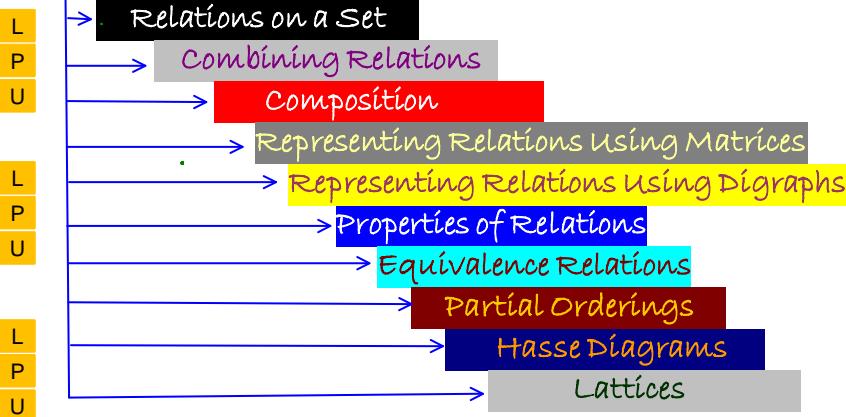


L
P
U

Relation



L Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.

P U $A = \{1, 2, 3, 4\}$ $B = \{0, 5, 10, 15\}$

L P U $A \times B = \{(a, b) \mid a \in A, b \in B\}$

$A \times B = \{(1, 0), (1, 5), (1, 10), (1, 15), (2, 0), (2, 5), (2, 10), (2, 15), (3, 0), (3, 5), (3, 10), (3, 15), (4, 0), (4, 5), (4, 10), (4, 15)\}$

$$\begin{aligned} n(A) &= m \\ n(B) &= n \\ n(A \times B) &= m \cdot n \end{aligned}$$

L P U $R = \{(a, b) \mid a^2 + b^2 \leq 20, \forall a \in A, \forall b \in B\}$

L P U $R = \{(1, 0), (2, 0), (3, 0), (4, 0)\}$ $R \subseteq A \times B$.

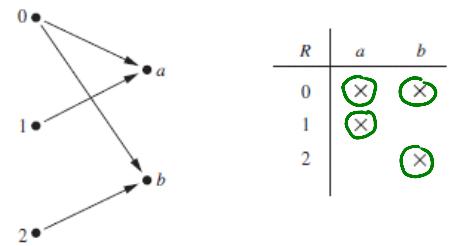
L P U $A = \{a, b, c\}, B = \{1, 2, 3\}$

L P U $R = \{(x_1, x_2) \mid x_1 + x_2 \leq 4, x_1 = \text{Position of elements of set } A \text{ in Alphabet Syst.}$

L P U $x_2 \in B$

L P U $\begin{array}{l} a=1 \\ b=2 \\ c=3 \end{array} R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

L Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. Then $\{(0, a), (0, b), (1, a), (2, b)\}$ is a relation from A to B.

L
P
UL
P
UL
P
U

$$A \times B \neq B \times A, \quad n(A \times B) = n(B \times A)$$

A relation on a set A is a relation from A to A .

In other words, a relation on a set A is a subset of $A \times A$.

$$R \subseteq A \times A$$

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation

$$R = \{(a, b) \mid a \text{ divides } b\}?$$

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

$$\text{If } A \text{ has } n \text{ elements, then } n(A \times A) = n^2$$

$$A = \{\text{AYush, Sourav, Peer, Aditi, Garima}\}$$

$$H = \{5 \cdot 6, 5 \cdot 3, 5 \cdot 7, 5 \cdot 7, 5 \cdot 9\}$$

$$R = \{(a, b) \mid \text{height}(a) - \text{height}(b) = 0.2, \forall a \in A, \forall b \in A\}$$

$$R = \{(Garima, Aditi), (Garima, Peer)\}$$

$$\text{Domain : } a \quad R = \{(a, b)\} \subseteq A \times B$$

$$\text{Range : } b \quad a \in A, b \in B$$

$$\text{No. of relations from set } A \text{ to } B = 2^{m \cdot n}$$

$$m = n(A) \quad A = \{a, b\} \quad A \times B = \{(a, 1), (b, 1)\}$$

$$n = n(B) \quad B = \{1\} \quad R_1 = \{(a, 1)\}$$

$$R_2 = \{(b, 1)\}$$

$$R_3 = \{(a, 1), (b, 1)\}$$

$$R_4 = \emptyset$$

$$\text{How many relations are there on a set with } n \text{ elements?} = 2^n$$

$$Q: \text{If } A = \{a, b, c\}$$

$$\text{Total No. of relations from } A \text{ to } A = 2^9$$

Q: Let set A has 5 elements & set B has 6 elements
 then cardinalities (No. of elements)

$$n(A \times B) = 5 \cdot 6 = 30 \rightarrow \text{No. of relation from } A \text{ to } B = 2^{30}$$

$$n(A \times A) = 25 \quad \text{No. of relation from } B \text{ to } B = 2^{36}$$

$$n(B \times B) = 36 \quad \text{No. of relation from } A \text{ to } B = 2^{30}$$

Combining Relations

L Because relations from A to B are subsets of $A \times B$, two relations from A to
 P B can be combined in any way two sets can be combined.
 U

$$R_1 \cap R_2, \quad R_1 \cup R_2, \quad R_1 - R_2, \quad R_2 - R_1$$

L $R_1 = \{(a,a), (a,b), (b,c), (b,b)\}$

P $R_2 = \{(a,b), (a,d), (b,d), (d,d)\}$

U $\underline{R_1 \cup R_2} = \{(a,a), (a,b), (b,c), (b,b), (a,d), (b,d), (d,d)\}$

L $\underline{R_1 \cap R_2} = \{(a,b)\}$

P $R_1 - R_2 = \{(a,a), (b,c), (b,b)\}$

U $R_2 - R_1 =$

L $R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2$
 P $= \{(a,a), (b,c), (b,b), (a,d), (b,d), (d,d)\}$

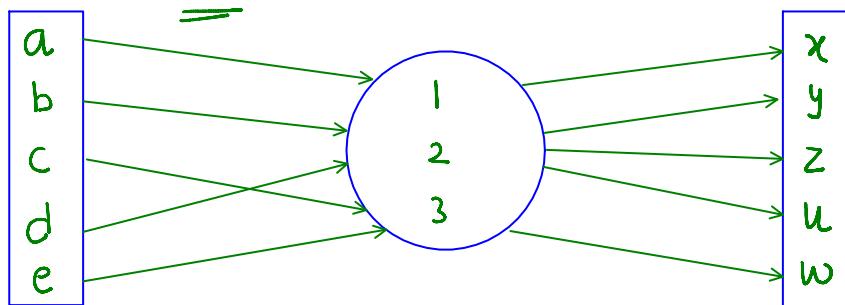
U

Composition

L Let R be a relation from a set A to a set B and S a relation from B to a
 P set C. The composite of R and S is the relation consisting of ordered
 U pairs (a, c) , where $a \in A, c \in C$, and for which there exists an element b
 in B such that $(a, b) \in R$ and $(b, c) \in S$. We denote the composite of R

L and S by $\underline{S \circ R}$
 P $S \circ R = \{(1,1)(b,2)(c,3)(d,2)(e,3)\}$
 U

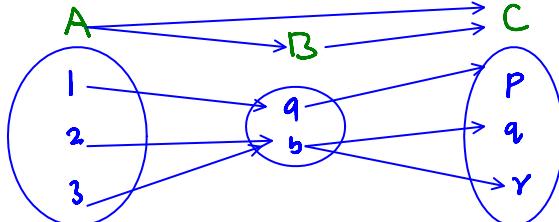
$$S = \{(1,n)(2,y)(2,z)(2,u)(3,w)\}$$

L
P
UL
P
U

$$S \circ R = (a, x) (b, y) (b, z) (b, u) (c, w) (d, y) (d, z) (d, u) (e, w)$$

$\overset{R}{\curvearrowleft}$ $\overset{S}{\curvearrowleft}$

$$(1, P) (2, a) (2, r) (3, a) \\ (3, r)$$



$$R_1 = \{(1, a) (2, a) (3, b)\}$$

$$R_2 = \{(a, P) (b, a) (b, r)\}$$

Composing the Parent Relation with Itself

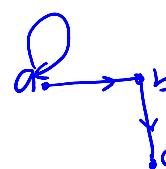
$$R = \{(a, a) (a, b) (b, c)\}$$

$$R = R^2 = R^3 = \dots = R^n$$

$$R_1 = \{(a, b)\}$$

$$R_2 = \{(b, 1) (b, 2)\}$$

$$R_2 \circ R_1 = \underline{(a, 1)} \underline{(a, 2)}$$



Representing Relations Using Matrices

Representing Relations Using Digraphs

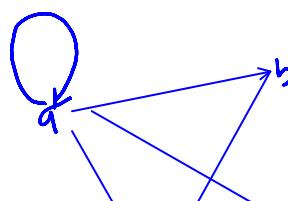
$$M_R = \begin{cases} 1 & \text{if } a R b \quad \forall a \in A, b \in B \\ 0 & \text{if } a \not R b \quad \forall a \in A, b \in B \end{cases}$$

A directed graph, or digraph, consists of a set V of vertices (or nodes) together with a set E of ordered pairs of elements of V called edges (or arcs).

The vertex a is called the initial vertex of the edge (a, b) , and the vertex b is called the terminal vertex of this edge.

$$A = \{a, b, c, d\}$$

$$R = \{(a, b), (a, c), (a, a'), (a, d), (b, c)\}$$



	a	b	c	d
a	1	1	1	1
b	0	0	1	0
c	0	0	0	0

L
P
U

L
P
U # Two cities are related if they have less distance b/w them

L
P
U

$$\text{dist}(n) = \begin{cases} 1 & \text{if } \text{dist}(x_1, x_2) \leq 20 \text{ Km.} \\ \frac{(100-n)}{80} & \text{if } 20 \text{ Km.} < \text{dist}(x_1, x_2) \leq 100 \\ 0 & \text{if } \text{dist}(x_1, x_2) > 100 \end{cases}$$

L
P
U

$$\text{dist}(a, b) = \text{dist}(a, c) \leq 20 \text{ Km.}$$

L
P
U

$$\frac{50}{100} = 0.5 = \frac{100-n}{80} \Rightarrow \frac{1}{2} = \frac{100-n}{80} \Rightarrow 40 = 100-n$$

L
P
U

$$n = 60 \text{ Km.}$$

L
P
U

$$A = 100$$

$$B \leq 100$$

$$C > 100$$

L
P
U Suppose that the relations R_1 and R_2 on a set A are represented by the matrices

L
P
U

$$R_1 = \begin{pmatrix} a & b & c \\ 1 & 0 & 1 \\ b & 0 & 0 \\ c & 0 & 1 \end{pmatrix}, \quad R_2 = \begin{pmatrix} a & b & c \\ 1 & 0 & 1 \\ b & 0 & 1 \\ c & 1 & 0 \end{pmatrix}$$

L
P
U What are the matrices representing $R_1 \cup R_2$, $R_1 \cap R_2$ and $R_2 \circ R_1$?

L
P
U

$$R_1 \cup R_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}, \quad R_1 \cap R_2 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

L
P
U

$$R_1 = \{(a,a), (a,c), (b,a), (c,b)\}$$

$$R_2 = \{(a,a), (a,c), (b,b), (b,c), (c,a)\}$$

$$R_2 \circ R_1 = \begin{vmatrix} a & b & c \\ 1 & 0 & 1 \\ b & 1 & 0 \\ c & 0 & 1 \end{vmatrix}$$

L
P
U Suppose that the relation R on a set is represented by the matrix

MR =

L
P
U 110

L
P
U 111

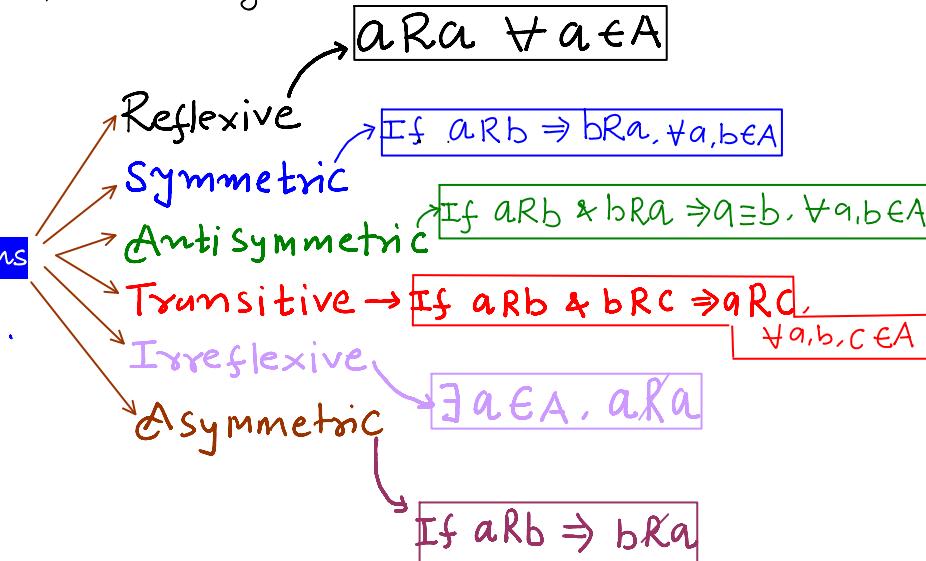
L
P
U 011

L
P
U

Is R reflexive, symmetric, and/or antisymmetric?

L
P
ULet $a, b, c \in A$ L
P
U

Properties of Relations

L
P
UL
P
U

Reflexive : Let R be the relation defined on set A then it is said to be reflexive if $aRa \quad \forall a \in A$

L
P
U

$$A = \{1, 2, 3, 4\}, R = \{(1,1) (1,2) (2,2) (3,3) (4,4)\}$$

L
P
U

$$R = \{(1,1) (1,2) (2,2) (3,3) (2,1)\} \text{ not reflexive}$$

L
P
U

$\exists a \in A$ s.t. aRa say $\underline{\underline{4}}$

L
P
U

$$A = \{1, 2, 3\} \quad R = \{(a, b) \mid a^2 + b^2 \leq 8\}$$

L
P
U

$$R = \{(1,1) (1,2) (2,1) (2,2)\}$$

not reflexive $(3,3) \notin R$

L
P
U

Symmetric : Let R be a relation defined on set A & it is said to be symmetric if aRb then $bRa \quad \forall a, b \in A$

L
P
U

$$A = \{\text{Ayush, Sourav, Peer, Aditi, Granna}\}$$

$$R = \{s.c \quad s.z \quad s.7 \quad s.f \quad s.g\}$$

O - One to one, many to one, universal

L
P

L
P
U

$$R = \{(a, b) \mid \text{height}(a) - \text{height}(b) = 0.2, \forall a \in A, \forall b \in A\}$$

R = {(Garima, Aditi), (Garima, Peer)}

L
P
U

not reflexive, not symmetric

$$\text{Garima}(H) - \text{Aditi}(H) = 0.2$$

$$\text{Aditi}(H) - \text{Garima}(H) \neq 0.2$$

L
P
U

$$A = \{a, b, c\} \quad R = \{(a, a), (b, b), (a, b), (c, c), (b, a)\}$$

Reflexive yes ✓

Symmetric yes ✓

L
P
U

$$S = \{\text{Ayush}, \text{Garima}, \text{Aditi}\}$$

L
P
U

$$R = \{(a, b) \mid a \text{ can Spend } \geq 1000 \text{ on } b, \forall a, b \in S\}$$



L
P
U

Antisymmetric Let R be the relation defined on set A then it is said to be antisymmetric if aRb & bRa, then a ≡ b

Hypo.

L
P
U

If $aRb \& bRa$ then $a \equiv b$

I	II	Con.
aRb & bRa		$a \equiv b$

L
P
U

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

L
P
U

Q: $A = \{a, b, c\} \quad R = \{(a, a), (a, b), (b, b)\}$

L
P
U

 reflexive No, $(c,c) \notin R$
 symmetric. No $(b,a) \notin R$
 Antisymmetric yes if $aRa \& aRb \Rightarrow a = b$
 if $bRb \& bRb \Rightarrow b = b$
 if $aRb \& \boxed{\quad} \Rightarrow$
 Hypo. Incomp.

L
P
U $A = \{1, 2, 3, 4\}$ $R = \{(1,1), (2,2), (3,3), (4,4)\}$

Reflexive - yes

Symmetric - yes

Antisymmetric yes

if $1R1 \& 1R1 \Rightarrow 1 = 1$

Same for 2, 3 & 4,

L
P
U $S = \{1, 2, 3\}$ $R = \{(1,1), (1,2), (2,1)\}$

Reflexive - NO $(2,2) \notin R$

Symmetric - yes

Antisymmetric - NO $(1,2)$ $1 = 1, (1,2) 1 \neq 2$

if $\boxed{1R2 \& 2R1} \Rightarrow 1 \neq 2$

Hypo. Present Con. Absent

L
P
U

 L
P
U $A = \{a, b, c, d\}$ $R = \{(a,b), (a,a), (b,b), (b,c)\}$

Reflexive, NO. $(c,c) \notin R$

Symmetric NO $(b,a) \notin R$

Antisymmetric: (a,b) , if $aRb \& \boxed{\quad} \Rightarrow$
 Incomplete Hypo.

(a,a) if $\underline{aRa} \& \underline{aRb} \Rightarrow a = a$

(b,b) if $bRb \& bRb \Rightarrow b = b$

(b,c) , if $bRc \& \boxed{\quad} \Rightarrow$

L
P
U

yes

CR5

L P U Transitive : Let R be a relation defined on a set A . & if
 $aRb \text{ & } bRc \Rightarrow aRc$
L P Hypo. Con.

$$L \quad A = \{a, b, c\} \quad R = \{(a, b)\}$$

- reflexive - NO
 - symmetric - NO
 - Anti symmetric - yes if $aRb \wedge bRa \Rightarrow$
 - transitive - yes if $aRb \wedge bRc \Rightarrow aRc$

$$L \quad A = \{1, 2, 3, 4\} \quad R = \left\{ \left(\begin{smallmatrix} 1 & 1 \\ \underline{\underline{1}} & 1 \end{smallmatrix} \right), \left(\begin{smallmatrix} 1 & 2 \\ \underline{\underline{1}} & 2 \end{smallmatrix} \right), (2, 3), (3, 1) \right\}$$

- Ref x
 - Symm x
 - Antisymm ✓
 - Transitive

If  if $1 \rightarrow 2 \rightarrow 3$

(1.1) Hypo ✓ Conv ✓ +reas.

(1.2) Hypo ✓ C.n. x not. trans.

Not tragic

$A = \{ \text{All the Student of KIACJ} \}$

$$R = \{(a, b) \mid a \text{ is older than } b\}$$

Refl
 Symm

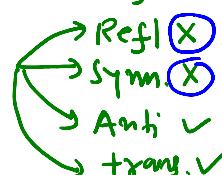
Say Age of AYush 20 yr

Satyanarayana

Lovish 19 yr.

Granma 18 yrs

Aditi 17 yr



$$18 > 17 > 16$$

$$R = \{(Aryush, Satyam) (Aryush, Lovish) (Aryush, Garima) \\ (Aryush, Aditi) (Satyam, Aditi) (Lovish, Satyam) \\ (Lovish, Garima) (Lovish, Aditi) (Garima, Aditi)\}$$

Q: $A = \{x \mid x \in \mathbb{N}\}$

$$R = \{(a, b) \mid a|b \wedge a, b \in A\}$$

$$R = \{(1, 1) (1, 2) (1, 3) \dots , \\ (2, 2) (2, 4) (2, 6) \dots , \\ ; \}$$

Reflexive - Yes every No. divides it self

Symmetric - No Counter example $1R2$ but $2R1$

Transitive - yes

Antisymmetric - yes

Irreflexive - No If (x, y) & $\boxed{}$ \Rightarrow

Asymmetric - yes

$$(x, y) (y, z) (z, x) (z, y) \\ x \neq z$$

GATE | GATE-CS-2009 | Question 4

Consider the binary relation $R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$.

Which one of the following is TRUE?

- (A) R is symmetric but NOT antisymmetric \times not Symm. $(y, x) \notin R$
- (B) R is NOT symmetric but antisymmetric \times if $x R y \& \boxed{y \neq z} \Rightarrow x = y$
- (C) R is both symmetric and antisymmetric \times if $x R y \& \boxed{y R z} \Rightarrow x = z$
- (D) R is neither symmetric nor antisymmetric \checkmark if $x R z \& \boxed{z R x} \Rightarrow x \neq z$ not Anti

GATE | GATE CS 1999 | Question 1

The number of binary relations on a set with n elements is:

- (A) n^2
- (B) 2^n
- (C) ~~2^{n^2}~~
- (D) None of the above

Counting of Relation

Let a set has n elements then the no. of different relation on set A are

Relation	2^{n^2}	$n = 10$
		2^{100}

Relation	2^{n^2}	2^{100}
Reflexive	$2^{n(n-1)}$	2^{90}
Symmetric	$2^{\frac{n(n+1)}{2}}$	2^{55}
Antisymmetric	$2^n \cdot 3^{\frac{n(n-1)}{2}}$	
Asymmetric	$3^{\frac{n(n-1)}{2}}$	
Irreflexive	$2^{n(n-1)}$	

Equivalence Relations:

A relation on a set A is called an equivalence relation if it is reflexive, symmetric, and transitive.

Partial Orderings:

A relation R on a set S is called a partial ordering or partial order if it is reflexive, antisymmetric, and transitive.

A set S together with a partial ordering R is called a partially ordered set, or poset, and is denoted by (S, R) . Members of S are called elements of the poset.

Which of these relations on $\{0, 1, 2, 3\}$ are equivalence/partial ordered relations?

- a) $\{(0,0), (1,1), (2,2), (3,3)\}$
 Reflexive Symmetric Antisymmetric Transitive

E	P
✓	✓

- b) $\{(0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3)\}$
 not reflexive so not Equi. not Partial

X	X
---	---

- c) $\{(0,0), (1,1), (1,2), (2,1), (2,2), (3,3)\}$
 reflexive, symmetric, not Antisym, transitive

✓	X
---	---

- d) $\{(0,0), (1,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,3)\}$
 reflexive, symmetric, not Antisym, not trans.

X	X
---	---

- e) $\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,2), (3,3)\}$
 reflexive, not symmetric, not Anti, not transitive

X	X
---	---

Which of these relations on the set of all people are equivalence/partial ordered Relations?

- a) $\{(a,b) \mid a \text{ and } b \text{ are the same age}\}$
 reflexive, symmetric, Antisymmetric, transitive

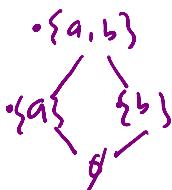
✓	X
---	---

- b) $\{(a,b) \mid a \text{ and } b \text{ have the same parents}\}$

✓	X
---	---

c) $\{(a, b) \mid a \text{ and } b \text{ speak a common language}\}$
 reflexive, symmetric, not Anti-symmetric, transitive \checkmark \times

$$A = \{(a, b), \subseteq\}$$



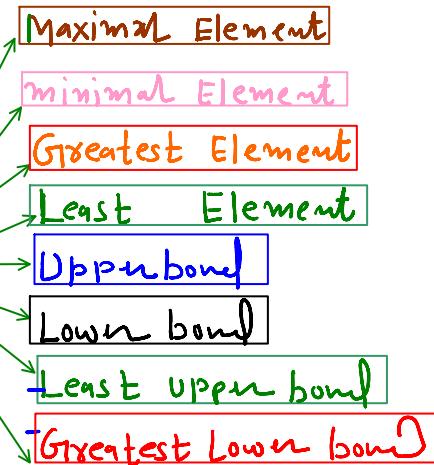
Hasse Diagrams of POSET

$$S = \{2, 3, 6\} \checkmark$$

$$R = \{(a, b) \mid a | b\} \checkmark$$

$$R = \{(2, 1), (2, 2), (3, 1), (3, 2), (6, 1), (6, 2)\}$$

$$\text{Poset} = \{(2, 3, 6), |\}\checkmark$$



Consider an arbitrary poset, $\langle X, \leq \rangle$ (not necessarily finite). Given any element, $a \in X$, the following situations are of interest:

For no $b \in X$ do we have $b < a$. We say that a is a **minimal element** (of X).

For no $b \in X$ do we have $a < b$. We say that a is a **maximal element** (of X).

An element, $b \in X$, is the **least element of A** iff $b \in A$ and $b \leq a$ for all $a \in A$.

An element, $m \in X$, is the **greatest element of A** iff $m \in A$ and $a \leq m$ for all $a \in A$.

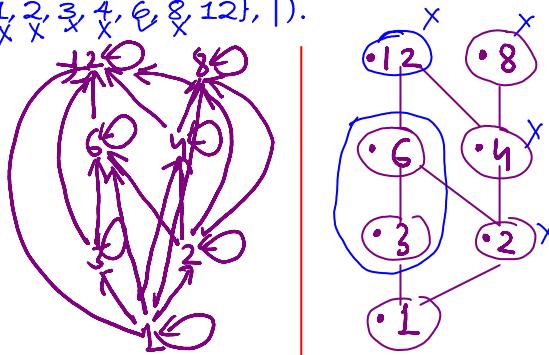
An element, $m \in X$, is an **upper bound of A** iff $a \leq m$ for all $a \in A$.

be any subset of X . An element, $b \in X$, is a **lower bound of A** iff $b \leq a$ for all $a \in A$.

An element, $b \in X$, is the **greatest lower bound of A** iff the set of lower bounds of A is nonempty and if b is the greatest element of this set.

An element, $m \in X$, is the **least upper bound of A** iff the set of upper bounds of A is nonempty and if m is the least element of this set.

Hasse diagram of $(\{1, 2, 3, 4, 6, 8, 12, 24\}, |)$.



$$\text{Maximal} = \{12, 8\}, \text{ Greatest} = \emptyset$$

$$\text{minimal} = \{1\} \quad \text{least} = \{1\}$$

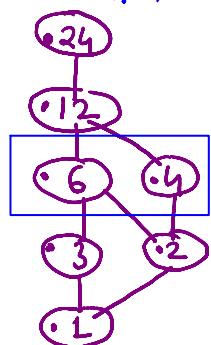
$$\text{Upper bound}(6, 3) = \{6, 12\}$$

$$\text{Lower bound}(6, 3) = \{3, 1\}$$

$$\text{Least Upper bound of } \{6, 3\} = \text{Least}\{6, 12\} = \{6\}$$

$$\text{Greatest Lower bound of } \{6, 3\} = \text{greatest}\{3, 1\} = \{3\}$$

$$(S, \leq) = \{(1, 2, 3, 4, 6, 12, 24), |\}\checkmark$$



$$\text{Maximal} = \{24\}$$

$$\text{Minimal} = \{1\}$$

$$\text{greatest} = \{24\}$$

$$\text{least} = \{1\}$$

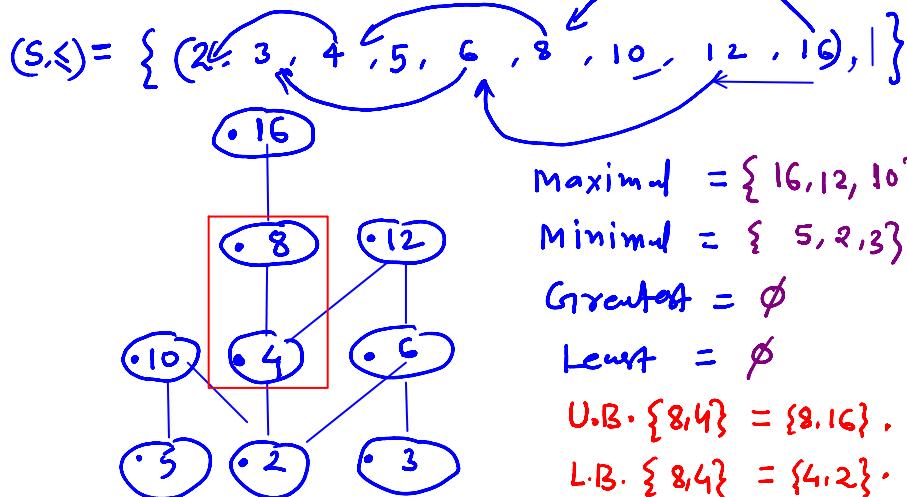
$$\text{Upper bound of } \{6, 4\} = \{12, 24\}$$

$$\text{Lower bound of } \{6, 4\} = \{2, 1\}$$

$$\text{Least Upper bound of } \{6, 4\} = \text{Least}\{12, 24\} = \{12\}$$

$$\text{Greatest Lower bound of } \{6, 4\} = \text{greatest}\{2, 1\} = \{2\}$$

$=\{2\}$



Maximal = $\{16, 12, 10\}$

Minimal = $\{5, 2, 3\}$

Greatest = \emptyset

Least = \emptyset

$$U.B. \{8, 4\} = \{8, 16\}, L.U.B. \{8, 4\} = \{8\}$$

$$L.B. \{8, 4\} = \{4, 2\}, G.L.B. \{8, 4\} = \{4\}$$

$$(S, \leq) = \{(a, b, c), \leq\}$$

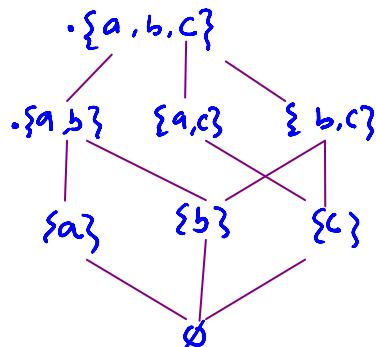
$$\text{Max} = \{\{a, b, c\}\}$$

$$\text{Min} = \emptyset$$

$$G = \{\{a, b, c\}\}$$

$$L = \emptyset$$

Upper bound
of $\{\{a, b, c\}, \{a, c\}\}$
is $\{a, b, c\}$



$$(\{1, 2, 3, 4\}, \leq).$$

$$\text{Max} = \{4\}$$

$$\text{min} = \{1\}$$

$$\text{Great.} = \{4\}$$

$$\text{Least} = \{1\}$$

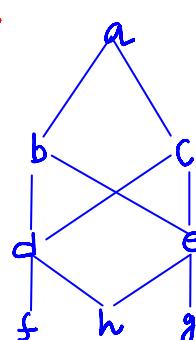
4
3
2
1

$$\text{Upper bound of } \{4, 3, 2\} = \{4\}$$

$$\text{Lower bound of } \{4, 3, 2\} = \{2\}$$

$$\text{Least Upper bound of } \{4, 3, 2\} = \{4\}$$

$$\text{greatest Lower bound of } \{4, 3, 2\} = \{2\}$$



a) Find the maximal elements. $\{a\}$

b) Find the minimal elements. $\{f, g, h\}$

c) Is there a greatest element? $\{a\}$

d) Is there a least element? No

e) Find all upper bounds of $\{a, b, c\}$. $\{a\}$

f) Find the least upper bound of $\{a, b, c\}$, if it exists. $\{a\}$

g) Find all lower bounds of $\{f, g, h\}$. \emptyset

h) Find the greatest lower bound of $\{f, g, h\}$, if it exists. \emptyset



Maximal

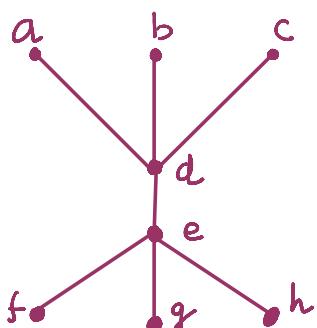
The elements of poset which are not having any further path in upward direction

minimal

The elements of poset which does not have any further path in downward direction

Greatest \rightarrow Unique element, If more than 1 maximal then NO greatest & If Single maximal then it is the greatest

*Least \rightarrow Unique element, If more than 1 minimal then NO least & If Single minimal then it is the least



Maximal = $\{a, b, c\}$

Minimal = $\{f, g, h\}$

Greatest = \emptyset

Least = \emptyset

Upper bound of $\{a, b, c\} = \emptyset$

Lower bound of $\{a, b, c\}$
= $\{d, e, f, g, h\}$

Greatest lower bound $\{a, b, c\} = \{d\}$

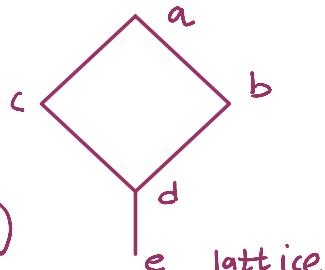
Least upper bound $\{a, b, c\} = \emptyset$

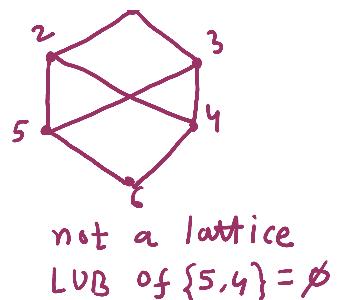
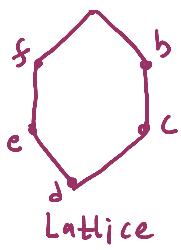
Lattices

A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice.



a
b
c
not a lattice
(More than 1 maximal)
LUB $\{a, b\} = \emptyset$





Determine whether the posets $(\{1, 2, 3, 4, 5\}, |)$ and $(\{1, 2, 4, 8, 16\}, |)$ are lattices.