Status Talk

Integration of Multi-Perspective and Non-Monotonic Reasoning

Piotr Gorczyca
Computational Logic Group, TU Dresden
Dresden, February 5th, 2026

Supervisor:
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Fachrefent:
Prof. Dr. Markus Krötzsch

Motivation: Procedural and Declarative Approaches

According to the NIH:

[PCOS is diagnosed in the presence of] ***Oligo-ovulation***
and clinical and/or biological signs of ***hyperandrogenism****,*
and [assuming the] ***exclusion of other aetiologies****.*

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Hyperandrogenism

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National Institutes of Health [S1] criteria statement	European Society for Human Reproduction and Embryology/American Society for Reproductive Medicine [S2] statement	Androgen Excess Society [S3] statement
Oligo-ovulation [B] and clinical and/or biochemical signs of hyperandrogenism [A], and exclusion of other aetiologies [X]	Two out of three of: oligo-ovulation and/or anovulation [A], clinical and/or biochemical signs of hyperandrogenism [B], or polycystic ovaries [C], and exclusion of other aetiologies [X]	Hyperandrogenism (hirsutism and/or hyperandrogenaemia) [A], [and] ovarian dysfunction (oligoanovulation [A] and/or polycystic ovaries [C]), and exclusion of other androgen excess related disorders [Y]

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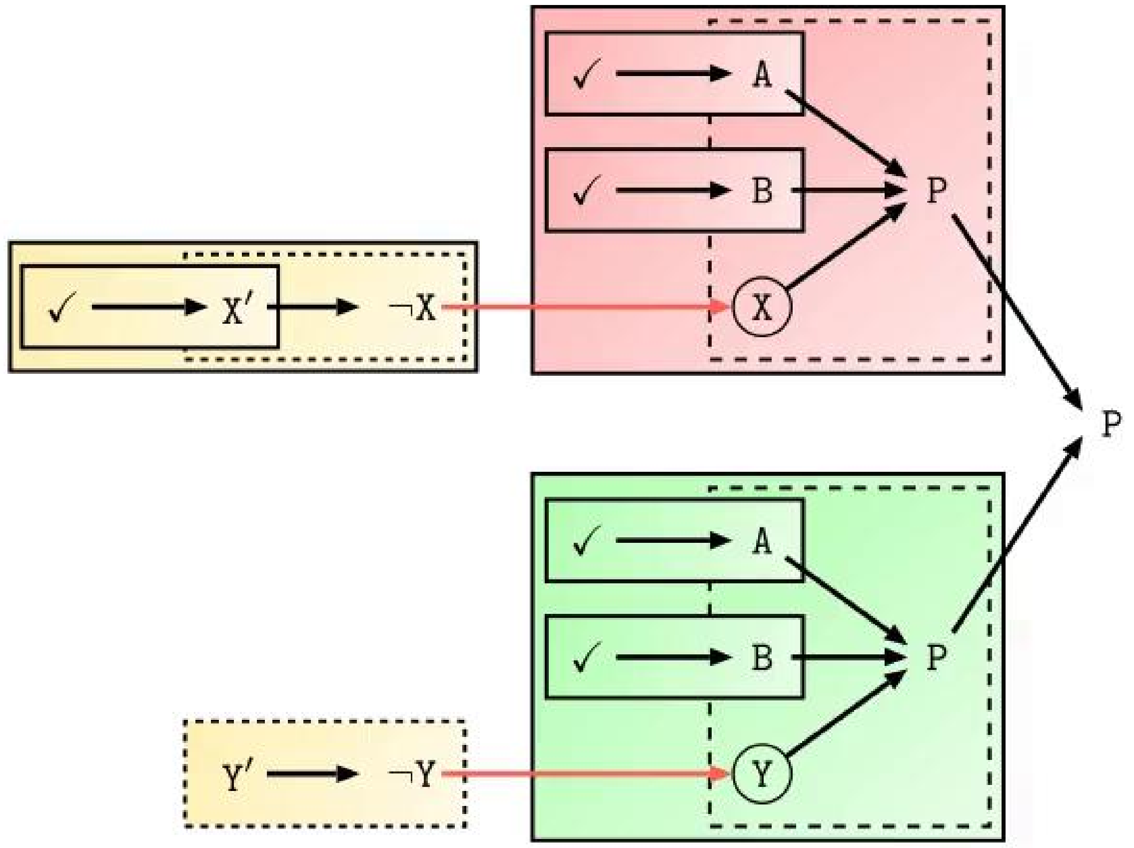

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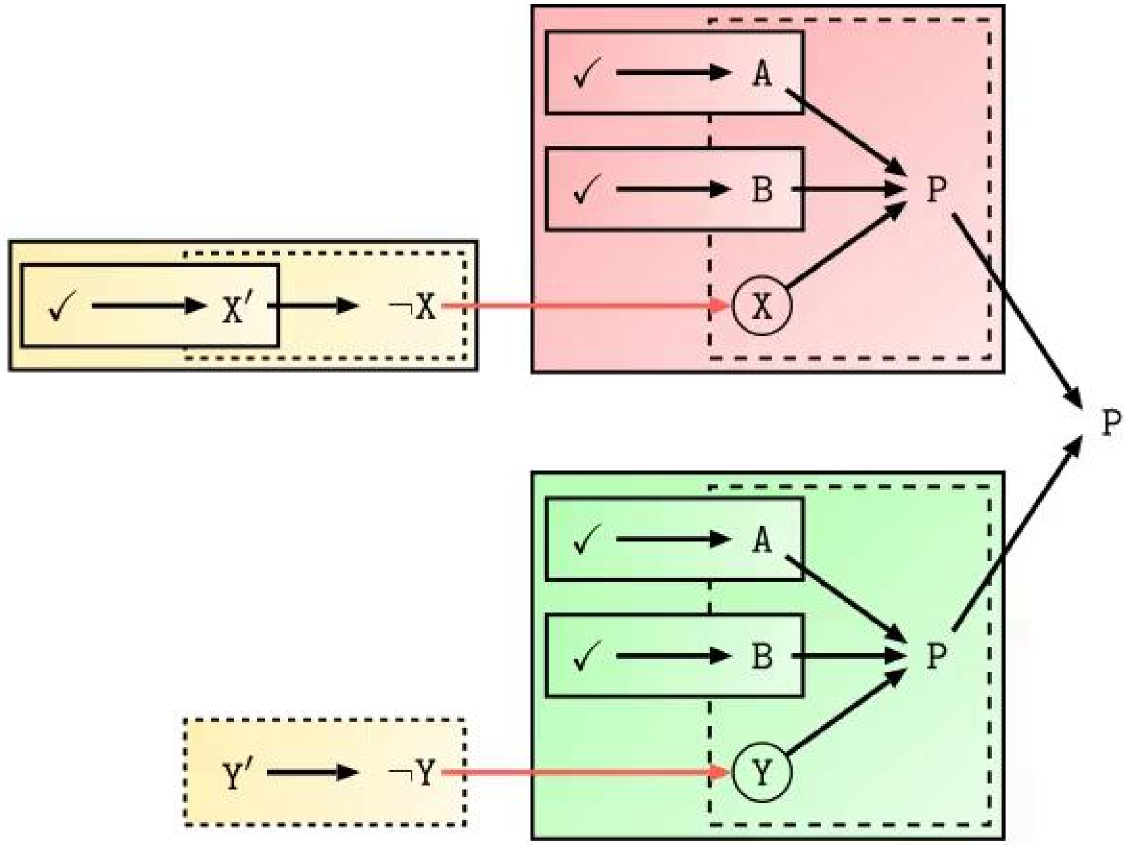

$$\Box_{S1}\left[\frac{A \wedge B : X}{P}\right]$$

$$\Box_{S2}\left[\frac{(A \wedge B) \vee (A \wedge C) \vee (B \wedge C) : X}{P}\right]$$

$$\Box_{S3}\left[\frac{A \wedge (B \vee C) : Y}{P}\right]$$

$A \wedge C$

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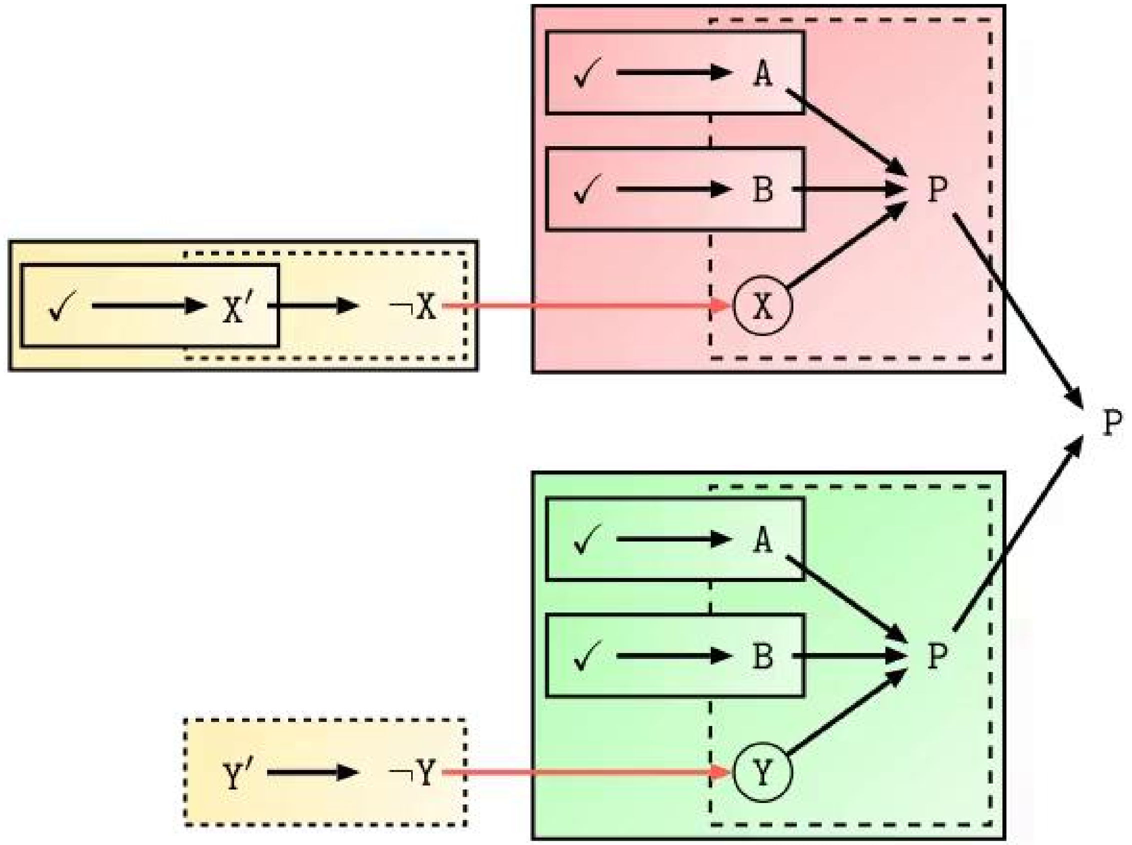

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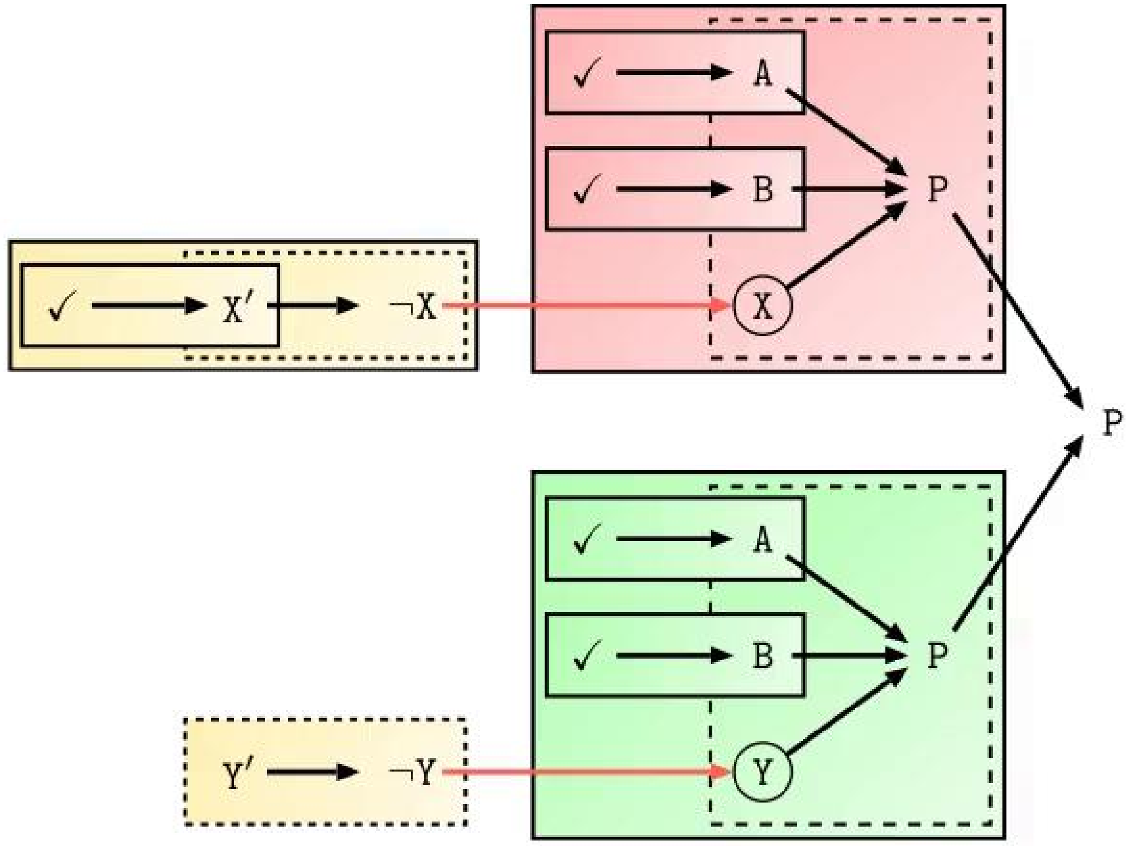

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Outline

Part I: Argumentation Games in Multi-Shot ASP

Part II: Generalisations of S4F and Standpoint Logic

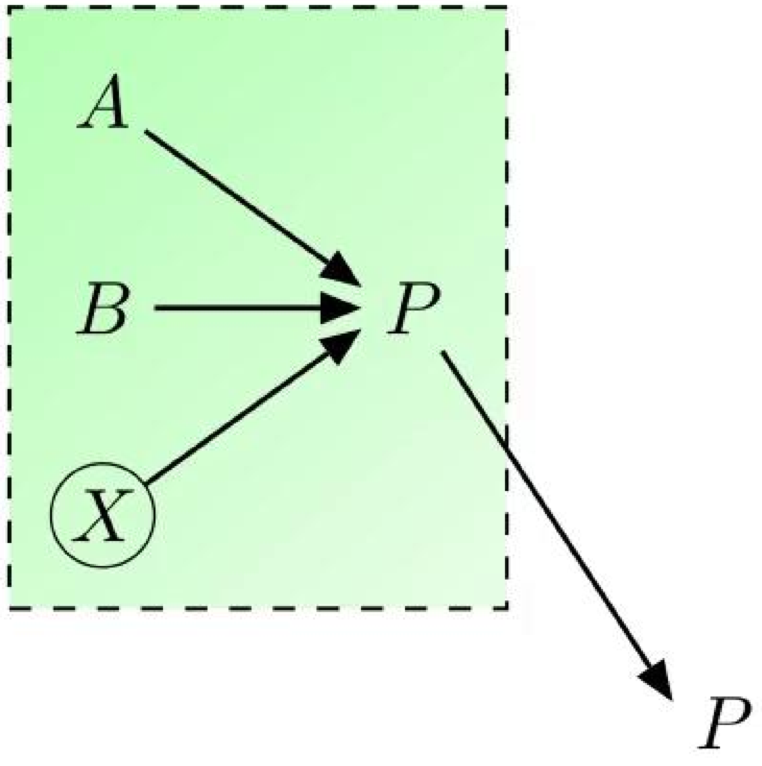
– Product of Standpoint and S4F Logic: Standpoint S4F Logic

– Fusion of S4F and Standpoint Logic: S4F Standpoint Logic

Part I: Argumentation Games in Multi-Shot ASP

(Structured) Argumentation Games

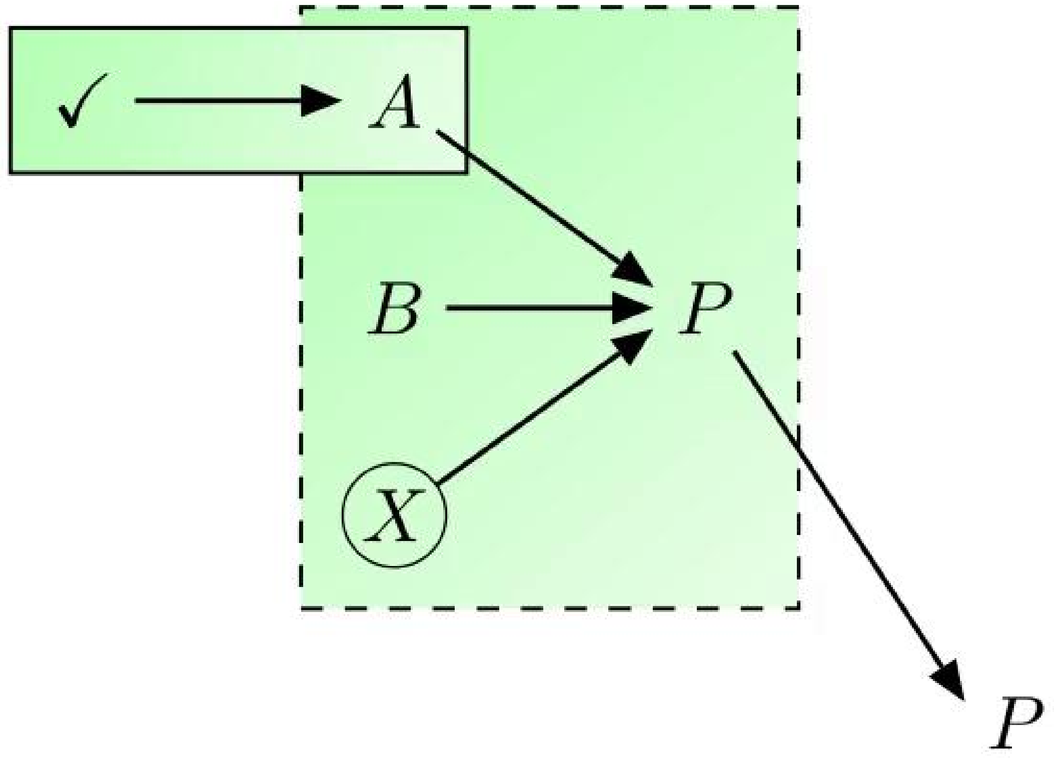
Conclude P, if A, B and assuming X.



(Structured) Argumentation Games

Conclude P, if A, B and assuming X.

A is true.

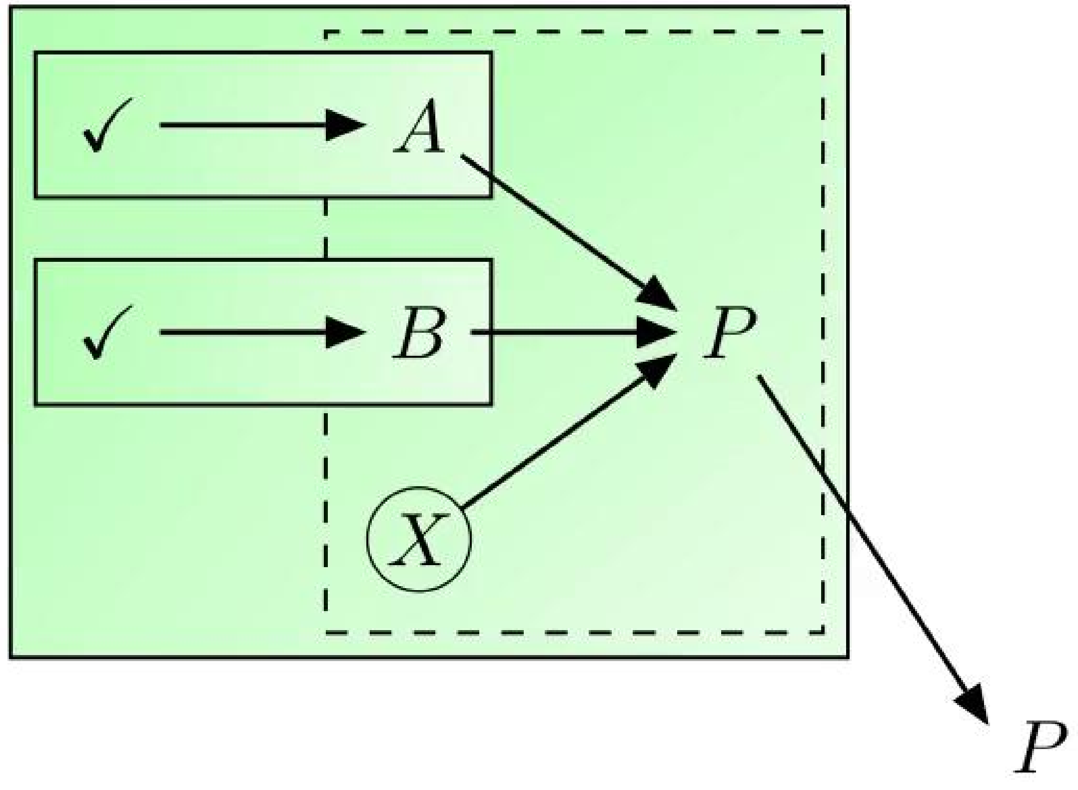


(Structured) Argumentation Games

Conclude P, if A, B and assuming X.

A is true.

B is true.



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A is true.

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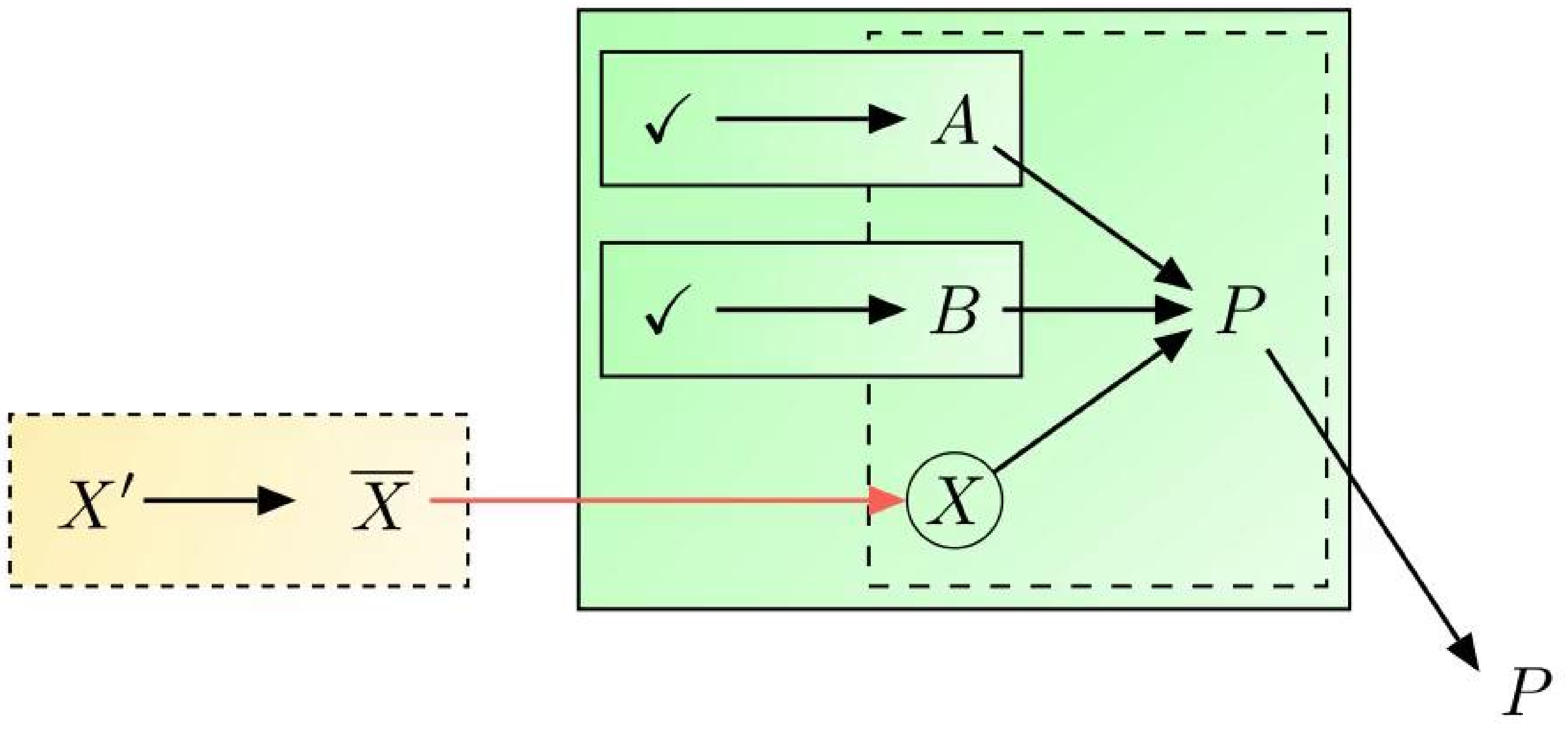
Conclude $\overline{X}$ if X'.



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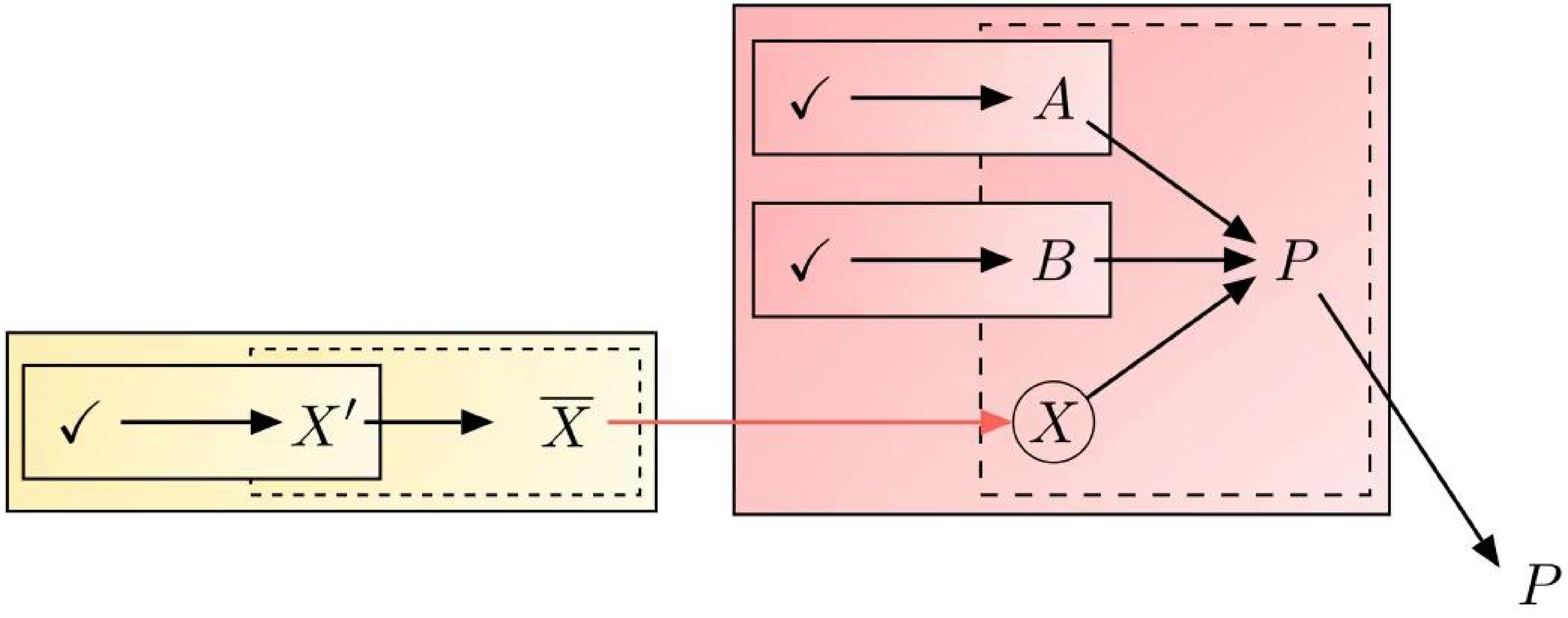

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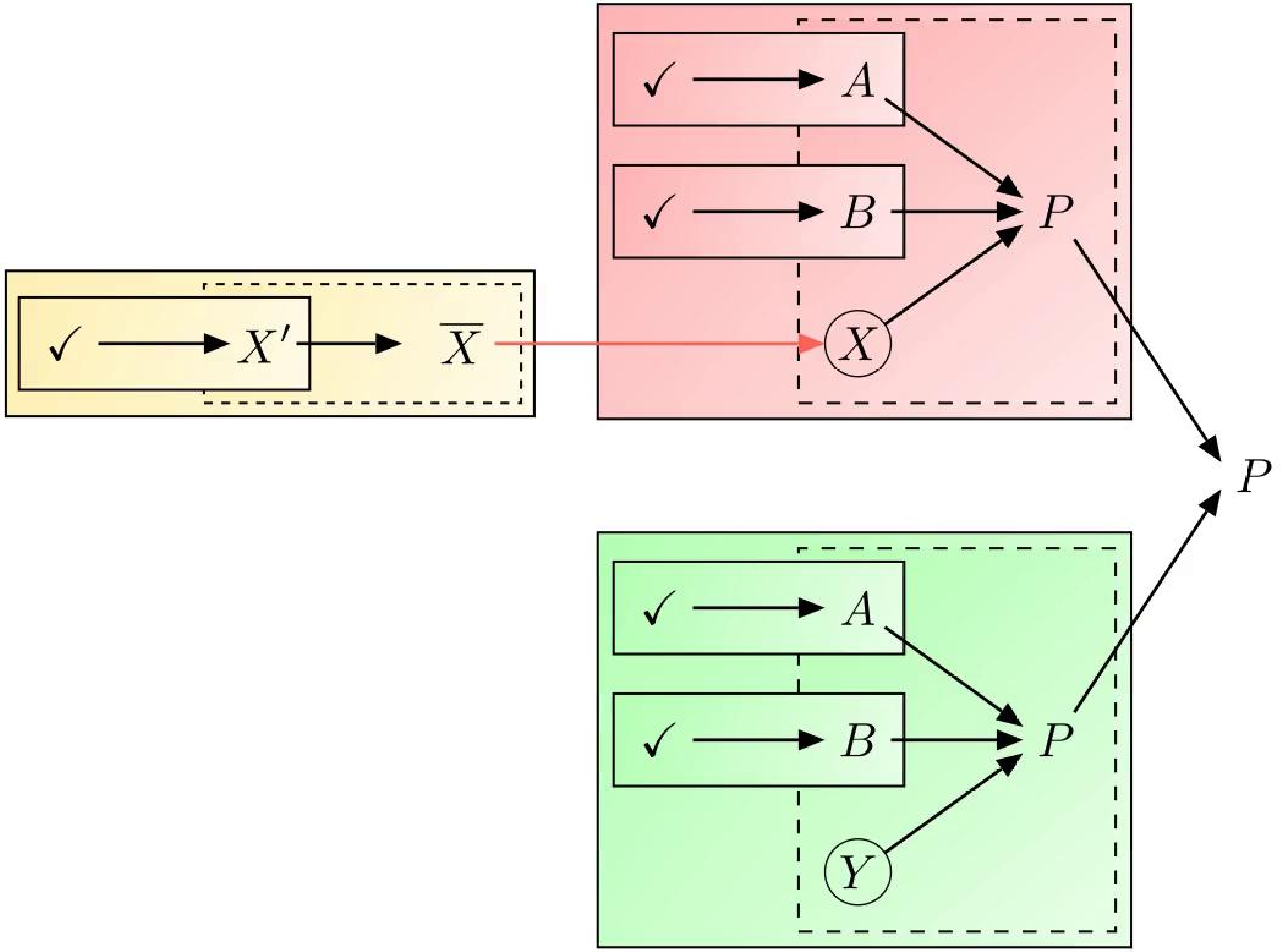
Conclude P, if A, B and assuming Y.



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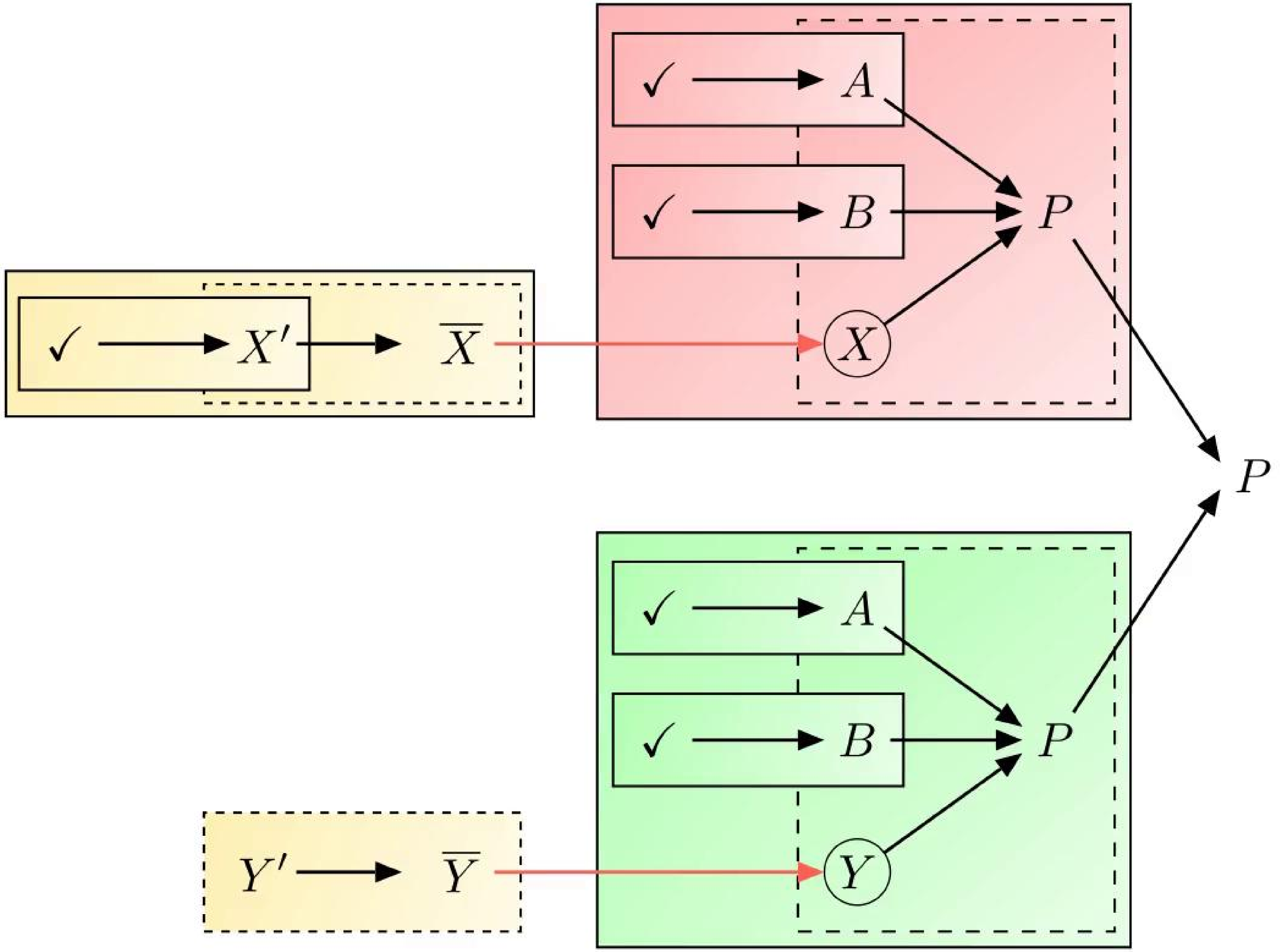

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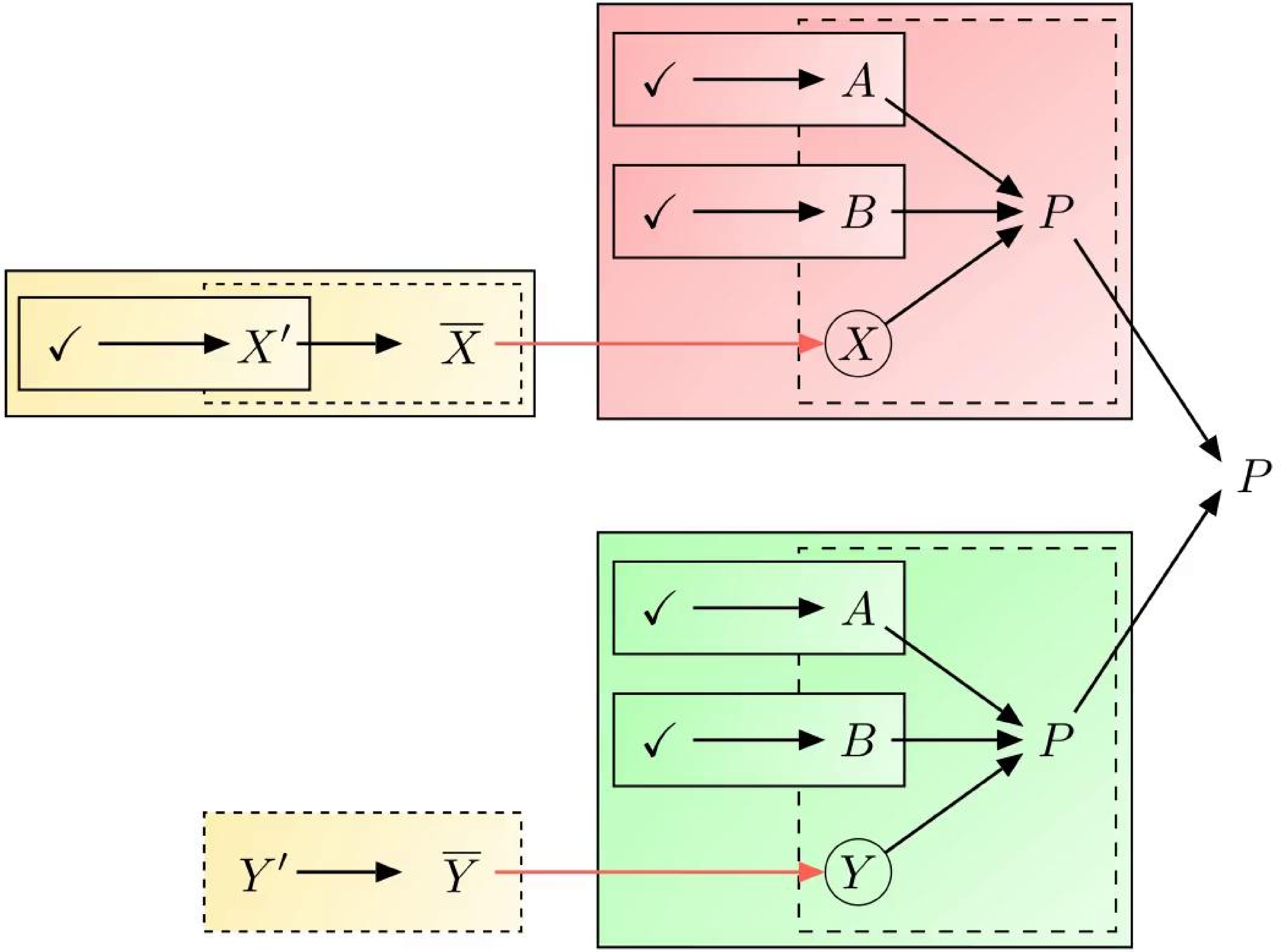

Argument Games: Implementations

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$(\mathbb{P} \cap \mathscr{L})^{\downarrow} = \{s \in \mathbb{P} \cap \mathscr{L} \mid \neg \exists h \leftarrow B \in \mathbb{P} \textit{ with } h = s\}$	Played unexpanded statements of the proponent
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$\mathbb{P}^{*} = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^{*}\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^{*} \text{ with } h = s\}$	Complete played pieces of the proponent
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$\mathbb{B}_{S}^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^{-}) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_{S}^{!/-} \cap \mathscr{R} \textit{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid h \in \mathbb{B}_{S}^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
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- **Problem**: imperative implementations hard to maintain and grow in complexity with each added feature
- **Idea**: declarative ASP implementation; using multi-shot grounding to avoid full re-grounding; first use of multi-shot ASP for argument games

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Declarative ASP Implementation

$$\mathcal{R} = \{p \leftarrow a, b, x\}$$
$$\mathcal{A} = \{x\}$$

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```
% Framework encoding
head(1,p).
body(1,a). body(1,b). body(1,x).
assumption(x).
```

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$\mathscr{J}_{\mathbb{P}}^{\sim} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Remaining rules blocked for the proponent
$(\mathbb{P} \cap \mathscr{L})^{\downarrow} = \{s \in \mathbb{P} \cap \mathscr{L} \mid \neg\exists h \leftarrow B \in \mathbb{P} \textit{ with } h = s\}$	Played unexpanded statements of the proponent
$(\mathbb{B} \cap \mathscr{L})^{\uparrow\uparrow} = \{s \in (\mathbb{B} \cap \mathscr{L}) \mid \neg\exists h \leftarrow B \in (\mathscr{J}_{\mathbb{B}} \setminus \mathscr{J}_{\mathbb{B}}^{-}) \textit{ with } h = s\}$	Played fully expanded statements
$\mathbb{B}^{-} = (\mathbb{B} \cap \mathscr{C}) \cup \{s \in (\mathbb{B} \cap \mathscr{L})^{\uparrow\uparrow} \setminus \mathscr{A} \mid \neg\exists h \leftarrow B \in (\mathbb{B} \cap \mathscr{R}) \setminus \mathbb{B}^{-} \textit{ with } h = s\} \cup \{h \leftarrow B \in \mathbb{B} \cap \mathscr{R} \mid B \cap \mathbb{B}^{-} \neq \emptyset\}$	Played blocked pieces
$\mathbb{P}^{*} = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^{*}\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^{*} \text{ with } h = s\}$	Complete played pieces of the proponent
$\mathbb{B}^{*/-} = (\mathbb{B} \cap (\mathscr{A} \setminus \mathscr{C})) \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid B \subseteq \mathbb{B}^{*/-}\} \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap (\mathscr{L} \setminus \mathscr{A}) \mid \exists h \leftarrow B \in \mathbb{B}^{*/-} \textit{ with } h = s\}$	Unblocked complete played pieces of the opponent
$\mathbb{B}_{S}^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^{-}) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_{S}^{!/-} \cap \mathscr{R} \textit{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid h \in \mathbb{B}_{S}^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
$\mathscr{A}^{!} = \mathscr{A} \cap \mathbb{B}_{\mathscr{D}}^{!/-}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin \mathbb{B}^{*/-}\}$	Currently defended assumptions

Declarative ASP Implementation

$\mathcal{R} = \{p \leftarrow a, b, x\}$

$\mathcal{A} = \{x\}$

```
% Framework encoding
head(1,p).
body(1,a). body(1,b). body(1,x).
assumption(x).
```

Table 10. Auxiliary notation for rule-based flexible dispute derivations. All notions w.r.t. a dispute state $(\mathbb{B}, \mathbb{P})$.

Notation	Description
$\mathscr{D} = \mathbb{P} \cap \mathscr{A}$	Defenses
$\mathscr{C} = \{u \in \mathscr{A} \mid \overline{u} \in \mathbb{P}\}$	Culprits
$\mathscr{J}_{\mathbb{B}} = \mathscr{R} \setminus \mathbb{B}$	Remaining rules for the opponent
$\mathscr{J}_{\mathbb{P}} = \mathscr{R} \setminus \mathbb{P}$	Remaining rules for the proponent
$\mathscr{J}_{\mathbb{B}}^{-} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \mid B \cap \mathscr{C} \neq \emptyset\}$	Blocked remaining rules
$\mathscr{J}_{\mathbb{P}}^{\sim} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Remaining rules blocked for the proponent
$B \cap \mathbb{B}^{-} \neq \emptyset\}$	
$\mathbb{P}^* = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^*\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^* \text{ with } h = s\}$	Complete played pieces of the proponent
$\mathbb{B}^{*/-} = (\mathbb{B} \cap (\mathscr{A} \setminus \mathscr{C})) \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid B \subseteq \mathbb{B}^{*/-}\} \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap (\mathscr{L} \setminus \mathscr{A}) \mid \exists h \leftarrow B \in \mathbb{B}^{*/-} \text{ with } h = s\}$	Unblocked complete played pieces of the opponent
$\mathbb{B}_S^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^{-}) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_S^{!/-} \cap \mathscr{R} \text{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid h \in \mathbb{B}_S^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
$\mathscr{A}^{!} = \mathscr{A} \cap \mathbb{B}_{\mathscr{D}}^{!/-}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin \mathbb{B}^{*/-}\}$	Currently defended assumptions

Complete Proponent’s Statements:

$$\mathbb{P}^* := (\mathbb{P} \cap \mathcal{A}) \cup \{h \mid h \leftarrow B \in (\mathbb{P} \cap \mathcal{R}) \text{ and } B \subseteq \mathbb{P}^*\}$$

Declarative ASP Implementation

$$\mathcal{R} = \{p \leftarrow a, b, x\}$$
$$\mathcal{A} = \{x\}$$

```
% Framework encoding
head(1,p).
body(1,a). body(1,b). body(1,x).
assumption(x).
```

```
% Complete Proponent's Statements
comPropSt(T,S) :- step(T), propSt(T-1,S), assumption(S).
comPropSt(T,H) :- step(T), propRule(T-1,RuleID), head(RuleID,H),
                  comPropSt(T-1,B) : body(RuleID, B).
```

Table 10. Auxiliary notation for rule-based flexible dispute derivations. All notions w.r.t. a dispute state $(\mathbb{B}, \mathbb{P})$.

Notation	Description
$\mathscr{D} = \mathbb{P} \cap \mathscr{A}$	Defenses
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$\mathscr{J}_{\mathbb{P}} = \mathscr{R} \setminus \mathbb{P}$	Remaining rules for the proponent
$\mathscr{J}_{\mathbb{B}}^{-} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{B}} \mid B \cap \mathscr{C} \neq \emptyset\}$	Blocked remaining rules
$\mathscr{J}_{\mathbb{P}}^{\sim} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Remaining rules blocked for the proponent
$B \cap \mathbb{B}^{-} \neq \emptyset\}$	
$\mathbb{P}^* = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^*\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^* \text{ with } h = s\}$	Complete played pieces of the proponent
$\mathbb{B}^{*/-} = (\mathbb{B} \cap (\mathscr{A} \setminus \mathscr{C})) \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid B \subseteq \mathbb{B}^{*/-}\} \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap (\mathscr{L} \setminus \mathscr{A}) \mid \exists h \leftarrow B \in \mathbb{B}^{*/-} \textit{ with } h = s\}$	Unblocked complete played pieces of the opponent
$\mathbb{B}_S^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^{-}) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_S^{!/-} \cap \mathscr{R} \textit{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid h \in \mathbb{B}_S^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
$\mathscr{A}^{!} = \mathscr{A} \cap \mathbb{B}_{\mathscr{D}}^{!/-}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin \mathbb{B}^{*/-}\}$	Currently defended assumptions

Complete Proponent's Statements:

$$\mathbb{P}^* := (\mathbb{P} \cap \mathcal{A}) \cup \{h \mid h \leftarrow B \in (\mathbb{P} \cap \mathcal{R}) \text{ and } B \subseteq \mathbb{P}^*\}$$

Declarative ASP Implementation

$\mathcal{R} = \{p \leftarrow a, b, x\}$

$\mathcal{A} = \{x\}$

```
% Framework encoding
head(1,p).
body(1,a). body(1,b). body(1,x).
assumption(x).
```

```
% Complete Proponent's Statements
comPropSt(T,S) :- step(T), propSt(T-1,S), assumption(S).
comPropSt(T,H) :- step(T), propRule(T-1,RuleID), head(RuleID,H),
                  comPropSt(T-1,B) : body(RuleID, B).
```

```
% Initialise the step counter
#const max_step = 99.
step(1..max_step).
```

Table 10. Auxiliary notation for rule-based flexible dispute derivations. All notions w.r.t. a dispute state $(\mathbb{B}, \mathbb{P})$.

Notation	Description
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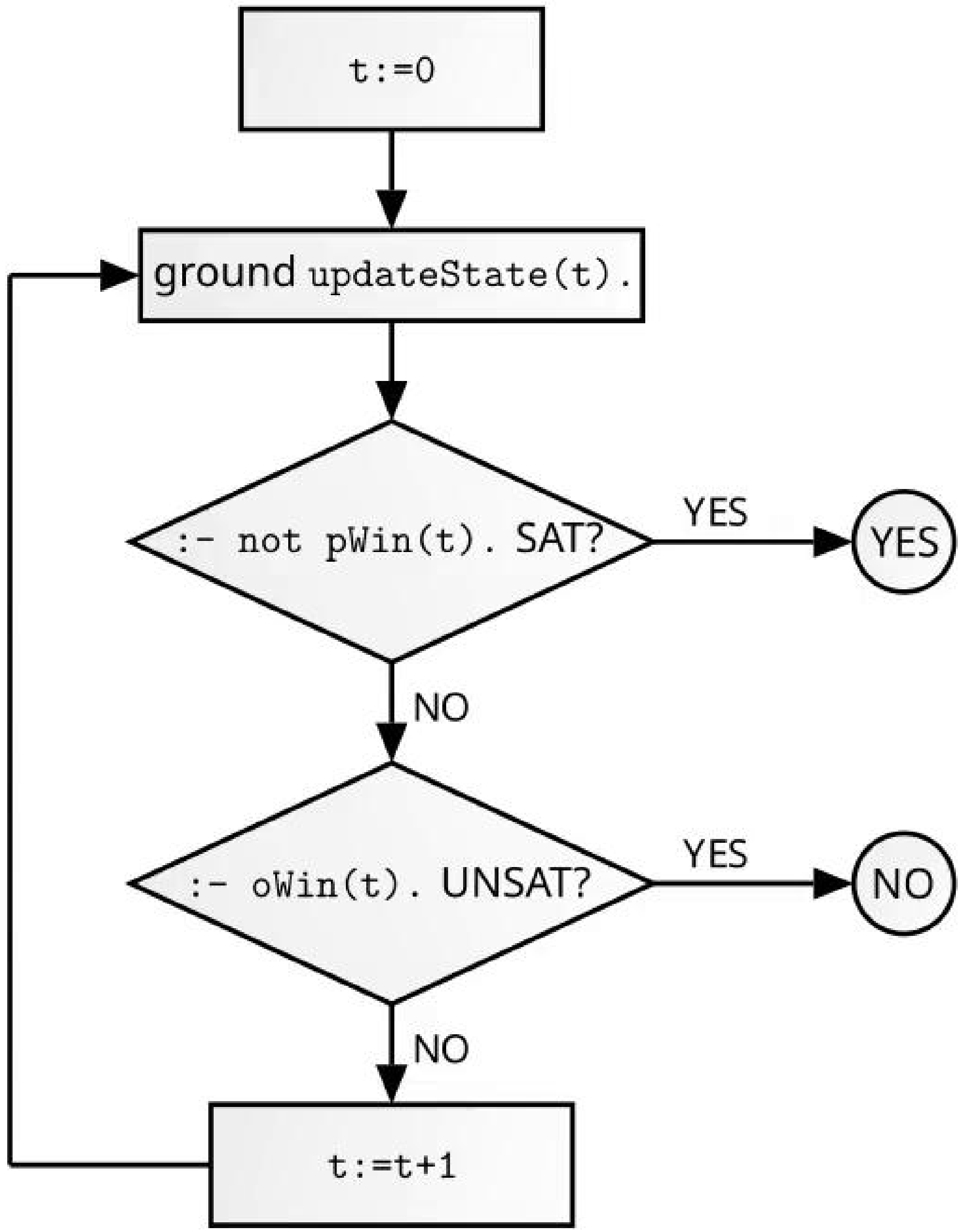
$\mathscr{J}_{\mathbb{P}}^{\sim} = \{h \leftarrow B \in \mathscr{J}_{\mathbb{P}} \mid (\{h\} \cup B) \cap (\overline{B} \cup \mathscr{C} \cup \overline{\mathscr{D}}) \neq \emptyset\}$	Remaining rules blocked for the proponent
$B \cap \mathbb{B}^{-} \neq \emptyset\}$	
$\mathbb{P}^* = (\mathbb{P} \cap \mathscr{A}) \cup \{h \leftarrow B \in (\mathbb{P} \cap \mathscr{R}) \mid B \subseteq \mathbb{P}^*\} \cup \{s \in (\mathbb{P} \cap (\mathscr{L} \setminus \mathscr{A})) \mid \exists h \leftarrow B \in \mathbb{P}^* \text{ with } h = s\}$	Complete played pieces of the proponent
$\mathbb{B}^{*/-} = (\mathbb{B} \cap (\mathscr{A} \setminus \mathscr{C})) \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid B \subseteq \mathbb{B}^{*/-}\} \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap (\mathscr{L} \setminus \mathscr{A}) \mid \exists h \leftarrow B \in \mathbb{B}^{*/-} \textit{ with } h = s\}$	Unblocked complete played pieces of the opponent
$\mathbb{B}_S^{!/-} = ((\mathbb{B} \setminus \mathbb{B}^{-}) \cap S) \cup \{s \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{L} \mid \exists h \leftarrow B \in \mathbb{B}_S^{!/-} \cap \mathscr{R} \textit{ with } s \in B\} \cup \{h \leftarrow B \in (\mathbb{B} \setminus \mathbb{B}^{-}) \cap \mathscr{R} \mid h \in \mathbb{B}_S^{!/-}\}$	Unblocked pieces supporting statements in $S \subseteq \mathscr{L}$
$\mathscr{A}^{!} = \mathscr{A} \cap \mathbb{B}_{\mathscr{D}}^{!/-}$	Candidates for culprits
$\mathscr{I} = \{u \in \mathscr{A} \setminus \mathscr{C} \mid \overline{u} \notin \mathbb{B}^{*/-}\}$	Currently defended assumptions

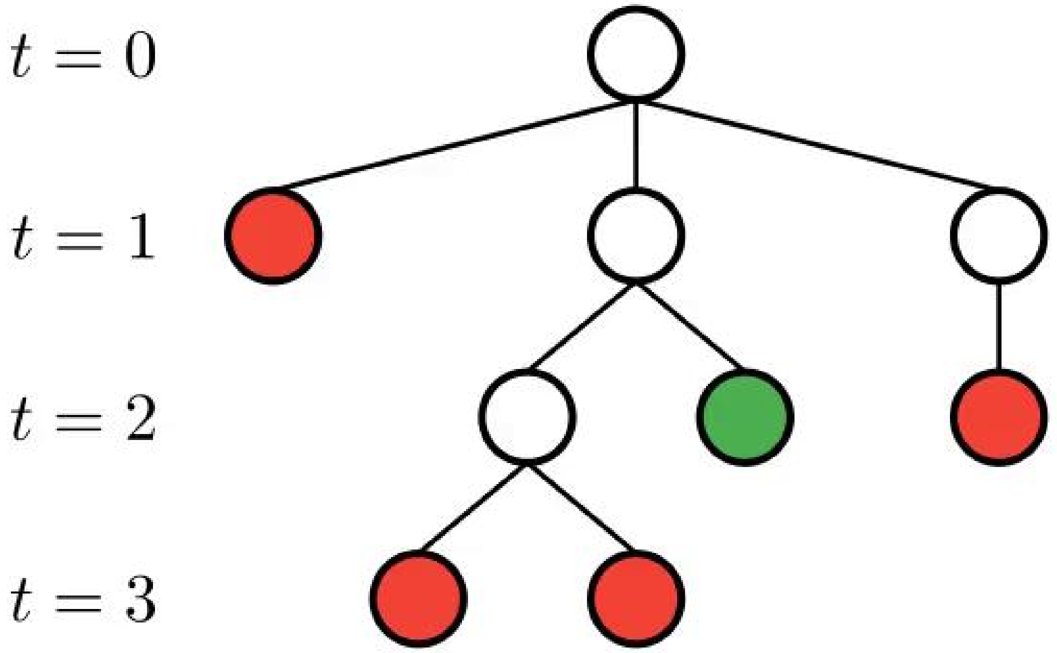
Complete Proponent's Statements:

$$\mathbb{P}^* := (\mathbb{P} \cap \mathcal{A}) \cup \{h \mid h \leftarrow B \in (\mathbb{P} \cap \mathcal{R}) \text{ and } B \subseteq \mathbb{P}^*\}$$

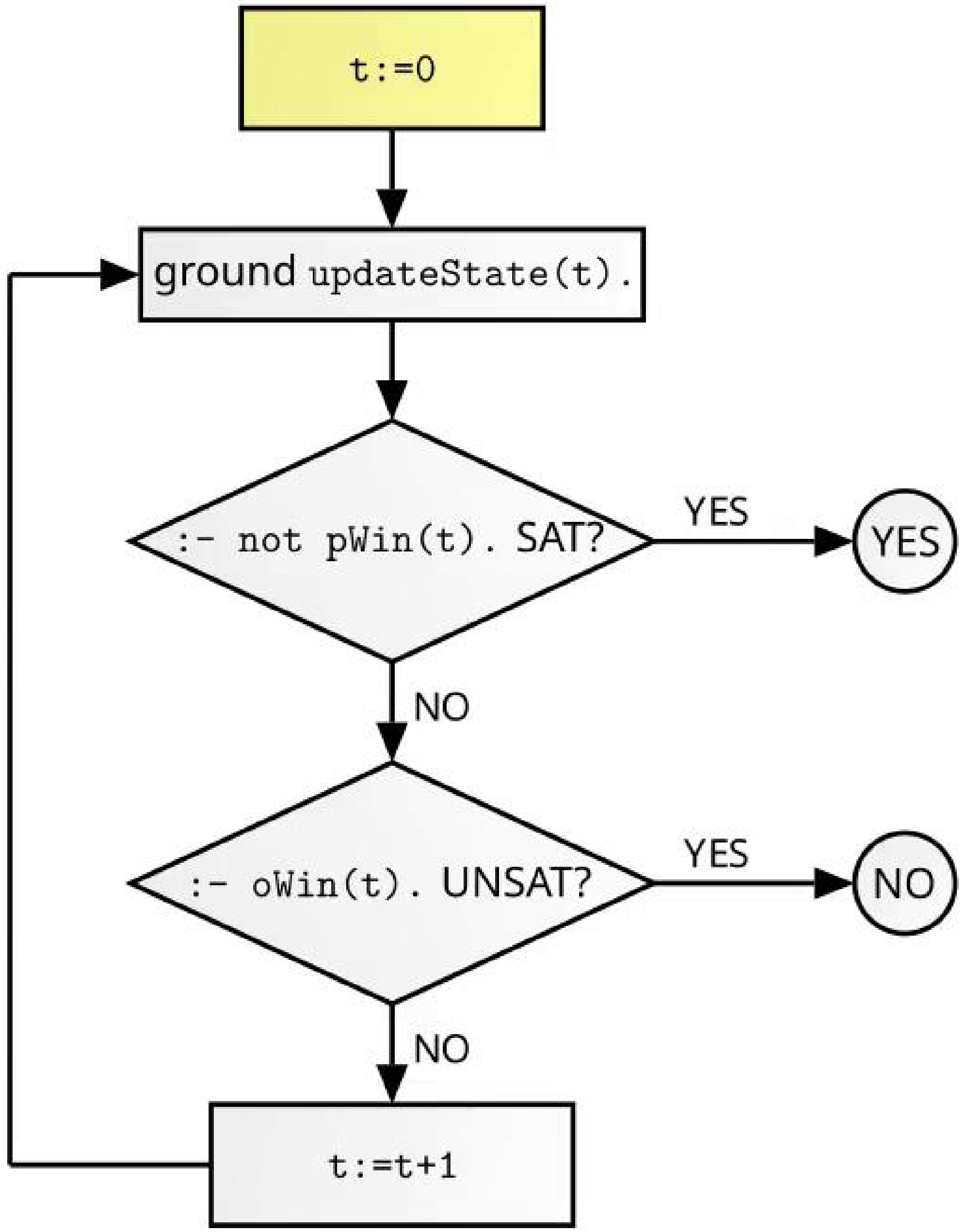
Multi-Shot ASP: Dynamic Solving & Grounding



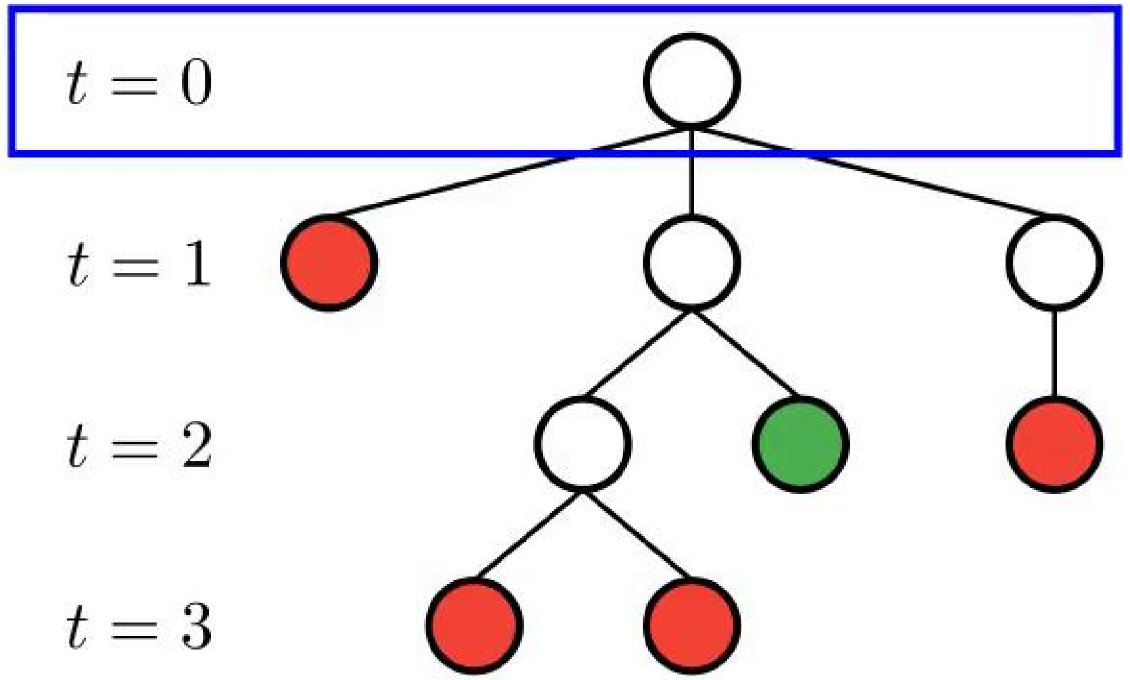

```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



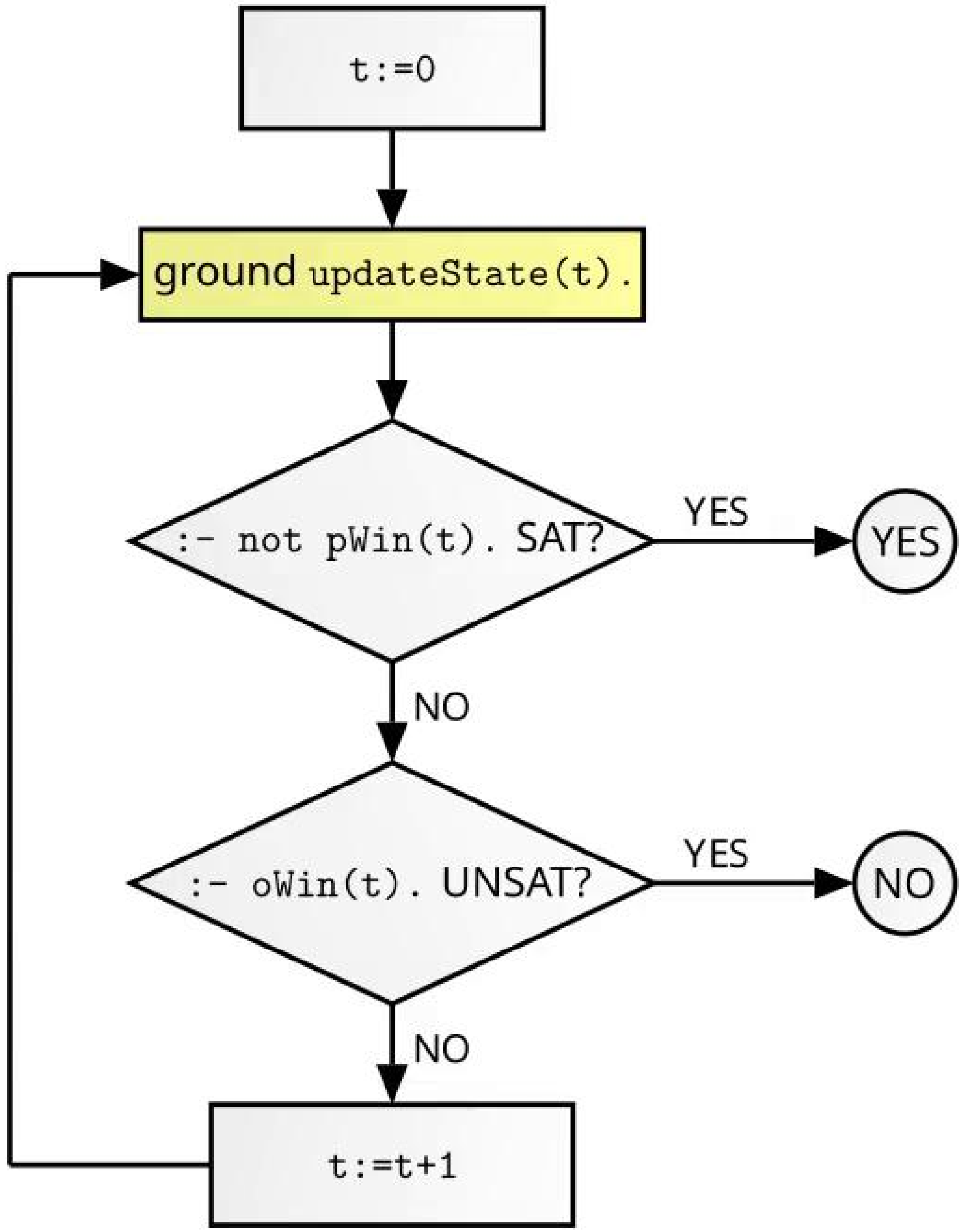
Multi-Shot ASP: Dynamic Solving & Grounding



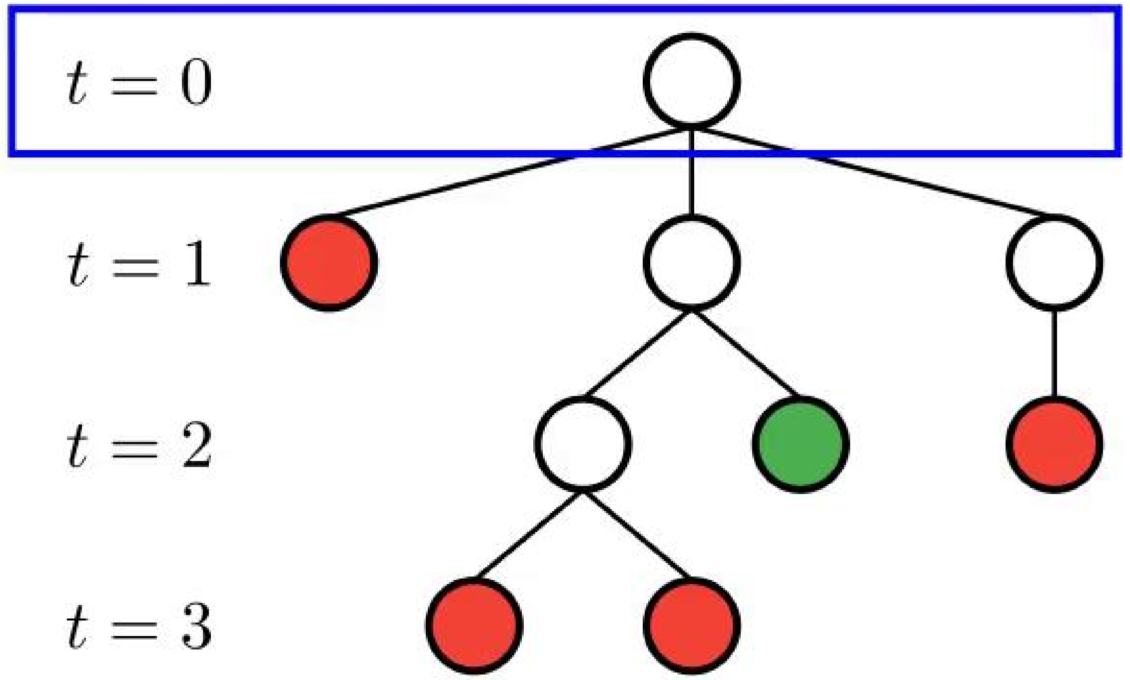

```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



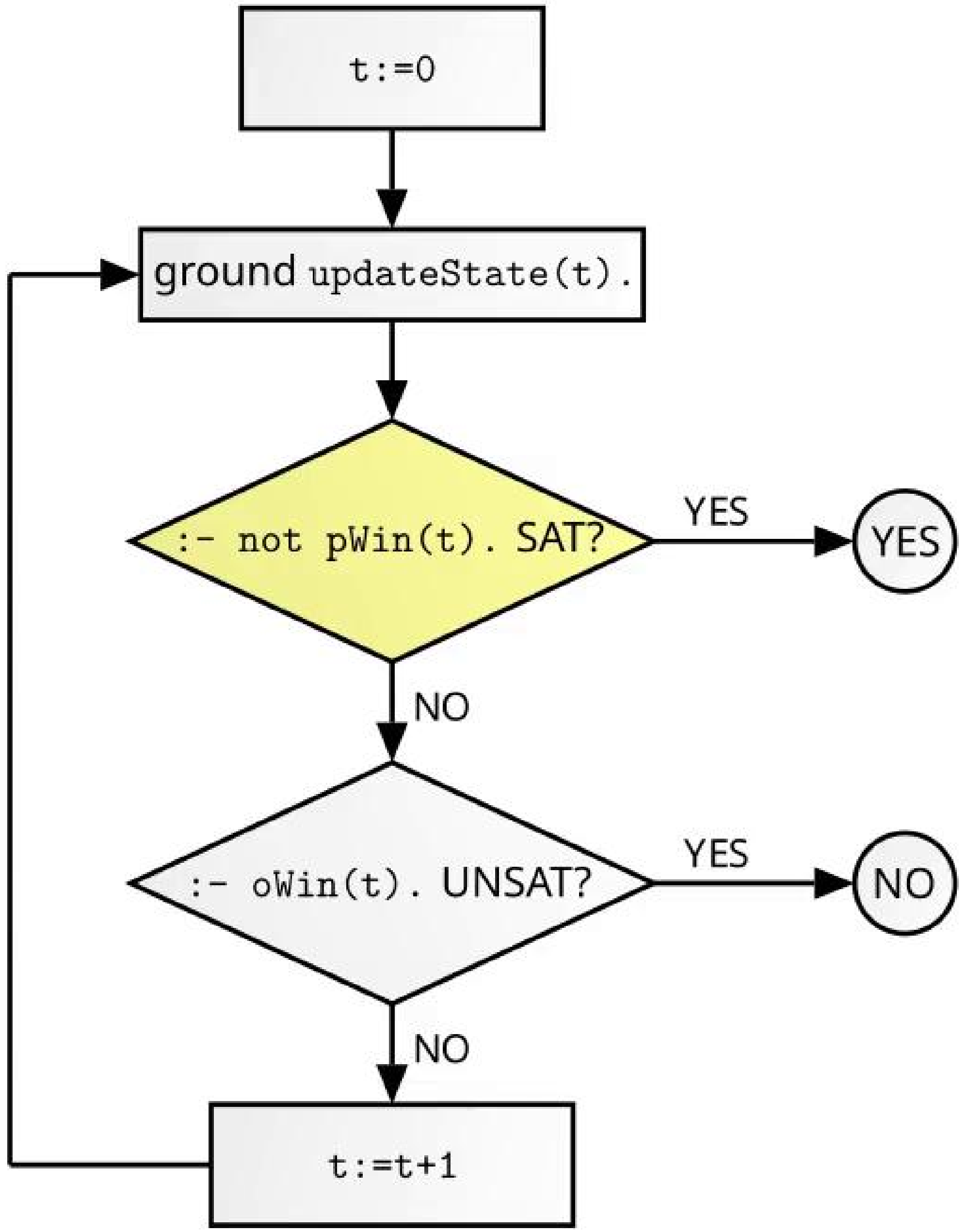
Multi-Shot ASP: Dynamic Solving & Grounding




```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



Multi-Shot ASP: Dynamic Solving & Grounding




```
pWin(t) :- termination(t), not possibleMove(t,o,_).
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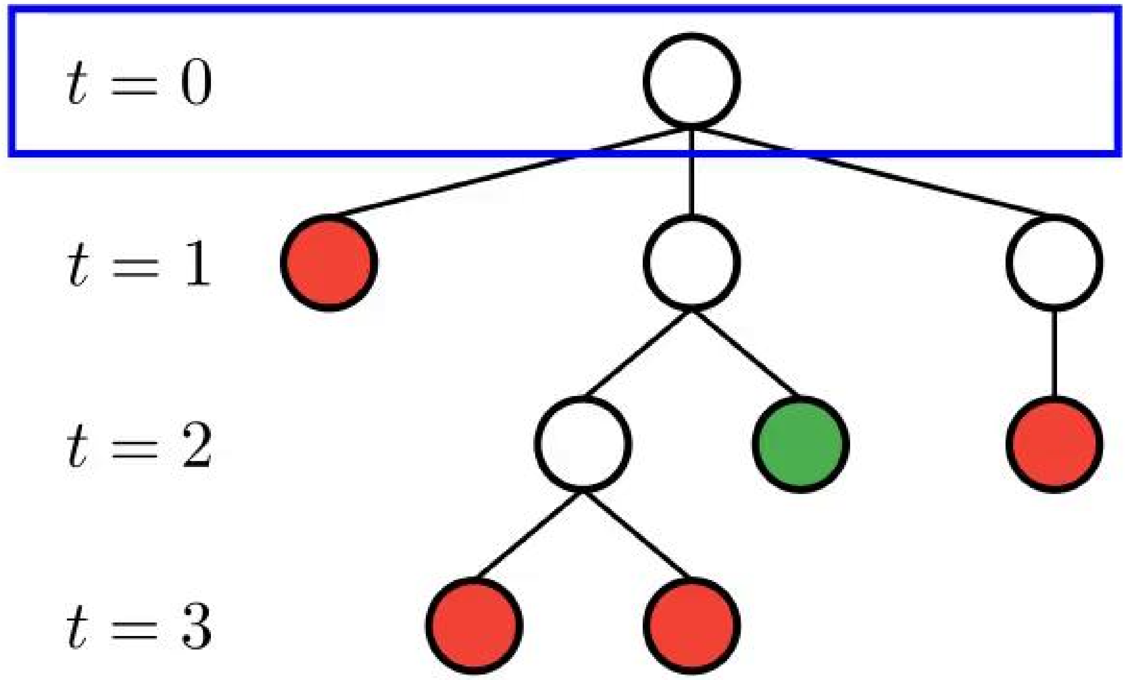
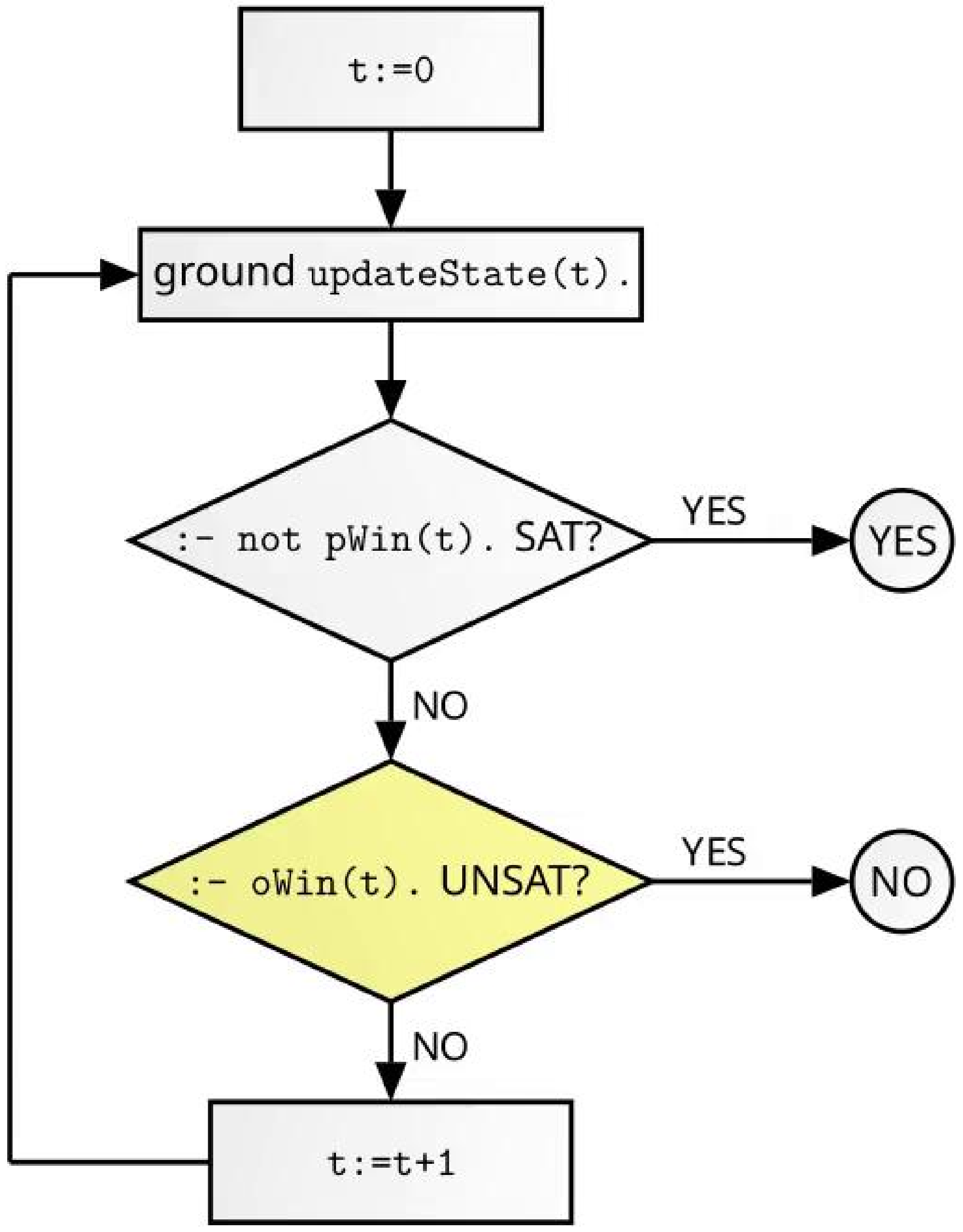
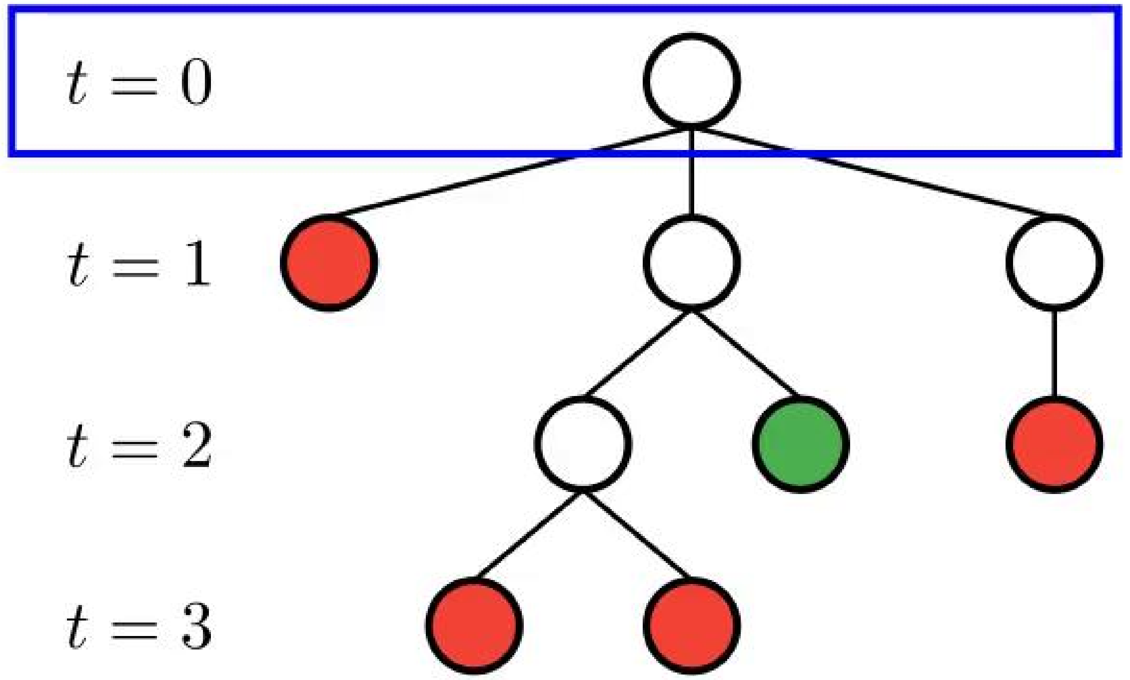


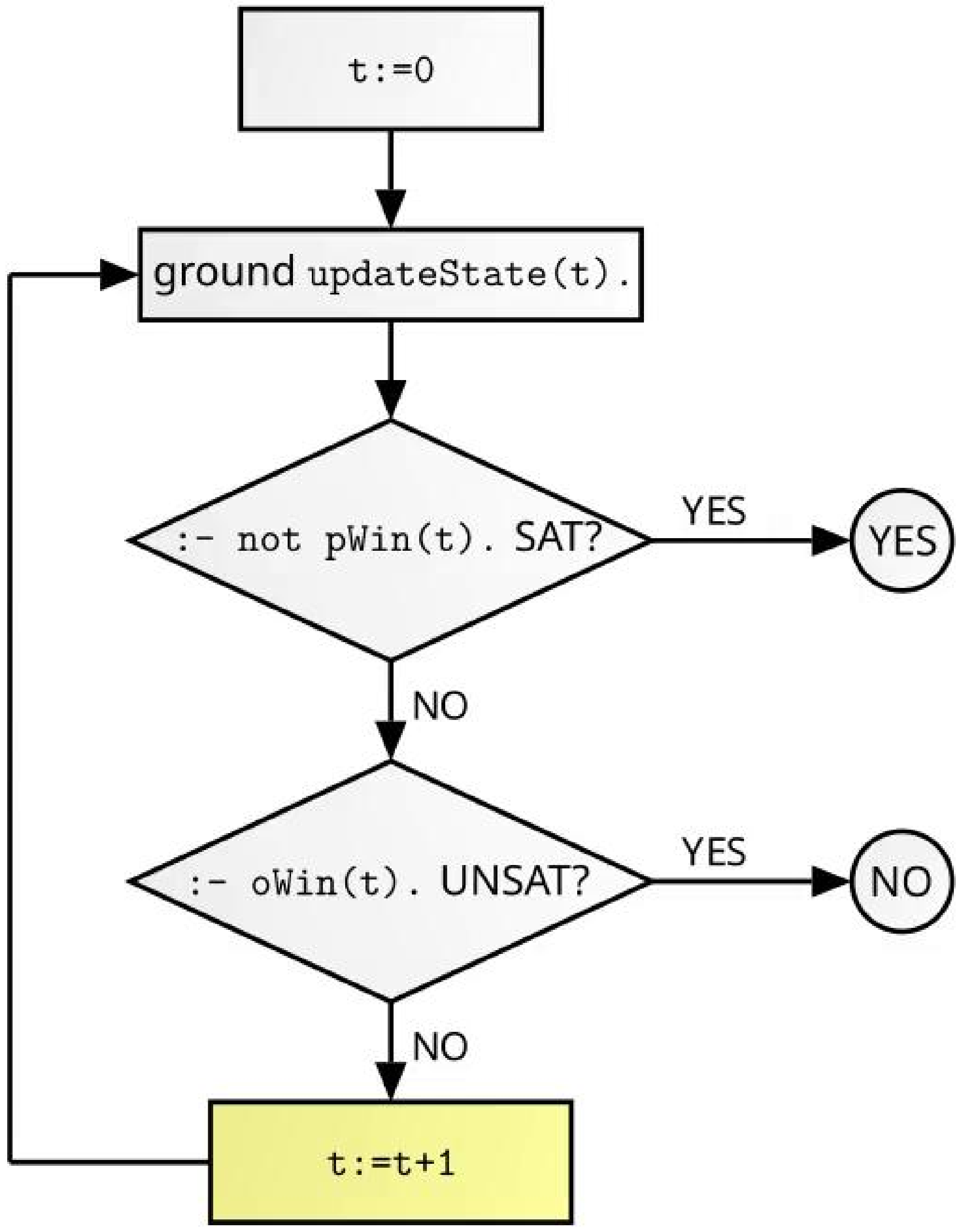
Multi-Shot ASP: Dynamic Solving & Grounding




```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



Multi-Shot ASP: Dynamic Solving & Grounding




```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
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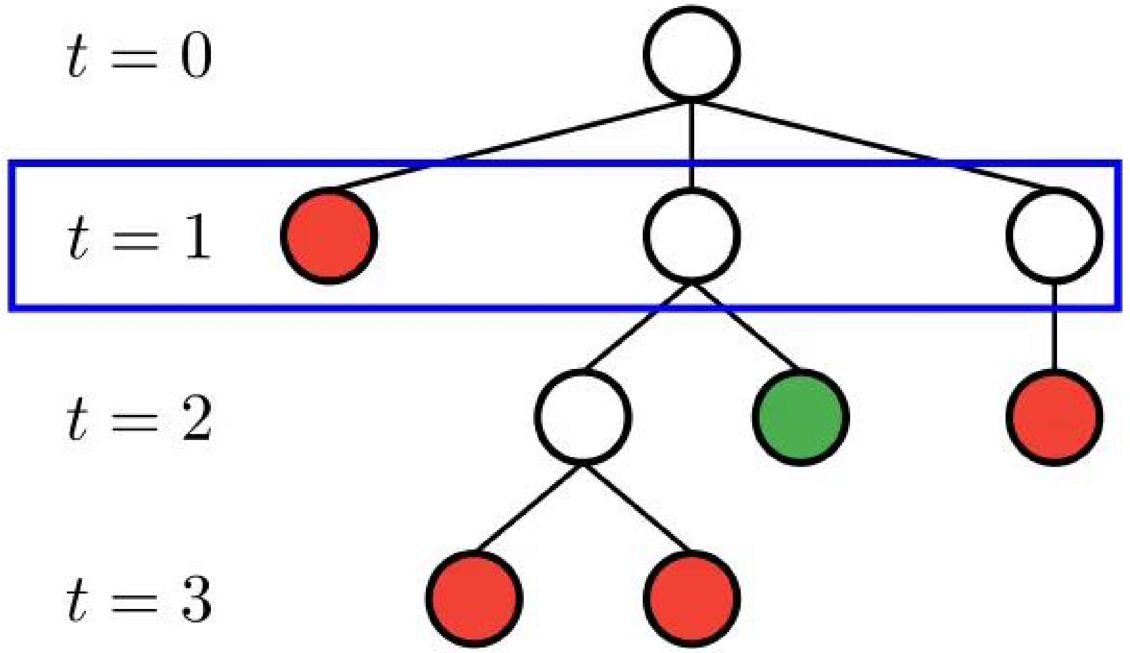

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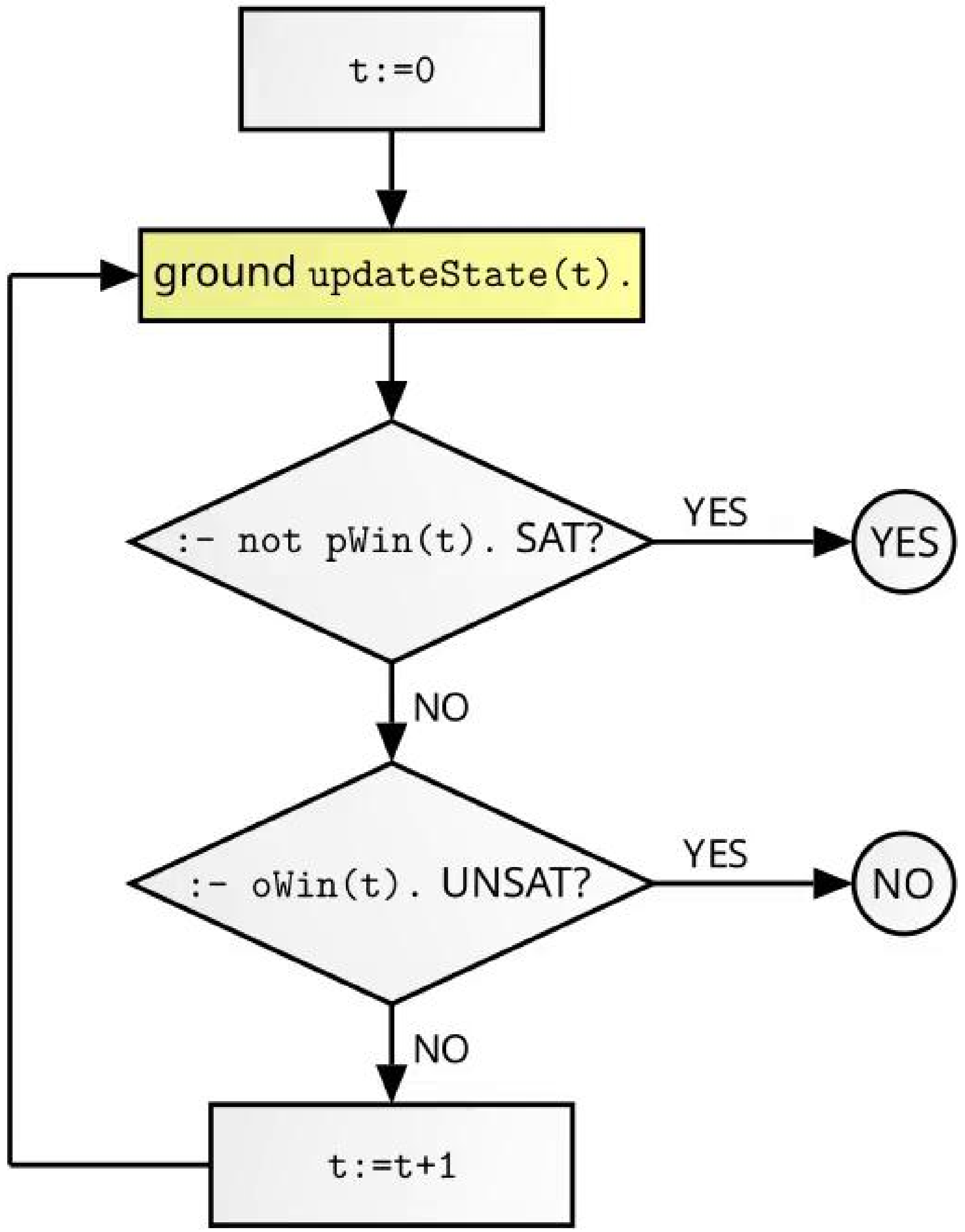


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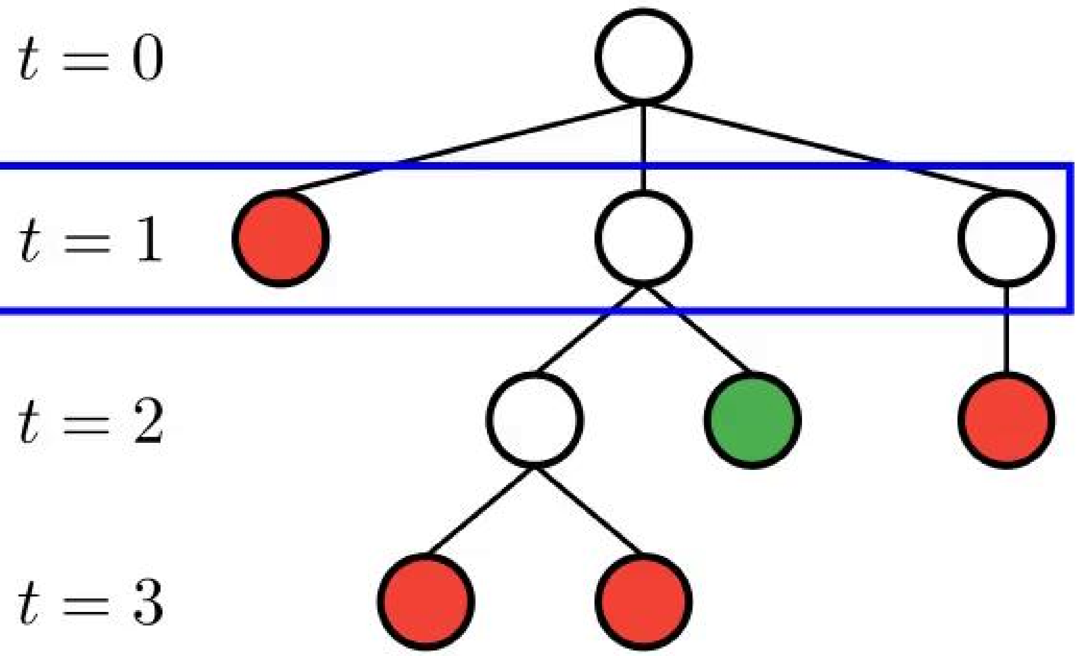

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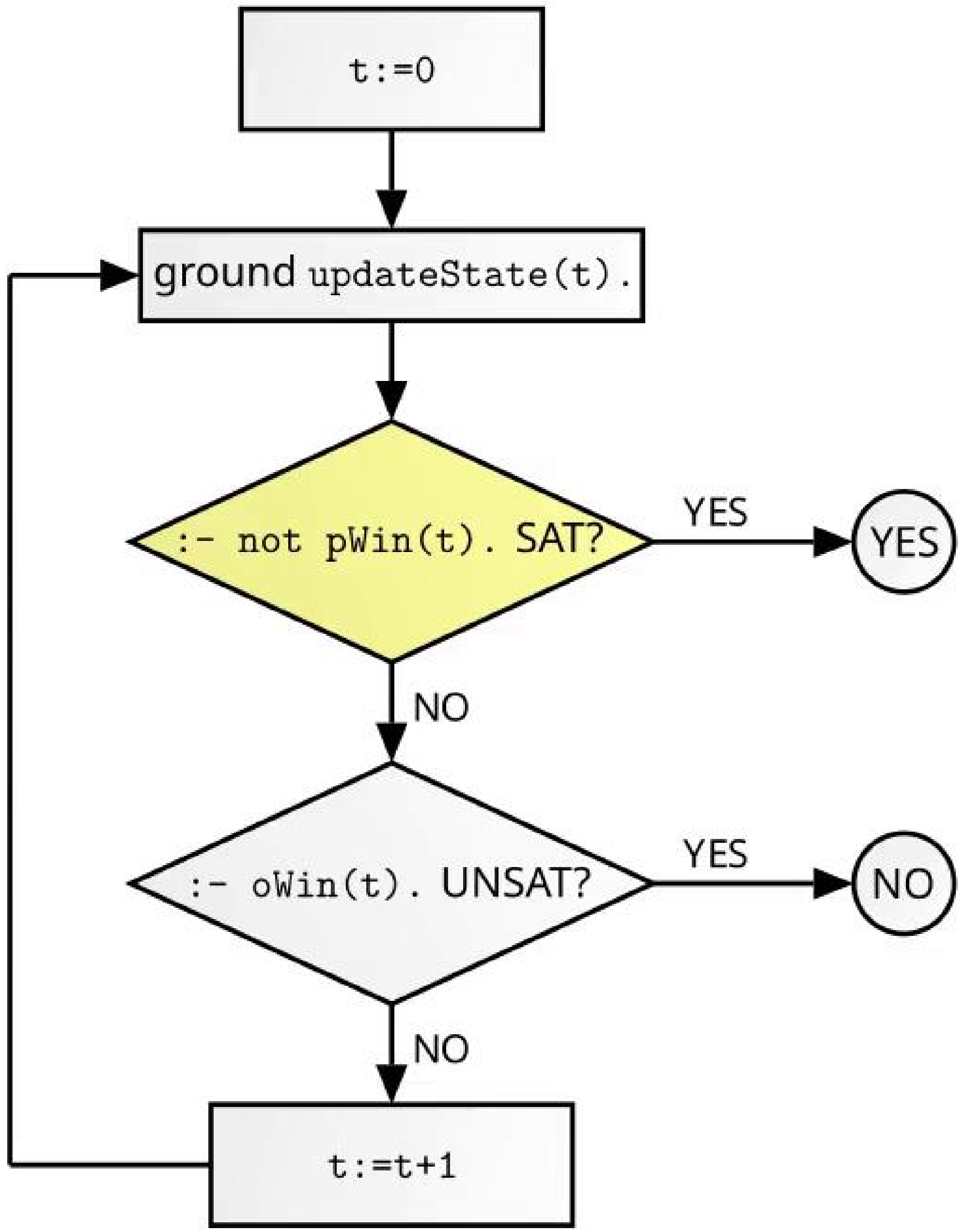


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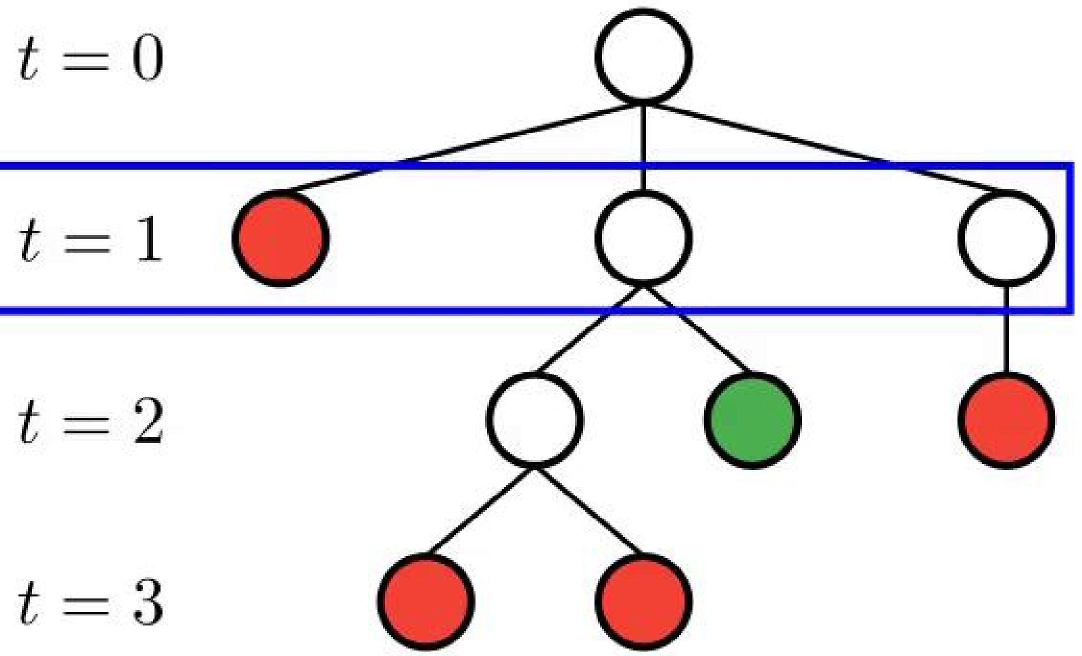
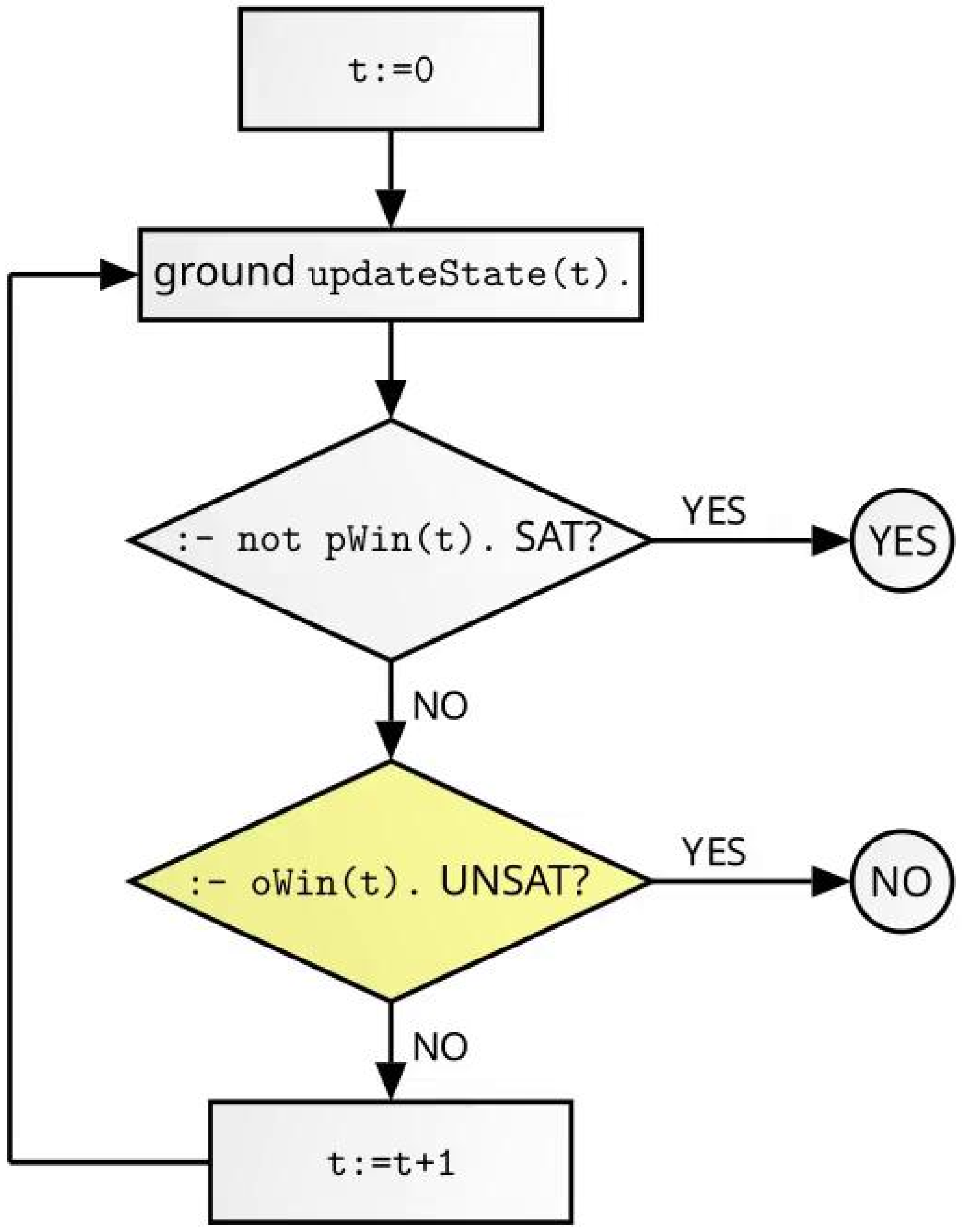


Multi-Shot ASP: Dynamic Solving & Grounding

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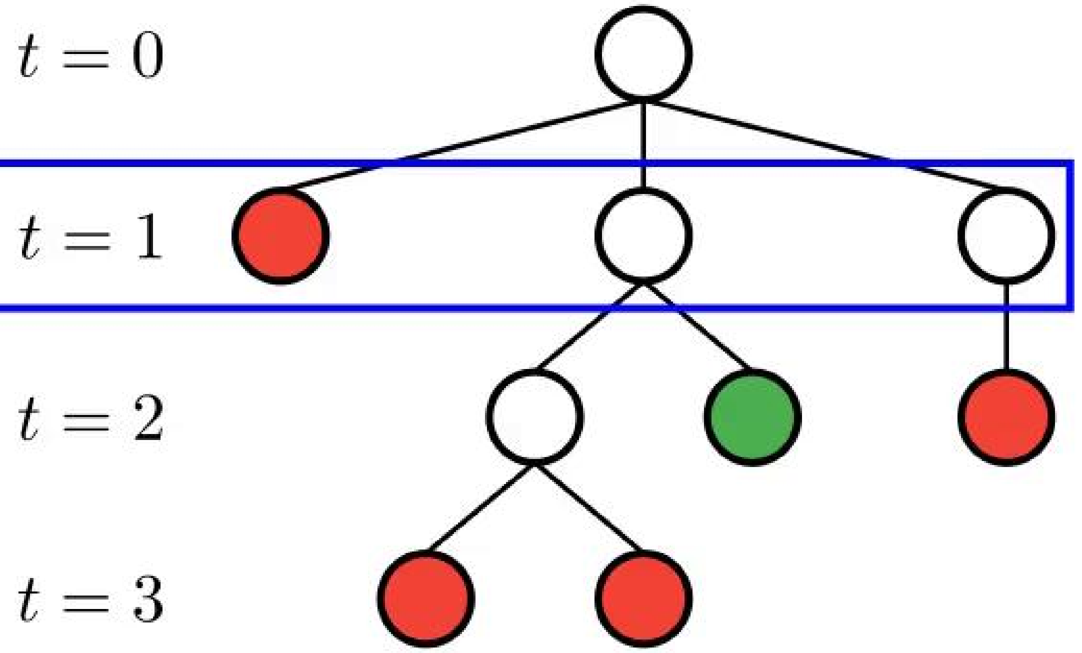

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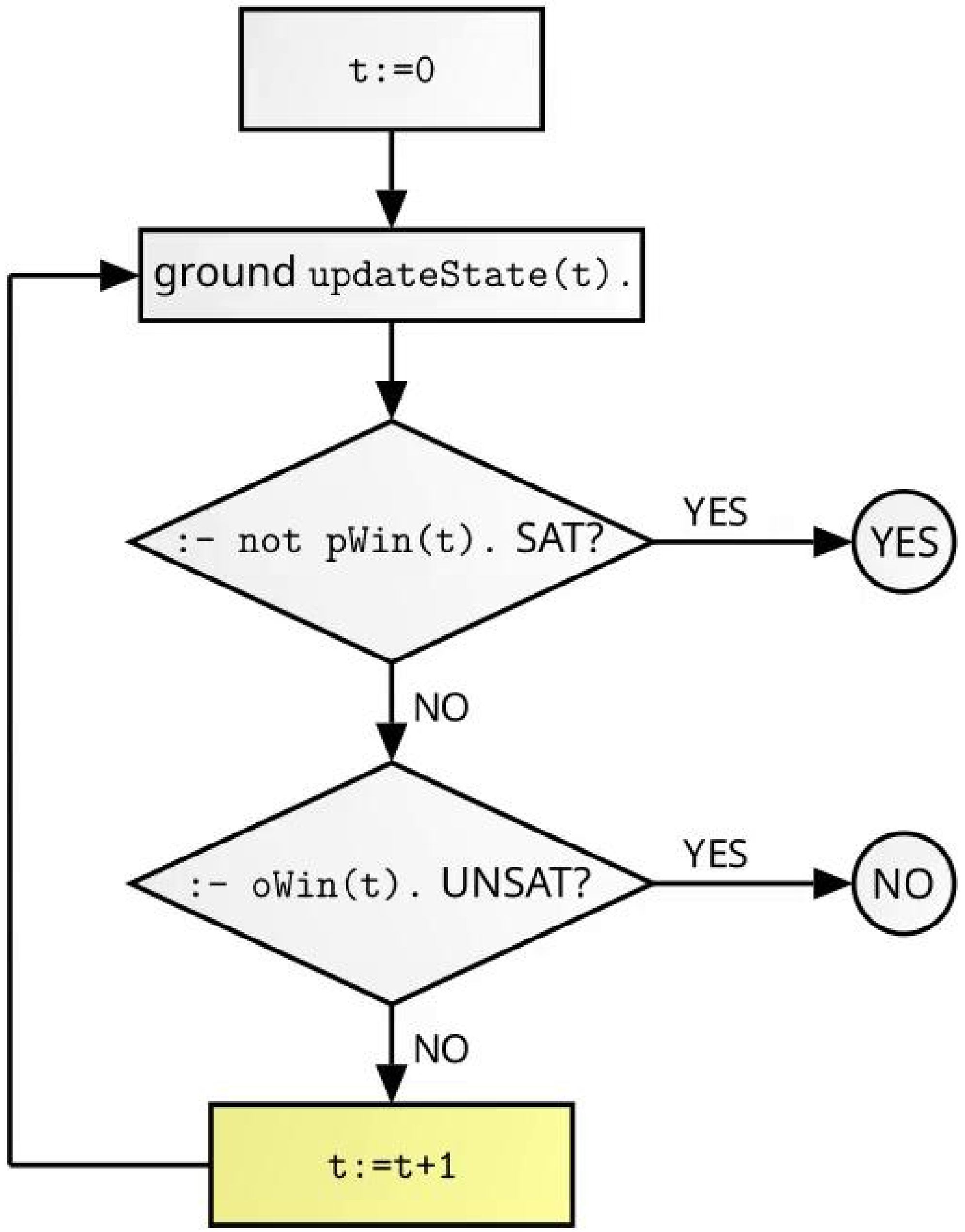


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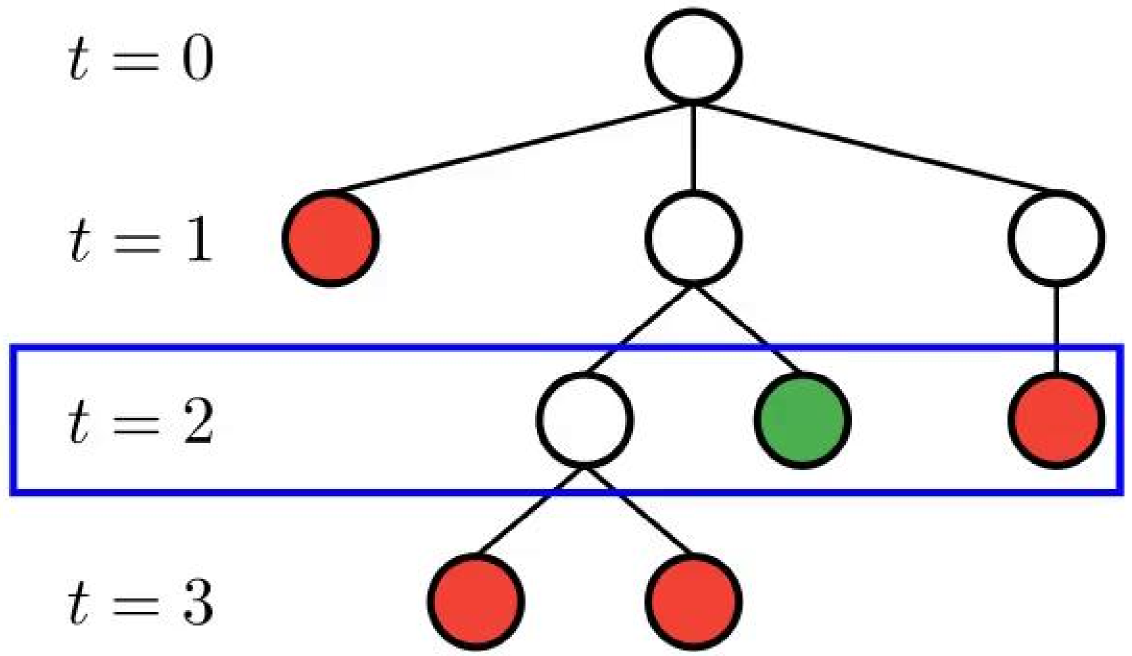

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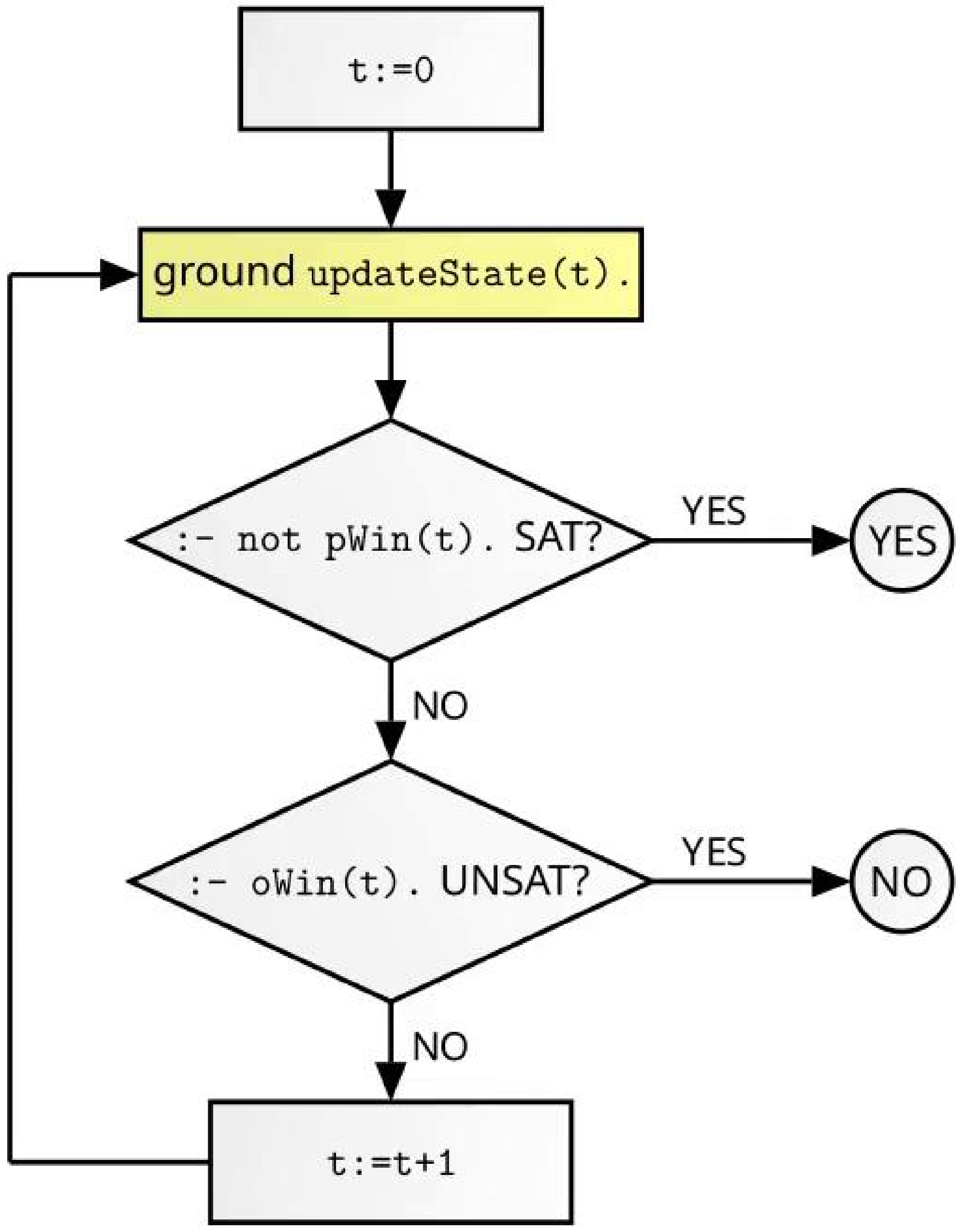


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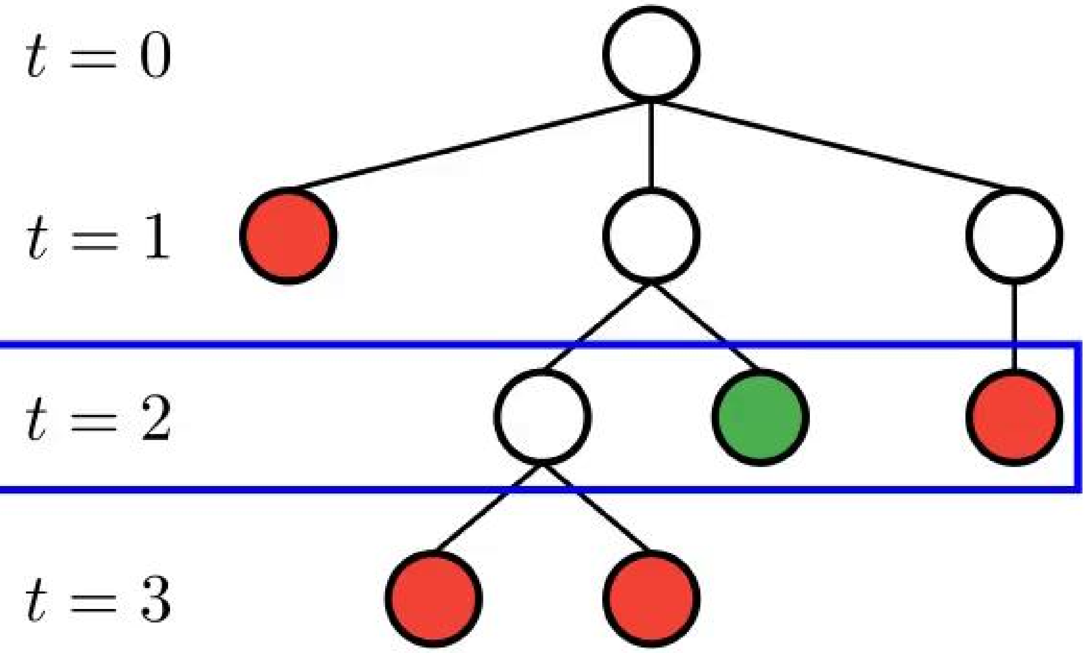

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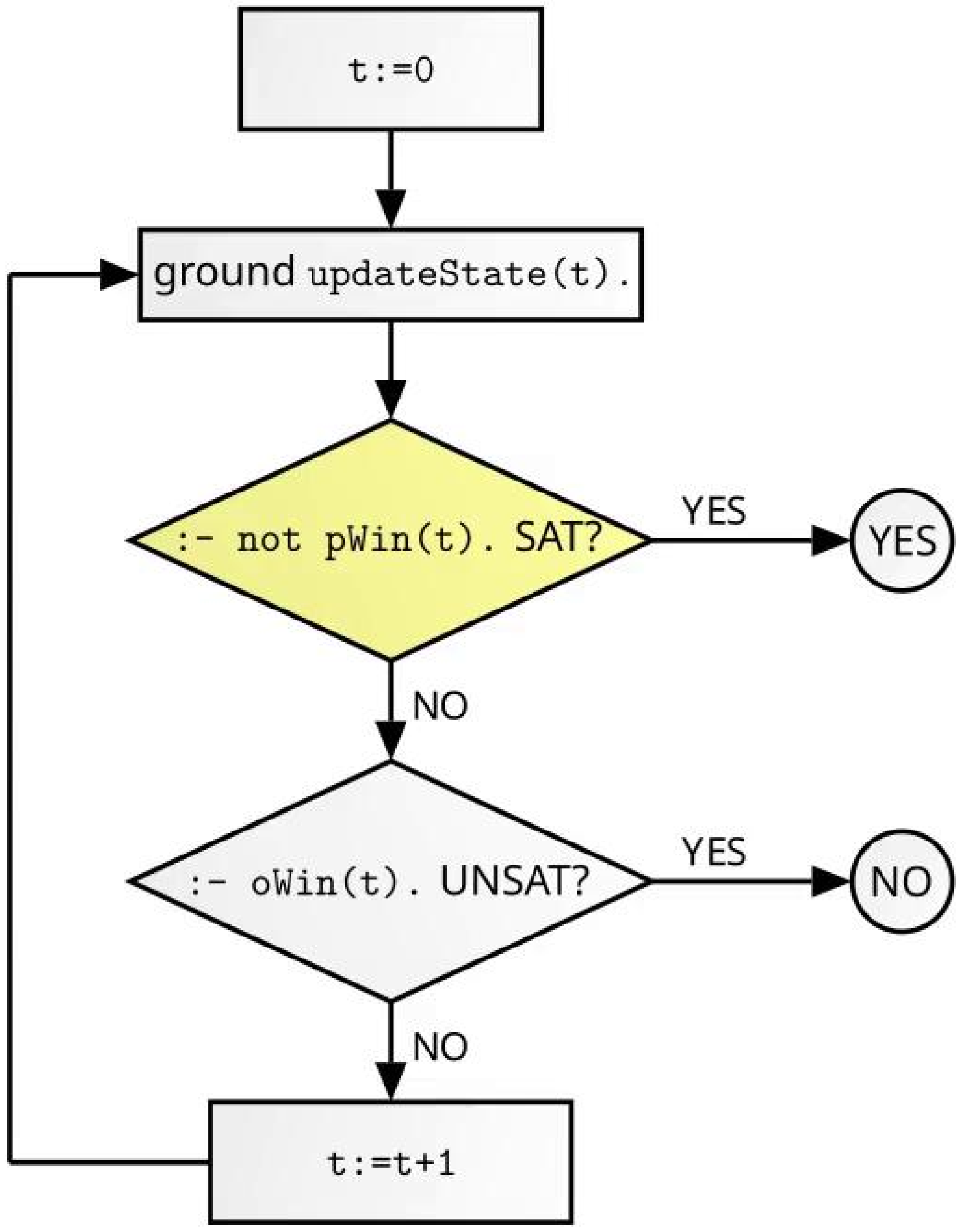


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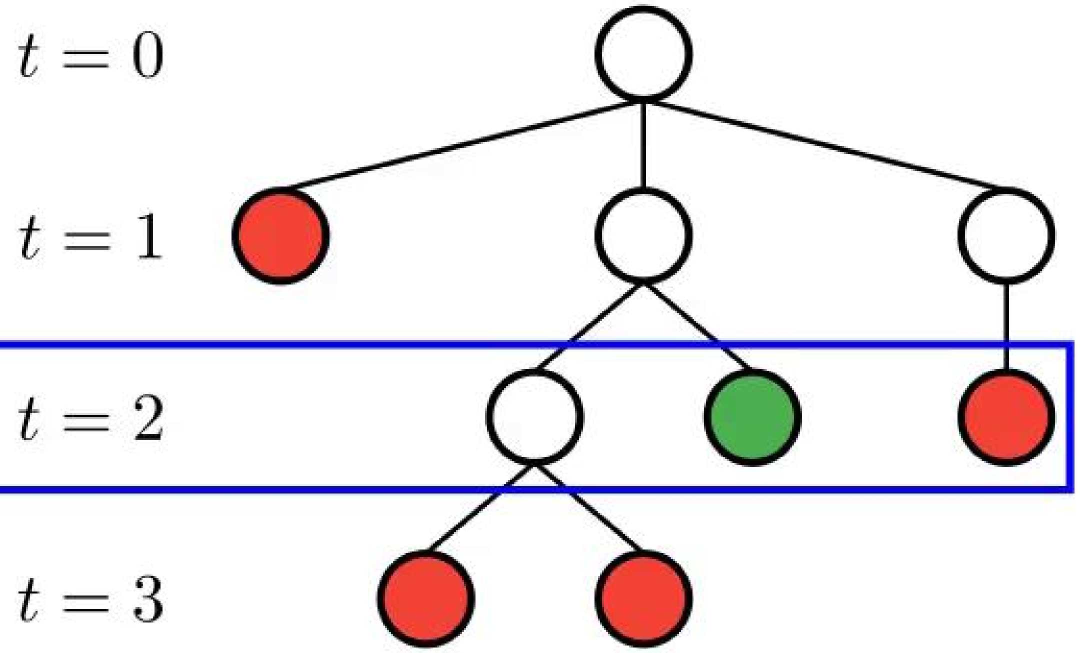

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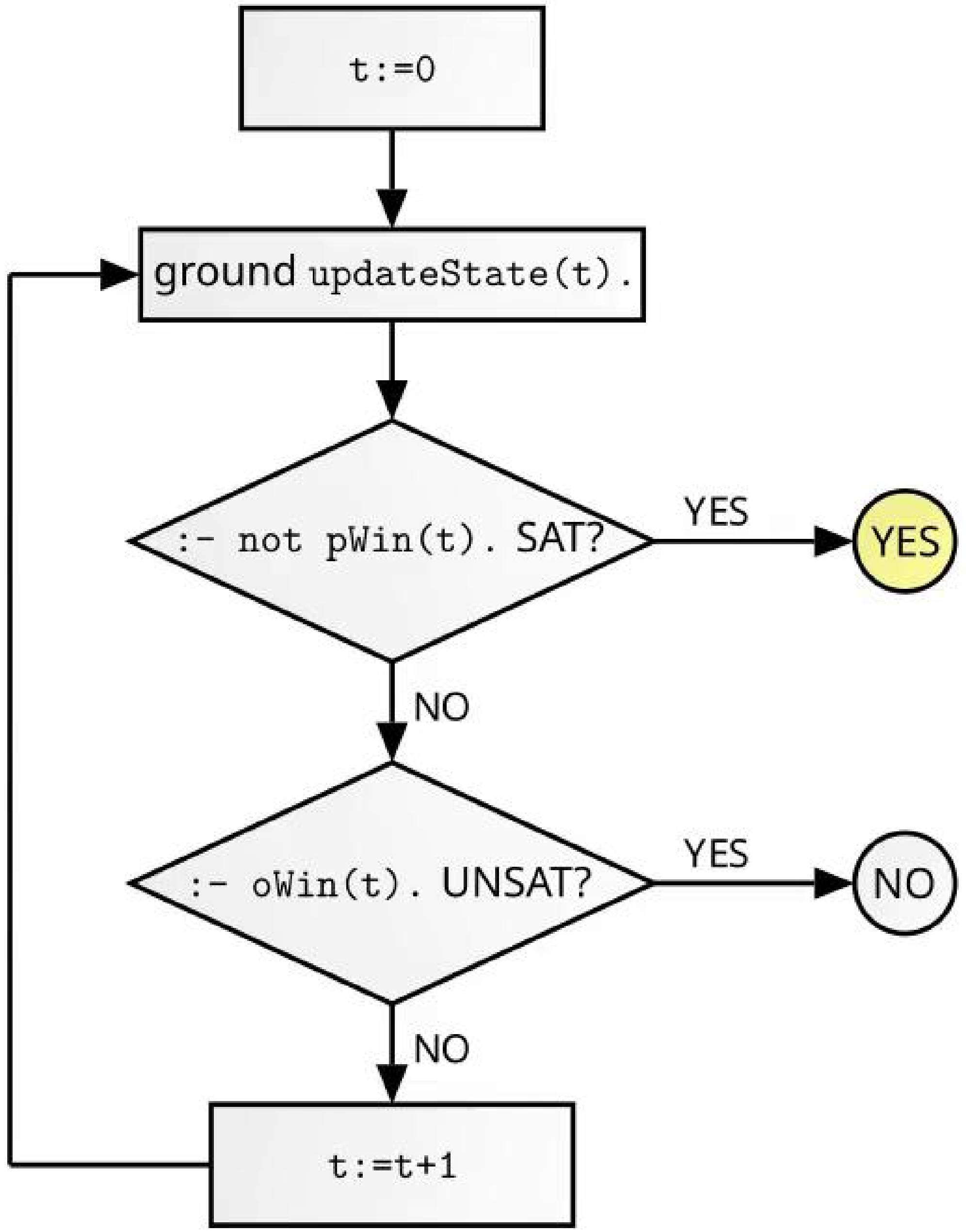


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pWin(t) :- termination(t), not possibleMove(t,o,_).
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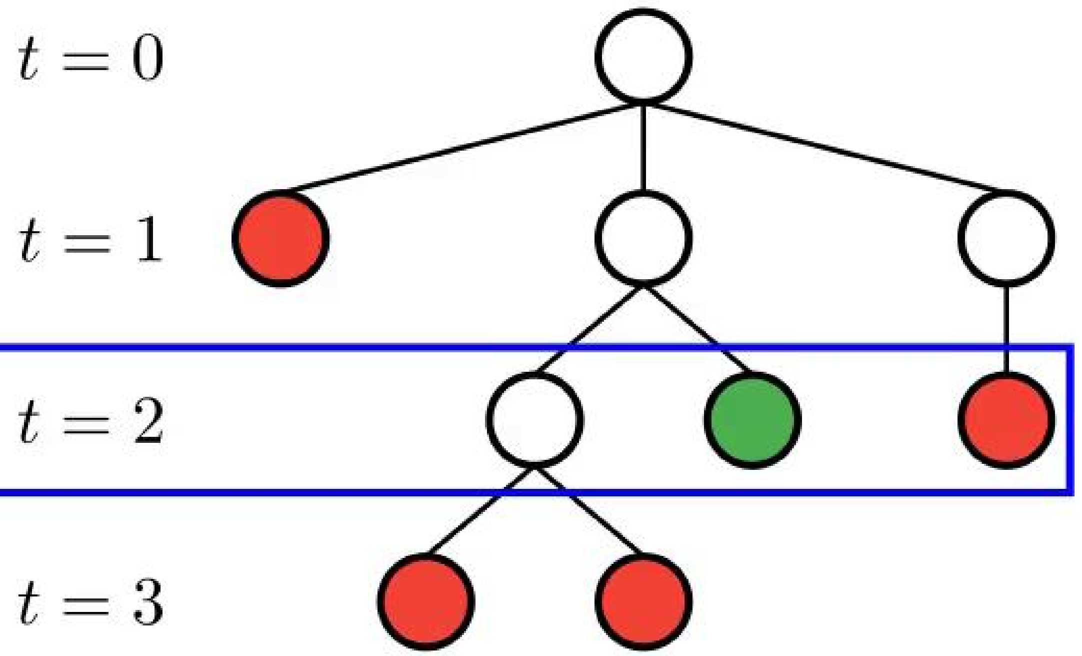
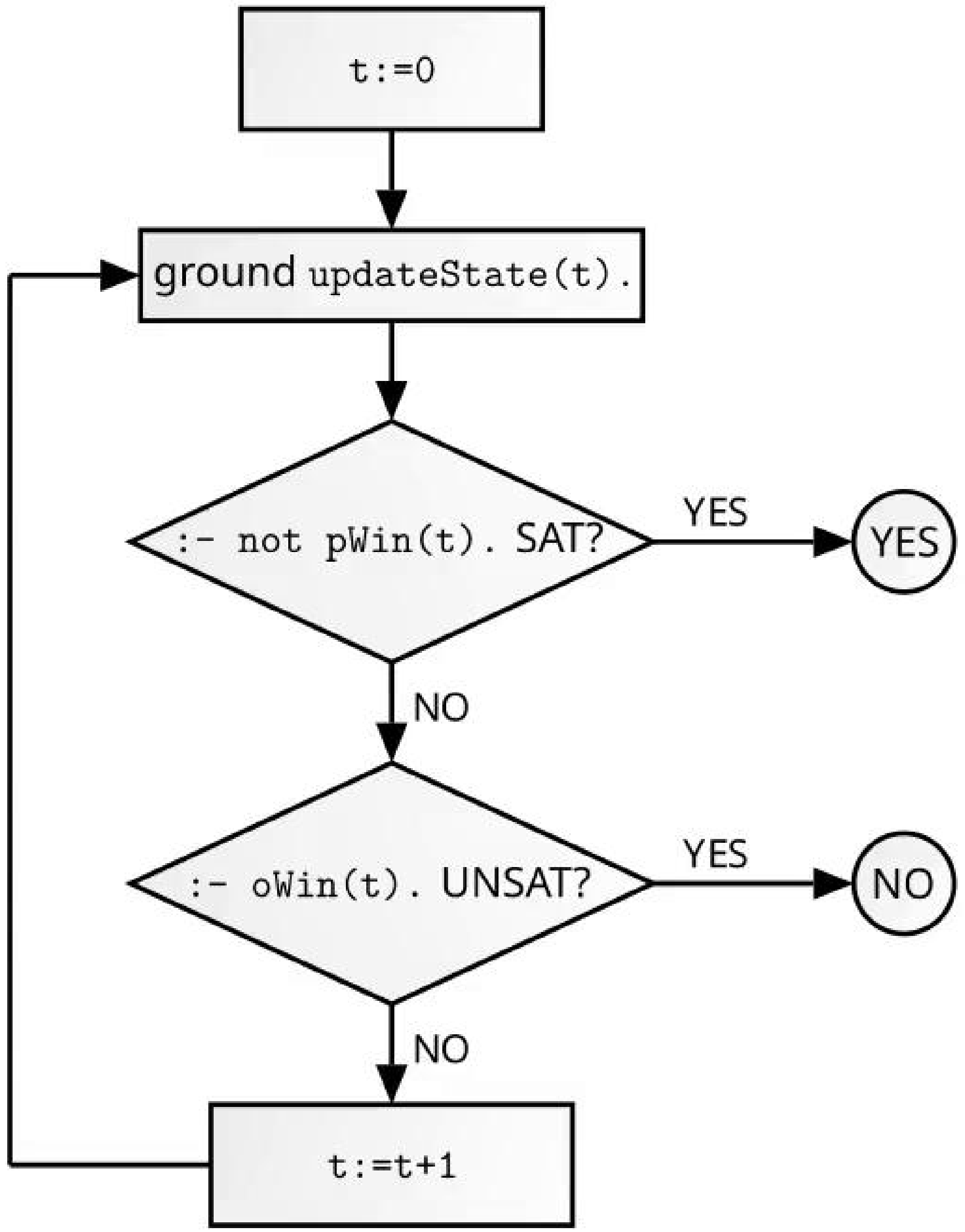


Multi-Shot ASP: Dynamic Solving & Grounding

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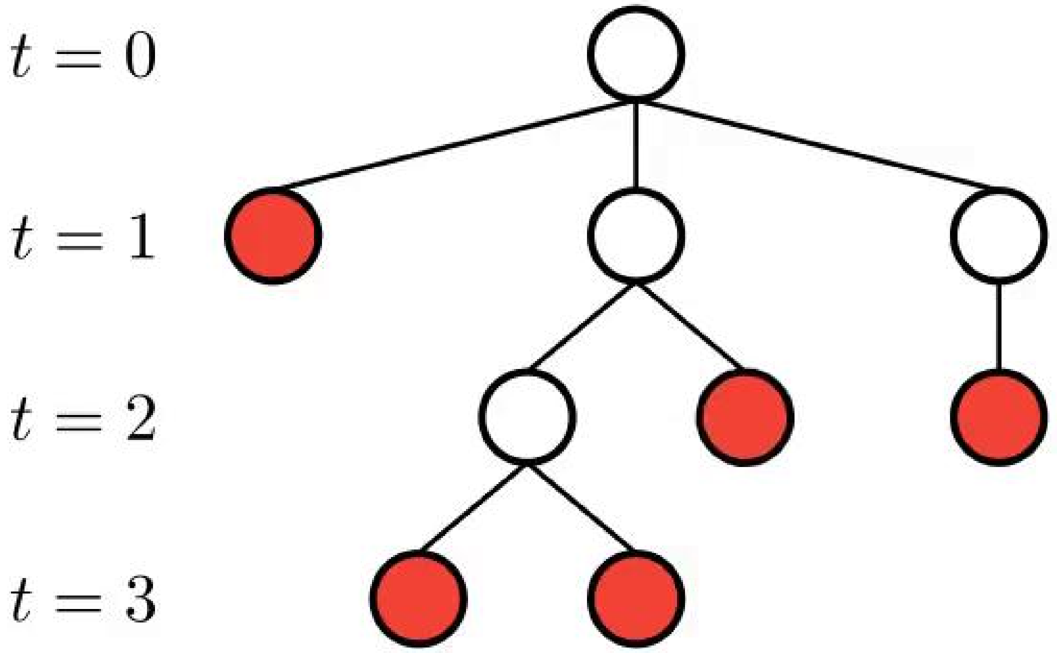

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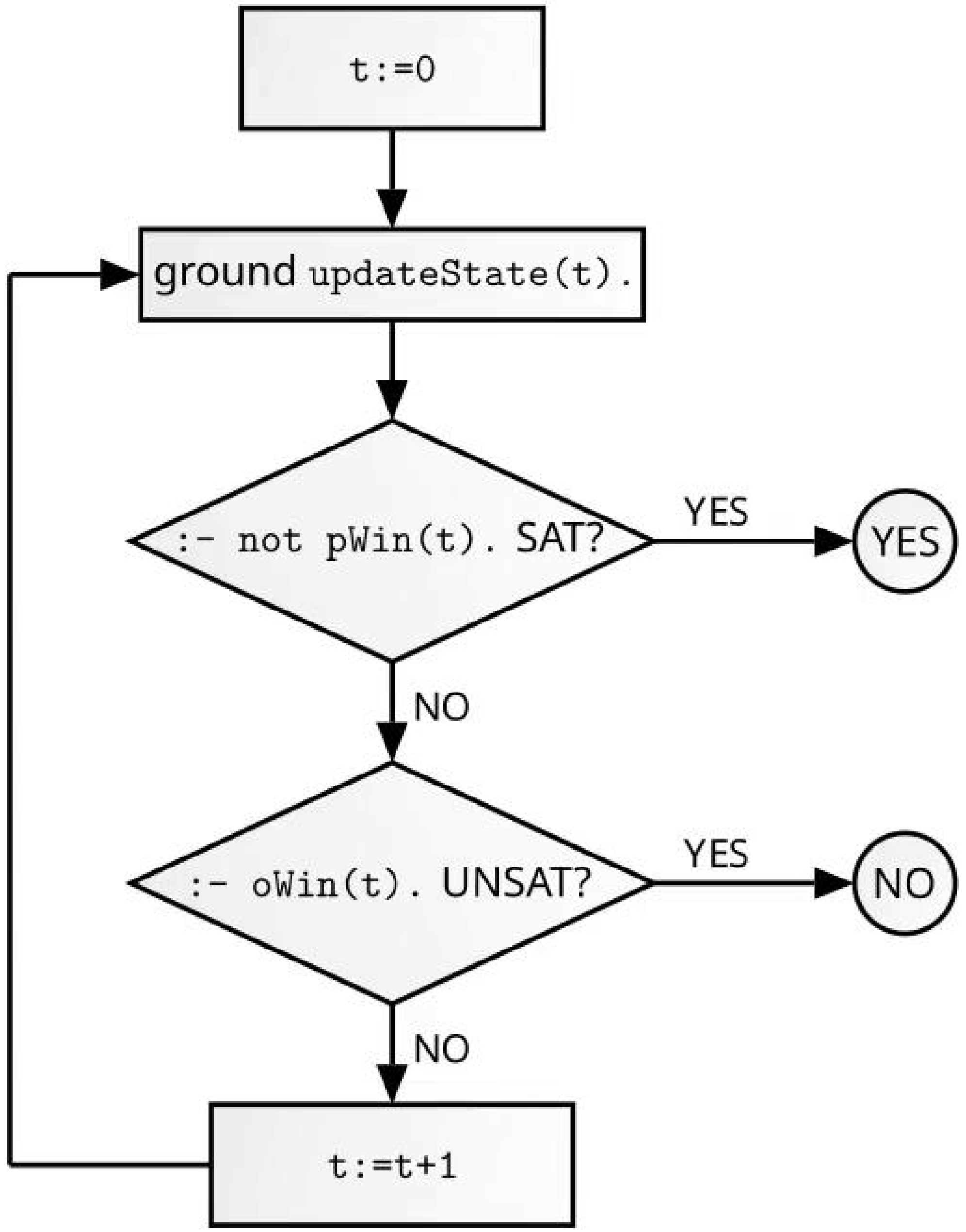


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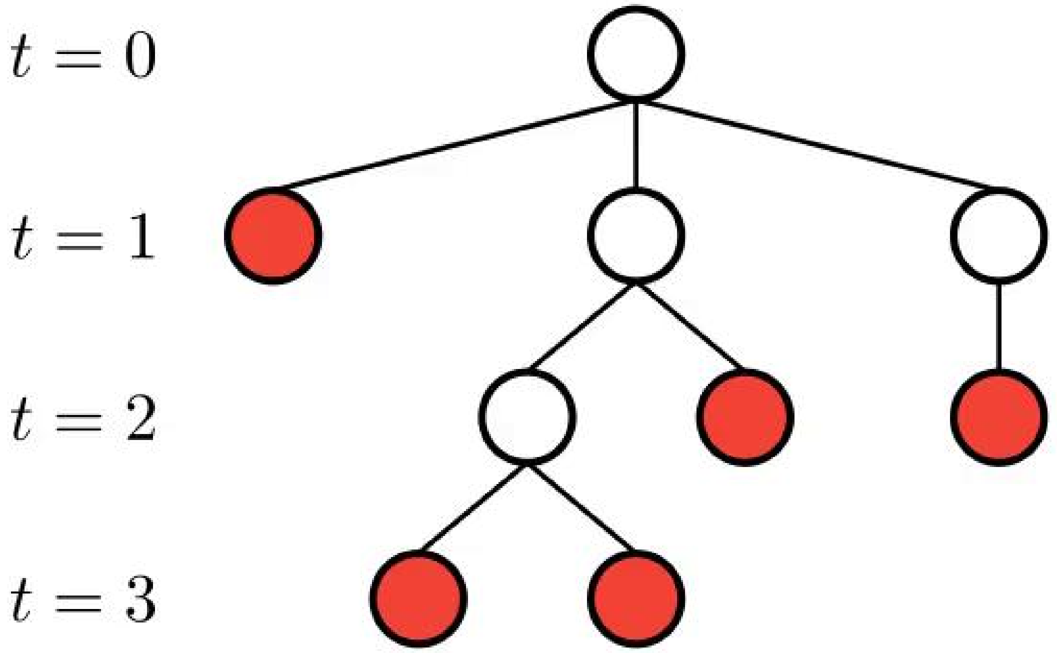

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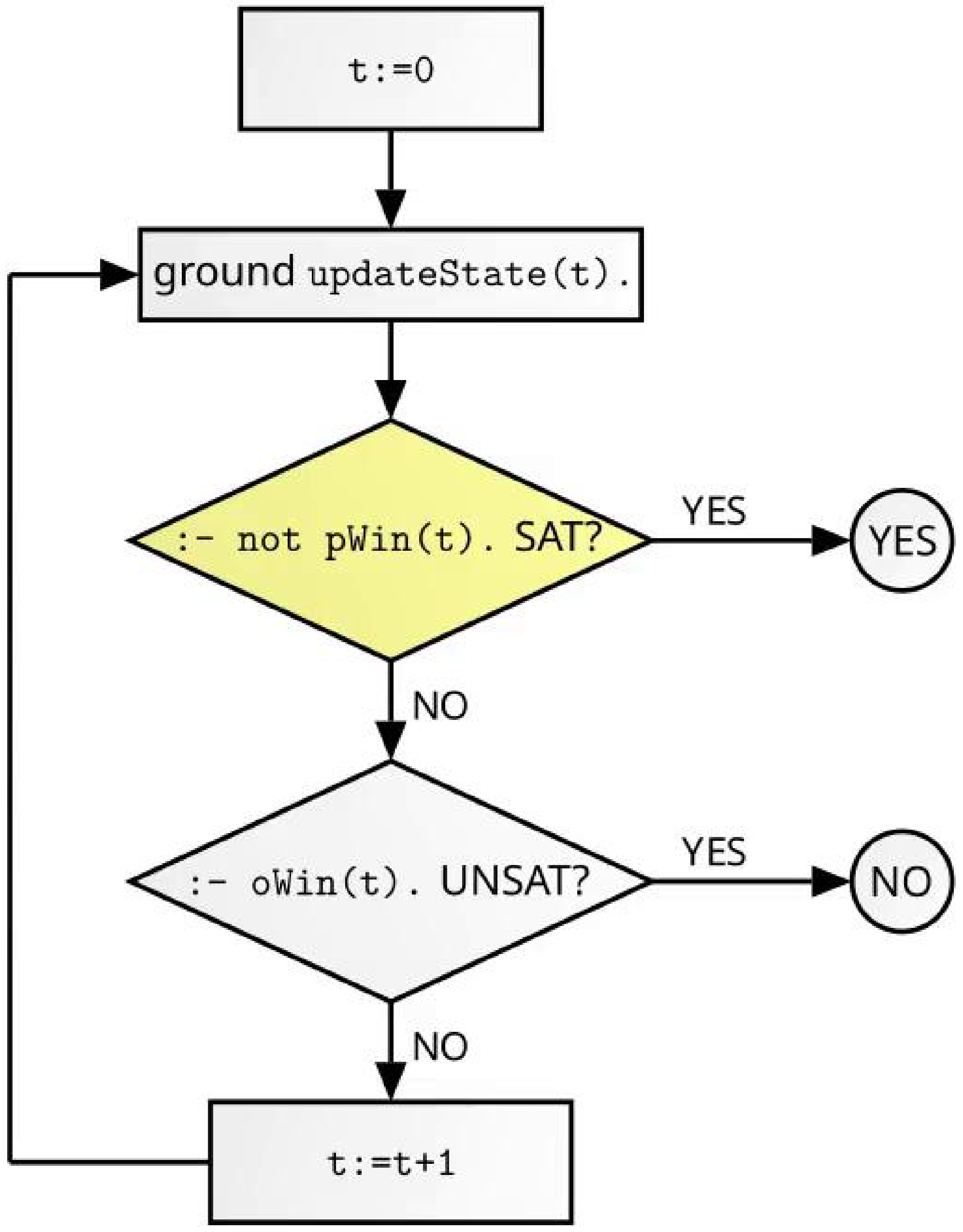
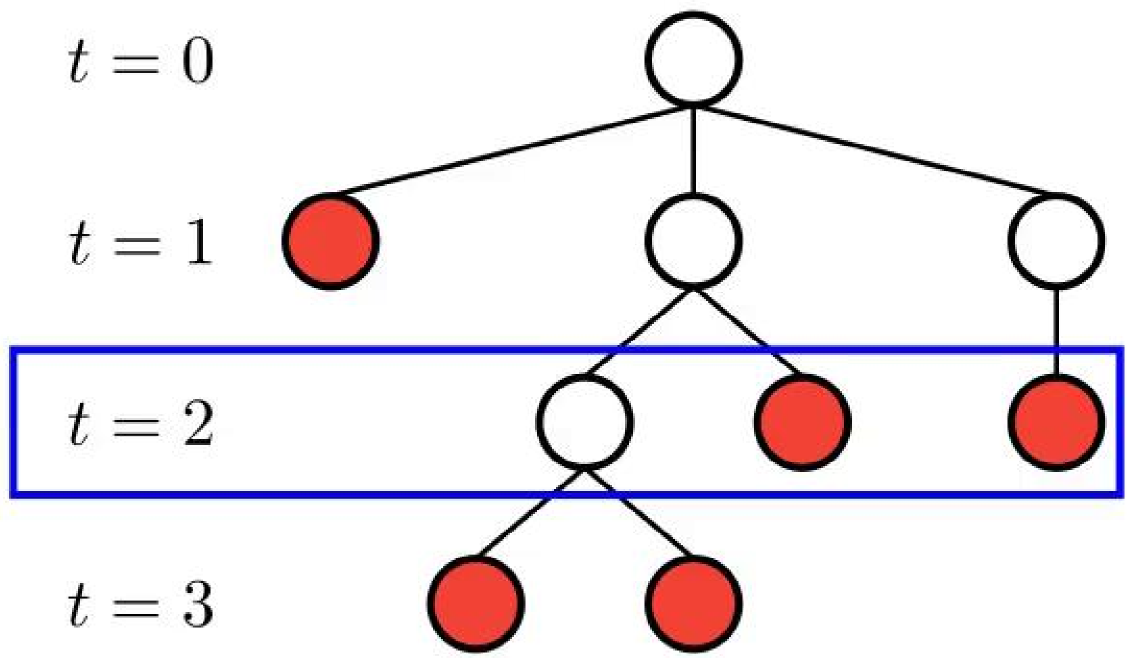
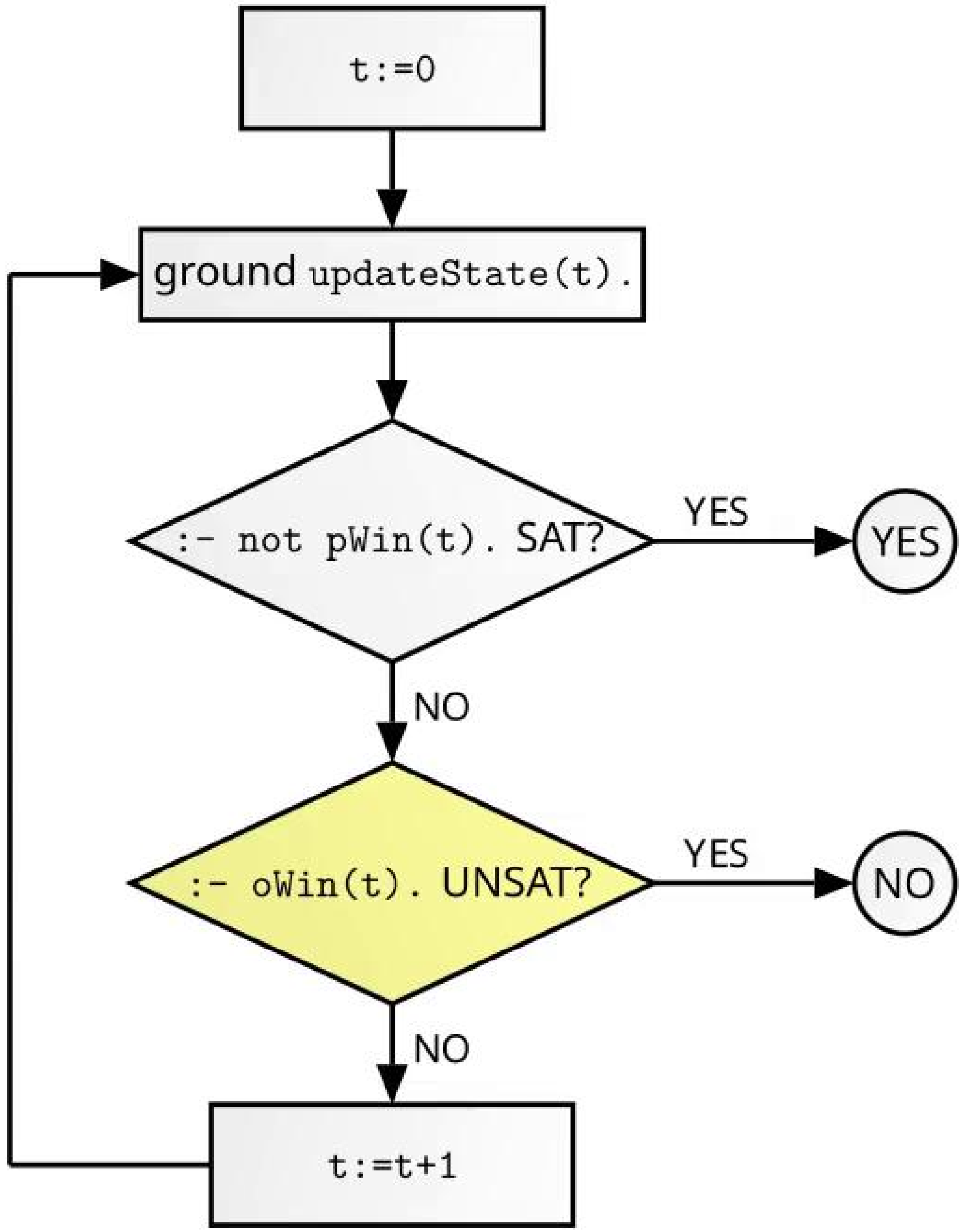


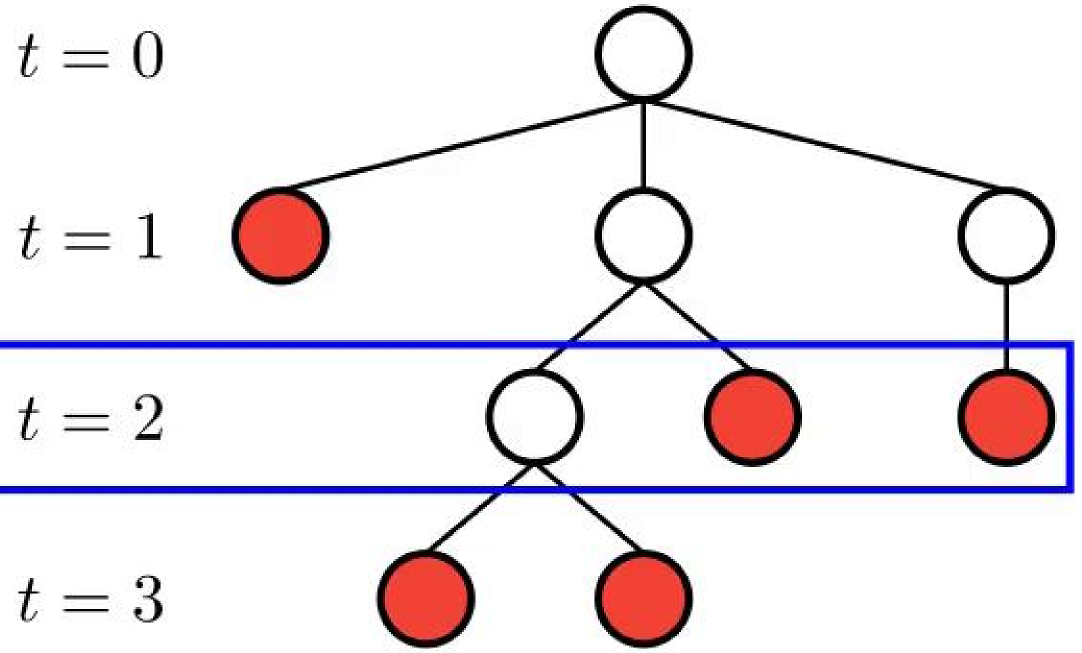

```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



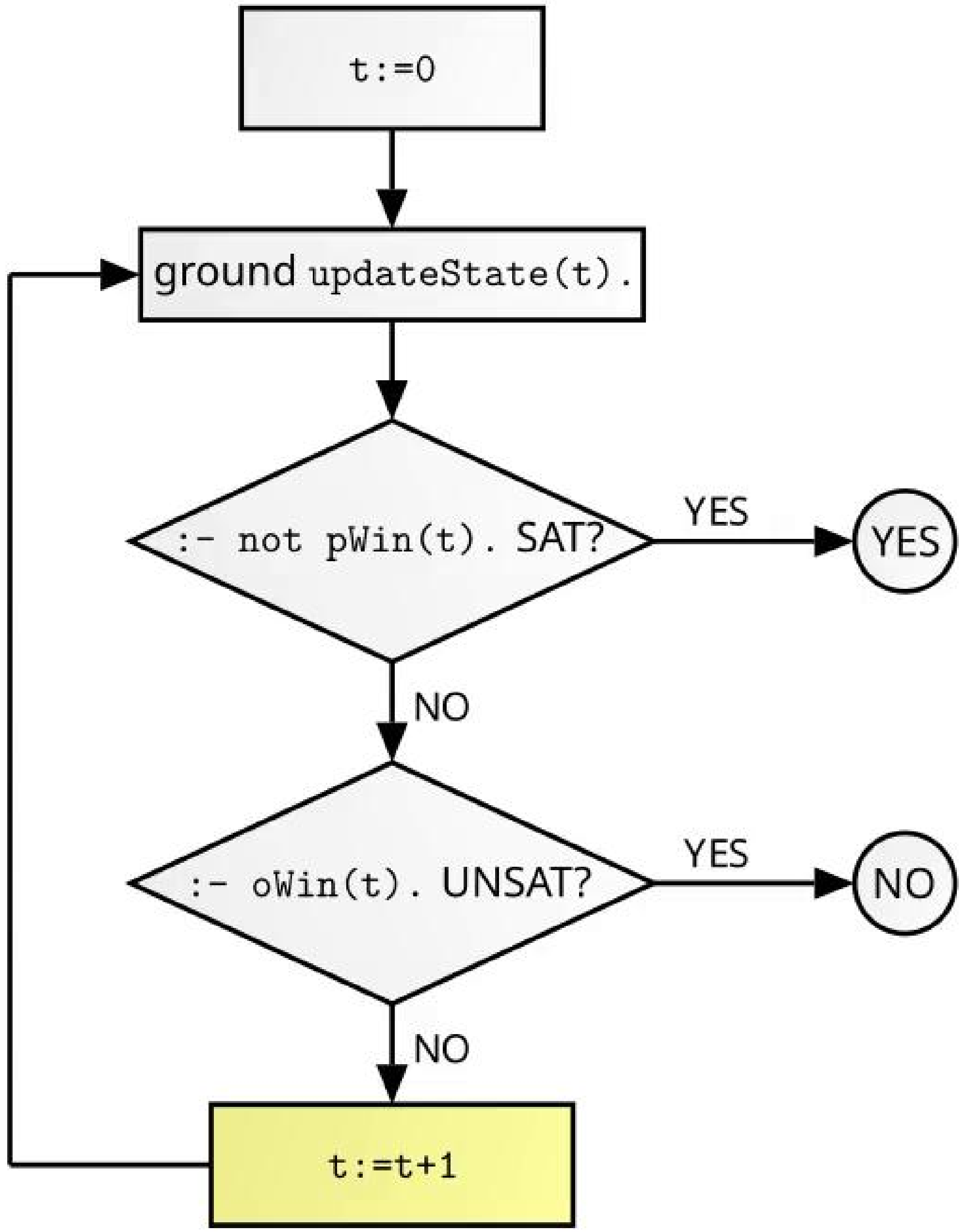
Multi-Shot ASP: Dynamic Solving & Grounding




```
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



Multi-Shot ASP: Dynamic Solving & Grounding

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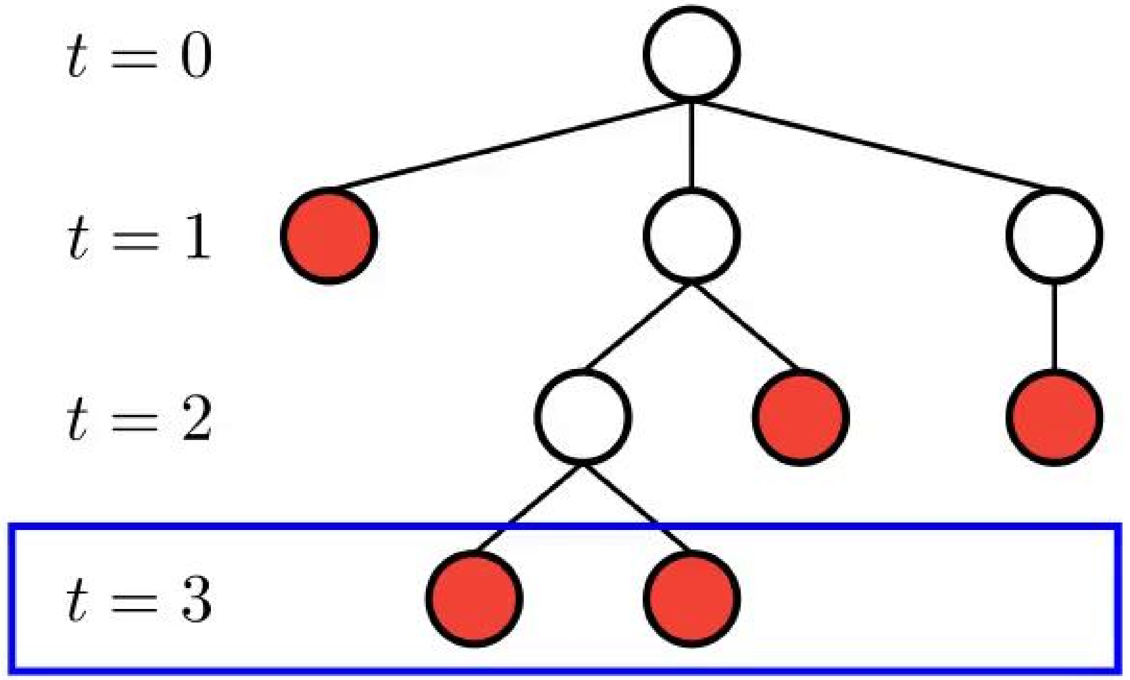
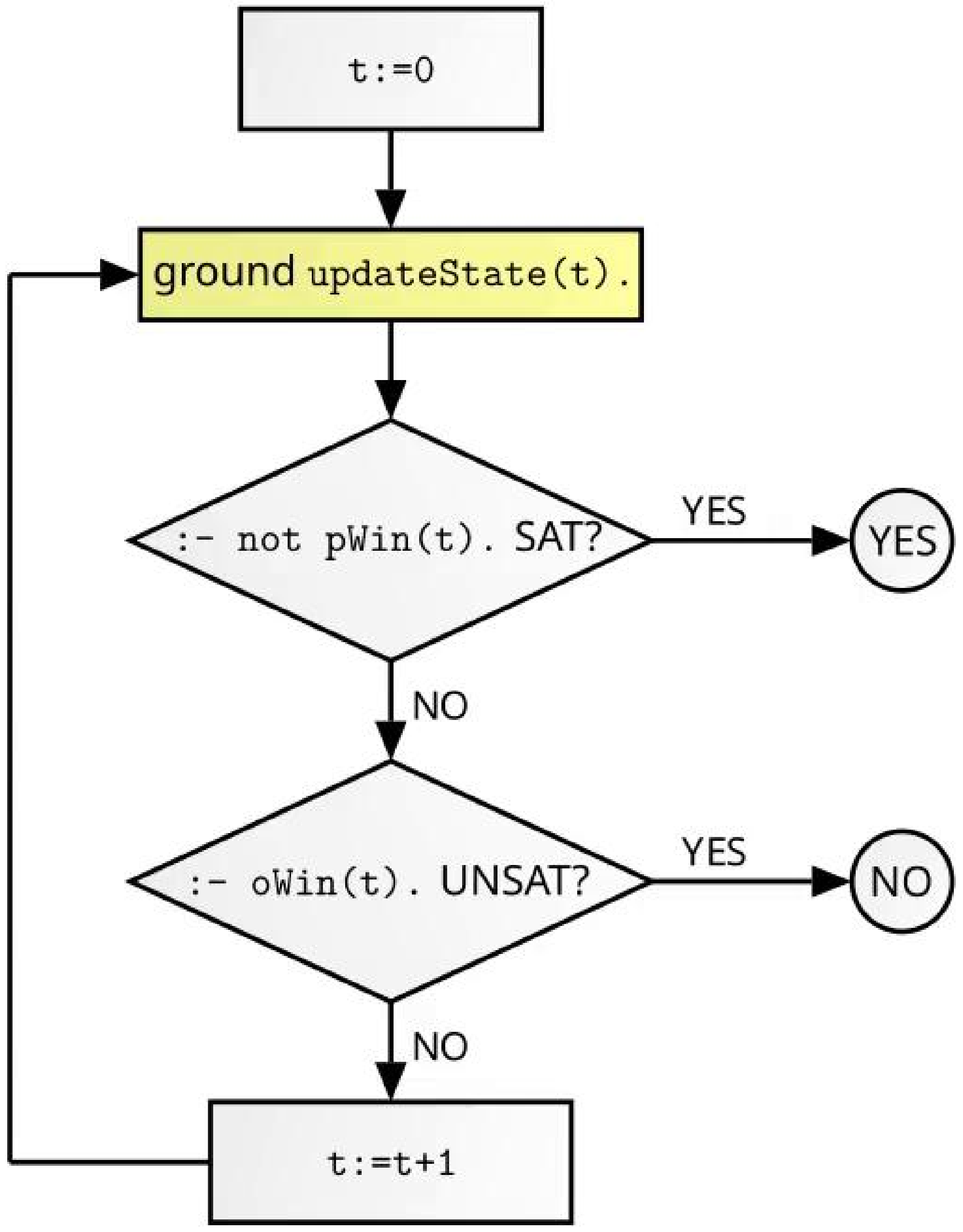
pWin(t) :- termination(t), not possibleMove(t,o,_).
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



Multi-Shot ASP: Dynamic Solving & Grounding

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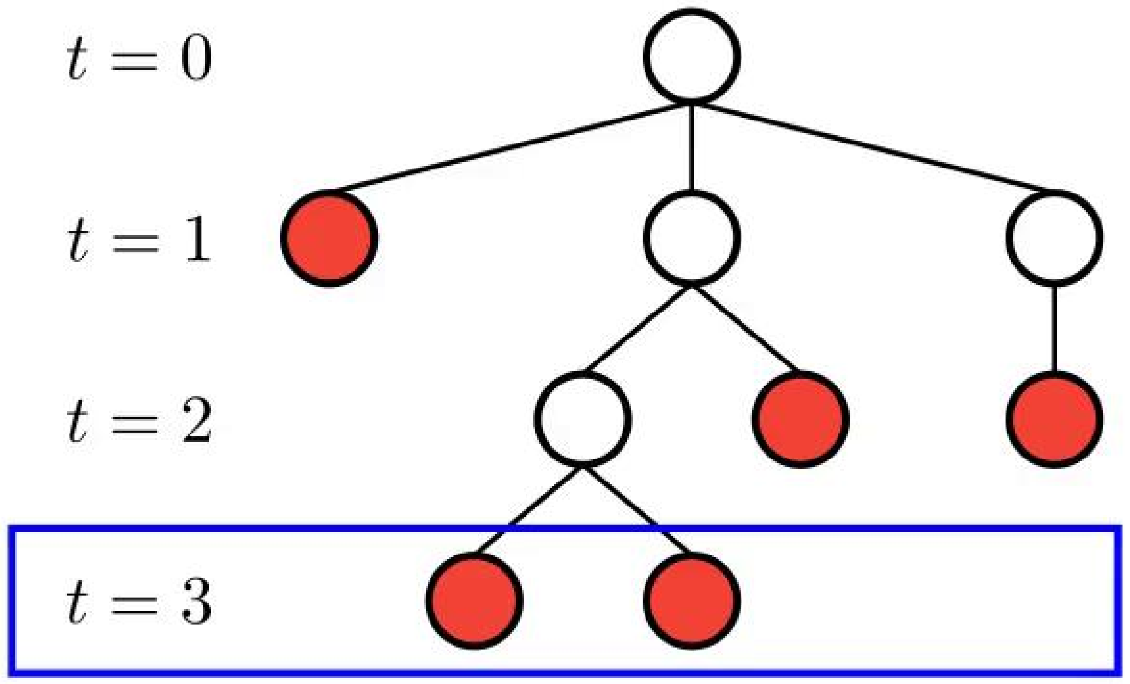
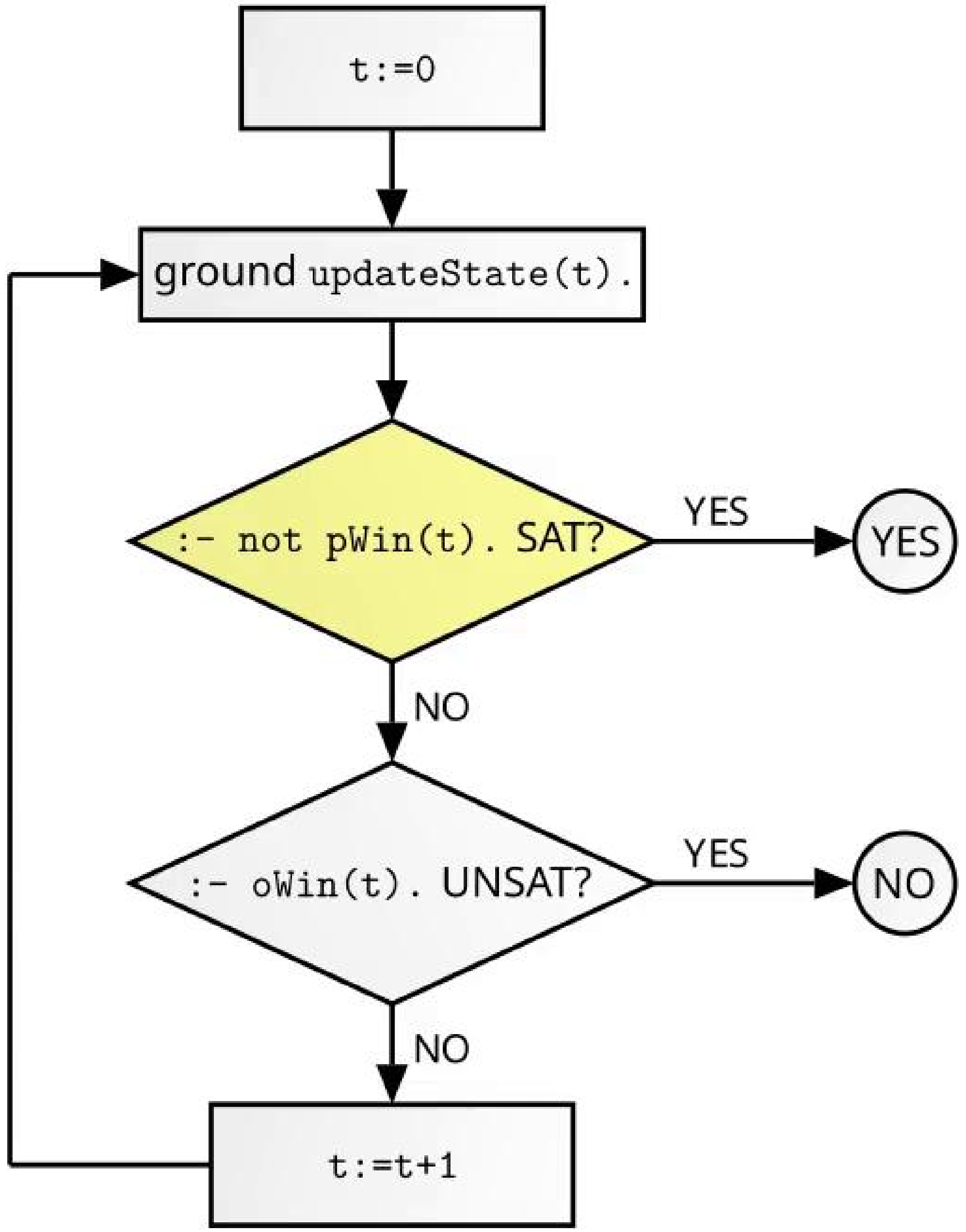
```



Multi-Shot ASP: Dynamic Solving & Grounding

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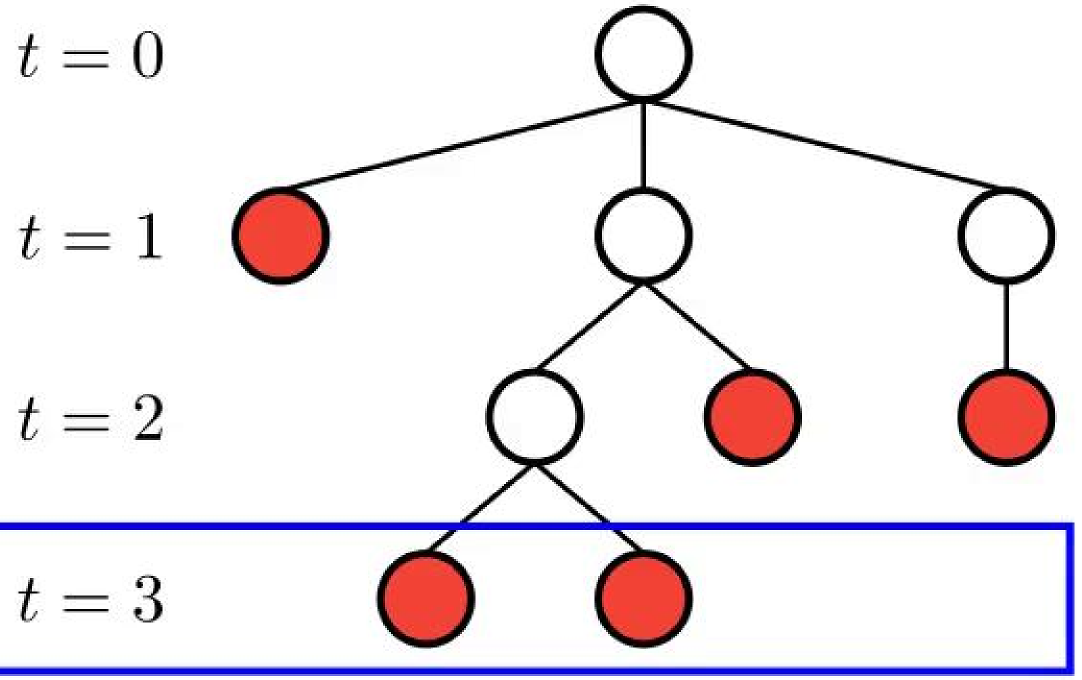
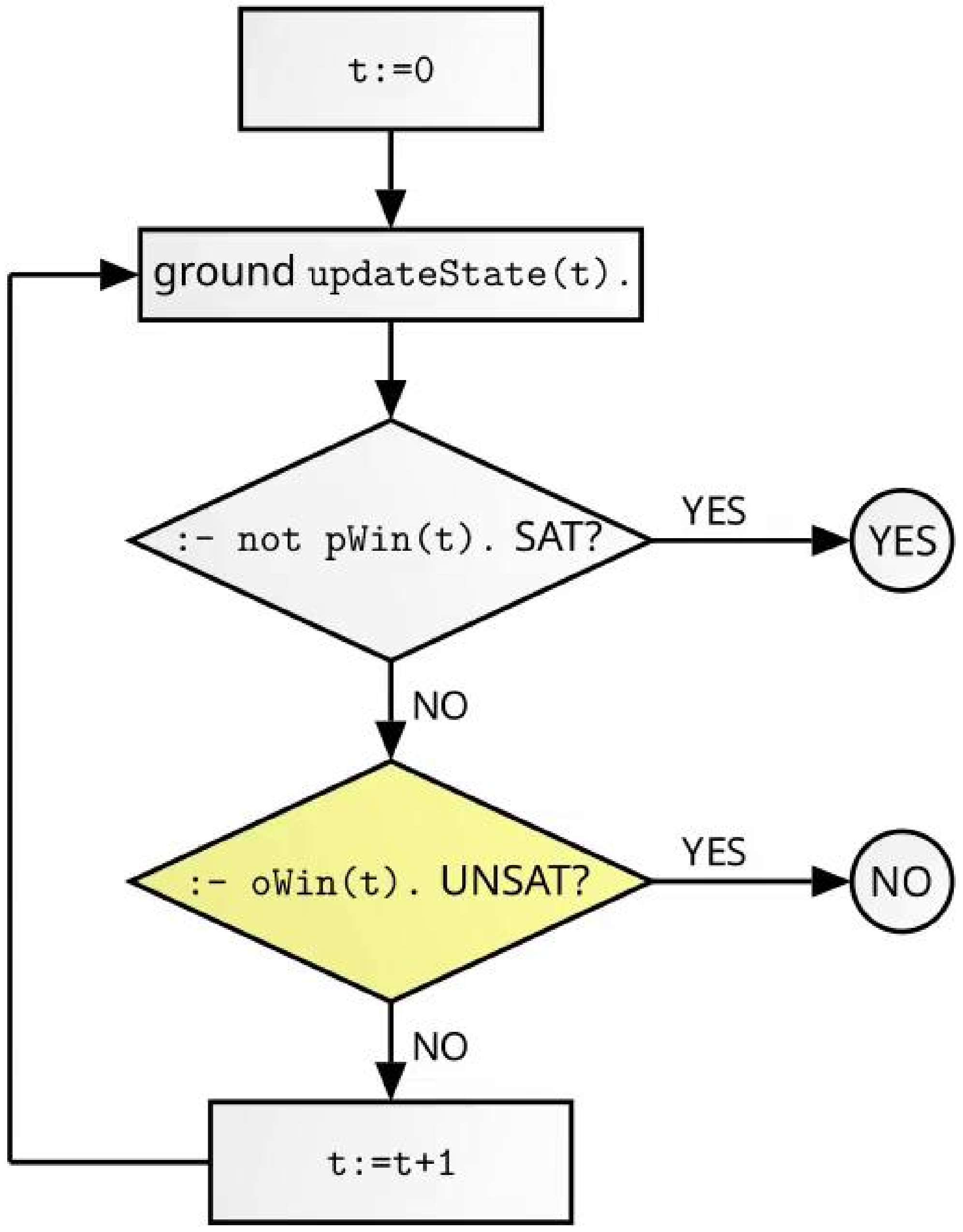
```



Multi-Shot ASP: Dynamic Solving & Grounding

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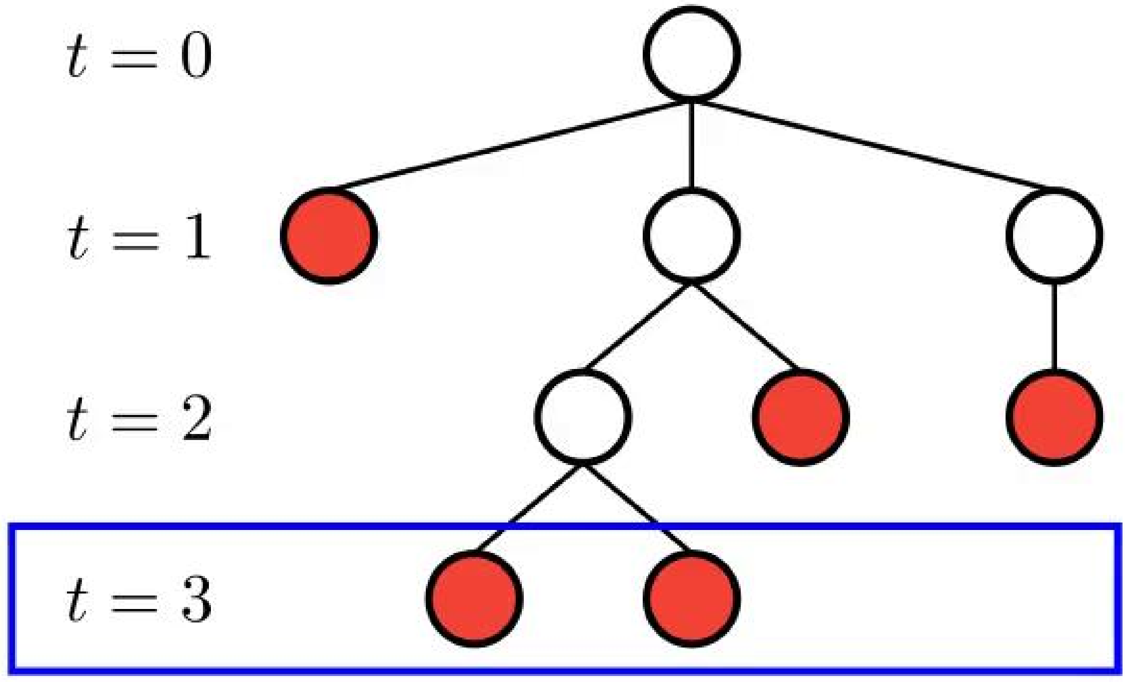
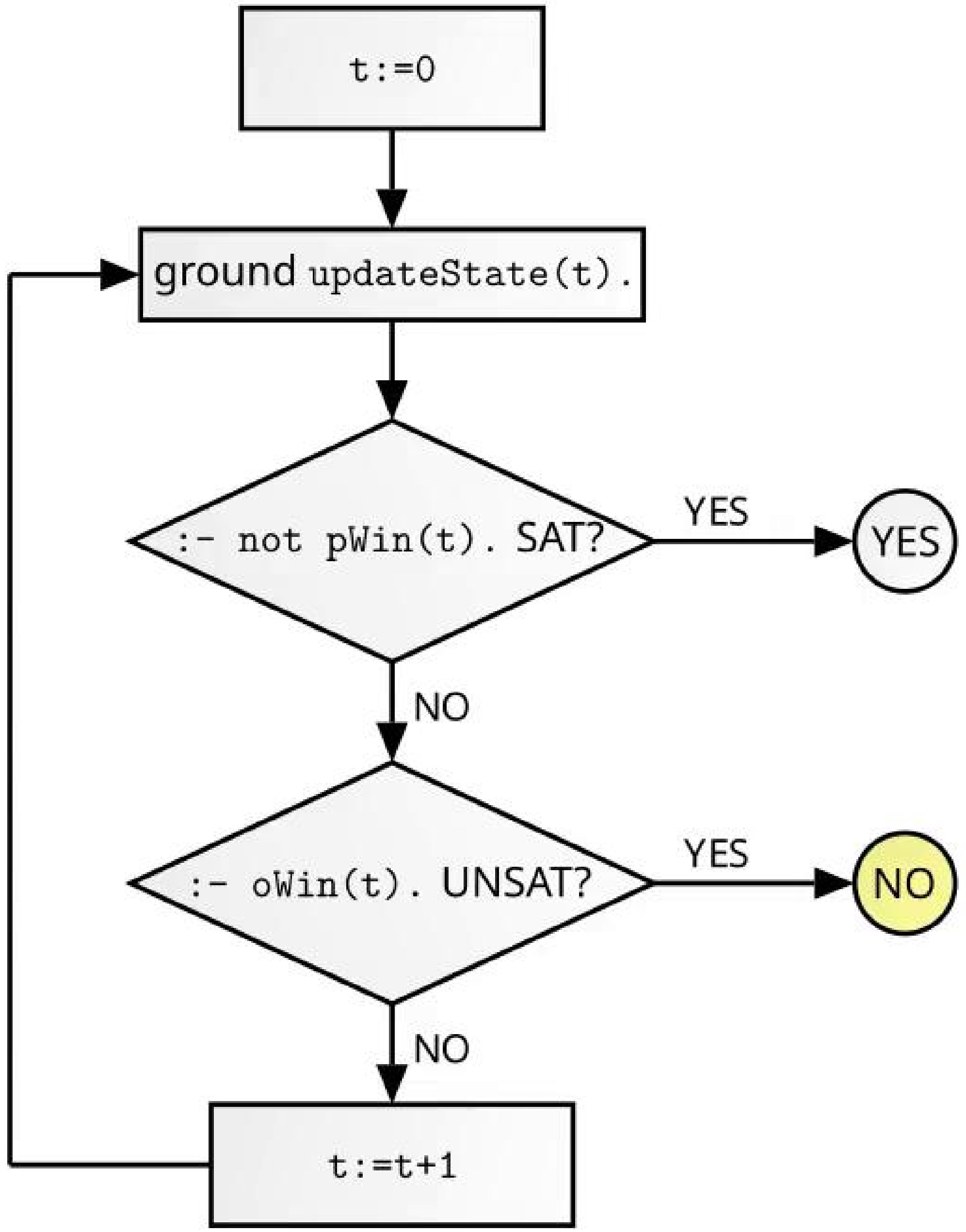
oWin(t) :- not termination(t), not possibleMove(t,p,_).
```



Multi-Shot ASP: Dynamic Solving & Grounding

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pWin(t) :- termination(t), not possibleMove(t,o,_).
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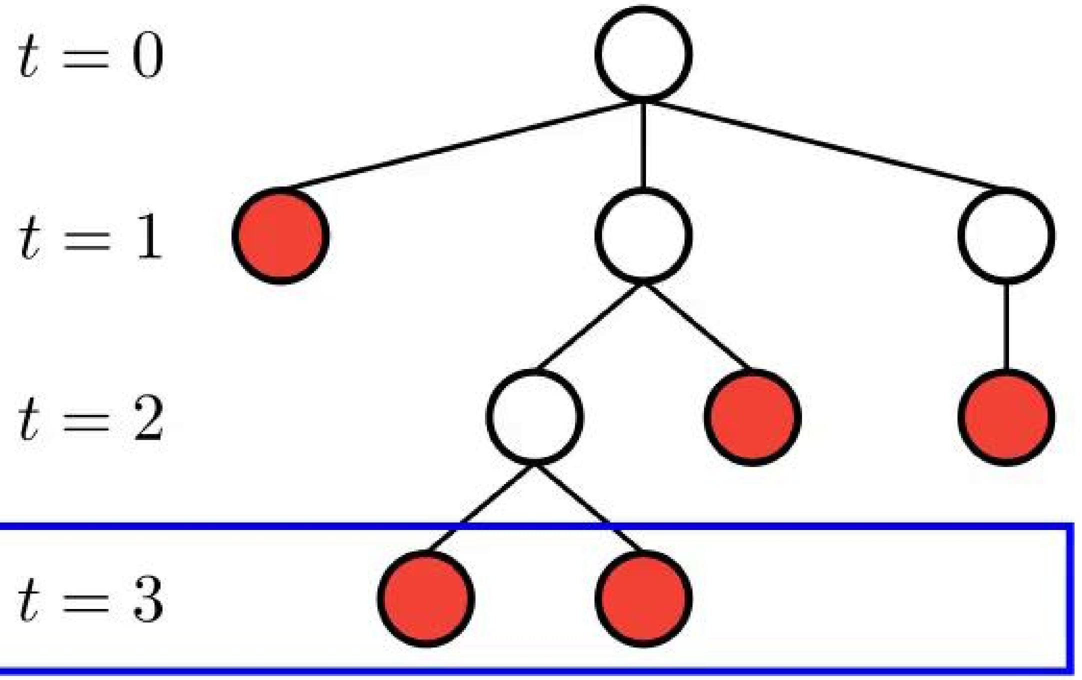
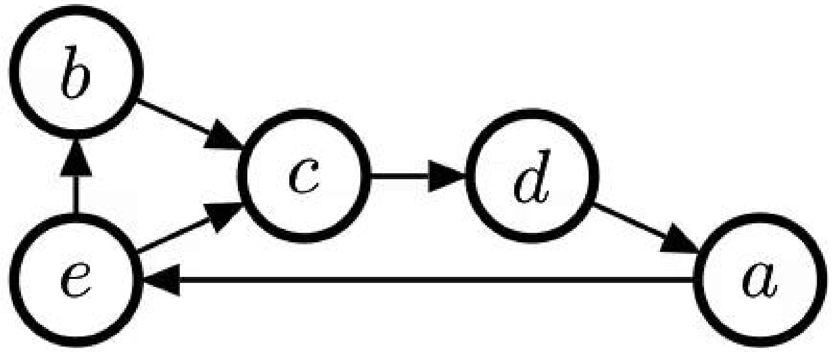
```



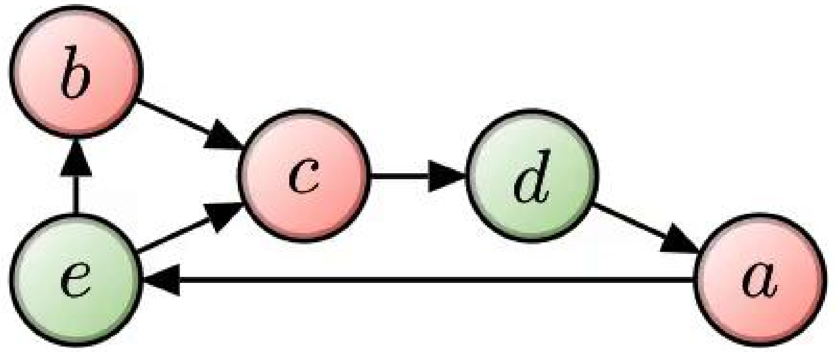
Computing Disputes via ASP: Abstract and ABA AFs



Output:

```
m(0,p,d).
m(1,o,c).
m(2,p,e).
```

Computing Disputes via ASP: Abstract and ABA AFs



Output:

```
m(0,p,d).
m(1,o,c).
m(2,p,e).
```

Computing Disputes via ASP: Abstract and ABA AFs

$$\mathcal{A} = \{a, b, c, d, e, \bar{e}, f\}$$
$$\mathcal{R} = \{s \leftarrow d, p, a; \;\; p \leftarrow \bar{c}; \;\; \bar{c} \leftarrow f;$$
$$\bar{a} \leftarrow b, t; \;\; \bar{d} \leftarrow e; \;\; t \leftarrow c \;\; \}$$

Computing Disputes via ASP: Abstract and ABA AFs

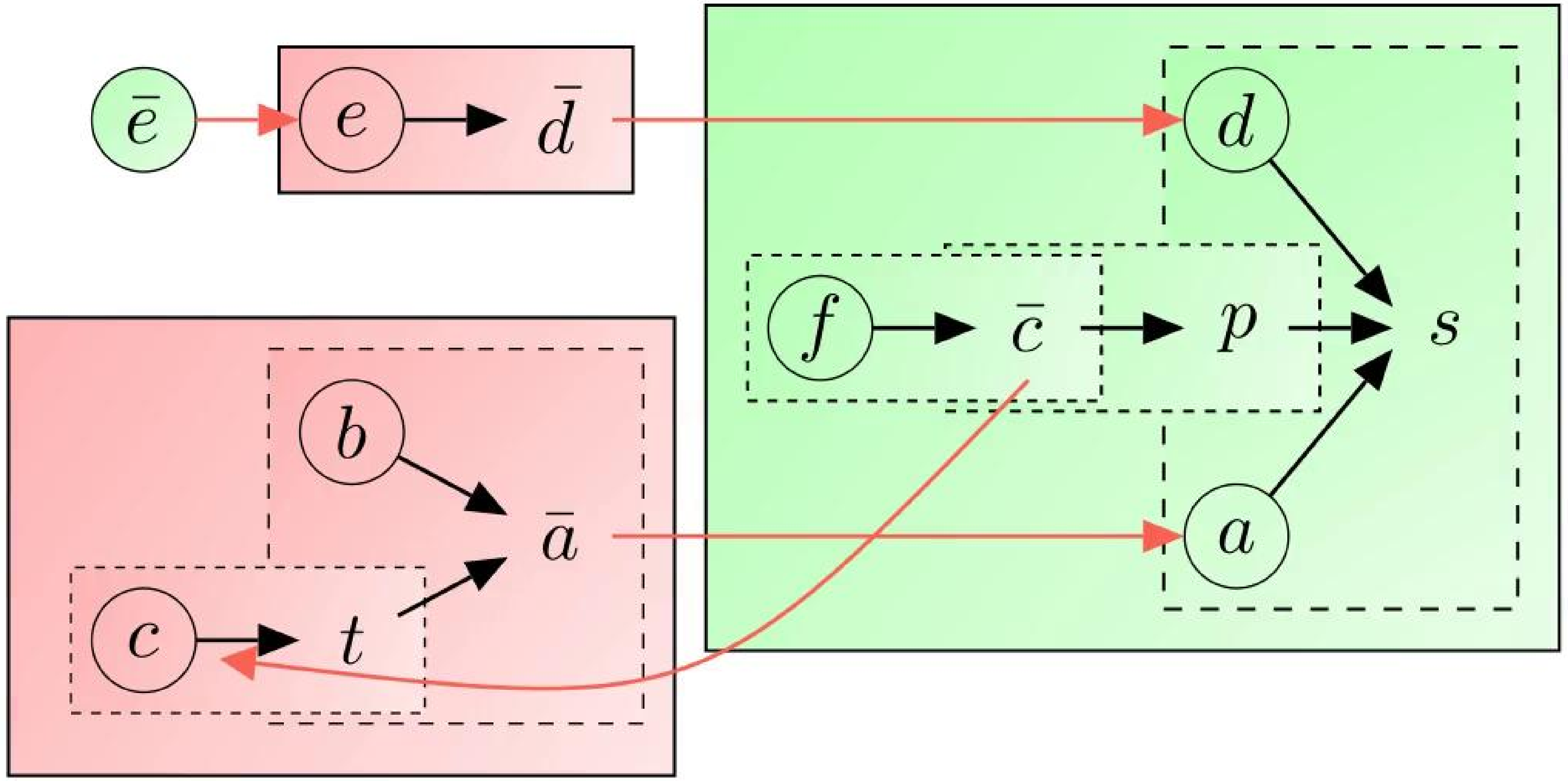
$$\mathcal{A} = \{a, b, c, d, e, \bar{e}, f\}$$
$$\mathcal{R} = \{s \leftarrow d, p, a;\ \ p \leftarrow \bar{c};\ \ \bar{c} \leftarrow f;$$
$$\bar{a} \leftarrow b, t;\ \ \bar{d} \leftarrow e;\ \ t \leftarrow c\ \ \}$$

Output:

m(1,p,pb1,$s \leftarrow d, p, a$).
m(2,o,ob2,$\bar{a} \leftarrow b, t$).
m(2,o,ob2,$\bar{d} \leftarrow e$).
m(3,o,ob1,$t \leftarrow c$).
m(4,p,pf2,$\bar{e}$).
m(5,p,pb1,$p \leftarrow \bar{c}$).
m(6,p,pb1,$\bar{c} \leftarrow f$).

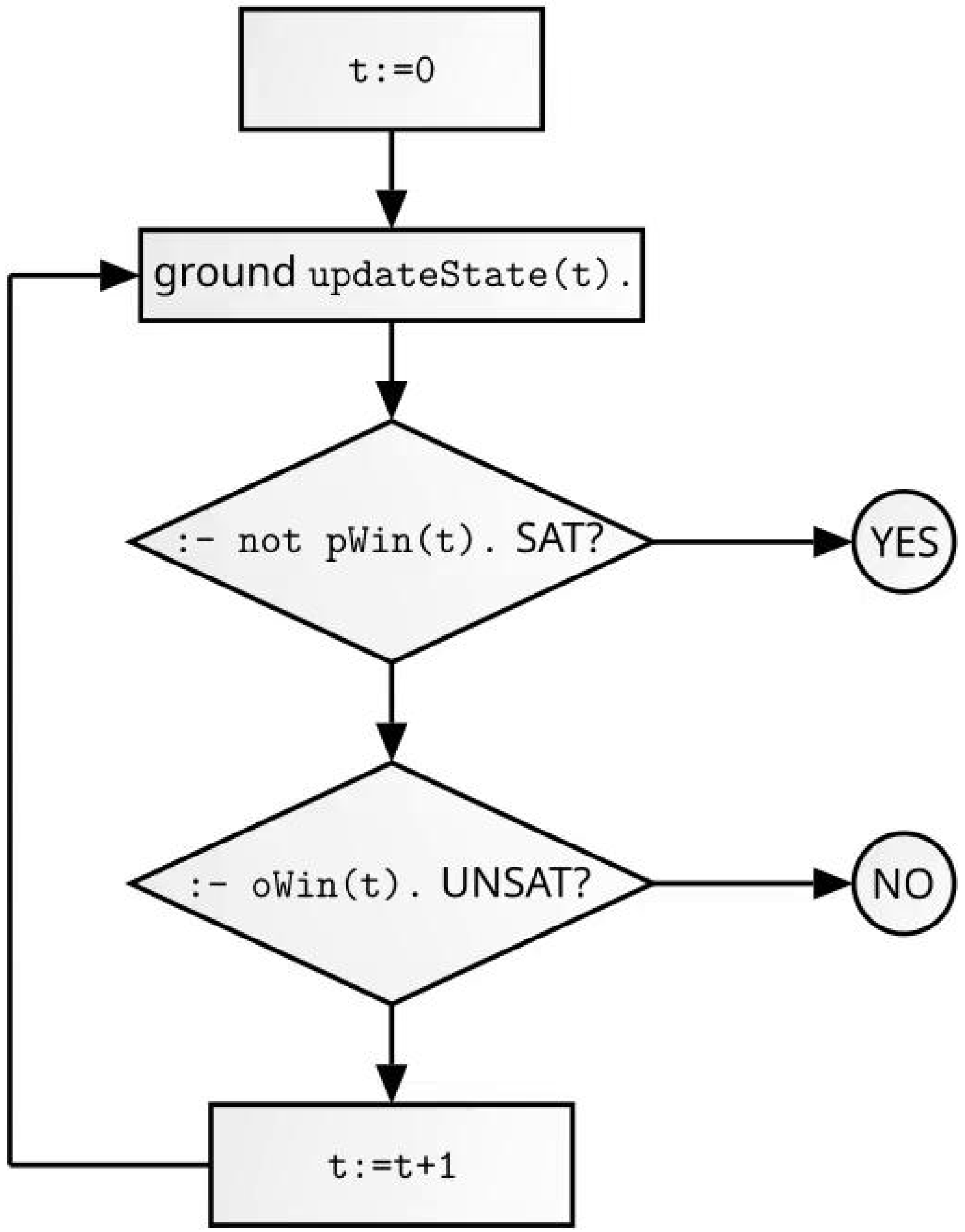
Computing Disputes via ASP: Abstract and ABA AFs

$$\mathcal{A} = \{a, b, c, d, e, \bar{e}, f\}$$
$$\mathcal{R} = \{s \leftarrow d, p, a;\ \ p \leftarrow \bar{c};\ \ \bar{c} \leftarrow f;$$
$$\bar{a} \leftarrow b, t;\ \ \bar{d} \leftarrow e;\ \ t \leftarrow c\ \ \}$$



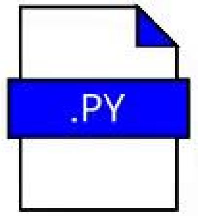
Output:
m(1,p,pb1,$s \leftarrow d, p, a$).
m(2,o,ob2,$\bar{a} \leftarrow b, t$).
m(2,o,ob2,$\bar{d} \leftarrow e$).
m(3,o,ob1,$t \leftarrow c$).
m(4,p,pf2,$\bar{e}$).
m(5,p,pb1,$p \leftarrow \bar{c}$).
m(6,p,pb1,$\bar{c} \leftarrow f$).

MS-DIS: System Architecture & Extensibility



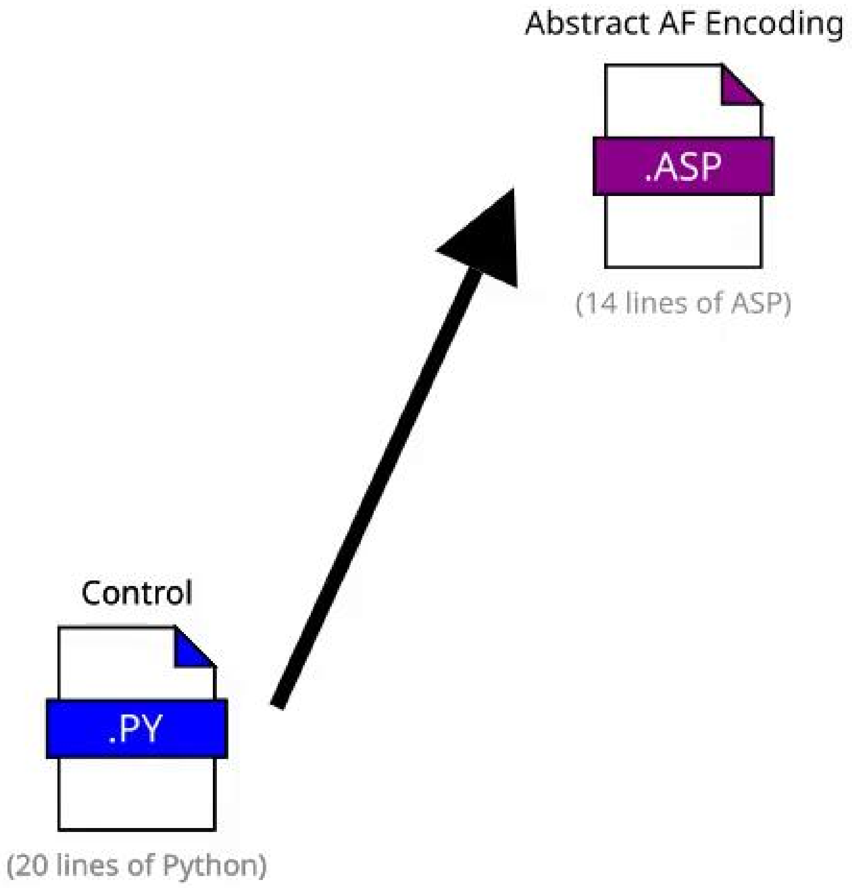
MS-DIS: System Architecture & Extensibility

Control

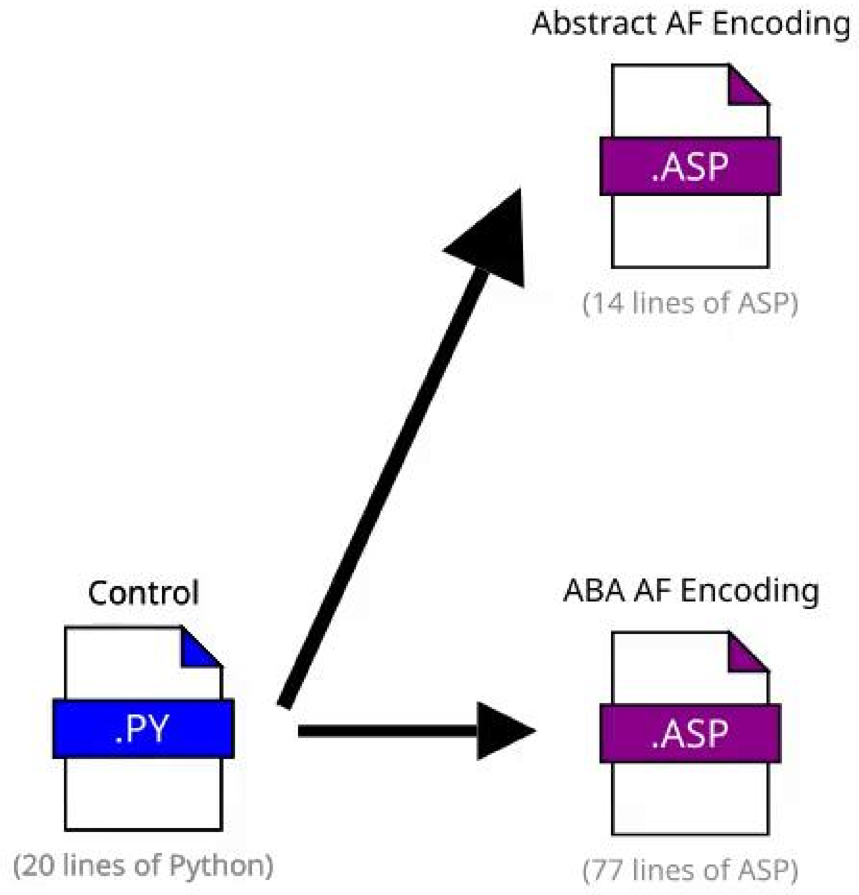


(20 lines of Python)

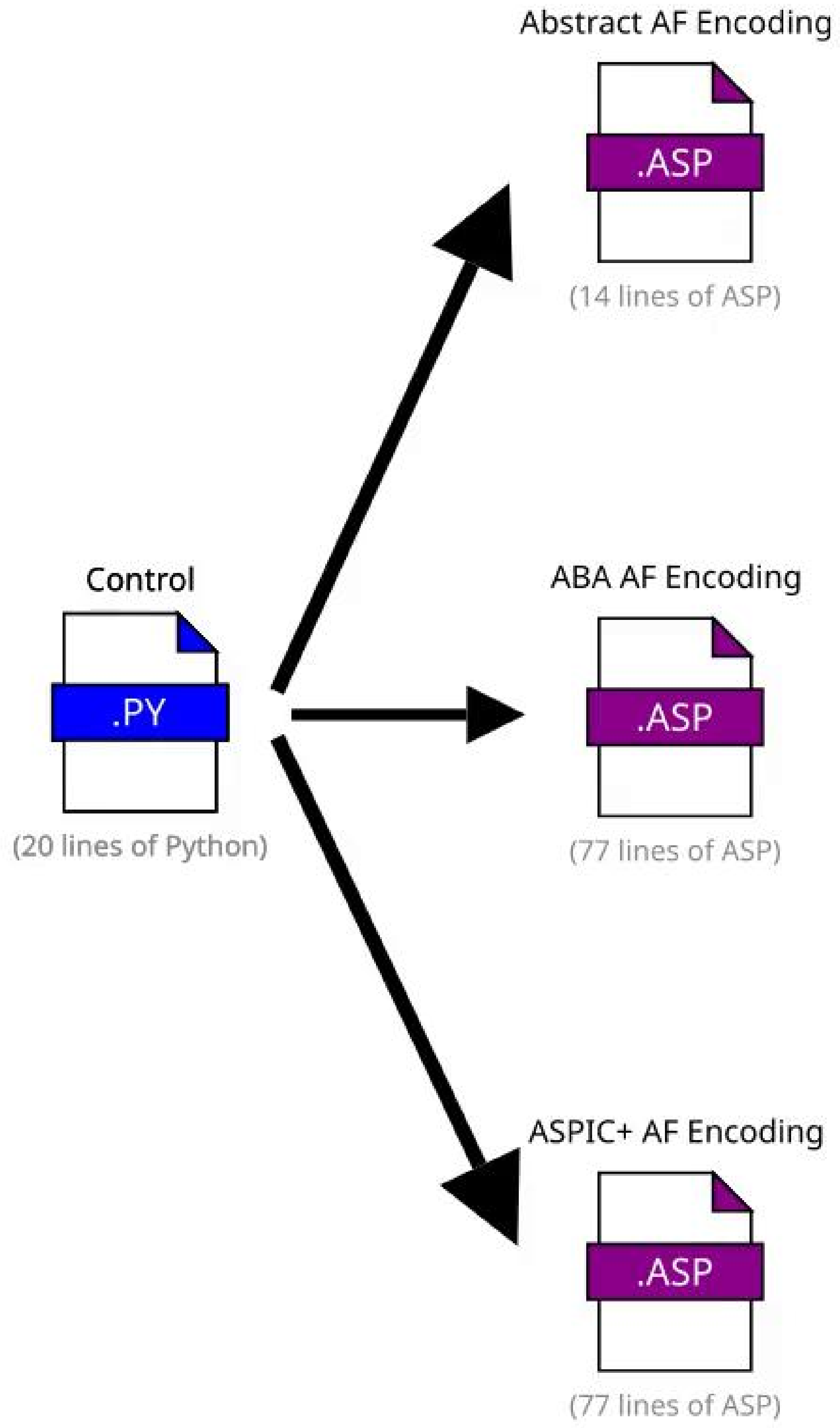
MS-DIS: System Architecture & Extensibility



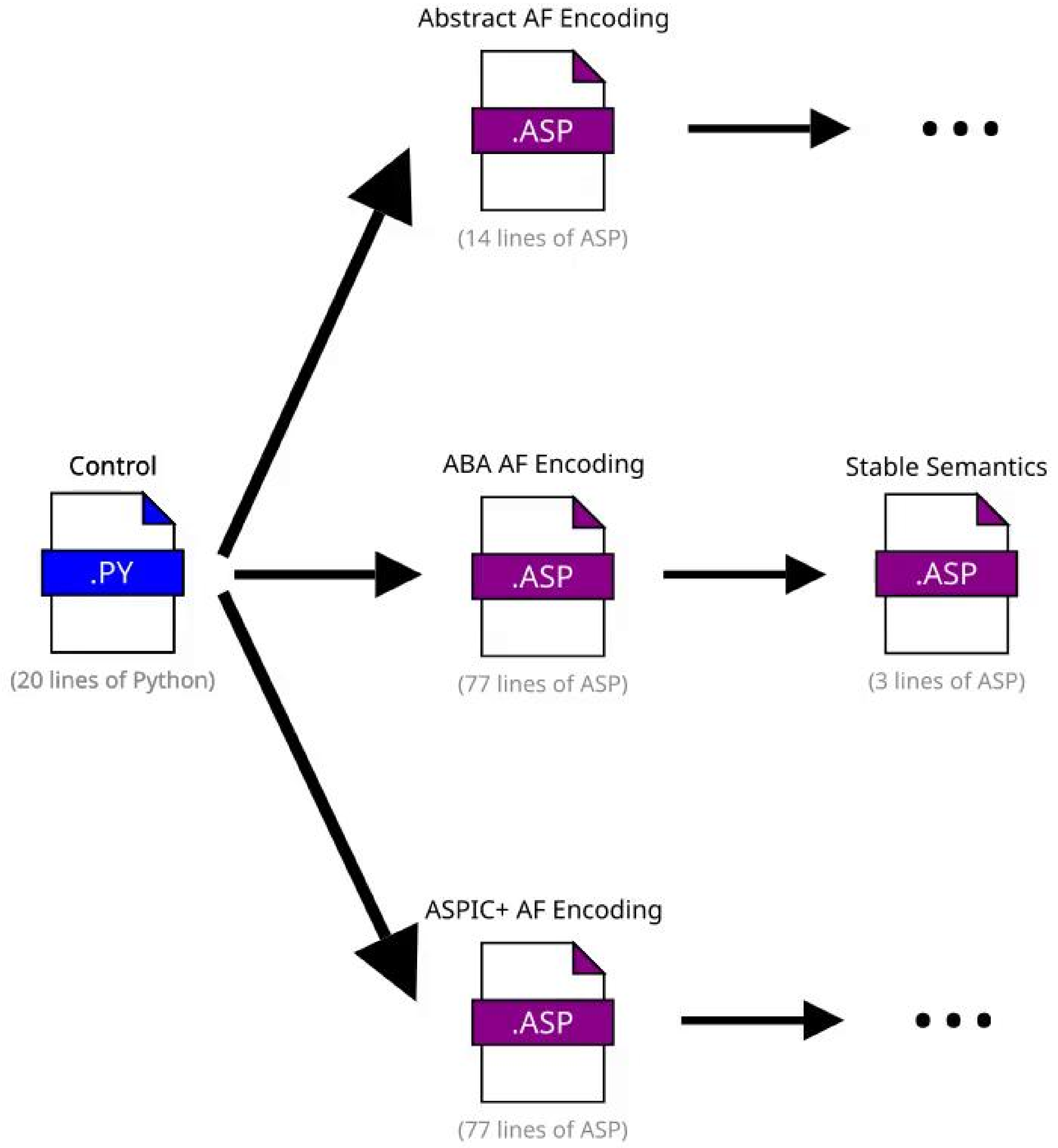
MS-DIS: System Architecture & Extensibility



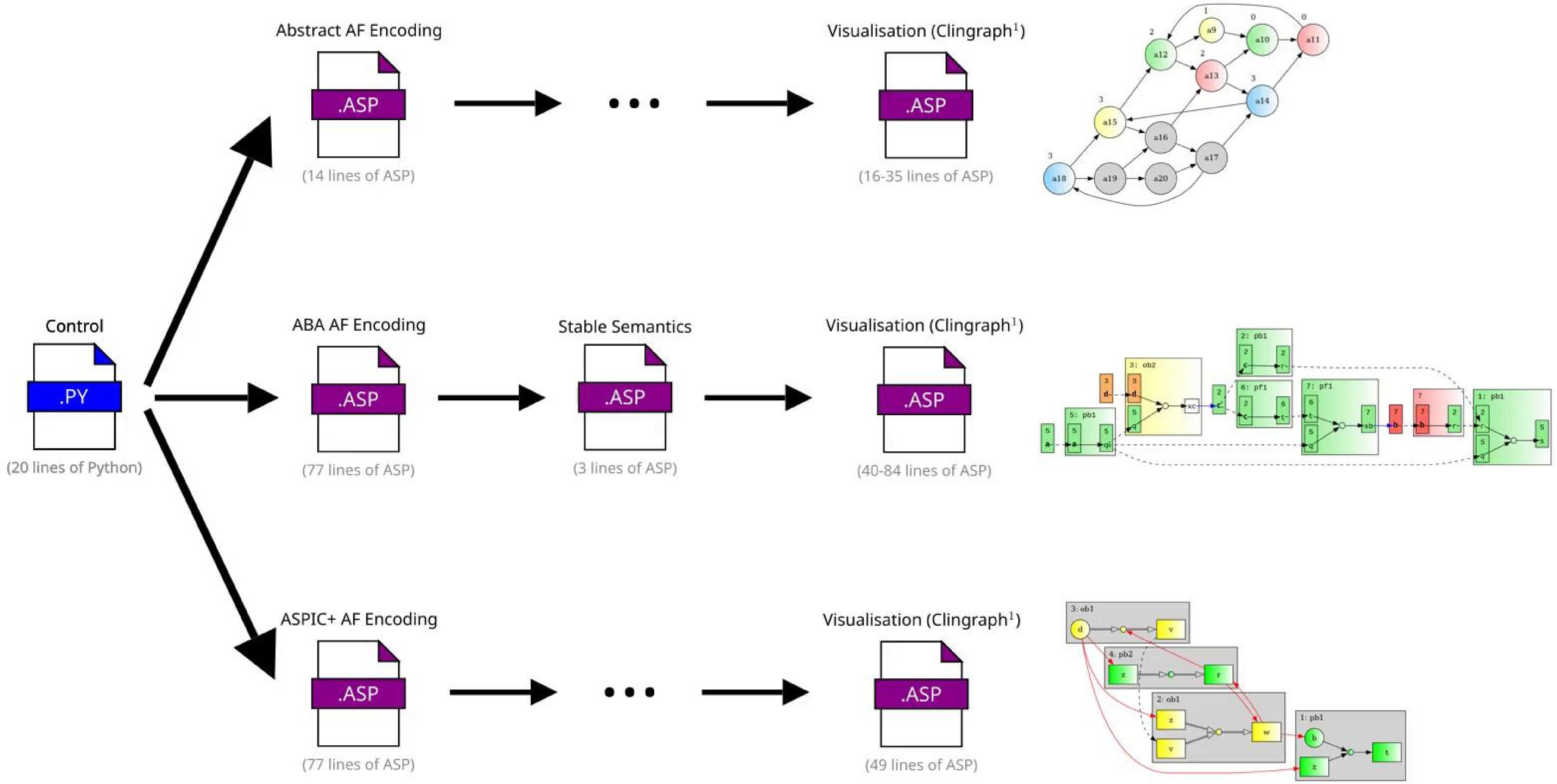
MS-DIS: System Architecture & Extensibility



MS-DIS: System Architecture & Extensibility

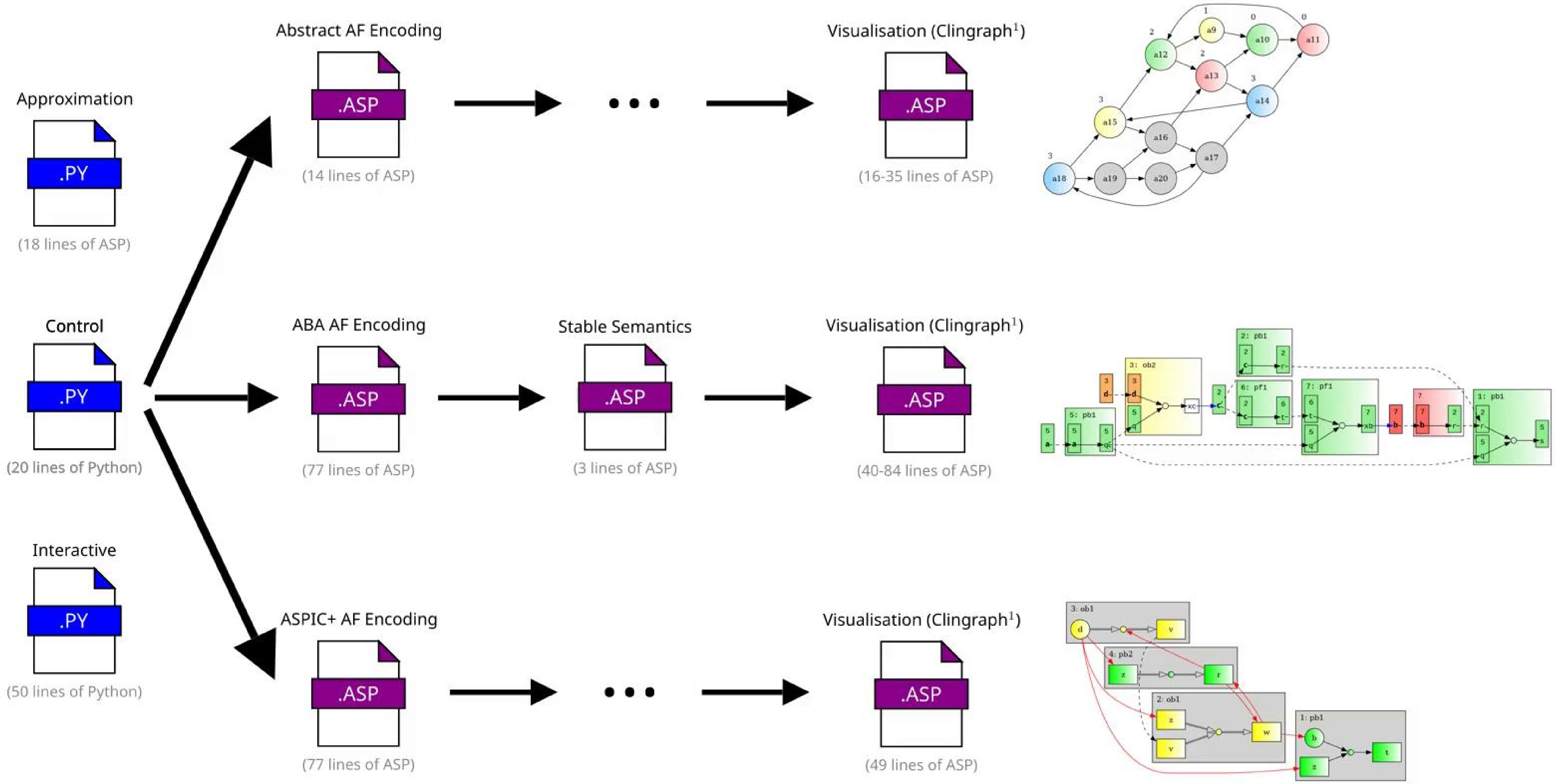


MS-DIS: System Architecture & Extensibility



[1] Susana Hahn, Orkunt Sabuncu, Torsten Schaub, Tobias Stolzmann. *Clingraph: A System for ASP-based Visualization.* TPLP (2024).

MS-DIS: System Architecture & Extensibility



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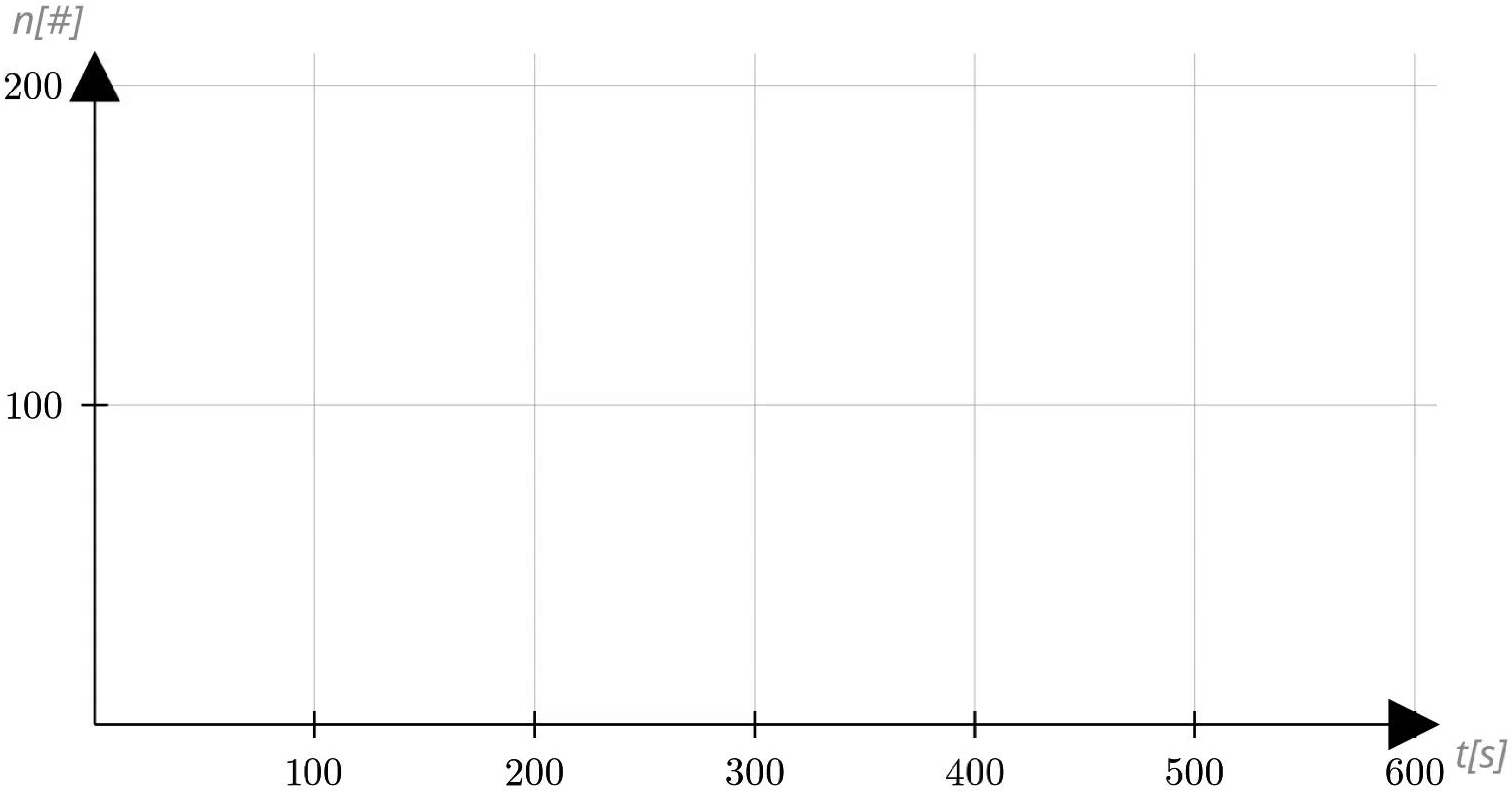
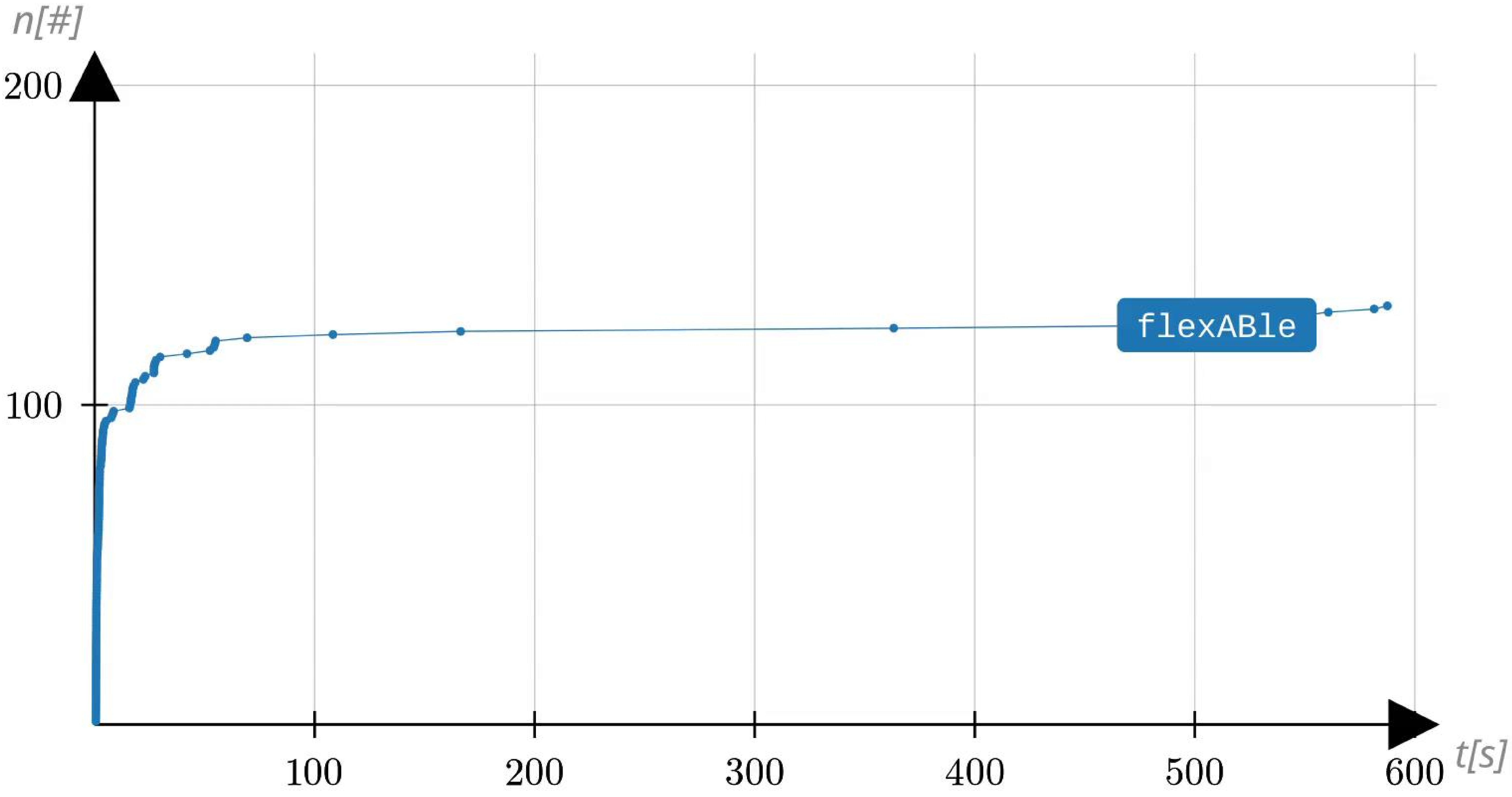
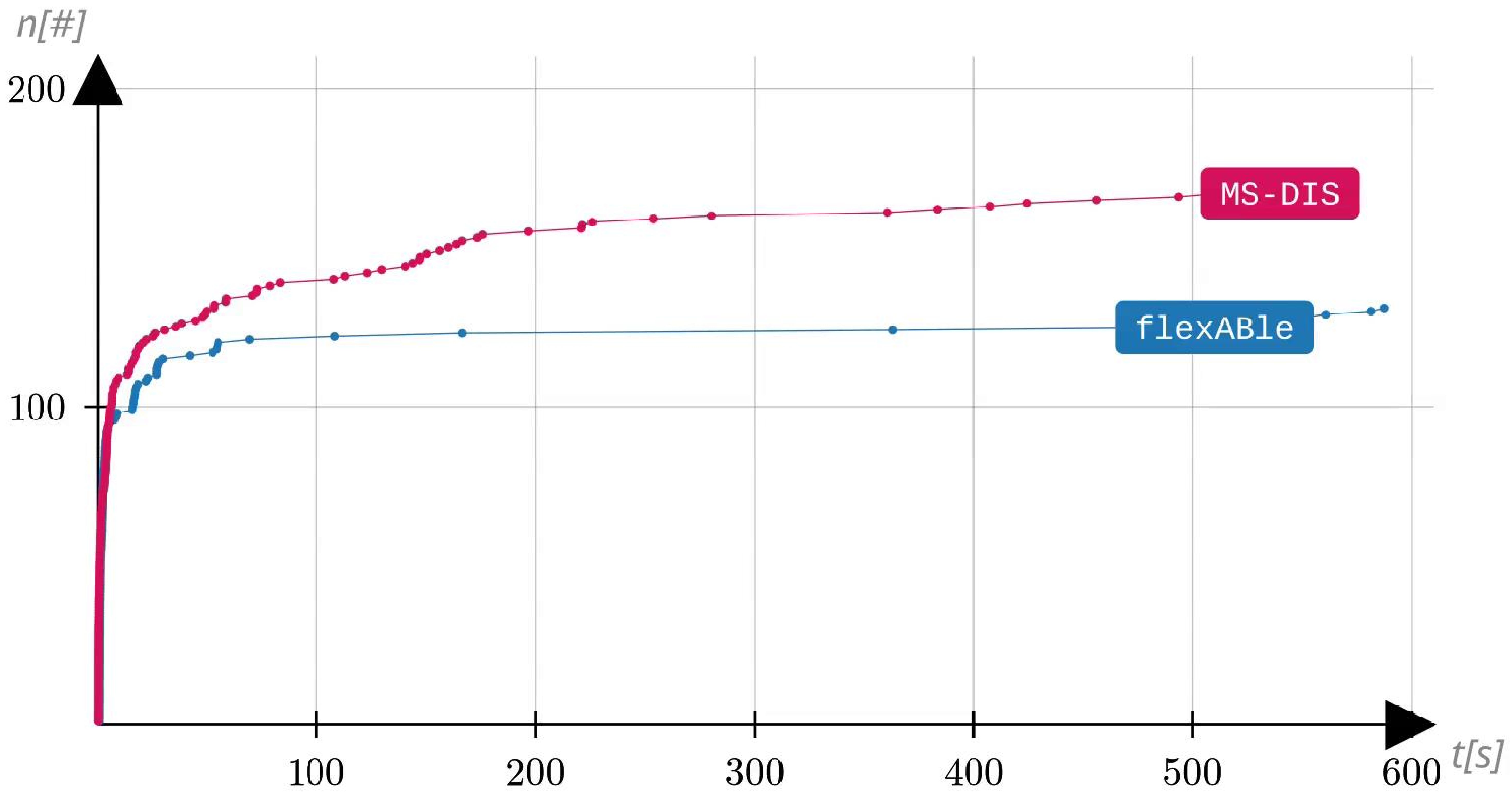
Evaluation



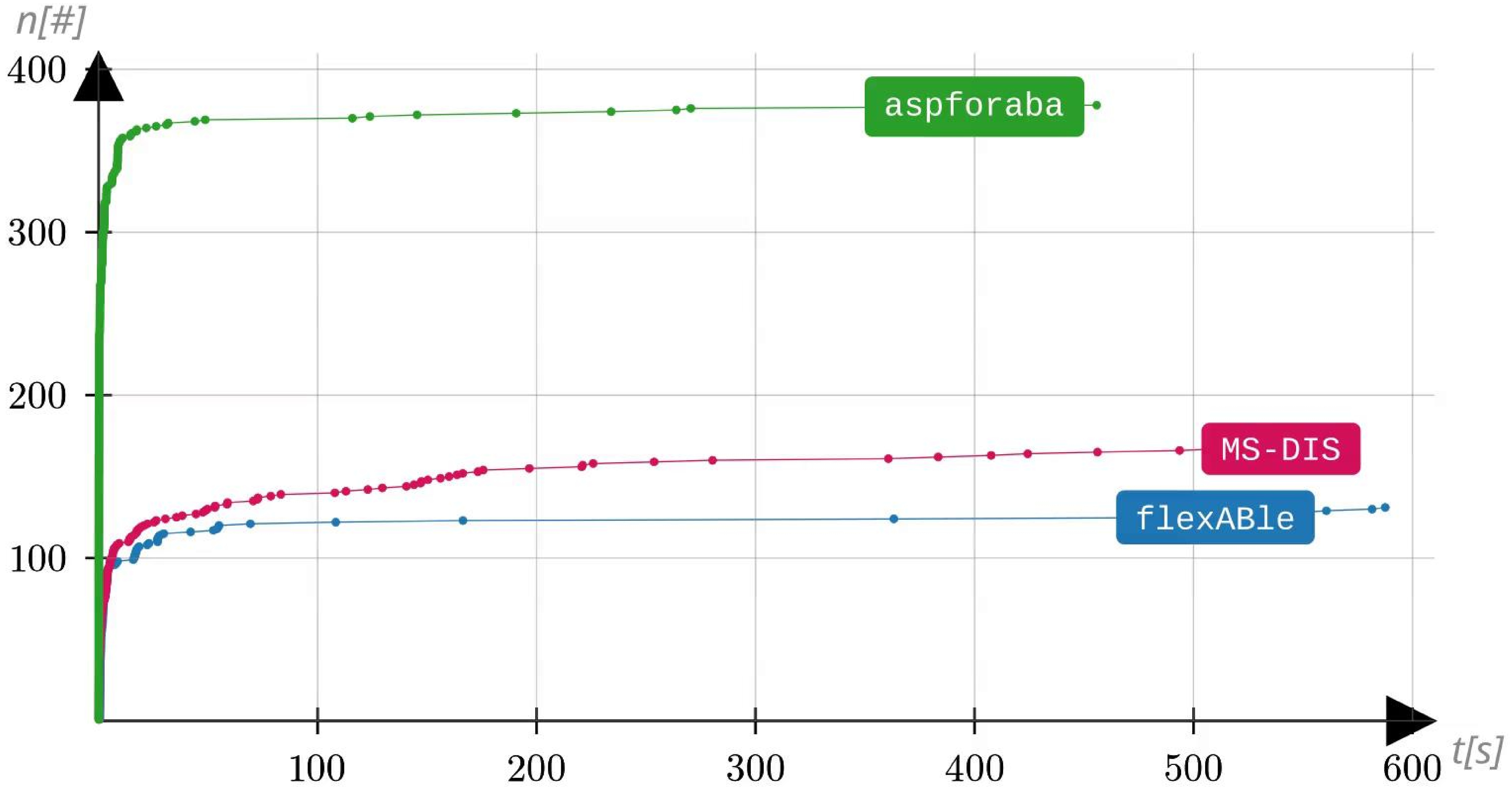
Evaluation



Evaluation

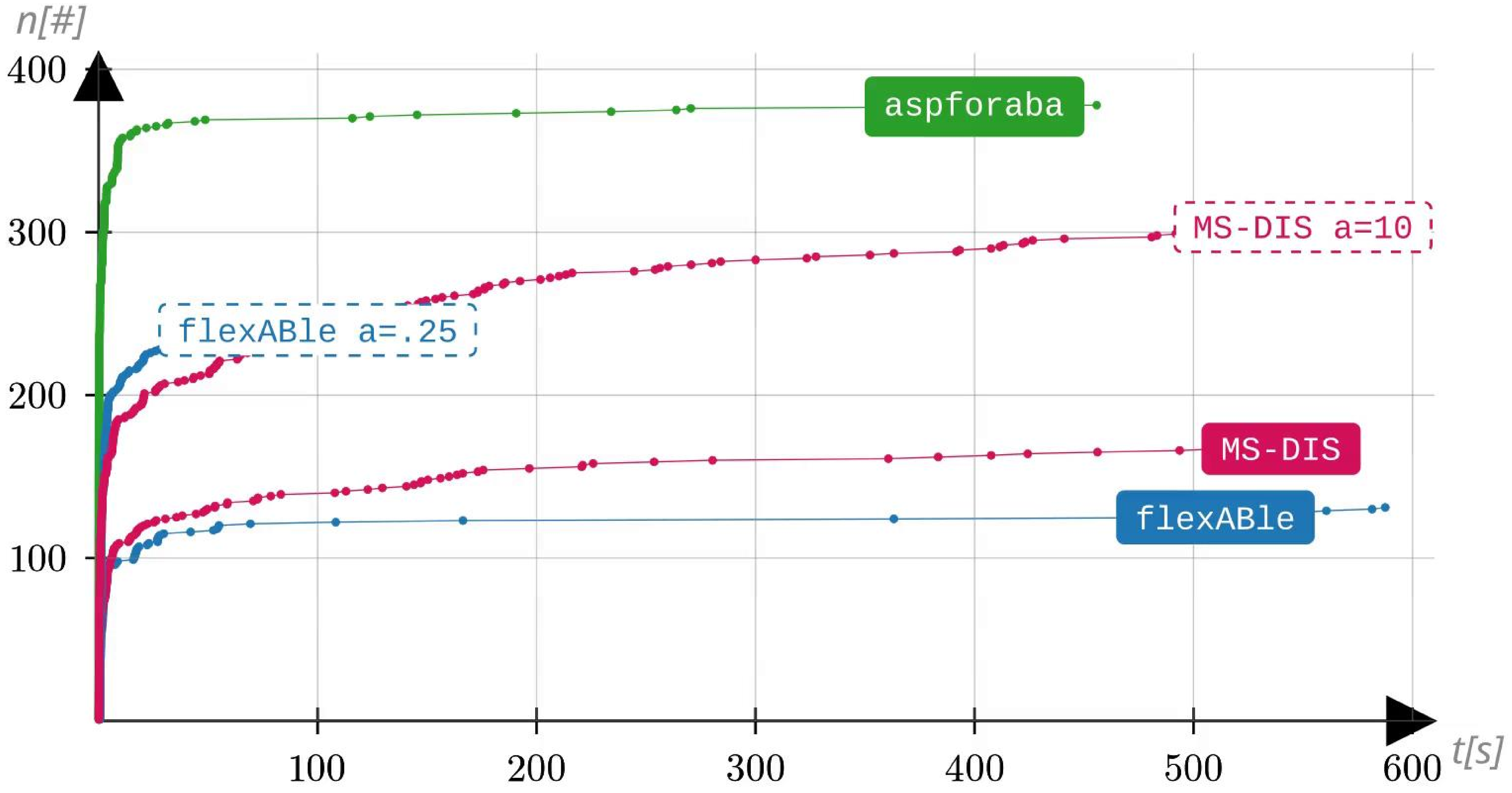


Evaluation



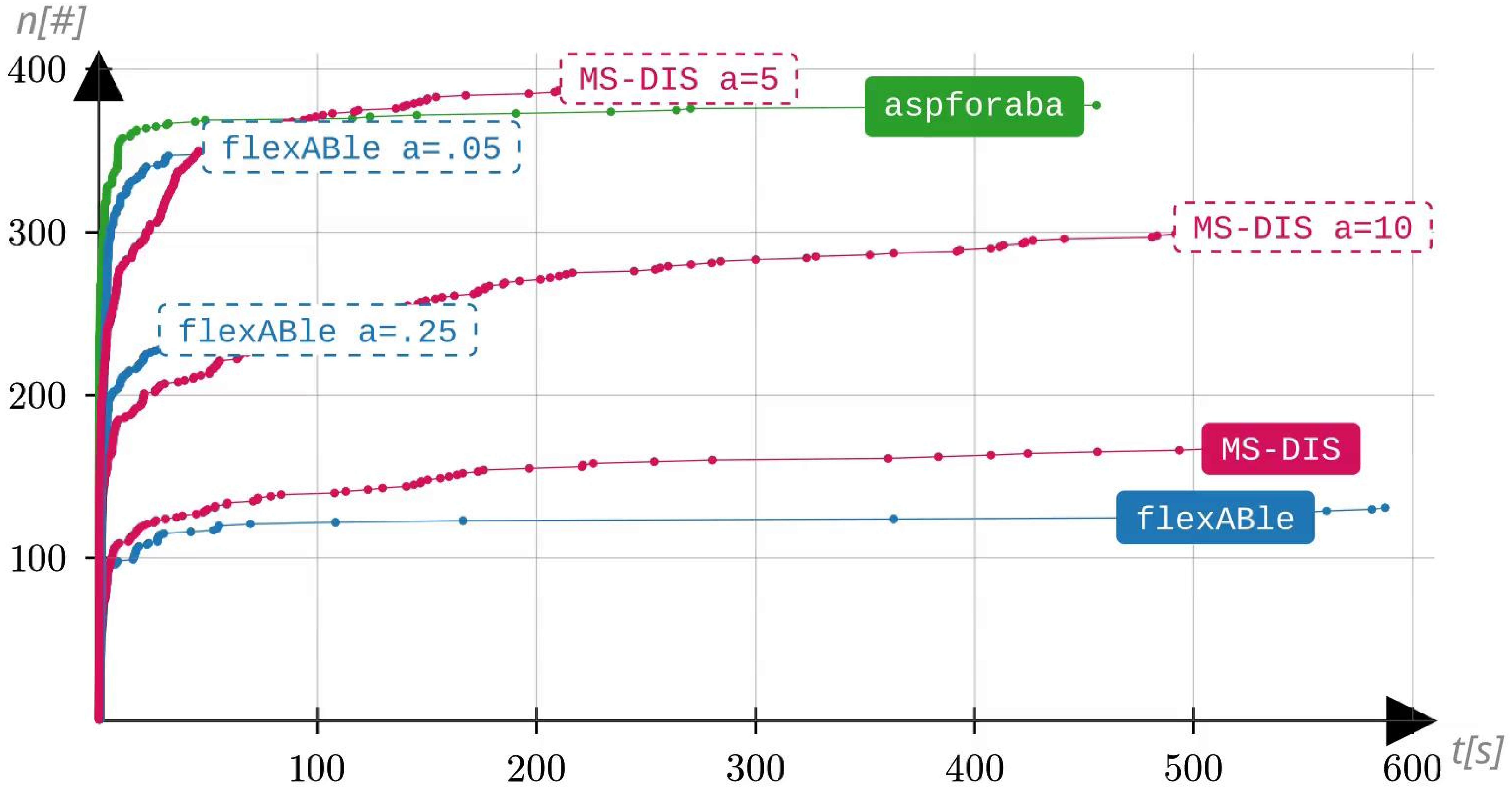
n[#]
400
300
200
100
aspforaba
MS-DIS
flexABle
100
200
300
400
500
600
t[s]

Evaluation



n[#]
400
300
200
100
aspforaba
MS-DIS a=10
flexABle a=.25
MS-DIS
flexABle
100
200
300
400
500
600
t[s]

Evaluation

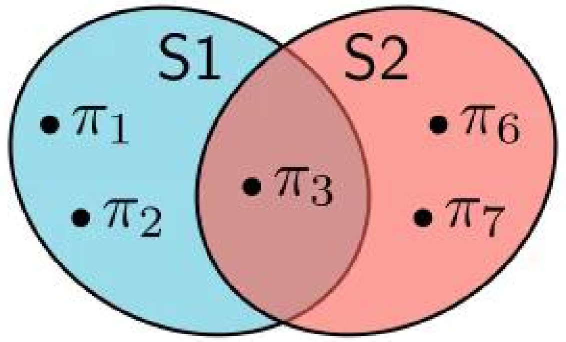


n[#]
400
300
200
100
100
200
300
400
500
600
t[s]
MS-DIS a=5
aspforaba
flexABle a=.05
MS-DIS a=10
flexABle a=.25
MS-DIS
flexABle

Part II: Generalisations of S4F and Standpoint Logic

Background: Component Logics

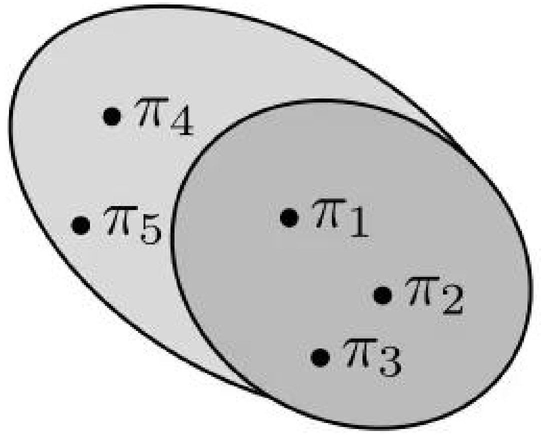
Standpoint Logic
[Gómez Álvarez & Rudolph, 2021]



$$\mathcal{M}, \pi \Vdash \Box_{\mathsf{s}}\varphi \quad :\Longleftrightarrow \quad \mathcal{M}, \pi' \Vdash \varphi \text{ for all } \pi' \in \sigma(s)$$

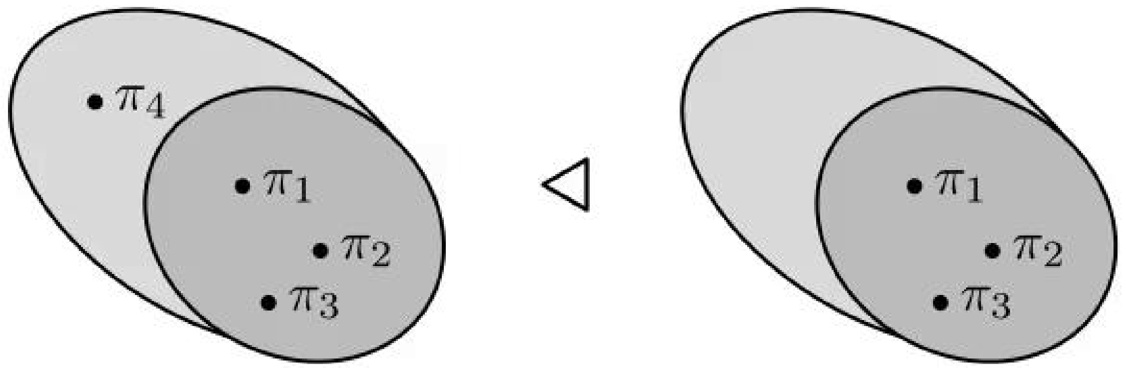
$$\mathcal{M}, \pi \Vdash \mathsf{s} \preccurlyeq \mathsf{u} \quad :\Longleftrightarrow \quad \sigma(\mathsf{s}) \subseteq \sigma(\mathsf{u})$$

Modal Logic S4F
[Segerberg, 1971]

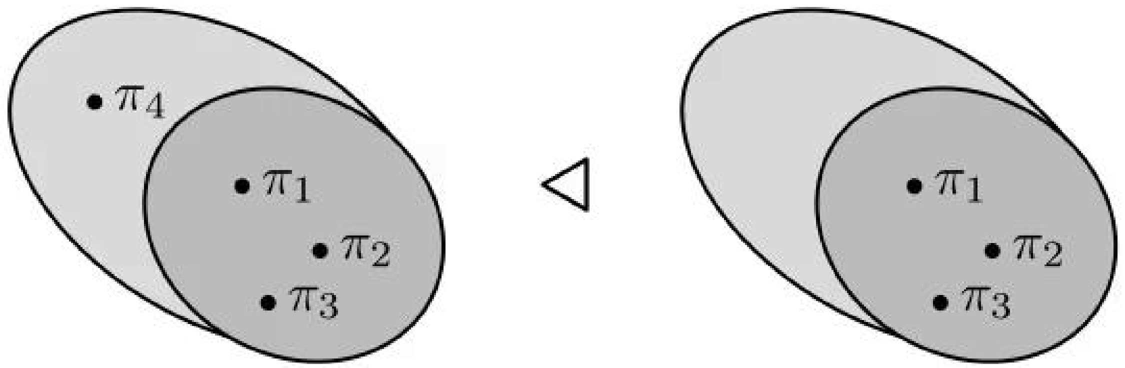


$$\mathcal{M}, \pi \Vdash \mathbf{K}\varphi \quad :\Longleftrightarrow \quad \begin{cases} \mathcal{M}, \pi' \Vdash \varphi \text{ for all } \pi' \in V \cup W \text{ if } \pi \in V, \\ \mathcal{M}, \pi' \Vdash \varphi \text{ for all } \pi' \in W \text{ otherwise} \end{cases}$$

Background: Non-Monotonicity in S4F

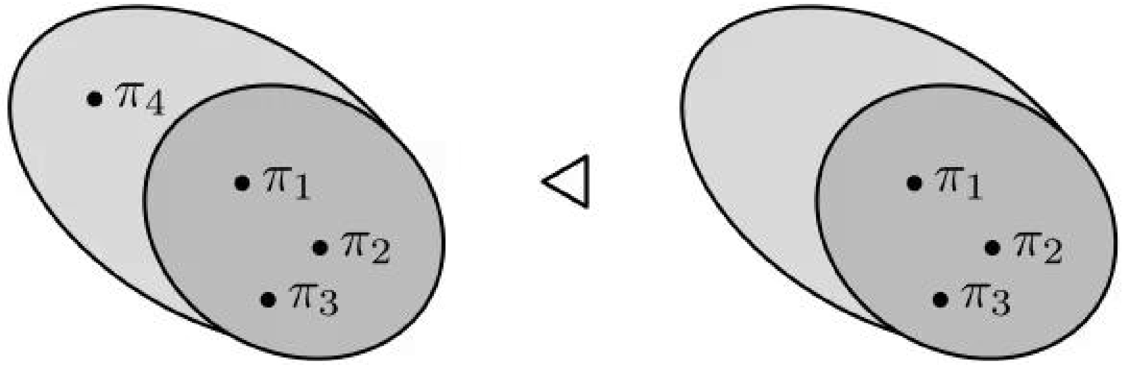


Background: Non-Monotonicity in S4F



$\mathcal{M}$ is a **minimal model of** $T \subseteq \mathcal{L}_{\mathbf{K}}$ iff:
1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \nVdash T$

Background: Non-Monotonicity in S4F



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1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \nVdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\rightsquigarrow \ (\mathbf{K}\phi \wedge \mathbf{K}\neg\mathbf{K}\neg\psi') \to \mathbf{K}\psi$

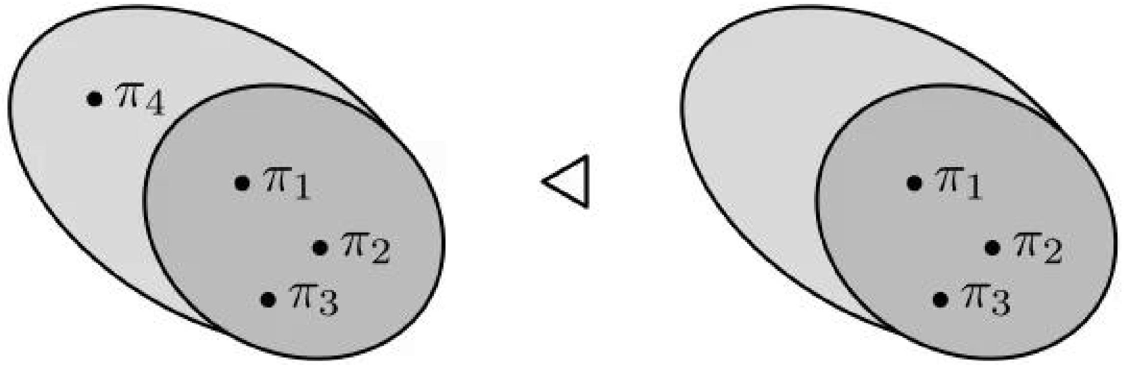
Logic Programs: $p_0 \leftarrow p_1, \texttt{not}\, p_2$

$\rightsquigarrow \ (\mathbf{K}p_1 \wedge \mathbf{K}\neg\mathbf{K}p_2) \to \mathbf{K}p_0$

Argumentation: (A, R) and $a \in A, (a, b) \in R$

$\rightsquigarrow \ \mathbf{K}\neg\mathbf{K}\neg a \to \mathbf{K}a, \quad \mathbf{K}a \to \mathbf{K}\neg b$

Background: Non-Monotonicity in S4F



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2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \nVdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\rightsquigarrow \ (\mathbf{K}\phi \wedge \mathbf{K}\neg\mathbf{K}\neg\psi') \rightarrow \mathbf{K}\psi$

Logic Programs: $p_0 \leftarrow p_1, \mathtt{not}\, p_2$

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Argumentation: (A, R) and $a \in A, (a, b) \in R$

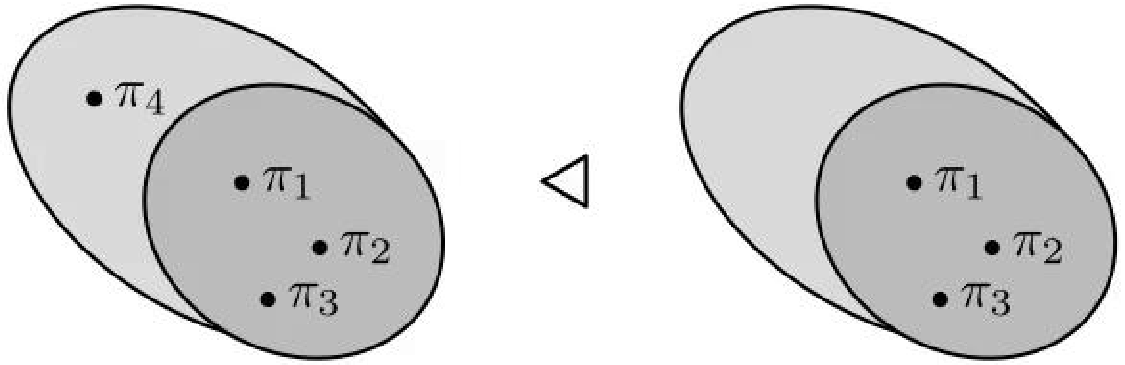
$\rightsquigarrow \ \mathbf{K}\neg\mathbf{K}\neg a \rightarrow \mathbf{K}a, \quad \mathbf{K}a \rightarrow \mathbf{K}\neg b$

[Schwarz & Truszczynski, 1993]

Represent $\mathcal{M}$ by a partition (Φ, Ψ) of $\{\mathbf{K}\phi \mid \mathbf{K}\phi \in \mathit{Subf}(T)\}$.

Let $\Theta = A \cup \{\neg\mathbf{K}\phi \mid \phi \in \Phi\} \cup \{\mathbf{K}\psi \mid \psi \in \Psi\} \cup \Psi$

Background: Non-Monotonicity in S4F



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1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \nVdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\rightsquigarrow$ $(\mathbf{K}\phi \wedge \mathbf{K}\neg\mathbf{K}\neg\psi') \rightarrow \mathbf{K}\psi$

Logic Programs: $p_0 \leftarrow p_1, \texttt{not}\, p_2$

$\rightsquigarrow$ $(\mathbf{K}p_1 \wedge \mathbf{K}\neg\mathbf{K}p_2) \rightarrow \mathbf{K}p_0$

Argumentation: (A, R) and $a \in A, (a, b) \in R$

$\rightsquigarrow$ $\mathbf{K}\neg\mathbf{K}\neg a \rightarrow \mathbf{K}a, \quad \mathbf{K}a \rightarrow \mathbf{K}\neg b$

[Schwarz & Truszczynski, 1993]

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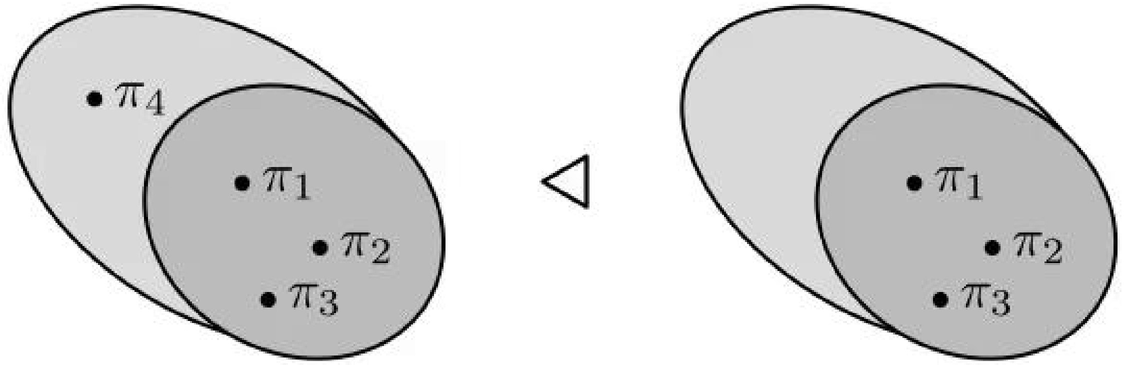
Check if:

Background: Non-Monotonicity in S4F



$\mathcal{M}$ is a **minimal model of** $T \subseteq \mathcal{L}_{\mathbf{K}}$ iff:
1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \not\Vdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\rightsquigarrow$ $(\mathbf{K}\phi \land \mathbf{K}\neg\mathbf{K}\neg\psi') \rightarrow \mathbf{K}\psi$

Logic Programs: $p_0 \leftarrow p_1, \texttt{not}\, p_2$

$\rightsquigarrow$ $(\mathbf{K}p_1 \land \mathbf{K}\neg\mathbf{K}p_2) \rightarrow \mathbf{K}p_0$

Argumentation: (A, R) and $a \in A, (a, b) \in R$

$\rightsquigarrow$ $\mathbf{K}\neg\mathbf{K}\neg a \rightarrow \mathbf{K}a, \quad \mathbf{K}a \rightarrow \mathbf{K}\neg b$

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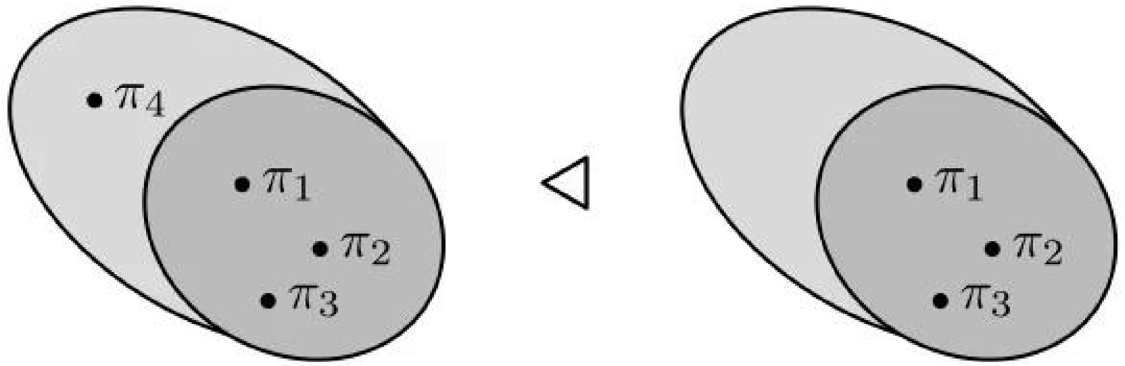
Let $\Theta = A \cup \{\neg\mathbf{K}\phi \mid \phi \in \Phi\} \cup \{\mathbf{K}\psi \mid \psi \in \Psi\} \cup \Psi$

Check if:

1. Guessed beliefs can all be true?

Θ is prop. consistent

Background: Non-Monotonicity in S4F



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1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \lhd \mathcal{M}$: $\mathcal{M}' \nVdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\leadsto$ $(\mathbf{K}\phi \wedge \mathbf{K}\neg\mathbf{K}\neg\psi') \rightarrow \mathbf{K}\psi$

Logic Programs: $p_0 \leftarrow p_1, \texttt{not}\, p_2$

$\leadsto$ $(\mathbf{K}p_1 \wedge \mathbf{K}\neg\mathbf{K}p_2) \rightarrow \mathbf{K}p_0$

Argumentation: (A, R) and $a \in A, (a, b) \in R$

$\leadsto$ $\mathbf{K}\neg\mathbf{K}\neg a \rightarrow \mathbf{K}a, \quad \mathbf{K}a \rightarrow \mathbf{K}\neg b$

[Schwarz & Truszczynski, 1993]

Represent $\mathcal{M}$ by a partition (Φ, Ψ) of $\{\mathbf{K}\phi \mid \mathbf{K}\phi \in \mathit{Subf}(T)\}$.

Let $\Theta = A \cup \{\neg\mathbf{K}\phi \mid \phi \in \Phi\} \cup \{\mathbf{K}\psi \mid \psi \in \Psi\} \cup \Psi$

Check if:

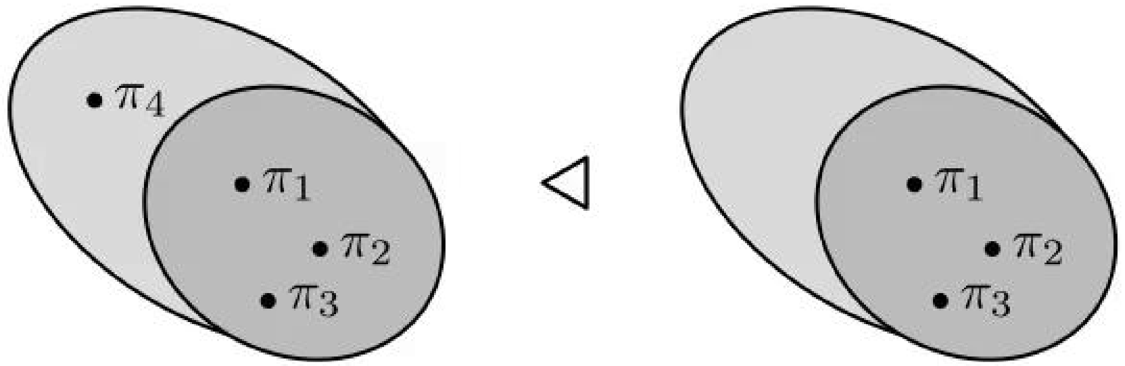
1. Guessed beliefs can all be true?

Θ is prop. consistent

2. Rejected beliefs don't follow from the accepted ones?

$\forall \varphi \in \Phi$: $\Theta \nvdash \varphi$

Background: Non-Monotonicity in S4F



$\mathcal{M}$ is a **minimal model of** $T \subseteq \mathcal{L}_{\mathbf{K}}$ iff:
1. $\mathcal{M} \Vdash T$
2. for all $\mathcal{M}'$ s.t. $\mathcal{M}' \triangleleft \mathcal{M}$: $\mathcal{M}' \nVdash T$

Default Logic: $\dfrac{\phi : \psi'}{\psi}$

$\rightsquigarrow$ $(\mathbf{K}\phi \wedge \mathbf{K}\neg\mathbf{K}\neg\psi') \rightarrow \mathbf{K}\psi$

Logic Programs: $p_0 \leftarrow p_1, \texttt{not}\, p_2$

$\rightsquigarrow$ $(\mathbf{K}p_1 \wedge \mathbf{K}\neg\mathbf{K}p_2) \rightarrow \mathbf{K}p_0$

Argumentation: (A, R) and $a \in A, (a, b) \in R$

$\rightsquigarrow$ $\mathbf{K}\neg\mathbf{K}\neg a \rightarrow \mathbf{K}a, \quad \mathbf{K}a \rightarrow \mathbf{K}\neg b$

[Schwarz & Truszczynski, 1993]

Represent $\mathcal{M}$ by a partition (Φ, Ψ) of $\{\mathbf{K}\phi \mid \mathbf{K}\phi \in \mathit{Subf}(T)\}$.

Let $\Theta = A \cup \{\neg\mathbf{K}\phi \mid \phi \in \Phi\} \cup \{\mathbf{K}\psi \mid \psi \in \Psi\} \cup \Psi$

Check if:

1. Guessed beliefs can all be true?

Θ is prop. consistent

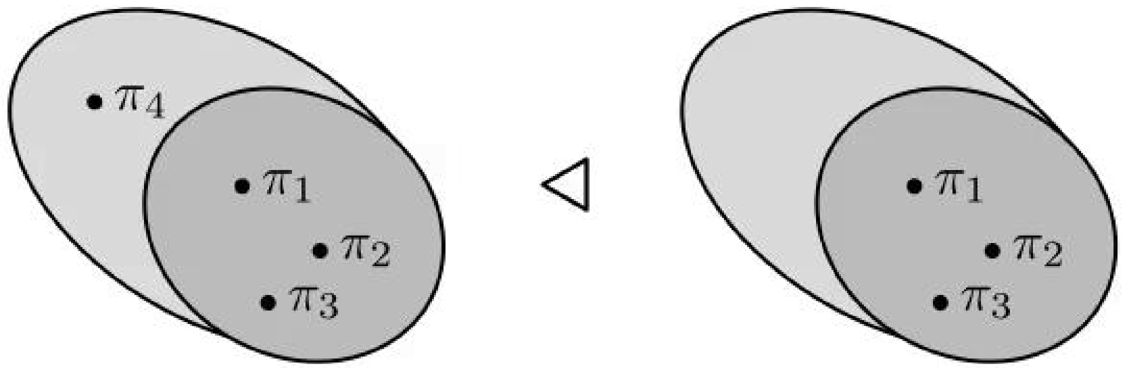
2. Rejected beliefs don't follow from the accepted ones?

$\forall\varphi \in \Phi$: $\Theta \nvdash \varphi$

3. Structure is minimal?

$\forall\psi \in \Psi$: $A \cup \{\neg\mathbf{K}\varphi \mid \varphi \in \Phi\} \vdash_{\text{S4F}} \psi$

Background: Non-Monotonicity in S4F



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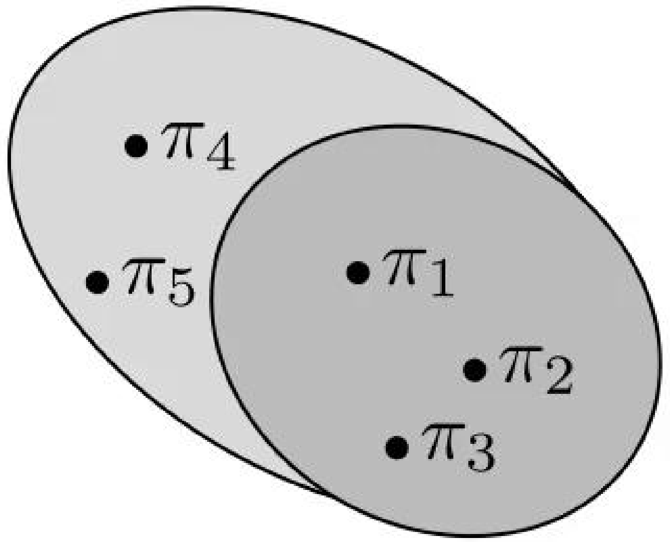
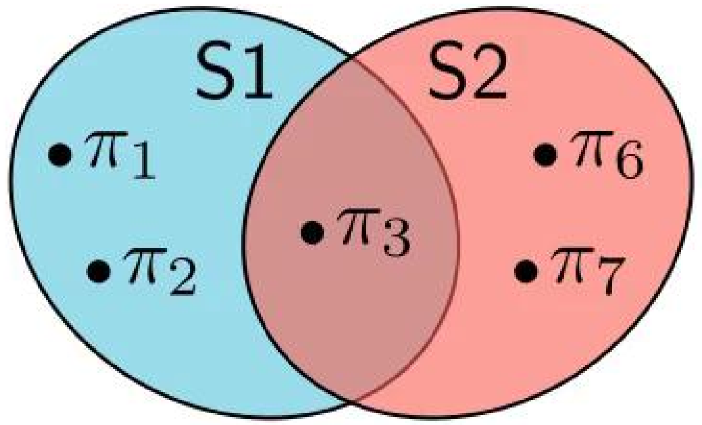
S4F and Standpoint Logic: Product and Fusion

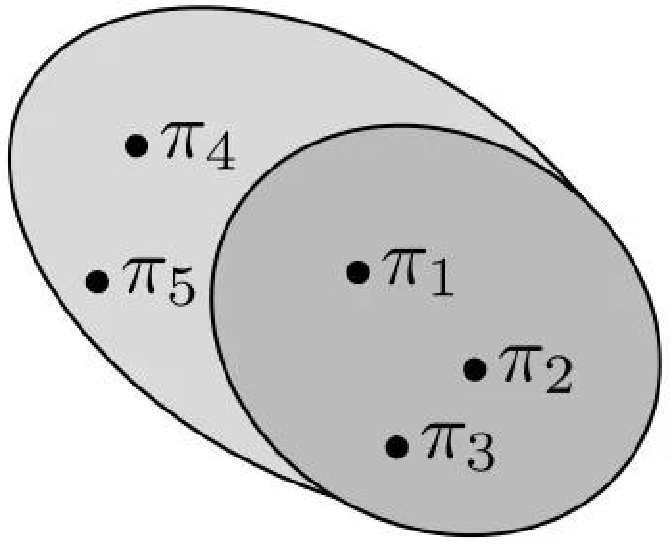


S4F



Standpoint Logic

S4F and Standpoint Logic: Product and Fusion



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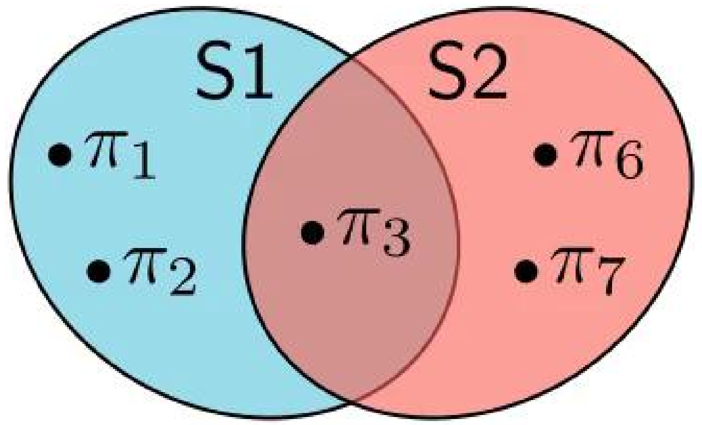

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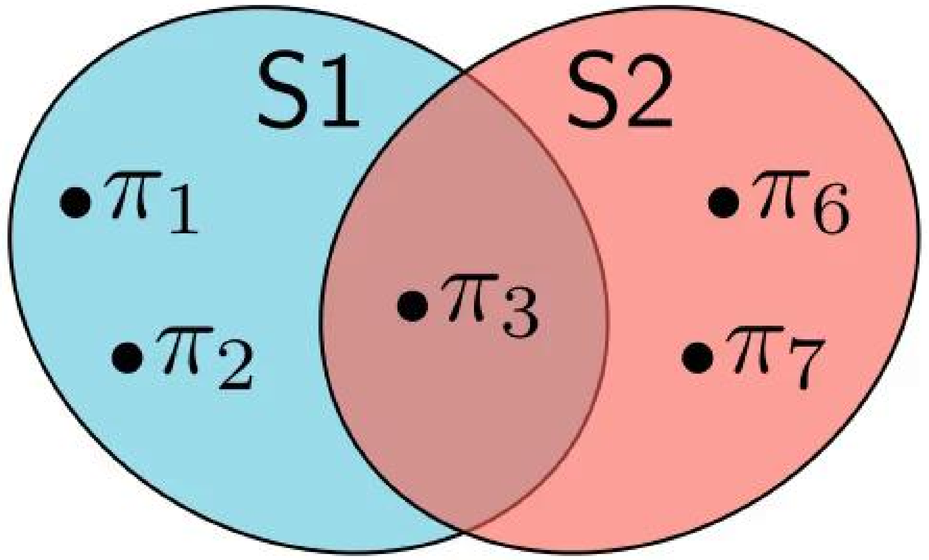

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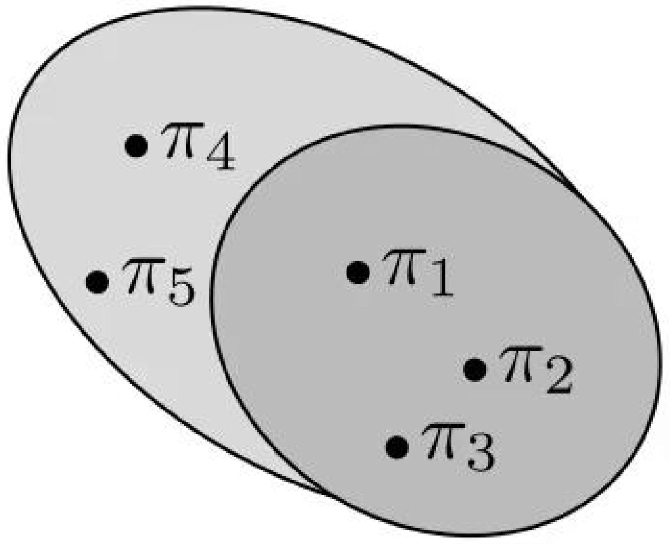

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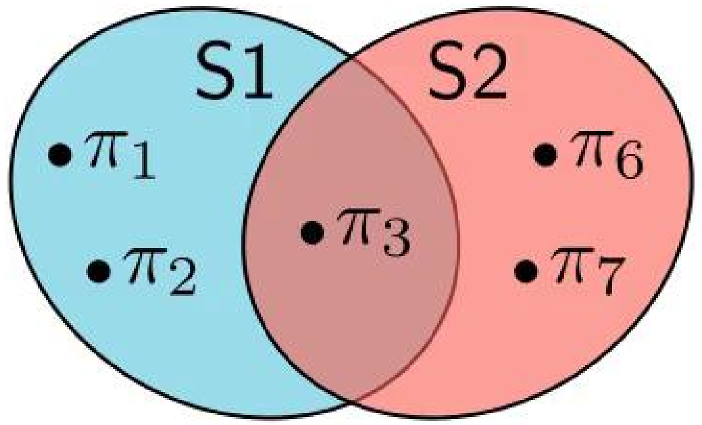

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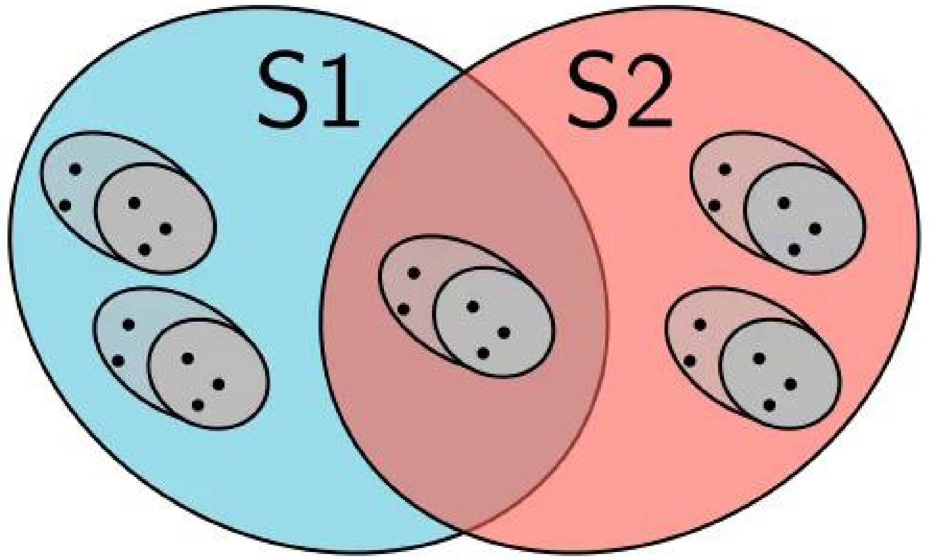
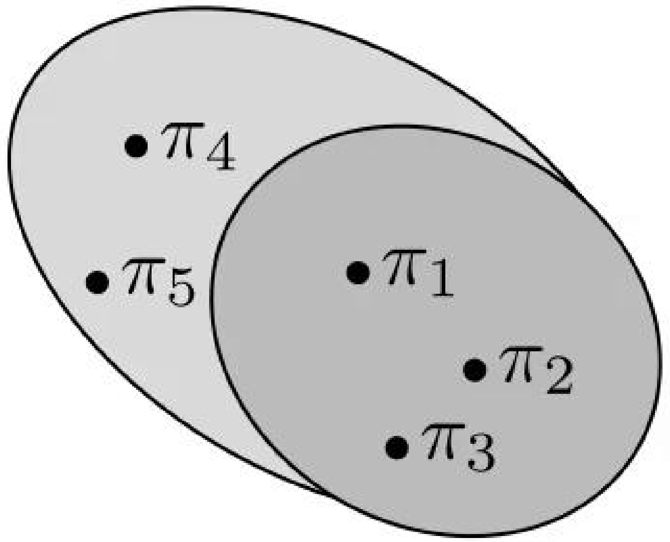


Standpoint S4F

S4F and Standpoint Logic: Product and Fusion



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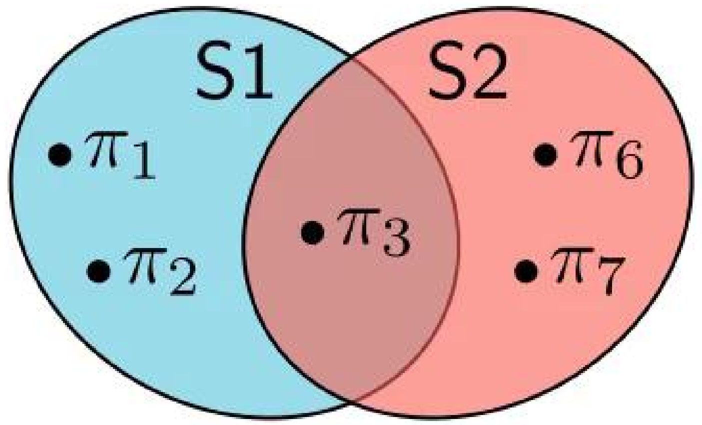

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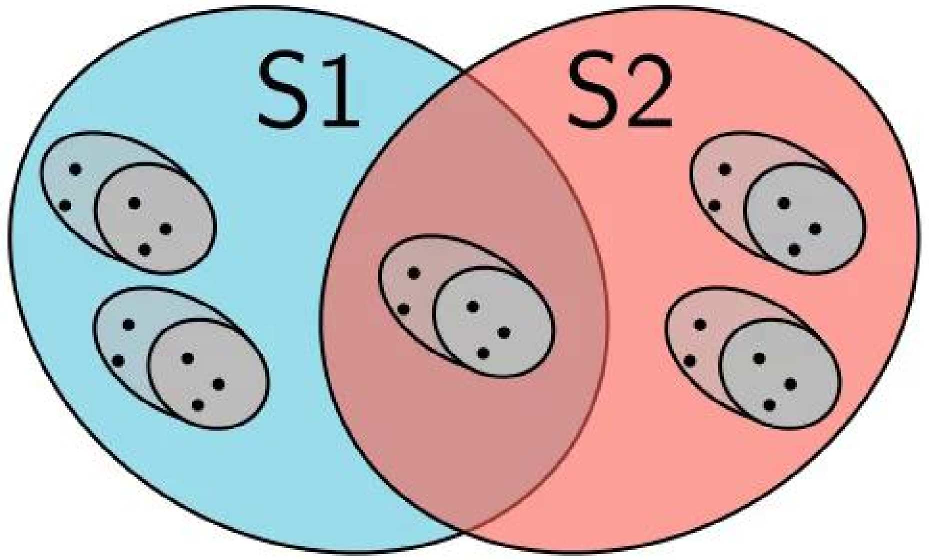

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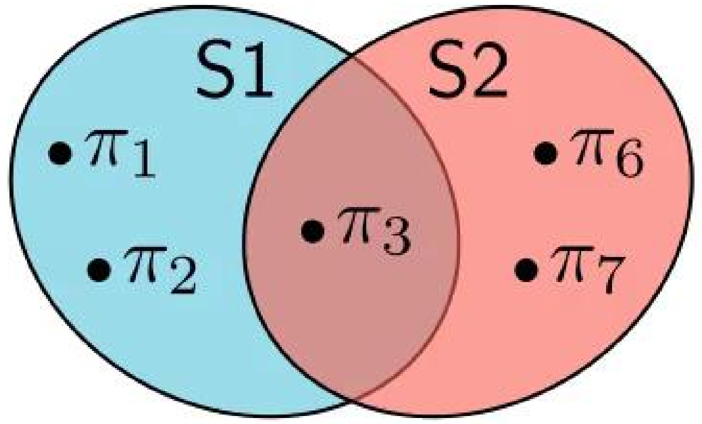

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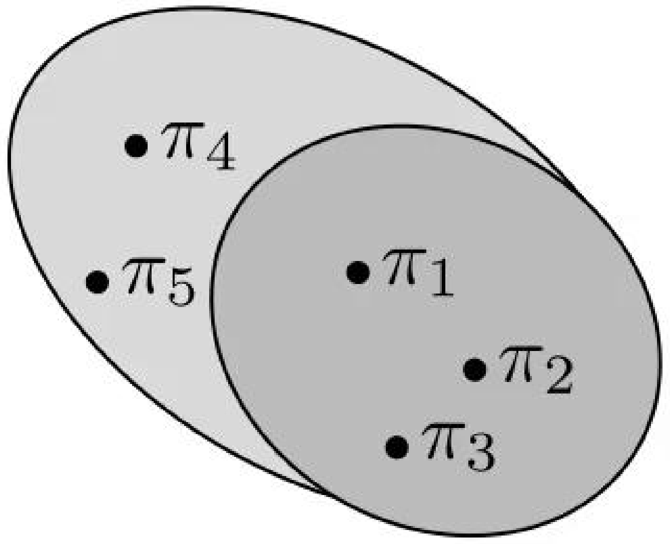

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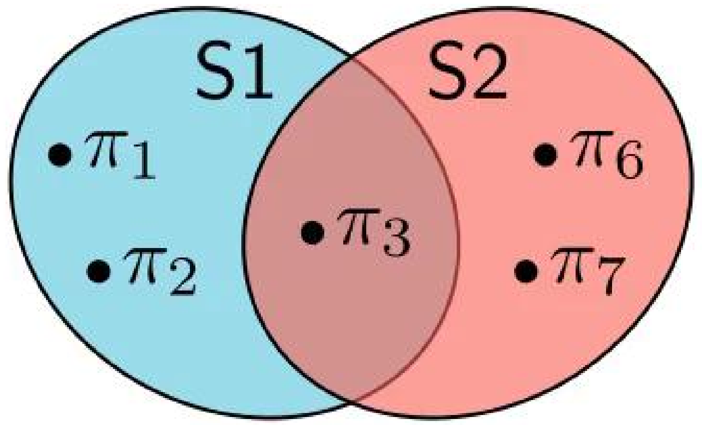

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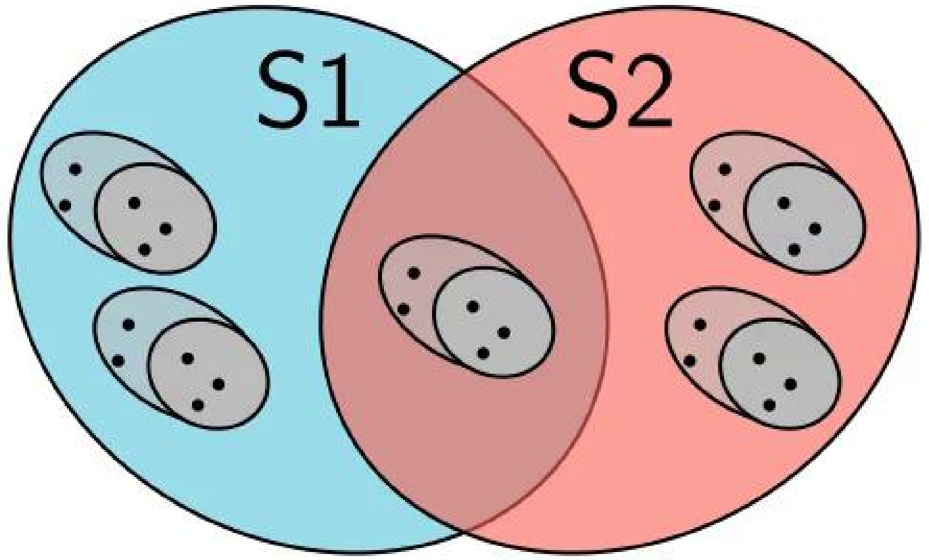

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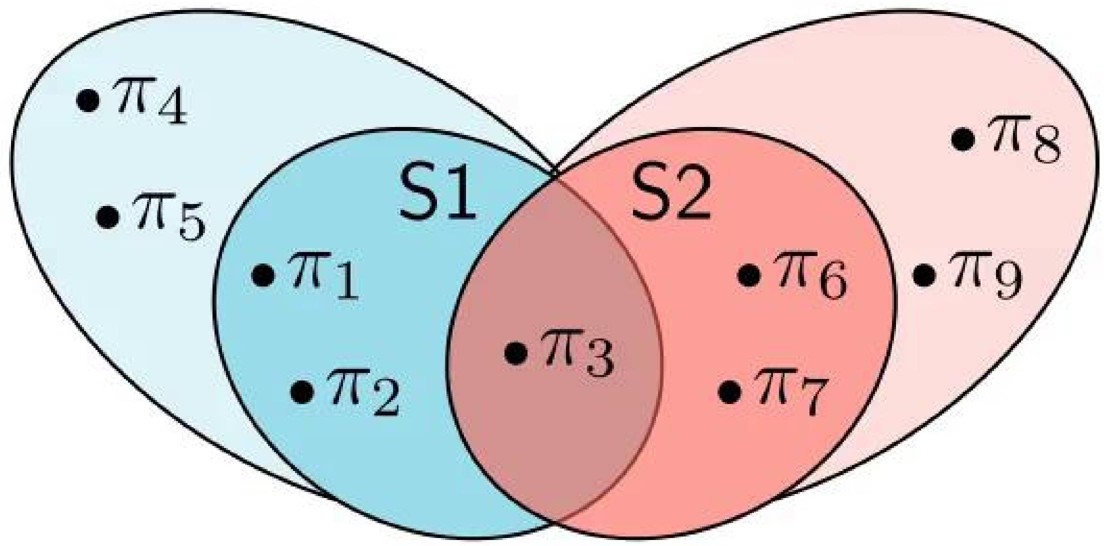


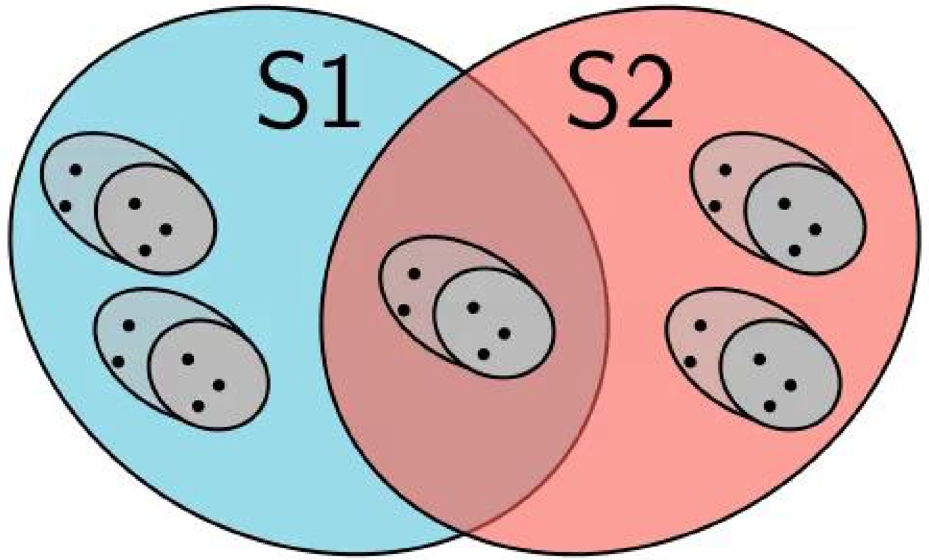
S4F Standpoint Logic

Product of Standpoint and S4F Logic: Standpoint S4F Logic

Standpoint S4F: Syntax & Semantics

Language $\mathcal{L}_{\mathbf{K}}$:

$\psi ::= \mathbf{K}a \mid \neg\psi \mid \psi \wedge \psi \mid \mathbf{K}\psi$



Standpoint S4F: Syntax & Semantics

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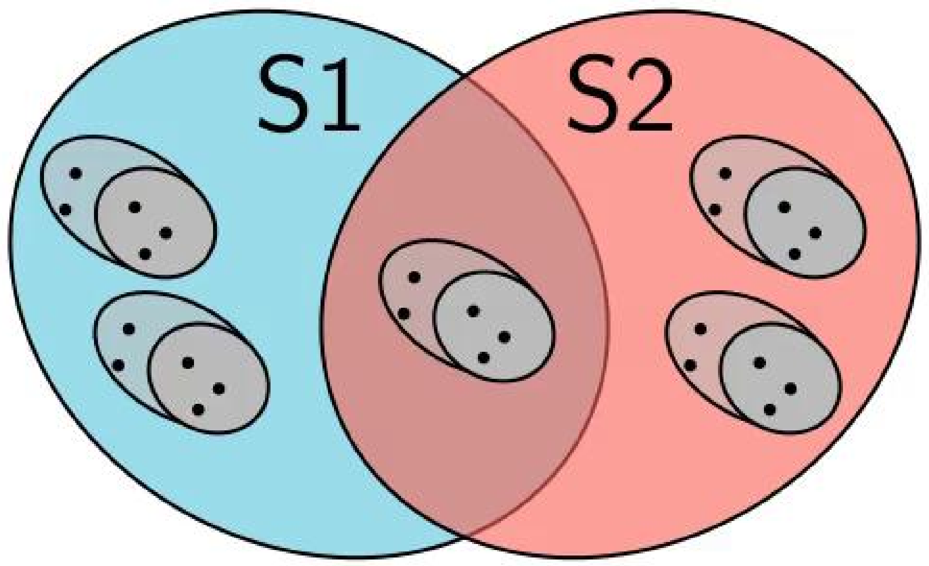
An $\mathbb{S}$S4F **simple** theory T:

Each $\varphi \in T$ is of the form $\varphi = \Box_{\mathsf{s}}\psi$ or $\Diamond_{\mathsf{s}}\psi$ with $\psi \in \mathcal{L}_{\mathbf{K}}$.



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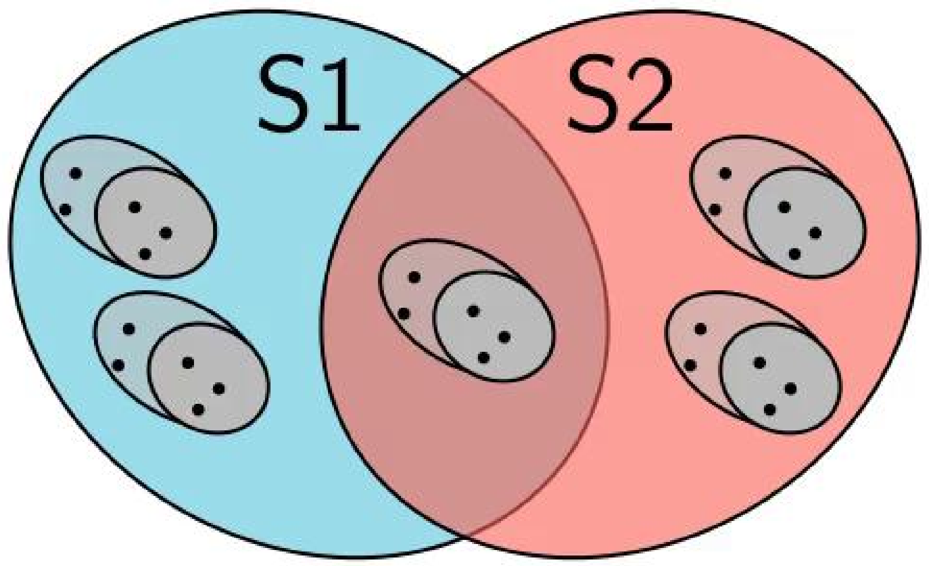

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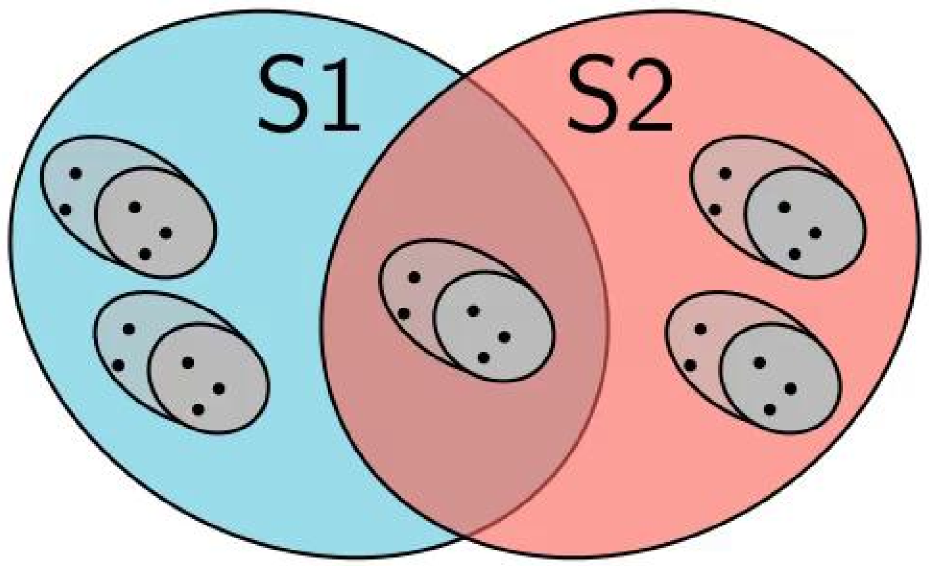

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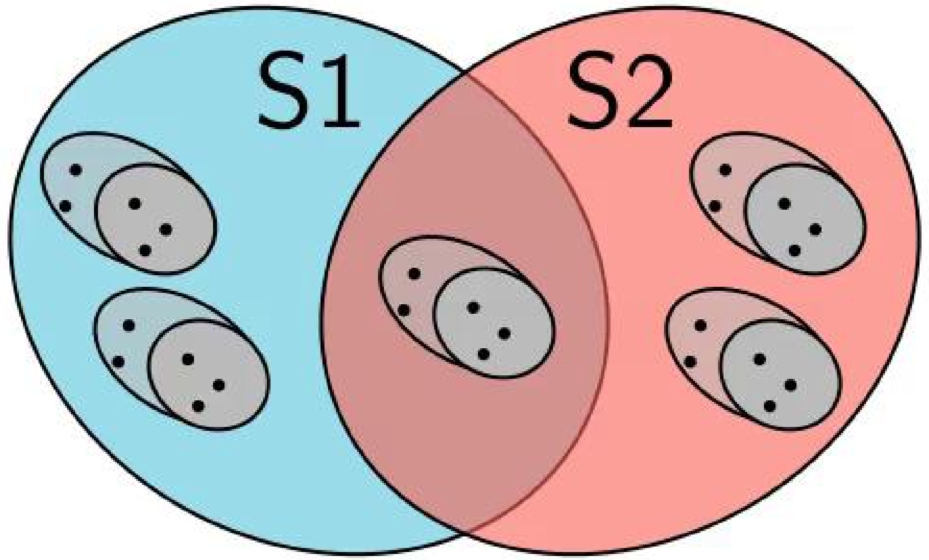

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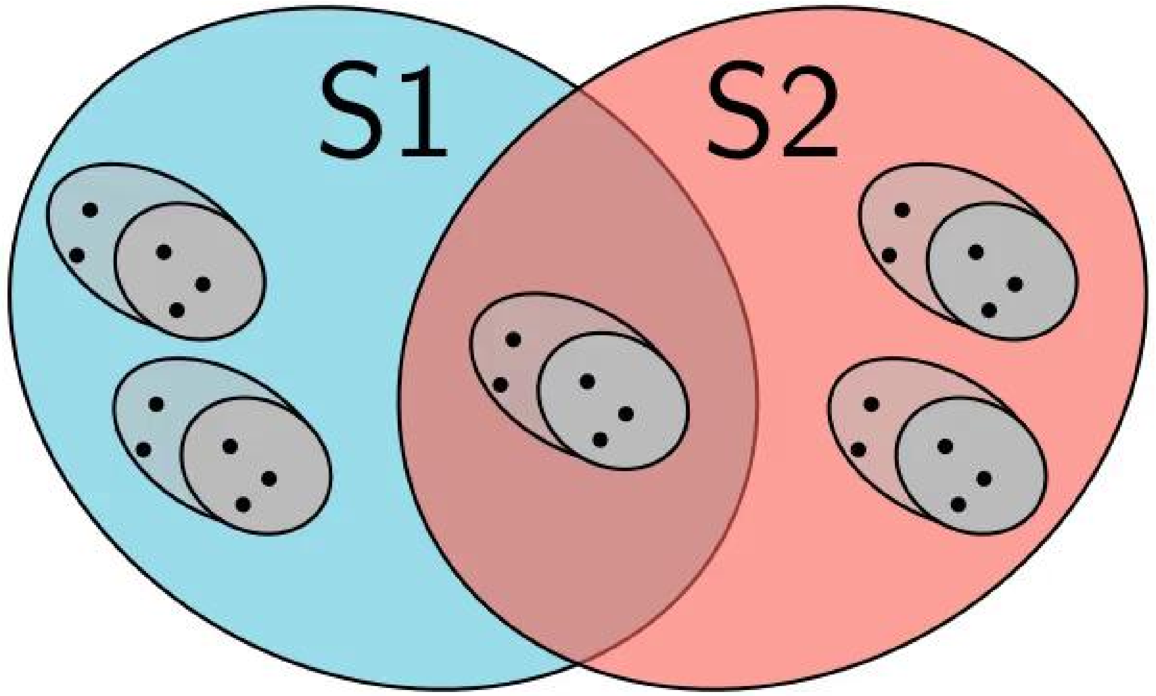
$$\mathfrak{S}, \pi, w \Vdash \Box_{\mathsf{s}}\varphi \quad :\Longleftrightarrow \quad \mathfrak{S}, \pi', w' \Vdash \varphi \text{ for all } \pi' \in \sigma(s) \text{ and } w' \in \zeta_o(\pi') \cup \zeta_i(\pi')$$

$$\mathfrak{S}, \pi, w \Vdash \mathbf{K}\varphi \quad :\Longleftrightarrow \quad \begin{cases} \mathfrak{S}, \pi, w' \Vdash \varphi \text{ for all } w' \in \zeta_o(\pi) \cup \zeta_i(\pi) & \text{if } w \in \zeta_o(\pi) \\ \mathfrak{S}, \pi, w' \Vdash \varphi \text{ for all } w' \in \zeta_i(\pi) & \text{if } w \in \zeta_i(\pi) \end{cases}$$

Standpoint S4F: Non-Monotonicity



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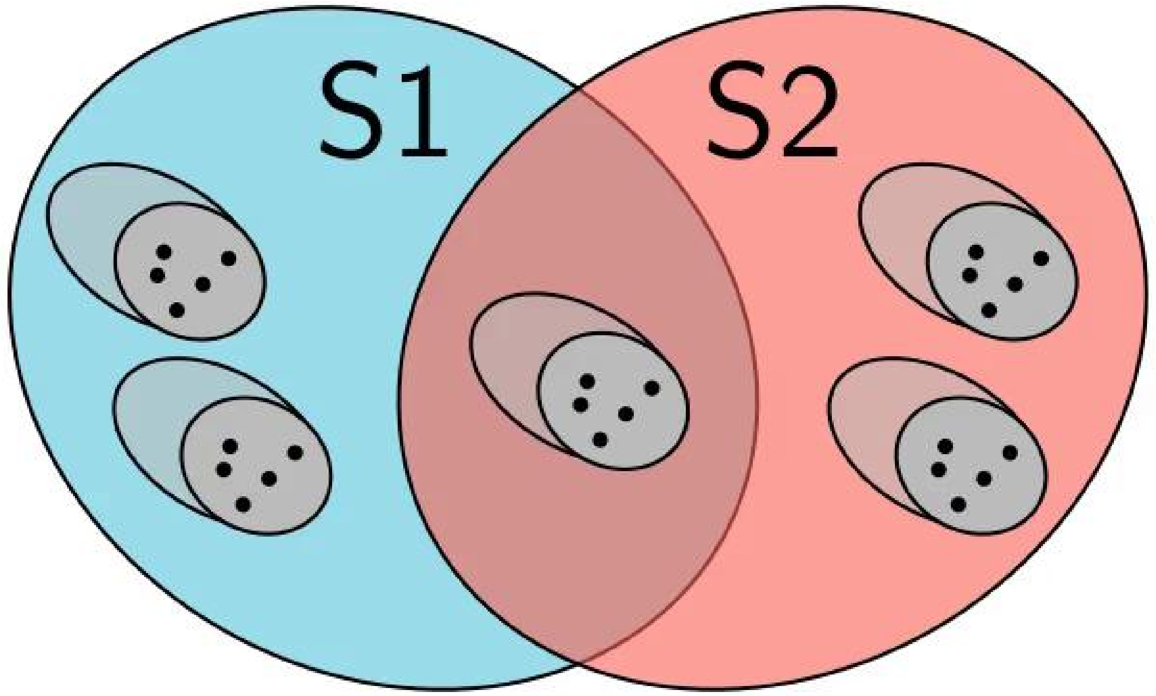

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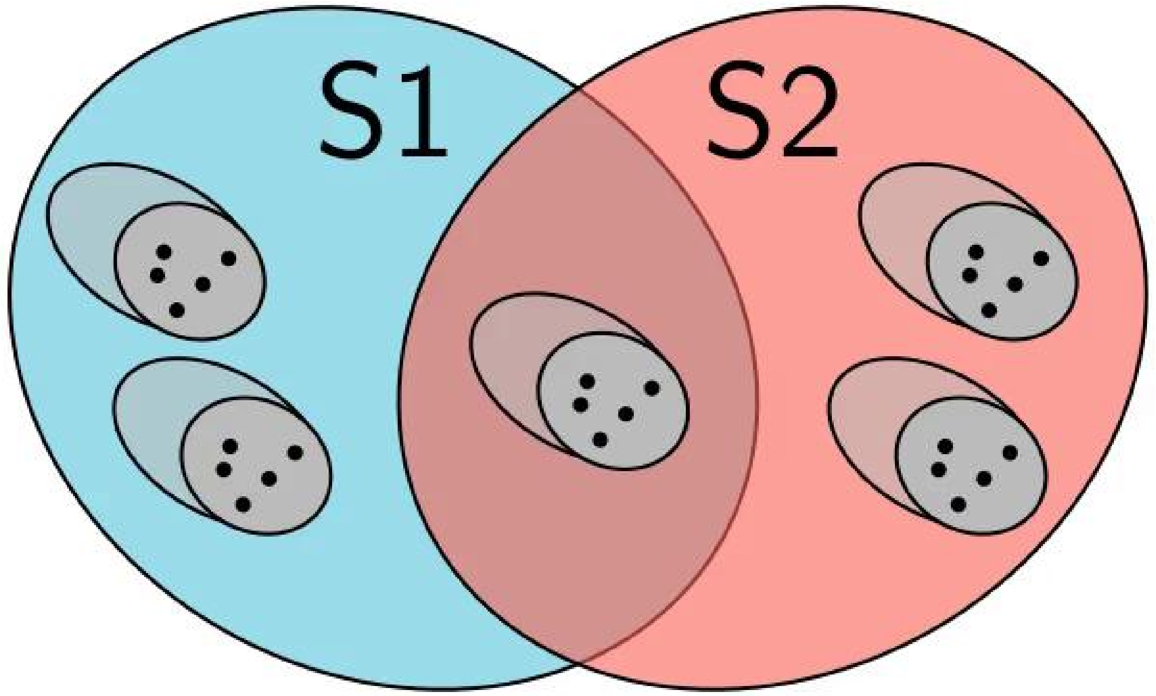
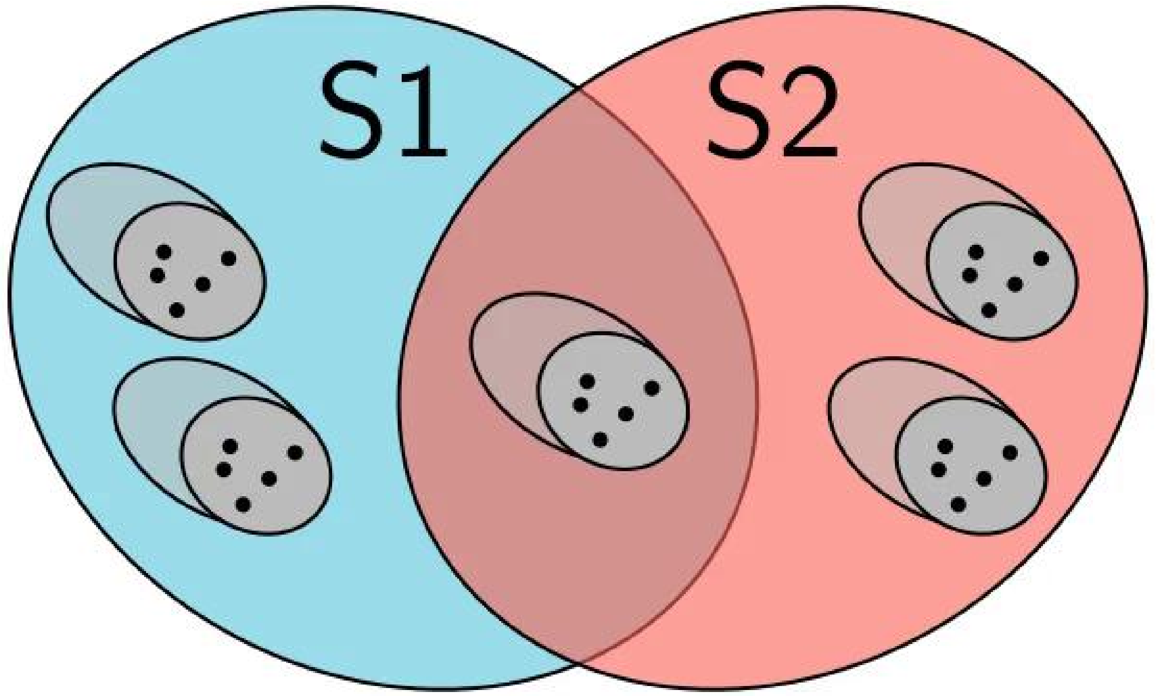


Minimal model:

Assert the local S4F theory T_π at each π such that $T_\pi \subseteq \{\varphi \mid \Box_{\mathsf{s}}\varphi \in T, \pi \in \sigma(\mathsf{s})\}$ (due to simple theories and small model property), then apply the Schwarz & Truszczyński (1993) procedure.

Standpoint S4F: Non-Monotonicity



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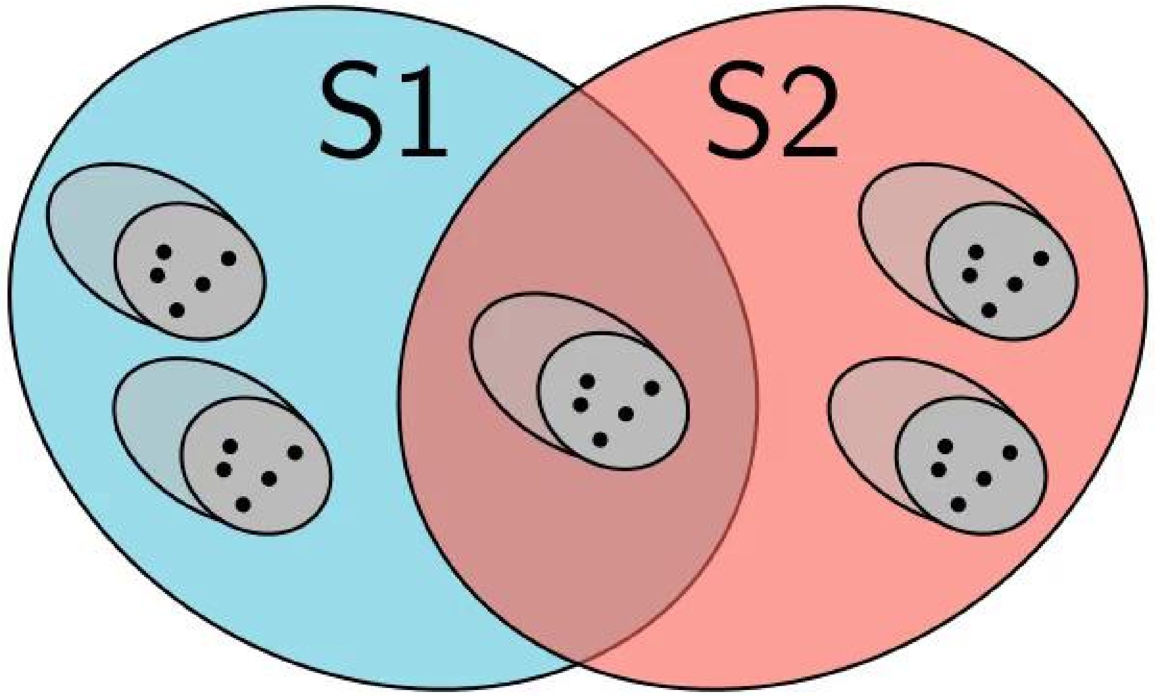
"DOES A SIMPLE $\mathbb{S}$S4F THEORY T HAVE A MINIMAL MODEL?" is Σ_2^P-complete.

Standpoint S4F: Non-Monotonicity



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Problem:

$\{\Box_{\mathsf{s}}\mathbf{K}a, \Box_{\mathsf{u}}\mathbf{K}b\} \approx\!\!\!\!|_{\mathsf{cred}}\ \Box_{\mathsf{s}}\mathbf{K}b$

$\mathfrak{S}$ with $\sigma(\mathsf{s}) = \sigma(\mathsf{u}) = \{\pi\}$ and $\gamma(\pi) = \{a, b\}$ is a minimal model

"Does a simple $\mathbb{S}$S4F theory T have a minimal model?" is Σ_2^P-complete.

Standpoint S4F: Beyond Simple Theories

Beyond "atomic" standpoint modalities:

$\{\Box_{\mathsf{s}}\mathbf{K}a \vee \neg\Box_{\mathsf{s}}\mathbf{K}a\} \not\approx_{\mathsf{cred}} \Box_{\mathsf{s}}\mathbf{K}a$

Standpoint S4F: Beyond Simple Theories

Beyond “atomic” standpoint modalities:

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Unrestricted syntax $\mathcal{L}_{\mathbf{K}}^{\Box}$:

$$\varphi ::= a \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathbf{K}\varphi \mid \Box_{\mathsf{s}}\varphi$$

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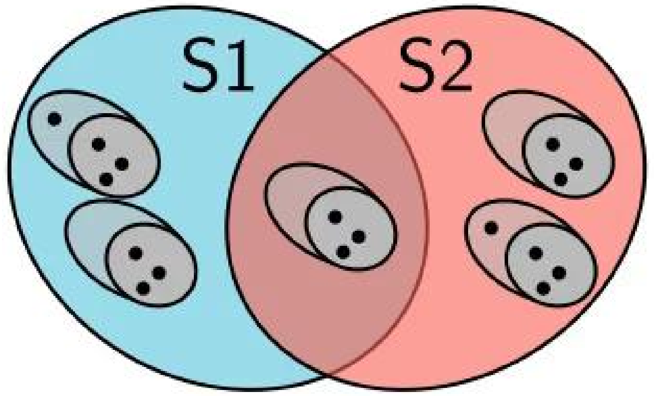

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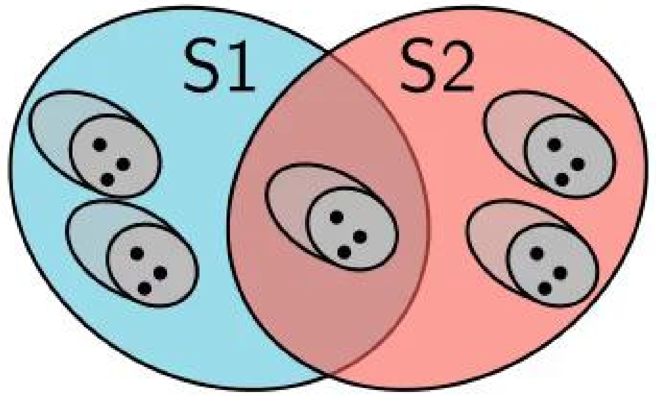

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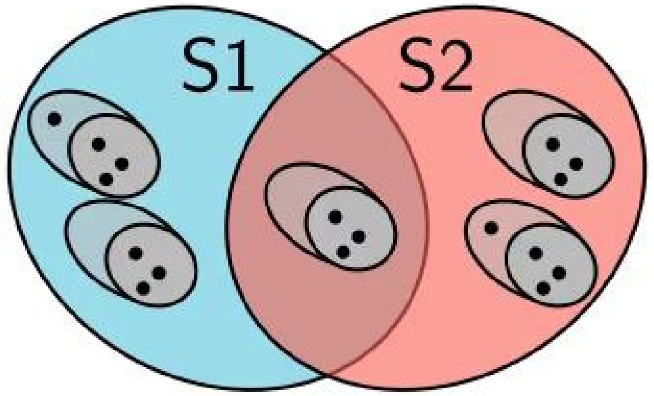

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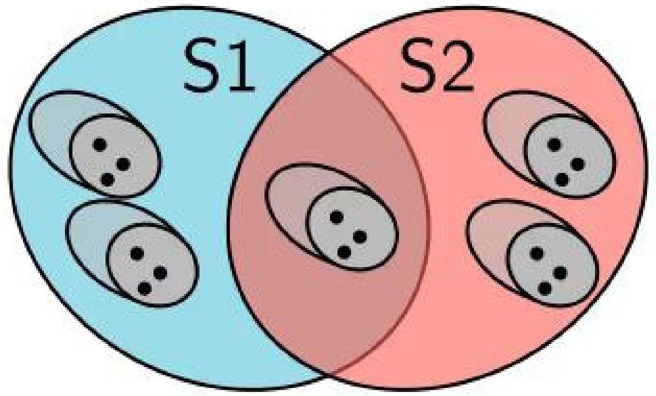

"DOES $T \subseteq \mathcal{L}^{\Box}_{\mathbf{K}}$ HAVE A MINIMAL MODEL?" is Σ^P_3-hard.

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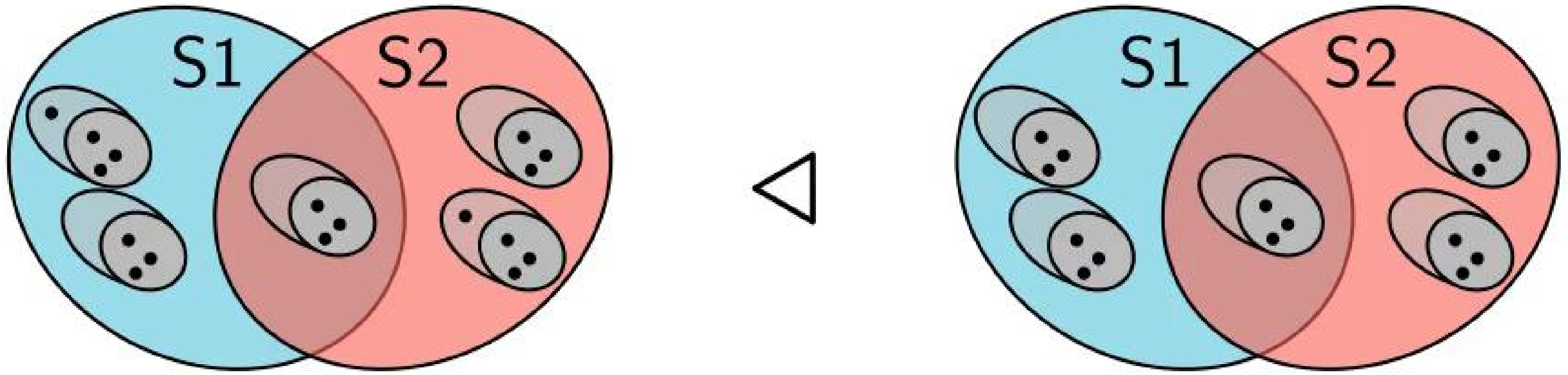

"DOES $T \subseteq \mathcal{L}_{\mathbf{K}}^{\Box}$ HAVE A MINIMAL MODEL?" is Σ_3^P-hard.

$$\Phi := \exists s_1, \ldots, s_m \forall p_1, \ldots, p_n \exists r_1, \ldots, r_l \Psi$$

$$\begin{aligned}
trans(\Phi) := {} & (\mathbf{K}p_1 \vee \mathbf{K}\neg p_1) \wedge \ldots \wedge (\mathbf{K}p_1 \vee \mathbf{K}\neg p_n)) && \wedge\ (4.1)\\
& (\neg((p_1 \leftrightarrow \Box_* q_1) \wedge \ldots \wedge (p_n \leftrightarrow \Box_* q_n)) \vee (p_1 \wedge \ldots \wedge p_n)) && \wedge\ (4.2)\\
& (\mathbf{MK}q_1 \wedge \ldots \wedge \mathbf{MK}q_n) && \wedge\ (4.3)\\
& ((\Box_* s_1 \vee \Box_* \neg s_1) \wedge \ldots \wedge (\Box_* s_m \vee \Box_* \neg s_m)) && \wedge\ (4.4)\\
& ((\mathbf{K}r_1 \vee \mathbf{K}\neg r_1) \wedge \ldots \wedge (\mathbf{K}r_l \vee \mathbf{K}\neg r_l)) && \wedge\ (4.5)\\
& \Diamond_*(p_1 \wedge \ldots \wedge p_n) && \wedge\ (4.6)\\
& \Psi && (4.7)
\end{aligned}$$

Fusion of S4F and Standpoint Logic: S4F Standpoint Logic

S4F Standpoint: Syntax & Semantics

Language $\mathcal{L}_{\mathbb{S}}^{\preceq}$:

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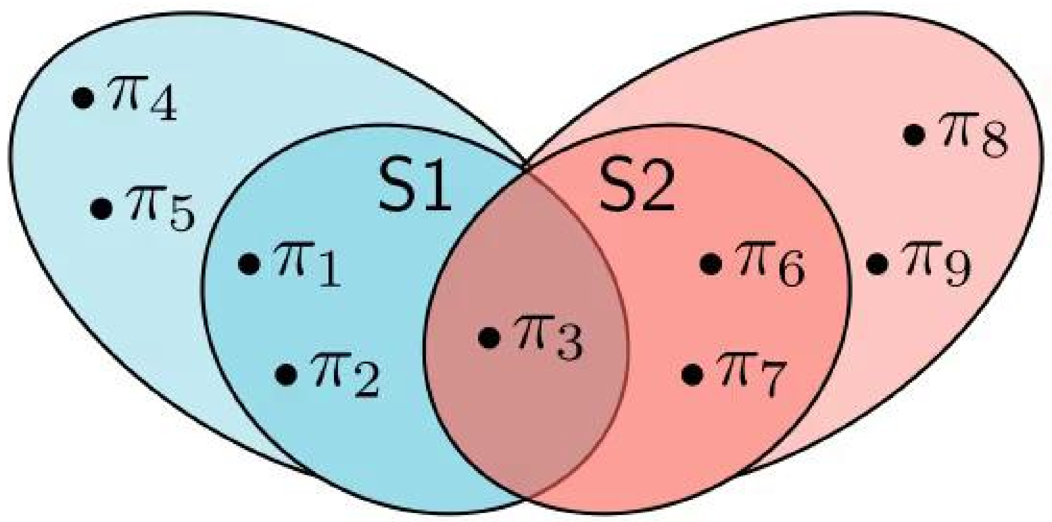

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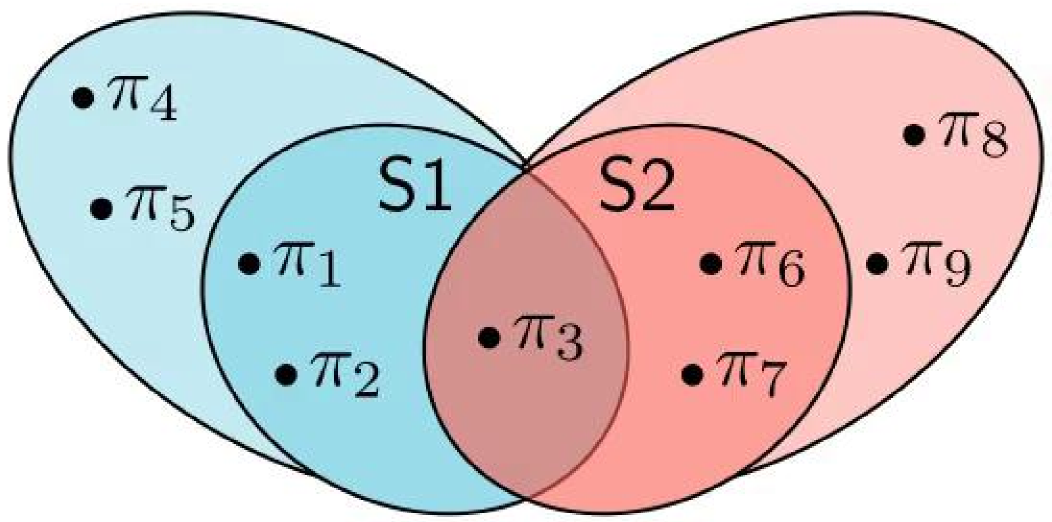

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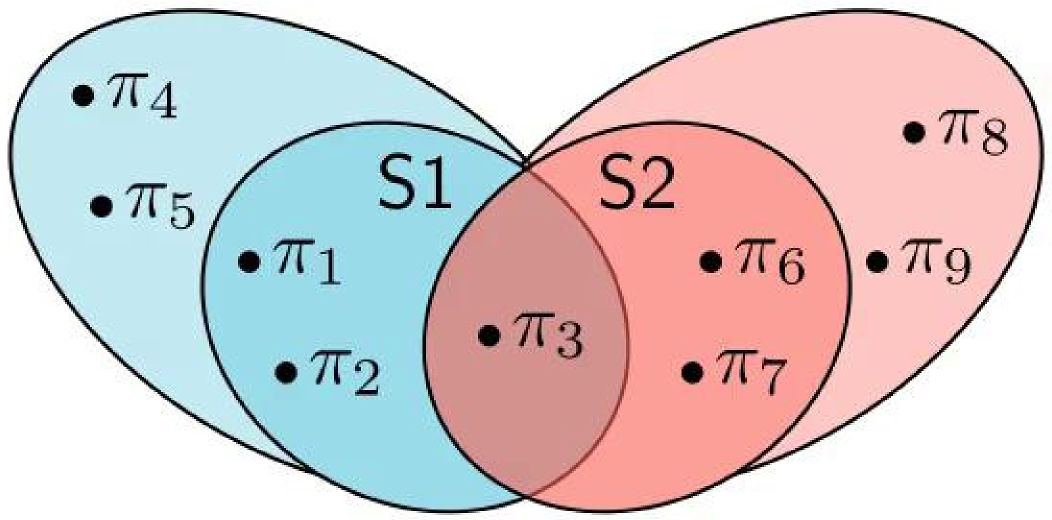

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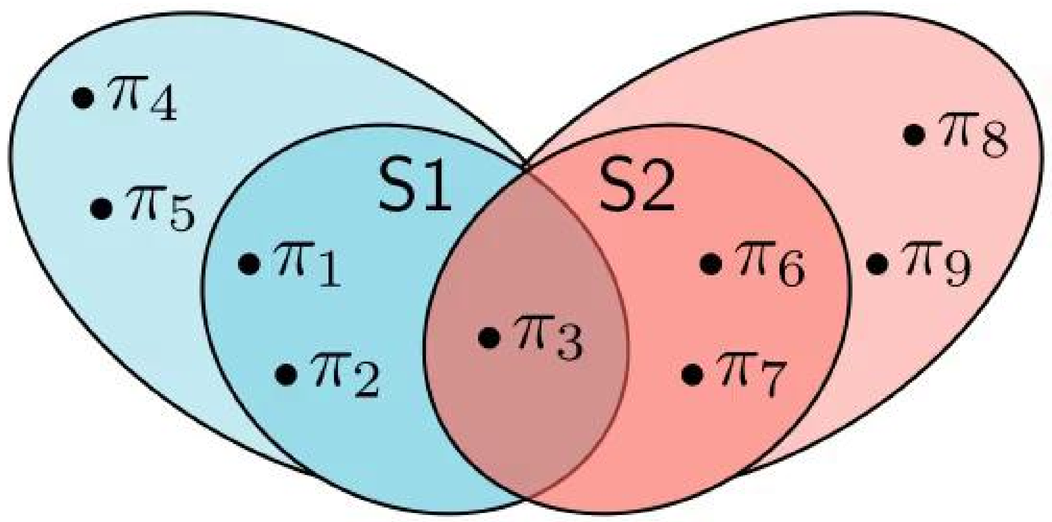

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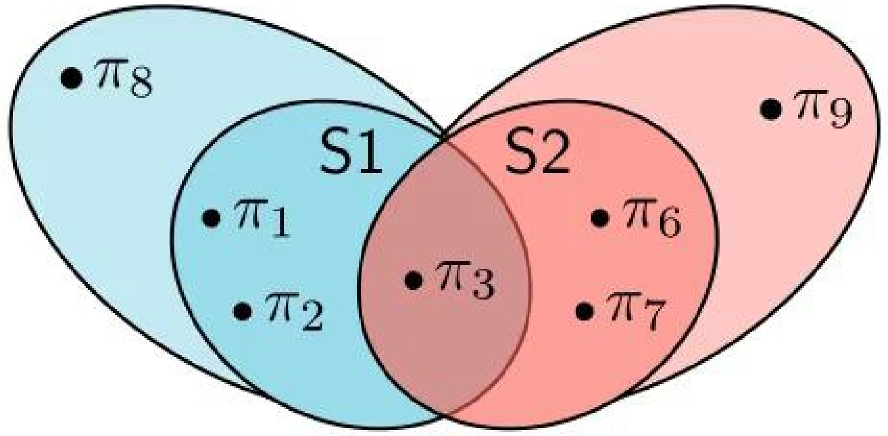
$$\mathfrak{F}, \pi \Vdash \Box_{\mathsf{s}}\psi \quad :\Longleftrightarrow \quad \begin{cases} \mathfrak{F}, \pi' \Vdash \psi \text{ for all } \pi' \in \sigma(s) \cup \tau(s) & \text{if } \pi \in \tau(*) \\ \mathfrak{F}, \pi' \Vdash \psi \text{ for all } \pi' \in \sigma(s) & \text{otherwise} \end{cases}$$

$$\mathfrak{F}, \pi \Vdash \mathsf{s} \preceq \mathsf{u} \quad :\Longleftrightarrow \quad \sigma(s) \subseteq \sigma(u) \text{ and } \tau(s) \subseteq \tau(u)$$

S4F Standpoint: Non-Monotonicity



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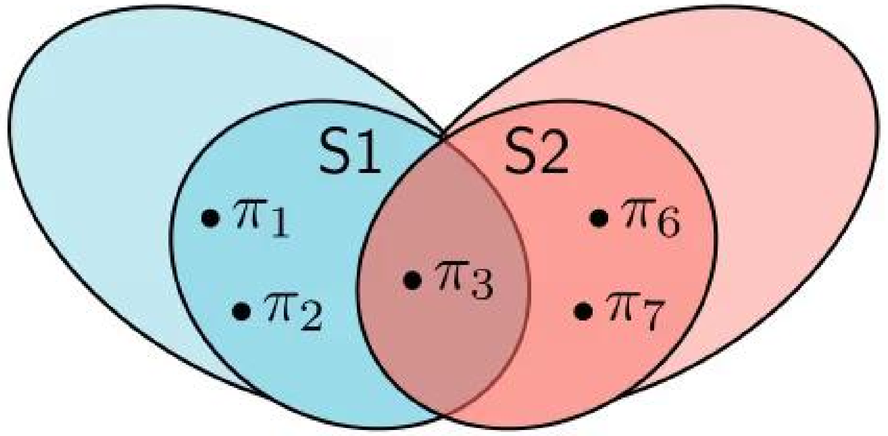

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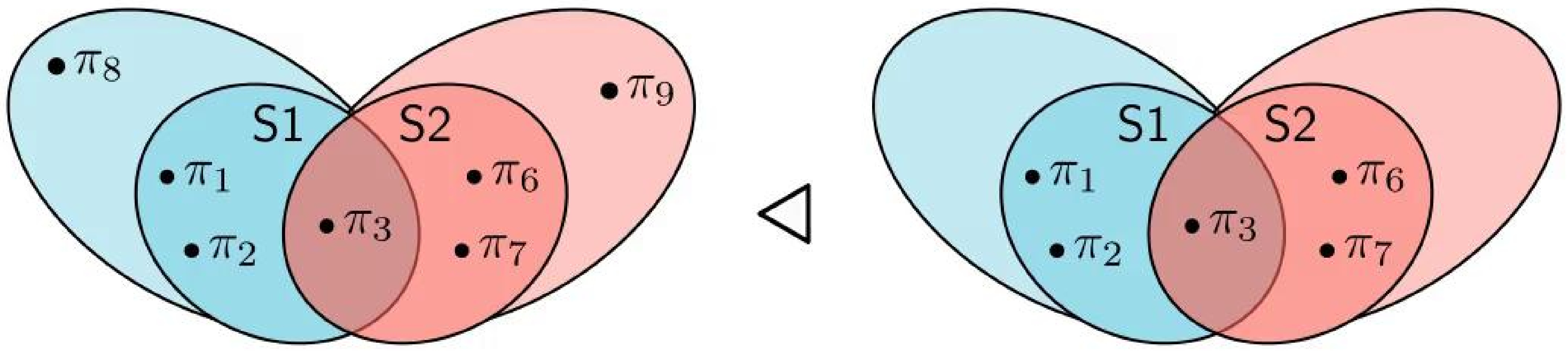

Minimal model:

Represent $\mathfrak{F}$ by a partition (Φ, Ψ) of $\{\Box_{\mathsf{s}}\phi \mid \Box_{\mathsf{s}}\phi \in Subf(T)\}$, then adapt the Schwarz & Truszczynski (1993) procedure.

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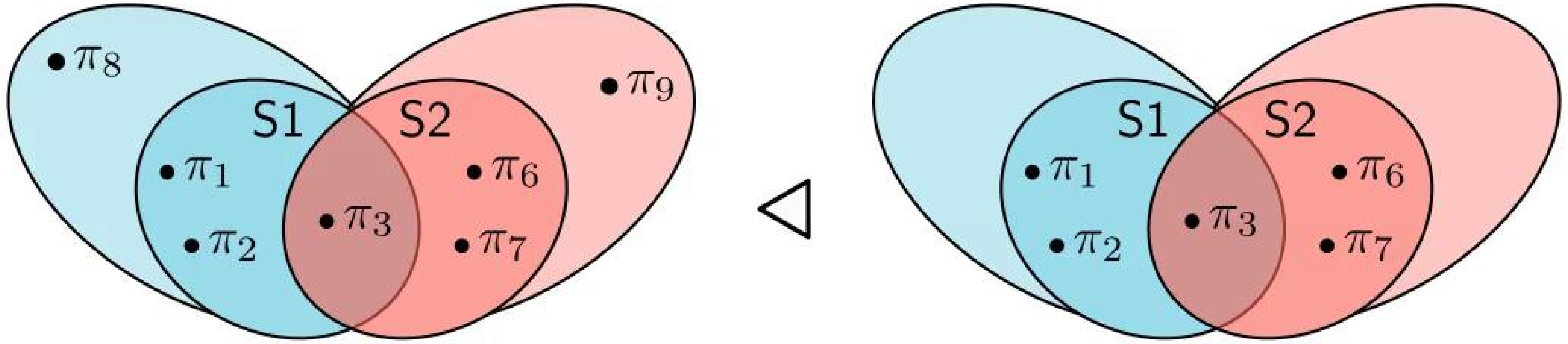

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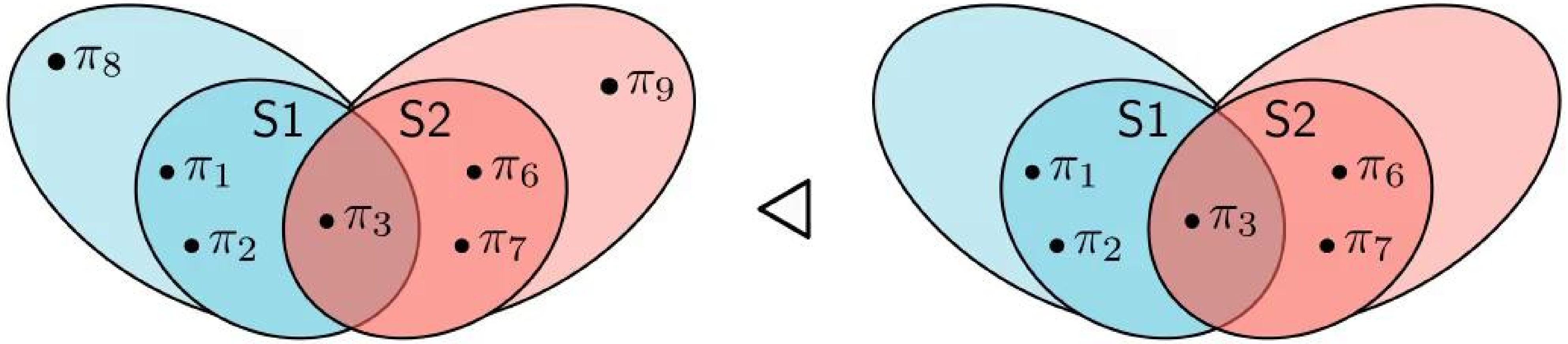

$\square_{S1}\left[\frac{\mathtt{A} \wedge \mathtt{B} : \mathtt{X}}{\mathtt{P}}\right]$

$\square_{S2}\left[\frac{(\mathtt{A} \wedge \mathtt{B}) \vee (\mathtt{A} \wedge \mathtt{C}) \vee (\mathtt{B} \wedge \mathtt{C}) : \mathtt{X}}{\mathtt{P}}\right]$

$\square_{S3}\left[\frac{\mathtt{A} \wedge (\mathtt{B} \vee \mathtt{C}) : \mathtt{Y}}{\mathtt{P}}\right]$

$\mathtt{A} \wedge \mathtt{C}$

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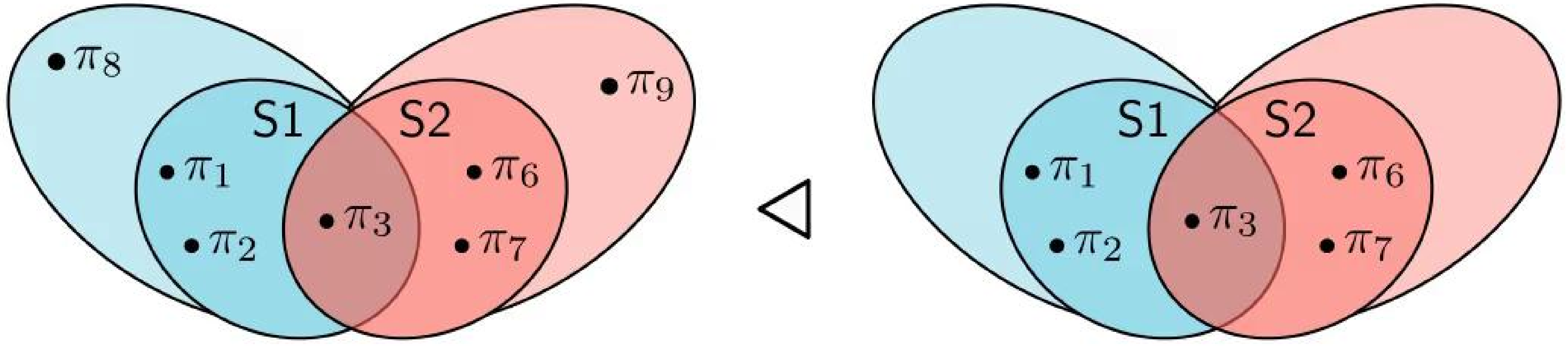

$$\left.\begin{array}{l}
\Box_{\mathsf{S1}}\left[\dfrac{\mathtt{A} \wedge \mathtt{B} : \mathtt{X}}{\mathtt{P}}\right] \\[2ex]
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\Box_{\mathsf{S3}}\left[\dfrac{\mathtt{A} \wedge (\mathtt{B} \vee \mathtt{C}) : \mathtt{Y}}{\mathtt{P}}\right] \\[2ex]
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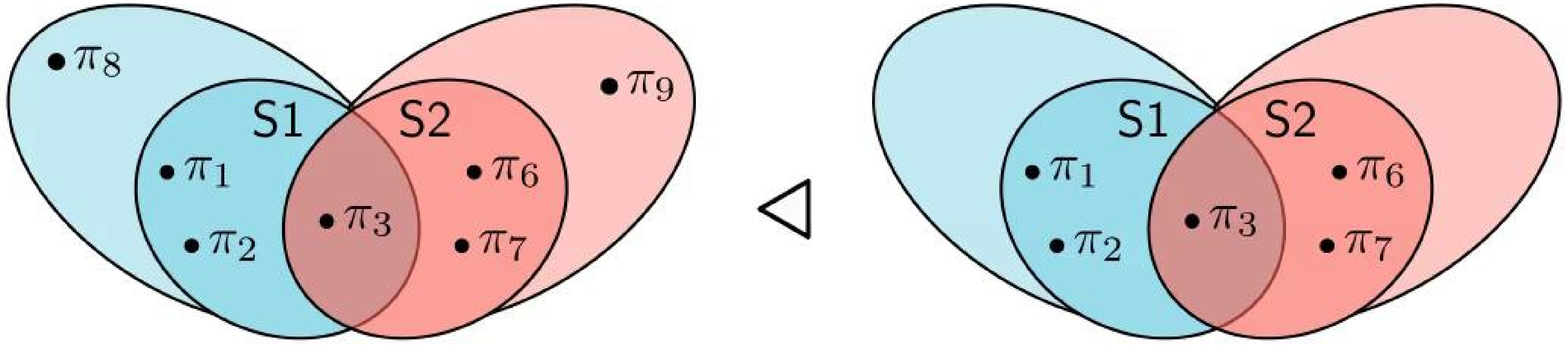

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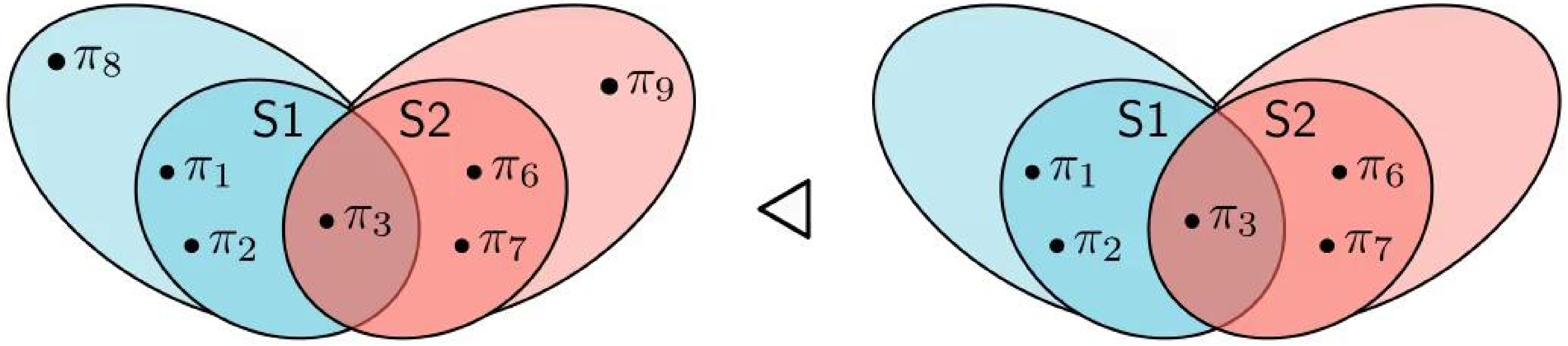

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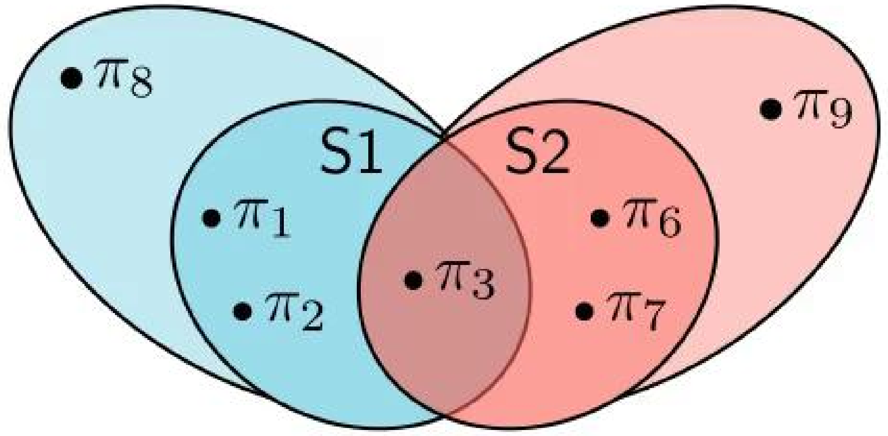

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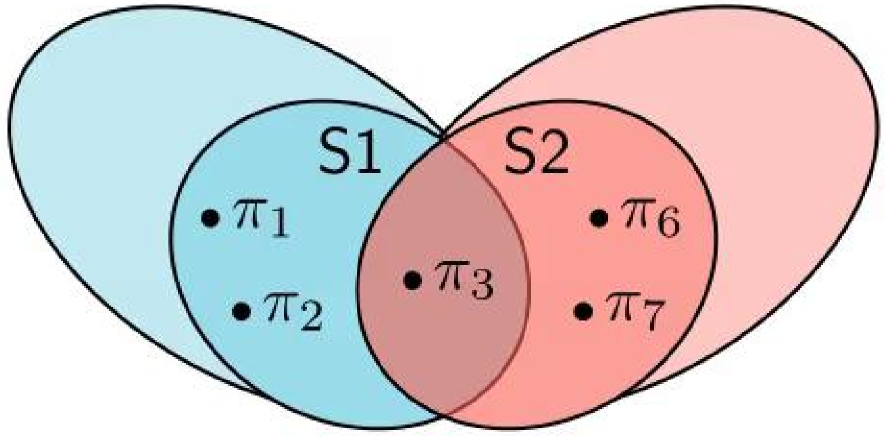

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Additional results:

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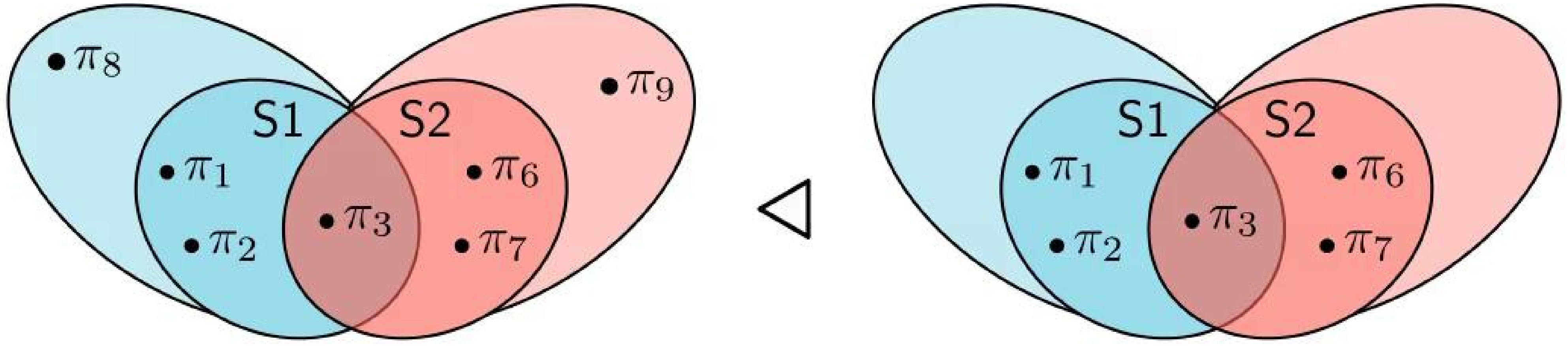

$$\left.\begin{array}{l} \Box_{S1}\left[\dfrac{\mathtt{A} \land \mathtt{B} : \mathtt{X}}{\mathtt{P}}\right] \\ \Box_{S2}\left[\dfrac{(\mathtt{A} \land \mathtt{B}) \lor (\mathtt{A} \land \mathtt{C}) \lor (\mathtt{B} \land \mathtt{C}) : \mathtt{X}}{\mathtt{P}}\right] \\ \Box_{S3}\left[\dfrac{\mathtt{A} \land (\mathtt{B} \lor \mathtt{C}) : \mathtt{Y}}{\mathtt{P}}\right] \\ \mathtt{A} \land \mathtt{C} \\ \neg\mathtt{Y} \end{array}\right\} \models \{\Box_{S2}\mathtt{P}, \xcancel{\Box_{S3}\mathtt{P}}\}$$

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"Does $T \subseteq \mathcal{L}_{\mathbb{S}}^{\preceq}$ have a minimal model?" is Σ_2^P-complete.

Additional results:

- Credulous and sceptical reasoning

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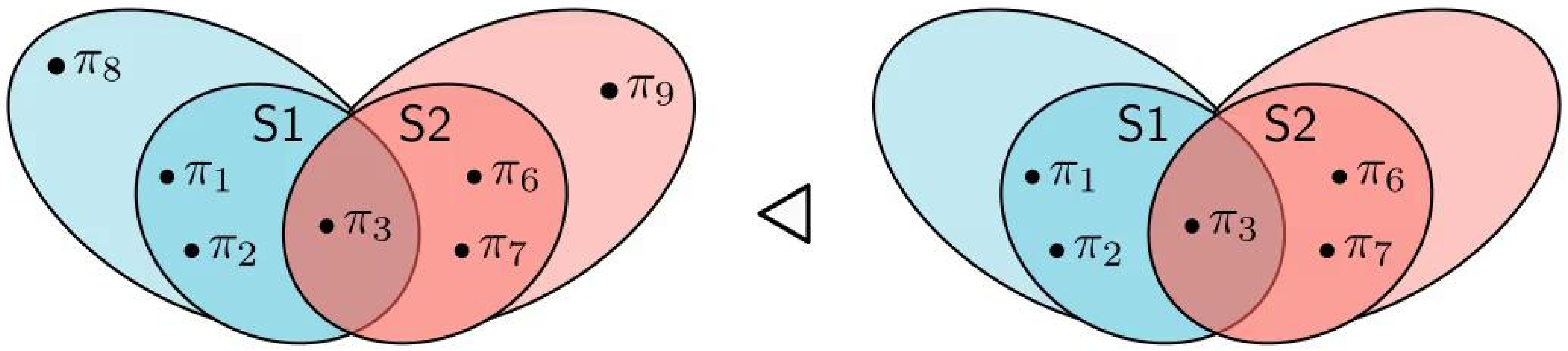

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Additional results:

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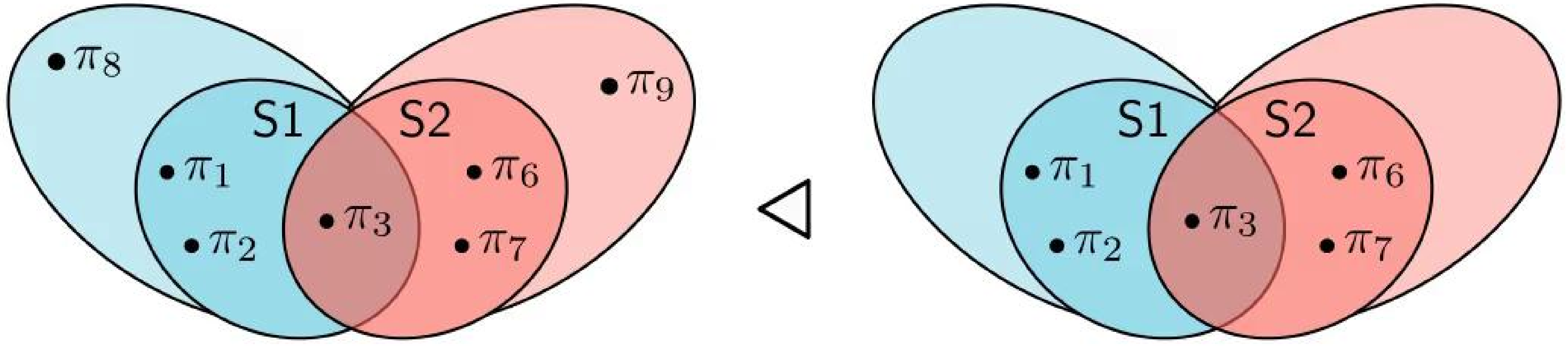
- Expansions

S4F Standpoint: Non-Monotonicity



$$\left.\begin{array}{l}
\Box_{S1}\left[\dfrac{\mathtt{A \land B : X}}{\mathtt{P}}\right] \\[2ex]
\Box_{S2}\left[\dfrac{\mathtt{(A \land B) \lor (A \land C) \lor (B \land C) : X}}{\mathtt{P}}\right] \\[2ex]
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Additional results:

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- Expansions
- Disjunctive ASP implementation

Summary

Two approaches to multi-perspective, non-monotonic reasoning:

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Two approaches to multi-perspective, non-monotonic reasoning:

Procedural (ASP / argumentation)

- First multi-shot ASP implementation of argument games.
- Modular and declarative design, separating formal definitions (ASP) from procedural control (Python).
- Extensible: interactivity, visualisation, additional semantics.
- Outperforms existing dispute-based ABA systems (e.g. `flexABle`).

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- Extensible: interactivity, visualisation, additional semantics.
- Outperforms existing dispute-based ABA systems (e.g. `flexABle`).

Declarative (Generalising S4F and Standpoint Logic)

- Explored multiple approaches to integrating standpoint modalities into S4F.
- Adding standpoint modalities does not increase complexity (in monotonic or non-monotonic case).
- Yields standpoint-enhanced variants of non-monotonic reasoning formalisms (default logic, argumentation, logic programs…)

Publications Summary

Dissertation publications

- Piotr Gorczyca, Hannes Straß. *"Non-Monotonic S4F Standpoint Logic."* (AAAI 2026)
- Martin Diller, Piotr Gorczyca. *"ABA Disputes in ASP: Advancing Argument Games through Multi-Shot Solving."* PRIMA (2025).
- Piotr Gorczyca, Hannes Straß. *"Adding Standpoint Modalities to Non-Monotonic S4F: Preliminary Results."* (NMR 2024)

Publications (outside the dissertation scope)

- Piotr Gorczyca, Dörthe Arndt, Martin Diller, Jochen Hampe, Georg Heidenreich, Pascal Kettmann, Markus Krötzsch, Stephan Mennicke, Sebastian Rudolph, Hannes Straß. *"Supporting Risk Management for Medical Devices via the Riskman Ontology and Shapes."* SEMANTiCS (2025).

Backup: S4F Standpoint

Credulous and sceptical reasoning

Let $T \subseteq \mathcal{L}_{\mathbb{S}}$ be a theory, $\xi \in \mathcal{L}_{\mathbb{S}}$ a formula, and atom $z \in \mathcal{A}$ not in $T \cup \{\xi\}$.

1. $T \approx_{\text{cred}} \xi$ iff T^{ξ}_{cred} has a minimal model, where

$T^{\xi}_{\text{cred}} := T \cup \{(\Box_* \neg \Box_* z \wedge \Box_* \neg \Box_* \xi) \to \Box_* z\}$

2. $T \not\approx_{\text{scep}} \xi$ iff T^{ξ}_{scep} has a minimal model, where

$T^{\xi}_{\text{scep}} := T \cup \{(\Box_* \neg \Box_* z \wedge \neg \Box_* \neg \Box_* \xi) \to \Box_* z\}$

Expansions

$U \subseteq \mathcal{L}_{\mathbb{S}}$ is an expansion of T iff $U = \{\psi \in \mathcal{L}_{\mathbb{S}} \mid T \cup \{\neg \Box_{\mathsf{s}} \phi \mid \Box_{\mathsf{s}} \phi \in \mathcal{L}_{\mathbb{S}} \setminus U\} \models_{\mathbb{S}} \psi\}$.

$\mathfrak{F}$ is a minimal model of T iff $Th(\mathfrak{F}) := \{\psi \in \mathcal{L}_{\mathbb{S}} \mid \mathfrak{F} \Vdash \psi\}$ is an expansion of T.

Backup: Comparison

Standpoint Normal Logic Program $P_{\mathbb{S}}$

$\Box_{\mathsf{s}}[p \leftarrow p_1, \ldots, p_n,\ \text{not}\ p_{n+1}, \ldots,\ \text{not}\ p_{n+m}]$

ABA framework $F(P_{\mathbb{S}})$

$\mathcal{R}:\ \{p^{\mathsf{s}} \leftarrow p_1^{\mathsf{s}}, \ldots, p_n^{\mathsf{s}},\ \text{not}\ p_{n+1}^{\mathsf{s}}, \ldots,\ \text{not}\ p_{n+m}^{\mathsf{s}}; \ldots\}$

$\mathcal{A}:\ \{\text{not}\ p_{n+1}^{\mathsf{s}}, \ldots, \text{not}\ p_{n+m}^{\mathsf{s}}; \ldots\}$

$\mathcal{C}:\ \overline{\text{not}\ p^{\mathsf{s}}} = p^{\mathsf{s}}$ for all $\text{not}\ p^{\mathsf{s}} \in \mathcal{A}$

M is an answer set of $P_{\mathbb{S}} \iff \{\,\text{not}\ p^{\mathsf{s}} \mid \Box_{\mathsf{s}} p \notin M\,\}$ is a stable extension of $F(P_{\mathbb{S}})$