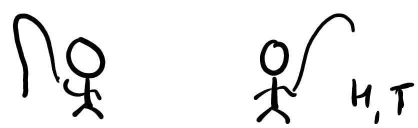
Thursday, January 13, 2022 01:47 PM

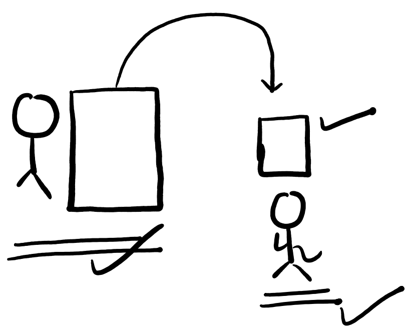
Lecture 3

In statistics and probability theory, independent events are two events wherein the occurrence of one event does not affect the occurrence of another event or events.

The simplest example of such events is tossing two coins.



The outcome of tossing the first coin cannot influence the outcome of tossing the second coin.



Example: Tossing a coin.

Each toss of a coin is a perfect isolated thing.

What it did in the past will not affect the current toss.

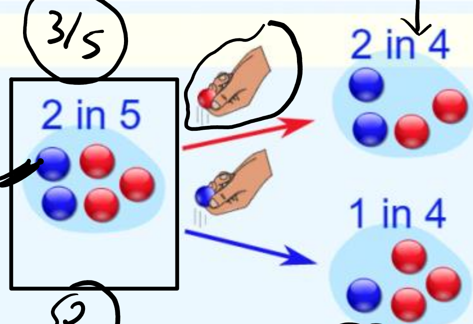
The chance is simply 1-in-2, or 50%, just like ANY toss of the coin.

So each toss is an **Independent Event**.

Red

3/5

Example: Marbles in a Bag



$\frac{2}{4}$

2 blue and 3 red marbles are in a bag.

What are the chances of getting a blue marble?

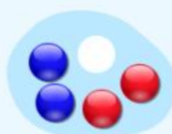
The chance is **2 in 5**

$\frac{2}{5}$

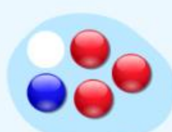
1/4

But after taking one out the chances change!

So the next time:



if we got a **red** marble before, then the chance of a blue marble next is **2 in 4**



if we got a **blue** marble before, then the chance of a blue marble next is **1 in 4**

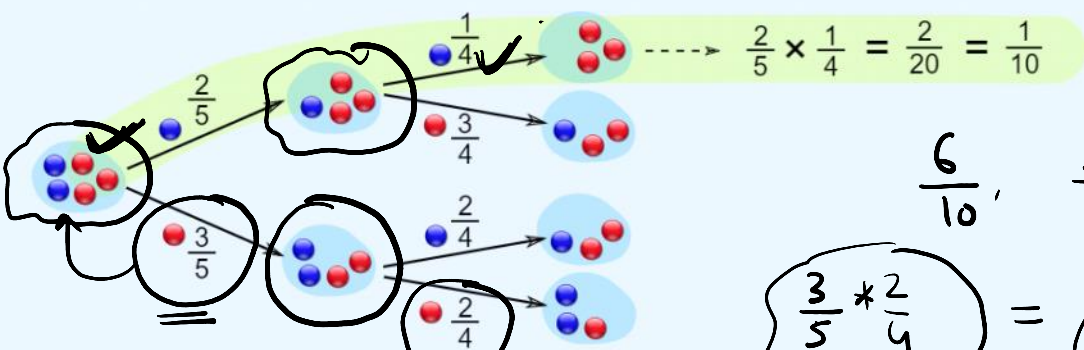
Replacement

Note: if we **replace** the marbles in the bag each time, then the chances do **not** change and the events are [independent]:

- **With** Replacement: the events are **Independent** (the chances don't change)
- **Without** Replacement: the events are **Dependent** (the chances change)

Now we can answer questions like **"What are the chances of drawing 2 blue marbles?"** without replacemnt

Answer: it is a **2/5 chance** followed by a **1/4 chance**:



$\frac{6}{10}$, $\frac{3}{4}$, $\frac{6}{20}$

$\frac{3}{5} * \frac{2}{4} = \frac{6}{20}$

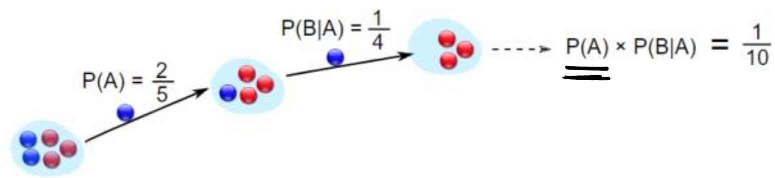
2

Did you see how we multiplied the chances? And got 1/10 as a result.

The chances of drawing 2 blue marbles is 1/10

$$P(B|A) = 1/4$$

So the probability of getting **2 blue marbles** is:



And we write it as

"Probability Of" "Given"

$$P(A \cap B) \quad P(\underbrace{A}_{\text{Event A}} \text{ and } \underbrace{B}_{\text{Event B}}) = P(A) \times P(B \mid A)$$

*"Probability of **event A and event B** equals*
*the probability of **event A** times the probability of **event B given event A**"*

$C \cap S$

$S|C$

And we have another useful formula:

$$\boxed{P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}} = \frac{P(A \cap B)}{P(A)}$$

*"The probability of **event B given event A** equals*
*the probability of **event A and event B** divided by the probability of **event A***

70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry.

70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry.

What percent of those who like Chocolate also like Strawberry?

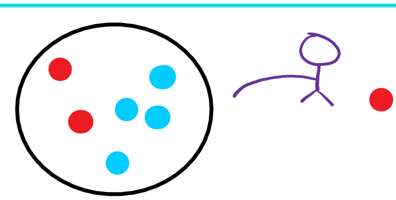
$P(E_1) = \frac{70}{100}$

$P(E_2) =$

$P(E_1 \cap E_2) = \frac{35}{100}$

50% of your friends who like Chocolate also like Strawberry
40% of your friends who like Chocolate also like Strawberry
60% of your friends who like Chocolate also like Strawberry
40% of your friends who like Chocolate also like Strawberry

A bag contains 3 red marbles and 4 blue marbles.



Two marbles are drawn at random without replacement.

If the first marble drawn is red, what is the probability the second marble is blue?

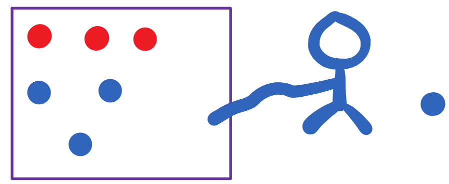
A $\cancel{\frac{3}{7}}$

B $\frac{1}{2}$

C $\frac{4}{7}$

D $\frac{2}{3}$

A bag contains 3 red marbles and 4 blue marbles.



Two marbles are drawn at random without replacement.

If the first marble drawn is blue, what is the probability the second marble is also blue?

A $\frac{3}{7}$

B $\frac{1}{2}$ $= \frac{3}{6} = 1/2$

C $\frac{4}{7}$

D $\frac{2}{3}$

Two cards are chosen at random without replacement from a pack of 52 playing cards.

If the first card chosen is an Ace, what is the probability the second card chosen is a King?

A $\frac{4}{13}$

B $\frac{1}{17}$

C $\frac{4}{51}$

D $\frac{1}{13}$

51

Two cards are chosen at random without replacement from a pack of 52 playing cards.

If the first card chosen is an Ace, what is the probability the second card chosen is also an Ace?

A $\frac{4}{13}$

B $\frac{1}{17}$

C $\frac{4}{51}$

D $\frac{1}{13}$

$\square \rightarrow$ Ace

4 → 1, 3

$\frac{3}{51}$ $\frac{1}{17}$

$\frac{3}{51}$

$$\frac{\frac{7\times 6}{\cancel{2}}}{\frac{12\times 11}{\cancel{2}}}$$

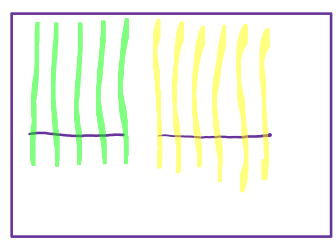
$$^{12}C_2 = \frac{12!}{2! * 10!} = \frac{12\times 11}{2}$$

$$^{7}C_2 = \frac{7\times 6}{2!}$$

A box contains 5 green pencils and 7 yellow pencils.

Two pencils are chosen at random from the box without replacement.

What is the probability they are both yellow?



A $\frac{7}{22}$

B $\frac{49}{144}$

C $\frac{6}{11}$

D $\frac{7}{12}$

E_1: choosing a yellow pencil first

E_2: choosing a yellow pencil - second

$P(E_1) = \frac{7}{}$

12

$$P(E_2|E_1) = \frac{6}{11}$$

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

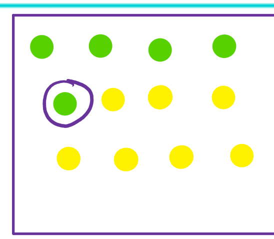
$$\Rightarrow \quad P(E_1 \cap E_2) = P(E_1) * P(E_2|E_1)$$

$$= \frac{7}{\cancel{12}} * \frac{\cancel{6}}{11} = \frac{7}{22}$$

A box contains 5 green pencils and 7 yellow pencils.

Two pencils are chosen at random from the box without replacement.

What is the probability they are different colors?



A $\frac{35}{144}$

B $\frac{35}{132}$ $\quad$ 35/132

C $\frac{35}{121}$

D $\frac{35}{66}$ ✓ $\quad \frac{70}{132}$

A: choosing yellow pencil first $\quad P(A) = \frac{7}{12}$

B: choosing green pencil second $\quad P(B|A) = \frac{5}{11}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(B|A) * P(A)$$

$$= \frac{5}{11} * \frac{7}{12}$$

$= \dfrac{35}{11 \times 12}$

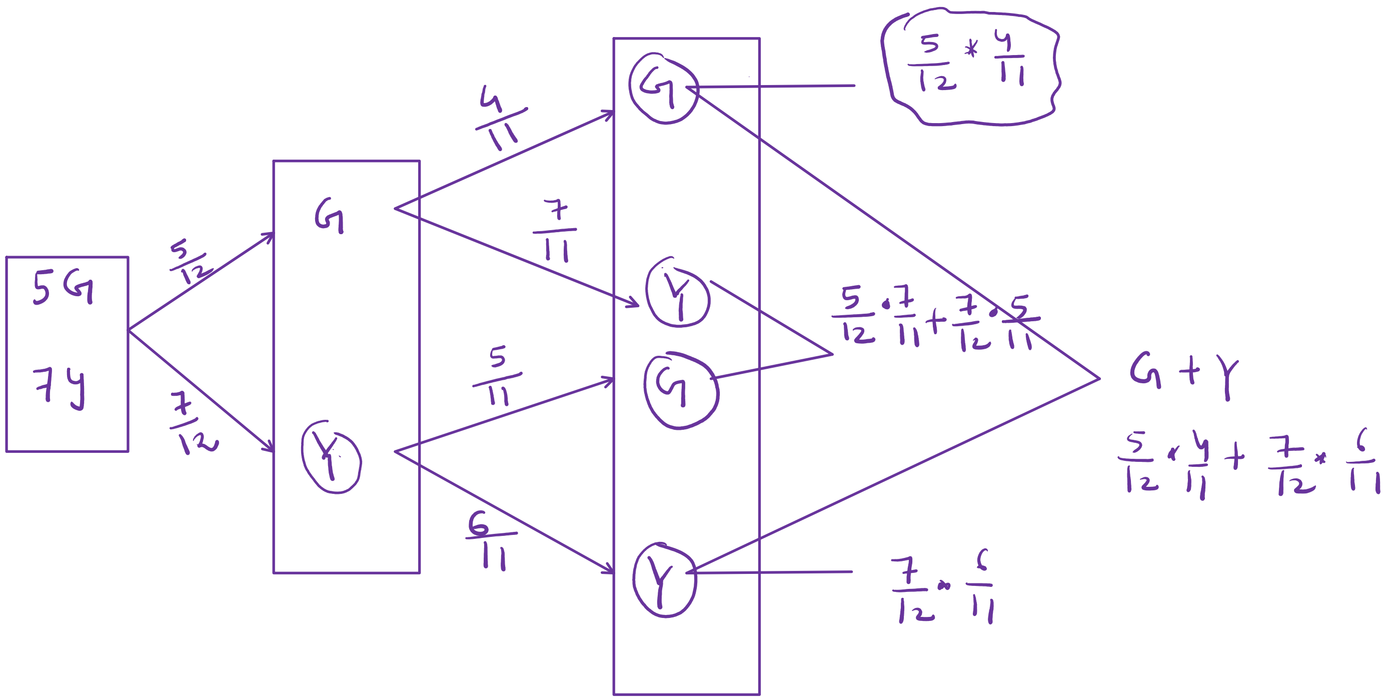
C: choosing a green first $P(C)\ \dfrac{5}{12}$

D: choosing a yellow second $P(D|C) = \dfrac{7}{11}$

$$P(D|C) = \frac{P(C \cap D)}{P(C)} \Rightarrow P(C \cap D) = P(C) * P(D|C)$$

$$= \frac{5}{12} * \frac{7}{11} = \frac{35}{11 \times 12}$$

$$\overset{\frac{35}{132}}{YG} + \overset{\frac{35}{132}}{GY} = \frac{70}{132}$$



In Exton School, 60% of the boys play baseball, and 24% of the boys play baseball and football.

What percent of those that play baseball also play football?

A 25%	B 36%
~~C 40%~~	D 84%

$E_1 : B \rightarrow \frac{60}{100}$

$E_2 : F -$

$B \cap F = \frac{24}{100}$

$$P(F|B) = \frac{P(B \cap F)}{P(B)} = \frac{24}{60}$$

A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good?

$\frac{5}{9}$

A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random (without replacement). Find the probability that

(i) both are good,

(ii) both have major defects,

(iii) at least 1 is good,

(iv) at most 1 is good,

(v) exactly 1 is good,

(vi) neither has major defects

(vii) neither is good.