



# Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

Sec: Sr.Super60, Elite,Target & LIIT BT's

Paper -2(Adv-2024-P2-Model)

Date: 27-04-2025

Time: 02.00Pm to 05.00Pm

GTA-30

Max. Marks: 180

## KEY SHEETS

### MATHEMATICS

1	C	2	C	3	B	4	B	5	AB	6	AD
7	AB	8	8	9	5	10	2	11	5	12	2
13	8	14	12	15	12	16	3027	17	6054		

### PHYSICS

18	D	19	D	20	A	21	C	22	ACD	23	ABCD
24	BCD	25	9	26	2	27	3	28	8	29	6
30	2	31	$0.16 - 0.17$	32	-0.33	33	0.18	34	$3.46 - 3.48$		

### CHEMISTRY

35	C	36	B	37	B	38	C	39	AB	40	ABCD
41	ACD	42	4	43	16	44	20	45	15	46	0
47	2	48	3	49	12	50	387	51	12		



## SOLUTIONS

### MATHEMATICS

1. Let coordinate of  $Q$  is  $(x_1, -4 - x_1)$

The image of  $R(12,0)$  w.r.t. line  $x + y = 2 \therefore$  Coordinate of  $P = (2, -10)$

Now slope of  $PQ \times$  slope of line  $(5x + y = 0)$  is  $-1$

$$\therefore \frac{-10 + 4 + x_1}{2 - x_1} \times (-5) = -1 \quad \therefore x_1 = \frac{32}{6} = \frac{16}{3}$$

$$\therefore \text{Coordinate of } Q = \left( \frac{16}{3}, -\frac{28}{3} \right)$$

2. Required area  $= \int_0^6 [x] + \sqrt{x - [x]} dx$

$$= \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx + \int_3^4 3 dx + \int_4^5 4 dx + \int_5^6 5 dx + 6 \int_0^1 \sqrt{x} dx = 19$$

3.  $A = (-1 + \lambda, 2 + \lambda, 1 - 2\lambda)$

$$B = \left( 1 + \mu, \frac{8}{3} + 2\mu, -3 - \mu \right)$$

$$O, A \text{ and } B \text{ are collinear} \Rightarrow \frac{-1 + \lambda}{1 + \mu} = \frac{2 + \lambda}{\frac{8}{3} + 2\mu} = \frac{1 - 2\lambda}{-3 - \mu} \Rightarrow \mu = \frac{1}{3}, \lambda = 3, A = (2, 5, -5),$$

$$B = \left( \frac{4}{3}, \frac{10}{3}, -\frac{10}{3} \right) \quad AB^2 = 6$$

4. Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AA^T = \begin{bmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{bmatrix}$$

$$a^2 + b^2 = 2$$

$$c^2 + d^2 = 2$$

$$ac + bd = 0$$

$$\det(A - I) = \begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = (a-1)(d-1) - bc$$

$$(1, 1, 1, 1), (1, 1, -1, -1), (1, -1, 1, 1), (1, -1, -1, -1), \quad (-1, 1, 1, 1), (-1, 1, -1, -1)$$

Six cases.

5.  $|A - xI| = 0 \Rightarrow \begin{vmatrix} 2-x & 1 & 3 \\ 1 & 1-x & 2 \\ 3 & 1 & 1-x \end{vmatrix} = 0$

$$x^3 - 4x^2 - 7x = -3 \Rightarrow A^3 - 4A^2 - 7A = -3I$$



$$A \left\{ -\frac{1}{3}A^2 + \frac{4}{3}A + \frac{7}{3}I \right\} = 1$$

$$A^{-1} = -\frac{1}{3}A^2 + \frac{4}{3}A + \frac{7}{3}I$$

$$\alpha = -\frac{1}{3}, \beta = \frac{4}{3}, \gamma = \frac{7}{3}$$

6.  $f(x)$  is continuous at  $x=0$  and  $f'(0)=0$

$$\Rightarrow \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} f(0-h) = f(0) = 0$$

$$\text{and L.H.D at } x=0 \text{ is } \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = 0$$

$$\text{and R.H.D at } x=0 \text{ is } 0 \Rightarrow \text{L.H.D} = \text{R.H.D at } x=0 \Rightarrow f'(0) = 0$$

$$7. w = \frac{z-1}{z-2}$$

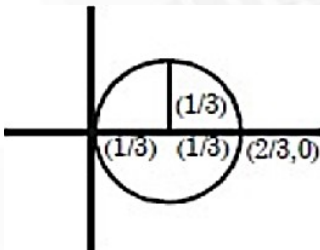
$$z = \frac{2w-1}{w-1}$$

$$|z|=1 \Rightarrow (w-1)(\bar{w}-1) = (2w-1)(2\bar{w}-1)$$

$$3|w|^2 - w - \bar{w} = 0$$

$$3(x^2 + y^2) - 2x = 0$$

$$\left(x - \frac{1}{3}\right)^2 + y^2 = \frac{1}{9}$$



8. Let  $a^2 = \tan \alpha$ ,  $b^2 = \tan \beta$  and  $c^2 = \tan \gamma$

$$\text{Then, } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \Rightarrow \sin^2 \alpha = \cos^2 \beta + \cos^2 \gamma$$

Use  $AM \geq GM$

$$\sin^2 \alpha = \cos^2 \beta + \cos^2 \gamma = 2 \left( \cos^2 \beta \cos^2 \gamma \right)^{1/2} \Rightarrow \sin^2 \alpha \geq 2 \cos \beta \cos \gamma$$

$$\text{Similarly, } \sin^2 \beta \geq 2 \cos \alpha \cos \gamma \text{ and } \sin^2 \gamma \geq 2 \cos \alpha \cos \beta$$

Multiplying 1, 2, 3

$$\sin^2 \alpha \sin^2 \beta \sin^2 \gamma \geq 8 \cos^2 \alpha \cos^2 \beta \cos^2 \gamma$$

$$\Rightarrow \tan^2 \alpha \tan^2 \beta \tan^2 \gamma \geq 8 \Rightarrow \tan \alpha \tan \beta \tan \gamma \geq 2\sqrt{2}$$

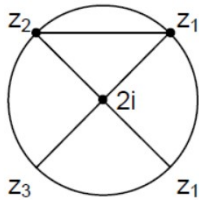
$$\Rightarrow a^2 b^2 c^2 \geq 2\sqrt{2} \text{ minimum value } 2\sqrt{2} = 2.82$$



$$9. \quad \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{\frac{k}{n^2} + \frac{k^2}{n^3}}{\sqrt{1 + \frac{k}{n^2} + \frac{k^2}{n^3} + 1}} \right) = \frac{1}{2} \int_0^1 (x + x^2) dx = \frac{5}{12}$$

$$10. \quad (z - 2i)^4 = 1 + i$$

$$z - 2i = (\sqrt[4]{2}) e^{i \left( \frac{2k\pi + \pi/4}{4} \right)} \quad \text{Put } k = 0, 1, 2, 3$$



$$\text{Minimum value of } \sum_{i=1}^4 |z - z_i| = 4 \cdot 2^{\frac{1}{8}} = 2^{2 + \frac{1}{8}} = 2^{\frac{17}{8}}$$

$$11. \quad \text{Let } N = \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} \sum_{c=1}^{\infty} \frac{ab(3a+c)}{4^{a+b+c} (a+b)(b+c)(c+a)} \Rightarrow 6N = 3 \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} \sum_{c=1}^{\infty} \frac{1}{4^{a+b+c}}$$

$$N = \frac{1}{2} \left( \sum_{a=1}^{\infty} \frac{1}{4^a} \right)^3 = \frac{1}{54} \Rightarrow 600N = \frac{100}{9} = 11.11$$

$$12. \quad (x-1)(x-n) < 0$$

$$x \in (1, n) \quad \therefore 2n - 11 = n - 2 \Rightarrow n = 9$$

$$\cot^{-1}(\cot 9) = 9 - 2\pi$$

$$13. \quad \text{Let } A = (h, k)$$

$$\text{Equation of normal to the parabola is } y = -tx + 2at + at^3 \Rightarrow k = -th + 2at + at^3$$

$$\text{or } at^3 + (2a - h)t - k = 0 \Rightarrow t_1 + t_2 + t_3 = 0$$

$$\text{Now } P = (at_1^2, 2at_1), Q = (at_2^2, 2at_2), R = (at_3^2, 2at_3)$$

$$\text{Equation of } C_1 \text{ is } (x-h)(x-at_1^2) + (y-k)(y-2at_1) = 0$$

$$\text{Equation of } C_2 \text{ is } (x-h)(x-at_2^2) + (y-k)(y-2at_2) = 0$$

$$\text{Equation of the common chord is } (t_1 + t_2)x + 2y - h(t_1 + t_2) - 2k = 0$$

$$\Rightarrow m_1 = -\left( \frac{t_1 + t_2}{2} \right) = \frac{t_3}{2}$$

$$\text{Also the equation of tangent through } R \text{ is } t_3 y = x + at_3^2 \Rightarrow m_2 = \frac{1}{t_3} \quad \text{So } m_1 m_2 = \frac{1}{2}$$



14. Diagonal elements must be  $\langle 0, 0, 0 \rangle$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

Now above diagonal element we can place

case 1:  $0, 1, -1$  by  $3!$  ways

case 2:  $0, 1, 1$  by  $\frac{3!}{2!}$  ways

case 3:  $0, -1, -1$  by  $\frac{3!}{2!}$  ways

$\therefore$  Total number of skew symmetric matrix = 12

15.  $\Delta = \begin{bmatrix} k_1 & a & b \\ a & k_2 & c \\ b & c & k_3 \end{bmatrix}$  be the corresponding matrix from set A. Its determinant value

$$\Delta = k_1 k_2 k_3 - (k_1 c^2 + k_2 b^2 + k_3 a^2) + 2abc \text{ case 1 : all diagonal element must be 0}$$

$$\Rightarrow \Delta = 0$$

so all matrix is singular case 2 : Only one diagonal element is 0 say

$$k_1 = 0$$

$$\Rightarrow \Delta = 2abc - k_2 b^2 - k_3 a^2$$

$a = b = 0$	$a = c = 0$	$b = c = 0$
$\Delta = 0$	$\Delta = k_2 b^2$	$\Delta = -a^2 k_3$
	here two	here two
	non-singular	non-singular
	matrix possible	matrix possible

$\therefore$  Total 4 matrix when  $k_1 = 0$

4 matrix when  $k_2 = 0$  and  $k_3 = 0 \therefore$  Total = 12

16.  $\int_0^{2018} f(x) dx = 3 \left( \frac{1}{2} (2018)(1009) \right)$

17.  $\int_0^{2018} g(x) dx = 2018 \int_0^1 g(x) dx = \frac{2018}{4} = \frac{1009}{2}$



## PHYSICS

18. When bullet-block system reaches its peak height, its speed is zero. Therefore it's momentum, angular momentum is not conserved. Also collision is perfectly inelastic

19. Recall the acceleration of an object rolling without slipping down an inclined plane is given by

$$I = \beta mr^2$$



$$a = \frac{g \sin \theta}{1 + \beta}$$

where the object has moment of inertia  $I = \beta mr^2$

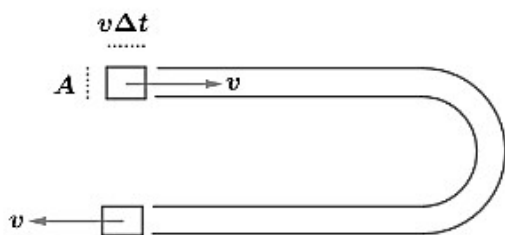
Objects A, B, and D are all solid balls with  $\beta = 2/5$  so they reach the bottom at the same time. Object C is a hollow sphere with  $\beta = 2/3$  so it has a smaller angular acceleration than a solid sphere. Hence, it takes longer to reach the bottom. Thus,  $T_C > T_A = T_B = T_D$

20. For the in-phase situation, the separation between the masses is constant, so the force of A on B is constant. In that case, only one spring exerts a varying force, and then the angular frequency must be given by  $\omega_1 = \sqrt{k/m}$

For the out of phase situation, the separation between the objects is not constant, but there exists a point on the connecting spring that does not move. As such, either object can be thought of moving under the influence of two springs in parallel, one with length  $L$ , the other with length  $L/2$ . That is equivalent to a spring with constant  $k$  in parallel with a spring of constant  $2k$ . The net spring constant is then  $k + 2k = 3k$ .

As such, the frequency is given by  $\omega_2 = \sqrt{3k/m} = \sqrt{3}\omega_1$

21. First, we study the force on one U-tube:



$$F = \frac{\Delta p}{\Delta t} = \frac{(\Delta m)v - (\Delta m)(-v)}{\Delta t} = 2 \frac{(\Delta m)v}{\Delta t} = 2\rho A v^2$$

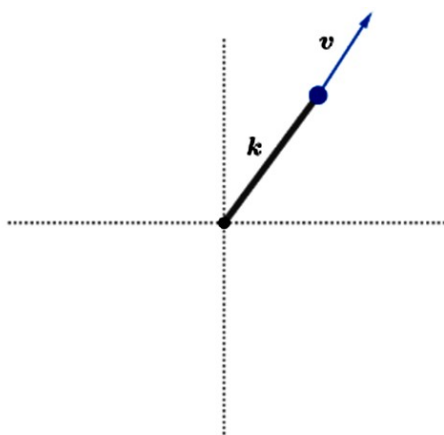
By Newton's 3rd law, this is also the force the water exerts on the U-tube. Going to the tube assembly of two U-tubes, since the net force is zero, we balance forces from each side:



$$F_1 = 2\rho A v^2 = 2\rho A' v'^2 = F_2$$

$$v' = v \sqrt{\frac{A}{A'}} = \sqrt{2}v$$

22.



Since  $\vec{r} = x\vec{i} + y\vec{j}$ , we have  $\vec{F} = -k\vec{r}$  so we can identify the spring constant as  $k = 8 \text{ N/m}$ .

The period of a mass-spring system is given by  $T = 2\pi\sqrt{\frac{m}{k}}$

The particle first returns to the origin in half the period,

$$v_{\max} = A\omega \text{ and } a_{\max} = A\omega^2 \quad \frac{T}{2} = \pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{2 \text{ kg}}{8 \text{ N/m}}} = 1.57 \text{ s}$$

$$23. \int_{P_1}^{P_2} \vec{B} \cdot d\vec{\ell} = \left( \frac{45^\circ}{360^\circ} \mu_0 I \right) + \left( \frac{45^\circ}{360^\circ} \mu_0 I \right) + \left( \frac{30^\circ}{360^\circ} \mu_0 I \right) - \left( \frac{15^\circ}{360^\circ} \mu_0 I \right) = \frac{7}{24} \mu_0 I$$

24. Conceptual

25. Draw a free-body diagram for each bead; let  $F_N$  be the (inward) normal force exerted by the hoop on the bead. Let  $\theta$  be the angular position of the bead, measured from the top of the hoop, and let the hoop have radius  $r$ . We see that

$$F_N + mg \cos \theta = m \frac{v^2}{r}$$

$$F_N = m \frac{v^2}{r} - mg \cos \theta$$

The (downward) vertical component  $F_{Ny}$  is given by  $F_{Ny} = F_N \cos \theta$

From Newton's third law, the two beads together exert an upwards vertical force on the hoop given by

$$F_u = 2F_{Ny}$$

$$F_u = 2m \cos \theta \left( \frac{v^2}{r} - g \cos \theta \right)$$

noting that the beads clearly reach the same position at the same time.

Meanwhile, when each bead is at a position  $\theta$  it has moved through a vertical distance  $r(1 - \cos \theta)$ . Thus from energy conservation,

$$\frac{1}{2}mv^2 = mgr(1 - \cos \theta)$$

$$\frac{v^2}{r} = 2g(1 - \cos \theta)$$





Inserting this into the previous result,

$$F_u = 2m\cos\theta(2g(1 - \cos\theta) - g\cos\theta)$$

$$F_u = 2mg(2\cos\theta - 3\cos^2\theta)$$

If the beads ever exert an upward force on the hoop greater than  $m_h g$ , the hoop will leave the ground; i.e., the condition for the hoop to remain in contact with the ground is that for all  $\theta$ ,

$$F_u \leq m_h g$$

We can replace the left side by its maximum value. Letting  $s = \cos\theta$ ,

$$F_u = 2mg(2s - 3s^2)$$

$$\frac{d}{ds} F_u = 2mg(2 - 6s)$$

The derivative is zero at  $s = \frac{1}{3}$ , where  $F_{u(\max)} = \frac{2}{3}mg$

So our condition is

$$\frac{2}{3}mg \leq m_h g$$

$$\frac{m}{m_h} \leq \frac{3}{2}$$

26. When speed is maximum, then  $F_{\text{electrostatics}} = F_{\text{friction}} \Rightarrow \frac{Kq^2}{r^2} = \mu mg \Rightarrow r = 3 \text{ m}$

$\therefore \text{C is correct}$

$$\frac{Kq^2}{1} = \frac{Kq}{2}^2 + 2 \times \frac{1}{2}mv^2 + \mu mg(3 - 1) \Rightarrow v = 2 \text{ m/s}$$

27. In equilibrium, torque about any point must be zero. Consider torque about the bottom most point.

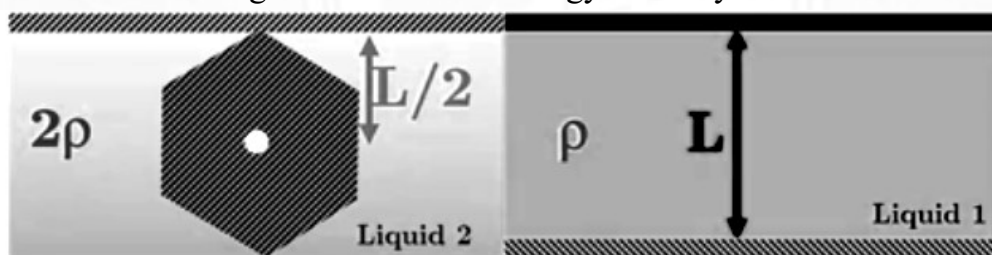
28.

$$\vec{B} = \mu_0 \epsilon_0 (\vec{v} \times \vec{E}) = \mu_0 \epsilon_0 (-\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})$$

$$\vec{B} = 2(\hat{i} - \hat{k})\mu_0 \epsilon_0$$

$$B = 2\sqrt{2}\mu_0 \epsilon_0$$

29. Work done = Change in Mechanical Energy of the system



$$\Delta W = \Delta PE_2 + \Delta PE_1 + \Delta PE_{\text{cube}} = 2Mg \times \frac{L}{2} + Mg \times L - 4Mg \times 2L$$

$$\Delta W = -MLL$$

30. Since the new bulb emits the same spectrum of light, the emitted power is simply proportional to the surface area, where we ignore the surface area of the ends as directed,

$$P \propto 2\pi aL \propto aL \text{ If the resistivity of the filament is } \rho, \text{ the resistance is } R = \frac{\rho L}{A} = \frac{\rho L}{\pi a^2}.$$





Therefore, the power is also given by

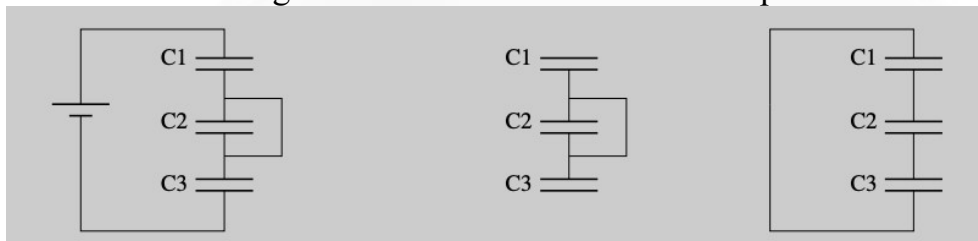
$$P = \frac{V^2}{R} = \frac{V^2 \pi a^2}{\rho L} \propto \frac{a^2}{L}$$

Thus if the new power is  $nP$ , we may only satisfy both equations if the new radius and length are  $a' = n^{2/3} a$ ,  $L' = n^{1/3} L$ .

31-32.

We treat the plates as three capacitors in series. Each has an identical capacitance  $C$ .

The figure below then show the three steps.



Let the final charge on the top plate of each capacitor also be labelled as  $q_1$ ,  $q_2$ , and  $q_3$ .

The last figure implies that  $V_1 + V_2 + V_3 = 0$

By symmetry, we have  $V_1 = V_3$  so  $2V_1 = -V_2$

By charge conservation between the bottom plate of  $C_1$  and the top plate of  $C_2$  we have

$$-q_0 = -q_1 + q_2$$

But  $q = CV$ , so  $-\frac{1}{2}V_0 = -V_1 + V_2$

Combining the above we get

$$-\frac{1}{2}V_0 = \frac{1}{2}V_2 + V_2$$

$$-\frac{1}{3}V_0 = V_2$$

Finally, solving for  $V_1$ , we get  $V_1 = V_0/6$ .

33-34.

i. As the rod accelerates downward, the rate of change in flux increases, increasing the emf and hence the current through the fuse. This produces an upward force on the rod, so if the fuse were indestructible, the rod would eventually reach terminal velocity. Hence the smallest mass that can break the fuse is the one which breaks it just as it reaches terminal velocity.

The maximal possible force on the rod comes when the fuse is about to break,

$$F = ILB = (5.0 \text{ A})(0.3 \text{ m})(1.2 \text{ T}) = 1.8 \text{ N}$$

This force must be exerted at terminal velocity, so its magnitude is  $mg$ , and

$$m = \frac{F}{g} = 0.18 \text{ kg}$$

ii. We must relate the force on the rod to the rod's velocity. The emf in the circuit is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = BLv$$

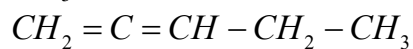
where  $v$  is the speed of the rod, so using Ohm's law,  $F = \frac{\mathcal{E}LB}{R} = \frac{B^2 L^2 v}{R}$

Rearranging to solve for  $v$ , we have  $v = \frac{FR}{B^2 L^2} = \frac{(1.8 \text{ N})(0.25 \Omega)}{(1.2 \text{ T})^2 (0.3 \text{ m})^2} = 3.5 \text{ m/s}$



## CHEMISTRY

35. Ans. C



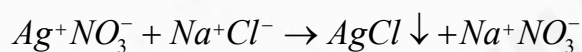
36. Ans. B



$$79 = [2(O - F) + 2(O - H)] - [(O = O) + 2(H - F)]$$

(Reaction is exothermic)

37. Ans. B

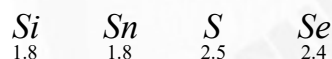


38. Ans. D

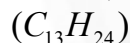
Inter planer distance in much smaller range.

39. Ans. A, B

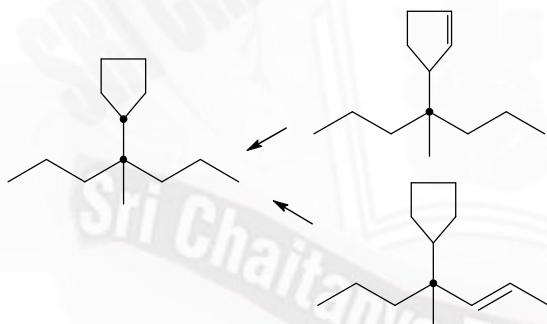
P 2.1



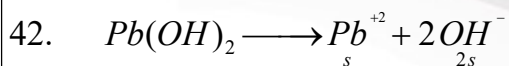
40. Ans. A, B, C, D



$$DU = 2$$



41. Ans. Factual



$$K_{sp} = (s)(2s)^2$$

$$= 4s^3$$

$$= 4 \times 10^{-12}$$

$$(Pb^{+2})(OH^-)^2 = 4 \times 10^{-12}$$

$$(s')(10^{-8}) = 4 \times 10^{-12}$$

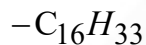
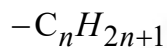
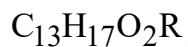
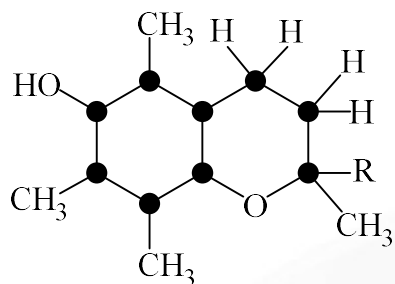
$$(s') = 4 \times 10^{-4}$$

$$pOH = 4$$

$$[OH^-] = 10^{-4}$$



43. Sol.



44. a = 4

b = 8

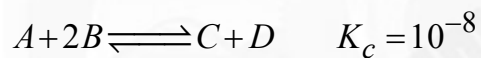
c = 3

d = 5

$$a + b + c + d = 20$$

45.  $1 + 2 + 3 + 4 + 5 = 15$

46. Sol.



$$0 \quad 0 \quad 2 \quad 4$$

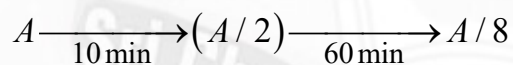
$$2 \quad 4 \quad x \quad 2$$

$$K_c = 10^{-8} = \frac{(x)(2)}{(2)(4)^2}$$

$$x = 1.6 \times 10^{-7}$$

$$1 + 6 - 7 = 0$$

47.  $A \longrightarrow 2B$



2<sup>nd</sup> order

48. Sol.



		1	2
		5	6
		3	5 Si 14
		13	
Cu 29	4	Ga 31	32
30			
6	7	8	9
47		48	49 50
10		11	12
79	Hg 80	81	82
13	14	15	FI 114
111	112	113	

49. Sol.  
 $x = 2$   
 $y = 3$   
 $z = 2$   
 $x \times y \times z = 12$

50. Proline

Isoelectric point (pH = 6.3)

51. (Yellow)  
[3]