DIGITAL ELECTRONICS: ECE 213

Topic: Number System

UNIT: I

Lecture No.: 1

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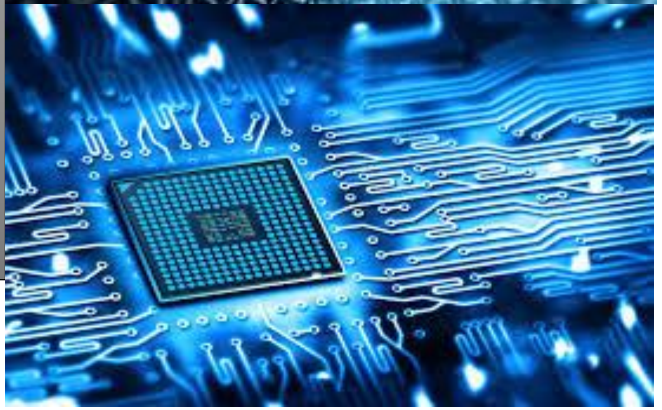
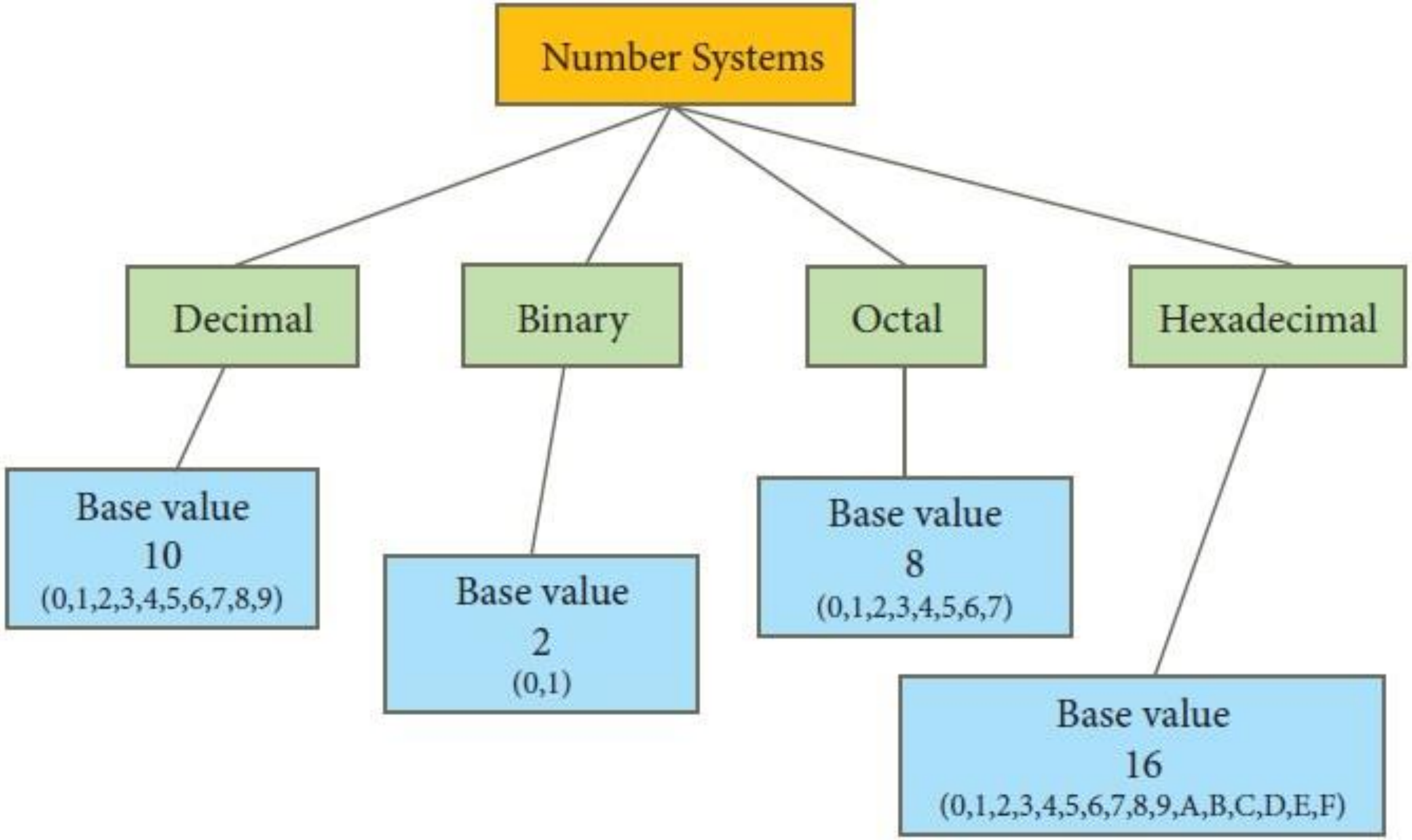


Types of Number System



Number System: Base Conversion with Fractions

1. Decimal to Binary Conversion:

Step 1: Continuously multiply by 2 until fractional part is 0.

or

Multiply by 2 until you observe that the fractional part is either repeating or never ending.

Step 2: Note down the integer part corresponding to each multiplication.

Step 3: The pattern of 1's and 0's at the integer column represented from top to bottom represents the fractional part into an equivalent binary number.

$Convert(0.6875)_{10}$ *to binary?*

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

Number System: Base Conversion with Fractions

1. Decimal to Binary Conversion:

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or

Multiply by 2 until you observe that the fractional part is either repeating or never ending.

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Step 3: The pattern of 1's and 0's at the integer column represented from top to bottom represents the fractional part into an equivalent binary number.

Convert $(0.4674)_{10}$ *to binary?*

$$\begin{array}{r} 0.4674 \\ \underline{\times 2} \\ 0.9348 \\ \underline{\times 2} \\ 1.8696 \\ \underline{\times 2} \\ 1.7392 \\ \underline{\times 2} \\ 1.4784 \\ \underline{\times 2} \\ 0.9568 \\ \underline{\times 2} \\ 1.8136 \end{array}$$

The binary equivalent of $(0.4674)_{10}$ is $(.011101)_2$

Try Yourself

Q1. Convert the following into Binary equivalents?

$a)$ $(0.65)_{10}$

$b)$ $(34.625)_{10}$

Number System: Base Conversion with Fractions

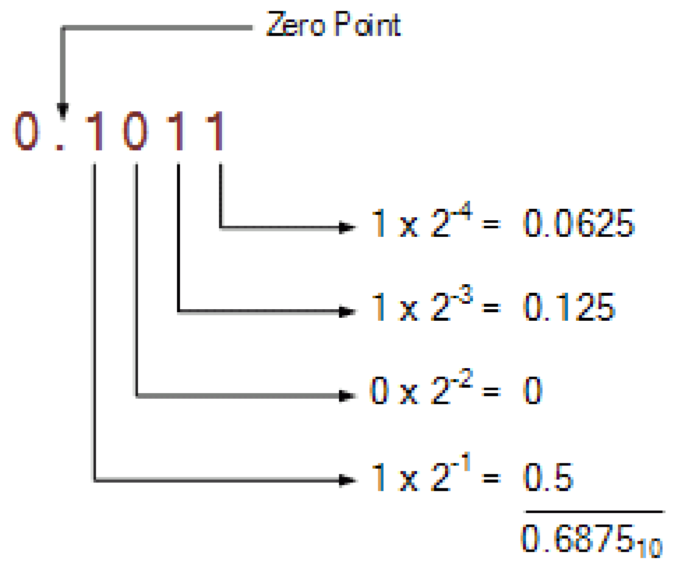
2. Binary to Decimal Conversion:

Step 1: Multiply with the powers of 2^{-n} where n=1,2,3,… starting from the MSB.

Step 2: Add all the decimal numbers corresponding to the coefficients.

Step 3: The sum obtained would be the equivalent decimal number representation of the given binary number.

Convert $(0.1011)_2$ *to decimal?*



Number System: Base Conversion with Fractions

2. Binary to Decimal Conversion:

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Step 2: Add all the decimal numbers corresponding to the coefficients.

Step 3: The sum obtained would be the equivalent decimal number representation of the given binary number.

Convert $(0.11011)_2$ *to decimal?*

$$(0.11011)_2 = 1\times 2^{-1} + 1\times 2^{-2} + 0\times 2^{-3} + 1\times 2^{-4} + 1\times 2^{-5}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{0}{8} + \frac{1}{16} + \frac{1}{32}$$

$$= 0.5 + 0.25 + 0 + 0.0625 + 0.03125$$

$$= (0.84375)_{10}$$

Try Yourself

Q1. Convert the following into Decimal equivalents?

$a)\ (1011.10011)_2$

$b)\ (1100101.001101)_2$

Types of Number System

3. Octal Number System:

- The octal number system has a radix of 8 and therefore has eight distinct digits.
- The independent digits are 0, 1, 2, 3, 4, 5, 6 and 7.
- The place values for the different digits in the octal number system are $8^0, 8^1, 8^2$, and so on (for the integer part) and $8^{-1}, 8^{-2}, 8^{-3}$, and so on (for the fractional part).

Types of Number System

4. Hexadecimal Number System:

- The hexadecimal number system is a radix-16 number system.
- The independent digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.
- The place values for the different digits in the hexadecimal number system are $16^0, 16^1$, 16^2, and so on (for the integer part) and $16^{-1}, 16^{-2}, 16^{-3}$, and so on (for the fractional part).
- The decimal equivalent of A, B, C, D, E and F are 10, 11, 12, 13, 14 and 15 respectively.

Types of Number System: COMPARIOSN

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

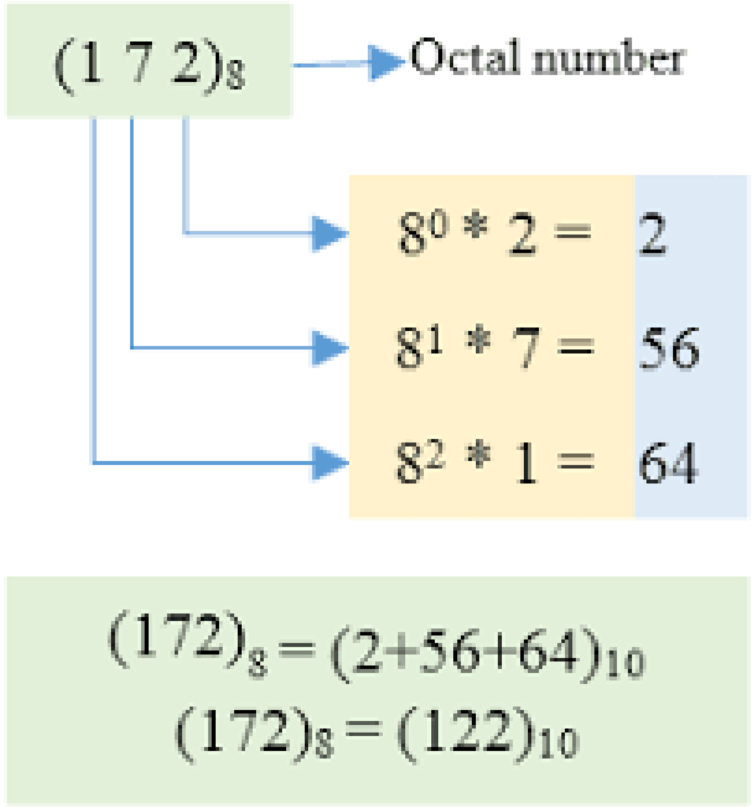
Number System: Base Conversion

3. Octal to Decimal:

- The method is quite similar to the transformation of any binary number to decimal, the only difference is that in this case the 2 will be replaced by 8 and everything else i.e. the methods will remain same.

Convert (214)8 into decimal

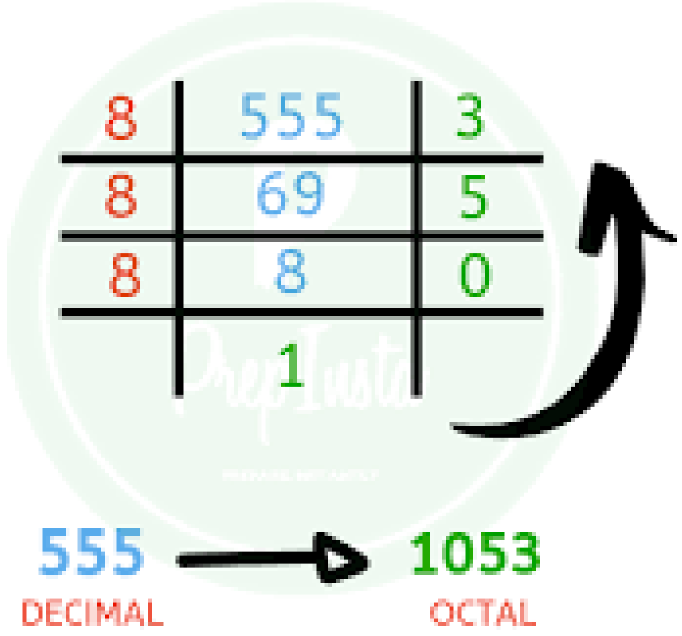
- We have to start multiplying the digits of the number from LSB side with increasing powers of 8 staring from 0 and finally summing up all the products.

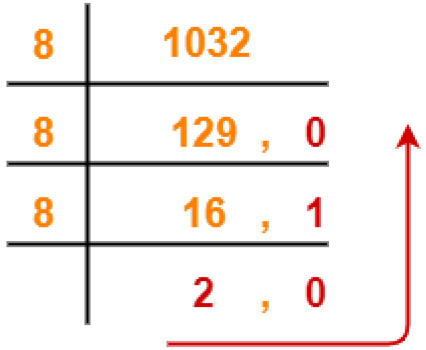


Number System: Base Conversion

4. Decimal to Octal :

- The method is quite similar to the transformation of any decimal number to binary, the only difference is that in this case the 2 will be replaced by 8 and everything else i.e. the methods will remain same.





QUICK QUIZ (POLL)

Q. Convert the following to decimal: $(214)_8$

a) 140
b) 141
c) 142
d) 130

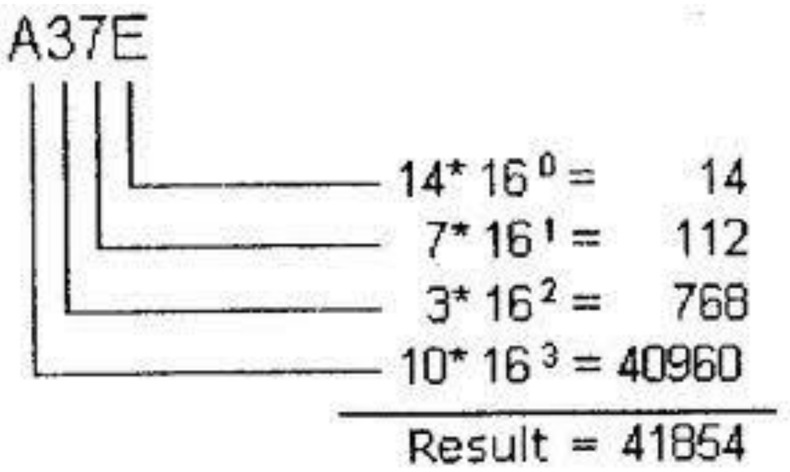
Practice Question

Q. Convert the following to octal: $(0.513)_{10}$

Number System: Base Conversion

5. Hexadecimal to Decimal:

- The method is quite similar to the transformation of any binary number to decimal, the only difference is that in this case the 2 will be replaced by 16 and everything else i.e. the methods will remain same.
- we have to start multiplying the digits of the number from LSB side with increasing powers of 16 staring from 0 and finally summing up all the products.



QUICK QUIZ (POLL)

Q. Convert the following to decimal: $(1\mathrm{E}2)_8$

a) 480

b) 483

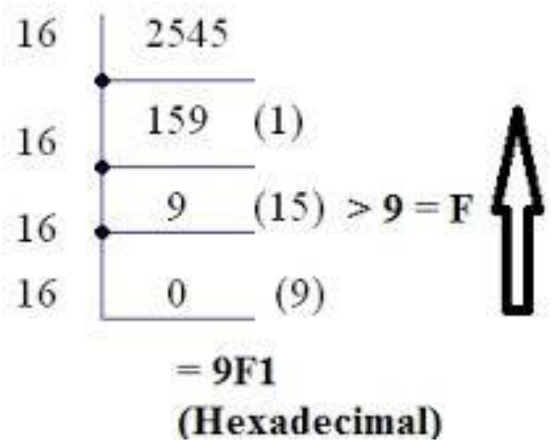
c) 482

d) 484

Number System: Base Conversion

6. Decimal to Hexadecimal:

- The method is quite similar to the transformation of any binary number to decimal, the only difference is that in this case the 2 will be replaced by 16 and everything else i.e. the methods will remain same.



QUICK QUIZ (POLL)

Q. Convert the following to hexadecimal: $(571)_{16}$

$a)$ $B32$

b) 23B

c) 23A

d) 240

Number System: Base Conversion

7,8. Octal and Hexadecimal into Binary:

STEPS:

1. **Convert each digit individually into binary.**
2. **Use 3 bits for Octal and 4 bits for Hexadecimal.**

$$\begin{array}{lccccccc} (673.124)_8 = & (110 & 111 & 011 & \cdot & 001 & 010 & 100)_2 \\ & 6 & 7 & 3 & & 1 & 2 & 4 \end{array}$$

and

$$\begin{array}{lccccc} (306.D)_{16} = & (0011 & 0000 & 0110 & \cdot & 1101)_2 \\ & 3 & 0 & 6 & & D \end{array}$$

Number System: Base Conversion

9,10. Binary into Octal and Hexadecimal :

The conversion from binary to octal is easily accomplished by partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group. The following example illustrates the procedure:

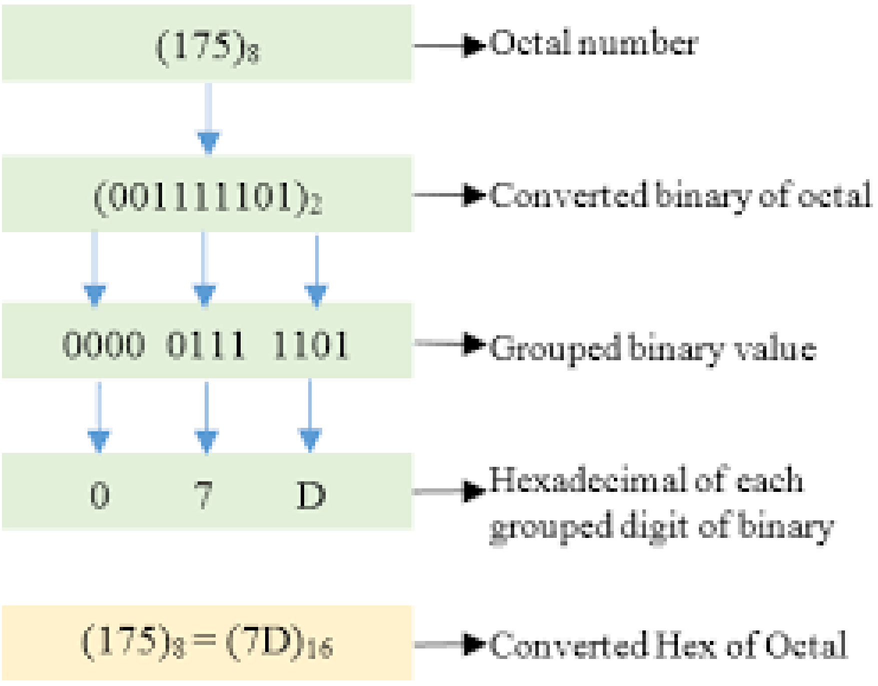
$$\begin{array}{cccccccccc} (10 & 110 & 001 & 101 & 011 & \cdot & 111 & 100 & 000 & 110)_2 = (26153.7406)_8 \\ 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6 \end{array}$$

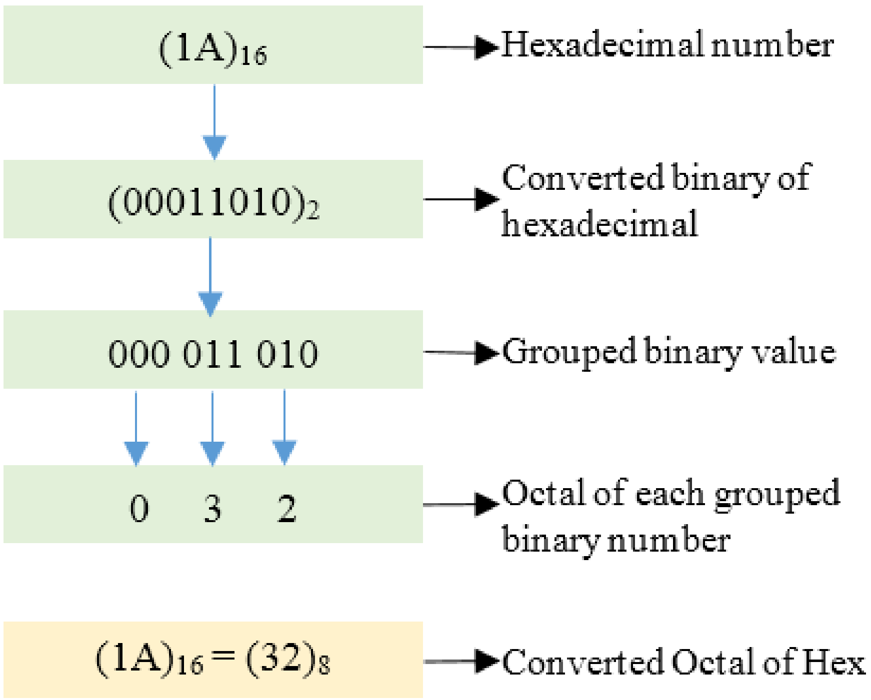
Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of *four* digits:

$$\begin{array}{ccccccc} (10 & 1100 & 0110 & 1011 & \cdot & 1111 & 0010)_2 = (2C6B.F2)_{16} \\ 2 & C & 6 & B & & F & 2 \end{array}$$

Number System: Base Conversion

11,12. Octal to Hexadecimal and vice-versa.:





Try Yourself

Q1. Convert the following in other three base systems?

$a)\ (250)_{10}$

$b)\ (266)_{10}$

Explanation Slide

a) $(250)_{10}$
b) $(266)_{10}$

Try Yourself

Q2. Convert the following in other three base systems?

a) $(10110)_2$

b) $(10011.10111)_2$

Explanation Slide

a) $(10110)_2$
b) $(10011.10111)_2$

QUICK QUIZ (POLL)

Q. Convert the following to octal: $(10111011001)_2$

$a)$ 5661

b) 5662

c) 2731

d) 05D9

Try Yourself

Q3. Convert the following in other three base systems?

$a)$ $(123)_8$

$b)$ $(462.625)_8$

Explanation Slide

$a)$ $(123)_8$
$b)$ $(462.625)_8$

Try Yourself

Q4. Convert the following in other three base systems?

$a)\ (ECE)_{16}$

$b)\ (1E9.C2)_{16}$

Explanation Slide

a) $(ECE)_{16}$
b) $(1E9.C2)_{16}$

DIGITAL ELECTRONICS: ECE 213

Topic: Complement of Numbers

UNIT I: Number Systems

Lecture No.: 2

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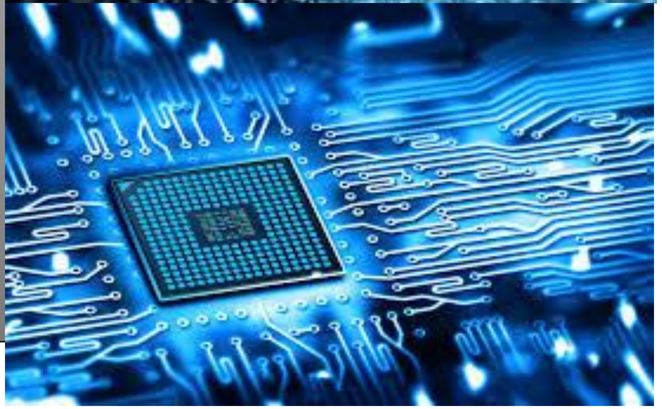


Binary Addition

❑RULES:

Addition		Result	Carry
0 + 0	=	0	0
0 + 1	=	1	0
1 + 0	=	1	0
1 + 1	=	0	1

Example:

Binary Addition

	Binary	*Decimal*
A	110011	51
B	1101	13
A + B	1000000	64

Binary Addition

❑RULES:

Addition		Result	Carry
0 + 0	=	0	0
0 + 1	=	1	0
1 + 0	=	1	0
1 + 1	=	0	1

Try Yourself

11001.101 + 101100.011

Binary Subtraction

❑RULES:

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Example:

X		229		1 1 1 0 0 1 0 1
Y	-	46	-	0 0 1 0 1 1 1 0
		183		1 0 1 1 0 1 1 1

❑RULES:

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Try Yourself

36 -12 in Binary

QUICK QUIZ (POLL)

Q: 011010 - 001100

a) 11110
b) 01110
c) 01010
d) 01101

Complement of Numbers

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.
- Leads to simpler, less expensive circuits to implement the operations.

Complement of Numbers

Complements

Radix Complement
(r’s complement)

Diminished Radix Complement
{(r-1)’s complement}

10’s complement

2’s complement

8’s complement

16’s complement

9’s complement

1’s complement

7’s complement

F’s complement

Complement of Numbers

Diminished Radix Complement	Radix Complement
Given a number N in base r having n digits, the (r - 1)’s complement of N, i.e., its diminished radix complement, is defined as: $$(\boldsymbol{r^n} - \mathbf{1}\ ) - \boldsymbol{N}$$	Given a number N in base r having n digits, the (r - 1)’s complement of N, i.e., its diminished radix complement, is defined as: $$\boldsymbol{r^n} - \boldsymbol{N}$$

Complement of Numbers

9’s complement

2- Find the 9’s complement of 546700 and 12389

The 9’s complement of 546700 is 999999 - 546700= 453299

and the 9’s complement of 12389 is 99999- 12389 = 87610.

	9	9	9	9	9	9
-	5	4	6	7	0	0
	4	5	3	2	9	9

		9	9	9	9	9
-		1	2	3	8	9
		8	7	6	1	0

10’s complement

Find the 10’s complement of 546700 and 12389

The 10’s complement of 546700 is 1000000 - 546700= 453300

and the 10’s complement of 12389 is

100000 - 12389 = 87611.

Notice that it is the same as 9’s complement + 1.

	1	0	0	0	0	0	0
-		5	4	6	7	0	0
		4	5	3	3	0	0

	1	0	0	0	0	0
-		1	2	3	8	9
		8	7	6	1	1

Complement of Numbers

QUICK QUIZ (POLL)

Q: 10’s complement of 6384 is:

a) 3615

b) 3616

c) 10000

d) 1000

QUICK QUIZ (POLL)

Q: 9’s complement of 012398 is:

a) 87601

b) 87602

c) 987601

d) 987602

Complement of Numbers

1's complement

For example:

- if $n = 4$, we have $2^4 = 10000_2$ and $2^4 - 1 = 1111_2$. Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1.
- However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively.

Therefore, the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.

The 1's complement of 1011000 is 0100111.

The 1's complement of 0101101 is 1010010.

2's complement

Example:

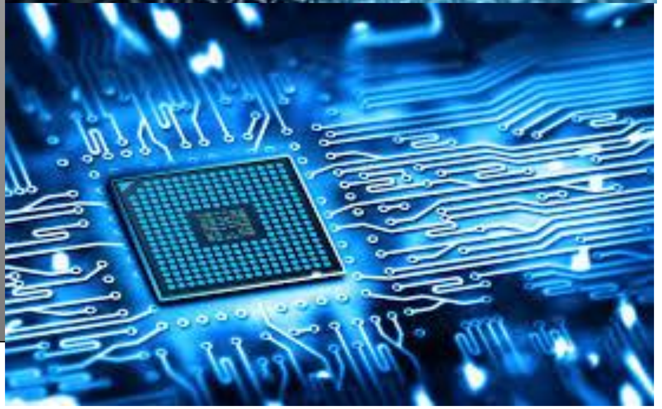
The 2's complement of binary 101100 is 010011 + 1 = 010100 and is obtained by **adding 1 to the 1's-complement value**

DIGITAL ELECTRONICS: ECE 213

Topic: Complement of Numbers

UNIT I: Number Systems

Lecture No.: 3



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Binary Addition

❑RULES:

Addition		Result	Carry
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0 + 1	=	1	0
1 + 0	=	1	0
1 + 1	=	0	1

Try Yourself

11001.101 + 101100.011

Binary Subtraction

❑RULES:

A	B	A-B	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Example:

$$
\begin{array}{lrr}
X & 229 & 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1 \\
Y\ - & \underline{46} & -\ \underline{0\ 0\ 1\ 0\ 1\ 1\ 1\ 0} \\
 & 183 & 1\ 0\ 1\ 1\ 0\ 1\ 1\ 1
\end{array}
$$

RULES:

A	B	A-B	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Try Yourself

36 -12 in Binary

QUICK QUIZ (POLL)

Q: 011010 - 001100

a) 11110

b) 01110

c) 01010

d) 01101

Complement of Numbers

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.
- Leads to simpler, less expensive circuits to implement the operations.

Complement of Numbers

1's complement

2's complement

One's Complement

Invert all bits. Each 1 becomes a 0, and each 0 becomes a 1.

Original Value		One's Complement
0 1	→	1 0
1010 1111	→	0101 0000
11110000 10100011	→	00001111 01011100
11110000 10100101	→	00001111 01011010

0 0 0 1 0 1 0 0 → Binary number

1 1 1 0 1 0 1 1 → One's complement

$$\begin{array}{r} 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1 \\ +\quad 1 \\ \hline 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0 \end{array} \rightarrow \text{2s complement}$$

Complement of Numbers

Summary:

Step 1: One's Complement

Example: Find the one's complement of 1001101

Number	1001101
Invert bits	0110010

Step 2: Two's Complement

Adding **1** to the **one's complement** of a binary digit

Example: Find the two's complement of 1001101

Number	1001101
One's complement	0110010
Add 1	+ 1
Two's complement	0110011

Complement of Numbers

QUICK QUIZ (POLL)

Q: 2’s complement of 1101100 is:

a) 0010011

b) 0011100

c) 1000000

d) 0010100

SUBTRACTION with Complements

Procedure:

The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Forget Borrow concept!

SUBTRACTION with Complements (Decimals)

Practice Problem:

Using 10’s complement, subtract $6369 - 70610$

SUBTRACTION with Complements (Binary)

Practice Problem:

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

SUBTRACTION with Complements (Binary)

Practice Problem:

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

SUBTRACTION with Complements (Binary)

Practice Problem:

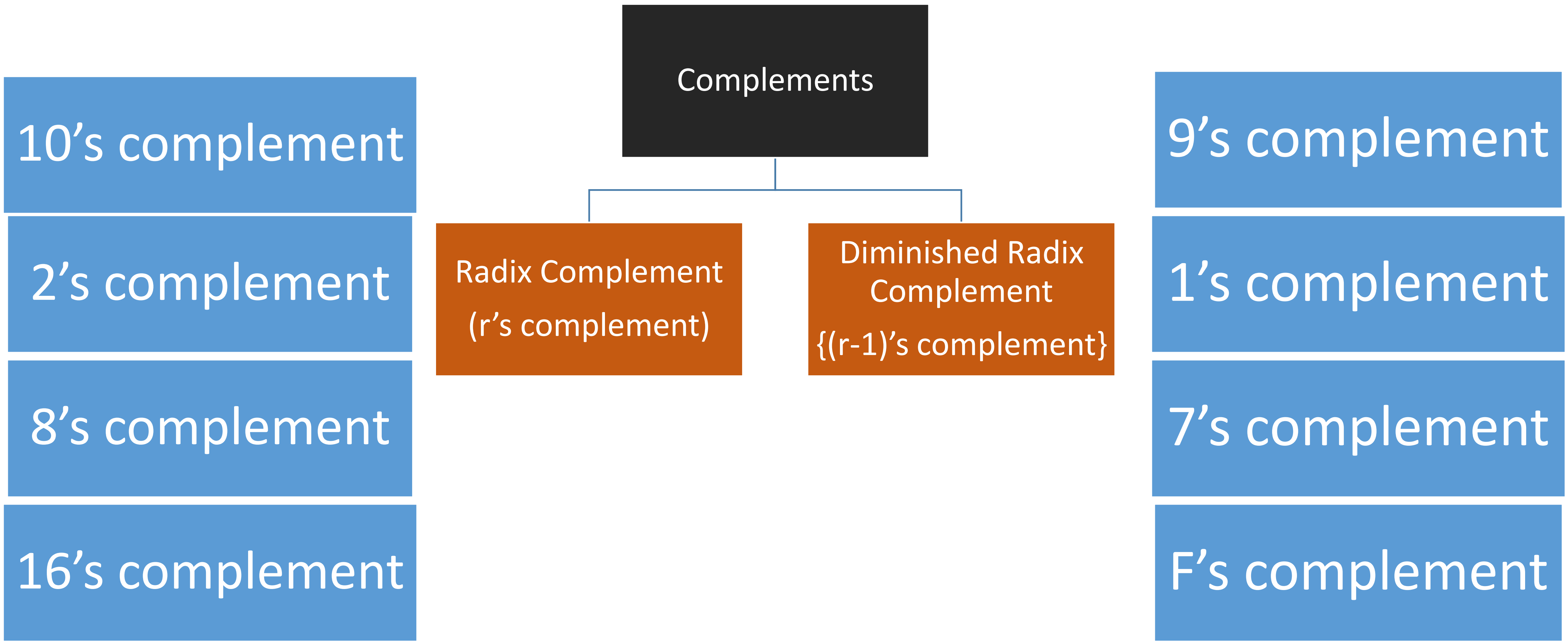
Repeat the previous question using 1's complement method?

SUBTRACTION with Complements

Try Yourself:

1. Perform $185 - 263$ using 9’s and 10’s complement method?
2. Perform $265 - 172$ using 9’s and 10’s complement method?
3. Perform $101100 - 100110$ using 1’s and 2’s complement method?
4. Perform $100110 - 110001$ using 1’s and 2’s complement method?

Complement of Numbers



Complement of Numbers

7's complement

Given a number: 43
Method 1: Formula

$$(r^n - 1) - N$$
$$(8^2 - 1) - 43$$
$$77 - 43$$
$$= 34$$

8's complement

Given a number: 43
Method 1: Formula

$$r^n - N$$
$$8^2 - 43$$
$$100 - 43$$
$$= 35$$

Method 2: 8's Comp = 7's Comp + 1

$$= 34 + 1$$
$$= 35$$

Complement of Numbers

QUICK QUIZ (POLL)

Q: 8’s complement of 634 is:

a) 143

b) 366

c) 144

d) 365

Complement of Numbers

F's complement

Given a number: 4A9
Method 1: Formula

$$(\boldsymbol{r^n - 1}) - \boldsymbol{N}$$
$$(16^3 - 1) - 4A9$$
$$FFF - 4A9$$
$$= B56$$

16's complement

Given a number: 4A9
Method 1: Formula

$$\boldsymbol{r^n - N}$$
$$16^3 - 4A9$$
$$1000 - 4A9$$
$$= B57$$

Method 2: 16's Comp = F's Comp + 1

$$= \mathrm{B56} + 1$$
$$= B57$$

Complement of Numbers

QUICK QUIZ (POLL)

Q: 16’s complement of 7B is:

a) 83

b) 84

c) 85

d) 86

SUBTRACTION with Complements

Try Yourself:

1. Find 7's and 8's complement of 124?
2. Find 7's and 8's complement of 362?
3. Find F's and 16's complement of $F20$?
4. Find F's and 16's complement of $1A6$?

DIGITAL ELECTRONICS: ECE 213

Topic: Signed Number Representation

UNIT I: Number Systems

Lecture No.: 4

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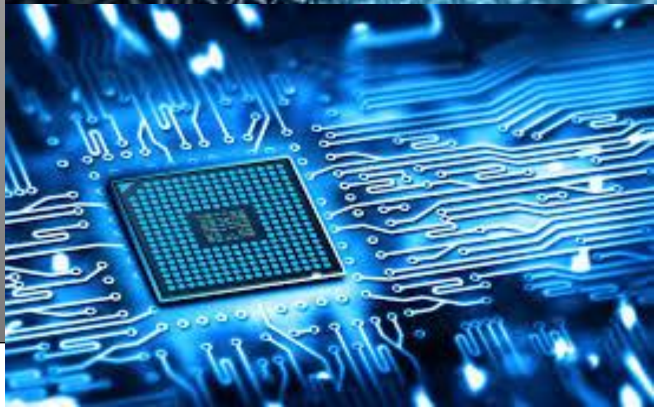
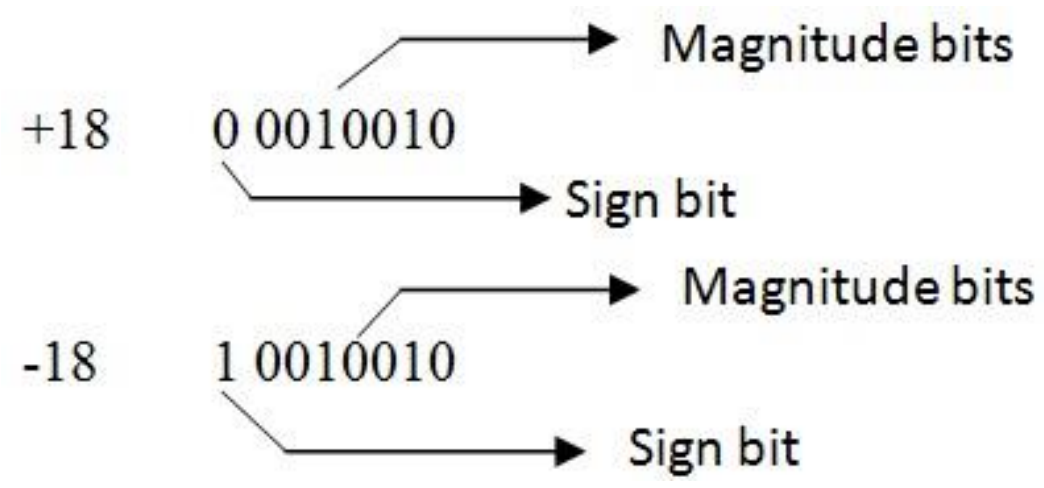


Signed Binary Numbers

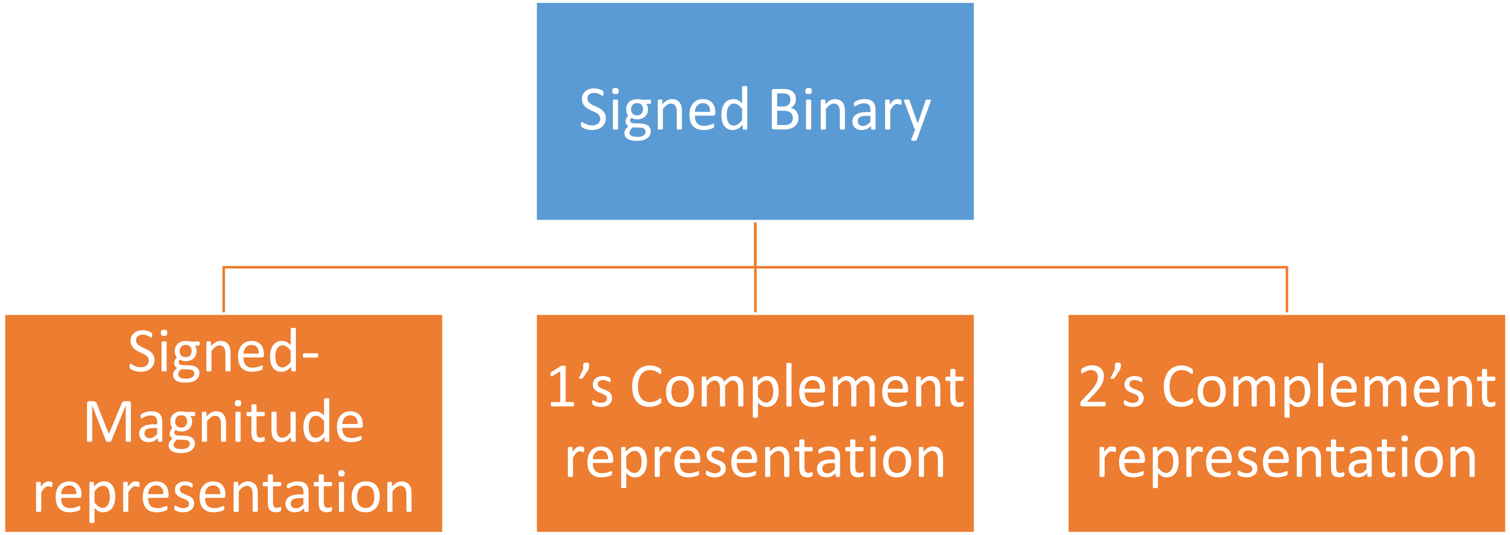
- Positive integers (including zero) can be represented as unsigned numbers. However, to represent negative integers, we need a notation for negative values.
- In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign.
- Because of hardware limitations, computers must represent everything with binary digits. It is customary to represent the sign with a bit placed in the leftmost position of the number.
- The convention is to make the sign bit 0 for positive and 1 for negative.

Signed Binary Numbers

- If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number.
- If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number.



Signed Binary Numbers



	+9	0	0	0	0	1	0	0	1
SMS	- 9	1	0	0	0	1	0	0	1
1’s C	- 9	1	1	1	1	0	1	1	0
2’s C	- 9	1	1	1	1	0	1	1	1

Why is 2’s Complement representation better?

- The signed-magnitude system is used in ordinary arithmetic, but is awkward when employed in computer arithmetic because of the separate handling of the sign and the magnitude. Therefore, the signed-complement system is normally used.
- The 1’s complement imposes some difficulties and is seldom used for arithmetic operations. It is useful as a logical operation, since the change of 1 to 0 or 0 to 1 is equivalent to a logical complement operation.

Why is 2's Complement representation better?

Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Signed Numbers: Arithmetic Addition

- The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.

- In contrast, the rule for adding numbers in the signed-complement system does not require a comparison or subtraction, but only addition.
- **PROCEDURE:**

The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits. **A carry out of the sign-bit position is discarded.**

Signed Numbers: Arithmetic Addition

- **Examples:**

1. $(+13) + (+6)$
2. $(+13) + (-6)$
3. $(-13) + (+6)$
4. $(-13) + (-6)$

NOTE:
- **Negative numbers must be initially in 2's-complement form. and If the sum obtained after the addition is negative, it is in 2's-complement form.**
- **Carry out of the sign-bit position is discarded, and negative results are automatically in 2's-complement form.**

Signed Numbers: Arithmetic Addition

- **Examples:**

1. $(+13) + (+6)$
2. $(+13) + (-6)$
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Concept of Overflow

- In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum. If we start with two n-bit numbers and the sum occupies n + 1 bits, we say that an overflow occurs.
- Overflow is a problem in computers because the number of bits that hold a number is finite, and a result that exceeds the finite value by 1 cannot be accommodated

Signed Numbers: Arithmetic Subtraction

- **PROCEDURE:**

Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit). **A carry out of the sign-bit position is discarded**

- This procedure is adopted because a subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed, as is demonstrated by the following relationship:

$$(\pm A) - (+B) = (\pm A) + (-B);$$
$$(\pm A) - (-B) = (\pm A) + (+B).$$

DIGITAL ELECTRONICS: ECE 213

Topic: Signed Number Representation

UNIT I: Number Systems

Lecture No.: 5

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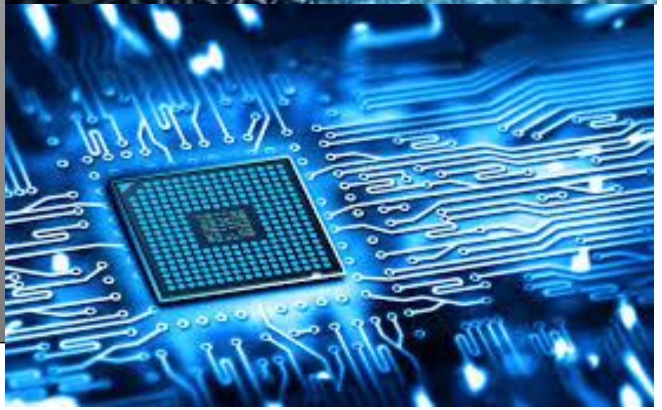

Concept of Overflow

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Binary Codes

- Any discrete element of information that is distinct among a group of quantities can be represented with a binary code (i.e., a pattern of 0's and 1's). The codes must be in binary because, in today's technology, only circuits that represent and manipulate patterns of 0's and 1's can be manufactured economically for use in computers.
- Although the minimum number of bits required to code 2^n distinct quantities is n, there **is no maximum number** of bits that may be used for a binary code.

Types of Binary Code

- ASCII Character code
- Binary Coded Decimal (BCD) code
- Excess-3 Code
- Biquinary Code
- 1 out of 10 code
- Gray Code

Binary Coded Decimal (BCD) code

- ❖Although the binary number system is the most natural system for a computer because it is readily represented in today's electronic technology, most people are more accustomed to the decimal system.
- ❖A binary code that distinguishes among 10 elements must contain at least four bits, but 6 out of the 16 possible combinations remain unassigned.
- ❖Different binary codes can be obtained by arranging four bits into 10 distinct combinations.
- ❖The code most commonly used for the decimal digits is the straight binary assignment. This scheme is called binary-coded decimal and is commonly referred to as BCD.

Binary Coded Decimal (BCD) code

❖ Table here gives the four-bit code for one decimal digit. A number with k decimal digits will require 4k bits in BCD.

❖ Decimal 396 is represented in BCD with 12 bits as:

0011 1001 0110, with each group of 4 bits representing one decimal digit.

❖ A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9.

❖ A BCD number greater than 10 looks different from its equivalent binary number, even though both contain 1's and 0's.

❖ Moreover, the binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

QUICK QUIZ (POLL)

The decimal number 10 is represented in its BCD form as ___________?

a) 10100000

b) 00001010

c) 00010000

d) 00101011

QUICK QUIZ (POLL)

A three digit decimal number requires _________ for representation in the conventional BCD format.

a) 3 bits

b) 6 bits

c) 12 bits

d) 24 bits

BCD Addition Rules

Case 1: Sum < 10, Carry = 0; Then the answer is correct

Case 2: Sum >= 10, Carry = 0; add 6 to obtain the correct BCD

Case 3: Sum < 10, Carry = 1; add 6 to obtain the correct BCD

After obtaining a valid BCD sum, if there is an end carry then pad with zero to the left to make it a 4-bit BCD number

BCD Addition Rules

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Case 3: Sum < 10, Carry = 1; add 6 to obtain the correct BCD

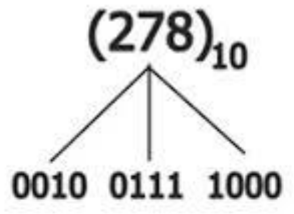
After obtaining a valid BCD sum, if there is an end carry then pad with zero to the left to make it a 4-bit BCD number

$$
\begin{array}{r}
4 \\
+5 \\
\hline
9
\end{array}
\qquad
\begin{array}{r}
0100 \\
+0101 \\
\hline
1001
\end{array}
\qquad
\begin{array}{r}
4 \\
+8 \\
\hline
12
\end{array}
\qquad
\begin{array}{r}
0100 \\
+1000 \\
\hline
1100 \\
+0110 \\
\hline
10010
\end{array}
\qquad
\begin{array}{r}
8 \\
+9 \\
\hline
17
\end{array}
\qquad
\begin{array}{r}
1000 \\
1001 \\
\hline
10001 \\
+0110 \\
\hline
10111
\end{array}
$$

BCD Addition Rules

Try Yourself:

1. Perform 248 + 876 using BCD rules?



Therefore, $(278)_{10}$ = 0010 0111 1000

QUICK QUIZ (POLL)

Carry out BCD subtraction for (68) – (61) using 10’s complement method?

a) 00000111

b) 01110000

c) 100000111

d) 011111000

Excess-3 code

- ❖The excess-3 code (or XS3) is a non-weighted code used to express code used to express decimal numbers.
- ❖It is a self-complementary code and numerical system.
- ❖It is particularly significant for arithmetic operations as it overcomes shortcoming encountered while using 8421 BCD code to add two decimal digits whose sum exceeds 9. Excess-3 arithmetic uses different algorithm than normal BCD or binary positional number system.
- ❖It is obtained by adding 3 (0011) to each decimal digit.

Excess-3 code

Digit Desimal	Input BCD				Output Excess-3			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

Advantage:

- The complement of a binary number can be obtained from that number by replacing 0's with 1's and 1's with 0's.
- The sum of binary number and its complement is always equal to decimal 9.

QUICK QUIZ (POLL)

The excess-3 code for 597 is given by ___________?

a) 100011001010

b) 100010100111

c) 010110010111

d) 010110101101

Bi-quinary, 2421 and 1-out-of-10 codes

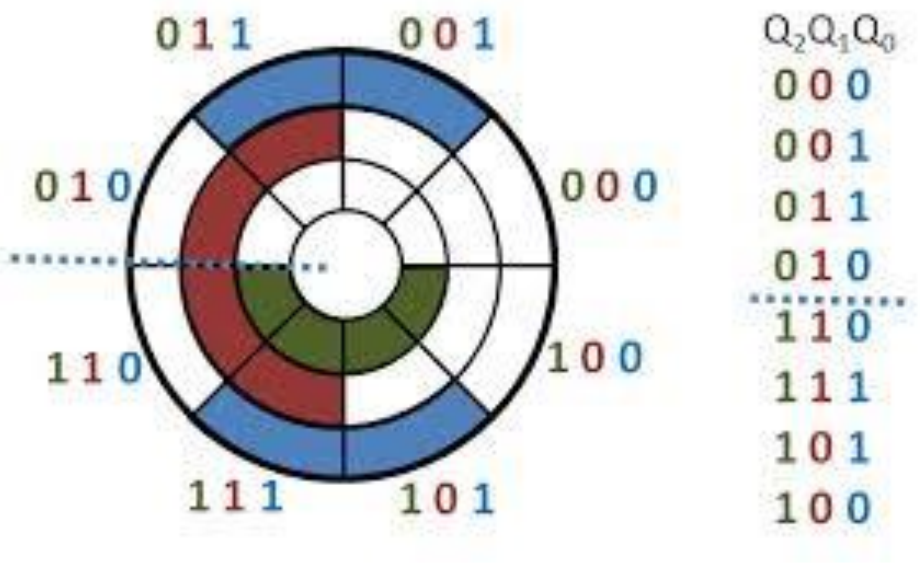
Decimal digit	BCD (8421)	2421	Excess-3	Biquinary	1-out-of-10
0	0000	0000	0011	0100001	1000000000
1	0001	0001	0100	0100010	0100000000
2	0010	0010	0101	0100100	0010000000
3	0011	0011	0110	0101000	0001000000
4	0100	0100	0111	0110000	0000100000
5	0101	1011	1000	1000001	0000010000
6	0110	1100	1001	1000010	0000001000
7	0111	1101	1010	1000100	0000000100
8	1000	1110	1011	1001000	0000000010
9	1001	1111	1100	1010000	0000000001
Unused code words					
	1010	0101	0000	0000000	0000000000
	1011	0110	0001	0000001	0000000011
	1100	0111	0010	0000010	0000000101
	1101	1000	1101	0000011	0000000110
	1110	1001	1110	0000101	0000000111
	1111	1010	1111	. . .	. . .

Gray Code

- The output data of many physical systems are quantities that are continuous. These data must be converted into digital form before they are applied to a digital system. Continuous or analog information is converted into digital form by means of an analog-to-digital converter.
- It is sometimes convenient to use the Gray code shown in Table 1.6 to represent digital data that have been converted from analog data. The advantage of the Gray code over the straight binary number sequence is that only one bit in the code group changes in going from one number to the next.
- The Gray code is used in applications in which the normal sequence of binary numbers generated by the hardware may produce an error or ambiguity during the transition from one number to the next.

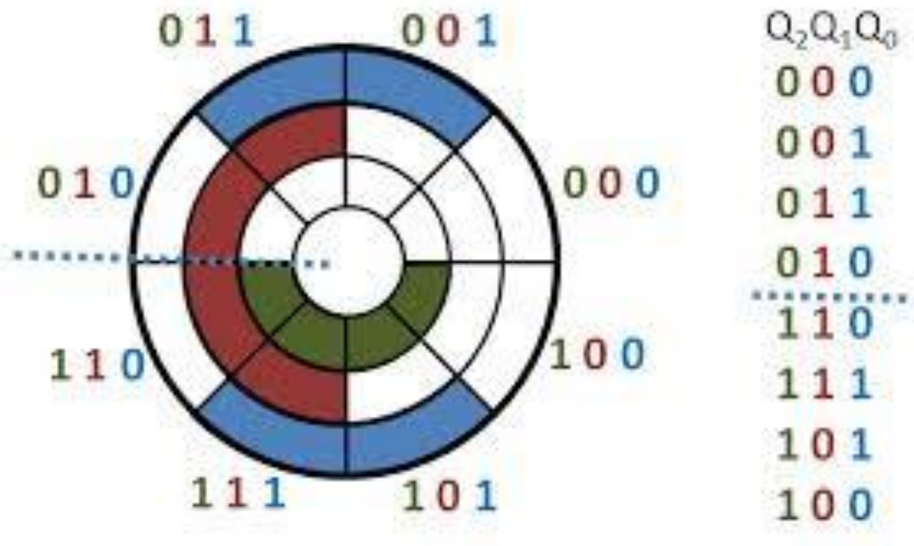
Gray Code

Binary				Gray Code			
b_3	b_2	b_1	b_0	g_3	g_2	g_1	g_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0

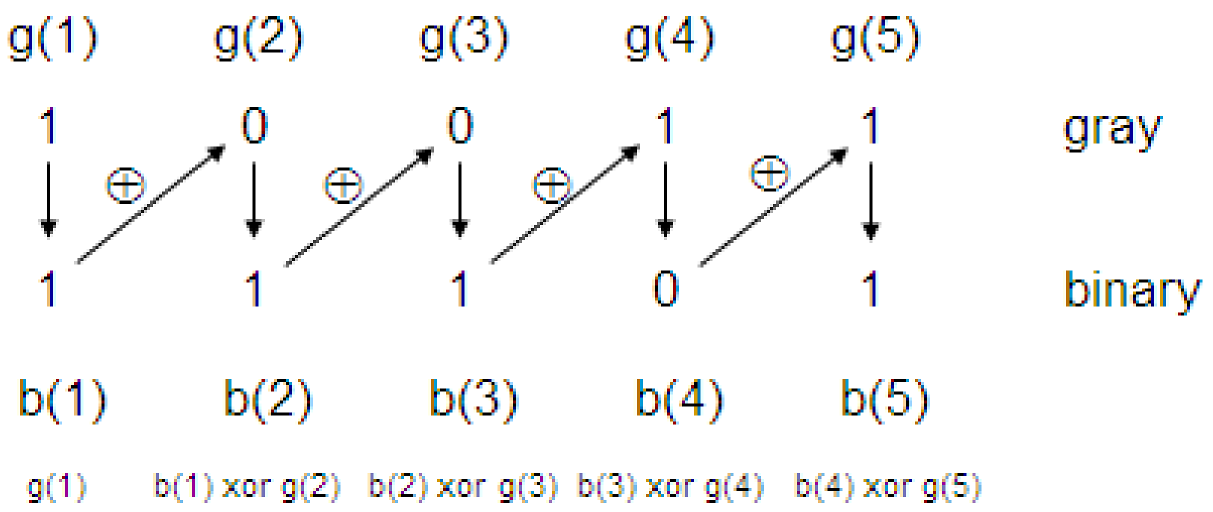


Binary to Gray Conversion

Decimal	Binary			Gray		
	b2	b1	b0	g2	g1	g0
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	0	1
7	1	1	1	1	0	0



Gray to Binary Conversion



Practice Questions:

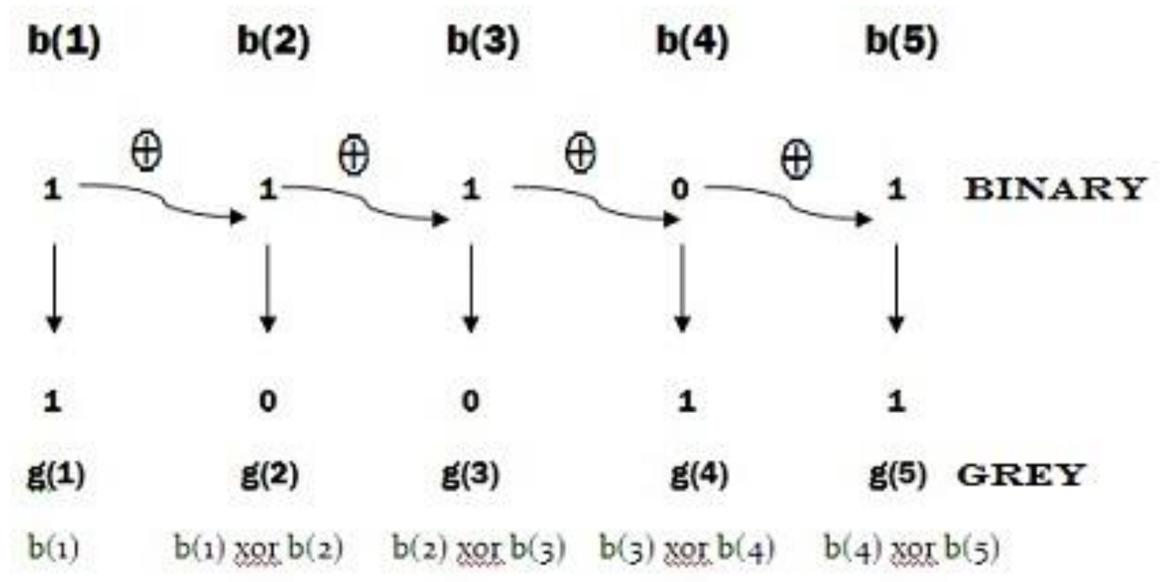
Qsn: Convert following binary numbers into gray codes?

i. 11010
ii. 1001100111
iii. 10101100

Qsn: Convert following gray codes into binary numbers?

i. 1001110
ii. 11001011
iii. 0100100111

Binary to Gray Conversion



QUICK QUIZ (POLL)

Why is the Gray code more practical to use when coding the position of a rotating shaft?

a) All digits change between counts

b) Two digits change between counts

c) Only one digit changes between counts

d) Alternate digit changes between counts

ASCII Character Code

- Many applications of digital computers require the handling not only of numbers, but also of other characters or symbols, such as the letters of the alphabet.
- An alphanumeric character set is a set of elements that includes the 10 decimal digits, the 26 letters of the alphabet, and a number of special characters.
- Stands for: American Standard Code for Information Interchange (ASCII).
- It uses **seven bits** to code 128 characters.
- EBCDIC Code: It is **an eight-bit** character encoding used mainly on IBM mainframe and IBM midrange computer operating systems (1963).
- EBCDIC = Extended Binary Coded Decimal Interchange Code

ASCII Character Code

American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	—	o	DEL

QUICK QUIZ (POLL)

What does ASCII stand for?

a) American Standard Code for Information Interchange

b) American Scientific Code for Information Interchange

c) American Scientific Code for Interchanging Information

d) American Standard Code for Interchanging Information

QUICK QUIZ (POLL)

The representation of the number 8 in binary in ASCII-8 format __________?

a) 00111000

b) 01001000

c) 1000

d) 00011000

DIGITAL ELECTRONICS: ECE 213

Topic: Codes

UNIT I: Number Systems

Lecture No.: 6

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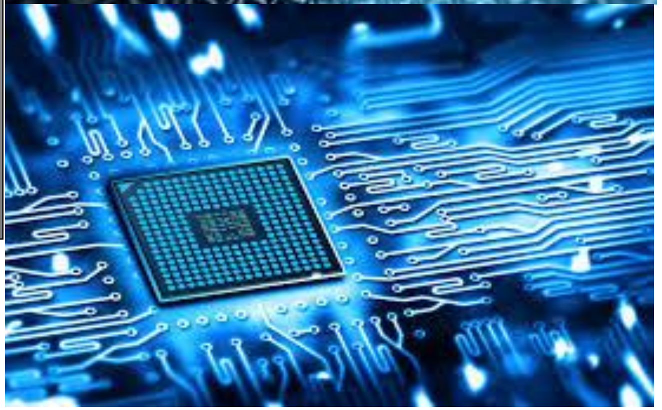

Binary Codes

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Types of Binary Code

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- Excess-3 Code
- Biquinary Code
- 1 out of 10 code
- Gray Code

Binary Coded Decimal (BCD) code

- ❖Although the binary number system is the most natural system for a computer because it is readily represented in today's electronic technology, most people are more accustomed to the decimal system.
- ❖A binary code that distinguishes among 10 elements must contain at least four bits, but 6 out of the 16 possible combinations remain unassigned.
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❖Table here gives the four-bit code for one decimal digit. A number with k decimal digits will require 4k bits in BCD.

❖Decimal 396 is represented in BCD with 12 bits as:

0011 1001 0110, with each group of 4 bits representing one decimal digit.

❖A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9.

❖ A BCD number greater than 10 looks different from its equivalent binary number, even though both contain 1's and 0's.

❖Moreover, the binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

QUICK QUIZ (POLL)

The decimal number 10 is represented in its BCD form as ___________?

a) 10100000

b) 00001010

c) 00010000

d) 00101011

QUICK QUIZ (POLL)

A three digit decimal number requires _________ for representation in the conventional BCD format.

a) 3 bits

b) 6 bits

c) 12 bits

d) 24 bits

Excess-3 Arithmetic

Addition

1. While adding two XS-3 numbers, if there is NO carry generated subtract 0011 from the sum term.
2. While adding two XS-3 numbers, if there carry is generated add 0011 to the sum term.

Subtraction

1. If there is no borrow from the next four bits, add 0011 to the difference term.
2. If there is a borrow from the next four bits, subtract 0011 from the difference term.

QUICK QUIZ (POLL)

What does ASCII stand for?

a) American Standard Code for Information Interchange

b) American Scientific Code for Information Interchange

c) American Scientific Code for Interchanging Information

d) American Standard Code for Interchanging Information

QUICK QUIZ (POLL)

The representation of the number 8 in binary in ASCII-8 format __________?

a) 00111000

b) 01001000

c) 1000

d) 00011000

Error Detection Code

What if an error is detected?

- One possibility is to request retransmission of the message on the assumption that the error was random and will not occur again.
- If, after a number of attempts, the transmission is still in error, a message can be sent to the operator to check for malfunctions in the transmission path.

DIGITAL ELECTRONICS: ECE 213

Topic: Codes

UNIT I: Number Systems

Lecture No.: 7

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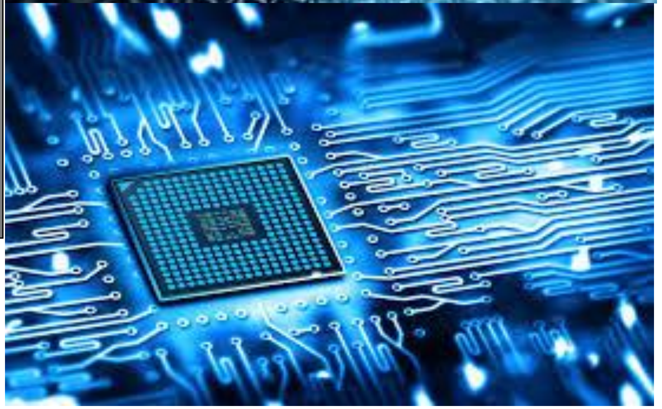


Error Detection Code

- To detect errors in data communication and processing, an eighth bit is sometimes added to the ASCII character to indicate its parity.
- A parity bit is an extra bit included with a message to make the total number of 1’s either even or odd.

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

- In each case, we insert an extra bit in the leftmost position of the code to produce an even number of 1’s in the character for even parity or an odd number of 1’s in the character for odd parity. In general, one or the other parity is adopted, with even parity being more common.

Error Detection Code

- The parity bit is helpful in detecting errors during the transmission of information from one location to another.
- The eight-bit characters that include parity bits are transmitted to their destination.
- The parity of each character is then checked at the receiving end.
- If the parity of the received character is not even, then at least one bit has changed value during the transmission.
- This method detects one, three, or any odd combination of errors in each character that is transmitted.
- An even combination of errors, however, goes undetected, and additional error detection codes may be needed to take care of that possibility.

Error Detection and Correction Code: (Hamming Code)

Practice Question?

- Consider the code word received be the receiver as following, and find errors if any?

1. 001100010100
2. 101110010100
3. 001110010100

QUICK QUIZ (POLL)

Which one of the following bit cannot be the parity position?

a) 4th bit

b) 8th bit

c) 12th bit

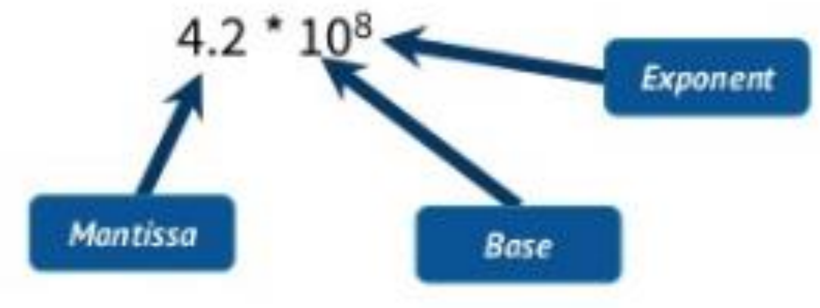
d) 16th bit

Floating Point Representation

- To represent very large numbers, many bits are required.
- Integer and fractions both parts need to be stored. E.g., 28.645
- Floating point system is based on scientific notation.

❑ A floating point number has two parts:

1. Mantissa (magnitude of the number)
2. Exponent (number of places decimal/binary point has to move)



Floating Point Representation

- The format for storing floating point numbers is defined by IEEE/ANSI Standard 754.
- The three forms are:

1. Single Precision (32 bits).
2. Double Precision (64 bits).
3. Extended Precision (80 bits).

Floating Point Representation

Single Precision **Floating Point Representation**

- MSB 1-bit signifies sign bit.
- Exponent is represented with 8 bits.
- 23 bits assigned for mantissa/Fraction.

Floating Point Representation

Biased Exponent:

- Biased exponent is obtained by **adding 127** to the actual exponent.

PURPOSE: to express very small and very large numbers without requiring a separate sign bit for the exponents.

IMPORTANT POINT:

1. The biased exponent allows a range of actual exponent values from -126 to +128.

QUICK QUIZ (POLL)

Single precision floating point representation format consists of?

a) 23 bits

b) 24 bits

c) 32 bits

d) 64 bits