UTAH STATE UNIVERSITY, COMPUTER SCIENCE DEPARTMENT

CS 7910 Computational Complexity
Homework 1

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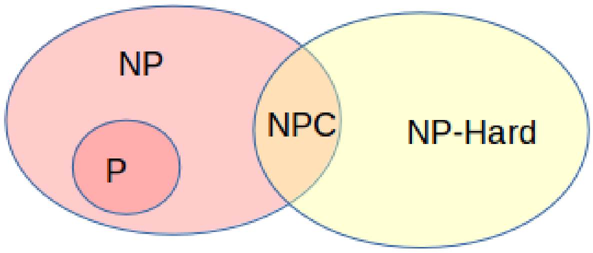

Figure 1: P, NP, NP-Complete (NPC) and NP-Hard problems

1. *For each of the following statements, please tell whether it is true and briefly explain your answer.*

   a) *If there exists a polynomial time algorithm that can solve the circuit satisfiability problem, then every problem in NPC can be solved in polynomial time.*

   The circuit satisfiability ($CSAT$) problem is NP-Complete. This means that

   i. $CSAT \in NP$ and

   ii. for all problems $Y \in NP$, $Y \leq_P CSAT$

   This means that every problem Y in NP can be reduced to $CSAT$. Or in other words, $CSAT$ is at least as hard as the hardest problem in NP. The set $NP-Complete$ is a subset of NP as shown in figure 1. So $CSAT$ is at least as hard as any $NP-Complete$ problem. This means that if if a polynomial time algorithm can be used to solve $CSAT$, then every other $NP-Complete$ problem can be solved in polynomial time since it would be easier or at most as difficult as $CSAT$.

b) *If no polynomial time algorithm can solve the SAT problem, then no polynomial time algorithm can solve the circuit satisfiability problem.*

The $CSAT$ problem can be reduced to the SAT problem as we saw in class. This means that $CSAT \leq_P SAT$. i.e. the SAT problem is at least as hard if not harder than $CSAT$. If no polynomial time algorithm can be used to solve the SAT problem, it does not follow from this that no polynomial time algorithm can be used to solve the $CSAT$ problem. However since all NP problems can be reduced to an $NP-Complete$ problem, and since we know that $CSAT$ is an $NP-Complete$ problem, it follows that $SAT \leq_P CSAT$. Since no polynomial time algorithm can be used to solve the SAT problem, it follows that no polynomial time algorithm can be used to solve the $CSAT$ problem which is at least as hard as the SAT problem.

c) *NP is the union of P and NPC.*

If $P = NP$ and since $NPC \subseteq NP$, in this case $NP = P \cup NPC$. But it is unlikely that $P = NP$. Since NPC problems are the hardest NP problems, the relation $NP = P \cup NPC$ can only be satisfied if $NP - P = NPC$. i.e. if each NP problem that cannot be solved in polynomial time can be reduced to any other NP problem in polynomial time.

d) *If there is an NPC problem A that is also in P, then NP is the union of P and NPC.*

e) *If no problem of NPC is in P, then NP is the union of P and NPC.*

f) *If NP is the union of P and NPC, then $P = NP$.*

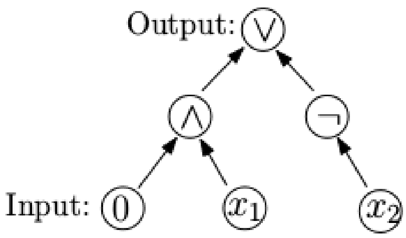

Figure 2: An instance of the circuit satisfiability problem

2. *In class we studied a polynomial time problem reduction from the circuit satisfiability problem to the SAT problem. The following figure shows an instance of the circuit satisfiability problem. Give the SAT problem instance constructed from it based on the problem reduction we studied in class, i.e., give the Boolean variables and the clauses for the SAT problem instance.*