# HASHING

# Let’s consider an Example

Suppose, a company with 250 employees, assign a 5-digit employee number to each employee which is used as primary key in company’s employee file.

- We can use employee number as a address of record in memory.
- The search will require no comparisons at all.
- Unfortunately, this technique will require space for 1,00,000 memory locations, where as fewer locations would actually be used.
- So, this trade off for time is not worth the expense.

# What is the SOLUTION to this problem? HASHING!!

- Idea of **using a key to determine address of a record**, but it must be modified so that large amount of memory space is not wasted.
- This modification takes the form of a function H from the set of keys, K into set of memory address, L.

H: K⟶L , Is called a *Hash Function*

# But there's a Problem!!
# COLLISION

- It may be possible that *function H may not yield distinct values* .
- That means, it is possible that two different keys **k1** and **k2** may yield same hash address.
- This situation is called *Collision*.
- Some method must be used to resolve it!

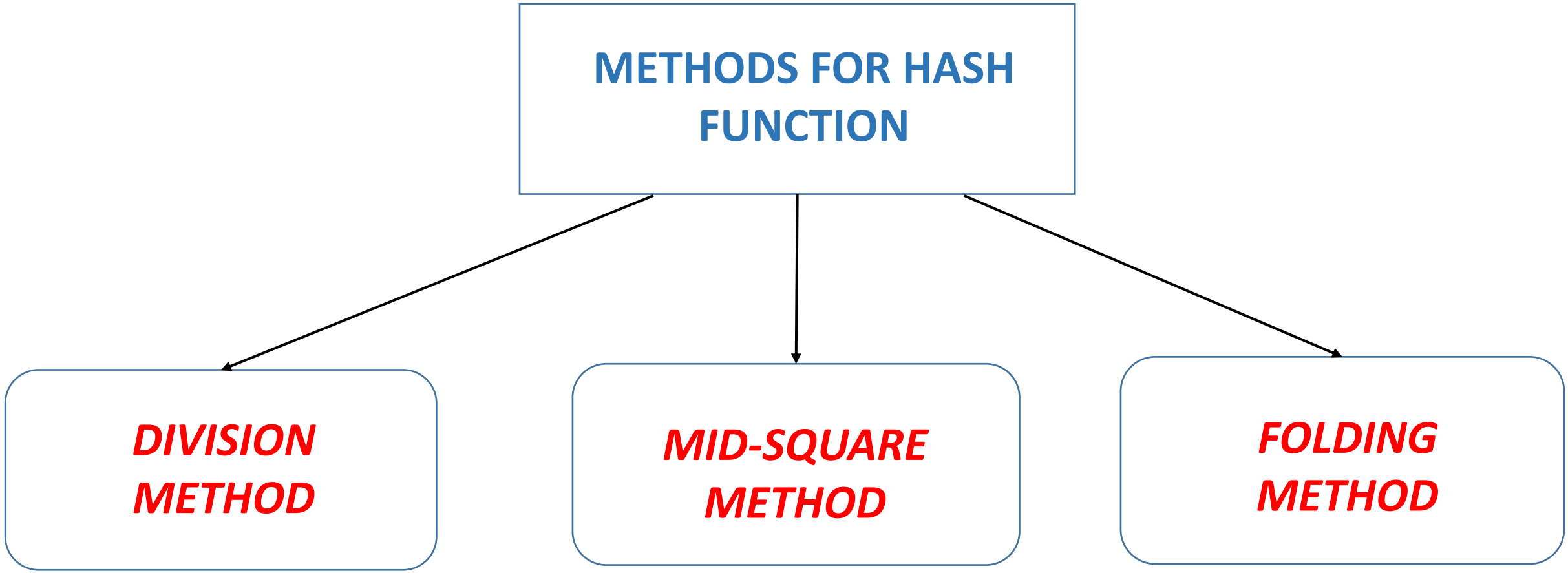
# Hash Function

Principal criteria used in selecting a hash function H: K⟶L are :

1) Function ‘H’ should be easy and quick to compute.

2) The function ‘H’ should as far as possible, i.e. uniformly distribute the hash address through out the set L so that there are minimum number of collision.

# Types of Hash Function



# 1. Division Method

- Choose a number *m* larger than number of keys in k, *n*
- Number *m* is either a prime number or a number without small divisor, since it minimizes number of collisions.
- Hash function H is defined by :

**H(k) = k(mod m)** or

**H(k) = k(mod m) + 1**

where *k(mod m)* denotes the reminder when k is divided by m.

The second formula is used when we want a hash address to range from 1 to m rather than 0 to m-1.

# Example

- Consider a company, with 68 employees, assigns a 4-digit employee number to each employee. Suppose L consists of 100 two-digit address: 00, 01, 02 , …….99.
- Consider Employee numbers: **3205, 7148, 2345**
- Choose a prime number *m* close to 99, let's take m = 97
- Apply H(k) = k(mod m) to all the above numbers.
- **H(3205)=4** (dividing 3205 by 97 gives remainder of 4)
- So, for 3205 address will be 4+1=5
- Similarly, **H(7148)=67+1** and **H(2345)=17+1** [taking addresses from range 1 to m]

# 2. Mid-Square Method

- The key *k* is squared.
- Then the hash function H is defined by **H(k) = $\ell$ ,** where $\ell$ is obtained by deleting digits from both end of $k^2$.
- **Example:**

| | | | |
|---|---|---|---|
| k = | 3205 | 7148 | 2345 |
| $k^2$ = | 10272025 | 51093904 | 5499025 |
| H(k)= | 72 | 93 | 99 |

# 3. Folding Method

- The key k is partitioned into number of parts k1,k2…kr, where each part, except possible the last, has same number of digits as required address.
- Then parts are added together, ignoring the last carry.
- Hash function, **H(k) = k1 + k2 + k3 + ….. + kr**

# Example

- Chopping the key into two parts  and yields following hash addresses:

i. H(3205)=32+05=37
ii. H(7148)=71+48=19
iii. H(2345)=23+45=68

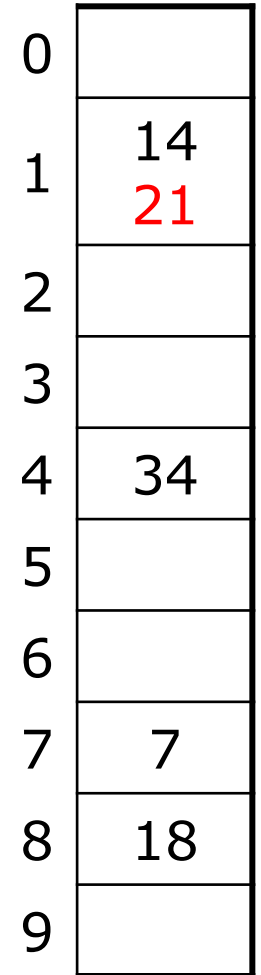
# Another variation

i. H(3205)=32+50=82
ii. H(7148)=71+84=55
iii. H(2345)=23+54=77

# What is Collision?

- A collision occurs when two values in the set hash to the same value.
- It means we want to add new record *r* with key *k* to our file *F*, but memory location address h(k) is already occupied.
- **Collision**: That event when2 hash table elements map into the same slot in the array.
- Example: add 41, 34, 7, 18, then 21
  - 21 hashes into the same slot as 41!
  - 21 should not replace 41 in the hash table; they should both be there

**Collision resolution**: means for fixing collisions in a hash table.



# Solution for Collision (Collision Resolution)

- There are two general ways to resolve collisions :

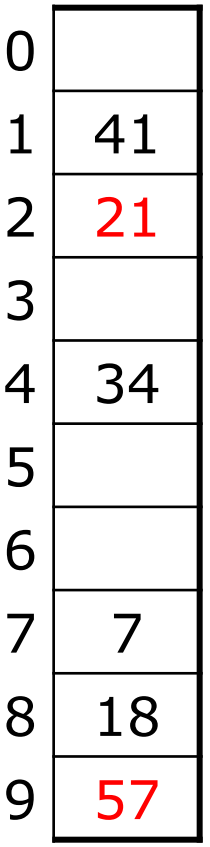
  - Open addressing (array method)
    - ❑*Quadratic Probing*
    - ❑*Double hashing(Re-hashing)*
    - ❑*Linear Probing*

  - Separate Chaining (linked list method)

# Open Addressing: Linear probing and Modifications

**Linear Probing**: resolving collisions in slot *i* by putting the colliding element into the next available slot (i+1, i+2, ...).

- Example: add 41, 34, 7, 18, then 21, then 57
  - 21 collides (41 is already there), so we search ahead until we find empty slot 2
  - 57 collides (7 is already there), so we search ahead twice until we find empty slot 9



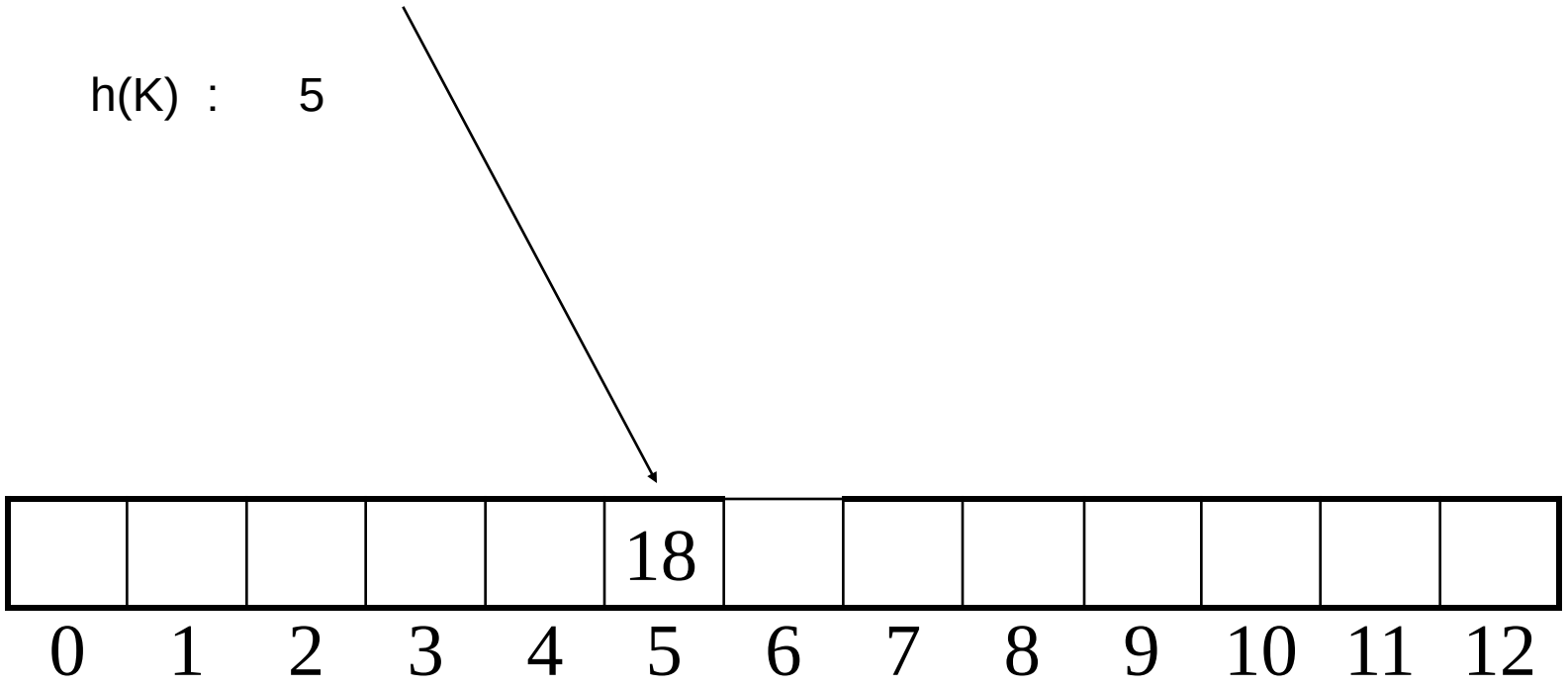

- Lookup algorithm becomes slightly modified; we have to loop now until we find the element or an empty slot.
- What happens when the table gets mostly full?

# Linear Probing

h(K) = K mod 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5

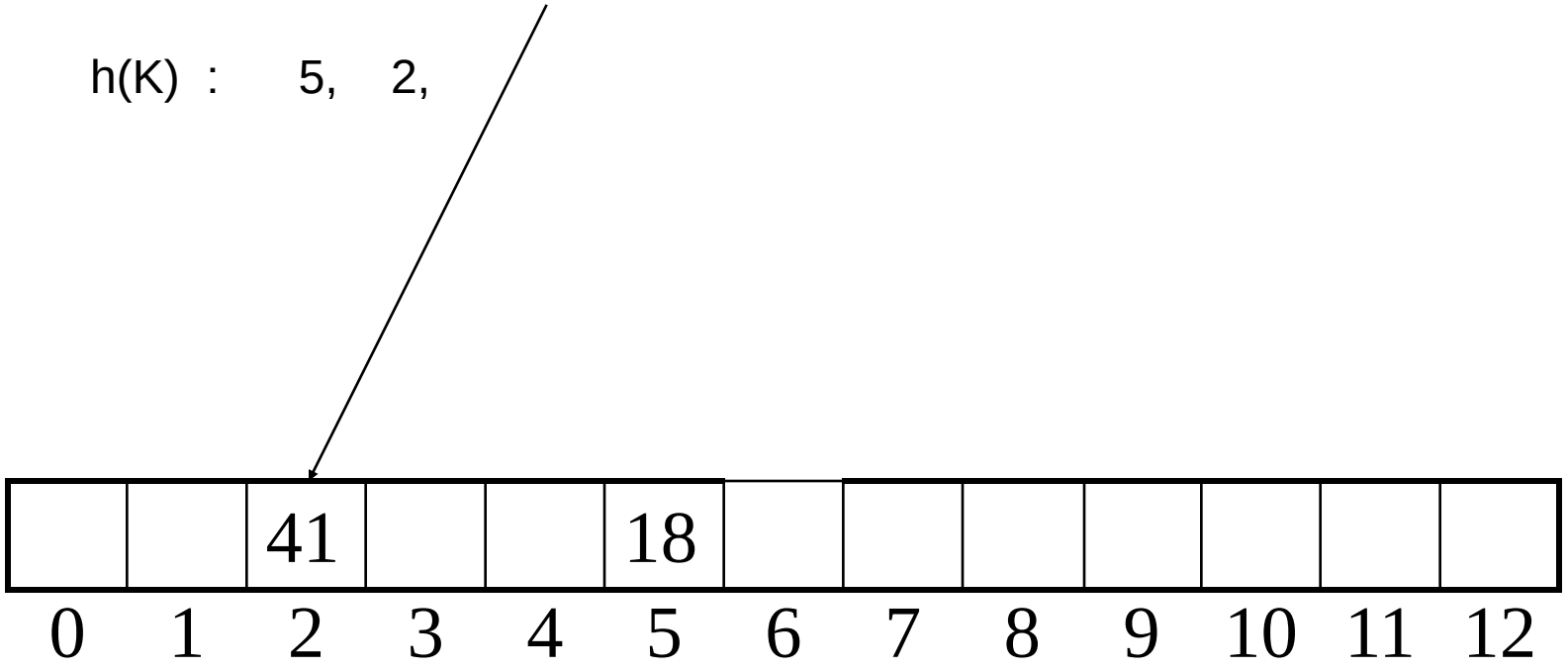


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2,

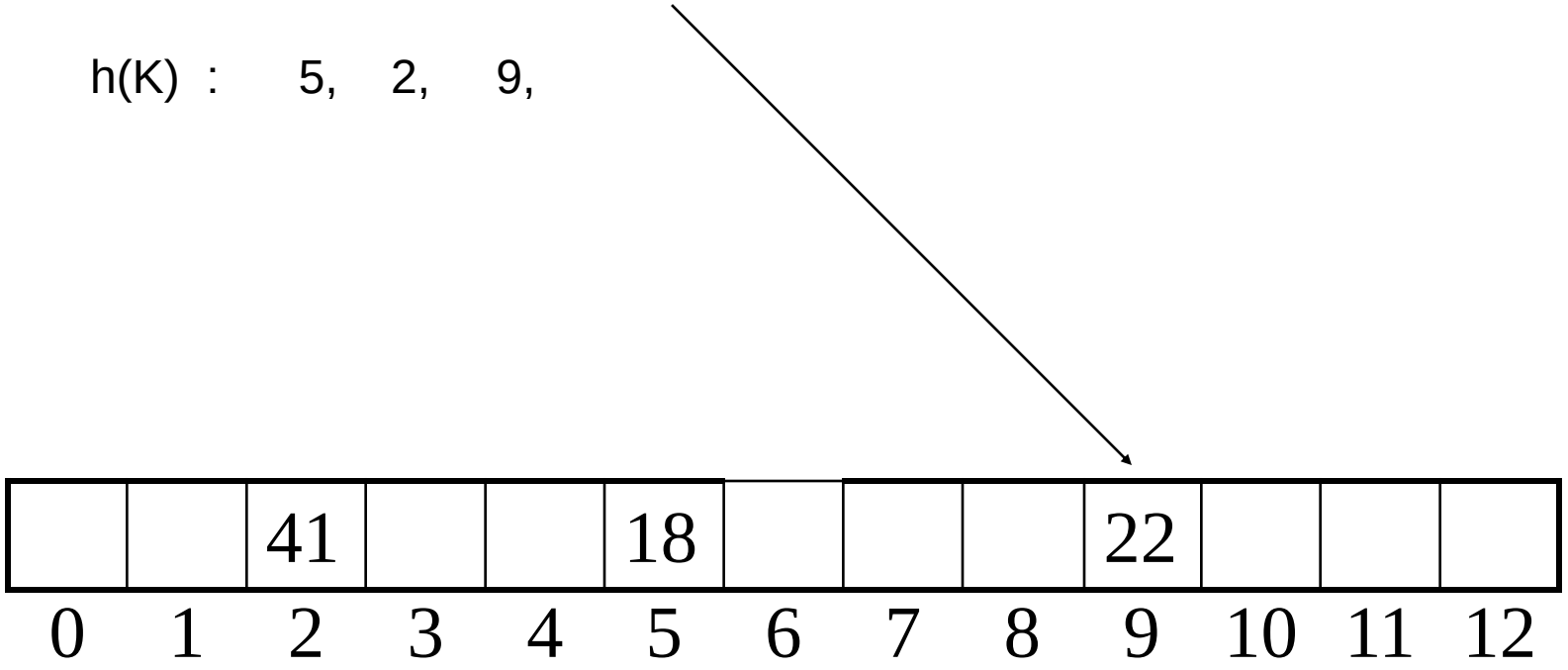


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9,

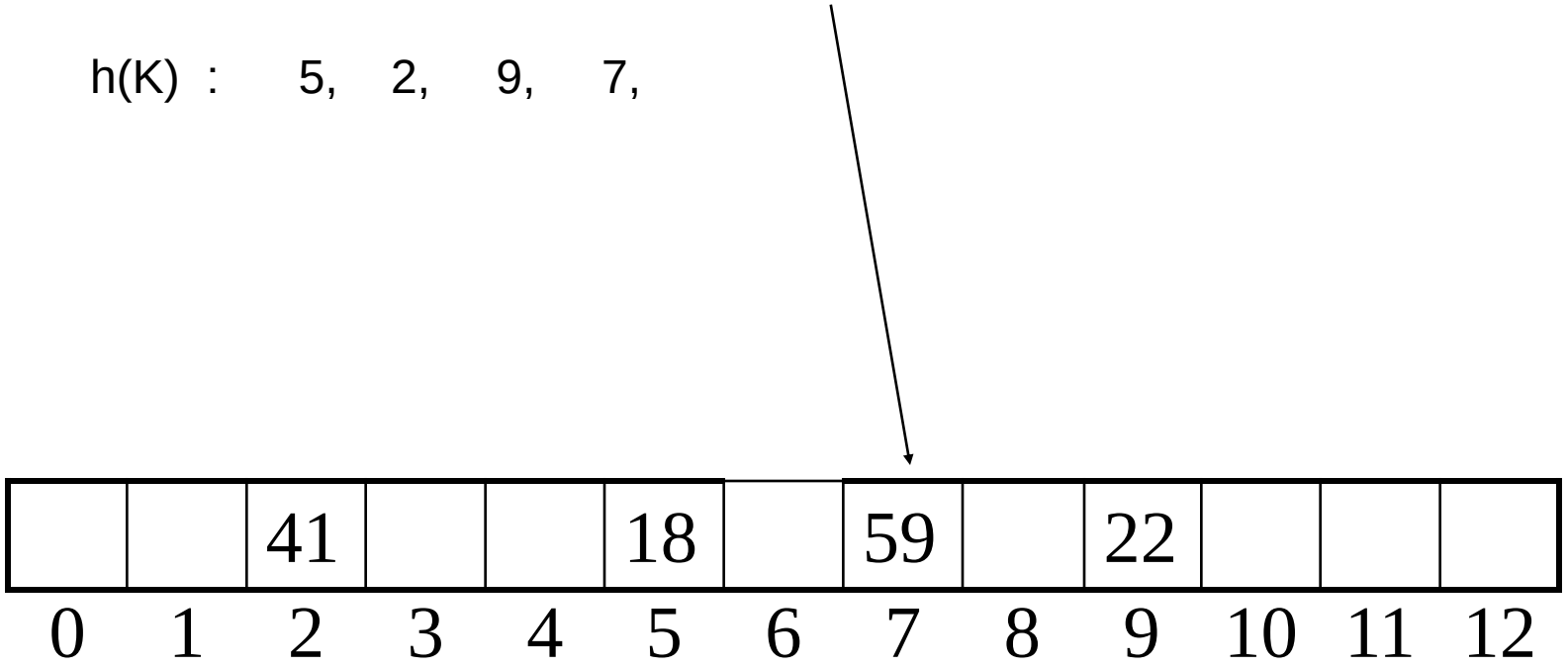


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7,

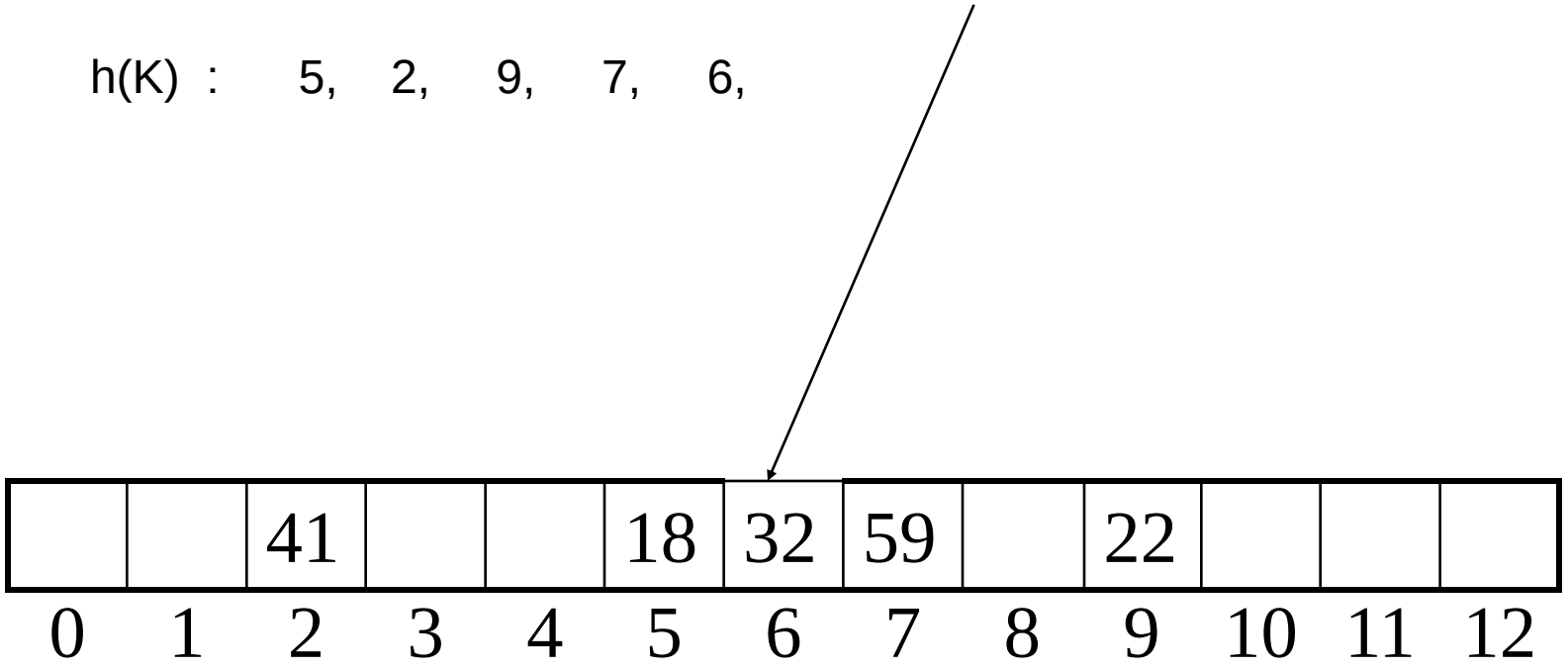


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7, 6,

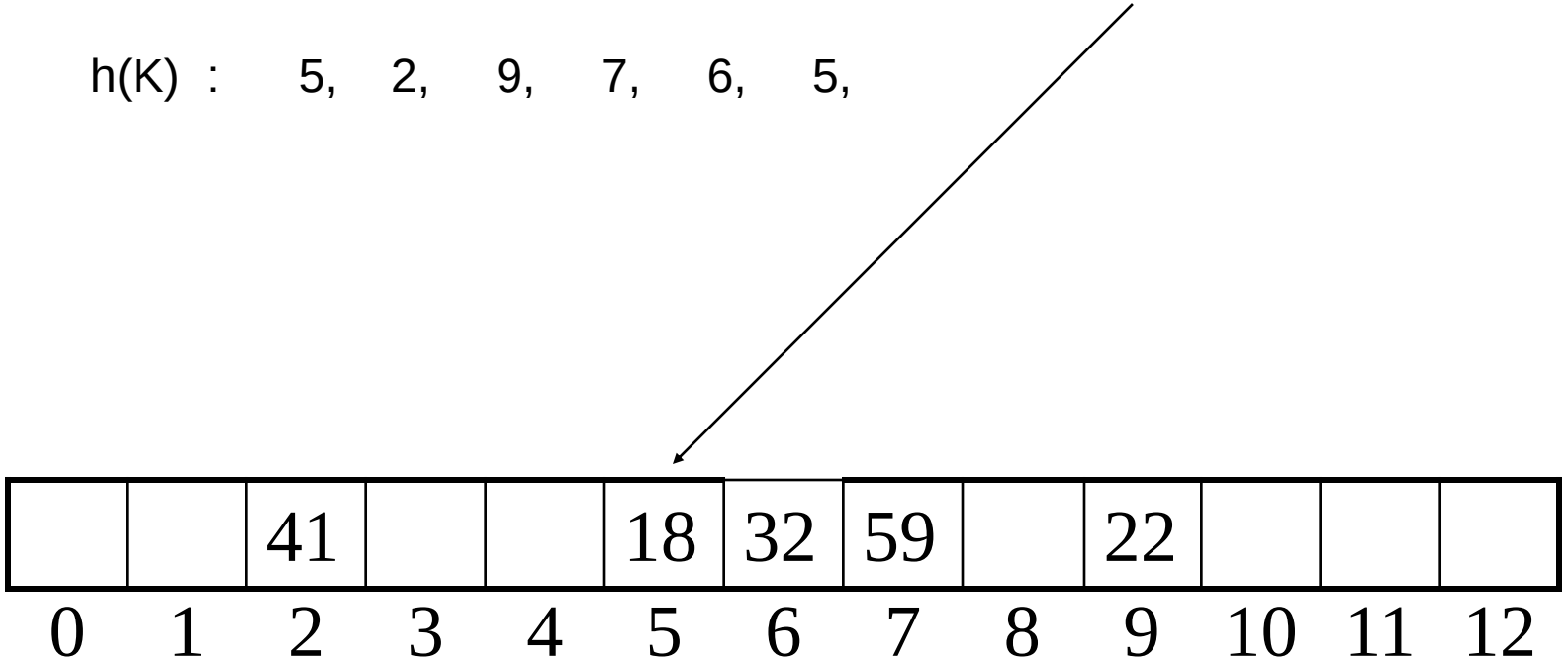


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7, 6, 5,



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h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

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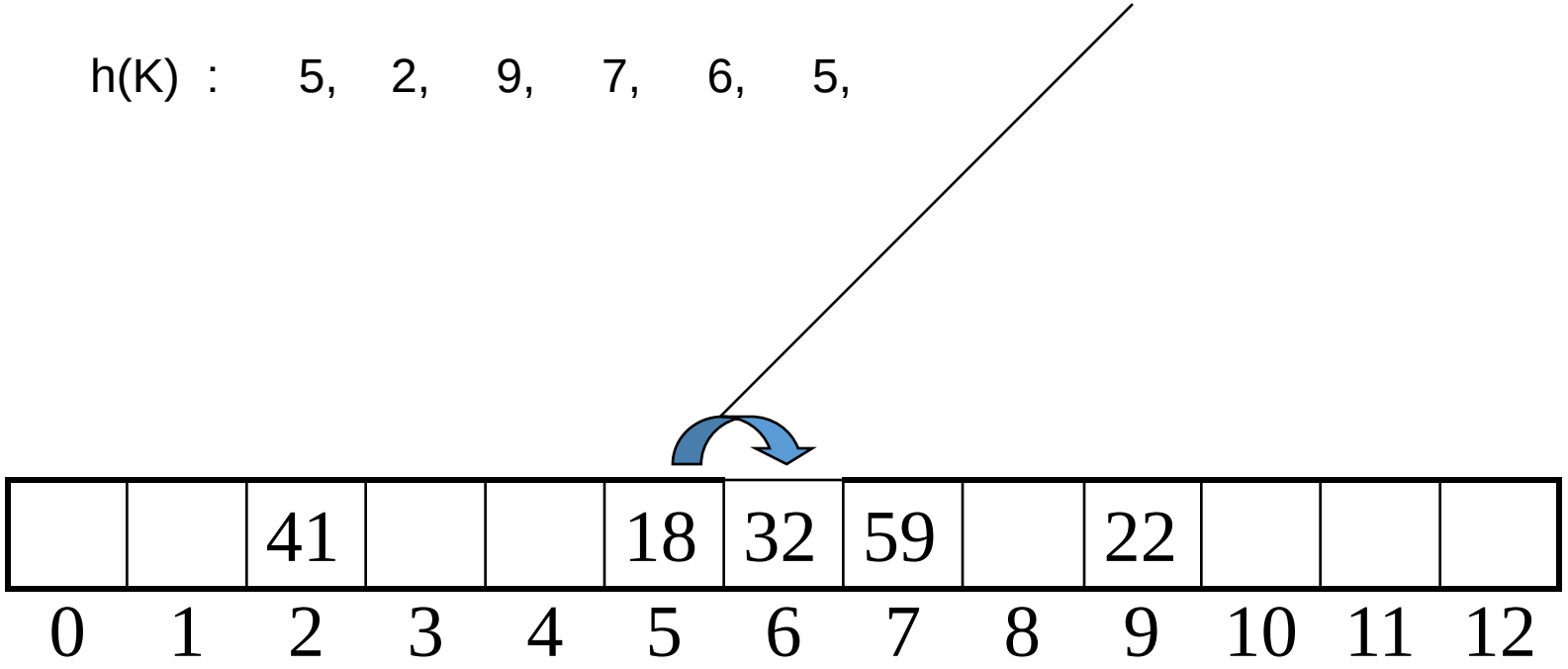

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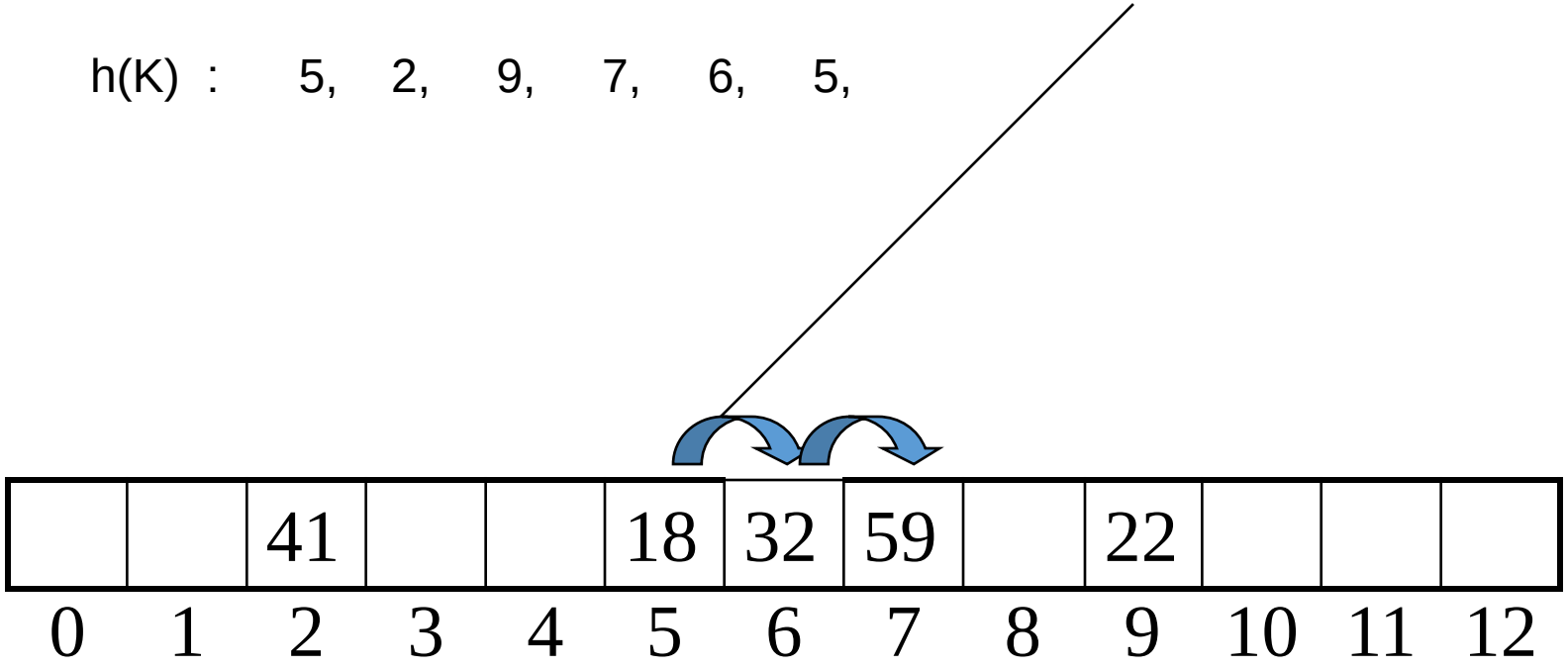
Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7, 6, 5,

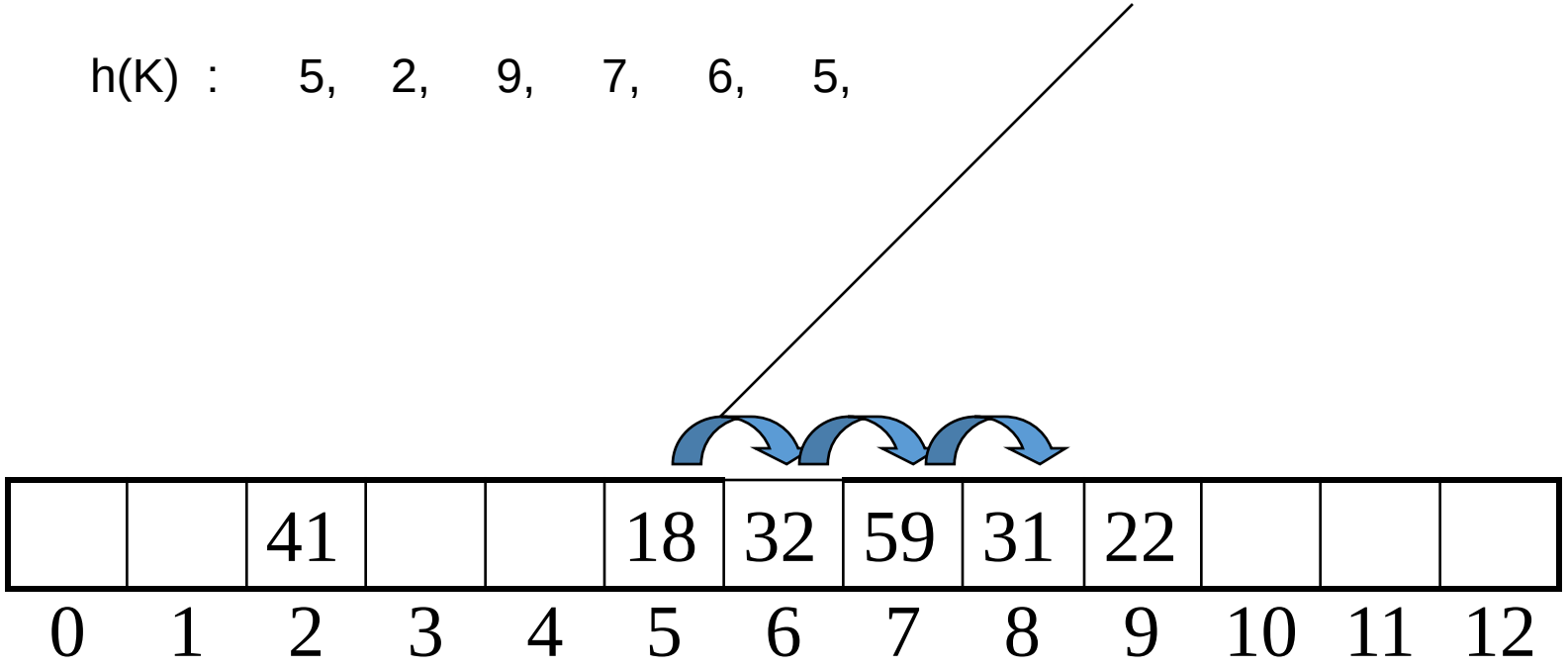


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7, 6, 5,

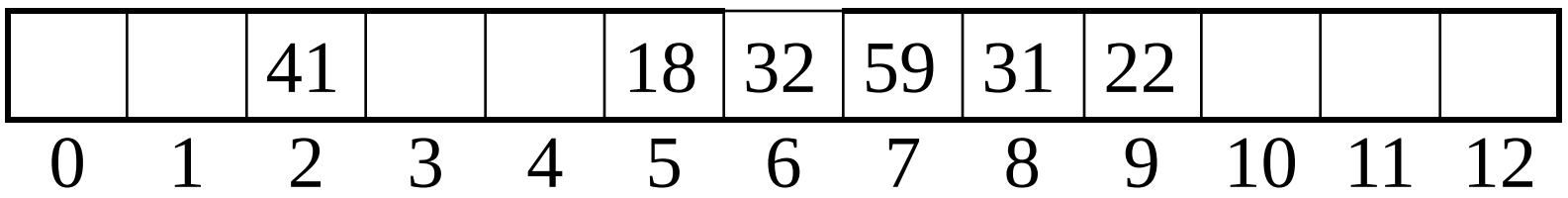


# Linear Probing

h(K) = K % 13

Insert: 18, 41, 22, 59, 32, 31, 73

h(K) : 5, 2, 9, 7, 6, 5, 8



# Linear Probing

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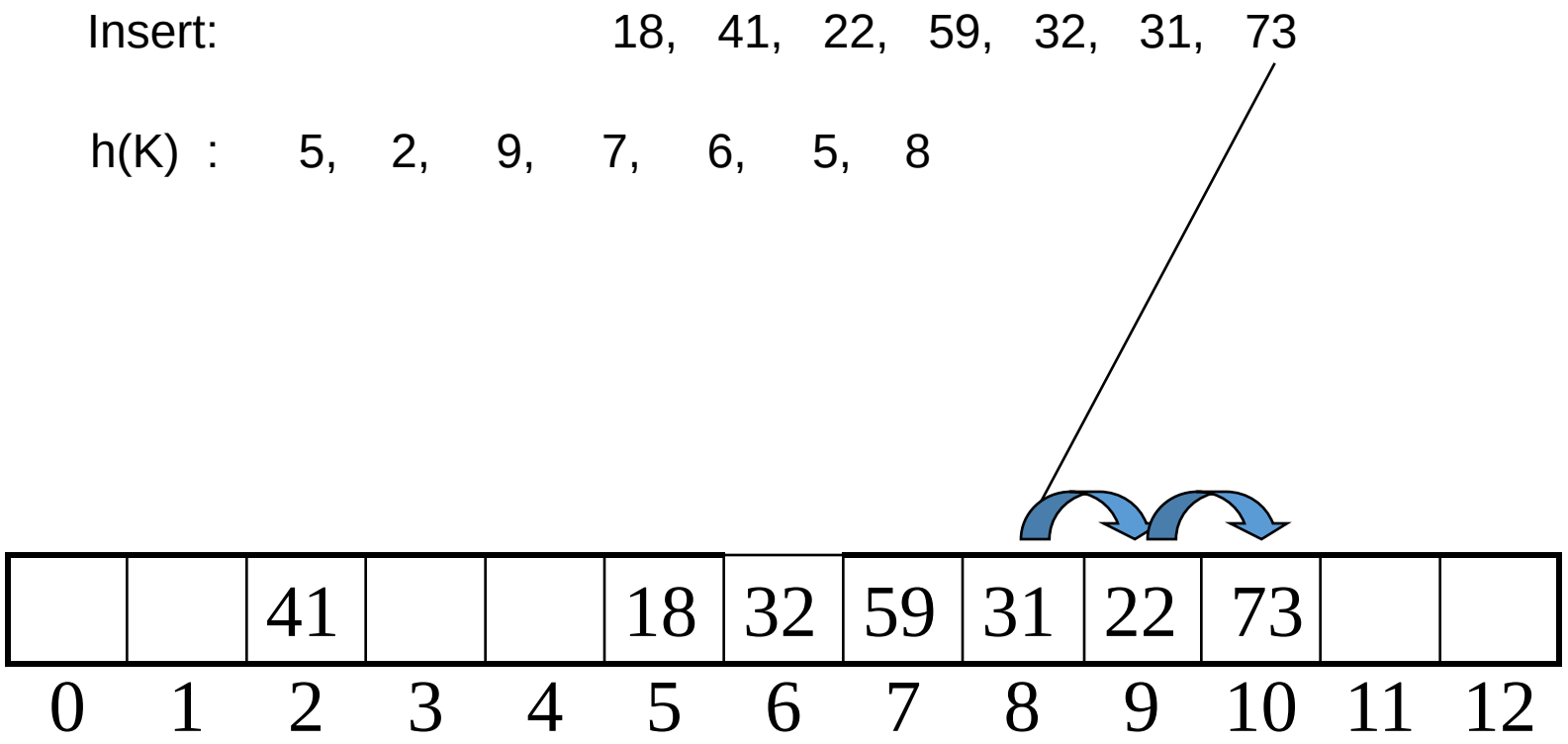
h(K) : 5, 2, 9, 7, 6, 5, 8



# Open Addressing: Quadratic probing & Double Hashing

- Disadvantage of linear probing: Records tend to cluster, i.e. , appear next to one another, when the load factor > 50%.

| | |
|---|---|
| 0 | 49 |
| 1 | |
| 2 | 58 |
| 3 | 79 |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | 18 |
| 9 | 89 |

The two technique that minimize the clustering are :

1. **Quadratic probing**: Suppose the record R with key K has the hash address H(k) = h. Then instead of searching the locations with address h, h+1, h+2, …….. , we search the location with address

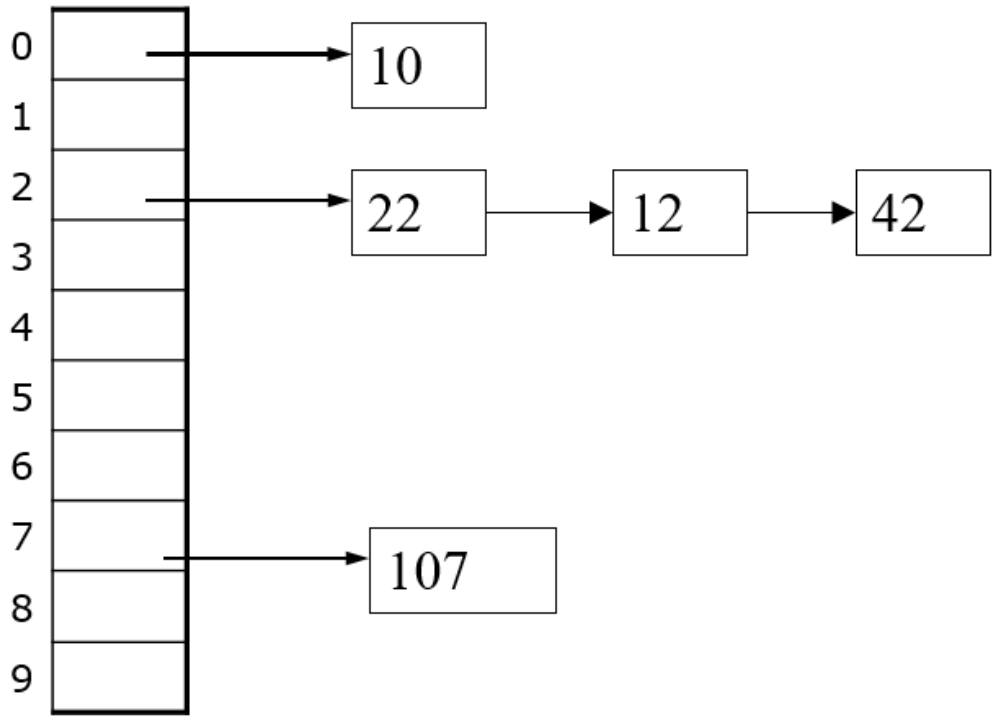
   **h, h+1, h+4, h+9, ……., h+$i^2$.**

2. **Double hashing**: Here the second hash function H' is used for resolving a collision. Suppose a record R with key k has a hash address H(k) = h and H'(k) = h' ≠ m. Then, we linearly search the locations with address

   **h, h+h', h+2h', h+3h', ……., h+nh'**

# Separate Chaining

- It involves maintaining 2 tables in memory.
- First of all, as before, there is a table T in memory which contains the records in F, except that T now has an additional field LINK which is used so that all record in **T** with same hash address **h** may be linked together to form a linked list. Second, there is a hash address table LIST which contain pointers to linked lists in T.
- Suppose a new record R with key k is added to the file F. **We place R in the first available location in the table T and then add R to the linked list with pointer LIST[H(k)].**
- All keys that map to the same hash value are kept in a linked list.



# Difference between Open Addressing and Chaining

| | SEPARATE CHAINING | OPEN ADDRESSING |
|---|---|---|
| 1. | Chaining is Simpler to implement. | Open Addressing requires more computation. |
| 2. | In chaining, Hash table never fills up, we can always add more elements to chain. | In open addressing, table may become full. |
| 3. | Chaining is Less sensitive to the hash function or load factors. | Open addressing requires extra care for to avoid clustering and load factor. |
| 4. | Chaining is mostly used when it is unknown how many and how frequently keys may be inserted or deleted. | Open addressing is used when the frequency and number of keys is known. |
| 5. | Cache performance of chaining is not good as keys are stored using linked list. | Open addressing provides better cache performance as everything is stored in the same table. |
| 6. | Wastage of Space (Some Parts of hash table in chaining are never used). | In Open addressing, a slot can be used even if an input doesn’t map to it. |
| 7. | Chaining uses extra space for links. | No links in Open addressing |