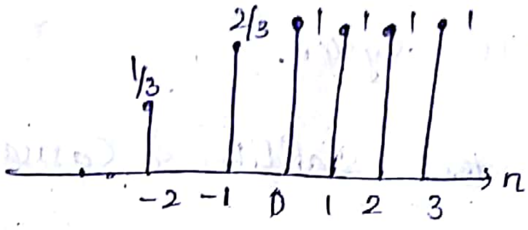
DSP

① $x[n] = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

ⓐ

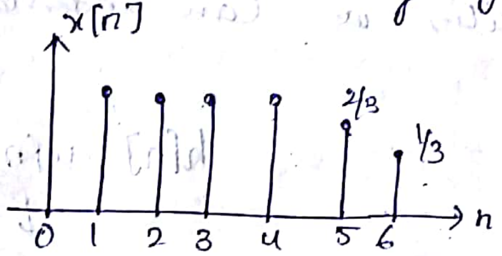
$x[n] = \{0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, \dots\}$

$x[n] =$



ⓑ

(i) first fold and then delay by 4 samples,

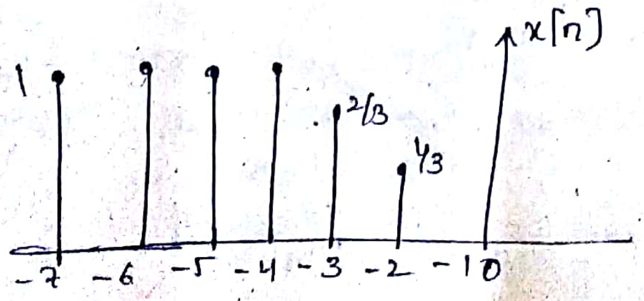


(ii) first delay $x[n]$ and then fold [4 samples delay]

$x[n] = \{ \underset{\uparrow}{0}, 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0 \dots \}$

folding

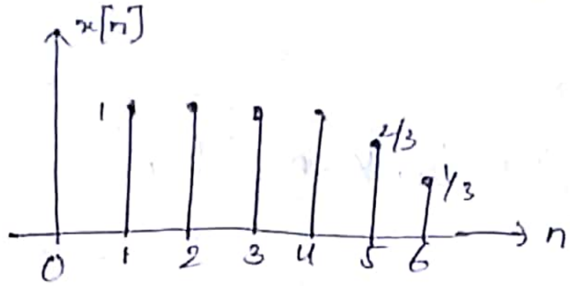
$x(-n-4) = \{ 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \underset{\uparrow}{0}, \dots \}$



ⓒ a $x(-n+4)$

$x(n+4) = \{ 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, \dots \}$

$= \{ 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \dots \}$



(d) $x(-n+k) \Rightarrow$ we will get this result by first shifting by k units and then folding $x(n+k)$.

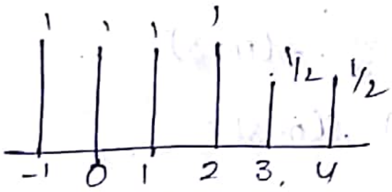
(e) $x[n]$ in terms of $\delta[n]$ and $u[n]$

$$x[n] = \frac{1}{3}\delta[n+2) + \frac{2}{3}\delta[n+1] + 1\cdot\delta[n] + 1\cdot\delta(n-1) + 1\cdot\delta(n+2)$$
$$+ 1\cdot\delta(n-3)$$

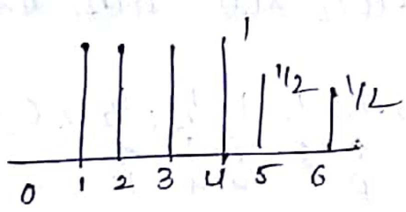
(or)

$$x[n] = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

(2) Given, $x[n]$.



(a) $x[n-2]$,

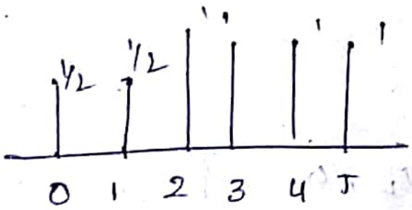


(b) $x[4-n] \Rightarrow x[-n+4]$

$x[n] = \{0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0\}$

$x[n+4] = \{0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0\}$

$x[-n+4] = \{0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0\}$



(c) $x[n] \cdot u[2-n]$

$x[n] = \{0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$,

$u[n] = \{0, 1, 1, 1, \ldots\}$

$u(n+2) = \{0, 1, 1, 1, 1, 1, \ldots\}$

$u(-n+2) = \{0, 1, 1, 1, 1, 0, 0 \ldots\}$

So,

$x(n) \cdot u(n-2) = \{0, 1, 1, 1, 1, 0, 0 \ldots\}$

(e) $x(n-1) \cdot \delta(n-3)$

$x(n-1) = \{0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$

$\delta(n-3) = \{0, 0, 0, 0, 1, 0, 0 \ldots\}$

$x(n-1) \cdot \delta(n-3) = x(n-1+3) \cdot \delta(n-3)$

$= x(n+2) \cdot \delta(n-3)$

$= \{\ldots 0, 0, 0, 1, 0 \ldots\}$

(f) $x(n^2) = \{\ldots 0, x(0), x(1), x(4), x(9), \ldots\}$

$= \{\ldots 0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$

0 1 4 9 16

(g) $x_e(n) = ?$

$$\frac{x[n] + x[-n]}{2}$$

$x[n] = \{0, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$

$x[-n] = \{0, 1/2, 1/2, 1, 1, 1, 1, 0 \ldots\}$

$x[n] + x[-n] = \{1/2, 1/2, 1, 2, 2, 2, 1, 1/2, 1/2, 0\}$

$$\frac{x[n]+x[-n]}{2} = \left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0 \ldots\right\}$$

(h) $x_0[n] = ?$

$$x[n] - x[-n] = \left\{-\frac{1}{2}, -\frac{1}{2}, -1, 0, 0, 0, \frac{1}{2}, \frac{1}{2}, 0 \ldots\right\}$$

$$\frac{x[n]-x[-n]}{2} = \left\{-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{4}, \frac{1}{4}, 0 \ldots\right\}$$

3) (a) $u[n] = \{0, 1, 1, 1, 1 \ldots\}$

$u[n-1] = \{0, 0, 1, 1, 1 \ldots\}$

$u[n] - u[n-1] = \{0, 1, 0, 0 \ldots\} = \delta(n)$

(b) $u(n) = \sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \delta(k) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$$

(4) $x[n] = \{2, 3, 4, 5, 6\}$

yes decomposition is unique, while.

$x_e(n) \Rightarrow x(-n) = x[n]$

$x_0[n] \Rightarrow x[-n] = -x[n]$

$x_e[n] = ?$, $x_0(e) = ?$

$x[n] = \{2, 3, 4, 5, 6\}$

$x[-n] = \{6, 5, 4, 3, 2\}$

$x_e[n] = \frac{x[n]+x[-n]}{2} = \{4, 4, 4, 4, 4\}$

$x_0[n] = \frac{x[n]-x[-n]}{2} = \{-2, -1, 0, 1, 2\}$

(5) Given, $y[n] = T[x[n]] = x[n^2]$

Time invariant condition = delay i/p response = delay of delay i/p response

$x[n-n_0] \Rightarrow y_1[n] = x[(n-n_0)^2]$

delay o/p for i/p $x[n]$ is

$y[n] \xrightarrow{\text{delay}} y[n-n_0] = x[n^2-n_0]$

So delay i/p response $\neq$ delayed output so system is time invariant

(b) Given,

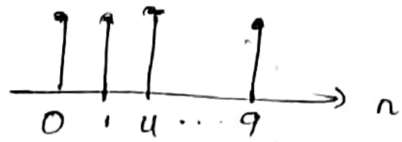
$$x[n] = \begin{cases} 1, & 0 \leq n \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) $x[n] = \{0, 1, 1, 1, 1, 0 \ldots\}$

(ii) $y[n] = T[x[n]] = x[n^2]$

$= \{0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0\}$

So, $y[n] =$



(iii) $y_2[n] = y[n-2]$

$= \{0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0 \ldots\}$

(iv) $y_2[n] = x[n-2]$

$= \{0, 0, 1, 1, 1, 1, 0 \ldots\}$

(v) $y_2[n] = T[x_2[n]]$

$= \{0, 1, 1, 1, 0, 0, 0 \ldots\}$

(vi) $y_2[n] \neq y_2[n]$

So system is time variant

c) $y(n) = x(n) - x(n-1)$

(ii) $y(n) = \{0, 1, 1, 1, 1, 0 \dots\} - \{0, 1, 1, 1, 1, 0 \dots\}$

$y[n] = \{0, 1, 0, 0, 0, -1, 0 \dots\}$

(iii) $y[n-2] = \{0, 0, 1, 0, 0, 0, -1, 0 \dots\} = y_2(n)$

(iv) $x[n-2] = \{0, 0, 1, 1, 1, 1, 0 \dots\} = x_2[n]$

(v) $y_2[n] = T[x_2[n]]$

$y_2[n] = x_2(n) - x_2[n-1]$

$= \{0, 0, 1, 1, 1, 1, 0\} - \{0, 0, 0, 1, 1, 1, 1, 0 \dots\}$

$= \{0, 0, 1, 0, 0, 0, -1, 0 \dots\}$

(vi) $y_2'(n) = y_2(n)$ = Time invariant

(d) $y[n] = n x[n]$

(i) $x[n] = \{0, 1, 1, 1, 1, 0 \dots\}$

(ii) $y[n] = n x[n] = \{0, 0, 1, 2, 3, 0 \dots\}$

(iii) $y[n-2] = y_2'[n] = \{0, 0, 0, 1, 2, 3, 0 \dots\}$

(iv) $x[n-2] = \{0, 0, 1, 1, 1, 1, 0 \dots\} = x_2[n]$

(v) $y_2[n] = T[x_2[n]] \Rightarrow n x_2[n]$

$= \{0, 0, 2, 3, 4, 5, 0 \dots\}$

(vi) $y_2(n) \neq y_2'(n)$ = Time Variant.

(f) (a) $y[n] = \cos[x(n)]$

→ Static, non linear, time invariant, Causal stable

(b) $y[n] = \sum_{k=-\infty}^{n+1} x[k]$

→ static linear, time invariant, non causal, unstable.

(c) $y[n] = x[n] \cdot \cos[\omega_0 n]$

→ static, nonlinear, time invariant, causal, stable

(d) $y[n] = x[-n+2]$

⇒ dynamic, linear, time invariant, non causal, stable

(e) $y[n] = \text{Trun}[y(n)]$

→ static, non linear, time invariant, causal, stable.

(f) $y[n] = \text{Round}[x(n)]$

→ static, non linear, time invariant, causal, stable

(g) $y[n] = |x[n]|$

→ static, nonlinear, time invariant, causal stable

(h) $y(n) = x(n) \cdot u(n)$

→ static linear, time variant, causal, stable.

(i) $y[n] = x[n] + n\, x(n+1)$

→ dynamic, linear, time variant, non causal, unstable

(j) $y(n) = x(2n)$

→ Dynamic, linear, time variant, non causal, unstable.

k) $y[n] = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$

→ static, nonlinear, time invariant, causal, stable

l) $y(n) = x[n]$

→ Dynamic, linear, time invariant, non causal, stable

m) $y[n] = \text{Sign}[x[n]]$

→ static, non linear, time invariant, causal, stable

n) $y[n] = x_0[n]$ - $\longrightarrow$ ideal Sampling

$\rightarrow$ static, linear, time invariant, Causal, Stable

2.9.

ⓐ $y[n] = \sum_{k=-\infty}^{n} h(k) \cdot x(n-k)$, $x[n] = 0$, $n<0$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k) \cdot x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k) \cdot x(n-k)$$

$$= \sum_{k=-\infty}^{n} h(k)\, x(n-k) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

$$= y(n) + \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k)$$

for BIBO s/m $\lim_{n\to\infty} |h(n)| = 0$

$$\therefore \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k) = 0 \text{ and}$$

$$\lim_{n\to\infty} y(n+N) = y(N),$$

ⓑ Let $x(n) = x_0(n) + a\,u(n)$, where a is Const and $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$

then, $y[n] = a \sum_{k=0}^{\infty} h(k) \cdot u(n-k) + \sum_{k=0}^{\infty} h(k) \cdot x_0(n-k)$

$$= a \sum_{k=0}^{n} h(k) + y_0(n)$$

clearly, $\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_1^2(n) < \infty$, Hence

$$\lim_{n\to\infty} |y_0(n)| = 0$$

ⓒ $y(n) = \sum_k h(k) \cdot x(n-k)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k) \cdot x(n-k) \right]^2$$

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$$\sum_k \sum_l h(k)\, h(l) \cdot \sum_n x(n-k) \cdot x(n-l)$$

But,

$$\sum_n x(n-k) \cdot x(n-l) \leq \sum_n x^2(n) = E_x$$

$$\therefore \sum_n y^2(n) \leq E_x \sum_k |h(k)| \cdot \sum_l |h(l)|$$

for BIBO s/m, $\sum_k |h(k)| \leq M$

Hence, $E_y \leq M^2 E_x$, so that

$E_y < 0$ if $E_x \leq 0$ //

2.10:- $x_2[n] = \{0, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, 1, 0, 2\}$

$x_3(n) = \{0, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, 2, 1\}$

for time invariant systems for $y_2(n)$ should have only 3 elements and $y_3(n)$ should have 4 elements.

So, it is non linear.

2.11:- $x_1(n) + x_2(n) = \delta(n)$

system is linear, the impulse response of system is $y_1(n) + y_2(n) = \{0, 3, -1, 2, 1\}$

if the system, were time invariant, the response to $x_3(n)$ would be $\{3, 2, 1, 3, 1\}$. But this is not a Case.

2.12: (a) Any weighted linear combination of signals $x_i(n)$,

$i = 1, 2 \dots N$

(b) Any $x_i(n-k)$ where k is any integer and

$i = 1, 2 \dots N$.

2.13: Response of LTI s/m.

$$y(n) = \sum_{n=-\infty}^{\infty} h(n)\, x(k-n)$$

for BIBO stability,

$$|y(n)| \leq \sum_{n=-\infty}^{\infty} |h(n)|\, |x(k-n)|$$

↳ max value of signal let M_x

$$\infty \leq \sum_{n=-\infty}^{\infty} M_x\, |h(n)|$$

for stable, condition is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_b \leq \infty$$

2.14: (a) Causal system ⇔ o/p becomes non zero after the i/p becomes non zero

Hence $x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$

(b) $$y(n) = \sum_{k=\infty}^{m} h(k) \cdot x(n-k)$$ where $x(n) = 0$ for $n < 0$

if $h(k) = 0$ for $k < 0$ then

$$y(n) = \sum_{k=0}^{n} h(k) \cdot x(n-k)$$ and hence $y(n) = 0$ for $n < 0$.

(or)

$$\sum_{-\infty}^{n} h(k).x(n-k) \Rightarrow h(k) = 0, \quad k<0, \text{ when } y(n)=0$$

2.15

$$\sum_{n=M}^{N} a^n$$

Series will be $a^m, a^{m+1}, \dots a^N$

for $a \neq 1$

by sum of N terms in GP $\Rightarrow \dfrac{a^m - a^{n+1}}{1-a}$

$\left[\because \text{first term} = a^m, \quad r = \dfrac{a^{m+1}}{a^m} = a \right]$

for $a = 1$

Series will be $\underbrace{1, 1, \dots 1}$

(b) $\sum_{n=0}^{\infty} a^n \Rightarrow$ it is GP and given $|a|<1 \dots$

Series will be .. $1, a, a^2 \dots$

first term $r = a$

and $r<1$ ($\because |a|<1$), so,

So, $\sum_{n=0}^{\infty} a^n = \dfrac{a}{1-r} = \dfrac{1}{1-a}$

2.16:- Given, $y(n) = x(n) * h(n)$

$$y(n) = \sum_k h(k).x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k).x(n-k)$$

$$= \sum_k h(k) . \sum_{n=-\infty}^{\infty} x(n-k)$$

$$= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

① $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$\sum y(n) = \sum x(n) * h(n)$

$= \sum_k h(n) \sum_n x(n)$

$= 5(1+2+4) \Rightarrow 35$

② $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

$y(n) = (1+2-1) \cdot (1+2-1) = 4$

③ $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{1/2, 1/2, 1, 1/2\}$

$\sum y(n) = (0+1-2+3-4)\left(\frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}\right)$

$= (-2)(2.5) = -5$

④ $x[n] = \{1, 2, 3, 4, 5\}$, $h[n] = (1)$

$\sum y(n) = (1+2+3+4+5) \cdot (1)$

$= 15$

⑤ $x(n) = \{1, -2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

$\sum y(n) = (1+(-2)+3) \cdot (0+0+1+1+1+1)$

$= (2)(4) = 8$

⑥ $x(n) = \{0, 0, 1, 1, 1, 1\}$, $h(n) = (1, 2, 3\}$

$\sum y(n) = (1+1+1+1) \cdot (1-2+3) = 8$

⑦ $y(n) = \{0, 1, 4, -3)$, $h(n) = \{1, 0, -1, -1\}$

$\sum y(n) = (0+1+4-3) \cdot (1+0-1-1)$

$= (2)(-1) = -2$

⑧ $x(n) = (1, 1, 2)$, $h(n) = u(n)$

$\sum y(n) = (1+1+2) \cdot \sum_{k=0}^{\infty} (1) \Rightarrow 4 \cdot \infty = \infty$

⑨ $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

$\sum y(n) = [1+1+0+1+1], [1-2-3+4]$

$= (4) \cdot 0 = 0$

⑩ $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$\sum y(n) = (1+2+0+2+1)(1+2+0+2+1)$

$= 6 \times 6 = 36$

⑪ $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

$= \sum x(n) = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^n ; \sum h(n) = \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$

So, $\sum x(n) = \frac{1}{1-\frac{1}{2}} \Rightarrow 2$ (where $a = 1$, $r = \frac{1}{2}$)

$\sum h(n) = \frac{1}{1-\frac{1}{4}} \Rightarrow \frac{4}{3}$ ($a = 1$, $r = 1/4$)

$\sum y(n) = \sum x(n) \sum h(n) = 2 \times \frac{4}{3} = \frac{8}{3}$

2.17:- ⓐ $x(n) = \{1, 1, 1, 1\}$

$h(n) = \{6, 5, 4, 3, 2, 1\}$

$y(n) = \sum_{k=0}^{n} x(k) \cdot h(n-k)$

$y(0) = x(0) \cdot h(0) = 6$

$y(1) = x(0) \cdot h(1) + x(1) \cdot h(0) = 11$

$y(2) = x(0) \cdot h(2) + x(1) \cdot h(1) + x(2) \cdot h(0) = 15$

$y(3) = x(0)\, h(3) + x(1)\, h(2) + x(2) \cdot h(1) + x(3) \cdot h(0) = 18$

$y(4) = x(0) \cdot h(4) + x(1) \cdot h(3) + x(2)\, h(2) + x(4)\, h(1)$
$+ x(4) \cdot h(0) = 14$

$y(5) = x(0).h(5) + x(1).h(4) + x(2).h(3) + x(3).h(2)$

$+ x(4).h(1) + x(5).h(0) = 10$

$y(6) = x(1).h(5) + x(2)h(4) + x(3)h(3) = 6$

$y(7) = x(2)h(5) + x(3)h(4) = 3$

$y(8) = x(3)h(5) = 1$

$y(n) = 0, \quad n \geq 9$

So, $y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

Simillarly as in ⓐ,

ⓑ $y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

ⓒ $y(n) = \{1, 2, 2, 2, 1\}$

ⓓ $y(n) = \{1, 2, 2, 2, 1\}$

2.18:- given,

$$x(n) = \begin{cases} \frac{1}{3}n; & 0 \leq n \leq 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1; & -2 \leq n \leq 2 \\ 0; & \text{elsewhere.} \end{cases}$$

$y(n) = x(n) * h(n)$

$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$= \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

h(n) \ x(n)	0	$\frac{1}{3}$	$\frac{2}{3}$	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
→ 1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

(b) $x(n) = \frac{1}{3} n \left[u(n) - u(n-7)\right]$

$h(n) = u(n+2) - u(n-3)$.

$y(n) = x(n) * h(n)$

$= \frac{1}{3} n \left[u(n) - u(n-7)\right] * \left[u[n+2] - u[n-3]\right]$

$= \frac{1}{3} n \left[u(n) * u(n+2) - u(n)\, u(n-3) - u(n-7) + u(n+2) + u(n-7)\, u(n-3) \right]$

$= \frac{1}{3} \delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3)$

$+ \frac{20}{3}\delta(n-4) + 6\delta(n-5) + 5\delta(n-6) + 5\delta(n-6)$

$+ \frac{11}{6}\delta(n-7) + \delta(n-8)$,,

2.20:

(a) $131 \times 122 = 15982$

(b) $\{1,3,1\} * \{1,2,2\}$

$= \{1, 5, 9, 8, 2\}$

(c) $(1+3z+z^2)(1+2z+2z^2) = (1+5z+9z^2+8z^3+2z^4$

(d) $(1.31)(12.2) = 15.982$

(e) There are different ways to perform convoluti

2.21 (a) $a^n u(n)$

$h(n) = b^n u(n)$

$y(n) = x(n) * h(n)$ where $a = b$
$a \neq b$

Case(i) $a \neq b$

$$y(n) = \sum_{k=0}^{n} a^k (u(k)) \cdot b^{n-k} \cdot u(n-k)$$

$$= b^n \sum_{k=0}^{n} (a b^{-1})^k$$

$$= b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

$\rightarrow GP = 1, \frac{a}{b}, \left(\frac{a}{b}\right)^2, \ldots$

$$= b^n \left[\frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \frac{a}{b}} \right]$$

$$= b^n \left[\frac{\frac{b^{n+1} - a^{n+1}}{b^{n+1}}}{\frac{b-a}{b}} \right]$$

$$= \cancel{b^{n+1}} \left[\frac{b^{n+1} - a^{n+1}}{\cancel{(b^{n+1})} (b-a)} \right]$$

$$y[n] = \frac{b^{n+1} - a^{n+1}}{b - a}$$

Case(ii) $a = b$

$$y(n) = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k, \quad \text{for } a = b \Rightarrow \frac{a}{b} = 1$$

$$= b^n \sum_{k=0}^{n} 1$$

$$\boxed{y(n) = b^n (n+1) \cdot u(n)}$$

(b) $x(n) = \{1, 1, 1, 1\}$

$h(n) = \{1, -1, 0, 91, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

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(c) $x(n) = \{1, 1, 1, 1, 1, 0, -1\}$

$y(n) = \{1, 2, 3, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

(d) $x(n) = \{1, 1, 1, 1, 1\}$

$h(n) = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$ where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2-22:- $y(n) = n\,y(n-1) + x(n)$ for $n \geq 0$

and also given $y(-1) = 0$

If $y_1(n) = n\,y_1(n-1) + x_1(n)$ &

$y_2(n) = n\,y_2(n-1) + x_2(n)$ then

$x(n) = a x_1(n) + b x_2(n)$

produces o/p as

$y(n) = n\,y(n-1) + x(n)$ where $y(n) = a y_1(n) + b y_2(n)$

Hence s/m is linear. If the input is $x(n-1)$ we have

$y(n-1) = (n-1)\,y(n-2) + x(n-1)$. But

$y(n-1) = n y(n-2) + x(n-1)$

Hence s/m is time variant. If $x(n) = u(n)$. then $|x(n)| \leq 1$. But for this bounded i/p, the o/p is

$y(0) = 1$, $y(1) = 1+1 = 2$, $y(2) = 2 \times 2 + 1 = 5$ ....

which is unbounded, Hence system is unstable

2.25:-

with $x(n) = 0$, we have

$$y(n-1) + \frac{4}{3} y(n-1) = 0$$

$$y(-1) = -\frac{4}{3} y(-2)$$

$$y(0) = (-4/3)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2) \leftarrow \text{Zero i/p response.}$$

2.26:- Given homogeneous eqn,

$$y(n) = \frac{-5}{6} y(n-1) + \frac{1}{6} \cdot y(n-2) = 0$$

The characteristic eqn is

$$\lambda^n - \frac{5}{6} \lambda^{n-1} + \frac{1}{6} \lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right] = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$$

roots, $\lambda = \frac{1}{2}, \frac{1}{3}$

Hence,

$$y_h(n) = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{3}\right)^n \rightarrow (1)$$

particular soln to

$$x(n) = 2^n u(n) \text{ is}$$

$$y_p(n) = k(2^n) \cdot u(n) \rightarrow (2)$$

Substituting into differential eqn.

$$k \cdot 2^n u(n) - \frac{5}{6} k \cdot 2^{n-1} u(n-1) + k \frac{1}{6} 2^{n-2} u(n-2)$$

$$= 2^n \cdot u(n)$$

for $n=2$

$$4k - \frac{5k}{3} + \frac{k}{6} = 4 \Rightarrow k = 8/5$$

$\therefore$ The total sol'n is

$y(n) = y_p(n) + y_h(n)$ {from eq(1) & eq(2)}

$= \frac{8}{5} 2^n u(n) + C_1 \left(\frac{1}{2}\right)^n u(n) + C_2 \left(\frac{1}{3}\right)^n u(n)$

Also assume, $y(-2) = y(-1) = 0$, then,

$y(0) = 1 \quad (\because y(0) = 5/6\, y(0-1) - \frac{1}{6} y(0-2) + x(0))$

&

$x(0) = 2\, u(0) = 1$

&

$y(1) = \frac{5}{6} y(0) + 2 = \frac{17}{6}$.

Thus,

$\frac{8}{5} + C_1 + C_2 = 1 \Rightarrow C_1 + C_2 = -3/5$

$\frac{16}{5} + \frac{1}{2} C_1 + \frac{1}{3} C_2 \Rightarrow \frac{17}{6} \Rightarrow 3C_1 + 2C_2 = -11/5$

$\therefore$

$C_1 = -1, \quad C_2 = 2/5$

Total solution is

$y[n] = \left[\left(\frac{8}{5}\right) 2^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right] u(n)$

2.27) Given eqn,

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

to the i/p $x(n) = 4^n u(n)$

Characteristic equation is

$\lambda(n) - 3\lambda(n-1) - 4\lambda(n-2) = 0$

$\lambda^{(n-2)} [\lambda^2 - 3\lambda - 4] = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda+1) - 4(\lambda+1) = 0$

$\lambda = -1, \quad \lambda = 4$

so, $y_h(n) = C_2(-1)^n + C_1(4)^n$

particular solution is

$y_p(n) = k n 4^n u(n)$

Then

$k n 4^n u(n) = 3k[n-1] 4^{n-1} u(n-1) - 4k(n-2) u(n-2)$

$= 4^n u(n) + 2(4)^n + u(n-1)$

for $n=2$,

$k(32-12) = 4^2+8 = 24 \quad \therefore k = 6/5$

Total solution is,

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + C_1 4^n + C_2(-1)^n\right] u(n)$

now to solve C_1 and C_2, assume that $y(-1) = y(-2) = 0$

Then, $y(0) = 1$ and,

$y(1) = 3$

$y(0) + 4 + 2 = 9.$

Here. $C_1 + C_2 = 1$ and

$\frac{24}{5} + 4C_1 - C_2 = 9$

$4C_1 - C_2 = 11/5$

Therefore,

$C_1 = \frac{26}{25}$ and $C_2 = -1/25$

The total soln is

$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$

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2.28:- from 2.27

$\lambda = 4, -1$. Hence

$y_h(n) = C_1 4^n + C_2 (-1)^n$

when $x(n) = \delta(n)$, we find that

$y(0) = 1$ and

$y(1) = 3$, $y(0) = 2$

$y(1) = 5$

Hence $C_1 + C_2 = 1$ and $4C_1 - C_2 = 5$

By solving $C_1 = 6/5$ and $C_2 = -1/5$. There fore

$h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)]$

2.29:-

(a) $L_1 = N_1 + M_1$ & $L_2 = N_2 + M_2$

(b) partial overlap from left

low $N_1 + M_1$; high $N_1 + M_2 - 1$

full overlap: low $N_1 + M_2$, high : $N_2 + M_1$

partial overlap from right:-

low $N_2 + M_2 + 1$; high $N_2 + M_2$

(c) $x(n) = \{1, 1, 1, 1, 1, 1, 1\}$

$h(n) = \{2, 2, 2, 2\}$

$N_1 = -2$, $N_2 = 4$, $M_1 = -1$, $M_2 = 2$

partial overlap from left : $n = -3$, $n = -1$, $L_1 = -3$

full overlap $\Rightarrow n = 0$, $n = 3$

partial overlap from right ; $n = 4$, $n = 6$, $L_2 = 6$

2.30:
(a) $y(n) = 0.6y(n-1) - 0.08y(n-2)\ x(n)$

The eqn is

$\lambda^2 - 0.6\lambda + 0.08 = 0.$

$\lambda = 0.2, 0.4$. Hence.

$y_h(n) = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{2}{5}\right)^n$

& with $x(n) = \delta(n)$. The intial Conditions are.

$y(0) = 1 \qquad (\because x(0) = 1)$

$y(1) - 0.6\,y(0) = 0 \qquad \Rightarrow y(1) = 0.6$

Hence, $c_1 + c_2 = 1$ (for $y(0)$)

$\frac{1}{5}c_1 + \frac{2}{5} = 0.6$ (for $(y(1))$)

$c_1 = -1, \quad c_2 = 3$

$\therefore \quad h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n)$

& step response, $x(n) = u(n)$. The intial conditions are,

$$s(n) = \sum_{k=0}^{n} h(n-k), \quad n \geq 0$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2^{n+1}}{5} - 1\right] - \frac{1}{0.16}\left[\frac{1^{n+1}}{5} - 1\right]\right\}u(n)$$

2.31:- Given, $h(n) = \begin{cases} \left(\frac{1}{2}\right)^n & ; 0 \leq n \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

$x(n) = ?$

$y(n) = \{1, 2, 2.5, 3, 3, 2, 1, 0 \ldots\}$

$h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$

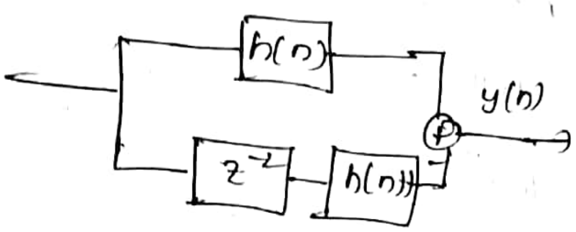
$x(0) \cdot h(0) = y(0) \Rightarrow x(0) = \frac{y(0)}{h(0)} = 1$

$\frac{1}{2} x(0) + x(1) = y(1) \Rightarrow x(1) = 3/2$

By continuing this process, we obtain

$x(n) = \{1, 3/2, 3/2, 7/4, 3/2, \dots\}$

2.33:-



given,

$x(n) = u(n+5) - u(n-10)$, $h(n) = a^n u(n)$, for $-1<a<1$

first we need to determine

$y(n) = x(n) * h(n)$

where $x(n) = u(n)$

$$= \sum_{k=0}^{n} x(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{n} 1 \cdot a^{n-k}$$

$$= \frac{a^{n+1} - 1}{a-1}, \quad n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$ we have the the

response

$$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5)$$

$$- \frac{a^{n-9} - 1}{a-1} u(n-10)$$

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from fig,

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

Hence. $$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$- \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-12)$$

2.34:- $h[n] = [u(n) - u(n-m)]/m$

$$g(n) = \sum_{k=-\infty}^{\infty} u(k) . h(n-k)$$

$$= \sum_{k=0}^{\infty} h.(n-k) \Rightarrow \begin{cases} \frac{n+1}{m} & , n<m \\ 1 & , n \geq m \end{cases}$$

2.35:- given,

$$h(n) = \begin{cases} a^n & ; n \geq 0 \text{ n even} \\ 0 & , \text{ otherwise} \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0 \text{ even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$ for stable condition is $|a|<1$,,

$$= \frac{a^{n+1}-1}{a-1} , n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$, we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

from given fig,

$y(n) = x(n) * h(n) - x(n) * h(n-2)$

Hence $y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$

$- \frac{a^{n-4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-14)$

2.34:- $h(n) = [u(n) - u(n-m)]/m$

$g(n) = \sum_{k=-\infty}^{\infty} u(k) \cdot h(n-k)$

$= \sum_{k=0}^{\infty} 1 \cdot h(n-k) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m \end{cases}$

2.35:- given, $h(n) = \begin{cases} a^n; & n \geq 0, n \text{ even} \\ 0 & \text{otherwise} \end{cases}$

$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0, \text{even}}^{\infty} |a|^n$

$= \sum_{n=0}^{\infty} |a|^{2n}$

$= \frac{1}{1-|a|^2}$, for stable conditions is $|a|<1$

2.36:- we have response $h(n) = a^n u(n)$, then result will be (∵

$y'(n) = \sum_{k=0}^{n} u(n-k) h(n)$

$= \sum_{k=0}^{n} 1 \cdot a^n$

$= \frac{1-a^{n+1}}{1-a} u(n).$

note,

for $x(n) = u(n) - u(n-10)$

result obtained is

$$y(n) = \frac{1-a^{n+1}}{1-a} u(n) - \frac{1-a^{n-9}}{1-a} u(n-10)$$

2.37:- Given,

$$h(n) = \left(\frac{1}{2}\right)^n u(n) \qquad a = 1/2$$

$$i/p \rightarrow x(n) = \begin{cases} 1 ; & 0 \le n \le 10 \\ 0 ; & \text{Otherwise} \end{cases}$$

$$x(n) = u(n) - u(n-10)$$

So,
$$y(n) = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-1/2} u(n) - \frac{1-\left(\frac{1}{2}\right)^{n-9}}{1-1/2} u(n-10)$$

$$= 2\left[\left(\left(1-\left(\frac{1}{2}\right)^{n+1}\right)u(n)\right) - \left(1-\left(\frac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

2.38:- Given, $h(n) = \left(\frac{1}{2}\right)^n u(n)$

i/p = (a) $x(n) = 2^n u(n)$

(b) $x(n) = u(-n)$

(a) $$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$= \sum_{k=0}^{n} x(n-k)\, h(k)$$

$$= \sum_{k=0}^{n} 2^{n-k} \cdot \left(\frac{1}{2}\right)^k$$

$$= 2^n \left[\sum_{k=0}^{n} \left(\frac{1}{4}\right)^k\right]$$

$$= 2^n \left[1-\left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \left[2^n - 2^n \cdot \frac{1}{4^n} \cdot \frac{1}{4}\right]\frac{4}{3}$$

$$= \left(2^n - \left(\tfrac{1}{2}\right)^n \cdot \tfrac{1}{4}\right)\tfrac{4}{3}$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \quad n<0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left[\frac{1-\left(\frac{1}{2}\right)^n}{1/2}\right]$$

$$= 2\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

2.39:- Given,

$$\left.\begin{aligned} h_1(n) &= \delta(n) - \delta(n-1) \\ h_2(n) &= h(n) \\ h_3(n) &= u(n) \end{aligned}\right\} \Rightarrow \text{Cascade.}$$

(or) $h(n) = ?$

$$h(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n)$$

$$= h(n)$$

(b) No

2.40:-

(a) $x(n)\,\delta(n-n_0) = x(n_0) = x(n)$ value at n_0 is only obtained in the result, rest of part will be zero.

Since $\delta(n-n_0) = 1$ only at n_0.

(b) $x(n) * \delta(n-n_0) = x(n-n_0) \Rightarrow$ we obtain shifted version seq of $x(n)$ while $\delta(n-n_0)$ is moving throughout $x(n)$ during Convolution.

(b) $y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$= h(n) * x(n)$

linearity:- $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

Then $x(n) = \alpha x_1(n) + \beta x_2(n) \rightarrow$

$y(n) = h(n) * x(n)$

$y(n) = h(n) * (\alpha x_1(n) + \beta x_2(n))$

$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$

Time invariance:-

$x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_2(n) = h(n) * x(n-n_0)$

$= \sum_{k} h(k) \cdot x(n-n_0-k)$

$= y(n-n_0)$

(c) $y(n) = x(n-n_0)$

$h(n) = \delta(n-n_0)$

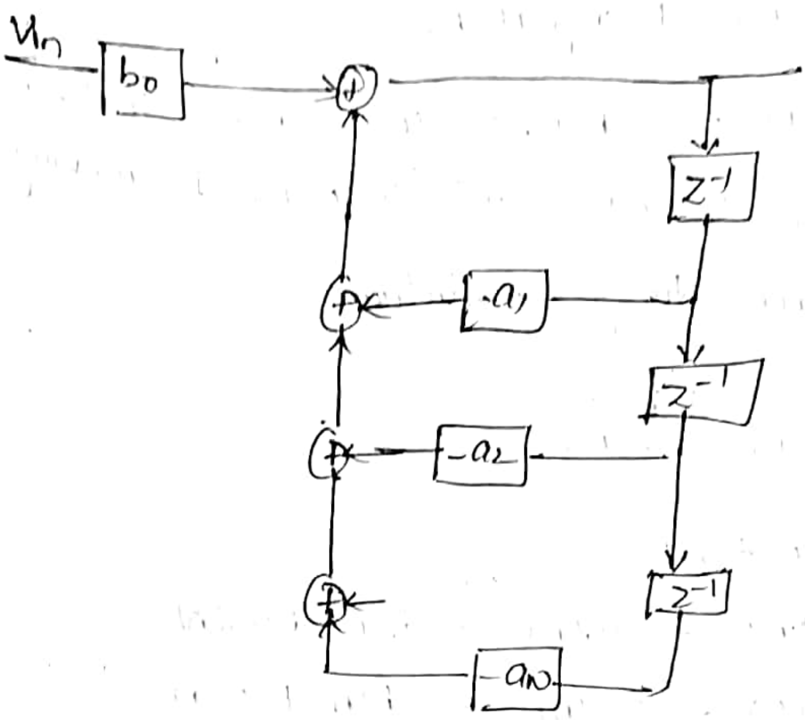
2.40:

Given functions,

(a) $\delta(n) = -a\,\delta(n-1) - a_2\,\delta(n-2) \cdots - a_N \delta(n-N) + \cdots$

(b) $\delta(n) = \frac{1}{b_0}\left[\delta(n) + a_1\,\delta(n-1) + a_2(\delta-2) + \cdots a_N\,\delta(n-N)\right.$

(c)



2.42: Zero state response of the system

$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$; $x(n) = \{1, 2, 3, 4, 4\}$

$y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$

$y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2(x-4) \Rightarrow 1$

$y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) \Rightarrow 3/2$

$y(0) = -\frac{1}{2}y(-1) + x(0) + 2x(-2) \Rightarrow \frac{11}{4}$

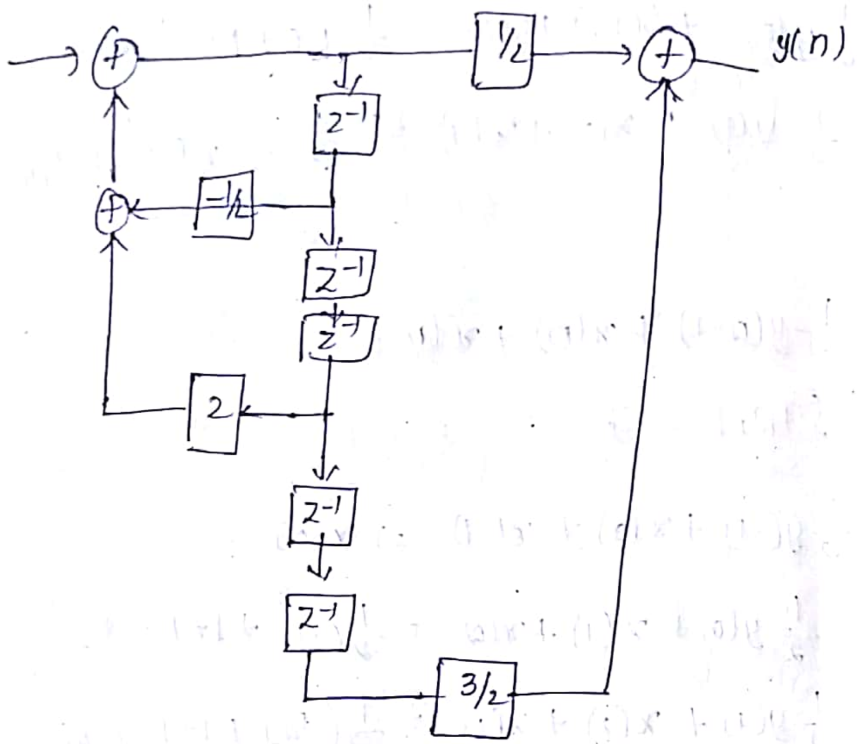
$y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}$

$\therefore$ $y(n) =$

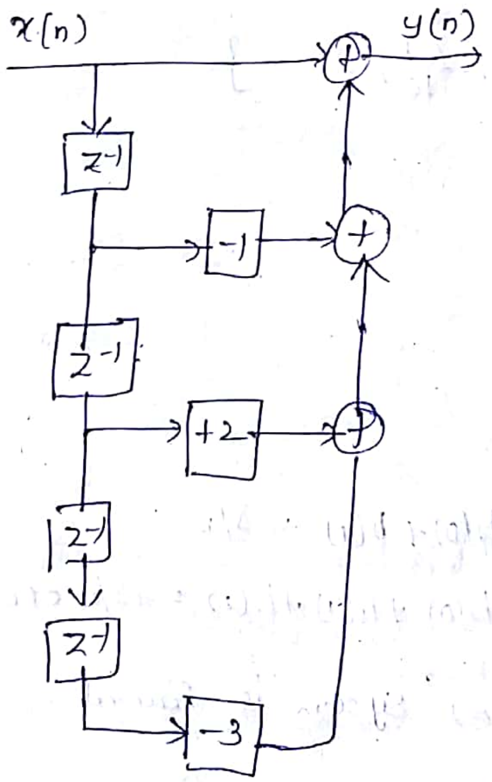
2.43:

(a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

$$y(n) = \frac{x(n) + 3x(n-5) + 4y(n-3) - y(n-1)}{2}$$



(b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$



2.44:

(a) $y(n) = \frac{1}{2} y(n+1) + x(n) + x(n-1)$

$x(n) = \{1, 0, 0, \dots\}$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2} + 0 + 1 = 3/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) \Rightarrow \frac{1}{2} \cdot \frac{3}{2} + 0 + 0 = 3/4$

Slly,

(b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

(c) $x(n) = \{1, 1, 1 \dots\}$

$y(0) = \frac{1}{2} y(-1) + x(0) + x(-1) \Rightarrow x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2}(1) + 1 + 1 = 5/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{1}{2}(5/2) + 1 + 1 = 13/4$

Slly,

$y(n) = \{1, 5/2, 13/4, \frac{29}{8}, \frac{61}{16}, \dots\}$

(d) $y(n) = u(n) * h(n)$

$= \sum_k u(k) h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1, \quad y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = 13/4$ etc.

(e) from a) $h(n) = 0 \Rightarrow$ System is Causal

$\sum_{n=0}^{\infty} |h(n)| = 1 + 3/2 (1 + 1/2 + 1/4 + \dots)$

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$= 1 + 3/2 \left[\frac{1}{1 - 1/2} \right]$

$= 4 \Rightarrow$ System is stable.

2.45: given

$$y(n) = a y(n-1) + b x(n)$$

(a) $\sum_{n=0}^{\infty} h(n) = 1$

$\Rightarrow h(n) = b a^n . u(n)$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

(b) $s(n) = \sum_{k=0}^{n} h(n-k)$

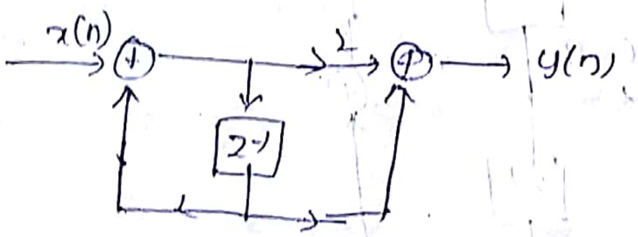
$$= b \left[\frac{1-a^{n+1}}{1-a} \right] u(n)$$

$$s(\infty) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

(c) $b = 1-a$ in both cases

2.46:



$$y(n) = 0.8\, y(n-1) + 2x(n) + 3x(n-1)$$

$$y(n) = 0.8\, y(n-1) + 2x(n) + 3x(n-1)$$

Characteristic equation is

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = c(0.8)^n$$

Let us first consider the response of the system.

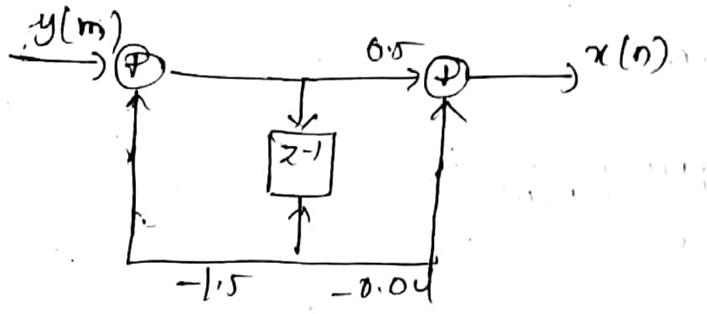
$y(n) - 0.8y(n-1) = x(n)$

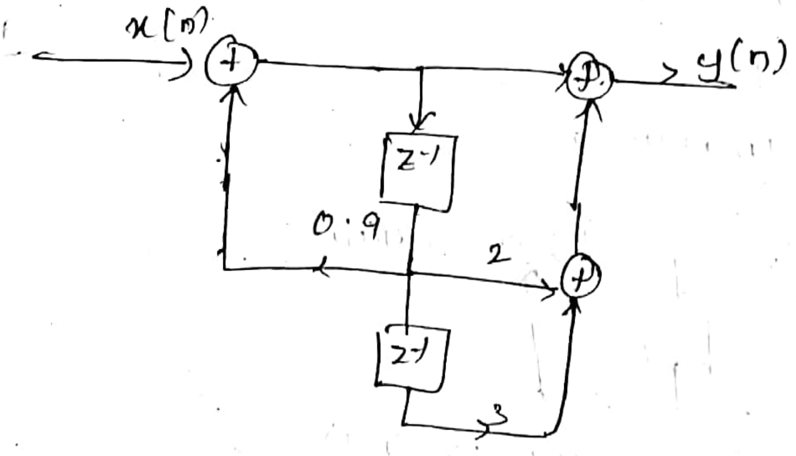
to $x(n) = \delta(n)$, since $y(0) = 1$, it follows that $C = 1$, then impulse response of the signal s/m is $h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$

(b) The inverse system is characterized by the diff eqn.

$x(n) = -1.5x(n-1) + \frac{1}{2} y(n) - 0.4y(n-1)$





(c) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

for $x(n) = \delta(n)$, we have

$y(0) = 1$

$y(1) = 2.9$

$y(2) = 5.61$

$y(3) = 5.049$

$y(4) = 4.514$

$y(5) = 4.090$

(B) Zero state response.

$s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

(C) $h(n) = (0.9)^n u(n) + 7(0.9)^{n-1} + 3(0.9)^{n-2} u(n-2)_4$

2.118:-

(a) $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$ we have.

$h(n) = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, 2/3, 2/3, 2/3 \ldots\}$

(b) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$ we obtain

$h(n) = \{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256} \ldots\}$

(c) $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774 \ldots\}$

(d) All three s/m are IIR

(e) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

The chorocteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$

$\lambda = 0.8, 0.6$ so

$y_h(n) = C_1 (0.8)^n + C_2 (0.6)^n$ for $x(n) = \delta(n)$, we have,

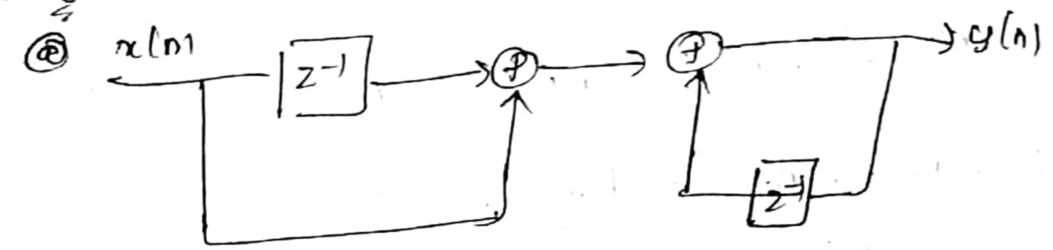
$C_1 + C_2 = 1$ & $0.8C_1 + 0.6C_2 = 1.4$

$C_1 = 4, \quad C_2 = -3$

$h[n] = [4(0.8)^n - 3(0.6)^n]\, u(n)$

2.(4):-

(a)



$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

for $y(n) - \frac{1}{2} y(n-1) = \delta(n)$, The soln is

$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

(b) $h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

2.(5):-

(a) $x_1(n) = \{1, 2, 4\}$, $h_1(n) = \{1, 1, 1, 1, 1\}$

Convolution $= \sum_{k=0}^{n} x_1(n) \cdot h(n-k)$

$y_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$

Correlation $= r_1(n) = \{1, 3, 7, 7, 6, 4\}$

(b) $x_2(n) = \{0, 1, -2, 3, -4\}$, $h_2(n) = \{1/2, 1, 2, 1, 1/2\}$

Correlation $= \{1/2, 0, 3/2, -2, 1/2, -6, 3/2, -3/4 \ldots\}$

Correlation $r_2(n) = \{-\frac{1}{2}, 0, 3/1, -2, 1/2, -6, -5/2, -2\}$

note: Convolution Correlation $[\because h_2(-n) = h(n)]$

Convolution $y_3(n) = \{c_1, 11, 20, 30, 20, 11, 4\}$

Correlation $r_1(n) = \{1, 4, 10, 20, 25, 24, 16\}$

(c) Convolution $y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

Correlation $r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$

and $h_4(-n) = h_3(n+3)$

hence $r_4(n) = y_3(n+3)$

2.52:- $x(n) = \{1, 3, 3, 1\}$

Zero state response of $x(n)$ is $y(n) = [1, 4, 6, 4, 1]$

Impulse response.

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$h_0 = 1, h_1 = 1$

2.54:-

$y(n) = ?$ for $n \geq 0$

given 2nd order D.eqn $\Rightarrow y(n) - 4y(n-1) + 4y(n-2)$

$= x(n) - x(n-1)$

i/p $= x(n) = (-1)^n u(n)$

and initial Conditions $y(-1) = y(-2) = 0$

Homogeneous eqn

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$\lambda = 2, 2.$$

$$y_h(n) = C_1 2^n + C_2 2^n$$

$$y_p(n) = y_p\ k(-1)^n u(n)$$

Substituting $y_p(n)$ into eq, we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for $n = 2$,

$$k + 4k + 4k = 1 + 1$$

$$k(1 + 4 + 4) = 2$$

$$k = 2/9$$

$$y(n) = \left[C_1 2^n + C_2 n 2^n\right) \frac{2}{9}(-1)^n\Big] u(n)$$

from intial conditions, we obtain

$y(0) = 1$, $y(1) = 2$

$C_1 + \frac{2}{9} = 1$, $C_1 = 7/9$

$2C_1 = 2C_2 - 2/9 = 2 \Rightarrow C_2 = 1/3$

So,

$$y[n] = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

2.55:

differential equations

$$y(n) - 4y(n-1) + 4(y(n-2)) = x(n) - x(n-1);$$

$$x(n) = \text{impulse}$$

from previous problem

$$h[n] = [c_1 2^n + c_2 n 2^n]\, u(n)$$

with $y(0) = 1$

$y(1) = 3$, we have

$$\because\ y(0) - 4\cancel{y(-1)}^{0} + 4\cancel{y(-2)}^{0} = x(0) - \cancel{x(-1)}^{0} \quad (\text{for impulse})$$

$$y(0) = 1 - 0$$

$$n=1;\ y(1) = 4y(0) - 4\cancel{y(-1)}^{0} + \cancel{x(1)}^{0} - x(0)$$

$$= 4 - 1 = 3$$

$$c_1 = 1$$

$$2c_1 + 2c_2 = 3$$

$$c_2 = 1/2$$

Thus $h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

2.56:

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

2.57: Let $h(n)$ be the impulse response of system

$$\delta(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = \delta(k) - \delta(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\delta(k) - \delta(k-1) \right) x(n-k)$$

2.58:

$$x(n) = \begin{cases} 1 & , n_0 - N \le n \le n_0 + N \\ 0 & , \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1 & , -N \le n \le N \\ 0 & , \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{k=-\infty}^{\infty} x(n) \cdot x(n-l)$$

The range of non zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N$$

which implies

$$-2N \le l \le 2N$$

for given shift l, the no. of terms in the summation for which both $x(n)$ and $x(n-l)$ are non zero is $2N+1-|l|$ and this value of each term is 1. Hence.

$$\gamma_{xx}(l) = \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

for $\gamma_{xy}(l)$, we have

$$\gamma_{xy}(l) = \begin{cases} 2N+1-|l-n_0| & , n_0-2N \le l \le n_0+2N \\ 0 & , \text{otherwise} \end{cases}$$

2.57 $$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$\gamma_{xx}(-3) = x(0)\, x(3) = 1$

$\gamma_{xx}(-2) = x(0)\, x(2) + x(1)\, x(3) = 3$

$\gamma_{xx}(-1) = x(0)\, x(1) + x(1)\, x(2) + x(2)\, x(3) = 5$

$\gamma_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

and $\gamma_{xx}(l) = \gamma_{xx}(-l)$, so,

$\gamma_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

(b) $$\gamma_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$$

we obtain

$\gamma_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$.

we observe that,

$y(n) = x(-n+3)$, which is equivalent ~~to~~ reversing the sequence $x(n)$. This has not changed auto correlation sequence.

2.60:

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \begin{cases} 2N+1-|l| , & -2N \le l \le 2N \\ 0 , & \text{otherwise} \end{cases}$$

$$\gamma_{xx}(0) = 2N+1$$

$\therefore$ Normalized autocorrelation is

$$\rho_{xx}(l) = \frac{1}{2N+1}\left[2N+1-|l|\right] , \quad -2N \le l \le 2N$$

$$= 0 , \text{ otherwise.}$$

2.61:

$$x(n) = \delta(n) + r_1 \delta(n-k_1) + r_2\, \delta(n-k_2)$$

k_1, k_2 are always

r_1, r_2 are reflection coefficients.

(a) $\gamma_{xx}(l) = ?$

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) . x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} \left[\delta(n) + r_1 \delta(n-k_1) + r_2\, \delta(n-k_2)\right] *$$

$$\left[\delta(n-l) + r_1 \delta(n-l-k_1) + r_2 \delta(n-l-k_2)\right]$$

$$= (1+r_1^2+r_2^2)\, \gamma_{xx}(l) + r_1 \left[\gamma_{xx}(l+k_1) + \gamma_{xx}(l-k_1)\right]$$

$$+ r_2 \left[\gamma_{xx}(l+k_2) + \gamma_{xx}(l-k_2)\right]$$

$$+ r_1 r_2 \left[\gamma_{xx}(l+k_1-k_2) + \gamma_{xx}(l+k_2-k_1)\right]$$

(b) $\gamma_{xx}(\ell)$ has peaks at $\ell = 0, \pm k_1, \pm k_2$ and $\pm(k_1 + k_2)$. Suppose $k_1 < k_2$, then we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

c) If $r_2 = 0$, the peaks occur at $\ell = 0$, $\ell = \pm k_1$. Then it is easy to obtain r_1 and k_1

2.64: