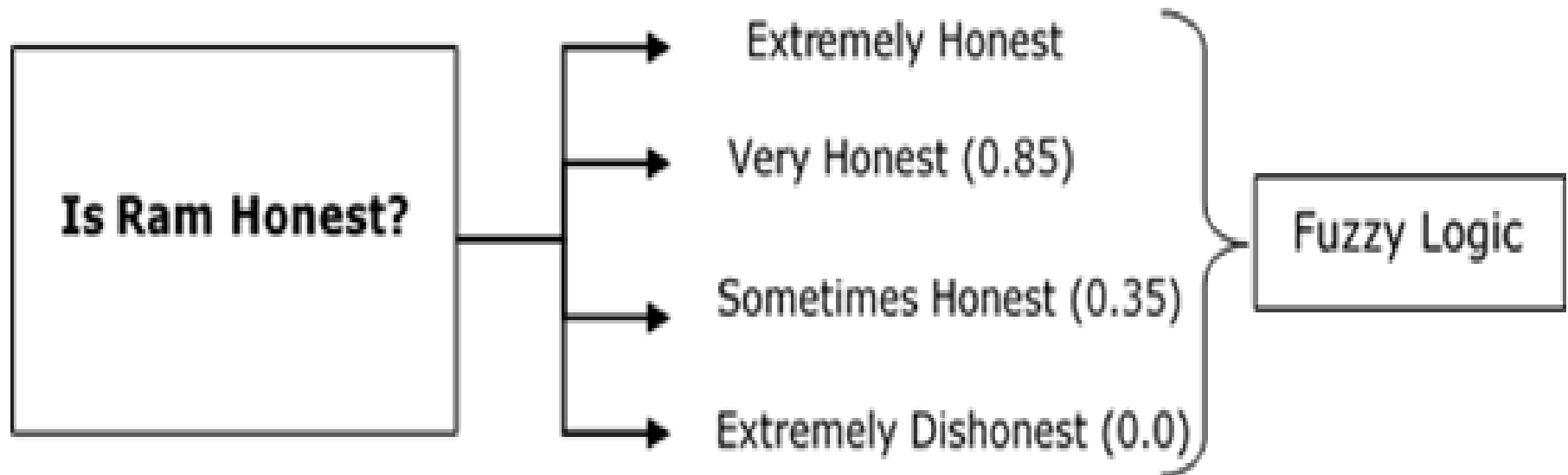
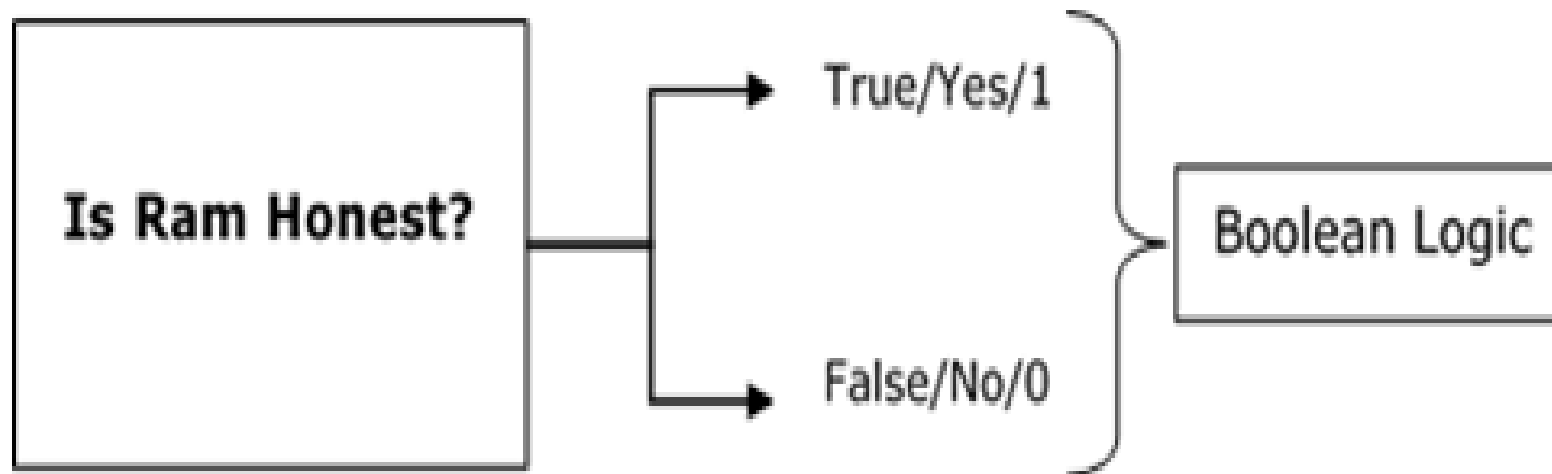


Fuzzy Logic

Fuzzy means VAGUENESS.

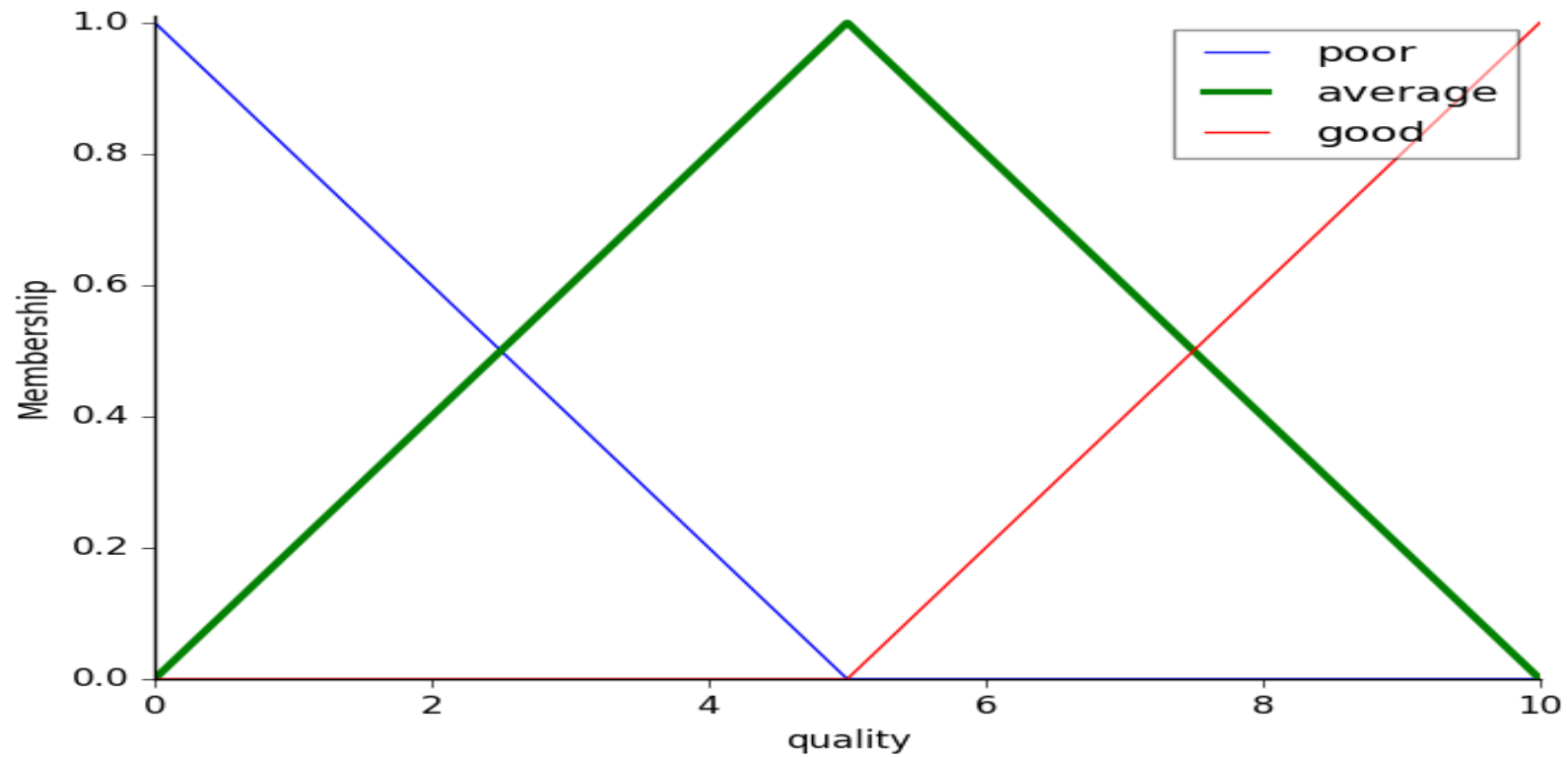
Introduced by Lofti A. Zadeh (1965)

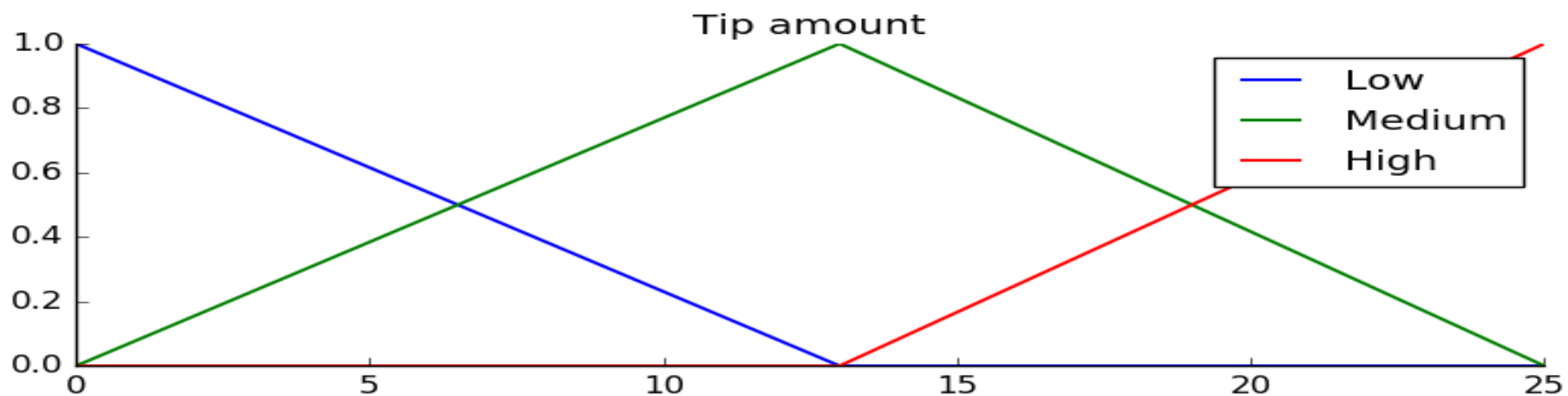
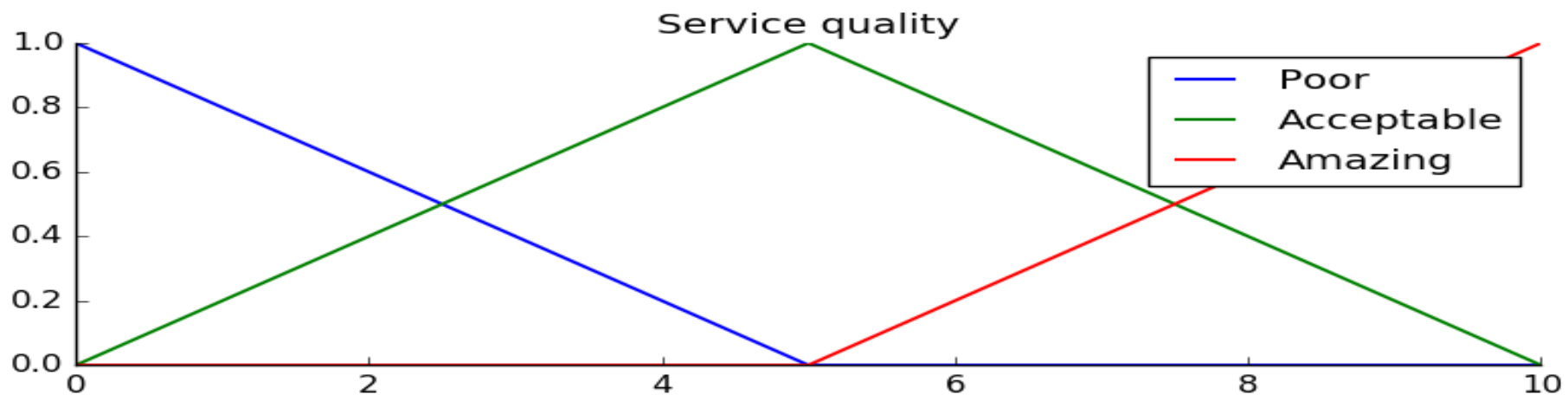
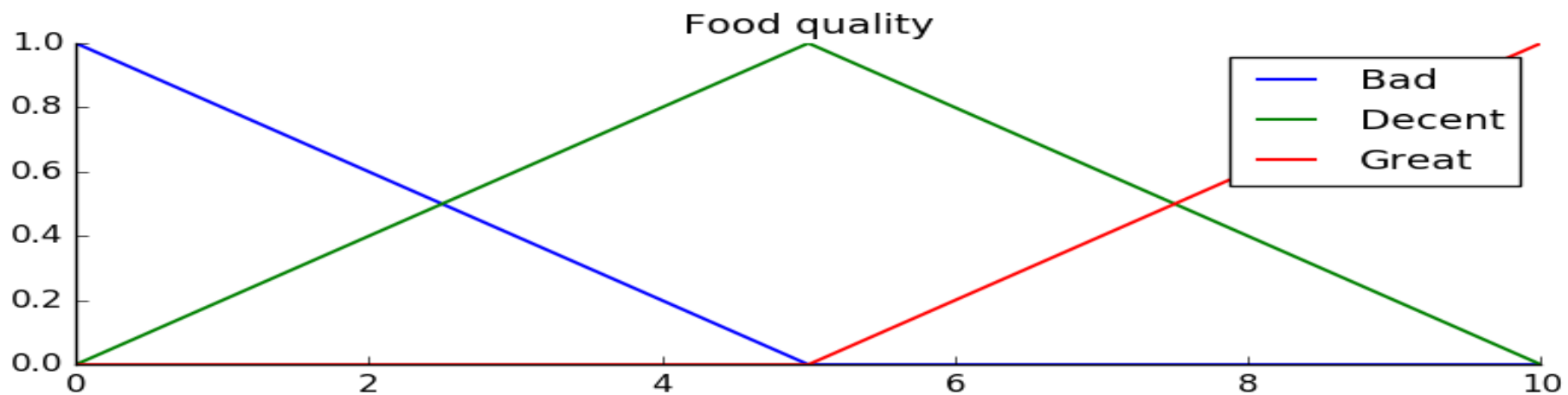


FUZZY RULES

1. IF the *service* was good *or* the *food quality* was good, THEN the tip will be high.
2. IF the *service* was average, THEN the tip will be medium.
3. IF the *service* was poor *and* the *food quality* was poor THEN the tip will be low.
4. IF the *service* was poor *and* the *food quality* was good THEN the tip will be average.
5. IF the *service* was poor *and* the *food quality* was average THEN the tip will be average.
6. IF the *service* was average *and* the *food quality* was average THEN the tip will be average.

Input 1 (Service) Input 2 (Food Quality) Output (Tip)





Example

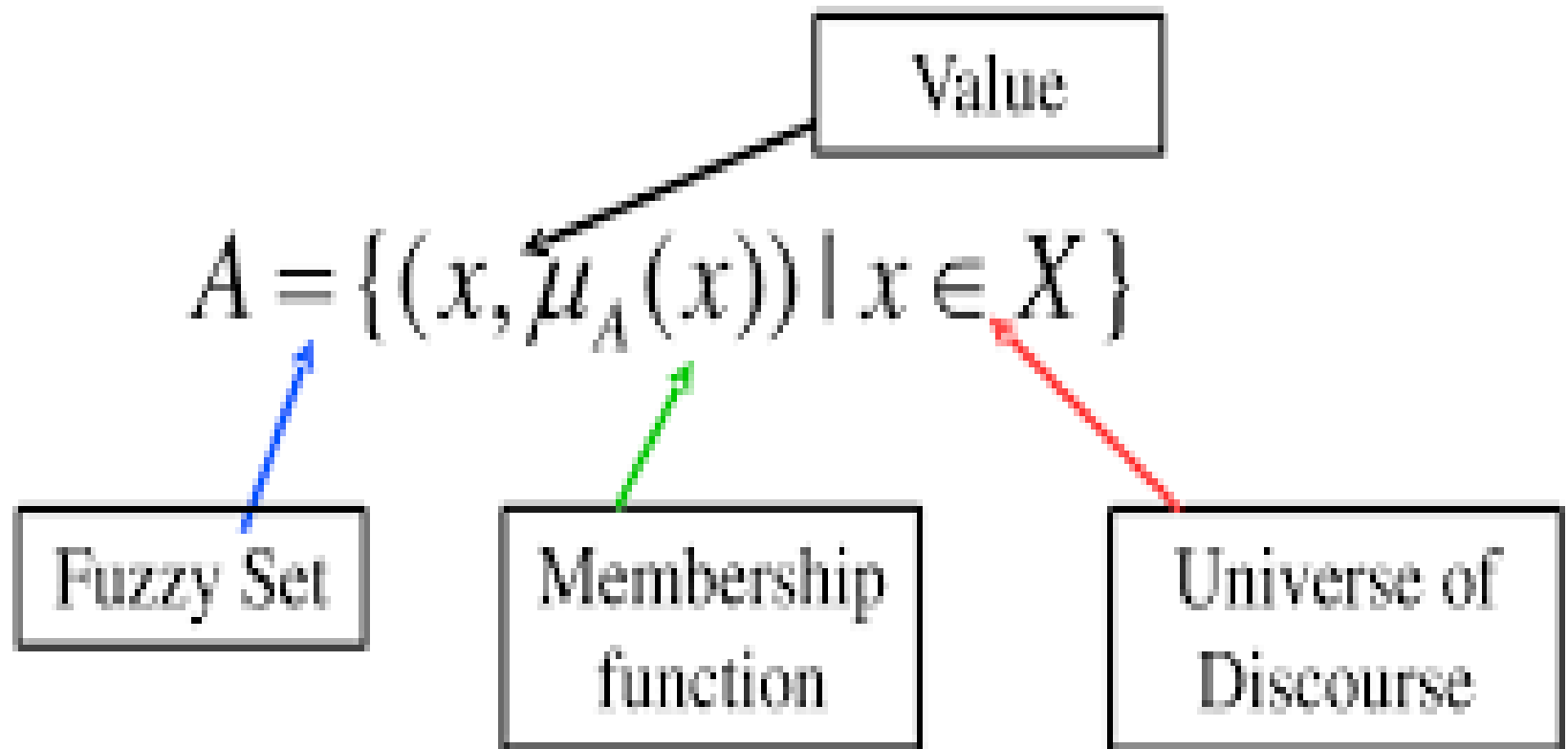
SERVICE

BAD $F1 = \{ (0, 1), (1, 0.8), (2, 0.6), (3, 0.5), (4, 0.4), (5, 0) \}$

AVERAGE F2 = { }

GOOD $F3 = \{$ $\}$

- A fuzzy set can be represented by an ordered set of pairs:



RULE TABLE

FUZZY RELATIONS

$$A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\} \text{ and } B = \{(b_1, 0.5), (b_2, 0.6)\}$$

$$R = A \times B = \begin{array}{cc} & \begin{matrix} b_1 & b_2 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \left[\begin{array}{cc} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{array} \right] \end{array}$$

Fuzzy Set Operation (Continue)

Example:

$$A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\} \quad B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$$

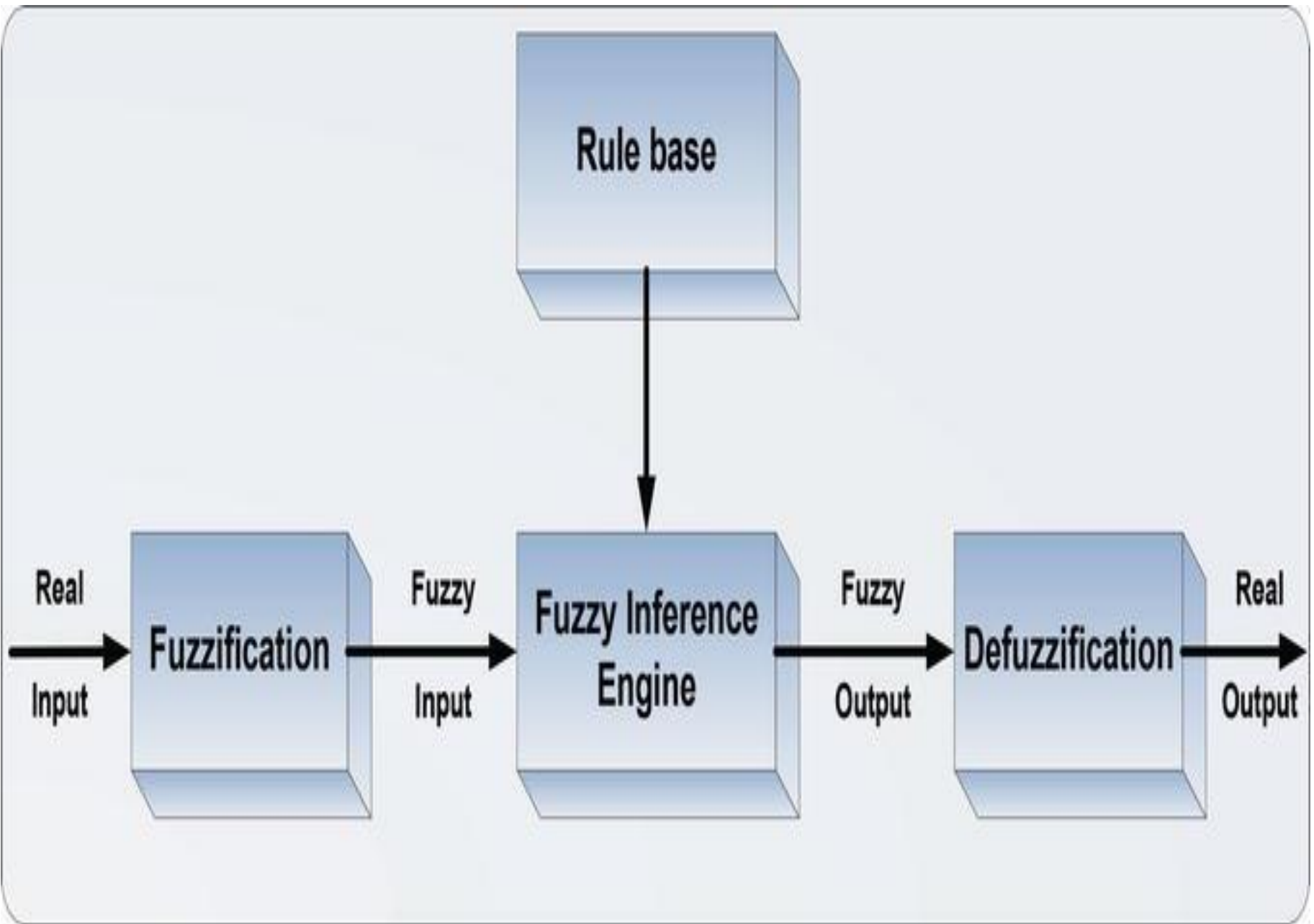
Intersection:

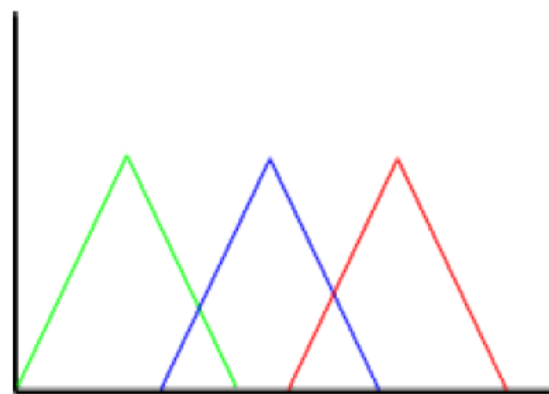
$$A \cap B = \{(x_1, 0.5), (x_2, 0.2), (x_3, 0)\}$$

Because

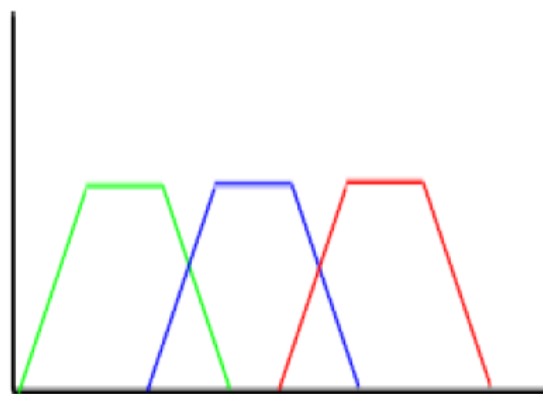
$$\begin{aligned}\mu_{A \cap B}(x_1) &= \min(\mu_A(x_1), \mu_B(x_1)) \\ &= \min(0.5, 0.8) \\ &= 0.5\end{aligned}$$

$$\mu_{A \cap B}(x_2) = 0.2 \quad \text{and} \quad \mu_{A \cap B}(x_3) = 0$$

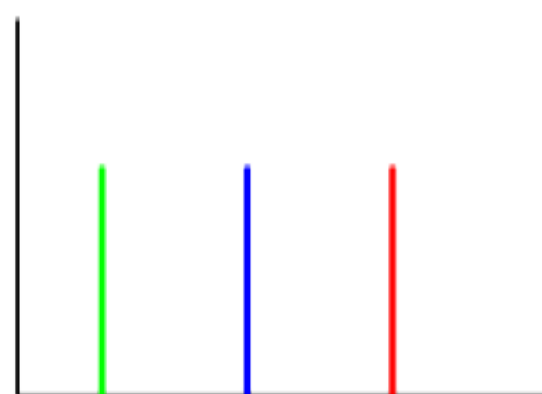




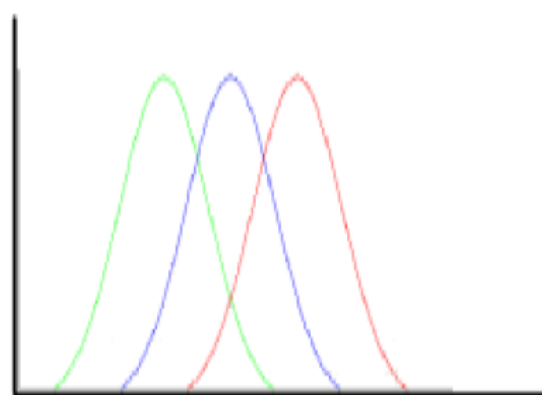
triangular



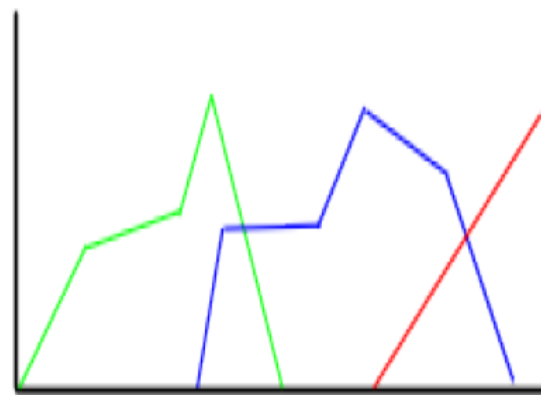
trapezoidal



singleton



Gaussian



Piecewise linear

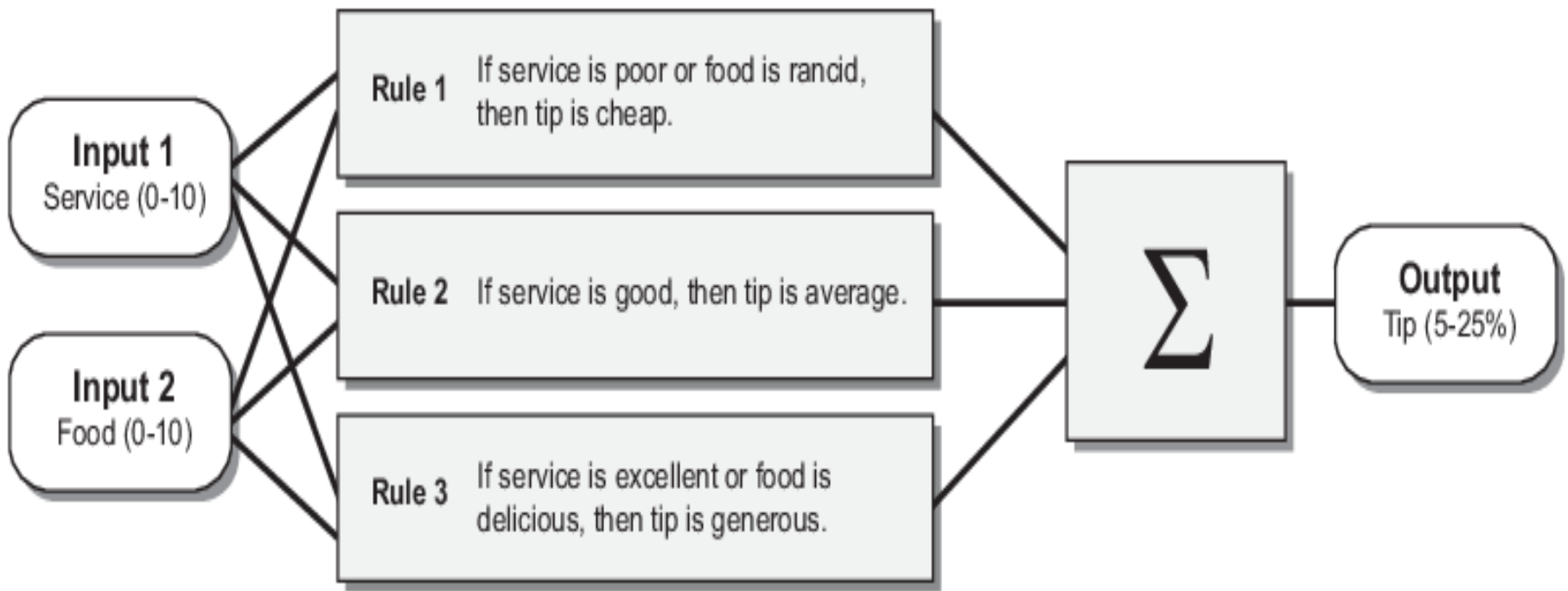
METHODS OF MEMBERSHIP VALUE ASSIGNMENT

The various methods of assigning membership values are:

- Intuition,
- Inference,
- Rank ordering,
- Angular fuzzy sets,
- Neural networks,
- Genetic algorithm,
- Inductive reasoning.

Dinner for Two

a 2 input, 1 output, 3 rule system

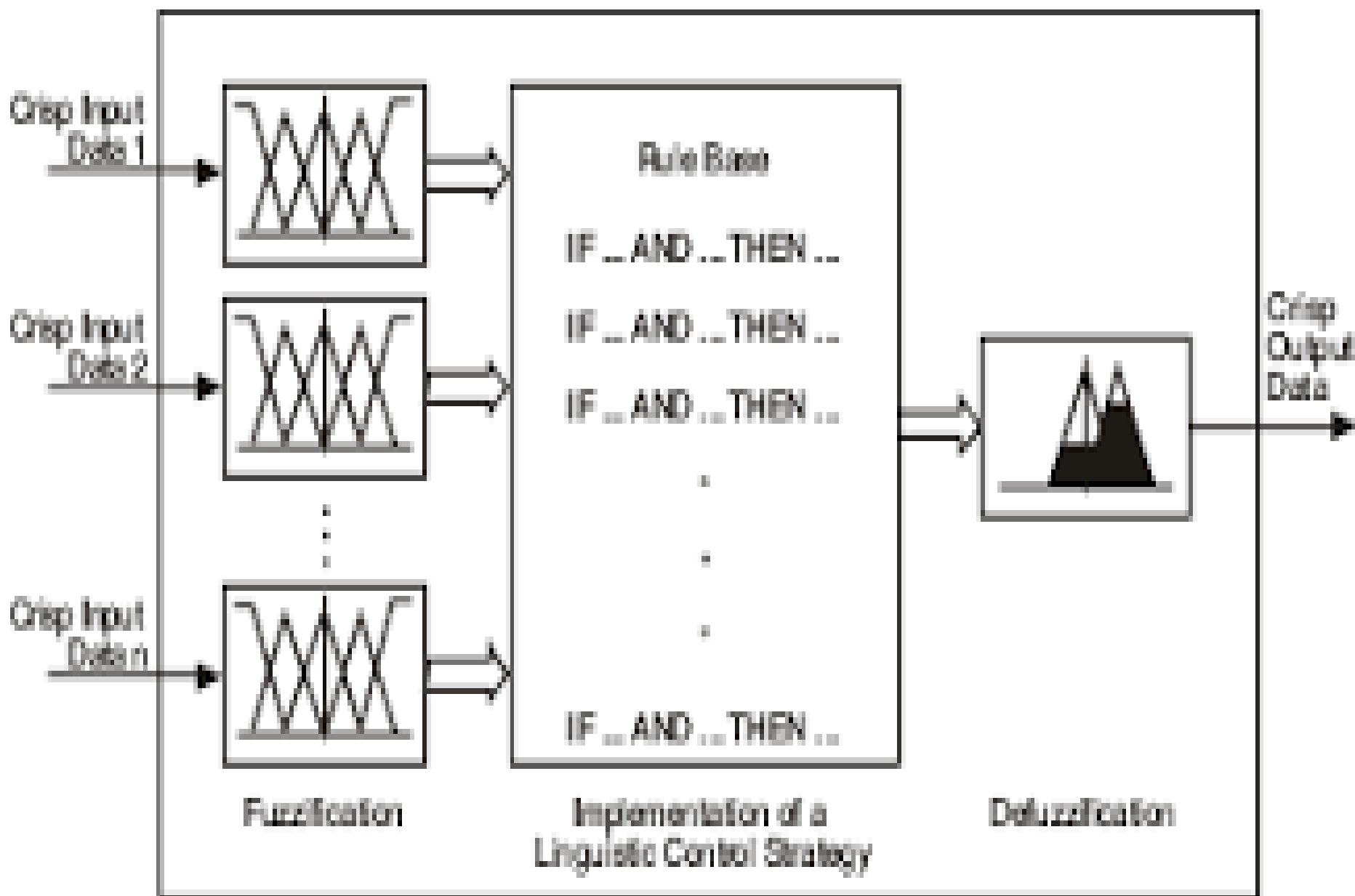


The inputs are crisp (non-fuzzy) numbers limited to a specific range.

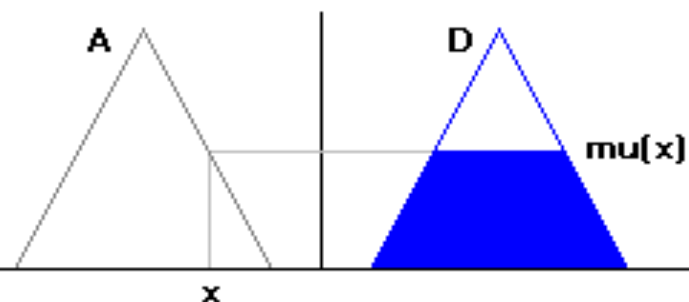
All rules are evaluated in parallel using fuzzy reasoning.

The results of the rules are combined and distilled (defuzzified).

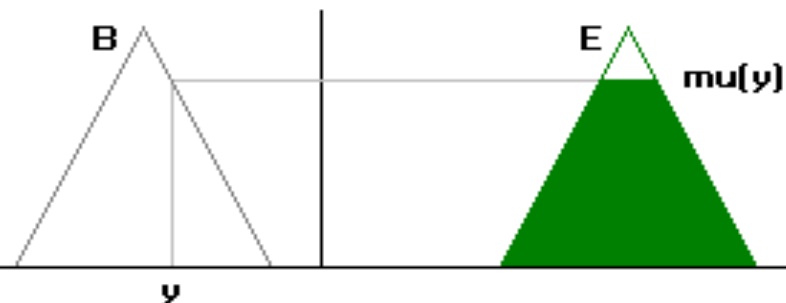
The result is a crisp (non-fuzzy) number.



rule 1: IF x IS A THEN n IS D:



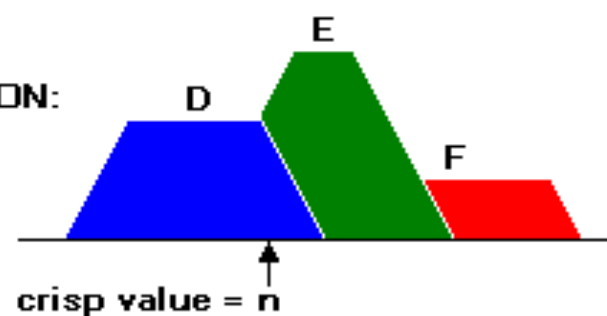
rule 2: IF y IS B THEN n IS E:



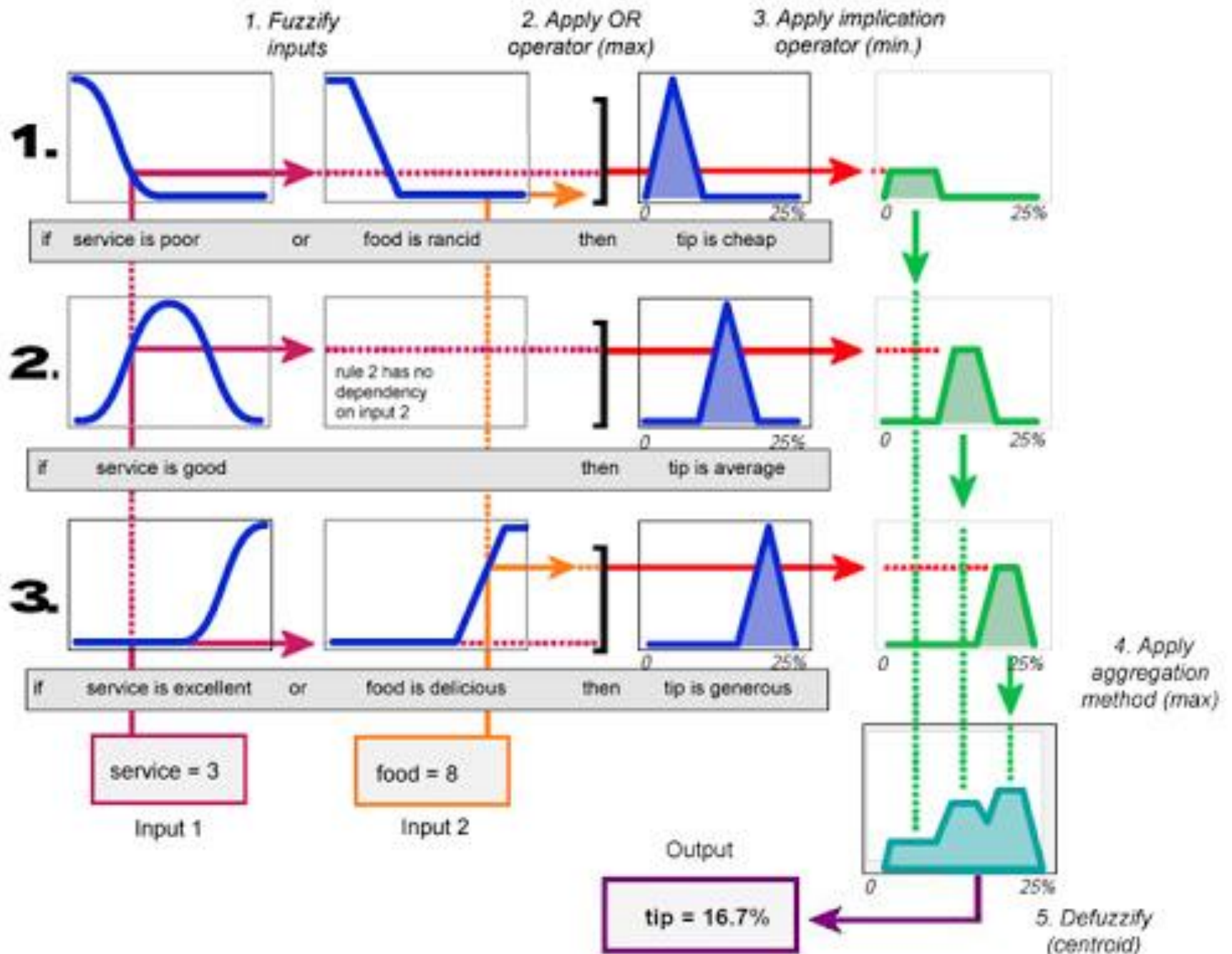
rule 3: IF z IS C THEN n IS F:







DEFUZZIFICATION:



CENTROID DEFUZZIFICATION
USING MAX-MIN INFERENCE



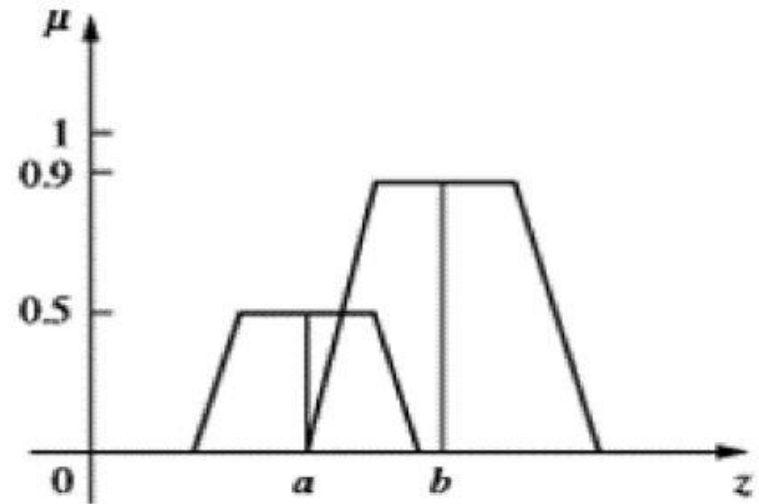
DEFUZZIFICATION METHODS

-  **Lambda-cut method**
-  **Weighted average method**
-  **Maxima methods**
-  **Centroid methods**

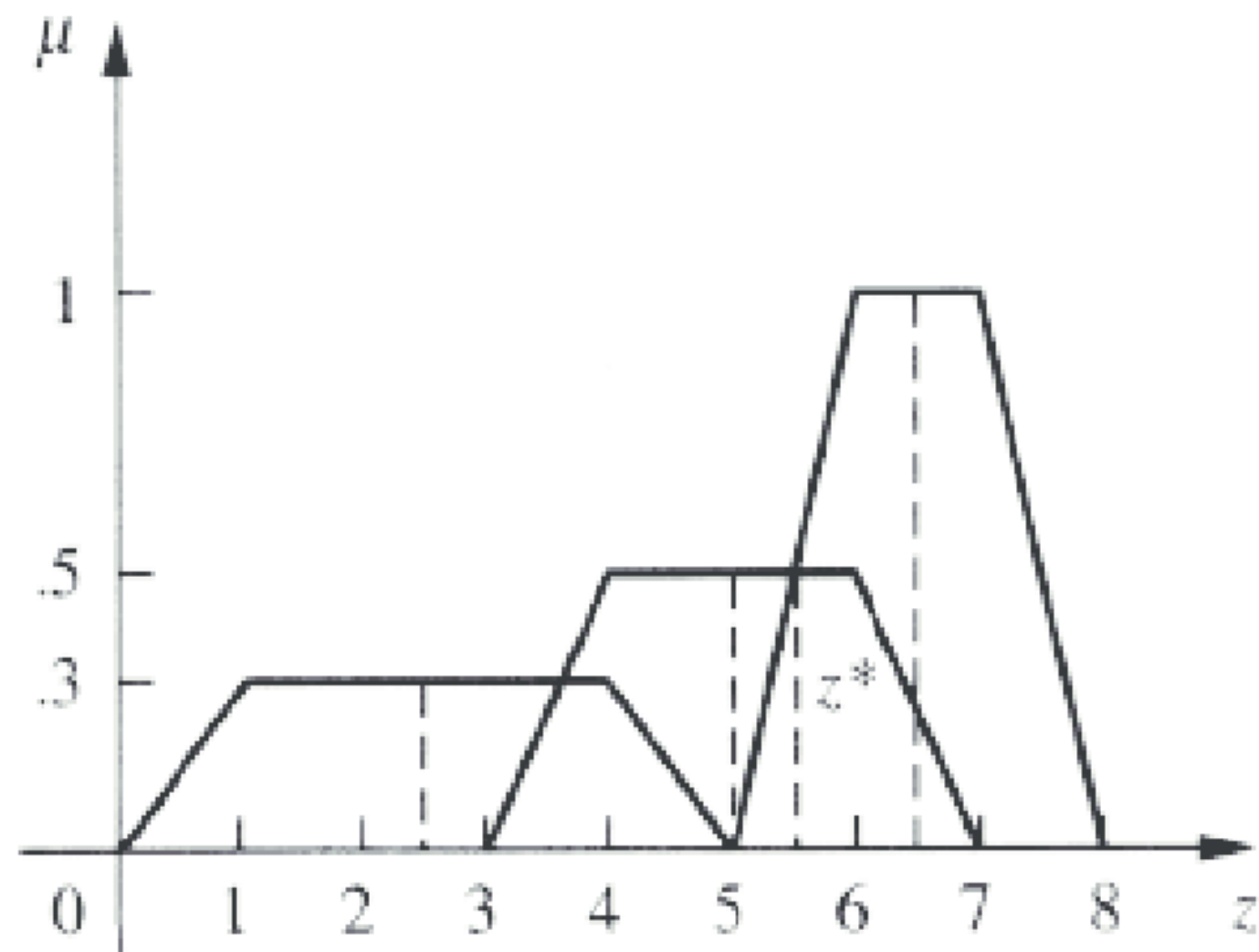
Lambda cut Method

Defuzzification

Weighted
Average
Method

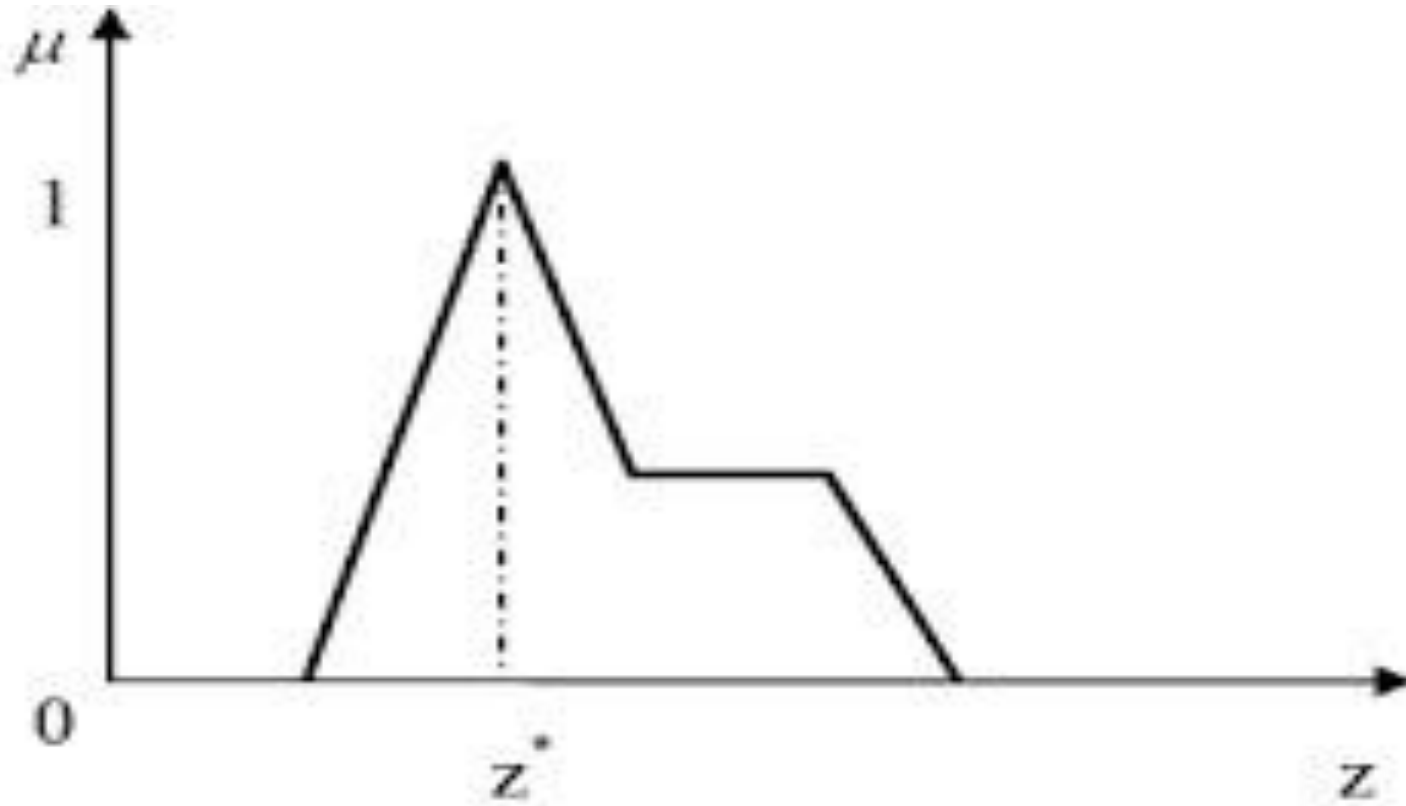


$$z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

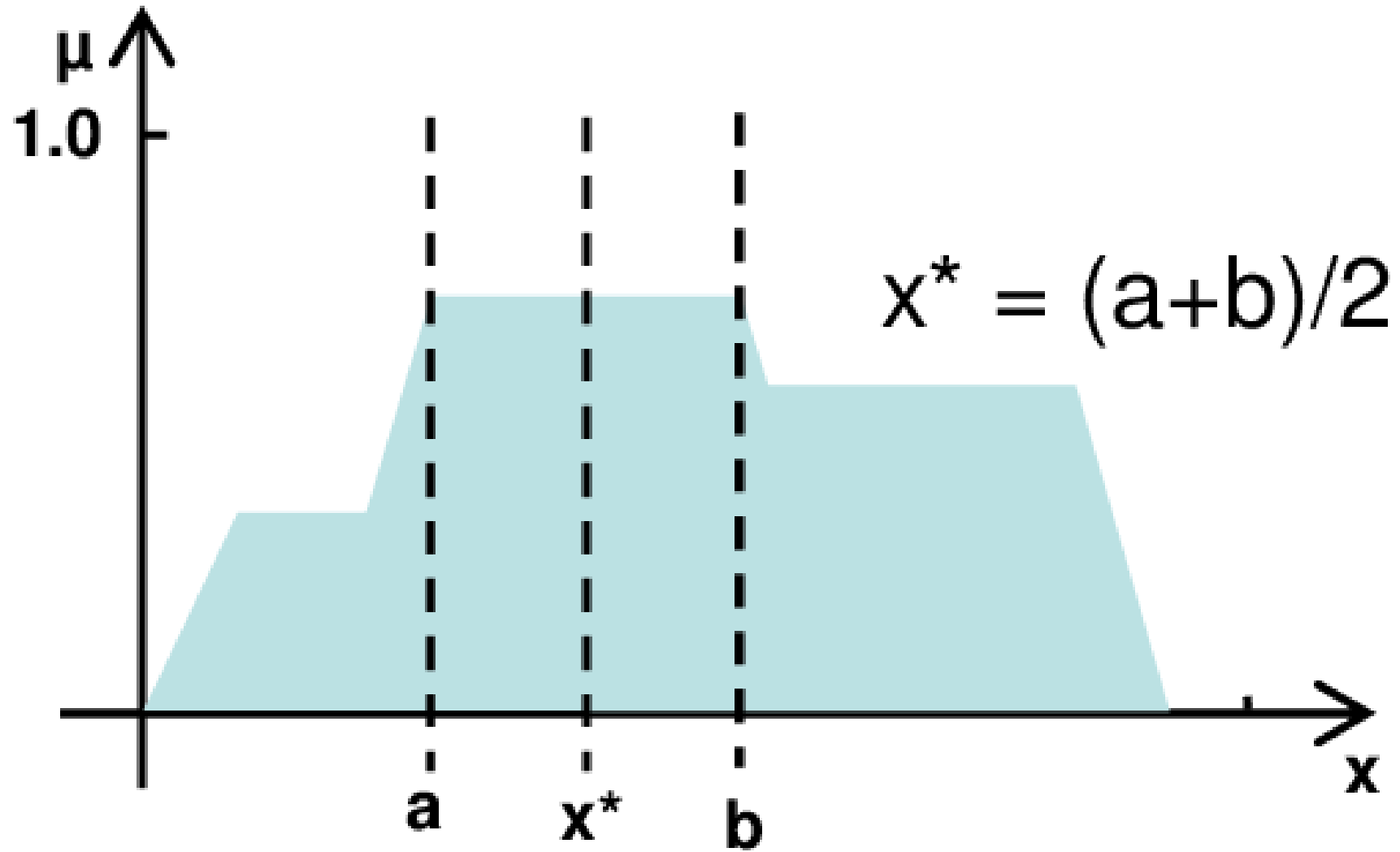


$$z^* = \frac{(.3 \times 2.5) + (.5 \times 5) + (1 \times 6.5)}{.3 + .5 + 1} = 5.41 \text{ meters}$$

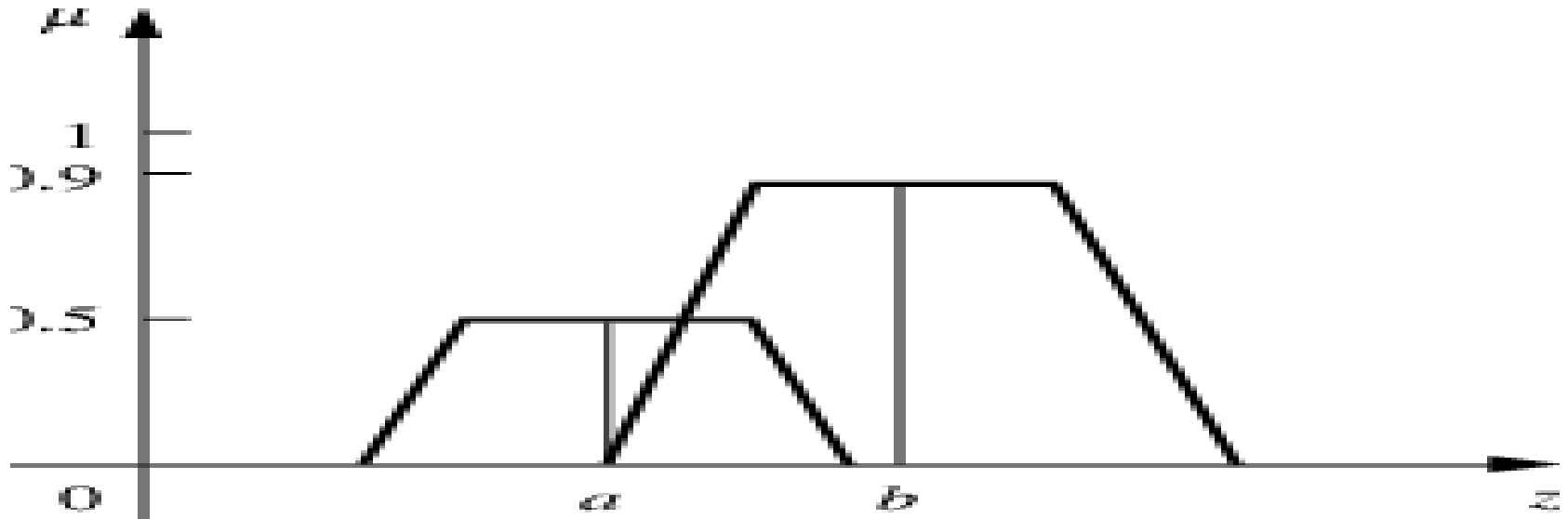
MAXIMA METHOD



MEAN OF MAXIMA



MAXIMA METHODS VS MEAN OF MAXIMA



MAXIMA METHOD \Rightarrow

b

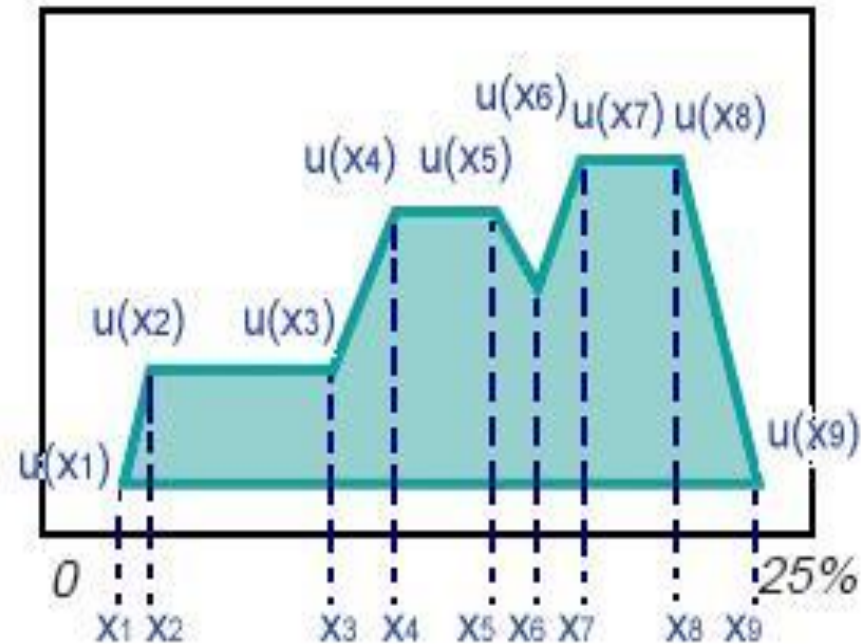
MEAN OF MAXIMA \Rightarrow

$(a + b) / 2$

center of area method (COA)

**5. Defuzzify the
aggregate output
(centroid)**

$$g = \frac{\sum_{i=1}^9 x_i \cdot u(x_i)}{\sum_{i=1}^9 u(x_i)} = 16,7$$



tip= 16,7%

**Result of
defuzzification**

MAMDANI INFERENCE SYSTEM

THANK YOU