CS 3130 **Design and Analysis of Algorithms** **Fall 2018**

HOMEWORK #2 [50 points].

Due date: Tuesday, October 9.

(Homework will be collected before the lecture!)

A. **Read Chapter 3 and do the following exercises**:

1. ***Explain***, if 3^{n+1} is: $O(3^n)$; $\Omega(3^n)$; $\Theta(3^n)$.
2. ***Explain***, if 3^{3n} is: $O(3^n)$; $\Omega(3^n)$; $\Theta(3^n)$.
3. Fill in the table below by indicating whether the function $f(n)$ is O, o, Ω, ω, or Θ of $g(n)$. Your answer should be the table with either "yes" or "no" in each cell.
Show all the necessary calculations! Answers without proof will not count!

$f(n)$	$g(n)$	O	o	Ω	ω	Θ
8^n	$8^{n/2}$					
$\sqrt{n^k}$ (k is an even positive integer)	2^n					
$\lg(n^n)$	$\lg(n!)$					

4. Problem 3 – 3 (a), pp.61 – 62.

NOTE: (1) from 30 functions listed in the book, do this exercise only for the functions, positions of which are marked by 'X' in the table below:

		X	X	X	
X	X	X	X		
X		X		X	X
X		X	X	X	X
		X	X	X	

(2) Please, rank exponential functions according to their bases.

(3) Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) \in \Theta(g(n))$.

(4) Explanations are not required.

B. **Read Chapter 4 and handout on recursion. Do the following exercises**:

5. Use the method of backward substitution (telescoping) to solve the recurrence:
$T(n) = 2T(n-1)+1$; $T(1) = 5$.
6. For the recurrence $T(n) = \begin{cases} 1 \ \ if \ \ n = 1 \\ 3T\left(\frac{n}{3}\right)+1 \ \ if \ \ n > 1 \end{cases}$

show that $T(n) \in \Theta(n)$. (Make sure that you prove two facts: $T(n) \in O(n)$ and $T(n) \in \Omega(n)$). Consider only $n = 3^k$. *Hint*: use the idea of subtracting the lower-power term (see p.85).

7. Use the recursion tree method to find a big-Θ estimate for the solution of the recurrence
$T(n) = 3T\left(\frac{n}{4}\right)+n$.
8. Exercise 4.5 – 1, p.96.

9. Answer the following questions (justify your answers!):

(a) [2 points] For the given version of a QUICKSORT algorithm, are arrays made up of all equal elements the worst-case input, the best-case input or neither?

(b) [2 points] For the given version of a QUICKSORT algorithm, are strictly decreasing arrays the worst-case input, the best-case input or neither?

10. (a) Show exactly the progress of the QUICKSORT algorithm with the median-of-3 partition described in the textbook for the array: <A, B, C, D, E, F>. Explain if strictly increasing arrays are the worst-case input, the best-case input or neither for the given version of a QUICKSORT algorithm with the median-of-3 partition.
    (b) Show exactly the progress of the QUICKSORT algorithm with the median-of-3 partition described in the textbook for the array: <F, E, D, C, B, A>. Explain if strictly decreasing arrays are the worst-case input, the best-case input or neither for the given version of a QUICKSORT algorithm with the median-of-3 partition.

Problem for 3 extra points.

There is one fake coin among 65 identically looking coins (the fake coin is lighter). In a balance scale, you can compare any two sets of coins. Develop the algorithm to determine a fake coin using not more than 4 comparisons.