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Assignment 05

QUESTIONS:

1. Consider a symmetric game where each player must decide whether to hunt or forage for food. If both players decide to hunt, they successfully capture an animal for meat, and they each get a payoff of 10. If one player hunts and one for ages, the hunter will be injured by the prey and get payoff 0, while the forager will gather some food and get payoff 5. If both players forage, they will help each other find food faster and each will get payoff 7.
a. Draw the matrix representation of this game.
b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.
c. Identify any pure strategy Nash equilibria. Explain your answer.

Answer:

(a) Matrix Representation of the given game:

		Player 2	
		Hunt	Forage
Player 1	Hunt	10, 10	0, 5
	Forage	5, 0	7, 7

(b) No, none of the player has a strictly dominant strategy because if player 2 chooses to hunt, player 1 will get better payoff if he also chooses to hunt and if player 2 chooses to forage, player 1 will get better payoff if he chooses to forage and vice versa. In either case, there is no strictly dominant strategy for any players to choose.

(c) Two Nash Equilibria (hunt, hunt) and (forage, forage) because these strategies are best response to each and other as explained below.

Player 1:
Hunt is the best response to Hunt by Player 2
Forage is the best response to Forage by Player 2
Player 2:
Hunt is the best response to Hunt by Player 1
Forage is the best response to Forage by Player 1

2.You (player 1) are browsing the WWW on a computer and things are frustratingly slow. A popup appears that promises increased speed if you click on it. It will manage this by using an alternative protocol to standard TCP. A classmate (player 2) is having the same experience and has seen the same popup. If you both stay with TCP, you will each experience a 1 ms delay (a payoff of -1). If one player stays with TCP and the other goes with the alternative, the TCP player will experience a 4 ms delay and the alternative player will experience zero delay. If you both switch to the alternative protocol, you will each experience a 3 ms delay.
a. Draw the matrix representation of this game.
b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.
c. Identify any pure strategy Nash equilibria. Explain your answer.

Answer:

(a) Matrix Representation of the given game:

		Player 2	
		TCP	Alternative
Player 1	TCP	-1, -1	-4, 0
	Alternative	0, -4	-3, -3

(b) Both players have strictly dominant strategy because irrespective of player 2's strategy, player 1 should go with alternative to get better payoff. Same as player 2 choose alternative to get better payoff regardless of player 1's strategies.

(c) There are two Nash Equilibria (Alternative, TCP) and (Alternative, Alternative) because they are best response to each and other.

Player 1:
Alternative is the best response to TCP by Player 2
Alternative is the best response to Alternative by Player 2

Player 2:
Alternative is the best response to TCP by Player 1
Alternative is the best response to Alternative by Player 1

3.Two drivers are playing chicken, the game where they drive straight toward each other and see if one or both will swerve to avoid a crash. If both drivers swerve, the game is a tie and each gets payoff 0. If one swerves and one stays straight, the driver who swerves is declared a "chicken" and gets payoff -5, while the driver who stays straight wins and gets payoff +5. If both drivers continue straight, they will crash and both get payoff -50.
a. Draw the matrix representation of this game.
b. Does either (or both) player(s)have a strictly dominant strategy? Explain your answer.
c. Identify any pure strategy Nash equilibria. Explain your answer.

Answer:

(a) Matrix Representation of the given game:

		Player 2	
		Swerve	Stay Straight
Player 1	Swerve	0, 0	-5, 5
	Stay Straight	5, -5	-50, -50

(b) No, none of the player has a strictly dominant strategy because if player 2 chooses to stay straight, player 1 will get better payoff if he chooses to swerve and if player 2 chooses to swerve, player 1 will get better payoff if he chooses to stay straight and vice versa. In either case, there is no strictly dominant strategy for any players to choose.

(c) Two Nash Equilibria (Swerve, Stay Straight) and (Stay Straight, Swerve) because of the below reason:

Player 1:
Stay Straight is the best response to Swerve by Player 2
Swerve is the best response to Stay Straight by Player 2

Player 2:
Stay Straight is the best response to Swerve by Player 1
Swerve is the best response to Stay Straight by Player 1

4. Draw the matrix representation of the game Rock Paper Scissors Lizard Spock. Use 0 for tie, 1 for win, -1 for lose. Note: Rock Paper Scissors Lizard Spock (from the Big Bang Theory show): As Sheldon explains: "Scissors cuts paper, paper covers rock, rock crushes lizard, lizard poisons Spock, Spock smashes scissors, scissors decapitates lizard, lizard eats paper, paper disproves Spock, Spock vaporizes rock, and as it always has, rock crushes scissors."

Answer:

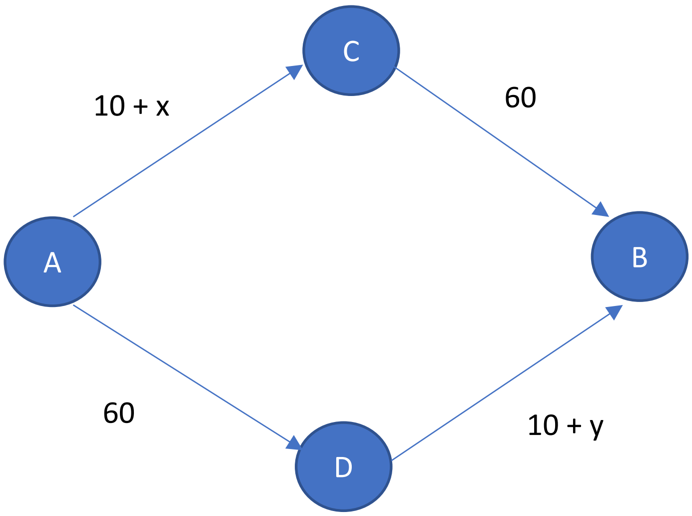
(a) Matrix Representation of the game Rock Paper Scissors Lizard Spock:

	Rock	**Paper**	**Scissors**	**Lizard**	**Spock**
Rock	0,0	-1, 1	1, -1	1, -1	-1, 1
Paper	1, -1	0, 0	-1, 1	-1, 1	1, -1
Scissors	-1, 1	1, -1	0, 0	1, -1	-1, 1
Lizard	-1, 1	1, -1	-1, 1	0, 0	1, -1
Spock	1, -1	-1, 1	1, -1	-1, 1	0, 0

5.There are 80 cars which begin in city A and must travel to city B. There are two routes between city A and city B:

Route I: begins with a local street leaving city A, which requires a travel time in minutes equal to 10 plus the number of cars which use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of cars which use this highway.
Route II: begins with a highway leaving city A. This highway takes one hour of travel time regardless of how many cars use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 10 plus the number of cars which use the street.

a. Draw the network described above and label the edges with the travel time in minutes needed to move along the edge, in terms of x. Let x be the number of travelers who use Route I. The network should be a directed graph as all roads are one-way.



In above figure,

b. What would be the travel time per car if all cars chose to use Route I?

If all cars chose to use Route I, the travel time would be T= 10 + x + 60 = 10 + 80 + 60 = **150 mins**

c. Assume that cars simultaneously chose which route to use. Find the Nash equilibrium value of x.

Nash equilibrium value of **x = 40** and total travelling time would be 10 + 40 + 60 = 110 mins

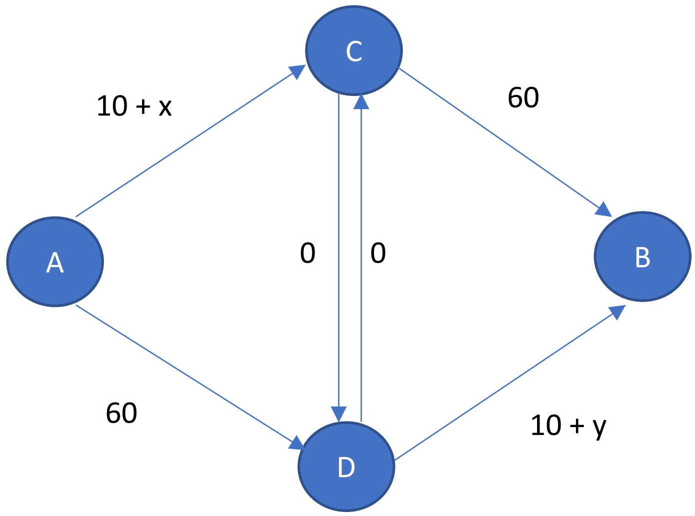
d. Explain your answer to part c.

Here, the only Nash equilibrium is equal balance of cars i.e. 40 cars on each route I and route II because if there was an uneven balance of cars, it will lead to more congested route to switch.

Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. The new road is very short and takes no travel time. This adds two new routes.

Route III: consists of the local street leaving city A (on Route I), the new road, and the local street into city B (on Route II).

Route IV: consists of the highway leaving city A (on Route II), the new road, and the highway leading into city B (on Route I).



e. What would the travel time be per car if all cars chose Route III?

If all cars chose Route III, the travel time would be T= 10 + x + 0 + 10 + y = 20 + 80 + 80 = **180 mins**
Here, y = 80 because all 80 cars going through D to B also.

f. What would the travel time be per car if all cars chose Route IV?

If all cars chose Route III, the travel time would be T= 60 + 0 + 60 = **120 mins**

g. What happens to total travel time as a result of the availability of the new road?

We can see from above calculations (b, e and f), total travel time is increasing if all cars choose Route III, while total travel of all cars is reducing if they choose route IV

h. If you can assign travelers to routes, it's possible to reduce total travel time relative to what it was before the new road was built. That is, the total travel time of the 80 cars can be reduced (below that in the original Nash equilibrium from part c) by assigning cars to routes. There are many assignments of routes that will accomplish this. Find one. Explain why your reassignment reduces total travel time by giving the number of cars assigned to each of the Routes I, II, III, IV, the travel time per car on each route, and the total travel time of the 80 cars.

Answer:

I will star with assigning 20 cars on Route I, 40 cars on Route IV and 20 cars on Route II. This assignment of total 80 cars will lead to total travelling time = 10 + 20 + 60 = 90 mins.

6.Consider an auction in which there is one seller who wants to sell one unit of a good and a group of bidders who are each interested in purchasing the good. The seller will run a sealed-bid, second-price auction. There will be around a dozen other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm's value for the good is c.
a. What bid should your firm submit? Explain your answer.
b. How does your bid depend on the number of other bidders who show up?

Answer:

(a) My firm should submit the bid of value **c** because we know as per second price auction, if true value for the item is v and my bid is the winner with also value v and b is the bid value at second place, then the payoff is v – b. So, I can make a positive payoff always if I win. In the same way, here my firm's true value for the good is c, my strategy would be c as my bid value.

(b) My bid doesn't depend on the number of other bidders who show up. I can have my independent bid value irrespective of total number of bidders.

7.A seller wants to sell one unit of a good to some bidders, by running a sealed-bid second-price auction. Assume that there are two bidders who have independent, private values which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. (If there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.)
a. Show that the seller's expected revenue is 6/4.
b. Suppose that there are three bidders who have independent, private values which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. What is the seller's expected revenue in this case? Explain your answer.
c. Briefly explain why changing the number of bidders affects the seller's expected revenue.

Answer:

(a) Here, we have given that there are two bidders, who have independent private values of 1 or 3 with probability of ½. So, we can have four total combinations of values for two bidders as show below.

Moreover, seller is running a sealed-bid second-price auction. So, selling price would be second highest bid's value and in case of tie, the selling price would be the highest common value, that winner has to pay.

Bidder 1	**Bidder 2**	**Selling Price**
1	1	1
1	3	1
3	1	1
3	3	3

From above table, we can say that total expected revenue = (1 + 1 + 1 + 3) / 4 = **6/4 = 1.5**

(b) Here, the situation is same as (a) but instead of two bidders, now we have three bidders. So, total combinations of values that each bidder can have is 8 as specified in below table with respective combination's selling price.

Bidder 1	**Bidder 2**	**Bidder 3**	**Selling Price**
1	1	1	1
1	1	3	1
1	3	1	1
1	3	3	3
3	1	1	1
3	1	3	3
3	3	1	3
3	3	3	3

So, revenue that seller would expect is (1 + 1 + 1 + 3 + 1 + 3 + 3 + 3) / 8 = **16/8 = 2**

(C) We can see from (a) and (b) part, as number of bidders are increasing, seller's expected revenue is also increasing because here private values are only two and as number of bidders increases, we can have more combination values, which will increase total selling price as well.