3.3

2 $x\frac{dy}{dx}+y=x^2y^2 \quad y(1)=1$

$$\frac{dy}{dx}+\frac{y}{x}=xy^2$$

$$u=y^{1-2}\to y=u^{-2}\to\frac{dy}{dx}=-u^{-2}\frac{du}{dx}$$

$$-u^{-2}\frac{du}{dx}+\frac{u^{-1}}{x}=xu^{-2}\to\frac{du}{dx}-\frac{1}{x}u=-x$$

$$p(x)\to I(x)=e^{\int\frac{-1}{x}dx}=e^{-\ln|x|}=x^{-1}$$

$x>0$

$$\frac{d}{dx}(x^{-1}u)=-1\to\int\frac{d}{dx}(x^{-1}u)dx=\int-1\,dx\to x^{-1}u=-x+C\to y^{-1}=-x^2+Cx$$

$$(1)^{-1}=-(1)^2+C(1)\to C=2\to y^{-1}=-x^2+2x$$

errors:

$y(2)=\text{und}\to$ VA @ $x=2$

absolute error = |actual value − approx| = |2.305 − und| = und

∞?

relative (%) error = $\frac{\text{absolute error}}{|\text{actual value}|}\times 100=\frac{\text{und}}{\text{und}}100=$ und

③ $\frac{dy}{dx}=y-2e^{-x}$ $y(0)=1$

$\frac{dy}{dx}-y=-2e^{-x}$
$\hookrightarrow P(x)=-1 \rightarrow I(x)=e^{\int -1\,dx}=e^{-x}$

$\frac{d}{dx}(e^{-x}y)=-2e^{-2x}$

$\int \frac{d}{dx}(e^{-x}y)\,dx=\int -2e^{-2x}dx \rightarrow e^{-x}y=e^{-2x}+C \rightarrow y=e^{-x}+Ce^{x}$

$1=e^{-(0)}+Ce^{(0)} \rightarrow C=0 \rightarrow y=e^{-x}$

Via Euler's method, $y(6)\approx-7.5$. This doesnt agree with the theoretical value of $y(6)=0.00248$. This is because each step taken adds an increasing amount of error. Since it took 60 steps to reach $y(6)$ from $y(0)$ with $h=0.1$, the value for $y(6)$ calculated by Euler's method will be drastically off from the theoretical value.

$y(0)=0.9 \rightarrow 0.9=e^{-(0)}+Ce^{(0)} \rightarrow C=-0.1 \rightarrow y=e^{-x}-0.1e^{x}$

if $y(0)=0.9$, the term $-0.1e^{x}$ dominates as x gets larger; this term makes $y\rightarrow-\infty$ as x gets larger. Without this term, the numerical solution in the IVP has $y\rightarrow0$ instead.

The curve predicted by Euler's method reaches no such asymptote that would otherwise limit its outputs in the way the original IVP does. With $y(0)=1$, $y'<0$ for all x. Euler's method simply continues to reach even larger negative outputs each step it takes when theoretical values get closer to 0 in every equivalent step.

(4) $\frac{dy}{dx} = y^2$, $y(0) = 1$

step	$y(1) \approx$
$h = 0.2$	4.1
$h = 0.1$	6.1
$h = 0.05$	9.55

the approximations for $y(1)$ increase as step size decreases

$$\frac{dy}{dx} = y^2 \rightarrow \int y^{-2}\,dy = \int dx \rightarrow -y^{-1} = x + c$$

$$-(1)^{-1} = (0) + C \rightarrow C = -1$$

$$-y^{-1} = x - 1 \rightarrow y = \frac{1}{-x+1} \rightarrow y = \frac{1}{-(1)+1} = \text{und}$$

Euler's method doesnt work because y has an vertical asymptote at $x=1$.