CS 3130 **Design and Analysis of Algorithms** **Spring 2016**

K E Y T O Q U I Z #2

[6 points]

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1. [3 points] Assuming that $f(n) > 0$ and $g(n) > 0$, fill in the blanks in each definition:

- $f(n)$ is $O(g(n))$ if <u>there exist constants C and n_0 such that $f(n) < Cg(n)$ for all $n > n_0$</u>.

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- $f(n)$ is $\Omega(g(n))$ if <u>there exist constants C and n_0 such that $f(n) > Cg(n)$ for all $n > n_0$</u>.

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- $f(n)$ is $\Theta(g(n))$ if <u>there exist constants C_1, C_2 and n_0 such that $C_1 g(n) f(n) < C_2 g(n)$ for all $n > n_0$</u>.

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- $f(n)$ is $o(g(n))$ if <u>for **every** constant C there exists a constant n_0 such that $f(n) < Cg(n)$ for all $n > n_0$</u>.

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- $f(n)$ is $\omega(g(n))$ if <u>for **every** constant C there exists a constant n_0 such that $f(n) > Cg(n)$ for all $n > n_0$</u>.

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2. [4 points] No proof is required. Given the function $f(n) = n^2 + 2n - 3$, indicate if each of the following statements is true or false:

$f(n)$ is $O(n^2)$	**True**	$f(n)$ is $\Omega(n^2)$	**True**	$f(n)$ is $\Theta(n^2)$	**True**
$f(n)$ is $O(n^3)$	**True**	$f(n)$ is $\Omega(n^3)$	**False**	$f(n)$ is $\Theta(n^3)$	**False**
$f(n)$ is $O(n)$	**False**	$f(n)$ is $\Omega(n)$	**True**	$f(n)$ is $\Theta(n)$	**False**

3. [1 point] Determine, which function grows faster: $\log_7(n^5)$ or $\log_5(n^7)$. Show your work!

<u>Solution.</u>

Since $\lim\limits_{n\to\infty} \frac{\log_5(n^7)}{\log_7(n^5)} = \lim\limits_{n\to\infty} \frac{7\log_5 n}{5\log_7 n} = \frac{7}{5}\lim\limits_{n\to\infty} \frac{\frac{\log_7 n}{\log_7 5}}{\log_7 n} = \frac{7}{5\log_7 5} = const \neq 0$, these two functions have the <u>same order of growth</u>.