

UNIT-4

Bubble sort

In bubble sort, each element is compared with its adjacent element. If first element is larger than the second element then the position of the element are interchanged otherwise it is not changed.

Path(2)

10 14 2 11 6

10 14 2 11 6

10 14 2 11 6

10 14 14 11 6

10 2 14 14 6

10 2 14 6 14

Path(2)

10 2 11 6 14

2 10 11 6 14

2 10 11 6 14

2 10 6 11 14

2 10 6 11 14

Path(3) → 2 10 6 11 14

2 10 6 11 14

2 6 10 11 14 (exit)

2 6 10 11 14

2 6 10 11 14

Path ①

Ex ② 4 1 5 6 2

1 5 6 2

4 (5) 6 2

4 4 (5) 6 2

1 4 (5) 6 2

1 4 5 (6) 2

1 4 5 2 6

Path ②

(1) 4 5 2 6

1 (4) 5 2 6

1 4 (5) 2 6

1 4 2 (5) 6

1 4 2 5 6

Path ③

Ex ① 4 2 5 6

1 (4) 2 5 6

1 2 4 5 6

Ex ③ 32, 51, 27, 85, 66, 23, 13, 15, 57

Path ③ 2 7, 51, 27, 85, 66, 23, 13, 15, 57

32 (51) 27 85 66 23 13 15, 57

32 27 (51) 85 66 23 13 15, 57

32 27 51 85 66 23 13 15, 57

32 27 51 66 85 23, 13 15, 57

32 27, 51, 66 23 (85) 13 15, 57

32 27 51 66 23 13 15 (85) 57

32 27 51 66 23 13 15 57 85

32 27 51 66 23 13 15 57 85

Path 2

(32) 27, 51, 66, 23, 13, 15, 57, 85

27 (32) 51 66 23 13 15 57 85

27 32 (51) 66 23 10 15 57 85

27 32 51 (66) 23 13 15 57 85

27 32 51 23 (66) 73 15 57 85

27 32 51 23 13 (66) 15 57 85

27 32 51 23 13 15 (66) 57 85

27 32 51 23 13 15 57 (66) 85

Path 3

(27) 32 51 23 13 15 57 66 85

27 (32) 51 23 13 15 57 66 85

27 32 (51) 23 13 15 57 66 85

27 32 23 (51) 13 15 57 66 85

27 32 23 13 (51) 15 57 66 85

27 32 23 13 15 (51) 57 66 85

Path 4

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27 (32) 23 13 15 51 57 66 85

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27 23 13 (32) 15 51 57 66 85

27 23 13 15 (32) 51 57 66 85

Path 5

(27) 23 13 15 32 51 57 66 85

23 (27) 13 15 32 51 57 66 85

23 13 (27) 15 32 51 57 66 85

23 13 15 27 32 51 57 66 85

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32

51

57

66

85

(11)

77

33

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55

(Pass 1)
3

33

77

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Pass 2

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Pass 3

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88

Pass 4

(1) 33 22 44 55 66 77 88
 11 33 22 44 55 66 77 88
 11 22 33 44 55 66 77 88

Selection sort $\Rightarrow O(n^2)$

(1) 77 33 11 88 22 66 55
 (k) | min

91 33 44 77 88 90 66 55
 | | | | min

11 22 44 77 88 90 66 55
 | | | | min

11 22 33 77 88 90 66 55
 | | | | min

11 22 33 44 88 77 66 55
 | | | | min

11 22 33 44 66 88 77 55
 | | | | min

11 22 33 44 66 77 88 55
 | | | | min

(2) 66 33 11 90 55 88 77
 (k) | min

90 33 11 66 55 88 77
 | | | min

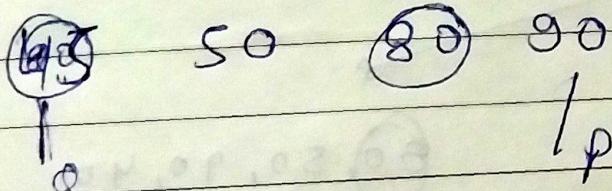
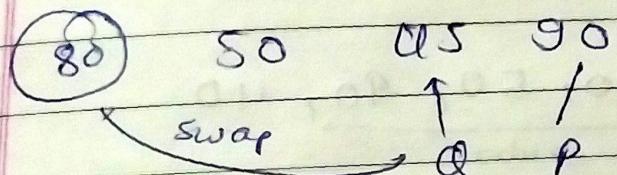
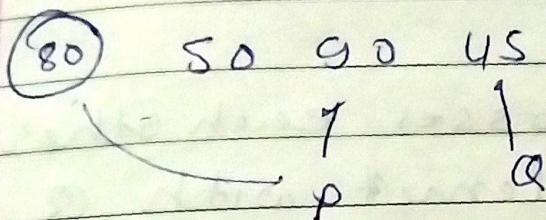
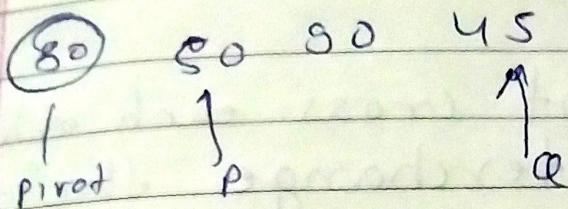
90 33 11 66 55 88 77
 | | | min

90 31 22 66 55 88 77
 | | | min

90 31 22 66 55 88 77
 | | | min

20 31 33 40 61 55 88

Quick sort,



45 50 80 90

35 50 15 25 80 20 90 45
Pivot 45

Quick sort works on divide and conquer root. In this we divide bigger problem into smaller subproblems. P increases or move toward RHS & stop only when it will get the element greater than the pivot element.

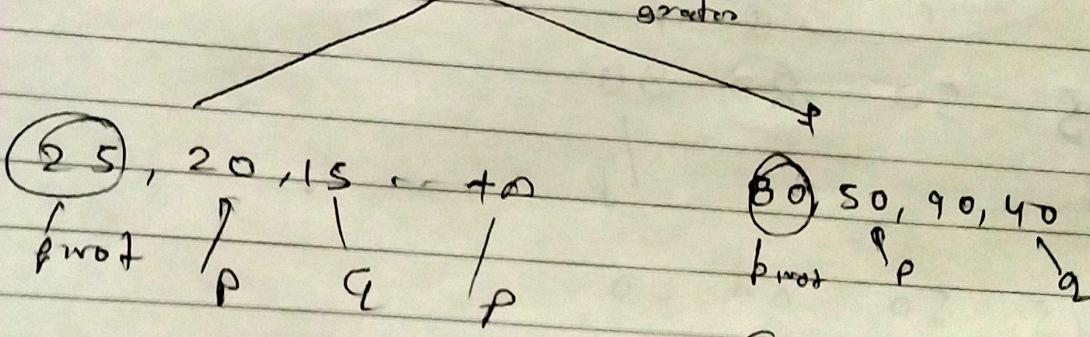
equals to 35 (pivot).

q decreases or move towards L.H.S and stops only when it will get the element lesser than the pivot element equal to 35 (pivot).

If P & Q , are not cross each other then swap or interchange their values.

check if P & Q crosses each other then replace pivot element with Q .

$25, 20, 15 \quad (35) \quad 80, 50, 90, 40$



15 20 25

(80) 50 90 40
P Q

(80) 50 40 90
1 P

= 40 50 80 90

combined as

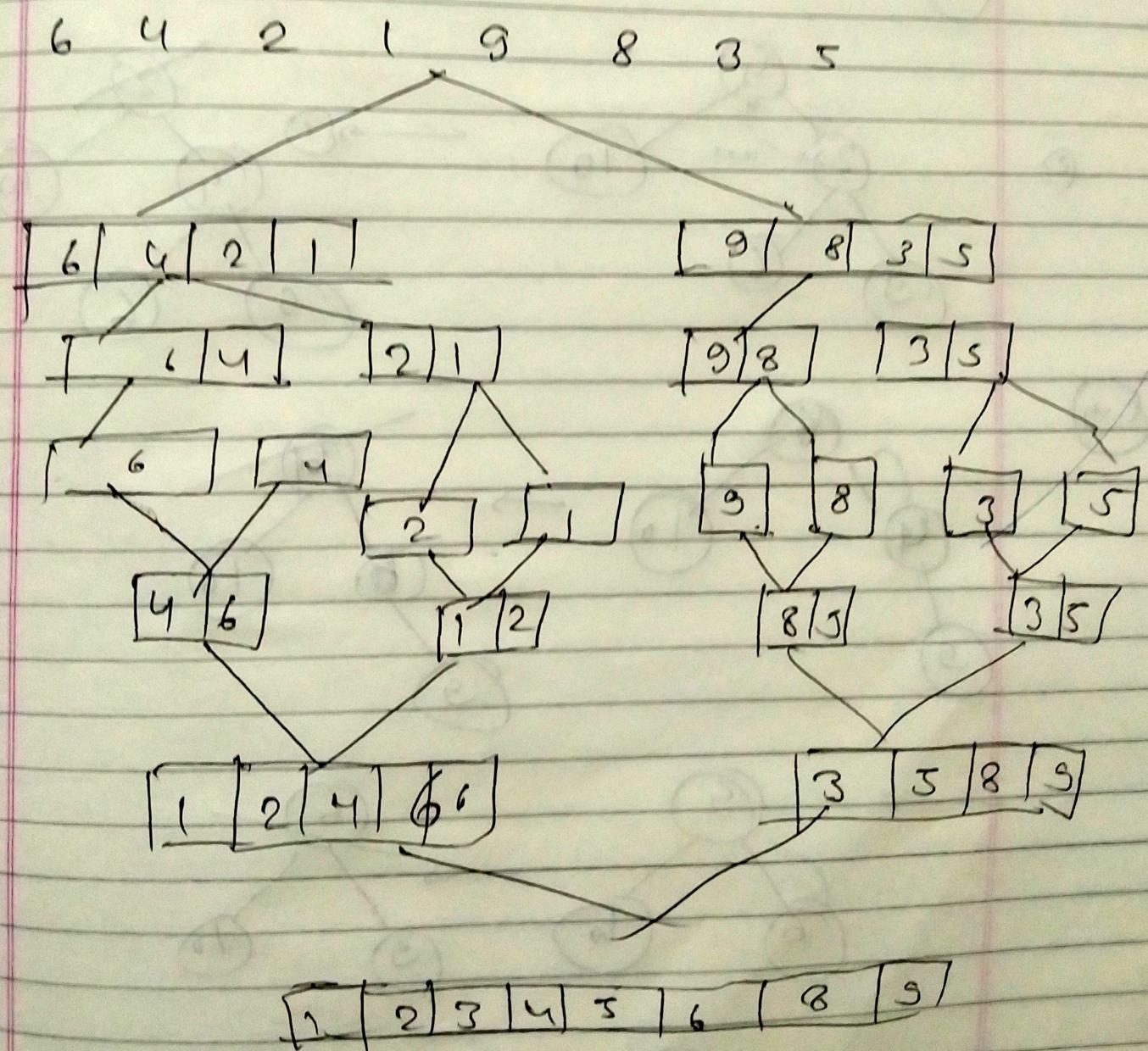
15 20 25 40 50 80 90

Best case $\Theta(n \log n)$
Worst case $\Theta(n^2)$

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Merge sort

It is a sorting algorithm that works by dividing any array into smaller subarray. Sorting each subarray then merge the sorted sub arrays back together to form the final sorted array.

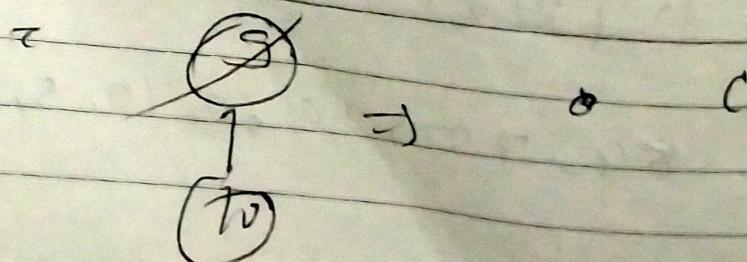
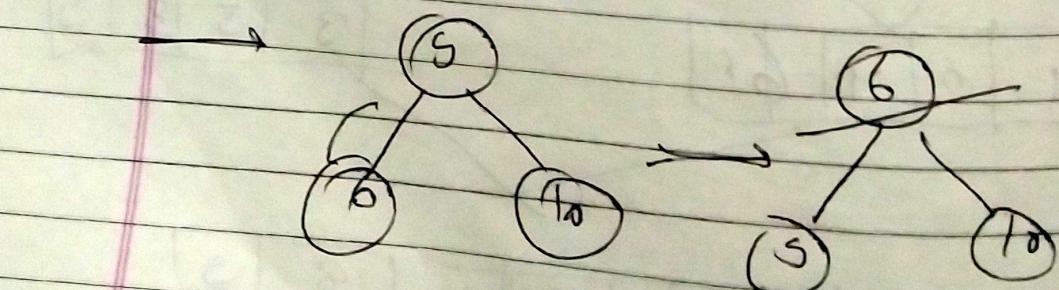
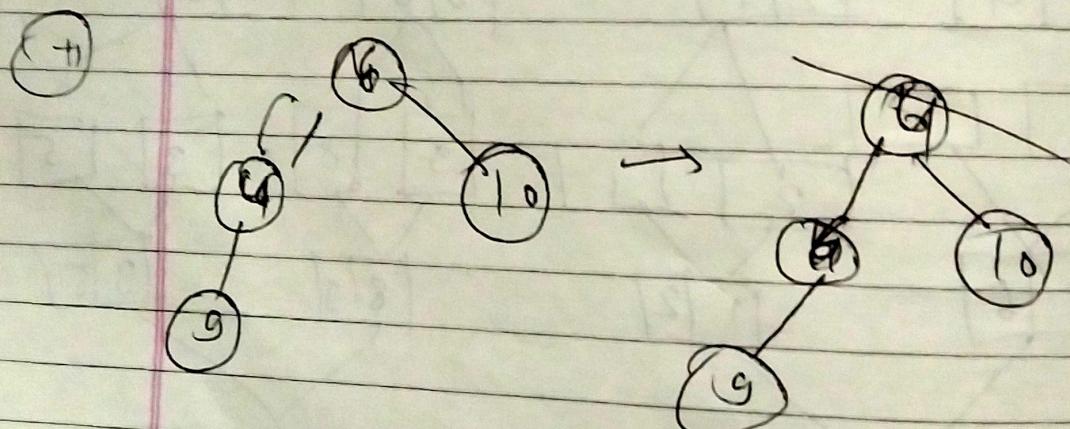
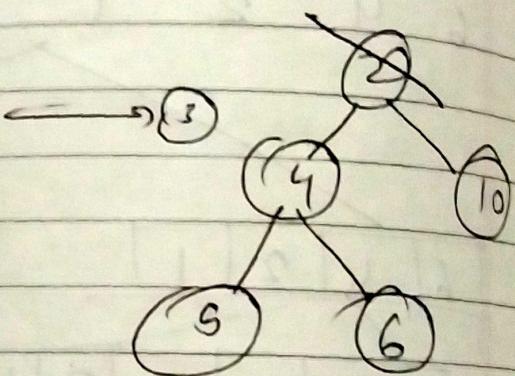
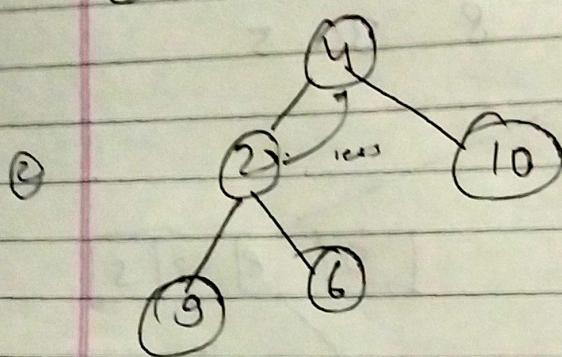
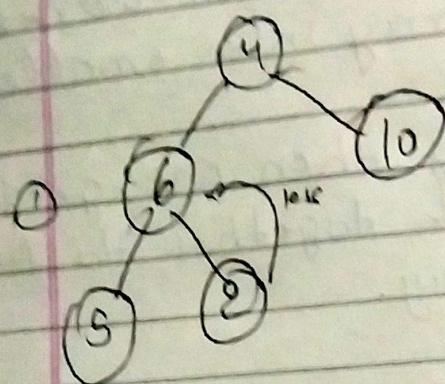


(Ex) ①

42, 84, 75, 20, 60, 10, 5, 30

~~# Heap sort \rightarrow Complexity $O(n \log(n))$~~

4 6 10, 9, 2

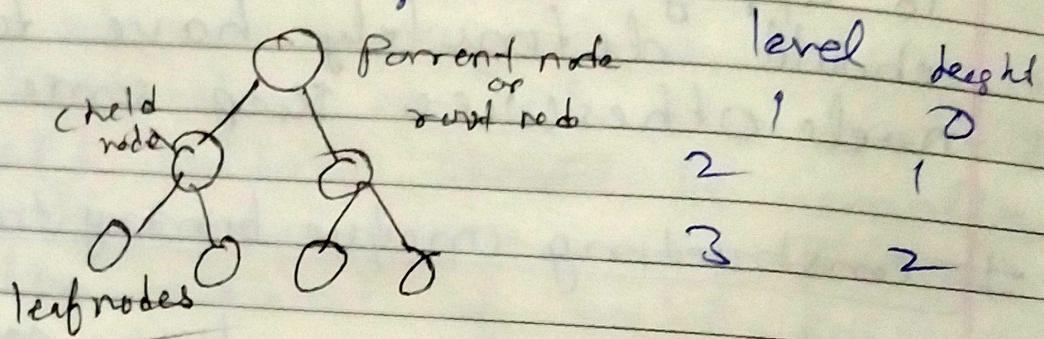


Tree → Tree is a linear data structure used to store data element.

Types → ① Binary tree
② An Array tree

① Binary tree -

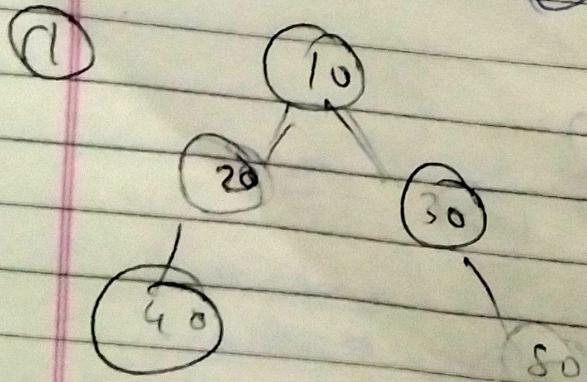
A tree in which every node can contain almost two child node is called a Binary tree.



height - The longest path from the root node to leaf node is called height of tree.
 $\text{height} = \text{level} - 1$

Tree Traversal →

- ① Preorder
- ② Postorder
- ③ Postorder



Preorder = 10, 20, 40, 30, 50

Inorder = 40, 20, 10, 30, 50

Postorder = 40, 20, 50, 30, 10

(sorted element)

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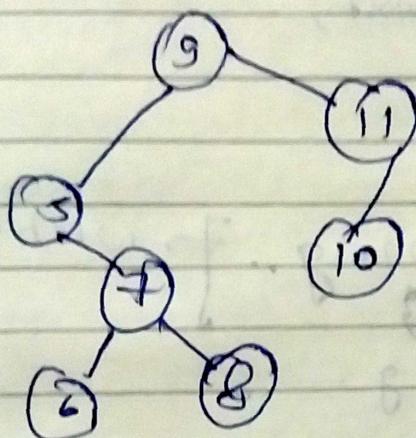
11 Binary search tree

① Preorder = Root, left, right 0

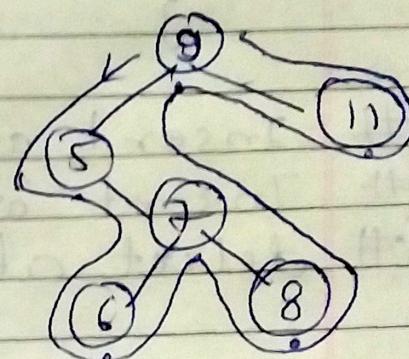
② Inorder = left, Root, right 8

③ Postorder = left, right, Root 0

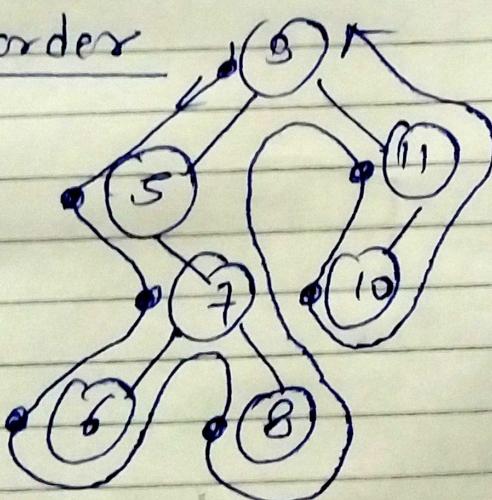
0, 5, 7, 8, 6, 11, 10



④ Inorder

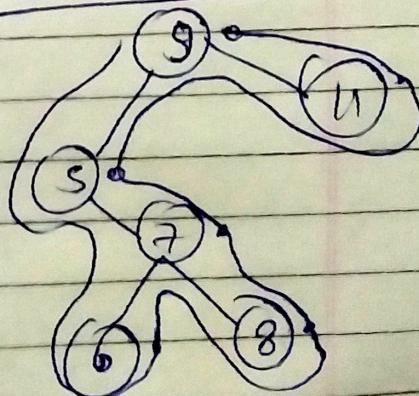


⑤ Preorder



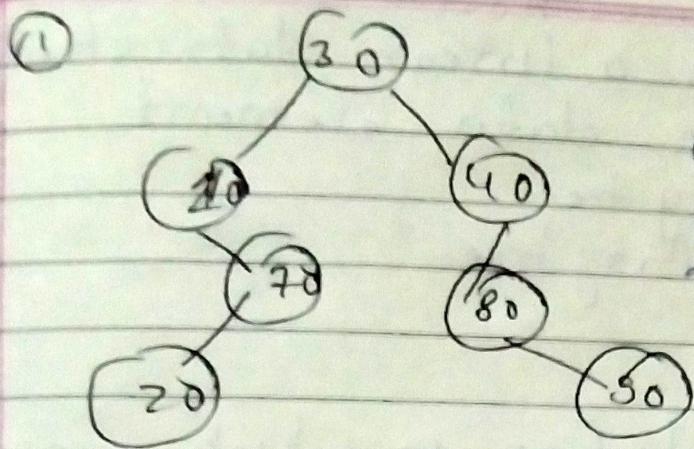
5, 6, 7, 8, 9, 11

postorder



9, 5, 7, 6, 8, 11, 10

6, 8, 7, 5, 11, 9



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(1st visit)
 Preorder = 30, 10, 20, 20, 40, 80, 50
 (2nd visit) Preorder = 10, 20, 20, 70, 80, 50, 40
 (3rd visit) Postorder = 20, 10, 90, 80, 40, 20

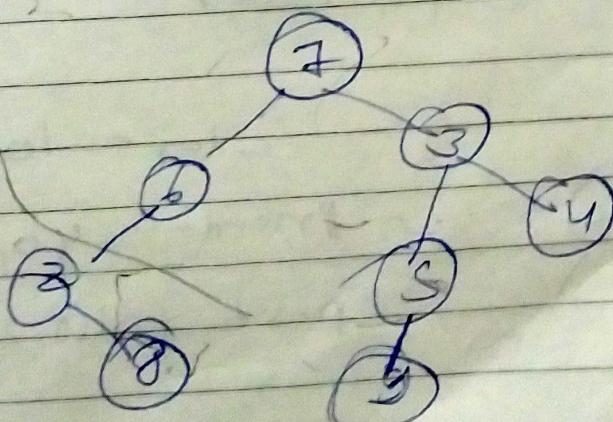
To apply the about techniques every node should definitely have two child nodes otherwise keep some dummy node

Constructing unique binary tree

A unique binary tree can be constructed if Any two traversal are given and one of them should in order.
 Ex Any two traversal are given

(B) postorder : 8, 2, 6, 5, 5, 4, 3, 7

inorder : 2, 8, 6, 7, 5, 5, 3, 4



Construct the unique binary with the two trees

Post: 10, 5, 23, 22, 27, 25, 15, 50, 95, 60, 40, 29

In: 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 95

Binary search tree (BST)

BST is also a binary tree but with the restriction that all the elements of left subtree should be smaller than the root node.

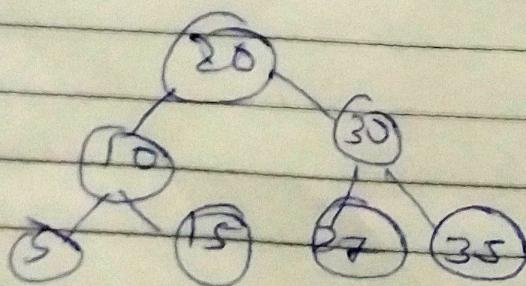
All the elements of right subtree should be greater than root node.

All the elements in BST should be unique.

BST is used for searching an element.

(a) Construct the BST with the given elements

→ 20, 10, 15, 30, 35, 27, 5



Preorder - 20, 10, 5, 15, 30, 27, 35

Inorder → 5, 10, 15, 20, 27, 30, 35

Postorder → 5, 10, 15, 27, 35, 30, 20

If NOTE :-

- # Inorder Traversal of a BST is always sorted order
- # last element in the preorder and last element in the inorder are same.
(This is true for only complete binary tree)

Time complexity for a BST =

Best case $O(\log n)$

worst $\rightarrow O(n)$

height

Stack data structure.

It is linear data structure

It work on LIFO

insertion & deletion operations from one end
of stack data structure.

① register stack

② memory stack

Conversion of Infix into Postfix, prefix

Q.1 # Infix $\Rightarrow a + b * c$

$$\Rightarrow a + b c$$

s.1 \checkmark Postfix $\Rightarrow + a * b c$

\checkmark Postfix $\Rightarrow a + b * c$,

$$\Rightarrow a b c * +$$

$$\Rightarrow a + [bc *]$$

$$\Rightarrow a [bc *] +$$

$$\Rightarrow abc * +$$

Q.2 # $(A + B) * c / D$ converse into prefix.

s.1 \checkmark Prefix -

$(A + B) * / c D$ $+ AB * c D$ $* (A + B) c D$ $/ * + A B c D$	$[+ AB) * c / D$ <p style="text-align: center;">left to right</p> $[* + A B c] / D$ $/ * + A B c D)$
--	---

+ Postfix \Rightarrow

$(AB +) * c / D$

$(AB +) c * / D$

$(AB +) c * D /$

Q:-

Postfix \Rightarrow $(a+b)$ $<\text{operator}> <\text{operand } 1> <\text{operator}> <\text{operand } 2>$ $(a+b/c) \star a - b/c$

$$\begin{aligned} & a + b * c / D \uparrow e \uparrow f * a - b / c \\ \Rightarrow & a + b * c / D \uparrow (e f \uparrow) * a - b / c \\ \Rightarrow & a + b * c / (D e f \uparrow \uparrow) * a - b / c \\ \Rightarrow & a + (b c \uparrow) / (D e f \uparrow \uparrow) * a - b / c \\ \Rightarrow & a + (b c \uparrow D e f \uparrow \uparrow \uparrow) * a - b / c \\ \Rightarrow & a + (b c \uparrow D e f \uparrow \uparrow / a \uparrow) - b / c \\ \Rightarrow & a + (b c \uparrow D e f \uparrow \uparrow / a \uparrow) - (b c /) \\ \Rightarrow & [\cancel{(a b c \uparrow D e f \uparrow \uparrow / a \uparrow)}] \cancel{+} (b c /) \\ \Rightarrow & \cancel{+} (a b c \uparrow D e f \uparrow \uparrow / a \uparrow) \cancel{+} (b c /) \\ \Rightarrow & a b c \uparrow D e f \uparrow \uparrow / a \uparrow + b c / - \quad [\underline{\text{Answer}}] \end{aligned}$$

Homeworks \Rightarrow

{

- Q1 $(A+B) \uparrow ((C+D))$
- Q2 $A \uparrow \cdot B + C \uparrow D$
- Q3 $A + B + C + D$
- Q4 $K + M - N + (O \uparrow P) + W / U / V \uparrow T + Q$
Postfix

$$\textcircled{1} \quad (A+B) * (C+D)$$

prefix -

$$*(A+B)(C+D)$$

$$*(+AB)(C+D)$$

$$*(+AB)(+CD)$$

$$\Rightarrow *+AB+CD$$

$$(AB+) * (CD+)$$

$$(AB+)(CD+) *$$

$$AB+CD+ *$$

$$\textcircled{2} \quad A * B + C * D$$

Prefix

$$(*AB)+C*D$$

$$(*AB)+(*CD)$$

$$+(*AB)(*CD)$$

$$+*AB*C*D$$

Postfix

$$(AB*)+C*D$$

$$(AB*)+(CD*)$$

$$(AB*)(CD*)+$$

$$AB*C*D*+$$

$$\textcircled{3} \quad A+B+C+D$$

Prefix

$$(+AB)+C+D$$

$$[+(+AB)(C)]+D$$

$$[+(+(+AB)(C))D]$$

$$+++ABCD$$

Postfix

$$= (AB+) + C + D$$

$$\Rightarrow (\overset{\text{on}}{AB} + \overset{\text{on}}{C} +) + \overset{\text{on}}{D}$$

$$\Rightarrow (AB + C + D) +$$

$$AB + C + D +$$

$$④ \text{ And } K + L - M^* N + (O \uparrow P) * w / u / v * T + Q$$

$$K + L - M^* N + (O P \uparrow) * w / u / v * T + Q$$

$$K + L - (M N \star) + (O P \uparrow) * w / u / v * T + Q$$

$$K + L - (M N \star) + (O P \uparrow (w \star)) / u / v * T + Q$$

$$K + L - (M N \star) + (O P \uparrow w \star u /) / v * T + Q$$

$$K + L - (M N \star) + (O P \uparrow w \star u / v /) * T + Q$$

$$K + L - (M N \star) + (O P \uparrow w \star u / v / T \star) + Q$$

$$(K + L) - (M N \star) + (O P \uparrow w \star u / v / T \star) + Q$$

$$(K L + M N \star -) + (O P \uparrow w \star u / v / T \star) + Q$$

$$(K L + M N \star - O P \uparrow w \star u / v / T \star +) + Q$$

$$K L + M N \star - O P \uparrow w \star u / v / T \star + Q + \underline{\underline{Ans}}$$

$$S \quad a * (b + c)$$

\Rightarrow Prefix

$$a * (+bc)$$

$$(* a + bc)$$

Algorithm in form of table

I/P	TDS	
H	L	PUSH
L	H	POP
E	E	PUSH(R-L)
E	E	POP(L-R)
C	OP+	PUSH
OP+	C	PUSH
C	C	PUSH

⑩ Postfix

$$a * (bc +)$$

$$(ab + c) *$$

$$abc + *$$



Operands A, B, C, X, Y, P, Q

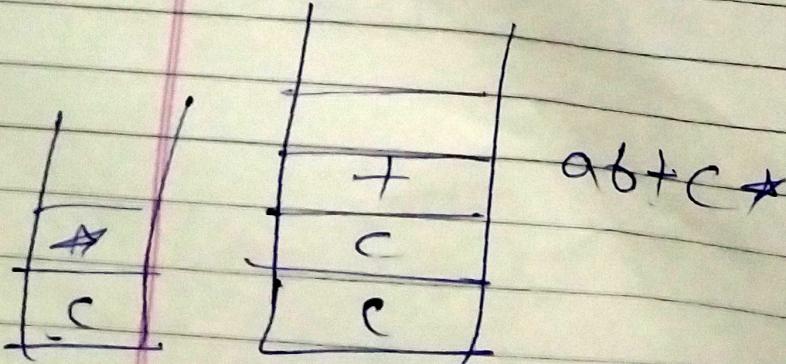
Operators +, -, *, /, %

Q1 $((a+b)*c)$

Soln Postfix

$$((ab+) * c)$$

$$= (ab + c *)$$



Q2 # $a + b * (c / d * e f) * (a - b) / c$

Post.

$$\Rightarrow a + b * (c / d * e f) * (a - b) / c$$

$$\Rightarrow a + b * (c / (d * e f)) * (a - b) / c$$

$$\Rightarrow a + b * (c / (d * e f)) * (a - b) / c$$

$$\Rightarrow a + b * (c / (d * e f)) * (a - b) / c$$

$$⑥ a \star b - c$$

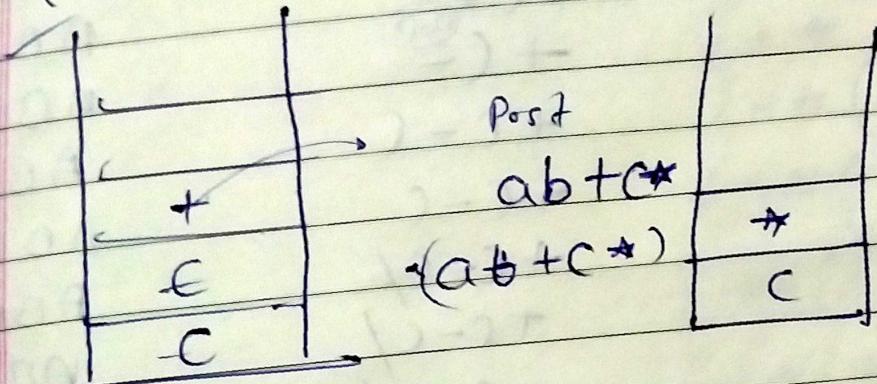
$$(2) a \star b + c$$

$$(3) a \uparrow b \uparrow c$$

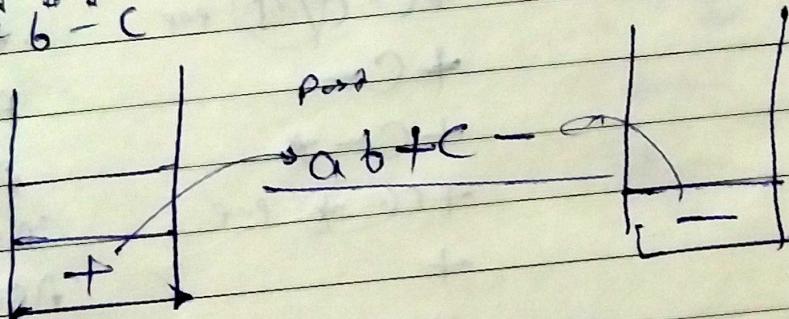
$$(3) a \star (b + c)$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ ((a+b) \star c) \end{matrix}$$

③



$$① a \star b - c$$



$$Q_1 A + (B \star C - (D/E \uparrow F) \star G) \star H$$

$A + B * C - (D / E \uparrow F) * G \downarrow H$

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Q. 1

~~SYMBOL~~

STACK

Postfix

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ABC*D/F/*/a/

*

+ *

ABC*D/F/*/*

H

+ *

ABC*D/F/*/*H

$$\frac{mn}{n} = mH$$

$$\frac{mn}{n} = mH$$

$$\frac{mn}{n} = mH$$

~~2nd P~~ $(A+B)/C * (D+E) - F$

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Q. 3 SYMBOL

STACK

Postfix

({	
A	{	A
+	{ +	A
B	{ +	AB
/	{ + /	AB
C	{ + / { ^{pop}	ABC
*	{ + * {	ABC/
({ + * (ABC/
D	{ + * (ABC/D
+	{ + * (+	ABC/D
E	{ + * (+	ABC/DE
)	{ + * ^{pop}	ABC/DE +
-	{ + -	ABC/DE + *
F	{ + - ^{pop}	ABC/DE + * F
)		

$A + C / B$

⑦ $(A+B)/C * (D+E) - F$

Prefix \Rightarrow

$$\begin{aligned} &\Rightarrow (A+B)/C * (+DE) - F \\ &\Rightarrow (A+/(BC)) * (+DE) - F \\ &\Rightarrow (A+[(/BC+DE)]) - F \\ &\Rightarrow (+ A * BC + DE) - F \\ &\Rightarrow - + A * / BC + DEF \end{aligned}$$

$$\begin{aligned} &(A+B/C * (+DE) - F) \\ &(A+B/- * (C+DE) - F) \\ &(A+/(B * (C+DE)) - F) \\ &A+(-/B * C+DEP) \\ &+ A - / B * C + DEP \end{aligned}$$

$$\text{Prefix} = \frac{(A+B)C * (D+E)-F}{F-(E+D)*C/B+A} \Rightarrow$$

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SYMBOL

STACK

POSTFIX

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$\Rightarrow FED + C * B / - A +$

Reverse:

$+ A - / B * C + DEF$

$$\# A + (B * C - (D/E \uparrow F) * G) * H$$

$$A + (B * C - (D/(E \uparrow F)) * G) * H$$

$$\Rightarrow A + ((BC) - (D/(E \uparrow F)) * G) * H$$

$$\Rightarrow A + ((BC) - (D/(E \uparrow F) * G)) * H$$

$$\Rightarrow A + ((BC) - (D/(E \uparrow F) * G) * H)$$

$$\Rightarrow A + ((BC) - (D/(E \uparrow F) * G) * H)$$

$$\Rightarrow A + ((BC) - (D/(E \uparrow F) * G) * H) \quad \text{Proved that}$$

$$\# H^*(C^*(F \uparrow E/D) - C^*B) + A$$

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HGPPTD

HGPBTD /

HGFED / *

HGFED / *C

HGFED / *C*

HGFED / *C B

HGFED / *C B **

HGFED / *C B * *

HGFED / *C B * A

+ A * - BC *

+ A * - *BC * / D 1 EFGH

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~~a + b + c / d~~ $\leftarrow f \rightarrow a - b / c$
~~c / b - a \leftarrow f \uparrow e \uparrow d / c \star b + a~~

c	/	c
l	/	e6
b	+	c6/
-	-	c6/9
a	-*	e6/9
+	-*	c6/9f
f	- * ↑	c6/9f
↑	- * ↑	c6/afe
e	- * 1 ↑	c6/afe1
↑	- * ↑ ↑	c6/afe10
d	- *	c6/afe10M*
/	- /	c6/afe10M*c
c	- *	c6/afe10M*c
+	- *	c6/afe10M*c
b	- *	c6/afe10M*c/b
+	- *	c6/afe10M*c/b*-
a	- *	c6/afe10M*c/b*-a
-	-	c6/afe10M*c/b*-a
	o	

$\Rightarrow \cancel{a} - \cancel{b} / c \leftarrow \cancel{d} \cancel{e} \cancel{f} a / b c$

$\Rightarrow \cancel{a} - \cancel{b} / c \leftarrow \cancel{d} \cancel{e} \cancel{f} a / b c$

linear search \Rightarrow

key = 6?

0	1	2	3	4	5	6
4	9	3	6	11	15	10

Linear search is a very simple search algorithm in this type of search a sequential search is made over all items one by one.

every item is checked & if a match is found then that particular item is returned otherwise search continue till the end of data collection.

① best case $O(1)$

② worst case $O(n)$

Binary search \Rightarrow

0	1	2	3	4	5	6	7	8	9
10	15	18	25	29	32	37	40	50	60

(sorted ascending order)

target 37

elements are arranged in sorted form.

Time complexity
 $O(\log n)$

$$\text{mid} = \frac{l+r}{2}$$

$$\text{mid} = \frac{0+9}{2} = 4.5 = 4 \quad \# \text{ conditions occurs}$$

① $A[\text{mid}] == \text{Target}$
return;

② $A[\text{mid}] < \text{Target}$

$$l = \text{mid} + 1;$$

③ $A[\text{mid}] > \text{Target}$

$$r = \text{mid} - 1;$$

	l	r	mid	
①	0	9	4	
②	5	9	7	
③	5	6	5	
④	6	6	6	

10	15	18	25	29	32	37	40	50	60
l	1	2	3	4	5	6	7	8	r

10	15	18	25	29	32	37	40	50	60
l	1	2	3	4	5	6	7	8	r

Stack \Rightarrow

A stack is a container of objects that are inserted & removed according to last in first out (LIFO). Principle.

Object can be inserted at any time. Inserting an item known as "pushing".

It allows insertion & deletion operation from one end of the stack data structure, that is top.

A stack is a logical concept that consists of a set of similar elements.

It is an abstract data type that holds an ordered linear sequence of items.

It is used for evaluating expression with operands & operation (Infix to postfix conversion)

Queues \Rightarrow

A queue is different from a stack that its insertion & removal operation according to first in first out (FIFO) principle. Elements may be inserted at any time. But only element which has been in the queue the longest may be removed.

Elements are inserted at the rear (enqueued) & removed from the front (dequeued).

It is a linear data structure that is open at both ends.

It is used to manage threads in multithreading & implementing priority queuing.