Denoising Images - Optimization for Big Data Project

Gopi Manthena 0030191751, Karthik Sajeev - 0030028666 , Mihir Bhatia - 0029923247

1 Introduction

Images are widely used in various fields, and are usually contaminated by noise during acquisition, transmission and compression. Consequently, real-life images are often degraded with noise and there is often a need for image denoising techniques. Although several techniques have been discussed in the literature, most tend to work well only for specific types of noise removal problems.

In our project, we implement the image denoising method from [2], which is regarded as a highly underconstrained problem. This method removes both additive and multiplicative noise from the image. The method would also allow us to preserve both the global and local structure of the image. This is done by combining reconstruction and learning based approaches using pixel-level and patch-based filtering methods. To capture global structure of the image, Laplacian Schatten p-norm is used. To reduce the multiplicative noise approximately to additive noise, a sparse term is introduced.

Moreover, a new approach to solving the optimization problem of dictionary learning called MOD-AK-SVD (Method of Optimal Directions - Approximate K - SVD) is proposed which is a combination of K-SVD [1] and MOD

2 Formulation

Image denoising may be regarded as a two-stage optimization problem, which splits the objective function into convex and non-convex parts. The former is solved using alternating optimization and the gradient descent method, while the latter uses the proposed dictionary learning MOD-AK-SVD approach. Our image denoising problem been formulated as:

$$\min \quad G(X,S) + \mu H(X,\Phi,\Omega) \tag{1}$$

G is the function that helps reconstruct global structure, H is the function that preserve local structure, μ is a penalty parameter

In the global function G we represent our noisy image Y as:

$$Y = X + S + E \tag{2}$$

X = Clean image, S = Matrix containing globally sparse noise, E = Remaining noise

$$G(X,S) = ||Y - X - S||_F^2 + \lambda_1 ||LX||_{S_p}^p + \lambda_2 ||S||_1 \tag{3}$$

L = Laplacian operator (high pass filter), λ_1, λ_2 = penalty parameters, $||.||_{S_p}$ = Schatten p-norm

$$H(X,\Phi,\Omega) = \sum_{i=1}^{N} ||R_i X - \Phi\omega_i||_2^2 \quad s.t. \quad ||\omega_i||_0 \leq T \tag{4}$$

Ω = sparse coefficient matrix, Φ = dictionary, R_i extracts the i^{th} path from X such that column vector $x_i = R_i X$, $\omega_i = i^{th}$ column of Ω, T = parameter that controls sparsity of representation

We minimize the two functions G and H separately using two different methods (described in section 3.1 and 3.2)

3 Possible Approaches

From [2] we use the following approaches to solve the formulated optimization problem

- **Global Structure Reconstruction Stage-** where gradient descent is used to reconstruct the global features of the image while denoising
- **Dictionary Learning Stage -** where the newly proposed MOD-AK-SVD is used to preserve local structure

3.1 Global Structure Reconstruction Stage

This method pertains to $G(X, S)$ in the formulation above, and can be considered as a pixel-level filtering technique. As the objective function is convex, the gradient descent method is used for optimization. An alternating optimization strategy is used for X and S, where one is fixed and the other updated until convergence is attained.

$$S_t^{s+1} \leftarrow \arg\min_S \quad ||Y - X_t^{(s)} - S||_F^2 + \lambda_2 ||S||_1 \tag{5}$$

$$X_t^{s+1} \leftarrow \arg\min_X ||Y - X - S_t^{(s+1)}||_F^2 + \lambda_1 \mathbf{Tr}[(LX)^T(LX) + \delta^2 I]^{p/2} + \mu ||X - X_2^t||_F^2 \tag{6}$$

A smoothing parameter δ is used to smooth the Laplacian Schatten norm term. X_1^t and X_2^t are obtained at the t^{th} step my minimizing the modified G and H functions.

3.2 Dictionary Learning Stage

In the dictionary learning stage, we attempt to minimize $H(X, \Phi, \Omega)$. This is non-convex part of our minimization problem. Here we use a patch based filtering technique. For this stage too, we use a alternating optimization strategy where we fix the value of 2 variables and update the 3^{rd} one. Moreover, since the l_0 minimization is an NP-Hard problem methods such as basis pursuit (BP), matching pursuit (MP), orthogonal matching pursuit (OMP) and focal underdetermined system solver(FOCUSS) can be used. In our project we use OMP which greedily selects dictionary atoms sequentially and is faster than MPA.

In [2], the above mentioned techniques are combined into an algorithm called Laplacian Schatten p-norm and Learning Algorithm (LSLA-p). There are 2 implementations of LSLA depending on the value of p which gives us LSLA-1 and LSLA-2 for $p = 1$ and $p = 2$ respectively.

References

[1] Michal Aharon, Michael Elad, Alfred Bruckstein, et al. K-svd: An algorithm for designing overcomplete dictionaries for sparse representation. *IEEE Transactions on signal processing*, 54(11):4311, 2006.

[2] Shuting Cai, Zhao Kang, Ming Yang, Xiaoming Xiong, Chong Peng, and Mingqing Xiao. Image denoising via improved dictionary learning with global structure and local similarity preservations. *Symmetry*, 10(5):167, 2018.