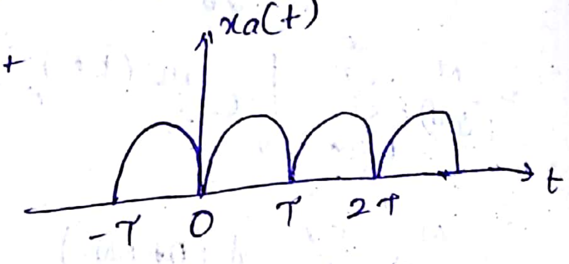
4^{th} chapter Solutions

Problems:-

4.1) Consider the full wave rectified sinusoid

(a) determine the spectrum $x_a(F)$



$$x_a(t) = \sum_{k=-\infty}^{\infty} C_k \, e^{j2\pi k f_0 t}$$

Let $F_0 = \frac{1}{T}$

$$= \sum_{k=-\infty}^{\infty} C_k \, e^{j2\pi k \frac{t}{T}}$$

$$C_k = \frac{1}{T} \int_0^T A \sin\left(\frac{\pi t}{T}\right) e^{-j2\pi k \frac{t}{T}} \, dt$$

$$= \frac{1}{T} \int_0^T A \, \frac{e^{\frac{j\pi t}{T}} - e^{\frac{-j\pi t}{T}}}{2j} \cdot e^{-j2\pi k \frac{t}{T}} \, dt$$

$$= \frac{A}{2j \cdot T} \int_0^T \left(e^{\frac{j\pi t}{T}} - e^{\frac{-j\pi t}{T}}\right) e^{\frac{-j2\pi k t}{T}} \, dt$$

$$= \frac{A}{2j \cdot T} \int_0^T \left(e^{\frac{j\pi t}{T}} \cdot e^{\frac{-j2\pi k t}{T}}\right) - \left(e^{\frac{-j\pi t}{T}} \cdot e^{\frac{-j2\pi k t}{T}}\right) dt$$

$$= \frac{A}{2j \cdot T} \int_0^T e^{j\pi(1-2k)\frac{t}{T}} dt - e^{-j\pi(1+2k)\frac{t}{T}} \, .dt$$

$$= \frac{A}{2j \cdot T} \left[\int_0^T e^{j\pi(1-2k)\frac{t}{T}} \, dt - \int_0^T e^{-j\pi(1+2k)\frac{t}{T}} \, dt \right]$$

$$= \frac{A}{2j \cdot T} \left\{ \left[\frac{e^{j\pi(1-2k)\frac{t}{T}}}{j\pi(1-2k)\frac{1}{T}} \right]_0^T - \left[\frac{e^{-j\pi(1+2k)\frac{t}{T}}}{-j\pi(1+2k)\frac{1}{T}} \right]_0^T \right\}$$

$$= \frac{A}{2j \cdot T} \left[\frac{e^{j\pi(1-2k)} - e^0}{j\pi(1-2k) \cdot \frac{1}{T}} - \frac{e^{-j\pi(1+2k)} - e^0}{-j\pi(1+2k) \cdot \frac{1}{T}} \right]$$

$$\frac{A}{2j\pi}\cdot\frac{T}{j\pi}\left[\frac{e^{j\pi(1-2k)}-1}{(1-2k)}+\frac{e^{-j\pi(1+2k)}-1}{(1+2k)}\right]$$

$$=-\frac{A}{2\pi}\left[\frac{-1-1}{(1-2k)}+\frac{-1-1}{(1+2k)}\right]$$

$$=-\frac{A}{2\pi}\left[-2\left(\frac{1}{1-2k}+\frac{1}{1+2k}\right)\right]$$

$$=\frac{A}{\pi}\left[\frac{1}{1-2k}+\frac{1}{1+2k}\right]$$

$$=\frac{A}{\pi}\left[\frac{1+\cancel{2k}+1-\cancel{2k}}{1+\cancel{2k}-\cancel{2k}-4k^2}\right]$$

$$=\frac{2A}{(1-4k^2)\pi}$$

$$X_a(F)=\int_{-\infty}^{\infty}x_a(t)\,e^{-j2\pi Ft}\,dt$$

$$=\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}C_k\,e^{j2\pi kF_0t}\cdot e^{-j2\pi Ft}\,dt$$

$$=\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}C_k\,e^{-j2\pi(F-kF_0)t}\,dt$$

$$=\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}C_k\,e^{-j2\pi\left(F-\frac{k}{T}\right)t}\,dt$$

$$=\sum_{k=-\infty}^{\infty}C_k\int_{-\infty}^{\infty}e^{-j2\pi\left(F-\frac{k}{T}\right)t}\,dt$$

$$=\sum_{k=-\infty}^{\infty}C_k\cdot\delta\left(F-\frac{k}{T}\right)$$

(b) Compute the power of signal.

$$P_x = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left(A \sin\frac{\pi t}{T}\right)^2 dt$$

$$= \frac{A^2}{T}\int_0^T \sin^2\frac{\pi t}{T}\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos 2\left(\frac{\pi}{T}\right)t}{2}$$

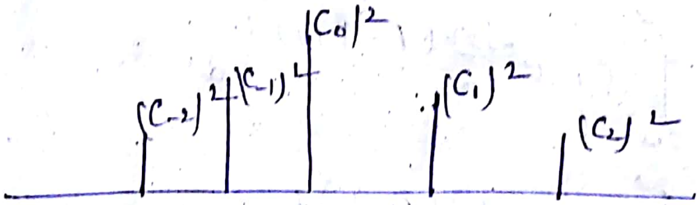
$$= \frac{A^2}{T}\int_0^T \frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{T}\right)t}{2}$$

$$= \frac{A^2}{T}\left[\frac{T}{2} - \left[\frac{\cos 2\pi}{2} - 0\right]\right]$$

$$= \frac{A^2}{\cancel{T}}\cdot\frac{\cancel{T}}{2} - 0$$

$$= \frac{A^2}{2}$$

(c) plot the power spectral density.



(d) Check the ~~ex~~ validity of parseval's relation for signal.

$$P_x = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$= \sum_{k=-\infty}^{\infty}\left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(4k^2-1)}$$

$$\frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(4k^2-1)^2}$$

$$\frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\Big|_{k=0} + 2\sum_{k=1}^{\infty}\frac{1}{(4k^2-1)^2}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1+\frac{2}{3^2}+\frac{2}{15^2}+\cdots\right]$$

$$\simeq \frac{4A^2}{\pi^2}[1.231]$$

$$\simeq 0.5A^2 = \frac{A^2}{2}.$$

4.2) Compute and sketch the magnitude and phase spectrum for following signals $(a>0)$

ⓐ $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t<0 \end{cases}$

$$X_a(F) = \int_0^{\infty} A e^{-at} \cdot e^{-j2\pi Ft}\, dt$$

$$= A\int_0^{\infty} e^{-(a+2\pi fj)t}\, dt$$

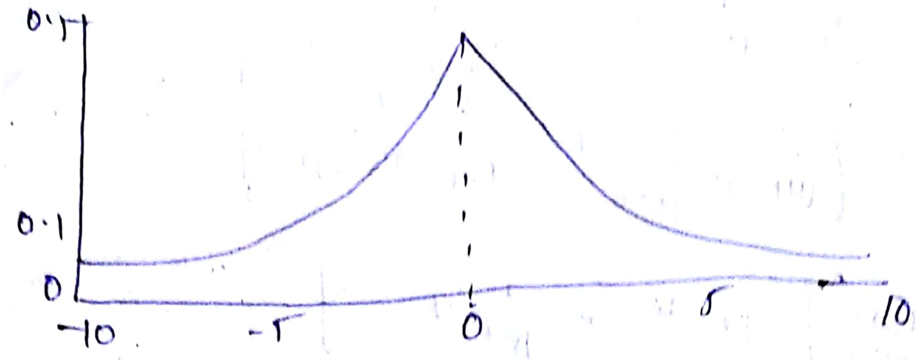
$$= A\left[\frac{e^{-(a+2\pi fj)}}{-(a+2\pi f j)}\right]_0^{\infty}$$

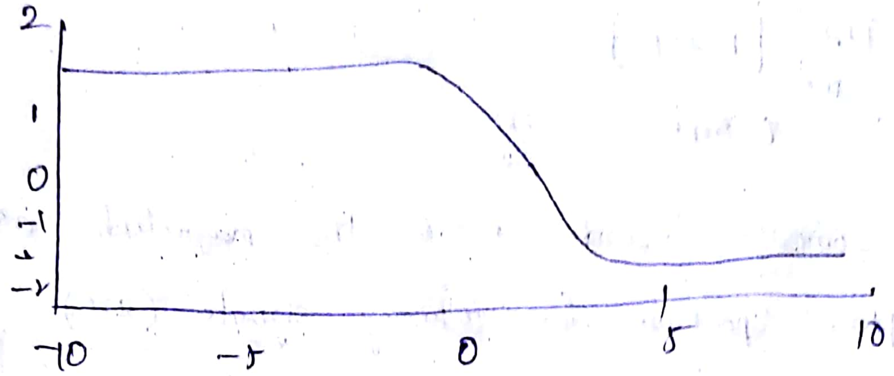
$$= A \cdot \frac{1}{a+2\pi fj}$$

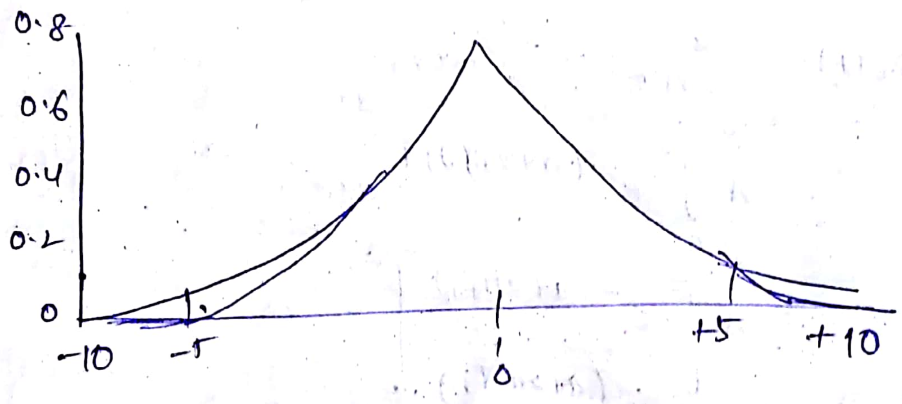
$$= \frac{a}{a+2\pi f}$$

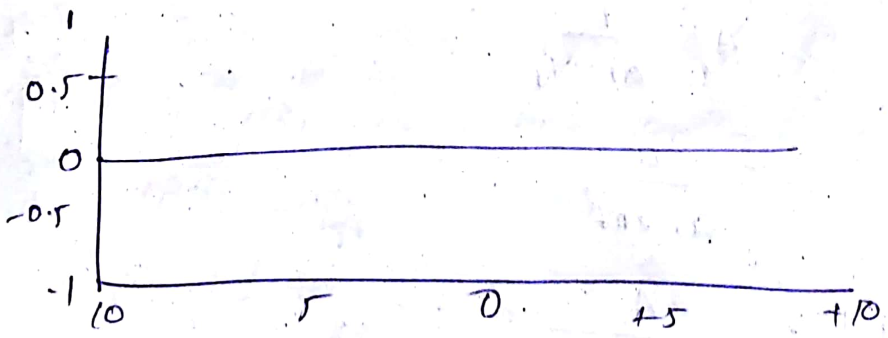
$$|X_a(F)| = \frac{A}{\sqrt{a^2+(2\pi f)^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi f}{a}\right).$$


0.5
0.1
0
-10
-5
0
5
10


2
1
0
-1
-2
-10
-5
0
5
10


0.8
0.6
0.4
0.2
0
-10
-5
0
+5
+10


1
0.5
0
-0.5
-1
10
5
0
+5
+10

(b) $x_a(t) = Ae^{-a|t|}$

$$x_a(F) = \int_{-\infty}^{\infty} Ae^{-a|t|} \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} Ae^{at} e^{-j2\pi ft} + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi ft}$$

$$= \int_{0}^{\infty} Ae^{-at} e^{j2\pi ft} + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi ft}$$

$$= A \int_{0}^{\infty} e^{-(-j2\pi f + a)t} + A \int_{0}^{\infty} e^{-(a+j2\pi f)t}$$

$$= A \left[\frac{e^{-(a-j2\pi f)t}}{-(a-j2\pi f)} \right]_0^{\infty} + A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_0^{\infty}$$

$$= A \left[\frac{1}{a-j2\pi f} \right] + A \left[\frac{1}{a+j2\pi f} \right].$$

$$= \frac{Aa + \cancel{Aj2\pi f} + Aa - \cancel{Aj2\pi f}}{a^2 - (j2\pi f)^2}$$

$$= \frac{2Aa}{a^2 - (j2\pi f)^2} = \frac{2Aa}{a^2 + (2\pi f)^2}$$

$$|X_a(F)| = X_a(F)$$

$$\angle X_a(F) = \tan^{-1}\left(\cancel{0}\right)$$

$$= 0$$

4.3) Consider the signal.

$$x(t) = \begin{cases} 1 - \frac{|t|}{\tau}, & |t| \le \tau \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine and sketch its magnitude and phase Spectra - $|x_a(F)|$ and $\angle x_a(F)$.

$$X_a(F) = \int_{-T}^{0}\left(1+\frac{t}{T}\right)e^{-j2\pi Ft}dt + \int_{0}^{T}\left(1-\frac{t}{T}\right)e^{-j2\pi Ft}dt$$

first find

Let

$$y(t) = \left\{\frac{1}{T};\ -T \le t \le T\right.$$

$$Y(F) = \int_{-T}^{0}\frac{1}{T}e^{-j2\pi ft}dt + \int_{0}^{T}-\frac{1}{T}e^{-j2\pi ft}dt$$

$$= -2\,\frac{\sin^2\pi ft}{j\pi ft}$$

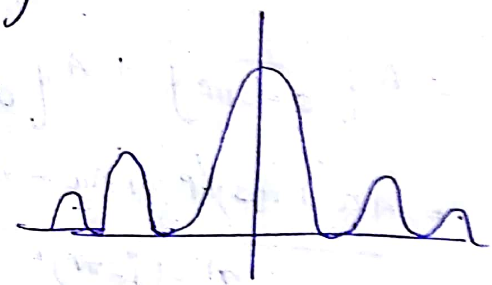
$$X(F) = \frac{1}{j2\pi f}\,y(f)$$

$$= T\left[\frac{\sin\pi fT}{\pi fT}\right]^2$$

$$|X(F)| = T\left(\frac{\sin\pi fT}{Af\,T}\right)^2$$

$$\angle X_a(F) = 0$$

$$T\cdot\left(\frac{\sin\pi fT}{\pi fT}\right)^2$$



Let $f = t/T$

$$\left(\frac{\sin\pi t}{\pi t}\right)^2 = \mathrm{sinc}^2(t)$$

(b) Create a periodic signal $x_p(t)$ with fundamental period $T_p \ge 2T$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$. What are the fourier coefficients C_k for the Signal $x_p(t)$?

Sol:

$$C_k = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} x(t)\,e^{-j2\pi kt/T_p}\,dt$$

$$= \frac{1}{T_p}\int_{-T}^{0}\left(1+\frac{t}{T}\right)e^{-j2\pi kt/T_p}\,dt$$

$$+ \int_{0}^{T}\left(1-\frac{t}{T}\right)e^{-j2\pi kt/T_p}\,dt$$

$$= \frac{T}{T_p}\left[\frac{\sin \pi k T/T_p}{\pi k T/T_p}\right]^2$$

(c) Using the results in parts a and b show that

$$c_k = \left(\frac{1}{T_p}\right) x_a\left(\frac{k}{T_p}\right)$$

$$\frac{1}{T_p} \cdot x_a\left(\frac{k}{T_p}\right)$$

$$\frac{1}{T_p} \cdot T \cdot \left(\frac{\sin \pi \cdot \frac{k}{T_p} \cdot T}{\pi \cdot \frac{k}{T_p} \cdot T}\right)^2$$

$$\frac{T}{T_p}\left(\frac{\sin \pi k T/T_p}{\pi k T/T_p}\right)^2 = c_k$$

$$c_k = \frac{1}{T_p} x_a\left(\frac{k}{T_p}\right)$$

4.4) Consider the following periodic signal.

$$x(n) = \{ \ldots\ldots \quad 1, 0, 1, 2, 3, 2, 1, 0, 1 \ldots \}$$

(a) Sketch the signal $x(n)$ and its magnitude and phase spectra.

1 0 1 2 3 2 1 0 1

$N = 6$

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

$$= \frac{1}{6}\sum_{n=0}^{5} x(n) e^{-j2\pi \frac{kn}{6}}$$

for $n=0 \longrightarrow x(0)\, e^{\frac{-j2\pi k(0)}{6}} = 3 \times 1 = 3$

$n=1 \longrightarrow x(1)\, e^{\frac{-j2\pi k(1)}{6}} = 2e^{-j2\pi \frac{k}{6}}$

$n=2 \longrightarrow x(2)\, e^{\frac{-j2\pi k(2)}{6}} = e^{-j2\pi \frac{k}{3}}$

$n=3 \longrightarrow x(3)\, e^{-\frac{j2\pi k}{6}(3)} = 0$

$n=4 \longrightarrow x(4)\, e^{-\frac{j2\pi k}{6}(4)} = 1 \cdot e^{-j4\pi \frac{k}{3}}$

$n=5 \longrightarrow x(5)\, e^{-\frac{j2\pi k}{6}(5)} = 2 \cdot e^{-\frac{j2\pi k}{6}}$

$$= \frac{1}{6}\left[3 + 2e^{\frac{-j2\pi k}{6}} + e^{\frac{-j2\pi k}{3}} + 0 + e^{\frac{-j4\pi k}{3}} + 2e^{\frac{-j10\pi k}{6}}\right]$$

for $k=0$

$$= \frac{1}{6}\left[3+2+1+0+1+2\right]$$

$$= \frac{1}{6}[9] = \frac{9}{6}$$

for $k=1$

$$= \frac{1}{6}\left[3+2e^{\frac{-j\pi 2}{6}} + e^{\frac{-j2\pi}{3}} + 0 + e^{\frac{-j4\pi}{3}} + 2e^{\frac{-j10\pi}{6}}\right]$$

$$= \frac{1}{6}\left[3+2e^{-j\pi/3} + e^{-j\frac{2\pi}{3}} + 0 + e^{\frac{-j4\pi}{3}} + 2e^{-j5\pi/3}\right]$$

$$= \frac{1}{6}\Big[3+2\left(\cos\left(\frac{\pi}{3}\right) - j\sin\left(\frac{\pi}{3}\right)\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)$$

$$+ \cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right) + 2\left(\cos\left(\frac{5\pi}{3}\right) - j\sin\left(\frac{5\pi}{3}\right)\right)\Big]$$

$$= \frac{4}{6}.$$

for $k=2$; $C_2 = 0$

$k=3$; $C_3 = 1/6$

$k=4$, $C_4 = 6$

$k=5$; $C_5 = 4/6$

(b) Using the results in part (a). Verify parsevals relation by computing the power in the time and frequency domain.

$$P_t = \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left[1^2+6^2+1^2+2^2+3^2+2^2\right]$$

$$= \frac{1}{6}[19] = \frac{19}{6}$$

$$P_f = \sum_{n=0}^{5} |c(n)|^2$$

$$= \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2$$

$$= \frac{114}{36} = \frac{19}{6}$$

4.5) Consider signal

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

(a) Determine and sketch its PDS.

$$y(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

$$= 2 + 2\left[\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right] + \frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2}$$

$$+ \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{\frac{-j\pi n}{2}}$$

$$+ \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$N = 8$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-\frac{j\pi k n}{4}}$$

$$x(n) = \left\{\frac{11}{2},\ 2 + \frac{3}{4}\sqrt{2},\ 1,\ 2 - \frac{3}{4}\sqrt{2} + \frac{1}{2},\ 2 - \frac{3}{4}\sqrt{2},\right.$$

$$\left. 1,\ 2 + \frac{3}{4}\sqrt{2}\right\}$$

$C_0 = 2,\ C_1 = C_7 = 1,\ C_2 = C_6 = 1/2,\ C_3 = C_5 = 1/4,\ C_4 = 0$

(b) Evaluate the power of signal.

$$\sum_{n=0}^{7} x(C_k)^2$$

$$= \left[2^2 + 1^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

$$= \left[4 + 2 + 1/2 + 1/8\right] = \frac{53}{8}$$

4 6) Determine and sketch the magnitude and phase spectra of following periodic signals

(a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

$$= 4\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right]$$

$$= 4\left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)}\right]$$

$N = 6$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{\frac{-j2\pi k n}{6}}$$

$$= \frac{4}{6}\sum_{n=0}^{5}\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right] e^{\frac{-j2\pi kn}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3}\right]$$

$$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3}\right] e^{-j2\pi k k/3}$$

$C_0 = 0$, $C_1 = -j2\pi e^{-j2\pi/3}$, $C_2 = C_3 = C_4 = 0$, $C_5 = 9$

$\angle C_1 = \frac{5\pi}{6}$, $\angle C_5 = -\frac{5\pi}{6}$, $\angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$

(b) $x(n) = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n$

$N = 15$

$\cos\frac{2\pi}{3}n$

$$\frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right]$$

$$e^{-j\frac{2\pi}{3}n} = e^{-\frac{2f\pi k n}{N}}$$

$k = \frac{N}{3} = 5$

$15 - 5 = 10$

$\sin\frac{2\pi}{5}n$

$$\frac{1}{2j}\left[e^{\frac{j2\pi 5}{5}n} - e^{\frac{-j2\pi}{5}n}\right]$$

$k = \frac{N}{5} = \frac{15}{5} = 3$

$15 - 3 = 12$

$$C_1k = \begin{cases} \frac{1}{2} ; k = 5, 10 \\ 0 ; \text{elsewhere} \end{cases} \qquad C_2k = \begin{cases} \frac{1}{2j} ; k = 3 \\ -\frac{1}{2j} ; k = 12 \\ 0 ; \text{Otherwise.} \end{cases}$$

$$C_k = C_1k + C_2k = \begin{cases} \frac{1}{2j} , k = 3 \\ \frac{1}{2} ; k = 5 \\ \frac{1}{2} , k = 10 \\ -\frac{1}{2j} , k = 12 \\ 0 , \text{otherwise} \end{cases}$$

⑥ $x(n) = \cos\frac{2\pi}{3} \cdot \sin\frac{2\pi}{5} n$

$$\cos a \sin b = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$$

$$= \frac{1}{2}\left[\sin\frac{10\pi n + 6\pi n}{15} - \sin\frac{10\pi n - 6\pi n}{15}\right]$$

$$= \frac{1}{2}\left[\sin\frac{16\pi n}{15} - \sin\frac{4\pi n}{15}\right]$$

$$= \frac{1}{2}\left[\frac{e^{j\frac{16\pi n}{15}} - e^{-j\frac{16\pi n}{15}}}{2j}\right] - \frac{1}{2}\left[\frac{e^{\frac{4\pi n j}{15}} \quad e^{-j\frac{4\pi n}{15}}}{2j}\right]$$

$$= \frac{1}{4j}\left(e^{\frac{j16\pi n}{15}} - e^{-\frac{j16\pi n}{15}}\right) - \frac{1}{4j}\left(e^{\frac{4\pi n j}{15}} - e^{-j\frac{4\pi n}{15}}\right)$$

$$\frac{8}{15} = \frac{k}{N}$$

$k = 8 \rightarrow \frac{1}{4j}$

$15 - 8 = 7 \rightarrow -\frac{1}{4j}$

$4 - 2k$

$k = 2$

$-\frac{1}{4j} \Rightarrow k = 2$

$15 - 2 = 13 \rightarrow \frac{1}{4j}$

$$C_k = \begin{cases} \frac{1}{4j} ; 8, 13 \\ -\frac{1}{4j} ; 7, 2 \\ 0 , \text{Otherwise} \end{cases}$$

(d) $x(n) = \{ \ldots -2, -1, 0, 1, 2, -2, -1, 0, 1, 2 \ldots \}$

$N = 5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$= \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{\frac{-j2\pi kn}{5}}$$

$$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2e^{\frac{-j4\pi k}{5}} - 2e^{-\frac{j6\pi k}{5}} - e^{\frac{-j8\pi k}{5}}\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

For plotting k values

$k = 0; \quad C_0 = 0$

$k = 1; \quad C_1 = \frac{2j}{5}\left[-\sin\frac{2\pi}{5} - 2\sin\frac{4\pi}{5}\right]$

$k = 2; \quad C_2 = \frac{2j}{5}\left[-\sin\frac{4\pi}{6} - 2\sin\frac{8\pi}{5}\right]$

$C_3 = -C_2$

$C_4 = -C_1$

(e) $x(n) = \{ \ldots -1, 2, 1, 2, -1, 0, -1, 2, 1, 2 \ldots \}$

$N = 6$

$N = 6$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$= \frac{1}{6}\left[1 + 2e^{\frac{-j\pi k}{3}} - e^{\frac{-j2\pi k}{3}} - e^{\frac{-j4\pi k}{3}} + 2e^{\frac{-j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$$

$C_0 = 1/2; \quad C_1 = 2/3, \quad C_2 = 0, \quad C_3 = -5/6, \quad C_4 = 0,$

$C_5 = 2/3.$

(f) $x(n) = \{ \ldots 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0 \ldots \}$

$\underbrace{\hspace{4em}}_{N=5}$

$N = 5$

$$C_k = \frac{1}{5} \sum_{n=0}^{4} x(n) e^{\frac{-j2\pi kn}{5}}$$

$$= \frac{1}{5} \left[1 + e^{\frac{-j2\pi k}{5}} \right]$$

$$= \frac{2}{5} \cos\left(\frac{\pi k}{5}\right) e^{\frac{-j2\pi k}{5}}$$

$C_0 = 2/5$

$$C_1 = \frac{2}{5} \cos\left(\frac{\pi}{5}\right) e^{-j\pi/5}$$

$$C_2 = \frac{2}{5} \cos\left(\frac{2\pi}{5}\right) e^{\frac{-j2\pi}{5}}$$

$$C_3 = \frac{2}{5} \cos\left(\frac{2\pi}{5}\right) e^{-j3\pi/5}$$

$$C_4 = \frac{2}{5} \cos\left(\frac{4\pi}{5}\right) e^{-j\frac{4\pi}{5}}$$

(g) $x(n) = 1, \quad -\infty < n < \infty$

$N = 1$

$C_k = x(0) = 1$

(or)

$C_0 = 1$

(h) $x(n) = (-1)^n, \quad -\infty < n < \infty$

$N = 2$

$$C_k = \frac{1}{2} \sum_{n=0}^{1} x(n) e^{-j2\pi nk}$$

$$= \frac{1}{2} \left(1 - e^{-j2\pi k}\right)$$

$\therefore C_0 = 0; \ C_1 = 1$

4.7) Determine the periodic signals $x(n)$ with fundamental period $N=8$ if their Fourier Coefficients are given by.

(a) $C_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

$$x(n) = \sum_{n=0}^{7} C_k e^{\frac{j2\pi nk}{N}}$$

Let,

$$C_k = e^{\frac{j2\pi pk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi pk}{N}} \cdot e^{\frac{j2\pi pk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{N}}$$

It gives 8; when $p=-n$

0; when $p \neq n$

$$\therefore C_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{-\frac{j2\pi k}{8}}\right] - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{-\frac{j6\pi k}{8}}\right]$$

$$x(n) = 4\delta(n-1) + 4\delta(n-1) - 4j\delta(n-1) - 4j\delta(n+3)$$
$$+ 4j\delta(n-3) \quad ; \quad -3 \leq n \leq 5.$$

(b)
$$C_k = \begin{cases} \sin\frac{k\pi}{3} & ; 0 \leq k \leq 6 \\ 0 & ; k=7 \end{cases}$$

$C_0 = 0$, $C_1 = \frac{\sqrt{3}}{2}$, $C_2 = +\frac{\sqrt{3}}{2}$, $C_3 = 0$; $C_4 = -\frac{\sqrt{3}}{2}$,

$C_5 = -\frac{\sqrt{3}}{2}$, $C_6 = C_7 = 0$

$$x(n) = \sum_{k=0}^{7} C_k e^{\frac{j2\pi nk}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n 2}{4}} - e^{-\frac{j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\frac{\sin \pi n}{2} + \sin\frac{\pi n}{4}\right] e^{\frac{j\pi(3n-2)}{4}}$$

(f) $C_k = \{ \dots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, \frac{1}{4}, 0 \dots \}$

$$x(n) = \sum_{k=-3}^{4} C_k e^{\frac{j2\pi nk}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{2} e^{-\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}} + \frac{1}{4} e^{\frac{-j3\pi n}{4}}$$

$$= 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$$

4.10) . Determine the signals having the following fourier transforms.

(a) $X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 \le |\omega| \le \pi \end{cases}$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\omega_0} X(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} X(\omega) e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^{-\omega_0} 1 \cdot e^{j\omega n} d\omega + \int_{\omega_0}^{\pi} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left[\left[\frac{e^{-j\omega n}}{jn} \right]_{-\pi}^{\omega_0} + \left[\frac{e^{j\omega n}}{jn} \right]_{\omega_0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{jn} \right]$$

$$= \frac{1}{2\pi} \left[2 \cdot \frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2 \cdot \frac{e^{j\pi n} - e^{-j\pi n}}{2jn} \right]$$

$$= \frac{1}{\cancel{2}\pi} \left[\frac{\cancel{2}}{n} - \sin \omega_0 n + 2 \cancel{\frac{\sin \omega \pi n}{2jn}} \right]$$

$$= -\frac{\sin \omega_0 n}{n\pi} \; ; n \neq 0$$

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for $n=0$

from eq①

$$\frac{1}{2\pi}(\pi-\omega_0)+\frac{1}{2\pi}(\pi-\omega_0)$$

$$= \frac{(\pi-\omega_0)+(\pi-\omega_0)}{}$$

$$= \frac{2(\pi-\omega_0)}{2\pi}$$

$= (\pi-\omega_0)$; when $n=0$

ⓑ $X(\omega) = \cos^2\omega$

$$= \left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{j\omega})^2+2e^{j\omega}.e^{-j\omega}+(e^{-j\omega})^2\right]$$

$$= \frac{1}{4}\left[e^{j2\omega}+2+e^{-j2\omega}\right]$$

$$= \frac{1}{4}e^{j2\omega}+\frac{1}{2}+\frac{1}{4}e^{-j2\omega}$$

I.F.T↓

$$= \frac{1}{4}\delta(n+2)+\delta(n)\frac{1}{2}+\frac{1}{4}\delta(n-2)$$

$$= \frac{1}{4}\left[\delta(n+2)+2\delta(n)+\delta(n-2)\right]$$

ⓒ $X(\omega) = \begin{cases} 1 ; & \omega-\frac{\delta\omega}{2} \le |\omega| \le \omega_0+\frac{\omega_0}{2} \\ 0 ; & \text{elsewhere.} \end{cases}$

$$\omega_0-\frac{\delta\omega}{2} \le \omega \le \omega_0+\frac{\delta\omega}{2}$$

$$\omega_0-\frac{\delta\omega}{2} \le -\omega \le \omega_0+\frac{\delta\omega}{2}$$

$$-\omega_0+\frac{\delta\omega}{2} \le \omega \le -\omega_0-\frac{\delta\omega}{2}$$

Consider limits $\omega_0-\frac{\delta\omega}{2} \le \omega \le \omega_0+\frac{\delta\omega}{2}$

$$\frac{1}{2\pi}\int_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}} 1 \cdot e^{j\omega n} \cdot d\omega$$

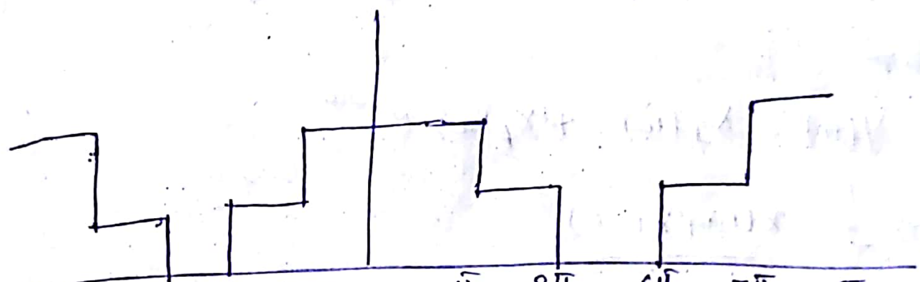
$$\frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}}$$

$$\frac{2}{2\pi} \cdot \frac{e^{j(\omega_0+\frac{\delta\omega}{2})n} - e^{j(\omega_0-\frac{\delta\omega}{2})n}}{2jn}$$

$$\delta\omega \frac{2}{\pi}\left[\frac{\sin\left(\frac{\delta\omega}{2}n\right)}{n\frac{\delta\omega}{2}}\right] e^{jn\omega_0}$$

$$\delta\omega \cdot \frac{2}{\pi} \cdot \sin\left(\frac{\delta\omega}{2}n\right) \cdot e^{jn\omega_0}$$

The signal shown in fig



$$\frac{1}{2\pi}\int_{-\pi}^{\pi} x(\omega)\, e^{j\omega n}\, d\omega$$

Let consider limits 0 to π.

$$2 \cdot \frac{1}{2\pi}\left[\int_0^{\pi/8} 2e^{j\omega n}\, d\omega + \int_{\pi/8}^{2\pi/8} e^{j\omega n}\, d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\, d\omega + \int_{7\pi/8}^{\pi} 2\, e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/8} 2\cos\omega n\, d\omega + \int_{\pi/8}^{3\pi/8} \cos\omega n\, d\omega + \int_{6\pi/8}^{7\pi/8} \cos\omega n\, d\omega + \int_{7\pi/8}^{\pi} 2\cdot\cos\omega n\, d\omega\right]$$

$$\frac{1}{\pi}\left[\left[-2\sin wn\right]_0^{\pi/8} + \left[-\sin wn\right]_{\pi/8}^{3\pi/8} + \left[-\sin wn\right]_{6\pi/8}^{7\pi/8} + \left[-2\sin wn\right]_{7\pi/8}^{\pi}\right]$$

$$\frac{1}{\pi}\left[-2\sin\frac{\pi n}{8} - \sin\frac{3\pi}{8}n + \sin\frac{\pi}{8}n - \sin\frac{7\pi}{8}n + \sin\frac{6\pi}{8}n - 2\sin\pi n + 2\sin\frac{7\pi}{8}\right]$$

$$\therefore \frac{1}{\pi}\left[\sin\frac{7\pi}{8}n - \frac{\sin \pi n}{8} + \sin\frac{6\pi}{8}n - \sin\frac{3\pi}{8}n\right]$$

4.11) Consider the signal.

$$x(n) = \{\underset{-3}{1}, \underset{-2}{0}, \underset{-1}{-1}, \underset{0}{2}, \underset{1}{3}\}$$

with fourier transform $X(\omega) = X_R(\omega) + jX_I(\omega)$.

Determine and sketch the signal $y(n)$ with fourier transform.

$$Y(\omega) = X_I(\omega) + X_R(\omega)\cdot e^{j2\omega}$$

$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

$n=0$; $\quad x_e(n) = \frac{x(0)+x(-0)}{2} = \frac{2+2}{2} = 2$

$n=1$; $\quad \frac{x(1)+x(-1)}{2} = \frac{3+(-1)}{2} = 1$

$n=1$; $\quad \frac{x(-1)+x(1)}{2} = 1$

$n=2$; $\quad \frac{x(2)+x(-2)}{2} = 0$

$n=3$; $\quad \frac{x(3)+x(-3)}{2} = \frac{0+1}{2} = 1/2$

$n=-2$; $\quad \frac{x(-2)+x(2)}{2} = 0$

$n=-3$; $\quad \frac{x(-3)+x(3)}{2} = \frac{1+0}{2} = 1/2$

$$x_e(n) = \{1/2, 0, 1, 2, 1, 0, 1/2\}$$

lly we get:

$$x_o(n) = \{ 1/2, 0, -2, 0, 2, 0, 1/2 \}$$

from $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-jn\omega}$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-jn\omega}$$

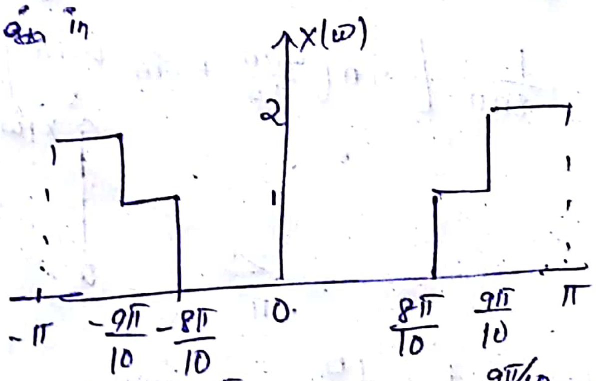
$$Y(\omega) = X_I(\omega) + X_R(\omega) e^{jn_2\omega}$$

$$= \frac{x_o(n)}{j} + x_e(n) \longrightarrow \text{from IFT of } X(\omega)$$

$$= -x_o(n) + x_e(n+2)$$

$$= \left\{ \tfrac{1}{2}, 0, 1, \tfrac{-j}{2}, 2, 5+\tfrac{j}{2}, 0, \tfrac{1}{2} - j2, 0, \tfrac{j}{2} \right\}$$

4.12) Determine the signal $x(n)$ if its fourier transform is given in



$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{9\pi}{10}} 2 \cdot e^{jn\omega}\, d\omega + \int_{-\frac{9\pi}{10}}^{-\frac{3\pi}{10}} 1 \cdot e^{jn\omega}\, d\omega + \int_{\frac{8\pi}{10}}^{9\pi/10} e^{jn\omega}\, d\omega \right.$$

$$\left. + 2 \int_{\frac{9\pi}{10}}^{\pi} e^{jn\omega}\, d\omega \right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{jn\omega}}{jn}\right)_{\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}}\right.$$

$$\left. + 2\left[\frac{e^{jn\omega}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[2\left[e^{-jn\frac{8\pi}{10}} - e^{-jn\pi}\right] + e^{jn-\frac{8\pi}{10}} - e^{-jn\frac{9\pi}{10}}\right.$$

$$\left. + e^{jn\frac{9\pi}{10}} - e^{jn\frac{8\pi}{10}} + 2e^{jn\pi} - 2e^{jn\frac{9\pi}{10}}\right]$$

$$= \frac{1}{2\pi jn}\left[2e^{-jn\frac{9\pi}{10}} - 2e^{-jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{jn\frac{9\pi}{10}} + e^{jn\frac{9\pi}{10}}\right.$$

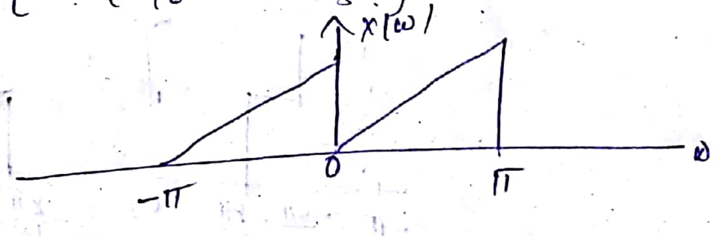
$$\left. - e^{jn\frac{8\pi}{10}} + 2e^{+jn\frac{9\pi}{10}}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jn\frac{9\pi}{10}} - e^{+jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{+jn\pi} + e^{-jn\frac{8\pi}{10}}\right.$$

$$\left. - e^{jn\frac{8\pi}{10}}\right]$$

$$= \frac{1}{8n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) - \sin\left(\frac{\overset{4}{\cancel{8}}n\pi}{\cancel{10}_{5}}\right) + \sin n\pi^{\,0}\right]$$

$$= \frac{1}{8n\pi}\left[\sin\left(\frac{9\pi n}{10}\right) + \sin\frac{4n\pi}{5}\right]$$



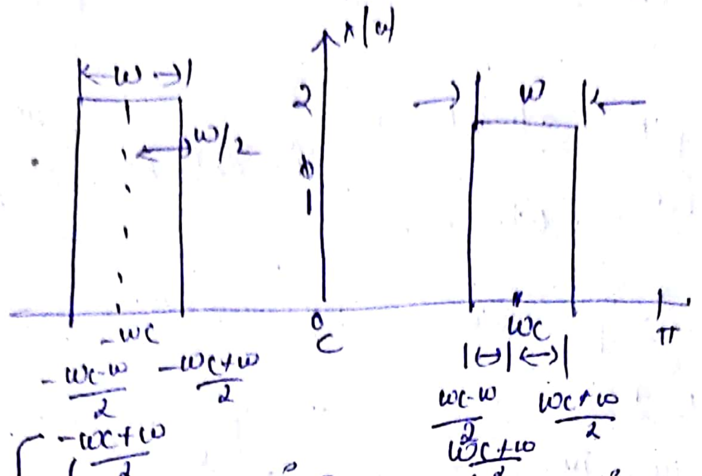
$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} x(\omega)\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} x(\omega) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi} + 1\right) e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi} + 1\right)$$

$$= \frac{1}{2\pi}\left\{\left[\frac{\omega}{jn\pi} e^{j\omega n}\right]_{-\pi}^{\pi} + \left(\frac{e^{j\omega n}}{jn}\right)_{\pi}^{0}\right]$$

$\frac{1}{\pi n} \sin\frac{\pi n}{2} e^{\frac{-jn\pi}{2}}$



$$x(n) = \frac{1}{2\pi}\left[\int_{\frac{-wc-w}{2}}^{\frac{-wc+w}{2}} 2e^{jnw}\,dw + \int_{wc-w/2}^{\frac{wc+w}{2}} 2e^{jnw}\,dw\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{e^{jnw}}{jn}\right]_{\frac{-wc-w}{2}}^{-wc+\frac{w}{2}} + \left[\frac{e^{jnw}}{jn}\right]_{wc-\frac{w}{2}}^{wc+\frac{w}{2}}\right]$$

$$= \frac{1}{j\pi n}\left[e^{jn(-wc+\frac{w}{2})} - e^{jn(-wc-\frac{w}{2})} + e^{jn(wc+\frac{w}{2})} - e^{-jn(wc-\frac{w}{2})}\right]$$

$$= \frac{1}{j\pi n}\left[e^{jn(-wc+\frac{w}{2})} - e^{jn(-wc-\frac{w}{2})} + e^{-jn(-wc-\frac{w}{2})} - e^{-jn(\frac{w}{2}-wc)}\right]$$

$$= \frac{2}{n\pi}\left[\sin\left(\frac{w}{2}-wc\right)n - \sin\left(-wc-\frac{w}{2}\right)n\right]$$

$$= \frac{2}{n\pi}\left[-\sin\left(wc-\frac{w}{2}\right)n + \sin\left(wc+\frac{w}{2}\right)n\right]$$

$$= \frac{2}{n\pi}\left[\sin\left(wc+\frac{w}{2}\right)n - \sin\left(wc-\frac{w}{2}\right)n\right]$$

4.13) given the fourier transform of signal.

$$x(n) = \begin{cases} 1 & ; -m \le n \le m \\ 0 & ; \text{otherwise} \end{cases}$$

was shown to be

$$x(w) = 1+2\sum_{n=1}^{m} \cos wn$$

then show that the fourier transform of

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$$x_1(n) = \begin{cases} 1 ; & 0 \le n \le m \\ 0 ; & \text{otherwise} \end{cases} \quad \text{is } x_1(\omega) = \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}$$

$$x_2(n) = \begin{cases} 1 ; & -m \le n \le -1 \\ 0 ; & \text{otherwise} \end{cases} \quad \text{is } x_2(\omega) = \frac{e^{j\omega} - e^{j\omega(m+1)}}{1-e^{j\omega}}$$

$$X_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$1+e^{-j\omega}+e^{j\omega 2}+e^{-j\omega 3}+ \dots - \left[e^{-j\omega(m+1)} + e^{-j\omega(m+2)} \right]$$

$$\frac{1}{1-e^{-j\omega}} - \frac{e^{-j\omega(m+1)}}{1-e^{-j\omega}} \qquad \frac{e^{j\omega(m+1)}}{1-e^{-j\omega}}$$

$$\frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}$$

$$x_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n}$$

$$= \sum_{n=1}^{m} e^{j\omega n}$$

$$= \frac{1-e^{j\omega n}}{1-e^{j\omega}} \cdot e^{j\omega}$$

$$X(\omega) = X_1(\omega) + x_2(\omega)$$

$$= \frac{1-e^{-j\omega}(m+1)}{1-e^{-j\omega}} + \frac{1-e^{j\omega m}}{1-e^{j\omega}} \cdot e^{j\omega}$$

$$= \frac{1+e^{j\omega}-e^{-j\omega}-1-e^{-j\omega(m+1)}-e^{j\omega(m+1)}+e^{j\omega m}+e^{-j\omega m}}{2-e^{-j\omega}-e^{j\omega}}$$

$$= \frac{2\cos\omega m - 2\cos\omega(m+1)}{2-2\cos\omega}$$

$$= \frac{2\sin(\omega n + \omega/2)\cos\frac{\omega}{2}}{2\sin^2\omega/2}$$

$$= \frac{\sin\left(m+\frac{1}{2}\right)\omega}{\sin\left(\omega/2\right)}$$

proved that $1+2\sum_{n=1}^{m}\cos\omega n = \dfrac{\sin\left(m+\frac{1}{2}\right)\omega}{\sin\left(\omega/2\right)}$

4.14 : Consider the signal.

$$x(n) = \{-1, 2, -3, 2, -1\}$$

with fourier transform $X(\omega)$. Compute the following quantities, without explicity computing $X(\omega)$:

ⓐ $X(0)$

$$X(\omega)\big|_{\omega=0} = \sum x(n)\, e^{-j\omega n}$$

$$x(0) = -3 \cdot e^{0} = -3$$

ⓑ $\angle X(\omega) = \pi$ for all ω

ⓒ $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

$$x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, d\omega$$

$$\int_{-\pi}^{\pi} X(\omega)\, d\omega = 2\pi\, x(0)$$

$$= 2\pi\,(-3)$$

$$= -6\pi$$

ⓓ $X(\pi) =$

$$X(\pi) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jn\omega}$$

$$= \sum_{n} e^{-jn\pi} \cdot x(n)$$

$$= \sum_{n} \left[\cos(n\pi) - j\,\cancel{\sin(n\pi)}^{\,0}\right] x(n)$$

$$= \sum_{n} (-1)^{n}\, x(n)$$

for $n=0$ $\quad (-1)^{0}\, x(0) = -3$

$n=1$ $\quad (-1)^{1}\, x(1) \Rightarrow -1.2 = -2$

$n=2$ $\quad (-1)^{2}\, x(2) = -1 = -1$

$n=-1$ $\quad (-1)^{-1}\, x(-1) \Rightarrow -2$

$n=-2 \quad (-1)^2 x(-2) = \not{b} = -1$

$\Rightarrow -3-2-1-2-1$

$\Rightarrow -3-4-2$

$\Rightarrow -9.$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

we know

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 = \sum_n (x(n))^2$$

$$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$$

$$= 1+4+9+4+1$$

$$= 19$$

$$\int_{-\pi}^{\pi} |X(\omega)|^2 = 2\pi \cdot 19$$

$$= 38\pi$$

4.15) The center of gravity of a signal $x(n)$ is defined as

$$C = \frac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

and provides a measure of the "time delay" of the signal.

a) Express c in terms of $X(\omega)$

We know

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{0}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)$$

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we know from differentiation in 'ω' domain multiplies 'n' with x(n)

$$n\,x(n) \xrightarrow{FT} j\frac{dX(\omega)}{d\omega}$$

$$-jn\,x(n) \xleftrightarrow{FT} \frac{dX(\omega)}{d\omega}$$

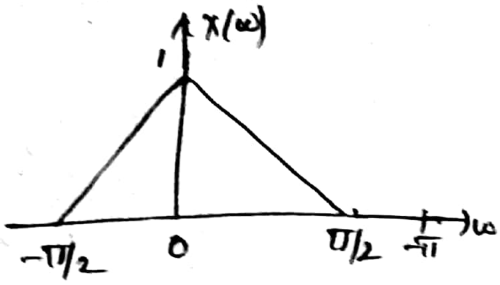
$$\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} -jn\,x(n)e^{-jn\omega}\,d\omega$$

$$= -j\sum_{n=-\infty}^{\infty} -jn\,x(n)e^{-jn\omega}\,d\omega$$

$$\frac{j\,dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n\,x(n)\,e^{-jn\omega}\,d\omega$$

$$C = \frac{j\frac{dX(\omega)}{d\omega}\Big|_{\omega=0}}{X(0)}$$

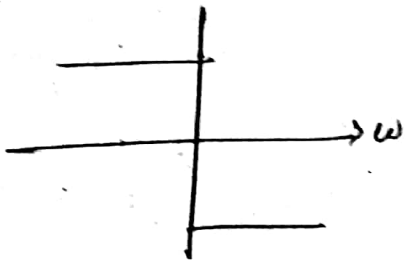
(b) Compute c for the signal x(n) whose fourier transform is shown



from given figure $X(0)=1$

$$C = \frac{\frac{j\,dX(\omega)}{d\omega}}{X(0)}$$

$$= \frac{0}{1} = 0$$



4.16) Consider the fourier transform pair

$$a^n u(n) \xleftrightarrow{FT} \frac{1}{1-ae^{-j\omega}} \quad |a|<1$$

Use the differentiation in frequency theorem and introduction to show the

$$x(n) = \frac{(n+l-1)!}{\quad} \quad a^n u(n) \xleftarrow{F} X(\omega) = \frac{1}{\quad}$$

Let $l = k+1$

$$x(n) = \frac{(n+k+l-1)!}{n!\,(k+l-1)!} a^n u(n)$$

$$= \frac{(n+k)!}{n!\,k!} a^n u(n)$$

$$= \frac{(n+k)(n+k-1)!}{n!\,(k-1)!} a^n u(n)$$

Let $x_k(n) = \dfrac{(n+k-1)!}{n!\,(k-1)!} a^n u(n)$

$$x_{k+1}(n) = \frac{n+k}{k} x_k(n)$$

$$X_{k+1}(\omega) = \sum_{n=-\infty}^{\infty} \frac{n+k}{k} x_k(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{n}{k} x_k(n) + x_k(n)\right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + X_k(\omega)$$

$$= \frac{1}{k} j \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^{k}}$$

4.17) Let $x(n)$ be a arbitary signal, not necessarily real valued, with fourier transform $X(\omega)$. Express the fourier transforms of following signals in terms of $X(\omega)$

(a) $x^*(n)$

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} [x(n) e^{-j\omega n}]^*$$

$$X(-\omega)^*$$

(b) $x^*(-n)$

$$\sum_{n=-\infty}^{\infty} x^*(-n) e^{-j\omega n}$$

replace '-n' with 'n'

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty} (x(n) e^{-j\omega n})$$

$$X^*(\omega)$$

c) $y(n) = x(n) - x(n-1)$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

$$= X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

let $l = n-1$ [dummy variable]

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega(l+1)}$$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l)\, e^{-j\omega l} . e^{-j\omega}$$

$$= X(\omega) - e^{-j\omega} \sum_{l=-\infty}^{\infty} x(l)\, e^{-j\omega l}$$

Replace l by n

$$= X(\omega) - e^{-j\omega} \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= X(\omega) - e^{-j\omega} X(\omega)$$

$$= X(\omega)[1 - e^{-j\omega}]$$

(d) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

$$= y(n) - y(n-1)$$

$$= x(n)$$

$$X(\omega) = Y(\omega)[1 - e^{-j\omega}]$$

$$Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}$$

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(f) $y(n) = x(2n)$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x(2n)e^{-jn\omega}$$

Let $l = 2n$

$$= \sum_{l=-\infty}^{\infty} x(l)\, e^{-j\frac{l}{2}\omega}$$

$$= \sum_{l=-\infty}^{\infty} x(l)e^{-jl\frac{\omega}{2}}$$

$$= X\left(\frac{\omega}{2}\right)$$

(g) $y(n) = \begin{cases} x(n/2) ; & 'n' \text{ even} \\ 0 ; & 'n' \text{ odd} \end{cases}$

$$Y(\omega) = \sum_{n} x\left(\frac{n}{2}\right) e^{jn\omega}$$

Let $n = 2l$

$$= \sum_{n} x\left(\frac{2l}{2}\right)e^{j2ln}$$

$$= \sum_{n} x(l)e^{j2l\omega}$$

$$= X(2\omega)$$

4.18) Determine and sketch the fourier transform $X_1(\omega)$, $X_2(\omega)$ and $X_3(\omega)$ the following signals.

(a) $x_1(n) = \{1, 1, 1, 1, 1\}$

$$\sum_{n=-\infty}^{\infty} x_1(n)e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} x_2(n)e^{-j\omega n}$$

for $n = -2;\ 1 \cdot e^{j2\omega} = e^{j2\omega}$

$n = -1;\ 1 \cdot e^{j\omega} = e^{j\omega}$

$n = 0;\ e^{0} = 1$

$n = 1;\ 1 \cdot e^{j\omega} = \frac{1}{e^{j\omega}}$

$n=2;\ = e^{-2j\omega}$

$e^{j2\omega} + e^{j\omega} + e^{-j\omega} + 1 + e^{-j2\omega}$

$= 2\cos(2\omega) + 2\cos(\omega) + 1$

(b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

for $n=-2$, $1 \cdot e^{j2\omega} = e^{j2\omega}$

$n=-1$ $\quad 1 \cdot e^{j4\omega} = e^{j4\omega}$

$n=0$ $\quad = e^{0} = 1$

$n=2$ $\quad = 1 \cdot e^{j\omega 2} = e^{-j2\omega}$

$n=-4$ $\quad e^{-j4\omega} = e^{-j4\omega}$

$= e^{j2\omega} + e^{j4\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 2\cos(2\omega) + 2\cos(4\omega) + 1$

(c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

for $n=-6$; $e^{j6n} = e^{j6\omega}$

$n=-3$; $1 \times e^{j3n} = e^{j3n}$

$n=0$; $1 \times e^{j0} = 1$

$n=3$; $1 \times e^{-j3n} = e^{-j3\omega}$

$n=6$; $1 \times e^{-j6n} = e^{-j6\omega}$

$= e^{j6\omega} + e^{j3\omega} + e^{-j3\omega} + e^{-j6\omega} + 1$

$= 2\cos(6\omega) + 2\cos(3\omega) + 1$

If is there any relation b/w $x_1(\omega)$, $x_2(\omega)$ and $x_3(\omega)$? what is physical meaning.

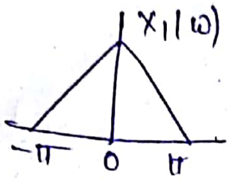
$x_1(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$

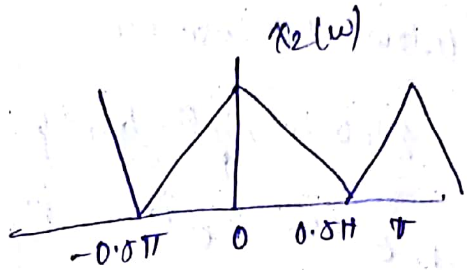
$x_3(\omega) = 2\cos(2\omega) + 2\cos(4\omega) + 1$

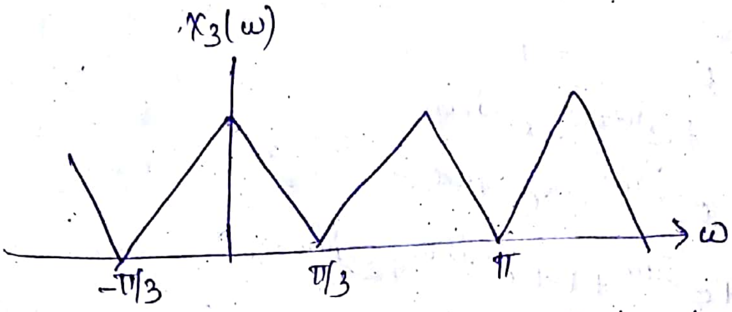
$$\boxed{x_4(\omega) = x_1(2\omega)}$$

$x_3(\omega) = 2\cos(6\omega) + 2\cos(8\omega) + 1$

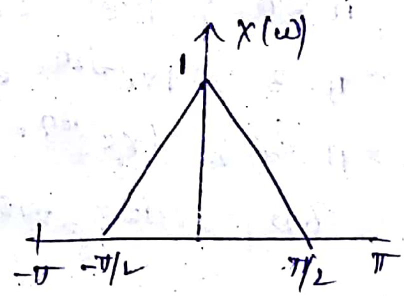
$\Rightarrow \boxed{X_3(\omega) = x_1(3\omega)}$







4.19) Let $x(n)$ be a signal with fourier transform as shown. Determine and sketch the fourier transforms of following signals. note: that these signals sequences are obtained by amplitude modulation of a carrier $\cos\omega_c n$ or $\sin\omega_c n$ by the sequence $x(n)$

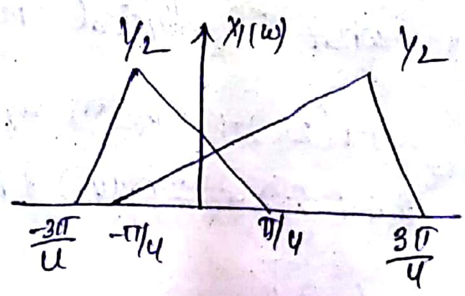


ⓐ $x_1(n) = x(n) \cdot \cos\left(\frac{\pi n}{4}\right)$

we know

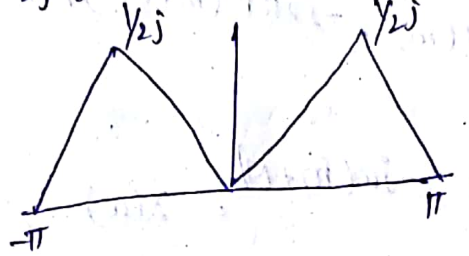
$$x(n)\cos(\omega_0 n) \xrightarrow{FT} \frac{1}{2}\left[x(\omega - \omega_0) + X(\omega + \omega_0)\right]$$

$$X_1(\omega) = \frac{1}{2}\left[x\left(\omega + \frac{\pi}{4}\right) + x\left(\omega - \pi/4\right)\right]$$



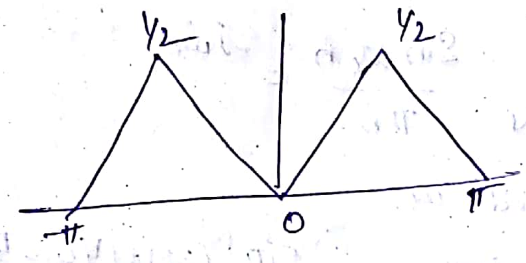
(b) $x_2(n) = x(n)\sin(\pi/2)$

$$x_2(\omega) = \frac{1}{2j}\left[X(\omega+\pi/2) + X(\omega-\pi/2)\right]$$



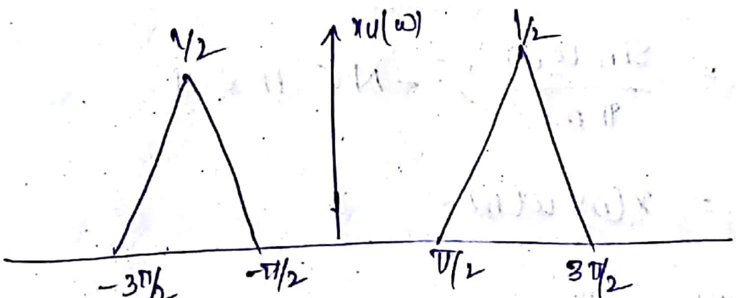
(c) $x_3(n) = x(n)\cdot\cos\left(\frac{n}{2}\pi\right)$

$$x_3(n) = \frac{1}{2}\left[X(\omega-\pi/2) + X\left(\omega+\frac{\pi}{2}\right)\right]$$



(d) $x_4(n) = x(n)\cos(\pi n)$

$$X_4 = \frac{1}{2}\left[X(\omega-\pi) + X(\omega+\pi)\right]$$



4.20:- Consider an aperiodic signal $x(n)$ with FT $X(\omega)$. Show that the Fourier series coefficients C_k of the periodic signal

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

are given by

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right) \quad k = 0, 1, \ldots N-1$$

$$C_k^y = \frac{1}{N}\sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N}.$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} x(n-lN) \right] e^{-j2\pi k n / N}$$

$$= \frac{1}{N} \sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-l-lN} x(m)\, e^{-j2\pi k(m+lN)/N}$$

But $\sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m)\, e^{-j\omega(m+N)} = X(\omega)$

$$C_k y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$$

4.21) prove that

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n}.$$

may be expressed as

$$X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin[(2N+1)(\omega-\theta/2)]}{\sin[\omega-\theta/2]} d\theta$$

let

$$x_N(n) = \frac{\sin \omega_c n}{\pi n} ; \quad -N \le n \le N$$

$$= x(n)\, w(n)$$

where $x(n) = \dfrac{\sin \omega_c n}{\pi n} \quad -\infty < n \le \infty$

$w(n) = 1 ; \quad -N \le n \le N$

$= 0 ; \quad$ otherwise

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta) \cdot W(\omega-\theta)\, d\theta$$

$$= \int_{-\omega_c}^{\omega_c} \frac{\sin(2N+1)(\omega-\theta)/2}{\sin(\omega-\theta)/2} \cdot d\theta$$

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4.22: A signal $x(n)$ has the following fourier transform

$$x(\omega) = \frac{1}{1-ae^{-j\omega}}$$

Determine the fourier transforms of the following signals

ⓐ $x(2n+1)$

$$\sum_{l=-\infty}^{\infty} x(2n+1)\, e^{-j\omega n}$$

Let $2n+1 = l$

$$\sum_{l=-\infty}^{\infty} x(l)\, e^{-j\omega\left[\frac{l-1}{2}\right]}$$

$$\sum_{l=-\infty}^{\infty} x(l) e^{-\frac{j\omega l}{2}} \cdot e^{\frac{j\omega}{2}}$$

$$e^{j\omega/2} \sum_{l=-\infty}^{\infty} x(l)\, e^{-\frac{j\omega l}{2}}$$

$$e^{j\omega} \cdot \sum_{l=-\infty}^{\infty} x(n)\, e^{-\frac{j\omega}{2} n}$$. Replace

$$= e^{\frac{j\omega}{2}} x\left(\frac{\omega}{2}\right)$$

$$e^{\frac{j\omega}{2}} \cdot \frac{1}{1-ae^{-j\left(\frac{\omega}{2}\right)}}$$

$$= \frac{e^{\frac{j\omega}{2}}}{1-ae^{-j\left(\frac{\omega}{2}\right)}}$$

ⓑ $e^{\frac{\pi j}{2}} x(n+2)$

$$e^{j2\omega} \cdot x\left(\omega - \frac{\pi j}{2}\right)$$

$$x(n) \longleftrightarrow X(\omega)$$

$$x(n+2) \longleftrightarrow e^{j2\omega} X(\omega)$$

$$e^{\frac{\pi j}{2}} x(n+2) \longleftrightarrow e^{j2\omega} X\left(\omega - \frac{\pi j}{2}\right)$$

$e^{j2\omega} \cdot X\left(\omega - \frac{\pi j}{2}\right)$

(c) $x(-2n)$

$x(n) \longleftrightarrow X(\omega)$

$x(2n) \longleftrightarrow X\left(\frac{\omega}{2}\right)$

$x(-2n) \longleftrightarrow X\left(-\frac{\omega}{2}\right)$

(d) $x(n)\cos(0.3\pi n)$

$x(n)\cos\omega_0 n \longleftrightarrow \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$

$x(n)\cos(0.3\pi n) \longleftrightarrow \frac{1}{2}\left[X(\omega+0.3\pi) + X(\omega-0.3\pi)\right]$

(e) $x(n) * x(n-1)$

$X(\omega) \cdot e^{j\omega} \cdot X(\omega)$

$X^2(\omega)e^{-j\omega}$

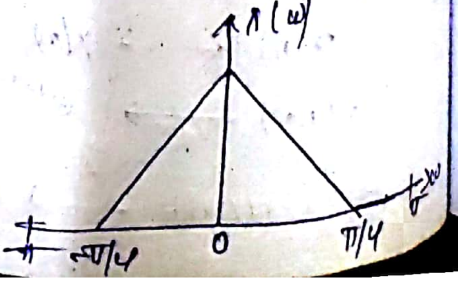
(f) $x(n) * x(-n)$

$X(\omega) \cdot X^*(\omega)$

$\frac{1}{1-ae^{-j\omega}} \cdot \frac{1}{1-ae^{j\omega}}$

$\frac{1}{1-ae^{j\omega}-ae^{-j\omega}+a^2e^0} = \frac{1}{1+a^2-2a\cos\omega}$

4.23) From a discrete time signal $x(n)$ with fourier transform $X(\omega)$ show in fig determine and sketch the fourier transform of the following signals



$y_1(n) = x(n)s(n)$

$s(n) = \{\ldots 0,1,0,1,0,1,0,1\}$

(a) $y(n) = \begin{cases} x(n), & 'n' \text{ even} \\ 0, & 'n' \text{ odd} \end{cases}$

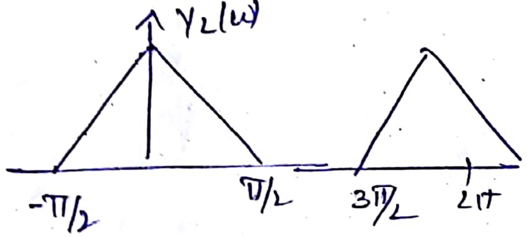
(b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$Y_2(\omega) = \sum_n x_2(n) e^{-j\omega n}$

$= \sum_n x(2n) e^{j\omega n}$

$= X\left(\frac{\omega}{2}\right)$



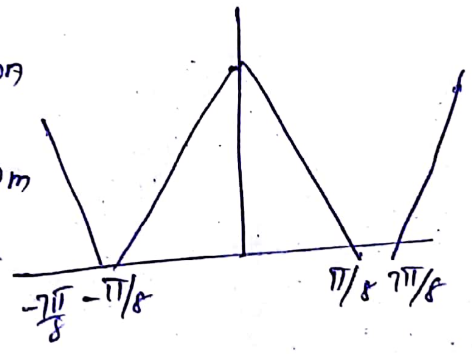
c) $y_3(n) = \begin{cases} x(n/2), & 'n' \text{ even} \\ 0, & 'n' \text{ odd} \end{cases}$

$Y_3(n) = \sum_n y_3(n) e^{-j\omega n}$

$= \sum_{n \text{ even}} x\left(\frac{n}{2}\right) e^{-j\omega n}$

$= \sum_m x(m) e^{-j2\omega m}$

$= X(2\omega)$



(e) $y_1(n) = \begin{cases} y_2(n/2); & 'n' \text{ even} \\ 0; & 'n' \text{ odd} \end{cases}$

$Y_1(\omega) = Y_2(2\omega)$