CGPA BOOSTER

Unit 3

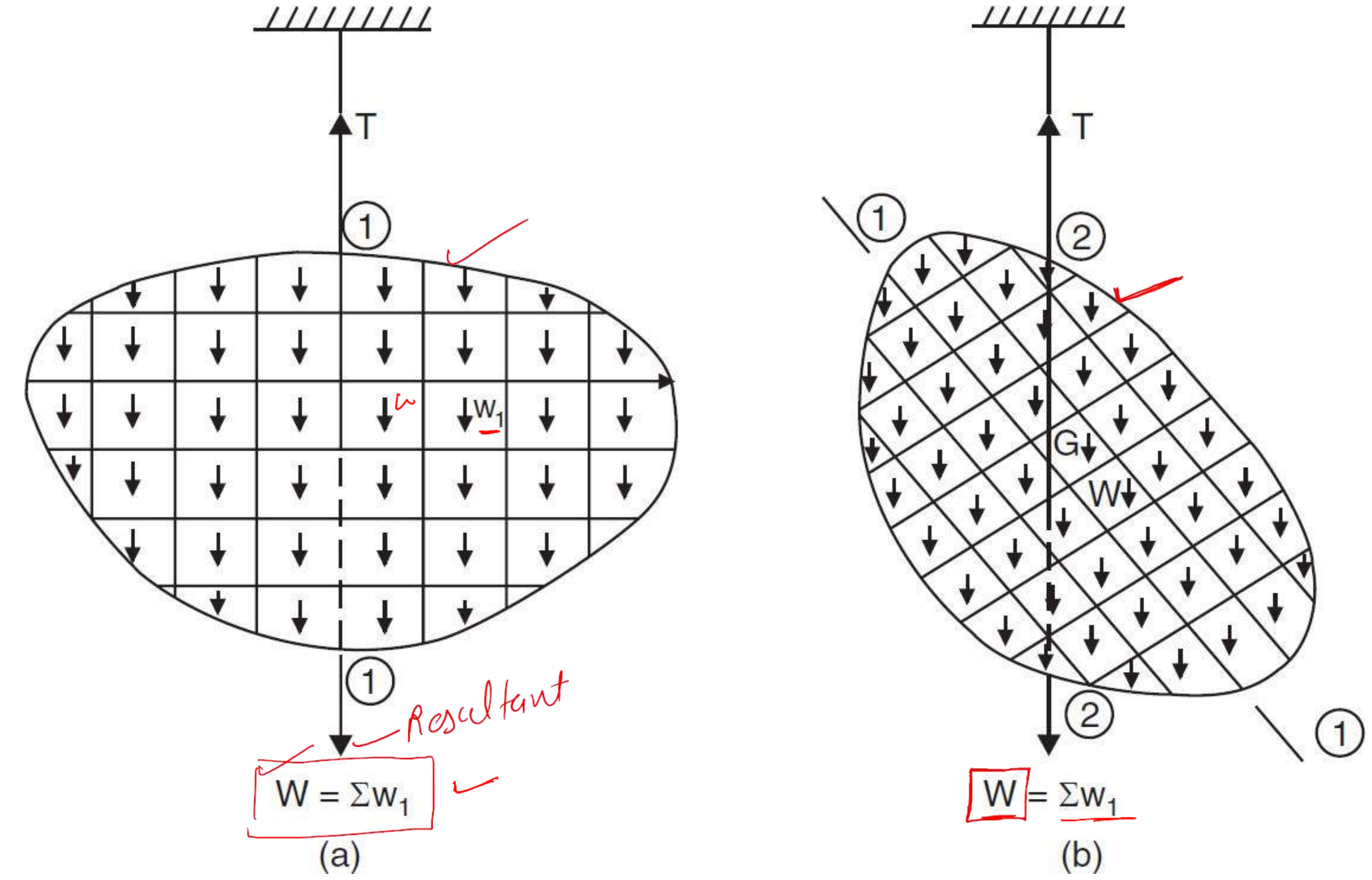
Centroid and Moment of Inertia

Introduction

- A rigid body is collection of particles and earth exerts a force of attraction on each of the particle.
- The force with which the body is attracted towards earth is called force of gravity or weight of the body (W).
- Thus weight of the body is distributed over the entire volume of the object.
- It is convenient to find resultant of distributed force system so that all the forces may be assumed to be concentrated on a point.

Centre of Gravity

magnitude direction → Position



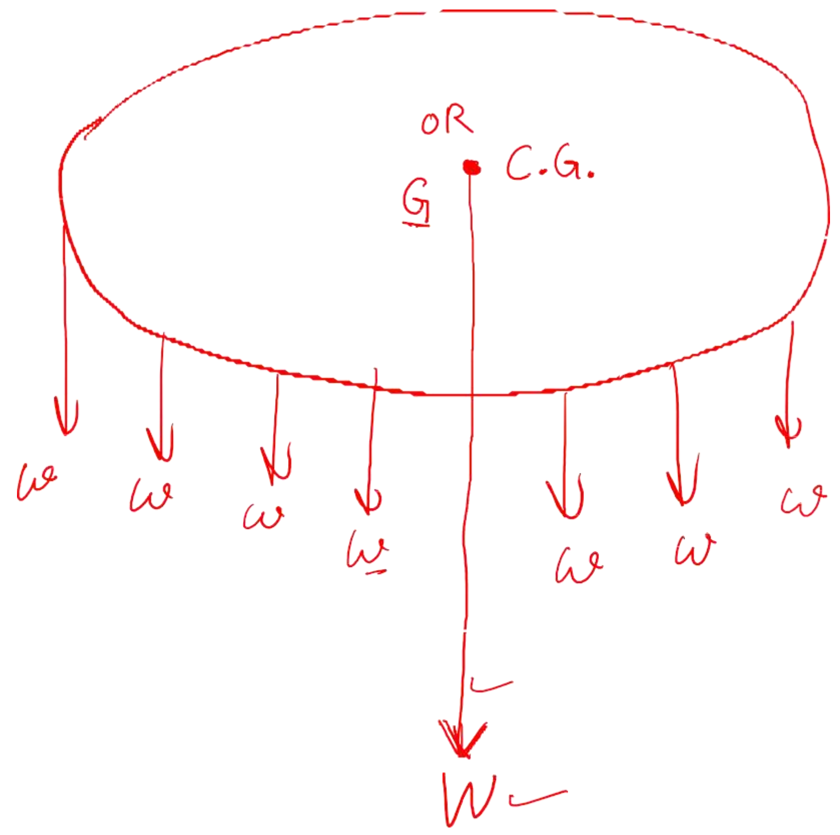
Centre of Gravity

Vol. (3D) —(2D) Area

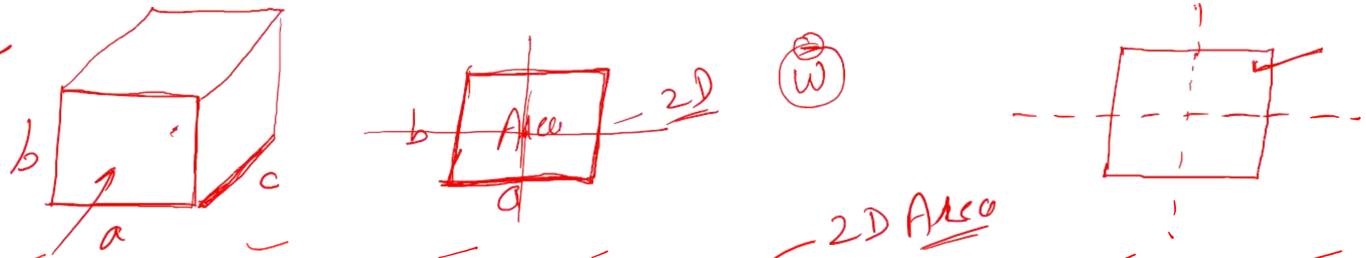
- The point through which the resultant of these parallel forces (weights) acts is known as **Centre of Gravity.**

OR

- The point at which the whole weight of the body is supposed to act is called the **Centre of Gravity.**



Centroid



2D Area

- The point at which the whole area of the lamina is supposed to act is called **Centroid**.

Note: -

3D $\boxed{(V, m, W)}$

- The term Centre of gravity is used for solid bodies. If we imagine that the thickness of this body is uniformly reduced, the position of CG will remain unchanged, although the total weight is reduced. If the body is assumed to be infinitely thin, it becomes a figure having an area only but no weight, is called Lamina.
- Both centre of gravity and centroid are denoted by 'G' or 'CG'.
- A body has only one centre of gravity or centroid.

Determination of Centroid of Plane Bodies

2B

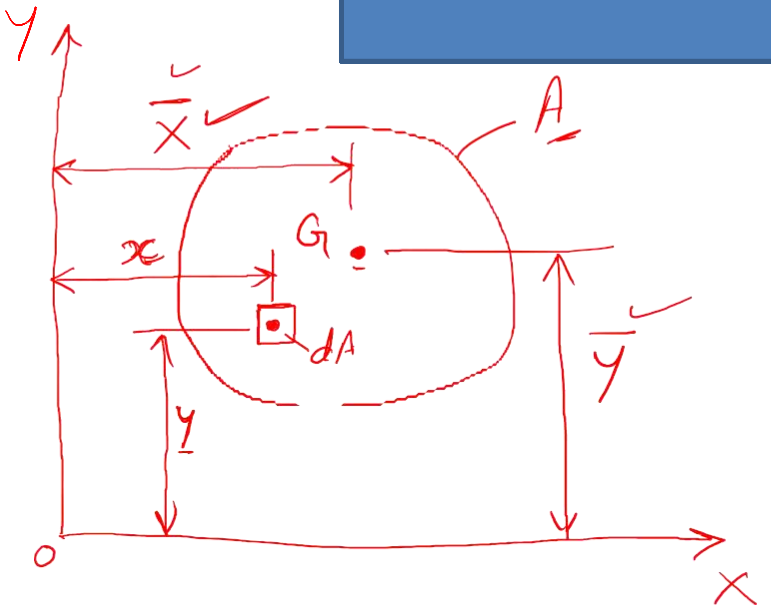
Applying method of moments: →

moment of elementry area about oy = $x\,dA$

moment of total area about oy = $\int x\,dA$

$$A\bar{X} = \int x\,dA = \sum_{i=1}^{n} a_i x_i$$

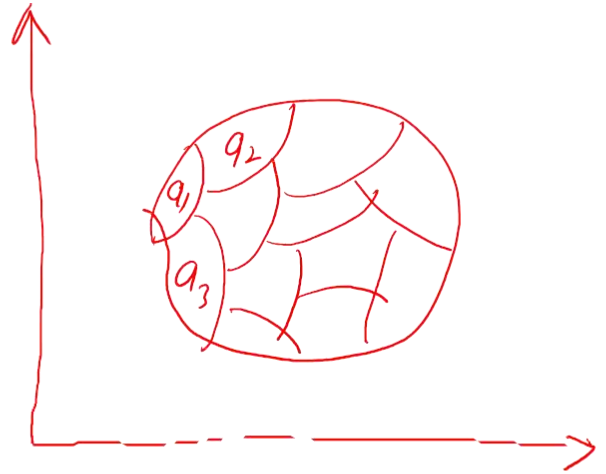
$$\bar{X} = \frac{\sum_{i=1}^{n} a_i x_i}{A} = \sum_{i=1}^{n} \frac{a_i x_i}{a_i}$$



Similarly

$$\boxed{\bar{Y} = \sum_{i=1}^{n} \frac{a_i y_i}{a_i}}$$

Location of G is $(\bar{X}, \bar{Y})$



Determination of Centroid of Plane Bodies

- **Rectangle**

Area of elementry strip = $b\,dy$.

moment of elementry strip about AB = $b\,dy \times y$.

Moment of total area about AB =

$$b \times d \times \bar{y} = \int_0^d b\,dy \times y$$

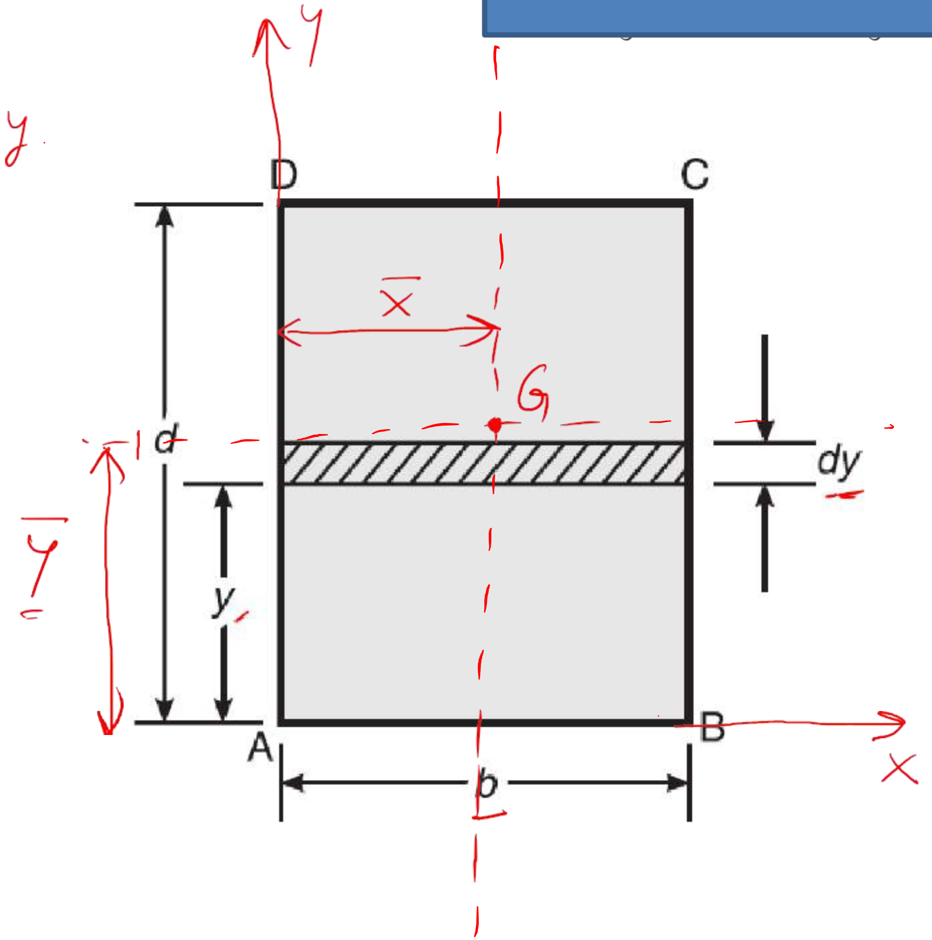
$$= b\left[\frac{y^2}{2}\right]_0^d$$

$$\cancel{b} \times d \times \bar{y} = \frac{\cancel{b}\,d^{\cancel{2}}}{2}$$

$$\boxed{\bar{y} = \frac{d}{2}}$$

Similarly

$$\boxed{\bar{x} = \frac{b}{2}}$$



Recap--

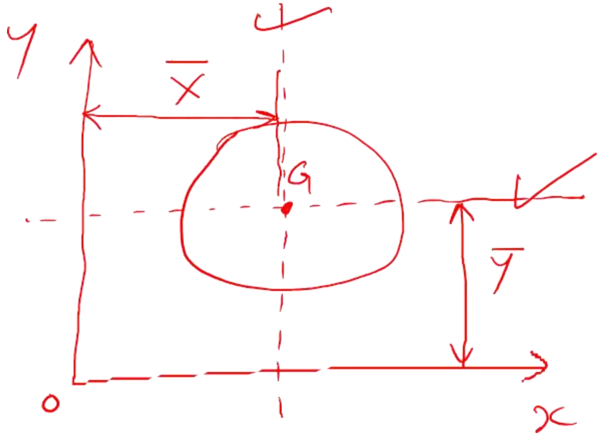
Lamina :→ Only area & no weight

Centroid :→ 2-D → Geometric centre of an area

Centre of gravity :→ Centre of mass of solid body.

Location of centroid :→ $(\bar{x}, \bar{y})$

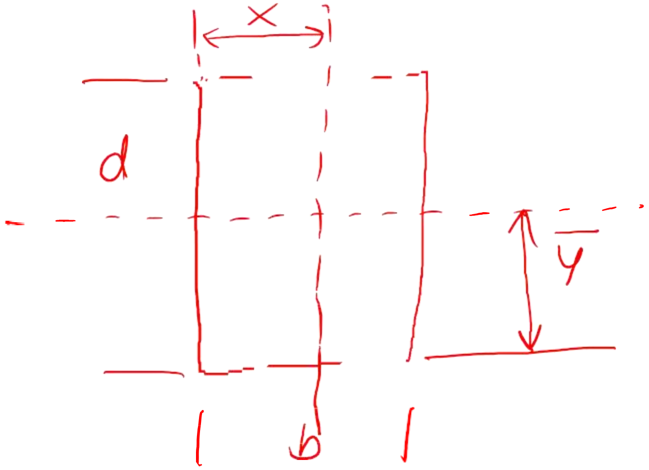
$$\bar{x} = \sum_{i=1}^{n} \frac{a_i x_i}{a_i} \quad ; \quad \bar{y} = \sum_{i=1}^{n} \frac{a_i y_i}{a_i}$$



note :- $\bar{x}$ – is the distance of vertical centroidal axis from y-axis.

$\bar{y}$ – is the distance of horizontal centroidal axis from x-axis.

$$\bar{x} = \frac{b}{2} \quad ; \quad \bar{y} = \frac{d}{2}$$ for Rectangle



Find CG.

Sol: Due to symmetry. $\bar{X} = \frac{150}{2} = 75 \text{ mm}$

$$\bar{y} = \frac{\sum_{i=1}^{n} a_i y_i}{a_i} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$a_1 = 150 \times 50 = 7500 \text{ mm}^2$

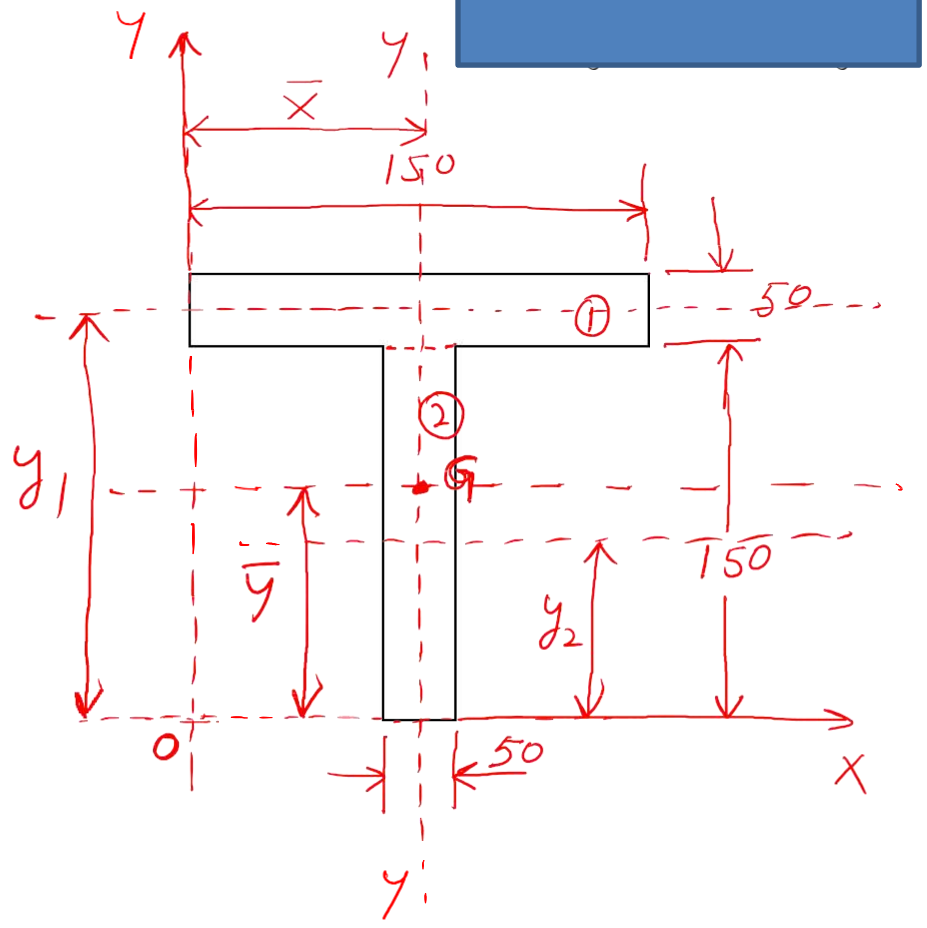
$a_2 = 7500 \text{ mm}^2$

$y_1 = 175 \text{ mm}$; $y_2 = 75 \text{ mm}$.

$$\bar{y} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500}$$

$$\boxed{\bar{y} = 125 \text{ mm.}}$$

$(\bar{X}, \bar{Y}) = (75, 125)$



All dim. are in mm.

Find CG

Sol: Due to symmetry

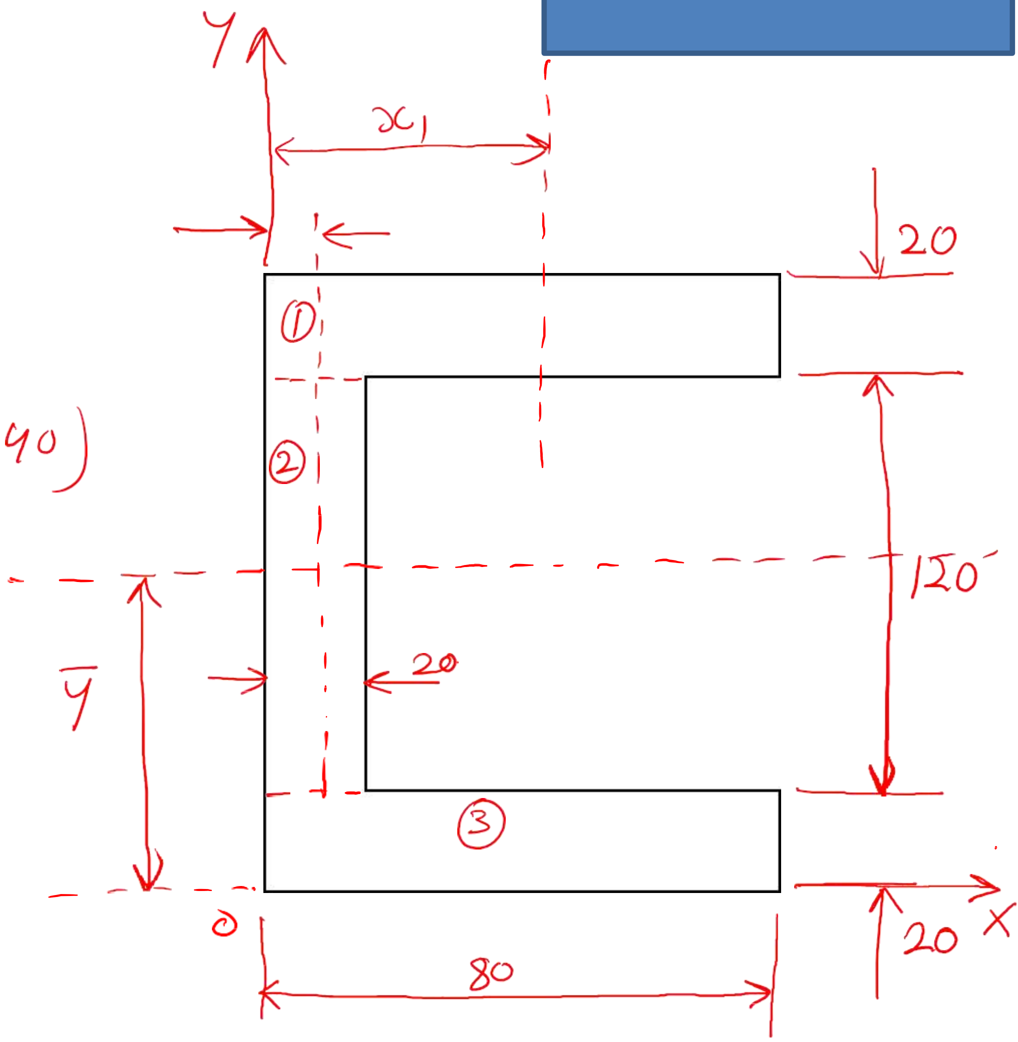
$$\bar{y} = \frac{160}{2} = 80\,mm.$$

$a_1 = 1600\,mm^2;\ a_2 = 2400\,mm^2;\ a_3 = 1600\,mm^2.$

$x_1 = x_3 = 40\,mm;\ x_2 = 10.$

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(1600 \times 40) + (2400 \times 10) + (1600 \times 40)}{1600 + 2400 + 1600}$$

$$\boxed{\bar{X} = 27.14\,mm.}$$ Ans



Find CG

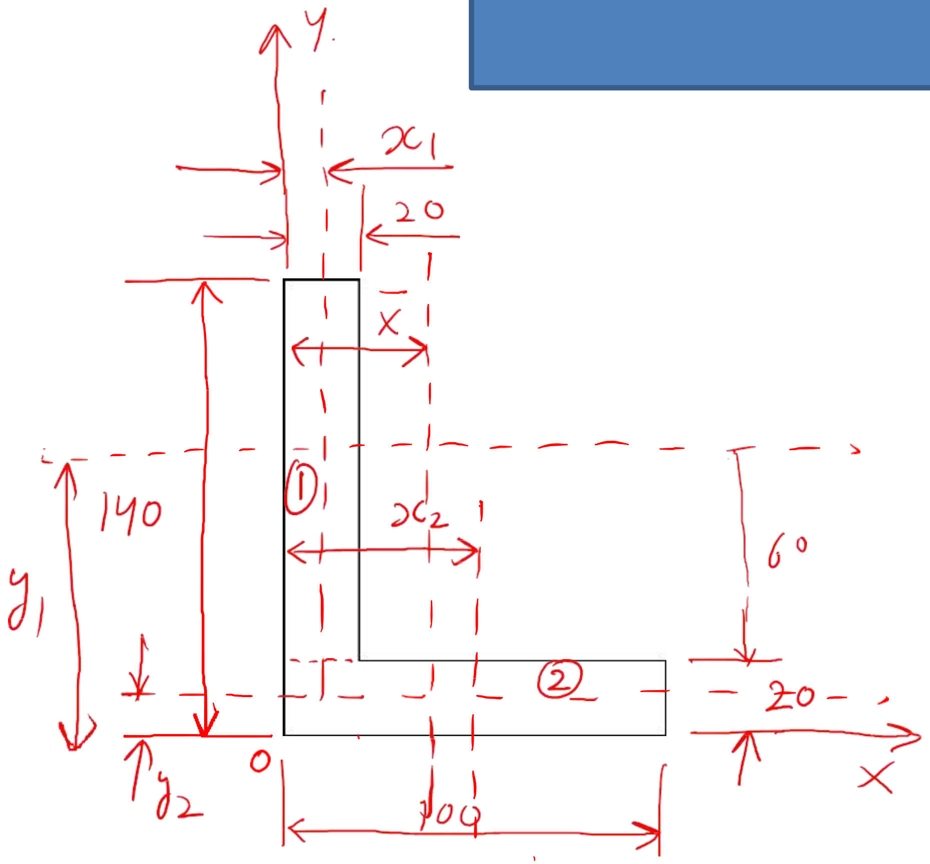
$x_1 = 10$ mm | $y_1 = 80$ mm | $a_1 = 2400$ mm^2

$x_2 = 50$ mm | $y_2 = 10$ mm | $a_2 = 2000$ mm^2

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 28.18 \text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 48.18 \text{ mm}$$

C.G is located at (28.18, 48.18) from O Ans



Find CG

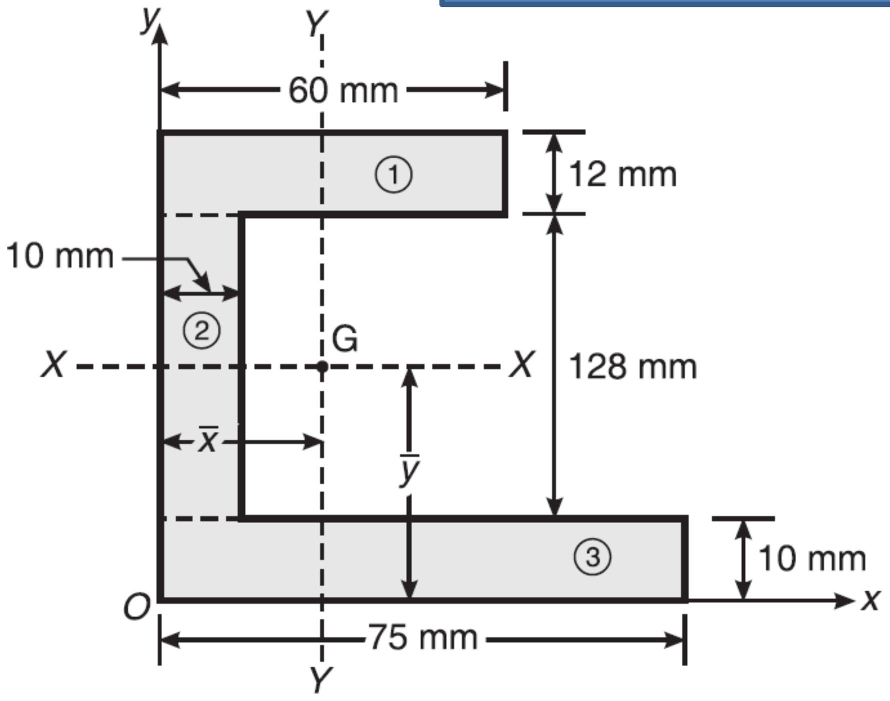
$a_1 = 720\ mm^2$ | $x_1 = 30\ mm$ | $y_1 = 144\ mm$

$a_2 = 1280\ mm^2$ | $x_2 = 5\ mm$ | $y_2 = 74\ mm$

$a_3 = 750\ mm^2$ | $x_3 = 37.5\ mm$ | $y_3 = 5\ mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = 20.409\ mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = 73.509\ mm$$ Ans



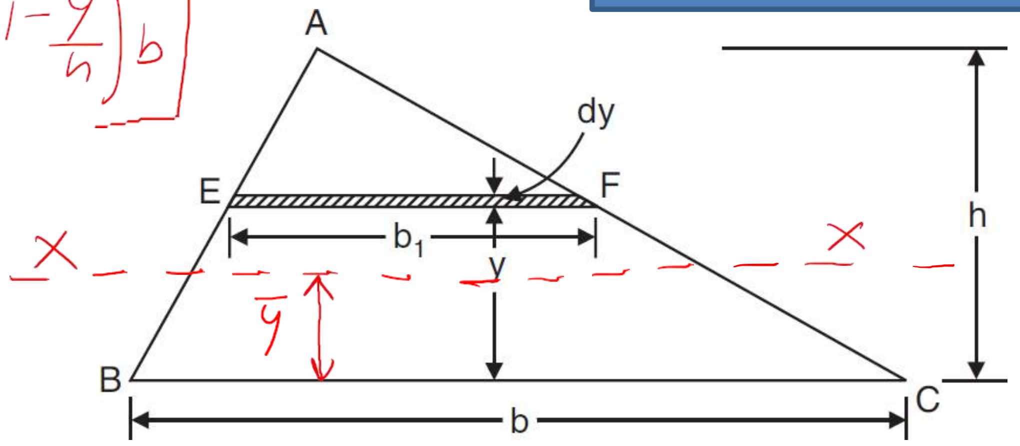
Centroid of a Triangle

Area of elementary strip $= b_1\,dy$

$$= \left(1-\frac{y}{h}\right) b\,dy$$

[In ΔABC & ΔAEF

$$\frac{b_1}{b} = \frac{h-y}{h} \Rightarrow b_1 = \left(1-\frac{y}{h}\right) b$$]



Moment of elementary area about $BC = \left(1-\frac{y}{h}\right) b\,dy \times y$

Moment of total area about $BC = \int_0^h \left(1-\frac{y}{h}\right) b\,dy \times y$

$$= b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$\frac{1}{2} \times \cancel{b} \times \cancel{h} \times \bar{y} = \frac{\cancel{b}h^{\cancel{2}}}{6}$$

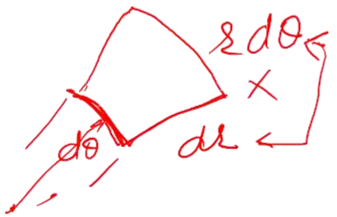
$$\boxed{\bar{y} = \frac{h}{3}}$$

[C.G is at $\frac{h}{3}$ from base

OR

at $\frac{2h}{3}$ from apex]

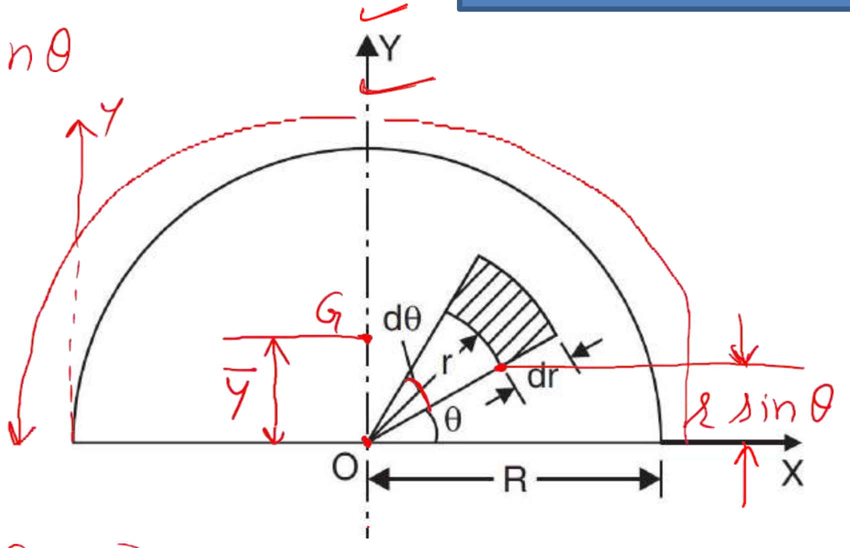
Centroid of a Semicircle



Area of element $= r\,d\theta \times dr$

Moment of elemental area about x-axis $= r\,d\theta \times dr \times r\sin\theta$

Moment of total area about x-axis $= \int_0^{\pi}\int_0^{R} r^2\, d\theta\, dr \sin\theta$

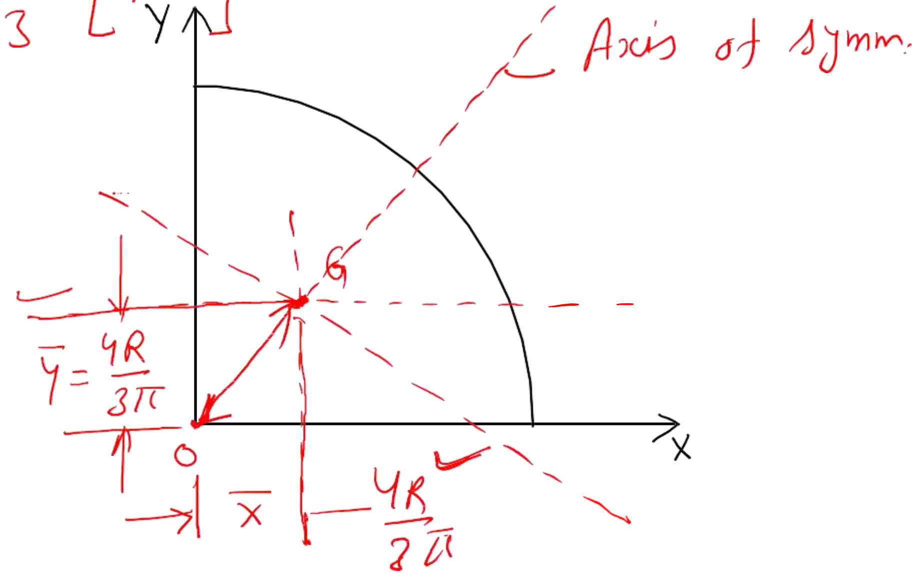


$$= \int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R \sin\theta\, d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi} = \frac{R^3}{3}\left[1+1\right]$$

$$\frac{\pi}{2}\times \cancel{R^2}\times\bar{y} = \frac{2R^{\cancel{3}}}{3}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}} = 0.424R \text{ OR } 0.212D$$



Find CG

$a_1 = 3200\ mm^2$

$a_2 = 5026.54\ mm^2$

$x_1 = \frac{2 \times 80}{3} = 53.33\ mm$

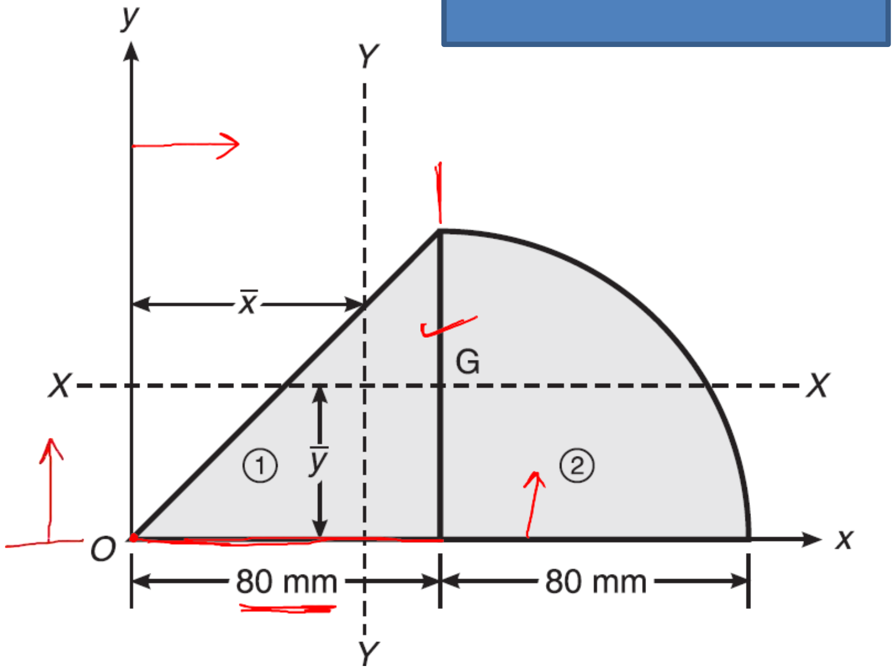
$x_2 = 80 + \frac{4 \times 80}{3\pi} = 113.95\ mm$

$y_1 = \frac{80}{3} = 26.67\ mm$; $y_2 = \frac{4 \times 80}{3\pi} = 33.95\ mm$

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 90.36\ mm$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 31.11\ mm$$

Ans



Find CG from the apex of the triangle :→

$\bar{X} = \frac{60}{2} = 30\ mm.$

$a_1 = \frac{1}{2} \times 60 \times \frac{60\sqrt{3}}{2} = 1558.84\ mm^2$

$a_2 = 3600\ mm^2$

$y_1 = 60 + \frac{1}{3} \times \frac{60\sqrt{3}}{2} = 77.32\ mm.$

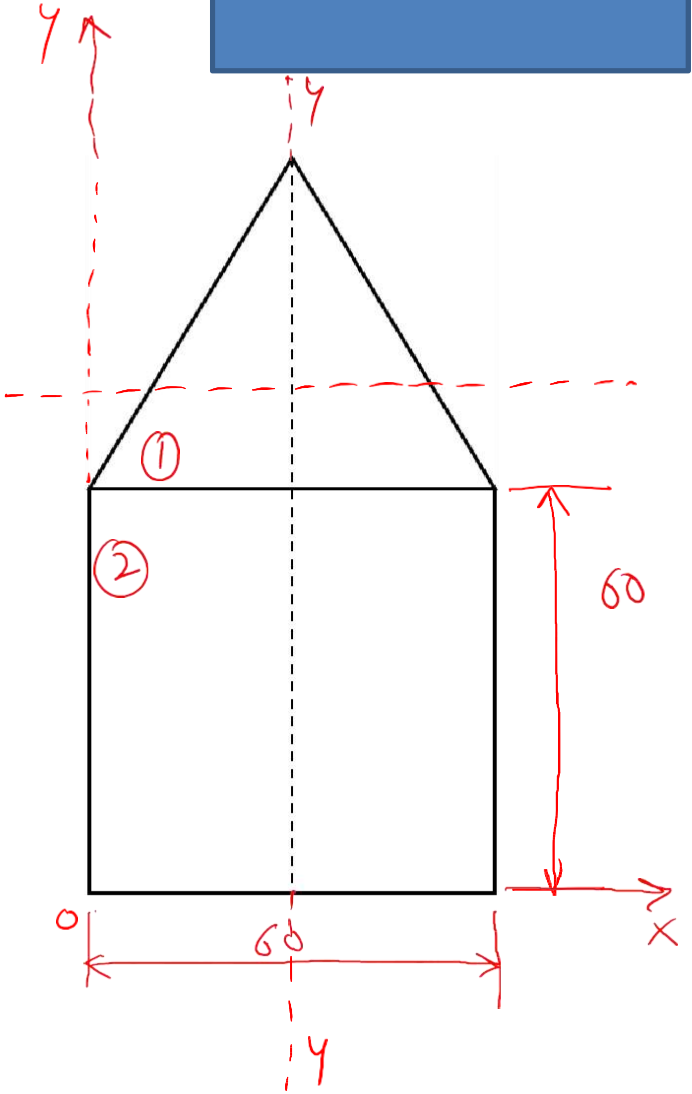
$y_2 = 30\ mm.$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 44.29\ mm.$$

Distance of CG from apex = total height $- \bar{y}$

$$= \left[60 + \frac{60\sqrt{3}}{2}\right] - 44.29$$

$$= 67.67\ mm \quad \underline{\text{Ans}}$$



Centroid of Remainder

Sol:

① − ②

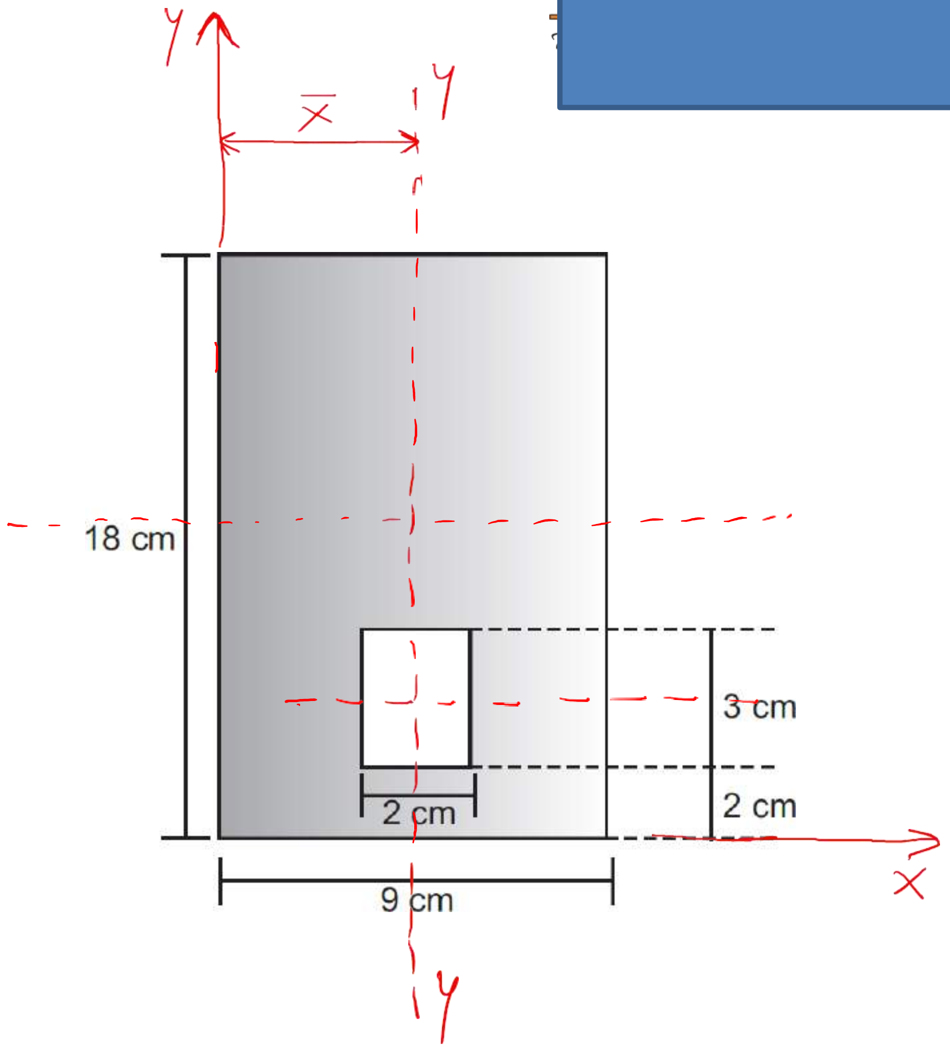
$$\bar{x} = \frac{9}{2} = 4.5 \text{ cm.}$$

$a_1 = 162 \text{ cm}^2$ | $y_1 = 9 \text{ cm.}$

$a_2 = 6 \text{ cm}^2$ | $y_2 = 3.5 \text{ cm}$

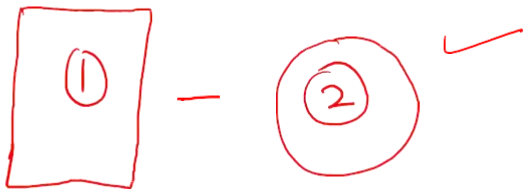
$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$\boxed{\bar{y} = 9.21 \text{ cm}}$$ Ans



Find CG of Remainder Area

Sol:

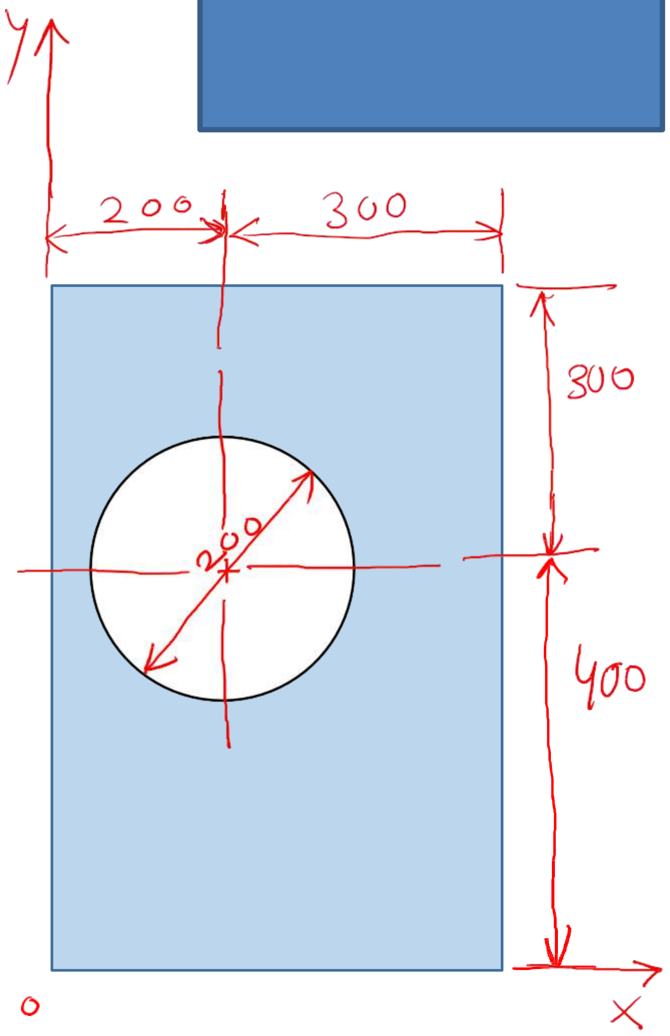


$a_1 = 350000\ mm^2$ | $x_1 = 250\ mm$ | $y_1 = 350\ mm$

$a_2 = 31415.92\ mm^2$ | $x_2 = 200\ mm$ | $y_2 = 400\ mm$

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = 254.93\ mm$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = 345.06\ mm \quad \text{Ans}$$



Centroid of Remainder

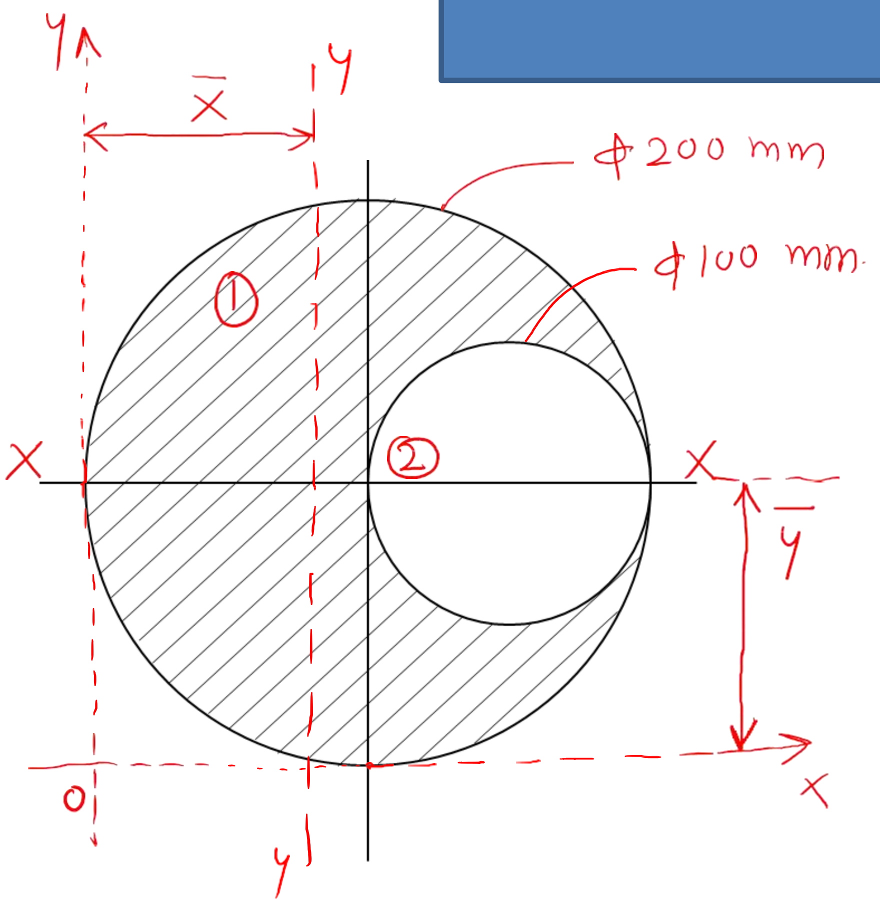
Sol: $\bar{y} = 100$ mm.

$a_1 = 31415.92\ mm^2$ | $x_1 = 100$ mm

$a_2 = 7853.98\ mm^2$ | $x_2 = 150$ mm

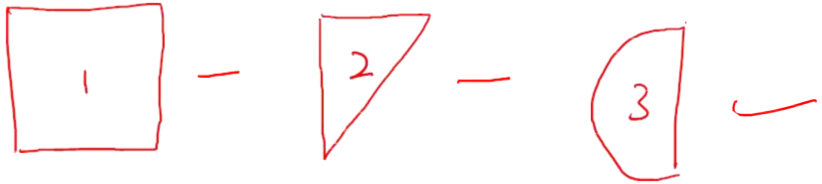
$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$\bar{x} = 83.33 \text{ mm}$$



Centroid of Remainder

Sol:



$a_1 = 42000\ mm^2$ | $x_1 = 150\ mm$

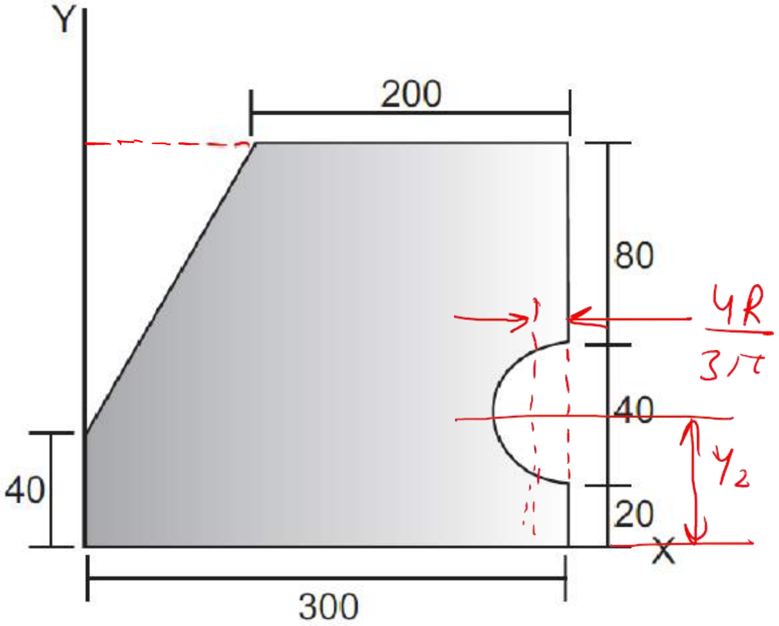
$a_2 = 5000\ mm^2$ | $x_2 = \frac{100}{3} = 33.33\ mm$

$a_3 = 628.31\ mm^2$ | $x_3 = 300 - \frac{4 \times 20}{3\pi} = 291.51\ mm$

$y_1 = 70\ mm$; $y_2 = 40 + \frac{2 \times 100}{3} = 106.67\ mm$; $y_3 = 40\ mm$.

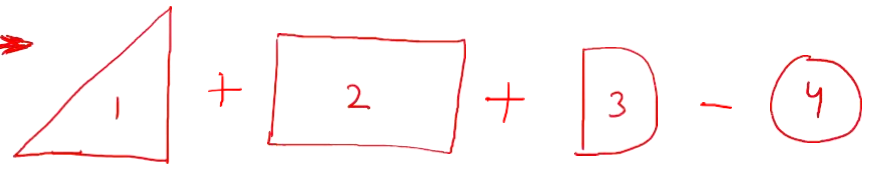
$$\bar{X} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3}{a_1 - a_2 - a_3} = 163.59\ mm.$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2 - a_3 y_3}{a_1 - a_2 - a_3} = 65.47\ mm. \quad \text{Ans.}$$



Centroid of Remainder

Sol:→

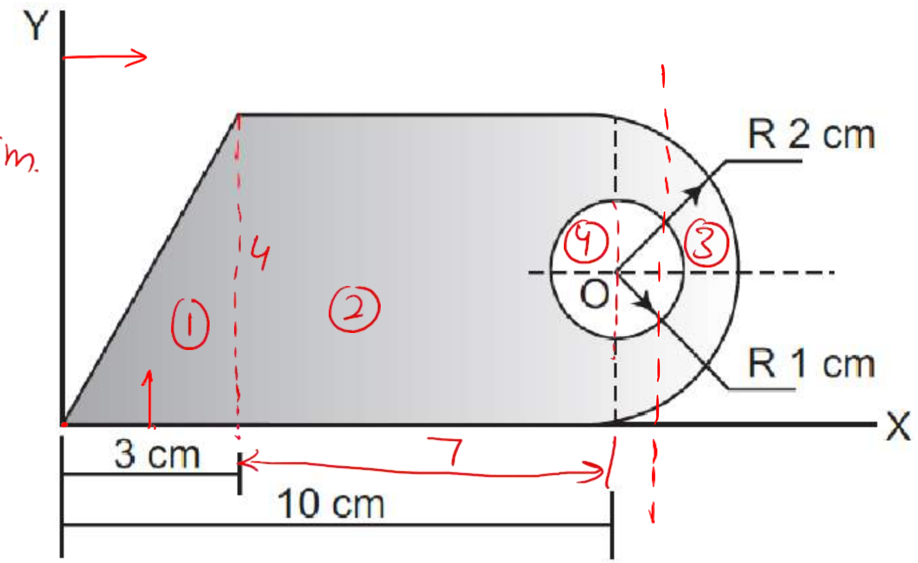


$a_1 = 6\ cm^2$ | $x_1 = \frac{2\times 3}{3} = 2\ cm$ | $y_1 = \frac{4}{3} = 1.33\ cm$

$a_2 = 28\ cm^2$ | $x_2 = 3 + \frac{7}{2} = 6.5\ cm$ | $y_2 = y_3 = y_4 = 2\ cm$

$a_3 = 6.28\ cm^2$ | $x_3 = 10 + \frac{4\times 2}{3\pi} = 10.84\ cm$

$a_4 = \pi\ cm^2$ | $x_4 = 10\ cm$

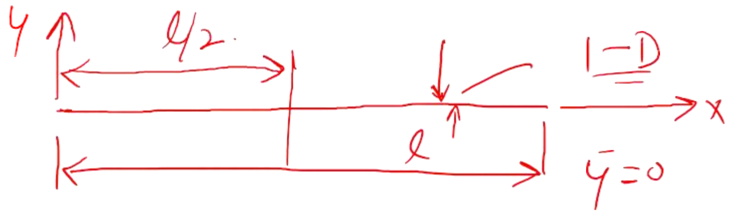


$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3 - a_4x_4}{a_1 + a_2 + a_3 - a_4} = 6.21\ cm.$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 - a_4y_4}{a_1 + a_2 + a_3 - a_4} = 1.89\ cm.$$

Ans

Centroid of Wire Frame

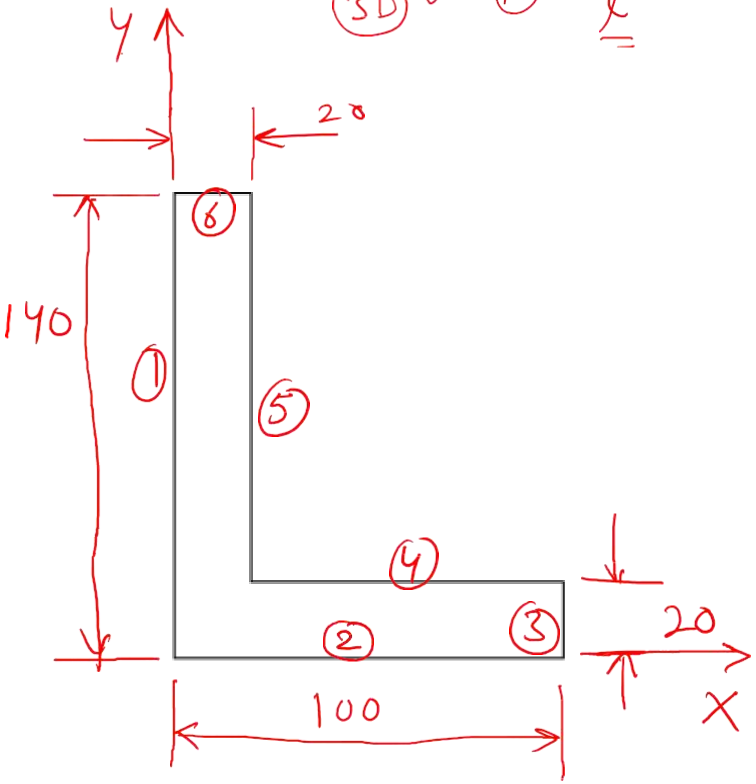


3D V – A – l

Seg.	l_i	x_i	y_i	$l_i x_i$	$l_i y_i$
1	140	0	70		
2	100	50	0		
3	20	100	10		
4	80	60	20		
5	120	20	80		
6	20	10	140		
	Σl_i			$\Sigma l_i x_i$	$\Sigma l_i y_i$

$$\bar{X} = \sum_{i=1}^{n} \frac{l_i x_i}{l_i} = 30\text{ mm}$$

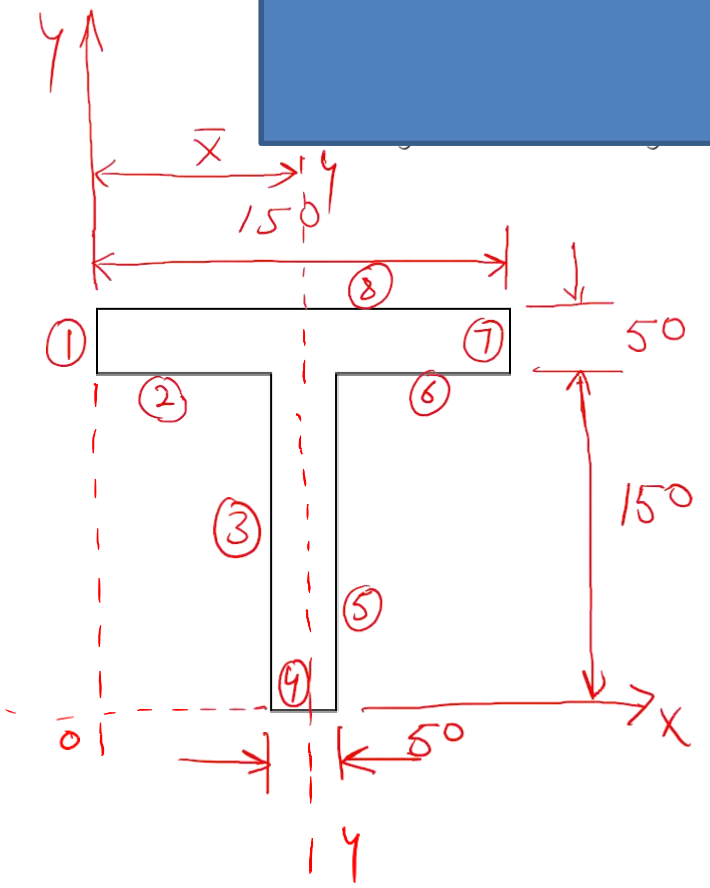
$$\bar{y} = \sum_{i=1}^{n} \frac{l_i y_i}{l_i} = 50\text{ mm}$$



Sol: $\bar{X} = \frac{150}{2} = 75\,mm.$

Seg	l_i	y_i
1	50	175
2	50	150
3	150	75
4	50	0
5	150	75
6	50	150
7	50	175
8	150	200

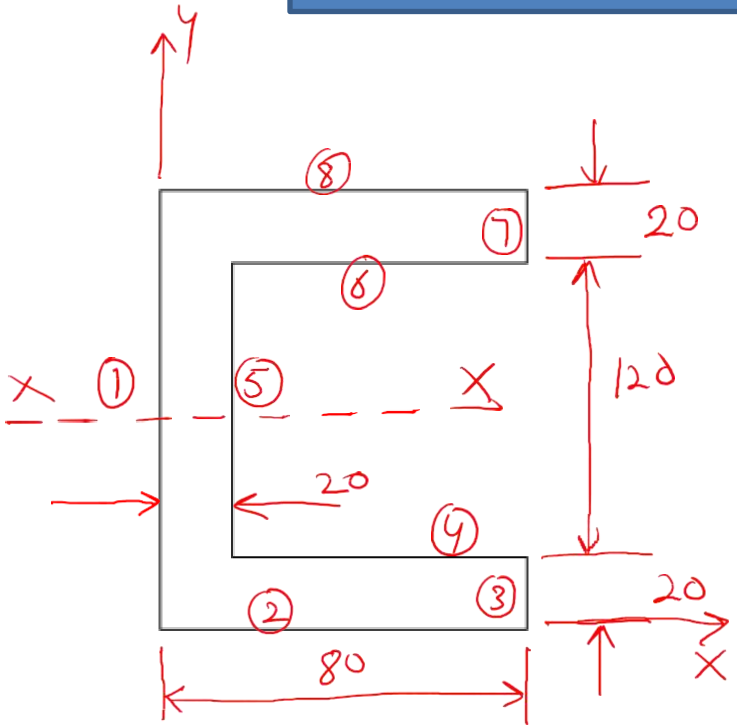
$$\bar{Y} = \frac{\sum_{i=1}^{n} l_i y_i}{l_i} = 121.43\ mm. \quad \text{Ans}$$



Sol: $\bar{y} = \frac{160}{2} = 80$ mm.

Seg	l	x
1	160	0
2	80	40
3	20	80
4	60	50
5	120	20
6	60	50
7	20	80
8	80	40

$$\bar{X} = \frac{\sum_{i=1}^{n} l_i x_i}{l_i} = 30 \text{ mm}$$ Ans

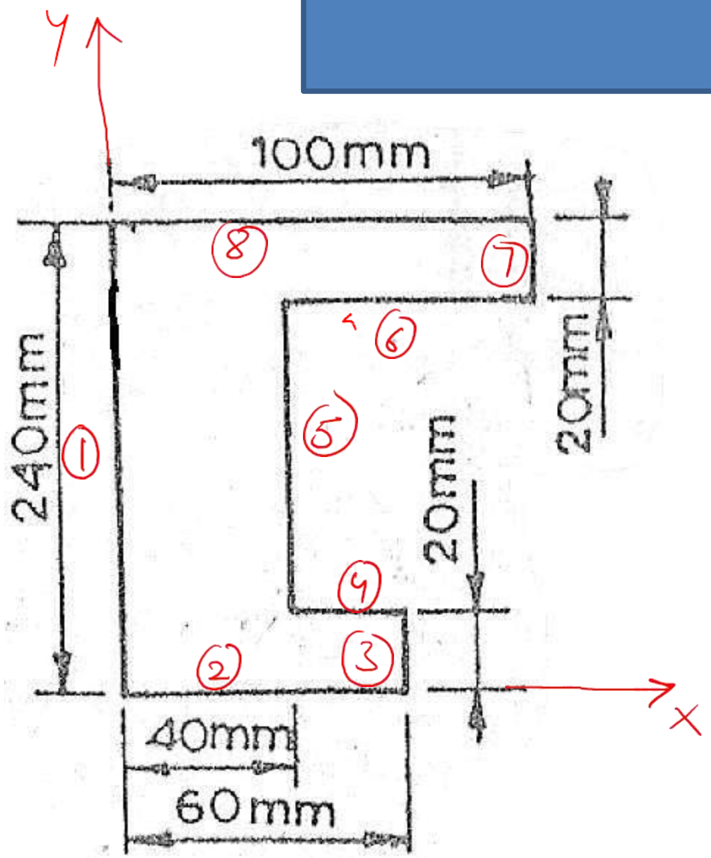


Sol:

Seg	l_i	x_i	y_i
1	240	0	120
2	60	30	0
3	20	60	10
4	20	50	20
5	200	40	120
6	60	70	220
7	20	100	230
8	100	50	240

$\bar{X} = 32.22$ mm.

$\bar{Y} = 132.22$ mm   Ans

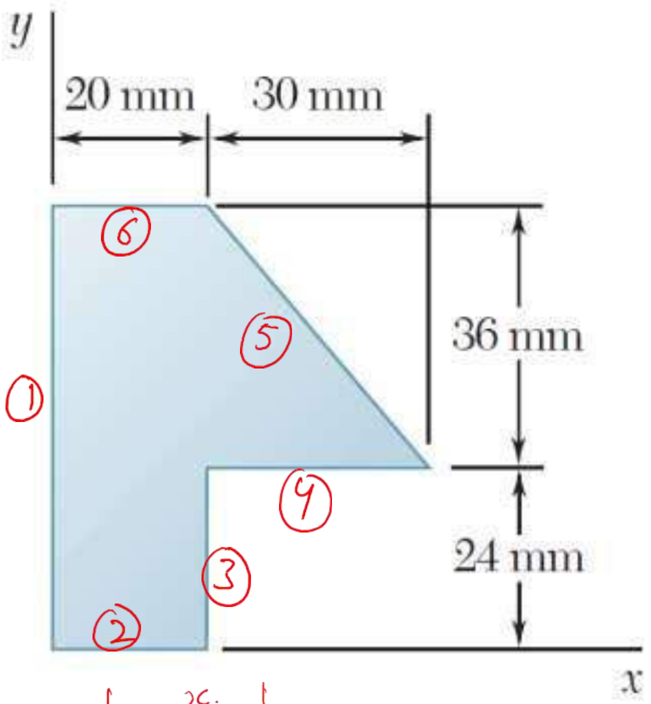


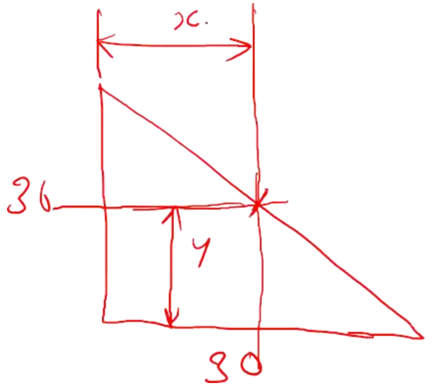
Sol:

Seg	l_i	x_i	y_i
1	60	0	30
2	20	10	0
3	24	20	12
4	30	35	24
5	46.86	35	42
6	20	10	60

$\bar{X} = 17.77$ mm

$\bar{Y} = 29.75$ mm Ans



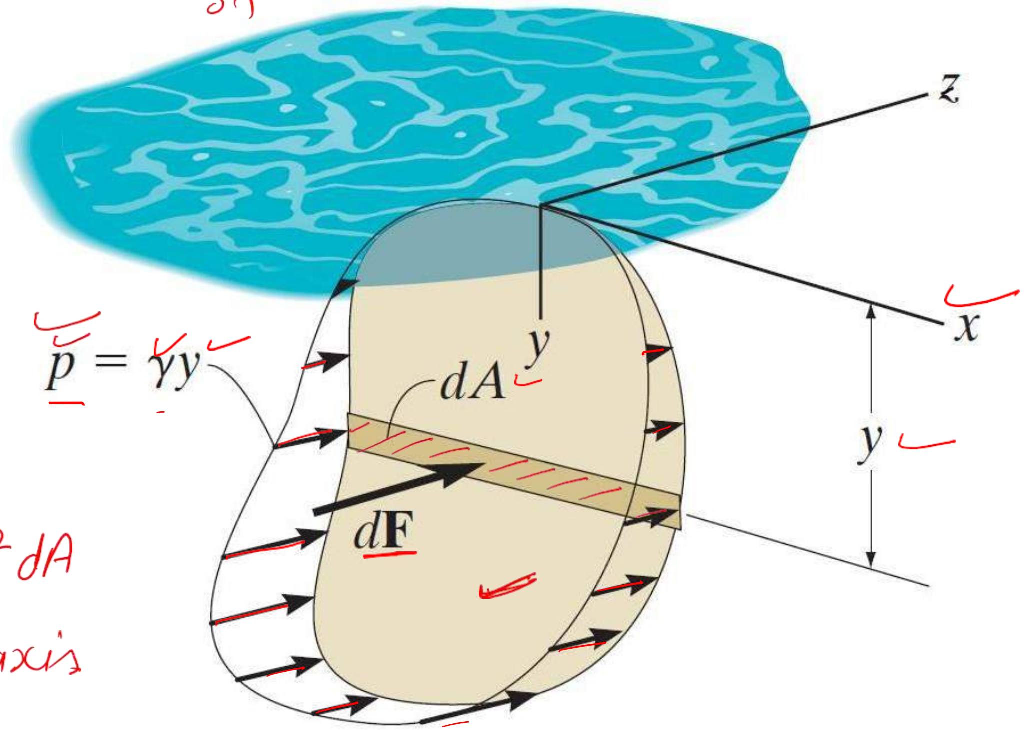


Introduction

$\int y\,dA \times y = \int y^2\,dA$

second moment of ar.

- Whenever a distributed load acts perpendicular to an area and its intensity varies linearly, the calculation of the moment of the loading about an axis will involve an integral of the form $\boxed{\int y^2 dA}$.



$p \rightarrow$ pressure; $\gamma =$ specific weight of liquid.

y – depth inside the liquid.

Force on elemental area $dF = p \times dA = \gamma y\,dA$

Moment of dF about x-axis 'dM' $= dF \times y = \gamma y^2 dA$

moment of total force on entire area about x-axis

$$M = \gamma \int y^2 dA$$

— also called second moment of area.

MoI

$\sum_{i=1}^{n} y_i^2 a_i =$

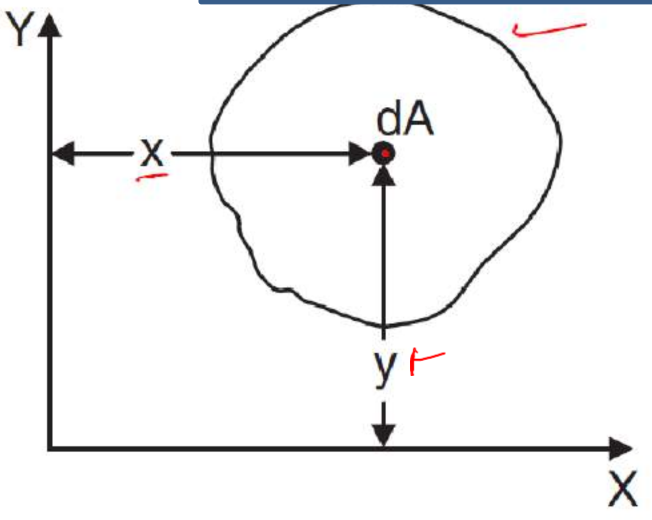
Introduction

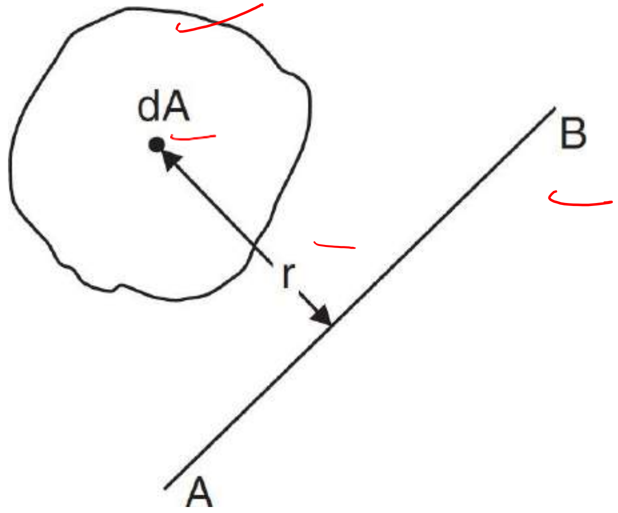
- Therefore, (Moment of Inertia of a plane area w.r.t. any axis is the sum of the products of each elementary area and square of its distance from the axis.
- It is also known as **Second Moment of Area.** MOI or 'I'

Moment of Inertia about X-axis $I_{xx} = \int y^2\, dA = \sum_{i=1}^{n} a_i y_i^2$

" " " " y-axis $I_{yy} = \int x^2\, dA = \sum_{i=1}^{n} a_i x_i^2$

$I_{AB} = \sum r^2\, dA$





Radius of Gyration

$$I = \sum r^2 \, dA$$

$r \rightarrow$ Radius of Gyration $\rightarrow$ distance of a point of C.G from given axis.

$\downarrow$

Rotation

$$\boxed{r = \sqrt{\frac{I}{A}}}$$

Theorem of Perpendicular Axis.

$$r^2 = x^2 + y^2$$

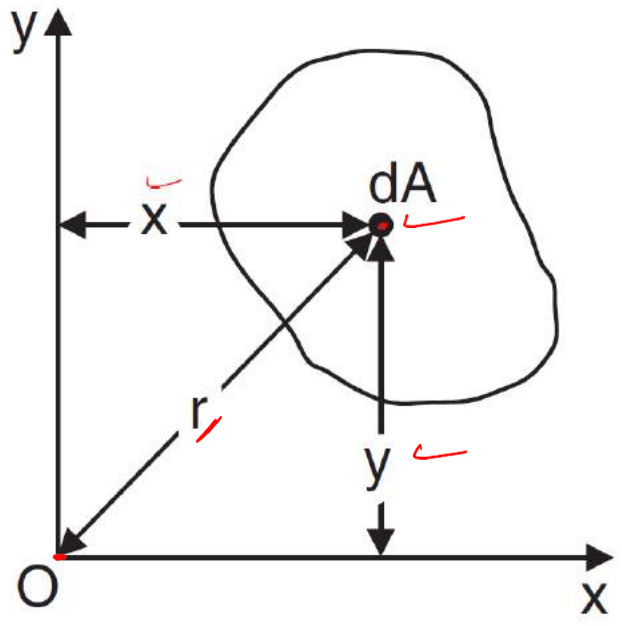
$$\underbrace{r^2 dA} = (x^2 + y^2)\, dA$$

$$I_{zz} = x^2 dA + \underline{y^2 dA}$$

$$\boxed{I_{zz} = I_{yy} + I_{xx}}$$

(I_p) or (J)

Polar moment of inestra.



Theorem of Parallel Axis.

$\boxed{I_{xx} \text{ or } I_{yy}}$

Area of elementry strip = dA

$$I_{xx} = \sum dA\, y^2$$

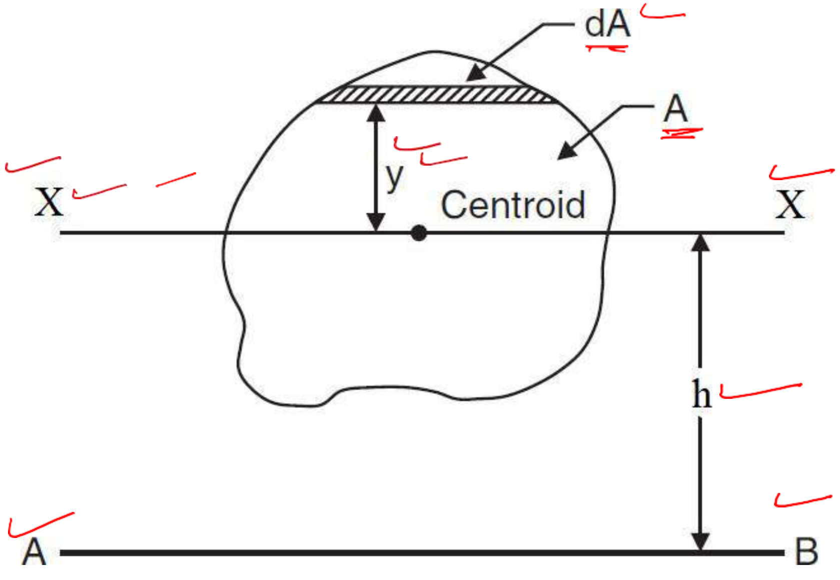
$$I_{AB} = \sum dA\,(y+h)^2$$

$$= \sum dA\,(y^2+h^2+2yh)$$

$$= \sum dA\, y^2 + \sum dA\, h^2 + \cancel{\sum dA\, y \cdot 2h} \to 0$$

$$\boxed{I_{AB} = I_{xx} + Ah^2}$$

(MoI about any axis AB || to centroidal axis) = (MoI of the area about its own centroidal axis) + (Total area × square of distance b/w centroidal axis and give axis)



$$\sum 2yh\, dA \to 0$$

$$= 2h \cancel{\left(\sum y\, dA\right)}$$

MOI of Plane Areas

Area of elementry strip = $\underbrace{b \times dy}$.

Second moment of area about x-x axis = $b \times dy \times y^2$

Total second moment of area of whole section about XX axis

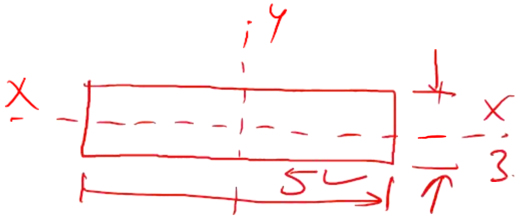
$$I_{xx} = \int_{-d/2}^{d/2} b y^2 \, dy = b \int_{-d/2}^{d/2} y^2 \, dy = b \left[\frac{y^3}{3} \right]_{-d/2}^{d/2}$$

$$= \frac{b}{3} \left[\frac{d^3}{8} + \frac{d^3}{8} \right]$$

$$\boxed{I_{xx} = \frac{b d^3}{12}}$$

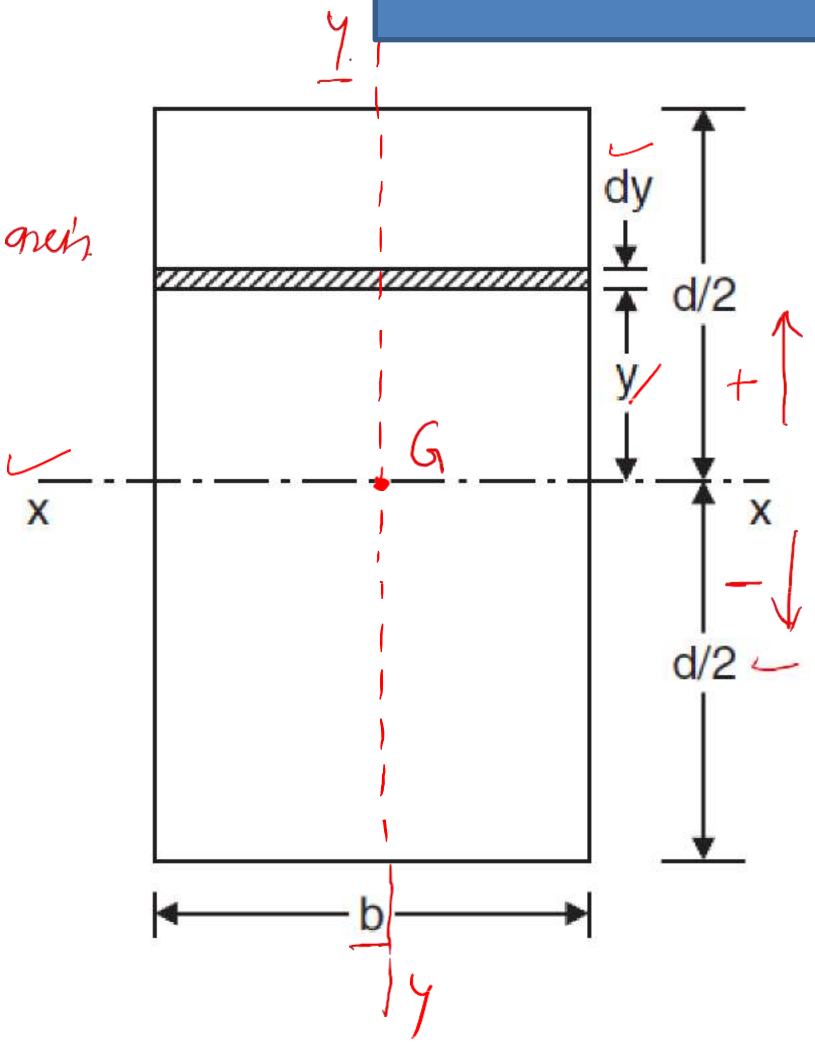
Similarly

$$\boxed{I_{yy} = \frac{d b^3}{12}}$$



$$I_{xx} = \frac{5 \times 3^3}{12}$$

$$I_{yy} = \frac{3 \times 5^3}{12}$$



Recap---

$$I = \Sigma dA K^2$$

K = Radius of Gyration → effective distance from C.G to given axis.

⊥ axis theorem → $I_{zz} = I_{xx} + I_{yy}$ → Polar MOI

∥ axis theorem = $I_{AB} = I_{xx} + Ah^2$

MOI of Rectangle $I_{xx} = \frac{bd^3}{12}$

$$I_{yy} = \frac{db^3}{12}$$

Sol: $\bar{X} = 25\ mm$ $\bar{Y} = 37.5\ mm$

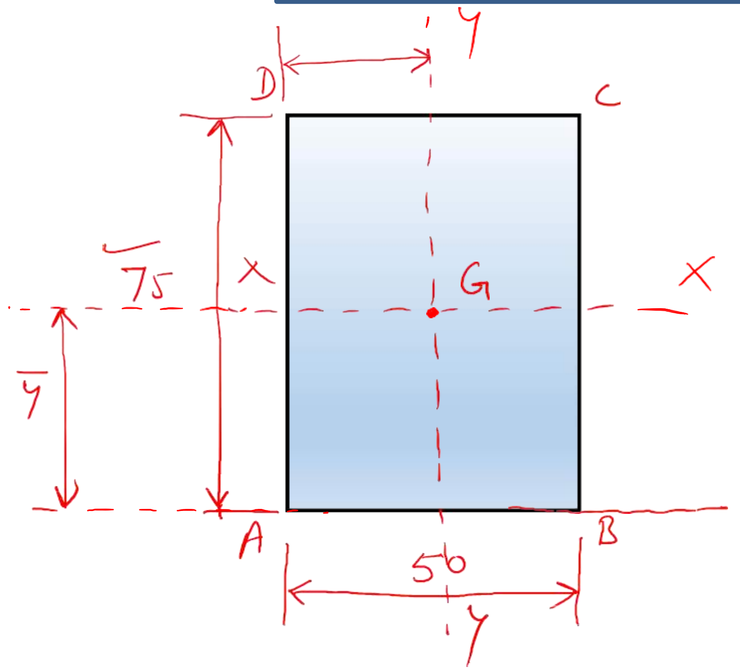
$$I_{xx} = \frac{50 \times 75^3}{12} = 1757812.5\ mm^4 \ ; \ I_{yy} = \frac{75 \times 50^3}{12} = 781250\ mm^4$$

$$I_{AB} = I_{xx} + Ah^2 = 1757812.5 + (75 \times 50 \times 37.5^2)$$

$$= 7031250\ mm^4$$

$$I_{AD} = I_{yy} + Ah^2 = 781250 + (75 \times 50 \times 25^2)$$

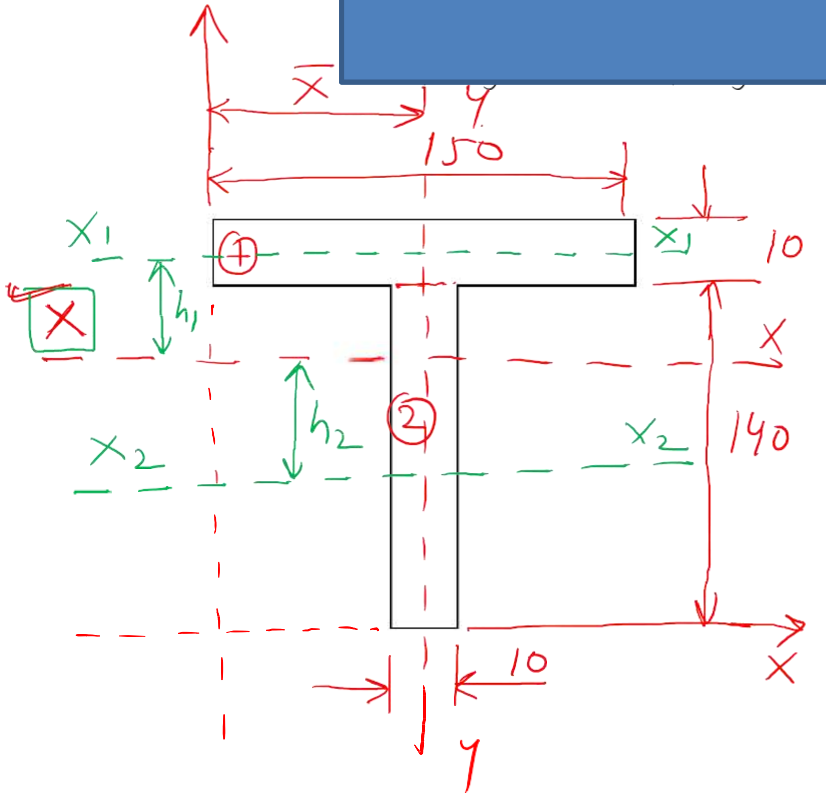
$$= 3125000\ mm^4$$



$$I_{zz} = I_{xx} + I_{yy}$$

$$= 2539062.5\ mm^4$$

Find I_{xx} & I_{yy} $\bar{x} = 75$ mm $\bar{y} = 108.8$ mm



$$I_{xx} = \bar{I}_{xx_1} + I_{xx_2}$$

$$= \left\{ \frac{150 \times 10^3}{12} + \left[1500 \times (145 - 108.8)^2\right] \right\}$$

$$+ \left\{ \frac{10 \times 140^3}{12} + \left[1400 \times (108.8 - 70)^2\right] \right\}$$

$$\boxed{I_{xx} = 6\,372\,443 \text{ mm}^4}$$

$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12}$$

$$I_{yy} = 2824167 \text{ mm}^4$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = 46.88 \text{ mm}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = 31.21 \text{ mm}$$

|| axis th.

$$I = I_{xx} + Ah^2$$

Find I_{xx} & I_{yy} $\quad \bar{X} = 28.2\ mm \quad \bar{Y} = 38.2\ mm$

$$I_{xx} = I_{xx_1} + I_{xx_2} = \left\{\frac{15\times105^3}{12} + \left[1575\times(67.5-38.2)^2\right]\right\}$$

$$+$$

$$\left\{\frac{100\times15^3}{12} + \left[1500\times(38.2-7.5)^2\right]\right\}$$

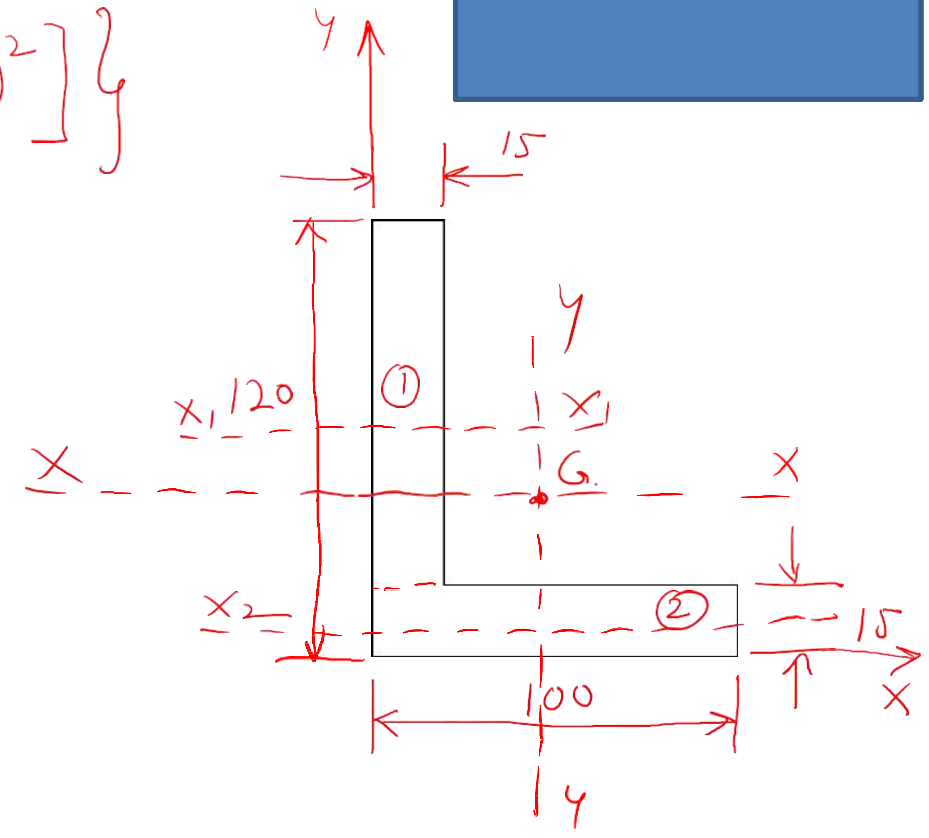
$$\boxed{I_{xx} = 4241013\ mm^4}$$

$$I_{yy} = I_{yy_1} + I_{yy_2} = \left\{\frac{105\times15^3}{12} + \left[1575\times(28.2-7.5)^2\right]\right\}$$

$$+$$

$$= \left\{\frac{15\times100^3}{12} + \left[1500\times(50-28.2)^2\right]\right\}$$

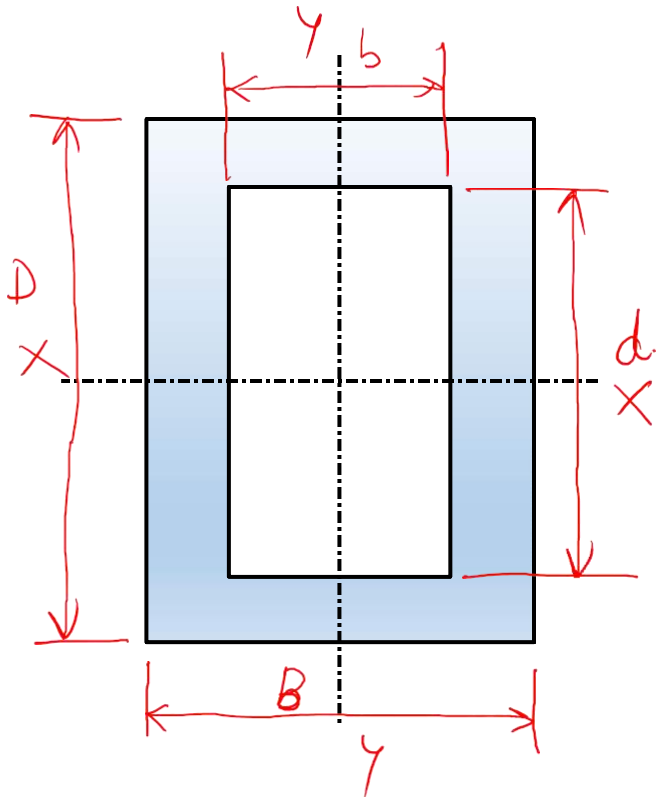
$$\boxed{I_{yy} = 2667263\ mm^4}$$ Ans



MOI of hollow rectangular section

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I_{yy} = \frac{DB^3}{12} - \frac{db^3}{12}$$



MOI of Circular Section

Area of elementry ring $= 2\pi r\, dr$

MoI of elementry area about Z-Z axis $= 2\pi r\, dr\, r^2$

MoI of whole area about Z-Z axis =

$$I_{zz} = \int_0^R 2\pi r^3\, dr = 2\pi\left[\frac{r^4}{4}\right]_0^R = \frac{2\pi R^4}{4}$$

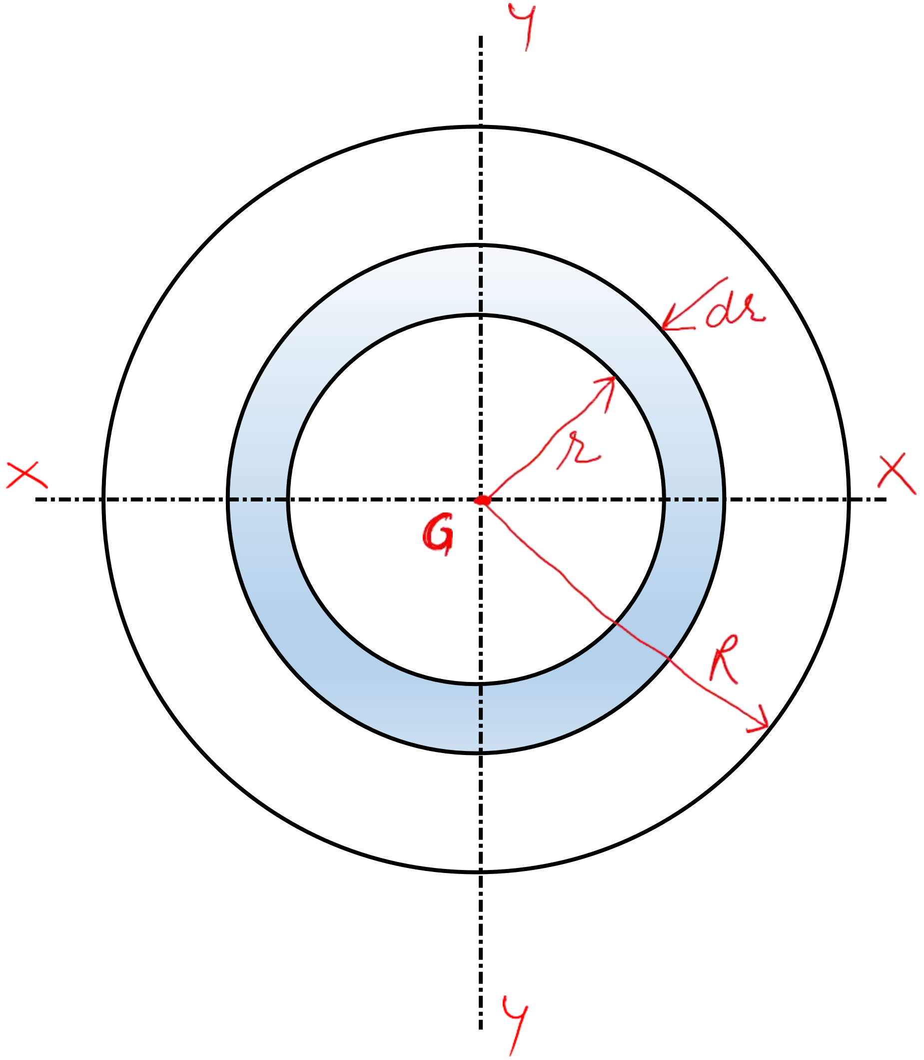
$$I_{zz} = \frac{\pi R^4}{2} = \frac{\pi}{32} D^4 \quad \left[\because R = \frac{D}{2}\right]$$

<u>From ⊥ axis theorem</u>

$$I_{zz} = I_{xx} + I_{yy} \quad [\because I_{xx} = I_{yy}]$$

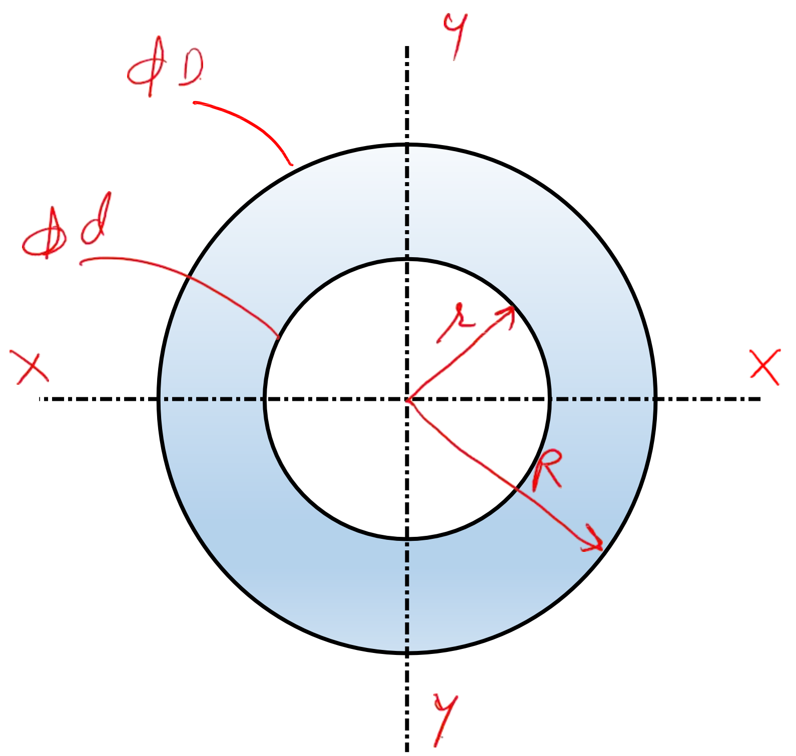
$$I_{zz} = 2 I_{xx}$$

$$\boxed{I_{xx} = \frac{\pi}{4} R^4 \text{ OR } \frac{\pi}{64} D^4 = I_{yy}}$$



MOI of hollow Circular Section

$$I_{xx} = I_{yy} = \frac{\pi}{64}\left(D^4 - d^4\right)$$



Find I_{xx} and I_{yy}.

$\boxed{1} - 2$

Sol: $\bar{X} = 100\text{ mm}$; $\bar{Y} = 129.14\text{ mm}$.

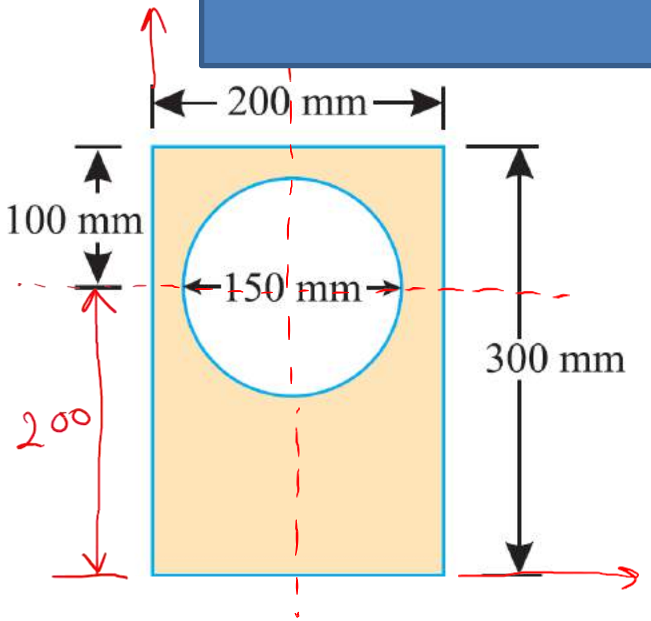
$$I_{xx} = I_{xx_1} - I_{xx_2} = \left\{\frac{200 \times 300^3}{12} + 60000(150 - 129.14)^2\right\} - \left\{\frac{\pi}{64} \times 150^4 + 17671.45(200 - 129.14)^2\right\}$$

$$\boxed{I_{xx} = 362527069.9\text{ mm}^4}$$

$$I_{yy} = I_{yy_1} - I_{yy_2}$$

$$= \frac{300 \times 200^3}{12} - \frac{\pi}{64} 150^4$$

$$\boxed{I_{yy} = 175149511.2\text{ mm}^4}$$



$a_1 = 60000$

$a_2 = 17671.45$

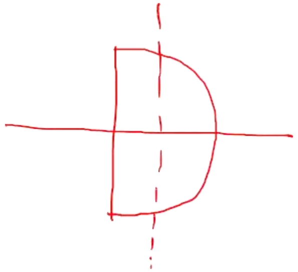
$y_1 = 150$

$y_2 = 200$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} =$$

MOI of a Semi-circle

$$I_{AB} = \underbrace{I_{yy}} = \frac{1}{2}\times\frac{\pi}{64}D^4 = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$$

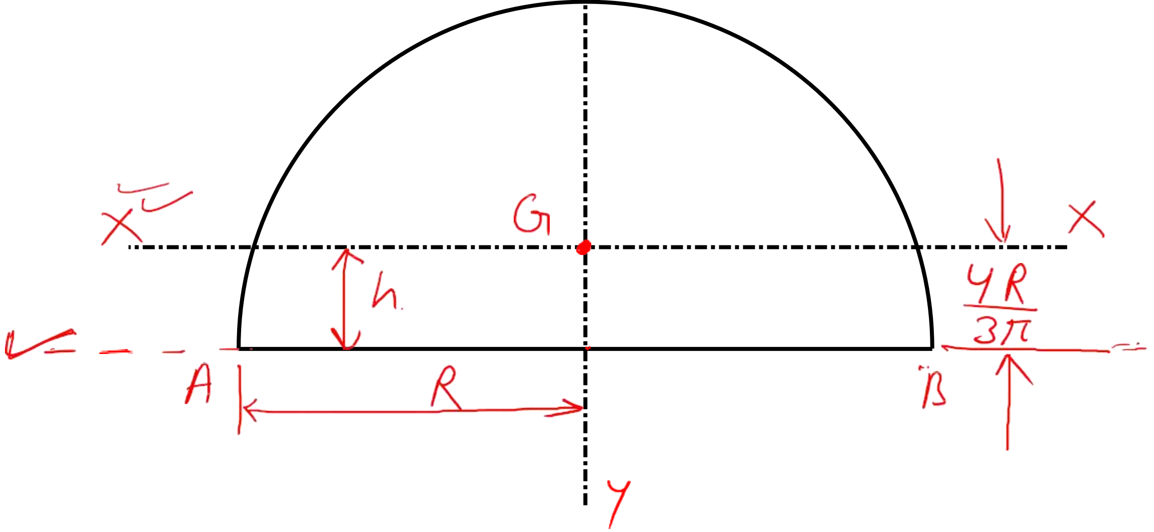


$$I_{AB} = I_{xx} + Ah^2 \quad \text{(// axis theorem).}$$

$$\frac{\pi R^4}{8} = I_{xx} + \frac{\pi}{2}R^2\left(\frac{4R}{3\pi}\right)^2$$

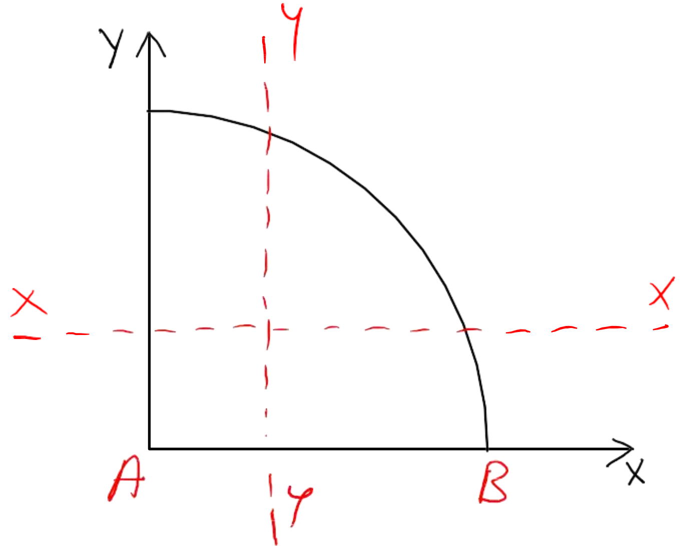
$$I_{xx} = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi} = R^4\underbrace{\left[\frac{\pi}{8} - \frac{8}{9\pi}\right]}$$

$$\boxed{I_{xx} = 0.11\,R^4}$$



$$I_{xx} = I_{yy} = 0.055\,R^4.$$

$$I_{AB} = \frac{\pi}{16}R^4$$



Find I_{xx} & I_{yy} | Sol: $\bar{x} = 60\text{ mm}$; $\bar{y} = 75\text{ mm}$.

$\boxed{1} - 2\,\boxed{2}$

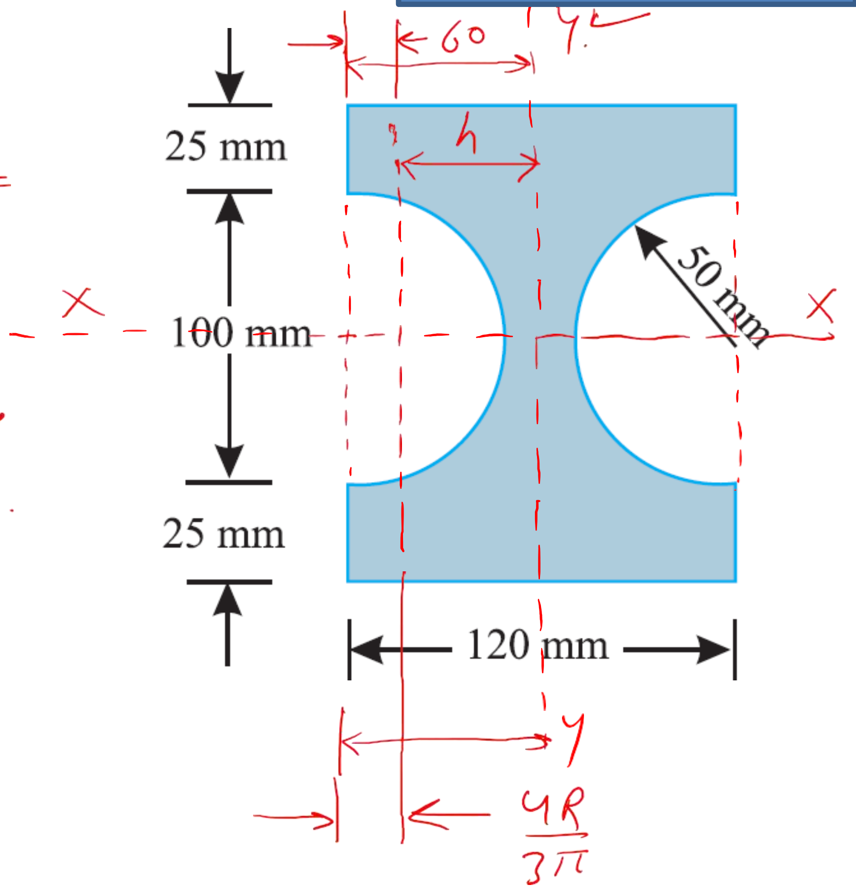
$$I_{xx} = I_{xx_1} - 2\, I_{xx_2}$$

$$= \frac{120 \times 150^3}{12} - 2 \times \left[\frac{\pi}{8}\, 50^4\right] = 28841261.48\text{ mm}^4 \quad \text{Ans}$$

$$I_{yy} = I_{yy_1} - 2 I_{yy_2}$$

$$= \frac{150 \times 120^3}{12} - 2\left\{0.11 \times 50^4 + \left(\frac{\pi}{2} \times 50^2 \times \left[60 - \frac{4 \times 50}{3\pi}\right]^2\right)\right\}$$

$$I_{yy} = 8413889.604\text{ mm}^4 \quad \text{Ans}$$



MOI of Triangular Section

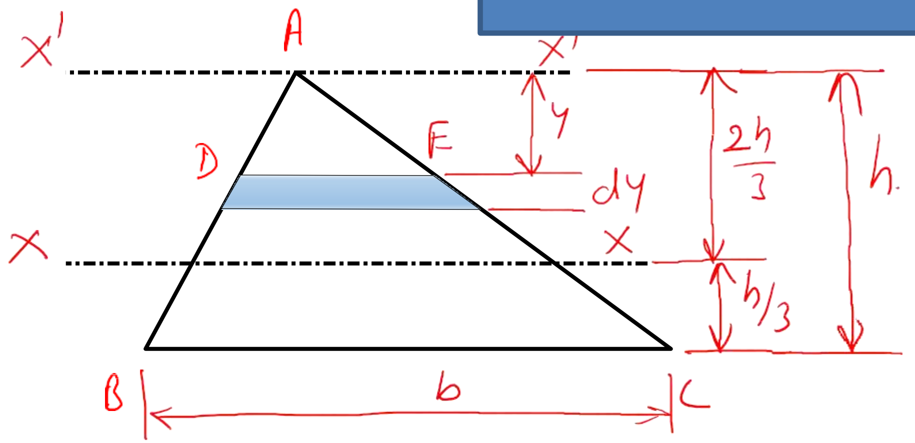
Width of elementary strip $= \frac{b}{h} \times y$

Area of elementary strip $= \frac{b}{h} \times y \times dy$

MoI of elementary area about $X'-X'$ axis $= \frac{b}{h} \times y \times dy \times y^2$

MoI of whole area about $X'-X'$ axis.

$$I_{x'x'} = \int_0^h \frac{b}{h} y^3 \, dy = \frac{b}{h}\left[\frac{y^4}{4}\right]_0^h = \frac{bh^3}{4}$$



$$I_{x'x'} = I_{xx} + A\left(\frac{2h}{3}\right)^2$$

$$\frac{bh^3}{4} = I_{xx} + \frac{bh}{2}\left[\frac{4h^2}{9}\right]$$

$$I_{xx} = \frac{bh^3}{4} - \frac{2\,bh^3}{9}$$

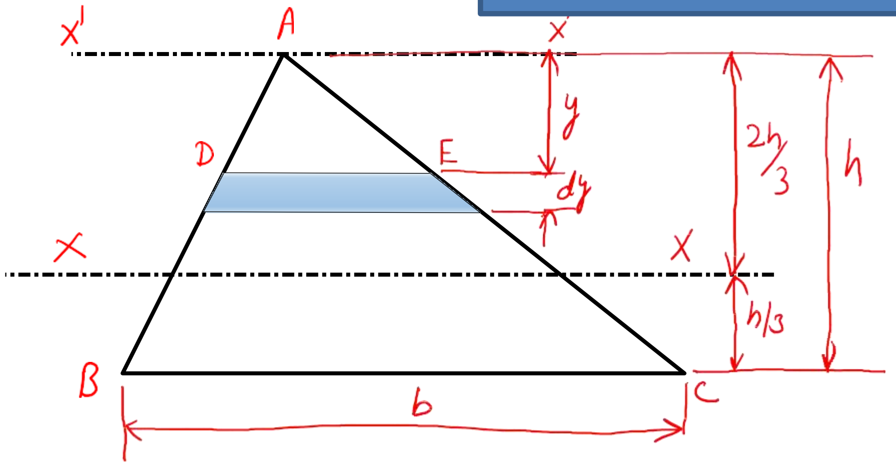
$$\boxed{I_{xx} = \frac{bh^3}{36}}$$

MOI of Triangular Section

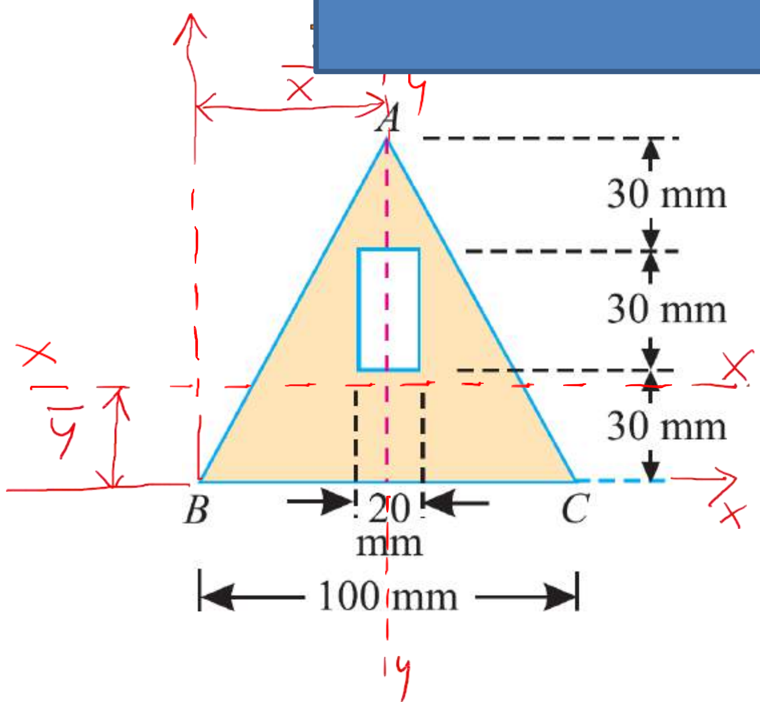
$$I_{BC} = I_{xx} + A\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{bh}{2} \times \frac{h^2}{9}$$

$$\boxed{I_{BC} = \frac{bh^3}{12}}$$



Determine the moment of inertia of the section about X-X axis, Y-Y axis and Base BC



Sol. △ 1 − □ 2 | $\bar{x} = 50$ mm ; $\bar{y} = 27.69$ mm.

$a_1 = 4500$ mm² ; $y_1 = 30$ mm

$a_2 = 600$ mm² ; $y_2 = 45$ mm

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$= \left\{\frac{100\times 90^3}{36} + 4500(30-27.69)^2\right\} - \left\{\frac{20\times 30^3}{12} + 600(45-27.69)^2\right\}$$

$$\boxed{I_{xx} = 1824230.79 \text{ mm}^4} \quad \text{Ans}$$

Determine the moment of inertia of the section about X-X axis, Y-Y axis and Base BC

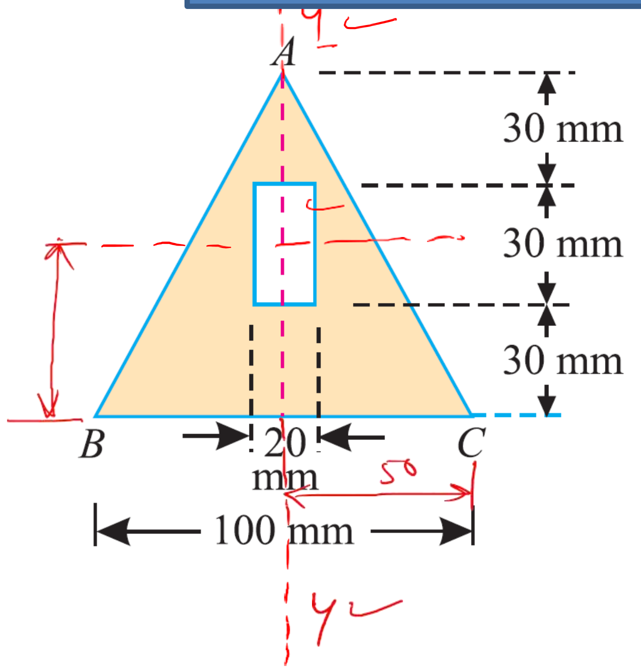
$$I_{yy} = I_{yy_1} - I_{yy_2}$$

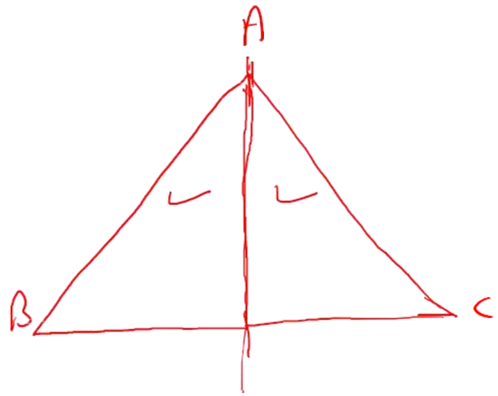
$$= 2\left\{\frac{90 \times 50^3}{12}\right\} - \frac{30 \times 20^3}{12} = 1855000\ mm^4$$

$$I_{BC} = I_{BC_1} - I_{BC_2}$$

$$= \frac{100 \times 90^3}{12} - \left\{\frac{20 \times 30^3}{12} + 600 \times 45^2\right\}$$

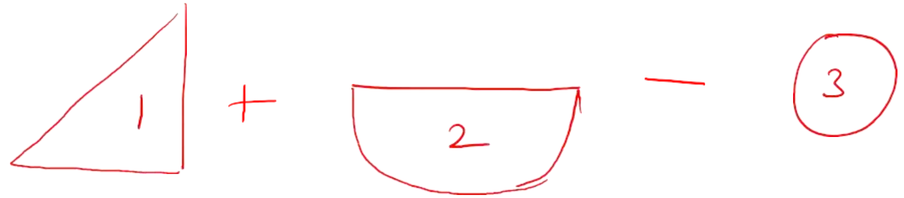
$$\boxed{I_{BC} = 4815000\ mm^4}$$





Find moment of inertia of the shaded area about the axis AB.

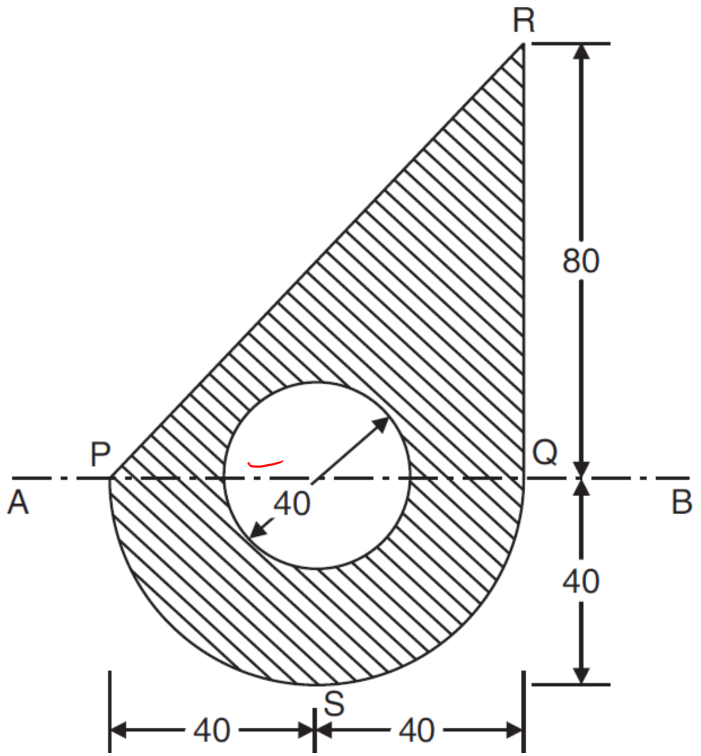
Sol.



$$I_{AB} = I_{AB_1} + I_{AB_2} - I_{AB_3}$$

$$= \left(\frac{80 \times 80^3}{12}\right) + \left(\frac{\pi}{8} \times 40^4\right) - \left(\frac{\pi}{64} \times 40^4\right)$$

$$\boxed{I_{AB} = 4292979.276 \text{ mm}^4}$$ Ans



Find I_{xx} & I_{yy} for Remainder

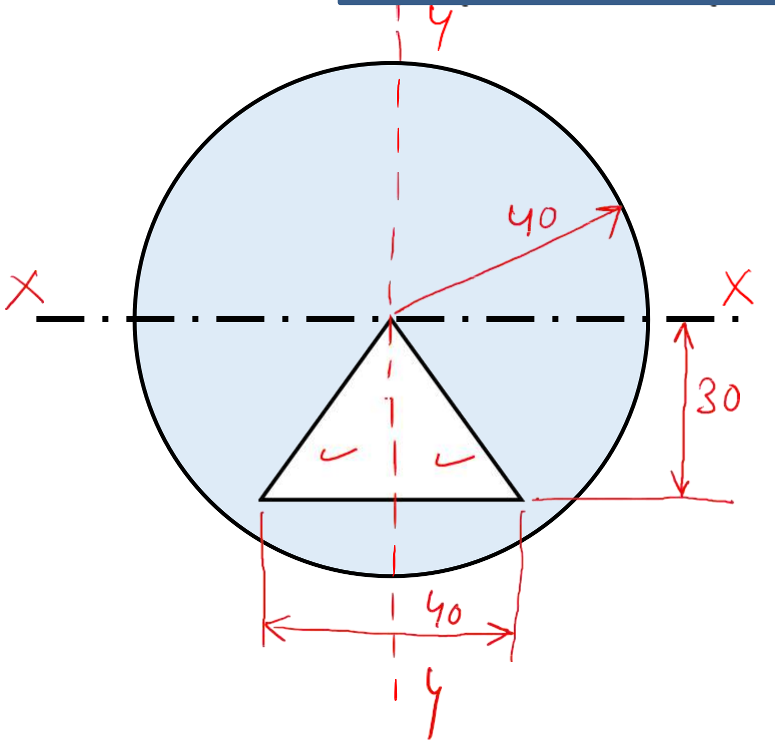
Sol. $I_{xx} = I_{xx_1} - I_{xx_2}$.

$$= \left(\frac{\pi}{64} \times 80^4\right) - \left(\frac{40 \times 30^3}{4}\right) = 1740619.3 \text{ mm}^4 \quad \text{Ans}$$

$I_{yy} = I_{yy_1} - I_{yy_2}$.

$$= \frac{\pi}{64} \times 80^4 - 2\left[\frac{30 \times 20^3}{12}\right]$$

$$\boxed{I_{yy} = 1970619.2 \text{ mm}^4} \quad \text{Ans}$$



Find I_{AC}

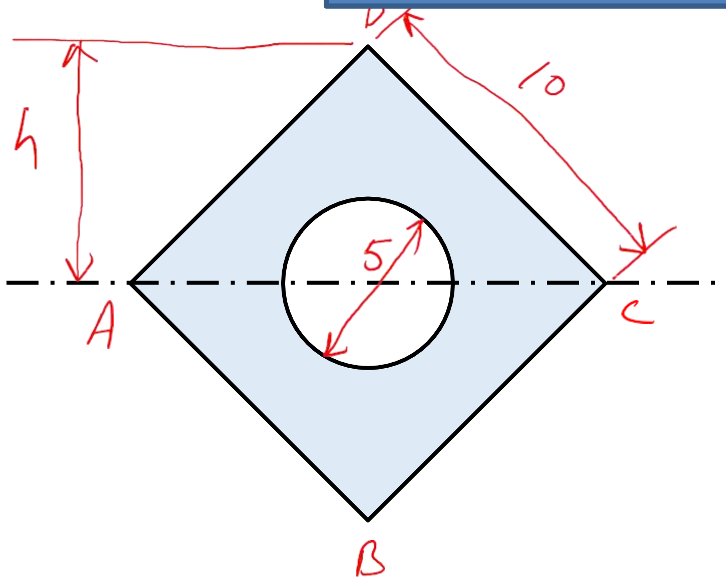
Sol: $2 \triangle_{1} - (2)$

$AC = 14.14\ cm.$, $h = 7.07\ cm.$

$$I_{AC} = 2 I_{AC_1} - I_{AC_2}.$$

$$= 2\left[\frac{14.14 \times 7.07^3}{12}\right] - \frac{\pi}{64} \times 5^4$$

$$\boxed{I_{AC} = 802.15\ cm^4}$$ Ans



dim in cm

Find CG

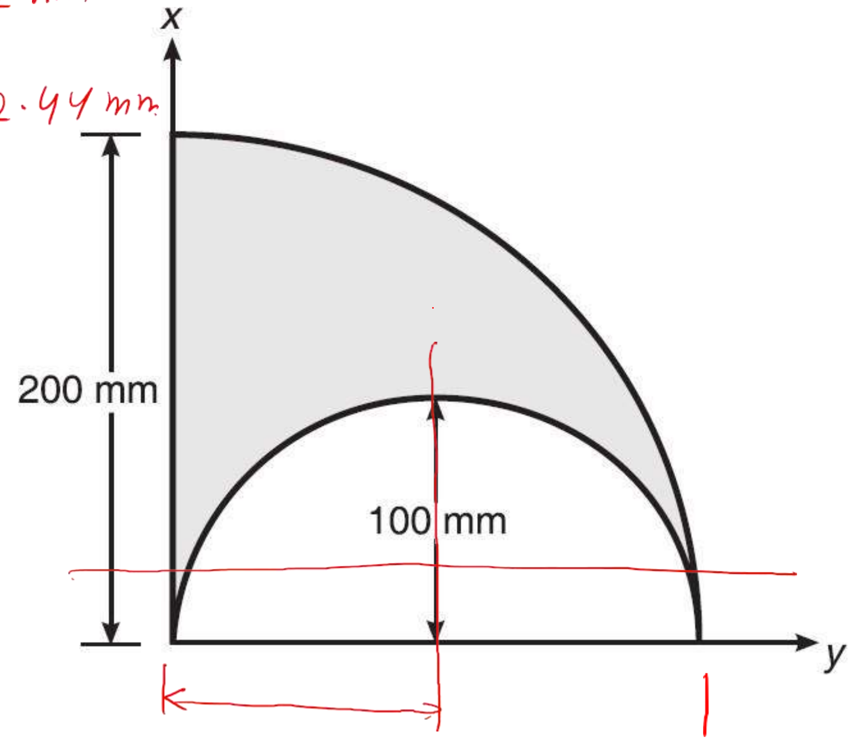
Soln:

$a_1 = \frac{\pi}{4} \times 200^2 = 31415.926\ mm^2$ | $x_1 = y_1 = \frac{4 \times 200}{3\pi} = 84.882\ mm$

$a_2 = \frac{\pi}{2} \times 100^2 = 15707.96$ | $x_2 = 100\ mm;\ y_2 = \frac{4 \times 100}{3\pi} = 42.44\ mm$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = 69.76\ mm^4$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = 127.32\ mm^4$$ Ans



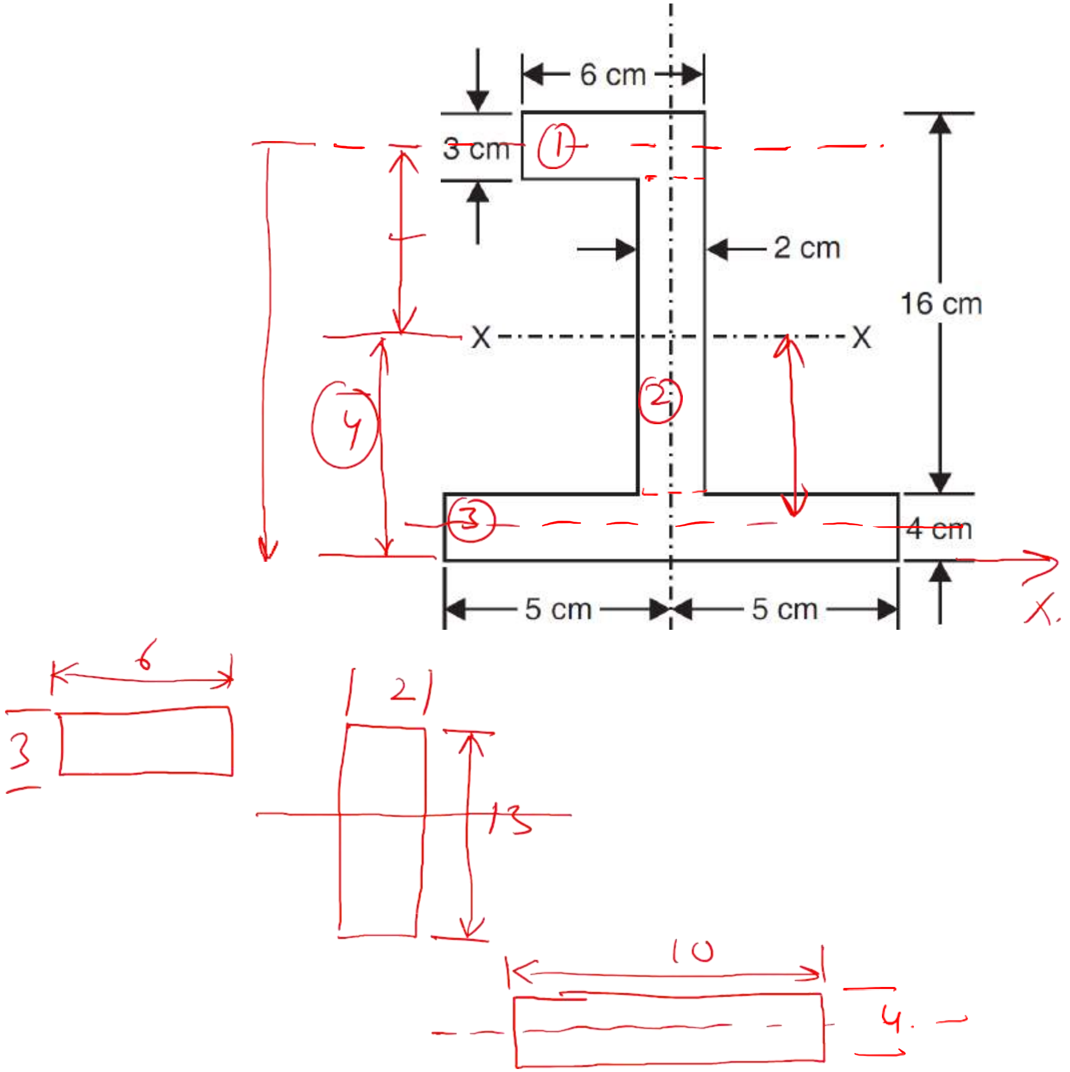
Find I_{xx}

Sol: $\bar{y} = \dfrac{(18\times 18.5)+(26\times 10.5)+(40\times 2)}{18+26+40} = 8.16\,cm$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= \left\{\frac{6\times 3^3}{12} + 18(18.5-8.16)^2\right\} + \left\{\frac{2\times 13^3}{12} + 26(10.5-8.16)^2\right\} + \left\{\frac{10\times 4^3}{12} + 40(8.16-2)^2\right\}$$

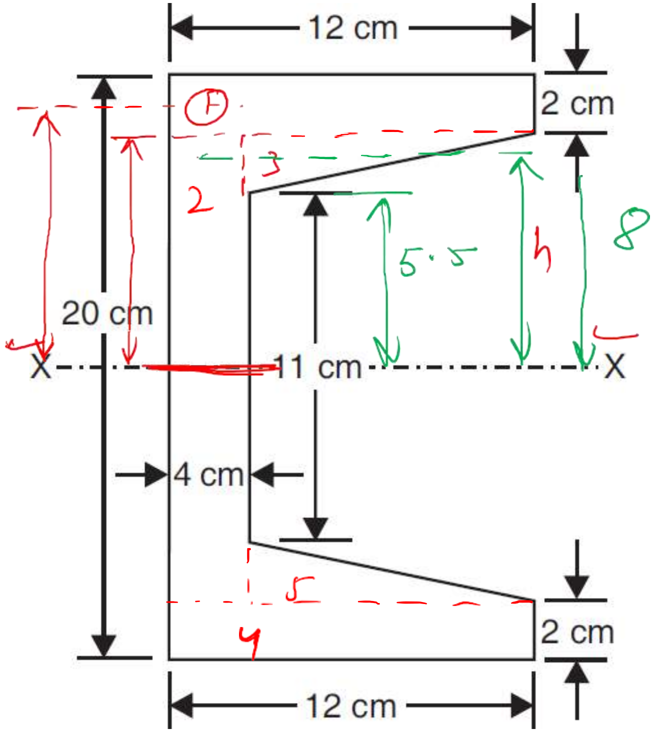
$$\boxed{I_{xx} = 4017.67\,cm^4}$$ Ans



Find I_{xx}.

$$I_{xx} = 2\left\{\left[\frac{12\times 2^3}{12} + 24\times 9^2\right] + \left[\frac{4\times 8^3}{12} + 32\times 4^2\right] + \left[\frac{8\times 2.5^3}{36} + 10\times 7.16^2\right]\right\}$$

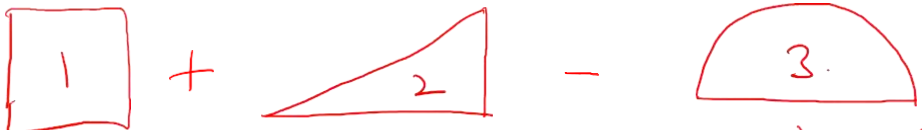
$$\boxed{I_{xx} = 6301.59\ cm^4}$$



$8 - \frac{2.5}{3}$

Find the least radius of gyration

Sol:



$$\bar{X} = \frac{(5500 \times 50) + (1500 \times 66.66) - (981.747 \times 75)}{5500 + 1500 - 981.747}$$

$\bar{X} = 50.074$ mm ; $\bar{Y} = 39.602$ mm.

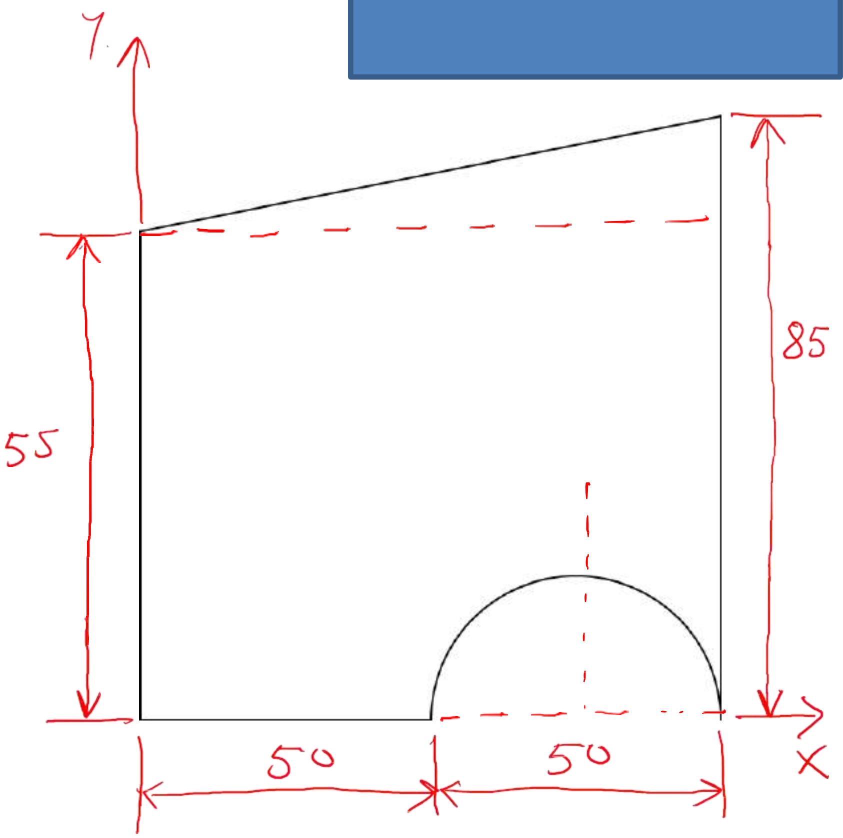
$$I_{xx} = I_{xx_1} + I_{xx_2} - I_{xx_3}$$

$$= \left\{\frac{100 \times 55^3}{12} + 5500\,(39.602 - 27.5)^2\right\}$$

$$+ \left\{\frac{100 \times 30^3}{36} + 1500\,(65 - 39.602)^2\right\}$$

$$- \left\{(0.11 \times 25^4) + 981.74\left(39.602 - \frac{4 \times 25}{3\pi}\right)^2\right\}$$

$$\boxed{I_{xx} = 2366423.411 \text{ mm}^4}$$



Find the least radius of gyration

$$I_{yy} = I_{yy_1} + I_{yy_2} - I_{yy_3}$$

$$= \left\{\frac{55 \times 100^3}{12} + 5500\,(50.074 - 50)^2\right\}$$

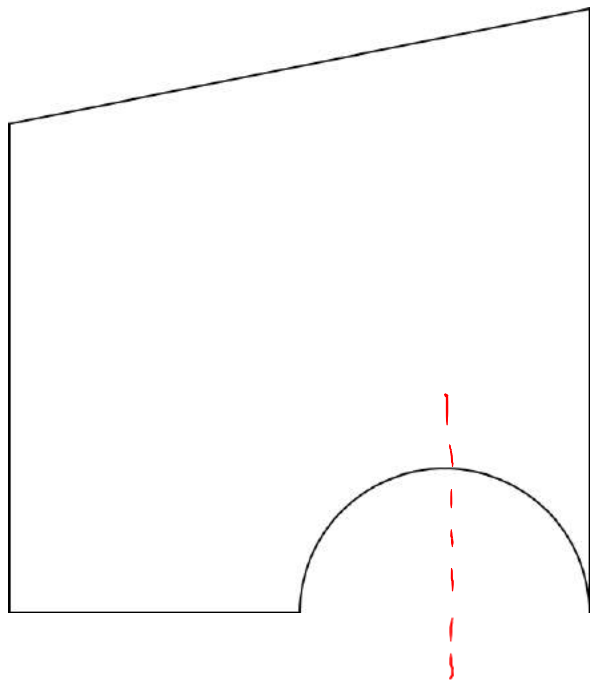
$$+$$

$$\left\{\frac{30 \times 100^3}{36} + 1500\,(66.66 - 50.074)^2\right\}$$

$$-$$

$$\left\{\frac{\pi}{8} \times 25^4 + 981.747\,(75 - 50.074)^2\right\}$$

$$\boxed{I_{yy} = 5065977\ mm^4}$$



$$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

$$\boxed{K_{xx} = 19.829\ mm}$$