

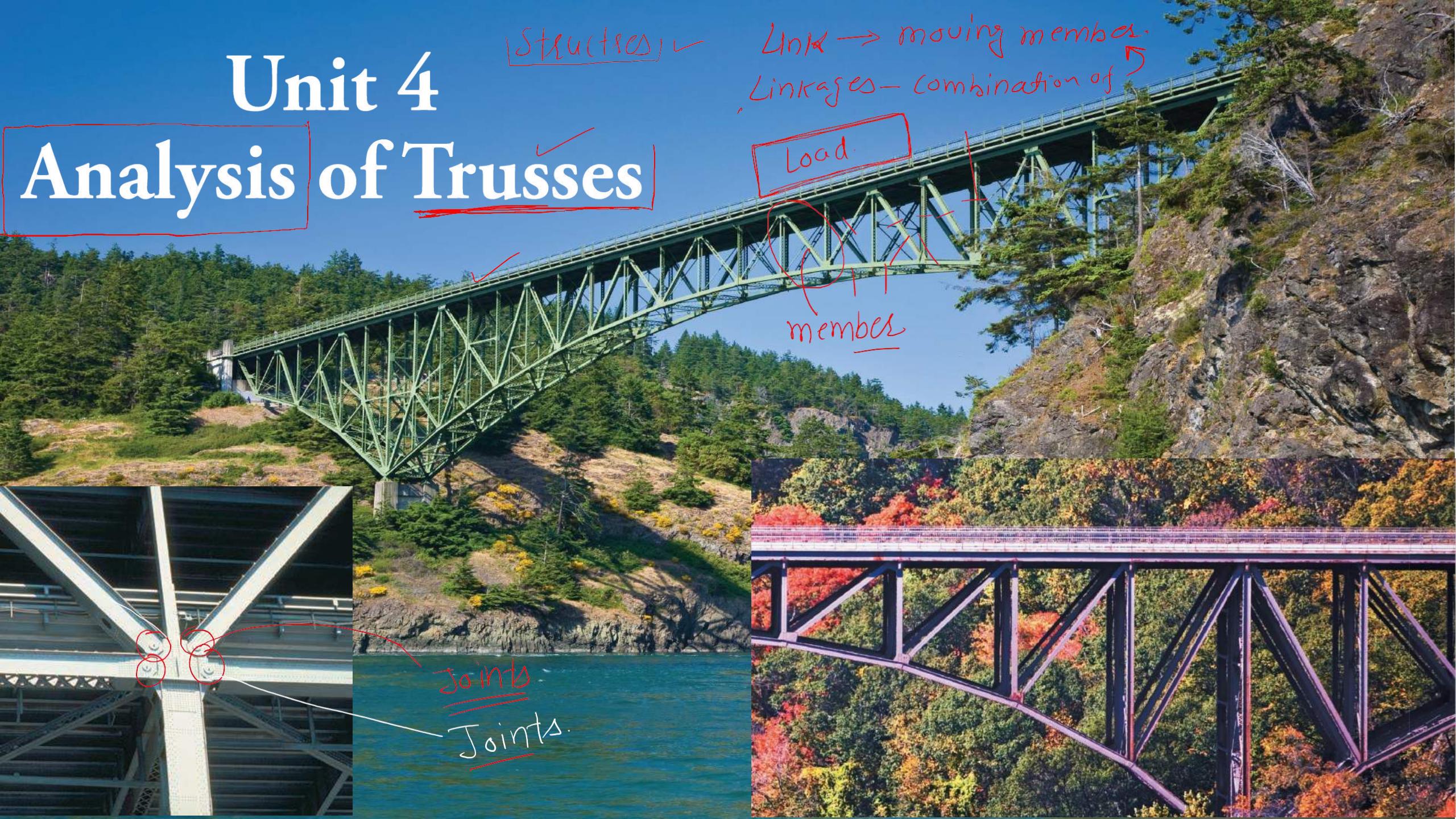


Unit 4

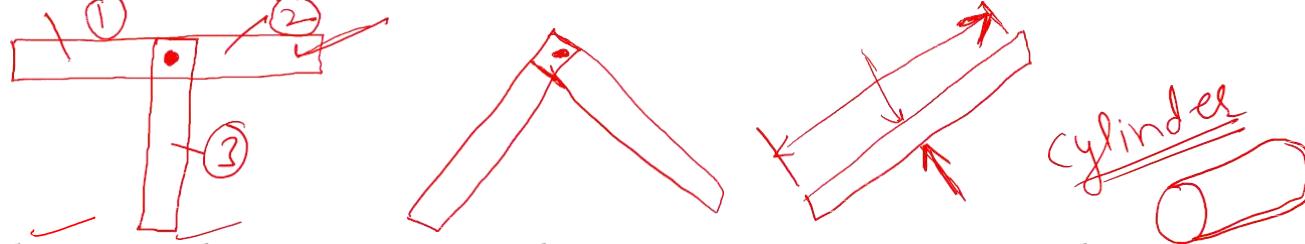
Analysis of Trusses

Structures → Link → moving members.

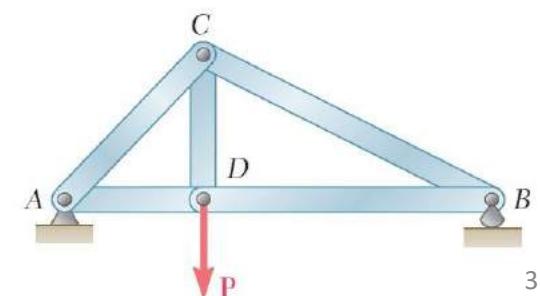
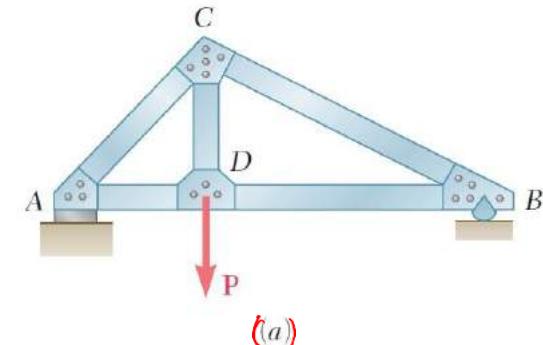
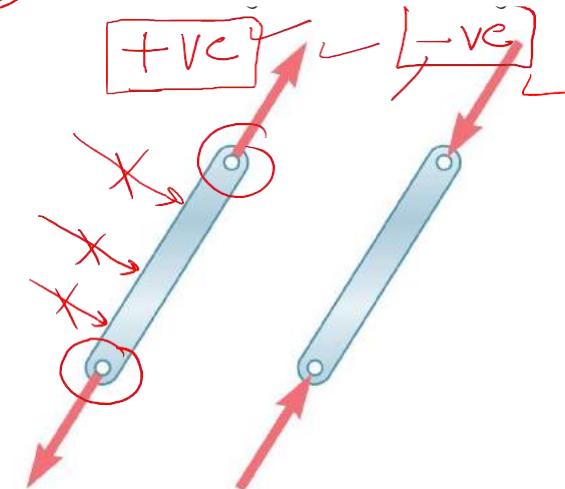
Linkages – combination of 5



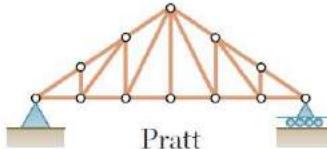
Introduction



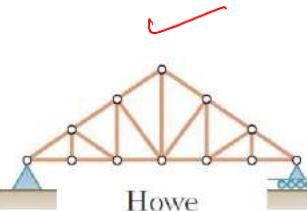
- A truss consists of straight members connected at joints. Truss members are connected at their extremities only; thus no member is continuous through a joint.
- The members of a truss are slender and can support little lateral load; all loads, therefore, must be applied to the various joints, and not to the members themselves.
structural frame or frame
- Most actual structures are made of several trusses joined together to form a space framework. Each truss is designed to carry those loads which act in its plane and thus may be treated as a two-dimensional structure.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.



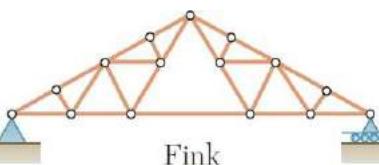
Types of Trusses



Pratt

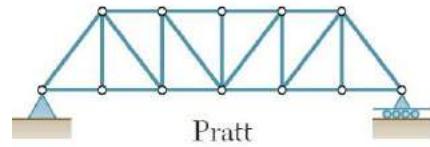


✓
Howe

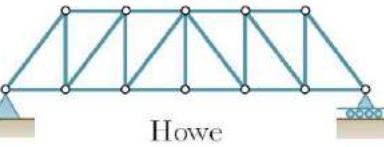


Fink

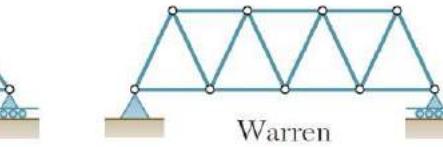
Typical Roof Trusses



Pratt



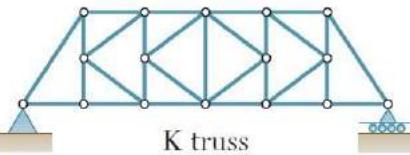
Howe



Warren

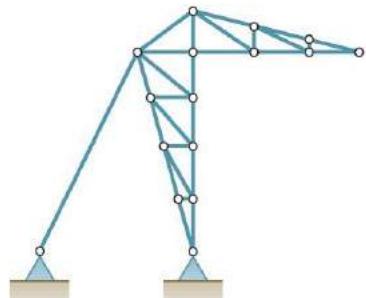


Baltimore

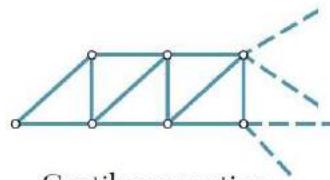


K truss

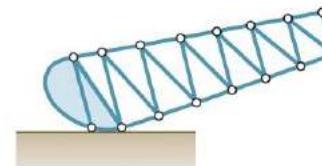
Typical Bridge Trusses



Stadium



Cantilever portion
of a truss



Bascule

Other Types of Trusses

Types of Trusses/Frames

- A pin-jointed truss which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a **perfect truss**.
- A truss is said to be **deficient** if the number of members in it are less than that required for a perfect truss. Such trusses cannot retain their shape when loaded.
- A truss is said to be **redundant** if the number of members in it are more than that required in a perfect truss. Such trusses cannot be analyzed by making use of the equations of equilibrium alone. Thus, a redundant truss is statically indeterminate.

$$n = 2j - 3$$

n = No. of members

j = No. of joints

If $LHS = RHS$ (Perfect)

$LHS < RHS$ (Deficient)

$LHS > RHS$ (Redundant)

(1) $3 = (2 \times 3) - 3$

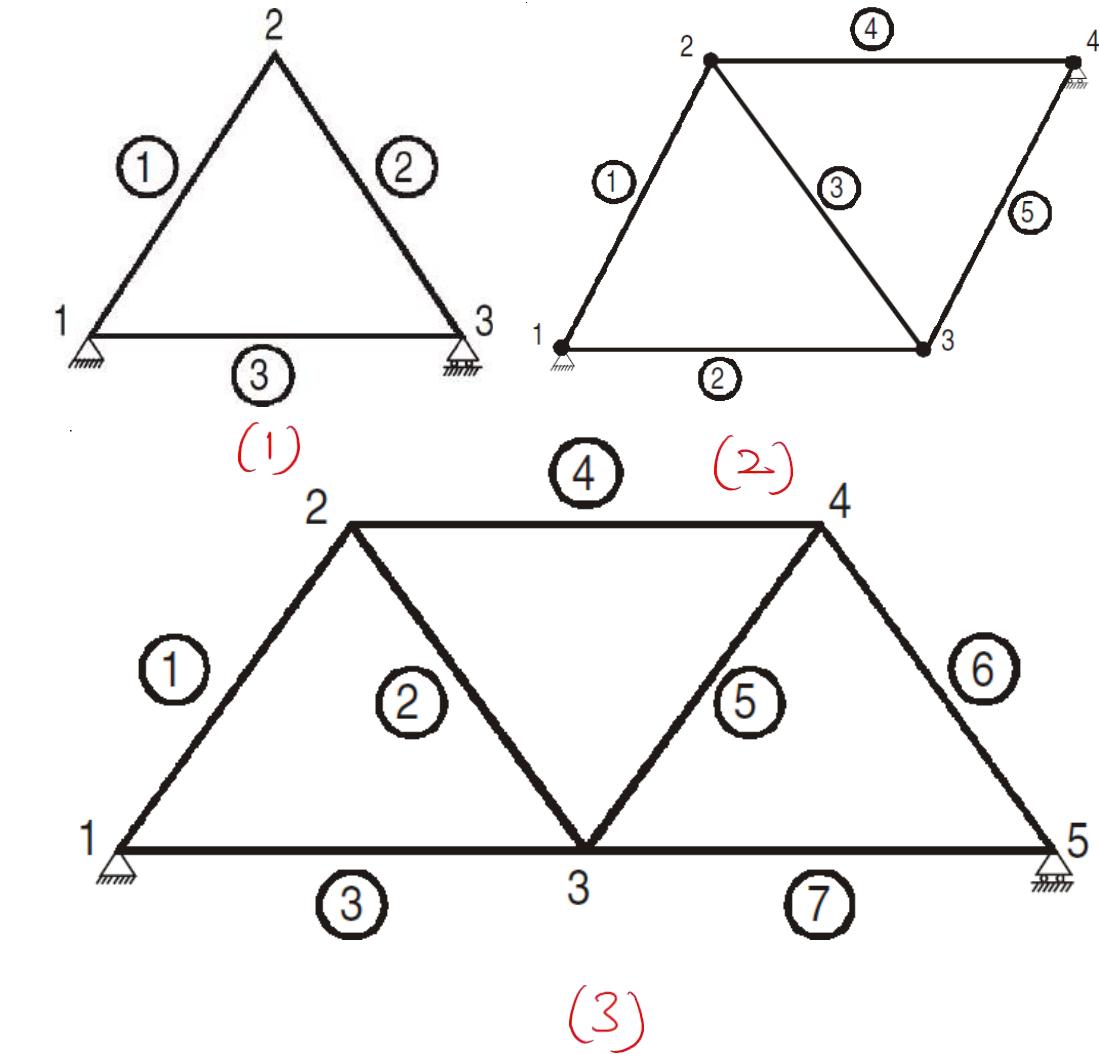
$3 = 3$ (Perfect)

(2) $5 = (2 \times 4) - 3$

$5 = 5$ (Perfect).

(3) $7 = (2 \times 5) - 3$

$7 = 7$ (Perfect)



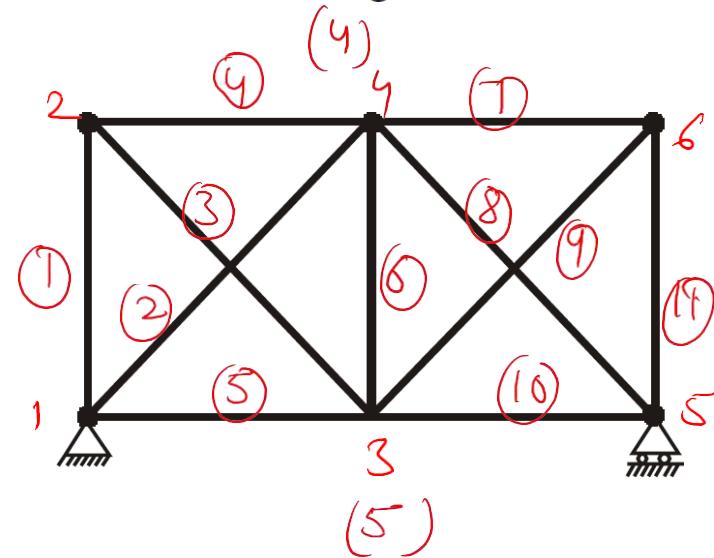
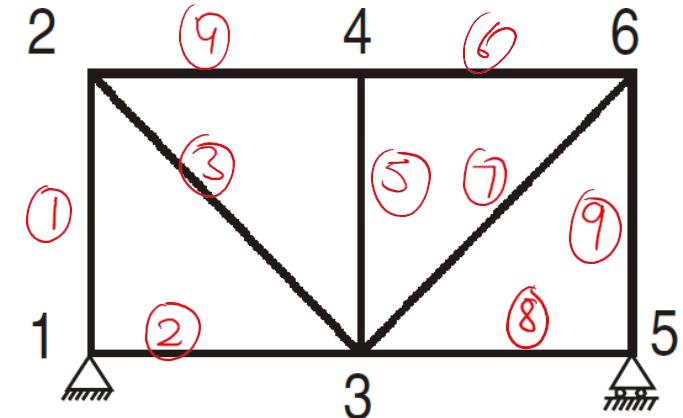
$$(4) \quad 9 = (2 \times 6) - 3$$

$9 = 9$ (Perfect).

$$(5) \quad 11 = (2 \times 6) - 3$$

$$11 = 9$$

LHS > RHS (Redundant)



Find the forces in all the members.

Sol: $R_B + R_C = 10 \text{ kN}$

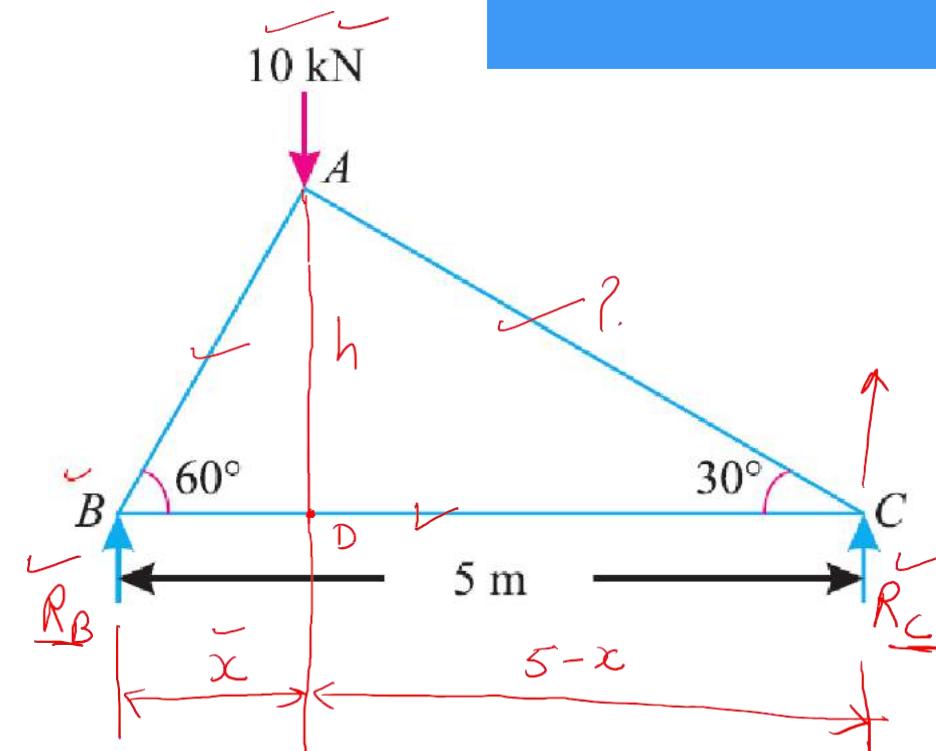
Taking moments about B

$$\begin{aligned} CW &= ACW \\ (10 \times 1.25) &= R_C \times 5 \\ R_C &= 2.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} P_{AB} &\quad 7.5 + P_{AB} \sin 60 = 0 \\ P_{AB} &= -8.66 \text{ kN} \\ P_{BC} + P_{AB} \cos 60 &= 0 \\ P_{BC} &= 4.33 \text{ kN} \end{aligned}$$

$$\begin{aligned} P_{AC} &\quad 2.5 + P_{AC} \sin 30 = 0 \\ P_{AC} &= -5 \text{ kN} \\ P_{BC} & \quad 2.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum F &= 0 \\ \sum M &= 0 \end{aligned}$$



$$\frac{h}{x} = \tan 60 \quad \text{---(1)}$$

$$\frac{h}{5-x} = \tan 30 \quad \text{---(2)}$$

$$x = 1.25 \text{ m.}$$

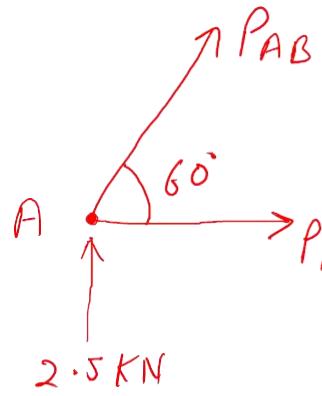
$$\text{Sol} \quad R_A + R_D = 6 \text{ kN}$$

Taking moments about A

$$CW = ACW$$

$$(2 \times 1.5) + (4 \times 4.5) = R_D \times 6 \quad | \quad R_A = 6 - R_D$$

$$R_D = 3.5 \text{ kN} \quad | \quad R_A = 2.5 \text{ kN}$$

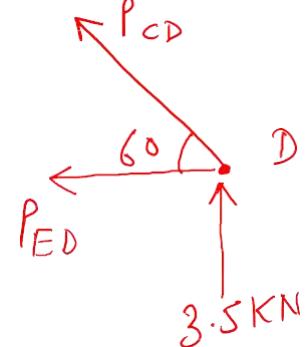


$$2.5 + P_{AB} \sin 60 = 0$$

$$P_{AB} = -2.88 \text{ kN}$$

$$P_{AE} + P_{AB} \cos 60 = 0$$

$$P_{AE} = 1.44 \text{ kN}$$

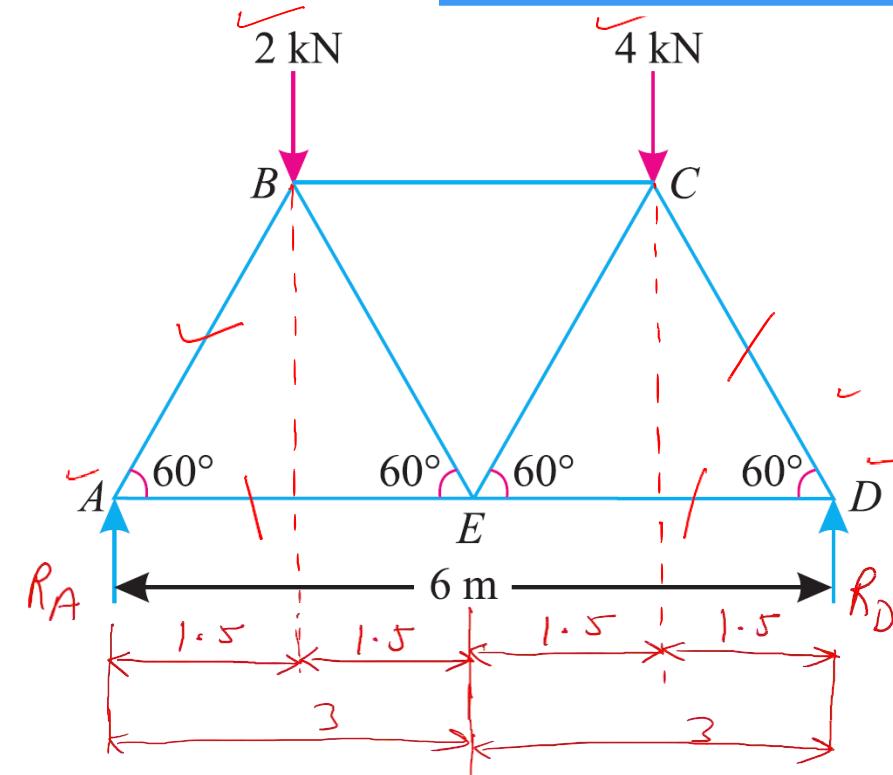


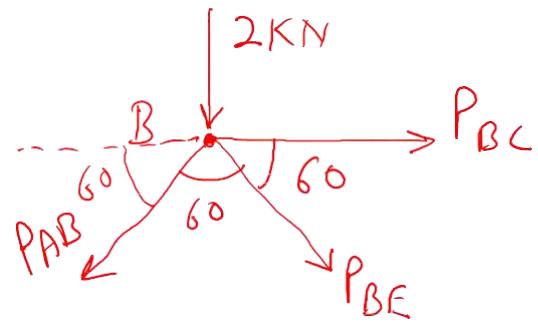
$$3.5 + P_{CD} \sin 60 = 0$$

$$P_{CD} = -4.04 \text{ kN}$$

$$P_{ED} + P_{CD} \cos 60 = 0$$

$$P_{ED} = 2.02 \text{ kN}$$



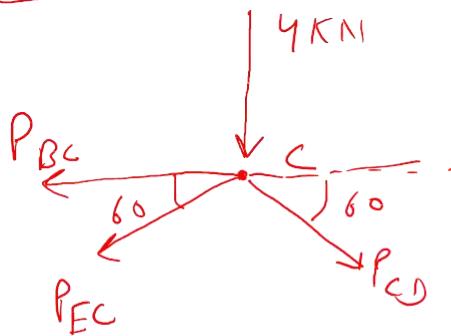


$$2 + P_{AB} \sin 60 + P_{BE} \sin 60 = 0$$

$$P_{BE} = 0.577 \text{ kN}$$

$$P_{BC} + P_{BE} \cos 60 = P_{AB} \cos 60$$

$$P_{BC} = -1.732 \text{ kN}$$

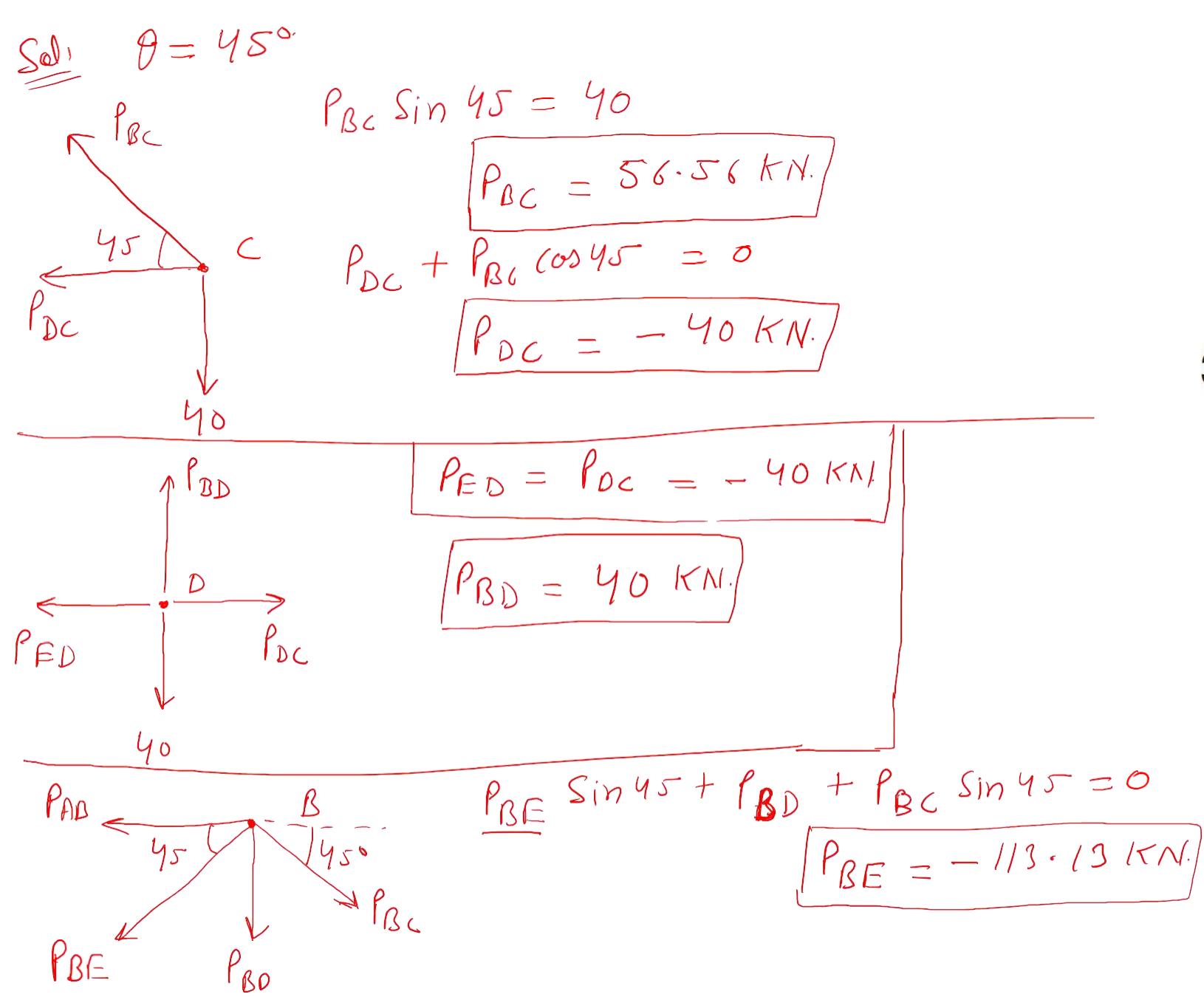


$$4 + P_{EC} \sin 60 + P_{CD} \sin 60 = 0$$

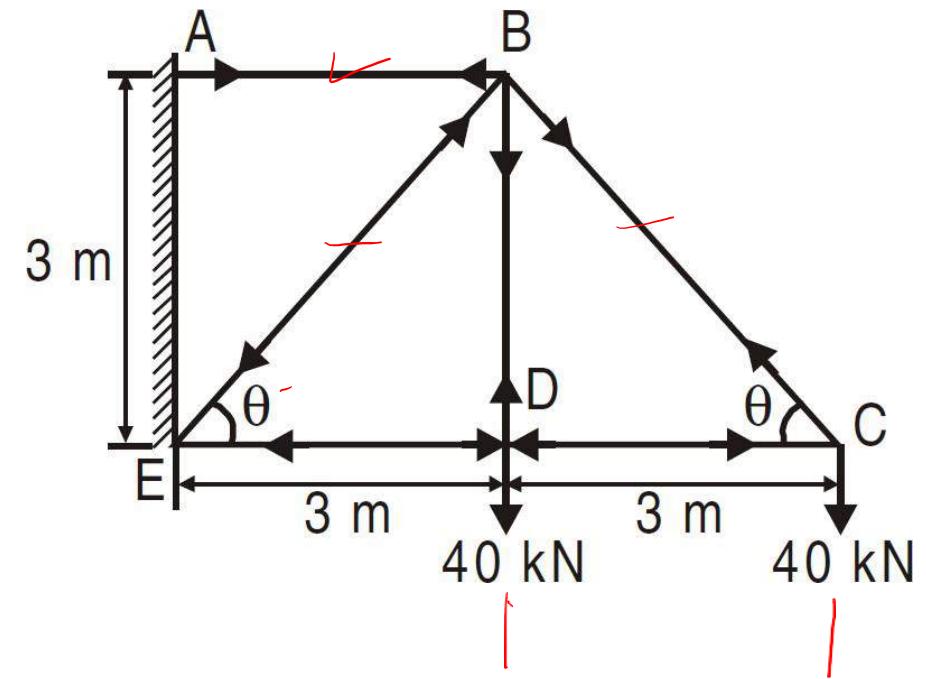
$$P_{EC} = -0.577 \text{ kN}$$



Simply supported frame



fixed Support



$$P_{AB} + P_{BE} \cos 45^\circ = P_{BC} \cos 45^\circ$$

$$P_{AB} = 120 \text{ kN}$$

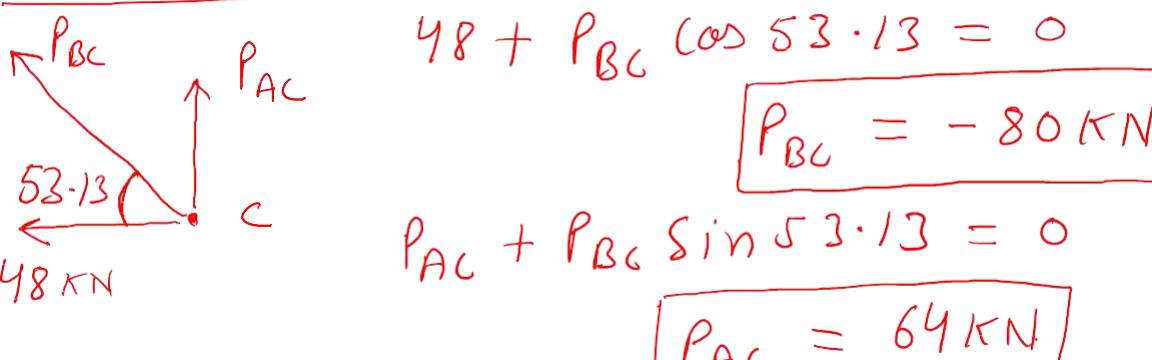
$$\text{SOL: } R_{Ay} = 84 \text{ kN} ; R_{Ax} + R_{Cx} = 0$$

Taking moments about A

$$CW = ACW$$

$$R_{Cx} \times 5.25 = 84 \times 3$$

$$R_{Cx} = 48 \text{ kN} ; R_{Ax} = -48 \text{ kN}$$

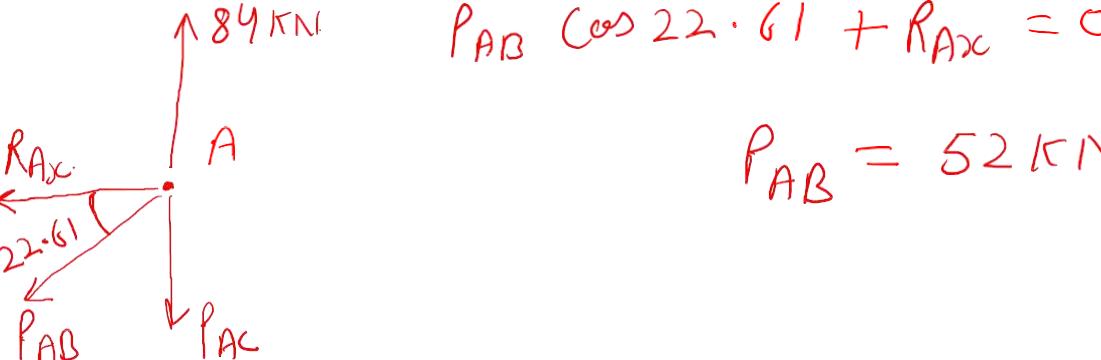


$$48 + P_{Bc} \cos 53.13 = 0$$

$$P_{Bc} = -80 \text{ kN}$$

$$P_{Ac} + P_{Bc} \sin 53.13 = 0$$

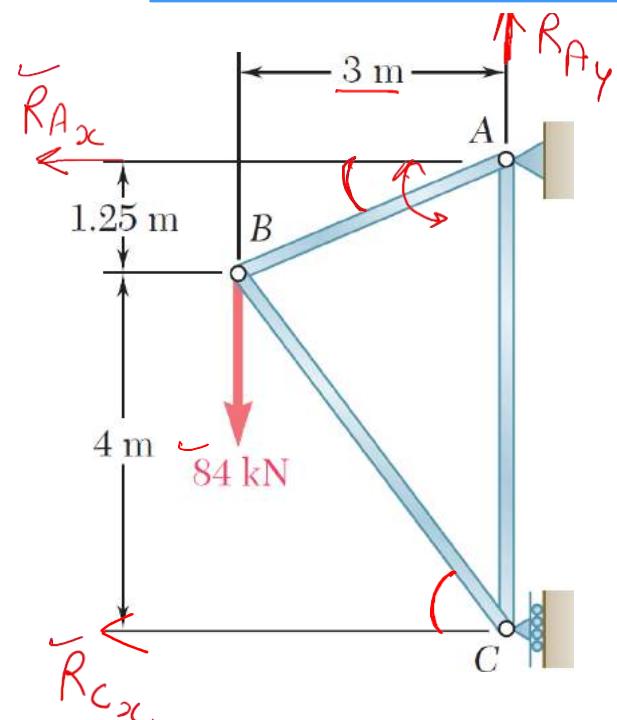
$$P_{Ac} = 64 \text{ kN}$$



$$P_{AB} \cos 22.61 + R_{Ax} = 0$$

$$P_{AB} = 52 \text{ kN}$$

at A Hinged support
at C Roller support

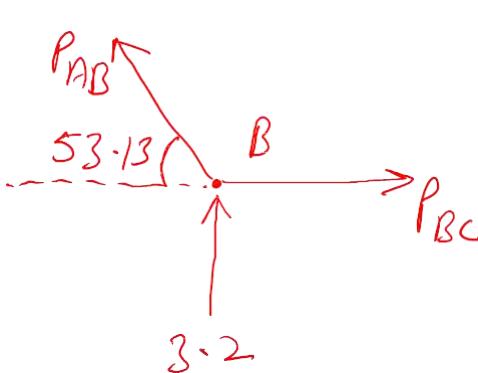


Sol: $R_{Cx} = 0$; $R_{By} + R_{Cy} = 1.92 \text{ kN}$.

Taking moments about B

$$CW = ACW$$

$$0 = (1.92 \times 3) + (R_{Cy} \times 4.5) \rightarrow R_{Cy} = -1.28 \text{ kN}, R_{By} = 3.2 \text{ kN}$$



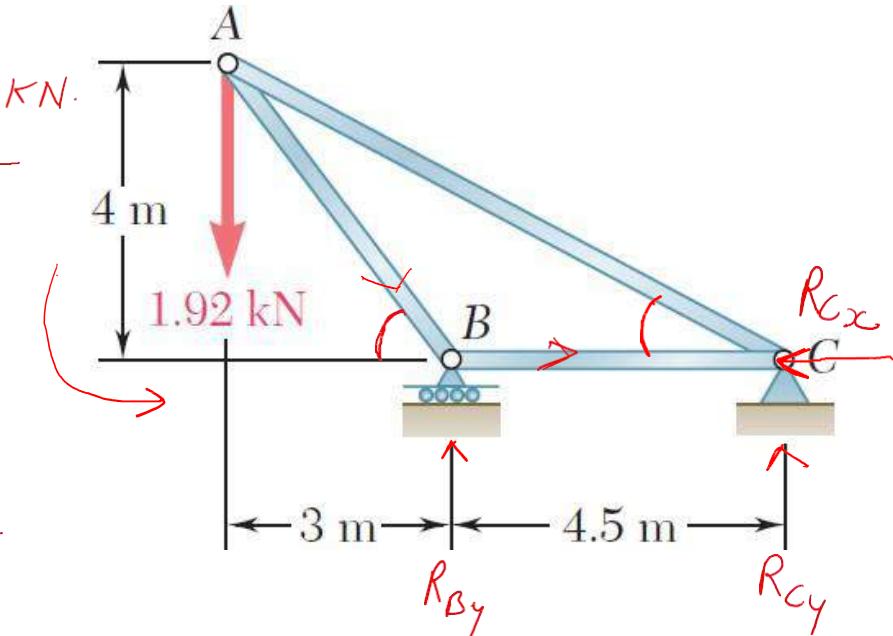
$$3.2 + P_{AB} \sin 53.13 = 0$$

$$\boxed{P_{AB} = -4 \text{ kN}}$$

$$P_{BC} = P_{AB} \cos 53.13 = -2.4 \text{ kN}$$

$$P_{AC} \sin 28.07 = 1.28 \text{ kN}$$

$$\boxed{P_{AC} = 2.72 \text{ kN}}$$



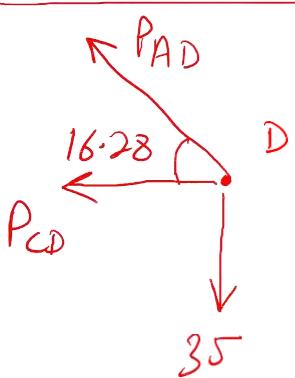
$$\text{Sol: } R_{Bx} = 48 \text{ kN} ; R_{By} + R_{Cy} = 35 \text{ kN.}$$

Taking moments about B

$$Cw = Aw$$

$$(48 \times 0.7) + (35 \times 4.8) = R_{Cy} \times 2.4.$$

$$R_{Cy} = 84 \text{ kN} ; R_{By} = -49 \text{ kN.}$$

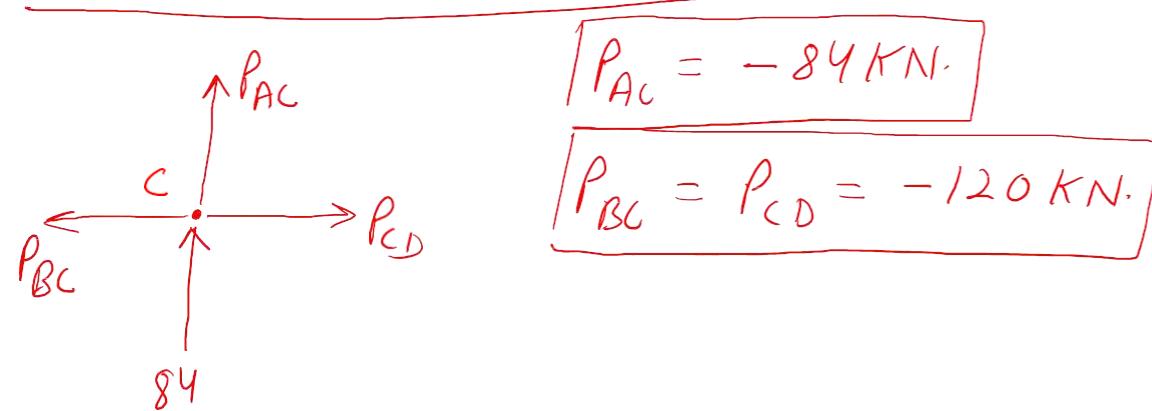


$$P_{AD} \sin 16.28 = 35$$

$$P_{AD} = 125 \text{ kN.}$$

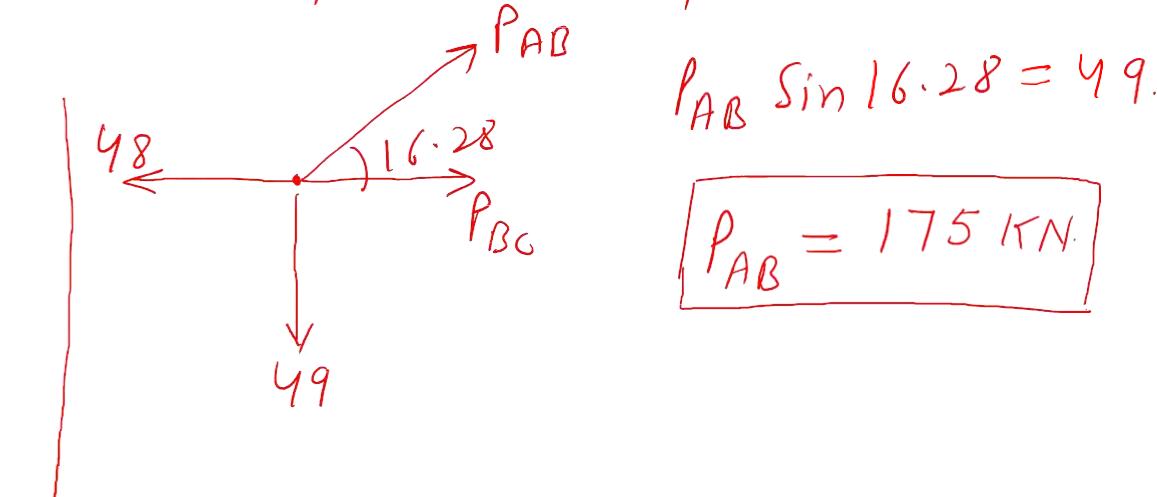
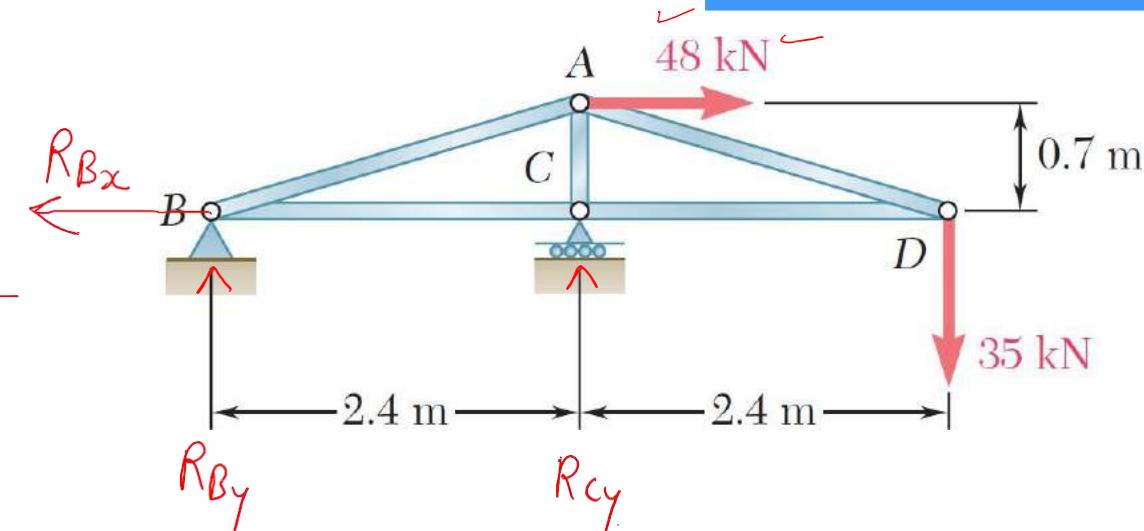
$$P_{CD} + P_{AD} \cos 16.28 = 0$$

$$P_{CD} = -120 \text{ kN.}$$



$$P_{AC} = -84 \text{ kN.}$$

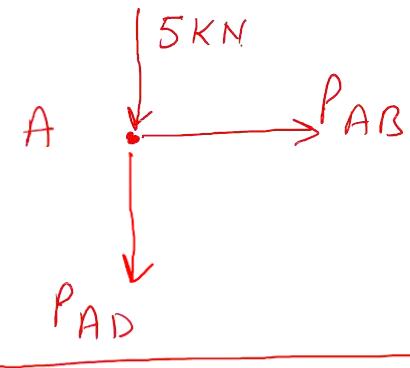
$$P_{BC} = P_{CD} = -120 \text{ kN.}$$



$$P_{AB} \sin 16.28 = 49.$$

$$P_{AB} = 175 \text{ kN.}$$

$$\text{Sol: } R_D = R_F = 21 \text{ kN}$$



$$P_{BC} = P_{AB} = 0 \quad [\text{zero force member}]$$

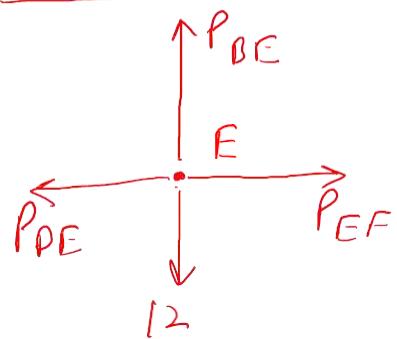
$$P_{AD} = -5 \text{ kN}$$

$$21 + P_{AD} + P_{DB} \sin 28.07^\circ = 0$$

$$P_{DB} = -34 \text{ kN} = P_{BF}$$

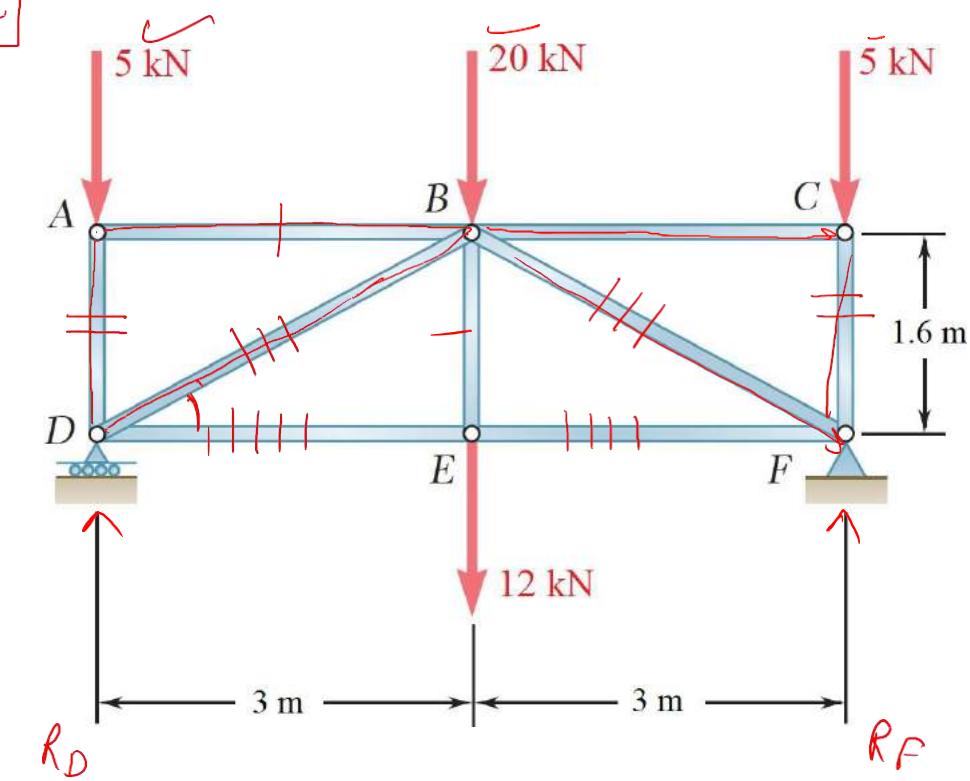
$$P_{DE} + P_{DB} \cos 28.07^\circ = 0$$

$$P_{DE} = 30 \text{ kN} = P_{EF}$$



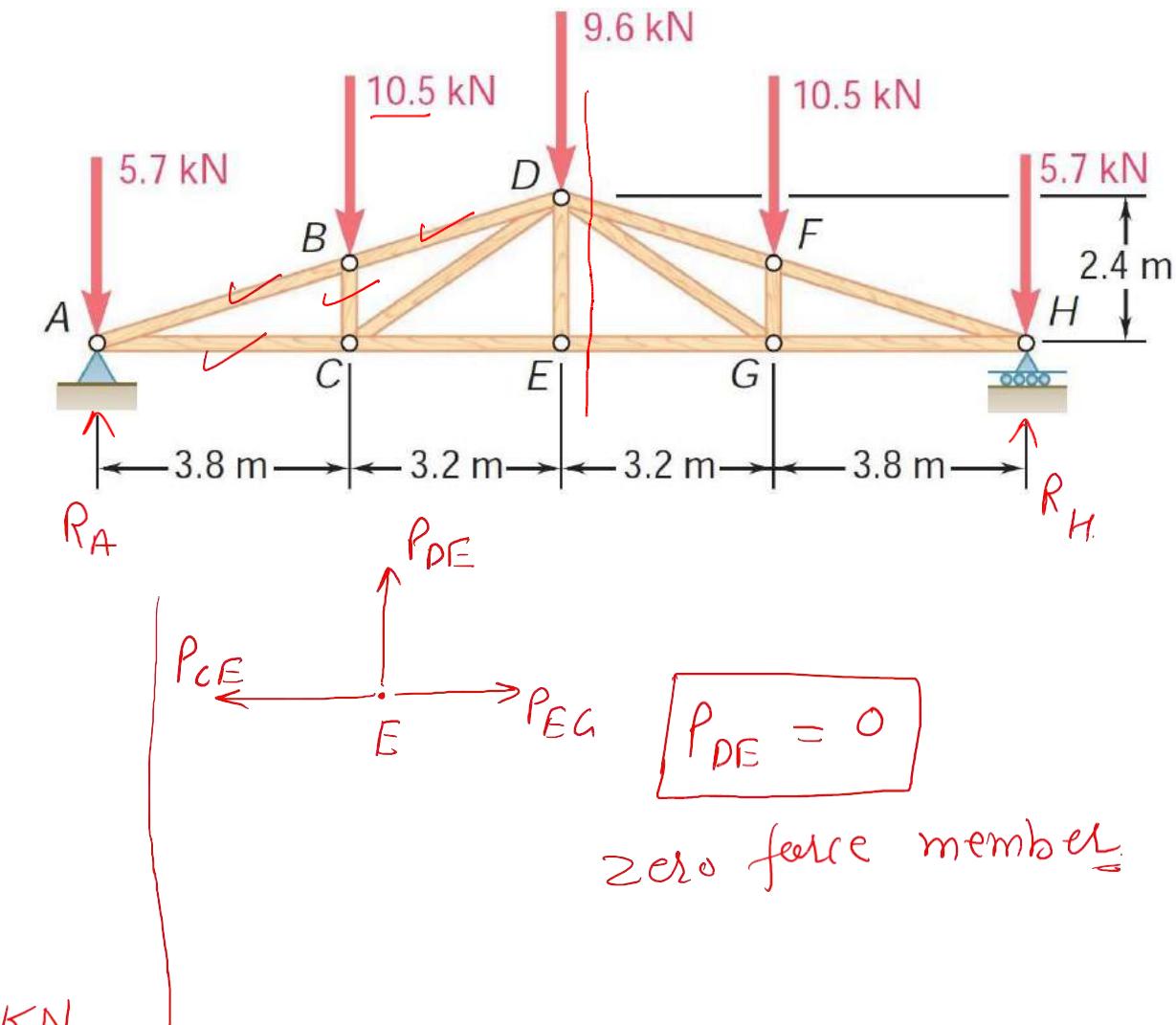
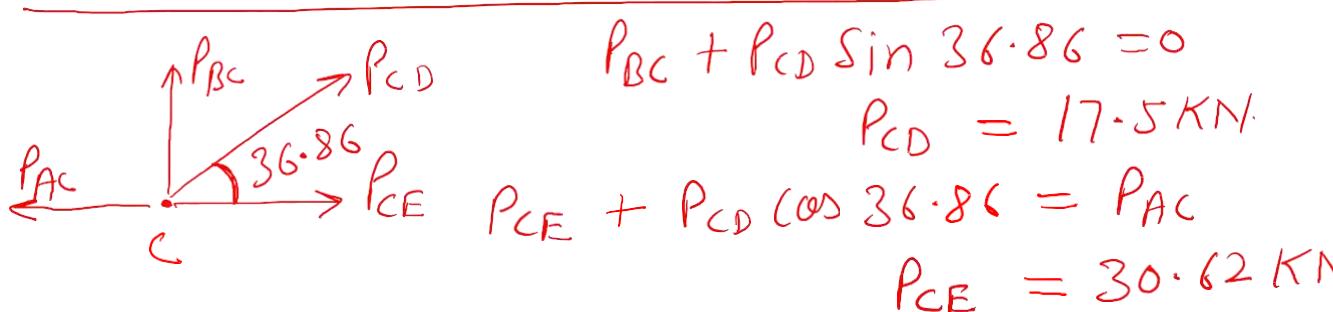
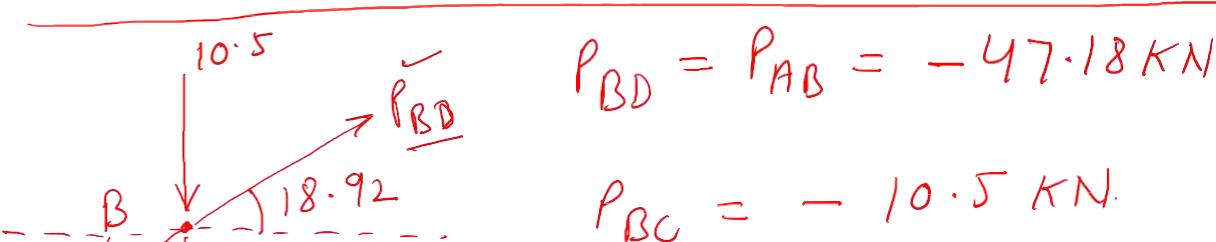
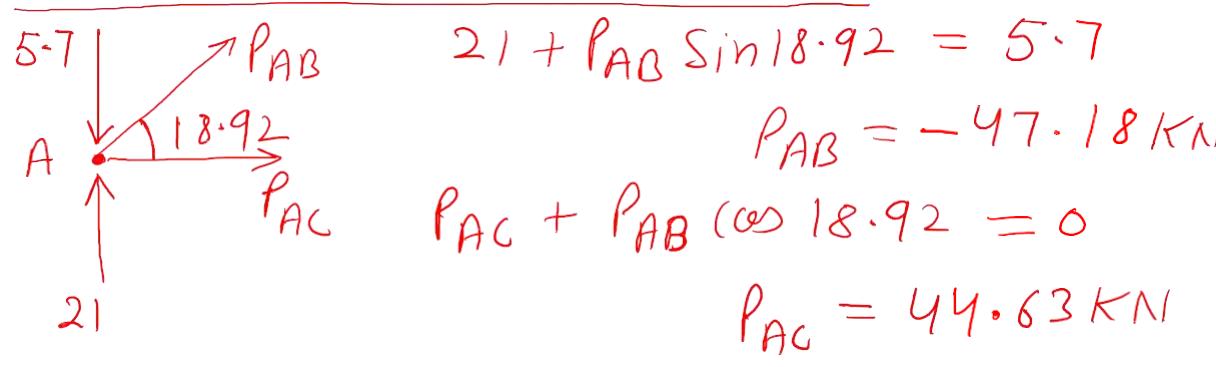
$$P_{BE} = 12 \text{ kN}$$

Symmetric frame



Sol: Frame is symmetric.

$$R_A = R_H = 21 \text{ kN}$$



zero force member

Method of Sections

- Find Forces in Members 1, 2 and 3

$$\text{Sol: } R_A + R_B = 1500 \text{ N}$$

Taking moments about A
 $(\omega = A\omega)$

$$(1500 \times a) = R_B \times 4a$$

$$R_B = 375 \text{ N} \quad ; \quad R_A = 1125 \text{ N}$$

$$\sum M_m = 0$$

$$(P_1 \cos 36.86 \times 1.5a) + (375 \times 2a) = 0$$

$$P_1 = -625 \text{ N} \quad \text{Ans}$$

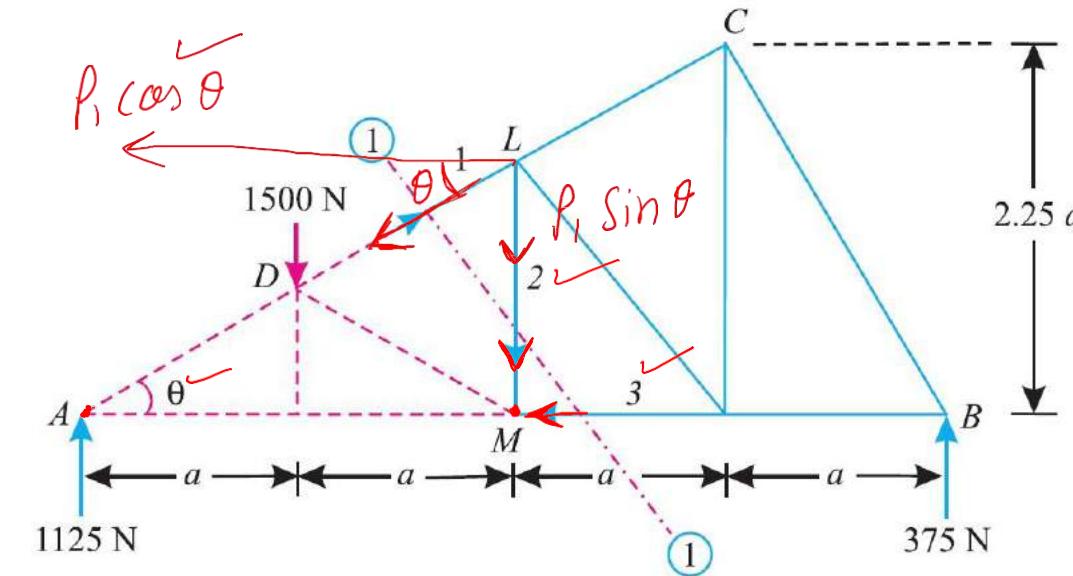
$$\sum P_y = 0$$

$$375 - P_1 \sin 36.86 - P_2 = 0$$

$$P_2 = 750 \text{ N} \quad \text{Ans}$$

$$\theta = 36.86^\circ \quad LM = 1.5a$$

$$\sum M = 0, \sum P_y = 0; \sum P_x = 0$$



$$\sum P_x = 0$$

$$-P_1 \cos 36.86 - P_3 = 0$$

$$P_3 = 500 \text{ N} \quad \text{Ans}$$

Find forces in Members BC, GC and GF

Sol: $R_A + R_E = 26 \text{ kN}$

Taking moments about A

$$ACW = \omega$$

$$(R_E \times 6) = (8 \times 1.5) + (6 \times 2) + (12 \times 4)$$

$$R_E = 12 \text{ kN} ; R_A = 14 \text{ kN}$$

$$\sum M_G = 0$$

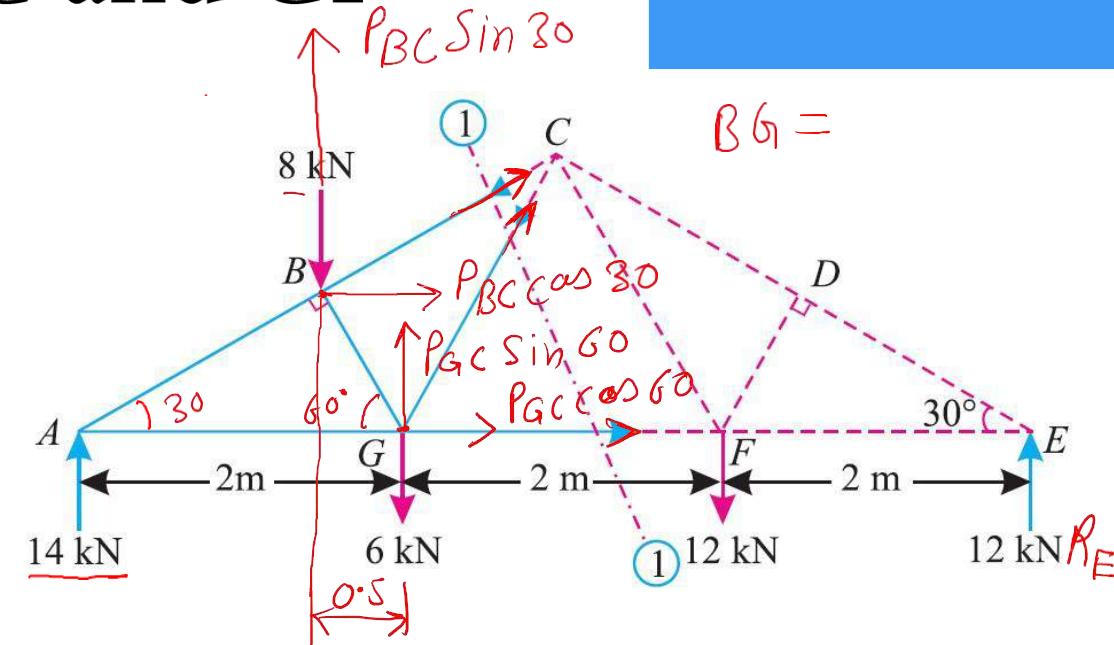
$$(8 \times 0.5) - (14 \times 2) - (P_{BC} \times 2 \cos 60) = 0$$

$$P_{BC} = -24 \text{ kN}$$

$$\sum P_y = 0$$

$$14 + P_{BC} \sin 30 + P_{GC} \sin 60 - 8 - 6 = 0$$

$$P_{GC} = 13.85 \text{ kN}$$



$$BG =$$

$$\sum P_x = 0$$

$$P_{BC} \cos 30 + P_{GC} \cos 60 + P_{GF} = 0$$

$$P_{GF} = 13.85 \text{ kN}$$

Determine the forces in the members FH , HG and GI

$$\text{Sol: } R_A = R_O = 35 \text{ kN}$$

$$\sum M_G = 0$$

$$(10 \times 2) + (10 \times 6) + (10 \times 10) - (35 \times 12) - (P_{FH} \times 3.46) = 0$$

$$P_{FH} = -69.36 \text{ kN}$$

$$\sum P_y = 0$$

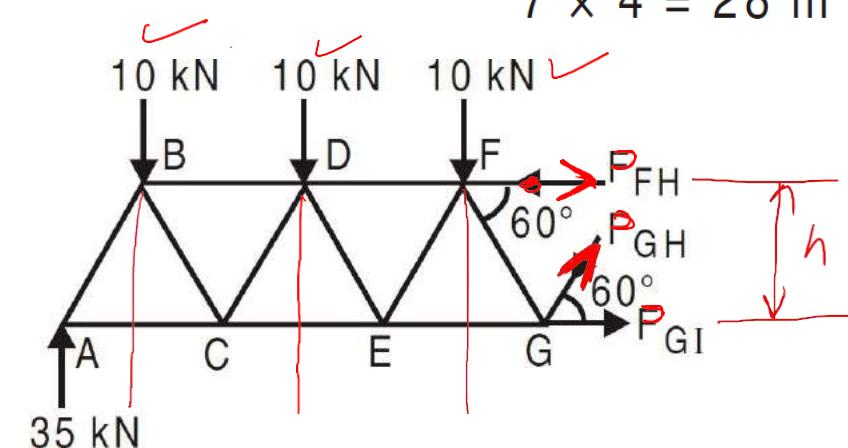
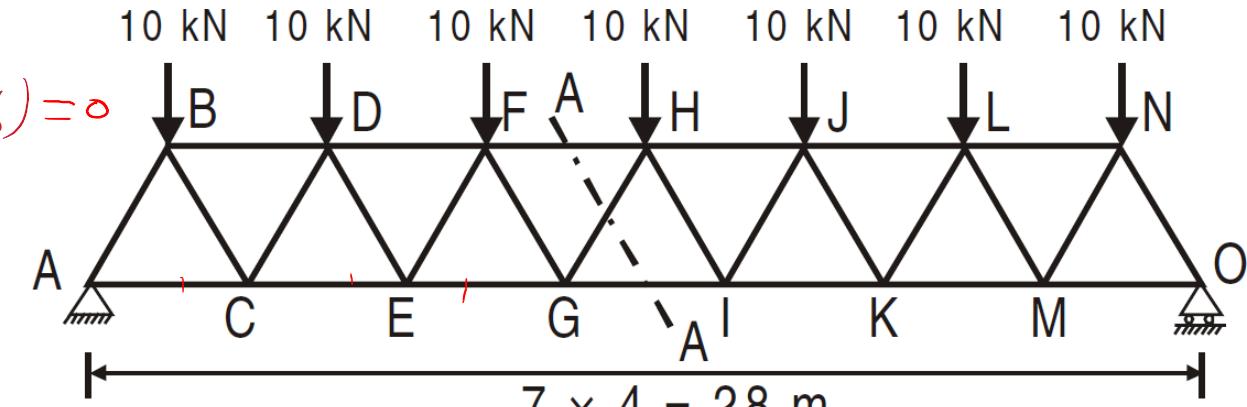
$$35 + P_{GH} \sin 60 - 30 = 0$$

$$P_{GH} = -5.77 \text{ kN}$$

$$\sum P_x = 0$$

$$P_{FH} + P_{GH} \cos 60 + P_{GI} = 0$$

$$P_{GI} = 72.245 \text{ kN}$$



The length of each truss member in figure is 7 m. Find the forces in members CD, DE & EF using method of sections.

$$\text{Sol: } R_{Ax} = 0; R_{Ay} + R_{By} = 4000 \text{ kN}$$

Taking moments about A

$$A(\omega) = C(\omega)$$

$$R_{By} \times 35 = (1200 \times 7) + (1000 \times 14) + (800 \times 21) R_{Ax} \\ + (1000 \times 28)$$

$$R_{By} = 1920 \text{ kN.} \quad ; \quad R_{Ax} = 2080 \text{ kN}$$

$$\sum M_E = 0$$

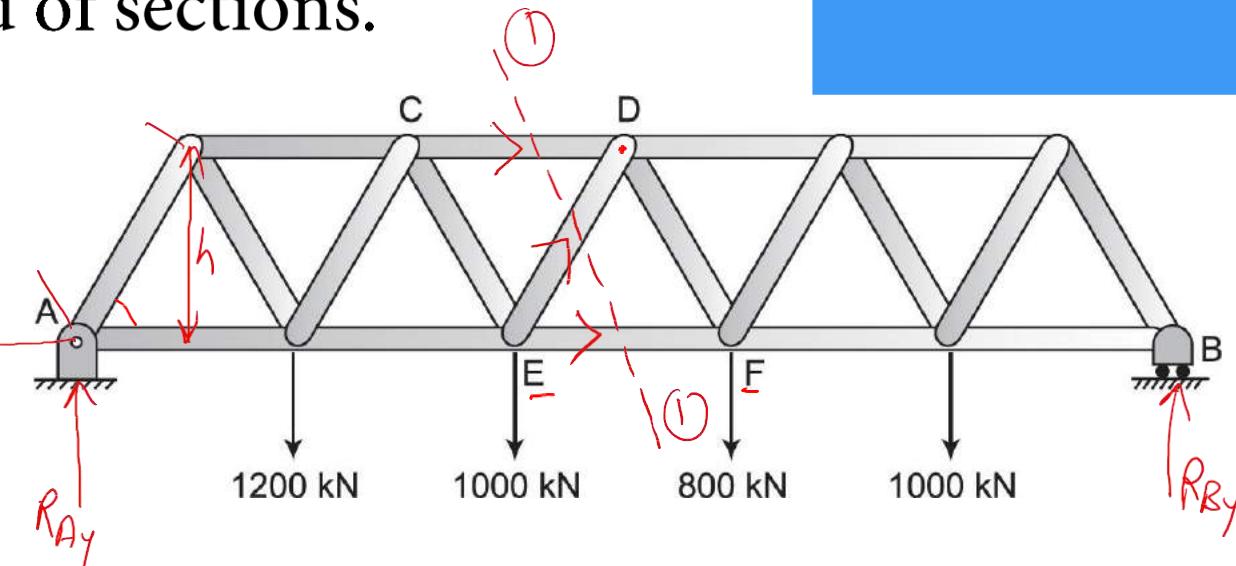
$$(1200 \times 7) - (2080 \times 14) - (P_{CD} \times 7 \sin 60) = 0$$

$$P_{CD} = -3418 \text{ kN}$$

$$\sum F_y = 0$$

$$2080 + P_{DE} \sin 60 - 1200 - 1000 = 0$$

$$P_{DE} = 138.56 \text{ kN}$$



$$\sum P_x = 0$$

$$P_{EF} + P_{DE} \cos 60 + P_{CD} = 0$$

$$P_{EF} = 3348.72 \text{ kN}$$

Determine the force in members CF , EF , and EG .

$$\text{S.d. } R_{Kx} = 0; R_{Ay} + R_{Ky} = 18$$

Taking moments about A
 $A(\omega) = C(\omega)$

$$R_{Ky} \times 4.8 = (4 \times 0.8) + (4 \times 1.6) + (3 \times 2.4) + (2 \times 3.2) + (2 \times 4) + (1 \times 4.8)$$

$$R_{Ky} = 7.5 \text{ kN} ; R_{Ay} = 10.5 \text{ kN}$$

$$\sum M_E = 0$$

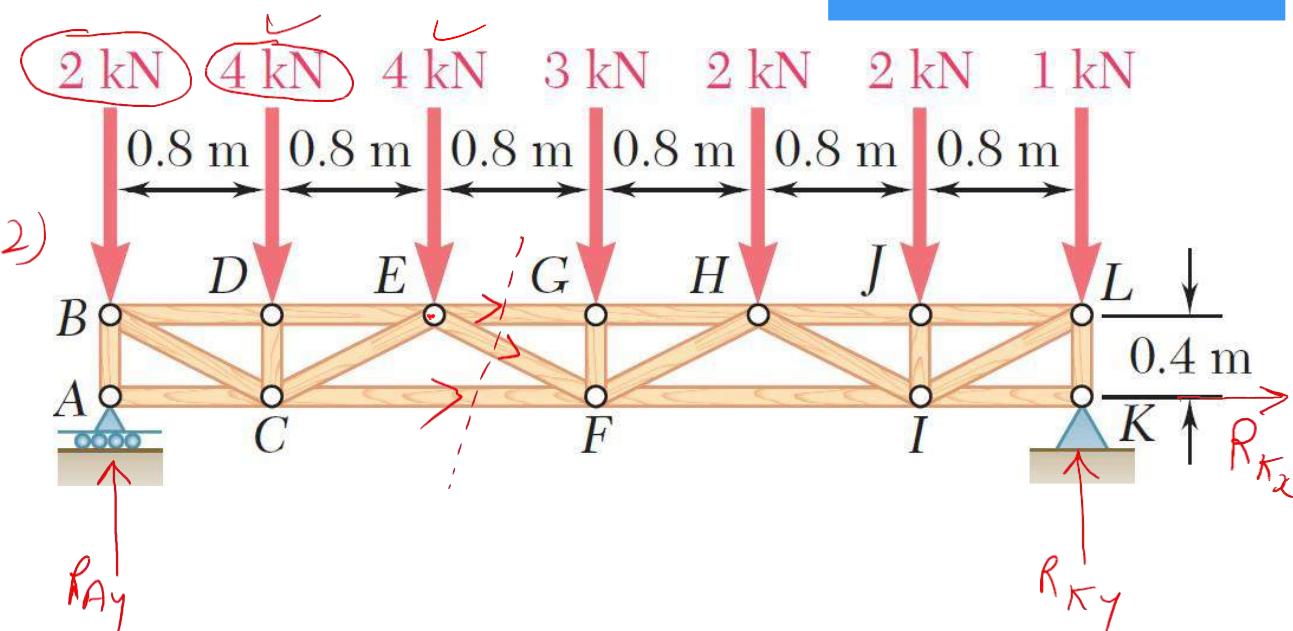
$$(4 \times 0.8) + (2 \times 1.6) + (P_{CF} \times 0.4) - (10.5 \times 1.6) = 0$$

$$P_{CF} = 26 \text{ kN}$$

$$\sum P_y = 0$$

$$10.5 - 2 - 4 - 4 - P_{EF} \sin 26.56^\circ = 0$$

$$P_{EF} = 1.118 \text{ kN}$$



$$\sum P_{xc} = 0$$

$$P_{EG} + P_{EF} \cos 26.56^\circ + P_{CF} = 0$$

$$P_{EG} = -271 \text{ kN}$$

Determine the force in members BD and DE of the truss shown

$$\text{SOL: } \sum M_E = 0$$

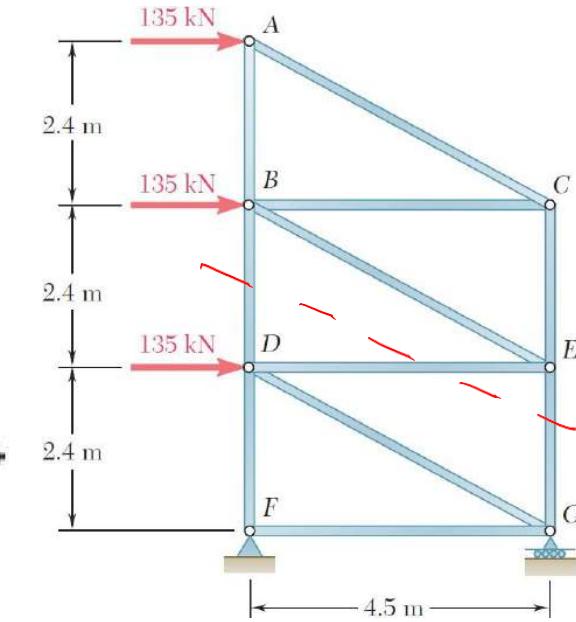
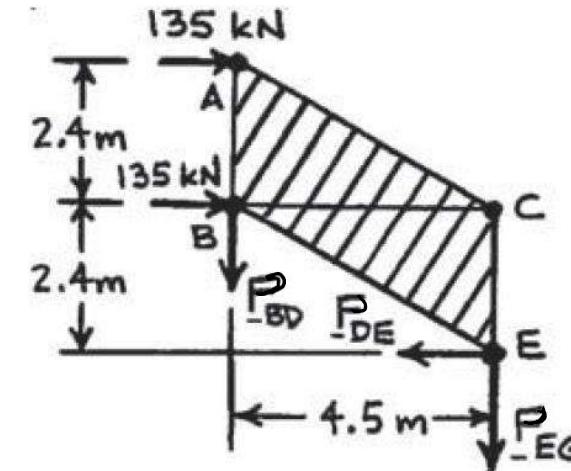
$$(P_{BD} \times 4.5) - (135 \times 2.4) - (135 \times 4.8) = 0$$

$$P_{BD} = 216 \text{ kN}$$

$$\sum P_x = 0$$

$$(2 \times 135) - P_{DE} = 0$$

$$P_{DE} = 270 \text{ kN}$$



Determine the force in members DG and EG of the truss shown

$$\begin{aligned} \text{S.t.} \quad \sum P_x &= 0 \\ (135 \times 3) + P_{DG} \cos 28.07 &= 0 \end{aligned}$$

$$\sum M_D = 0$$

$$(-135 \times 2.4) - (135 \times 4.8) - (P_{EG} \times 4.5) = 0$$

$$P_{EG} = -216 \text{ kN}$$

