Shift operator:

$$E(f(x)) = f(x+h)$$

$$E(c) = c$$

$$E^2(c) = c$$

Forward difference operator

$$\Delta f(x) = f(x+h) - f(x)$$

R1 $\Delta(c) = 0$

R2 $\Delta(x) = 1$

R3 $\Delta x^2 = 2x+1$

Δ doesn't behave like derivative operator.

factorial polynomial :-

1. $x^{[0]} = x^0 = 1$
2. $x^{[1]} = x$
3. $x^{[2]} = x(x-1)$

   $\cancel{x^{[2]} = x^2 - x}$
4. $x^{[3]} = x(x-1)(x-2)$

$\cancel{x^{[0]} = x^0 = 1}$

$\cancel{x^{[1]} = x}$

$1 = x^{[0]}$

$x = x^{[1]}$

$x^2 = x^{[2]} + x = x^{[2]} + x^{[1]}$

we want check the behavior of Δ when it is applied on factorial polynomial.

1. $\Delta(c) = 0$
2. $\Delta x^{[1]} = \Delta(x) = 1$

$f(x) = x^2 - x$

$\Delta f(x) = f(x+1) - f(x)$

② $\Delta x^{[1]} = \Delta(x) = 1$

③ $\Delta x^{[2]} = \Delta\, x(x-1)$

$= \Delta\left[x^2 - x\right]$

$f(x) = x^2 - x$

$\Delta f(x) = f(x+1) - f(x)$

$= (x+1)^2 - (x+1) - \left[x^2 - x\right]$

$= \cancel{x^2} + \cancel{1} + 2x - \cancel{x} - \cancel{1} - \cancel{x^2} + \cancel{x}$

$= 2x = 2x^{[1]}$

$\Delta x^{[2]} = 2x^{[1]}$

Δ behaves like derivative operator when it is applied on factorial polynomial

— X —

Solution of non-Homogeneous recurrence relation:

Consider a recurrence relation:

$a_{n+2} + a_{n+1} + a_n = f(n)$

$E^2(a_n) + E(a_n) + a_n = f(n)$

$(E^2 + E + 1)\, a_n = f(n)$

$\boxed{G(E) \cdot a_n = f(n)} \longrightarrow *$

$\boxed{a_n = \frac{1}{G(E)} f(n)}$

∴ This is the particular Solⁿ of the recurrence relation.

$* \quad (E^2 + E + 1)\, a_n = f(n)$

$G(E)\, a_n = f(n)$

$a_n = \frac{1}{G(E)} f(n)$

Case 1 When $f(n)$ is a Constant.

$\boxed{f(n) = c}$

$= c + c + c$

$= \boxed{3}c$

$= G(1)\,c$

$\therefore G(E)\, f(n) = G(1) \cdot c$

Case 1 When $f(n)$ is a constant.

$$\boxed{f(n) = c}$$

Consider $G(E).f(n) = [E^2+E+1]c$

$$= E^2(c) + E(c) + c$$

$$\therefore G(E)\,f(n) = G(1).c$$

$$\frac{1}{G(1)} f(n) = \frac{1}{G(E)}.c$$

$$\boxed{\frac{1}{G(1)} f(n) = \frac{1}{G(E)}.f(n)}$$

Q1) Find the P.S. of $\frac{1}{E^2+5E+2}(10)$

Sol^n $P.S = \frac{1}{E^2+5(E)+2}(10)$

$$= \frac{1}{(1)^2+5(1)+2}(10)$$

$$= \frac{1}{1+5+2}10 = \frac{10}{8} = \frac{5}{4}$$

Q2 $\frac{1}{E^2-5E}(100)$

$$= \frac{1}{(1)^2-5(1)}(100)$$

$$= \frac{100}{-4} = -25$$

Case 2 $(E^2+E+1)a_n = f(n)$

$$G(E)\,a_n = f(n)$$

$$a_n = \frac{1}{G(E)}.f(n)$$

$$f(n) = \alpha^n$$

Consider $G(E).f(n) = [E^2+E+1]\alpha^n$

$$= E^2(\alpha^n) + E(\alpha^n) + \alpha^n$$

$$= \alpha^{n+2} + \alpha^{n+1} + \alpha^n$$

$$\Rightarrow [\alpha^2 + \alpha + 1]\alpha^n$$

$$= G(\alpha).\alpha^n$$

$$G(E).f(n) = G(\alpha).\alpha^n$$

$$G(E)\,f(n) = G(\alpha).f(n)$$

$$\frac{1}{G(\alpha)} f(n) = \frac{1}{G(E)} f(n)$$

$$\frac{1}{G(E)} f(n) = \frac{1}{G(\alpha)} f(n)$$

$$- \alpha \quad + \alpha \quad + \alpha$$

$$= [\alpha^2 + \alpha + 1]\alpha^n =$$

$$\frac{1}{G(E)} f(n) = \frac{1}{G(\alpha)} f(n)$$

Q) Find the P.S of $\frac{1}{E^2+5E+3}(2^n)$

Sol^n P.S $= \frac{1}{E^2+5E+3}(2^n)$

$$= \frac{1}{(2)^2+5(2)+3} 2^n = \frac{1}{4+10+3} 2^n$$

$$= \frac{1}{17} 2^n$$

— x —

Case 3 If $f(n)$ is a polynomial (n, n^2, n^3, n^2+n)

$$P.S = \frac{1}{G(E)} f(n)$$

[Replace E by $(1+\Delta)$]

$$= \frac{1}{G(1+\Delta)} f(n)$$

$$= \frac{1}{G(1+\Delta)} [F[n^{[1]}]]$$

$f(n) = n^2$

$F[n^{[1]}] = n^{[2]} + n^{[1]}$

After this, apply binomial expansion.

— x —

Case 4 $f(n) = \alpha^n n^2$

$$P.S = \frac{1}{G(E)} \cdot f(n)$$

$$(\alpha^n \; n^2)$$

$$= \alpha^n \left[\frac{1}{G[\alpha(1+\Delta)]} n^{[2]} + n^{[1]} \right]$$

Apply Binomial expansion,

$$\frac{}{G(E)}$$

$$= \frac{1}{G(E)}\left(\alpha^n \cdot n^2\right)$$

$$= \alpha^n \left[\frac{1}{G(\alpha E)} n^2\right]$$

$$= \alpha^n \left[\frac{1}{G(\alpha(1+\Delta))} n^2\right]$$

Apply Binomial expansion, then it gives you particular Soln.

— X —

$f(n)$	P.S $\frac{1}{G(E)} f(n)$
c	$\frac{1}{G(E)}(c) = \frac{1}{G(1)} c$
α^n	$\frac{1}{G(E)} \alpha^n = \frac{1}{G(\alpha)} \alpha^n$
polynomial	$\frac{1}{G(E)} f(n) = \frac{1}{G(1+\Delta)} F[n^*]$
$\alpha^n g(n)$	$\frac{1}{G(E)} f(n) = \alpha^n \frac{1}{G(\alpha(1+\Delta))} g[n^*]$