Assignment-6

PHY617/473-Computational Physics

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Question 1.
Given the data points
$x = [1, 2, 3, 4, 5]$
$y = [13, 15, 12, 9, 13]$
(a) Calculate the second order derivatives (y'') analytically in all x points using the condition that first order derivatives are continous in each boundary. [**5 marks**]

(b) Use those second order derivatives (y'') and determine the natural cubic spline interpolant at $x = 3.4$. [**5 marks**]

Question 2. The spectrum of a star is well approximated as a blackbody. While studying the blackbody radiation most of you have encountered the Planck function. Integration of the Planck function over wavelength essentially gives the stellar brightness which is given below:

$$B = 2hc^2 \int_0^\infty \frac{1}{\lambda^5} \frac{1}{exp\left(\frac{hc}{\lambda k_B T}\right) - 1} d\lambda$$

Assuming $x = \frac{hc}{\lambda k_B T}$, the integration changes to

$$B = \frac{2(k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

Calculate the Integration part of this function using trapezoidal rule. Note that this integral has an analytic solution

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

Compare your numerically estimated result with the analytical solution. [**2 + 8 marks**]
Hints: Consider a transformation from x to z by assuming $z = \frac{x}{c+x}$ to make the integral finite. Here c is chosen to be close to the maximum of the integrand. This transformation maps the interval $x \in [0, \infty]$ to $z \in [0, 1]$

Question 3. Consider the integral

$$E(x) = \int_0^x e^{-t^2} dt.$$

1. Write a program to calculate $E(x)$ for values of x from 0 to 3 in steps of 0.1. Choose for yourself what method you will use for performing the integral and a suitable number of slices. [**7 marks**]
2. When you are convinced your program is working, extend it further to make a graph of $E(x)$ as a function of x. [**3 marks**]

Note that there is no known way to perform this particular integral analytically, so numerical approaches are the only way forward.