Homework 1

Started: Feb 8 at 2pm

Quiz Instructions

You may want to refer to the material:

https://github.com/wangshusen/CS583A-2019Spring/blob/master/Reading/MatrixCalculus.pdf

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Question 1 — 3 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $\|\mathbf{x}\|_2^2$ (i.e., the squared ℓ_2-norm of $\mathbf{x}$)?

Question 2 — 3 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $\|\mathbf{x}\|_1$ (i.e., the squared ℓ_1-norm of $\mathbf{x}$)?

Question 3 — 3 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $\|\mathbf{x}\|_{0.5}$ (i.e., the squared ℓ_p-norm of $\mathbf{x}$ with $p = 0.5$)?

Question 4

3 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ and $\mathbf{a} = [0, 9, -3, -2, 1]$ be 5-dimensional vectors.

What is the inner product $\mathbf{a}^T\mathbf{x}$?

Question 5

3 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $\|\mathbf{x}\|_\infty$ (i.e., the squared ℓ_∞-norm of $\mathbf{x}$)?

Question 6

3 pts

Define the matrix $\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix}$.

The matrix $\mathbf{A}^T\mathbf{A}$ is symmetric.

- True

False

Question 7

3 pts

Define the matrix $\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix}$.

The matrix $\mathbf{A}$ is symmetric.

True

False

Question 8

3 pts

Define the following matrix and vector:

$\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$.

What is the first entry of the vector $\mathbf{Ab}$?

Question 9

3 pts

Define the matrix $\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix}$.

What is $\|\mathbf{A}\|_F^2$ (i.e., the squared Frobenius norm of $\mathbf{A}$)?

Question 10

3 pts

Define the matrix $\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix}$.

What is $tr(\mathbf{A}^T\mathbf{A})$(i.e., the trace of$\mathbf{A}^T\mathbf{A}$)?

Question 11

12 pts

Let $\mathbf{x} = [x_1, x_2, x_3]$ and $y = \frac{x_1^2}{2} + \log_e(x_2) - \frac{x_1}{x_3}$.

Question: What is the value of $\frac{\partial y}{\partial \mathbf{x}}$ at $\mathbf{x} = \left[9, 1, \frac{1}{2}\right]$?

Answer: It is the vector [, ,].

Hint: The value of $\frac{\partial \log_e(z)}{\partial z}$ at $z = 1$ is $\left.\frac{\partial \log_e(z)}{\partial z}\right|_{z=1} = \left.\frac{1}{z}\right|_{z=1} = 1.$

Question 12

8 pts

A dataset has 100 positive samples and 100 negative samples. Furthermore,

#True Positive = 63,

#False Negative = 37,

#True Negative = 72,

#False Positive = 28.

What is the **False Positive Rate**?

Question 13

8 pts

A dataset has 100 positive samples and 100 negative samples. Furthermore,

#True Positive = 24,

#False Negative = 76,

#True Negative = 12,

#False Positive = 88.

What is the **True Positive Rate**?

Question 14

5 pts

Let x be a scalar variable and $f(x) = 3x^2 + 36x - 8$. Let $x^\star$ be the optimal solution to the problem $\min_x f(x)$.

What is the value of $x^\star$? (Hint: f is a convex function; use the first-order optimality condition.)

Question 15

5 pts

Let x be a scalar variable and $f(x) = -2x + 10$.

What is the value of $\min_x f(x) \quad s.t. \ -2 \leq x \leq 5$?

Question 16

32 pts

Are all of the following statements true?

- On Jan 31, the class will be canceled.
- On Jan 31 and Feb 28, the classes and office hours will be canceled.
- There is a quiz on Feb 28.

(Note: This question is a bonus for paying attention to the schedule.)

- True
- False

Not saved Submit Quiz