Assignment-4

PHY617/473-Computational Physics
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Explain the algorithm you are using for each question.

Question 1.
Planck's radiation law tells us that the intensity of radiation per unit area and per unit wavelength λ from a black body at temperature T is

$$I(\lambda) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda k_B T} - 1},$$

where h is Planck's constant, c is the speed of light, and k_B is Boltzmann's constant.

1. Show by differentiating that the wavelength λ at which the emitted radiation is strongest is the solution of the equation
$$5e^{-hc/\lambda k_B T} + \frac{hc}{\lambda k_B T} - 5 = 0.$$
Make the substitution $x = hc/\lambda k_B T$ and hence show that the wavelength of maximum radiation obeys the Wien displacement law [**2 marks**]:
$$\lambda = \frac{b}{T},$$
where the so-called Wien displacement constant is $b = hc/k_B x$, and x is the solution to the nonlinear equation
$$5e^{-x} + x - 5 = 0.$$
2. Write a program to solve this equation to an accuracy of $\epsilon = 10^{-6}$ using the binary search method, and hence find a value for the displacement constant. [**6 marks**]
3. The displacement law is the basis for the method of optical pyrometry, a method for measuring the temperatures of objects by observing the color of the thermal radiation they emit. The method is commonly used to estimate the surface temperatures of astronomical bodies, such as the Sun. The wavelength peak in the Sun's emitted radiation falls at $\lambda = 502\,\text{nm}$. From the equations above and your value of the displacement constant, estimate the surface temperature of the Sun. [**2 marks**]

Question 2. Consider the degree-six polynomial

$$P(x) = 924x^6 - 2772x^5 + 3150x^4 - 1680x^3 + 420x^2 - 42x + 1$$

There is no general formula for the roots of a polynomial of degree six, but one can find them easily enough using a computer.

a) Make a plot of $P(x)$ from $x = 0$ to $x = 1$ and by inspecting it find rough values for the six roots of the polynomial-the points at which the function is zero. **[3 marks]**

b) Write a Python program to solve for the positions of all six roots to at least ten decimal places of accuracy, using Newton-raphson method. **[7 marks]**

Question 3. (a) Find roots on an interval with the bisection method for $f(x) = x^2 - 4x + e^{-x}$. **[4 marks]**

(b) Use Newton-Raphson method to find roots for the same function. Compare the iteration numbers for both methods. Which one is efficient? **[4 +2 marks]**

Question 4. **[10 marks]**

BK-7 is a type of common optical crown glass. Its index of refraction n varies as a function of wavelength; for shorter wavelengths n is larger than for longer wavelengths, and thus violet light is refracted more strongly than red light, leading to the phenomenon of dispersion. The index of refraction is tabulated in the attached data file "refractive_index.txt".

Let us suppose that we wish to find the index of refraction at a wavelength of $5000°\mathring{A}$. Unfortunately, that wavelength is not found in the table, and so we must estimate it from the values in the table. We must make some assumption about how n varies between the tabular values. Presumably it varies in a smooth sort of way and does not take wild excursions between the tabulated values.

Write a function that will linearly interpolate the tabular data for the index of refraction of BK-7 and return a value for n for wavelengths between $3511°\mathring{A}$ and $23254°\mathring{A}$.