CS3130 Homework 2

Phillip Janowski (pajmc2@mail.umsl.edu)

October 7, 2018

Question 1.

Explain, if 3^{n+1} is: $O(3^n); \Omega(3^n); \Theta(3^n)$.
For $O(3^n)$
$0 \leq 3^{n+1} \leq C * 3^n$
$3^{n+1} = 3 * 3^n = C * 3^n \therefore 3^{n+1} \in O(3^n)$
For $\Omega(3^n)$
$3^{n+1} \geq C * 3^n$
$3^{n+1} = 3 * 3^n = C * 3^n \therefore 3^{n+1} \in \Omega(3^n)$
NOT DONE $\downarrow$
For $\Theta(3^n)$
$C_1 * 3^n \leq 3^{n+1} \leq C_2 * 3^n$
$3^{n+1} = 3 * 3^n = C * 3^n \therefore 3^{n+1} \in \Theta(3^n)$

Question 2.

Explain, if 3^{3n} is: $O(3^n); \Omega(3^n); \Theta(3^n)$.
$3^{3n} \notin O(3^n), \in \Omega(3^n), \notin \Theta(3^n)$
3^{3n} will always grow faster than any $C * 3^n$ therefore it will only be in $\Omega(3^n)$
$f(n) \geq C * g(n)$

Question 3.

$f(n)$	$g(n)$	O	o	Ω	ω	Θ
8^n	$8^{n/2}$	No	No	Yes	Yes	No
$\sqrt{n^k}$(k being an even integer)	2^n	Yes	Yes	No	Yes	No
$\log(n^n)$	$ln(n!)$					

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} C \neq 0 & f(n) \in \Theta(g(n)) \leftrightarrow g(n) \in \Theta(f(n)) \\ 0 & f(n) \in o(g(n)) \rightarrow f(n) \in O(g(n)) \\ \infty & f(n) \in \omega(g(n)) \rightarrow f(n) \in \Omega(g(n)) \end{cases}$$

$f(n) \in \omega(g(n)) \leftrightarrow f(n) \in o(g(n))$
$\lim_{x\to\infty} \frac{8^x}{8^{x/2}} \rightarrow \lim_{x\to\infty} 8^{x/2} = \infty \therefore 8^n \in \Omega(8^{x/2}), \in \omega(8^{n/2})$
$\lim_{x\to\infty} \frac{\sqrt{x^k}}{2^x}$
$\sqrt{1x^k}$ results in a polynomial function and 2^n is exponential
$\therefore \lim_{x\to\infty} \frac{\sqrt{x^k}}{2^x} = 0$ and is $o(g(n)), \omega(g(n))$ and $O(g(n))$

Question 4.

$T(n) = 2T(n-1) + 1; T(1) = 5.$
$T(n) = 2T(n-1) + 1 = 2(2T(n-2)) + 1 = 2^2T(n-2) + 2 + 1$
$= 2^3T(T(n-3) + 2^2 + 2 + 1 = 2^4T(n-4) + 2^3 + 2^2 + 2 + 1$
$= ... = 2^{n-1}T(1) + 2^{n-2} + 2^{n-3} + ... + 2 + 1$
$= 2^{n-1} * 5 + (1 + 2 + 2^2 + ... + 2^{n-2}) = 2^{n-1} * 5 + \frac{2^{n-1}-1}{1}$
$= 6 * 2^{n-1} - 1$

Question 5.

For the recurrence $T(n) = \begin{cases} 1 & n = 1 \\ 3T(\frac{n}{3} + 1 & n > 1 \end{cases}$ where $= 3^k$