MXN501

Stochastic Modelling

QUT

Acknowledgement of Country

- QUT acknowledges the Turrbal and Yugara as the First Nations owners of the lands where QUT now stands. We recognise that these lands have always been places of teaching, research and learning.

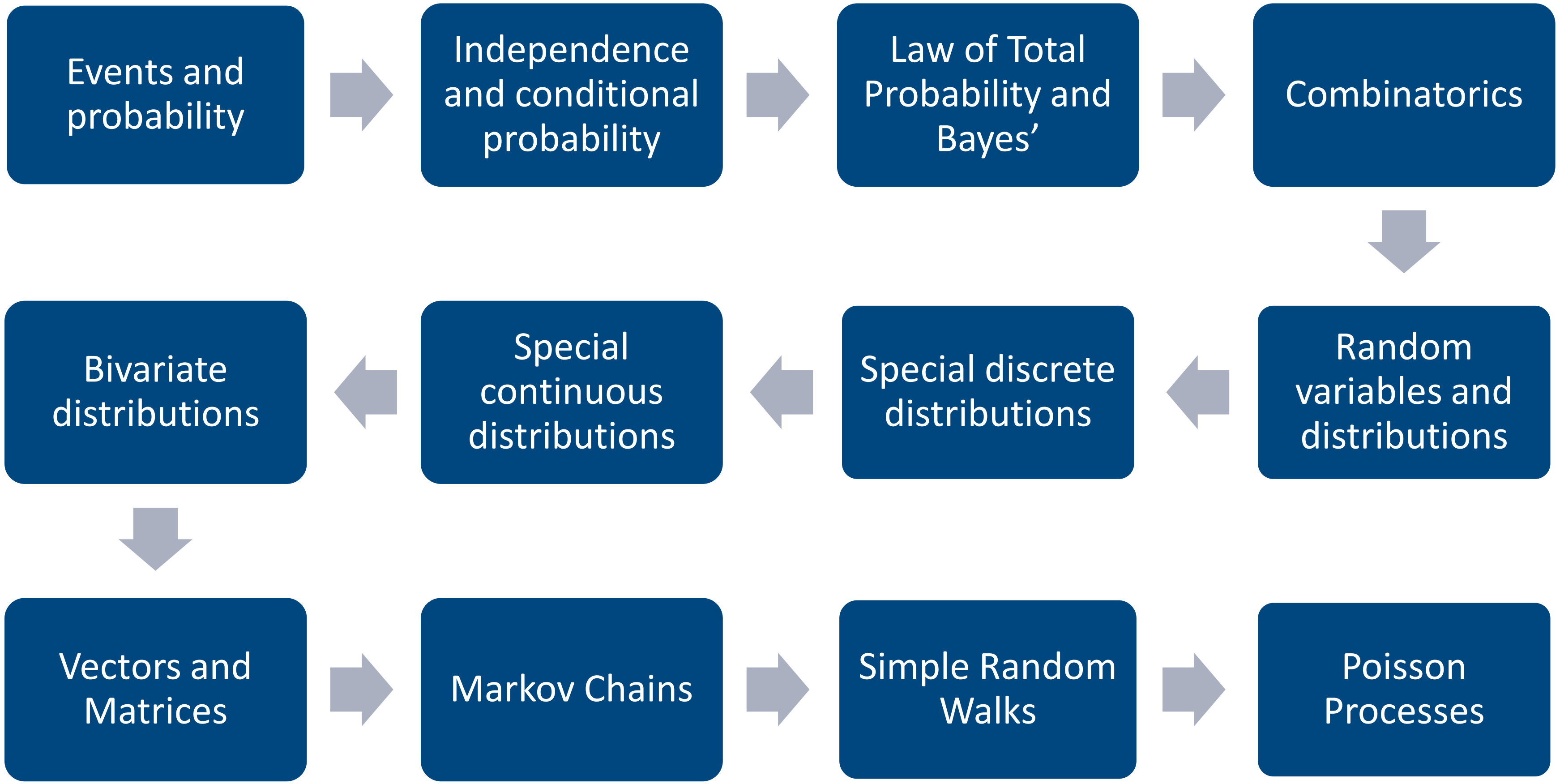
- QUT acknowledges the important role Aboriginal and Torres Strait Islander people play within the QUT community.

Teaching Team

Lecturer, Unit Coordinator
Dr Aiden Price

Workshop Tutor
Lily Zhang

Unit Roadmap



Assessment

- Online Quizzes (weekly) | Individual | Formative (0%)

  Designed to provide students an opportunity to answer questions alone. Completing quizzes is recommended to ensure a continued understanding of unit content.

- Problem Solving Task 1 (PST1) | Individual | Summative (10%)
- Problem Solving Task 2 (PST2) | Individual | Summative (10%)

  Individual assessment pieces for the semester designed as a way (a) to test student knowledge of course content and (b) to provide real-world scenarios where this type of knowledge could be applied.

- Group Assignment (GA) | Groups (3-4 members) | Summative (20%)

  Designed as an opportunity to assess students' capability to apply course knowledge in an open problem which enables creativity, while also building communication and coordination skills with your peers.

- Final Exam | Individual | Summative (60%)

Feedback

We’d love your feedback throughout the semester, through both formal and informal surveys, to ensure that we’re providing you with the best learning experience.

We’ll also give you feedback throughout the semester based on your performance on the formative and summative assessment tasks.

Extra Help

LTU_STIMulate Blackboard (link here)

STIMulate in-person | GP Library level 2 | KG Library level 3

Email Unit Coordinator: a11.price@qut.edu.au

Slack workspace: https://s22022mxn501.slack.com (invite link here).

INTRODUCTION TO PROBABILITY

MXN501

Stochastic Modelling

QUT

Unit Roadmap

1. Events and probability
2. Independence and conditional probability
3. Law of Total Probability and Bayes’
4. Combinatorics
5. Random variables and distributions
6. Special discrete distributions
7. Special continuous distributions
8. Bivariate distributions
9. Vectors and Matrices
10. Markov Chains
11. Simple Random Walks
12. Poisson Processes

Outline

- Context
- Events
- Venn Diagrams
- Random Variables
- Notation

Why do we need probability?

- Probability can allow us to estimate the likelihood of an event.
- An event is an outcome.

  E.g., Let A be the event that a coin flip lands on 'tails'.

- We can use probability in:
  - Marketing
  - Genetics
  - Insurance
  - Weather
  - Dungeons and Dragons
  - Gambling (Not recommended!)

Events

- Other examples of events:
  - Rolling a six
  - Your bus being late
  - The weather being sunny
  - Passing an exam
- We typically use a capital letter to refer to an event.
- When discussing the probability of an event, we use $\mathrm{Pr}(\ )$or P( ).

Random Variables

- A random variable is a variable that will take a random value.
- Random variables are often denoted with capital letters.
- **Do not confuse random variables with events!**
  - Let X be a random variable representing the result of a die roll.
  - $X = 2$, $X = 5$, and $X \leq 3$ are all events
- Random variables can be discrete (taking specific values), or continuous (taking real values, any values within a range).

Key Concepts

- The probability of an event occurring must be between 0 and 1 (inclusive)
  - If the probability of an event is 0, then the event will never happen.
  - If the probability of an event is 1, then the event will always happen.
- Sum of a set of <u>mutually exclusive</u> and <u>completely exhaustive</u> events is 1
- Something has to happen, even if that something is actually that nothing happened.

Notation

- Consider two events, A and B.
- The probability of A is $\Pr(A)$.
- NOT A, or the complement of A, is $\bar{A}$.
- A AND B, or the intersection of A and B, is $A \cap B$.
- A OR B, or the union of A and B, is $A \cup B$.

  Note that, in probability theory, the union operation represents "inclusive or", i.e., A itself, B itself, or both A and B.

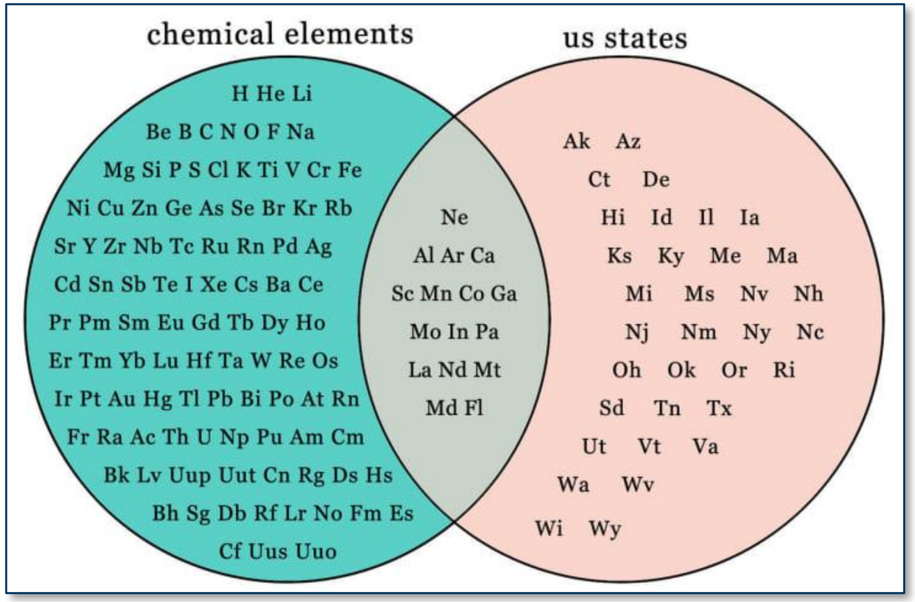
- $\Pr(A|B)$ is an example of conditional probability and represents the probability of A happening, given that B definitely happens.

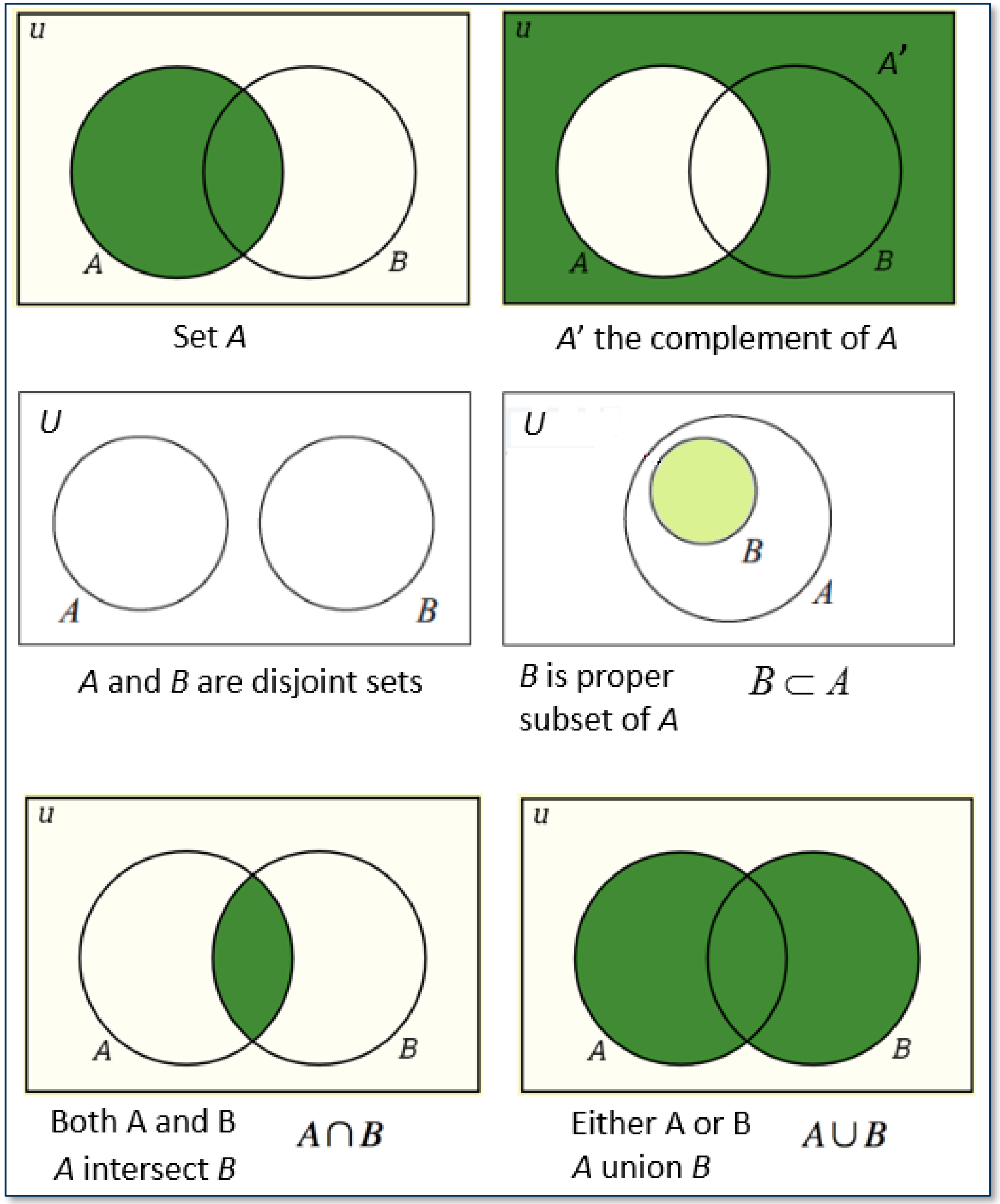
Explain the Errors

1. If the probability that an ore contains uranium is 0.128, the probability that it does not contain uranium is 0.62.
2. A company is working on the construction of two shopping centres. The probability that the larger of the two centres will be completed on time is 0.35; the probability that both shopping centres will be completed on time is 0.42.
3. The probability that a student will get a 7 in a particular subject is 0.32; the probability that they will get either a 6 or a 7 is 0.27.
4. The probability that a family has a TV is 0.89, the probability that a family has a dishwasher is 0.61, and the probability that they have both is 0.41.

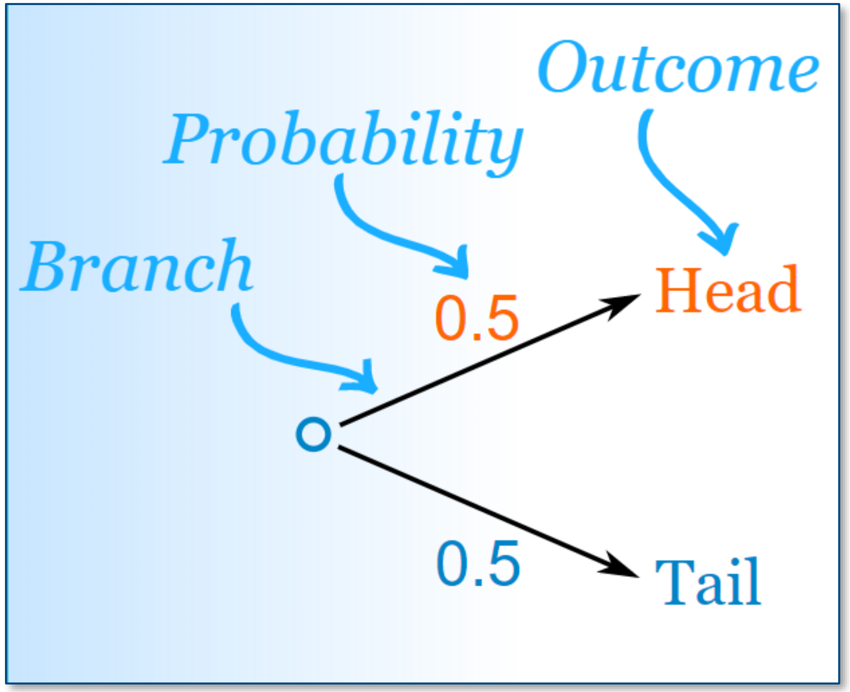
Venn Diagrams

- Venn diagrams are often use to represent a set of events.
- We will use Venn diagrams here to represent common probability rules.





Tree Diagrams



- Another way to visualise events and probability is through tree diagrams.
- We start at the origin ‘o’, and have a branch for every possible event.
- Each branch leads to an outcome.
- We write the probability along each branch.

Tree Diagram Example

You have a jar of mixed lollies. The jar contains 5 “Fantales” and 3 “Minties”. You randomly grab a lolly. You then randomly shuffle the bag and grab another lolly.

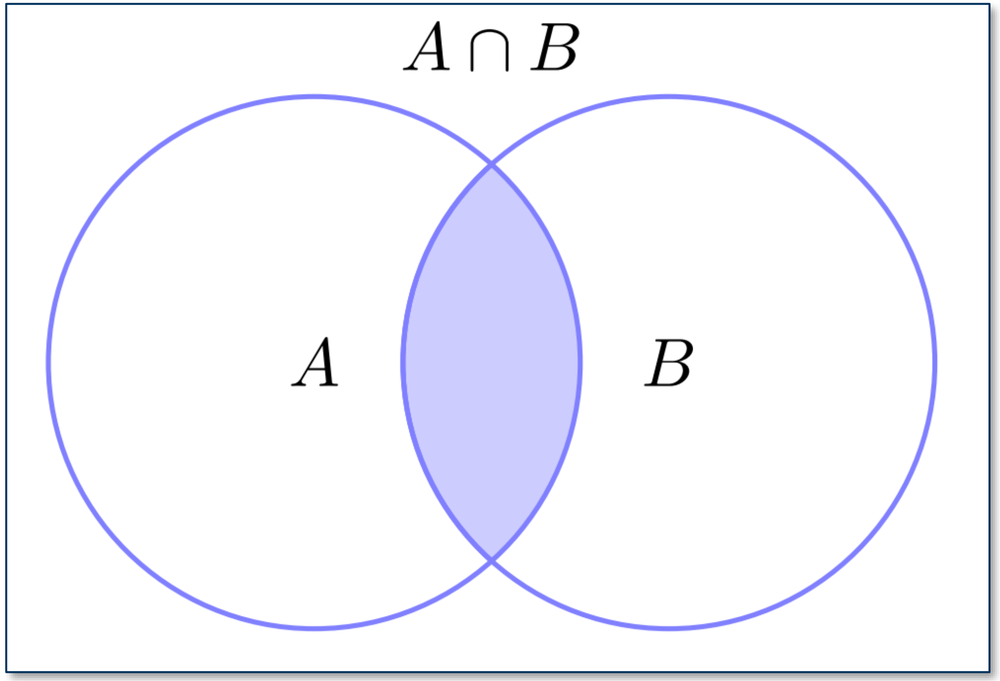
1. Construct a tree diagram for the above scenario.
2. What’s the probability that you pick two Fantales?
3. What’s the probability that you pick two Minties?
4. What’s the probability that you pick one of each?

Joint Events

- Frequently, we are modelling scenarios with more than one event that can happen at the same time. These are called joint events.
- For example:
  - Consider two different types of faults in a vehicle where either one or both faults could occur.
  - A circuit board with multiple components that could fail independently of each other.
  - People buying a product that has been advertised.

Intersection

- An <u>intersection</u> is when two events both occur simultaneously.
- It is calculated as the shared area of the probability space.
- Intersections are represented using the **AND** operator: $A \cap B$.



Intersection Rules

Iff (*if and only if*) events A and B are independent, then:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B).$$

Intersection Example

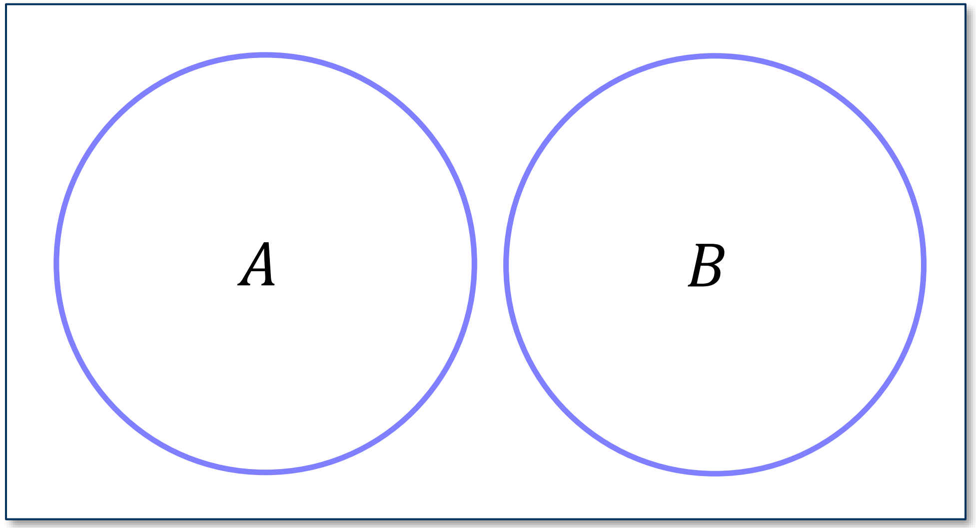
Lily’s cats, Darwin and Moseley, each accept pats independently of each other. The probability that Darwin accepts a pat on a given day is 0.4, whilst the probability that Moseley accepts a pat is 0.2. What is the probability that both Darwin and Moseley will accept pats?

Disjoint Events

- Disjoint events are events that can never occur simultaneously, i.e.,

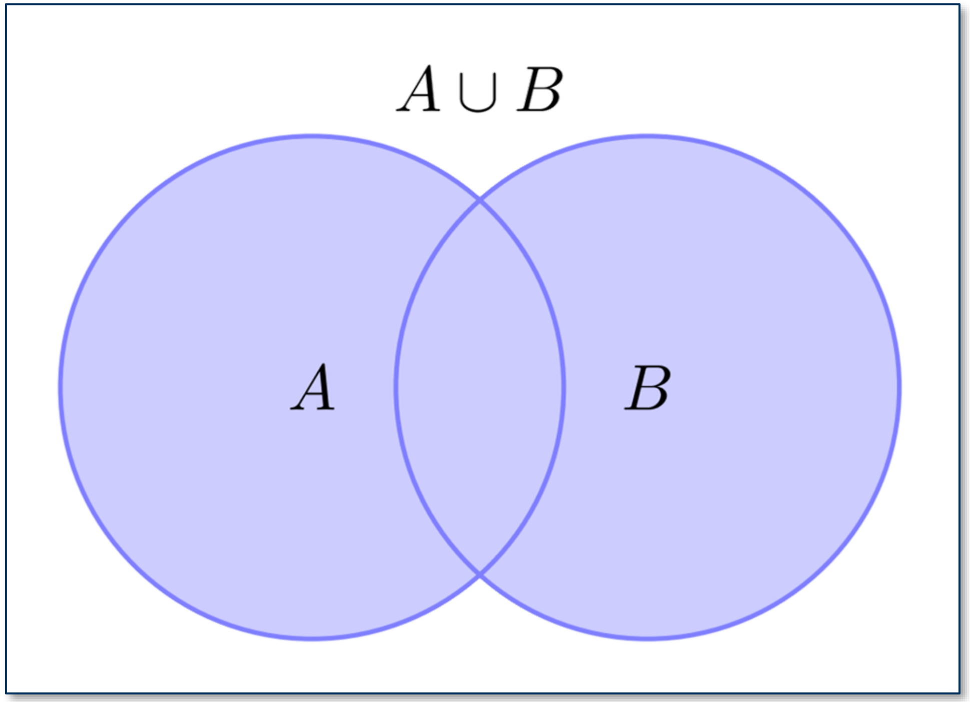
$$\Pr(A \cap B) = 0$$

- Disjoint events are also referred to as mutually exclusive events.



Union

- A <u>union</u> of events is when we are interested in *at least one event occurring* (i.e., inclusive or).
- It is represented with the **OR** operator: $A \cup B$



Union Rules

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ (addition rule for non-disjoint events)

$\Pr(A \cup B) = \Pr(A) + \Pr(B)$ (addition rule for disjoint events)

Union Example 1

Lily's cats, Darwin and Moseley, each accept pats independently of each other. The probability that Darwin accepts a pat on a given day is 0.4, whilst the probability that Moseley accepts a pat is 0.2. What is the probability that Darwin or Moseley will accept pats?

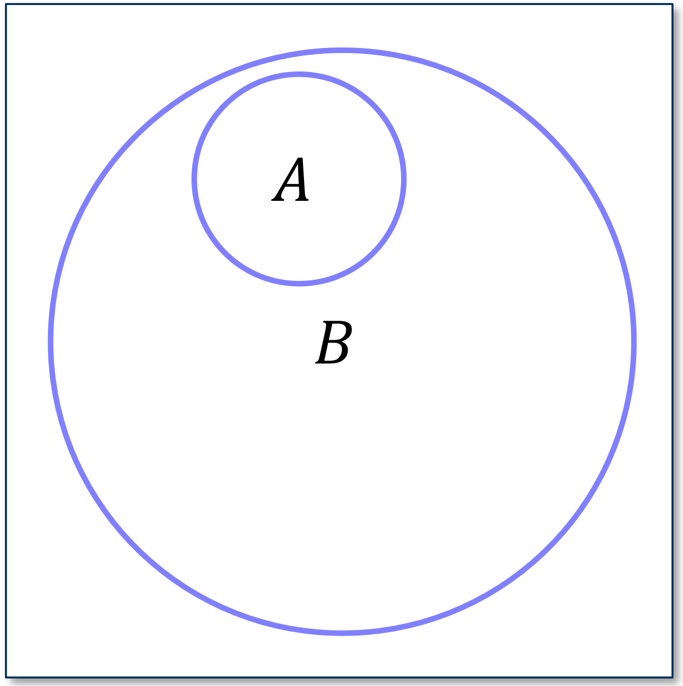
Union Example 2

Lily’s cats, Darwin and Moseley, sometimes accept pats, **but never on the same day**. The probability that Darwin accepts a pat on a given day is 0.4, whilst the probability that Moseley accepts a pat is 0.2. What is the probability that Darwin or Moseley will accept pats?

Subset

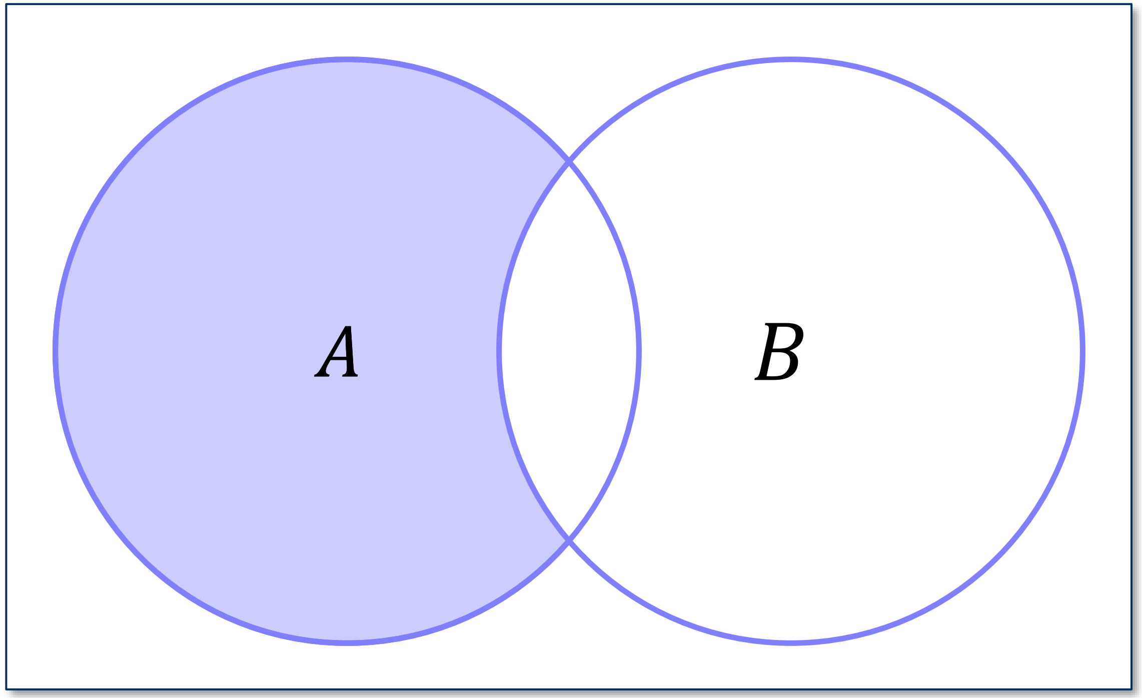
- If $A \subset B$ (read A is a subset of B), then

$$\Pr(A) \leq \Pr(B).$$



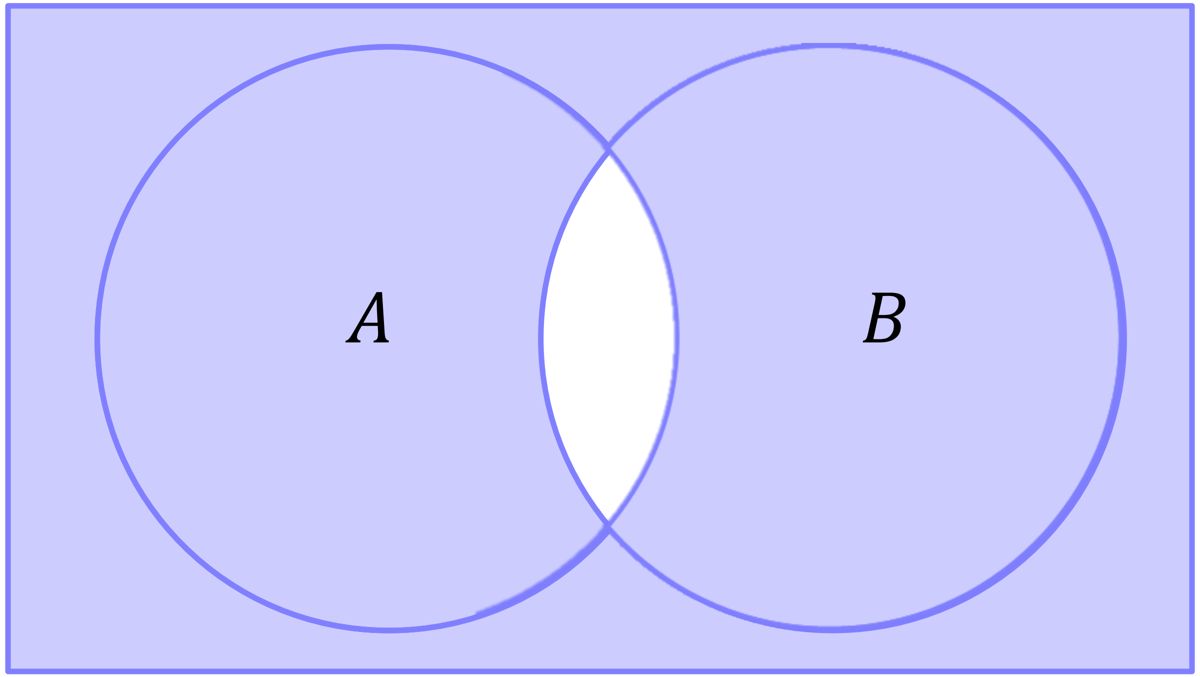
More Rules

$$\Pr(A \cap \bar{B}) = \Pr(A) - \Pr(A \cap B)$$

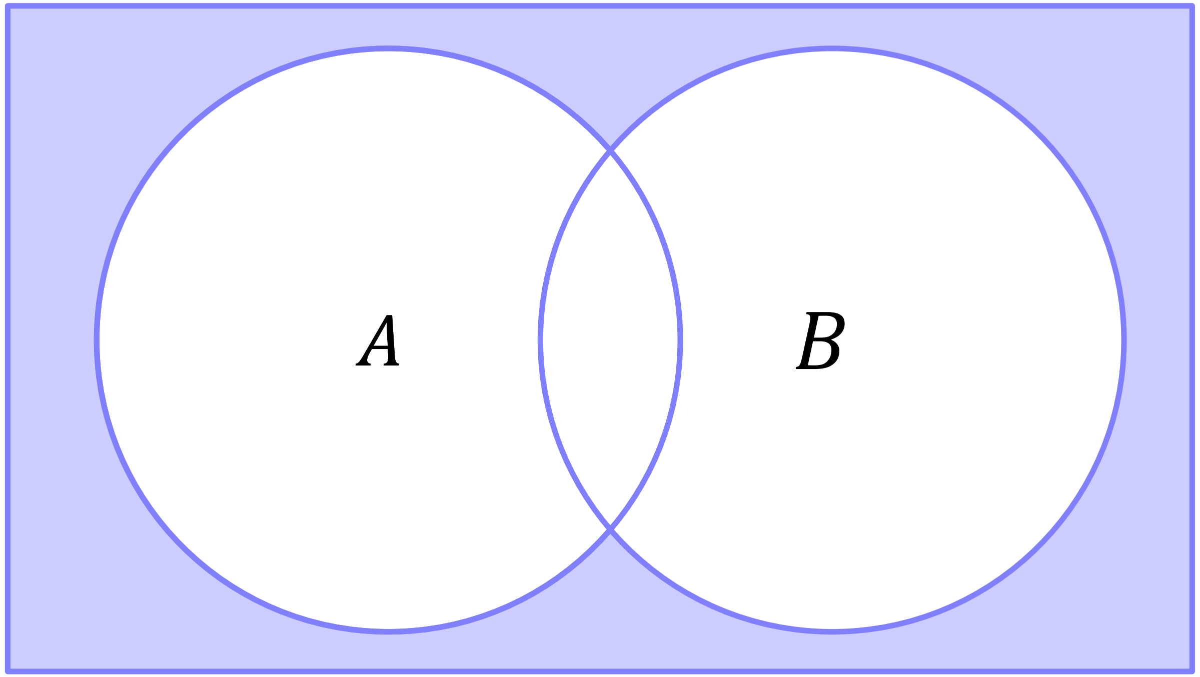


De Morgan's Law

$$\Pr(\overline{A \cap B}) = 1 - \Pr(A \cap B) = \Pr(\bar{A} \cup \bar{B})$$



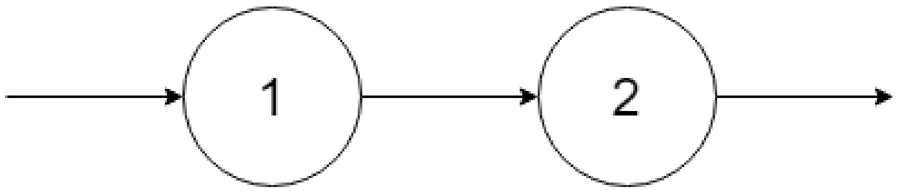
$$\Pr(\overline{A \cup B}) = 1 - \Pr(A \cup B) = \Pr(\bar{A} \cap \bar{B})$$



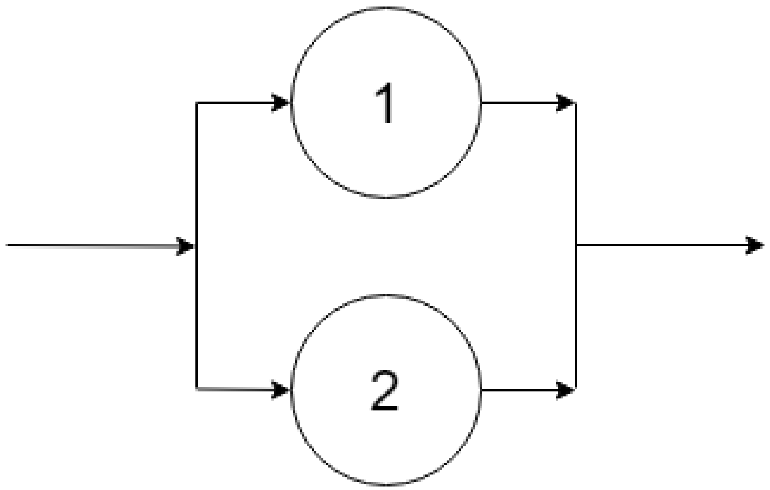
Simple Circuit Example

A signal can pass through the circuit if there is a working path from start to finish. Component i functions with probability p_i, *independently of other components*. What is the probability that the circuits below work?

(1)

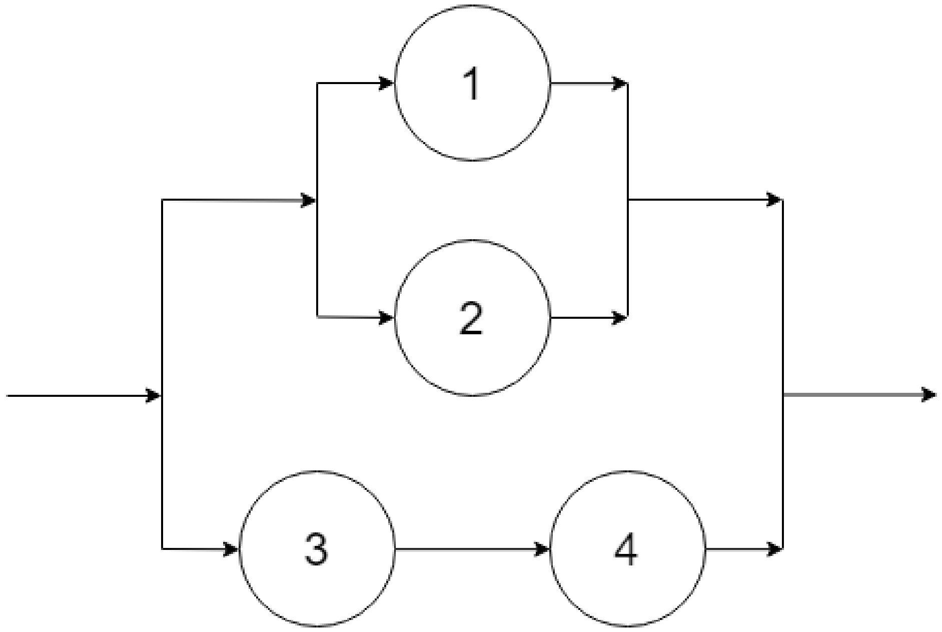


(2)



Another Circuit Example

Component i functions with probability p, independently of other components. What is the probability that the circuit below works?



Exercise

Verify that the solutions

$$2p - 2p^3 + p^4$$

and

$$1 - 2(1-p)^3 + (1-p)^4$$

are equivalent.

Summary

- Events are things of interest that happen.
- Probabilities are the likelihood of events occurring.
- Probability spaces are a tidy way to communicate events, outcomes and probabilities.
- Sum of all probabilities is 1.
- Probabilities must be between 0 and 1.
- The compliment is the scenario where an event does not occur.
- The union is “this **or** that”.
- The intersection is “this **and** that”.

Basic Rules

Here are a few basic probability rules:

$0 \leq \Pr(A) \leq 1$	(definition)
If $A \subset B$ then $\Pr(A) \leq \Pr(B)$	(subset)
$\Pr(\bar{A}) = 1 - \Pr(A)$	(complement)
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	(union)
$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap \bar{B})$	(total probability)

Additional Resources

Online Resources:

- Khan Academy
- MathsIsFun
- YouTube (!)

Readings:

Also available via readings on Blackboard.

- Probability for Dummies: Coming to Terms with Probability
- Probability for Dummies: Chapter 3