

# CS 7910 Computational Complexity

## Assignment 7

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Gopal Menon

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1. **(10 points)** We have studied a 2-approximation algorithm for the vertex cover problem in class. Give an example of a graph such that the size of the vertex cover computed by our algorithm is always equal to  $2 * OPT$ , where  $OPT$  is the size of the vertex cover in an optimal solution.

This actually proves that the approximation ratio 2 is “tight” for our algorithm.

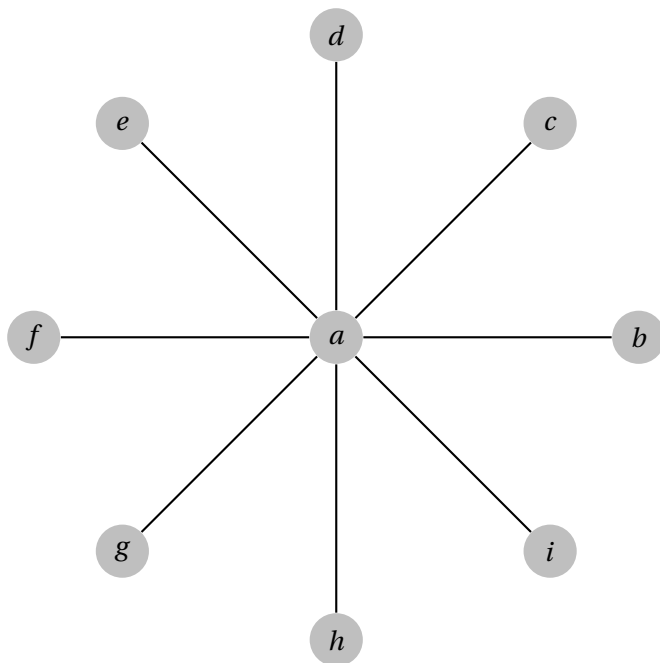


Figure 1: An example of a star shaped graph

Figure 1 shows an example of a star shaped graph where node  $a$  is connected to every other node by an edge. In such a graph, the set of vertices with just the node  $a$  will be

the vertex cover since the node will cover all the edges. If we use the 2-approximation algorithm, the vertex cover will include the two vertices in a randomly selected edge. So in such a graph, the 2-approximation algorithm will always return a vertex set of size 2, which will be twice the optimum solution of size 1.