

A hazards approach to the biometric analysis of infant mortality

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Outline

- Bourgeois-Pichat (1951): Biometric analysis of IM (background).
- Hazard-based alternative.
- Theoretical considerations
- Real-world examples.

Bourgeois-Pichat and causes of death

- Two categories:
 - **Endogenous**: *inherited, delivery, etc.*
 - **Exogenous**: *accidental, infectious diseases, etc.*
- How to differentiate between the two categories without information of causes of death?

Bourgeois-Pichat's **biometric model**.

The biometric model

Two postulates:

- 1 **Endogenous** deaths only occur during the **neonatal** period (0–28 days).
- 2 On a specific time scale, **exogenous** infant mortality is **uniformly** distributed.

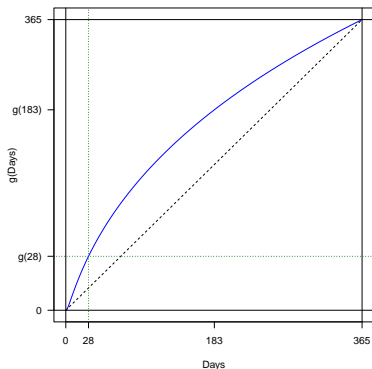
The log-cube transform

$$g(t) = C \log^3(t + 1), \quad 0 < t \leq 365.$$

where C is a normalizing constant:

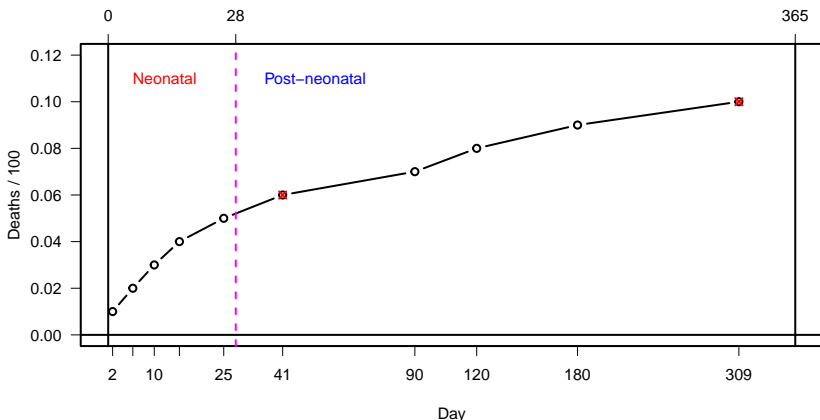
$$C = \frac{365}{\log^3(365 + 1)}$$

Note: C is **not** part of B-P's original definition.



Demonstration of the B-P plot

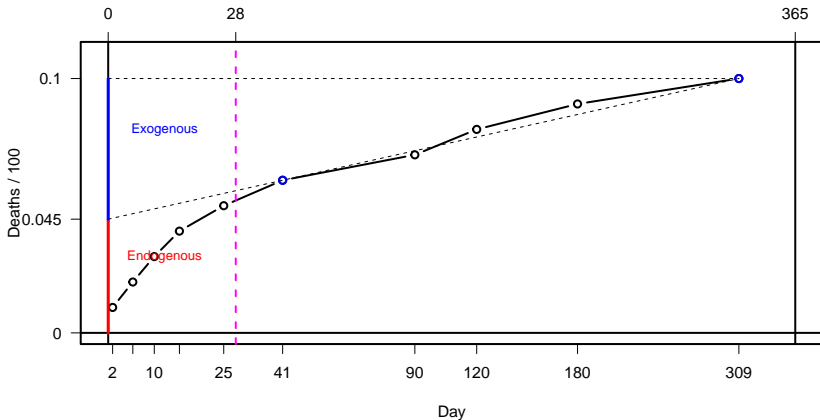
Assume 100 births, of which 90 survives infancy, ten death ages:
2, 6, 10, 15, 25, 41, 90, 120, 180, 309.



Calculation (no ties): $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \dots, \frac{10}{100}$.

Demonstration of the B-P plot II

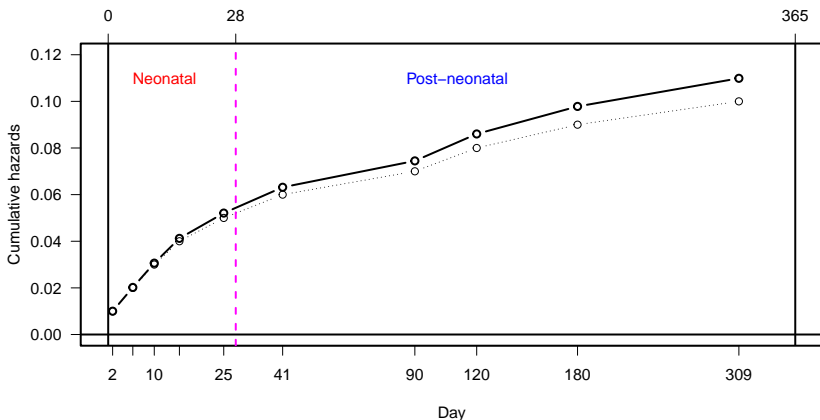
Assume 100 births, of which 90 survives infancy, ten death ages:
2, 6, 10, 15, 25, 41, 90, 120, 180, 309.



Calculation (no ties): $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \dots, \frac{10}{100}$.

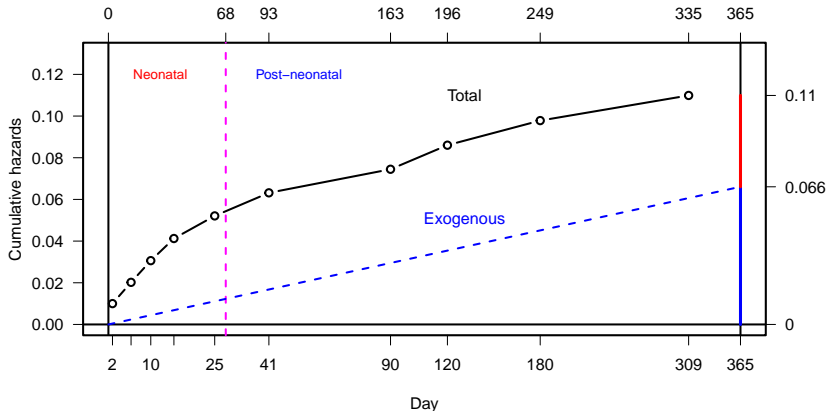
The hazards plot

Assume 100 births, of which 90 survives infancy, ten death ages:
2, 6, 10, 15, 25, 41, 90, 120, 180, 309 (same as before).



Calculation (no ties): $\frac{1}{100}, \frac{2}{99}, \frac{3}{98}, \dots, \frac{10}{91}$.

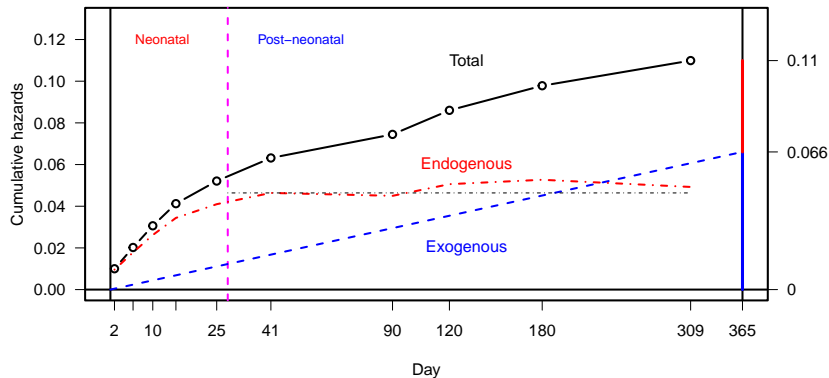
The hazards plot II



$$\text{Exogenous slope} = \frac{\#(\text{post deaths})}{\text{post exposure}} = \frac{5}{27626} \approx 0.00018$$

$$\text{Post exposure} = 90(365 - 68) + (93 - 68) + (163 - 68) + \dots + (335 - 68) \approx 27626$$

Adding the endogenous mortality

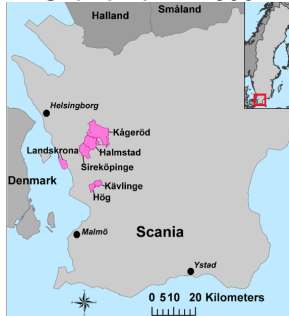


Earlier work

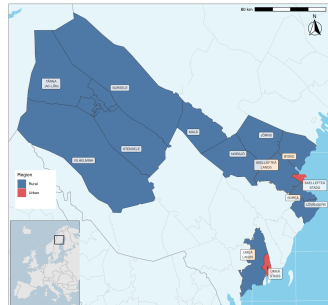
- Knodel & Kintner (1977).
 - Discusses effects of [breastfeeding](#) habits.
- Wrigley (1977).
 - Problems with [data recording](#).
- Lynch, Greenhouse & Brändström (1998).
 - Mentions "[constant hazard](#)".
 - Suggest [linear regression](#) methods.
- Bengtsson (1999).
 - Data [quality](#).
- Manfredini (2004).
 - Deals with effect of [climate](#) variation.

Study areas

Skåne 1711–1800

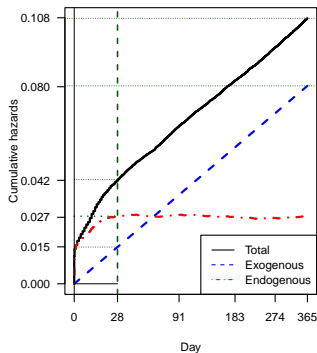


Västerbotten 1801–1950

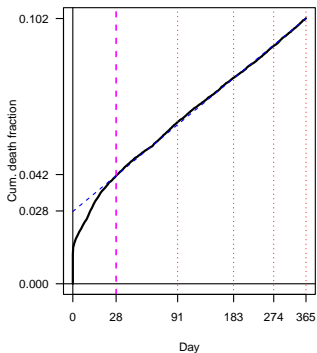


Västerbotten 1861-1890

Hazards method

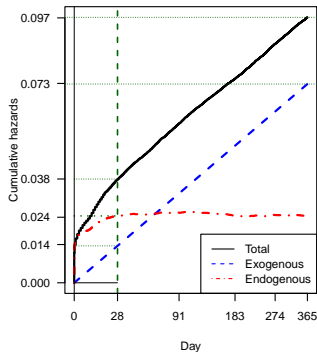


B-P method

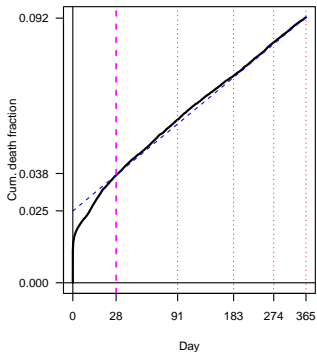


Västerbotten 1921–1950

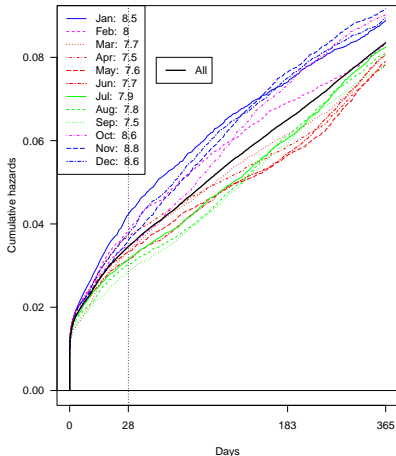
Hazards method



B-P method



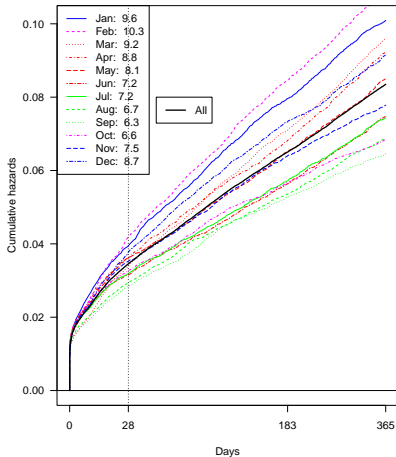
Cohort effect of birth month, Västerbotten



Conclusion:

Post-neonatal linearity is a population property, not an individual.

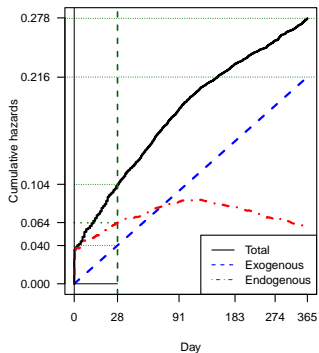
Period effect of month, Västerbotten



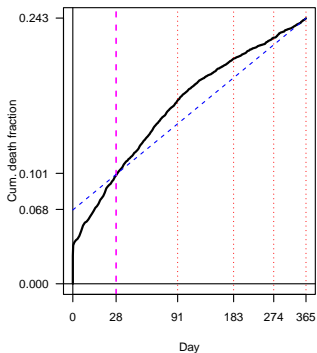
Looks quite proportional!

Scania 1711–1800

Hazards method

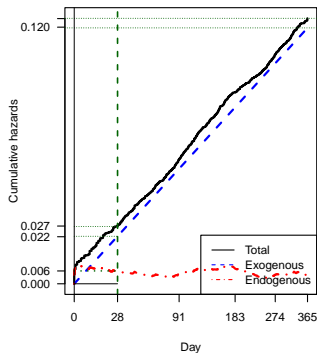


B-P method

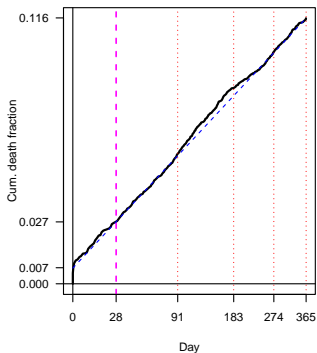


Skellefteå 1801–1820

Hazards method



B-P method



Biometric analysis on the natural time scale

Given that the exogenous mortality follows an Exponential distribution on the log-cube scale, it follows that the **cumulative hazards** function H on the **natural** time scale is

$$H(t; \lambda) = \lambda \log^3(t + 1), \quad t \geq 0,$$

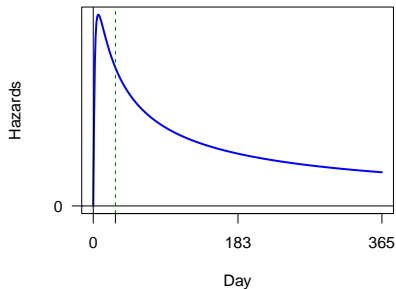
and it follows that the **hazard** function h is given by

$$h(t; \lambda) = \lambda \frac{3 \log^2(t + 1)}{t + 1}, \quad t \geq 0,$$

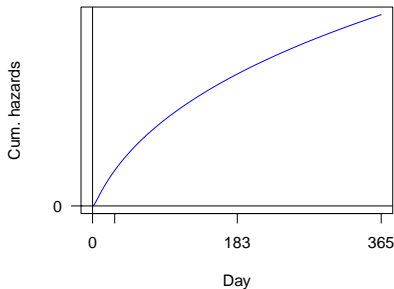
As λ varies, this constitutes a **proportional hazards** family of distributions.

Exogenous mortality on the normal time scale

Hazard function



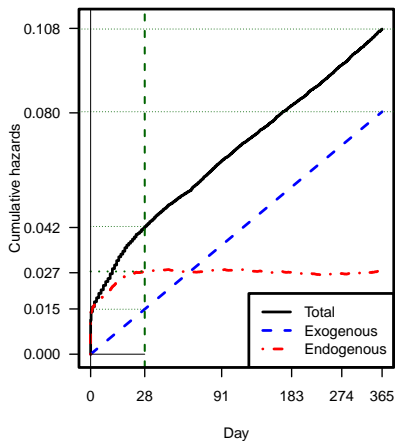
Cumulative hazards



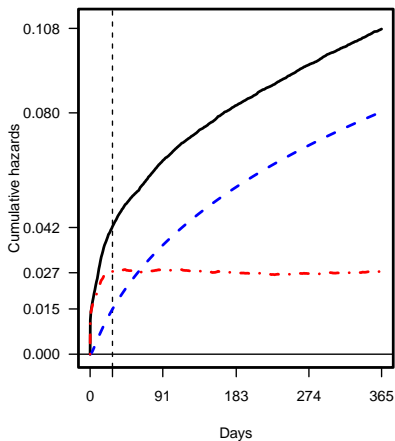
Natural vs. log-cube time scales

Västerbotten 1861–1890.

Log-cube

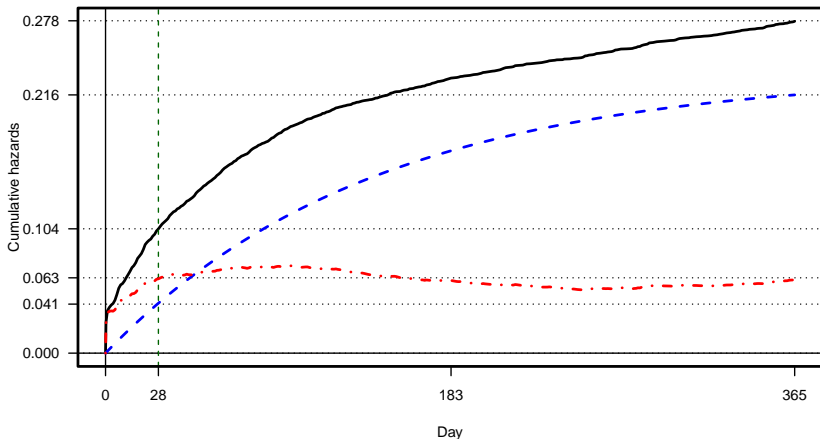


Natural



Natural time scale and Gompertz exogenous distribution

Skåne 1711–1800



Conclusions

- **Advantages** of the hazards method.
 - Fits naturally into modern survival analysis, allows **censored** and **truncated data**, and
 - The **exogenous** component forms a class of **proportional hazards** distributions.
- **Restrictions** for both models.
 - Models for **populations**, not for individuals.
 - **Less useful** for traditional **survival analysis**.
 - The **log-cube** transform is **not** universal.
- **Suggestions**.
 - Work on the **natural** time scale, if access to a computer.
 - Try **standard** distributions for **exogenous** mortality.
 - **Remember** that we are dealing with **competing risks**!
 - Requires special care.