

L  
P  
U

## Lecture : 6

Predicates

and

Quantifiers

L  
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U

Consider the statement :

Every computer connected to the university network is functioning properly

L  
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Propositional Logic can't adequately express the meaning of statement

(I)

Students of KIIT FG are responding through poll

Let  $x$ : student  
 $P(x)$ :  $x$  is responding through poll

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No rule from the propositional logic allow us to

conclude the truth of "The first computer of 34-310

is working properly"

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$x^2 \leq 10$  proposition - ?  
NO  
Subject

$x$   
P(2) is Proposition  
P(3) is Proposition  
one arrayed  
 $x^2 \leq 10$  P(4) is Proposition  
 $\underline{x^2 \leq 10}$

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Predicate : The property of subject

All Dogs are Mammal

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Propositional function : Subject follow the given property  
↓  
If a value is assigned to the variable  $x$  then  $P(x)$  become the Proposition

has  
Truth Value  
T  
F

$x^2 + y^2 \leq 10$   
Proposition X  
Predicate ✓

$P(x, y) : x^2 + y^2 \leq 10$

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L  
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$P : \{T, F\}$   
 $Q : \{T, F\}$

Subjects  
 $x, y$   
Property  
 $P(1, 2) \checkmark T$   
 $P(2, 3) \checkmark F$   
two arrayed

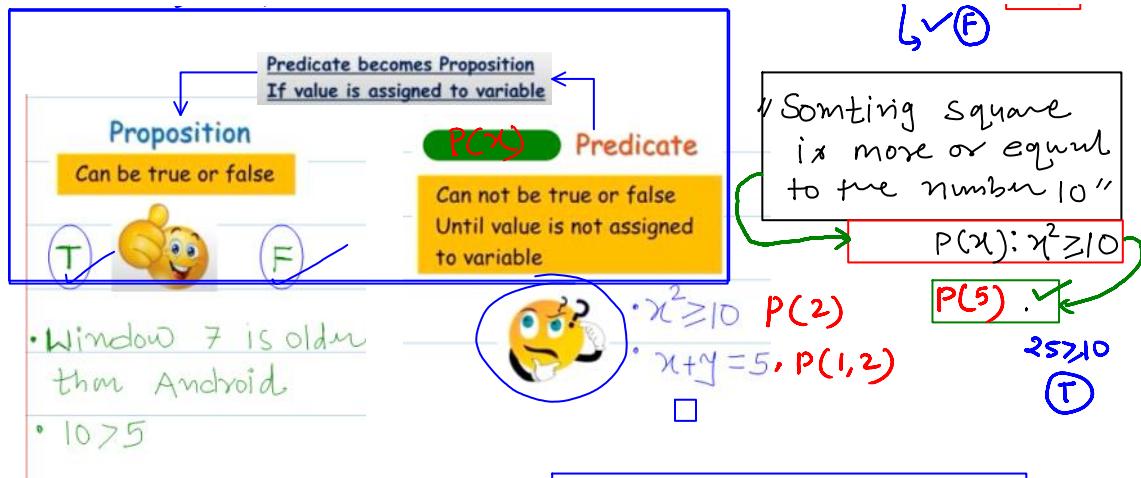
L  
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Predicate becomes Proposition  
If value is assigned to variable

L

L  
P  
U

L✓P

L  
P  
UL  
P  
UL  
P  
UL  
P  
U

▷  $P(x, y, z, p) : x^2 + y^2 \neq z + p$

one Arrayed  
Two Arrayed  
Three Arrayed  
Four Arrayed

Is it a Proposition - ?

**predicate**

$P(1, 2, 3, 4) \checkmark$

$P(5, 4, 3, 1) \checkmark$

X

? **All** the student of **KI9FG** **Scored at least A grade**

(in MTH 165)

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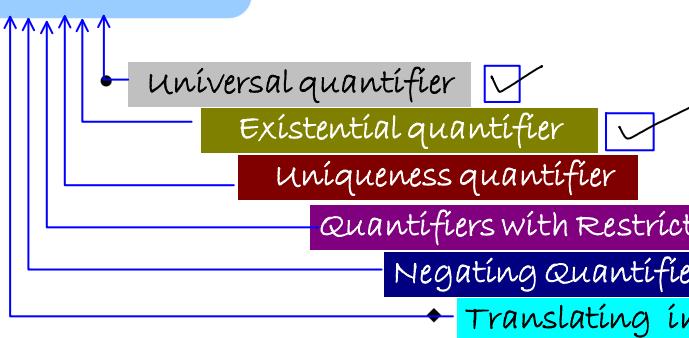
WITH OUT ASSIGNING  
ANY PROPOSITIONAL VALUE TO  
A PREDICATE, CAN WE MAKE  
IT A PROPOSITION - ?



yes / No

L  
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UL  
P  
U

**Quantifiers**



A predicate becomes a proposition when we assign it fixed values.

However, another way to make a predicate into a proposition is to quantify it.

That is, the predicate is true (or false) for all possible values in the universe of discourse or for some value(s) in the universe of discourse.

Section :  $\forall x \in K19FG$  "Students are responding through the poll"

L  
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Such quantification can be done with two quantifiers: the universal quantifier and the existential quantifier.

$$\forall x \in K19FG. P(x)$$

$\exists x$ , student  $\rightarrow$

$P(x)$ :  $x$  is responding

$P(Kannal)$  ✓

$P(Abhishek)$  ✓

$P(Rushree)$  ✓

: "Some is less than 5"

$x$ : Some thing

$x \in (-\infty, \infty)$

$P(x)$ :  $x < 5$

$x = 2, P(2): 2 < 5$  yes ✓ T  
 $x = 10, P(10): 10 < 5$ , yes F

QUANTIFIER

$$\boxed{\forall x \in K19FG. P(x)}$$

## QUANTIFIER

A quantifier is an operator used to create a proposition from a propositional function. Another way, if we do not assign values to  $P(x)$

$$P(x) \quad x < 5$$

Values not exceeding 4

### UNIVERSAL QUANTIFIER

$P(x)$  is true for all values of  $x$  in the universe of discourse

Universe of discourse

(i.e., range)

FOR ALL

$$\forall \forall x P(x)$$

What is the truth value of  $\forall x P(x)$  where  $P(x)$  is the statement " $x^2 < 10$ " and the universe of discourse consists of positive integers not exceeding 4?

$$\{0, 1, 2, 3, 4\}$$

F

L  
P  
U

What is the truth value of  $\exists x P(x)$ , where  $P(x)$  is the statement " $x^2 < 10$ " and the universe of discourse consists of positive integers not exceeding 4?

$$\exists x, P(x) \checkmark$$

$$1^2 < 10 \checkmark$$

$$\begin{array}{l} 0^2 < 10 \\ 1^2 < 10 \checkmark \\ 2^2 < 10 \checkmark \\ 3^2 < 10 \checkmark \\ 4^2 < 10 \times \end{array}$$

L  
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The universal quantification of  $P(x)$  is the statement " $P(x)$  for all values of  $x$  in the domain."

The notation  $\forall x P(x)$  denotes the universal quantification of  $P(x)$ .

Here  $\forall$  is called the universal quantifier. We read  $\forall x P(x)$  as "for all  $x P(x)$ " or "forever  $x P(x)$ ".

"An element for which  $P(x)$  is false is called a counterexample to  $\forall x P(x)$ ".

$$\boxed{\forall x \in D, P(x)}$$

L  
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U

Let  $P(x)$  be the statement " $x + 1 > x$ ." What is the truth value of the quantification  $\forall x P(x)$ , where the domain consists of all real numbers?

subject :  $x$

$$(-\infty, \dots, -3, -2, -1, 0, 1, 2, 3, \dots \infty)$$

L  
P  
U

property :  $x + 1 > x$

$$3 + 1 > 3 \text{ } \textcircled{T}$$

L  
P  
U

domain :  $(-\infty, \infty)$

$$-5 + 1 > -5$$

$$\text{ } \textcircled{F}$$

$$\forall x \in (-\infty, \infty) P(x) \text{ is } \underline{\text{true}}$$

L  
P  
U

Statement	When True?	When False?
$\forall x P(x) \rightarrow$	$P(x)$ is true for every $x$ . $\checkmark$	
$\exists x P(x) \rightarrow$	There is an $x$ for which $P(x)$ is true.	There is an $x$ for which $P(x)$ is false. $P(x)$ is false for every $x$ .

$$\cancel{x \in 15, 1}$$

L  
P  
U

Let  $Q(x)$  be the statement " $x < 2$ ." What is the truth value of the quantification  $\forall x Q(x)$ , where the domain consists of all real numbers?

L  
P  
U

$$\begin{array}{l} x \\ \checkmark \\ \therefore x < 2 \\ (-\infty, \infty) \end{array}$$

$$\boxed{\forall x \in D, Q(x)}$$

$$\textcircled{F}$$

$$\exists x \in D, \underline{Q(x)}$$

$$\textcircled{T}$$

L  
P  
U

What is the truth value of  $\exists x (x^2 \geq x)$  if the domain consists of all real numbers? What is the truth value of this statement if the domain consists of all integers?

$$\begin{aligned} x \\ x^2 \geq x \\ \Phi(x) : x^2 \geq x \end{aligned}$$

$$\begin{aligned} \exists x \in (-\infty, \infty) \Phi(x) &\rightarrow T \\ \exists x \in (-\infty, \infty) \Phi(x) &\rightarrow F \end{aligned}$$

L
P
U

The existential quantification of  $P(x)$  is the proposition "There exists an element  $x$  in the domain such that  $P(x)$ ."

We use the notation  $\exists x P(x)$  for the existential quantification of  $P(x)$ . Here  $\exists$  is called the existential quantifier.

$$\boxed{\exists x \in D P(x)}$$

L
P
U

Let  $P(x)$  denote the statement " $x > 3$ ." What is the truth value of the quantification  $\exists x P(x)$ , where the domain consists of all real numbers?

L
P
U

Let  $Q(x)$  denote the statement " $x = x + 1$ ." What is the truth value of the quantification  $\exists x Q(x)$ , where the domain consists of all real numbers?

$$\begin{aligned} 2 &= 2+1 \\ 0.5 &= 0.5+1 \end{aligned}$$

$\circlearrowleft F$

L
P
U

the uniqueness quantifier, denoted by  $\exists!$  or  $\exists !$ . The notation  $\exists ! x P(x)$ , [or  $\exists x P(x)$ ] states "There exists a unique  $x$  such that  $P(x)$  is true."

CA-I  $x$ : Student of KIITEG,  $P(n)$ : n scored  $\geq 25$

L
P
U

- Paper was very easy :  $\forall n, P(n)$

- Paper was difficult :  $\exists n P(n)$

- Paper was very difficult :  $\exists ! n P(n)$

L
P
U

What do the statements  $x < 0$  ( $x^2 > 0$ ),  $y \neq 0$  ( $y^3 \neq 0$ ), and  $z > 0$  ( $z^2 = 2$ ) mean, where the

domain in each case consists of the real numbers?

L
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U

$\forall x < 0$  s.t.  $x^2 > 0$

$\forall y \neq 0$  s.t.  $y^3 \neq 0$

$\exists z > 0$  s.t.  $z^2 = 2$

$\begin{aligned} \exists x \in \text{All the student of KIITEG} &\rightarrow (-\infty, \infty) \\ \text{Placement} &\rightarrow (-\infty, 0) \\ \text{C.gpa of } n \geq 6.5 &\rightarrow (0, \infty) \end{aligned}$

L
P
U

~~X~~

\* ~ -

→ go for placement

L  
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## Negating Quantified Expressions

We will often want to consider the negation of a quantified expression.

For instance, consider

the negation of the statement

"Every student in your class has taken a course in calculus."

This statement is a universal quantification, namely,

$\forall x P(x)$ ,

$x \neg P(x)$ .

$\neg [\text{all the students got } A^+]$   
There exists a student who got B

L  
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U

L  
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U

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	<u><math>\forall x \neg P(x)</math></u>	For every $x$ , $P(x)$ is false.	There is an $x$ for which $P(x)$ is true.
$\neg \forall x P(x)$	<u><math>\exists x \neg P(x)</math></u>	There is an $x$ for which $P(x)$ is false.	$P(x)$ is true for every $x$ .

L  
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$\neg [\text{There exists one student who got } A^+]$  all the students got  $< A^+$

What are the negations of the statements  $\forall x (x^2 > x)$  and  $\forall x (x^2 = x)$ ?

$\forall x : x^2 > x$  Domain  $\rightarrow \mathbb{R} (-\infty, \infty)$

$\neg [\forall x (x^2 > x)] = \exists x (x^2 \leq x) \quad \mathbb{Z} -3, -2, \underline{\textcircled{0}} \textcircled{1}, 2, 3$

$x^2 > 0 \quad x$   $x^2 \geq 0$

$x = 0 \quad 0^2 \neq 0$   
 $x = 0.25 \quad 0.25 \neq 0.25$

L  
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L  
P  
U

What are the negations of the statements

"There is an honest politician"

$x$ : Politician

L  
P  
U

$P(x)$ :  $x$  is honest

$\neg (\exists x P(x)) \equiv \forall x \neg P(x)$

$\neg (\forall x P(x)) \equiv \exists x \neg P(x)$

All the Politicians are not honest

Some politicians are not honest.

L  
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U

$\neg x(P(x) \rightarrow Q(x))$  and  $\neg x(P(x) \neg Q(x))$  are logically equivalent.

$P(x)$ :  $x^2 \geq 0 \quad x \in \mathbb{R}$

$Q(x)$ :  $x+1 > 5 \quad x \in \mathbb{R}$

$\neg [\text{if } \underline{x^2 \geq 0} \text{ then } x+1 > 5]$   
 $= \exists x x^2 \geq 0 \text{ and } x+1 \leq 5$

L  
P  
U

L  
P  
U

L  
P  
U

P: The all trials are completed for Covid-19 vaccine

q: The Production of vaccine begin in Sep

$$\neg(\forall x(P \rightarrow q)) \equiv \exists x[P \wedge \neg q]$$

L  
P  
U

L  
P  
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### GATE | GATE CS 2013 | Question 65

Last Updated: 17-10-2013

What is the logical translation of the following statement?

"None of my friends are perfect."

- (A)  $\exists x(F(x) \wedge \neg P(x))$  X
- (B)  $\exists x(\neg F(x) \wedge P(x))$
- (C)  $\exists x(\neg F(x) \wedge \neg P(x))$  ✓
- (D)  $\neg \exists x(F(x) \wedge P(x))$

- X(B)  $\exists x(\neg F(x) \wedge P(x))$
- (D)  $\neg \exists x(F(x) \wedge P(x))$

$P(x)$ :  $x$  is perfect

$F(x)$ :  $x$  is my friend

L  
P  
U

•  $P(x) \wedge F(x)$

$x$  is my friend & he is Perfect

•  $\exists x(P(x) \wedge F(x))$

There exist some persons which are perfect & my friend

•  $\forall x(P(x) \wedge F(x))$   
all my friend are present)

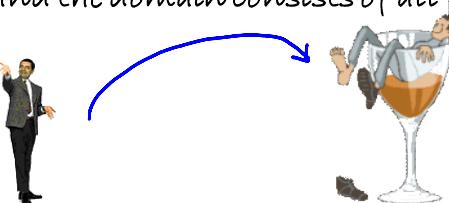
L  
P  
U

Translate these statements into English, where

$C(x)$  is "x is a comedian" and  $F(x)$  is "x is funny"

L  
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and the domain consists of all people.



Domain = {People}

L  
P  
U

$x$ : person

L  
P  
U

$C(x)$ :  $x$  is Comedian

L  
P  
U

$F(x)$ :  $x$  is funny

- a)  $\forall x(C(x) \rightarrow F(x))$
- b)  $\forall x(C(x) \wedge F(x))$
- c)  $\exists x(C(x) \rightarrow F(x))$
- d)  $\exists x(C(x) \wedge F(x))$

"For all  $x$  if  $x$  is comedian then he/she is funny"

L  
P  
U

a) All the comedian are funny

L  
P  
U

b) For all  $x$ ,  $x$  is comedian and funny  
 All the person are comedians & funny

L  
P  
U

c) There exist some  $x$  such that  
 if  $x$  is comedian then he/she

is funny  
" Some comedian are funny "

d) Some Person are Comedian & funny.

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

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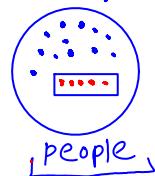
First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.  $\wedge$

$$\exists x \text{ Hindi}(x)$$

$$\exists x (\text{class}(y) \wedge \text{Hindi}(x))$$

section  $\rightarrow$  64  
your class



L  
P  
U

(b) Everyone in your class is friendly.  $\rightarrow$

$$\forall x \text{ Friend}(x)$$

$$\forall x (\text{class}(y) \rightarrow \text{Friend}(x))$$

$\wedge$   
 $\rightarrow$

L  
P  
U

c) There is a person in your class who was not born in California.

$$\exists x \text{ nBorn}(x)$$

$$\exists x (\text{class}(x) \wedge \text{nBorn}(x))$$

L  
P  
U

d) A student in your class has been in a movie.

$$\exists x \text{ Movie}(x)$$

$$\exists x (\text{class}(x) \wedge \text{Movie}(x))$$

L  
P  
U

e) No student in your class has taken a course in logic programming.

Let  $P(x) = "x \text{ is a baby,"}$   
 $Q(x) = "x \text{ is logical.}"$

illogical  $\rightarrow Q(x)$

L  
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U

$R(x) = "x \text{ is able to manage a crocodile,"}$

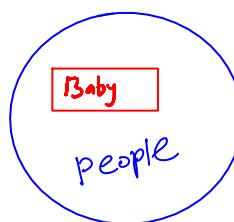
$S(x) = "x \text{ is despised,"}$

be the statements

Suppose that the domain consists of all people.

Express each of these statements

using quantifiers; logical connectives.



L  
P  
U

a) Babies are illogical. ---  $\forall x (P(x) \rightarrow \neg Q(x))$

b) Nobody is despised who can manage a crocodile. ---

c) Illogical persons are despised. ---  $\forall x (\neg Q(x) \rightarrow S(x))$

d) Babies cannot manage crocodiles. ---  $\forall x (P(x) \rightarrow \neg R(x))$

L  
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L  
P  
U

L  
P  
U

$$\begin{aligned}
 & x \in \mathbb{N} \leq 4 \\
 & x = \{ \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4} \} \quad \checkmark \\
 & P(x) : x^2 = 16 \rightarrow \exists! x P(x) \\
 & P(x) : \boxed{x+1 > 4} \quad \text{---} \\
 & \quad \begin{array}{l} 1+1 > 4 \times \checkmark \\ 2+1 > 4 \times \checkmark \\ 3+1 > 4 \times \checkmark \\ 4+1 > 4 \checkmark \end{array} \\
 & P(x) : \boxed{x^2 + 1 > x} \\
 & \quad \begin{array}{l} 1^2 + 1 > 1 \checkmark \\ 2^2 + 1 > 2 \checkmark \\ 3^2 + 1 > 3 \checkmark \\ 4^2 + 1 > 4 \checkmark \end{array} \\
 & \quad \bullet \exists! x P(x) \\
 & \quad \bullet \exists x P(x) \\
 & \quad \bullet \underline{\forall x P(x)}
 \end{aligned}$$

a unique  $\equiv$   
 Some  $\exists!$   $\subseteq \exists$   
 All  $\checkmark$