

**Assignment-5**  
PHY617/473-Computational Physics  
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Explain the algorithm you are using for each question.

**Question 1 [10 marks]**

Write a Python function that represents the mathematical function

$$f(x) = 2.0 * \exp(-x^2) \cos(2\pi x)$$

on a mesh consisting of  $q+1$  equally spaced points on  $[-1,1]$  and return 1) the interpolated function value at  $x = -0.45$  and 2) the error in the interpolated value. Call the function and write out the error for  $q = 2, 4, 8, 16$ . Use linear interpolation technique.

**Question 2.**

Imagine we have  $n+1$  measurements of some quantity  $y$  that depends on  $x : (x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ . We may think of  $y$  as a function of  $x$  and ask what  $y$  is at some arbitrary point  $x$  not coinciding with any of the points  $x_0, \dots, x_n$ . It is not clear how  $y$  varies between the measurement points, but we can make assumptions or models for this behavior using interpolation. One way to solve the interpolation problem is to fit a continuous function that goes through all the  $n+1$  points and then evaluate this function for any desired  $x$ . A candidate for such a function is the polynomial of degree  $n$  that goes through all the points. It turns out that this polynomial can be written as

$$p_L(x) = \sum_{k=0}^n y_k L_k(x), \quad (1)$$

where

$$L_k(x) = \prod_{i=0, i \neq k}^n \frac{x - x_i}{x_k - x_i}. \quad (2)$$

The polynomial  $p_L(x)$  is known as Lagrange's interpolation formula, and the points  $(x_0, y_0), \dots, (x_n, y_n)$  are called interpolation points.

a) Make functions  $p_L(x, xp, yp)$  and  $L_k(x, k, xp, yp)$  that evaluate  $p_L(x)$  and  $L_k(x)$  by Eq. 1 and Eq. 2, respectively, at the point  $x$ . The arrays  $xp$  and  $yp$  contain the  $x$  and  $y$  coordinates of the  $n+1$  interpolation points, respectively. That is,  $xp$  holds  $x_0, \dots, x_n$ , and  $yp$  holds  $y_0, \dots, y_n$ . **[5 marks]**

b) To verify the program, we observe that  $L_k(x_k) = 1$  and that  $L_k(x_i) = 0$  for  $i \neq k$ , implying that  $p_L(x_k) = y_k$ . That is, the polynomial  $p_L$  goes through all the points  $(x_0, y_0), \dots, (x_n, y_n)$ . Write a function `test_p_L(xp, yp)` that computes  $|p_L(x_k) - y_k|$  at all the interpolation points  $(x_k, y_k)$  and checks that the value is approximately zero. Call `test_p_L` with  $xp$  and  $yp$  corresponding to 5 equally spaced points along the curve  $y = \sin(x)$  for  $x \in [0, \pi]$ . Thereafter, evaluate  $p_L(x)$  for an  $x$

in the middle of two interpolation points and compare the value of  $p_L(x)$  with the exact one. [5 marks]

### Question 3

(a) Use Neville's method to compute  $y$  at  $x = \pi/4$  from the data points [5 marks]

$$x = [0, 0.5, 1, 1.5, 2]$$

$$y = [-1.00, 1.75, 4.00, 5.75, 7.00]$$

(b) Interpolate the above datapoints to the new  $x$  values between 0 to 2 with an interval of 0.2 using Lagrange's polynomial method. [5 marks]

(c) Do the same using Neville's method. Plot the original data, interpolated data using Lagrange method and Neville's method in a same graph. [5 marks]