University of Burgundy

Master of Science in Computer Vision – 2nd Year

MSFT Module - SFM + IMU

Report on 4 Points vs 2+1 Points Algorithms

by

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1. Introduction

Structure from Motion (SFM) is a hot topic now-a-days in computer vision. There are many classical SFM methods, and now the question is can we improve the classical methods including some prior knowledge.

In SFM, the motion of camera is significant. The affine transformation (rotation and translation) of camera from one frame to other is important to construct the structure. So, in order to know the extrinsic parameters (rotation and translation) we need to estimate the essential matrix, E. So, in order to estimate essential matrix, we need feature points between two views of camera. Many algorithms have been developed to estimate the essential matrix like 8 point algorithm, 5 point algorithm etc.

So, if we know few rotation angles of camera, can we reduce the number of feature points required to estimate the complete motion of camera? as we know some prior information about the motion of camera, we can reduce the feature points required for the estimation of essential matrix.

If the points lie on a plane then we need to estimate the homography between two planes.

$$H = R - \frac{tn^T}{d}$$

where, H - Homography

R - Rotation between two camera views

t - translation between two camera views

n - normal to plane of points

d - distance between plane and camera1

2. 4 Points vs 2+1 Points

In 4 points algorithm, the hypothesis is that the points belong to a plane. So, there exists a homography H as explained in above formula.

$$p' \cong Hp$$

$$p' \times Hp = 0$$

If we consider upto a scale then homography, 'H' has 8 unknown terms. From above equation each point gives 2 equations, so 4 points give 8 equations. So, 8 equations in 8 unknowns can be solved to estimate H.

In 2 points algorithm, the hypothesis is that the points belong to a vertical plane. So, there exists a homography H. Here, we suppose that the camera is filming the ground and that the ground is a flat surface. So, between the views of the ground we have a homography, H.

The goal of this practical is to compare the 4 points and 2+1 point algorithms in estimation of homography.

We consider 50 points randomly distributed in a plane of equation $N^T X_w + d = 0$ in the world frame (O_w, X_w, Y_w, Z_w).

There are two camera positions O_{c1} and O_{c2}. The image points are P_i.

Test1 : Example with different data

The camera1 is kept at origin and camera2 is rotated and translated by 20degrees and 10 respectively for testing. The theoretical and estimated homography is calculated for 4 points and 2+1 points algorithm. As there was no noise in data and IMU the results of theoretical and estimated homography match each other perfectly as shown in figs.1 and 2.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% TEST 1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%% Example with different data %%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Theoretical Homography

H =

    1.0253    0.3732   -0.4172
   -0.2456    1.0718    0.3492
    0.4865   -0.2456    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (4 Point Algorithm)

H4pt =

    1.0253    0.3732   -0.4172
   -0.2456    1.0718    0.3492
    0.4865   -0.2456    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fig. 1 MATLAB result for test1 4 point algorithm

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
True Theoretical Homography (2 angles known)

TrueHomography =

    0.9589    0.3490   -0.0262
   -0.3490    0.9589   -0.0122
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (2+1 Point Algorithm)

EstimatedH =

    0.9589    0.3490   -0.0262
   -0.3490    0.9589   -0.0122
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fig. 2 MATLAB result for test1 2+1 point algorithm

Test2 : Example with Noise

The camera1 is kept at origin and camera2 is rotated and translated by 20degrees and 10 units respectively and the noise is added to image points of camera2 with standard deviation of 0 to 1 pixels, for testing. The theoretical and estimated homography is calculated for 4 points and 2+1 points algorithm. As there was noise in data, the results of theoretical and estimated homography did not match each other perfectly and the error (SSD) is as shown in figs.3-5.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% TEST 2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%% Example with Noise %%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Theoretical Homography

H =

    1.0253    0.3732   -0.4172
   -0.2456    1.0718    0.3492
    0.4865   -0.2456    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (4 Point Algorithm)

H4pt =

    1.0248    0.3732   -0.4164
   -0.2454    1.0707    0.3495
    0.4863   -0.2458    1.0000
```

Fig. 3 MATLAB result for test2 4 point algorithm

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
True Theoretical Homography (2 angles known)

TrueHomography =

    0.9589    0.3490   -0.0262
   -0.3490    0.9589   -0.0122
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (2+1 Point Algorithm)

EstimatedH =

    0.9584    0.3489   -0.0255
   -0.3489    0.9584   -0.0119
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fig. 4 MATLAB result for test2 2+1 point algorithm

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Error(SSD) in 4 point estimation, with noise is : 2.2095e-06
Error(SSD) in 2 point estimation, with noise is : 1.0087e-06
```

Fig. 5 MATLAB result for test2 Errors

As we can see from the above errors that the error in estimation of homography using 2+1 point algorithm where two angles are known is less as compared to error in estimation of homography using 4 point algorithm.

Test 3 : Example with Noise on IMU information

The camera1 is kept at origin and camera2 is rotated and translated by 20degrees and 10 units respectively, noise is added to image points of camera2 with standard deviation of 0 to 1 pixels and white noise in IMU of 0 to 2 degrees is added, for testing. The theoretical and estimated homography is calculated for 4 points and 2+1 points algorithm. As there was noise in data, the results of theoretical and estimated homography did not match each other perfectly and the error (SSD) is as shown in figs.6-8.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% TEST 3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%% Example with Noise on IMU information %%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Theoretical Homography

H =

    1.0270    0.3871   -0.4264
   -0.2501    1.0742    0.3669
    0.5023   -0.2564    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (4 Point Algorithm)

H4pt =

    1.0258    0.3878   -0.4263
   -0.2499    1.0751    0.3671
    0.5042   -0.2557    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fig. 6 MATLAB result for test3, 4 point algorithm

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
True Theoretical Homography (2 angles known)

TrueHomography =

    0.9548    0.3599   -0.0263
   -0.3599    0.9548   -0.0119
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Estimated Homography (2+1 Point Algorithm)

EstimatedH =

    0.9550    0.3604   -0.0264
   -0.3604    0.9550   -0.0118
         0         0    1.0000

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fig. 7 MATLAB result for test3, 2+1 point algorithm

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Error(SSD) in 4 point estimation, with noise in IMU is : 6.8026e-06
Error(SSD) in 2 point estimation, with noise in IMU is : 5.981e-07
```

Fig. 8 MATLAB result for test3, Errors

As we can see from the above errors that the error in estimation of homography using 2+1 point algorithm where two angles are known is less as compared to error in estimation of homography using 4 point algorithm even if the IMU information is corrupted.

3. Conclusion

In this practical, I had compared the classical 4 point and 2+1 point algorithms to estimate the homography. i had made three tests to compare both algorithms, with noise in image points and with noise in IMU information. The 2+1 point algorithm with known rotation angles works well when compared to 4 point algorithm.