# Relation

- Relations on a Set
- Combining Relations
- Composition
- Representing Relations Using Matrices
- Representing Relations Using Digraphs
- Properties of Relations
- Equivalence Relations
- Partial Orderings
- Hasse Diagrams
- Lattices

Let A and B be sets. A binary relation from A to B is a subset of A × B.

$A = \{1, 2, 3, 4\}$ $\quad B = \{0, 5, 10, 15\}$

$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$

$$A \times B = \{(1,0)\ (1,5)\ (1,10)\ (1,15)\ (2,0)\ (2,5)\ (2,10)\ (2,15)\ (3,0)\ (3,5)\ (3,10)\ (3,15)\ (4,0)\ (4,5)\ (4,10)\ (4,15)\}$$

$n(A) = m$
$n(B) = n$
$n(A \times B) = m \cdot n$

$R = \{(a,b) \mid \boxed{a^2 + b^2 \le 20},\ \forall a \in A,\ \forall b \in B\}$

$R = \{(1,0)\ (2,0)\ (3,0)\ (4,0)\}$ $\quad R \subseteq A \times B.$

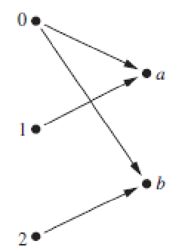
$A = \{a, b, c\}$, $B = \{1, 2, 3\}$

$R = \{(x_1, x_2) \mid x_1 + x_2 \le 4\}$, $\quad x_1$ = Position of elements of set A in Alphabet Syst.

$x_2 \in B$

$\boxed{a = 1,\ b = 2,\ c = 3}$ $\quad R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (3,1)\}$

Let A = {0, 1, 2} and B = {a, b}. Then {(0, a), (0, b), (1, a), (2, b)} is a relation from A to B.




| R | a | b |
|---|---|---|
| 0 | × | × |
| 1 | × | |
| 2 | | × |

$\boxed{A \times B \neq B \times A}$ , $\boxed{n(A\times B) = n(B \times A)}$

A relation on a set A is a relation from A to A.

In other words, a relation on a set A is a subset of A × A.

$$R \subseteq A \times A$$

Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

If A has n elements then $\boxed{n(A \times A) = n^2}$

A = { Ayush, Sourav, Peer, Aditi, Garima }

H = { 5.6 5.3 5.7 5.7 5.9 }

$R = \{(a,b) \mid \text{height}(a) - \text{Height}(b) = 0.2,\ \forall a \in A,\ \forall b \in A\}$

R = { (Garima, Aditi) (Garima, Peer) }

Domain : a

Range : b

$R = \{(a,b)\} \subseteq A \times B$

$a \in A,\ b \in B$

$\boxed{\text{No. of relation from set } A \text{ to } B = 2^{m \cdot n}}$

$\boxed{\begin{array}{l} m = n(A) \\ n = n(B) \end{array}}$

A = {a, b}

B = {1}

$A \times B = \{(a,1)(b,1)\}$

$R_1 = \{(a,1)\}$

$R_2 = \{(b,1)\}$

$R_3 = \{(a,1)(b,1)\}$

$R_4 = \emptyset$

No. of relation A to B $= 2^{2\cdot1} = 4$

How many relations are there on a set with n elements? $= \boxed{2^{n^2}}$

Q: If A = {a, b, c}

Total No. of relation from A to A $= 2^9$

Q: Let set A has 5 elements & set B has 6 elements

then cardinalities (No. of elements)

$n(A\times B) = 5\cdot 6 = 30$ , No. of relation from A to A $= 2^{25}$

$n(A\times A) = 25$ No. of relation from B to B $= 2^{36}$

$n(B\times B) = 36$ No. of relation from A to B $= 2^{30}$

## Combining Relations



Because relations from A to B are subsets of A × B, two relations from A to B can be combined in any way two sets can be combined.

R1 R2, R1 ∩ R2, R1 R2, R1 − R2, R2 − R1



$R_1 = \{(a,a)\ (a,b)\ (b,c)\ (b,b)\}$

$R_2 = \{(a,b)\ (a,d)\ (b,d)\ (d,d)\}$



$\underbrace{R_1 \cup R_2} = \{(a,a)\ (a,b)\ (b,c)\ (b,b)\ (a,d)\ (b,d)\ (d,d)\}$



$\underbrace{R_1 \cap R_2} = \{(a,b)\}$

$R_1 - R_2 = \{(a,a), (b,c), (b,b)\}$



$R_2 - R_1 =$

$R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2$

$= \{(a,a)\ (b,c)\ (b,b)\ (a,d)\ (b,d)\ (d,d)\}$





## Composition



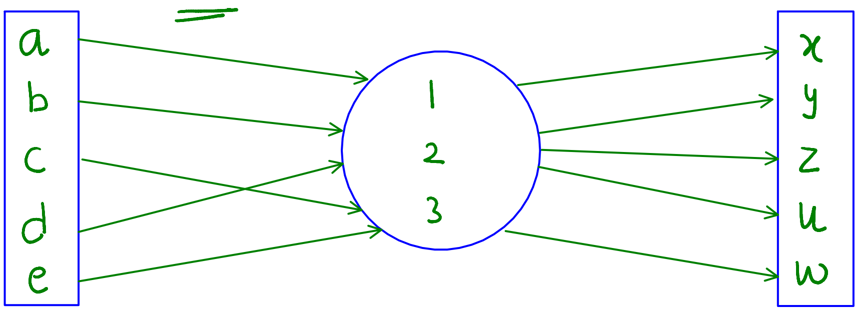
Let R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pairs (a, c), where a ∈ A, c ∈ C, and for which there exists an element b ∈ B such that $\boxed{(a, b) \in R}$ and $\boxed{(b, c) \in S}$. We denote the composite of R and S by $\underline{S \circ R}$.



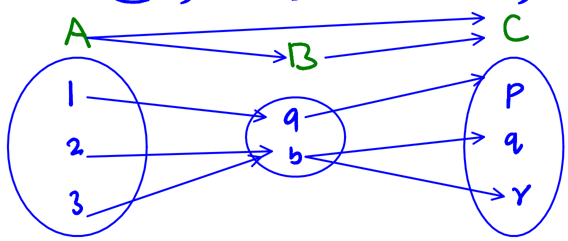
$S \circ R$

$R = \{(a,1)(b,2)\ (c,3)\ (d,2)\ (e,3)\}$

$S = \{(1,x)(2,y)\ (2,z)\ (2,u)\ (3,w)\}$




$S \circ R = (a,x)\ (b,y)\ (b,z)\ (b,u)\ (c,w)\ (d,y)\ (d,z)\ (d,u)\ (e,w)$




$R \circ S \qquad S \circ R$

$(1,p)\ (2,q)(2,r)(3,q)$
$(3,r)$

$R_1 = \{(1,a)\ (2,a)\ (3,b)\}$

$R_2 = \{(a,p)\ (b,q)(b,r)\}$

$R_1 = \{(a,b)\}$

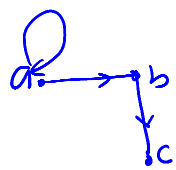
$R_2 = \{(b,1)(b,2)\}$

$R_2 \circ R_1 = (a,1)(a,2)$

Composing the Parent Relation with Itself

$R = \{(a,a)\ (a,b)\ (b,c)\}$

$R = R^2 = R^3 = \dots, = R^n$




## Representing Relations Using Matrices

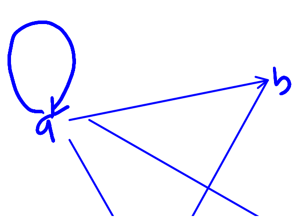
## Representing Relations Using Digraphs

$$M_R = \begin{cases} 1 & \text{if } aRb \ \forall a \in A, b \in B \\ 0 & \text{if } a \not R b \ \forall a \in A, b \in B \end{cases}$$

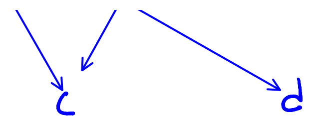
A directed graph, or digraph, consists of a set V of vertices (or nodes) together with a set E of ordered pairs of elements of V called edges (or arcs). The vertex a is called the initial vertex of the edge (a, b), and the vertex b is called the terminal vertex of this edge.

$A = \{a, b, c, d\}$

$R = \{(a,b),\ (a,c)\ (a,a)\ (a,d)\ (b,c)\}$




| | a | b | c | d |
|---|---|---|---|---|
| a | 1 | 1 | 1 | 1 |
| b | 0 | 0 | 1 | 0 |
| c | 0 | 0 | 0 | 0 |

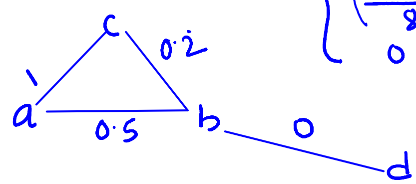


$d \,|\, 0 \quad 0 \quad 0 \quad 0$

# Two cities are related if they have less distance b/w them

$\to (0-100)\%$

$$dist(x) = \begin{cases} 1 & \text{if } dist(x_1,x_2) \le 20 \text{ Km.} \\ \left(\frac{100-x}{80}\right) & \text{if } 20\text{ km} < dist(x_1,x_2) \le 100 \\ 0 & \text{if } dist(x_1,x_2) > 100 \end{cases}$$




$dist.(a,b) =$

$dist(a,c) \le 20$ K.m..

$$\frac{50}{100} = 0.5 = \frac{100-x}{80} \Rightarrow \frac{1}{2} = \frac{100-x}{80}$$

$$\Rightarrow 40 = 100 - x$$

$$\boxed{x = 60 \text{ km.}}$$

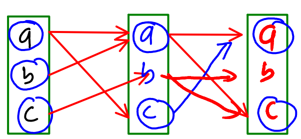
$A = 100$

$B \le 100$

$C > 100$

Suppose that the relations R1 and R2 on a set A are represented by the matrices

$$R_1 = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 1 & 0 & 0 \\ c & 0 & 1 & 0 \end{array}, \quad R_2 = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 0 \end{array}$$




What are the matrices representing R1 U R2, R1 ∩ R2 and R2 o R1?

$$R_1 \cup R_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}, \qquad R_1 \cap R_2 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$R_1 = \{(a,a)\ (a,c), (b,a)\ (c,b\}$ ✓

$R_2 = \{(a,a)\ (a,c). (b,b)\ (b,c), (c,a)\}$ ✓

$$R_2 \circ R_1 = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 1 \end{array}$$ ✓

Suppose that the relation R on a set is represented by the matrix

MR =

1 1 0

1 1 1

0 1 1

Is R reflexive, symmetric, and/or antisymmetric?

Let $a, b, c \in A$

**Properties of Relations**

- Reflexive → $\boxed{aRa \ \forall a \in A}$
- Symmetric → $\boxed{\text{If } aRb \Rightarrow bRa, \forall a, b \in A}$
- Antisymmetric → $\boxed{\text{If } aRb \ \& \ bRa \Rightarrow a = b, \forall a, b \in A}$
- Transitive → $\boxed{\text{If } aRb \ \& \ bRc \Rightarrow aRc, \ \forall a, b, c \in A}$
- Irreflexive → $\boxed{\exists a \in A, a \not R a}$
- Asymmetric → $\boxed{\text{If } aRb \Rightarrow b \not R a}$

Reflexive: Let R be the relation defined on set A then it is said to be reflexive if $aRa \ \forall a \in A$

$A = \{1, 2, 3, 4\}$, $R = \{(1,1) (1,2) (2,2) (3,3) (4,4)\}$

$R = \{(1,1) (1,2) (2,2) (3,3) (2,1)\}$ not reflexive

$\exists a \in A$ s.t. $\underline{a \not R a}$ say 4

$A = \{1, 2, 3\}$ $R = \{(a,b) \mid a^2 + b^2 \le 8$

$R = \{(1,1) (1,2) (2,1) (2,2)\}$

not reflexive $(3,3) \notin R$

Symmetric: Let R be a relation defined on set A & it is said to be symmetric if $aRb$ then $bRa$ $\forall a, b \in A$

$A = \{$ Ayush, Sourav, Peer, Aditi, Garima $\}$

$H = \{$ 5.6 5.3 5.7 5.7 5.9 $\}$

$R = \{(a,b) \mid \text{height}(a) - \text{Height}(b) = 0.2,\ \forall a \in A, \forall b \in A\}$

$R = \{(\text{Garima}, \text{Aditi})\ (\text{Garima}, \text{Peer})\}$

not reflexive, not symmetric

Garima (H) − Aditi (H) = 0.2

Aditi (H) − Garima (H) ≠ 0.2

$A = \{a, b, c\}$ $R = \{(a,a)\ (b,b)\ (a,b)\ (c,c)\ (b,a)\}$

Reflexive yes ✓

Symmetric yes ✓

$S = \{\text{Ayush}, \text{Garima}, \text{Aditi}\}$

$R = \{(a,b) \mid a$ can spend ₹1000 on $b$, $\forall a, b \in S\}$

Ayush $\xrightarrow{₹1000}$ Garima

Garima $\xrightarrow{₹100}$ Ayush

not symmetric

Antisymmetric Let R be the relation defined on set A then it is said to be antisymmetric if $aRb$ & $bRa$, then $a \equiv b$

| Hypo. | | Con. |
|---|---|---|
| If $aRb$ (I) & $bRa$ (II) | then | $a \equiv b$ |

| p | q | p → q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Q: $A = \{a, b, c\}$ $R = \{(a,a)\ (a,b)\ (b,b)\}$

reflexive NO, $(c,c) \notin R$

symmetric. NO $(b,a) \notin R$

Antisymmetric yes if $aRa$ & $aRa \Rightarrow a=a$ ✓

if $bRb$ & $bRb \Rightarrow b=b$ ✓

if $aRb$ & $\boxed{\phantom{xx}} \Rightarrow$

Hypo. Incomp.

$A=\{1,2,3,4\}$ $R=\{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

Reflexive - yes

symmetric - yes

Antisymmetric yes

if $1R1$ & $1R1 \Rightarrow 1=1$

Same for 2, 3 & 4,

$S=\{1,2,3\}$ $R=\{(1,1)\ (1,2)\ (2,1)\}$

Reflexive -NO $(2,2) \notin R$

Symmetric - yes

Antisymmetric - No $(1,1)$ $1=1$, $(1,2)$ $1 \neq 2$

if $\boxed{1R2 \ \& \ 2R1} \Rightarrow 1 \neq 2$

Hypo. Present Con. Absent

$A=\{a,b,c,d\}$ $R=\{(a,b)\ (a,a)\ (b,b)\ (b,c)\}$

Reflexive, No. $(c,c) \notin R$

Symmetric NO $(b,a) \notin R$

Antisymmetric: $(a,b)$, if $aRb$ & $\boxed{\phantom{xx}} \Rightarrow$

Incomplete Hypo.

$(a,a)$ if $\underline{aRa}$ & $\underline{aRa} \Rightarrow a=a$

$(b,b)$ if $bRb$ & $bRb \Rightarrow b=b$

$(b,c)$, if $bRc$ & $\boxed{\phantom{xx}} \Rightarrow$

yes CRb

**Transitive :** Let R be a relation defined on a set A. & if

$\boxed{aRb \ \& \ bRc} \Rightarrow \boxed{aRc}$

Hypo. Con.

$A = \{a, b, c\}$ $R = \{(a, b)\}$

- reflexive — NO
- Symmetric — NO
- Anti symmetric - yes
- transitive - yes

if aRb & $\boxed{\phantom{bR}} \Rightarrow$

if aRb & $\boxed{bR} \Rightarrow$

$A = \{1, 2, 3, 4\}$ $R = \{(1, 1)\ (1, 2)\ (2, 3)\ (3, 1)\}$

- Ref x
- Symm x
- Antisymm ✓
- Transitive

If $1 \to 1 \to 1$, $1 \to 1 \to 2$

if $1 \to 2 \to 3$

(1, 1) Hypo ✓ Con ✓ trans.

(1, 2) Hypo ✓ Con. X not. trans.

Not transitive

A = { All the Student of K19CJ }

R = {(a, b) | a is older than b

- Refl X
- Symm. X
- Anti ✓
- trans. ✓

Say Age of

| | |
|---|---|
| Ayush | 20 yr |
| Satyam | 18 yr |
| Lovish | 19 yr |
| Garima | 18 yr |
| Aditi | 17 yr |

$18 > 17 > 16$

R = { (Ayush, Satyam) (Ayush, Lovish) (Ayush, Garima)
(Ayush, Aditi) (Satyam, Aditi) (Lovish, Satyam)
(Lovish, Garima) (Lovish, Aditi) (Garima, Aditi) }

Q: $A = \{x \mid x \in \mathbb{N}\}$
$R = \{(a, b) \mid a|b \ \forall a \in A, b \in A$

$R = \{ (1,1) (1,2) (1,3) \dots,$
$(2,2) (2,4) (2,6) \dots,$
$\vdots \}$

Reflexive - Yes  every No. divides itself
Symmetric - No  Counter example $1R2$ but $2 \not R 1$
Transitive - yes
Antisymmetric - yes
Irreflexive - No
Asymmetric - yes

if $(x,y)$ & $\boxed{\phantom{xx}}$ $\Rightarrow$

$(x,y)\ (x,z)\ (z,x)\ (z,y)$
$x \neq z$

GATE | GATE-CS-2009 | Question 4
Consider the binary relation $R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$.
Which one of the following is TRUE?
(A) R is symmetric but NOT antisymmetric ✗
(B) R is NOT symmetric but antisymmetric ✗
(C) R is both symmetric and antisymmetric ✗
(D) R is neither symmetric nor antisymmetric ✓

not Symm. $(y,x) \notin R$
✓ if $xRy$ & $\boxed{\cancel{yRx}}$ $\Rightarrow x = y$
✗ if $xRz$ & $\boxed{zRx}$ $\Rightarrow x \neq z$
not Anti

GATE | GATE CS 1999 | Question 1
The number of binary relations on a set with n elements is:
(A) $n^2$
(B) $2^n$
(C) $2^{n^2}$
(D) None of the above

$\boxed{\text{Counting of Relation}}$

Let a set has n elements then the no. of different relation on set A are

$n = 10$

| | |
|---|---|
| Relation | $2^{n^2}$ |

$2^{100}$

| | |
|---|---|
| Relation | $2^{n^2}$ |
| Reflexive | $2^{n(n-1)}$ |
| Symmetric | $2^{\frac{n(n+1)}{2}}$ |
| Antisymmetric | $2^n \cdot 3^{\frac{n(n-1)}{2}}$ |
| Asymmetric | $3^{\frac{n(n-1)}{2}}$ |
| Irreflexive | $2^{n(n-1)}$ |

$2^{100}$

$2^{90}$

$2^{55}$

**Equivalence Relations:**
relation on a set A is called an equivalence relation if it is $\boxed{\text{reflexive,}}$ $\boxed{\text{symmetric,}}$ and $\boxed{\text{transitive.}}$

**Partial Orderings:**
A relation R on a set S is called a partial ordering or partial order if it is $\boxed{\text{reflexive, antisymmetric, and transitive.}}$

A set S together with a partial ordering R is called a partially ordered set, or poset, and is denoted by (S, R). Members of S are called elements of the poset.

Which of these relations on {0, 1, 2, 3} are equivalence/partial ordered relations?

| | | E | P |
|---|---|---|---|
| a) {(0, 0), (1, 1), (2, 2), (3, 3)} | Reflexive Symmetric Antisymmetric Transitive | ✓ | ✓ |
| b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)} | not reflexive so not Equi. not Partial | X | X |
| c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} | reflexive, Symmetric, not Antisymm, transitive | ✓ | X |
| d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} | reflexive, symmetric, not Antisymm, not trans. | X | X |
| e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)} | reflexive, not symmetric, not Anti, not transitive | X | X |

Which of these relations on the set of all people are equivalence /partial ordered Relations?

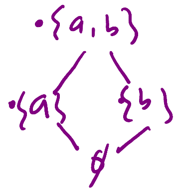
Shreya → Kamal, Rajshee
18 ← 18 18

| | | E | P |
|---|---|---|---|
| a) {(a, b) a and b are the same age} | reflexive, Symmetric, ~~Antisymmetric~~ X, transitive | ✓ | X |
| b) {(a, b) a and b have the same parents} | reflexive, Symmetric, not Anti, transitive | ✓ | X |

c) {(a, b)  a and b speak a common language}

reflexive, symmetric, not Anti, transitive ✓ | x

A = {(a, b), ⊆}




Hasse Diagrams of POSET

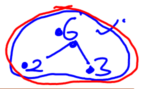
- Maximal Element
- minimal Element
- Greatest Element
- Least Element
- Upper bound
- Lower bound
- Least upper bound
- Greatest Lower bound

$S = \{2, 3, 6\}$ ✓

$R = \{(a, b) \mid a|b\}$ ✓

$R = \{(2,2)\ (2,6)\ (3,3)\ (3,6)\ (6,6)\}$

Poset = {(2, 3, 6), |} ✓

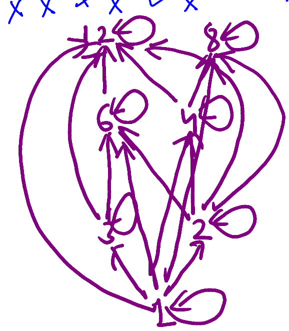


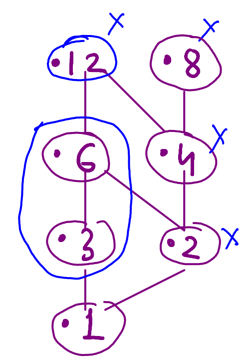

Consider an arbitrary poset, $\langle X \le \rangle$ (not necessarily finite). Given any element, $a \in X$, the following situations are of interest:

| |
|---|
| For **no** $b \in X$ do we have $b < a$. We say that $a$ is a *minimal element* (of $X$). |
| For **no** $b \in X$ do we have $a < b$. We say that $a$ is a *maximal element* (of $X$). |
| An element, $b \in X$, is the *least element of A* iff $b \in A$ and $b \le a$ for all $a \in A$. |
| An element, $m \in X$, is the *greatest element of A* iff $m \in A$ and $a \le m$ for all $a \in A$. |
| An element, $m \in X$, is an *upper bound of A* iff $a \le m$ for all $a \in A$. be any subset of $X$. An element, $b \in X$, is a *lower bound of A* iff $b \le a$ for all $a \in A$. |
| An element, $b \in X$, is the *greatest lower bound of A* iff the set of lower bounds of $A$ is nonempty and if $b$ is the greatest element of this set. |
| An element, $m \in X$, is the *least upper bound of A* iff the set of upper bounds of $A$ is nonempty and if $m$ is the least element of this set. |

Hasse diagram of

({1, 2, 3, 4, 6, 8, 12}, |).




Maximal = {12, 8} , Greatest = ∅

minimal = {1}  least = {1}

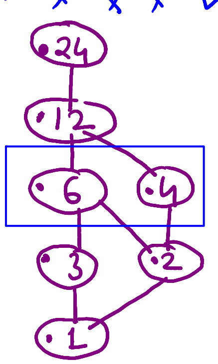
Upper bound (6, 3) = {6, 12}

Lower bound (6, 3) = {3, 1}

Least upper bound of (6, 3) = Least {6, 12} = {6}

Greatest lower bound of (6, 3) = greatest {3, 1} = {3}

(S, ≤) = {(1, 2, 3, 4, 6, 12, 24), |}




Maximal = {24}

Minimal = {1}

greatest = {24}

least = {1}

Upper bound of {6, 4} = {12, 24}

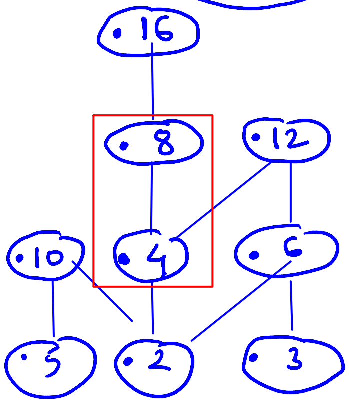
Lower bound of {6, 4} = {2, 1}

Least upper bound of {6, 4} = Least {12, 24}
= {12}

Greatest lower bound of {6, 4} = greatest {2, 1}
= {2}

$=\{2\}$

$(S,\preccurlyeq) = \{(2, 3, 4, 5, 6, 8, 10, 12, 16), |\}$




Maximal $= \{16, 12, 10\}$

Minimal $= \{5, 2, 3\}$

Greatest $= \emptyset$

Least $= \emptyset$

U.B. $\{8,4\} = \{8, 16\}$. L.U.B. $\{8,4\} = \{8\}$

L.B. $\{8,4\} = \{4, 2\}$. G.L.B $\{8,4\} = \{4\}$

$(S,\leq) = \{(a, b, c), \subseteq\}$

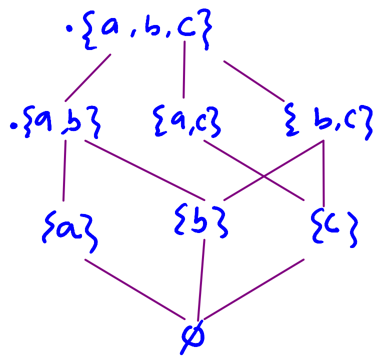
Max $= \{\{a,b,c\}\}$

min $= \emptyset$

G $= \{\{a,b,c\}\}$

L $= \emptyset$

Upper bound of $\{\{a,b\},\{a,c\}\}$ is $\{a,b,c\}$




$(\{1, 2, 3, 4\}, \leq)$.


Max $= \{4\}$

min $= \{1\}$

Great. $= \{4\}$

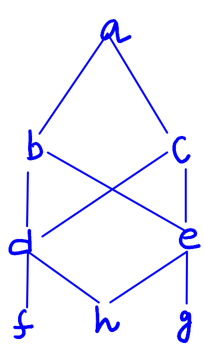
Least $= \{1\}$

Upper bound of $\{4, 3, 2\} = \{4\}$

Lower bound of $\{4, 3, 2\} = \{2\}$

Least upper bound of $\{4,3,2\} = \{4\}$

greatest lower bound of $\{4,3,2\} = \{2\}$




a) Find the maximal elements. $\{a\}$

b) Find the minimal elements. $\{f, h, g\}$

c) Is there a greatest element? $\{a\}$

d) Is there a least element? NO

e) Find all upper bounds of $\{a, b, c\}$. $\{a\}$

f) Find the least upper bound of $\{a, b, c\}$, if it exists. $\{a\}$

g) Find all lower bounds of $\{f, g, h\}$. $\emptyset$

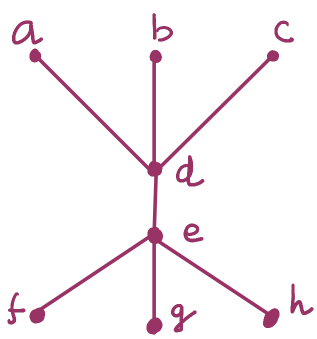
h) Find the greatest lower bound of {f, g, h}, if it exists. $\phi$


\# Maximal → The elements of poset which are not having any further path in upward direction

\# minimal → The elements of poset wich does not have any further path in downward direction

\# Greatest → Unique element, If more than 1 maximal then No greatest & If single maximal then it is the greatest

\#Least → Unique element, If more than 1 minimal then No least & If single minimal then it is the least




Maximal = {a, b, c}

Minimal = {f, g, h}

Greatest = $\phi$

Least = $\phi$

Upper bound of {a, b, c} = $\phi$

Lower bound of {a, b, c}
= {d, e, f, g, h}

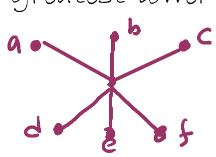
Greatest lower bound {a, b, c} = {d}

Least upper bound {a, b, c} = $\phi$

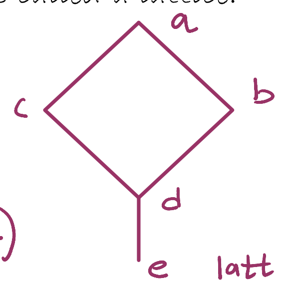
## Lattices

A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice.

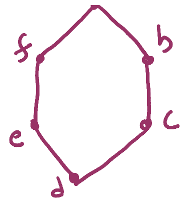

lattice



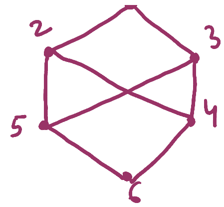

not a lattice
(More than 1 maximal)
LUB {a, b} = $\phi$




lattice




Lattice




not a lattice

LUB of $\{5,4\} = \emptyset$

Determine whether the posets $(\{1, 2, 3, 4, 5\}, \mid)$ and $(\{1, 2, 4, 8, 16\}, \mid)$ are lattices.