

DIGITAL ELECTRONICS: ECE 213

**Topic: Adders, Subtractors, Decoders
and Multiplexer**

**UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families**

Lecture No.: 15

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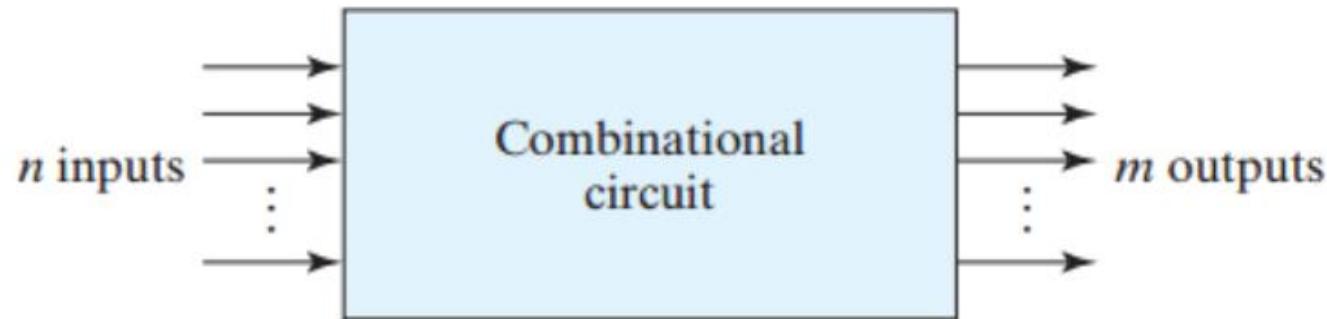
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Introduction to Combinational Circuits

- Combinational logic is a type of digital logic which is implemented by Boolean circuits, where the **output is a pure function of the present inputs only**.

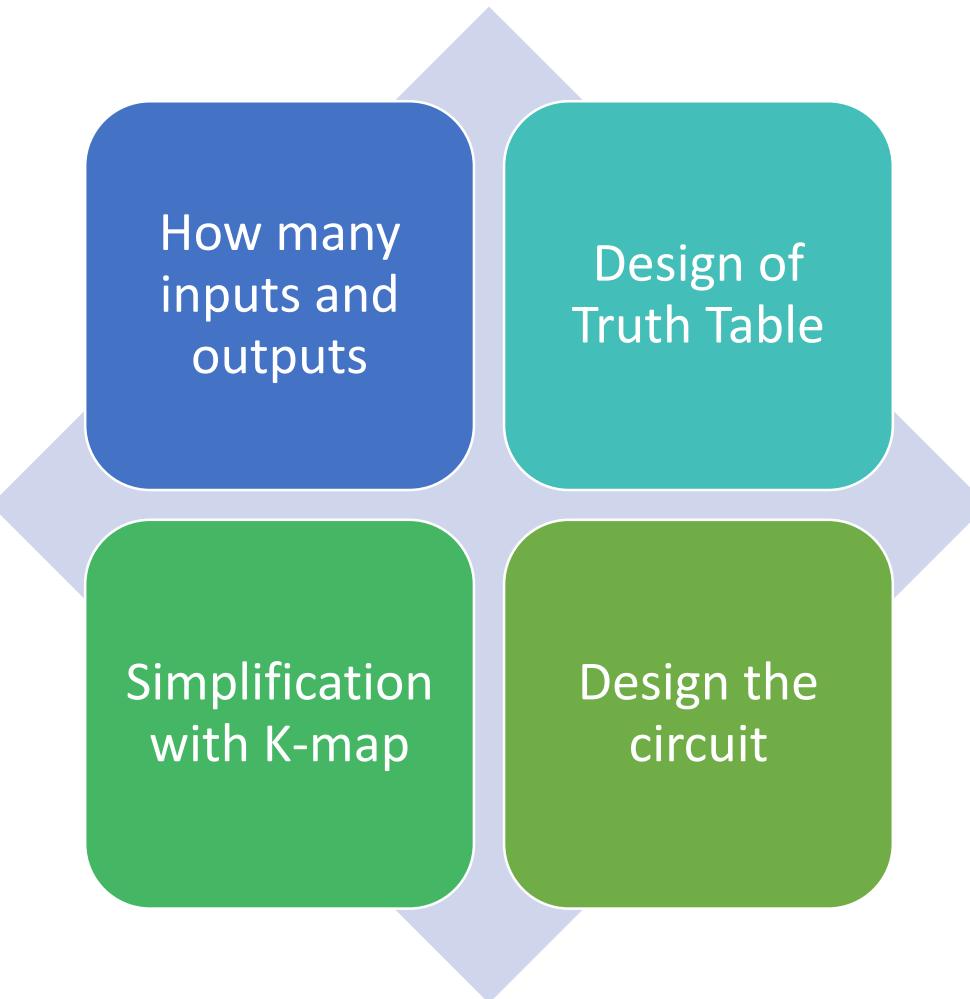


- The combinational circuit do not use any memory.
- The previous state of input does not have any effect on the present state of the circuit.

Design Procedure

1. From the specifications of the circuit, determine the required number of inputs and outputs and assign a symbol to each.
2. Derive the truth table that defines the required relationship between inputs and outputs.
3. Obtain the simplified Boolean functions for each output as a function of the input variables.
4. Draw the logic diagram and verify the correctness of the design (manually or by simulation).

Design Procedure: BCD to Excess-3 Code Conversion



Design Procedure: BCD to Excess-3 Code Conversion

STEP: 1 and 2

Truth Table for Code Conversion Example

Input BCD				Output Excess-3 Code			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Design Procedure: BCD to Excess-3 Code Conversion

Design Procedure: BCD to Excess-3 Code Conversion

STEP: 3

	<i>AB</i>	<i>CD</i>	00	01	11	10	<i>C</i>
	<i>AB</i>	<i>CD</i>	<i>m</i> ₀	<i>m</i> ₁	<i>m</i> ₃	<i>m</i> ₂	<i>m</i> ₂
	<i>AB</i>	<i>CD</i>	00	1			1
	<i>AB</i>	<i>CD</i>	01	1	<i>m</i> ₅	<i>m</i> ₇	1
<i>A</i>	<i>AB</i>	<i>CD</i>	11	X	<i>m</i> ₁₃	<i>m</i> ₁₅	X
	<i>AB</i>	<i>CD</i>	10	1	<i>m</i> ₉	<i>m</i> ₁₁	X
							<i>D</i>
							<i>z</i> = <i>D'</i>

	<i>AB</i>	<i>CD</i>	00	01	11	10	<i>C</i>
	<i>AB</i>	<i>CD</i>	<i>m</i> ₀	<i>m</i> ₁	<i>m</i> ₃	<i>m</i> ₂	<i>m</i> ₂
	<i>AB</i>	<i>CD</i>	00	1			1
	<i>AB</i>	<i>CD</i>	01	1	<i>m</i> ₅	<i>m</i> ₇	1
<i>A</i>	<i>AB</i>	<i>CD</i>	11	X	<i>m</i> ₁₃	<i>m</i> ₁₅	X
	<i>AB</i>	<i>CD</i>	10	1	<i>m</i> ₉	<i>m</i> ₁₁	X
							<i>D</i>
							<i>y</i> = <i>CD</i> + <i>C'D'</i>

	<i>AB</i>	<i>CD</i>	00	01	11	10	<i>C</i>
	<i>AB</i>	<i>CD</i>	<i>m</i> ₀	<i>m</i> ₁	<i>m</i> ₃	<i>m</i> ₂	<i>m</i> ₂
	<i>AB</i>	<i>CD</i>	00		1	1	1
	<i>AB</i>	<i>CD</i>	01	1			
<i>A</i>	<i>AB</i>	<i>CD</i>	11	X	<i>m</i> ₁₃	<i>m</i> ₁₅	X
	<i>AB</i>	<i>CD</i>	10	1	<i>m</i> ₉	<i>m</i> ₁₁	X
							<i>D</i>
							<i>x</i> = <i>B'C</i> + <i>B'D</i> + <i>BC'D'</i>

	<i>AB</i>	<i>CD</i>	00	01	11	10	<i>C</i>
	<i>AB</i>	<i>CD</i>	<i>m</i> ₀	<i>m</i> ₁	<i>m</i> ₃	<i>m</i> ₂	<i>m</i> ₂
	<i>AB</i>	<i>CD</i>	00				
	<i>AB</i>	<i>CD</i>	01		1	1	1
<i>A</i>	<i>AB</i>	<i>CD</i>	11	X	<i>m</i> ₁₃	<i>m</i> ₁₅	X
	<i>AB</i>	<i>CD</i>	10	1	<i>m</i> ₉	<i>m</i> ₁₁	X
							<i>D</i>
							<i>w</i> = <i>A</i> + <i>BC</i> + <i>BD</i>

Design Procedure: BCD to Excess-3 Code Conversion

STEP: 3

$$z = D'$$

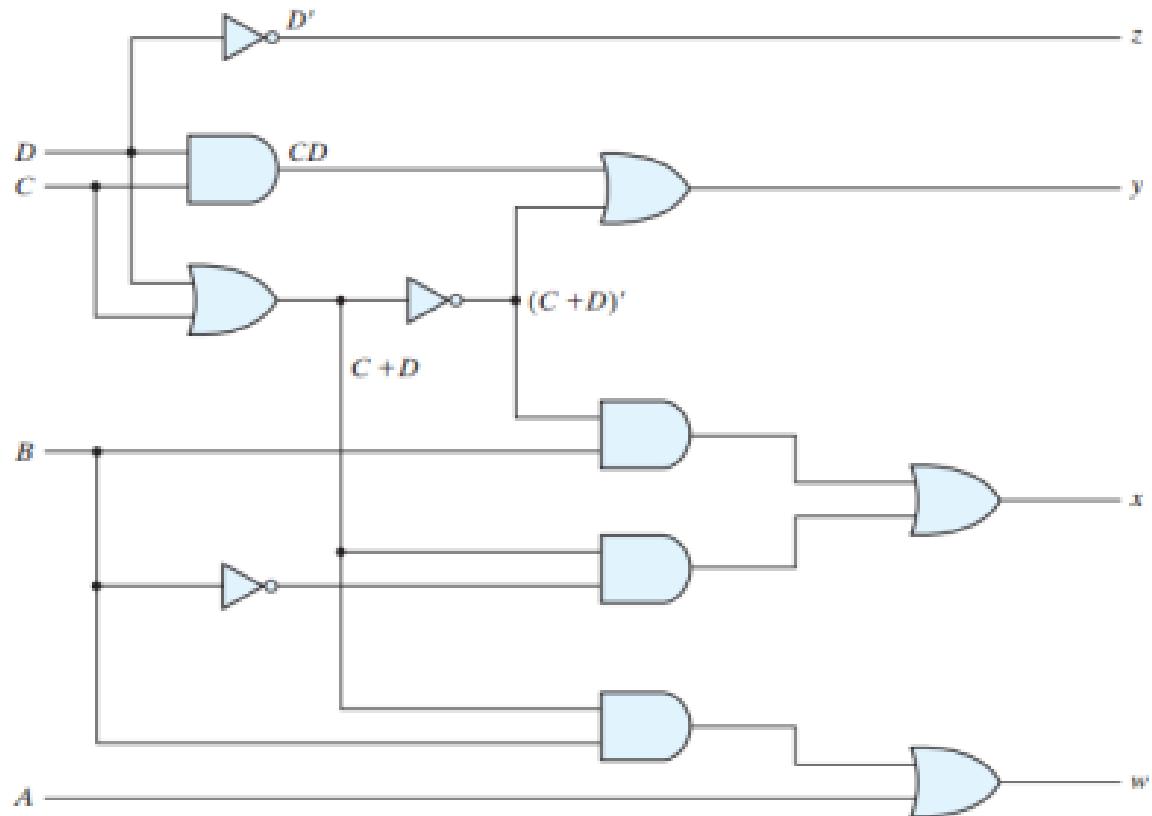
$$y = CD + C'D' = CD + (C + D)'$$

$$\begin{aligned}x &= B'C + B'D + BC'D' = B'(C + D) + BC'D' \\&= B'(C + D) + B(C + D)'\end{aligned}$$

$$w = A + BC + BD = A + B(C + D)$$

Design Procedure: BCD to Excess-3 Code Conversion

STEP: 4



$$z = D'$$

$$y = CD + C'D' = CD + (C + D)'$$

$$\begin{aligned}x &= B'C + B'D + BC'D' = B'(C + D) + BC'D' \\&= B'(C + D) + B(C + D)'\end{aligned}$$

$$w = A + BC + BD = A + B(C + D)$$

Half Adder

- ❖ A combinational logic circuit with two inputs and two outputs.
- ❖ The half adder circuit adds two single bits without any carry. This circuit has two outputs carry and sum.



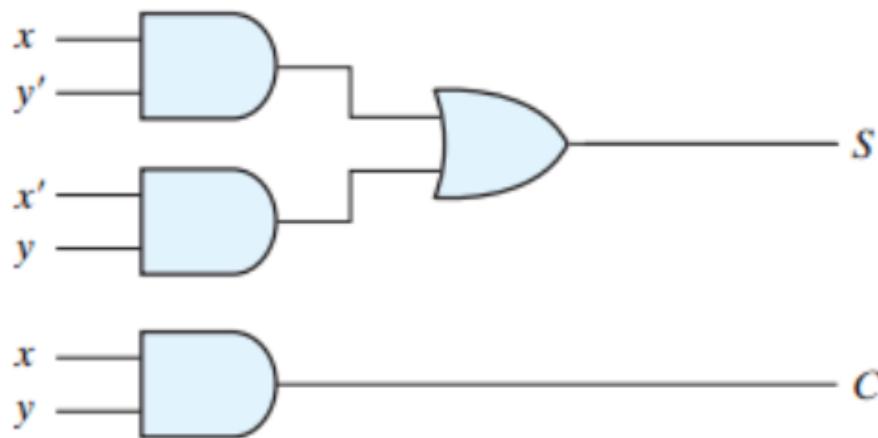
Half Adder

x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

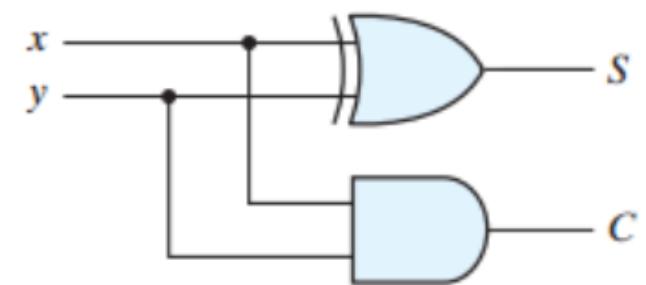
Half Adder

Half Adder

x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



$$(a) S = xy' + x'y \\ C = xy$$



$$(b) S = x \oplus y \\ C = xy$$

QUICK QUIZ (POLL)

If A and B are the inputs of a half adder, the sum is given by _____

- a) A AND B
- b) A OR B
- c) A XOR B
- d) A EX-NOR B

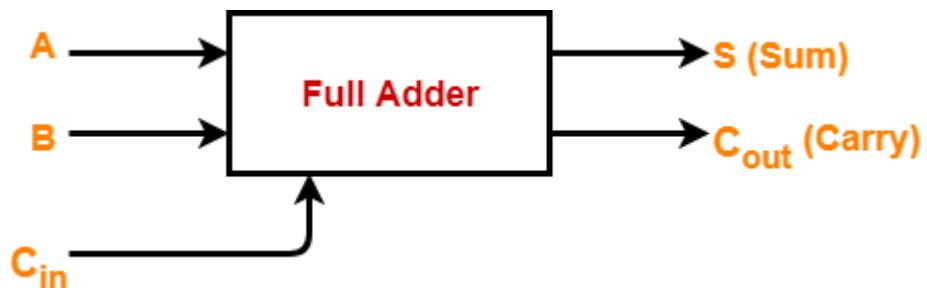
QUICK QUIZ (POLL)

Total number of inputs in a half adder is _____

- a) 2
- b) 3
- c) 4
- d) 1

Full Adder

- ❖ A combinational logic circuit with 3 inputs and 2 outputs.
- ❖ The Full adder circuit adds 3 bits including carry.



A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\overline{ABC} + \overline{AB}\overline{C} + A\overline{B}\overline{C} + ABC$$

$$\overline{A}(\overline{BC} + B\overline{C}) + A(\overline{B}\overline{C} + BC)$$

$$\overline{A}(B \oplus C) + A(B \ominus C)$$

$$Let B \oplus C = D$$

$$\overline{AD} + A\overline{D}$$

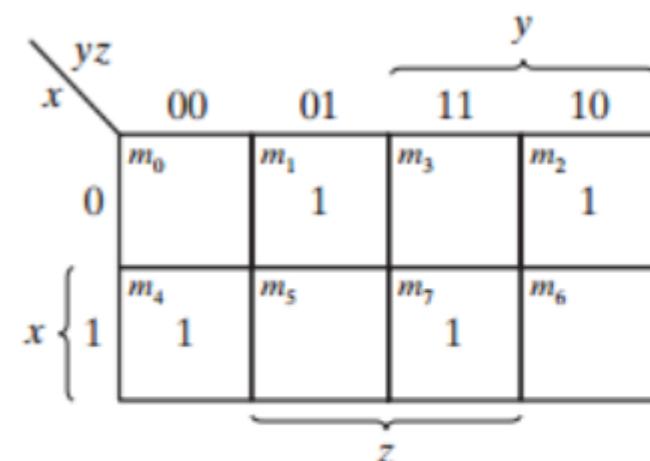
$$A \oplus D$$

$$A \oplus B \oplus C$$

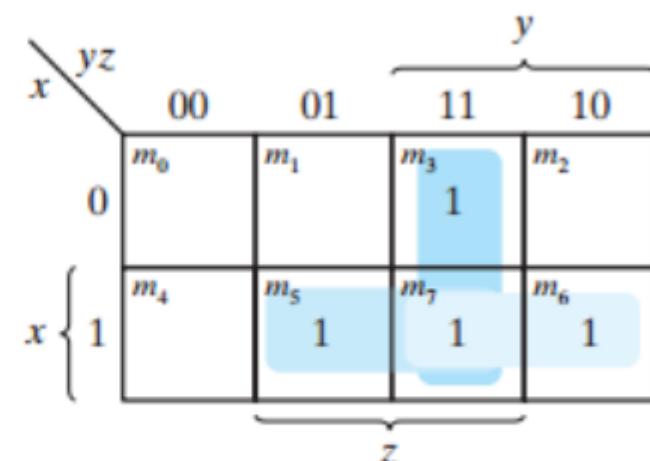
Full Adder

Full Adder

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

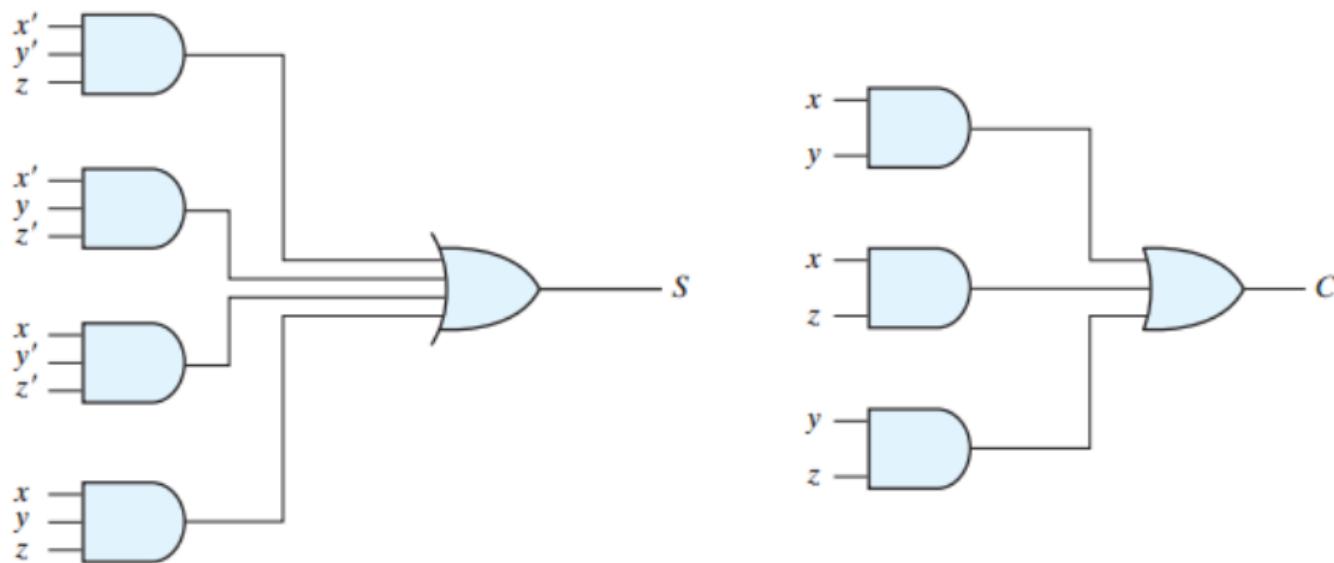


$$(a) S = x'y'z + x'yz' + xy'z' + xyz$$



$$(b) C = xy + xz + yz$$

Full Adder

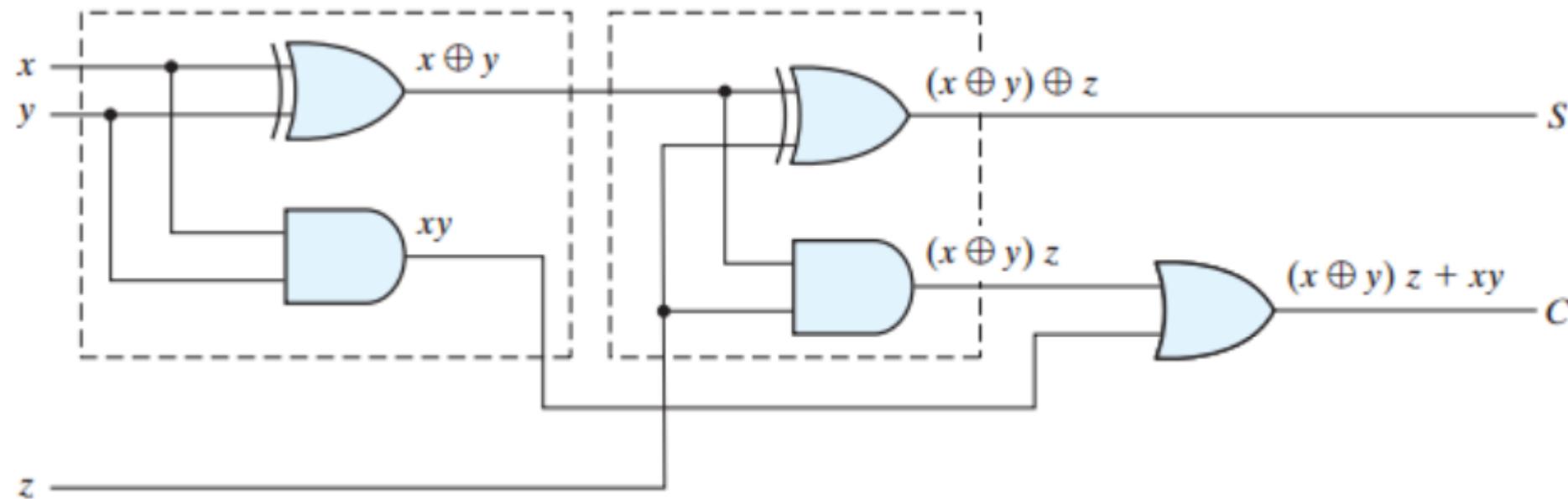


QUICK QUIZ (POLL)

If A, B and C are the inputs of a full adder then the carry is given by

- a) A AND B OR (A OR B) AND C
- b) A OR B OR (A AND B) C
- c) (A AND B) OR (A AND B)C
- d) A XOR B XOR (A XOR B) AND C

Implementation of Full Adder using Half Adders



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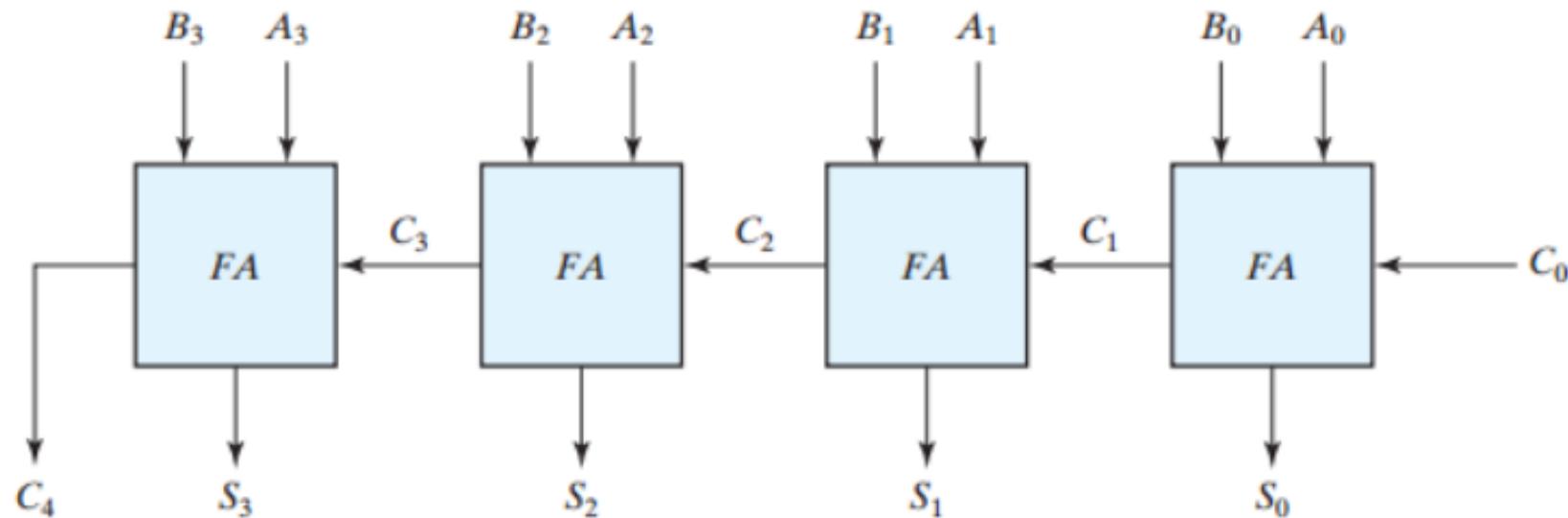
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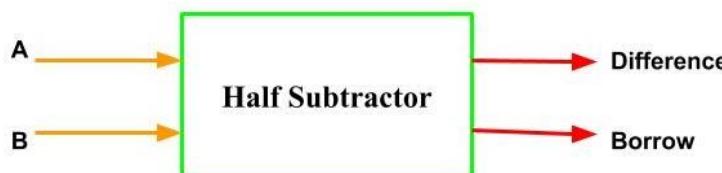


Implementation of Ripple Carry Adder (RCA):4-bits



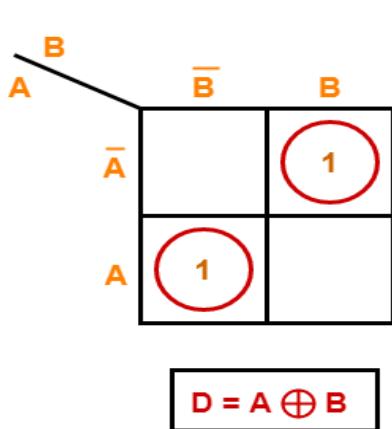
Half Subtractor

- ❖ Combinational circuit perform binary subtraction
- ❖ accepts 2 inputs and provides two outputs: Difference and Borrow

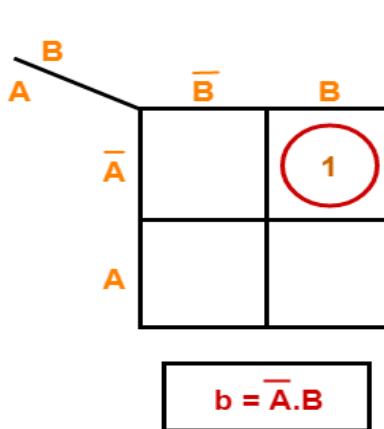


Inputs		Outputs	
A	B	D (Difference)	b (Borrow)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

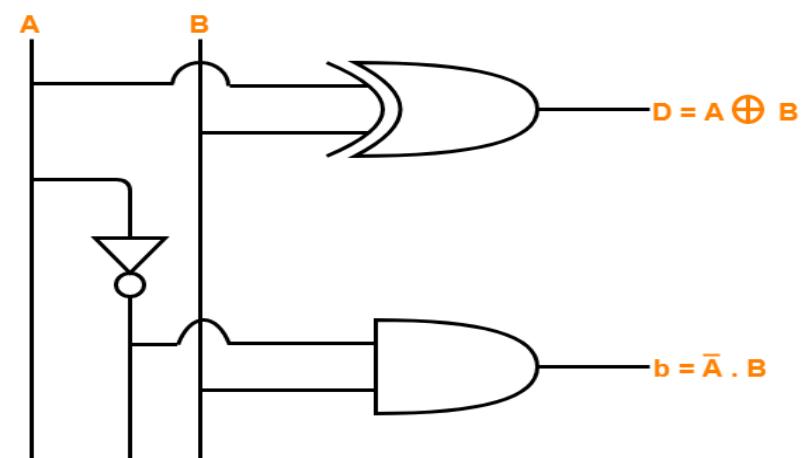
For D:



For b:



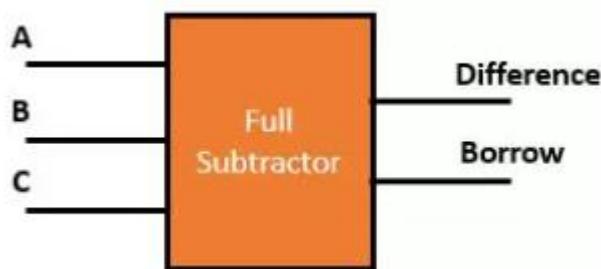
K Maps



Half Subtractor Logic Diagram

Full Subtractor

- ❖ Performs subtraction of 3 bits
- ❖ This circuit has three inputs and two outputs.
- ❖ The three inputs A, B and C, denote the minuend, subtrahend, and previous borrow, respectively.
- ❖ The two outputs are: D and Bout



$$\begin{aligned} \text{Difference} &= \overline{A} \overline{B} C + \overline{A} B \overline{C} + A \overline{B} \overline{C} + ABC \\ &= C (\overline{A} \overline{B} + AB) + \overline{C} (\overline{A} B + A \overline{B}) \\ &= C (A \oplus B) + \overline{C} (A \oplus B) \\ &= C (A \overline{\oplus} B) + \overline{C} (A \oplus B) \\ &= C \oplus (A \oplus B) \end{aligned}$$

Input			Output	
A	B	C	Difference	Borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$\text{Sum}(A, B, C) = \sum m (1, 2, 4, 7)$$

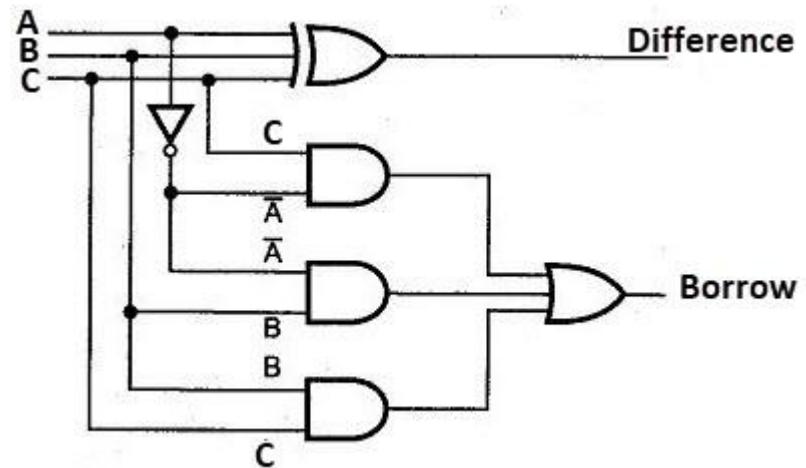
$$\text{Bout}(A, B, C) = \sum m (1, 2, 3, 7)$$

Full Subtractor

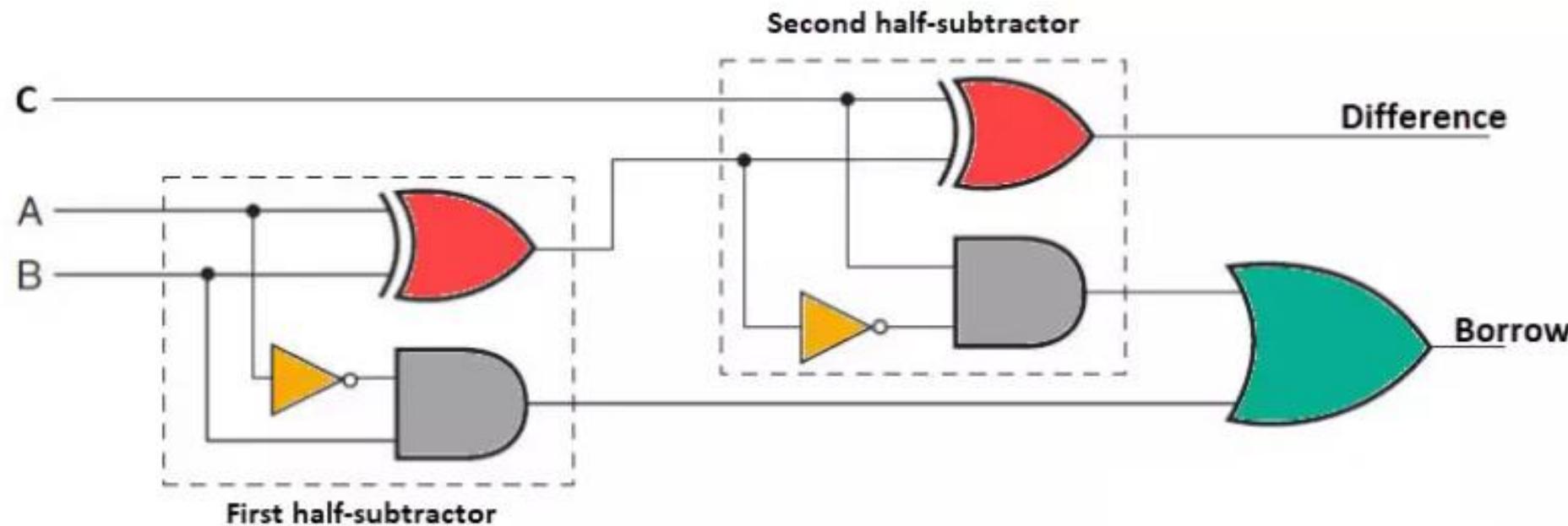
BC	00	01	11	10
A	0	1	1	1
o	0	0	1	0
1	0	0	1	0

$$\begin{aligned}Borrow &= \overline{A} \overline{B} C + \overline{A} B \overline{C} + \overline{A} BC + ABC \\&= \overline{A} B + \overline{A} C + BC\end{aligned}$$

$$\begin{aligned}Bout &= A'B'C + A'BC' + A'BC + ABC \\&= C(AB + A'B') + A'B(C + C') \\&= C(A \text{ XNOR } B) + A'B \\&= C(A \text{ XOR } B)' + A'B\end{aligned}$$



Implementation of Full Subtractor using Half Subtractor



QUICK QUIZ (POLL)

The difference between half adder and full adder is _____

- a) Half adder has two inputs while full adder has four inputs
- b) Half adder has one output while full adder has two outputs
- c) Half adder has two inputs while full adder has three inputs
- d) All of the Mentioned

QUICK QUIZ (POLL)

How many AND, OR and EXOR gates are required for the configuration of full adder?

- a) 1, 2, 2
- b) 2, 1, 2
- c) 3, 1, 2
- d) 4, 0, 1

Try Yourself

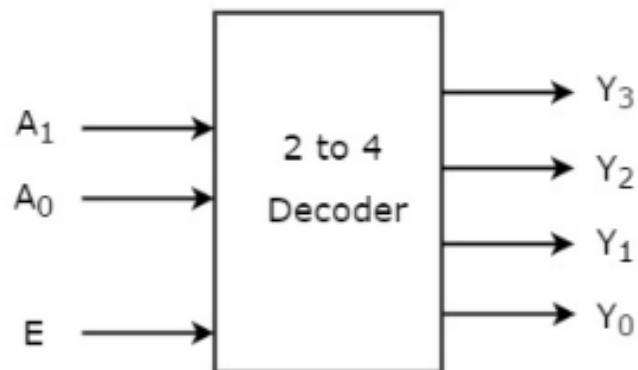
Design a 3 input digital circuit which produces high output for ODD decimal equivalent?

Decoder

What is a Decoder?

- ❖ A combinational circuit that has ‘n’ input lines and maximum of 2^n output lines.
- ❖ One of these outputs will be active High based on the combination of inputs present, when the decoder is enabled.
- ❖ The outputs of the decoder are min terms of ‘n’ input variables lines when it is enabled

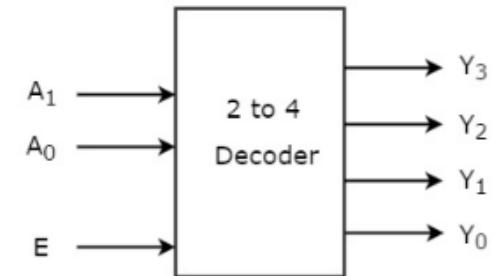
2x4 Decoder



Enable	Inputs		Outputs			
	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

2x4 Decoder

Enable	Inputs		Outputs			
	A ₁	A ₀	Y ₃	Y ₂	Y ₁	Y ₀
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0



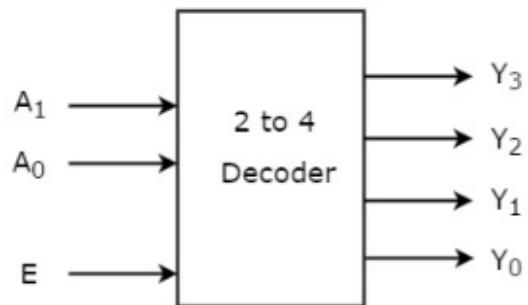
$$Y_3 = E \cdot A_1 \cdot A_0$$

$$Y_2 = E \cdot A_1 \cdot A_0'$$

$$Y_1 = E \cdot A_1' \cdot A_0$$

$$Y_0 = E \cdot A_1' \cdot A_0'$$

2x4 Decoder



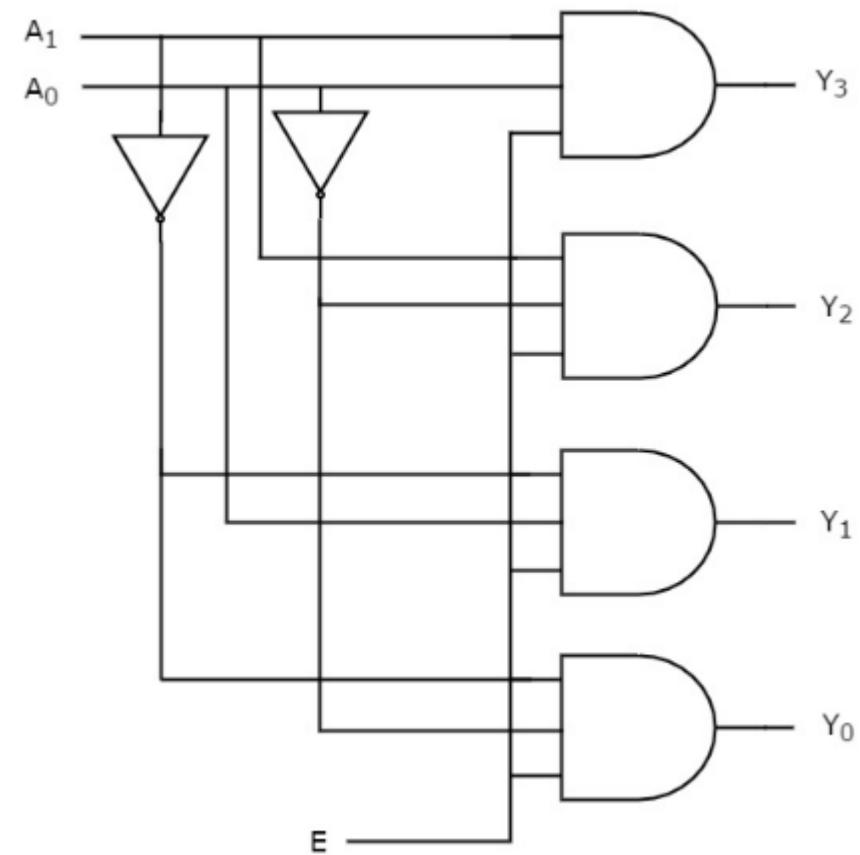
$$Y_3 = E \cdot A_1 \cdot A_0$$

$$Y_2 = E \cdot A_1 \cdot A_0'$$

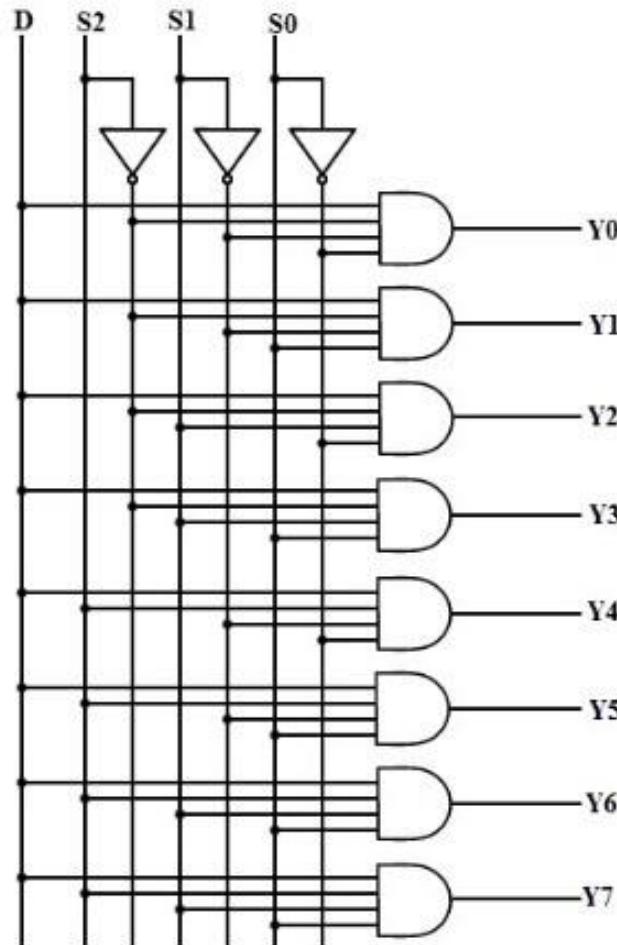
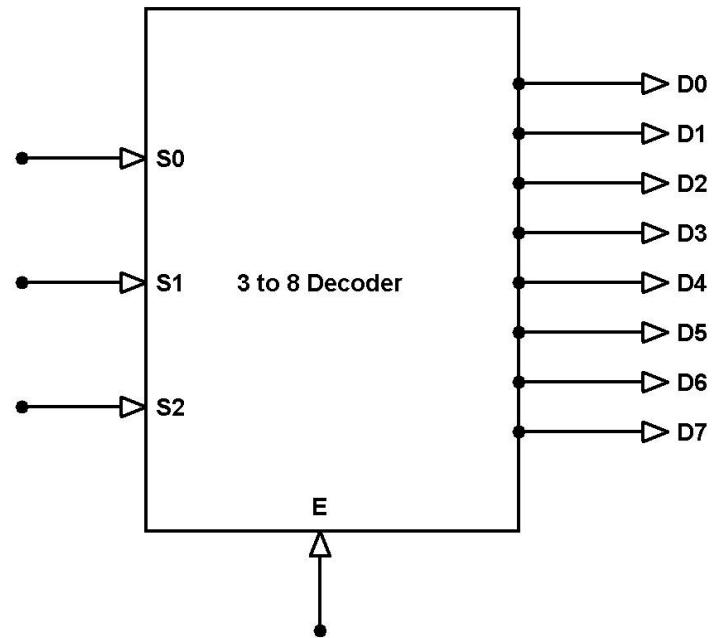
$$Y_1 = E \cdot A_1' \cdot A_0$$

$$Y_0 = E \cdot A_1' \cdot A_0'$$

Therefore, the outputs of 2 to 4 decoder are nothing but the **min terms** of two input variables A_1 & A_0 , when enable, E is equal to one. If enable, E is zero, then all the outputs of decoder will be equal to zero.



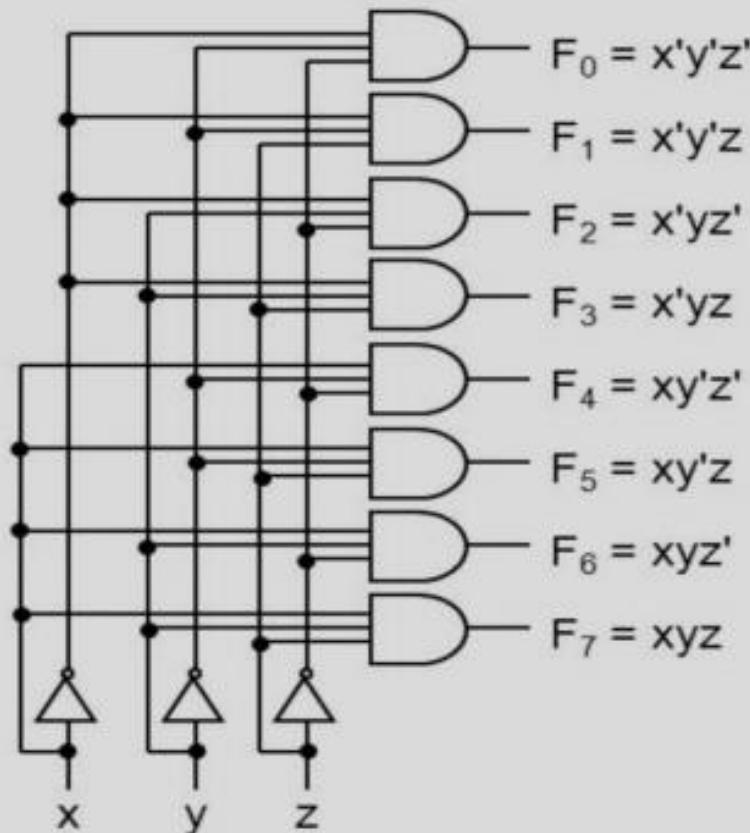
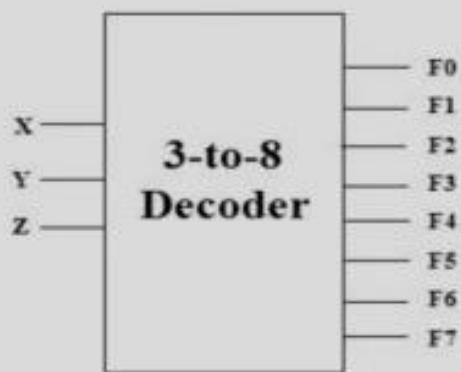
3x8 Decoder



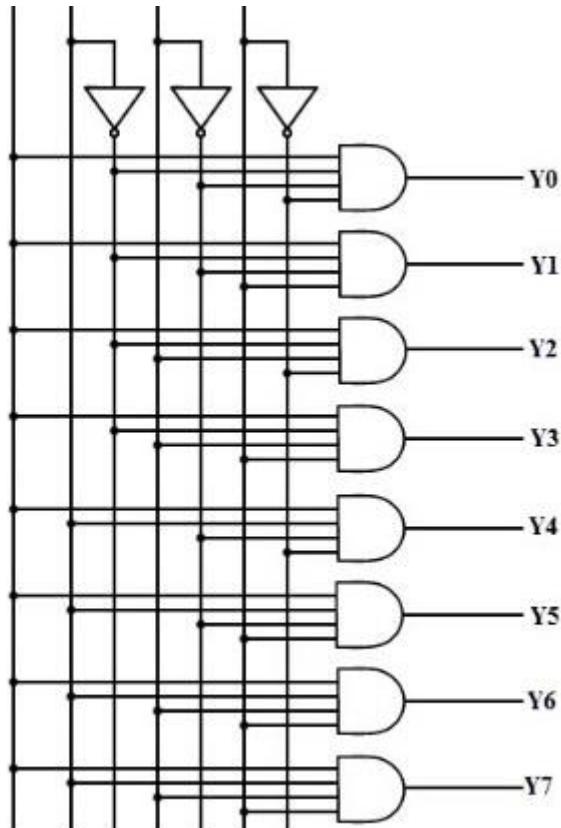
3x8 Decoder: Active High

Truth Table:

x	y	z	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



3x8 Decoder: Active Low



Inputs			Outputs							
A	B	C	Y ₀	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

Full Adder Design using 3:8 Decoder

Full Adder Truth table

INPUTS			OUTPUTS	
A	B	C _{in}	SUM	CARRY OUT
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

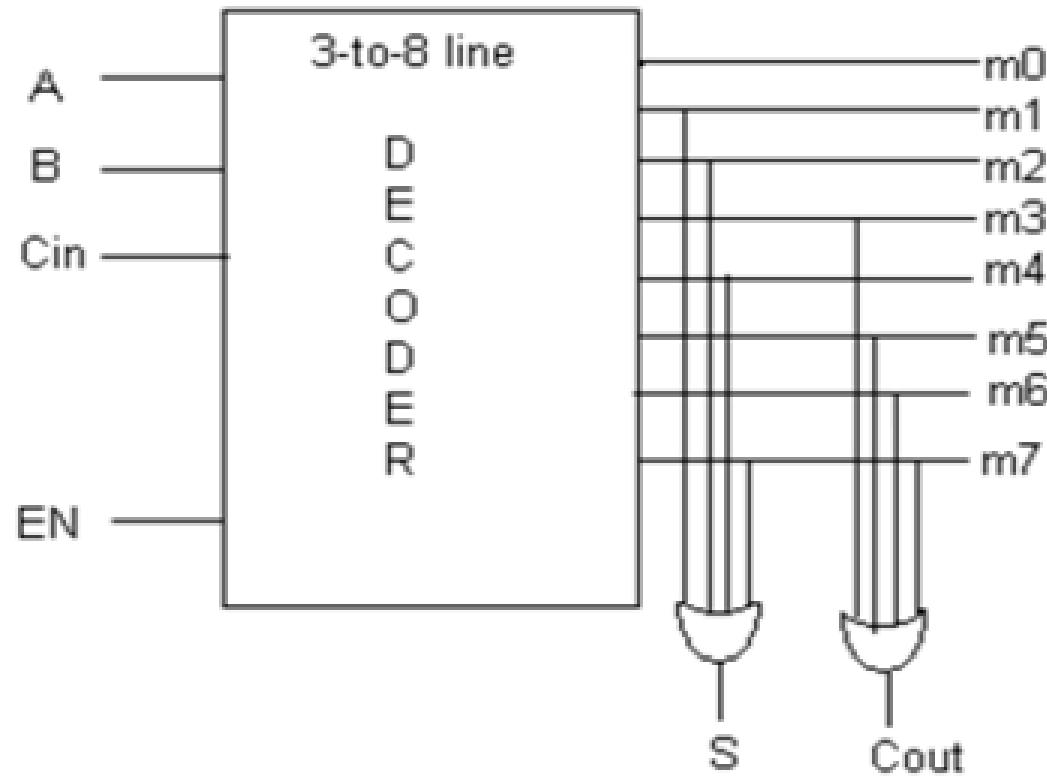
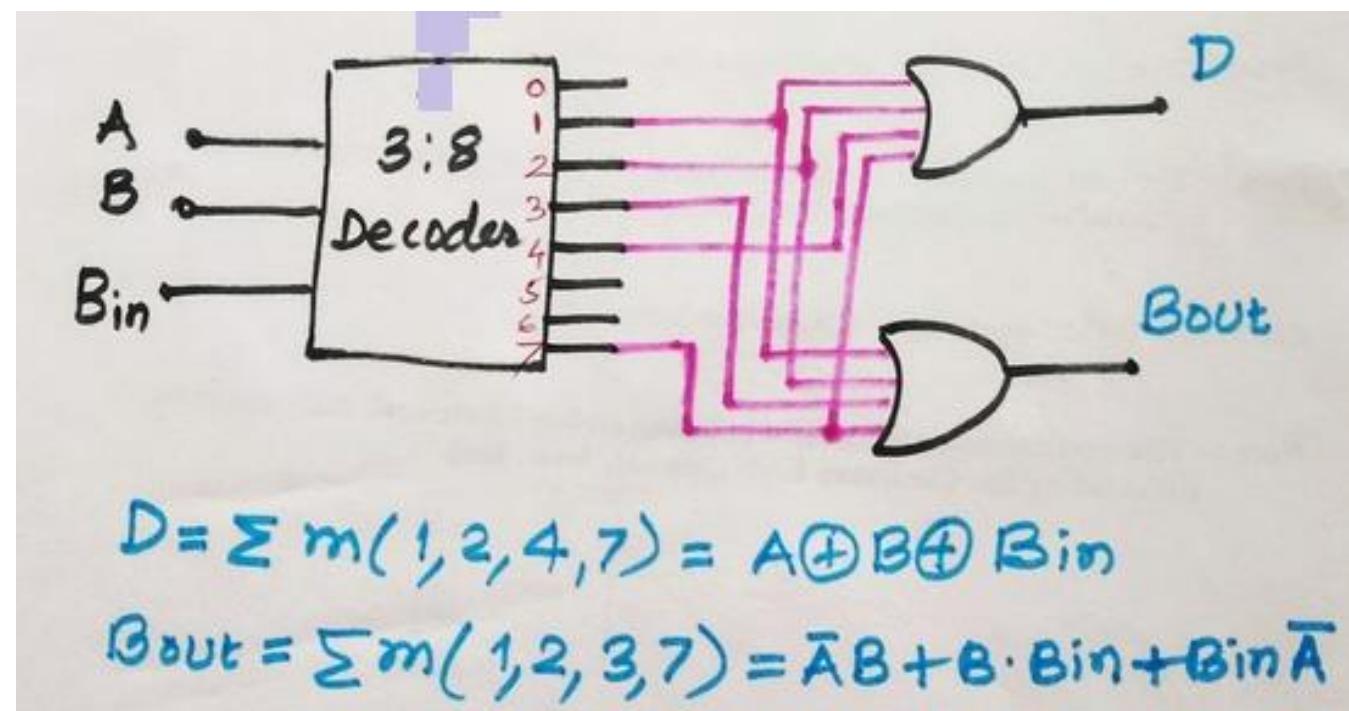


Fig1: Full Adder Implementation using 3:8 decoder

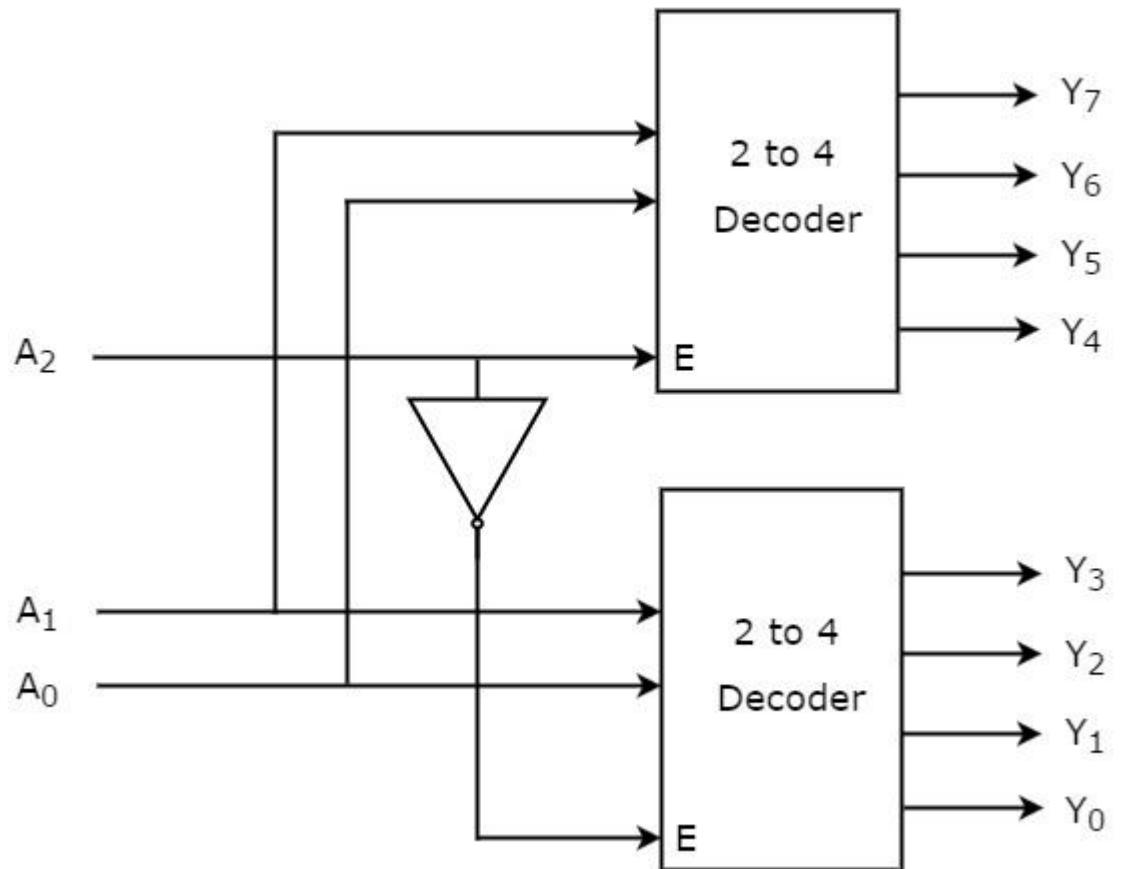
Full Subtractor Design using 3:8 Decoder

Inputs			Outputs	
A	B	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



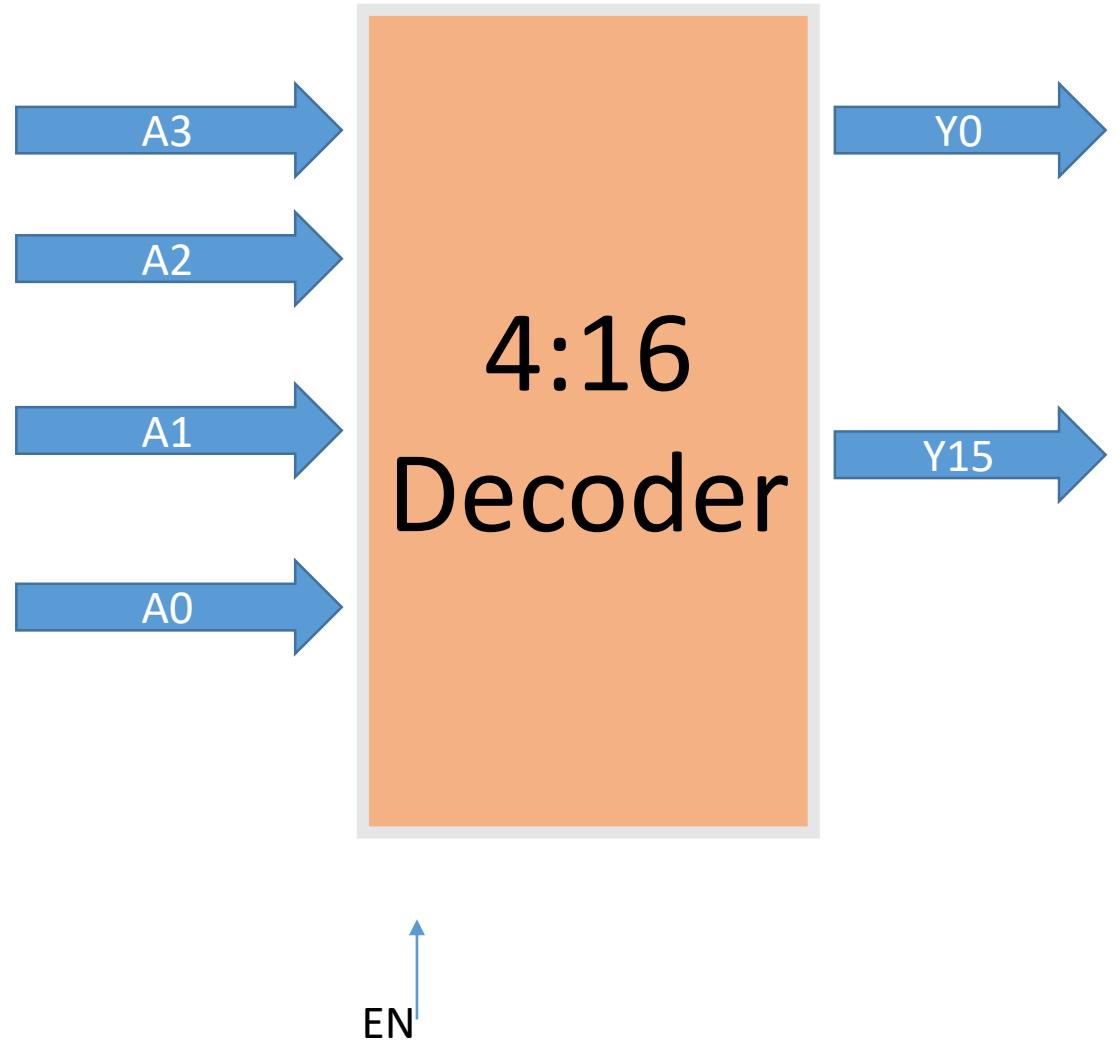
3:8 Decoder using 2:4 Decoder

A ₂	A ₁	A ₀	HIGH OUTPUT
0	0	0	Y ₀
0	0	1	Y ₁
0	1	0	Y ₂
0	1	1	Y ₃
1	0	0	Y ₄
1	0	1	Y ₅
1	1	0	Y ₆
1	1	1	Y ₇



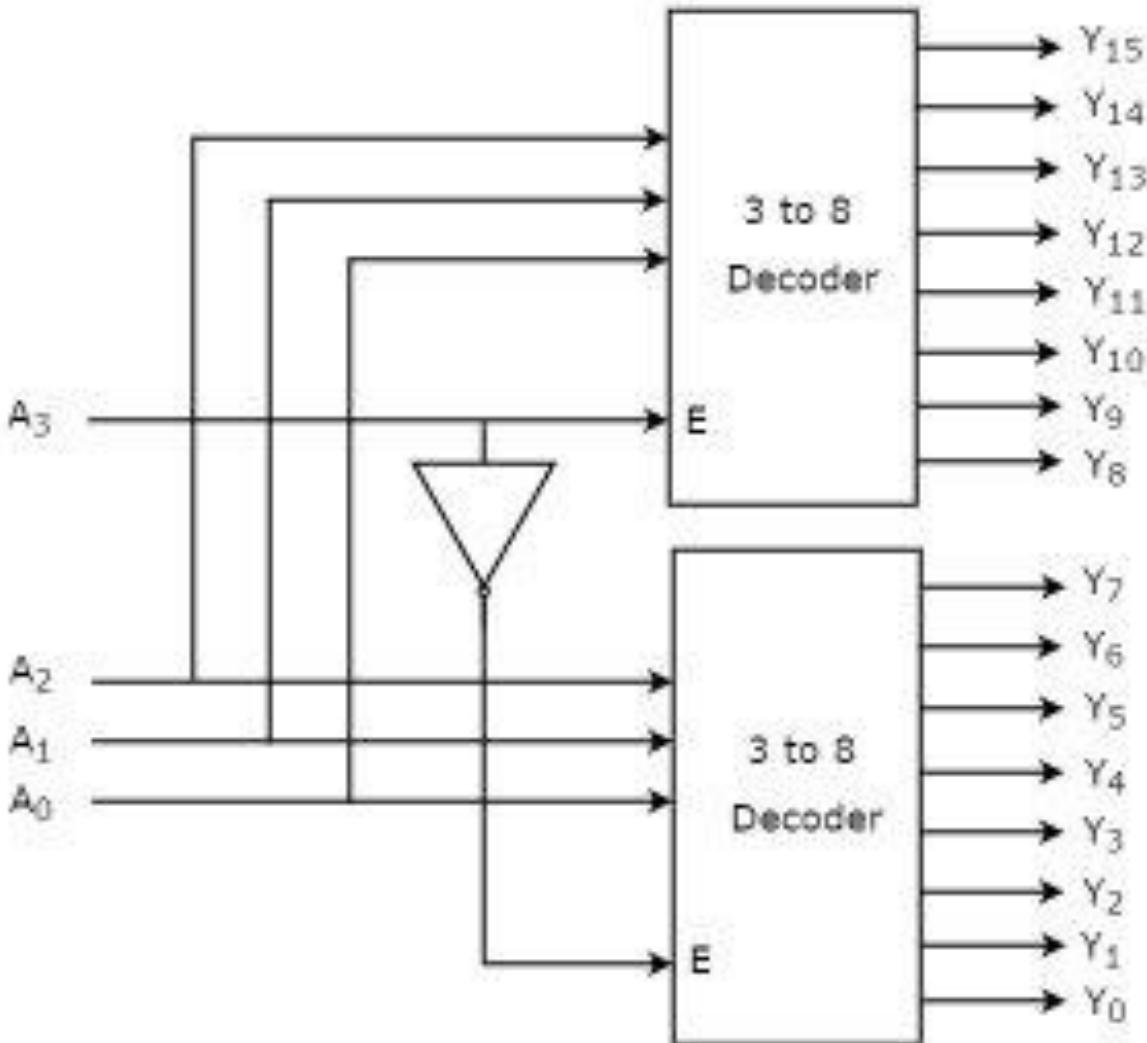
4:16 Decoder

A3	A2	A1	A0	HIGH OUTPUT
0	0	0	0	Y0
0	0	0	1	Y1
0	0	1	0	Y2
0	0	1	1	Y3
0	1	0	0	Y4
0	1	0	1	Y5
0	1	1	0	Y6
0	1	1	1	Y7
1	0	0	0	Y8
1	0	0	1	Y9
1	0	1	0	Y10
1	0	1	1	Y11
1	1	0	0	Y12
1	1	0	1	Y13
1	1	1	0	Y14
1	1	1	1	Y15



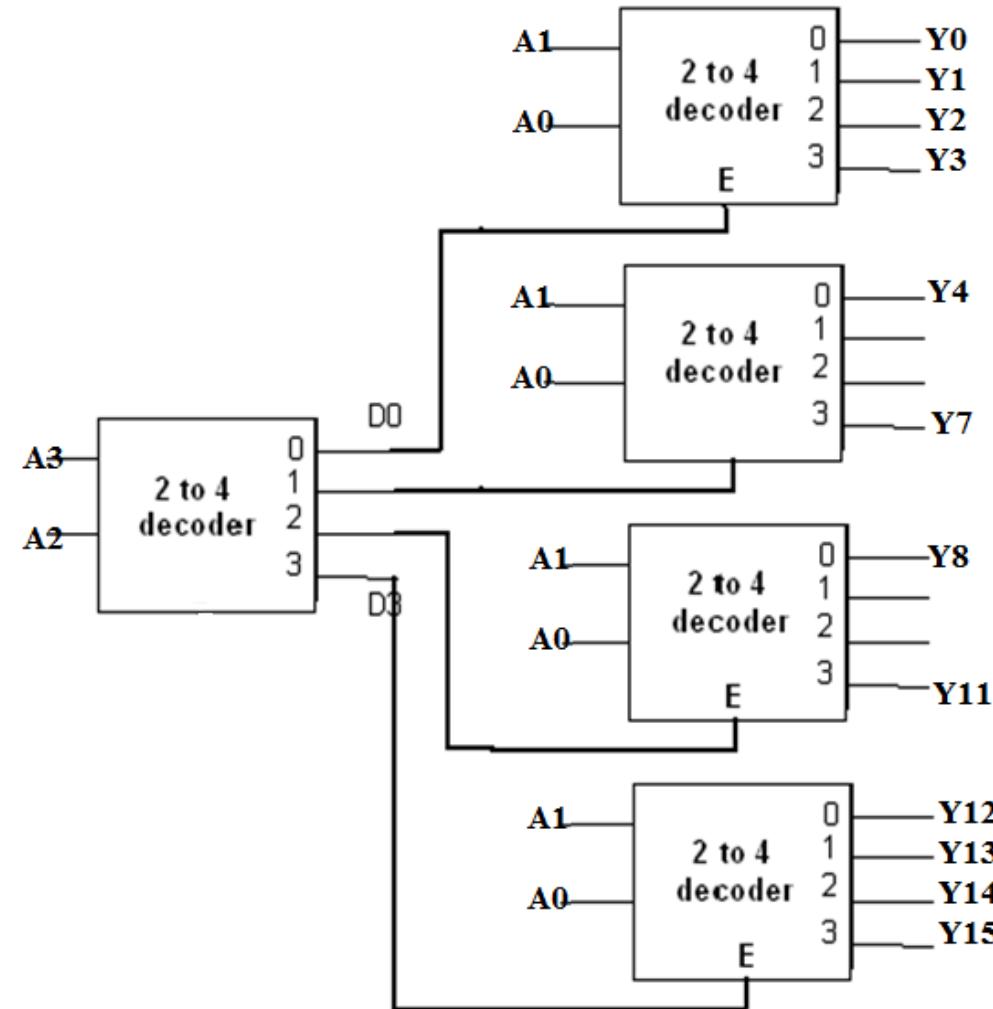
4:16 Decoder using 3:8 Decoder

A3	A2	A1	A0	HIGH OUTPUT
0	0	0	0	Y0
0	0	0	1	Y1
0	0	1	0	Y2
0	0	1	1	Y3
0	1	0	0	Y4
0	1	0	1	Y5
0	1	1	0	Y6
0	1	1	1	Y7
1	0	0	0	Y8
1	0	0	1	Y9
1	0	1	0	Y10
1	0	1	1	Y11
1	1	0	0	Y12
1	1	0	1	Y13
1	1	1	0	Y14
1	1	1	1	Y15



4:16 Decoder using 2:4 Decoder

A3	A2	A1	A0	HIGH OUTPUT
0	0	0	0	Y0
0	0	0	1	Y1
0	0	1	0	Y2
0	0	1	1	Y3
0	1	0	0	Y4
0	1	0	1	Y5
0	1	1	0	Y6
0	1	1	1	Y7
1	0	0	0	Y8
1	0	0	1	Y9
1	0	1	0	Y10
1	0	1	1	Y11
1	1	0	0	Y12
1	1	0	1	Y13
1	1	1	0	Y14
1	1	1	1	Y15



DIGITAL ELECTRONICS: ECE 213

**Topic: Adders, Subtractors, Decoders
and Multiplexer**

**UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families**

Lecture No.: 17

Prepared By: Irfan Ahmad Pindoo

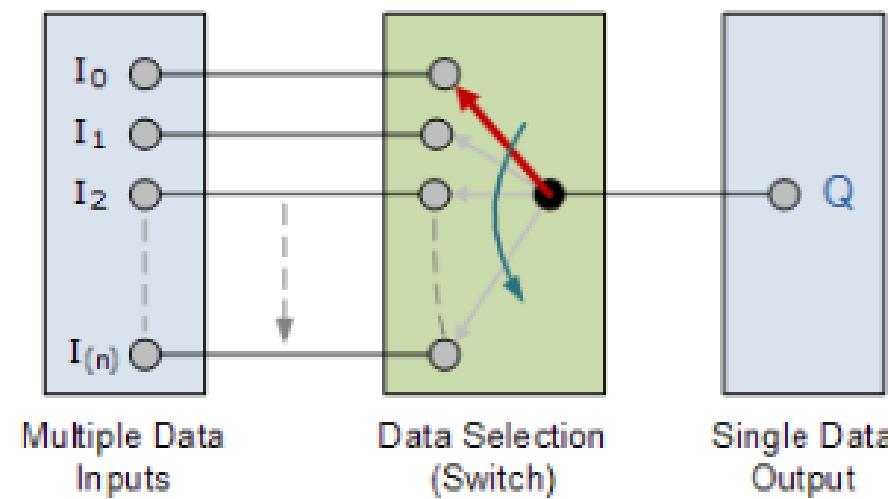
Assistant Professor
VLSI Design, ECE

School of Computer Science and Engineering



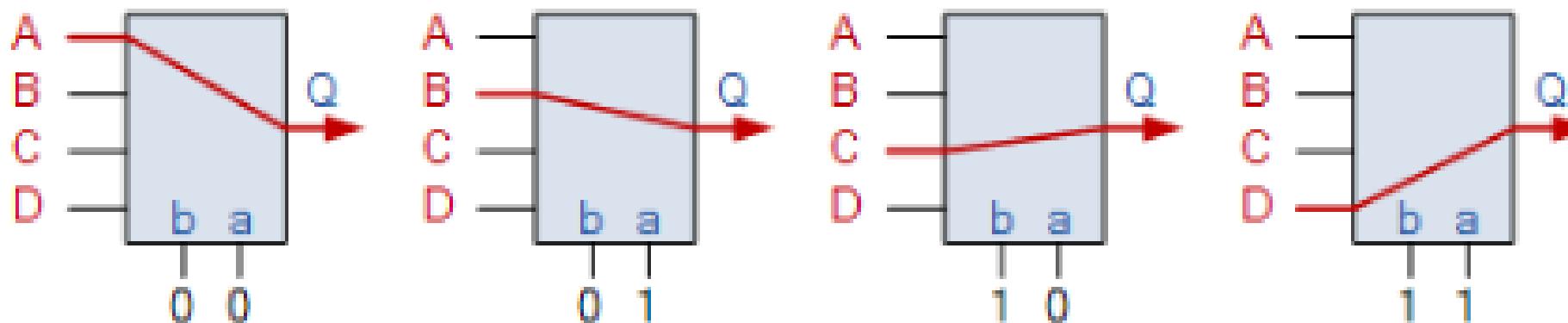
Multiplexer

- The *multiplexer*, shortened to “MUX” or is a **combinational logic** circuit designed to switch **one of several input lines** through to a **single common output line** by the application of a **control signal**.
- Multiplexers operate like very fast acting multiple position rotary switches connecting or controlling multiple input lines called “channels” one at a time to the output.



Multiplexer

- **Multiplexer** is a combinational circuit that has maximum of 2^n data inputs, ‘n’ selection lines and single output line. One of these data inputs will be connected to the output based on the values of selection lines.
- Since there are ‘n’ selection lines, there will be 2^n possible combinations of zeros and ones. So, each combination will select only one data input. Multiplexer is also called as **Mux**.

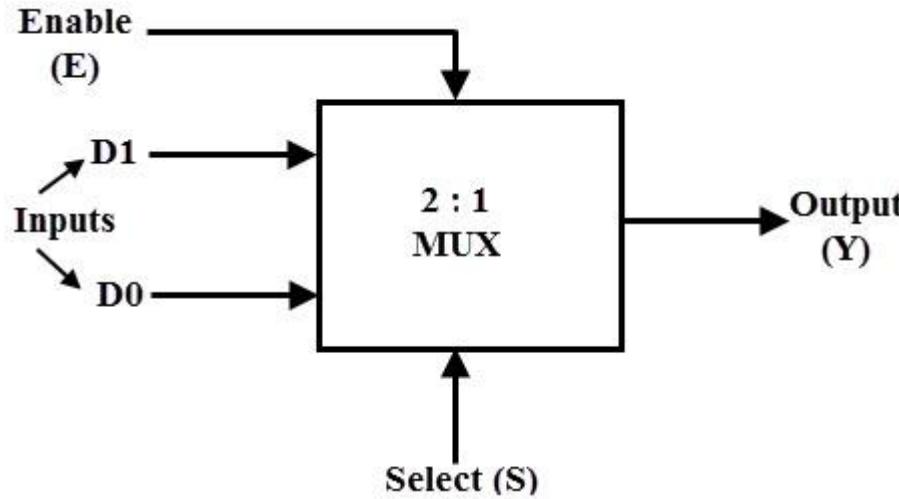


QUICK QUIZ (POLL)

Which combinational circuit is renowned for selecting a single input from multiple inputs & directing the binary information to output line?

- a) Data Selector
- b) Data distributor
- c) Both data selector and data distributor
- d) DeMultiplexer

2:1 Multiplexer



$$Y = D_0 \bar{S} + D_1 S$$

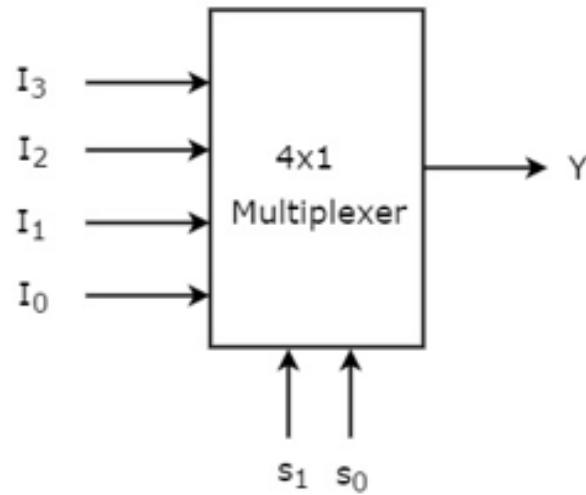
Select	Inputs		Output
0	0	0	0
0	0	1	1
1	1	0	1
1	1	1	1

QUICK QUIZ (POLL)

What is the function of an enable input on a multiplexer chip?

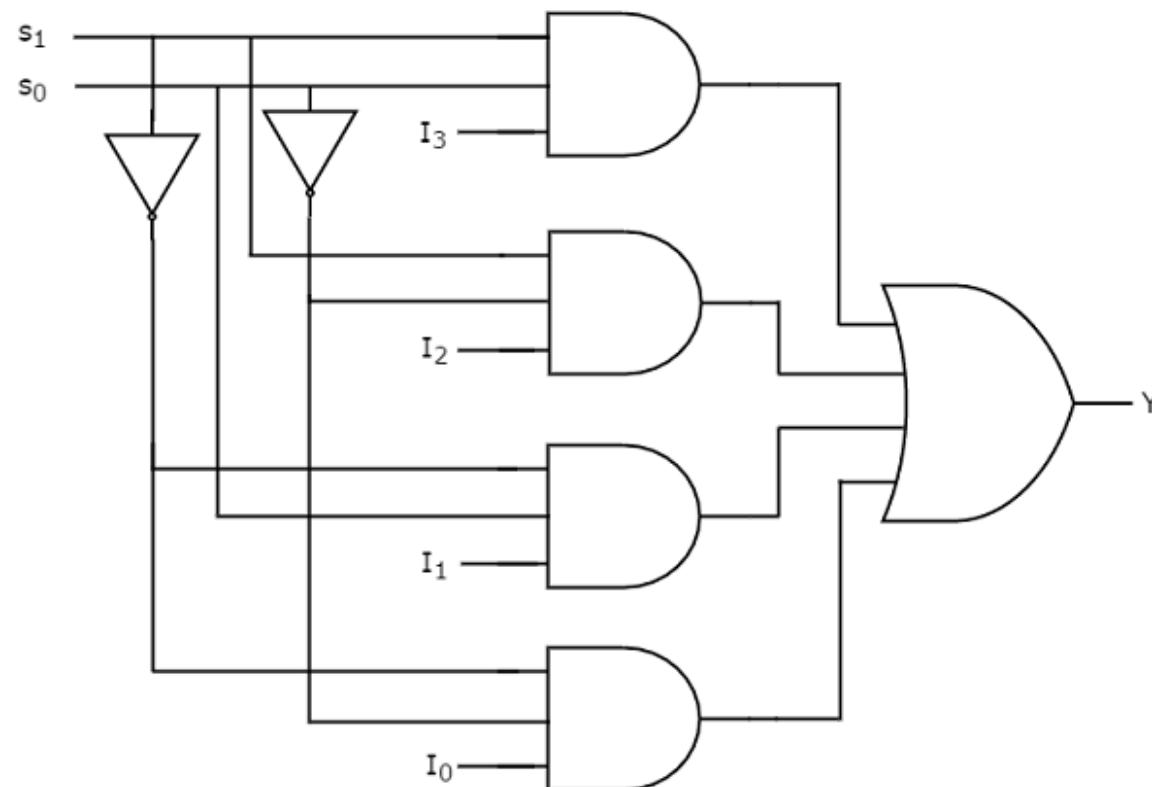
- a) To apply Vcc
- b) To connect ground
- c) To active the entire chip
- d) To active one half of the chip

4:1 Multiplexer



Selection Lines		Output
S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

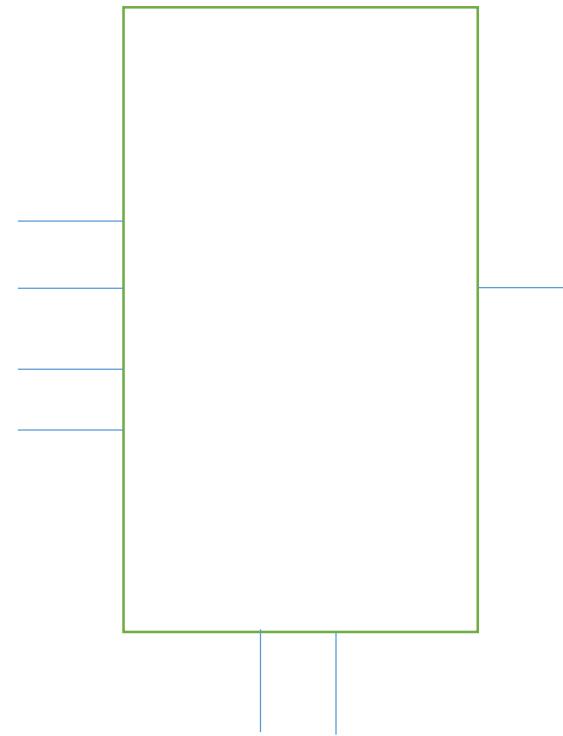
$$Y = S_1' S_0' I_0 + S_1' S_0 I_1 + S_1 S_0' I_2 + S_1 S_0 I_3$$



Boolean Function Implementation using Multiplexer

Implement $f = \sum(1, 3, 5, 6)$ using 4:1 MUX?

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



Boolean Function Implementation using Multiplexer

TRY YOURSELF

Implement $f = \sum(0, 2, 4, 5, 6)$ using 4:1 MUX?

Full Adder Implementation using Multiplexer

TRY YOURSELF

Logic Gates Implementation using Multiplexer

AND GATE

A	B	OUT
0	0	0
0	1	0
1	0	0
1	1	1

OUT = 0
when A = 0

OUT = B
when A = 1



OR GATE

A	B	OUT
0	0	0
0	1	1
1	0	1
1	1	1

OUT = B
when A = 0

OUT = 1
when A = 1

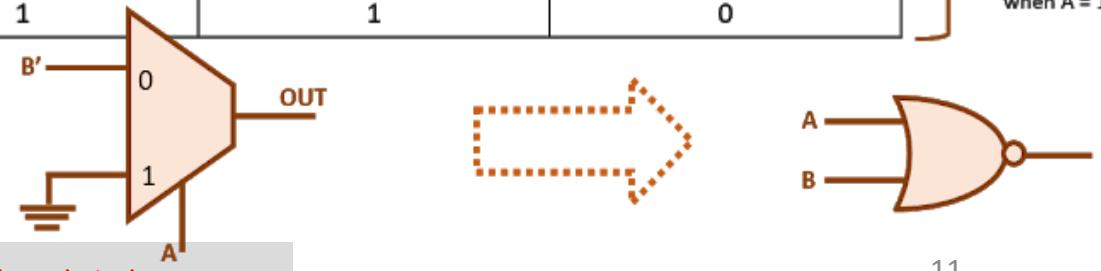


NOR GATE

A	B	OUT
0	0	1
0	1	0
1	0	0
1	1	0

OUT = B'
when A = 0

OUT = 0
when A = 1



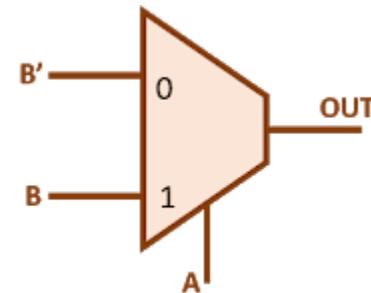
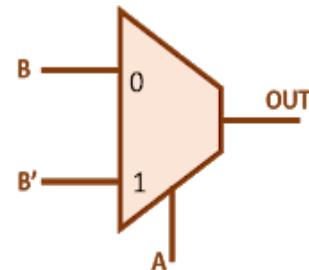
Logic Gates Implementation using Multiplexer

XOR GATE

A	B	OUT
0	0	0
0	1	1
1	0	1
1	1	0

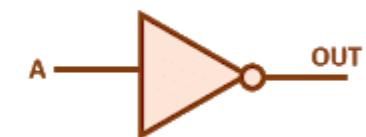
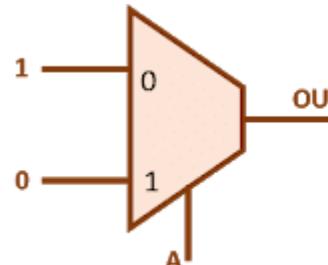
XNOR GATE

A	B	OUT
0	0	1
0	1	0
1	0	0
1	1	1



NOT GATE

A	OUT
0	1
1	0

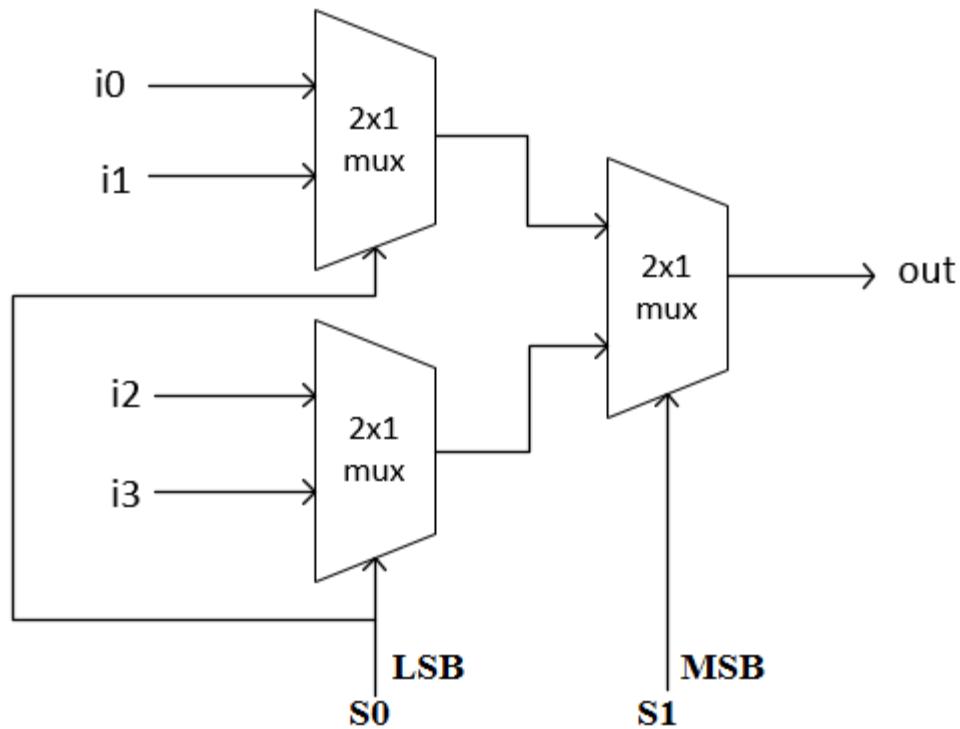


Multiplexer Trees

Multiplexers with higher number of inputs can be implemented by cascading two or more multiplexers with less number of inputs.

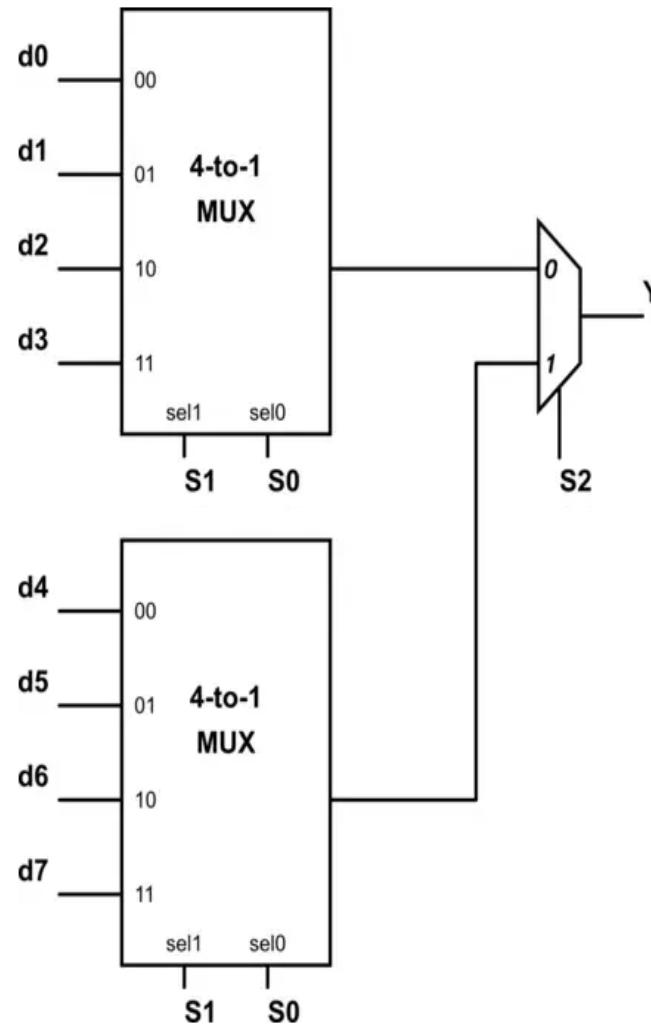
4:1 MUX using 2:1 MUX

S1 (MSB)	S0(LSB)	Out
0	0	I0
0	1	I1
1	0	I2
1	1	I3



8:1 MUX using 4:1 MUX

Select Data Inputs			Output
S_2	S_1	S_0	Y
0	0	0	D_0
0	0	1	D_1
0	1	0	D_2
0	1	1	D_3
1	0	0	D_4
1	0	1	D_5
1	1	0	D_6
1	1	1	D_7



8:1 MUX: Example

Question: Implement the Boolean function $f(A, B, C, D) = \sum(1, 3, 4, 11, 12, 13, 14, 15)$ using Multiplexer?

8:1 MUX: Example

Question: Implement the Boolean function $f(A, B, C, D) = \sum(1, 3, 4, 11, 12, 13, 14, 15)$ using Multiplexer?

DIGITAL ELECTRONICS: ECE 213

Topic: Demultiplexers and Encoders
UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families

Lecture No.: 18

Prepared By: Irfan Ahmad Pindoo

Assistant Professor

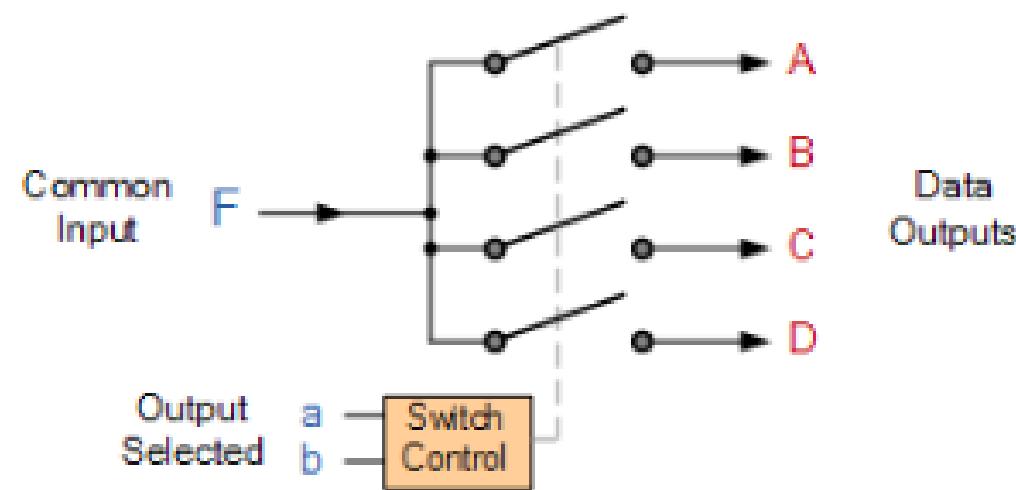
VLSI Design, ECE

School of Computer Science and Engineering



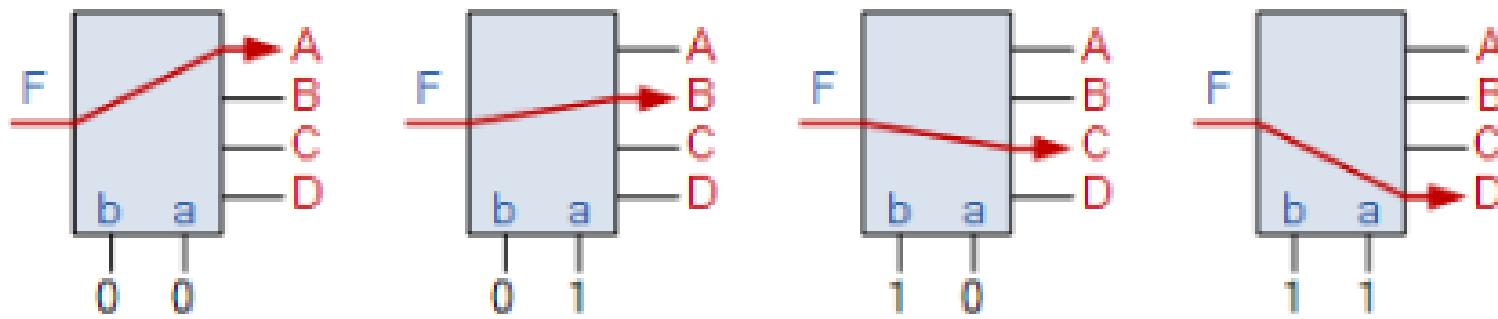
De-Multiplexer

- De-Multiplexer is a combinational circuit that performs the reverse operation of Multiplexer.
- It has single input, 'n' selection lines and maximum of 2^n outputs.
- The input will be connected to one of these outputs based on the values of selection lines.

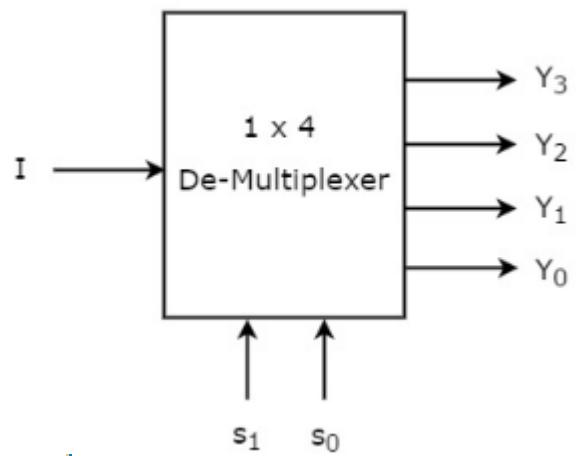


De-Multiplexer

- The **demultiplexer** converts a serial data signal at the input to a parallel data at its output lines as shown below.



1:4 De-Multiplexer



$$Y_3 = s_1 s_0 I$$

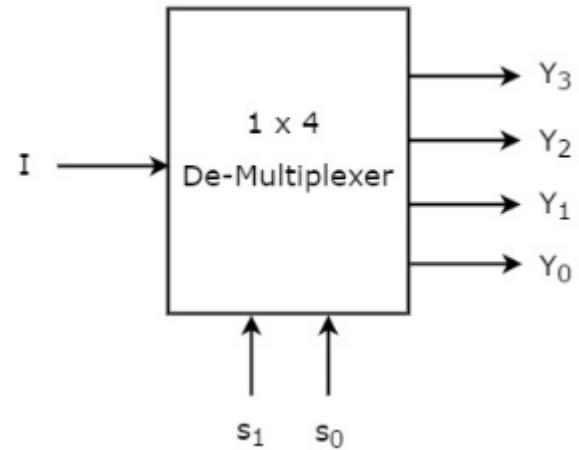
$$Y_2 = s_1 s_0' I$$

$$Y_1 = s_1' s_0 I$$

$$Y_0 = s_1' s_0' I$$

Selection Inputs		Outputs			
s_1	s_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

1:4 De-Multiplexer



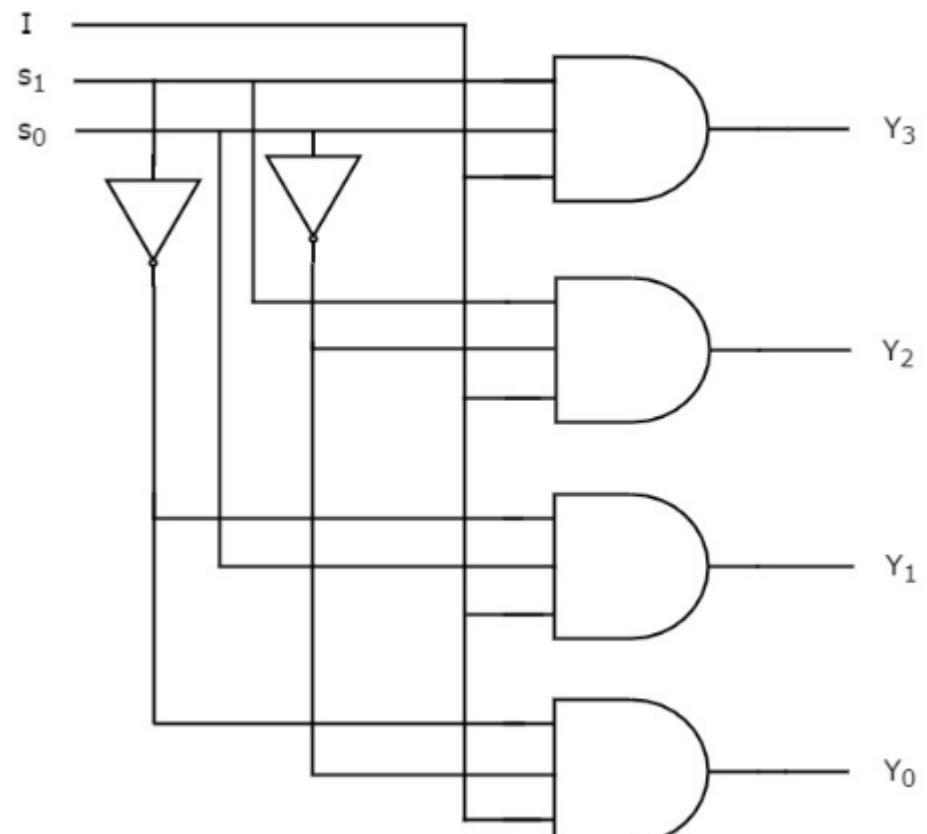
$$Y_3 = s_1 s_0 I$$

$$Y_2 = s_1 s_0' I$$

$$Y_1 = s_1' s_0 I$$

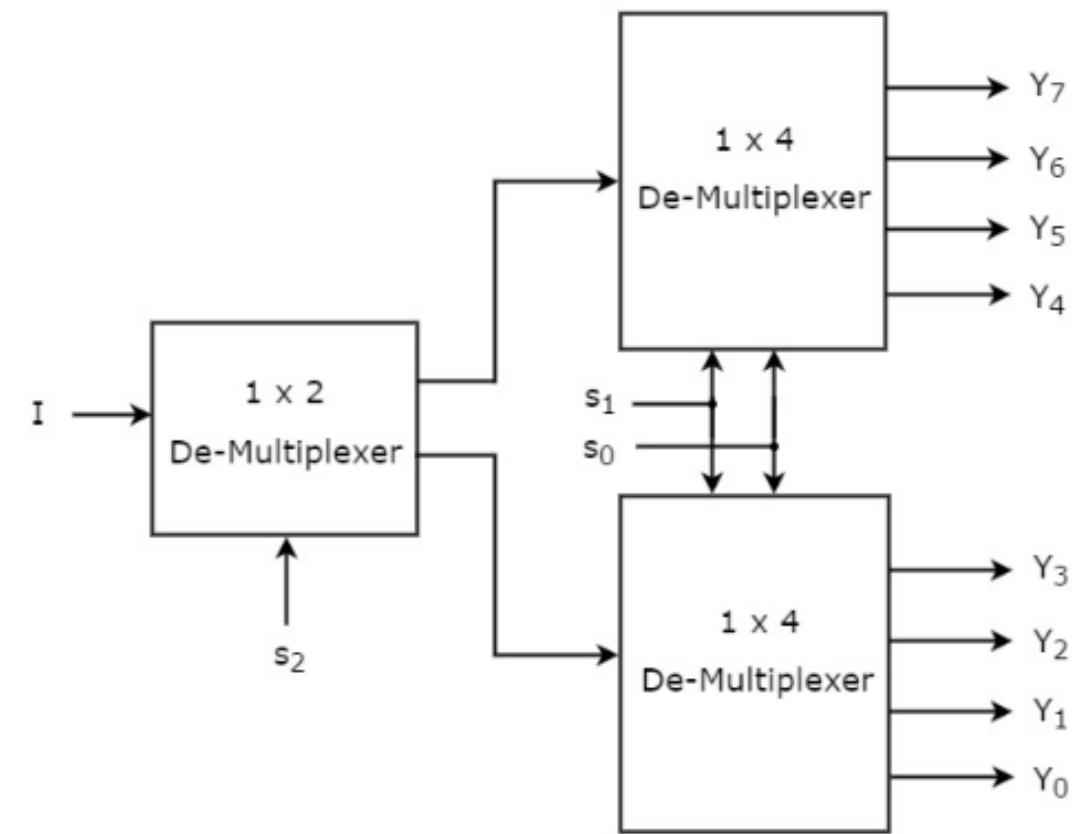
$$Y_0 = s_1' s_0' I$$

Selection Inputs		Outputs			
s_1	s_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0



1:8 De-MUX using 1:4 De-MUX

Selection Inputs			Outputs							
s_2	s_1	s_0	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	1	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0



QUICK QUIZ (POLL)

To implement 1:8 de-mux how many 1:2 de-mux would be required?

- a) 2
- b) 4
- c) 5
- d) 6

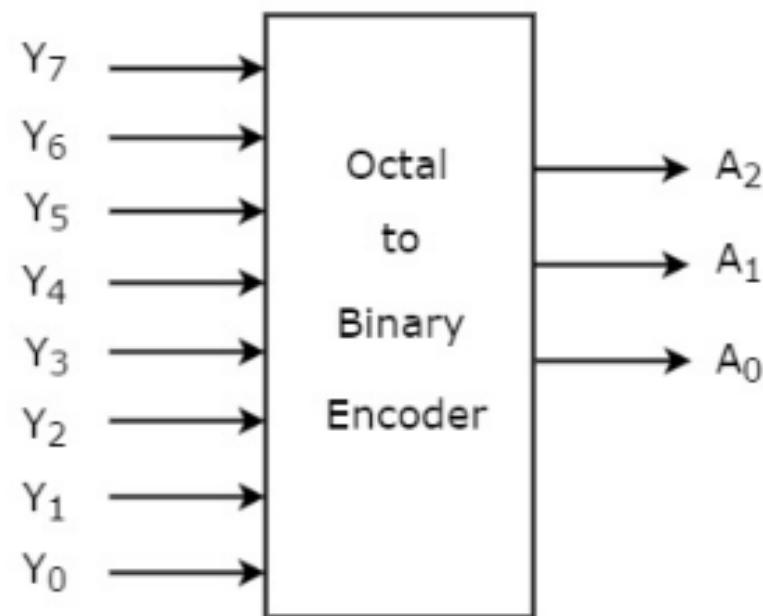
Encoder

- An encoder is a combinational circuit that performs the inverse operation of a decoder.
- An encoder has 2^n (or fewer) input lines and n output lines.
- The output lines, **as an aggregate**, generate the **binary code** corresponding to the input value.

Encoder: Example

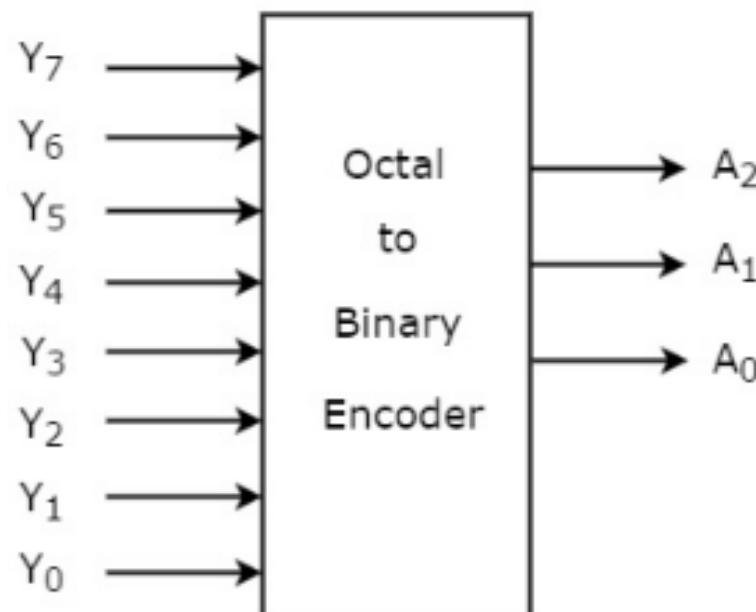
Qsn: Design an octal to binary encoder?

Octal to binary Encoder has eight inputs, Y_7 to Y_0 and three outputs A_2 , A_1 & A_0 . Octal to binary encoder is nothing but 8 to 3 encoder. The block diagram of octal to binary Encoder is shown in the following figure.



Encoder: Example

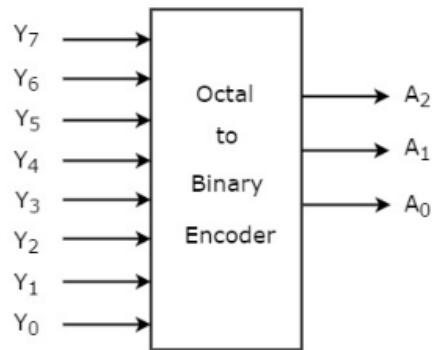
Qsn: Design an octal to binary encoder?



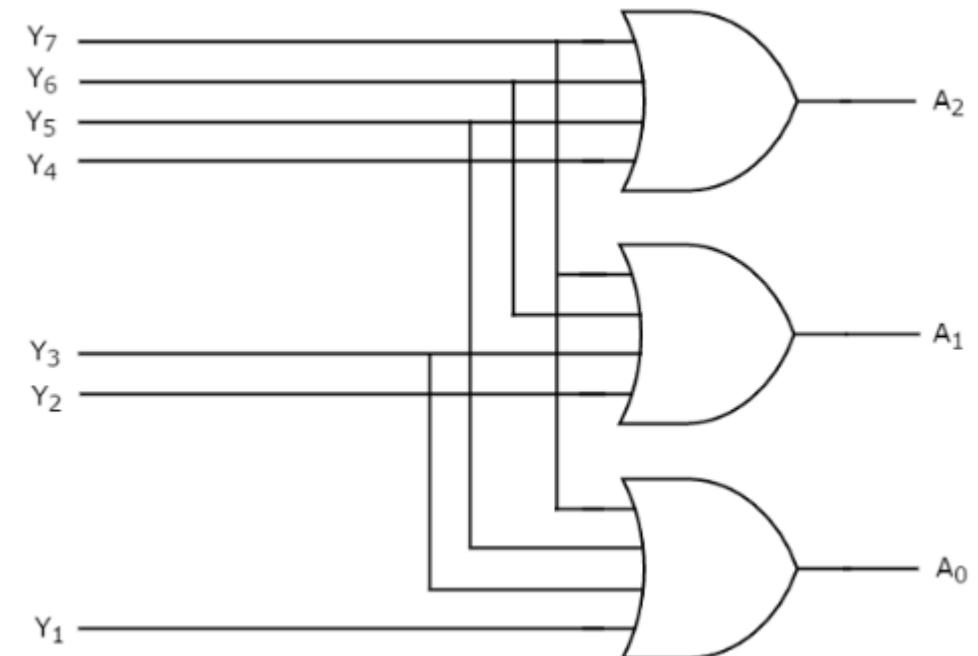
Inputs								Outputs		
Y ₇	Y ₆	Y ₅	Y ₄	Y ₃	Y ₂	Y ₁	Y ₀	A ₂	A ₁	A ₀
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

Encoder: Example

Qsn: Design an octal to binary encoder?



Inputs								Outputs		
Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0	A_2	A_1	A_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1



$$A_2 = Y_7 + Y_6 + Y_5 + Y_4$$

$$A_1 = Y_7 + Y_6 + Y_3 + Y_2$$

$$A_0 = Y_7 + Y_5 + Y_3 + Y_1$$

QUICK QUIZ (POLL)

How is an encoder different from a decoder?

- a) The output of an encoder is a binary code for 1-of-N input
- b) The output of a decoder is a binary code for 1-of-N input
- c) The output of an encoder is a binary code for N-of-1 output
- d) The output of a decoder is a binary code for N-of-1 output

QUICK QUIZ (POLL)

How many outputs will a decimal-to-BCD encoder have?

- a) 4
- b) 8
- c) 12
- d) 16

DIGITAL ELECTRONICS: ECE 213

**Topic: Priority Encoders, Comparator
and Parity Generator and Checker**

**UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families**

Lecture No.: 19

Prepared By: Irfan Ahmad Pindoo
Assistant Professor
VLSI Design, ECE

School of Computer Science and Engineering



Limitations of Encoder

- There is an ambiguity, when all outputs of encoder are equal to zero. Because, it could be the code corresponding to the inputs, when only least significant input is one or when all inputs are zero.
- If more than one input is active High, then the encoder produces an output, which may not be the correct code. For example, if both Y3 and Y6 are '1', then the encoder produces 111 at the output. This is neither equivalent code corresponding to Y3, when it is '1' nor the equivalent code corresponding to Y6, when it is '1'.
- So, to overcome these difficulties, we should **assign priorities** to each input of encoder. Then, the output of encoder will be the binary code corresponding to the active High inputs, which has higher priority. This encoder is called as **priority encoder**.

Priority Encoder

- A priority encoder is an encoder circuit that includes the priority function. The operation of the priority encoder is such that **if two or more inputs** are equal to 1 at the **same time**, the input having the **highest priority will take precedence**.
- In addition to the two outputs x and y , the circuit has **a third output designated by V** ; this is a **valid bit indicator** that is set to 1 when one or more inputs are equal to 1. If all inputs are 0, there is no valid input and V is equal to 0.
- The other two outputs are not inspected when V equals 0 and are specified as don't-care conditions.

Priority Encoder

Designing of 4X2 Priority Encoder

Truth Table of a Priority Encoder

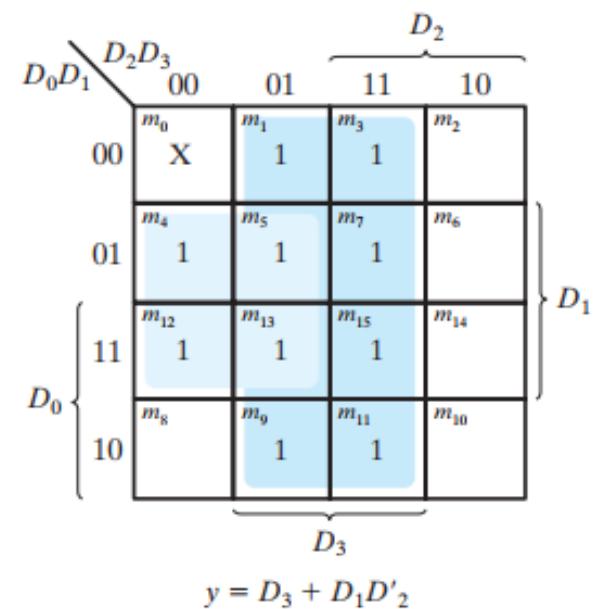
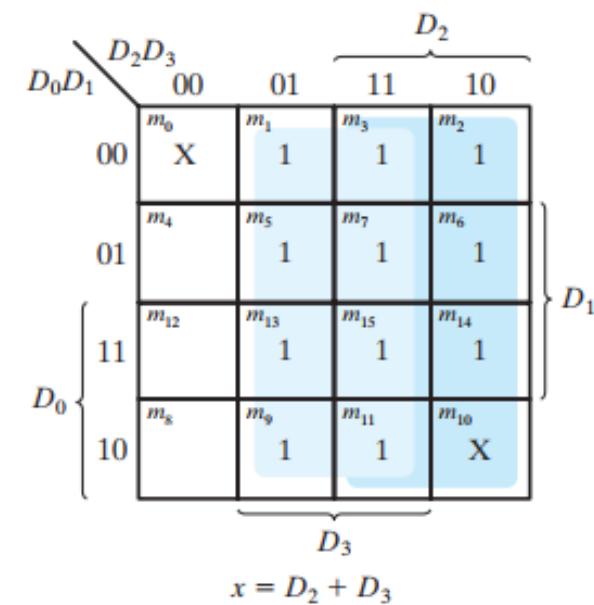
Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	v
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

Priority Encoder

Designing of 4X2 Priority Encoder

Truth Table of a Priority Encoder

Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

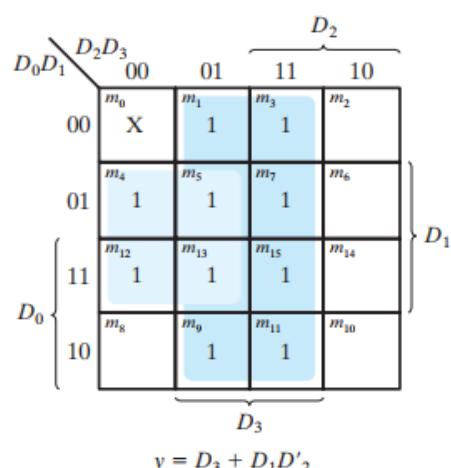
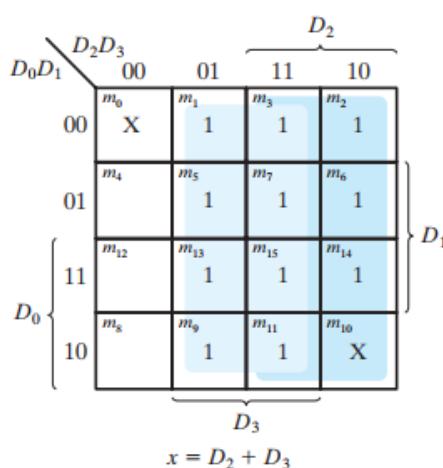


Priority Encoder

Designing of 4X2 Priority Encoder

Truth Table of a Priority Encoder

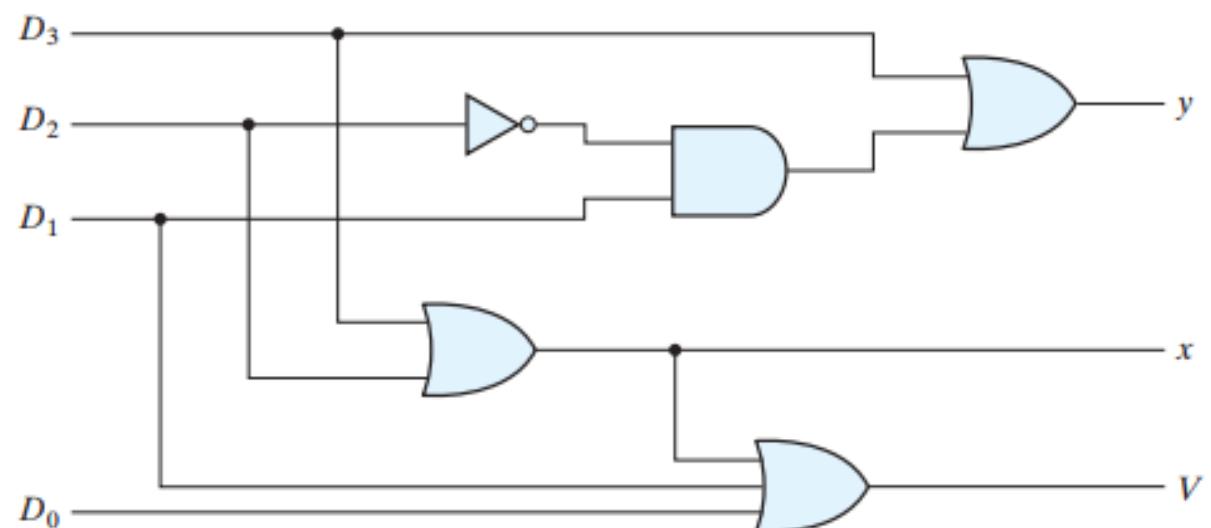
Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1



$$x = D_2 + D_3$$

$$y = D_3 + D_1 D'_2$$

$$V = D_0 + D_1 + D_2 + D_3$$



Magnitude Comparator

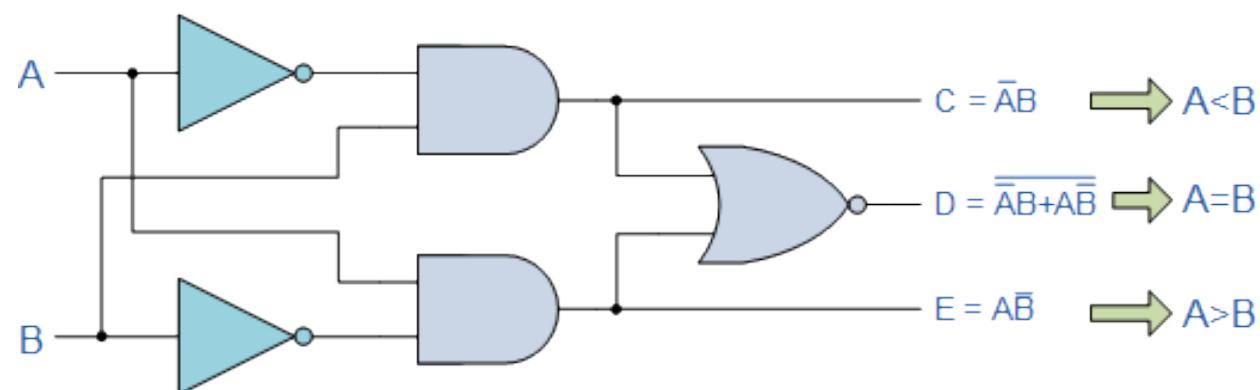
- Another common and very useful combinational logic circuit is that of the Digital Comparator circuit.
- Digital or Binary Comparators are made up from standard AND, NOR and NOT gates that compare the digital signals present at their input terminals and produce an output depending upon the condition of those inputs.



1-Bit Magnitude Comparator

- Another common and very useful combinational logic circuit is that of the Digital Comparator circuit.
- Digital or Binary Comparators are made up from standard AND, NOR and NOT gates that compare the digital signals present at their input terminals and produce an output depending upon the condition of those inputs.

A	B	$A < B$	$A = B$	$A > B$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0



2-Bit Magnitude Comparator

Step 1:

Determine number of inputs and outputs.

Step 2 and 3:

INPUT				OUTPUT		
A1	A0	B1	B0	A<B	A=B	A>B
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

2-Bit Magnitude Comparator

Step 2 and 3:

Step 4: K Maps

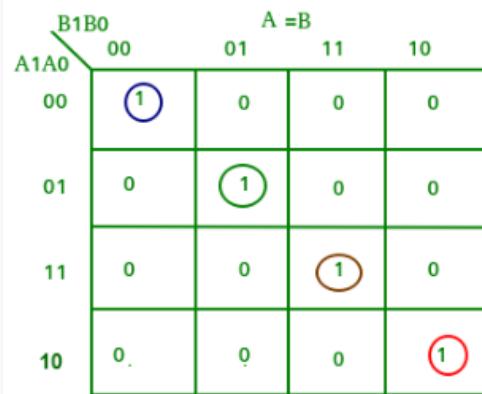
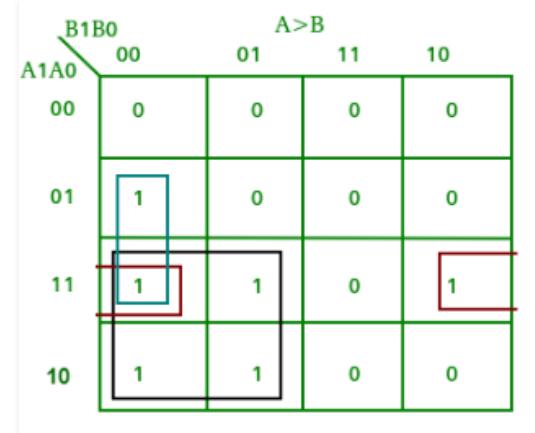
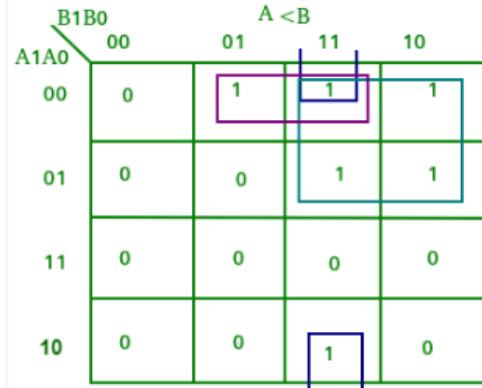
INPUT				OUTPUT		
A1	A0	B1	B0	A<B	A=B	A>B
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

2-Bit Magnitude Comparator

Step 2 and 3:

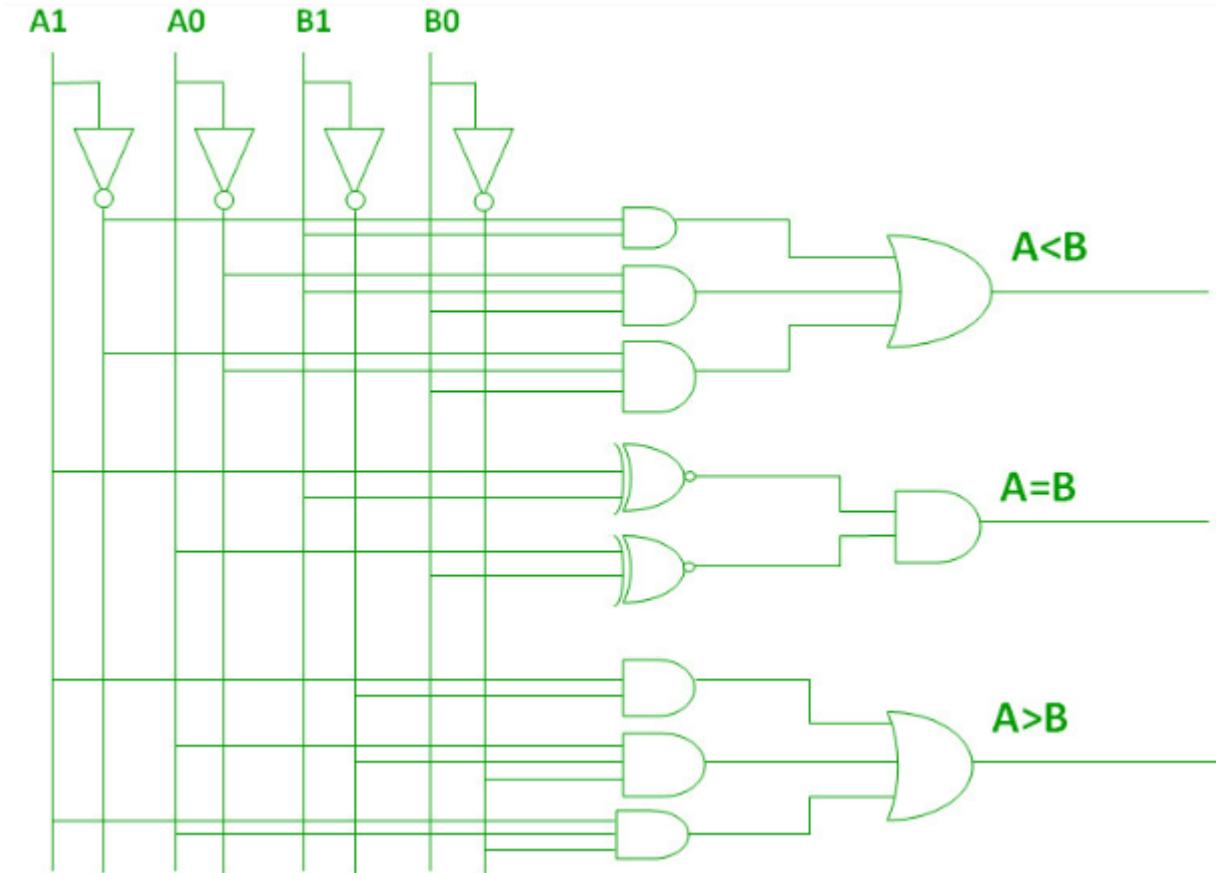
INPUT				OUTPUT		
A1	A0	B1	B0	A < B	A = B	A > B
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

Step 4: K Maps



2-Bit Magnitude Comparator

Step 5: Logic Circuit



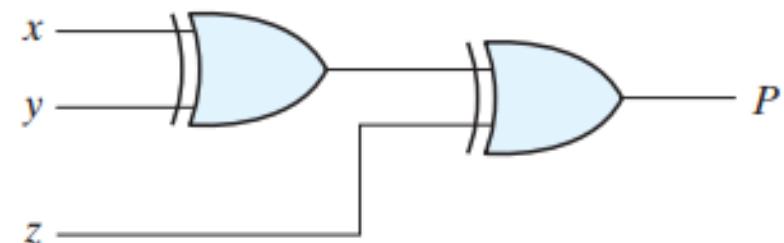
Parity Generation and Checking

- ❑ Exclusive-OR functions are very useful in systems requiring error detection and correction codes.
- ❑ A parity bit is an extra bit included with a binary message to make the number of 1's either odd or even.
- ❑ The message, including the parity bit, is transmitted and then checked at the receiving end for errors.
- ❑ The circuit that generates the parity bit in the transmitter is called a parity generator. The circuit that checks the parity in the receiver is called a parity checker.

Parity Generation and Checking

Even-Parity-Generator Truth Table

Three-Bit Message			Parity Bit
x	y	z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



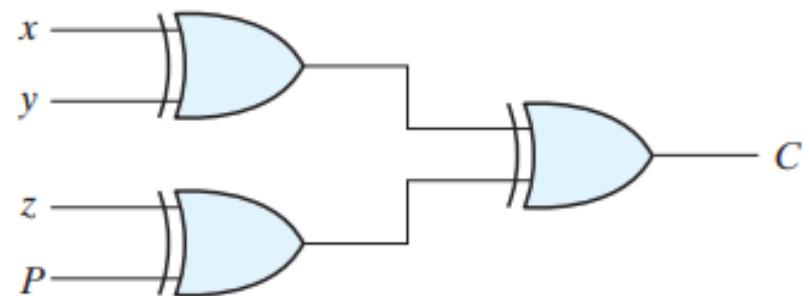
(a) 3-bit even parity generator

$$P = x \oplus y \oplus z$$

Parity Generation and Checking

Even-Parity-Checker Truth Table

Four Bits Received				Parity Error Check
x	y	z	P	C
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



(b) 4-bit even parity checker

$$C = x \oplus y \oplus z \oplus P$$

DIGITAL ELECTRONICS: ECE 213

Topic: Digital IC Logic Families
UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families

Lecture No.: 20

Prepared By: Irfan Ahmad Pindoo

Assistant Professor

VLSI Design, ECE

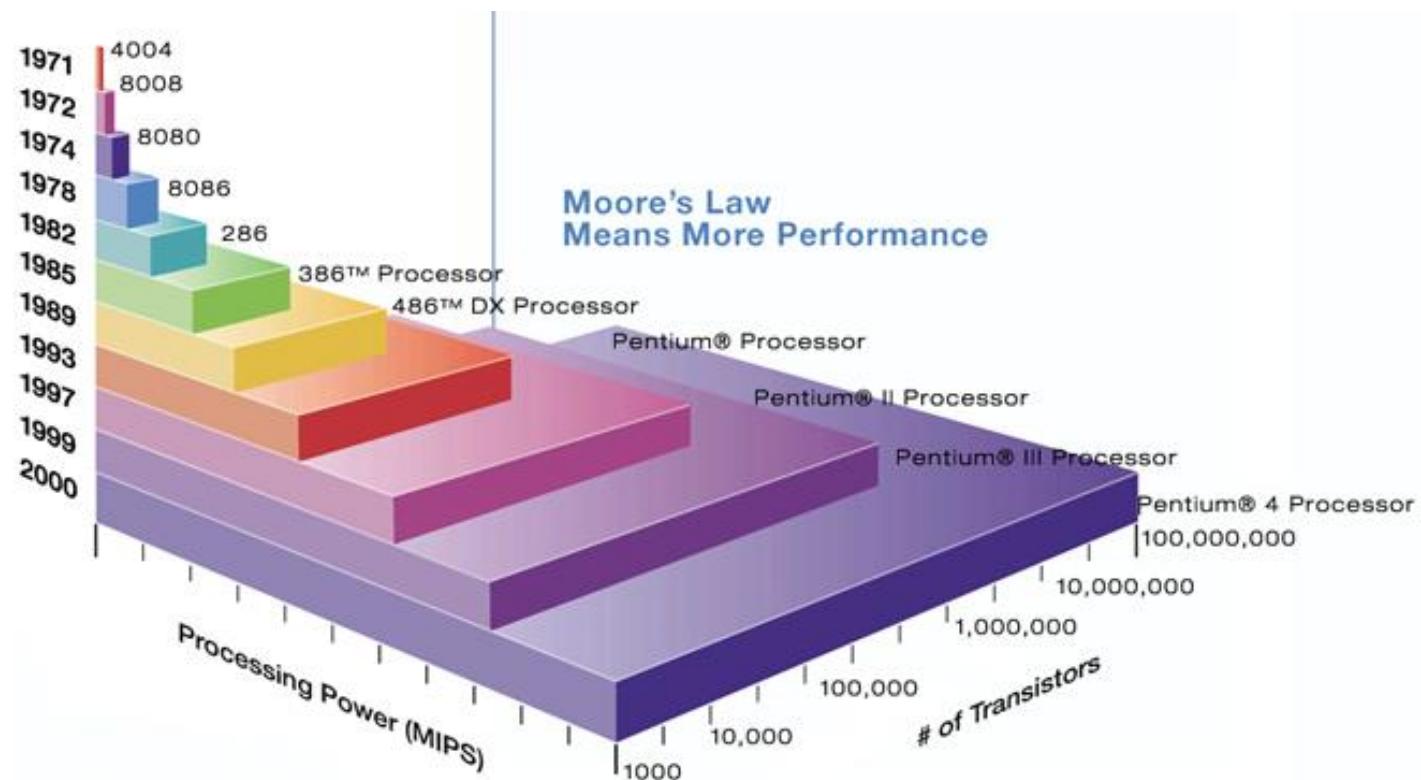
School of Computer Science and Engineering



DIGITAL IC LOGIC FAMILIES:

Moore's Law

- A prediction made by Moore (a co-founder of Intel) in 1965: "... a number of transistors to double every 2 years."



Logic Families

OBSOLETE ONES:

1. Diode Logic.
2. Diode Transistor Logic (DTL).
3. Resistor Transistor Logic (RTL).

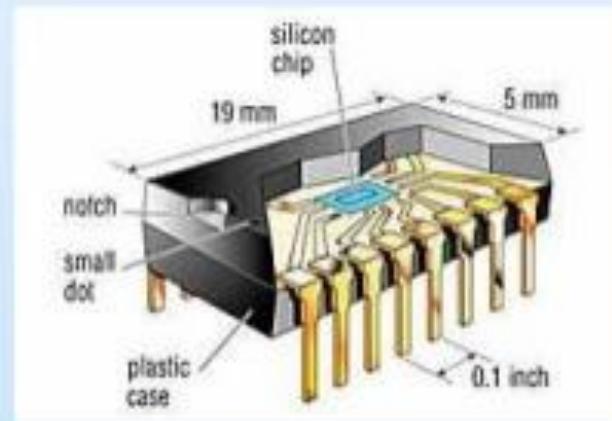
CURRENT ONES:

1. TTL (Transistor Transistor Logic)
2. ECL (Emitter Coupled Logic)
3. CMOS (Complementary Metal Oxide Semiconductor)

Characteristics of an Ideal Logic Family

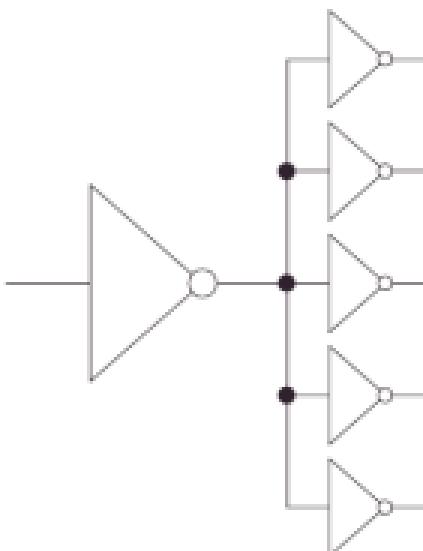
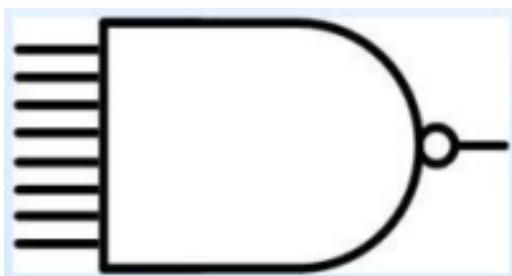
The most important parameters for evaluating and comparing logic families include :

- ◆ Logic Levels
- ◆ Power Dissipation
- ◆ Propagation delay
- ◆ Noise margin
- ◆ Fan-out (loading)

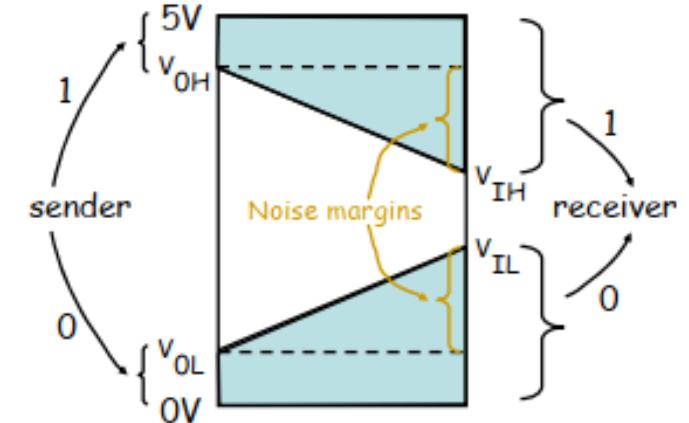


Characteristics of Logic Family

- Fan in : The number of inputs that the gate can handle properly without disturbing the output level.
- Fan out : The number of inputs that can be driven simultaneously by the output without disturbing the output level.
- Noise immunity : Noise immunity is the ability of the logic circuit to tolerate the noise voltage.



Hold the sender to tougher standards!



$$\begin{aligned}\text{"1" noise margin: } & V_{IH} - V_{OH} \\ \text{"0" noise margin: } & V_{IL} - V_{OL}\end{aligned}$$

As illustrated in Figure the *noise margin for a logical 0* is given by

$$NM_0 = V_{IL} - V_{OL}$$

and the *noise margin for a logical 1* is given by

$$NM_1 = V_{OH} - V_{IH}$$

Characteristics of Logic Family

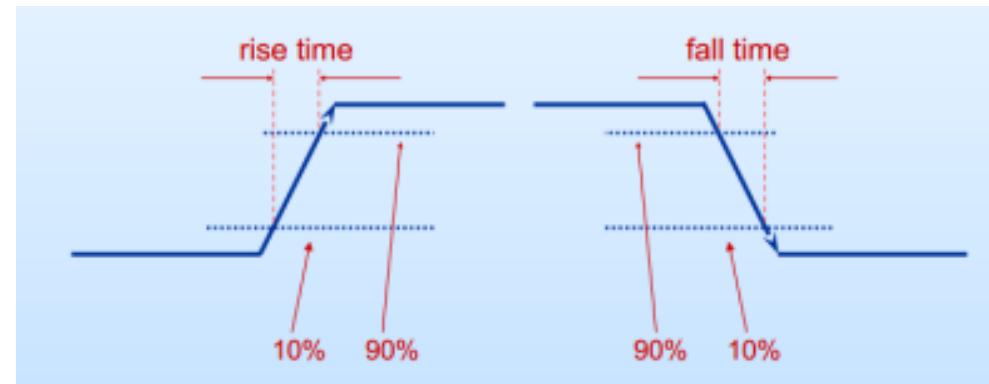
➤ Noise Margin : The quantitative measure of noise immunity is called noise margin.

➤ Propagation Delay : The propagation delay of gate is the average transition delay time for the signal to propagate from input to output

➤ Threshold Voltage : The voltage at which the circuit changes from one state to another state

➤ Operating Speed : The speed of operation of the logic gate is the time that elapses between giving input and getting output.

➤ Power Dissipation : The power dissipation is defined as power needed by the logic circuit.



QUICK QUIZ (POLL)

If a logic circuit has a fan out of 4 then the circuit

- A. 4 input
- B. has 4 outputs
- C. can drive maximum of 4 inputs
- D. gives output 4 times the input

QUICK QUIZ (POLL)

The rise time (t_r) is the time it takes for a pulse to rise from its _____ point up to its _____ point. The fall time (t_f) is the length of time it takes to fall from the _____ to the _____ point.

- A. 10%, 90%, 90%, 10%
- B. 90%, 10%, 10%, 90%
- C. 20%, 80%, 80%, 20%
- D. 10%, 70.7%, 70.7%, 10%

QUICK QUIZ (POLL)

A RTL consists of

- A. Resistor ,transistor and inductors
- B. Resistors,diodes and bipolar junction transistor
- C. Resistors,capacitors and diodes
- D. Resistors and transistors



TTL IC Family

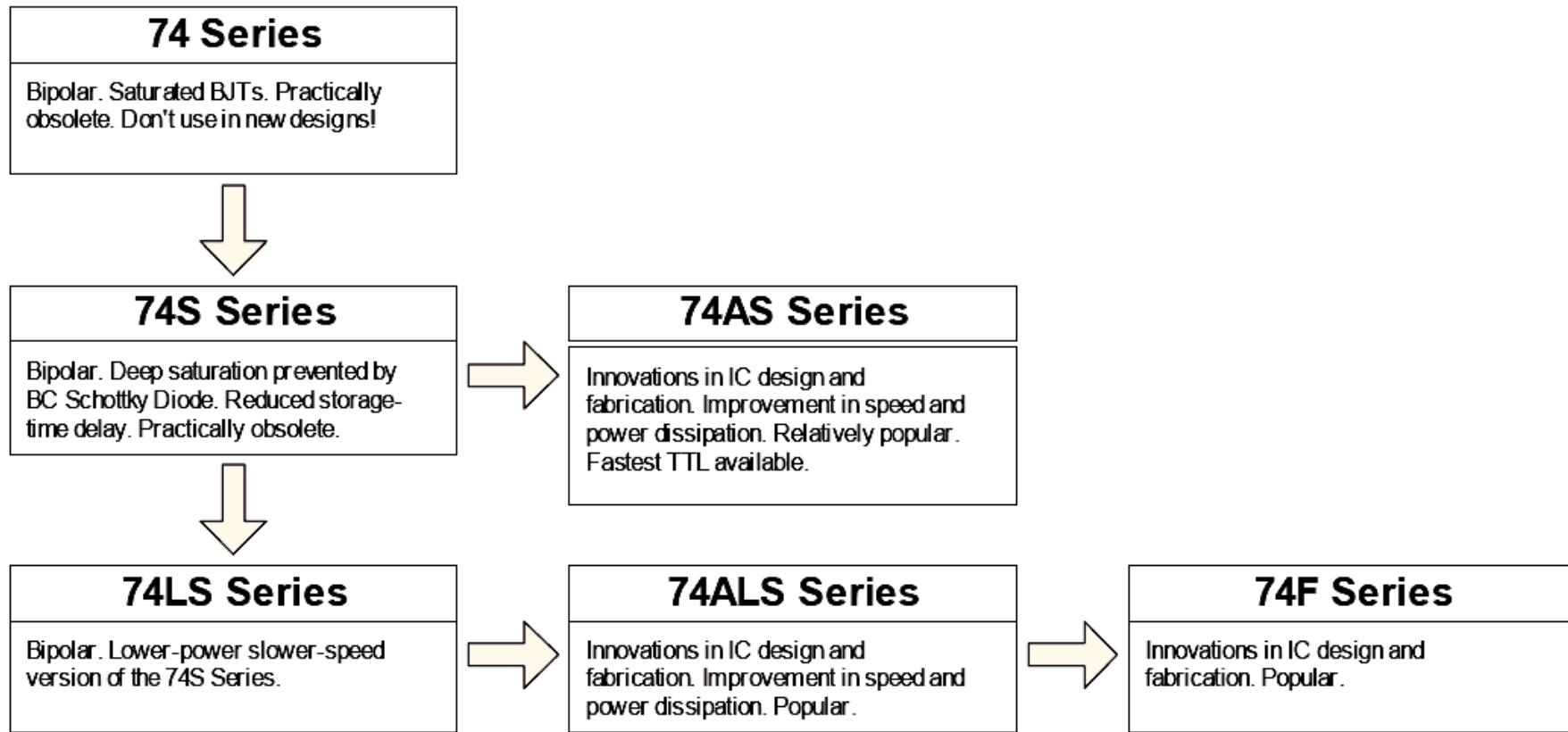
- In Transistor-Transistor logic or just TTL, logic gates are built only around transistors
- TTL was developed in 1965
- Through the years basic TTL has been improved to meet performance requirements. There are many versions or families of TTL. For example
 - Standard TTL
 - High Speed TTL (twice as fast, twice as much power)
 - Low Power TTL (1/10 the speed, 1/10 the power of "standard" TTL)
 - Schottky TTL etc. (for high-frequency uses)

TTL IC Family

- Transistor-Transistor Logic Families:

- 74L Low power
- 74H High speed
- 74S Schottky
- 74LS Low power Schottky
- 74AS Advanced Schottky
- 74ALS Advance Low power Schottky

TTL IC Family Evolution



Legacy: don't use
in new designs

Widely used today

ECL IC Family

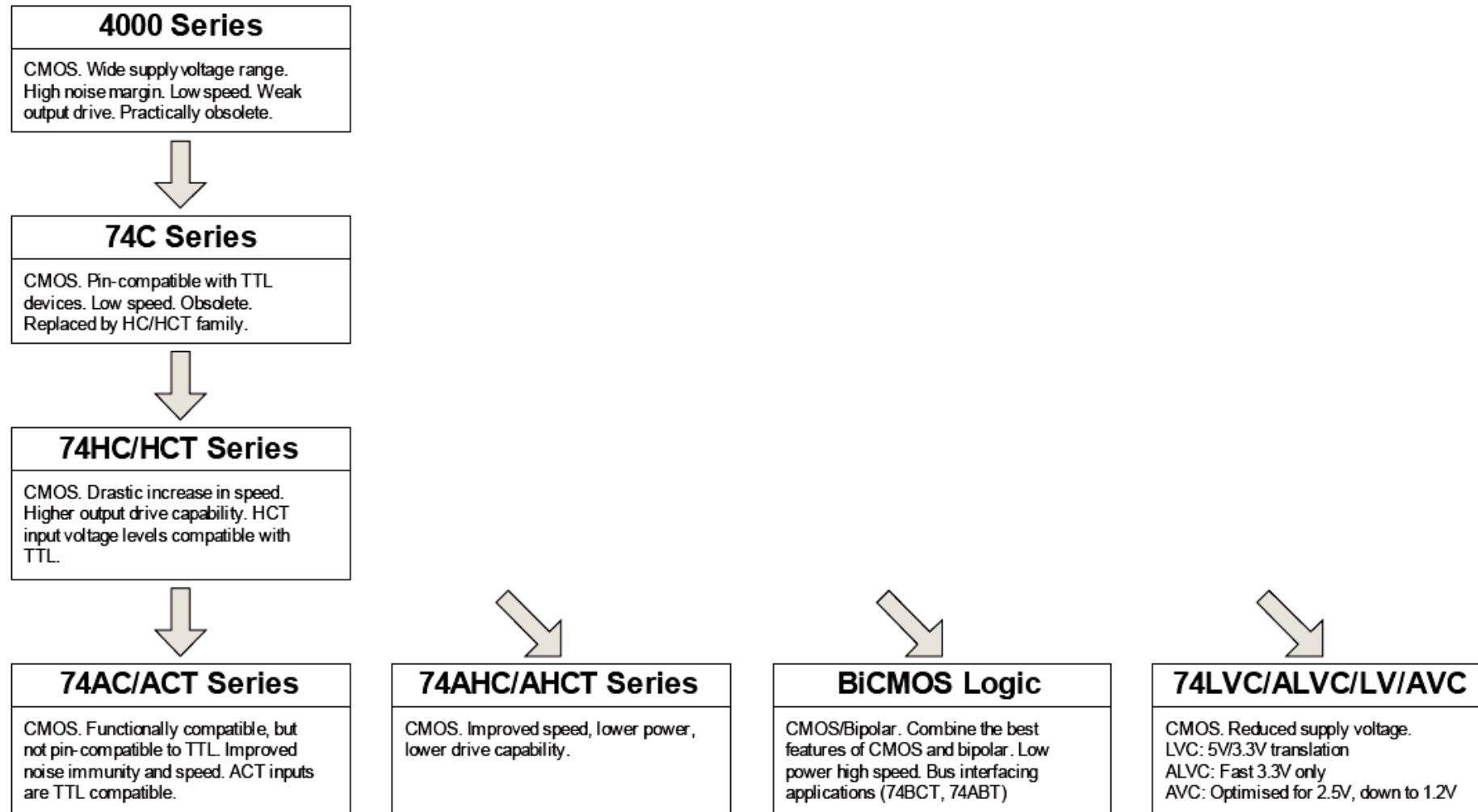
- PROS: Fastest logic family available ($\sim 1\text{ns}$)
- CONS: low noise margin and high power dissipation
- Operated in emitter coupled geometry (recall differential amplifier or emitter-follower), transistors are biased and operate near their Q-point (never near saturation!)
- Logic levels. “0”: -1.7V . “1”: -0.8V
- Such strange logic levels require extra effort when interfacing to TTL/CMOS logic families

CMOS IC Family

Complementary MOS (CMOS)

- Other variants: NMOS, PMOS (obsolete)
- Very low static power consumption
- Scaling capabilities (large integration all MOS)
- Full swing: rail-to-rail output

CMOS Family Evolution



QUICK QUIZ (POLL)

The standard TTL gates are marketed as ____ series.

- a) 80
- b) 82
- c) 74
- d) 08

QUICK QUIZ (POLL)

. The full form of ECL is _____

- a) Emitter-collector logic
- b) Emitter-complementary logic
- c) Emitter-coupled logic
- d) Emitter-cored logic

QUICK QUIZ (POLL)

Which logic is the fastest of all the logic families?

- a) TTL
- b) ECL
- c) HTL
- d) DTL

TTL vs CMOS

● TTL

- faster (some versions)
- strong drive capability
- rugged

● CMOS

- lower power consumption
- simpler to make
- greater packing density
- better noise immunity

QUICK QUIZ (POLL)

Which of the following is not the advantage of MOS gates?

- a) Low power dissipation
- b) Small size
- c) Good immunity to noise
- d) High switching speeds

DIGITAL ELECTRONICS: ECE 213

Topic: Digital IC Logic Families
UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families

Lecture No.: 21

Prepared By: Irfan Ahmad Pindoo

Assistant Professor

VLSI Design, ECE

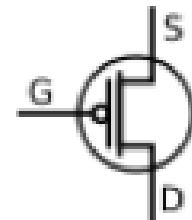
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CMOS as a Switch

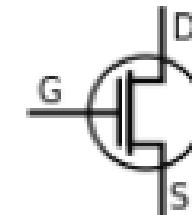
PMOS

ON → Gate input Low
OFF → Gate input High



NMOS

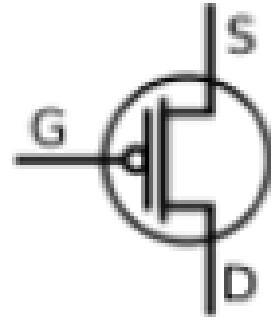
OFF → Gate input Low
ON → Gate input High



QUICK QUIZ (POLL)

If the input applied to the gate is GND, what would be the output of the following transistor?

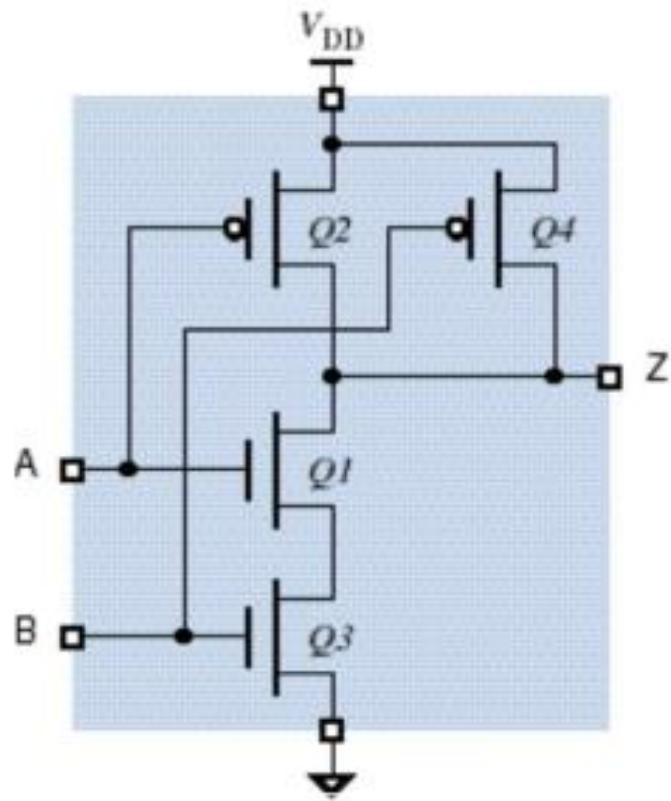
- a) ON state
- b) OFF state
- c) GND
- d) None of these



CMOS as a NOT Gate

<i>Truth Table</i>		<i>Integrated Circuit</i>													
<table border="1"><thead><tr><th>V_{in}</th><th>T_1</th><th>T_2</th><th>V_{out}</th></tr></thead><tbody><tr><td>0</td><td>off</td><td>on</td><td>1</td></tr><tr><td>1</td><td>off</td><td>off</td><td>0</td></tr></tbody></table>		V_{in}	T_1	T_2	V_{out}	0	off	on	1	1	off	off	0		
V_{in}	T_1	T_2	V_{out}												
0	off	on	1												
1	off	off	0												

CMOS as a NAND Gate



A	B	<i>Q1</i>	<i>Q2</i>	<i>Q3</i>	<i>Q4</i>	Z
L	L	off	on	off	on	H
L	H	off	on	on	off	H
H	L	on	off	off	on	H
H	H	on	off	on	off	L



CMOS as a NOR Gate

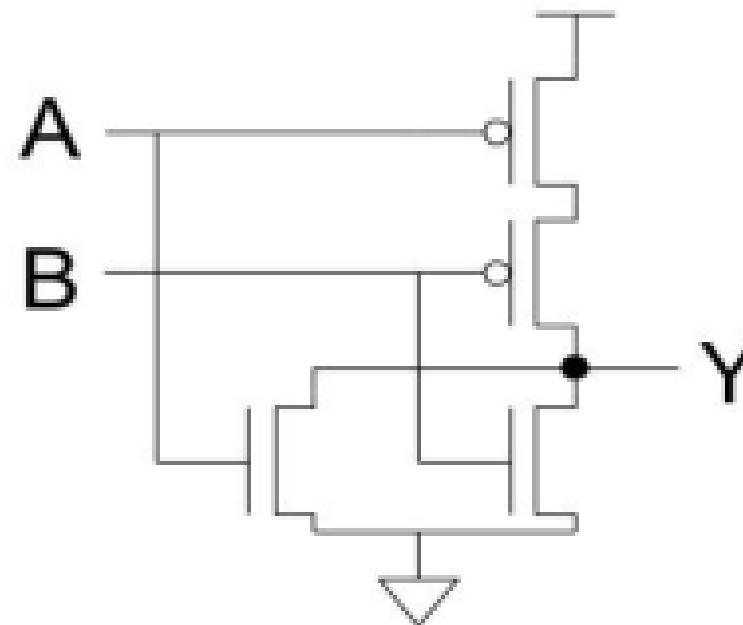
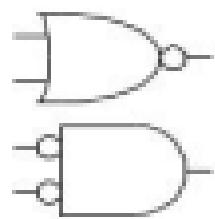
A B Y

0 0 1

0 1 0

1 0 0

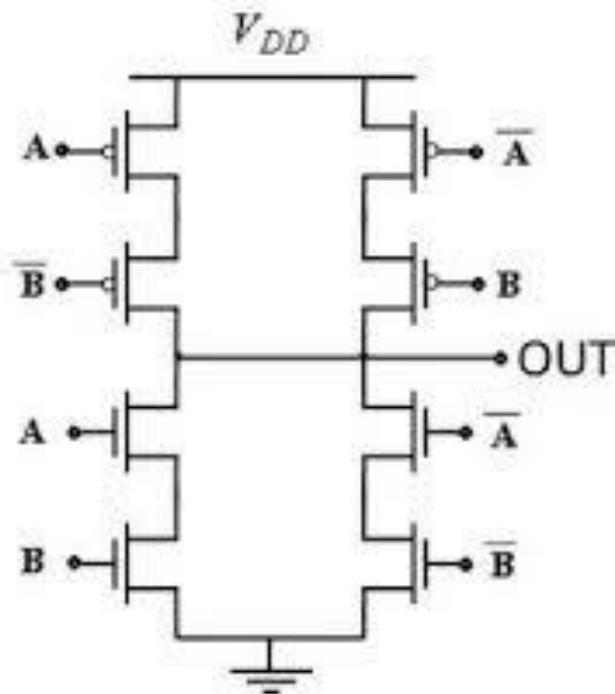
1 1 0



QUICK QUIZ (POLL)

Identify the logic gate provided below?

- a) AND gate
- b) OR gate
- c) XNOR gate
- d) XOR gate



Comparison of Logic Families

Parameter	CMOS	TTL	ECL
Basic gate	NAND/NOR	NAND	OR/NOR
Fan-out	>50	10	25
Power per gate (mW)	1 @ 1 MHz	1 - 22	4 - 55
Noise immunity	Excellent	Very good	Good
t_{PD} (ns)	1 - 200	1.5 – 33	1 - 4

DIGITAL ELECTRONICS: ECE 213

Topic: PRACTICE QUESTIONS.
UNIT III: Introduction to
Combinational Logic Circuits and Logic
Families

Lecture No.: 22

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PRACTICE QUESTIONS:

PRACTICE QUESTIONS:

Q. Design a combinational circuit that converts:

- a) 4-bit binary to Gray code.
- b) 4-bit Gray code to binary

PRACTICE QUESTIONS:

Q. Design a combinational circuit that converts:

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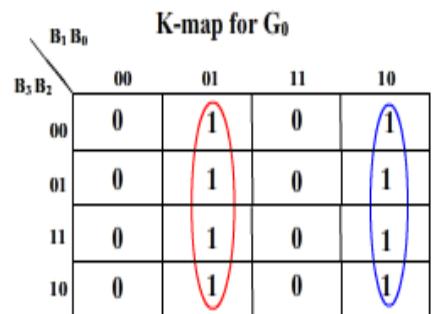
Binary				Gray Code			
b ₃	b ₂	b ₁	b ₀	g ₃	g ₂	g ₁	g ₀
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0

PRACTICE QUESTIONS:

Q. Design a combinational circuit that converts:

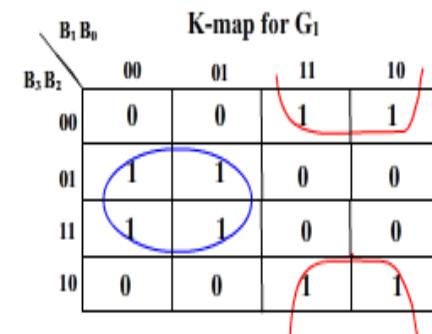
- a) 4-bit binary to Gray code.
- b) 4-bit Gray code to binary

Binary				Gray Code			
b ₃	b ₂	b ₁	b ₀	g ₃	g ₂	g ₁	g ₀
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0



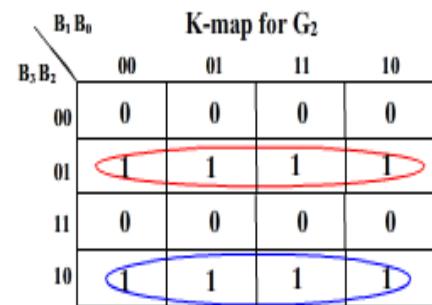
$$G_0 = B'_1 B_0 + B_1 B'_0$$

$$G_0 = B_0 \oplus B_1$$



$$G_1 = B'_1 B_2 + B_1 B'_2$$

$$G_1 = B_1 \oplus B_2$$



$$G_2 = B'_3 B_2 + B_3 B'_2$$

$$G_2 = B_2 \oplus B_3$$

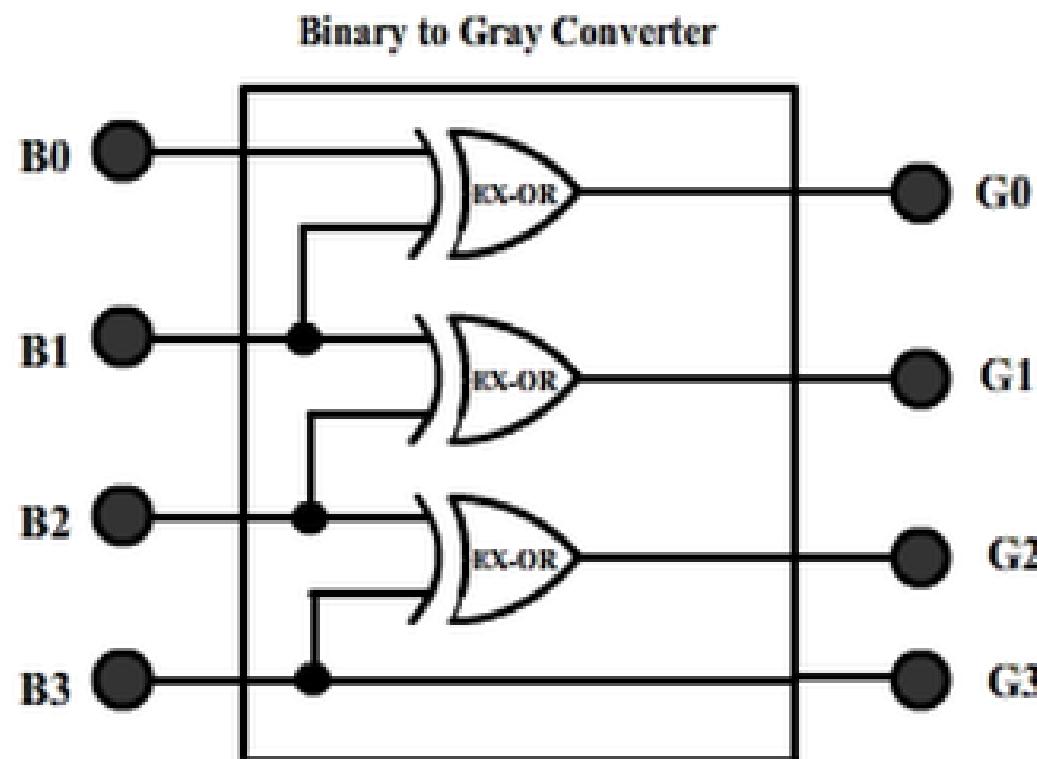
And, G₃ = B₃

PRACTICE QUESTIONS:

Q. Design a combinational circuit that converts:

- a) 4-bit binary to Gray code.
- b) 4-bit Gray code to binary

Binary				Gray Code			
b ₃	b ₂	b ₁	b ₀	g ₃	g ₂	g ₁	g ₀
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0



PRACTICE QUESTIONS:

Implement the Boolean function $F = xy + x'y' + y'z$ using:

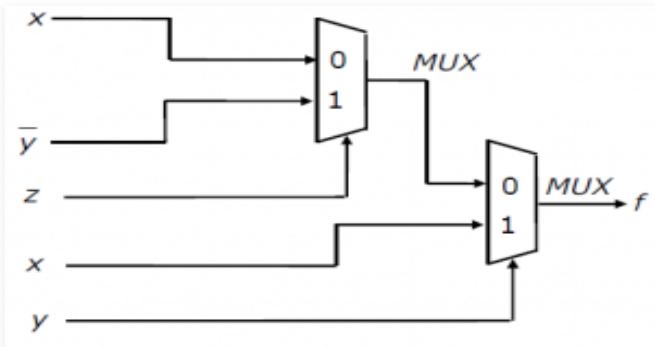
- a) Decoder
- b) Multiplexer

PRACTICE QUESTIONS:

Implement the Boolean function $F = xy + x'y' + y'z$ using:

- a) Decoder
- b) Multiplexer

GATE Problem

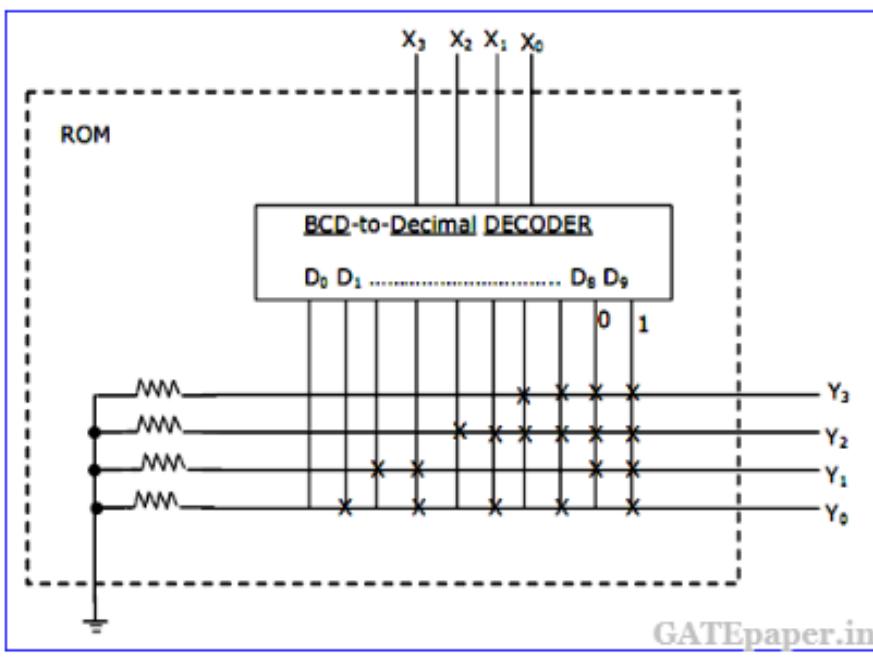


Consider the circuit above. Which one of the following options correctly represents $f(x, y, z)$?

- A $xz' + xy + y'z$
- B $xz' + xy + (yz)'$
- C $xz + xy + (yz)'$
- D $xz + xy' + y'z$

GATE Problem

If the inputs X_3, X_2, X_1, X_0 to the ROM in the figure are 8-4-2-1 BCD numbers, then the outputs $Y_3 Y_2 Y_1 Y_0$ are



- a. Gary code numbers
- b. 2-4-2-1 BCD numbers
- c. Excess-3 code numbers
- d. None of the above

GATE Problem

In the following truth table, V = 1 if and only if the input is valid.

Inputs				Outputs		
D ₀	D ₁	D ₂	D ₃	X ₀	X ₁	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

What function does the truth table

represent?

- a) Multiplexer b) Encoder c) Decoder d) Priority Encoder

PRACTICE QUESTIONS:

Find essential prime implicants of following functions, using K-map:

- (a) $F(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$
- (b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$
- (c)* $F(A, B, C, D) = \Sigma(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$			
$\bar{w}x$			
wx			
$w\bar{x}$			

PRACTICE QUESTIONS:

Find essential prime implicants of following functions, using K-map:

- (a) $F(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$
- (b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$
- (c)* $F(A, B, C, D) = \Sigma(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			
$\bar{A}B$			
AB			
$A\bar{B}$			