CS3130 Homework 4

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Question 1.

8.2 - 1
Evaluate $\sum_{k=1}^{n} \lg k$
Upper Bound $\lg(n!) = O(n \lg n)$
$\lg(n!) = O(n \lg n)$
$\sum_{k=1}^{n} \lg k \leq \sum_{k=1}^{n} \lg n$
$= n \lg n$
Lower Bound $\lg(n!) = \Omega(n \lg n)$
$\sum_{k=1}^{n} \lg k = \sum_{k=1}^{n/2} + \sum_{k=n/2+1}^{n} \lg k \leq \sum_{k=1}^{n/2} \lg 1 + \sum_{k=n/2+1}^{n} \lg(n/2)$
$= 0 + n/2 * \lg(n/2)$
$\therefore \lg(n!) = O(n \lg n)$

Question 2.

8.3 - 1
$n!/2$
All comparison sort algorithms are at worst $\Omega(n \lg n)$ or $n!/2 \leq n! \leq r \leq 2h$
$h \geq \lg(n!/2) = \lg(n!) - 1$
$= \Theta(n \lg n) - 1 = \Theta(n \lg n) = \Theta(n \lg n)$
$= \Theta(n \lg n) - n = \Theta(n \lg n)$
$\therefore$ There is no comparison sort with linear run time for $n!/2$ of the inputs of array length n
$1/n$
$(1/n) * n! \leq n! \leq r \leq 2^h$
$h \geq \lg(n!/n) = \lg(n!) - \lg n$
$= \Theta(n \lg n) - \lg n = \Theta(n \lg n)$
$\therefore$ There is no comparison sort with linear run time for 1/n of the inputs of array length n
$1/2^n$

$(1/2^n)n! \leq n! \leq r \leq 2^h$
$h \geq \lg(n!/2^n) = \lg(n!) - n$
$= \Theta(n \lg n) - n = \Theta(n \lg n)$
$\therefore$ There is no comparison sort with linear run time for $1/2^n$ of the inputs of array length n

Question 3.

3. Find the lower and upper boundaries for the height of a 2-3-4 tree with n nodes.
The uppper bound of the height is every node having two children and the lower bound would be the height of a tree with every node having four children
Upper $2^0 + 2^1 + 2^2 + ... + 2^h \geq n \rightarrow \lg n$
Lower $4^0 + 4^1 + 4^2 + ... + 4^h \geq n \rightarrow \frac{1}{2} \lg n$

Question 4.

4. Assume that at some point of a construction, a 2-3-4 tree has only m 3-nodes. How many different red-black trees will correspond to that tree?
For a 2-3-4 tree with 3 nodes, there anywhere between 3 and 12 values, which will result in in red-black trees with 3 to 12 nodes, meaning there are 9 different red-black trees that the 2-3-4 tree can correspond to.