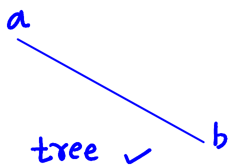
Tree : A tree is a $\boxed{\text{connected undirected}}$ graph with $\boxed{\text{no simple circuit}}$

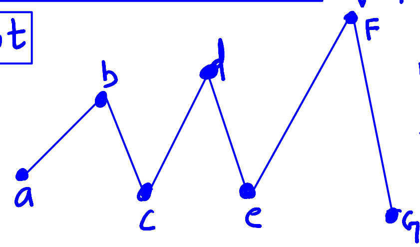
V=2
E=1



tree ✓

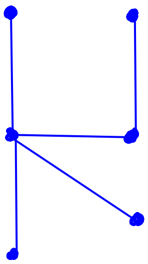
V=1
E=0

tree



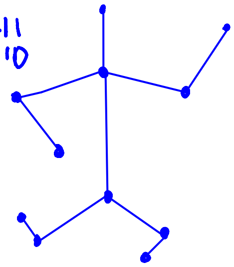
V=7
E=6

V=6
E=5

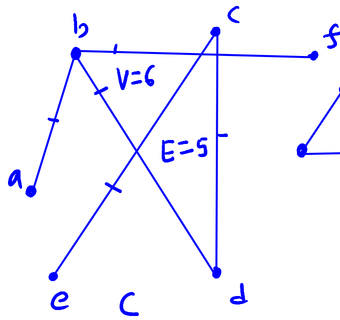


A

V=11
E=10

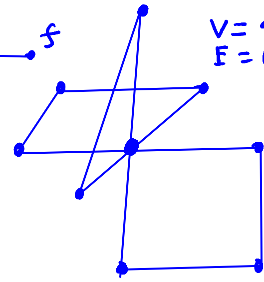


B



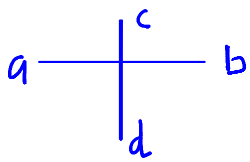
C

V=9
E=11 ✗



D

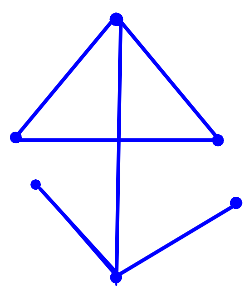
	A	B	C	D
Simple	✓	✓	✓	✓
Connected	✓	✓	✓	✓
Undirected	✓	✓	✓	✓
Simple circuit	✗	✗	✗	✓
Tree	✓	✓	✓	✗



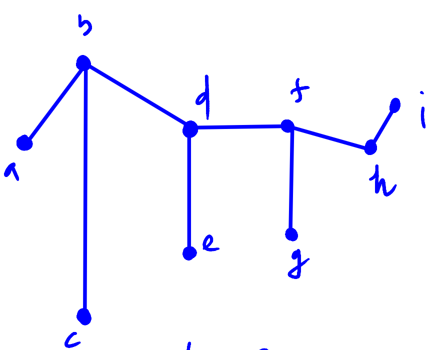
Not a tree

; not connected :

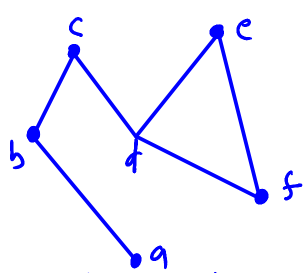
Simple ✓
undirected ✓
Circuit ✗
Connected ✗



not a tree
: Circuit



tree



Not a tree

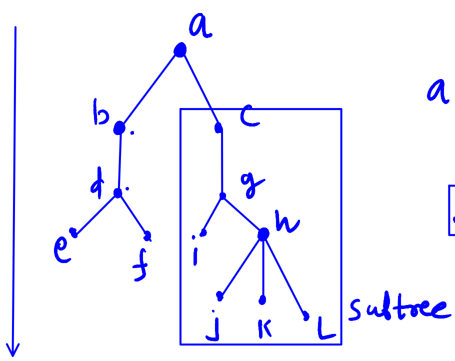
Theorem : An undirected graph is a tree iff There is a unique simple path b/w any two of it's vertices two of it's vertices

Theorem : A tree with n vertices has $n-1$ edges

Q: Let G be a simple connected undirected graph with no simple circuit & has 10 edges then $|V|$ in G

$\boxed{V = 11}$

Rooted tree A rooted tree is a tree in which one vertex has been designated as a root. Every edge is directed away from the root node



a is the root Node

Subtree : tree which is subgraph of G

$\boxed{G \text{ is a tree}}$

Rooted Node : A Node assigned as a root {a}

Parent Node : {a, b, c, d, g, h} $\boxed{\text{degree} > 1}$

Child Node : {b, c, d, e, f, g, h, j, K, L} [Every Node rather than root]

Siblings : {b, c} {e, f} {i, h} {J, K, L} [Same parent]

Ancestors J : – h, g, c, a [the vertices in the Path from j to a]

Descendants, c : g, h, i, J, K, L,

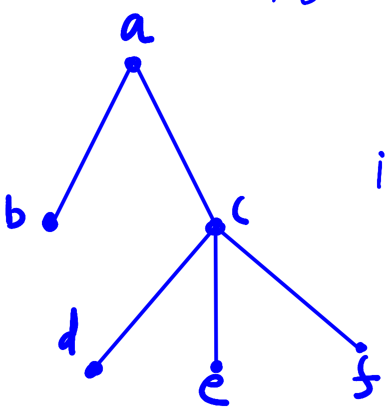
Leaf : {e, f, i, J, K, L} [that has no child]

Internal Node : that have some child (Apart from leaf)

Sub tree : ✓

Binary tree
Left child
Right child

$\boxed{\text{M-ary tree}}$: A rooted tree is called an m-ary tree if every internal vertex has not more than $\boxed{m \text{ child}}$

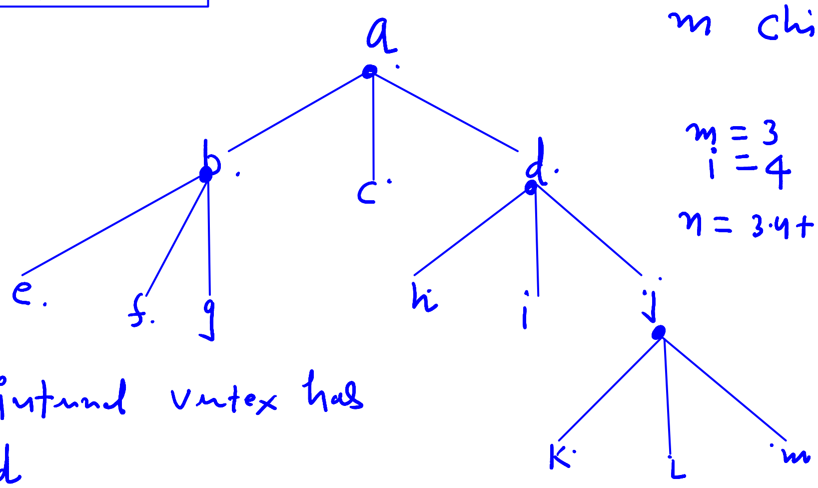


internal vertices $\{\underset{2}{a}, \underset{\underline{\underline{3}}}{c}\}$

3-ary tree

$\underline{\underline{2}} + \underline{\underline{4}} = n$

$\boxed{\text{Full-m-ary tree}}$: Every internal vertex has exaclty m child



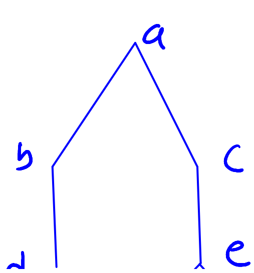
$m = 3$
$i = 4$
$n = 3 \cdot 4 + 1 = 13$

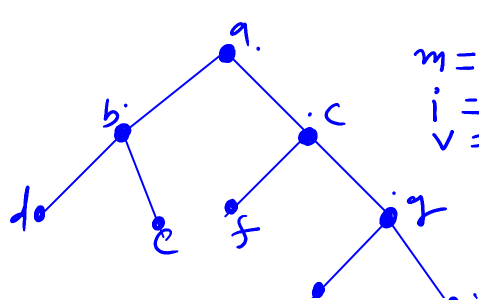
Every internal vertex has 3 child

Full - 3 - ary tree

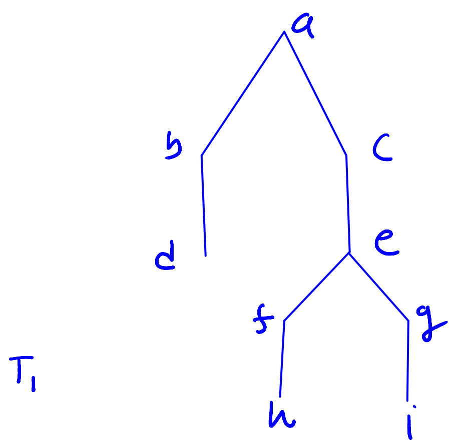
If m=2 ary tree $\boxed{\text{Binary tree}}$ [Every Internal vertex has max 2 child]

If m=2 : Full Binary tree or full 2-ary tree [Every Internal vertex has exactly 2 child]



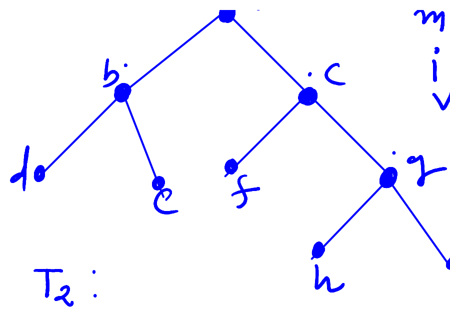


$m = 2$
$i = 4$
$v = 9$



T_1

2-ary tree / Binary tree



$m = 2$
$i = 4$
$v = 9$

T_2 : Full Binary tree
2-full ary tree or

$\underline{n} = \underline{m} \cdot \underline{i} + 1$
$9 = 2 \cdot 4 + 1$

Left child
Right child { Do by your own }

A full m-ary tree with i $\boxed{\text{internal vertices}}$ Contains $\boxed{n = mi + 1}$ vertices

m - ary
n - vertices
i = internal vertices

imp.

verification: In - tree

$m = 2$
$i = 4$
$n = \underline{2} \times \underline{4} + \underline{1} = \underline{9}$

m - ary tree : m ✓
Internal vertices : i ✓
leaf / leaves : l ✓
Total vertices : n

Relation b/w
n, m, i, l

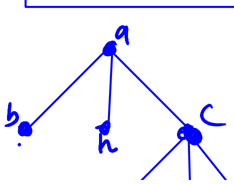
A full m-ary tree with n vertices has

$\boxed{i = \frac{n-1}{m}}$ internal vertices

$\boxed{m = \frac{n-1}{i}}$ ary - tree

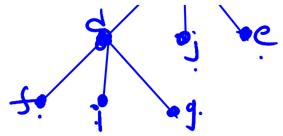
★★★ imp.

$\boxed{l = \frac{(m-1)n + 1}{m}}$



$m = 3$
$i = 3$
$v = 10$

In any a full m-ary tree if $m = 3$
$i = 3$



3-full-ary

$\boxed{l = 7}$

$\boxed{l = 2}$

$n = m \cdot i + 1$

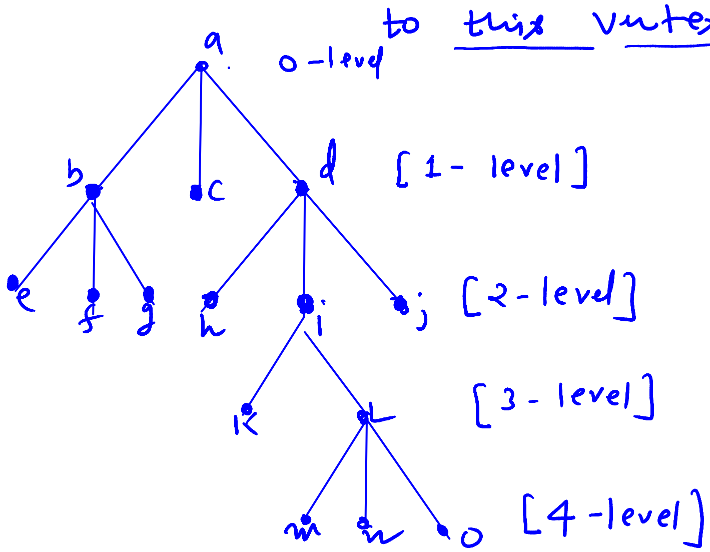
$\boxed{n = 10}$

$l = \frac{(m-1)n+1}{m}$

$l = \frac{2 \cdot 10 + 1}{3}$

$\boxed{l = 7}$

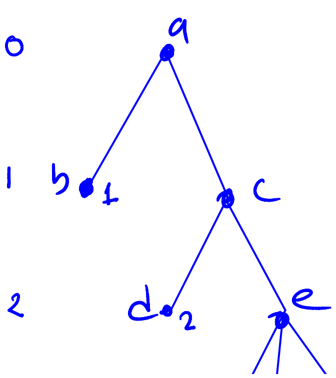
level of vertex: In a rooted is the length of the unique Path from the root to this vertex



$\boxed{H(T) = 4}$

Height: Max No. of level

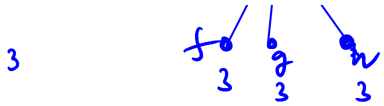
Balanced tree: A rooted tree (m-ary) of height h is balanced if all the leaves are at h level or h-1.



$\boxed{h = 3}$

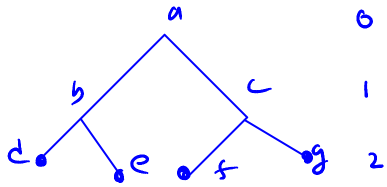
3^{rd} or 2^{nd} level

All the leaves {b, d, f, g, h} are not at 3^{th} level or



2nd level

Not a balanced



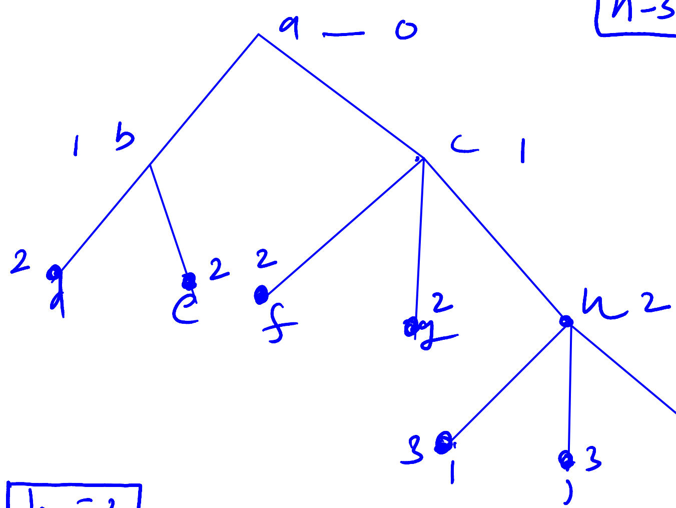
$\boxed{h=3}$

0-level

1-level

2-level

3-level

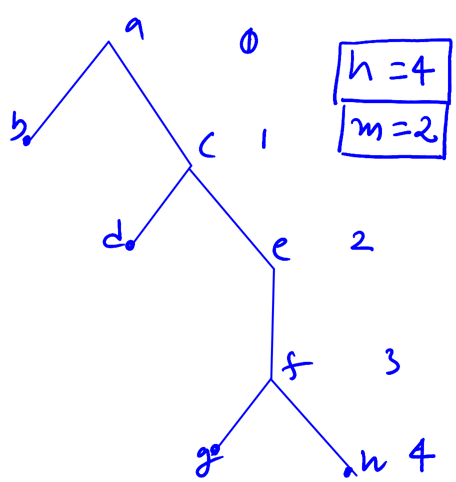


$\boxed{h=3}$

All the leaves are situated at 3 level/2 level ✓ [Balanced tree]

There are atmost m^h leaves in m-ary tree of height h

Q: A 2-ary tree with 2 height can have Max 2^2 leaves

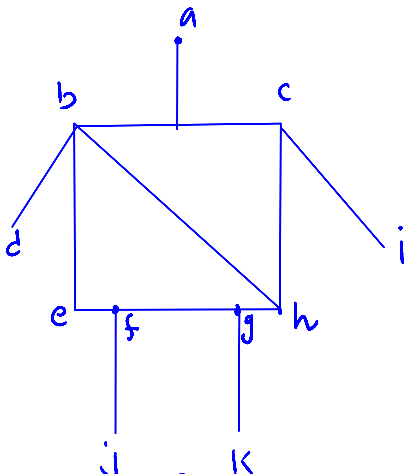


$\boxed{h=4}$
$\boxed{m=2}$

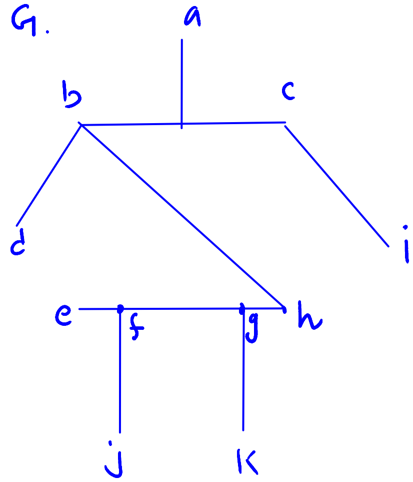
$2^4 = \underline{\underline{16}}$

Spanning tree

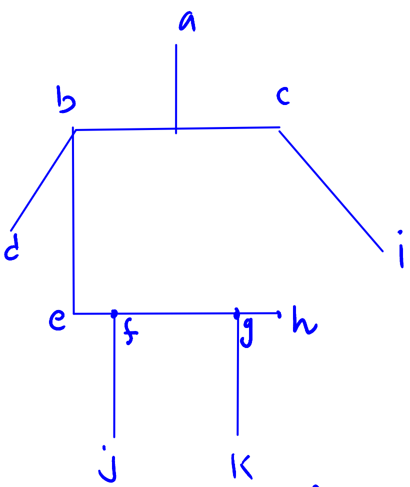
Let G be a $\boxed{\text{Simple graph}}$. A Spanning tree of G is a Subgraph of G that is a $\boxed{\text{tree}}$ $\boxed{\text{Containing Every}}$ vertex of G.



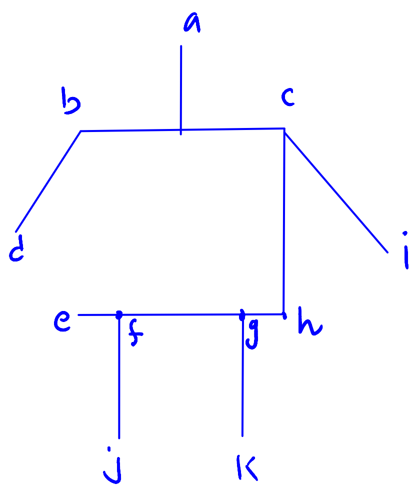
G



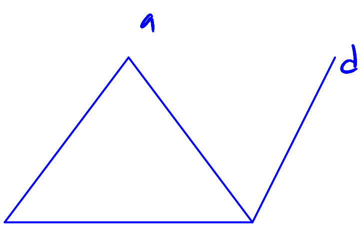
Spanning tree of G

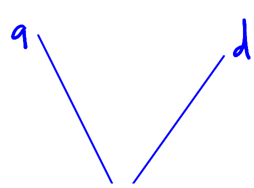


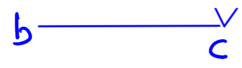
Spanning tree of G



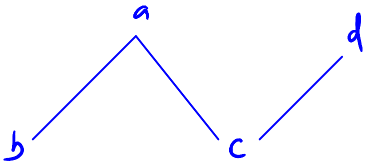
Spanning tree of G



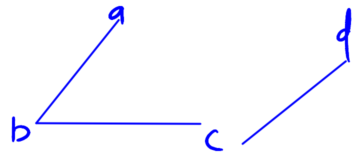




Spanning tree of G



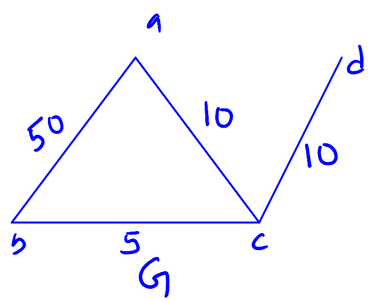
Spanning tree

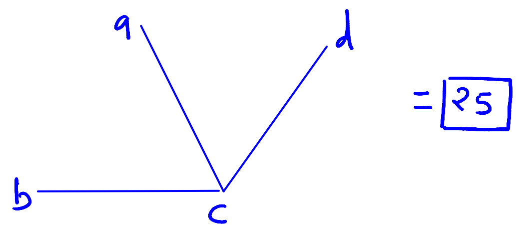


Spanning tree

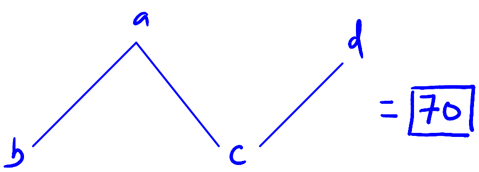
Minimum Spanning tree

A minimum spanning tree in a Connected weighted graph is a Spanning tree that has smallest possible sum of weighted edge.

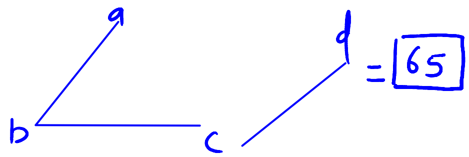




Min. Spanning tree of G



Spanning tree



Spanning tree

Prim's Algorithm for M.S.T.

- Let G be a Connected undirected weighted graph
- Select $e_1 = \boxed{\text{minimum weighted edge}}$
- select e_2 with Minimum weighted adjacent edge to e_1 that is
- select not forming any circuit

- select not forming any circuit

select e_3 ⊣|| ——— e_1, e_2

e_4 ⊣|| ——— e_1 or e_2 or e_3

$\boxed{\text{Kruskal Algorithm}}$

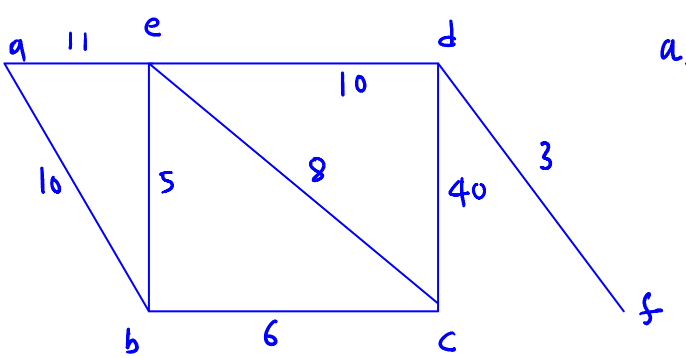
G be $\boxed{\text{Connected weighted}}$ graph

for $i = 1$ to $n-1$

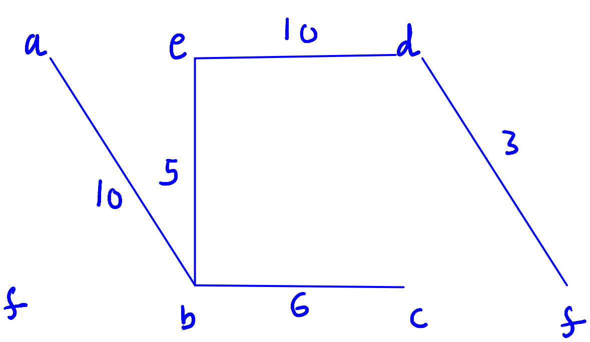
begin e = Any edge in G with smallest weight that does not form a cycle when its added to Tree

T = tree: All possible edges are added

end



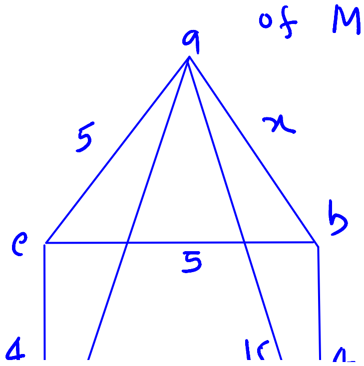
KRUSKAL'S



34

weight	Edge		
3	df	1	3
5	eb	1	5
6	bc	1	6
8	~~ec~~	0	0
10	ab ed	2	20
11	~~ae~~	0	0
40	~~dc~~	0	0
			34

for what value of x The No. of M.S.T are maximized

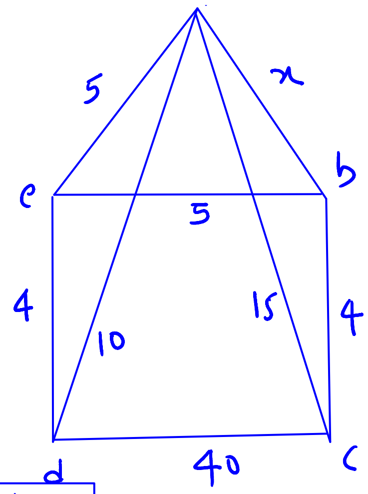


$x = 9$.

A) 4

B) 3

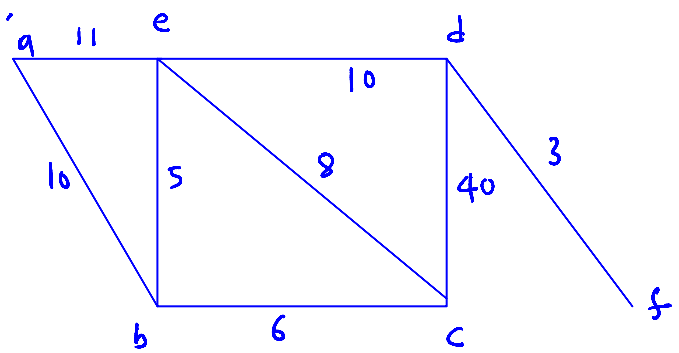
C) 5 ✓



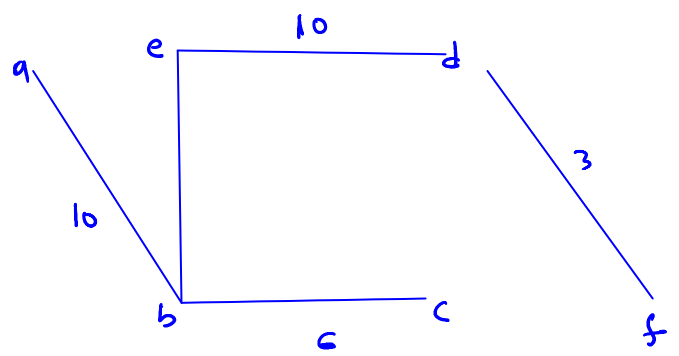
$x = 9.$

A) 4
B) 3
C) 5 ✓
d) 6

$\boxed{\text{Prim's algorithm}}$



weight	Edge
3	df ✓
10	ed ✓
40	dc ✗
11	ae ✗
5	eb ✓
8	ec ✗
6	bc ✓
10	ab ✓



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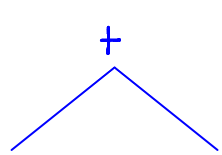
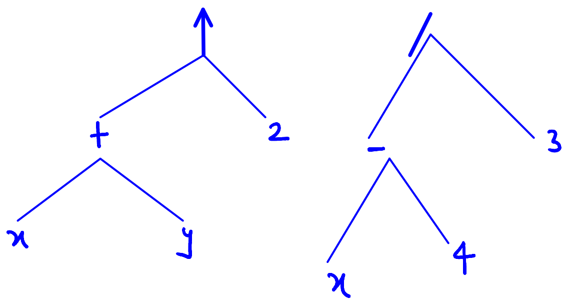
$\boxed{\text{Prefix Post fix Infix Notation}}$

$((x+y)\uparrow 2) + ((x-4)/3)$: Expression





Post fix : Left $\longrightarrow$ Right $\longrightarrow$ Parent

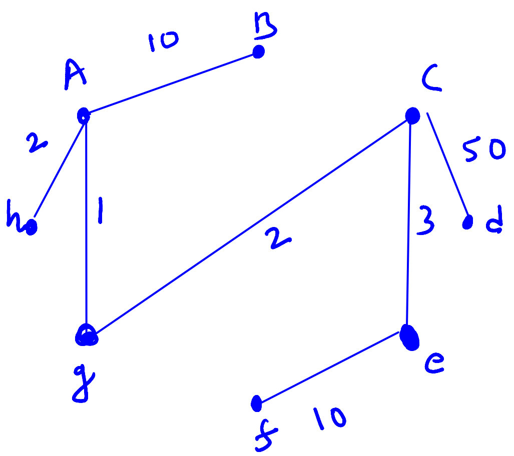
$x\,y + 2 \uparrow x\,4 - 3 / +$

Prefix : Parent $\longrightarrow$ left $\longrightarrow$ Right

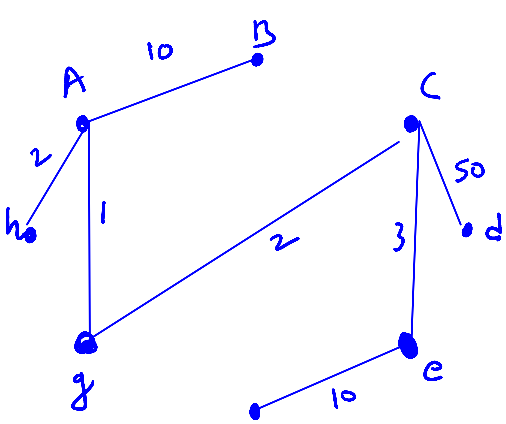
$+ \uparrow + x\,y\,2 / - x\,4\,3$

Infix : Left $\longrightarrow$ Parent $\longrightarrow$ Right

$x + y \uparrow 2 + x - 4 / 3$



30-sec.

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f