Topic 1: Events and Probability

Expressing events

1. Three tests are available to assist in the identification of a substance which may be one of 6 substances; any number of the three tests may be used. The first test is positive only in the presence of substances 1, 2, 3 or 4; the second test is positive only in the presence of substances 3, 4 or 5; and the third test is positive only in the presence of substances 2, 5 or 6. Let E_i denote the event that the ith test is positive, $i = 1, 2, 3$. Using set notation and the minimum number of tests, denote the events that a substance under test is

(a) substance 1,

(b) substance 2,

(c) substance 6.

Name any substances which cannot be identified by the tests.

2. There are two technicians for maintenance of certain copiers, and as jobs arrive, they are assigned to technicians so that there are two queues of jobs. Let A_i denote the event that there are at most i jobs in the first technician's schedule, and B_i the event that there are at most i jobs in the second technician's schedule.

(a) Give expressions for the following events:

i. there is at most 1 job in the system

ii. there are at most 2 jobs in total in the system.

(b) Explain briefly (one sentence) why $A_1 A_2 = A_1$

3. Outside of a bank exists two automatic teller machines (ATMs), each with a queue of people waiting to use an ATM. Let A_i be the event "there are at least i people in the queue at ATM A". Let B_i be the event "there are at least i people in the queue at ATM B". For this question, we define the number of people waiting in the queue at an ATM to also include the person being served at the ATM. Express the following events only in terms of A_i and B_i

(a) There are at least 5 people in the queue for ATM A.

(b) There are fewer than 4 people in the queue for ATM B.

(c) There are no more than 6 people in the queue for ATM A.

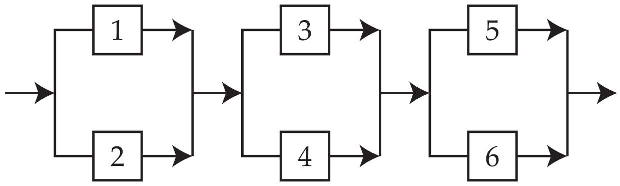
(d) There are exactly 3 people in the queue for ATM B.

(e) There are at least 4 people queueing at the ATMs outside the bank.

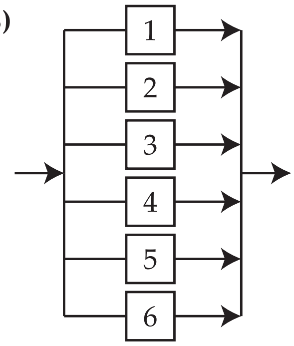
(f) There are more people queueing for ATM A than for ATM B.

4. Within a network design, six identical components can be arranged according to either (A) or (B) below. Let W_i be the event component i functions.

(A)



(B)



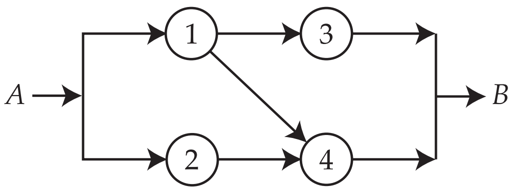
Give an expression for the events that

(a) (A) functions, and

(b) (B) functions.

5. An electronic system with four components is represented by the diagram below. The system functions correctly if and only if it is possible to trace a path of functioning components from A to B. Let W_i be the event that component i functions.

Give an expression for the event that the system functions correctly.



6. In a diving competition, there are 2 judges who each give a score in integers out of 10. Let A_i be the event the first judge gives at least i out of 10, and B_i be the event the second judge gives at least i out of 10. The final score is taken to be the average of the two scores. Is the following correct for the event that a competitor scores at least 9 out of 10?

$$A_9 B_9 \cup A_{10} B_8 \cup A_8 B_{10}$$

7. In an assessment moderation scheme, two moderators independently consider a school's rating of a student portfolio. If both moderators agree with the rating, it is ratified. If at least one moderator disagrees with the rating, two other independent moderators also consider the rating. If at least two of the total of four moderators agree with the rating, it is ratified, otherwise it is returned to the school for further consideration and discussion.

Let A_i be the event that the i^{th} moderator agrees with the school's rating. Use set notation to denote the event that the school's rating is ratified.

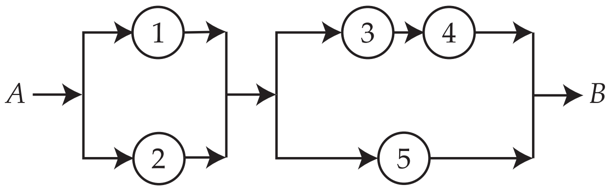
8. *(From Probability and Statistics for Engineering and the Sciences, Jay L. Devore, Ninth Edition)*

Use Venn diagrams to verify the following two relationships for any two events A and B (These are called De Morgan's laws): '

(a) $\overline{A \cup B} = \overline{A}\ \overline{B}$,

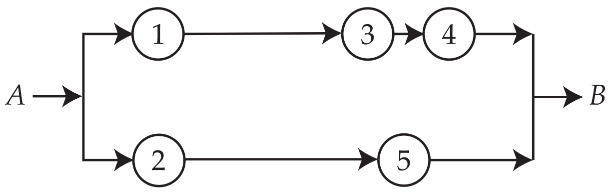
(b) $\overline{AB} = \overline{A} \cup \overline{B}$.

9. A system with five components is represented by the diagram below. The system functions if and only if it is possible to trace a path of functioning components from A to B. Let W_i be the event that component i functions.



(a) Give an expression for the event that the system functions.

(b) An alternative configuration is given below. Give an expression for the event that this system functions. Hence show that the configuration below is never more reliable (that is, never has a greater probability of functioning) than the configuration above, no matter what is assumed about the individual reliabilities or how they relate to each other. (That is, you do not need to consider probabilities at all).



Probability

10. *(From Probability and statistics for Engineering and the Sciences, Jay L. Devore, Ninth Edition)*

Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store.

(a) If the probability that at most one of these purchases an electric dryer is 0.428, what is the probability that at least two purchase an electric dryer?

(b) If P(all five purchase gas) = 0.116 and P(all five purchase electric) = 0.005, what is the probability that at least one of each type is purchased?

11. Hydraulic assemblies for landing gear created by an aircraft revamp facility are examined for faults. History demonstrates that 15% have faults in the shafts, 10% have faults in the bushings, and 4% have faults in both the shafts and the bushings. If one such assembly is randomly selected, find the probability that it has the following characteristics.

(a) Only a bushing fault;

(b) A shaft or bushing fault;

(c) Only one of the two types of faults;

(d) No faults in either shafts or bushings.

12. *(From Probability and statistics for Engineering and the sciences, Jay L. Devore, Ninth Edition)*

Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = 0.3$ and $P(B) = 0.5$.

(a) Why is it not the case that $P(A) + P(B) = 1$?

(b) Calculate $P(\overline{A})$

(c) Calculate $P(A \cup B)$

(d) Calculate $P(\overline{A}\ \overline{B})$

13. The table below gives the frequencies of eye colours observed in a group of fathers and children. If A is the event "Father's eye colour light" and B is the event "Child's eye colour light" estimate the probabilities of the events A, B, $A\ \ B$, and $A \cup$ B.

	Father's eye colour		
Childs eye colour	**Light**	**Dark**	
Light	471	148	**619**
Dark	151	230	**381**
	622	**378**	**1000**

Also estimate the probability that the child of a dark-eyed father will also be dark. What is wrong scientifically with this question?