From Bayes' theorem,

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2)}$$

Now $P(A_1) = P(A_2) = \dfrac{1}{2}$

$$P\left(\frac{B}{A_1}\right) = {}^5C_1\left(\frac{3}{4}\right)^4 \times \left(\frac{1}{4}\right)^1 = \frac{405}{1024}$$

$$P\left(\frac{B}{A_2}\right) = {}^5C_1\left(\frac{1}{4}\right)^4 \times \left(\frac{3}{4}\right)^1 = \frac{15}{1024}$$

Substituting these values, we get

$$P\left(\frac{A_1}{B}\right) = \frac{(405/1024)\times(1/2)}{[1/2][(405/1024)+(15/1024)]} = \frac{405}{420} = 0.964$$

SOLVED GATE PREVIOUS YEARS' QUESTIONS

1. Let $P(E)$ denotes the probability of the event E. Given $P(A) = 1$, $P(B) = 1/2$, the values of $P(A/B)$ and $P(B/A)$, respectively, are

(a) 1/4, 1/2 (b) 1/2, 1/4
(c) 1/2, 1 (d) 1, 1/2

(GATE 2003, 1 Mark)

Solution: We are given,

$$P(A) = 1 \text{ and } P(B) = 1/2$$

Both events are independent.

So,

$P(A \cap B) = P(A) \cdot P(B) = 1 - 1/2 = 1/2$

$$P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$P(B \mid A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Ans. (d)

2. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(a) 100% (b) 50%
(c) 49% (d) None of these

(GATE 2003, 1 Mark)

Solution: Here, $n = 2$

$x = 0$ (no defective)

$P = P(\text{defective}) = 3/10$

So,

$$P(x=0) = {}^2C_0\left(\frac{3}{10}\right)^0\left(1-\frac{3}{10}\right)^2 = 0.49 = 49\%$$

Ans. (c)

3. A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is

(a) $\dfrac{1}{90}$ (b) $\dfrac{1}{2}$
(c) $\dfrac{19}{90}$ (d) $\dfrac{2}{9}$

(GATE 2003, 2 Marks)

Solution: Probability of drawing two red balls

$= P(\text{first is red})$
$\times P(\text{second is red given that first is red})$

$$= \frac{5}{10}\times\frac{4}{9} = \frac{2}{9}$$

Ans. (d)

4. A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$, respectively, denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x) f_2(x)dx$
(c) $\int_0^1 f_1(x)f_2(t-x)dx$ (d) $\max\{f_1(t), f_2(t)\}$

(GATE 2003, 2 Marks)

Solution: Let the time taken for first and second modules be represented by x and y and total time $= t$.
$\therefore$ $t = x + y$ is a random variable.

Now the joint density function,

$$g(t) = \int_0^t f(x,y)\,dx = \int_0^t f(x,t-x)\,dx = \int_0^t f_1(x)f_2(t-x)\,dx$$

which is also called as convolution of f_1 and f_2, abbreviated as $f_1 * f_2$.

Therefore, the correct answer is option (c).

Ans. (c)

5. If a fair coin is tossed four times, then what is the probability that two heads and two tails will result?

(a) 3/8 (b) 1/2

(c) 5/8 (d) 3/4

(GATE 2004, 1 Mark)

Solution: Coin is tossed four times. Hence, the total outcomes $= 2^4 = 16$.

Favorable outcomes (the condition getting 2 heads and 2 tails) = {HHTT, HTHT, HTTH, TTHH, THTH, THHT} = 6

Hence, the required probability $= \frac{6}{16} = \frac{3}{8}$

Ans. (a)

6. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is

(a) 0.240 (b) 0.200

(c) 0.040 (d) 0.008

(GATE 2004, 2 Marks)

Solution: As all three gates are independent probability (gate 2 and gate 3 fail | gate 1 failed)

$= P$(gate 2 and gate 3 fail)

$= P$(gate 2) $\times$ P(gate 3)

$= 0.2 \times 0.2 = 0.04$

Ans. (c)

7. An examination paper has 150 multiple choice questions of one mark each, with each question having four choices. Each incorrect answer fetches –0.25 marks. Suppose 1000 students choose all their answers randomly with uniform probability. The total sum of the expected marks obtained by all these students is

(a) 0 (b) 2550

(c) 7525 (d) 9375

(GATE 2004, 2 Marks)

Solution: Let the marks obtained per question be a random variable X.

Its probability distribution table is given as follows:

X	1	–0.25
$P(X)$	1/4	3/4

Expected marks per question

$$= E(x)$$
$$= \sum X\, P(X)$$
$$= 1 \times \frac{1}{4} + (-0.25) \times \frac{3}{4}$$
$$= \frac{1}{4} - \frac{3}{16}$$
$$= \frac{1}{16} \text{ marks}$$

Total marks expected for 150 questions $= \frac{1}{16} \times 150 = \frac{17}{8}$ marks per student.

Total expected marks of 1000 students $= \frac{17}{8} \times 1000 = 9375$ marks.

Ans. (d)

8. Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probability. The probability that the hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is

(a) ${}^n\mathrm{C}_d/2^2$ (b) ${}^n\mathrm{C}_d/2^d$

(c) $d/2^n$ (d) $1/2^d$

(GATE 2004, 2 Marks)

Solution: If hamming distance between two n bit strings is d, we are asking d out of n trials to be a success (success here means that the bits are different). So, this is a binomial distribution with n trials and d successes and the probability of success is

$$P = 2/4 = 1/2$$

(Because out of the four possibilities {(0, 0), (0, 1), (1, 0), (1, 1)}, only two, (0, 1) and (1, 0), are success.)

So,

$$P(X = d) = {}^n\mathrm{C}_d (1/2)^d (1/2)^{n-d} = \frac{{}^n\mathrm{C}_d}{2^n}$$

Ans. (a)

9. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card is not replaced?

(a) $\frac{1}{5}$ (b) $\frac{1}{52}$

(c) $\frac{1}{169}$ (d) $\frac{1}{221}$

(GATE 2004, 2 Marks)

Solution: Problems can be solved by hypergeometric distribution as follows:

$$P(X=2) = \frac{{}^{4}C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{1}{221}$$

Ans. (d)

10. A point is randomly selected with uniform probability in the $x-y$ plane within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, the expected value of p^2 is

(a) 2/3 (b) 1

(c) 4/3 (d) 5/3

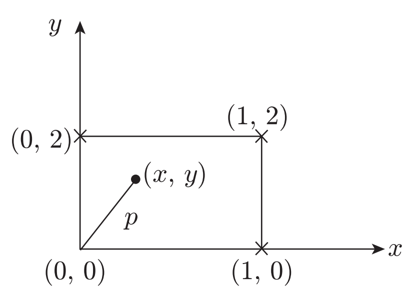
(GATE 2004, 2 Marks)

Solution: Length of position vector of point,

$$p = \sqrt{x^2 + y^2}$$

$$p^2 = x^2 + y^2$$

$$E(p^2) = E(x^2 + y^2) = E(x^2) + E(y^2)$$



Now x and y are uniformly distributed $0 \le x \le 1$ and $0 \le y \le 2$.

Probability density function of $x = \frac{1}{1-0} = 1$

Probability density function of $y = \frac{1}{2-0} = 1/2$

$$E(x^2) = \int_0^1 x^2 p(x)dx = \int_0^1 x^2 \cdot 1 \cdot dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$E(y^2) = \int_0^2 y^2 p(y)dy = \int_0^2 y^2 \cdot 1/2 \cdot dy = \left[\frac{y^3}{6}\right]_0^2$$

$$= \frac{8}{6} = \frac{4}{3}$$

$$\therefore E(p^2) = E(x^2) + E(y^2)$$

$$= \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

Ans. (d)

11. Let $f(x)$ be the continuous probability density function of a random variable X. The probability that $a < X \le b$ is

(a) $f(b-a)$ (b) $f(b) - f(a)$

(c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$

(GATE 2005, 1 Mark)

Solution: If $f(x)$ is the continuous probability density function of a random variable X, then

$$P(a < x \le b) = P(a \le x \le b) = \int_a^b f(x)dx$$

In general, we say that X is a uniform random variable on the interval (a, b) if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$

As $f(x)$ is a constant, all values of x between α and β are equally likely (uniform).

Ans. (c)

12. If P and Q are two random events, then the following is TRUE:

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$

(b) Probability $(P \cup Q) \ge$ Probability (P) + Probability (Q)

(c) If P and Q are mutually exclusive, then they must be independent

(d) Probability $(P \cap Q) \le$ Probability (P)

(GATE 2005, 1 Mark)

Solution: Option (a) is false because P and Q are independent

$$p(P \cap Q) = p(P) * p(Q)$$

which need not be zero.

Option (b) is false because

$$p(P \cup Q) = p(P) + p(Q) - p(P \cap Q)$$

$$\therefore p(P \cup Q) \le p(P) + p(Q)$$

Option (c) is false because independence and mutually exclusion are unrelated properties.

Now, as $P \cap Q \subseteq P$

$$\Rightarrow n(P \cap Q) \leq n(P)$$
$$\Rightarrow pr(P \cap Q) \leq pr(P)$$

Hence, option (d) is true.

Ans. (d)

13. A fair dice is rolled twice. The probability that an odd number will follow an even number is

(a) 1/2 (b) 1/6
(c) 1/3 (d) 1/4

(GATE 2005, 1 Mark)

Solution: Probability of getting an odd number $P_o = 3/6 = 1/2$

Probability of getting an even number $P_e = 3/6 = 1/2$

As both events are independent of each other,

$$P_{(o/e)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Ans. (d)

14. Which one of the following statements is NOT true?

(a) The measure of skewness is dependent upon the amount of dispersion
(b) In a symmetric distribution, the values of mean, mode and median are the same
(c) In a positively skewed distribution: mean > median > mode
(d) In a negatively skewed distribution: mode > mean > median

(GATE 2005, 1 Mark)

Solution: Options (a), (b), (c) are true but option (d) is not true because in a negatively skewed distribution, mode > median > mean.

Ans. (d)

15. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly two of the chosen items are defective is

(a) 0.0036 (b) 0.1937
(c) 0.2234 (d) 0.3874

(GATE 2005, 1 Mark)

Solution: This problem can be done using binomial distribution because population is infinite.
Probability of defective item, $p = 0.1$

Probability of non-defective item, $q = 1 - p = 1 - 0.1 = 0.9$

Probability that exactly two of the chosen items are defective

$$= {}^{10}C_2 (p)^2 \, (q)^8 = {}^{10}C_2 \, (0.1)^2 (0.9)^8 = 0.1937$$

Ans. (b)

16. A single die is thrown twice. What is the probability that the sum is neither 8 nor 9?

(a) $\frac{1}{9}$ (b) $\frac{5}{36}$
(c) 1/4 (d) 3/4

(GATE 2005, 2 Marks)

Solution: Sample space $(S) = (6)^2 = 36$
Total ways in which sum is either 8 or 9 = (2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (5, 3), (5, 4), (6, 2), (6, 3)

Hence, favorable outcomes = 27. Therefore,

Probability of sum neither being 8 nor 9 = 27/36 = 3/4

Ans. (d)

17. A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is

(a) 1/8 (b) 1/2
(c) 3/8 (d) 3/4

(GATE 2005, 2 Marks)

Solution: We know that

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$\therefore$ P(2 heads in 3 tosses | first toss is head)

$$= \frac{P(\text{2 heads in 3 tosses and first toss is head})}{P(\text{first toss is head})}$$

P(first toss is head) = 1/2

P(2 head in 3 tosses and first toss is head) = $P(\text{HHT}) + P(\text{HTH}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$\therefore \quad \text{Required probability} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Ans. (b)

18. The value of the integral $I = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{x^2}{8}\right) dx$ is

(a) 1 (b) π
(c) 2 (d) 2π

(GATE 2005, 2 Marks)

Solution: We have

$$I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{\left(-x^2/8\right)}dx$$

$$\text{Comparing with } \frac{1}{\sqrt{2\pi\sigma}}\int_{-\infty}^{\infty} e^{\frac{(x-\mu)^2}{2\sigma^2}}\,dx = 1$$

We can provide μ and σ any value.

Here, putting $\mu = 0$

$$2\int_0^{\infty}\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{x^2}{2\sigma^2}} = 1$$

Given,

$$-\frac{x^2}{8} = -\frac{x^2}{2\sigma^2}$$

$\Rightarrow \sigma = 2$,

Now putting $\sigma = 2$ in the above equation, we get

$$\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{8}} = 1$$

Ans. (a)

19. A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

(a) $\frac{1}{5}$ (b) $\frac{1}{25}$
(c) $\frac{20}{99}$ (d) $\frac{19}{495}$

(GATE 2006, 1 Mark)

Solution: The problem can be solved by hyper-geometric distribution,

$$P(X=2) = \frac{{}^{20}C_2 \times {}^{80}C_2}{{}^{100}C_2} = \frac{19}{495}$$

Ans. (d)

20. A probability density function is of the form $p(x) = Ke^{-\alpha|x|}$, $x \in (-\infty, \infty)$. The value of K is

(a) 0.5 (b) 1
(c) 0.5α (d) α

(GATE 2006, 1 Mark)

Solution:

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} K\,e^{-\alpha|x|}\,dx = 1$$

$$\int_{-\infty}^{0} K\,e^{\alpha x}\,dx + \int_0^{\infty} K\,e^{-\alpha x}\,dx = 1$$

$$\Rightarrow \frac{K}{\alpha}\left(e^{\alpha x}\right)_{-\infty}^{0} + \frac{K}{-\alpha}\left(e^{-\alpha x}\right)_0^{\infty} = 1$$

$$\Rightarrow \frac{K}{\alpha} + \frac{K}{\alpha} = 1$$

$$2K = \alpha$$

$$\Rightarrow K = 0.5\alpha$$

Ans. (c)

21. Two fair dice are rolled and the sum r of the numbers turned up is considered:

(a) $P(r > 6) = (1/6)$
(b) $P(r/3 \text{ is an integer}) = (5/6)$
(c) $P(r = 8 \mid r/4 \text{ is an integer}) = (5/9)$
(d) $P(r = 6 \mid r/5 \text{ is an integer}) = (1/18)$

(GATE 2006, 2 Marks)

Solution: If two fair dice are rolled, the probability distribution of r where r is the sum of the numbers on each dice is given by

γ	2	3	4	5	6	7	8	9	10	11	12
$P(\gamma)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The above table has been obtained by taking all different ways of obtaining a particular sum. For example, a sum of 5 can be obtained by (1, 4), (2, 3), (3, 2) and (4, 1).

$$P(x=5) = 4/36$$

Now let us consider option (a),

$$P(r > 6) = P(r > 7)$$

$$= \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{21}{36} = \frac{7}{12}$$

Therefore, option (a) is wrong.

Consider option (b),

$P(r/3 \text{ is an integer}) = P(r = 3) + P(r = 6) + P(r=9) + P(r=12) = \frac{2}{36} + \frac{5}{36} + \frac{4}{36} + \frac{1}{36} = \frac{12}{36} = \frac{1}{3}$

Therefore, option (b) is wrong.

Consider option (c),

$$P(r = 8 \mid r/4 \text{ is an integer}) = \frac{1}{36}$$

Now, $P(r/4 \text{ is an integer}) = P(r = 4) + P(r = 8) + P(r = 12) = \frac{3}{36} + \frac{5}{36} + \frac{1}{36} = \frac{9}{36} = \frac{1}{4}$

$P(r = 8 \text{ and } r/4 \text{ is an integer}) = P(r = 8) = 5/36$

$$P(r = 8 \mid r/4 \text{ is an integer}) = \frac{\frac{5}{36}}{\frac{1}{4}} = \frac{20}{36} = \frac{5}{9}$$

Hence, option (c) is correct.

Ans. (c)

22. Three companies X, Y and Z supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are tabulated below:

Company	% of Computer	Probability of Being Defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

Given that a computer is defective, the probability that it was supplied by Y is

(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

(GATE 2006, 2 Marks)

Solution:
$S \rightarrow$ supply by Y, $d \rightarrow$ defective

Probability that the computer was supplied by Y, if the product is defective

$$P(S/d) = \frac{P(S \cap d)}{P(d)}$$

$$P(S \cap d) = 0.3 \times 0.02 = 0.006$$

$$P(d) = 0.6 \times 0.1 + 0.3 \times 0.02 + 0.1 \times 0.03 = 0.015$$

$$P(S/d) = \frac{0.006}{0.015} = 0.4$$

Ans. (d)

23. For each element in a set of size $2n$, an unbiased coin is tossed. The $2n$ coins tossed are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is

(a) $\binom{2n}{n}\Big/4^n$ (b) $\binom{2n}{n}\Big/2^n$

(c) $1\Big/\binom{2n}{n}$ (d) $\frac{1}{2}$

(GATE 2006, 2 Marks)

Solution: The probability that exactly n elements are chosen is equal to the probability of getting n heads out of $2n$ tosses, that is

$$= {}^{2n}\mathrm{C}_n (1/2)^n (1/2)^{2n-n} \text{ (Binomial formula)}$$

$$= {}^{2n}\mathrm{C}_n (1/2)^n (1/2)^n$$

$$= \frac{{}^{2n}\mathrm{C}_n}{2^{2n}} = \frac{{}^{2n}\mathrm{C}_n}{\left(2^2\right)^n} = \frac{{}^{2n}\mathrm{C}_n}{4^n}$$

Ans. (a)

24. There are 25 calculators in a box. Two of them are defective. Suppose five calculators are randomly picked for inspection (i.e., each has the same chance of being selected). What is the probability that only one of the defective calculators will be included in the inspection?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(GATE 2006, 2 Marks)

Solution: As population is finite, hypergeometric distribution is applicable:

$$P(1 \text{ defective in 5 calculators}) = \frac{{}^{2}C_1 \times {}^{23}C_4}{{}^{25}C_5} = \frac{1}{3}$$

Ans. (b)

25. Consider the continuous random variable with probability density function,

$$f(t) = 1 + t \text{ for } -1 \leq t \leq 0$$
$$= 1 - t \text{ for } 0 \leq t \leq 1$$

The standard deviation of the random variable is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{6}}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

(GATE 2006, 2 Marks)

Solution: Mean is given by

$$\mu_t = E(t)\int_{-\infty}^{\infty} t \cdot f(t)dt = \int_{-1}^{1} t \cdot f(t)\,dt$$

$$= \int_{-1}^{0} t(1+t)dt + \int_{0}^{1} t(1-t)\,dt$$

$$= \left[\frac{t^2}{2} + \frac{t^3}{3}\right]_{-1}^{0} + \left[\frac{t^2}{2} - \frac{t^3}{3}\right]_{0}^{1}$$

$$= -\left[\frac{1}{2} - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] - \left[\frac{1}{6} + \frac{1}{6}\right] = 0$$

$$\text{Variance } = E(t^2) - [E(t)]^2$$

$$= \int_{-\infty}^{\infty} t^2 f(t)dt - [E(t)]^2 = \int_{-\infty}^{\infty} t^2 f(t)dt - (0)^2$$

$$= \int_{-1}^{0} (t^2 + t^3)dt + \int_{0}^{1} t^2(1-t)dt$$

$$= \left[\frac{t^3}{3} + \frac{t^4}{4}\right]_{-1}^{0} + \left[\frac{t^3}{3} - \frac{t^4}{4}\right]_{0}^{1}$$

$$= -\frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

$$\text{Standard deviation } = \sqrt{\text{Variance}} = \frac{1}{\sqrt{6}}$$

Ans. (b)

26. A class of first year B.Tech. students is composed of four batches A, B, C and D, each consisting of 30 students. It is found that the sessional marks of students in Engineering Drawing in batch C have a mean of 6.6 and standard deviation of 2.3. The mean and standard deviation of the marks for the entire class are 5.5 and 4.2, respectively. It is decided by the course instructor to normalize the marks of the students of all batches to have the same mean and standard deviation as that of the entire class. Due to this, the marks of a student in batch C are changed from 8.5 to

(a) 6.0 (b) 7.0
(c) 8.0 (d) 9.0

(GATE 2006, 2 Marks)

Solution: Let the mean and standard deviation of the students of batch C be μ_c and σ_c, respectively, and the mean and standard deviation of the entire class of first year students be μ and σ, respectively. Now given, $\mu_c = 6.6$, $\sigma_c = 2.3$, $\mu = 5.5$ and $\sigma = 4.2$

In order to normalize batch C to entire class, the normalized score (Z scores) must be equated. As

$$Z = \frac{x-\mu}{\sigma} = \frac{x-5.5}{4.2}$$

$$Z_c = \frac{x_c - \mu_c}{\sigma_c} = \frac{8.5-6.6}{2.3}$$

Equating these two and solving, we get

$$\frac{8.5-6.6}{2.3} = \frac{x-5.5}{4.2}$$

$$\Rightarrow \qquad x = 8.969 \simeq 9.0$$

Ans. (d)

27. Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3, …, 20. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation?

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$
(c) $\frac{9!}{20!}$ (d) None of these

(GATE 2007, 2 Marks)

Solution: Number of permutations with '2' in the first place = 19!

Number of permutations with '2' in the second place = $10 \times 18!$

Number of permutations with '2' in the third place $= 10 \times 9 \times 17!$

and so on until '2' is in the eleventh place. After that, it is not possible to satisfy the given condition, because there are only 10 odd numbers available to fill before '2'. So, the desired number of permutations which satisfies the given condition is

$$19! + 10 \times 18! + 10 \times 9 \times 17! + 10 \times 9 \times 8 \times 16! + \cdots + 10! \times 9!$$

Now, the probability of this happening is given by

$$\frac{19! + 10 \times 18! + 10 \times 9 \times 17! + \cdots + 10! \times 9!}{20!}$$

which is clearly not options (a), (b) or (c).

Therefore, correct option is (d), that is, none of these.

Ans. (d)

28. An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is

(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06

(GATE 2007, 2 Marks)

Solution: Let A denote the event of failing in Paper 1 and B denote the event of failing in Paper 2. Then we are given that

$P(A) = 0.3$

$P(B) = 0.2$

$P(A/B) = 0.6$

Probability of failing in both,

$P(A \cap B) = P(B) * P(A/B) = 0.2 * 0.6 = 0.12$

Ans. (c)

29. A loaded dice has the following probability distribution of occurrences

Dice value	1	2	3	4	5	6
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$

If three identical dice as the above are thrown, the probability of occurrence of values 1, 5 and 6 on the three dice is

(a) same as that of occurrence of 3, 4, 5

(b) same as that of occurrence of 1, 2, 5

(c) 1/128

(d) 5/8

(GATE 2007, 2 Marks)

Solution:

Dice value	1	2	3	4	5	6
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$

As the dice are independent,

$$P(1,\,5,\,6) = \frac{1}{4}\times\frac{1}{8}\times\frac{1}{4} = \frac{1}{128}$$

$$P(3,\,4,\,5) = \frac{1}{8}\times\frac{1}{8}\times\frac{1}{8} = \frac{1}{512}$$

$$P(1,\,2,\,5) = \frac{1}{4}\times\frac{1}{8}\times\frac{1}{8} = \frac{1}{256}$$

Therefore, option (c), $P(1, 5 \text{ and } 6) = \dfrac{1}{128}$ is correct.

Ans. (c)

30. If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is

(a) 0.1517 (b) 0.1867

(c) 0.2666 (d) 0.3646

(GATE 2007, 2 Marks)

Solution:

$$\text{C.V.} = \frac{\sigma}{\mu} = \frac{8.8}{33} = 0.2666$$

Ans. (c)

31. Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE?

(a) $E(XY) = E(X)E(Y)$

(b) $\text{Cov}(X,Y) = 0$

(c) $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

(d) $E(X^2Y^2) = (E(X))^2(E(Y))^2$

(GATE 2007, 2 Marks)

Solution:

Option (a) is true.

Option (b) is true.

Option (c) is true.

Option (d) is false.

Because $E(X^2Y^2) = (E(X))^2(E(Y))^2$

However, as X is not independent of X,

$$E(X^2) \neq [E(X)]^2$$

$$\therefore E(X^2Y^2) = E(X^2)E(Y^2) \neq [E(X)]^2\left[E(Y^2)\right]^2$$

Ans. (d)

32. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times?

(a) 1/4 (b) 3/8

(c) 1/2 (d) 3/4

(GATE 2008, 1 Mark)

Solution: We know that

$$P = P(H) = 0.5$$

Probability of getting head exactly 3 times is

$$P(X=3) = {}^4C_3(0.5)^1 = \frac{1}{4}$$

Ans. (a)

33. A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport, further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively, would be

(a) 0.45, 0.30 and 0.25

(b) 0.45, 0.25 and 0.30

(c) 0.45, 0.55 and 0.00

(d) 0.45, 0.35 and 0.20

(GATE 2008, 2 Marks)

Solution: We are given that $P(\text{Car}) = 0.45$

Now, probability of choosing a public transport = 0.55

Furthermore, probability of commuting by a bus = 0.55

Hence, $P(\text{Bus}) = 0.55 \times 0.55 = 0.30$

Now, probability of commuting by a metro = 0.45

$$P(\text{Metro}) = 0.55 \times 0.45 = 0.25$$

Ans. (a)

34. Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday?

(a) 0.24 (b) 0.36
(c) 0.4 (d) 0.6

(GATE 2008, 2 Marks)

Solution: Let C denote computer science study and M denote maths study.

Now by rule of total probability, we total up the desired branches and get the answer as shown below:

P(C on Monday and C on Wednesday) = P(C on Monday, C on Tuesday and C on Wednesday) + P(C on Monday, M on Tuesday and C on Wednesday)

$$\begin{aligned} &= 1\times 0.6\times 0.4 + 1\times 0.4\times 0.4 \\ &= 0.24 + 0.16 \\ &= 0.40 \end{aligned}$$

Ans. (c)

35. If probability density function of a random variable X is

$$\begin{aligned} f(X) &= X^2 \text{ for } -1 \le X \le 1 \\ &= 0 \text{ for any other value of } x \end{aligned}$$

then the percentage probability $P\left(-\frac{1}{3} < x^2 < \frac{1}{3}\right)$ is

(a) 0.247 (b) 2.47
(c) 24.7 (d) 247

(GATE 2008, 2 Marks)

Solution: Given

$$\begin{aligned} f(x) &= x^2 \text{ for} -1 < x < 1 \\ &= 0 \text{ elsewhere} \end{aligned}$$

$$\begin{aligned} P\left(-\frac{1}{3} < x < \frac{1}{3}\right) &= \int_{-\frac{1}{3}}^{\frac{1}{3}} f(x)dx = \int_{-\frac{1}{3}}^{\frac{1}{3}} x^2 dx = \left[\frac{x^3}{3}\right]_{-\frac{1}{3}}^{\frac{1}{3}} \\ &= \frac{2}{81} \end{aligned}$$

The probability expressed in percentage,

$$P = \frac{2}{81}\times 100 = 2.469\% = 2.47\%$$

Ans. (b)

36. Let X be a random variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean −1 and variance unknown. If $P(X \le -1) = P(Y \ge 2)$, the standard deviation of Y is

(a) 3 (b) 2
(c) $\sqrt{2}$ (d) 1

(GATE 2008, 2 Marks)

Solution: Given, $\mu_x = 1$, $\sigma_x^2 = 4 \Rightarrow \sigma_y = 2$

Also given, $\mu_y = -1$ and σ_y is unknown

Given, $P(X \le -1) = P(Y \ge 2)$

Converting into standard normal variates.

$$P\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = P\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$

$$P\left(z \le \frac{-1-1}{2}\right) = P\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$P(z \le -1) = P\left(z \ge \frac{3}{\sigma_y}\right) \quad \text{(i)}$$

Now, we know that in standard normal distribution,

$$P(z \le -1) = P(z \ge 1) \quad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we can say that

$$\frac{3}{\sigma_y} = 1 \Rightarrow \sigma_y = 3$$

Ans. (a)

37. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

(GATE 2009, 1 Mark)

Solution: Probably of only the first two tosses being heads = P(H, H, T, T, T, …, T)

As each toss is independent, the required probability $= P(H)\times P(H)\times[P(T)]^8$

$$=\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^{10}$$

Ans. (c)

38. If two fair coins are flipped and at least one of the outcomes is known to be a head, then what is the probability that both outcomes are heads?

(a) 1/3 (b) 1/4
(c) 1/2 (d) 2/3

(GATE 2009, 1 Mark)

Solution: Let A be the event of head in one coin and B be the event of head in second coin.
The required probability is

$$P(A\cap B\mid A\cup B)=\frac{P[(A\cap B)\cap(A\cup B)]}{P(A\cup B)}=\frac{P(A\cap B)}{P(A\cup B)}$$

$$P(A\cap B)=P(\text{both coin heads})=P(\text{H, H})=\frac{1}{4}$$

P(HH, HT, TH) = 3/4

So, required probability $=\dfrac{\frac{1}{4}}{\frac{3}{4}}=\dfrac{1}{3}$

Ans. (a)

39. If three coins are tossed simultaneously, the probability of getting at least one head is

(a) 1/8 (b) 3/8
(c) 1/2 (d) 7/8

(GATE 2009, 1 Mark)

Solution: Here, total outcomes = 2^3 = 8.

The probability of getting zero heads when three coins are tossed = 1/8 (when all three are tails).

Hence, the probability of getting at least one head when three coins are tossed = 1 − 1/8 = 7/8.

Ans. (d)

40. An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even-numbered face is the same. If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3?

(a) 0.453 (b) 0.468
(c) 0.485 (d) 0.492

(GATE 2009, 2 Marks)

Solution: It is given that

$$P(\text{odd}) = 0.9P(\text{even})$$
$$\sum p(x)=1$$

Now,

$$P(\text{odd}) + P(\text{even}) = 1$$
$$0.9P(\text{even}) + P(\text{even}) = 1$$
$$P(\text{even}) = 1/1.9 = 0.5263$$

Now, it is given that P(any even face) is same

$$\Rightarrow P(2)=P(4)=P(6)$$

Now,

$P(\text{even}) = P(2)$ or $P(4)$ or $P(6) = P(2) + P(4) + P(6)$

$\therefore$ $P(2) = P(4) = P(6) = 1/3P(\text{even})$
$= 1/3(0.5263) = 0.1754$

It is given that

$P(\text{even} \mid \text{face} > 3) = 0.75$

$$\Rightarrow \frac{P(\text{even}\cap\text{face}>3)}{P(\text{face}>3)}=0.75$$

$$\Rightarrow \frac{P(\text{face}=4,6)}{P(\text{face}>3)}=0.75$$

$$\Rightarrow P(\text{face}>3)=\frac{P(\text{face}=4,6)}{0.75}=\frac{P(4)+P(6)}{0.75}=\frac{0.1754+0.1754}{0.75}=0.4677\simeq 0.468$$

Ans. (b)

41. The standard deviation of a uniformly distributed random variable between 0 and 1 is

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

(GATE 2009, 2 Marks)

Solution: $\sigma = \sqrt{\frac{(\beta - \alpha)^2}{12}} = \sqrt{\frac{(1-0)^2}{12}} = \frac{1}{\sqrt{12}}$

Ans. (a)

42. The standard normal probability function can be approximated as

$$F_{(X_N)} = \frac{1}{1 + \exp(-1.7255\ x_n |X_n|^{0.12})}$$

where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm, respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

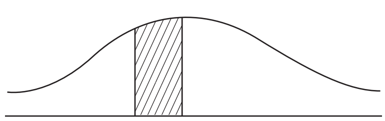
(a) 66.7% (b) 50.0%
(c) 33.3% (d) 16.7%

(GATE 2009, 2 Marks)

Solution: Here $\mu = 102$ cm and $\sigma = 27$ cm.

$$P(90 \le x \le 102) = P\left(\frac{90-102}{27} \le x \le \frac{102-102}{27}\right)$$
$$= P(-0.44 \le x \le 0)$$

This area is shown as follows:



The shaded area in the above figure is given by

$$F(0) - F(-0.44) = \frac{1}{1+\exp(0)}$$
$$= \frac{1}{1+\exp[-1.7255(-0.44)(0.44)0.12)]}$$
$$= 0.5 - 0.3345 = 1.1655 \simeq 16.55\%$$

Hence, the closest answer is 16.7%.

Ans. (d)

43. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

(a) 2/315 (b) 1/630
(c) 1/1260 (d) 1/2520

(GATE 2010, 2 Marks)

Solution: Box contains 2 washers, 3 nuts and 4 bolts.

Probability of drawing 2 washers first $= \left(\frac{2}{9} \times \frac{1}{8}\right)$

Probability of drawing 3 nuts after drawing 2 washers $= \left(\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right)$

Probability of drawing 4 bolts after drawing 4 bolts $= \left(\frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}\right)$

P(2 washers, then 3 nuts, then 4 bolts)

$$= \left(\frac{2}{9} \times \frac{1}{8}\right) \times \left(\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right) \times \left(\frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}\right) = \frac{1}{1260}$$

Ans. (c)

44. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7
(c) 1/2 (d) 4/7

(GATE 2010, 2 Marks)

Solution: We have to calculate,

$$P(\text{II is red} \mid \text{I is white}) = \frac{P(\text{II is red and I is white})}{P(\text{I is white})}$$
$$= \frac{P(\text{I is white and II is red})}{P(\text{I is white})}$$

Probability of first removed ball being white $= \frac{4}{7}$

Probability of second removed ball being red $= \frac{3}{6}$

$$P(\text{II is red} \mid \text{I is white}) = \frac{\frac{4}{7} \times \frac{3}{6}}{\frac{4}{7}} = \frac{3}{6} = \frac{1}{2}$$

Ans. (c)

45. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) $pq + (1-p)(1-q)$ (b) $(1-q)p$
(c) $(1-p)q$ (d) pq

(GATE 2010, 2 Marks)

Solution: The probability of a faulty assembly of any computer $= p$

The probability of a computer being declared faulty from a faulty assembly $= pq$

The probability of a non-faulty assembly of any computer $= 1 - q$

The probability of a computer being declared faulty from a non-faulty assembly $= (1 - p)(1 - q)$

Required probability $= pq + (1 - p)(1 - q)$

Ans. (a)

46. A fair coin is tossed independently four times. The probability of the event "the number of times heads show up is more than the number of times tails show up" is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{5}{16}$

(GATE 2010, 2 Marks)

Solution: Coin is tossed 4 times. Hence, the total outcomes $= 2^4 = 16$.

Favorable outcomes (the number of times heads show up is more than the number of times tails show up) = {HHHH, HHTH, HHHT, HTHH, THHH} = 5

Hence, the required probability $= \frac{5}{16}$.

Ans. (d)

47. There are two containers, with one containing 4 red and 3 green balls and the other containing 3 blue and 4 green balls. One ball is drawn at random from each container. The probability that one of the ball is red and the other is blue will be

(a) $\frac{1}{7}$ (b) $\frac{9}{49}$

(c) $\frac{12}{49}$ (d) $\frac{3}{7}$

(GATE 2011, 1 Mark)

Solution: Probability that one of the ball is red $= \frac{4}{7}$

Probability that one of the ball is blue $= \frac{3}{7}$

Probability that one of the ball is red and the other is blue $= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$

Ans. (c)

48. A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is

(a) 2/36 (b) 2/6

(c) 5/12 (d) 1/2

(GATE 2011, 1 Mark)

Solution: In the first toss, results can be 1, 2, 3, 4, 5.

For 1, the second toss results can be 2, 3, 4, 5, 6.

For 2, the second toss results can be 3, 4, 5, 6.

For 3, the second toss results can be 4, 5, 6.

For 4, the second toss results can be 5, 6.

For 5, the second toss results can be 6.

The required probability

$$= \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{4}{6} + \frac{1}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{2}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{5}{12}$$

Ans. (c)

49. If the difference between the expectation of the square of a random variable $\left(E[x]\right)^2$ and the square of the expectation of the random variable $\left(E[x]\right)^2$ is denoted by R, then

(a) $R = 0$ (b) $R < 0$

(c) $R \geq 0$ (d) $R > 0$

(GATE 2011, 1 Mark)

Solution: $V(x) = E(x^2) - [E(x)]^2 = R$

where $V(x)$ is the variance of x.

As variance is σ_x^2 and hence never negative, $R \geq 0$.

Ans. (c)

50. A deck of five cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(a) $\frac{1}{5}$ (b) $\frac{4}{25}$

(c) 1.4 (d) $\frac{2}{5}$

(GATE 2011, 2 Marks)

Solution: The five cards are $\{1, 2, 3, 4, 5\}$

Sample space $= 5 \times 4$

Favorable outcome $= P\{(2,1),(3,2),(4,3),(5,4)\}$

$$= \frac{4}{5 \times 4} = \frac{1}{5}$$

Ans. (a)

51. Consider the finite sequence of random values $X = [x_1, x_2, \ldots, x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$ where a and b are positive constant. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is incorrect?

(a) Index position of mode of X in X is the same as the index position of mode of Y in Y.
(b) Index position of median of X in X is the same as the index position of median of Y in Y.
(c) $\mu_y = a\mu_x + b$
(d) $\sigma_y = a\sigma_x + b$

(GATE 2011, 2 Marks)

Solution: Standard deviation is affected by scale but not by shift of origin.

So, $y_i = ax_i + b$

$\Rightarrow$ $\sigma_y = a\sigma_x$

and not $\sigma_y = a\sigma_x + b$

Ans. (d)

52. An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is

(a) $\dfrac{1}{32}$ (b) $\dfrac{13}{32}$
(c) $\dfrac{16}{32}$ (d) $\dfrac{31}{32}$

(GATE 2011, 2 Marks)

Solution: Here, total outcomes $= 2^5 = 32$.

The probability of getting zero heads when three coins are tossed = 1/32 (when all five are tails).

Hence, the probability of getting at least one head when three coins are tossed = 1 − 1/32 = 31/32.

Ans. (d)

53. Consider a random variable X that takes values +1 and −1 with probability 0.5 each. The values of the cumulative distribution function $F(x)$ at $x = -1$ and +1 are

(a) 0 and 0.5 (b) 0 and 1
(c) 0.5 and 1 (d) 0.25 and 0.75

(GATE 2012, 1 Mark)

Solution: The probability distribution table of the random variable is

x	−1	+1
$P(x)$	0.5	0.5

The cumulative distribution function $F(x)$ is the probability up to x as follows:

x	−1	+1
$F(x)$	0.5	1.0

Hence, the values of cumulative distributive function are 0.5 and 1.0.

Ans. (c)

54. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(a) <50% (b) 50%
(c) 75% (d) 100%

(GATE 2012, 1 Mark)

Solution: The annual precipitation is normally distributed with $\mu = 1000$ mm and $\sigma = 200$ mm.

$$P(x > 1200) = P\left(z > \frac{1200-1000}{200}\right)$$
$$= P(z > 1)$$

where z is the standard normal variate.

In normal distribution,

$$P(-1 < z < 1) \approx 0.68$$

($\simeq$ 68% of data is within one standard deviation of mean)

$$P(0 < z < 1) = \frac{0.68}{2} = 0.34$$

So, $P(z > 1) = 0.5 - 0.34 = 0.16 \simeq 16\%$

which is < 50%.

Ans. (a)

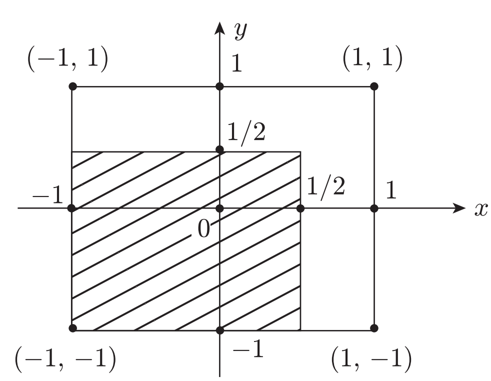
55. Two independent random variables X and Y are uniformly distributed in the interval [−1, 1]. The probability that max[X, Y] is less than 1/2 is

(a) 3/4 (b) 9/16
(c) 1/4 (d) 2/3

(GATE 2012, 1 Mark)

Solution: $-1 < x < 1$ and $-1 \leq y \leq 1$ is the entire rectangle.

The region in which maximum of (x, y) is less than $\frac{1}{2}$ is shown below as the shaded region inside the rectangle.



$$P\left(\max|x,y| < \frac{1}{2}\right) = \frac{\text{Area of shaded region}}{\text{Area of entire rectangle}}$$

$$= \frac{\frac{3}{2}\times\frac{3}{2}}{2\times 2} = \frac{9}{16}$$

Ans. (b)

56. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(GATE 2012, 2 Marks)

Solution: Probability that the number of toss is odd = Probability of the number of toss is 1, 3, 5, 7, …

Probability that the number of toss is 1 = Probability of getting head in first toss = 1/2

Probability that the number of toss is 3 = Probability of getting tail in first toss, tail in second toss and head in third toss

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

Probability that the number of toss is 5 = P(T, T, T, T, H) = $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

So, probability that the number of tosses are odd

$$= \frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots$$

Sum of infinite geometric series with

$$a = \frac{1}{2} \quad \text{and} \quad r = \frac{1}{4} = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

Ans. (c)

57. Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2 or 3, the die is rolled a second time. What is the probability that the total sum of values that turn up is at least 6?

(a) 10/21 (b) 5/12
(c) 2/3 (d) 1/6

(GATE 2012, 2 Marks)

Solution: $P(\text{sum} > 6) = P(6 \text{ on first throw}) + P(1, 5) + P(1, 6) + P(2, 4) + P(2, 5) + P(2, 6) + P(3, 3) + P(3, 4) + P(3, 5) + P(3, 6)$

$$= \frac{1}{6}+\frac{9}{36} = \frac{15}{36} = \frac{5}{12}$$

Ans. (b)

58. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

(a) 1/32 (b) 2/32
(c) 3/32 (d) 6/32

(GATE 2012, 2 Marks)

Solution: As negative and positive are equally likely, the distribution of number of negative values is binomial with $n = 5$ and $p = 1/2$.

Let X represents the number of negative values in 5 trials.

$P(\text{at most 1 negative value}) = P(x \le 1)$

$= P(x = 0) + P(x = 1)$

$$= {}^5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{5}\right)^5 + {}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 = \frac{6}{32}$$

Hence, required probability is $\frac{6}{32}$.

Ans. (d)

59. A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains 1 red ball and 2 black balls is

(a) 1/20 (b) 1/12
(c) 3/10 (d) 1/2

(GATE 2012, 2 Marks)

Solution: P(1 red and 2 black)

$$= \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{60}{120} = \frac{1}{2}$$

Ans. (d)

60. A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P\{X > 1\}$ is

(a) 0.368 (b) 0.5
(c) 0.632 (d) 1.0

(GATE 2013, 1 Mark)

Solution:

$$P = \int_1^\infty f(x)dx = \int_1^\infty e^{-x}dx = e^{-x}\Big|_\infty^1 = e^{-1} = 0.368$$

Ans. (a)

61. Suppose p is the number of cars per minute passing through a certain road junction around

5 PM, and p has Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(a) $8/(2e^3)$ (b) $9/(2e^3)$

(c) $17/(2e^3)$ (d) $26/(2e^3)$

(GATE 2013, 1 Mark)

Solution: Poisson formula for (p = x) is given as $\dfrac{e^{-x}\lambda^x}{x!}$.

λ: mean of Poisson distribution = 3 (given)

Probability of observing fewer than 3 cars = (p = 0) + (p = 1) + (p = 2)

$$= \frac{e^{-3}\lambda^0}{0!} + \frac{e^{-3}\lambda^1}{1!} + \frac{e^{-3}\lambda^2}{2!} = \frac{17}{2e^3}$$

Ans. (c)

62. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is

(a) 0.5

(b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0

(d) 1.0

(GATE 2013, 1 Mark)

Solution:

$$P(x<0) = P\left(\frac{x-\mu}{\sigma} < \frac{0-\mu}{\sigma}\right) - P(Z < -0.5)$$
$$= P(Z > 0.5) = 0.5 - P(0 < Z < 0.5)$$

Hence, the probability is greater than zero and less than 0.5.

Ans. (b)

63. Find the value of λ such that function $f(x)$ is valid probability density function,

$$f(x) = \lambda(x-1)(2-x) \text{ for } 1 \le x \le 2$$
$$= 0 \text{ otherwise}$$

(GATE 2013, 2 Marks)

Solution: $$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$f(x) = \begin{Bmatrix} \lambda(-x^2+3x-2) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{Bmatrix}$$

$$\therefore \int_1^2 \lambda(-x^2+3x-2)dx = 1$$

$$\Rightarrow \lambda\left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x\right]_1^2 = 1$$

$$\Rightarrow \lambda\left[-\left(\frac{8}{3}-\frac{1}{3}\right) + \frac{3}{2}(4-1) - 2(2-1)\right] = 1$$

$$\Rightarrow \lambda\left[-\frac{7}{9} + \frac{9}{2} - 2\right] = 1$$

$$\Rightarrow \lambda\left[\frac{-14+27-12}{6}\right] = 1$$

$$\Rightarrow \lambda = \frac{6}{1} = 6$$

$$\lambda = 6$$

64. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p then $100p$ = ________.

(GATE 2014, 1 Mark)

Solution: If p is the probability that system is functional means at least three systems are working, p will be calculated as:

p for at least 3 systems are working

$$= \frac{\text{All 4 systems working} + \text{3 systems working}}{\text{Number of ways to pick 4 systems from 10}}$$

p that 3 systems are working

= (Number of systems not selected)
$\times$(Number of ways to pick 3 from 4 selected systems)
$= 6 \times 4 \times 3 \times 2 \times 4$

Therefore,

p for at least 3 systems are working

$$= \frac{(4\times3\times2\times1) + (6\times4\times3\times2\times4)}{10\times9\times8\times7} = 0.1192$$

So, $100p$ = 11.92

Ans. 11.92

65. Each of the nine words in the sentence '**The quick brown fox jumps over the lazy dog**' is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The *expected* length of the word drawn is ________. (The answer should be rounded to one decimal place.)

(GATE 2014, 1 Mark)

Solution: The given 9 words are as follows:

The, Quick, Brown, Fox, Jumps, Over, The, Lazy, Dog
Words with length 3 are THE, FOX, THE, DOG (4)
Words with length 4 are OVER, LAZY (2)
Words with length 5 are QUICK, BROWN, JUMPS (3)
Probability for drawn 3 length words = 4/9
Probability for drawn 4 length words = 2/9
Probability for drawn 5 length words = 3/9

Expected word length

$$= \left\{\left(3\times\frac{4}{9}\right)+\left(4\times\frac{2}{9}\right)+\left(5\times\frac{3}{9}\right)\right\} = 3.88$$

Ans. 3.88

66. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random has a sibling is ________.

(GATE 2014, 1 Mark)

Solution: Let E_1 = one child family, E_2 = two children family and A = picking a child. Then by Bayes' theorem, required probability is given by

$$P(E_2/A) = \frac{\frac{1}{2}x}{\frac{1}{2}\cdot\frac{x}{2}+\frac{1}{2}x} = \frac{2}{3} = 0.667$$

where x is the number of families.

Ans. 0.667

67. Let X_1, X_2 and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1$ is the largest$\}$ is ________.

(GATE 2014, 1 Mark)

Solution: Given that three random variables X_1, X_2, and X_3 are uniformly distributed on [0, 1], we have the following possible values:

$X_1 > X_2 > X_3$
$X_1 > X_3 > X_2$
$X_2 > X_1 > X_3$
$X_2 > X_3 > X_1$
$X_3 > X_1 > X_2$
$X_3 > X_2 > X_1$

Since all the three variables are identical, the probabilities for all the above inequalities are same. Hence, the probability that X_1 is the largest is $P\{X_1$ is the largest$\} = 2/6 = 1/3 = 0.33$

Ans. 0.33

68. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, $E[X]$, is ________.

(GATE 2014, 1 Mark)

Solution: $X = 1, 3, 5,\ldots, 99 \Rightarrow n = 50$ (number of observations)
Therefore

$$E(x) = \frac{1}{n}\sum_{i=j}^{n} x_i = \frac{1}{50}[1+3+5+\cdots+99]$$

$$= \frac{1}{50}\cdot(50)^2 = 50$$

Ans. 50

69. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(a) 0.067 (b) 0.073 (c) 0.082 (d) 0.091

(GATE 2014, 1 Mark)

Solution: P[fourth head appears at the tenth toss] $= P$[getting 3 heads in the first 9 tosses and one head at tenth toss]

$$= \left[{}^9C_3\cdot\left(\frac{1}{2}\right)^9\right]\left[\frac{1}{2}\right] = \frac{21}{256} = 0.082$$

Ans. (c)

70. Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ________.

(GATE 2014, 1 Mark)

Solution:

$$X - N(0.1) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, -\infty < x < \infty$$

Therefore

$$E\{|x|\} = \int_{-\infty}^{\infty} |x|\, f(x)$$

$$= \frac{1}{\sqrt{2\pi}}x^2\int_0^{\infty} xe^{-x^2/2}dx = \frac{2}{\sqrt{2\pi}}\int_0^{\infty} e^{-u}du$$

$$= \sqrt{\frac{2}{\pi}} = 0.797 \simeq 0.8$$

Ans. 0.8

71. Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is ________.

(GATE 2014, 1 Mark)

Solution: $P(n) = K\cdot n$ where n = 1 to 6. We know that,

$$\sum_n P(x) = 1 \Rightarrow K[1+2+3+4+5+6] = 1$$

$$\Rightarrow K = \frac{1}{21}$$

Therefore, required probability is $P(3) = 3K = \frac{1}{7}$.

72. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years, respectively, then the value of k is ________.

(GATE 2014, 1 Mark)

Solution: Density of the random variable is $f(x) = kx^2$. Since $f(x)$ is a PDF

$$f(x) = \begin{cases} kx^2, & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

$$\int_1^2 f(x)dx = 1 \Rightarrow k\left[\frac{x^3}{3}\right]_1^2 = 1 \Rightarrow k = \frac{3}{7} = 0.428 \simeq 0.43$$

73. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good

(a) $\frac{7}{20}$ (b) $\frac{42}{125}$ (c) $\frac{25}{29}$ (d) $\frac{5}{9}$

(GATE 2014, 1 Mark)

Solution: The probability of selecting two parts from 25 is ${}^{25}C_2$. The probability of selecting two good parts is ${}^{(25-10)}C_2$.

So, the required probability is $\frac{{}^{15}C_2}{{}^{25}C_2} = \frac{105}{300} = \frac{7}{20}$

Ans. (a)

74. A group consists of equal number of men and women. Of this group, 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is ________.

(GATE 2014, 1 Mark)

Solution: Let M be the event of person selected is a man, W be the event of person selected is a woman, E be the event of person selected being employed and U be the event of person selected being unemployed.

Given that $P(\text{M}) = 0.5$, $P(\text{W}) = 0.5$, $P(\text{U/M}) = 0.2$ and $P(\text{U/W}) = 0.5$

The probability of selecting an unemployed person is

$P(\text{U}) = P(\text{M}) \times (P(\text{U/M}) + P(\text{W}) \times (P(\text{U/W}) = 0.5 \times 0.2 + 0.5 \times 0.5 = 0.35$

So, the probability of selecting an employed person $= 1 - P(U) = 1 - 0.35 = 0.65$

Ans. 0.65

75. A nationalized bank has found that the daily balance available in its savings account follows a normal distribution with a mean of ₹ 500 and a standard deviation of ₹ 50. The percentage of savings account holders, who maintain an average daily balance more than ₹ 500 is ________.

(GATE 2014, 1 Mark)

Solution: Given that mean (μ) = ₹ 500 and standard deviation (σ) = ₹ 50

If x follows the normal distribution, then the standard normal variable,

$$z = \frac{x-\mu}{\sigma} = \frac{500-500}{50} = 0$$

Hence, if $x > 500$ then $z > 0$ and so the percentage of savings account holders, who maintain an average daily balance more than ₹ 500 is ≈50%.

Ans. 50%

76. The probability density function of evaporation E on any day during a year in a watershed is given by

$$f(E) = \begin{cases} \frac{1}{5} & 0 \le E \le 5\,\text{mm/day} \\ 0 & \text{otherwise} \end{cases}$$

The probability that E lies in between 2 and 4 mm/day in a day in the watershed is (in decimal) ________.

(GATE 2014, 1 Mark)

Solution: The probability density function of E on any day is given by

$$f(E) = \begin{cases} \frac{1}{5} & 0 \le E \le 5 \text{ mm/day} \\ 0 & \text{otherwise} \end{cases}$$

$$P(2 < E < 4) = \int_2^4 f(E)dE = \int_2^4 \frac{1}{5}dE = \frac{1}{5}[E]_2^4$$

$$= \frac{1}{5}(4-2) = \frac{2}{5} = 0.4$$

Ans. 0.4

77. A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes:

(i) Head, (ii) Head, (iii) Head, (iv) Head.

The probability of obtaining a ‘Tail’ when the coin is tossed again is

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

(GATE 2014, 1 Mark)

Solution: Since the coin is unbiased, the probability of getting a Tail is $\frac{1}{2}$.

Ans. (b)

78. If $\{x\}$ is a continuous, real-valued random variable defined over the interval $(-\infty, +\infty)$ and its occurrence is defined by the density function given as $f(x) = \frac{1}{\sqrt{2\pi + b}} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where ‘a’ and ‘b’ are the statistical attributes of the random variable $\{x\}$. The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi + b}} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx$ is

(a) 1 (b) 0.5 (c) π (d) $\frac{\pi}{2}$

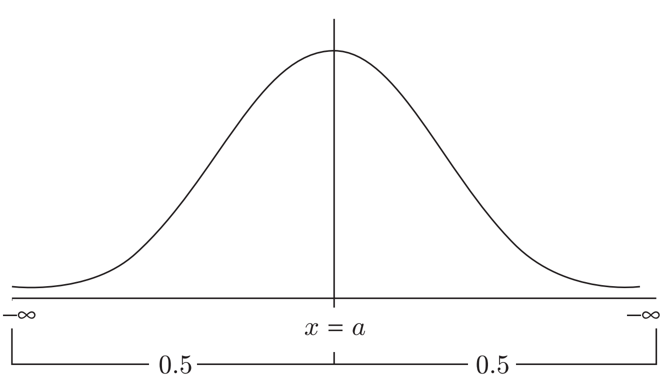
(GATE 2014, 1 Mark)

Solution: We have

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{a} f(x)dx = 0.5$$



Ans. (b)

79. Given that x is a random variable in the range $[0, \infty]$ with a probability density function $\frac{e^{\frac{x}{2}}}{K}$, the value of the constant K is ________.

(GATE 2014, 1 Mark)

Solution: We have,

$$\int_{0}^{\infty} \frac{e^{-x/2}}{K} = 1$$

$$\Rightarrow \frac{1}{K} \cdot \frac{e^{-x/2}}{(-1/2)}\Bigg|_{0}^{\infty} = 1$$

$$\Rightarrow \frac{-2}{K}(0-1) = 1 \Rightarrow \frac{2}{K} = 1 \Rightarrow K = 2$$

80. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $X/1296$. The value of X is ________.

(GATE 2014, 2 Marks)

Solution: Sum 22 comes with following possibilities:
$6 + 6 + 6 + 4 = 22$ (numbers can be arranged in 4 ways)
$6 + 6 + 5 + 5 = 22$ (numbers can arranged in 6 ways)
Therefore, the total ways $X = 4 + 6 = 10$

Ans. 10

81. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is ________.

(GATE 2014, 2 Marks)

Solution: Let A event $\rightarrow$ Divisible by 2, B event $\rightarrow$ Divisible by 3, C event $\rightarrow$ Divisible by 5. Then,
$P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$
$P(A) = 50/100$, $P(B) = 33/100$, $P(C) = 20/100$
$P(A \cap B) = 16/100$, $P(B \cap C) = 6/100$,
$P(A \cap C) = 10/100$, $P(A \cap B \cap C) = 3/100$
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
$P(A \cup B \cup C) = (50 + 33 + 20 - 16 - 6 - 10 + 3)/100 = 74/100$
$P(A' \cap B' \cap C') = 1 - 74/100 = 26/100 = 0.26$

Ans. 0.26

82. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(.)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is ________.

(GATE 2014, 2 Marks)

Solution: Maximum value of $P(A)\ P(B) = ?$

$$P(A)P(B) = P(A)[1 - P(A)]$$

Let $P(A) = x$

$$f(x) = x - x^2$$

$$f'(x) = 1 - 2x = 0$$

So, $x = 1/2$ and $f''(x) = -2$ and x is maximum at 1/2. Hence

$$P(A)P(B) = 0.25$$

Ans. 0.25

83. A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is ________.

(GATE 2014, 2 Marks)

Solution: Let the first toss be head. Let x denotes the number of tosses (after getting first head) to get first tail.
We can summarize the event as:

Event (After getting the first H)	x	Probability, $p(x)$
T	1	1/2
HT	2	1/2 × 1/2 = 1/4
HHT and so on …	3	1/8

$$E(x) = \sum_{x=1}^{\infty} xp(x) = 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8} + \cdots$$

Let,

$$S = 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8} + \cdots \quad \text{(i)}$$

$$\Rightarrow \frac{1}{2}S = \frac{1}{4} + 2\times\frac{1}{8} + 3\times\frac{1}{16} + \cdots \quad \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i) we get

$$\left(1-\frac{1}{2}\right)S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$\Rightarrow \frac{1}{2}S = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 \Rightarrow S = 2 \Rightarrow E(x) = 2$$

Hence, the expected number of tosses (after first head) to get first tail is 2 and same can be applicable if first toss results in tail.
Thus, the average number of tosses is 1 + 2 = 3.

Ans. 3

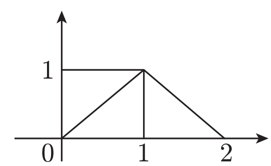
84. Let x_1, x_2 and x_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{x_1 + x_2 \le x_3\}$ is ________.

(GATE 2014, 2 Marks)

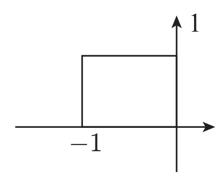
Solution: Given x_1, x_2 and x_3 be independent and identically distributed with uniform distribution on [0, 1]. Let $z = x_1 + x_2 = x_3$. Then

$$P\left\{x_1 + x_2 \le x_3\right\} = P\left\{x_1 + x_2 - x_3 \le 0\right\}$$
$$= P\left\{z \le 0\right\}$$

Let us find probability density function of random variable z.
Since z is summation of three random variables x_1, x_2 and $-x_3$, overall PDF of z is convolution of the PDF of x_1, x_2 and $-x_3$.
PDF of $\{x_1 + x_2\}$ is



PDF of $\{-x_3\}$ is



$$P\left\{z \le 0\right\} = \int_{-1}^{0} \frac{(z+1)^2}{2}dz = \left.\frac{(z+1)^3}{6}\right|_{-1}^{0} = \frac{1}{6} = 0.16$$

Ans. 0.16

85. Parcels from sender S to receiver R pass sequentially through two post offices. Each post office has a probability 5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post office is ________.

(GATE 2014, 2 Marks)

Solution: Parcel will be lost if
(a) It is lost by the first post office
(b) It is passed by first post office but lost by the second post office

$$P(\text{parcel is lost}) = \frac{1}{5} + \frac{4}{5}\times\frac{1}{5} = \frac{9}{25}$$

P(parcel lost by second post office if it passes first post office) = P(Parcel passed by first post office) × P(Parcel lost by second post office) $= \frac{4}{5}\times\frac{1}{5} = \frac{4}{25}$

P(parcel lost by second post office | parcel lost) = $\dfrac{4/25}{9/25} = \dfrac{4}{9} = 0.44$

Ans. 0.44

86. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n-3)$ is

(a) 2^{-n} (b) 0 (c) ${}^{n}C_{n-3}2^{-n}$ (d) 2^{-n+3}

(GATE 2014, 2 Marks)

Solution: Let X be the difference between number of heads and tails.

Take $n = 2 \Rightarrow S = \{HH, HT, TH, TT\}$ and $X = -2$, 0, 2. Here, $n - 3 = -1$ is not possible.

Take $n = 3 \Rightarrow S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and $X = -3, -1, 1, 3$.

Here, $n - 3 = 0$ is not possible.

Similarly, if a coin is tossed n times, then the difference between heads and tails is $n - 3$, which is possible.

Thus, required probability is 0.

Ans. (b)

87. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(0.5 < x < 5)$ is ________.

(GATE 2014, 2 Marks)

Solution: The probability density function of the random variable is

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$$P(0.5 < x < 5) = \int_{0.5}^{5} f(x)\,dx$$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx + \int_{4}^{5} f(x)\,dx$$

$$= (0.2)(x)_{0.5}^{1} + (0.1)(x)_{1}^{4} + 0$$

$$= 0.1 + 0.3 = 0.4$$

88. The mean thickness and variance of silicon steel laminations are 0.2 mm and 0.02, respectively. The varnish insulation is applied on both die sides of the laminations. The mean thickness of one side insulation and its variance are 0.1 mm and 0.01, respectively. If the transformer core is made using 100 such varnish coated laminations, the mean thickness and variance of the core, respectively, are

(a) 30 mm and 0.22 (b) 30 mm and 2.44
(c) 40 mm and 2.44 (d) 40 mm and 0.24

(GATE 2014, 2 Marks)

Solution: Mean thickness of laminations = 0.2 mm

Mean thickness of one side varnish = 0.1 mm

Mean thickness of varnish including both sides = 0.2 mm

Therefore, mean thickness of one lamination varnished on both sides = 0.4 mm

Thickness of the stack of 100 such laminations = $0.4 \times 100 = 40$ mm

If there are 100 laminations with mean thickness = 0.2 mm and variance = 0.1 mm, then we can write the following expressions:

$$(d_1 - 0.2)^2 + (d_2 - 0.2)^2 + \ldots + (d_{100} - 0.2)^2/100 = 0.02$$
$$(x_1 - 0.1)^2 + (x_2 - 0.1)^2 + \ldots + (x_{100} - 0.1)^2 = 0.01$$

Assuming all thicknesses to be equal to d and each side lamination varnish thickness to be equal to x, the two equations reduce to:

$$d = 0.3414 \text{ and } x = 0.2$$

Each lamination thickness therefore including steel and two-side varnish coating thicknesses = 0.3414 + 0.4 = 0.7414 mm.

Variance of overall core is therefore given by

$$100 \times (0.7414 - 0.4)^2/100 = 0.1166$$

Therefore, none of the given answers matches the correct answer.

89. In the following table, x is a discrete random variable and $p(x)$ is the probability density. The standard deviation of x is

x	1	2	3
$P(x)$	0.3	0.6	0.1

(a) 0.18 (b) 0.36 (c) 0.54 (d) 0.6

(GATE 2014, 2 Marks)

Solution: From the given table, we have

Mean (μ) $= \sum xP(x) = 1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1 = 1.8$

$\sum x^2P(x) = 1 \times 0.3 + 4 \times 0.6 + 9 \times 0.1 = 3.6$

Variance $= \sum x^2P(x) - \left[\sum xP(x)\right]^2 = 3.6 - (1.8)^2 = 0.36$

Standard deviation $= \sqrt{\text{Variance}} = \sqrt{0.36} = 0.6$

Ans. (d)

90. Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is ________.

(GATE 2014, 2 Marks)

Solution: The probability of each colour appearing only two times on the dice is $p = 2/6 = 1/3$ and the probability of obtaining red colour at the top face is $q = 1-(1/3)$.

Therefore, the required probability of obtaining red colour on top face of the dice at least twice is

$$p(x \geq 2) = {}^3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 + {}^3C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0 = \frac{7}{27}$$

91. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day, respectively, are

(a) 1 and 1/3
(b) 1/3 and 1
(c) 1 and 4/3
(d) 1/3 and 4/3

(GATE 2014, 2 Marks)

Solution: From the question, we have

x	0	1	2
$P(x)$	1/6	2/3	1/6

where x is the number of defective pieces.

Mean (μ) $= \sum xP(x) = 0 \times (1/6) + 1 \times (2/3) + 2 \times (1/6) = 1$

$\sum x^2P(x) = 0 \times (1/6) + 1 \times (2/3) + 4 \times (1/6) = 4/3$

Variance $= \sum x^2P(x) - \left[\sum xP(x)\right]^2 = 4/3 - (1)^2 = 1/3$

Ans. (a)

92. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(a) 0.029 (b) 0.034 (c) 0.039 (d) 0.044

(GATE 2014, 2 Marks)

Solution: For Poisson distribution, $P(\lambda) = \dfrac{e^{-\lambda}\lambda^n}{n!}$. Given that $\lambda = 5.2$ and $n < 2$, so

$$P(\lambda) = \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-1}\lambda^1}{1!} = e^{-5.2}(1+5.2)$$
$$= 0.00552 \times 6.2 = 0.034$$

Ans. (b)

93. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ________.

(GATE 2014, 2 Marks)

Solution:

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \lambda = 5$$
$$P(x<4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$
$$= \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!}$$
$$= e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right] = 0.265$$

Ans. 0.265

94. An observer counts 240 vehicles/h at a specific highway location. Assume that the vehicle arrival at the location is Poission distributed, the probability of having one vehicle arriving over a 30-second tome interval is ________.

(GATE 2014, 2 Marks)

Solution: Average number of vehicles per hour,

$$\lambda = 240/\text{h} = \frac{240}{60}/\min = 4/\min = 2/30\,\text{s}$$

$$P(x=1) = \frac{e^{-2}\cdot 2}{1!} = 0.27$$

Ans. 0.27

95. Consider the following two normal distributions

$$f_1(x) = \exp\left(-\pi x^2\right)$$

$$f_2(x) = \frac{1}{2\pi}\exp\left\{-\frac{1}{4\pi}\left(x^2+2x+1\right)\right\}$$

If μ and σ denote the mean and standard deviation, respectively, then

(a) $\mu_1 < \mu_2$ and $\sigma_1^2 < \sigma_2^2$

(b) $\mu_1 < \mu_2$ and $\sigma_1^2 > \sigma_2^2$

(c) $\mu_1 > \mu_2$ and $\sigma_1^2 < \sigma_2^2$

(d) $\mu_1 > \mu_2$ and $\sigma_1^2 > \sigma_2^2$

(GATE 2014, 2 Marks)

Solution: We are given that

$$f_1(x) = e^{-\pi x^2}$$

Comparing with $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, we get

$$\mu_1 = 0 \text{ and } \sigma_1 = \frac{1}{\sqrt{2\pi}}$$

Now, $f_2(x) = \dfrac{1}{2\pi}e^{-\frac{1}{4\pi}\left(x^2+2x+1\right)} = \dfrac{1}{2\pi}e^{-\frac{1}{4\pi}(x+1)^2}$

Comparing with $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ we get

$$\mu_2 = -1 \text{ and } \sigma_2 = \sqrt{2\pi}$$

So

$$\mu_1 > \mu_2 \text{ and } \sigma_1^2 < \sigma_2^2$$

Ans. (c)

96. In rolling of two fair dice, the outcome of an experiment is considered to be the sum of the numbers appearing on the dice. The probability is highest for the outcome of ________.

(GATE 2014, 2 Marks)

Solution: We can form the table as follows:

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Where x is a random variable and denotes the sum of the numbers appearing on the dice.

$P(x)$ = corresponding probabilities

Thus, the probability is highest for the outcome '7,' i.e. $\dfrac{6}{36}$.

Ans. 0.1667

97. Suppose A and B .are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which of the following statement is FALSE?

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A/B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A}) + P(\bar{B})$

(GATE 2015, 1 Mark)

Solution: We know that A and B are independent, then

$$P(A \cap B) = P(A) \cdot P(B)$$

A and B are independent then $P(A/B) = P(A)$ and $P(B/A) = P(B)$.

Also, if A and B are independent then $\bar{A}$ and $\bar{B}$ are also independent, that is,

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$$

Thus, (a), (b) and (d) are correct. Hence, (c) is false.

Ans. (c)

98. The variance of the random variable X with probability density function $f(x) = \dfrac{1}{2}|\,x\,|\,e^{-|x|}$ is ________.

(GATE 2015, 1 Mark)

Solution: Given that $f(x) = \dfrac{1}{2}|x|e^{-|x|}$ is probability density function of random variable X.

$$V(x) = E(x^2) - \{E(x)\}^2$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)dx = \int_{-\infty}^{\infty}\frac{1}{2}|x|e^{|x|}x dx = 0$$

(∵ the function is odd)

$$E(x)^2 = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{\infty} x^2\frac{1}{2}|x|e^{-|x|}dx$$

$$= \frac{2}{3}\int_{2}^{\infty} x^3 e^{-x}dx \text{ (∵ function is even)}$$

$$= 3! = 6$$

Ans. 6

99. A random variable X has the probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If the expected value $E[X] = 2/3$, then $Pr[X < 0.5]$ is ______.

(GATE 2015, 1 Mark)

Solution: We have

$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

So, $$\int_0^1 (a + bx)dx = 1$$

$$a + \frac{b}{2} = 1 \Rightarrow 2a + b = 2 \qquad \text{(i)}$$

Given that,

$$E[X] = \frac{2}{3} = \int_0^1 x[a + bx]dx$$

$$\frac{2}{3} = \frac{a}{2} + \frac{b}{3} \Rightarrow 3a + 2b = 4 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $a = 0$ and $b = 2$

$$\Pr[X < 0.5] = \int_0^{0.5} f(x)dx = 2\int_0^{0.5} xdx = 0.25$$

Ans. 0.25

100. Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

(a) 5/11 (b) 1/2 (c) 7/13 (d) 6/11

(GATE 2015, 1 Mark)

Solution: Probability of getting 6 $= \frac{6}{36} = \frac{1}{6}$

That is, probability of A wins the game $= \frac{1}{6}$

Probability of A not wins the game $= 1 - \frac{1}{6} = \frac{5}{6}$

Probability of B wins the game $= \frac{1}{6}$

Probability of B not winning the game $= \frac{5}{6}$

If A starts the game, probability that A wins the game $= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})$.

$P(\bar{B})P(A) + \cdots$

$$= \frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{1}{6} + \cdots$$

$$= \frac{1}{6}\left[1 + \frac{5}{6}\frac{5}{6} + \frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{5}{6} + \cdots\right] = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \cdots\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \left(\frac{5}{6}\right)^2}\right] = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Ans. (d)

101. If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, the value of $P(Y/X)$ is

(a) $\frac{1}{4}$ (b) $\frac{4}{25}$ (c) $\frac{1}{3}$ (d) $\frac{29}{50}$

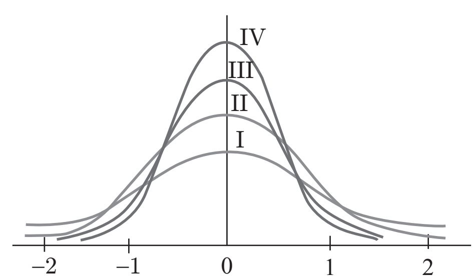
(GATE 2015, 1 Mark)

Solution:

$$P(Y/X) = \frac{P(X \cap Y)}{P(X)} = \frac{1/12}{1/4} = \frac{1}{3}$$

Ans. (c)

102. Among the four normal distribution with probability density functions as shown below, which one has the lowest variance?



(a) I (b) II (c) III (d) IV

(GATE 2015, 1 Mark)

Solution: The correct answer is option (d).

Ans. (d)

103. Consider the following probability mass function (p.m.f) of a random variable X.

$$p(x, q) = \begin{cases} q, & \text{if } X = 0 \\ 1 - q, & \text{if } X = 1 \\ 0, & \text{otherwise} \end{cases}$$

If $q = 0.4$, the variance of X is ______.

(GATE 2015, 1 Mark)

Solution: We are given,

$$p(x,q)=\begin{cases} q, & \text{if } X=0 \\ 1-q, & \text{if } X=1 \\ 0, & \text{otherwise} \end{cases}$$

Given $q = 0.4$, hence

$$p(x,q)=\begin{cases} 0.4, & \text{if } X=0 \\ 0.6, & \text{if } X=1 \\ 0, & \text{otherwise} \end{cases}$$

X	0	1
$p(X=x)$	0.4	0.6

Required value $= V(X) = E(X^2) - \{E(X)\}^2$

$$E(X) = \sum x_i p_i = 0\times 0.4 + 1\times 0.6 = 0.6$$
$$E(X^2) = \sum x_i^2 p_i = 0^2\times 0.4 + 1^2\times 0.6 = 0.6$$
$$V(X) = 0.6 - (0.6)^2 = 0.6 - 0.36 = 0.24$$

Ans. 0.24

104. Let X and Y denote the sets containing 2 and 20 distinct objects, respectively, and F denote the set of all possible functions defined from X to Y. Let f be randomly chosen from F. The probability of f being chosen from F. The probability of f being one-to-one is ________.

(GATE 2015, 2 Marks)

Solution: $|X| = 2, |Y| = 20$

Number of functions from X to Y is 20^2, that is 400 and number of one-one functions from X to Y is $20\times 19 = 380$.

Therefore, probability of a function f being one-one is $\dfrac{380}{400} = 0.95$.

Ans. 0.95

105. A random binary wave $y(t)$ is given by

$$y(t)=\sum_{n=-\infty}^{\infty} X_n p(t-nT-\phi)$$

where $p(t) = u(t) - u(t-T)$, $u(t)$ is the unit-step function and ϕ is an independent random variable with uniform distribution in $[0, T]$. The sequence $\{X_n\}$ consists of independent and identically distributed binary valued random variables with $P\{X_n = +1\} = P\{X_n = -1\} = 0.5$ for each n.

The value of the autocorrelation $R_{yy}\left(\frac{3T}{4}\right) \triangleq E\left[y(t)y\left(t-\frac{3T}{4}\right)\right]$ equals ________.

(GATE 2015, 2 Marks)

Solution: We are given,

$$y(t)=\sum_{n=-\infty}^{\infty} X_n p(t-nT-\phi) \quad R_{yy(z)} = \left[1-\frac{|\tau|}{T}\right]$$

$$R_{yy}\left(\frac{3T}{4}\right) = \left[1-\frac{3\pi/4}{\pi}\right] = \frac{1}{4} = 0.25$$

Ans. 0.25

106. The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% marks in it is 5%. Given that a student passes the examination, the probability that the student gets above 90% marks is

(a) $\dfrac{1}{18}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{2}{9}$ (d) $\dfrac{5}{18}$

(GATE 2015, 2 Marks)

Solution: Let A be the event that a student passes the exam and B be the event that student gets above 90% marks.

We are given,

$$P(A) = 0.2; P(A\cap B) = 0.05$$

Required probability is $P(B/A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{0.05}{0.2} = \dfrac{1}{4}$

Ans. (b)

107. The probability of obtaining at least two 'SIX' in throwing a fair dice 4 times is

(a) 425/432 (b) 19/144

(c) 13/144 (d) 125/432

(GATE 2015, 2 Marks)

Solution: From the given data, $n = 4$ and $p = 1/6$

$$\Rightarrow q = 1-\frac{1}{6} = \frac{5}{6}$$

$$p(x\ge 2) = 1-p(x<2) = 1-[p(x=0)+p(x=1)]$$
$$= 1-\left[4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 + 4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3\right] = \frac{19}{144}$$

Ans. (b)

108. For probability density function of a random variable, x is

$$f(x) = \frac{x}{4}(4 - x^2) \text{ for } 0 \le x \le 2$$
$$= 0 \text{ otherwise}$$

The mean μ_x of the random variable is ________.

(GATE 2015, 2 Marks)

Solution: We are given

$$f(x) = \frac{x}{4}(4 - x^2),\ 0 \le x \le 2$$

Mean $= \mu_x = E(x)$

$$= \int_0^2 x f(x)dx = \int_0^2 x\left(\frac{x}{4}\right)(4 - x^2)dx$$

$$= \frac{1}{4}\int_0^2 (4x^2 - x^4)dx = \frac{1}{4}\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{1}{4}\left[4.\frac{8}{3} - \frac{32}{5}\right] = \frac{32}{4}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= 8\left[\frac{2}{15}\right] = \frac{16}{15} = 1.0667$$

Ans. 1.0667

109. The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up 3 are defective is

(a) 0.001 (b) 0.057 (c) 0.107 (d) 0.3

(GATE 2015, 2 Marks)

Solution: Let p be probability of a thermistor is defective = 0.1

$$q = 1 - p = 1 - 0.1 = 0.9$$
$$n = 10$$

Let X be the random variable which is number of defective pieces.
Required probability = $P(x = 3)$

$$= {}^{10}C_3 p^3 q^7 = {}^{10}C_3 (0.1)^3 (0.9)^7 = 0.057$$

Ans. (b)

110. The probability-density function of a random variable x is $p_x(x) = e^{-x}$ for $x \ge 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{3x/4}$ is ________.

(GATE 2015, 2 Marks)

Solution: Probability density function of x is given as

$$p_x(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

and $$g_x(x) = e^{\frac{3x}{4}}$$

$$E[g_x(x)] = \int_0^\infty f_x(x) g_x(x) dx$$

$$= \int_0^\infty e^{-x} e^{\frac{3x}{4}} dx = \int_0^\infty e^{\frac{3x}{4} - x} dx = \int_0^\infty e^{-\frac{x}{4}} dx$$

$$= \left[\frac{e^{-x/4}}{-1/4}\right]_0^\infty = -4(e^{-\infty} - 1) = -4(0 - 1) = 4$$

Ans. 4

111. A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is ________.

(GATE 2016, 1 Mark)

Solution: It is given that

$$f(x) = \begin{cases} 1/x^2, & \text{for } a \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

That is,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_a^1 \frac{1}{x^2} dx = 1$$

$$\Rightarrow \left[-\frac{1}{x}\right]_a^1 = 1$$

$$\Rightarrow \frac{1}{a} - 1 = 1$$

$$\Rightarrow a = \frac{1}{2} = 0.5$$

Ans. 0.5

112. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is ________.

(GATE 2016, 1 Mark)

Solution:

(i) **Type 1 LED bulb:** The probability that the bulb is of Type 1 and it lasts more than 100 hours is

$$\frac{1}{2}\times 0.7$$

(ii) **Type 2 LED bulb:** The probability that the bulb is of Type 2 and it lasts for more than 100 hours is

$$\frac{1}{2}\times 0.4$$

Therefore, the probability that an LED bulb chosen uniformly at random and lasts more than 100 hours is

$$\left(\frac{1}{2}\times 0.7\right)+\left(\frac{1}{2}\times 0.4\right)=0.55$$

Ans. 0.55

113. The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is ________.

(GATE 2016, 1 Mark)

Solution: For this Poisson distribution,

$$\text{Mean}=\text{Variance}=2$$

However, we know that variance is

$$\sigma^2=m_2-m_1^2 \qquad \text{(i)}$$

where m_2 is the second central moment and m_1 is the mean. From Eq. (i), we get

$$\lambda=2-\lambda^2$$

$$\lambda^2+\lambda-2=0\Rightarrow(\lambda+2)\ (\lambda-1)=0$$

Therefore,

$$\lambda=-2 \text{ or } \lambda=1$$

Since variance (λ) is positive, $\lambda=1$.

Ans. 1

114. The probability of getting a 'Head' in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a 'Head' is obtained. If the tosses are independent, then the probability of getting 'Head' for the first time in the fifth toss is ________.

(GATE 2016, 1 Mark)

Solution: We have

$$P(\text{Head})=0.3=p \text{ and } P(\text{Tail})=0.7=q$$

The probability of getting 'head' for the first time in the fifth toss:

$$P(\text{Tail})\cdot P(\text{Tail})\cdot P(\text{Tail})\cdot P(\text{Tail})\cdot P(\text{Head})=(0.7)^4\ (0.3)=0.07203$$

Ans. 0.07203

115. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(a) $\sqrt{\mu}$ (b) μ^2 (c) μ (d) $1/\mu$

(GATE 2016, 1 Mark)

Solution: For Poisson distribution,

$$\sigma^2=\mu$$

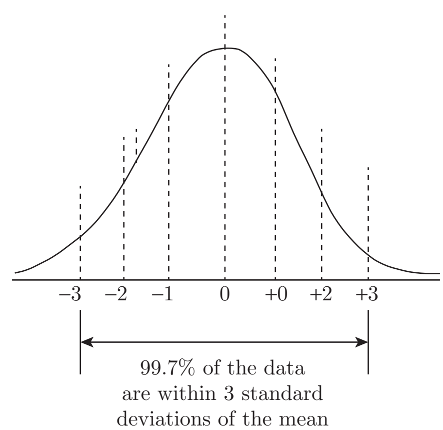
$$\Rightarrow \sigma=\sqrt{\mu}$$

Ans. (a)

116. The area (in percentage) under standard normal distribution curve of random variable Z within limits from −3 to +3 is ______.

(GATE 2016, 1 Mark)

Solution: A standard normal curve (as shown in figure) has 68% are in limits −1 to +1, 95% area is limits −2 to +2 and 99.7% area in limits −3 to +3.



Ans. 99.7

117. Type II error in hypothesis testing is

(a) acceptance of the null hypothesis when it is false and should be rejected
(b) rejection of the null hypothesis when it is true and should be accepted
(c) rejection of the null hypothesis when it is false and should be rejected
(d) acceptance of the null hypothesis when it is true and should be accepted

(GATE 2016, 1 Mark)

Solution: In hypothesis testing, type II error has acceptance of null hypothesis when it is false and should be rejected.

Ans. (a)

118. X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7 (b) 0.5 (c) 0.4 (d) 0.3

(GATE 2016, 1 Mark)

Solution: $P(X) = 0.4$ and $P(X \cup Y^C) = 0.7$

$$P(X) + P(Y^C) - \left\{P(X \cap Y^C)\right\} = 0.7$$

$$P(X) + P(Y^C) - P(X)P(Y^C) = 0.7$$

$$0.4 + P(Y^C) - 0.4\ P(Y^C) = 0.7$$

$$0.4 + 0.6\ P(Y^C) = 0.7$$

$$P(Y^C) = \frac{0.7 - 0.4}{0.6}$$

$$P(Y^C) = 0.5$$

Now,

$$P(Y^C) = 1 - P(Y)$$

Therefore,

$$P(Y) = 1 - 0.5 = 0.5$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$
$$= 0.4 + 0.5 - 0.4 \times 0.5$$
$$= 0.7$$

Ans. (a)

119. Consider the following experiment.

Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places) ________.

(GATE 2016, 2 Marks)

Solution: The probability, that the output of the given experiment is Y, can be written as follows:

$$P(Y) = \left(\frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$$

$$= \frac{1/4}{1 - (1/4)} = \frac{1}{3} = 0.33$$

1st time	HEADS	HEADS	TAILS	TAILS
2nd time	HEADS	TAILS	HEADS	TAILS
		N	Y	

Ans. 0.33

120. Two random variables x and y are distributed according to

$$f_{x,y}(x, y) = \begin{cases} (x + y), & 0 \le x \le 1 \quad 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(X+Y \le 1)$ is ________.

(GATE 2016, 2 Marks)

Solution: We have

$$P(x + y \le 1) = \int_{x=0}^{1} \int_{y=0}^{1} f_{xy}(x,y)\ dx\ dy$$

$$\Rightarrow \int_{x=0}^{1} \int_{0}^{1-x} (x + y) dx\ dy = \int_{x=0}^{1} \left(xy + \frac{y^2}{2}\right)_0^{1-x} dx$$

$$= \int_0^1 \left(x(1-x) + \frac{(1-x)^2}{2}\right) dx$$

$$= \frac{x}{2} - \frac{x^3}{6}\Big|_0^1 = 0.33$$

Ans. 0.33

121. Candidates were asked to come to an interview with 3 pens each. Black, blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is ________.

(GATE 2016, 2 Marks)

Solution: Let Black = a
Blue = b
Green = c
Red = d

For all three pens to be of the same colour, four cases are possible:

aaa

bbb

ccc

ddd

Let first pen be a.

Next, choose 1 from 4 = ${}^4C_1$

Next, choose 1 from 4 = $^4C_1$

$$= a \cdot {}^4C_1 \cdot {}^4C_1 = 16 \text{ cases}$$

Similarly

$$b \cdot {}^4C_1 \cdot {}^4C_1 = 16 \text{ cases}$$
$$c \cdot {}^4C_1 \cdot {}^4C_1 = 16 \text{ cases}$$
$$d \cdot {}^4C_1 \cdot {}^4C_1 = 16 \text{ cases}$$

Total 64 cases. Therefore, probability $= \dfrac{4}{64} = \dfrac{1}{6}$ or 0.167

Ans. 0.167

122. Let the probability density function of a random variable, X, be given as

$$f_X(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$$

where $u(x)$ is the unit step function.

Then the value of 'a' and Prob$\{X \leq 0\}$, respectively, are

(a) $2, \dfrac{1}{2}$ (b) $4, \dfrac{1}{2}$

(c) $2, \dfrac{1}{4}$ (d) $4, \dfrac{1}{4}$

(GATE 2016, 2 Marks)

Solution: For the given function:

$$f(x) = \begin{cases} ae^{4x}, & x < 0 \\ \dfrac{3}{2}e^{-3x}, & x \geq 0 \end{cases}$$

$$\int_{\infty}^{+\infty} f_x(x) = 1$$

Therefore,

$$\int_{-\infty}^{0} ae^{4x}dx + \int_{0}^{\infty} \frac{3}{2}e^{-3x}dx = 1$$

$$\left.\frac{ae^{4x}}{4}\right|_{-\infty}^{0} + \frac{3}{2}\left(\frac{-1}{3}\right)e^{-3x}\Big|_{0}^{\infty} = 1$$

$$\frac{a}{4} + \frac{1}{2} = 1 \Rightarrow a = 2$$

Therefore,

$$P(X \leq 0) = \int_{-\infty}^{0} 2e^{4x}dx = \frac{1}{2}$$

Ans. (a)

123. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is ______.

(GATE 2016, 2 Marks)

Solution: P(screw is defective) = 0.1

P(that a packet would have to be replaced) = P(one screw is found defective) + P(two screws defective) + $\cdots$ + P(five screws are defective)

$$= 1 - P(\text{no screw is defective})$$
$$= 1 - 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9$$
$$= 0.4095$$

Ans. 0.4095

124. Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(a) $\dfrac{16}{5525}$ (b) $\dfrac{64}{2197}$ (c) $\dfrac{3}{13}$ (d) $\dfrac{8}{16575}$

(GATE 2016, 2 Marks)

Solution: Total ways in which three cards can be drawn

$$= {}^{52}C_3$$

Number of ways in which a king, a queen and a jack can be drawn

$$= {}^4C_1 \times {}^4C_1 \times {}^4C_1$$

Therefore, the required probability

$$= \frac{4 \times 4 \times 4}{\dfrac{52 \times 51 \times 50}{6}}$$

$$= \frac{384}{132600} = \frac{16}{5525}$$

Ans. (a)

125. Probability density function of a random variable X is given below

$$f(x) = \begin{cases} 0.25 & \text{if } 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

$P(X \leq 4)$ is

(a) $\dfrac{3}{4}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

(GATE 2016, 2 Marks)

Solution: $P(X \le 4) = \int_0^4 f(x)\,dx$

$$= \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx$$

$$= 0 + \int_1^4 0.25\,dx = [0.25x]_1^4$$

$$= 0.25\,[4-1] = 0.25 \times 3$$

$$= 0.75 = \frac{3}{4}$$

Ans. (a)

126. If $f(x)$ and $g(x)$ are two probability density functions.

$$f(x) = \begin{cases} \dfrac{x}{a}+1 & : \; -a \le x < 0 \\ -\dfrac{x}{a}+1 & : \; 0 \le x \le a \\ 0 & : \; \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} -\dfrac{x}{a} & : \; -a \le x < 0 \\ \dfrac{x}{a} & : \; 0 \le x \le a \\ 0 & : \; \text{otherwise} \end{cases}$$

Which one of the following statements is true?

(a) Mean of $f(x)$ and $g(x)$ are same: Variance of $f(x)$ and $g(x)$ are same
(b) Mean of $f(x)$ and $g(x)$ are same: Variance of $f(x)$ and $g(x)$ are different
(c) Mean of $f(x)$ and $g(x)$ are different: Variance of $f(x)$ and $g(x)$ are same
(d) Mean of $f(x)$ and $g(x)$ are different: Variable of $f(x)$ and $g(x)$ are different

(GATE 2016, 2 Marks)

Solution: Mean of $f(x)$ = Mean of $g(x)$
Variance of $f(x) \neq$ Variance of $g(x)$

Ans. (b)

127. Let X be a Gaussian random variable with mean 0 and variance σ^2. Let $Y = \max(X, 0)$, where max (a, b) is the maximum of a and b. The median of Y is __________.

(GATE 2017, 1 Mark)

Solution: As X is a Gaussian random variable, the distribution of x is $N(0, \sigma^2)$.

Given $Y = \max(X, 0)$

$$= \begin{cases} X, & \text{if } 0 < X < \infty \\ 0, & \text{if } -\infty < X \le 0 \end{cases}$$

Therefore, median remains at 0, as it is a positional average.

Hence, median $(Y) = 0$.

Ans. (0.0)

128. An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball in the second draw is

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{6}{9}$

(GATE 2017, 1 Mark)

Solution: Let the first ball be red. Then P(red) in the second draw = 4/9.
Let the first ball be black. Then P(red) in the second draw = 5/9.
Therefore, probability

$$= \frac{1}{2}\times\frac{4}{9} + \frac{1}{2}\times\frac{5}{9} = \frac{1}{2}$$

Ans. (a)

129. Two resistors with nominal resistance values R_1 and R_2 have additive uncertainties ΔR_1 and ΔR_2, respectively. When these resistances are connected in parallel, the standard deviation of the error in the equivalent resistance R is

(a) $\pm\sqrt{\left\{\dfrac{\partial R}{\partial R_1}\Delta R_1\right\}^2 + \left\{\dfrac{\partial R}{\partial R_2}\Delta R_2\right\}^2}$

(b) $\pm\sqrt{\left\{\dfrac{\partial R}{\partial R_2}\Delta R_1\right\}^2 + \left\{\dfrac{\partial R}{\partial R_1}\Delta R_2\right\}^2}$

(c) $\pm\sqrt{\left\{\dfrac{\partial R}{\partial R_1}\right\}^2 \Delta R_2 + \left\{\dfrac{\partial R}{\partial R_2}\right\}^2 \Delta R_1}$

(d) $\pm\sqrt{\left\{\dfrac{\partial R}{\partial R_1}\right\}^2 \Delta R_1 + \left\{\dfrac{\partial R}{\partial R_2}\right\}^2 \Delta R_2}$

(GATE 2017, 1 Mark)

Solution:

$$\sigma_{\text{res}} = \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 \sigma_2^2}$$

$$= \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 \Delta R_1^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 \Delta R_2^2}$$

Ans. (a)

130. A six-face dice is rolled a large number of times. The mean value of the outcome is ____.

(GATE 2017, 1 Mark)

Solution: The probabilities corresponding to the outcomes are 1/6, 2/6, 3/6, 4/6, 5/6, 6/6.

Therefore, the mean value of the outcome is 21/6 = 3.5.

Ans. 3.5

131. Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is ______.

(GATE 2017, 1 Mark)

Solution: Total number of outcomes when two coins are tossed is 4 and sample space is

$$S = \{HH, HT, TH, TT\}$$
$$n(S) = 4$$

Favourable outcomes for existence of at least one head (H) are only three. Thus, required probability is 3/4 = 0.75.

Ans. 0.75

132. A sample of 15 data is as follows: 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

(a) 4 (b) 13 (c) 17 (d) 20

(GATE 2017, 1 Mark)

Solution: The mode is the value of the data which occurred most of time. For the given data, mode is 17.

Ans. (c)

133. The number of parameters in the univariate exponential and Gaussian distributions, respectively, are

(a) 2 and 2 (b) 1 and 2
(c) 2 and 1 (d) 1 and 1

(GATE 2017, 1 Mark)

Solution: Normal or Gaussian distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

There are two parameters, i.e. μ and σ, in Gaussian distribution.
The PDF of an exponential distribution is

$$f(x) = \lambda e^{-\lambda x}; \qquad x = 0$$

where $\lambda > 0$ is the parameter of distribution.

Therefore, the total parameters are

Exponential = 1

Gaussian = 2

Ans. (b)

134. Vehicles arriving at an intersection from one of the approach roads follow the Poisson distribution. The mean rate of arrival is 900 vehicles per hour. If a gap is defined as the time difference between two successive vehicle arrivals (with vehicles assumed to be points), the probability (up to four decimal places) that the gap is greater than 8 seconds is _________.

(GATE 2017, 1 Mark)

Solution: Probability of time headway being greater than 8 s is

$$P(n \geq 8) = e^{-8\lambda} = e^{-8\times\frac{900}{360}} = e^{-2} = 0.1353$$

Ans. (0.1353)

135. A two-faced fair coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes: H, H, H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realizations of H, H and H, would be ______.

(GATE 2017, 1 Mark)

Solution:

$$\text{Probability of getting (H, H, H, H)} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{16}$$

$$\text{Probability of getting (H, H, H)} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

The given condition is getting next H after (H, H, H). Therefore,

$$\text{Required probability} = \frac{\frac{1}{16}}{\frac{1}{8}} = 0.5$$

Ans. (0.5)

136. If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)^2]$ equals ______.

(GATE 2017, 2 Marks)

Solution:

In Poisson distribution, if X is a random variable, then

$$E(X) = \text{Mean} = \lambda = 5$$