NARAYANA IIT ACADEMY INDIA

Sec: OSR.IIT_*CO-SC | OGTA-3(P1) | Date: 10-03-25
Time: 3HRS | 2023_P1 | Max. Marks: 180

# KEY SHEET

## MATHEMATICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **ABC** | 2 | **AC** | 3 | **BCD** | 4 | **A** | 5 | **A** |
| 6 | **A** | 7 | **B** | 8 | **1** | 9 | **6** | 10 | **495** |
| 11 | **31** | 12 | **8** | 13 | **23** | 14 | **B** | 15 | **D** |
| 16 | **A** | 17 | **D** | | | | | | |

## PHYSICS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 18 | **ACD** | 19 | **AC** | 20 | **BCD** | 21 | **D** | 22 | **A** |
| 23 | **C** | 24 | **A** | 25 | **2** | 26 | **5** | 27 | **3** |
| 28 | **2** | 29 | **60** | 30 | **2** | 31 | **A** | 32 | **B** |
| 33 | **C** | 34 | **B** | | | | | | |

## CHEMISTRY

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|---|---|---|---|---|---|---|---|---|---|
| 35 | **CD** | 36 | **ABC** | 37 | **ABD** | 38 | **B** | 39 | **C** |
| 40 | **D** | 41 | **B** | 42 | **4** | 43 | **12** | 44 | **85** |
| 45 | **20** | 46 | **1140** | 47 | **39** | 48 | **D** | 49 | **B** |
| 50 | **D** | 51 | **A** | | | | | | |

# SOLUTIONS
## MATHS

1. $f(x)=\sin x+px+q$

$f'(x)=\cos x+p$

If $p>1$ then $f(x)$ is increasing. So only one real root, which is positive if $q<0$ and negative if $q>0$.

If $p<-1 f(x)$ is decreasing so only one real root, which is negative if $q<0$

2. $u.\cdot\vec{w}=\cos\alpha\cos\gamma+\sin\alpha\sin\gamma=\cos(\alpha-\gamma)$

$\vec{v}\cdot\vec{w}=\cos(\beta-\gamma),\vec{u}\cdot\vec{v}=\cos(\alpha-\beta)$

Let $A=\alpha-\gamma, B=\beta-\gamma, \alpha-\beta=A-B$

Now, $\vec{u}\cdot\vec{w}=\vec{v}\cdot\vec{w}$

$\Rightarrow \cos A=\cos B \Rightarrow B=2m\pi+A$ where $m\in I$.

But $-2\pi<A, B<2\pi$

$\therefore B=A, B=-A, B=2\pi-A$ or $B=-2\pi+A$

when $B=A$ or $B=-2\pi+A$

we get $\cos A=-\sqrt{3}\cos 0$ or $-\sqrt{3}\cos(-2\pi)$

$\Rightarrow \cos A=-\sqrt{3}$. Not possible

when $B=-A$ or $B=2\pi-A$, we get $\cos A=-\sqrt{3}\cos(2A)$

$\Rightarrow 2\sqrt{3}\cos^2 A+\cos A-\sqrt{3}=0$

$$\Rightarrow \cos A=\frac{-1\pm\sqrt{1+4(2\sqrt{3})(\sqrt{3})}}{2(2\sqrt{3})}$$

$$=\frac{-1\pm 5}{4\sqrt{3}}=\frac{1}{\sqrt{3}},\frac{\sqrt{3}}{2}$$

Now, $\vec{u}\cdot\vec{v}=\cos(2A)$

$$=2\cos^2 A-1=\begin{cases}-\frac{1}{3} & \text{when } \cos A=\frac{1}{\sqrt{3}}\\ \frac{1}{2} & \text{when } \cos A=-\frac{\sqrt{3}}{2}\end{cases}$$

3. $3PQP^{-1}+3P=2P^{-1}QP+2P$

$\Rightarrow 3PQP+3PPP^{-1}=2P^{-1}QP+2P^{-1}PP$

$\Rightarrow |3P(Q+P)P^{-1}|=|2P^{-1}(Q+P)P|$

$\Rightarrow 3^n|\mathbf{P}+Q|=2^n|\mathbf{P}+Q|$

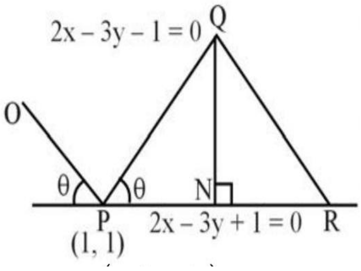
$\Rightarrow |P+Q|=0$

(c) $3PQP^{-1}-3P^{-1}QP=-P^{-1}QP-P$

$=-P^{-1}QP-P^{-1}PP$

$\Rightarrow 3^n|PQP^{-1}-P^{-1}QP|=(-1)^n|Q+P|=0$

4.




Let $Q = \left(t, \dfrac{2t-1}{3}\right)$

$PQ = QN\operatorname{cosec}\theta$

$\Rightarrow t = \dfrac{47}{13}, -1$

For $t = -1, Q$ lies on $OP$ produced which is not possible

$\Rightarrow Q \equiv \left(\dfrac{47}{13}, \dfrac{27}{13}\right)$

Let $R = \left(\dfrac{3\alpha - 1}{2}, \alpha\right)$

Slope of QR = slope of OP $\Rightarrow \alpha = 53/13$

5. $S_1 < 0 \Rightarrow m^2 - 3m + 2 < 0 \Rightarrow m \in (1,2)$

$(2m - 1 - 2)\left(2\left(\dfrac{3}{2}\right) - 0 - 2\right) < 0 \Rightarrow m < \dfrac{3}{2}$

hence, $m \in \left(1, \dfrac{3}{2}\right)$

6. $h'(x) = (2x^2 - \ln x)g(x)$

$g'(x) = \dfrac{1}{\ln x^3}3x^2 - \dfrac{1}{\ln x^2}2x$

$g'(x) = \dfrac{x(x-1)}{\ln x} > 0 \forall x > 1; g(x) > g(1) \Rightarrow g(x) > 0 \forall x > 1$

For $h(x)$ is increasing

$h'(x) > 0 \Rightarrow 2x^2 - \ln x > 0$ as $(g(x) > 0)$

Let $H(x) = 2x^2 - \ln x$

$H'(x) = 4x - \dfrac{1}{x} = \dfrac{4x^2 - 1}{x} > 0$ where $x > 1$

$H(x) > H(1) \Rightarrow H(x) > 2$

$\therefore h'(x) > 0 \forall x \in (1, \infty)$

$\therefore h(x)$ is increasing on $(1, \infty)$

7. The given equation is $x^3 + y^3 = a^3$ .Differentiating both sides w.r.t. $x$ we have $\dfrac{dy}{dx} = \dfrac{-x^2}{y^2}$

$\left(\dfrac{dy}{dx}\right)_{(x',y')} = -\left(\dfrac{x'}{y'}\right)^2$ …….(i)

Equation of the tangent at ( $x', y'$ ) is $y - y' = -\left(\frac{x'}{y'}\right)^2 (x - x')$

$(x')^2 x + (y')^2 y = (x')^3 + (y')^3$

This passes through $(x_1, y_1)$. Therefore

$(x')^2 x_1 + (y')^2 y_1 = (x')^3 + (y')^3 = a^3 = x_1^3 + y_1^3$

As $(x_1, y_1)$ lies on the curve. Hence $x_1(x'^2 - x_1^2) = -y_1(y'^2 - y_1^2) - \frac{x_1(x'+x_1)}{y_1(y'+y_1)} = \frac{y'-y_1}{x'-x_1}$

$=$ Slope of the line joining $(x', y')$ and $(x_1, y_1)$

Which is the slope of the tangent at ( $x', y'$ ). So from equation (i)

$\frac{-x_1(x'+x_1)}{y_1(y'+y_1)} = -\left(\frac{x'}{y'}\right)^2$

$x_1 x'(y')^2 + x_1^2 (y')^2 = y_1 y' x'^2 + y_1^2 x'^2$

$x'y'(x_1 y' - x' y_1) = -(x_1 y' - x y_1)(x_1 y' + x y_1) \ldots$(ii)

Suppose $x_1 y' - x' y_1 = 0$ so that $\frac{x'}{x_1} = \frac{y'}{y_1} = \lambda$

Hence, $x' = \lambda x_1, y' = \lambda y_1$ so that

$a_3 = x'^3 + y'^3 = \lambda^3 (x_1^3 + y_1^3) = \lambda^3 a^3$

Which implies $\lambda = 1$ and hence $x' = x_1$ and $y_1 = y'$ a contradiction. Therefore $x_1 y' - x' y_1 \neq 0$ .Hence from equation (ii)

$x'y' = -(x_1 y' + x' y_1)$

Dividing both sides with $x'y'$ we have $\frac{x_1}{x'} + \frac{y_1}{y'} = -1$

8. Put $x^2 = X \Rightarrow x dx = \frac{dX}{2}$

$y^2 = Y \Rightarrow y dy = \frac{dY}{2}$

$(2X + 3Y - 7)\frac{dX}{2} - (3X + 2Y - 8)\frac{dY}{2} = 0$

$\frac{dY}{dX} = \frac{2X + 3Y - 7}{3X + 2Y - 8}$

Reduce it to variable separable by substituting

$X = X' + h, Y = Y' + k$

We get the solution $(x^2 + y^2 - 3) = (x^2 - y^2 - 1)^5 C$

9. $|z| - \left|\frac{5}{z}\right| \leqslant \left|z - \frac{5}{z}\right| \Rightarrow \left||z| - \frac{5}{|z|}\right| \leqslant 4$

Let $|z| = t$

$-4 \le t - \frac{5}{t} \leqslant 4 \Rightarrow -4t \leqslant t^2 - 5 \leqslant 4t$

Now, $t^2 + 4t - 5 \ge 0$ $t^2 - 4t - 5 \le 0$

$(t+5)(t-1)\geq 0\,(t-5)(t+1)\leq 0$

$\Rightarrow t\in[1,\infty)\ t\in[0,5]$ (as $t$ is positive)

$\therefore t\in[1,5]$

10. $\left(\frac{x^4+x^2+1}{x^2-x+1}-\frac{x^2+1}{x}\right)^{12}=\left(x^2-\frac{1}{x}\right)^{12}$

$\Rightarrow 9^{th}$ term is free from '$x$'

11. The key is to count backwards. First choose the pair which you pick on Wednesday in 5 ways. Then there are four pairs of socks for you to pick a pair of on Tuesday and don't want to pick a pair. Since there are 4 pairs, the number of ways to do this is $\binom{8}{2}-4$. Then, there are two pairs and two nonmatching socks for you to pick from on Monday a total of 6 socks. Since you don't want to pick a pair, the number of ways to do this $\binom{6}{2}-2$. Thus the answer is

$$\frac{(5)\left(\binom{8}{2}-4\right)\left(\binom{6}{2}-2\right)}{\binom{10}{2}\binom{8}{2}\binom{6}{2}}=\frac{26}{315}$$

$$\frac{m+n}{11}=\frac{315+26}{11}=\frac{341}{11}=31$$

12. $\because$ Elements of determinant can be 1 or -1

$\therefore$ First two places of first row can be filled in $2\times 2=4$ ways and last place can be filled in only one way to product be 1

$\therefore$ First row can be made in 4 ways

Similarly, second row can be made in 4 ways and third row can be made in 1 ways only

$\therefore m=16$

$$\because |P|=\begin{vmatrix}3 & -2 & 3\\ 2 & -2 & 3\\ 0 & -1 & 1\end{vmatrix}=1$$

$\because \operatorname{adj}(\operatorname{adj}\mathbf{P})=|\mathbf{P}|^{n-2}.\mathbf{P}=P\Rightarrow \operatorname{tr}(\operatorname{adj}(\operatorname{adj}\mathbf{P}))=\operatorname{tr}(\mathbf{P})=2=n$

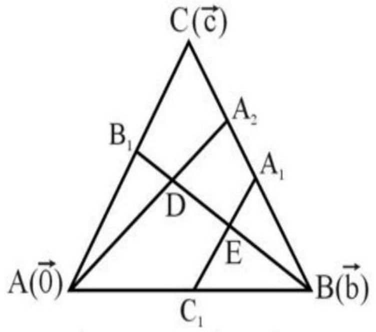
$\because \frac{m}{n}=\frac{16}{2}=8$

13. Let the position vectors of $A,B$ and $C$ be $\vec{O},\vec{b}$ and $\vec{c}$-respectively.

We have, $\overrightarrow{AC_1}=\frac{\vec{b}}{2},\overrightarrow{AB_1}=\frac{\vec{c}}{2},\overrightarrow{AA_1}=\frac{\vec{b}+\vec{c}}{2},\overrightarrow{AA_1}=\frac{3\vec{c}+\vec{b}}{4}$ Equation of lines $BB_1,AA_2$ and $C_1\ A_1$ are respectively

$\vec{r}=\vec{b}+\lambda_1\left(\frac{\vec{c}}{2}-\vec{b}\right)\quad \vec{r}=\lambda_2\frac{3\vec{c}+\vec{b}}{4}$ and $\vec{r}=\frac{\vec{b}}{2}+\lambda_3\left(\frac{\vec{c}}{2}\right)$

For point $D$, we have $\vec{b}+\lambda_1\left(\frac{\vec{c}}{2}-\vec{b}\right)=\lambda_2\left(\frac{3\vec{c}+\vec{b}}{4}\right)$




$$\Rightarrow \vec{b}\left(1-\lambda_1-\frac{\lambda_2}{4}\right)+\frac{\vec{c}}{4}(2\lambda_1-3\lambda_2)=\vec{0}$$

$$\Rightarrow \lambda_1=\frac{6}{7},\lambda_2=\frac{4}{7}$$

$$\Rightarrow \overrightarrow{AD}=\frac{3\vec{c}+\vec{b}}{7}$$

For point E we have $\vec{b}+\lambda_1\left(\frac{\vec{c}}{2}-\vec{b}\right)=\frac{\vec{b}}{2}+\frac{\lambda_3}{2}\vec{c}$

$$\Rightarrow \lambda_1=\lambda_3=\frac{1}{2}\Rightarrow=\frac{2\vec{b}+\vec{c}}{4}$$

Now, $\overrightarrow{EA_1}=\frac{3\vec{c}+\vec{b}-2\vec{b}-\vec{b}}{4}=\frac{2\vec{c}-\vec{b}}{4}$,

$$\overrightarrow{DA_1}=\frac{\vec{b}+\vec{c}}{2}-\frac{3\vec{c}+\vec{b}}{7}=\frac{5\vec{b}+\vec{c}}{14}$$

Area of the quadrilateral $EA_1\ A_2D=\frac{1}{2}\left|\overrightarrow{EA_2}\times\overrightarrow{DA_1}\right|$

$$=\frac{1}{112}\left|\left(2\vec{c}-\vec{b}\right)\times\left(5\vec{b}+\vec{c}\right)\right|=\frac{1}{112}\left|10\vec{c}\times\vec{b}-\vec{b}\times\vec{c}\right|$$

$$=\frac{11}{112}\left|\vec{c}\times\vec{b}\right|=\frac{11}{56}\times\frac{1}{2}\left|\vec{c}\times\vec{b}\right|=\frac{11}{56}\ \text{(area of }\triangle ABC\text{ )}$$

Thus required ratio of is $\frac{11}{56}$

14. (P) $\frac{dy}{dx}=\frac{-y\sin x}{1+\cos x}$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{-\sin x}{1+\cos x}dx$$

$$\Rightarrow \ln y=\ln\left(1+\cos x\right)+\ln C$$

$$\Rightarrow y=\left(1+\cos x\right)C$$

$$\because y\left(\frac{\pi}{2}\right)=-1$$

$$\therefore y=-\left(1+\cos x\right)\Rightarrow y\left(0\right)=-2$$

(Q) At $x=\frac{-\pi}{2},-1,0,1,\frac{\pi}{2},f\left(x\right)$ is non-derivable

(R) Since $e^{\ln(x+1)}=x+1$, thus $e^{\ln(x+1)}=|y|$

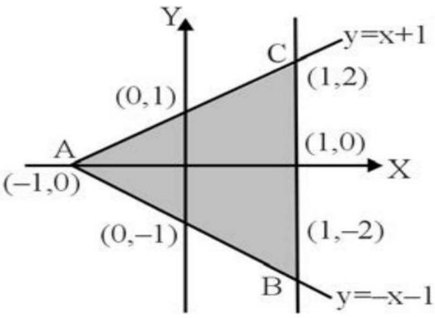
$\Rightarrow x+1 \geqslant |y|$

$\Rightarrow \mathrm{x}+1<\quad )0., \mathrm{x} \geqslant 1$ or $(\mathrm{x}+1) \geqslant 0$

and $-(x+1) \leqslant y \leqslant (x+1)$.

Thus the region represented by inequalities $-(\mathrm{x}+1)$

$<\mathrm{y}<(\mathrm{x}+1)$ and $|\mathrm{x}| \leqslant 1., -1 \leqslant \mathrm{x} \leqslant 1$ is as shown below




Thus required area is the area of $\triangle ABC = \frac{1}{2}(4)(2)=4$ square units

(S) $\left(\sec^{-1} x+\operatorname{cosec}^{-1} x\right)^{3}-3 \sec^{-1} x \operatorname{cosec}^{-1} x\left(\sec^{-1} x+\operatorname{cosec}^{-1} x\right)\left(\frac{\pi}{2}\right)^{3}$

$\Rightarrow \frac{\pi^{3}}{8}-3 \sec^{-1} \times \operatorname{cosec}^{-1} x\left(\frac{\pi}{2}\right)=\frac{7 \pi^{3}}{8}$

$\Rightarrow \sec^{-1} \mathrm{x}=\pi$ or $\sec^{-1} \mathrm{x}=\frac{-\pi}{2}$

$\Rightarrow \mathrm{x}=-1$

15. (P) sinAsin2 Asin3 Asin4 A

$=a x^{2}+b x^{3}+c x^{4}+d x^{5}$

$a+b+c+d=f(1)$

Put $\mathrm{x}=1$

$\Rightarrow \sin^{2} \mathrm{~A}=1$

$A=(2 n+1) \frac{\pi}{2}$

$\Rightarrow \sin \mathrm{A} \cdot \sin 2 \mathrm{~A} \cdot \sin 3 \mathrm{~A} \cdot \sin 4 \mathrm{~A}=0$

(Q) $2 \sin x \cos x=\frac{1}{\sqrt{2}}$

$\Rightarrow \sin 2 x=\frac{1}{\sqrt{2}} \Rightarrow 2 \mathrm{x}=\frac{\pi}{4}, \frac{3 \pi}{4}$

$\Rightarrow \mathrm{x}=\frac{\pi}{8}, \frac{3 \pi}{8}$

$\Rightarrow \mathrm{d}=\frac{3 \pi}{8}-\frac{\pi}{8}=\frac{\pi}{4}$

(R) $4 \cos x\left(2-3 \sin^{2} x\right)+(\cos 2 x+1)=0$

$\Rightarrow 4 \cos \mathrm{x}\left(-1+3 \cos^{2} \mathrm{x}\right)+2 \cos^{2} \mathrm{x}=0 \Rightarrow 2 \cos x\left[-2+6 \cos^{2} x+\cos x\right]=0$

$\Rightarrow 2 \cos \mathrm{x}(2 \cos \mathrm{x}-1)(3 \cos \mathrm{x}+2)=0 \Rightarrow \cos x=0, \frac{1}{2}, \frac{-2}{3}$

In $x\in\left[0,\frac{\pi}{2}\right]\Rightarrow x=\frac{\pi}{2},\frac{\pi}{3}$

Least deference between the roots $=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

(S) $f(x)=\sqrt{\sin^4x+4\cos^2x}-\sqrt{\cos^4x+4\sin^2x}$

$=\sqrt{\sin^4x+4-4\sin^2x}-\sqrt{\cos^4x+4-4\cos^2x}=2-\sin^2x-\left(2-\cos^2x\right)$

$=\cos^2x-\sin^2x=\cos2x$

$g(\sin2t)=\sin t+\cos t$

$(g(\sin2t))^2=1+\sin2t\Rightarrow(g(\sin2t))^2=1+\sin2t$

$\Rightarrow g(\sin2t)=\sqrt{1+\sin2t}$

$\therefore g(x)=\sqrt{1+x},-1\leqslant x\leqslant1$

Now, $g\{f(x)\}=g(\cos2x)$

$=\sqrt{1+\cos2x}=\sqrt{2}|\cos x|$

$\therefore$ Range of $g\{f(x)\}$ is $\left[0,\sqrt{2}\right]$

$\Rightarrow a^2+b^2=2$

16. (P) $\lim_{h\to0^+}2\int_0^1\frac{hdx}{h^2+x^2}=\lim_{h\to0^+}\left[2\frac{h}{h}\tan^{-1}\frac{x}{h}\right]_{0^+}^1=\pi$

(Q) Put $x-2=x,y+1=y$

The required region is defined by $1\leqslant|x|+|y|\leqslant2$

Required area $=(2\sqrt{2})^2-(\sqrt{2})^2=6$

(R) $\int_0^{110\pi}[\sin x+\cos x]dx=55\int_0^{2\pi}[\sin x+\cos x]dx=55\int_{\frac{\pi}{4}}^{\frac{9\pi}{4}}\left[\sqrt{2}\sin\theta\right]d\theta=-55\pi$

(S) $$\frac{\left(\frac{x-2y+3}{\sqrt{5}}\right)^2}{4}+\frac{\left(\frac{2x+y+1}{\sqrt{5}}\right)^2}{\frac{1}{4}}=1$$

$a^2=4,b^2=\frac{1}{4}\Rightarrow e=\frac{\sqrt{15}}{4}$

$k^2=4a^2e^2=4\times4\times\frac{15}{16}=15$

17. (P) $9-a^2>5$

$\Rightarrow-2<a<2\Rightarrow$ number of integral values $=3$

(Q) $b+c=a^3-a\ \&\ bc=a^2$

$x^2-\left(a^3-a\right)a+a^2=0$ has two roots $b\ \&\ c$

as $b\ \&\ c$ are real so

$D\geq0$

$\left(a^3-a\right)^2-4a^2\geq0$

$a^2\left[\left(a^2-1\right)^2-4\right]\ge 0 \Rightarrow a^2\left[a^4-2a^2-3\right]\ge 0$

$a^2\left(a^2+1\right)\left(a^2-3\right)\ge 0$

$a^2-3\ge 0$

$a^2\ge 3\Rightarrow$ Least value of $a^2$ is 3 .

(R) number of terms in $(a+b+c-d)^n$ is ${}^{n+3}C_3$

and the sum of coefficients $=2^n$

So $2^n\ {}^{n+3}C_3$ which is true for $n>6$

(S) $\lim_{x\to\infty}\left(8x^3+ax^2\right)^{\frac{1}{3}}-bx \quad =\lim_{x\to\infty}(2x)\left(1+\frac{a}{8x}\right)^{\frac{1}{3}}-bx$

$=\lim_{x\to\infty}2x\left(1+\frac{a}{24x}\right)-bx=\lim_{x\to\infty}(2-b)x+\frac{a}{12}$

for limit to exists $2-b=0$

$\Rightarrow b=2\ \&\ \frac{a}{12}=1\Rightarrow a=12$

$a-3b=6$

## PHYSICS

18. $\Rightarrow dI=kr^2\left(2\pi r dr\right)$

$\Rightarrow I=2\pi k\int_0^a r^3 dr$

$\Rightarrow I=\frac{1}{2}\pi k a^4$

Further, field for $r>a$ is $B=\frac{\mu_0 I}{2\pi r}=\frac{\mu_0 k a^4}{4r}$

and field for $r<a$ is $B=\frac{\mu_0\left(\frac{\pi k A^4}{2}\right)}{2\pi r}=\frac{1}{4}\mu_0 k r^3$

19. When number of electrons increases, then number of photons emitted also increases, thereby increasing the intensity. Energy, wavelength and frequency of photon are related to potential difference.

20. Work done by $\vec{F}_1$ is $W_1=\int_{P_1}^{P_2}F_1\cos\theta ds$

Here, $ds=(6)d(-2\theta)=-12\ d\theta$ and $F_1=20$ N

$\therefore W_1=-240\int_{x/4}^{0}\cos\theta d\theta=240\sin\frac{\pi}{4}=120\sqrt{2}$ J

$\vec{F}_1$ is conservative because it is always directed

towards a fixed point $P_2$.

$W_2=F_2\left(OP_2\right)=(30)(6)=180$ J and

$W_3=\vec{F}_1\int_0^{6(\pi/2)}F_3 ds=\int_0^{3\pi}15ds$

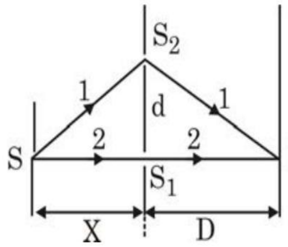
$=[15x]_0^{3\pi}=45\pi$

21. Acceleration $a = \frac{dv}{dt} \quad \Rightarrow a = 2bt$

For block to just slide on the plate, we have

$\mu mg = m(2bt) \Rightarrow t = \frac{\mu g}{2\,b}$

22. Refer to figure, to reach point $P$, wave 1 has to travel a path $(SS_2 + S_2P)$ while wave 2 has to travel a path $(SS_1 + S_1P)$. Therefore, when the waves arrive at $P$, the path difference is




$\delta = (SS_2 + S_2P) - (SS_1 + S_2P)$

Now, in triangle $SS_2\ S_1$, we have

$SS_2 = (x^2 + d^2)^{1/2} = \left(1 + \frac{d^2}{x^2}\right)^{1/2} = \left(1 + \frac{d^2}{2x^2}\right)(\because d << x)$

Similarly, $S_2P = (D^2 + d^2)^{1/2} = D\left(1 + \frac{d^2}{2D^2}\right)$

$(\because d << D)$

Also $(SS_1 + S_1P) = x + D$, Using these in Eq.

(1), we have

$\Delta = x\left(1 + \frac{d^2}{2x^2}\right) + D\left(1 + \frac{d^2}{2D^2}\right) - (x + D)$

$= x + \frac{d^2}{2x} + D + \frac{d^2}{2D} - x - D$ or $\Delta = \frac{d^2}{2}\left(\frac{1}{x} + \frac{1}{D}\right)$

In order to have a dark fringe at $P, \Delta = \frac{\lambda}{2}$.

Hence $\frac{\lambda}{2} = \frac{d^2}{2}\left(\frac{1}{x} + \frac{1}{D}\right)$

or $d = \left[\frac{\lambda x D}{x + D}\right]^{1/2}$

Putting $x = \frac{D}{2}$ in Eq. (2), we find that the correct option is (A).

24. Maximum speed is attained when the entire energy stored in inductor is converted into kinetic energy of rod.

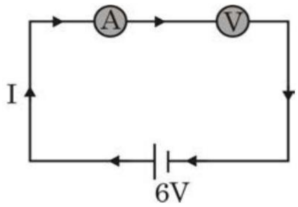
$\Rightarrow \frac{1}{2}\, L\left(\frac{E}{R}\right)^2 = \frac{1}{2} mv^2$

$\Rightarrow v = \left(\frac{E}{R}\right)\sqrt{\frac{L}{m}}$

25. Let $R_1 =$ resistance of ammeter and $R_2 =$ combined resistance of ammeter and voltmeter

In the first case,current inthe circuit is given by $I = \frac{6}{R_2}$

and voltage across voltmeter is given by




$V = 6 -$ (voltage across ammeter) $\Rightarrow V = 6 - IR_1 \Rightarrow V = 6 - \frac{6R_1}{R_2}$

In the second case reading of ammeter becomes two times, i.e., the total resistance becomes half whereas the resistance of ammeter remains unchanged.

Hence, $I' = \frac{6}{R_2/2} = \frac{12}{R_2}$

and $V' = 6 - (I')R_1 \Rightarrow V' = 6 - \frac{12R_1}{R_2}$

Further, it is given that $V' = \frac{V}{2} \Rightarrow 6 - \frac{12R_1}{R_2} = 3 - \frac{3R_1}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{1}{3}$

Substituting this value in equation (4), we have $V' = 6 - (12)\left(\frac{1}{3}\right) \quad \Rightarrow V' = 2\text{ V}$

26. For an adiabatic process $TV^{\gamma-1} =$ constant

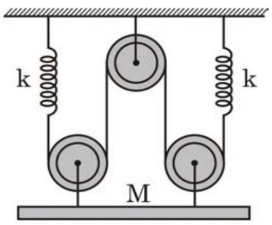
$\Rightarrow TV^{\gamma-1} = \left(\frac{T}{2}\right)(5.66\text{ V})^{\gamma-1} \Rightarrow (5.66\text{ V})^{\gamma-1} = 2$

$\Rightarrow (\gamma - 1)\log_e(5.66) = \log_e(2)$

$\Rightarrow \gamma - 1 = 0.4 \Rightarrow \gamma = 1.4$

Since, $\gamma = 1 + \frac{2}{f} = 1.4$

27.




Net restoring force is

$F = -8x$

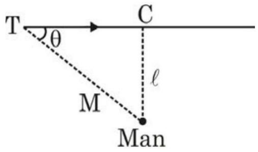
$\Rightarrow Ma = -8kx$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{8k}{M}} = \frac{1}{\pi}\sqrt{\frac{2k}{M}}$

$\Rightarrow x = 2$

29. Let the velocity of truck at T when it blows the whistle be $v_s$.Then




$$600=\left(\frac{v}{v-v_s\cos\theta}\right)500$$

During this time, speed of truck gets doubled, so $2v_s = v_s + at \Rightarrow v_s = at$

Now, $vt = AM = \frac{\ell}{\sin\theta}$.

Also, $AB=\ell\cot\theta = v_s t+\frac{1}{2}at^2$

$$\Rightarrow \frac{\ell\cos\theta}{\sin\theta}=at^2+\frac{1}{2}at^2=\frac{3}{2}at^2 \Rightarrow \frac{a\ell}{v}=\left(\frac{2}{3}\sin\theta\cos\theta\right)v$$

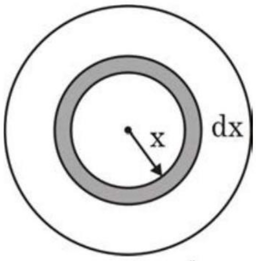
From equation (1), we get

$$\frac{6}{5}=\frac{u}{v-at\cos\theta}=\frac{v}{v-\frac{a\ell}{v}\left(\frac{\cos\theta}{\sin\theta}\right)} \quad \left\{\because t=\frac{\ell}{v\sin\theta}\right\}$$

$$\Rightarrow \frac{6}{5}=\frac{v}{v-\left(\frac{2}{3}\sin\theta\cos\theta\right)\left(\frac{\cos\theta}{\sin\theta}\right)v} \quad \Rightarrow \cos\theta=\frac{1}{2}\Rightarrow \theta=60^\circ$$

30. $E=\frac{x}{2}\frac{dB}{dt}\Rightarrow E=\frac{3Kxt^2}{2}$

$$\Rightarrow d\tau=(dq)Ex=\left(\frac{3\,KKt^2}{2}\right)\left(\frac{2\pi x dx}{\pi r^2}\right)qx$$




$$\Rightarrow \tau=\frac{3Kt^2q}{r^2}\int_0^r x^3dx \Rightarrow \tau=\frac{3Kqt^2}{r}r^2$$

torque due to friction force

$$d\tau=\mu gxdm=\mu gx(2\pi xdx)\frac{m}{\pi r^2}$$

$$\Rightarrow \tau=2\mu g\frac{m}{r^2}\int_0^r x^2dx=\frac{2}{3}\mu mgr \Rightarrow \frac{3Kqt^2r^2}{4}=\frac{2}{3}\mu mgr$$

$$\Rightarrow t=\sqrt{\frac{8\mu mg}{9Kqr}}$$

Substituting values, we get $t = 2$ s

34. disc gives uniform E so force on dipole is zero.
Electric field inside a sheet varies linely.

# CHEMISTRY

35. $V = \dfrac{nRT}{P}$

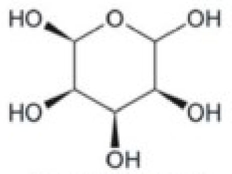
$V_A = 200R, V_C = 400R, V_B = 400R$

Obtaining equation for AB process $P = \dfrac{400R}{600R - V}$

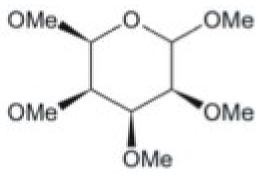
$w_{AB} = -\int Pdv = -400R\ln 2$

$\Delta S = nC_V \ln\dfrac{T_2}{T_1} + nR\ln\dfrac{V_2}{V_1} = nCp\ln\dfrac{T_2}{T_1} + nR\ln\dfrac{P_1}{P_2} = 0.2$ L.atm

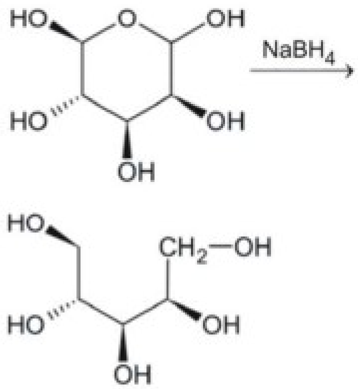
36. Since anomeric carbon is a chiral carbon, so given sugar can exist in two anomeric pyranose forms. Since it is in hemiacetal, so it can reduce to tollen's reagent to silver mirror.



(Reducing sugar)



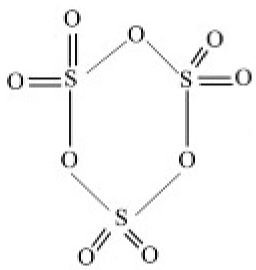
(Non-reducing because anomeric -OH is not - free



40. On increasing volume concentration decreases.

41. Q must be an ester. Addition of ethyl magnesium chloride to an alkyl acetate gives II, formate ester gives III and propionic ester gives IV. I can be produced from acetone and not from ester.

43.



x = 6, y = 6

44. Let KI solution is M molar so $\text{m.mol} = 20\times \text{M}$

Given, $\text{KIO}_{3:}\ \frac{1}{10}\text{M} \Rightarrow 20\times \text{M} = \frac{1}{10}\times 30$

$\Rightarrow$ Molarity of $\text{KI} = 0.15\text{M}$

$\text{KIO}_3 + 2\text{KI} + 6\text{HCl} \rightarrow 3\text{IC}\ell + 3\text{kC}\ell + 3\text{H}_2\text{O}$

m.mol: 510

$\Rightarrow \text{AgI} = 0.3\times 50 - 10 = 5\ \text{m.mol} \Rightarrow \%\text{AgNO}_3 = 85\%$

45. $2\left[\text{Au(CN)}_2\right]^- + \text{Zn} \rightarrow 2\text{Au} + \left[\text{Zn(CN)}_4\right]^{2-}$

$E^\circ = -0.60 - (-1.26) = 0.66\ \text{V}$

Since $E^\circ$ value for the reduction of complexed $\text{Ag}^+$ ion to Ag is higher than that of complexed $\text{Au}^+$ ion into Au. So, $\text{Ag}^+$ ion will be reduced first.

Mole of Zn added $= \frac{78}{65} = 1.2$

Moles of $\left[\text{Ag(CN)}_2\right]^- = 0.003\times 500 = 1.5$

Moles of $\left[\text{Au(CN)}_2\right]^- = 0.002\times 500 = 1.0$

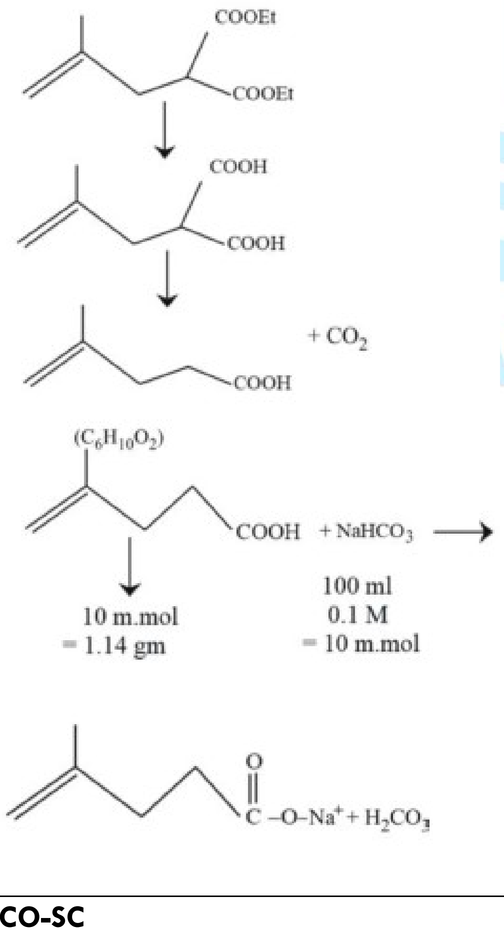
Moles of Zn used to reduce 1.5 moles of $\left[\text{Ag(CN)}_2\right]^- = 0.75$

Moles of $\left[\text{Au(CN)}_2\right]^-$ reduced by remaining 0.45 moles of $\text{Zn} = 0.90$

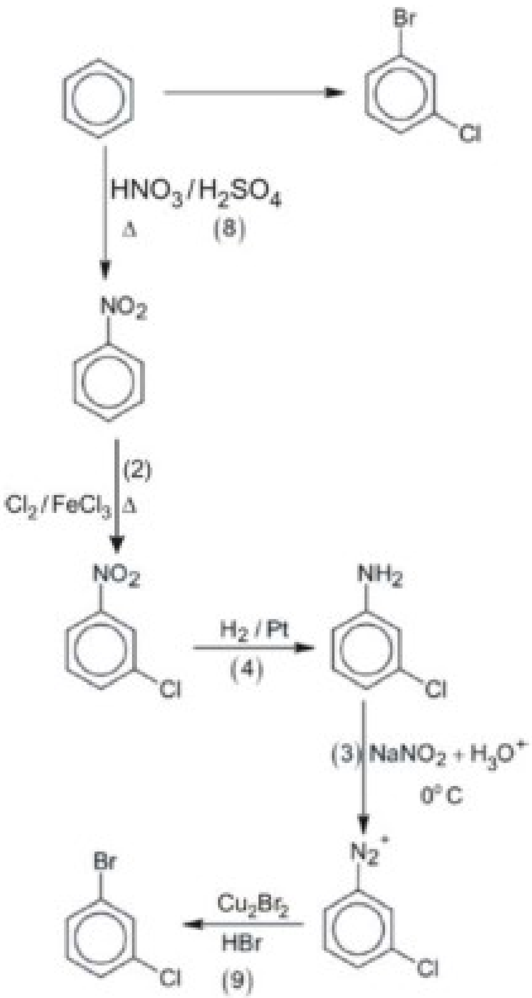
Moles of $\left[\text{Au(CN)}_2\right]^-$ left in the solution $= 0.10$

Concentration of $\left[\text{Au(CN)}_2\right]^-$ left in the solution $= \frac{0.10}{500} = 2\times 10^{-4}\text{M} = \text{x}$

46.



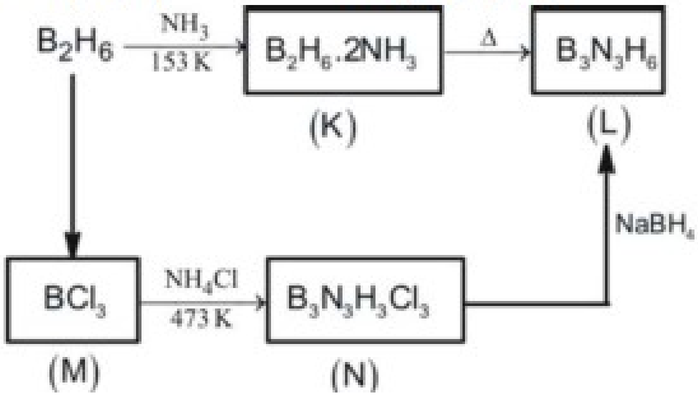
47.




48.

K : $[BH_2(NH_3)_2][BH_4]$ M: $BCl_3$

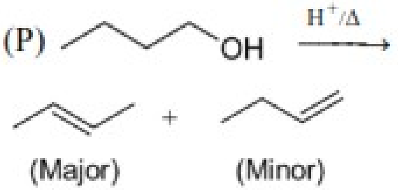
L : $B_3N_3H_6$ N:$B_3N_3H_3Cl_3$

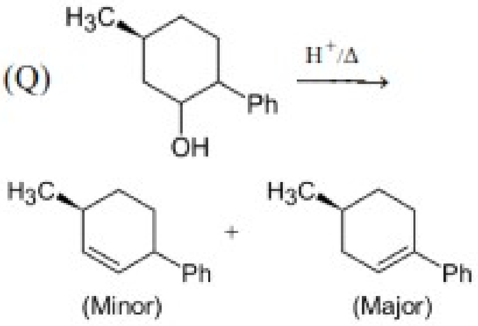


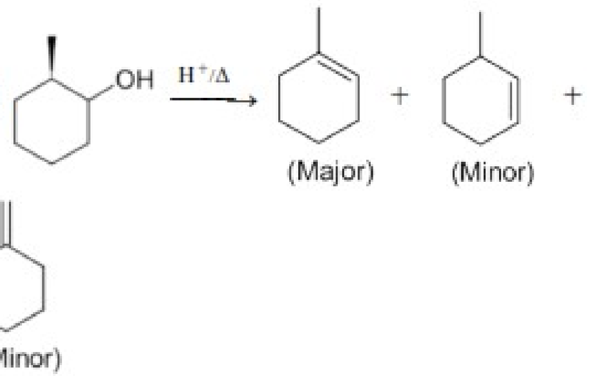

49. $I^{st}$ order: $t_{1/2} = \dfrac{0.693}{k}, k = \dfrac{2.303}{t}, \log\dfrac{a}{a-x}$

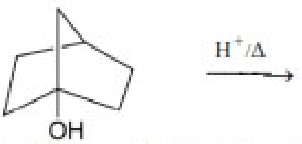
$3 : t_{1/2} = \dfrac{2}{k}\left(\sqrt{a} - \sqrt{\dfrac{a}{2}}\right)$

50.

(P) 


(Q) 


(R) 


(S) 

cannot undergo dehydration as bridge head C-atom can never be $sp^2$-hybridised.