CS 3130 **Design and Analysis of Algorithms** **Fall 2018**

KEY TO QUIZ #4

[14 points]

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(1) [1 point] Express $3^{\log_4 N}$ as the power of N (that is, N raised to some power).

Answer: $3^{\log_4 N} = N^{\log_4 3}$

(2) [1 point] Express $\log_5 16$ through binary logarithms.

Answer: $\log_5 16 = \dfrac{\lg 16}{\lg 5} = \dfrac{4}{\lg 5}$

(3) [4 points] ***Explain,*** *which function grows faster:*

(a) $\log_3(n^7)$ or $\log_7(n^3)$

Solution.

Since $\lim\limits_{n\to\infty} \dfrac{\log_3(n^7)}{\log_7(n^3)} = \lim\limits_{n\to\infty} \dfrac{7\log_3 n}{3\log_7 n} = \dfrac{7}{3}\lim\limits_{n\to\infty} \dfrac{\dfrac{\log_7 n}{\log_7 3}}{\log_7 n} =$

$= \dfrac{7}{3}\lim\limits_{n\to\infty} \dfrac{1}{\log_7 3} = const \neq 0$,

these two functions have the <u>same order of growth</u>: $\log_3(n^7)$ is $\Theta(\log_7(n^3))$

(b) $\ln(n!)$ or $\ln(F_n)$, where F_n is the n-th Fibonacci number.

Solution.

Since F_n is $\Theta(\phi^n) \Rightarrow \ln(F_n)$ is $\Theta(n)$.

Also: $\ln(n!)$ is $\Theta(n \lg n)$. Since $\ln(n!)$ grows faster than any linear function, $\ln(n!)$ is $\omega(\ln(F_n))$,

or: $\ln(n!)$ grows faster than $\ln(F_n)$.

4. [2 points] Fill in blanks in the following definitions:

(a) In a full binary tree, each node has **<u>exactly 0 or two children.</u>**

(b) In a complete binary tree, all leaves are **<u>located at the same depth</u>** and each internal node has **<u>exactly 2 children.</u>**

5. [3 points] A complete binary tree has 256 leaves. Find the following:

(a) The number of internal nodes.	(b) The height of a tree.	(c) The total number of links=
$n = L - 1 = 255$	$h = \lg L = 8$	$= N - 1 = 256 + 255 - 1 = 510$

6. [3 points] Rank the following functions from the slowest growing to the fastest growing (explanations are not required): 2^n, $\sqrt[3]{n}$, $\ln(n!)$, $n^{\log_9 3}$, 5^n, $\log_3(n)^4$, $\log_4(n^3)$

Answer:

$$\{\ \log_3(n)^4,\ \log_4(n^3)\ \},\ \sqrt[3]{n},\ n^{\log_9 3},\ \ln(n!),\ 2^n,\ 5^n.$$

Note that $\log_3(n)^4 = 4\log_3 n$, which is different from $(\log_3 n)^4$.