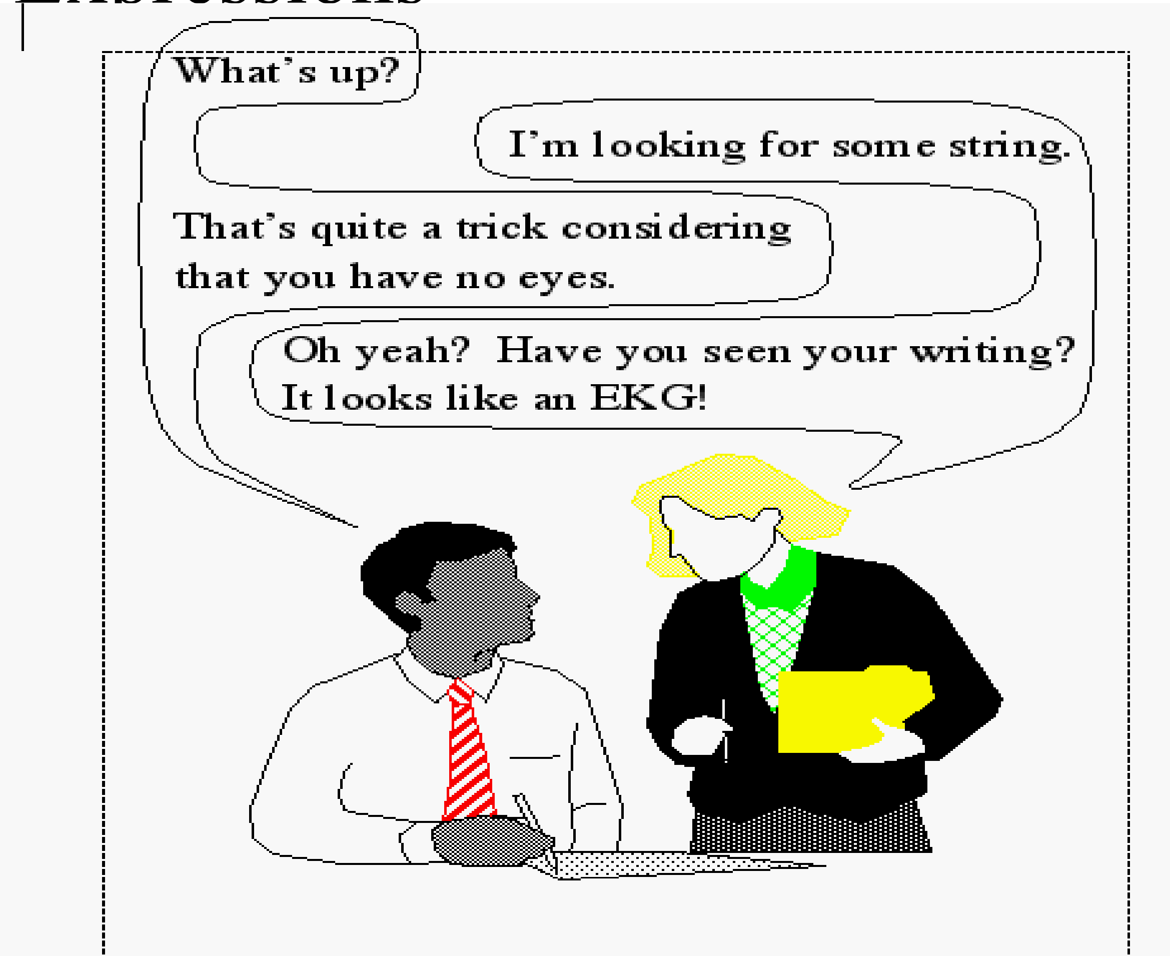
Strings Matching Algorithms

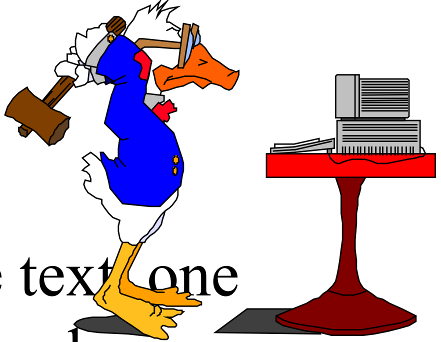
- Brute Force,Rabin-Karp, Knuth-Morris-Pratt
- Regular Expressions



String Searching

- The previous slide is not a great example of what is meant by "String Searching." Nor is it meant to ridicule people without eyes....
- The object of string searching is to find the location of a specific text pattern within a larger body of text (e.g., a sentence, a paragraph, a book, etc.).
- As with most algorithms, the main considerations for string searching are speed and efficiency.
- There are a number of string searching algorithms in existence today, but the three we shall review are Brute Force,Rabin-Karp, and Knuth-Morris-Pratt.

Brute Force



- The Brute Force algorithm compares the pattern to the text, one character at a time, until unmatching characters are found

```
TWO ROADS DIVERGED IN A YELLOW WOOD
ROADS
TWO ROADS DIVERGED IN A YELLOW WOOD
 ROADS
TWO ROADS DIVERGED IN A YELLOW WOOD
  ROADS
TWO ROADS DIVERGED IN A YELLOW WOOD
   ROADS
TWO ROADS DIVERGED IN A YELLOW WOOD
    ROADS
```

Compared characters are *italicized*.

Correct matches are in **boldface** type.

- The algorithm can be designed to stop on either the first occurrence of the pattern, or upon reaching the end of the text.

Brute Force Pseudo-Code

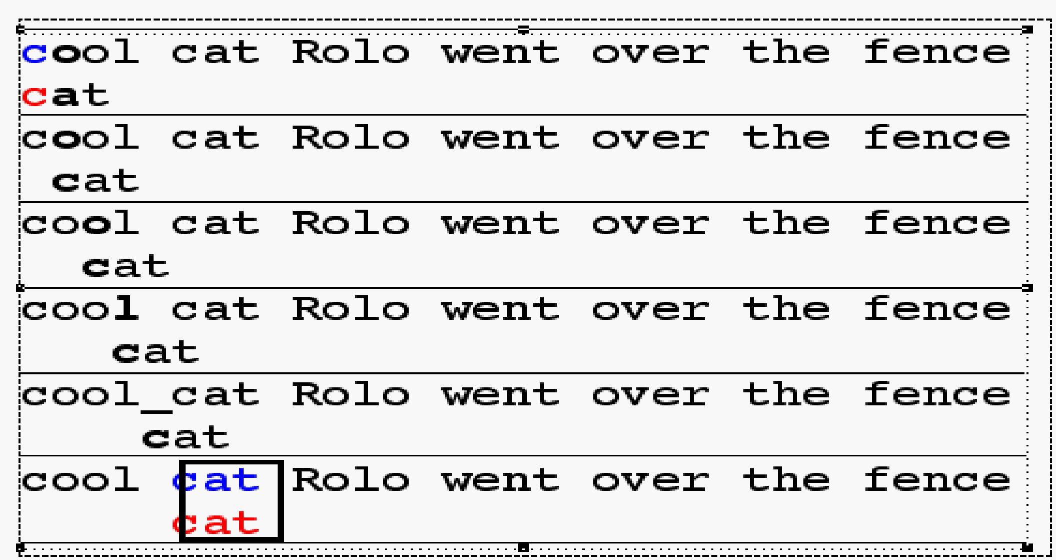
- Here’s the pseudo-code

do if (text letter == pattern letter)

compare next letter of pattern to next letter of text

else move pattern down text by one letter

while (entire pattern found or end of text)



Brute Force-Complexity

- Given a pattern M characters in length, and a text N characters in length...
- Worst case: compares pattern to each substring of text of length M. For example, M=5.
- This kind of case can occur for image data.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
2) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
3) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
4) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
5) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
....
N) AAAAAAAAAAAAAAAAAAAAAAAAA*AAAAH*
5 comparisons made *AAAAH*

Total number of comparisons: M (N-M+1)
Worst case time complexity: O(MN)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- Best case if pattern found: Finds pattern in first M positions of text. For example, M=5.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAA **5 comparisons made**

Total number of comparisons: M
Best case time complexity: O(M)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- Best case if pattern not found: Always mismatch on first character. For example, M=5.

```
1) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
   OOOOH      1 comparison made
2) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
    OOOOH      1 comparison made
3) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
     OOOOH      1 comparison made
4) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
      OOOOH      1 comparison made
5) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
       OOOOH      1 comparison made
 ...
N) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
          1 comparison made      OOOOH
```

Total number of comparisons: N

Best case time complexity: O(N)

Rabin-Karp

- The Rabin-Karp string searching algorithm calculates a **hash value** for the pattern, and for each M-character subsequence of text to be compared.
- If the hash values are unequal, the algorithm will calculate the hash value for next M-character sequence.
- If the hash values are equal, the algorithm will do a Brute Force comparison between the pattern and the M-character sequence.
- In this way, there is only one comparison per text subsequence, and Brute Force is only needed when hash values match.
- Perhaps an example will clarify some things...

Rabin-Karp Example

- Hash value of “AAAAA” is 37
- Hash value of “AAAAH” is 100

1) AAAAAAAAAAAAAAAAAAAAAAAAAAAH
   AAAAH
   $37 \neq 100$ **1 comparison made**
2) AAAAAAAAAAAAAAAAAAAAAAAAAAAH
    AAAAH
   $37 \neq 100$ **1 comparison made**
3) AAAAAAAAAAAAAAAAAAAAAAAAAAAH
     AAAAH
   $37 \neq 100$ **1 comparison made**
4) AAAAAAAAAAAAAAAAAAAAAAAAAAAH
      AAAAH
   $37 \neq 100$ **1 comparison made**

...

N) AAAAAAAAAAAAAAAAAAAAAAAAAAAH
   AAAAH
   5 comparisons made $100 = 100$

Rabin-Karp Algorithm

pattern is M characters long

hash_p=hash value of pattern

hash_t=hash value of first M letters in body of text

```
do
    if (hash_p == hash_t)
            brute force comparison of pattern
            and selected section of text
        hash_t= hash value of next section of text, one character over
while (end of text        or
        brute force comparison == true)
```

Rabin-Karp

- Common Rabin-Karp questions:

  "What is the hash function used to calculate values for character sequences?"

  "Isn't it time consuming to hash very one of the M-character sequences in the text body?"

  "Is this going to be on the final?"

- To answer some of these questions, we'll have to get mathematical.

Rabin-Karp Math

- Consider an M-character sequence as an M-digit number in base b, where b is the number of letters in the alphabet. The text subsequence t[i .. i+M-1] is mapped to the number

$$x(i) = t[i] \cdot b^{M-1} + t[i+1] \cdot b^{M-2} + \ldots + t[i+M-1]$$

- Furthermore, given x(i) we can compute x(i+1) for the next subsequence t[i+1 .. i+M] in constant time, as follows:

$$x(i+1) = t[i+1] \cdot b^{M-1} + t[i+2] \cdot b^{M-2} + \ldots + t[i+M]$$

$x(i+1) = x(i) \cdot b$	Shift left one digit
$- \; t[i] \cdot b^{M}$	Subtract leftmost digit
$+ \; t[i+M]$	Add new rightmost digit

- In this way, we never explicitly compute a new value. We simply adjust the existing value as we move over one character.

Rabin-Karp Math Example

- Let’s say that our alphabet consists of 10 letters.
- our alphabet = a, b, c, d, e, f, g, h, i, j
- Let’s say that “a” corresponds to 1, “b” corresponds to 2 and so on.

The hash value for string “cah” would be ...

$$3*100 + 1*10 + 8*1 = 318$$

Rabin-Karp Mods

- If M is large, then the resulting value (~bM) will be enormous. For this reason, we hash the value by taking it **mod** a prime number **q**.
- The **mod** function (% in Java) is particularly useful in this case due to several of its inherent properties:

  [(x mod q) + (y mod q)] mod q = (x+y) mod q

  (x mod q) mod q = x mod q
- For these reasons:

h(i)=((t[i]· bM-1 mod q) +(t[i+1]· bM-2 mod q) + ...

+(t[i+M-1] mod q))mod q

h(i+1) =(h(i) · b mod q

Shift left one digit

-t[i] · b^M mod q

Subtract leftmost digit

+t[i+M] mod q)

Add new rightmost digit

mod q

Rabin-Karp Complexity

- If a sufficiently large prime number is used for the *hash function*, the hashed values of two different patterns will usually be distinct.
- If this is the case, searching takes O(N) time, where N is the number of characters in the larger body of text.
- It is always possible to construct a scenario with a worst case complexity of O(MN). This, however, is likely to happen only if the prime number used for hashing is small.

The Knuth-Morris-Pratt Algorithm

- The Knuth-Morris-Pratt (KMP) string searching algorithm differs from the brute-force algorithm by keeping track of information gained from previous comparisons.
- A failure function (f) is computed that indicates how much of the last comparison can be reused if it fails.
- Specifically, f is defined to be the longest prefix of the pattern P[0,..,j] that is also a suffix of P[1,..,j]

  -Note: **not** a suffix of P[0,..,j]
- Example:-value of the
- KMP failure function:

j	0	1	2	3	4	5
$P[j]$	a	b	a	b	a	c
$f(j)$	0	0	1	2	3	0

- This shows how much of the beginning of the string matches up to the portion immediately preceding a failed comparison.

  -if the comparison fails at (4), we know the a,b in positions 2,3 is identical to positions 0,1

The KMP Algorithm (contd.)

- the KMP string matching algorithm: Pseudo-Code

Algorithm KMPMatch(T,P)

Input: Strings T (text) with n characters and P (pattern) with m characters.

Output: Starting index of the first substring of T matching P, or an indication that P is not a substring of T.

Algorithm

```
f ← KMPFailureFunction(P) {build failure function}
    i ← 0
    j ← 0
    while i < n do
        if P[j] = T[i] then
            if j = m - 1 then
                return i - m - 1  {a match}
            i ← i + 1
            j ← j + 1
        else if j > 0 then  {no match, but we have advanced}
            j ← f(j-1)  {j indexes just after matching prefix in P}
        else
            i ← i + 1
    return "There is no substring of T matching P"
```

The KMP Algorithm (contd.)

- The KMP failure function: Pseudo-Code

Algorithm KMPMatch(T,P)

  Input: String P (pattern) with m characters

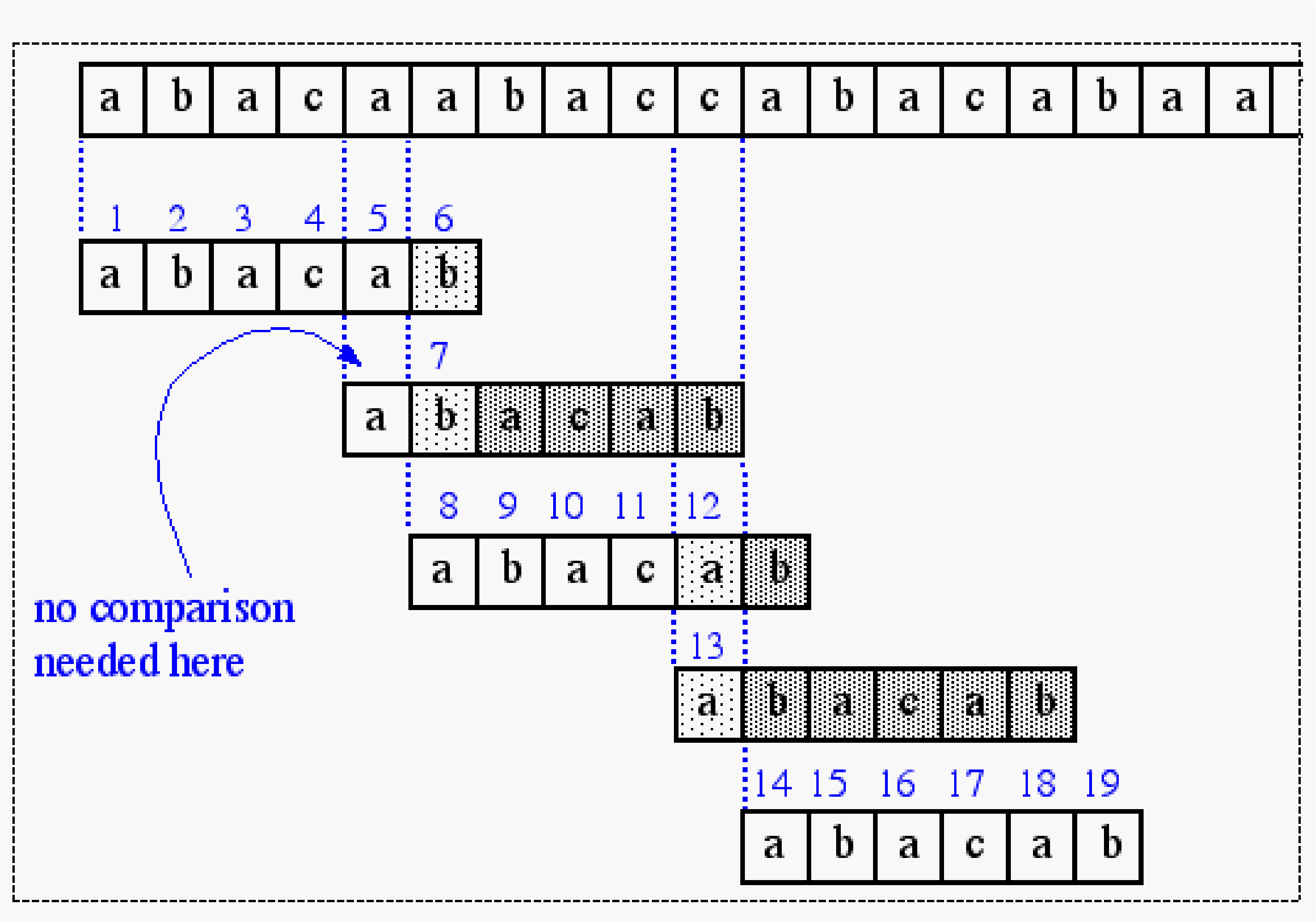
  Output: The failure function f for P, which maps j to the length of the longest prefix of P that is a suffix of P[1,..,j]

Algorithm

```
f ← KMPFailureFunction(P) {build failure function}
    i ← 0
    j ← 0
    while i ≤ m-1 do
        if P[j] = T[i] then
            if j = m - 1 then
              { we have matched j+1 characters}
            f(i) ← j + 1
            i ← i + 1
            j ← j + 1
        else if j > 0 then
            j ← f(j-1) {j indexes just after matching prefix in P}
        else {there is no match}
            f(i) ← 0
            i ← i + 1
```

The KMP Algorithm (contd.)

- A graphical representation of the KMP string searching algorithm



The KMP Algorithm (contd.)

- Time Complexity Analysis
- define k = i - j
- In every iteration through the while loop, one of three things happens.
  1) if T[i] = P[j], then i increases by 1, as does j k remains the same.
  2) if T[i] != P[j] and j > 0, then i does not change and k increases by at least 1, since k changes from i - j to i - f(j-1)
  3) if T[i] != P[j] and j = 0, then i increases by 1 and k increases by 1 since j remains the same.
- Thus, each time through the loop, either i or k increases by at least 1, so the greatest possible number of loops is 2n
- This of course assumes that f has already been computed.
- However, f is computed in much the same manner as KMPMatch so the time complexity argument is analogous. KMPFailureFunction is O(m)
- Total Time Complexity: O(n + m)

Regular Expressions

- notation for describing a set of strings, possibly of infinite size
- ε denotes the empty string
- ab + c denotes the set {ab, c}
- a* denotes the set {ε, a, aa, aaa, ...}
- Examples

  (a+b)* all the strings from the alphabet {a,b}

  b*(ab*a)*b* strings with an even number of a's

  (a+b)*sun(a+b)* strings containing the pattern "sun"

  (a+b)(a+b)(a+b)a 4-letter strings ending in a