KI9CJ KI9FG KI9EG KIJEN

Saturday, August 01, 2020
10:11 PM

Lecture 1

$3^2 = 8$ not tre

The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.
Besides the importance of logic in understanding mathematical reasoning, logic has numerous applications to computer science. **These rules are used in the design of computer circuits, the construction of computer programs, the verification of the correctness of programs, and in many other ways.** Furthermore, software systems have been developed for constructing some, but not all, types of proofs automatically.
We will discuss how to understand and how to construct correct mathematical arguments.
the basic building blocks of logic—propositions.

[See in the last Page]

Propositions: A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Examples

1. Washington, D.C., is the capital of the United States of America.
2. $1 + 1 = 2$.
3. $2 + 2 = 3$.
4. Covid 19 was first identified in China

True / False

$x^2+1 = 5$ if $x=2$

yes $\rightarrow$ T

Are they Proposition?		Propositional value
1	Yes	T
2	Yes	T
3	Yes	T
4	Yes	T

1) • Answer this question
2) • what are you doing
3) • Read this carefully
4) • $x + 2 = 15$ — if $x = 3 \longrightarrow$ F ✓, $x = 13 \longrightarrow$ T ✓
5) • $5 + 7 = 10$
6) • $\boxed{x + y = z}$ ✓ — if $x=1$, $y=2$, $z=3$ T — $x=4$, $y=5$, $z=10$ F
7) • LPU is situated in Himachalpradesh (F)

N.D.S, N.T.N.F., PwF TV

NDS.– not declarative sentence ✓

NTNF neither true nor false

NDS.– not declarative sentence ✓

NTNF – Neither true nor false ✓

PWF/TV: Proposition with true/False Value ✓

*The truth value of a proposition is true, denoted by T, if it is a true proposition.

*The truth value of a proposition is false, denoted by F, if it is a false proposition.

$x = 3$ then $x^2 = 9$ T
$x^2 \neq 9$ F

Definition 1: Let p be a proposition. The negation of p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement "It is not the case that p."
The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

p: "Michael's PC runs Linux"
The negation is
$\sim p$: "It is not the case that Michael's PC runs Linux."
more simply expressed as
$\sim p$: "Michael's PC does not run Linux."

$\boxed{x \geq 32 \text{ Gb}}$

p: "Vandana's Smartphone has at least 32 GB of memory"
$\sim p$ "It is not the case that Vandana's Smartphone has at least 32 GB of memory."

$x \ngeq 32$

"Vandana's Smartphone does not have at least 32 GB of memory"

"Vandana's Smartphone has less than 32 GB of memory."
$x < 32$

p: The Reciever reached to sender's Locatin

P: The solution for tower of Hanoi game is 2^n (n is the no. of disc)

P_3: $\boxed{2+10=1} \to F$

My name is not Pratush

1) 2^n+1 ✓

2) 2^n-1 ✓

3) 3^n ✓

P: I am fine ; ~P : I am not fine

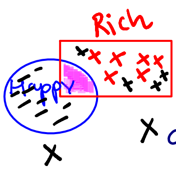
Q: Find the Negation of the Following

1) You are either rich or happy

Let P: You are rich

Q: You are happy

P	Q
✓	×
×	✓
✓	✓
×	×



a) You are Neither happy nor rich

b) You are Happy as well as rich

c) both

If not cleared don't worry & see the truth table $\sim(P \vee q) \equiv \sim P \wedge \sim q$

Q: Let $x, y \subseteq \mathbb{R}$ [$\mathbb{R}$ set of real No.]

P: $x \oplus y = x^2 + y^2$ is Commutative

Now check the truth value of P

$\boxed{\oplus}$ is the binary operation

Let $x = 2$, $y = 3$

$x \oplus y = x^2 + y^2 \Rightarrow 2^2 + 3^2 = 13$

$y \oplus x = y^2 + x^2 \Rightarrow 3^2 + 2^2 = 13$

Commutative : $x \oplus y = y \oplus x$

P: T ✓ yes $x \oplus y$ is comm.

$\sim P$ = $x \oplus y$ is not comm. & F

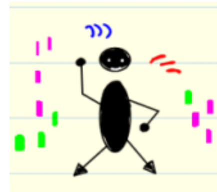
P: T ✓ yes $x \oplus y$ is comm.

$\sim P$ = $x \oplus y$ is not comm. & (F)

Q: Rocky is the rockstar of the class

a) not the rock

b) Rocky is the Dancer



$\sim P$: Rocy is the Dancer

$\sim(\sim P)$:

P: $x = 5$ | $x = 6$

$\sim P$: $x \neq 5$ | $\sim(x=6)$

$\sim$ not Ryesh

P: My name is Ryesh] T

$\sim[\sim P]$ My name is SALMANKHAN

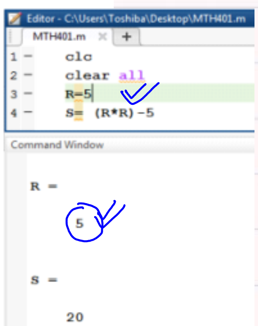
Look the following commands

$R=5$ —— $\boxed{A}$ —— $P = R^2 - 5$

(I) IF $R=5$ then Evaluate $y = R^2 - 5$

Last page L-1:

$\boxed{R=5}$ then $\boxed{y = R^2 - 5}$



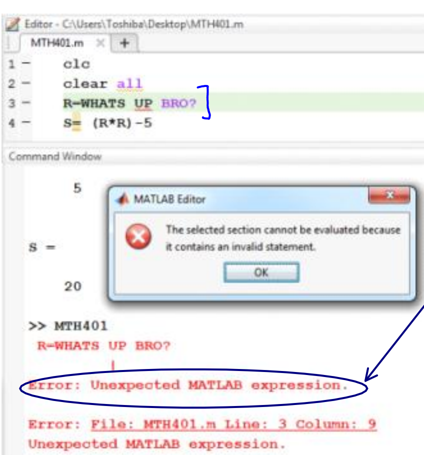
```
Editor - C:\Users\Toshiba\Desktop\MTH401.m
MTH401.m
1 - clc
2 - clear all
3 - R=5
4 - S= (R*R)-5

Command Window
R =
     5
S =
    20
```

- $R = 5;$
- $y = (R * 2) - 5;$

logical operation (* −)

Run the Program

Result: Executed & y is Evaluated as 20



```
Editor - C:\Users\Toshiba\Desktop\MTH401.m
MTH401.m
1 - clc
2 - clear all
3 - R=WHATS UP BRO?
4 - S= (R*R)-5

Command Window
     5
S =
    20
>> MTH401
R=WHATS UP BRO?
Error: Unexpected MATLAB expression.

Error: File: MTH401.m Line: 3 Column: 9
Unexpected MATLAB expression.
```

MATLAB Editor: The selected section cannot be evaluated because it contains an invalid statement. OK

If R= what's up Bro?

$\boxed{\text{only Numeric Entries}}$

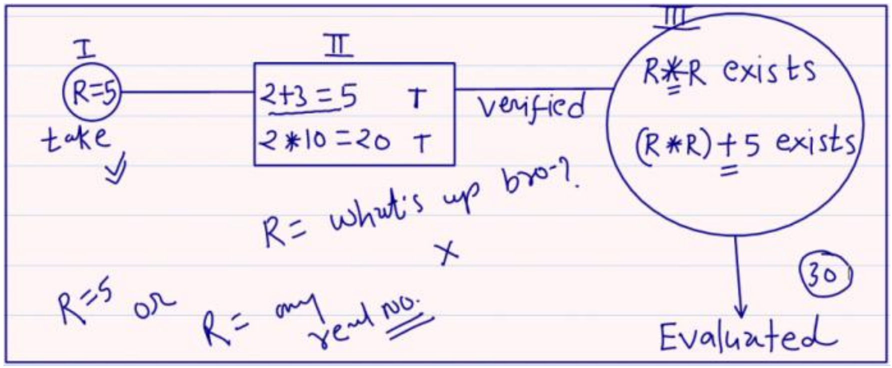
A?

2

Error: File: MTH401.m Line: 3 Column: 9
Unexpected MATLAB expression.

■ If $R=5$ & $\boxed{\text{if } 2+3=5 \text{ \& } 2*10=20}$ then $y=R^2+5$

A ↷



☑ A: if $(10+5=15)$ And $(6+4=10)$

R=5 —— A: verified —— P Executed $(P=R^2-5)$

☑ A: if LPU is situated in punjab   P: Executed = ?

Garima : 53 $5+3 \rightarrow 08$ say x

Abhi : 06 $0+6 \rightarrow 06$ say y

$$P: x^2+y^2=100$$

x: Sum of the digits for Garima's roll No.

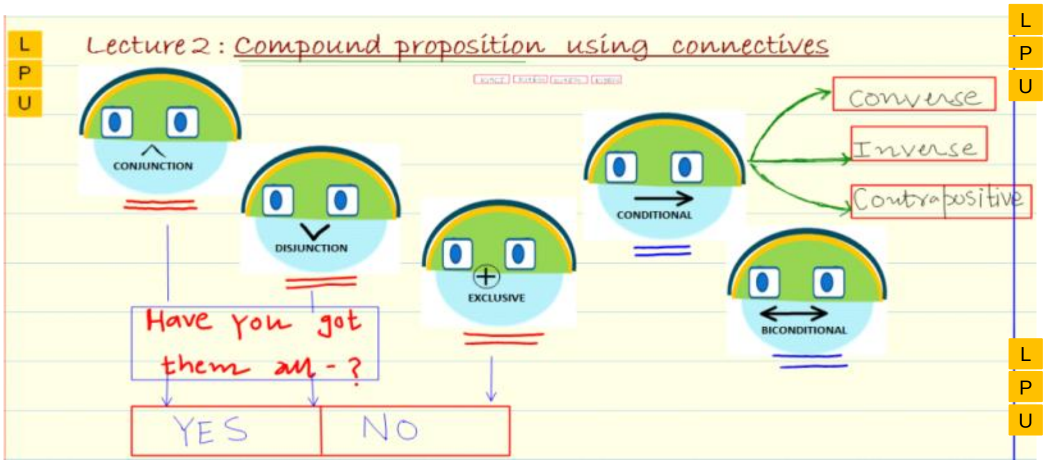
y: Sum of the digits for Abhi's roll No.

then truth value of P is T   I   II

$$P: x^2+y^2=100$$

P	~P
T	F

F $\sim P: x^2+y^2 \neq 100$ T



The negation operator constructs a new proposition from a single existing proposition.

We will now introduce the logical operators that are used to form new propositions from two or more existing propositions.

These logical operators are also called connectives.

Definition 2 :
Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q."
The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

P: $R+1 = 5 \to F$
q: $R^2+1 = 26 \to T$
$F \wedge T = F$
S : Not executed

Find the conjunction of the propositions P and Q

P: "Rebecca's PC has more than 16 GB free hard disk space".

Q: "The processor in Rebecca's PC runs faster than 1 GHz."

Solution: The conjunction of these propositions, $p \wedge q$, is the proposition
$p \wedge q$
"Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz."
"Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz."

For this conjunction to be true, both conditions given must be true. It is false when one or both of these conditions are false.

$p \wedge q$ p: Laxmi scored A+
q: Rajesh scored F

Definition 3

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\wedge$ both p, q T

$\vee$ atleast one T

$\oplus$ Exactly one

$\wedge \rightarrow$

$\vee \rightarrow$

$\oplus \rightarrow$

p: T. Arora scored A grade in MTH401 O

q: T. Arora Placed in Microsoft T

$p \wedge q$: T. Arora scored A grade in MTH and she placed in microsoft
Executed-?

$p \vee q$: Executed-?

p: student who have taken calculus can take take this class

q: Student who have taken Computer science Can take this class

$p \vee q$ (Inclusive or) student who have taken MTH401 or computer science can take this class

$\boxed{\text{Exclusive or } \oplus}$ student who have taken MTH401 or

Computer Science, <u>but not both</u>

Can enroll in this class : MTH401 / CSE

XOR : Either P or q is true [not both]

$P \oplus q$ is true

$R = 5$

$P: R+1=5$

$q: R^2+1=26$

$P: F$

$q: T$

$P \oplus q = T$

$S = R-2$

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$\wedge$ Both T✓

$\vee$ atleast 1 T✓

$\oplus$ Exactly 1 T✓

$\wedge$

$\vee$

$\oplus$

Now Check whether the following Commands can be Executed ? (using P, q, r)

If $R = 5$;

P: $\boxed{R+1=5}$ (F), $\boxed{q: R^2+1=26}$ (T), $\boxed{r: R+1=6}$ (T)

$\boxed{P \text{ and } q} \longrightarrow S = R-1$

Executed - ?

Input → $\boxed{T}$ → S

$P: R+1=5 \Rightarrow F$

$q: T$

$F \wedge T = F$

S not be executed

$F \vee T = T$ ✓

$\boxed{P \vee q} \longrightarrow S = R^3 - 25$ Executed, if yes

$S = 125 - 25$

then $S =$ 100, 20, 90, NE

$\boxed{q \vee r} \longrightarrow S = \frac{R+1}{2}$

If Yes then

$S =$ a) 3 ✓

b) 2

c) N.E.

$\boxed{P \oplus q} \longrightarrow S = R-2$

$F \oplus T = T$ ✓

Practice questions on proposition and negation:

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

b) Miami is the capital of Florida.

c) $2 + 3 = 5$. yes T

d) $5 + 7 = 10$. yes F

e) $x + 2 = 11$. No

f) Answer this question. No

2. Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.

b) What time is it?

c) There are no black flies in Maine.

d) $4 + x = 5$.

e) The moon is made of green cheese.

f) $2^n \geq 100$.

3. What is the negation of each of these propositions?

a) Linda is younger than Sanjay.

b) Mei makes more money than Isabella.

c) Moshe is taller than Monica.

d) Abby is richer than Ricardo.

4. What is the negation of each of these propositions?

a) Janice has more Facebook friends than Juan.

b) Quincy is smarter than Venkat.

c) Zelda drives more miles to school than Paola.

d) Briana sleeps longer than Gloria.

KMCJ | K19FG | K19EG | K19EN

Lecture :3

Conditional Statements

→ Converse,

→ Contra-positive

→ Inverse

→ Bi-conditional

Covid-19 vaccine tracker, July 30: Russian vaccine to be ready by August 12

Coronavirus (COVID-19) vaccine tracker update: A Bloomberg report said the Gamaleya vaccine was likely to get "conditional registration" in August, meaning it would be approved for use, even as phase-III trials are carried out.

https://indianexpress.com/article/explained/covid-19-vaccine-tracker-russia-oxford-moderna-6530179/

Conditional Statements:

Let p and q be propositions. The conditional statement

$\boxed{p \to q}$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise.

In the conditional statement $p \to q$, p is called the hypothesis (or premise) and q is called the consequence.

$\underset{T}{p} \to \underset{F}{q}$

If $\boxed{\text{I will pass}}$ then $\boxed{\text{I will get a new bike}}$

p q

$p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Hypo true but Con. F

p: PM Modi will win the election

q: He will decrease the tax

Let p: All clinical trials are completed

q: The production of vaccine begin in September

$p \to q$: If all clinical trials are completed then the production of vaccine begin in September

$p \to q$

Let $R = 5$, $\boxed{\text{if } \boxed{2+3=5} \text{ then } \boxed{S = R^2+1}}$

p: $2+3=5$

q: S is Executed

p	q	$p \to q$
T	T	T

If $2+3=5$ then $S = 26$

$\boxed{\text{if } \boxed{2+3=6} \text{ then } \boxed{S=R^2+1}}$

p: F, q: F, $p \to q$: T

Let P: $\boxed{2+3=6}$ ✓

q: S executed

If $2+3=6$ then $S \neq 26$

P: (F), q: F , P→q : T

If $2+3 \neq 5$ then S can't be executed

take R=S

P: 2+3= 5]

q : S is executed]

if $\boxed{2+3=5}$ T $\boxed{S = \frac{R^2-1}{R-5}}$

P	q	P→q
T	F	F

If 2+3 = 5 then S can't be Executed

P: 2+3 = 5

q: S is executed

Because conditional statements play such an essential role in mathematical reasoning, a variety of terminology is used to express p → q.

- ☑ "if p, then q" : If 2+3 = 5 then S is executed
- ☐ "p implies q" : 2+3 = 5 ⇒ S is executed
- ☐ "if p, q" : If 2+3 = 5, S is executed
- ☐ "p only if q" : 2+3 = 5 only if S is executed
- ☐ "p is sufficient for q": 2+3 = 5 is Sufficient for S is executed
- ☐ "a sufficient condition for q is p" ↳ A Sufficient Condition for "S is executed is 2+3 = 5
- ☐ "q if p"
- ☐ "q whenever p"
- ☐ "q when p"
- ☐ "q is necessary for p"
- ☐ "a necessary condition for p is q"
- ☐ "q follows from p"
- ☐ "q unless ¬p"
- ☐ "q provided that p"

P: x+y = 18

q: Saloni & Dimple are good Friend

P → q

q → P → (T)

Saloni - 67 = 13

Dimple - 05 = 5

$P \xrightarrow{100} q$, $q \xrightarrow{10} P$

- The proposition q → p is called the converse of p → q.
- The contra-positive of p → q is the proposition ¬q → ¬p.
- The proposition ¬p → ¬q is called the inverse of p → q.

If Yash is not happy then 1+1 ≠ 2

P: 1+1 = 2, q = Yashray is happy

P: All the clinical trials are completed p: All the clinical trial's are completed

q: The Production of vaccine begin in Sep

p: $2+3=5$ $p \to q$: if $2+3=5$ then S is exe

q: S is executed

1. if S is executed then $2+3=5$
2. if S is not executed then $2+3 \neq 5$
3. if $2+3 \neq 5$ then S is not executed

Converse — ? If Saloni & Dimple are good friend then $x+y=18$

Contrapositive — ? [Saloni & Dimple are not good friend then $x+y \neq 18$] / is

Inverse — ? if $x+y \neq 18$ then Saloni & Dimple are not good F.

If I will Score A^+ in MTH 401 then my father will be happy

Conditional Statements

Converse,

Contra-positive

Inverse

Bi-conditional statement

p: $2+8=10$

q: I am happy

If You will study hard then you get A^+

Lecture 4

1. Bi-conditional statement
2. Truth Tables of Compound Propositions
3. Precedence of Logical Operators
4. Exercises

① Bi-conditional statement

Let p and q be propositions. The bi-conditional statement $\boxed{p \leftrightarrow q}$ is the proposition "p if and only if q." The statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

T T
F F

p	q	$p \leftrightarrow q$
T	T	T

Note that the statement $p \leftrightarrow q$ is true when both the conditional statements $p \to q$ and

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Note that the statement $p \leftrightarrow q$ is true when both the conditional statements $p \to q$ and $q \to p$ are true and is false otherwise.

There are some other common ways to express $p \leftrightarrow q$:

- "p is necessary and sufficient for q"
- "if p then q, and conversely"
- "p iff q." "p exactly when q."

P: Suraj is shining
q: He got A+

The last way of expressing the bi-conditional statement $p \leftrightarrow q$ uses the abbreviation "iff" for "if and only if."

- $p \leftrightarrow q$ has exactly the same truth value as $(p \to q) \wedge (q \to p)$.

$P \leftrightarrow q$: $P \to q$, $q \to P$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Let p: "You can take the flight," q: "You buy a ticket."
Then $p \leftrightarrow q$: "You can take the flight if and only if you buy a ticket."

- This statement is true if p and q are either both true or both false, that is, if you buy a ticket, and can take the flight

$P \leftrightarrow q$

$P \to q$, $q \to P$

- if you do not buy a ticket and you cannot take the flight.

It is false when p and q have opposite truth values

- when you do not buy a ticket, but you can take the flight
- when you buy a ticket but you cannot take the flight

$P \leftrightarrow q$

Poll

P: 2+2=4

Q: I learn MATLAB

① 2+2≠4 iff I learn MATLAB

F

② 2+2=4 iff I learn MATLAB

Ⓐ T F

Ⓑ F F

2

Truth Tables of Compound Propositions

- $(p \quad \neg q) \to (p \quad q)$.
- $(p \to q) \wedge (q \to p) = p \leftrightarrow q$

P: 2+2=4
q: You are happy

∧∨
∧∨
∨∨
→∨

- p ¬p

p	q	~q	p∨~q	p∧q	(p∨~q)→p∧q
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

p	~p	p∧~p
T	F	F
T	F	F
F	T	F
F	T	F

T
F

3

Precedence of Logical Operators.

Operator	*Precedence*
¬	1
∧	2
∨	3
→	4
↔	5

p: 1+1=2
q: 5 evaluated
r = I am fine

- $p \vee q \wedge r$ means $p \vee (q \wedge r)$ rather than $(p \vee q) \wedge r$
- $p \wedge q \vee r$ means $(p \wedge q) \vee r$ rather than $p \wedge (q \vee r)$

p: 1+1=2
q: 5 is Evaluated
r: You are a nice person
$p \vee (q \wedge r)$

4.1

Exercises

∧ ∨ → ↔

p→q
q→p

Determine whether these biconditionals are true or false.

if 1+1=2 then 2+2=4
if 2+2=4 then 1+1=2

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$. → T F
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$. → F
c) $1 + 1 \neq 3$ if and only if monkeys can fly. T
d) $0 > 1$ if and only if $2 > 1$. T

F F

4.2

Determine whether each of these conditional statements is true or false.

→ T → F F

a) If $1 + 1 = 2$, then $2 + 2 = 5$. T F = F
b) If $1 + 1 = 3$, then $2 + 2 = 4$. F T T
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$. T/F

F F T

4.3

p: I bought a lottery ticket this week. T
q: I won the million dollar jackpot. T
Express each of these propositions as an English sentence.

a) $\neg p$ b) $\boxed{p \vee q}$ c) $\boxed{p \rightarrow q}$
d) $p \wedge q$ e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$
g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$

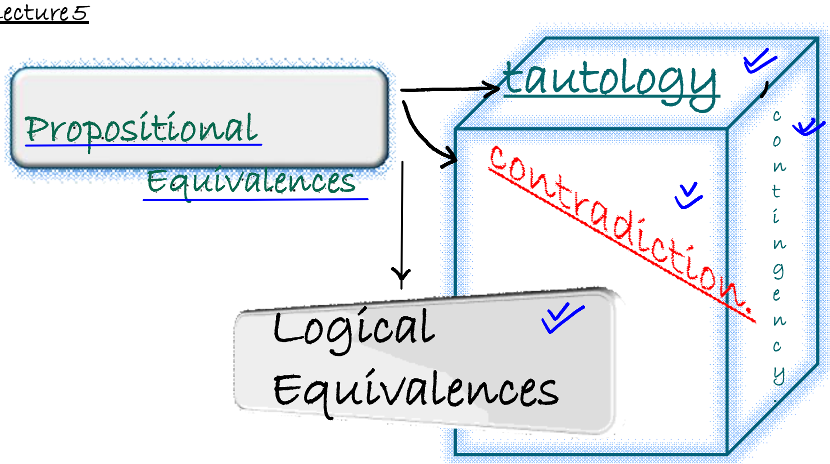
h)

4.4

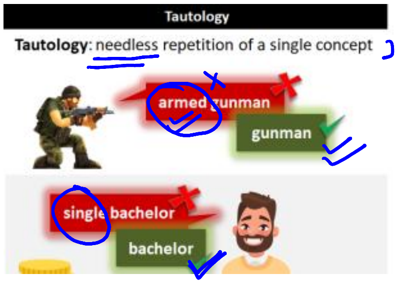
Let p and q be the propositions
p: You drive over 65 miles per hour.
q: You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations). ¬p, p, ¬q

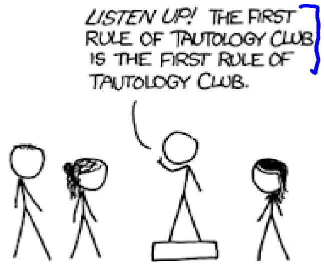
a) You do not drive over 65 miles per hour. ¬p
b) You drive over 65 miles per hour, but you do not get a speeding ticket. p ¬q p∨¬q , p∧q , p∧¬q
c) You will get a speeding ticket if you drive over 65 miles per hour. p→q q→p , p→q , p∧q , p∨q
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket. ¬p→¬q ¬q→¬p , ¬p→¬q
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket
f) You get a speeding ticket, but you do not drive over 65 miles per hour
g) Whenever you get a speeding ticket, you are driving over 65 miles per hour

Lecture 5



Equivalences





GRAMMAR

- A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology.
- A compound proposition that is always false is called a contradiction.
- A compound proposition that is neither a tautology nor a contradiction is called a contingency.

p	~p	p∨~p
T	F	T
F	T	T

- p ∨ ¬p T
- p ∧ ¬p F
- (p ∧ q) → p

p	~p	p∧~p
T	F	F
F	T	F

p	q	p∧q	(p∧q)→p
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Logical Equivalences: Compound propositions that have the same truth values in all possible cases are called logically equivalent.

De Morgan's Laws.

- ¬(p q) ≡ ¬p ¬q
- ¬(p q) ≡ ¬p ¬q
- Show that ¬(p q) and ¬p ¬q are logically equivalent.
- Show that p → q and ¬p q are logically equivalent.
- Show that p (q r) and (p q) (p r) are logically equivalent
- Show that (p q) → (p q) is a tautology.

p	q	p∧q	~(p∧q)	~p	~q	~p∨~q
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

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2 $2^2 = 4$ {T, F}

P: You get A+ in MTH

q: You know about the Linux

p→q, ~p∨q

P	q	P→q	~p	~p∨q
T	T	T	F	T

T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

L P U

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

L P U

L P U

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Some Rule

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logical Equivalence without truth table

Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

$$\sim(p\to q) = \sim(\sim p \vee q)$$
$$= \sim(\sim p) \wedge \sim q$$
$$= p \wedge \sim q$$

Show that ¬ (p ∨ (¬p ∧ q)) and ¬p ∧ ¬q are logically equivalent

LHS $\sim(p\vee(\sim p\wedge q)) = \sim p \wedge \sim(\sim p \wedge q)$ ← De morgan's Law

$= \sim p \wedge (\sim(\sim p) \vee \sim q)$

$= \sim p \wedge (p \vee \sim q)$

Distributive law

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q) \vee F$

Identity Law

$= \boxed{\sim p \wedge \sim q}$

p	~p	p∧~p
T	F	F
F	T	F