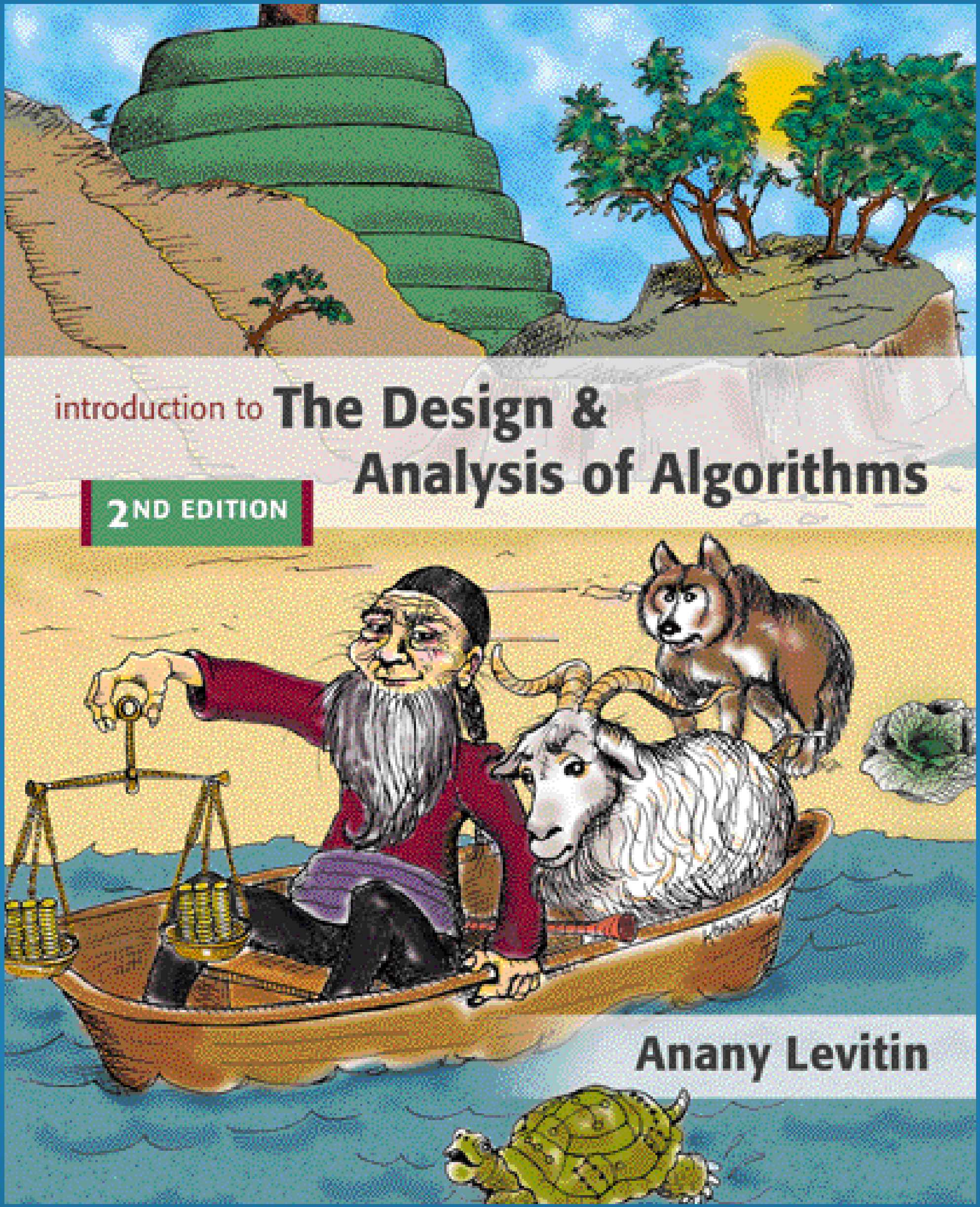
Chapter 3

Brute Force



Brute Force

A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved

Examples:

1. Computing a^n ($a > 0$, n a nonnegative integer)

2. Computing $n!$

3. Multiplying two matrices

4. Searching for a key of a given value in a list

Brute-Force Sorting Algorithm

Selection Sort **Scan the array to find its smallest element and swap it with the first element. Then, starting with the second element, scan the elements to the right of it to find the smallest among them and swap it with the second elements. Generally, on pass i ($0 \le i \le n$-2), find the smallest element in $A[i..n-1]$ and swap it with $A[i]$:**

$$A[0] \le \;.\;\;.\;\;.\; \le A[i-1] \;\mid\; A[i], \;.\;\;.\;\;.\;, A[min], \;.\;\;.\;\;., A[n-1]$$

in their final positions

Example: 7 3 2 5

Analysis of Selection Sort

ALGORITHM *SelectionSort*$(A[0..n-1])$
    //Sorts a given array by selection sort
    //Input: An array $A[0..n-1]$ of orderable elements
    //Output: Array $A[0..n-1]$ sorted in ascending order
    for $i \leftarrow 0$ **to** $n-2$ **do**
        $min \leftarrow i$
        for $j \leftarrow i+1$ **to** $n-1$ **do**
            if $A[j] < A[min]$ $min \leftarrow j$
        swap $A[i]$ and $A[min]$

Time efficiency: $\Theta(n\hat{}2)$

Space efficiency: $\Theta(1)$, so in place

Stability: yes

Brute-Force String Matching

- ***pattern***: a string of *m* characters to search for
- ***text***: a (longer) string of *n* characters to search in
- problem: find a substring in the text that matches the pattern

Brute-force algorithm

Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until
  - all characters are found to match (successful search); or
  - a mismatch is detected

Step 3 While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

Examples of Brute-Force String Matching

1. **Pattern:** 001011
   Text: 10010101101001100101111010

2. **Pattern:** happy
   Text: It is never too late to have a happy childhood.

Pseudocode and Efficiency

ALGORITHM *BruteForceStringMatch*($T[0..n-1]$, $P[0..m-1]$)

```
//Implements brute-force string matching
//Input: An array T[0..n − 1] of n characters representing a text and
//        an array P[0..m − 1] of m characters representing a pattern
//Output: The index of the first character in the text that starts a
//          matching substring or −1 if the search is unsuccessful
for i ← 0 to n − m do
    j ← 0
    while j < m and P[j] = T[i + j] do
        j ← j + 1
    if j = m return i
return −1
```

Time efficiency: **$\Theta(mn)$ comparisons (in the worst case)**

Why?

Brute-Force Polynomial Evaluation

Problem: Find the value of polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$$

at a point $x = x_0$

Brute-force algorithm

```
p ← 0.0
for i ← n downto 0 do
    power ← 1
    for j ← 1 to i do        //compute x^i
        power ← power * x
    p ← p + a[i] * power
return p
```

Efficiency: $\Sigma_{0 \le i \le n}\, i = \Theta(n^2)$ **multiplications**

Polynomial Evaluation: Improvement

We can do better by evaluating from right to left:

<u>Better brute-force algorithm</u>

$p \leftarrow a[0]$
$power \leftarrow 1$
for $i \leftarrow 1$ **to** n **do**
  $power \leftarrow power * x$
  $p \leftarrow p + a[i] * power$
return p

Efficiency: **$\Theta(n)$ multiplications**

Horner's Rule is another linear time method.

Closest-Pair Problem

Find the two closest points in a set of *n* points (in the two-dimensional Cartesian plane).

Brute-force algorithm

Compute the distance between every pair of distinct points and return the indexes of the points for which the distance is the smallest.

Closest-Pair Brute-Force Algorithm (cont.)

```
ALGORITHM BruteForceClosestPoints(P)
    //Input: A list P of n (n ≥ 2) points P_1 = (x_1, y_1), ..., P_n = (x_n, y_n)
    //Output: Indices index1 and index2 of the closest pair of points
    dmin ← ∞
    for i ← 1 to n − 1 do
        for j ← i + 1 to n do
            d ← sqrt((x_i − x_j)^2 + (y_i − y_j)^2) //sqrt is the square root function
            if d < dmin
                dmin ← d; index1 ← i; index2 ← j
    return index1, index2
```

Efficiency: **Θ(n^2) multiplications (or sqrt)**

How to make it faster? Using divide-and-conquer!

Brute-Force Strengths and Weaknesses

- **Strengths**
  - wide applicability
  - simplicity
  - yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)

- **Weaknesses**
  - rarely yields efficient algorithms
  - some brute-force algorithms are unacceptably slow
  - not as constructive as some other design techniques

Exhaustive Search

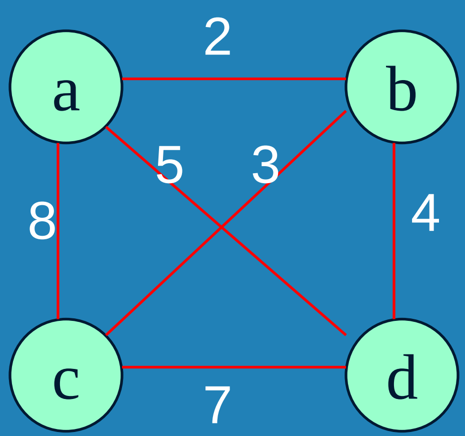
A brute force solution to a problem involving search for an element with a special property, usually among combinatorial objects such as permutations, combinations, or subsets of a set.

Method:

- generate a list of all potential solutions to the problem in a systematic manner (see algorithms in Sec. 5.4)
- evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far
- when search ends, announce the solution(s) found

Example 1: Traveling Salesman Problem

- Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city
- Alternatively: Find shortest *Hamiltonian circuit* in a weighted connected graph
- Example:



How do we represent a solution (Hamiltonian circuit)?

TSP by Exhaustive Search

Tour	Cost
a→b→c→d→a	2+3+7+5 = 17
a→b→d→c→a	2+4+7+8 = 21
a→c→b→d→a	8+3+4+5 = 20
a→c→d→b→a	8+7+4+2 = 21
a→d→b→c→a	5+4+3+8 = 20
a→d→c→b→a	5+7+3+2 = 17

Efficiency: $\Theta((n-1)!)$

Chapter 5 discusses how to generate permutations fast.

Example 2: Knapsack Problem

Given n items:

- weights: $w_1 \quad w_2 \ \dots \ w_n$
- values: $v_1 \quad v_2 \ \dots \ v_n$
- a knapsack of capacity W

Find most valuable subset of the items that fit into the knapsack

Example: Knapsack capacity W=16

item	weight	value
1	2	$20
2	5	$30
3	10	$50
4	5	$10

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

Efficiency: $\Theta(2^n)$

Each subset can be represented by a binary string (bit vector, Ch 5).

Example 3: The Assignment Problem

There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

	Job 0	Job 1	Job 2	Job 3
Person 0	9	2	7	8
Person 1	6	4	3	7
Person 2	5	8	1	8
Person 3	7	6	9	4

Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

How many assignments are there? n!

Pose the problem as one about a cost matrix: cycle cover in a graph

Assignment Problem by Exhaustive Search

$$C = \begin{matrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{matrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
	etc.

(For this particular instance, the optimal assignment can be found by exploiting the specific features of the number given. It is: 2,1,3,4)

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time <u>only on very small instances</u>

- In some cases, there are much better alternatives!
  - Euler circuits
  - shortest paths
  - minimum spanning tree
  - assignment problem

The Hungarian method runs in O(n^3) time.

- In many cases, exhaustive search or its variation is the only known way to get exact solution