Function

A function relates an input to an output.

Definition

A relation " f “from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

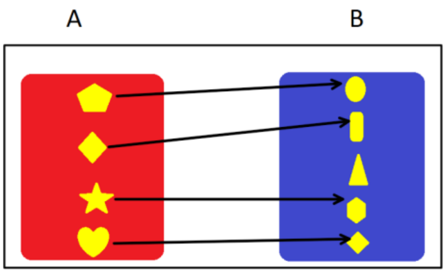

Figure 4:The Function f Maps A to B

In the function one-to-many" is not allowed, but "many-to-one" is allowed it can be checked by vertical line test.

In Figure 5 it can be seen that the vertical line crosses the left curve at more than one point it means there are two outputs (y values) for the same input (x value) which is not allowed in the function. The right curve has exactly one output (y value) for every input(x value)

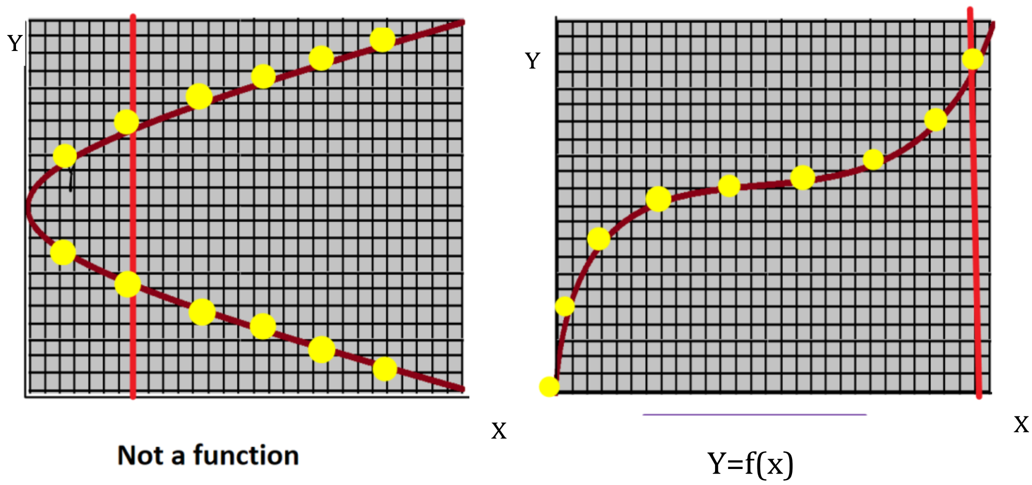

Figure 5: Vertical line test for function

Domain, Codomain, Range

If f is a function from A to B, we say that A is the domain of f and B is the codomain of f. If f (a) = b, we say that b is the image of a and a is a preimage of b.

The range, or image, of f is the set of all images of elements of A.

Example:

Let A= {a, b, c} and B= {1, 2, 3, 4} $and\ f: A \to B = \{(a, 1), (b, 2), (c, 3)\}$ then the domain is {a, b, c} ,the codomain is {1, 2, 3, 4}, the range is {1, 2, 3}

> Let A and B are two finite sets having m and n elements respectively then total no. of function from set A to B is n^m.

Types of the function

Constant function

The function f is called constant function if every element of A has the same image in B. Range of a constant function is a singleton set.

Example:

Let A = {x, y, u, v}, B = {2, 3, 5, 7, 11}. The function $f: A \to B$ defined by f (x) = 5 for every x belonging to A is a constant function.

Identity function

The function f: R → R defined by y = f (x) = x for each x ∈ R is called the identity function. That is, an identity function maps each element of A into itself.

Let A be the set of real numbers (R). The function $f: R \to R$ be defined by f (x) = x for all x belonging to R is the identity function on R.

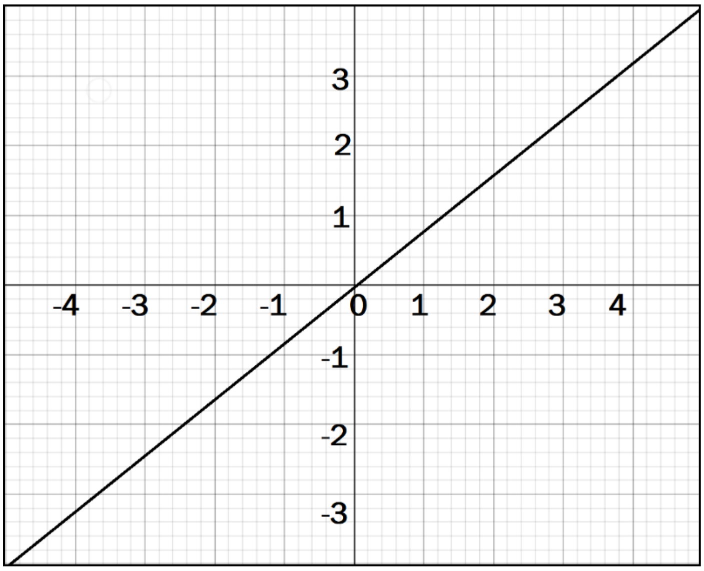

Figure 6: Graph of the identity function

The Modulus Function

The modulus of any number gives us the magnitude of that number. Using the modulus operation, we can define the modulus function as follows: $f(x)=|x|$.

$$f(x) = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$$

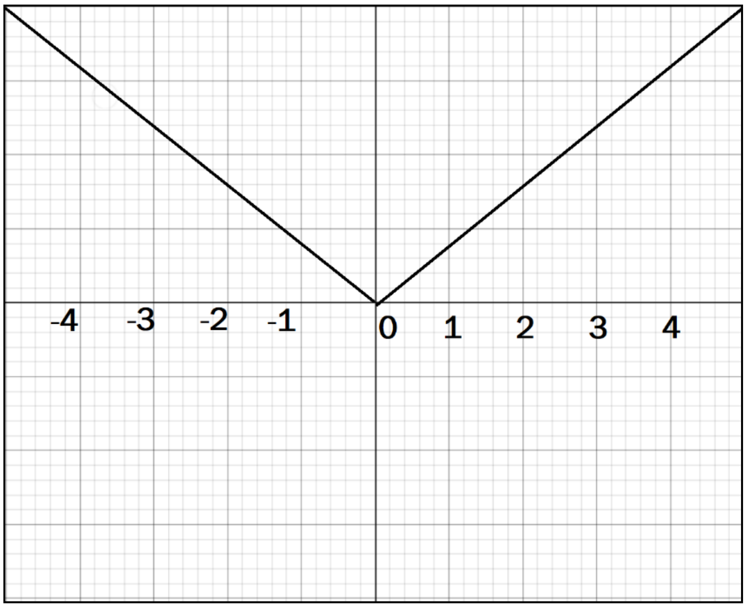

Figure 7:Graph of Modulus function

Greatest integer function

The real function *f: R ⟶ R defined by f (x) = [x], x ∈R* assumes the value of the greatest integer less than or equal to x, is called the greatest integer function.

Example:

[5.5] =5

Polynomial Function

A real valued function f: R ⟶ R defined by $y = f(x) = a_0x^0 + a_1x^1 + a_2x^2 + \cdots, a_nx^n$. where n ∈ N,
$a_0, a_1, a_2 \cdots a_n \in R$ and for each x ∈ R, is called Polynomial functions.

Example:

$y = f(x) = 5x^0 + 10x^1 + 15x^2$, $\forall x \in [0,10]$.

One-one function

A function f: A ⟶ B is defined to be one-one (or injective) if distinct elements of A have a different image in B. $i.e.,\ for\ every\ x_1, x_2 \in X, f(x_1) = f(x_2)$ $implies\ x_1 = x_2$

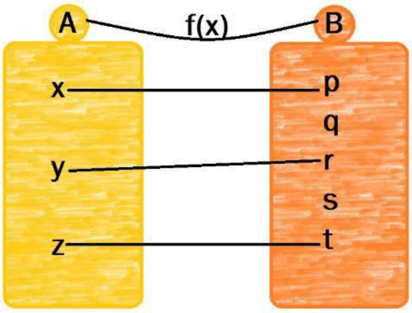

Figure 8:A One-to-One Function.

Example:

$f(x) = x^3$ one-one where f: R→R

Many-one

A function f: A → B is defined to be many one if there exist two or more than 2 elements in set A has
the same image in set B

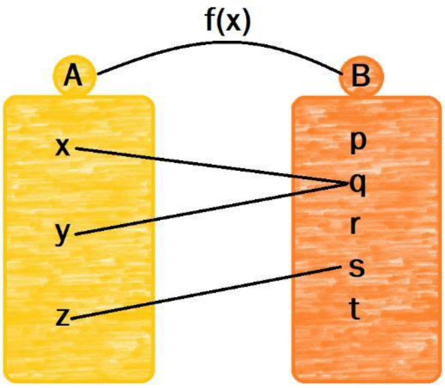

Figure 9: Many one function

Example:

$f(x) = Sinx$ Many-one where f: R→R .

Onto

A function f: A → B is defined to be onto (or surjective), if every element of B is the image of some
element of A under f.

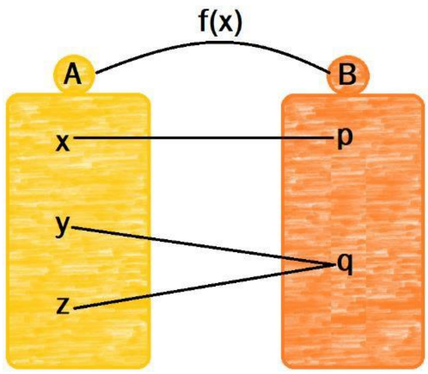

Figure 10: An onto function

Example:

Let f be the function from *{P, Q, R, S} to {10, 20, 30}* defined by
f (P) = 10, f (Q) = 20, f (R) = 10, and f (S) = 30 is an onto function.

One-one and onto

A function f: A → B is said to be one-one and onto (or bijective), if f is both one-one and onto.

Example:

$$g(x) = \begin{cases} x - 1, x \text{ is odd} \\ x + 1, x \text{ is even} \end{cases}$$

1. **Inverse Functions:**
   Bijection function are also known as invertible function because they have inverse function property. The inverse of bijection f is denoted as f^{-1}. It is a function which assigns to b, a unique element a such that f(a) = b. hence f^{-1} (b) = a.

Strictly Increasing and Strictly decreasing functions: A function f is strictly increasing if f(x) > f(y) when x>y. A function f is strictly decreasing if f(x) < f(y) when x<y.
Increasing and decreasing functions: A function f is increasing if f(x) ≥ f(y) when x>y. A function f is decreasing if f(x) ≤ f(y) when x<y.
Function Composition: let g be a function from B to C and f be a function from A to B, the composition of f and g, which is denoted as fog(a)= f(g(a)).

Properties of function composition:

1. fog ≠ gof
2. If f and g both are one to one function, then fog is also one to one.
3. If f and g both are onto function, then fog is also onto.

4. If f and fog both are one to one function, then g is also one to one.
5. If f and fog are onto, then it is not necessary that g is also onto.
6. $(fog)^{-1} = g^{-1} o f^{-1}$

Some Important Points:

1. A function is one to one if it is either strictly increasing or strictly decreasing.
2. one to one function never assigns the same value to two different domain elements.
3. For onto function, range and co-domain are equal.
4. If a function f is not bijective, inverse function of f cannot be defined.