CS 7910 Computational Complexity Assignment 2

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1. In class we have studied a polynomial-time reduction from ≤ 3-SAT to 3SAT. Consider the following ≤ 3-SAT problem instance of four clauses over five variables. $C_1 = x_1 \vee x_2, C_2 = x_2 \vee x_3, C_3 = x_4, C_4 = x_3 \vee x_5$. Construct a problem instance of 3SAT by following the polynomial-time reduction we studied in class.

   We can create four clauses equivalent to the ≤ 3-SAT clauses:

   $$C_a = x_1 \vee x_2 \vee y_1$$
   $$C_b = x_2 \vee x_3 \vee y_1$$
   $$C_c = x_4 \vee y_1 \vee y_2$$
   $$C_d = \overline{x_3} \vee x_5 \vee y_1$$

   In order to guarantee the values of y_1 and y_2 to be zero, we can create the following additional clauses:

   $$C_i = (\overline{y_1} \vee y_3 \vee y_4) \wedge (\overline{y_1} \vee \overline{y_3} \vee y_4) \wedge (\overline{y_1} \vee y_3 \vee \overline{y_4}) \wedge (\overline{y_1} \vee \overline{y_3} \vee \overline{y_4})$$
   $$C_j = (\overline{y_2} \vee y_3 \vee y_4) \wedge (\overline{y_2} \vee \overline{y_3} \vee y_4) \wedge (\overline{y_2} \vee y_3 \vee \overline{y_4}) \wedge (\overline{y_2} \vee \overline{y_3} \vee \overline{y_4})$$

   In each of the clauses C_i and C_j, whatever value y_3 and y_4 are assigned, for the clauses to be true, y_1 and y_2 have to be 0 or $false$.

2. In class we proved that the problem 3SAT is NP-Complete. In this exercise we consider a related problem, called MAX-3SAT. The following is the problem.

   Given a set of k clauses $C_1, C_2, ..., C_k$ over a set of n variables $x_1, x_2, ..., x_n$ such that each clause C_i has exactly three literals (or terms), the goal of MAX-3SAT is to compute an assignment for all variables such that the number of satisfied clauses is *maximized*.

   Please answer the following questions.

   a) Clearly, the problem MAX-3SAT is an optimization problem. Please give the decision version of the problem.

b) Prove the decision version of MAX-3SAT is NP-Complete. (Hint: reduce 3SAT to it)