Image Denoising with global structure and local similarity preservations

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Overview

Introduction

- Implement denoising method used in [2] to remove additive and multiplicative noise from images
- Preserve local and global structure of image using pixel-level and patch-based filtering methods
- Perform dictionary learning based on the MOD-AK-SVD (Method of Optimal Directions - Approximate K - Singular Value Decomposition) approach

Introduction (contd.)

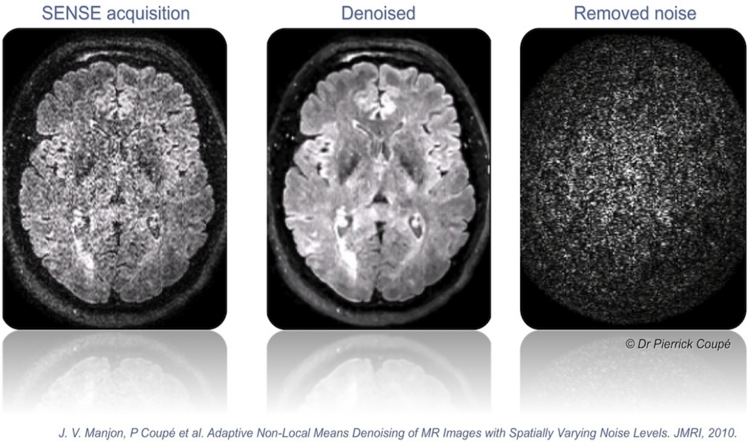

Figure: Example

Formulation

Image denoising may be regarded as a two-stage optimization problem. Our image denoising problem has been formulated as:

$$\min \quad G(X,S) + \mu H(X,\Phi,\Omega) \tag{1}$$

$$Y = X + S + E \tag{2}$$

X = Clean image, S = Matrix containing globally sparse noise,
E = Remaining noise

Formulation (contd.)

$$G(X,S) = ||Y - X - S||_F^2 + \lambda_1 ||LX||_{S_p}^p + \lambda_2 ||S||_1 \quad (3)$$

L = Laplacian operator (high pass filter), λ_1, λ_2 = penalty parameters, $||.||_{S_p}$ = Schatten p-norm

$$H(X,\Phi,\Omega) = \sum_{i=1}^{N} ||R_i X - \Phi\omega_i||_2^2 \quad s.t. \quad ||\omega_i||_0 \leq T \quad (4)$$

Ω = sparse coefficient matrix, Φ = dictionary, R_i extracts the i^{th} patch from X such that column vector $x_i = R_i X$, T = parameter that controls sparsity of representation

Approach

Global Structure Reconstruction Stage

Gradient descent is used to reconstruct the global features of the image while denoising

Dictionary Learning Stage

Proposed method MOD-AK-SVD [3] is used to preserve local structure of the image (patch-wise)

Global Structure Reconstruction Stage

$$\tilde{G} = G + \mu ||X - X_2^t||_F^2 \tag{5}$$

- Pixel-level filtering
- High pass filter using Laplacian operator to obtain filtered image containing global structures of image
- Objective function convex- gradient descent method used
- Alternating strategy to update X and S

Dictionary Learning Stage

$$\tilde{H} = H + \frac{1}{\mu}||X - X_1^{t+1}||_F^2 \quad s.t.||\omega_i||_0 \leq T \tag{6}$$

- Patch-based filtering
- Non-convex part
- Alternating optimization strategy for three variables: X, Φ, Ω
- Orthogonal Matching Pursuit (OMP) is used to deal with l_0 minimization and MOD-AK-SVD [3] is used for updating the dictionary

Algorithm

Algorithm 1 The Laplacian Schatten p-norm and Learning Algorithm (LSLA-p).

Require: Noisy Image: Y;
Penalty parameter: $\lambda_1, \lambda_2, \mu$;
Smoothing parameter: δ;
Stopping tolerence: ϵ_1, ϵ_2;
Clearn Image X;

1: Initialize $\Phi, X_2^0 = 0, S = 0$
2: $t = 0$; a = 1; $j = 1; \Delta_1 = \epsilon_1 + 1; \Delta_2 = \epsilon_2 + 1$

Figure: Initalization

Algorithm (contd.)

repeat

$s = 0$

while $\Delta_2 \geq \epsilon_2$ & $s \leq s_{\max}$ **do**

$S_t^{s+1} \leftarrow \arg\min_S \|Y - X_t^s - S\|_F^2 + \lambda_2 \|S\|_1$ (17)

$X_t^{s+1} \leftarrow \arg\min_X \|Y - X - S_t^{s+1}\|_F^2 + \lambda_1 Tr[(\mathbb{L}X)^T(\mathbb{L}X) + \delta^2 I]^{p/2} + \mu \|X - X_2^t\|_F^2$ (18)

$\Delta_2 = \min\left\{ \|X_t^{s+1} - X_t^s\|_F^2, \|S_t^{s+1} - S_t^s\|_F^2 \right\}$

$s = s + 1$

end while

Figure: Update X and S

Algorithm (contd.)

$X_1^{t+1} = X_t^{(s)}$

$X_2^{t+1} \leftarrow \arg\min_X \frac{1}{\mu}\|X - X_1^{t+1}\|_F^2 + \sum_{i=1}^{N} \|R_i X - \Phi\omega_i\|_2^2 \quad s.t. \|\omega_i\|_0 \leq T$ (21)

$\Delta_1 = \min\left\{\|X_1^{t+1} - X_1^t\|_F^2, \|X_2^{t+1} - X_2^t\|_F^2\right\}$

$t = t + 1$

until $\Delta_1 \leq \epsilon_1$ or $t \geq t_{\max}$

return $X = X_2^t$

Sparse Coding Stage: $\Omega = \text{OMP}(X, \Phi, k_0)$

Update Dictionary Φ

Figure: Update X and Ω

Algorithm (contd.)

Update Dictionary Φ

repeat

$X = \left(\frac{1}{\mu} I + \sum_{i=1}^{N} \mathbf{R}_i^T \mathbf{R}_i\right)^{-1} \left(\frac{1}{\mu} X_1^t + \sum_{i=1}^{N} \mathbf{R}_i^T \Phi \omega_i\right)$ (27)

$E = X - \Phi\Omega$

repeat

$E_j = E + \varphi_j \omega_{\bar{j}}$

$\varphi_j = E_j(:, t_j)\omega_{\bar{j}}^T(t_j)$ **where** $t_j = \{i : \omega_{\bar{j}}(i) \neq 0\}$

$\omega_{\bar{j}}(t_j) = \varphi_j^T E_j(:, t_j)$

$E = E_j - \varphi_j \omega_{\bar{j}}$

j++

until j = K

a++

until a = A

Figure: Dictionary update Φ

Critical Observations

Initialization of X unclear

Initialize X with noisy image Y

Patch extraction operator R_i ambiguous

Coded a general function to extract patches of size $\sqrt{n}x\sqrt{n}$ based on other sources

Values of s_{max} and t_{max} not mentioned

Tweaked s_{max} and t_{max} to balance out time taken and convergence of algorithm

Parameter Tuning

- Six levels of λ_1 tested: 20, 10, 5, 1, 0.1, and 0.01.
- Best output obtained for $\lambda_1 = 0.1$

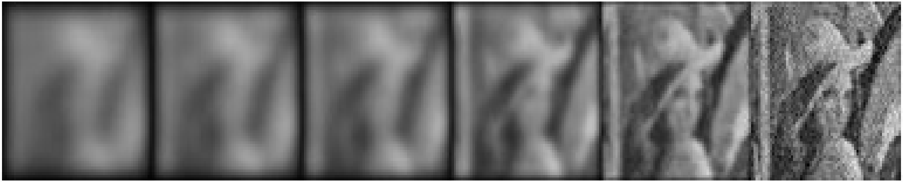

Figure: Output images for different levels of λ_1

Parameter Tuning (contd.)

- Six levels of λ_2 tested: 10, 1, 0.1, 0.01, 0.05, and 0.001.
- No change in output

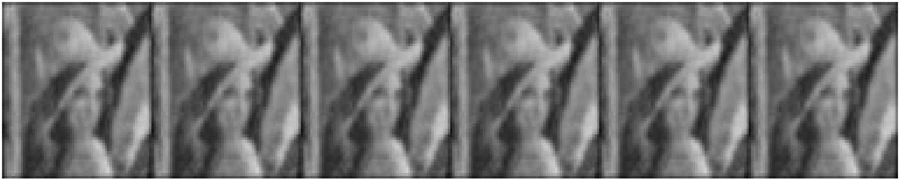

Figure: Output images for different levels of λ_2

Results

Figure: For different noise variance levels : $\{0.001, 0.005, 0.01, 0.02, 0.05\}$

Results (contd.)

PSNR - Peak Signal To Noise Ratio

Noise Variance	0.001	0.005	0.01	0.02	0.05
PSNR - noisy image	-18.21	-25.13	-28.21	-30.95	-34.49
PSNR - denoised image	-26.25	-26.52	-29.227	-27.22	-29.56

SSIM - Structural Similarity

Noise Variance	0.001	0.005	0.01	0.02	0.05
SSIM - noisy image	0.7765	0.5733	0.4621	0.3596	0.2359
SSIM - denoised image	0.6989	0.651	0.5906	0.5589	0.4864

References

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[2] Shuting Cai, Zhao Kang, Ming Yang, Xiaoming Xiong, Chong Peng, and Mingqing Xiao. **Image-denoising via improved dictionary learning with global structure and local similarity preservations**.Symmetry, 10(5):167, 2018.

[3] Shuting Cai, Shaojia Weng, Binling Luo, Daolin Hu, Simin Yu, and Shuqiong Xu. **A dictionary-learning algorithm based on method of optimal directions and approximate k-svd.** InControl Con-ference (CCC), 2016 35th Chinese, pages 69576961. IEEE, 2016.

Thank You