UNIT 2: AC CIRCUITS

(Lecture 8 to 14 + Tutorial 4 to 6)

Prepared By:

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Outcome: Understand the fundamental behaviour and notations of AC circuits and solve AC circuit problems

Fundamentals of A.C. circuits : Alternating current and voltage, concept of notations (i, v, I, V), definitions of amplitude, phase, phase difference, RMS value and average value of an AC signal, complex representation of impedance, steady state analysis of ac circuits consisting of RL, RC and RLC (series), resonance in series RLC circuit, power factor and power calculation in RL, RC and RLC circuits, three-phase circuits- numbering and interconnection (delta or mesh connection) of three phases, relations in line and phase voltages and currents in star and delta

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 8

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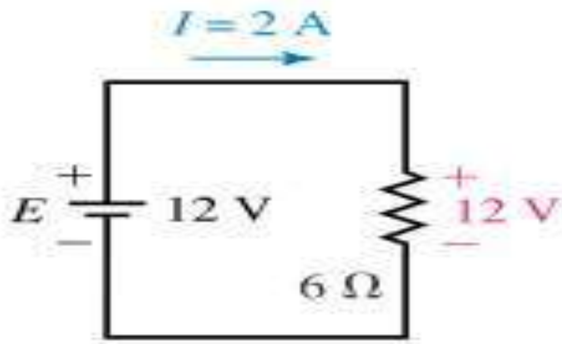
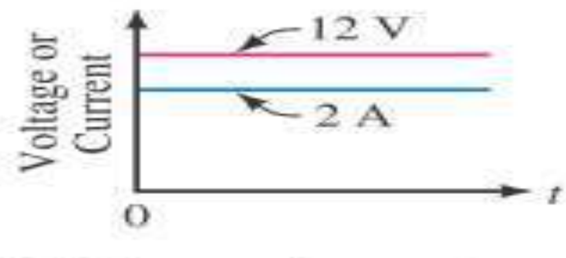
AC Fundamentals

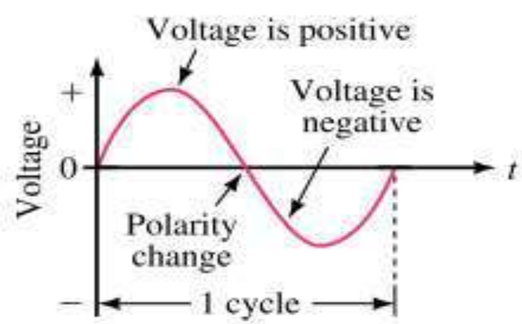
- Previously you learned that DC sources have fixed polarities and constant magnitudes and thus produce currents with constant value and unchanging direction





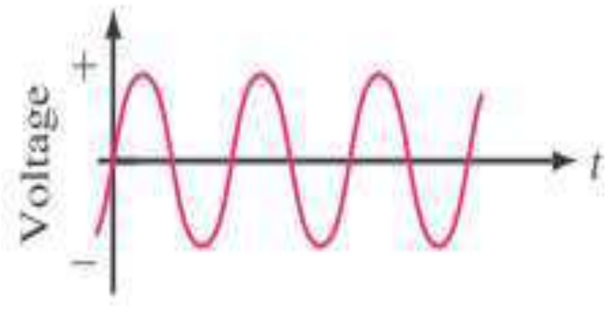
(b) Voltage and current versus time for dc

- In contrast, the voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.



➢ Sinusoidal ac Voltage

One complete variation is referred to as a cycle.

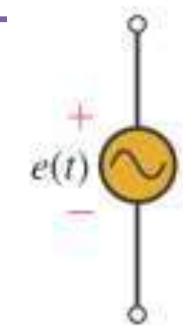


(b) A continuous stream of cycles

Starting at zero,
the voltage increases to a positive peak amplitude,
decreases to zero,
changes polarity,
increases to a negative peak amplitude,
then returns again to zero.

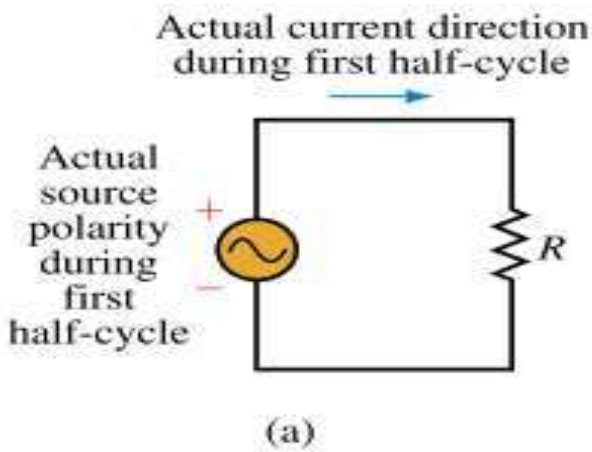
- Since the waveform repeats itself at regular intervals, it is called a **periodic signal**.

➢ Symbol for an ac Voltage Source

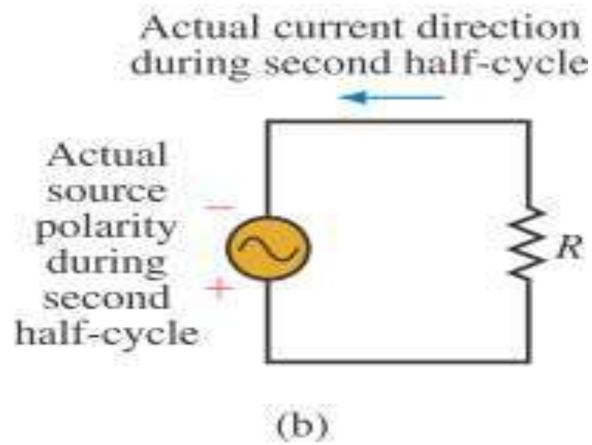


Lowercase letter e is used
to indicate that the voltage varies with time.

Sinusoidal ac Current



(a)



(b)

- During the first half-cycle, the source voltage is positive
- Therefore, the current is in the clockwise direction.

- During the second half-cycle, the voltage polarity reverses
- Therefore, the current is in the counterclockwise direction.

- Since current is proportional to voltage, its shape is also sinusoidal

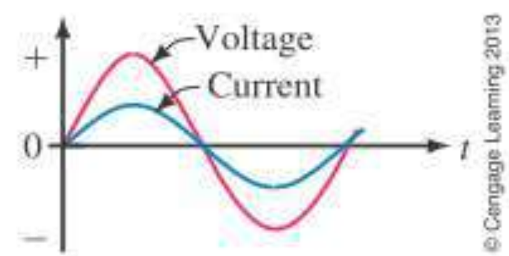


FIGURE 15–5 Current has the same wave-shape as voltage.

Quick Quiz (Poll 1)

The frequency of domestic power supply in India is

(A) 200 Hz

(B) 100 Hz

(C) 60 Hz

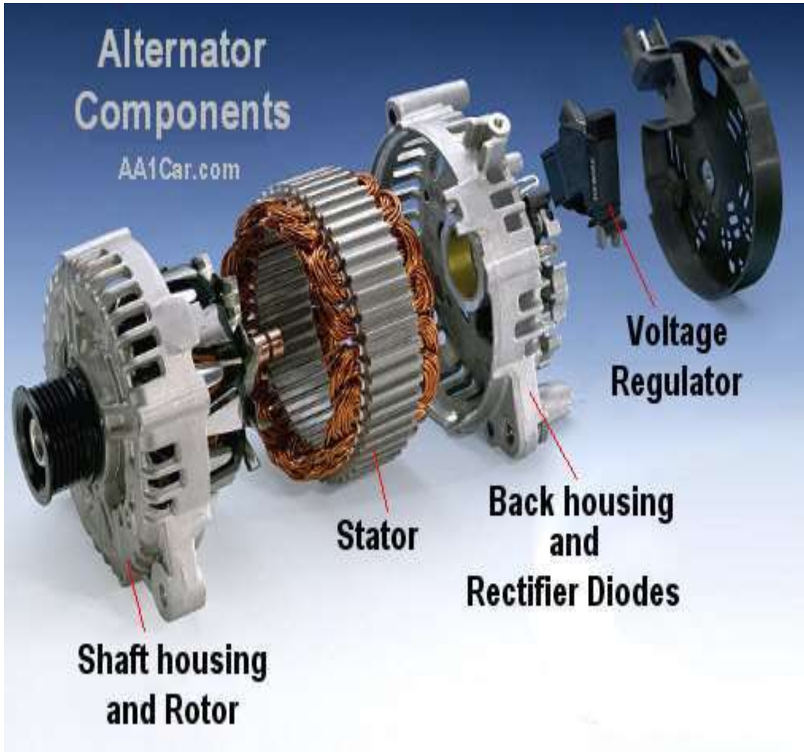
(D) 50 Hz

GENERATION OF AC VOLTAGE

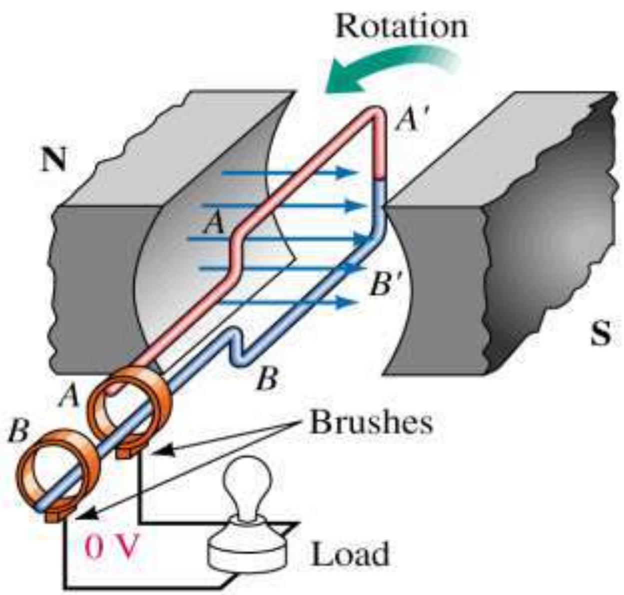
An **alternator** is an electrical generator that converts mechanical energy to electrical energy in the form of alternating current

Principle: A conductor moving relative to a magnetic field develops an electromotive force (EMF) in it. (Faraday's Law).

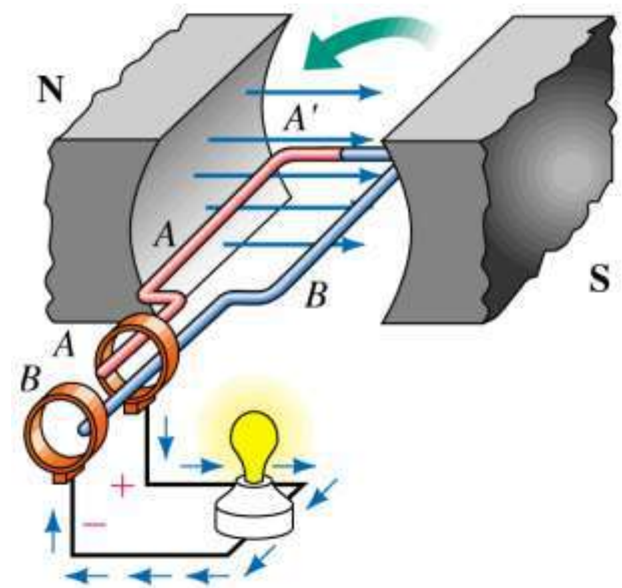
This emf reverses its polarity when it moves under magnetic poles of opposite polarity.



Generating AC Voltages

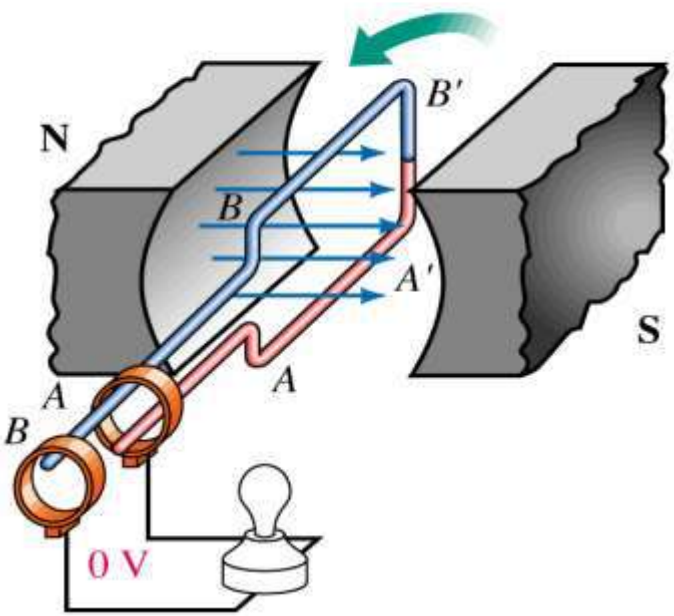


(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.

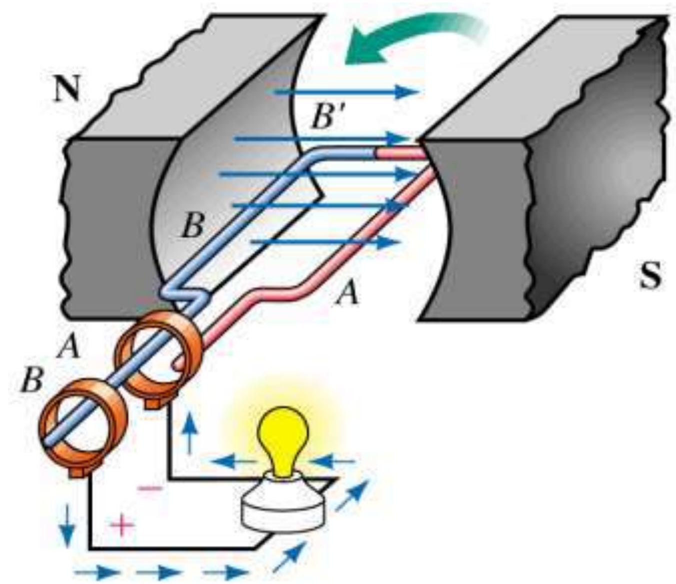


(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

Generating AC Voltages



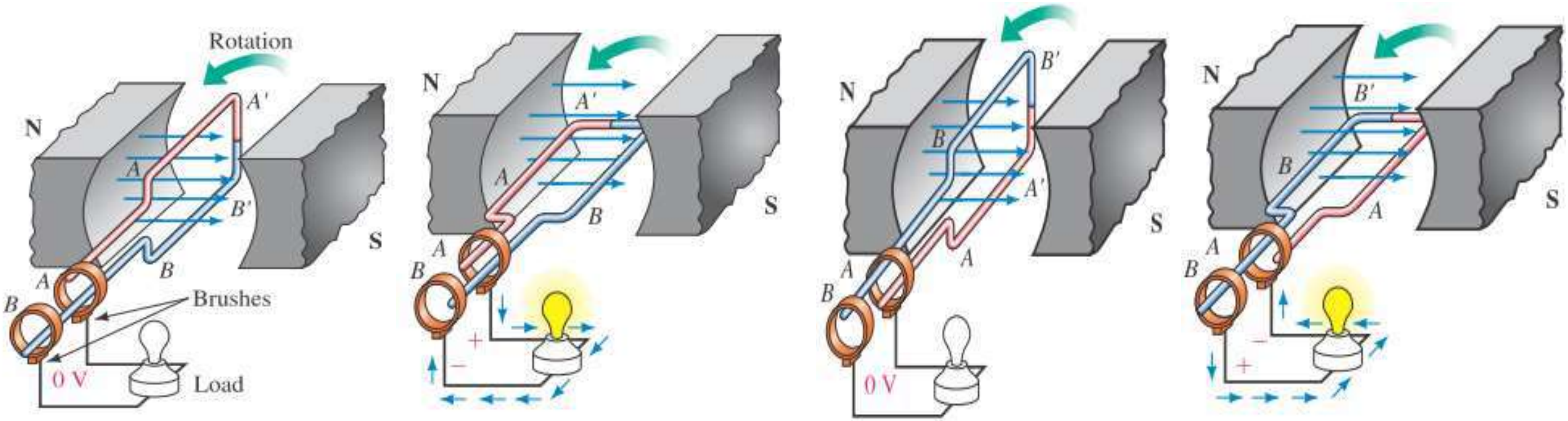
(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.



(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.

Generating ac Voltages (Method A)

- One way to generate an ac voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field



(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.

(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

(d) 270° Position: Voltage polarity has reversed, therefore, current direction has also reversed.

- The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut

- its polarity is dependent on the direction the coil sides move through the field.

Generating ac Voltages

➢ Since the coil rotates continuously, the voltage produced will be a repetitive,

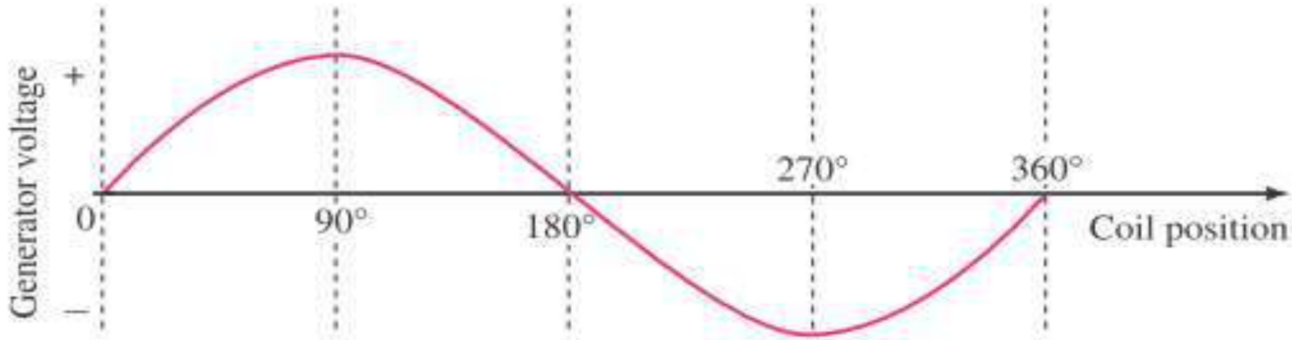


FIGURE 15–7 Coil voltage versus angular position.

Time Scales

➢ Often we need to scale the output voltage in time.

➢ The length of time required to generate one cycle depends on the velocity of rotation.

600 revolutions in 1 minute = 600 rev / 60 s
= 10 revolutions in 1 second.

The time for 1 revolution = one-tenth of a second
= 100 ms

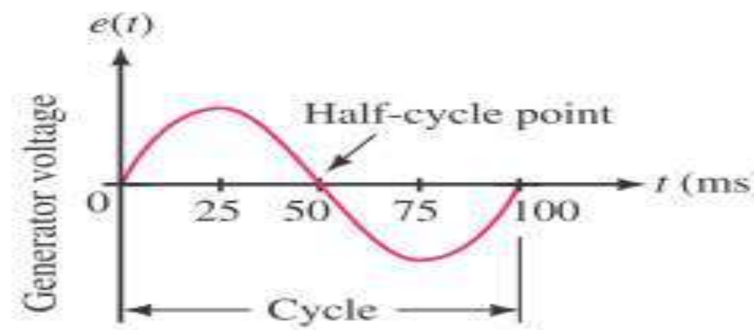


FIGURE 15–8 Cycle scaled in time. At 600 rpm, the cycle length is 100 ms.

Generating ac Voltages (Method-2)

- AC waveforms may also be created electronically using function (or signal) generators.
- With function generators, you are not limited to sinusoidal ac. gear.

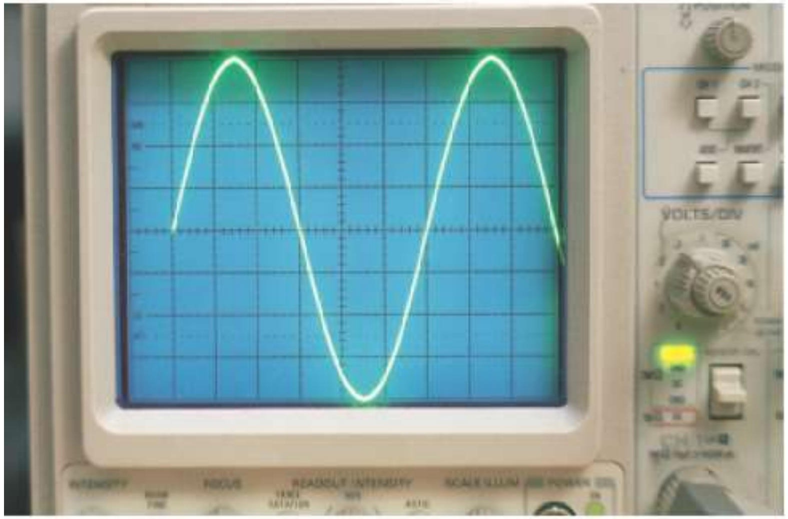
- The unit of Figure can produce a variety of variable-frequency waveforms, including sinusoidal, square wave, triangular, and so on.
- Waveforms such as these are commonly used to test electronic

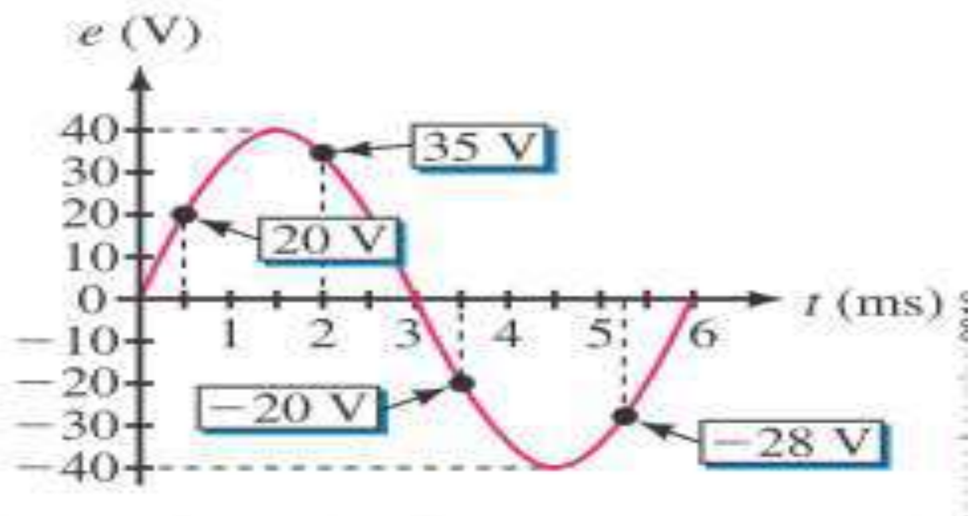
(a) Model 4045 Function Generator (b) Sample waveforms

0 Electronic function generators provide a variety of variable-frequency, variable-amplitude waveforms.

Instantaneous Value

- As the coil voltage changes from instant to instant. The value of voltage at any point on the waveform is referred to as its **instantaneous value.**





(b) Values scaled from the photograph

- The voltage has a peak value of 40 volts
- The cycle time of 6 ms.

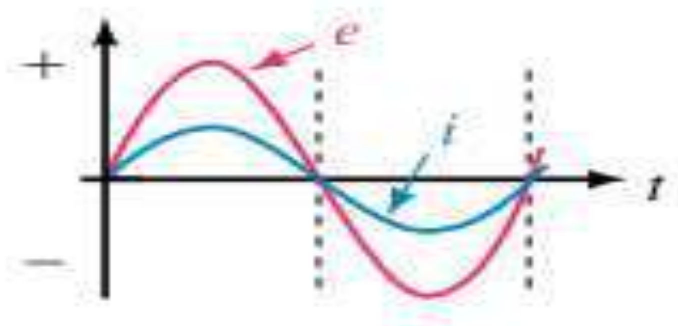
✓ at t = 0 ms, the voltage is zero.
✓ at t=0.5 ms, the voltage is 20V.

Voltage and Current Conventions for ac

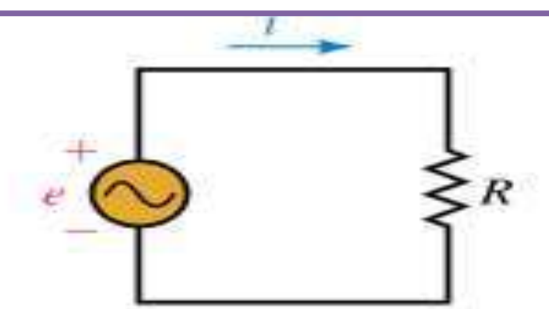
- First, we assign reference polarities for the source and a reference direction for the current.

- We then use the convention that, when e has a positive value, its actual polarity is the same as the reference polarity, and when e has a negative value, its actual polarity is opposite to that of the reference.

- For current, we use the convention that when i has a positive value, its actual direction is the same as the reference arrow,
- and when i has a negative value, its actual direction is opposite to that of the reference.

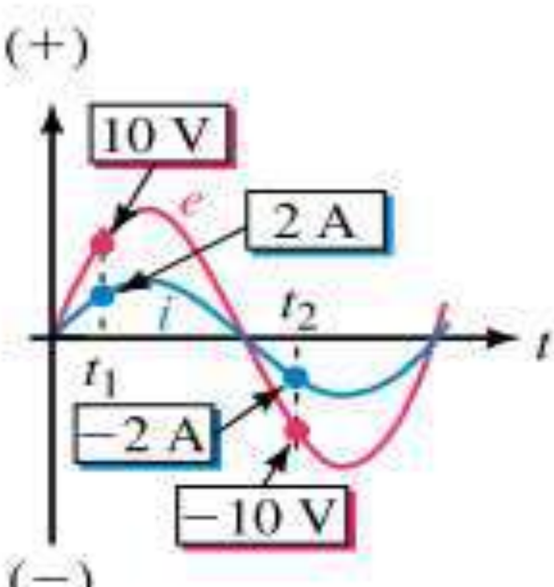


(b) During the first half-cycle, voltage polarity and current direction are as shown in (a). Therefore, e and i are positive. During the second half-cycle, voltage polarity and current direction are opposite to that shown in (a). Therefore, e and i are negative.

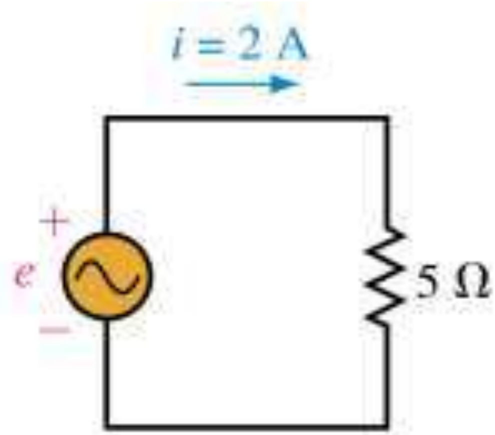


(a) References for voltage and current.

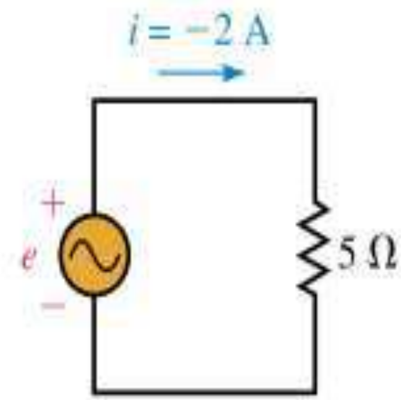
Voltage and Current Conventions for ac



(a)



(b) Time t_1: $e = 10$ V and $i = 2$ A. Thus voltage and current have the polarity and direction indicated

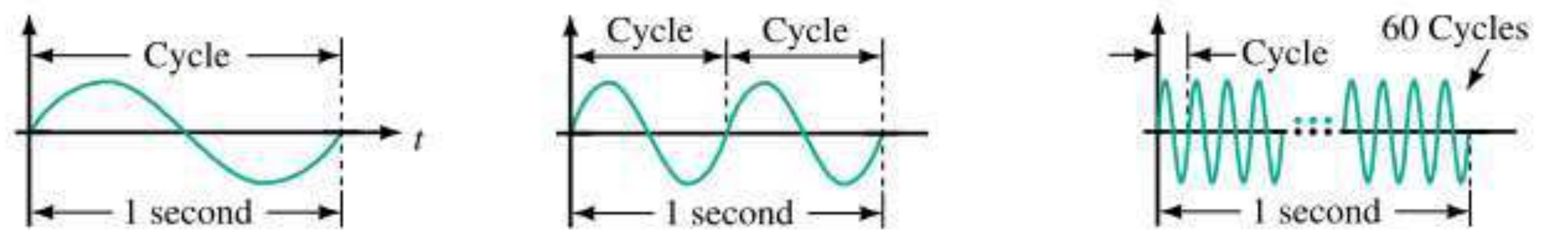


(c) Time t_2: $e = -10$ V and $i = -2$ A. Thus, voltage polarity is opposite to that indicated and current direction is opposite to the arrow direction.

Attributes of Periodic Waveforms

- Periodic waveforms (i.e., waveforms that repeat at regular intervals), regardless of their wave shape, may be described by a group of attributes such as:
  - ✓ **Frequency, Period, Amplitude, Peak value.**

Frequency: The number of cycles per second of a waveform is defined



(a) 1 cycle per second = 1 Hz (b) 2 cycles per second = 2 Hz (c) 60 cycles per second = 60 Hz

- Frequency is denoted by the lower-case letter f.
- In the SI system, its unit is the hertz (Hz, named in honor of pioneer researcher Heinrich Hertz, 1857–1894).

$$1\ \text{Hz} = 1\ \text{cycle per second}$$

Attributes of Periodic Waveforms

- **Period:**

- The period, T, of a waveform, is the duration of one cycle.

- It is the inverse of frequency.

$$T = \frac{1}{f} \quad \text{(s)}$$

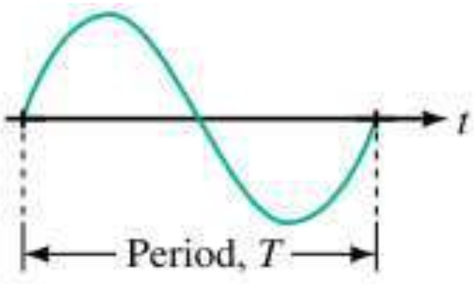


FIGURE 15–15 Period T is the duration of one cycle, measured in seconds.

- The period of a waveform can be measured between any two corresponding points (Often it is measured between zero points because they are easy to establish on an oscilloscope trace).

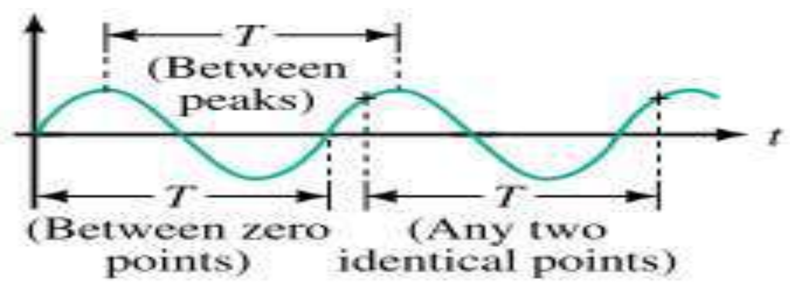


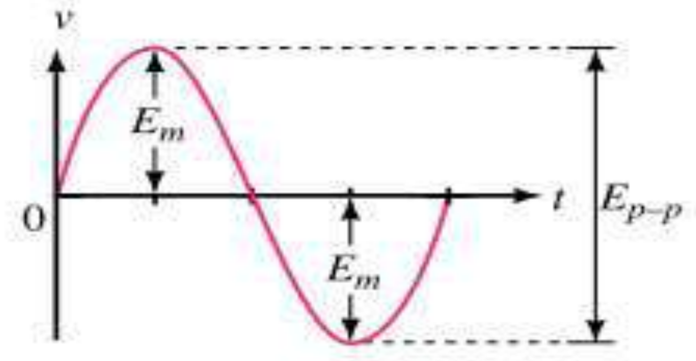
FIGURE 15–17 Period may be measured between any two corresponding points.

Attributes of Periodic Waveforms

Amplitude , Peak-Value, and Peak-to-Peak Value

Amplitude (Em):

The amplitude of a sine wave is the distance from its average to its peak.



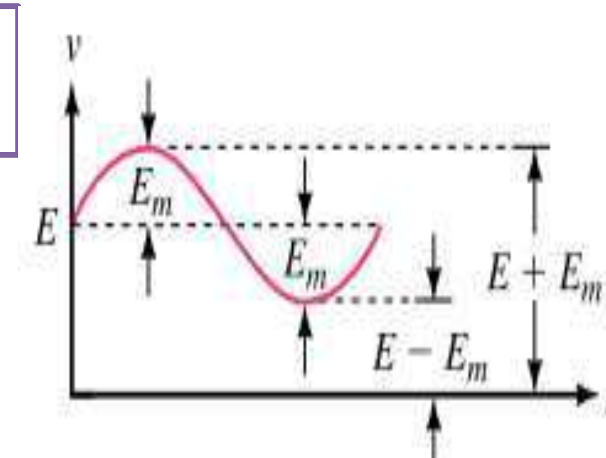
(a)

Peak-to-Peak Value (Ep-p):

It is measured between minimum and maximum peaks.

Peak Value

The peak value of a voltage or current is its maximum value with respect to zero.



In this figure : Peak voltage = E + Em

Quick Quiz (Poll 2)

Peak to peak value of a sine wave is

a. Equal to the maximum or phase value of sine wave
b. Twice the maximum or phase value of sine wave
c. Half of the maximum or phase value of sine wave
d. Four times the maximum or phase value of sine wave

Quick Quiz (Poll 3)

The most common waveforms of ac is

a. Square
b. Triangular
c. Sinusoidal
d. Saw tooth

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 9
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The Basic Sine Wave Equation

The voltage produced by the previously described generator is:

$$e = E_m \sin \alpha \quad (V)$$

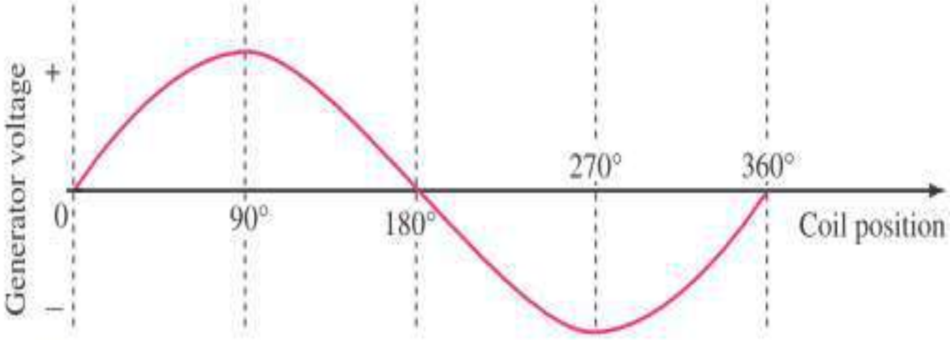


FIGURE 15–7 Coil voltage versus angular position.

- Em: the maximum coil voltage and
- α : the instantaneous angular position of the coil.

> ➢ For a given generator and rotational velocity, Em is constant.)
> ➢ Note that a 0° represents the horizontal position of the coil and that one complete cycle corresponds to 360°.

Angular Velocity (ω)

The rate at which the generator coil rotates is called its angular velocity

If the coil rotates through an angle of 30° in one second, its angular velocity is 30° per second.

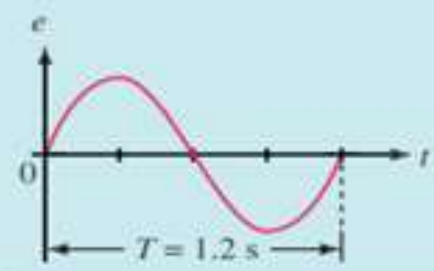
➢ When you know the angular velocity of a coil and the length of time that it has rotated, you can compute the angle through which it has turned using:

$$\alpha = \omega t$$

EXAMPLE 15–6 If the coil of Figure 15–23 rotates at $\omega = 300°/\text{s}$, how long does it take to complete one revolution?

Solution One revolution is 360°. Thus,

$$t = \frac{\alpha}{\omega} = \frac{360 \text{ degrees}}{300 \dfrac{\text{degrees}}{\text{s}}} = 1.2 \text{ s}$$

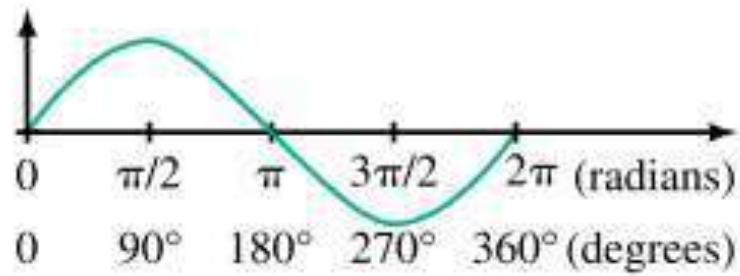


Since this is one period, we should use the symbol T. Thus, $T = 1.2$ s, as in Figure 15–25.

Radian Measure

- In practice, q is usually expressed in radians per second,
- Radians and degrees are related by :

$$2\pi \text{ radians} = 360^\circ$$



(b) Cycle length scaled in degrees and radians

For Conversion:

$$\alpha_{\text{radians}} = \frac{\pi}{180^\circ} \times \alpha_{\text{degrees}}$$

$$\alpha_{\text{degrees}} = \frac{180^\circ}{\pi} \times \alpha_{\text{radians}}$$

Relationship between ω, T, and f

- Earlier you learned that one cycle of sine wave may be represented as either:

$$\alpha = 2\pi \text{ rads} \qquad t = T \text{ s}$$

- Substituting these into:

$$\alpha = \omega t$$

$$\omega T = 2\pi \quad \text{(rad)}$$

$$\omega = \frac{2\pi}{T} \quad \text{(rad/s)}$$

$$\omega = 2\pi f \quad \text{(rad/s)}$$

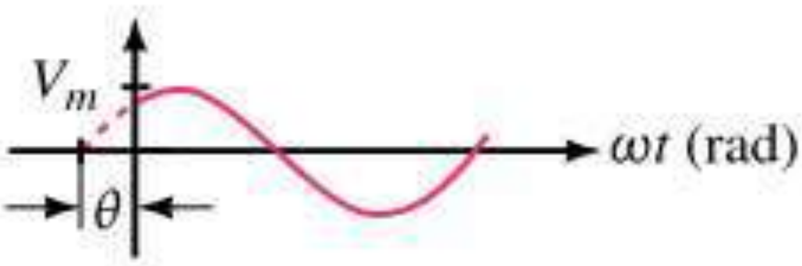
Sinusoidal Voltages and Currents as Functions of Time:

- We could replace the angle α as:

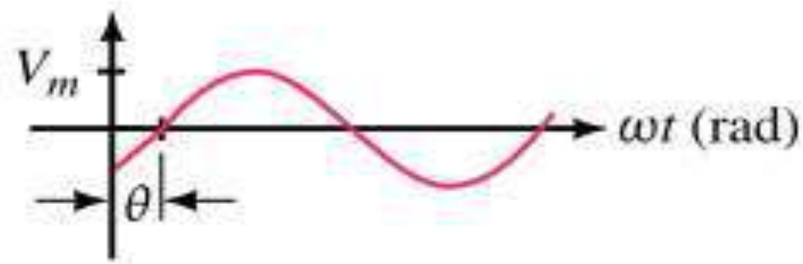
$$e = E_m \sin \omega t$$

Voltages and Currents with Phase Shifts

- If a sine wave does not pass through zero at t =0 s, it has a phase shift.
- Waveforms may be shifted to the left or to the right



(a) $v = V_m \sin(\omega t + \theta)$



(b) $v = V_m \sin(\omega t - \theta)$

Quick Quiz (Poll 1)

The time period or periodic time T of an alternating quantity is the time taken in seconds to complete

a. one cycle

b. alternation

c. none of the above

d. Half cycle

Quick Quiz (Poll 2)

The time period of an alternating quantity is 0.02 second. Its frequency will be

a. 25 Hz

b. 50 Hz

c. 100 Hz

d. 0.02 Hz

Quick Quiz (Poll 3)

The angular frequency of an alternating quantity is a mathematical quantity obtained by multiplying the frequency f of the alternating quantity by a factor

a. $\pi/2$

b. π

c. 2π

d. 4π

Introduction to Phasors

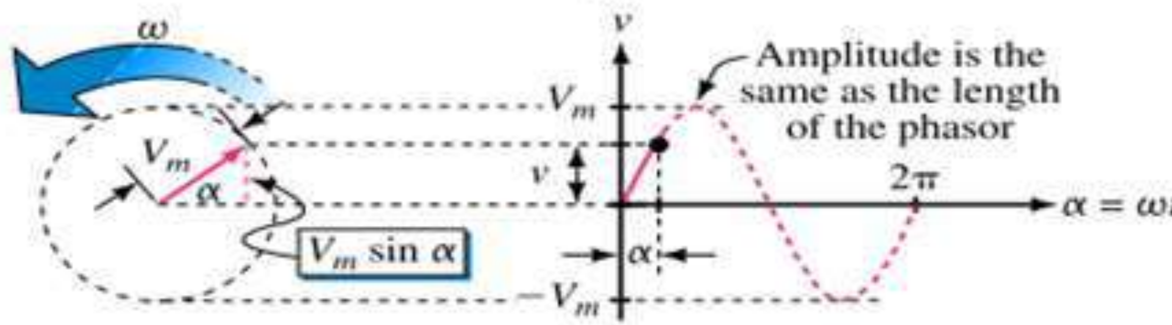
- A phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities.
- To get at the idea, consider the red line of length **Vm** shown in Figure :

The vertical projection of this line (indicated in dotted red) is :

$$v = V_m \sin \alpha.$$

- By assuming that the phasor rotates at angular velocity of ω rad/s in the counterclockwise direction

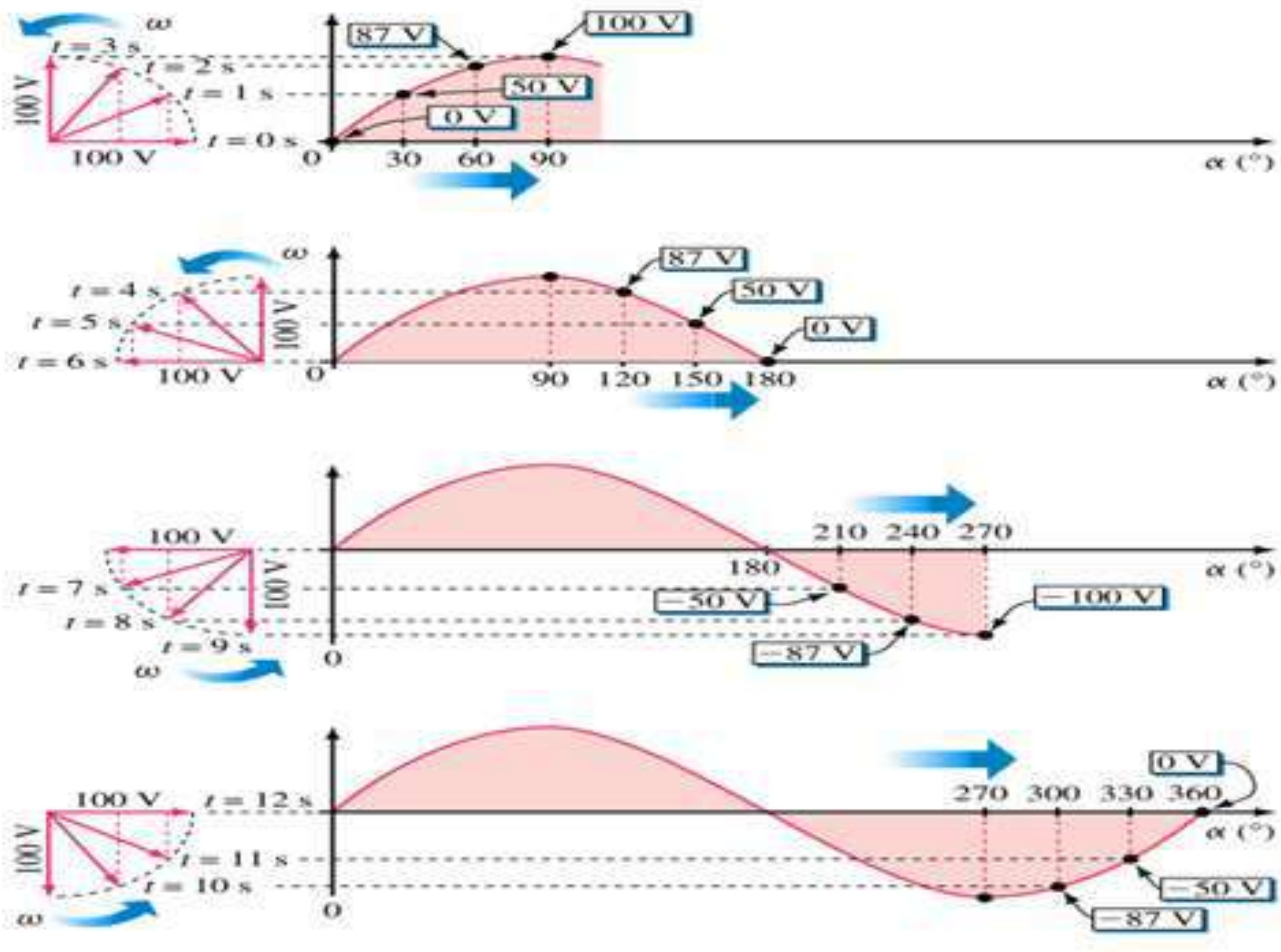
$$v = V_m \sin \omega t$$



(a) Phasor (b) Resulting sine wave

FIGURE 15–34 As the phasor rotates about the origin, its vertical projection creates a sine wave. (Figure 15–35 illustrates the process.)

Introduction to Phasors



EXAMPLE 15–15 Draw the phasor and waveform for current $i = 25 \sin \omega t$ mA for $f = 100$ Hz.

Solution The phasor has a length of 25 mA and is drawn at its $t = 0$ position, which is zero degrees as indicated in Figure 15–36. Since $f = 100$ Hz, the period is $T = 1/f = 10$ ms.

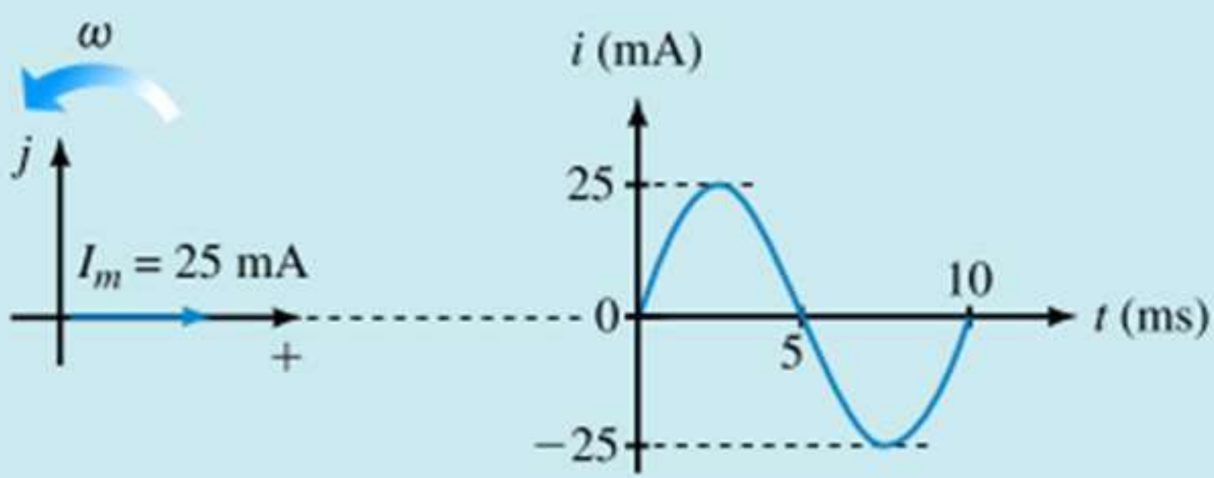


FIGURE 15–36 The reference position of the phasor is its $t = 0$ position.

Shifted Sine Waves Phasor Representation

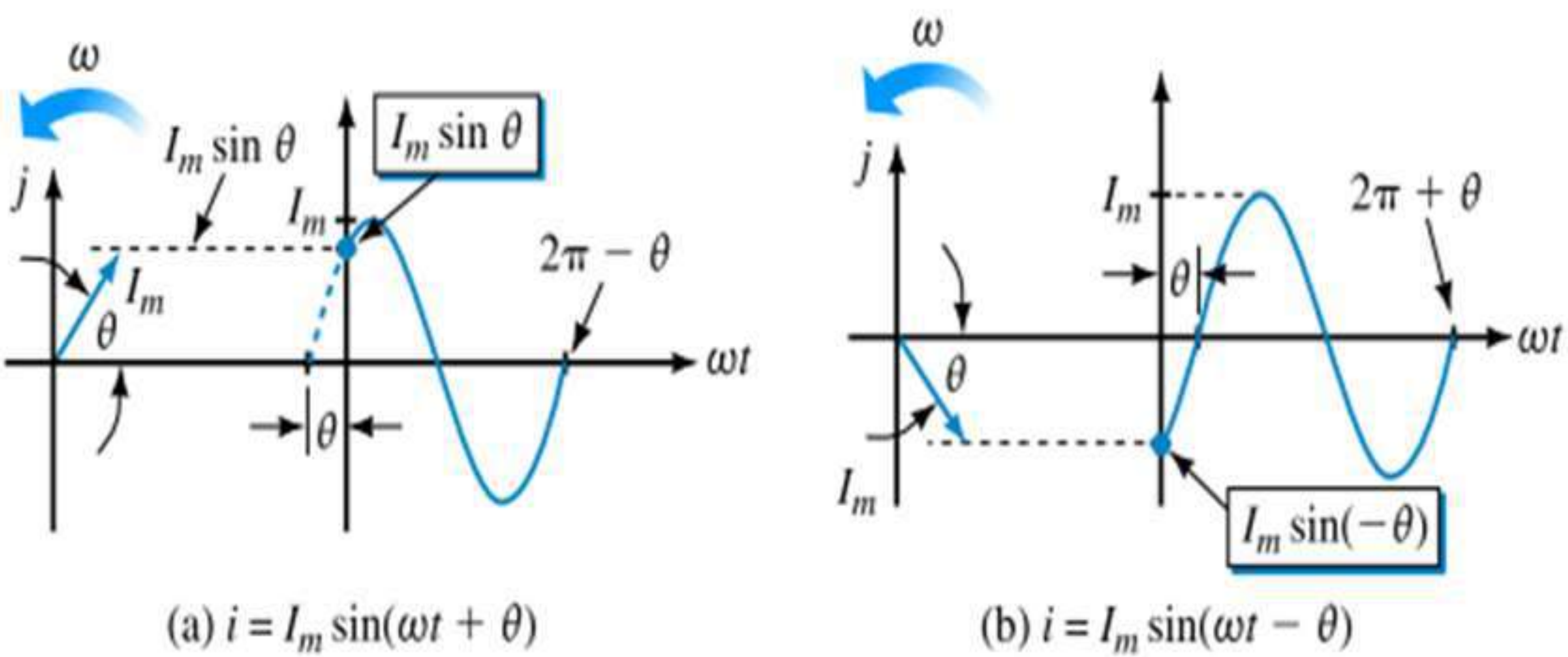


FIGURE 15–37 Phasors for shifted waveforms. Angle θ is the position of the phasor at $t = 0$ s.

The equation for an ac voltage is given as $v = 0.04 \sin(2000t + 60°)$ V. Determine the frequency, the angular frequency, and the instantaneous voltage when $t = 160\ \mu s$. What is the time represented by a 60° phase angle?

Solution Comparing the given equation with the general equation given in Eq. 9.3, we get

$$\omega = 2\pi f = \mathbf{2000\ rad/s} \qquad \text{and} \qquad f = \frac{\omega}{2\pi} = \frac{2000}{2\pi} = \mathbf{318.3\ Hz}$$

The instantaneous value of the voltage at $t = 160\ \mu s$ is

$$v = 0.04 \sin(2000 \times 160 \times 10^{-6}\ \text{rad} + 60°)\ \text{V} = 0.04 \sin(0.32\ \text{rad} + 60°)\ \text{V}$$

$$= 0.04 \sin\left(0.32 \times \frac{180°}{\pi} + 60°\right)\ \text{V} = 0.04 \sin(18.3° + 60°)\ \text{V} = 0.0392\ \text{V} = \mathbf{39.2\ mV}$$

$$\text{Time period, } T = \frac{1}{f} = \frac{1}{318.3} = 3.14\ \text{ms}$$

Thus, full-cycle of 360° corresponds to 3.14 ms. Therefore, the angle 60° corresponds to

$$t = \frac{60°}{360°} \times 3.14\ \text{ms} = \mathbf{0.52\ ms}$$

Quick Quiz (Poll 4)

A phasor is

A. A line which represents the magnitude and phase of an alternating quantity

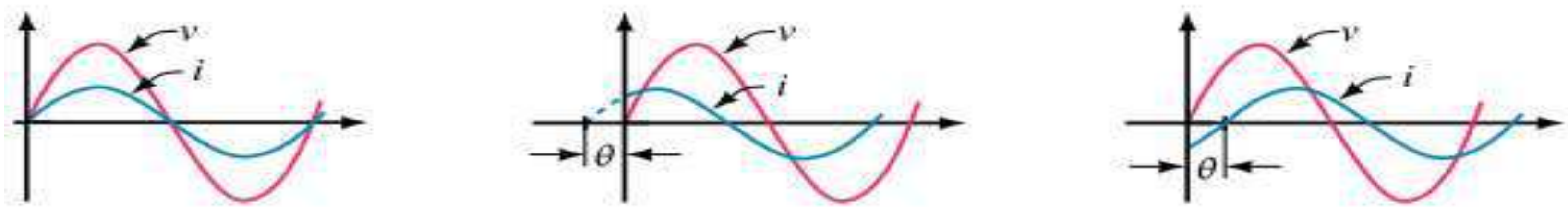
B. A line representing the magnitude and direction of an alternating quantity

C. A colored tag or band for distinction between different phases of a 3-phase supply

D. An instrument used for measuring phases of an unbalanced 3-phase load

Phasor Difference

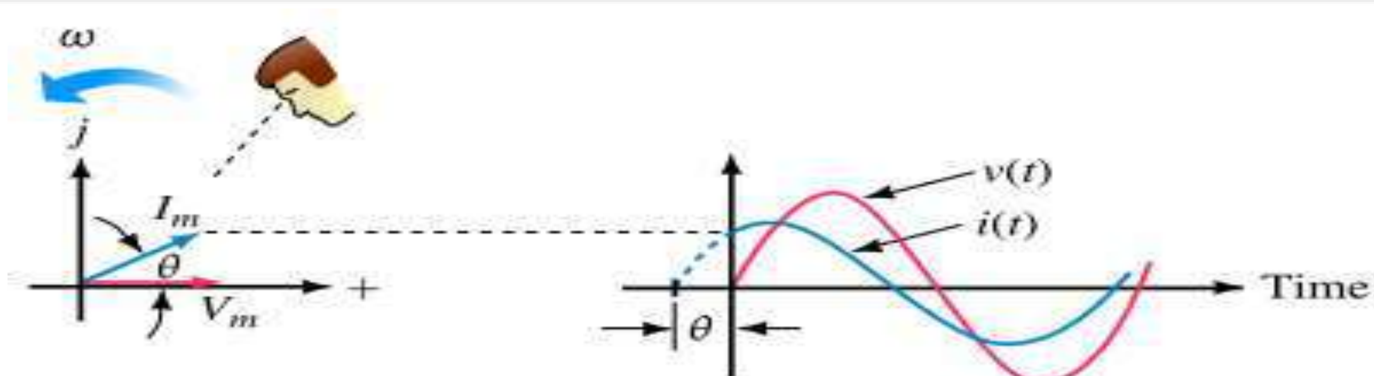
- Phase difference refers to the angular displacement between different waveforms of the same frequency.



(a) In phase (b) Current leads (c) Current lags

FIGURE 15–40 Illustrating phase difference. In these examples, voltage is taken as reference.

- The terms lead and lag can be understood in terms of phasors. If you observe phasors rotating as in Figure, the one that you see passing first is leading and the other is lagging.



(a) I_m leads V_m (b) Therefore, $i(t)$ leads $v(t)$

Quick Quiz (Poll 5)

If the phase angle ϕ is positive then the phase difference is said to a ___________ phase difference

A additive

B Leading

C Lagging

D None of the above.

Quick Quiz (Poll 6)

If the phase difference between them is equal to zero, the two AC voltages (or any two AC quantities) are said to be ___________.

A in phase

B out of phase

C in phase opposition

D None of the above

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 10
Prepared By:
Krishan Arora
Assistant Professor and Head

AC Waveforms and Average Value

- Since ac quantities constantly change its value, we need one single numerical value that truly represents a waveform over its complete cycle.

Average Values:

- To find the average of a set of marks for example, you add them, then divide by the number of items summed.
- For waveforms, the process is conceptually the same. You can sum the instantaneous values over a full cycle, then divide by the number of points used.
- The trouble with this approach is that waveforms do not consist of discrete values.

Average in Terms of the Area Under a Curve:

$$\text{average} = \frac{\text{area under curve}}{\text{length of base}}$$

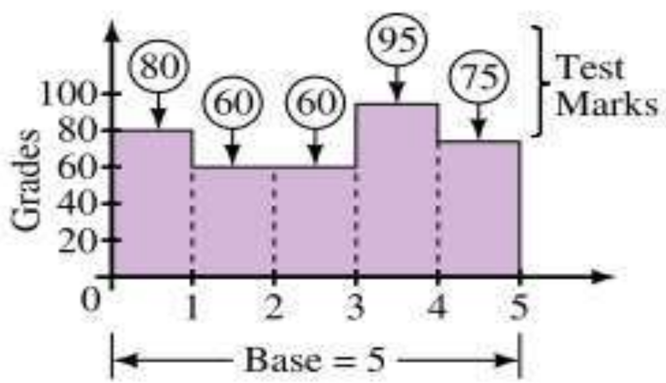


FIGURE 15–50 Determining average by area.

$$\text{average} = (80 + 60 + 60 + 95 + 75)/5 = 74$$

Or use area

$$\frac{(80 \times 1) + (60 \times 2) + (95 \times 1) + (75 \times 1)}{5} = 74$$

AC Waveforms and Average Value

- To find the average value of a waveform, divide the area under the waveform by the length of its base.
- Areas above the axis are counted as positive, while areas below the axis are counted as negative.
- This approach is valid regardless of waveshape.

- Average values are also called **dc values**, because dc meters indicate average values rather than instantaneous values.

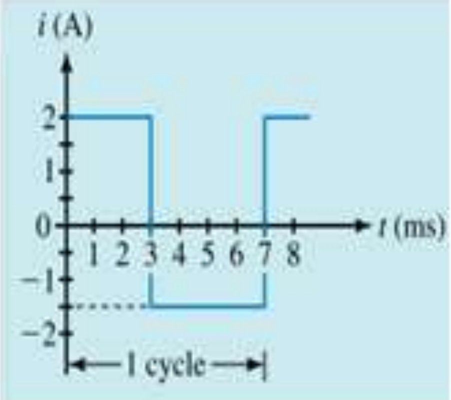
AC Waveforms and Average Value

EXAMPLE 15–23

a. Compute the average for the current waveform of Figure 15–51.

b. If the negative portion of Figure 15–51 is −3 A instead of −1.5 A, what is the average?

c. If the current is measured by a dc ammeter, what will the ammeter indicate?



Solution

a. The waveform repeats itself after 7 ms. Thus, $T = 7$ ms and the average is

$$I_{avg} = \frac{(2\text{ A} \times 3\text{ ms}) - (1.5\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \frac{6-6}{7} = 0\text{ A}$$

b. $I_{avg} = \dfrac{(2\text{ A} \times 3\text{ ms}) - (3\text{ A} \times 4\text{ ms})}{7\text{ ms}} = \dfrac{-6\text{ A}}{7} = -0.857\text{ A}$

c. A dc ammeter measuring (a) will indicate zero, while for (b) it will indicate −0.857 A.

Derivation of Average Value

The average value of A.C. is the average over one complete cycle and is clearly zero, because there are alternately equal positive and negative half cycles.
Alternating current is represented as $I = I_0 \sin \omega t$

$$I_{\text{mean}} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt}$$

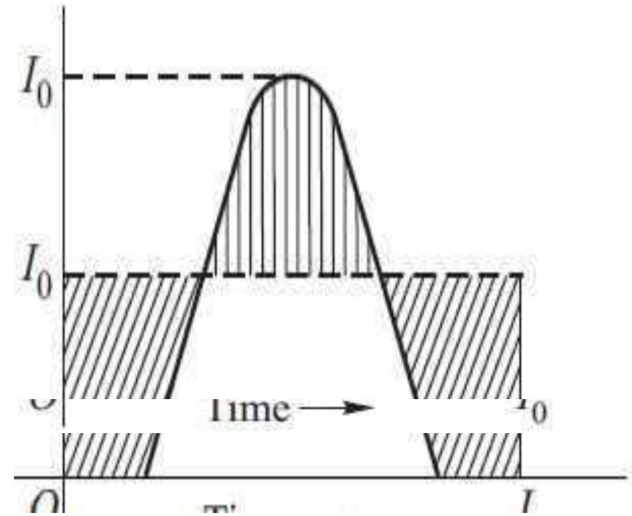
$$= \frac{I_0}{T/2} \cdot \frac{1}{\omega} \left[-\cos \omega t \right]_0^{T/2}$$

$$= \frac{2I}{T} \cdot \frac{T}{2\pi} \left[\cos 0^\circ - \cos \frac{\omega T}{2} \right]$$

$$= \frac{I_0}{\pi} \left[\cos 0^\circ - \cos \frac{\omega}{2} \cdot \frac{2\pi}{\omega} \right]$$

$$= \frac{2I_0}{\pi} = \frac{2}{\pi} \times \text{Peak value of current}$$

Similarly, $E_{\text{mean}} = \dfrac{2E_0}{\pi} = \dfrac{2}{\pi} \times \text{Peak value of voltage}$



Root Mean Square Value

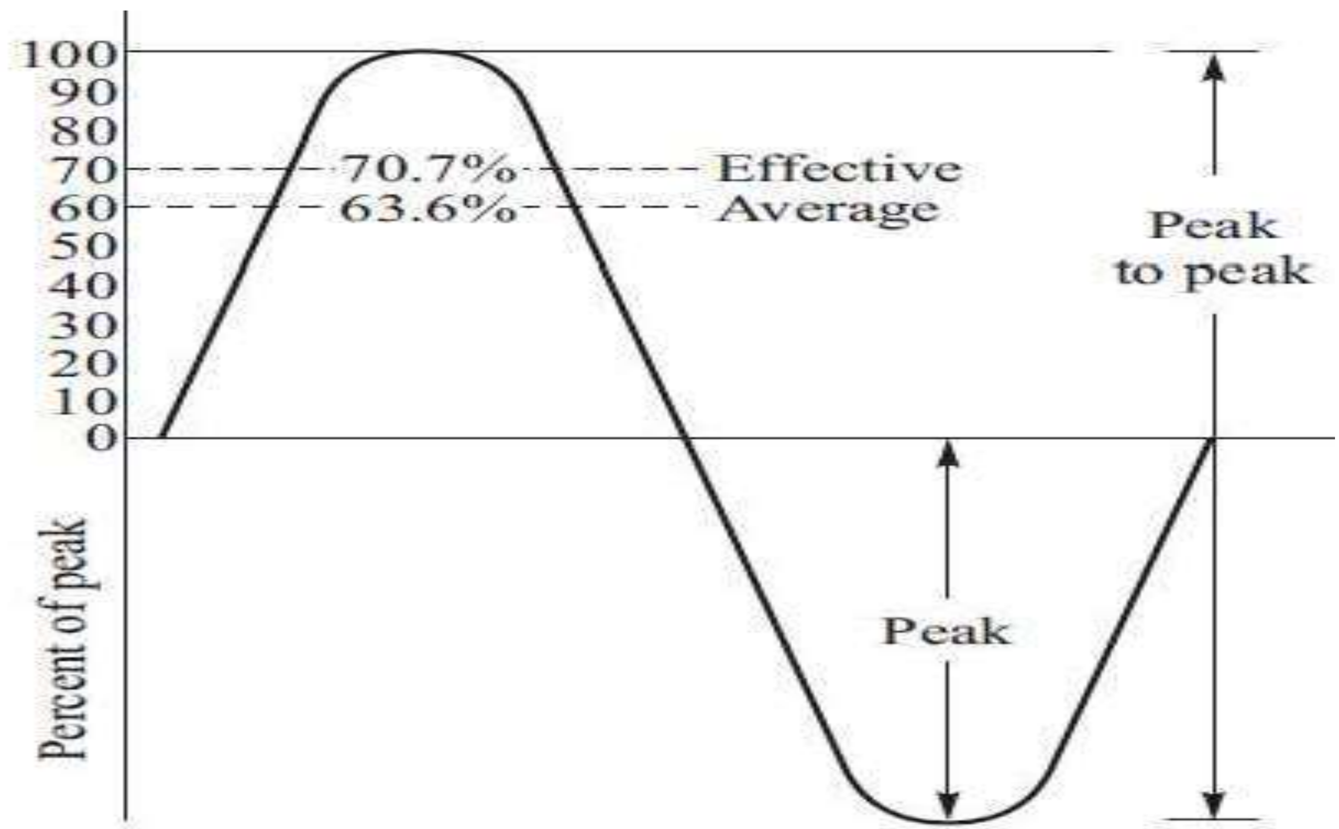
Circuit currents and voltage in A.C. circuits are generally stated as root-mean-square or rms values rather than by quoting the maximum values. The root-mean-square for a current is defined by

$$I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{avg}}}$$

That is, you take the square of the current and average it, then take the square root. When this process is carried out for a sinusoidal current

$$\left[I_m^2 \sin^2 \omega t\right]_{\text{avg}} = \frac{I_m^2}{2} \quad \text{so} \quad I_{\text{rms}} = \sqrt{\left(I^2\right)_{\text{avg}}} = \frac{I_m}{\sqrt{2}}$$

Derivation of RMS Value



Form Factor and Peak Factor

The ratio of the effective value to the average value is known as the *form factor* of a waveform of any shape (sinusoidal or nonsinusoidal). Thus,

Form factor, $$K_f = \frac{V_{rms}}{V_{av}} \tag{9.12}$$

The ***peak factor*** or ***crest factor*** or ***amplitude factor*** of a waveform is defined as the ratio of its peak (or maximum) value to its rms value. Thus,

Peak factor, $$K_p = \frac{V_m}{V_{rms}} \tag{9.13}$$

Let us calculate these two factors for ***a sinusoidal voltage waveform***,

$$K_f = \frac{V_{rms}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{0.707\,V_m}{0.637\,V_m} = \mathbf{1.11}$$

And $$K_p = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$$

Quick Quiz (Poll 1)

What is referred as the average value in AC opearation?

a) Average of all values of an alternating quantity.
b) Average of all values of the phase sequences.
c) Average of all values of the (+)ve and (-)ve half.
d) Average of all values of an alternating quantity over a complete cycle.

Quick Quiz (Poll 2)

What is form factor?

a) Average value / R.M.S. value.
b) Average value / Peak value.
c) Instantaneous value / Average value.
d) R.M.S. value / Average value.

POWER IN AC CIRCUITS

TRUE POWER:

The actual amount of power being used, or dissipated, in a circuit is called *true power*, and it is measured in watts (symbolized by the capital letter P, as always)

REACTIVE POWER:

We know that reactive loads such as inductors and capacitors dissipate zero power, yet the fact that they drop voltage and draw current gives the deceptive impression that they actually *do* dissipate power. This “phantom power” is called *reactive power*, and it is measured in a unit called *Volt-Amps-Reactive* (VAR), rather than watts. The mathematical symbol for reactive power is (unfortunately) the capital letter Q.

APPARENT POWER:

The combination of reactive power and true power is called *apparent power*, and it is the product of a circuit’s voltage and current, without reference to phase angle. Apparent power is measured in the unit of *Volt-Amps* (VA) and is symbolized by the capital letter S.

Active Power, it is the true power which is actually consumed in the circuit. We can say that it is the product of voltage and current and power factor.

Reactive power: It is the product of voltage current and sin of the phase angle.

Apparent power: It is the product of voltage and current.

P = true power $\quad P = I^2R \quad P = \dfrac{E^2}{R}$

*Measured in units of **Watts***

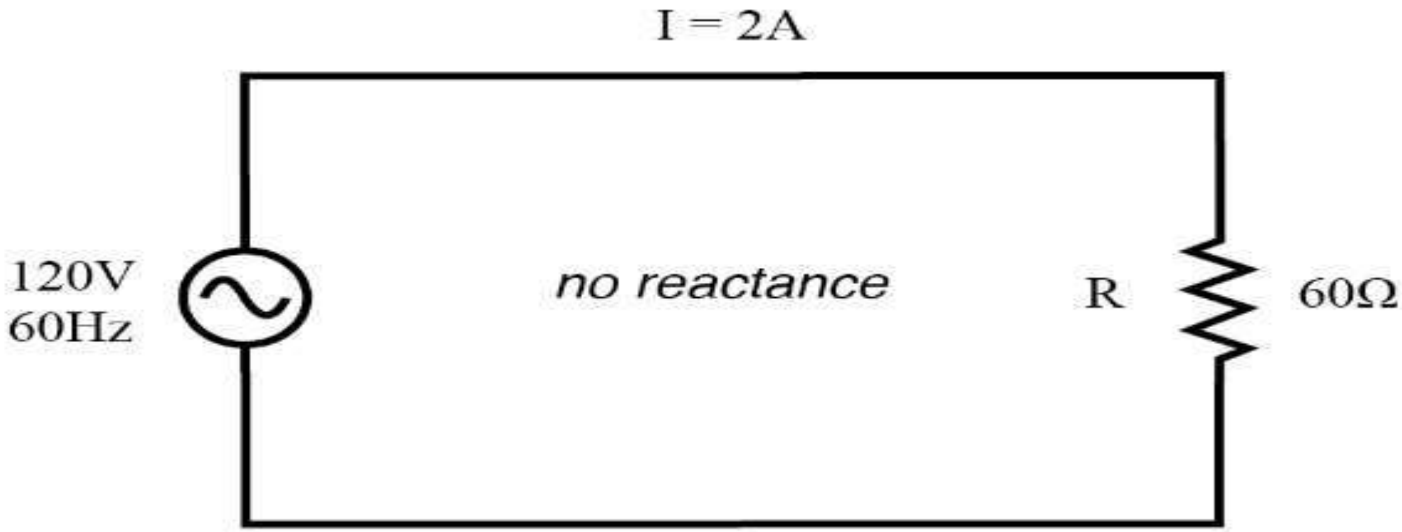
Q = reactive power $\quad Q = I^2X \quad Q = \dfrac{E^2}{X}$

*Measured in units of **Volt-Amps-Reactive (VAR)***

S = apparent power $\quad S = I^2Z \quad S = \dfrac{E^2}{Z} \quad S = IE$

*Measured in units of **Volt-Amps (VA)***

For Resistive Load

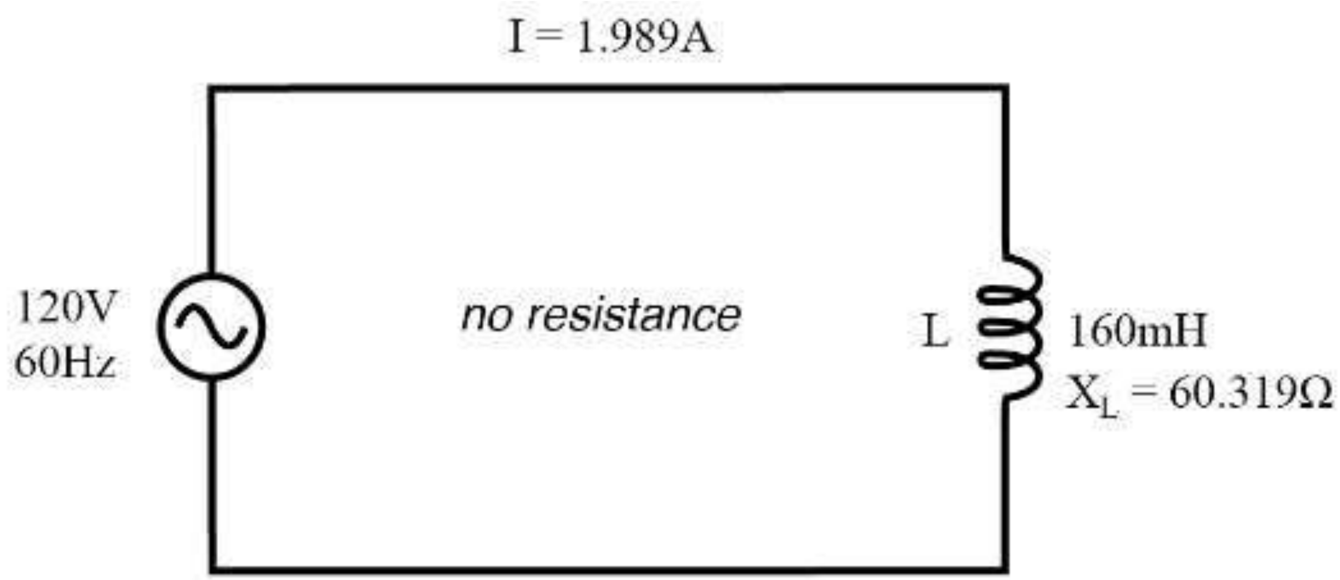


P = true power = I^2R = 240W

Q = reactive power = I^2X = 0 VAR

S = apparent power = I^2Z = 240VA

For Reactive Load

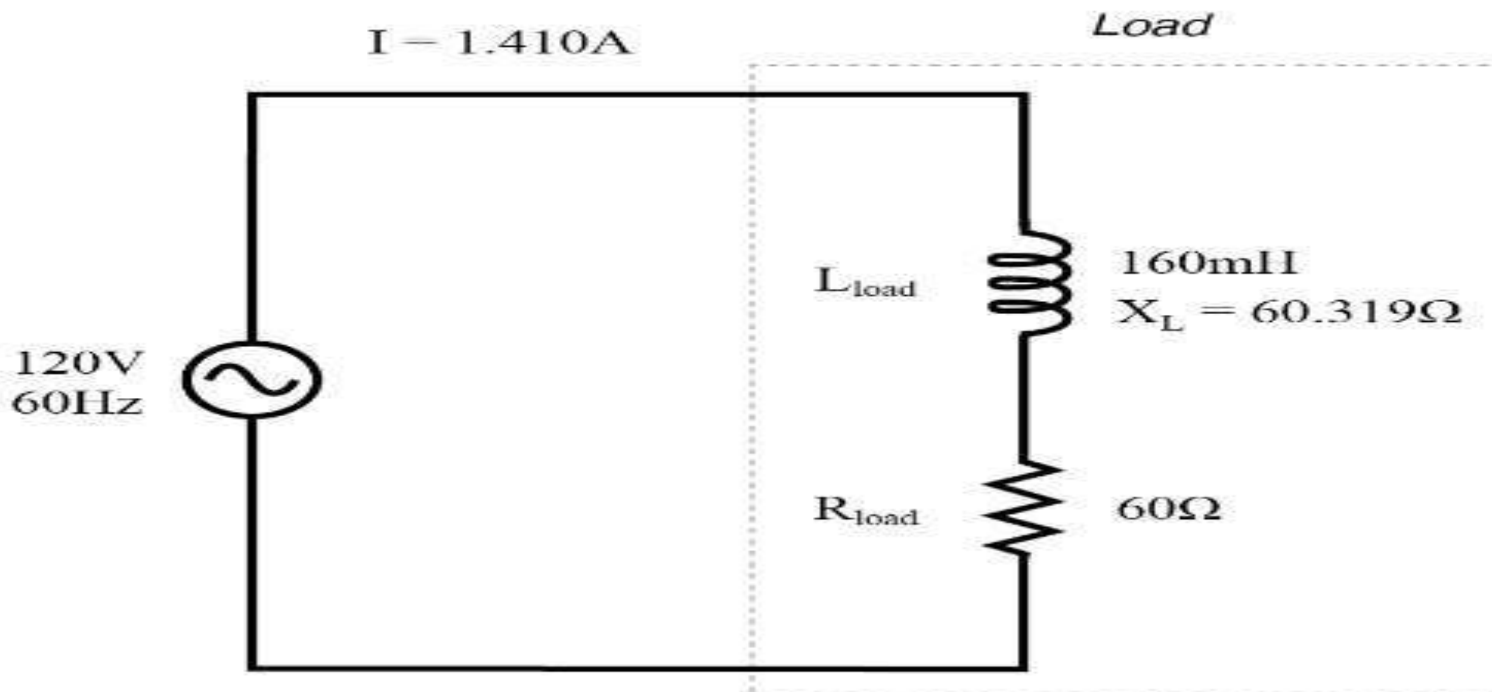


P = true power = I^2R = 0W

Q = reactive power = I^2X = 238.73VAR

S = apparent power = I^2Z = 238.73VA

For Resistive/Reactive Load



P = true power = I^2R = 119.365W

Q = reactive power = I^2X = 119.998VAR

S = apparent power = I^2Z =169.256VA

Example

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55 \sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1 \sin (\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Solution Here, the phase angle, $\theta = \pi/5$

The rms value of the voltage, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{55}{\sqrt{2}} = 38.89$ V

The rms value of the current, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{6.1}{\sqrt{2}} = 4.31$ A

Therefore, using Eq. 9.36, the average power is given as

$$P_{av} = VI\cos\theta = 38.89 \times 4.31 \times \cos \pi/5$$
$$= 167.62 \times 0.809 = \mathbf{135.6\ W}$$

The apparent power is

$$P_a = VI = 38.89 \times 4.31 = \mathbf{167.62\ VA}$$

Note that the apparent power is expressed in volt amperes (VA) and not in watts (W), since it is not a power in reality.

The instantaneous power at $\omega t = 0.3$ is given by Eq. 9.35, as

$$p = VI\cos\theta - VI\cos(2\omega t - \theta)$$
$$= 135.6 - 167.62 \times \cos(2 \times 0.3 - \pi/5) = \mathbf{-31.95\ VA}$$

The power factor is given as

$$pf = \cos\theta = \cos \pi/5 = 0.809 = \mathbf{80.9\ \%}$$

Quick Quiz (Poll 3)

Reactive power is expressed in?

a) Watts (W)

b) Volt Amperes Reactive (VAR)

c) Volt Ampere (VA)

d) No units

Quick Quiz (Poll 4)

- Active Power is defined by

A) VI cos ϕ

B) VI sin ϕ

C) VI

D) All of the above

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

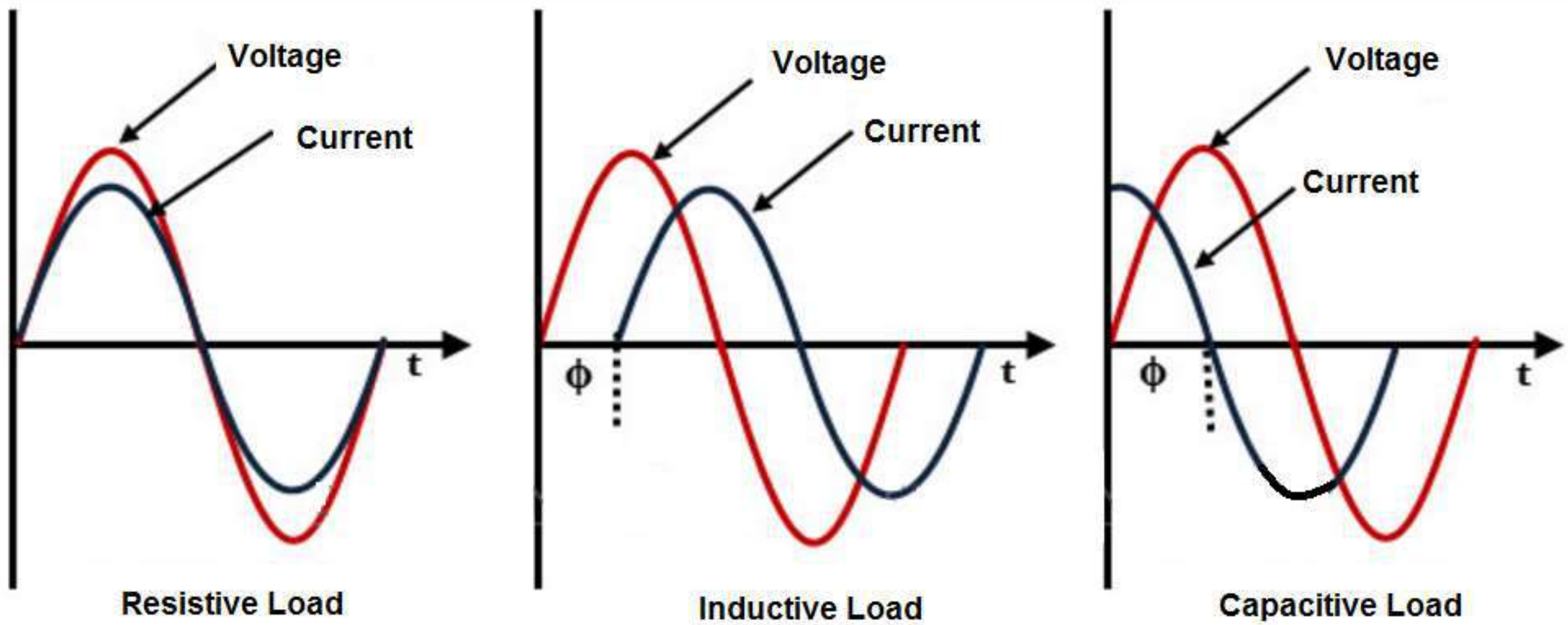
Lecture 11

Prepared By:
Krishan Arora
Assistant Professor and Head

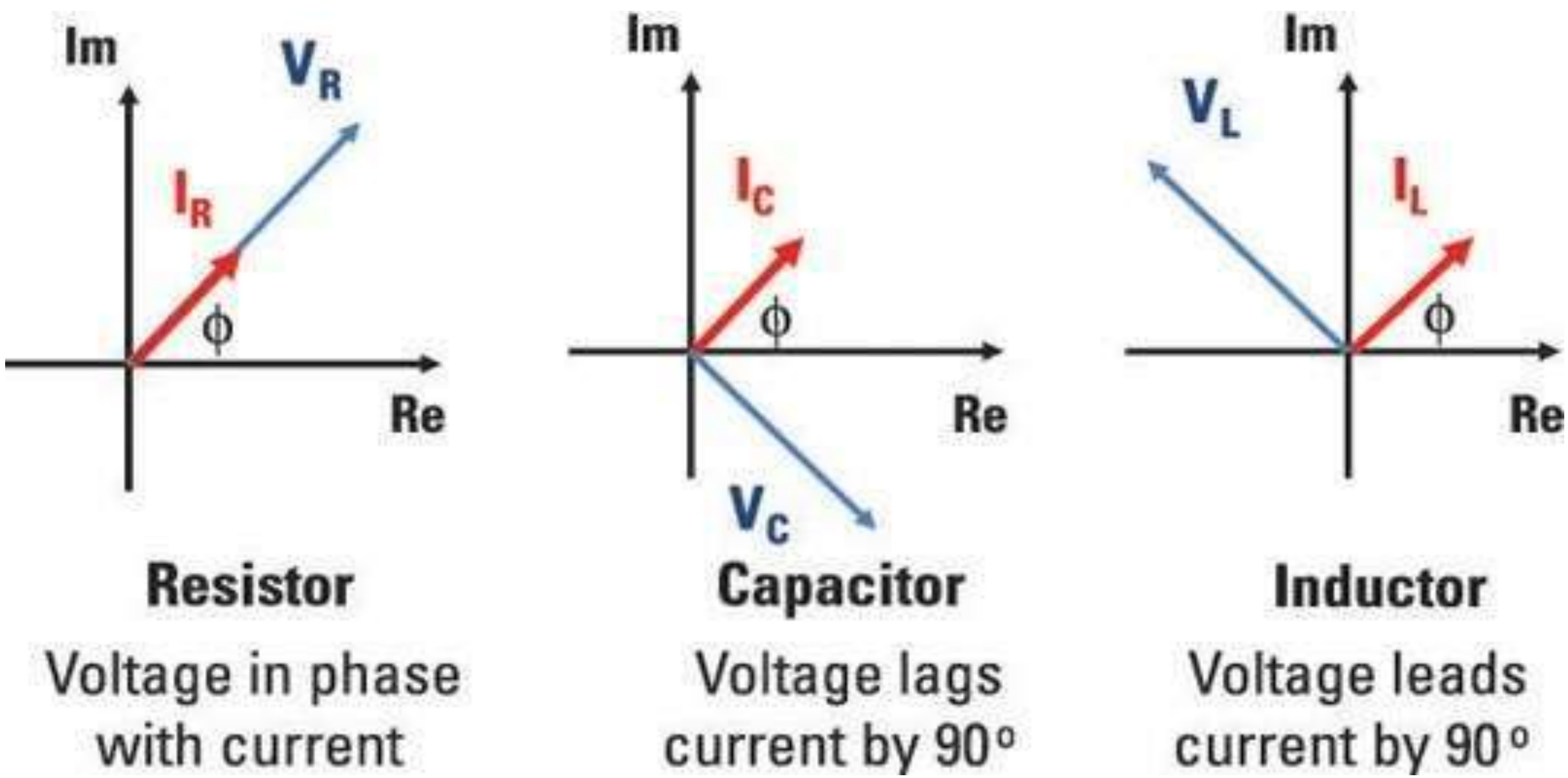
AC Circuits

- An AC circuit consists of a combination of circuit elements and a power source.
- The power source provides an alternating voltage, Δv.
- Notation note:
  - Lower case symbols will indicate instantaneous values.
  - Capital letters will indicate fixed values.

Current and Voltages in Resistive, Inductive and Capacitive Circuits



Phasor Diagram for Purely Resistive, Capacitive and Inductive Circuits



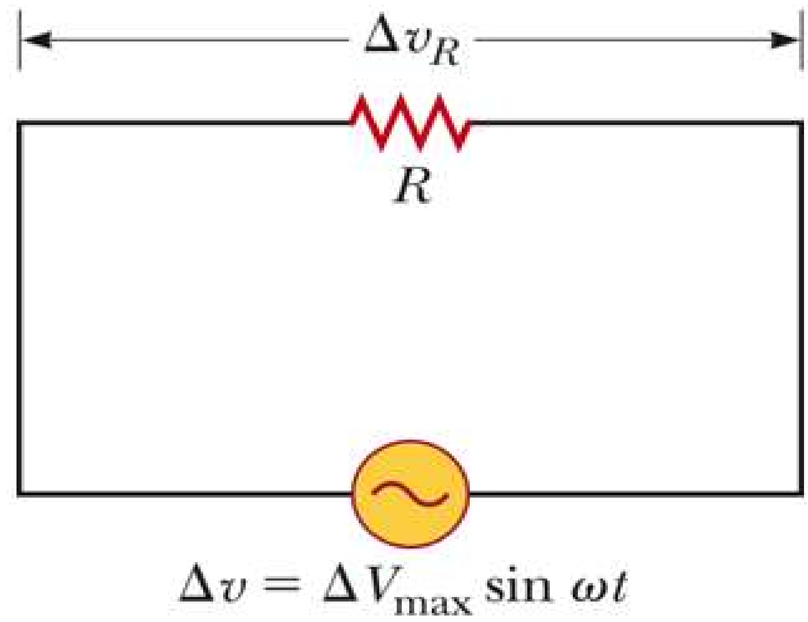
Resistors in an AC Circuit

Consider a circuit consisting of an AC source and a resistor.

The AC source is symbolized by 

$\Delta v_R = \Delta V_{max} = V_{max} \sin \omega t$

Δv_R is the instantaneous voltage across the resistor.



Resistors in an AC Circuit, cont.

The instantaneous current in the resistor is

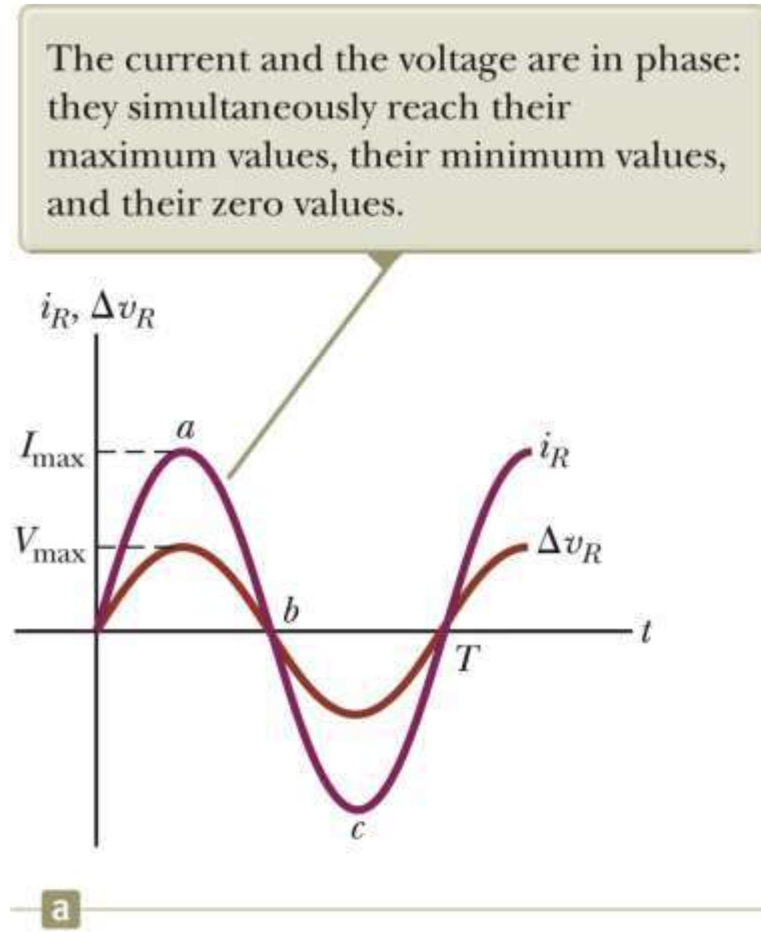
$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

The instantaneous voltage across the resistor is also given as

$$\Delta v_R = I_{max} R \sin \omega t$$

Resistors in an AC Circuit, final

- The graph shows the current through and the voltage across the resistor.
- The current and the voltage reach their maximum values at the same time.
- The current and the voltage are said to be *in phase.*
- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.
- The direction of the current has no effect on the behavior of the resistor.
- Resistors behave essentially the same way in both DC and AC circuits.



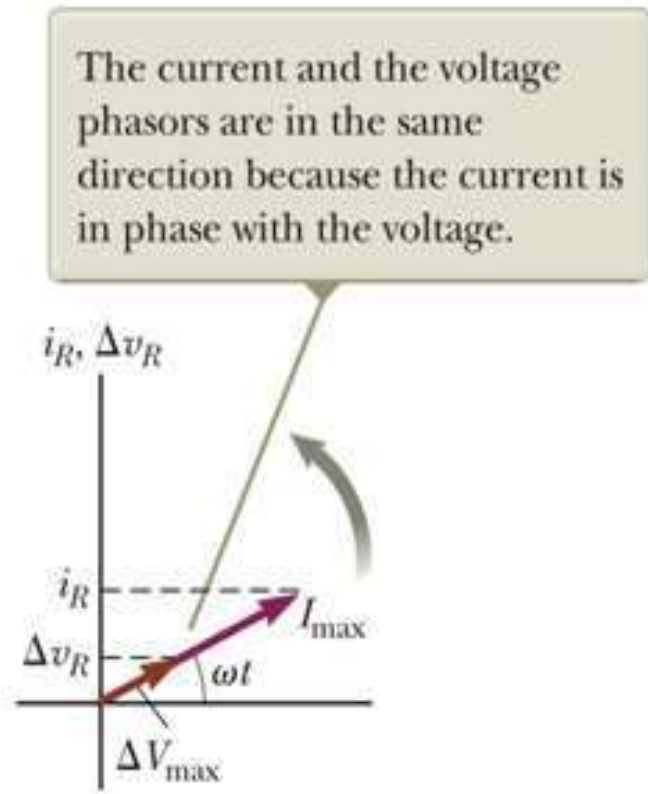
Phasor Diagram

To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.

The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.



Power

The rate at which electrical energy is delivered to a resistor in the circuit is given by

- $P = i^2 R$
  - i is the *instantaneous current*.
  - The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
  - The maximum current occurs for a small amount of time.
- The average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

Quick Quiz (Poll 1)

- Find the value of the instantaneous voltage if the resistance is 2 ohm and the instantaneous current in the circuit is 5A.
  a) 5V
  b) 2V
  c) 10V
  d) 2.5V

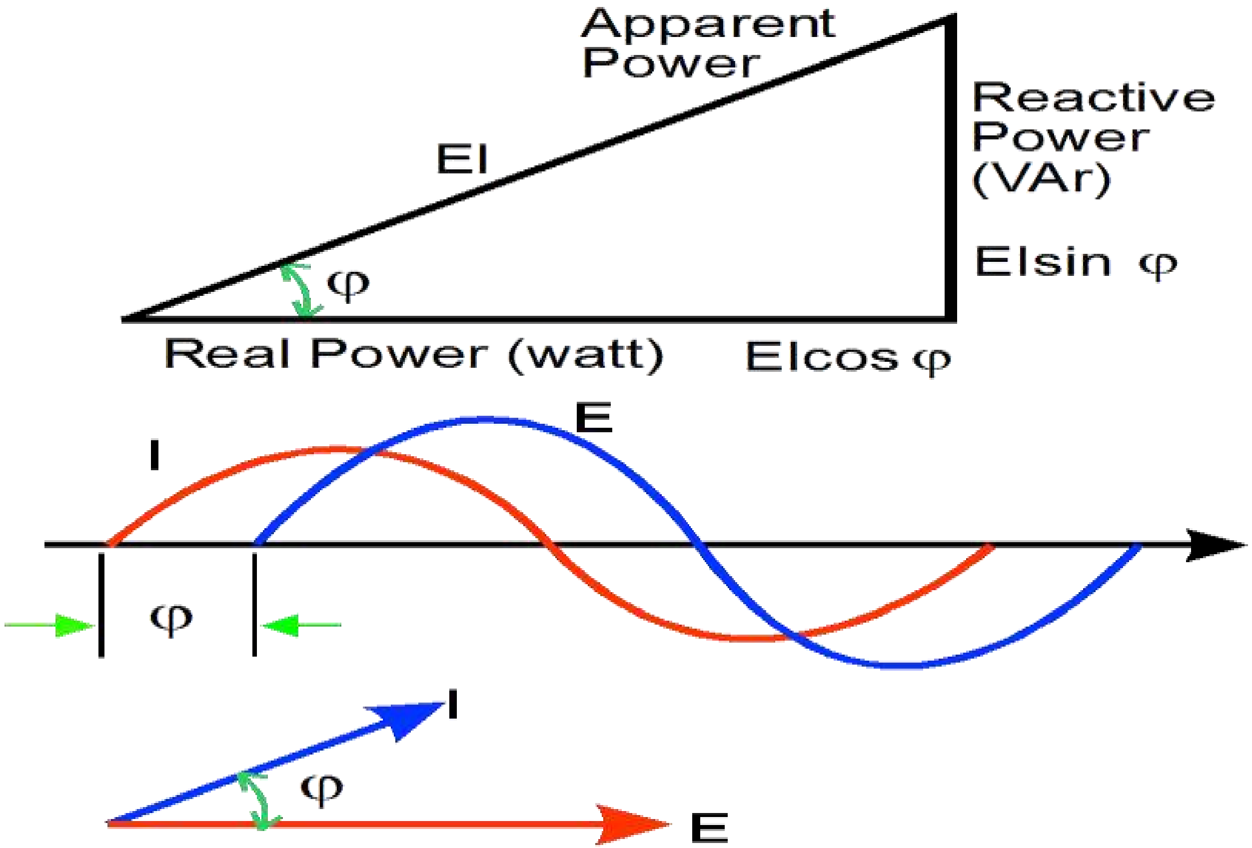
Quick Quiz (Poll 2)

- The correct expression for the instantaneous current in a resistive circuit is?
  a) $i = Vm(\sin \omega t)/R$
  b) $i = Vm(\cos \omega t)/R$
  c) $i = V(\sin \omega t)/R$
  d) $i = V(\cos \omega t)/R$

Concept of Power Factor

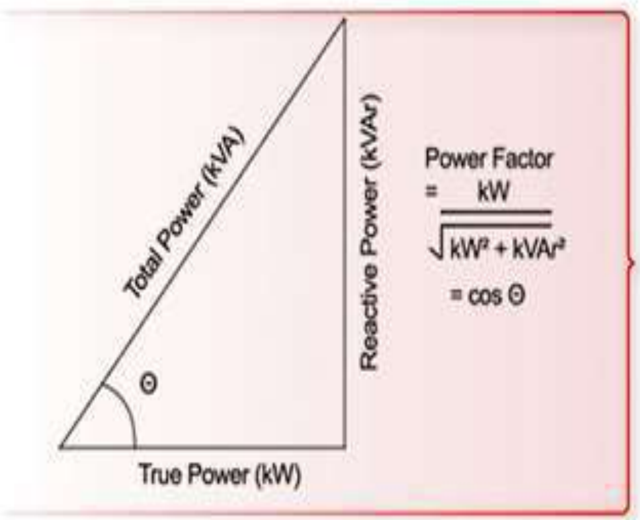
- Ratio of real power to apparent power is called the power factor, F_p
- $F_p = P/S = \cos\theta$
- Angle θ is angle between voltage and current

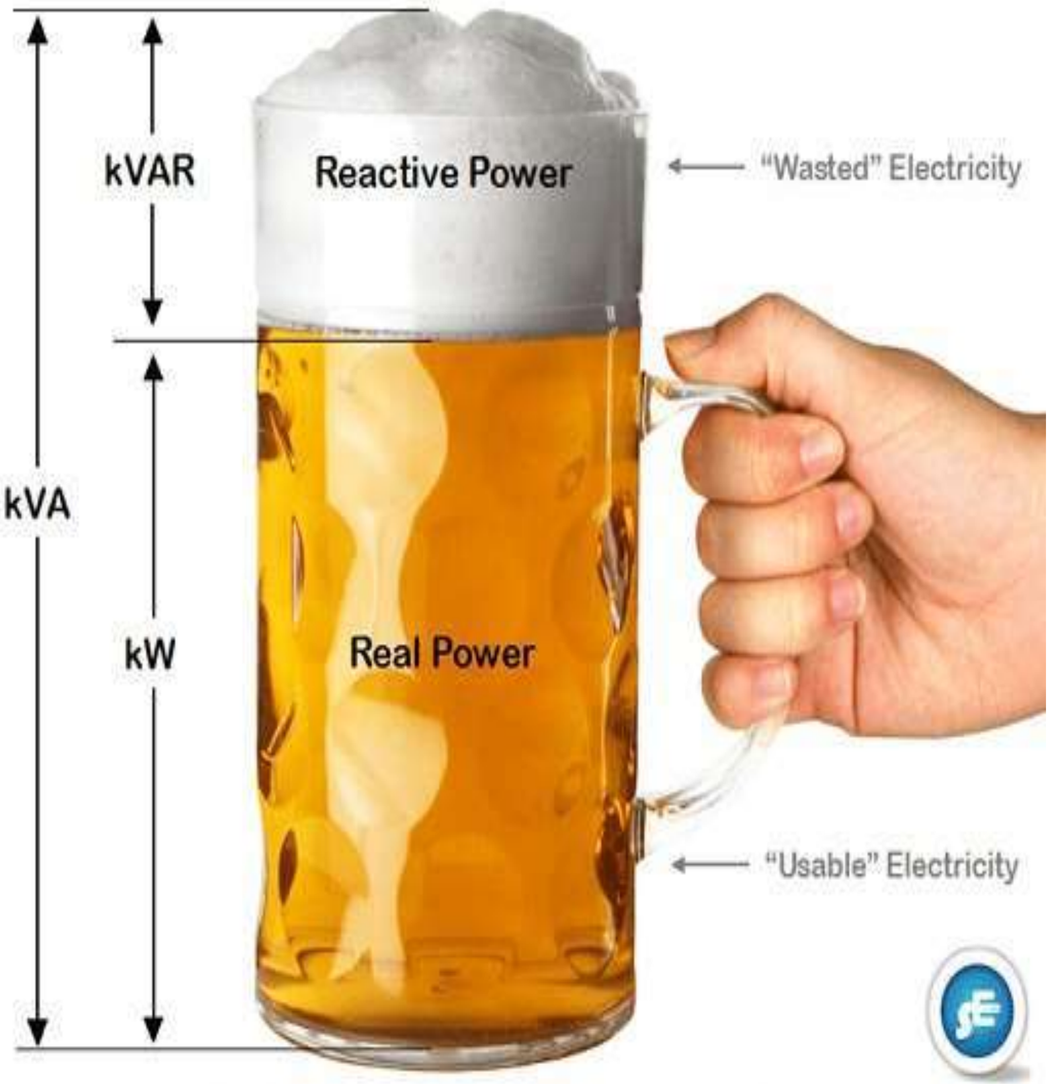
POWER TRIANGLE



What is Power Factor?

Power Factor is the percentage of apparent power that does real work. Understand Power Factor using Beer Mug Analogy.





SIGNIFICANCE OF Mvar

REACTIVE POWER (MVAR)
Beer: Foam
Electricity: Unable to do work

APPARENT POWER
Beer: Full glass
Electricity: Available from utility

REAL POWER (MW)
Beer: Drinkable
Electricity: Able to do work

It is the Active Power that contributes to the energy consumed, or transmitted. Reactive Power does not contribute to the energy. It is an inherent part of the "total power" which is often referred as "Useless Power".

Power Factor

- For pure resistance $\theta = 0°$
- For inductance, $\theta = 90°$
- For capacitance, $\theta = -90°$
- For a circuit containing a mixture, θ is somewhere between 0° and $\pm$90°

Power Factor

- Unity power factor
  - For a purely resistive circuit, the power factor will be one
- For load containing resistance and inductance
  - Power factor will be less than one and lagging
  - Current lags the voltage
- For a circuit containing resistance and capacitance
  - F_p is less than one and is leading

Power Factor Correction

- A load with a small power factor can draw a large current
- Can be alleviated by
  - Cancelling some or all reactive components of power by adding reactance of opposite type to the circuit
    - This is power factor correction

Power Factor Correction

- Industrial customers may pay a penalty for low power factors due to large currents required for highly reactive loads

Quick Quiz (Poll 3)

What is maximum value of power factor?

a. 0.5
b. 1
c. 1.5
d. 0.95

Quick Quiz (Poll 4)

For which among the following consumers is penalty imposed for low power factor?

a. Residential and commercial consumers.
b. Industrial consumers.
c. Agricultural consumers.
d. All of the above.

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 12

Prepared By:
Krishan Arora
Assistant Professor and Head

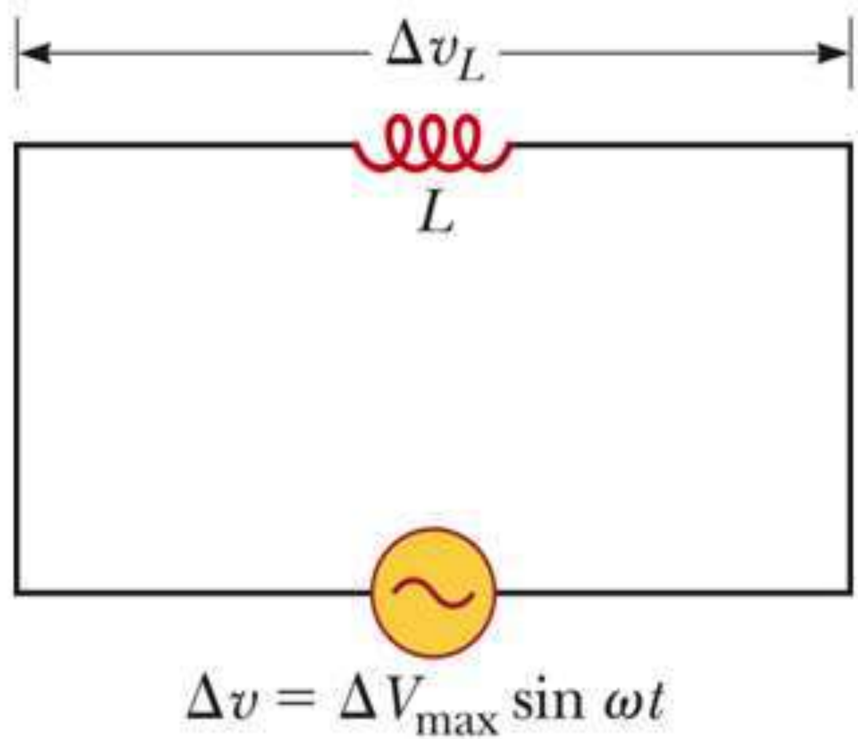
Inductors in an AC Circuit

Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\, sin\omega t$$



Current in an Inductor

The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{\max}}{L} \int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\omega L}$$

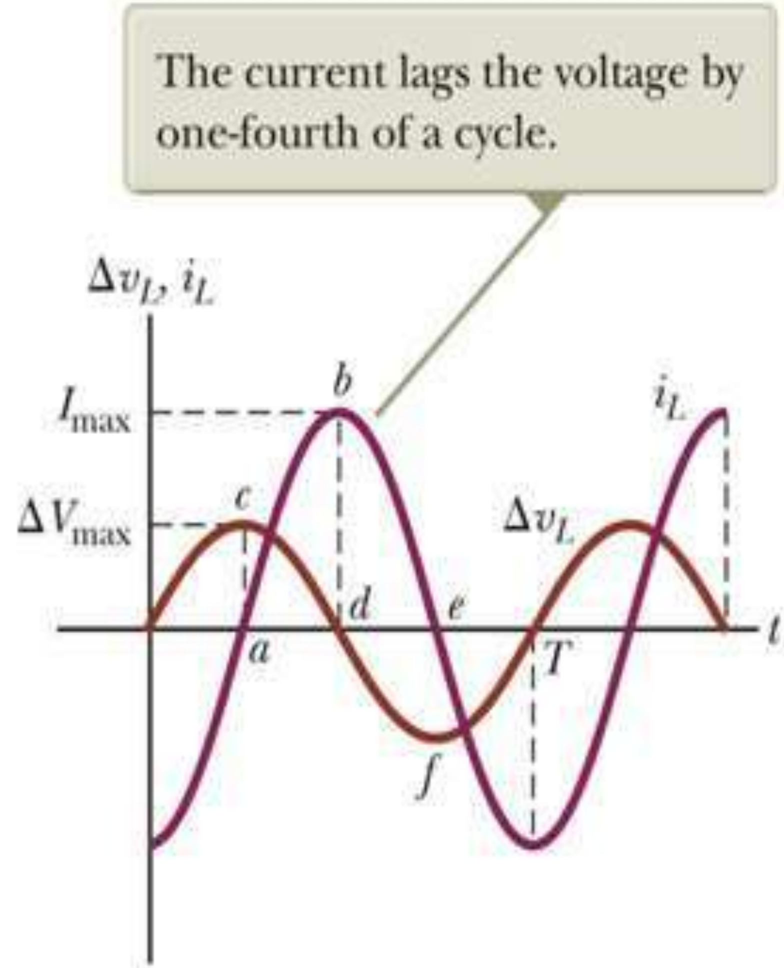
This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)$ rad = 90^{o}.

Phase Relationship of Inductors in an AC Circuit

The current is a maximum when the voltage across the inductor is zero.

- The current is momentarily not changing

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$).

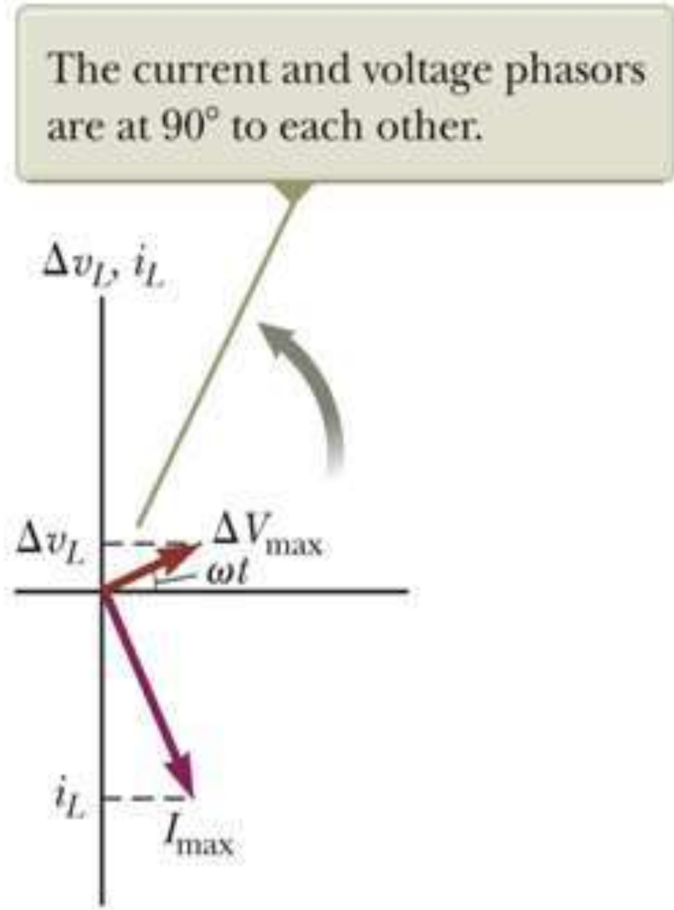


Phasor Diagram for an Inductor

The phasors are at 90° with respect to each other.

This represents the phase difference between the current and voltage.

Specifically, the current lags behind the voltage by 90°.



Inductive Reactance

The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.

Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.

The factor is the **inductive reactance** and is given by:

- $X_L = \omega L$

Inductive Reactance, cont.

Current can be expressed in terms of the inductive reactance:

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} \ or\ I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{X_L}$$

As the frequency increases, the inductive reactance increases

- This is consistent with Faraday's Law:
  - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

Voltage Across the Inductor

The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt}$$

$$= -\Delta V_{\max} \sin \omega t$$

$$= -I_{\max} X_L \sin \omega t$$

Quick Quiz (Poll 1)

- The two quantities are said to be in phase with each other when

a. the phase difference between two quantities is zero degree or radian

b. each of them pass through zero values at the same instant and rise in the same direction

c. each of them pass through zero values at the same instant but rises in the opposite directions

d. Both (a) or (b)

Quick Quiz (Poll 2)

- The inductive reactance of a circuit .... with the increase in supply frequency

a. increases

b. decreases

c. remains unchanged

d. unpredictable

Capacitors in an AC Circuit

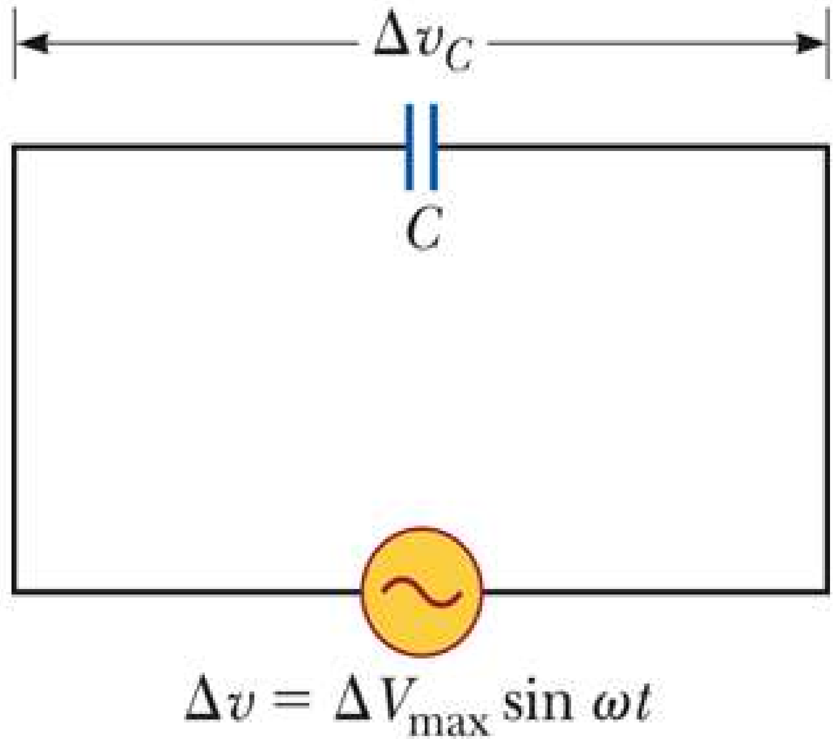
The circuit contains a capacitor and an AC source.

Kirchhoff's loop rule gives:

$\Delta v + \Delta v_C = 0$ and so

$\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$

- Δv_C is the instantaneous voltage across the capacitor.



Capacitors in an AC Circuit, cont.

The charge is $q = C\Delta V_{max} \sin \omega t$

The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

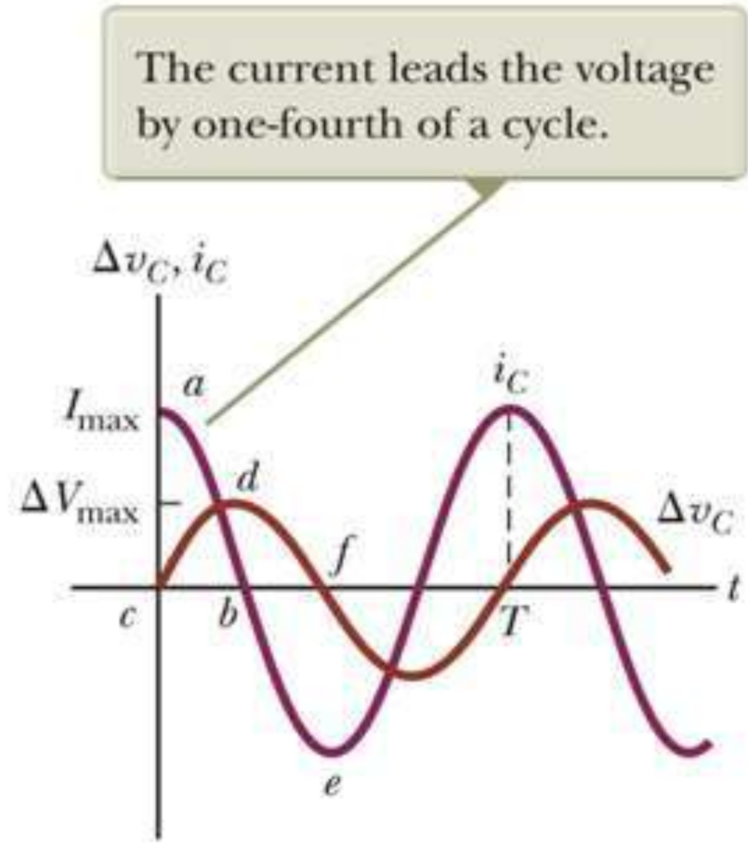
$$\text{or } i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current is $\pi/2$ rad = 90^o out of phase with the voltage

More About Capacitors in an AC Circuit

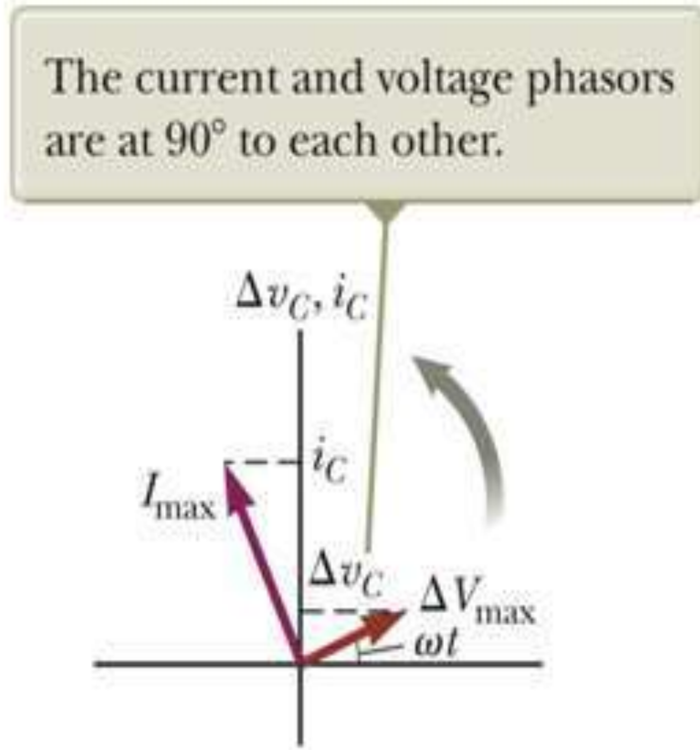
The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

The current leads the voltage by 90°.



Phasor Diagram for Capacitor

The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°.



Capacitive Reactance

The maximum current in the circuit occurs at cos ωt = 1 which gives

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{max} = \frac{\Delta V_{max}}{X_C}$$

Voltage Across a Capacitor

The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$.

As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.

As the frequency approaches zero, X_C approaches infinity and the current approaches zero.

- This would act like a DC voltage and the capacitor would act as an open circuit.

Quick Quiz (Poll 3)

- In a pure capacitive circuit, the current will

a. lag behind the voltage by 90 degree

b. lead behind the voltage by 90 degree

c. remains in phase with voltage

d. none of the above

Quick Quiz (Poll 4)

- A phasor is a line which represents the

a. rms value and phase of an alternating quantity

b. average value and phase of an alternating quantity

c. magnitude and direction of an alternating quantity

d. none of the above

UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 13

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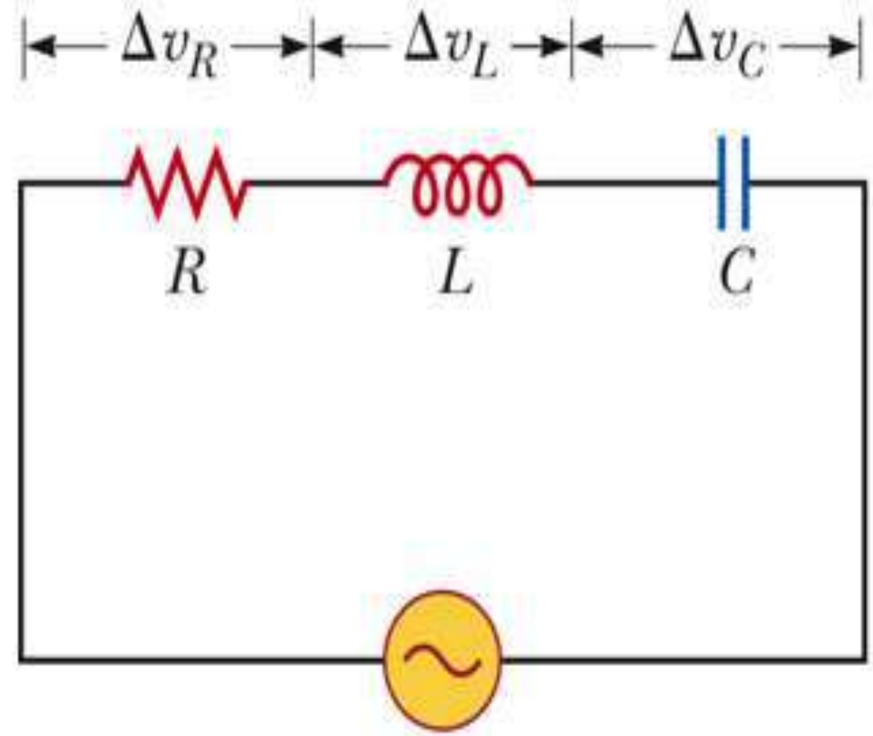
The *RLC* Series Circuit

The resistor, inductor, and capacitor can be combined in a circuit.

The current and the voltage in the circuit vary sinusoidally with time.



The *RLC* Series Circuit, cont.

The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.

The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.

- φ is the **phase angle** between the current and the applied voltage.

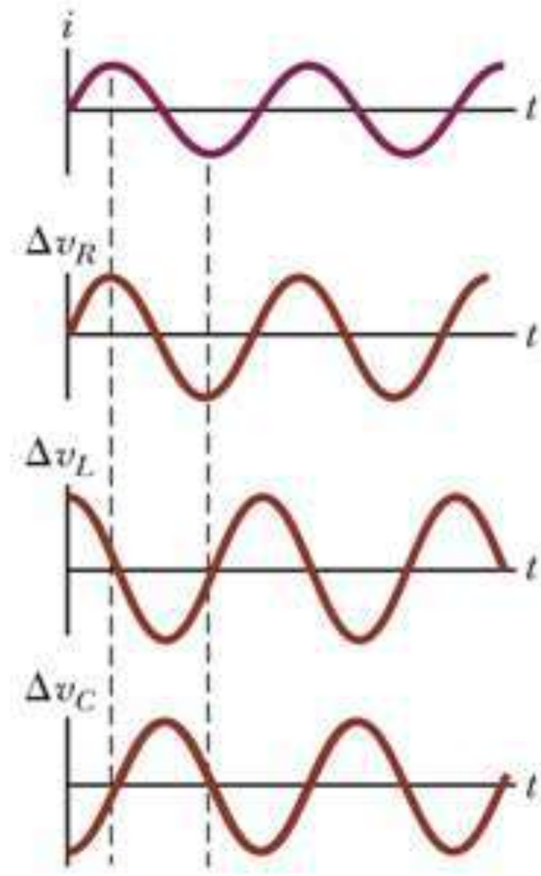
Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.

i and *v* Phase Relationships – Graphical View

The instantaneous voltage across the resistor is in phase with the current.

The instantaneous voltage across the inductor leads the current by 90° .

The instantaneous voltage across the capacitor lags the current by 90° .



i and *v* Phase Relationships – Equations

The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$.

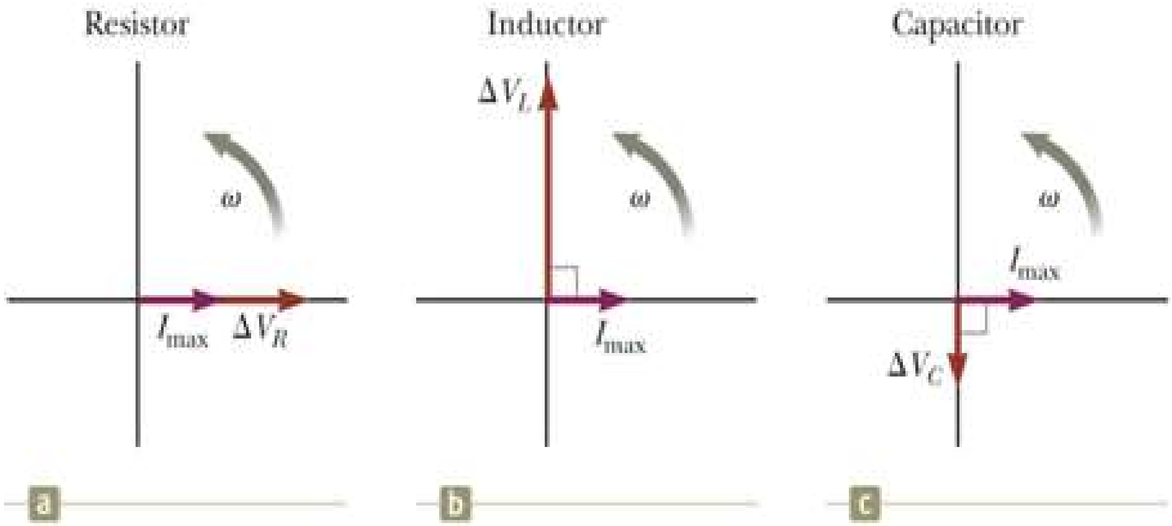
ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_L$.

ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_C$.

The sum of these voltages must equal the voltage from the AC source.

Because of the different phase relationships with the current, they cannot be added directly.

Phasor Diagrams



To account for the different phases of the voltage drops, vector techniques are used.

Remember the phasors are rotating vectors

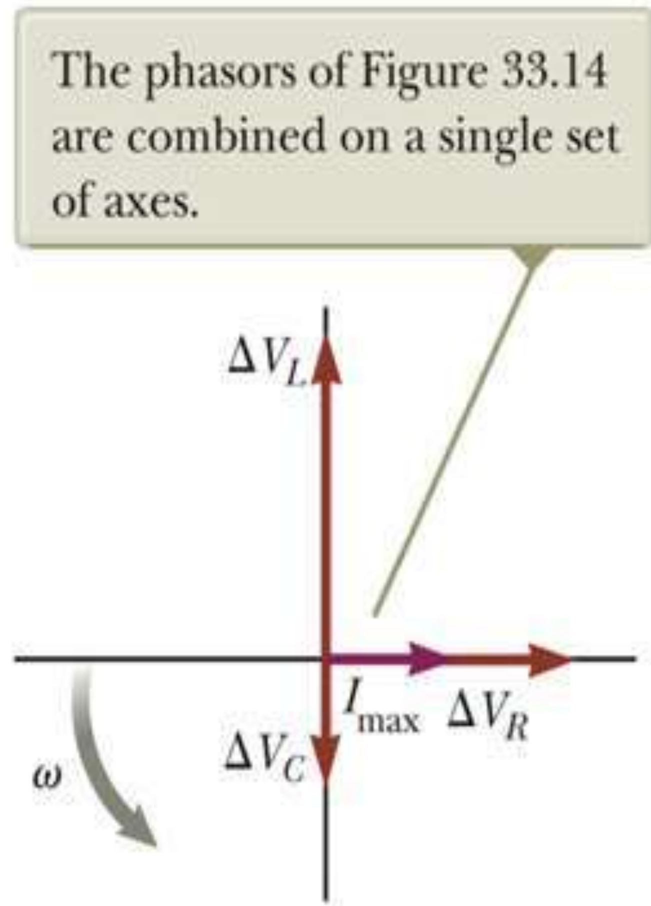
The phasors for the individual elements are shown.

Resulting Phasor Diagram

The individual phasor diagrams can be combined.

Here a single phasor I_{max} is used to represent the current in each element.

- In series, the current is the same in each element.



Vector Addition of the Phasor Diagram

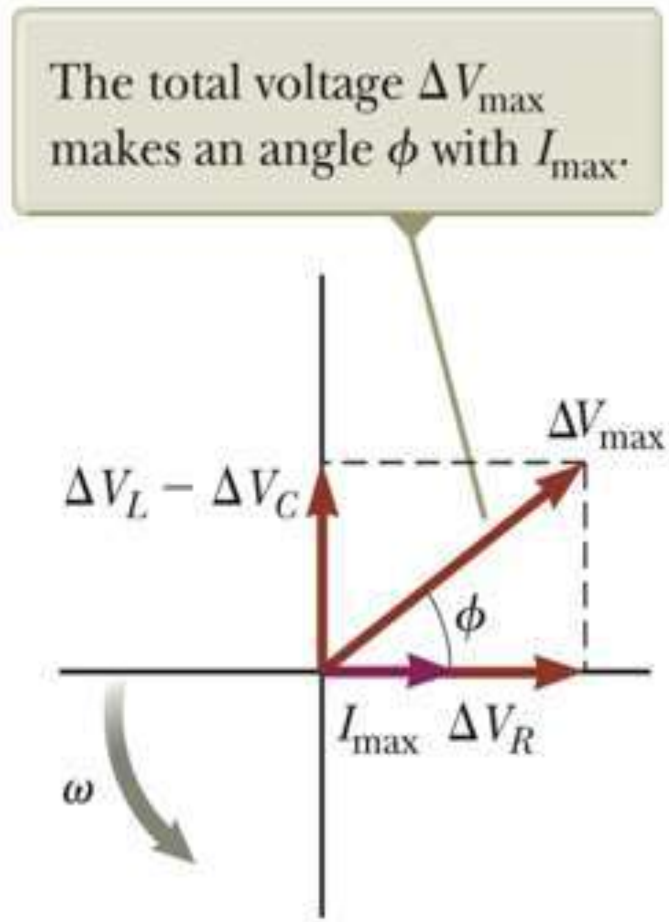
Vector addition is used to combine the voltage phasors.

ΔV_L and ΔV_C are in opposite directions, so they can be combined.

Their resultant is perpendicular to ΔV_R.

The resultant of all the individual voltages across the individual elements is Δv_{max}.

- This resultant makes an angle of φ with the current phasor I_{max}.



Total Voltage in RLC Circuits

From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$

Quick Quiz (Poll 1)

In a series RLC circuit, the phase difference between the current in the capacitor and the current in the inductor is?

a) 0^0

b) 90^0

c) 180^0

d) 360^0

Quick Quiz (Poll 2)

In a series RLC circuit, the phase difference between the current in the circuit and the voltage across the capacitor is?

a) 0^0

b) 90^0

c) 180^0

d) 360^0

Impedance

The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

- Impedance has units of ohms

Phase Angle

The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle can be positive or negative and determines the nature of the circuit.

Determining the Nature of the Circuit

If ϕ is positive

- $X_L > X_C$ (which occurs at high frequencies)
- The current lags the applied voltage.
- The circuit is *more inductive than capacitive.*

If ϕ is negative

- $X_L < X_C$ (which occurs at low frequencies)
- The current leads the applied voltage.
- The circuit is *more capacitive than inductive.*

If ϕ is zero

- $X_L = X_C$
- The circuit is *purely resistive.*

Power in an AC Circuit

The average power delivered by the AC source is converted to internal energy in the resistor.

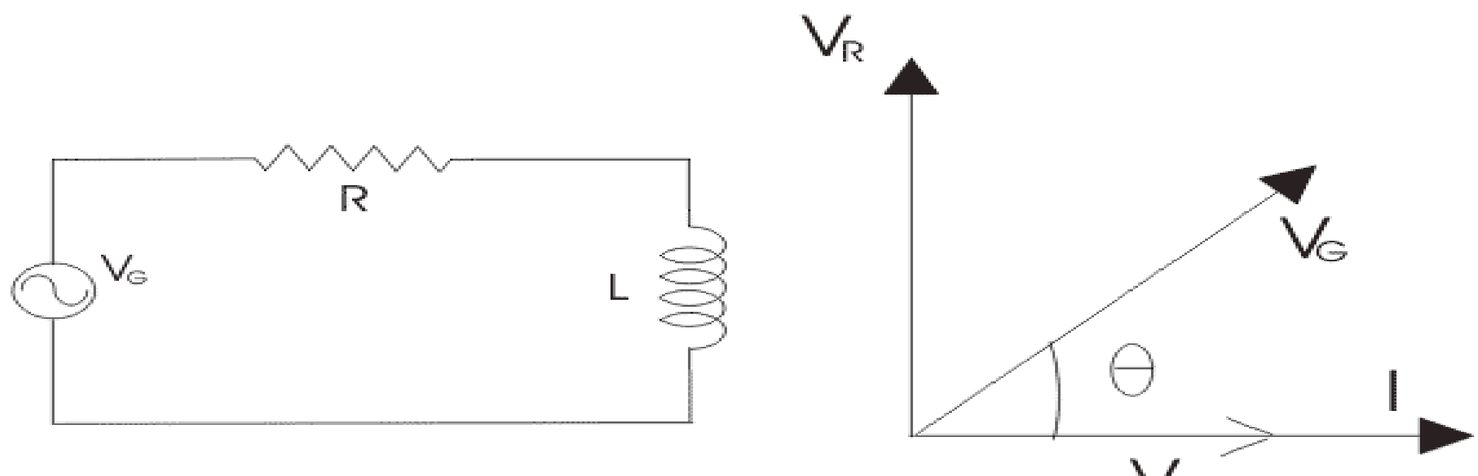
- $P_{avg} = ½\ I_{max}\,\Delta V_{max} \cos\phi = I_{rms}\Delta V_{rms} \cos\phi$
- $\cos\phi$ is called the power factor of the circuit

We can also find the average power in terms of R.

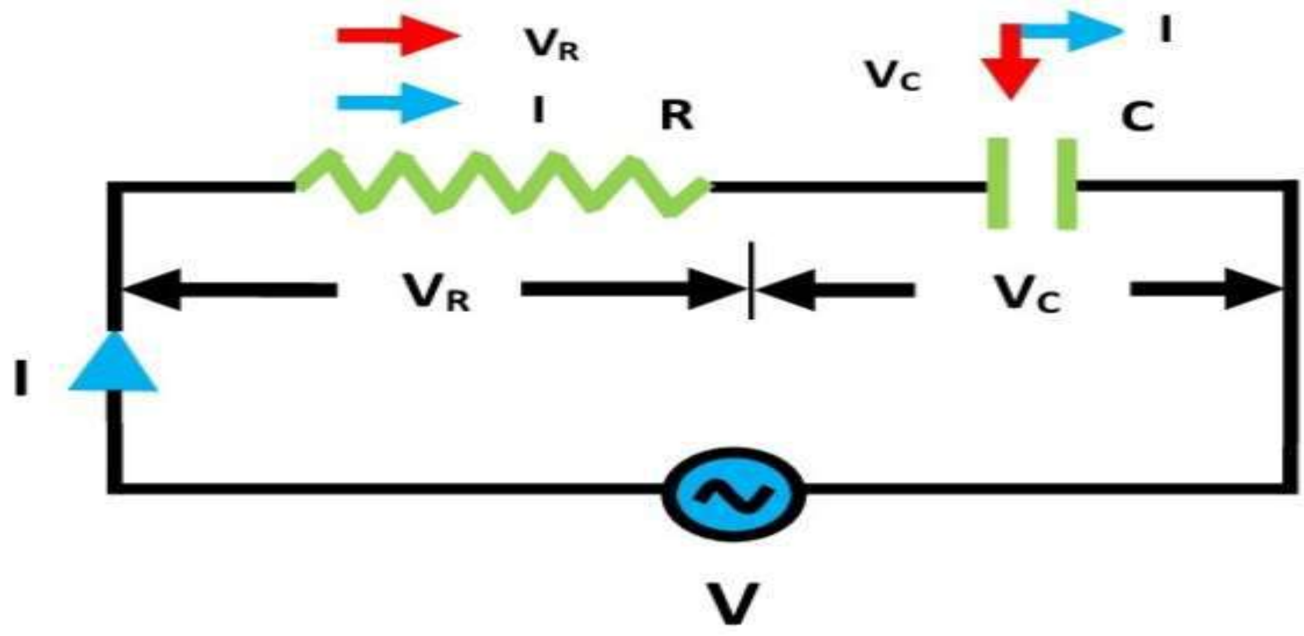
- $P_{avg} = I^2_{rms}R$

When the load is purely resistive, $\phi = 0$ and $\cos\phi = 1$

- $P_{avg} = I_{rms}\,\Delta V_{rms}$



(a) (b)



Resonance in an AC Circuit

Resonance occurs at the frequency ω_0 where the current has its maximum value.

- To achieve maximum current, the impedance must have a minimum value.
- This occurs when $X_L = X_C$
- Solving for the frequency gives

$$\omega_o = 1/\sqrt{LC}$$

The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.

The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.

At the resonance frequency, the current is in phase with the applied voltage.

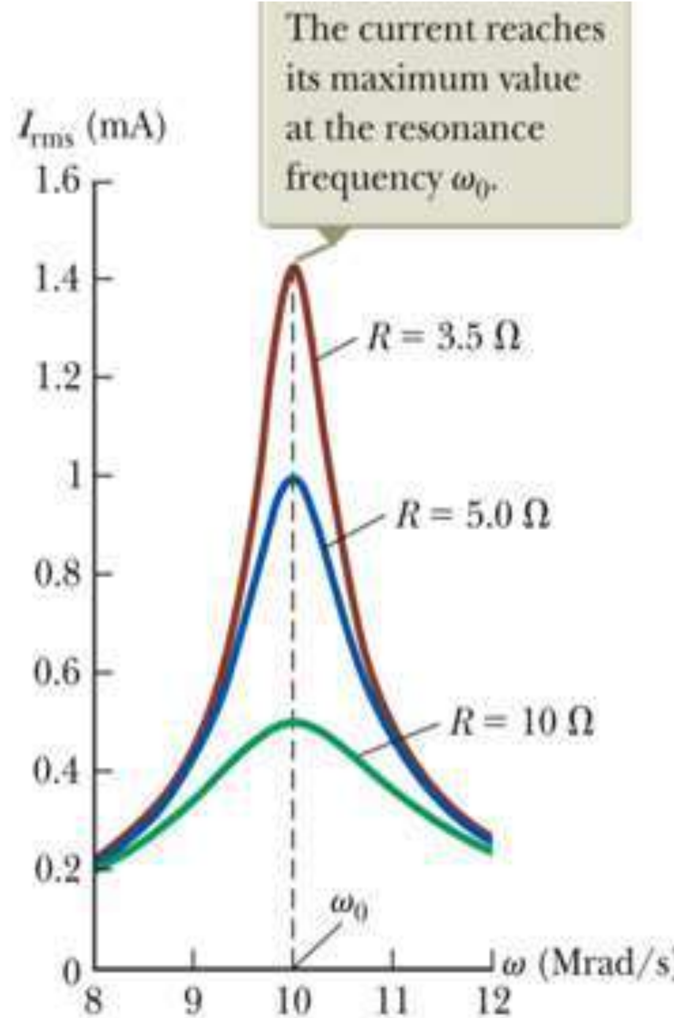
Resonance, cont.

Resonance occurs at the same frequency regardless of the value of *R*.

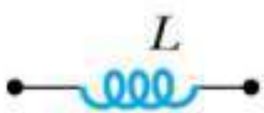
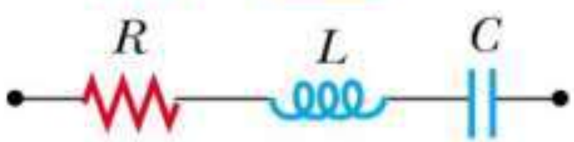
As *R* decreases, the curve becomes narrower and taller.

Theoretically, if $R = 0$ the current would be infinite at resonance.

- Real circuits always have some resistance.



Summary of Circuit Elements, Impedance, Phase Angles

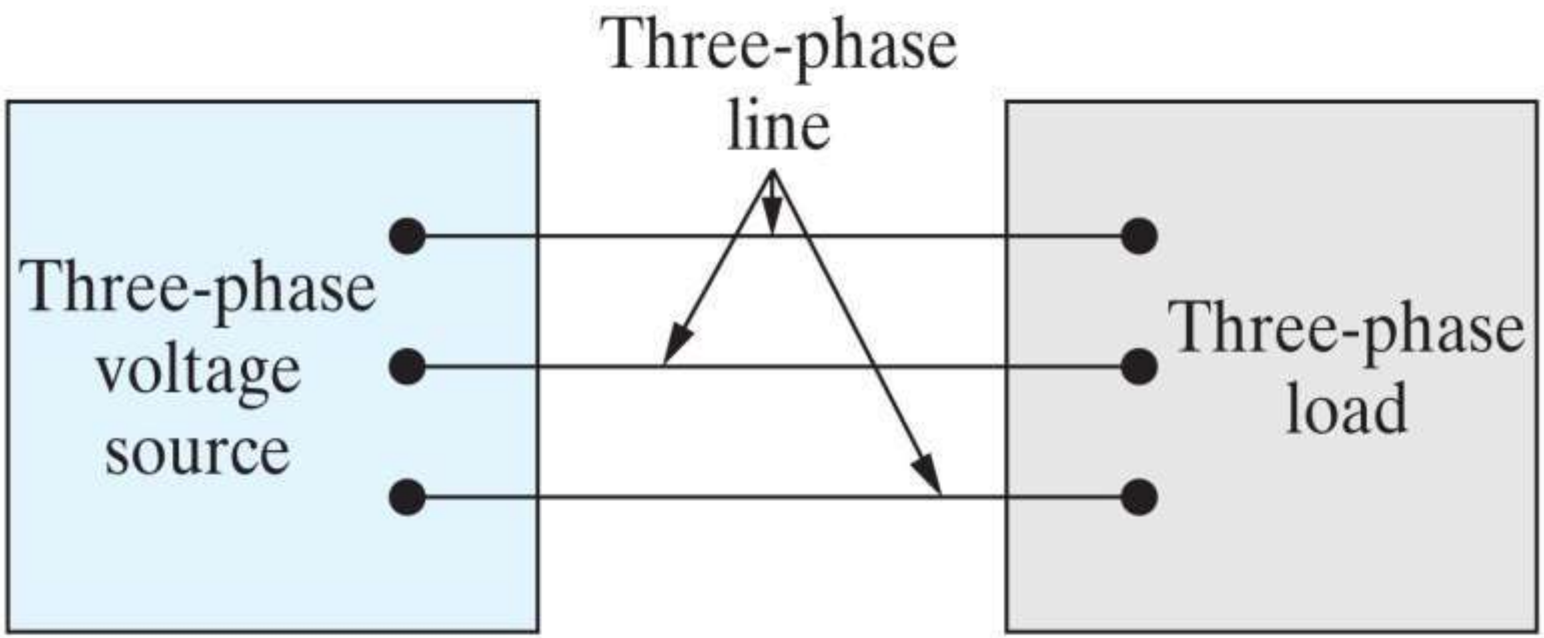
	$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$	$\tan\phi=\dfrac{X_L-X_C}{R}$
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2+X_C^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2+X_L^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2+(X_L-X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Quick Quiz (Poll 3)

- __________ the resonant frequency, the current in the inductor lags the voltage in a series RLC circuit.
  a) Above
  b) Below
  c) Equal to
  d) Depends on the circuit

THREE PHASE Connections

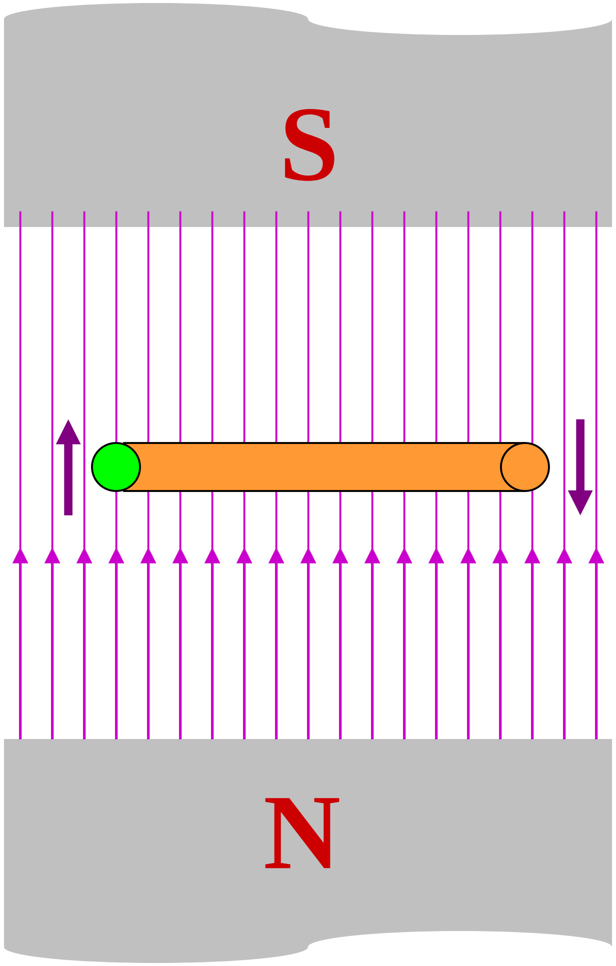
Basic Three-Phase Circuit

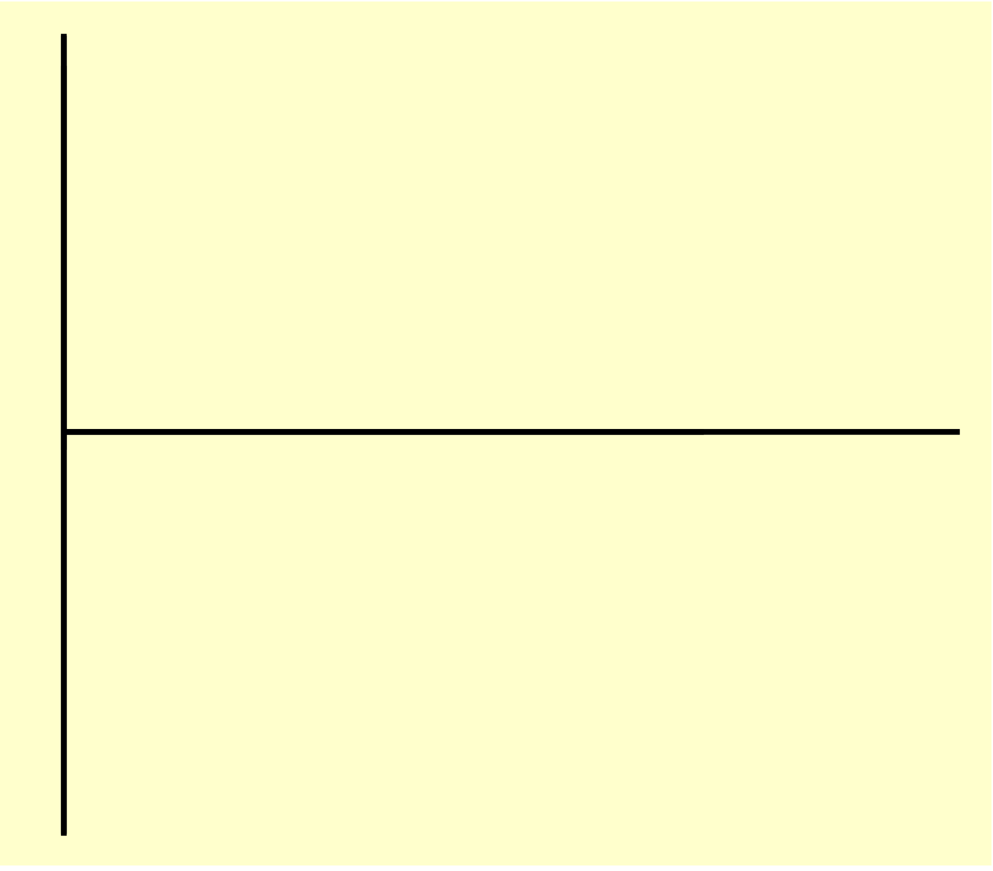


What is Three-Phase Power?

- Three sinusoidal voltages of equal amplitude and frequency out of phase with each other by 120°. Known as “balanced”.
- Phases are labeled A, B, and C. or R,Y and B.
- Phases are sequenced as A, B, C (positive) or A, C, B (negative).

GENERATING A SINGLE PHASE

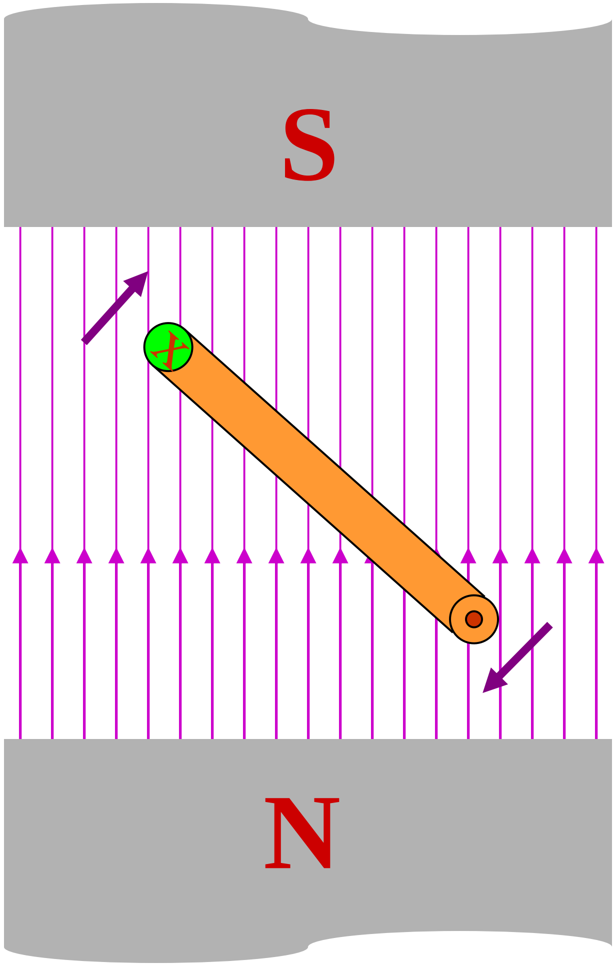


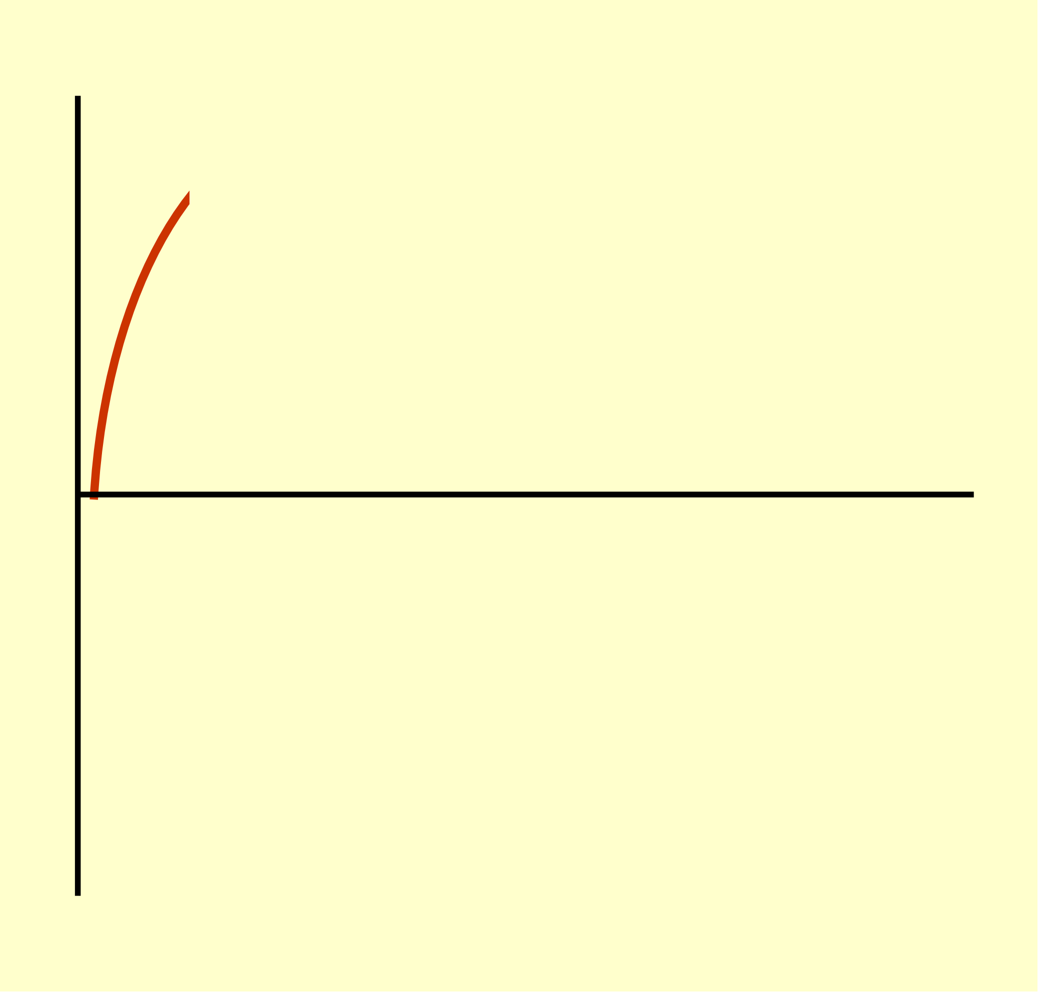


Motion is parallel to the flux.

No voltage is induced.

GENERATING A SINGLE PHASE

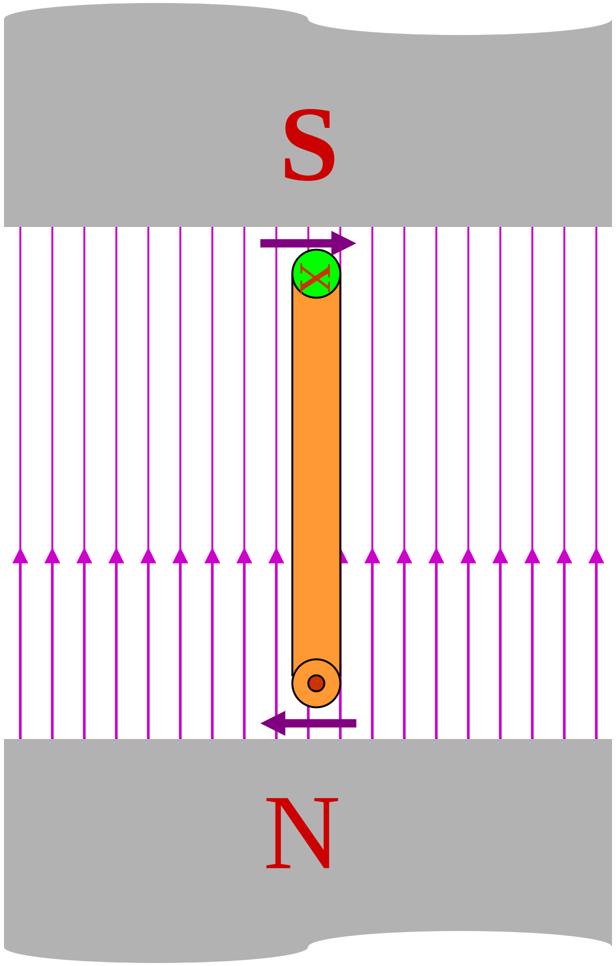


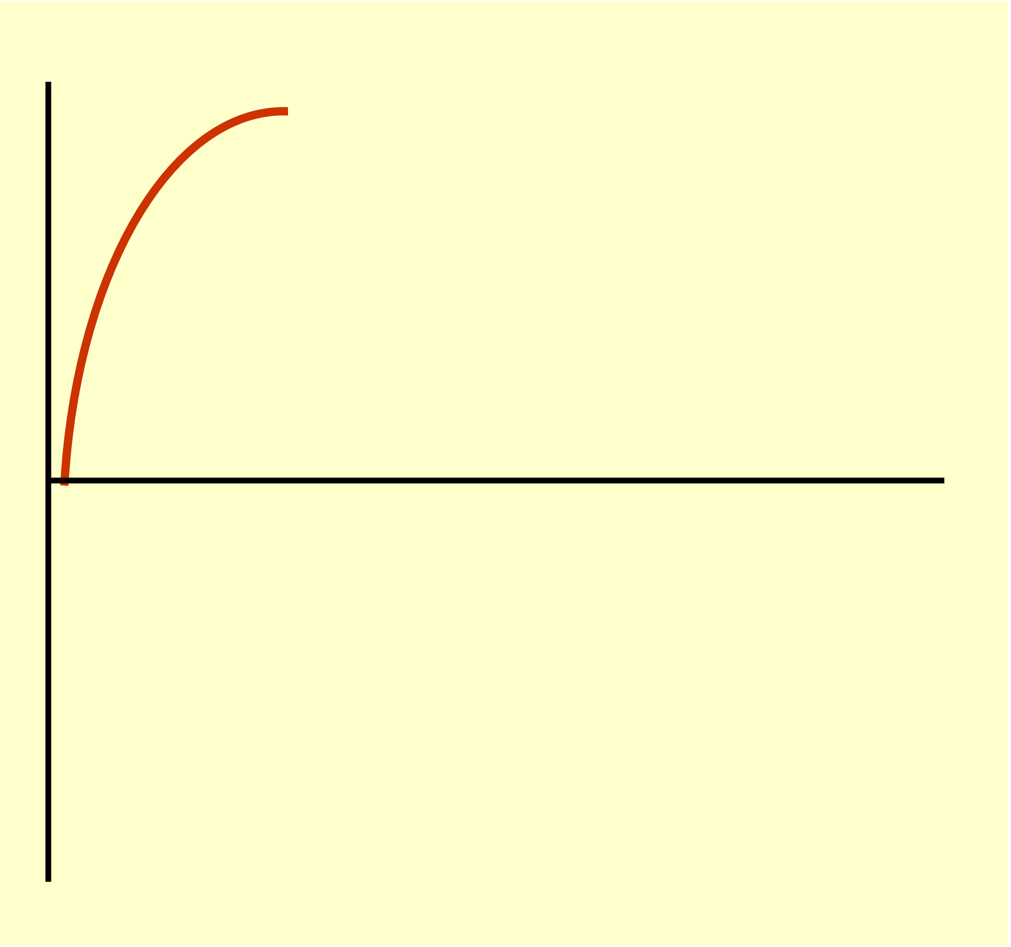


Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

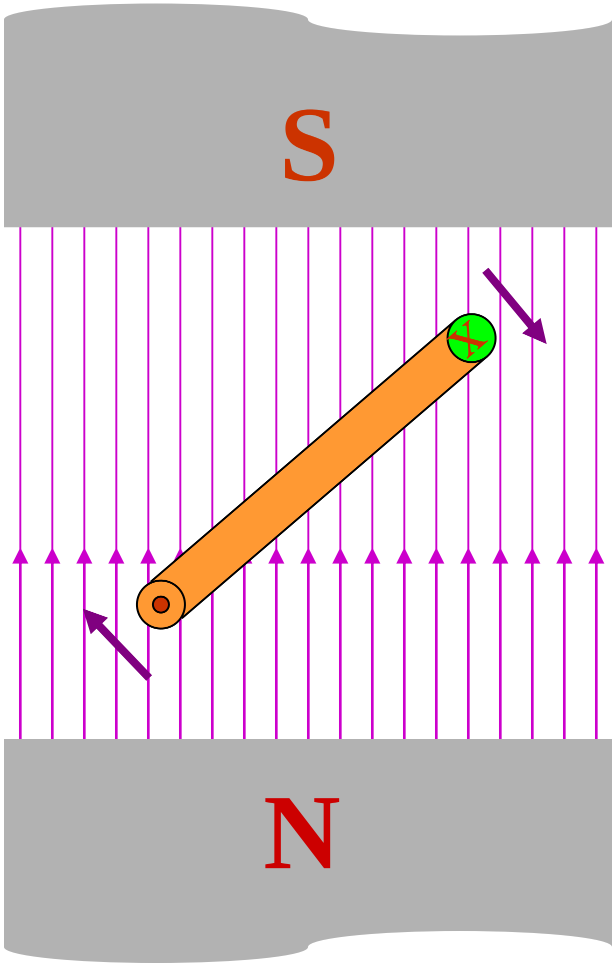


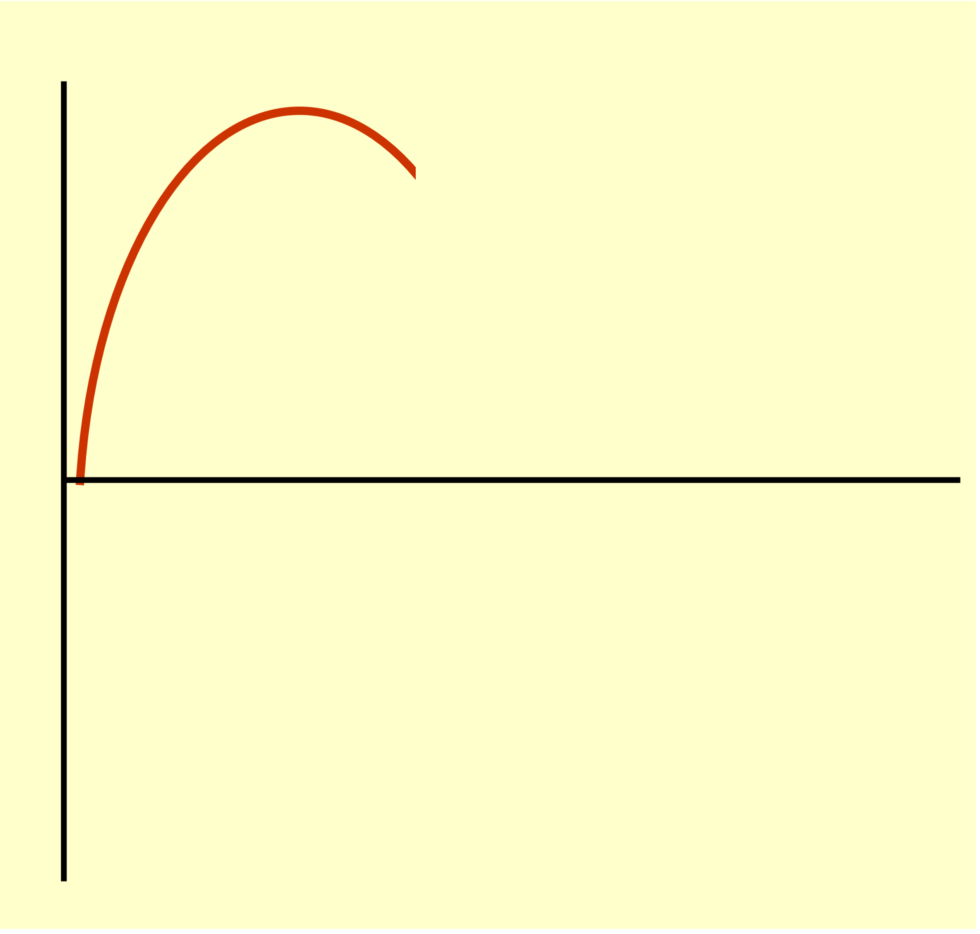


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE

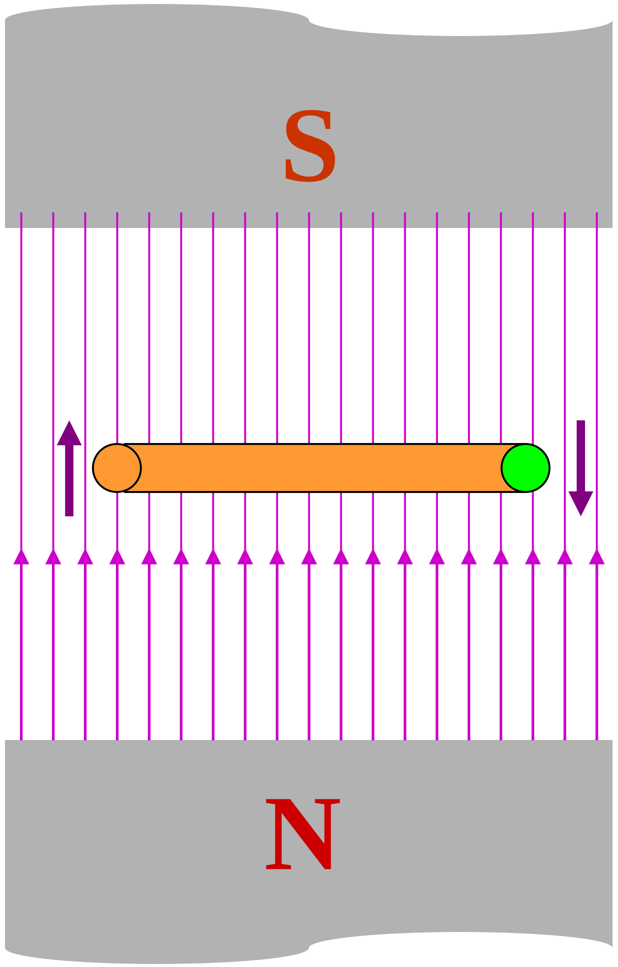


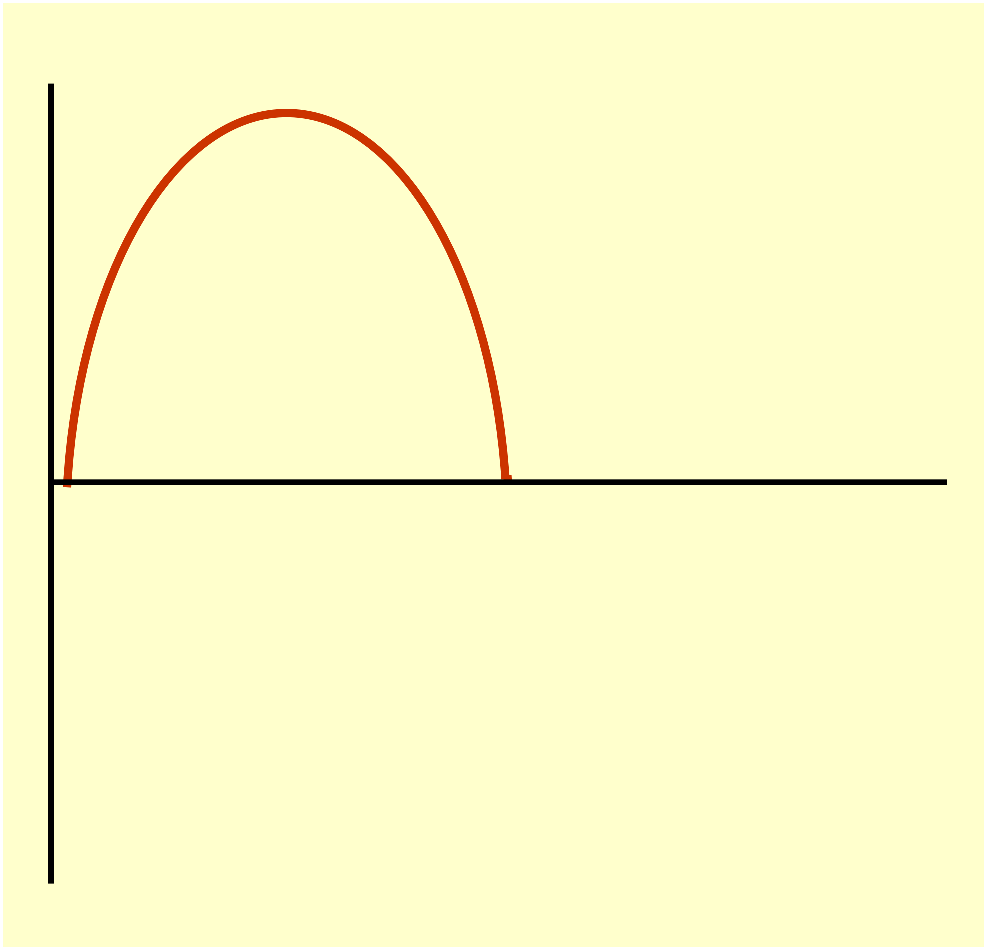


Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

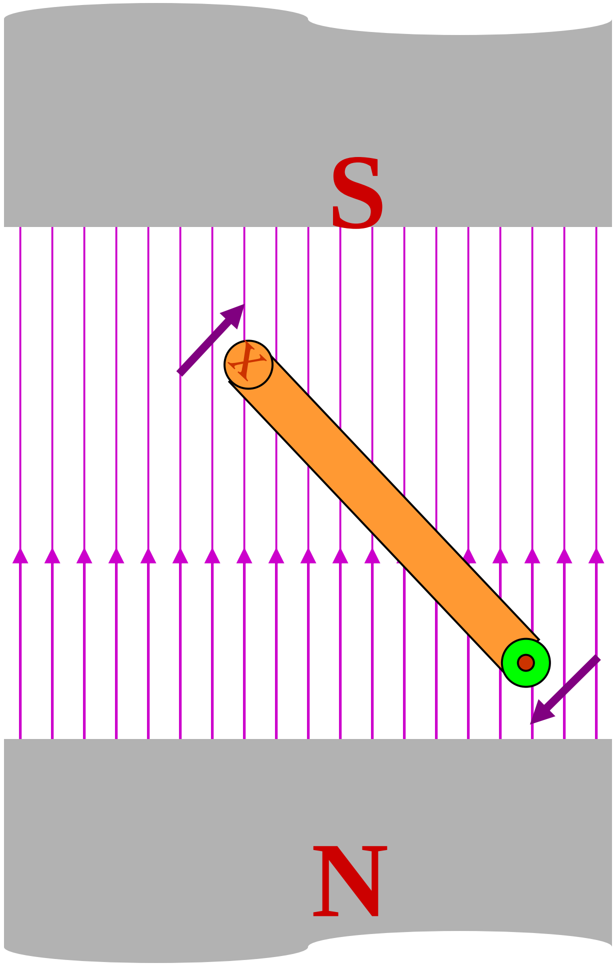


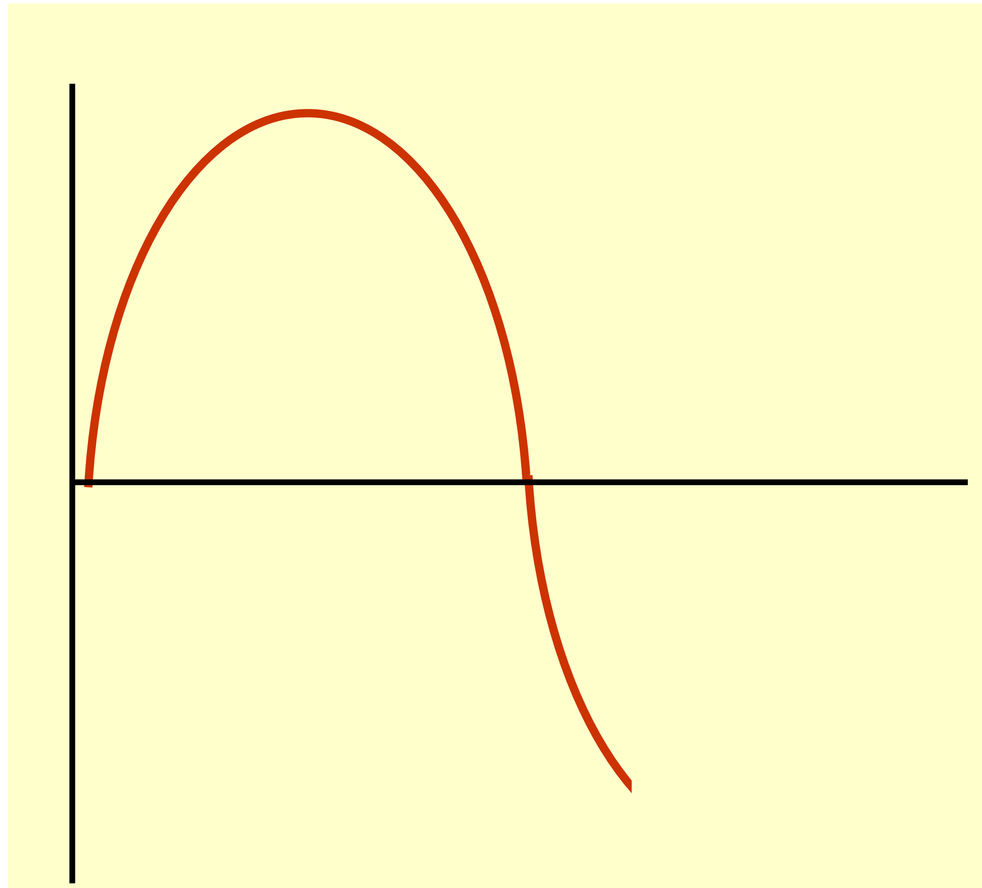


Motion is parallel to flux.

No voltage is induced.

GENERATING A SINGLE PHASE



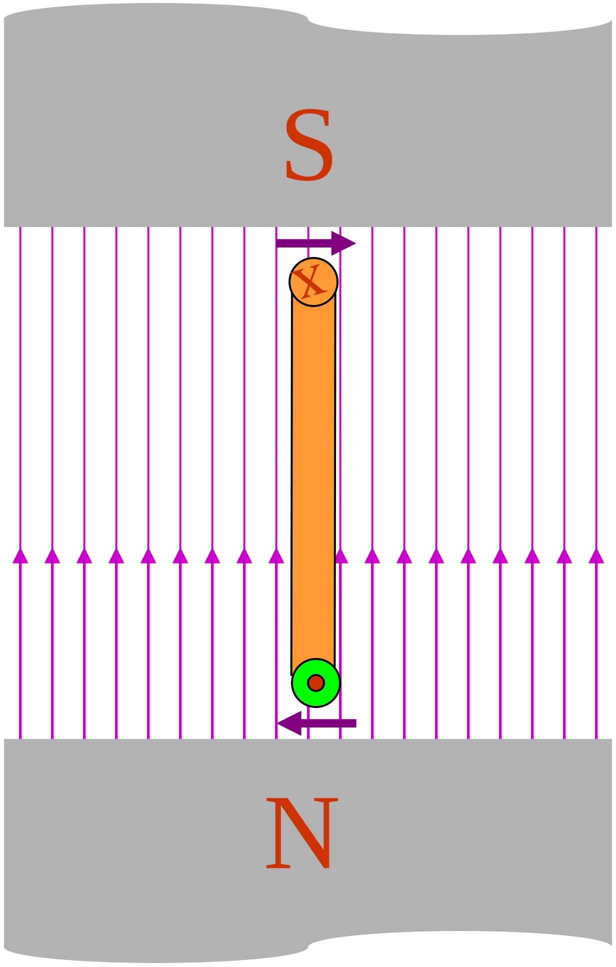


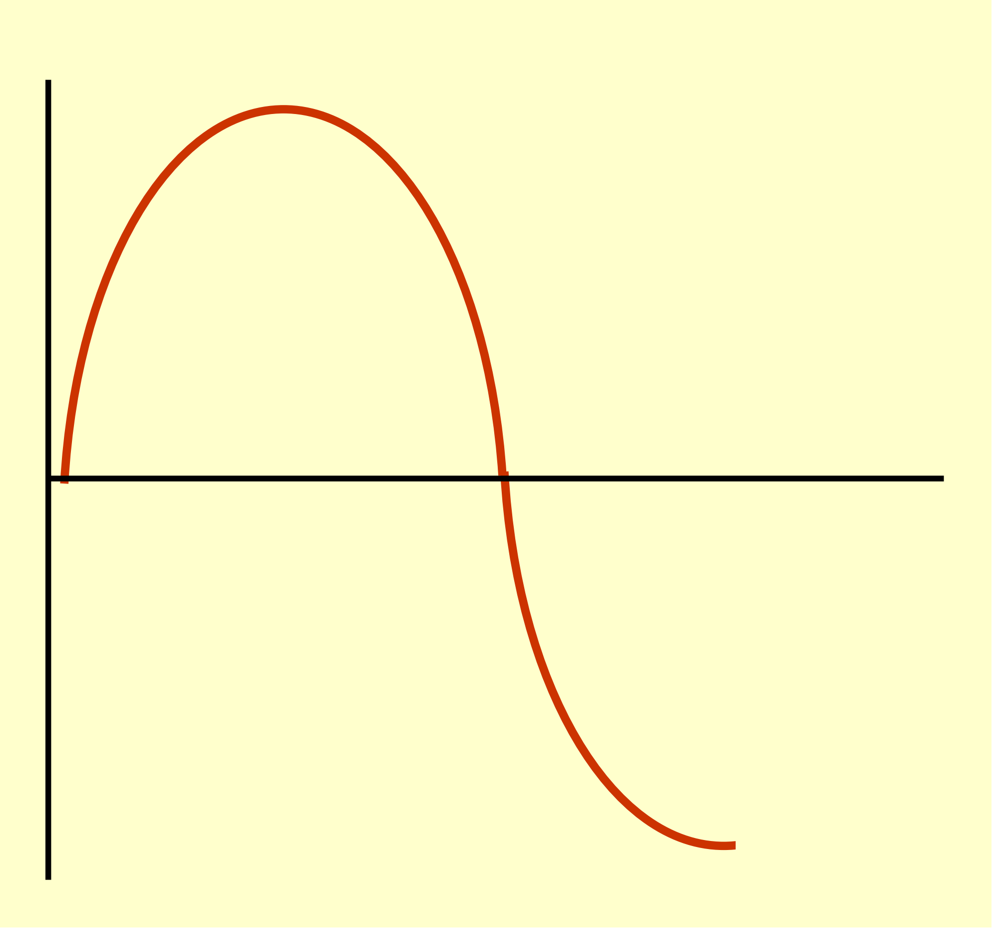
Notice current in the conductor has reversed.

Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

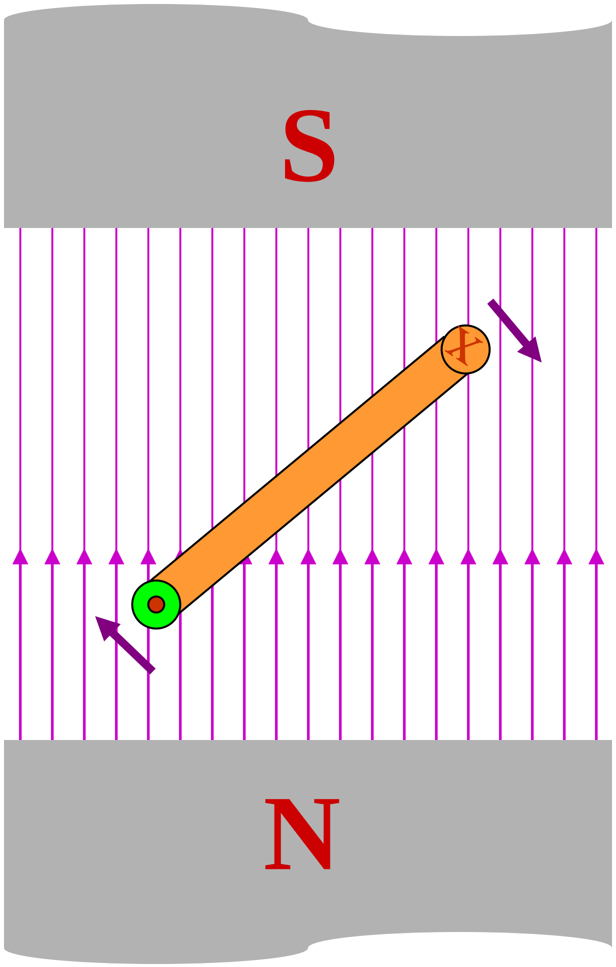


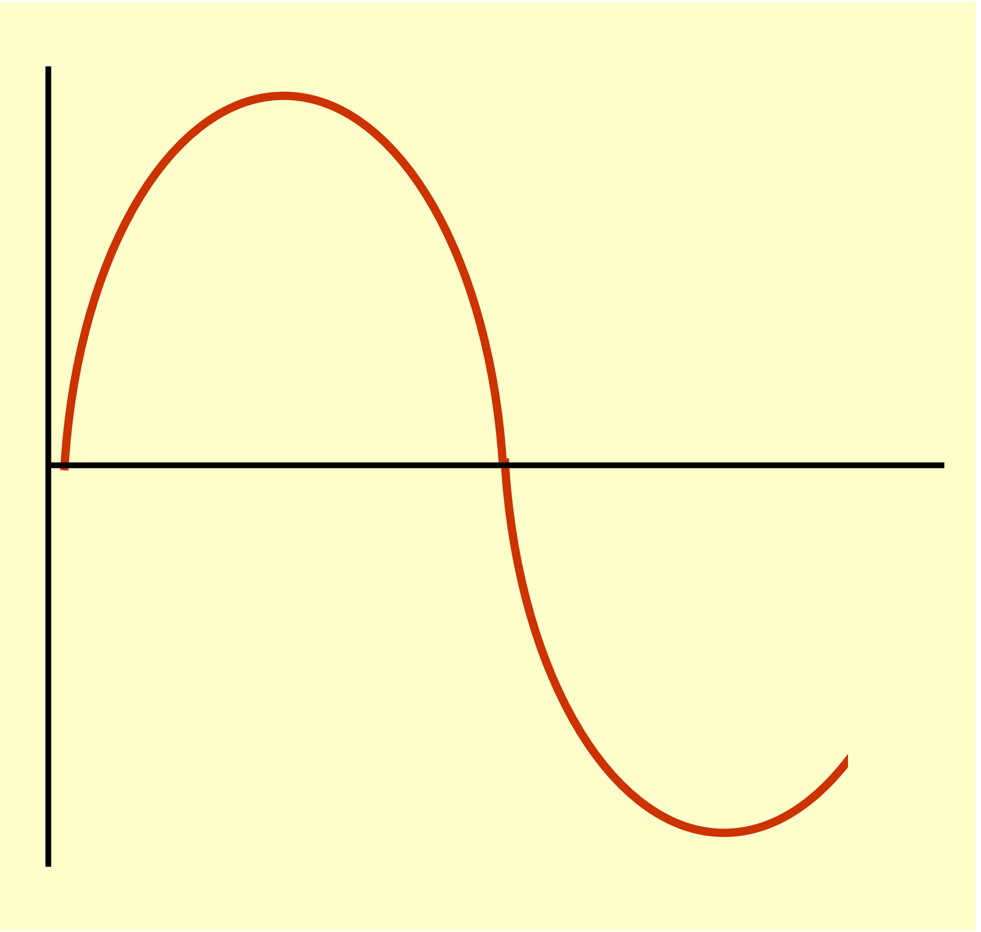


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE

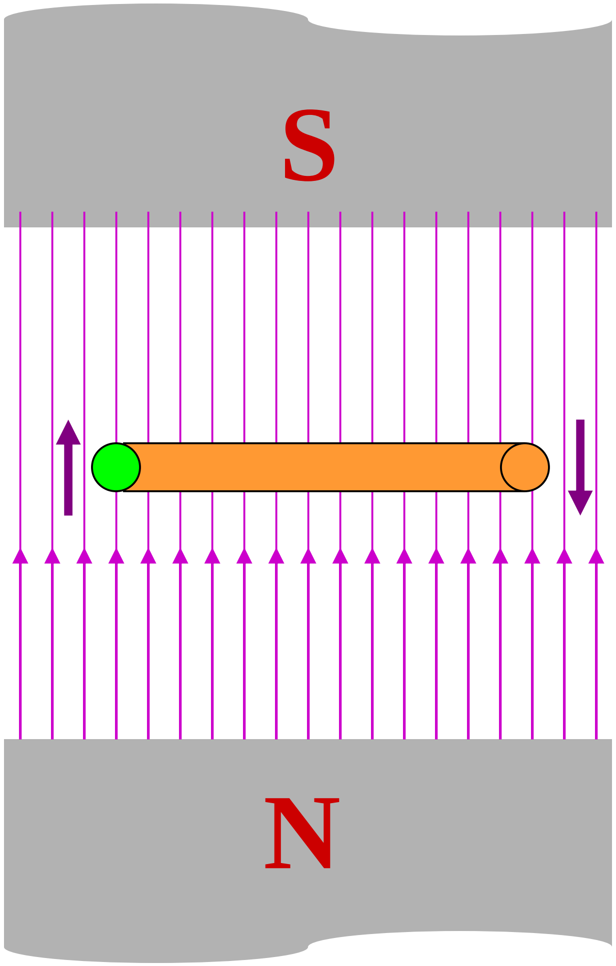


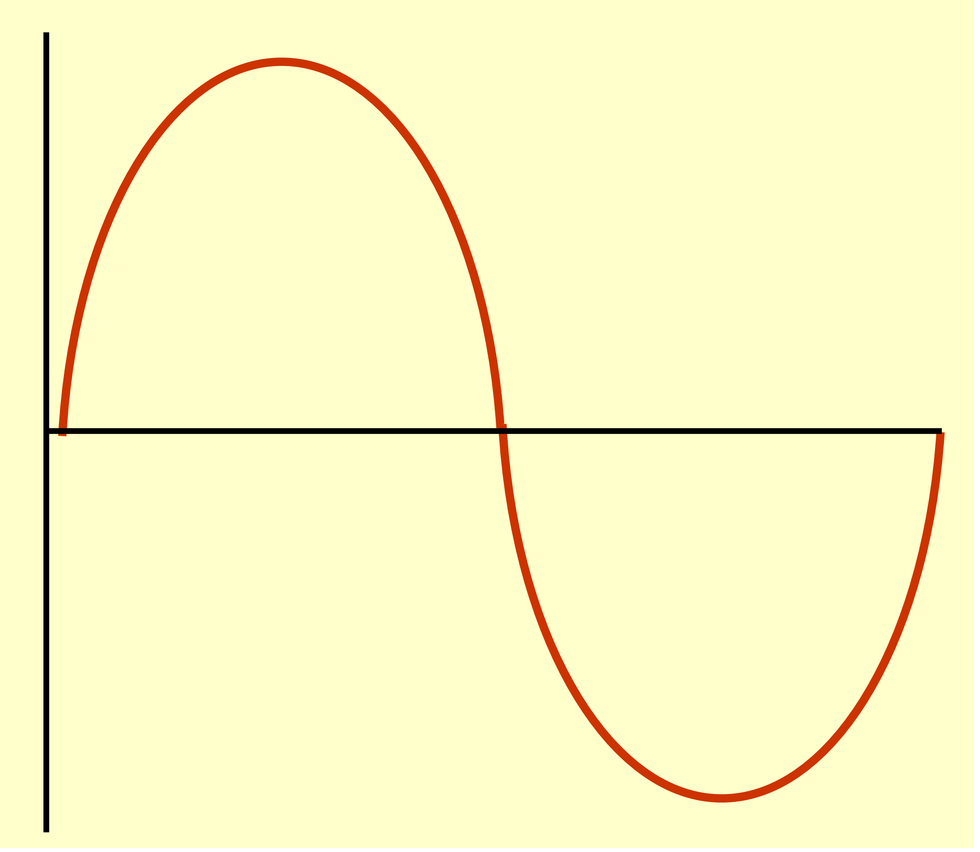


Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE





Motion is parallel to flux.

No voltage is induced.

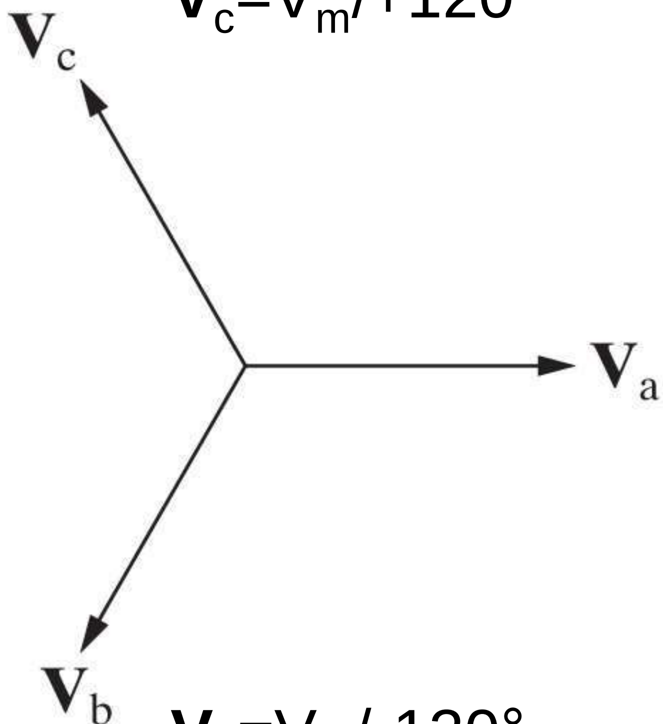
Ready to produce another cycle.

Three phase system

- 4 wires
  - 3 “active” phases, A, B, C
  - 1 “ground”, or “neutral”
- Color Code
  - Phase A Red
  - Phase B Yellow
  - Phase C Blue
  - Neutral Black

Phasor (Vector) Form for abc

V$_c$=V_m/+120°

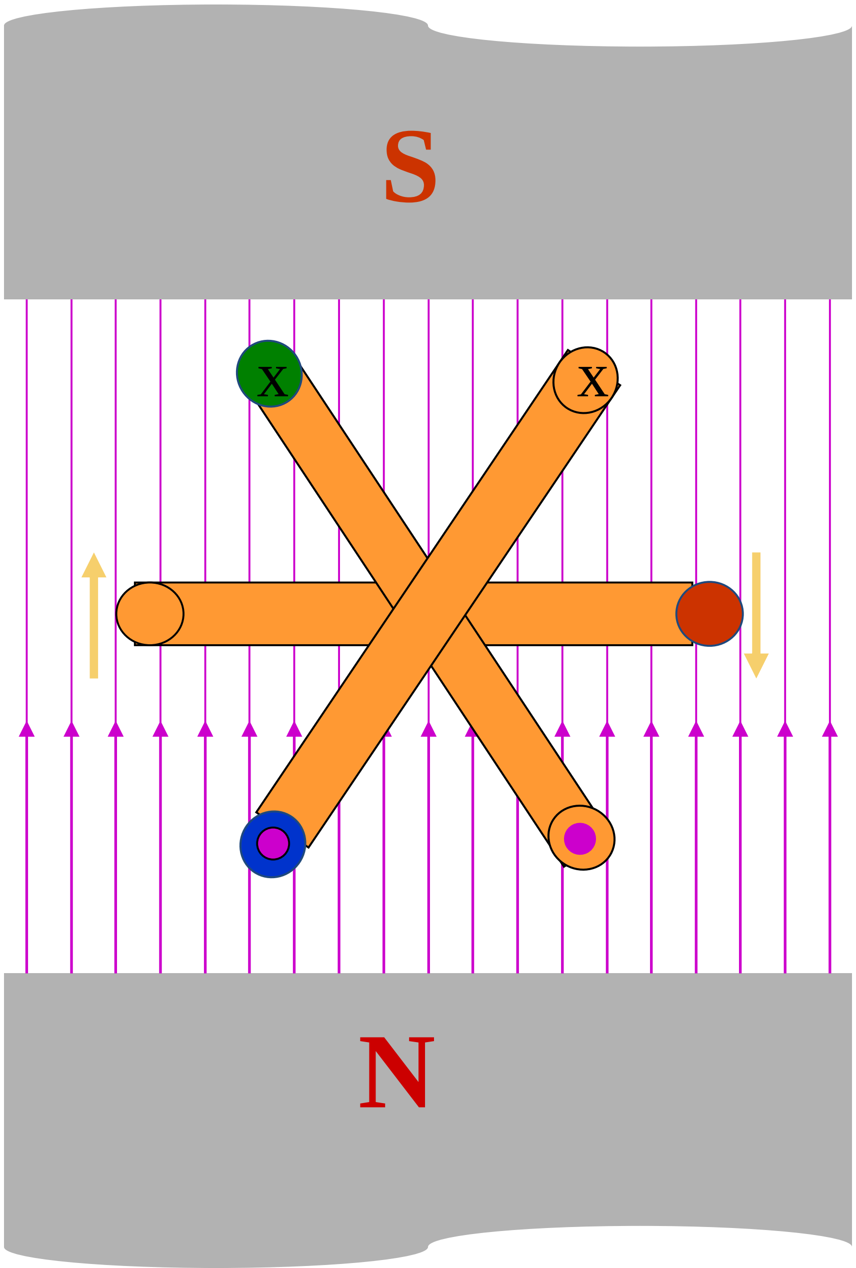


V$_a$=V_m/0°

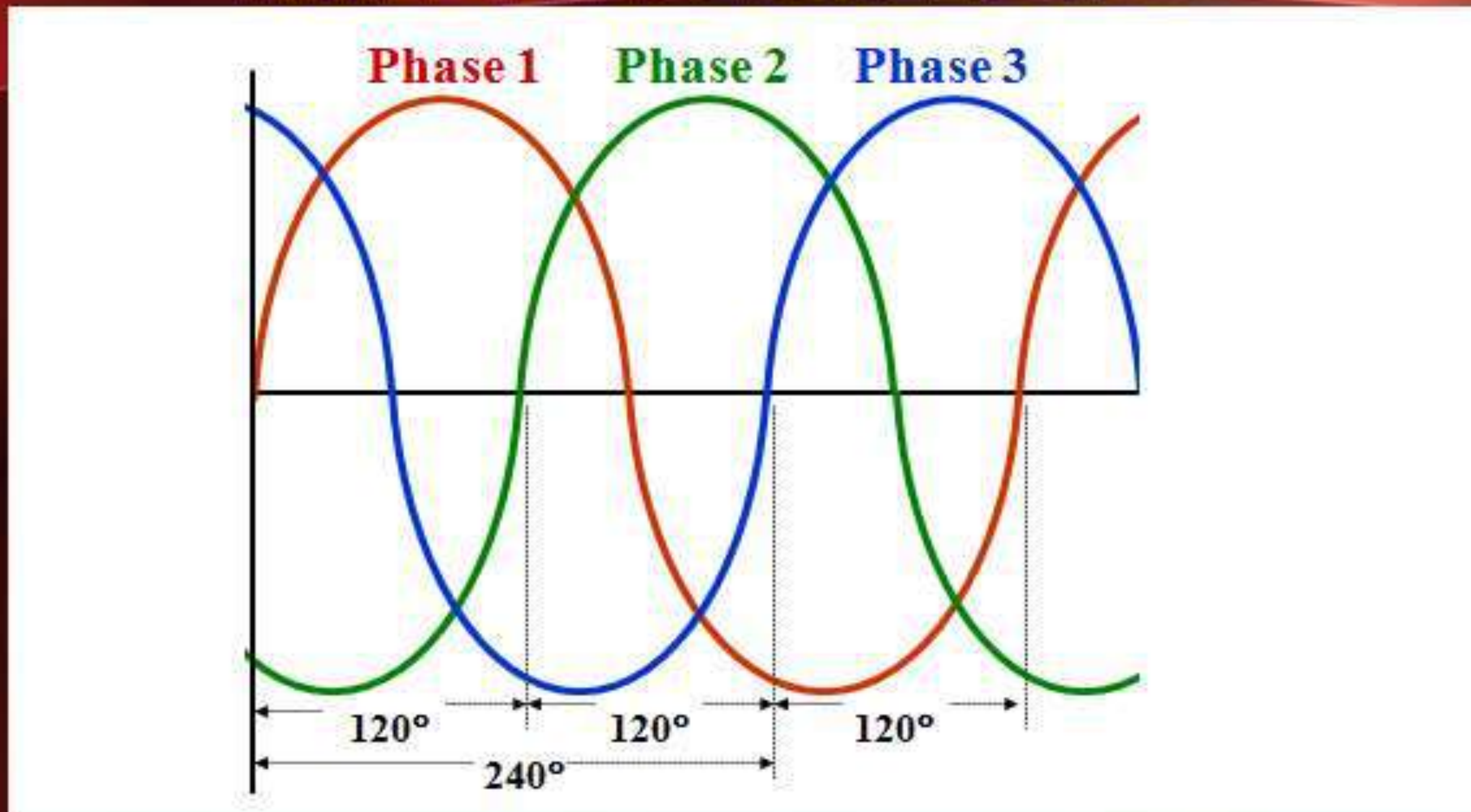
V$_b$=V_m/-120°

Note that KVL applies .... **V**$_a$+**V**$_b$+**V**$_c$=0

GENERATION OF THREE-PHASE AC



THREE-PHASE WAVEFORM



Phase 2 lags phase 1 by 120°.

Phase 3 lags phase 1 by 240°.

Phase 2 leads phase 3 by 120°.

Phase 1 lags phase 3 by 120°.

THREE PHASE SYSTEM

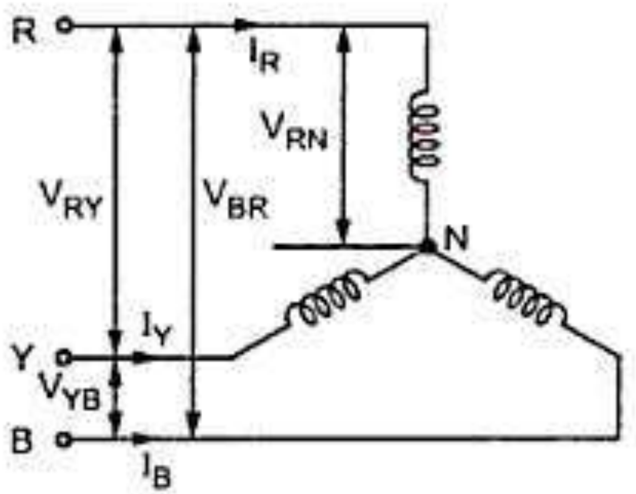
BASICS

Line voltage V_L= voltage between lines

Phase voltage Vph= voltage between a line and neutral

THREE PHASE SYSTEM

BALANCED STAR

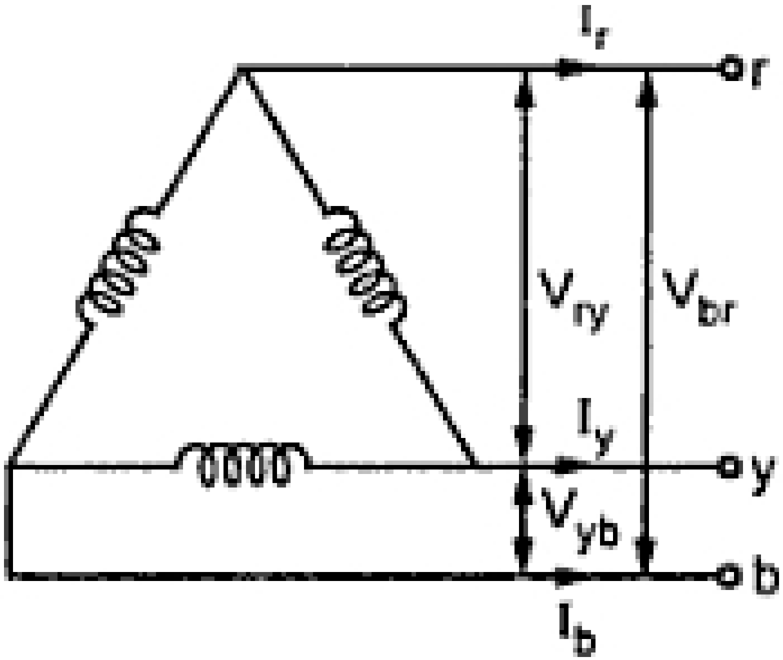


Line Voltage $V_L = \sqrt{3}\, V_{ph}$

Line current $I_L = I_{ph}$

THREE PHASE SYSTEM

BALANCED DELTA



Line Voltage $V_L = V_{ph}$

Line current $I_L = \sqrt{3}\, I_{ph}$

Quick Quiz (Poll 1)

- Power in a Three Phase Circuit = ___________.

a) $P = 3\ V_{Ph}\ I_{Ph}\ Cos\Phi$

b) $P = \sqrt{3}\ V_L\ I_L\ Cos\Phi$

c) Both a & b.

d) None of The Above

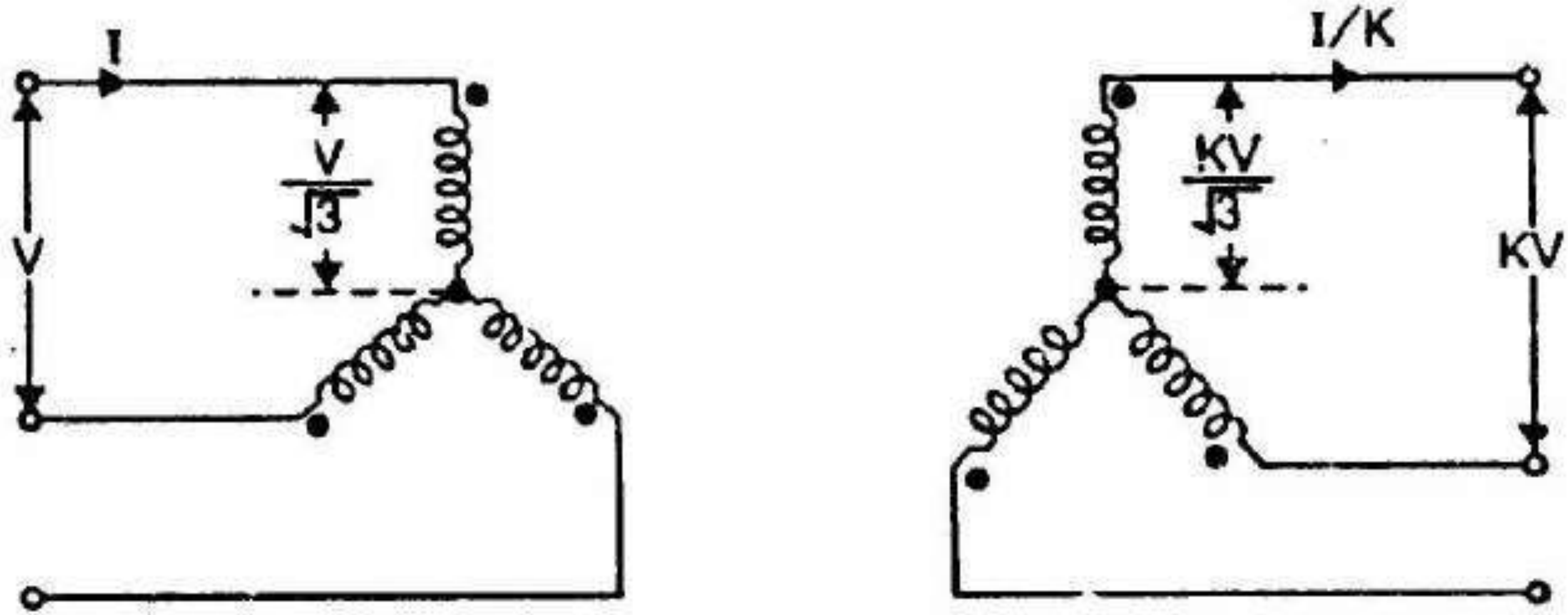
3 phase Transformer connections

By connecting three single phase transformers

1. **Star- Star connection**
2. **Delta- Delta connection**
3. **Star – Delta connection**
4. **Delta – Star connection**

$$\text{Phase transformation ratio, } K = \frac{\text{Secondary phase voltage}}{\text{Primary phase voltage}} = \frac{N_2}{N_1}$$

Star- Star connection



Y – Y Connection

- This connection satisfactory only in balanced load otherwise neutral point will be shifted.

Star- Star connection

Advantages

1.Requires less turns per winding ie cheaper

Phase voltage is 1/√3 times of line voltage

2.Cross section of winding is large i.e stronger to bear stress during short circuit

Line current is equal to phase current

3. Less dielectric strength in insulating materials

phase voltage is less

Star- Star connection

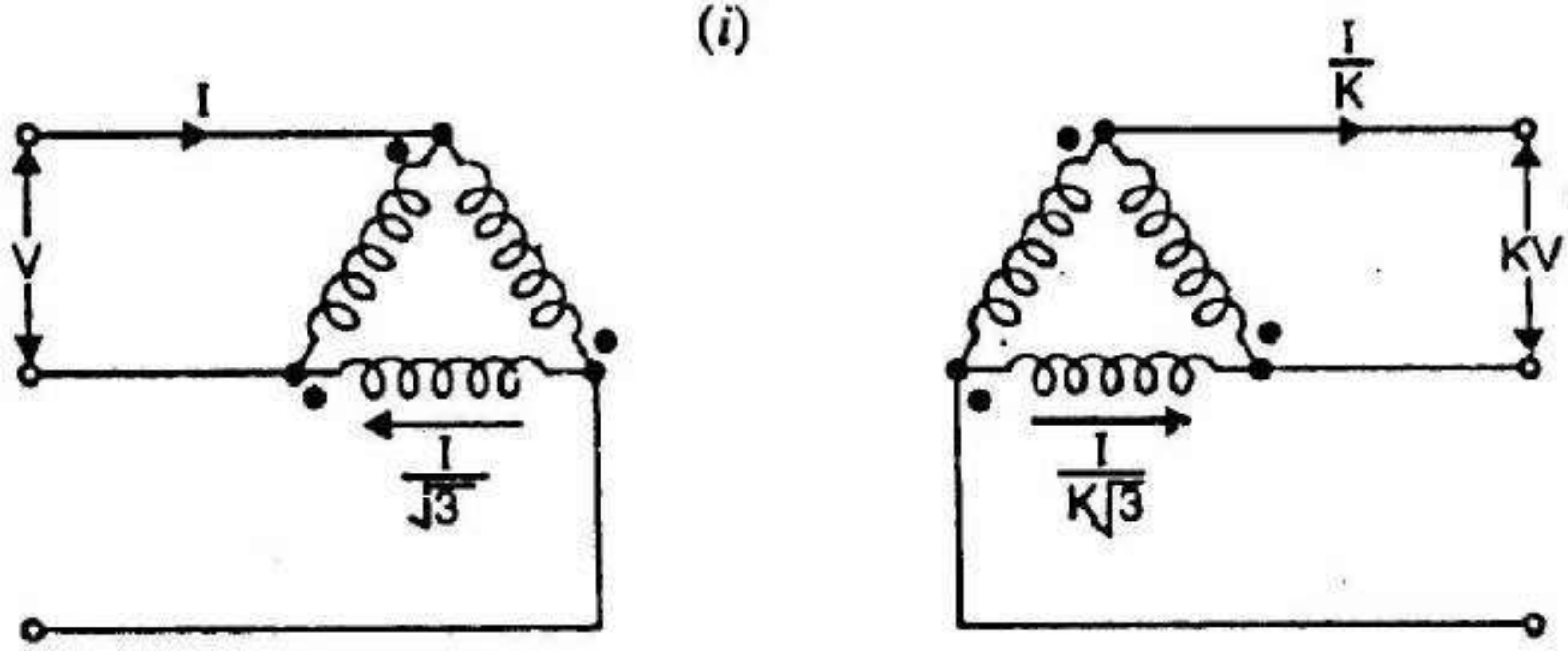
Disadvantages

1.If the load on the secondary side unbalanced then the shifting of neutral point is possible

2.The third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages

3. Magnetizing current of transformer has 3rd harmonic component

Delta - Delta connection



Δ – Δ Connection

- This connection is used for moderate voltages

Delta - Delta connection

Advantages

1. System voltages are more stable in relation to unbalanced load
2. If one t/f is failed it may be used for low power level ie V-V connection
3. No distortion of flux ie 3rd harmonic current not flowing to the line wire

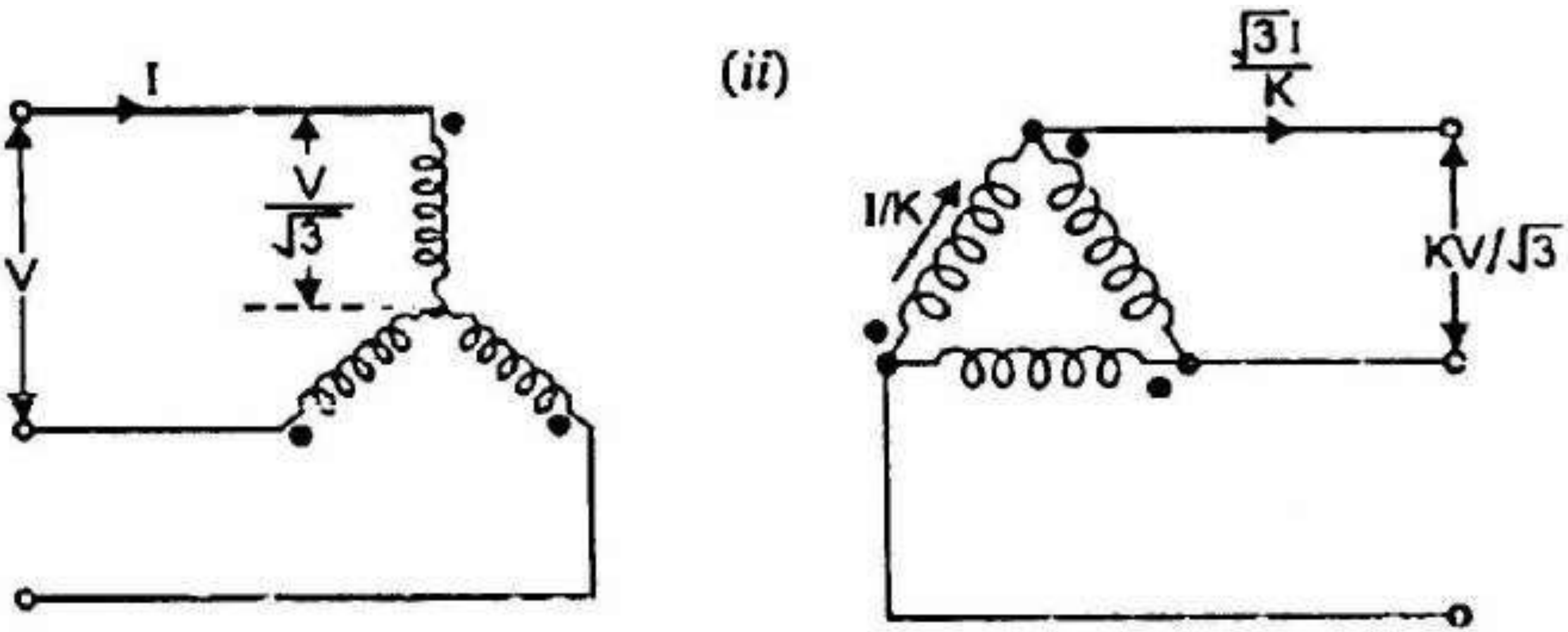
Delta - Delta connection

Disadvantages

1. **Compare to Y-Y require more insulation**

2. **Absence of star point ie fault may severe**

Star- Delta connection



Y – Δ Connection

- Used to step down voltage i.e end of transmission line

Star- Delta connection

Advantages

1. The primary side is star connected. Hence fewer number of turns are required. This makes the connection economical

2. The neutral available on the primary can be earthed to avoid distortion.

3. Large unbalanced loads can be handled satisfactory.

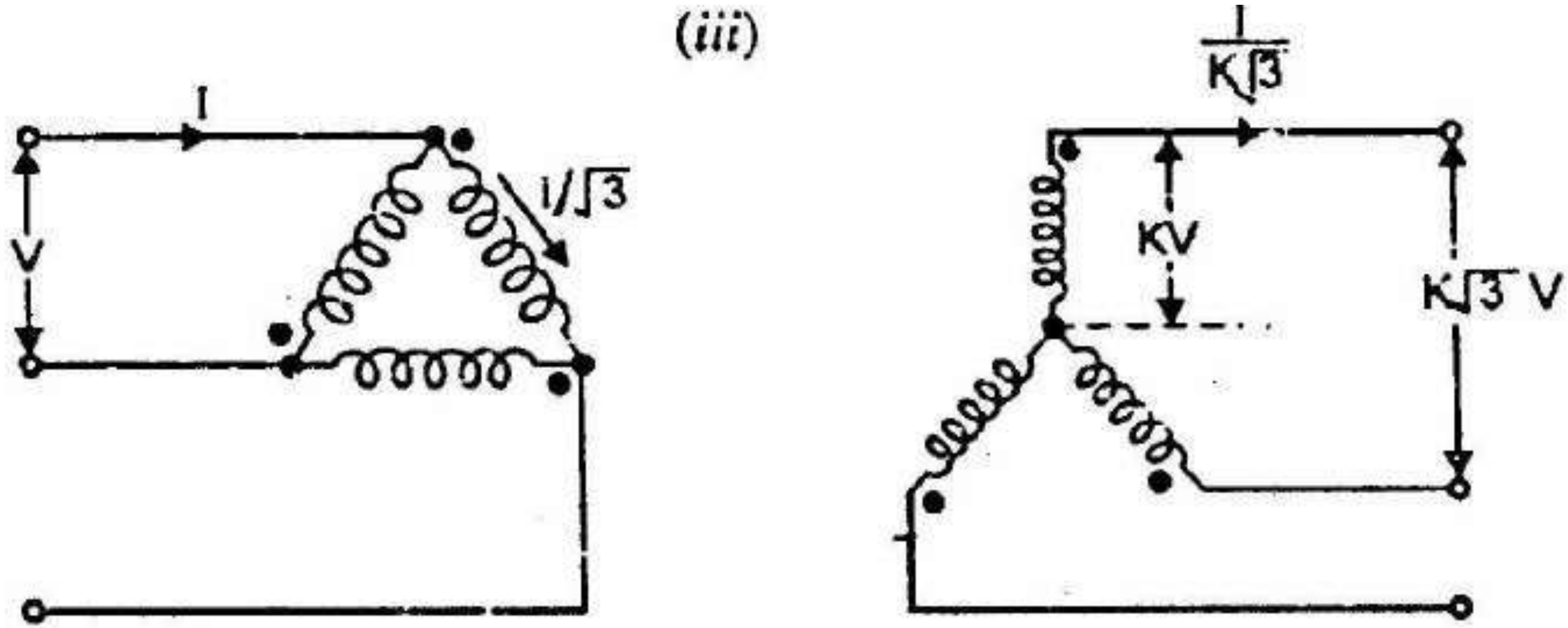
Star- Delta connection

Disadvantages

The secondary voltage is not in phase with the primary. (30 ^{o} phase difference)

Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.

Delta - Star connection



Δ – Y Connection

- **This connection is used to step up voltage ie. Beginning of high tension line**

Delta - Star connection

Features

- secondary Phase voltage is 1/√3 times of line voltage
-
- neutral in secondary can be grounded for 3 phase 4 wire system
- Neutral shifting and 3rd harmonics are there
- Phase shift of 30^{o} between secondary and primary currents and voltages

Tutorial 4

A sinusoidal voltage is given by v= 20 sin ωt volts. (a) At what angle will the instantaneous value of voltage be the 10 V? (b) What is the maximum value of the voltage and at what angle?

Solution

(*a*) The angle can be determined by using the given equation,

$$10 = 20 \sin \omega t \quad \Rightarrow \quad \theta = \omega t = \sin^{-1}(10/20) = 30°$$

(*b*) The maximum value is $V_m = 20$ **V**. This occurs twice in one cycle when $\sin \omega t = \pm 1$, that is at the instants when $\omega t = 90°$ or $270°$.

Example 2

An alternating current of frequency 60 Hz has a maximum value of 12A (a) Write down the equation for its instantaneous value. (b) Calculate the value of current after 1/360 seconds. (c) Find the time taken to reach 9.6A for the first time.

Solution

(*a*) The angular frequency, $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s. Therefore, the equation for the instantaneous value is given as

$$i = 12 \sin 377t \text{ A} \qquad (i)$$

(*b*) The value of current at $t = 1/360$ second is

$$i = 12 \sin\left(377 \times \frac{1}{360} \times \frac{180^\circ}{\pi}\right) \text{A} = \mathbf{10.4\ A}$$

(*c*) Putting $i = 9.6$ A in Eq. (*i*), we have

$$9.6 = 12 \sin\left(377t \times \frac{180^\circ}{\pi}\right) \Rightarrow 377t \times \frac{180^\circ}{\pi} = 53.13^\circ$$

$$\therefore \qquad t = \frac{53.13\pi}{377 \times 180} = \mathbf{2.46\ ms}$$

Example 3

Determine the phase difference of the sinusoidal current i1= 4sin (100Πt + 30degree) Amp with respect to current i2= 6sin (100Πt) Amp. In terms of time and draw the phasor diagram to represent the two phasors.

Solution Comparing the expressions of the given sinusoids with the standard form, we get

$$\omega = 2\pi f = 100\pi \;\Rightarrow\; f = 50\text{ Hz} \;\Rightarrow\; T = \frac{1}{f} = \frac{1}{50} = 0.02\text{ s} = 20\text{ ms}$$

Thus, the time for full revolution (360°) for either of the phasors is 20 ms. The time taken to cover 30° is

$$t = (20\text{ ms}) \times \frac{30^\circ}{360^\circ} = \mathbf{1.67\ ms}$$

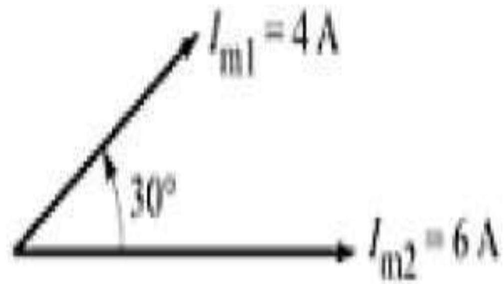


Fig. 9.8 *Phasor diagram.*

The phasor i_1 leads the phasor i_2 by 30° (or 1.67 ms). Taking i_2 as the reference phasor, the phasor i_1 can be drawn 30° leading i_2. The phasor diagram is drawn in Fig. 9.8, in terms of the maximum values of the two sinusoidal currents.

Example 4

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55 \sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1 \sin(\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Solution Here, the phase angle, $\theta = \pi/5$

The rms value of the voltage, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{55}{\sqrt{2}} = 38.89$ V

The rms value of the current, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{6.1}{\sqrt{2}} = 4.31$ A

Therefore, using Eq. 9.36, the average power is given as

$$\begin{aligned} P_{av} &= VI\cos\theta = 38.89 \times 4.31 \times \cos \pi/5 \\ &= 167.62 \times 0.809 = \mathbf{135.6\ W} \end{aligned}$$

The apparent power is

$$P_a = VI = 38.89 \times 4.31 = \mathbf{167.62\ VA}$$

Note that the apparent power is expressed in volt amperes (VA) and not in watts (W), since it is not a power in reality.

The instantaneous power at $\omega t = 0.3$ is given by Eq. 9.35, as

$$\begin{aligned} p &= VI\cos\theta - VI\cos(2\omega t - \theta) \\ &= 135.6 - 167.62 \times \cos(2 \times 0.3 - \pi/5) = \mathbf{-31.95\ VA} \end{aligned}$$

The power factor is given as

$$pf = \cos\theta = \cos \pi/5 = 0.809 = \mathbf{80.9\ \%}$$

Example 5

Determine the average and rms value of current given by i= 10 +5 cos314t Amp.

Solution The given current is seen to be the combination of a dc current of 10 A and a sinusoidal current of peak value 5 A. The average value of the sinusoidal current being zero, the average of the overall current would be the same as the dc current. That is, I_{av} = **10 A.**

The rms value can easily be found by adding the rms values of the two components on square basis,

$$I_{rms} = \sqrt{(10)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{112.5} = \mathbf{10.6\ A}$$

IMPORTANT FORMULAE

- $f = \dfrac{1}{T}$
- Sinusoidal current, $i = I_m \sin \omega t$, where $\omega = 2\pi f$ or $\omega = \dfrac{2\pi}{T}$
- For a sinusoidal ac and for a full-wave-rectified wave,

$$I_{av} = \frac{2I_m}{\pi}, \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}.$$

- Form factor, $K_f = \dfrac{V_{rms}}{V_{av}} = 1.11$ (for sinusoidal waveform)
- Peak factor, $K_p = \dfrac{V_m}{V_{rms}} = 1.414$ (for sinusoidal waveform)

Property	*Resistance*	*Inductance*	*Capacitance*
Current	V/R	V/X_L	V/X_C
Frequency dependency	Independent	$X_L \propto f$	$X_C \propto (1/f)$
Power	$VI = I^2R = V^2/R$	Zero	Zero
Phase difference	0°	90° lagging	90° leading
Reactance	R	$jX_L = j\omega L = j2\pi fL$	$-jX_C = 1/j\omega C = 1/j2\pi fC$

Unit 2 Problems

Tutorial 5

Example 1

An ac voltage is mathematically expressed as $v = 141.42\sin(157.08t + \Pi/2)$ volts. Find its (a) effective value (b) frequency (c) periodic time.

An ac voltage is mathematically expressed as

$$v = 141.42 \sin (157.08t + \pi/2) \text{ Volts}$$

$$v = V_m \sin (\omega t + \theta)$$

Effective value $= \dfrac{V_m}{\sqrt{2}} = \dfrac{141.42}{\sqrt{2}} = 100V$

$$\omega = 157.08$$

$$2\pi f = 157.08$$

$$f = \frac{157.08}{2\pi} = \underline{25 HZ}$$

$$T = \frac{1}{f} = \frac{1}{25} = \underline{0.04 \text{ sec}}$$

Example 2 Polar Notation Problem

- An AC current denoted by a phasor in complex plane as I= 4+j3 Amp. Is flowing through a resistor of 10 ohm . Determine the power consumed by the resistor.

$$r = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{b}{a}$$

Solution

Solution Let us first express the current **I** in the polar form,

$$\mathbf{I} = I_r + jI_i = 4 + j3 = \sqrt{4^2 + 3^2}\angle\tan^{-1}(3/4) = 5\angle 36.87° \text{ A}$$

Thus, we find that the magnitude (the rms value) of the given current is 5 A. Therefore, the power consumed is

$$P = I^2R = 5^2 \times 10 = \mathbf{250\ W}$$

Problem on Rectangular and Polar calculations

Two phasors **A** and **B** are given as **A** = 3 + j1, and **B** = 4 + j3. Calculate the values of (a) **A** + **B**; (b) **A** – **B**; (c) **AB**; (d) **A**/**B**. Express the results in both polar and rectangular coordinates.

[**Ans.** (a) $7 + j4 = 8.06\angle 29.7°$;
(b) $-1 - j2 = 2.24\angle -116.57°$;
(c) $15.8\angle 55.3° = 8.99 + j12.99$;
(d) $0.632\angle -18.44° = 0.6 - j0.02$]

Addition subtraction and Multiplication

$A = 3 + j1 \qquad B = 4 + j3$

a) $A + B = 3 + j1 + 4 + j3 = 7 + j4 = 8.06\angle 29.7^\circ$

b) $A - B = 3 + j1 - 4 - j3 = -1 - j2 = 2.24\angle -116.57^\circ$

c) $AB = (3 + j1)(4 + j3) = 15.8\angle 55.3^\circ = 8.99 + j12.99$

d) $\frac{A}{B} = \frac{3 + j1}{4 + j3} = 0.632\angle -18.44^\circ = 0.6 - j0.02$

Addition subtraction and Multiplication

Addition, Subtraction and Multiplication For these operations, just use ordinary algebra plus two more rules: (1) keep real and imaginary parts separate, and (2) treat j^2 as -1. For example, whenever we add complex numbers, we add the real parts and the imaginary parts separately:

$$\mathbf{z}_1 + \mathbf{z}_2 = (3 + j4) + (-7 - j3) = (3 - 7) + j(4 - 3) = -4 + j1$$

and similarly for subtraction. Thus, complex numbers are added and subtracted like vectors in a plane. This is one of the few properties common between complex numbers and vectors.

Similarly, multiplication of $\mathbf{z}_1$ and $\mathbf{z}_2$ is

$$\begin{aligned}\mathbf{z}_1\mathbf{z}_2 &= (3 + j4)(-7 - j3) = 3(-7) + j4(-j3) + 3(-j3) + j4(-7)\\ &= -21 - j^2 12 - j9 - j28 = -21 + 12 - j9 - j28 = (-21 + 12) - j(9 + 28)\\ &= -9 - j37\end{aligned}$$

Division

Division and Conjugation Division requires a trick to get the results in standard form:

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{3+j4}{-7-j3} = \frac{3+j4}{-7-j3} \times \frac{-7+j3}{-7+j3}$$

$$= \frac{(3)(-7)+(j4)(j3)+(3)(j3)+(j4)(-7)}{(-7)^2-(j3)^2} = \frac{-21-12+j9-j28}{49+9}$$

$$= \frac{-33-j19}{58} = -\frac{33}{58} - j\frac{19}{58}$$

Problem on Representation of sin wave equations

Obtain the sum of the three voltages,

$$v_1 = 147.3\cos(\omega t + 98.1°)\ \text{V}, \quad v_2 = 294.6\cos(\omega t - 45°)\ \text{V} \quad \text{and} \quad v_3 = 88.4\sin(\omega t + 135°)\ \text{V}$$

Solution We plot the above phasors in complex plane, in terms of their peak values. First, we write the voltages in terms of sine functions. Since, $\sin(90° + \theta) = \cos\theta$, we can write

$$v_1 = 147.3\sin(90° + \omega t + 98.1°)\ \text{V} = 147.3\sin(\omega t + 188.1°)\ \text{V}$$

$$v_2 = 294.6\sin(90° + \omega t - 45°)\ \text{V} = 294.6\sin(\omega t + 45°)\ \text{V}$$

and

$$v_3 = 88.4\sin(\omega t + 135°)\ \text{V}$$

Problem on XL and XC calculation

(a) What reactance will be offered (i) by an inductor of 0.2 H, (ii) by a capacitance of 10 μF, to an ac voltage source of 10V, 100 Hz? (b) What, if the frequency of the source is changed to 140 Hz?

Solution

(*a*) (*i*) $X_L = 2\pi fL = 2\pi \times 100 \times 0.2 = \mathbf{125.66\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 100 \times 10 \times 10^{-6}} = \mathbf{159.15\ \Omega}$

(*b*) (*i*) $X_L = 2\pi fL = 2\pi \times 140 \times 0.2 = \mathbf{175.9\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 140 \times 10 \times 10^{-6}} = \mathbf{113.7\ \Omega}$

Problem on Resonance Frequency

A coil having resistance & inductance of 5Ω & $32mH$, respectively is connected in series with a 796-pF capacitor. Determine resonance frequency of the circuit

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

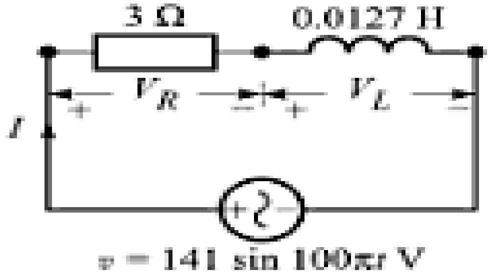
$$= \frac{1}{2\pi\sqrt{32\times10^{-3}\times796\times10^{-12}}}$$

$$= 31.53\ KHz$$

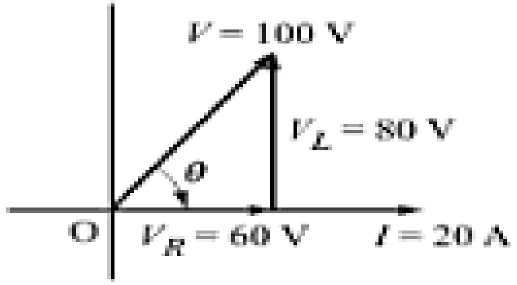
Problem on Series RL circuit

For the series *RL* circuit shown in Fig. 10.2*a*,

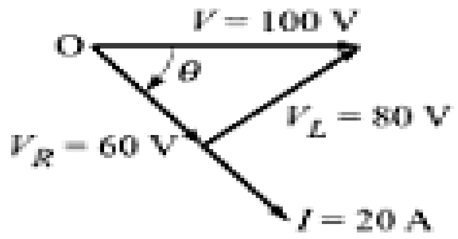
(*a*) Calculate the rms value of the steady state current and the relative phase angle.
(*b*) Write the expression for the instantaneous current.
(*c*) Find the average power dissipated in the circuit.
(*d*) Determine the power factor.
(*e*) Draw the phasor diagram.



(*a*) The circuit.



(*b*) Phasor diagram.



(*c*) Phasor diagram redrawn.

Fig. 10.2 *A series RL circuit.*

$$\mathbf{V} = V\angle 0^\circ = \frac{V_m}{\sqrt{2}}\angle 0^\circ = \frac{141}{\sqrt{2}}\angle 0^\circ = 100\angle 0^\circ = 100 + j0 \text{ volts}$$

The impedance, $\mathbf{Z} = R + j\omega L = 3 + j100\pi \times 0.0127 = 3 + j4 = 5\angle 53.1^\circ$ ohms

$\therefore$ Current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{100\angle 0^\circ}{5\angle 53.1^\circ} = \mathbf{20\angle{-53.1^\circ}\ A}$

Thus, the rms value of the steady state current is 20 A, and the phase angle is 53.1° lagging.

(*b*) The expression for the instantaneous current can be written as

$$i = 20\sqrt{2}\sin(100\pi t - 53.1^\circ) = \mathbf{28.28\sin(100\pi t - 53.1^\circ)\ A}$$

(*c*) Average power, $P = VI\cos\theta = 100 \times 20 \times \cos 53.1^\circ =$ **1200 W**

Or, $$P = I^2R = (20)^2\, 3 = \mathbf{1200\ W}$$

(*d*) $pf = \cos\theta = \cos 53.1^\circ =$ **0.6 lagging**. Alternatively,

$$pf = \frac{\text{Average power}}{\text{Apparent power}} = \frac{P}{VI} = \frac{1200}{100 \times 20} = \mathbf{0.6\ lagging}$$

(*e*) Taking the current as reference, the phasor diagram is drawn in Fig. 10.2*b*, where

$$I = 20 \text{ A}; \quad V_R = IR = 20 \times 3 = 60 \text{ V}; \quad V_L = IX_L = 20 \times 4 = 80 \text{ V} \quad \text{and} \quad V = 100 \text{ V}$$

The same phasor diagram is redrawn in Fig. 10.2*c*, by rotating it clockwise by an angle 53.1°, so that the applied voltage becomes the reference phasor.

Problem on Power and RC circuit

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Solution Taking current as the reference phasor, $\mathbf{I} = 0.9\angle 0°$ A.

Impedance,	$\mathbf{Z} = 120 - j250 = \mathbf{277.3\angle -64.4°\ \Omega}$
Power factor,	$pf = \cos\theta = \cos(-64.4°) = \mathbf{0.432\ leading}$
Supply voltage,	$\mathbf{V} = \mathbf{IZ} = (0.9\angle 0°)(277.3\angle -64.4°) = \mathbf{249.6\angle -64.4°\ V}$
Voltage across resistor,	$V_R = IR = (0.9\angle 0°) \times 120 = \mathbf{108\angle 0°\ V}$
Voltage across capacitor,	$V_C = IX_C = (0.9\angle 0°)(250\angle -90°) = \mathbf{225\angle -90°\ V}$
Apparent power,	$P_{app} = VI = 249.6 \times 0.9 = \mathbf{224.6\ VA}$
Actual power,	$P_a = VI\cos\theta = 249.6 \times 0.9 \times \cos 64.4° = \mathbf{97.06\ W}$
Reactive power,	$P_r = VI\sin\theta = 249.6 \times 0.9 \times \sin 64.4° = \mathbf{202.58\ VAR}$

Tutorial 6

Example

When a two element series circuit is connected across an ac source of frequency 50 Hz, it offers an impedance Z = (10+j10) ohm. Find the values of two elements?

Solution

The nature of the impedance indicates that the circuit is inductive.

$$\mathbf{Z} = R + jX_L = (10 + j10)\ \Omega$$

$$R = 10\ \Omega \quad \text{and} \quad X_L = 10\ \Omega \Rightarrow L = \frac{X_L}{2\pi f} = \frac{10}{2\pi \times 50} = 0.0318\ \text{H} = \mathbf{31.8\ mH}$$

Example

When a two element parallel circuit is connected across an ac source of frequency 50 Hz, it offers an impedance Z = (10-j10) ohm. Find the values of two elements?

$$\mathbf{Y} = G + jB = (\text{conductance}) + j(\text{susceptance})$$

$$\mathbf{Y} = (G + jB) = \left(\frac{1}{R} + j\omega C\right)$$

Solution

Solution The nature of the impedance indicates that the circuit is capacitive. Since the circuit has two elements connected in parallel, we find the admittance of the circuit.

$$\mathbf{Y} = (G + jB) = \frac{1}{\mathbf{Z}} = \frac{1}{10 - j10} = \frac{1}{14.14\angle -45^\circ} = 0.0707\angle 45^\circ \text{ S} = (0.05 + j0.05) \text{ S}$$

$$\therefore \quad G = 0.05 \text{ S} \Rightarrow R = \frac{1}{G} = \frac{1}{0.05} = 20\ \Omega \quad \text{and} \quad B = 0.05 \text{ S} \Rightarrow C = \frac{B}{\omega} = \frac{0.05}{2\pi f} = 159\ \mu\text{F}$$

Example

- A circuit consists of a resistance R in series with a capacitive reactance of 60 ohm. Determine the value of R for which the power factor of the circuit is 0.8.

Solution

Solution The power factor, $\cos\theta = 0.8$. Therefore, we get

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(0.8)^2} = 0.6 \quad \Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = \frac{3}{4}$$

From the impedance triangle, we know that

$$\tan\theta = \frac{X_C}{R} \quad \Rightarrow \quad \frac{3}{4} = \frac{X_C}{R} \quad \text{or} \quad \frac{3}{4} = \frac{60}{R} \quad \Rightarrow \quad R = \mathbf{80\ \Omega}$$

Example

A series RLC circuit has R= 12 ohm, L = 0.15 H and C = 100 μF. It is connected to an ac source of voltage 100 V, whose frequency can be varied. Determine (a) the resonant frequency of the source at which the current supplied by it is maximum, (b) the value of this current.

Solution

Solution

(*a*) The current becomes maximum at the resonant frequency of the circuit,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.15 \times 100 \times 10^{-6}}} = \mathbf{41.09\ Hz}$$

(*b*) The maximum current supplied by the source is

$$I_0 = \frac{V}{R} = \frac{100}{12} = \mathbf{8.3\ A}$$

Example

- An iron choke takes 4 Amp current when connected to a 20 V dc supply. When connected to 65 V, 50 Hz ac supply, it takes 5 Amp. Current. Calculate (a) resistance and inductance of the coil (b) the power factor (c) the power drawn by the coil.

Solution

Solution

(*a*) The iron choke has both resistance R and inductance L. When connected to a dc supply, only its resistance is effective in limiting the current; when connected to an ac supply, its impedance becomes effective in limiting the current. Hence,

$$R = \frac{20\text{ V}}{4\text{ A}} = 5\ \Omega \quad \text{and} \quad Z = \frac{65\text{ V}}{5\text{ A}} = 13\ \Omega \Rightarrow X_L = \sqrt{Z^2 - R^2} = 12\ \Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{12}{2\pi \times 50} = 0.0382\text{ H} = \mathbf{38.2\ mH}$$

(*b*) The power factor, $pf = \cos\theta = \frac{R}{Z} = \frac{5}{13} = \mathbf{0.38}$ **(lagging)**

(*c*) The power, $P = VI\cos\theta = 65 \times 5 \times 0.38 = \mathbf{123.5\ W}$

Problem on Power and RC circuit

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

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Reactive power,	$P_r = VI\sin\theta = 249.6 \times 0.9 \times \sin 64.4° = \mathbf{202.58\ VAR}$