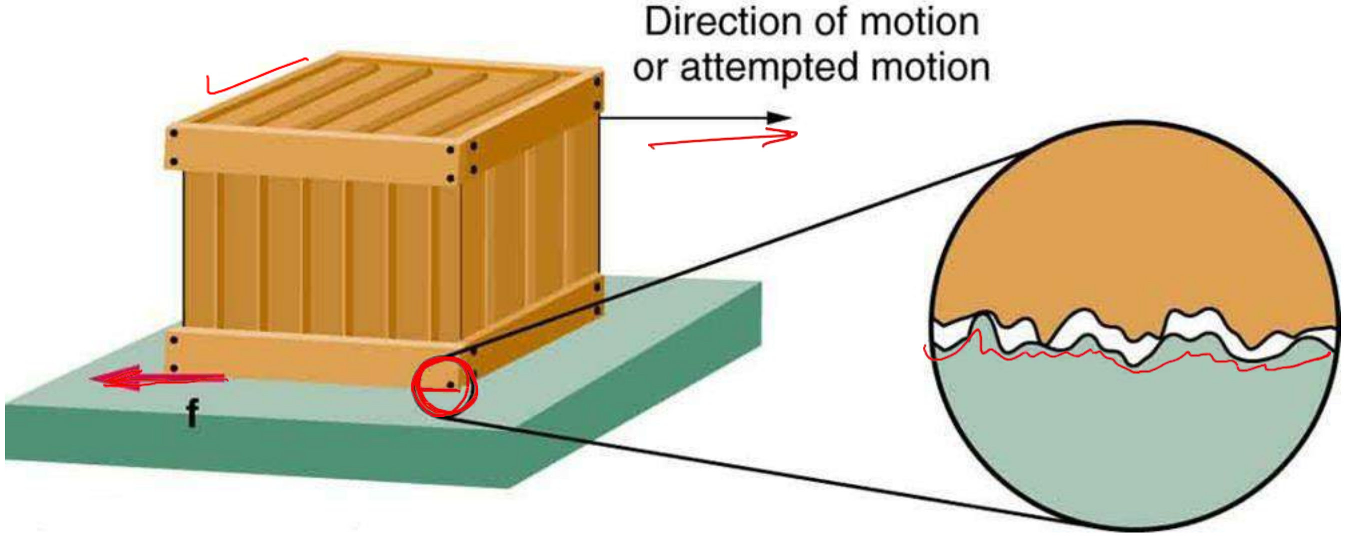
CGPA
BOOSTER

Unit 2
Friction

Introduction

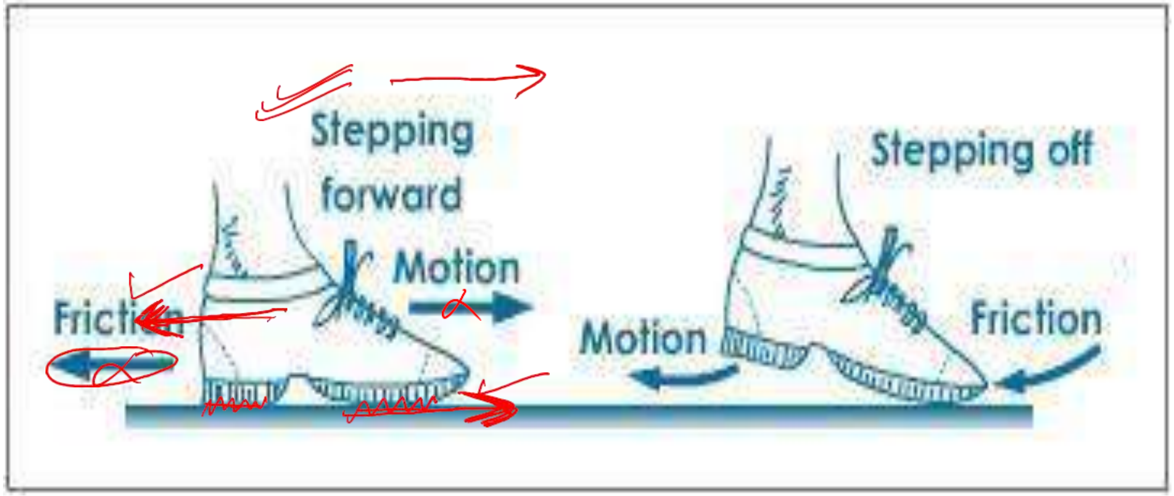
- In practice, the surfaces of bodies are never perfectly smooth. If we see a smooth surface under a microscope, it will be found that the surface has roughness and irregularities.

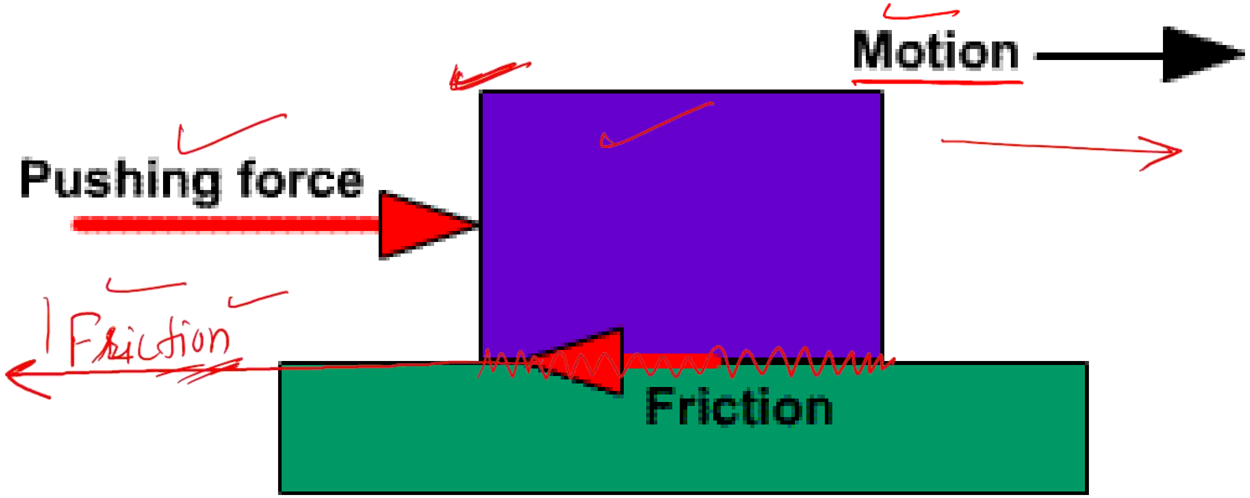


- Therefore, when a body moves or tends to move over another body, the irregularities of the surface of one body get interlocked with that of the surface of another body.

Introduction

- This interlocking action of the irregularities produces a force which opposes the motion of one body over the other.
- Such an opposing force which comes into play along the surfaces of contact of the two bodies is called **force of friction** or simply **Friction.**





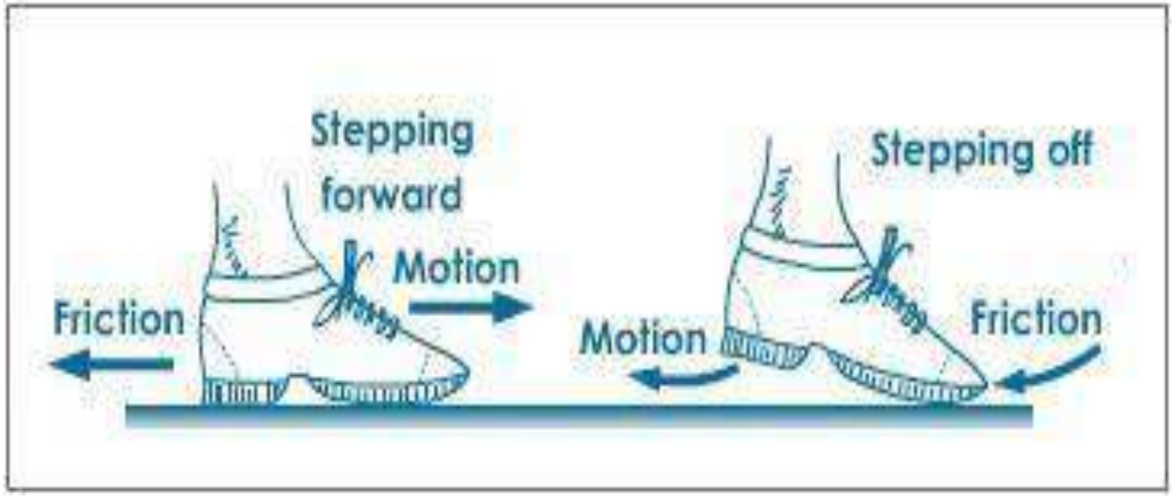
Introduction

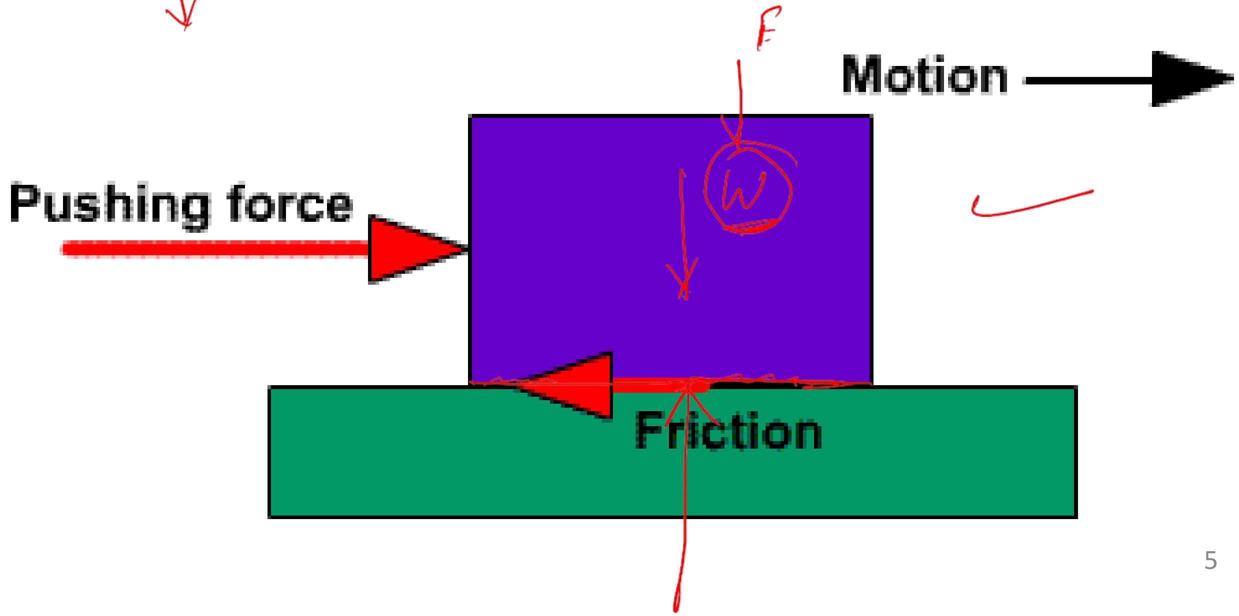
- The force of friction is tangential to the contacting surfaces and always acts opposite to the direction of motion of the body.

- The magnitude of friction between two surfaces depends upon:

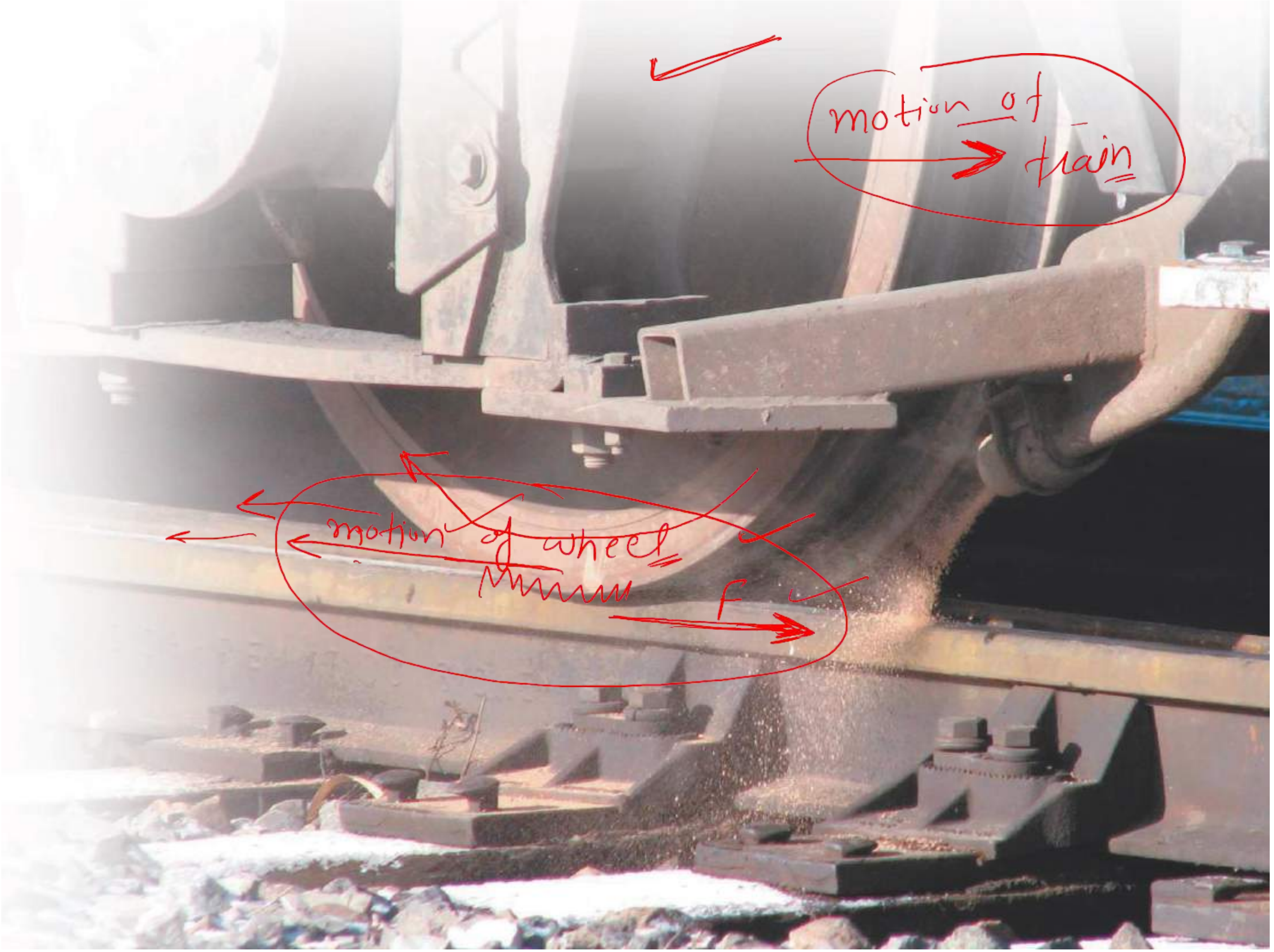
  1. Nature of Material of surfaces — Roughness OR Irregularities
  2. Pressure between surfaces — F/A

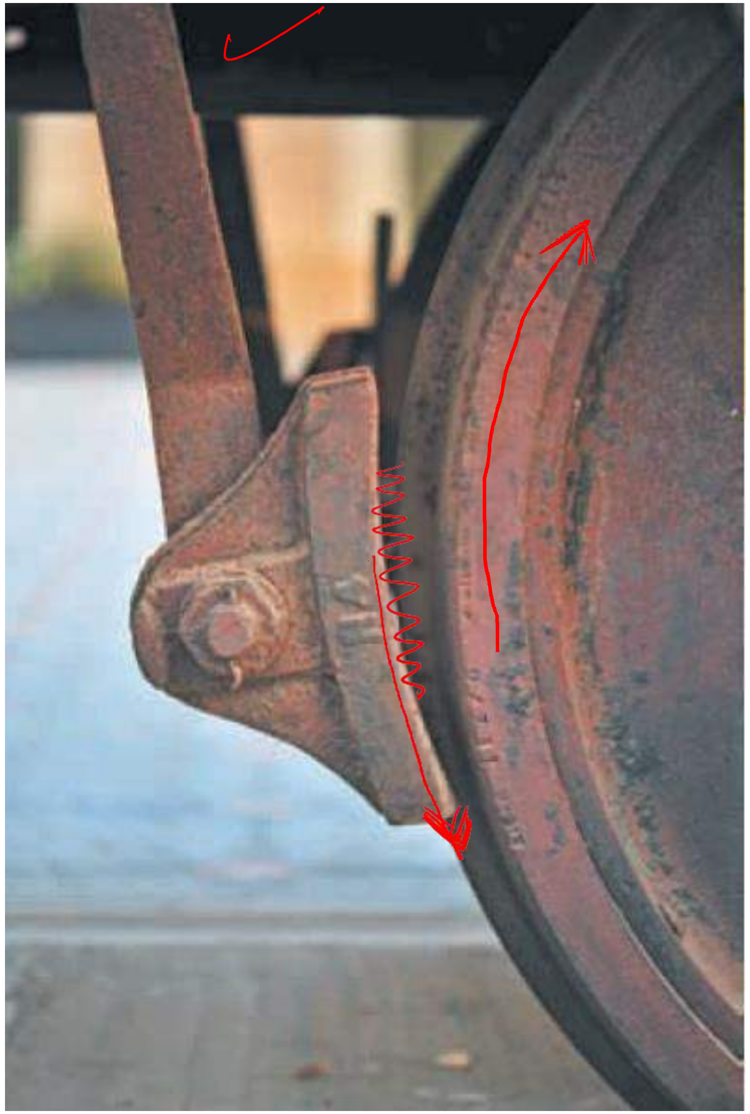
① ②



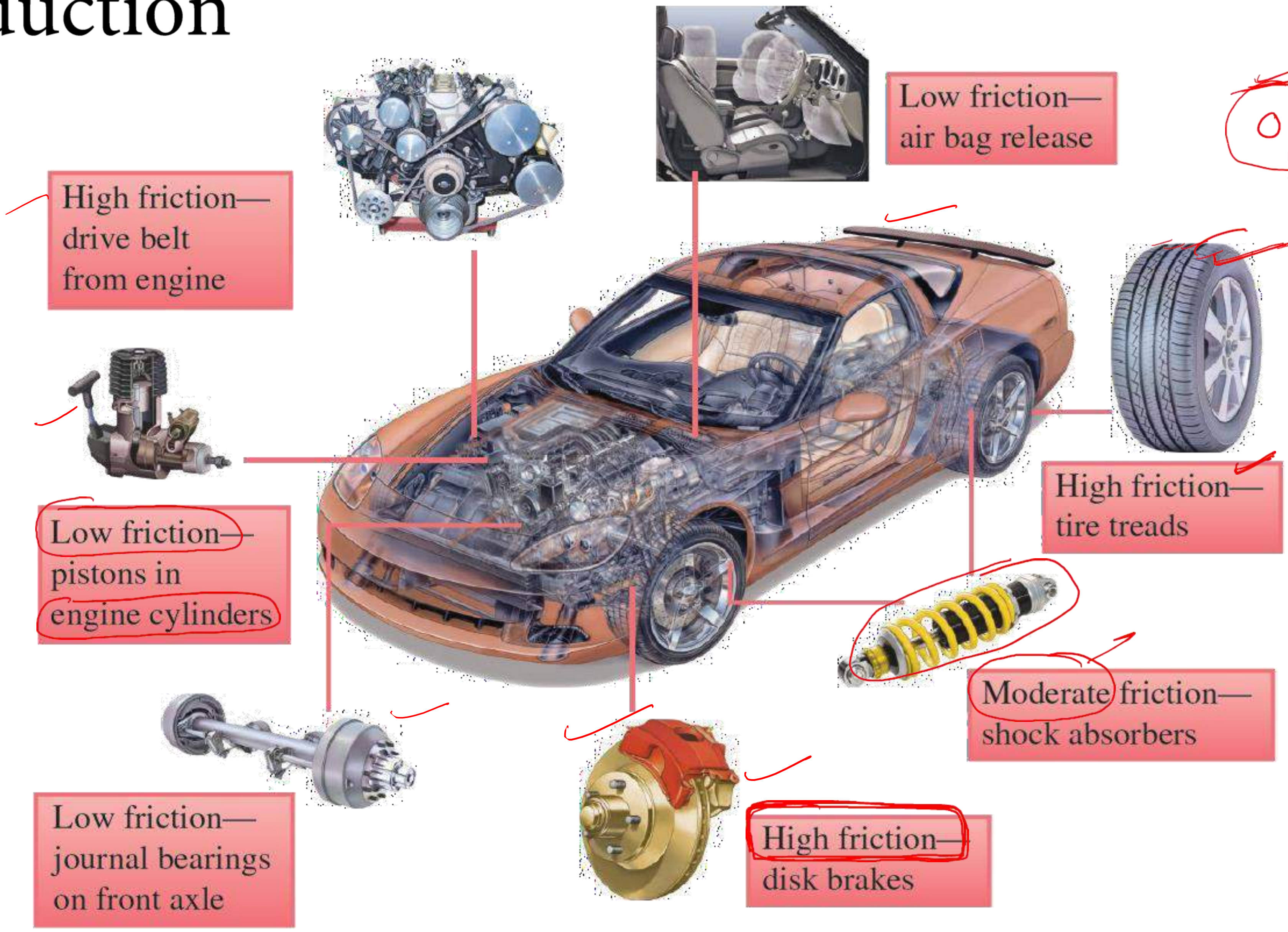


Introduction



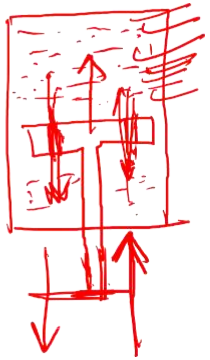


Introduction



Optimum friction

grip



Effects of Friction

- **Useful Effects:** Walking, Skating, smooth and safe movement of vehicles, efficient working of machine parts such as: Belts & ropes, nut & bolt, brakes etc., Writing with chalk, pencil, pen etc.

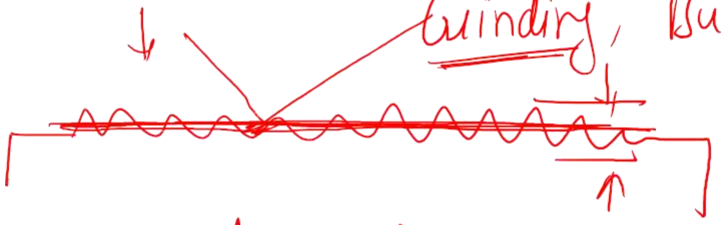
- **Harmful Effects:**
  - It causes loss of power in machines.
  - It causes wear and tear of moving parts of machines, cutting tools, road surfaces, rails etc.
  - It causes more tractive resistance to the movement of vehicles thus causing more consumption of fuel.

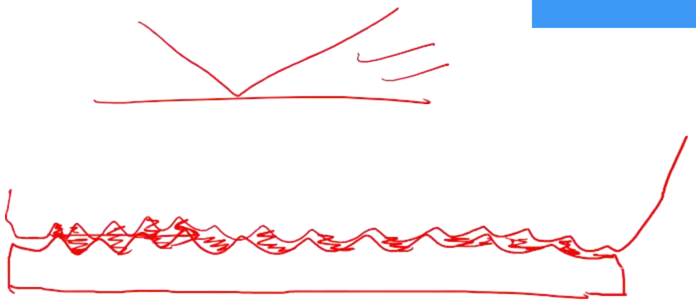
tractor

Methods of reducing friction

- Polishing

Grinding, Buffing





- Lubrication

viscosity of lubricant

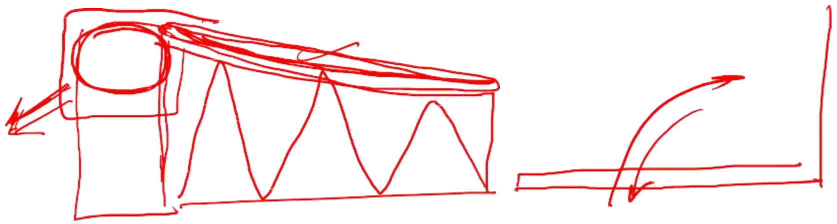
- Using bearings
- Using alloys

iron steel alloy Alloy wheel

- Avoiding moisture

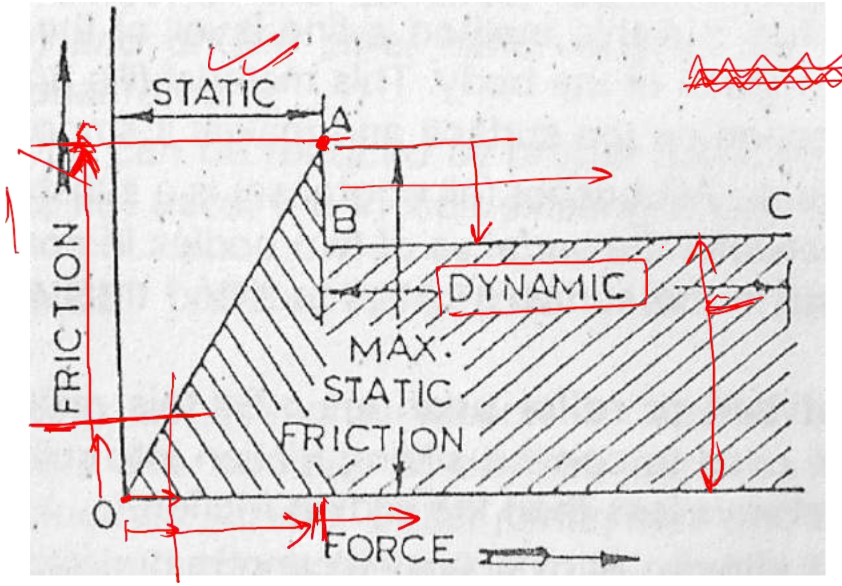
LONDON Bridge

optimum value of friction



Types of Friction

- **Static Friction:** The friction offered by the surfaces subjected to external force until there is no motion between them is called static friction.
- **Dynamics Friction:** The friction experienced by a body when it is in motion over another body is called dynamic friction.



D.F < S.F.

Types of Friction

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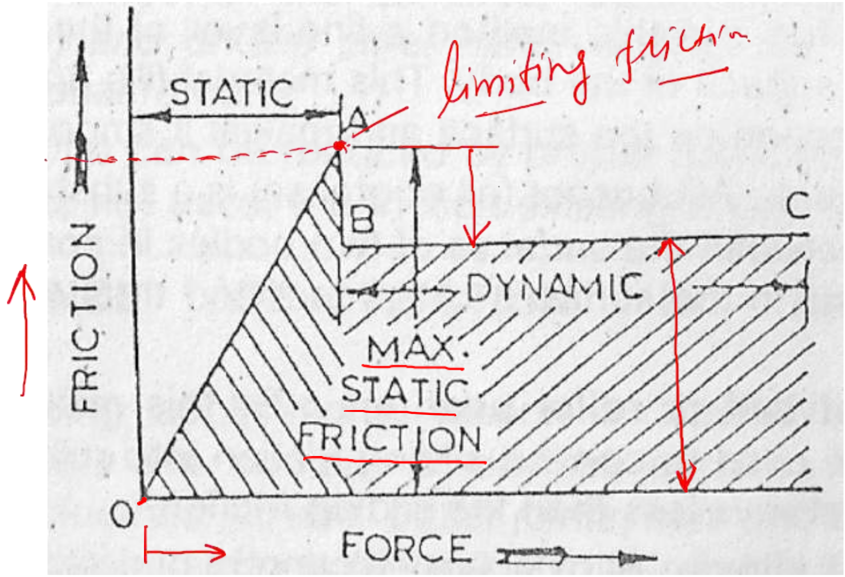


D.F < S.F.

Types of Friction

$\boxed{F = \mu N}$

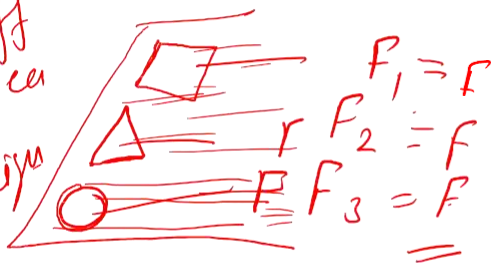
Dry friction

wet friction

- Dynamic Friction is further divided into two parts:

Laws of Friction

diff area & same weight



- **Laws of Static Friction:**
  - The force of friction always acts in the direction opposite to that in which body tends to move.
  - The magnitude of friction between two surfaces depends upon:

1. Nature of Material of surfaces
2. Pressure between surfaces

  - The magnitude of the force of friction is exactly equal to the applied force which tends to move the body.
  - The Force of friction is independent of the area of contact between two surfaces.
  - The ratio of maximum friction and normal reaction between two surfaces remain constant.

Laws of Friction

D.F < S.F.

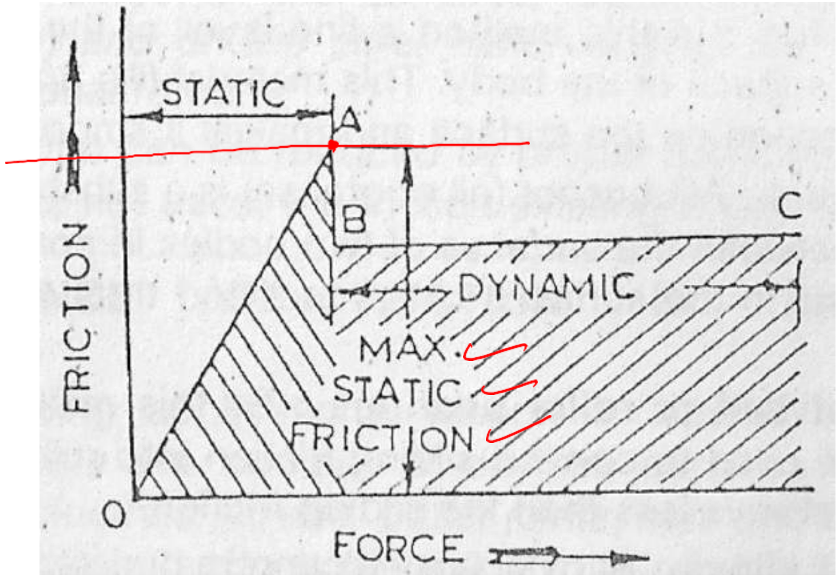
- **Laws of Dynamic Friction or Kinetic Friction:**
  - The force of friction always acts in a direction opposite to that in which the body is moving.
  - The force of friction is directly proportional to the normal reaction between the two contact surfaces. But the ratio is slightly less than the maximum friction.
  - The force of friction remains constant for moderate speeds, but it decreases with the increase of speed.

Limiting Friction

- The maximum value of frictional force which comes into play, when a body is just on the point of start of motion over the surface of another body is called Limiting Friction.

Impending motion

- Limiting friction is the maximum static friction.



Limiting Friction

Since block is in equilibrium under W, N, P & F

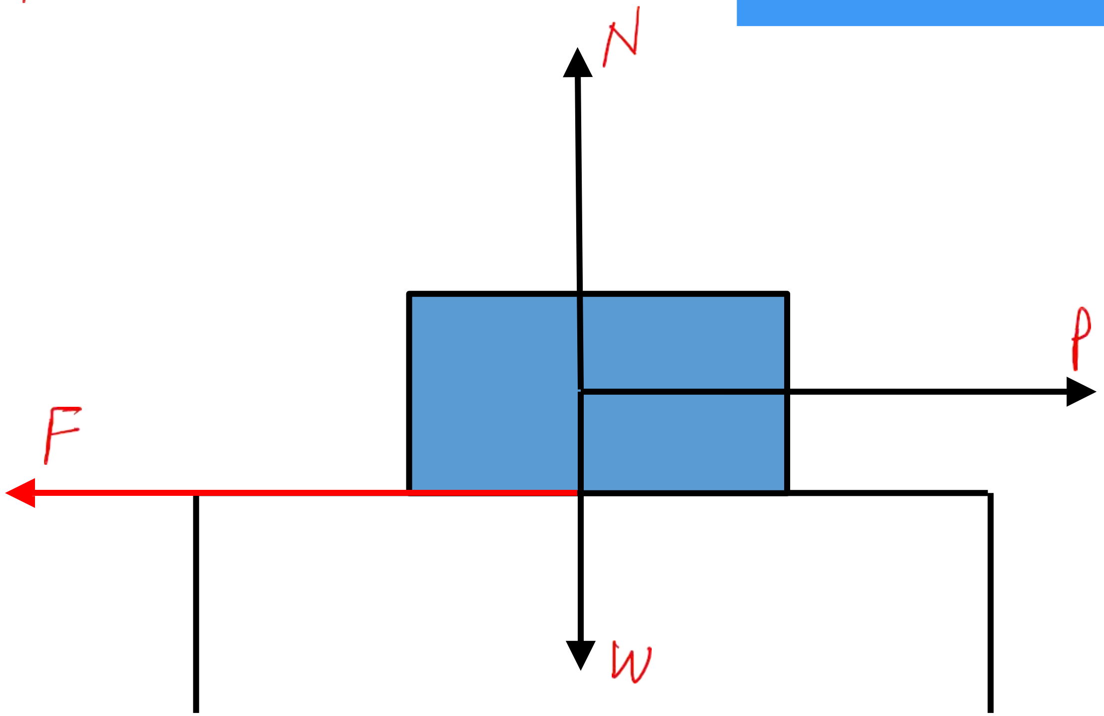
$\therefore$ $W = N$ and $P = F$

F_m = limiting friction.

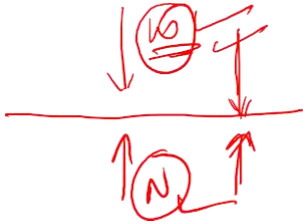
till $P < F_m$ $\rightarrow$ No motion.

when $P = F_m$ $\rightarrow$ Impending motion.

when $P > F_m$ $\rightarrow$ motion.



Co-efficient of Friction



- According to laws of friction, limiting friction is directly proportional to the normal reaction, i.e.

$F_m \propto N$

$F_m = \mu N$

Pressure b/w surface

Nature of material of surfaces

$$\boxed{\frac{F_m}{N} = \mu_s}$$

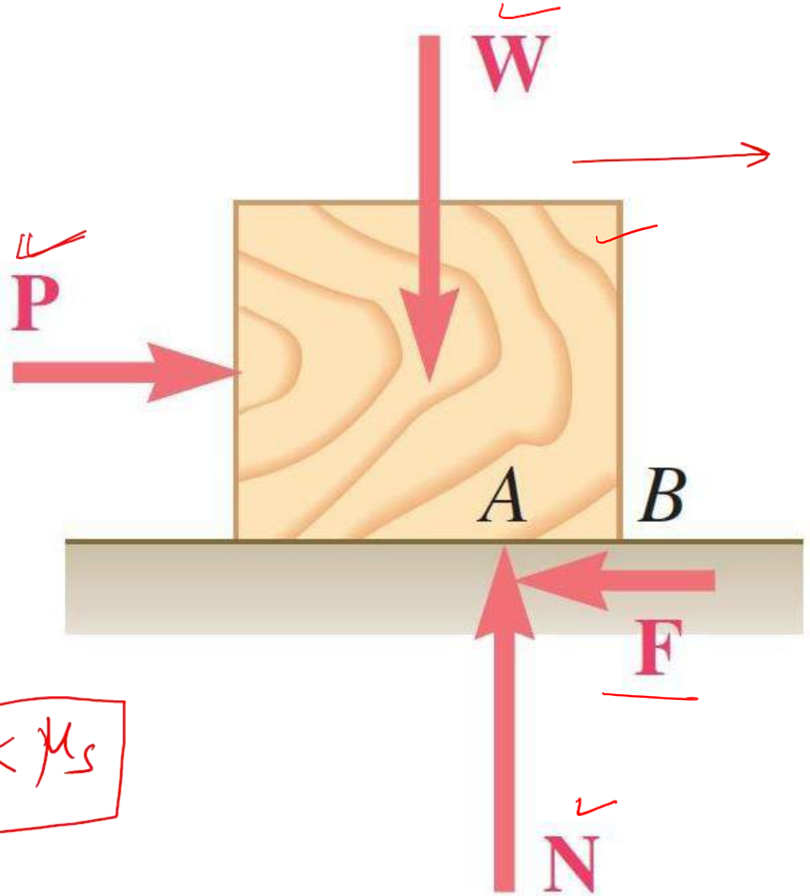
co-efficient of static friction

when $P > F_m$

$$\boxed{F_K = \mu_K N}$$

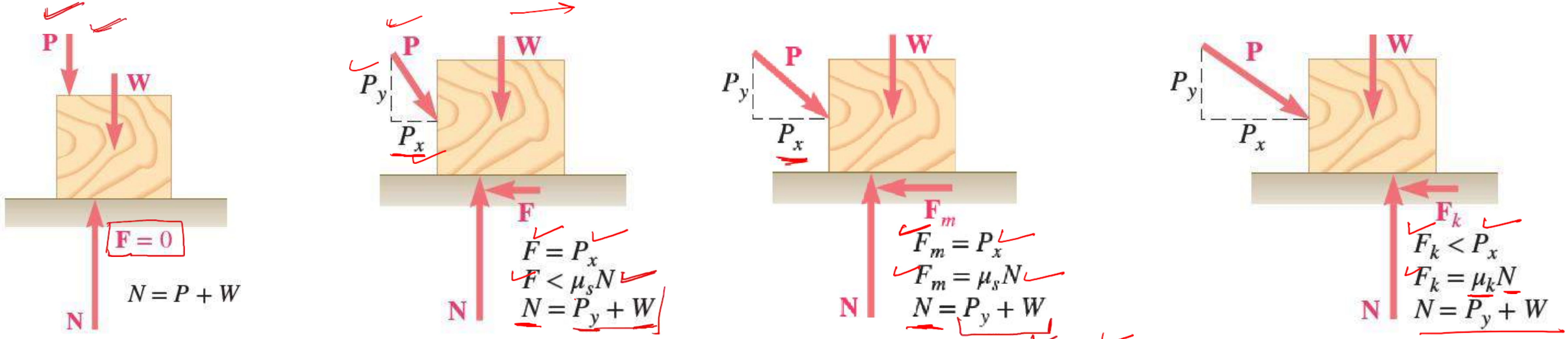
co-eff. of dynamic or kinetic friction

$$\boxed{\mu_K < \mu_s}$$



Co-efficient of Friction $\boxed{\frac{F_m}{N} = \mu_s}$

$F_k < F_s$, $\mu_k < \mu_s$



(a) No friction ($P_x = 0$)   (b) No motion ($P_x < F_m$)   (c) Motion impending $\longrightarrow$ ($P_x = F_m$)   (d) Motion $\longrightarrow$ ($P_x > F_k$)

Angle of Friction

- It is sometimes convenient to replace the normal force **N** and the friction force **F** by their resultant **R**.

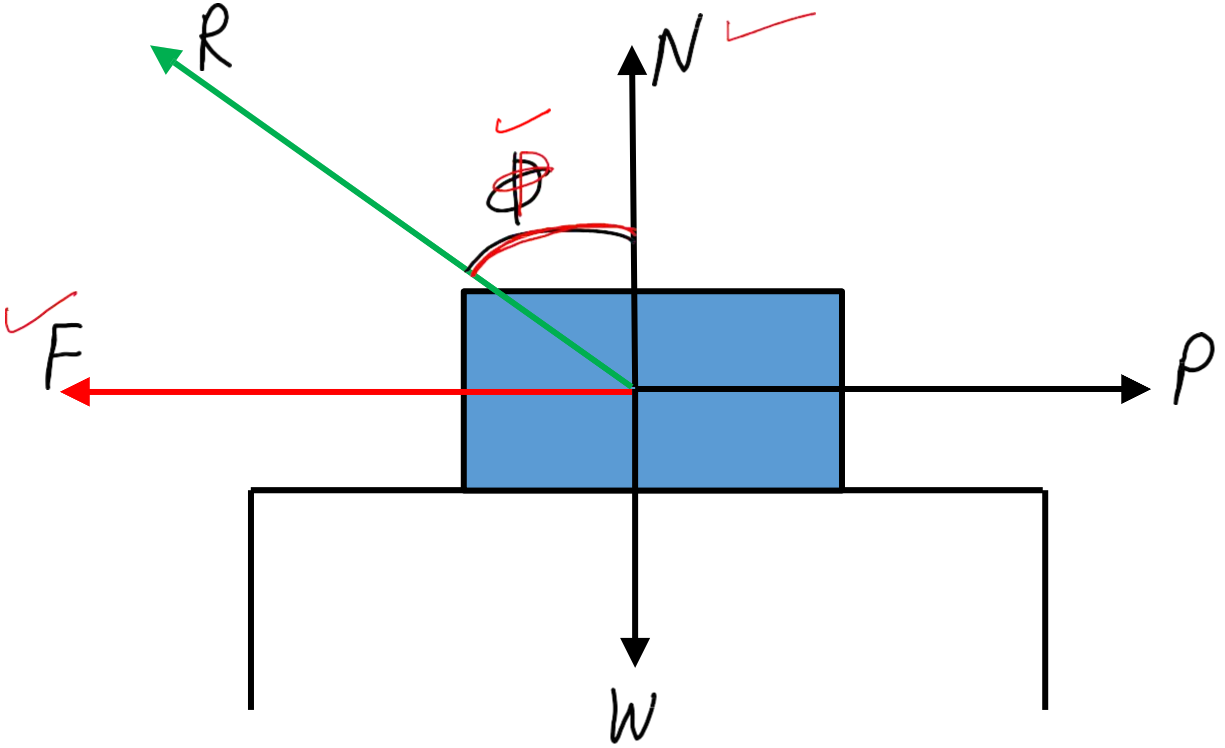
$\tan\phi = \frac{F}{N} = \mu$

$\boxed{\phi = \tan^{-1}(\mu)}$

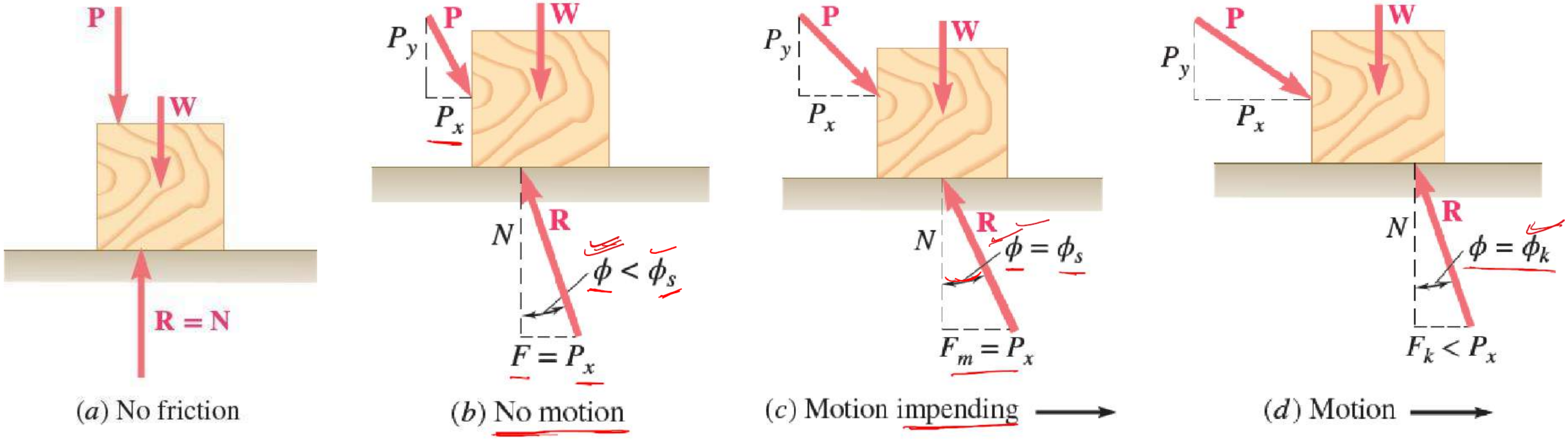
$\boxed{\phi_s = \tan^{-1}\mu_s}$

OR

$\boxed{\phi_K = \tan^{-1}\mu_K}$



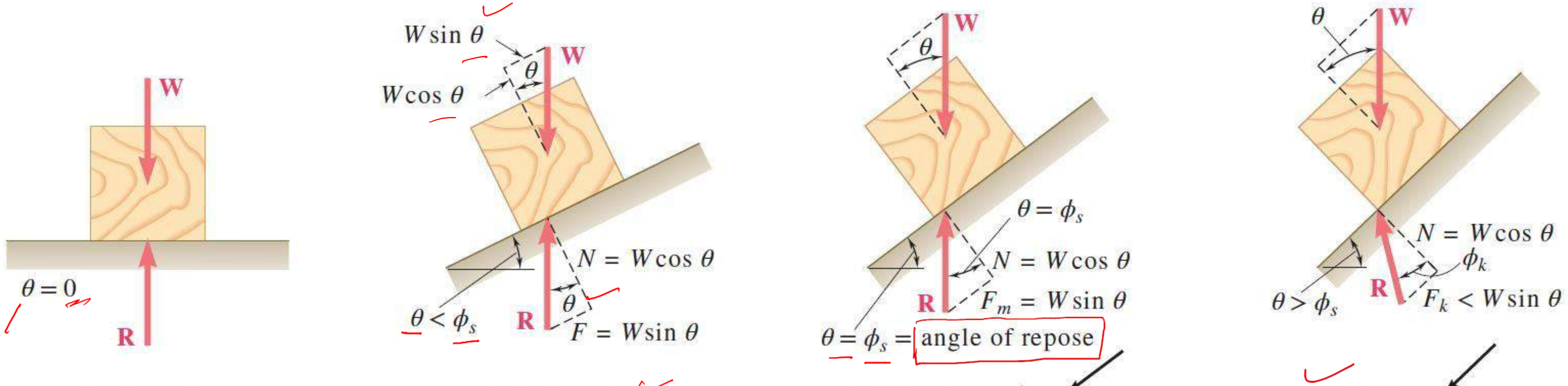
Angle of Friction



Angle of Repose

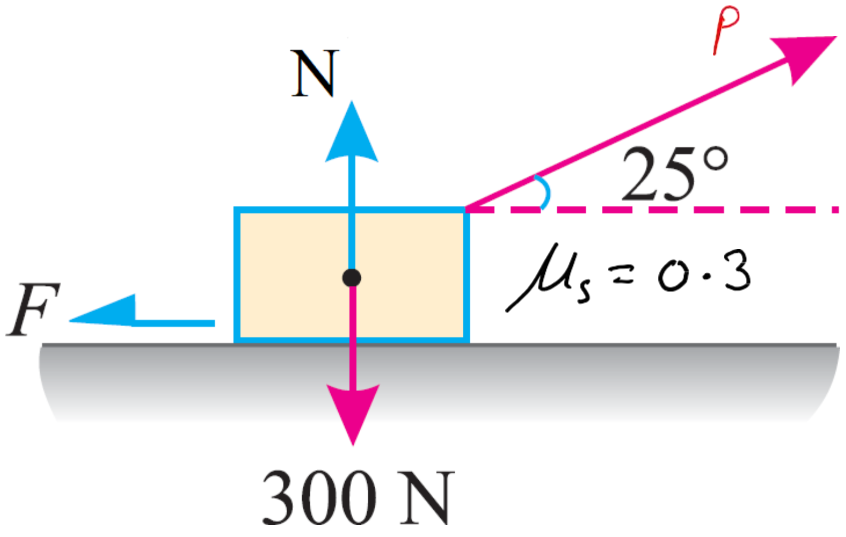
- The angle which an inclined plane makes with horizontal, when a body placed on it is just on the point of moving down is called **Angle of Repose**.

$$\mu_s = \frac{F_m}{N} = \frac{W \sin\theta}{W \cos\theta} = \tan\theta \Rightarrow \mu_s = \tan\theta \Rightarrow \tan\phi_s \Rightarrow \boxed{\theta = \phi_s}$$



(a) No friction (b) No motion (c) Motion impending (d) Motion

Find the magnitude of the force which can move the body.



Impending motion

Sol:

$$F = P\cos 25 = \mu_s N$$

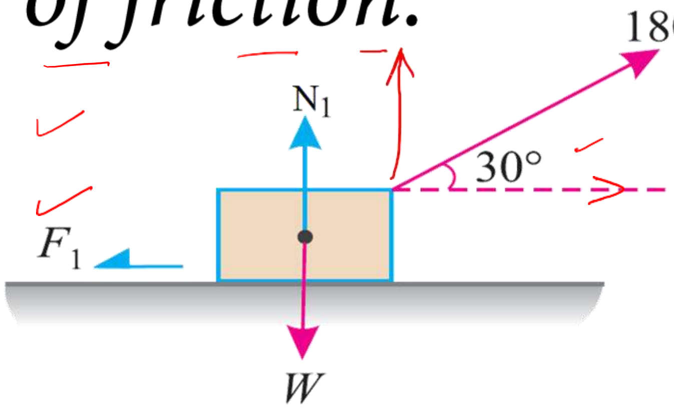
$$P = \frac{\mu_s N}{\cos 25} = \frac{0.3(300 - P\sin 25)}{\cos 25}$$

$$P\cos 25 = 0.3\,(300 - P\sin 25)$$

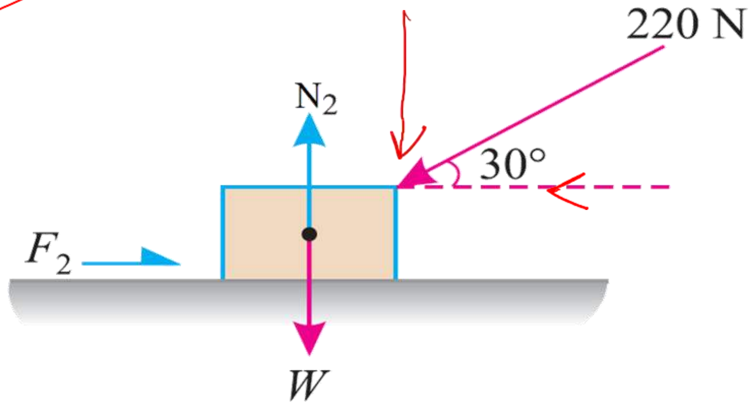
$$P(\cos 25^\circ + 0.3\sin 25) = 0.3 \times 300$$

$$\boxed{P = 87.11\ N}$$ Ans

Determine the weight of the body and the coefficient of friction.



(*a*) Pull of 180 N



(*b*) Pull of 220 N

$F_1 = 180 \cos 30 = 155.9\ N$

$155.9 = \mu N_1 = \mu(W - 180 \sin 30)$

$155.9 = \mu(W - 90)$ — (1)

$F_2 = 220 \cos 30 = 190.52\ N$

$190.52 = \mu N_2 = \mu(W + 220 \sin 30)$

$190.52 = \mu(W + 110)$ (2)

from (1) & (2)

$$\frac{155.9}{190.52} = \frac{\cancel{\mu}(W-90)}{\cancel{\mu}(W+110)}$$

$W = 991 N$

$\mu = 0.173$

Equilibrium of a body on Rough Inclined Plane

Case 1 Force is acting parallel to plane.

(a) motion is up the plane

$$P = W\sin\theta + F = W\sin\theta + \mu N$$

$$= W\sin\theta + \mu(W\cos\theta)$$

$$= W(\sin\theta + \mu\cos\theta) \qquad [\mu = \tan\phi]$$

$$= W(\sin\theta + \tan\phi\cos\theta)$$

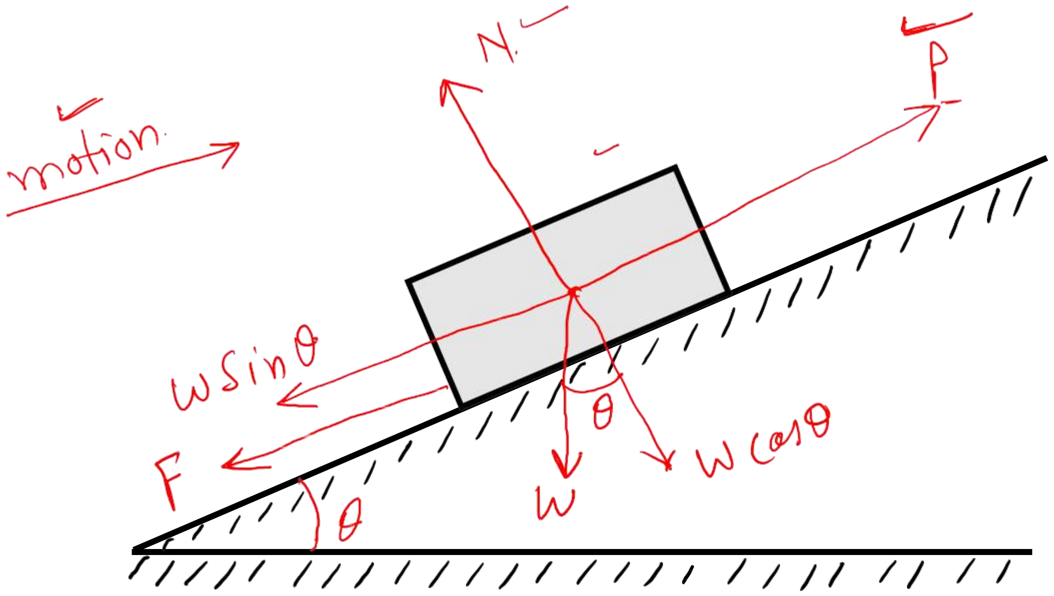
$$= W\left(\sin\theta + \frac{\sin\phi}{\cos\phi}\cos\theta\right)$$

$$= W\frac{(\sin\theta\cos\phi + \sin\phi\cos\theta)}{\cos\phi}$$

$$\boxed{P = W\frac{\sin(\theta+\phi)}{\cos\phi}}$$

[Max force required to maintain equilibrium]

Angle of friction



Equilibrium of a body on Rough Inclined Plane

(b) motion down the plane

$$P + F = W \sin\theta$$

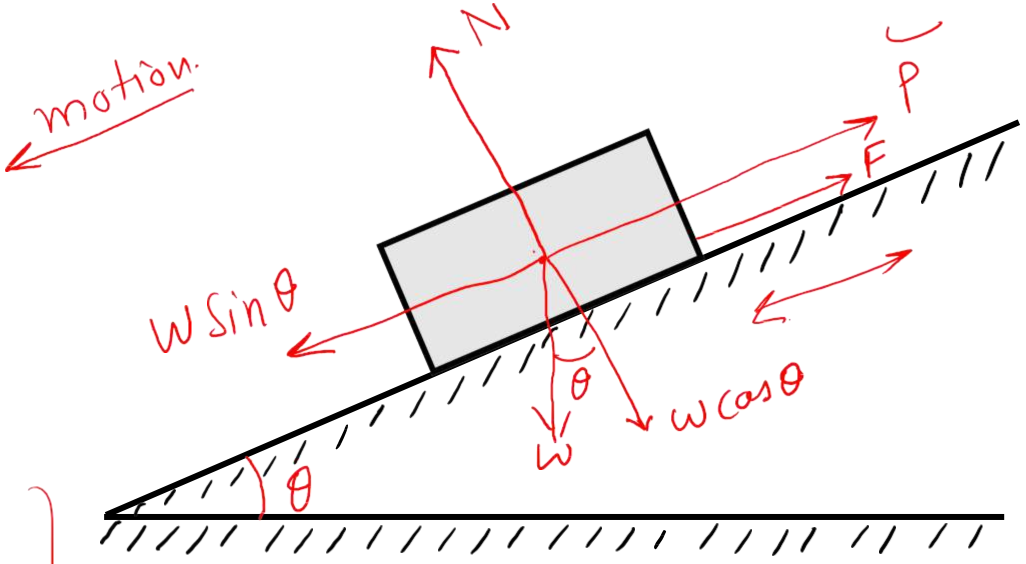
$$P = W\sin\theta - \mu N = W\sin\theta - \mu(W\cos\theta)$$

$$= W(\sin\theta - \mu\cos\theta)$$

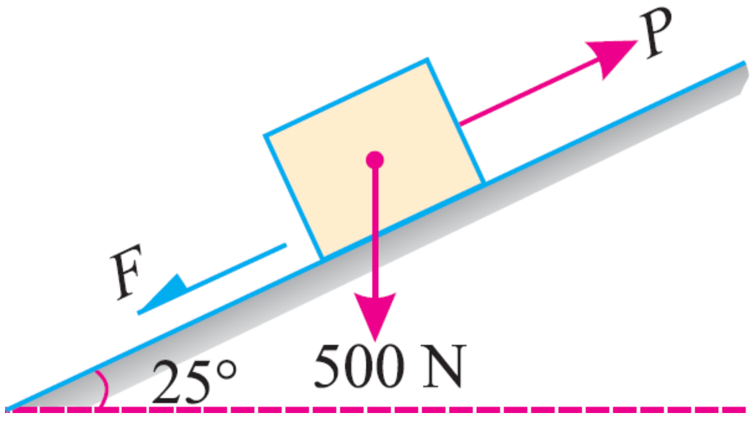
$$= W\frac{(\sin\theta\cos\phi - \sin\phi\cos\theta)}{\cos\phi}$$

$$\boxed{P = W\frac{\sin(\theta - \phi)}{\cos\phi}}$$

[Min. force required to maintain equilibrium]



The body is supported by an effort (P) parallel to the plane. Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°



$\phi = 20°$

Sol:

$$P_{min} = \frac{W(\sin\theta - \phi)}{\cos\phi} = 500\,\frac{\sin(25-20)}{\cos 20}$$

$$\boxed{P_{min.} = 46.4\,N}$$

$$P_{max} = W\,\frac{\sin(\theta+\phi)}{\cos\phi} = 500\,\frac{\sin(25+20)}{\cos 20}$$

$$\boxed{P_{max} = 376.24\,N}$$

Equilibrium of a body on Rough Inclined Plane

Case 2 :- Force is acting Horizontally.

(a) motion up the plane

$(\mu = \tan\phi) \to \frac{\sin\phi}{\cos\phi}$

$P\cos\theta = W\sin\theta + F = W\sin\theta + \mu N.$

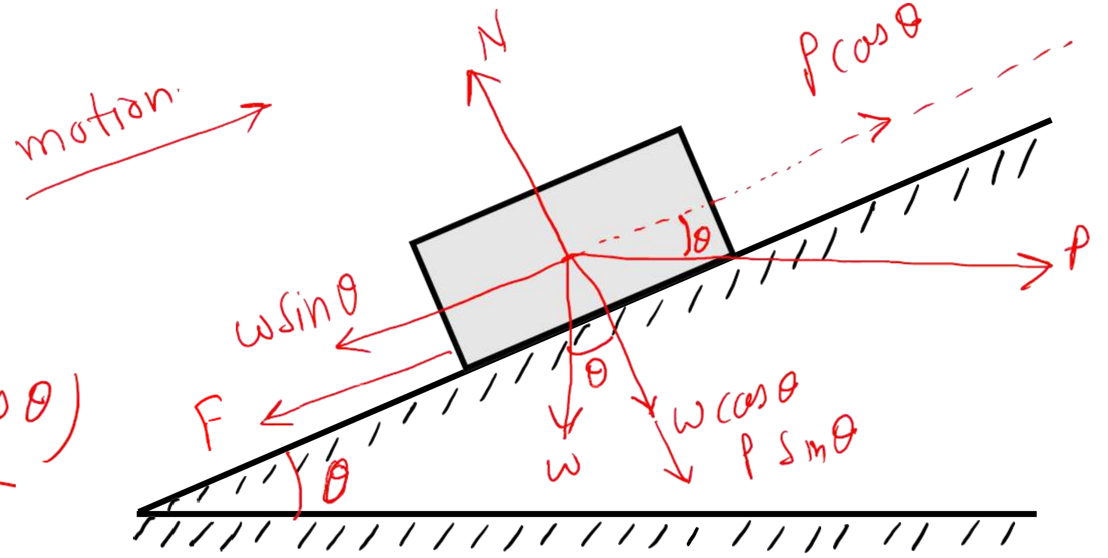
$P\cos\theta = W\sin\theta + \mu(W\cos\theta + P\sin\theta)$

$P(\cos\theta - \mu\sin\theta) = W(\sin\theta + \mu\cos\theta)$

$P\frac{(\cos\theta\cos\phi - \sin\phi\sin\theta)}{\cos\phi} = W\frac{(\sin\theta\cos\phi + \sin\phi\cos\theta)}{\cos\phi}$

$P\cos(\theta+\phi) = W\sin(\theta+\phi)$

$$\boxed{P = W\tan(\theta+\phi)}$$



Equilibrium of a body on Rough Inclined Plane

(b) motion down the plane

$$P\cos\theta + F = W\sin\theta$$

$$P\cos\theta + \mu N = W\sin\theta$$

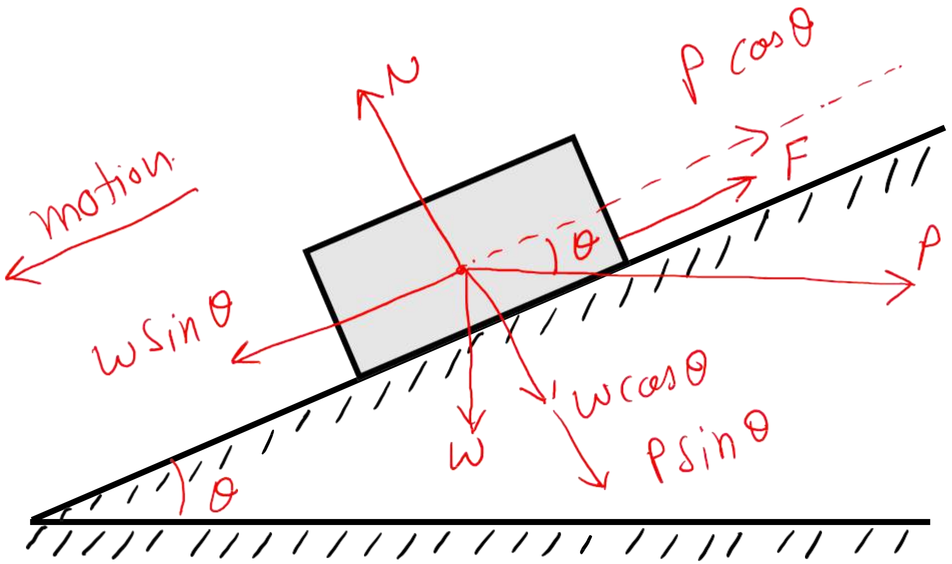
$$P\cos\theta + \mu(W\cos\theta + P\sin\theta) = W\sin\theta$$

$$P(\cos\theta + \mu\sin\theta) = W(\sin\theta - \mu\cos\theta)$$

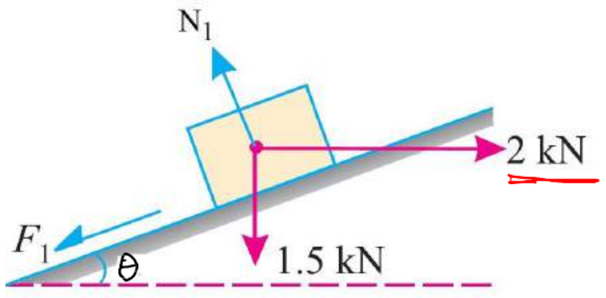
$$P\frac{(\cos\theta\cos\phi + \sin\phi\sin\theta)}{\cos\phi} = W\frac{(\sin\theta\cos\phi - \sin\phi\cos\theta)}{\cos\phi}$$

$$P\cos(\theta-\phi) = W\sin(\theta-\phi)$$

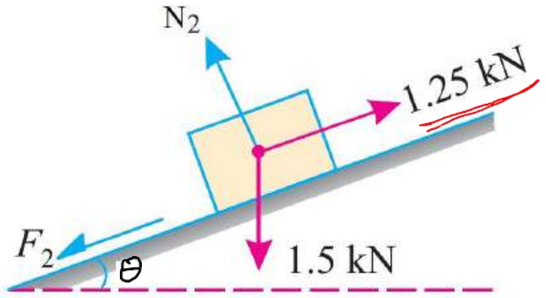
$$\boxed{P = W\tan(\theta-\phi)}$$



A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.



(*a*) Horizontal force



(*b*) Force parallel to the plane

(a) $P = W \tan(\theta + \phi)$

$2 = 1.5 \tan(\theta + \phi)$

$(\theta + \phi) = 53.13$ — (1)

(b) $P = \dfrac{W \sin(\theta + \phi)}{\cos \phi}$

$1.25 = \dfrac{1.5 \sin(53.13)}{\cos \phi}$

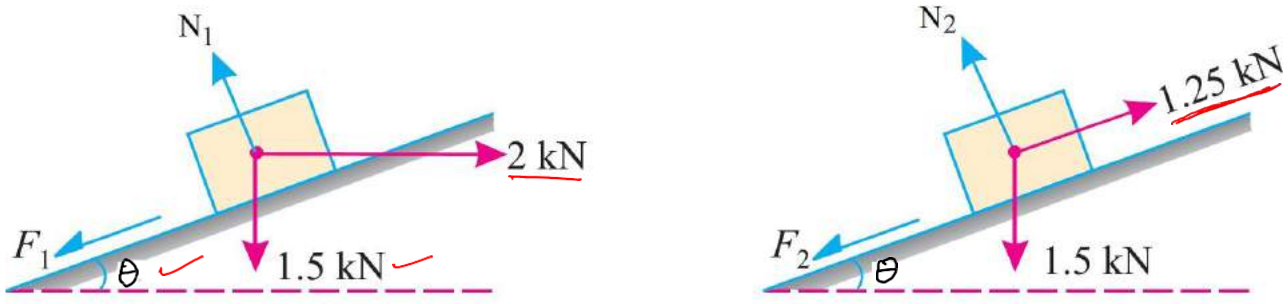
$\boxed{\phi = 16.3^\circ}$

$\boxed{\theta = 36.83^\circ}$ Ans

$\mu = \tan \phi$

$\boxed{\mu = 0.292}$ Ans

A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.



(*a*) Horizontal force (*b*) Force parallel to the plane

Sol: (a) $P = W\tan(\theta+\phi)$

$2 = 1.5\tan(\theta+\phi)$

$(\theta+\phi) = 53.13^\circ$ — (1)

(b) $P = \dfrac{W\sin(\theta+\phi)}{\cos\phi}$

$1.25 = \dfrac{1.5\sin 53.13^\circ}{\cos\phi}$

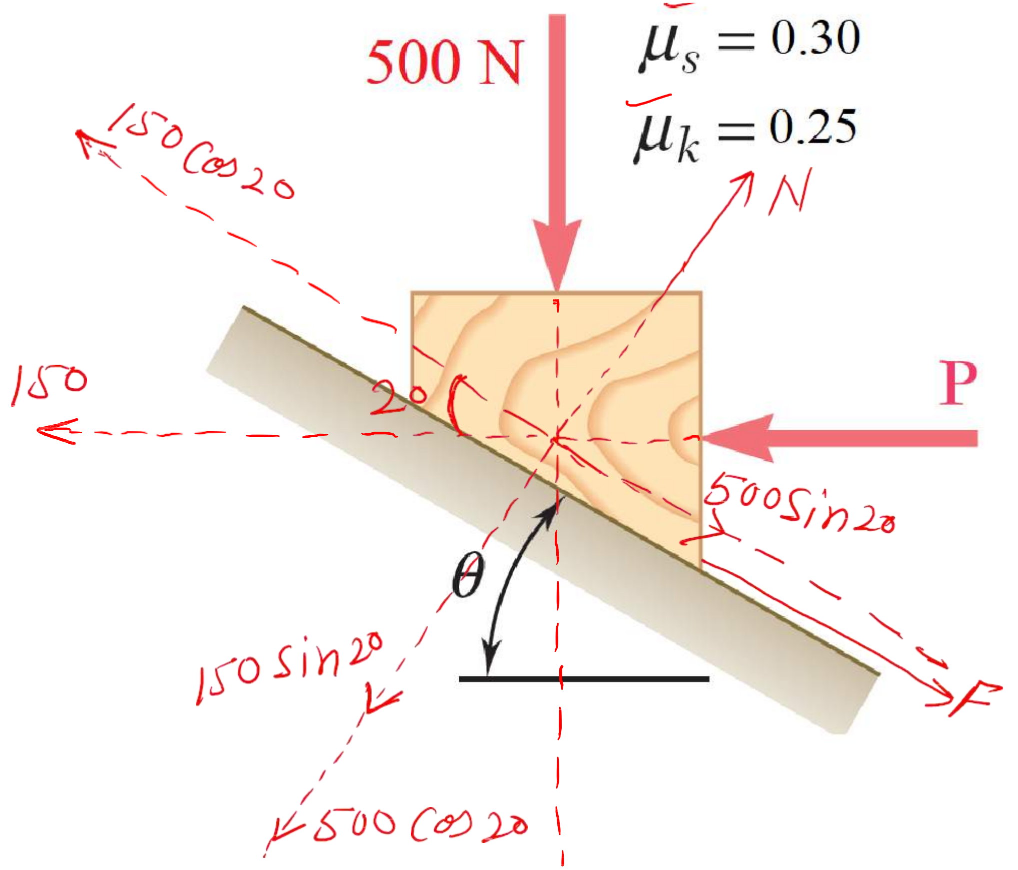
$\phi = 16.3^\circ$

$\theta = 36.83^\circ$ Ans

$\mu = \tan\phi$

$\boxed{\mu = 0.292}$ Ans

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 150$ N and $\theta = 20°$



Sol:

$$F = 150\cos 20 - 500\sin 20 = -30.056\ N$$

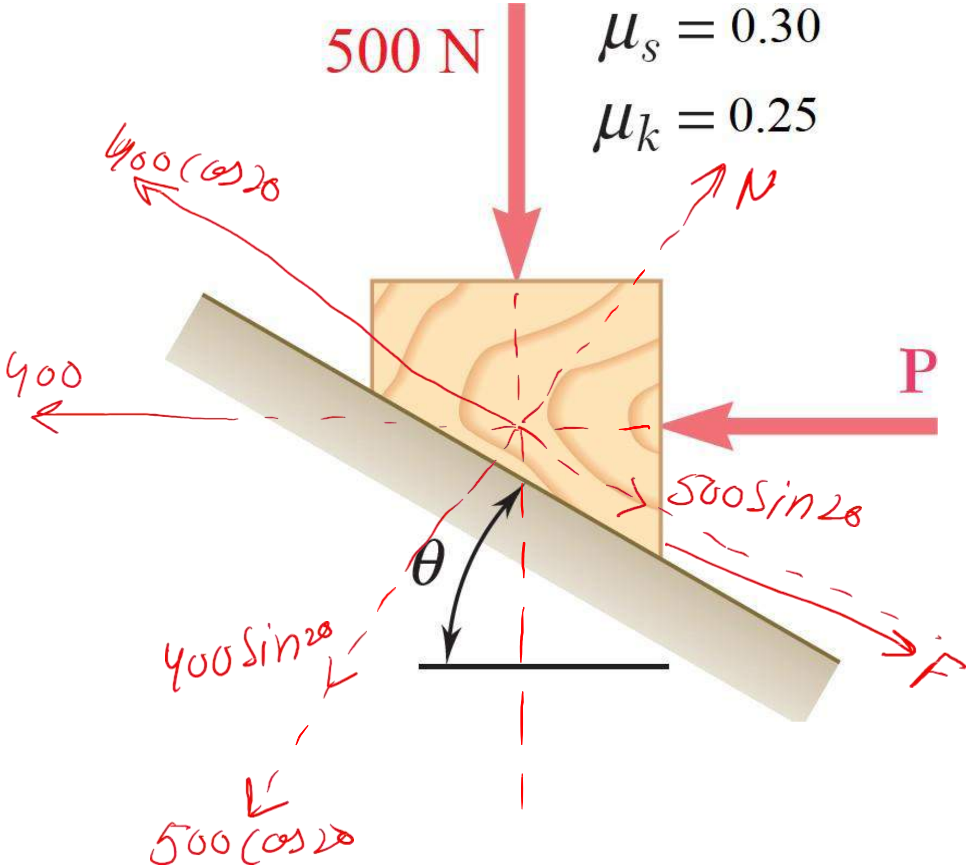
$$F_m = \mu_s N = 0.3(500\cos 20 + 150\sin 20)$$

$$F_m = 156.34\ N$$

$\boxed{F < F_m}$ Block is in equilibrium

$\boxed{F = 30.056\ N}$ Ans

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 400$ N and $\theta = 20°$



Sol:

$$F = 400\cos 20 - 500\sin 20 = 204.87\ N$$

$$F_m = \mu_s N = 0.3\,(500\cos 20 + 400\sin 20)$$

$$F_m = 182\ N$$

$$\boxed{F > F_m}$$

Block is moving up the plane.

$$F_k = \mu_k N = 0.25\,(500\cos 20 + 400\sin 20)$$

$$\boxed{F_k = 151.7N}$$ Ans

Equilibrium of a body on Rough Inclined Plane

Case 3 Force acting inclined to plane.

(a) motion up the plane

$$P\cos\beta = W\sin\theta + F$$

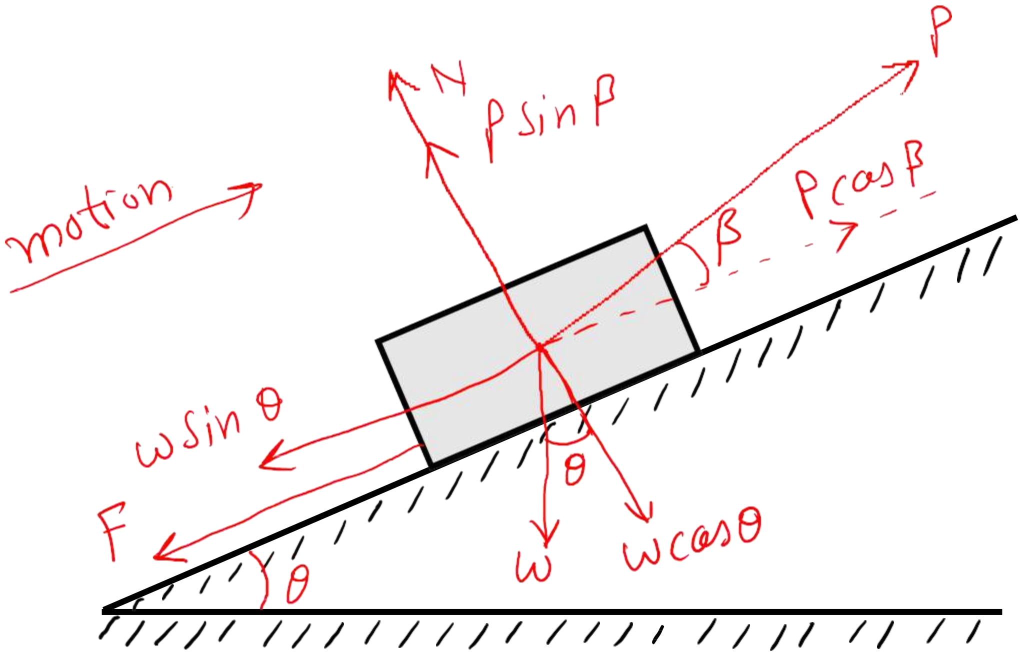
$$P\cos\beta = W\sin\theta + \mu(W\cos\theta - P\sin\beta)$$

$$P(\cos\beta + \mu\sin\beta) = W(\sin\theta + \mu\cos\theta)$$

$$\frac{P(\cos\beta\cos\phi + \sin\phi\sin\beta)}{\cancel{\cos\phi}} = \frac{W(\sin\theta\cos\phi + \sin\phi\cos\theta)}{\cancel{\cos\phi}}$$

$$P\cos(\beta - \phi) = W\sin(\theta + \phi)$$

$$\boxed{P = \frac{W\sin(\theta + \phi)}{\cos(\beta - \phi)}} \; P_{max.}$$



Equilibrium of a body on Rough Inclined Plane

(b) motion down the plane

$$P\cos\beta + F = W\sin\theta$$

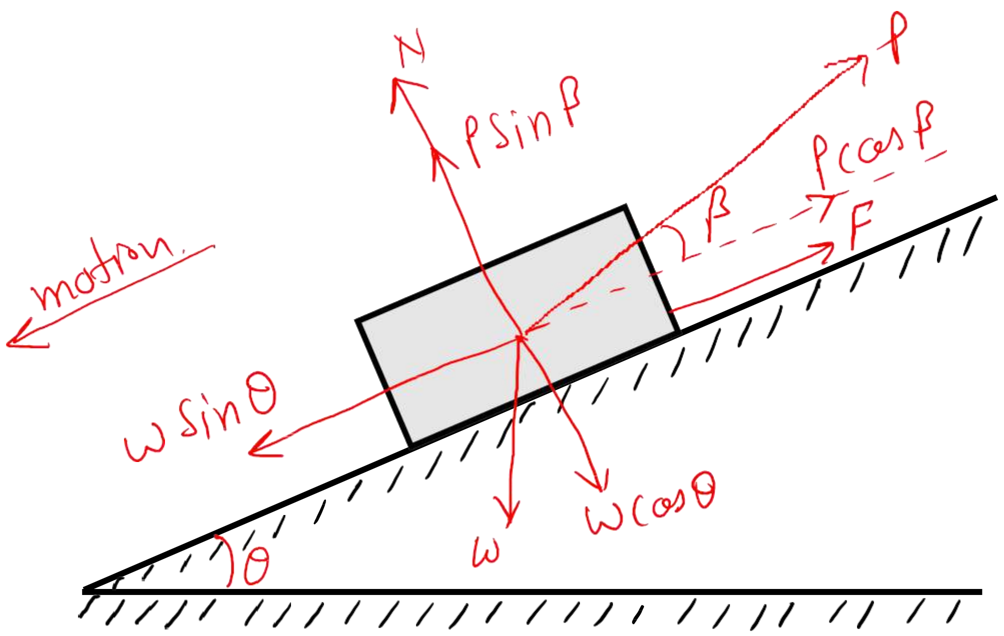
$$P\cos\beta + \mu(W\cos\theta - P\sin\beta) = W\sin\theta$$

$$P(\cos\beta - \mu\sin\beta) = W(\sin\theta - \mu\cos\theta)$$

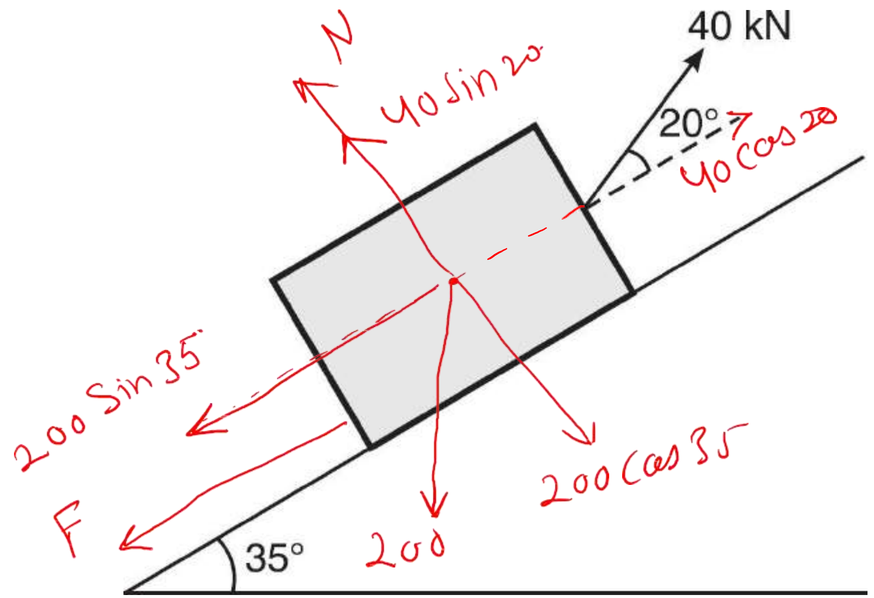
$$\frac{P(\cos\beta\cos\phi - \sin\phi\sin\beta)}{\cos\phi} = \frac{W(\sin\theta\cos\phi - \sin\phi\cos\theta)}{\cos\phi}$$

$$P\cos(\beta+\phi) = W\sin(\theta-\phi)$$

$$\boxed{P = \frac{W\sin(\theta-\phi)}{\cos(\beta+\phi)}}$$ P_{min}



A body of weight 200 kN is acted upon by a force of 40 kN as shown. If the coefficient of friction between the inclined plane and the body is 0.3, check whether the body is in equilibrium or not.



Sol:

$$F = 40\cos 20 - 200\sin 35 = -77.13\text{ KN}$$

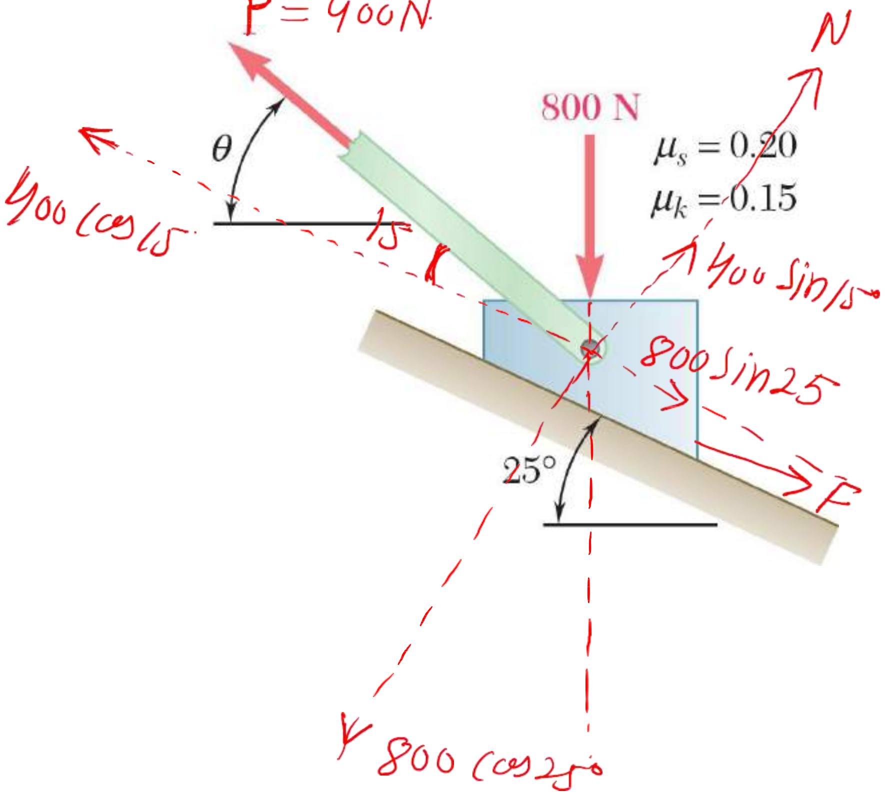
$$F_m = \mu_s N = 0.3(200\cos 35 - 40\sin 20)$$

$$F_m = 45.04\text{ KN}$$

$$\boxed{F > F_m}$$

The block is moving down the plane.

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 40°$ and $P = 400$ N.



Sol:

$$F = 400\cos 15 - 800\sin 25 = 48.27\ \text{N}$$

$$F_m = \mu_s N = 0.2\,(800\cos 25 - 400\sin 15)$$

$$F_m = 124.3\ \text{N}$$

$\boxed{F < F_m} \rightarrow$ equilibrium

$\boxed{F = 48.27\ \text{N} \searrow}$ Ans

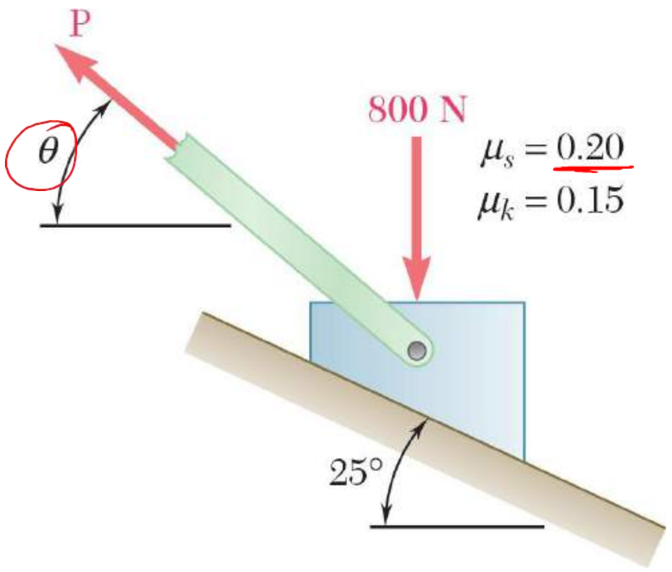
Determine the range of P for which equilibrium can be maintained. Take $\theta = 40°$

Sol: $P_{max} = \dfrac{W \sin(\theta + \phi)}{\cos(\beta - \phi)} = \dfrac{800 \sin(25 + 11.3)}{\cos(15 - 11.3)}$

$\boxed{P_{max} = 474.6\ N}$ Ans

$P_{min} = \dfrac{W \sin(\theta - \phi)}{\cos(\beta + \phi)} = \dfrac{800 \sin(25 - 11.3)}{\cos(15 + 11.3)}$

$\boxed{P_{min} = 211.34\ N}$ Ans

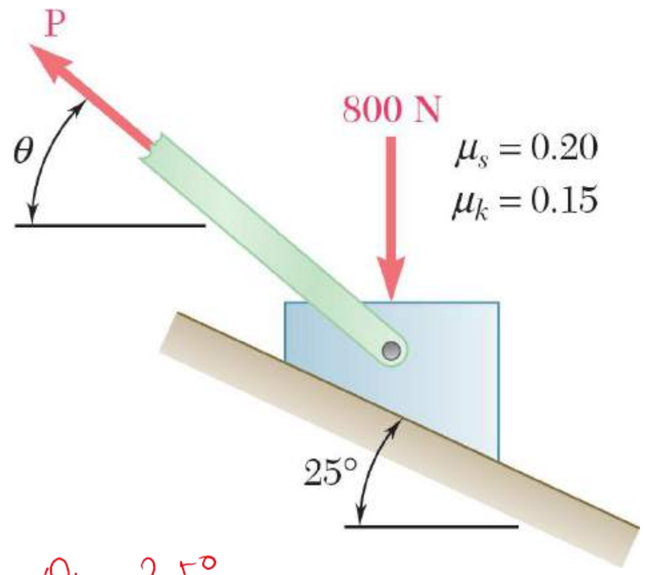


$\theta = 25°$

$\phi = \tan^{-1} \mu_s = 11.3°$

$\beta = 40 - 25 = 15°$

Determine the range of P for which equilibrium can be maintained. Take θ = 40°



$\theta = 25^\circ$

$\phi = \tan^{-1}\mu_s = 11.3^\circ$

$\beta = 40 - 25 = 15^\circ$

Sol

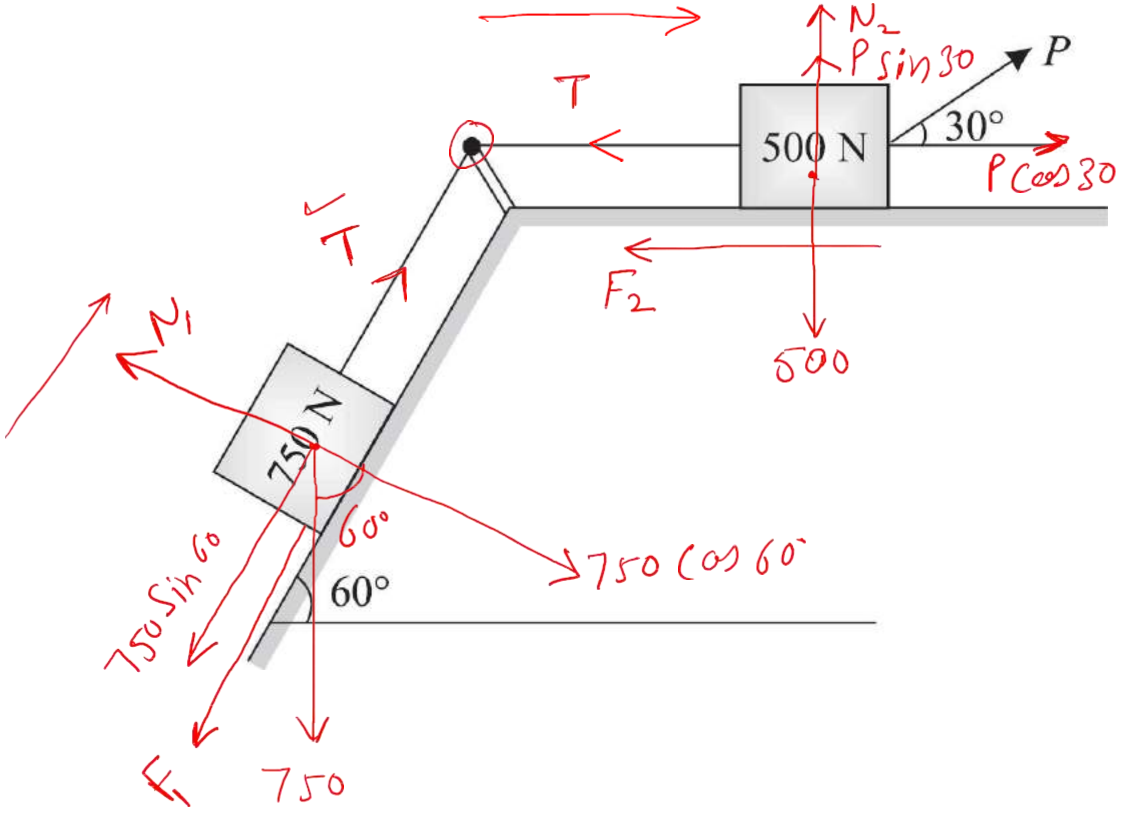
$$P_{max} = \frac{W \sin(\theta + \phi)}{\cos(\beta - \phi)} = \frac{800 \sin(25 + 11.3)}{\cos(15 - 11.3)}$$

$$\boxed{P_{max} = 474.6\ N}$$ Ans

$$P_{min} = \frac{W \sin(\theta - \phi)}{\cos(\beta + \phi)} = \frac{800 \sin(25 - 11.3)}{\cos(15 + 11.3)}$$

$$\boxed{P_{min} = 211.34 N}$$ Ans

What is the value of P in the system to cause the motion to impend? Assume the pulley is smooth and coefficient of friction between the other contact surfaces is 0.2.



Sol: For 750 N block

$N_1 = 750 \cos 60 = 375\,N$

$F_1 = \mu N_1 = 0.2 \times 375 = 75\,N$

$T = 750 \sin 60 + F_1 = 724.5\,N$

For 500 N block.

$N_2 = 500 - P \sin 30 = 500 - 0.5P$

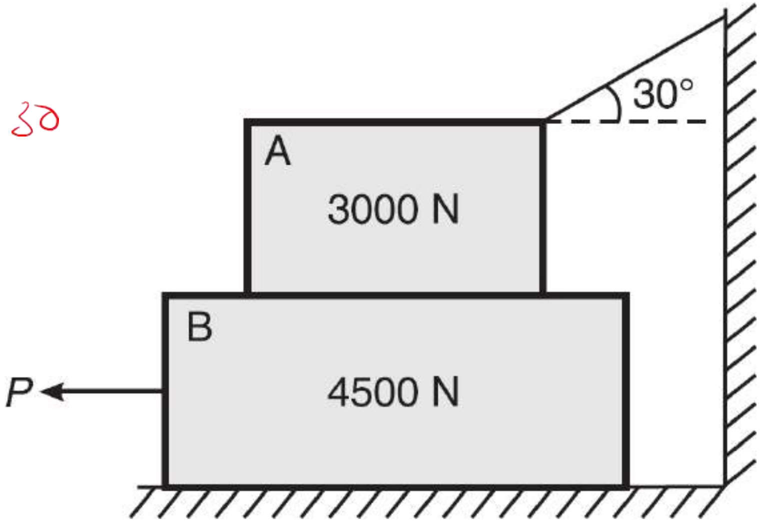
$F_2 = \mu N_2 = 0.2\,(500 - 0.5P)$

$F_2 = 100 - 0.1P$

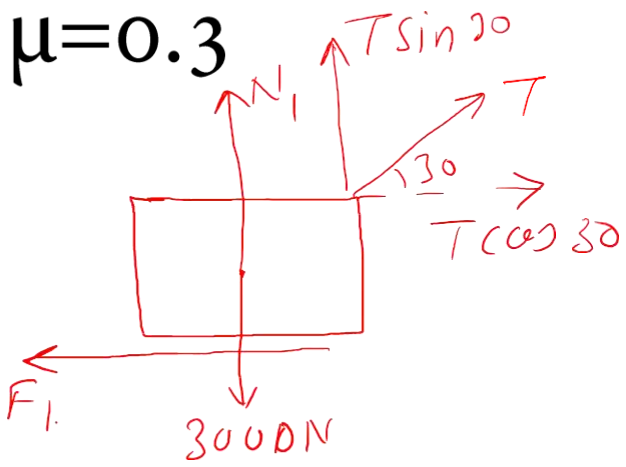
$P \cos 30 = T + F_2 = 724.5 + (100 - 0.1P)$

$\boxed{P = 853.5\,N}$ Ans

Find the horizontal force P required to just move the block to the left. Assume $\mu=0.3$



Sol: 3000 N block is in equilibrium



$$N_1 = (3000 - T\sin 30)$$

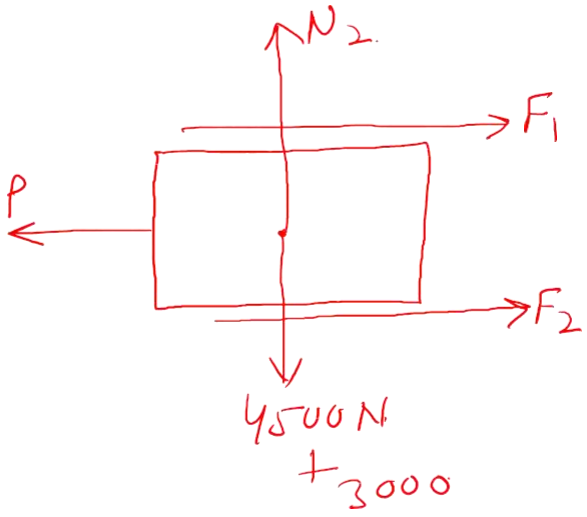
$$\boxed{N_1 = 3000 - 0.5T}$$

$$T\cos 30 = F_1 = 0.3(3000 - 0.5T)$$

$$T = \frac{900}{1.016} = \underline{886\,N} \text{ Ans}$$

$$N_1 = \underline{2557\,N} \; ; \; F_1 = 767.1\,N$$

For block B



$$N_2 = 4500 + N_1 = 7057\,N$$

$$F_2 = \mu N_2 = 2117.1\,N$$

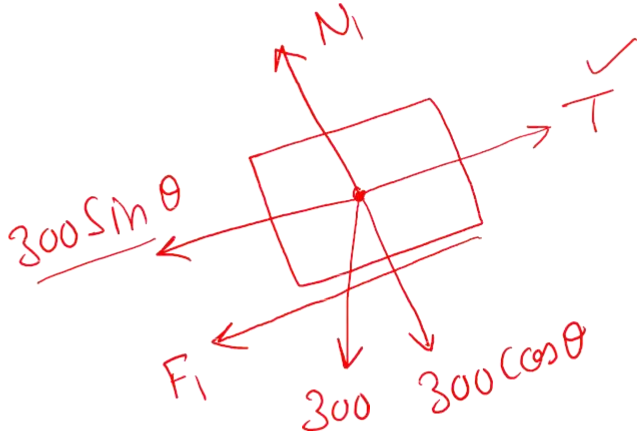
$$P = F_1 + F_2 = 2884.2\,N \text{ Ans}$$

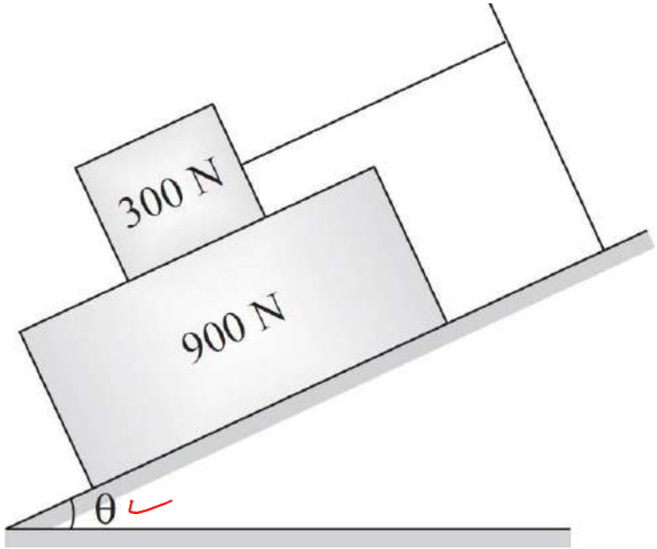
What should be the value of θ which will make the motion of 900 N block down the plane to impend? The coefficient of friction for all contact surfaces is 1/3.

Sol: 300 N block is in equilibrium

$N_1 = 300\cos\theta$

$F_1 = \mu N_1 = \frac{1}{3} \times 300\cos\theta = 100\cos\theta$





For 900 N

$N_2 = 1200\cos\theta$

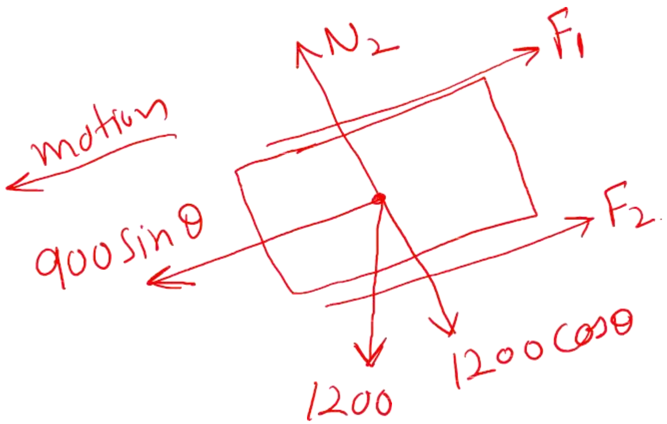
$F_2 = \frac{1}{3} \times 1200\cos\theta = 400\cos\theta$

$900\sin\theta = F_1 + F_2$

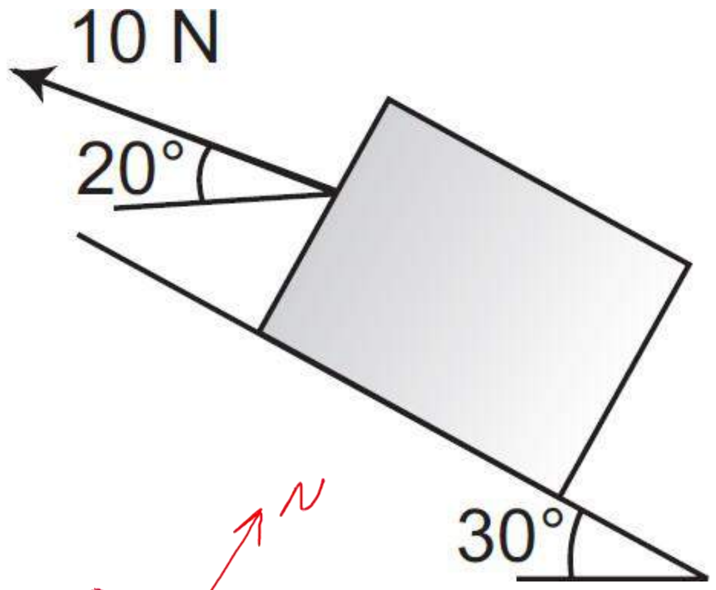
$900\sin\theta = 500\cos\theta$

$\tan\theta = \frac{500}{900}$

$\boxed{\theta = 29.05^\circ}$ Ans



The 10 N force is required to produce impending motion of the block up the inclined surface. If the weight of the block is 5 kg, determine the coefficient of static friction (μ_s) between the block and the inclined surface.



Sol:

$$F = 10\cos 10 - 49.05\sin 30^\circ$$

$$\boxed{F = -14.67\ N}$$

$$F = \mu_s N$$

$$14.67 = \mu_s(49.05\cos 30 + 10\sin 10)$$

$$\boxed{\mu_s = 0.33}$$ Ans