Assignment-10

PHY617/473-Computational Physics

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Question1: Write a program to solve the differential equation:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + x + 5 = 0$$

using the leapfrog method. Solve from $t = 0$ to $t = 50$ in steps of $h = 0.001$ with initial condition $x = 1$ and $\mathrm{d}x/\mathrm{d}t = 0$. Make a plot of your solution showing x as a function of t. [**10 marks**]

Question2: Orbit of the Earth

Use the Verlet method to calculate the orbit of the Earth around the Sun. The equations of motion for the position $\vec{r} = (x, y)$ of the planet in its orbital plane are given in the vector form below:

$$\frac{\mathrm{d}^2 \vec{r}}{\mathrm{d}t^2} = -GM\frac{\vec{r}}{r^3},$$

where $G = 6.6738 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ is Newton's gravitational constant and $M = 1.9891 \times 10^{30}\,\mathrm{kg}$ is the mass of the Sun.

The orbit of the Earth is not perfectly circular, the planet being sometimes closer to and sometimes further from the Sun. When it is at its closest point, or *perihelion*, it is moving precisely tangentially (i.e., perpendicular to the line between itself and the Sun) and it has distance $1.4710 \times 10^{11}\,\mathrm{m}$ from the Sun and linear velocity $3.0287 \times 10^4\,\mathrm{m\,s}^{-1}$.

1. Write a program to calculate the orbit of the Earth using the Verlet method, with a time-step of $h = 1$ hour. Make a plot of the orbit, showing several complete revolutions about the Sun. The orbit should be very slightly, but visibly, non-circular.[**4 marks**]
2. The gravitational potential energy of the Earth is $-GMm/r$, where $m = 5.9722 \times 10^{24}\,\mathrm{kg}$ is the mass of the planet, and its kinetic energy is $\frac{1}{2}mv^2$ as usual. Modify your program to calculate both of these quantities at each step, along with their sum (which is the total energy), and make a plot showing all three as a function of time on the same axes. You should find that the potential and kinetic energies vary visibly during the course of an orbit, but the total energy remains constant.[**4 marks**]
3. Now plot the total energy alone without the others and you should be able to see a slight variation over the course of an orbit. Because you're using the Verlet method, however, which conserves energy in the long term, the energy should always return to its starting value at the end of each complete orbit.[**2 marks**]

Question3: Motion of a particle in an EM Field

Find out the motion of a charged particle in an electromagnetic field which is under the influence of the Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Consider the constant EMF field of the form $\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$ and $\vec{B} = B\hat{z}$.

Construct the equation of motion of the charged particle of mass m and charge q in x, y and z direction and solve those using your favourite ODE solving method for the following cases.

1. Solve the equation of motion and plot 3D trajectory of the charged particle with only constant magnetic field $\vec{B} = B\hat{z}$ where $qB/m = 1$ and initial conditions are given below: [**3 marks**]

$$v(\vec{0}) = 1.0\hat{y} + 0.1\hat{z}, r(\vec{0}) = 1.0\hat{x}$$

2. Solve the equation of motion with constant magnetic field $\vec{B} = B\hat{z}$ and a constant electric field $\vec{E} = E_x\hat{x} + E_y\hat{y}$, where $qE_x/m = qE_y/m = 0.1$. Take initial conditions as before. Plot the trajectory.[**3 marks**]
3. Do the same exercise with varying magnetic field $\vec{B} = B_y\hat{y} + B_z\hat{z}$ with $qB_y/m = -0.02y$, $qB_z/m = 1 + 0.02z$. Assume initital condition on velocity and position as before. Plot 3d trajectory of the particle.[**4 marks**]