

SEQUENCE AND SERIES

Number series tests present numerical sequences that follow a logical rule which is based on elementary arithmetic. An initial sequence is given from which the rule is to be deduced. You are then asked to predict the next number that obeys the rule. The difficulty level of these questions can increase in two ways; first, the logic behind the sequence becomes less trivial and demands attention and creativity; second, the missing number can be positioned at an early stage, thus preventing you from deciphering the hidden rule by looking only at the previous numbers in the sequence.

Different types of Number Series

There are some formats of series which are given in Exams.

Perfect Square Series: This Types of Series are based on square of a number which is in same order and one square number is missing in that given series.

Example: 441, 484, 529, 576?

Answer: $441 = 21^2$, $484 = 22^2$, $529 = 23^2$, $576 = 24^2$, $625 = 25^2$.

Perfect Cube Series: This Types of Series are based on cube of a number which is in same order and one cube number is missing in that given series

Example : 1331, 1728, 2197, ?

Answer : 11^3 , 12^3 , 13^3 , 14^3

Geometric Series: This type of series are based on ascending or descending order of numbers and each successive number is obtain by multiplying or dividing the previous number with a fixed number.

Example : 5, 45, 405, 3645,?

Answer: $5 \times 9 = 45$, $45 \times 9 = 405$, $405 \times 9 = 3645$, $3645 \times 9 = 32805$.

Two stage Type Series: A two tier Arithmetic series is one in which the differences of successive numbers themselves form an arithmetic series.

Example 4: i. 3, 9, 18, 35, 58,——

ii. 6, 9, 17, 23,———

Mixed Series: This type of series are more than one different order are given in a series which arranged in alternatively in a single series or created according to any non-conventional rule. This mixed series Examples are describes in separately.

Examples: 11, 24, 50, 102, 206, ?

Answer: $11 \times 2 = 22 + 2 = 24$,

$24 \times 2 = 48 + 2 = 50$,

$50 \times 2 = 100 + 2 = 102$,

$102 \times 2 = 204 + 2 = 206$,

$206 \times 2 = 412 + 2 = 414$.

So the missing number is 414.

Letter Series

- A series of single, pairs of groups or combination of letters and numerals is given.
- The terms of the series form a certain pattern as regards the position of the letters in the English alphabet.
- You have to decipher the pattern and accordingly, find the missing term or wrong term in the given series.

Examples:

1. A, C, F, J, ?, ?

Sol: A (B) C, C (D, E) F, F (G, H, I) J, J (K, L, M, N) O, O (P, Q, R, S, T) U

So, the next terms are O, U.

2. BMO, EOQ, HQS, ?

Sol: See all the first letters of the given series.

B (C, D) E, E (F, G) H, H (I, J) K

Now, see all the second letters of the given series.

M (N) O, O (P) Q, Q (R) S

Now, see all the third letters of the given series.

O (P) Q, Q (R) S, S (T) U

So, the next term will be KSU.

Alpha-Numeric Series: It is a jumbled combination of Alphabetic and Numeric series.

Examples:

1. Z1A, X2D, V6G, T21J, R88M, ?

Sol: The series formed by the numerals 1, 2, 6, 21, 88, ... follow the pattern

$x \ 1 + 1, x \ 2 + 2, x \ 3 + 3, x \ 4 + 4, \dots$

So, numeral in the desired term = $88 \times 5 + 5 = 445$

Observe the first letters of all the terms.

Z (Y) X, X (W) V, V (U) T, T (S) R, R (Q) P

Observe the second letters of all the terms.

A (B, C) D, D (E, F) G, G (H, I) J, J (K, L) M, M (N, O) P

So, the next term in the series will be P445P.

2. Find the odd man out from the following series.

G4T, J10R, M20P, P43N

Sol: Observe the pattern followed by the first letter in all the terms.

G (H, I) J, J (K, L) M, M (N, O) P

Observe the pattern followed by the second letter in all the terms.

T (S) R, R (Q) P, P (O) N

Observe the pattern followed by the numerals in all the terms.

Here, if $x \ 2 + 1$ rule is satisfied then second term should be J9R.

This rule can be applied to whole series except the second term.

So, J10R is the odd man.

Continuous Pattern Series:

- This type of series usually consists of a small letters which follow a certain pattern.
- But some letters will be missing from the series.
- These missing letters are then given in a proper sequence as one of the choices.
- You have to choose the correct alternative.

Example:

1. aab _ aaa _ bba _

1) baa 2) abb 3) bab 4) aab 5) bbb

Sol:

Step 1: Fill the first blank space by 'b' so that you can have two a's followed by two b's.

Step 2: Fill the second blank space either by 'a' so that you have four a's followed by two b's or 'b' so that you have three a's followed by three b's.

Step 3: The last space must be filled by 'a'.

Step 4: So, now you can have two possible answers: 'baa' and 'bba'. But, only baa appears in the choices. Thus 1 is the answer.

Step 5: In case, you have both the possible answers in the choices, you have to choose the one that forms a more prominent pattern, which is aabb/aaabbb/aa. Thus, your answer should be 'bba'.

Arithmetic, Geometric and Harmonic Progressions

Arithmetic Progression

An arithmetic progression is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference "d".

The general form of an Arithmetic Progression is $a, a + d, a + 2d, a + 3d$ and so on.

- The n th term of an AP series is $T_n = a + (n - 1) d$,
where T_n = n th term and a = first term. Here d = common difference = $T_n - T_{n-1}$.
- Sum of first n terms of an AP: $S = [2a + (n - 1)d]/2$
- The sum of n terms is also equal to the formula $= n/2[a + l]$,
where l is the last term.
- $T_n = S_n - S_{n-1}$, where T_n = n th term
- When three quantities are in AP, the middle one is called as the arithmetic mean of the other two. If a, b and c are three terms in AP then $b = (a+c)/2$

For example, 3,7,11,15,19 is an Arithmetic Progression in which
constant difference = $7-3 = 11-7 = 15-11 = 4$, and first term = 3

Geometric Progression

A geometric progression is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio.

The general form of a GP is a, ar, ar^2, ar^3 and so on.

- The n th term of a GP series is $T_n = ar^{n-1}$,
where a = first term and r = common ratio = T_n/T_{n-1} .
- The formula applied to calculate sum of first n terms of a GP: $S_n = a[r^n - 1]/[r - 1]$
- The sum of infinite terms of a GP series $S_\infty = a/(1-r)$ where $0 < r < 1$.
- When three quantities are in GP, the middle one is called as the geometric mean of the other two. If a, b and c are three quantities in GP and b is the geometric mean of a and c i.e. $b = \sqrt{ac}$

For example, 4,8,16,32,64 is a in which

First term $a = 4$ and r (common ratio) = $8/4 = 16/8 = 32/16 = 2$

Harmonic Progression

A series of terms is known as a HP series when their reciprocals are in arithmetic progression.

- The n th term of a HP series is $T_n = 1/[a + (n - 1) d]$.

In order to solve a problem on Harmonic Progression, one should make the corresponding AP series and then solve the problem.

- n th term of H.P. = $1/(\text{nth term of corresponding A.P.})$
- If three terms a, b, c are in HP, then $b = 2ac/(a+c)$.

For Example: $1/a, 1/(a+d), 1/(a+2d)$, and so on are in HP because $a, a + d, a + 2d$ are in AP.

Relation between Means of AP, GP and HP

$$AM = (a+b)/2$$

$$GM = \sqrt{ab}$$

$$HM = 2ab/a+b$$

By solving these equations, we can say

$$GM^2 = AM * HM$$

Standard Series

The sum of first n natural numbers = $\sum n = 1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$

The sum of squares of first n natural numbers = $\sum n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

The sum of cubes of the first n natural number

$$= \sum n^3 = 1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

Sum of first n odd natural numbers = $1 + 3 + 5 + \dots + \text{up to } n \text{ terms} = n^2$.

Sum of first n even natural numbers = $2 + 4 + 6 + \dots + \text{up to } n \text{ terms} = n(n+1)$.

Example 1: Find the nth term of the A.P. 5, 9, 13, 17,

- (1) $4n+3$ (2) $4n+2$ (3) $4n+1$ (4) $5n+1$

Solution: (3) In the given A.P.,

we have $a = 5$, $d = 9 - 5 = 4$

using $a_n = a + (n-1)d$, we get

$$= 5 + (4-1)6 \Rightarrow a_n = 4n+1$$

Example 2: How many terms are there in the A.P. given by 15, 21, 27,, 279

- (1) 85 (2) 55 (3) 43 (4) 45

Solution: (4) let the A.P. contain n terms, then

we have $a = 15$, $d = 21 - 15 = 6$ $a_n = 279$

using $a_n = a + (n-1)d$, we get

$$\Rightarrow 279 = 15 + (n-1)6$$

$$\Rightarrow 6n = 270 \Rightarrow n = 45$$

Example 3: Find the 7th term of the G.P. 4, 8, 16,

- (1) 256 (2) 156 (3) 254 (4) 264

Solution: (1) In the given G.P., $a = 4$, $r = \frac{8}{4} = 2$

Using $a_n = ar^{n-1}$, where $n = 7$

$$a_7 = 4 * 2^{7-1} = 4 * 2^6 = 256$$

Example 4: Consider the sequence where nth term is $t_n = \frac{n}{(n+2)}$, $n = 1, 2, \dots$

The value of $t_3 * t_4 * t_5 * \dots * t_{53}$ equals:

- A. $\frac{2}{495}$
B. $\frac{2}{477}$
C. $\frac{12}{55}$
D. $\frac{1}{1485}$
E. $\frac{1}{2970}$

Solution: Answer is option A

$$t_3 * t_4 * t_5 * \dots * t_{53} = \frac{3}{5} * \frac{4}{6} * \frac{5}{7} * \dots * \frac{53}{55}$$

$$= \frac{3 * 4 * (5 * 6 * 7 * \dots * 53)}{(5 * 6 * 7 * \dots * 53) * 54 * 55}$$

$$= \frac{3 * 4}{54 * 55} = \frac{2}{495}$$

LEVEL - I

Q(1-5) Find the missing number in the given series.

1. 2, 7, 10, 22, 18, 37, 26, ?

- A. 42 B. 52 C. 46 D. 62 E. None of these

2. 279936, 46656, 7776, 1296, 216, ?

- A. 60 B. 46 C. 36 D. 66 E. None of these

3. 12, 38, 116, 350, 1052, ?

- A. 1800 B. 2200 C. 2800 D. 3158 E. None of these

4. 46080, 3840, 384, 48, 8, 2, ?

- A. 1 B. $\frac{1}{64}$ C. $\frac{1}{8}$ D. 2 E. None of these

5. 5, 28, 57, 88, 125

- A. 156 B. 147 C. 166 D. 186 E. None of these

Q (6-10). Find the wrong number in the following series.

6. 13, 18, 25, 30, 37, 40

- A. 25 B. 30 C. 37 D. 40 E. None of these

7. 15, 46, 63, 71, 75, 77, 78

- A. 15 B. 46 C. 63 D. 71 E. 75

8. 2, 3, 12, 37, 86, 166, 288

- A. 37 B. 86 C. 166 D. 288 E. 12

9. 15, 16, 34, 105, 424, 2124, 12756

- A. 16 B. 34 C. 105 D. 424 E. 2124

10. 3, 7, 15, 27, 63, 127, 255

- A. 7 B. 15 C. 27 D. 63 E. 127

11. Look at this series: 7, 10, 8, 11, 9, 12, ... What number should come next?

- A. 7 B. 10 C. 12 D. 13

12. Look at this series: 53, 53, 40, 40, 27, 27, ... What number should come next?

- A. 12 B. 14 C. 27 D. 53

13. Look at this series: 21, 9, 21, 11, 21, 13, 21, ... What number should come next?

- A. 14 B. 15 C. 21 D. 23

14. Look at this series: 1.5, 2.3, 3.1, 3.9, ... What number should come next?

- A. 4.2 B. 4.4 C. 4.7 D. 5.1

15. Look at this series: 14, 28, 20, 40, 32, 64, ... What number should come next?

LEVEL – II

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|------------------------------------|----------|----------|----------|
| 1. C, E, H, L, Q, ? | | | |
| A. R | B. W | C. U | D. X |
| 2. H, V, G, T, F, R, E, P, ? | | | |
| A. K, L | B. D, N | C. C, D | D. L, K |
| 3. EV, JQ, OL, ? | | | |
| A. TH | B. TG | C. FT | D. US |
| 4. prt bdf, hjl, npr | | | |
| A. uwy | B. vya | C. vxz | D. axy |
| 5. C-3, E-6, G-12, I-24, K-48, ? | | | |
| A. S-48 | B. M-96 | C. L-96 | D. O-48 |
| 6. Q1F, S2E, U6D, W21C, ? | | | |
| A. Z88B | B. Y66B | C. Y88B | D. Y44B |
| 7. 2Z5, 7Y7, 14X9, 23W11, 34V13, ? | | | |
| A. 27U24 | B. 47U15 | C. 45U15 | D. 47V14 |
| 8. P3C, R5F, T8I, V12L, ? | | | |
| A. Y17O | B. X17M | C. X17O | D. X16O |

9. J2Z, K4X, I7V, ?, H16R, M22P

- A. L11S B. L12T C. L11T D. L12S

10. A, CD, GHI, ?, UVWXY

- A. LMNO B. MNO C. MNOP D. NOPQ

11. dc__abc_cb_abcb_abaa

- A. bacaa B. badaa C. cadaa D. bbcaa

12. ab__ea_bc_aaabc_aa_b

- A. cdaabb B. cdabab C. caadaa D. cdadaa

13. bcp_bc__qbcdpq_bcd__qr

- A. qdprpq B. qdpqr C. qdprep D. qdpepr

14. 21 : 3 :: 574 : ?

- A. 23 B. 82 C. 97 D. 113

15. 1 : 1 :: 25 : ?

- A. 26 B. 125 C. 240 D. 625

16. 121 : 12 :: 25 : ?

- A. 1 B. 2 C. 6 D. 7

17. Finding the odd-pair, (95 - 82) , (69 - 56) , (55 - 42) , (48 - 34)

- A. 95 - 82 B. 69 - 56 C. 55 - 42 D. 48 - 34

18. Finding the odd-pair, (3 - 5) , (5 - 3) , (6 - 2) , (7 - 3)

- A. 3 - 5 B. 5 - 3 C. 6 - 2 D. 7 - 3

19. Choose or find odd number: 45 , 99 , 109 , 126 , 207

- A. 45 B. 99 C. 109 D. 126

20. AYBXCW, FTGSHR, KOLNMM, ?

- A. PQRSTU B. PJQIRH C. PIQHRQ D. PDQIRH