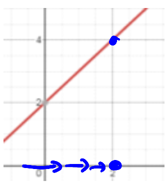
Limit

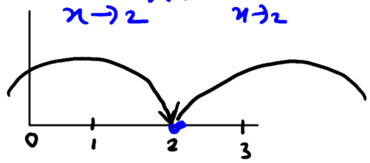
Let us consider the function $f(x)$. Now if x is approaches a value say $\boxed{x=c}$ then the LER is the value or limit of $f(x)$ as $x \to c$, $\boxed{\lim_{x\to c} f(x) = L}$

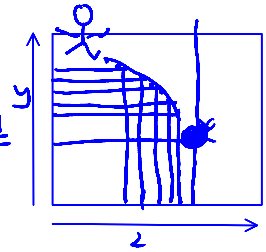
$f(x) = \dfrac{x^2-4}{x-2} = \dfrac{4-4}{2-2} \doteq \dfrac{0}{0}$ $\qquad \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$\boxed{f(2)}$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} x+2 = \underline{4}$



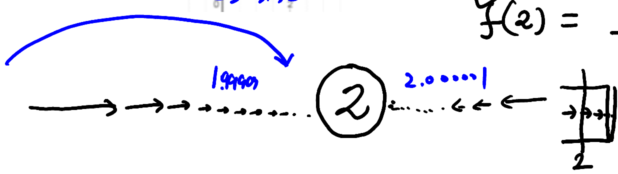




at $x = 2$
$y = 3$

$f(x) = \dfrac{x^2-4}{x-2}$

$f(2) = \dfrac{0-0}{0-0} = \dfrac{0}{0}$



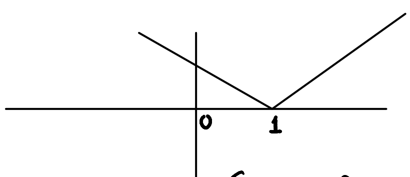
$f(x)$, c $\qquad \lim_{x\to c^-} f(x) = L^-$

$x \to c$ $\qquad \lim_{x\to c^+} f(x) = L^+$

if $\boxed{L^- = L^+}$

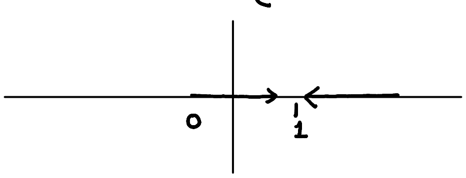
Q: $f(x) = \begin{cases} \dfrac{|x-1|}{x-1} & x \neq 1 \\ 0 & \boxed{x = 1} \end{cases}$ $\qquad \lim_{x\to 1} f(x) = ?$

$f(1) = 0$



$|x-1| = \begin{cases} -(x-1) & x \le 1 \\ (x-1) & x > 1 \end{cases}$

$f(x) = \begin{cases} \dfrac{-(x-1)}{x-1} & x < 1 & (A) \\ \dfrac{x-1}{x-1} & x > 1 & (B) \\ 0 & x = 1 & (C) \end{cases}$



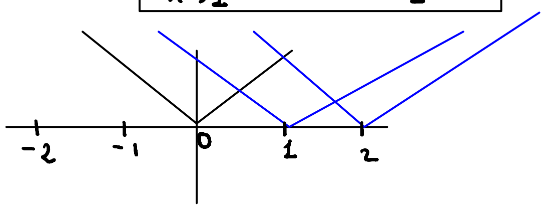
$\lim_{x\to 1^-} \dfrac{-\cancel{(x-1)}}{\cancel{x-1}} = -1$

$\lim_{x\to 1^+} \dfrac{\cancel{x-1}}{\cancel{x-1}} = 1$

$\boxed{\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)}$

$x \to 1^-$ $\quad x-1$

$$\boxed{\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)}$$



$|x|$

$|x-1|$

$|x-2|$

Some Imp. results $\qquad$ **L-Hopital rule** $\quad \frac{0}{0} \quad \frac{\infty}{\infty}$

1) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

2) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

3) $\lim_{x\to 0} \cos x = 1$

4) $\boxed{{}^{*}\lim_{x\to 0} (1+x)^{1/x} = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x} = e}$ $\quad \boxed{100\%}$

5) $\lim_{x\to 0} \frac{\log(1+x)}{x} = 0$

6) $\lim_{x\to\infty} \frac{\log x}{x^m} = 0 \quad (m>0)$

If $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ or $\pm\infty$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$

if $g'(x) \neq 0$

Q: $\lim_{x\to 0} \frac{\sin(\frac{5}{3}x)}{\frac{x}{3}} = \frac{\sin(0)}{0} = \frac{0}{0}$ Viable form $= \frac{0}{0}$

$\lim_{x\to 0} \frac{\cos \frac{4}{3}x \cdot \frac{4}{3}}{1} = \frac{4}{3} \cdot \lim_{x\to 0} \boxed{\cos \frac{4}{3}x}$

2008

Q: $\lim_{x\to 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$ $\quad \frac{0}{0}$

A 0

B $\frac{1}{2}$ ✓

C $-\frac{1}{2}$

D Lim Does not exist

$\Rightarrow \frac{8 - 24 + 22 - 6}{4 - 12 + 8} = \frac{0}{0}$

$\Rightarrow \lim_{x\to 2} \boxed{\frac{3x^2 - 12x + 11}{2x - 6}} = \frac{12 - 24 + 11}{4 - 6} = \frac{-1}{-2}$ $\qquad \frac{-1}{-2}$

Q: $\lim_{x\to\infty} \boxed{\frac{x + \sin x}{x - \cos x}}$ $\quad \frac{\infty}{\infty}$

A 1

B 0

C -1

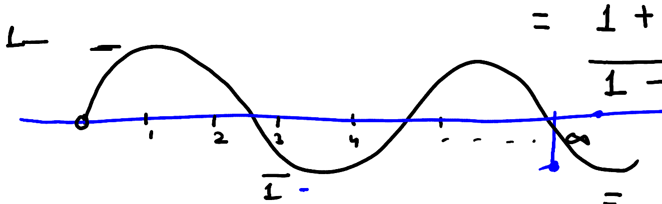
D.N.E

$\lim_{x\to\infty} \frac{x\left[1 + \frac{\sin x}{x}\right]}{x\left[1 - \frac{\cos x}{x}\right]} = \frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{\sin x}{x}}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{\cos x}{x}}$

$$x\left[1-\frac{\cos x}{x}\right] \qquad \lim_{x\to\infty} 1 - \lim_{x\to\infty}\frac{\cos x}{x}$$

$$\frac{\lim_{x\to\infty}\left[1+\frac{\sin x}{x}\right]}{\lim_{x\to\infty} 1-\frac{\cos x}{x}} = \frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty}\frac{\sin x}{x}}{\lim_{x\to\infty} 1 - \lim_{x\to\infty}\frac{\cos x}{x}}$$

$$= \frac{1 + \lim_{x\to\infty}\frac{\sin x}{x}}{1 - \lim_{x\to\infty}\frac{\cos x}{x}}$$



$$= \frac{1+\frac{t}{\infty}}{1-\frac{R}{\infty}} = \frac{1+0}{1-0} = 1$$

$$\lim_{x\to\infty}\sin x = t \in [-1,1]$$

$$\cos\infty = R \in [-1,1]$$

$\cos 0 = 1$

$\boxed{\sin 0 = 0}$

Imp

$$\lim_{x\to 0}\frac{1-\cos 2x}{x} \qquad \frac{1-1}{0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{0+\sin 2x\cdot 2}{1} \qquad 2\cdot\lim_{x\to 0}\sin 2x = 0$$

Gate 2008

$$\lim_{x\to 8}\frac{x^{1/3}-2}{x-8} = \frac{8^{1/3}-2}{8-8} = \frac{(2^3)^{1/3}-2}{0}$$

$$\lim_{x\to 8}\frac{\frac{1}{3}\cdot x^{1/3-1}-0}{1-0} = \frac{1}{12}$$

Gate

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} \qquad 1$$

GATE 2007

$$\lim_{x\to 0}\frac{e^x-\left(1+x+x^2/2\right)}{x^3}$$

A - 0
B - 1/6
C - 1/3
D - 1

$$\lim_{x\to 0} x^{\sin x} = 1 \qquad \text{Let } x^{\sin x} = S(x)$$

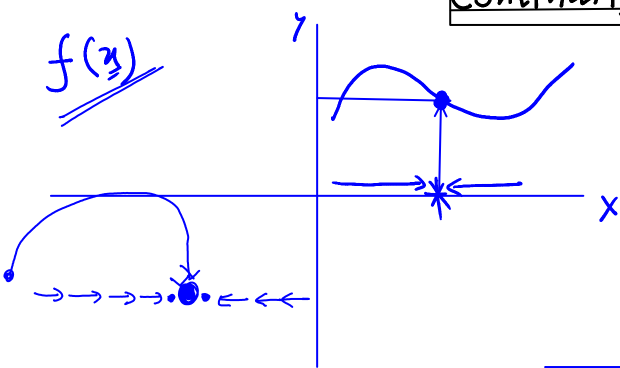
$$\sin x\cdot\log x = \log S(x)$$

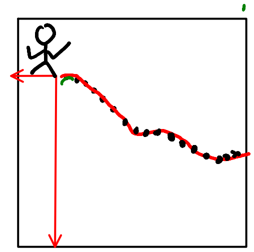
$$\lim_{x\to 0} S(x) = ? \qquad e^{\sin x\cdot\log x} = S(x)$$

$x \to 0$

Continuity

$f(x)$

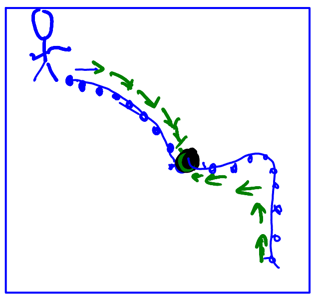


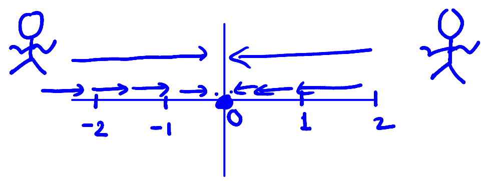


1) $\lim_{x \to c} f(x)$ exists

2) $f(c)$ exists

3) $\boxed{\lim_{x \to c} f(x) = f(c)}$





Gate 2018

$$F(x) = \begin{cases} \dfrac{\sin x}{x} + 10 \cos x & , x \neq 0 \\ K & , x = 0 \end{cases}$$

The value of λ s.t. $f(x)$ is continuous at $\boxed{x=0}$

A - 0

B - 1

~~C~~ - 2

D = 5

$\boxed{f(0) = \lim_{x \to 0} f(x)}$

$\lambda = \lim_{x \to 0} \boxed{\frac{\sin x}{x} + \cos x} = 2$

$\boxed{\lim_{x \to 0} \frac{\sin x}{x}} + \boxed{\lim_{x \to 0} \cos x}$

$1 + 1 = 2$

$\boxed{f(0) = \lambda}$, $\boxed{\lim_{x \to 0} f(x) = 2}$

$f(0) = \lim_{x \to 0} f(x)$

$\boxed{\lambda = 2}$

ISRO

$$f(x) = \begin{cases} \boxed{\frac{1}{2}+x} & \boxed{0 \le x < \frac{1}{2}} \\ \boxed{1} & \boxed{x = \frac{1}{2}} \\ \boxed{\frac{3}{2}-x} & \boxed{\frac{1}{2} < x \le 1} \end{cases}$$

A> $f(\frac{1}{2})$ exists ✓ $f(\frac{1}{2}) = 1$ exists ✓

B> $\lim_{x\to\frac{1}{2}} f(x)$ exists ✓ exist ✓

C> $f(x)$ is Continuous at $x = \frac{1}{2}$

1> only A is true

2> only A, B are true

3> C is

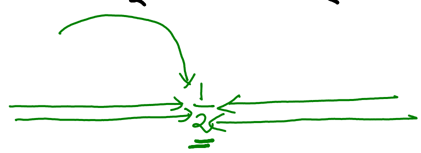
4> None of A, B & C is true

$$\lim_{x\to\frac{1}{2}} f(x) = ?$$

$$\lim_{x\to\frac{1}{2}^-} f(x) = \lim_{x\to\frac{1}{2}^-} \frac{1}{2}+x = \frac{1}{2}+\frac{1}{2} = 1$$

$$\lim_{x\to\frac{1}{2}^+} f(x) = \frac{3}{2}-x = \frac{3}{2}-\frac{1}{2} = \frac{2}{2} = 1$$

(=)



$$\boxed{f(\tfrac{1}{2}) = \lim_{x\to\frac{1}{2}} f(x)}$$

$1 = 1$

limit exists $\xrightarrow{\text{may be}}$ continuous

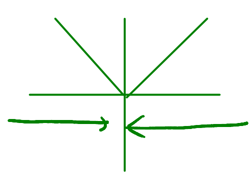
continuous $\xrightarrow{100\%}$ limit exists

Q:

$f(x) = 2x - |x|$

$f(x)$ is continuous at $x = 0$ → True / False

$$|x| = \begin{cases} -x & x < 0 \\ x & x \ge 0 \end{cases}$$



$$f(x) = \begin{cases} 2x-(-x) & x < 0 \\ 2x-(x) & x \ge 0 \end{cases} \Rightarrow f(x) = \begin{cases} 3x & x < 0 \\ x & x \ge 0 \end{cases}$$

$$\underline{\lim_{x\to 0} f(x)} \begin{cases} \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = \underline{\underline{0}} \\ \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 3x = \underline{\underline{0}} \end{cases} \; (=) \to \text{exists}$$

$f(0) = \underline{\underline{0}}$, exists

$$\boxed{f(0) = \lim_{x\to 0} f(x)}$$ yes continuous.

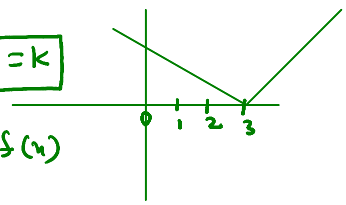
Q: what is the value of K s.t. the

$$\boxed{f(x) = \begin{cases} \dfrac{x^2-9}{x-3} + |x-3| & x \neq 3 \\ K & x = 3 \end{cases}}$$

is continuous ✓ at $\boxed{x=3}$ ✓

$x-3$ 1) 0 2) 3 ~~3)~~ 6 4) None

$$\boxed{f(3) = \lim_{x\to 3} f(x)} \qquad \boxed{f(3) = K}$$

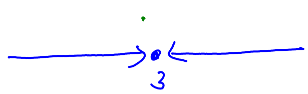


$$\lim_{x\to 3} f(x) = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

$$\boxed{\frac{x^2-9}{x-3} = \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}}$$

$$f(x) = \begin{cases} x+3-(x-3) & x<3 \\ K & x=3 \\ x+3+(x-3) & x>3 \end{cases}$$

$$f(x) = \begin{cases} 6 & \boxed{x<3} ✓ \\ K & x=3 \\ 2x & x>3 ✓ \end{cases}$$



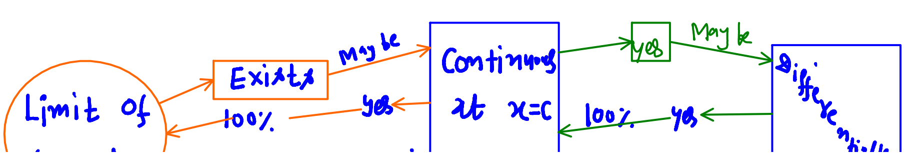
$$\lim_{x\to 3} f(x) = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

$$\boxed{\lim_{x\to 3^-} f(x) = 6} \qquad \boxed{\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x = 6}$$

$\lim_{x\to 3} f(x) = 6$ $\underline{\text{exists}}$ ✓

$f(3)$ exists $\Rightarrow$ $\boxed{f(3) = k}$

$f(3) = \lim_{x\to 3^+} f(x) \Rightarrow \boxed{K = 6}$



$f(x)$ at c → $\boxed{\text{not exists}}$ $\xrightarrow{100\%}$ not → not Cont. $\xrightarrow{100\%}$ not $\boxed{\text{at } x=c}$

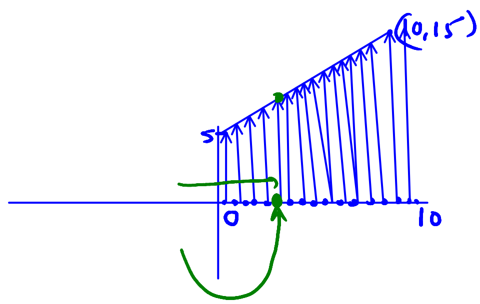
If $f(x)$ is differentiable at $x=c$ then $\lim_{x\to c} f(x)$ → exist / not exist

$(a,b) \underset{\subseteq}{\text{———}} [a,b]$

(a,b) a .... b

$[a,b]$

A function $f(x)$ is $\boxed{\text{continuous}}$ at $x\in(a,b)$ means it is continuous at every point of (a,b)



$f(x) = x+5$

$x \in (0,10)$

A function $f(x)$ is continuous at $x_0 \in [a,b]$ means

$\Rightarrow$ $f(x)$ is continuous at $x_0 \in (a,b)$

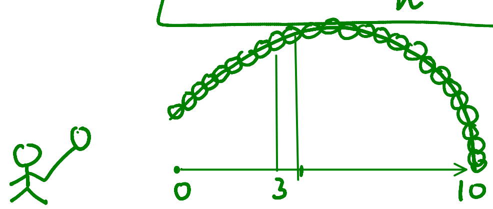
$\Rightarrow$ $f(x)$ is also continuous at $x_0 = a$, $x_0 = b$

$$\boxed{f(x_0) = \lim_{x\to x_0} f(x) \qquad x_0 \in [a,b]}$$

Differentiability

Let $f(x)$ be a real valued function defined on an open interval $\boxed{(a,b)}$ & it is said to be differentiable at $x=c$, $c\in(a,b)$

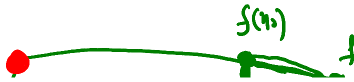
If $\boxed{\lim_{h\to c} \dfrac{f(c+h)-f(c)}{h}}$ exists $\quad h = c+\delta - c$



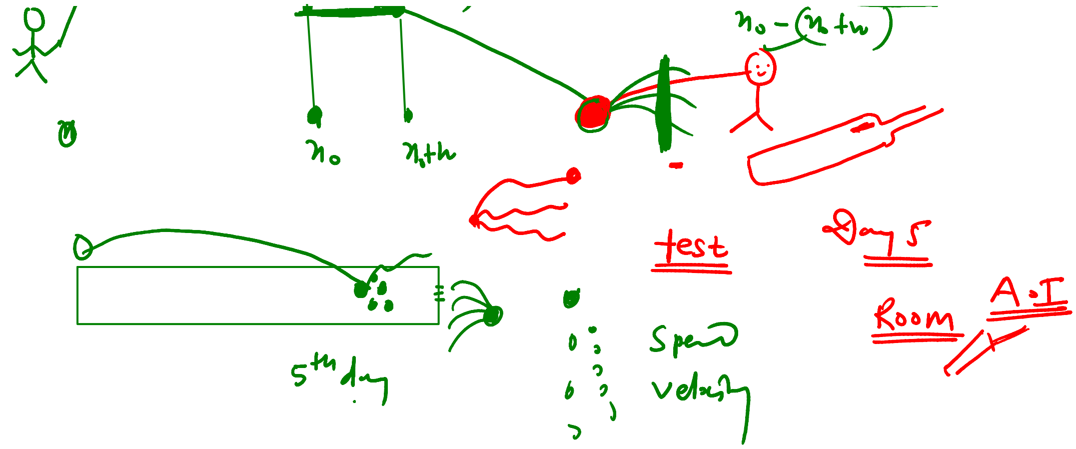
$c \in (0,10)$

$c = 3$

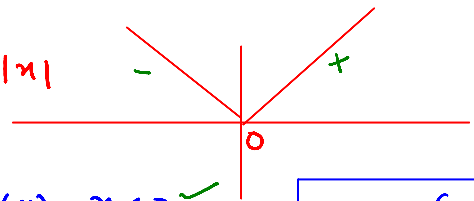
$c+h = 3+0.001$



$\tan\theta = \dfrac{f(x_0) - f(x_0+h)}{}$



Discuss the differentiability of $\boxed{f(x) = x|x|}$ at $x = 0$



$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} x(-x) & x < 0 \\ x \cdot x & x \geq 0 \end{cases} \Rightarrow \boxed{f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}}$$

Now To check the differentiability at $x = 0$

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \quad \text{exist}$$

$$f(0) = 0$$

$$f(0+h) = f(h) = \begin{cases} \boxed{-h^2} & h < 0 \\ \boxed{h^2} & h \geq 0 \end{cases}$$

$$\lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - 0}{h} = \lim_{h \to 0} \frac{f(h)}{h}$$

$$\lim_{h \to 0} \frac{f(h)}{h}$$

$$\lim_{h \to 0^-} \frac{f(h)}{h} = \lim_{h \to 0^-} \frac{-h^2}{h} = \lim_{h \to 0} -h = 0$$

$$\lim_{h \to 0^+} \frac{f(h)}{h} = \lim_{h \to 0^+} \frac{h^2}{h} = \lim_{h \to 0} h = 0$$

Hence $f(x) = x|x|$ is differentiable at $\boxed{x = 0}$

Q: $|x|$ is differentiable at $x = 0$

A) $\lim_{x \to 0} |x|$ exists $\boxed{\text{correct}}$

B) $|x|$ is continuous at $x = 0$ $\boxed{\text{correct}}$

C) $|x|$ is differentiable at $x = 0$

c) $|x|$ is differentiable at $x=0$

$$|x| = \begin{cases} -x & x<0 \\ x & x \geq 0 \end{cases}$$

Ⓐ $\lim_{x\to 0} |x|$: $\lim_{x\to 0^-} -x = 0$, $\lim_{x\to 0^+} x = 0$ ⊜

Ⓑ $|x|$ is continuous at $x=0$

$f(x) = |x|$ then $\boxed{f(0) = \lim_{x\to 0} f(x)}$ verified

Ⓒ $|x|$ is differentiable at $x=0$

$f(x) = |x|$

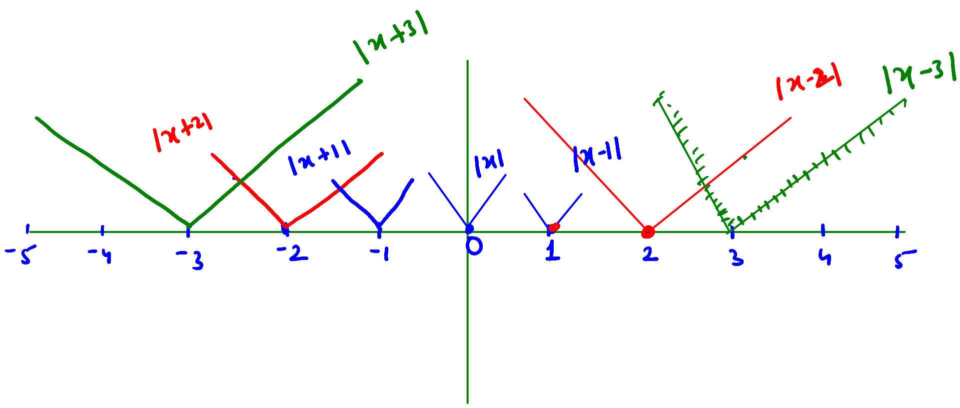
$\boxed{f(0) = 0}$

$f(h) = |h|$

$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$ should exists

$$\lim_{h\to 0} \frac{|h|}{h} = \begin{cases} \lim_{h\to 0^-} \frac{-h}{h} = -1 \\ \lim_{h\to 0^+} h/h = 1 \end{cases} \neq$$

$|x|$ is not differentiable at $x=0$



$c \in \mathbb{R}$

$|x-c|$

$\lim_{x\to c} |x-c|$ exists

$|x-c|$ is continuous every where

$|x-c|$ is continuous at $x=c$

$|x-c|$ is differentiable every where there $\underline{\underline{c}}$

$|x-c|$ is not diff. at $x=c$.

$\boxed{\text{Newton - Leibnitz formula}}$

Let $I(x) = \int_{\phi(x)}^{\psi(x)} f(t)\,dt$

$$\boxed{\frac{d}{dx} I(x) = \psi'(x)\cdot f(\psi(x)) - \phi'(x)\, f(\phi(x))}$$

1) $I(x) = \int_0^{x^2} \cos t^2\, dt = ?$

3) $\lim_{x\to 0} \frac{\int_0^x \cos t\, dt}{x}$ 1

2) $\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2\, dt}{x\cdot \sin x} = 1$

4) $\int_1^x \frac{\sin t}{t}\, dt = I(x)$

$\lim_{x\to\infty} I'(x)$

5) $\lim_{x\to 0} \frac{\int_x^{x^2} \sin t\, dt}{x^2}$

1) $I'(x) = 2x\cdot \cos(x^2)^2 - 0\cdot[\cos(0^2)] = 2x\cos x^4$

3) $I = \lim_{x\to 0} \frac{\int_0^x \cos t}{x}$ $\frac{0}{0}$ $\Rightarrow \lim_{x\to 0} \frac{\frac{d}{dx}\int_0^x \cos t\, dt}{\frac{d}{dx} x} = \lim_{x\to 0} \frac{1\cdot\cos x - 0}{1}$

$\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2\, dt}{x\sin x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{2x\cos(x^2)^2 - 0\cos 0^2}{x\cos x + \sin x}$ $\frac{0}{0}$

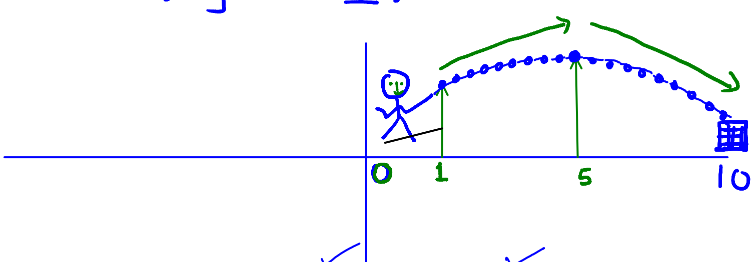
$\lim_{x\to 0} \frac{2[x\cdot\sin x^4\cdot 4x^3 + \cos x^4\cdot 1]}{-x\sin x + \cos x + \cos x}$

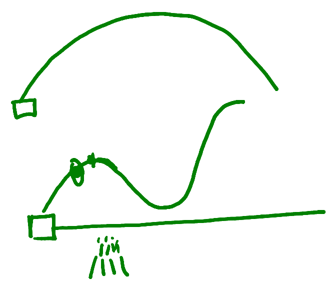
$\frac{2[0+1]}{0+1+1} = \frac{2}{2} = 1$

$\boxed{\text{Maxima \& minima of } f(x)}$

- Monotonic function

A function $f(x)$ is said to be $\boxed{\text{Monotonic}}$ on I if it is either increasing or decreasing on I.





Monotonic → $[1,5]$ or $[5,10]$
$[1,5]$ → Incr
$[5,10]$ → Dec.

$[1 \;\; 10]$ → Increasing & decreasing not monotonic

$\boxed{\text{Condition for monotonic function}}$

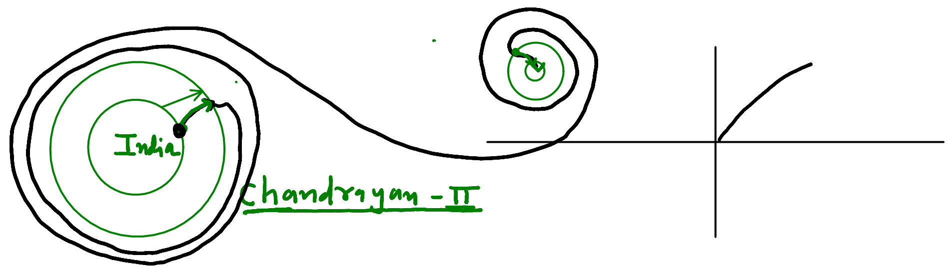
- A function $f(x)$ is increasing in $[a,b]$ if $\boxed{f'(x) \geq 0}$, $\forall x \in [a,b]$
- A function $f(x)$ is strictly increasing in $[a,b]$ if $\boxed{f'(x) > 0}$, $\forall x \in [a,b]$
- A function $F(x)$ is decreasing in $[a,b]$ if $\boxed{f'(x) \leq 0}$, $\forall x \in [a,b]$
- A function $F(x)$ is strictly decreasing in

$[a,b]$ if $\boxed{f'(x) < 0}$, $\forall\ x \in [a,b]$

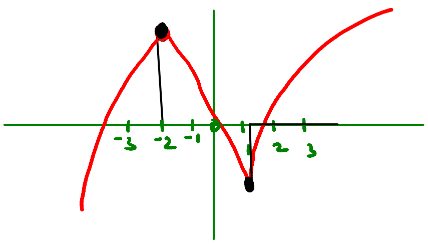
$f(x)$ $\quad \mathbb{R} \to \mathbb{R}$

$f(x,y)$ $\quad \mathbb{R}^2 \to \mathbb{R}$

$f(x,y,z)$ $\quad \mathbb{R}^3 \to \mathbb{R}$



Q: The interval in which $f(x) = 2x^3 + 3x^2 - 12x + 1$ is increasing



$f'(x) = 0$ [rate change is zero]

$6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$ h

Critical Point

$x = -2, 1$

$6x^2 + 6x - 12 = 0$

$\Rightarrow \quad x^2 + x - 2 = 0$

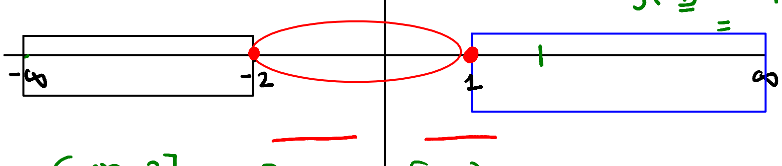
$x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = -2, 1$

$x = -2$ and $x = 1$ are the critical point

$\boxed{(-\infty \ \infty)}$ ✓

$f(x) = 2x^3 + 3x^2 - 12x + 1$

✓ $f(-4) = -128 + 48 + 48 + 1$



$(-\infty, -2]$ $\quad [-2 \ \ 1]$ $\quad [1 \ \ \infty)$

$f(x) = 2x^3 + 3x^2 - 12x + 1$

$f'(x) = 6x^2 + 6x - 12$ $\quad$, $f'(-2) = 0$

$(-\infty, -2)$ ✓ $\quad x = -3 \Rightarrow 6(-3)^2 + 6\cdot(-3) - 12$

$54 - 18 - 12$

$54 - 30 = 24 > 0$

Strictly increasing

$(-\infty \ -2]$ $\quad x = -2$

$6(-2)^2 + 6(-2) - 12$

$24 - 12 - 12 = 0$ ✓

$\boxed{f(x) \geq 0}$ ✓

$f(x)$ is strictly increasing in $(-\infty, -2)$ ✓

$f(x)$ is increasing in $(-\infty, -2]$

$[-2, 1]$ $\qquad f'(x) = 6x^2 + 6x - 12$

$(-2, 1)$ $\qquad f'(-1) = 6(-1)^2 + 6(-1) - 12$

$= 6 - 6 - 12$

$= \boxed{-12 < 0}$

- Since $f'(x) \leq 0$ in $[-2, 1]$ then $f(x)$ is decreasing
- $f(x)$ is strictly decreasing in $(-2, 1)$

$[1, \infty)$ $\quad x = 2$ $\quad f'(x) = \boxed{6x^2 + 6x - 12}$

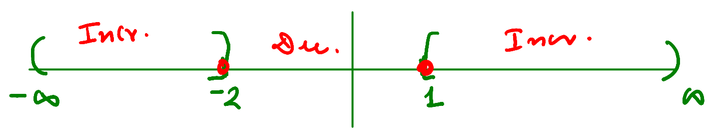
$f'(2) = 6 \cdot 4 + 6 \cdot 2 - 12$

$= 24 - 12 - 12$

$= 24 > 0$ ✓

$f(x)$ is increasing in $[1, \infty)$

$f(x)$ is strictly increasing in $(1, \infty)$



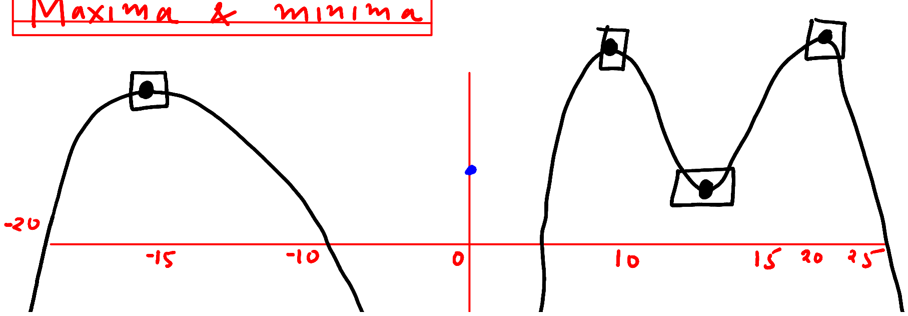
Increasing $(-\infty, -2] \cup [1, \infty)$ ✓ ≥ 0

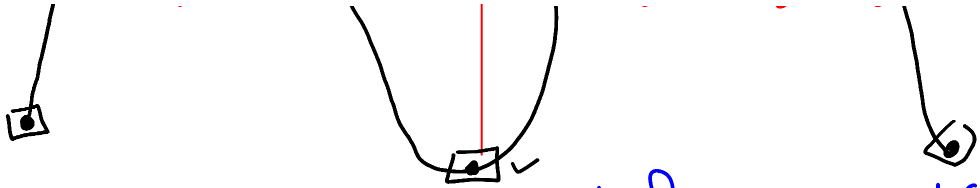
Strictly increasing $(-\infty, -2) \cup (1, \infty)$ > 0

decreasing $[-2, 1]$ ≤ 0

strictly decreasing $(-2, 1)$ < 0

Maxima & minima





Let $f(x)$ be a real valued function defined on I $\quad I = [a,b]$ or (a,b)

I Find the $f'(x)$

II Solve for x s.t. $f'(x) = 0$ & find the critical point c

III Do check the critical point $c \in I$

IV Find $f''(x)$

V If $f''(c) < 0$ then f has local Maxima at $x = c$

VI If $f''(c) > 0$ then f has local minima at $x = c$

⊙ If $f''(c) = 0$ & $f'''(c) \neq 0$ f has Neither Maxima nor minima

☆ If $I = [a,b]$

$x = a$, $x = b$, $x = c_i$ $\quad i = 1, 2, 3 \dots$

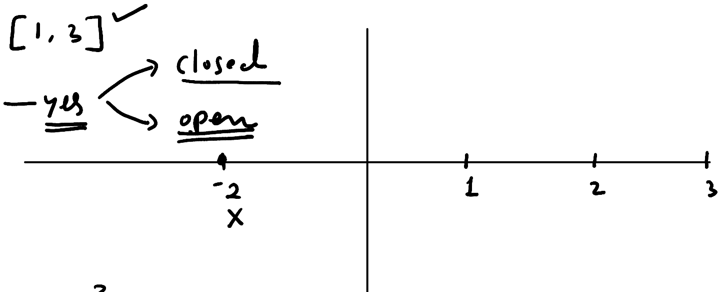
Absolute maxima $= \max\{f(a), f(b), f(c_i)\}$

Absolute minima $= \min\{f(a), f(b), f(c_i)\}$

Q: Find the Maxima & minima for $f(x) = 2x^3 - 24x + 107$ in $[1,3]$ $\quad [1,3]$

$I = [1,3]$

Interval — yes → closed, open



- $f'(x) = 6x^2 - 24$
- $f'(x) = 0 \Rightarrow 6x^2 - 24 = 0 \Rightarrow x^2 = 4 \Rightarrow \boxed{x = \pm 2}$

$x = 2, -2$ are the critical point

- $x = 2$ is the only critical point which belongs to $[1 \quad 3]$ $\quad [1 \quad 2 \quad 3]$

- $x = 1$, $\boxed{x = 2}$, $x = 3$

- $f''(x) = 12x$ at $x = 2$, $f''(2) > 0$

  $12 \cdot 2 = 24$

  function has Local minima at $\boxed{x = 2}$

- Absolute Maxima $= \text{Max}\{ f(1), f(3) \; f(2) \}$

  Absolute minima $= \min\{ f(1) \; f(3) \; f(2) \}$

critical
$$\begin{cases} f(1) = 2 \cdot 1^3 - 24 \cdot 1 + 107 = 85 \\ f(2) = 2 \cdot 2^3 - 24 \cdot 2 + 107 = 75 \\ f(3) = 2 \cdot 3^3 - 24 \cdot 3 + 107 = 89 \end{cases}$$

F(x) has Absolute Maxima at $x = 3$ is 89

f(x) has Absolute minima at $x = 2$ is 75

Q: $f(x) = x^3 - 6x^2 + 9x$ in $(-5 \quad 5)$

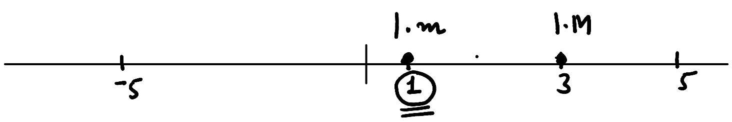
$[-5 \quad 5]$

$f'(x) = 3x^2 - 12x + 9$

$f'(c) = 3c^2 - 12c + 9 = 0 \qquad c = \dfrac{12 \pm \sqrt{144 - 108}}{6}$

$c = \dfrac{12 \pm \sqrt{36}}{6} = \dfrac{12 \pm 6}{6} = 3, 1$

$x = \dfrac{12 \pm 6}{6} = \boxed{3, 1}$



$f''(x) = 6x - 12$

$f''(1) = 6 \cdot 1 - 12 = -6 < 0$ local maxima at $\boxed{x = 1}$

$f''(3) = 6 \cdot 3 - 12 = 6 > 0$ local minima at $x = 3$

$x^3 - 6x^2 + 9x$

$f(1) = 1^3 - 6 \cdot 1^2 + 9 = 4$, f has local maxima at $x = 1$ the v

$f(3) = 3^3 - 6 \cdot 9 + 9 \cdot 3 = 27 - 54 + 27 = 0$ Local minimum value is

$[-5 \quad 5]$

$-125 - 6.25 - 9.5$

$-125 - 6.25 - 45$

$125 - 6.25 + 45$ 125

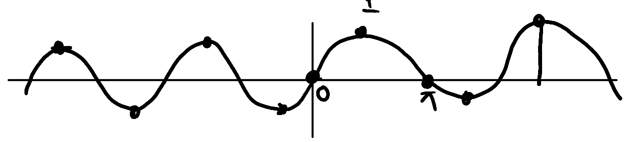
$f(-5) = \boxed{-320}$ ✓

$f(1) = 4$ ✓

$f(3) = 0$ ✓

$f(5) = \boxed{20}$ ✓

$f(x) = \sin x \quad (-\infty \ \infty)$



$f(x) = \sin x$

$f'(x) = \cos x$

$\cos c = 0$

$\boxed{c = \pi/2}$

$[0 \quad \pi]$ 1 0

$[0 \quad 2\pi]$ ✓

$[0 \quad \pi/2 \quad \pi]$

0 1 0

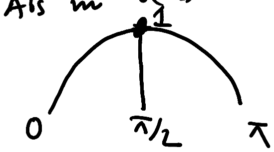
LM = 1

Lm = 1

Abs M

Abs m

$f'(\pi/2) = 0$



$f(x) = 5x^2 - 10x + 100$ ✓

$f'(x) = 10x - 10 = 0$

$\boxed{x=1}$

$f'(1) = 0$

$(-\infty \ 1]$ let $x = 0$

$f'(0) = -10 < 0$

$\boxed{(-\infty \ 1] \quad f'(x) \le 0}$ ✓ decreasing

$\boxed{(-\infty \ 1) \quad f'(x) < 0}$ strictly decreasing

$[1, \infty) \quad f'(x) = 10x - 10$

$f'(2) = 20 - 10 = 10 > 0$

$f'(1) = 0$

$[1, \infty) \quad f'(x) \ge 0$ Increasing

$(1, \infty) \quad f'(x) > 0$ strictly increasing

$\mathbb{R} - \{1\}$

$x = 2$

- $(-\infty \ 1) \cup (1 \ \infty)$ ✗
- $x = 1$ ✗ ✓
- $(-\infty \ 1]$ ✓ dec.
- $[1, \infty)$ ✗✗
- $(-\infty \ -1)$ ✗
- $(-\infty \ 1)$ ✓

$\boxed{\cdot (1, \infty)}$ ✗

$\cdot (-\infty \ \infty)$ ✗

10

$f(x) = \sin x$

Increasing

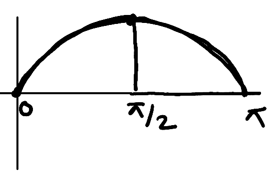
$f'(x) = \cos x$

A $[0 \quad \pi/2]$ I

B $[\pi/2 \quad \pi]$ D

C $(0, \ \pi/4]$ ✓

D $(0 \quad \pi/2]$ S.I.



$[0, \pi/2]$ $f'(x) \geq 0$

$(0, \pi/2]$

Increasing

$f(x) = \cos x$

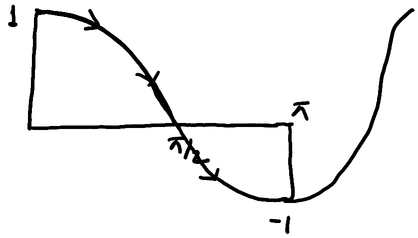
A $[0 \quad \pi/2]$

B $(0 \quad \pi/2)$

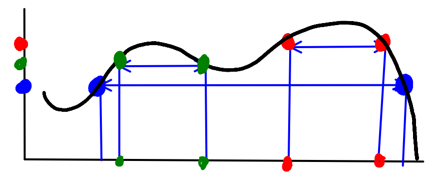
C $[\pi/2 \quad \pi]$

D $(\pi/2 \quad \pi)$

$[\pi \quad 3\pi/2)$

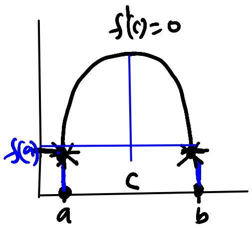


$\boxed{\text{Mean Value Theorem}}$



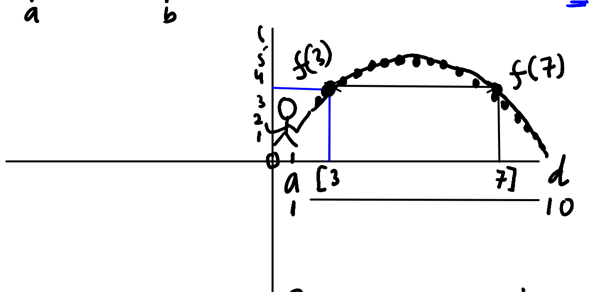
· $\boxed{\text{Rolle's Theorem}}$

Rolle's Theorem : Let $f(x)$ be



1) Continuous in $[a, b]$ ✓
2) differentiable in (a, b) ✓
3) $f(a) = f(b)$ ✓

then $\exists$ a $c \in [a, b]$ s.t. $f'(c) = 0$



$f(x) = x^2 - 6x + 5$ is cont. $[2, 4]$ $c = 3 = \frac{4+2}{2}$

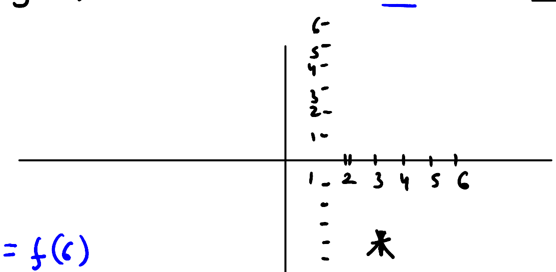
is diff. $(2, 4)$

do verify the rolle's theorem

$f(2) = 4 - 12 + 5 = -3$ ✓

$f(4) = 16 - 24 + 5 = -3$ ✓

$\boxed{f(2) = f(4)}$



$f'(x) = 2x - 6$

$f'(c) = 2c - 6 = 0$

$\boxed{c = \frac{6}{2} = 3}$

$f(0) = f(6)$

$\exists\, c \in [0, 6],\ f'(c) = 0$

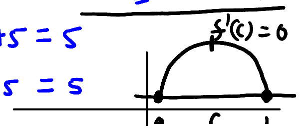
$c = 3 \in [2, 4]$ ✓

$\frac{4+2}{2}$

$I = [0, 6]$

$f(0) = 0^2 - 6 \cdot 0 + 5 = 5$

$f(6) = 6^2 - 6 \cdot 6 + 5 = 5$



$f'(x) = \underline{2x-6}$

$f'(c) = \underline{2c-6} = 0 \Rightarrow \boxed{c=3}$

a	c	b

$0, 6 \; \frac{0+6}{2} = 3$

Q: $f(x) = x^3 - 6x^2 + ax + b$ hold's the Rolle's Theorem in $[1, 3]$ find $a = ?$ $b = ?$

$\boxed{f(1) = f(3) = 0}$

$f(1) = f(3)$

$f(1) = 1 - 6 + a + b \Rightarrow f(1) = a + b - 5$

$f(3) = 27 - 54 + 3a + b \Rightarrow f(3) = 3a + b - 27$

$$\boxed{\overset{f(1)}{a+b-5} = \overset{f(3)}{3a+b-27}} \Rightarrow 2a = +27 - 5 = 22$$

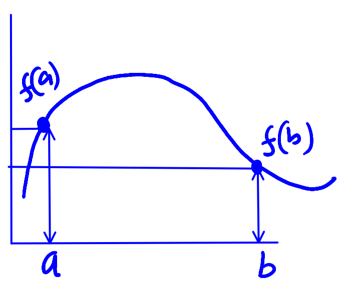
$\boxed{a = 11}$

$f(1) = 0 \Rightarrow$

$f(1) = 11 + b - 5 = b + 6$

$f(3) = 33 + b - 27 = b + 6$

Mean Value Theorem



$f(x)$ is

a) continuous in $[a, b]$

b) differentiable in (a, b)

then $\exists$ a c $a < c < b$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Ex

$f(x) = x^2 - 5x + 6$ $\quad [1, 3]$

$1 - 5 + 6$

$f(b) = 9 - 15 + 6 = 0$ $\quad b = 3$

$f(a) = 1 - 5 + 6 = 2$ $\quad a = 1$

$f'(c) = \underline{2c - 5}$

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 2}{3 - 1} = \boxed{-1}$$

$f'(x) = 2x - 5$

$f'(c) = \underline{2c - 5 = -1} \Rightarrow 2c = 4$

$\boxed{c = 2}$

Let f be differentiable for all x. If f(1) = −2 [redacted] Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for all $x \in [1, 6]$, then the minimum value of $f(6)$ is

(1) 4   (2) 2

(3) 8 ✓   (4) None of these

$x \in [1, 6]$

$f(1) = -2$

$f(6) = ?$

$f'(x) \geq 2$

$f(1) = f(6)$ Rolle's Th.

$f'(c) = 0$

Not applicable

$f'(c) = \dfrac{f(6) - f(1)}{6-1}$

$f'(c) = \dfrac{f(6)+2}{5}$

$f(6) = ?$

$f(1) = -2$

$f'(x) \geq 2$

$\dfrac{f(6)+2}{5} \geq 2$

$f(6) + 2 \geq 10$

$\boxed{f(6) \geq 8}$ ✓

$f(x)$ is diff.

& $f'(x) = \underline{10}$

$f'(c) = f(6$

$f(x)$ is diff

$f'(x) \leq 5$, $f'(c)$

• Integration

• MCQ's of Unit 4 :
- Limit
- Continuity
- diff.
- Increasing decreasing
- Maxima, minima 03
- Rolles Th.
- L.M.V.T.

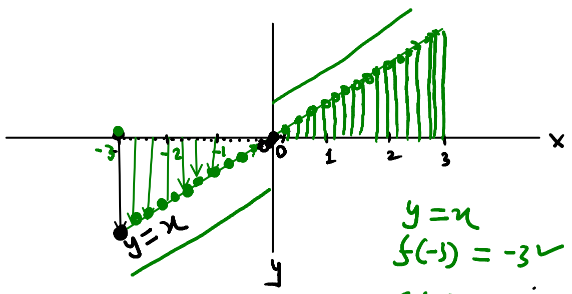
$[-x \ \ x]$

$y = x$

10 ✓

100 ✓

1000 ✓



$x \in [-3, 3]$ ✓

$y = f(x) = x$

$f(-3) = \boxed{-3}$ ✓

$f(-2) = \boxed{-2}$

$f(-1) = \boxed{-1}$ ✓

$f(0) = 0$

$f(1) = \boxed{1}$ ✓

$f(2) = \boxed{2}$

$f(3) = \boxed{3}$ ✓

$y = x$

$f(-3) = -3$ ✓

$f(3) = 3$ ✓

Integration of $y = x$

$$\int_{-3}^{3} x\,dx = \frac{1}{n}\sum_{i=1}^{n} f(x_i) \quad n \to \infty$$

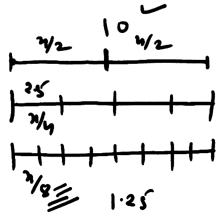
$$\int_{-3}^{3} x\,dx = \left.\frac{x^2}{2}\right|_{-3}^{3} = \frac{9}{2} - \left(\frac{9}{2}\right) = 0$$

$\sqrt{4 - x^2}$ ✓ $\int^{1} x^2 dx = \left.\dfrac{x^3}{3}\right|^{1} = \dfrac{1}{}$ −

$\boxed{y = x^2}$ $\quad \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} =$

$y = x^2$

0.33333 $\quad [0 \quad 1]$



$\times \quad \frac{10}{4}$

$[0 \quad 1/2] = \frac{x^3}{3}\Big|_0^{1/2} = \frac{1}{3}\cdot\frac{1}{8} = \frac{1}{24} = 0.0416666$

GATE 2017

$$I = \int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$$

$\frac{\pi^3}{24}$

$\frac{\pi^3}{12}$

$\frac{\pi^3}{48}$

$\pi^3/64$

We know that $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

let $\sin^{-1}x = H$ $\quad \boxed{0 \to 1}$

$\frac{1}{\sqrt{1-x^2}} = \frac{dH}{dx}$ $\quad \boxed{\frac{dx}{\sqrt{1-x^2}} = dH}$

$$\boxed{I = \int_0^{\pi/2} H^2\,dH} = \frac{H^3}{3}\Big|_0^{\pi/2} = \frac{1}{3}\left[\frac{\pi}{2}\right]^3 - \frac{1}{3}(0)^3$$

$$= \frac{\pi^3}{3\cdot 8} = \boxed{\frac{\pi^3}{24}}$$

$$\int_1^0 \frac{(-\cos^{-1}x)^2}{\sqrt{1-x^2}}\,d$$

$$I = \int_0^\infty \frac{(\tan^{-1}x)^2}{1+x^2}\,dx =$$

Let $\tan^{-1}x = N \Rightarrow \frac{d}{dx}(\tan^{-1}x) = \frac{dN}{dx}$

$\frac{dx}{1+x^2} = dN \qquad \frac{1}{1+x^2} = \frac{dN}{dx}$

$$\int_0^\infty \frac{\tan^{-1}x}{1+x^2}\,dx = \boxed{\int_0^{\pi/2} N^2\,dN} = \frac{N^3}{3}\Big|_0^{\pi/2} = \frac{\pi^3}{8\cdot 3} = \boxed{\frac{\pi^3}{24}}$$

Q: $\int_0^{\pi/2} (\sin x + \cos x)\,dx = -\cos x + \sin x\Big|^{\pi/2}$

$-0 + 1 - [(-\cos 0) - \sin 0]$

$1 + 1 = 2$

GATE 2017 : 2 mark

GATE 2017 : 2 mark

Q: The tangent to curve represented by $y = x\ln x$ is required to have 45° inclination with x-axis. The coordinates of the tangent point

A) (1,0)

B) (0,1)

C) (1,1)

D) $(\sqrt{2}, \sqrt{2})$

$y = x \cdot \log x$ — (I)

$\frac{dy}{dx} = \tan 45^\circ$

$\frac{dy}{dx} = 1$

$\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$ — (III)

$\frac{dy}{dx} = \boxed{1 + \log x = 1}$

$\log x = 0$

$e^{\log x} = e^0$

$\boxed{x = 1}$

$y = x\log x = 1 \cdot \log 1 = 0$

$\boxed{y = 0}$

(1, 0)

$(-\infty, 0)$

GATE 2017 : 2 Mark

$$\boxed{g(x) = \begin{cases} -x & x < 1 \\ x+1 & x \geq 1 \end{cases}} \quad \& \quad \boxed{f(x) = \begin{cases} 1-x & x \leq 0 \\ x^2 & x > 0 \end{cases}}$$

Consider the composition f o g $(f \circ g)(x) = f(g(x))$

Then The No. of discontinuities in $(f \circ g)(x)$ on $(-\infty, 0)$

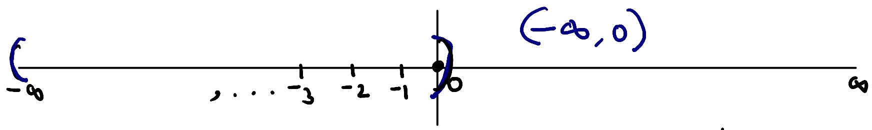
A) 0

B) 1

C) 2

D) 4

$(-\infty, 0)$



$f(x) = 1 - x$

$g(x) = -x$

$(f \circ g)(x) = f(g(x))$

$= f(-x)$

$(f \circ g)x = 1 - (-x)$

$= \boxed{1 + x}$

$x + 1$ is continuous $(-\infty, 0)$

Gate 2017 : 4

Q: The minimum value of function $f(x) = \frac{1}{3} \cdot x(x^2 - 3)$ in the interval $\boxed{-100 \leq x \leq 100}$ occurs at

$x =$ _____

$[\cancel{-100}, 100]$

$$f(x) = \frac{x^3-3x}{3} \quad ; \quad f'(x) = \frac{3x^2-3}{3} = 0$$

$$\Rightarrow x^2-1=0 \Rightarrow \boxed{x = \pm 1}$$

$$f''(x) = \frac{6x}{3} \quad \text{at } \boxed{x=1} \quad f''(1) > 0 \quad \text{Local minima}$$

$$x=-1 \quad f''(-1) < 0 \quad \text{Local Maxima}$$

-100 ... -1 ... 0 ... 1 ... 100

$$f(x) = \frac{1}{3}(x^3-3x)$$

$$f(-100) = \frac{1}{3}\left((-100)^3+300\right) = -333233.33333$$

$$f(-1) = \frac{1}{3}(-1+3) = +\frac{2}{3}$$

$$f(1) = \frac{1}{3}(1-3) = -2/3$$

$$f(100) = 333233.3333$$

ISRO 2017 : 2 marks

$$\lim_{x\to 1} \frac{x^7-2x^5+1}{x^3-3x^2+2} \qquad \lim_{x\to 1} \frac{x^7-2x^5+1}{x^3-3x^2+2}$$

$$1 \quad \frac{1-2+1}{1-3+2} \quad \frac{0}{0}$$

$$\lim_{x\to 1} \frac{7x^6-10x^4}{3x^2-6x} \Rightarrow 1 \quad \frac{7-10}{3-6} = \frac{-3}{-3} = 1$$

$$\lim_{x\to 0} \frac{x^3-\sin x}{x} =$$

0

3

1

$\boxed{-1}$

$$0 \quad \frac{0-0}{0} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{3x^2-\cos x}{1}$$

$$\Rightarrow \frac{0-1}{1} = \boxed{-1}$$

(2.) Gate 2017

Q: Let x be a continuous variable defined over $(-\infty,\infty)$ & $f(x) = e^{-x-e^{-x}}$ then

$I = \underbrace{\int f(x)\,dx}$ → $e^{e^{-x}}$, ~~$e^{-e^{-x}}$~~, $e^{-e^{x}}$, e^{-x}

$$f(x) = e^{-x-e^{-x}} = e^{-x}\cdot e^{-e^{-x}} = \frac{e^{-x}}{e^{e^{-x}}}$$

$$I = \int \frac{e^{-x}}{e^{e^{-x}}}\,dx$$

let $e^{-x} = t$

$$e^{-x}\cdot(-1) = \frac{dt}{dx}$$

$$\boxed{-e^{-x}\,dx = dt}$$

$$I = -\int \frac{dt}{e^{t}} = -\int e^{-t}\,dt = -\left[\frac{e^{-t}}{-1}\right] = e^{-t}$$

$$\boxed{I = e^{-e^{-x}}}$$

Q: $\lim_{x\to 0} \frac{\tan x}{x^2 - x}$ $\frac{0}{0}$

$$\frac{\sec^2 x}{2x-1} = \frac{1}{-1} = -1$$

Gate 2017

Let x be continuous variable defined over $(-\infty, \infty)$ ✓ & $\boxed{f(x) = e^{-x-e^{-x}}}$. The $I = \int f(x)\,dx$

A) $e^{e^{-x}}$

~~B)~~ $e^{-e^{-x}}$ /

C) $e^{-e^{x}}$

D) e^{-x}

$$I = \int e^{-x-e^{-x}}\,dx$$

$$= \int e^{-x}\cdot e^{-e^{-x}}\,dx$$

$$= \int \frac{e^{-x}}{e^{e^{-x}}}\,dx$$

Let $e^{-x} = s \Rightarrow \underline{-e^{-x}\,dx = ds}$

$$I = -\int \frac{ds}{e^{s}} = -\int e^{-s}\,ds$$

e^{-x}

$$I = -\frac{[e^{-s}]}{-1} = e^{-s} = (e^{-t})$$

B : $e^{-e^{-x}}$ $\qquad \frac{d}{dx} e^{-e^{-x}} = +\underline{\underline{e^{-e^{-x}} \cdot e^{-x}}} = e^{-x-e^{-x}}$

Gate 2016

$$\boxed{\int_0^1 \frac{dx}{\sqrt{1-x}} =}$$

1
2
3
4

x: $0 = \sin^2\theta \Rightarrow \theta = 0$; $1 = \sin^2\theta \Rightarrow \theta = \pi/2$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

let $x = \sin^2\theta$

$dx = 2\sin\theta \cdot \cos\theta \, d\theta$

$0 = \sin^2\theta$

$0 = \sin\theta$

$$\int \frac{2\sin\theta \cdot \cos\theta \, d\theta}{\sqrt{1-\sin^2\theta}} = \int_{\pi/2} \frac{2\sin\theta \cdot \cos\theta \, d\theta}{\sqrt{\cos^2\theta}}$$

$$= \int \frac{2\sin\theta \cdot \cancel{\cos\theta}}{\cancel{\cos\theta}} d\theta = \int_0 2\sin\theta \, d\theta$$

$$= 2\left[-\cos\theta\right]_0^{\pi/2} = 2\left[-\cos\pi/2 + \cos 0\right]$$

$$= 2[-0+1] = \boxed{2}$$

Q: $\boxed{\int_{1/\pi}^{2/\pi} \frac{\cos\left(\frac{1}{x}\right) dx}{x^2}}$ $\qquad$ GATE 2015 $\quad$ 2: mark

Let $\frac{1}{x} = K$ $\quad$ $x = 2/\pi \Rightarrow K = \pi/2$; $1/\pi = x \Rightarrow K = \pi$

$-\frac{1}{x^2} dx = dK$

$$-\int_{\pi}^{\pi/2} \cos K \, dK$$

$\pi/2 \qquad \pi$

$$\int_{\pi/2}^{\pi} \cos K \, dK = \sin K \Big|_{\pi/2}^{\pi} = \sin\pi - \sin\pi/2$$

$$= 0 - 1$$

$$= \boxed{-1}$$

$$\boxed{\int_{\alpha}^{\beta} f(x)dx = \int_{\alpha}^{\beta} f(\alpha+\beta-x)\,dx}$$

Q: $\underline{\underline{I}} = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} dx$ —— ® , $I = \int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)}+\sqrt{1+2-x}} dx$

$$I = \int_1^2 \frac{\sqrt{3+x}}{\sqrt{x}+\sqrt{3+x}} \quad \text{— (5)}$$

$$2I = \int_1^2 \left(\frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} + \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} \right) dx$$

$$2I = \int_1^2 1\,dx \Rightarrow \boxed{I = \frac{1}{2}}$$

2

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}\,dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right) = \left(\frac{\pi}{2}-x\right)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$\frac{\frac{\pi}{3}-\frac{\pi}{6}}{2}$$

Q: $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$ — (1)

$$I = \int_0^{\pi/2} \frac{\sin[0+\pi/2-x]}{\sin(\pi/2-x)+\cos(\pi/2-x)}\,dx = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx \quad \text{—(2)}$$

$$2I = \int_0^{\pi/2} \left(\frac{\sin x}{\sin x+\cos x} + \frac{\cos x}{\cos x+\sin x} \right) dx = \int_0^{\pi/2} 1\,dx = x\Big|_0^{\pi/2}$$

$$2I = \pi/2 \Rightarrow \boxed{I = \pi/4}$$

Q: $\boxed{I = \int_0^{\pi/2} \frac{\sin^{3/2} x\,dx}{\sin^{3/2} x + \cos^{3/2} x} = \pi/4}$

Q $I \quad \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\,dx = \pi/20 \qquad n \in \mathbb{R}$

Q: Gate : 2016

$$\lim_{x\to 0} \frac{\log_e(1+4x)}{e^{3x}-1} =$$

A) 0
B) $\frac{1}{12}$
C) $4/3$
D) 1

M/m

Q: The optimum value of the function $f = x^2-4x+2$ is ______ Gate 2016 (1 marks)

A 2
B −2
C = 4
D = −4

$f = x$

$\frac{df}{dx} =$

Q: Gate : 2016 : 2 marks

A) 0
B) ∞

lim

$\lim_{x\to\infty} \sqrt{x^2+x-1} - x$

OR

$\lim_{x\to\infty}$

B) ∞
C) $1/2$ ✓
D) $-\infty$

$x \to \infty \quad t = \frac{1}{x}$ $\boxed{\frac{1}{\infty}}$

$t \to 0$

let $x = \frac{1}{t}$

$\lim_{t\to 0} \sqrt{\frac{1}{t^2} + \frac{1}{t} - 1} - \frac{1}{t}$

$\lim_{t\to 0} \sqrt{\frac{1+t-t^2}{t^2}} - \frac{1}{t}$

Q: Gate 2015 : 1 mark

At $\underline{\underline{x=0}}$, $|x|$ has

A) Max
B) min:
C) Neither M nor m
D) None

Q: Gate 2015 : 1 mark

$$\lim_{x\to 0} \frac{1-\cos(x^2)}{2x}$$

Q: $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{10x} =$

e^{-2x}
$\cancel{e^{10}}$ ✓
1
e^2

$\underline{\underline{\lim_{x\to\infty}}} \left(1+\frac{1}{x}\right)^{x} = e$

$\lim_{x\to\infty} \left[\left(1+\frac{1}{x}\right)^{x}\right]^2$

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x} \cdot \left(1+\frac{1}{x}\right)^{x}$

$e \cdot e = e^2$

Q: $\lim_{x\to 0} \left(\frac{-\sin x}{2\sin x + x\cos x}\right)$ is ____

A) $\frac{1}{3}$ $\cancel{B)}$ $-\frac{1}{3}$ C) $\frac{2}{3}$ D) $-\frac{2}{3}$

Q: Gate 2014 : 2 marks

$$\int_0^{\pi} x^2 \cos x \, dx =$$

-2π
π
$-\pi$
2π ✓

Q: $\boxed{\int_0^{\pi/2} x^3 \cdot \sin(x^4)\, dx}$ $\frac{1}{4}$

let. $x^4 = s$ $\to 0$, $\to \frac{\pi^4}{16}$

$4x^3 dx = ds$

$x^3 dx = \frac{ds}{4}$

$\int_0^{\frac{\pi^4}{16}}$

$-c$ ____

Q: $$\int_{\pi/6}^{\pi/3} \frac{1}{\sqrt{\cot x}+1}\, dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{\frac{\sqrt{\cos x}}{\sqrt{\sin x}}+1}\, dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx \quad \text{—(1)}$$

$$\frac{\pi}{6} \quad \frac{\sqrt{\cos x}}{\sqrt{\sin x}}+1 \qquad\qquad \frac{\pi}{6} \quad \sqrt{\cos x}+\sqrt{\sin x}$$

$$\frac{\pi}{6}+\frac{\pi}{3} = \frac{2\pi+\pi}{6} = \frac{3\pi}{6} = \pi/2$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}}\,dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \text{— (ii)}$$

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} dx = x\Big|_{\pi/6}^{\pi/3} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$$

$$\boxed{I = \pi/12}$$

$$I = \int_{0}^{\pi/2} \frac{1}{\sqrt{\tan x}+1}\,dx \qquad 2I = \int_0^{\pi/2} dx \qquad \boxed{I = \pi/4}$$

$$\pi/4$$