CGPA BOOSTER

# Unit 6
# Plane Kinematics and Kinetics of Rigid bodies

# Introduction

- Earlier, you studied relations of motion, assuming then that the body could be considered as a particle, i.e., that its mass could be concentrated in one point and that all forces acted at that point.
- The shape of the body, as well as the exact location of the points of application of the forces, will now be taken into account.
- You will also be concerned not only with the motion of the body as a whole but also with the motion of the body about its mass center.
- Our approach will be to consider rigid bodies as made of large numbers of particles and to use the results obtained for the motion of systems of particles.

# Introduction

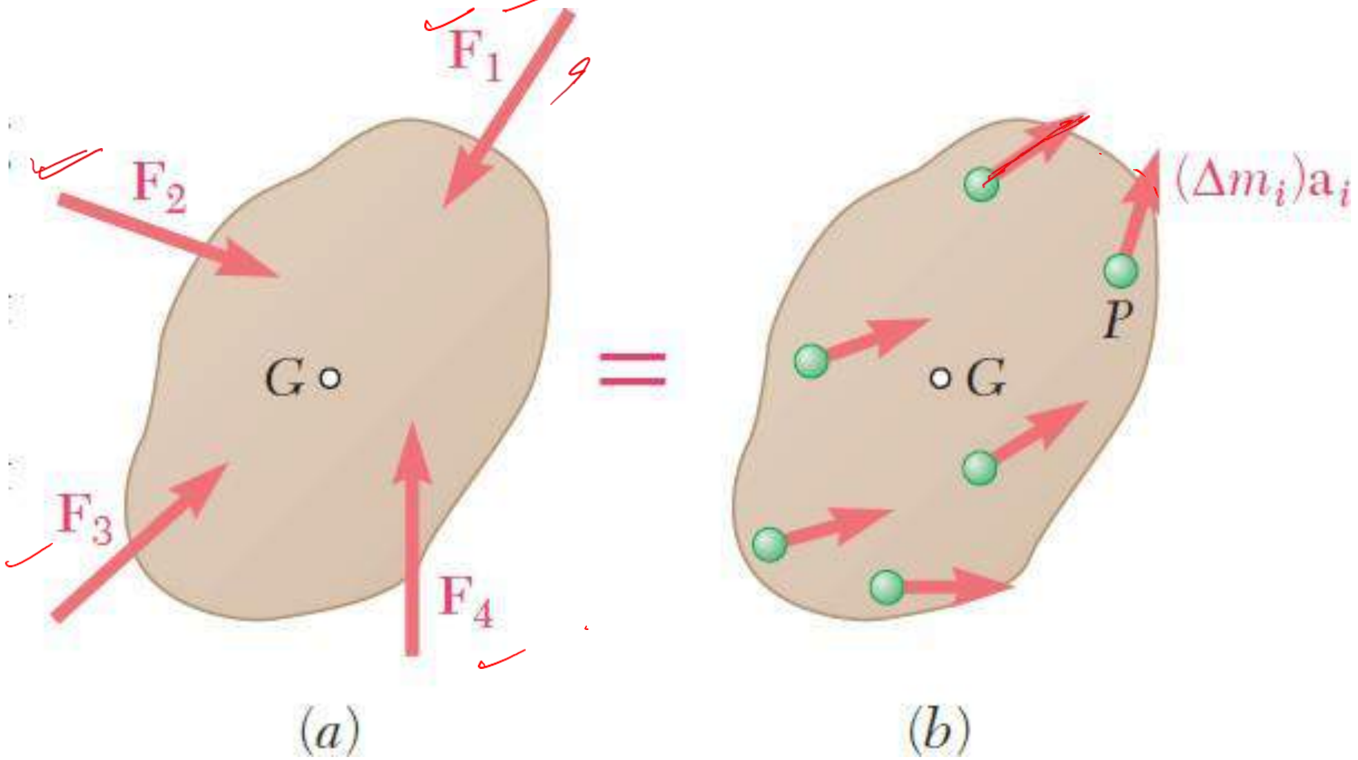
- The results derived in this chapter will be limited in two ways:

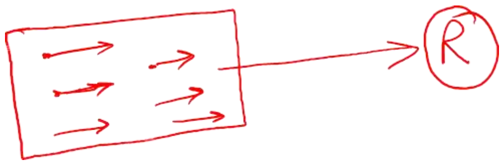
  (1) They will be restricted to the *plane motion* 2D of rigid bodies, i.e., to a motion in which each particle of the body remains at a constant distance from a fixed reference plane.

  (2) The rigid bodies considered will consist only of plane slabs and of bodies which are symmetrical with respect to the reference plane.

# D'ALEMBERT'S PRINCIPLE

- *The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body.*



# D'ALEMBERT'S PRINCIPLE

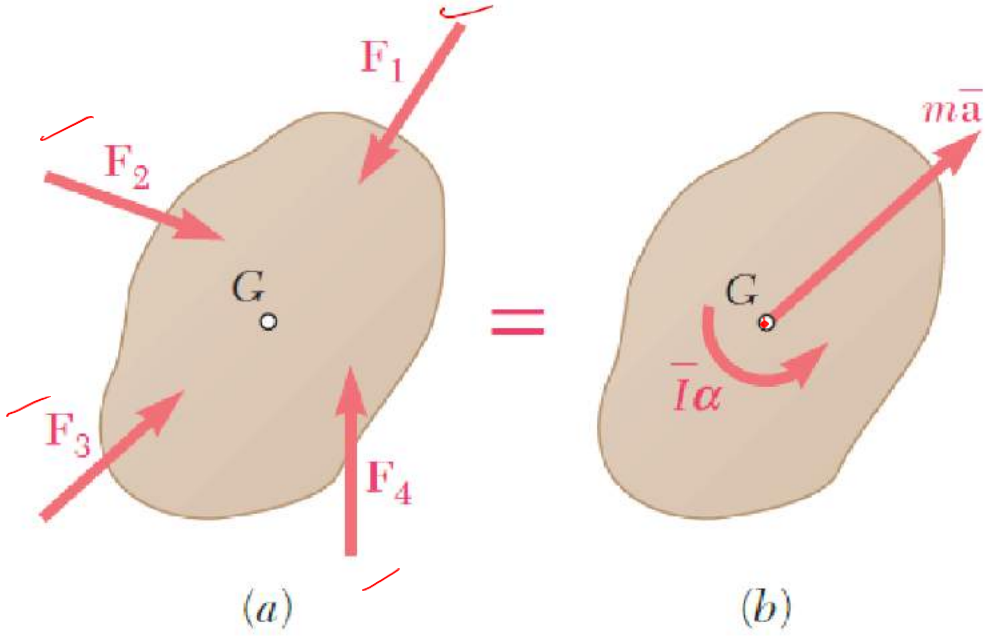
- *If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force.*

$F = ma$

$M_G = I\alpha$

MoI

angular acceleration




$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma M_G = \bar{I}\alpha$$

*A body of mass 7.5 kg is moving with a velcoity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.*

Sol.

$$v = v_0 + at$$

$$F = ma$$

$$a = \frac{F}{m} = \frac{15\,N}{7.5} = 2\ m/s^2$$

$$v = 12 + (2 \times 2)$$

$$\boxed{v = 5.2\ m/s}$$

*A vehicle, of mass 500 kg, is moving with a velocity of 25 m/s. A force of 200 N acts on it for 2 minutes. Find the velocity of the vehicle:*
*(1) when the force acts in the direction of motion, and*
*(2) when the force acts in the opposite direction of the motion.*

Sol:

$$a = \frac{F}{m} = \frac{200}{500} = 0.4 \ m/s^2$$

(1) $v = v_0 + at$

$= 25 + (0.4 \times 120)$

$$\boxed{v = 73 \ m/s}$$

(ii) $v = v_0 + at$ $\quad [a = -0.4 \ m/s^2]$

$= 25 - (0.4 \times 120)$

$$\boxed{v = -23 \ m/s}$$

*A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a velocity of 15 m/s. How long the body will take to stop?*

Sol. $F = -50\,N$ ; $m = 20\,kg$ , $v_0 = 15\,m/s$ , $v = 0\,m/s$

$$a = \frac{F}{m} = -\frac{50}{20} = -2.5\,m/s^2$$

$$v = v_0 + at$$

$$0 = 15 - (2.5 \times t)$$

$$\boxed{t = 6\,s}$$ Ans

*A multiple unit electric train has 800 tonnes mass. The resistance to motion is 100 N per tonne of the train mass. If the electric motors can provide 200 kN tractive force, how long does it take to accelerate the train to a speed of 90 km/hr from rest.*

Sol: $V_0 = 0$ m/s ; $V = 90$ km/h $= 25$ m/s ; $F_R = 100$ N/ton $= 80000$ N $= 80$ KN.

$F_T = 200$ KN.

$$F = F_T - F_R = 200 - 80 = 120 \text{ KN}.$$

$$a = \frac{F}{m} = \frac{120}{800} = 0.15 \text{ m/s}^2$$

$$v = \cancel{v_0}^{0} + at$$

$$25 = 0.15 \times t$$

$$\boxed{t = 166.7 \text{ s}}$$ Ans

*A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of the water. Neglect the resistance of air.*

Sol: Motion from tower to water surface.

$$v^2 - \cancel{v_0^2}^{\,0} = 2gh$$

$$v = \sqrt{2 \times 9.81 \times 20}$$

$$\boxed{v = 19.8 \text{ m/s}}$$

Motion under water.

$v_0 = 19.8$ ; $h = 2$ m ; $v = 0$.

$$v^2 - v_0^2 = 2ah$$

$$0 - 19.8^2 = 2a \times 2$$

$$\boxed{a = -98 \text{ m/s}^2}$$

$$F_R = ma$$

$$= 60 \times (-98)$$

$$\boxed{F_R = -5880 \text{ N}}$$ Ans

*At a certain instant, a body of mass 10 kg, falling freely under the force of gravity, was found to be falling at the rate of 20 m/s. What force will stop the body in (i) 2 seconds and (ii) 2 metres?*

Sol. (i) $t = 2\,s.$

$$v = v_0 + at$$
$$0 = 20 + a \times 2.$$
$$a = -10\ m/s^2$$
$$a_f = a + g = -10 - 9.81$$
$$a_f = -19.81\ m/s^2$$
$$F = m \times a_f$$
$$= 10 \times (-19.81).$$
$$\boxed{F = -198.1\,N}$$
$$F = 198.1\,N$$

(ii) $h = 2\,m$

$$v^2 = v_0^2 + 2ah.$$
$$0 = 20^2 + 2a \times 2.$$
$$a = -100\ m/s^2.$$
$$a_f = a + g = -100 - 9.81$$
$$a_f = -109.81\ m/s^2.$$
$$F = m \times a_f = 10 \times (-109.81)$$
$$\boxed{F = -1098.1\,N.}$$
$$F = 1098.1\,N \quad \text{Ans}$$

# D'ALEMBERT'S PRINCIPLE

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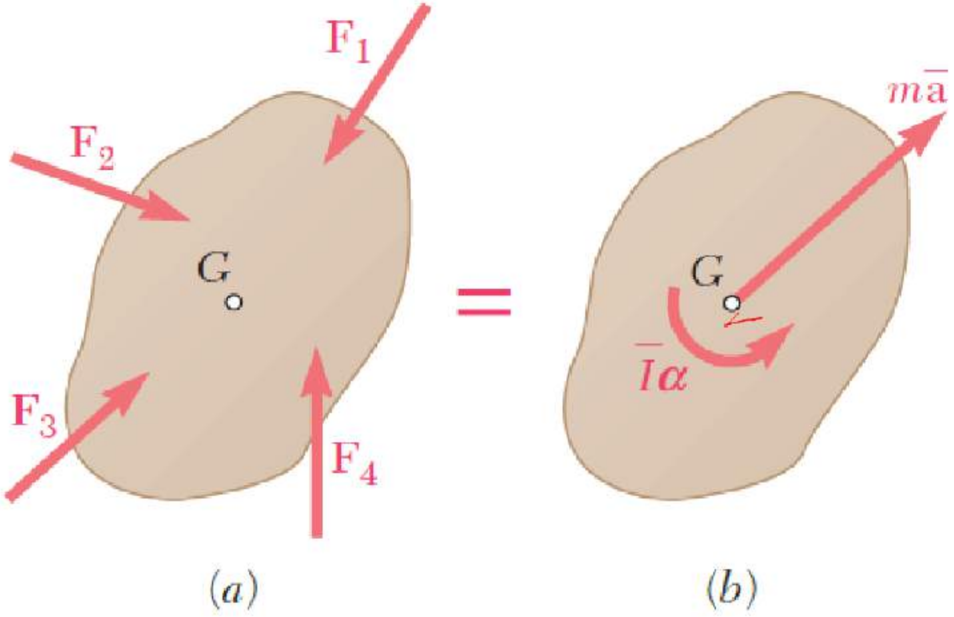
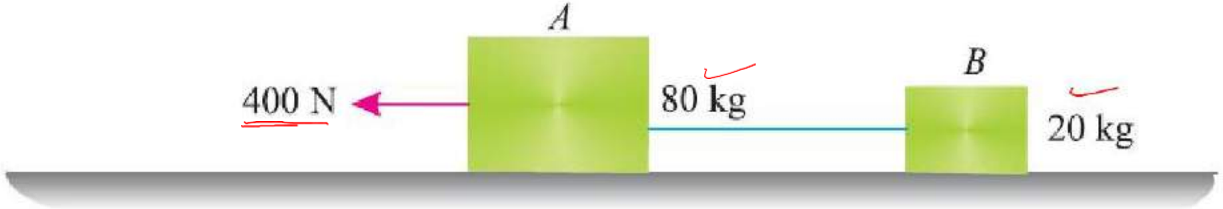



$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma M_G = \bar{I}\alpha$$

*Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. The coefficient of friction between the sliding surfaces of the bodies and the plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.*




Sol: For body A

Resultant horizontal force

$P = 400 - T - F$

$= 400 - T - \mu N$

$= 400 - T - 0.3(80 \times 9.81)$

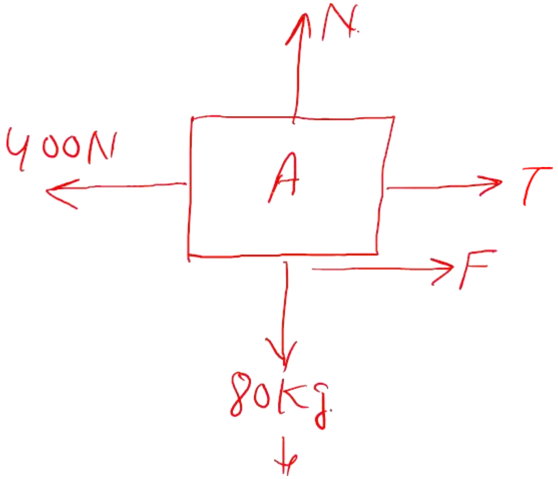
$\boxed{P = 164.56 - T}$ —(1)

also $P = m_A a = 80a$ —(2)

from (1) & (2)

$164.56 - T = 80a$

$\boxed{T = 164.56 - 8a}$ —(3)



for body B

Resultant horizontal force.

$$P = T - F = T - 0.3(20 \times 9.81)$$

$$\boxed{P = T - 58.86}$$

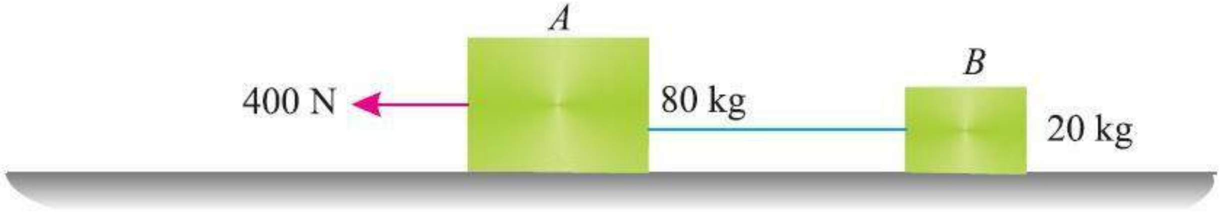
also $P = m_B a = 20a$

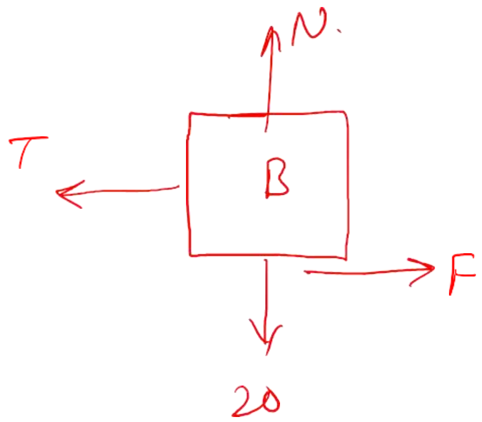
$$\boxed{T = 58.86 + 20a} \quad \text{— (4)}$$

equating (3) & (4)

$$\boxed{a = 1.06 \ m/s^2}$$

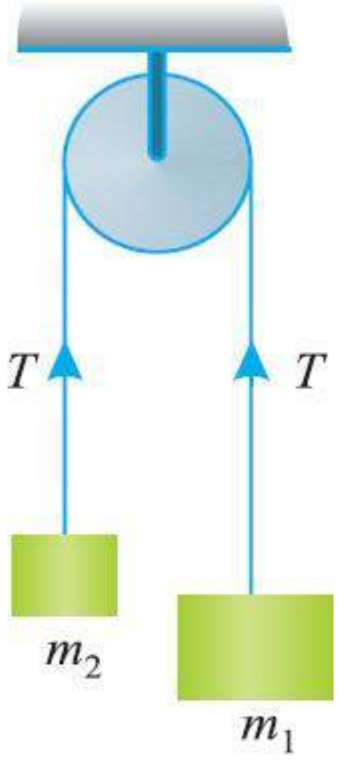
and $\boxed{T = 80N}$ Ans



# MOTION OF TWO BODIES CONNECTED BY A STRING AND PASSING OVER A SMOOTH PULLEY

Consider two bodies of masses $m_1$ and $m_2$ kg respectively connected with inextensible string whose own weight is neglected and passing over a smooth pulley. Let $m_1 > m_2$, $m_1$ will move downwards and $m_2$ upwards with same acceleration. For smooth pulley tension in string will be same on both sides.




for body 1 Resultant downward force.

$$P = m_1 g - T$$

also $P = m_1 a$

$$\therefore \boxed{T = m_1 g - m_1 a} \quad \text{— (1)}$$

# MOTION OF TWO BODIES CONNECTED BY A STRING AND PASSING OVER A SMOOTH PULLEY

for body 2 Resultant upward force.

$$P = T - m_2 g$$

also $P = m_2 a$.

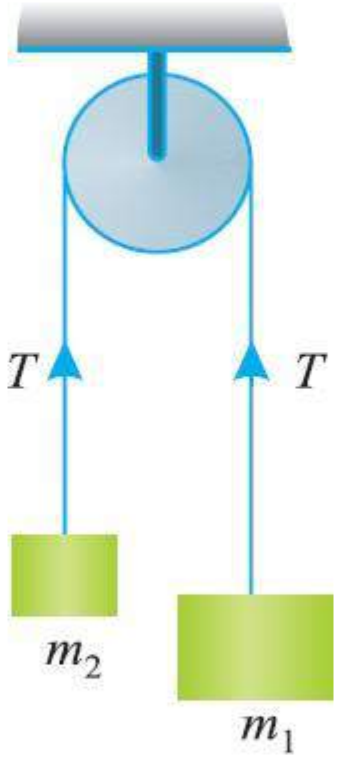
$$\therefore \boxed{T = m_2 g + m_2 a} \quad \text{— (2)}$$

equating (1) & (2)

$$\boxed{a = \frac{g(m_1 - m_2)}{m_1 + m_2}}$$

and $T = m_2(a + g)$.

$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$



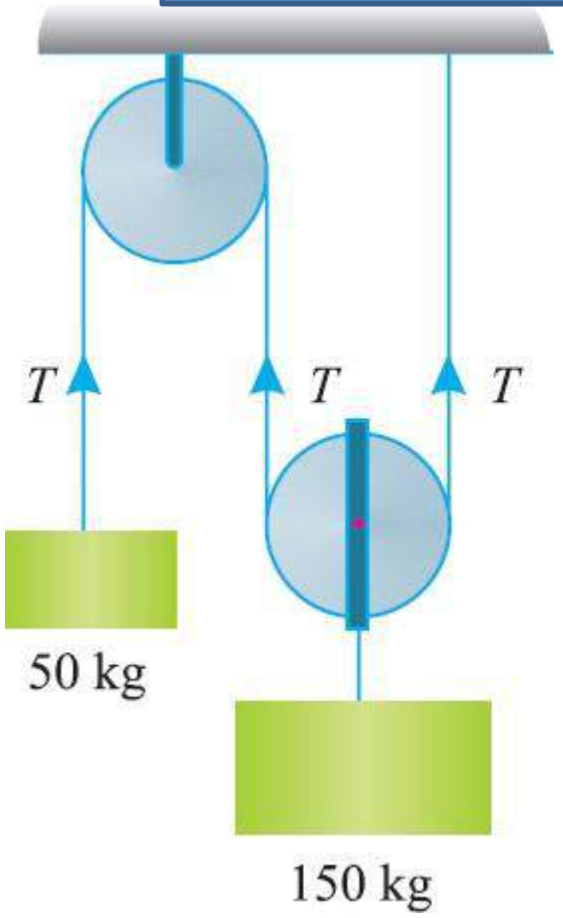
*Two bodies of masses 45 and 30 kg are hung to the ends of a rope, passing over a frictionless pulley. With what acceleration the heavier mass comes down? What is the tension in the string?*

Sol:

$$a = \frac{g(m_1 - m_2)}{m_1 + m_2} = \frac{9.81(45-30)}{45+30} = 1.962 \text{ m/s}^2 \quad \text{Ans}$$

$$T = \frac{2 m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 45 \times 30 \times 9.81}{45+30} = 353.16 \text{ N} \quad \text{Ans}$$

*Determine the tension in the strings and accelerations of two blocks of mass 150 kg and 50 kg connected by a string and a frictionless and weightless pulley as shown in Fig.*




Sol: for body 1

Resultant downward force

$P = 150g - 2T$

also $P = 150a$.

$\therefore 2T = 150g - 150a$. — (1)

for body 2

Resultant upward force.

$P = T - 50g$.

also $P = 50 \times 2a = 100a$.

$T = 50g + 100a$

OR

$2T = 100g + 200a$ (2)

equating (1) & (2).

$a_1 = 1.4\ m/s^2$ (for body 1).

for body 2 = $a_2 = 2a_1 = 2.8\ m/s^2$

$\boxed{T = 630.75 N}$ Ans

# *Find the acceleration of the three masses*

Sol: $a_{15} = \dfrac{g(m_1 - m_2)}{m_1 + m_2} = \dfrac{9.81(15-10)}{15+10} = 1.962\ m/s^2$ (downwards)

$a_6 = a_4 = 1.962\ m/s^2$ (upwards).

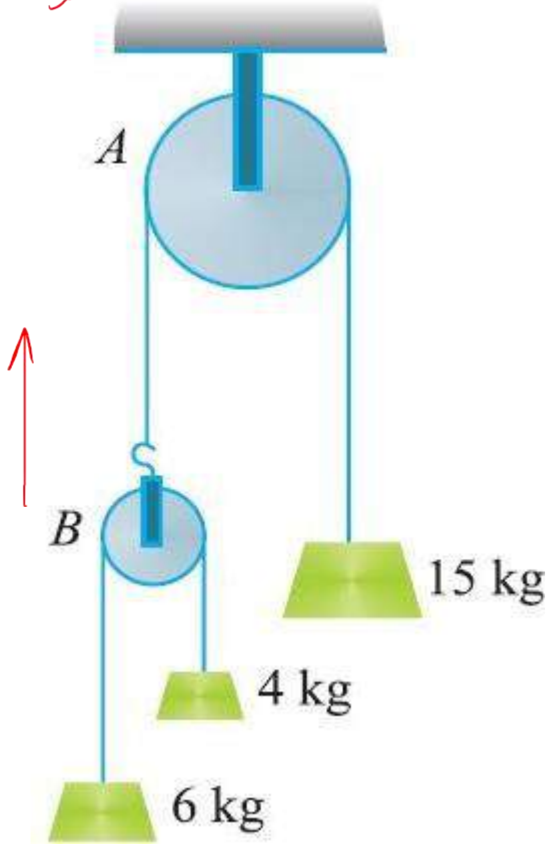
for Pulley B

$a_6 = \dfrac{9.81(6-4)}{6+4} = 1.962\ m/s^2$ (downwards).

net $\boxed{a_6 = 0}$

$a_4 = 1.962\ m/s$ (upward)

Net $a_4 = 1.962 + 1.962 = 3.924\ m/s^2$ (upwards).



Two weights 90N and 80 N respectively are attached by cord that passes over a frictionless pulley. If the weights start from rest, find the distance travelled by 90N weight in 6 seconds. Also find the tension in rope and pressure on the pulley.

Sol:
$$a = \frac{g(w_1 - w_2)}{w_1 + w_2} = \frac{9.81(90-80)}{90+80} = 0.577\ m/s^2$$

$$y = \cancel{v_0 t} + \frac{1}{2}at^2$$
$$= \frac{1}{2} \times 0.577 \times 6^2$$
$$\boxed{y = 10.386\ m.}$$

$$T = \frac{2 m_1 m_2 g}{m_1 + m_2} = \frac{2 \times \frac{w_1}{\cancel{g}} \times \frac{w_2}{\cancel{g}} \times \cancel{g}}{\frac{1}{\cancel{g}}(w_1 + w_2)}$$

$$T = \frac{2 w_1 w_2}{w_1 + w_2} = \frac{2 \times 90 \times 80}{90+80} = 84.7N.$$

Total pressure on Pulley.

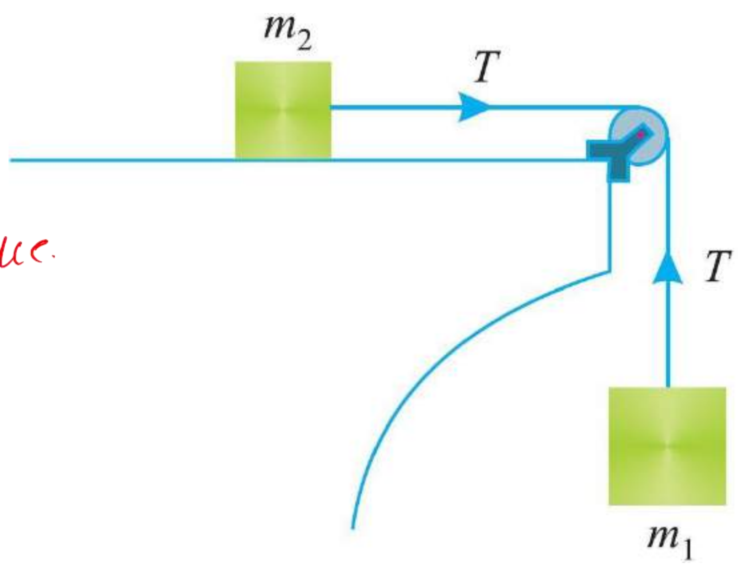
$$R = 2T = 169.4\ N.\ \text{Ans}$$

→ Smooth horizontal plane; → Smooth Pulley.

→ Inextensible string

→ $m_1$ hung free and $m_2$ placed on horizontal plane

→ velocity and acceleration of $m_1$ & $m_2$ are equal.




for body 1

Resultant downward force

$P = m_1 g - T$

also $P = m_1 a$.

$\boxed{T = m_1 g - m_1 a}$ — (1)

for body 2 Resultant horizontal force

$P = T$

also $P = m_2 a$.

$\boxed{T = m_2 a}$ — (2)

equating (1) & (2)

$$\boxed{a = \frac{m_1 g}{m_1 + m_2}}$$

$$\boxed{T = \frac{m_1 m_2 g}{m_1 + m_2}}$$

## → Rough horizontal plane, smooth pulley

for body 1

$$\boxed{T = m_1 g - m_1 a} \quad -(1)$$

for body 2

Resultant horizontal force

$$P = T - F = T - \mu N = T - \mu m_2 g.$$

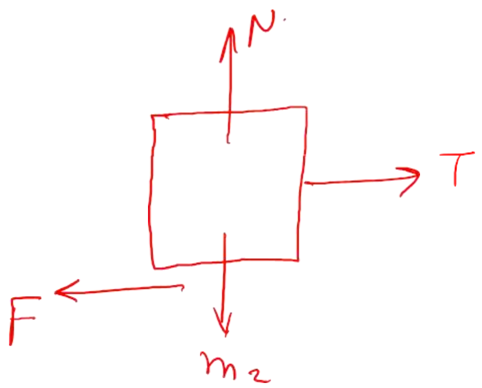
also $\quad P = m_2 a$

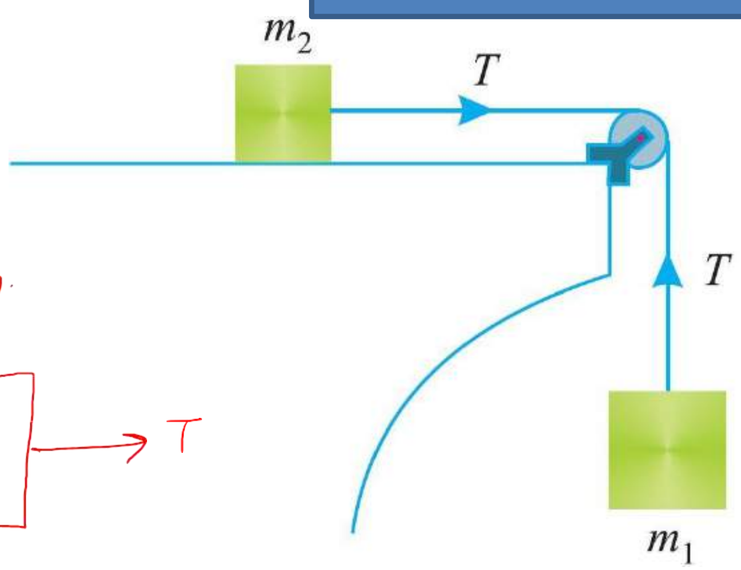
$$\boxed{T = m_2 a + \mu m_2 g.} \quad -(2)$$

equating (1) & (2).

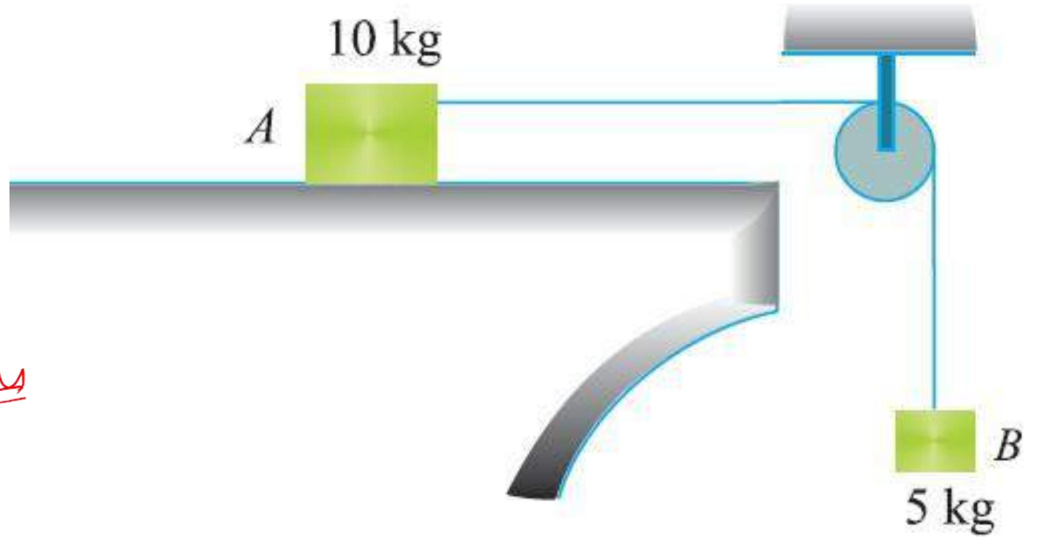
$$\boxed{a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2}}$$

$$\boxed{T = \frac{m_1 m_2 g (1+\mu)}{m_1 + m_2}}$$



*Find the acceleration of a solid body A and tension in the string, assuming the string to be inextensible and smooth horizontal plane.*

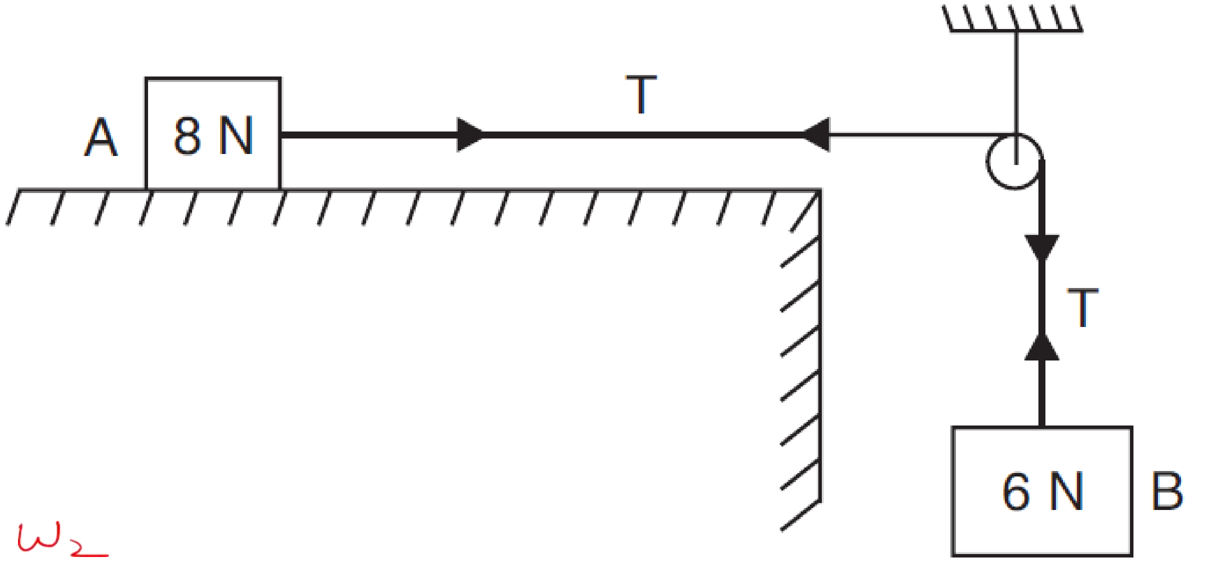



Sol:

$$a = \frac{m_1 g}{m_1 + m_2} = \frac{5 \times 9.81}{5 + 10} = 3.27 \ m/s^2$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{5 \times 10 \times 9.81}{5 + 10} = 32.7 \ N \ \text{Ans}$$

Find the acceleration of a solid body A of weight 8 N, when it is being pulled by another body of weight 6 N along a smooth horizontal plane as shown in Fig. Also find Tension in the thread.




Sol:

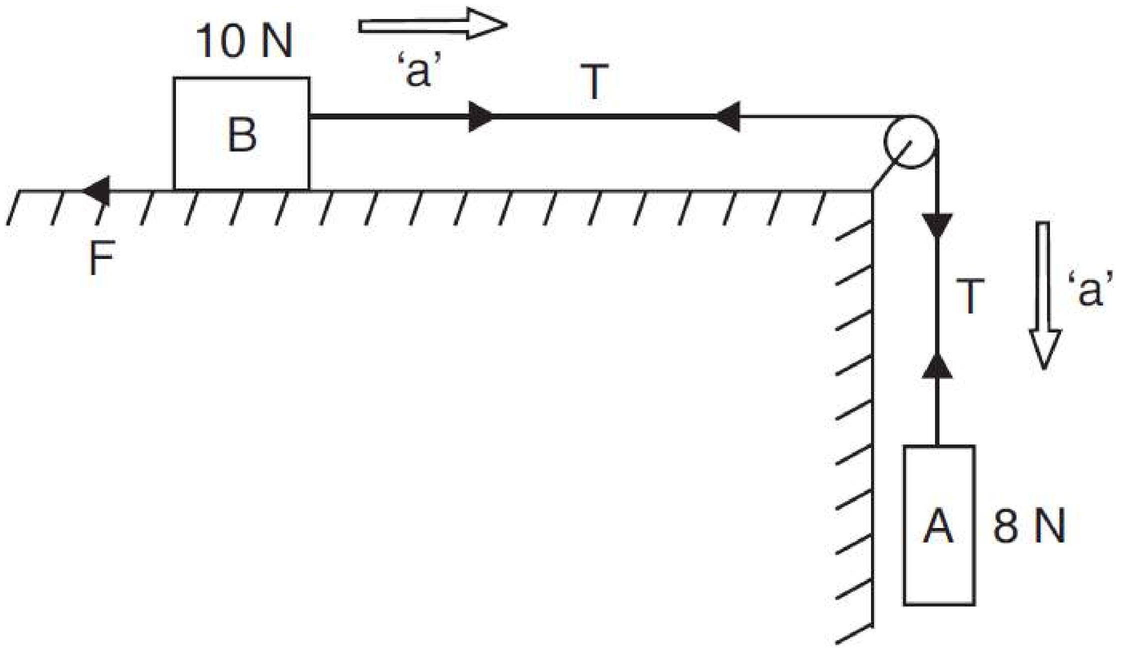
$$a = \frac{w_1 g}{w_1 + w_2} = \frac{6 \times 9.81}{6+8}$$

$$\boxed{a = 4.2\ m/s^2}$$ Ans

$$T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{\frac{1}{g^2} w_1 w_2 g}{\frac{1}{g}(w_1 + w_2)} = \frac{w_1 w_2}{w_1 + w_2}$$

$$\boxed{T = 3.42\ N}$$ Ans

Two blocks shown in Fig. and co-efficient of friction between the block A and horizontal plane, μ = 0.2. If the system is released, from rest and the block A falls through a vertical distance of 1.5 m, what is the velocity acquired by it ?




Sol. $a = \dfrac{g(w_1 - \mu w_2)}{w_1 + w_2} = \dfrac{9.81[8-(0.2\times 10)]}{8+10}$

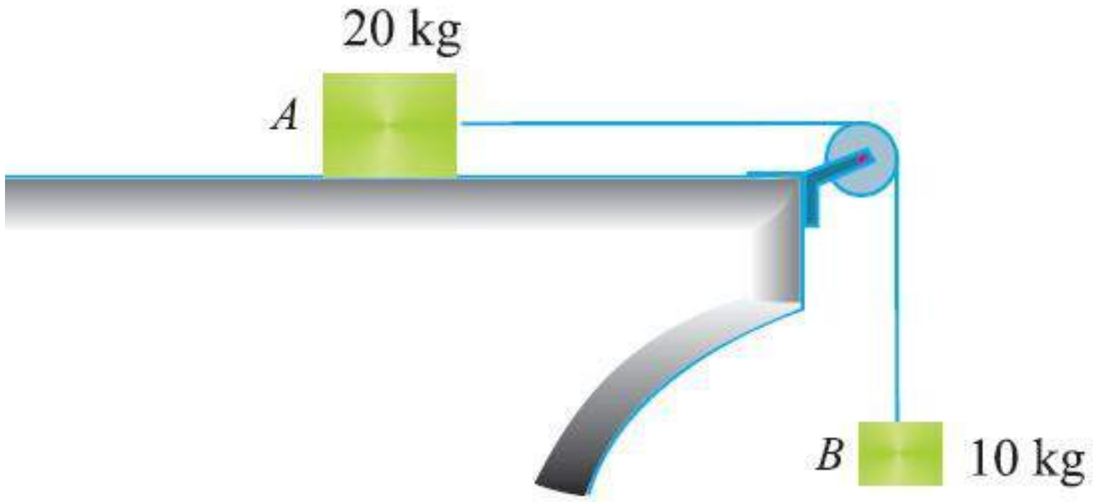
$\boxed{a = 3.27\ m/s^2}$ Ans

$v^2 - \cancel{v_0^2}^{\,0} = 2ah$

$v = \sqrt{2\times 3.27\times 1.5}$

$\boxed{v = 3.13\ m/s}$ Ans

*Two blocks have masses A = 20 kg and B = 10 kg and the coefficient of friction between the block A and the horizontal plane, μ = 0.25. If the system is released, from rest, and the block B falls through a vertical distance of 1m, what is the velocity acquired by it? Neglect the friction in the pulley and the extension of the string.*




Sol.

$$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2} = \frac{9.81\,[10 - (0.25 \times 20)]}{10+20}$$

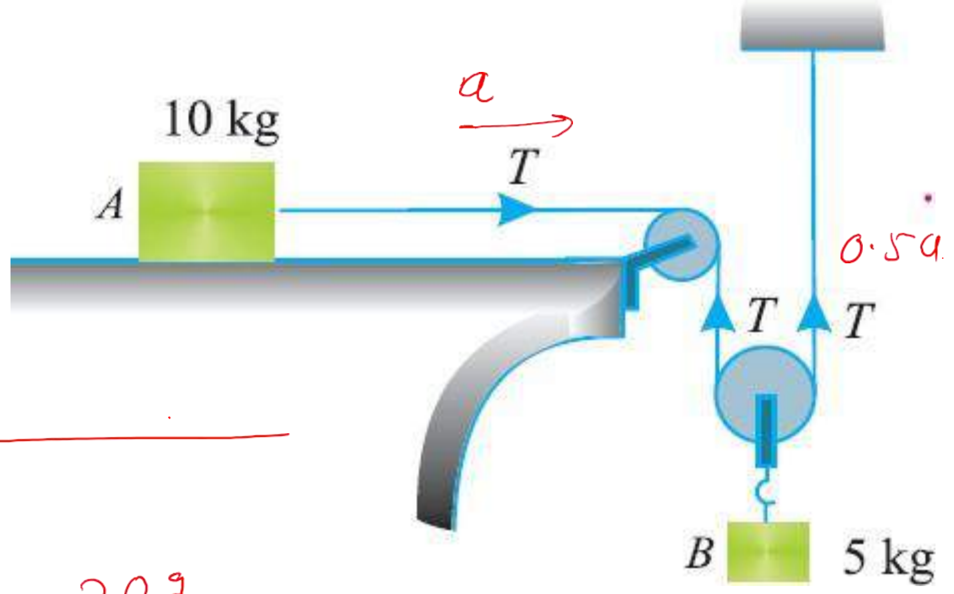
$$\boxed{a = 1.635\ m/s^2}$$

$$v^2 - \cancel{u_0}^{\,2} = 2ah$$

$$v = \sqrt{2 \times 1.635 \times 1}$$

$$\boxed{v = 1.8\ m/s}$$ Ans

*When the 10 kg block is released, it moves with an acceleration of g/9. Determine the value of coefficient of friction between the block and the table.*




Sol: body A   Resultant horizontal force

$$P = T - F = T - 10\mu g$$

also $$P = m_A a = 10 \times \frac{g}{9}$$

$$\therefore \; T = 10\mu g + \frac{10}{9} g \quad -(1) \Rightarrow 2T = 20\mu g + \frac{20g}{9} \quad -(1)$$

for body B   Resultant downward force

$$P = 5g - 2T$$

also $$P = m_B a = 5 \times 0.5a = \frac{5g}{18}$$

$$2T = 5g - \frac{5g}{18} \quad -(2)$$

equating (1) & (2).

$$20\mu g = 5g - \frac{5g}{18} - \frac{20g}{9}.$$

$$\boxed{\mu = 0.125}$$ Ans

When the 20 N body is released, it moves with an acceleration of g/5. Determine the value of co-efficient of friction between the block and the table.

Sol: for body 1

Resultant horizontal force

$P = T - F = T - \mu w_1$

also $P = m_1 a = \frac{w_1}{g} \times a$

$T = \mu w_1 + \frac{w_1}{g} a$ —

$T = 20\mu + \frac{20}{g} \times \frac{g}{5}$

$\boxed{T = 20\mu + 4}$ —(1)

for body 2 Resultant downward force

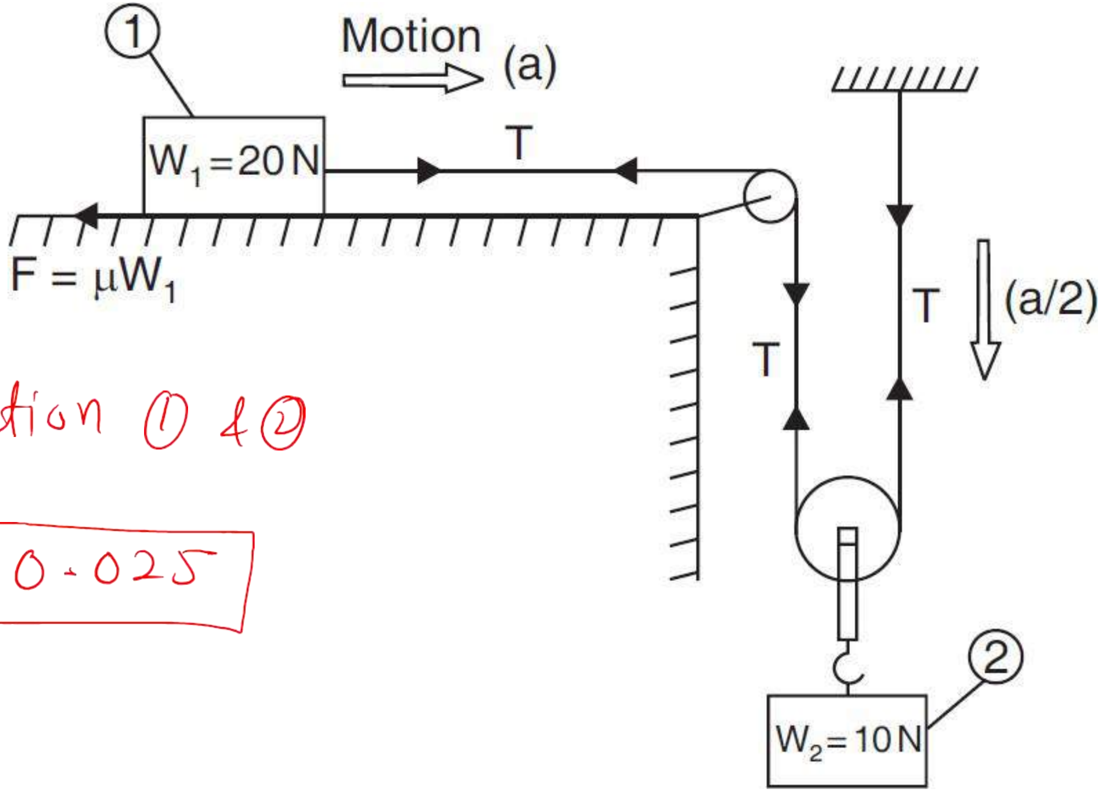
$P = w_2 - 2T$

also $P = m_2 \frac{a}{2} = \frac{w_2}{g} \times \frac{a}{2} = \frac{w_2}{g} \times \frac{g}{10}$

$\boxed{T = 4.5}$ —(2)

equation (1) & (2)

$\boxed{\mu = 0.025}$



## → Smooth inclined plane

for body 1

Resultant downward force

$P = m_1 g - T$

also $P = m_1 a$

$\therefore T = m_1 g - m_1 a$ — (1)

for body 2

Resultant upward force along the plane

$P = T - m_2 g \sin\alpha$

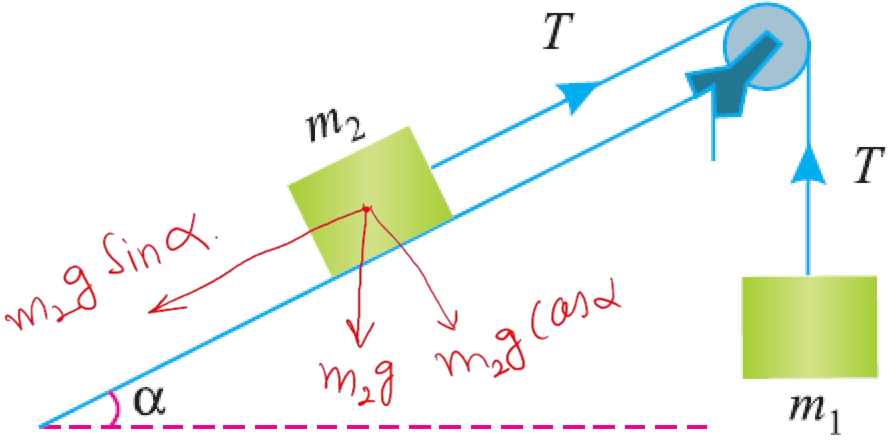
also $P = m_2 a$

$T = m_2 a + m_2 g \sin\alpha$ — (2)

equating (1) & (2)

$$a = \frac{g(m_1 - m_2 \sin\alpha)}{m_1 + m_2}$$

$$\boxed{T = \frac{g\, m_1 m_2 (1 + \sin\alpha)}{m_1 + m_2}}$$



→ Rough inclined plane

for body 1 $\quad T = m_1 g - m_1 a$ — (1)

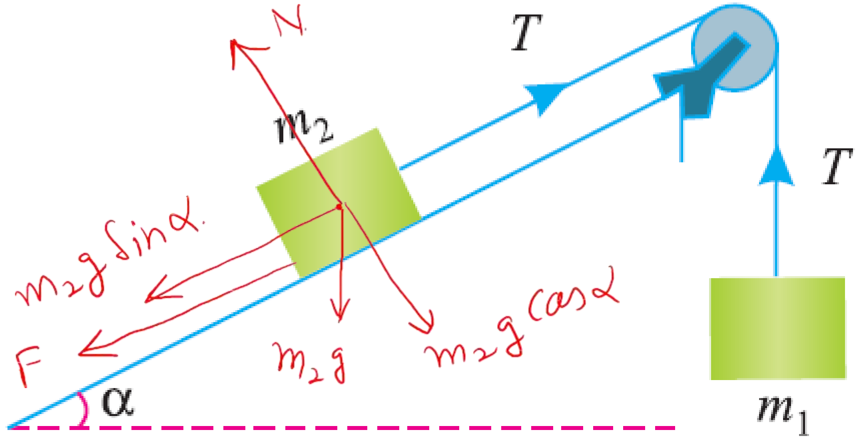
for body 2 Resultant upward force along the plane.

$$P = T - (m_2 g \sin\alpha) - (\mu\, m_2 g \cos\alpha)$$

also $P = m_2 a$.

$$T = m_2 a + m_2 g \sin\alpha + \mu\, m_2 g \cos\alpha \quad \text{— (2)}$$

equating (1) & (2)




$$\boxed{a = \frac{g\left[m_1 - m_2(\sin\alpha + \mu\cos\alpha)\right]}{m_1 + m_2}} \; ; \; \boxed{T = \frac{m_1 m_2 g\,(1 + \sin\alpha + \mu\cos\alpha)}{m_1 + m_2}}$$

*A body of mass 150 kg, rests on a rough plane inclined at 10° to the horizontal. It is pulled up the plane, from rest, by means of a light flexible rope running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a man of 80 kg at the end. If the coefficient of friction for the plane and the body is 0.2, find*

*(i) the tension in the rope,*

*(ii) the acceleration in m/s2, with which the body moves up the plane, and*

*(iii) the distance in metres moved by the body in 4 seconds starting from rest.*

Sol:

$$T = \frac{m_1 m_2 g(1+\sin\alpha + \mu\cos\alpha)}{m_1+m_2} = \frac{80\times 150\times 9.81(1+\sin 10 + 0.2(\cos 10))}{80+150}$$

$$\boxed{T = 701.5\,N} \quad \text{Ans}$$

$$a = \frac{g\left[m_1 - m_2(\sin\alpha + \mu\cos\alpha)\right]}{m_1+m_2} = \frac{9.81\left[80 - 150(\sin 10 + 0.2(\cos 10))\right]}{80+90}$$

$$\boxed{a = 1.04\,m/s^2} \quad \text{Ans}$$

$$d = \cancel{v_0 t}^{\,0} + \frac{1}{2}at^2$$

$$= \frac{1}{2}\times 1.04\times 4^2$$

$$\boxed{d = 8.32\,m} \quad \text{Ans}$$

*Determine the resulting motion of the body A assuming the pulleys to be smooth and weightless. If the system starts from rest, determine the velocity of the body A after 10 seconds.*

Sol: for body A   Resultant upward force along the plane.

$$P = T - (10 \times 9.81 \sin 30) - (0.2 \times 10 \times 9.81 \cos 30)$$

$$P = T - 66$$

also $P = 10a$

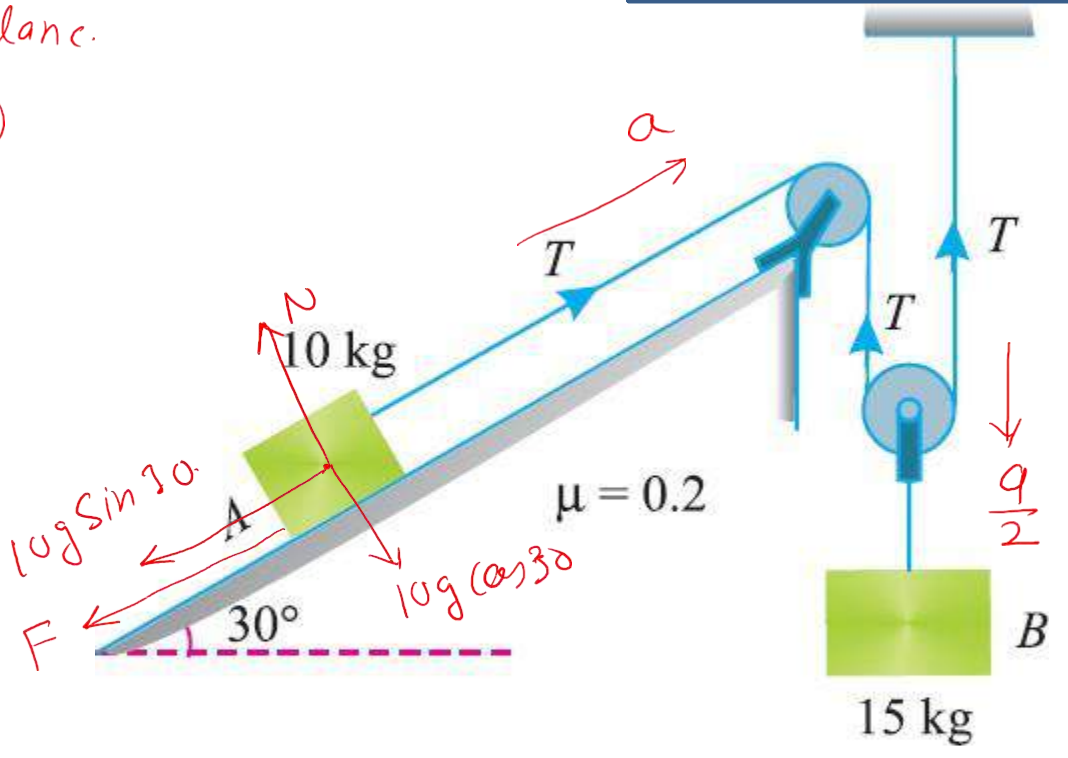
$\therefore\ T = 66 + 10a \Rightarrow 2T = 132 + 20a$ — (1)

for body B   Resultant downward force.

$$P = (15 \times 9.81) - 2T$$

also $P = 15 \times \frac{a}{2} = 7.5a$.

$\therefore\ 2T = (15 \times 9.81) - 7.5a$ — (2)



equating (1) & (2)

$$a = 0.55\ m/s^2.$$

velocity after 10 s.

$$v = \cancel{v_0}^{\,0} + at$$

$$= 0.55 \times 10$$

$$\boxed{v = 5.5\ m/s}$$ Ans

*Determine the acceleration of body B and the tension in the string supporting body A.*

Sol: for body B Resultant upward force along the plane.

$P = 2T - 750 \sin 36.86 - 0.2 \times 750 \cos 36.86$

$P = 2T - 570$

also $P = \frac{750}{9.81} a = 76.45a.$

$2T = 570 + 76.45a. \quad -(1)$

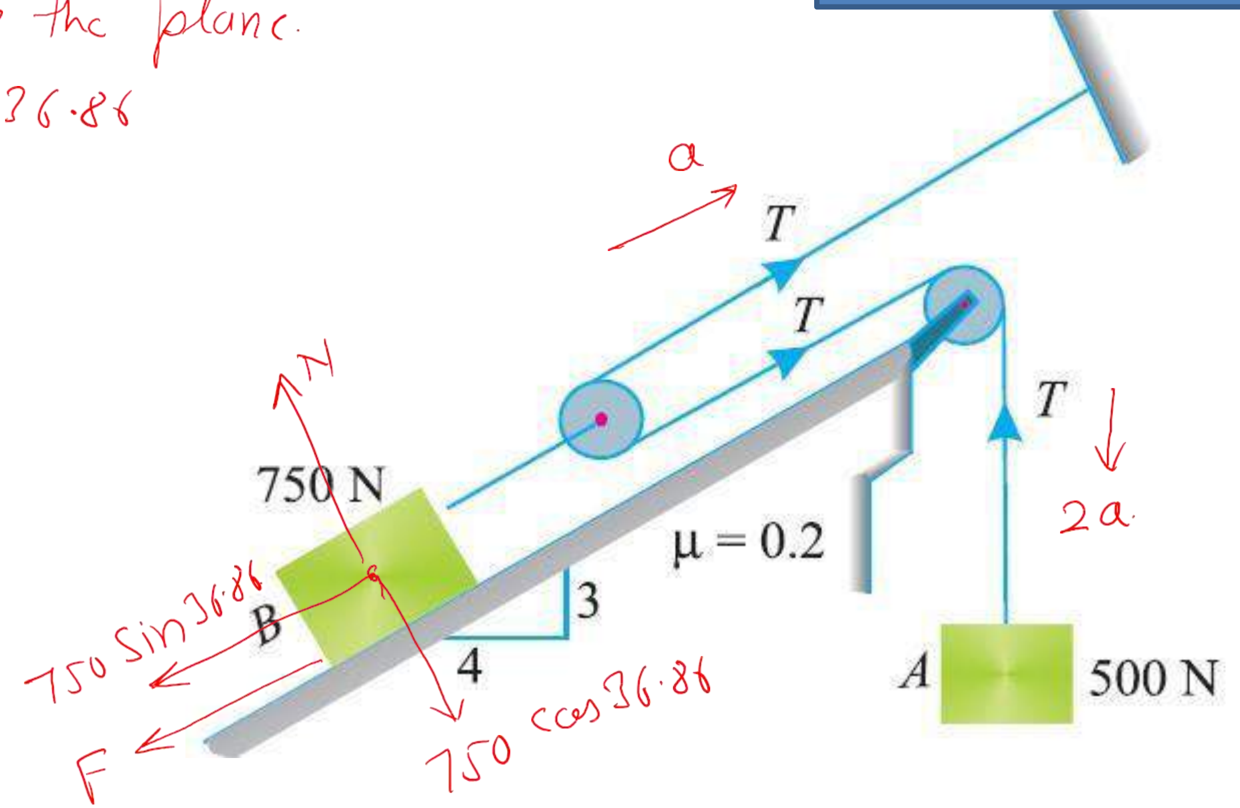
for body B Resultant downward force.

$P = 500 - T.$

also $P = \frac{500}{9.81} \times 2a = 102a$

$T = 500 - 102a.$

$2T = 1000 - 204a - (2)$



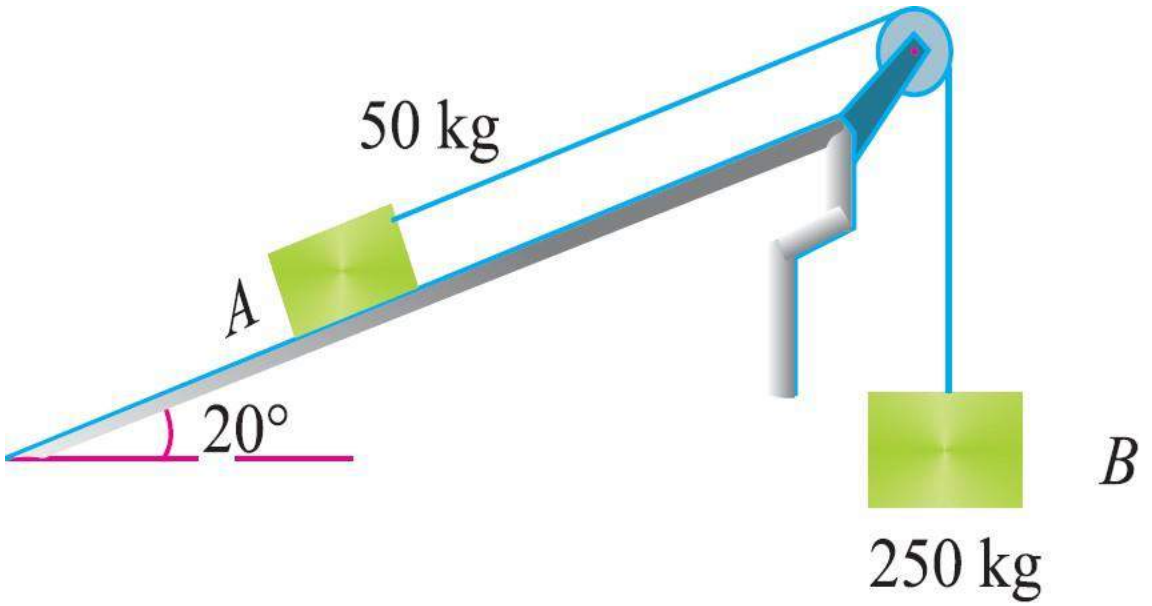
equating (1) & (2)

$$\boxed{a = 1.53\ m/s^2}$$ Ans

and

$$\boxed{T = 343.94\ N}$$

If the system is released from rest, with what velocity 250 kg mass will touch the floor, if initially it was 1.6 m higher than the floor level. Take coefficient of friction between 50 kg mass and inclined surface as 0.2.




Sol

$$a = \frac{g[m_1 - m_2(\sin\alpha + \mu\cos\alpha)]}{m_1 + m_2}$$

$$= \frac{9.81[250 - 50(\sin 20 + 0.2\cos 20)]}{300}$$

$$\boxed{a = 7.3\ m/s^2}$$

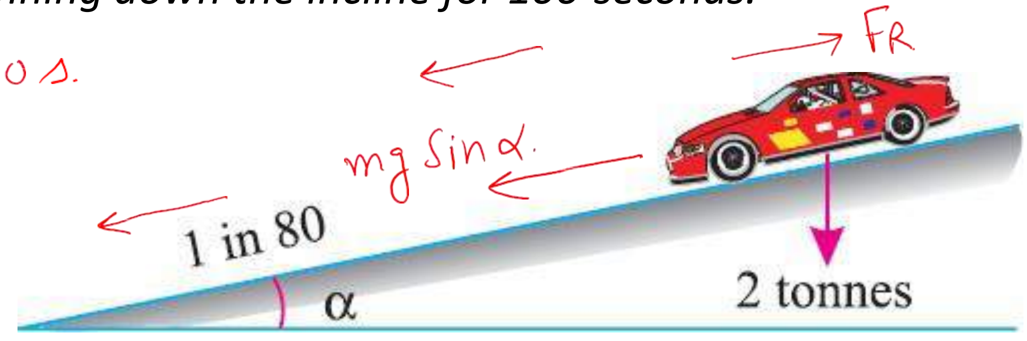
$$v^2 = \cancel{v_0^2}^{\,0} + 2ah$$

$$v = \sqrt{2 \times 7.3 \times 1.6}$$

$$\boxed{v = 4.83\ m/s}$$ Ans

# Motion on Inclined Planes

*A vehicle of mass 2 tonnes has a frictional resistance of 50 N/tonne. As one instant, the speed of this vehicle at the top of an incline was observed to be 36 km.p.h.* ***Find the speed of the vehicle after running down the incline for 100 seconds.***




Sol: Slope $\sin\alpha = \frac{1}{80} = 0.0125$ ; $m = 2$ tonnes ; $t = 100\,s$.

$F_R = 50\,N/\text{tonne} = 100\,N$ ; $v_0 = 36\,km/h = 10\ m/s$.

Net downward force along the plane.

$P = mg\sin\alpha - F_R = (2000 \times 9.81 \times 0.0125) - 100$

$\boxed{P = 145.25\,N}$

also $P = ma$.

$145.25 = 2000 \times a$

$\boxed{a = 0.0725\,m/s^2}$

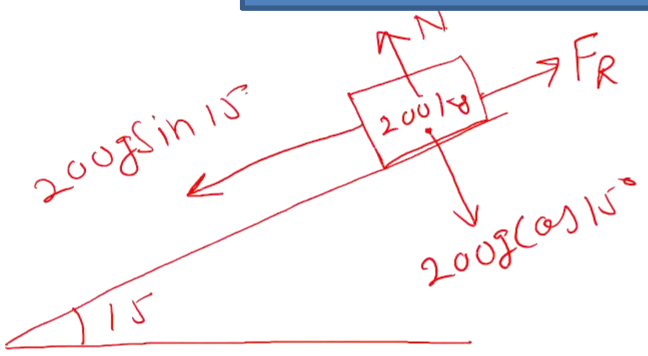
$v = v_0 + at$

$= 10 + (0.0725 \times 100)$

$\boxed{v = 17.25\,m/s}$ OR $\boxed{v = 62.1\ Km/h}$ Ans

*A body of mass 200 kg is initially stationary on a 15º inclined plane. What distance along the incline must the body slide before it reaches a speed of 10 m/s? Take coefficient of friction between the body and the plane as 0.1.*

Sol: Net downward force along the Plane = Sliding force − friction force




$$P = (200 \times 9.81 \times \sin 15) - (0.1 \times 200 \times 9.81 \cos 15)$$

$$\boxed{P = 318.28\ N}$$

also $P = ma \Rightarrow a = \frac{P}{m} = \frac{318.28}{200} = 1.59\ m/s^2$

then

$$v^2 - \cancel{v_0^2} = 2ad.$$

$$10^2 = 2 \times 1.59 \times d$$

$$\boxed{d = 31.44\ m}$$ Ans

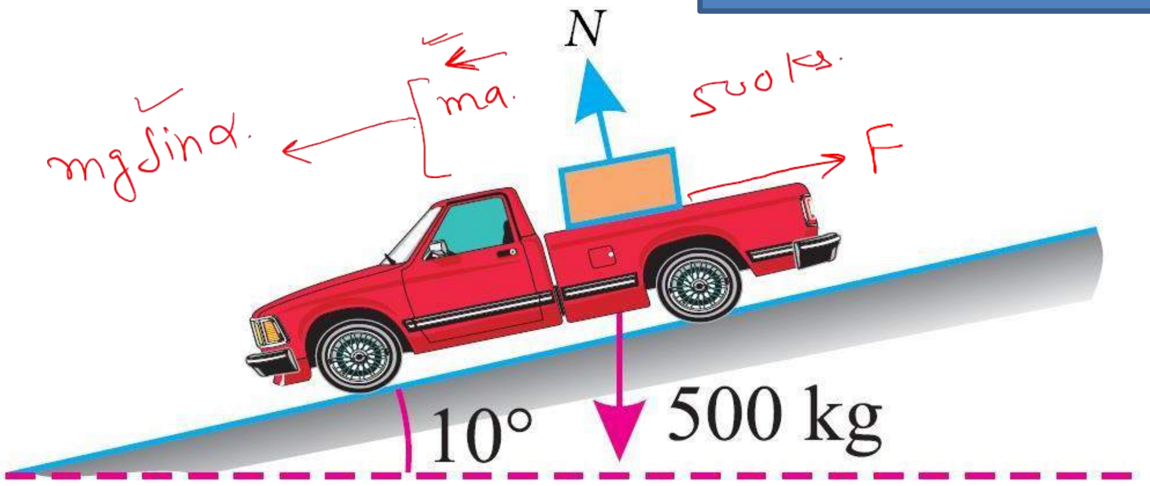
*A truck is moving down a 10º incline when the driver applies brakes, with the result that the truck decelerates at a steady rate of 1 m/s². Investigate whether a 500 kg placed on the truck will slide or remain stationary relative to the truck. Assume the coefficient of friction between the truck surface and the load as 0.4. What will be the factor of safety against slipping for this load?*

Sol: Net force which can cause sliping.

$$P = mg\sin\alpha + ma$$

$$= (500 \times 9.81 \times \sin 10) + (500 \times 1)$$

$$\boxed{P = 1351.74\ N}$$




Friction force $F = \mu N = 0.4 \times 500 \times 9.81 \times \cos 10$

$$\boxed{F = 1932.19\ N}$$

$F > P$; the load is stationary

$$FoS = \frac{F}{P} = \frac{1932.19}{1351.74}$$

$$\boxed{FoS = 1.429}$$

Work done by a force $W = P \times d$ (N-m) or J

Power $p$ = Rate of doing work.

$$p = \frac{W}{t} \quad \left(\frac{N\text{-}m}{s}\right) \text{ or } \frac{J}{s} \text{ or } W \text{ (watt)}.$$

$$\text{or } p = \frac{P \times d}{t} = P \times v \text{ (W)}.$$

*Calculate the work done in pulling up a block of mass 200 kg for 10 m on a smooth plane inclined at an angle of 15° with the horizontal.*

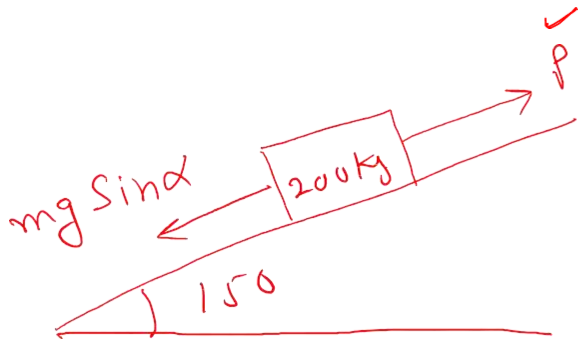
Sol

$W = P \times d$

$= mg \sin\alpha \times d$

$= 200 \times 9.81 \times \sin 15 \times 10$

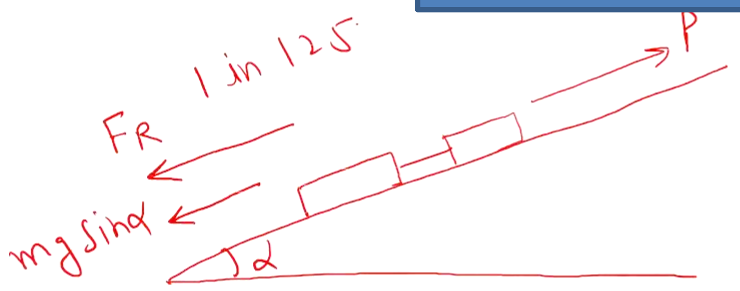
$W = 5078 \text{ J}$ or $5.078 \text{ KJ}$ Ans



*A locomotive and train together has a mass of 200 t and tractive resistance 100 N per tonne. If the train can move up a grade of 1 in 125 with a constant speed of 28·8 km.p.h. find the power of the locomotive. Also find the speed, which the train can attain, on a level track, with the same tractive resistance and power of the locomotive.*

Sol:→ $m = 200$ tonnes ; $F_R = 100\,N/\text{tonne} = 100 \times 200 = 20\,KN$

$\sin\alpha = \frac{1}{125} = 0.008$ ; $v = 28.8\,Km/h = \underline{8}\,m/s$




Net force up the plane

$P = mg\sin\alpha + F_R = (200 \times 9.81 \times 0.008) + 20$

$\boxed{P = 35.7\,KN}$

$p = P \times v$

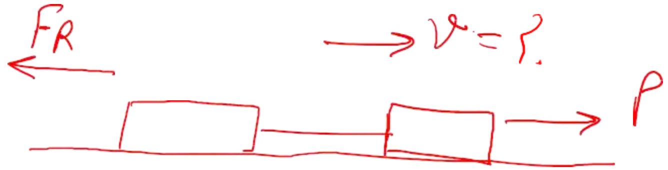
$p = 35.7 \times 8$

$\boxed{p = 286\,KW}$

$P = F_R = 20\,KN$

$p = P \times v$

$v = \frac{p}{P} = \frac{286}{20}$

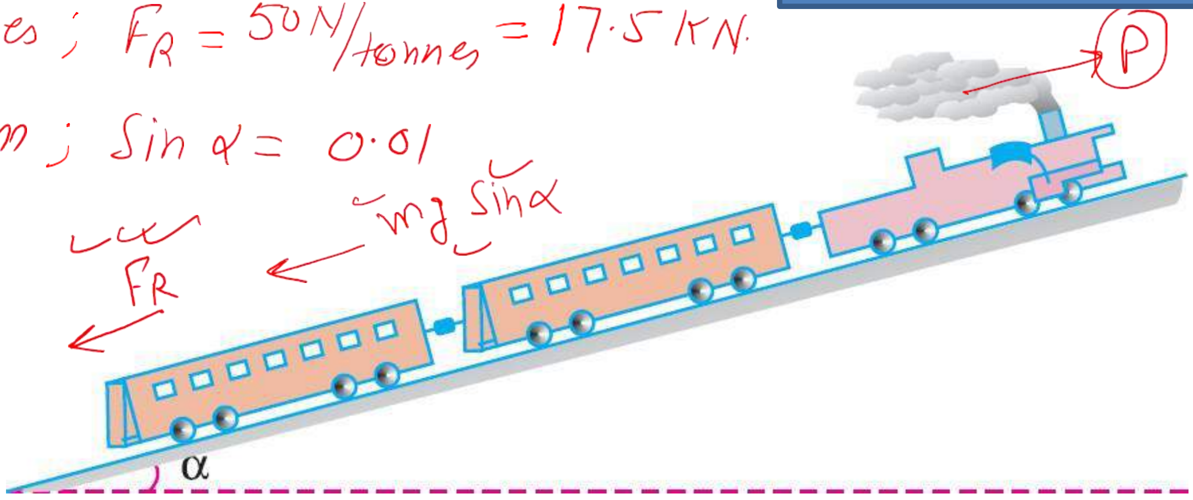



$v = 14.3\,m/s$ or $v = 51.48\,Km/h$ Ans

*An engine of mass 50 tonnes pulls a train of mass 300 tonnes up an incline of 1 in 100. The train starts from rest and moves with a constant acceleration against a total resistance of 50 newtons per tonnes. If the train attains a speed of 36 km.p.h. in a distance of 1 kilometre, find power of the engine.*

Sol: $m_1 = 50$ tonnes; $m_2 = 300$ tonnes; $m = 350$ tonnes; $F_R = 50\,N/\text{tonnes} = 17.5\,kN$

$v_0 = 0\,m/s$; $v = 36\,km/h = 10\,m/s$; $d = 1000\,m$; $\sin\alpha = 0.01$




$$v^2 = v_0^2 + 2ad$$

$$10^2 = 2 \times a \times 1000$$

$$\boxed{a = 0.05\,m/s^2}$$

Net upward force

$$P = F_R + mg\sin\alpha + ma$$

$$P = 17.5 + (350 \times 9.81 \times 0.01) + (350 \times 0.05)$$

$$\boxed{P = 69.335\,KN}$$

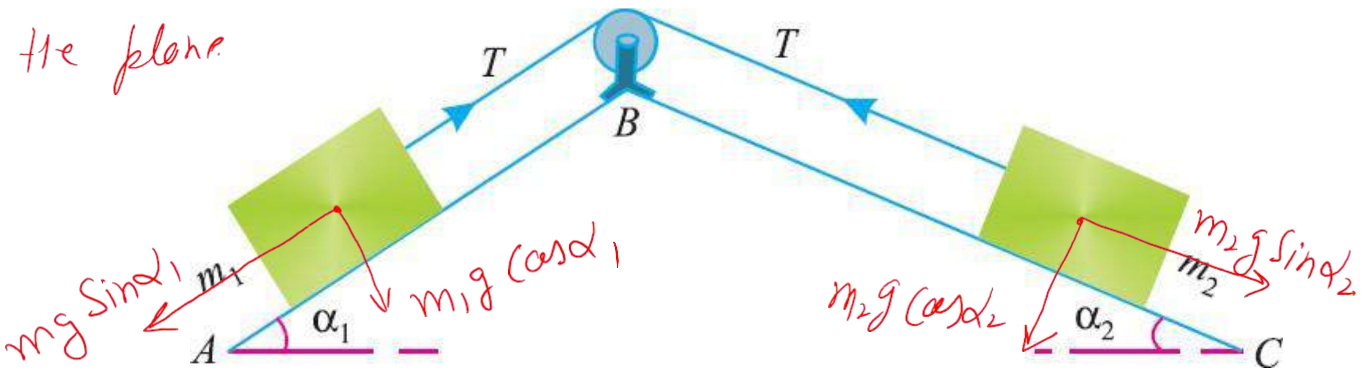
$$p = P \times v = 69.335 \times 10$$

$$\boxed{p = 693.35\,KN}$$

An

# MOTION OF TWO BODIES, CONNECTED BY A STRING AND LYING ON SMOOTH INCLINED PLANES

Let $m_1$ coming down along the plane.




for body 1 Resultant downward force along the plane

$$P = m_1 g \sin\alpha_1 - T$$

also $P = m_1 a$

$$\therefore T = m_1 g \sin\alpha_1 - m_1 a \quad -(1)$$

for body 2 Resultant upward force along the plane.

$$P = T - m_2 g \sin\alpha_2.$$

also $P = m_2 a.$

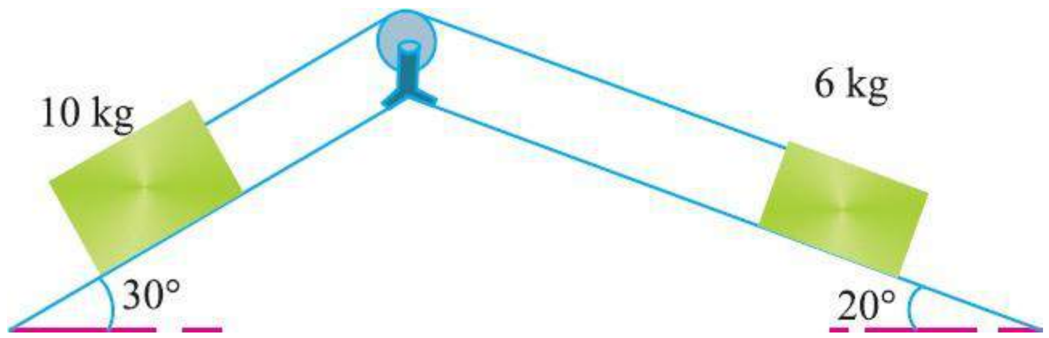
$$\therefore T = m_2 g \sin\alpha_2 + m_2 a \quad -(2)$$

equating (1) & (2)

$$\boxed{a = \frac{g(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)}{m_1 + m_2}}$$

$$\boxed{T = \frac{m_1 m_2 g (\sin\alpha_1 + \sin\alpha_2)}{m_1 + m_2}}$$

Find the acceleration and tension in the string.




Sol:

$$a = \frac{g(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)}{m_1 + m_2}$$

$$= \frac{9.81(10 \sin 30 - 6 \sin 20)}{10 + 6}$$

$$\boxed{a = 1.81\ m/s^2}$$ Ans

$$T = \frac{m_1 m_2 g(\sin\alpha_1 + \sin\alpha_2)}{m_1 + m_2} = \frac{10 \times 6 \times 9.81(\sin 30 + \sin 20)}{10 + 6}$$

$$\boxed{T = 31 N}$$ Ans

# If both the bodies are released simultaneously, what distance do they move in 3 seconds?

Sol: for body B

Resultant downward force

$P = (15 \times 9.81 \times \sin 60) - T$

$P = 127.43 - T$

also $P = 15a$

$\therefore T = 127.43 - 15a$ — (1)

for body A

Resultant downward force

$P = (25 \times 9.81 \times \sin 20) + T$

$P = 83.88 + T$

also $P = 25a$

$\therefore T = 25a - 83.88$ — (2)

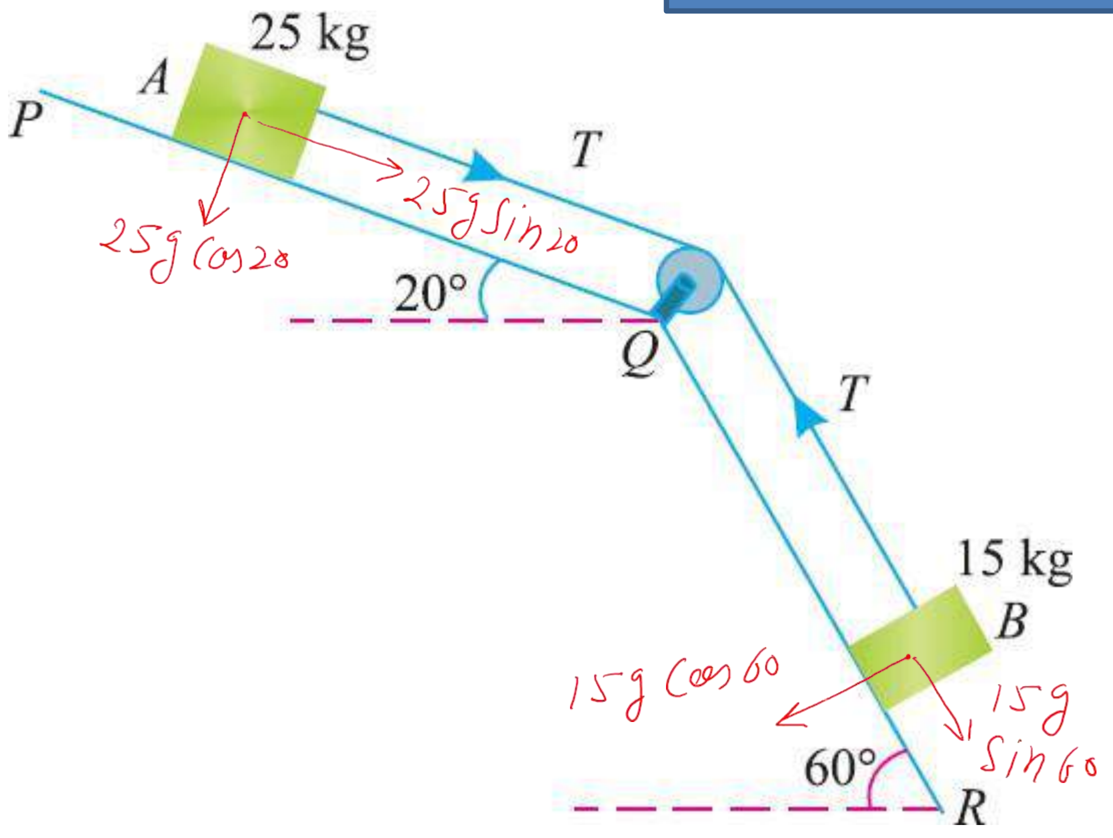
equating (1) & (2)

$\boxed{a = 5.28\ m/s^2}$ Ans

$d = \cancel{v_0 t}^{\,0} + \frac{1}{2} a t^2$

$= \frac{1}{2} \times 5.28 \times 3^2$

$\boxed{d = 23.76\ m}$ Ans



# If the coefficient of friction be 0.3 find the resulting acceleration.

for body 1 Resultant downward force along the plane

$P = [(15 \times 9.81 \times \sin 30) - (0.3 \times 15 \times 9.81) \times (\cos 30)] - T$

$P = 35.34 - T$

also $P = 15a$.

$\therefore T = 35.34 - 15a$ — (1)

for body 2

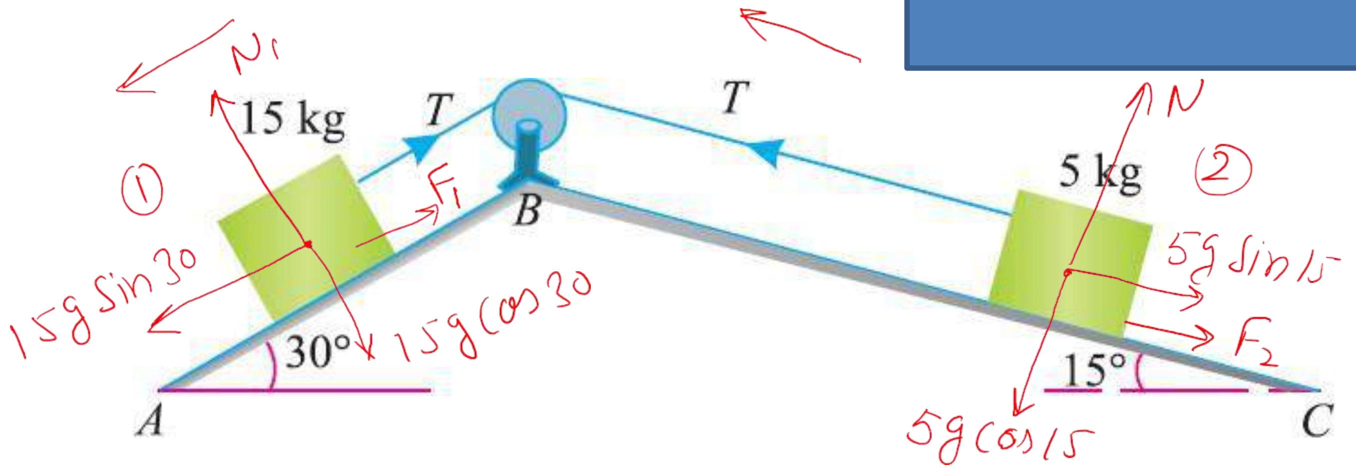
Resultant upward force

$P = T - (5 \times 9.81 \times \sin 15) - (0.3 \times 5 \times 9.81 \cos 15)$

$P = T - 27$

also $P = 5a$

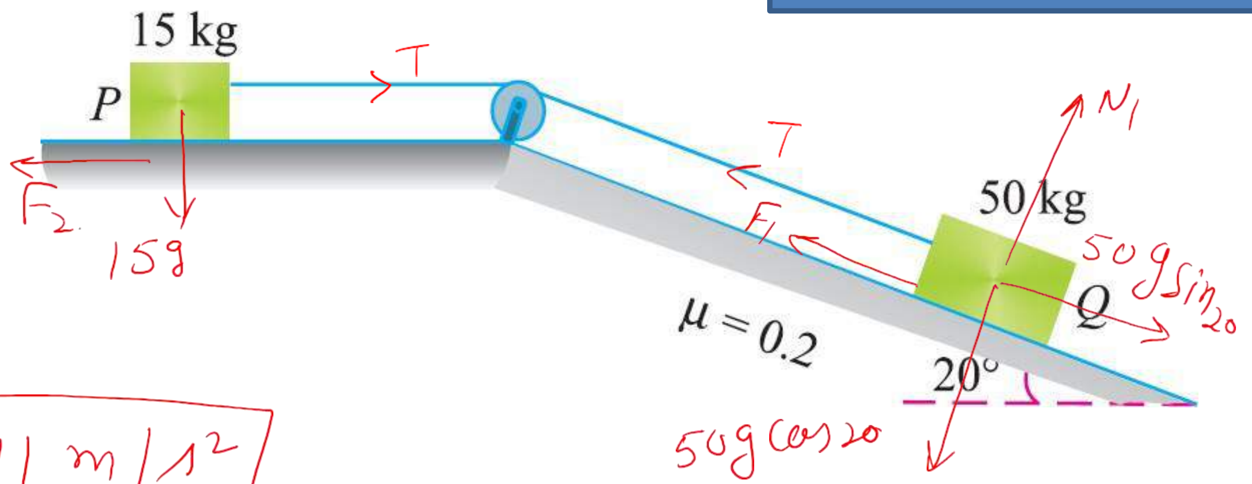
$\therefore T = 27 + 5a$ — (2)




equating (1) & (2)

$\boxed{a = 0.42\ m/s^2}$ Ans

Find acceleration of two bodies and tension in the string.




for body Q Resultant downward force

$P = (50 \times 9.81 \times \sin 20) - (0.2 \times 50 \times 9.81) \times (\cos 20) - T$

$P = 75.57 - T$

also $P = 50a$.

$\therefore T = 75.57 - 50a$ — (1)

for body P Resultant horizontal force

$P = T - 0.2 \times 15 \times 9.81$

$P = T - 29.43$

also $P = 15a$.

$T = 29.43 + 15a$ — (2)

equation (1) & (2)

$\boxed{a = 0.71 \ m/s^2}$

$\boxed{T = 40.07 \ N}$

Ans

**Energy** :→ Capacity to do work. units (J)

**Types of Energy** :- Electrical, Chemical, Thermal, Mechanical etc...

**Mechanical Energy** :→

(1) **Potential** :→ Energy possessed by a body, for doing work, due to its position.

→ A body raised to some height

→ compressed air in a cylinder.

→ Compressed spring or extended spring

(2) **Kinetic Energy** :→ Energy possessed by a body, for doing work, by virtue of its mass and velocity (or motion), which has been brought to rest by uniform retardation.

$$W = F \times d$$

$$= m \times a \times d$$

$$\boxed{W = \frac{m v_0^2}{2}}$$

$$\cancel{v^2}^{\,0} - v_0^2 = 2ad$$

$$-ad = -\frac{v_0^2}{2}$$

$$ad = \frac{v_0^2}{2}$$

This work is due to K.E.

A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of the water. Neglect the air resistance.

Sol: from tower to water surface

$$P.E = mgh = 60 \times 9.81 \times 20 = 11772 \text{ N-m} \quad -(2)$$

$$\longleftrightarrow$$

under water

work done by resistive force of water.

$$W = P \times h$$

$$W = 2P \quad -(2)$$

equate (1) & (2)

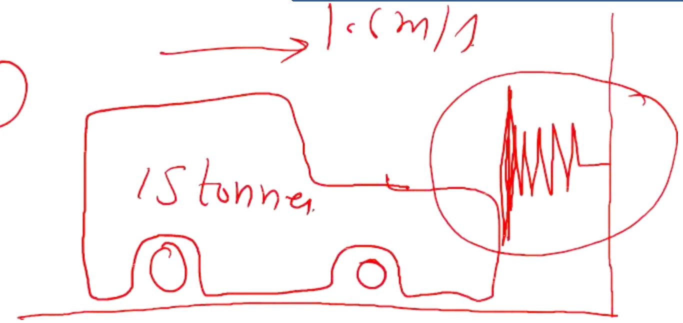
$$11772 = 2P$$

$$\boxed{P = 5886\,N}$$ Ans

*A truck of mass 15 tonnes travelling at 1.6 m/s impacts with a buffer spring, which compresses 1.25 mm per kN. Find the maximum compression of the spring.*

Sol: $m = 15$ tonnes; $v = 1.6$ m/s; $K = \frac{1}{1.25} = 0.8$

$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} \times 15 \times 1.6^2 = 19.2\ kJ = 19200\ kN\text{-}mm$ — (1)




$F = Kx$

work of force F

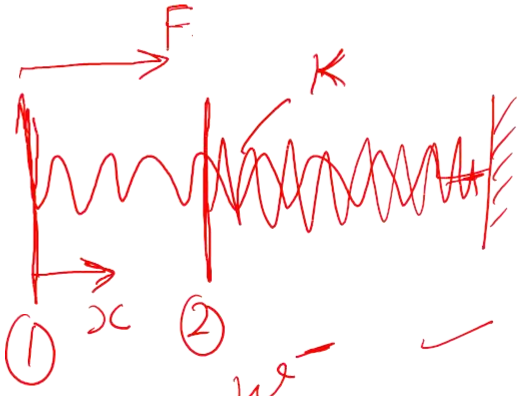
$\delta W = F\,dx$

Integrate from (1) & (2)

$$W_{1-2} = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} Kx\,dx$$

$$W_{1-2} = \frac{1}{2} K x_2^2 - \frac{1}{2} K x_1^2 \Rightarrow K\left(\frac{x_2^2 - x_1^2}{2}\right)$$

OR $\boxed{W_{1-2} = \frac{F}{2} \times \Delta x}$ Avg load × distance




$k = \frac{W}{\delta}$

Compressive load = $Kx = 0.8x$

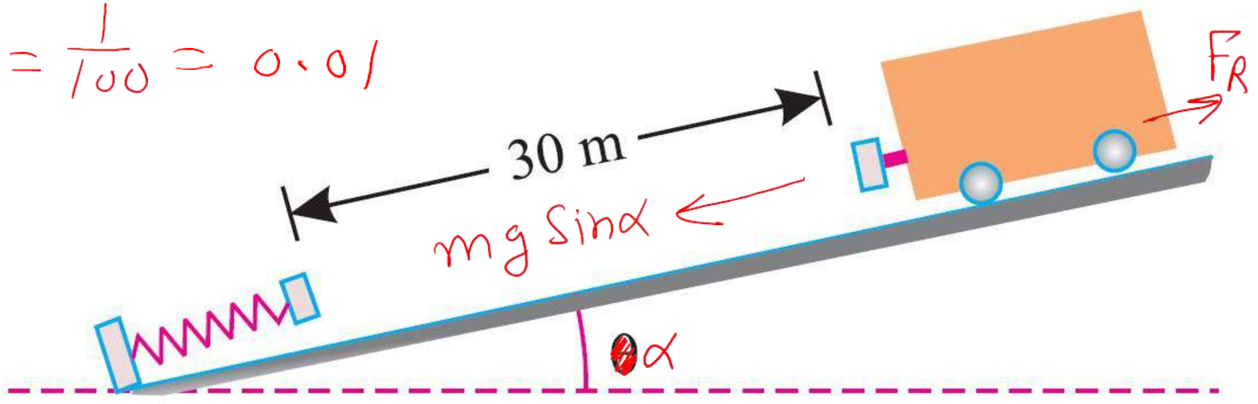
then $W_{1-2} = \boxed{\frac{0.8x}{2}} \times x = 0.4x^2$ KN-mm. — (2)

equating (1) & (2).

$19200 = 0.4x^2$

$\boxed{x = 219.089}$ Ans

A wagon of mass 50 tonnes, starts from rest and travels 30 metres down a 1% grade and strikes a post with bumper spring as shown in Fig. If the rolling resistance of the track is 50 N/t, find the velocity with which the wagon strikes the post. Also find the amount by which the spring will be compressed, if the bumper spring compreses 1 mm per 20 kN force.




Sol. $m = 50\,t$ ; $V_0 = 0\ m/s$ ; $d = 30\ m$ ; $\sin\alpha = \frac{1}{100} = 0.01$

$F_R = 50\ N/t = 2500\ N = 2.5\ kN$ ; $K = 20$

Net downward force

$$P = mg\sin\alpha - F_R$$

$$= (50 \times 9.81 \times 0.01) - 2.5$$

$$\boxed{P = 2.4\ kN}$$

also $P = ma = 50a$

$$a = \frac{P}{m} = 0.048\ m/s^2$$

$$v^2 = \cancel{v_0^2}^{\,0} + 2ad$$

$$v = \sqrt{2 \times 0.048 \times 30}$$

$$\boxed{v = 1.7\ m/s}$$

$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} \times 50 \times 1.7^2 = 72.25 \text{ KN-m}$

$K.E. = 72250 \text{ KN-mm.}$ — (1)

Compressive load $= Kx = 20x$

Work by comp. load on spring.

$W_{1-2} =$ Avg load $\times$ deflection.

$W_{1-2} = \frac{20x}{2} \times x$ — (2)

equating (1) & (2)

$72250 = 10x^2$

$\boxed{x = 85 \text{ mm}}$ Ans

An automobile weighing 4 tonnes is driven down a 5° incline at a speed of 60 km/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 1.5 kN. Determine the distance traveled by the automobile as it comes to a stop.


$m = 4t$ ; $F_R = 1.5$ KN ; $v_0 = 60$ Km/h $= 16.66$ m/s.

Net downward force.

$$P = mg\sin\alpha - F_R$$

$$= (4 \times 9.81 \times \sin 5) - 1.5$$

$$P = 1.9 \text{ KN.}$$

$$W_{1-2} = P \times d$$

$$W_{1-2} = 1.9d \quad -(1)$$

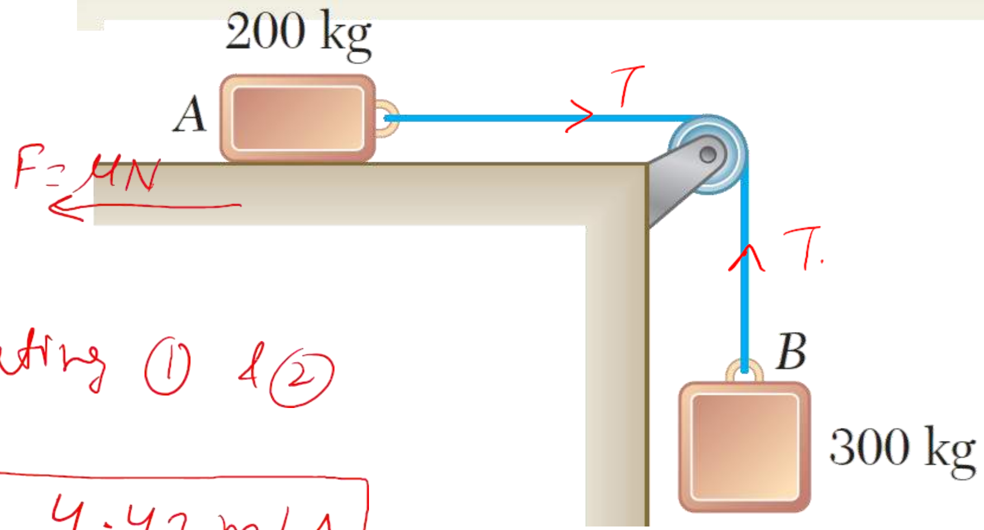
$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times 4 \times 16.66^2 = 555.11 \text{ KJ} \quad -(2)$$

equating (1) & (2).

$$1.9d = 555.11$$

$$\boxed{d = 292.16 \text{ m}}$$ Ans

Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block A after it has moved 2 m. Assume that the coefficient of kinetic friction between block A and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.




Sol: for A $\quad W_{1-2} = K.E.$

$$P \times d = \frac{1}{2} m v^2$$

$$(T - 0.25 \times 200 \times 9.81) \times 2 = \frac{1}{2} \times 200 \times v^2$$

$$2T - 981 = 100 v^2 \quad \text{— (1)}$$

for B $\quad W_{1-2} = K.E.$

$$[(300 \times 9.81) - T] \times 2 = \frac{1}{2} \times 300 \times v^2$$

$$5886 - 2T = 150 v^2 \quad \text{— (2)}$$

equating (1) & (2)

$$\boxed{v = 4.43 \text{ m/s}}$$ Ans

A 4-kg stone is dropped from a height h and strikes the ground with a velocity of 25 m/s. (a) Find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped. (b) From what height h1 must a 1 kg stone be dropped so it has the same kinetic energy?

Sol: $K.E. = \frac{1}{2} m v^2 = \frac{1}{2} \times 4 \times 25^2 = 1250 \text{ N-m}$.

a) $W_{1-2} = 4 \times 9.8 \times h = 39.24 h$.

$W_{1-2} = K.E.$

$h = \frac{1250}{39.24}$

$h = 31.85 \text{ m}$ Ans

b) $W_{1-2} = 1 \times 9.81 \times h_1$

$W_{1-2} = K.E.$

$h_1 = \frac{1250}{9.81}$

$h_1 = 127.42 \text{ m.}$ Ans

A 1 kg stone is dropped from a height h and strikes the ground with a velocity of 25 m/s. (a) Find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped, (b) Solve Part a assuming that the same stone is dropped on the moon. (Acceleration of gravity on the moon = 1.62 $m/s^2$.)

Sol:

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times 1 \times 25^2 = 312.5\ J.$$

(a) On earth

$$W_{1-2} = 1 \times 9.81 \times h$$

$$W_{1-2} = K.E$$

$$h = \frac{312.5}{9.81} = 31.85\ m. \quad \text{Ans}$$

b) On moon.

$$W_{1-2} = 1 \times 1.62 \times h.$$

$$W_{1-2} = K.E$$

$$h = \frac{312.5}{1.62} = 192.9\ m \quad \text{Ans}$$

Determine the maximum theoretical speed that may be achieved over a distance of 100 m by a car starting from rest, assuming there is no slipping. The coefficient of static friction between the tires and pavement is 0.75, and 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (a) front-wheel drive, (b) rear-wheel drive.

Sol: (a) Front wheel drive

$N = 0.6\,W$

$F_R = \mu N = 0.75 \times 0.6\,W$

$F_R = 0.45\,W\ (N)$

$W_{1-2} = F_R \times d = 0.45\,W \times 100$

$W_{1-2} = 45\,W\ (J)$ — (1)

$K.E. = \frac{1}{2} \times \frac{W}{g} \times v^2$

$= \frac{1}{2} \times \frac{W}{9.81} \times v^2$ — (2)

equatin (1) & (2)

$W_{1-2} = K.E.$

$45\,\cancel{W} = \frac{1}{2} \times \frac{\cancel{W}}{9.81} \times v^2$

$v = \sqrt{2 \times 9.81 \times 45}$

$\boxed{v = 29.72\ m/s}$ Ans

(b) Rear wheel drive

$N = 0.4\,W$

$F_R = 0.7 \times 0.4\,W = 0.3\,W$

$W_{1-2} = F_R \times d = 0.3\,W \times 100$

$W_{1-2} = 30\,W\ (J)$ — (1)

$K.E = \frac{1}{2} \times \frac{W}{g} \times v^2$ — (2)

equating (1) & (2)

$W_{1-2} = K.E.$

$30\,\cancel{W} = \frac{1}{2} \times \frac{\cancel{W}}{9.81} \times v^2$

$v = \sqrt{2 \times 9.81 \times 30}$

$\boxed{v = 24.26\ m/s}$ Ans

# Kinetics of Rotation

Torque = Turning moment of force.

$T = P \times l$ (Force x $\perp$ distance)

Work done by torque.

$W = T \times \theta$ (Torque x Angular displacement)

K.E. of Rotation = $\frac{1}{2} \times I \times \omega^2$ [I - mass MOI ; $\omega$ = Angular velocity].

also $T = I\alpha$ [$\alpha$ - angular acceleration]

Angular Momentum = $\underline{\underline{I\omega}}$

*A flywheel of mass 8 tonnes starts from rest, and gets up a speed of 180 r.p.m. in 3 minutes. Find the average torque exerted on it, if the radius of gyration of the flywheel is 60 cm.*

Sol: $m = 8$ tonnes, $\omega_0 = 0$ rad/s; $N = 180$ rpm; $\omega = \frac{2\pi N}{60} = 6\pi$ rad/s; $t = 180$ s; $K = 0.6$ m.

$$I = mK^2 = 8000 \times 0.6^2 = 2880 \text{ Kg-m}^2$$

$$\omega = \cancel{\omega_0}^{0} + \alpha t$$

$$6\pi = \alpha \times 180$$

$$\alpha = 0.105 \text{ rad/s}^2$$

$$T = I\alpha$$

$$= 2880 \times 0.105$$

$$\boxed{T = 302.4 \text{ N-m}}$$ Ans

A flywheel of mass 400 kg and radius of gyration 1 m losses its speed from 300 r.p.m. to 240 r.p.m. in 120 seconds. Determine the retarding torque acting on it.

Sol: $m = 400\,Kg$ ; $K = 1\,m$ ; $\omega_0 = 10\pi$ rad/s ; $\omega = 8\pi$ rad/s ; $t = 120\,s$

$$\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega - \omega_0}{t} = \frac{8\pi - 10\pi}{120} = -0.052\ rad/s^2$$

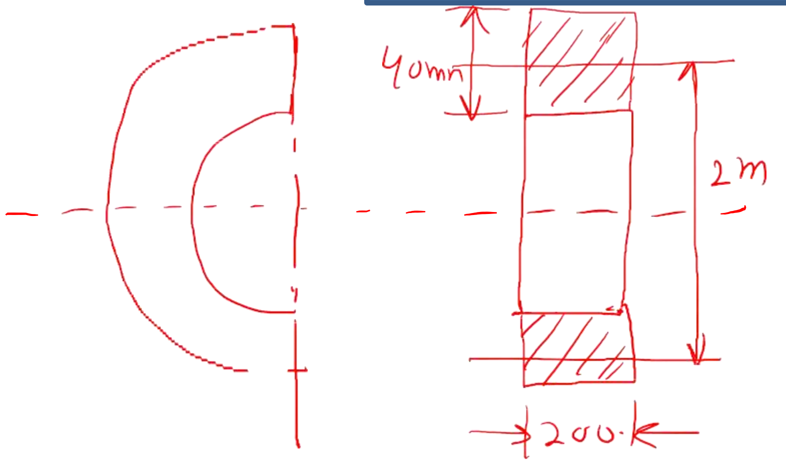
$$I = mK^2 = 400 \times 1^2 = 400\ Kg\text{-}m^2$$

$$T = I\alpha$$

$$= 400 \times (-0.052)$$

$$\boxed{T = -20.8\ N\text{-}m}$$ Ans

*A flywheel is made up of steel ring 40 mm thick and 200 mm wide plate with mean diameter of 2 metres. If initially the flywheel is rotating at 300 r.p.m., find the time taken by the wheel in coming to rest due to frictional couple of 100 N-m. Take mass density of the steel as 7900 kg/m³.*




Sol: Volume of flywheel $= \pi D \times A$

$= \pi \times 2 \times 0.04 \times 0.2$

$V = 0.05\ m^3$

mass of flywheel $= \rho \times V$

$= 7900 \times 0.05$

$m = 395\ kg.$

$N_0 = 300\ rpm;\ \omega_0 = 10\pi\ rad/s.$

$\omega = 0\ rad/s.$

$I = mk^2 = 395 \times 1^2$

$I = 395\ kg\text{-}m^2$

$T = I\alpha.$

$-100 = 395 \times \alpha.$

$\alpha = -0.253\ rad/s^2$

$\omega^0 = \omega_0 + \alpha t$

$0 = 10\pi - (0.253\, t)$

$\boxed{t = 124.17\, s}$ Ans

A constant torque of 2 kN-m is exerted on a crankshaft to start the engine. The flywheel has a mass of 1800 kg and radius of gyration 1 m. If there is a resisting torque of 1 kN-m, find the speed of the engine after 1 minute.

Sol: $T = 2\,kN\text{-}m$ ; $\omega_0 = 0\ rad/s$ ; $m = 1800\ kg$ ; $K = 1\ m$ ; $T_R = 1\,kN\text{-}m$ ; $t = 60\ s$

Net torque $T_{Net} = T - T_R = 1\,kN\text{-}m$

$$I = mK^2 = 1800 \times 1^2 = 1800\ kg\text{-}m^2$$

$$T_{Net} = I\alpha \Rightarrow \alpha = \frac{T_{Net}}{I} = \frac{1000}{1800} = 0.55\ rad/s^2$$

$$\omega = \cancel{\omega_0}^{0} + \alpha t = 0.55 \times 60 = 33.33\ rad/s$$

also $\omega = \frac{2\pi N}{60}$

$$\boxed{N = 318.3\ rpm}$$

A retarding torque of 600 N-m is applied to a flywheel rotating at 240 r.p.m. Find the moment of inertia of the flywheel, if it comes to rest in 100 seconds.

Sol: $T = -600\, N\text{-}m$ ; $\omega_0 = 8\pi$ rad/s ; $\omega = 0$ rad/s ; $t = 100$ s.

$$\omega = \omega_0 + \alpha t$$

$$0 = 8\pi + (\alpha \times 100)$$

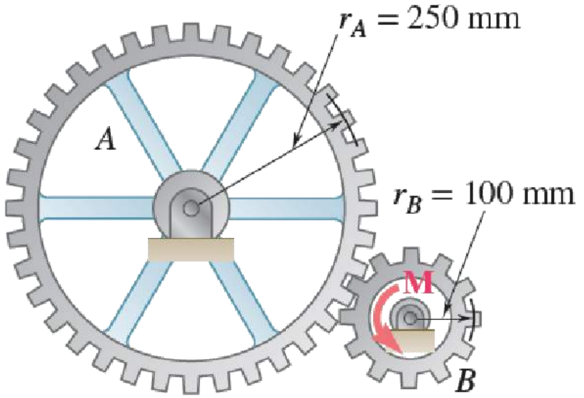
$$\alpha = -0.25 \text{ rad/s}^2$$

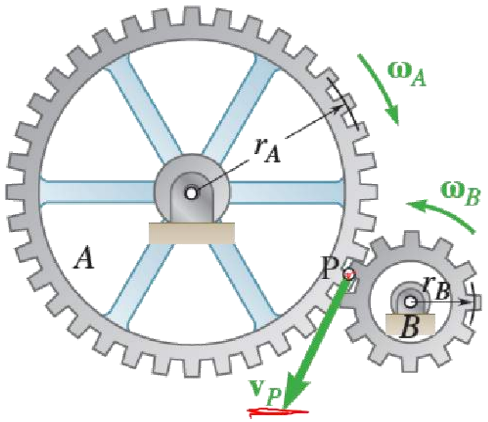
$$T = I\alpha$$

$$I = \frac{-600}{-0.25}$$

$$\boxed{I = 2400 \text{ Kg-m}^2}$$ Ans

Gear $A$ has a mass of 10 kg and a radius of gyration of 200 mm; gear $B$ has a mass of 3 kg and a radius of gyration of 80 mm. The system is at rest when a couple **M** of magnitude 6 N·m is applied to gear $B$. Neglecting friction, determine (*a*) the number of revolutions executed by gear $B$ before its angular velocity reaches 600 rpm, (*b*) the tangential force that gear $B$ exerts on gear $A$.




$$v_P = r_A \omega_A = r_B \omega_B \Rightarrow \omega_A = \frac{r_B}{r_A}\omega_B = \frac{100}{250}\omega_B = 0.4\,\omega_B$$

$N_B = 600$ rpm; $\omega_B = 62.83$ rad/s ; $\omega_A = 25.13$ rad/s.

$$K.E. = \frac{1}{2} I_A \omega_A^2 + \frac{1}{2} I_B \omega_B^2 = 164.2\text{ J.}$$

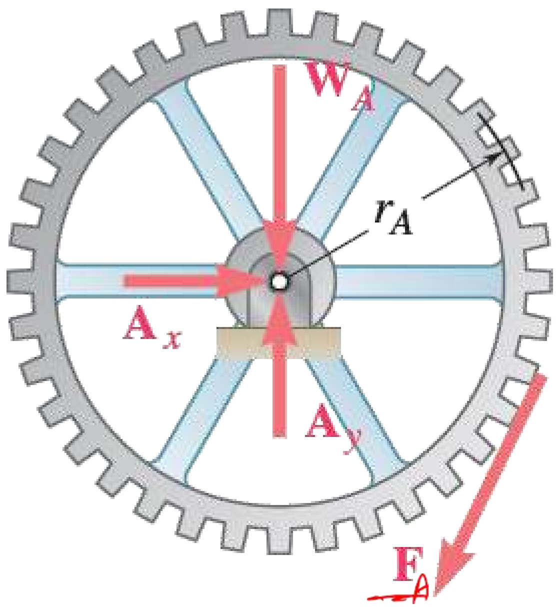
$$W = M\theta_B = 6\theta_B$$

$$W = K.E.$$

$$\theta_B = \frac{164.2}{6}$$

$$\theta_B = 27.36\text{ rad}$$

$$n_B = \frac{\theta_B}{2\pi} = 4.35\text{ rev} \quad \text{Ans}$$




$K.E$ of gear $A = \frac{1}{2} m_A \times \omega_A^2 = 126.3\ J.$

$$W = T_A \theta_A = F_A \times r_A \times \theta_A$$

$$= F_A \times r_B \times \theta_B$$

$$W = F_A \times 0.1 \times 27.36$$

$$W = K.E$$

$$F_A = \frac{126.3}{0.1 \times 27.36} = 46.16\,N \quad \text{Ans}$$