Rain or Shine? Optimal Utility Pricing under Extreme Weather

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As climate change amplifies extreme weather conditions, water utilities face increasingly volatile demand and revenue, challenging the ability of fixed Increasing Block Prices (IBP) to simultaneously ensure revenue feasibility, promote conservation, and protect low-income consumers. This paper tests whether price alone can achieve these competing policy goals under such conditions. Using granular household data from Austin, TX, and a structural demand model enhanced with satellite imagery-derived vegetation index, I find that the presence of high-quantity consumers across all income levels undermines the effectiveness of IBP. Furthermore, higher-income households—who are both weather-sensitive and surprisingly price-elastic—complicate the utility's ability to achieve its distributional objectives while meeting conservation and revenue targets. Empirically, when extreme weather shifts the demand curve rightward (e.g., during a drought), the conservation constraint becomes binding, imposing a disproportionate welfare loss on the lowest-income stratum, averaging \$60.92 per household per month. Conversely, when demand shifts leftward (e.g., during rainy periods), revenue concerns become binding, also imposing a disproportionate loss on the same group, averaging \$63.65 per household per month. These results highlight the necessity of complementary policies to achieve distributional goals. For example, encouraging households to zeroscaping/xeriscaping by 20% could generate \$57.35 in welfare per household per month for the lowest-income stratum, almost nullifying the welfare loss from the conservation policy's shadow cost.[*] *Keywords: Ramsey Pricing, Multi-part Tariff, Demand Volatility, Distributional Effect*

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I. Introduction

The increasing variability of weather patterns, exacerbated by climate change, poses a significant challenge to the pricing of essential utility services, particularly water. Urban water utilities face a complex balancing act, primarily managed through their rate structures: they must ensure financial sustainability through cost recovery, promote resource conservation amid dwindling supplies, and maintain equitable access for all consumers, especially low-income households. This study focuses on these challenges within the context of Austin, Texas—a rapidly growing metropolitan area known for its highly variable precipitation. The primary objective of this paper is to develop and empirically implement an optimal water pricing framework, grounded in Ramsey principles and tailored for an Increasing Block Price (IBP) tariff, that explicitly accounts for and remains resilient while achieving policy goals related to financial feasibility, conservation, and distributional equity under extreme weather patterns.

The theoretical underpinning for many utility pricing schemes stems from Ramsey (1927), who proposed "second-best" solutions to cover large fixed costs while minimizing societal welfare loss, typically through an inverse elasticity rule. In practice, utilities often adopt IBP tariffs, where prices rise with consumption, to encourage conservation among high-volume users while ensuring affordability for basic needs. However, the efficacy of IBP structures is predicated on the assumption that high consumption correlates with high income and low price elasticity—an assumption severely tested by extreme weather. A critical operational constraint is that utility prices are typically preset for an entire year, lacking the flexibility to adjust to monthly or seasonal weather variations. This means a single, static IBP structure must bear the burden of achieving all policy goals, from mitigating extreme usage during dry months to offsetting revenue loss during wet months, all while addressing equity concerns. This paper first offers reduced-form evidence of consumer sensitivity to precipitation and then structurally estimates demand under nonlinear IBP using a Discrete/Continuous Choice (DCC) model.

Previous literature has explored the complex nature of estimating public utility demand and the welfare impacts of IBP optimization. Hewitt and Hanemann (1995) and Olmstead, Hanemann and Stavins (2007) use DCC models to estimate price elasticity in urban water demand. Some studies, such as Castro-Rodríguez, María Da-Rocha and Delicado (2002) and Nataraj and Hanemann (2011), have shown that demand is responsive and that specific IBP changes can generate welfare improvements. In particular, Szabo (2015), examining South Africa's free water policy, demonstrated that nonlinear pricing alone can improve welfare and conservation goals without relying on ad-hoc subsidies. Conversely, other research highlights the potential shortcomings of relying heavily on IBP as a policy instrument. Ito (2014) found that consumers appear to react more to average prices than to the marginal prices of multiple tiers, while Ruijs (2009) and Echeverri (2023), while acknowledging that consumers respond to IBP changes, concluded that IBP restructuring tends to favor higher-income households.

This paper fills a gap in the literature by exploring the effectiveness of IBP as a price instrument under the strain of extreme weather. I align closely with Wolak (2016), who employed a Ramsey-style model to determine an optimal IBP structure for welfare maximization. However, to my knowledge, this paper is the first to develop and empirically apply a Ramsey-style optimal pricing model that uniquely incorporates extreme weather—both in terms of mean shifts and variance shifts—to "stress test" the efficacy of IBP. Furthermore, the demand estimation is enhanced by the novel integration of high-resolution (10m $\times$ 10m) satellite-derived Normalized Difference Vegetation Index (NDVI) data. My descriptive analysis and the structural DCC model reveal that, contrary to common assumptions, high-quantity users exist across all income strata and that higher-income households can exhibit greater price elasticity, particularly when weather pushes the demand curve rightward. This finding introduces significant challenges for the utility relying solely on IBP to achieve its distributional goals under extreme weather, while conserving resources and addressing the revenue concern.

This research also relates to the industrial organization literature on firms' responses to weather-induced demand volatility, such as Lin, Schmid and Weisbach (2017) and Baumgartner et al. (2022), and to work on the social consequences of real-time electricity pricing by Holland and Mansur (2008) and the heterogeneities of social marginal cost by Borenstein and Bushnell (2022). Though also accounting for resource conservation, this paper extends the existing paper by adding the specific concern regarding the distributional effects of pricing, on top of the environmental concerns, by exploiting the unpredictability of urban residential water demand through extreme weather.

Lastly, this paper contributes to a growing literature evaluating the distributional effects of climate-related policies (Parry and Williams III (2010), Goulder et al. (2019), Känzig (2023))in both the field of industrial organization and macroeconomics, which has often found such policies to be regressive. Instead of targeting a specific green policy, this paper extends this strand of literature by evaluating the social shadow costs of a specific policy goal set to be achieved by the utility pricing, and disentangling and quantifying the regressiveness caused by the policy constraints on utility pricing from the regressiveness caused by exogenous weather shifts towards lower-income households.

I find that under extreme weather, price alone struggles to improve, or even maintain, distributional equity due to binding conservation and revenue constraints. When demand shifts rightward due to extreme drought or high weather variance, the conservation constraint imposes an average welfare loss of \$60.92 and \$74.87 per household per month for the lowest-income stratum, respectively. Conversely, when demand shifts leftward due to heavy rain or low weather variance, revenue concerns impose an average welfare loss of \$63.65 and \$67.19 per household per month for the same group. These estimated shadow costs can inform the design of complementary policies to achieve distributional objectives. For example, a 20% household zeroscaping/xeriscaping effort could generate \$57.35 in welfare per household per month for the lowest-income stratum, almost nullifying the welfare loss from the conservation policy's shadow cost.

The remainder of this paper is structured as follows. Section II provides background on water utility pricing in Austin, TX, discusses the inherent challenges posed by weather volatility, describes the datasets utilized, and presents descriptive evidence of how weather deviations affect water consumption across different income strata. Section III details the development of the structural demand model, a Discrete/Continuous Choice model, explains the estimation strategy incorporating household characteristics, weather variables, and NDVI data, and presents the key estimation results, including price and income elasticities and their heterogeneity. Section IV establishes the theoretical Ramsey pricing model, extends it to incorporate weather stochasticity and a risk-averse revenue preference from the utility, and outlines the empirical model used to determine optimal IBP parameters under these conditions. Section V presents the counterfactual analysis, where I estimate optimal prices and their welfare consequences under various shifts in precipitation patterns (both mean and variance shifts) and simulations, comparing outcomes under linear versus concave revenue constraints, and exploring the optimal level of concavity to balance welfare stability and welfare levels for lower-income households. Finally, Section VI concludes the paper by summarizing the main findings, discussing their broader policy implications for water resource management in an era of climate change, and suggesting avenues for future research.

II. Water Utility Pricing in Austin, TX

In this section, I introduce water utility pricing in Austin, TX, and detail the challenges water utilities generally face under extreme weather conditions. This includes the current pricing structure and the policy goals utilities set to achieve, noting in particular that the price structure is often predicated on an assumed correlation between household income and consumption levels/price elasticities. Subsequently, I present the datasets used and examine whether these assumptions could hold, and spotlight the challenges faced by the utility's pricing decision under extreme weather.

A. Utility Pricing and Increasing Block Pricing (IBP)

A utility company, typically a natural monopoly with large fixed costs, faces a classic challenge outlined in Ramsey's pricing problem (from Ramsey (1927)): pricing at marginal cost, though efficient, would not cover its total costs, rendering the "first-best" solution infeasible. Ramsey proposed a "second-best" solution to meet the company's financial requirements while minimizing the reduction in social welfare. This price-setting solution is known as the inverse elasticity rule, where prices are higher for consumers with lower price elasticity, as inelastic demand causes smaller reductions in consumption and thus smaller welfare distortions. In the case of a water utility, where major costs are fixed (e.g., reservoir management, purification, and distribution infrastructure) while the marginal cost per household is relatively small, Ramsey pricing helps ensure the utility's financial viability while maximizing consumer welfare.

In addition to ensuring cost recovery, the utility must also meet its policy goals. Typically, the utility must consider two major policy objectives: 1) **resource conservation** and 2) the **distributional effect** of the price. With limited natural resources, the utility is encouraged to reduce total consumption—a constraint that has become more pressing with dwindling supply caused by climate change. On the other hand, the utility must also ensure that water remains affordable for low-income consumers' basic needs. Therefore, the optimal price must strike a balance: it cannot be so high as to be unaffordable for low-income households, nor so low as to jeopardize revenue and encourage excessive consumption.

A common practical solution is to adopt a multi-part tariff with Increasing Block Pricing (IBP), which categorizes consumers into tiers based on their consumption. For all tiers, the final bill consists of a fixed charge (or access fee) to cover fixed costs and a volumetric charge (or usage fee) based on consumption to cover marginal costs. In an IBP structure, both the fixed and volumetric charges typically increase with higher tiers. For the volumetric charge, consumers pay a lower marginal price on consumption up to each kink point and a higher marginal price only on the amount exceeding it. This structure encourages high-quantity users to conserve, while lower-quantity users can benefit from a low marginal price (often below marginal cost). Predicated on the assumption that higher-quantity users typically have higher incomes and lower elasticities, IBP can be interpreted as an empirical application of Ramsey pricing that also addresses the policy goals of conservation and equity.

Thus far, the discussion of IBP applies to both electricity and water utilities. If I add different weather patterns, particularly precipitation, the pricing decision for water utilities becomes even more complex. Unlike electricity, urban household water usage is highly correlated to precipitation for single-family homes (as opposed to just cyclicality from seasons), where lawn watering constitutes a major portion of water consumption, far larger than essential usage like cooking, showering, and washing. Low precipitation (dry months) encourages much higher water usage, while high precipitation (wet months) reduces usage as households may not need to use extra water for lawn watering. Compared to electricity utilities, which may need to account for seasonal demand fluctuations, water utilities must prepare for months with unpredictable high or low precipitation. For some cities, such as Austin, this can occur randomly throughout the year.[1] For water utilities, it is important to design the IBP with the policy goals of conservation and maintaining financial viability for all weather situations. This creates another price-setting challenge for utilities, as the price typically cannot change from month to month. While economists typically suggest dynamic pricing, such as that used in ride-sharing services, to combat volatile demand, in the case of utility pricing, the price must be preset for the entire year and cannot change within a certain time period. This inability to adjust prices in response to weather stochasticity is specifically what this paper addresses.[2]

B. Water Utility Pricing in Austin, TX

For this paper, I use water utility transaction data from Austin, TX. The water utility in Austin is managed solely by Austin Water, a public entity and natural monopoly. All operations—from water supply management in Lake Travis to purification, transport, wastewater processing, and price setting—are managed by Austin Water. This means

[1]For Austin, and many cities in Texas, high rainfall months could happen in any months between March to June, and September to November. These months could equally have low rainfall due to some years with a longer summer season.

[2]Some may question the reason behind this inability to implement dynamic pricing, and there are deeper reasons that need further discussion in political economy. There is simply a lack of industrial convention to deploy dynamic pricing in water utilities on a monthly basis. The unpredictability of which month will be wet or dry also makes it difficult to proactively set up a stable pricing rule (such as price discrimination by summer and winter months).

Austin Water acts as a social planner aiming to maximize welfare while pursuing policy goals, making a Ramsey pricing framework highly suitable for analysis.

Austin, and Texas in general, are known for extreme and unpredictable precipitation. A report by Nielsen-Gammon et al. (2020) noted that future precipitation trends in Texas are likely to be dominated by largely unpredictable natural variability and a projected increase in the intensity and frequency of extreme rainfall events. This makes the price-setting challenge more acute, as it is difficult to predict which months will be wet or dry, rendering preset price discrimination based on seasonality nearly impossible. In Austin, as shown in Figure A1 using NOAA data, recent precipitation has become more erratic compared to the 30-year average. This trend, combined with Austin's rapid population growth, makes residential water conservation an increasingly pressing issue.[3]

The problem faced by Austin Water is not unique. Many U.S. cities face the combined pressures of extreme weather, growing populations, and dwindling natural water supplies due to climate change.[4] This paper aims to test whether a Ramsey-style pricing solution can achieve the policy goals of revenue recovery, conservation, and distributional equity under extreme weather. I utilize a panel dataset from Austin Water for approximately $120,000$ households from May 2018 to December 2019, public data from the Travis County Appraisal District, weather data from NOAA, and high-resolution (10m $\times$ 10m) satellite imagery capturing household vegetation health. These data sources are detailed in Section III.B.

Austin Water's current pricing uses a five-tier IBP structure, detailed in Table A1. This design aims to improve distributional outcomes and encourage conservation by setting higher marginal prices and fixed payments for higher-quantity users. This is based on the assumption that higher-quantity users are correlated with higher income levels, have lower price elasticities, and can afford higher prices. To better measure distributional effects, I segment households into five income strata, aligning with the five-tier price structure, to analyze how policies affect different economic groups (see Figure A.A1).

C. *Descriptive and Reduced-Form Evidence*

To study the impact of extreme weather on the utility's price-setting problem and the challenges its policy goals present, I will first test the assumption of a correlation between income and quantity.

The scatterplot[5] shows more instances of high-quantity users (defined here as those consuming above 20k gallons, the cutoff between tier 4 and tier 5) in the highest income stratum ($>$ \$100k monthly household income). However, many households in lower strata also consume well above this threshold. This undermines the premise that lower marginal prices in lower tiers effectively target low-income households for distributional benefits. This omnipresence of high-quantity consumers across all income strata poses a challenge for using IBP to achieve equity goals.

[3] According to recent census data, the Austin metro area is one of the fastest-growing regions in the U.S.

[4] The EPA confirms that hourly rainfall rates and the intensity of heavy precipitation events have increased across most of the U.S. since the 1970s.

[5] The scatterplot caps quantity at 250k Gallons for better visualization.

Figure 1. : Quantity by Income for each Income Strata

In addition, to understand the impact of weather, I examine the correlation between weather changes and quantity changes from May 2018 to December 2019. To account for seasonality, I calculated the monthly difference in precipitation (Δ Precipitation) and temperature (Δ Temperature) relative to their 30-year averages (1990-2020). Using panel data, I then calculated the percentage deviation of a household's monthly consumption from its historical average for that same month. I am particularly interested in how monthly weather deviations influence this quantity deviation and how the effect differs across income strata.

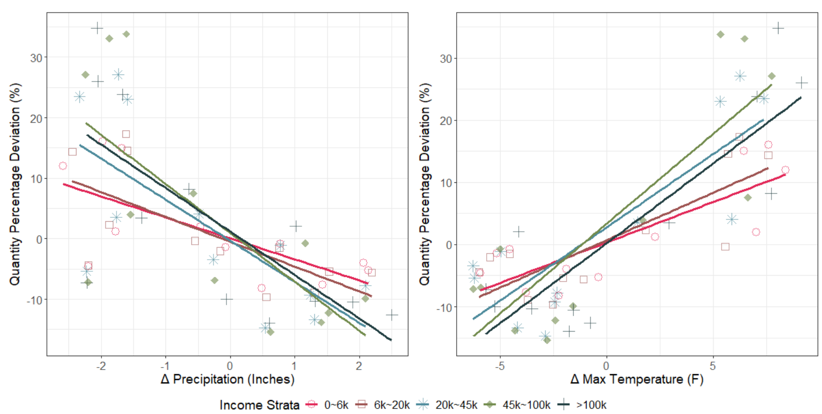

Figure 2. : Quantity Change (%) by Weather Deviation for each Income Strata

We can see that higher-than-usual precipitation typically leads to lower-than-usual consumption. However, the percentage deviation is greater for high-income households, likely reflecting higher demand for outdoor water use. A similar, though less pronounced, effect appears to exist for temperature. A reduced-form analysis of Δ Precipitation, Δ Temperature, and their interactions with income strata on quantity deviation (see Table A3) confirms these patterns.

Based on the results, higher income strata tend to have a lower baseline percentage deviation from their usual consumption (when weather differences and marginal price are zero) compared to the lowest income group. However, higher income strata ($>$ 6k) show a stronger negative response to increases in precipitation difference. The linear response to temperature difference is less clear-cut across strata in this model; while the base effect for the reference group is positive, higher income groups show interactions that temper this positive effect, resulting in less positive linear sensitivity compared to the low-income strata. Due to this unclear effect from the reduced-form analysis, coupled with the intrinsic correlation between precipitation and single-family home water usage, I will focus on the impact of changing precipitation for the rest of the paper and observe the shifts in consumer behavior.

This heightened weather sensitivity among higher-income households is somewhat counterintuitive and challenges the assumption that they have uniformly low price elasticity. This creates potential complexities for designing an optimal pricing structure that can meet all policy goals under extreme weather. To accurately evaluate counterfactual prices and revenue risks, it is crucial to measure how behavior changes in response to price, precipitation, income, and their interactions. Therefore, a structural model is needed to incorporate the full vector of household characteristics—a necessity given the limited price variation in the dataset—followed by an empirical Ramsey model that considers weather stochasticity to estimate the optimal IBP.

III. Demand Model and Estimation

In this section, I develop the demand model for the water utility to estimate the parameters needed for optimal price calculation and counterfactual analysis. As noted earlier, residential water demand typically faces an IBP structure, meaning that the nonlinear budget constraint generated must be accounted for. Therefore, I utilize a Discrete/Continuous Choice (DCC) model estimated via Maximum Likelihood to estimate the conditional demand probabilities for each pricing tier for each household, thereby accurately estimating the demand parameters.

Each household chooses between water consumption and a numeraire good subject to its budget constraint. The pricing scheme creates a nonlinear budget constraint, and I focus exclusively on the IBP structure in this paper. I assume a log-log functional form relating demand to prices and income, which is a common assumption in the water demand estimation literature.

The first paper to introduce a demand model handling piece-wise linear budget constraints arising from price nonlinearity was Burtless and Hausman (1978) in the context of labor supply. Dubin and McFadden (1984), Hanemann (1984) and Hewitt and Hanemann (1995) laid the groundwork for applying the DCC model to residential utility demand (with Hewitt and Hanemann (1995) specifically addressing water demand). Our approach largely follows this established model structure, with minor modifications, to maintain interpretability.

A. *Demand Model*

The demand model assumes each household consumes both water (w) and a numeraire good (Y) (with price $= 1$), and the household's total monthly income is I. Suppose for an IBP, the marginal price for each tier k is p_k, the fixed payment is A_k, and the cutoff point between tier k and $k+1$ is q_k. Due to the nature of IBP and suppose there are a total of K tiers, $p_{k+1} \geq p_k$, $A_{k+1} \geq A_k$, $q_{k+1} \geq q_k$ $\forall k \in \{0, 1, \ldots, K\}$. Together $\{p_k, q_k, A_k\}_{k=1}^{K}$ fully identify the nonlinear pricing structure. Throughout this section, the demand for every month should have a subscript of t, but for simplicity, the time-level subscript will be omitted.

Condition on the household choosing the optimal tier to be $k \leq K$ and the starting price $p_0 = 0$, the household's budget constraint is:

(1)
$$
\begin{aligned}
I &= A_1 + p_1 w + Y \quad (\text{if } k = 1) \\
I &= A_k + p_1 q_1 + p_2(q_2 - q_1) + \cdots + p_{k-1}(q_{k-1} - q_{k-2}) + p_k(w - q_{k-1}) + Y \quad (\text{if } k > 1) \\
&= A_k + \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j + p_k w + Y
\end{aligned}
$$
$$
p_k w + Y = I - A_k - \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j
$$

Therefore, given the IBP pricing structure, the only way to maintain the typical utility maximization problem with budget constraint is to add the additional term on the income such that the Virtual Income is equal to $I + d_k$, where the correction term d_k is defined as:

$$
\begin{aligned}
d_k &= -A_1 \quad (\text{if } k = 1) \\
d_k &= -A_k - \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j \quad (\text{if } k > 1)
\end{aligned}
\tag{2}
$$

The stipulation of using Virtual Income instead of just income is to make sure the utility maximization problem behaves as if the household is facing the marginal price p_k for all the quantities she consumes. To visually illustrate, consider a simpler model with 2 tiers without loss of generality:

Since consuming at a higher tier will grant paying the lower quantity portion at a lower marginal price, the correction term d_k (which will be > 0 in the higher tier given that $p_k \leq p_{k+1}$ $\forall k$ and A_k is not too large) has an income effect on the overall budget set. This correction term will allow us to solve for the utility maximization problem using

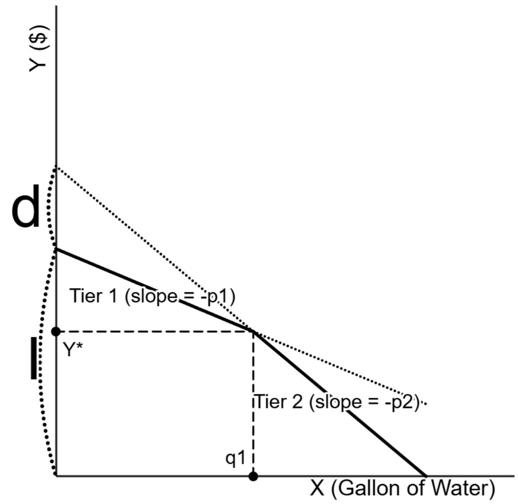

Figure 3. : Virtual Income $I + d_2$ when $K = 2$

the marginal price for each tier without causing any trouble.

Define the utility function for the household between water and numeraire good to be $U(w,I)$. Condition on tier k being the optimal choice for the household, define the conditional water demand function to be $g(p_k,I)$, where the functional form will be introduced later. Then the conditional indirect utility function for tier k is:

$$
\begin{aligned}
V(p_k,I) &= \max_{w} U(w, I - p_k w) \\
&= U(g(p_k,I), I - p_k g(p_k,I))
\end{aligned} \tag{3}
$$

For general U and budget sets, there will be a case where the household will have multiple optimal tiers such that the conditional indirect utility function is not unique. However, fortunately, given the case of IBP, Hausman (1979)'s theorem showed that if the budget set is convex, the optimal tier will be unique. In addition, the concavity of the utility function U will make sure the optimal consumption point $g(p_k,I)$ resides in the specific quantity boundaries of each tier. Therefore, without loss of generality, the unconditional indirect utility function for the total tier $K = 2$ is defined as:

$$
V(p,I) = \begin{cases} V(p_1,I) & \text{if } q_1 \geq g(p_1,I) \\ V(p_2, I + d_2) & \text{if } q_1 < g(p_2, I + d_2) \\ U(q_1, I - A_2 - p_1 q_1) & \text{if } g(p_2, I + d_2) \leq q_1 < g(p_1, I) \end{cases} \tag{4}
$$

The form of the unconditional indirect utility function can be expanded towards a more general case. I would like to highlight the case of $g(p_2, I + d_2) \leq q_1 < g(p_1, I)$ to make sure the Incentive Compatibility constraints are satisfied throughout the entire quantity line. The reason for the inclusion of this case is explained in detail in Section A.A2.

So far, I have covered the decision process of the household's water demand, in which they solve for a utility maximization problem and solve for $g(p_k, I)$ for all k, and choose the k that maximizes their utilities only with the assumption that the demand function is concave without imposing any functional form on the demand function. A typical functional form used by recent literature of water demand is a log-log demand function[6], I will use the same approach as the data of water quantity w_i is very right-skewed with only positive values. Therefore, a better parametric assumption for the distribution of w_i should be log normal, which will induce a log-log demand function. With vectors of household characteristics X and weather Z, the conditional indirect utility function for tier k has the functional form of:

$$V(p_k, I+d_k, \beta) = -\exp(\beta_1' X + \beta_2' Z + c + \varepsilon)\frac{p_k^{1-\alpha}}{1-\alpha} + \frac{(I+d_k)^{1-\rho}}{1-\rho} \tag{5}$$

This functional form, combined with Roy's Identity, will give us the log-log demand form for tier k:

$$\log(g(p_k, I+d_k)) = \log(w_k) = \beta_1' X + \beta_2' Z - \alpha \log p_k + \rho \log(I+d_k) + c \tag{6}$$

where d_k is defined in Equation 2, α is the log price effect, ρ is the log virtual income effect, and c is the constant. The observed log consumption includes unobservables ε: $\log(w) = \log(w_k) + \varepsilon$. Following the same parametric assumption of Olmstead, Hanemann and Stavins (2007), the unobservables have two terms $\varepsilon = \eta + \nu$. $\eta \sim N(0, \sigma_\eta^2)$ represents the household-level heterogeneity observed by households (e.g., preferences), while $\nu \sim N(0, \sigma_\nu^2)$ represents the household-level perception/optimization error that is unobserved by households even ex-post. The idea is that it is nearly impossible for a household to precisely consume the water quantity (w_k) they would ideally demand given their preferences. For instance, there will always be extra cold water down the drain when she demands warm and hot water, for example. For all households, they are risk-neutral with $E[\nu] = 0\ \forall k$. Neither the error terms are observed by the econometrician, hence the parametric assumption. In addition, this parametric assumption can allow us to derive a closed-form likelihood function for MLE. Given the structure of the error

[6] A couple of relatively recent examples of using this functional form for water demand are Hewitt and Hanemann (1995), Olmstead, Hanemann and Stavins (2007), and Wolak (2016). Others, like Szabo (2015) used a linear demand function.

term, the unconditional ex post water demand after the realization of the error term is:

(7)

$$\log(w) = \begin{cases} \log(w_1) + \eta + \nu & \text{if } \eta \le \log(q_1) - \log(w_1) \\ \log(q_1) + \nu & \text{if } \log(q_1) - \log(w_1) < \eta \le \log(q_1) - \log(w_2) \\ \log(w_2) + \eta + \nu & \text{if } \log(q_1) - \log(w_2) < \eta \le \log(q_2) - \log(w_2) \\ \log(q_2) + \nu & \text{if } \log(q_2) - \log(w_2) < \eta \le \log(q_2) - \log(w_3) \\ \dots & \\ \log(q_{K-1}) + \nu & \text{if } \log(q_{K-1}) - \log(w_{K-1}) < \eta \le \log(q_{K-1}) - \log(w_K) \\ \log(w_K) + \eta + \nu & \text{if } \log(q_{K-1}) - \log(w_K) < \eta \end{cases}$$

and the likelihood function for the observed water demand w_i for household i (omitting i subscript below for brevity) given the parametric assumption of η, ν is:

(8)

$$f(w_i|X,Z) = \sum_{k=1}^{K} \left[\frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \phi(s_k)(\Phi(r_k) - \Phi(n_k)) + \frac{1}{\sigma_\nu} \phi(u_k)(\Phi(m_k) - \Phi(t_k)) \right]$$

where

$$t_k = (\log(q_k) - \log(w_k))/\sigma_\eta$$
$$r_k = (t_k \sigma_\eta - \rho_s s_k \sqrt{\sigma_\eta^2 + \sigma_\nu^2})/(\sigma_\eta \sqrt{1 - \rho_s^2})$$
$$\rho_s = \sigma_\eta / \sqrt{\sigma_\eta^2 + \sigma_\nu^2}$$
$$s_k = (\log(w_i) - \log(w_k))/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$$
$$n_k = (m_{k-1} \sigma_\eta - \rho_s s_k \sqrt{\sigma_\eta^2 + \sigma_\nu^2})/(\sigma_\eta \sqrt{1 - \rho_s^2})$$
$$m_k = (\log(q_k) - \log(w_{k+1}))/\sigma_\eta$$
$$u_k = (\log(w_i) - \log(q_k))/\sigma_\nu$$

where Φ is standard normal cdf and ϕ is standard normal pdf.

and $q_0 = 0, q_K = \infty, w_k = g(p_k, I + d_k)$ is defined implicitly via Equation 6

Empirically, this likelihood function is used to calculate the likelihood of the observed water consumption w_i given the model parameters $(\alpha, \rho, \beta_1', \beta_2', c, \sigma_\eta, \sigma_\nu)$ and data $(X_i, Z_i, p_k, q_k, A_k, I_i)$.[7] The derivation of the likelihood function is explained in a 2-tier case in Section A.A2 without loss of generality.

[7] In the actual empirical model, I increase the flexibility by adding interaction term of price effect and income effect, therefore α is a function depends on parameter β_3', β_4', c_α and data X_α, Z_α, and ρ is a function depends on parameter β_5', c_ρ and data X_ρ, Z_ρ. The details will be discussed in Section III.B.

B. Empirical Model and Data

In this subsection, I outline the empirical model, based on the demand function (Equation 6) and the likelihood function (Equation 8), and describe the data used for maximum likelihood estimation.

The core dataset consists of panel data detailing monthly water transactions for single-family households in Austin, TX, from 2018-2019. These data include payments, total water usage, the billing date for each month, and the household's longitude and latitude. The data were provided by Austin Water, the public utility monopoly responsible for water services in Austin.

To operationalize the demand function, I incorporate supplemental datasets. These include two variables in the household characteristics matrix (X): the number of bathrooms per household and the time-variant household Normalized Difference Vegetation Index (NDVI). The number of bathrooms serves as a proxy for indoor water usage, while NDVI accounts for outdoor water usage, which is typically larger in volume and more sensitive to weather variations. The application of NDVI to water demand analysis was introduced by Wolak (2016). It aims to approximate lawn watering habits, as higher NDVI values indicate healthier vegetation, suggesting greater watering efforts by the household. Details on constructing and interpreting NDVI can be found in Section A.A2. For the time-variant weather matrix (Z), I include the monthly average maximum daily temperature, the monthly interquartile range (IQR) of maximum daily temperature, total monthly precipitation, and the monthly IQR of total precipitation.[8]

To enhance model flexibility, I allow the log price effect (α) and the log virtual income effect (ρ) to depend on household characteristics and weather conditions, thereby generating interaction terms. Specifically, α depends on the number of bedrooms, household NDVI, monthly average maximum daily temperature, and total monthly precipitation. ρ depends on the number of bedrooms, NDVI, and the number of heavy water-use appliances (including pools, hot tubs, sprinkler systems, fountains, and car washes).[9]

All other household characteristics, except for NDVI, were collected from public data provided by the Travis County Appraisal District (TCAD). I use data from 2018 to match the time frame of the transaction data. The TCAD data also include addresses, which were matched via geo-spatial analysis with the longitude and latitude data from Austin Water. I also collected lot size data from TCAD. Weather data were collected from the National Oceanic and Atmospheric Administration (NOAA), using data from approximately 120 weather stations in Austin. Since NDVI and weather data are collected on a calendar month basis, while household water billing cycles do not typically align with calendar months, I prorate both the weather and NDVI data to match each household's billing cycle.[10] Household income data were estimated using zipcode-level aver-

[8] Note that both IQRs account for the spread of the data within each month, which differs slightly from what I mean by growing weather variance, namely the increasing variance of weather patterns within a year.

[9] In practice, I used $\alpha = \exp(\beta_3' X_\alpha + \beta_4' Z_\alpha + c_\alpha)$, and $\rho = \beta_5' X_\rho + c_\rho$ to maximize the bias-variance trade-off of the estimation results. Regardless of the functional form, the expansion of both α and ρ is to provide flexibility in identifying heterogeneity of price effect and virtual income effect of the households, through the interaction terms.

[10] This practice was first introduced by Train et al. (1984).

age homeowner income data from the IRS combined with normalized household value data from TCAD, under the assumption that the normalized variance of household values within each zipcode mirrors the variance of household income.

After matching all datasets and eliminating outliers, the final sample consists of $127,323$ households with $2,345,742$ transaction records. Summary statistics can be found in Section A.A2.

C. Estimation Result and Identification

After conducting the maximum likelihood estimation using Equation 8 with $k = 5$ and the data discussed in Section III.B, the final estimation results can be found in Table A5.

With the panel data, both time-variant and time-invariant variables provide significant heterogeneity among households and across different months. This helps identify the parameters associated with household characteristics, income, and their interaction terms. Moreover, for weather variables, instead of using weather data for the entire city, I found the closest weather station for each household to introduce small variations, providing heterogeneity to help identify the weather parameters. Nevertheless, identifying the price effect and its interaction terms presents a challenge, as there is little or no price variation (neither through time nor geography) in the data. Another concern highlighted in past literature (e.g., Borenstein (2009); Ito (2014)) is that households may not react to the observed marginal price, as prices and the total payment amount are typically observed only after the billing cycle ends; instead, they might respond to the average price they face.

Fortunately, both of these identification concerns regarding price parameters in the DCC model have been addressed by Olmstead (2009). She demonstrated that each household, for each month, is optimizing over the entire price schedule. Consequently, the econometrician can recover the parameter estimates, along with the probabilities that households consume on each of their budget segments and at each kink point, directly from the DCC model results. In addition, the DCC model is estimated without ever determining the 'observed' marginal price of consumption – all the prices and the kink points enter the likelihood function, regardless of where consumption is actually observed. Olmstead (2009) also pointed out that price elasticity estimation has smaller bias when using the DCC model if variation in demand is driven primarily by household preferences (η) instead of the perception error (ν), and our estimation results align with this requirement, showing that σ_η is much larger than σ_ν. Furthermore, the model generates less bias if the existing price jumps between different tiers are sufficiently salient. Although there is no definite conclusion on how salient the price jump is needed to minimize the estimation bias, the marginal price differences between tiers in Austin are among the largest in the US[11].

To further address the price elasticity identification challenge, I also utilize data from a small category of consumers enrolled in the Consumer Assistance Program (CAP).

[11]See `https://bseacd.org/conservation-based-rate-structures/` for "Conservation-Based Rate Structures" - Barton Springs/Edwards Aquifer Conservation District

Based on certain income requirements, these consumers enjoy lower marginal prices (see Section A1). Although the variation is small, these consumers provide a small degree of price variation to aid in the identification of price elasticity.

Another potential endogeneity issue involves the use of NDVI, as the NDVI of a given calendar month could be the result of water usage in that same period. Therefore, I used a lagged term for NDVI, meaning that water demand this month is dependent on the previous month's NDVI. The idea is that households decide how much water to consume this month based on the health of their lawn vegetation last month. A higher previous month's NDVI may indicate the household takes more care of its lawn, helping to account for variations in outdoor water usage.

Based on the identification strategy discussed above, I now discuss the price and income elasticity derived from the DCC model. Due to the nonlinear pricing structure, I cannot directly use α and ρ to calculate price and income elasticities. Instead, by following Olmstead, Hanemann and Stavins (2007), I use a simulation-based approach. For a small number ξ, I calculate how expected demand changes in response to a small change in the status quo marginal price (p_0), holding income (I) and other factors constant.[12] The formula for price elasticity is as follows, where the same structure applies to income elasticity:

$$\text{Price Elasticity: } (E[g((1+\xi)p_0,I)] - E[g(p_0,I)])/(\xi E[g(p_0,I)]) \tag{9}$$

The median price elasticity is estimated to be -0.395. Compared to previous literature that uses the structural DCC model for Increasing Block Pricing (IBP) in developed countries, our estimate is within the range of Pint (1999)'s estimations ($-1.24 \sim -0.04$) and more strictly, Olmstead, Hanemann and Stavins (2007)'s estimations ($-0.59 \sim -0.33$). Our estimate is more inelastic compared to Hewitt and Hanemann (1995)'s ($-1.63 \sim -1.57$). This could potentially be due to their data, which includes a limited number of households in the summer months in Denton, TX (where the weather seems less extreme based on their data). Our estimate is also close to that of Olmstead (2009) (reporting -0.609 for a specific DCC model specification, using data from 11 cities with diverse weather patterns) and Strand and Walker (2005) ($-0.3 \sim -0.1$, using data from 17 cities in Central America, where water demand was primarily for indoor usage). The median income elasticity is estimated to be 0.112, which is close to estimations from previous literature for developed countries, such as Olmstead, Hanemann and Stavins (2007) ($0.1786 \sim 0.1865$) and Olmstead (2009) (0.1865).

To further explore the heterogeneities of price elasticities across income strata, I plot the price elasticities for all 5 income strata by precipitation.

Across different strata, I could see that more elastic households in general correlated

[12] Additionally, due to the flexibility of the price elasticity function, I limit the sample for empirical analysis to the year 2019, as opposed to from demand estimation, I used data from Jun. 2018 to Dec. 2019. This approach helps avoid overestimating the humid summer months of 2018.

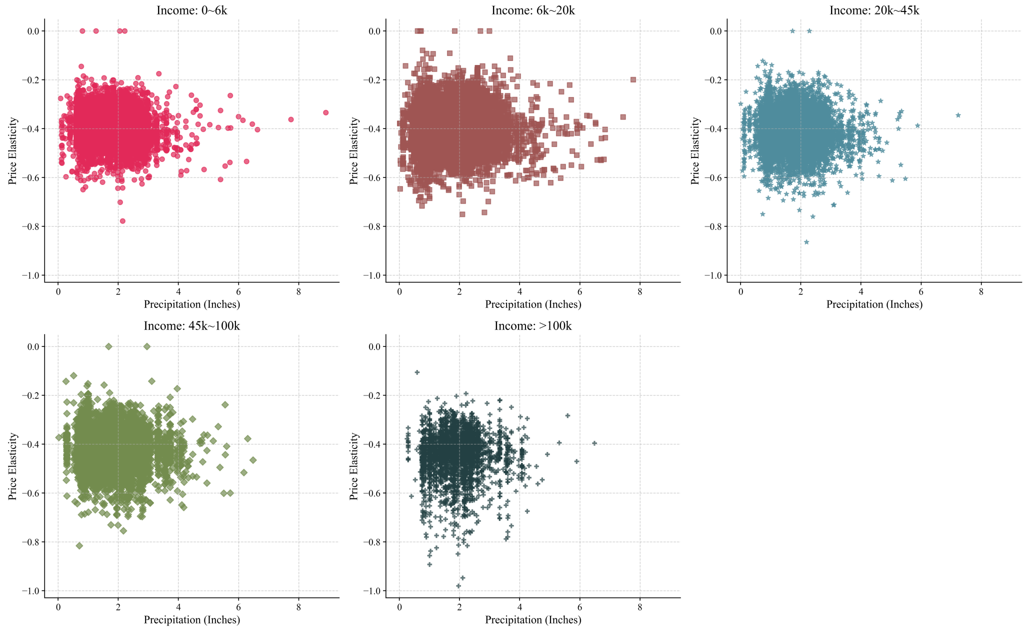

Figure 4. : Price Elasticity and Precipitation by Income Strata

with lower precipitation, and most of them are in the highest stratum.[13] This fits the reduced-form evidence from Section II.C, as higher income strata tend to be more sensitive towards precipitation decreases. Furthermore, I can see that regardless of how precipitation changes, lower income strata (especially monthly income of $6k \sim 20k$) have more inelastic demand.

The structural estimation of price elasticities paints a better picture for the price-setting utility, and further explains the data pattern I saw from the reduced-form evidence in Figure 2 that higher income households not just consume more when precipitation is low, but have more elastic demand as well. This counterintuitive result is likely due to the interaction with different precipitation patterns: when it rains a lot, few households across strata have excess outdoor water consumption that responds to price, but when it is dry, households with the preferences of greener lawns and larger pool usage are more responsive to price change, and these households tend to have higher income. In addition, higher-income households are more likely to be equipped with better technology to precisely control their water usage [14], which will also contribute to higher price elasticities. This, combined with the omnipresence of high quantity consumers across all income strata (see Figure 1), adds extra challenges to the equity goal of the optimal price design

[13]In the picture, it is evident that there is household-level heterogeneity in precipitation. This is because I gather the data from various weather stations in Austin and calculate the household-level weather data by a weighted average of the nearest weather stations.

[14]Like smart meter, smart sprinkler, etc.

from the utility under different weather patterns due to the inverse elasticity rule.

D. How weather shifts the Demand Curve?

From the structural demand model, I have estimated the heterogeneity of price elasticities across income strata and weather conditions, and observed that the highest income stratum has plenty of households with high elasticities under dry weather. In this subsection, I will introduce two alternative weather conditions: 1) dry weather, where the precipitation decreases by 0.25 inches for every month compared to the status quo weather, and 2) rainy weather, where the precipitation increases by 0.25 inches for every month compared to the status quo weather. This will allow us to observe the shift in the demand curve more directly before introducing price changes. I only changed precipitation and kept all other weather variables unchanged. In addition, all other house characteristics and prices remain the same as their status quo counterparts. I will then compare the new quantities and payments through the alternative weather and compare them to the status quo. I will limit the data to the lowest income stratum ($0 \sim 6$k) and the highest income stratum (> 100 k) for a cleaner picture of comparing the heterogeneities between income strata[15].

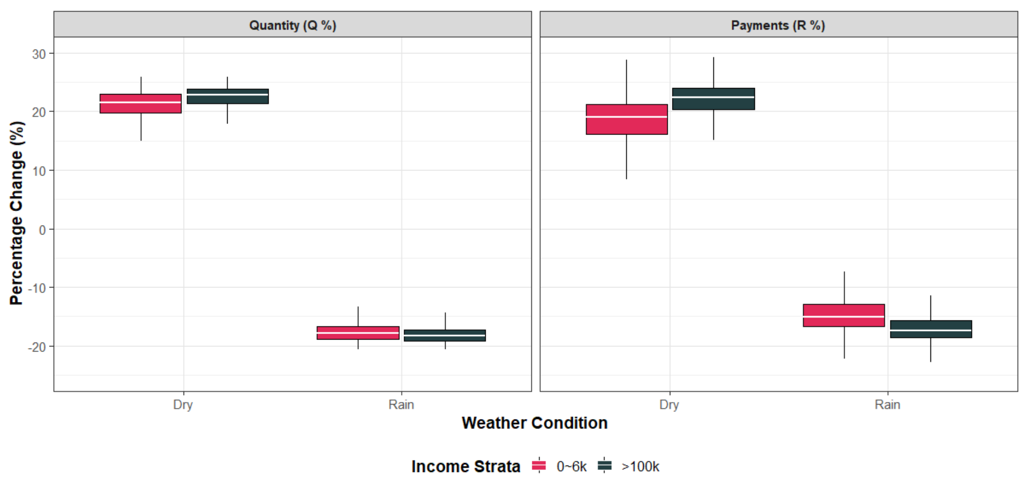

Figure 5. : Quantity and Payments Change between Weather Conditions by Income Strata

When the weather becomes drier, less precipitation will shift the demand curve rightward, causing quantity and payments to increase. The highest income stratum has a slightly higher percentage increase in their quantity, aligning with the evidence I saw

[15]This is also due to the counterintuitive data pattern where high quantity users exist in the lowest income stratum, and the high elasticity users in the highest income stratum

from Figure 2. This means that in order to achieve a better conservation goal for the utilities, the status quo price is too low, at least in the higher tier, to better curb the excessive demand under dry weather conditions, while the financial feasibility concern is minimized. When the weather becomes more rainy, more precipitation will shift the demand curve leftward, causing quantity and payments to decrease. In this case, both income strata have similar percentage decreases in quantity, but for payments, the highest stratum has a slightly higher percentage decrease, likely due to their base quantities, and payments are higher on average. This means under rainy weather, the conservation goal is easy to achieve, while financial feasibility concerns are more pressing. The status quo price is also too low in this case, as the utilities would need to raise the price to increase their revenue.

Noted that both the pressures of pushing the price upwards are caused by the policy goals set by the utilities. When the demand curve shifts rightwards, the conservation goal will require the price to increase at least at higher tiers. When the demand curve shifts leftwards, the concern of financial feasibility will also require the price to increase. Balancing both goals with a single price will be challenging for the utilities and will certainly create a shadow cost, which will inevitably decrease welfare. Based on the counter-intuitive price elasticity results in Figure 4 and the existence of high quantity households in lower income strata in Figure A5, it is reasonable to question if the price itself can achieve both policy goals, as well as improve the distributional effect. In the next section, I will formally define the empirical optimal price model and further implications and caveats of the challenge created by price elasticities interacted with weather across different income strata can be found in Section A.A2.

IV. Ramsey Pricing Model

In this section, I establish the theoretical basis for the Ramsey pricing problem (from Ramsey (1927)) faced by a natural monopoly water utility company with the policy constraints it is facing. Finally, I introduce the empirical model used to estimate the optimal Ramsey pricing under different extremes of weather conditions.

As previously established, Ramsey proposed the "second-best" solution to address a company's financial requirements while minimizing the reduction in overall social welfare, employing the inverse elasticity rule. Water utility companies, particularly in urban metropolitan areas with high fixed costs, often adopt the Ramsey format of IBP within a multi-part tariff structure to meet financial requirements and policy goals of conservation and equity. For simplicity, when discussing the theoretical basis for the Ramsey problem, I treat the entire pricing structure as a single price p, as opposed to the pricing vector of IBP in a multi-part tariff. In the empirical model, I reintroduce the multi-part IBP to derive more realistic solutions.

The theoretical Ramsey model, introduced by Ramsey (1927), is extended to include additional policy constraints in the generalized Ramsey model, as discussed in literature such as Coady and Drèze (2002). For the empirical model, I largely follow the setup of the conservation constraint for the Ramsey problem as presented in Wolak (2016). To empirically measure consumer welfare, I adopt the framework and definition of equivalence variation for nonlinear budget constraints from Hausman (1981), Reiss and White (2005), and Ruijs (2009).

A. Ramsey Pricing Model with Conservation Constraint

In this subsection, I briefly showcase the changes from the classic Ramsey model by using a simplified pricing model with the price p, chose by the water utulity to maximize consumer welfare, subject to two constraints: 1) annual revenue ($R(p)$) is greater than or equal to an exogenous total annual cost (C) that is independent of demand (simplifying for large fixed costs),[16] and 2) total annual quantity ($q(p)$) is less than or equal to an exogenous annual quantity upper bound ($\bar{Q}$).

$$\begin{aligned} &\max_{p} \quad CS(p) \\ &\text{s.t. } R(p) - C \geq 0 \\ &\text{and } q(p) \leq \bar{Q} \end{aligned} \tag{10}$$

Solving this Lagrangian yields the following result:

$$\begin{aligned} \underbrace{\frac{p-C}{p}}_{\text{Markup}} &= \underbrace{\frac{\lambda-1}{\lambda}\frac{1}{\varepsilon}}_{\text{Ramsey Rule}} + \underbrace{\frac{\mu}{\lambda p}}_{\text{Conservation Penalty}} \\ p^* &= \frac{C+\mu/\lambda}{1-\frac{\lambda-1}{\lambda}\frac{1}{\varepsilon}} \end{aligned} \tag{11}$$

The classic Ramsey rule states that the markup is proportional to the inverse of price elasticity (ε), but the optimal markup here includes an additional term that serves as a penalty for the conservation constraint. The optimal price depends on λ (the Lagrangian multiplier for the revenue constraint), μ (the Lagrangian multiplier for the conservation constraint), the exogenous cost C, and the price elasticity ε. If μ increases, the shadow price of the conservation constraint increases, and the utility has a greater incentive to raise the markup. If $\mu = 0$, the conservation constraint is not binding, and the standard Ramsey rule is recovered. If C increases, the price will need to increase. If ε increases, demand becomes more elastic, and the price will need to decrease, which is consistent with the standard Ramsey rule. If I denote $k = \frac{\lambda-1}{\lambda}$, the price can be expressed as $p = \frac{C+\mu(1-k)}{1-\frac{k}{\varepsilon}} = \frac{C+\mu}{1-\frac{k}{\varepsilon}} - \frac{1}{k-\frac{1}{\varepsilon}}$. If λ increases, k (which is less than 1) will also increase. The first term increases. The second term, being minus a negative number since $k - \frac{1}{\varepsilon} < 0$, also increases. This means that when λ increases, the price will increase.

[16] In reality, the total cost of maintaining urban water is not related to demand fluctuation; regardless of the amount of water consumed, maintenance and transportation fees do not change, making this a reasonable assumption.

B. Empirical Model

In this subsection, I develop the empirical model used to estimate the optimal price derived from Equation 10. The empirical model's goal is to extend the Ramsey model into a more realistic setting. [17] To keep consistency with the demand model, the optimal price ($\mathbf{p}$) consists of three vectors: marginal prices ($\{p_k\}_{k=1}^5$), kink points ($\{q_k\}_{k=1}^4$), and fixed payments ($\{A_k\}_{k=1}^5$). To maintain Austin Water's current pricing structure of 5 tiers with 4 kink points, $\mathbf{p}$ has a total of 14 parameters, and I assume the pricing structure does not change[18]. The optimal price, which depends on weather Z, maximizes the annual total equivalence variation (EV) weighted by household income I to boost the distributional effect.[19] To account for weather-level stochasticity, ideally, I would need to utilize the real distributions of weather to derive E_Z. However, it is challenging to estimate this real distribution of weather consistently by using either data from very recent years (which lack sufficient data points for empirical distribution derivation) or long-term data (which are affected by climate change trends). Instead, I assume the utility faces an expectation based on its prior[20], and then calculates the optimal price based on its weather expectation. Later, I will perform a robustness check by adding a Monte-Carlo disturbance to simulate weather prediction error and generate a distribution of Z. Further details on simulating the distribution of Z can be found in Section A.A3. Aside from stochasticity from Z, the demand estimation using MLE (Equation 8) also introduces household-month-level demand stochasticity from ε_h, as the econometrician observes neither η_h nor ν_h. I use Monte-Carlo simulations for both to generate their distributions.[21]

$$
\begin{aligned}
\mathbf{p}^*(Z) = \underset{(\mathbf{p},q,A)}{\arg\max} \sum_h \Big[EV_h(Z;\mathbf{p},\mathbf{p}_0,I)/I \Big] - \lambda \cdot \max(0, C - R_h(Z;\mathbf{p})) \Bigg] \\
\text{s.t.} \quad P\left(\sum_h q_h(Z_h;\mathbf{p}) \le \bar{Q} \right) \ge 0.95
\end{aligned} \tag{12}
$$

Note that for the two exogenous thresholds: $C = \sum R(Z_0;\mathbf{p}_0)$ and $\bar{Q} = \sum q(Z_0;\mathbf{p}_0)$. Both Z_0 and $\mathbf{p}_0$ represent the status quo weather and price. This means I compare the counterfactual revenue and quantity to their status quo counterparts. In particular, due to the lack of detailed information on costs, I use the status quo revenue as the benchmark to evaluate financial viability. Utilities usually ensure their revenue is just enough to cover the annual cost to avoid excessive welfare distortion, which makes this a valid assumption. I further discuss the validity of this assumption in this case in Section A.A3. For the

[17] The model is partially based on Wolak (2016).

[18] This includes the assumption that each marginal price and fixed payment will be non-decreasing in tiers.

[19] As pointed out by Feldstein (1972), for public pricing to account for distributional equity, the price setting should set the welfare weight according to the social marginal utility of income. In addition, a logarithmic social utility of inequality and $\varepsilon = 1$ for the Atkinson Index (Atkinson (1970)) result in a weighting proportional to 1/I.

[20] In reality, utilities would form a general trend of next year's weather based on phases of the El Niño-Southern Oscillation (ENSO). For example, next year's weather will be drier in general, causing precipitation to be lower in the mean.

[21] I will generate η_h on the household level to avoid overfitting.

conservation constraint, I introduce chance-constrained programming to provide more flexibility in price setting. It essentially requires that the probability of the counterfactual annual total quantity for all households being smaller than the status quo annual total quantity is greater than 0.95. The structures of both constraints are adopted from Wolak (2016). Unlike the typical Ramsey pricing setting, I set up the revenue constraint as a revenue loss term $-\lambda \cdot \max(0, \text{loss})$, such that when the revenue requirement is not met, it incurs a cost to the entire economy, but does not necessarily award any positive value when the requirement is met. The goal is to capture the utilities' surcharge mechanisms or debt-service coverage rules that make large revenue losses more costly. The parameter λ controls the weight between the monetary value of welfare, measured by the weighted equivalent variation, and the loss of revenue for the utilities. Mathematically, it is similar to the Ramsey model, but empirically, this provides more flexibility as, instead of a hard constraint, the utilities are allowed to have some annual losses.

The equivalence variation ($EV_h(Z;\mathbf{p},\mathbf{p}_0,I)$) for water with a nonlinear budget constraint is calculated by first obtaining the expenditure function from the indirect utility function and then calculating the welfare effects from the price change. The framework is developed by Hausman (1981) and Reiss and White (2005), and the formal definition is from Ruijs (2009).

Based on the indirect utility function $V(\mathbf{p}, I + d_k)$ from Equation 3, the expenditure function $e(\mathbf{p}, u)$ for a household choosing tier k is:

$$e(p_k, u) = \left[(1-\rho) \left(u + \exp(\beta_1' X + \beta_2' Z + c + \varepsilon) \frac{p_k^{1-\alpha}}{1-\alpha} \right) \right]^{\frac{1}{1-\rho}} \tag{13}$$

If a household's counterfactual demand falls within tier k, rather than at a kink point q_k, its equivalent variation is:

$$EV(p_0, p, I) = e(p, V'(p)) - d_k^0 - I \tag{14}$$

where $V'(p)$ is the new counterfactual utility generated by the new price p, and d_k^0 is the virtual income correction term from the original price scheme, i.e., $d_k = -A_k - \sum_{j=1}^{k-1}(p_j - p_{j+1})q_j$. The equivalent variations are adjusted for the amount of subsidies received due to the nonlinearities in the budget set. Note that the expenditure function ($e(p_k, u)$) is used to solve for the virtual income from the indirect utility function, instead of I, hence there is no correction term d_k here. If the price does not change, $EV(p_0, p_0, I) = 0$ and $e(V_0, p_0) = I + d_k^0$.

On the other hand, when the predicted consumption is at a kink point q_k, the above definition does not apply as these consumers are not technically facing the new marginal price from their chosen tier.[22] The equivalent variation is generated from $\bar{p}$, where $\bar{p}$ is the price at which the demand function (from Equation 6) generates the result q_k. If I denote $\mathscr{A} = \beta_1 X + \beta_2 Z + c + \varepsilon$, and $\bar{V}I$ denotes the corresponding virtual income, the

[22] To see more details of the additional case, see Section A.A2.

idea is to solve for $(\bar{p}, \bar{V}I)$ as a substitution effect where $(\bar{p}, \bar{V}I)$ generates the same utility. This means: $\begin{cases} V(p_k, I + d_k^0) & = V(\bar{p}, \bar{V}I) \\ \log(q_k) & = \mathcal{A} - \alpha \log(\bar{p}) + \rho \log(\bar{V}I) \end{cases}$. I can solve for $\bar{p}$:

$$\log(q_k) = \mathcal{A} - \alpha \log(\bar{p}) + \frac{\rho}{1-\rho} \log[\exp(\mathcal{A}) \frac{1-\rho}{1-\alpha} (\bar{p}^{1-\alpha} - p_k^{1-\alpha}) + (I + d_k^0)^{1-\rho}]$$

For this demand function and indirect utility function, I need to solve this nonlinear function numerically to obtain $\bar{p}$. Then, the equivalent variation for predicted demand at the kink point q_k is:

$$EV(p_0, p, I) = e(\bar{p}, V'(p)) - (\bar{p} - p_0) q_k^0 - d_k^0 - I \tag{15}$$

V. Counterfactual Analysis

In this section, I analyze the welfare effects of counterfactual optimal prices, calculated using Equation 12, under various counterfactual precipitation patterns (while assuming all other weather patterns remain unchanged). As the utility faces different future weather patterns (Z) when setting prices, I derive these optimal prices (and their associated welfare) for different precipitation scenarios.

I begin with the status quo precipitation (Z^0) and generate two types of counterfactual precipitation patterns: 1) **Mean Shift**: $Z' = Z^0 \pm \zeta_1$, where $\zeta_1 \in [-0.25, 0.25]$. The generated Z' will have the same variance as Z^0 but different means, and 2) **Variance Shift**: $Sd(Z') = Sd(Z^0) \cdot \zeta_2$, where $\zeta_2 \in [0.75, 1.25]$. The generated Z' will have the same median as Z^0 but different variances. The latter pattern is achieved by scaling the standard deviation from Z^0 nonparametrically while maintaining the same median, and then clipping the result at 0. [23] As discussed in Section II.C, I focus solely on changes in precipitation, assuming temperature remains at its status quo (as well as the IQR of temperature and IQR of precipitation, which measure within-month variability, do not change). Throughout the counterfactual analysis, I also assume household characteristics and income remain constant, with the exception of NDVI. Counterfactual NDVI is accounted for through a reduced-form analysis, as discussed in Section A.A4. Utilizing the counterfactual weather and NDVI, alongside constant household characteristics and income, I employ the derived empirical model from Equation 12 to calculate the optimal price. Details on the optimization procedure can be found in Section A.A4.

After calculating the optimal prices, I quantify the associated welfare by comparing the new and status quo values for: revenue ($R\%$, based on total revenues), quantity ($Q\%$, based on total quantities), and equivalent variation to income (EV/I, derived from the new prices). These measures capture the changes from both the price (which includes policy constraints) and the weather, and are calculated for all five income strata. In addition, I estimate the change from only the price and weather without the policy constraints

[23] Although it looks like precipitation is more likely to follow a right-skewed log-normal distribution because it never falls below zero, a parametric assumption will make scaling by the factor ζ_2 imprecise. As stated by meteorologists Heredia et al. (2018), it is more realistic to treat it nonparametrically when generating synthetic high-variance data.

to estimate the shadow cost of the policy constraints.

A. Welfare Results - Mean Shift (ζ_1)

In this section, I will first present the result of shifting the mean of the precipitation by adding $\zeta_1 \in [-0.25, 0.25]$ to the status quo weather. This means the utility will expect the average precipitation next year to change by ζ_1 and the variance will not change. I implement the optimal pricing with $\lambda = 0.5$ for the loss function, and the resulting menu of optimal prices solved from both scenarios can be found in Section A.A4. Based on their prior of ζ_1, they can choose a specific optimal IBP from the menu.

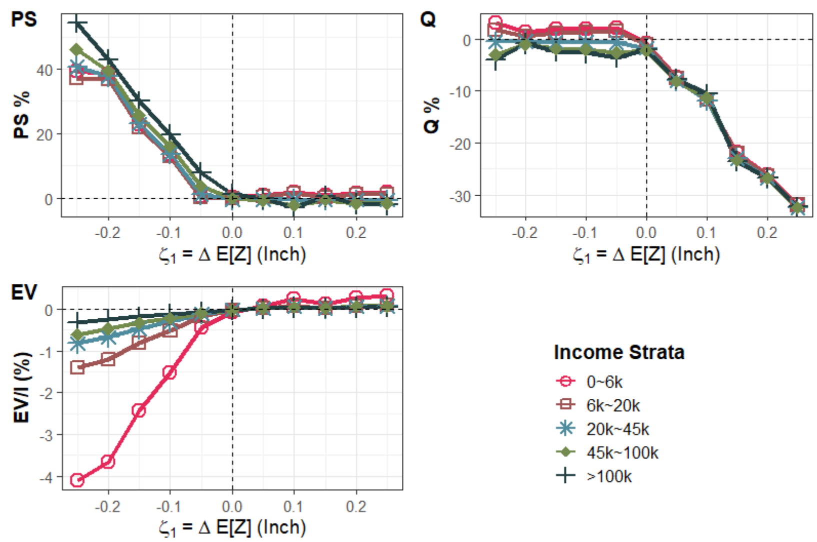

Figure 6. : Welfare from the Optimal Price in Shifting Mean ($\zeta_1 \in [-0.25, 0.25]$)

When analyzing **shifting means (ζ_1)**, Figure 6 showcases the general welfare results for precipitation expected to be, on average, drier ($\zeta_1 \leq 0$) and rainier ($\zeta_1 \geq 0$). When $\zeta_1 \leq 0$, the quantity constraint becomes binding while the revenue constraint becomes slack. Even with price increases only for higher tiers (see Figure A7) and accounting for EV/I in the optimization objective, average lower-income households generally exhibit a larger welfare loss (in dollar value relative to income) compared to income strata above \$20k, which show minimal EV/I loss. Conversely, when $\zeta_1 \geq 0$ (i.e., when predicted weather is rainier), the revenue constraint is more binding, and the quantity constraint is slack.

There are a few reasons why the optimal prices, even with the added weight in EV/I, still hit lower-income households harder: 1) Even though the water utility is not generally expensive, it will naturally represent a higher proportion of income for lower-income households, and any price or weather change will incur a larger shift towards EV/I[24]. 2) By the inverse elasticity rule from the Ramsey problem, the price should decrease for high-elasticity consumers and vice versa. This does not help with distributional effects, as many high-elasticity households are in the highest income stratum and consume high quantities.[25]. All of these factors place boundaries on adjusting the marginal price of higher tiers. 3) There is a non-negligible number of lower-income households that are high-quantity consumers. [26]. These households are often in a much higher tier than their income might suggest, and they do not have particularly high elasticities. [27]. This compromises the premise that a higher marginal price in a higher tier would help with distributional effects and the equity goal. To further see evidence of the welfare effect for high-quantity consumers in the lowest stratum, see Section A.A4. All these effects actively work against the premise that IBP could improve distributional effects, and this effect is worsened when weather pushes the demand curve rightward. This generally fits the result from the previous literature that the price changes of IBP in general favor higher-income households from Ruijs, Zimmermann and van den Berg (2008), Echeverri (2023), and Wichman (2024). Policymakers should take that into consideration when using IBP as the instrument to achieve policy goals. [28]

B. Welfare Results - Variance Shift (ζ_2)

When analyzing **shifting variances (ζ_2)**, Figure 7 showcases the general welfare results for $\zeta_2 \leq 1$ and $\zeta_2 \geq 1$. When $\zeta_2 \leq 1$ (i.e., when variance decreases), the revenue constraint becomes binding while the conservation constraint becomes slack. This is due to the shape of the precipitation distribution: as variance decreases, the distribution becomes less right-skewed, resulting in more rain during months that were originally drier and fewer extremely dry months. Through this change, welfare generally increases, more so for the lower-income stratum ($0 \sim \$6k$), but not substantially for all strata collectively. Conversely, when $\zeta_2 > 1$ (i.e., when variance increases), the quantity constraint becomes binding while the revenue constraint becomes slack. Given the now more extreme weather patterns, welfare decreases for all strata, disproportionately affecting lower-income strata (similar to the situation when $\zeta_1 \leq 0$). Although lower-income strata typically consume less (resulting in a smaller revenue increase compared to higher-income strata), the impact of higher precipitation variance remains more dominant for them.

[24]Similar patterns have been pointed out by Ruijs, Zimmermann and van den Berg (2008) using data in Sao Paolo, Brazil.

[25]See Figures 4 and A5

[26]They are usually consuming more than 20k Gallons, which is the status quo kink point between the 4th and 5th tier.

[27]See Figure A5

[28]In practice, many utilities in the US have income-based benefits like the CAP program in Austin. The empirical analysis measures the amount of compensation that should be implemented across different ζ_1. In the literature, both Ruijs, Zimmermann and van den Berg (2008) and Wichman (2024) have pointed out that IBP itself does not effectively target the lower stratum. There needs to be more active progressive measures.

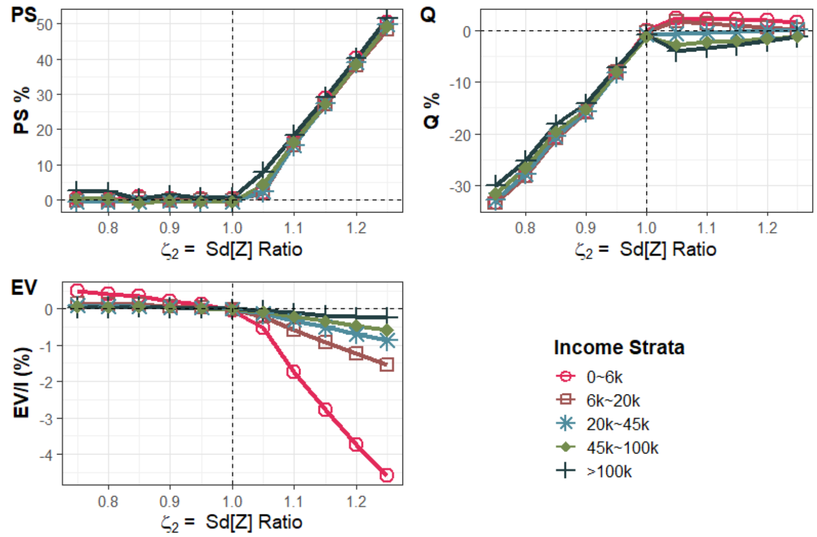

Figure 7. : Welfare from the Optimal Price in Shifting Variance ($\zeta_2 \in [0.75, 1.25]$)

C. Shadow Cost of the Policy Constraints

To extend previous literature on the distributional effect of IBP, the counterfactual analysis further proves that in the context of extreme weather patterns, while the pricing optimization needs to achieve the conservation goal in dry months and maintain revenue feasibility in rainy months, it is difficult to improve, or even maintain, the distributional effect, specifically when weather shifts the demand rightward. In order to fully separate the welfare loss from the shadow cost of the policy constraints and the welfare loss from the weather itself, I calculate welfare using the status quo price $\mathbf{p}_0$ under all counterfactual weather conditions, to estimate the shadow cost of the binding policy constraint, specifically for the lowest income stratum (as they have the biggest swing in welfare). The welfare result for all counterfactual weather patterns can be found in Section A.A4[29].

For shifting ζ_1, I have already found that consumers have higher quantity when $\zeta_1 < 0$, and lower payments when $\zeta_1 > 0$. This means the binding conservation constraint will generate a shadow cost, which in turn will cost welfare when $\zeta_1 < 0$, and the revenue loss function will generate a shadow cost, which in turn will cost welfare when $\zeta_1 > 0$. For the most extreme condition when $\zeta_1 = -0.25$, the welfare for the lowest stratum is -2.41% on average, which is higher compared to the welfare result (-4.11%) from Figure 6. This means the shadow cost of the binding conservation constraint incurs an average welfare loss of \$60.92 per month per household in the lowest stratum. On the other hand, when $\zeta_1 = 0.25$, the welfare for the lowest stratum is 2.18% on average, which is higher compared to the welfare result (0.32%) from Figure 6. This means the

[29]Similar to what I have done in Section III.D, but for a vector of counterfactual weathers.

shadow cost of the higher revenue loss incurs an average welfare loss of \$63.65 per month per household in the lowest stratum.

For shifting ζ_2, I have also concluded that for $\zeta_2 < 1$, due to the right-skewness of the precipitation data, it behaves similarly to $\zeta_1 > 0$, and for $\zeta_2 > 1$, it behaves similarly to $\zeta_1 < 0$. This means I will be measuring the shadow cost of the revenue loss function when $\zeta_2 < 1$ and one of the conservation constraints when $\zeta_2 > 1$. For the most extreme condition when $\zeta_2 = 0.75$, the welfare for the lowest stratum is 2.45% on average, which is higher compared to the welfare result (0.48%) from Figure 6. This means the shadow cost of the higher revenue loss incurs an average welfare loss of \$67.19 per month per household in the lowest stratum. On the other hand, when $\zeta_2 = 1.25$, the welfare for the lowest stratum is -2.48% on average, which is higher compared to the welfare result (-4.60%) from Figure 6. This means the shadow cost of the conservation constraint incurs an average welfare loss of \$74.87 per month per household in the lowest stratum.

D. *Zeroscaping for the Lowest Income Stratum*

The analysis so far has concluded that under extreme weather, policy constraints on optimal prices cause greater welfare losses for the lowest income stratum. Since the policy constraints are essential for the optimal utility price, the best way to mitigate the welfare imbalance is to use an additional policy, specifically targeted at the lowest income stratum. Many factors contribute to the heterogeneity of consumption quantity, and I have found that income, typically assumed to be a key driver, is not a high-quality predictor. In a reduced-form analysis, I analyzed the qualitative trend of predicting high-quantity consumers and found that larger houses, higher care of vegetation (NDVI), more bathrooms, and larger pool/spa/hot tub areas are statistically significant predictors of high-quantity users. However, only NDVI becomes more pronounced when precipitation is low. Therefore, a natural policy to test is zeroscaping/xeriscaping – reducing NDVI. The result of the reduced-form analysis can be found here in Table A7.

For the counterfactual NDVI, I first take half of the status quo NDVI for the lowest income stratum, which essentially means households now remove roughly half of their lawns[30]. I then redid the exercise of the counterfactual optimal pricing for all counterfactual weather patterns. Since the NDVI only changes for the lowest stratum, I will only show the welfare changes for that stratum. I will first show the result of changing ζ_1, and then ζ_2.

Unsurprisingly, by halving the status quo NDVI, the lowest stratum consumes less and pays less. This results in an increase in welfare across different weather conditions, but more so when $\zeta_1 < 0$, where the demand curve is shifting rightward. In the most extreme weather condition of $\zeta_1 = -0.25$, zeroscaping improves EV/I for the lowest stratum from -4.11% to -0.62%. This corresponds to roughly \$121.58 per household per month on average. Note that this is the immediate welfare effect of zeroscaping and

[30]Note that I only modified NDVI> 0; if the status quo NDVI is already negative, meaning their lawn vegetation is below the level of bare soil, I did not change their counterfactual NDVI.

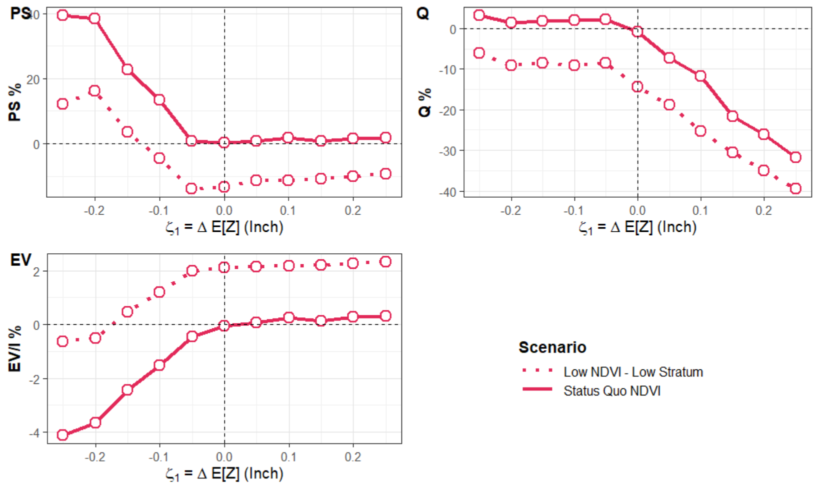

Figure 8. : Welfare Comparison of Halving NDVI ($\zeta_1 \in [-0.25, 0.25]$)

does not count the long-term benefits, which would presumably be higher[31]. On the other hand, for the result of changing ζ_2:

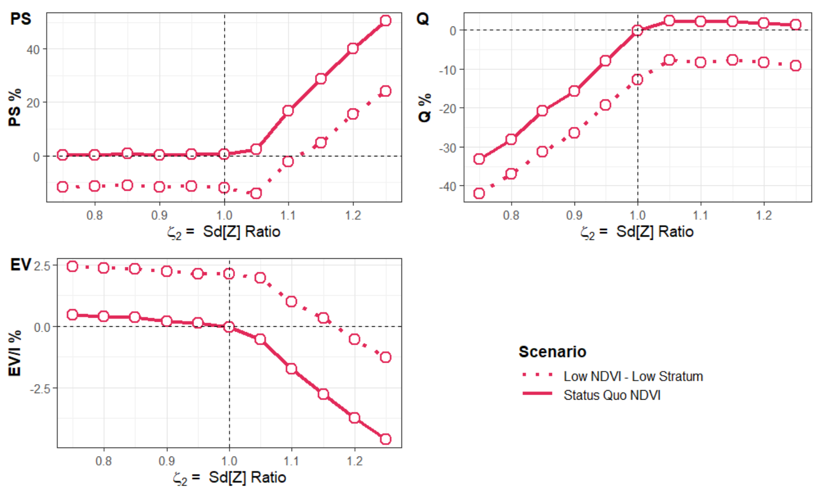

Figure 9. : Welfare Comparison of Halving NDVI ($\zeta_2 \in [0.75, 1.25]$)

Similar to the pattern observed with optimal prices in Figure 6 and Figure 7, this is almost a mirror image of Figure 8. When the demand curve is shifting rightward in the

[31]This is because a one-time zeroscaping would provide an almost permanent shift in the demand curve in the long run, meaning the future welfare for each month would be increased.

cases of $\zeta_2 > 1$, zeroscaping improves welfare for the lowest stratum. For the extreme case of $\zeta_2 = 1.25$, zeroscaping improves EV/I for the lowest stratum from -4.60% to -1.26%. This corresponds to roughly \$116.46 per household per month on average, and just like the previous cases, this measures the immediate welfare jump from zeroscaping.

In order to further investigate the value of zeroscaping, I implemented multiple NDVI reduction values and calculated the corresponding welfare changes to compare different levels of zeroscaping. The reduction value ranges from 0 to 0.9, where 0 represents the status quo NDVI, and 0.9 represents reducing the status quo NDVI by 0.9, which roughly translates to removing 90% of a lawn. I will share the welfare change in terms of EV/I for both extreme cases where the demand curve shifts rightward due to extreme weather, and then compare how much welfare in terms of dollar value is gained by zeroscaping. The results are as follows:

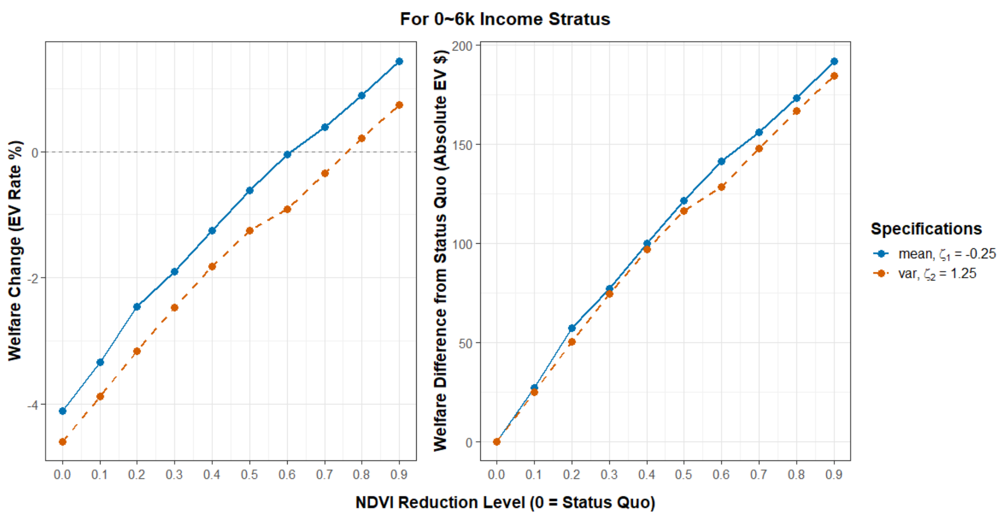

Figure 10. : Welfare Result under Extreme Weather Conditions by NDVI reduction level $\in [0, 0.9]$

From the graph, I can first see the general increasing trend as the NDVI reduction level is higher for both the case of $\zeta_1 = -0.25$ and the case of $\zeta_2 = 1.25$. For $\zeta_1 = -0.25$, reducing NDVI by 0.2 will generate \$57.35 per household per month, which almost nullifies the welfare loss from the shadow cost of the conservation constraint. In addition, reducing by 0.6 will set EV/I to be roughly 0, which means both the welfare loss from the shadow cost of the conservation constraint and the impact from extreme weather are nullified. For $\zeta_2 = 1.25$, due to the higher price across tiers in general, welfare is lower, meaning I would need higher NDVI reduction levels to reach the same amount of welfare. In general, across all reduction levels of NDVI, the $\zeta_2 = 1.25$ scenario generates \$6.06 less per household per month on average, showcasing that higher variance is arguably a "worse" extreme weather condition in terms of welfare for the lowest income stratum. The same levels of zeroscaping result in slightly lower welfare compared to the $\zeta_1 = -0.25$ scenario, but could still be considered as a policy solution to improve the distributional effect.

VI. Conclusion

In this paper, I estimate the optimal Inclining Block Price (IBP) tariffs for water utilities using a Ramsey-style pricing model that incorporates extreme precipitation patterns. My initial reduced-form analysis reveals that deviations in precipitation are correlated with greater volatility in water consumption for higher-income households. This evidence highlights a significant price-setting challenge for utilities, particularly concerning their distributional goals.

I then employ a structural Discrete-Continuous Choice (DCC) model, applied to the piecewise-linear budget constraint, which incorporates a satellite-derived vegetation health index (NDVI) for household lawns. This allows for a more granular understanding of outdoor water usage in single-family households—a substantial portion of consumption that is most sensitive to precipitation.

Subsequently, I develop an empirical Ramsey model that includes both revenue recovery and quantity conservation constraints to determine the optimal IBP structure. Given that prices are set well before weather events occur, I explore optimal pricing under various weather scenarios, including shifts in the mean and variance of precipitation from the status quo. The analysis shows that when precipitation averages decrease or when volatility increases, the demand curve shifts rightward. This places more pressure on the now-binding conservation constraint, compounding the welfare loss caused by the extreme weather itself. Notably, the lowest-income stratum experiences the highest welfare losses. My findings confirm existing literature that current IBP tariffs used by utilities generally favor high-income households due to mismatches between household income and consumption levels. Furthermore, high-income households demonstrate greater price sensitivity during periods of low precipitation, further undermining the intended distributional effects of IBP.

This paper provides empirical measurements of the shadow cost of policy constraints in the utility's optimal rate design by comparing welfare outcomes with and without the conservation and revenue requirements. I find that the lowest-income stratum experiences the largest welfare loss from these shadow costs, particularly under extreme weather. During conditions that shift the demand curve rightward, such as extreme drought or high precipitation variance, the conservation constraint becomes binding. This imposes an average welfare loss of \$60.92 and \$74.87 per household per month, respectively, on the lowest-income stratum. Conversely, during periods of extreme rain or low precipitation variance, revenue loss concerns make the revenue constraint binding, leading to average welfare losses of \$63.65 and \$67.19 per household per month, respectively, for this same group. Notably, the shadow costs of these constraints do not cause significant welfare differences for other income strata. This finding underscores that price alone is an insufficient instrument for achieving a utility's multifaceted policy goals, as both conservation and revenue objectives weaken the intended distributional effect under extreme weather.

This result has a profound policy implication: under extreme weather, using price as the sole instrument to achieve a utility's policy goals is not feasible. Additional policies are necessary to achieve distributional objectives. I focus on zeroscaping/xeriscaping (i.e., reducing lawn water needs and thus NDVI) as a straightforward policy to curb demand during extreme weather conditions when the demand curve is pushed rightward. Under extremely dry conditions, reducing NDVI by 0.2 could generate $57.35 in welfare per household per month, almost nullifying the welfare loss from the shadow cost of the conservation constraint. While high precipitation variance causes a larger welfare reduction overall, making the effect of zeroscaping slightly smaller, it remains highly effective.

Looking ahead, I plan to expand this research in two directions. First, I will model consumption smoothing programs. The existence of "Budget Billing" and similar programs reflects consumer risk aversion and a preference for stable monthly payments, particularly among lower-income individuals. This preference acts as a form of insurance against weather stochasticity and interacts significantly with the utility's pricing optimization problem. Second, I will endogenize vegetation changes (NDVI). I currently treat lagged NDVI and any reduction from zeroscaping as exogenous variables. However, consumers make long-term decisions about landscaping based on potential welfare gains. Endogenizing this decision-making process would allow for a more precise estimation of the long-term welfare benefits of xeriscaping.

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APPENDIX

All plots of this paper have utilized the software "Colorgorical" to choose a more clear and aesthetically pleasing color scheme. Gramazio, Laidlaw and Schloss (2017).

A1. Water Utility Pricing in Austin, TX

PRECIPITATION TREND IN AUSTIN

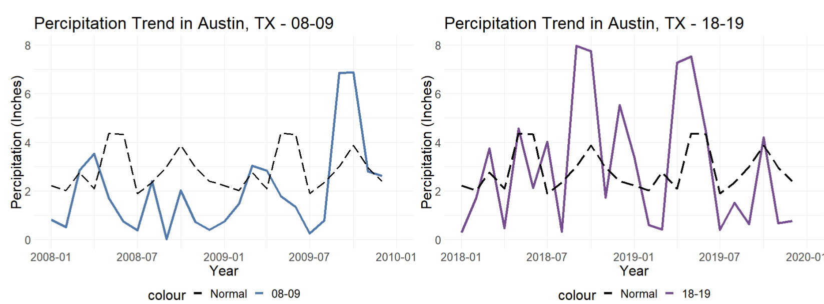

Figure A1. : Precipitation Trends of 2008-2009 vs. 2018-2019

CURRENT PRICING STRUCTURE

TIER AND INCOME DISTRIBUTION

Based on the status quo price and status quo quantity distribution, I divide the household's monthly income into 5 different strata. Since both the quantity and income are

Table A1—: Pricing Structure

Kink Points (kGallon)	Fixed Charge ($)	Marginal Price ($)	
		Non-CAP	CAP
0-2	8.5	3.09	2.42
2-6	10.8	5.01	4.1
6-11	16.5	8.54	6.72
11-20	37	12.9	11.56
>20	37	14.41	14.26

Note: Each fixed charge is composed of fixed payment and meter charge. The fixed payment depends on final consumption quantity, and the meter charge assumes a 5/8 meter size, which is the most common residential meter size. The marginal price includes volume charge, reserved fund charge and community benefit charge. These itemized charges are all billed per 1,000 gallons, meaning they are essentially part of the marginal price. Volume charge depends on the amount of final consumption quantity while the other two are billed per 1,000 gallons. Reserved fund surcharge goes into a restricted reserve fund to offset water service revenue shortfalls that may impact operations and services. Community benefit charge is only billed to Non-CAP consumers to fund the CAP.

very right-skewed, only for this plot, I filtered out all households with monthly household income bigger than $\$250,000$, and quantities higher than $100,000$ gallons. These households are not filtered out in demand estimation and counterfactual analysis, but are simply filtered out for the picture below for visualization purposes.

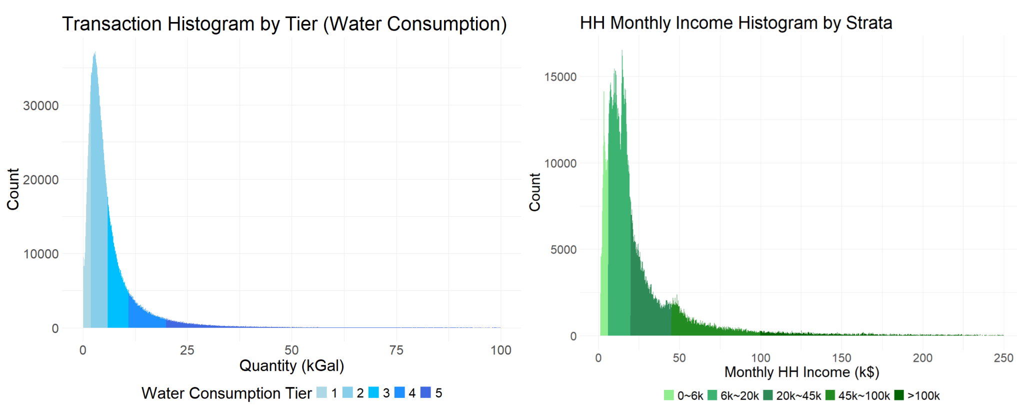

Figure A2. : Monthly Water Quantity Distribution vs. Income Distribution

Descriptive and Reduced Form Evidence

In the interest of seeing how different income strata will react to abnormal weather changes and the descriptive evidence of Figure 2, I performed the following OLS:

$$\Delta q = \beta_0 + \beta_1 \Delta Z_T + \beta_2 \Delta Z_P + \beta_3 \text{Income} * \Delta Z_T + \beta_4 \text{Income} * \Delta Z_P + \alpha p + \varepsilon$$

Table A2—: Monthly Water Quantity Distribution vs. Income Distribution

Income Strata (\$)	Percentage	Tiers (kGal)	Percentage
$0 \sim 6k$	0.146	$0 \sim 2k$	0.159
$6k \sim 20k$	0.489	$2k \sim 6k$	0.503
$20k \sim 45k$	0.216	$6k \sim 11k$	0.200
$45k \sim 100k$	0.104	$11k \sim 20k$	0.0927
$> 100k$	0.0441	$> 20k$	0.0456

Note: Just like the utility targeting different quantity levels of households. This division of household monthly income into different strata is without loss of generality and serves as a way to gauge the distributional effect incurred by any price change.

where Income is the income strata defined in Table A2. ΔZ for both precipitation and temperature is the difference between observed Z_m for month m and the corresponding 30 year average. $\Delta q = \frac{q_m - \bar{q_m}}{\bar{q_m}}$, which is the quantity deviation from the average quantity for the specific household for a specific month between 2016-2020 in percentage terms.

Table A3—: Regression Results for Quantity Deviation

Variable	Estimate	Std. Error	t value	$\Pr(>\|t\|)$
(Intercept)	-0.4182	0.0008490	-492.609	< 2e-16 ***
ΔZ_P	-0.01668	0.0003582	-46.555	< 2e-16 ***
ΔZ_T	0.004787	0.0001438	33.283	< 2e-16 ***
income_strata: 6k∼20k	-0.02104	0.0007867	-26.739	< 2e-16 ***
income_strata: 20k∼45k	-0.08237	0.0008990	-91.627	< 2e-16 ***
income_strata: 45k∼100k	-0.1673	0.001096	-152.710	< 2e-16 ***
income_strata: >100k	-0.2476	0.001513	-163.593	< 2e-16 ***
p	0.07486	0.00008868	844.089	< 2e-16 ***
$\Delta Z_P \times$income_strata: 6k∼20k	0.001576	0.0004098	3.846	0.00012 ***
$\Delta Z_P \times$income_strata: 20k∼45k	-0.009682	0.0004614	-20.986	< 2e-16 ***
$\Delta Z_P \times$income_strata: 45k∼100k	-0.01923	0.0005504	-34.933	< 2e-16 ***
$\Delta Z_P \times$income_strata: >100k	-0.01874	0.0007744	-24.198	< 2e-16 ***
$\Delta Z_T \times$income_strata: 6k∼20k	0.00009410	0.0001657	0.568	0.57012
$\Delta Z_T \times$income_strata: 20k∼45k	0.0004584	0.0001880	2.439	0.01474 *
$\Delta Z_T \times$income_strata: 45k∼100k	-0.002830	0.0002234	-12.667	< 2e-16 ***
$\Delta Z_T \times$income_strata: >100k	-0.002834	0.0003005	-9.428	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.4014 on 2351610 degrees of freedom
Multiple R-squared: 0.263, Adjusted R-squared: 0.263
F-statistic: 5.596e+04 on 15 and 2351610 DF, p-value: < 2.2e-16

In this model, controlling for differences in precipitation, maximum temperature, and the marginal price, the relationship between income strata and the percentage difference in water consumption is seen through: 1) for baseline differences (when Δ weather and price are zero), compared to the lowest income strata (0-6k), most higher income strata show a statistically significant lower baseline percentage difference in water consumption from their usual amount. This does not mean higher income groups use less water overall; it means their deviation from their own usual quantity, under these specific baseline conditions, is lower than the deviation for the reference group. This effect is likely influenced by the strong role of price in this model, as marginal price might be correlated with income or consumption levels that influence which price tier is reached. 2) The linear effect of precipitation difference: the interaction terms between income strata and ΔZ_P are largely significant. This means that the linear rate at which Δq changes for every unit increase in precipitation difference varies significantly across income strata. In particular, higher income strata (20k-45k, 45k-100k, $>$100k) show increasingly more negative interaction terms (-0.009682, -0.01923, -0.01874). This means the negative impact of the additional precipitation difference on consumption deviation is increasingly stronger as income rises in these groups. 3) the linear effect of temperature difference: many interaction terms between income strata and ΔZ_T are also significant, indicating that the linear rate at which Δq changes for every unit increase in temperature difference varies significantly across income strata. However, the relationship is less clear-cut compared to the one from precipitation. Compared to the reference group (where the effect of ΔZ_T is 0.004787, meaning a 0.48 percentage point increase in consumption difference for every unit increase in temperature difference), the 6k-20k stratum's interaction is not significant (0.0000941), suggesting its linear temperature sensitivity is not statistically different from the reference group in this model. The 20k-45k stratum has a significant positive interaction (0.0004584), making its positive temperature effect slightly stronger (0.004787 + 0.0004584 = 0.0052454). The 45k-100k and $>$100k strata have significant negative interaction terms (-0.002830 and -0.002834). This means that although the base effect of temperature difference is positive, the additional positive effect seen in higher income groups in the first model is now appearing as a reduction in the sensitivity compared to the reference group's base sensitivity (0.004787 - 0.002830 = 0.001957 for 45k-100k; 0.004787 - 0.002834 = 0.001953 for $>$100k).

This difference between temperature and precipitation prompts the research to be more focused on changing precipitation. In addition, a structural model is used to estimate the demand on the panel data to show how price, precipitation, and the interaction term with income strata will affect the demand.

A2. Demand Model and Estimation

ADDITIONAL CASE IN THE UNCONDITIONAL INDIRECT UTILITY FUNCTION

When constructing the unconditional indirect utility function from the conditional ones, the case where consumption occurs exactly at a tier boundary (kink point), rather than strictly within a tier, may require clarification. Continuing with the two-tier scenario ($K = 2$) without loss of generality, this case arises when the household's optimal consumption calculated using the tier 1 price, $g(p_1, I)$, would exceed the tier 1 limit

q_1, *and* the optimal consumption calculated using the tier 2 price and virtual income, $g(p_2, I + d_2)$, would fall below q_1. That is, the condition is $g(p_2, I + d_2) \leq q_1 < g(p_1, I)$ (assuming $d_1 = 0$ or defined appropriately). To satisfy the Incentive Compatibility (IC) constraint (i.e., ensure the chosen consumption is utility-maximizing given the full budget set), the household optimally consumes exactly at the kink point q_1. The diagram below illustrates this:

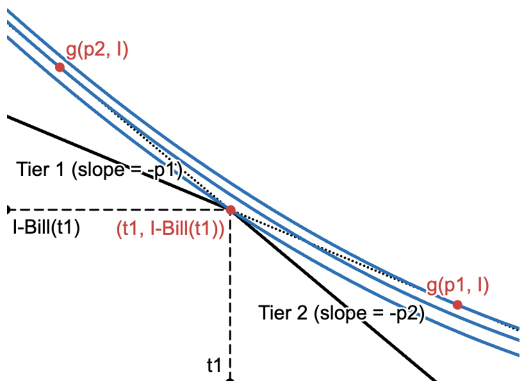

Figure A3. : The household consumes exactly at the kink $(q_1, I - \mathrm{Bill}(q_1))$

As shown, under the condition $g(p_2, I + d_2) \leq q_1 < g(p_1, I)$, the optimal (incentive compatible) choice for this household is to consume the bundle corresponding to the kink point, $(q_1, I - \mathrm{Bill}(q_1))$. Here, $\mathrm{Bill}(q_1)$ represents the total water bill incurred when consuming exactly q_1 units (e.g., typically $A_2 + p_1 q_1$ in a two-tier system where A_2 is the fixed charge associated with entering tier 2). If consumption at the kink were not allowed as an option in the model, such consumers would be incorrectly assigned to consume within one of the adjacent tiers, violating the true optimum.

Generalizing, for any tier boundary q_k where $k \in \{1, \ldots, K-1\}$, some households might satisfy the condition $g(p_{k+1}, I + d_{k+1}) \leq q_k < g(p_k, I + d_k)$, for whom the optimal consumption point is exactly $(q_k, I - \mathrm{Bill}(q_k))$. In our empirical application to Austin, which has 5 tiers ($K = 5$), there are 4 such kink points (q_1, q_2, q_3, q_4), requiring the model to allow for consumption exactly at these quantities to ensure incentive compatibility across the full range of consumption.

Likelihood Function

Without loss of generality, I explain the derivation of the likelihood function for a two-tier case ($K = 2$); the approach generalizes to K tiers. Conditional on the ex-ante optimal choice (based on the known preference shock η but ignoring the ex-post error ν) being tier 1 ($k^* = 1$), the condition is $\log w_1 + \eta \leq \log q_1$. Similarly, $k^* = 2$ if $\log w_2 + \eta > \log q_1$. The remaining possibility, ensuring incentive compatibility and covering all η, is consumption at the kink q_1, which occurs if $\log w_2 + \eta \leq \log q_1 < \log w_1 + \eta$. Rearranging these conditions on η and adding the ex-post error ν (unobserved by the household when choosing the tier/kink), the observed log-consumption $\log(w)$ is:

$$\log(w) = \begin{cases} \log(w_1) + \eta + \nu & \text{if } \eta \leq \log(q_1) - \log(w_1) \\ \log(q_1) + \nu & \text{if } \log(q_1) - \log(w_1) < \eta \leq \log(q_1) - \log(w_2) \\ \log(w_2) + \eta + \nu & \text{if } \eta > \log(q_1) - \log(w_2) \end{cases}$$

which is the two-tier special case of Equation 7. Note that since the household does not observe ν when making its choice, ν affects the final observed consumption in all scenarios.

Let $f(\cdot)$ denote a probability density function (pdf). The overall likelihood for an observation $\log w$ is the sum of the contributions from these three mutually exclusive and exhaustive scenarios ($L = L_1 + L_{kink} + L_2$).

Case 1: $k^* = 1$ ($\eta \leq \log(q_1) - \log(w_1)$) The observed log consumption is $\log w = \log w_1 + \eta + \nu$. The contribution to the likelihood depends on the joint pdf of $(\eta + \nu, \eta)$, integrated over the relevant range of η:

$$L_1 = \int_{-\infty}^{\log q_1 - \log w_1} f_{\nu+\eta,\eta}(\log w - \log w_1, \eta)\, d\eta$$

Case 2: $k^* = 2$ ($\eta > \log(q_1) - \log(w_2)$) The observed log consumption is $\log w = \log w_2 + \eta + \nu$. The likelihood contribution is:

$$L_2 = \int_{\log q_1 - \log w_2}^{\infty} f_{\nu+\eta,\eta}(\log w - \log w_2, \eta)\, d\eta$$

Case 3: Kink Consumption ($\log(q_1) - \log(w_1) < \eta \leq \log(q_1) - \log(w_2)$) The observed log consumption is $\log w = \log q_1 + \nu$. The likelihood contribution depends on the joint pdf of (ν, η), integrated over the relevant range of η:

$$L_{kink} = \int_{\log q_1 - \log w_1}^{\log q_1 - \log w_2} f_{\nu,\eta}(\log w - \log q_1, \eta)\, d\eta$$

If I assume $\eta \sim N(0, \sigma_\eta^2)$ and $\nu \sim N(0, \sigma_\nu^2)$, and that they are independent, these integrals can be solved in closed form. Let $\phi(\cdot)$ and $\Phi(\cdot)$ be the standard normal pdf and cdf, respectively. The joint distribution of $(\eta, \nu + \eta)$ is bivariate normal. Let $\rho_s = \mathrm{Corr}(\eta, \nu + \eta) = \sigma_\eta / \sqrt{\sigma_\eta^2 + \sigma_\nu^2}$.

Evaluating the first integral (L_1):

$$\begin{aligned}
L_1 &= \int_{-\infty}^{\log q_1 - \log w_1} f_{\nu+\eta}(\log w - \log w_1) f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_1)\, d\eta \\
&= f_{\nu+\eta}(\log w - \log w_1) \int_{-\infty}^{\log q_1 - \log w_1} f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_1)\, d\eta \\
&= \frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \phi\left(\frac{\log w - \log w_1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}}\right) \Phi\left(\frac{(\log q_1 - \log w_1)/\sigma_\eta - \rho_s(\frac{\log w - \log w_1}{\sqrt{\sigma_\eta^2+\sigma_\nu^2}})}{\sqrt{1-\rho_s^2}}\right) \\
&\equiv \frac{\phi(s_1)}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \Phi(r_1)
\end{aligned}$$

where $s_1 = (\log w - \log w_1)/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$ and $r_1 = (t_1^* - \rho_s s_1)/\sqrt{1-\rho_s^2}$ with $t_1^* = (\log q_1 - \log w_1)/\sigma_\eta$.

Similarly, evaluating the integral for the second case (L_2):

$$\begin{aligned}
L_2 &= f_{\nu+\eta}(\log w - \log w_2) \int_{\log q_1 - \log w_2}^{\infty} f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_2)\, d\eta \\
&= f_{\nu+\eta}(\log w - \log w_2) \left[1 - \Phi\left(\frac{(\log q_1 - \log w_2)/\sigma_\eta - \rho_s(\frac{\log w - \log w_2}{\sqrt{\sigma_\eta^2+\sigma_\nu^2}})}{\sqrt{1-\rho_s^2}}\right)\right] \\
&\equiv \frac{\phi(s_2)}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} (1 - \Phi(n_2))
\end{aligned}$$

where $s_2 = (\log w - \log w_2)/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$ and $n_2 = (m_1 - \rho_s s_2)/\sqrt{1-\rho_s^2}$ with $m_1 = (\log q_1 - \log w_2)/\sigma_\eta$.

Finally, evaluating the integral for the third case (kink consumption, L_{kink}), using the independence of ν and η:

$$\begin{aligned}
L_{kink} &= \int_{t_1^*\sigma_\eta}^{m_1\sigma_\eta} f_\nu(\log w - \log q_1) f_\eta(\eta)\, d\eta = f_\nu(\log w - \log q_1) \int_{t_1^*\sigma_\eta}^{m_1\sigma_\eta} f_\eta(\eta)\, d\eta \\
&= \frac{1}{\sigma_\nu} \phi\left(\frac{\log w - \log q_1}{\sigma_\nu}\right) \left[\Phi\left(\frac{m_1\sigma_\eta}{\sigma_\eta}\right) - \Phi\left(\frac{t_1^*\sigma_\eta}{\sigma_\eta}\right)\right] \\
&= \frac{\phi(u_1)}{\sigma_\nu} (\Phi(m_1) - \Phi(t_1^*))
\end{aligned}$$

where $u_1 = (\log w - \log q_1)/\sigma_\nu$.

Summing the three components gives the likelihood for the two-tier case:

$$
\begin{aligned}
L &= L_1 + L_{kink} + L_2 \\
&= \frac{\phi(s_1)}{\sqrt{\sigma_\eta^2 + \sigma_v^2}}\Phi(r_1) + \frac{\phi(u_1)}{\sigma_v}(\Phi(m_1) - \Phi(t_1^*)) + \frac{\phi(s_2)}{\sqrt{\sigma_\eta^2 + \sigma_v^2}}(1 - \Phi(n_2))
\end{aligned}
$$

Using the general definitions $t_k^* = (\log q_k - \log w_k)/\sigma_\eta$, $m_k = (\log q_k - \log w_{k+1})/\sigma_\eta$, $s_k = (\log w - \log w_k)/\sqrt{\sigma_\eta^2 + \sigma_v^2}$, $u_k = (\log w - \log q_k)/\sigma_v$, $r_k = (t_k^* - \rho_s s_k)/\sqrt{1-\rho_s^2}$, $n_k = (m_{k-1} - \rho_s s_k)/\sqrt{1-\rho_s^2}$, and applying boundary conditions ($m_0 \to -\infty \implies n_1 \to -\infty \implies \Phi(n_1) = 0$; $q_2 \to \infty \implies t_2^* \to \infty \implies r_2 \to \infty \implies \Phi(r_2) = 1$), this derived likelihood function matches the general form given in Equation 8 for $K = 2$.

NDVI

The use of the Normalized Difference Vegetation Index (NDVI) for estimating water demand was introduced by Wolak (2016). The essential idea is to quantify vegetation health for specific geographical locations – in this case, the health of lawns associated with single-family homes. Higher NDVI values indicate healthier vegetation, suggesting greater lawn care efforts.

NDVI is calculated from satellite imagery using the difference between the reflectance in the near-infrared (NIR) and red light bands. Healthy plants typically reflect more NIR light and absorb more red light (appearing green), while less healthy or stressed plants reflect less NIR and more red light (appearing yellow or brown). The index is typically calculated as (NIR - Red) / (NIR + Red), normalized to a range between -1 and +1, where higher values indicate healthier, denser vegetation. Values near -1 often correspond to water bodies. Values around 0 typically represent bare soil or sparse vegetation, and values approaching +1 indicate dense, healthy vegetation like forests or well-maintained lawns.

The raw data I use is the Sentinel-2 Surface Reflectance dataset,[32] providing imagery with a 10m x 10m spatial resolution (pixel size). Since typical residential lots in Texas are larger than this pixel size, this higher resolution allows for a more precise calculation of NDVI within each lot compared to the 30m x 30m resolution used by Wolak (2016). Sentinel-2 images are collected frequently (on average, every 5 days for a given location), though not always regularly, resulting in multiple images per month. Some images, however, contain areas obscured by clouds. To address this, I employ cloud masking and create monthly composite images.[33] The cloud masking algorithm utilizes bands of the images, such as the "Cloud Probability" or "QA". These bands indicate the likelihood of cloud cover or contain bitwise flags for clouds and shadows. To create a cloud-free composite, I apply these masks to filter out cloudy pixels from multiple images over all images of a calendar month of the same area, then merge the remaining clear-sky pixels

[32] If using Google Earth Engine to access data, please refer to Google Earth Engine Sentinel-2 Surface Reflectance.

[33] Details for cloud masking in Google Earth Engine can be found here Cloud Masking - gee map.

into a seamless, cloud-free composite.

To visually illustrate the process of calculating NDVI in Figure A4, I take a small sample from the Brentwood neighborhood in Austin, near the intersection of W Koenig Lane and Burnet Road. While detecting subtle vegetation differences can be difficult in the raw satellite image, the corresponding NDVI visualization clearly showcases the variations in vegetation health between households.

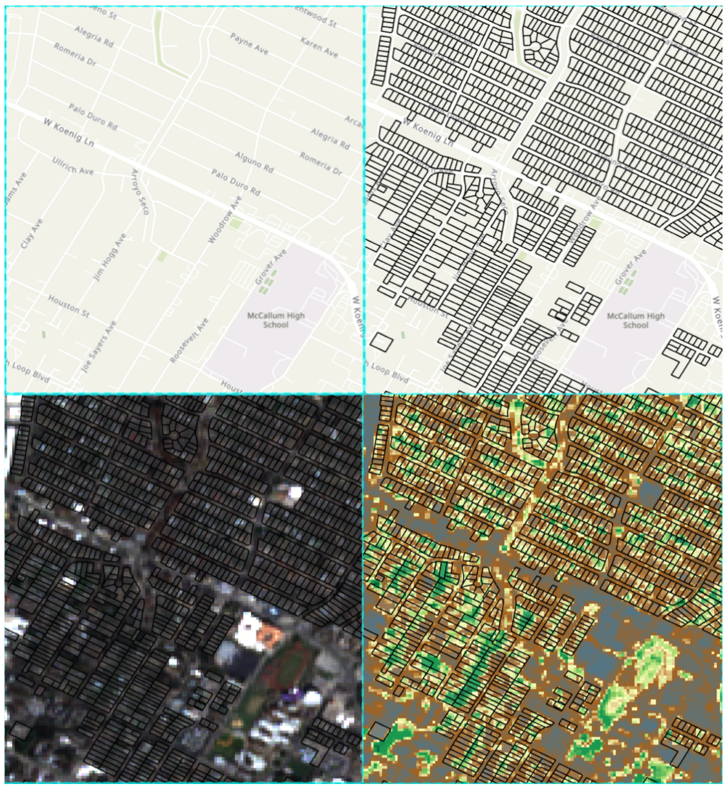

Figure A4. : The Process of Calculating NDVI

Note: From Top Left to Bottom Right: OpenStreetMap base map; OpenStreetMap with household lot shapefiles overlaid; Sentinel-2 Raw Image mosaic with shapefiles overlaid; and final NDVI visualization derived from Sentinel-2 with shapefiles overlaid.

Summary Statistics

After matching the panel data with TCAD records, filtering outliers, and removing households located outside Travis County (and thus likely outside the Austin Water service area), the final dataset primarily covers the period from May 2018 to December 2019. It is important to note that this data includes households on slightly different payment plans. Specifically, some households meeting certain income requirements are eligible for CAP, which offers lower marginal prices. The inclusion of CAP households provides valuable price variation for model identification, despite their small number. The summary statistics can be found in Table A4.[34]

Table A4—: Summary Statistics

Parameter Name	Min	1st Quartile	Median	Mean	3rd Quartile	Max	N
Heavy Water Appliances	0	0	0	0.1637	0	12	127320
Bedrooms	1	1	1	1.439	1	31	127320
Bathrooms	0.1	2	2	2.427	2.5	34	127320
Lot Size (Acre)	0.02136	0.15018	0.19060	0.27396	0.25309	278.25085	127320
Household Monthly Income (k$)	1.2	8.642	15.320	29.295	27.765	5057.051	127320
NDVI	-0.3564	0.3266	0.4038	0.3988	0.4766	0.7729	2351626
Mean Max Temp (F)	56.24	68.98	83.74	82.18	95.04	101.65	2351626
IQR Max Temp (F)	2	4.984	9.499	9.904	14.217	25.334	2351626
Total Precip (Inches)	0	1.009	2.550	2.967	4.349	13.819	2351626
IQR Precip (Inches)	0	0.004724	0.034655	0.158315	0.238235	2.434279	2351626
Quantity (kGal)	0.1	2.6	4.3	6.537	7.4	1275.8	2351626
Payment ($)	8.426	19.118	27.436	57.746	53.074	17577.749	2351626

Note: Heavy Water Appliances include pool, hot tub, sprinkler system, fountain, and car wash station/area. In the real estate industry, a full bathroom requires a sink, a tub, a shower, and a toilet. If it only has 3/4 of fixtures, it will constitute a 3/4 bathroom, and only 2/4 fixtures will constitute a half bathroom.

MLE Estimation Result

Given the model with the interaction terms, the MLE estimates the parameters $(\beta_1', \beta_2', \beta_3', \beta_4', \beta_5', c, c_\alpha, c_\rho, \sigma_\eta, \sigma_\nu)$ from data $(X_i, Z_i, X_{\alpha,i}, Z_{\alpha,i}, X_{\rho,i}, p_k, q_k, A_k, I_i, w_i)$, with a total of 18 parameters. All data measuring price or payment are scaled from nominal value to real dollar value in January 2017 using the Federal Reserve Economic Database Gross Domestic Product (GDP) deflator from St Louis Federal Reserve Bank. [35] The estimation results are listed in Table A5:

Most results of MLE fit the intuition qualitatively. The standard error is calculated by the inverse of the matrix of the sum of the outer products of the observation-by-observation gradient of the log-likelihood for each household evaluated at the maximum likelihood parameter estimates, a method introduced by Hall, Hall and Hausman (1974).

Price Elasticities of Income Strata

From Figure A5, I can conclude that, as opposed to common belief, higher strata will be less elastic; the higher income households are actually more elastic across different consumption levels. In addition, there are a decent number of lower-income households that have pretty high consumption levels with not very high elasticities. Both the mismatches between income and quantity, and income and elasticity, explain the reduced-form evidence from 2, and weather is one of the factors that causes these mismatches.

[34]Overall, approximately 60% of households and 55.84% of transactions from the original data are used for the demand estimation analysis. Among the final sample, 99.57% are non-CAP consumers, and 0.43% are CAP consumers.

[35]See Fred-GDPDEF

Table A5—: MLE Estimation Results

Parameter Name	Estimate	Standard Error	Parameter Name	Estimate	Standard Error
Bathroom	1.16	(9.20E-04)	Average High Temp	0.00218	(2.04E-05)
NDVI	1.19	(0.011)	IQR High Temp	-0.0189	(1.46E-04)
Constant	0.731	(0.0017)	Total Precipitation	-0.941	(1.34E-04)
Price * bedroom	0.0486	(3.94E-04)	IQR Precipitation	0.12	(0.005)
Price * NDVI	-0.0404	(0.00465)	Income * Heavy Water Appliances	-0.0718	(4.81E-04)
Price * Avg High Temp	-0.0168	(2.47E-05)	Income * Bedroom	-0.0158	(5.34E-05)
Price * Total Prcp	-0.0363	(4.23E-04)	Income * NDVI	-0.0692	(3.91E-04)
Price	0.688	(0.00193)	Income	0.162	(1.67E-04)
σ_η	2.56	(8.45E-04)	σ_ν	4.71E-04	(2.25E-03)

Note: Price * bedroom represents the interaction term inside α, measuring how the price effect is changed through the number of bedrooms. The same goes for other interaction terms for price and income.

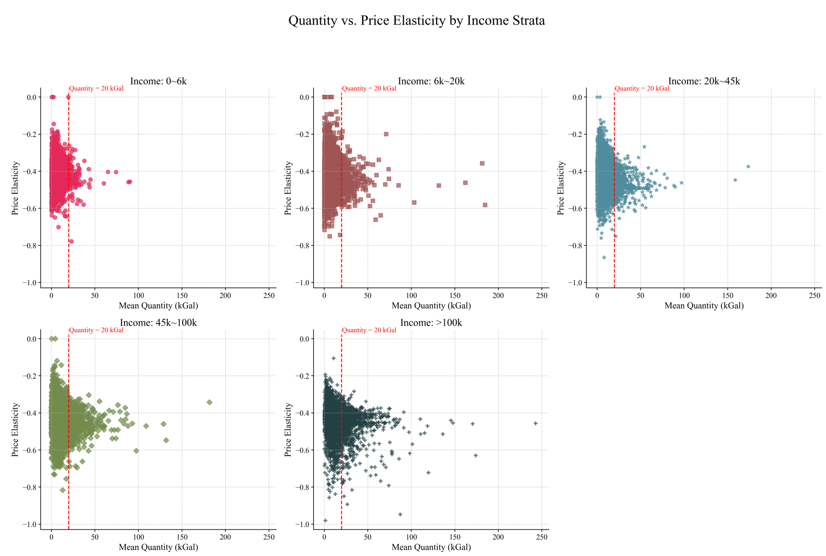

Figure A5. : Price Elasticities v. Mean Quantity by Income Strata

This can be a potential mechanism explaining the findings from previous literature [36] where they conclude that IBP optimization will likely favor higher income strata. Essentially, I would like the utilities to be aware of the price sensitivities of higher income strata, as the higher paying consumers are not actually "reliably" contributing towards the revenue, as well as the existence of the low elasticity-high quantity households in the lower income strata, as these consumers who should be benefiting from the equity goal, could potentially suffer from other competing policy goals.

One important caveat for this study (and further discussion of distributional results) is that the household income I calculated actually stemmed from the correlated house value. Even though it has been proven to be generally true[37], it is a strong assumption to assume the same distribution of house value and household income within the same zipcode. Some households in the lower strata may simply be low maintenance on their houses, old, or get a low appraisal value, and are actually having a high income.

A3. Ramsey Pricing Model

Status Quo Revenue as the Threshold

I will explore the validity of using the status quo revenue as the lower bound for counterfactual revenue in this section. Due to the lack of detailed cost breakdown data, it would be difficult to establish a supply-side cost model. However, by using aggregate cost information, I can make some inferences. In order to study the financial viability of Austin Water, I need to ensure that total revenue can cover total costs. However, total water service revenue is only a portion of Austin Water's overall revenue stream. Furthermore, the revenue I observe in this data represents only single-home residential water revenue, which itself is only a portion of total water service revenue. Nonetheless, water service revenue is typically the largest revenue stream ($> 70\%$), and single-home housing usage usually constitutes the largest part of total water service revenue ($> 50\%$). In particular, single-home housing usage is most susceptible to precipitation variation because multi-home housing is typically in condominiums, which do not have lawns, and commercial usage usually does not vary with changes in precipitation. Therefore, it is valid to use single-home housing to study the impact of weather, and compare the total revenue from single-home housing usage to some threshold to measure the revenue risk faced by the utility due to weather variation.

However, the lack of detailed cost information makes it hard to estimate the cost generated solely by single-home housing. The total costs from water services include operation, labor, debt service requirements, and funds transferred to the city government. None of these can be separated and attributed solely to single-home housing usage.

Looking at the seasonal trends from quarterly financial reports of water service revenue in Figure A6 and cost, I can see that the cost for the whole year is relatively stable. The only peak is usually around the summer months when the water supply is lower and requires more water from the reservoir. The main reason for negative profit in water

[36]Ruijs, Zimmermann and van den Berg (2008), Echeverri (2023), and Wichman (2024)
[37]See Zhang (2016), and Kim (2020)

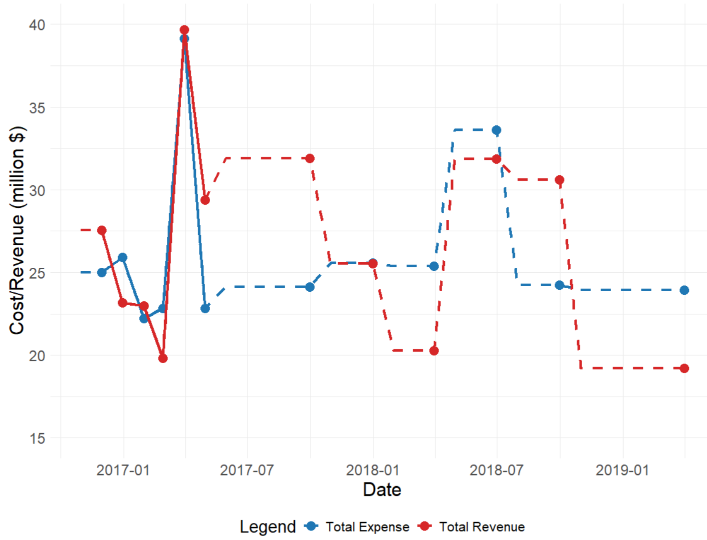

Figure A6. : Total Cost and Revenue from Water Service

Note: Austin Water releases its financial report irregularly, but on average, every 3 months. During some time between January 2017 and May 2017, they released the report every month. Hence, some data have a monthly frequency, and some do not.

service is that revenues in some months fall below the typical cost level during non-peak months. Therefore, the financial risk faced by the utility is largely due to revenue volatility between months. Therefore, it is reasonable to set up a revenue lower bound for comparison with counterfactual revenues. I could set up an intricate cost model to estimate this lower bound, but the status quo revenue (which is the revenue of the year 2019, and the weather variation in 2019 was quite standard) serves as a good enough benchmark.

Simulate Weather Distribution

It is tricky to evaluate E_Z without the knowledge of the real distribution of Z. This is due to 1) if limited to recent years of Z data, there are too few data points to empirically generate the real distribution of Z, and 2) if expanding to long-term data of Z, the weather data 30 years ago does not share the macro climate trends of the recent weather data. Unfortunately, truly predicting the weather is out of the scope of this paper, but I offer an alternative solution to estimate the optimal price based on the prior of the utility. In addition, in one of the specifications, I used a Monte Carlo method to simulate weather prediction errors and the optimal result does not deviate that much.

The counterfactual weather is generated from a prior shift from the utility. If the utility thinks next year's weather is on average more rainy, then $\zeta_1 > 0$ and vice versa. If the utility thinks next year's weather is on average higher in variance, then $\zeta_2 > 1$. The utility will treat this counterfactual Z as the static weather and then calculate the optimal price.

Adding a Monte Carlo small log-normal disturbance will simulate the prediction error and the result is more realistic. I generate a small log-normal disturbance with $\mu = 0$ and $\sigma = 1$. The $\sigma = 1$ is calculated from the standard deviation of the strictly positive precipitation data from the recent 5 years (2014-2018). This means the utility first predicts a general trend of precipitation data, it could be either $Z' = Z \pm \zeta_1$ or $Z'/Z = \zeta_2$, and then in order to account for prediction errors, Z' is perturbed by a small log-normal error term to simulate the stochasticity of Z' based on the utility's prediction.

The Monte Carlo process is calculated as follows: for each simulation s, the utility will calculate both the objective ($CS(Z^s) - \lambda \cdot \max(0, C - R^s(Z^s))$) and the conservation constraint ($P(\sum q(Z^s) \leq \bar{Q}) \geq 0.95$) and take the average of all numbers of simulations. Essentially, the utility is maximizing over the average objective for all the simulations, while making sure the average quantity for all simulations is below the threshold. One might question why not average over all simulated months, with a total number of simulated data points to be $s \cdot 12$. It is hard to imagine the utility optimizing on a monthly basis, even though it could generate more data points. On the other hand, it is reasonable to assume that the utility would need to check the projected overall welfare and projected revenue condition by the end of their financial year. Averaging the yearly objective for all simulations essentially calculated the expected objective for the utility under the weather stochasticity. Due to the long running time, I eventually chose $s = 25$ as a start. The resulting optimal prices do not change significantly compared to Figure 6 and Figure 7. This showcases that the baseline model gives a consistent measurement of the first moment of weather stochasticity.

A4. Counterfactual Analysis

Counterfactual NDVI

As NDVI measures the health of vegetation, it would not make sense to keep it unchanged through shifting counterfactual precipitation. To isolate the effect of precipitation on NDVI, I conducted an OLS using the following formula, where P represents precipitation and T represents daily max temperature:

$$NDVI = \beta_0 + \beta_1 P_{\text{sum}} + \beta_2 P_{\text{iqr}} + \beta_3 T_{\text{mean}} + \beta_4 T_{\text{iqr}} + \beta_5 I$$

Note that both the mean and IQR here refer to the within-month mean and IQR. For this paper, I focus on the variance of the monthly sum of precipitation (P_{sum}); therefore, while keeping all the other variables unchanged, the parameter of interest will be β_1. The OLS results are as follows:

This regression captures the impact of precipitation on vegetation health ecologically. The only control variable included is income to capture the size of the lawn for each household. This means for every change in precipitation in inches, the NDVI will increase by 0.008, keeping demand and income the same. Through the calculation of counterfactual NDVI, I can calculate the counterfactual demand, welfare, etc.

Table A6—: OLS Estimation Results of Precipitation on NDVI

Variable	Estimate	Std. Error	t value	Pr($>|t|$)
(Intercept)	0.1149	0.0010	92.940	< 2e-16 ***
P_{sum}	0.0080	0.000045	177.803	< 2e-16 ***
P_{iqr}	−0.0118	0.0004	−29.460	< 2e-16 ***
T_{mean}	0.0030	0.000012	256.693	< 2e-16 ***
T_{iqr}	0.0013	0.000032	40.768	< 2e-16 ***
I (in millions)	1.449	1.3	114.940	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Multiple R-squared: 0.127, Adjusted R-squared: 0.127
F-statistic: 3.935e+04 on 5 and 1,356,918 DF, p-value: < 2e-16

Optimal Price Procedure

Given the counterfactual weather and NDVI, household characteristics, and income, I will use Equation 12 to solve for the optimal price of 14 parameters. All parameters are part of IBP; therefore, I set all prices to be larger than 0, and force each step to be increasing for both marginal prices and fixed payment (meaning the difference between each tier will at least be \$0.01). The optimization algorithm I used is COBYQA, a derivative-free optimization solver designed to supersede COBYLA to solve for the bounded optimization process. In essence, COBYQA is a trust-region SQP method based on quadratic models obtained by underdetermined interpolation.[38]. Of all the constrained optimization algorithms, COBYQA consistently can provide reliable results as long as the initial value is within the valid range with respect to constraints.

Since I am dealing with price optimization, I do not require very refined results. I chose the initial searching radius of the algorithm to be 1.0, and the final radius for convergence tolerance to be 0.01 (as the lowest price difference will be in cents). I also utilize putting the loss function in the objective function in addition to the conservation constraint to avoid optimizing on 2 constraints at the same time (since otherwise, without the loss function, both constraints will need to be implemented). I choose to set the initial value as if each price jump is the same between tiers to intensify the certain benefit of more salient price jumps between certain tiers. Since COBYQA is good at finding relatively local results, I choose the initial value to be close to the status quo IBP.

Aside from weather stochasticity, I set up a Monte-Carlo simulation for ε using the result from Table A5. Note that there are two levels of stochasticity with unobserved taste (η) and perception error (ν). The perception error is purely random and not controlled by the household, so for each household for each month, I will generate a simulation for ν_{mh}. As for η, I assume each household has an unchanged $\bar{\eta}_h$ across all months to avoid overfitting and simulate the results. I have tried using η_{mh}, the counterfactual welfare

[38]The algorithm is developed by Ragonneau (2022), Ragonneau and Zhang (2025)

generates very little difference between specifications.

PRICE RESULT - MEAN SHIFT

Since the 5-tier IBP includes both marginal prices and fixed payments, I argue the most effective way to visualize the pricing structure is by plotting total payment (in dollars) versus quantity (in thousand gallons). Note that all plots are truncated at 50 k gallons to focus on the changes around the majority of the data points. In practice, a decent number of consumers use significantly more than 50 k gallons. In this subsubsection, I will only focus on ζ_1 and only showcase the result of $\zeta_1 = -0.25$ and $\zeta_1 = 0.25$. The results of non-extreme weather conditions in between fit the general trend and are omitted for aesthetics. All price plots include three benchmarks derived from the status quo. The status quo price (shown with a dot-dash line) reflects the current pricing structure used by Austin Water. The status quo flat price is constructed by averaging the five marginal prices and five fixed payments, resulting in a linear pricing structure. I also include the distribution of the status quo quantity.

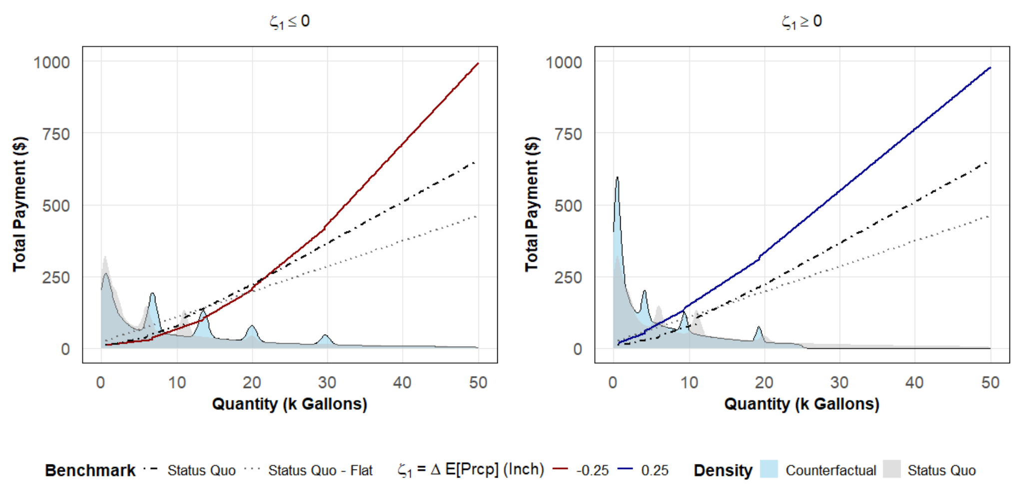

Figure A7. : Optimal Prices - ζ_1

When $\zeta = -0.25$, the entire quantity distribution shifts to the right. When there are no revenue concerns and the goal of welfare maximization, the prices are lower for lower tiers. However, for the higher tier, due to the conservation constraint becoming more binding, the price will have to increase to encourage conservation with a trade-off of some welfare losses. On the other hand, when $\zeta = 0.25$, the entire quantity distribution shifts to the left. With a more pressing revenue concern, the utility will have to raise prices across all tiers in order to ensure financial feasibility.

Price Result - Variance Shift

When changing ζ_2 (the ratio of weather standard deviation compared to the status quo), I will only focus on the extreme cases of $\zeta_2 = 0.75$ and $\zeta_2 = 1.25$.

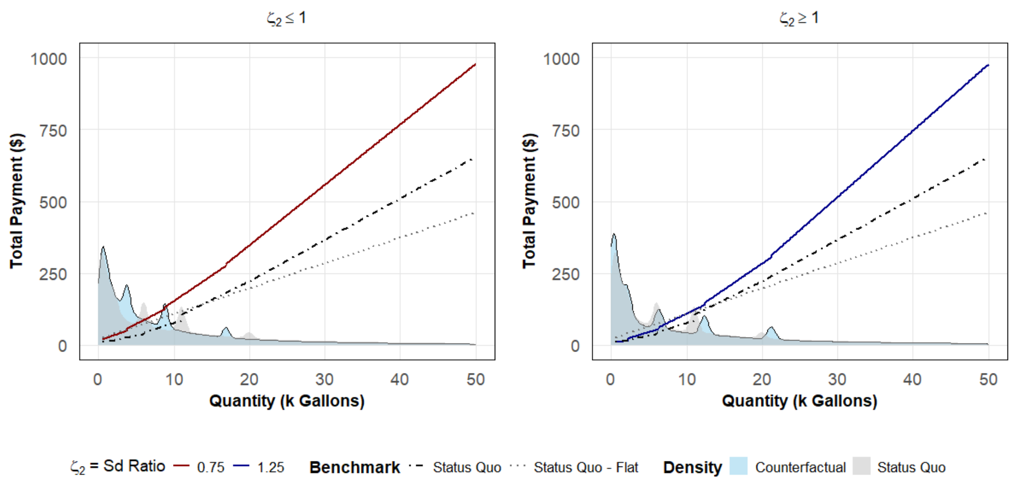

Figure A8. : Optimal Prices - ζ_2

When $\zeta = 0.75$, the quantity distribution shifts towards the median. However, since the status quo (and the counterfactual) quantity distribution is right-skewed, the quantity in general decreases. This will result in a similar situation of $\zeta_1 > 0$ where the revenue losses are more pressing, which will in turn result in an increase in prices across all tiers. When $\zeta = 1.25$, the quantity distribution becomes more spread out. Due to the right-skewness of the data, the quantity in general increases. This will result in a similar situation of $\zeta_1 < 0$ where the conservation constraint is binding. However, what is different from the cases of $\zeta_1 < 0$ is the decreasing quantity in even drier months, making the price for lower tiers higher due to the financial risks during the now even drier months.

Status Quo Price - Weather Change

In order to separate the welfare loss from weather itself and the welfare loss from the shadow cost of the policy constraints, I calculate the welfare using the status quo price vector $\mathbf{p}_0$ under all counterfactual weather conditions ($\zeta_1 \in [-0.25, 0.25], \zeta_2 \in [0.75, 1.25]$). Here are the welfare result for the mean shift ($\zeta_1 \in [-0.25, 0.25]$) and variance shift ($\zeta_2 \in [0.75, 1.25]$):

I can see the clear shift of the demand curve through the weather and how it's affecting both total quantity and total payments from all strata for both cases of shifting mean and variance. Without the optimal pricing procedure considering the policy constraints, it

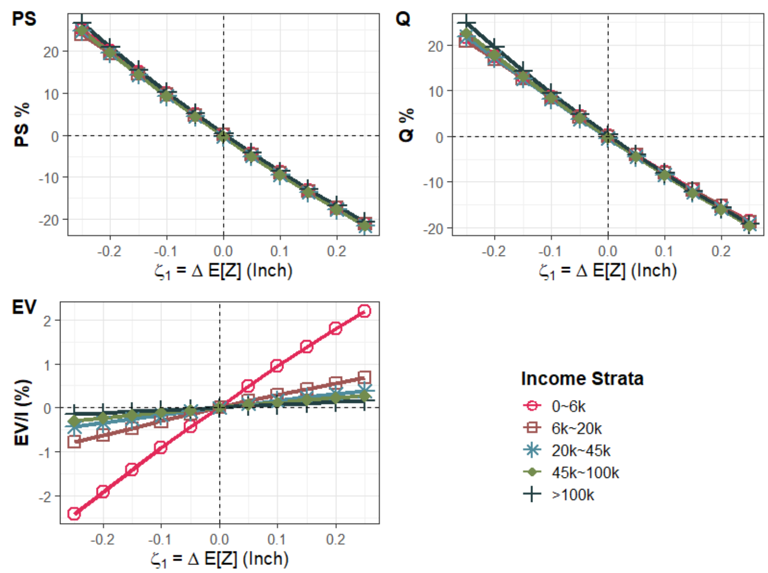

Figure A9. : Welfare from Status Quo Price in Shifting Mean ($\zeta_1 \in [-0.25, 0.25]$)

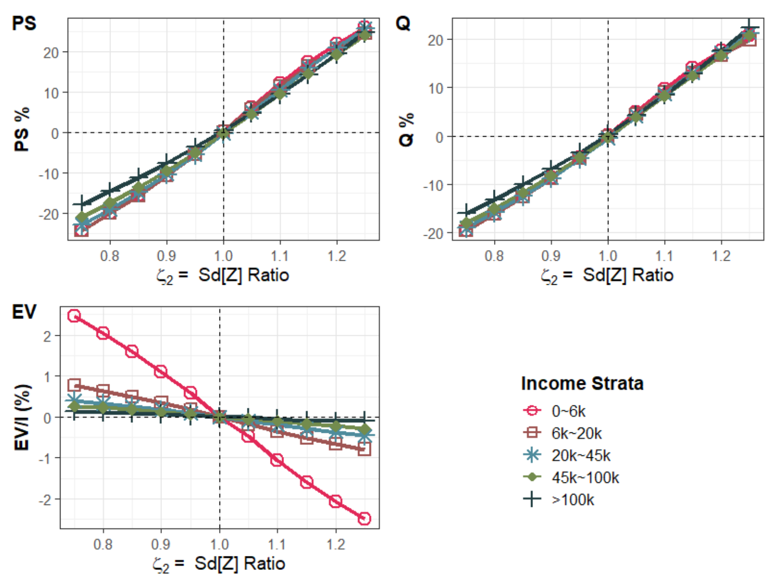

Figure A10. : Welfare from Status Quo Price in Shifting Variance ($\zeta_2 \in [0.75, 1.25]$)

will be difficult to curb consumption when $\zeta_1 < 0$ and lower bound the revenue when $\zeta_1 > 0$. However, the upside of not including the policy constraint is an increase in welfare, in particular, a significant increase in welfare for the lowest income stratum.

HIGH QUANTITY CONSUMERS

There are plenty of high quantity consumers, but what truly makes the equity goals of the optimal price hard to achieve is the existence of high quantity consumers in lower tiers. I have already explored the existence of these consumers in the data, but here I would like to specifically show the welfare loss incurred by these customers in the lowest stratum (0$\sim$6k) and compare it to the highest stratum (>100k). I will focus on the specific weather pattern of $\zeta_1 = -0.25$ where the welfare for the lowest stratum is the lowest.

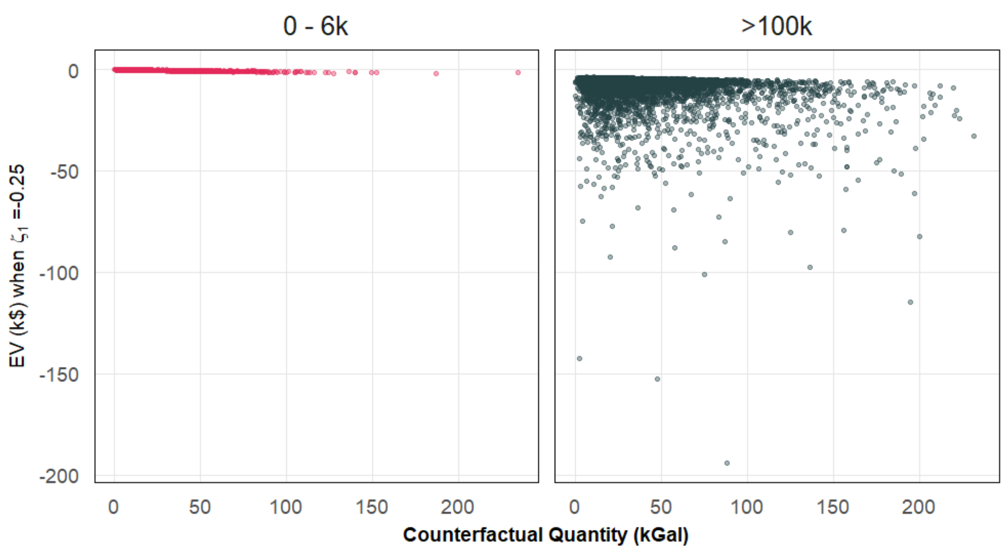

Figure A11. : EV when - $\zeta_1 = -0.25$, $0 \sim 6$k and >100k strata

From this picture, I can clearly see that the magnitude of welfare losses is higher in the higher income strata across different quantity levels. However, if I measure EV/I, which is a fairer measurement of welfare across income strata:

Even though the higher income stratum loses more welfare, the high quantity users (those higher than 20k gallons), and their relatively low elasticities, cause lower welfare in extreme weather conditions for the lowest income stratum.

To separate the heterogeneities of the quantity, and since income turns out to be a problematic predictor of quantity. I performed a reduced-form logit regression to figure

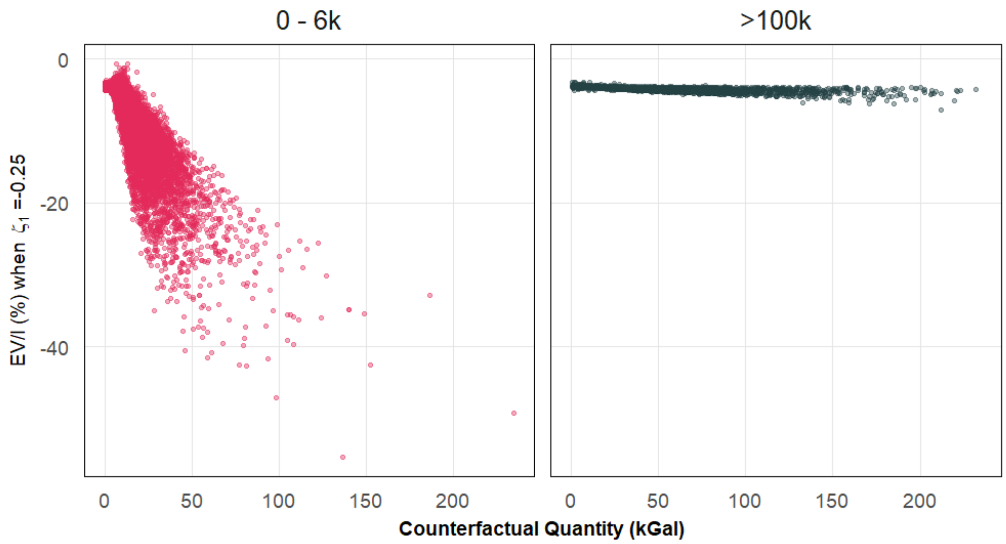

Figure A12. : EV/I when - $\zeta_1 = -0.25$, $0 \sim 6$k and >100k strata

out the source of the heterogeneity. I define high quantity transactions as > 20k gallon and only limit to the lowest income stratum. I include some predictors from the demand model, as well as variables like HVAC category (a categorical variable labeling the size of the house), lawn percentage (a continuous variable measuring the percentage of the lot size not counted as living areas), and the interactions with precipitation. Here is the equation and the result.

$$\begin{aligned}
\mathbb{1}\{\text{High Quantity}\} = &\ \text{house value} + \text{precipitation} + \text{NDVI} + \text{bathroom} + \text{bedroom} \\
&+ \text{spa area} + \text{heavy water app} + \text{HVAC category} + \text{lawn percentage} \\
&+ \text{precipitation} \times \text{house value} + \text{precipitation} \times \text{NDVI} \\
&+ \text{precipitation} \times \text{bathroom} + \text{precipitation} \times \text{bedroom} \\
&+ \text{precipitation} \times \text{spa area} + \text{precipitation} \times \text{heavy water app} \\
&+ \text{precipitation} \times \text{HVAC category} + \text{precipitation} \times \text{lawn percentage}
\end{aligned}$$

For households in the lowest-income stratum. Higher living area, more attention to lawn (NDVI), more bathrooms, and larger pool/hot tub areas (in sqft) are all strong predictors of increased water use. House value has a negative coefficient. Based on my assumption to correlate house value with house income, and the conclusion that income is not a strong predictor of heterogeneity, this is not entirely surprising. This showcases that some households with low house value (which means low income) have very high

Table A7—: Logit Regression Model of Predicting High Quantity for Low-Income Households

Main Effects				Interaction Terms			
Predictor	**Est.**	**Std. Err.**	**z val.**	**Predictor**	**Est.**	**Std. Err.**	**z val.**
(Intercept)	-1.031***	0.138	-7.458	Precipitation : house value	0.002***	0.000	21.639
house value (k$)	-0.004***	0.000	-38.242	Precipitation : NDVI	-0.369**	0.125	-2.965
Precipitation (Inch)	-2.345***	0.138	-17.061	Precipitation : bathroom	-0.066***	0.017	-3.902
NDVI	5.294***	0.129	41.048	Precipitation : bedroom	0.002	0.015	0.112
bathroom	0.209***	0.019	10.892	Precipitation : spa area	-0.001***	0.000	-10.222
bedroom	-0.016	0.016	-0.996	Precipitation : heavy water app	0.110	0.152	0.723
spa area (sqft)	0.003***	0.000	13.203	Precipitation : HVAC Med-Low	-0.104**	0.033	-3.101
heavy water app (sqft)	-0.091	0.199	-0.458	Precipitation : HVAC Med-High	0.037	0.039	0.967
HVAC Med-Low	0.414***	0.032	12.944	Precipitation : HVAC High	0.128**	0.048	2.691
HVAC Med-High	0.696***	0.042	16.721	Precipitation : lawn percentage	0.472**	0.155	3.036
HVAC High	1.009***	0.057	17.739				
lawn percentage	0.212	0.156	1.361				

Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1. Est. = Estimate, Std. Err. = Std. Error, z val. = z value.

quantities. The influence of these factors changes with precipitation: the impact of vegetation is most pronounced in dry conditions, while the effect of lawn percentage and the consumption gap between the largest and smallest homes becomes more significant in wetter climates. Therefore, for the cases of dry weather conditions, where welfare loss is the largest, NDVI is a good predictor for the heterogeneities of high water consumption, and zeroscaping is a natural additional policy to remedy the impact from NDVI. Further studies could be extended towards the impact of bathrooms and larger pool/hot tub areas and their related policy impacts.