Rain or Shine? Optimal Utility Pricing under Changing Weather

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As climate change amplifies weather extremes, water utilities face increasingly volatile demand and revenue, challenging the ability of fixed Increasing Block Prices (IBP) to simultaneously ensure revenue stability, promote conservation, and protect low-income consumers. This paper demonstrates how utilities can strategically manage this uncertainty through optimal, risk-averse IBP design. Using granular household data from Austin, TX, and a structural demand model enhanced with satellite-derived NDVI imagery, we find that higher-income consumers—who are both weather-sensitive and surprisingly price-elastic—substantially contribute to revenue volatility. Our key contribution is to show that adopting a concave (risk-averse) revenue preference results in pricing that reduces welfare volatility for low-income households in response to weather shocks. Specifically, while concave (risk-averse) pricing increases the average welfare loss for the lowest income stratum from $-7.62\% \sim -7.15\%$ *under linear (risk-neutral) pricing to* $-8.05\% \sim -7.71\%$*, it reduces the standard deviation of welfare outcomes by* $0.46 \sim 0.50$ *percentage points. All welfare measures are expressed in monetary terms as equivalent variation relative to income. This quantifiable trade-off—accepting higher average welfare losses in exchange for lower volatility—is essential for policy makers to design targeted and preemptive compensation for lower income households based on their risk aversion, particularly when future weather conditions are difficult to forecast at the time of price-setting.* *Keywords: Ramsey Pricing, Multi-part Tariff, Risk Aversion, Distributional Effect*

I. Introduction

The increasing variability of weather patterns, exacerbated by climate change, poses a significant and growing challenge to the effectiveness of pricing essential utility services, particularly water. Urban water utilities are tasked with a complex balancing act, primarily managed through pricing: they must ensure their financial sustainability through cost recovery, promote resource conservation in the face of potentially dwindling supplies, and maintain equitable access for all consumers, especially low-income households. This study focuses on these challenges within the specific context of Austin, Texas, a rapidly growing metropolitan area known for its erratic precipitation trends. The primary objective of this paper is to develop and empirically implement an optimal water utility pricing framework, grounded in Ramsey pricing principles and tailored for Increasing Block Pricing (IBP), that explicitly accounts for and remains resilient to such weather stochasticity.

The theoretical underpinning for widely adopted utility pricing schemes often stems from Ramsey (1927), who proposed ”second-best” solutions to cover large fixed costs while minimizing societal welfare loss, typically through an inverse elasticity rule. In practice, many water utilities adopt multi-part tariffs with Increasing Block Pricing (IBP), where prices rise with consumption tiers, aiming to encourage conservation among high-volume users while ensuring affordability for basic needs. However, the efficacy of IBP structures (and their ability to achieve policy goals) is severely tested by weather volatility, especially for water utilities where a significant portion of residential demand, such as lawn watering, is directly correlated with precipitation. A critical operational constraint is that utility prices are typically preset for an entire year, lacking the flexibility to adjust dynamically to monthly or even seasonal weather variations—a specific challenge this paper addresses. This paper first offers reduced-form evidence of price sensitivity to precipitation changes and then structurally estimates demand under nonlinear IBP using a Discrete/Continuous Choice (DCC) model. Identifying true price elasticity, however, remains a complex econometric problem. While previous research has extended Ramsey models to include policy constraints and explored welfare measurement under nonlinear prices, the distributional effects of IBP often reveal that these structures may inadvertently favor higher-income households, a concern amplified by unpredictable weather.

Previous literature has explored the complex nature of estimating public utility demand and the welfare impacts of changes to IBP. Research by Hewitt and Hanemann (1995) and Olmstead, Hanemann and Stavins (2007) demonstrates the validity of using DCC models to estimate price elasticity in urban water demand using household-level data. On one hand, literature such as Castro-Rodríguez, María Da-Rocha and Delicado (2002) and Nataraj and Hanemann (2011) has shown that, under certain price structures, demand is responsive, and specific changes in IBP can generate welfare improvements. In particular, Szabo (2015), examining South Africa’s free water policy, demonstrated that free allowances are not always necessary; welfare and conservation goals can be improved using nonlinear pricing alone, rather than relying on ad-hoc policy adjustments. Conversely, many literature pointed out the potential issues on over relying using IBP as an instrument for policy. Ito (2014) questioned whether consumers genuinely respond to complex IBP structures and found out they seem to react more to average prices instead of the marginal prices of multiple tiers, while Ruijs (2009) and Echeverri (2023), even though believed the consumers will respond to IBP changes, highlighted its potential shortcomings regarding distributional effects, and concluded that changing IBP structure in general favors higher income households more.

This paper fills the gap in the literature to explore the effectiveness of IBP as a price instrument in fulfilling its policy goals with the consideration of weather stochasticity, in particular its distributional goal. We align closely with Wang and Wolak (2022), who employed a Ramsey-style model to determine the optimal IBP for welfare maximization and reduction of revenue risks arising from household-level demand stochasticity. Nevertheless, to our knowledge, this paper is the first to develop and empirically apply a Ramsey-style optimal pricing model that uniquely incorporates weather stochasticity (specifically precipitation variability) and allows for utility revenue risk aversion via a concave revenue constraint, while simultaneously addressing the equity goals of a social planner. Furthermore, our demand estimation is enhanced by the novel integration of high-resolution (10m x 10m) satellite-derived Normalized Difference Vegetation Index (NDVI) data. While Wang and Wolak (2022) also utilized this approach, our study offers a more granular estimation of factors affecting outdoor water usage and further examines NDVI's interaction with weather changes across different income levels for single-family homes. Utilizing this novel data, our structural DCC model reveals that, contrary to some common assumptions, higher-income households can exhibit greater price elasticity, particularly in response to precipitation shifts. This finding introduces significant revenue uncertainty for the utility and further complicates the already challenging task of achieving equitable distributional impacts with IBP.

Another strand of related literature concerns the stability of welfare and revenue for policymakers and social planners, particularly in environmental economics. Kind, Botzen and Aerts (2017) incorporated risk aversion and income distribution into cost-benefit analysis of flood risk management, highlighting the need for targeted investments in resilience measures for low-income populations. Schlee and Smith (2019) calculated the ex-ante compensation an individual would require to accept uncertainty, noting that the degree of compensation depends on factors such as the level of risk aversion and the variance in the environmental service measure associated with the policy. Specifically about the risks of social planner/policy maker, Valentini and Vitale (2019) and DeCanio, Manski and Sanstad (2022) both showcased when facing deep climate uncertainty, the social planner should adopt more aggressive strategies to mitigate risk, to avoid worst-case climate scenarios. This paper extends this strategy the price setting problem in one-sided market in Industrial Organization. In particular, we introduce concavity into the utility's revenue constraint within the Ramsey framework to stabilize welfare outcomes (measured by equivalent variation) for the lowest income stratum across diverse weather scenarios. This approach highlights a trade-off: while average welfare might reduce, the increased predictability facilitates the design of more precise and reliable targeted compensation schemes. We find that for the lowest income stratum, under specific scenarios of decreased mean precipitation or increased precipitation variance, a moderate—and optimal—level of risk aversion (CRRA function with $\gamma = 0.25$) generates a welfare loss from $-8.05\% \sim -7.71\%$, which compared to the welfare loss of $-7.62\% \sim -7.15\%$, is an additional loss of 0.495 percentage point on average. Nevertheless, the concavity also reduce the standard deviation of this welfare loss by $0.46\% \sim 0.50\%$ compared to risk neutral. All welfare is measured as the equivalent variation from the price change relative to their income. Crucially, this paper offers an empirical quantification of the trade-off between welfare levels and welfare stability for vulnerable consumer groups under climate uncertainty. This leads to actionable policy recommendations advocating for preemptive, transparent, and progressive compensation mechanisms to complement existing IBP structures.

The remainder of this paper is structured as follows. Section II provides background on water utility pricing in Austin, TX, discusses the inherent challenges posed by weather volatility, describes the datasets utilized, and presents descriptive evidence of how weather deviations affect water consumption across different income strata. Section III details the development of the structural demand model, a Discrete/Continuous Choice model, explains the estimation strategy incorporating household characteristics, weather variables, and NDVI data, and presents the key estimation results, including price and income elasticities and their heterogeneity. Section IV establishes the theoretical Ramsey pricing model, extends it to incorporate weather stochasticity and a risk-averse revenue preference from the utility, and outlines the empirical model used to determine optimal IBP parameters under these conditions. Section V presents the counterfactual analysis, where we estimate optimal prices and their welfare consequences under various shifts in precipitation patterns (both mean and variance shifts) and simulations, comparing outcomes under linear versus concave revenue constraints, and exploring the optimal level of concavity to balance welfare stability and welfare levels for lower-income households. Finally, Section VI concludes the paper by summarizing the main findings, discussing their broader policy implications for water resource management in an era of climate change, and suggesting avenues for future research.

II. Water Utility Pricing in Austin, TX

In this section, we first introduce water utility pricing in Austin, TX within the context of this research, detailing the challenges U.S. water utility companies generally face with growing weather volatility. This includes the current pricing structure and the policy goals utilities need to achieve. After that, we present the datasets used and discuss how higher or lower than usual precipitation/temperature could potentially affect utility pricing decisions.

A. Utility Pricing and Increasing Block Pricing (IBP)

A utility company, typically considered a natural monopoly with large fixed costs, faces a challenge according to Ramsey's pricing problem (from Ramsey (1927)): pricing at marginal cost, though efficient, would not cover its total cost, rendering the "first-best" solution infeasible. Ramsey proposed the "second-best" solution to address the company's financial requirements while minimizing the reduction in overall social welfare. This price-setting solution is known as the inverse elasticity rule, where price increases with lower price elasticity, as inelastic demand causes smaller reductions in the quantity consumed and thus smaller distortions. In the case of water utility companies (especially in large cities), where the major costs are fixed costs including water management in reservoirs, cleaning, and transporting to each household, and the marginal cost per household is relatively smaller, Ramsey pricing ensures the financial viability of the utility company while maximizing consumer welfare.

In addition to ensuring price can recover costs, the utility must also ensure its policy goals are satisfied. Typically, there are two major policies they need to consider: 1) resource conservation, and 2) the distributional effect of the price. Utilities, with their limited natural resources, are typically encouraged to reduce total consumption, and with climate change, this constraint is more pressing. On the other hand, utilities also need to ensure the price is not set too high for lower-income consumers to afford resources for their basic needs. Thus, for the price to recover costs, the optimal price cannot be set too high such that lower-income households find it unaffordable, nor can it be too low such that revenue is insufficient and overall consumption is excessive.

A typical solution in practice for utility companies is to adopt a multi-part tariff mechanism with Increasing Block Pricing (IBP), which categorizes consumers into different tiers based on their total consumption. For all tiers, the final bill consists of a fixed charge (or access fee), paid regardless of consumption quantity to cover fixed costs, and a volumetric charge (or usage fee), which depends on the consumption quantity to cover marginal costs. The higher the tier, the higher both the fixed charge and the volumetric charge. For the volumetric charge, consumers in higher tiers pay a lower marginal price for consumption up to the kink point and a higher marginal price only for the excess amount above the kink point. Through the increasing block structure, consumers in higher tiers are encouraged to consume less to reduce the quantity for which they pay a higher marginal price. Meanwhile, consumers in lower tiers can benefit from a low marginal price (which is typically lower than marginal cost) to ensure equity.

Thus far, the discussion of IBP applies to both electricity and water utilities. If we add different weather patterns, particularly precipitation, the pricing decision for water utilities becomes even more complex. Unlike electricity, urban household water usage is highly correlated to precipitation for single-family homes (as opposed to just cyclicality from seasons), where lawn watering constitutes a major portion of water consumption, far larger than essential usage like cooking, showering, and washing. Low precipitation (dry months) encourages much higher water usage, while high precipitation (wet months) reduces usage as households may not need to use extra water for lawn watering. Compared to electricity utilities, where they possibly need to account for seasonal demand fluctuation, water utilities will need to prepare for some months that will have unpredictable high or low precipitation, and for some cities like Austin, this could happen randomly in many months throughout the year.[1] For water utilities, it is important to design the IBP with the policy goals of conservation, to maintain financial viability for all weather situations. This creates another price-setting challenge for utilities, as the price typically cannot change from month to month. While economists typically suggest dynamic pricing, such as that used in ride-sharing services, to combat volatile demand, in the case of utility pricing, the price must be preset for the entire year and cannot change within a certain time period. This inability to adjust prices in response to weather stochasticity is specifically what this paper addresses.[2]

[1]For Austin, and many cities in Texas, high rainfall months could happen in any months between March to June, and September to November. These months could equally have low rainfall due to some years with a longer summer season.

[2]Some may question the reason behind this inability to implement dynamic pricing, and there are deeper reasons that need further discussion in political economy. There is simply a lack of industrial convention to deploy dynamic pricing in water utilities on a monthly basis. The unpredictability of which month will be wet or dry also makes it difficult to proactively set up a stable pricing rule (such as price discrimination by summer and winter months).

B. Water Utility Pricing in Austin, TX

For this paper, we use water utility transaction data from Austin, TX. The water utility in Austin is solely managed by Austin Water, a natural monopoly public entity. From water supply in Lake Travis (a natural lake serving as a reservoir approximately 20 miles west of Austin) to maintenance/cleaning procedures, transportation, wastewater processing, and price setting, all are managed by Austin Water. This means that when setting prices, Austin Water acts like a social planner aiming to maximize welfare for the entire economy while pursuing the policy goals resource conservation and equity distribution. The lack of competition makes using Ramsey pricing as a price-setting rule very suitable.

Austin, and Texas in general, are known for erratic and unpredictable precipitation trends. A report by Nielsen-Gammon et al. (2020) noted that future precipitation trends in TX are likely to be dominated by natural variability, which is largely unpredictable. The report also discusses the projected increase in the intensity and frequency of extreme rainfall events. This makes the price-setting challenge discussed previously more pressing, as it becomes even harder to predict which months will be wet or dry, rendering preset price discrimination on wet/dry months nearly impossible. Utilities must utilize one pricing structure for the entire year to satisfy all their policy goals. In the case of Austin, using weather data directly from the National Oceanic and Atmospheric Administration (NOAA), as shown in Figure A1, we can observe that precipitation becomes more erratic compared to a 30-year average. This growing trend is one of the reasons why demand becomes more volatile and makes price-setting decisions more complex. Moreover, Austin is a fast-growing metropolitan area and residential water conservation is a more pressing issue. According to the recent census, the Austin metro area is the second-fastest growing region in the U.S., [3] meaning that with limited natural water supply, utilities need to use pricing to set stricter conservation goals.

The problem faced by Austin Water is by no means limited only to the local context. Many other cities in the U.S. will face more erratic precipitation, growing urban populations, and lower natural water supply in the near future due to climate change[4]. This paper aims to find a Ramsey-style pricing solution in this specific setting for water utilities in general, such that pricing can cover costs during a certain time frame with more erratic weather patterns, and achieving policy goals. We utilize panel data provided by Austin Water for approximately $120,000$ households from May 2018 to Dec 2019, and the public data from the Travis County Appraisal District for household characteristics and house value (which, after normalization and combination with zip code income data from the IRS, generates household income data), weather data from NOAA, and highly refined satellite image data (10m by 10m) capturing the health of household vegetation. These data sources are discussed in detail in Section III.B.

Austin Water's current pricing adopts an IBP structure with 5 different tiers. The current pricing structure can be found in Table A1. Although not directly targeting a

[3]https://www.austintexas.gov/news/new-census-data-austin-metro-slips-top-spot-remains-one-nations-fastest-growing-regions

[4]EPA confirms that hourly rainfall rates and the intensity of heavy precipitation events have increased across the U.S. since 1970. https://www.epa.gov/climate-indicators/climate-change-indicators-heavy-precipitation

specific income level, Austin Water sets the current pricing to have higher marginal prices and higher fixed payments for higher quantity users to improve distributional effects and encourage resource conservation. This is based on the assumption that higher quantity users are usually correlated with higher income levels, who typically consume more as they can afford these prices and have lower price elasticities (and we will conclude later during unexpected precipitation, these households actually tend to have higher price elasticities). Based on the status quo consumption pattern, I divide all households into 5 different strata to match the status quo quantity distribution (see Figure A.A1), to better measure the distributional effect of any estimation of the optimal Ramsey price.

C. *Descriptive and Reduced-Form Evidence*

This subsection presents the data patterns for weather change and quantity change from May 2018 to December 2019. To account for seasonality, we calculated the monthly difference in precipitation (Δ Precipitation) compared to its 30-year average (1990-2020), and similarly for temperature (Δ Temperature). Using panel data of household transactions, we derived the average household quantity between 2016 and 2020 for that specific month and compared it to the corresponding monthly quantity from May 2018 to December 2019 (Quantity Deviation in Percentage). We are particularly interested in how monthly weather deviations influence the percentage deviation of quantity from its historical monthly average, and how this effect differs across income strata.

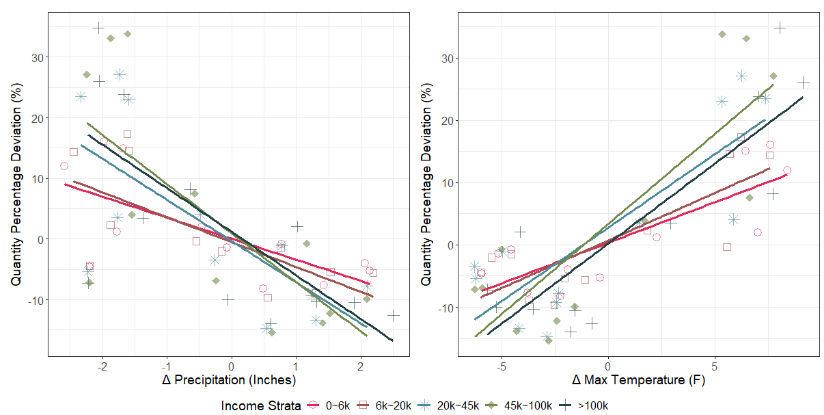

Figure 1. : Quantity Change(%) by Weather Deviation for each Income Strata

We can see that for precipitation, higher than usual amounts typically lead to lower than usual consumption. However, the percentage deviation is higher for high-income households, as they typically have higher demand for outdoor usage with larger backyards and more resources for lawn watering. This effect still seems to exist for temperature, but is less obvious. We also performed a reduced-form analysis of Δ Precipitation and Δ Temperature and their interaction terms with income strata on the percentage quantity deviation. The results can be found in Table A3.

Based on the results, higher income strata tend to have a lower baseline percentage deviation from their usual consumption (when weather differences and marginal price are zero) compared to the lowest income group. However, higher income strata (> 6k) show a stronger negative response to increases in precipitation difference. The linear response to temperature difference is less clear-cut across strata in this model; while the base effect for the reference group is positive, higher income groups show interactions that temper this positive effect, resulting in less positive linear sensitivity compared to the low-income strata. Due to this unclear effect from the reduced-form analysis, coupled with the intrinsic correlation between precipitation and single-family home water usage, we focus on the effect of changing precipitation as a representation of erratic weather throughout this paper and observe the shifts in consumer behavior.

The descriptive and reduced-form evidences suggest a counter-intuitive pattern that possibly due to their higher proportional usage of outdoor water, unusual precipitation changes causes more behavioral changes for higher income households. This adds extra unpredictability and revenue risks as higher-income households will typically have higher demand and pay more. So far, we have kept price unchanged, it is crucial to estimate the price elasticity especially for higher income households as in order to evaluate counterfactual prices and the revenue risks, it is important to accurately measure the behavioral changes caused by price, precipitation, income and their interactions. Therefore, a structural model is needed to incorporate the full vector of household characteristics (especially we also do not observe huge price changes in our dataset), and furthermore, an empirical model of Ramsey problem considering weather-level stochasticity to estimate the optimal IBP.

III. Demand Model and Estimation

In this section, we develop the demand model for water utility to estimate the parameters needed for optimal price calculation and counterfactual analysis. As noted earlier, residential water demand typically faces an IBP structure, meaning that the nonlinear budget constraint generated must be accounted for. Therefore, we utilize a Discrete/Continuous Choice (DCC) model estimated via Maximum Likelihood to estimate the conditional demand probabilities for each pricing tier for each household, thereby accurately estimating the demand parameters.

Each household chooses between water consumption and a numeraire good subject to its budget constraint. The pricing scheme creates a nonlinear budget constraint, and we focus exclusively on the IBP structure in this paper. We assume a log-log functional form relating demand to prices and income, which is a common assumption in the water demand estimation literature.

The first paper to introduce a demand model handling piece-wise linear budget constraints arising from price nonlinearity was Burtless and Hausman (1978) in the context of labor supply. Dubin and McFadden (1984), Hanemann (1984) and Hewitt and Hanemann (1995) laid the groundwork for applying the DCC model to residential utility demand (with Hewitt and Hanemann (1995) specifically addressing water demand). Our approach largely follows this established model structure, with minor modifications, to maintain interpretability.

A. *Demand Model*

The demand model assumes each household consumes both water (w) and a numeraire good (Y) (with price = 1) and the household's total monthly income is I. Suppose for an IBP, the marginal price for each tier k is p_k, the fixed payment is A_k, and the cutoff point between tier k and $k+1$ is q_k. Due to the nature of IBP and suppose there are a total of K tiers, $p_{k+1} \geq p_k, A_{k+1} \geq A_k, q_{k+1} \geq q_k \; \forall k \in \{0, 1, \ldots, K\}$. Together $\{p_k, q_k, A_k\}_{k=1}^{K}$ fully identify the nonlinear pricing structure. Throughout this section, the demand for every month should have a subscript of t, but for simplicity, the time-level subscript will be omitted.

Condition on the household choosing the optimal tier to be $k \leq K$ and the starting price $p_0 = 0$, the household's budget constraint is:

(1)

$$
\begin{aligned}
I &= A_1 + p_1 w + Y \quad (\text{if } k = 1) \\
I &= A_k + p_1 q_1 + p_2(q_2 - q_1) + \cdots + p_{k-1}(q_{k-1} - q_{k-2}) + p_k(w - q_{k-1}) + Y \quad (\text{if } k > 1) \\
&= A_k + \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j + p_k w + Y
\end{aligned}
$$

$$
p_k w + Y = I - A_k - \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j
$$

Therefore, given the IBP pricing structure, the only way to maintain the typical utility maximization problem with budget constraint is to add the additional term on the income such that the Virtual Income is equal to $I + d_k$, where the correction term d_k is defined as:

$$
\begin{aligned}
d_k &= -A_1 \quad (\text{if } k = 1) \\
d_k &= -A_k - \sum_{j=1}^{k-1} (p_j - p_{j+1}) q_j \quad (\text{if } k > 1)
\end{aligned}
\tag{2}
$$

The stipulation of using Virtual Income instead of just income is to make sure the utility maximization problem behaves as if the household is facing the marginal price p_k for all the quantities she consumes. To visually illustrate, consider a simpler model with 2 tiers without loss of generality:

Since consuming at higher tier will grant paying the lower quantity portion in a lower marginal price, therefore correction term d_k (which will be > 0 in higher tier given that $p_k \leq p_{k+1} \; \forall k$ and A_k is not too large) has an income effect to overall budget set. This correction term will allow us solve for the utility maximization problem using the marginal price for each tier without causing any trouble.

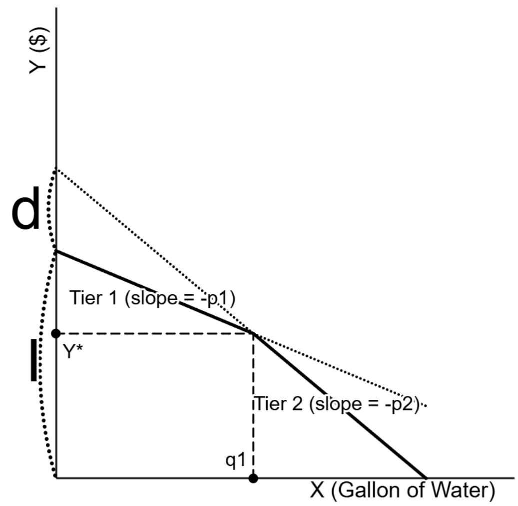

Figure 2. : Virtual Income $I+d_2$ when $K=2$

Define the utility function for the household between water and numeraire good to be $U(w,I)$. Condition on tier k being the optimal choice for the household, define the conditional water demand function to be $g(p_k,I)$, which the functional form will be introduced later. Then the conditional indirect utility function for tier k is:

$$\begin{aligned} V(p_k,I) &= \max_w U(w,I-p_k w) \\ &= U(g(p_k,I),I-p_k g(p_k,I)) \end{aligned} \tag{3}$$

For general U and budget sets, there will be a case where the household will have multiple optimal tiers such that the conditional indirect utility function is not unique. However, fortunately, given the case of IBP, Hausman (1979)'s theorem showed that if the budget set is convex, the optimal tier will be unique. In addition, the concavity of the utility function U will make sure the optimal consumption point $g(p_k,I)$ resides in the specific quantity boundaries of each tier. Therefore, without loss of generality, the unconditional indirect utility function for the total tier $K=2$ is defined as:

$$V(p,I) = \begin{cases} V(p_1,I) & \text{if } q_1 \geq g(p_1,I) \\ V(p_2,I+d_2) & \text{if } q_1 < g(p_2,I+d_2) \\ U(q_1,I-A_2-p_1q_1) & \text{if } g(p_2,I+d_2) \leq q_1 < g(p_1,I) \end{cases} \tag{4}$$

The form of the unconditional indirect utility function can be expanded towards a more general case. We would like to highlight the case of $g(p_2,I+d_2) \leq q_1 < g(p_1,I)$ to make sure the Incentive Compatibility constraints satisfy throughout the entire quantity line. The reason for inclusion of this case is explained in detail in Section A.A2.

So far, we have covered the decision process of the household's water demand, in which they solve for a utility maximization problem and solve for $g(p_k, I)$ for all k, and choose the k that maximizes their utilities only with the assumption that the demand function is concave without imposing any functional form on the demand function. A typical functional form used by recent literature of water demand is a log-log demand function[5], we will use the same approach as the data of water quantity w_i is very right-skewed with only positive values. Therefore, a better parametric assumption for the distribution of w_i should be log normal, which will induce a log-log demand function. With vectors of household characteristics X and weather Z, the conditional indirect utility function for tier k has the functional form of:

$$V(p_k, I+d_k, \beta) = -\exp(\beta_1' X + \beta_2' Z + c + \varepsilon) \frac{p_k^{1-\alpha}}{1-\alpha} + \frac{(I+d_k)^{1-\rho}}{1-\rho} \tag{5}$$

This functional form, combined with Roy's Identity, will give us the log-log demand form for tier k:

$$\log(g(p_k, I+d_k)) = \log(w_k) = \beta_1' X + \beta_2' Z - \alpha \log p_k + \rho \log(I+d_k) + c \tag{6}$$

where d_k is defined in Equation 2, α is the log price effect, ρ is the log virtual income effect, and c is the constant. The observed log consumption includes unobservables ε: $\log(w) = \log(w_k) + \varepsilon$. Following the same parametric assumption of Olmstead, Hanemann and Stavins (2007), the unobservables have two terms $\varepsilon = \eta + \nu$. $\eta \sim N(0, \sigma_\eta^2)$ represents the household-level heterogeneity observed by households (e.g., preferences), while $\nu \sim N(0, \sigma_\nu^2)$ represents the household-level perception/optimization error that is unobserved by households even ex-post. The idea is that it is nearly impossible for a household to precisely consume the water quantity (w_k) they would ideally demand given their preferences. For instance, there will always be extra cold water down the drain when she demands warm and hot water, for example. For all households, they are risk-neutral with $E[\nu] = 0 \ \forall k$. Both error terms are not observed by the econometrician, hence the parametric assumption. In addition, this parametric assumption can allow us to derive a closed-form likelihood function for MLE. Given the structure of the error term,

[5] A couple of relatively recent examples of using this functional form for water demand are Hewitt and Hanemann (1995), Olmstead, Hanemann and Stavins (2007), and Wang and Wolak (2022). Others like Szabo (2015) used a linear demand function.

the unconditional ex post water demand after the realization of the error term is:

(7)

$$\log(w) = \begin{cases} \log(w_1) + \eta + \nu & \text{if } \eta \leq \log(q_1) - \log(w_1) \\ \log(q_1) + \nu & \text{if } \log(q_1) - \log(w_1) < \eta \leq \log(q_1) - \log(w_2) \\ \log(w_2) + \eta + \nu & \text{if } \log(q_1) - \log(w_2) < \eta \leq \log(q_2) - \log(w_2) \\ \log(q_2) + \nu & \text{if } \log(q_2) - \log(w_2) < \eta \leq \log(q_2) - \log(w_3) \\ \dots & \\ \log(q_{K-1}) + \nu & \text{if } \log(q_{K-1}) - \log(w_{K-1}) < \eta \leq \log(q_{K-1}) - \log(w_K) \\ \log(w_K) + \eta + \nu & \text{if } \log(q_{K-1}) - \log(w_K) < \eta \end{cases}$$

and the likelihood function for the observed water demand w_i for household i (omitting i subscript below for brevity) given the parametric assumption of η, ν is:

(8)

$$f(w_i|X,Z) = \sum_{k=1}^{K} \left[\frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \phi(s_k)(\Phi(r_k) - \Phi(n_k)) + \frac{1}{\sigma_\nu} \phi(u_k)(\Phi(m_k) - \Phi(t_k)) \right]$$

where

$$t_k = (\log(q_k) - \log(w_k))/\sigma_\eta$$

$$r_k = (t_k\sigma_\eta - \rho_s s_k \sqrt{\sigma_\eta^2 + \sigma_\nu^2})/(\sigma_\eta \sqrt{1 - \rho_s^2})$$

$$\rho_s = \sigma_\eta / \sqrt{\sigma_\eta^2 + \sigma_\nu^2}$$

$$s_k = (\log(w_i) - \log(w_k))/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$$

$$n_k = (m_{k-1}\sigma_\eta - \rho_s s_k \sqrt{\sigma_\eta^2 + \sigma_\nu^2})/(\sigma_\eta \sqrt{1 - \rho_s^2})$$

$$m_k = (\log(q_k) - \log(w_{k+1}))/\sigma_\eta$$

$$u_k = (\log(w_i) - \log(q_k))/\sigma_\nu$$

where Φ is standard normal cdf and ϕ is standard normal pdf.

and $q_0 = 0, q_K = \infty, w_k = g(p_k, I + d_k)$ is defined implicitly via Equation 6

Empirically, this likelihood function is used to calculate the likelihood of the observed water consumption w_i given the model parameters $(\alpha, \rho, \beta_1', \beta_2', c, \sigma_\eta, \sigma_\nu)$ and data $(X_i, Z_i, p_k, q_k, A_k, I_i)$.[6] The derivation of the likelihood function is explained in a 2 tier case in Section A.A2 without loss of generality.

[6]In the actual empirical model, we increase the flexibility by adding interaction term of price effect and income effect, therefore α is a function depends on parameter β_3', β_4', c_α and data X_α, Z_α, and ρ is a function depends on parameter β_5', c_ρ and data X_ρ, Z_ρ. The details will be discussed in Section III.B.

B. Empirical Model and Data

In this subsection, we outline the empirical model, based on the demand function (Equation 6) and the likelihood function (Equation 8), and describe the data used for maximum likelihood estimation.

The core dataset consists of panel data detailing monthly water transactions for single-family households in Austin, TX, from 2018-2019. These data include payments, total water usage, the billing date for each month, and the household's longitude and latitude. The data were provided by Austin Water, the public utility monopoly responsible for water services in Austin.

To operationalize the demand function, we incorporate supplemental datasets. These include two variables in the household characteristics matrix (X): the number of bathrooms per household and the time-variant household Normalized Difference Vegetation Index (NDVI). The number of bathrooms serves as a proxy for indoor water usage, while NDVI accounts for outdoor water usage, which is typically larger in volume and more sensitive to weather variations. The application of NDVI to water demand analysis was introduced by Wang and Wolak (2022). It aims to approximate lawn watering habits, as higher NDVI values indicate healthier vegetation, suggesting greater watering efforts by the household. Details on constructing and interpreting NDVI can be found in Section A.A2. For the time-variant weather matrix (Z), we include the monthly average maximum daily temperature, the monthly interquartile range (IQR) of maximum daily temperature, total monthly precipitation, and the monthly IQR of total precipitation.[7]

To enhance model flexibility, we allow the log price effect (α) and the log virtual income effect (ρ) to depend on household characteristics and weather conditions, thereby generating interaction terms. Specifically, α depends on the number of bedrooms, household NDVI, monthly average maximum daily temperature, and total monthly precipitation. ρ depends on the number of bedrooms, NDVI, and the number of heavy water-use appliances (including pools, hot tubs, sprinkler systems, fountains, and car washes).[8]

All other household characteristics, except for NDVI, were collected from public data provided by the Travis County Appraisal District (TCAD). We use data from 2018 to match the time frame of the transaction data. The TCAD data also include addresses, which were matched via geo-spatial analysis with the longitude and latitude data from Austin Water. We also collected lot size data from TCAD. Weather data were collected from the National Oceanic and Atmospheric Administration (NOAA), using data from approximately 120 weather stations in Austin. Since NDVI and weather data are collected on a calendar month basis, while household water billing cycles do not typically align with calendar months, we prorate both the weather and NDVI data to match each household's billing cycle.[9] Household income data were estimated using zipcode-level

[7]Note that both IQRs account for the spread of the data within each month, which differs slightly from what we mean by growing weather variance, namely the increasing variance of weather patterns within a year.

[8]In practice, we used $\alpha = \exp(\beta_3' X_\alpha + \beta_4' Z_\alpha + c_\alpha)$, and $\rho = \beta_5' X_\rho + c_\rho$ to maximize the bias-variance trade-off of the estimation results. Regardless of the functional form, the expansion of both α and ρ is to provide flexibility of identifying heterogeneity of price effect and virtual income effect of the households, through the interaction terms.

[9]This practice was first introduced by Train et al. (1984).

average homeowner income data from the IRS combined with normalized household value data from TCAD, under the assumption that the normalized variance of household values within each zipcode mirrors the variance of household income.

After matching all datasets and eliminating outliers, the final sample consists of $127,323$ households with $2,345,742$ transaction records. Summary statistics can be found in Section A.A2.

C. Estimation Result and Identification

After conducting the maximum likelihood estimation using Equation 8 with $k = 5$ and the data discussed in Section III.B, the final estimation results can be found in Table A5.

With the panel data, both time-variant and time-invariant variables provide significant heterogeneity among households and across different months. This helps identify the parameters associated with household characteristics, income, and their interaction terms. Moreover, for weather variables, instead of using weather data for the entire city, we found the closest weather station for each household to introduce small variations, providing heterogeneity to help identify the weather parameters. Nevertheless, identifying the price effect and its interaction terms presents a challenge as there is little or no price variation (neither through time nor geography) in the data. Another concern highlighted in past literature (e.g., Borenstein (2009); Ito (2014)) is that households may not react to the observed marginal price, as prices and the total payment amount are typically observed only after the billing cycle ends; instead, they might respond to the average price they face.

Fortunately, both of these identification concerns regarding price parameters in the DCC model have been addressed by Olmstead (2009). She demonstrated that each household, for each month, is optimizing over the entire price schedule. Consequently, the econometrician can recover the parameter estimates, along with the probabilities that households consume on each of their budget segments and at each kink point, directly from the DCC model results. In addition, the DCC model is estimated without ever determining the 'observed' marginal price of consumption – all the prices and the kink points enter the likelihood function, regardless of where consumption is actually observed. Olmstead (2009) also pointed out that price elasticity estimation has smaller bias when using the DCC model if variation in demand is driven primarily by household preferences (η) instead of the perception error (ν), and our estimation results align with this requirement, showing that σ_η is much larger than σ_ν. Furthermore, the model generates less bias if the existing price jumps between different tiers are sufficiently salient. Although there is not definitely conclusion on how salient the price jump is needed to minimize the estimation bias, the marginal price differences between tiers in Austin are among the largest in the US[10].

To further address the price elasticity identification challenge, we also utilize data from a small category of consumers enrolled in the Consumer Assistance Program (CAP).

[10]See `https://bseacd.org/conservation-based-rate-structures/` for "Conservation-Based Rate Structures" - Barton Springs/Edwards Aquifer Conservation District

Based on certain income requirements, these consumers enjoy lower marginal prices (see Section A1). Although the variation is small, these consumers provide a small degree of price variation to aid in the identification of price elasticity.

Another potential endogeneity issue involves the use of NDVI, as the NDVI of a given calendar month could be the result of water usage in that same period. Therefore, we used a lagged term for NDVI, meaning that water demand this month is dependent on the previous month's NDVI. The idea is that households decide how much water to consume this month based on the health of their lawn vegetation last month. A higher previous month's NDVI may indicate the household takes more care of its lawn, helping to account for variations in outdoor water usage.

Based on the identification strategy discussed above, we now discuss the price and income elasticity derived from the DCC model. Due to the nonlinear pricing structure, we cannot directly use α and ρ to calculate price and income elasticities. Instead, by following Olmstead, Hanemann and Stavins (2007), we use a simulation-based approach. For a small number ξ, we calculate how expected demand changes in response to a small change in the status quo marginal price (p_0), holding income (I) and other factors constant.[11] The formula for price elasticity is as following, where the same sturcture applies to income elasticity:

$$\text{Price Elasticity: } (E[g((1+\xi)p_0,I)] - E[g(p_0,I)])/(\xi E[g(p_0,I)]) \tag{9}$$

The median price elasticity is estimated to be -0.395. Compared to previous literature that uses the structural DCC model for Increasing Block Pricing (IBP) in developed countries, our estimate is within the range of Pint (1999)'s estimations ($-1.24 \sim -0.04$) and more strictly, Olmstead, Hanemann and Stavins (2007)'s estimations ($-0.59 \sim -0.33$). Our estimate is more inelastic compared to Hewitt and Hanemann (1995)'s ($-1.63 \sim -1.57$). This could potentially be due to their data, which includes a limited number of households in the summer months in Denton, TX (where the weather seems less extreme based on their data). Our estimate is also close to that of Olmstead (2009) (reporting -0.609 for a specific DCC model specification, using data from 11 cities with diverse weather patterns) and Strand and Walker (2005) ($-0.3 \sim -0.1$, using data from 17 cities in Central America, where water demand was primarily for indoor usage). The median income elasticity is estimated to be 0.112, which is close to estimations from previous literature for developed countries, such as Olmstead, Hanemann and Stavins (2007) ($0.1786 \sim 0.1865$) and Olmstead (2009) (0.1865).

To further explore how changes in precipitation affect consumer behavior, we plot the price elasticities for all 5 income strata by precipitation.

Across different strata, we could see more elastic households in general correlated

[11] Additionally, due to the flexibility of the price elasticity function, we limit the sample for empirical analysis to the year 2019, as opposed to from demand estimation, we used data from Jun.2018-Dec.2019. This approach helps avoid overestimating the humid summer months of 2018.

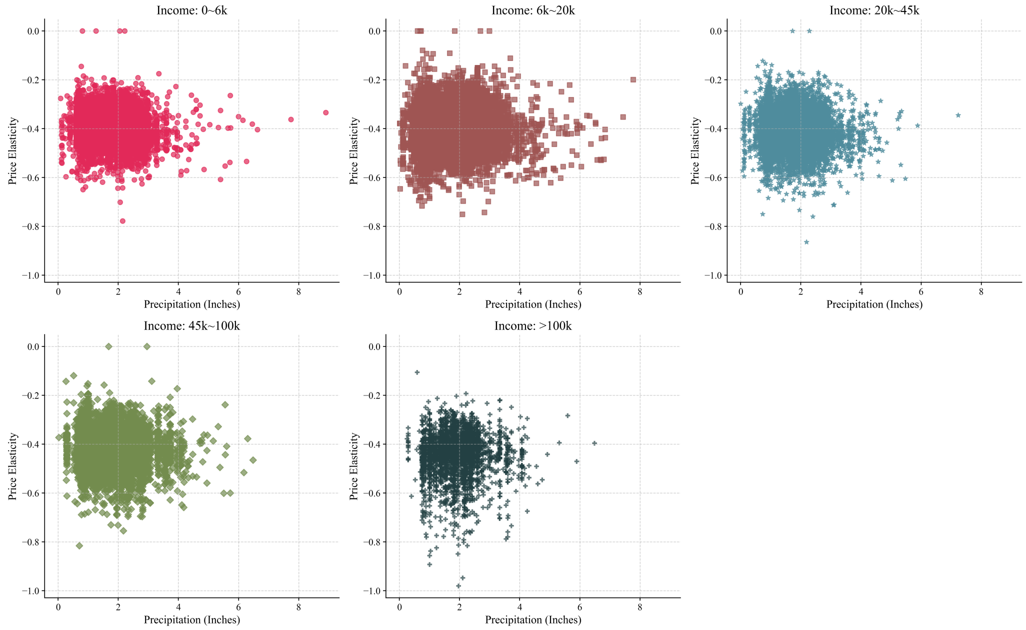

Figure 3. : Price Elasticity and Precipitation by Income Strata

with lower precipitation, and most of them are in the highest stratum.[12] This fits the reduce-form evidence from Figure 1, as higher income strata tends to be more sensitive towards precipitation decreases. Furthermore, we can see that regardless of how precipitation changes, lower income strata (especially monthly income of $6k \sim 20k$) have more inelastic demand.

The structural estimation of price elasticities paints a better picture for the price-setting utility, as opposed to common belief, it is actually higher income households having more elastic demand, where some of the lower income households have more inelastic demand. This is potentially due to the fact that higher income strata is more likely to decrease their quantities to lower tier and this behaviors in the data is being captured by the structural model as part of price elasticities.[13] This adds extra challenges to the equity goal of the optimal price design from the utility as the marginal price differences in IBP are supposed to target different income strata. If the higher income strata are more elastic, with changes in precipitation, the utility runs more revenue risks as higher income households (which are also higher paying) will be sensitive to switching to lower tiers.

This fits the pattern described by Ruijs (2009), Echeverri (2023) and Wichman (2024)

[12]In the picture, it is evident that there are household-level heterogeneity in precipitation. This is because we gather the data from various weather stations in Austin and calculate the household level weather data by weighted average of the nearest weather stations.

[13]As explained by Olmstead (2009) by the identification of the structural DCC model.

as they pointed out the IBP design in water utility was intrinsically better off for higher income strata. We went a step further to conclude that one potential reason why IBP is more favorable towards higher income households as they have more "room" to switching to lower tier and captured by the structural DCC model. While for lower strata as they will stick to consuming at the lowest tier, they will have less choices and entirely depends on the marginal price of the lowest tier. This is further discussed in Section A.A2 with some caveat discussions.

IV. Ramsey Pricing Model

In this section, we establish the theoretical basis for the Ramsey pricing problem (from Ramsey (1927)) faced by a natural monopoly water utility company. We then introduce weather stochasticity into the optimal price determination, considering revenue risk aversion. In addition, we introduce the empirical model used to determine the optimal price under conditions of shifting precipitation variance.

As previously established, Ramsey proposed the "second-best" solution to address a company's financial requirements while minimizing the reduction in overall social welfare, employing the inverse elasticity rule. Water utility companies, particularly in urban metropolitan areas with high fixed costs, often adopt the Ramsey format of IBP within a multi-part tariff structure to meet financial requirements and policy goals of conservation and equity. For simplicity, when discussing the theoretical basis for the Ramsey problem, we treat the entire pricing structure as a single price p, as opposed to the pricing vector of IBP in a multi-part tariff. In the empirical model, we reintroduce the multi-part IBP to derive more realistic solutions.

The theoretical Ramsey model, introduced by Ramsey (1927), is extended to include additional policy constraints in the generalized Ramsey model, as discussed in literature such as Coady and Drèze (2002). For the empirical model, we largely follow the setup of the conservation constraint for the Ramsey problem as presented in Wang and Wolak (2022). To empirically measure consumer welfare, we adopt the framework and definition of equivalence variation for nonlinear budget constraints from Hausman (1981), Reiss and White (2005), and Ruijs (2009).

A. Ramsey Pricing Model with Conservation Constraint

In this subsection, we briefly showcase the changes from the classic Ramsey model by using a simplified pricing model with only the price p. We assume the water utility chooses price p to maximize consumer welfare, subject to two constraints: 1) annual revenue ($R(p)$) is greater than or equal to an exogenous total annual cost (C) that is independent of demand (simplifying for large fixed costs),[14] and 2) total annual quantity

[14]In reality, the total cost of maintaining urban water is not related to demand fluctuation; regardless of the amount of water consumed, maintenance and transportation fees do not change, making this a reasonable assumption.

$(q(p))$ is less than or equal to an exogenous annual quantity upper bound ($\bar{Q}$).

$$\begin{aligned} \max_{p} \quad & CS(p) \\ \text{s.t. } & R(p) - C \geq 0 \\ \text{and } & q(p) \leq \bar{Q} \end{aligned} \tag{10}$$

Solving this Lagrangian yields the following result:

$$\begin{aligned} \underbrace{\frac{p-C}{p}}_{\text{Markup}} &= \underbrace{\frac{\lambda-1}{\lambda}\frac{1}{\varepsilon}}_{\text{Ramsey Rule}} + \underbrace{\frac{\mu}{\lambda p}}_{\text{Conservation Penalty}} \\ p^* &= \frac{C+\mu/\lambda}{1-\frac{\lambda-1}{\lambda}\frac{1}{\varepsilon}} \end{aligned} \tag{11}$$

The classic Ramsey rule states that the markup is proportional to the inverse of price elasticity (ε), but the optimal markup here includes an additional term that serves as a penalty for the conservation constraint. The optimal price depends on λ (the Lagrangian multiplier for the revenue constraint), μ (the Lagrangian multiplier for the conservation constraint), the exogenous cost C, and the price elasticity ε. If μ increases, the shadow price of the conservation constraint increases, and the utility has a greater incentive to raise the markup. If $\mu = 0$, the conservation constraint is not binding, and the standard Ramsey rule is recovered. If C increases, the price will need to increase. If ε increases, demand becomes more elastic, and the price will need to decrease, which is consistent with the standard Ramsey rule. If we denote $k = \frac{\lambda-1}{\lambda}$, the price can be expressed as $p = \frac{C+\mu(1-k)}{1-\frac{k}{\varepsilon}} = \frac{C+\mu}{1-\frac{k}{\varepsilon}} - \frac{1}{k-\frac{1}{\varepsilon}}$. If λ increases, k (which is less than 1) will also increase. The first term increases. The second term, being minus a negative number since $k - \frac{1}{\varepsilon} < 0$, also increases. This means that when λ increases, the price will increase.

B. Ramsey Pricing Model with Stochasticity

So far, we have presented the model in a deterministic setting with no stochasticity induced by weather. In this subsection, we consider a yearly pricing decision where the price is pre-set and fixed throughout the year, and a weather variable Z for the next year is unpredictable and stochastic.[15] The utility must consider maximizing the total yearly consumer welfare ($\sum CS_m$), where the subscript m represents the month. As showcased by Figure 3, the level of the weather Z_m matters for elasticity which is crucial to measure the effects of counterfactual pricing. The fact that many high income households have more elastic demand makes it even more important to consider revenue risks faced by the utility as they can generate bigger revenue swings.

[15]The time period can be extended to be longer than a year, which is typically what utilities do. Arguably, the longer the time period with a fixed price, the harder it is to satisfy all the constraints. Therefore, we choose one year as a representative period for the utility to revisit and change its price without loss of generality.

In this process, we assume the utility has a belief/prediction about Z_m for next year. Depending on their confidence in their prediction, they can choose between a risk-neutral and a risk-averse revenue constraint. Generally, utilities typically have some qualitative predictions of weather trends for the next year, e.g., a larger-than-average precipitation variance, but a precise prediction of precipitation for each month is rarely achievable a year in advance.[16] The utility will set up both the revenue constraint and the conservation constraint similar to the deterministic case. However, depending on their risk aversion, which is influenced by the confidence in their predictions of Z_m, they will consider adding concavity to the revenue constraint to curb the spread of $R_m(Z_m)$, and reducing revenue risks. A concave constraint on the distribution of $R_m(Z_m)$ means the utility prefers the distribution of R_m for a random realization of Z_m for the next year to have a lower variance. To simplify the price optimization without deviating too much from the status quo, we will assume the utility only impose concavity in the revenue constraint.

When the utility is confident in its prediction of Z_m, the expected monthly revenue $E[R_m(p,Z_m)]$ is enough to measure financial viability. In this case, as long as the average monthly revenue is higher than the exogenous monthly average fixed cost, the utility will at least break even annually. Combining this with the conservation constraint, we set up the Ramsey problem for the risk-neutral utility company:

$$
\begin{aligned}
\max_{p} \quad & E[CS_m(p,Z_m)] \\
\text{s.t. } & E[R_m(p,Z_m)] - C/12 \geq 0 \\
\text{and } & E[q_m(p,Z_m)] \leq \bar{Q}/12
\end{aligned}
\tag{12}
$$

Assuming the exogenous total annual cost and quantity upper bound are constant with respect to Z_m, the utility essentially ensures that 1) the expected monthly revenue resulting from this price is not too small, and 2) the average monthly quantity resulting from this price is not too large. This provides the utility with some revenue and quantity flexibility. For wet months, the utility might incur losses, while for dry months, the conservation constraint might not be met.

When the utility is not confident in its prediction of Z_m for the next year, it may face increasing revenue risks. When precipitation pattern changes, some months could have revenues dropping below the thresholds, partially caused by the high sensitivity of high income households. If the weather variance is very high, consecutive months of revenue losses would not be desirable. In this case, the utility should consider setting up a concave revenue constraint to curb the spread of the realization of the monthly revenue ($R_m(Z_m)$) so that it's not too far away from the threshold. By introducing a concave and increasing function of revenue, $f(R)$, into the constraint, the higher the concavity of the function f, the more risk-averse the utility, implying a more prominent preference for lower variance in $R_m(Z_m)$. It is worth noting that the concave revenue constraint is a more strict con-

[16]Predictability comes more from slowly changing boundary conditions and large-scale climate phenomena such as ENSO (El Niño/La Niña). Current technology is good at predicting the state of ENSO or the likely average conditions over a whole season (e.g., winter will likely be warmer than average), but it doesn't provide the precision needed for the average conditions of a specific month a year in advance.

straint compared to the linear one by Jensen's Inequality (see Section A.A3). Solving the same pricing problem with the new revenue constraint $E_m[f(R_m(p,Z_m))] - f(C/12) \geq 0$, we obtain:

$$
\begin{aligned}
\frac{pE[f'(R(p,Z_m))] - \frac{1}{12}C}{p} &= \frac{\lambda E[f'(R(p,Z_m))] - 1}{\lambda}\frac{1}{\bar{\varepsilon}} + \frac{\mu}{\lambda p} \\
p &= \frac{C + \mu/\lambda}{E[f'(R(p,Z_m))] - \frac{1}{\bar{\varepsilon}}(E[f'(R(p,Z_m))] - \frac{1}{\lambda})}
\end{aligned}
\tag{13}
$$

where $\bar{\varepsilon}$ is the expected price elasticity unconditional on Z_m. $\bar{\varepsilon} = \frac{\partial E[q(Z_m)|p]}{\partial p}\frac{p}{E[q(Z_m)|p]}$, and $E[q(Z_m)|p] = \int E[q|p, Z_m = z]g_Z(z)dz$, where $g_Z(z)$ is the pdf of Z. Due to the additional term and the functional form of demand, there is no closed-form solution for the optimal price even without the IBP. Therefore, we turn to an empirical method to estimate the optimal IBP for different counterfactuals of Z_m and measure its welfare and distributional effects.

It is worth noting that we keep the quantity constraint and the concavity of welfare unchanged, even though both will fluctuate with Z_m. Typically, for conservation goals, the utility only needs to ensure their annual aggregate goals are met.[17] In addition, considering purely the monthly fluctuations for the utility company, the more important constraint is the revenue constraint, as the core issue of revenue risks caused by Z_m is that higher income household (who contributes the most in R_m) is more elastic, and a consecutive low draws of R_m can significantly impact annual revenue viability. The concavity of the revenue constraint ensures that the revenue collected in wet months (with low quantity) and dry months (with high quantity) do not differ that much. Since payment from each transaction r_{mh} is highly correlated to quantity q_{mh} and to some degree cs_{mh}, lower variance in monthly revenue should also lead to lower variance in CS_m and q_m, making the concavity of the revenue constraint a more direct way of addressing revenue risks without posing too many additional restrictions.

As for not adding concavity to the CS term to account for consumer-level month-to-month risk aversion, there are several reasons: 1) With regard to weather stochasticity, it is hard to imagine urban single-home residential households making specific weather pattern predictions and planning ahead, 2) Water utility payments remain a relatively smaller portion of monthly total income, especially for single-home residential households in developed countries, and 3) it is justifiable that many households would prefer smooth month-to-month payments for their utility bills.[18]. In this paper, we simply lack the data to endogenize this preference for smooth monthly payment for utility bills.

[17] Empirically, a risk aversion on conservation goals requires an *additional* constraint on top of the existing conservation constraint due to Jensen's Inequality of the concave function. This means the satisfaction of the concave conservation constraint does not automatically guarantee the satisfaction of the linear conservation constraint (actually the reverse is true).

[18] In many U.S. cities, utilities offer 'Budget Billing,' where a predetermined amount is charged monthly regardless of consumption, and the utility periodically reviews the billing annually to ensure financial viability. Consumers may prefer this type of billing for easy budgeting. Austin Water also offers this type of billing, but in this study, budget billing is largely ignored at the household level due to the lack of information on which consumers utilize it.

C. Empirical Model

In this subsection, we develop the empirical model used to estimate the optimal price derived from Equation 12. The empirical model's goal is to address the within-year revenue risk aversion of the price setter when solving a Ramsey pricing model, and the risk considered is primarily from weather stochasticity.[19] The optimal price ($\mathbf{p}$) consists of three vectors: marginal prices ($\{p_k\}_{k=1}^5$), kink points ($\{q_k\}_{k=1}^4$), and fixed payments ($\{A_k\}_{k=1}^5$). To maintain Austin Water's current pricing structure of 5 tiers with 4 kink points, $\mathbf{p}$ has a total of 14 parameters and we assume the pricing structure does not change[20]. The optimal price, which depends on weather Z, maximizes the annual total equivalence variation (EV) weighted by household income I to boost the distributional effect.[21] In order to account for weather-level stochasticity, ideally we would need to utilize the real distributions of weather to derive E_Z. However, it is not consistent to either use data of very recent years (not enough data point to empirically derive the distribution), or use long term data (climate change affects long term data trend). We ended up adding a Monte-Carlo disturbance to simulate weather prediction error to simulate a distribution of Z. Further details of simulating the distribution of Z could be found in Section A.A3. Aside from stochasticity from Z, the demand estimation using MLE (Equation 8) also introduces household-month-level demand stochasticity from ε_{hm}, as the econometrician observes neither η_{hm} nor ν_{hm}. We use Monte-Carlo simulations for both to generate their distributions.[22] Household-level stochasticity, though important, is not the main focus of this paper, hence will be omitted in the written version of the model for simplicity. All values with the subscript m is accounted for all transactions h happen in the month m.

(14)

$$\mathbf{p}^*(Z) = \underset{(p,q,A)}{\arg\max} \quad E_Z\left[\sum_m \Big[EV_m(Z_m;\mathbf{p},\mathbf{p}_0,I)/I\Big] - \lambda \cdot \max(0, C - f^{-1}\overline{f(R_m(Z_m;\mathbf{p}))} \cdot 12)\right]$$

$$\text{s.t.} \quad E_Z\left[P\left(\sum_m q_m(Z_m;\mathbf{p}) \leq \bar{Q})\right)\right] \geq 0.95$$

Noted that for two exogenous thresholds: $C = \sum_m R_m(Z_0;\mathbf{p}_0)$ and $\bar{Q} = \sum_m q_m(Z_0;\mathbf{p}_0$. Both Z_0 and $\mathbf{p}_0$ represent the status quo weather and price. This means we compare the counterfactual revenue and quantity to their status quo counterparts. In particular, due to the lack of detailed information on costs, we use the status quo revenue as the benchmark to evaluate financial viability. Utilities usually ensure their revenue is just enough to cover the annual cost to avoid excessive welfare distortion, which makes this a

[19]The model is partially based on Wang and Wolak (2022).

[20]This includes the assumption that each marginal prices and fixed payment will be non-decreasing in tiers.

[21]As pointed out by Feldstein (1972), in order for the public pricing to account for distributional equity, the price setting should set the welfare weight according to the social marginal utility of income. In addition, a logarithmic social utility of inequality and $\varepsilon = 1$ for the Atkinson Index (Atkinson (1970)) result in a weighting proportional to 1/I.

[22]We will generate η_h on the household level to avoid overfitting.

valid assumption. We further discuss the validity of this assumption in our case in Section A.A3. For the conservation constraint, we introduce a chance-constrained programming to provide more flexibility in price setting. It essentially requires that the probability of the counterfactual annual total quantity for all households being smaller than the status quo annual total quantity is greater than 0.95. The structures of both constraints are adopted from Wang and Wolak (2022). Unlike typically Ramsey pricing setting, we set up the revenue constraint as a revenue loss term $-\lambda \cdot \max(0, \text{loss})$, such that when the revenue requirement is not met, it incurs a cost to the entire economy, but does not necessarily award any positive value when the requirement is met. The parameter λ controls the weight between the monetary value of welfare, measured by the weighted equivalent variation, and the monetary value of the loss of revenue preference for the utilities. Mathematically, it is similar to the Ramsey model, but empirically this provides more flexibilities as instead of a hard constraint, the utilities is allowed to have some annual losses, albeit incur a negative welfare.

The added increasing and concave function f is used to curb the variance of monthly revenue ($R_m(Z_m)$) so that the utility faces less likelihood of multiple extreme months of revenues within a year. This potential loss will be evaluated under a simulated distribution of Z to generate the expected value of revenue risks while setting the price. Hence, when the utility has low confidence in Z_m, they should set up a risk-adjusted 'preference' for monthly revenue ($f(R_m)$) to be larger than some threshold. If the utility takes the average 'preference' of monthly revenue $\overline{f(R_m)}$, the f^{-1} part[23] simply translates the 'utils' of the utility's preference for risk-adjusted monthly revenue into dollars, and multiplying by 12 scales up the result into the annual, economy-wide monetary value of this risk-adjusted preference on the revenue. The higher the variance of R_m and the higher the concavity of the function f, the lower the 'preference' of risk-adjusted monthly revenue, and the harder it is to satisfy the constraint. We can observe that if $f(x) = f^{-1}(x) = x$ is a linear function, the constraint simply becomes the linear constraint: $C - \overline{R_m} \cdot 12$. This added concavity is not against the typical assumption of $R \geq C$ and in fact a stricter constraint (with proof provided in Section A.A3). Empirically, we choose Constant Relative Risk Aversion (CRRA) function $f(x) = \begin{cases} x^{1-\gamma}/(1-\gamma) & \gamma > 0, \gamma \neq 1 \\ \log(x) & \gamma = 1 \end{cases}$ as the concave function.[24] Here γ represents the level of risk aversion of the utility; the higher the γ, the more the utility dislikes a larger spread of R_{mh} and the more they seek to improve the distributional effects.

The equivalence variation ($EV_{mh}(Z_m; \mathbf{p}, \mathbf{p}_0, I)$) for water with a nonlinear budget constraint is calculated by first obtaining the expenditure function from the indirect utility function and then calculating the welfare effects from the price change. The framework is developed by Hausman (1981) and Reiss and White (2005), and the formal definition is from Ruijs (2009).

[23] f^{-1} exists because f is an increasing function.

[24] We have also tried 1) simple concave lose function $-\lambda \cdot \sum_m [\max(0, C/12 - R_m)^2]$ to adjust against far away R_m, and 2) Constant Absolute Risk Aversion (CARA). Both concave function generates worse welfare results and 1) in particular took longer to converge.

Based on the indirect utility function $V(\mathbf{p}, I+d_k)$ from Equation 3, the expenditure function $e(\mathbf{p}, u)$ for a household choosing tier k is:

$$e(p_k, u) = \left[(1-\rho)\left(u + \exp(\beta_1' X + \beta_2' Z + c + \varepsilon)\frac{p_k^{1-\alpha}}{1-\alpha}\right)\right]^{\frac{1}{1-\rho}} \tag{15}$$

If a household's counterfactual demand falls within tier k, rather than at the kink point q_k, their equivalent variation is:

$$EV(p_0, p, I) = e(p, V'(p)) - d_k^0 - I \tag{16}$$

where $V'(p)$ is the new counterfactual utility generated by the new price p, and d_k^0 is the virtual income correction term from the original price scheme, i.e., $d_k = -A_k - \sum_{j=1}^{k-1}(p_j - p_{j+1})q_j$. The equivalent variations are adjusted for the amount of subsidies received due to the nonlinearities in the budget set. Note that the expenditure function ($e(p_k, u)$) is used to solve for the virtual income from the indirect utility function, instead of I, hence there is no correction term d_k here. If the price does not change, $EV(p_0, p_0, I) = 0$ and $e(V_0, p_0) = I + d_k^0$.

On the other hand, when the predicted consumption is at the kink point q_k, the above definition does not apply as these consumers are not technically facing the new marginal price from their chosen tier.[25] The equivalent variation is generated from $\bar{p}$, where $\bar{p}$ is the price at which the demand function (from Equation 6) generates the result q_k. If we denote $\mathcal{A} = \beta_1 X + \beta_2 Z + c + \varepsilon$, and $\bar{V}I$ denotes the corresponding virtual income, the idea is to solve for $(\bar{p}, \bar{V}I)$ as a substitution effect where $(\bar{p}, \bar{V}I)$ generates the same utility. This means: $\begin{cases} V(p_k, I + d_k^0) & = V(\bar{p}, \bar{V}I) \\ \log(q_k) & = \mathcal{A} - \alpha\log(\bar{p}) + \rho\log(\bar{V}I) \end{cases}$. We can solve for $\bar{p}$:

$$\log(q_k) = \mathcal{A} - \alpha\log(\bar{p}) + \frac{\rho}{1-\rho}\log[\exp(\mathcal{A})\frac{1-\rho}{1-\alpha}(\bar{p}^{1-\alpha} - p_k^{1-\alpha}) + (I + d_k^0)^{1-\rho}]$$

s For this demand function and indirect utility function, we need to solve this nonlinear function numerically to obtain $\bar{p}$. Then, the equivalent variation for predicted demand at the kink point q_k is:

$$EV(p_0, p, I) = e(\bar{p}, V'(p)) - (\bar{p} - p_0)q_k^0 - d_k^0 \ - I \tag{17}$$

V. Counterfactual Analysis

In this section, we analyze the welfare effects of counterfactual optimal prices, calculated using Equation 14, under various counterfactual precipitation patterns (while assuming all other weather patterns unchanged). As the utility faces uncertainty in predicting future weather (Z_m) when setting prices, we derive these optimal prices (and their

[25] To see more details of the additional case, see Section A.A2.

associated welfare) for different precipitation scenarios.

We begin with the status quo monthly precipitation (Z_m^0) and generate two types of counterfactual precipitation patterns: 1) **Mean Shift**: $Z_m' = Z_m^0 \pm \zeta_1$, where $\zeta_1 \in [-0.25, 0.25]$. The generated Z_m' will have the same variance as Z_m^0 but different means, and 2) **Variance Shift**: $Sd(Z_m') = Sd(Z_m^0) \cdot \zeta_2$, where $\zeta_2 \in [0.75, 1.25]$. The generated Z_m' will have the same median as Z_m^0 but different variances. The latter pattern is achieved by scaling the standard deviation from Z_m^0 nonparametrically while maintaining the same median, and then clipping the result at 0. [26] As discussed in Section II.C, we focus solely on changes in precipitation, assuming temperature remains at its monthly status quo (as well as the IQR of temperature and IQR of precipitation, which measure within-month variability, do not change). Throughout the counterfactual analysis, we also assume household characteristics and income remain constant, with the exception of NDVI. Counterfactual NDVI is accounted for through a reduced-form analysis, as discussed in Section A.A4. Utilizing the counterfactual weather and NDVI, alongside constant household characteristics and income, we employ the derived empirical model from Equation 14 to calculate the optimal price. Details on the optimization procedure can be found in Section A.A4.

After calculating the optimal prices, we quantify the associated welfare by comparing the new and status quo values for: consumer surplus ($CS\%$, based on total indirect utilities), revenue ($R\%$, based on total revenues), quantity ($Q\%$, based on total quantities), and equivalent variation to income (EV/I, derived from the new prices). These measures are calculated for all five income strata.

A. *Welfare Results - Mean Shift (ζ_1)*

In this section, we will first present the result of shifting the mean of the precipitation by adding $\zeta_1 \in [-0.25, 0.25]$ to the status quo weather. This means the utility will expect the average precipitation next year to change by ζ_1 and the variance will not change. We implement the optimal pricing with $\gamma = 0.3$ for the CRRA function and $\lambda = 0.5$ for the loss function. We then compare this to a linear revenue constraint with the same λ for the loss function. The resulting menu of the optimal prices solved from both scenarios can be found in Section A.A4. Based on their predictions of ζ_1 and their confidence level, they can choose a specific optimal IBP from the menu.

When analyzing **shifting means (ζ_1)**, Figure 4 showcases the general welfare results for precipitation expected to be, on average, drier ($\zeta_1 \leq 0$) and more humid ($\zeta_1 \geq 0$), along with comparisons between linear and concave revenue constraints. Under a linear constraint, when $\zeta_1 \leq 0$, the quantity constraint becomes binding while the revenue constraint becomes slack. Even with price increases only for higher tiers (see Figure A7) and accounting for EV/I in the optimization objective, average lower-income households generally exhibit a larger welfare loss (in dollar value relative to income) compared to income strata above \$20k, which show minimal EV/I loss. Conversely, when $\zeta_1 \geq 0$ (i.e., when predicted weather is more humid), the revenue constraint is more binding.

[26] Although it looks like precipitation is more likely to follow a right-skewed log-normal distribution because it never falls below zero, a parametric assumption will make scaling by the factor ζ_2 imprecise. As stated by meteorologists Heredia et al. (2018), it is more realistic to treat it nonparametrically when generating synthetic high-variance data.

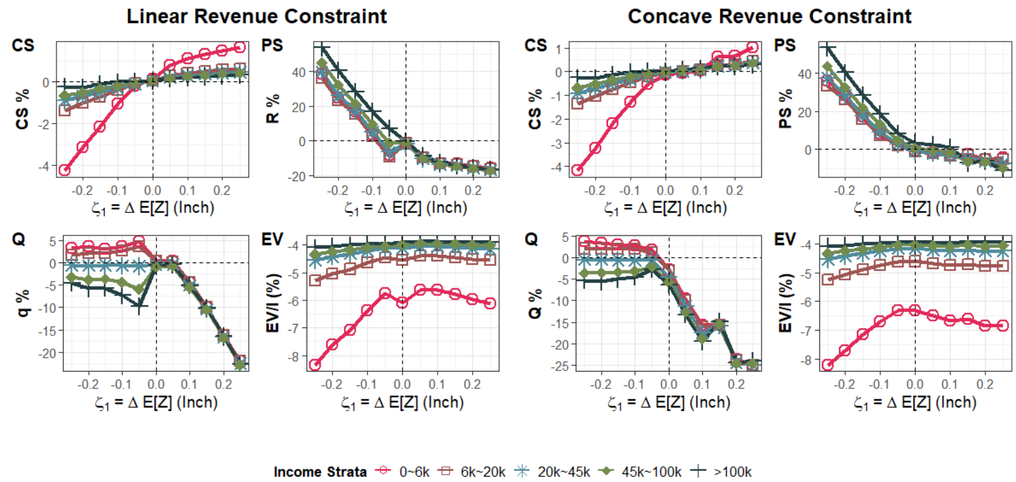

Figure 4. : Welfare from the Optimal Price in Shifting Mean ($\zeta_1 \in [-0.25, 0.25]$)

However, with the setup of the loss function, the average revenue is below the status quo threshold, and the quantity constraint is slack. Even though CS graph shows an increase in CS for the lowest stratum (possibly due to the fact that the baseline CS for the lowest stratum is not high), the EV graph still indicates greater welfare losses for average lower-income households.

There are a few reasons why the optimal prices, even with the added weight in EV/I, still hit lower-income households harder: 1) Even though water utility is not generally expensive, it will naturally represent a higher proportion of income for lower-income households, and any price change will incur a larger shift towards EV/I[27]. 2) By the inverse elasticity rule from the Ramsey problem, the price should decrease for high-elasticity consumers and vice versa. This does not help with distributional effects, as many high-elasticity households are in the highest income stratum and consume high quantities.[28]. All of these factors place boundaries on adjusting the marginal price of higher tiers. 3) There is a non-negligible number of lower-income households that are high-quantity consumers. [29]. These households are often in a much higher tier than their income might suggest, and they do not particularly have high elasticities. [30]. This compromises the premise that a higher marginal price in a higher tier would help with distributional effects and the equity goal. To further see evidence of the welfare effect

[27]Similar patterns have been pointed out b Ruijs, Zimmermann and van den Berg (2008) by using data in Sao Paolo, Brazil.

[28]See Figures 3 and A5

[29]They are usually consuming more than 20k Gallons, which is the status quo kink point between the 4th and 5th tier.

[30]See Figure A5

for high-quantity consumers in the lowest stratum, see Section A.A4. All these effects actively work against the premise that IBP could improve distributional effects, and this effect is worsened under shifting precipitation patterns. This generally fits the literature, as price changes of IBP in general favor higher-income households. Policymakers should consider implementing additional compensation for the lowest income stratum to improve distributional effects. [31]

This imbalance of welfare impacts does not improve under a concave constraint; in fact, it worsens. This is because the concave revenue constraint is *stricter*, leading to generally higher revenues and lower quantities (as showcased by the figure). Unlike linear constraints, where the revenue constraint would be well below the status quo when $\zeta_1 \geq 0$, revenues are higher and in general binding with the threshold. For the quantity constraint, the quantity decrease when $\zeta_1 \geq 0$ is larger compared to the linear case due to higher prices, but still in general binding when $\zeta_1 \leq 0$. Regarding welfare, there are very small differences between linear and concave revenue bound when $\zeta_1 \leq 0$, as in both cases the quantity constraint is dominant, which is the same for both cases. When $\zeta_1 \geq 0$, the lower-income stratum unsurprisingly has lower welfare for the concave revenue constraint, while other strata are less impacted, with the highest income strata experiencing almost no adverse effects (more evident in EV graphs). However, the purpose of the concave constraint is to ensure that for every simulated year, the overall $R_m(Z_m)$ every month does not deviate too far from the threshold, so that when the utility is unsure of their predictions of ζ_1, the welfare will be relatively stable. According to Figure 4, this is generally true (in particular when $\zeta_1 < 0$ when both constraints are actively participating in the optimization process), especially for the lowest income stratum, which has the largest welfare swing across ζ_1. However, this comes at the cost that the overall welfare level for the lowest income stratum is lower due to the concavity. One of the main contributions of this empirical analysis is to measure the trade-off between concavity (bringing welfare stability) and welfare levels, particularly for the lowest stratum. This trade off fits the patterns discussed by Kind, Botzen and Aerts (2017) and Schlee and Smith (2019), showcasing that even regardless of individual level risk aversion, the risk aversion from social planner with equity goals would still facing a choice between the lesser of two evils: lower welfare or higher welfare instability. As discussed before, policymakers should consider compensation for the lowest stratum, but a hard-to-predict ζ_1 would mean hard-to-predict compensation preemptively. Even if welfare is lower, more stable welfare will make targeted compensation towards the lowest income stratum easier regardless of the realization of ζ_1, plus additional funds to compensate from the stricter concave revenue constraint.

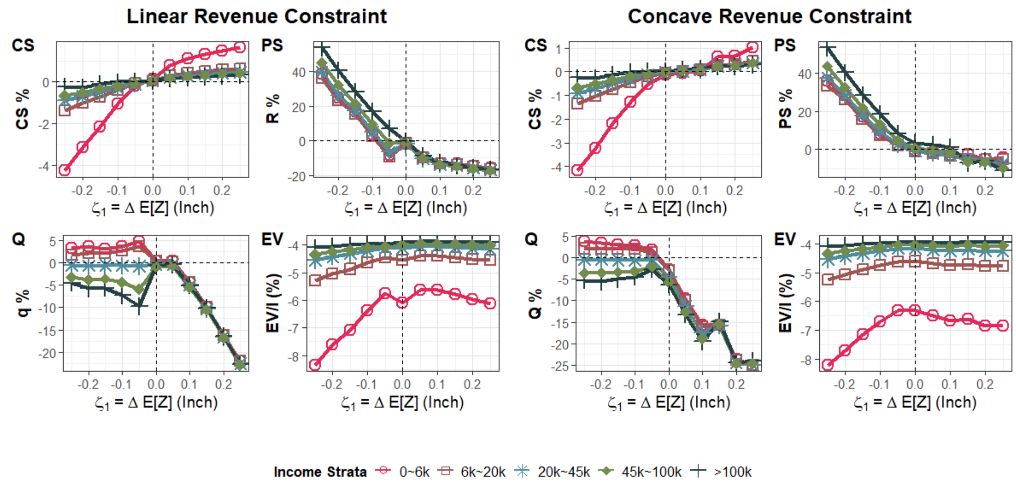

Figure 5. : Welfare from the Optimal Price in Shifting Variance ($\zeta_2 \in [0.75, 1.25]$)

B. Welfare Results - Variance Shift (ζ_2)

When analyzing **shifting variances (ζ_2)**, Figure 5 showcases the general welfare results for $\zeta_2 \leq 1$ and $\zeta_2 \geq 1$, and the comparisons between linear and concave revenue constraints. Under the linear revenue constraint, when $\zeta_2 \leq 1$ (i.e., when variance decreases), the revenue constraint becomes binding while the conservation constraint becomes slack. This is due to the shape of the precipitation distribution: as variance decreases, the distribution becomes less right-skewed, resulting in more humid conditions during months that were originally drier and fewer extremely dry months. Through this change, CS generally increases, more so for the lower income stratum ($0 \sim \$6k$), but not substantially for all strata collectively. Conversely, when $\zeta_2 > 1$ (i.e., when variance increases), the quantity constraint becomes binding while the revenue constraint becomes slack. Given the now more extreme weather patterns, CS decreases for all strata, disproportionately affecting lower-income strata (similar to the situation when $\zeta_1 \leq 0$). Although lower-income strata typically consume less (resulting in a smaller revenue increase compared to higher-income strata), the impact of higher precipitation variance remains more dominant for them. For the concave revenue constraint with $\gamma = 0.3$, a similar situation arises where the revenue constraint becomes stricter across different ζ_2 values, while quantity constraints become binding only in a few cases when ζ_2 is quite large. Regarding

[31] In practice, many utilities in the US have income-based benefits like the CAP program in Austin. The empirical analysis measures the amount of compensation that should be implemented across different ζ_1. In the literature, both Ruijs, Zimmermann and van den Berg (2008) and Wichman (2024) have pointed out that IBP itself does not effectively target the lower stratum. There needs to be more active progressive measures.

welfare, similar to the cases of shifting ζ_1, the lower-income strata generally experience lower welfare for similar reasons.

For both cases of shifting ζ_1 and ζ_2, the concave bound generally results in lower welfare levels; however, its benefit lies in **greater welfare stability**, especially for the lowest stratum. As discussed above, more stable welfare, even with a lower level, would make direct compensation from the policymaker easier to implement without knowing ζ_1 or ζ_2. This is particularly true when $\zeta_1 \leq 0$ and $\zeta_2 \geq 1$, both cases where the quantity constraint is not slack and actively participates in the optimization process. On the other hand, implementing a linear revenue constraint when either ζ_1 or ζ_2 is unknown would run the risk of over- or under-compensation for the lowest stratum, as the welfare levels across different ζ_1 and ζ_2 are quite different, causing social inefficiency.

Existing literature like Ruijs, Zimmermann and van den Berg (2008), Echeverri (2023), and Wichman (2024) already concluded that price changes in IBP generally favor higher-income strata. The counterfactual analysis further proves that in shifting weather patterns, the welfare imbalance between strata *worsens*, and both the high elasticities of the higher-income strata, and the high-quantity consumers in the lower-income strata are contributing factors. In addition, we agree with the literature that active progressive compensation is needed in the case of shifting precipitation patterns. By adding concavity towards $R_m(Z_m)$ and ensuring the utility has extra financial protection, the utility also gains relatively more stable welfare across different weather patterns, making it less important to predict specific ζ_1 and ζ_2. To justify the welfare transfer from CS to revenue, the utility could exploit the more stable welfare for a more precise targeted compensation to improve efficiency.

C. *Optimal Concavity*

So far, we have established the trade-off of implementing a concave revenue constraint for the optimal Ramsey pricing for the utility. The utility could sacrifice the welfare level (usually more evident for the lowest stratum) for welfare stability across different weather patterns. Since using IBP alone generally makes it harder to achieve the distributional effect, and the utility will need to transfer some compensation to the lowest stratum regardless, welfare stability makes it easier for the utility to measure the compensation, irrespective of the realization of the weather pattern. In this section, we will explore different levels of **concavity (γ)** in the CRRA function and see which γ can maintain the balance between welfare level and stability. Since only the lowest stratum has the largest swing, we will only focus on the equivalence variation-income ratio (EV/I) for the lowest income stratum ($0 \sim 6$k).

Since different weather patterns (ζ_1 and ζ_2) impose different constraints in the optimization process, we will divide it into four cases: 1) $\zeta_1 < 0$: precipitation on average lower, with both constraints more binding; 2) $\zeta_1 > 0$: precipitation on average higher, with revenue constraints more binding; 3) $\zeta_2 < 1$: precipitation less volatile, with revenue constraints more binding; and 4) $\zeta_2 > 1$: precipitation more volatile, with both constraints more binding. For all four cases, we will measure EV/I for all weather patterns and calculate the mean, standard deviation, and its 5th percentile. Note that when $\gamma = 0$, even though it does not make mathematical sense for the CRRA function, it signifies a linear constraint in this scenario.

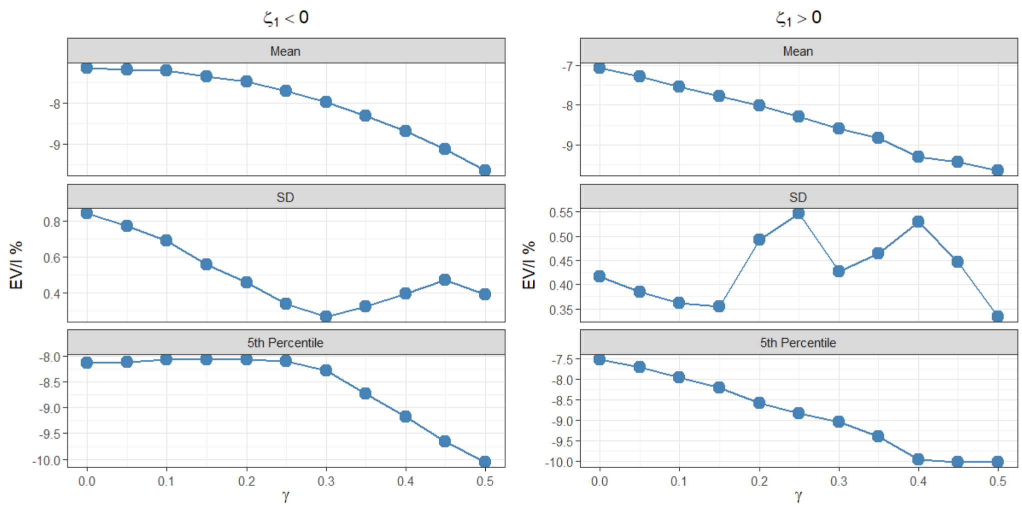

Figure 6. : Statistics of $EV/I\%$ ratio vs γ - ζ_1

When $\zeta_1 < 0$, both constraints are more binding. The mean EV/I becomes increasingly decreasing, but the standard deviation (SD) of EV/I decreases much faster. In the meantime, the floor (5th percentile of EV/I) will maintain at a certain level but starts to decrease when γ becomes larger. This is because the floor of EV/I is from when ζ_1 is low, and when γ is small, the quantity constraint is the dominant constraint. When γ becomes larger, the revenue constraint becomes more dominant even in the driest condition, pushing down the overall welfare along with the 5th percentile. In drier conditions with higher demand, an always-binding quantity constraint means prices are set to maximize welfare given the capped quantity. In this context, the concave revenue constraint effectively works to smooth out the revenue variability that arises from hitting capacity limits or from high, but still fluctuating, demands. The prices actively try to manage the revenue stream at the capped quantity, and the concave function helps them do that, leading to the observed reduction in welfare volatility (SD). However, it is interesting that this reduction only works for moderate concavity ($\gamma \leq 0.3$). When the constraint becomes overly concave, the optimizer is now forced to implement increasingly aggressive pricing strategies to achieve even tiny, marginal reductions in revenue variance. These extreme pricing adjustments might severely distort consumer behavior or strain the system's ability to respond efficiently. In addition, in the case of high concavity, the prices might become so compressed or so high for certain tiers that they make welfare highly sensitive to small shifts in demand caused by the weather, leading to larger welfare swings.

When $\zeta_1 > 0$, with the quantity constraint not binding, both the mean and the 5th percentile of EV/I decrease with higher γ, which is unsurprising. However, the SD does not become smaller with higher γ. In fact, concavity seems to be less effective for SD when $\zeta_1 > 0$. This is because when demand is already low and the revenue constraint is binding, the model's ability to further smooth revenue variability through pricing might be limited. Prices might be primarily driven by the need to simply hit the revenue target, rather than fine-tuning for stability (this could be the additional reason why for $\zeta_1 < 0$ and $\gamma > 0.3$, which the quantity constraint is no longer bound and only the revenue constraint matters, higher concavity does not lead to lower SD). This also points out the asymmetric demand response towards prices, as it is easier to curb demand and revenue when it is high than to stimulate it when it is low.

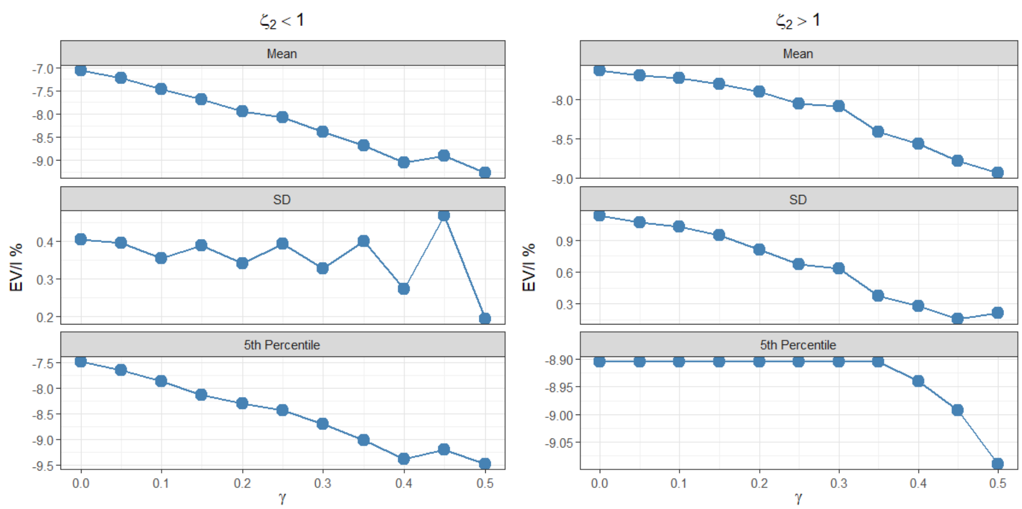

Figure 7. : Statistics of $EV/I\%$ ratio vs γ - ζ_2

In the case of ζ_2, shifting the variance of precipitation: as we discussed before, less extreme precipitation is effectively similar to $\zeta_1 > 0$, while more extreme precipitation is effectively similar to $\zeta_1 < 0$. Even though both the mean and 5th percentile decrease with higher γ, for $\zeta_2 > 1$, the SD of EV/I decreases monotonically, making it more effective to implement concavity to curb welfare volatility. For $\zeta_2 < 1$, it still exhibits some non-monotonicity due to similar reasons, as the model's ability to further smooth revenue variability through pricing is limited by the always-binding revenue constraint.

Therefore, for concavity to be effective, it is best implemented when both revenue and quantity constraints are active (in particular, when the quantity constraint is more binding), i.e., when $\zeta_1 < 0$ and $\zeta_2 > 1$. Fortunately, due to climate change, these two scenarios with drier and more volatile weather are becoming more prominent, meaning concavity will be more useful. As for the **optimal level of** γ, it seems that **moderate concavity** ($\gamma \in [0.25, 0.3]$) is the sweet spot, as it can effectively curb welfare variance but does not decrease the mean and the 5th percentile by too much. For moderate concavity, the quantity constraint is still binding, making sure the revenue constraint will be evaluated at the quantity capacity, and the concavity of the revenue constraint is actively smoothing the revenue.

VI. Conclusion

In this paper, we estimate the optimal Inclining Block Pricing (IBP) for water utilities using a Ramsey-like pricing model that accounts for precipitation stochasticity. Our initial reduced-form analysis revealed that weather stochasticity disproportionately affects higher-income strata, manifesting as high price elasticities during unusual precipitation events. This evidence highlights a significant price-setting challenge for utilities, particularly concerning their distributional goals.

We then employed a structural Discrete-Continuous Choice Model (DCC) on the piecewise linear budget constraint, incorporating satellite imagery of household lawn vegetation health. This allowed for a more granular understanding of the outdoor water usage in single-home households, which accounts for a substantial portion of consumption most easily affected by precipitation. Our findings confirm existing literature that current IBP pricing used in utility generally favors high-income strata as aggregately, these income groups are more sensitive to price. To maintain the current pricing structure while improving distributional equity, we suspect that an income-based welfare compensation scheme targeting lower-income strata is essential.

Subsequently, we developed an empirical Ramsey model that incorporates both revenue recovery and quantity conservation constraints to determine optimal IBP. Given that prices are set well before weather events occur, we explore how utilities can manage revenue variance by adjusting the concavity of their revenue constraints based on their confidence in weather predictions and their risk aversion. Our analysis shows that a moderate concavity in the revenue constraint—particularly when precipitation averages decrease or when it becomes more volatile (situations where the quantity constraint is more likely to be binding)—can significantly reduce the welfare volatility for the lowest income stratum across various weather patterns. However, this also leads to a decrease in their average welfare. This enhanced welfare predictability allows utilities to implement more precise income-based targeted compensation regardless of actual weather realization. In essence, by introducing some concavity into their revenue constraint, utilities don't need highly precise weather forecasts to reduce the welfare standard deviation for the lowest stratum, thereby simplifying the design of compensation.

This paper provides novel empirical measurements of the trade-off between welfare standard deviation and welfare levels. Specifically, when precipitation mean reduces, our analysis indicates that for a moderate risk aversion ($\gamma = 0.25$ compared to risk neutrality), the lowest income stratum will experience an additional welfare loss of $325,244.2

(0.56% of their total income)[32]. However, this comes with a significant reduction in the standard deviation of monetary welfare loss by $290,396.7 (0.50% of their total income). Similarly, when precipitation variance increases, a moderate risk aversion leads to an additional welfare loss of $249,741.1 (0.43% of total income)[33] for the lowest income stratum, but a reduction in welfare standard deviation by $267,164.9 (0.46% of their total income). In addition, our empirical measurement demonstrates that a moderate risk aversion (CRRA function with $\gamma \in [0.25, 0.3]$) can achieve the smallest average welfare loss alongside the biggest drop in welfare standard deviation for the lowest income stratum, particularly in scenarios of decreasing precipitation mean and increasing precipitation variance.

The measurement of this trade-off has profound policy implications. To minimize climate-induced welfare losses, policymakers should move beyond solely relying on increasingly difficult precise weather forecasts to trigger reactive measures or relying only on nonlinear IBP to achieve equity goals. Instead, the focus should shift to establishing preemptive, transparent, and progressive compensation mechanisms that complement existing pricing structures. Such schemes could involve expanding current Consumer Assistance Programs (CAPs)[34] to include predefined compensation amounts for low-income households when certain weather thresholds are met, moving away from retrospective damage assessments. It could also involve developing subsidized micro-insurance programs, extending current consumption smoothing programs like "Budget Billing" into government-backed or subsidized micro-insurance programs for low-income households, specifically designed to buffer against weather-related income fluctuations. Furthermore, integrating climate risk considerations into existing social welfare programs, allowing for dynamic adjustments in benefits or eligibility criteria in response to anticipated weather patterns.

Looking ahead, we plan to expand this research into two possible directions to more accurately measure weather-induced price elasticity: 1) Modeling Consumption Smoothing Programs: The existence of "Budget Billing" and similar consumption smoothing programs indicates a consumer preference for stable monthly utility payments, particularly among lower-income individuals. This preference, which acts as an existing insurance against weather stochasticity, isn't explicitly modeled in this paper but is highly interactive with the utility's risk aversion and their pricing and compensation scheme designs. Future research will explore integrating this consumer preference. 2) Endogenizing Vegetation Changes (NDVI): We currently treat lagged NDVI as an exogenous variable solely affected by weather. However, consumers can actively change their vegetation, for example, by adopting xeriscaping or zeroscaping to semi-permanently reduce water demand and lessen their susceptibility to weather stochasticity. Endogenizing NDVI into the demand model will yield more accurate measurements of price elasticity and provide deeper insights into the price-setting challenges faced by utilities.

[32] -7.15% vs. -7.71%

[33] -7.62% vs. -8.05%

[34] CAP is not unique in Austin, but a common program for lower income households for both water and electricity bills so that they can enjoy lower price. Similar programs can be found across the US in different names.

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APPENDIX

All plots of this paper have utilized the software "Colorgorical" to choose a more clear and aesthetically pleasing color scheme. Gramazio, Laidlaw and Schloss (2017).

A1. Water Utility Pricing in Austin, TX

PRECIPITATION TREND IN AUSTIN

CURRENT PRICING STRUCTURE

TIER AND INCOME DISTRIBUTION

Based on status quo price and status quo quantity distribution, we divide the household monthly income into 5 different strata. Since both the quantity and income are very right-skewed, only for this plot, we filtered out all households with monthly household income

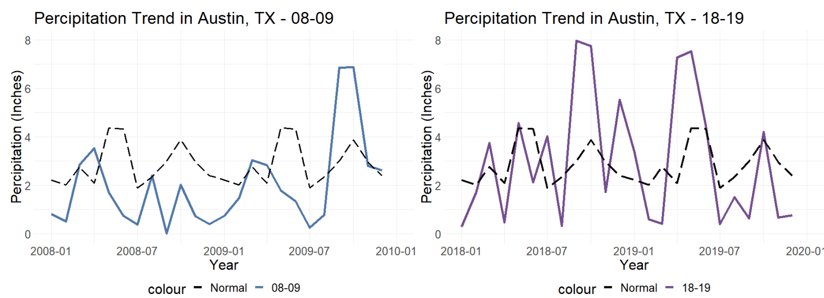

Figure A1. : Precipitation Trends of 2008-2009 vs. 2018-2019

Table A1—: Pricing Structure

Kink Points (kGallon)	Fixed Charge ($)	Marginal Price ($)	
		Non-CAP	CAP
0-2	8.5	3.09	2.42
2-6	10.8	5.01	4.1
6-11	16.5	8.54	6.72
11-20	37	12.9	11.56
>20	37	14.41	14.26

Note: Each fixed charge is composed of fixed payment and meter charge. The fixed payment depends on final consumption quantity, and the meter charge assumes a 5/8 meter size, which is the most common residential meter size. The marginal price includes volume charge, reserved fund charge and community benefit charge. These itemized charges are all billed per 1,000 gallons, meaning they are essentially part of the marginal price. Volume charge depends on the amount of final consumption quantity while the other two are billed per 1,000 gallons. Reserved fund surcharge goes into a restricted reserve fund to offset water service revenue shortfalls that may impact operations and services. Community benefit charge is only billed to Non-CAP consumers to fund the CAP.

bigger than $\$250,000$, and quantities higher than $100,000$ gallons. These households are not filtered out in demand estimation and counterfactual analysis, simply filtered out for the picture below for visualization purposes.

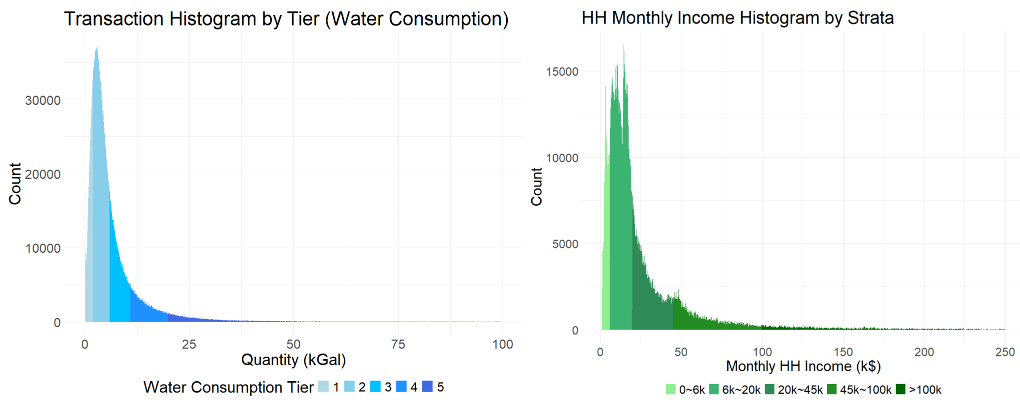

Figure A2. : Monthly Water Quantity Distribution vs. Income Distribution

Table A2—: Monthly Water Quantity Distribution vs. Income Distribution

Income Strata ($)	Percentage	Tiers (kGal)	Percentage
$0 \sim\ 6k$	0.146	$0 \sim\ 2k$	0.159
$6k \sim\ 20k$	0.489	$2k \sim\ 6k$	0.503
$20k \sim\ 45k$	0.216	$6k \sim\ 11k$	0.200
$45k \sim\ 100k$	0.104	$11k \sim\ 20k$	0.0927
$> 100k$	0.0441	$> 20k$	0.0456

Note: Just like the utility targeting different quantity levels of households. This division of household monthly income into different strata is without loss of generality and serves as a way to gauge the distributional effect incurred by any price change.

Descriptive and Reduced Form Evidence

In the interest of seeing how different income strata will react to abnormal weather changes and the descriptive evidence of Figure 1, we performed the following OLS:

$$\Delta q = \beta_0 + \beta_1 \Delta Z_T + \beta_2 \Delta Z_P + \beta_3 \text{Income} * \Delta Z_T + \beta_4 \text{Income} * \Delta Z_P + \alpha p + \varepsilon$$

where Income is the income strata defined in Table A2. ΔZ for both precipitation and temperature is the difference between observed Z_m for month m and the corresponding 30 year average. $\Delta q = \frac{q_m - \bar{q}_m}{\bar{q}_m}$ which is the quantity deviation from the average quantity for the specific household for specific month between 2016-2020 in percentage term.

Table A3—: Regression Results for Quantity Deviation

Variable	Estimate	Std. Error	t value	Pr(> $\|t\|$)
(Intercept)	−0.4182	0.0008490	−492.609	< 2e-16 ***
ΔZ_P	−0.01668	0.0003582	−46.555	< 2e-16 ***
ΔZ_T	0.004787	0.0001438	33.283	< 2e-16 ***
income_strata: 6k∼20k	−0.02104	0.0007867	−26.739	< 2e-16 ***
income_strata: 20k∼45k	−0.08237	0.0008990	−91.627	< 2e-16 ***
income_strata: 45k∼100k	−0.1673	0.001096	−152.710	< 2e-16 ***
income_strata: >100k	−0.2476	0.001513	−163.593	< 2e-16 ***
p	0.07486	0.00008868	844.089	< 2e-16 ***
$\Delta Z_P \times$income_strata: 6k∼20k	0.001576	0.0004098	3.846	0.00012 ***
$\Delta Z_P \times$income_strata: 20k∼45k	−0.009682	0.0004614	−20.986	< 2e-16 ***
$\Delta Z_P \times$income_strata: 45k∼100k	−0.01923	0.0005504	−34.933	< 2e-16 ***
$\Delta Z_P \times$income_strata: >100k	−0.01874	0.0007744	−24.198	< 2e-16 ***
$\Delta Z_T \times$income_strata: 6k∼20k	0.00009410	0.0001657	0.568	0.57012
$\Delta Z_T \times$income_strata: 20k∼45k	0.0004584	0.0001880	2.439	0.01474 *
$\Delta Z_T \times$income_strata: 45k∼100k	−0.002830	0.0002234	−12.667	< 2e-16 ***
$\Delta Z_T \times$income_strata: >100k	−0.002834	0.0003005	−9.428	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.4014 on 2351610 degrees of freedom
Multiple R-squared: 0.263, Adjusted R-squared: 0.263
F-statistic: 5.596e+04 on 15 and 2351610 DF, p-value: < 2.2e-16

In this model, controlling for differences in precipitation, maximum temperature, and the marginal price, the relationship between income strata and the percentage difference in water consumption is seen through: 1) for baseline differences (when Δ weather and price are zero), compared to the lowest income strata (0-6k), most higher income strata show a statistically significant lower baseline percentage difference in water consumption from their usual amount. This does not mean higher income groups use less water overall; it means their deviation from their own usual quantity, under these specific baseline conditions, is lower than the deviation for the reference group. This effect is likely influenced by the strong role of price in this model, as marginal price might be correlated with income or consumption levels that influence which price tier is reached. 2) the linear effect of precipitation difference: the interaction terms between income strata and ΔZ_P are largely significant. This means that the linear rate at which Δq changes for every unit increase in precipitation difference varies significantly across income strata. In particular, higher income strata (20k-45k, 45k-100k, >100k) show increasingly more negative interaction terms (-0.009682, -0.01923, -0.01874). This means the negative impact of additional precipitation difference on consumption deviation is increasingly stronger as income rises in these groups. 3) the linear effect of temperature difference: many interaction terms between income strata and ΔZ_T are also significant, indicating that the linear rate at which Δq changes for every unit increase in temperature difference varies significantly across income strata. However, the relationship is less clear-cut compared to the one from precipitation. Compared to the reference group (where the effect of ΔZ_T is 0.004787, meaning a 0.48 percentage point increase in consumption difference for every unit increase in temperature difference): The 6k-20k stratum's interaction is not significant (0.0000941), suggesting its linear temperature sensitivity is not statistically different from the reference group in this model. The 20k-45k stratum has a significant positive interaction (0.0004584), making its positive temperature effect slightly stronger (0.004787 + 0.0004584 = 0.0052454). The 45k-100k and >100k strata have significant negative interaction terms (-0.002830 and -0.002834). This means that although the base effect of temperature difference is positive, the additional positive effect seen in higher income groups in the first model is now appearing as a reduction in the sensitivity compared to the reference group's base sensitivity (0.004787 - 0.002830 = 0.001957 for 45k-100k; 0.004787 - 0.002834 = 0.001953 for >100k).

This difference between temperature and precipitation prompts the research to be more focused on changing precipitation. In addition, a structural model to estimate the demand on the panel data to show how price, precipitation, and the interaction term with income strata will affect the demand.

A2. Demand Model and Estimation

ADDITIONAL CASE IN THE UNCONDITIONAL INDIRECT UTILITY FUNCTION

When constructing the unconditional indirect utility function from the conditional ones, the case where consumption occurs exactly at a tier boundary (kink point), rather than strictly within a tier, may require clarification. Continuing with the two-tier scenario ($K = 2$) without loss of generality, this case arises when the household's optimal consumption calculated using the tier 1 price, $g(p_1, I)$, would exceed the tier 1 limit q_1, *and* the optimal consumption calculated using the tier 2 price and virtual income, $g(p_2, I + d_2)$, would fall below q_1. That is, the condition is $g(p_2, I + d_2) \leq q_1 < g(p_1, I)$ (assuming $d_1 = 0$ or defined appropriately). To satisfy the Incentive Compatibility (IC) constraint (i.e., ensure the chosen consumption is utility-maximizing given the full budget set), the household optimally consumes exactly at the kink point q_1. The diagram below illustrates this:

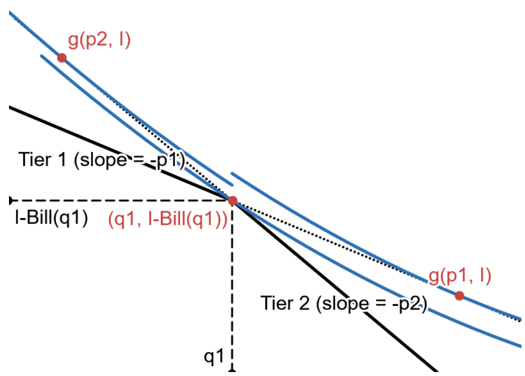

Figure A3. : The household consumes exactly at the kink $(q_1, I - \text{Bill}(q_1))$

As shown, under the condition $g(p_2, I + d_2) \leq q_1 < g(p_1, I)$, the optimal (incentive compatible) choice for this household is to consume the bundle corresponding to the kink point, $(q_1, I - \text{Bill}(q_1))$. Here, $\text{Bill}(q_1)$ represents the total water bill incurred when consuming exactly q_1 units (e.g., typically $A_2 + p_1 q_1$ in a two-tier system where A_2 is the fixed charge associated with entering tier 2). If consumption at the kink were not allowed as an option in the model, such consumers would be incorrectly assigned to consume within one of the adjacent tiers, violating the true optimum.

Generalizing, for any tier boundary q_k where $k \in \{1, \ldots, K-1\}$, some households might satisfy the condition $g(p_{k+1}, I + d_{k+1}) \leq q_k < g(p_k, I + d_k)$, for whom the optimal consumption point is exactly $(q_k, I - \text{Bill}(q_k))$. In our empirical application to Austin, which has 5 tiers ($K = 5$), there are 4 such kink points (q_1, q_2, q_3, q_4), requiring the model to allow for consumption exactly at these quantities to ensure incentive compatibility across the full range of consumption.

Likelihood Function

Without loss of generality, we explain the derivation of the likelihood function for a two-tier case ($K = 2$); the approach generalizes to K tiers. Conditional on the ex-ante optimal choice (based on the known preference shock η but ignoring the ex-post error ν) being tier 1 ($k^* = 1$), the condition is $\log w_1 + \eta \leq \log q_1$. Similarly, $k^* = 2$ if $\log w_2 + \eta > \log q_1$. The remaining possibility, ensuring incentive compatibility and covering all η, is consumption at the kink q_1, which occurs if $\log w_2 + \eta \leq \log q_1 < \log w_1 + \eta$. Rearranging these conditions on η and adding the ex-post error ν (unobserved by the household when choosing the tier/kink), the observed log-consumption $\log(w)$ is:

$$\log(w) = \begin{cases} \log(w_1) + \eta + \nu & \text{if } \eta \leq \log(q_1) - \log(w_1) \\ \log(q_1) + \nu & \text{if } \log(q_1) - \log(w_1) < \eta \leq \log(q_1) - \log(w_2) \\ \log(w_2) + \eta + \nu & \text{if } \eta > \log(q_1) - \log(w_2) \end{cases}$$

which is the two-tier special case of Equation 7. Note that since the household does not observe ν when making its choice, ν affects the final observed consumption in all scenarios.

Let $f(\cdot)$ denote a probability density function (pdf). The overall likelihood for an observation $\log w$ is the sum of the contributions from these three mutually exclusive and exhaustive scenarios ($L = L_1 + L_{kink} + L_2$).

Case 1: $k^* = 1$ ($\eta \leq \log(q_1) - \log(w_1)$) The observed log consumption is $\log w = \log w_1 + \eta + \nu$. The contribution to the likelihood depends on the joint pdf of $(\eta + \nu, \eta)$, integrated over the relevant range of η:

$$L_1 = \int_{-\infty}^{\log q_1 - \log w_1} f_{\nu+\eta,\eta}(\log w - \log w_1, \eta)\, d\eta$$

Case 2: $k^* = 2$ ($\eta > \log(q_1) - \log(w_2)$) The observed log consumption is $\log w = \log w_2 + \eta + \nu$. The likelihood contribution is:

$$L_2 = \int_{\log q_1 - \log w_2}^{\infty} f_{\nu+\eta,\eta}(\log w - \log w_2, \eta)\, d\eta$$

Case 3: Kink Consumption ($\log(q_1) - \log(w_1) < \eta \leq \log(q_1) - \log(w_2)$) The observed log consumption is $\log w = \log q_1 + \nu$. The likelihood contribution depends on the joint pdf of (ν, η), integrated over the relevant range of η:

$$L_{kink} = \int_{\log q_1 - \log w_1}^{\log q_1 - \log w_2} f_{\nu,\eta}(\log w - \log q_1, \eta)\, d\eta$$

If we assume $\eta \sim N(0, \sigma_\eta^2)$ and $\nu \sim N(0, \sigma_\nu^2)$, and that they are independent, these integrals can be solved in closed form. Let $\phi(\cdot)$ and $\Phi(\cdot)$ be the standard normal pdf and cdf, respectively. The joint distribution of $(\eta, \nu + \eta)$ is bivariate normal. Let $\rho_s =$

$\text{Corr}(\eta, \nu+\eta) = \sigma_\eta / \sqrt{\sigma_\eta^2 + \sigma_\nu^2}$.

Evaluating the first integral (L_1):

$$
\begin{aligned}
L_1 &= \int_{-\infty}^{\log q_1 - \log w_1} f_{\nu+\eta}(\log w - \log w_1) f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_1)\, d\eta \\
&= f_{\nu+\eta}(\log w - \log w_1) \int_{-\infty}^{\log q_1 - \log w_1} f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_1)\, d\eta \\
&= \frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \phi\left(\frac{\log w - \log w_1}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}}\right) \Phi\left(\frac{(\log q_1 - \log w_1)/\sigma_\eta - \rho_s(\frac{\log w - \log w_1}{\sqrt{\sigma_\eta^2+\sigma_\nu^2}})}{\sqrt{1-\rho_s^2}}\right) \\
&\equiv \frac{\phi(s_1)}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} \Phi(r_1)
\end{aligned}
$$

where $s_1 = (\log w - \log w_1)/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$ and $r_1 = (t_1^* - \rho_s s_1)/\sqrt{1-\rho_s^2}$ with $t_1^* = (\log q_1 - \log w_1)/\sigma_\eta$.

Similarly, evaluating the integral for the second case (L_2):

$$
\begin{aligned}
L_2 &= f_{\nu+\eta}(\log w - \log w_2) \int_{\log q_1 - \log w_2}^{\infty} f_{\eta|\nu+\eta}(\eta \,|\, \log w - \log w_2)\, d\eta \\
&= f_{\nu+\eta}(\log w - \log w_2) \left[1 - \Phi\left(\frac{(\log q_1 - \log w_2)/\sigma_\eta - \rho_s(\frac{\log w - \log w_2}{\sqrt{\sigma_\eta^2+\sigma_\nu^2}})}{\sqrt{1-\rho_s^2}}\right)\right] \\
&\equiv \frac{\phi(s_2)}{\sqrt{\sigma_\eta^2 + \sigma_\nu^2}} (1 - \Phi(n_2))
\end{aligned}
$$

where $s_2 = (\log w - \log w_2)/\sqrt{\sigma_\eta^2 + \sigma_\nu^2}$ and $n_2 = (m_1 - \rho_s s_2)/\sqrt{1-\rho_s^2}$ with $m_1 = (\log q_1 - \log w_2)/\sigma_\eta$.

Finally, evaluating the integral for the third case (kink consumption, L_{kink}), using the independence of ν and η:

$$
\begin{aligned}
L_{kink} &= \int_{t_1^*\sigma_\eta}^{m_1\sigma_\eta} f_\nu(\log w - \log q_1) f_\eta(\eta)\, d\eta = f_\nu(\log w - \log q_1) \int_{t_1^*\sigma_\eta}^{m_1\sigma_\eta} f_\eta(\eta)\, d\eta \\
&= \frac{1}{\sigma_\nu} \phi\left(\frac{\log w - \log q_1}{\sigma_\nu}\right) \left[\Phi\left(\frac{m_1\sigma_\eta}{\sigma_\eta}\right) - \Phi\left(\frac{t_1^*\sigma_\eta}{\sigma_\eta}\right)\right] \\
&= \frac{\phi(u_1)}{\sigma_\nu} (\Phi(m_1) - \Phi(t_1^*))
\end{aligned}
$$

where $u_1 = (\log w - \log q_1)/\sigma_\nu$.

Summing the three components gives the likelihood for the two-tier case:

$$L = L_1 + L_{kink} + L_2$$
$$= \frac{\phi(s_1)}{\sqrt{\sigma_\eta^2 + \sigma_v^2}}\Phi(r_1) + \frac{\phi(u_1)}{\sigma_v}(\Phi(m_1) - \Phi(t_1^*)) + \frac{\phi(s_2)}{\sqrt{\sigma_\eta^2 + \sigma_v^2}}(1 - \Phi(n_2))$$

Using the general definitions $t_k^* = (\log q_k - \log w_k)/\sigma_\eta$, $m_k = (\log q_k - \log w_{k+1})/\sigma_\eta$, $s_k = (\log w - \log w_k)/\sqrt{\sigma_\eta^2 + \sigma_v^2}$, $u_k = (\log w - \log q_k)/\sigma_v$, $r_k = (t_k^* - \rho_s s_k)/\sqrt{1-\rho_s^2}$, $n_k = (m_{k-1} - \rho_s s_k)/\sqrt{1-\rho_s^2}$, and applying boundary conditions ($m_0 \to -\infty \implies n_1 \to -\infty \implies \Phi(n_1) = 0$; $q_2 \to \infty \implies t_2^* \to \infty \implies r_2 \to \infty \implies \Phi(r_2) = 1$), this derived likelihood function matches the general form given in Equation 8 for $K = 2$.

NDVI

The use of the Normalized Difference Vegetation Index (NDVI) for estimating water demand was introduced by Wang and Wolak (2022). The essential idea is to quantify vegetation health for specific geographical locations – in this case, the health of lawns associated with single-family homes. Higher NDVI values indicate healthier vegetation, suggesting greater lawn care efforts.

NDVI is calculated from satellite imagery using the difference between the reflectance in the near-infrared (NIR) and red light bands. Healthy plants typically reflect more NIR light and absorb more red light (appearing green), while less healthy or stressed plants reflect less NIR and more red light (appearing yellow or brown). The index is typically calculated as (NIR - Red) / (NIR + Red), normalized to a range between -1 and +1, where higher values indicate healthier, denser vegetation. Values near -1 often correspond to water bodies. Values around 0 typically represent bare soil or sparse vegetation, and values approaching +1 indicate dense, healthy vegetation like forests or well-maintained lawns.

The raw data we use is the Sentinel-2 Surface Reflectance dataset,[35] providing imagery with a 10m x 10m spatial resolution (pixel size). Since typical residential lots in Texas are larger than this pixel size, this higher resolution allows for a more precise calculation of NDVI within each lot compared to the 30m x 30m resolution used by Wang and Wolak (2022). Sentinel-2 images are collected frequently (on average, every 5 days for a given location), though not always regularly, resulting in multiple images per month. Some images, however, contain areas obscured by clouds. To address this, we employ cloud masking and create monthly composite images.[36] The cloud masking algorithm utilizes bands of the images, such as the "Cloud Probability" or "QA". These bands indicate the likelihood of cloud cover or contain bitwise flags for clouds and shadows. To create a cloud-free composite, we apply these masks to filter out cloudy pixels from multiple images over all images of a calendar month of the same area, then merge the remaining

[35]If using Google Earth Engine to access data, please refer to Google Earth Engine Sentinel-2 Surface Reflectance.
[36]Details for cloud masking in Google Earth Engine can be found here Cloud Masking - gee map.

clear-sky pixels into a seamless, cloud-free composite.

To visually illustrate the process of calculating NDVI in Figure A4, we take a small sample from the Brentwood neighborhood in Austin, near the intersection of W Koenig Lane and Burnet Road. While detecting subtle vegetation differences can be difficult in the raw satellite image, the corresponding NDVI visualization clearly showcases the variations in vegetation health between households.

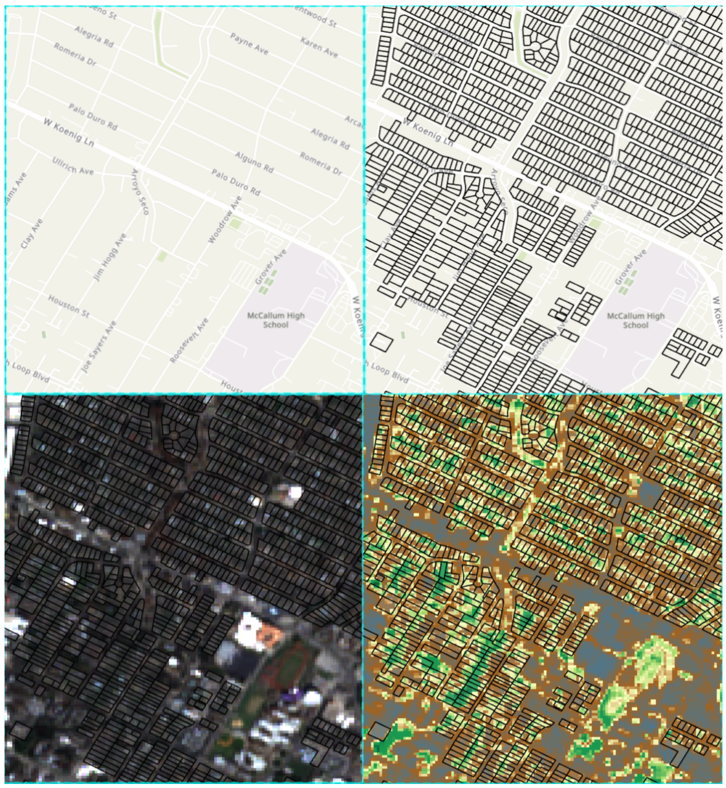

Figure A4. : The Process of Calculating NDVI

Note: From Top Left to Bottom Right: OpenStreetMap base map; OpenStreetMap with household lot shapefiles overlaid; Sentinel-2 Raw Image mosaic with shapefiles overlaid; and final NDVI visualization derived from Sentinel-2 with shapefiles overlaid.

Summary Statistics

After matching the panel data with TCAD records, filtering outliers, and removing households located outside Travis County (and thus likely outside the Austin Water service area), the final dataset primarily covers the period from May 2018 to December 2019. It is important to note that this data includes households on slightly different payment plans. Specifically, some households meeting certain income requirements are eligible for CAP, which offers lower marginal prices. The inclusion of CAP households provides valuable price variation for model identification, despite their small number. The summary statistics can be found in Table A4.[37]

Table A4—: Summary Statistics

Parameter Name	Min	1st Quartile	Median	Mean	3rd Quartile	Max	N
Heavy Water Appliances	0	0	0	0.1637	0	12	127320
Bedrooms	1	1	1	1.439	1	31	127320
Bathrooms	0.1	2	2	2.427	2.5	34	127320
Lot Size (Acre)	0.02136	0.15018	0.19060	0.27396	0.25309	278.25085	127320
Household Monthly Income (k$)	1.2	8.642	15.320	29.295	27.765	5057.051	127320
NDVI	-0.3564	0.3266	0.4038	0.3988	0.4766	0.7729	2351626
Mean Max Temp (F)	56.24	68.98	83.74	82.18	95.04	101.65	2351626
IQR Max Temp (F)	2	4.984	9.499	9.904	14.217	25.334	2351626
Total Precip (Inches)	0	1.009	2.550	2.967	4.349	13.819	2351626
IQR Precip (Inches)	0	0.004724	0.034655	0.158315	0.238235	2.434279	2351626
Quantity (kGal)	0.1	2.6	4.3	6.537	7.4	1275.8	2351626
Payment ($)	8.426	19.118	27.436	57.746	53.074	17577.749	2351626

Note: Heavy Water Appliances include pool, hot tub, sprinkler system, fountain, and car wash station/area. In the real estate industry, a full bathroom requires a sink, a tub, a shower, and a toilet. If it only has 3/4 of fixtures, it will constitute a 3/4 bathroom, and only 2/4 fixtures will constitute a half bathroom.

MLE Estimation Result

Given the model with the interaction terms, the MLE estimates the parameters ($\beta'_1, \beta'_2, \beta'_3, \beta'_4, \beta'_5, c, c_\alpha, c_\rho, \sigma_\eta, \sigma_\nu$) from data ($X_i, Z_i, X_{\alpha,i}, Z_{\alpha,i}, X_{\rho,i}, p_k, q_k, A_k, I_i, w_i$), with a total of 18 parameters. All data measuring price or payment are scaled from nominal value to real dollar value in January 2017 using the Federal Reserve Economic Database Gross Domestic Product (GDP) deflator from St Louis Federal Reserve Bank. [38] The estimation results are listed in Table A5:

Most results of MLE fit the intuition qualitatively. The standard error is calculated by the inverse of the matrix of the sum of the outer products of the observation-by-observation gradient of the log-likelihood for each household evaluated at the maximum likelihood parameter estimates, a method introduced by Hall, Hall and Hausman (1974).

Price Elasticities of Income Strata

From Figure A5, we can conclude that as opposed to common belief, that higher strata will be less elastic, due to the fact that the DCC model fully captures the impact of weather (on quantity and through the interaction with price), the higher income households are actually more elastic and tend to switch down to lower tiers. We think this could

[37]Overall, approximately 60% of households and 55.84% of transactions from the original data are used for the demand estimation analysis. Among the final sample, 99.57% are non-CAP consumers, and 0.43% are CAP consumers.

[38]See Fred-GDPDEF

Table A5—: MLE Estimation Results

Parameter Name	Estimate	Standard Error	Parameter Name	Estimate	Standard Error
Bathroom	1.16	(9.20E-04)	Average High Temp	0.00218	(2.04E-05)
NDVI	1.19	(0.011)	IQR High Temp	-0.0189	(1.46E-04)
Constant	0.731	(0.0017)	Total Precipitation	-0.941	(1.34E-04)
Price * bedroom	0.0486	(3.94E-04)	IQR Precipitation	0.12	(0.005)
Price * NDVI	-0.0404	(0.00465)	Income * Heavy Water Appliances	-0.0718	(4.81E-04)
Price * Avg High Temp	-0.0168	(2.47E-05)	Income * Bedroom	-0.0158	(5.34E-05)
Price * Total Prcp	-0.0363	(4.23E-04)	Income * NDVI	-0.0692	(3.91E-04)
Price	0.688	(0.00193)	Income	0.162	(1.67E-04)
σ_η	2.56	(8.45E-04)	σ_v	4.71E-04	(2.25E-03)

Note: Price * bedroom represents the interaction term inside α, measuring how the price effect is changed through the number of bedrooms. The same goes for other interaction terms for price and income.

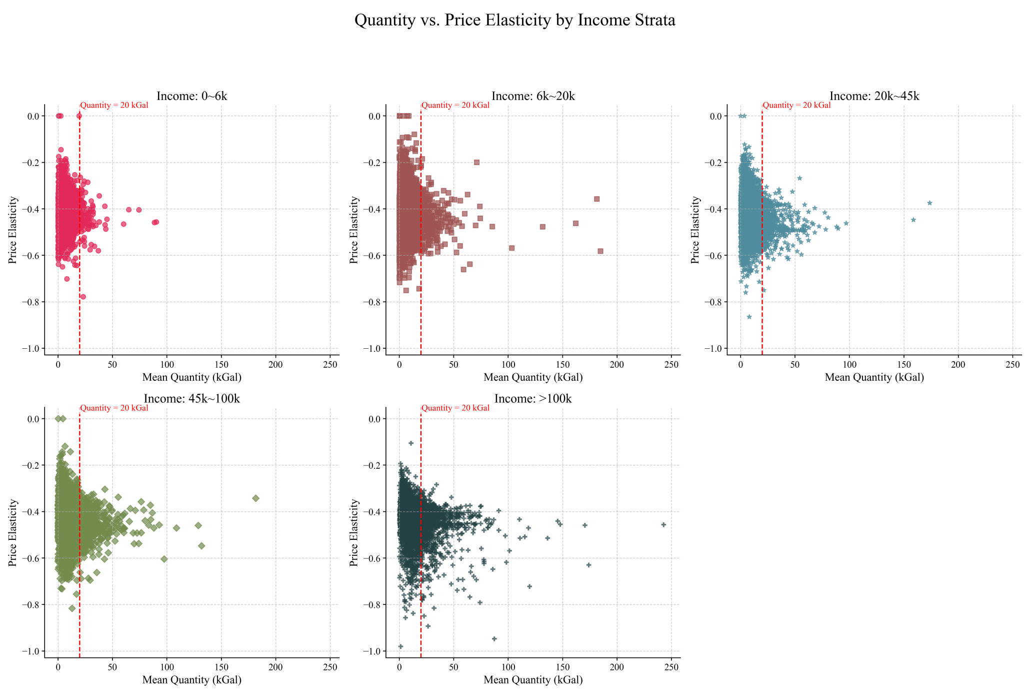

Figure A5. : Price Elasticities v. Mean Quantity by Income Strata

Note: These show usually higher income households will consume more, but not without conditions. The higher consumption households are actually more elastic due to the fact that the demand model captures the changes through precipitation.

potentially explain the result from previous literature[39], as the higher income households have more choices with all different tiers. Any marginal price change in IBP could potentially be beneficial as they could just switch down to lower tiers. As for lower income households, even though fewer households are above the 20k Gallon threshold (the kink point between tier 4 and tier 5), many of them essentially have fewer choices in terms of their demands. Another potential explanation is that for higher income households, they are more likely to have smart water devices to accurately measure water consumption (like smart water sprinklers etc.) which makes their demand easier to control. The precise reason behind this difference unfortunately cannot be answered by this paper due to data limitations, albeit being important. Essentially, we would like the utilities to be aware of the price sensitivities of higher income strata as the higher paying consumers are not actually "reliably" contributing towards the revenue, which could translate to more revenue risks as precipitation changes.

One important caveat for this study (and further discussion of distributional results) is that the household income we calculated is actually stemmed from the correlated house value. Even though it has been proven to be generally true[40], it is a strong assumption to assume the same distribution of house value and the household income within the same zipcode. Some households in the lower strata may simply be low maintenance on their houses or get a low appraisal value and are actually having a high income.

A3. Ramsey Pricing Model

STATUS QUO REVENUE AS THE THRESHOLD

We will explore the validity of using the status quo revenue as the lower bound for counterfactual revenue in this section. Due to the lack of detailed cost breakdown data, it would be difficult to establish a supply-side cost model. However, by using aggregate cost information, we can make some inferences. In order to study the financial viability of Austin Water, we need to ensure total revenue can cover total cost. However, total water service revenue is only a portion of Austin Water's overall revenue stream. Furthermore, the revenue we observe in this data represents only single-home residential water revenue, which itself is only a portion of total water service revenue. Nonetheless, water service revenue is typically the largest revenue stream ($> 70\%$), and single-home housing usage usually constitutes the largest part of total water service revenue ($> 50\%$). In particular, single-home housing usage is most susceptible to precipitation variation because multi-home housing is typically in condominiums, which do not have lawns, and commercial usage usually does not vary with changes in precipitation. Therefore, it is valid to using single-home housing to study the impact from weather, and compare the total revenue from single-home housing usage to some threshold to measure the revenue risk faced by the utility due to weather variation.

However, the lack of detailed cost information makes it hard to estimate the cost generated solely by single-home housing. The total costs from water services include op-

[39] Ruijs, Zimmermann and van den Berg (2008), Echeverri (2023), and Wichman (2024)
[40] See Zhang (2016), and Kim (2020)

eration, labor, debt service requirements, and funds transferred to the city government. None of these can be separated and attributed solely to single-home housing usage.

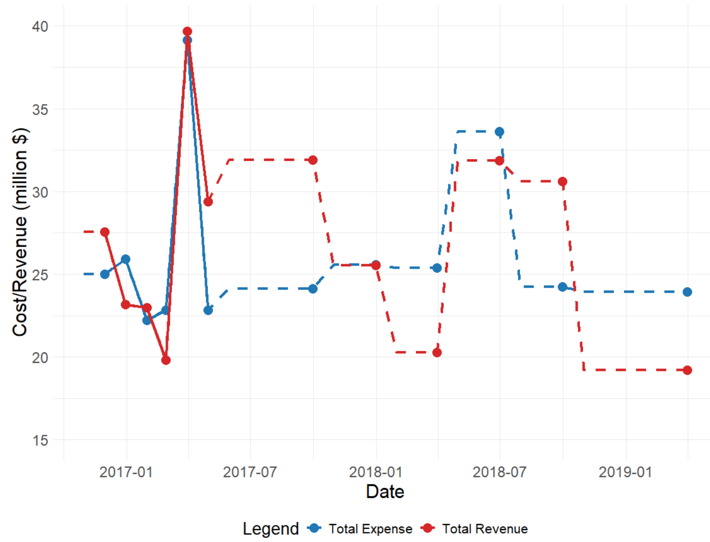

Figure A6. : Total Cost and Revenue from Water Service

Note: Austin Water releases its financial report irregularly, but on average, every 3 months. During some time between Jan.2017 and May.2017, they released the report every month. Hence, some data have a monthly frequency, and some do not.

Looking at the seasonal trends from quarterly financial reports of water service revenue in Figure A6 and cost, we can see that the cost for the whole year is relatively stable. The only peak is usually around summer months when water supply is lower and requires more water from the reservoir. The main reason for negative profit in water service is that revenues in some months fall below the typical cost level during non-peak months. Therefore, the financial risk faced by the utility is largely due to revenue volatility between months. Therefore, it is reasonable to set up a revenue lower bound for comparison with counterfactual revenues. We could set up an intricate cost model to estimate this lower bound, but the status quo revenue (which is the revenue of the year 2019, and the weather variation in 2019 was quite standard) serves as a good enough benchmark.

Simulate Weather Distribution

It is tricky to evaluate E_Z without the knowledge of the real distribution of Z. This is due to 1) if limited to recent years of Z data, there are too few data points to empirically generate the real distribution of Z, and 2) if expanding to long term data of Z, the weather data 30 years ago does not share the macro climate trends of the recent weather data.

Therefore, we choose to combine these two approaches and use Monte Carlo methods to simulate the weather data.

As precipitation data is positive and right skewed, we generate a small log-normal disturbance with $\mu = 0$ and $\sigma = 1$. The $\sigma = 1$ is calculated from the standard deviation of the strictly positive precipitation data from the recent 5 years (2014-2018). This means the utility first predicts a general trend of precipitation data, it could be either $Z' = Z \pm \zeta_1$ or $Z'/Z = \zeta_2$, and then in order to account for prediction errors, Z' is perturbed by a small log normal error terms to simulate the stochasticity of Z' based on the utility's prediction.

The Monte Carlo process is calculated as follows: for each simulated year s, the utility will calculate both the objective $(CS(Z_m^s) - \lambda \cdot \max(0, C - f^{-1}\overline{R_m^s(Z_m^s)}) \cdot 12)$ and the conservation constraint $(P(\sum_m q_m(Z_m^s) \leq \bar{Q}) \geq 0.95)$ and take the average of all numbers of simulations. Essentially, the utility is maximizing over the average objective for all the simulated years, while making sure the average quantity for all the simulated years is below the threshold. One might question why not average over all simulated months, as all of the CS_m, Q_m, and R_m are calculated on a monthly basis, with a total number of simulated data points to be $s \cdot 12$. It is hard to imagine the utility optimizing on a monthly basis, even though it could generate more data points. Applying concavity to a Monte Carlo simulation for all $s12$ data seems to be too strict. On the other hand, it is reasonable to assume that the utility would need to check the projected overall welfare and projected revenue condition by the end of their financial year. Averaging the yearly objective for all simulations essentially calculated the expected objective for the utility under the weather stochasticity. Due to the long running time, we eventually chose $s = 25$ as a start.

Concavity and Revenue Constraint

In the Ramsey model with stochasticity, we start from the linear constraint where $R_m \geq C/12$ such that the monthly average revenue is greater than or equal to a threshold. After introducing the concave and increasing function f, it is valid to question whether the new constraint $(f^{-1}f(\bar{R}_{mh}) \cdot N_{mh} \geq C)$ ensures $R \geq C$. PROOF:

$$\begin{aligned}
f^{-1}f(\bar{R}_m) \cdot 12 &\geq C \\
f(\bar{R}_m) &\geq f\left(\frac{C}{12}\right) \\
f(E[R_m]) \geq E[f(R_m)] &\geq f\left(\frac{C}{12}\right) \quad \text{(By Jensen's Inequality)} \\
E[R_m] &\geq \frac{C}{12} \\
R_m &\geq C
\end{aligned}$$

In general, the revenue constraint with a concave function inside the expectation is a stronger condition as it adds punishment when the variance of R_{mh} is too large. This means the revenue required to satisfy the concave constraint will usually need to be higher.

A4. Counterfactual Analysis

COUNTERFACTUAL NDVI

As NDVI measures the health of vegetation, it would not make sense to keep it unchanged through shifting counterfactual precipitation. To isolate the effect of precipitation on NDVI, we conducted an OLS using the following formula where P represents precipitation and T represents daily max temperature:

$$NDVI = \beta_0 + \beta_1 P_{\text{sum}} + \beta_2 P_{\text{iqr}} + \beta_3 T_{\text{mean}} + \beta_4 T_{\text{iqr}} + \beta_5 I$$

Note that both the mean and IQR here refer to the within-month mean and IQR. For this paper, we focus on the variance of monthly sum of precipitation (P_{sum}), therefore while keeping all the other variables unchanged, the parameter of interest will be β_1. The OLS results are as follows:

Table A6—: OLS Estimation Results of Precipitation on NDVI

Variable	Estimate	Std. Error	t value	$\Pr(>\lvert t \rvert)$
(Intercept)	0.1149	0.0010	92.940	< 2e-16 ***
P_{sum}	0.0080	0.000045	177.803	< 2e-16 ***
P_{iqr}	−0.0118	0.0004	−29.460	< 2e-16 ***
T_{mean}	0.0030	0.000012	256.693	< 2e-16 ***
T_{iqr}	0.0013	0.000032	40.768	< 2e-16 ***
I (in millions)	1.449	1.3	114.940	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Multiple R-squared: 0.127, Adjusted R-squared: 0.127
F-statistic: 3.935e+04 on 5 and 1,356,918 DF, p-value: < 2e-16

This regression captures the impact of precipitation on vegetation health ecologically. The only control variable included is income to capture the size of the lawn for each household. This means for every change in precipitation in inches, the NDVI will increase by 0.008, keeping demand and income the same. Through the calculation of counterfactual NDVI, we can calculate the counterfactual demand, and welfare etc.

OPTIMAL PRICE PROCEDURE

Given the counterfactual weather and NDVI, and household characteristics and income, we will use Equation 14 to solve for the optimal price of 14 parameters. All parameters are part of IBP, therefore, we set all prices to be larger than 0, and force each step to be increasing for both marginal prices and fixed payment (meaning the difference between each tier will at least be $0.01). The optimization algorithm we used is COBYQA, a derivative-free optimization solver designed to supersede COBYLA to solve for the bounded optimization process. In essence, COBYQA is a trust-region SQP method based on quadratic models obtained by underdetermined interpolation.[41]. Of all the constrained optimization algorithms, COBYQA consistently can provide reliable results as long as the initial value is within the valid range with respect to constraints.

Due to the fact that we are dealing with price optimization, luckily, we do not require very refined results. We choose the initial searching radius of the algorithm to be 1.0, and the final radius for convergence tolerance to be 0.01 (as the lowest price difference will be in cents). We also utilize putting the loss function in the objective function in addition to the conservation constraint to avoid optimizing on 2 constraints at the same time (since otherwise, without the loss function, both constraints will need to be implemented). We choose to set the initial value as if each price jump is the same between tiers to intensify the certain benefit of more salient price jumps between certain tiers. Since COBYQA is good at finding relatively local results, we choose the initial value to be close to the status quo IBP.

Aside from weather stochasticity, we set up a Monte-Carlo simulation for ε using the result from Table A5. Note that there are two levels of stochasticity with unobserved taste (η) and perception error (ν). The perception error is purely random and not controlled by the household, so for each household for each month, we will generate a simulation for ν_{mh}. As for η, we assume each household has an unchanged $\bar{\eta}_h$ across all months to avoid overfitting and simulate the results. We have tried using η_{mh}, the counterfactual welfare generates very little difference between specifications.

PRICE RESULT - MEAN SHIFT

Since the 5-tier IBP includes both marginal prices and fixed payments, we believe the most effective way to visualize the pricing structure is by plotting total payment (in dollars) versus quantity (in thousand gallons). Note that all plots are truncated at 25 kGallons to focus on the changes around the four kink points. In practice, a decent number of consumers use significantly more than 25 kGallons. In this subsubsection, we will only focus on ζ_1 and we will separate the scenarios by whether $\zeta_1 \leq 0$ (drier conditions) or $\zeta_1 \geq 0$ (more humid conditions). All price plots include two benchmark cases derived from the status quo. The status quo price (shown with a dot-dash line) reflects the current pricing structure used by Austin Water. The status quo flat price is constructed by averaging the five marginal prices and five fixed payments, resulting in a linear pricing structure. We first present the optimal pricing results under the linear revenue constraint, and then show corresponding plots under the concave revenue constraint.

When $\zeta_1 \leq 0$ and the quantity constraint is binding, prices become steeper beyond $q > 20$ kGallons to incentivize conservation. The drier the condition, the more the utility must charge to reinforce the increasingly pressing conservation goal. When $\zeta_1 \geq 0$ and the revenue constraint becomes binding – due to higher precipitation and reduced usage – the utility must raise prices across all q levels, rather than only in the higher tiers. However, despite the increase in prices, total payments often do not rise because consumers

[41]The algorithm is developed by Ragonneau (2022), Ragonneau and Zhang (2025)

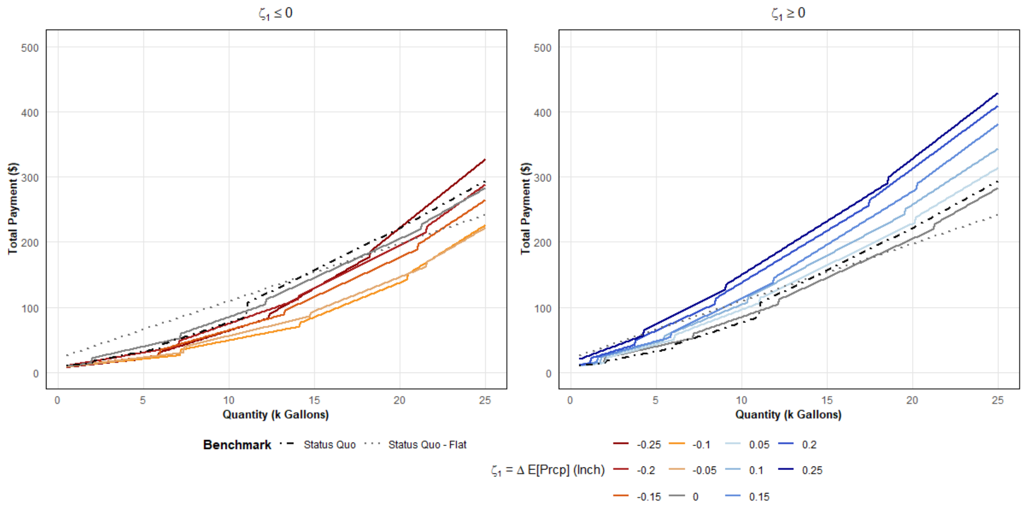

Figure A7. : Optimal Prices under Linear Revenue Constraint - ζ_1

generally use less water.

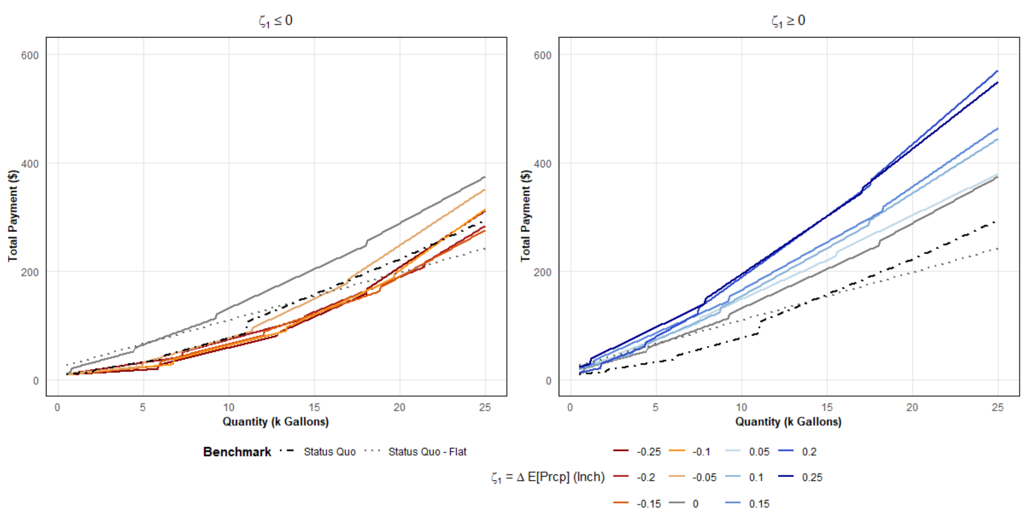

Figure A8. : Optimal Prices under Concave ($\gamma = 0.25$) Revenue Constraint - ζ_1

Under the concave revenue constraint ($\gamma = 0.25$), when $\zeta_1 \leq 0$, all prices under different ζ_1 are more condensed and less differentiated. Even though the quantity constraint is mostly binding, the concave revenue constraint is still actively working against a larger spread of monthly revenue. Due to the binding quantity constraint and just like the linear case, the pricing is steeper for all ζ_1 beyond $q > 20$. When $\zeta_1 \geq 0$, the marginal prices are in general higher for all ζ_1s as the concave constraint is stricter, hence the total payments are higher compared to the linear case. The difference between whether $\zeta_1 \leq 0$ or ≥ 0 creates an interesting dichotomy primarily due to whether the quantity constraint is binding or slack. The interaction between the quantity and revenue constraint (linear or concave) makes the concave constraint produce more stable results regardless of the actual ζ_1. This means even if the utility has low confidence in their prediction of ζ_1, if they opt for a concave constraint, the counterfactual price (and the induced welfare) will be within a smaller range.

Price Result - Variance Shift

When changing ζ_2 (the ratio of weather standard deviation compared to the status quo), we will separate the scenarios by whether $\zeta_2 \leq 1$ (less volatile conditions) or $\zeta_2 \geq 1$ (more volatile conditions). We first present the optimal pricing results under the linear revenue constraint, and then show corresponding plots under the concave revenue constraint.

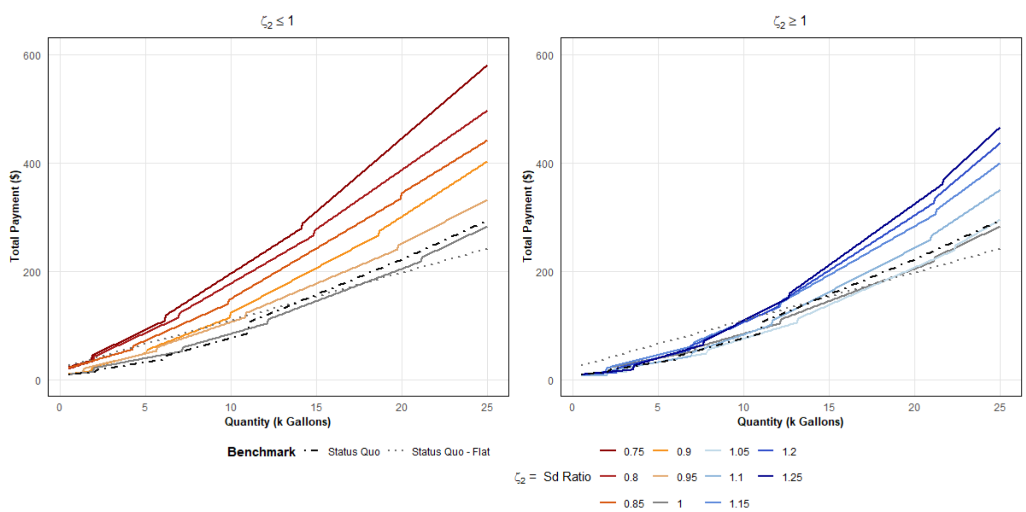

Figure A9. : Optimal Prices under Linear Revenue Constraint - ζ_2

When $\zeta_2 \leq 1$, the weather becomes less volatile. As precipitation becomes less right-skewed (as observed in the current data), more months are wetter than usual, reducing overall consumption and tightening the revenue constraint. In response, the utility raises prices across all tiers. Conversely, when $\zeta_2 \geq 1$ and the weather becomes more volatile, precipitation becomes more right-skewed, increasing the frequency of drier months. This raises consumption and tightens the quantity constraint (note it is more binding), prompting the utility to raise prices – especially in the higher tiers.

Under the concave revenue constraint ($\gamma = 0.25$), when $\zeta_2 \leq 1$, the pricing compared

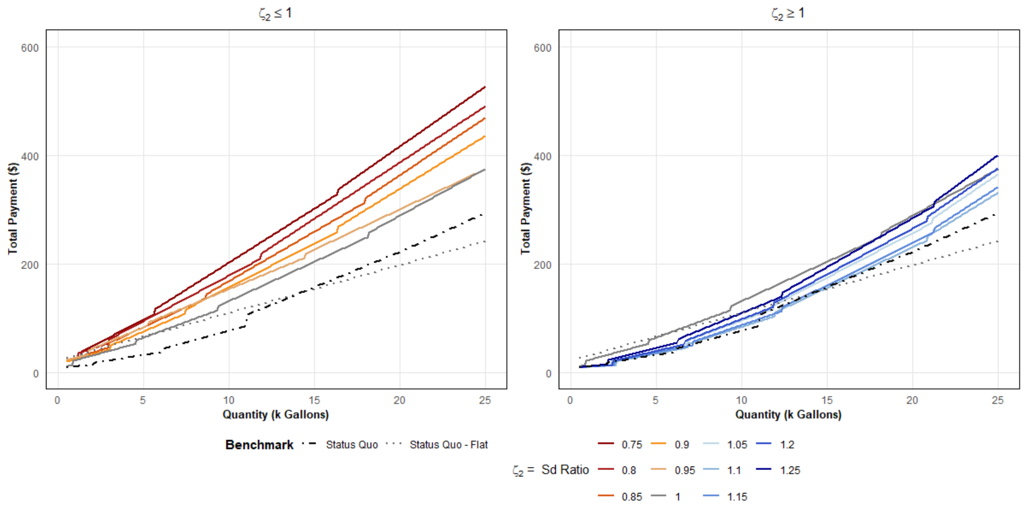

Figure A10. : Optimal Prices under Concave ($\gamma = 0.25$) Revenue Constraint - ζ_2

to the linear case looks a bit linear, but mostly due to a more strict and binding revenue constraint (and the quantity constraint being slack), pricing in general is higher. When $\zeta_2 \geq 1$, as the quantity constraint is no longer slack, the interaction between the quantity and the revenue constraint makes the pricing regardless of ζ_2 (as long as it's bigger than 1) very similar to each other. This means regardless of the utility's prediction of ζ_2, if they opt for a concave revenue constraint, it will generate counterfactual pricing and welfare results within a smaller range, making the overall welfare prediction easier.

High Quantity Consumers

There are plenty of high quantity consumers, but what truly makes the equity goals of the optimal price hard to achieve is the existence of high quantity consumers in lower tiers. Here we will present evidence of that, showcasing the welfare result of price changes specifically focusing on the lowest stratum (0∼6k) and the highest stratum (>100k). We will focus on the specific weather pattern of $\zeta_1 = 0.25$ where the welfare for the lowest stratum is the lowest. When weather is quite humid, there is extra incentive to raise the price. The concave constraint in this subsubsection is when $\gamma = 0.25$ and $\lambda = 0.5$.

Even though the majority of transactions are concentrated right under 0 with low quantities, there are plenty of households consuming very high quantities (well above 20) and they are the ones who suffer huge welfare loss. In comparison, for the concave constraint, the welfare seems to be worse as the welfare, especially for high quantity consumers, becomes worse. In addition, originally for the linear constraint, some transactions have positive EV due to switching to lower tiers. Now, since each tier has less salient jumps, the number of transactions that deviate to lower tiers (and have welfare gains) is much

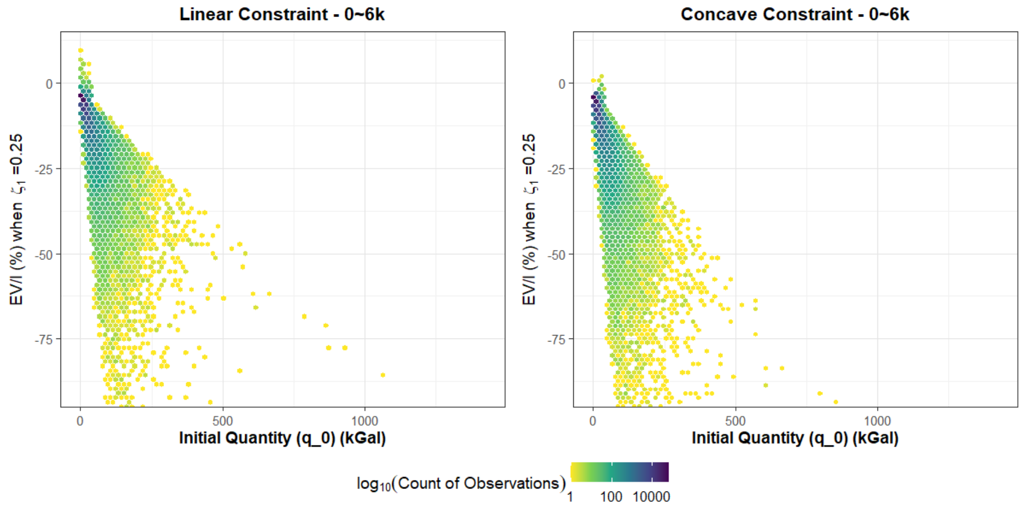

Figure A11. : EV/I of linear and concave revenue constraint - $\zeta_1 = 0.25$, $0 \sim 6$k stratum

Note: This is a geom hex plot for transactions (in the level of *hm*) where the darker color represents higher number of observations.

lower.

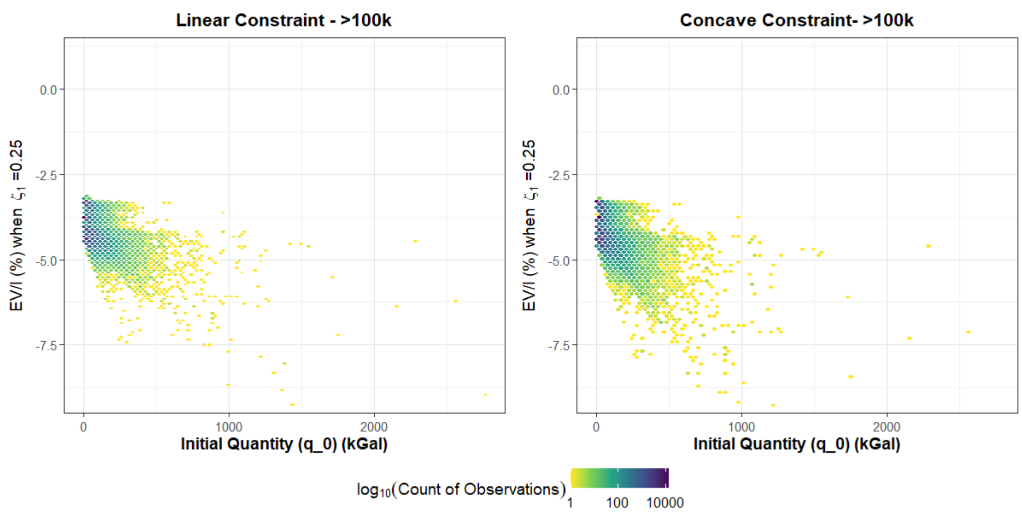

Figure A12. : EV/I of linear and concave revenue constraint - $\zeta_1 = 0.25$, > 100k stratum

For the highest stratum, even though there are more very high quantity transactions and well above the lowest stratum, since they are already making high income, the welfare loss from the price increase is relatively small and very similar between the linear and concave constraint.