LUNAR SOLUTION ELP
version ELP/MPP02

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1 Introduction

ELP/MPP02 is a semi-analytical solution for the orbital motion of the Moon. It is built on the basis of the lunar theories ELP 2000-82 (Chapront-Touzé, Chapront, 1983) and ELP 2000-85 (Chapront-Touzé, Chapront, 1988) and contains the same components including :

- the Main Problem 'Moon, Earth and Sun' using kleperian orbit for the Earth-Moon barycenter EMB;
- the direct planetary perturbations due to the action of the planets on the Moon;
- the indirect planetary perturbations induced by the deviation of EMB from a kleperian orbit;
- the Earth's figure perturbation including nutational motion of the Earth;
- the Moon's figure perturbations including the effects lunar potential up to the third order;
- the relativistics effects;
- the tidal perturbations.

The differences between ELP/MPP02 and the previous ELP versions concern mainly the use of the new planetary perturbations MPP01 (Bidart, 2001). The construction of ELP/MPP02 is explained in (Chapront and Francou, 2003) and explanatory comments about the original ELP2000-82 series are given in the technical note 'Lunar solution ELP, version 2000-82B'.

When using a ELP solution, we have to distinguish between the theory itself whose series are available on data files from the parameters or constants used in the numerical evaluation of the theory. Some of these constants have 'nominal values' which are corrected with adjustments resulting from the fit of the solution to observations. The partial derivatives given together with the series of the Main Problem allow to do these corrections. For ELP/MPP02, we propose two ways:

- to use corrections obtained by the fit to Laser Lunar Ranging data (LLR) provided since 1970;
- to use corrections obtained by the fit to the numerical integration DE405 of the Jet Propulsion Laboratory (JPL) used as an observing model; in this case some additive corrections are also applied to secular values of the lunar angles for approaching closer the JPL Ephemeris DE406 over 6000 years (Standish, 1998).

This note decribes the data files containing the series of the lunar solution ELP/MPP02 which is separated in 2 parts: the Main Problem and the various perturbations brought together, for the 3 geocentric coordinates: Longitude (V), Latitude (U) and Distance (r). It gives also all the specifications allowing the use of these series: arguments, corrections to the constants and coordinate systems. Informations about the computation of lunar ephemerides are also provided. The last section gives more details about the construction of the ELP solutions and their improvements.

2 Data files

2.1 Names and contents

`ELP_MAIN.S1` :	Main Problem.	Longitude.	Fourier series.
`ELP_MAIN.S2` :	Main Problem.	Latitude.	Fourier series.
`ELP_MAIN.S3` :	Main Problem.	Distance.	Fourier series.
`ELP_PERT.S1` :	Perturbations.	Longitude.	Poisson series.
`ELP_PERT.S2` :	Perturbations.	Latitude.	Poisson series.
`ELP_PERT.S3` :	Perturbations.	Distance.	Poisson series.

2.2 Description of records

2.2.1 Files for the Main Problem: ELP_MAIN.S1, ELP_MAIN.S2, ELP_MAIN.S3

They contain the Fourier series of the longitude, the latitude and the distance. The first record contains the title of series and the number of terms. Each following record contains one periodic term.
The formulation is:

$$\sum_{\{i\}} A_{\{i\}} \left\{ \begin{array}{c} sin \\ cos \end{array} \right\} (i_1 D + i_2 F + i_3 l + i_4 l') \quad with \quad \{i\} = (i_1, i_2, i_3, i_4) \tag{1}$$

Longitude and latitude are sine series (files ELP_MAIN.S1 and ELP_MAIN.S2) and the distance is cosine series (file ELP_MAIN.S3). The coefficients $A_{\{i\}}$ are expressed in arcsecond for the longitude and the latitude, in kilometer for the distance; $\{i\}$ is a set of integers. The expression of the Delaunay arguments D, F, l, l' are given in subsection 3.1.
Each record gives: $i_1, i_2, i_3, i_4, A_{\{i\}}, B_j^{[i]} (j = 1...6)$ with the FORTRAN format: 4i3,2x,f13.5,6f12.2.
The six quantities $B_j^{[i]}$ are the derivatives of $A_{\{i\}}$ with respect to six constants $\sigma_j = (m, \Gamma, E, e', \alpha, \mu)$ (see subsection 4.3.1).

$$B_j^{[i]} \text{ are dimensionless: } \quad \frac{\partial A_{\{i\}}}{\partial \sigma_j} \text{ for longitude and latitude and } \quad a_0 \frac{\partial}{\partial \sigma_j} \left(\frac{A_{\{i\}}}{a_0} \right) \text{ for distance.}$$

2.2.2 Files for the perturbations: ELP_PERT.S1, ELP_PERT.S2, ELP_PERT.S3

They contain Poisson series of the longitude, the latitude and the distance. The records are collected in groups which correspond to the same time power n ($n = 0, ..., 4$). Each group is preceded by a "header record" which contains the title of the series and its number of terms. Each other record contains one Poisson term. For a given time power n the formulation is:

$$t^n \sum_{\{i\}} [S_{\{i\}} sin\phi + C_{\{i\}} cos\phi] \quad with \quad \{i\} = (i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}, i_{13}) \tag{2}$$

$$with: \quad \phi = i_1 D + i_2 F + i_3 l + i_4 l' + i_5 Me + i_6 V + i_7 T + i_8 Ma + i_9 J + i_{10} S + i_{11} U + i_{12} N + i_{13} \zeta$$

The coefficients $S_{\{i\}}$ and $C_{\{i\}}$ are given in arcsecond for the longitude and the latitude, in kilometer for the distance. The expressions of the arguments $D, F, l, l', Me, V, T, Ma, J, S, U, N, \zeta$ are provided in section 3; $\{i\}$ is a set of integers. Each record gives the values of: $S, C, i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}, i_{13}$ with the FORTRAN format: 5x,2d20.13,13i3.

3 Arguments

3.1 Fundamental arguments

Delaunay arguments D, F, l, l' are :

$$\begin{array}{lcl} D & = & W_1 - T + 180° \\ F & = & W_1 - W_3 \\ l & = & W_1 - W_2 \\ l' & = & T - \varpi' \end{array} \tag{3}$$

with the Moon and Earth-Moon arguments:
W_1, the mean mean longitude of the Moon,
W_2, the mean longitude of the lunar perigee,
W_3, the mean longitude of the lunar ascending node,
T, the heliocentric mean longitude of the Earth-Moon barycenter,
ϖ', the mean longitude of the perihelion of the Earth-Moon barycenter,

W_1, W_2 and W_3 are angles of the inertial mean ecliptic of date referred to the departure point γ'_{2000} (see subsection 5.1). T and ϖ' are angles of the inertial mean ecliptic of J2000 referred to the inertial mean equinox

γ^I_{2000} of J2000. The secular developments of these arguments are polynomial functions of time under the general formulation: $a = a^{(0)} + a^{(1)}t + a^{(2)}t^2 + a^{(3)}t^3 + a^{(4)}t^4$. Their 'nominal values' before corrections are given in Table 1. The mean motions of the Moon and the Sun (coefficient of t in W_1 and T) are also denoted respectively as ν and n': $\nu = W_1^{(1)}$, $n' = T^{(1)}$.

Table 1: Secular developments of the Moon and Earth-Moon arguments before corrections.

W_1	=	218°18′59.955 71"	+	1 732 559 343.736 04"	t	−	6.808 4"	t^2	+	0.006 604"	t^3	−	0.000 031 69"	t^4
W_2	=	83°21′11.674 75"	+	14 643 420.317 1"	t	−	38.263 1"	t^2	−	0.045 047"	t^3	+	0.000 213 01"	t^4
W_3	=	125°02′40.398 16"	−	6 967 919.538 3"	t	+	6.359 0"	t^2	+	0.007 625"	t^3	−	0.000 035 86"	t^4
T	=	100°27′59.138 85"	+	129 597 742.293 0"	t	−	0.020 2"	t^2	+	0.000 009"	t^3	+	0.000 000 15"	t^4
ϖ'	=	102°56′14.457 66"	+	1 161.243 42"	t	+	0.529 265"	t^2	−	0.000 118 14"	t^3	+	0.000 011 379"	t^3

t is barycentric time TDB in Julian centuries from J2000.

The argument ζ is deduced from W_1 by: $\zeta = W_1 + (p + \Delta p)t$ where p is the IAU 1976 precession constant: 5029.0966"/cy and Δp an additive correction from (Herring et al., 2002): -0.29965"/cy.

3.2 Planetary arguments

The planetary arguments $Me, V, Ma, J, S, U,\ N$ are the linear parts of the mean mean longitudes of the planets Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune from the planetary theory VSOP2000 (Moisson, 2000) under the formulation: $\lambda = \lambda^{(0)} + \lambda^{(1)}t$; t is barycentric time TDB in Julian centuries from J2000 (Julian Date 2 451 545.0).

Table 2: Expressions of the planetary arguments.

(Me)rcury	${\lambda^{(0)}}_{Me}$	=	252°15′03.216 919"	${\lambda^{(1)}}_{Me}$	=	538 101 628.688 88 "/cy
(V)enus	${\lambda^{(0)}}_{V}$	=	181°58′44.758 419"	${\lambda^{(1)}}_{V}$	=	210 664 136.457 77 "/cy
(Ma)rs	${\lambda^{(0)}}_{Ma}$	=	355°26′03.642 778"	${\lambda^{(1)}}_{Ma}$	=	68 905 077.659 36 "/cy
(J)upiter	${\lambda^{(0)}}_{J}$	=	34°21′05.379 392"	${\lambda^{(1)}}_{J}$	=	10 925 660.573 35 "/cy
(S)aturn	${\lambda^{(0)}}_{S}$	=	50°04′38.902 495"	${\lambda^{(1)}}_{S}$	=	4 399 609.336 32 "/cy
(U)ranus	${\lambda^{(0)}}_{U}$	=	314°03′04.354 234"	${\lambda^{(1)}}_{U}$	=	1 542 482.578 45 "/cy
(N)eptune	${\lambda^{(0)}}_{N}$	=	304°20′56.808 371"	${\lambda^{(1)}}_{N}$	=	786 547.897 00 "/cy

4 Correction to the constants

4.1 List of the fitted constants

4.1.1 The constants of the Moon

- $W_1^{(0)}$, mean mean longitude of the Moon at J2000,
- $W_2^{(0)}$, mean longitude of lunar perigee at J2000,
- $W_3^{(0)}$, mean longitude of lunar node at J2000,
- $W_1^{(1)} = \nu$, sidereal mean motion of the Moon,
- $W_2^{(1)}$, mean motion of lunar perigee,
- $W_3^{(1)}$, mean motion of lunar node,
- $W_1^{(2)}$, coefficient of the quadratic term of the lunar mean mean longitude,
- Γ, half coefficient of $sinF$ in latitude,
- E, half coefficient of $sinl$ in longitude.

Remark: $W_1^{(2)}$ yields the tidal part of the coefficient of the quadratic term of the mean longitude, $W_1^{(2,T)}$, which is half the tidal secular acceleration.

4.1.2 The constants of the Earth-Moon barycenter

- $T^{(0)}$, mean mean longitude of Earth-Moon barycenter at J2000,
- $T^{(1)} = n'$, sidereal motion of the Earth-Moon barycenter,
- $\varpi'^{(0)}$, mean longitude of perihelion of Earth-Moon barycenter at J2000,
- e', eccentricity of the heliocentric orbit of the Earth-Moon barycenter at J2000.

4.2 Values of the corrections to the nominal values

The notation ELP/MPP02(405) means that the constants are derived from the fit to JPL numerical integration DE405 over the time span [1950-2060]. The notation ELP/MPP02(LLR) means that the constants are derived from the fit to LLR observations made between 1970 and 2001 (14500 normal points with 4 terrestrial observing stations and 4 lunar reflectors). The values of the corrections to the constants induced by these fits are given in Table 3.

Table 3: Corrections to constants given by the 2 fits DE405 and LLR.

Corrections	ELP/MPP02(405)	ELP/MPP02(LLR)
$\Delta W_1^{(0)}$	$-0.070\ 08$	$-0.105\ 25$
$\Delta W_2^{(0)}$	$+0.207\ 94$	$+0.168\ 26$
$\Delta W_3^{(0)}$	$-0.072\ 15$	$-0.107\ 60$
$\Delta W_1^{(1)} = \Delta\nu$	$-0.351\ 06$	$-0.323\ 11$
$\Delta W_2^{(1)}$	$+0.080\ 17$	$+0.080\ 17$
$\Delta W_3^{(1)}$	$-0.043\ 17$	$-0.043\ 17$
$\Delta W_1^{(2)}$	$-0.037\ 43$	$-0.037\ 94$
$\Delta\Gamma$	$+0.000\ 85$	$+0.000\ 69$
ΔE	$-0.000\ 06$	$+0.000\ 05$
$\Delta T^{(0)}$	$-0.000\ 33$	$-0.040\ 12$
$\Delta T^{(1)} = \Delta n'$	$+0.007\ 32$	$+0.014\ 42$
$\Delta\varpi'^{(0)}$	$-0.007\ 49$	$-0.048\ 54$
$\Delta e'$	$+0.002\ 24$	$+0.002\ 26$

Units: arcsecond except for $\Delta W_1^{(1)}$, $\Delta W_2^{(1)}$, $\Delta W_3^{(1)}$, $\Delta T^{(1)}$ in arcsecond/cy, and $\Delta W_1^{(2)}$ in arcsecond/cy^2.

4.3 Implementation of the corrections

4.3.1 Corrections to the coefficients of periodic series

In the formulation (1), the coefficients $A_{\{i\}}$ of periodic series in the Main Problem have to be corrected in accordance with the selected fit (DE405 or LLR).
These corrections $\delta A_{\{i\}}$ are deduced from the corrections to the fitted constants (ν, Γ, E, e', n') given in Table 4: the numerical parts come from a previous fit of ELP to the JPL Ephemeris DE200 and the corrections Δ are given in Table 3.

Table 4: Corrections to the constants ν, Γ, E, e', n'

$$\begin{aligned}
\delta\nu &= 0.556\ 04''/cy &+& \Delta W_1^{(1)} \\
\delta\Gamma &= -0.080\ 66'' &+& \Delta\Gamma \\
\delta E &= 0.017\ 89'' &+& \Delta E \\
\delta e' &= -0.128\ 79'' &+& \Delta e' \\
\delta n' &= -0.064\ 2\ ''/cy &+& \Delta n'
\end{aligned}$$

We use also the derivatives $B_j^{[i]}$ of $A_{\{i\}}$ with respect to the constants $\sigma_j = (m, \Gamma, E, e', \alpha)$ given in the files for the Main Problem (see subsection 2.2.1).

- *Correction to the coefficients in longitude and latitude:*

$$\delta A_{\{i\}} = -m\left(B_1^{[i]} + \frac{2}{3}\frac{\alpha}{m}B_5^{[i]}\right)\frac{\delta\nu}{\nu} + \left(B_1^{[i]} + \frac{2}{3}\frac{\alpha}{m}B_5^{[i]}\right)\frac{\delta n'}{\nu} + (B_2^{[i]}\delta\Gamma + B_3^{[i]}\delta E + B_4^{[i]}\delta e')$$

- *Correction to the coefficients in distance:*

$$\delta A_{\{i\}} = -m\left(B_1^{[i]} + \frac{2}{3}\frac{\alpha}{m}B_5^{[i]} + \frac{2}{3}\frac{A_{\{i\}}}{m}\right)\frac{\delta\nu}{\nu} + \left(B_1^{[i]} + \frac{2}{3}\frac{\alpha}{m}B_5^{[i]}\right)\frac{\delta n'}{\nu} + (B_2^{[i]}\delta\Gamma + B_3^{[i]}\delta E + B_4^{[i]}\delta e')$$

with:

$$\begin{aligned} m &= n'/\nu \quad (= 0.074\ 801\ 329), \\ \alpha &= a_0/a' \quad (= 0.002\ 571\ 881). \end{aligned}$$

a_0 and a' are the kleperian semi-major axis of the Moon and the Earth-Moon barycenter. They are respectively related to ν and n' by: $\nu^2 a_0^3 = G(m_T + m_L)$ and $n'^2 a'^3 = G(m_S + m_T + m_L)$. m_S, m_T and m_L are respectively Sun, Earth and Moon masses. The last term in the 2 equations have to be expressed in radian.

4.3.2 Corrections to the coefficients in the secular developments of arguments

Some coefficients of the Moon and Earth-Moon arguments have to be corrected in accordance with the selected fit (DE405 or LLR):

$$\begin{array}{lll} W_1^{(0)} + \Delta W_1^{(0)} & \quad & W_1^{(1)} + \Delta W_1^{(1)} \\ W_2^{(0)} + \Delta W_2^{(0)} & & W_2^{(1)} + \Delta W_2^{(1)} + \delta W_2^{(1)} \\ W_3^{(0)} + \Delta W_3^{(0)} & & W_3^{(1)} + \Delta W_3^{(1)} + \delta W_2^{(1)} \\ T^{(0)} + \Delta T^{(0)} & & T^{(1)} + \Delta T^{(1)} \\ \varpi'^{(0)} + \Delta\varpi'^{(0)} & & W_1^{(2)} + \Delta W_1^{(2)} \end{array}$$

The 'nominal values' of these coefficients are given in Table 1 and their corresponding corrections in Table 3.

The corrections $\delta W_2^{(1)}$ and $\delta W_3^{(1)}$ for the mean motions of perigee and node of the Moon, $W_2^{(1)}$ and $W_3^{(1)}$, are infered from the corrections $\Delta W_1^{(1)}, \Delta T^{(1)}, \Delta\Gamma, \Delta E, \Delta e'$ given in Table 3 with the formulation:

$$\delta W_{(i=2,3)}^{(1)} = \left[\frac{W_i^{(1)}}{\nu} - m\left(B'_{i,1} + \frac{2}{3}\frac{\alpha}{m}B'_{i,5}\right)\right]\Delta W_1^{(1)} + \left(B'_{i,1} + \frac{2}{3}\frac{\alpha}{m}B'_{i,5}\right)\Delta T^{(1)} + \nu(B'_{i,2}\Delta\Gamma + B'_{i,3}\Delta E + B'_{i,4}\Delta e')$$

$B'_{2,j}$ and $B'_{3,j}$ (for j=1,...,5) are the derivatives $\dfrac{\partial}{\partial\sigma'_j}\dfrac{W_2^{(1)}}{\nu}$ and $\dfrac{\partial}{\partial\sigma'_j}\dfrac{W_3^{(1)}}{\nu}$ with respect to the constants $\sigma'_j = (\nu, \Gamma, E, e', n')$ (see Table 5). The last term of the equation has to be expressed in radian and the values of m and α are given in 4.3.1.

Table 5: Derivatives $B'_{i,j}$ used for the corrections $\delta W_2^{(1)}$ and $\delta W_3^{(1)}$.

j	Constants σ'_j	$B'_{2,j} = \dfrac{\partial}{\partial\sigma'_j}\dfrac{W_2^{(1)}}{\nu}$	$B'_{3,j} = \dfrac{\partial}{\partial\sigma'_j}\dfrac{W_3^{(1)}}{\nu}$
1	ν	+0.311 079 095	−0.103 837 907
2	Γ	−0.004 482 398	+0.000 668 287
3	E	−0.001 102 485	−0.001 298 072
4	e'	+0.001 056 062	−0.000 178 028
5	n'	+0.000 050 928	−0.000 037 342

4.3.3 Additive corrections to secular terms

When adding the corrections given Table 6 to secular terms of the arguments W_1, W_2, W_3 the solution ELP/MPP02 approaches closely the JPL Ephemeris DE406 on a long range (a few seconds over 6 millennia). These corrections have to be used together with the corrections resulting from the fit to DE405 ephemeris, ELP/MPP02(405).

Table 6: Corrections to secular terms of W_1, W_2, W_3.

Mean longitude of the Moon	t^3 term	$\Delta W_1^{(3)}$	=	$-0.000\ 188\ 65$ "$/cy^3$
Mean longitude of the Moon	t^4 term	$\Delta W_1^{(4)}$	=	$-0.000\ 010\ 24$ "$/cy^4$
Mean longitude of the perigee	t^2 term	$\Delta W_2^{(2)}$	=	$+0.004\ 706\ 02$ "$/cy^2$
Mean longitude of the perigee	t^3 term	$\Delta W_2^{(3)}$	=	$-0.000\ 2521\ 3$ "$/cy^3$
Mean longitude of the node	t^2 term	$\Delta W_3^{(2)}$	=	$-0.002\ 610\ 70$ "$/cy^2$
Mean longitude of the node	t^3 term	$\Delta W_3^{(3)}$	=	$-0.000\ 107\ 12$ "$/cy^3$

5 Coordinate systems

5.1 Inertial mean ecliptic frames

The departure point γ'_{2000} is the point of the inertial mean ecliptic of date defined by:

$$N\gamma'_{2000} = N\gamma^I_{2000}$$

where γ^I_{2000} is the inertial mean equinox of J2000 and N the node of the inertial mean ecliptic of date and of J2000 (see Fig.1).

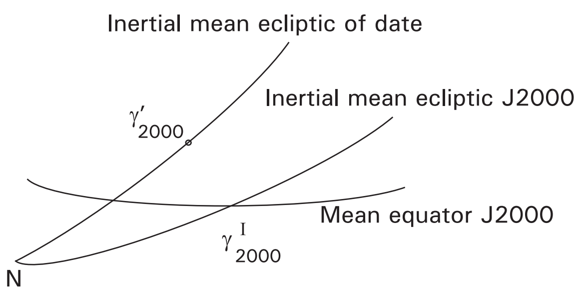

Fig. 1 Position of the departure point.

The natural coordinate system of the lunar theories ELP consists of the inertial mean ecliptic of date and departure point γ'_{2000}.

In this coordinate system and taking into account the evaluation of arguments and the corrections of constants induced by the selected fit (DE405, LLR) as explained in this note, the geocentric coordinates of the Moon (V, U, r) are:

Longitude:	V	=	[periodic series (`ELP_MAIN.S1`)	+ Poisson series (`ELP_PERT.S1`)]	$+ W_1$
Latitude:	U	=	[periodic series (`ELP_MAIN.S2`)	+ Poisson series (`ELP_PERT.S2`)]	
Distance:	r	=	[periodic series (`ELP_MAIN.S3`)	+ Poisson series (`ELP_PERT.S3`)]	$\times r_{a0}$

with:

$$r_{a0} = \frac{a_0(DE405)}{a_0(ELP)} = \frac{384747.961370173}{384747.980674318}$$

In the following expressions, t is barycentric time TDB in Julian centuries from J2000.

- *Longitude and latitude V_d, U_d referred to the inertial mean ecliptic and equinox of date:*

$$V_d \ = \ V \ + \ p_A \ + \ \Delta p \ t \qquad U_d \ = \ U$$

p_A is the accumulated precession between J2000 and the date derived from (Laskar, 1986) after the truncating of the series and Δp is a correction to the precession constant from (Herring et al., 2002):

$$p_A = 5\ 029.096\ 6"t + 1.112\ 0"t^2 + 0.000\ 077"t^3 - 0.000\ 023\ 53"t^4 \qquad \Delta p = -0.29965"/cy.$$

- *Rectangular coordinates $x^E_{2000}, y^E_{2000}, z^E_{2000}$ referred to the inertial mean ecliptic and equinox of J2000:*

$$\begin{pmatrix} x^E_{2000} \\ y^E_{2000} \\ z^E_{2000} \end{pmatrix} = \begin{pmatrix} 1-2P^2 & 2PQ & 2P\sqrt{1-P^2-Q^2} \\ 2PQ & 1-2Q^2 & -2Q\sqrt{1-P^2-Q^2} \\ -2P\sqrt{1-P^2-Q^2} & 2Q\sqrt{1-P^2-Q^2} & 1-2P^2-2Q^2 \end{pmatrix} \begin{pmatrix} r\cos V\cos U \\ r\sin V\cos U \\ r\sin U \end{pmatrix}$$

P and Q are issued from (Laskar, 1986) reproduced here up to degree five:

$$\begin{aligned} P \ = \ & 0.101\ 803\ 91\ \ 10^{-4}\ t\ + 0.470\ 204\ 39\ 10^{-6}\ t^2 - 0.541\ 736\ 7\ 10^{-9}\ t^3 \\ & -0.250\ 794\ 8\ \ 10^{-11}\ t^4 + 0.463\ 486\ \ 10^{-14}\ t^5 \\ Q \ = \ & -0.113\ 469\ 002\ 10^{-3}\ t\ + 0.123\ 726\ 74\ 10^{-6}\ t^2 + 0.126\ 541\ 7\ 10^{-8}\ t^3 \\ & -0.137\ 180\ 8\ \ 10^{-11}\ t^4 - 0.320\ 334\ \ 10^{-14}\ t^5 \end{aligned}$$

5.2 Position of the inertial mean ecliptic J2000

The inertial mean ecliptic J2000 can be positioned with respect to the following 'equatorial' frames (R):

ICRS	:	International Celestial Reference System;
MCEP	:	Frame linked to the Mean Celestial Ephemeris Pole of J2000 (CEP);
JPL405	:	Frame defined by the JPL numerical integration DE405.

The position angles are illustrated in Fig.2 with:

$\gamma^I_{2000}(R)$	:	Ascending node of the inertial mean ecliptic J2000 on the equator of R;
$\epsilon(R)$	:	Inclination of the inertial mean ecliptic on the equator of R;
$o(R)$	:	Origin of right ascensions in the equator of R;
$\phi(R)$	:	Arc between $o(R)$ and $\gamma^I_{2000}(R)$;
$\psi(R)$	:	Arc between γ^I_{2000} and $(ICRS)\gamma^I_{2000}(R)$.

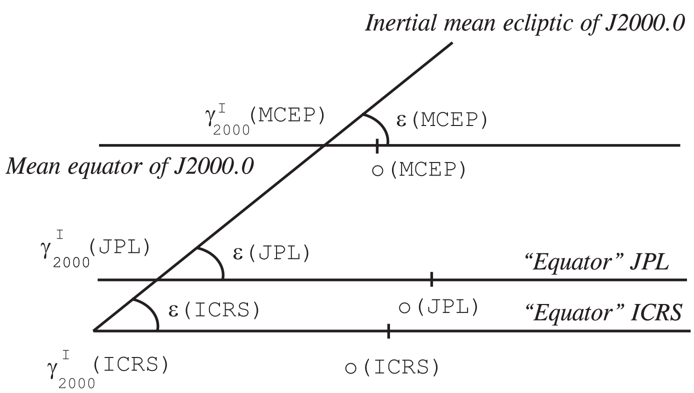

Fig. 2 Relative positions of the mean inertial ecliptic of J2000 with respect to ICRS, MCEP and JPL405.

We gather in Table 7 the values of the position angles ϕ, ϵ and ψ.
They are deduced from fits of ϕ and ϵ to observations: LLR observations when (R) is ICRS or MCEP frame, and DE405 Ephemeris used as an observing model when (R) is JPL405.

For ICRS, the fit takes into account the Precession-Nutation matrix $P \times N$ computed with the IERS conventions 1996, in particular the daily values provided by the IERS.

For MCEP frame, the fit takes into account the matrix $P \times N$ provided by analytical solutions with a correction to the IAU 1976 constant of precession. The same method is used for JPL405 frame and for determining the corrected constants given in Table 3.

Taking as a basic reference the mean mean longitude of the Moon deduced from the fit in the ICRS: W_1(ICRS), the angles $\psi(MCEP)$ and $\psi(JPL405)$ are obtained with the differences:

$$W_1(ICRS) - W_1(MCEP) \quad or \quad W_1(ICRS) - W_1(JPL405)$$

$W_1(MCEP)$ (resp. $W_1(JPL405)$) is the mean mean longitude of the Moon deduced from the fit in the MCEP (resp. JPL405) frame for an epoch which depends of the time distribution of the observations used in the fits.

Table 7: Position angles of the inertial mean ecliptic of J2000 with respect to equatorial celestial frames R.

R	$\epsilon - 23°26'21"$	ϕ	ψ	Mean Epoch
ICRS	0.41100 ± 0.00005	-0.05542 ± 0.00011		Dec 1994
MCEP	0.40564 ± 0.00009	-0.01460 ± 0.00015	0.0445 ± 0.0003	Dec 1994
JPL405	0.40960 ± 0.00001	-0.05028 ± 0.00001	0.0064 ± 0.0003	Jan 1990

Units: arcsecond. The uncertainties are formal errors .

6 Practical computation

The FORTRAN subroutine ELPMPP02 allows to compute rectangular geocentric lunar coordinates (positions and velocities) referred to the inertial mean ecliptic and equinox of J2000. It uses the six data files which contain the series of the solution and takes into account the specifications presented in this paper. Check values are given in Tables 8.

The inputs are:
- The date (TDB): tj in days from J2000 (double precision).
- The logical unit of the data files: lu (integer).
- The index of the corrections: $icor$ (integer).
  $icor = 1$: the constants are fitted to LLR observations.
  $icor = 2$: the constants are fitted to JPL Ephemeris DE405.

The outputs are:
- The table of rectangular coordinates: $xyz(6)$ (double precision),
  positions $x^E_{2000}, y^E_{2000}, z^E_{2000}$ (km) and velocities $\dot{x}^E_{2000}, \dot{y}^E_{2000}, \dot{z}^E_{2000}$ (km/day)
  referred to the inertial mean ecliptic and equinox of J2000.
- The error index: $ierr$ (integer).
  If $ierr = 0$, there is no signaled error.
  If $ierr \neq 0$, there is an error in reading the data files.

The subroutine ELPMPP02 calls 3 subroutines:
- INITIAL : Initialization of the constants of the solution,
- READFILE: Reading the 6 files containing the ELP/MPP02 series,
- EVALUATE: Computation of the geocentric coordinates of the Moon.

The subroutines INITIAL and READFILE are used at the first call to the subroutine ELPMPP02 and each time the index of corrections ($icor$) is changing.

The subroutine ELPMPP02 uses 2 common blocks:
- ELPCST : Constants useful for the evaluation of the series.
- ELPSER : Series of the semi-analytical solution ELP/MPP02,

The constants are initialized by INITIAL and used in READFILE and EVALUATE.

The series are read by READFILE and used in EVALUATE.
The names of the 6 data files that contain the ELP/MPP02 series are given by the parameter 'filename'. If the files are not stored in the current directory, this parameteter have to include the access path in the file names.

Table 8.a: Lunar geocentric rectangular coordinates in km and km/day (ecliptic and equinox of J2000). Fit to LLR observations.

Julian Date	Date (0h TDB)	x^E_{2000} / $\dot{x}^E_{2000}$	y^E_{2000} / $\dot{y}^E_{2000}$	z^E_{2000} / $\dot{z}^E_{2000}$
JD 2 444 239.5	1980 Jan 01	43 890. 282 40	381 188. 727 45	-31 633. 381 65
		-87 516. 197 48	13 707. 664 44	2 754. 221 24
JD 2 446 239.5	1985 Jun 23	-313 664. 596 45	212 007. 266 74	33 744. 751 20
		-47 315. 912 81	-75 710. 875 01	-1 475. 628 69
JD 2 448 239.5	1990 Dec 14	-273 220. 060 67	-296 859. 768 22	-34 604. 357 00
		60 542. 327 59	-58 162. 316 74	2 270. 886 91
JD 2 450 239.5	1996 Jun 05	171 613. 142 80	-318 097. 337 50	31 293. 548 24
		83 266. 779 90	42 585. 830 28	-1 695. 826 11
JD 2 452 239.5	2001 Nov 26	396 530. 006 35	47 487. 922 49	-36 085. 309 03
		-12 664. 286 94	83 512. 757 19	1 507. 367 56

Table 8.a: Lunar geocentric rectangular coordinates in km and km/day (ecliptic and equinox of J2000). Fit to DE405 ephemeris + Corrections of secular terms.

Julian Date	Date (0h TDB)	x^E_{2000} / $\dot{x}^E_{2000}$	y^E_{2000} / $\dot{y}^E_{2000}$	z^E_{2000} / $\dot{z}^E_{2000}$
JD 2 500 000.5	2132 Sep 01	274 034. 591 03	252 067. 536 89	-18 998. 755 19
		-62 463. 613 38	65 693. 963 92	6 595. 328 90
JD 2 300 000.5	1585 Feb 01	353 104. 313 59	-195 254. 118 08	34 943. 545 92
		39 543. 136 78	74 373. 180 70	-700. 653 51
JD 2 100 000.5	1037 Jun 28	-19 851. 276 74	-385 646. 177 17	-27 597. 661 34
		87 539. 407 44	-7 599. 684 84	-4 960. 443 60
JD 1 900 000.5	489 Dec 02	-370 342. 792 54	-37 574. 255 33	-4 527. 918 40
		12 255. 287 46	-89 710. 975 08	7 649. 442 85
JD 1 700 000.5	-58 May 08	-164 673. 047 20	367 791. 713 29	31 603. 980 27
		-75 884. 688 15	-35 802. 265 58	-4 239. 598 95

7 A few comments about ELP/MPP02

The solution ELP/MPP02 is issued from a generation of ELP solutions which goes back to the 1980s. The first one, ELP 2000-82 (Chapront- Touzé and Chapront, 1983), put the bases of the complete form of the semi-analytical theory used today. This initial version was built with the planetary solution VSOP82 (Bretagnon, 1982) and fitted to the JPL Ephemeris DE200/LE200.

The next one, ELP 2000-85 (Chapront- Touzé and Chapront, 1988), extended the length of validity of the solution up to historical times with high degree in time for the secular motions. The version ELP 2000-82B is the export version of ELP 2000-85 with some improvements in constants and parameters (masses, gravitational parameters, tidal coefficients, ...).

With ELP 2000-96, first comparisons to LLR observations have been done by adding numerical complements to ELP on the basis of JPL Ephemeris DE245. In 1998, an analytically completed theory of the libration of M. Moons has been performed (Chapront et al., 1999) which improved noticeably the analysis of LLR observations.

The present version, ELP/MPP02, takes into account new planetary perturbations in the orbital motion of the Moon, MPP01 (Bidart, 2001), which were built using the planetary solutions VSOP2000 (Moisson, 2000) and compared to the JPL Ephemeris DE403. MPP02 is an improvement of MPP01 (moving perigee, secular arguments, selected terms in planetary perturbations, level of truncation,...). Besides, ELP/MPP02 has been compared to the ephemeris issued from the JPL numerical integration DE405 built on the basis modern observations of high accuracy, in particular VLBI. Some changes are brought on secular motions of the lunar mean mean longitude (in particular tidal acceleration) and the mean longitudes of perigee and node of the Moon. The new ELP solution has been also compared on the long range version of DE405, DE406, over 6 millennia centered on 2000.

The fits of ELP/MPP02 have been realized in two steps.

• At first, the solution has been fitted to DE405 ephemeris over the time interval [+1950;+2060]. The method is similar to the previous ELP versions where the JPL ephemerides were used as observing models (DE200, DE245, DE403). We then obtained the solution ELP/MPP02(405) with the corrections of constants listed in Table 3 (column 2) .
To reduce the residuals with DE406 on a long range, some additive corrections to secular parameters of lunar arguments are also provided. Note that it is not an analytical improvement of the solution but only a fit which keeps ELP/MPP02(405) closer to DE406 over 6000 years. The largest differences on this time interval are about 3.5" in longitude, 0.8" in latitude and 1.5 km in distance.

• Secondly, the ephemeris issued from ELP/MPP02(405) has been compared to LLR observations provided by the american and french observing stations with the 4 operating reflectors landed on the Moon ground since 1970. We obtained a first solution fitted to LLR data: ELP/MPP02(LLR).
Then, for the following comparisons to LLR observations we used as ephemeris: ELP/MPP02(LLR)+ρ_{405}, with: ρ_{405} = DE405 - ELP/MM02(405). Taking advantage the numerical complements ρ_{405} which are insensible to the change of constants at the centimeter level, this new ephemeris maintains both the precision of an numerical integration and the advantage of an analytical formulation. It is used for the determination of various constants such as lunar and solar parameters, lunar tidal acceleration, position of the dynamical reference frame, correction to the precession constant, free libration parameters, positions of the LLR stations and lunar reflectors. Thus, we obtained the present solution ELP/MPP02(LLR) with the corrections to constants listed in Table 3 (column 3).

Comparing ELP/MPP02(LLR) to the last export ELP version, ELP 2000-82B, on the interval [+1950;+2060] a factor 3 has been won for the longitude and the latitude and 8 for the distance.
Comparing ELP/MPP02(LLR) to DE405/DE406, the maximum of the differences are:
for the interval [+1950;+2060], 0".06 in longitude, 0".003 in latitude, 4 meters in distance,
for the interval [+1500;+2500], 0".6 in longitude, 0".05 in latitude, 50 meters in distance,
for the interval [−3000;+3000], 50." in longitude, 5." in latitude, 10 kilometers in distance,
The ephemeris ELP/MPP02(LLR)+ρ_{405} gives post-fit residuals which decrease from 6 cm to 3 cm for the LLR data provided since 1987.

ELP/MPP02(LLR) is henceforth the new analytical version of ELP solution.

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