Assignment-8

PHY617/473-Computational Physics

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Question 1: Differentiation of Unequally Spaced Data . As shown in Figure-1, a temperature gradient can be measured down into the soil. The heat flux at the soil-air interface can be computed with Fourier's law,

$$q(z=0) = -k\rho C \frac{dT}{dz}|_{z=0}$$

where q = heat flux (W/m2), k = coefficient of thermal diffusivity in soil ($\sim 3.5 \times 10^{-7}$ $\mathrm{m^2/s}$), ρ = soil density ($\approx$ 1800 $\mathrm{kg/m^3}$), and C = soil specific heat ($\approx$840 J/(kg $°C$)). Note that a positive value for flux means that heat is transferred from the air to the soil. Use numerical differentiation to evaluate the gradient at the soil-air interface and employ this estimate to determine the heat flux into the ground. **[10 marks]**

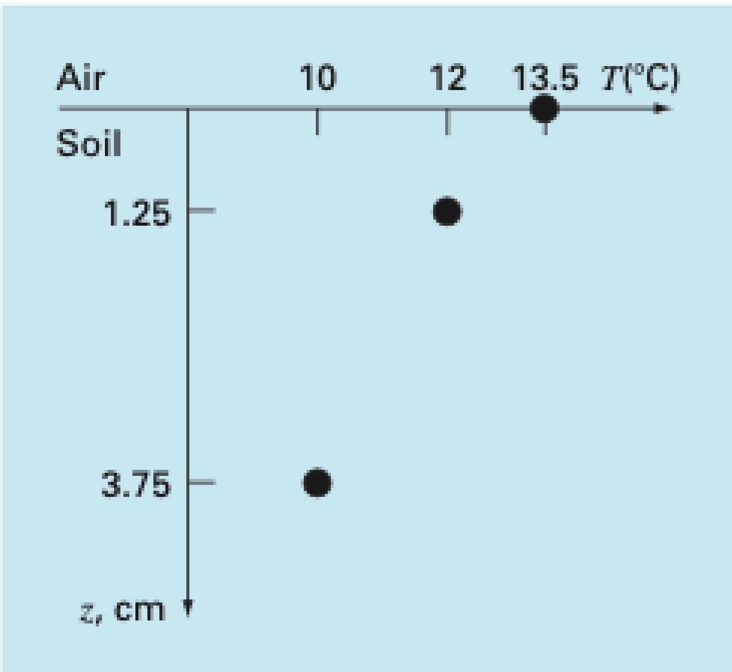

Figure 1: Temperature versus depth into the soil.

Question 2.
It is often the case that the frictional force on an object will increase as the object moves faster. A fortunate example of this is a parachutist; the role of the parachute is to produce a frictional force due

to air drag, which is larger than would normally be the case without the parachute. The physics of air drag will be discussed in more detail in the next chapter. Here we consider a very simple example in which the frictional force depends on the velocity. Assume that the velocity of an object obeys an equation of the form

$$\frac{dv}{dt} = a - bv \tag{1}$$

where a and b are constants. You could think of a as coming from an applied force, such as gravity, while b arises from friction. Note that the frictional force is negative (we assume that $b > 0$), so that it opposes the motion, and that it increases in magnitude as the velocity increases. Use the Euler method to solve equation 1 for v as a function of time. A convenient choice of parameters is $a = 10$ and $b = 1.0$. You should find that v approaches a constant value at long times; this is called the terminal velocity. [**6 marks**]

Do the same calculation using Fourth-Order Runge-Kutta Method and comment on the number of steps that you need to achieve terminal velocity in comparison to the Euler method. [**4 marks**]

Question 3.
The Lorenz equations

One of the most celebrated sets of differential equations in physics is the Lorenz equations:

$$\frac{dx}{dt} = \sigma(y - x), \qquad \frac{dy}{dt} = rx - y - xz, \qquad \frac{dz}{dt} = xy - bz,$$

where σ, r, and b are constants. (The names σ, r, and b are odd, but traditional—they are always used in these equations for historical reasons.)

These equations were first studied by Edward Lorenz in 1963, who derived them from a simplified model of weather patterns. The reason for their fame is that they were one of the first incontrovertible examples of *deterministic chaos*, the occurrence of apparently random motion even though there is no randomness built into the equations.

1. Write a program preferably using Fourth-Order Runge-Kutta method to solve the Lorenz equations for the case $\sigma = 10$, $r = 28$, and $b = \frac{8}{3}$ in the range from $t = 0$ to $t = 50$ with initial conditions $(x, y, z) = (0, 1, 0)$. Have your program make a plot of y as a function of time. Note the unpredictable nature of the motion. [**7 marks**]
2. Make a plot of z against x. You should see a picture of the famous "strange attractor" of the Lorenz equations, a lop-sided butterfly-shaped plot that never repeats itself. [**3 marks**]

Question 4 .
Consider a radioactive decay problem involving two types of nuclei, A and B, with populations $N_A(t)$ and $N_B(t)$. Suppose that type A nuclei decay to form type B nuclei, which then also decay, according to the differential equations

$$\frac{dN_A}{dt} = \frac{N_B}{\tau} - \frac{N_A}{\tau}, \tag{2}$$

$$\frac{dN_B}{dt} = \frac{N_A}{\tau} - \frac{N_B}{\tau} \tag{3}$$

where for simplicity we have assumed that the two types of decay are characterized by the same time constant, τ. Solve this system of equations for the numbers of nuclei, N_A and N_B, as functions of time. Consider different initial conditions, such as $N_A = 100$, $N_B = 0$, etc., and take $\tau = 1$s. Show that your numerical results are consistent with the idea that the system reaches a steady state in which N_A and N_B are constant. In such a steady state, the time derivatives dN_A/dt and dN_B/dt should vanish. [**10 marks**]