# A hazards approach to the biometric analysis of infant mortality

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# Outline

- Jean Bourgeois-Pichat (1951): Biometric analysis of infant mortality (background).
- Hazard-based alternative.
- Theoretical considerations
- Real-world examples.

# Bourgeois-Pichat and causes of death

- Two categories:
  - Endogenous: *inherited*, *delivery*, etc.
  - Exogenous: *accidental*, *infectious diseases*, etc.
- How to differentiate between the two categories without information of causes of death?

Bourgeois-Pichat's biometric model.

# The biometric model

Two postulates:

1. Endogenous deaths only occur during the neonatal period (0–28 days).
2. On a specific time scale, exogenous infant mortality is uniformly distributed.
    - The log-cube time scale.

# The log-cube transform

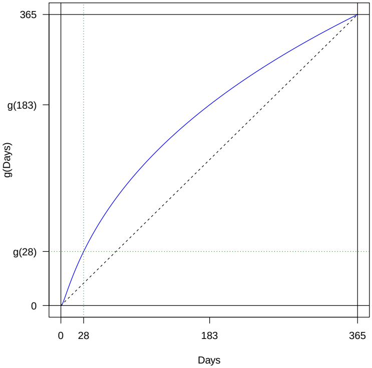
$$g(t) = C \log^3(t+1), \; 0 < t \leq 365.$$

where $C$ is a normalizing constant:

$$C = \frac{365}{\log^3(365+1)}$$

$t$ is age in days.

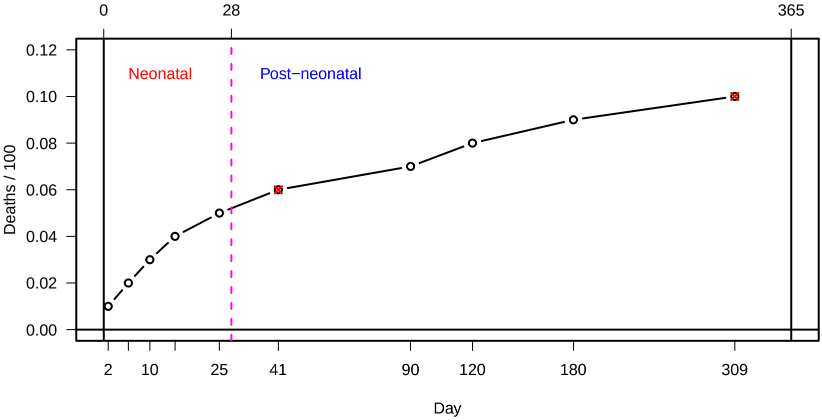
*Note:* $C$ is not part of B-P's original definition.



# Demonstration of the B-P plot

Assume 100 births, of which 90 survives infancy, ten death ages:

2, 6, 10, 15, 25, 41, 90, 120, 180, 309.

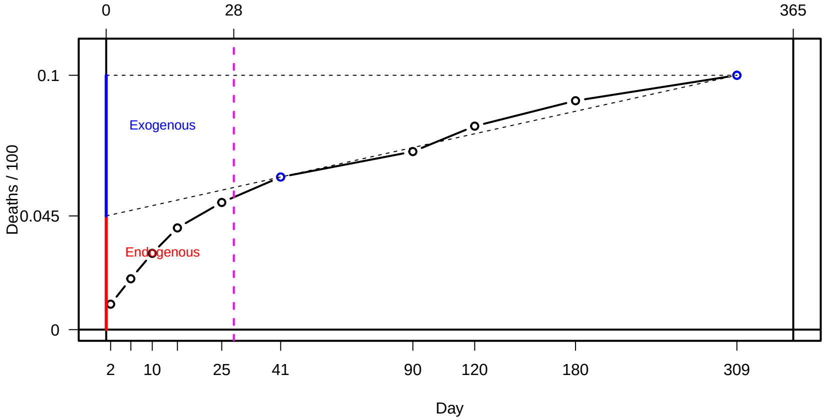



Calculation (no ties): $(2, \frac{1}{100}), (6, \frac{2}{100}), (10, \frac{3}{100}), \ldots, (309, \frac{10}{100})$.

# Demonstration of the B-P plot II

Assume 100 births, of which 90 survives infancy, ten death ages:

2, 6, 10, 15, 25, 41, 90, 120, 180, 309.

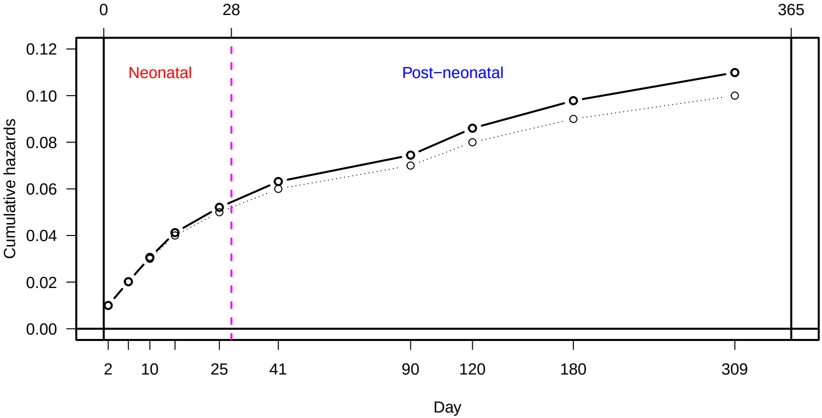



Calculation (no ties): $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \ldots, \frac{10}{100}$.

# The hazards plot

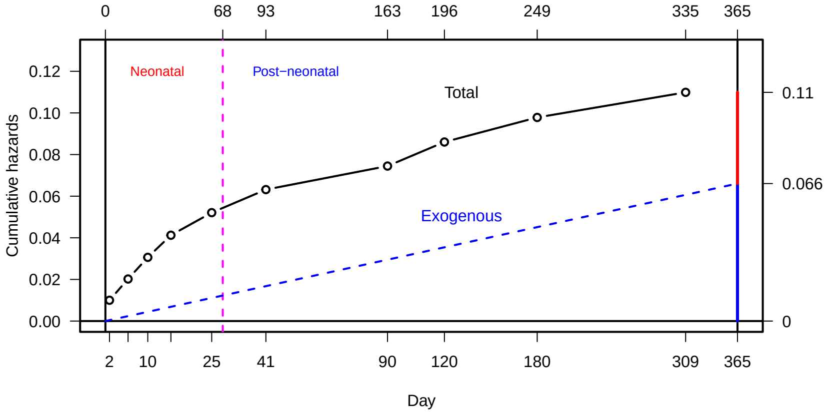
Assume 100 births, of which 90 survives infancy, ten death ages:

2, 6, 10, 15, 25, 41, 90, 120, 180, 309 (same as before).




Calculation (no ties): $\frac{1}{100}, \frac{2}{99}, \frac{3}{98}, \ldots, \frac{10}{91}$.

# The hazards plot II

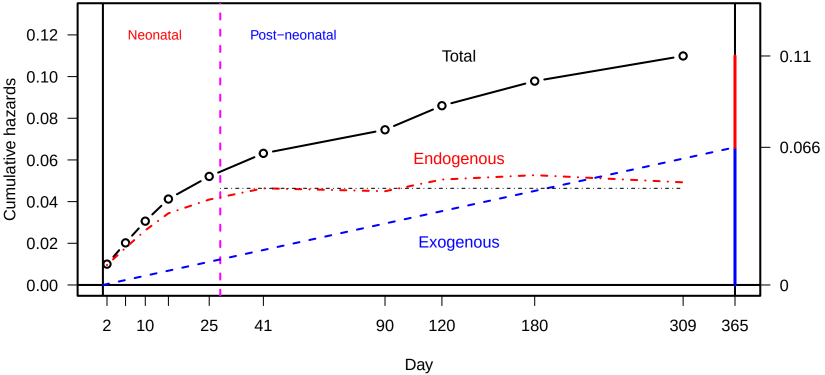



$$\text{Exogenous slope} = \frac{\#(\text{post deaths})}{\text{post exposure}} = \frac{5}{27626} \approx 0.00018$$

Post exposure $=$

$$90(365-68)+(93-68)+(163-68)+\cdots+(335-68) \approx 27626$$

By subtraction: Endogenous = Total − Exogenous

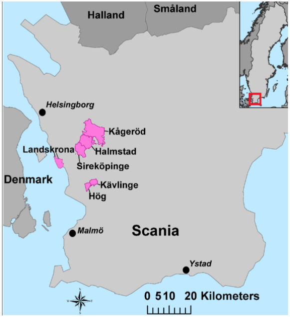


# Earlier work

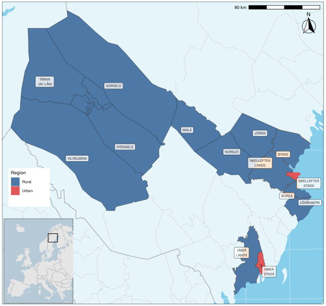
- Knodel & Kintner (1977).
  - Discusses effects of breastfeeding habits.
- Wrigley (1977).
  - Problems with data recording.
- Lynch, Greenhouse & Brändström (1998).
  - Mentions "constant hazard".
  - Suggest linear regression methods.
- Bengtsson (1999).
  - Data quality.
- Manfredini (2004).
  - Deals with effect of climate variation.

# Study areas

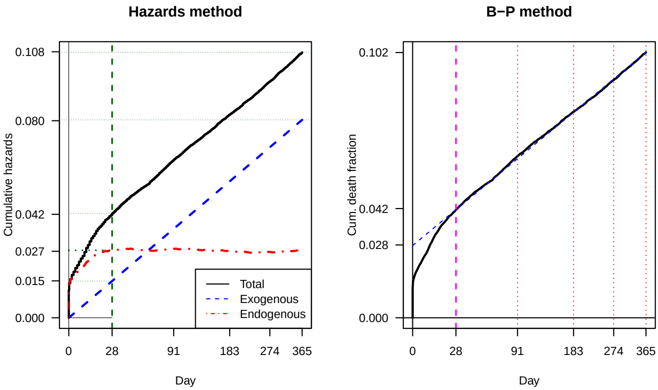
Skåne 1711–1800




Västerbotten 1801–1950



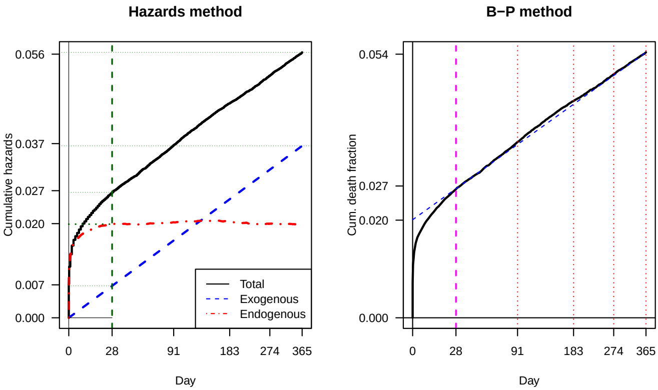
# Västerbotten 1861-1890




Total mortality: $1 - \exp(-0.106) \approx 0.101$.

Excellent fits!

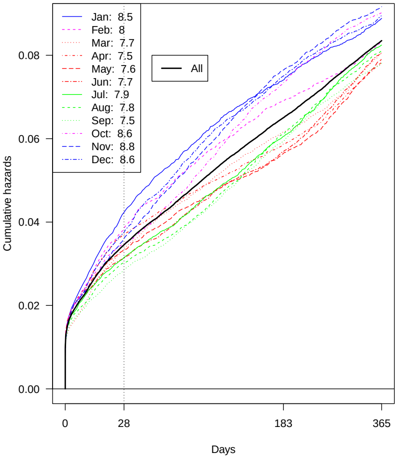
# Västerbotten 1921–1950




Total mortality: $1 - \exp(-0.056) \approx 0.054$.

Excellent fits!

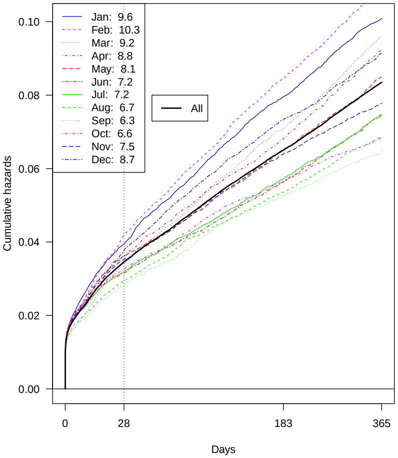
# Cohort effect of birth month, Västerbotten




Conclusion:

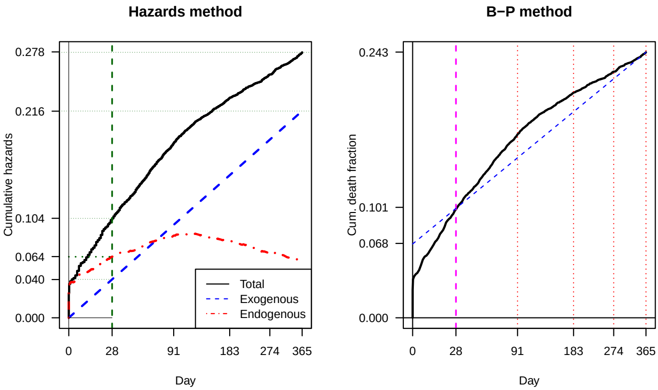
Post-neonatal linearity is a population property, not an individual.

# Period effect of month, Västerbotten




Looks quite proportional!

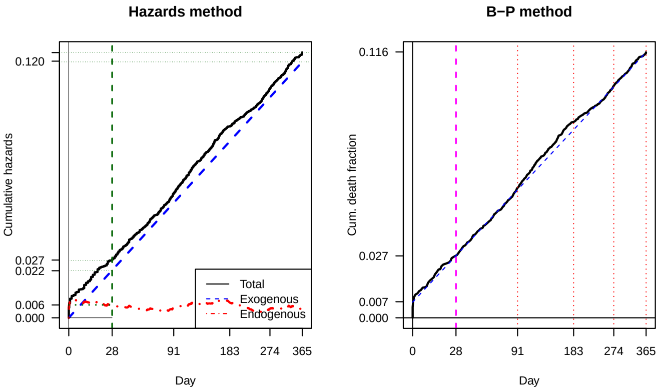
# Scania 1711–1800




The log-cube model does not fit at all, despite high-quality data!

Too old time period?

# Skellefteå 1801–1820




- Reasonable linear post-neonatal fit.
- But no room for endogenous mortality.
- Many early deaths missing.

# Biometric analysis on the natural time scale

Given that the exogenous mortality follows an Exponential distribution on the log-cube scale, it follows that the cumulative hazards function $H$ on the natural time scale is

$$H(t; \lambda) = \lambda \log^3(t+1), \; t \ge 0,$$

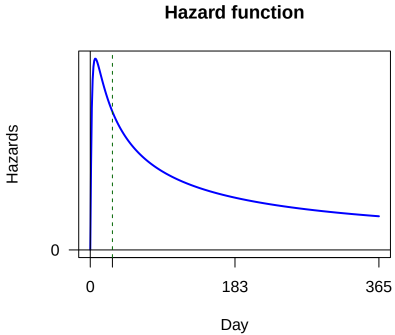
and it follows that the hazard function $h$ is given by

$$h(t; \lambda) = \lambda \frac{3 \log^2(t+1)}{t+1}, \; t \ge 0,$$

As $\lambda$ varies, this constitutes a proportional hazards family of distributions.

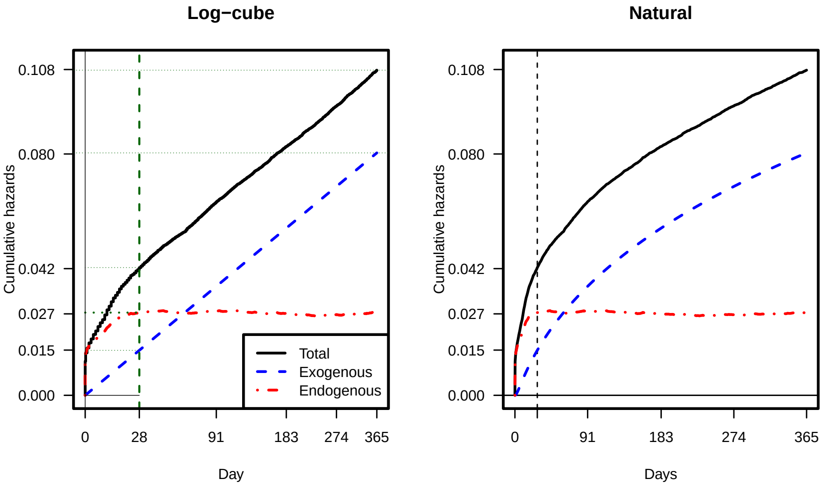
NOTE: $H(t; \lambda)$ is the log-cube transform!

# Exogenous mortality on the natural time scale



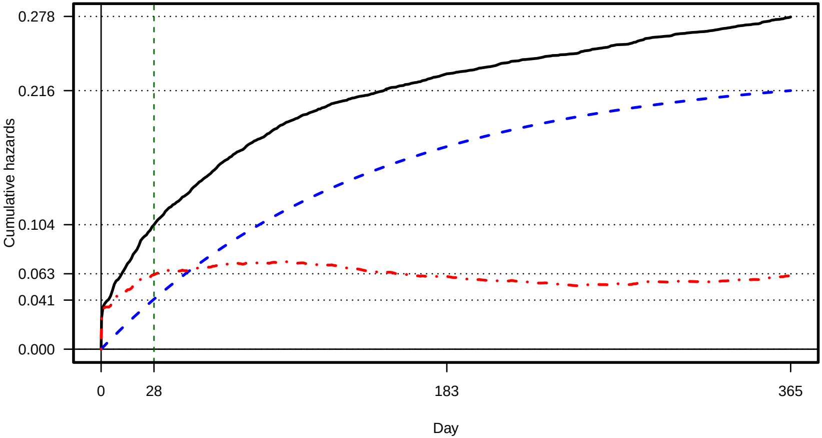
# Natural vs. log-cube time scales

Västerbotten 1861–1890.



# Natural time scale and Gompertz exogenous distribution

Skåne 1711–1800.




Decent fit if we skip the log-cube distribution!

# Conclusions

- Advantages of the hazards method.
  - Fits naturally into modern survival analysis, allows censored and truncated data, and
  - The exogenous component forms a class of proportional hazards distributions.
  - So, theoretically more appropriate.
- Restrictions for both models.
  - Models for populations, not for individuals.
  - Less useful for traditional survival analysis.
  - The log-cube transform is not universal.

# Suggestions for future work

- Work on the natural time scale, if
  - access to a computer, and
  - suitable software.
- Try standard distributions for exogenous mortality, eg.
  - Weibull,
  - Gompertz.
- Remember that we are dealing with competing risks!
  - Requires special care.

**R** code for this paper:

https://github.com/goranbrostrom/ESSHC_23