4.2

☆ Rewrite as a matrix equation:

$$\frac{dx_1}{dt} = -\frac{1}{50}x_1$$

$$\frac{dx_2}{dt} = \frac{1}{50}x_1 - \frac{2}{75}x_2$$

$$\frac{dx_3}{dt} = \frac{2}{75}x_2 - \frac{1}{25}x^3$$

(System is stable b/c eigenvalues came out negative. Computes exponential Decay over time)

Transition to matrix A: $\frac{dx}{dt} = Ax$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} -\frac{1}{50} & 0 & 0 \\ \frac{1}{50} & -\frac{2}{75} & 0 \\ 0 & \frac{2}{75} & -\frac{1}{25} \end{bmatrix}$

$\det(A-\lambda I)=0$

☆ Matrix solution: $\lambda_1 = -0.04$, $\lambda_2 = -0.0267$, $\lambda_3 = -0.02$ (Eigenvalues)

$$V = \begin{bmatrix} 0 & 0 & 0.1961 \\ 0 & 0.4472 & 0.5883 \\ 1 & 0.8944 & 0.7845 \end{bmatrix}$$

(Eigenvectors in columns of V)

1.) use the eigenvalue
2.) compute $A-\lambda I$
3.) solve $(A-\lambda I)\vec{v} = \vec{0}$
4.) Extract the eigen vector from solution Set

Diagonal Matrix D

$$D = \begin{bmatrix} -0.04 & 0 & 0 \\ 0 & -0.0267 & 0 \\ 0 & 0 & -0.02 \end{bmatrix}$$

☆ Checking with eigenvector Decomposition

$$V^{-1}AV = D = \begin{bmatrix} -0.04 & 0 & 0 \\ 0 & -0.0267 & 0 \\ 0 & 0 & -0.02 \end{bmatrix}$$

(Verified the decomposition by checking $V^{-1}AV = D$. Shows we can solve the system using $X(t) = Ve^{Dt}V^{-1}x_0$)

Since the Product gives a diagonal matrix D, this confirms that the matrix A is diagonalizable.