CS3130 Homework 1

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September 6, 2018

Question 1.

(a) Find $gcd(213486, 5423)$

$213486/5243 = 39R1989$
$213486 = 5423 * 39 + 1989$
$213486/1989 = 107R663$
$213486 = 1989 * 107 + 663$
$213486/663 = 332R0$

$\therefore gcd(213486, 5423) = 663$

(b) Estimate approximately how many times faster it will be to find $gcd(213486, 5423)$ with the help of the Euclid's algorithm compared with the algorithm based on checking consecutive integers from minm, n down to gcd(m, n) (see the algorithm #2 from the handout). You may only count the number of modulus divisions of the largest integer by different divisors.

$min(213486, 5423) = 5423$
$m/t \neq 0$ where $m = 213486$ and $t = 5423$
$t - 1 = 5422$
$m - gcd(213486, 5423) = 4760$ steps
4760 steps /4 steps = 1190 times faster

Question 2.

2.1) Prove the formula on which Euclid's algorithm is based: $gcd(m, n) = gcd(n, m \text{ mod } n)$ for every pair of positive integers m and n
Solution:
We know $gcd(m, 0) = m$ and $gcd(0, n) = n$

if $m = n * I + R$ where I is some integer and $N \neq 0$
then $gcd(m, n) = gcd(n, R)$ where remainder $R = m \bmod n$
$\therefore gcd(m, n) = gcd(n, m \bmod n)$

2.2) 2.1-4
Given two array's A and B, length n, are added together and stored in an $(n+1)$ length array, C

```
carried = 0
for i = n-1 down to 0
     C[i+1] = {A[i] + B[i] + carried} mod 2
     carried = {A[i] + B[i] + carried} / 2
C[0] = carried
```

2.3) 2.2-2

(a)
```
for i = 0 to n - 1
     smallest = 0
     smallestElement = 0
     for 0 = i to n -1
          if A[j]< A[j+1]
               smallest = A[j]
               smallestElement = j
     A[smallestElement] = j
     A[i] = smallest
```

(b) The loop is invariant because at the beginning of each for loop, the sorted part of the array changes in size with each iteration and only have the i-1 smallest elements

(c) The loop only needs to run for n-1 times because the last element will already be sorted

(d) The run time of this loop is $\theta(n^2)$, because the inner loop runs each of its lines n times and the out loop runs its lines, including the inner loop, n times. $\therefore \theta(n^2)$

Question 3.

Modify the psuedo code on p.40 to count swaps for the bubble sort

```
i = 0
swap = 1
while swap != 0 and i < A.length
     swap = 0
          for j = A.length down to i + 1
               if A[j] < A[j-1]
                    exchange A[j] < A[j-1]
                    swap++
                    i++
```

Question 4.

Handwritten and Stapled to the packet

(a) Insertion Sort

R	E	A	L	I	T	Y	S	H	O	W	Comparisons
R	E	A	L	I	T	Y	S	H	O	W	1
E|	R	A	L	I	T	Y	S	H	O	W	2
A	E|	R	L	I	T	Y	S	H	O	W	2
A	E	L|	R	I	T	Y	S	H	O	W	3
A	E	I	L|	R	T	Y	S	H	O	W	1
A	E	I	L	R|	T	Y	S	H	O	W	1
A	E	I	L	R	T|	Y	S	H	O	W	3
A	E	I	L	R	S	T|	Y	H	O	W	7
A	E	H	I	L	R	S	T|	Y	O	W	5
A	E	H	I	L	O	R	S	T|	Y	W	2
A	E	H	I	L	O	R	S	T	W|	Y	27 total

(b) Selection Sort

R	E	A	L	I	T	Y	S	H	O	W	Comparisons
R	E	A	L	I	T	Y	S	H	O	W	10
A|	E	R	L	I	T	Y	S	H	O	W	9
A	E|	R	L	I	T	Y	S	H	O	W	8
A	E	H|	L	I	T	Y	S	R	O	W	7
A	E	H	I|	L	T	Y	S	R	O	W	6
A	E	H	I	L|	T	Y	S	R	O	W	5
A	E	H	I	L	O|	Y	S	R	T	W	4
A	E	H	I	L	O	R|	S	Y	T	W	3
A	E	H	I	L	O	R	S|	Y	T	W	2
A	E	H	I	L	O	R	S	T|	Y	W	1
A	E	H	I	L	O	R	S	T	W|	Y	55 total

(c) Bubble Sort With Swap Count

R	E	A	L	I	T	Y	S	H	O	W	Comparisons	Swaps
R	E	A	L	I	T	Y	S	H	O	W	10	8
E	A	L	I	R	T	S	H	O	W	|Y	9	4
A	E	I	L	R	S	H	O	T	|W	Y	8	2
A	E	I	L	R	H	O	S	|T	W	Y	7	2
A	E	I	L	H	O	R	|S	T	W	Y	6	1
A	E	I	H	L	O	|R	S	T	W	Y	5	1
A	E	H	I	L	|O	R	S	T	W	Y	4	0
A	E	H	I	L	O	R	S	T	W	Y	49 total	14 total

(d) Bubble Sort Without Swap Count

R	E	A	L	I	T	Y	S	H	O	W	Comparisons	Swaps
R	E	A	L	I	T	Y	S	H	O	W	10	8
E	A	L	I	R	T	S	H	O	W	|Y	9	4
A	E	I	L	R	S	H	O	T	|W	Y	8	2
A	E	I	L	R	H	O	S	|T	W	Y	7	2
A	E	I	L	H	O	R	|S	T	W	Y	6	1
A	E	I	H	L	O	|R	S	T	W	Y	5	1
A	E	H	I	L	|O	R	S	T	W	Y	4	0
A	E	H	I	|L	O	R	S	T	W	Y	3	0
A	E	H	|I	L	O	R	S	T	W	Y	2	0
A	E	|H	I	L	O	R	S	T	W	Y	1	0
A	E	H	I	L	O	R	S	T	W	Y	55 total	14 total

Question 5.

(a)
```
i = 0
while i < 3n
    print"CS3130"
        i = i + 3
```

$= \sum_{i=0}^{(3n-3)/3} 1 = (n-1+0-1)+3(3n-0+1)$

$$= n$$

(b)
```
for i = 0 to n
    for j = 0 to n
        print"CS3130"
```

$= \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} 1 = \sum_{i=0}^{n-1}(n-1-0+1) = n\sum_{i=0}^{n-1} 1$
$= n(n-1-0+1)$

$$= n^2$$

(c)
```
for i = 0 to n
     for j = i to n
          print"CS3130"
```

$$= \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} 1 = \sum_{i=0}^{n-1} (n-1-i+1) = n \sum_{i=0}^{n-1} 1 - \sum_{i=0}^{n-1} i$$

$$= n(n-1-0+1) - \frac{(0+n-1)}{2}(n-1-0+1) = n^2 - \frac{n^2-n}{2}$$

$$= \frac{n^2+n}{2}$$

Question 6.

2.1-3 Find v in Array A[ ]

```
i = 0
while i < A.length and v != A[i]
     i++
if A[i] != v
     i = null
```

This loop is invariant because at the beginning we have $i = 0$ at ininitialization. Maintenance, because it maintains while $i < length$ of A or $v \neq A[i]$ and is true before i and i + 1. And Termination, because it only terminates when v is found in A or when we run out of elements in A to check

Question 7.

The average performance of the linear search is $O(\frac{n}{2})$. The worst performance would happen from v being in the $A.length$th position of the array and the best would happen from v being in the 1st position of the array. In a set of completely random runs the average number of comparisons would eventually converge on $\frac{n}{2}$.