Z-test

Test of significance for single mean

We know that if $x_i \, (i = 1, 2 \ldots n)$ is a random sample of size n from a normal population with mean μ and variance σ^2, then the sample mean is distributed normally with mean μ and variance $\frac{\sigma^2}{n}$, ie $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

However this result holds ie $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, even in random sampling from non-normal population provided the sample size n is large (Usually greater than 30)

Thus for large samples, the standard normal variate corresponding to $\bar{x}$ is

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Under the null hypothesis H_0, that the sample has been drawn from a population with mean μ and variance σ^2, ie. there is no significant difference between the sample mean $(\bar{x})$ and population mean (μ), the test statistics (for large sample) is

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

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Critical values of Z at commonly used level of significance for both two tailed and single-tailed test

Critical value (Z_α)	Level of significance 1%	5%	10%
Two tailed test	$\|Z_\alpha\| = 2.58$	$\|Z_\alpha\| = 1.96$	$\|Z_\alpha\| = 1.645$
Right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Ex ① A sample of 900 members has mean 3.4 cms and standard deviation 2.61 cms. Is the sample from a large population of mean 3.25 cm and standard deviation 2.61 cm?

If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

Sol: Null hypothesis (H_0): The sample has been drawn from the population with mean $\mu = 3.25$ cms, and S.D $\sigma = 2.61$ cms.

Alternative hypothesis

$H_1: \mu \neq 3.25$ (Two tailed)

Test statistics

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}} = \frac{0.15 \times 30}{2.61}$$

$$= 1.73$$

Since $|Z| < 1.96$, we conclude that the data don't provide us any evidence against the null hypothesis (H_0) which may, therefore be accepted at 5% level of significance

95% fiducial limit for population mean μ are

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 3.40 \pm 1.96 \frac{(2.61)}{30}$$

$$= 3.40 \pm 0.1705$$

ie 3.5705 and 3.2295

98% fiducial limit for population mean μ are

$$\bar{x} \pm 2.33 \frac{\sigma}{\sqrt{n}} = 3.40 \pm 2.33 \times \frac{2.61}{30}$$

$$= 3.40 \pm 0.2027$$

ie 3.6027 and 3.1973

Ex (2) A sample of 100 students is taken from a large population. The mean height of the student in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm. and the S.D is 10 cm ?

Sol Null hypothesis : H_0 : The sample has been drawn from population with mean height 165 cm, ie $\mu = 165$ cms

Alternative hypothesis

$H_1 : \mu \neq 165$ cms (Two tailed)

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{160 - 165}{\frac{10}{\sqrt{100}}} = -5$$

$Z = -5$

Now $|Z| = 5 > 1.96$

Thus the null hypothesis is rejected at 5% level of significance.

So it is not correct to assume that the sample has been drawn from a population with mean height 165 cms.

Ex (13) The mean breaking strength of the cables supplied by a manufacturer is 1800 with a S.D of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. In order to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance.

Sol: $\bar{x} = 1850, \ n = 50, \ \mu = 1800, \ \sigma = 100$

Null hypothesis $H_0 : \bar{x} = \mu$

Alternate hypothesis $H_1 : \bar{x} > \mu$ (Right tailed)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1850 - 1800}{100/\sqrt{50}} = 3.54$$

Here $|Z| > 2.33$

Null hypothesis is rejected and alternate hypothesis is accepted

So we conclude that the data suggest that there is increase in breaking strength.