DMS

UNIT-1 (Discrete mathematic structure)

Discrete mathematic isnot the name of branch of mathematic like calculus, algebra, number theory rather it is a common feature or a description of a set of branches of math that all have the common feature which are discrete rather than contineous.

It is used for logic making and problem solving capability.

Preposition ⇒ It is declarative sentence which are either true or false but not both.

#1 2 * 2 > 4

(Preposition)

#2 Wish you happy life.

(It is not a preposition because true and false is not certain.

Truth value ⇒ If any preposition is true, then it is denoted by T.

If the value of preposition is false then it is denoted by F.

Eg (1) 1 is less than 3 T

14 is odd number F

Type of preposition ⇒ (1) Simple preposition

(2) Compound preposition

Simple ~~pro~~position – In preposition having only one subject & one predicate is called simple preposition.

Eg 2 is even number.

Compound preposition – When two or more simple preposition combines by various connectivities into a single composite sentence is called compound preposition.

Eg. ΔA is equilateral iff it is all sides are equals.

Negation : It's denoted by $\sim p$ & $\neg p$.

Eg p : today is saturday

$\sim p$: today isnot saturday.

~~#~~ logic connectives :

The particular word and symbol used to join two or more single preposition into a single form are called logical connectives.

①	AND/conjunction/join	$\wedge$	$p \wedge q$
②	OR/disjunction/meet	$\vee$	$p \vee q$
③	Negation	$\sim$ or $\neg$	$\sim p / \neg p$
④	Equivalent	$\equiv$	$p \equiv q$
⑤	Conditional if...then'	$\Rightarrow$	$p \Rightarrow q$
⑥	Biconditional (iff)	$\Leftrightarrow$	$p \Leftrightarrow q$
⑦	NAND (NOT+AND)	$\uparrow$	$p \uparrow q$
⑧	NOR (NOT+OR)	$\downarrow$	$p \downarrow q$
⑨	XOR	$\oplus$	$p \oplus q$

① **And / conjunction** :- Any two preposition can be combined by the word "AND" to form compound preposition said to be "AND" conjunction.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

When both are true, $p \wedge q$ is always true otherwise $p \wedge q$ is false.

② **OR / disjunction** :- Any two preposition can be combined by the word "OR" to form compound preposition said to be "OR" disjunction.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

If both condition is false than $p \vee q$ is false. otherwise in any condition it will be true.

③ **Negation** :- The negation of any given preposition p is the preposition where truth value opposite to p.

p	$\sim p$ / $\neg p$
T	F
F	T

④ **Equivalent** :- When truth value of two compound preposition are same.

$\boxed{\sim(p \vee q) \equiv \sim p \wedge q}$

(5) Implication / conditional :-

An implication is denoted by $p \rightarrow q$ / $p \Rightarrow q$. It's the preposition in which if p then q. It is false only when p is true and q is false.

p	q	$p \rightarrow q$ / $p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(6) Biconditional (if & only if (iff)) :-

It is denoted by $p \Longleftrightarrow q$ is a biconditional logical operation which is true when p & q are same i.e both are true or both are false. both must be equal for truth value.

p	q	$p \Longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautology ⇒

It is the formula which is always true for every value of its propositional variable.

A preposition is said to be tautology if it contain only T (truth) in the last column of its truth table.

Q.1 Prove that $[(A \to B) \wedge A] \to B$ is a tautology.

Ans

A	B	$A \to B$	$[(A \to B) \wedge A]$	$[(A \to B) \wedge A] \to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is tautology.

Contradiction ⇒

It is formula which is always false for every value of its prepositional variable.

A proposition is said to be contradiction if it contain only false in the last column

Q.1 Prove that $(A \vee B) \wedge [(\sim A) \wedge (\sim B)]$ is a contradiction.

Ans

A	B	$A \vee B$	$\sim A$	$\sim B$	$[(\sim A) \wedge (\sim B)]$	$(A \vee B) \wedge [(\sim A) \wedge (\sim B)]$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

Contigency ⇒

It is formula which contain both some true value or some false value.

① Prove $(A \vee B) \wedge (\sim A)$ is a contigency.

A	B	$\sim A$	$A \vee B$	$(A \vee B) \wedge (\sim A)$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Q.1 $[(A \vee \sim B) \wedge (\sim A \vee \sim B)] \vee B$ what is it?

Ans

A	B	~A	~B	(~A ∨ ~B)	(A ∨ ~B)	(~A ∨ ~B) ∧ (A ∨ ~B)	with ∨B
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

This expression is tautology.

Q.2 Prove that preposition $P \vee \sim(q \wedge r)$ and $(P \vee \sim q) \vee (\sim r)$ are equivalent.

Ans

P	q	r	~q	~r	q ∧ r	~(q ∧ r)	P ∨ ~(q ∧ r)	P ∨ ~q	(P ∨ ~q) ∨ (~r)
T	T	T	F	F	T	F	T	T	T
T	T	F	F	T	F	T	T	T	T
T	F	T	T	F	F	T	T	T	T
F	T	T	F	F	T	F	F	F	F
T	F	F	T	T	F	T	T	T	T
F	F	T	T	F	F	T	T	T	T
F	T	F	F	T	F	T	T	F	T
F	F	F	T	T	F	T	T	T	T

Proved.

Q.3 following is tautology, contradiction & contigency.

$Q \vee (P \wedge \sim Q) \vee (\sim P \vee \sim Q)$

Ans

P	Q	~P	~Q	~P ∨ ~Q	P ∧ ~Q	Q ∨ (P ∧ ~Q)	Q ∨ (P ∧ ~Q) ∨ (~P ∨ ~Q)
T	T	F	F	F	F	T	T
T	F	F	T	T	T	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	F	T

This is tautology.

Q.4 Is the formula $p \to (p \wedge (q \to p))$ a tautology?

Ans

p	q	$q \to p$	$p \wedge (q \to p)$	$p \to (p \wedge (q \to p))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Yes it is tautology

Q.5 Check these are tautology, contradiction & contigency

(a) $(\sim p \vee r) \to (p \vee \sim r)$

(b) $(p \leftrightarrow q) \vee (p \leftrightarrow \sim q)$

(c) $r \to (p \wedge \sim r)$

Answer (a) $(\sim p \vee r) \to (p \vee \sim r)$

p	r	$\sim p$	$\sim r$	$\sim p \vee r$	$p \vee \sim r$	$(\sim p \vee r) \to (p \vee \sim r)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

contigency

(b) $(p \leftrightarrow q) \vee (p \leftrightarrow \sim q)$

p	q	$p \leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$	$(p \leftrightarrow q) \vee (p \leftrightarrow \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Tautology.

(c) $r \to (p \wedge \neg r)$

r	p	$p \wedge \neg r$	$r \to (p \wedge \neg r)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

contigency

Q.6 Check $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

Ans

p	q	r	$p \wedge q$	$q \Rightarrow r$	$p \wedge q \Rightarrow r$	$p \Rightarrow (q \Rightarrow r$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
F	T	T	F	T	T	T
T	F	F	F	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Equivalent

Q.7 check $p \vee (p \to q) \vee \{\neg(p \vee q)\}$

Ans

p	q	$p \to q$	$p \vee q$	$\neg(p \vee q)$	$p \vee (p \to q)$	$p \vee (p \to q) \vee (\neg(p \vee q))$
T	T	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	F	T	T
F	F	T	F	T	T	T

Tautology

Q.8. What is this?

$(P \vee (Q \vee R)) \longleftrightarrow [((P \to Q) \to Q) \to R) \to R]$

Ans

P	Q	R	$Q \vee R$	$P \vee (Q \vee R)$	$P \to Q$	$(P \to Q) \to Q$	$((P \to Q) \to Q) \to R$	$() \to R$	$() \leftrightarrow ()$
T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T	T
F	T	T	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F	T	T
F	T	F	T	T	T	T	F	T	T
F	F	T	T	T	T	F	T	T	T
F	F	F	F	F	T	F	T	F	T

tautology

Converse, inverse & contrapositive of conditional statement

① **Converse** :- let $p \to q$ be a conditional statement then it converse will be

$q \to p$

Eg → If $\underbrace{x \text{ is labour}}_{p}$, then $\underbrace{\text{he is poor}}_{q}$.

$p \longrightarrow q$

Converse is $q \to p$ it mean in term of statement
if He is poor than x is labour

$q \longrightarrow p$

② **Inverse** → let $p \to q$ be a conditional statement then it converse will be

$\sim p \to \sim q$.

Eg. If $\underbrace{\text{krishna will read book}}_{p}$ then $\underbrace{\text{he will get knowledge}}_{q}$

$p \longrightarrow q$

Inverse: $\sim p \to \sim q$

If krishna will not read book than he will not get knowledge

Contrapositive ⇒ let $p \to q$ will be a conditional statement then its contrapositive will be $\sim q \to \sim p$.

Eg. If $f(x)$ is differential then it is contineous.
If $f(x)$ is not contineous, then it is not differential.

Some basic law / Logic equivalance ⇒

Identity laws $p \wedge T \equiv p$, $p \vee F \equiv p$

① Idempotent law $p \wedge p = q$ $p \vee q = p$

② commutative law $p \vee q = q \vee p$ $p \wedge q = q \wedge p$

③ Associative law $(p \vee q) \vee r = p \vee (q \vee r)$
$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

④ De-morgans law ⇒ $\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$

⑤ $p \to q \equiv \sim p \vee q$ / $p \Longleftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

let $A(P_1, P_2 \cdots P_n)$ be a statement formula then construction of truth table may not be practical always. so we consider alternate procedure known as reduction to normal form.

① disjunction normal form ⇒
It is a statement form which consist of disjunction between conjunction.

① $(p \wedge q) \vee r$

② $(p \wedge \sim q) \vee (p \wedge \sim r) \vee (p \wedge r)$

It is (sum of product) method.

⑥ Absorption law

① $p \vee (p \wedge q) \equiv p$

② $p \wedge (p \vee q) \equiv p$

Q.1 DNF form of $(p \to q) \wedge (\sim p \wedge q)$

Answer ⇒ We know $p \to q = \sim p \vee q$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

apply distributive law

$[\sim p \wedge (\sim p \wedge q)] \vee [q \wedge (\sim p \wedge q)]$

$[\sim p \wedge q] \vee [q \wedge \sim p \wedge q]$

$[\underset{\text{product}}{\sim p \wedge q}] \underset{\text{sum}}{\vee} [\underset{\text{product}}{q \wedge \sim p}]$ DNF FORM.

② Conjunction normal form (CNF) ⇒

The statement formed which consist of conjunction between disjunction.

Ex $\sim p \wedge q$

or $\underset{\text{sum}}{(\sim p \vee q)} \underset{\text{product}}{\wedge} \underset{\text{sum}}{(\sim p \vee r)}$ [products of sums]

Q.1 CNF Form $(p \wedge q) \vee [(\sim p \wedge q \wedge r)]$

Ans $[p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$

$[\underset{T=1}{(p \vee \sim p)} \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge \underset{q}{(q \vee q)} \wedge (q \vee r)]$

$[(p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]$

sum product sum

Q.2 Find CNF form of $p \wedge (p \to q)$

Ans $p \to q = \sim p \vee r$

$p \wedge (\sim p \vee r)$

$= \underset{F}{(p \wedge \sim p)} \vee (p \wedge r)$

⇒ F

⇒ $p \wedge r$

Q.3 obtain CNF form $(\sim p \to r) \wedge (q \leftrightarrow p)$

Ans $(\sim p \to r) \wedge (\sim q \vee p) \wedge (\sim p \vee q)]$

$[(\sim(\sim p) \vee r)] \wedge [(\sim q \vee p) \wedge (\sim p \vee q)]$

$[(p \vee r)] \wedge [(\sim q \vee p) \wedge (\sim p \vee q)]$ CNF form

Q.4 obtain DNF form $p \vee (\sim p \to (q \vee (q \to \sim r)))$

Ans $p \vee [(\sim p) \to (q \vee (\sim q \vee \sim r))]$

$p \vee [(\sim p) \to [(q \vee \sim q) \vee (q \vee \sim r)]$

$p \vee [(\sim p) \to [q \vee (q \vee \sim r)]$

$p \vee [\sim(\sim p) \vee [q \vee (q \vee \sim r)]$

$p \vee [p \vee [q \vee (q \vee \sim r)]$

$p \vee [p \vee (q \vee \sim r)]$

$p \vee [p \vee q \vee \sim r]$

$[p \vee q \vee \sim r]$ DNF

rule of Inference =

Infrence → Deriving conclusion from evidences.

rule of infrence = These are template for valid arguement.

Argument - It is process by which a conclusion is obtain from the given set of premises.

Premises = (Given group of preposition is called premises)

Conclusion = The preposition get by given premise is called conclusion.

Ex. I will become famous or I will ~~be~~ a writter

I will not be a writter.

conclusion : I will become famous

valid arguement =

let $P_1 P_2 \cdots P_n$ are premises & q is conclusion then the arguement is valid only iff $A(P_1, P_2 \cdots P_n) \rightarrow Q$ is tautology.

falancy arguement = An arguement is called falacy if it is not valid.

Ex If it rain, Ram will be sick.

It didnot rain

conclusion : Ram was not sick

Here $P \equiv$ if it rain

$Q \equiv$ Ram will be sick.

$\sim Q \equiv$ Ram was not sick

$\sim P =$ ~~It~~ didnot Rain

expression $[(P \rightarrow Q) \wedge \sim P] \rightarrow \sim Q$

P	Q	P→Q	~P	(P→Q) ∧ ~P	~Q	[(P→Q)∧(~P)] → ~Q
T	T	T	F	F	F	T
F	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

falacy

Quantifiers

These are words that refers to quantities such as some or all.
It tells: for how many elements a given predicate is true.
It is used to express the quantities without giving an exact number
Ex all some, few, many.
(1) Universal
(2) Existential

Universal	Existential
(1) The phrase "for all" is called universal quantifier	(1) the phrase "for all" there exist is called existential quantifier
(2) It is denoted by $\forall$	(2) It is denoted by $\exists$
(3) for all x / $\forall x$ represent for all x, for each x, for every x	$\exists$ represent there exist an x, for some x, there is at least one x
(4) All human being are mortal	(4) Some human being are mortal
(5) $P(x)$: x is mortal $\forall x\, P(x)$ $\forall x \in U (P(x))$	(5) $P(x)$: x is mortal $\exists x\, P(x)$

Conditions for universal :

① $\forall x \rightarrow$ True

When all elements in the domain are true.

② $\forall x \rightarrow$ false

At least one false in the domain

$\forall(x) P(x) \rightarrow$ false

Ex) Domain :- Z (set the +ve integer)

open statement :- $x^2 \geq 0$

$P(x) : x^2 \geq 0$ Predicate form

$\forall(x) P(x)$

$\forall(x)(x^2 \geq 0)$ ⇒ True

Condition for existential.

① $\exists(x) P(x) \longrightarrow$ True

When we have atleast one true.

② $\exists x P(x) \rightarrow$ false

When all element in the domain are false

Ex $D = \{1, 2, 3, 4\}$

$P(x) : x^2 = 4$

$\exists x P(x)$

$\exists x (x^2 = 4)$

Note :- Negation of a quantifiers.

① $\neg \forall x P(x) \equiv \exists x \neg P(x)$

② $\neg \exists x P(x) \equiv \forall x \neg P(x)$

Nested quantifiers. When we used one in the scope of other.

Domain: $\{1,2,3\}$

$\forall x\,(x^2+y^2 \geq 0)$

$(1+y^2 \geq 0) \rightarrow$ Not valuable

Type 1: $\forall x \forall y$

Ex. $P(x,y): x \times y \leq 16$

$\forall x \forall y \Rightarrow$ True, when all edges must true, & false when atleast one edge is false

$\boxed{\forall x \forall y = \forall y \forall x}$

Type-2: $\exists x \exists y$

Ex $P(x,y): x \times y = 4$

$\exists x \exists y$ will be true, when atleast one edge is true & false when all edges are false.

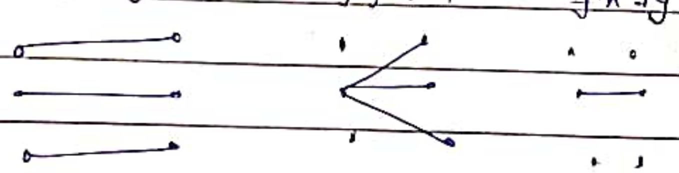
$\boxed{\exists x \exists y = \exists y \exists x}$

Type-3: $\forall x \exists y$

eg D: set of integer$(\mathbb{Z})$

$P(x,y): \forall x \exists y\,(x+y = 10)$

$\forall x \exists y$ $\quad\quad$ $\exists y \forall x$ $\quad\quad$ $\exists x \exists y$



Important inferance rule

① $A \rightarrow B$ $[(A \rightarrow B) \wedge A] \rightarrow B$ modus Pones

A

$\therefore B$

② $A \vee B$ $[(A \vee B) \wedge \neg A] \rightarrow B$ (disjunction syllogism)

$\neg A$

$\therefore B$

③ $A \rightarrow B$ $[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$

$B \rightarrow C$ Hypothetical Syllogism

$\therefore A \rightarrow C$

④ P $P \rightarrow P \vee Q$ (Addition)

$\therefore P \vee Q$

⑤ $P \wedge Q$ $P \wedge Q \rightarrow P$ (simplification)

$\therefore P$

⑥ $P \vee Q$

$\neg Q \vee R$ Resolution

$\therefore P \vee R$

Predicate logic (first order logic) ⇒

It is a generalization of Prepositional logic that allows us to express or infer arguement in infinite models.

Ex. All man are mortal

Some bird cannot fly

Some terminology:

(1) open statement: x is an even no ? generalize

(2) is even no

input 1 → $\boxed{\text{open statement}}$ → simple statement 1

input 2 → → simple statement 2

(3) Domain :- →

D = {0, 1, 2 ...}

D = set of integer

p(x) : x is even

Predicate

subject

subject + Predicate → It stay same

It may change

Ex $\underbrace{\text{Ram}}_{\text{subject}}$ $\underbrace{\text{is eating pizza}}_{\text{Predicate}}$

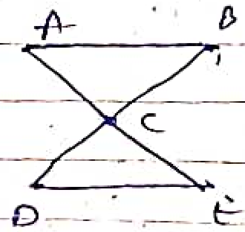
DMS

It is a mathematical structure used to

Graph ⇒ Model pair relation b/w object from certain collection.

A graph is a mathematical structure it consist vertex & edges is called the graph.

$G(V, E)$



A, B, C, D, E (node / vertex)

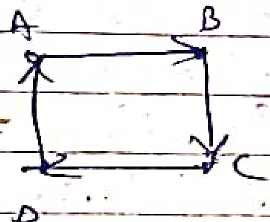
AB, BC, AC, DE, DC, CE (Edges)

Basic terminology

Type of graph ⇒ on the basis of edges whether they consist of ordered or unorderd pair of vertices graph can be categories in three

- Directed graph
- Undirected graph
- Mixed graph

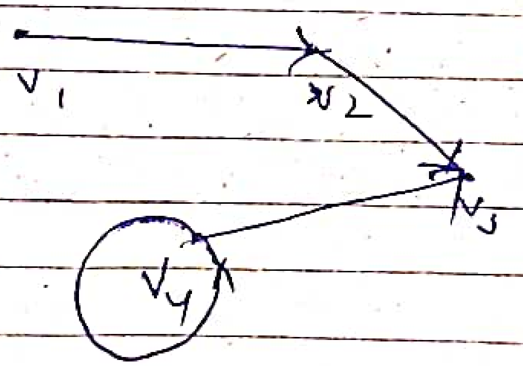
① Directed graph ⇒



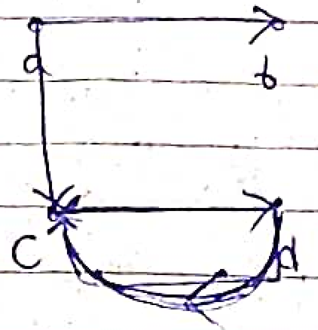
A diagram consist of a direction set of vertex (V) the and set of edge (E) that consist a direction and order pair of element.

Eg. following the graph of diagram

① $G = \{V_1, V_2, V_3, V_4\}, \{(V_1, V_2)(V_2, V_3)(V_4, V_4) (V_4, V_3)\}$



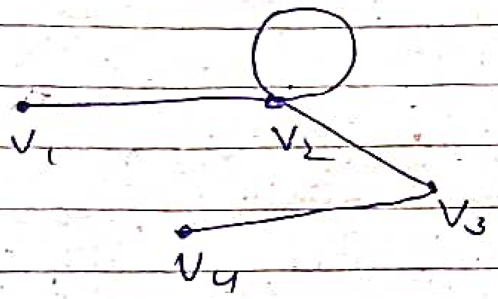
Eg ② : $G = \{a, b, c, d, e\} \{(a, b) (a, c)) (c, d), (d, c)\}$



Undirected graph ⇒

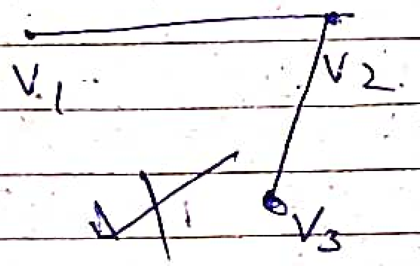
A diagram consist of set of vertex & set of edge that consist a undirected & unorderd pair of element.

Eg ① $G = \{v_1, v_2, v_3, v_4\}\} \{(v_1, v_2) (V_2, v_3) (v_3, v_2) (v_3 v_4)\}$

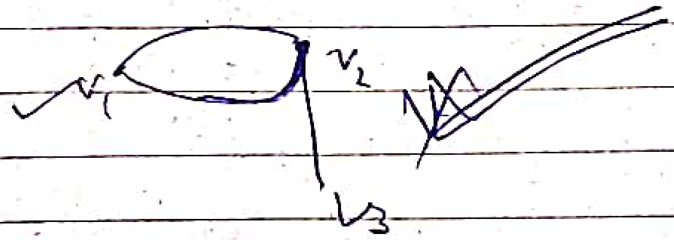


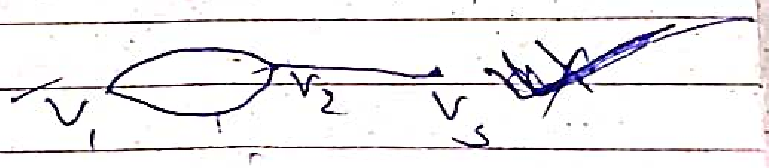
② $G = \{v_1, v_2, v_3\} \{(v_1 v_2) (V_1, v_2) (v_3, v_3)\}$

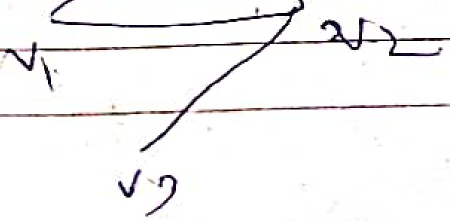
①



②





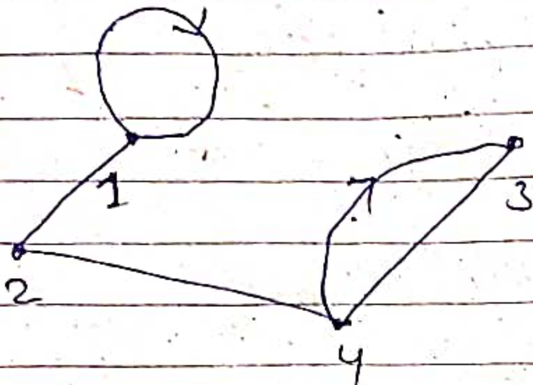


(5)

Mixed graph ⇒

A graph G equal to (V, E) is set to be a mixed graph if some of edge are directed & some of edges are undirected.

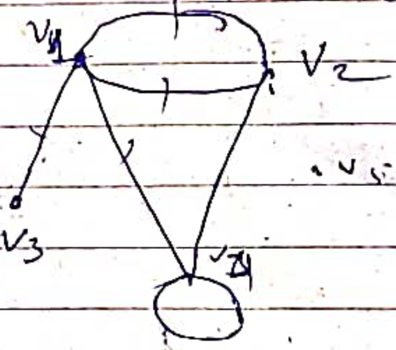
(Eg) $G = \{1,2,3,4\}$, $\{(1,1)\ (3,4)\ (1,2)\ (4,3)\}$ is a mixed gr.



Degree of vertex

⇒ let G be a graph & v be a vertex of G. The degree of v written deg(v) or d(v) is the no of edge.

(1)



$V_1 = 4$

$V_2\ 3$

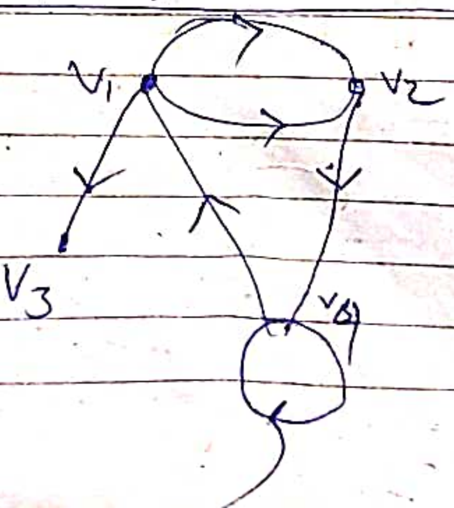
$V_3 = 1$

$V_5 = 0$

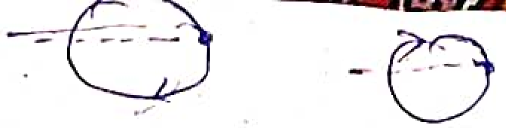
$V_4 = 3$

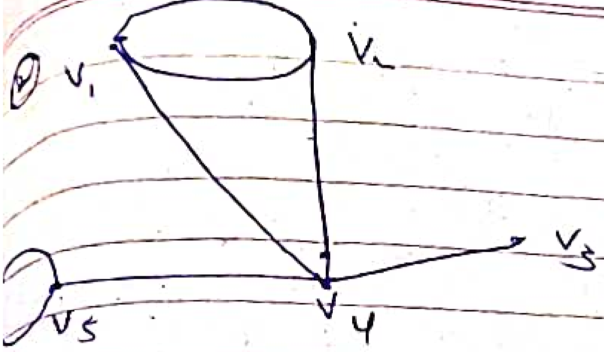
degree of vertex in Digraph (outgoing) (incoming)

(2)

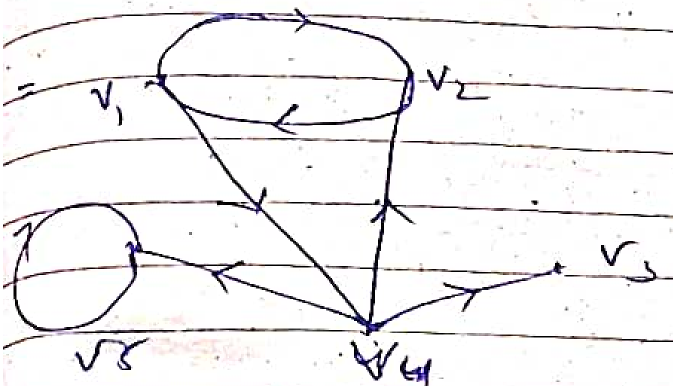


outgoing	incoming	Total
$V_1^+ = 3$	$V_1^- = 1$	$V_1 = 4$
$V_2^+ = 1$	$V_2^- = 2$	$V_2 = 3$
$V_3^+ = 0$	$V_3^- = 1$	$V_3 = 1$
$V_4^+ =$	$V_4^- =$	
$V_5^+ =$	$V_5^- =$	





$v_1 = 3$
$v_2 = 3$
$v_4 = 4$
$v_3 = 1$
$v_5 = 2$



$v_1^+ = 2$	$v_1^- = 1$
$v_2^+ = 1$	$v_2^- = 2$
$v_3^+ = 0$	$v_3^- = 1$
$v_4^+ = 3$	$v_4^- = 1$
$v_5^+ = 0$	$v_5^- = 2$

Note

Each loop on vertex contributes 2 to its degree.

- A vertex of degree one is known as pendant vertex.

degree of vertex in Digraph

The edges in a digraph start from vertex & terminate at other vertex.

- The indegree of a vertex v denoted by $\deg^-(v)$ is the no of edge terminating at v.
- The outdegree of a vertex v denoted by $\deg^+(v)$ is no of edge starting from v

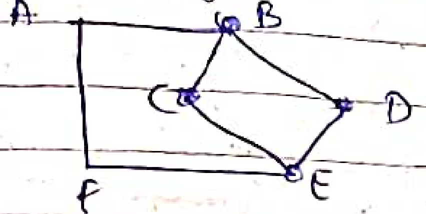
Note ⇒

$$\sum_{u \in V} \deg^-(u) + \sum_{u \in V} \deg^+(u) = 2m$$

Note → simple graph has atleast two vertices of same degree & A simple graph cannot have degree of all vertices distinct.

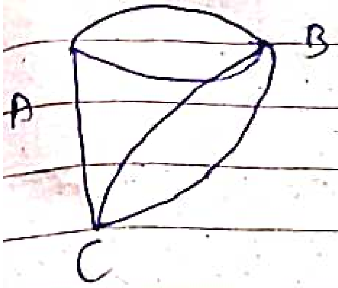
Simple graph ⇒ In simple graph. atleast one no. of should be repeated in the degree sequence.

A simple graph is not contain a any self/ loop parallel edges

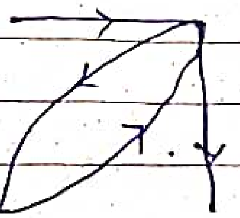


Multigraph ⇒

Any graph which contain multiple edges or a ~~loop~~ is called multigraph

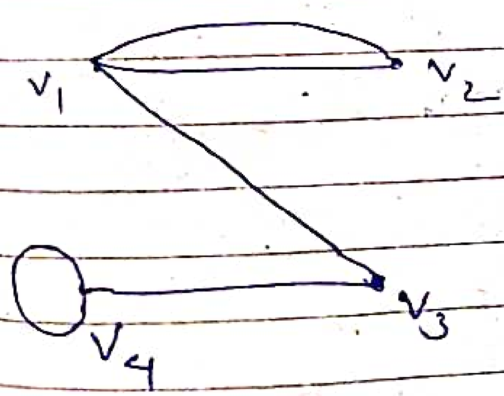


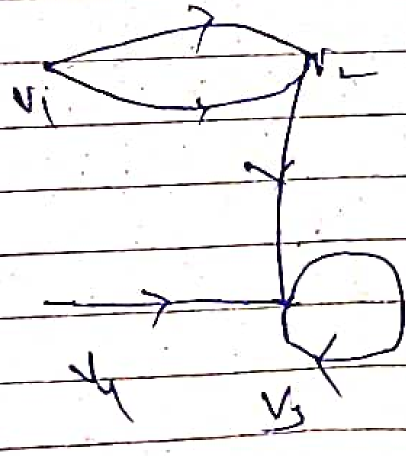
Undirected multi graph



Directed multi graph

Pseudograph ⇒ A graph in which include has loop & multiple edge both are allowed.





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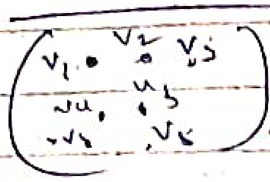
Type of (simple) graph ⇒

1. ⇒ Null graph



⇒ Regular graph

⇒ Cyclic graph

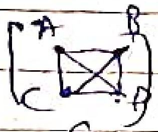
Wheel graph

Bipartite graph

Complete Bipartite graph

Null graph ⇒ A graph which contain isolated node only is called null graph. A graph which contain zero edges is called Null graph Ex $\cdot v_1 \cdot v_2$

Regular graph



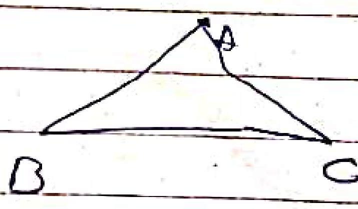
A=3
B=3
C=3
D=3

A graph G which contain all vertex are of equal degree

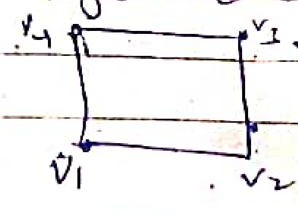
⇒ $\boxed{q = \frac{p \times r}{2}}$ p = vertices, q = edge, r = regular

NOTE ⇒ Every null graph is a regular graph of degree zero.

Cyclic graph ⇒ $(n \geq 3)$ consist of n vertices & n edges.



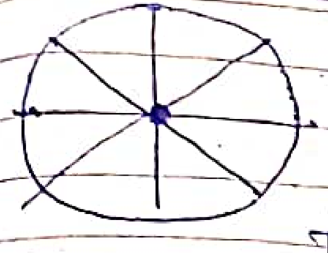
A cyclic graph $(n \geq 3)$ consist of n vertex v_1, v_2, v_3 ----- v_n & connected a cyclic order is called a cyclic graph



Every cyclic C_n is two regular graph with n vertices

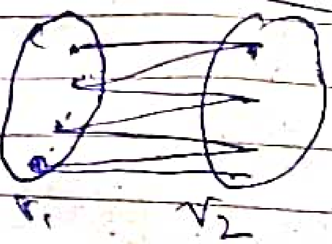
(There are 2n edge in wheel)

Wheels graph ⇒ It is denoted by (W_n) of n vertices can obtained by adding additional vertex.



A graph which must have any diagonal is called wheel graph. It is connecting the new vertex to each of the n vertex in C_n by new edge.

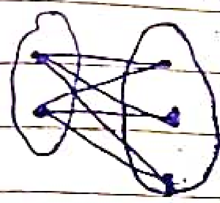
Bipartite graph ⇒ $\boxed{V_1 \cap V_2 = \phi}$



A simple graph G is called Bipartite if vertex set can be parted in two part of subset V_1, V_2.

$V_1 \cup V_2$ is called Bipartite of G

A complete graph - A simple graph G with n vertices
Bipartite
is said to be complete if there is an edge b/w every pair of distinct edge this is denoted by K_n



K_n has exactly $n(n-1)/2$ edge

Operation on graph ⇒

Union (LCM)

(U)

The union of graph $G(V_1 E_1)$ G_2 $E(V_2, E_2)$ is a graph $G(V, E)$ where $V = V_1 \cup V_2$ & $E = E_1 \cup E_2$ & it is denoted by $G_1 \cup G_2$

$G_1 = \{a, b, c, d, e\}$

$G_2 = \{a, b, c, d, f\}$

$G_1 \cup G_2 = \{a, b, c, d, e, f\}$

$G_1 \cap G_2 = \{a, b, c, d\}$

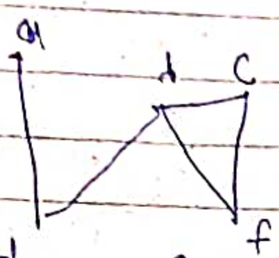
(1) $\{(a, b)\ (a, c)(a, d)\ (b, e)(c, d)\} = G_1$

(2) $\{(a, d)\ (c, d)\ (a, f)\ (b, a)\} = G_2$

$G_1 \cup G_2 = \{(a, d)(c, d)(b, a)(a, c)(b, e)(a, f)\}$

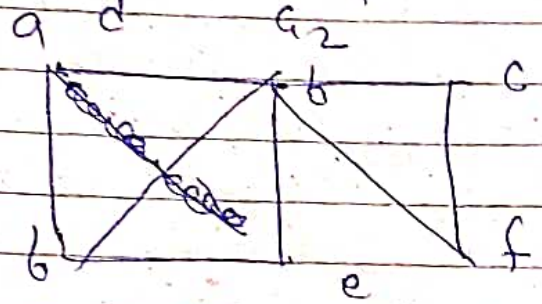
$G_1 \cap G_2 = \{(a, d)\ (c, d)(a, b)\}$

Ex



G_2

$G_1 \cup G_2 \rightarrow$



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Intial & Terminal vertices in directed graph

let $e = (v, w)$ be a directed edge in a graph $G(V, E)$

$$v \xrightarrow{e} w$$

(Intial) (Terminal)

Isolated vertices :

A vertex $v \in V$ is said to be isolated if it is not incident with any edge Ex V_7

Adjacent vertices :

Two vertices v & w of G said to be adjacent if there exist an edge $e(v, w) \in E$ that connect v & w

(V_1, V_2) (V_2, V_5) (V_5, V_4)

Adjacent edge :

Two edges are said to be adjacent if they have common end vertex.

(e_1, e_2) (e_2, e_3)

loop :

If there is an edge e that connect a vertex v to itself i.e $e = (u, v)$ such an edge called a loop at vertex ~~v~~ & ~~w~~ is adjacent to itself Ex e_7

Parallel or multiple edges

If a pair of vertaces is connected by more than one edge than these edges are called parallel or multple edge

Ex- e_2 & e_3 both connect v_2 & v_4 i.e.

$e_2 = e_3 = \{v_2 - v_4\}$

order & size of graph

no. of vertices in a graph is called its order & the no. of edge is called its size.

If order & size of a graph are finite, the graph is said to be finite graph

If no of vertices & no. of edge are became infinite, the graph is said to be infinite graph

IMP

Euler Handshaking theorm.

The sum of degree of all the vertices of a graph is twice the no. of edge i.e If there are vertices $V_1, V_2, \cdots V_k$ & n edge then

$d(v_1) + d(v_2) + \cdots + d(V_k) = 2n$

(4) degree of a vertex v $deg(v) = deg^-(v) + deg^+(v)$

Note ① sum of degree's of all the vertice in a graph is an even integer

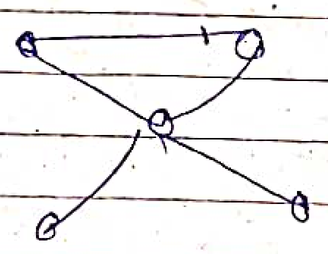
② The no. of vertices of odd degree is an undirected graph is always even

③ A loop contributes 1 to the indegree & to the outdegree.

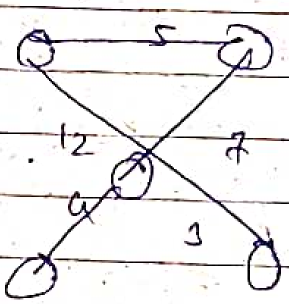
weight graph :

It is a graph in which each branch is given a numerical weight. A weighted graph is therefore a special type of labeled graph in which the labels are number

①



undirected & unweighted



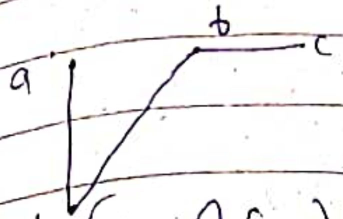
undirected + weighted.

Isomorphic graph

When Two graph which contain the same no of graph vertices connected in the same way are said to be isomorphic.

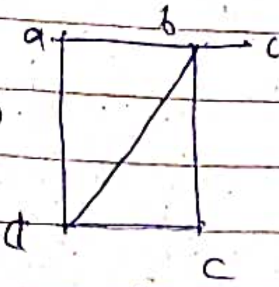
Intersection → The intersection of two graph $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ is a graph $G = (V, E)$

$V = V_1 \cap V_2$ & $E = E_1 \cap E_2$ & it denoted $(G_1 \cap G_2)$



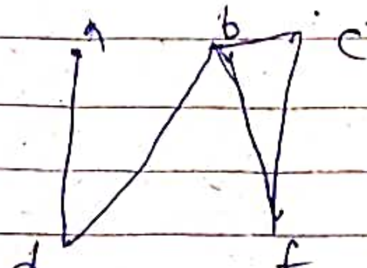
$(G_1 \cap G_2)$

If (1)



G_1

(2)

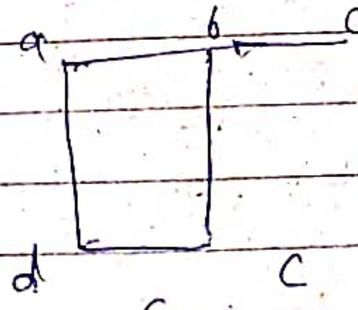


G_2

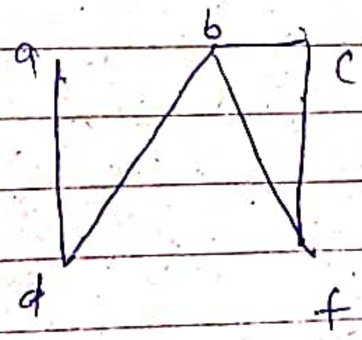
Ring-sum → The Ring sum of two graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ is a graph $G(V, E)$

$V = V_1 \cup V_2$ & E is the set of edge either in G_1 or G_2 but not in both the ring-sum of two graphs G_1 & G_2 is $G_2 \oplus G_2$

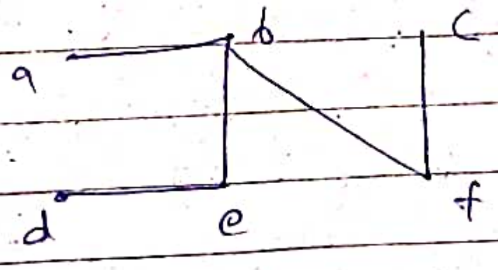
(1)



G_1



$[G_1 \oplus G_2]$



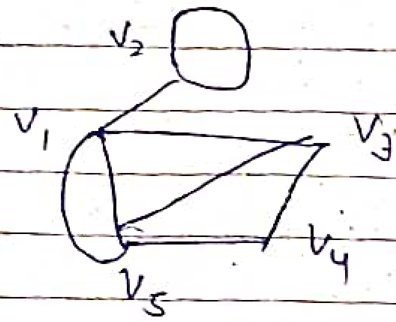
Matrix representation of a graph.

IMP

① Adjacency matrix.

let $G(V, E)$ be a graph with n vertices where $V = \{v_1, v_2, \ldots v_n\}$ the adjacency matrix A_G is an $n \times n$ matrix such that (i, j) the entry a_{ij} of A_G is the number of edge from v_i to v_j

Ex a_{ij} = The number of edge from v_i to v_j



$$A_G \quad \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1 & 1 & 0 & 2 \\ v_2 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 1 & 0 & 0 & 1 & 1 \\ v_4 & 0 & 0 & 1 & 0 & 1 \\ v_5 & 2 & 0 & 1 & 1 & 0 \end{array}$$

Adjacency matrix A_G passes/has following properties:-

① If A_G doesnot contain any loop & all the diagonal element of A_G are 0

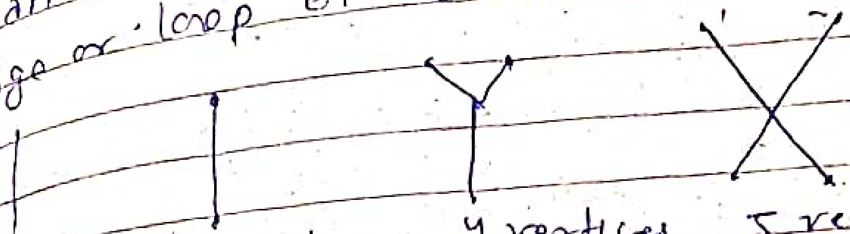
② If G doesnot contain parallel edge the each element of A_G is either 0 or 1

③ A_G is symmetric matrix if the graph is undirected since an undirected edge (a, b) (connect a to b & b to a) can be considered as two edge (a, b) & (b, a)

④ A_G of a simple graph consist of only zero & 1's & all diagonal element are zero.

Trees :-

A Tree is a connected graph without any circuits or we can say that it is a connected acyclic graph. It A. tree is a tree cannot involve multiple mean that a tree or cannot have simple circuits. edge or loop.

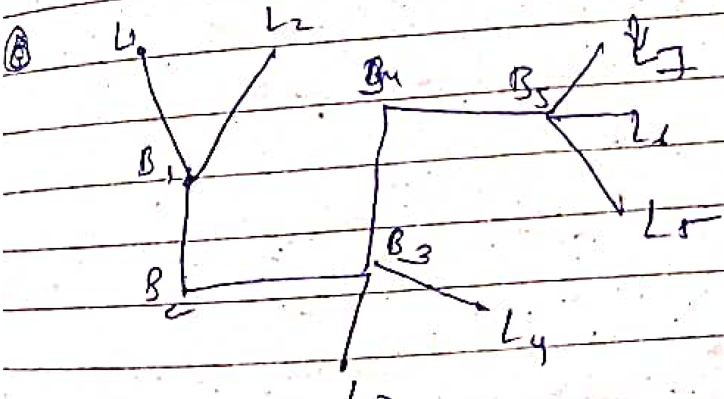


2 vertices. 3 vertic. 4 vertices 5 vertices

Terminologies :-

① leaf :- In a tree a vertex of degree 1 is called leaf. The leaf is also known as Terminal node & pandant vertex.

$G_1 = L_1, L_2, L_3, L_4, L_5, L_6, L_7$



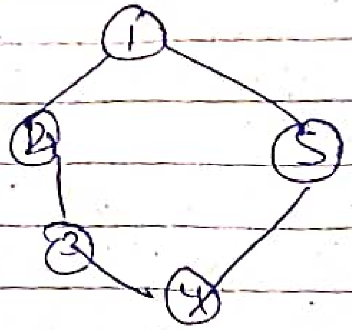
② Branch node :- In a tree, a vertex of degree > 1 is said to be branch node or Internal node

eg. B_1, B_2, B_3, B_4, B_5.

IMP

Spanning tree

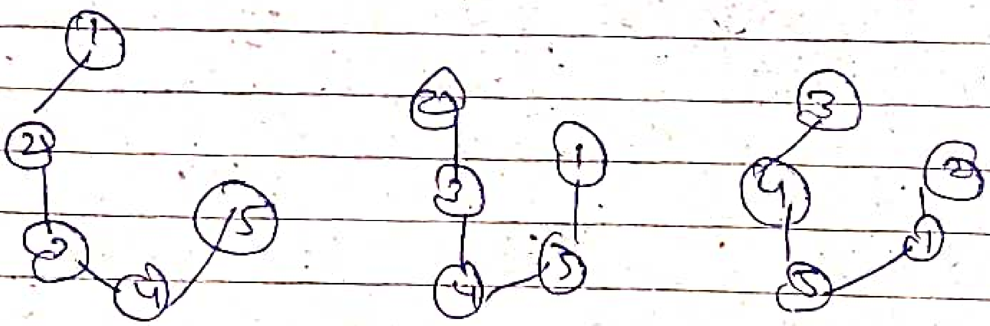
A sub graph (T) of a connected graph G is said to be a spanning tree if T is a tree and T include all the vertices of G



G (V, E)

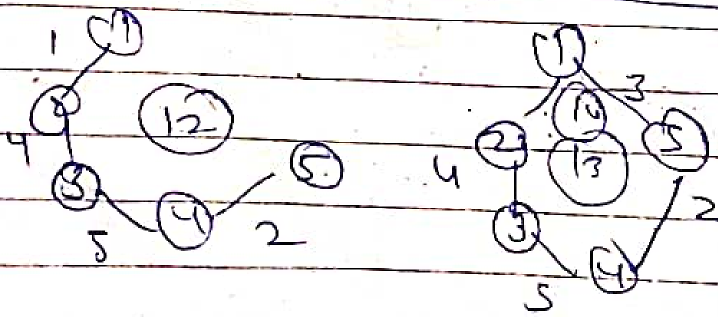
The spanning tree of this graph is denoted by G' (V', E') where V' = V & E' ⊂ E

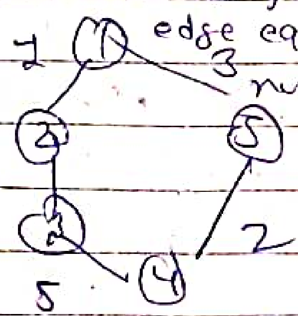
$$\boxed{E' = |V| - 1}$$



$$\boxed{\text{Possible spanning tree} = n^{n-2}}$$ & 4

Minimum spanning tree ⇒ It is subset of graph with same no. of vertices as graph & edge equal to the number of vertices - 1





G (V, E)

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Edge have weights assigned in the graph.

⇒ out of all the spanning tree constructed from the graph. a graph having min sum of weight is called min ST.

⇒ If each edge have distinct weight. then there will only unique MST of that graph

Properties of spanning tree

(1) ST should not contain a cycle

(2) A ST should not be disconnected (Removing one edge from the ST will make its disconnected)

(3) Adding 1 edge to the ST may create a loop.

(4) A complete undirected graph can have n^{n-2} ST

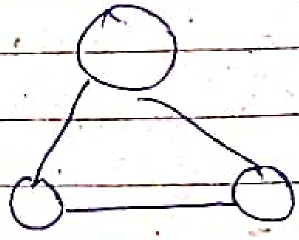
(5) every connected & undirected graph has atleast one ST

(6) A disconnected graph doesnot have any spanning tree

(7) From a complete graph, by removing max$(e-n+1)$ edge we can construct a ST.

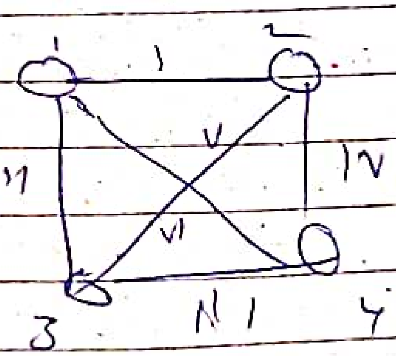
&

(1)



max$(3-3+1) = 1$

(2)



max $= (6-4+1) = $ max(3)

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Rooted tree

It is a directed tree has extectly one mode or vertex called root whose incoming degree is 0 & all other vertices have incoming degree one then the tree is called rooted tree.

IMP
~~Prism~~ algo.

It is a greedy algo that is used to find minimum spanning tree from a graph. It finds the subset of edge that includes every vertex of the graph such that sum of weight of edge can be minimized.

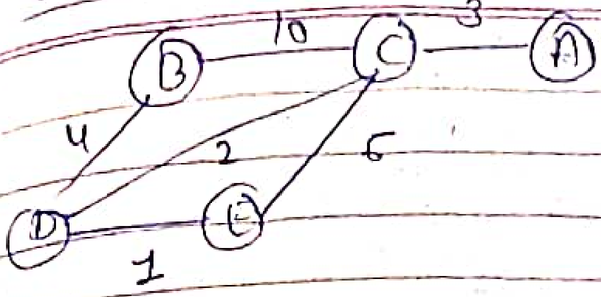
It start with single node & explore all the adjacent node with all connecting edge at every step. The edges with the terminal weight causing no cycle in graph got selected

We have to intialize an MST with randomly chosen vertex

(2) We have to find all edge that connect the tree in above step with the new vertices. from the edge found, select minimum edge and add it to the tree

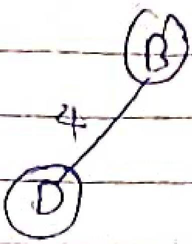
(3) Repeat step 2 until minimum spanning tree is formed.

weight graph

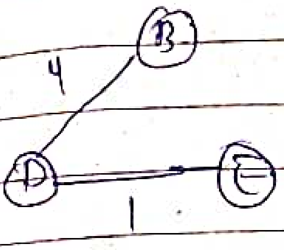


choose B

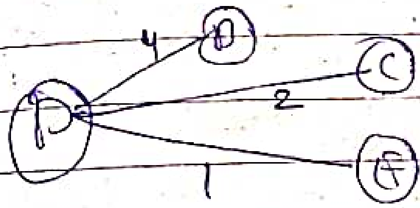
2) shortest edge from B



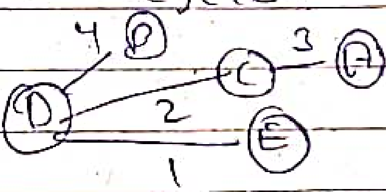
3) choose edge the minimum weight among all other edge



4)



5) We cannot select the edge CE, it would create cycle.



Time complexity

⇒ Adjacent matrix, linear searching $O(|V|^2)$

In generally $O(E \log V)$.

E = Edge
V = vertices

Kruskal algo.

It is the concept that is introduced in the graph theory of discrete mathematic.
It is used to discover the shortest path between two points in a connected weight graph.

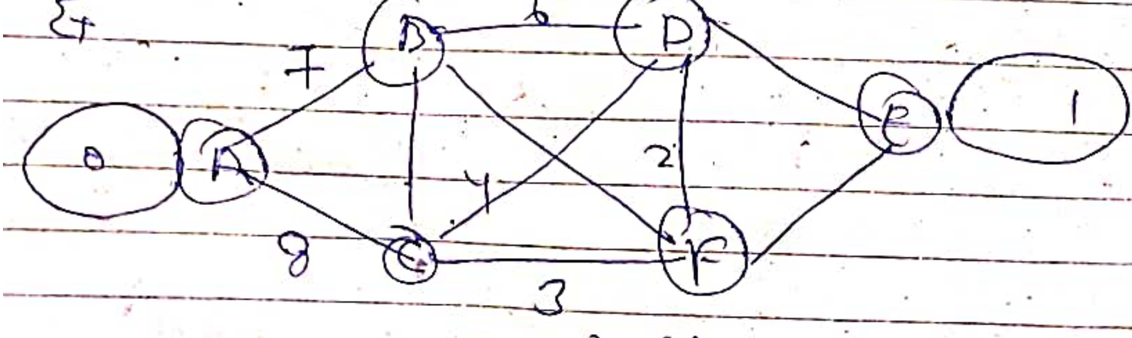
This algo convert a given graph into forest considered each node as a seperate tree.

It is used to generate a minimum spanning tree for a given graph.

steps ⇒ Create MST using Krushal ⇒

(1) Sort all edge in increasing order of their edge weights.
(2) Pick the smallest edge.
(3) check if the new edge creats a cycle or loop.
(4) If the doesnt from cycle, then include that edge in MST otherwise, discard it.
(5) Repeat from step 2 until it includes |V| - 1 edge in MST.

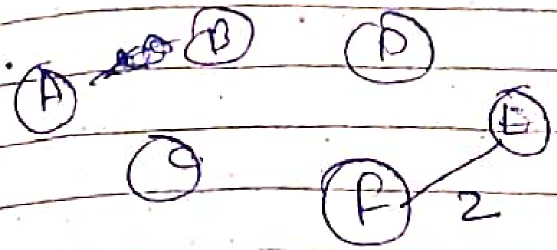
Ex.



Graph G(V, E)

Removing parallel edge or loops from graph

1



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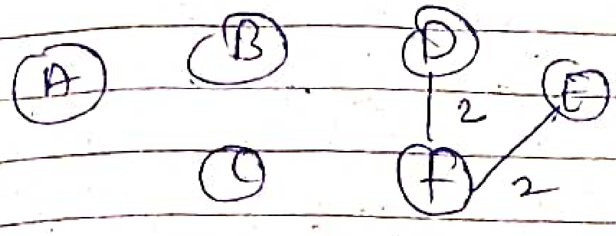

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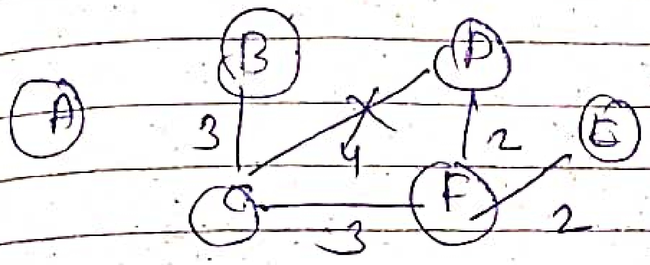

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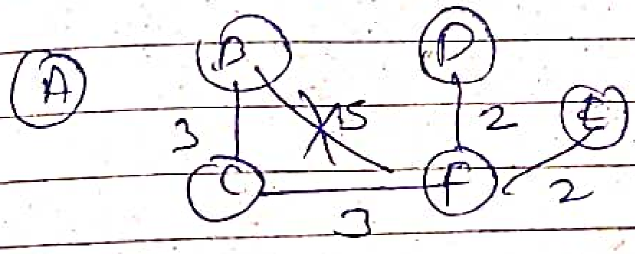

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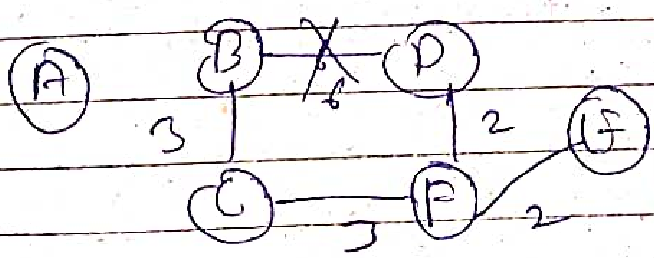

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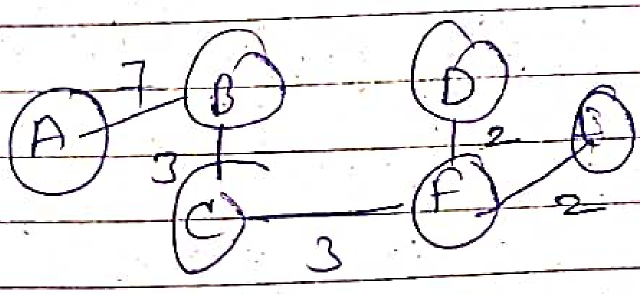
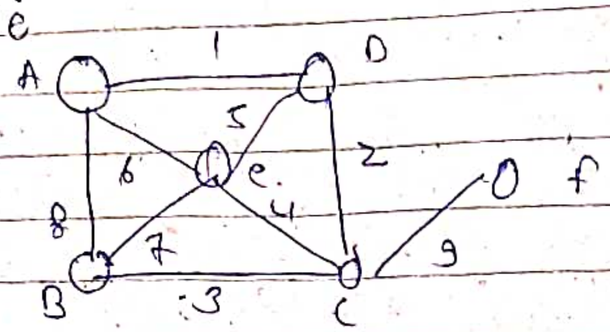


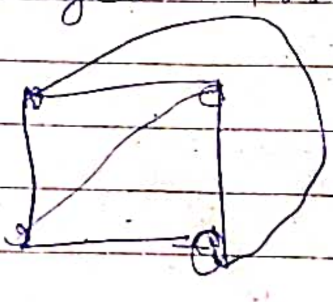
Hamiltonian graph.

A connected graph G is called Hamiltonian graph if there is a cycle which include every vertex of G & cycle is called Hamiltonian cycle



Euler graph: An Eulerian circuit traverses every edge in a graph exactly once but many repeat vertices, while a Hamiltonian circuit visits each vertex in graph exactly one but many repeat edges

Planar graph- It is a graph that can be embedded in the plane. It can be drawn on the plane in such a way that it edges intersect only at their endpoint.



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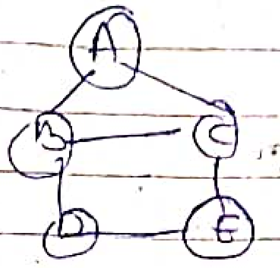
Walk :- It can be defined as a sequence of edge & vertice of a graph.
When we have a graph & traverse it then that traverse will be known as a walk.
The no of edges which is covered in walk will be known as length of the walk.



1) Edge can be repeated
2) vertex can be repeated

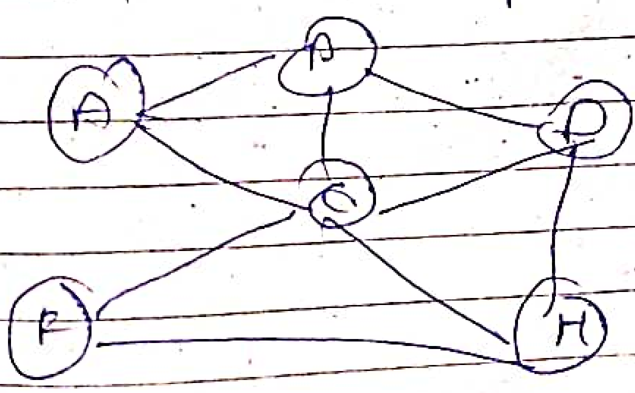
Ex) A B C E D (length = 4)
D B A C E D C (length = 7)

1) open walk ⇒ the starting vertex & ending vertex must be different. In open walk, the length of the walk must be more than 0.

2) closed walk :- starting vertex & ending vertex must be identical & length must be more than 0.

Circuit :- It can be described as closed walk when no edge is allowed to repeat. vertex can be repeated.
1) Edge cannot be repeated
2) vertex can be repeated

Eg



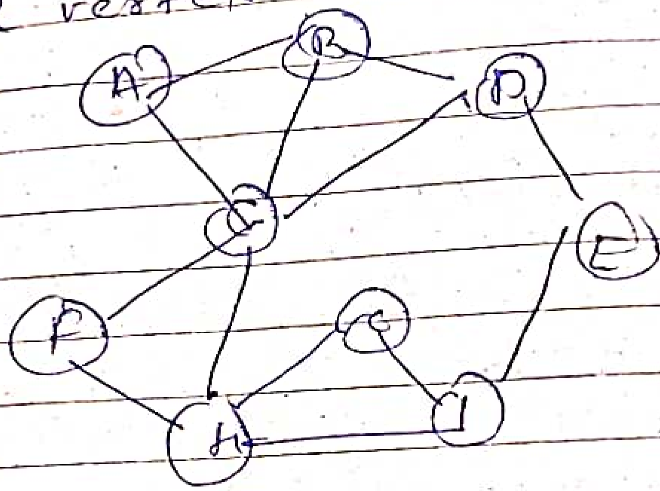
Check A, B, D, C, F, H, C, A

Cycle :- A closed path in the graph theory is also known as cycle.

It is a type of closed walk where neither edge nor vertice are allowed to repeat

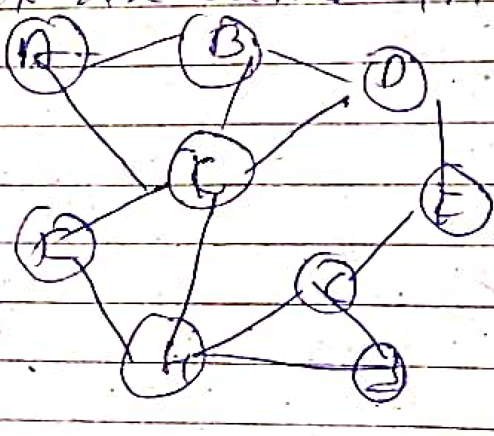
edge & vertex cannot repeated

(1) Edge & vertex



Cycle : A, B, D, C, A

Path :- It is a type of open walk where edge & vertices is not allowed to repeat. There a possibility that only the starting vertex & end vertex are same in a path.



Path F, H, C, A, B, D

Unit : 2

★ Set Theory, Relations, functions.

→ A Set is Collⁿ of well defined objects / things / elements / members.

Ex:- $A = \{a, e, i, o, u\}$.

→ Set is represented by Capital letters and elements and objects is represented by Small letter.

Note:- Order is not important in Set.

Repetation is not allowed.

★ Note:-

$$\boxed{\phi \subseteq A \subseteq U}$$

$\phi \subseteq$

Types of Set :-

1. Empty Set (ϕ) or { }.
2. Universal Set (U).
3. finite Set.
4. Infinite Set.
5. Singleton Set.

(1) Empty Set :-

A Set having no. element.

It is denoted by ϕ or { }

(2) Universal Set :- Set of all elements in the Set is k/as Universal.

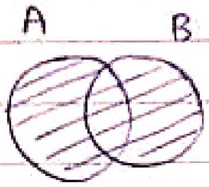
(3) finite Set :- A Set which has finite no. of elements.

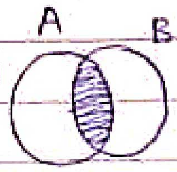
(4) Infinite Set :- A Set which has infinite no. of elements

(5) Singleton Set :- Set which has only one element.

Operation of Set :-

① Union ($\cup$)
② Intersection ($\cap$)
③ Difference ($-$).

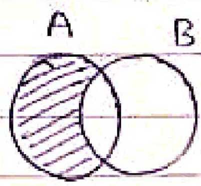
① 

② 

③ Set difference :-

$A = \{1, 2, 3\}$, $B = \{2, 3\}$.

$A - B = \{1\}$



B se Jii Ke Bal L

Subset :-

$A = \{a, e, i\}$ $\longrightarrow$ $A \subseteq B$ here.

$B = \{a, e, i, o, u\}$

A Set is Called Subset of Set B, if $\forall\, x \in A$ implies $x \in B$.

It is expressed $A \subseteq B$.

Ex:-

$A = \{\underset{e1}{x}, \underset{e2}{\{x\}}, \underset{e3}{\{x, \{x\}\}}\}$

$x \in A$

$\{x\} \in A$

$\{x, \{x\}\} \in A$

- element $\in$ Set
- Set 1 $\subseteq$ Set 2

Power Set :- The family of all Subsets of a Set A is called the Power Set of A.

Ex:- $A = \{a, b\}$ 2^A.

$P(A) = \{\{\}, \{a\}, \{b\}, \{a,b\}\}$

Cardinality :- Number of elements in the set is K/as Cardinality of Set.

$A = \{1, 2, 3\}$

$|A| = 3$.

Ex:- $A = \{1,2,3\}$

$|A| = 3$

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

14|12|22

~~$A = \{1, 3, 5, 6\}$~~

~~$P(A)$~~

$A = \{x \mid x$ is odd, +ve integer, $x < 10\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{x \mid x$ is an even integer, $x > 0\}$

(It means B is the set of x such that x is greater than 0. denote the set B whose elements are +ve integers.

note that letter usually x is used to denote ~~typical~~ typical member of the set & the vertical line (|) read as "such that and the comma" an

* Continue of Subset.....

Two sets are equal if they have Both same elements or equivalently if each is contained in the other that is –

$$A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A$$

$A = \{1, 3, 4, 7, 8, 9\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 3\}$

1. Is $C \subseteq A$? Yes
2. Is $C \subseteq B$? Yes
3. If $B \subseteq A$? No.
4. If $A \subseteq B$? No.

Note :-

* Theorem :-

Let A, B, C be any sets then

1. $A \subseteq A$
2. If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
3. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

N :- Set of natural no.

Z :- The set of all integers.

Q :- The set of all Rational no.'s

R :→ Real no.'s

C :→ Complex no.'s.

* Disjoint Sets

→ Two set are ~~~~ said to be Disjoint if they have no elements in Common.

$A = \{1, 2\}$
$B = \{4, 5, 6\}$
$C = \{5, 6, 7, 8\}$

→ If A and B are disjoint then neither is subset of other {unless one is the empty set}.

Disjoint Union :-

If $S = A \cup B$ and $A \cap B = \phi$ then S is called the disjoint Union.

Property ① every element x in $A \cap B \in$ both A and B. hence, $x \in A$ and $x \in B$.

thus, $A \cap B$ is a subset of A and of B.

$$A \cap B \subseteq A \text{ and } A \cap B \subseteq B.$$

② An element x belongs to the Union $A \cup B$ if $x \in A$ or $x \in B$. hence, ~~et~~ every element in belongs to $A \cup B$ and every element in B belongs to $A \cup B$.

$$A \subseteq A \cup B \text{ and } B \subseteq A \cup B.$$

$$\boxed{\begin{array}{l} A\cap B \subseteq A \subseteq A\cup B \\ A\cap B \subseteq B \subseteq A\cup B \end{array}}$$

Operations :-

① Compliment - The absolute Compliment / Compliment of Set A denoted by A^c or $\bar{A}$ or A', is the set of elements which belongs to U but which not belong to A. i.e.

$$A' = \{x \mid x\in U, x\notin A\}$$

② Difference - $(A-B\ ,\ A\backslash B,\ A\sim B)$

③ Symmetric Difference - Symmetric difference of Set A and B denoted by $A\oplus B$, Consist of those elements which belong to A or B, but not Both.

$$A\oplus B = (A\cup B)/(A\cap B) = (A\cup B)-(A\cap B).$$

OR

$$A\oplus B = (A-B)\cup(B-A)$$

$(A\cup B)$ -

Q:- Suppose,

$U = N = \{1,2,3, ----\}$

$A = \{1,2,3,4,5\}$, $B = \{3,4,5,6,7\}$, $E = \{2,4,6..\}$

$C = \{2,3,8,9\}$

find ① A^c or A'.

② B'

③ E'

④ $A-B$

⑤ $A-C$

⑥ $B-C$

⑦ $A-E$

⑧ $B-A$

⑨ $C-A$

⑩ $C-B$

⑪ $E-A$

⑫ $A\oplus B$

⑬ $B\oplus C$

⑭ $A\oplus C$

⑮ $A\oplus E$

1) $A' = \{5, 6, 7, \ldots\}$
2) $B' = \{1, 2, 8, 9, \ldots\}$
3) $E' = \{1, 3, 5, 7, \ldots\}$

4) $A - B = \{1, 2\}$
5) $A - C = \{1, 4\}$
6) $B - C = \{4, 5, 6, 7\}$
7) $A - E = \{1, 3\}$
8) $B - A = \{5, 6, 7\}$
9) $C - A = \{8, 9\}$
10) $C - B = \{2, 8, 9\}$
11) $E - A = \cancel{\{2, 4, 6, 8, 10, \ldots\}}$ $\{6\}$
12) $A \oplus B = \{1, 2, 5, 6, 7\}$
13) $B \oplus C = \{2, 8, 9, 4, 5, 6, 7\}$
14) $A \oplus C = \{1, 4, 8, 9\}$
15) $A \oplus E = \cancel{\{1, 3, 6, 8, \ldots\}}$ $\{1, 6\}$

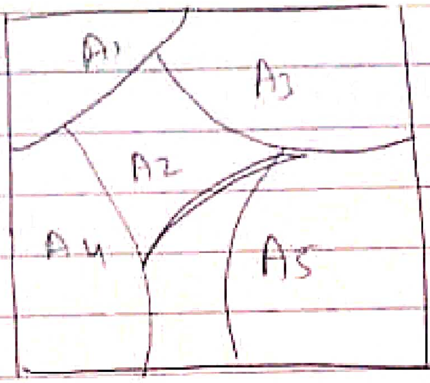
Partitions

1) Let S be a not empty set.
The Partition of S is a subdivision of S in non-overlapping, non empty subset.

) Precisely a partition of S is colln $\{A_i\}$ of non empty set of S such that:
~~each~~ each ~~a~~ a in S $\in$ one of the A_i.

). The sets of $\{A_i\}$ are mutually disjoint i.e. if $A_j \neq A_k$ then $A_j \cap A_k = \phi$.

(5) The Subsets in the Partition is Called Cells.



Consider the following Coll^n of Subset of
$S = \{1, 2, ---, 8, 9\}$

(i) $[\{1,3,5\}, \{2,6\}, \{4,8,9\}]$ ×

(ii) $[\{1,3,5\}, \{2,4,6,8\}, \{5,7,9\}]$ ×

(iii) $[\{1,3,5\}, \{2,4,6,8\}, \{7,9\}]$ Partition ✓

19|12|22

★ Algebra of Sets :-

(1) Idempotent law (1a) $A \cup A = A$ ~~2b~~ (1b) $A \cap A = A$

(2) Associative law (2a) $(A \cup B) \cup C = A \cup (B \cup C)$ (2b) $(A \cap B) \cap C = A \cap (B \cap C)$

(3) Commutative law (3a) $A \cup B = B \cup A$ (3b) $A \cap B = B \cap A$

(4) Distributive law (4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(4b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(5) Identity law

(5a) $A \cup \phi = A$ (5b) $A \cap U = A$

~~(6) Involution law~~

(6a) $A \cup U = U$ (6b) $A \cap \phi = \phi$

~~(7) Complement law~~

~~(7) $(A^c)^c = A$~~

⑥ Involution law

$$(A^c)^c = A$$

⑦ Complement law

ⓐ $A \cup A^c = U$ ⓑ $A \cap A^c = \phi$

ⓒ $U^c = \phi$ ⓓ $\phi^c = U$

⑧ De - Morgan's law

ⓐ $(A \cup B)^c = A^c \cap B^c$

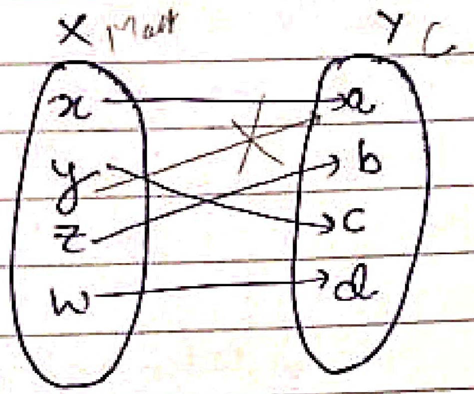
ⓑ $(A \cap B)^c = A^c \cup B^c$.

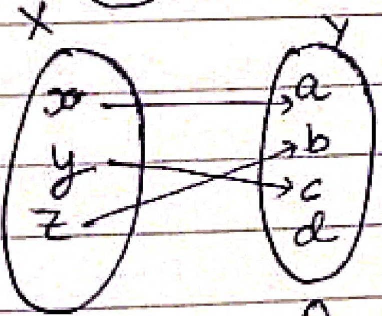
Functions :-

Let X and Y are two sets.
A rule in which every element of x is assigned to unique element of Y and then this rule is called funⁿ.

It is written as

$$f : X \longrightarrow Y$$





Domain Co - Domain

$\{a, b, c, d\}$ = Co - Domain.

$\{a, b, d\}$ = f (range)

* Domain & Co-Domain

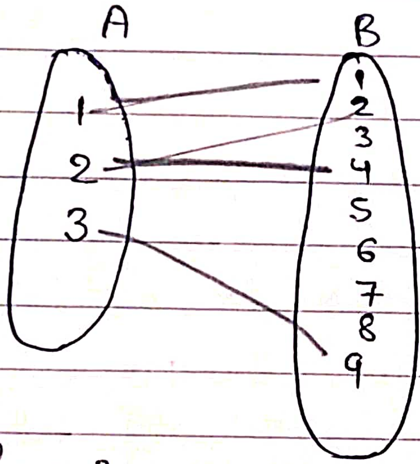
If ~~F~~ f is a funⁿ from a to b then a is Called domain and b is Called Co-domain.

* Range :-

The set of all those element of b which are related with elements of a is Called the range f.

$$\boxed{\text{range } f \subseteq B}$$

Ex:-



$f = x^2$

Domain $= \{1, 2, 3\}$

Range $= \{1, 4, 9\}$

Co-Domain $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

* Types of Functions :-

① One - One :- Let $f : A \rightarrow B$ be a funⁿ and it is Called one-one. if

Or

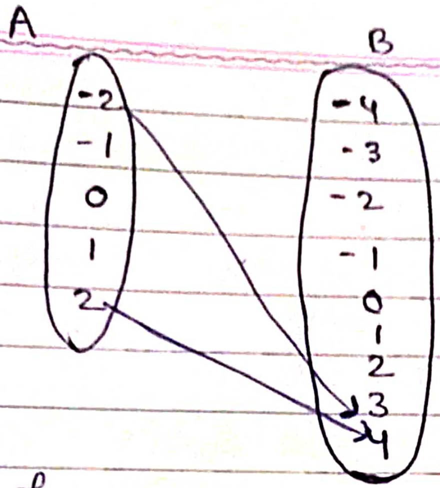
Injective

$x \neq y \Rightarrow f(x) \neq f(y)$ for $x, y \in$

Or

if $f(x) = f(y) \Rightarrow x = y$

Ex :-



$f = x^2$

$= \quad -2 \neq 2$

$f(-2) = f(2)$

Ex :- $f(x) = x+1$ $\Rightarrow$ one - one.

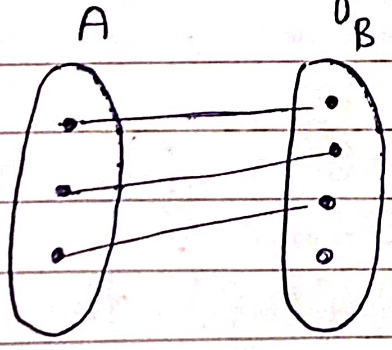
$y = x+1$

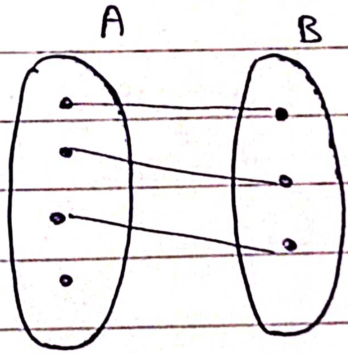
allowed ! | Not allowed !

① $a \longrightarrow f(a)$ $a \longrightarrow f$

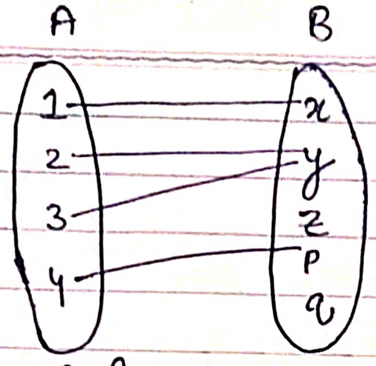
$b \longrightarrow f(b)$ $b \longrightarrow f$

②

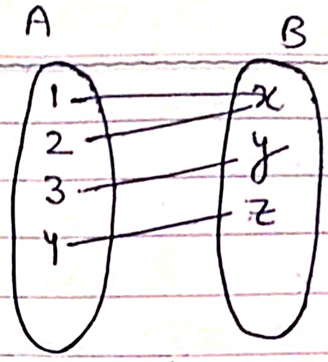




② Onto / Surjective :- A $f : A \to B$ is called onto if range $f = B$ (Co- Domain)



Onto ✗



Onto ✓

16/1/23

Bijective / 1:1 Correspondence funⁿ :-
Let $f : A \rightarrow B$ be a funⁿ & it is called bijective if f is one-one & onto.

★ Some special functions :-

→ Let x be any real funⁿ then x lies b/w two integers called the floor & Ceiling of x.

① Floor functions :-
→ This funⁿ gives the largest nearest integer ~~value~~ of the specified value which are less than the value of x.
→ It is denoted by $\lfloor x \rfloor$. Called the floor of x.
→ It denotes the greatest integer that does not exceed x. i.e.

$$\lfloor x \rfloor = \{\max\{a \in \mathbb{Z},\ a \leq x\}\}.$$

Ex :- $\lfloor 3.14 \rfloor = 3$

$\lfloor \sqrt{5} \rfloor = 2$

$\lfloor -8.5 \rfloor = -9$

$\lfloor 7 \rfloor = 7$

② Ceiling function :-

→ In this ^the^ Smallest Successive integer is written i.e. the Ceiling fun~ of real no. of x is the least integer that is greater than or equal to the given number x. ~~that is~~.

→ It is denoted by $\lceil x \rceil$.

i.e. $\lceil x \rceil = \min \{a \in Z, a \geq x\}$

Ex:- $\lceil 3.14 \rceil = 4$ $\qquad$ $\lceil -8.5 \rceil = -8$

$\lceil \sqrt{5} \rceil = 3$ $\qquad$ $\lceil 7 \rceil = 7$

*Note :-

→ If x is itself an integer then.
$\lfloor x \rfloor = \lceil x \rceil$, otherwise $\lfloor x \rfloor + 1 = \lceil x \rceil$

③ Integer fun~ :-
and absolute Value fun~

→ Let x be any real no., the integer Value of x written as Int (x) Converts x into a integer by deleting the fractional Part of the no..

Int $(3.14) = 3$ $\qquad$ Int $(7) = 7$

Int $(\sqrt{5}) = 2$

Int $(-8.5) = -8$

absolute Value fun~ :- The absolute Value of real no. x is written as ABS(x) or $|x|$ is defined the greater of x or $-x$.

Hence, ABS$(0) = 0$ & for $x \neq 0$, ABS$(x) = x$ or ABS$(x) = -x$ depending on whether the x is Positive or negative.

ex:- $|-15| = 15$

$|7| = 7$

$|-3.33| = 3.33$

→ $|x| = |-x|$ and for $x \neq 0$, x is ⊕ve.

④ Remainder funⁿ :-

→ Let K be any integer & let M be a +ve integer then K (mod M) will denote the integer remainder when K is divided by M.

→ K (mod M) is the unique int r such that $K = Mq + r$, where $0 \le r < M$.

→ when K is +ve simply divided K by M to obtain remainder r.

Ex:- 25 (mod 7) = 4

25 (mod 5) = 0

35 (mod 11) = 2

→ When K is −ve, divided |K| by M to obtain a remainder r' then K (mod M) = M − r' when $r' \neq 0$

ex:- −26 (mod 7) = 2

Exponential funⁿ :- For integer exponents (where m is a +ve integer) the following definition says :-

$a^m = (a \cdot a \cdot a \cdots\cdots (m \text{ times}))$

$a^0 = 1$

$a^{-m} = \frac{1}{a^m}$

→ exponents are extended to include all rational no.'s by defining all rational no. $\frac{m}{n}$ i.e.

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m.$$

Ex:- $2^4 = 16$

$2^{-4} = \frac{1}{16}$

$125^{2/3} = 5^{3(2/3)} = 25$

⑥ logarithmic funⁿ:-

→ logarithms are related to exponents as follows:-

1) Let b is be a +ve number, the logarithm of any +ve no. x to the base b written as,

$\log_b x$ represents the exponent to which we must be raised to obtain x. i.e.

$y = \log_b x$ and $b^y = x$ are equivalent to each other.

ex:- $\log_2 8 = \cancel{8}\,3$ $\qquad \log_2 64 = 6.$

Note:-

For any ~~to~~ base b we have $b^0 = 1$ & $b^1 = b$

hence. $\log_b 1 = 0$ & $\log_b b = 1$

→ The logarithm of the (−ve) number & the log (0) ~~is~~ are not defined.

logarithms

Common ($\log_{10}$)

Natural ($\log_e$)

Binary ($\log_2$)

→ Some text
$\ln x$ for $\log_e x$
~~log~~ $\lg x$ or $\log x$ for $\log_2 x$.

→ The term $\log x$ by itself usually means $\log_{10} x$ but it is also used for $\log_e x$ in advanced Mathematical ~~text~~ & for $\log_2 x$ in Computer Science text.

→ ~~The term $\log x$ by itself usually means~~

→ Frequently we will require only the floor or the Ceiling of Binary logarithm, this can be obtained by looking at Power of 2..

$$\lfloor \log_2 100 \rfloor = 6$$

$$\lceil \log_2 1000 \rceil = 10.$$

→ Exponential & logarithmic fun^s are inverses of each other.

17|1|23

★ Composition of fun^n :-

- The Composition of fun^n is an operation where two fun^n's say f and g generates a new fun^n h in such a way that.

$$h(x) = g(f(x)).$$

- It means that here fun^n g is applied to the fun^n of x. Basically, a fun^n is applied to the result of another fun^n.

Ex :- $f(x) = x^2$ & $g(x) = 3x$

$(f \circ g)(x)$ or $f[g(x)]$

$= f(3x)$

$= (3x)^2 = 9x^2.$

(2) $f(x) = 2x+1$ & $g(x) = -x^2$

$(g \circ f)(x)$ for $x = 2$.

$g(f(x)) = -(2x+1)^2$

$= g(2x+1) = -(4x^2+1+4x)$

$= -(16+1+8)$

$= -25$

(3) If there are three fun^n's such as $f(x) = x$, $g(x) = 2x$ & $h(x) = 3x$ find the composition of these fun^n's such as $f(g(h(x)))$ for $x = -1$

$$f(g(h(x))) = f(g(3x)) = f(6x) = 6x = -6$$

④ $f(x) = x^2 - 6x + 2$, $g(x) = -2x$

~~Let x~~ find $f[g(x)]$ at $x = -1$

$\therefore f(-2x) =$

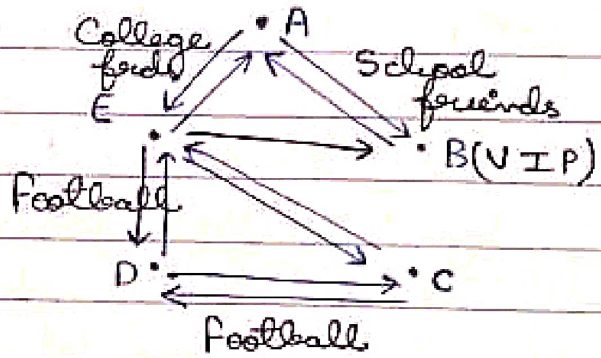
$= 4x^2 + 12x + 2$

$= 4 - 12 + 2 = -6$ Ans

Relations

* Representation of a relation :-

① Graphical representation :-

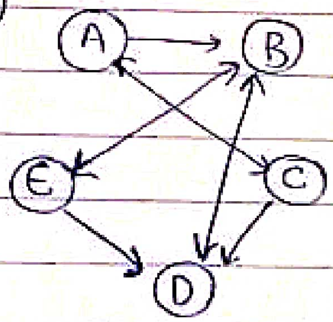


- The Set May not always be People and the relation may not always be knowing or not Knowing each other.

- The element of the Set Can be anything.

② Matrix representation :- assuming the fact that everyone Knows itself.

	A	B	C	D	E
A	1	1	1	0	0
B	0	1	0	1	1
C	1	0	1	1	0
D	0	1	0	1	0
E	0	1	0	1	1



- One way of representing a relation was using arrows. (Graphical representation). another way is Called Matrix representation.

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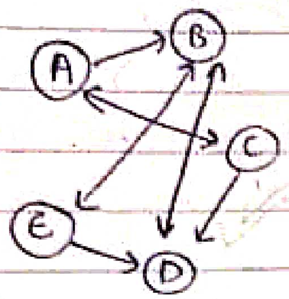
③ Set Representation of a relation :-



$R = \{(A,B)\ (A,C), (A,A), (B,D), (B,E), (B,B), (C,A), (C,D), (C,C), (D,B), (D,D), (E,B), (E,D), (E,E)\}$

- Any relation on S is a Subset of S×S, & ~~vice~~ vice versa.
- Any Subset of S×S is a relation on S.

Ex:- $A = \{1,2,3\}$, $B = \{a,b,c,d\}$

$A \times B = ?$

$= \{(1,a),(1,b),(1,c)(1,d),(2,a),(2,b),(2,c),(2,d),(3,a)(3,b),(3,c),(3,d)\}$

$\Rightarrow \boxed{A \times B \neq B \times A}$ as per $(1,a)$ & $(a,1)$ are two diff^n elements.

Ex:- Let,

$S = \{1,2,3,4,5,6\}$. A relation

R on the Set S is defined as

$R = \{(a,b) \mid a+b \leq 4\}$. What are the elements of R.

$\Rightarrow \{(1,2)(1,3)(2,1)(3,1)(1,1)(2,2)\}$

★ Types of Relation :-

① Reflexive Relation :- If a Relation Contains all Possible (x, x) for all Values of x from A.
(or)
If a every element of Set A maps to itself then Set A is a reflexive relation.

Matrix representation of Reflexive relation
A Matrix representation of a relation that is reflexive will have all the diagonal entries as 1.

~~Condition for relation to be reflexive.~~

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i, i) entry equals to 1, for every i.

② Symmetric relation :- A relation R, on a Set A such that $(a_1, a_2) \in R$ and also if $(a_2, a_1) \in R$ then the relation is Called a Symmetric relation.

eg :- $\{(1,2), (2,1)\}$

Matrix representation of Symmetric relation

$$\begin{array}{c|cc} & a & b \\ \hline a & & 1 \\ b & 1 & \end{array} \qquad \begin{array}{c|cc} & i & j \\ \hline i & & 1/0 \\ j & 1/0 & \end{array}$$

mirror reflection.

i.e. $(i,j)^{th}$ entry & $(j,i)^{th}$ entry will either be 1 or 0.

Q) Let $R = \{(a,b) \mid a,b \in N, b = a^2\}$ Is R reflexive
$(1,1) \in R$
$(2,2) \notin R$ So, not reflexive.

Q) Let $R = \{(x,y) \mid x,y \in I, x^2 = y^2\}$ Is R reflexive
or
Is R symmetric.
~~(-1,1), (-2,2), (-3,-3) .... so,~~
~~(1,1)~~ $\{(1,1),(1,-1),(-2,2),(-1,1),(2,2)\}$

Q) Let $R = \{(a,b) \mid a,b \in N, a \times b = 14\}$ Is R is symmetric?
$R = \{(1,14),(14,1),(7,2),(2,7)\}$

③ Transitive Relation :-
→ A Relation R on a set A. Such that the elements $a_1, a_2, a_3 \in A$. Such that $(a_1,a_2) \in R$, $(a_2,a_3) \in R$ then it implies that $(a_1,a_3) \in R$ as well. Such a relation is called a transitive relation.

① $\{(1,2),(2,3)\}$ ×
② $\{(1,1),(2,3),(3,1),(2,1)\}$ ✓

) Antisymmetric relation :- A Relation R on a set A is antisymmetric if $(a,b) \in R$ then $(b,a) \notin R$ unless $a = b$.

K

Ex:- ① $R = \{(2,1), (2,3), (1,1)\}$ ✓
② $R = \{(2,3), (3,2), (2,2)\}$ ✗
③ $R = \{(1,1), (2,2), (3,3)\}$ ✓

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Q) Consider a relation on the set of integers

$R = \{(a,b) \mid a+b=0\}$ Is R transitive ? Is R Symmetric ?

$R = \{(0,0)\ (1,-1), (2,-2), (-2,2)\ (-1,1) \text{-----}\}$

$\cancel{(1,1)}\ \cancel{(2,2)}$

Q) Consider a relation R

$R = \{(n, n+1 \mid n \in N\}$ Is R antisymmetric ✓

$(1,2)\ (2,1)$

$R = \{(1,2), (2,3), (3,4) \text{---}\}$

Note :-

→ Antisymmetric is not same as not Symmetric.

Ex: $A = \{1,2,3,4,5\}$

$R = \{(1,2), (2,1), (3,4)\}$ Not Symmetric ~~as well as~~ but ~~no~~ antisymmetric.

Matrix representation of antisymmetric

$$= \begin{bmatrix} \searrow & 0 \\ 1 & \end{bmatrix}$$

* Condⁿ for relation to be reflexive in terms of matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \leq \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Identity matrix (I) (M)

$$I \leq M$$

* Condⁿ for relation to be Symmetric.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M = M^{\text{Transpose}}$$

* Condⁿ for relation to be anti Symmetric.

$$M \cap M^{\text{Transpose}} \leq I$$

5) Equivalence relation :- Consider a non-empty set S. a relation R on S is an equivalence relation if

1) R is reflexive.
2) R is transitive.
3) R is Symmetric. i.e.
1) for every $x \in S$, xRx.
3) if xRy then yRx.

(c) If xRy and yRz then xRz.

The general idea behind an equivalence relation is that it is a classification of objects which are in some way "alike".

Operations on relation :-

- Let us consider two Relations R_1 & R_2 from set A to B.
- These relations can be combined in a way, two sets can be combined.

Q) $A = \{1,2,3\}$ and $B = \{3,4,5\}$

$R_1 = \{(1,3), (1,4), (3,3)\}$

$R_2 = \{(2,3), (2,4), (3,3), (2,5)\}$

find out :

(i) $R_1 \cup R_2$ =

(ii) $R_1 \cap R_2$

(iii) $R_1 - R_2$

(iv) $R_2 - R_1$

(v) $R_1 \oplus R_2$

(i) $(1,3), (1,4), (3,3), (2,3), (2,4), (2,5)$

(ii) $(3,3)$

(iii) $(1,3), (1,4)$

(iv) $(2,3), (2,4), (2,5)$

(v) $(R_1 \cup R_2) - (R_1 \cap R_2) = (1,3), (1,4), (2,3), (2,4), (2,5)$.

$A = \{1,2,3\}$

Q) $R = \{(1,1),(2,2),(3,3),(3,4)\}$

$S = \{(1,1),(2,2),(3,3),(2,3)\}$

find out :-

1. $R \cup S$
2. $R \cap S$
3. $R \oplus S$
4. $R - S$

the outcomes are reflexive or irreflexive.

1. $R \cup S = \{(1,1)\ (2,2)\ (3,4)(3,3),(2,3)\}$ = reflexive.
2. $R \cap S = \{(1,1)(2,2)\ (3,3)\}$ = reflexive.
3. $R \oplus S = \{(3,4),(2,3)\}$ = irreflexive.
4. $R - S = \{(3,4)\}$ = irreflexive.

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Closure of Relations :-

1. Reflexive Closure :-

$A = \{1,2,3\}$

$R_{re}^{+} = \{(1,1),(2,2),(2,3)\},\ \underbrace{(3,3)}\}$ → Relative

Let say, we have relation R, Clearly the relation R is not Reflexive.

Reflexive Closure of a relation R on a set A is the smallest reflexive relation of the set A that contains R.

Reflexive Closure of R is usually denoted by R_{re}^{+}.

Formally we can say,

$$R_{re}^{+} = R \cup \{(a,a) \mid a \in A\}.$$

Q:- Let R be the relation on the set.

1) $A = \{0,1,2,3\}$. Containg the ordered Pair $(0,1),(1,1),(1,2),(2,0),(2,2),(3,0)$.

Find the Reflexive Closure of R. ~~(1,0) (2,1) (0,2) (0,3)~~

$$R_{\alpha}^{+} = \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0), (0,0), (3,3)\}$$

(2) Symmetric Closure :-

- Symmetric Closure of Relation R on a Set A is the Smallest Symmetric relation on a Set A, that Contains R.
- It is usually denoted by R_s^{+}.

$$R_s^{+} = R \cup \{(b,a) \mid (a,b) \in R\}.$$

eg (1) :-

$$R_s^{+} = \{(0,1)(1,1)(1,2)(2,0)(2,2)(3,0)(1,0)(2,1)(0,2)(0,3)\}$$

(3) Transitive Closure :-

- Transitive Closure of a Relation R on a Set A is the Smallest Transitive relation on a Set A, that Contains
- It is usually denoted by R_t^{+}.
- Formally we can write,

$$R_t^{+} = R \cup \{(a,c) \mid (a,b) \in R \wedge (b,c) \in R\}$$

eg (1) :- $R = \{(1,1)(2,3), (3,1)\}$

$$R_t^{+} = \{(1,1), (2,3), (3,1), (2,1)\}$$

(3)

ILo
Bm

★ **WARSHALL'S Algorithm :-**

- Warshall's algorithm is Considered as efficient method in finding Transitive Closure of a Relation

eg:- By using WA finding the Transitive Closure of Set A = {1,2,3,4},

$$R = \{(2,1),(2,3),(3,1),(3,4),(4,1),(4,3)\}$$

$$R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & \cancel{0}\,1 \\ 3 & 1 & 0 & \cancel{0}\,1 & 1 \\ 4 & 1 & 0 & 1 & \cancel{0}\,1 \end{array}$$

- The no. of steps required to find the transitive Closure of relation depends on the no. of element in Set A.

Step I => $C_1 = \{2,3,4\}$ $R_1 = \phi$

$C_1 \times R_1 = \phi.$, means no new addition

- In Step 1 we will consider I Column of & I Row of above matrix that is, C_1 & R_1.

write all Positions where 1 is present in Column 1

i.e. $C_1 = \{2,3,4\}$

also, write all Position where 1 is Present in Row 1

i.e. $R_1 = \phi$.

Now Take the Cross Product of C_1 and R_1 i.e.

$C_1 \times R_1 = \phi$.

∴ No new additions to be done.

Step II :- $C_2 = \phi$ $R_2 = \{1, 3\}$

$C_2 \times R_2 = \phi$

$\therefore$ no, new additions.

Step III :- $C_3 = \{2, 4\}$ $R_3 = \{1, 4\}$

$C_3 \times R_3 = \{(2,1)(2,4)(4,1)(4,4)\}$

we now have to Place 1 on these Positions.

Step 4 :- $C_4 = \{2, 3, 4\}$, $R_4 = \{1, 3, 4\}$

$C_4 \times R_4 = \{(2,1)(2,3)(2,4)(3,1)(3,3)(3,4),(4,1),$
$(4,3)(4,4)\}.$

UNIT 4

Q1. Give a brief overview on different types of algebraic structures.

Ans1. There are various types of algebraic structure, described as follows:

Semigroup

Monoid

Group

Abelian group

⇒ SEMI GROUP : Suppose there is an algebraic structure $(G, *)$ which will be known as semigroup if it satisfies the following condition :-

- closure : The operation $*$ is a closed operation on G that means $(a * b)$ belongs to set G for all $a, b \in$
- Associative : The operation $*$ shows an association operation between a, b and c that means $a*(b*c) = (a*b)*c$ for all a, b, c in G.

Examples of semigroup are (Matrix, *) and (set of integers, +)

⇒ MONOID : A monoid is a semigroup, but it contains an extra identity element (E or e). An algebraic structure $(G, *)$ will be known as a monoid if it satisfies the following condition :

- closure : G is closed under operation $*$ that means $(a * b)$ belongs to set G for all $a, b \in$
- Associative : operation $*$ shows an association operation between a, b and c that means $a*(b*c) = (a*b)*c$ for

All a, b, c in G

- Identity Element: There must be an identity in set G that means $a*e = e*a = a$ for all x.

⇒ GROUP: A Group is a monoid, but it contains an extra inverse element which is denoted by 1. An algebraic structure (G, *) will be known as a group if it satisfies the following condition :-

- Closure: G is closed under operation * that means $(a*b)$ belongs to set G for a, b ∈
- Associative: * shows an association operation between a, b and c that means $a*(b*c) = a*(b*c)$ for all a, b, c in G
- Identity element: There must be an identity in set G that means $a*e = e*a = a$ for all a, b, c in G.
- Inverse Element: It contains an inverse element that means $a*a^{-1} = a^{-1}*a = e$ for a e.

⇒ ABELIAN GROUP: An abelian group is a group, but it contains commutative law. An algebraic structure (G, *) will be known as abelian group if it satisfies the above mentioned four conditions of closure, associative, identity element inverse element along with commutative law which says that $a*b = b*a$ such that a, b belongs to G

Q2. What do you mean by groups? What are the certain properties associated with it?

Ans2: A group, G, is a finite or infinite set of components / factors unitedly through a binary operation or group operation that jointly meets the four primary properties of the group, i.e.-

- Closure
- Associativity
- Identity
- Inverse property

⇒ Some certain properties associated with group are as given below:

1. Uniqueness of Identity in a group.
2. The inverse of an element in a group is unique.
3. If an element of G having an inverse as a^{-1} then, the a^{-1} is the inverse of a, similarly a is the inverse of a^{-1} such that: $(a^{-1})^{-1} = a$
4. $$(ab)^{-1} = b^{-1}a^{-1}$$
5. Generalized reversal law: $(abc \dots p)^{-1} = p^{-1} \dots c^{-1} b^{-1} a^{-1}$
6. Cancellation law -

   i) $ab = ac \Rightarrow b = c$ $[\because a, b, c \in G]$

   ii) $ba = ca \Rightarrow b = c$
7. If $a, b \in G$ then, equations $ax = b$ and $ya = b$ have a unique solution in G

Q3: Briefly explain the following :-

(i) Semi group

An algebraic structure $(G, *)$ is a called a semigroup if the binary operation $*$ satisfies the associative and closure property in a non-empty set.

$[G_1]$ $(a^* b)^* c = a^*(b^* c)$

$\forall$ $a \in G$

(ii) Monoid group

A semigroup is called a monoid group after if there exists identity element e in the set $[G_2]$ such that

$[G_2]$ $e^* a = a^* e = a$

$\forall$ $a \in G$

(iii) Permutation group:

The set of permutations S_n of a non void set A where permutation of A is a function $f : A \rightarrow A$ which is one-one and onto.

The number of elements in finite set G is called the degree of Permutation.

Let G have n elements then P_n is called a set of all permutations of degree n.

P_n is also called symmetric group of degree n.

P_n is also denoted by S_n

The number of elements in P_n or S_n is $n!$

(iv) Cyclic group:

A cyclic group is group in which every element is a power of some fixed element. A cyclic group G can be generated by a single element a so that every element in G has the form a^i for some integer i. we denote the cyclic group of order n by Z_n, since additive ~~of~~ group of Z_n is a cyclic group of order n.

(V) Factor group:

Let H be a normal subgroup of G. Then it can be verified that the cosets of G relative to H form a group. This group is called the quotient group or factor group of G relative to H and is denoted by G/H.

(Vi) Abelian group:

An abelian group is a group in which law of composition is commutative, i.e. the group law satisfies
$g \cdot h = h \cdot g$ for any g, h in the group
For example, the set of integers with group operation of addition i.e. $1+2 = 2+1$.

Q4. Define Subgroups? Briefly explain Normal subgroup.

Ans4. A subgroup is a subset of group that itself is a group. That means if H is a non empty subset of group G, then H is called the subgroup of G if H is a group.

Normal subgroup - A normal subgroup H of a group G is a subgroup of G which satisfies the similarity transformation with any fixed arbitrary element in G. If G is an abelian group and x is an arbitrary element of G, then Hx is a right coset of H in G and xH is a left coset in G since G is abelian $xH = Hx$. However it is possible that even though G is not an abelian group, it satisfies $xH = Hx$. In this case subgroups of G are said to be normal subgroups or invariant subgroups or distinguished subgroups.

A subgroup H of a group G is a normal subgroup $\Leftrightarrow xHx^{-1}$ $xHx^{-1} \subseteq H$ for every x G, where x may or may not be in H.

Normal subgroups are sometimes also referred to as self-conjugates. Denoted as $H \triangleleft G$

Q5. Discuss about Homomorphism, Isomorphism of Groups.

Ans5. Let $(G, *)$ and (H, Δ) be groups. A function $f: G \to H$ doesn't necessarily tell us anything about the relationship between G and H as groups unless we insist that it interacts in some specific way with the group operations $*$ and Δ. We define a group homomorphism $G \to H$ to be a function which respects the group structure on G and H in the following sense:

- A group homomorphism $f: G \to H$ is a function such that for all $x, y \in G$ we have:-

$$f(x*y) = f(x) \Delta f(y)$$

- A group isomerism is a group homomorphism which is a bijection.

Example - The identity map $G \to G$ is an isomorphism

- Let $G, *$ and H, Δ be any groups. The map $f: G \to H$ given by $f(g) = e_H$ for all $g \in G$ is a group homomorphism.

Q6. Write short notes on-

(i) Rings and fields

⇒ A Ring is a set R which is closed under two operatio[ns] + and × and satisfying the following properties-

① R is an abelian group under +

② Associativity of × - for every a, b, c ∈ R

③ Distributive Property - For every a, b, c ∈ R the following identities hold-

$$a \times (b+c) = (a \times b) + (a \times c)$$

and

$$(b+c) \times a = b \times a + c \times a$$

⇒ A Field is a set F which is closed under two operations + and × such that

① F is an abelian group + and

② F - {0} (the set F without additive identity 0) is an abelia[n] group under ×

(ii) Cosets

Coset is a subset of mathematical group consisting of all the products obtained by multiplying fixed element of group by each of elements of given subgroups, either on right or left. Cosets are basic tool in study of groups suppose A is a group and B is a subgroup of A, and is an element of A then

$aB = \{ab : b \text{ an element of } B\}$ is left coset of B in A.

$Ba = \{ba : b \text{ an element of } B\}$ is right coset of B in A.