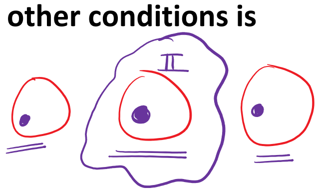
Lecture 4

Tuesday, January 18, 2022 02:09 PM

Bayes' theorem describes the probability of occurrence of an event related to any condition. It is also considered for the case of conditional probability.

Bayes theorem is also known as the formula for the probability of "causes".

For example: if we have to calculate the probability of taking a blue ball from the second bag out of three different bags of balls, where each bag contains three different color balls viz. red, blue, black. In this case, the probability of occurrence of an event is calculated depending on other conditions is known as conditional probability.



Let $E_1, E_2, \ldots, E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \ldots, E_n$ have nonzero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

$$\boxed{P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{k=1}^{n} P(E_k)P(A|E_k)}}$$

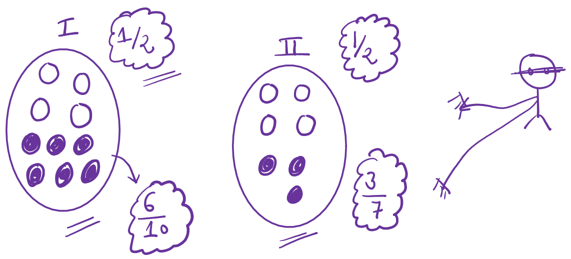
If A and B are two events, then the **formula for Bayes theorem** is given by:

P(A|B) = P(A∩B)/P(B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black.
Find the probability that it was drawn from Bag I.

Let E1 be the event of choosing bag I, E2 the event of choosing bag II, and A be the event of drawing a black ball.



$$\frac{\boxed{\frac{1}{2} * \frac{6}{10}}}{\boxed{\frac{1}{2} * \frac{6}{10} + \frac{1}{2} * \frac{3}{7}}}$$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$0 \cdot 10, \quad 0 \cdot 5$

You are planning a picnic today, but the morning is cloudy

- Oh no! 50% of all rainy days start off cloudy!
- But cloudy mornings are common (about 40% of days start cloudy)
- And this is usually a dry month (only 3 of 30 days tend to be rainy, or 10%)

What is the chance of rain during the day?

$$\frac{0 \cdot 5 \times 0 \cdot 10}{0 \cdot 4}$$

$\frac{2}{3} \cdot \frac{1}{3}$ $\qquad$ $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$

A man is known to speak the truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually a four.

A: Truth (4)

E_1: 4 comes

E_2: 4 does not come

$$P(E_1|A) = \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \cdot \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}} \quad \frac{\frac{2}{18}}{\frac{7}{18}} = 2/7$$

60 X | 40 Y

In a factory, machine X produces 60% of the daily output and machine Y produces 40% of the daily output.

2% of machine X's output is defective, and 1.5% of machine Y's output is defective.

One day, an item was inspected at random and found to be defective. What is the probability that it was produced by machine X?

A $\frac{2}{3}$	B $\frac{1}{2}$
C $\frac{1}{3}$	D $\frac{1}{50}$

$$= \frac{\frac{60}{100} \cdot \frac{2}{100}}{\frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1.5}{100}} = \frac{\frac{120}{10000}}{\frac{120}{10000} + \frac{600}{10000 \times 10}}$$

$$= \frac{120}{10000} \times \frac{10000}{120 + 60} = \frac{120}{180} = \frac{2}{3}$$

$$\frac{5 \times 95}{5 \times 95 + 15 \times 95}$$

0

$$\frac{5}{100} \times \frac{95}{100} + \frac{.15}{100} \times \frac{95}{100}$$

A test for a disease gives a correct positive result with a probability of 0.95 when the disease is present, but gives an incorrect positive result (false positive) with a probability of 0.15 when the disease is not present.

If 5% of the population has the disease, and Jean tests positive to the test, what is the probability Jean really has the disease?

A 0.05	B 0.25

really has the disease?

A 0.05

B 0.25

C 0.33

D 0.5

=> E_1 => corr.
E_2 => false

$$\frac{(0.95)\left(\frac{5}{100}\right)}{(0.95)\left(\frac{5}{100}\right)+(0.15)\left(\frac{95}{100}\right)}$$

A supermarket buys light globes (light bulbs) from three different manufacturers - Brightlight (35%), Glowglobe (20%) and Shinewell (45%).

In the past, the supermarket has found that 1% of Brightlight's globes are faulty, and that 1.5% of each of Glowglobe's and Shinewell's globes are faulty.

A customer buys a globe without looking at the manufacturer's name - in other words, it's a random choice. When she gets home, she finds the globe is faulty.

What is the probability she chose a Shinewell's globe?

A 0.23

B 0.26

C 0.45

D 0.51

Let

A_1 = The globe was a Brightlight's. $P(A_1) = 0.35$

A_2 = The globe was a Glowglobe's. $P(A_2) = 0.20$

A_3 = The globe was a Shinewell's. $P(A_3) = 0.45$

and B = A globe chosen at random is faulty.

$$P(A_3|B) = \frac{0.45\times 0.15}{0.45\times 0.15 + 0.20\times 0.15 + 0.35\times 0.01}$$

A glazier buys his glass from four different manufacturers - Clearglass (10%), Strongpane (25%), Mirrorglass (30%) and Reflection (35%).

In the past, the glazier has found that 1% of Clearglass' product is cracked, 1.5% of Strongpane's product is cracked, and 2% of Mirrorglass' and Reflection's products are cracked.

The glazier removes the protective covering from a sheet of glass without looking at the manufacturer's name - in other words, it's a random choice. He finds the glass is cracked. What is the probability it was made by Mirrorglass?

$\frac{.35}{100}$

A 0.06	B 0.21
C 0.34	D 0.39

Let

A_1 = The glass was from Clearglass.

A_2 = The glass was from Strongpane.

A_3 = The glass was from Mirrorglass

A_4 = The glass was from Reflection.

and B = A glass chosen at random is cracked.

Dr. Foster remembers to take his umbrella with him 80% of the days.
It rains on 30% of the days when he remembers to take his umbrella, and it rains on 60% of the days when he forgets to take his umbrella.

What is the probability that he remembers his umbrella when it rains?

A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{4}{5}$

$(U) = 0.80$ $P(R|\ \ =$

$P(\bar{U}) = 0.20$ $P(R\ U)$