CGPA

BOOSTER

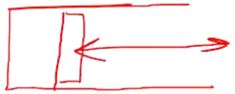
Unit 5
Introduction to Dynamics

motion.

Introduction

- Dynamics includes:
  - *Kinematics*: study of the motion (displacement, velocity, acceleration, & time) without reference to the cause of motion (i.e. *regardless of forces*).
  - *Kinetics*: study of the forces acting on a body, and the resulting motion caused by the given forces.
    - *Rectilinear* motion: position, velocity, and acceleration of a particle as it moves along a **straight line**.
    - *Curvilinear* motion: position, velocity, and acceleration of a particle as it moves along a **curved line.**
    - Circular Motion:
    - Rotational Motion:

Introduction



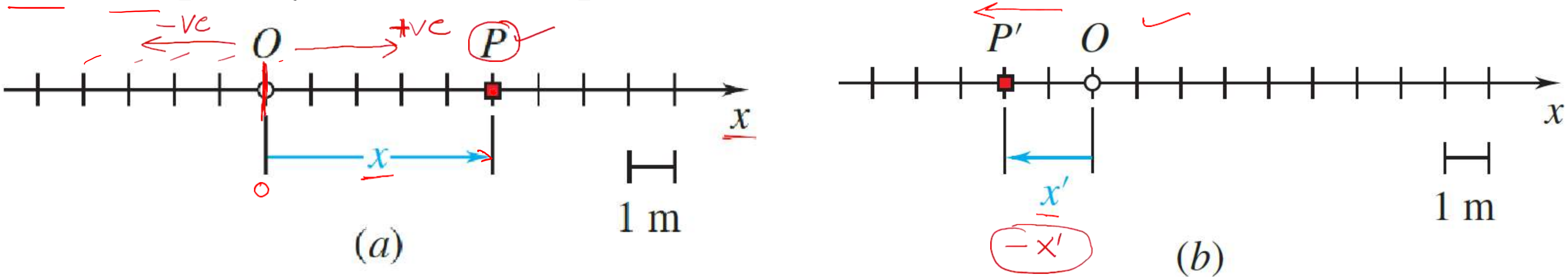
- A body is said to be in motion, if it changes its position, with respect to its surroundings.
- The rate of change of its position w.r.t. its surroundings is called Speed. The speed of a body is irrespective of its direction and is, thus, a scalar quantity.
- The rate of change of displacement, with respect to its surroundings, in a particular direction, is called velocity. As the velocity is always expressed in particular direction, therefore it is a vector quantity.
- The rate of change of its velocity is known as acceleration. It is said to be positive, when the velocity of a body increases with time, and negative when the velocity decreases with time. The negative acceleration is also called retardation. It may be uniform or variable.

Introduction

- If a body moves in such a way that its velocity changes in equal magnitudes in equal intervals of time, it is said to be moving with a uniform acceleration.
- If a body moves in such a way, that its velocity changes in unequal magnitudes in equal intervals of time, it is said to be moving with a variable acceleration.

Rectilinear Motion: Position, Velocity & Acceleration

- A particle moving along a straight line is said to be in **rectilinear motion**.
- The only variables we need to describe this motion are the time, t, and the distance along the line, x, as a function of time.
- With these variables, we can define the particle's position, velocity, and acceleration, which completely describe the particle's motion.



Rectilinear Motion: Position, Velocity & Acceleration

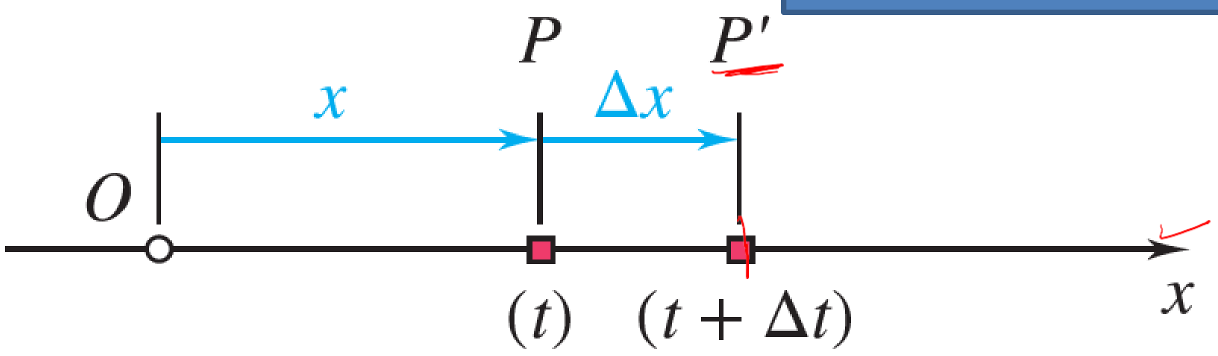
Average velocity $= \dfrac{\Delta x}{\Delta t}$ (m/s)

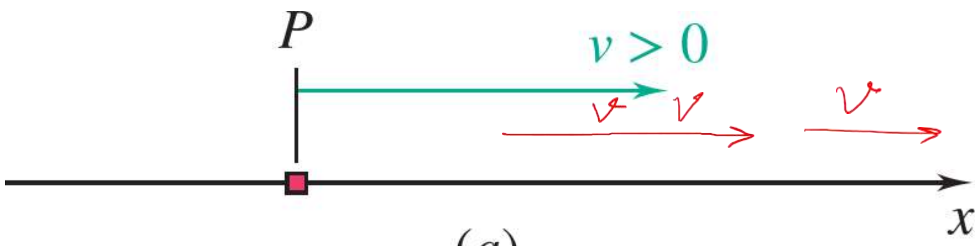
Instant velocity $= \lim\limits_{t \to 0} \dfrac{\Delta x}{\Delta t}$

$$\boxed{v = \frac{dx}{dt}}$$

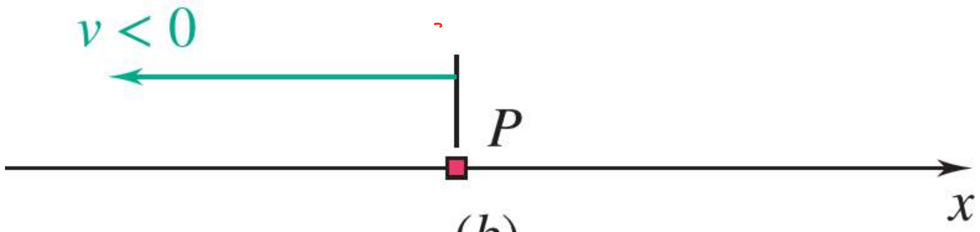
+ve v indicates, x is increasing.

-ve value v indicates x is decreasing



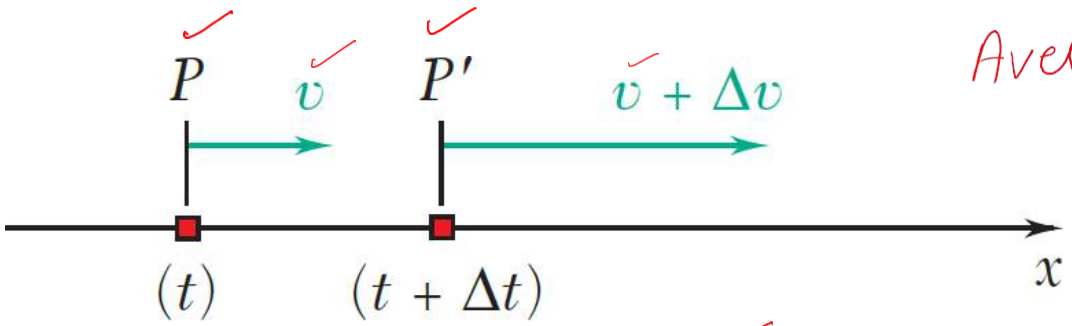


(a)



(b)

Rectilinear Motion: Position, Velocity & Acceleration



Average acceleration $= \frac{\Delta v}{\Delta t}$

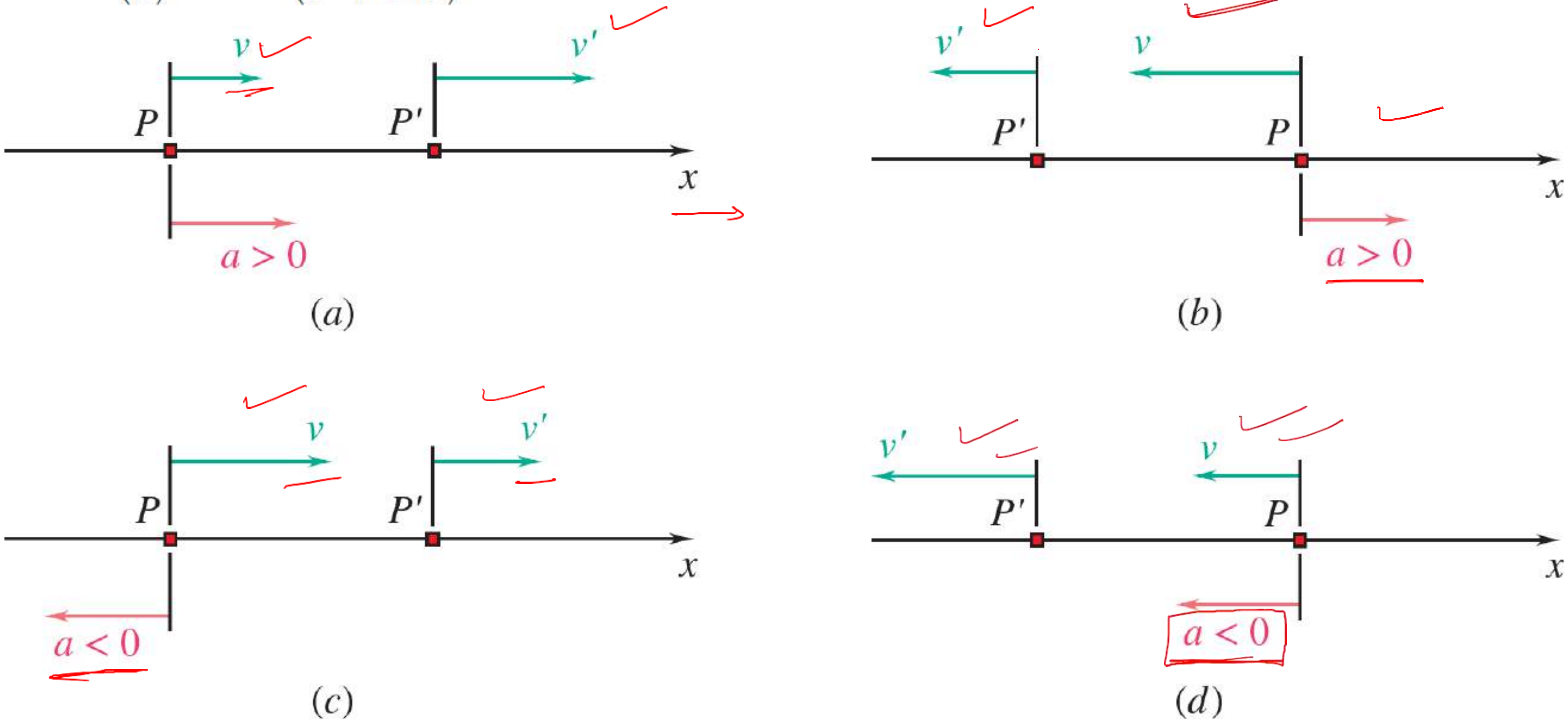
Instant acc. $(a) = \lim_{t \to 0} \frac{\Delta v}{\Delta t}$

$\boxed{a = \frac{dv}{dt}}$ m/s^2

$\boxed{a = \frac{d^2x}{dt^2}}$

$\boxed{a = \frac{dv}{dx} \times \frac{dx}{dt}}$

$\boxed{a = v \times \frac{dv}{dx}}$



Determining the Motion of a Particle

- Recall, *motion* is defined if position x is known for all time t.

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2x}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

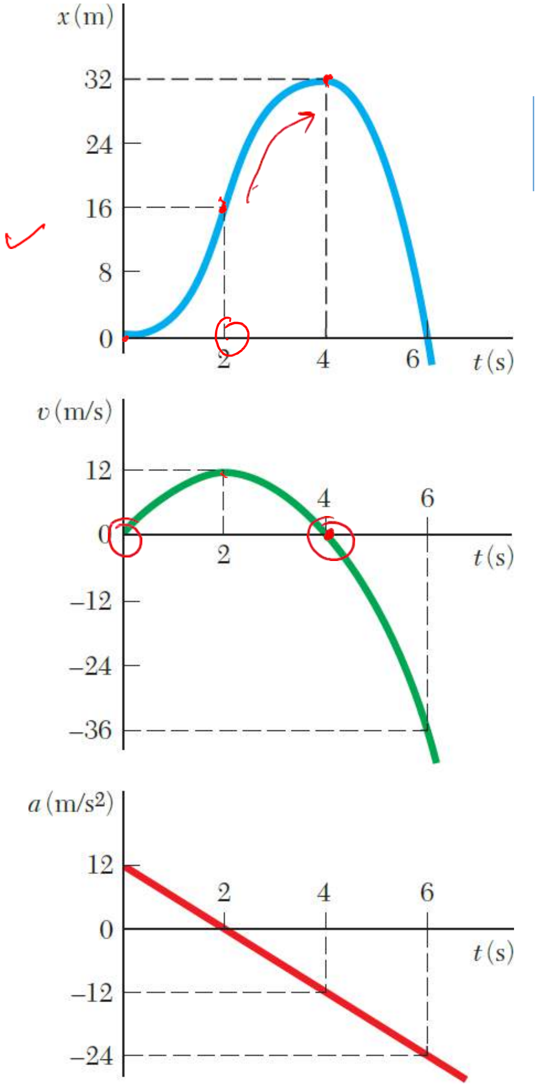
- If the acceleration is given, we can determine velocity and position by two successive integrations.
- Three classes of motion may be defined for:
  - acceleration given as a function of *time*, $a = f(t)$ v, x
  - acceleration given as a function of *position*, $a = f(x)$, v, t
  - acceleration given as a function of *velocity*, $a = f(v)$, x, t

Example: Let Position of particle is defined by.

$$x = 6t^2 - t^3$$

then $v = \frac{dx}{dt} = 12t - 3t^2$

also $a = \frac{dv}{dt} = 12 - 6t$



at $t = 0$; $x = 0\,m$; $v = 0\,m/s$; $a = 12\,m/s^2$

at $t = 2s$; $x = 16m$; $v = 12\,m/s$; $a = 0\,m/s^2$

at $t = 4s$; $x = 32m$; $v = 0\,m/s$; $a = -12\,m/s^2$

at $t = 6s$ $x = 0\,m$; $v = -36\,m/s$, $a = -24\,m/s^2$

Determining the Motion of a Particle

- **Acceleration given as a function of *time*, $a = f(t)$:**

$$v = \frac{dx}{dt} \quad \Rightarrow dx = v\,dt \quad \Rightarrow \int^{x} dx = \quad {}^{t}$$

- **Acceleration given as a function of *position*, $a = f(x)$:**

$$a = f(x) = v\frac{dv}{dx} \quad \Rightarrow v\,dv = f(x)\,dx \quad \Rightarrow \int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx \quad \Rightarrow \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\,dx$$

$$v = \frac{dx}{dt} \quad \Rightarrow \frac{dx}{v} = dt \quad \Rightarrow \int_{x_0}^{x} \frac{dx}{v} = \int_{0}^{t} dt$$

Determining the Motion of aParticle

- **Acceleration given as a function of velocity, *a* = *f*(*v*):**

$$a = f(v) = \frac{dv}{dt} \quad \Rightarrow \frac{dv}{\quad} = dt \quad \Rightarrow {}^{v} \quad dv \quad {}^{t} \quad {}^{v}$$

$$a = f(v) = v\frac{dv}{dx} \quad \Rightarrow dx = \frac{vdv}{f(v)} \quad \Rightarrow \int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{vdv}{f(v)} \quad \Rightarrow x - x_0 = \int_{v_0}^{v} \frac{vdv}{f(v)}$$

Uniform Rectilinear Motion

Uniform rectilinear motion ⇨ acceleration = 0 ⇨ velocity = constant

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} v\,dt$$

$$\boxed{x - x_0 = vt}$$

$$\boxed{x = x_0 + vt}$$

$x \rightarrow$ final Position.
x_0 = initial Position
v = final velocity
v_0 = initial velocity.
a = acceleration
t = time

Uniformly Accelerated Rectilinear Motion

acceleration = constant

Uniformly accelerated motion ⇒

$$\frac{dv}{dt} = a = \text{const} \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt \Rightarrow \boxed{v - v_0 = at} \Rightarrow \boxed{v = v_0 + at}$$

$$\frac{dx}{dt} = v_0 + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t (v_0 + at)\,dt = \boxed{x - x_0 = v_0 t + \frac{1}{2}at^2} \Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$$v\frac{dv}{dx} = a = \text{const} \Rightarrow \int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a\,dx \Rightarrow \frac{v^2 - v_0^2}{2} = a(x - x_0) = \boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

OR

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

A car starting from rest is accelerated at the rate of 0.4 m/s². Find the distance covered by the car in 20 seconds.

Sol: $v_0 = 0$; $a = 0.4\,m/s^2$; $t = 20\,s$, $x - x_0 = ?$

$$x - x_0 = \cancel{v_0}^{\,0} t + \frac{1}{2} a t^2$$

$$x - x_0 = \frac{0.4 \times 20^2}{2}$$

$$\boxed{x - x_0 = 80\,m}$$ Ans

A train travelling at 27 km/h is accelerated at the rate of 0.5 m/s². What is the distance travelled by the train in 12 seconds ?

Sol: $v_0 = 27\ km/h = 7.5\ m/s$; $a = 0.5\ m/s^2$; $t = 12\ s.$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$= (7.5 \times 12) + \frac{1}{2} \times 0.5 \times 12^2$$

$$\boxed{x - x_0 = 126\ m}$$

A scooter starts from rest and moves with a constant acceleration of 1.2 m/s². Determine its velocity, after it has travelled for 60 m.

Sol: $v_0 = 0$; $a = 1.2\ m/s^2$, $x - x_0 = 60\ m$

$$v^2 - \cancel{v_0^2}^{\,0} = 2a(x - x_0)$$

$$v = \sqrt{2 \times 1.2 \times 60}$$

$$\boxed{v = 12\ m/s}$$

On turning a corner, a motorist rushing at 20 m/s, finds a child on the road 50 m ahead. He instantly stops the engine and applies brakes, so as to stop the car within 10 m of the child. Calculate (i) retardation, and (ii) time required to stop the car.

$$\cancel{v^2}^{0} - v_0^2 = 2a(x - x_0)$$

$$a = \frac{-20^2}{2 \times 40} = -5\ m/s^2 \quad \text{Ans}$$

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = 4s \quad \text{Ans}$$

A motor car takes 10 seconds to cover 30 meters and12 seconds to cover 42 m. Find the uniform acceleration of the car and its velocity at the end of 15 s.

Sol: at $t = 10\,s$ $\quad x - x_0 = 30\,m$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$\boxed{30 = 10 v_0 + 50 a} \quad -(1)$$

at $t = 12\,s$ $\quad x - x_0 = 42\,m.$

$$\boxed{42 = 12 v_0 + 72 a} \quad -(2)$$

(1) × 6 − (2) × 5 →

$$30 = 60 a.$$

$$\boxed{a = 0.5\ m/s^2}$$

$$v_0 = 0.5\ m/s.$$

$$v = v_0 + at$$

$$v = 0.5 + (0.5 \times 15)$$

$$\boxed{v = 8\ m/s}$$

A motorist enters a freeway at 45 km/h and accelerates uniformly to 99 km/h. From the odometer in the car, the motorist knows that she traveled 0.2 km while accelerating. Determine (*a*) the acceleration of the car, (*b*) the time required to reach 99 km/h.

Sol: $v_0 = 45\,km/h = 12.5\ m/s \mid x_0 = 0$

$v = 99\,km/h = 27.5\ m/s \mid x = 200\,m.$

$$v^2 - v_0^2 = 2a(x - \cancel{x_0}).$$

$$a = \frac{v^2 - v_0^2}{2x} = \frac{27.5^2 - 12.5^2}{2 \times 200}$$

$$\boxed{a = 1.5\ m/s^2}$$

$$v = v_0 + at$$

$$t = \frac{27.5 - 12.5}{1.5}$$

$$\boxed{t = 10\,s}$$

A stone is thrown upwards with a velocity of 4.9 m/s from a bridge. If it falls down in water after 2 s, then find the height of the bridge from the water level.

Sol.

$v = v_0 + at$

$0 = 4.9 - 9.81\,t$

$\boxed{t = 0.5\,s.}$

in $t = 0.5$ hight gained

$y - y_0 = v_0 t + \frac{1}{2}at^2$

$h_1 = 1.22\,m$

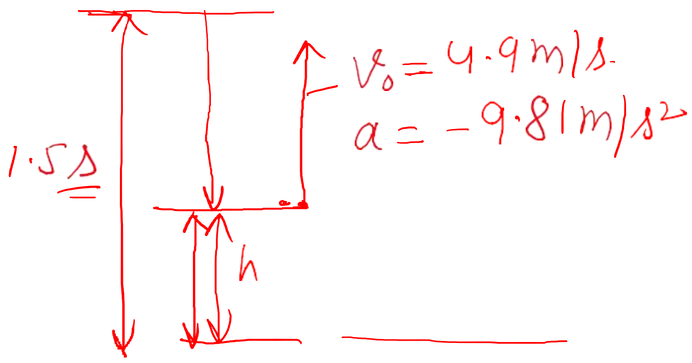
$h_2 = \cancel{v_0 t} + \frac{1}{2}at^2$

$= \qquad -4.905 \times 1.5^2$

$h_2 = 11.03\,m$

$h_2 - h_1 = 11.03 - 1.22\,m$

$\boxed{h_2 - h_1 = 9.82\,m}$



OR

height of bridge $= v_0 t + \frac{1}{2}at^2 = (4.9 \times 2) - 4.905 \times 2^2$

$= 9.82\,m$

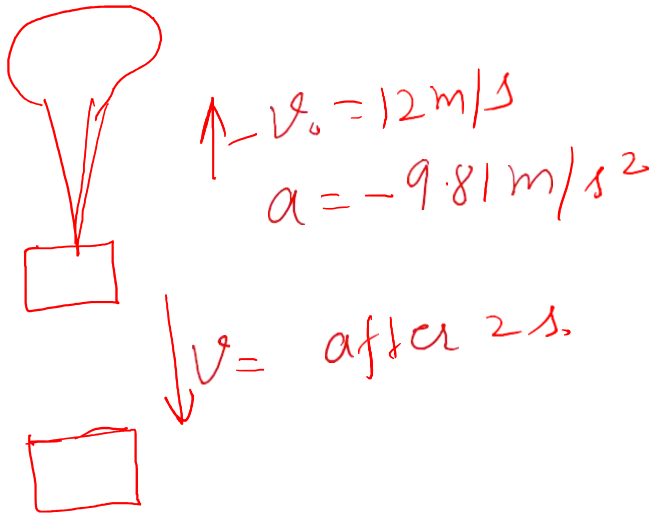
A packet is dropped from a balloon which is going upwards with a velocity 12 m/s. Calculate the velocity of the packet after 2 seconds.

Sol.

$$v = v_0 + at$$

$$= 12 - 9.81 \times 2$$

$$\boxed{v = -7.62\ m/s}$$ Ans



A bullet is fired vertically upwards with a velocity of 80 m/s. To what height will the bullet rise above the point of projection ?

$$\cancel{v^2}^{0} - v_0^2 = 2a(y - y_0)$$

$$-80^2 = 2(-9.81)(y - y_0)$$

$$\boxed{y - y_0 = 326.2\ m}$$ Ans

A body is released from a great height falls freely towards earth. Another body is released from the same height exactly one second later. Find the separation between both the bodies, after two seconds of the release of second body.

Sol: Total flight time of $B_2 = 2\,s$.

$B_1 = 3\,s$

$-y_1 = \cancel{v_0}^{0} t + \frac{1}{2}at^2$

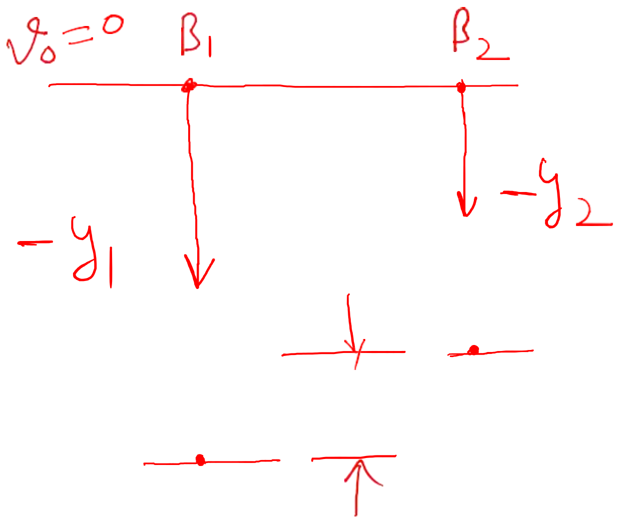
$= -4.905 \times 3^2$

$+y_1 = +44.145\ m$

$-y_2 = \cancel{v_0}^{0} t + \frac{1}{2}at^2$

$-y_2 = -4.905 \times 2^2$

$+y_2 = +19.62\ m$



$\boxed{y_1 - y_2 = 24.525\,m}$ Ans

A stone is thrown vertically upwards, from the ground, with a velocity 49 m/s. After 2 seconds, another stone is thrown vertically upwards from the same place. If both the stone strike the ground at the same time, find the velocity, with which the second stone was thrown upwards.

Sol: for S_1 at max height $v = 0$

$v = v_0 + at$

$0 = 49 - 9.81t$

$\boxed{t = 5s}$

in another 5s stone 1 reaches to ground.

Total flight time of $S_1 = 10s$

Total flight time for stone 2 = 8s

time to gain max height = 4s

$\cancel{v}^{0} = v_0 + at$

$0 = v_0 - 9.81 \times 4$

$\boxed{v_0 = 39.24 \text{ m/s}}$ Ans

A stone is dropped from the top of a tower 50 m high. At the same time, another stone is thrown upwards from the foot of the tower with a velocity of 25 m/s. When and where the two stones cross each other ?

Sol.

$-y_1 = \cancel{v_0}\, t^{0} + \frac{1}{2} a t^2$

$\boxed{\cancel{+} y_1 = \cancel{+} 4.905 t^2}$

$y_2 = v_0 t + \frac{1}{2} a t^2$

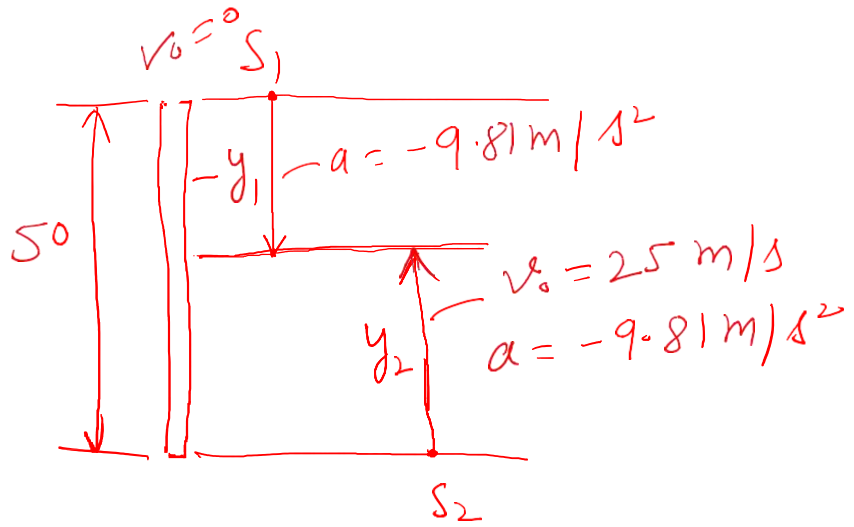
$50 - y_1 = 25t - 4.905 t^2$

$50 - 4.905 t^2 = 25t - 4.905 t^2$

$\boxed{t = 2\,s}$

$y_1 = 4.905 \times 2^2$

$\boxed{y_1 = 19.62\,m}$ or $\boxed{y_2 = 30.38\,m}$ Ans



A stone, dropped into a well, is heard to strike the water after 4 seconds. Find the depth of well, if velocity of the sound is 350 m/s.

Sol.

$-h = \cancel{v_0 t}^{\,0} + \frac{1}{2} a t^2$

$\boxed{\cancel{-}h = +4.905\, t^2}$

Distance travelled by sound = Depth of well

$v_s t_s = 4.905\, t^2$

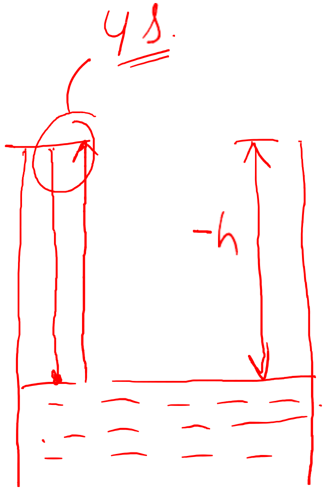
$t_s = \dfrac{4.905\, t^2}{350} = 0.014\, t^2$

$t + t_s = 4$

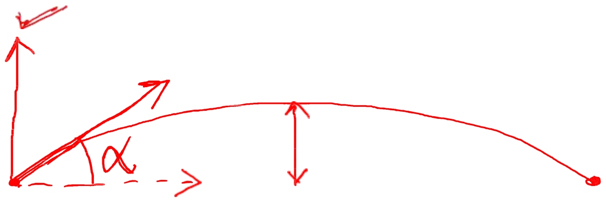
$t + 0.014\, t^2 = 4$

$\boxed{t = 3.8\,s}$

$\boxed{h = 4.905 \times 3.8^2 = 70.82\, m}$

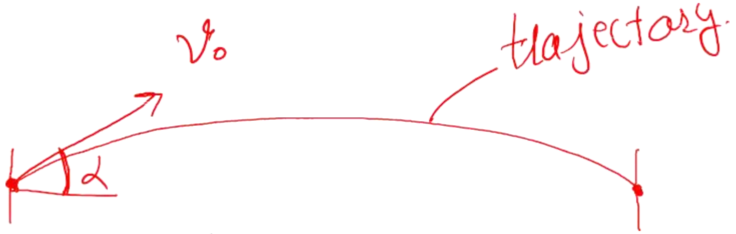


INTRODUCTION



- Whenever a particle is projected upwards at a certain angle (but not vertical), we find that the particle traces some path in the air and falls on the ground at a point, other than the point of projection.
- If we study the motion of the particle, we find that the velocity, with which the particle was projected, has two components namely vertical and horizontal.
- The function of the vertical component is to project the body vertically upwards, and that of the horizontal is to move the body horizontally in its direction.
- The combined effect of both the components is to move the particle along a parabolic path. A particle, moving under the combined effect of vertical and horizontal forces, is called a *projectile*.
- It may be noted that the vertical component of the motion is always subjected to gravitational acceleration, whereas the horizontal component remains constant.

IMPORTANT TERMS



- *Trajectory.* The path, traced by a projectile in the space, is known as trajectory.
- *Velocity of projection.* The velocity, with which a projectile is projected, is known as the velocity of projection.
- *Angle of projection.* The angle, with the horizontal, at which a projectile is projected, is known as the angle of projection.
- *Time of flight.* The total time taken by a projectile, to reach maximum height and to return back to the ground, is known as the time of flight.
- *Range.* The distance, between the point of projection and the point where the projectile strikes the ground, is known as the *range*. It may be noted that the range of a projectile may be horizontal or inclined.

An aircraft, moving horizontally at 108 km/hr at an altitude of 1000 m towards a target on the ground, releases a bomb which hits it. Estimate the horizontal distance of the aircraft from the target, when it released the bomb. Calculate also the direction and velocity with which the bomb hits the target. Neglect air friction.

Sol. $v_x = 108\ km/h = 30\ m/s$

$v_{oy} = 0\ m/s$; $h = -1000\ m.$

$h = \cancel{v_{oy}t}^{\,0} + \frac{1}{2}at^2$

$-1000 = -4.905\,t^2$

$\boxed{t = 14.3\,s}$

$x - x_0 = v_x \times t$

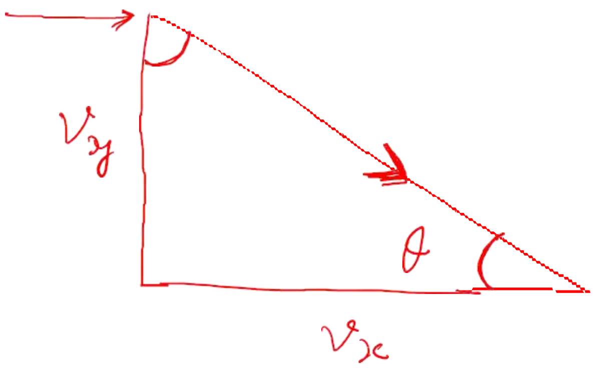
$= 30 \times 14.3$

$\boxed{x - x_0 = 429\ m}$

$v_y = \cancel{v_{oy}}^{\,0} t + at$

$= -9.81 \times 14.3$

$\boxed{v_y = -140.28\ m/s}$

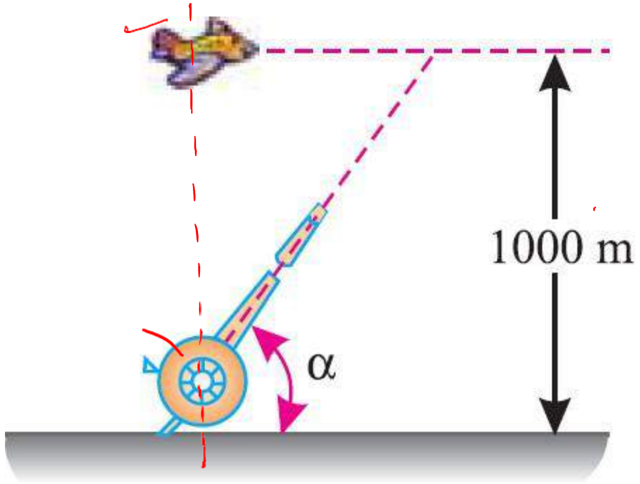


$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = -77.92$ with horizontal.

Resultant velocity $= \sqrt{v_x^2 + v_y^2}$

$= 143.45\ m/s$ Ans

An aeroplane is flying on a straight level course at 200 km per hour at a height of 1000 metres above the ground. An anti-aircraft gun located on the ground fires a shell with an initial velocity of 300 m/s, at the instant when the plane is vertically above it. At what inclination, to the horizontal, should the gun be fired to hit the plane ? What time after firing, the gun shell will hit the plane ? What will then be the horizontal distance of the plane from the gun ?



Sol: for Plane

$$\boxed{AB = V_x \times t = 55.55\, t} \quad -(1)$$

for Shell

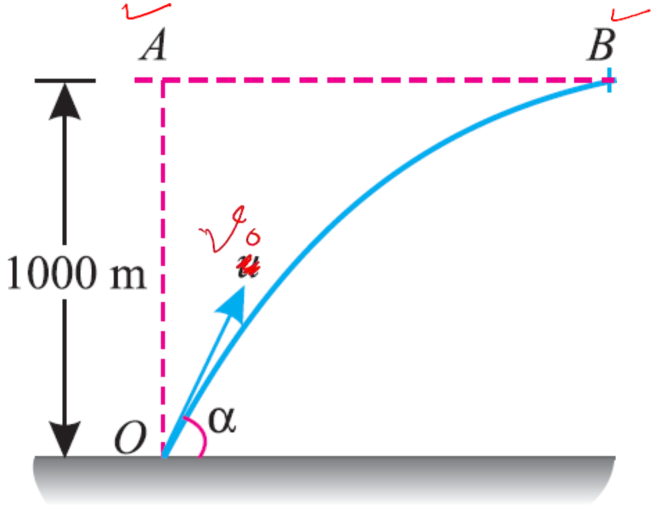
$$V_{ox} = V_0 \cos\alpha = 300 \cos\alpha$$

$$AB = V_{ox} \times t = 300 \cos\alpha \cdot t \quad -(2)$$

equating (1) & (2)

$$300 \cos\alpha \cdot t = 55.55\, t$$

$$\boxed{\alpha = 79.33^\circ}$$



$v_{oy} = v_s \sin\alpha = 300 \sin 79.33 = 295$ m/s

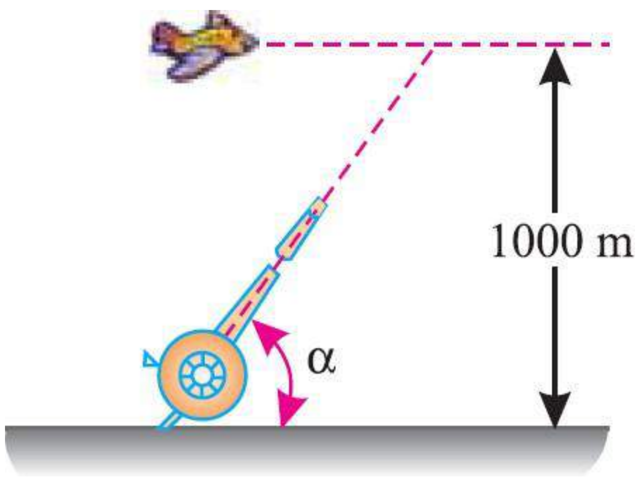
Rise of shell $OA = v_{oy}\, t + \frac{1}{2} a t^2$

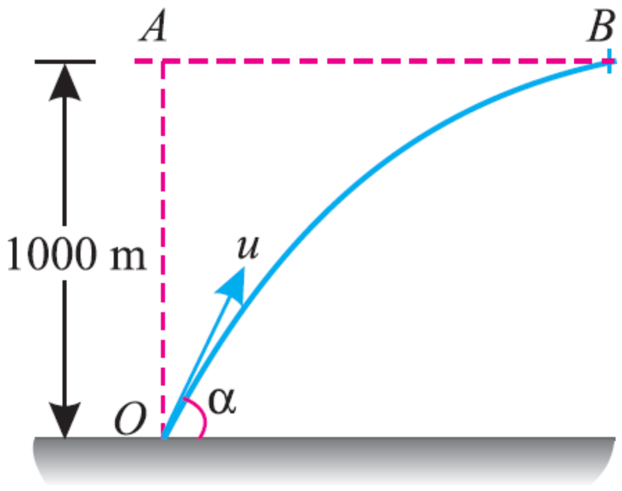
$$1000 = 295t - 4.905t^2$$

$$\boxed{t = 3.6\ s}$$

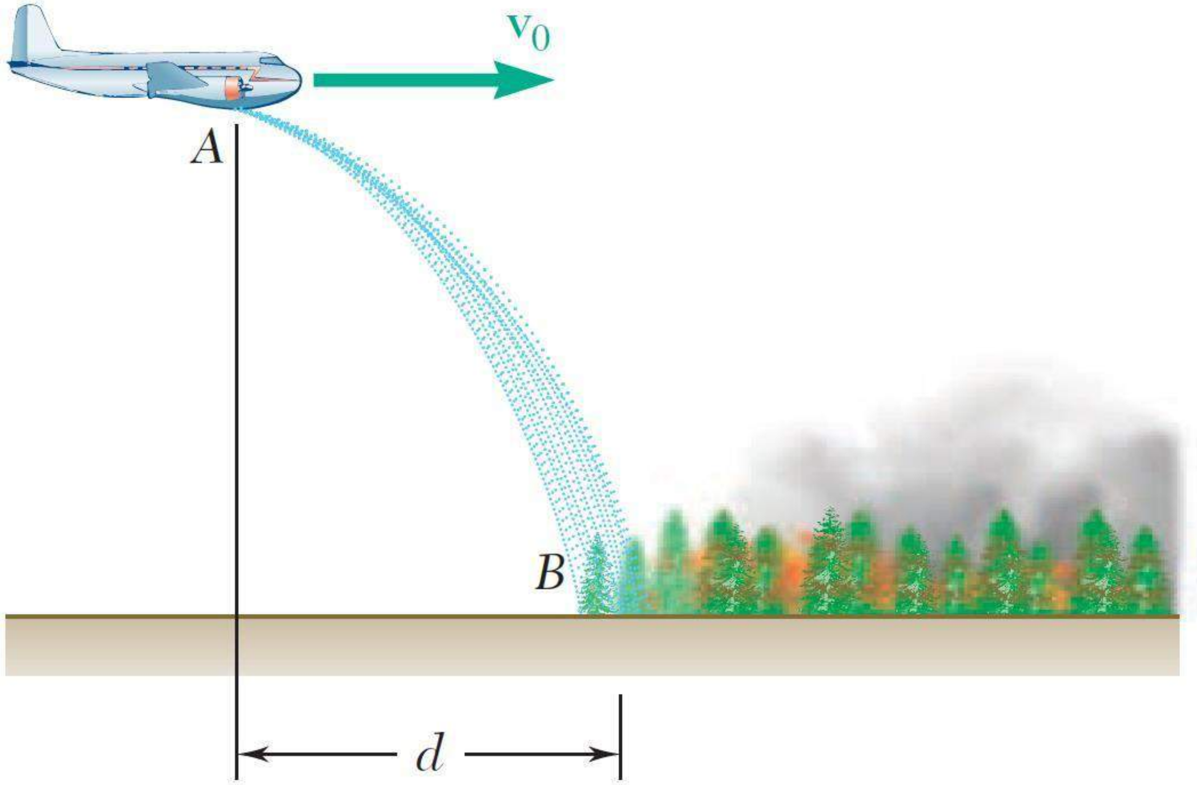
$$AB = 55.55 \times 3.6$$

$\boxed{AB = 200\ m}$ Ans





An airplane used to drop water on brushfires is flying horizontally in a straight line at 315 km/h at an altitude of 80 m. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.



Sol.

$$h = \cancel{v_{0y}}\overset{0}{t} + \frac{1}{2} a t^2$$

$$-80 = -4.905\, t^2$$

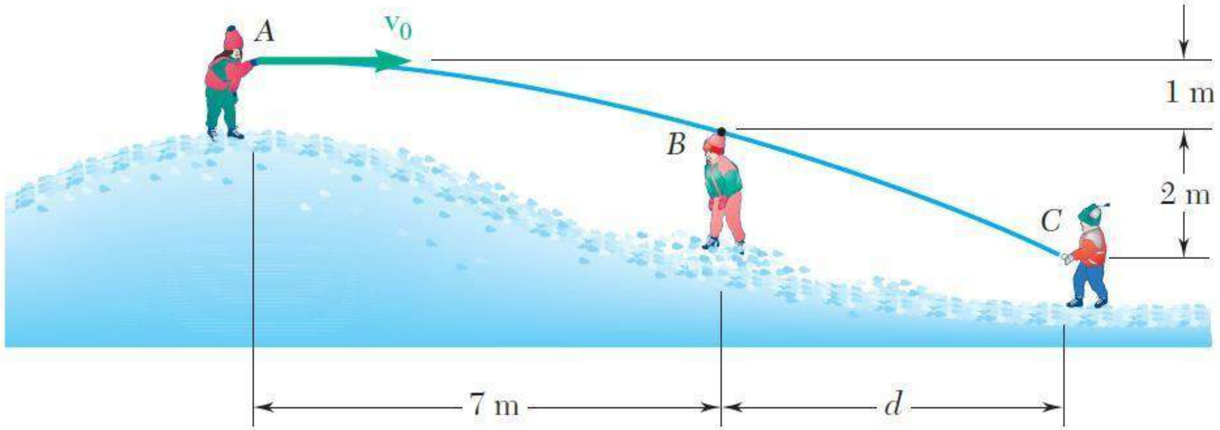
$$\boxed{t = 4.03\, s}$$

$$d = v_{0x} \times t$$

$$= 87.5 \times 4.03$$

$$\boxed{d = 353\, m}$$ Ans

Three children are throwing snowballs at each other. Child A throws a snowball with a horizontal velocity $\mathbf{v}_0$. If the snowball just passes over the head of child B and hits child C, determine (*a*) the value of v_0, (*b*) the distance d.



Sol: Vertical motion AB

$$y - y_0 = v_{0y}t + \frac{1}{2}at^2$$

$$-1 = -4.905\,t^2$$

$$\boxed{t_{AB} = 0.45\,s}$$

Horizontal motion AB

$$x - x_0 = v_0 t$$

$$7 = v_0 \times 0.45$$

$$\boxed{v_0 = 15.5\ m/s}$$

Vertical motion AC

$$-3 = -4.905\,t^2$$

$$\boxed{t_{AC} = 0.78\,s}$$

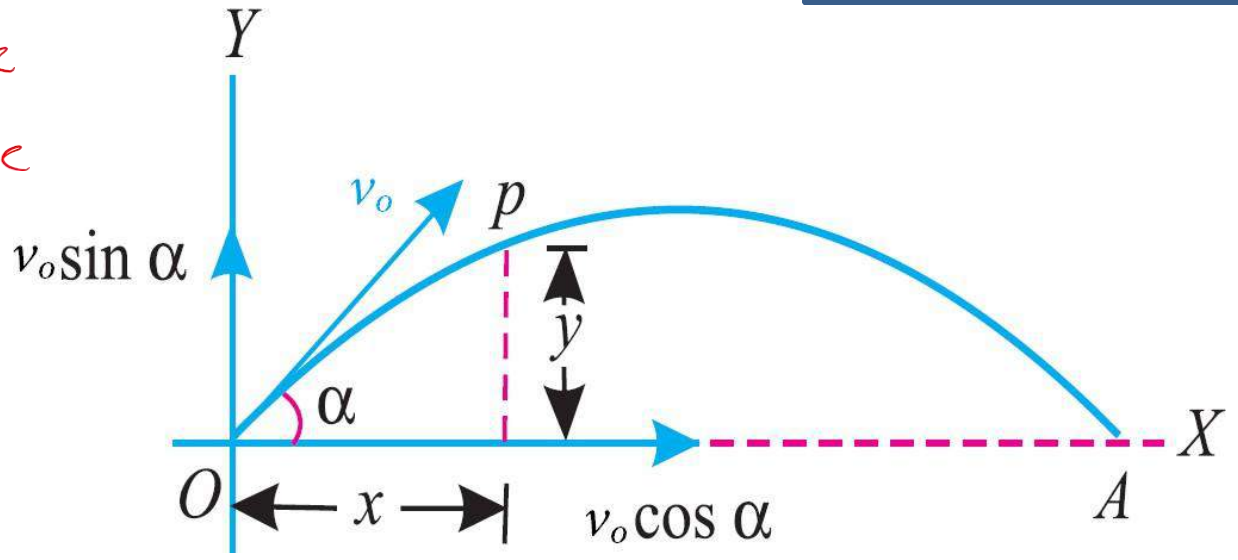
Horizontal motion AC

$$7 + d = 15.5 \times 0.75$$

$$\boxed{d = 5.129\ m}$$ Ans

MOTION OF A PROJECTILE

Cosider a particle projected from O to A at a Certain angle 'α' with horizontal. The particle will move along a certain path OPA in the air and finally fall down at A.



Now Horizontal velocity component

$$V_{0x} = V_0 \cos\alpha.$$

vertical velocity component

$$V_{0y} = V_0 \sin\alpha.$$

$$\left[\begin{array}{l} a = -9.81\ m/s^2 \\ a = -g \end{array}\right]$$

$$y = V_{0y} t + \frac{1}{2} a t^2$$

$$\boxed{y = V_0 \sin\alpha . t - \frac{1}{2} g t^2} \quad - (1)$$

MOTION OF A PROJECTILE

$x = V_{0x} t = V_0 \cos\alpha \cdot t$

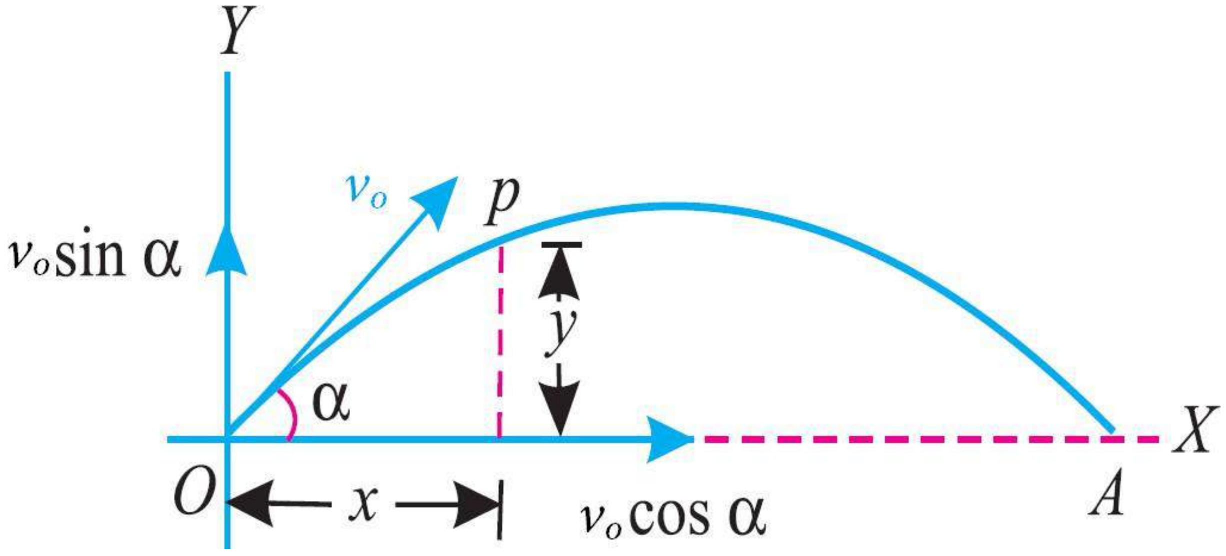
$$\boxed{t = \frac{x}{V_0 \cos\alpha}}$$

$$\therefore y = V_0 \sin\alpha \cdot \left(\frac{x}{V_0 \cos\alpha}\right) - \frac{1}{2} \times g \times \left(\frac{x}{V_0 \cos\alpha}\right)^2$$

$$\boxed{y = x \tan\alpha - \frac{g x^2}{2 V_0^2 \cos^2\alpha}}$$

This is equation of parabala.

Therefore, the path of projectile is parabala.



MOTION OF A PROJECTILE

again from eqn (1)

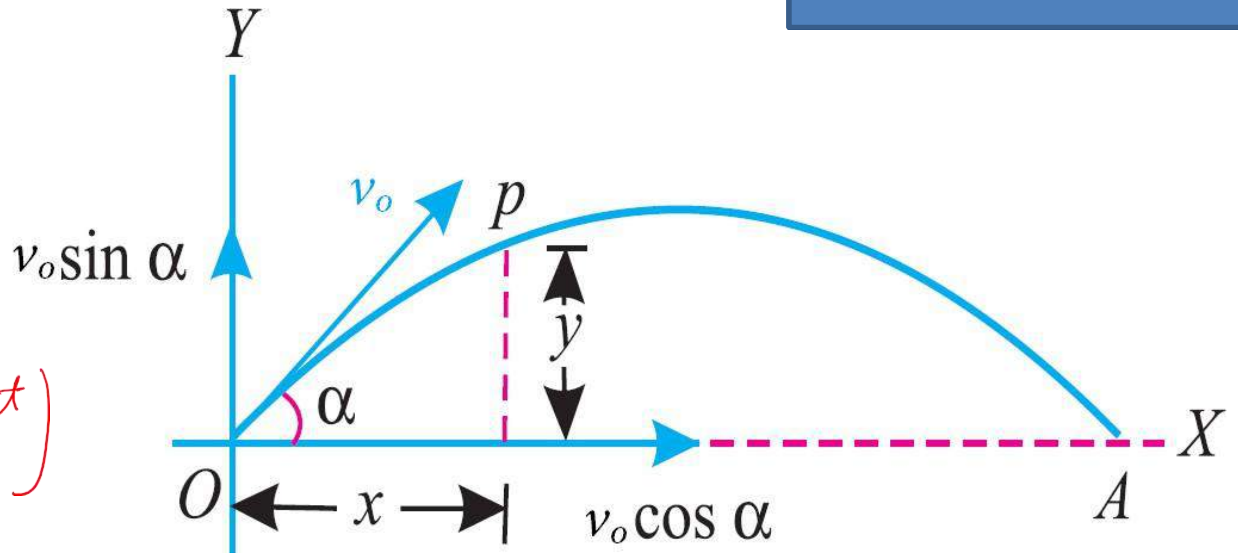
$$y = v_0 \sin\alpha . t - \frac{1}{2} g t^2$$

after completion of path from 'O' to 'A' in time 't', $y = 0$

$$0 = v_0 \sin\alpha . t - \frac{1}{2} g t^2$$

$$\boxed{t = \frac{2 v_0 \sin\alpha}{g}}$$

(Total flight time)



$$x = v_0 \cos\alpha . t = v_0 \cos\alpha \times \frac{2 v_0 \sin\alpha}{g}$$

$$x = \frac{2 v_0^2 \sin\alpha \cos\alpha}{g} \Rightarrow \boxed{x = \frac{v_0^2 \sin 2\alpha}{g}}$$

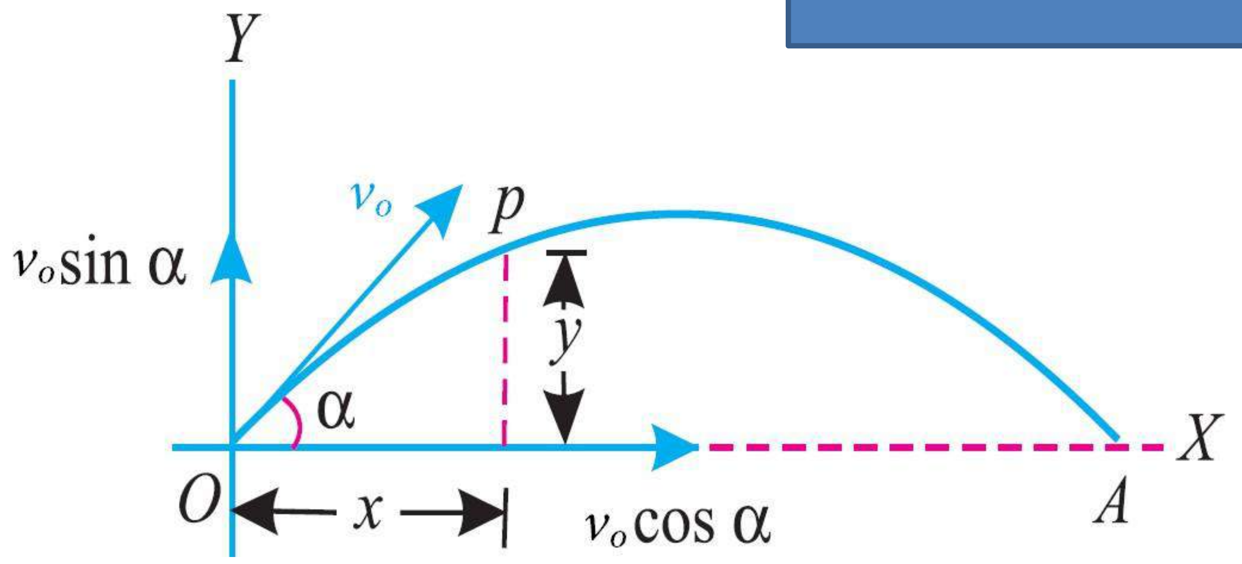
[Horizontal Range of Projectile].

MOTION OF A PROJECTILE

for give velocity of projectile, range will be max. when $\sin 2\alpha = 1$

$2\alpha = 90$ or $\boxed{\alpha = 45^\circ}$

$$x_{max} = \frac{v_0^2 \sin 90}{g} = \frac{v_0^2}{g} \quad \left[\text{Max. Range of Projectile}\right].$$



Now initial vertical velocity component

$v_{0y} = v_0 \sin\alpha$ —(1)

final velocity.

$v_y = 0$ —(2).

Average velocity of (1) & (2).

$$\boxed{v_{avg} = \frac{v_0 \sin\alpha}{2}}$$

MOTION OF A PROJECTILE

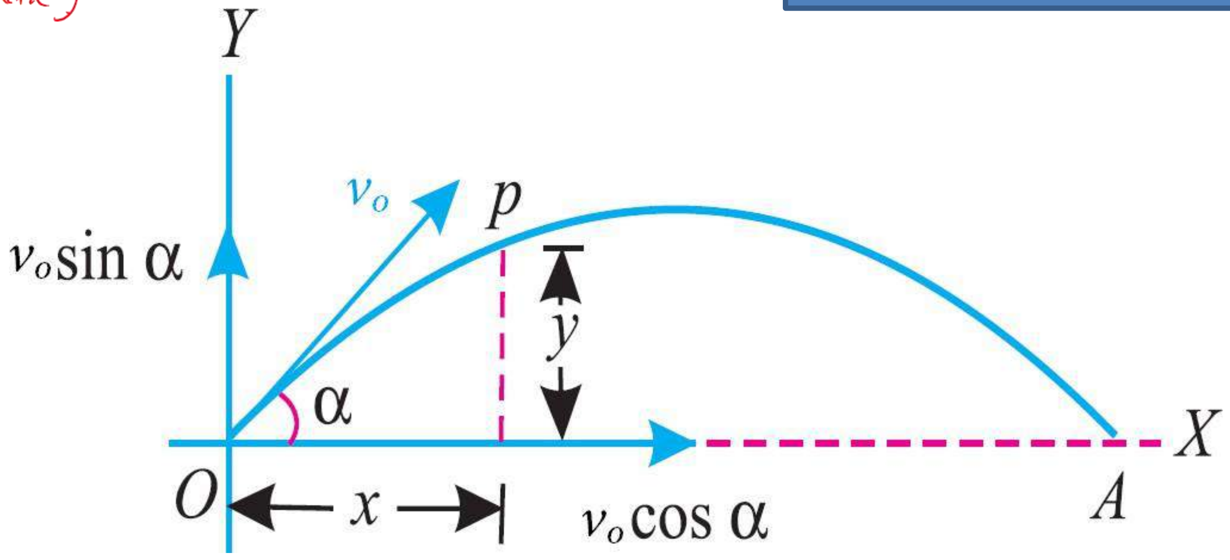
Time required to reach max height (y_{max})

is $\frac{v_0 \sin\alpha}{g}$.

$y_{max} = v_{avg} \times t$.

$= \frac{v_0 \sin\alpha}{2} \times \frac{v_0 \sin\alpha}{g}$.

$\boxed{y_{max} = \frac{v_0^2 \sin^2\alpha}{2g}}$ [Max. height of Projectile].



A projectile is fired upwards at an angle of 30° with a velocity of 40 m/s. Calculate the time taken by the projectile to reach the ground, after the instant of firing.

Sol:

flight time $t = \dfrac{2V_o \sin\alpha}{g}$

$$= \frac{2 \times 40 \sin 30}{9.81}$$

$$\boxed{t = 4.077 \, s}$$

A ball is projected upwards with a velocity of 15 m/s at an angle of 25° with the horizontal. What is the horizontal range of the ball ?

Sol:

Horizontal Range $x = \frac{V_0^2 \sin 2\alpha}{g} = \frac{15^2 \sin(50)}{9.81}$

$\boxed{x = 17.6\ m}$ Ans

A bullet is fired with a velocity of 100 m/s at an angle of 45° with the horizontal. How high the bullet will rise ?

Sol:

$$y_{max} = \frac{V_0^2 \sin^2\alpha}{2g}$$

$$= \frac{100^2 \times \sin^2 45}{2 \times 9.81}$$

$$\boxed{y_{max} = 255\ m}$$

If a particle is projected inside a horizontal tunnel which is 5 m high with a velocity of 60 m/s, find the angle of projection and the greatest possible range.

Sol:

$$y_{max} = \frac{v_0^2 \sin^2\alpha}{2g}$$

$$5 = \frac{60^2 \sin^2\alpha}{2g}$$

$$\boxed{\alpha = 9.5^\circ}$$ Ans

Range $$x = \frac{v_0^2 \sin 2\alpha}{g}$$

$$= \frac{60^2 \sin(2\times 9.5)}{9.81}$$

$$\boxed{x = 119.6\ m}$$ Ans

A body is projected at such an angle that the horizontal range is three times the greatest height. Find the angle of projection.

Sol.

$$\frac{\cancel{v_0^2}\sin 2\alpha}{\cancel{g}} = 3 \times \frac{\cancel{v_0^2}\sin^2\alpha}{2\cancel{g}}$$

$$2\sin\alpha\cos\alpha = 1.5\sin^2\alpha$$

$$2\cos\alpha = 1.5\sin\alpha$$

$$\tan\alpha = \left(\frac{2}{1.5}\right)$$

$$\boxed{\alpha = 53.13^\circ} \text{ Ans}$$

A particle is thrown with a velocity of 5 m/s at an elevation of 60° to the horizontal. Find the velocity of another particle thrown at an elevation of 45° which will have (a) equal horizontal range, (b) equal maximum height, and (c) equal time of flight.

Sol. (a) $\frac{v_{0_1}^2 \sin 2\alpha_1}{\cancel{g}} = \frac{v_{0_2}^2 \sin 2\alpha_2}{\cancel{g}}$

$\boxed{v_{0_2} = 4.65 \text{ m/s}}$

(b) $\frac{v_{0_1}^2 \sin^2\alpha_1}{\cancel{2g}} = \frac{v_{0_2}^2 \sin^2\alpha_2}{\cancel{2g}}$

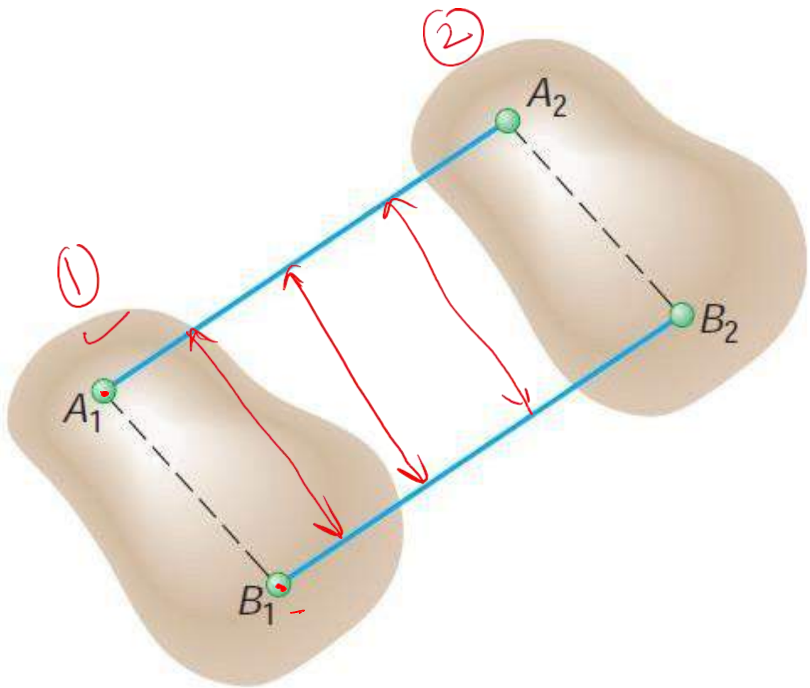
$\boxed{v_{0_2} = 6.12 \text{ m/s}}$

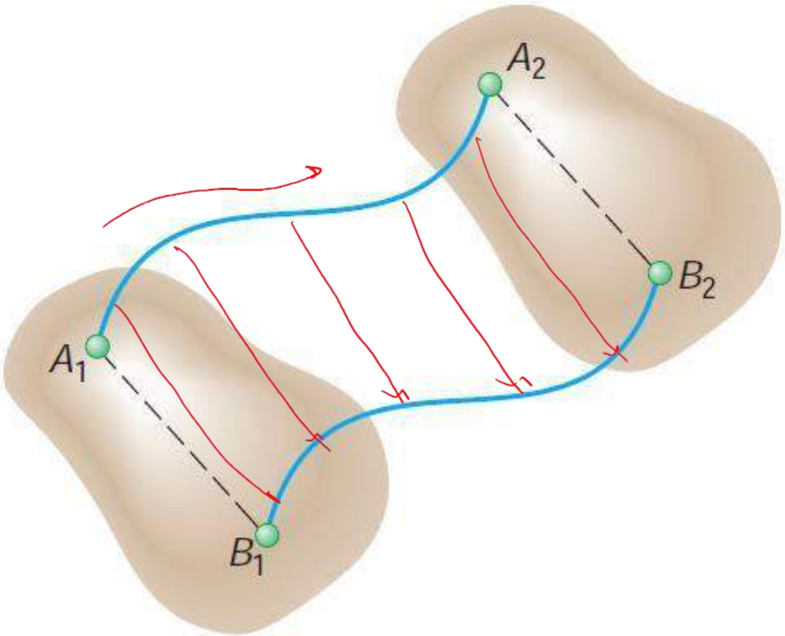
(c) $\frac{\cancel{2} v_{0_1} \sin\alpha_1}{\cancel{g}} = \frac{\cancel{2} v_{0_2} \sin\alpha_2}{\cancel{g}}$

$\boxed{v_{0_2} = 6.12 \text{ m/s}}$

Types of Motion

- *Translation*: In a translation all the particles forming the body move along parallel paths.

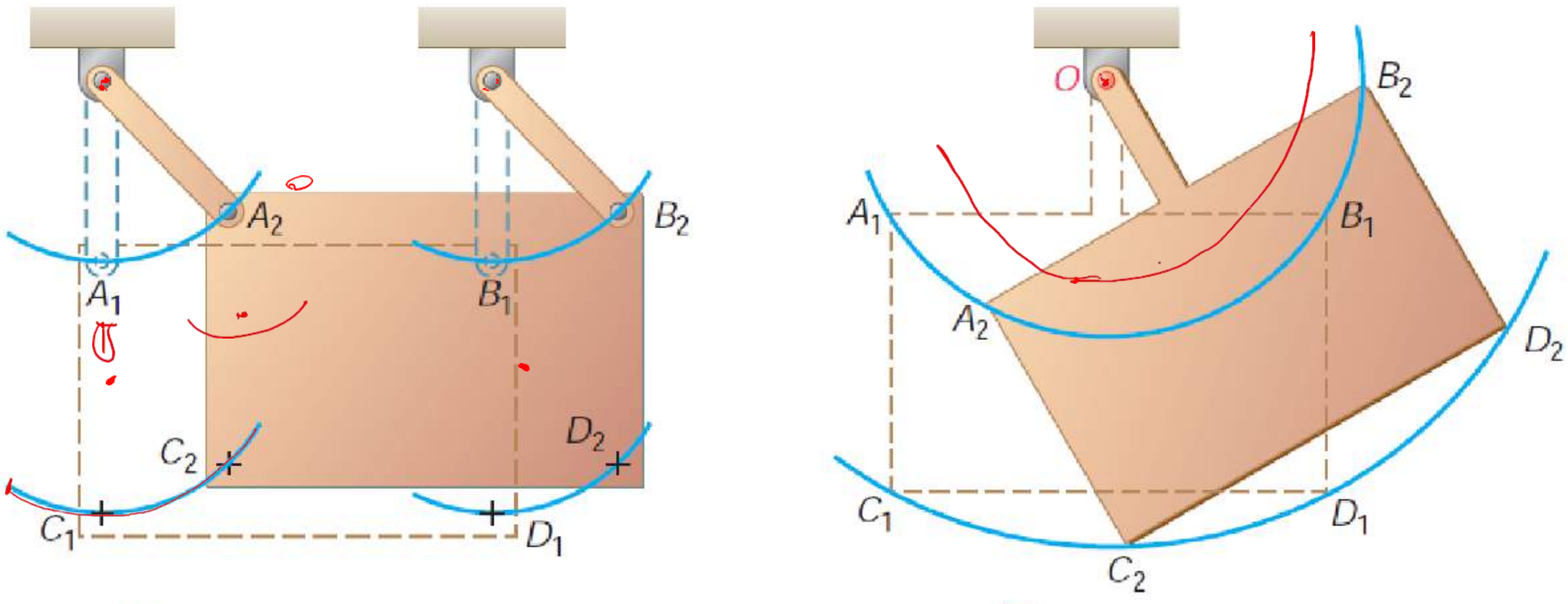




- If the paths of motion of the particles are straight lines, the motion is said to be a *rectilinear translation and* if the paths are curved lines, the motion is a *curvilinear translation*.

Types of Motion

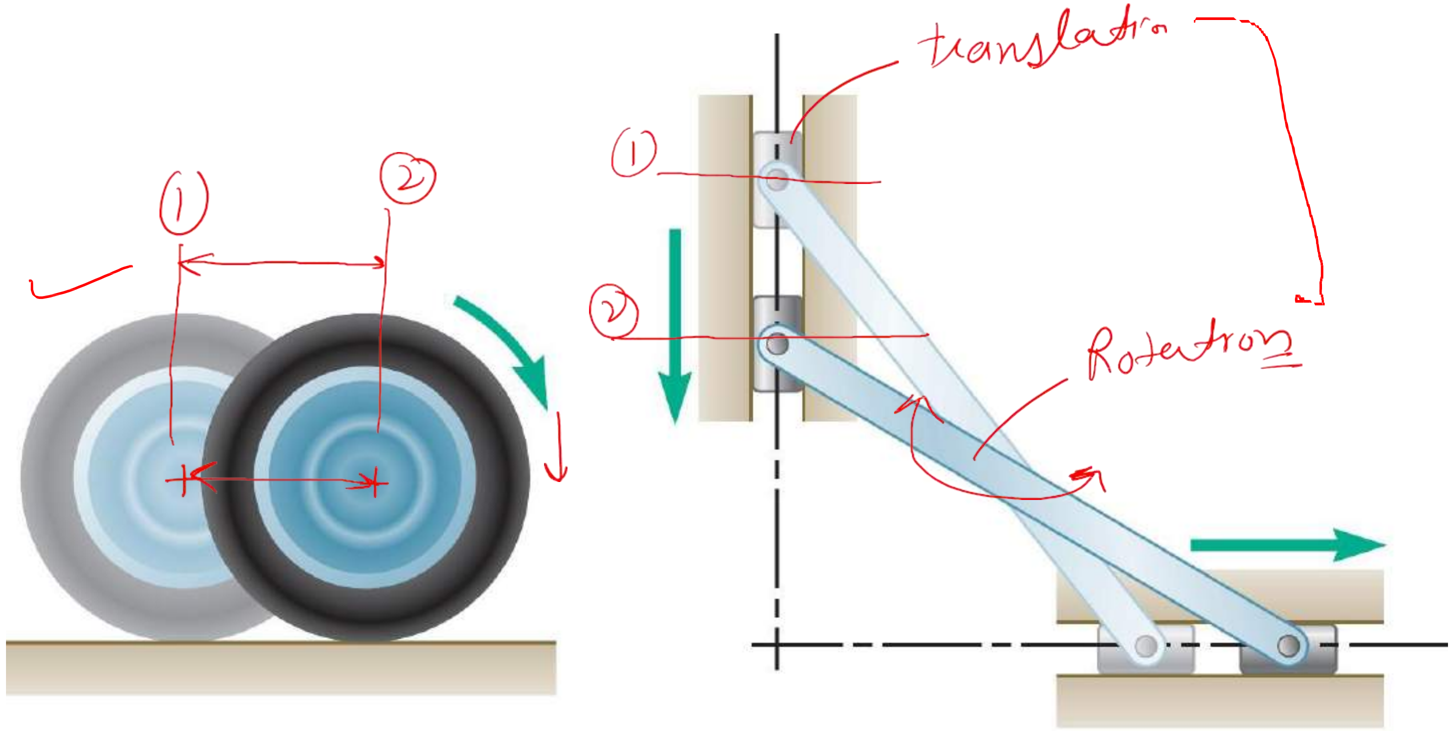
- ***Rotation About a Fixed Axis***: In this motion, the particles forming the rigid body move in parallel planes along circles centered on the same fixed axis.
- Rotation should not be confused with certain types of curvilinear translation.



(a) Curvilinear translation (b) Rotation

Types of Motion

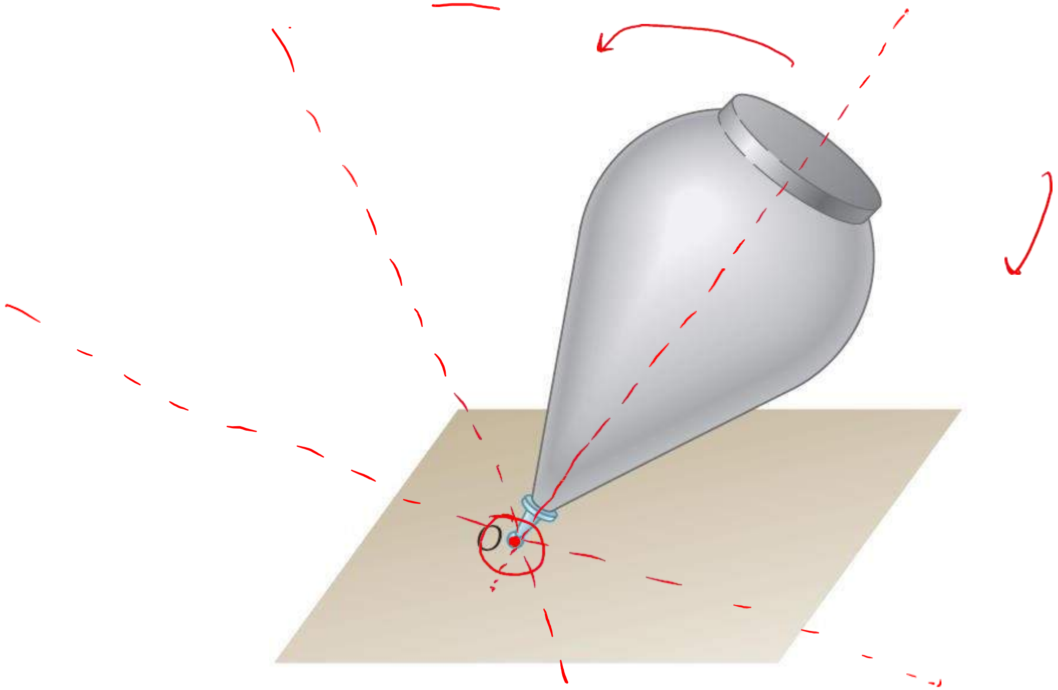
- ***General Plane Motion***: Any plane motion that is neither a rotation nor a translation is referred to as a general plane motion.



(*a*) Rolling wheel (*b*) Sliding rod

Types of Motion

- ***Motion About a Fixed Point***: The three-dimensional motion of a rigid body attached at a fixed point *O*.



- ***General Motion***. Any motion of a rigid body that does not fall in any of the categories above is referred to as a general motion.

Rotational Motion

ROTATION ABOUT A FIXED AXIS

- Some bodies like pulley, shafts, flywheels etc., have motion of rotation (*i.e.*, angular motion) which takes place about the geometric axis of the body.
- The angular velocity of a body is always expressed in terms of revolutions described in one minute, *e.g.*, if at an instant the angular $\boxed{\text{velocity}}$ (speed) of rotating body in N r.p.m. (*i.e.* revolutions per min) the corresponding angular velocity ω (in rad/s) may be found out as:

$$\begin{aligned} 1 \text{ revolution/min} &= 2\pi \text{ rad/min} \\ \therefore \quad N \text{ revolutions/min} &= 2\pi N \text{ rad/min} \\ \text{and angular velocity} \quad \omega &= 2\pi N \text{ rad/min} \\ &= \frac{2\pi N}{60} \text{ rad/sec} \end{aligned}$$

IMPORTANT TERMS

$a =$ α

- ***Angular velocity.*** It is the rate of change of angular displacement of a body, and is expressed in r.p.m. (revolutions per minute) or in radian per second. It is, usually, denoted by ω (omega).
- ***Angular acceleration.*** It is the rate of change of angular velocity and is expressed in radian per second per second (rad/s²) and is usually, denoted by α. It may be constant or variable.
- ***Angular displacement.*** It is the total angle, through which a body has rotated, and is usually denoted by θ. Mathematically, if a body is rotating with a uniform angular velocity (ω) then in *t* seconds, the angular displacement $\boxed{\theta = \omega t}$

ROTATION UNDER CONSTANT ANGULAR ACCELERATION

Consider a particle rotating about its axis:

Let ω_0 = Initial angular velocity (rad/s)

ω = Final angular velocity (rad/s)

t = time taken by the particle to change its velocity from ω_0 to ω (s).

α = Angular acceleration (rad/s^2)

θ = Total angular displacement (rad).

ROTATION UNDER CONSTANT ANGULAR ACCELERATION

Let the angular velocity of the particle has increased uniformly from ω_0 to ω in time 't' at the rate of 'α'

then $\boxed{\omega = \omega_0 + \alpha t}$ or $\boxed{t = \frac{\omega - \omega_0}{\alpha}}$

Average angular velocity $= \frac{\omega_0 + \omega}{2}$

Total angular displacement 'θ' = Avg. angular velocity x time.

$$\theta = \frac{\omega_0 + \omega}{2} \times t = \frac{\omega_0 + \omega_0 + \alpha t}{2} \times t$$

$$\theta = \left(\frac{2\omega_0 + \alpha t}{2}\right) \times t$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$$

OR

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right) \times \left(\frac{\omega - \omega_0}{\alpha}\right)$$

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha\theta}$$

A flywheel starts from rest and revolves with an acceleration of 0.5 rad/ sec². What will be its angular velocity and angular displacement after 10 seconds.

Sol. $\omega_0 = 0$; $\alpha = 0.5\ rad/s^2$, $t = 10s$.

$$\omega = \cancel{\omega_0}^{0} + \alpha t$$

$$\omega = 0.5 \times 10$$

$$\boxed{\omega = 5\ rad/s}$$ Ans

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{5^2}{2 \times 0.5}$$

$$\boxed{\theta = 25\ rad}$$ Ans

$$n = \frac{\theta}{2\pi} = \quad rev.$$

A wheel increases its speed from 45 r.p.m. to 90 r.p.m. in 30 seconds. Find (a) angular acceleration of the wheel, and (b) no. of revolutions made by the wheel in these 30 seconds.

Sol: $N_0 = 45\ rpm \rightarrow \omega_0 = 1.5\pi$ rad/s | $t = 30$ s.

$N = 90\ rpm \rightarrow \omega = 3\pi$ rad/s.

$\omega = \omega_0 + \alpha t$

$3\pi = 1.5\pi + (\alpha \times 30)$

$\alpha = \frac{1.5\pi}{30} = 0.05\pi$ rad/s^2

or

$\boxed{\alpha = 0.157\ rad/s^2}$ Ans

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= (1.5\pi \times 30) + \frac{1}{2}(0.05\pi \times 30^2)$

$\theta = 67.5\pi = 212$ rad.

$n = \frac{\theta}{2\pi} = 33.75$ rev. Ans

A flywheel is making 180 r.p.m. and after 20 sec it is running at 120 r.p.m. How many revolutions will it make and what time will elapse before it stops, if the retardation is uniform ?

Sol: $N_0 = 180$ rpm $\rightarrow$ $\omega_0 = 6\pi$ rad/s | $t = 20$ s.

$N = 120$ rpm $\rightarrow$ $\omega = 4\pi$ rad/s.

$\omega = \omega_0 + \alpha t$

$4\pi = 6\pi + (\alpha \times 20)$

$\boxed{\alpha = -0.1\pi \text{ rad/s}^2}$

OR

$\alpha = -0.314$ rad/s

from 180 rpm – 0 rpm

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$-6\pi^2 = 2 \times (-0.1\pi \times \theta)$

$\theta = 180\pi$ rad.

$n = \frac{\theta}{2\pi} = 90$ rev. Ans

$\omega = \omega_0 + \alpha t$

$0 = 6\pi - 0.1\pi \times t$

$\boxed{t = 60 \text{ s}}$ Ans

A flywheel increases its speed from 30 r.p.m. to 60 r.p.m. in 10 seconds. Calculate (*i*) the angular acceleration ; and (*ii*) no. of revolutions made by the wheel in these 10 seconds.`

Sol. $N_0 = 30$ rpm $\rightarrow \omega_0 = \pi$ rad/s

$N = 60$ rpm $\rightarrow \omega = 2\pi$ rad/s

$t = 10$ s.

$\omega = \omega_0 + \alpha t$

$2\pi = \pi + (\alpha \times 10)$

$\boxed{\alpha = \frac{\pi}{10} \text{ rad/s}^2}$

Ans

$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$

$= \frac{(2\pi)^2 - (\pi)^2}{2 \times \left(\frac{\pi}{10}\right)}$

$\theta = 15\pi$ rad.

$n = \frac{\theta}{2\pi} = 7.5$ rev. Ans

A wheel, starting form rest, is accelerated at the rate of 5 rad/s2 for a period of 10 seconds. It is then made to stop in the next 5 seconds by applying brakes. Find (*a*) maximum velocity attained by the wheel, and (*b*) total angle turned by the wheel.

Sol: $\omega_0 = 0$; $\alpha = 5$ rad/s^2.

$t = 10$ s.

$\omega = \cancel{\omega_0}^{0} + \alpha t$

$= 5 \times 10$

$\boxed{\omega = 50 \text{ rad/s}}$ Ans

$\theta_1 = \dfrac{\omega^2 - \cancel{\omega_0^2}^{0}}{2\alpha}$

$\theta_1 = \dfrac{50^2}{2 \times 5}$

$\boxed{\theta_1 = 250 \text{ rad}}$

for next 5s $\omega_0 = 50$ rad/s ; $\omega = 0$

$\omega = \omega_0 + \alpha t$

$0 = 50 + (\alpha \times 5)$

$\boxed{\alpha = -10 \text{ rad/s}^2}$

$\theta_2 = \dfrac{\cancel{\omega^2}^{0} - \omega_0^2}{2\alpha} = \dfrac{-50^2}{-2 \times 10} = 125$ rad.

Total angle turned

$\theta_1 + \theta_2 = 250 + 125 = 375$ rad. Ans

A wheel is running at a constant speed of 360 r.p.m. At what constant rate, in rad/s^2, its motion must be retarded to bring the wheel to rest in (*i*) 2 minutes, and (*ii*) 18 revolution

Sol. (1) for 120 s. $N_0 = 360$ rpm $-$ $\omega_0 = 12\pi$ rad/s.

$\cancel{\omega}^{0} = \omega_0 + \alpha t$

$0 = 12\pi + \alpha \times 120$

$\boxed{\alpha = -0.1\pi \text{ rad/s}^2}$ Ans

$\boxed{\alpha = -0.314 \text{ rad/s}^2}$

(ii) for 18 rev. $\theta = 36\pi$

$\cancel{\omega^2}^{0} - \omega_0^2 = 2\alpha\theta$

$\alpha = \dfrac{-(12\pi)^2}{2 \times 36\pi} = -2\pi \text{ rad/s}^2$

$\boxed{\alpha = -6.28 \text{ rad/s}^2}$ Ans

A wheel rotating about a fixed axis at 24 r.p.m. is uniformly accelerated for 70 seconds, during which time it makes 50 revolution. Find (*i*) angular velocity at the end of this interval, and (*ii*) time required for the speed to reach 150 r.p.m.

Sol: $N_0 = 24$ rpm $\rightarrow \omega_0 = 0.8\pi$ rad/s ; $t = 70$ s ; $n = 50$ revs $\Rightarrow \theta = 100\pi$

(i) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$100\pi = (0.8\pi \times 70) + \frac{1}{2} \times \alpha \times 70^2$

$\alpha = 0.0179\pi$ rad/s²

$\boxed{\alpha = 0.056 \text{ rad/s}^2}$

$\omega = \omega_0 + \alpha t$

$= 0.8\pi + (0.0179\pi \times 70)$

$\omega = 2.053\pi$ rad/s.

$\boxed{\omega = 6.45 \text{ rad/s}}$ Ans

(ii) for 150 rpm

$\omega = 5\pi$ rad/s.

$\omega = \omega_0 + \alpha t$

$5\pi = 0.8\pi + (0.0179\pi \times t)$

$t = 234.63$ s

$t = 3$m 54.63 s. Ans

A wheel rotates for 5 seconds with a constant angular acceleration and describes 80 radians. It then rotates with a constant angular velocity in the next 5 seconds and describes 100 radians. Find the initial angular velocity and angular acceleration of the wheel.

Sol: $t = 5\,s$; $\theta = 80$ rad.

$$\theta = \frac{\omega_0 + \omega}{2} \times t$$

$$(\omega_0 + \omega) = \frac{80 \times 2}{5}$$

$$\boxed{\omega_0 + \omega = 32} \quad (1)$$

$$\theta = \omega t$$

$$\omega = \frac{\theta}{t} = \frac{100}{5}$$

$$\omega = 20 \text{ rad/s.}$$

from (1)

$$\boxed{\omega_0 = 12 \text{ rad/s}}$$ Ans

$$\omega = \omega_0 + \alpha t$$

$$20 = 12 + (\alpha \times 5)$$

$$\boxed{\alpha = 1.6 \text{ rad/s}^2}$$ Ans

A shaft is uniformly accelerated from 10 rev/s to 18 rev/s in 4 seconds. The shaft cotinues to accelerate at this rate for the next 8 seconds. Thereafter the shaft rotates with a uniform angular speed. Find the total time to complete 400 revolutions.

Sol. $N_0 = 10\ rps \rightarrow \omega_0 = 20\pi\ rad/s.$

$N = 18\ rps \rightarrow \omega = 36\pi\ rad/s.$

$t = 4s,\ \theta = 800\pi\ rad.$

for first 4s.

$\omega = \omega_0 + \alpha t.$

$36\pi = 20\pi + (\alpha \times 4)$

$\alpha = 4\pi\ rad/s^2.$

$\theta_1 = \omega_0 t + \frac{1}{2}\alpha t^2$

$= (20\pi \times 4) + \frac{1}{2} \times 4\pi \times 4^2$

$\boxed{\theta_1 = 112\pi\ rad}$

for Next 8s. $\rightarrow \omega_0 = 36\pi\ rad/s.$

$\omega = \omega_0 + \alpha t$

$\omega = 36\pi + 4\pi \times 8 = 68\pi\ rad/s.$

$\theta_2 = \omega_0 t + \frac{1}{2}\alpha t^2 = (36\pi \times 8) + \frac{1}{2} 4\pi \times 8^2$

$\theta_2 = 416\pi\ rad.$

Now $\omega = 68\pi\ rad/s.$

$\theta_3 = 800\pi - \theta_1 - \theta_2 = 272\pi$

$\theta_3 = \omega t$

$t = \frac{\theta_3}{\omega} = 4s.$

$\boxed{T = (4 + 8 + 4) = 16s.}$ Ans