# Concepts of Physics

H C Verma

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# CONCEPTS OF PHYSICS

[VOLUME 1]

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*Dedicated to*
*Indian Philosophy & Way of Life*
*of which*
*my parents were*
*an integral part*

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## FOREWORD

A few years ago I had an occasion to go through the book *Calculus* by L V Terasov. It unravels intricacies of the subject through a dialogue between Teacher and Student. I thoroughly enjoyed reading it. For me this seemed to be one of the few books which teach a difficult subject through inquisition, and using programmed concept for learning. After that book, Dr Harish Chandra Verma's book on physics, *CONCEPTS OF PHYSICS* is another such attempt, even though it is not directly in the dialogue form. I have thoroughly appreciated it. It is clear that Dr Verma has spent considerable time in formulating the structure of the book, besides its contents. I think he has been successful in this attempt. Dr Verma's book has been divided into two parts because of the size of the total manuscript. There have been several books on this subject, each one having its own flavour. However, the present book is a totally different attempt to teach physics, and I am sure it will be extremely useful to the undergraduate students. The exposition of each concept is extremely lucid. In carefully formatted chapters, besides problems and short questions, a number of objective questions have also been included. This book can certainly be extremely useful not only as a textbook, but also for preparation of various competitive examinations.

Those who have followed Dr Verma's scientific work always enjoyed the outstanding contributions he has made in various research areas. He was an outstanding student of Physics Department of IIT Kanpur during his academic career. An extremely methodical, sincere person as a student, he has devoted himself to the task of educating young minds and inculcating scientific temper amongst them. The present venture in the form of these two volumes is another attempt in that direction. I am sure that young minds who would like to *learn physics in an appropriate manner* will find these volumes extremely useful.

I must heartily congratulate Dr Harish Chandra Verma for the magnificent job he has done.

**Y R Waghmare**
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# PREFACE

### Why a new book ?

Excellent books exist on physics at an introductory college level so why a new one ? Why so many books exist at the same level, in the first place, and why each of them is highly appreciated ? It is because each of these books has the privilege of having an author or authors who have *experienced* physics and have their own method of communicating with the students. During my years as a physics teacher, I have developed a somewhat different methodology of presenting physics to the students. *Concepts of Physics* is a translation of this methodology into a textbook.

### Prerequisites

The book presents a calculus-based physics course which makes free use of algebra, trigonometry and co-ordinate geometry. The level of the latter three topics is quite simple and high school mathematics is sufficient. Calculus is generally done at the introductory college level and I have assumed that the student is enrolled in a concurrent first calculus course. The relevant portions of calculus have been discussed in Chapter 2 so that the student may start using it from the beginning.

Almost no knowledge of physics is a prerequisite. I have attempted to start each topic from the zero level. A receptive mind is all that is needed to use this book.

### Basic philosophy of the book

The motto underlying the book is *physics is enjoyable.*

Being a description of the nature around us, physics is our best friend from the day of our existence. I have extensively used this aspect of physics to introduce the physical principles starting with common day occurrences and examples. The subject then appears to be friendly and enjoyable. I have taken care that numerical values of different quantities used in problems correspond to real situations to further strengthen this approach.

### Teaching and training

The basic aim of physics teaching has been to let the student know and understand the principles and equations of physics and their applications in real life.

However, to be able to use these principles and equations correctly in a given physical situation, one needs further training. A large number of *questions and solved and unsolved problems* are given for this purpose. Each question or problem has a specific purpose. It may be there to bring out a subtle point which might have passed unnoticed while doing the text portion. It may be a further elaboration of a concept developed in the text. It may be there to make the student react when several concepts introduced in different chapters combine and show up as a physical situation and so on. Such tools have been used to develop a culture: *analyse the situation, make a strategy to invoke correct principles and work it out.*

### Conventions

I have tried to use symbols, names, etc., which are popular nowadays. SI units have been consistently used throughout the book. SI prefixes such as *micro, milli, mega,* etc., are used whenever they make the presentation more readable. Thus, $20\ \mu F$ is preferred over $20 \times 10^{-6}$ F. Co-ordinate sign convention is used in geometrical optics. Special emphasis has been given to dimensions of physical quantities. Numerical values of physical quantities have been mentioned with the units even in equations to maintain dimensional consistency.

I have tried my best to keep errors out of this book. I shall be grateful to the readers who point out any errors and/or make other constructive suggestions.

**H C Verma**

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## ACKNOWLEDGEMENTS

The work on this book started in 1984. Since then, a large number of teachers, students and physics lovers have made valuable suggestions which I have incorporated in this work. It is not possible for me to acknowledge all of them individually. I take this opportunity to express my gratitude to them. However, to Dr S B Mathur, who took great pains in going through the entire manuscript and made valuable comments, I am specially indebted. I am also beholden to my colleagues Dr A Yadav, Dr Deb Mukherjee, Mr M M R Akhtar, Dr Arjun Prasad, Dr S K Sinha and others who gave me valuable advice and were good enough to find time for fruitful discussions. To Dr T K Dutta of B E College, Sibpur I am grateful for having taken time to go through portions of the book and making valuable comments.

I thank my student Mr Shailendra Kumar who helped me in checking the answers. I am grateful to Dr B C Rai, Mr Sunil Khijwania & Mr Tejaswi Khijwania for helping me in the preparation of rough sketches for the book.

Finally, I thank the members of my family for their support and encouragement.

**H C Verma**

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## TO THE STUDENTS

Here is a brief discussion on the organisation of the book which will help you in using the book most effectively. The book contains 47 chapters divided in two volumes. Though I strongly believe in the underlying unity of physics, a broad division may be made in the book as follows:

Chapters 1–14: Mechanics
15–17: Waves including wave optics
18–22: Optics
23–28: Heat and thermodynamics
29–40: Electric and magnetic phenomena
41–47: Modern physics

Each chapter contains a description of the physical principles related to that chapter. It is well supported by mathematical derivations of equations, descriptions of laboratory experiments, historical background, etc. There are "in-text" solved examples. These examples explain the equation just derived or the concept just discussed. These will help you in fixing the ideas firmly in your mind. Your teachers may use these in-text examples in the classroom to encourage students to participate in discussions.

After the theory section, there is a section on *Worked Out Examples.* These numerical examples correspond to various thinking levels and often use several concepts introduced in that chapter or even in previous chapters. You should read the statement of a problem and try to solve it yourself. In case of difficulty, look at the solution given in the book. Even if you solve the problem successfully, you should look into the solution to compare it with your method of solution. You might have thought of a better method, but knowing more than one method is always beneficial.

Then comes the part which tests your understanding as well as develops it further. *Questions for Short Answer* generally touch very minute points of your understanding. It is not necessary that you answer these questions in a single sitting. They have great potential to initiate very fruitful dicussions. So, freely discuss these questions with your friends and see if they agree with your answer. Answers to these questions are not given for the simple reason that the answers could have cut down the span of such discussions and that would have sharply reduced the utility of these questions.

There are two sections on multiple-choice questions, namely OBJECTIVE I and OBJECTIVE II. There are four options following each of these questions. Only one option is correct for OBJECTIVE I questions. Any number of options, zero to four, may be correct for OBJECTIVE II questions. Answers to all these questions are provided.

Finally, a set of numerical problems are given for your practice. Answers to these problems are also provided. The problems are generally arranged according to the sequence of the concepts developed in the chapter but they are not grouped under section-headings. I don't want to bias your ideas beforehand by telling you that this problem belongs to that section and hence use that particular equation. You should yourself look into the problem and decide which equations or which methods should be used to solve it. Many of the problems use several concepts developed in different sections of the chapter. Many of them even use the concepts from the previous chapters. Hence, you have to plan out the strategy after understanding the problem.

Remember, no problem is difficult. Once you understand the theory, each problem will become easy. So, don't jump to exercise problems before you have gone through the theory, the worked-out problems and the objectives. Once you feel confident in theory, do the exercise problems. The exercise problems are so arranged that they gradually require more thinking.

I hope you will enjoy *Concepts of Physics*.

**H C Verma**

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**CHAPTER 1**

# INTRODUCTION TO PHYSICS

## 1.1 WHAT IS PHYSICS ?

The nature around us is colourful and diverse. It contains phenomena of large varieties. The winds, the sands, the waters, the planets, the rainbow, heating of objects on rubbing, the function of a human body, the energy coming from the sun and the nucleus …… there are a large number of objects and events taking place around us.

Physics is the study of nature and its laws. We expect that all these different events in nature take place according to some basic laws and *revealing these laws of nature from the observed events* is physics. For example, the orbiting of the moon around the earth, falling of an apple from a tree and tides in a sea on a full moon night can all be explained if we know the Newton's law of gravitation and Newton's laws of motion. Physics is concerned with the basic rules which are applicable to all domains of life. Understanding of physics, therefore, leads to applications in many fields including bio and medical sciences.

The great physicist Dr R. P. Feynman has given a wonderful description of what is "understanding the nature". Suppose we do not know the rules of chess but are allowed to watch the moves of the players. If we watch the game for a long time, we may make out some of the rules. With the knowledge of these rules we may try to understand why a player played a particular move. However, this may be a very difficult task. Even if we know all the rules of chess, it is not so simple to understand all the complications of a game in a given situation and predict the correct move. Knowing the basic rules is, however, the minimum requirement if any progress is to be made.

One may guess at a wrong rule by partially watching the game. The experienced player may make use of a rule for the first time and the observer of the game may get surprised. Because of the new move some of the rules guessed at may prove to be wrong and the observer will frame new rules.

Physics goes the same way. The nature around us is like a big chess game played by Nature. The events in the nature are like the moves of the great game. We are allowed to watch the events of nature and guess at the basic rules according to which the events take place. We may come across new events which do not follow the rules guessed earlier and we may have to declare the old rules inapplicable or wrong and discover new rules.

Since physics is the study of nature, it is real. No one has been given the authority to frame the rules of physics. We only *discover* the rules that are operating in nature. Aryabhat, Newton, Einstein or Feynman are great physicists because from the observations available at that time, they could guess and frame the laws of physics which explained these observations in a convincing way. But there can be a new phenomenon any day and if the rules discovered by the great scientists are not able to explain this phenomenon, no one will hesitate to change these rules.

## 1.2 PHYSICS AND MATHEMATICS

The description of nature becomes easy if we have the freedom to use mathematics. To say that the gravitational force between two masses is proportional to the product of the masses and is inversely proportional to the square of the distance apart, is more difficult than to write

$$F \propto \frac{m_1 m_2}{r^2} \cdot \qquad \dots \quad (1.1)$$

Further, the techniques of mathematics such as algebra, trigonometry and calculus can be used to make predictions from the basic equations. Thus, if we know the basic rule (1.1) about the force between two particles, we can use the technique of integral calculus to find what will be the force exerted by a uniform rod on a particle placed on its perpendicular bisector.

Thus, mathematics is the language of physics. Without knowledge of mathematics it would be much more difficult to discover, understand and explain the

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laws of nature. The importance of mathematics in today's world cannot be disputed. However, mathematics itself is not physics. We use a language to express our ideas. But the idea that we want to express has the main attention. If we are poor at grammar and vocabulary, it would be difficult for us to communicate our feelings but while doing so our basic interest is in the feeling that we want to express. It is nice to board a deluxe coach to go from Delhi to Agra, but the sweet memories of the deluxe coach and the video film shown on way are next to the prime goal of reaching Agra. "To understand nature" is physics, and mathematics is the deluxe coach to take us there comfortably. This relationship of physics and mathematics must be clearly understood and kept in mind while doing a physics course.

## 1.3 UNITS

Physics describes the laws of nature. This description is quantitative and involves measurement and comparison of physical quantities. To measure a physical quantity we need some standard unit of that quantity. An elephant is heavier than a goat but exactly how many times ? This question can be easily answered if we have chosen a standard mass calling it a *unit mass*. If the elephant is 200 times the unit mass and the goat is 20 times we know that the elephant is 10 times heavier than the goat. If I have the knowledge of the unit length and some one says that Gandhi Maidan is 5 times the unit length from here, I will have the idea whether I should walk down to Gandhi Maidan or I should ride a rickshaw or I should go by a bus. Thus, the physical quantities are quantitatively expressed in terms of a unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives how many times of the standard unit and the second part gives the name of the unit. Thus, suppose I have to study for 2 hours. The numeric part 2 says that it is 2 *times* of the unit of time and the second part *hour* says that the unit chosen here is an hour.

### Who Decides the Units ?

How is a standard unit chosen for a physical quantity ? The first thing is that it should have international acceptance. Otherwise, everyone will choose his or her own unit for the quantity and it will be difficult to communicate freely among the persons distributed over the world. A body named *Conférence Générale des Poids et Mesures* or CGPM also known as *General Conference on Weight and Measures* in English has been given the authority to decide the units by international agreement. It holds its meetings and any changes in standard units are communicated through the publications of the Conference.

### Fundamental and Derived Quantities

There are a large number of physical quantities which are measured and every quantity needs a definition of unit. However, not all the quantities are independent of each other. As a simple example, if a unit of length is defined, a unit of area is automatically obtained. If we make a square with its length equal to its breadth equal to the unit length, its area can be called the unit area. All areas can then be compared to this standard unit of area. Similarly, if a unit of length and a unit of time interval are defined, a unit of speed is automatically obtained. If a particle covers a unit length in unit time interval, we say that it has a unit speed. We can define a set of *fundamental quantities* as follows :

(a) the fundamental quantities should be independent of each other, and

(b) all other quantities may be expressed in terms of the fundamental quantities.

It turns out that the number of fundamental quantities is only seven. All the rest may be derived from these quantities by multiplication and division. Many different choices can be made for the fundamental quantities. For example, one can take speed and time as fundamental quantities. Length is then a derived quantity. If something travels at unit speed, the distance it covers in unit time interval will be called a unit distance. One may also take length and time interval as the fundamental quantities and then speed will be a derived quantity. Several systems are in use over the world and in each system the fundamental quantities are selected in a particular way. The units defined for the fundamental quantities are called *fundamental units* and those obtained for the derived quantities are called the *derived units*.

Fundamental quantities are also called base quantities.

### SI Units

In 1971 CGPM held its meeting and decided a system of units which is known as the *International System of Units*. It is abbreviated as SI from the French name *Le Systéme International d'Unités*. This system is widely used throughout the world.

Table (1.1) gives the fundamental quantities and their units in SI.

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**Table 1.1 : *Fundamental or Base Quantities***

| *Quantity* | *Name of the Unit* | *Symbol* |
|---|---|---|
| Length | **metre** | m |
| Mass | **kilogram** | kg |
| Time | **second** | s |
| Electric Current | **ampere** | A |
| Thermodynamic Temperature | **kelvin** | K |
| Amount of Substance | **mole** | mol |
| Luminous Intensity | **candela** | cd |

Besides the seven fundamental units two supplementary units are defined. They are for plane angle and solid angle. The unit for plane angle is *radian* with the symbol *rad* and the unit for the solid angle is *steradian* with the symbol *sr.*

**SI Prefixes**

The magnitudes of physical quantities vary over a wide range. We talk of separation between two protons inside a nucleus which is about $10^{-15}$ m and the distance of a quasar from the earth which is about $10^{26}$ m. The mass of an electron is $9{\cdot}1 \times 10^{-31}$ kg and that of our galaxy is about $2{\cdot}2 \times 10^{41}$ kg. The CGPM recommended standard prefixes for certain powers of 10. Table (1.2) shows these prefixes.

**Table 1.2 : *SI prefixes***

| *Power of 10* | *Prefix* | *Symbol* |
|---|---|---|
| 18 | **exa** | E |
| 15 | **peta** | P |
| 12 | **tera** | T |
| 9 | **giga** | G |
| 6 | **mega** | M |
| 3 | **kilo** | k |
| 2 | **hecto** | h |
| 1 | **deka** | da |
| – 1 | **deci** | d |
| – 2 | **centi** | c |
| – 3 | **milli** | m |
| – 6 | **micro** | μ |
| – 9 | **nano** | n |
| – 12 | **pico** | p |
| – 15 | **femto** | f |
| – 18 | **atto** | a |

## 1.4 DEFINITIONS OF BASE UNITS

Any standard unit should have the following two properties :

(a) *Invariability* : The standard unit must be invariable. Thus, defining distance between the tip of the middle finger and the elbow as a unit of length is not invariable.

(b) *Availability* : The standard unit should be easily made available for comparing with other quantities.

CGPM decided in its 2018 meeting that all the SI base quantities will be defined in terms of certain universal constants and these constants will be assigned fixed numerical values by definition. In this case both the criteria of invariability and availability are automatically satisfied. The new definitions became operative since 20th May 2019. We give below the definitions of the these quantities. The fixed values given to the universal constants will appear in the definitions only. The definitions carry certain physical quantities and concepts that are beyond the scope of this book but you need not worry about it.

**Second**

1 second is the time that makes the unperturbed ground state hyperfine transition frequency $\Delta\nu_{Cs}$ to be 9192631770 when expressed in the unit Hz which is equal to $s^{-1}$.

**Metre**

1 metre is the length that makes the speed of light in vacuum to be 299792458 when expressed in the unit $m{\cdot}s^{-1}$, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.

**Kilogram**

1 kilogram is the mass that makes the Planck's constant $h$ to be $6.62607015 \times 10^{-34}$ when expressed in the unit J·s which is equal to $kg{\cdot}m^2\ s^{-1}$, where the metre and the second are defined in terms of $c$ and $\Delta\nu_{Cs}$.

**Ampere**

1 ampere is the current which makes the elementary charge $e$ to be $1.602176634 \times 10^{-19}$ when expressed in the unit C which is equal to A·s, where the second is defined in terms of $\Delta\nu_{Cs}$.

**Kelvin**

1 kelvin is the temperature that makes the Boltzmann constant to be $1.380649 \times 10^{-23}$ when expressed in the unit $J{\cdot}K^{-1}$ which is equal to $kg{\cdot}m^2{\cdot}s^2{\cdot}K^{-1}$, where kilogram, metre and second are defined in terms of $h$, $c$ and $\Delta\nu_{Cs}$.

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**Mole**

1 mole of a substance is defined to contain exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant $N_A$ when expressed in the unit $mol^{-1}$ and is called Avogadro number.

**Candela**

The candela is the SI unit of luminous intensity. 1 candela is the luminous intensity that makes the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}$ Hz, $K_{cd}$ to be 683 when expressed in the unit $lm \cdot W^{-1}$ which is equal to $cd \cdot sr \cdot kg^{-1} m^{2} s^{3}$, where kilogram, metre and second are defined in terms of $h$, $c$ and $\Delta \nu_{Cs}$.

## 1.5 DIMENSION

All the physical quantities of interest can be derived from the base quantities. When a quantity is expressed in terms of the base quantities, it is written as a product of different powers of the base quantities. The exponent of a base quantity that enters into the expression, is called the *dimension of the quantity in that base.* To make it clear, consider the physical quantity force. As we shall learn later, force is equal to mass times acceleration. Acceleration is change in velocity divided by time interval. Velocity is length divided by time interval. Thus,

$$
\begin{aligned}
force &= mass \times acceleration \\
&= mass \times \frac{velocity}{time} \\
&= mass \times \frac{length/time}{time} \\
&= mass \times length \times (time)^{-2}. \qquad \ldots (1.2)
\end{aligned}
$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero. Note that in this type of calculation the magnitudes are not considered. It is equality of the type of quantity that enters. Thus, change in velocity, initial velocity, average velocity, final velocity all are equivalent in this discussion, each one is length/time.

For convenience the base quantities are represented by one letter symbols. Generally, mass is denoted by M, length by L, time by T and electric current by I. The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitudes. Thus equation (1.2) may be written as $[force] = MLT^{-2}$.

Such an expression for a physical quantity in terms of the base quantities is called the *dimensional formula*. Thus, the dimensional formula of force is $MLT^{-2}$. The two versions given below are equivalent and are used interchangeably.

(a) The dimensional formula of force is $MLT^{-2}$.

(b) The dimensions of force are 1 in mass, 1 in length and –2 in time.

***Example 1.1***

*Calculate the dimensional formula of energy from the equation* $E = \frac{1}{2} mv^2$.

***Solution*** **:** Dimensionally, $E = mass \times (velocity)^2$, since $\frac{1}{2}$ is a number and has no dimension.

$$\therefore \quad [E] = M \times \left(\frac{L}{T}\right)^2 = ML^2 T^{-2}.$$

## 1.6 USES OF DIMENSION

### A. Homogeneity of Dimensions in an Equation

An equation contains several terms which are separated from each other by the symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This is another way of saying that one can add or subtract similar physical quantities. Thus, a velocity cannot be added to a force or an electric current cannot be subtracted from the thermodynamic temperature. This simple principle is called the *principle of homogeneity of dimensions* in an equation and is an extremely useful method to check whether an equation may be correct or not. If the dimensions of all the terms are not same, the equation must be wrong. Let us check the equation

$$x = ut + \frac{1}{2} at^2$$

for the dimensional homogeneity. Here $x$ is the distance travelled by a particle in time $t$ which starts at a speed $u$ and has an acceleration $a$ along the direction of motion.

$$[x] = L$$

$$[ut] = velocity \times time = \frac{length}{time} \times time = L$$

$$
\begin{aligned}
\left[\frac{1}{2} at^2\right] &= [at^2] = acceleration \times (time)^2 \\
&= \frac{velocity}{time} \times (time)^2 = \frac{length/time}{time} \times (time)^2 = L
\end{aligned}
$$

Thus, the equation is correct as far as the dimensions are concerned.

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#### Limitation of the Method

Note that the dimension of $\frac{1}{2}at^2$ is same as that of $at^2$. Pure numbers are dimensionless. Dimension does not depend on the magnitude. Due to this reason the equation $x = ut + at^2$ is also dimensionally correct. Thus, a dimensionally correct equation need not be actually correct but a dimensionally wrong equation must be wrong.

***Example 1.2***

*Test dimensionally if the formula $t = 2\pi\sqrt{\frac{m}{F/x}}$ may be correct, where t is time period, m is mass, F is force and x is distance.*

**Solution :** The dimension of force is $\mathrm{MLT^{-2}}$. Thus, the dimension of the right-hand side is

$$\sqrt{\frac{\mathrm{M}}{\mathrm{MLT^{-2}/L}}} = \sqrt{\frac{1}{\mathrm{T^{-2}}}} = \mathrm{T}.$$

The left-hand side is time period and hence the dimension is T. The dimensions of both sides are equal and hence the formula may be correct.

### B. Conversion of Units

When we choose to work with a different set of units for the base quantities, the units of all the derived quantities must be changed. Dimensions can be useful in finding the conversion factor for the unit of a derived physical quantity from one system to other. Consider an example. When SI units are used, the unit of pressure is 1 pascal. Suppose we choose 1 cm as the unit of length, 1 g as the unit of mass and 1 s as the unit of time (this system is still in wide use and is called CGS system). The unit of pressure will be different in this system. Let us call it for the time-being 1 CGS pressure. Now, how many CGS pressure is equal to 1 pascal ?

Let us first write the dimensional formula of pressure.

We have $$P = \frac{F}{A}.$$

Thus, $$[P] = \frac{[F]}{[A]} = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}} = \mathrm{ML^{-1}T^{-2}}$$

so, $$1 \text{ pascal} = (1\text{ kg})\,(1\text{ m})^{-1}\,(1\text{ s})^{-2}$$

and $$1 \text{ CGS pressure} = (1\text{ g})\,(1\text{ cm})^{-1}\,(1\text{ s})^{-2}$$

Thus, $$\frac{1\text{ pascal}}{1\text{ CGS pressure}} = \left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^{-1}\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2} = (10^3)(10^2)^{-1} = 10$$

or, $$1 \text{ pascal} = 10 \text{ CGS pressure}.$$

Thus, knowing the conversion factors for the base quantities, one can work out the conversion factor for any derived quantity if the dimensional formula of the derived quantity is known.

### C. Deducing Relation among the Physical Quantities

Sometimes dimensions can be used to deduce a relation between the physical quantities. If one knows the quantities on which a particular physical quantity depends and if one guesses that this dependence is of product type, method of dimension may be helpful in the derivation of the relation. Taking an example, suppose we have to derive the expression for the time period of a simple pendulum. The simple pendulum has a bob, attached to a string, which oscillates under the action of the force of gravity. Thus, the time period may depend on the length of the string, the mass of the bob and the acceleration due to gravity. We assume that the dependence of time period on these quantities is of product type, that is,

$$t = kl^a m^b g^c \qquad \ldots \ (1.3)$$

where $k$ is a dimensionless constant and $a$, $b$ and $c$ are exponents which we want to evaluate. Taking the dimensions of both sides,

$$\mathrm{T} = \mathrm{L}^a \mathrm{M}^b (\mathrm{LT^{-2}})^c = \mathrm{L}^{a+c}\mathrm{M}^b\mathrm{T}^{-2c}.$$

Since the dimensions on both sides must be identical, we have

$$a + c = 0$$
$$b = 0$$
and $$-2c = 1$$
giving $$a = \frac{1}{2},\ b = 0 \text{ and } c = -\frac{1}{2}.$$

Putting these values in equation (1.3)

$$t = k\sqrt{\frac{l}{g}}. \qquad \ldots \ (1.4)$$

Thus, by dimensional analysis we can deduce that the time period of a simple pendulum is independent of its mass, is proportional to the square root of the length of the pendulum and is inversely proportional to the square root of the acceleration due to gravity at the place of observation.

#### Limitations of the Dimensional Method

Although dimensional analysis is very useful in deducing certain relations, it cannot lead us too far. First of all we have to know the quantities on which a particular physical quantity depends. Even then the method works only if the dependence is of the product type. For example, the distance travelled by a uniformly accelerated particle depends on the initial velocity $u$, the acceleration $a$ and the time $t$. But the method of dimensions cannot lead us to the correct expression for $x$ because the expression is not of

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product type. It is equal to the sum of two terms as
$$x = ut + \frac{1}{2}at^2.$$
Secondly, the numerical constants having no dimensions cannot be deduced by the method of dimensions. In the example of time period of a simple pendulum, an unknown constant $k$ remains in equation (1.4). One has to know from somewhere else that this constant is $2\pi$.

Thirdly, the method works only if there are as many equations available as there are unknowns. In mechanical quantities, only three base quantities length, mass and time enter. So, dimensions of these three may be equated in the guessed relation giving at most three equations in the exponents. If a particular quantity (in mechanics) depends on more than three quantities we shall have more unknowns and less equations. The exponents cannot be determined uniquely in such a case. Similar constraints are present for electrical or other nonmechanical quantities.

## 1.7 ORDER OF MAGNITUDE

In physics, we come across quantities which vary over a wide range. We talk of the size of a mountain and the size of the tip of a pin. We talk of the mass of our galaxy and the mass of a hydrogen atom. We talk of the age of the universe and the time taken by an electron to complete a circle around the proton in a hydrogen atom. It becomes quite difficult to get a feel of largeness or smallness of such quantities. To express such widely varying numbers, one uses the *powers of ten* method.

In this method, each number is expressed as $a \times 10^b$ where $1 \le a < 10$ and $b$ is a positive or negative integer. Thus the diameter of the sun is expressed as $1{\cdot}39 \times 10^9$ m and the diameter of a hydrogen atom as $1{\cdot}06 \times 10^{-10}$ m. To get an approximate idea of the number, one may round the number $a$ to 1 if it is less than or equal to 5 and to 10 if it is greater than 5. The number can then be expressed approximately as $10^b$. We then get the *order of magnitude* of that number. Thus, the diameter of the sun is *of the order of* $10^9$ m and that of a hydrogen atom is *of the order of* $10^{-10}$ m. More precisely, the exponent of 10 in such a representation is called the order of magnitude of that quantity. Thus, the diameter of the sun is 19 *orders of magnitude larger* than the diameter of a hydrogen atom. This is because the order of magnitude of $10^9$ is 9 and of $10^{-10}$ is $-10$. The difference is $9 - (-10) = 19$.

To quickly get an approximate value of a quantity in a given physical situation, one can make an *order of magnitude calculation.* In this all numbers are approximated to $10^b$ form and the calculation is made.

Let us estimate the number of persons that may sit in a circular field of radius 800 m. The area of the field is
$$A = \pi r^2 = 3{\cdot}14 \times (800 \text{ m})^2 \approx 10^6 \text{ m}^2.$$
The average area one person occupies in sitting $\approx 50 \text{ cm} \times 50 \text{ cm} = 0{\cdot}25 \text{ m}^2 = 2{\cdot}5 \times 10^{-1} \text{ m}^2 \approx 10^{-1} \text{ m}^2$. The number of persons who can sit in the field is
$$N \approx \frac{10^6 \text{ m}^2}{10^{-1} \text{ m}^2} = 10^7.$$

Thus of the order of $10^7$ persons may sit in the field.

## 1.8 THE STRUCTURE OF WORLD

Man has always been interested to find how the world is structured. Long long ago scientists suggested that the world is made up of certain indivisible small particles. The number of particles in the world is large but the varieties of particles are not many. Old Indian philosopher Kanadi derives his name from this proposition (In Sanskrit or Hindi *Kana* means a small particle). After extensive experimental work people arrived at the conclusion that the world is made up of just three types of ultimate particles, the proton, the neutron and the electron. All objects which we have around us, are aggregation of atoms and molecules. The molecules are composed of atoms and the atoms have at their heart a nucleus containing protons and neutrons. Electrons move around this nucleus in special arrangements. It is the number of protons, neutrons and electrons in an atom that decides all the properties and behaviour of a material. Large number of atoms combine to form an object of moderate or large size. However, the laws that we generally deduce for these macroscopic objects are not always applicable to atoms, molecules, nuclei or the elementary particles. These laws known as *classical physics* deal with large size objects only. When we say a particle in classical physics we mean an object which is small as compared to other moderate or large size objects and for which the classical physics is valid. It may still contain millions and millions of atoms in it. Thus, a particle of dust dealt in classical physics may contain about $10^{18}$ atoms.

Twentieth century experiments have revealed another aspect of the construction of world. There are perhaps no ultimate indivisible particles. Hundreds of elementary particles have been discovered and there are free transformations from one such particle to the other. Nature is seen to be a well-connected entity.

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## *Worked Out Examples*

**1.** *Find the dimensional formulae of the following quantities :*
*(a) the universal constant of gravitation G,*
*(b) the surface tension S,*
*(c) the thermal conductivity k and*
*(d) the coefficient of viscosity η.*
*Some equations involving these quantities are*

$$F = \frac{Gm_1 m_2}{r^2}, \qquad S = \frac{\rho g r h}{2},$$

$$Q = k\frac{A(\theta_2 - \theta_1)t}{d} \quad \text{and} \quad F = -\eta A\frac{v_2 - v_1}{x_2 - x_1}$$

*where the symbols have their usual meanings.*

**Solution :** (a) $F = G\dfrac{m_1 m_2}{r^2}$

or, $G = \dfrac{Fr^2}{m_1 m_2}$

or, $[G] = \dfrac{[F]L^2}{M^2} = \dfrac{MLT^{-2}.L^2}{M^2} = M^{-1}L^3T^{-2}$.

(b) $S = \dfrac{\rho g r h}{2}$

or, $[S] = [\rho]\,[g]L^2 = \dfrac{M}{L^3}\cdot\dfrac{L}{T^2}\cdot L^2 = MT^{-2}$.

(c) $Q = k\dfrac{A(\theta_2 - \theta_1)t}{d}$

or, $k = \dfrac{Qd}{A(\theta_2 - \theta_1)t}$.

Here, $Q$ is the heat energy having dimension $ML^2T^{-2}$, $\theta_2 - \theta_1$ is temperature, $A$ is area, $d$ is thickness and $t$ is time. Thus,

$$[k] = \frac{ML^2T^{-2}L}{L^2KT} = MLT^{-3}K^{-1}.$$

(d) $F = -\eta A\dfrac{v_2 - v_1}{x_2 - x_1}$

or, $MLT^{-2} = [\eta]L^2\dfrac{L/T}{L} = [\eta]\dfrac{L^2}{T}$

or, $[\eta] = ML^{-1}T^{-1}$.

**2.** *Find the dimensional formulae of*
*(a) the charge Q,*
*(b) the potential V,*
*(c) the capacitance C, and*
*(d) the resistance R.*
*Some of the equations containing these quantities are*
*Q = It, U = VIt, Q = CV and V = RI;*
*where I denotes the electric current, t is time and U is energy.*

**Solution :** (a) $Q = It$. Hence, $[Q] = IT$.

(b) $U = VIt$

or, $ML^2T^{-2} = [V]IT$ or, $[V] = ML^2I^{-1}T^{-3}$.

(c) $Q = CV$

or, $IT = [C]ML^2I^{-1}T^{-3}$ or, $[C] = M^{-1}L^{-2}I^2T^4$.

(d) $V = RI$

or, $R = \dfrac{V}{I}$ or, $[R] = \dfrac{ML^2I^{-1}T^{-3}}{I} = ML^2I^{-2}T^{-3}$.

**3.** *The SI and CGS units of energy are joule and erg respectively. How many ergs are equal to one joule ?*

**Solution :** Dimensionally, Energy $= mass \times (velocity)^2$

$$= mass \times \left(\frac{length}{time}\right)^2 = ML^2T^{-2}.$$

Thus, 1 joule $= (1\text{ kg})\,(1\text{ m})^2(1\text{ s})^{-2}$

and 1 erg $= (1\text{ g})\,(1\text{ cm})^2\,(1\text{ s})^{-2}$

$$\frac{1\text{ joule}}{1\text{ erg}} = \left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^2\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= \left(\frac{1000\text{ g}}{1\text{ g}}\right)\left(\frac{100\text{ cm}}{1\text{ cm}}\right)^2 = 1000 \times 10000 = 10^7.$$

So, 1 joule $= 10^7$ erg.

**4.** *Young's modulus of steel is* $19 \times 10^{10}\text{ N/m}^2$. *Express it in dyne/cm*$^2$. *Here dyne is the CGS unit of force.*

**Solution :** The unit of Young's modulus is $\text{N/m}^2$.

This suggests that it has dimensions of $\dfrac{Force}{(distance)^2}$.

Thus, $[Y] = \dfrac{[F]}{L^2} = \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

$\text{N/m}^2$ is in SI units.

So, $1\text{ N/m}^2 = (1\text{ kg})(1\text{ m})^{-1}(1\text{ s})^{-2}$

and $1\text{ dyne/cm}^2 = (1\text{ g})(1\text{ cm})^{-1}(1\text{ s})^{-2}$

so, $$\frac{1\text{ N/m}^2}{1\text{ dyne/cm}^2} = \left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^{-1}\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= 1000 \times \frac{1}{100} \times 1 = 10$$

or, $1\text{ N/m}^2 = 10\text{ dyne/cm}^2$

or, $19 \times 10^{10}\text{ N/m}^2 = 19 \times 10^{11}\text{ dyne/cm}^2$.

**5.** *If velocity, time and force were chosen as basic quantities, find the dimensions of mass.*

**Solution :** Dimensionally, $Force = mass \times acceleration$

$$= mass \times \frac{velocity}{time}$$

or, $mass = \dfrac{force \times time}{velocity}$

or, $[mass] = FTV^{-1}$.

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**6.** *Test dimensionally if the equation* $v^2 = u^2 + 2ax$ *may be correct.*

***Solution*** : There are three terms in this equation $v^2$, $u^2$ and $2ax$. The equation may be correct if the dimensions of these three terms are equal.

$$[v^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \mathrm{L}^2\mathrm{T}^{-2};$$

$$[u^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \mathrm{L}^2\mathrm{T}^{-2};$$

and $$[2ax] = [a]\,[x] = \left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\mathrm{L} = \mathrm{L}^2\mathrm{T}^{-2}.$$

Thus, the equation may be correct.

**7.** *The distance covered by a particle in time t is given by* $x = a + bt + ct^2 + dt^3$; *find the dimensions of a, b, c and d.*

***Solution*** : The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = *length*, each of the remaining four must have the dimension of length.

Thus, $[a] = length = \mathrm{L}$

$[bt] = \mathrm{L}$, or, $[b] = \mathrm{LT}^{-1}$

$[ct^2] = \mathrm{L}$, or, $[c] = \mathrm{LT}^{-2}$

and $[dt^3] = \mathrm{L}$, or, $[d] = \mathrm{LT}^{-3}$.

**8.** *If the centripetal force is of the form* $m^a v^b r^c$, *find the values of a, b and c.*

***Solution*** : Dimensionally,

$$Force = (Mass)^a \times (velocity)^b \times (length)^c$$

or, $\mathrm{MLT}^{-2} = \mathrm{M}^a(\mathrm{L}^b\mathrm{T}^{-b})\,\mathrm{L}^c = \mathrm{M}^a\mathrm{L}^{b+c}\mathrm{T}^{-b}$

Equating the exponents of similar quantities,

$a = 1,\ b + c = 1,\ -b = -2$

or, $a = 1,\ b = 2,\ c = -1$ or, $F = \dfrac{mv^2}{r}$.

**9.** *When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.*

***Solution*** : Suppose the formula is $F = k\,\eta^a r^b v^c$.

Then, $$\mathrm{MLT}^{-2} = [\mathrm{ML}^{-1}\mathrm{T}^{-1}]^a\,\mathrm{L}^b\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^c$$
$$= \mathrm{M}^a\mathrm{L}^{-a+b+c}\mathrm{T}^{-a-c}.$$

Equating the exponents of M, L and T from both sides,

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2$$

Solving these, $a = 1$, $b = 1$, and $c = 1$.
Thus, the formula for $F$ is $F = k\eta rv$.

**10.** *The heat produced in a wire carrying an electric current depends on the current, the resistance and the time. Assuming that the dependence is of the product of powers type, guess an equation between these quantities using dimensional analysis. The dimensional formula of resistance is* $\mathrm{ML}^2\mathrm{I}^{-2}\mathrm{T}^{-3}$ *and heat is a form of energy.*

***Solution*** : Let the heat produced be $H$, the current through the wire be $I$, the resistance be $R$ and the time be $t$. Since heat is a form of energy, its dimensional formula is $\mathrm{ML}^2\mathrm{T}^{-2}$.

Let us assume that the required equation is

$$H = kI^a R^b t^c,$$

where $k$ is a dimensionless constant.
Writing dimensions of both sides,

$$\mathrm{ML}^2\mathrm{T}^{-2} = \mathrm{I}^a(\mathrm{ML}^2\mathrm{I}^{-2}\mathrm{T}^{-3})^b\,\mathrm{T}^c$$
$$= \mathrm{M}^b\,\mathrm{L}^{2b}\,\mathrm{T}^{-3b+c}\,\mathrm{I}^{a-2b}$$

Equating the exponents,

$$b = 1$$
$$2b = 2$$
$$-3b + c = -2$$
$$a - 2b = 0$$

Solving these, we get, $a = 2$, $b = 1$ and $c = 1$.

Thus, the required equation is $H = kI^2Rt$.

□

## QUESTIONS FOR SHORT ANSWER

**1.** The metre is defined as the distance travelled by light in $\dfrac{1}{299{,}792{,}458}$ second. Why didn't people choose some easier number such as $\dfrac{1}{300{,}000{,}000}$ second? Why not 1 second?

**2.** What are the dimensions of :
(a) volume of a cube of edge $a$,
(b) volume of a sphere of radius $a$,
(c) the ratio of the volume of a cube of edge $a$ to the volume of a sphere of radius $a$?

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**3.** Suppose you are told that the linear size of everything in the universe has been doubled overnight. Can you test this statement by measuring sizes with a metre stick ? Can you test it by using the fact that the speed of light is a universal constant and has not changed ? What will happen if all the clocks in the universe also start running at half the speed ?

**4.** If all the terms in an equation have same units, is it necessary that they have same dimensions ? If all the terms in an equation have same dimensions, is it necessary that they have same units ?

**5.** If two quantities have same dimensions, do they represent same physical content ?

**6.** It is desirable that the standards of units be easily available, invariable, indestructible and easily reproducible. If we use foot of a person as a standard unit of length, which of the above features are present and which are not ?

**7.** Suggest a way to measure :
(a) the thickness of a sheet of paper,
(b) the distance between the sun and the moon.

## OBJECTIVE I

**1.** Which of the following sets cannot enter into the list of fundamental quantities in any system of units ?
(a) length, mass and velocity,
(b) length, time and velocity,
(c) mass, time and velocity,
(d) length, time and mass.

**2.** A physical quantity is measured and the result is expressed as $nu$ where $u$ is the unit used and $n$ is the numerical value. If the result is expressed in various units then
(a) $n \propto$ size of $u$ (b) $n \propto u^2$
(c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$.

**3.** Suppose a quantity $x$ can be dimensionally represented in terms of M, L and T, that is, $[x] = M^a L^b T^c$. The quantity mass
(a) can always be dimensionally represented in terms of L, T and $x$,
(b) can never be dimensoinally represented in terms of L, T and $x$,
(c) may be represented in terms of L, T and $x$ if $a = 0$,
(d) may be represented in terms of L, T and $x$ if $a \neq 0$.

**4.** A dimensionless quantity
(a) never has a unit, (b) always has a unit,
(c) may have a unit, (d) does not exist.

**5.** A unitless quantity
(a) never has a nonzero dimension,
(b) always has a nonzero dimension,
(c) may have a nonzero dimension,
(d) does not exist.

**6.** $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$.
The value of $n$ is
(a) 0 (b) –1
(c) 1 (d) none of these.
You may use dimensional analysis to solve the problem.

## OBJECTIVE II

**1.** The dimensions $ML^{-1}T^{-2}$ may correspond to
(a) work done by a force
(b) linear momentum
(c) pressure
(d) energy per unit volume.

**2.** Choose the correct statement(s) :
(a) A dimensionally correct equation may be correct.
(b) A dimensionally correct equation may be incorrect.
(c) A dimensionally incorrect equation may be correct.
(d) A dimensionally incorrect equation may be incorrect.

**3.** Choose the correct statement(s) :
(a) All quantities may be represented dimensionally in terms of the base quantities.
(b) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
(c) The dimension of a base quantity in other base quantities is always zero.
(d) The dimension of a derived quantity is never zero in any base quantity.

## EXERCISES

**1.** Find the dimensions of
(a) linear momentum,
(b) frequency and
(c) pressure.

**2.** Find the dimensions of
(a) angular speed $\omega$, (b) angular acceleration $\alpha$,
(c) torque $\Gamma$ and (d) moment of interia $I$.
Some of the equations involving these quantities are

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$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$, $\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$, $\Gamma = F.r$ and $I = mr^2$.
The symbols have standard meanings.

**3.** Find the dimensions of
(a) electric field $E$, (b) magnetic field $B$ and
(c) magnetic permeability $\mu_0$.
The relevant equations are
$F = qE$, $F = qvB$, and $B = \frac{\mu_0 I}{2\pi a}$;
where $F$ is force, $q$ is charge, $v$ is speed, $I$ is current, and $a$ is distance.

**4.** Find the dimensions of
(a) electric dipole moment $p$ and
(b) magnetic dipole moment $M$.
The defining equations are $p = q.d$ and $M = IA$;
where $d$ is distance, $A$ is area, $q$ is charge and $I$ is current.

**5.** Find the dimensions of Planck's constant $h$ from the equation $E = h\nu$ where $E$ is the energy and $\nu$ is the frequency.

**6.** Find the dimensions of
(a) the specific heat capacity $c$,
(b) the coefficient of linear expansion $\alpha$ and
(c) the gas constant $R$.
Some of the equations involving these quantities are
$Q = mc(T_2 - T_1)$, $l_t = l_0[1 + \alpha(T_2 - T_1)]$ and $PV = nRT$.

**7.** Taking force, length and time to be the fundamental quantities find the dimensions of
(a) density, (b) pressure,
(c) momentum and (d) energy.

**8.** Suppose the acceleration due to gravity at a place is 10 m/s$^2$. Find its value in cm/(minute)$^2$.

**9.** The average speed of a snail is 0·020 miles/hour and that of a leopard is 70 miles/hour. Convert these speeds in SI units.

**10.** The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data : Specific gravity of mercury = 13·6, Density of water = $10^3$ kg/m$^3$, $g$ = 9·8 m/s$^2$ at Calcutta. Pressure = $h\rho g$ in usual symbols.

**11.** Express the power of a 100 watt bulb in CGS unit.

**12.** The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How many microcenturies did you sleep yesterday ?

**13.** The surface tension of water is 72 dyne/cm. Convert it in SI unit.

**14.** The kinetic energy $K$ of a rotating body depends on its moment of inertia $I$ and its angular speed $\omega$. Assuming the relation to be $K = kI^a\omega^b$ where $k$ is a dimensionless constant, find $a$ and $b$. Moment of inertia of a sphere about its diameter is $\frac{2}{5}Mr^2$.

**15.** Theory of relativity reveals that mass can be converted into energy. The energy $E$ so obtained is proportional to certain powers of mass $m$ and the speed $c$ of light. Guess a relation among the quantities using the method of dimensions.

**16.** Let $I$ = current through a conductor, $R$ = its resistance and $V$ = potential difference across its ends. According to Ohm's law, product of two of these quantities equals the third. Obtain Ohm's law from dimensional analysis. Dimensional formulae for $R$ and $V$ are $ML^2I^{-2}T^{-3}$ and $ML^2T^{-3}I^{-1}$ respectively.

**17.** The frequency of vibration of a string depends on the length $L$ between the nodes, the tension $F$ in the string and its mass per unit length $m$. Guess the expression for its frequency from dimensional analysis.

**18.** Test if the following equations are dimensionally correct :
(a) $h = \frac{2S\cos\theta}{\rho r g}$, (b) $v = \sqrt{\frac{P}{\rho}}$,
(c) $V = \frac{\pi P r^4 t}{8\eta l}$, (d) $\nu = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$;
where $h$ = height, $S$ = surface tension, $\rho$ = density, $P$ = pressure, $V$ = volume, $\eta$ = coefficient of viscosity, $\nu$ = frequency and $I$ = moment of inertia.

**19.** Let $x$ and $a$ stand for distance. Is $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ dimensionally correct ?

□

## ANSWERS

### OBJECTIVE I

1. (b) 2. (d) 3. (d) 4. (c) 5. (a) 6. (a)

### OBJECTIVE II

1. (c), (d) 2. (a), (b), (d) 3. (a), (b), (c)

### EXERCISES

1. (a) $MLT^{-1}$ (b) $T^{-1}$ (c) $ML^{-1}T^{-2}$
2. (a) $T^{-1}$ (b) $T^{-2}$ (c) $ML^2T^{-2}$ (d) $ML^2$
3. (a) $MLT^{-3}I^{-1}$ (b) $MT^{-2}I^{-1}$ (c) $MLT^{-2}I^{-2}$
4. (a) LTI (b) $L^2I$
5. $ML^2T^{-1}$
6. (a) $L^2T^{-2}K^{-1}$ (b) $K^{-1}$ (c) $ML^2T^{-2}K^{-1}(mol)^{-1}$

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7. (a) $FL^{-4}T^2$ (b) $FL^{-2}$ (c) FT (d) FL
8. $36 \times 10^5$ cm/(minute)$^2$
9. 0·0089 m/s, 31 m/s
10. $10 \times 10^4$ N/m$^2$, $10 \times 10^5$ dyne/cm$^2$
11. $10^9$ erg/s
12. 1·9 microcenturies
13. 0·072 N/m
14. $a = 1$, $b = 2$
15. $E = kmc^2$
16. $V = IR$
17. $\frac{k}{L}\sqrt{\frac{F}{m}}$
18. all are dimensionally correct
19. no

□

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**CHAPTER 2**

# PHYSICS AND MATHEMATICS

Mathematics is the language of physics. It becomes easier to describe, understand and apply the physical principles, if one has a good knowledge of mathematics. In the present course we shall constantly be using the techniques of algebra, trigonometry and geometry as well as vector algebra, differential calculus and integral calculus. In this chapter we shall discuss the latter three topics. Errors in measurement and the concept of significant digits are also introduced.

## 2.1 VECTORS AND SCALARS

Certain physical quantities are completely described by a numerical value alone (with units specified) and are added according to the ordinary rules of algebra. As an example the mass of a system is described by saying that it is 5 kg. If two bodies one having a mass of 5 kg and the other having a mass of 2 kg are added together to make a composite system, the total mass of the system becomes 5 kg + 2 kg = 7 kg. Such quantities are called *scalars*.

The complete description of certain physical quantities requires a numerical value (with units specified) as well as a direction in space. Velocity of a particle is an example of this kind. The magnitude of velocity is represented by a number such as 5 m/s and tells us how fast a particle is moving. But the description of velocity becomes complete only when the direction of velocity is also specified. We can represent this velocity by drawing a line parallel to the velocity and putting an arrow showing the direction of velocity. We can decide beforehand a particular length to represent 1 m/s and the length of the line representing a velocity of 5 m/s may be taken as 5 times this unit length. Figure (2.1) shows representations of several velocities in this scheme. The front end (carrying the arrow) is called the head and the rear end is called the tail.

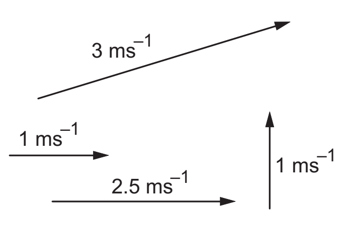


Figure 2.1

Further, if a particle is given two velocities simultaneously its resultant velocity is different from the two velocities and is obtained by using a special rule. Suppose a small ball is moving inside a long tube at a speed 3 m/s and the tube itself is moving in the room at a speed 4 m/s along a direction perpendicular to its length. In which direction and how fast is the ball moving as seen from the room ?

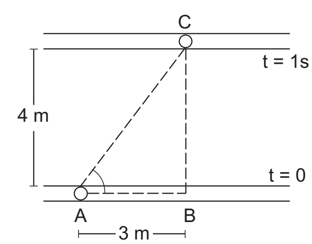


Figure 2.2

Figure (2.2) shows the positions of the tube and the ball at $t$ = 0 and $t$ = 1 s. Simple geometry shows that the ball has moved 5 m in a direction $\theta = 53°$ from the tube. So the resultant velocity of the ball is 5 m/s along this direction. The general rule for finding the resultant of two velocities may be stated as follows.

Draw a line $AB$ representing the first velocity with $B$ as the head. Draw another line $BC$ representing the second velocity with its tail $B$ coinciding with the head of the first line. The line $AC$ with $A$ as the tail and $C$ as the head represents the resultant velocity. Figure (2.3) shows the construction.

The resultant is also called the sum of the two velocities. We have added the two velocities $AB$ and $BC$ and have obtained the sum $AC$. This rule of addition is called the "triangle rule of addition".

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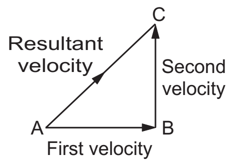


Figure 2.3

The physical quantities which have magnitude and direction and which can be added according to the triangle rule, are called *vector quantities*. Other examples of vector quantities are force, linear momentum, electric field, magnetic field etc.

The vectors are denoted by putting an arrow over the symbols representing them. Thus, we write $\overrightarrow{AB}$, $\overrightarrow{BC}$ etc. Sometimes a vector is represented by a single letter such as $\vec{v}$, $\vec{F}$ etc. Quite often in printed books the vectors are represented by bold face letters like **AB**, **BC**, **v**, **f** etc.

If a physical quantity has magnitude as well as direction but does not add up according to the triangle rule, it will not be called a vector quantity. Electric current in a wire has both magnitude and direction but there is no meaning of triangle rule there. Thus, electric current is not a vector quantity.

## 2.2 EQUALITY OF VECTORS

Two vectors (representing two values of the same physical quantity) are called equal if their magnitudes and directions are same. Thus, a parallel translation of a vector does not bring about any change in it.

## 2.3 ADDITION OF VECTORS

The triangle rule of vector addition is already described above. If $\vec{a}$ and $\vec{b}$ are the two vectors to be added, a diagram is drawn in which the tail of $\vec{b}$ coincides with the head of $\vec{a}$. The vector joining the tail of $\vec{a}$ with the head of $\vec{b}$ is the vector sum of $\vec{a}$ and $\vec{b}$. Figure (2.4a) shows the construction. The same rule

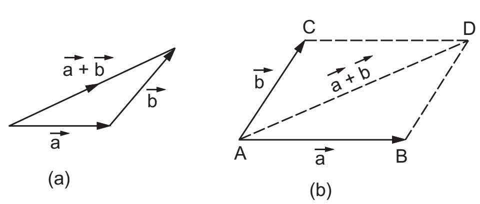


Figure 2.4

may be stated in a slightly different way. We draw the vectors $\vec{a}$ and $\vec{b}$ with both the tails coinciding (figure 2.4b). Taking these two as the adjacent sides we complete the parallelogram. The diagonal through the common tails gives the sum of the two vectors. Thus, in figure, (2.4b) $\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$.

Suppose the magnitude of $\vec{a} = a$ and that of $\vec{b} = b$. What is the magnitude of $\vec{a} + \vec{b}$ and what is its direction ? Suppose the angle between $\vec{a}$ and $\vec{b}$ is $\theta$. It is easy to see from figure (2.5) that

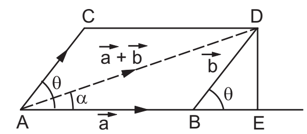


Figure 2.5

$$AD^2 = (AB + BE)^2 + (DE)^2$$

$$= (a + b\cos\theta)^2 + (b\sin\theta)^2$$

$$= a^2 + 2ab\cos\theta + b^2.$$

Thus, the magnitude of $\vec{a} + \vec{b}$ is

$$\sqrt{a^2 + b^2 + 2ab\cos\theta}. \qquad \ldots (2.1)$$

Its angle with $\vec{a}$ is $\alpha$ where

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a + b\cos\theta}\cdot \qquad \ldots (2.2)$$

***Example 2.1***

*Two vectors having equal magnitudes A make an angle θ with each other. Find the magnitude and direction of the resultant.*

***Solution*** **:** The magnitude of the resultant will be

$$B = \sqrt{A^2 + A^2 + 2AA\cos\theta}$$

$$= \sqrt{2A^2(1 + \cos\theta)} = \sqrt{4A^2\cos^2\frac{\theta}{2}}$$

$$= 2A\cos\frac{\theta}{2}\cdot$$

The resultant will make an angle $\alpha$ with the first vector where

$$\tan\alpha = \frac{A\sin\theta}{A + A\cos\theta} = \frac{2A\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2A\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

or, $\quad \alpha = \dfrac{\theta}{2}$

Thus, the resultant of two equal vectors bisects the angle between them.

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## 2.4 MULTIPLICATION OF A VECTOR BY A NUMBER

Suppose $\vec{a}$ is a vector of magnitude $a$ and $k$ is a number. We define the vector $\vec{b} = k\,\vec{a}$ as a vector of magnitude $|ka|$. If $k$ is positive the direction of the vector $\vec{b} = k\,\vec{a}$ is same as that of $\vec{a}$. If $k$ is negative, the direction of $\vec{b}$ is opposite to $\vec{a}$. In particular, multiplication by (–1) just inverts the direction of the vector. The vectors $\vec{a}$ and $-\vec{a}$ have equal magnitudes but opposite directions.

If $\vec{a}$ is a vector of magnitude $a$ and $\vec{u}$ is a vector of unit magnitude in the direction of $\vec{a}$, we can write $\vec{a} = a\vec{u}$.

## 2.5 SUBTRACTION OF VECTORS

Let $\vec{a}$ and $\vec{b}$ be two vectors. We define $\vec{a} - \vec{b}$ as the sum of the vector $\vec{a}$ and the vector $(-\vec{b})$. To subtract $\vec{b}$ from $\vec{a}$, invert the direction of $\vec{b}$ and add to $\vec{a}$. Figure (2.6) shows the process.

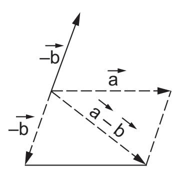


Figure 2.6

***Example 2.2***

*Two vectors of equal magnitude* 5 unit *have an angle* 60° *between them. Find the magnitude of* (a) *the sum of the vectors and* (b) *the difference of the vectors.*

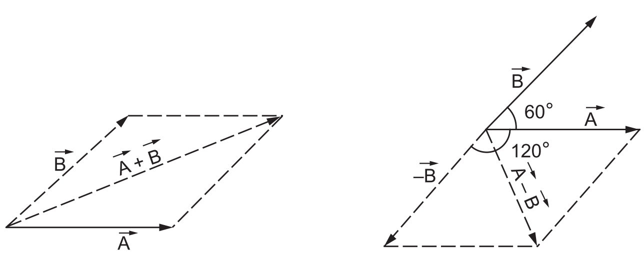


Figure 2.7

***Solution*** **:** Figure (2.7) shows the construction of the sum $\vec{A} + \vec{B}$ and the difference $\vec{A} - \vec{B}$.

(a) $\vec{A} + \vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$. Both have a magnitude of 5 unit and the angle between them is 60°. Thus, the magnitude of the sum is

$$|\vec{A} + \vec{B}| = \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \cos 60°}$$
$$= 2 \times 5 \cos 30° = 5\sqrt{3} \text{ unit.}$$

(b) $\vec{A} - \vec{B}$ is the sum of $\vec{A}$ and $(-\vec{B})$. As shown in the figure, the angle between $\vec{A}$ and $(-\vec{B})$ is 120°. The magnitudes of both $\vec{A}$ and $(-\vec{B})$ is 5 unit. So,

$$|\vec{A} - \vec{B}| = \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \cos 120°}$$
$$= 2 \times 5 \cos 60° = 5 \text{ unit.}$$

## 2.6 RESOLUTION OF VECTORS

Figure (2.8) shows a vector $\vec{a} = \overrightarrow{OA}$ in the $X$-$Y$ plane drawn from the origin $O$. The vector makes an angle $\alpha$ with the $X$-axis and $\beta$ with the $Y$-axis. Draw perpendiculars $AB$ and $AC$ from $A$ to the $X$ and $Y$ axes respectively. The length $OB$ is called the projection of $\overrightarrow{OA}$ on $X$-axis. Similarly $OC$ is the projection of $\overrightarrow{OA}$ on $Y$-axis. According to the rules of vector addition

$$\vec{a} = \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{OC}.$$

Thus, we have resolved the vector $\vec{a}$ into two parts, one along $OX$ and the other along $OY$. The magnitude of the part along $OX$ is $OB = a\cos\alpha$ and the magnitude of the part along $OY$ is $OC = a\cos\beta$. If $\vec{i}$ and $\vec{j}$ denote vectors of unit magnitude along $OX$ and $OY$ respectively, we get

$$\overrightarrow{OB} = a\cos\alpha\,\vec{i} \text{ and } \overrightarrow{OC} = a\cos\beta\,\vec{j}$$

so that $\quad \vec{a} = a\cos\alpha\,\vec{i} + a\cos\beta\,\vec{j}.$

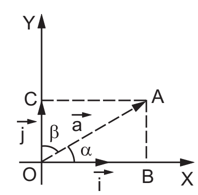


Figure 2.8

If the vector $\vec{a}$ is not in the $X$-$Y$ plane, it may have nonzero projections along $X,Y,Z$ axes and we can resolve it into three parts i.e., along the $X$, $Y$ and $Z$ axes. If $\alpha$, $\beta$, $\gamma$ be the angles made by the vector $\vec{a}$ with the three axes respectively, we get

$$\vec{a} = a\cos\alpha\,\vec{i} + a\cos\beta\,\vec{j} + a\cos\gamma\,\vec{k} \qquad \dots (2.3)$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the unit vectors along $X$, $Y$ and $Z$ axes respectively. The magnitude $(a\cos\alpha)$ is called the component of $\vec{a}$ along $X$-axis, $(a\cos\beta)$ is called the component along $Y$-axis and $(a\cos\gamma)$ is called the component along $Z$-axis. In general, the component of a vector $\vec{a}$ along a direction making an angle $\theta$ with it

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is $a$ cosθ (figure 2.9) which is the projection of $\vec{a}$ along the given direction.

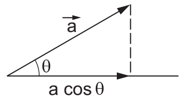


Figure 2.9

Equation (2.3) shows that any vector can be expressed as a linear combination of the three unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$.

***Example 2.3***

*A force of* 10·5 N *acts on a particle along a direction making an angle of* 37° *with the vertical. Find the component of the force in the vertical direction.*

***Solution*** : The component of the force in the vertical direction will be

$$F_{\perp} = F\cos\theta = (10{\cdot}5\text{ N})\,(\cos 37°)$$

$$= (10{\cdot}5\text{ N})\frac{4}{5} = 8{\cdot}40\text{ N}.$$

We can easily add two or more vectors if we know their components along the rectangular coordinate axes. Let us have

$$\vec{a} = a_x\,\vec{i} + a_y\,\vec{j} + a_z\,\vec{k}$$

$$\vec{b} = b_x\,\vec{i} + b_y\,\vec{j} + b_z\,\vec{k}$$

and $$\vec{c} = c_x\,\vec{i} + c_y\,\vec{j} + c_z\,\vec{k}$$

then

$$\vec{a} + \vec{b} + \vec{c} = (a_x + b_x + c_x)\,\vec{i} + (a_y + b_y + c_y)\,\vec{j} + (a_z + b_z + c_z)\,\vec{k}.$$

If all the vectors are in the $X$-$Y$ plane then all the $z$ components are zero and the resultant is simply

$$\vec{a} + \vec{b} + \vec{c} = (a_x + b_x + c_x)\,\vec{i} + (a_y + b_y + c_y)\,\vec{j}.$$

This is the sum of two mutually perpendicular vectors of magnitude $(a_x + b_x + c_x)$ and $(a_y + b_y + c_y)$. The resultant can easily be found to have a magnitude

$$\sqrt{(a_x + b_x + c_x)^2 + (a_y + b_y + c_y)^2}$$

making an angle α with the $X$-axis where

$$\tan\alpha = \frac{a_y + b_y + c_y}{a_x + b_x + c_x}.$$

## 2.7 DOT PRODUCT OR SCALAR PRODUCT OF TWO VECTORS

The dot product (also called scalar product) of two vectors $\vec{a}$ and $\vec{b}$ is defined as

$$\vec{a}\cdot\vec{b} = ab\cos\theta \qquad \ldots\ (2.4)$$

where $a$ and $b$ are the magnitudes of $\vec{a}$ and $\vec{b}$ respectively and θ is the angle between them. The dot product between two mutually perpendicular vectors is zero as cos90° = 0.

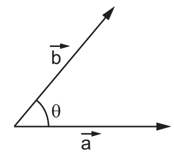


Figure 2.10

The dot product is commutative and distributive.

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\vec{a}\cdot(\vec{b} + \vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}.$$

***Example 2.4***

*The work done by a force $\vec{F}$ during a displacement $\vec{r}$ is given by $\vec{F}\cdot\vec{r}$. Suppose a force of* 12 N *acts on a particle in vertically upward direction and the particle is displaced through* 2·0 m *in vertically downward direction. Find the work done by the force during this displacement.*

***Solution*** : The angle between the force $\vec{F}$ and the displacement $\vec{r}$ is 180°. Thus, the work done is

$$W = \vec{F}\cdot\vec{r}$$

$$= Fr\cos\theta$$

$$= (12\text{ N})(2{\cdot}0\text{ m})(\cos 180°)$$

$$= -\,24\text{ N–m} = -\,24\text{ J}.$$

**Dot Product of Two Vectors in terms of the Components along the Coordinate Axes**

Consider two vectors $\vec{a}$ and $\vec{b}$ represented in terms of the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ along the coordinate axes as

$$\vec{a} = a_x\,\vec{i} + a_y\,\vec{j} + a_z\,\vec{k}$$

and $$\vec{b} = b_x\,\vec{i} + b_y\,\vec{j} + b_z\,\vec{k}.$$

Then

$$\begin{aligned}\vec{a}\cdot\vec{b} &= (a_x\,\vec{i} + a_y\,\vec{j} + a_z\,\vec{k})\cdot(b_x\,\vec{i} + b_y\,\vec{j} + b_z\,\vec{k})\\ &= a_x b_x\,\vec{i}\cdot\vec{i} + a_x b_y\,\vec{i}\cdot\vec{j} + a_x b_z\,\vec{i}\cdot\vec{k}\\ &\quad + a_y b_x\,\vec{j}\cdot\vec{i} + a_y b_y\,\vec{j}\cdot\vec{j} + a_y b_z\,\vec{j}\cdot\vec{k}\\ &\quad + a_z b_x\,\vec{k}\cdot\vec{i} + a_z b_y\,\vec{k}\cdot\vec{j} + a_z b_z\,\vec{k}\cdot\vec{k} \qquad \ldots\ \text{(i)}\end{aligned}$$

Since, $\vec{i}$, $\vec{j}$ and $\vec{k}$ are mutually orthogonal,

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we have $\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{i} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = \vec{k} \cdot \vec{j} = 0.$

Also, $\vec{i} \cdot \vec{i} = 1 \times 1 \cos 0 = 1.$

Similarly, $\vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1.$

Using these relations in equation (i) we get

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$

## 2.8 CROSS PRODUCT OR VECTOR PRODUCT OF TWO VECTORS

The cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a} \times \vec{b}$ is itself a vector. The magnitude of this vector is

$$|\vec{a} \times \vec{b}| = ab \sin\theta \qquad \dots \quad (2.5)$$

where $a$ and $b$ are the magnitudes of $\vec{a}$ and $\vec{b}$ respectively and $\theta$ is the smaller angle between the two. When two vectors are drawn with both the tails coinciding, two angles are formed between them (figure 2.11). One of the angles is smaller than 180°

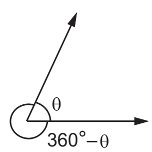


Figure 2.11

and the other is greater than 180° unless both are equal to 180°. The angle $\theta$ used in equation (2.5) is the smaller one. If both the angles are equal to 180°, $\sin\theta = \sin 180° = 0$ and hence $|\vec{a} \times \vec{b}| = 0$. Similarly if $\theta = 0$, $\sin\theta = 0$ and $|\vec{a} \times \vec{b}| = 0$. The cross product of two parallel vectors is zero.

The direction of $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. Thus, it is perpendicular to the plane formed by $\vec{a}$ and $\vec{b}$. To determine the direction of arrow on this perpendicular several rules are in use. In order to avoid confusion we here describe just one rule.

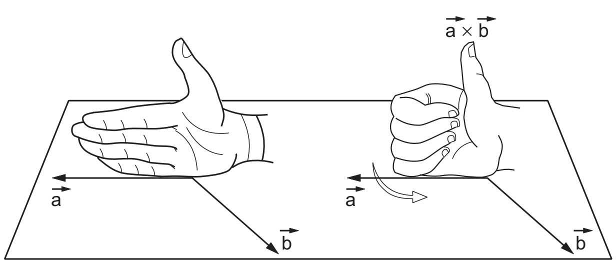


Figure 2.12

Draw the two vectors $\vec{a}$ and $\vec{b}$ with both the tails coinciding (figure 2.12). Now place your stretched right palm perpendicular to the plane of $\vec{a}$ and $\vec{b}$ in such a way that the fingers are along the vector $\vec{a}$ and when the fingers are closed they go towards $\vec{b}$. The direction of the thumb gives the direction of arrow to be put on the vector $\vec{a} \times \vec{b}$.

This is known as the *right hand thumb rule*. The left handers should be more careful in using this rule as it must be practiced with right hand only.

Note that this rule makes the cross product noncommutative. In fact

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}.$$

The cross product follows the distributive law

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}.$$

It does not follow the associative law

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}.$$

When we choose a coordinate system any two perpendicular lines may be chosen as $X$ and $Y$ axes. However, once $X$ and $Y$ axes are chosen, there are two possible choices of $Z$-axis. The $Z$-axis must be perpendicular to the $X$-$Y$ plane. But the positive direction of $Z$-axis may be defined in two ways. We choose the positive direction of $Z$-axis in such a way that

$$\vec{i} \times \vec{j} = \vec{k}.$$

Such a coordinate system is called a *right handed system*. In such a system

$$\vec{j} \times \vec{k} = \vec{i} \quad \text{and} \quad \vec{k} \times \vec{i} = \vec{j}.$$

Of course $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0.$

***Example 2.5***

*The vector $\vec{A}$ has a magnitude of 5 unit, $\vec{B}$ has a magnitude of 6 unit and the cross product of $\vec{A}$ and $\vec{B}$ has a magnitude of 15 unit. Find the angle between $\vec{A}$ and $\vec{B}$.*

***Solution*** : If the angle between $\vec{A}$ and $\vec{B}$ is $\theta$, the cross product will have a magnitude

$$|\vec{A} \times \vec{B}| = AB \sin\theta$$

or, $\quad 15 = 5 \times 6 \sin\theta$

or, $\quad \sin\theta = \frac{1}{2}.$

Thus, $\quad \theta = 30°$ or, 150°.

**Cross Product of Two Vectors in terms of the Components along the Coordinate Axes**

Let $\quad \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$

and $\quad \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}.$

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Then $\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$

$$= a_x b_x \vec{i} \times \vec{i} + a_x b_y \vec{i} \times \vec{j} + a_x b_z \vec{i} \times \vec{k}$$
$$+ a_y b_x \vec{j} \times \vec{i} + a_y b_y \vec{j} \times \vec{j} + a_y b_z \vec{j} \times \vec{k}$$
$$+ a_z b_x \vec{k} \times \vec{i} + a_z b_y \vec{k} \times \vec{j} + a_z b_z \vec{k} \times \vec{k}$$
$$= a_x b_y \vec{k} + a_x b_z(-\vec{j}) + a_y b_x(-\vec{k}) + a_y b_z(\vec{i})$$
$$+ a_z b_x(\vec{j}) + a_z b_y(-\vec{i})$$
$$= (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j}$$
$$+ (a_x b_y - a_y b_x)\vec{k}.$$

**Zero Vector**

If we add two vectors $\vec{A}$ and $\vec{B}$, we get a vector. Suppose the vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes but opposite directions. What is the vector $\vec{A} + \vec{B}$ ? The magnitude of this vector will be zero. For mathematical consistency it is convenient to have a vector of zero magnitude although it has little significance in physics. This vector is called zero vector. The direction of a zero vector is indeterminate. We can write this vector as $\vec{0}$. The concept of zero vector is also helpful when we consider vector product of parallel vectors. If $\vec{A} \parallel \vec{B}$, the vector $\vec{A} \times \vec{B}$ is zero vector. For any vector $\vec{A}$,

$$\vec{A} + \vec{0} = \vec{A}$$

$$\vec{A} \times \vec{0} = \vec{0}$$

and for any number $\lambda$,

$$\lambda \vec{0} = \vec{0}.$$

## 2.9 DIFFERENTIAL CALCULUS : $\frac{dy}{dx}$ AS RATE MEASURER

Consider two quantities $y$ and $x$ interrelated in such a way that for each value of $x$ there is one and only one value of $y$. Figure (2.13) represents the graph

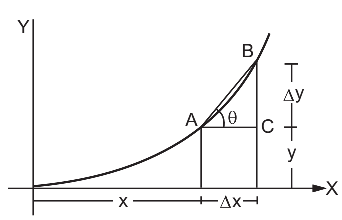


Figure 2.13

of $y$ versus $x$. The value of $y$ at a particular $x$ is obtained by the height of the ordinate at that $x$. Let $x$ be changed by a small amount $\Delta x$, and the corresponding change in $y$ be $\Delta y$. We can define the "rate of change" of $y$ with respect to $x$ in the following way. When $x$ changes by $\Delta x$, $y$ changes by $\Delta y$ so that the rate of change seems to be equal to $\frac{\Delta y}{\Delta x}$ · If $A$ be the point $(x, y)$ and $B$ be the point $(x + \Delta x, y + \Delta y)$, the rate $\frac{\Delta y}{\Delta x}$ equals the slope of the line $AB$. We have

$$\frac{\Delta y}{\Delta x} = \frac{BC}{AC} = \tan\theta.$$

However, this cannot be the precise definition of the rate. Because the rate also varies between the points $A$ and $B$. The curve is steeper at $B$ than at $A$. Thus, to know the rate of change of $y$ at a particular value of $x$, say at $A$, we have to take $\Delta x$ very small. However small we take $\Delta x$, as long as it is not zero the rate may vary within that small part of the curve. However, if we go on drawing the point $B$ closer to $A$ and everytime calculate $\frac{\Delta y}{\Delta x} = \tan\theta$, we shall see that as $\Delta x$ is made smaller and smaller the slope $\tan\theta$ of the line $AB$ approaches the slope of the tangent at $A$. This slope of the tangent at $A$ thus gives the rate of change of $y$ with respect to $x$ at $A$. This rate is denoted by $\frac{dy}{dx}$ · Thus,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \cdot$$

For small changes $\Delta x$ we can approximately write

$$\Delta y = \frac{dy}{dx} \Delta x.$$

Note that if the function $y$ increases with an increase in $x$ at a point, $\frac{dy}{dx}$ is positive there, because both $\Delta y$ and $\Delta x$ are positive. If the function $y$ decreases with an increase in $x$, $\Delta y$ is negative when $\Delta x$ is positive. Then $\frac{\Delta y}{\Delta x}$ and hence $\frac{dy}{dx}$ is negative.

***Example 2.6***

*From the curve given in figure (2.14) find* $\frac{dy}{dx}$ *at* $x = 2$, 6 *and* 10.

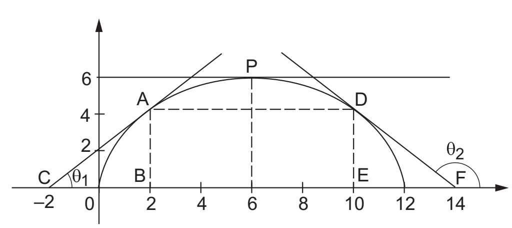


Figure 2.14

***Solution*** : The tangent to the curve at $x = 2$ is $AC$. Its slope is $\tan\theta_1 = \frac{AB}{BC} = \frac{5}{4}$ ·

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Thus, $\frac{dy}{dx} = \frac{5}{4}$ at $x = 2$.

The tangent to the curve at $x = 6$ is parallel to the $X$-axis.

Thus, $\frac{dy}{dx} = \tan\theta = 0$ at $x = 6$.

The tangent to the curve at $x = 10$ is $DF$. Its slope is

$$\tan\theta_2 = \frac{DE}{EF} = -\frac{5}{4}\cdot$$

Thus, $\frac{dy}{dx} = -\frac{5}{4}$ at $x = 10\cdot$

If we are given the graph of $y$ versus $x$, we can find $\frac{dy}{dx}$ at any point of the curve by drawing the tangent at that point and finding its slope. Even if the graph is not drawn and the algebraic relation between $y$ and $x$ is given in the form of an equation, we can find $\frac{dy}{dx}$ algebraically. Let us take an example.

The area $A$ of a square of length $L$ is $A = L^2$.

If we change $L$ to $L + \Delta L$, the area will change from $A$ to $A + \Delta A$ (figure 2.15).

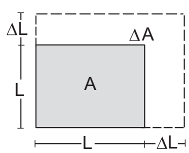


Figure 2.15

$$A + \Delta A = (L + \Delta L)^2$$
$$= L^2 + 2L\,\Delta L + (\Delta L)^2$$

or, $$\Delta A = 2L(\Delta L) + (\Delta L)^2$$

or, $$\frac{\Delta A}{\Delta L} = 2L + \Delta L.$$

Now if $\Delta L$ is made smaller and smaller, $2L + \Delta L$ will approach $2L$.

Thus, $$\frac{dA}{dL} = \lim_{\Delta L \to 0} \frac{\Delta A}{\Delta L} = 2L.$$

Table (2.1) gives the formulae for $\frac{dy}{dx}$ for some of the important functions. $\frac{dy}{dx}$ is called the differential coefficient or derivative of $y$ with respect to $x$.

**Table 2.1 : $\frac{dy}{dx}$ *for some common functions***

| $y$ | $\frac{dy}{dx}$ | $y$ | $\frac{dy}{dx}$ |
|---|---|---|---|
| $x^n$ | $nx^{n-1}$ | $\sec x$ | $\sec x \tan x$ |
| $\sin x$ | $\cos x$ | $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cos x$ | $-\sin x$ | $\ln x$ | $\frac{1}{x}$ |
| $\tan x$ | $\sec^2 x$ | $e^x$ | $e^x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ | | |

Besides, there are certain rules for finding the derivatives of composite functions.

(a) $\frac{d}{dx}(cy) = c\frac{dy}{dx}$ ($c$ is a constant)

(b) $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

(c) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(d) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(e) $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

With these rules and table 2.1 derivatives of almost all the functions of practical interest may be evaluated.

***Example 2.7***

*Find $\frac{dy}{dx}$ if $y = e^x \sin x$.*

***Solution*** : $y = e^x \sin x$.

So $$\frac{dy}{dx} = \frac{d}{dx}(e^x \sin x) = e^x\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(e^x)$$
$$= e^x \cos x + e^x \sin x = e^x(\cos x + \sin x).$$

## 2.10 MAXIMA AND MINIMA

Suppose a quantity $y$ depends on another quantity $x$ in a manner shown in figure (2.16). It becomes maximum at $x_1$ and minimum at $x_2$.

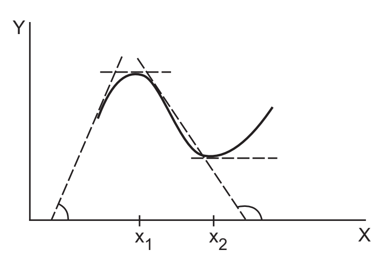


Figure 2.16

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At these points the tangent to the curve is parallel to the $X$-axis and hence its slope is $\tan\theta = 0$. But the slope of the curve $y$-$x$ equals the rate of change $\frac{dy}{dx}\cdot$ Thus, at a maximum or a minimum,

$$\frac{dy}{dx} = 0.$$

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. Thus, $\frac{dy}{dx}$ decreases at a maximum and hence the rate of change of $\frac{dy}{dx}$ is negative at a maximum i.e.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0 \text{ at a maximum.}$$

The quantity $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is the rate of change of the slope. It is written as $\frac{d^2y}{dx^2}\cdot$ Thus, the condition of a maximum is

$$\left.\begin{array}{l}\dfrac{dy}{dx} = 0\\[2ex] \dfrac{d^2y}{dx^2} < 0\end{array}\right\} \text{— maximum.} \qquad \ldots \quad (2.6)$$

Similarly, at a minimum the slope changes from negative to positive. The slope increases at such a point and hence $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$. The condition of a minimum is

$$\left.\begin{array}{l}\dfrac{dy}{dx} = 0\\[2ex] \dfrac{d^2y}{dx^2} > 0\end{array}\right\} \text{— minimum.} \qquad \ldots \quad (2.7)$$

Quite often it is known from the physical situation whether the quantity is a maximum or a minimum. The test on $\frac{d^2y}{dx^2}$ may then be omitted.

***Example 2.8***

*The height reached in time t by a particle thrown upward with a speed u is given by*

$$h = ut - \frac{1}{2}gt^2$$

*where* $g = 9{\cdot}8 \text{ m/s}^2$ *is a constant. Find the time taken in reaching the maximum height.*

***Solution*** : The height $h$ is a function of time. Thus, $h$ will be maximum when $\frac{dh}{dt} = 0$. We have,

$$h = ut - \frac{1}{2}gt^2$$

or,
$$\frac{dh}{dt} = \frac{d}{dt}(ut) - \frac{d}{dt}\left(\frac{1}{2}gt^2\right)$$

$$= u\frac{dt}{dt} - \frac{1}{2}g\frac{d}{dt}(t^2)$$

$$= u - \frac{1}{2}g(2t) = u - gt.$$

For maximum $h$,

$$\frac{dh}{dt} = 0$$

or, $\quad u - gt = 0 \quad$ or, $\quad t = \frac{u}{g}\cdot$

## 2.11 INTEGRAL CALCULUS

Let $PQ$ be a curve representing the relation between two quantities $x$ and $y$ (figure 2.17). The point $P$ corresponds to $x = a$ and $Q$ corresponds to $x = b$. Draw perpendiculars from $P$ and $Q$ on the $X$-axis so as to cut it at $A$ and $B$ respectively. We are interested in finding the area $PABQ$. Let us denote the value of $y$ at $x$ by the symbol $y = f(x)$.

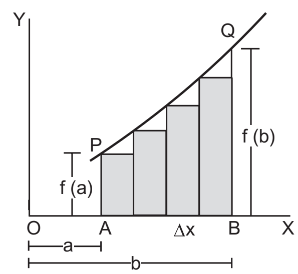


Figure 2.17

Let us divide the length $AB$ in $N$ equal elements each of length $\Delta x = \frac{b-a}{N}\cdot$ From the ends of each small length we draw lines parallel to the $Y$-axis. From the points where these lines cut the given curve, we draw short lines parallel to the $X$-axis. This constructs the rectangular bars shown shaded in the figure. The sum of the areas of these $N$ rectangular bars is

$$I' = f(a)\,\Delta x + f(a + \Delta x)\,\Delta x + f(a + 2\Delta x)\,\Delta x + \ldots$$
$$\ldots + f[a + (N-1)\,\Delta x]\,\Delta x\,.$$

This may be written as

$$I' = \sum_{i=1}^{N} f(x_i)\Delta x \qquad \ldots \quad (2.8)$$

where $x_i$ takes the values $a, a + \Delta x, a + 2\Delta x, .., b - \Delta x$.

This area differs slightly from the area $PABQ$. This difference is the sum of the small triangles formed just under the curve. Now the important point is the following. As we increase the number of intervals $N$, the vertices of the bars touch the curve $PQ$ at more points and the total area of the small triangles decreases. As $N$ tends to infinity ($\Delta x$ tends to zero

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because $\Delta x = \frac{b-a}{N}$) the vertices of the bars touch the curve at infinite number of points and the total area of the triangles tends to zero. In such a limit the sum (2.8) becomes the area $I$ of $PABQ$. Thus, we may write,

$$I = \lim_{\Delta x \to 0} \sum_{i=1}^{N} f(x_i) \Delta x.$$

The limit is taken as $\Delta x$ tends to zero or as $N$ tends to infinity. In mathematics this quantity is denoted as

$$I = \int_a^b f(x)\, dx$$

and is read as the integral of $f(x)$ with respect to $x$ within the limits $x = a$ to $x = b$. Here $a$ is called the lower limit and $b$ the upper limit of integration. The integral is the sum of a large number of terms of the type $f(x)\,\Delta x$ with $x$ continuously varying from $a$ to $b$ and the number of terms tending to infinity.

Let us use the above method to find the area of a trapezium. Let us suppose the line $PQ$ is represented by the equation $y = x$.

The points $A$ and $B$ on the $X$-axis represent $x = a$ and $x = b$. We have to find the area of the trapezium $PABQ$.

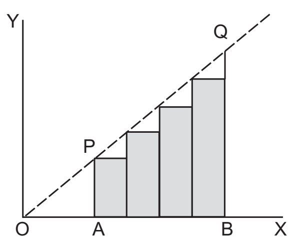


Figure 2.18

Let us divide the length $AB$ in $N$ equal intervals. The length of each interval is $\Delta x = \frac{b-a}{N}$. The height of the first shaded bar is $y = x = a$, of the second bar is $y = x = a + \Delta x$, that of the third bar is $y = x = a + 2\,\Delta x$ etc. The height of the $N$th bar is $y = x = a + (N-1)\Delta x$. The width of each bar is $\Delta x$, so that the total area of all the bars is

$$I' = a\Delta x + (a + \Delta x)\,\Delta x + (a + 2\Delta x)\,\Delta x + \dots + [a + (N-1)\Delta x]\Delta x$$

$$= \big[a + (a + \Delta x) + (a + 2\Delta x) + \dots + \{a + (N-1)\Delta x\}\big]\Delta x \qquad \dots (2.9)$$

This sum can be written as

$$I' = \sum_{i=1}^{N} x_i\, \Delta x$$

where $\Delta x = \frac{b-a}{N}$ and $x_i = a,\ a + \Delta x, \dots b - \Delta x$.

As $\Delta x \to 0$ the total area of the bars becomes the area of the shaded part $PABQ$.

Thus, the required area is

$$I = \lim_{\Delta x \to 0} \sum_{i=1}^{N} x_i\, \Delta x$$

$$= \int_a^b x dx. \qquad \dots \text{(i)}$$

Now the terms making the series in the square bracket in equation (2.9) are in arithmetic progression so that this series may be summed up using the formula $S = \frac{n}{2}(a + l)$. Equation (2.9) thus becomes

$$I' = \frac{N}{2}[a + \{a + (N-1)\Delta x\}]\Delta x$$

$$= \frac{N\Delta x}{2}\,[2a + N\Delta x - \Delta x]$$

$$= \frac{b-a}{2}\,[2a + b - a - \Delta x]$$

$$= \frac{b-a}{2}\,[a + b - \Delta x].$$

Thus, the area $PABQ$ is

$$I = \lim_{\Delta x \to 0} \left[\frac{b-a}{2}\right][a + b - \Delta x]$$

$$= \frac{b-a}{2}\,(a + b)$$

$$= \frac{1}{2}\,(b^2 - a^2). \qquad \dots \text{(ii)}$$

Thus, from (i) and (ii)

$$\int_a^b x\, dx = \frac{1}{2}\,(b^2 - a^2).$$

In mathematics, special methods have been developed to find the integration of various functions $f(x)$. A very useful method is as follows. Suppose we wish to find

$$\int_a^b f(x)\, dx = \lim_{\Delta x \to 0} \sum_{i=1}^{N} f(x_i)\, \Delta x$$

where $\Delta x = \frac{b-a}{N}$; $x_i = a,\ a + \Delta x, \dots b - \Delta x$.

Now look for a function $F(x)$ such that the derivative of $F(x)$ is $f(x)$ that is, $\frac{dF(x)}{dx} = f(x)$. If you can find such a function $F(x)$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)\,;$$

$F(b) - F(a)$ is also written as $[F(x)]_a^b$.

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$F(x)$ is called the indefinite integration or the antiderivative of $f(x)$. We also write $\int f(x)\,dx = F(x)$. This may be treated as another way of writing $\frac{dF(x)}{dx} = f(x)$ ·

For example, $\frac{d}{dx}\left(\frac{1}{2}x^2\right) = \frac{1}{2}\frac{d}{dx}(x^2) = \frac{1}{2}\cdot 2x = x.$

Thus,
$$\int_a^b x\,dx = \left[\frac{1}{2}x^2\right]_a^b$$
$$= \left(\frac{1}{2}b^2\right) - \left(\frac{1}{2}a^2\right)$$
$$= \frac{1}{2}(b^2 - a^2)$$
as deduced above.

Table (2.2) lists some important integration formulae. Many of them are essentially same as those given in table (2.1).

**Table 2.2 : *Integration Formulae***

| $f(x)$ | $F(x) = \int f(x)\,dx$ | $f(x)$ | $F(x) = \int f(x)\,dx$ |
|---|---|---|---|
| $\sin x$ | $-\cos x$ | $x^n (n \neq -1)$ | $\frac{x^{n+1}}{n+1}$ |
| $\cos x$ | $\sin x$ | $\frac{1}{x}$ | $\ln x$ |
| $\sec^2 x$ | $\tan x$ | $\frac{1}{x^2 + a^2}$ | $\frac{1}{a}\tan^{-1}\frac{x}{a}$ |
| $\operatorname{cosec}^2 x$ | $-\cot x$ | $\frac{1}{\sqrt{a^2 - x^2}}$ | $\sin^{-1}\frac{x}{a}$ |
| $\sec x \tan x$ | $\sec x$ | | |
| $\operatorname{cosec} x \cot x$ | $-\operatorname{cosec} x$ | | |

Some useful rules for integration are as follows:

(a) $\int c\, f(x)\,dx = c\int f(x)\,dx$ where $c$ is a constant

(b) Let $\int f(x)\,dx = F(x)$

then $\int f(cx)\,dx = \frac{1}{c}F(cx).$

(c) $\int [\,f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx.$

***Example 2.9***

*Evaluate* $\int_3^6 (2x^2 + 3x + 5)\,dx.$

***Solution*** : $\int (2x^2 + 3x + 5)\,dx$
$$= \int 2x^2 dx + \int 3x\,dx + \int 5\,dx$$
$$= 2\int x^2 dx + 3\int x\,dx + 5\int x^0 dx$$
$$= 2\frac{x^3}{3} + 3\frac{x^2}{2} + 5\frac{x^1}{1}$$
$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + 5x.$$

Thus, $\int_3^6 (2x^2 + 3x + 5)\,dx = \left[\frac{2}{3}x^3 + \frac{3}{2}x^2 + 5x\right]_3^6$
$$= \frac{2}{3}(216 - 27) + \frac{3}{2}(36 - 9) + 5(6 - 3)$$
$$= 126 + 40{\cdot}5 + 15 = 181{\cdot}5.$$

## 2.12 SIGNIFICANT DIGITS

When a measurement is made, a numerical value is read generally from some calibrated scale. To measure the length of a body we can place a metre scale in contact with the body. One end of the body may be made to coincide with the zero of the metre scale and the reading just in front of the other end is noted from the scale. When an electric current is measured with an ammeter the reading of the pointer on the graduation of the ammeter is noted. The value noted down includes all the digits that can be directly read from the scale and one doubtful digit at the end. The doubtful digit corresponds to the eye estimation within the smallest subdivision of the scale. This smallest subdivision is known as the *least count* of the instrument. In a metre scale, the major graduations are at an interval of one centimetre and ten subdivisions are made between two consecutive major graduations. Thus, the smallest subdivision measures a millimetre. If one end of the object coincides with the zero of the metre scale, the other end may fall between 10·4 cm and 10·5 cm mark of the scale (figure 2.19). We can estimate the distance between the 10·4 cm mark and the edge of the body as follows.

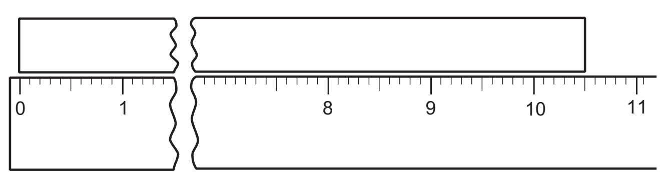


Figure 2.19

We mentally divide the 1 mm division in 10 equal parts and guess on which part is the edge falling. We may note down the reading as 10·46 cm. The digits 1, 0 and 4 are certain but 6 is doubtful. All these digits are called *significant digits*. We say that the length is measured up to four significant digits. The rightmost or the doubtful digit is called the *least significant digit* and the leftmost digit is called the *most significant digit*.

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There may be some confusion if there are zeroes at the right end of the number. For example, if a measurement is quoted as 600 mm and we know nothing about the least count of the scale we cannot be sure whether the last zeros are significant or not. If the scale had marking only at each metre then the edge must be between the marks 0 m and 1 m and the digit 6 is obtained only through the eye estimation. Thus, 6 is the doubtful digit and the zeros after that are insignificant. But if the scale had markings at centimetres, the number read is 60 and these two digits are significant, the last zero is insignificant. If the scale used had markings at millimetres, all the three digits 6, 0, 0 are significant. To avoid confusion one may report only the significant digits and the magnitude may be correctly described by proper powers of 10. For example, if only 6 is significant in 600 mm we may write it as $6 \times 10^{2}$ mm. If 6 and the first zero are significant we may write it as $6{\cdot}0 \times 10^{2}$ mm and if all the three digits are significant we may write it as $6{\cdot}00 \times 10^{2}$ mm.

If the integer part is zero, any number of continuous zeros just after the decimal part is insignificant. Thus, the number of significant digits in 0·0023 is two and in 1·0023 is five.

## 2.13 SIGNIFICANT DIGITS IN CALCULATIONS

When two or more numbers are added, subtracted, multiplied or divided, how to decide about the number of significant digits in the answer? For example, suppose the mass of a body $A$ is measured to be 12·0 kg and of another body $B$ to be 7·0 kg. What is the ratio of the mass of $A$ to the mass of $B$? Arithmetic will give this ratio as

$$\frac{12{\cdot}0}{7{\cdot}0} = 1{\cdot}714285\ldots$$

However, all the digits of this answer cannot be significant. The zero of 12·0 is a doubtful digit and the zero of 7·0 is also doubtful. The quotient cannot have so many reliable digits. The rules for deciding the number of significant digits in an arithmetic calculation are listed below.

1. In a multiplication or division of two or more quantities, the number of significant digits in the answer is equal to the number of significant digits in the quantity which has the minimum number of significant digits. Thus, $\frac{12{\cdot}0}{7{\cdot}0}$ will have two significant digits only.

The insignificant digits are dropped from the result if they appear after the decimal point. They are replaced by zeros if they appear to the left of the decimal point. The least significant digit is rounded according to the rules given below.

If the digit next to the one rounded is more than 5, the digit to be rounded is increased by 1. If the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged. If the digit next to the one rounded is 5, then the digit to be rounded is increased by 1 if it is odd and is left unchanged if it is even.

2. For addition or subtraction write the numbers one below the other with all the decimal points in one line. Now locate the first column from left that has a doubtful digit. All digits right to this column are dropped from all the numbers and rounding is done to this column. The addition or subtraction is now performed to get the answer.

***Example 2.10***

*Round off the following numbers to three significant digits (a)* 15462*, (b)* 14·745*, (c)* 14·750 *and (d)* 14·650 $\times 10^{12}$.

***Solution*** : (a) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore, be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

(b) The third significant digit in 14·745 is 7. The number next to it is less than 5. So 14·745 becomes 14·7 on rounding to three significant digits.

(c) 14·750 will become 14·8 because the digit to be rounded is odd and the digit next to it is 5.

(d) $14{\cdot}650 \times 10^{12}$ will become $14{\cdot}6 \times 10^{12}$ because the digit to be rounded is even and the digit next to it is 5.

***Example 2.11***

*Evaluate* $\frac{25{\cdot}2 \times 1374}{33{\cdot}3}\cdot$ *All the digits in this expression are significant.*

***Solution*** : We have $\frac{25{\cdot}2 \times 1374}{33{\cdot}3} = 1039{\cdot}7838....$

Out of the three numbers given in the expression 25·2 and 33·3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounding 1039·7838... to three significant digits, it becomes 1040. Thus, we write

$$\frac{25{\cdot}2 \times 1374}{33{\cdot}3} = 1040.$$

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***Example 2.12***

*Evaluate* $24{\cdot}36 + 0{\cdot}0623 + 256{\cdot}2.$

**Solution :**

$$\begin{array}{r} 24{\cdot}36 \\ 0{\cdot}0623 \\ \underline{256{\cdot}2\phantom{000}} \end{array}$$

Now the first column where a doubtful digit occurs is the one just next to the decimal point (256·2). All digits right to this column must be dropped after proper rounding. The table is rewritten and added below

$$\begin{array}{r} 24{\cdot}4 \\ 0{\cdot}1 \\ \underline{256{\cdot}2} \\ 280{\cdot}7 \end{array}$$

The sum is 280·7.

## 2.14 ERRORS IN MEASUREMENT

While doing an experiment several errors can enter into the results. Errors may be due to faulty equipment, carelessness of the experimenter or random causes. The first two types of errors can be removed after detecting their cause but the random errors still remain. No specific cause can be assigned to such errors.

When an experiment is repeated many times, the random errors are sometimes positive and sometimes negative. Thus, the average of a large number of the results of repeated experiments is close to the true value. However, there is still some uncertainty about the truth of this average. The uncertainty is estimated by calculating the standard deviation described below.

Let $x_1, x_2, x_3, \ldots, x_N$ are the results of an experiment repeated $N$ times. The standard deviation $\sigma$ is defined as

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

where $\bar{x} = \frac{1}{N}\sum_i x_i$ is the average of all the values of $x$.

The best value of $x$ derived from these experiments is $\bar{x}$ and the uncertainty is of the order of $\pm\sigma$. In fact $\bar{x} \pm 1{\cdot}96\,\sigma$ is quite often taken as the interval in which the true value should lie. It can be shown that there is a 95% chance that the true value lies within $\bar{x} \pm 1{\cdot}96\,\sigma$.

If one wishes to be more sure, one can use the interval $\bar{x} \pm 3\,\sigma$ as the interval which will contain the true value. The chances that the true value will be within $\bar{x} \pm 3\,\sigma$ is more that 99%.

All this is true if the number of observations $N$ is large. In practice if $N$ is greater than 8, the results are reasonably correct.

***Example 2.13***

*The focal length of a concave mirror obtained by a student in repeated experiments are given below. Find the average focal length with uncertainty in* $\pm\sigma$ *limit.*

| No. of observation | focal length in cm |
|---|---|
| 1 | 25·4 |
| 2 | 25·2 |
| 3 | 25·6 |
| 4 | 25·1 |
| 5 | 25·3 |
| 6 | 25·2 |
| 7 | 25·5 |
| 8 | 25·4 |
| 9 | 25·3 |
| 10 | 25·7 |

***Solution*** : The average focal length $\bar{f} = \frac{1}{10}\sum_{i=1}^{10} f_i$

$$= 25{\cdot}37 \approx 25{\cdot}4.$$

The calculation of $\sigma$ is shown in the table below:

| $i$ | $f_i$ cm | $f_i - \bar{f}$ cm | $(f_i - \bar{f})^2$ cm$^2$ | $\Sigma(f_i - \bar{f})^2$ cm$^2$ |
|---|---|---|---|---|
| 1 | 25·4 | 0·0 | 0·00 | |
| 2 | 25·2 | – 0·2 | 0·04 | |
| 3 | 25·6 | 0·2 | 0·04 | |
| 4 | 25·1 | – 0·3 | 0·09 | |
| 5 | 25·3 | – 0·1 | 0·01 | 0·33 |
| 6 | 25·2 | – 0·2 | 0·04 | |
| 7 | 25·5 | 0·1 | 0·01 | |
| 8 | 25·4 | 0·0 | 0·00 | |
| 9 | 25·3 | – 0·1 | 0·01 | |
| 10 | 25·7 | 0·3 | 0·09 | |

$$\sigma = \sqrt{\frac{1}{10}\sum_i (f_i - \bar{f})^2} = \sqrt{0{\cdot}033\ \text{cm}^2} = 0{\cdot}18\ \text{cm}$$

$$\cong 0{\cdot}2\ \text{cm}.$$

Thus, the focal length is likely to be within (25·4 ± 0·2 cm) and we write

$$f = (25{\cdot}4 \pm 0{\cdot}2)\ \text{cm}.$$

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## *Worked Out Examples*

**1.** *A vector has component along the X-axis equal to* 25 unit *and along the Y-axis equal to* 60 unit. *Find the magnitude and direction of the vector.*

***Solution*** **:** The given vector is the resultant of two perpendicular vectors, one along the $X$-axis of magnitude 25 unit and the other along the $Y$-axis of magnitude 60 units. The resultant has a magnitude $A$ given by

$$A = \sqrt{(25)^2 + (60)^2 + 2 \times 25 \times 60 \cos 90°}$$

$$= \sqrt{(25)^2 + (60)^2} = 65.$$

The angle $\alpha$ between this vector and the $X$-axis is given by

$$\tan\alpha = \frac{60}{25}.$$

**2.** *Find the resultant of the three vectors shown in figure (2-W1).*

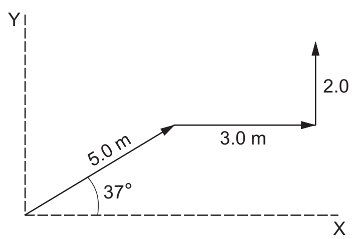


Figure 2-W1

***Solution*** **:** Take the axes as shown in the figure.

The $x$-component of the 5·0 m vector = 5·0 m cos37°

= 4·0 m,

the $x$-component of the 3·0 m vector = 3·0 m

and the $x$-component of the 2·0 m vector = 2·0 m cos90°

= 0.

Hence, the $x$-component of the resultant

= 4·0 m + 3·0 m + 0 = 7·0 m.

The $y$-component of the 5·0 m vector = 5·0 m sin37°

= 3·0 m,

the $y$-component of the 3·0 m vector = 0

and the $y$-component of the 2·0 m vector = 2·0 m.

Hence, the $y$-component of the resultant

= 3·0 m + 0 + 2·0 m = 5·0 m.

The magnitude of the resultant vector

$$= \sqrt{(7\cdot0 \text{ m})^2 + (5\cdot0 \text{ m})^2}$$

$$= 8\cdot6 \text{ m}.$$

If the angle made by the resultant with the $X$-axis is $\theta$, then

$$\tan\theta = \frac{y\text{-component}}{x\text{-component}} = \frac{5\cdot0}{7\cdot0} \quad \text{or,} \quad \theta = 35\cdot5°.$$

**3.** *The sum of the three vectors shown in figure (2-W2) is zero. Find the magnitudes of the vectors $\vec{OB}$ and $\vec{OC}$.*

***Solution*** **:** Take the axes as shown in the figure

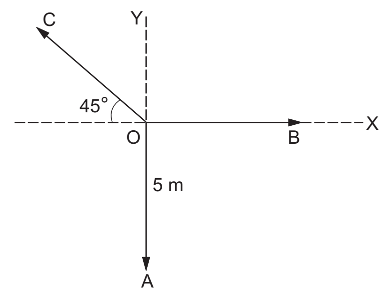


Figure 2-W2

The $x$-component of $\vec{OA} = (OA)\cos 90° = 0$.

The $x$-component of $\vec{OB} = (OB)\cos 0° = OB$.

The $x$-component of $\vec{OC} = (OC)\cos 135° = -\frac{1}{\sqrt{2}} OC$.

Hence, the $x$-component of the resultant

$$= OB - \frac{1}{\sqrt{2}} OC. \qquad \ldots \text{(i)}$$

It is given that the resultant is zero and hence its $x$-component is also zero. From (i),

$$OB = \frac{1}{\sqrt{2}} OC. \qquad \ldots \text{(ii)}$$

The $y$-component of $\vec{OA} = OA \cos 180° = -OA$.

The $y$-component of $\vec{OB} = OB \cos 90° = 0$.

The $y$-component of $\vec{OC} = OC \cos 45° = \frac{1}{\sqrt{2}} OC$.

Hence, the $y$-component of the resultant

$$= \frac{1}{\sqrt{2}} OC - OA \qquad \ldots \text{(iii)}$$

As the resultant is zero, so is its $y$-component. From (iii),

$$\frac{1}{\sqrt{2}} OC = OA, \quad \text{or,} \quad OC = \sqrt{2}\, OA = 5\sqrt{2} \text{ m}.$$

From (ii), $OB = \frac{1}{\sqrt{2}} OC = 5$ m.

**4.** *The magnitudes of vectors $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ in figure (2-W3) are equal. Find the direction of $\vec{OA} + \vec{OB} - \vec{OC}$.*

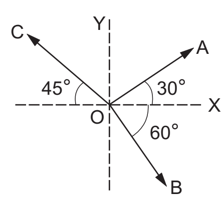


Figure 2-W3

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***Solution*** : Let $OA = OB = OC = F$.

$x$-component of $\overrightarrow{OA} = F\cos 30° = F\frac{\sqrt{3}}{2}$.

$x$-component of $\overrightarrow{OB} = F\cos 60° = \frac{F}{2}$.

$x$-component of $\overrightarrow{OC} = F\cos 135° = -\frac{F}{\sqrt{2}}$.

$x$-component of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$$= \left(\frac{F\sqrt{3}}{2}\right) + \left(\frac{F}{2}\right) - \left(-\frac{F}{\sqrt{2}}\right)$$

$$= \frac{F}{2}(\sqrt{3} + 1 + \sqrt{2}).$$

$y$-component of $\overrightarrow{OA} = F\cos 60° = \frac{F}{2}$.

$y$-component of $\overrightarrow{OB} = F\cos 150° = -\frac{F\sqrt{3}}{2}$.

$y$-component of $\overrightarrow{OC} = F\cos 45° = \frac{F}{\sqrt{2}}$.

$y$-component of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$$= \left(\frac{F}{2}\right) + \left(-\frac{F\sqrt{3}}{2}\right) - \left(\frac{F}{\sqrt{2}}\right)$$

$$= \frac{F}{2}(1 - \sqrt{3} - \sqrt{2}).$$

Angle of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with the $X$-axis

$$= \tan^{-1}\frac{\frac{F}{2}(1 - \sqrt{3} - \sqrt{2})}{\frac{F}{2}(1 + \sqrt{3} + \sqrt{2})} = \tan^{-1}\frac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})}.$$

**5.** *Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in figure (2-W4). Radius of the circle is $R$.*

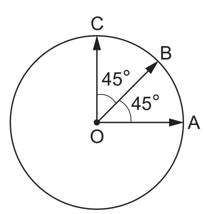


Figure 2-W4

***Solution*** : $OA = OC$.

$\overrightarrow{OA} + \overrightarrow{OC}$ is along $\overrightarrow{OB}$ (bisector) and its magnitude is

$$2R\cos 45° = R\sqrt{2}.$$

$(\overrightarrow{OA} + \overrightarrow{OC}) + \overrightarrow{OB}$ is along $\overrightarrow{OB}$ and its magnitude is

$$R\sqrt{2} + R = R(1 + \sqrt{2}).$$

**6.** *The resultant of vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$ (figure 2-W5). Find the angle AOB.*

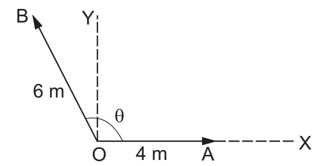


Figure 2-W5

***Solution*** : Take the dotted lines as $X$, $Y$ axes.

$x$-component of $\overrightarrow{OA} = 4$ m, $x$-component of

$$\overrightarrow{OB} = 6 \text{ m} \cos\theta.$$

$x$-component of the resultant $= (4 + 6\cos\theta)$ m.

But it is given that the resultant is along $Y$-axis. Thus, the $x$-component of the resultant $= 0$

$4 + 6\cos\theta = 0$ or, $\cos\theta = -2/3$.

**7.** *Write the unit vector in the direction of $\vec{A} = 5\vec{i} + \vec{j} - 2\vec{k}$.*

***Solution*** : $|\vec{A}| = \sqrt{5^2 + 1^2 + (-2)^2} = \sqrt{30}$.

The required unit vector is $\frac{\vec{A}}{|\vec{A}|}$

$$= \frac{5}{\sqrt{30}}\vec{i} + \frac{1}{\sqrt{30}}\vec{j} - \frac{2}{\sqrt{30}}\vec{k}.$$

**8.** *If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ show that $\vec{a} \perp \vec{b}$.*

***Solution*** : We have $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$

$$= \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

$$= a^2 + b^2 + 2\,\vec{a}\cdot\vec{b}.$$

Similarly,

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$= a^2 + b^2 - 2\,\vec{a}\cdot\vec{b}.$$

If $\quad |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$,

$$a^2 + b^2 + 2\,\vec{a}\cdot\vec{b} = a^2 + b^2 - 2\,\vec{a}\cdot\vec{b}$$

or, $\quad \vec{a}\cdot\vec{b} = 0$

or, $\quad \vec{a} \perp \vec{b}$.

**9.** *If $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find the angle between $\vec{a}$ and $\vec{b}$.*

***Solution*** : We have $\quad \vec{a}\cdot\vec{b} = ab\cos\theta$

or, $\quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}$

where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$.

Now $\quad \vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

$$= 2\times 4 + 3\times 3 + 4\times 2 = 25.$$

Also $\quad a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$= \sqrt{4 + 9 + 16} = \sqrt{29}$$

and $\quad b = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$.

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Thus, $\cos\theta = \dfrac{25}{29}$

or, $\theta = \cos^{-1}\left(\dfrac{25}{29}\right).$

**10.** *If* $\vec{A} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, $\vec{B} = \vec{i} + 2\vec{k}$ *and* $\vec{C} = \vec{j} - \vec{k}$ *find* $\vec{A}\cdot(\vec{B}\times\vec{C})$.

***Solution :*** $\vec{B}\times\vec{C} = (\vec{i} + 2\vec{k})\times(\vec{j} - \vec{k})$

$$= \vec{i}\times(\vec{j} - \vec{k}) + 2\vec{k}\times(\vec{j} - \vec{k})$$
$$= \vec{i}\times\vec{j} - \vec{i}\times\vec{k} + 2\vec{k}\times\vec{j} - 2\vec{k}\times\vec{k}$$
$$= \vec{k} + \vec{j} - 2\vec{i} - 0 = -2\vec{i} + \vec{j} + \vec{k}.$$
$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (2\vec{i} - 3\vec{j} + 7\vec{k})\cdot(-2\vec{i} + \vec{j} + \vec{k})$$
$$= (2)(-2) + (-3)(1) + (7)(1)$$
$$= 0.$$

**11.** *The volume of a sphere is given by*

$$V = \frac{4}{3}\pi R^3$$

*where R is the radius of the sphere. (a) Find the rate of change of volume with respect to R. (b) Find the change in volume of the sphere as the radius is increased from* 20·0 cm *to* 20·1 cm. *Assume that the rate does not appreciably change between R* = 20·0 cm *to R* = 20·1 cm.

***Solution :*** (a) $V = \dfrac{4}{3}\pi R^3$

or, $\dfrac{dV}{dR} = \dfrac{4}{3}\pi\dfrac{d}{dR}(R)^3 = \dfrac{4}{3}\pi\cdot 3R^2 = 4\pi R^2.$

(b) At $R$ = 20 cm, the rate of change of volume with the radius is

$$\frac{dV}{dR} = 4\pi R^2 = 4\pi(400\text{ cm}^2)$$
$$= 1600\pi\text{ cm}^2.$$

The change in volume as the radius changes from 20·0 cm to 20·1 cm is

$$\Delta V = \frac{dV}{dR}\Delta R$$
$$= (1600\pi\text{ cm}^2)(0{\cdot}1\text{ cm})$$
$$= 160\pi\text{ cm}^3.$$

**12.** *Find the derivative of the following functions with respect to x. (a)* $y = x^2\sin x$, *(b)* $y = \dfrac{\sin x}{x}$ *and (c)* $y = \sin(x^2)$.

***Solution :***

(a) $y = x^2\sin x$

$$\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(x^2)$$
$$= x^2\cos x + (\sin x)(2x)$$
$$= x(2\sin x + x\cos x).$$

(b) $y = \dfrac{\sin x}{x}$

$$\frac{dy}{dx} = \frac{x\dfrac{d}{dx}(\sin x) - \sin x\left(\dfrac{dx}{dx}\right)}{x^2}$$
$$= \frac{x\cos x - \sin x}{x^2}.$$

(c) $\dfrac{dy}{dx} = \dfrac{d}{dx^2}(\sin x^2)\cdot\dfrac{d(x^2)}{dx}$

$$= \cos x^2(2x)$$
$$= 2x\cos x^2.$$

**13.** *Find the maximum or minimum values of the function* $y = x + \dfrac{1}{x}$ *for* $x > 0$.

***Solution :*** $y = x + \dfrac{1}{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1})$$
$$= 1 + (-x^{-2})$$
$$= 1 - \frac{1}{x^2}.$$

For $y$ to be maximum or minimum,

$$\frac{dy}{dx} = 0$$

or, $1 - \dfrac{1}{x^2} = 0$

Thus, $x = 1$ or $-1$.

For $x > 0$ the only possible maximum or minimum is at $x = 1$. At $x = 1$, $y = x + \dfrac{1}{x} = 2$.

Near $x = 0$, $y = x + \dfrac{1}{x}$ is very large because of the term $\dfrac{1}{x}$. For very large $x$, again $y$ is very large because of the term $x$. Thus $x = 1$ must correspond to a minimum. Thus, $y$ has only a minimum for $x > 0$. This minimum occurs at $x = 1$ and the minimum value of $y$ is $y = 2$.

**14.** *Figure (2-W6) shows the curve* $y = x^2$. *Find the area of the shaded part between x* = 0 *and x* = 6.

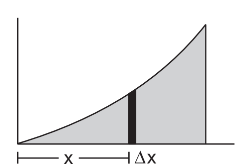


Figure 2-W6

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***Solution :*** The area can be divided into strips by drawing ordinates between $x = 0$ and $x = 6$ at a regular interval of $dx$. Consider the strip between the ordinates at $x$ and $x + dx$. The height of this strip is $y = x^2$. The area of this strip is $dA = y\,dx = x^2 dx$.

The total area of the shaded part is obtained by summing up these strip-areas with $x$ varying from 0 to 6. Thus

$$A = \int_0^6 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^6 = \frac{216 - 0}{3} = 72.$$

**15.** *Evaluate* $\int_0^t A \sin \omega t\,dt$ *where A and* $\omega$ *are constants.*

***Solution :*** $\int_0^t A \sin \omega t\,dt$

$$= A\left[\frac{-\cos \omega t}{\omega}\right]_0^t = \frac{A}{\omega}(1 - \cos \omega t).$$

**16.** *The velocity v and displacement x of a particle executing simple harmonic motion are related as*

$$v\frac{dv}{dx} = -\omega^2 x.$$

*At* $x = 0$, $v = v_0$. *Find the velocity v when the displacement becomes x.*

***Solution :*** We have

$$v\frac{dv}{dx} = -\omega^2 x$$

or, $$v\,dv = -\omega^2 x\,dx$$

or, $$\int_{v_0}^{v} v\,dv = \int_0^x -\omega^2 x\,dx \quad \ldots \text{(i)}$$

When summation is made on $-\omega^2 x\,dx$ the quantity to be varied is $x$. When summation is made on $v\,dv$ the quantity to be varied is $v$. As $x$ varies from 0 to $x$ the velocity varies from $v_0$ to $v$. Therefore, on the left the limits of integration are from $v_0$ to $v$ and on the right they are from 0 to $x$. Simiplifying (i),

$$\left[\frac{1}{2}v^2\right]_{v_0}^{v} = -\omega^2\left[\frac{x^2}{2}\right]_0^x$$

or, $$\frac{1}{2}(v^2 - v_0^2) = -\omega^2\frac{x^2}{2}$$

or, $$v^2 = v_0^2 - \omega^2 x^2$$

or, $$v = \sqrt{v_0^2 - \omega^2 x^2}.$$

**17.** *The charge flown through a circuit in the time interval between t and t + dt is given by* $dq = e^{-t/\tau}dt$, *where* $\tau$ *is a constant. Find the total charge flown through the circuit between* $t = 0$ *to* $t = \tau$.

***Solution :*** The total charge flown is the sum of all the $dq$'s for $t$ varying from $t = 0$ to $t = \tau$. Thus, the total charge flown is

$$Q = \int_0^\tau e^{-t/\tau}\,dt$$

$$= \left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^\tau = \tau\left(1 - \frac{1}{e}\right).$$

**18.** *Evaluate* $(21{\cdot}6002 + 234 + 2732{\cdot}10) \times 13$.

***Solution :***

| | | |
|---|---|---|
| 21.6002 | | 22 |
| 234 | ⇒ | 234 |
| 2732·10 | | 2732 |
| | | 2988 |

The three numbers are arranged with their decimal points aligned (shown on the left part above). The column just left to the decimals has 4 as the doubtful digit. Thus, all the numbers are rounded to this column. The rounded numbers are shown on the right part above. The required expression is $2988 \times 13 = 38844$. As 13 has only two significant digits the product should be rounded off after two significant digits. Thus the result is 39000.

□

## QUESTIONS FOR SHORT ANSWER

1. Is a vector necessarily changed if it is rotated through an angle ?
2. Is it possible to add two vectors of unequal magnitudes and get zero ? Is it possible to add three vectors of equal magnitudes and get zero ?
3. Does the phrase "direction of zero vector" have physical significance ? Discuss in terms of velocity, force etc.
4. Can you add three unit vectors to get a unit vector ? Does your answer change if two unit vectors are along the coordinate axes ?

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5. Can we have physical quantities having magnitude and direction which are not vectors ?

6. Which of the following two statements is more appropriate ?

(a) Two forces are added using triangle rule because force is a vector quantity.

(b) Force is a vector quantity because two forces are added using triangle rule.

7. Can you add two vectors representing physical quantities having different dimensions ? Can you multiply two vectors representing physical quantities having different dimensions ?

8. Can a vector have zero component along a line and still have nonzero magnitude ?

9. Let $\varepsilon_1$ and $\varepsilon_2$ be the angles made by $\vec{A}$ and $-\vec{A}$ with the positive $X$-axis. Show that $\tan\varepsilon_1 = \tan\varepsilon_2$. Thus, giving $\tan\varepsilon$ does not uniquely determine the direction of $\vec{A}$.

10. Is the vector sum of the unit vectors $\vec{i}$ and $\vec{j}$ a unit vector ? If no, can you multiply this sum by a scalar number to get a unit vector ?

11. Let $\vec{A} = 3\vec{i} + 4\vec{j}$. Write four vector $\vec{B}$ such that $\vec{A} \neq \vec{B}$ but $A = B$.

12. Can you have $\vec{A} \times \vec{B} = \vec{A} \cdot \vec{B}$ with $A \neq 0$ and $B \neq 0$ ? What if one of the two vectors is zero ?

13. If $\vec{A} \times \vec{B} = 0$, can you say that (a) $\vec{A} = \vec{B}$, (b) $\vec{A} \neq \vec{B}$ ?

14. Let $\vec{A} = 5\vec{i} - 4\vec{j}$ and $\vec{B} = -7{\cdot}5\vec{i} + 6\vec{j}$. Do we have $\vec{B} = k\vec{A}$ ? Can we say $\dfrac{\vec{B}}{\vec{A}} = k$ ?

## OBJECTIVE I

1. A vector is not changed if
(a) it is rotated through an arbitrary angle
(b) it is multiplied by an arbitrary scalar
(c) it is cross multiplied by a unit vector
(d) it is slid parallel to itself.

2. Which of the sets given below may represent the magnitudes of three vectors adding to zero ?
(a) 2, 4, 8 (b) 4, 8, 16 (c) 1, 2, 1 (d) 0·5, 1, 2.

3. The resultant of $\vec{A}$ and $\vec{B}$ makes an angle $\alpha$ with $\vec{A}$ and $\beta$ with $\vec{B}$,
(a) $\alpha < \beta$ (b) $\alpha < \beta$ if $A < B$
(c) $\alpha < \beta$ if $A > B$ (d) $\alpha < \beta$ if $A = B$.

4. The component of a vector is
(a) always less than its magnitude
(b) always greater than its magnitude
(c) always equal to its magnitude
(d) none of these.

5. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is
(a) along west (b) along east
(c) zero (d) vertically downward.

6. The radius of a circle is stated as 2·12 cm. Its area should be written as
(a) 14 $cm^2$ (b) 14·1 $cm^2$ (c) 14·11 $cm^2$ (d) 14·1124 $cm^2$.

## OBJECTIVE II

1. A situation may be described by using different sets of coordinate axes having different orientations. Which of the following do not depend on the orientation of the axes ?
(a) the value of a scalar (b) component of a vector
(c) a vector (d) the magnitude of a vector.

2. Let $\vec{C} = \vec{A} + \vec{B}$.
(a) $|\vec{C}|$ is always greater than $|\vec{A}|$
(b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
(c) $C$ is always equal to $A + B$
(d) $C$ is never equal to $A + B$.

3. Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and its resultant be $\vec{C}$.
(a) $C$ must be equal to $|A - B|$
(b) $C$ must be less than $|A - B|$
(c) $C$ must be greater than $|A - B|$
(d) $C$ may be equal to $|A - B|$.

4. The $x$-component of the resultant of several vectors
(a) is equal to the sum of the $x$-components of the vectors
(b) may be smaller than the sum of the magnitudes of the vectors
(c) may be greater than the sum of the magnitudes of the vectors
(d) may be equal to the sum of the magnitudes of the vectors.

5. The magnitude of the vector product of two vectors $|\vec{A}|$ and $|\vec{B}|$ may be
(a) greater than $AB$ (b) equal to $AB$
(c) less than $AB$ (d) equal to zero.

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## EXERCISES

**1.** A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the $X$-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

**2.** Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 unit each. If they are inclined to the $X$-axis at angles 30° and 60° respectively, find the resultant.

**3.** Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the $X$-axis at angles 45°, 135° and 315° respectively.

**4.** Let $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{i} + 4\vec{j}$. (a) Find the magnitudes of (a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ and (d) $\vec{a} - \vec{b}$.

**5.** Refer to figure (2-E1). Find (a) the magnitude, (b) $x$ and $y$ components and (c) the angle with the $X$-axis of the resultant of $\vec{OA}$, $\vec{BC}$ and $\vec{DE}$.

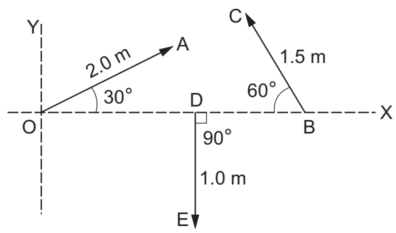


Figure 2-E1

**6.** Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

**7.** A spy report about a suspected car reads as follows. "The car moved 2·00 km towards east, made a perpendicular left turn, ran for 500 m, made a perpendicular right turn, ran for 4·00 km and stopped". Find the displacement of the car.

**8.** A carrom board (4 ft × 4 ft square) has the queen at the centre. The queen, hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. Find the magnitude of displacement of the queen (a) from the centre to the front edge, (b) from the front edge to the hole and (c) from the centre to the hole.

**9.** A mosquito net over a 7 ft × 4 ft bed is 3 ft high. The net has a hole at one corner of the bed through which a mosquito enters the net. It flies and sits at the diagonally opposite upper corner of the net. (a) Find the magnitude of the displacement of the mosquito. (b) Taking the hole as the origin, the length of the bed as the $X$-axis, its width as the $Y$-axis, and vertically up as the $Z$-axis, write the components of the displacement vector.

**10.** Suppose $\vec{a}$ is a vector of magnitude 4·5 unit due north. What is the vector (a) $3\vec{a}$, (b) $-4\vec{a}$?

**11.** Two vectors have magnitudes 2 m and 3 m. The angle between them is 60°. Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product.

**12.** Let $A_1 A_2 A_3 A_4 A_5 A_6 A_1$ be a regular hexagon. Write the $x$-components of the vectors represented by the six sides taken in order. Use the fact that the resultant of these six vectors is zero, to prove that
$\cos 0 + \cos\pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$.
Use the known cosine values to verify the result.

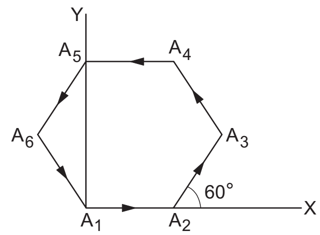


Figure 2-E2

**13.** Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$. Find the angle between them.

**14.** Prove that $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$.

**15.** If $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find $\vec{A} \times \vec{B}$.

**16.** If $\vec{A}$, $\vec{B}$, $\vec{C}$ are mutually perpendicular, show that $\vec{C} \times (\vec{A} \times \vec{B}) = 0$. Is the converse true ?

**17.** A particle moves on a given straight line with a constant speed $v$. At a certain time it is at a point $P$ on its straight line path. $O$ is a fixed point. Show that $\vec{OP} \times \vec{v}$ is independent of the position $P$.

**18.** The force on a charged particle due to electric and magnetic fields is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Suppose $\vec{E}$ is along the $X$-axis and $\vec{B}$ along the $Y$-axis. In what direction and with what minimum speed $v$ should a positively charged particle be sent so that the net force on it is zero ?

**19.** Give an example for which $\vec{A} \cdot \vec{B} = \vec{C} \cdot \vec{B}$ but $\vec{A} \neq \vec{C}$.

**20.** Draw a graph from the following data. Draw tangents at $x$ = 2, 4, 6 and 8. Find the slopes of these tangents. Verify that the curve drawn is $y = 2x^2$ and the slope of tangent is $\tan\theta = \frac{dy}{dx} = 4x$.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $y$ | 2 | 8 | 18 | 32 | 50 | 72 | 98 | 128 | 162 | 200 |

**21.** A curve is represented by $y = \sin x$. If $x$ is changed from $\frac{\pi}{3}$ to $\frac{\pi}{3} + \frac{\pi}{100}$, find approximately the change in $y$.

**22.** The electric current in a charging $R$–$C$ circuit is given by $i = i_0 e^{-t/RC}$ where $i_0$, $R$ and $C$ are constant parameters of the circuit and $t$ is time. Find the rate of change of current at (a) $t$ = 0, (b) $t = RC$, (c) $t = 10\,RC$.

**23.** The electric current in a discharging R–C circuit is given by $i = i_0 e^{-t/RC}$ where $i_0$, $R$ and $C$ are constant parameters and $t$ is time. Let $i_0 = 2{\cdot}00$ A, $R = 6{\cdot}00 \times 10^5\ \Omega$

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and $C = 0{\cdot}500\ \mu F$. (a) Find the current at $t = 0{\cdot}3$ s. (b) Find the rate of change of current at $t = 0{\cdot}3$ s. (c) Find approximately the current at $t = 0{\cdot}31$ s.

**24.** Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the $X$-axis with the ordinates at $x = 5$ and $x = 10$.

**25.** Find the area enclosed by the curve $y = \sin x$ and the $X$-axis between $x = 0$ and $x = \pi$.

**26.** Find the area bounded by the curve $y = e^{-x}$, the $X$-axis and the $Y$-axis.

**27.** A rod of length $L$ is placed along the $X$-axis between $x = 0$ and $x = L$. The linear density (mass/length) $\rho$ of the rod varies with the distance $x$ from the origin as $\rho = a + bx$. (a) Find the SI units of $a$ and $b$. (b) Find the mass of the rod in terms of $a$, $b$ and $L$.

**28.** The momentum $p$ of a particle changes with time $t$ according to the relation $\frac{dp}{dt} = (10\ \text{N}) + (2\ \text{N/s})t$. If the momentum is zero at $t = 0$, what will the momentum be at $t = 10$ s?

**29.** The changes in a function $y$ and the independent variable $x$ are related as $\frac{dy}{dx} = x^2$. Find $y$ as a function of $x$.

**30.** Write the number of significant digits in (a) 1001, (b) 100·1, (c) 100·10, (d) 0·001001.

**31.** A metre scale is graduated at every millimetre. How many significant digits will be there in a length measurement with this scale?

**32.** Round the following numbers to 2 significant digits. (a) 3472, (b) 84·16, (c) 2·55 and (d) 28·5.

**33.** The length and the radius of a cylinder measured with a slide callipers are found to be 4·54 cm and 1·75 cm respectively. Calculate the volume of the cylinder.

**34.** The thickness of a glass plate is measured to be 2·17 mm, 2·17 mm and 2·18 mm at three different places. Find the average thickness of the plate from this data.

**35.** The length of the string of a simple pendulum is measured with a metre scale to be 90·0 cm. The radius of the bob plus the length of the hook is calculated to be 2·13 cm using measurements with a slide callipers. What is the effective length of the pendulum? (The effective length is defined as the distance between the point of suspension and the centre of the bob.)

□

## ANSWERS

### OBJECTIVE I

1. (d) 2. (c) 3. (c) 4. (d) 5. (a) 6. (b)

### OBJECTIVE II

1. (a), (c), (d) 2. (b) 3. (c) 4. (a), (b), (d)
5. (b), (c), (d)

### EXERCISES

1. 5 m at 73° with $X$-axis
2. 20 cos15° unit at 45° with $X$-axis
3. 100 unit at 45° with $X$-axis
4. (a) 5 (b) 5 (c) $7\sqrt{2}$ (d) $\sqrt{2}$
5. (a) 1·6 m (b) 0·98 m and 1·3 m respectively (c) $\tan^{-1}(1{\cdot}32)$
6. (a) 180° (b) 90° (c) 0
7. 6·02 km, $\tan^{-1}\frac{1}{12}$.
8. (a) $\frac{2}{3}\sqrt{10}$ ft (b) $\frac{4}{3}\sqrt{10}$ ft (c) $2\sqrt{2}$ ft
9. (a) $\sqrt{74}$ ft (b) 7 ft, 4 ft, 3 ft
10. (a) 13·5 unit due north (b) 18 unit due south
11. (a) 3 m$^2$ (b) $3\sqrt{3}$ m$^2$
13. $\cos^{-1}(38/\sqrt{1450})$
15. $-6\vec{i} + 12\vec{j} - 6\vec{k}$
16. no
18. along $Z$-axis with speed $E/B$
21. 0·0157
22. (a) $\frac{-i_0}{RC}$ (b) $\frac{-i_0}{RCe}$ (c) $\frac{-i_0}{RCe^{10}}$
23. (a) $\frac{2{\cdot}00}{e}$ A (b) $\frac{-20}{3e}$ A/s (c) $\frac{5{\cdot}8}{3e}$ A
24. 1135
25. 2
26. 1
27. (a) kg/m, kg/m$^2$ (b) $aL + bL^2/2$
28. 200 kg–m/s
29. $y = \frac{x^3}{3} + C$
30. (a) 4 (b) 4 (c) 5 (d) 4
31. 1, 2, 3 or 4
32. (a) 3500 (b) 84 (c) 2·6 (d) 28
33. 43·7 cm$^3$
34. 2·17 mm
35. 92·1 cm

□

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CHAPTER 3

# REST AND MOTION : KINEMATICS

## 3.1 REST AND MOTION

When do we say that a body is at rest and when do we say that it is in motion ? You may say that if a body does not change its position as time passes it is at rest. If a body changes its position with time, it is said to be moving. But when do we say that it is not changing its position ? A book placed on the table remains on the table and we say that the book is at rest. However, if we station ourselves on the moon (the Appollo missions have made it possible), the whole earth is found to be changing its position and so the room, the table and the book are all continuously changing their positions. The book is at rest if it is viewed from the room, it is moving if it is viewed from the moon.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. Nothing is in absolute rest or in absolute motion. The moon is moving with respect to the book and the book moves with respect to the moon. Take another example. A robber enters a train moving at great speed with respect to the ground, brings out his pistol and says "Don't move, stand still". The passengers stand still. The passengers are at rest with respect to the robber but are moving with respect to the rail track.

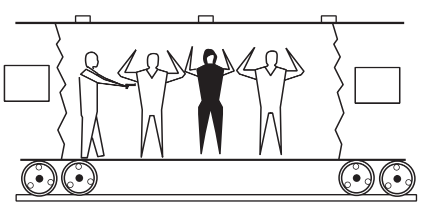

Figure 3.1

To locate the position of a particle we need a *frame of reference.* A convenient way to fix up the frame of reference is to choose three mutually perpendicular axes and name them *X-Y-Z* axes. The coordinates, $(x, y, z)$ of the particle then specify the position of the particle with respect to that frame. Add a clock into the frame of reference to measure the time. If all the three coordinates $x$, $y$ and $z$ of the particle remain unchanged as time passes, we say that the particle is at rest with respect to this frame. If any one or more coordinates change with time, we say that the body is moving with respect to this frame.

There is no rule or restriction on the choice of a frame. We can choose a frame of reference according to our convenience to describe the situation under study. Thus, when we are in a train it is convenient to choose a frame attached to our compartment. The coordinates of a suitcase placed on the upper berth do not change with time (unless the train gives a jerk) and we say that the suitcase is at rest in the train-frame. The different stations, electric poles, trees etc. change their coordinates and we say that they are moving in the train-frame. Thus, we say that "Bombay is coming" and "Pune has already passed".

In the following sections we shall assume that the frame of reference is already chosen and we are describing the motion of the objects in the chosen frame. Sometimes the choice of the frame is clear from the context and we do not mention it. Thus, when one says the car is travelling and the rickshaw is not, it is clear that all positions are measured from a frame attached to the road.

## 3.2 DISTANCE AND DISPLACEMENT

Suppose a particle is at $A$ at time $t_1$ and at $B$ at time $t_2$ with respect to a given frame (figure 3.2).

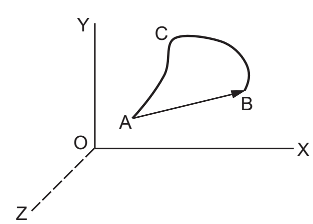


Figure 3.2

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During the time interval $t_1$ to $t_2$ the particle moves along the path $ACB$. The length of the path $ACB$ is called the *distance* travelled during the time interval $t_1$ to $t_2$. If we connect the initial position $A$ with the final position $B$ by a straight line, we get the *displacement* of the particle. The magnitude of the displacement is the length of the straight line joining the initial and the final position. The direction is from the initial to the final position. The displacement has both the magnitude as well as the direction. Further the displacements add according to the triangle rule of vector addition. Suppose a particle kept on a table is displaced on the table and at the same time the table is also displaced in the room. The net displacement of the particle in the room is obtained by the vector sum of the two displacements. Thus, displacement is a vector quantity. In contrast the distance covered has only a magnitude and is thus, a scalar quantity.

***Example 3.1***

*An old person moves on a semi-circular track of radius* 40·0 m *during a morning walk. If he starts at one end of the track and reaches at the other end, find the distance covered and the displacement of the person.*

**Solution** : The distance covered by the person equals the length of the track. It is equal to $\pi R = \pi \times 40{\cdot}0$ m $= 126$ m.

The displacement is equal to the diameter of the semi-circular track joining the two ends. It is $2R = 2 \times 40{\cdot}0 \text{ m} = 80$ m. The direction of this displacement is from the initial point to the final point.

## 3.3 AVERAGE SPEED AND INSTANTANEOUS SPEED

The average speed of a particle in a time interval is defined as the distance travelled by the particle divided by the time interval. If the particle travels a distance $s$ in time $t_1$ to $t_2$, the *average speed* is defined as

$$v_{av} = \frac{s}{t_2 - t_1} \cdot \qquad \dots (3.1)$$

The average speed gives the overall "rapidity" with which the particle moves in this interval. In a one-day cricket match, the average run rate is quoted as the total runs divided by the total number of overs used to make these runs. Some of the overs may be expensive and some may be economical. Similarly, the average speed gives the total effect in the given interval. The rapidity or slowness may vary from instant to instant. When an athelete starts running, he or she runs slowly and gradually

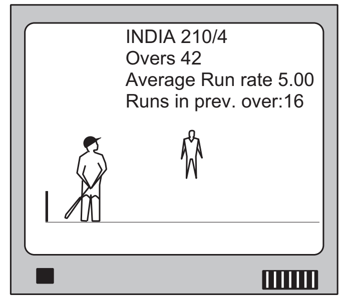


Figure 3.3

increases the rate. We define the *instantaneous speed* at a time $t$ as follows.

Let $\Delta s$ be the distance travelled in the time interval $t$ to $t + \Delta t$. The average speed in this time interval is

$$v_{av} = \frac{\Delta s}{\Delta t} \cdot$$

Now make $\Delta t$ vanishingly small and look for the value of $\frac{\Delta s}{\Delta t} \cdot$ Remember $\Delta s$ is the distance travelled in the chosen time interval $\Delta t$. As $\Delta t$ approaches 0, the distance $\Delta s$ also approaches zero but the ratio $\frac{\Delta s}{\Delta t}$ has a finite limit.

The instantaneous speed at a time $t$ is defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \dots (3.2)$$

where $s$ is the distance travelled in time $t$. The average speed is defined for a time interval and the instantaneous speed is defined at a particular instant. Instantaneous speed is also called "speed".

***Example 3.2***

*The distance travelled by a particle in time $t$ is given by $s = (2{\cdot}5 \text{ m/s}^2)\, t^2$. Find (a) the average speed of the particle during the time 0 to 5·0 s, and (b) the instantaneous speed at $t$ = 5·0 s.*

**Solution :** (a) The distance travelled during time 0 to 5·0 s is

$$s = (2{\cdot}5 \text{ m/s}^2)\,(5{\cdot}0 \text{ s})^2 = 62{\cdot}5 \text{ m}.$$

The average speed during this time is

$$v_{av} = \frac{62{\cdot}5 \text{ m}}{5 \text{ s}} = 12{\cdot}5 \text{ m/s}.$$

(b) $$s = (2{\cdot}5 \text{ m/s}^2)\, t^2$$

or, $$\frac{ds}{dt} = (2{\cdot}5 \text{ m/s}^2)\,(2\,t) = (5{\cdot}0 \text{ m/s}^2)\, t.$$

At $t$ = 5·0 s the speed is

$$v = \frac{ds}{dt} = (5{\cdot}0 \text{ m/s}^2)\,(5{\cdot}0 \text{ s}) = 25 \text{ m/s}.$$

If we plot the distance $s$ as a function of time (figure 3.4), the speed at a time $t$ equals the slope of

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the tangent to the curve at the time $t$. The average speed in a time interval $t$ to $t + \Delta t$ equals the slope of the chord $AB$ where $A$ and $B$ are the points on the

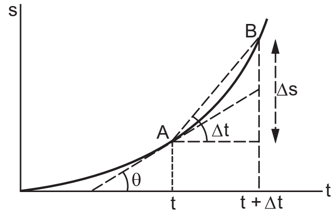


Figure 3.4

curve corresponding to the time $t$ and $t + \Delta t$. As $\Delta t$ approaches zero, the chord $AB$ becomes the tangent at $A$ and the average speed $\frac{\Delta s}{\Delta t}$ becomes the slope of the tangent which is $\frac{ds}{dt}$.

If the speed of the particle at time $t$ is $v$, the distance $ds$ travelled by it in the short time interval $t$ to $t + dt$ is $v\,dt$. Thus, $ds = v dt$. The total distance travelled by the particle in a finite time interval $t_1$ to $t_2$ can be obtained by summing over these small distances $ds$ as time changes from $t_1$ to $t_2$. Thus, the distance travelled by a particle in the time interval

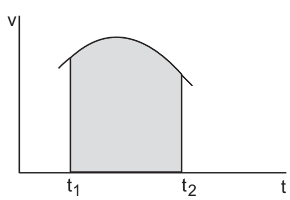


Figure 3.5

$t_1$ to $t_2$ is

$$s = \int_{t_1}^{t_2} v\, dt. \qquad \ldots \quad (3.3)$$

If we plot a graph of the speed $v$ versus time $t$, the distance travelled by the particle can be obtained by finding the area under the curve. Figure (3.5) shows such a speed-time graph. To find the distance travelled in the time interval $t_1$ to $t_2$ we draw ordinates from $t = t_1$ and $t = t_2$. The area bounded by the curve $v - t$, the $X$-axis and the two ordinates at $t = t_1$ and $t = t_2$ (shown shaded in the figure) gives the total distance covered.

The dimension of speed is $LT^{-1}$ and its SI unit is metre/second abbreviated as m/s.

***Example 3.3***

*Figure (3.6) shows the speed versus time graph for a particle. Find the distance travelled by the particle during the time $t$ = 0 to $t$ = 3 s.*

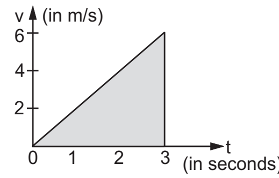


Figure 3.6

***Solution*** **:** The distance travelled by the particle in the time 0 to 3 s is equal to the area shaded in the figure. This is a right angled triangle with height = 6 m/s and the base = 3 s. The area is $\frac{1}{2}$ (base) (height) = $\frac{1}{2} \times$ (3 s) (6 m/s) = 9 m. Thus, the particle covered a distance of 9 m during the time 0 to 3 s.

## 3.4 AVERAGE VELOCITY AND INSTANTANEOUS VELOCITY

The *average velocity* of a particle in a time interval $t_1$ to $t_2$ is defined as its displacement divided by the time interval. If the particle is at a point $A$ (figure 3.7) at time $t = t_1$ and at $B$ at time $t = t_2$, the displacement in this time interval is the vector $\overrightarrow{AB}$. The average velocity in this time interval is then,

$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1}.$$

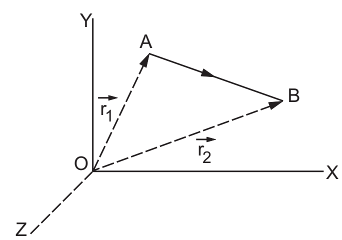


Figure 3.7

Like displacement, it is a vector quantity.

**Position vector :** If we join the origin to the position of the particle by a straight line and put an arrow towards the position of the particle, we get the *position vector* of the particle. Thus, the position vector of the particle shown in figure (3.7) at time $t = t_1$ is $\overrightarrow{OA}$ and that at $t = t_2$ is $\overrightarrow{OB}$. The displacement of the particle in the time interval $t_1$ to $t_2$ is

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1.$$

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The average velocity of a particle in the time interval $t_1$ to $t_2$ can be written as

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \cdot \qquad \dots (3.4)$$

Note that only the positions of the particle at time $t = t_1$ and $t = t_2$ are used in calculating the average velocity. The positions in between $t_1$ and $t_2$ are not needed, hence the actual path taken in going from $A$ to $B$ is not important in calculating the average velocity.

***Example 3.4***

*A table clock has its minute hand* 4·0 cm *long. Find the average velocity of the tip of the minute hand (a) between* 6·00 a.m. *to* 6·30 a.m. *and (b) between* 6·00 a.m. *to* 6·30 p.m.

***Solution :*** At 6·00 a.m. the tip of the minute hand is at 12 mark and at 6·30 a.m. or 6·30 p.m. it is 180° away. Thus, the straight line distance between the initial and final position of the tip is equal to the diameter of the clock.

Displacement = 2 $R$ = 2 × 4·0 cm = 8·0 cm.

The displacement is from the 12 mark to the 6 mark on the clock panel. This is also the direction of the average velocity in both cases.

(a) The time taken from 6·00 a.m. to 6·30 a.m. is 30 minutes = 1800 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8\cdot 0 \text{ cm}}{1800 \text{ s}} = 4\cdot 4 \times 10^{-3} \text{ cm/s.}$$

(b) The time taken from 6·00 a.m. to 6·30 p.m. is 12 hours and 30 minutes = 45000 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8\cdot 0 \text{ cm}}{45000 \text{ s}} = 1\cdot 8 \times 10^{-4} \text{ cm/s.}$$

The *instantaneous velocity* of a particle at a time $t$ is defined as follows. Let the average velocity of the particle in a short time interval $t$ to $t + \Delta t$ be $\vec{v}_{av}$. This average velocity can be written as

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

where $\Delta \vec{r}$ is the displacement in the time interval $\Delta t$. We now make $\Delta t$ vanishingly small and find the limiting value of $\frac{\Delta \vec{r}}{\Delta t}$. This value is instantaneous velocity $\vec{v}$ of the particle at time $t$.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \cdot \qquad \dots (3.5)$$

For very small intervals the displacement $\Delta \vec{r}$ is along the line of motion of the particle. Thus, the length $\Delta r$ equals the distance $\Delta s$ travelled in that interval. So the magnitude of the velocity is

$$v = \left| \frac{d\vec{r}}{dt} \right| = \frac{|d\vec{r}|}{dt} = \frac{ds}{dt} \qquad \dots (3.6)$$

which is the instantaneous speed at time $t$. Instantaneous velocity is also called the "velocity".

## 3.5 AVERAGE ACCELERATION AND INSTANTANEOUS ACCELERATION

If the velocity of a particle remains constant as time passes, we say that it is moving with uniform velocity. If the velocity changes with time, it is said to be accelerated. The acceleration is the rate of change of velocity. Velocity is a vector quantity hence a change in its magnitude or direction or both will change the velocity.

Suppose the velocity of a particle at time $t_1$ is $\vec{v}_1$ and at time $t_2$ it is $\vec{v}_2$. The change produced in time interval $t_1$ to $t_2$ is $\vec{v}_2 - \vec{v}_1$. We define the *average acceleration* $\vec{a}_{av}$ as the change in velocity divided by the time interval. Thus,

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \cdot \qquad \dots (3.7)$$

Again the average acceleration depends only on the velocities at time $t_1$ and $t_2$. How the velocity changed in between $t_1$ and $t_2$ is not important in defining the average acceleration.

*Instantaneous acceleration* of a particle at time $t$ is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \dots (3.8)$$

where $\Delta \vec{v}$ is the change in velocity between the time $t$ and $t + \Delta t$. At time $t$ the velocity is $\vec{v}$ and at time $t + \Delta t$ it becomes $\vec{v} + \Delta \vec{v}$. $\frac{\Delta \vec{v}}{\Delta t}$ is the average acceleration of the particle in the interval $\Delta t$. As $\Delta t$ approaches zero, this average acceleration becomes the instantaneous acceleration. Instantaneous acceleration is also called *"acceleration"*.

The dimension of acceleration is $LT^{-2}$ and its SI unit is metre/second$^2$ abbreviated as m/s$^2$.

## 3.6 MOTION IN A STRAIGHT LINE

When a particle is constrained to move on a straight line, the description becomes fairly simple. We choose the line as the $X$-axis and a suitable time instant as $t = 0$. Generally the origin is taken at the point where the particle is situated at $t = 0$. The position of the particle at time $t$ is given by its coordinate $x$ at that time. The velocity is

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$$v = \frac{dx}{dt} \qquad \dots (3.9)$$

and the acceleration is $a = \dfrac{dv}{dt}$ ... (3.10)

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \cdot \qquad \dots (3.11)$$

If $\frac{dx}{dt}$ is positive, the direction of the velocity is along the positive $X$-axis and if $\frac{dx}{dt}$ is negative, the direction, is along the negative $X$-axis. Similarly if $\frac{dv}{dt}$ is positive, the acceleration is along the positive $X$-axis and if $\frac{dv}{dt}$ is negative, the acceleration is along the negative $X$-axis. The magnitude of $v$ is speed. If the velocity and the acceleration are both positive, the speed increases. If both of them are negative then also the speed increases but if they have opposite signs, the speed decreases. When the speed decreases, we say that the particle is decelerating. Deceleration is equivalent to negative acceleration. An acceleration of 2·0 m/s$^2$ towards east is same as a deceleration of 2·0 m/s$^2$ towards west.

**Motion with Constant Acceleration**

Suppose the acceleration of a particle is $a$ and remains constant. Let the velocity at time 0 be $u$ and the velocity at time $t$ be $v$. Thus,

$$\frac{dv}{dt} = a, \quad \text{or,} \quad dv = a\,dt$$

or, $$\int_u^v dv = \int_0^t a\,dt.$$

As time changes from 0 to $t$ the velocity changes from $u$ to $v$. So on the left hand side the summation is made over $v$ from $u$ to $v$ whereas on the right hand side the summation is made on time from 0 to $t$. Evaluating the integrals we get,

$$[v]_u^v = a[t]_0^t$$

or, $$v - u = at$$

or, $$v = u + at. \qquad \dots (3.12)$$

Equation (3.12) may be written as

$$\frac{dx}{dt} = u + at$$

or, $$dx = (u + at)dt$$

or, $$\int_0^x dx = \int_0^t (u + at)dt.$$

At $t = 0$ the particle is at $x = 0$. As time changes from 0 to $t$ the position changes from 0 to $x$. So on the left hand side the summation is made on position from 0 to $x$ whereas on the right hand side the summation is made on time from 0 to $t$. Evaluating the integrals, the above equation becomes

$$[x]_0^x = \int_0^t u\,dt + \int_0^t at\,dt$$

or, $$x = u\int_0^t dt + a\int_0^t t\,dt$$

$$= u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or, $$x = ut + \frac{1}{2}at^2 \cdot \qquad \dots (3.13)$$

From equation (3.12),

$$v^2 = (u + at)^2$$

or, $$= u^2 + 2\,uat + a^2t^2$$

or, $$= u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$$

or, $$= u^2 + 2ax. \qquad \dots (3.14)$$

The three equations (3.12) to (3.14) are collected below in table 3.1. They are very useful in solving the problems of motion in a straight line with constant acceleration.

**Table 3.1**

| |
|---|
| $v = u + at$ |
| $x = ut + \frac{1}{2}at^2$ |
| $v^2 = u^2 + 2ax$ |

Remember that $x$ represents the position of the particle at time $t$ and not (in general) the distance travelled by it in time 0 to $t$. For example, if the particle starts from the origin and goes upto $x$ = 4 m, then turns and is at $x$ = 2 m at time $t$, the distance travelled is 6 m but the position is still given by $x$ = 2 m.

The quantities $u$, $v$ and $a$ may take positive or negative values depending on whether they are directed along the positive or negative direction. Similarly $x$ may be positive or negative.

***Example 3.5***

*A particle starts with an initial velocity* 2·5 m/s *along the positive x direction and it accelerates uniformly at the rate* 0·50 m/s$^2$. *(a) Find the distance travelled by it in the first two seconds. (b) How much time does it take to reach the velocity* 7·5 m/s ? *(c) How much distance will it cover in reaching the velocity* 7·5 m/s ?

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***Solution*** **:** (a) We have,

$$x = ut + \frac{1}{2}at^2$$

$$= (2{\cdot}5 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(0{\cdot}50 \text{ m/s}^2)(2 \text{ s})^2$$

$$= 5{\cdot}0 \text{ m} + 1{\cdot}0 \text{ m} = 6{\cdot}0 \text{ m}.$$

Since the particle does not turn back it is also the distance travelled.

(b) We have,

$$v = u + at$$

or, $$7{\cdot}5 \text{ m/s} = 2{\cdot}5 \text{ m/s} + (0{\cdot}50 \text{ m/s}^2)\, t$$

or, $$t = \frac{7{\cdot}5 \text{ m/s} - 2{\cdot}5 \text{ m/s}}{0{\cdot}50 \text{ m/s}^2} = 10 \text{ s}$$

(c) We have,

$$v^2 = u^2 + 2ax$$

or, $$(7{\cdot}5 \text{ m/s})^2 = (2{\cdot}5 \text{ m/s})^2 + 2(0{\cdot}50 \text{ m/s}^2)x$$

or, $$x = \frac{(7{\cdot}5 \text{ m/s})^2 - (2{\cdot}5 \text{ m/s})^2}{2 \times 0{\cdot}50 \text{ m/s}^2} = 50 \text{ m}.$$

***Example 3.6***

*A particle having initial velocity u moves with a constant acceleration a for a time t. (a) Find the displacement of the particle in the last* 1 *second. (b) Evaluate it for* $u = 5$ m/s, $a = 2$ m/s$^2$ *and* $t = 10$ s.

***Solution*** **:** (a) The position at time $t$ is

$$s = ut + \frac{1}{2}at^2$$

The position at time $(t - 1 \text{ s})$ is

$$s' = u(t - 1 \text{ s}) + \frac{1}{2}a(t - 1 \text{ s})^2$$

$$= ut - u(1 \text{ s}) + \frac{1}{2}at^2 - at(1 \text{ s}) + \frac{1}{2}a(1 \text{ s})^2$$

Thus, the displacement in the last 1 s is

$$s_t = s - s'$$

$$= u(1 \text{ s}) + at(1 \text{ s}) - \frac{1}{2}a(1 \text{ s})^2$$

or, $$s_t = u(1 \text{ s}) + \frac{a}{2}(2t - 1 \text{ s})(1 \text{ s}). \quad \ldots \text{(i)}$$

(b) Putting the given values in (i)

$$s_t = \left(5\frac{\text{m}}{\text{s}}\right)(1 \text{ s}) + \frac{1}{2}\left(2\frac{\text{m}}{\text{s}^2}\right)(2 \times 10 \text{ s} - 1 \text{ s})(1 \text{ s})$$

$$= 5 \text{ m} + \left(1\frac{\text{m}}{\text{s}^2}\right)(19 \text{ s})(1 \text{ s})$$

$$= 5 \text{ m} + 19 \text{ m} = 24 \text{ m}.$$

Sometimes, we are not careful in writing the units appearing with the numerical values of physical quantities. If we forget to write the unit of second in equation (i), we get,

$$s_t = u + \frac{a}{2}(2t - 1).$$

This equation is often used to calculate the displacement in the "$t$th second". However, as you can verify, different terms in this equation have different dimensions and hence the above equation is dimensionally incorrect. Equation (i) is the correct form which was used to solve part (b).

Also note that this equation gives the displacement of the particle in the last 1 second and not necessarily the distance covered in that second.

## Freely Falling Bodies

A common example of motion in a straight line with constant acceleration is free fall of a body near the earth's surface. If air resistance is neglected and a body is dropped near the surface of the earth, it falls along a vertical straight line. The acceleration is in the vertically downward direction and its magnitude is almost constant if the height is small as compared with the radius of the earth (6400 km). This magnitude is approximately equal to 9·8 m/s$^2$ or 32 ft/s$^2$ and is denoted by the letter $g$.

If we take vertically upward as the positive $Y$-axis, acceleration is along the negative $Y$-axis and we write $a = -g$. The equation (3.12) to (3.14) may be written in this case as

$$v = u - gt$$

$$y = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gy.$$

Here $y$ is the $y$-coordinate (that is the height above the origin) at time $t$, $u$ is the velocity in $y$ direction at $t = 0$ and $v$ is the velocity in $y$ direction at time $t$. The position of the particle at $t = 0$ is $y = 0$.

Sometimes it is convenient to choose vertically downward as the positive $Y$-axis. Then $a = g$ and the equations (3.12) to (3.14) become

$$v = u + gt$$

$$y = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gy.$$

***Example 3.7***

*A ball is thrown up at a speed of* 4·0 m/s. *Find the maximum height reached by the ball. Take* $g = 10$ m/s$^2$.

***Solution*** **:** Let us take vertically upward direction as the positive $Y$-axis. We have $u = 4{\cdot}0$ m/s and $a = -10$ m/s$^2$. At the highest point the velocity becomes zero. Using the formula.

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$$v^2 = u^2 + 2ay,$$

$$0 = (4{\cdot}0 \text{ m/s})^2 + 2(-10 \text{ m/s}^2)y$$

or, $$y = \frac{16 \text{ m}^2/\text{s}^2}{20 \text{ m/s}^2} = 0{\cdot}80 \text{ m}.$$

## 3.7 MOTION IN A PLANE

If a particle is free to move in a plane, its position can be located with two coordinates. We choose the plane of motion as the *X-Y* plane. We choose a suitable instant as $t = 0$ and choose the origin at the place where the particle is situated at $t = 0$. Any two convenient mutually perpendicular directions in the *X-Y* plane are chosen as the *X* and *Y*-axes.

The position of the particle at a time $t$ is completely specified by its coordinates $(x, y)$. The coordinates at time $t + \Delta t$ are $(x + \Delta x, y + \Delta y)$. Figure (3.8) shows the positions at $t$ and $t + \Delta t$ as *A* and *B* respectively. The displacement during the time interval $t$ to $t + \Delta t$ is

$$\Delta\vec{r} = \vec{AB} = \vec{AC} + \vec{CB}$$

$$= \Delta x\,\vec{i} + \Delta y\,\vec{j}$$

or, $$\frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\vec{j}.$$

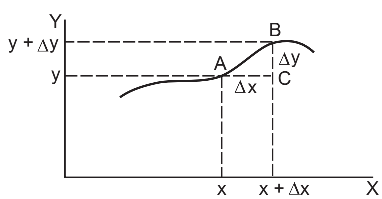


Figure 3.8

Taking limits as $\Delta t \to 0$

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j}. \quad \ldots (3.15)$$

Thus, we see that the $x$-component of the velocity is

$$v_x = \frac{dx}{dt} \quad \ldots (3.16)$$

and the $y$-component is

$$v_y = \frac{dy}{dt}. \quad \ldots (3.17)$$

Differentiating (3.15) with respect to time,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j}$$

Thus, the acceleration has components

$$a_x = \frac{dv_x}{dt} \quad \ldots (3.18)$$

and $$a_y = \frac{dv_y}{dt}. \quad \ldots (3.19)$$

We see that the $x$-coordinate, the $x$-component of velocity $v_x$ and the $x$-component of acceleration $a_x$ are related by

$$v_x = \frac{dx}{dt} \quad \text{and} \quad a_x = \frac{dv_x}{dt}.$$

These equations are identical to equations (3.9) and (3.10). Thus, if $a_x$ is constant, integrating these equations we get

$$\left.\begin{aligned} v_x &= u_x + a_x t \\ x &= u_x t + \frac{1}{2}a_x t^2 \\ v_x^2 &= u_x^2 + 2a_x x \end{aligned}\right| \quad \ldots (3.20)$$

where $u_x$ is the $x$-component of the velocity at $t = 0$. Similarly we have

$$v_y = \frac{dy}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

and if $a_y$ is constant,

$$\left.\begin{aligned} v_y &= u_y + a_y t \\ y &= u_y t + \frac{1}{2}a_y t^2 \\ v_y^2 &= u_y^2 + 2a_y y \end{aligned}\right| \quad \ldots (3.21)$$

The general scheme for the discussion of motion in a plane is therefore simple. The $x$-coordinate, the $x$-component of velocity and the $x$-component of acceleration are related by equations of straight line motion along the *X*-axis. Similarly the $y$-coordinate, the $y$-component of velocity and the $y$-component of acceleration are related by the equations of straight line motion along the *Y*-axis. The problem of motion in a plane is thus, broken up into two independent problems of straight line motion, one along the *X*-axis and the other along the *Y*-axis.

***Example 3.8***

*A particle moves in the X-Y plane with a constant acceleration of* $1{\cdot}5 \text{ m/s}^2$ *in the direction making an angle of 37° with the X-axis. At t = 0 the particle is at the origin and its velocity is* $8{\cdot}0$ m/s *along the X-axis. Find the velocity and the position of the particle at t = 4·0 s.*

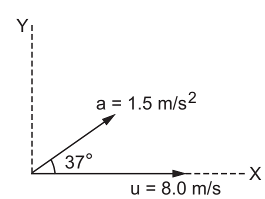


Figure 3.9

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***Solution :*** $a_x = (1{\cdot}5 \text{ m/s}^2)(\cos 37°)$

$$= (1{\cdot}5 \text{ m/s}^2) \times \frac{4}{5} = 1{\cdot}2 \text{ m/s}^2$$

and $a_y = (1{\cdot}5 \text{ m/s}^2)(\sin 37°)$

$$= (1{\cdot}5 \text{ m/s}^2) \times \frac{3}{5} = 0{\cdot}90 \text{ m/s}^2.$$

The initial velocity has components

$$u_x = 8{\cdot}0 \text{ m/s}$$

and $u_y = 0$

At $t = 0$, $x = 0$ and $y = 0$.

The $x$-component of the velocity at time $t = 4{\cdot}0$ s is given by

$$v_x = u_x + a_x t$$
$$= 8{\cdot}0 \text{ m/s} + (1{\cdot}2 \text{ m/s}^2)(4{\cdot}0 \text{ s})$$
$$= 8{\cdot}0 \text{ m/s} + 4{\cdot}8 \text{ m/s} = 12{\cdot}8 \text{ m/s}.$$

The $y$-component of velocity at $t = 4{\cdot}0$ s is given by

$$v_y = u_y + a_y t$$
$$= 0 + (0{\cdot}90 \text{ m/s}^2)(4{\cdot}0 \text{ s}) = 3{\cdot}6 \text{ m/s}.$$

The velocity of the particle at $t = 4{\cdot}0$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(12{\cdot}8 \text{ m/s})^2 + (3{\cdot}6 \text{ m/s})^2}$$
$$= 13{\cdot}3 \text{ m/s}.$$

The velocity makes an angle $\theta$ with the $X$-axis where

$$\tan\theta = \frac{v_y}{v_x} = \frac{3{\cdot}6 \text{ m/s}}{12{\cdot}8 \text{ m/s}} = \frac{9}{32}.$$

The $x$-coordinate at $t = 4{\cdot}0$ s is

$$x = u_x t + \frac{1}{2} a_x t^2$$
$$= (8{\cdot}0 \text{ m/s})(4{\cdot}0 \text{ s}) + \frac{1}{2}(1{\cdot}2 \text{ m/s}^2)(4{\cdot}0 \text{ s})^2$$
$$= 32 \text{ m} + 9{\cdot}6 \text{ m} = 41{\cdot}6 \text{ m}.$$

The $y$-coordinate at $t = 4{\cdot}0$ s is

$$y = u_y t + \frac{1}{2} a_y t^2$$
$$= \frac{1}{2}(0{\cdot}90 \text{ m/s}^2)(4{\cdot}0 \text{ s})^2$$
$$= 7{\cdot}2 \text{ m}.$$

Thus, the particle is at (41·6 m, 7·2 m) at 4·0 s.

## 3.8 PROJECTILE MOTION

An important example of motion in a plane with constant acceleration is the projectile motion. When a particle is thrown obliquely near the earth's surface, it moves along a curved path. Such a particle is called a *projectile* and its motion is called *projectile motion.* We shall assume that the particle remains close to the surface of the earth and the air resistance is negligible. The acceleration of the particle is then almost constant. It is in the vertically downward direction and its magnitude is $g$ which is about $9{\cdot}8 \text{ m/s}^2$.

Let us first make ourselves familiar with certain terms used in discussing projectile motion. Figure (3.10) shows a particle projected from the point $O$ with an initial velocity $u$ at an angle $\theta$ with the horizontal. It goes through the highest point $A$ and falls at $B$ on the horizontal surface through $O$. The point $O$ is called the *point of projection*, the angle $\theta$ is called the *angle of projection* and the distance $OB$ is called the *horizontal range* or simply *range*. The total time taken by the particle in describing the path $OAB$ is called the *time of flight.*

The motion of the projectile can be discussed separately for the horizontal and vertical parts. We take the origin at the point of projection. The instant when the particle is projected is taken as $t = 0$. The plane of motion is taken as the $X$-$Y$ plane. The horizontal line $OX$ is taken as the $X$-axis and the vertical line $OY$ as the $Y$-axis. Vertically upward direction is taken as the positive direction of the $Y$-axis.

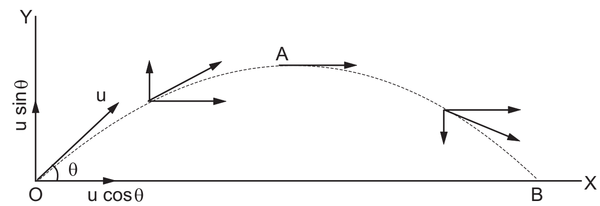


Figure 3.10

We have $u_x = u\cos\theta$ ; $a_x = 0$
$u_y = u\sin\theta$ ; $a_y = -g$.

### Horizontal Motion

As $a_x = 0$, we have

$$v_x = u_x + a_x t = u_x = u\cos\theta$$

and $$x = u_x t + \frac{1}{2} a_x t^2 = u_x t = ut\cos\theta.$$

As indicated in figure (3.10), the $x$-component of the velocity remains constant as the particle moves.

### Vertical Motion

The acceleration of the particle is $g$ in the downward direction. Thus, $a_y = -g$. The $y$-component of the initial velocity is $u_y$. Thus,

$$v_y = u_y - gt$$

and $$y = u_y t - \frac{1}{2} gt^2.$$

Also we have,

$$v_y^2 = u_y^2 - 2gy.$$

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The vertical motion is identical to the motion of a particle projected vertically upward with speed $u \sin\theta$. The horizontal motion of the particle is identical to a particle moving horizontally with uniform velocity $u \cos\theta$.

**Time of Flight**

Consider the situation shown in figure (3.10). The particle is projected from the point $O$ and reaches the same horizontal plane at the point $B$. The total time taken to reach $B$ is the time of flight.

Suppose the particle is at $B$ at a time $t$. The equation for horizontal motion gives

$$OB = x = ut \cos\theta$$

The $y$-coordinate at the point $B$ is zero. Thus, from the equation of vertical motion,

$$y = ut \sin\theta - \frac{1}{2} gt^2$$

or, $$0 = ut \sin\theta - \frac{1}{2} gt^2$$

or, $$t(u \sin\theta - \frac{1}{2} gt) = 0.$$

Thus, either $t = 0$ or, $t = \dfrac{2u \sin\theta}{g}$.

Now $t = 0$ corresponds to the position $O$ of the particle. The time at which it reaches $B$ is thus,

$$T = \frac{2u \sin\theta}{g} . \qquad \ldots (3.22)$$

This is the time of flight.

**Range**

The distance $OB$ is the horizontal range. It is the distance travelled by the particle in time $T = \dfrac{2u \sin\theta}{g}$. By the equation of horizontal motion,

$$x = (u\cos\theta)t$$

or, $$OB = (u \cos\theta)\left(\frac{2u \sin\theta}{g}\right)$$

$$= \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{g} . \qquad \ldots (3.23)$$

**Maximum Height Reached**

At the maximum height ($A$ in figure 3.10) the velocity of the particle is horizontal. The vertical component of velocity is thus, zero at the highest point. The maximum height is the $y$-coordinate of the particle when the vertical component of velocity becomes zero.

We have,

$$v_y = u_y - gt$$

$$= u \sin\theta - gt.$$

At the maximum height

$$0 = u \sin\theta - gt$$

or, $$t = \frac{u \sin\theta}{g} . \qquad \ldots (3.24)$$

The maximum height is

$$H = u_y t - \frac{1}{2} gt^2$$

$$= (u \sin\theta)\left(\frac{u \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{u \sin\theta}{g}\right)^2$$

$$= \frac{u^2 \sin^2\theta}{g} - \frac{1}{2}\frac{u^2 \sin^2\theta}{g}$$

$$= \frac{u^2 \sin^2\theta}{2g} . \qquad \ldots (3.25)$$

Equation (3.24) gives the time taken in reaching the maximum height. Comparison with equation (3.22) shows that it is exactly half the time of the flight. Thus, the time taken in ascending the maximum height equals the time taken in descending back to the same horizontal plane.

***Example 3.9***

*A ball is thrown from a field with a speed of* 12·0 m/s *at an angle of* 45° *with the horizontal. At what distance will it hit the field again ? Take* $g$ = 10·0 m/s$^2$.

***Solution*** **:** The horizontal range $= \dfrac{u^2 \sin 2\theta}{g}$

$$= \frac{(12 \text{ m/s})^2 \times \sin(2 \times 45°)}{10 \text{ m/s}^2}$$

$$= \frac{144 \text{ m}^2/\text{s}^2}{10·0 \text{ m/s}^2} = 14·4 \text{ m}.$$

Thus, the ball hits the field at 14·4 m from the point of projection.

## 3.9 CHANGE OF FRAME

So far we have discussed the motion of a particle with respect to a given frame of reference. The frame can be chosen according to the convenience of the problem. The position $\vec{r}$, the velocity $\vec{v}$ and the acceleration $\vec{a}$ of a particle depend on the frame chosen. Let us see how can we relate the position, velocity and acceleration of a particle measured in two different frames.

Consider two frames of reference $S$ and $S'$ and suppose a particle $P$ is observed from both the frames. The frames may be moving with respect to each other. Figure (3.11) shows the situation.

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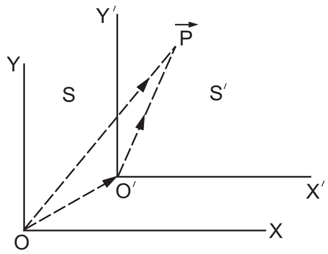


Figure 3.11

The position vector of the particle $P$ with respect to the frame $S$ is $\vec{r}_{P,S} = \overrightarrow{OP}$ . The position vector of the particle with respect to the frame $S'$ is $\vec{r}_{P,S'} = \overrightarrow{O'P}$ . The position of the frame $S'$ (the origin of frame $S'$ in fact) with respect to the frame $S$ is $OO'$.

It is clear that

$$\overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P} = \overrightarrow{O'P} + \overrightarrow{OO'}$$

or, $$\vec{r}_{P,S} = \vec{r}_{P,S'} + \vec{r}_{S',S} . \qquad \dots (3.26)$$

The position of the particle with respect to $S$ is equal to the position of the particle with respect to $S'$ plus the position of $S'$ with respect to $S$.

If we differentiate equation (3.26) with respect to time, we get

$$\frac{d}{dt}(\vec{r}_{P,S}) = \frac{d}{dt}(\vec{r}_{P,S'}) + \frac{d}{dt}(\vec{r}_{S',S})$$

or, $$\vec{v}_{P,S} = \vec{v}_{P,S'} + \vec{v}_{S',S} \qquad \dots (3.27)$$

where $\vec{v}_{P,S}$ is the velocity of the particle with respect to $S$, $\vec{v}_{P,S'}$ is the velocity of the particle with respect to $S'$ and $\vec{v}_{S',S}$ is the velocity of the frame $S'$ with respect to $S$. The velocity of the particle with respect to $S$ is equal to the velocity of the particle with respect to $S'$ plus the velocity of $S'$ with respect to $S$.

It is assumed that the meaning of time is same in both the frames. Similarly it is assumed that $\frac{d}{dt}$ has same meaning in both the frames. These assumptions are not correct if the velocity of one frame with respect to the other is so large that it is comparable to $3 \times 10^8$ m/s, or if one frame rotates with respect to the other. If the frames only translate with respect to each other with small velocity, the above assumptions are correct.

Equation (3.27) may be rewritten as

$$\vec{v}_{P,S'} = \vec{v}_{P,S} - \vec{v}_{S',S} . \qquad \dots (3.28)$$

Thus, if the velocities of two bodies (here the particle and the frame $S'$) are known with respect to a common frame (here $S$) we can find the velocity of one body with respect to the other body. The velocity of body 1 with respect to the body 2 is obtained by subtracting the velocity of body 2 from the velocity of body 1.

When we say that the muzzle velocity of a bullet is 60 m/s we mean the velocity of the bullet with respect to the gun. If the gun is mounted in a train moving with a speed of 20 m/s with respect to the ground and the bullet is fired in the direction of the train's motion, its velocity with respect to the ground will be 80 m/s. Similarly, when we say that a swimmer can swim at a speed of 5 km/h we mean the velocity of the swimmer with respect to the water. If the water itself is flowing at 3 km/h with respect to the ground and the swimmer swims in the direction of the current, he or she will move at the speed of 8 km/h with respect to the ground.

***Example 3.10***

*A swimmer can swim in still water at a rate* 4·0 km/h. *If he swims in a river flowing at* 3·0 km/h *and keeps his direction (with respect to water) perpendicular to the current, find his velocity with respect to the ground.*

***Solution*** **:** The velocity of the swimmer with respect to water is $\vec{v}_{S,R} = 4 \cdot 0$ km/h in the direction perpendicular to the river. The velocity of river with respect to the ground is $\vec{v}_{R,G} = 3 \cdot 0$ km/h along the length of the river. The velocity of the swimmer with respect to the ground is $\vec{v}_{S,G}$ where

$$\vec{v}_{S,G} = \vec{v}_{S,R} + \vec{v}_{R,G} .$$

Figure (3.12) shows the velocities. It is clear that,

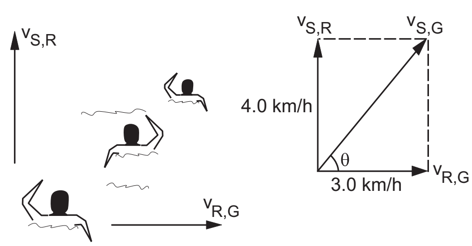


Figure 3.12

$$v_{S,G} = \sqrt{(4 \cdot 0 \text{ km/h})^2 + (3 \cdot 0 \text{ km/h})^2}$$

$$= 5 \cdot 0 \text{ km/h}$$

The angle θ made with the direction of flow is

$$\tan\theta = \frac{4 \cdot 0 \text{ km/h}}{3 \cdot 0 \text{ km/h}} = \frac{4}{3} .$$

***Example 3.11***

*A man is walking on a level road at a speed of* 3·0 km/h. *Rain drops fall vertically with a speed of* 4·0 km/h. *Find the velocity of the raindrops with respect to the man.*

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***Solution*** **:** We have to find the velocity of raindrops with respect to the man. The velocity of the rain as well as the velocity of the man are given with respect to the street. We have

$$\vec{v}_{rain,\,man} = \vec{v}_{rain,\,street} - \vec{v}_{man,\,street}\,.$$

Figure (3.13) shows the velocities.

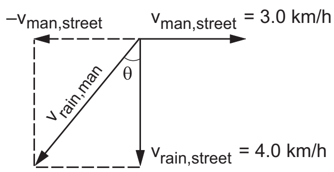


Figure 3.13

It is clear from the figure that

$$v_{rain,\,man} = \sqrt{(4{\cdot}0 \text{ km/h})^2 + (3{\cdot}0 \text{ km/h})^2}$$
$$= 5{\cdot}0 \text{ km/h}.$$

The angle with the vertical is θ, where

$$\tan\theta = \frac{3{\cdot}0 \text{ km/h}}{4{\cdot}0 \text{ km/h}} = \frac{3}{4}\,.$$

Thus, the rain appears to fall at an angle $\tan^{-1}(3/4)$ with the speed 5·0 km/h as viewed by the man.

The relation between the accelerations measured from two frames can be obtained by differentiating equation (3.27) with respect to time.

We have,

$$\frac{d}{dt}(\vec{v}_{P,S}) = \frac{d}{dt}(\vec{v}_{P,S'}) + \frac{d}{dt}(\vec{v}_{S',S})$$

or, $$\vec{a}_{P,S} = \vec{a}_{P,S'} + \vec{a}_{S',S}\,. \qquad \ldots (3.29)$$

*If S′ moves with respect to S at a uniform velocity,* $\vec{a}_{S',S} = 0$ *and so*

$$\vec{a}_{P,S} = \vec{a}_{P,S'}\,.$$

If two frames are moving with respect to each other with uniform velocity, acceleration of a body is same in both the frames.

## *Worked Out Examples*

**1.** *A man walks at a speed of* 6 km/hr *for* 1 km *and* 8 km/hr *for the next* 1 km. *What is his average speed for the walk of* 2 km ?

***Solution*** **:** Distance travelled is 2 km.

$$\text{Time taken} = \frac{1 \text{ km}}{6 \text{ km/hr}} + \frac{1 \text{ km}}{8 \text{ km/hr}}$$
$$= \left(\frac{1}{6} + \frac{1}{8}\right)\text{hr} = \frac{7}{24} \text{ hr}.$$

$$\text{Average speed} = \frac{2 \text{ km} \times 24}{7 \text{ hr}} = \frac{48}{7} \text{ km/hr}$$
$$\approx 7 \text{ km/hr}.$$

**2.** *The I.Sc. lecture theatre of a college is* 40 ft *wide and has a door at a corner. A teacher enters at* 12·00 noon *through the door and makes* 10 *rounds along the* 40 ft *wall back and forth during the period and finally leaves the class-room at* 12·50 p.m. *through the same door. Compute his average speed and average velocity.*

***Solution*** **:** Total distance travelled in 50 minutes = 800 ft.

$$\text{Average speed} = \frac{800}{50} \text{ ft/min} = 16 \text{ ft/min}.$$

At 12·00 noon he is at the door and at 12·50 pm he is again at the same door.

The displacement during the 50 min interval is zero.

Average velocity = zero.

**3.** *The position of a particle moving on X-axis is given by*

$$x = At^3 + Bt^2 + Ct + D.$$

*The numerical values of A, B, C, D are* 1, 4, –2 *and* 5 *respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at* $t$ = 4 s, *(c) the acceleration of the particle at* $t$ = 4 s, *(d) the average velocity during the interval* $t$ = 0 *to* $t$ = 4 s, *(e) the average acceleration during the interval* $t = 0$ to $t = 4$ s.

***Solution*** : (a) Dimensions of $x$, $At^3$, $Bt^2$, $Ct$ and $D$ must be identical and in this case each is length. Thus,

$$[At^3] = \text{L}, \quad \text{or,} \quad [A] = \text{LT}^{-3}$$
$$[Bt^2] = \text{L}, \quad \text{or,} \quad [B] = \text{LT}^{-2}$$
$$[Ct] = \text{L}, \quad \text{or,} \quad [C] = \text{LT}^{-1}$$

and $$[D] = \text{L}.$$

(b) $x = At^3 + Bt^2 + Ct + D$

or, $v = \dfrac{dx}{dt} = 3At^2 + 2Bt + C.$

Thus, at $t$ = 4 s, the velocity

$$= 3(1 \text{ m/s}^3)(16 \text{ s}^2) + 2(4 \text{ m/s}^2)(4 \text{ s}) + (-2 \text{ m/s})$$
$$= (48 + 32 - 2) \text{ m/s} = 78 \text{ m/s}.$$

(c) $v = 3At^2 + 2Bt + C$

or, $a = \dfrac{dv}{dt} = 6\,At + 2\,B.$

At $t = 4$ s, $a = 6(1 \text{ m/s}^3)(4 \text{ s}) + 2(4 \text{ m/s}^2) = 32 \text{ m/s}^2$.

(d) $x = At^3 + Bt^2 + Ct + D.$

Position at $t$ = 0 is $x = D$ = 5 m.

Position at $t$ = 4 s is

$$(1 \text{ m/s}^3)(64 \text{ s}^3) + (4 \text{ m/s}^2)(16 \text{ s}^2) - (2 \text{ m/s})(4 \text{ s}) + 5 \text{ m}$$
$$= (64 + 64 - 8 + 5) \text{ m} = 125 \text{ m}.$$

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Thus, the displacement during 0 to 4 s is
$125 \text{ m} - 5 \text{ m} = 120 \text{ m}.$

Average velocity $= \dfrac{120 \text{ m}}{4 \text{ s}} = 30 \text{ m/s}.$

(e) $v = 3At^2 + 2Bt + C.$

Velocity at $t = 0$ is $C = -2$ m/s.

Velocity at $t = 4$ s is $= 78$ m/s.

Average acceleration $= \dfrac{v_2 - v_1}{t_2 - t_1} = 20 \text{ m/s}^2.$

**4.** *From the velocity-time graph of a particle given in figure (3-W1), describe the motion of the particle qualitatively in the interval* 0 *to* 4 s. *Find (a) the distance travelled during first two seconds, (b) during the time* 2 s *to* 4 s, *(c) during the time* 0 *to* 4 s, *(d) displacement during* 0 to 4 s, *(e) acceleration at* $t = 1/2$ s *and (f) acceleration at* $t = 2$ s.

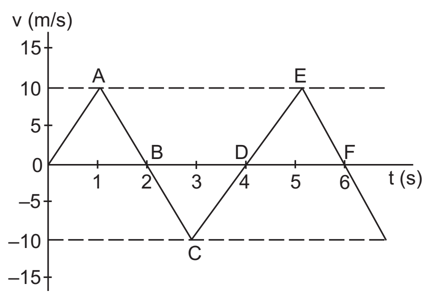


Figure 3-W1

***Solution*** **:** At $t = 0$, the particle is at rest, say at the origin. After that the velocity is positive, so that the particle moves in the positive $x$ direction. Its speed increases till 1 second when it starts decreasing. The particle continues to move further in positive $x$ direction. At $t = 2$ s, its velocity is reduced to zero, it has moved through a maximum positive $x$ distance. Then it changes its direction, velocity being negative, but increasing in magnitude. At $t = 3$ s velocity is maximum in the negative $x$ direction and then the magnitude starts decreasing. It comes to rest at $t = 4$ s.

(a) Distance during 0 to 2 s = Area of $OAB$

$$= \frac{1}{2} \times 2 \text{ s} \times 10 \text{ m/s} = 10 \text{ m}.$$

(b) Distance during 2 to 4 s = Area of $BCD$ = 10 m. The particle has moved in negative $x$ direction during this period.

(c) The distance travelled during 0 to 4s = 10 m + 10 m = 20 m.

(d) displacement during 0 to 4 s = 10 m + (– 10 m) = 0.

(e) at $t = 1/2$ s acceleration = slope of line $OA$ = 10 m/s$^2$.

(f) at $t = 2$ s acceleration = slope of line $ABC$ = – 10 m/s$^2$.

**5.** *A particle starts from rest with a constant acceleration. At a time t second, the speed is found to be* 100 m/s *and one second later the speed becomes* 150 m/s. *Find (a) the acceleration and (b) the distance travelled during the (t+1)$^{th}$ second.*

***Solution*** **:** (a) Velocity at time $t$ second is

$100 \text{ m/s} = a.(t \text{ second})$ ... (1)

and velocity at time $(t + 1)$ second is

$150 \text{ m/s} = a.(t + 1) \text{ second}.$ ... (2)

Subtracting (1) from (2), $a = 50 \text{ m/s}^2$

(b) Consider the interval $t$ second to $(t + 1)$ second,

time elapsed = 1 s

initial velocity = 100 m/s

final velocity = 150 m/s.

Thus, $(150 \text{ m/s})^2 = (100 \text{ m/s})^2 + 2(50 \text{ m/s}^2)\, x$

or, $x = 125$ m.

**6.** *A boy stretches a stone against the rubber tape of a catapult or 'gulel' (a device used to detach mangoes from the tree by boys in Indian villages) through a distance of* 24 cm *before leaving it. The tape returns to its normal position accelerating the stone over the stretched length. The stone leaves the gulel with a speed* 2·2 m/s. *Assuming that the acceleration is constant while the stone was being pushed by the tape, find its magnitude.*

***Solution*** **:** Consider the accelerated 24 cm motion of the stone.

Initial velocity = 0

Final velocity = 2·2 m/s

Distance travelled = 24 cm = 0·24 m

Using $v^2 = u^2 + 2ax$,

$$a = \frac{4{\cdot}84 \text{ m}^2/\text{s}^2}{2 \times 0{\cdot}24 \text{ m}} = 10{\cdot}1 \text{ m/s}^2.$$

**7.** *A police inspector in a jeep is chasing a pickpocket on a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pickpocket will be caught if* $v \geq \sqrt{2ad}$.

***Solution*** **:** Suppose the pickpocket is caught at a time $t$ after the motorcycle starts. The distance travelled by the motorcycle during this interval is

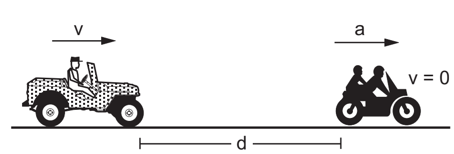


Figure 3-W2

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$$s = \frac{1}{2} at^2. \qquad \ldots \text{ (i)}$$

During this interval the jeep travels a distance

$$s + d = vt. \qquad \ldots \text{ (ii)}$$

By (i) and (ii),

$$\frac{1}{2} at^2 - vt + d = 0$$

or,
$$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}.$$

The pickpocket will be caught if $t$ is real and positive. This will be possible if

$$v^2 \geq 2ad \quad \text{or,} \quad v \geq \sqrt{2ad}.$$

**8.** *A car is moving at a constant speed of* 40 km/h *along a straight road which heads towards a large vertical wall and makes a sharp* 90° *turn by the side of the wall. A fly flying at a constant speed of* 100 km/h, *starts from the wall towards the car at an instant when the car is* 20 km *away, flies until it reaches the glasspane of the car and returns to the wall at the same speed. It continues to fly between the car and the wall till the car makes the* 90° *turn. (a) What is the total distance the fly has travelled during this period ? (b) How many trips has it made between the car and the wall ?*

***Solution*** : (a) The time taken by the car to cover 20 km before the turn is $\frac{20 \text{ km}}{40 \text{ km/h}} = \frac{1}{2} \text{h}$. The fly moves at a constant speed of 100 km/h during this time. Hence the total distance coverd by it is $100 \frac{\text{km}}{\text{h}} \times \frac{1}{2} \text{h} = 50 \text{ km}$.

(b) Suppose the car is at a distance $x$ away (at $A$) when the fly is at the wall (at $O$). The fly hits the glasspane at $B$, taking a time $t$. Then

$$AB = (40 \text{ km/h})t,$$

and
$$OB = (100 \text{ km/h})t.$$

Thus,
$$x = AB + OB$$
$$= (140 \text{ km/h})t$$

or,
$$t = \frac{x}{140 \text{ km/h}}, \text{ or } OB = \frac{5}{7} x.$$

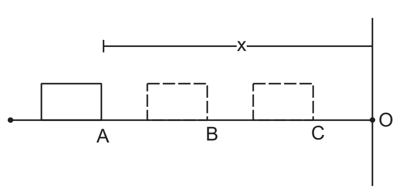


Figure 3-W3

The fly returns to the wall and during this period the car moves the distance $BC$. The time taken by the fly in this return path is

$$\left(\frac{5x/7}{100 \text{ km/h}}\right) = \frac{x}{140 \text{ km/h}}.$$

Thus,
$$BC = \frac{40x}{140} = \frac{2}{7}x$$

or,
$$OC = OB - BC = \frac{3}{7}x.$$

If at the beginning of the round trip (wall to the car and back) the car is at a distance $x$ away, it is $\frac{3}{7}x$ away when the next trip again starts.

Distance of the car at the beginning of the 1st trip = 20 km.

Distance of the car at the beginning of the 2nd trip

$$= \frac{3}{7} \times 20 \text{ km}.$$

Distance of the car at the beginning of the 3rd trip

$$= \left(\frac{3}{7}\right)^2 \times 20 \text{ km}.$$

Distance of the car at the beginning of the 4th trip

$$= \left(\frac{3}{7}\right)^3 \times 20 \text{ km}.$$

Distance of the car at the beginning of the $n$th trip

$$= \left(\frac{3}{7}\right)^{n-1} \times 20 \text{ km}.$$

Trips will go on till the car reaches the turn that is the distance reduces to zero. This will be the case when $n$ becomes infinity. Hence the fly makes an infinite number of trips before the car takes the turn.

**9.** *A ball is dropped from a height of* 19·6 m *above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive X-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with the ground.*

***Solution*** : Since the acceleration of the ball during the contact is different from *'g'*, we have to treat the downward motion and the upward motion separately.

For the downward motion : $a = g = 9{\cdot}8 \text{ m/s}^2$,

$$x = ut + \frac{1}{2}at^2 = (4{\cdot}9 \text{ m/s}^2)t^2.$$

The ball reaches the ground when $x$ = 19·6 m. This gives $t$ = 2 s. After that it moves up, $x$ decreases and at $t$ = 4 s, $x$ becomes zero, the ball reaching the initial point.

We have at $t = 0$, $x = 0$

$t = 1$ s, $x = 4{\cdot}9$ m

$t = 2$ s, $x = 19{\cdot}6$ m

$t = 3$ s, $x = 4{\cdot}9$ m

$t = 4$ s, $x = 0$.

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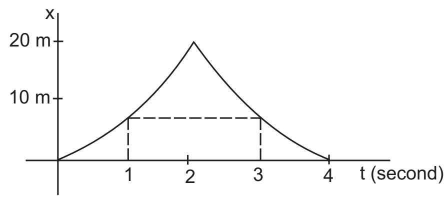


Figure 3-W4

**Velocity** : During the first two seconds,

$v = u + at = (9{\cdot}8 \text{ m/s}^2)t$

at $t = 0$ $\quad v = 0$

at $t = 1$ s, $\quad v = 9{\cdot}8$ m/s

at $t = 2$ s, $\quad v = 19{\cdot}6$ m/s.

During the next two seconds the ball goes upward, velocity is negative, magnitude decreasing and at $t = 4$ s, $v = 0$. Thus,

at $t = 2$ s, $\quad v = -19{\cdot}6$ m/s

at $t = 3$ s, $\quad v = -9{\cdot}8$ m/s

at $t = 4$ s, $\quad v = 0$.

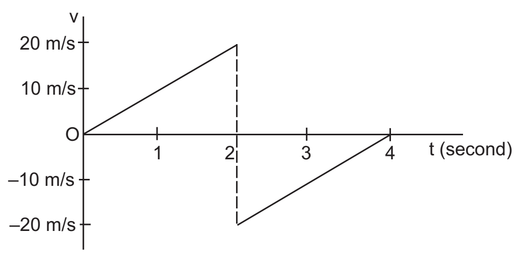


Figure 3-W5

At $t = 2$ s there is an abrupt change in velocity from 19·6 m/s to – 19·6 m/s. In fact this change in velocity takes place over a small interval during which the ball remains in contact with the ground.

**Acceleration :** The acceleration is constant $9{\cdot}8 \text{ m/s}^2$ throughout the motion (except at $t = 2$ s).

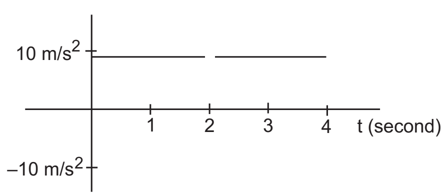


Figure 3-W6

**10.** *A stone is dropped from a balloon going up with a uniform velocity of* 5·0 m/s. *If the balloon was* 50 m *high when the stone was dropped, find its height when the stone hits the ground. Take* $g = 10 \text{ m/s}^2$.

***Solution*** **:** At $t = 0$, the stone was going up with a velocity of 5·0 m/s. After that it moved as a freely falling particle with downward acceleration $g$. Take vertically upward as the positive $X$-axis. If it reaches the ground at time $t$,

$$x = -50 \text{ m}, \quad u = 5 \text{ m/s}, \quad a = -10 \text{ m/s}^2.$$

We have $\qquad x = ut + \frac{1}{2}at^2$

or, $\qquad -50 \text{ m} = (5 \text{ m/s}).t + \frac{1}{2} \times (-10 \text{ m/s}^2)t^2$

or, $\qquad t = \frac{1 \pm \sqrt{41}}{2}$ s.

or, $\qquad t = -2{\cdot}7$ s $\quad$ or, $\;3{\cdot}7$ s.

Negative $t$ has no significance in this problem. The stone reaches the ground at $t = 3{\cdot}7$ s. During this time the balloon has moved uniformly up. The distance covered by it is

$$5 \text{ m/s} \times 3{\cdot}7 \text{ s} = 18{\cdot}5 \text{ m}.$$

Hence, the height of the balloon when the stone reaches the ground is 50 m + 18·5 m = 68·5 m.

**11.** *A football is kicked with a velocity of* 20 m/s *at an angle of* 45° *with the horizontal. (a) Find the time taken by the ball to strike the ground. (b) Find the maximum height it reaches. (c) How far away from the kick does it hit the ground ? Take* $g = 10 \text{ m/s}^2$.

***Solution*** **:** (a) Take the origin at the point where the ball is kicked, vertically upward as the $Y$-axis and the horizontal in the plane of motion as the $X$-axis. The initial velocity has the components

$$u_x = (20 \text{ m/s}) \cos 45° = 10\sqrt{2} \text{ m/s}$$

and $\qquad u_y = (20 \text{ m/s}) \sin 45° = 10\sqrt{2}$ m/s.

When the ball reaches the ground, $y = 0$.

Using $\qquad y = u_y t - \frac{1}{2}gt^2,$

$$0 = (10\sqrt{2} \text{ m/s})\, t - \frac{1}{2} \times (10 \text{ m/s}^2) \times t^2$$

or, $\qquad t = 2\sqrt{2}$ s $= 2{\cdot}8$ s.

Thus, it takes 2·8 s for the football to fall on the ground.

(b) At the highest point $v_y = 0$. Using the equation

$$v_y^2 = u_y^2 - 2gy,$$

$$0 = (10\sqrt{2} \text{ m/s})^2 - 2 \times (10 \text{ m/s}^2)\, H$$

or, $\qquad H = 10$ m.

Thus, the maximum height reached is 10 m.

(c) The horizontal distance travelled before falling to the ground is $x = u_x t$

$$= (10\sqrt{2} \text{ m/s})\,(2\sqrt{2} \text{ s}) = 40 \text{ m}.$$

**12.** *A helicopter on flood relief mission, flying horizontally with a speed u at an altitude H, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the packet be dropped ? The victim stands in the vertical plane of the helicopter's motion.*

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***Solution*** **:** The velocity of the food packet at the time of release is $u$ and is horizontal. The vertical velocity at the time of release is zero.

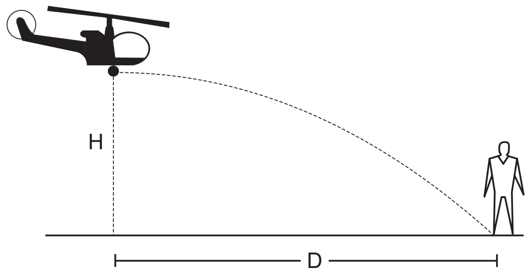


Figure 3-W7

**Vertical motion :** If $t$ be the time taken by the packet to reach the victim, we have for vertical motion,

$$H = \frac{1}{2} g t^2 \quad \text{or,} \quad t = \sqrt{\frac{2H}{g}} . \qquad \dots \text{(i)}$$

**Horizontal motion :** If $D$ be the horizontal distance travelled by the packet, we have $D = ut$. Putting $t$ from (i),

$$D = u\sqrt{\frac{2H}{g}} .$$

The distance between the victim and the packet at the time of release is

$$\sqrt{D^2 + H^2} = \sqrt{\frac{2u^2 H}{g} + H^2} .$$

**13.** *A particle is projected horizontally with a speed $u$ from the top of a plane inclined at an angle $\theta$ with the horizontal. How far from the point of projection will the particle strike the plane ?*

***Solution*** **:** Take $X,Y$-axes as shown in figure (3-W8). Suppose that the particle strikes the plane at a point $P$ with coordinates $(x, y)$. Consider the motion between $A$ and $P$.

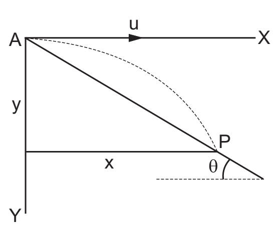


Figure 3-W8

Motion in $x$-direction :

Initial velocity $= u$

Acceleration $= 0$

$$x = ut. \qquad \dots \text{(i)}$$

Motion in $y$-direction :

Initial velocity $= 0$

Acceleration $= g$

$$y = \frac{1}{2} g t^2. \qquad \dots \text{(ii)}$$

Eliminating $t$ from (i) and (ii)

$$y = \frac{1}{2} g \frac{x^2}{u^2} .$$

Also $\quad y = x \tan\theta.$

Thus, $\dfrac{gx^2}{2u^2} = x \tan\theta$ giving $x = 0$, or, $\dfrac{2u^2 \tan\theta}{g}$.

Clearly the point $P$ corresponds to $x = \dfrac{2u^2 \tan\theta}{g}$,

then $y = x \tan\theta = \dfrac{2u^2 \tan^2\theta}{g}$.

The distance $AP = l = \sqrt{x^2 + y^2}$

$$= \frac{2u^2}{g} \tan\theta \sqrt{1 + \tan^2\theta}$$

$$= \frac{2u^2}{g} \tan\theta \sec\theta.$$

**Alternatively :** Take the axes as shown in figure 3-W9. Consider the motion between $A$ and $P$.

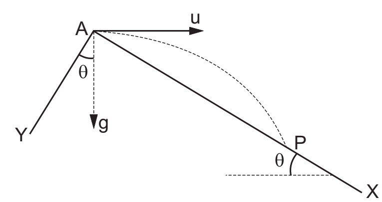


Figure 3-W9

Motion along the $X$-axis :

Initial velocity $= u \cos\theta$

Acceleration $= g \sin\theta$

Displacement $= AP$.

Thus, $AP = (u \cos\theta)\, t + \frac{1}{2} (g \sin\theta)\, t^2$. $\qquad \dots$ (i)

Motion along the $Y$-axis :

Initial velocity $= - u \sin\theta$

Acceleration $= g \cos\theta$

Displacement $= 0$.

Thus, $\quad 0 = - ut \sin\theta + \frac{1}{2} g t^2 \cos\theta$

or, $\quad t = 0, \ \dfrac{2u \sin\theta}{g \cos\theta}$.

Clearly, the point $P$ corresponds to $t = \dfrac{2u \sin\theta}{g \cos\theta}$.

Putting this value of $t$ in (i),

$$AP = (u \cos\theta)\left(\frac{2u \sin\theta}{g \cos\theta}\right) + \frac{g \sin\theta}{2}\left(\frac{2u \sin\theta}{g \cos\theta}\right)^2$$

$$= \frac{2u^2 \sin\theta}{g} + \frac{2u^2 \sin\theta \tan^2\theta}{g}$$

$$= \frac{2u^2}{g} \sin\theta \sec^2\theta = \frac{2u^2}{g} \tan\theta \sec\theta.$$

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**14.** *A projectile is fired with a speed u at an angle θ with the horizontal. Find its speed when its direction of motion makes an angle α with the horizontal.*

***Solution*** : Let the speed be $v$ when it makes an angle α with the horizontal. As the horizontal component of velocity remains constant,

$$v \cos\alpha = u \cos\theta$$

or, $$v = u \cos\theta \sec\alpha.$$

**15.** *A bullet is fired horizontally aiming at an object which starts falling at the instant the bullet is fired. Show that the bullet will hit the object.*

***Solution*** **:** The situation is shown in figure (3-W10). The object starts falling from the point $B$. Draw a vertical line $BC$ through $B$. Suppose the bullet reaches the line $BC$ at a point $D$ and it takes a time $t$ in doing so.

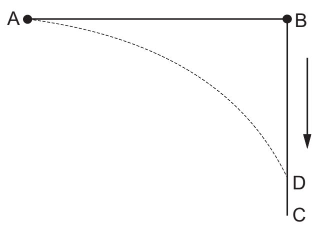


Figure 3-W10

Consider the vertical motion of the bullet. The initial vertical velocity = 0. The distance travelled vertically $= BD = \frac{1}{2} gt^2$. In time $t$ the object also travels a distance $\frac{1}{2} gt^2 = BD$. Hence at time $t$, the object will also be at the same point $D$. Thus, the bullet hits the object at point $D$.

**16.** *A man can swim in still water at a speed of* 3 km/h. *He wants to cross a river that flows at* 2 km/h *and reach the point directly opposite to his starting point. (a) In which direction should he try to swim (that is, find the angle his body makes with the river flow) ? (b) How much time will he take to cross the river if the river is* 500 m *wide ?*

***Solution*** **:** (a) The situation is shown in figure (3-W11). The $X$-axis is chosen along the river flow and the origin at the starting position of the man. The direction of the velocity of man with respect to ground is along the $Y$-axis (perpendicular to the river). We have to find the direction of velocity of the man with respect to water.

Let $\vec{v}_{r,g}$ = velocity of the river with respect to the ground
= 2 km/h along the $X$-axis

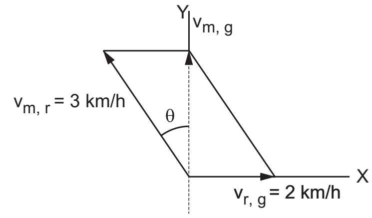


Figure 3-W11

$\vec{v}_{m,r}$ = velocity of the man with respect to the river
= 3 km/h making an angle θ with the $Y$-axis

and $\vec{v}_{m,g}$ = velocity of the man with respect to the ground along the $Y$-axis.

We have

$$\vec{v}_{m,g} = \vec{v}_{m,r} + \vec{v}_{r,g} . \quad \ldots \text{(i)}$$

Taking components along the $X$-axis

$$0 = -(3 \text{ km/h})\sin\theta + 2 \text{ km/h}$$

or, $$\sin\theta = \frac{2}{3} .$$

(b) Taking components in equation (i) along the $Y$-axis,

$$v_{m,g} = (3 \text{ km/h}) \cos\theta + 0$$

or, $$v_{m,g} = \sqrt{5} \text{ km/h}.$$

$$\text{Time} = \frac{\text{Displacement in } y \text{ direction}}{\text{Velocity in } y \text{ direction}}$$

$$= \frac{0{\cdot}5 \text{ km}}{\sqrt{5} \text{ km/h}} = \frac{\sqrt{5}}{10} \text{ h}.$$

**17.** *A man can swim at a speed of* 3 km/h *in still water. He wants to cross a* 500 m *wide river flowing at* 2 km/h. *He keeps himself always at an angle of* 120° *with the river flow while swimming.*
*(a) Find the time he takes to cross the river. (b) At what point on the opposite bank will he arrive ?*

***Solution*** **:** The situation is shown in figure (3-W12).

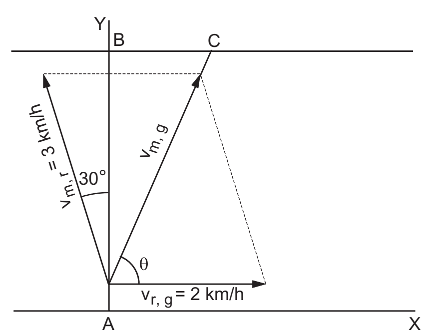


Figure 3-W12

Here $\vec{v}_{r,g}$ = velocity of the river with respect to the ground
$\vec{v}_{m,r}$ = velocity of the man with respect to the river
$\vec{v}_{m,g}$ = velocity of the man with respect to the ground.

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(a) We have,

$$\vec{v}_{m,g} = \vec{v}_{m,r} + \vec{v}_{r,g} \qquad \ldots \text{ (i)}$$

Hence, the velocity with respect to the ground is along $AC$. Taking $y$-components in equation (i),

$$\vec{v}_{m,g} \sin\theta = 3 \text{ km/h} \cos 30° + 2 \text{ km/h} \cos 90° = \frac{3\sqrt{3}}{2} \text{ km/h}.$$

Time taken to cross the river

$$= \frac{\text{displacement along the } Y\text{-axis}}{\text{velocity along the } Y\text{-axis}}$$

$$= \frac{1/2 \text{ km}}{3\sqrt{3}/2 \text{ km/h}} = \frac{1}{3/3} \text{ h}.$$

(b) Taking $x$-components in equation (i),

$$\vec{v}_{m,g} \cos\theta = -3 \text{ km/h} \sin 30° + 2 \text{ km/h}$$

$$= \frac{1}{2} \text{ km/h}.$$

Displacement along the $X$-axis as the man crosses the river

$$= (\text{velocity along the } X\text{-axis}) \cdot (\text{time})$$

$$= \left(\frac{1 \text{ km}}{2 \text{ h}}\right) \times \left(\frac{1}{3\sqrt{3}} \text{ h}\right) = \frac{1}{6\sqrt{3}} \text{ km}.$$

**18.** *A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.*

***Solution*** **:** When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure (3-W13b).

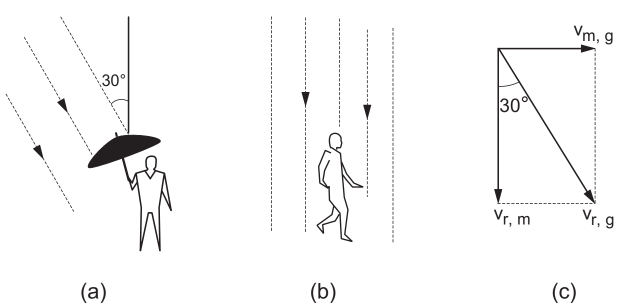


Figure 3-W13

Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground

and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man.

We have, $\quad \vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}. \qquad \ldots$ (i)

Taking horizontal components, equation (i) gives

$$v_{r,g} \sin 30° = v_{m,g} = 10 \text{ km/h}$$

or, $$v_{r,g} = \frac{10 \text{ km/h}}{\sin 30°} = 20 \text{ km/h},$$

Taking vertical components, equation (i) gives

$$v_{r,g} \cos 30° = v_{r,m}$$

or, $$v_{r,m} = (20 \text{ km/h}) \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3} \text{ km/h}.$$

**19.** *A man running on a horizontal road at* 8 km/h *finds the rain falling vertically. He increases his speed to* 12 km/h *and finds that the drops make angle 30° with the vertical. Find the speed and direction of the rain with respect to the road.*

***Solution*** **:**

We have $\quad \vec{v}_{rain,road} = \vec{v}_{rain,man} + \vec{v}_{man,road} \qquad \ldots$ (i)

The two situations given in the problem may be represented by the following figure.

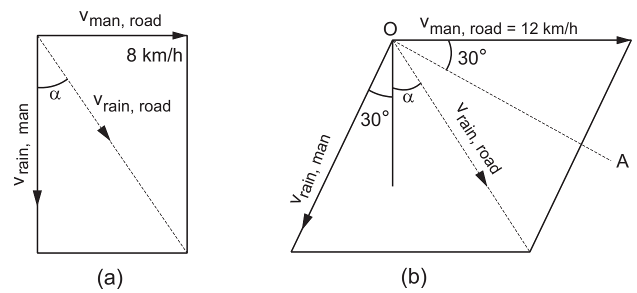


Figure 3-W14

$v_{rain,road}$ is same in magnitude and direction in both the figures.

Taking horizontal components in equation (i) for figure (3-W14a),

$$v_{rain,road} \sin\alpha = 8 \text{ km/h}. \qquad \ldots \text{ (ii)}$$

Now consider figure (3-W14b). Draw a line $OA \perp v_{rain,man}$ as shown.

Taking components in equation (i) along the line $OA$.

$$v_{rain,road} \sin(30° + \alpha) = 12 \text{ km/h} \cos 30°. \qquad \ldots \text{ (iii)}$$

From (ii) and (iii),

$$\frac{\sin(30° + \alpha)}{\sin\alpha} = \frac{12 \times \sqrt{3}}{8 \times 2}$$

or, $$\frac{\sin 30° \cos\alpha + \cos 30° \sin\alpha}{\sin\alpha} = \frac{3\sqrt{3}}{4}$$

or, $$\frac{1}{2} \cot\alpha + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$$

or, $$\cot\alpha = \frac{\sqrt{3}}{2}$$

or, $$\alpha = \cot^{-1} \frac{\sqrt{3}}{2}.$$

From (ii), $$v_{rain,road} = \frac{8 \text{ km/h}}{\sin\alpha} = 4\sqrt{7} \text{ km/h}.$$

**20.** *Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side d at t = 0. Each*

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*of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other ?*

***Solution*** **:** The motion of the particles is roughly sketched in figure (3-W15). By symmetry they will meet at the centroid $O$ of the triangle. At any instant the particles will form an equilateral triangle $ABC$ with the same centriod $O$. Concentrate on the motion of any one particle, say $A$. At any instant its velocity makes angle 30° with $AO$.

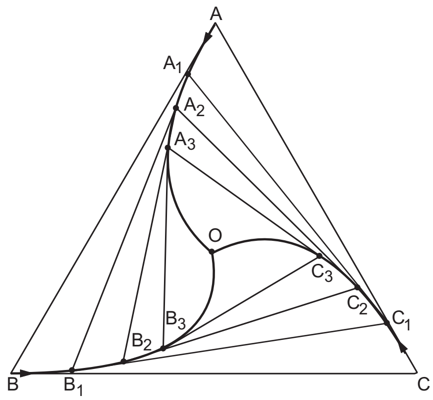


Figure 3-W15

The component of this velocity along $AO$ is $v\cos 30°$. This component is the rate of decrease of the distance $AO$. Initially,

$$AO = \frac{2}{3}\sqrt{d^2 - \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{3}}.$$

Therefore, the time taken for $AO$ to become zero

$$= \frac{d/\sqrt{3}}{v\cos 30°} = \frac{2d}{\sqrt{3}\,v \times \sqrt{3}} = \frac{2d}{3v}.$$

**Alternative** **:** Velocity of $A$ is $v$ along $AB$. The velocity of $B$ is along $BC$. Its component along $BA$ is $v\cos 60° = v/2$. Thus, the separation $AB$ decreases at the rate

$$v + \frac{v}{2} = \frac{3v}{2}.$$

Since this rate is constant, the time taken in reducing the separation $AB$ from $d$ to zero is

$$t = \frac{d}{\frac{3v}{2}} = \frac{2d}{3v}.$$

□

## QUESTIONS FOR SHORT ANSWER

1. Galileo was punished by the Church for teaching that the sun is stationary and the earth moves around it. His opponents held the view that the earth is stationary and the sun moves around it. If the absolute motion has no meaning, are the two viewpoints not equally correct or equally wrong ?
2. When a particle moves with constant velocity, its average velocity, its instantaneous velocity and its speed are all equal. Comment on this statement.
3. A car travels at a speed of 60 km/hr due north and the other at a speed of 60 km/hr due east. Are the velocities equal ? If no, which one is greater ? If you find any of the questions irrelevant, explain.
4. A ball is thrown vertically upward with a speed of 20 m/s. Draw a graph showing the velocity of the ball as a function of time as it goes up and then comes back.
5. The velocity of a particle is towards west at an instant. Its acceleration is not towards west, not towards east, not towards north and not towards south. Give an example of this type of motion.
6. At which point on its path a projectile has the smallest speed ?
7. Two particles $A$ and $B$ start from rest and move for equal time on a straight line. The particle $A$ has an acceleration $a$ for the first half of the total time and $2a$ for the second half. The particle $B$ has an acceleration $2a$ for the first half and $a$ for the second half. Which particle has covered larger distance ?
8. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement ?
9. A food packet is dropped from a plane going at an altitude of 100 m. What is the path of the packet as seen from the plane ? What is the path as seen from the ground ? If someone asks "what is the actual path", what will you answer ?
10. Give examples where (a) the velocity of a particle is zero but its acceleration is not zero, (b) the velocity is opposite in direction to the acceleration, (c) the velocity is perpendicular to the acceleration.
11. Figure (3-Q1) shows the $x$ coordinate of a particle as a function of time. Find the signs of $v_x$ and $a_x$ at $t = t_1$, $t = t_2$ and $t = t_3$.

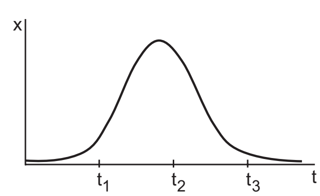


Figure 3-Q1

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**12.** A player hits a baseball at some angle. The ball goes high up in space. The player runs and catches the ball before it hits the ground. Which of the two (the player or the ball) has greater displacement ?

**13.** The increase in the speed of a car is proportional to the additional petrol put into the engine. Is it possible to accelerate a car without putting more petrol or less petrol into the engine ?

**14.** Rain is falling vertically. A man running on the road keeps his umbrella tilted but a man standing on the street keeps his umbrella vertical to protect himself from the rain. But both of them keep their umbrella vertical to avoid the vertical sun-rays. Explain.

## OBJECTIVE I

**1.** A motor car is going due north at a speed of 50 km/h. It makes a 90° left turn without changing the speed. The change in the velocity of the car is about
(a) 50 km/h towards west
(b) 70 km/h towards south-west
(c) 70 km/h towards north-west
(d) zero.

**2.** Figure (3-Q2) shows the displacement-time graph of a particle moving on the $X$-axis.

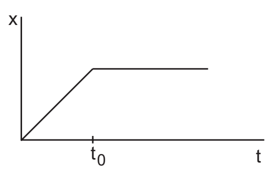


Figure 3-Q2

(a) the particle is continuously going in positive $x$ direction
(b) the particle is at rest
(c) the velocity increases up to a time $t_0$, and then becomes constant
(d) the particle moves at a constant velocity up to a time $t_0$, and then stops.

**3.** A particle has a velocity $u$ towards east at $t = 0$. Its acceleration is towards west and is constant. Let $x_A$ and $x_B$ be the magnitude of displacements in the first 10 seconds and the next 10 seconds
(a) $x_A < x_B$ (b) $x_A = x_B$ (c) $x_A > x_B$
(d) the information is insufficient to decide the relation of $x_A$ with $x_B$.

**4.** A person travelling on a straight line moves with a uniform velocity $v_1$ for some time and with uniform velocity $v_2$ for the next equal time. The average velocity $v$ is given by
(a) $v = \frac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$
(c) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (d) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$.

**5.** A person travelling on a straight line moves with a uniform velocity $v_1$ for a distance $x$ and with a uniform velocity $v_2$ for the next equal distance. The average velocity $v$ is given by
(a) $v = \frac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$
(c) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (d) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$.

**6.** A stone is released from an elevator going up with an acceleration $a$. The acceleration of the stone after the release is
(a) $a$ upward (b) $(g - a)$ upward
(c) $(g - a)$ downward (d) $g$ downward.

**7.** A person standing near the edge of the top of a building throws two balls $A$ and $B$. The ball $A$ is thrown vertically upward and $B$ is thrown vertically downward with the same speed. The ball $A$ hits the ground with a speed $v_A$ and the ball $B$ hits the ground with a speed $v_B$. We have
(a) $v_A > v_B$, (b) $v_A < v_B$ (c) $v_A = v_B$
(d) the relation between $v_A$ and $v_B$ depends on height of the building above the ground.

**8.** In a projectile motion the velocity
(a) is always perpendicular to the acceleration
(b) is never perpendicular to the acceleration
(c) is perpendicular to the acceleration for one instant only
(d) is perpendicular to the acceleration for two instants.

**9.** Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first ?
(a) the faster one (b) the slower one
(c) both will reach simultaneously
(d) depends on the masses.

**10.** The range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
(a) 25 m (b) 37 m (c) 50 m (d) 100 m.

**11.** Two projectiles $A$ and $B$ are projected with angle of projection 15° for the projectile $A$ and 45° for the projectile $B$. If $R_A$ and $R_B$ be the horizontal range for the two projectiles, then
(a) $R_A < R_B$ (b) $R_A = R_B$ (c) $R_A > R_B$
(d) the information is insufficient to decide the relation of $R_A$ with $R_B$.

**12.** A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

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(a) due north (b) 30° east of north
(c) 30° north of west (d) 60° east of north.

**13.** In the arrangement shown in figure (3-Q3), the ends $P$ and $Q$ of an inextensible string move downwards with uniform speed $u$. Pulleys $A$ and $B$ are fixed. The mass $M$ moves upwards with a speed
(a) $2u\cos\theta$ (b) $u/\cos\theta$ (c) $2u/\cos\theta$ (d) $u\cos\theta$.

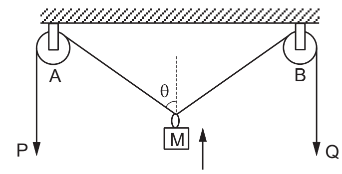

Figure 3-Q3

## OBJECTIVE II

**1.** Consider the motion of the tip of the minute hand of a clock. In one hour
(a) the displacement is zero
(b) the distance covered is zero
(c) the average speed is zero
(d) the average velocity is zero

**2.** A particle moves along the $X$-axis as

$$x = u(t - 2\text{ s}) + a(t - 2\text{ s})^2.$$

(a) the initial velocity of the particle is $u$
(b) the acceleration of the particle is $a$
(c) the acceleration of the particle is $2a$
(d) at $t$ = 2 s particle is at the origin.

**3.** Pick the correct statements :
(a) Average speed of a particle in a given time is never less than the magnitude of the average velocity.
(b) It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d}{dt}|\vec{v}| = 0$.
(c) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.
(d) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval. (Infinite accelerations are not allowed.)

**4.** An object may have
(a) varying speed without having varying velocity
(b) varying velocity without having varying speed
(c) nonzero acceleration without having varying velocity
(d) nonzero acceleration without having varying speed.

**5.** Mark the correct statements for a particle going on a straight line :
(a) If the velocity and acceleration have opposite sign, the object is slowing down.
(b) If the position and velocity have opposite sign, the particle is moving towards the origin.
(c) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
(d) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

**6.** The velocity of a particle is zero at $t$ = 0.
(a) The acceleration at $t$ = 0 must be zero.
(b) The acceleration at $t$ = 0 may be zero.
(c) If the acceleration is zero from $t$ = 0 to $t$ = 10 s, the speed is also zero in this interval.
(d) If the speed is zero from $t$ = 0 to $t$ = 10 s the acceleration is also zero in this interval.

**7.** Mark the correct statements :
(a) The magnitude of the velocity of a particle is equal to its speed.
(b) The magnitude of average velocity in an interval is equal to its average speed in that interval.
(c) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero.
(d) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

**8.** The velocity-time plot for a particle moving on a straight line is shown in the figure (3-Q4).

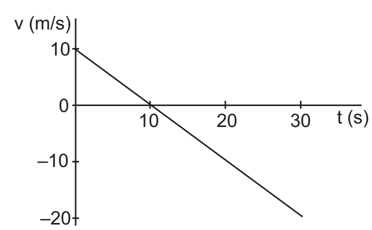

Figure 3-Q4

(a) The particle has a constant acceleration.
(b) The particle has never turned around.
(c) The particle has zero displacement.
(d) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s.

**9.** Figure (3-Q5) shows the position of a particle moving on the $X$-axis as a function of time.
(a) The particle has come to rest 6 times.
(b) The maximum speed is at $t$ = 6 s.
(c) The velocity remains positive for $t$ = 0 to $t$ = 6 s.
(d) The average velocity for the total period shown is negative.

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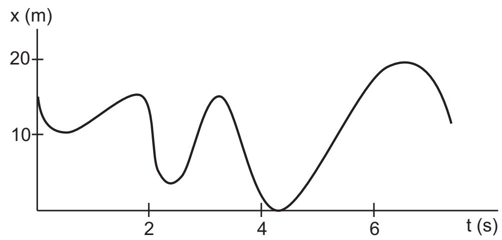


Figure 3-Q5

**10.** The accelerations of a particle as seen from two frames $S_1$ and $S_2$ have equal magnitude 4 m/s$^2$.
(a) The frames must be at rest with respect to each other.
(b) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.
(c) The acceleration of $S_2$ with respect to $S_1$ may either be zero or 8 m/s$^2$.
(d) The acceleration of $S_2$ with respect to $S_1$ may be anything between zero and 8 m/s$^2$.

## EXERCISES

**1.** A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field. (a) What distance he has to walk to reach the field? (b) What is his displacement from his house to the field?

**2.** A particle starts from the origin, goes along the $X$-axis to the point (20 m, 0) and then returns along the same line to the point (–20 m, 0). Find the distance and displacement of the particle during the trip.

**3.** It is 260 km from Patna to Ranchi by air and 320 km by road. An aeroplane takes 30 minutes to go from Patna to Ranchi whereas a delux bus takes 8 hours. (a) Find the average speed of the plane. (b) Find the average speed of the bus. (c) Find the average velocity of the plane. (d) Find the average velocity of the bus.

**4.** When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km. (a) What is the average speed of the car during this period? (b) What is the average velocity?

**5.** An athelete takes 2·0 s to reach his maximum speed of 18·0 km/h. What is the magnitude of his average acceleration?

**6.** The speed of a car as a function of time is shown in figure (3-E1). Find the distance travelled by the car in 8 seconds and its acceleration.

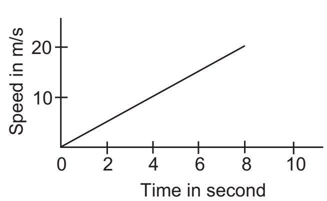


Figure 3-E1

**7.** The acceleration of a cart started at $t = 0$, varies with time as shown in figure (3-E2). Find the distance travelled in 30 seconds and draw the position-time graph.

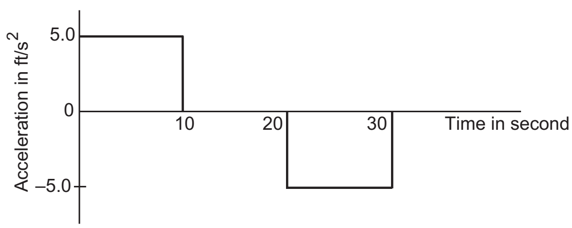


Figure 3-E2

**8.** Figure (3-E3) shows the graph of velocity versus time for a particle going along the $X$-axis. Find (a) the acceleration, (b) the distance travelled in 0 to 10 s and (c) the displacement in 0 to 10 s.

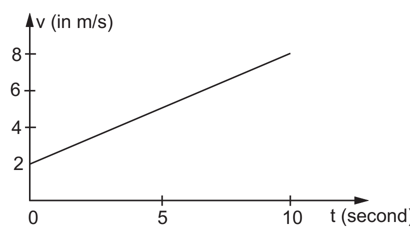


Figure 3-E3

**9.** Figure (3-E4) shows the graph of the $x$-coordinate of a particle going along the $X$-axis as a function of time. Find (a) the average velocity during 0 to 10 s, (b) instantaneous velocity at 2, 5, 8 and 12s.

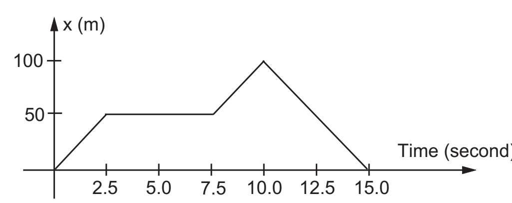


Figure 3-E4

**10.** From the velocity–time plot shown in figure (3-E5), find the distance travelled by the particle during the first 40

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seconds. Also find the average velocity during this period.

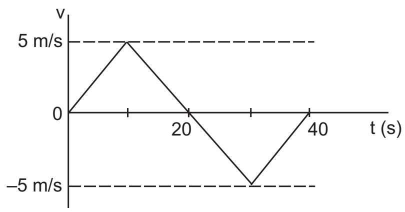


Figure 3-E5

**11.** Figure (3-E6) shows *x-t* graph of a particle. Find the time *t* such that the average velocity of the particle during the period 0 to *t* is zero.

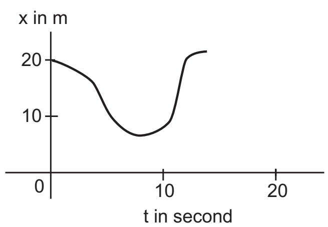


Figure 3-E6

**12.** A particle starts from a point *A* and travels along the solid curve shown in figure (3-E7). Find approximately the position *B* of the particle such that the average velocity between the positions *A* and *B* has the same direction as the instantaneous velocity at *B*.

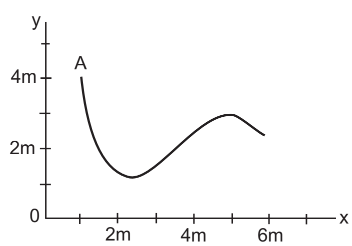


Figure 3-E7

**13.** An object having a velocity 4·0 m/s is accelerated at the rate of 1·2 m/s$^2$ for 5·0 s. Find the distance travelled during the period of acceleration.

**14.** A person travelling at 43·2 km/h applies the brake giving a deceleration of 6·0 m/s$^2$ to his scooter. How far will it travel before stopping ?

**15.** A train starts from rest and moves with a constant acceleration of 2·0 m/s$^2$ for half a minute. The brakes are then applied and the train comes to rest in one minute. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed.

**16.** A bullet travelling with a velocity of 16 m/s penetrates a tree trunk and comes to rest in 0·4 m. Find the time taken during the retardation.

**17.** A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of 5·0 cm before coming to rest. Find the deceleration.

**18.** A particle starting from rest moves with constant acceleration. If it takes 5·0 s to reach the speed 18·0 km/h find (a) the average velocity during this period, and (b) the distance travelled by the particle during this period.

**19.** A driver takes 0·20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6·0 m/s$^2$, find the distance travelled by the car after he sees the need to put the brakes on.

**20.** Complete the following table :

| Car Model | Driver *X*<br>Reaction time 0·20 s | Driver *Y*<br>Reaction time 0·30 s |
|---|---|---|
| *A* (deceleration on hard braking = 6·0 m/s$^2$) | Speed = 54 km/h<br>Braking distance<br>*a* = ..........................<br>Total stopping distance<br>*b* = .......................... | Speed = 72 km/h<br>Braking distance<br>*c* = ..........................<br>Total stopping distance<br>*d* = ........................... |
| *B* (deceleration on hard braking = 7·5 m/s$^2$) | Speed = 54 km/h<br>Braking distance<br>*e* = ..........................<br>Total stopping distance<br>*f* = .......................... | Speed = 72km/h<br>Braking distance<br>*g* = ..........................<br>Total stopping distance<br>*h* = .......................... |

**21.** A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h, crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike ?

**22.** A car travelling at 60 km/h overtakes another car travelling at 42 km/h. Assuming each car to be 5·0 m long, find the time taken during the overtake and the total road distance used for the overtake.

**23.** A ball is projected vertically upward with a speed of 50 m/s. Find (a) the maximum height, (b) the time to reach the maximum height, (c) the speed at half the maximum height. Take $g$ = 10 m/s$^2$.

**24.** A ball is dropped from a balloon going up at a speed of 7 m/s. If the balloon was at a height 60 m at the time of dropping the ball, how long will the ball take in reaching the ground ?

**25.** A stone is thrown vertically upward with a speed of 28 m/s. (a) Find the maximum height reached by the stone. (b) Find its velocity one second before it reaches the maximum height. (c) Does the answer of part (b) change if the initial speed is more than 28 m/s such as 40 m/s or 80 m/s ?

**26.** A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

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**27.** A healthy youngman standing at a distance of 7 m from a 11·8 m high building sees a kid slipping from the top floor. With what speed (assumed uniform) should he run to catch the kid at the arms height (1·8 m) ?

**28.** An NCC parade is going at a uniform speed of 6 km/h through a place under a berry tree on which a bird is sitting at a height of 12·1 m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform ?

**29.** A ball is dropped from a height. If it takes 0·200 s to cross the last 6·00 m before hitting the ground, find the height from which it was dropped. Take $g = 10 \text{ m/s}^2$.

**30.** A ball is dropped from a height of 5 m onto a sandy floor and penetrates the sand up to 10 cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform.

**31.** An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator.

**32.** A ball is thrown horizontally from a point 100 m above the ground with a speed of 20 m/s. Find (a) the time it takes to reach the ground, (b) the horizontal distance it travels before reaching the ground, (c) the velocity (direction and magnitude) with which it strikes the ground.

**33.** A ball is thrown at a speed of 40 m/s at an angle of 60° with the horizontal. Find (a) the maximum height reached and (b) the range of the ball. Take $g = 10 \text{ m/s}^2$.

**34.** In a soccer practice session the football is kept at the centre of the field 40 yards from the 10 ft high goalposts. A goal is attempted by kicking the football at a speed of 64 ft/s at an angle of 45° to the horizontal. Will the ball reach the goal post ?

**35.** A popular game in Indian villages is *goli* which is played with small glass balls called golis. The goli of one player is situated at a distance of 2·0 m from the goli of the second player. This second player has to project his goli by keeping the thumb of the left hand at the place of his goli, holding the goli between his two middle fingers and making the throw. If the projected goli hits the goli of the first player, the second player wins. If the height from which the goli is projected is 19·6 cm from the ground and the goli is to be projected horizontally, with what speed should it be projected so that it directly hits the stationary goli without falling on the ground earlier ?

**36.** Figure (3-E8) shows a 11·7 ft wide ditch with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motorbike be moving on the road so that it safely crosses the ditch ?

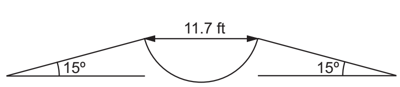


Figure 3-E8

Assume that the length of the bike is 5 ft, and it leaves the road when the front part runs out of the approach road.

**37.** A person standing on the top of a cliff 171 ft high has to throw a packet to his friend standing on the ground 228 ft horizontally away. If he throws the packet directly aiming at the friend with a speed of 15·0 ft/s, how short will the packet fall ?

**38.** A ball is projected from a point on the floor with a speed of 15 m/s at an angle of 60° with the horizontal. Will it hit a vertical wall 5 m away from the point of projection and perpendicular to the plane of projection without hitting the floor ? Will the answer differ if the wall is 22 m away ?

**39.** Find the average velocity of a projectile between the instants it crosses half the maximum height. It is projected with a speed $u$ at an angle $\theta$ with the horizontal.

**40.** A bomb is dropped from a plane flying horizontally with uniform speed. Show that the bomb will explode vertically below the plane. Is the statement true if the plane flies with uniform speed but not horizontally ?

**41.** A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of $1 \text{ m/s}^2$ and the projection velocity in the vertical direction is 9·8 m/s. How far behind the boy will the ball fall on the car ?

**42.** A staircase contains three steps each 10 cm high and 20 cm wide (figure 3-E9). What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane ?

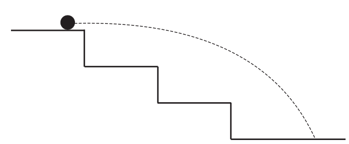

Figure 3-E9

**43.** A person is standing on a truck moving with a constant velocity of 14·7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58·8 m. Find the speed and the angle of projection (a) as seen from the truck, (b) as seen from the road.

**44.** The benches of a gallery in a cricket stadium are 1 m wide and 1 m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit ?

**45.** A man is sitting on the shore of a river. He is in the line of a 1·0 m long boat and is 5·5 m away from the centre of the boat. He wishes to throw an apple into the boat. If he can throw the apple only with a speed of 10 m/s, find the minimum and maximum angles of projection for successful shot. Assume that the point of

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projection and the edge of the boat are in the same horizontal level.

**46.** A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?

**47.** A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle $\theta$ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

**48.** Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

**49.** An aeroplane has to go from a point $A$ to another point $B$, 500 km away due 30° east of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (a) Find the direction in which the pilot should head the plane to reach the point $B$. (b) Find the time taken by the plane to go from $A$ to $B$.

**50.** Two friends $A$ and $B$ are standing a distance $x$ apart in an open field and wind is blowing from $A$ to $B$. $A$ beats a drum and $B$ hears the sound $t_1$ time after he sees the event. $A$ and $B$ interchange their positions and the experiment is repeated. This time $B$ hears the drum $t_2$ time after he sees the event. Calculate the velocity of sound in still air $v$ and the velocity of wind $u$. Neglect the time light takes in travelling between the friends.

**51.** Suppose $A$ and $B$ in the previous problem change their positions in such a way that the line joining them becomes perpendicular to the direction of wind while maintaining the separation $x$. What will be the time lag $B$ finds between seeing and hearing the drum beating by $A$ ?

**52.** Six particles situated at the corners of a regular hexagon of side $a$ move at a constant speed $v$. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (b) | 2. (d) | 3. (d) | 4. (a) | 5. (c) | 6. (d) |
| 7. (c) | 8. (c) | 9. (c) | 10. (d) | 11. (d) | 12. (a) |
| 13. (b) | | | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (a), (d) | 2. (c), (d) | 3. (a), (b), (c) |
| 4. (b), (d) | 5. (a), (b), (d) | 6. (b), (c), (d) |
| 7. (a) | 8. (a), (d) | 9. (a) |
| 10. (d) | | |

### EXERCISES

1. (a) 110 m (b) 50 m, $\tan^{-1}$ 3/4 north to east
2. 60 m, 20 m in the negative direction
3. (a) 520 km/h (b) 40 km/h
   (c) 520 km/h Patna to Ranchi
   (d) 32·5 km/h Patna to Ranchi
4. 32 km/h (b) zero
5. 2·5 m/s$^2$
6. 80 m, 2·5 m/s$^2$
7. 1000 ft
8. (a) 0·6 m/s$^2$ (b) 50 m (c) 50 m
9. (a) 10 m/s (b) 20 m/s, zero, 20 m/s, – 20 m/s
10. 100 m, zero
11. 12 s
12. $x$ = 5 m, $y$ = 3 m
13. 35 m
14. 12 m
15. (a) 2·7 km (b) 60 m/s (c) 225 m and 2·25 km
16. 0·05 s
17. 12·2 × 10$^5$ m/s$^2$
18. (a) 2·5 m/s (b) 12·5 m
19. 22 m
20. (a) 19 m (b) 22 m (c) 33 m (d) 39 m
    (e) 15 m (f) 18 m (g) 27 m (h) 33 m
21. 1·0 km
22. 2 s, 38 m
23. (a) 125 m (b) 5 s (c) 35 m/s
24. 4·3 s
25. (a) 40 m (b) 9·8 m/s (c) No
26. 44·1 m, 19·6 m and 4·9 m below the top
27. 4·9 m/s
28. 2·62 m
29. 48 m
30. 490 m/s$^2$
31. 20 ft/s$^2$
32. (a) 4·5 s (b) 90 m (c) 49 m/s, $\theta$ = 66° with horizontal
33. (a) 60 m (b) $80\sqrt{3}$ m
34. Yes
35. 10 m/s
36. 32 ft/s

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37. 192 ft
38. Yes, Yes
39. $u \cos\theta$, horizontal in the plane of projection
41. 2 m
42. 2 m/s
43. (a) 19·6 m/s upward
    (b) 24·5 m/s at 53° with horizontal
44. Sixth
45. Minimum angle 15°, maximum angle 75° but there is an interval of 53° between 15° and 75°, which is not allowed for successful shot
46. (a) 40 s (b) 80 m
47. (a) $\frac{10 \text{ minutes}}{\sin\theta}$ (b) 10 minutes
48. 2/3 km
49. (a) $\sin^{-1}$ (1/15) east of the line $AB$ (b) 50 min
50. $\frac{x}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right), \frac{x}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)$
51. $\frac{x}{\sqrt{v^2-u^2}}$
52. $2a/v$.

□

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CHAPTER 4

# THE FORCES

## 4.1 INTRODUCTION

Force is a word which we have all heard about. When you push or pull some object you exert a force on it. If you push a body you exert a force away from yourself; when you pull, you exert a force toward yourself. When you hold a heavy block in your hand you exert a large force; when you hold a light block, you exert a small force.

Can nonliving bodies exert a force ? Yes, they can. If we stand in a great storm, we feel that the wind is exerting a force on us. When we suspend a heavy block from a rope, the rope holds the block just as a man can hold it in air. When we comb our dry hair and bring the comb close to small pieces of paper, the pieces jump to the comb. The comb has attracted the paper pieces i.e. the comb has exerted force on the pieces. When a cork is dipped in water it comes to the surface; if we want to keep it inside water, we have to push it downward. We say that water exerts a force on the cork in the upward direction.

The SI unit for measuring the force is called a *newton*. Approximately, it is the force needed to hold a body of mass 102 g near the earth's surface. An accurate quantitative definition can be framed using Newton's laws of motion to be studied in the next chapter.

Force is an interaction between two objects. Force is exerted by an object $A$ on another object $B$. For any force you may ask two questions, (i) who exerted this force and (ii) on which object was this force exerted ? Thus, when a block is kept on a table, the table exerts a force on the block to hold it.

Force is a vector quantity and if more than one forces act on a particle we can find the resultant force using the laws of vector addition. Note that in all the examples quoted above, if a body $A$ exerts a force on $B$, the body $B$ also exerts a force on $A$. Thus, the table exerts a force on the block to hold it and the block exerts a force on the table to press it down. When a heavy block is suspended by a rope, the rope exerts a force on the block to hold it and the block exerts a force on the rope to make it tight and stretched. In fact these are a few examples of Newton's third law of motion which may be stated as follows.

**Newton's Third Law of Motion**

*If a body A exerts a force* $\vec{F}$ *on another body B, then B exerts a force* $-\vec{F}$ *on A, the two forces acting along the line joining the bodies.*

The two forces $\vec{F}$ and $-\vec{F}$ connected by Newton's third law are called *action-reaction pair*. Any one may be called 'action' and the other 'reaction'.

We shall discuss this law in greater detail in the next chapter.

The various types of forces in nature can be grouped in four categories :

(a) Gravitational, (b) Electromagnetic,
(c) Nuclear and (d) Weak.

## 4.2 GRAVITATIONAL FORCE

Any two bodies attract each other by virtue of their masses. The force of attraction between two point masses is $F = G\frac{m_1 m_2}{r^2}$, where $m_1$ and $m_2$ are the masses of the particles and $r$ is the distance between them. $G$ is a universal constant having the value $6{\cdot}67 \times 10^{-11}$ N-m $^2$/kg $^2$. To find the gravitational force on an extended body by another such body, we have to write the force on each particle of the 1st body by all the particles of the second body and then we have to sum up vectorially all the forces acting on the first body. For example, suppose each body contains just three particles, and let $\vec{F}_{ij}$ denote the force on the $i$ th particle of the first body due to the $j$ th particle of the second body. To find the resultant force on the first body (figure 4.1), we have to add the following 9 forces :

$$\vec{F}_{11},\ \vec{F}_{12},\ \vec{F}_{13},\ \vec{F}_{21},\ \vec{F}_{22},\ \vec{F}_{23},\ \vec{F}_{31},\ \vec{F}_{32},\ \vec{F}_{33}.$$

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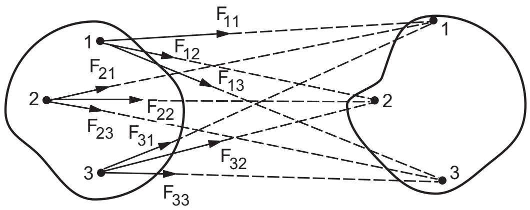


Figure 4.1

For large bodies having a large number of particles, we have to add quite a large number of forces. If the bodies are assumed continuous (a good approximation in our course), one has to go through the integration process for the infinite summation involved. However, the integration yields a particularly simple result for a special case which is of great practical importance and we quote it below. The proof of this result will be given in a later chapter.

*The gravitational force exerted by a spherically symmetric body of mass $m_1$ on another such body of mass $m_2$ kept outside the first body is $G\frac{m_1 m_2}{r^2}$, where r is the distance between the centres of the two bodies. Thus, for the calculation of gravitational force between two spherically symmetric bodies, they can be treated as point masses placed at their centres.*

**Gravitational Force on Small Bodies by the Earth**

The force of attraction exerted by the earth on other objects is called *gravity*. Consider the earth to be a homogeneous sphere of radius $R$ and mass $M$. The values of $R$ and $M$ are roughly 6400 km and $6 \times 10^{24}$ kg respectively. Assuming that the earth is spherically symmetric, the force it exerts on a particle of mass $m$ kept near its surface is by the previous result, $F = G\frac{Mm}{R^2}$. The direction of this force is towards the centre of the earth which is called the *vertically downward* direction.

The same formula is valid to a good approximation even if we have a body of some other shape instead of a particle, provided the body is very small in size as compared to the earth. The quantity $G\frac{M}{R^2}$ is a constant and has the dimensions of acceleration. It is called the *acceleration due to gravity*, and is denoted by the letter $g$ (a quantity much different from $G$). Its value is approximately $9{\cdot}8\ \mathrm{m/s^2}$. For simplicity of calculations we shall often use $g = 10\ \mathrm{m/s^2}$. We shall find in the next chapter that all bodies falling towards earth (remaining all the time close to the earth's surface) have this particular value of acceleration and hence the name acceleration due to gravity. Thus, the force exerted by the earth on a small body of mass $m$, kept near the earth's surface is $mg$ in the vertically downward direction.

The gravitational constant $G$ is so small that the gravitational force becomes appreciable only if at least one of the two bodies has a large mass. To have an idea of the magnitude of gravitational forces in practical life, consider two small bodies of mass 10 kg each, separated by 0·5 m. The gravitational force is

$$F = \frac{6{\cdot}7 \times 10^{-11}\ \mathrm{N\text{–}m^2/kg^2} \times 10^2\ \mathrm{kg^2}}{0{\cdot}25\ \mathrm{m^2}}$$

$$= 2{\cdot}7 \times 10^{-8}\ \mathrm{N}$$

a force needed to hold about 3 microgram. In many of the situations we encounter, it is a good approximation to neglect all the gravitational forces other than that exerted by the earth.

## 4.3 ELECTROMAGNETIC (EM) FORCE

Over and above the gravitational force $G\frac{m_1 m_2}{r^2}$, the particles may exert upon each other electromagnetic forces. If two particles having charges $q_1$ and $q_2$ are at rest with respect to the observer, the force between them has a magnitude

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where $\varepsilon = 8{\cdot}85419 \times 10^{-12}\ \mathrm{C^2/N\text{–}m^2}$ is a constant. The quantity $\frac{1}{4\pi\varepsilon_0}$ is $9{\cdot}0 \times 10^9\ \frac{\mathrm{N\text{–}m^2}}{\mathrm{C^2}}$.

This is called *Coulomb force* and it acts along the line joining the particles. If $q_1$ and $q_2$ are of same nature (both positive or both negative), the force is repulsive otherwise it is attractive. It is this force which is responsible for the attraction of small paper pieces when brought near a recently used comb. The electromagnetic force between moving charged paritcles is comparatively more complicated and contains terms other than the Coulomb force.

Ordinary matter is composed of electrons, protons and neutrons. Each electron has $1{\cdot}6 \times 10^{-19}$ coulomb of negative charge and each proton has an equal amount of positive charge. In atoms, the electrons are bound by the electromagnetic force acting on them due to the protons. The atoms combine to form molecules due to the electromagnetic forces. A lot of atomic and molecular phenomena result from electromagnetic forces between the subatomic particles (electrons, protons, charged mesons, etc.).

Apart from the atomic and molecular phenomena, the electromagnetic forces show up in many forms in

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daily experience. Some examples having practical importance given below.

### (a) Forces between Two Surfaces in Contact

When we put two bodies in contact with each other, the atoms at the two surfaces come close to each other. The charged constituents of the atoms of the two bodies exert great forces on each other and a measurable force results out of it. We say that the two bodies in contact exert forces on each other. When you place a book on a table, the table exerts an upward force on the book to hold it. This force comes from the electromagnetic forces acting between the atoms and molecules of the surface of the book and of the table.

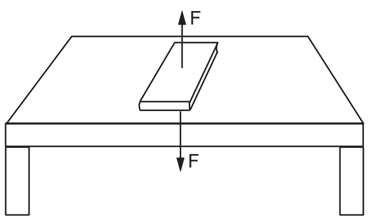


Figure 4.2

Generally, the forces between the two objects in contact are along the common normal (perpendicular) to the surfaces of contact and is that of a push or repulsion. Thus, the table pushes the book away from it (i.e., upward) and the book pushes the table downward (again away from it).

However, the forces between the two bodies in contact may have a component parallel to the surface of contact. This component is known as *friction*. We assume existence of frictionless surfaces which can exert forces only along the direction perpendicular to them. The bodies with smooth surfaces can exert only small amount of forces parallel to the surface and hence are close to frictionless surface. Thus, it is difficult to stay on a smooth metallic lamp-post, because it cannot exert enough vertical force and so it will not hold you there. The same is not true if you try to stay on the trunk of a tree which is quite rough. We shall often use the word smooth to mean frictionless.

The contact forces obey Newton's third law. Thus the book in figure (4.2) exerts a downward force $F$ on the table to press it down and the table exerts an equal upward force $F$ on the book to hold it there. When you stay on the trunk of a tree, it exerts a frictional upward force (frictional force because it is parallel to the surface of the tree) on you to hold you there, and you exert an equal frictional downward force on the tree.

### (b) Tension in a String or a Rope

In a tug of war, two persons hold the two ends of a rope and try to pull the rope on their respective sides. The rope becomes tight and its length is slightly increased. In many situations this increase is very small and goes undetected. Such a stretched rope is said to be in a state of tension.

Similarly, if a heavy block hangs from a ceiling by a string, the string is in a state of tension. The electrons and protons of the string near the lower end exert forces on the electrons and protons of the block and the resultant of these forces is the force exerted by the string on the block. It is the resultant of these electromagnetic forces which supports the block and prevents it from falling. A string or rope under tension exerts electromagnetic forces on the bodies attached at the two ends to *pull* them.

### (c) Force due to a Spring

When a metallic wire is coiled it becomes a spring. The straight line distance between the ends of a spring is called its length. If a spring is placed on a horizontal surface with no horizontal force on it, its length is called the *natural length*. Every spring has its own natural length. The spring can be stretched to increase its length and it can be compressed to decrease its length. When a spring is stretched, it pulls the bodies attached to its ends and when compressed, it pushes the bodies attached to its ends. If the extension or the compression is not too large, the force exerted by the spring is proportional to the change in its length. Thus, if the spring has a length $x$ and its natural length is $x_0$ the magnitude of the force exerted by it will be

$$F = k\left|x - x_0\right| = k\left|\Delta x\right|.$$

If the spring is extended, the force will be directed towards its centre and if compressed, it will be directed away from the centre. The proportionality constant $k$, which is the force per unit extension or compression, is called the *spring constant* of the spring. This force again comes into picture due to the electromagnetic forces between the atoms of the material.

The macroscopic bodies which we have to generally deal with are electrically neutral. Hence two bodies not in contact do not exert appreciable electromagnetic forces. The forces between the charged particles of the first body and those of the second body have both attractive and repulsive nature and hence they largely cancel each other. This is not the case with gravitational forces. The gravitational forces between the particles of one body and those of the other body are all attractive and hence they add to give an appreciable gravitational force in many cases. Thus, the gravitational force between the earth and a 1 kg

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block kept 100 m above the earth's surface is about 9·8 N whereas the electromagnetic force between the earth and this block is almost zero even though both these bodies contain a very large number of charged particles, the electrons and the protons.

***Example 4.1***

*Suppose the exact charge neutrality does not hold in a world and the electron has a charge* 1% *less in magnitude than the proton. Calculate the Coulomb force acting between two blocks of iron each of mass* 1 kg *separated by a distance of* 1 m. *Number of protons in an iron atom* = 26 *and* 58 kg *of iron contains* $6 \times 10^{26}$ *atoms.*

***Solution :*** Each atom of iron will have a net positive charge $26 \times 0{\cdot}01 \times 1{\cdot}6 \times 10^{-19}$ C on it in the assumed world. The total positive charge on a 1 kg block would be

$$\frac{6 \times 10^{26}}{58} \times 26 \times 1{\cdot}6 \times 10^{-21}\ \mathrm{C}$$

$$= 4{\cdot}3 \times 10^{5}\ \mathrm{C}.$$

The Coulomb force between the two blocks is

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{9{\cdot}0 \times 10^{9}\ \mathrm{N{-}m^2/C^2} \times (4{\cdot}3 \times 10^{5}\ \mathrm{C})^2}{(1\ \mathrm{m})^2}$$

$$= 9 \times 10^{9} \times 18{\cdot}49 \times 10^{10}\ \mathrm{N}$$

$$= 1{\cdot}7 \times 10^{21}\ \mathrm{N}.$$

A tremendous force indeed !

## 4.4 NUCLEAR FORCES

Each atom contains a certain number of protons and neutrons in its nucleus. The nucleus occupies a volume of about $10^{-44}\ \mathrm{m}^3$ whereas the atom itself has a volume of about $10^{-23}\ \mathrm{m}^3$. Thus, the nucleus occupies only $1/10^{21}$ of the volume of the atom. Yet it contains about 99·98% of the mass of the atom. The atomic nucleus of a non-radioactive element is a stable particle. For example, if both the electrons are removed from a helium atom, we get the bare nucleus of helium which is called an *alpha particle*. The alpha particle is a stable object and once created it can remain intact until it is not made to interact with other objects.

An alpha particle contains two protons and two neutrons. The protons will repel each other due to the Coulomb force and will try to break the nucleus. Neutrons will be silent spectators in this electromagnetic drama (Remember, neutron is an uncharged particle). Then, why does the Coulomb force fail to break the nucleus ? Can it be the gravitational attractive force which keeps the nucleus bound ? All the protons and the neutrons will take part in this attraction, but if calculated, the gravitational attraction will turn out to be totally negligible as compared to the Coulomb repulsion.

In fact, a third kind of force, altogether different and over and above the gravitational and electromagnetic force, is operating here. These forces are called *Nuclear forces* and are exerted only if the interacting particles are protons or neutrons or both. (There are some more cases where this force operates but we shall not deal with them.) These forces are largely attractive, but are short ranged. The forces are much weaker than the Coulomb force if the separation between the particles is more than say $10^{-14}$ m. But for smaller separation ($\approx 10^{-15}$ m) the nuclear force is much stronger than the Coulomb force and being attractive it holds the nucleus stable.

Being short ranged, these forces come into picture only if the changes within the nucleus are discussed. As bare nuclei are less frequently encountered in daily life, one is generally unaware of these forces. Radioactivity, nuclear energy (fission, fusion) etc. result from nuclear forces.

## 4.5 WEAK FORCES

Yet another kind of forces is encountered when reactions involving protons, electrons and neutrons take place. A neutron can change itself into a proton and simultaneously emit an electron and a particle called *antinutrino*. This is called $\beta^-$ decay. Never think that a neutron is made up of a proton, an electron and an antineutrino. A proton can also change into neutron and simultaneously emit a positron (and a neutrino). This is called $\beta^+$ decay. The forces responsible for these changes are different from gravitational, electromagnetic or nuclear forces. Such forces are called weak forces. The range of weak forces is very small, in fact much smaller than the size of a proton or a neutron. Thus, its effect is experienced inside such particles only.

## 4.6 SCOPE OF CLASSICAL PHYSICS

The behaviour of all the bodies of linear sizes greater than $10^{-6}$ m are adequately described on the basis of relatively a small number of very simple laws of nature. These laws are the Newton's laws of motion, Newton's law of gravitation, Maxwell's electromagnetism, Laws of thermodynamics and the Lorentz force. The principles of physics based on them is called the *classical physics*. The formulation of classical physics is quite accurate for heavenly bodies like the sun, the earth, the moon etc. and is equally good for the behaviour of grains of sand and the raindrops. However, for the subatomic particles much smaller

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than $10^{-6}$ m (such as atoms, nuclei etc.) these rules do not work well. The behaviour of such particles is governed by *quantum physics*. In fact, at such short dimensions the very concept of "particle" breaks down. The perception of the nature is altogether different at this scale. The validity of classical physics also depends on the velocities involved. The classical mechanics as formulated by Newton has to be considerably changed when velocities comparable to $3 \times 10^{8}$ m/s are involved. This is the speed of light in vacuum and is the upper limit of speed which material particle can ever reach. No matter how great and how long you apply a force, you can never get a particle going with a speed greater than $3 \times 10^{8}$ m/s. The mechanics of particles moving with these large velocities is known as *relativistic mechanics* and was formulated by Einstein in 1905.

Thus, classical physics is a good description of the nature if we are concerned with the particles of linear size > $10^{-6}$ m moving with velocities < $10^{8}$ m/s. In a major part of this book, we shall work within these restrictions and hence learn the techniques of classical physics. The size restriction automatically excludes any appreciable effects of nuclear or weak forces and we need to consider only the gravitational and electromagnetic forces. We might consider the subatomic particles here and there but shall assume the existence of gravitational and electromagnetic forces only and that classical physics is valid for these particles. The results arrived at by our analysis may only be approximately true because we shall be applying the laws which are not correct in that domain. But even that may play an important role in the understanding of nature. We shall also assume that the Newton's third law is valid for the forces which we shall be dealing with. In the final chapters we shall briefly discuss quantum physics and some of its important consequences.

## *Worked Out Examples*

**1.** *Figure (4-W1) shows two hydrogen atoms. Show on a separate diagram all the electric forces acting on different particles of the system.*

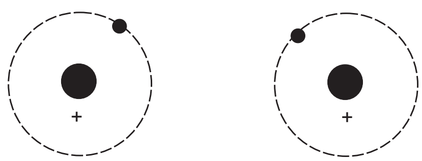


Figure 4-W1

***Solution*** **:** Each particle exerts electric forces on the remaining three particles. Thus there exist 4 × 3 = 12 forces in all. Figure (4-W2) shows them.

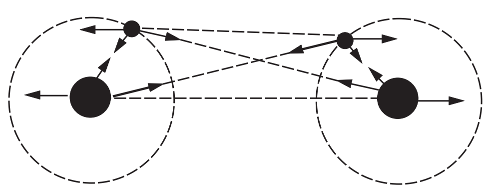

Figure 4-W2

**2.** *Figure (4-W3) shows two rods each of length $l$ placed side by side, with their facing ends separated by a distance $a$. Charges + $q$, − $q$ reside on the rods as shown. Calculate the electric force on the rod A due to the rod B. Discuss the cases when $l >> a$, $a >> l$.*

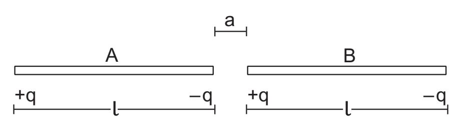


Figure 4-W3

***Solution*** **:** The force on the rod $A$ due to the charge $+q$ of the rod $B$

$$= -\frac{q^2}{4\pi\varepsilon_0 (l+a)^2} + \frac{q^2}{4\pi\varepsilon_0 a^2}$$

towards right. The force on this rod due to the charge $-q$

$$= \frac{q^2}{4\pi\varepsilon_0 (2l+a)^2} - \frac{q^2}{4\pi\varepsilon_0 (l+a)^2}$$

towards right.

The resultant force on the rod is

$$F = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{a^2} - \frac{2}{(l+a)^2} + \frac{1}{(2l+a)^2}\right] \text{towards right.}$$

If $l >> a$, the last two terms in the square bracket are negligible as compared to the first term. Then,

$$F \approx \frac{q^2}{4\pi\varepsilon_0 a^2}\,.$$

If $a >> l$

$$F \approx \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{a^2} - \frac{2}{a^2} + \frac{1}{a^2}\right] \approx 0\,.$$

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Two neutral objects placed far away exert only negligible force on each other but when they are placed closer they may exert appreciable force.

**3.** *Calculate the ratio of electric to gravitational force between two electrons.*

***Solution*** : The electric force $= \dfrac{e^2}{4\pi\varepsilon_0 r^2}$

and the gravitational force $= \dfrac{G(m_e)^2}{r^2}$.

The ratio is $\dfrac{e^2}{4\pi\varepsilon_0 G(m_e)^2}$

$$= \frac{9\times10^{9}\,\dfrac{\text{N–m}^2}{\text{C}^2}\times(1{\cdot}6\times10^{-19}\,\text{C})^2}{6{\cdot}67\times10^{-11}\,\dfrac{\text{N–m}^2}{\text{kg}^2}\times(9{\cdot}1\times10^{-31}\,\text{kg})^2} = 4{\cdot}17\times10^{42}.$$

□

## QUESTIONS FOR SHORT ANSWER

**1.** A body of mass $m$ is placed on a table. The earth is pulling the body with a force $mg$. Taking this force to be the action what is the reaction ?

**2.** A boy is sitting on a chair placed on the floor of a room. Write as many action-reaction pairs of forces as you can.

**3.** A lawyer alleges in court that the police has forced his client to issue a statement of confession. What kind of force is this ?

**4.** When you hold a pen and write on your notebook, what kind of force is exerted by you on the pen ? By the pen on the notebook ? By you on the notebook ?

**5.** Is it true that the reaction of a gravitational force is always gravitational, of an electromagnetic force is always electromagnetic and so on ?

**6.** Suppose the magnitude of Nuclear force between two protons varies with the distance between them as shown in figure (4-Q1). Estimate the ratio "Nuclear force/Coulomb force" for (a) $x$ = 8 fm (b) $x$ = 4 fm, (c) $x$ = 2 fm and (d) $x$ = 1 fm (1 fm = $10^{-15}$ m).

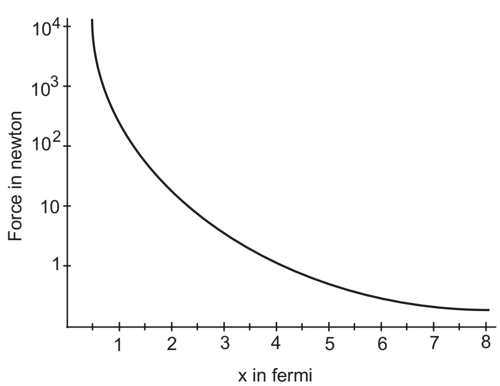


Figure 4-Q1

**7.** List all the forces acting on the block $B$ in figure (4-Q2).

**8.** List all the forces acting on (a) the pulley $A$, (b) the boy and (c) the block $C$ in figure (4-Q3).

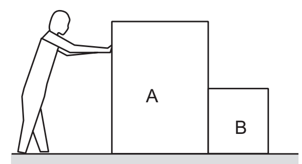


Figure 4-Q2

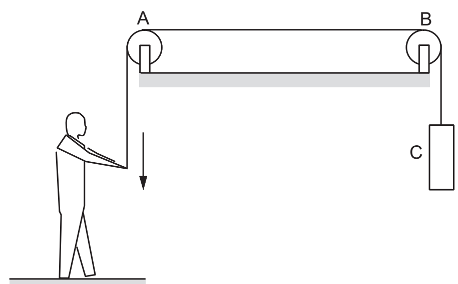


Figure 4-Q3

**9.** Figure (4-Q4) shows a boy pulling a wagon on a road. List as many forces as you can which are relevant with this figure. Find the pairs of forces connected by Newton's third law of motion.

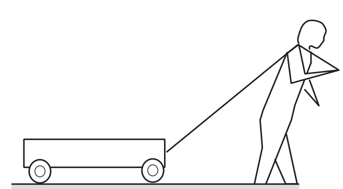

Figure 4-Q4

**10.** Figure (4-Q5) shows a cart. Complete the table shown below.

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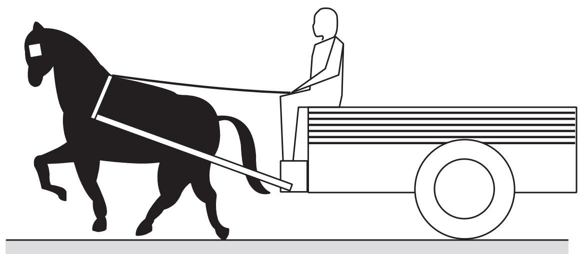

Figure 4-Q5

| Force on | Force by | Nature of the force | Direction |
|---|---|---|---|
| Cart | 1<br>2<br>3<br>: | | |
| Horse | 1<br>2<br>3<br>: | | |
| Driver | 1<br>2<br>3<br>: | | |

## OBJECTIVE I

**1.** When Neils Bohr shook hand with Werner Heisenberg, what kind of force they exerted ?
(a) Gravitational (b) Electromagnetic
(c) Nuclear (d) Weak.

**2.** Let $E$, $G$ and $N$ represent the magnitudes of electromagnetic, gravitational and nuclear forces between two electrons at a given separation. Then
(a) $N>E>G$ (b) $E>N>G$ (c) $G>N>E$ (d) $E>G>N$.

**3.** The sum of all electromagnetic forces between different particles of a system of charged particles is zero
(a) only if all the particles are positively charged
(b) only if all the particles are negatively charged
(c) only if half the particles are positively charged and half are negatively charged
(d) irrespective of the signs of the charges.

**4.** A 60 kg man pushes a 40 kg man by a force of 60 N. The 40 kg man has pushed the other man with a force of
(a) 40 N (b) 0 (c) 60 N (d) 20 N.

## OBJECTIVE II

**1.** A neutron exerts a force on a proton which is
(a) gravitational (b) electromagnetic
(c) nuclear (d) weak.

**2.** A proton exerts a force on a proton which is
(a) gravitational (b) electromagnetic
(c) nuclear (d) weak.

**3.** Mark the correct statements :
(a) The nuclear force between two protons is always greater than the electromagnetic force between them.
(b) The electromagnetic force between two protons is always greater than the gravitational force between them.
(c) The gravitational force between two protons may be greater than the nuclear force between them.
(d) Electromagnetic force between two protons may be greater than the nuclear force acting between them.

**4.** If all matter were made of electrically neutral particles such as neutrons,
(a) there would be no force of friction
(b) there would be no tension in the string
(c) it would not be possible to sit on a chair
(d) the earth could not move around the sun.

**5.** Which of the following systems may be adequately described by classical physics ?
(a) motion of a cricket ball
(b) motion of a dust particle
(c) a hydrogen atom
(d) a neutron changing to a proton.

**6.** The two ends of a spring are displaced along the length of the spring. All displacements have equal mangnitudes. In which case or cases the tension or compression in the spring will have a maximum magnitude ?
(a) the right end is displaced towards right and the left end towards left
(b) both ends are displaced towards right
(c) both ends are displaced towards left
(d) the right end is displaced towards left and the left end towards right.

**7.** Action and reaction
(a) act on two different objects
(b) have equal magnitude
(c) have opposite directions
(d) have resultant zero.

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## EXERCISES

**1.** The gravitational force acting on a particle of 1 g due to a similar particle is equal to $6{\cdot}67 \times 10^{-17}$ N. Calculate the separation between the particles.

**2.** Calculate the force with which you attract the earth.

**3.** At what distance should two charges, each equal to 1 C, be placed so that the force between them equals your weight ?

**4.** Two spherical bodies, each of mass 50 kg, are placed at a separation of 20 cm. Equal charges are placed on the bodies and it is found that the force of Coulomb repulsion equals the gravitational attraction in magnitude. Find the magnitude of the charge placed on either body.

**5.** A monkey is sitting on a tree limb. The limb exerts a normal force of 48 N and a frictional force of 20 N. Find the magnitude of the total force exerted by the limb on the monkey.

**6.** A body builder exerts a force of 150 N against a bullworker and compresses it by 20 cm. Calculate the spring constant of the spring in the bullworker.

**7.** A satellite is projected vertically upwards from an earth station. At what height above the earth's surface will the force on the satellite due to the earth be reduced to half its value at the earth station ? (Radius of the earth is 6400 km.)

**8.** Two charged particles placed at a separation of 20 cm exert 20 N of Coulomb force on each other. What will be the force if the separation is increased to 25 cm ?

**9.** The force with which the earth attracts an object is called the weight of the object. Calculate the weight of the moon from the following data : The universal constant of gravitation $G = 6{\cdot}67 \times 10^{-11}$ N–m $^2$/kg $^2$, mass of the moon $= 7{\cdot}36 \times 10^{22}$ kg, mass of the earth $= 6 \times 10^{24}$ kg and the distance between the earth and the moon $= 3{\cdot}8 \times 10^5$ km.

**10.** Find the ratio of the magnitude of the electric force to the gravitational force acting between two protons.

**11.** The average separation between the proton and the electron in a hydrogen atom in ground state is $5{\cdot}3 \times 10^{-11}$ m. (a) Calculate the Coulomb force between them at this separation. (b) When the atom goes into its first excited state the average separation between the proton and the electron increases to four times its value in the ground state. What is the Coulomb force in this state ?

**12.** The geostationary orbit of the earth is at a distance of about 36000 km from the earth's surface. Find the weight of a 120-kg equipment placed in a geostationary satellite. The radius of the earth is 6400 km.

□

## ANSWERS

### OBJECTIVE I

1. (b) 2. (d) 3. (d) 4. (c)

### OBJECTIVE II

1. (a), (c) 2. (a), (b), (c) 3. (b), (c), (d)
4. (a), (b), (c) 5. (a), (b) 6. (a), (d)
7. (a), (b), (c), (d)

### EXERCISES

1. 1 m
4. $4{\cdot}3 \times 10^{-9}$ C
5. 52 N
6. 750 N/m
7. 2650 km
8. 13 N
9. $2 \times 10^{20}$ N
10. $1{\cdot}24 \times 10^{36}$
11. (a) $8{\cdot}2 \times 10^{-8}$ N,(b) $5{\cdot}1 \times 10^{-9}$ N
12. 27 N

□

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**CHAPTER 5**

# NEWTON'S LAWS OF MOTION

Newton's laws of motion are of central importance in classical physics. A large number of principles and results may be derived from Newton's laws. The first two laws relate to the type of motion of a system that results from a given set of forces. These laws may be interpreted in a variety of ways and it is slightly uninteresting and annoying at the outset to go into the technical details of the interpretation. The precise definitions of mass, force and acceleration should be given before we relate them. And these definitions themselves need use of Newton's laws. Thus, these laws turn out to be definitions to some extent. We shall assume that we know how to assign mass to a body, how to assign the magnitude and direction to a force and how to measure the acceleration with respect to a given frame of reference. Some discussions of these aspects were given in the previous chapters. The development here does not follow the historical track these laws have gone through, but are explained to make them simple to apply.

## 5.1 FIRST LAW OF MOTION

*If the (vector) sum of all the forces acting on a particle is zero then and only then the particle remains unaccelerated (i.e., remains at rest or moves with constant velocity).*

If the sum of all the forces on a given particle is $\vec{F}$ and its acceleration is $\vec{a}$, the above statement may also be written as

"$\vec{a} = 0$ *if and only if* $\vec{F} = 0$".

Thus, if the sum of the forces acting on a particle is known to be zero, we can be sure that the particle is unaccelerated, or if we know that a particle is unaccelerated, we can be sure that the sum of the forces acting on the particle is zero.

However, the concept of rest, motion or acceleration is meaningful only when a frame of reference is specified. Also the acceleration of the particle is, in general, different when measured from different frames. Is it possible then, that the first law is valid in all frames of reference ?

Let us consider the situation shown in figure (5.1). An elevator cabin falls down after the cable breaks. The cabin and all the bodies fixed in the cabin are accelerated with respect to the earth and the acceleration is about 9·8 m/s$^2$ in the downward direction.

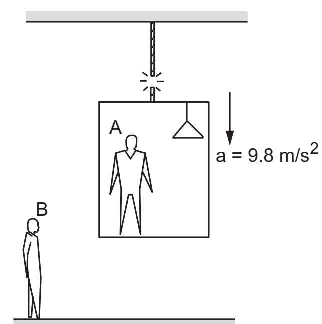


Figure 5.1

Consider the lamp in the cabin. The forces acting on the lamp are (a) the gravitational force $W$ by the earth and (b) the electromagnetic force $T$ (tension) by the rope. The direction of $W$ is downward and the directon of $T$ is upward. The sum is $(W - T)$ downward.

Measure the acceleration of the lamp from the frame of reference of the cabin. The lamp is at rest. The acceleration of the lamp is zero. The person $A$ who measured this acceleration is a learned one and uses Newton's first law to conclude that the sum of the forces acting on the particle is zero, i.e.,

$$W - T = 0 \text{ or, } W = T.$$

Instead, if we measure the acceleration from the ground, the lamp has an acceleration of 9·8 m/s$^2$. Thus, $a \neq 0$ and hence the person $B$ who measured this acceleration, concludes from Newton's first law that the sum of the forces is not zero. Thus, $W - T \neq 0$ or $W \neq T$. If $A$ measures acceleration and applies the first

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law he gets $W = T$. If $B$ measures acceleration and applies the same first law, he gets $W \neq T$. Both of them cannot be correct simultaneously as $W$ and $T$ can be either equal or unequal. At least one of the two frames is a bad frame and one should not apply the first law in that frame.

There are some frames of reference in which Newton's first law is valid. Measure acceleration from such a frame and you are allowed to say that "$\vec{a} = 0$ if and only if $\vec{F} = 0$". But there are frames in which Newton's first law is not valid. You may find that even if the sum of the forces is not zero, the acceleration is still zero. Or you may find that the sum of the forces is zero, yet the particle is accelerated. So the validity of Newton's first law depends on the frame of reference from which the observer measures the state of rest, motion and acceleration of the particle.

A frame of reference in which Newton's first law is valid is called an *inertial frame of reference*. A frame in which Newton's first law is not valid is called a *noninertial frame of reference*.

Newton's first law, thus, reduces to a definition of inertial frame. Why do we call it a law then? Suppose after going through this lesson, you keep the book on your table fixed rigidly with the earth (figure 5.2).

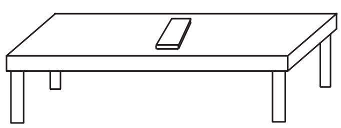

Figure 5.2

The book is at rest with respect to the earth. The acceleration of the book with respect to the earth is zero. The forces on the book are (a) the gravitational force $\vec{W}$ exerted by the earth and (b) the contact force $\vec{\mathcal{N}}$ by the table. Is the sum of $\vec{W}$ and $\vec{\mathcal{N}}$ zero? A very accurate measurement will give the answer "No". The sum of the forces is not zero although the book is at rest. The earth is not strictly an inertial frame. However, the sum is not too different from zero and we can say that the earth is an inertial frame of reference to a good approximation. Thus, for routine affairs, "$\vec{a} = 0$ if and only if $\vec{F} = 0$" is true in the earth frame of reference. This fact was identified and formulated by Newton and is known as Newton's *first law*. If we restrict that all measurements will be made from the earth frame, indeed it becomes a law. If we try to universalise this to different frames, it becomes a definition. We shall assume that unless stated otherwise, we are working from an inertial frame of reference.

***Example 5.1***

*A heavy particle of mass* 0·50 kg *is hanging from a string fixed with the roof. Find the force exerted by the string on the particle* (*Figure* 5.3). *Take* $g$ = 9·8 m/s$^2$.

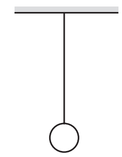

Figure 5.3

***Solution :*** The forces acting on the particle are

(a) pull of the earth, 0·50 kg × 9·8 m/s$^2$ = 4·9 N, vertically downward

(b) pull of the string, $T$ vertically upward.

The particle is at rest with respect to the earth (which we assume to be an inertial frame). Hence, the sum of the forces should be zero. Therefore, $T$ is 4·9 N acting vertically upward.

**Inertial Frames other than Earth**

Suppose $S$ is an inertial frame and $S'$ a frame moving uniformly with respect to $S$. Consider a particle $P$ having acceleration $\vec{a}_{P,S}$ with respect to $S$ and $\vec{a}_{P,S'}$ with respect to $S'$.

We know that,

$$\vec{a}_{P,S} = \vec{a}_{P,S'} + \vec{a}_{S',S}.$$

As $S'$ moves uniformly with respect to $S$,

$$\vec{a}_{S',S} = 0.$$

Thus,

$$\vec{a}_{P,S} = \vec{a}_{P,S'} \qquad \ldots \text{(i)}$$

Now $S$ is an inertial frame. So from definition, $\vec{a}_{P,S} = 0$, if and only if $\vec{F} = 0$ and hence, from (i), $\vec{a}_{P,S'} = 0$ if and only if $\vec{F} = 0$.

Thus, $S'$ is also an inertial frame. We arrive at an important result : *All frames moving uniformly with respect to an inertial frame are themselves inertial*. Thus, a train moving with uniform velocity with respect to the ground, a plane flying with uniform velocity with respect to a highway, etc., are examples of inertial frames. The sum of the forces acting on a suitcase kept on the shelf of a ship sailing smoothly and uniformly on a calm sea is zero.

**5.2 SECOND LAW OF MOTION**

*The acceleration of a particle as measured from an inertial frame is given by the (vector) sum of all the forces acting on the particle divided by its mass.*

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In symbols : $\vec{a} = \vec{F}/m$ or, $\vec{F} = m\,\vec{a}$. ... (5.2)

The inertial frame is already defined by the first law of motion. A force $\vec{F}$ acting on a particle of mass $m$ produces an acceleration $\vec{F}/m$ in it with respect to an inertial frame. This is a law of nature. If the force ceases to act at some instant, the acceleration becomes zero at the same instant. In equation (5.2) $\vec{a}$ and $\vec{F}$ are measured at the same instant of time.

## 5.3 WORKING WITH NEWTON'S FIRST AND SECOND LAW

Newton's laws refer to a particle and relate the forces acting on the particle with its acceleration and its mass. Before attempting to write an equation from Newton's law, we should very clearly understand which particle we are considering. In any practical situation, we deal with extended bodies which are collection of a large number of particles. The laws as stated above may be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing equations from Newton's laws is as follows :

**Step 1 : Decide the System**

The first step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

Consider the situation shown in figure (5.4). The block $B$ does not slip over $A$, the disc $D$ slides over the string and all parts of the string are tight.

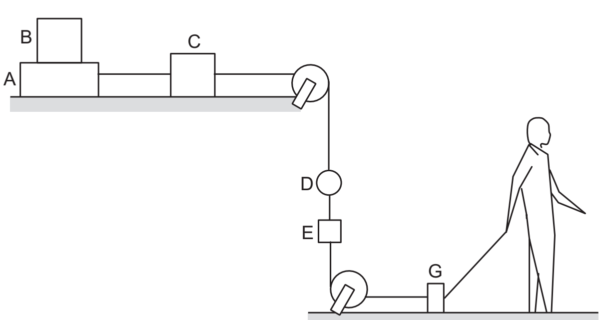


Figure 5.4

$A$ and $B$ move together. $C$ is not in contact with $A$ or $B$. But as the length of the string between $A$ and $C$ does not change, the distance moved by $C$ in any time interval is same as that by $A$. The same is true for $G$. The distance moved by $G$ in any time interval is same as that by $A$, $B$ or $C$. The direction of motion is also the same for $A$, $B$, $C$ and $G$. They have identical accelerations. We can take any of these blocks as a system or any combination of the blocks from these as a system. Some of the examples are ($A$), ($B$), ($A$ + $B$), ($B$ + $C$), ($A$ + $B$ + $C$), ($C$ + $G$), ($A$ + $C$ + $G$), ($A$ + $B$ + $C$ + $G$) etc. The distance covered by $E$ is also the same as the distance covered by $G$ but their directions are different. $E$ moves in a vertical line whereas $G$ in a horizontal line. ($E$ + $G$) should not be taken as a system. At least at this stage we are unable to apply Newton's law treating $E$ + $G$ as a single particle. As the disc $D$ slides over the string the distance covered by $D$ is not equal to that by $E$ in the same time interval. We should not treat $D$ + $E$ as a system. *Think carefully.*

**Step 2 : Identify the Forces**

Once the system is decided, make a list of the forces acting *on* the system due to all the objects other than the system. Any force applied *by* the system should not be included in the list of the forces.

Consider the situation shown in figure (5.5). The boy stands on the floor balancing a heavy load on his head. The load presses the boy, the boy pushes the load upward the boy presses the floor downward, the floor pushes the boy upward, the earth attracts the load downward, the load attracts the earth upward, the boy attracts the earth upward and the earth attracts the boy downward. There are many forces operating in this world. Which of these forces should we include in the list of forces ?

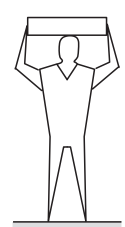

Figure 5.5

We cannot answer this question. Not because we do not know, but because we have not yet specified the system. Which is the body under consideration ? Do not try to identify forces before you have decided the system. Suppose we concentrate on the state of motion of the boy. We should then concentrate on the forces acting *on* the boy. The forces are listed in the upper half of table (5.1). Instead, if we take the load as the system and discuss the equilibrium of the load,

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the list of the forces will be different. These forces appear in the lower half of table (5.1).

**Table 5.1**

| System | Force exerted by | Magnitude of the force | Direction of the force | Nature of the force |
|---|---|---|---|---|
| | Earth | $W$ | Downward | Gravitational |
| Boy | Floor | $\mathscr{N}$ | Upward | Electro-magnetic |
| | Load | $\mathscr{N}_1$ | Downward | ,, |
| | Earth | $W'$ | Downward | Gravitational |
| Load | | | | |
| | Boy | $\mathscr{N}_1$ | Upward | Electro-magnetic |

One may furnish as much information as one has about the magnitude and direction of the forces. The contact forces may have directions other than normal to the contact surface if the surfaces are rough. We shall discuss more about it under the heading of friction.

**Step 3 : Make a Free Body Diagram**

Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin. The forces may lie along a line, may be distributed in a plane (coplanar) or may be distributed in the space (non-planar). We shall rarely encounter situations dealing with non-planar forces. For coplanar forces the plane of diagram represents the plane of the forces acting on the system. Indicate the magnitudes and directions of the forces in this diagram. This is called a *free body diagram*. The free body diagram for the example discussed above with the boy as the system and with the load as the system are shown in figure (5.6).

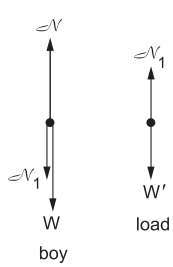


Figure 5.6

**Step 4 : Choose Axes and Write Equations**

Any three mutually perpendicular directions may be chosen as the $X$-$Y$-$Z$ axes. We give below some suggestions for choosing the axes to solve problems.

If the forces are coplanar, only two axes, say $X$ and $Y$, taken in the plane of forces are needed. Choose the $X$-axis along the direction in which the system is known to have or is likely to have the acceleration. A direction perpendicular to it may be chosen as the $Y$-axis. If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes. Write the components of all the forces along the $X$-axis and equate their sum to the product of the mass of the system and its acceleration. This gives you one equation. Write the components of the forces along the $Y$-axis and equate the sum to zero. This gives you another equation. If the forces are collinear, this second equation is not needed.

If necessary you can go to step 1, choose another object as the system, repeat steps 2, 3 and 4 to get more equations. These are called equations of motion. Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

The magnitudes of acceleration of different objects in a given situation are often related through kinematics. This should be properly foreseen and used together with the equations of motion. For example in figure (5.4) the accelerations of $C$ and $E$ have same magnitudes. Equations of motion for $C$ and for $E$ should use the same variable $a$ for acceleration.

***Example 5.2***

*A block of mass M is pulled on a smooth horizontal table by a string making an angle θ with the horizontal as shown in figure (5.7). If the acceleration of the block is a, find the force applied by the string and by the table on the block.*

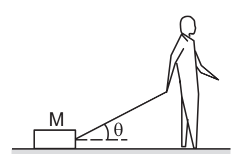


Figure 5.7

***Solution*** **:** Let us consider the block as the system.

The forces on the block are

(a) pull of the earth, $Mg$, vertically downward,

(b) contact force by the table, $\mathscr{N}$, vertically upward,

(c) pull of the string, $T$, along the string.

The free body diagram for the block is shown in figure (5.8).

The acceleration of the block is horizontal and towards the right. Take this direction as the $X$-axis and vertically upward direction as the $Y$-axis. We have,

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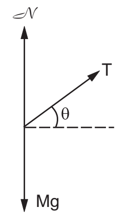


Figure 5.8

component of $Mg$ along the $X$-axis $= 0$

component of $\mathcal{N}$ along the $X$-axis $= 0$

component of $T$ along the $X$-axis $= T\cos\theta$.

Hence the total force along the $X$-axis $= T\cos\theta$.

Using Newton's law, $T\cos\theta = Ma$. ... (i)

Component of $Mg$ along the $Y$-axis $= -Mg$

component of $\mathcal{N}$ along the $Y$-axis $= \mathcal{N}$

component of $T$ along the $Y$-axis $= T\sin\theta$.

Total force along the $Y$-axis $= \mathcal{N} + T\sin\theta - Mg$.

Using Newton's law, $\mathcal{N} + T\sin\theta - Mg = 0$. ... (ii)

From equation (i), $T = \dfrac{Ma}{\cos\theta}\cdot$ Putting this in equation (ii)

$\mathcal{N} = Mg - Ma\tan\theta$.

## 5.4 NEWTON'S THIRD LAW OF MOTION

Newton's third law has already been introduced in chapter 4. "*If a body A exerts a force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A.*"

Thus, the force exerted by $A$ on $B$ and that by $B$ on $A$ are equal in magnitude but opposite in direction. This law connects the forces exerted by two bodies on one another. The forces connected by the third law act on two different bodies and hence will never appear together in the list of forces at step 2 of applying Newton's first or second law.

For example, suppose a table exerts an upward force $\mathcal{N}$ on a block placed on it. This force should be accounted if we consider the block as the system. The block pushes the table down with an equal force $\mathcal{N}$. But this force acts on the table and should be considered only if we take the table as the system. Thus, only one of the two forces connected by the third law may appear in the equation of motion depending on the system chosen. The force exerted by the earth on a particle of mass $M$ is $Mg$ downward and therefore, by the particle on the earth is $Mg$ upward. These two forces will not cancel each other. The downward force on the particle will cause acceleration of the particle and that on the earth will cause acceleration (how large ?) of the earth.

Newton's third law of motion is not strictly correct when interaction between two bodies separated by a large distance is considered. We come across such deviations when we study electric and magnetic forces.

**Working with the Tension in a String**

The idea of tension was qualitatively introduced in chapter 4. Suppose a block of mass $M$ is hanging through a string from the ceiling (figure 5.9).

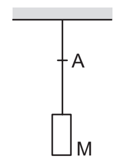


Figure 5.9

Consider a cross-section of the string at $A$. The cross-section divides the string in two parts, lower part and the upper part. The two parts are in physical contact at the cross-section at $A$. The lower part of the string will exert an electromagnetic force on the upper part and the upper part will exert an electromagnetic force on the lower part. According to the third law, these two forces will have equal magnitude. The lower part pulls down the upper part with a force $T$ and the upper part pulls up the lower part with equal force $T$. The common magnitude of the forces exerted by the two parts of the string on each other is called the tension in the string at $A$. What is the tension in the string at the lower end ? The block and the string are in contact at this end and exert electromagnetic forces on each other. The common magnitude of these forces is the tension in the string at the lower end. What is the tension in the string at the upper end ? At this end, the string and the ceiling meet. The string pulls the ceiling down and the ceiling pulls the string up. The common magnitude of these forces is the tension in the string at the upper end.

***Example 5.3***

*The mass of the part of the string below A in figure (5.9) is m. Find the tension of the string at the lower end and at A.*

***Solution*** **:** To get the tension at the lower end we need the force exerted by the string on the block.

Take the block as the system. The forces on it are

(a) pull of the string, $T$, upward,

(b) pull of the earth, $Mg$, downward,

The free body diagram for the block is shown in figure (5.10a). As the acceleration of the block is zero, these forces should add to zero. Hence the tension at the lower end is $T = Mg$.

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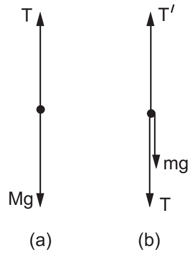


Figure 5.10

To get the tension $T$ at $A$ we need the force exerted by the upper part of the string on the lower part of the string. For this we may write the equation of motion for the lower part of the string. So take the string below $A$ as the system. The forces acting on this part are

(a) $T'$, upward, by the upper part of the string

(b) $mg$, downward, by the earth

(c) $T$, downward, by the block.

Note that in (c) we have written $T$ for the force by the block on the string. We have already used the symbol $T$ for the force by the string on the block. We have used Newton's third law here. The force exerted by the block on the string is equal in magnitude to the force exerted by the string on the block.

The free body diagram for this part is shown in figure (5.10b). As the system under consideration (the lower part of the string) is in equilibrium, Newton's first law gives

$$T' = T + mg$$

But $T = Mg$ hence, $T' = (M + m)g$.

***Example 5.4***

*The block shown in figure (5.11) has a mass M and descends with an acceleration a. The mass of the string below the point A is m. Find the tension of the string at the point A and at the lower end.*

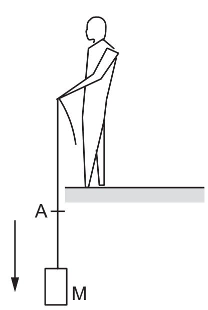


Figure 5.11

***Solution*** **:** Consider "the block + the part of the string below $A$" as the system. Let the tension at $A$ be $T$. The forces acting on this system are

(a) $(M + m)g$, downward, by the earth

(b) $T$, upward, by the upper part of the string.

The first is gravitational and the second is electromagnetic. We do not have to write the force by the string on the block. This electromagnetic force is by one part of the system on the other part. Only the forces acting on the system by the objects other than the system are to be included.

The system is descending with an acceleration $a$. Taking the downward direction as the $X$-axis, the total force along the $X$-axis is $(M + m)g - T$. Using Newton's law

$$(M + m)g - T = (M + m)a.$$

or, $$T = (M + m)\,(g - a). \qquad \dots \text{(i)}$$

We have omitted the free body diagram. This you can do if you can draw the free body diagram in your mind and write the equations correctly.

To get the tension $T'$ at the lower end we can put $m = 0$ in (i).

Effectively, we take the point $A$ at the lower end. Thus, we get $T' = M(g - a)$.

Suppose the string in ***Example 5.3*** or ***5.4*** is very light so that we can neglect the mass of the string. Then $T' = T$. The tension is then the same throughout the string. This result is of general nature. The tension at all the points in a string or a spring is the same provided it is assumed massless and no massive particle or body is connected in between.

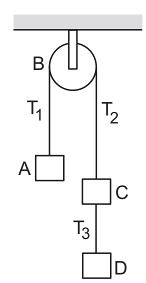


Figure 5.12

If the string in figure (5.12) is light, the tension $T_1$ of the string is same at all the points between the block $A$ and the pulley $B$. The tension $T_2$ is same at all the points between the pulley $B$ and the block $C$. The tension $T_3$ is same at all the points between the block $C$ and the block $D$. The three tensions $T_1$, $T_2$ and $T_3$ may be different from each other. If the pulley $B$ is also light, then $T_1 = T_2$.

## 5.5 PSEUDO FORCES

In this section we discuss the techniques of solving the motion of a body with respect to a noninertial frame of reference.

Consider the situation shown in figure (5.13). Suppose the frame of reference $S'$ moves with a

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constant acceleration $\vec{a}_0$ with respect to an inertial frame $S$. The acceleration of a particle $P$ measured with respect to $S'$ is $\vec{a}_{P,S'} = \vec{a}$ and that with respect to $S$ is $\vec{a}_{P,S}$. The acceleration of $S'$ with respect to $S$ is $\vec{a}_{S',S} = \vec{a}_0$.

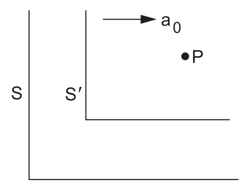


Figure 5.13

Since $S'$ is translating with respect to $S$ we have,

$$\vec{a}_{P,S'} = \vec{a}_{P,S} + \vec{a}_{S,S'} = \vec{a}_{P,S} - \vec{a}_{S',S}$$

or, $$\vec{a} = \vec{a}_{P,S} - \vec{a}_0$$

or, $$m\vec{a} = m\vec{a}_{P,S} - m\vec{a}_0$$

where $m$ is the mass of the particle $P$. Since $S$ is an inertial frame $m\vec{a}_{P,S}$ is equal to the sum of all the forces acting on $P$. Writing this sum as $\vec{F}$, we get

$$m\vec{a} = \vec{F} - m\vec{a}_0$$

or, $$\vec{a} = \frac{\vec{F} - m\vec{a}_0}{m}. \quad \ldots \quad (5.3)$$

This equation relates the acceleration of the particle and the forces acting on it. Compare it with equation (5.2) which relates the acceleration and the force when the acceleration is measured with respect to an inertial frame. The acceleration of the frame (with respect to an inertial frame) comes into the equation of a particle. Newton's second law $\vec{a} = \vec{F}/m$ is not valid in such a noninertial frame. An extra term $-m\vec{a}_0$ has to be added to the sum of all the forces acting on the particle before writing the equation $\vec{a} = \vec{F}/m$. Note that in this extra term, $m$ is the mass of the particle under consideration and $\vec{a}_0$ is the acceleration of the working frame of reference with respect to some inertial frame.

However, we people spend most of our lifetime on the earth which is an (approximate) inertial frame. We are so familiar with the Newton's laws that we would still like to use the terminology of Newton's laws even when we use a noninertial frame. This can be done if we agree to call $(-m\vec{a}_0)$ a *force* acting on the particle. Then while preparing the list of the forces acting on the particle $P$, we include all the (real) forces acting on $P$ by all other objects and also include an imaginary force $-m\vec{a}_0$. Applying Newton's second law will then lead to equation (5.3). Such correction terms $-m\,a_0$ in the list of forces are called *pseudo forces*. This so-called force is to be included in the list only because we are discussing the motion from a noninertial frame and still want to use Newton's second law as "total force equals mass times acceleration". If we work from an inertial frame, the acceleration $\vec{a}_0$ of the frame is zero and no pseudo force is needed. The pseudo forces are also called *inertial forces* although their need arises because of the use of noninertial frames.

***Example 5.5***

*A pendulum is hanging from the ceiling of a car having an acceleration $a_0$ with respect to the road. Find the angle made by the string with the vertical.*

***Solution*** **:** The situation is shown in figure (5.14a). Suppose the mass of the bob is $m$ and the string makes an angle $\theta$ with the vertical. We shall work from the car frame. This frame is noninertial as it has an acceleration $\vec{a}_0$ with respect to an inertial frame (the road). Hence, if we use Newton's second law we shall have to include a pseudo force.

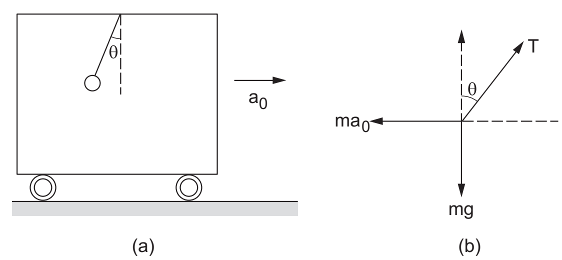


Figure 5.14

Take the bob as the system.

The forces are :

(a) $T$ along the string, by the string
(b) $mg$ downward, by the earth
(c) $ma_0$ towards left (pseudo force).

The free body diagram is shown in figure (5.14b). As the bob is at rest (remember we are discussing the motion with respect to the car) the force in (a), (b) and (c) should add to zero. Take $X$-axis along the forward horizontal direction and $Y$-axis along the upward vertical direction. The components of the forces along the $X$-axis give

$$T\sin\theta - m\,a_0 = 0 \quad \text{or,} \quad T\sin\theta = m\,a_0 \quad \ldots \quad \text{(i)}$$

and the components along the $Y$-axis give

$$T\cos\theta - mg = 0 \quad \text{or,} \quad T\cos\theta = mg. \quad \ldots \quad \text{(ii)}$$

Dividing (i) by (ii) $\tan\theta = a_0/g$.

Thus, the string makes an angle $\tan^{-1}(a_0/g)$ with the vertical.

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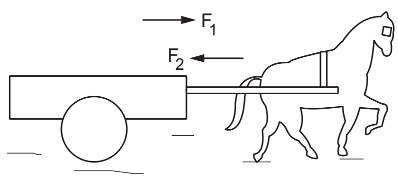
## 5.6 THE HORSE AND THE CART

A good example which illustrates the ideas discussed in this chapter is the motion of a cart pulled by a horse. Suppose the cart is at rest when the driver whips the horse. The horse pulls the cart and the cart accelerates forward. The question posed is as follows. The horse pulls the cart by a force $F_1$ in the forward direction. From the third law of motion the cart pulls the horse by an equal force $F_2 = F_1$ in the backward direction. The sum of these forces is, therefore, zero (figure 5.15). Why should then the cart accelerate forward ?




$F_1$ : Force on the cart by the horse

$F_2$ : Force on the horse by the cart

$F_1 = F_2 = F$

Figure 5.15

Try to locate the mistake in the argument. According to our scheme, we should first decide the system. We can take the horse as the system or the cart as the system or the cart and the horse taken together as the system. Suppose you take the cart as the system. Then the forces on the cart should be listed and the forces on the horse should not enter the discussion. The force on the cart is $F_1$ in the forward direction and the acceleration of the cart is also in the forward direction. How much is this acceleration ? Take the mass of the cart to be $M_C$. Is the acceleration of the cart $a = F_1/M_C$ in forward direction ? Think carefully. We shall return to this question.

Let us now try to understand the motion of the horse. This time we have to consider the forces on the horse. The forward force $F_1$ by the horse acts on the cart and it should not be taken into account when we discuss the motion of the horse. The force on the horse by the cart is $F_2$ in the backward direction. Why does the horse go in the forward direction when whipped ? The horse exerts a force on the cart in the forward direction and hence the cart is accelerated forward. But the cart exerts an equal force on the horse in the backward direction. Why is the horse not accelerated in backward direction ? (Imagine this situation. If the cart is accelerated forward and the horse backward, the horse will sit on the cart kicking out the driver and the passengers.) Where are we wrong ? We have not considered *all* the forces acting on the horse. The road pushes the horse by a force $P$ which has a forward component. This force acts on the horse and we must add this force when we discuss the motion of the horse. The horse accelerates forward if the forward component $f$ of the force $P$ exceeds $F_2$ (Figure 5.16). The acceleration of the horse is $(f - F_2)/M_h$. We should make sure that *all* the forces acting on the system are added. Note that the force of gravity acting on the horse has no forward component.

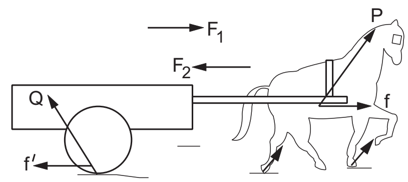



Figure 5.16

Going back to the previous paragraph the acceleration of the cart may not be $F_1/M_C$. The road exerts a force $Q$ on the cart which may have a backward component $f'$. The total force on the cart is $F_1 - f'$. The acceleration of the cart is then $a = \dfrac{F_1 - f'}{M_C}$ in the forward direction.

The forces $f$ and $f'$ are self adjustable and they so adjust their values that $\dfrac{F_1 - f'}{M_C} = \dfrac{f - F_2}{M_h}$. The acceleration of the horse and that of the cart are equal in magnitude and direction and hence they move together.

So, once again we remind you that only the forces on the system are to be considered to discuss the motion of the system and all the forces acting on the system are to be considered. Only then apply $\vec{F} = m\vec{a}$.

## 5.7 INERTIA

A particle is accelerated (in an inertial frame) if and only if a resultant force acts on it. Loosely speaking, the particle does not change its state of rest or of uniform motion along a straight line unless it is forced to do this. This unwillingness of a particle to change its state of rest or of uniform motion along a straight line is called as *inertia*. We can understand the property of inertia in more precise terms as follows. If equal forces are applied on two particles, in general, the acceleration of the particles will be different. The property of a particle to allow a smaller acceleration is called *inertia*. It is clear that larger the mass of the particle, smaller will be the acceleration and hence larger will be the inertia.

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## *Worked Out Examples*

**1.** *A body of mass m is suspended by two strings making angles $\alpha$ and $\beta$ with the horizontal. Find the tensions in the strings.*

***Solution*** : Take the body of mass $m$ as the system. The forces acting on the system are

(i) $mg$ downwards (by the earth),

(ii) $T_1$ along the first string (by the first string) and

(iii) $T_2$ along the second string (by the second string).

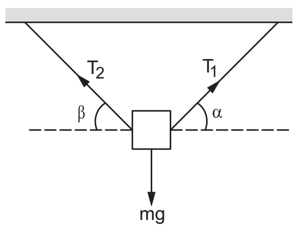


Figure 5-W1

These forces are shown in figure (5-W1). As the body is in equilibrium, these forces must add to zero. Taking horizontal components,

$$T_1 \cos\alpha - T_2 \cos\beta + mg \cos\frac{\pi}{2} = 0$$

or, $$T_1 \cos\alpha = T_2 \cos\beta. \quad \text{... (i)}$$

Taking vertical components,

$$T_1 \sin\alpha + T_2 \sin\beta - mg = 0. \quad \text{... (ii)}$$

Eliminating $T_2$ from (i) and (ii),

$$T_1 \sin\alpha + T_1 \frac{\cos\alpha}{\cos\beta} \sin\beta = mg$$

or, $$T_1 = \frac{mg}{\sin\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta} = \frac{mg\cos\beta}{\sin(\alpha+\beta)}.$$

From (i), $$T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}.$$

**2.** *Two bodies of masses $m_1$ and $m_2$ are connected by a light string going over a smooth light pulley at the end of an incline. The mass $m_1$ lies on the incline and $m_2$ hangs vertically. The system is at rest. Find the angle of the incline and the force exerted by the incline on the body of mass $m_1$ (figure 5-W2).*

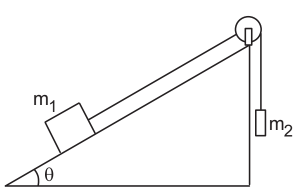


Figure 5-W2

***Solution*** : Figure (5-W3) shows the situation with the forces on $m_1$ and $m_2$ shown. Take the body of mass $m_2$ as the system. The forces acting on it are

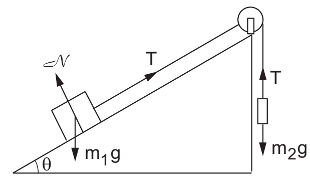


Figure 5-W3

(i) $m_2 g$ vertically downward (by the earth),

(ii) $T$ vertically upward (by the string).

As the system is at rest, these forces should add to zero.

This gives $$T = m_2 g. \quad \text{... (i)}$$

Next, consider the body of mass $m_1$ as the system. The forces acting on this system are

(i) $m_1 g$ vertically downward (by the earth),

(ii) $T$ along the string up the incline (by the string),

(iii) $\mathcal{N}$ normal to the incline (by the incline).

As the string and the pulley are all light and smooth, the tension in the string is uniform everywhere. Hence, same $T$ is used for the equations of $m_1$ and $m_2$. As the system is in equilibrium, these forces should add to zero.

Taking components parallel to the incline,

$$T = m_1 g \cos\left(\frac{\pi}{2} - \theta\right) = m_1 g \sin\theta. \quad \text{... (ii)}$$

Taking components along the normal to the incline,

$$\mathcal{N} = m_1 g \cos\theta. \quad \text{... (iii)}$$

Eliminating $T$ from (i) and (ii),

$$m_2 g = m_1 g \sin\theta$$

or, $$\sin\theta = m_2 / m_1$$

giving $$\theta = \sin^{-1}(m_2/m_1).$$

From (iii) $$\mathcal{N} = m_1 g \sqrt{1 - (m_2/m_1)^2}.$$

**3.** *A bullet moving at 250 m/s penetrates 5 cm into a tree limb before coming to rest. Assuming that the force exerted by the tree limb is uniform, find its magnitude. Mass of the bullet is 10 g.*

***Solution*** : The tree limb exerts a force on the bullet in the direction opposite to its velocity. This force causes deceleration and hence the velocity decreases from 250 m/s to zero in 5 cm. We have to find the force exerted by the tree limb on the bullet. If $a$ be the deceleration of the bullet, we have,

$$u = 250 \text{ m/s}, \; v = 0, \; x = 5 \text{ cm} = 0{\cdot}05 \text{ m}$$

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giving, $$a = \frac{(250 \text{ m/s})^2 - 0^2}{2 \times 0{\cdot}05 \text{ m}} = 625000 \text{ m/s}^2.$$

The force on the bullet is $F = ma = 6250$ N.

**4.** *The force on a particle of mass* 10 g *is* $(\vec{i}\,10 + \vec{j}\,5)$ N. *If it starts from rest what would be its position at time* $t = 5$ s ?

***Solution*** : We have $F_x = 10$ N giving

$$a_x = \frac{F_x}{m} = \frac{10 \text{ N}}{0{\cdot}01 \text{ kg}} = 1000 \text{ m/s}^2.$$

As this is a case of constant acceleration in $x$-direction,

$$x = u_x t + \frac{1}{2} a_x t^2 = \frac{1}{2} \times 1000 \text{ m/s}^2 \times (5 \text{ s})^2$$
$$= 12500 \text{ m}$$

Similarly, $a_y = \dfrac{F_y}{m} = \dfrac{5 \text{ N}}{0{\cdot}01 \text{ kg}} = 500 \text{ m/s}^2$

and $y = 6250$ m.

Thus, the position of the particle at $t = 5$ s is,

$$\vec{r} = (\vec{i}\,12500 + \vec{j}\,6250) \text{ m}.$$

**5.** *With what acceleration 'a' should the box of figure (5-W4) descend so that the block of mass M exerts a force* $Mg/4$ *on the floor of the box ?*

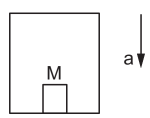


Figure 5-W4

***Solution*** : The block is at rest with respect to the box which is accelerated with respect to the ground. Hence, the acceleration of the block with respect to the ground is '$a$' downward. The forces on the block are

(i) $Mg$ downward (by the earth) and

(ii) $\mathcal{N}$ upward (by the floor).

The equation of motion of the block is, therefore

$$Mg - \mathcal{N} = Ma.$$

If $\mathcal{N} = Mg/4$, the above equation gives $a = 3g/4$. The block and hence the box should descend with an acceleration $3g/4$.

**6.** *A block 'A' of mass m is tied to a fixed point C on a horizontal table through a string passing round a massless smooth pulley B (figure 5-W5). A force F is applied by the experimenter to the pulley. Show that if the pulley is displaced by a distance x, the block will be displaced by 2x. Find the acceleration of the block and the pulley.*

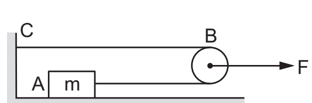


Figure 5-W5

***Solution*** : Suppose the pulley is displaced to $B'$ and the block to $A'$ (figure 5-W6). The length of the string is $CB + BA$ and is also equal to $CB + BB' + B'B + BA'$. Hence, $CB + BA' + A'A = CB + BB' + B'B + BA'$

or, $A'A = 2\,BB'$.

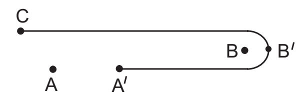


Figure 5-W6

The displacement of $A$ is, therefore, twice the displacement of $B$ in any given time interval. Diffrentiating twice, we find that the acceleration of $A$ is twice the acceleration of $B$.

To find the acceleration of the block we will need the tension in the string. That can be obtained by considering the pulley as the system.

The forces acting on the pulley are

(i) $F$ towards right by the experimenter,

(ii) $T$ towards left by the portion $BC$ of the string and

(iii) $T$ towards left by the portion $BA$ of the string.

The vertical forces, if any, add to zero as there is no vertical motion.

As the mass of the pulley is zero, the equation of motion is

$F - 2T = 0$ giving $T = F/2$.

Now consider the block as the system. The only horizontal force acting on the block is the tension $T$ towards right. The acceleration of the block is, therefore, $a = T/m = \dfrac{F}{2m}\cdot$ The acceleration of the pulley is $a/2 = \dfrac{F}{4m}\cdot$

**7.** *A smooth ring A of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley B and carries a block C of mass M* ( = 2 *m*) *as shown in figure (5-W7). At an instant the string between the ring and the pulley makes an angle* θ *with the rod. (a) Show that, if the ring slides with a speed v, the block descends with speed v* cos θ. *(b) With what acceleration will the ring start moving if the system is released from rest with* θ = 30° ?

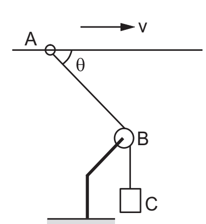


Figure 5-W7

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***Solution*** : (a) Suppose in a small time interval $\Delta t$ the ring is displaced from $A$ to $A'$ (figure 5-W8) and the block from $C$ to $C'$. Drop a perpendicular $A'P$ from $A'$ to $AB$. For small displacement $A'B \approx PB$. Since the length of the string is constant, we have

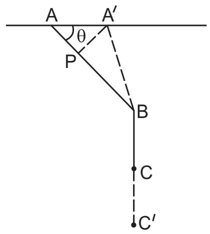


Figure 5-W8

$$AB + BC = A'B + BC'$$

or, $$AP + PB + BC = A'B + BC'$$

or, $$AP = BC' - BC = CC' \quad (\text{as } A'B \approx PB)$$

or, $$AA' \cos\theta = CC'$$

or, $$\frac{AA' \cos\theta}{\Delta t} = \frac{CC'}{\Delta t}$$

or, (velocity of the ring) cos θ = (velocity of the block).

(b) If the initial acceleration of the ring is $a$, that of the block will be $a \cos\theta$. Let $T$ be the tension in the string at this instant. Consider the block as the system. The forces acting on the block are

(i) $Mg$ downward due to the earth, and

(ii) $T$ upward due to the string.

The equation of motion of the block is

$$Mg - T = Ma \cos\theta. \quad \dots \text{(i)}$$

Now consider the ring as the system. The forces on the ring are

(i) $Mg$ downward due to gravity,

(ii) $\mathcal{N}$ upward due to the rod,

(iii) $T$ along the string due to the string.

Taking components along the rod, the equation of motion of the ring is

$$T \cos\theta = ma. \quad \dots \text{(ii)}$$

From (i) and (ii)

$$Mg - \frac{ma}{\cos\theta} = M a \cos\theta$$

or, $$a = \frac{M g \cos\theta}{m + M \cos^2\theta}.$$

Putting $\theta = 30°$, $M = 2m$ and $g = 9{\cdot}8 \text{ m/s}^2$; therefore

$$a = 6{\cdot}78 \text{ m/s}^2.$$

**8.** *A light rope fixed at one end of a wooden clamp on the ground passes over a tree branch and hangs on the other side (figure 5-W9). It makes an angle of* 30° *with the ground. A man weighing* (60 kg) *wants to climb up the rope. The wooden clamp can come out of the ground if an upward force greater than* 360 N *is applied to it. Find the maximum acceleration in the upward direction with which the man can climb safely. Neglect friction at the tree branch. Take* $g = 10 \text{ m/s}^2$.

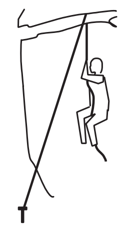

Figure 5-W9

***Solution*** : Let $T$ be the tension in the rope. The upward force on the clamp is $T \sin 30° = T/2$. The maximum tension that will not detach the clamp from the ground is, therefore, given by

$$\frac{T}{2} = 360 \text{ N}$$

or, $$T = 720 \text{ N}.$$

If the acceleration of the man in the upward direction is $a$, the equation of motion of the man is

$$T - 600 \text{ N} = (60 \text{ kg})\, a$$

The maximum acceleration of the man for safe climbing is, therefore

$$a = \frac{720 \text{ N} - 600 \text{ N}}{60 \text{ kg}} = 2 \text{ m/s}^2.$$

**9.** *Three blocks of masses* $m_1, m_2$ *and* $m_3$ *are connected as shown in the figure (5–W10). All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of* $m_1$.

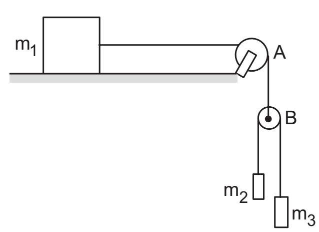


Figure 5-W10

***Solution*** : Suppose the acceleration of $m_1$ is $a_0$ towards right. That will also be the downward acceleration of the pulley $B$ because the string connecting $m_1$ and $B$ is constant in length. Also the string connecting $m_2$ and $m_3$ has a constant length. This implies that the decrease in the separation between $m_2$ and $B$ equals the increase in the separation between $m_3$ and $B$. So, the upward acceleration of $m_2$ with respect to $B$ equals the downward acceleration of $m_3$ with respect to $B$. Let this acceleration be $a$.

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The acceleration of $m_2$ with respect to the ground $= a_0 - a$ (downward) and the acceleration of $m_3$ with respect to the ground $= a_0 + a$ (downward).

These accelerations will be used in Newton's laws. Let the tension be $T$ in the upper string and $T'$ in the lower string. Consider the motion of the pulley $B$.

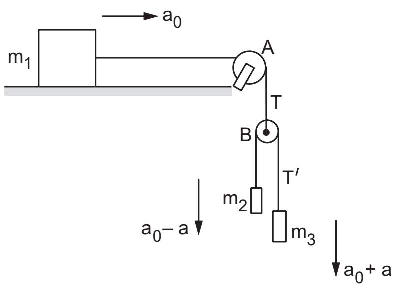


Figure 5-W11

The forces on this light pulley are
(a) $T$ upwards by the upper string and
(b) $2\,T'$ downwards by the lower string.
As the mass of the pulley is negligible,

$$2\,T' - T = 0$$

giving

$$T' = T/2. \qquad \ldots \text{(i)}$$

Motion of $m_1$ :

The acceleration is $a_0$ in the horizontal direction. The forces on $m_1$ are

(a) $T$ by the string (horizontal).

(b) $m_1 g$ by the earth (vertically downwards) and

(c) $\mathscr{N}$ by the table (vertically upwards).

In the horizontal direction, the equation is

$$T = m_1 a_0. \qquad \ldots \text{(ii)}$$

Motion of $m_2$ : acceleration is $a_0 - a$ in the downward direction. The forces on $m_2$ are

(a) $m_2 g$ downward by the earth and

(b) $T' = T/2$ upward by the string.

Thus, $$m_2 g - T/2 = m_2 (a_0 - a) \qquad \ldots \text{(iii)}$$

Motion of $m_3$ : The acceleration is $(a_0 + a)$ downward. The forces on $m_3$ are

(a) $m_3 g$ downward by the earth and

(b) $T' = T/2$ upward by the string. Thus,

$$m_3 g - T/2 = m_3 (a_0 + a). \qquad \ldots \text{(iv)}$$

We want to calculate $a_0$, so we shall eliminate $T$ and $a$ from (ii), (iii) and (iv).

Putting $T$ from (ii) in (iii) and (iv),

$$a_0 - a = \frac{m_2 g - m_1 a_0/2}{m_2} = g - \frac{m_1 a_0}{2\,m_2}$$

and $$a_0 + a = \frac{m_3 g - m_1 a_0/2}{m_3} = g - \frac{m_1 a_0}{2\,m_3}.$$

Adding, $$2a_0 = 2g - \frac{m_1 a_0}{2}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0 = g - \frac{m_1 a_0}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right] = g$$

or, $$a_0 = \frac{g}{1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)}.$$

**10.** *A particle slides down a smooth inclined plane of elevation θ, fixed in an elevator going up with an acceleration $a_0$ (figure 5-W12). The base of the incline has a length L. Find the time taken by the particle to reach the bottom.*

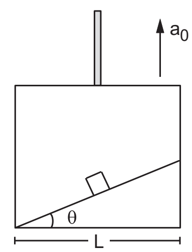


Figure 5-W12

***Solution*** : Let us work in the elevator frame. A pseudo force $ma_0$ in the downward direction is to be applied on the particle of mass $m$ together with the real forces. Thus, the forces on $m$ are (figure 5-W13)

(i) $\mathscr{N}$ normal force,

(ii) $mg$ downward (by the earth),

(iii) $ma_0$ downward (pseudo).

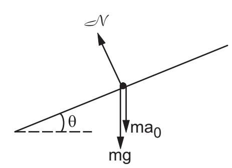


Figure 5-W13

Let $a$ be the acceleration of the particle with respect to the incline. Taking components of the forces parallel to the incline and applying Newton's law,

$$m\,g\,\sin\theta + ma_0 \sin\theta = m\,a$$

or, $$a = (g + a_0)\sin\theta.$$

This is the acceleration with respect to the elevator. In this frame, the distance travelled by the particle is $L/\cos\theta$. Hence,

$$\frac{L}{\cos\theta} = \frac{1}{2}(g + a_0)\sin\theta . t^2$$

or, $$t = \left[\frac{2\,L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}.$$

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**11.** *All the surfaces shown in figure (5-W14) are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.*

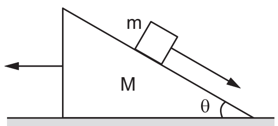


Figure 5-W14

***Solution*** : Let the acceleration of the prism be $a_0$ in the backward direction. Consider the motion of the smaller block from the frame of the prism.

The forces on the block are (figure 5-W15a)

(i) $\mathcal{N}$ normal force,
(ii) $mg$ downward (gravity),
(iii) $ma_0$ forward (pseudo).

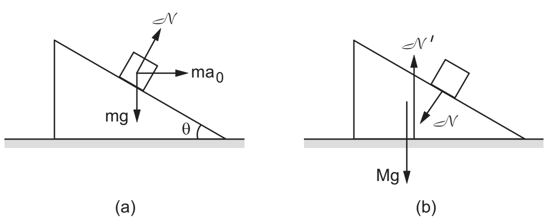


Figure 5-W15

The block slides down the plane. Components of the forces parallel to the incline give

$$ma_0 \cos\theta + mg \sin\theta = ma$$

or, $$a = a_0 \cos\theta + g \sin\theta. \quad \dots \text{(i)}$$

Components of the force perpendicular to the incline give

$$\mathcal{N} + ma_0 \sin\theta = mg \cos\theta. \quad \dots \text{(ii)}$$

Now consider the motion of the prism from the lab frame. No pseudo force is needed as the frame used is inertial. The forces are (figure 5-W15b)

(i) $Mg$ downward,
(ii) $\mathcal{N}$ normal to the incline (by the block),
(iii) $\mathcal{N}'$ upward (by the horizontal surface).

Horizontal components give,

$$\mathcal{N} \sin\theta = Ma_0 \quad \text{or,} \quad \mathcal{N} = Ma_0/\sin\theta. \quad \dots \text{(iii)}$$

Putting in (ii)

$$\frac{Ma_0}{\sin\theta} + ma_0 \sin\theta = mg \cos\theta$$

or, $$a_0 = \frac{m\,g \sin\theta \cos\theta}{M + m \sin^2\theta}.$$

From (i), $$a = \frac{m\,g \sin\theta \cos^2\theta}{M + m \sin^2\theta} + g \sin\theta$$

$$= \frac{(M+m)\,g \sin\theta}{M + m \sin^2\theta}.$$

□

## QUESTIONS FOR SHORT ANSWER

**1.** The apparent weight of an object increases in an elevator while accelerating upward. A moongphaliwala sells his moongphali using a beam balance in an elevator. Will he gain more if the elevator is accelerating up ?

**2.** A boy puts a heavy box of mass $M$ on his head and jumps down from the top of a multistoried building to the ground. How much is the force exerted by the box on his head during his free fall ? Does the force greatly increase during the period he balances himself after striking the ground ?

**3.** A person drops a coin. Describe the path of the coin as seen by the person if he is in (a) a car moving at constant velocity and (b) in a freely falling elevator.

**4.** Is it possible for a particle to describe a curved path if no force acts on it ? Does your answer depend on the frame of reference chosen to view the particle ?

**5.** You are riding in a car. The driver suddenly applies the brakes and you are pushed forward. Who pushed you forward ?

**6.** It is sometimes heard that inertial frame of reference is only an ideal concept and no such inertial frame actually exists. Comment.

**7.** An object is placed far away from all the objects that can exert force on it. A frame of reference is constructed by taking the origin and axes fixed in this object. Will the frame be necessarily inertial ?

**8.** Figure (5-Q1) shows a light spring balance connected to two blocks of mass 20 kg each. The graduations in the balance measure the tension in the spring. (a) What is the reading of the balance? (b) Will the reading change if the balance is heavy, say 2·0 kg ? (c) What will happen if the spring is light but the blocks have unequal masses ?

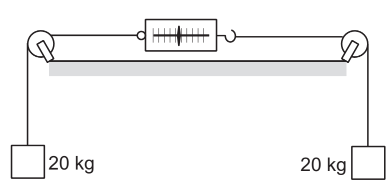


Figure 5-Q1

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**9.** The acceleration of a particle is zero as measured from an inertial frame of reference. Can we conclude that no force acts on the particle ?

**10.** Suppose you are running fast in a field when you suddenly find a snake in front of you. You stop quickly. Which force is responsible for your deceleration ?

**11.** If you jump barefooted on a hard surface, your legs get injured. But they are not injured if you jump on a soft surface like sand or pillow. Explain.

**12.** According to Newton's third law each team pulls the opposite team with equal force in a tug of war. Why then one team wins and the other loses ?

**13.** A spy jumps from an airplane with his parachute. The spy accelerates downward for some time when the parachute opens. The acceleration is suddenly checked and the spy slowly falls on the ground. Explain the action of parachute in checking the acceleration.

**14.** Consider a book lying on a table. The weight of the book and the normal force by the table on the book are equal in magnitude and opposite in direction. Is this an example of Newton's third law ?

**15.** Two blocks of unequal masses are tied by a spring. The blocks are pulled stretching the spring slightly and the system is released on a frictionless horizontal platform. Are the forces due to the spring on the two blocks equal and opposite ? If yes, is it an example of Newton's third law ?

**16.** When a train starts, the head of a standing passenger seems to be pushed backward. Analyse the situation from the ground frame. Does it really go backward ? Coming back to the train frame, how do you explain the backward movement of the head on the basis of Newton's laws ?

**17.** A plumb bob is hung from the ceiling of a train compartment. If the train moves with an acceleration '$a$' along a straight horizontal track, the string supporting the bob makes an angle $\tan^{-1}(a/g)$ with the normal to the ceiling. Suppose the train moves on an inclined straight track with uniform velocity. If the angle of incline is $\tan^{-1}(a/g)$, the string again makes the same angle with the normal to the ceiling. Can a person sitting inside the compartment tell by looking at the plumb line whether the train is accelerated on a horizontal straight track or it is going on an incline ? If yes, how ? If no, suggest a method to do so.

## OBJECTIVE I

**1.** A body of weight $w_1$ is suspended from the ceiling of a room through a chain of weight $w_2$. The ceiling pulls the chain by a force

(a) $w_1$ (b) $w_2$ (c) $w_1 + w_2$ (d) $\frac{w_1 + w_2}{2}$.

**2.** When a horse pulls a cart, the force that helps the horse to move forward is the force exerted by

(a) the cart on the horse (b) the ground on the horse
(c) the ground on the cart (d) the horse on the ground.

**3.** A car accelerates on a horizontal road due to the force exerted by

(a) the engine of the car (b) the driver of the car
(c) the earth (d) the road.

**4.** A block of mass 10 kg is suspended through two light spring balances as shown in figure (5-Q2).

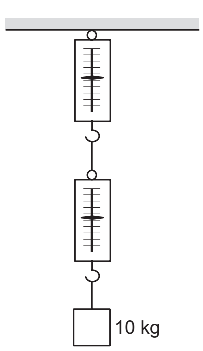


Figure 5-Q2

(a) Both the scales will read 10 kg.
(b) Both the scales will read 5 kg.
(c) The upper scale will read 10 kg and the lower zero.
(d) The readings may be anything but their sum will be 10 kg.

**5.** A block of mass $m$ is placed on a smooth inclined plane of inclination $\theta$ with the horizontal. The force exerted by the plane on the block has a magnitude

(a) $mg$ (b) $mg/\cos\theta$ (c) $mg\cos\theta$ (d) $mg\tan\theta$.

**6.** A block of mass $m$ is placed on a smooth wedge of inclination $\theta$. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude

(a) $mg$ (b) $mg/\cos\theta$ (c) $mg\cos\theta$ (d) $mg\tan\theta$.

**7.** Neglect the effect of rotation of the earth. Suppose the earth suddenly stops attracting objects placed near its surface. A person standing on the surface of the earth will

(a) fly up (b) slip along the surface
(c) fly along a tangent to the earth's surface
(d) remain standing.

**8.** Three rigid rods are joined to form an equilateral triangle $ABC$ of side 1 m. Three particles carrying charges 20 μC each are attached to the vertices of the triangle. The whole system is at rest in an inertial frame. The resultant force on the charged particle at $A$ has the magnitude

(a) zero (b) 3·6 N (c) $3\cdot6\sqrt{3}$ N (d) 7·2 N.

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**9.** A force $F_1$ acts on a particle so as to accelerate it from rest to a velocity $v$. The force $F_1$ is then replaced by $F_2$ which decelerates it to rest.
(a) $F_1$ must be equal to $F_2$ (b) $F_1$ may be equal to $F_2$
(c) $F_1$ must be unequal to $F_2$ (d) none of these.

**10.** Two objects $A$ and $B$ are thrown upward simultaneously with the same speed. The mass of $A$ is greater than the mass of $B$. Suppose the air exerts a constant and equal force of resistance on the two bodies.
(a) The two bodies will reach the same height.
(b) $A$ will go higher than $B$.
(c) $B$ will go higher than $A$.
(d) Any of the above three may happen depending on the speed with which the objects are thrown.

**11.** A smooth wedge $A$ is fitted in a chamber hanging from a fixed ceiling near the earth's surface. A block $B$ placed at the top of the wedge takes a time $T$ to slide down the length of the wedge. If the block is placed at the top of the wedge and the cable supporting the chamber is broken at the same instant, the block will
(a) take a time longer than $T$ to slide down the wedge
(b) take a time shorter than $T$ to slide down the wedge
(c) remain at the top of the wedge
(d) jump off the wedge.

**12.** In an imaginary atmosphere, the air exerts a small force $F$ on any particle in the direction of the particle's motion. A particle of mass $m$ projected upward takes a time $t_1$ in reaching the maximum height and $t_2$ in the return journey to the original point. Then
(a) $t_1 < t_2$ (b) $t_1 > t_2$ (c) $t_1 = t_2$ (d) the relation between $t_1$ and $t_2$ depends on the mass of the particle.

**13.** A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in a time $t_1$ if the elevator is stationary and in time $t_2$ if it is moving uniformly. Then
(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

**14.** A free $^{238}U$ nucleus kept in a train emits an alpha particle. When the train is stationary, a nucleus decays and a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes $x$ at time $t$ after the decay. If the decay takes place while the train is moving at a uniform velocity $v$, the distance between the alpha particle and the recoiling nucleus at a time $t$ after the decay as measured by the passenger is
(a) $x + v\,t$ (b) $x - v\,t$ (c) $x$
(d) depends on the direction of the train.

## OBJECTIVE II

**1.** A reference frame attached to the earth
(a) is an inertial frame by definition
(b) cannot be an inertial frame because the earth is revolving around the sun
(c) is an inertial frame because Newton's laws are applicable in this frame
(d) cannot be an inertial frame because the earth is rotating about its axis.

**2.** A particle stays at rest as seen in a frame. We can conclude that
(a) the frame is inertial
(b) resultant force on the particle is zero
(c) the frame may be inertial but the resultant force on the particle is zero
(d) the frame may be noninertial but there is a nonzero resultant force.

**3.** A particle is found to be at rest when seen from a frame $S_1$ and moving with a constant velocity when seen from another frame $S_2$. Mark out the possible options.
(a) Both the frames are inertial.
(b) Both the frames are noninertial.
(c) $S_1$ is inertial and $S_2$ is noninertial.
(d) $S_1$ is noninertial and $S_2$ is inertial.

**4.** Figure (5-Q3) shows the displacement of a particle going along the $X$-axis as a function of time. The force acting on the particle is zero in the region
(a) $AB$ (b) $BC$ (c) $CD$ (d) $DE$.

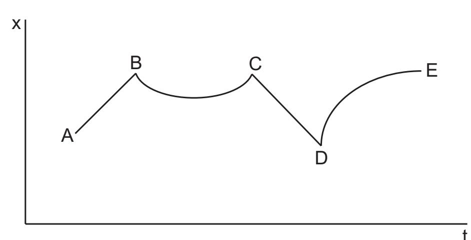

Figure 5-Q3

**5.** Figure (5-Q4) shows a heavy block kept on a frictionless surface and being pulled by two ropes of equal mass $m$. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let $F_1$ and $F_2$ be the magnitudes of the forces by the right rope and the left rope on the block respectively.

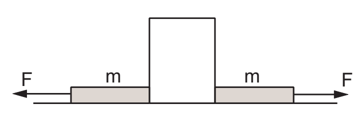

Figure 5-Q4

(a) $F_1 = F_2 = F$ for $t < 0$
(b) $F_1 = F_2 = F + mg$ for $t < 0$
(c) $F_1 = F,\ F_2 = F$ for $t > 0$
(d) $F_1 < F,\ F_2 = F$ for $t > 0$.

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**6.** The force exerted by the floor of an elevator on the foot of a person standing there is more than the weight of the person if the elevator is
(a) going up and slowing down
(b) going up and speeding up
(c) going down and slowing down
(d) going down and speeding up.

**7.** If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be
(a) going up with increasing speed
(b) going down with increasing speed
(c) going up with uniform speed
(d) going down with uniform speed.

**8.** A particle is observed from two frames $S_1$ and $S_2$. The frame $S_2$ moves with respect to $S_1$ with an acceleration $a$. Let $F_1$ and $F_2$ be the pseudo forces on the particle when seen from $S_1$ and $S_2$ respectively. Which of the following are not possible ?
(a) $F_1 = 0, \; F_2 \neq 0$ (b) $F_1 \neq 0, \; F_2 = 0$
(c) $F_1 \neq 0, \; F_2 \neq 0$ (d) $F_1 = 0, \; F_2 = 0$.

**9.** A person says that he measured the acceleration of a particle to be nonzero while no force was acting on the particle.
(a) He is a liar.
(b) His clock might have run slow.
(c) His meter scale might have been longer than the standard.
(d) He might have used noninertial frame.

## EXERCISES

**1.** A block of mass 2 kg placed on a long frictionless horizontal table is pulled horizontally by a constant force $F$. It is found to move 10 m in the first two seconds. Find the magnitude of $F$.

**2.** A car moving at 40 km/h is to be stopped by applying brakes in the next 4·0 m. If the car weighs 2000 kg, what average force must be applied on it ?

**3.** In a TV picture tube electrons are ejected from the cathode with negligible speed and reach a velocity of $5 \times 10^6$ m/s in travelling one centimeter. Assuming straight line motion, find the constant force exerted on the electron. The mass of the electron is $9{\cdot}1 \times 10^{-31}$ kg.

**4.** A block of mass 0·2 kg is suspended from the ceiling by a light string. A second block of mass 0·3 kg is suspended from the first block through another string. Find the tensions in the two strings. Take $g = 10 \text{ m/s}^2$.

**5.** Two blocks of equal mass $m$ are tied to each other through a light string and placed on a smooth horizontal table. One of the blocks is pulled along the line joining them with a constant force $F$. Find the tension in the string joining the blocks.

**6.** A particle of mass 50 g moves on a straight line. The variation of speed with time is shown in figure (5-E1). Find the force acting on the particle at $t = 2$, 4 and 6 seconds.

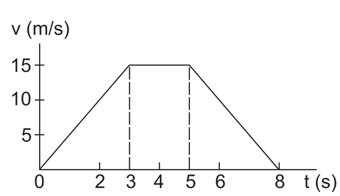


Figure 5-E1

**7.** Two blocks $A$ and $B$ of mass $m_A$ and $m_B$ respectively are kept in contact on a frictionless table. The experimenter pushes the block $A$ from behind so that the blocks accelerate. If the block $A$ exerts a force $F$ on the block $B$, what is the force exerted by the experimenter on $A$ ?

**8.** Raindrops of radius 1 mm and mass 4 mg are falling with a speed of 30 m/s on the head of a bald person. The drops splash on the head and come to rest. Assuming equivalently that the drops cover a distance equal to their radii on the head, estimate the force exerted by each drop on the head.

**9.** A particle of mass 0·3 kg is subjected to a force $F = -k\,x$ with $k$ = 15 N/m. What will be its initial acceleration if it is released from a point $x$ = 20 cm ?

**10.** Both the springs shown in figure (5-E2) are unstretched. If the block is displaced by a distance $x$ and released, what will be the initial acceleration?

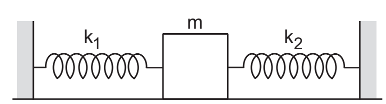


Figure 5-E2

**11.** A small block $B$ is placed on another block $A$ of mass 5 kg and length 20 cm. Initially the block B is near the right end of block $A$ (figure 5-E3). A constant horizontal force of 10 N is applied to the block $A$. All the surfaces are assumed frictionless. Find the time elapsed before the block $B$ separates from $A$.

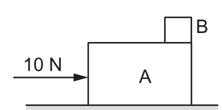


Figure 5-E3

**12.** A man has fallen into a ditch of width $d$ and two of his friends are slowly pulling him out using a light rope and two fixed pulleys as shown in figure (5-E4). Show that

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the force (assumed equal for both the friends) exerted by each friend on the rope increases as the man moves up. Find the force when the man is at a depth $h$.

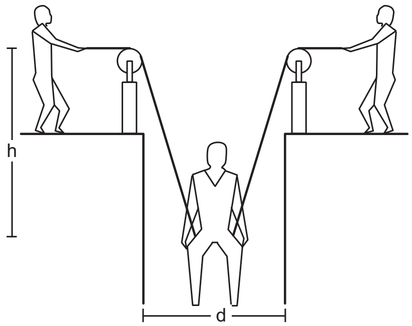


Figure 5-E4

**13.** The elevator shown in figure (5-E5) is descending with an acceleration of 2 m/s $^2$. The mass of the block $A$ is 0·5 kg. What force is exerted by the block $A$ on the block $B$ ?

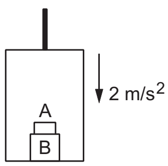


Figure 5-E5

**14.** A pendulum bob of mass 50 g is suspended from the ceiling of an elevator. Find the tension in the string if the elevator (a) goes up with acceleration 1·2 m/s $^2$, (b) goes up with deceleration 1·2 m/s $^2$, (c) goes up with uniform velocity, (d) goes down with acceleration 1·2 m/s $^2$, (e) goes down with deceleration 1·2 m/s $^2$ and (f) goes down with uniform velocity.

**15**. A person is standing on a weighing machine placed on the floor of an elevator. The elevator starts going up with some acceleration, moves with uniform velocity for a while and finally decelerates to stop. The maximum and the minimum weights recorded are 72 kg and 60 kg. Assuming that the magnitudes of the acceleration and the deceleration are the same, find (a) the true weight of the person and (b) the magnitude of the acceleration. Take $g$ = 9·9 m/s $^2$.

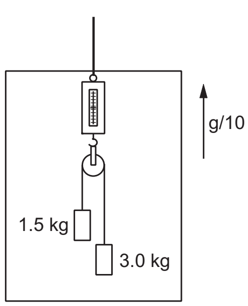


Figure 5-E6

**16.** Find the reading of the spring balance shown in figure (5-E6). The elevator is going up with an acceleration of $g/10$, the pulley and the string are light and the pulley is smooth.

**17.** A block of 2 kg is suspended from the ceiling through a massless spring of spring constant $k$ = 100 N/m. What is the elongation of the spring ? If another 1 kg is added to the block, what would be the further elongation ?

**18.** Suppose the ceiling in the previous problem is that of an elevator which is going up with an acceleration of 2·0 m/s $^2$. Find the elongations.

**19.** The force of buoyancy exerted by the atmosphere on a balloon is $B$ in the upward direction and remains constant. The force of air resistance on the balloon acts opposite to the direction of velocity and is proportional to it. The balloon carries a mass $M$ and is found to fall down near the earth's surface with a constant velocity $v$. How much mass should be removed from the balloon so that it may rise with a constant velocity $v$ ?

**20.** An empty plastic box of mass $m$ is found to accelerate up at the rate of $g/6$ when placed deep inside water. How much sand should be put inside the box so that it may accelerate down at the rate of $g/6$ ?

**21**. A force $\vec{F} = \vec{v} \times \vec{A}$ is exerted on a particle in addition to the force of gravity, where $\vec{v}$ is the velocity of the particle and $\vec{A}$ is a constant vector in the horizontal direction. With what minimum speed a particle of mass $m$ be projected so that it continues to move undeflected with a constant velocity ?

**22.** In a simple Atwood machine, two unequal masses $m_1$ and $m_2$ are connected by a string going over a clamped light smooth pulley. In a typical arrangement (figure 5-E7) $m_1$ = 300 g and $m_2$ = 600 g. The system is released from rest. (a) Find the distance travelled by the first block in the first two seconds. (b) Find the tension in the string. (c) Find the force exerted by the clamp on the pulley.

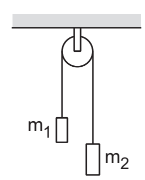


Figure 5-E7

**23.** Consider the Atwood machine of the previous problem. The larger mass is stopped for a moment 2·0 s after the system is set into motion. Find the time elapsed before the string is tight again.

**24**. Figure (5-E8) shows a uniform rod of length 30 cm having a mass of 3·0 kg. The strings shown in the figure are pulled by constant forces of 20 N and 32 N. Find the force exerted by the 20 cm part of the rod on the 10 cm part. All the surfaces are smooth and the strings and the pulleys are light.

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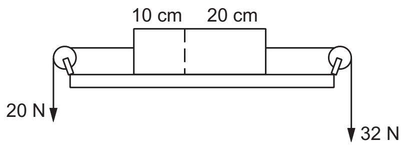


Figure 5-E8

**25.** Consider the situation shown in figure (5-E9). All the surfaces are frictionless and the string and the pulley are light. Find the magnitude of the acceleration of the two blocks.

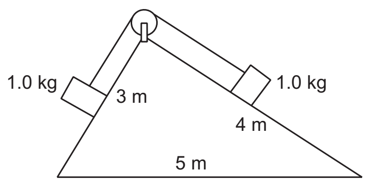


Figure 5-E9

**26.** A constant force $F = m_2 g/2$ is applied on the block of mass $m_1$ as shown in figure (5-E10). The string and the pulley are light and the surface of the table is smooth. Find the acceleration of $m_1$.

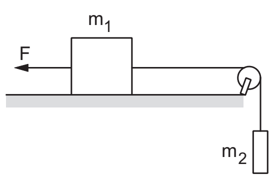


Figure 5-E10

**27.** In figure (5-E11) $m_1 = 5$ kg, $m_2 = 2$ kg and $F = 1$ N. Find the acceleration of either block. Describe the motion of $m_1$ if the string breaks but $F$ continues to act.

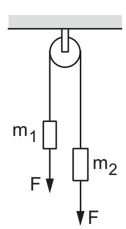


Figure 5-E11

**28.** Let $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg in figure (5-E12). Find the accelerations of $m_1$, $m_2$ and $m_3$. The string from the upper pulley to $m_1$ is 20 cm when the system is released from rest. How long will it take before $m_1$ strikes the pulley ?

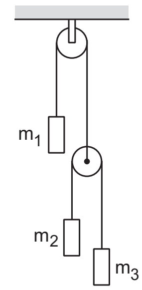


Figure 5-E12

**29.** In the previous problem, suppose $m_2 = 2{\cdot}0$ kg and $m_3 = 3{\cdot}0$ kg. What should be the mass $m$ so that it remains at rest ?

**30.** Calculate the tension in the string shown in figure (5-E13). The pulley and the string are light and all surfaces are frictionless. Take $g = 10 \text{ m/s}^2$.

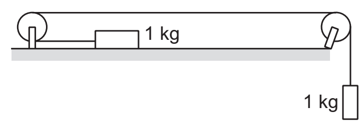


Figure 5-E13

**31.** Consider the situation shown in figure (5-E14). Both the pulleys and the string are light and all the surfaces are frictionless. (a) Find the acceleration of the mass $M$. (b) Find the tension in the string. (c) Calculate the force exerted by the clamp on the pulley $A$ in the figure.

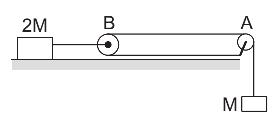


Figure 5-E14

**32.** Find the acceleration of the block of mass $M$ in the situation shown in figure (5-E15). All the surfaces are frictionless and the pulleys and the string are light.

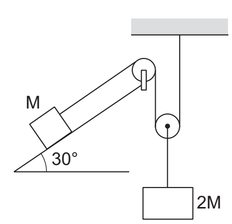


Figure 5-E15

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**33.** Find the mass $M$ of the hanging block in figure (5-E16) which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light.

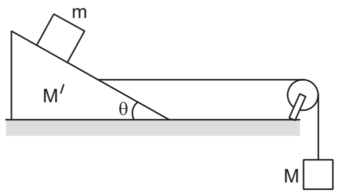


Figure 5-E16

**34.** Find the acceleration of the blocks $A$ and $B$ in the three situations shown in figure (5-E17).

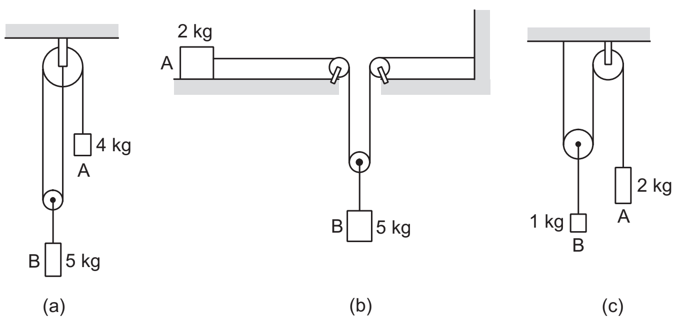


Figure 5-E17

**35.** Find the acceleration of the 500 g block in figure (5-E18).

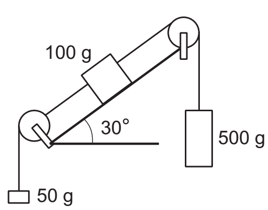


Figure 5-E18

**36.** A monkey of mass 15 kg is climbing on a rope with one end fixed to the ceiling. If it wishes to go up with an acceleration of 1 m/s $^2$, how much force should it apply to the rope? If the rope is 5 m long and the monkey starts from rest, how much time will it take to reach the ceiling?

**37.** A monkey is climbing on a rope that goes over a smooth light pulley and supports a block of equal mass at the other end (figure 5-E19). Show that whatever force the monkey exerts on the rope, the monkey and the block move in the same direction with equal acceleration. If initially both were at rest, their separation will not change as time passes.

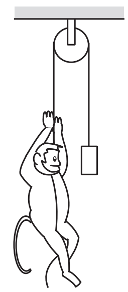

Figure 5-E19

**38.** The monkey $B$ shown in figure (5-E20) is holding on to the tail of the monkey $A$ which is climbing up a rope. The masses of the monkeys $A$ and $B$ are 5 kg and 2 kg respectively. If $A$ can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey $B$ with it? Take $g = 10$ m/s $^2$.

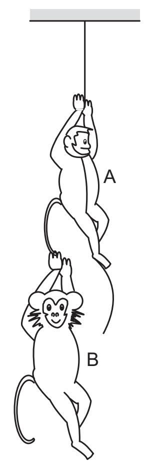


Figure 5-E20

**39.** Figure (5-E21) shows a man of mass 60 kg standing on a light weighing machine kept in a box of mass 30 kg. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the weight shown by the machine? What force should he exert on the rope to get his correct weight on the machine?

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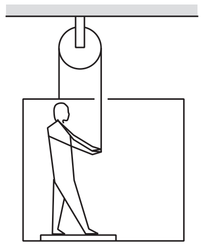

Figure 5-E21

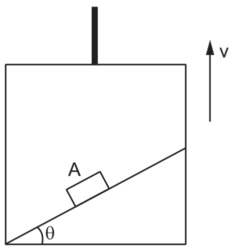


Figure 5-E22

**40.** A block $A$ can slide on a frictionless incline of angle $\theta$ and length $l$, kept inside an elevator going up with uniform velocity $v$ (figure 5-E22). Find the time taken by the block to slide down the length of the incline if it is released from the top of the incline.

**41.** A car is speeding up on a horizontal road with an acceleration $a$. Consider the following situations in the car. (i) A ball is suspended from the ceiling through a string and is maintaining a constant angle with the vertical. Find this angle. (ii) A block is kept on a smooth incline and does not slip on the incline. Find the angle of the incline with the horizontal.

**42.** A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of 12 m/s $^2$. Find the displacement of the block during the first 0·2 s after the start. Take $g = 10$ m/s $^2$.

□

## ANSWERS

### OBJECTIVE I

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (b) | 3. (d) | 4. (a) | 5. (c) | 6. (b) |
| 7. (d) | 8. (a) | 9. (b) | 10. (b) | 11. (c) | 12. (b) |
| 13. (a) | 14. (c) | | | | |

### OBJECTIVE II

| | | |
|---|---|---|
| 1. (b), (d) | 2. (c), (d) | 3. (a), (b) |
| 4. (a), (c) | 5. (a) | 6. (b), (c) |
| 7. (c), (d) | 8. (d) | 9. (d) |

### EXERCISES

1. 10 N
2. $3·1 \times 10^{4}$ N
3. $1·1 \times 10^{-15}$ N
4. 5 N and 3 N
5. $F/2$
6. 0·25 N along the motion, zero and 0·25 N opposite to the motion.
7. $F\left(1 + \frac{m_A}{m_B}\right)$
8. 1·8 N
9. 10 m/s $^2$
10. $(k_1 + k_2)\frac{x}{m}$ opposite to the displacement.
11. 0·45 s.
12. $\frac{mg}{4h}\sqrt{d^2 + 4h^2}$
13. 4 N
14. (a) 0·55 N (b) 0·43 N (c) 0·49 N (d) 0·43 N (e) 0·55 N (f) 0·49 N
15. 66 kg and 0·9 m/s $^2$
16. 4·4 kg
17. 0·2 m, 0·1 m
18. 0·24 m, 0·12 m
19. $2\left(M - \frac{B}{g}\right)$
20. $2m/5$
21. $mg/A$
22. (a) 6·5 m (b) 3·9 N (c) 7·8 N
23. 2/3 s
24. 24 N
25. $g/10$
26. $\frac{m_2 g}{2(m_1 + m_2)}$ towards right
27. 4·3 m/s $^2$, moves downward with acceleration $g + 0·2$ m/s $^2$

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28. $\frac{19}{29}g\,(up)$ , $\frac{17}{29}g$ (down) , $\frac{21}{29}g$ (down) , 0·25 s
29. 4·8 kg
30. 5 N
31. (a) $2g/3$ (b) $Mg/3$
    (c) $\sqrt{2}\,Mg/3$ at an angle of 45° with the horizontal
32. $g/3$ up the plane
33. $\frac{M' + m}{\cot\theta - 1}$
34. (a) $\frac{2}{7}g$ downward, $\frac{g}{7}$ upward
    (b) $\frac{10}{13}g$ forward, $\frac{5}{13}g$ downward
    (c) $\frac{2}{3}g$ downward, $\frac{g}{3}$ upward
35. $\frac{8}{13}g$ downward
36. 165 N, $\sqrt{10}$ s
38. between 70 N and 105 N
39. 15 kg, 1800 N
40. $\sqrt{\frac{2l}{g\sin\theta}}$
41. $\tan^{-1}(a/g)$ in each case
42. 20 cm

□

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**CHAPTER 6**

# FRICTION

## 6.1 FRICTION AS THE COMPONENT OF CONTACT FORCE

When two bodies are kept in contact, electromagnetic forces act between the charged particles at the surfaces of the bodies. As a result, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and hence the contact forces obey Newton's third law.

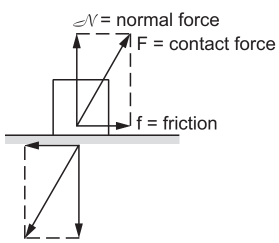


Figure 6.1

The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (Figure 6.1). The perpendicular component is called the *normal contact force* or *normal force* and the parallel component is called *friction.*

***Example 6.1***

*A body of mass* 400 g *slides on a rough horizontal surface. If the frictional force is* 3·0 N, *find (a) the angle made by the contact force on the body with the vertical and (b) the magnitude of the contact force. Take* $g = 10 \text{ m/s}^2$.

***Solution*** **:** Let the contact force on the block by the surface be $F$ which makes an angle $\theta$ with the vertical (figure 6.2).

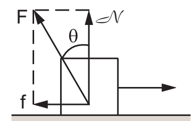


Figure 6.2

The component of $F$ perpendicular to the contact surface is the normal force $\mathcal{N}$ and the component of $F$ parallel to the surface is the friction $f$. As the surface is horizontal, $\mathcal{N}$ is vertically upward. For vertical equilibrium,

$$\mathcal{N} = Mg = (0{\cdot}400 \text{ kg})\,(10 \text{ m/s}^2) = 4{\cdot}0 \text{ N}.$$

The frictional force is $f = 3{\cdot}0$ N.

(a) $\qquad \tan\theta = \dfrac{f}{\mathcal{N}} = \dfrac{3}{4}$

or, $\qquad \theta = \tan^{-1}(3/4) = 37°.$

(b) The magnitude of the contact force is

$$F = \sqrt{\mathcal{N}^2 + f^2}$$

$$= \sqrt{(4{\cdot}0 \text{ N})^2 + (3{\cdot}0 \text{ N})^2} = 5{\cdot}0 \text{ N}.$$

Friction can operate between a given pair of solids, between a solid and a fluid or between a pair of fluids. Frictional force exerted by fluids is called *viscous force* and we shall study it in a later chapter. Here we shall study about the frictional forces operating between a pair of solid surfaces.

When two solid bodies slip over each other, the force of friction is called *kinetic friction*. When two bodies do not slip on each other, the force of friction is called *static friction.*

It is difficult to work out a reliable theory of friction starting from the electromagnetic interaction between the particles at the surface. However, a wide range of observations can be summarized in a small number of *laws of friction* which we shall discuss.

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## 6.2 KINETIC FRICTION

When two bodies in contact move with respect to each other, rubbing the surfaces in contact, the friction between them is called kinetic friction. The directions of the frictional forces are such that the relative slipping is opposed by the friction.

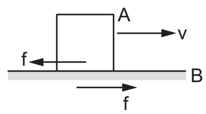

Figure 6.3

Suppose a body $A$ placed in contact with $B$ is moved with respect to it as shown in figure (6.3). The force of friction acting on $A$ due to $B$ will be opposite to the velocity of $A$ with respect to $B$. In figure (6.3) this force is shown towards left. The force of friction on $B$ due to $A$ is opposite to the velocity of $B$ with respect to $A$. In figure (6.3) this force is shown towards right. The force of kinetic friction opposes the relative motion. We can formulate the rules for finding the direction and magnitude of kinetic friction as follows :

### (a) Direction of Kinetic Friction

*The kinetic friction on a body $A$ slipping against another body $B$ is opposite to the velocity of $A$ with respect to $B$.*

It should be carefully noted that the velocity coming into picture is with respect to the body applying the force of friction.

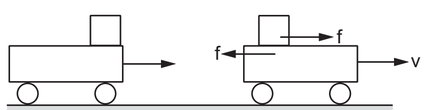

Figure 6.4

As another example, suppose we have a long box having wheels and moving on a horizontal road (figure 6.4). A small block is placed on the box which slips on the box to fall from the rear end. As seen from the road, both the box and the block are moving towards right, of course the velocity of the block is smaller than that of the box. What is the direction of the kinetic friction acting on the block due to the box ? The velocity of the block as seen from the box is towards left. Thus, the friction on the block is towards right. The friction acting on the box due to the block is towards left.

### (b) Magnitude of the Kinetic Friction

*The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write*

$$f_k = \mu_k \mathcal{N} \qquad \dots \quad (6.1)$$

*where $\mathcal{N}$ is the normal force*. The proportionality constant $\mu_k$ is called the *coefficient of kinetic friction* and its value depends on the nature of the two surfaces in contact. If the surfaces are smooth $\mu_k$ will be small, if the surfaces are rough $\mu_k$ will be large. It also depends on the materials of the two bodies in contact.

According to equation (6.1) the coefficient of kinetic friction does not depend on the speed of the sliding bodies. Once the bodies slip on each other the frictional force is $\mu_k \mathcal{N}$, whatever be the speed. This is approximately true for relative speeds not too large (say for speeds $< 10$ m/s).

We also see from equation (6.1) that as long as the normal force $\mathcal{N}$ is same, the frictional force is independent of the area of the surface in contact. For example, if a rectangular slab is slid over a table, the frictional force is same whether the slab lies flat on the table or it stands on its face of smaller area (figure 6.5)

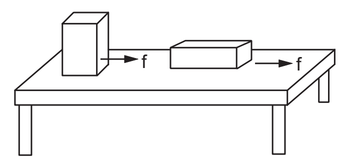

Figure 6.5

***Example 6.2***

*A heavy box of mass* 20 kg *is pulled on a horizontal surface by applying a horizontal force. If the coefficient of kinetic friction between the box and the horizontal surface is* 0·25*, find the force of friction exerted by the horizontal surface on the box.*

***Solution*** **:** The situation is shown in figure (6.6). In the vertical direction there is no acceleration, so

$$\mathcal{N} = Mg.$$

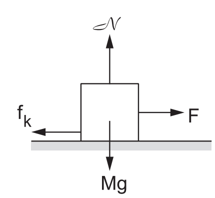

Figure 6.6

As the box slides on the horizontal surface, the surface exerts kinetic friction on the box. The magnitude of the kinetic friction is

$$f_k = \mu_k \mathcal{N} = \mu_k Mg$$

$$= 0{\cdot}25 \times (20 \text{ kg}) \times (9{\cdot}8 \text{ m/s}^2) = 49 \text{ N}.$$

This force acts in the direction opposite to the pull.

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## 6.3 STATIC FRICTION

Frictional forces can also act between two bodies which are in contact but are not sliding with respect to each other. The friction in such cases is called *static friction*. For example, suppose several labourers are trying to push a heavy almirah on the floor to take it out of a room (figure 6.7).

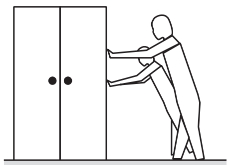

Figure 6.7

The almirah is heavy and even the most sincere effort by them is not able to slide it on the floor even by a millimeter. As the almirah is at rest the resultant force on the almirah should be zero. Thus, something is exerting a force on the almirah in the opposite direction. In this case, it is the floor which exerts a frictional force on the almirah. The labourers push the almirah towards left in figure (6.7) and the floor exerts a frictional force on the almirah towards right. This is an example of static friction.

How strong is this frictional force ? Suppose the almirah is pushed with a small force in the beginning and the force is gradually increased. It does not slide until the force applied is greater than a minimum value say $F$. The force of static friction is equal and opposite to the force exerted by the labourers as long as the almirah is at rest. This means that the magnitude of static friction adjusts its value according to the applied force. As the applied force increases, the frictional force also increases. The static friction is thus, self adjustable. It adjusts its magnitude (and direction) in such a way that together with other forces applied on the body, it maintains 'relative rest' between the two surfaces. However, the frictional force cannot go beyond a maximum. When the applied force exceeds this maximum, friction fails to increase its value and slipping starts. The maximum static friction that a body can exert on the other body in contact with it, is called *limiting friction*. This limiting friction is proportional to the normal contact force between the two bodies. We can write

$$f_{\max} = \mu_s \mathcal{N} \qquad \ldots (6.2)$$

where $f_{\max}$ is the maximum possible force of static friction and $\mathcal{N}$ is the normal force. The constant of proportionality is called the *coefficient of static friction* and its value again depends on the material and roughness of the two surfaces in contact. In general, $\mu_s$ is slightly greater than $\mu_k$. As long as the normal force is constant, the maximum possible friction does not depend on the area of the surfaces in contact.

Once again we emphasise that $\mu_s \mathcal{N}$ is the **maximum** possible force of static friction that **can** act between the bodies. The actual force of static friction may be smaller than $\mu_s \mathcal{N}$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest. Thus,

$$f_s \le f_{\max} = \mu_s \mathcal{N}. \qquad \ldots (6.3)$$

***Example 6.3***

*A boy* (30 kg) *sitting on his horse whips it. The horse speeds up at an average acceleration of* 2·0 m/s². *(a) If the boy does not slide back, what is the force of friction exerted by the horse on the boy ? (b) If the boy slides back during the acceleration, what can be said about the coefficient of static friction between the horse and the boy. Take* $g$ = 10 m/s².

***Solution :*** (a) The forces acting on the boy are

(i) the weight $Mg$.

(ii) the normal contact force $\mathcal{N}$ and

(iii) the static friction $f_s$.

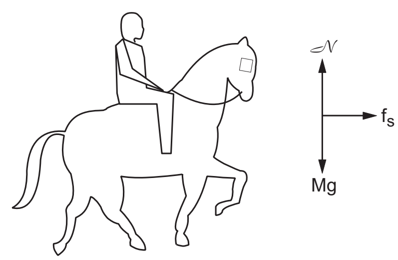


Figure 6.8

As the boy does not slide back, its acceleration $a$ is equal to the acceleration of the horse. As friction is the only horizontal force, it must act along the acceleration and its magnitude is given by Newton's second law

$$f_s = Ma = (30\text{ kg})\,(2{\cdot}0\text{ m/s}^2) = 60\text{ N}.$$

(b) If the boy slides back, the horse could not exert a friction of 60 N on the boy. The maximum force of static friction that the horse may exert on the boy is

$$f_s = \mu_s \mathcal{N} = \mu_s Mg$$
$$= \mu_s (30\text{ kg})\,(10\text{ m/s}^2) = \mu_s\, 300\text{ N}$$

where $\mu_s$ is the coefficient of static friction. Thus,

$$\mu_s (300\text{ N}) < 60\text{ N}$$

or,
$$\mu_s < \frac{60}{300} = 0{\cdot}20.$$

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