CS6190: Probabilistic Modeling Homework 1
Exponential Families, Conjugate Priors

Gopal Menon

29 January, 2018

Written Part

1. **Expectation of sufficient statistics:** Consider a random variable X from a continuous exponential family with natural parameter $\eta = (\eta_1, \ldots, \eta_n)$. Recall this means the pdf is of the form:

$$p(x) = h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right)$$

   (a) Show that $E\left[T(x)|\eta\right] = \nabla A(\eta) = \left(\frac{\partial A}{\partial \eta_1}, \ldots, \frac{\partial A}{\partial \eta_d}\right)$.

   Hint: Start with the identity $\int p(x)dx = 1$, and take the derivative with respect to η.

$$\begin{aligned}
\int p(x)dx &= 1 \\
\Rightarrow \int h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right) dx &= 1 \\
\nabla \left(\int h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right) dx \right) &= 0 \\
\int h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right) . \left(T(x) - \nabla A(\eta)\right) dx &= 0 \\
\int T(x) h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right) dx &= \nabla A(\eta) \int h(x) \exp\left(\eta \cdot T(x) - A(\eta)\right) dx \\
\int T(x) p(x) dx &= \nabla A(\eta) \int p(x) dx \\
E\left[T(x)\right] &= \nabla A(\eta)
\end{aligned}$$