, that without today's computer technology and software, the operational utility of the basic model is severely limited. Academic contemporaries of Markowitz therefore sought alternative ways to measure investment risk

This began with the realisation that the *total risk* of an investment (the standard deviation of its returns) within a diversified portfolio can be divided into *systematic* and *unsystematic* risk. You will recall that the latter can be eliminated entirely by efficient diversification. The other (also termed *market* risk) cannot. It therefore affects the overall risk of the portfolio in which the investment is included.

Since all rational investors (including management) interested in wealth maximisation should be concerned with individual security (or project) risk relative to the stock market as a whole, portfolio analysts were quick to appreciate the importance of systematic (market) risk. According to Tobin (1958) it represents the only risk that they will pay a premium to avoid.

Using this information and the assumptions of perfect markets with opportunities for risk-free investment, the required return on a risky investment was therefore redefined as the risk-free return, plus a premium for risk. This premium is not determined by the total risk of the investment, but only by its systematic (market) risk.

Of course, the systematic risk of an individual financial security (a company's share, say) might be higher or lower than the overall risk of the market within which it is listed. Likewise, the systematic risk for some projects may differ from others within an individual company. And this is where the theoretical development of the beta factor (β) and the Capital Asset Pricing Model (CAPM) fit into portfolio analysis.

We shall begin by defining the relationship between an individual investment's systematic risk and market risk measured by (β_j) its *beta* factor (or coefficient). Using *earlier notation* and continuing with the *equation numbering* from the *PTIA* text which ended with Equation (32):

$$(33) \qquad \beta_j = \frac{COV(j,m)}{VAR(m)}$$

This factor equals the covariance of an investment's return, relative to the market portfolio, divided by the variance of that portfolio.

As we shall discover, beta factors exhibit the following characteristics:

The market as a whole has a $\beta = 1$
A risk-free security has a $\beta = 0$
A security with systematic risk below the market average has a $\beta < 1$
A security with systematic risk above the market average has a $\beta > 1$
A security with systematic risk equal to the market average has a $\beta = 1$

> The significance of a security's β value for the purpose of stock market investment is quite straightforward. If overall returns are expected to fall (a *bear* market) it is worth buying securities with low β values because they are expected to fall less than the market. Conversely, if returns are expected to rise generally (a *bull* scenario) it is worth buying securities with high β values because they should rise faster than the market.

Ideally, beta factors should reflect *expectations* about the *future* responsiveness of security (or project) returns to corresponding changes in the market. However, without this information, we shall explain how individual returns can be compared with the market by plotting a *linear* regression line through *historical* data.

Armed with an operational measure for the market price of risk (β), in Chapter Two we shall explain the rationale for the Capital Asset Pricing Model (CAPM) as an alternative to Markowitz theory for constructing efficient portfolios.

For any investment with a beta of β_j, its expected return is given by the CAPM *equation*:

$$(34) \qquad r_j = r_f + (r_m - r_f) \beta_j$$

Similarly, because all the characteristics of systematic betas apply to a *portfolio*, as well as an *individual* security, any portfolio return (r_p) with a portfolio beta (β_p) can be defined as:

(35) $r_p = r_f + (r_m - r_f)\beta_p$

For a given a level of systematic risk, the CAPM determines the expected rate of return for any investment relative to its beta value. This equals the risk-free rate of interest, *plus* the product of a market risk premium and the investment's beta coefficient. For example, the mean return on equity that provides adequate compensation for holding a share is the value obtained by incorporating the appropriate equity beta into the CAPM equation.

> The CAPM can be used to estimate the expected return on a security, portfolio, or project, by investors, or management, who desire to eliminate unsystematic risk through efficient diversification and assess the required return for a given level of non-diversifiable, systematic (market) risk. As a consequence, they can tailor their portfolio of investments to suit their individual risk- return (utility) profiles.

Finally, in Chapter Two we shall validate the CAPM by reviewing the balance of empirical evidence for its application within the context of capital markets.

In Chapter Three we shall then focus on the CAPM's operational relevance for strategic financial management within a corporate capital budgeting framework, characterised by capital gearing. And as we shall explain, the stock market CAPM can be modified to derive a project discount rate based on the systematic risk of an individual investment. Moreover, it can be used to compare different projects across different risk classes.

At the end of Chapter Three, you should therefore be able to confirm that:

> The CAPM not only represents a viable alternative to managerial investment appraisal techniques using NPV wealth maximisation, mean-variance analysis, expected utility models and the WACC concept. It also establishes a mathematical connection with the seminal leverage theories of Modigliani and Miller (MM 1958 and 1961).

1.1 Beta, Systemic Risk and the Characteristic Line

Suppose the price of a share selected for inclusion in a portfolio happens to increase when the equity market rises. Of prime concern to investors is the extent to which the share's total price increased because of unsystematic (specific) risk, which is diversifiable, rather than systematic (market) risk that is not.

A practical solution to the problem is to isolate systemic risk by comparing past trends between individual share price movements with movements in the market as a whole, using an appropriate all-share stock market index.

So, we could plot a "scatter" diagram that correlates percentage movements for:

- The selected share price, on the vertical axis,
- Overall market prices using a relevant index on the horizontal axis.

The "spread" of observations equals unsystematic risk. Our line of "best fit" represents systematic risk determined by *regressing* historical share prices against the overall market over the time period. Using the statistical method of *least squares*, this linear regression is termed the share's *Characteristic Line.*

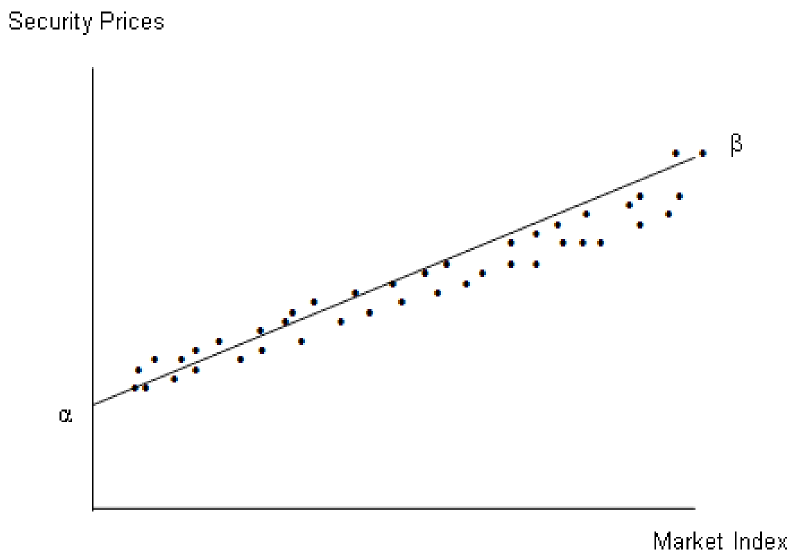

Figure 1.1: The Relationship between Security Prices and Market Movements The Characteristic Line

As Figure 1.1 reveals, the *vertical intercept* of the regression line, termed the *alpha factor* (α) measures the average percentage movement in share price if there is no movement in the market. It represents the amount by which an individual share price is greater or less than the market's systemic risk would lead us to expect. A positive alpha indicates that a share has outperformed the market and *vice versa.*

The *slope* of our regression line in relation to the horizontal axis is the *beta factor* (β) measured by the share's covariance with the market (rather than individual securities) divided by the variance of the market. This calibrates the *volatility* of an individual share price relative to market movements, (more of which later). For the moment, suffice it to say that the steeper the Characteristic Line the more volatile the share's performance and the higher its systematic risk. Moreover, if the slope of the Characteristic Line is very steep, β will be greater than 1.0. The security's performance is volatile and the systematic risk is high. If we performed a similar analysis for another security, the line might be very shallow. In this case, the security will have a low degree of systematic risk. It is far less volatile than the market portfolio and β will be less than 1.0. Needless to say, when β equals 1.0 then a security's price has "tracked" the market as a whole and exhibits *zero* volatility.

The beta factor has two further convenient statistical properties applicable to investors generally and management in particular.

First, it is a far simpler, computational proxy for the covariance (relative risk) in our original Markowitz portfolio model. Instead of generating numerous new covariance terms, when portfolio constituents (securities-projects) increase with diversification, all we require is the covariance on the additional investment relative to the efficient market portfolio.

Second, the Characteristic Line applies to investment *returns*, as well as *prices*. All risky investments with a market price must have an expected return associated with risk, which justify their inclusion within the market portfolio that all risky investors are willing to hold.

Activity 1

If you read different financial texts, the presentation of the Characteristic Line is a common source of confusion. Authors often define the axes differently, sometimes with prices and sometimes returns.

Consider Figure 1.2, where *returns* have been substituted for the *prices* of Figure 1.1. Does this affect our linear interpretation of alpha and beta?

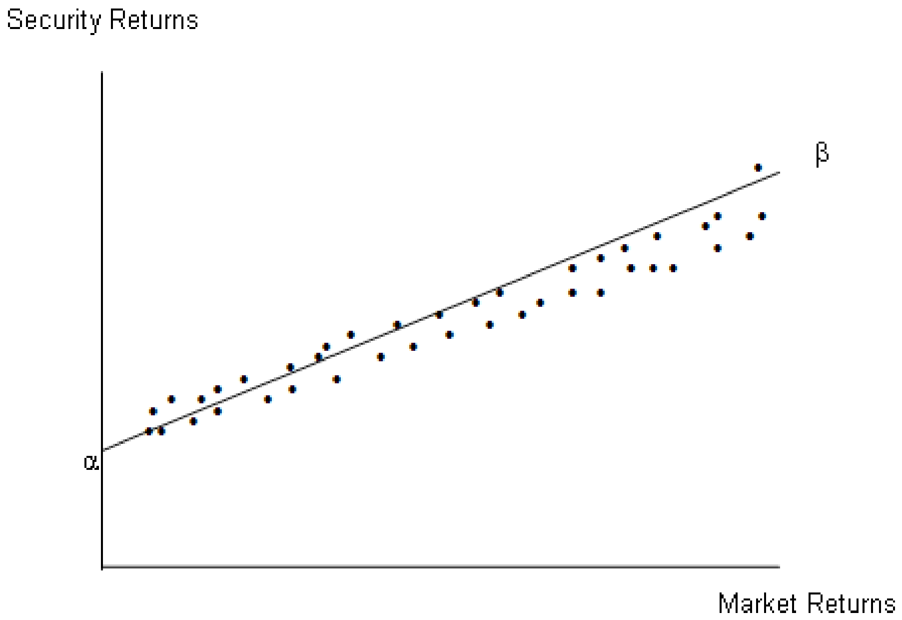

Figure 1.2: The Relationship between Security Returns and Market Returns The Characteristic Line

The substitution of returns for prices in the regression doesn't affect our interpretation of the graph, because returns obviously determine prices.

- The horizontal intercept (α) now measures the extent to which *returns* on an investment are greater or less than those for the market portfolio.
- The steeper the slope of the Characteristic Line, then the more volatile the return, the higher the systematic risk (β) and *vice versa*.

We began by graphing the security prices of risky investments and total market capitalisation using a stock market index because it serves to remind us that the development of Capital Market Theory initially arose from portfolio theory as a pricing model. However, because theorists discovered that returns (like prices) can also be correlated to the market, with important consequences for internal management decision making, as well as stock market investment, many modern texts focus on returns and skip pricing theory altogether.

Henceforth, we too, shall place increasing emphasis on returns to set the scene for Chapter Three. There our ultimate concern will relate to strategic financial management and an optimum project selection process derived from models of capital asset pricing using β factors for individual companies that provide the highest expected return in terms of investor attitudes to the risk involved.

1.2 The Mathematical Derivation of Beta

So far, we have only explained a beta factor (β) by reference to a *graphical* relationship between the pricing or return of an individual security's risk and overall market risk. Let us now derive *mathematical* formulae for β by adapting our *earlier notation* and continuing with the *equation numbering* from previous Chapters of the *PTIA* text.

Suppose an individual was to place all their investment funds in all the financial securities that comprise the global stock market in proportion to the individual value of each constituent relative to the market's total value.

The market portfolio has a variance of VAR(m) and the covariance of an individual security j with the market average is COV(j,m). So, the relative risk (the security's beta) denoted by β_j is given by our earlier equation:

$$(33) \qquad \beta_j = \frac{\mathrm{COV}(j,\mathrm{m})}{\mathrm{VAR(m)}}$$

Alternatively, we know from Chapter Two of the *PTIA* text that given the relationship between the covariance and the *linear* correlation coefficient, the covariance term in Equation (33) can be rewritten as:

$$\mathrm{COV}\ (j,\mathrm{m}) = \mathrm{COR}\ (j,\mathrm{m}).\ \sigma\, j\ \sigma\ \mathrm{m}$$

So, we can also define a theoretical value for beta as follows:

$$\beta_j = \frac{\mathrm{COR}(j,\mathrm{m})\ .\ \sigma\, j\ \sigma\ \mathrm{m}}{\sigma^2(\mathrm{m})}$$

And simplifying:

$$(36) \qquad \beta_j = \frac{\mathrm{COR}(j\ \mathrm{m})\ \sigma\, j}{\sigma\ (\mathrm{m})}$$

If information on the variance or standard deviation and covariance or correlation coefficient is readily available, the calculation of beta is extremely straightforward using either equation. Ideally, we should determine β using *forecast* data (in order to appraise *future* investments). In its absence, however, we can derive an *estimator* using least-squares regression. This plots a security's *historical* periodic return against the corresponding return for the appropriate market index.

$$\mathrm{r}_t = \frac{\text{Increase in the period's ex-div value per share + the dividend per share paid}}{\text{Share value at the beginning of the period}}$$

Obviously it needs to be adjusted for events such as bonus or rights issues and any capital reorganisation-reconstruction. Fortunately, because of their ease of calculation, β estimators are published regularly by the financial services industry for stock exchange listings world-wide. A particularly fine example is the London Business School Risk Management Service (LBSRMS) that supplies details of equity betas, which are also geared up (leveraged) according to the firm's capital structure (more of which later in Chapter Seven).

Given the universal, freely available publication of beta factors, considerable empirical research on their behaviour has been undertaken over a long period of time. So much so, that as a measure of systematic risk they are now known to exhibit another extremely convenient property (which also explains their popularity within the investment community).

Although alpha risk varies considerably over time, numerous studies (beginning with Black, Jensen and Scholes in 1972) have continually shown that beta values are more stable. They move only slowly and display a near *straight-line* relationship with their returns. The longer the period analysed, the better. The more data analysed, the better. Thus, betas are invaluable for efficient portfolio selection. Investors can tailor a portfolio to their specific risk-return (utility) requirements, aiming to hold *aggressive* stocks with a β in excess of one while the market is rising, and less than one (*defensive*) when the market is falling.

Activity 2

Explain the investment implications of a beta factor of 1.15 and a beta factor that is less than the market portfolio

A beta of 1.15 implies that if the underlying market with a beta factor of one were to rise by 10 per cent, then the stock may be expected to rise by 11.5 per cent. Conversely, a security with a beta of less than one would not be as responsive to market movements. In this situation, smaller systemic risk would mean that investors would be satisfied with a return that is below the market average. The market portfolio has a beta of one precisely because the covariance of the market portfolio with itself is identical to the variance of the market portfolio. Needless to say, a risk-free investment has a beta of zero because its covariance with the market is zero.

1.3 The Security Market Line

Let us pause for thought:

- *Total* risk comprises unsystematic and systematic risk.
- *Unsystematic* risk, unique to each company, can be eliminated by portfolio diversification.
- *Systematic* risk is undiversifiable and depends on the market as a whole.

These distinctions between total, unsystematic and systematic risk are vital to our understanding of the development of Modern Portfolio Theory (MPT). Not only do they validate beta factors as a measure of the only risk that investors will pay a premium to avoid. As we shall discover, they also explain the rationale for the Capital Asset Pricing Model (CAPM) whereby investors can assess the portfolio returns that satisfy their risk-return requirements. So, before we consider the CAPM in detail, let us contrast systemic beta analysis with basic portfolio theory that only considers total risk.

The linear relationship between *total* portfolio risk and expected returns, the *Capital Market Line* (CML) based on Markowitz efficiency and Tobin's Theorem, graphed in Chapter Four of *PTIA* does not hold for *individual* risky investments. Conversely, all the characteristics of systemic beta risk apply to portfolios *and* individual securities. The beta of a portfolio is simply the weighted average of the beta factors of its constituents.

This new relationship becomes clear if we reconstruct the CML (Figure 4.2 from Chapter Four of the *PTIA* text) to form what is termed the *Security Market Line* (SML). As Figure 1.3 illustrates, the expected return is still calibrated on the vertical axis but the SML substitutes systemic risk (β) for total risk (σ_p) on the horizontal axis of our earlier CML diagrams.

Once beta factors are calculated (not a problem) the SML provides a universal measure of risk that still adheres to *Markowitz efficiency* and his criteria for portfolio selection, namely:

Maximise return for a given level of risk Minimise risk for a given level of return

Like the CML, the SML still confirms that the *optimum* portfolio is the *market* portfolio. Because the return on a portfolio (or security) depends on whether it follows market prices as a whole, the closer the correlation between a portfolio (security) and the market index, then the greater will be its expected return. Finally, the SML predicts that both portfolios and securities with higher beta values will have higher returns and *vice versa*.

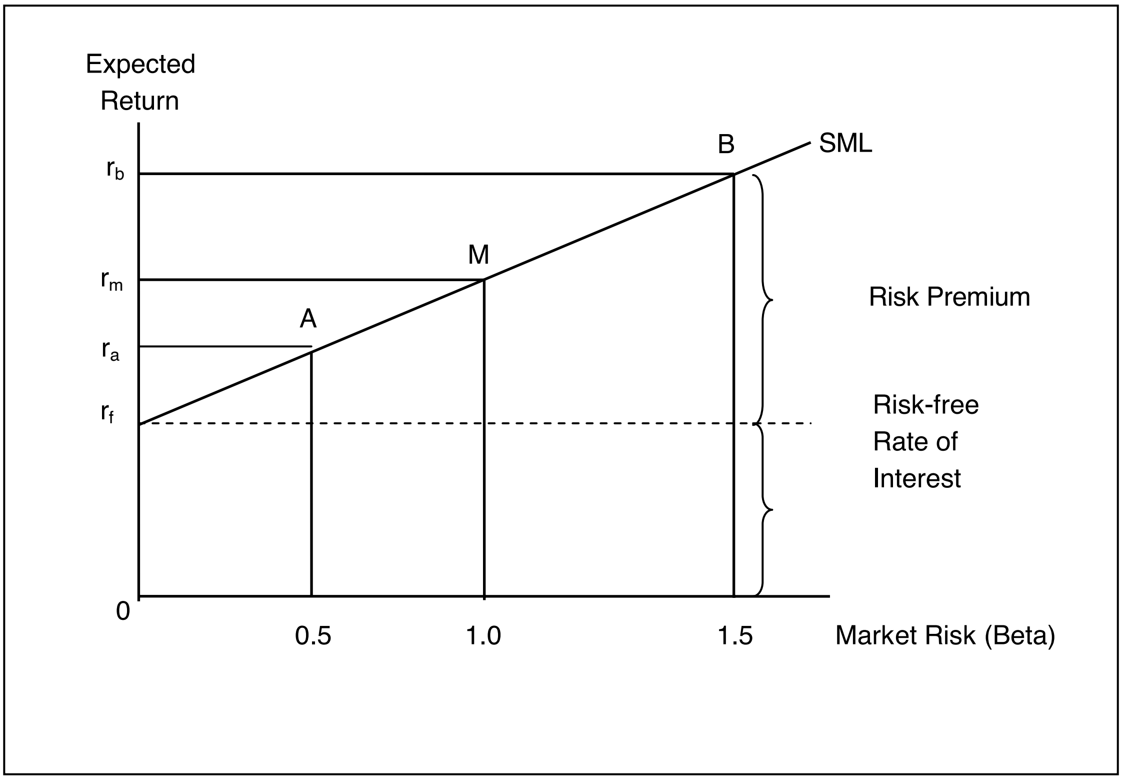

Figure 1.3: The Security Market Line

As Figure 1.3 illustrates, the expected risk-rate return of r_m from a balanced market portfolio (M) will correspond to a beta value of one, since the portfolio cannot be more or less risky than the market as a whole. The expected return on risk-free investment (r_f) obviously exhibits a beta value of zero.

Portfolio A (or anywhere on the line r_f-M) represents a *lending* portfolio with a mixture of risk and risk-free securities. Portfolio B is a *borrowing* or leveraged portfolio, because beyond (M) additional securities are purchased by borrowing at the risk-free rate of interest.

Review Activity

Given your knowledge of perfect capital markets, Fisher's Separation Theorem, stock market efficiency, mean-variance analysis, utility theory, Markowitz efficiency and Tobin's Capital Market Line (CML):

Briefly summarise what the Security Market Line (SML) offers rational, risk-averse individuals seeking a well-diversified portfolio of investments?

Summary and Conclusions

Throughout our analyses (based on the origins of portfolio theory, explained in *PTIA*) we have observed how rational, risk-averse individuals and companies operating in perfect markets with no "barriers to trade" can rank *individual* investments by interpreting their expected returns and standard deviations using the concept of expected utility to calibrate their risk-return attitudes. In this book (and our *PTIA* companion) we began with the same mean-variance efficiency criteria to derive optimum *portfolio* investments that can reduce risk (standard deviation) without impairing return. This culminated with Tobin's Theorem and the CML that incorporates borrowing and lending opportunities to define optimum "efficient" portfolio investment opportunities.

Unfortunately, the CML only calibrates total risk (σ_p) not all of which is diversifiable. Fortunately, the SML offers investors a lifeline, by discriminating between non-systemic and systemic risk. The latter is defined by a beta factor that measures relative (systematic) risk, which explains how rational investors with different utility (risk-return) requirements can choose an optimum portfolio by borrowing or lending at the risk-free rate.

We shall return to this topic in Chapter Two when risk is related to the expected return from an investment or portfolio using the CAPM.

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2 The Capital Asset Pricing Model (CAPM)

Introduction

Basic portfolio theory defines the expected return from a risky investment in general terms as the risk-free return, plus a premium for risk. However, we have observed that this premium is determined not by the overall risk of the investment but only by its systematic (market) risk.

(36) $$\beta_j = \frac{COR(j\ m)\ \sigma\ j}{\sigma\ (m)}$$

Using the geometry of the Security Market Line (SML) that determines the market risk premium (β), numerous academics, notably Sharpe (1963) followed by Lintner (1965), Treynor (1965) and Mossin (1966) were quick to develop (quite independently) the *Capital Asset Pricing Model* (CAPM) as a logical extension to basic portfolio theory.

Today, the CAPM is regarded by many as a superior model of security price behaviour to others based on wealth maximisation criteria with which you should be familiar. For example, unlike the dividend and earnings share valuation models of Gordon (1962) and Modigliani and Miller (1961) covered in our *SFM* and *SFME* texts (referenced in Chapter One) the CAPM explicitly identifies the risk associated with an ordinary share (common stock) as well as the future returns it is expected to generate. Moreover, the CAPM can also express investment returns in two forms.

For individual securities:

(34) $$r_j = r_f + (\ r_m - r_f)\ \beta_j$$

And because systemic betas apply to a *portfolio*, as well as an *individual* investment:

(35) $$r_p = r_f + (\ r_m - r_f)\ \beta_p$$

For a given a level of systematic risk, the CAPM determines the expected rate of return for any investment (security, project, or portfolio) relative to its beta value defined by the SML (a market index). As we shall discover, it also establishes whether individual securities, projects (or their portfolios) are under or over-priced relative to the market, (hence its name). The CAPM can therefore be used by investors or management, who desire to eliminate unsystematic risk through efficient diversification and assess the required return for a given level of non-diversifiable, systematic (market) risk. As a consequence, they can tailor their portfolio of investments to suit their individual risk-return (utility) profiles.

2.1 The CAPM Assumptions

The CAPM is a *single-period model*, which means that all investors make the same decision over the same time horizon. Expected returns arise from expectations over the same period.

The CAPM is a *single-index* model because systemic risk is prescribed entirely by *one* factor; the beta factor.

The CAPM is defined by random variables that are normally distributed, characterised by mean expected returns and covariances, upon which all investors agree.

Markowitz mean-variance efficiency criteria based on perfect markets still determine the optimum portfolio (P).

MAX: R(P), given δ(P)
MIN: δ(P), given R(P).

- All investments are infinitely divisible.
- All investors are rational and risk averse.
- All investors are price takers, since no individual, firm or financial institution is large enough to distort prevailing market values.
- All investors can borrow-lend without restriction at the risk-free market rate of interest.
- Transaction costs are zero and the tax system is neutral.
- There is a perfect capital market where all information is available and costless.

Table 2.1: The CAPM Assumptions

The application of the CAPM and beta factors is straight forward as far as stock market tactics are concerned. The model assumes that investors have three options when managing a portfolio:

i. To trade,
ii. To hold,
iii. To substitute, (*i.e.* securities for property, property for cash, cash for gold *etc.*).

A profitable trade is accomplished by buying (selling), undervalued (overvalued) securities relative to an appropriate measure of systematic risk, a global stock market index such as the FT/S&P World Index. If the market is "bullish" and prices are expected to rise generally, it is worth buying securities with high β values because they can be expected to rise faster than the market. Conversely, if markets are "bearish" and expected to fall, then securities with low beta factors are more attractive because they can be expected to fall less than prices overall.

To validate the CAPM, however, there are other assumptions (many of which should be familiar) that we will question later. For the moment, they are simply listed in Table 2.1 without comment to develop our analysis.

2.2 The Mathematical Derivation of the CAPM

Given the perfect market assumptions of the single period-index CAPM, consider an investor who initially places nearly all their funds in a portfolio reflecting the composition of the market. They subsequently invest the balance in security *j*. Using sequential numbering from previous equations, let us define R(P) the expected return on the revised portfolio as the weighted average of the expected returns of the individual components. This is given by adapting Equation (1) the basic formula for portfolio return from the *PTIA* text (remember?).

(37) $R(P) = x\, r_j + (1-x)\, r_m$

Where:

x = an extremely small proportion,
r_j = expected rate of return on security *j*,
r_m = expected rate of return on the market portfolio.

Subject to the original model's non-negativity constraints and requirements that sources of funds equal uses, the portfolio variance is also based on Equation (2) from the *PTIA* text:

(38) $VAR(P) = x^2\ VAR\ (r_j) + (1\text{-}x)^2\ VAR(r_m) + 2x\ (1\text{-}x)\ COV(r_j,r_m)$

The portfolio will be efficient if it has the lowest degree of risk for the highest expected return, given by the objective functions:

MAX: R(P), given VAR(P)

MIN: VAR(P), given R(P)

But note what has happened. By introducing security *j* into the market portfolio, the investor has altered the risk-return characteristics of their original portfolio. According to Sharpe and others, the *marginal return per unit of risk* is derived by:

i. *Differentiating* R(P) with respect to the investment in security *j*; Δ R(P)/ Δx,
ii. *Differentiating* VAR(P) with respect to the investment in security *j*; ΔVAR(P)/ Δ x.
iii. *Solving* $\dfrac{\Delta\ R(P)/\ \Delta x}{\Delta\ VAR(P)/\ \Delta x}$ as $x \rightarrow 0$

Since (iii) above simplifies to ΔR(P)/ ΔVAR(P) as x tends to zero, *the incremental return per unit of risk* is therefore given by:

(39) $$\frac{\Delta R(P)}{\Delta VAR(P)} = \frac{r_m - r_j}{2(1-\beta_j)\ VAR\ (r_m)} \quad \text{for } x \rightarrow 0$$

However, you will recall from our explanation of the SML in the *PTIA* text that an investor can either borrow or lend at the risk-free rate of interest (r_f) with a beta value of zero. So, by incorporating a risk-free investment or a liability (if x is negative) the incremental rate of return given by Equation (39) is established by substituting $r_j = r_f$ and $\beta_j = 0$ into the equation such that:

(40) $$\frac{\Delta R(P)}{\Delta VAR(P)} = \frac{r_m - r_f}{2VAR\ (r_m)}$$

In a perfectly competitive capital market, the *incremental* risk-return trade-off must be the same for all investors. So, Equations (39) and (40) are identical:

(41) $$\frac{r_m - r_j}{2(1-\beta_j)\ VAR(r_m)} \quad \text{is equivalent to} \quad \frac{r_m - r_f}{2\ VAR(r_m)}$$

Now, multiplying both sides of Equation (41) by the denominator on the left hand side and rearranging terms, Sharpe's *one* period, *single* factor Capital Asset Pricing Model (CAPM) for individual investments (explained earlier) is confirmed as follows:

(34) $r_j = r_f + (r_m - r_f) \beta_j$

And because systematic betas apply to a *portfolio*, as well as an *individual* investment we can define R(P) using our earlier notation

(35) $r_p = r_f + (r_m - r_f) \beta_p$

Remember, the CAPM is a *one period* model because the independent variables, r_f, r_m and β_j are assumed to remain constant over the time horizon. It is also a *single factor* model because systematic risk is prescribed entirely by the beta factor.

Equation (34) represents the expected rate of return on security *j*, which comprises a risk free return plus a premium for accepting market risk (the market rate minus the risk free rate), assuming that all correctly priced securities will lie on the SML. The market portfolio offers a premium $(r_m – r_f) \beta_j$ over the risk-free rate, r_f, which may differ from the *j*th security's risk premium measured by the beta factor β_j.

Thus, Sharpe's CAPM (like the others mentioned earlier, Lintner *et. al.*) enables an investor to establish whether individual securities (or portfolios) are under or over-priced, since the linear relationship between their expected rates of return and beta factors (systematic risk) can be compared with the SML (the market index).

2.3 The Relationship between the CAPM and SML

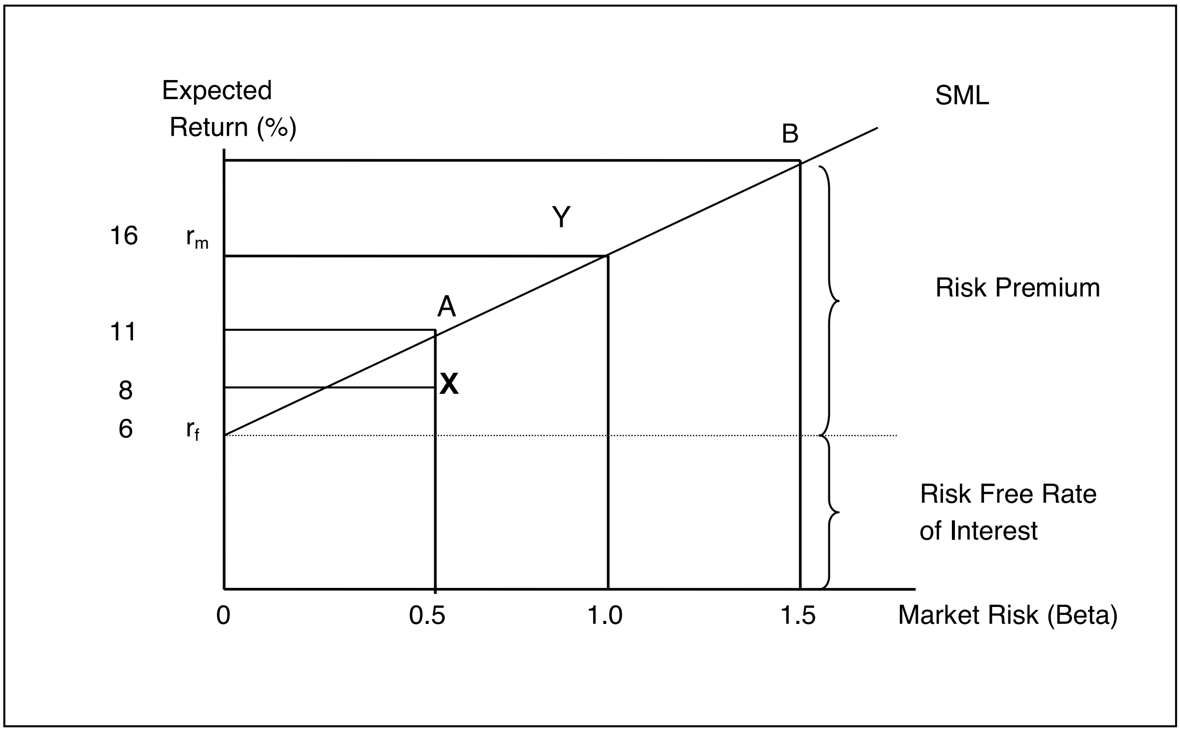

Figure 2.1: The CAPM and SML

Activity 1

Take a look at Figure 2.1. This is a reproduction of Figure 1.3, the Security Market Line (SML) explained in the *PTIA* text. At one extreme we have the expected return on risk-free investment (rf) with a beta value of zero. At the other, portfolio B is a *borrowing* or *leveraged* portfolio with a beta of 1.5, which contains securities purchased by borrowing at the risk-free rate of interest. However, superimposed on the new graph are other beta values associated with expected returns, one of which is defined by the point X.

Explain its portfolio implications for rational, risk-averse investors.

Suppose we are considering investing in the security denoted by X on the graph with an expected return of 8 per cent and a beta coefficient of 0.5. We can see that the return is too low for the risk involved and that the security is overpriced because X is located below the SML. Consequently, rational investors wishing to sell their holdings would need to drop their price and increase the return (yield) until it impinges upon the SML at point A.

Given the slope of the SML defined by a risk free rate of 6 per cent and a market return of 16 per cent from a risky balanced portfolio, Figure 2.1 illustrates why the new *equilibrium* rate of return A with a beta value of 0.5 should be 11%. You can confirm this using the CAPM model:

(34) $$r_j = r_f + (r_m - r_f)\,\beta_j$$

where the expected return equals the risk-free rate, plus the market rate minus the risk-free rate, multiplied by the beta factor.

$$\underline{11\%} = 6\% + (16\% - 6\%).0.5$$

It is also clear from Figure 2.1 why investing in a security such as Y is beneficial. Stocks above the line will be in great demand, so they will rise in price causing a fall in yield.

From our examination of the data we can therefore draw the following conclusions.

In theoretical efficient capital markets in equilibrium that assimilate all information concerning a security into its price, all securities (or portfolios) will lie on the SML.

Individual investors need not conform to the market portfolio. They need only determine how much systematic risk they wish to assume, leaving market forces to ensure that any security can be expected to yield the appropriate return for its beta.

2.4 Criticism of the CAPM

Like much else in modern financial theory, critics of the CAPM maintain that its assumptions are so restrictive as to invalidate its conclusions, notably investor rationality, perfect markets and linearity. Moreover, the CAPM is only a single-period model, based on estimates for the risk-free rate, market return and beta factor, which are all said to be difficult to determine in practice. Finally, the CAPM also assumes that investors will hold a well diversified portfolio. It therefore ignores unsystematic risk, which may be of vital importance to investors who do not. However, as we have emphasised elsewhere in our studies, the relevant question is whether a model works, despite its limitations?

Although there is evidence by Black (1993) to suggest that the CAPM does not work accurately for investments with very high or low betas, overstating the required return for the former and understating the required return for the latter (suggesting compensation for unsystematic risk) most tests validate the CAPM for a broad spectrum of beta values.

The beta-return characteristics of individual securities also hold for portfolios. In fact, the beta of a portfolio seems more stable because fluctuations among its constituents tend to cancel each other out.

Way back in 1972, Black, Jensen and Scholes analysed the New York Stock Exchange over a 35 year period by dividing the listing into 10 portfolios, the first comprising constituents with the lowest beta factors and so on. Based on time series tests and cross-sectional analyses they found that the intercept term was not equal to the risk-free rate of interest, r_f, (which they approximated by 30 day Treasury bills). However, their study revealed an almost *straight-line* relationship between a portfolio's beta and its average return.

Critics still maintained that beta will only be stable if a company's systematic risk remains the same because it continues in the same line of business. However, subsequent studies using historical data to establish the stability of beta over time confirmed that if beta factors are calculated from past observable returns this problem can be resolved.

- The longer the period analysed, the better.
- The more data, the better, which suggests the use of a *sector* beta, rather than a *company* beta.

As an alternative to the basic CAPM, Black (1972) also tested a *two-factor* model, which assumed that investors couldn't borrow at a risk free rate but at a rate, r_z, defined as the return on a portfolio with a beta value of zero. This is equivalent to a portfolio whose covariance with the market portfolio's rate of return is zero.

(42) $$r_j = r_z + (r_m - r_z) \beta_j$$

The Black two-factor model confirmed the study by Black, Jensen and Scholes (*op.cit.*) and that a zero beta portfolio with an expected return, r_z exceeds the risk free rate of interests, r_f.

Despite further modifications to the original model, which need not detain us here, (multi-factors, multi-periods) the CAPM in its traditional guise continues to attract criticism, particularly in relation to its fundamental assumptions.

For example, even if we accept that all investors can borrow or lend at the risk-free rate, it does not follow that r_f describes a risk-free investment in *real* terms. Future inflation rates are neither pre-determined, nor affect individuals equally.

Marginal adjustments to a portfolio's constituents may also be prohibited by substantial transaction costs that outweigh their future benefits.

The fiscal system can also be *biased* with differential tax rates on income and capital gains. So much so, that different investors will construct or subscribe to portfolios that minimise their personal tax liability (a *clientele* effect).

And what if the stock market is *inefficient*? As we have discussed at great length in our other *Bookboon* companion texts, investors can not only profit from legitimate data by paying for the privilege. With access to insider information, which may even anticipate global events (such as the 1987 crash, millennium dot. com. fiasco and 2007 meltdown) perhaps they can also destabilise markets.

Conversely, even if we assume that the market is *efficient*, it has not always responded to significant changes in information, ranging from patterns of dividend distribution, takeover activity and government policies through to global geo-political events. Why else do even professional *active* managed portfolio funds periodically under-perform relative to the market index? The only way to "beat" the market, or so the argument goes, is either through pure speculation or insider information. Otherwise, adopt a *passive* policy of "buy and hold" to track the market portfolio and hope for the best

Other forces are also at work to invalidate the CAPM. You will recall that the model implies that the optimum portfolio is the market portfolio, which lies on the Security Market Line (SML) with a beta factor of one. Individual securities and portfolios with different levels of risk (betas) can be priced because their expected rate of return and beta can be compared with the SML. In equilibrium, all securities will lie on the line, because those above or below are either under or over priced in relation to their expected return. Thus, market demand, or the lack of it, will elicit either a rise or fall on price, until the return matches that of the market.

However, we have a problem, namely how to define the market. It is frequently forgotten that the CAPM is a *linear mode*l based on *partial equilibrium analysis* that subscribes to the Modigliani-Miller (MM) *law of one price*. Based on their arbitrage process, (1958 and 1961) explained in our companion texts, you will recall that two similar assets must be valued equally. In other words, two portfolio constituents that contribute the same amount of risk to the overall portfolio are *close substitutes*. So, they should exhibit the same return. But what if an asset has no close substitute, such as the market itself? How do we establish whether the market is under or overvalued?

As Roll (1977) first noted, most CAPM tests may be invalid because all stock exchange indices are only a *partial* measure of the *true* global market portfolio. Explained simply, by definition the market portfolio should include every security world-wide.

To prove the point, Roll demonstrated that a change in the surrogate for the American stock market from the Standard and Poor 500 to the Wilshire 5000 could radically alter a security's expected return as predicted by the CAPM. Furthermore, if betas and returns derived from a stock market listing were unrelated, the securities might still be priced correctly relative to the global market portfolio. Conversely, even if the listing was efficient (shares with high betas did exhibit high returns) there is no obvious reason for assuming that each constituent's return is only affected by global systematic risk.

A further criticism of the CAPM is that however one defines the capital market, movements up and down are dominated by price changes in the securities of larger companies, Yet as Fama and French (1992) first observed, it is to these companies that institutional portfolio fund managers (active or passive) are attracted, though they may under-perform relative to smaller companies. Explained simply, fund managers with perhaps billions to spend are hostages to fortune, even in a "bull" scenario. They have neither the time, nor research budgets to scrutinise innumerable companies "neglected" by the market with small capitalisations based on little information.

Turning to "bear" markets characterised by rising systematic risk, multi-national portfolio fund managers still have little room for manoeuvre. According to Hill and Meredith (1994):

The first option is to liquidate all or part of a portfolio. However, if the whole portfolio were sold it could be difficult to dispose of a large fund quickly and efficiently without affecting the market. Unlike a private investor, total disposal may also be against the fund's trust deed. If only part of the portfolio was liquidated there is the further question of which securities to sell.

The second option is to reduce all holdings, to be followed by subsequent reinvestment when the market bottoms out. However, the fall in prices may have to be in excess of 2 per cent to cover transaction and commission costs.

Clearly, both alternatives may be untenable and impose significant constraints upon the opportunities to control risk. Indeed, those sceptical of portfolio management generally and the CAPM in particular, regard successful investment as a matter of luck rather than judgement, insider information, or unlikely economic circumstances where all prices move in unison.

Review Activity

Assuming the risk-free rate and expected return on the market portfolio for Muse plc are 10 per cent and 18 per cent respectively:

(1) Use the CAPM to calculate the expected returns on stocks with the following beta values:

$$\beta = 0, 0.5, 1.0, 1.5$$

(2) How would each stock fit into the investment plans for an actively managed portfolio?

(1) Using the data and Equation (34) to derive the expected returns, the CAPM reveals that if:

$$\beta = 0, 0.5, 1.0 \text{ or } 1.5$$

$$r_j = 10+(18 - 10)\beta = 10\%, 14\%, 18\% \text{ and } 22\%, \text{ respectively}$$

(2) The investment plans for an actively portfolio can be explained as follows.

With a beta value greater than one, a stock's expected return should "beat" the market and *vice versa.* A beta of one produces a return equal to the market return and a beta value of zero produces an expected return equal to the risk-free rate.

Thus, we can classify investment into three broad categories of risk for the purpose of "active" portfolio management:

$$\beta > 1.0 = \textit{Aggressive}$$
$$\beta < 1.0 = \textit{Defensive}$$
$$\beta = 1.0 = \textit{Neutral}$$

A portfolio manager's interest in each category of beta factor concerns the likely impact of changes in a market index on the share's expected return. Aggressive shares can be expected to outperform the market in either direction. If the return on the index is expected to rise, the returns on high beta shares will rise faster. Conversely, if the market is expected to fall, then their returns will fall faster. Defensive shares with beta values lower than one will obviously under-perform relative to the market in each direction. Neutral shares will tend to shadow it.

Hence, rather than adopt a passive policy of "buy and hold" by constructing a *tracker* fund representative of a stock market index, "active" portfolio managers will wish to pursue:

An *aggressive* investment strategy by moving into *high* beta shares when stock market returns are expected to rise (a bull market).

A *defensive* strategy based on *low* beta shares and even risk-free assets with zero betas, when the market is about to fall (a bear market).

Summary and Conclusions

If the capital market is so unpredictable that it is impossible for investors to beat it using the CAPM, it is important to remember that the operational usefulness of alternative mean-variance analyses and expected utility models explained at the very beginning of the *PTIA* text are also severely limited in their application. This is why the investment community turned to Markowitz portfolio theory and the Sharpe CAPM for inspiration. And why others refined these models into a coherent body of work now termed Modern Portfolio Theory (MPT) to facilitate the efficient diversification of investment.

Since the new millennium, despite the volatility of financial markets and their tendency to crash (or perhaps because of it) the portfolio objectives of investors remain the same:

> To eliminate unsystematic risk and to establish the optimum relationship between the systematic risk of a financial security, project, or portfolio, and their respective returns; a trade-off with which investors feels comfortable.

So to conclude our studies, what does the *single- period* model CAPM based on Markowitz efficiency contribute to Strategic Financial Management within the context of their *multi-period* investment, dividend and financing decisions, which previous models have failed to deliver?

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3 Capital Budgeting, Capital Structure and the CAPM

Introduction

So far, our study of Markowitz efficiency, beta factors and the CAPM has concentrated on the stock market's analyses of security prices and expected returns by financial institutions and private individuals. This is logical because it reflects the rationale behind the chronological development of Modern Portfolio Theory (MPT). But what about the impact of MPT on individual companies and their appraisal of capital projects upon which all investors absolutely depend? If management wish to maximise shareholder wealth, then surely a new project's expected return and systematic risk relative to the company's existing investment portfolio and stock market behaviour, like that for any financial security, is a vitally important consideration.

In this Chapter we shall explore the corporate applications of the CAPM by strategic financial management, namely:

- The derivation of a discount rate for the appraisal of capital investment projects on the basis of their systematic risk.
- How the CAPM can be used to match discount rates to the systematic risk of projects that differ from the current business risk of a firm.

Because the model can be applied to projects financed by debt as well as equity, we shall also establish a mathematical connection between the CAPM and the Modigliani-Miller (MM) theory of capital gearing based on their "law of one price" covered in my "Strategic Financial Management" (*SFM*) texts.

3.1 Capital Budgeting and the CAPM

As an alternative to calculating a firm's weighted average cost of capital (WACC) explained in the *SFM* texts, the theoretical derivation of a project discount rate using the CAPM and its application to NPV maximisation is quite straightforward. A risk-adjusted discount rate for the *j*th project is simply the risk-free rate added to the product of the market premium and the *project* beta, given by the following expression for the familiar CAPM equation:

(45) $r_j = r_f + (r_m - r_f) \beta_j$

The project beta (β_j) measures the *systematic* risk of a specific project (more of which later). For the moment, suffice it to say that in many textbooks the project beta is also termed an *asset* beta denoted by β_A.

We then derive the expected NPV by discounting the average net annual cash flows at the risk-adjusted rate from which the initial cost of the investment is subtracted, using a mathematical formulation that you should be familiar with.

$$(46) \quad NPV = \sum_{t=1}^{n} C_t / (1+r_j)^t - I_0$$

Individual projects are acceptable if:

$$NPV \geq 0$$

Collectively, projects that satisfy this criterion can also be ranked for selection according to the size of their NPV. Given:

$$NPV_A > NPV_B > \ldots NPV_N \text{ we prefer project A.}$$

So far, so good; but remember that CAPM project discount rates are still based on a number of simplifying assumptions. Apart from adhering to the traditional concept of perfect capital markets (Fisher's Separation Theorem) and mean-variance analysis (Markowitz efficiency) the CAPM is only a *single-period* model, whereas most projects are *multi-period* problems.

According to the CAPM, all investors face the same set of investment opportunities, have the same expectations about the future and make decisions within *one* time horizon. Any new investment made *now* will be realised *then*, next year (say) and a new decision made.

Given the assumptions of perfect markets characterised by random cash flow distributions, there is no theoretical objection to using a *single-period* model to generate an NPV discount rate for the evaluation of a firm's *multi-period* investment plans. The only constraints are that the risk-free rate of interest, the average market rate of return and the beta factor associated with a particular investment are *constant* throughout its life.

Unfortunately, in reality the risk-free rate, the market rate and beta are rarely constant. However the problem is not insoluble. We just substitute *periodic* risk-adjusted discount rates (now dated $r_{j\,t}$) for a constant r_j into Equation (46) for each future "state of the world", even if only one of the variables in Equation (45) changes. It should also be noted that the phenomenon of multiple discount rates combined with different economic circumstances is not unique to the CAPM. As we first observed in Part Two of *SFM*, it is common throughout NPV analyses.

On first acquaintance, it would therefore appear that the application of a CAPM return to capital budgeting decisions provides corporate financial management with a practical alternative to the WACC approach. A particular weakness of WACC is that it defines a single discount rate applicable to *all* projects, based on the assumptions that their acceptance doesn't change the company's risk or capital structure and is *marginal* to existing activities. In contrast, the CAPM rate varies from project to project, according to the systematic risk of each investment proposal. However, the CAPM still poses a number of problems that must be resolved if it is to be applied successfully, notably how to derive an appropriate *project* beta factor and how to measure the impact of *capital gearing* on its calculation.

For these reasons, we shall defer a comprehensive numerical example of investment appraisal and the CAPM until you attempt the Activities associated with this chapter, by which time we will have covered the issues involved.

3.2 The Estimation of Project Betas

For simplicity throughout previous chapters we have used a *general* beta factor (β) applicable to the *overall* systemic risk of portfolios, securities and projects. But now our analysis is becoming more focussed, *precise* notation and definitions are necessary to *discriminate* between systemic *business* and *financial* risk. Table 3.1 summarises the beta measures that we shall be using for future reference and also highlights a number of problems.

β = total *systematic* risk, which relates portfolio, security and project risk to *market* risk.
β_j = the *business* risk of a specific project (*project* risk) for investment appraisal.
β_E = the *published* equity beta for a company that incorporates business risk and systematic *financial* risk if the firm is geared.
β_A = the overall business risk of a firm's *assets* (projects). It also equals a company's *deleveraged* published beta (β_E) which measures business risk *free* from financial risk.
β_D = the beta value of debt (which obviously equals zero if it is risk-free).
β_{EU} and β_{EG} are the respective equity betas for similar all-share and geared companies.

Table 3.1: Beta Factor Definitions

When an all-equity company is considering a new project with the same level of risk as its current portfolio of investments, total systematic risk *equals* business risk, such that:

$$\beta = \beta_j = \beta_E = \beta_A = \beta_{EU}$$

When a company is funded by a combination of debt and equity, this series of equalities must be modified to incorporate a *premium* for systematic *financial* risk. As we shall discover, the equity beta (β_E) will be a *geared* beta reflecting business risk *plus* financial risk, which measures shareholder exposure to debt in their firm's capital structure. Thus, the equity beta of an all-share company is always lower than that for a geared firm with the same business risk.

$$\beta_{EU} < \beta_{EG}$$

Irrespective of a gearing problem, Table 3.1 reveals a further weakness of the CAPM. A company's asset beta (β_A) should produce a discount rate that is appropriate for evaluating projects with the same overall risk as the company itself. But what if a new project does not reflect the average risk of the company's assets? Then the use of β_A is no more likely to produce a correct investment decision than the use of a WACC calculation.

To illustrate the point, Figure 3.1 graphs the Security Market Line (SML) to show the required return on a project for different beta factors, with a company's WACC. The use of the overall cost of capital to evaluate projects whose risk differs from the company's average will be sub-optimal where the IRR of the project is in either of the two shaded sections. To calculate the correct CAPM discount rate using Equation (45), we must determine the project beta.

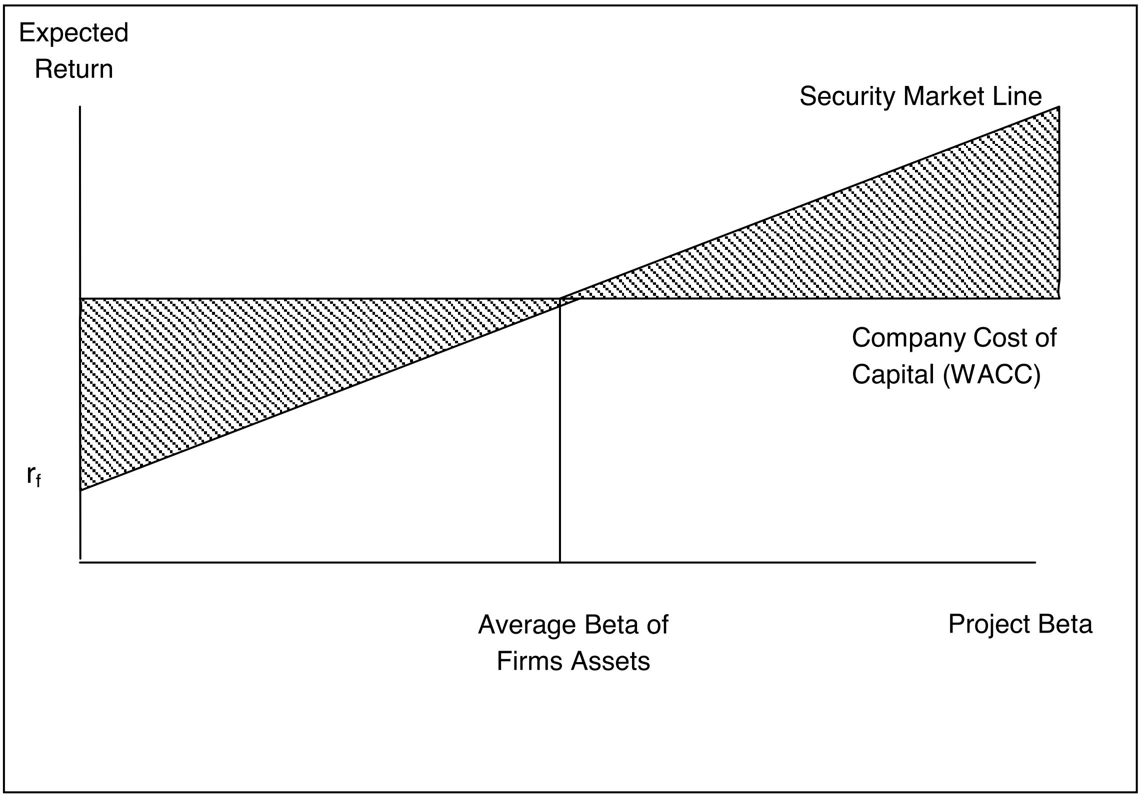

Figure 3.1: The SML, WACC and Project Betas

The company's average beta, shown in the diagram, provides a measure of risk for the firm's overall returns compared with that of the *market*. However, management's investment decision is whether or not to invest in a *project*. So, like the WACC, if the project involves diversification away from the firm's core activities, we must use a beta coefficient appropriate to that class of investment. The situation is similar to a stock market investor considering whether to purchase the shares of the *company*. The individual would need to evaluate the share's return by using the *market* beta in the CAPM.

Even if diversification is not contemplated, the project's beta factor may not conform to the *average* for the firm's assets. For example, the investment proposal may exhibit high *operational gearing (*the proportion of fixed to variable costs) in which case the project's beta will exceed the average for existing operations.

A serious conflict (the *agency* problem) can also arise for those companies producing few products, or worse still a single product, particularly if management approach their capital budgeting decisions based on self-interest and short-termism, rather than shareholder preferences. Shareholders with well-diversified corporate holdings who dominate such companies may prefer to see projects with high risk (high beta coefficients) to balance their own portfolios. Such a strategy may carry the very real threat of bankruptcy but in the event may have very little impact on their overall returns. For corporate management, the firm's employees and its suppliers, however, the policy may be economic suicide.

Fortunately, if a beta is required to validate the CAPM for project appraisal, help is at hand. Management can obtain factors for companies operating in similar areas to the proposed project by subscribing to the many commercial services that regularly publish beta coefficients for a large number of companies, world wide. Their listings also include stock exchange classifications for *industry* betas. These are calculated by taking the market average for quoted companies in the same industry. Research reveals that the measurement errors of individual betas cancel out when industry betas are used. Moreover, the larger the number of comparable beta constituents, the more reliable the industry factor.

So, if management wish to obtain an estimate for a project's beta, it can identify the industry in which the project falls, and use that industry's beta as the project's beta. This approach is particularly suitable for highly *diversified* and *divisionalised* companies because their WACC or market beta would be of little relevance as a discount rate for its divisional operations.

As an alternative to stock market data, management can also estimate a project's beta from first principles by calculating its *F-value*.

> The F-value of a project is rather like a beta factor in that it measures the variability of a project's performance, *relative* to the performance of an entity for which a beta value exists.
>
> The entity could be the industry in which the project falls, the firm undertaking the project, or a division within the firm that is responsible for the project.

A project's F-value is defined as follows:

$$(47) \qquad F = \frac{\text{Percentage change in the project's performance}}{\text{Percentage change in the "entity's" performance}}$$

As a result, we can obtain an estimate of a project's beta through one of three routes:

$$\left.\begin{array}{ll}
\text{(i)} & \dfrac{\text{\% change in the company's performance}}{\text{\% change in the industry's performance}} \times \beta_{\text{industry}} \\[2ex]
\text{(ii)} & \dfrac{\text{\% change in the project's performance}}{\text{\% change in the company's performance}} \times \beta_{\text{company}} \\[2ex]
\text{(iii)} & \dfrac{\text{\% change in the project's performance}}{\text{\% change in the division's performance}} \times \beta_{\text{division}}
\end{array}\right\} \beta_{\text{project}}$$

> **Activity 1**
>
> Let us suppose that a company's divisional management is considering a capital project, whose performance may be affected 15 per cent either way, depending on whether the division's overall performance rises or falls by 10 per cent. In other words, the project's profitability is expected to be more volatile than that of the division because of specific economic factors.
>
> Calculate the project's F-value and estimate the project's beta coefficient given the division's beta factor is 0.80.

Using Equation (47) we can calculate the F-value as follows:

$$F = 15\% / 10\% = \underline{1.5}$$

If the divisional beta value is 0.80, then the project beta (β_{project}) can be estimated as follows:

(% change in the project's performance / % change in the division's performance) × β_{division}

$$\beta_{\text{project}} = 1.5 \times 0.80 = \underline{1.2}$$

3.3 Capital Gearing and the Beta Factor

The CAPM defines an individual investment's risk relative to a well-diversified portfolio as *systematic risk*. Measured by the beta coefficient, it is the only risk a company or an investor will pay a premium to avoid. If you have read either of my portfolio texts (*PTFA* or *PTIA*), you will recall from Chapter Four (Figure 4.3) that it can be sub-divided into:

- *Business risk* that arises from the variability of a firm's earnings caused by market forces,
- *Financial risk* associated with dividend policies and capital gearing, both of which may amplify business risk

Without getting enmeshed in dividend policies, we shall accept the 1961 MM hypothesis that they are *irrelevant*. Based on their "law of one price" (covered in the *SFM* texts and for which there is considerable empirical support) *financial* risk should not matter in an all-equity company. Applied to the CAPM, the *systematic* risk of investors (who are all shareholders) can be defined by the *business* risk of the firm's underlying asset investments.

The *equity* beta of an unlevered (all-equity) firm equals an *asset* beta, which measures the business risk of all its investments relative to the market for ordinary shares (common stock). Using earlier notation:

$$\beta_{E\,U} = \beta_A$$

The CAPM return on project (r_j) is then defined by:

(48) $r_j = r_f + (r_m - r_f) \beta_A$

If there is no debt in the firm's capital structure, the company's asset (equity) beta equals the *weighted average* of its individual project betas (β_i) based on the market value of equity.

(49) $\beta_A = \Sigma\, w_i\, \beta_i = \beta_{EU}$

But what about companies who decide to fund future investments by gearing up, or the vast majority who already employ debt finance?

To make rational decisions, it would appear that management now require an asset beta to measure a firm's business risk that an ungeared equity beta can no longer provide. For example, an all-equity company may be considering a take-over that will be financed entirely by debt. To assess the acquisition's viability, management will now need to calculate their overall CAPM return on investment using an asset beta that reflects a *leveraged* financial mix of fixed interest on debt and dividends on shares.

Later in this chapter we shall resolve the dilemma using the predictions of MM's capital structure hypothesis (1958). Based on their law of one price, whereby similar firms with the same risk characteristics (except capital gearing) cannot sell at different prices, it confirms their dividend hypothesis, namely that financial policy is irrelevant. First, however, let us develop the CAPM, to illustrate the relationship between an asset beta and the equity and debt coefficients for a geared company.

When a firm is financed by a debt-equity mix, its earnings stream and associated risk is divided between the firm's shareholders and providers of corporate debt. The proportion of risk reflects the market values of debt and equity respectively, defined by the *debt-equity ratio*. So, the equity beta will be a *geared* equity beta. It not only incorporates business risk. It also determines shareholders' exposure to financial risk defined by the proportion of contractual, fixed interest securities in the capital structure. For this reason the equity beta of an unlevered company is always lower than the beta of a levered company.

Given a geared equity beta (β_E) and debt beta (β_D), the asset beta (β_A) for a company's investment in risky capital projects can be expressed as a weighted average of the two:

(50) $\beta_A = \beta_{E\,G}\,[V_E / (V_E + V_D)] + \beta_D\,[V_D / (V_E + V_D)]$

Where:

V_E and V_D are the *market* values of equity and debt, respectively,

V_E *plus* V_D define the firm's total market value (V).

> **Activity 2**
>
> A firm with respective market values of €60m and €30m for equity and debt has an equity beta of 1.5. The debt beta is zero.
>
> (1) Use Equation (50) to calculate the asset beta (β_A).
> (2) Explain a simplified mathematical structure of the calculation.

(1) The asset beta (β_A) calculation

(50) $\beta_A = \beta_{E\,G}\,[V_E / (V_E + V_D)] + \beta_D\,[V_D /(V_E + V_D)]$

$= 1.5\ [60/(60 + 30)] + 0\ [30/(60 + 30)] = \underline{1.0}$

(2) The mathematical structure of β_A.

When a company is financed by debt and equity, management need to derive an asset beta using the *weighted average* of its geared equity and debt components. The market values of debt and equity provide the weightings for the calculation. Note, however, that because the market risk of debt (β_D) was set to zero, the right hand side of Equation (50) disappears.

This is not unusual. As explained in *SFM*, debt has priority over equity's share of profits and the sale of assets in the event of liquidation. Thus, debt is more secure and if it is risk-free, there is no variance. So if β_D equals zero, our previous equation for an asset beta reduces to:

(51) $\beta_A = \beta_{EG}\,[V_E / (V_E + V_D)]$

For example, if a company has an equity beta of 1.20, a debt-equity ratio of 40 per cent and we assume that debt is risk-free, the asset beta is given by:

$$\beta_A = 1.20\ [100 / (100 + 40)] = \underline{0.86}$$

Perhaps you also recall from *SFM* that debt is also a *tax deductible* expense in many economies. If we incorporate this fiscal adjustment into the previous equations (where t is the tax rate) we can redefine the mathematical relationship between the asset beta and its geared equity and debt counterparts as follows.

(52) $\beta_A = \beta_{E\,G} \{V_E / [V_E + V_D(1\text{-}t)]\} + \beta_D \{[V_D(1\text{-}t) / (V_E + V_D(1\text{-}t))]\}$

(53) $\beta_A = \beta_{E\,G} \{V_E / [V_E + V_D(1\text{-}t)]\}$ if debt is risk-free

Despite the tax effect, our methodology for deriving a company's asset beta still reveals a *universal* feature of the CAPM that financial management can usefully adopt to assess individual projects.

Whenever risky investments are combined, the asset beta of the resultant portfolio is a *weighted average* of the debt and equity betas.

Activity 3

Consider a company with a current asset beta of 0.90. It accepts a project with a beta of 0.5 that is equivalent to 10 per cent of its corporate value after acceptance.

Confirm that:

1. The new (ex-post) beta coefficient of the company equals 0.86.
2. *The new project reduces the original* (*ex-ante*) risk of the firm's *existing portfolio*.

3.4 Capital Gearing and the CAPM

The CAPM defines a project's discount rate as a return equal to the risk-free rate of interest, plus the product of the market premium and the project's asset beta (a risk premium) to compensate for systematic (business) risk. However, we now know that the financial risk associated with capital gearing can also affect beta factors. So, the discount rate derived from the CAPM for investment appraisal must also be affected, but how?

Let us first consider a company funded entirely by equity that is considering a new project with the same level of risk as its existing activities. The firm's equity beta (β_{EU}) can be used as the project's asset beta (β_A) because the shareholders' return (K_e) equals the company's return (r_j) on a new project of equivalent risk. So, the project return that provides adequate compensation for holding shares in the company is the equity return (K_e) obtained by substituting the appropriate equity beta (β_E) into the familiar CAPM formula.

(54) $K_e = r_j = r_f + (r_m - r_f)\, \beta_{EU}$

The CAPM therefore offers management an important alternative to the derivation of project discount rates that use the traditional dividend and earnings valuation models explained in the *SFM* texts. In an *unlevered* (all-equity) firm, the *shareholders'* return (K_e) defines the *company's* cost of capital (K_U) as follows:

(55) $K_U = K_e = r_j = r_f + (r_m - r_f) \beta_{EU}$

The question we must now ask is whether Equation (55) has any parallel if the firm is geared.

The short answer is yes. Rather than use traditional dividend, earnings and interest models to derive a WACC (explained in *SFM*) we can substitute an appropriately *geared* asset beta for an *all-equity* beta into the CAPM to estimate the overall return on debt and equity capital for project appraisal.

(56) $K_G = r_j = r_f + (r_m - r_f) \beta_A$

3.5 Modigliani-Miller and the CAPM

Without debt in it capital structure, a company's asset beta equals its equity beta for projects of equivalent risk. However, according to MM's theory of capital structure (*op. cit.*) based on their "law of one price" and the *arbitrage* process, companies that are identical in every respect apart from their gearing should also have the same asset betas. Because their business risk is the same, the factors are not influenced by methods of financing. To summarise MM's position

> An ungeared company's asset beta equals its equity beta.
> A geared company's asset beta is lower than its equity beta.
> Irrespective of gearing, the asset beta for any company equals the equity beta of an ungeared company with the same business risk.
> The asset beta (equity beta) of an unlevered company can be used to evaluate projects in the same risk class without considering their finance.
>
> $$\beta_j = \beta_A = \beta_{EU} < \beta_{E\,G}$$

You will recall from your studies that MM's capital theory (like their dividend irrelevancy hypothesis) depends on perfect market assumptions. However, because these assumptions also underpin much else in finance (including the CAPM) for the moment we shall accept them. To illustrate the MM relationship between the beta factors of all-equity and geared companies with the same systemic business risk, let us begin with the following equation using our familiar notation in a taxless world.

(57) $$\beta_A = \beta b_{E\,U} = \beta_{E\,G}\,[V_E / (V_E + V_D)] + \beta_D\,[V_D / (V_E + V_D)]$$

If we now rearrange terms, divide through by V_E and solve for β_{EG}, the mathematical relationship between the geared and ungeared equity betas can be expressed as follows:

(58) $$\beta_{EG} = \beta_{E\,U} + (\beta_{EU} - \beta_D)\; V_D / V_E$$

This equation reveals that the equity beta in a geared company equals the equity beta for an all-share company in the same class of business risk, *plus* a premium for systemic financial risk. The premium represents the difference between the all-equity beta and debt beta multiplied by the debt-equity ratio. However, the important point is that the increase in the equity beta measured by the risk premium is exactly offset by a lower debt factor as the firm gears up leaving the asset beta unaffected. In other words, irrespective of leverage, the asset betas of the two firms are still identical and equal the equity beta of the ungeared firm.

$$\beta_A = \beta_{EU} < \beta_{E\,G}$$

For those of you familiar with MM's capital structure hypothesis, the parallels are striking. According to MM, the expected return on equity for a geared firm (K_{eG}) relative to the return (K_{eU}) for an all-share firm in a taxless world equals:

(59) $K_{eG} = K_{eU} + (K_{eU} - K_d) V_D / V_E$.

This states that the return for a geared firm equals an all-equity return for the same class of business risk, *plus* a financial risk premium defined by the difference between the all-equity return and the cost of debt multiplied by the debt-equity ratio. The premium compensates shareholders for increasing exposure to financial risk as a firm gears up. As we observed in *SFM*, however, because the cheaper cost of debt exactly offsets rising equity yields, the overall cost of capital (WACC) is unaffected. So, irrespective of leverage, all firms with the same business risk can use the cost of equity for an all-share firm as a project discount rate before considering methods of financing.

Turning to a world of taxation, where debt is a *tax-deductible* expense with a tax rate (t), we can redefine the equity beta of a geared company from Equation (58) as follows:

(60) $\beta_{EG} = \beta_{E\,U} + [(\beta_{EU} - \beta_D) (1\text{-}t) V_D / V_E]$

And if debt is risk-free with *zero* variance, so that β_D is zero, the formula simplifies to:

(61) $\beta_{EG} = \beta_{E\,U} + [(\beta_{EU} (1\text{-}t) V_D / V_E]$

Review Activity

To illustrate the union between MM and the CAPM, consider a leveraged company in an economy where interest is tax deductible at a 20 per cent corporate rate. 20 million ordinary shares are authorised and issued at a current market value of £2.00 each (*ex-div*). The equity beta is 1.5. Debt capital comprises £10 million, irredeemable 10 per cent loan stock, currently trading at par value.

Calculate the company's asset beta and briefly explain the result.

Since the equity beta for an *ungeared* company equals the asset beta for any company in the same risk class, we can use Equation (61) to solve for β_{EU} and hence β_A as follows.

First, define the market values of equity and debt:

V_E	= £2.00 x 20 million	= £40 million
V_D		= £10 million

Next, define the geared equity beta of 1.5 assuming that debt sold at par is risk-free ($\beta_D = 0$).

$$\begin{aligned}\beta_{EG} = 1.5 &= \beta_{EU} + [\beta_{EU}\,(1\text{-}0.2)\,(10/40)] \\ &= \beta_{EU}\,\{1 + [(1\text{-}0.2)\,(10/40)]\}\end{aligned}$$

Finally, rearrange terms to solve for β_{EU} and β_A .

$$\beta_A = \beta_{EU} = 1.5/1.2 = \underline{1.25}$$

The result is to be expected. The asset beta should be smaller than the geared equity beta (i.e. 1.25 < 1.5) since the systemic risk associated with the asset investment is only one component of the total risk associated with the shares. The asset beta measures business risk, whereas the geared beta measures business and financial risk

Summary and Conclusions

If management use the CAPM rather than a WACC to obtain a risk-adjusted discount rate for project appraisal, they need to resolve the following questions

Question:	**Is the business risk of a project equivalent to that for the company?**	
Answer:	YES	NO
Solution:	Use the company's current equity beta	Use an equity beta for similar companies with similar projects

Question:	**Is the chosen equity beta affected by capital gearing?**	
Answer:	YES	NO
Solution:	De-leverage "ungear" the equity beta to derive an asset beta	Use an equity beta equivalent to an asset beta if it is not affected by gearing

Having obtained an appropriate asset beta, the project discount rate may then be calculated using the CAPM formula.

(62) $r_j = r_f + (r_m - r_f)\ \beta_A$

According to MM's capital structure theory, the asset betas of companies, or projects, in the same class of business risk are identical irrespective of leverage. Higher equity betas are offset by lower debt betas, just as higher equity yields offset cheaper financing, as a firm gears up

Even in a taxed world, it is possible to establish a connection between MM and the CAPM. With tax, the MM cost of equity for a geared firm is given by:

(63) $K_{eG} = K_{eU} + [\ (K_{eU} - K_d\ )\ (1\text{-}t)\ V_D\ /\ V_E]$

According to the CAPM, the equity costs for an ungeared and geared firm are given by:

(64) $K_{eU} = r_j = r_f + (r_m - r_f)\ \beta_{EU}$

(65) $K_{eG} = r_j = r_f + (r_m - r_f)\ \beta_{EG}$

Where:

$$\beta_A = \beta_{EU} < \beta_{EG}$$

If we assume that the company's pre-tax cost of debt (K_d) in Equation (63) equals the risk-free rate (r_f) in Equations (64) and (65) we can write r_f for K_d in Equation (63). If we now substitute Equations (64) and (65) into Equation (63) rearrange terms and simplify the result, we can confirm our earlier equation for a geared equity beta:

(61) $$\beta_{EG} = \beta_{EU} + [\beta_{EU}(1\text{-}t)\, V_D / V_E] = \beta_{EU}\{1 + [(1\text{-}t)\,(10/40)]\}$$

For an application of this formula and the derivation of the cost of equity using the CAPM see Exercise 7.2 in the companion text "Portfolio Theory and Financial Analyses: Exercises" (PTFAE).

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4 Arbitrage Pricing Theory and Beyond

Introduction

Previous chapters have presented a series of mathematical models representing a body of work termed Modern Portfolio Theory (MPT) available to financial management when making strategic investment decisions. MPT was originally developed for use by investors in securities, primarily fund managers and professional analysts with the time, resources and expertise to implement the models and interpret their findings. Today, anybody with access to a computer, the appropriate software and a reasonable financial education can *model* quite complex tasks. Ultimately, however, it is people who should *interpret* the results and not the computer. One lesson to be learnt from the1987 stock market crash is the catastrophic effect of automated trading. Another from the 2007 meltdown and ongoing financial crises is that computer driven models can be so complex that hardly anybody understands what is going on anymore.

Like all financial theories, MPT should therefore be a guide to human action and not a substitute. And while the benefits of IT cannot be overstressed, you should always understand the financial model that underpins the computer program you are running. So, let us review the original purpose of MPT, notably the CAPM and then outline its subsequent development, notably *Arbitrage Pricing Theory* (APT).

4.1 Portfolio Theory and the CAPM

You will recall that portfolio theory was initially developed by Harry Markowitz in the early 1950s to explain how rational investors in perfect markets can minimise the risk of investment without comprising return by diversifying and building up an efficient portfolio of investments. The risk of each portfolio is measured by the variability of possible returns about the mean measured by the standard deviation. Investor risk-return attitudes can be expressed by indifference curves.

In 1958, John Tobin explained how the introduction of risk-free investments into Markowitz' theory further reduces the risk of a portfolio. According to Tobin, the Capital Market Line (CML) defines a new "efficient frontier" of investments for all investors.

Applied to project appraisal, Markowitz theory reveals that an individual project's risk is not as important as its effect on the portfolio's overall risk. So, whenever management evaluate a risky project they must correlate the individual project risk with that for the existing portfolio it will join to assess its suitability.

Without the benefit of today's computer technology, the mathematical complexity of the Markowitz model arising from its covariance calculations prompted other theorists to develop alternative approaches to efficient portfolio diversification. In the early 1960s by common consensus, the CAPM emerged as a means whereby investors in financial securities were able to reduce their total risk by constructing portfolios that discriminate between systematic (market risk) and unsystematic (specific) risk.

The CAPM (usually associated with its prime advocate William Sharpe) states that the return on a security or portfolio depends on whether their prices follow prices in the market as a whole by reference to a suitable index, such as the FT-SE 100. The closer the correlation between the price of either an individual security or a portfolio and this market proxy (measured by the beta factor) the greater will be their expected returns. Thus, if an investor knows the beta factor (relative risk) of a security or portfolio, their returns can be predicted with accuracy. Profitable trading of portfolios is then accomplished by buying (selling) undervalued (overvalued) securities relative to their systematic or market risk.

The CAPM also states that rational investors would choose to hold a portfolio that comprises the stock market as a whole. By definition, the market portfolio has a beta of one and is the most "efficient" in the sense that no other combination of securities would provide a higher return for the same risk. You will recall that it is a benchmark by which the CAPM establishes the Security Market Line (SML) in order to compare other beta factors and returns. From this linear relationship, rational investors can ascertain whether individual shares are underpriced or overpriced and determine other efficient portfolios that balance their personal preference for risk and return.

According to the CAPM:

Any security with the same risk as the market will have a beta of 1.0; half as risky it will have a beta of 0.5; twice as risky it will have a beta of two.

The required rate of return given by the CAPM formula is composed of the return on risk-free investments, *plus* a risk premium measured by the difference between the market return and the risk free rate multiplied by an appropriate beta factor. For example, using Equation (45) for an investment with a beta of β_j:

$$r_j = r_f + (r_m - r_f) \beta_j$$

If we use the CAPM for project appraisal, rather than stock market analysis, the procedure remains the same. Essentially, we are substituting an investment project for a security into a company's portfolio of investments, rather than a market portfolio. Risk relates to the cost of capital and management's objective is to obtain a discount rate to appraise individual projects.

4.2 Arbitrage Pricing Theory (APT)

So far, so good, but if we consider the purpose for which the CAPM was originally intended, namely stock market investment, it has limitations. As we observed in Chapter Two, even the most actively managed, institutional portfolio funds periodically underperform relative to the market as a whole.

Leaving aside the questionable assumptions that investors are rational, markets are efficient and prices perform a "random walk" (dealt with throughout the author's entire *bookboon* series) one early explanation of the variable performance of portfolios, institutional or otherwise, was provided by Roll's critique of the CAPM (1977).

According to Roll, it is not only impossible for the most discerning investor to establish the composition of the *true* market portfolio, but there is also no reason to assume that a security's expected return is only affected by systematic risk. In the same year, Firth (1977) also observed that if the stock market is so efficient at assimilating all relevant information into security prices, it is impossible to claim that it is either efficient or inefficient, since by definition there is no alternative measurement criterion.

Such criticisms are important, not because they invalidate the CAPM (most empirical tests support it). But because they gave credibility to an alternative approach to portfolio asset management and security price determination based on stock market efficiency presented by Ross (1976). This is termed *Arbitrage Pricing Theory* (APT).

Unlike the CAPM, which prices securities in relation to a *global market portfolio*, the APT possesses the advantage of pricing securities *in relation to each other*. The *single index* (beta factor) CAPM focuses upon an assumed *specific* linear relationship between betas and expected returns (systemic risk plotted by the SML). The APT is a *general* model that *subdivides* systematic risk into smaller components, which need not be specified in advance. These define the *Arbitrage Pricing Plane* (APP). Any macro-economic factors, including market sentiment, which impact upon investor returns may be incorporated into the APP (or ignored, if inconsequential.) For example, an unexpected change in the rate of inflation (purchasing power risk) might affect the price of securities generally. The advantage of the APT, however, is that it can be used to eliminate this risk specifically, such as a pension fund portfolio's requirement that it should be immune to inflation.

Statistical tests on the model, including those of Roll and Ross (1980), established that a *four factor* linear version of the APT is a more accurate predictor of security and portfolio returns than the *single factor* (index) CAPM. Specifically, their APT states that the expected return is directly proportional to its sensitivity to the following:

i. Interest rates,
ii. Inflation
iii. Industrial productivity,
iv. Investor risk attitudes.

The return equation for a four-factor APP conforms to the following *simple linear* relationship for the expected return on the *j* th security in a portfolio:

(66) $r_j = a + b_1 (r_1) + b_2(r_2) + b_3(r_3) + b_4 (r_4)$

Where:

$r_j =$ expected rate of return on security *j*,
$r_i =$ expected return on factor i, (i = 1,2,3,4),
$a =$ intercept,
$b_i =$ slope of r_i.

The expected risk premium on the *j*th security is defined as the difference between its expected return (r_j) and the risk-free fate of interest (r_f) associated with each factor's return (r_i) and the security's sensitivity to each of these factors (b_i). The four-factor equation is given by:

(67) $(r_j - r_f) = b_1 (r_1 - r_f) + b_2 (r_2 - r_f) + b_3 (r_3 - r_f) + b_4 (r_4 - r_f)$

Like the *specific* CAPM, the *general* APT is still a *linear* model. Theoretically, it assumes that unsystematic (unique) risk can be eliminated in a well-diversified portfolio, leaving only the portfolio's sensitivity to unexpected changes in *macro-economic* factors. Subsequent studies, such as Chen, Roll and Ross (1986) therefore focused upon identifying further significant factors and why the sensitivity of returns on a particular share to each factor will vary. However, the work of Dhrymes, Friend and Gultekin (1984) had already suggested that this line of research may be redundant. Their study concluded that as the number of portfolio constituents increases, a greater number of factors must be incorporated into the model. Thus, at the limit, the APT could be equivalent to the CAPM, which defines risk in terms of a *single* over-arching *micro-economic factor* relative to the return on the market portfolio.

For one of the first comprehensive reviews of the APT, which explains why even today it is not fully developed and its application has been less successful than the CAPM, you should read Elton, Gruber and Mei (1994). A more recent perspective on the APT is provided by Huberman and Wang (2005).

Summary and Conclusions

By now you appreciate that financial analysis is not an exact science and the theories upon which it is based may even be "bad" science. The fundamental problem is that real world economic decisions are characterised by uncertainty. By definition *uncertainty is non-quantifiable*. Yet, rather than bury their heads in sand, academics continue to defend financial models, such as the CAPM based on *simplifying assumptions* that *rationalise* a search for investment opportunities in the *chaotic* world we inhabit. See Fama and French (2003).

New mathematical theories and statistical models of investor *irrationality* and market *inefficiency,* characterised by *non-random* walks are being crystallised. These *post-modern* "Quants" reject the assumptions of a normal distribution of returns. See Peters (1991) for a comprehensive exposition. Scientific "catastrophe theory" is also being applied to stock market analysis to explain why "bull" markets crash *without warning*. See Varian (2007).

Academics and financial analysts are also returning to twentieth-century economic theorists for inspiration, from John Maynard Keynes to the *behaviouralists* who dispensed with the assumption that we can maximise anything.

Today's proponents of behaviouralism, such as Montier (2002) reject the *neo-classical* economic profit motive and the wealth maximisation objectives of twentieth-century finance They believe that finance is a blend of economics and psychology that determines how investor attitudes can determine financial decisions. Explained simply, investors do not appreciate what motivates them to make one choice, rather than another. Behavioural Finance therefore seeks to explain why individuals, companies, or institutions make mistakes and how to avoid them.

Suffice it to say, that much of the "new" Quants is so complex as to confuse most financial analysts, let alone individuals who wish to beat the market (think the millennium dot.com fiasco and the 2007 meltdown). Likewise, the "new" behavioural finance (just like the "new" behavioural economics of the1960s) seems to prefer "a sledgehammer to crack a walnut" (see Hill 1990).

As a parting shot, let us therefore return to *first principles* and *common sense* with a guide to your future studies or investment plans, which places Modern Portfolio Theory in a human context.

Ignore forecasts: Evidence suggests that predictions are invariably wrong. The behavioural trait to avoid is known as *anchoring*, whereby you latch on to uncertain data that is hopelessly wrong. Develop a strategy that does not depend on them.

Information Overload: The financial services industry believes that to "beat" the market they need to know more than everybody else. But empirical studies reveal too much information leads to overconfidence, rather than accuracy. So concentrate on an investment's "key" elements.

Overconfidence: Most investors overestimate their skills. Prepare a plan based on your risk-return profile and ability. Then stick to it.

Denial: Investors are more attracted to good news, rather than bad. Prior to the millennium, the market did not want the dot.com boom to fail. So, any information suggesting that techno-shares were overvalued was ignored. The lesson is not to be complacent.

Overreaction: Investors become optimistic in a rising (bull) market and pessimistic in a falling (bear) market. When a significant proportion of investors believe that the market will rise or fall it may be a signal that the opposite will happen.

Crowd Behaviour: People feel safer herded together, which is why investors mimic the behaviour of others and buy fashionable securities and funds. Speculative investors turn this to their own advantage by acquiring stocks that are cheap and unfashionable.

Selective Memory: Most investors tend to forget failure but remember success. To beat the market and keep ahead of the crowd, keep a record of your decisions (good or bad) and learn from your mistakes.

Ignore Current Market Sentiment and Noise: Today, most investors are doing the opposite. The average holding period for a share on the New York Stock Exchange is eleven months, compared with eight years in the 1950s.

Go for long-term investment: Over time, most shareholder returns come from dividends. *But remember* the expected return from a stock is equal to the dividend yield, *plus* any dividend growth, *plus* any changes in valuation that occur. The strategy to adopt is "value investing", where you buy stocks that are cheap with high dividend yields.

To summarise:

Short-term gain equals long-term pain: According to Patrick Hosking (2010) the global financial crisis, which has cost somewhere between one and five times the entire world's financial output, started with reckless bankers lending to poor Americans. Since 2007, other contributory factors have also been suggested for the meltdown. Central banks ignored rising asset prices, governments talked up a global economic boom and financial regulators still adhered to efficient market theory by using a light touch.

However, these are merely the consequences of a more fundamental problem, motivated by greed, referred to throughout our analyses (and the author's *bookboon* series), namely:

> The *principal-agency conflict* associated with inappropriate short-term, managerial reward structures that arise from a bonus culture and lack of corporate governance, first explained by Jensen and Meckling (1976).

As Hosking observes, these flawed incentives still exist today not just in banks, but also within financial institutions and companies whose management (agents) hold shares on behalf of their owners (principles). Management rarely accept responsibility if things go wrong, but always accept rewards, even if their strategies have no lasting value. Thus, managerial short-termism rarely coincides with the long-term income and capital aspirations of their shareholders.

If proof be needed that professional portfolio management has lost its way, let us conclude with two telling UK statistics from the *London School of Economics' Centre for the Study of Capital Market Disfunctionality.*

In *real* terms, pension fund returns grew by an average of *4.1* per cent per annum between 1963 and 2009, but by only *1.1* per cent a year in the last 10 years.

Unless corporate management are held personally responsible for their bonuses long after their receipt (perhaps a decade) it is therefore difficult to see how the rational objectives of efficient portfolio theory can ever match the rational expectations of a portfolio's clientele.

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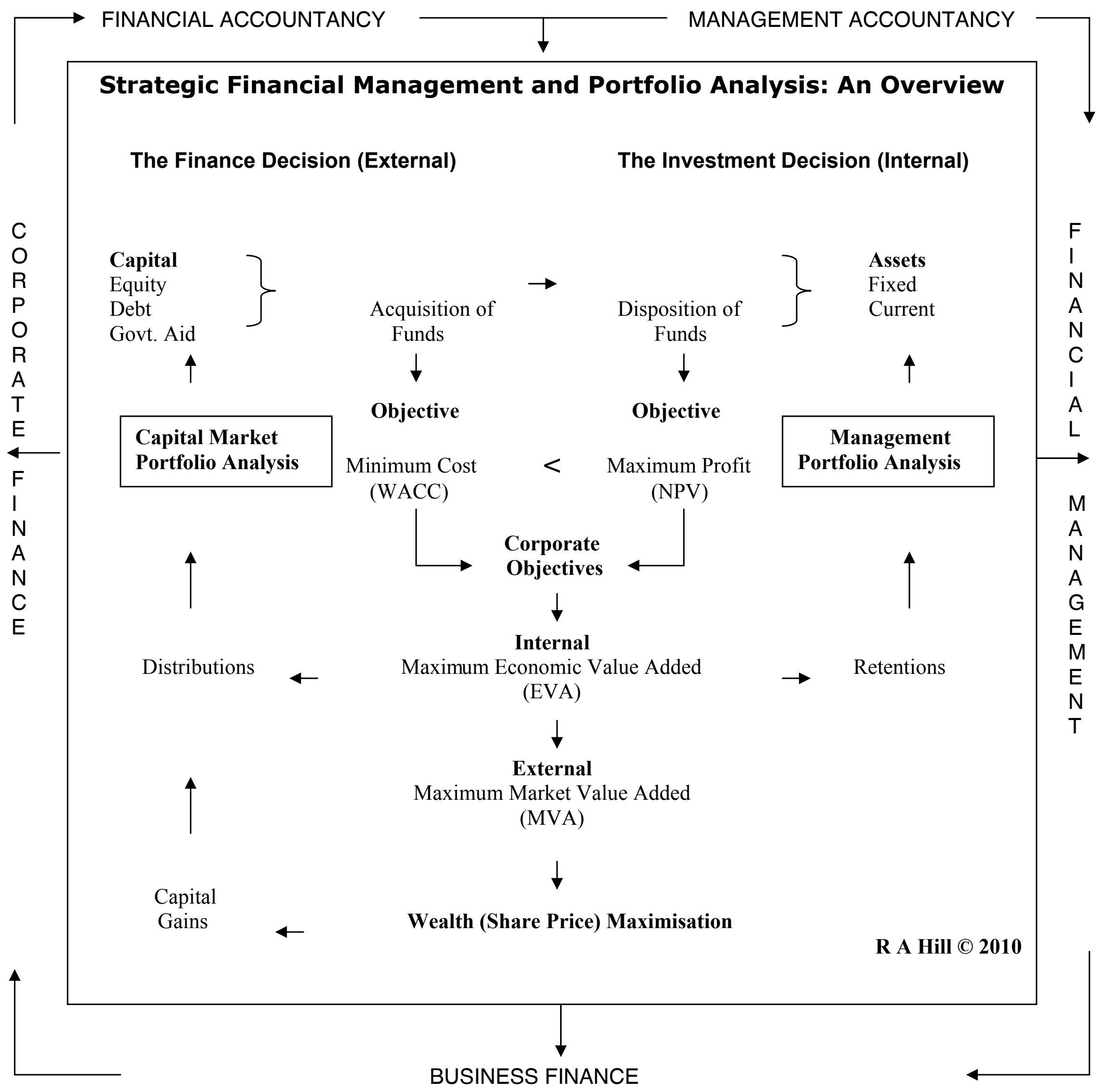
5 Appendix



FINANCIAL ACCOUNTANCY
MANAGEMENT ACCOUNTANCY
Strategic Financial Management and Portfolio Analysis: An Overview
The Finance Decision (External)
The Investment Decision (Internal)
CORPORATE FINANCE
FINANCIAL MANAGEMENT
Capital
Equity
Debt
Govt. Aid
Acquisition of Funds
Disposition of Funds
Assets
Fixed
Current
Objective
Objective
Capital Market Portfolio Analysis
Minimum Cost (WACC)
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Maximum Profit (NPV)
Management Portfolio Analysis
Corporate Objectives
Internal
Maximum Economic Value Added (EVA)
Distributions
Retentions
External
Maximum Market Value Added (MVA)
Capital Gains
Wealth (Share Price) Maximisation
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