Examples illustrating the notion of algorithm.

Below we consider three computational procedures for finding the greatest common divisor of two integers. (The greatest common divisor of two non-negative, not-both-zero integers `m` and `n`, denoted as `gcd(m, n)`, is defined as the largest integer that divides both `m` and `n` evenly, i.e., with a remainder of zero.)

1. Euclid's algorithm is based on applying repeatedly the equality $\gcd(m,n) = \gcd(n, \; m \mod n)$ (where $m \mod n$ is the remainder of the division of m by n) until $m \mod n$ is equal to 0. Since $\gcd(m,0)$ is `m` (why?) , the last value of `m` is also the greatest common divisor of the initial `m` and `n`. Here is a more structured description of this algorithm:
   (a) If `n` is zero, return the value of `m` as the answer and stop; otherwise, proceed to step (b);
   (b) Divide `m` by `n` and assign the value of the remainder to `r`;
   (c) Assign the value of `n` to `m` and the value of r to `n`. Go to step (a).
2. Consecutive integer checking algorithm for computing `gcd(m, n)`.
   (a) Assign the value of `min{m, n}` to `t`;
   (b) Divide `m` by `t`. If the remainder of this division is `0`, go to step (c), otherwise proceed to step (d);
   (c) Divide `n` by `t`. If the remainder of this division is `0`, return the value of `t` as the answer and stop, otherwise proceed to step (d);
   (d) Decrease the value of `t` by `1`; go to step (b).
3. 'Middle-school' procedure for computing `gcd(m, n)`.
   (a) Find the prime factors of `m`.
   (b) Find the prime factors of `n`.
   (c) Identify all the common factors in the two prime expansions found in steps (a) and (b). (If `p` is a common factor occurring p_m times in `m` and p_m times in `n`, it should be repeated $\min\{p_m, p_n\}$ times.)
   (d) Compute the product of all the common factors and return it as the greatest common divisor of the numbers given.

The third procedure, however, cannot be considered as an algorithm, because steps (a) and (b) are not defined unambiguously: they require a list of prime numbers. Also, step (c) does not say explicitly how to find common elements in two sorted lists.

Check yourself: Why will the first and the second algorithms eventually stop?
Will all three procedures work properly with all possible input values?