CS 5350/6350: Machine Learining Fall 2016

Homework 2

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1 Warm up: Feature expansion

1. Consider the following function that maps the examples in $\Re^2$ into a higher space $\Re^3$:

$$\phi : (x_1, x_2) \rightarrow (x_1, x_2, f_r(x_1, x_2))$$

This has the effect of raising the positively labelled examples in the newly introduced dimension. I'm not sure about the mathematical notation, but my intention is to map the function from two-dimensional to three-dimensional space.

2. In order to verify that this achieves a linear separation between the positive and negative examples, I will define a weight vector w which includes a bias term as follows:

$$w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The dot product of the weight vector transpose and an example (here the example has a dimension in addition to the added third dimension in order to accommodate the bias) will be

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ f_r(x_1, x_2) \end{bmatrix} = \begin{cases} +1 & 4x_1^4 + 16x_2^4 \leq r; \\ -1 & \text{otherwise} \end{cases}$$

This means that the weight vector w can be used to separate out the positive and negative labelled examples. The hyperplane that separates the positive and negative labelled examples will pass through the origin (since the bias is zero) and will be perpendicular to the weight vector w. The dot product of the weight vector and any example will give the distance of the example point from the linearly separating plane

(since it's a projection of the example vector on the weight vector) that separates the positive and negative examples. We can see that the dot product is +1 for positively labelled examples and is −1 for negatively labelled examples. Hence the weight vector w separates out the examples with different labels.

2 Mistake Bound Model of Learning

1. Each function f_r in the concept class $\mathcal{C}$ is defined by a radius r. Since $1 \leq r \leq 80$ and r is being compared with the sum of the squares of two integers, we need only consider integral values of r. So each function f_r in the concept class $\mathcal{C}$ that needs to be considered, will have a different integral value of r. So $|\mathcal{C}| = 80$.

2. [5 points] We need to check if the following equality is true

$$y^t = \begin{cases} +1 & (x_1^t)^2 + (x_2^t)^2 \leq r^2; \\ -1 & \text{otherwise} \end{cases}$$

   If it is not true then it means that the hypothesis f_r has made a mistake.

3. [10 points] Consider the case when the label is −1 and the prediction is +1 because $x_1^2 + x_2^2 \leq r^2$. In order to correct this, we need to update r to make it $x_1^2 + x_2^2 > r^2$ or $r = \left\lfloor \sqrt{x_1^2 + x_2^2 - 1} \right\rfloor$.

   Consider the case when the label is +1 and the prediction is −1 because $x_1^2 + x_2^2 > r^2$. In order to correct this, we need to update r to make it $r = \left\lceil \sqrt{x_1^2 + x_2^2 + 1} \right\rceil$.

   In both cases above, we need to consider only the positive value of the square root.

4. [20 points] Here is a mistake-driven learning algorithm, that is executed once for each training sample, to learn the function.

Algorithm 1 Mistake-Driven Learning Algorithm

```
1: procedure MISTAKE-DRIVEN LEARNING ALGORITHM(x_1, x_2, y)
2:     if x_1^2 + x_2^2 ≤ r^2 then
3:         if y == −1 then
4:             r = ⌊√(x_1^2 + x_2^2 − 1)⌋
5:     else
6:         if y == +1 then
7:             r = ⌈√(x_1^2 + x_2^2 + 1)⌉
```

   Here the algorithm receives as input the values of x_1, x_2 and the label y. It then uses these values to update the value of r that it maintains in its internal state. In the algorithm above, == represents the test for equals and = represents an assignment.

Since the correct function will use a value of r between 1 and 80, the worst case scenario for learning the correct function will be the case where all the functions with the incorrect value of r are first tried and the test data results in a wrong prediction in each such case. So the correct function will be the last one tried and will be found after making 79 (that is $|\mathcal{C} - 1|$) mistakes.

5. a. The set of hypotheses consistent with all examples seen so far can be defined by storing the upper and lower values of the range of r values that satisfy the examples seen so far.

   b. [5 points] At any point in the iteration of the halving algorithm, we can check and see if the value of r^2 corresponding to the lowest value of r in the range of r values in the top half of the ranges of r satisfies the following

$$y^t = \begin{cases} +1 & (x_1^t)^2 + (x_2^t)^2 \leq r^2; \\ -1 & \text{otherwise} \end{cases}$$

   c. [5 points] The halving algorithm that is executed once for each training sample, can be as follows:

Algorithm 2 Halving Algorithm

1: **procedure** HALVING ALGORITHM(x_1^t, x_2^t, y^t)
2: Construct sets R_1 and R_2 by splitting the sorted set R of remaining r values down the middle. The split is made such that for the case of odd number of r values, the set R_2 will contain one more element than R_1
3: **if** $|R_1| == |R_2|$ and $|R_1| \neq 1$ **then**
4: Remove largest vaue of r from R_1 and put it into R_2
5: r_t = minimum value of r in set R_2
6: **if** $(x_1^t)^2 + (x_2^t)^2 \leq r_t^2$ and $y^t == -1$ **then**
7: $R = R_1$
8: **if** $|R| == 1$ **then**
9: The function has been learnt and is f_r where r = the element in set R

In the above halving algorithm, in step 4, the set R_2 is made the majority set if it is not already one. In the case where there is only one element left in each split set, there is no change made. The majority step is used to make a prediction. If the prediction is wrong, the entire set is dropped from the list of potential values of r that will be considered to be the value used in the target function. This step where the majority set is checked is shown in line 6.

The halving algorithm discards at least half the functions from the hypothesis set each time its makes a mistake in the prediction. In the worst case, the algorithm discards exactly half the functions from the hypothesis set. This means that it uses at most $log_2 |\mathcal{C}|$ steps to arrive at the correct function. Here $\mathcal{C}$ is the concept class that the algorithm searches over. This means that the mistake bound (the number of steps in the worst case) is $log_2 80$.

3 The Perceptron Algorithm and Its Variants

3.1 The Task and Data

3.2 Algorithms

3.3 Experiments

1. The weight vector at the end of the run was $[0.0, 0.0, 1.0, 0.0, -1.0, 2.0]^T$. The number of mistakes made was 4.

2. 6-fold cross validation was run for finding the hyper-parameters for the Perceptron and Margin Perceptron. Random weight values between 0 and 1 were assigned in this and all subsequent experiments.

   For the Perceptron, a learning rate of 0.4 was selected after cross-validation. 1385 mistakes were made during the training process. The accuracy on the training set (**calculated in all cases of this assignment by adding true positives and true negatives and dividing by total number of training samples**) was 0.8141, and was 0.8115 on the testing set.

   For the Margin Perceptron, a learning rate of 0.1 and a μ value of 4.0 was selected after cross-validation. 2364 mistakes were made during the training process. The accuracy on the training set was 0.8435, and was 0.8398 on the testing set.

   Section 4 has the cross validation details for the hyper-parameters that were evaluated.

3. In this case, the data was shuffled for the case of multiple epochs before each subsequent epoch.

   For the Perceptron, a learning rate of 0.8 and 3 epochs was selected after cross-validation. 4025 mistakes were made during the training process. The accuracy on the training set was 0.8226, and was 0.8233 on the testing set.

   For the Margin Perceptron, a learning rate of 0.4, a μ value of 5.0 and 3 epochs was selected after cross-validation. 5847 mistakes were made during the training process. The accuracy on the training set was 0.8411, and was 0.8391 on the testing set.

   Section 5 has the cross validation details for the hyper-parameters that were evaluated.

   In the case of the Perceptron, cross-validation resulted in increased accuracy. For the case of Margin Perceptron, accuracy after cross-validation slightly decreased. Typically cross-validation should result in improved accuracy. However, it is possible that when the samples are not linearly separated, the accuracy just keeps shifting around the maximum possible value and never converges.

4. When the Aggressive Margin Perceptron was trained with no shuffling of data (for 1 epoch), cross-validation resulted in a μ of 5.0. 2702 mistakes were made during the training process. The accuracy on the training set was 0.8101, and was 0.8130 on the testing set.

When the training was done with shuffling of data before each subsequent epoch (run for 3 and 5 epochs), the number of epochs were also selected by cross-validation. The Aggressive Margin Perceptron used a μ of 1.0 and 5 epochs were selected. 12995 mistakes were made during the training process. The accuracy on the training set was 0.7137, and was 0.7007 on the testing set.

Section 6 has the cross validation details for the hyper-parameters that were evaluated.

4 Perceptron and Margin Perceptron Cross Validation Results

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Perceptron Epochs 1, learning rate 0.1 and average accuracy 0.8151
Perceptron Epochs 1, learning rate 0.2 and average accuracy 0.822
Perceptron Epochs 1, learning rate 0.3 and average accuracy 0.8195
Perceptron Epochs 1, learning rate 0.4 and average accuracy 0.825
Perceptron Epochs 1, learning rate 0.5 and average accuracy 0.822
Perceptron Epochs 1, learning rate 0.6 and average accuracy 0.8119
Perceptron Epochs 1, learning rate 0.7 and average accuracy 0.8212
Perceptron Epochs 1, learning rate 0.8 and average accuracy 0.8208
Perceptron Epochs 1, learning rate 0.9 and average accuracy 0.8198
Perceptron Epochs 1, learning rate 1.0 and average accuracy 0.8175

Margin Perceptron Epochs 1, myu 1.0, learning rate 0.1 and average accuracy 0.8153
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.2 and average accuracy 0.8222
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.3 and average accuracy 0.819
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.4 and average accuracy 0.8261
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.5 and average accuracy 0.8245
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.6 and average accuracy 0.8153
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.7 and average accuracy 0.8232
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.8 and average accuracy 0.8324
Margin Perceptron Epochs 1, myu 1.0, learning rate 0.9 and average accuracy 0.8264
Margin Perceptron Epochs 1, myu 1.0, learning rate 1.0 and average accuracy 0.8198
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.1 and average accuracy 0.8164
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.2 and average accuracy 0.8114
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.3 and average accuracy 0.8237
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.4 and average accuracy 0.8198
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.5 and average accuracy 0.8225
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.6 and average accuracy 0.8228
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.7 and average accuracy 0.824
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.8 and average accuracy 0.8181
Margin Perceptron Epochs 1, myu 2.0, learning rate 0.9 and average accuracy 0.8232
Margin Perceptron Epochs 1, myu 2.0, learning rate 1.0 and average accuracy 0.8197
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.1 and average accuracy 0.829
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.2 and average accuracy 0.8128
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.3 and average accuracy 0.8112
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.4 and average accuracy 0.8184
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Margin Perceptron Epochs 1, myu 3.0, learning rate 0.5 and average accuracy 0.8265
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.6 and average accuracy 0.8247
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.7 and average accuracy 0.8231
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.8 and average accuracy 0.8279
Margin Perceptron Epochs 1, myu 3.0, learning rate 0.9 and average accuracy 0.8208
Margin Perceptron Epochs 1, myu 3.0, learning rate 1.0 and average accuracy 0.8232
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.1 and average accuracy 0.8398
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.2 and average accuracy 0.8243
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.3 and average accuracy 0.8131
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.4 and average accuracy 0.82
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.5 and average accuracy 0.8181
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.6 and average accuracy 0.8179
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.7 and average accuracy 0.8256
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.8 and average accuracy 0.8228
Margin Perceptron Epochs 1, myu 4.0, learning rate 0.9 and average accuracy 0.8271
Margin Perceptron Epochs 1, myu 4.0, learning rate 1.0 and average accuracy 0.8228
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.1 and average accuracy 0.8396
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.2 and average accuracy 0.8243
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.3 and average accuracy 0.8072
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.4 and average accuracy 0.81
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.5 and average accuracy 0.8151
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.6 and average accuracy 0.8126
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.7 and average accuracy 0.8137
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.8 and average accuracy 0.8178
Margin Perceptron Epochs 1, myu 5.0, learning rate 0.9 and average accuracy 0.8201
Margin Perceptron Epochs 1, myu 5.0, learning rate 1.0 and average accuracy 0.8215
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5 Perceptron and Margin Perceptron with Epochs Cross Validation Results

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Perceptron Epochs 3, learning rate 0.1 and average accuracy 0.8164
Perceptron Epochs 3, learning rate 0.2 and average accuracy 0.8117
Perceptron Epochs 3, learning rate 0.3 and average accuracy 0.812
Perceptron Epochs 3, learning rate 0.4 and average accuracy 0.8169
Perceptron Epochs 3, learning rate 0.5 and average accuracy 0.8106
Perceptron Epochs 3, learning rate 0.6 and average accuracy 0.8186
Perceptron Epochs 3, learning rate 0.7 and average accuracy 0.8142
Perceptron Epochs 3, learning rate 0.8 and average accuracy 0.8187
Perceptron Epochs 3, learning rate 0.9 and average accuracy 0.8103
Perceptron Epochs 3, learning rate 1.0 and average accuracy 0.8033
Perceptron Epochs 5, learning rate 0.1 and average accuracy 0.7888
Perceptron Epochs 5, learning rate 0.2 and average accuracy 0.803
Perceptron Epochs 5, learning rate 0.3 and average accuracy 0.801
Perceptron Epochs 5, learning rate 0.4 and average accuracy 0.7996
Perceptron Epochs 5, learning rate 0.5 and average accuracy 0.7838
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Perceptron Epochs 5, learning rate 0.6 and average accuracy 0.801
Perceptron Epochs 5, learning rate 0.7 and average accuracy 0.798
Perceptron Epochs 5, learning rate 0.8 and average accuracy 0.7762
Perceptron Epochs 5, learning rate 0.9 and average accuracy 0.7985
Perceptron Epochs 5, learning rate 1.0 and average accuracy 0.8014

Margin Perceptron Epochs 3, myu 1.0, learning rate 0.1 and average accuracy 0.8373
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.2 and average accuracy 0.8307
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.3 and average accuracy 0.8317
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.4 and average accuracy 0.8296
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.5 and average accuracy 0.8228
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.6 and average accuracy 0.8212
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.7 and average accuracy 0.82
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.8 and average accuracy 0.8268
Margin Perceptron Epochs 3, myu 1.0, learning rate 0.9 and average accuracy 0.82
Margin Perceptron Epochs 3, myu 1.0, learning rate 1.0 and average accuracy 0.8172
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.1 and average accuracy 0.8356
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.2 and average accuracy 0.839
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.3 and average accuracy 0.8284
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.4 and average accuracy 0.8306
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.5 and average accuracy 0.8318
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.6 and average accuracy 0.8204
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.7 and average accuracy 0.83
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.8 and average accuracy 0.8253
Margin Perceptron Epochs 3, myu 2.0, learning rate 0.9 and average accuracy 0.8156
Margin Perceptron Epochs 3, myu 2.0, learning rate 1.0 and average accuracy 0.8215
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.1 and average accuracy 0.8359
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.2 and average accuracy 0.8402
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.3 and average accuracy 0.8298
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.4 and average accuracy 0.8356
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.5 and average accuracy 0.8354
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.6 and average accuracy 0.8331
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.7 and average accuracy 0.8317
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.8 and average accuracy 0.827
Margin Perceptron Epochs 3, myu 3.0, learning rate 0.9 and average accuracy 0.8236
Margin Perceptron Epochs 3, myu 3.0, learning rate 1.0 and average accuracy 0.8204
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.1 and average accuracy 0.8387
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.2 and average accuracy 0.8395
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.3 and average accuracy 0.8368
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.4 and average accuracy 0.8339
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.5 and average accuracy 0.8289
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.6 and average accuracy 0.8307
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.7 and average accuracy 0.8254
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.8 and average accuracy 0.8321
Margin Perceptron Epochs 3, myu 4.0, learning rate 0.9 and average accuracy 0.8276
Margin Perceptron Epochs 3, myu 4.0, learning rate 1.0 and average accuracy 0.824
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.1 and average accuracy 0.8374
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Margin Perceptron Epochs 3, myu 5.0, learning rate 0.2 and average accuracy 0.8395
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.3 and average accuracy 0.8362
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.4 and average accuracy 0.8406
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.5 and average accuracy 0.8289
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.6 and average accuracy 0.8332
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.7 and average accuracy 0.831
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.8 and average accuracy 0.832
Margin Perceptron Epochs 3, myu 5.0, learning rate 0.9 and average accuracy 0.8343
Margin Perceptron Epochs 3, myu 5.0, learning rate 1.0 and average accuracy 0.8354
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.1 and average accuracy 0.8117
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.2 and average accuracy 0.8108
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.3 and average accuracy 0.7854
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.4 and average accuracy 0.7801
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.5 and average accuracy 0.807
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.6 and average accuracy 0.7861
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.7 and average accuracy 0.7964
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.8 and average accuracy 0.7952
Margin Perceptron Epochs 5, myu 1.0, learning rate 0.9 and average accuracy 0.7849
Margin Perceptron Epochs 5, myu 1.0, learning rate 1.0 and average accuracy 0.7712
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.1 and average accuracy 0.8321
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.2 and average accuracy 0.8192
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.3 and average accuracy 0.8164
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.4 and average accuracy 0.8091
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.5 and average accuracy 0.8034
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.6 and average accuracy 0.805
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.7 and average accuracy 0.8153
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.8 and average accuracy 0.7977
Margin Perceptron Epochs 5, myu 2.0, learning rate 0.9 and average accuracy 0.7961
Margin Perceptron Epochs 5, myu 2.0, learning rate 1.0 and average accuracy 0.8038
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.1 and average accuracy 0.8301
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.2 and average accuracy 0.8284
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.3 and average accuracy 0.8179
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.4 and average accuracy 0.8139
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.5 and average accuracy 0.812
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.6 and average accuracy 0.8133
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.7 and average accuracy 0.798
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.8 and average accuracy 0.8081
Margin Perceptron Epochs 5, myu 3.0, learning rate 0.9 and average accuracy 0.793
Margin Perceptron Epochs 5, myu 3.0, learning rate 1.0 and average accuracy 0.8067
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.1 and average accuracy 0.8304
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.2 and average accuracy 0.8271
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.3 and average accuracy 0.8257
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.4 and average accuracy 0.8236
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.5 and average accuracy 0.8187
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.6 and average accuracy 0.8133
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.7 and average accuracy 0.8042
Margin Perceptron Epochs 5, myu 4.0, learning rate 0.8 and average accuracy 0.8162
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Margin Perceptron Epochs 5, myu 4.0, learning rate 0.9 and average accuracy 0.8155
Margin Perceptron Epochs 5, myu 4.0, learning rate 1.0 and average accuracy 0.8067
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.1 and average accuracy 0.8321
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.2 and average accuracy 0.8307
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.3 and average accuracy 0.8271
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.4 and average accuracy 0.8257
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.5 and average accuracy 0.8222
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.6 and average accuracy 0.82
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.7 and average accuracy 0.8186
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.8 and average accuracy 0.8092
Margin Perceptron Epochs 5, myu 5.0, learning rate 0.9 and average accuracy 0.8075
Margin Perceptron Epochs 5, myu 5.0, learning rate 1.0 and average accuracy 0.8108
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6 Aggressive Margin Perceptron Cross Validation Results

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Aggressive Margin Perceptron Epochs 1, myu 1.0 and average accuracy 0.7879
Aggressive Margin Perceptron Epochs 1, myu 2.0 and average accuracy 0.7894
Aggressive Margin Perceptron Epochs 1, myu 3.0 and average accuracy 0.7891
Aggressive Margin Perceptron Epochs 1, myu 4.0 and average accuracy 0.7893
Aggressive Margin Perceptron Epochs 1, myu 5.0 and average accuracy 0.7896

Aggressive Margin Perceptron Epochs 3, myu 1.0 and average accuracy 0.8111
Aggressive Margin Perceptron Epochs 3, myu 2.0 and average accuracy 0.81
Aggressive Margin Perceptron Epochs 3, myu 3.0 and average accuracy 0.8106
Aggressive Margin Perceptron Epochs 3, myu 4.0 and average accuracy 0.81
Aggressive Margin Perceptron Epochs 3, myu 5.0 and average accuracy 0.8098
Aggressive Margin Perceptron Epochs 5, myu 1.0 and average accuracy 0.8156
Aggressive Margin Perceptron Epochs 5, myu 2.0 and average accuracy 0.8147
Aggressive Margin Perceptron Epochs 5, myu 3.0 and average accuracy 0.8145
Aggressive Margin Perceptron Epochs 5, myu 4.0 and average accuracy 0.8141
Aggressive Margin Perceptron Epochs 5, myu 5.0 and average accuracy 0.8142
```